Title: Non-Gaussianities in New Ekpyrotic Cosmology

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Abstract:

Plan

1. Review of New Ekpyrotic Cosmology
    - Flatness and homogeneity problems (a few words)
    - Scale-invariant spectrum
    - The form of the potential
2. Connection with curvature perturbations
3. Non-gaussianities
    - The 3-point function
    - The 4-point function
4. Conclusion

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   - The 3-point function
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New Ekpyrotic Cosmology

We study a negative potential $V(\phi)$ and introduce two fast-roll parameters

$$\epsilon = \frac{1}{M_{Pl}^2}\left(\frac{V}{V_{,\phi}}\right)^2, \quad \eta = 1 - \frac{V_{,\phi\phi}V}{V_{,\phi}^2}$$

$$\epsilon, |\eta| \ll 1$$

The potential is nearly exponential

$$V(\phi) = -V_0 e^{-\sqrt{2/p}\frac{\phi}{M_{Pl}}}, \qquad p \ll 1$$

Solution

$$a(t) \sim (-t)^p, \qquad t \in (-\infty, 0), \qquad H = \frac{p}{t} \Longrightarrow$$

$H < 0$ and $|H|$ is increasing.

- the Universe is contracting
- the size of the Universe stays approximately constant
- the horizon $|H|^{-1}$ decreases

This solves the flatness and homogeneity problems (assuming the Universe is flat and homogeneous on the initial Hubble scale). $\Omega_k \sim \frac{1}{a^2 H^2}$ becomes much less than unity since $|H|^{-1}$ is decreasing. To solve the homogeneity problem it is enough to require that $N_{ek} \geq N_{rad}, \quad N_{ek} = \ln\left(\frac{H_{end}}{H_i}\right)$

Perturbations

Work in the spatially flat gauge $\psi = 0$.

Define Sasaki-Mukhanov variables

$$Q_I = \delta\phi_I + \frac{\dot{\phi}_I}{H}\psi = \delta\phi_I$$

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Eq. of motion (Fourier mode with wave vector k)

$$\ddot{Q}_I + 3H\dot{Q}_I + \frac{k^2}{a^2}Q_I + \sum_J \left[V_{,\phi_I\phi_J} - \frac{8\pi G}{a^2}\left(\frac{a^3}{H}\dot{\phi}_I\dot{\phi}_J\right)^{\cdot}\right] = 0$$

The comoving curvature perturbation

$$\mathcal{R} = H\sum_I \left(\frac{\dot{\phi}_I}{\sum_J \dot{\phi}_J^{\,2}}\right) Q_I = -\zeta$$

ζ is the curvature perturbation on uniform density hypersurfaces. In the case of a single field

$$\zeta = -H\frac{\delta\phi}{\dot{\phi}}$$

In the one-field ekpyrotic cosmology we have

$$\ddot{Q} + 3H\dot{Q} + \frac{k^2}{a^2}Q = 0$$

It follows that $Q \sim 1/\sqrt{k}$ - not scale invariant.

Consider two fields with exponential potentials (will forget about them soon)

$$V = -V_0 e^{-\sqrt{2/p}\frac{\phi_1}{M_{Pl}}} - V_0' e^{-\sqrt{2/q}\frac{\phi_2}{M_{Pl}}}, \qquad p, q \ll 1$$

$$a \sim (-t)^{p+q}, \qquad H = \frac{p+q}{t}$$

Now we study perturbations. Go to the conformal time

$$a d\tau = dt, \qquad u_1 = aQ_1, \qquad u_2 = aQ_2$$

We will get two coupled equations for u_1 and u_2. Diagonalize them by a rotation

$$u_+ = \cos\Theta u_1 + \sin\Theta u_2, \qquad u_- = \cos\Theta u_2 - \sin\Theta u_1$$

with the angle Θ given by $\tan\Theta = \sqrt{q/p}$. The result is

$$u_+'' + k^2 u_+ = 0, \qquad u_-'' + (k^2 - \frac{2}{\tau^2}) u_- = 0$$

u_- has a scale invariant spectrum!

In general, one can separate perturbations of two fields into **adiabatic** and **entropy** perturbations.

$$\delta\sigma = \cos\theta\delta\phi_1 + \sin\theta\delta\phi_2, \qquad \delta s = \cos\theta\delta\phi_2 - \sin\theta\delta\phi_1$$

$$\tan\theta = \frac{\dot{\phi}_2}{\dot{\phi}_1}$$

Apply it to ekpyrotic cosmology $\Longrightarrow \theta = \Theta$.

u_+ is the adiabatic perturbation

u_- is the entropy perturbation (the corresponding field stays constant during the evolution).

The equation for the entropy perturbation ($s = u_-/a$) can be written

$$\ddot{\delta s} + 3H\dot{\delta s} + \left(\frac{k^2}{a^2} + V_{,ss}\right)\delta s = 0$$

$$V_{,ss} = V_{,\sigma\sigma}$$

where $\sigma = u_+/a$ is the adiabatic field.

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More natural to work with field (ϕ, χ) which coincide with (σ, s) during ekpyrosis. This leads to the most general ekpyrotic potential.

- $V(\phi) \approx -e^{-\sqrt{2/p}\frac{\phi}{M_{Pl}}}, \qquad \epsilon, |\eta| \ll 1$
- $\chi = 0$
- $V_{,\phi\phi} = V_{,\chi\chi} \Longrightarrow$ scale invariant spectrum

$$V(\phi, \chi) = \mathcal{V}(\phi)\left(1 + \frac{1}{2}\frac{\mathcal{V}_{,\phi\phi}}{\mathcal{V}} f(\phi)\chi^2 + \ldots\right)$$

where the higher order terms in χ are irrelevant and $f(\phi)$ is approximately unity $f(\phi) \approx 1 + 3\delta, \qquad \delta \ll 1$

δ is an additional parameter just like ϵ, η. The spectral index is

$$n_s - 1 = 4(\epsilon - \eta - \delta) \Longrightarrow$$

One can have a red tilt for $\eta = 0$.

We will write the potential in the form

$$V(\phi,\chi) = -V_0 e^{-\phi/\Lambda}\left(1 + \frac{\chi^2}{2\Lambda^2} + \frac{\alpha_3}{3!}\frac{\chi^3}{\Lambda^3} + \frac{\alpha_4}{4!}\frac{\chi^4}{\Lambda^4} + \dots\right)$$

$$\Lambda = \sqrt{\epsilon} M_{Pl}, \epsilon \ll 1$$

The entropy perturbation $\delta\chi_k^{(0)}$ receives a scale invariant spectrum

$$\delta\chi_k^{(0)} = \frac{1}{\sqrt{2k}} e^{-ikt}(1 - \frac{i}{kt}) \sim -\frac{i}{\sqrt{2}tk^{3/2}}$$

$$< \delta\chi_{\vec{k}_1}^{(0)} \delta\chi_{\vec{k}_2}^{(0)} >= (2\pi)^3 \delta^3(\vec{k}_1 + \vec{k}_2)\mathcal{P}_\chi(k), \qquad k^3\mathcal{P}_\chi(k) = \frac{1}{2t^2}$$

Let us see how $\delta\chi$ is related to ζ **during** ekpyrosis. To derive the evolution of ζ at large scales we use the separate Universe approximation and the gauge $\delta\rho = 0$.

$$\dot{\zeta} = 2H\frac{\delta V}{\dot{\phi}^2 - 2\delta V}, \qquad \delta V = V(\phi,\chi) - V(\bar{\phi},\bar{\chi})$$

It is enough to solve it to order $(\delta\chi_k^{(0)})^3$. Ignore $\delta\phi$.

- To the linear order in $\delta\chi^{(0)}$ we have
  $\dot{\zeta} = 2H\frac{\dot{\theta}}{\dot{\phi}}\delta\chi^{(0)} = 0$ since $\theta = 0 \Longrightarrow \zeta$ is **NOT** scale invariant during ekpyrosis

- Go to the next order. At large scales we have
  $\ddot{\delta\chi} + V_{,\chi\chi}\delta\chi + \frac{1}{2}V_{,\chi\chi\chi}\delta\chi^2 = 0$
  Write $\delta\chi = \delta\chi^0 + \delta\chi^1$ and solve for $\delta\chi^{(1)}$ in terms of $\delta\chi^{(0)}$
  $\delta\chi = \delta\chi^{(0)} + \frac{\alpha_3}{4\Lambda}(\delta\chi^{(0)})^2 + \dots$
  This is enough to obtain ζ to the required order.

- Integrate the equation for ζ.
  $\zeta_{ek} = \frac{1}{2}\left(\frac{\delta\chi^{(0)}}{M_{Pl}^2}\right)^2 + \frac{5\alpha_3}{18\sqrt{\epsilon}}\left(\frac{\delta\chi^{(0)}}{M_{Pl}^2}\right)^3 + \dots$

To convert scale invariant $\delta\chi$ to ζ we need to move away from the ekpyrotic trajectory (this has to be done anyway since the ekpyrotic potential is unbounded from below).

To do this we add a correction to the potential which at the end of ekpyrosis moves χ away from $\chi = 0$.

To simplify integration during this phase we assume that the exit is fast in the Hubble time. Then the contribution to ζ during the exit is

$$\zeta \approx \mp 2\sqrt{\epsilon}\beta\frac{\delta\chi^{(0)}}{M_{Pl}}$$

- Scale invariant entropy perturbations are converted into ζ during the exit.
- The sign $\mp$ is related to the sign of the angle at which we exit.
- β is a **model-dependent** coefficient related to $|\Delta\theta|$.

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The power spectrum of ζ is given by

$$k^3\mathcal{P}_\zeta(k) = \frac{4\epsilon\beta^2}{M_{Pl}^2}k^3\mathcal{P}_\chi(k) = \beta^2\frac{H_{end}^2}{2\epsilon M_{Pl}^2}$$

which is identical to the inflationary result up to a model-dependent factor β^2. We will concentrate on the following choice of parameters (mostly to be consistent with NG)

$$\beta \sim \mathcal{O}(1), \qquad \epsilon \sim 10^{-2}, \qquad H_{end} \sim 10^{13} GeV$$

This corresponds to $k^3\mathcal{P}_\zeta(k) \sim 10^{-10}$ and GUT-scale reheat temperature.

The non-linear equation for ζ is

$$\zeta(x) = \zeta_c(x) + \frac{1}{8\epsilon\beta^2}\zeta_c^2(x) \mp \frac{\alpha_3}{288\epsilon^2\beta^3}\zeta_c^3(x) + \dots$$

Almost true: to study NG due to non-linearities in ζ_c it is sufficient to treat ζ_c as Gaussian. To study NG from the potential it is sufficient to set $\zeta = \zeta_c$.

NG from the Three-Point function

The three-point function NG are captured by f_{NL}. In terms of the Newtonian potential

$$\Phi = \Phi_g - f_{NL}\Phi_g^2$$

Positive f_{NL} means a deeper gravitational potential and more colder spots in the CMB. In terms of ζ

$$\zeta = \zeta_g + \frac{3}{5} f_{NL}\zeta_g^2$$

$$< \zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3} >= (2\pi)^3\delta^3(\vec{k}_1 + \vec{k}_2 + \vec{k}_3)B(k_1, k_2, k_3)$$

$$B(k_1, k_2, k_3) = \frac{6}{5} f_{NL}(\mathcal{P}_\zeta(k_1)\mathcal{P}_\zeta(k_2) + \text{permutations})$$

If f_{NL} is a constant, NG is of the local form. This turns out to be the case in New Ekpyrotic Cosmology. From the non-linear form of ζ we read off

$$f_{NL}^{conv} = \frac{5}{24\epsilon\beta^2}$$

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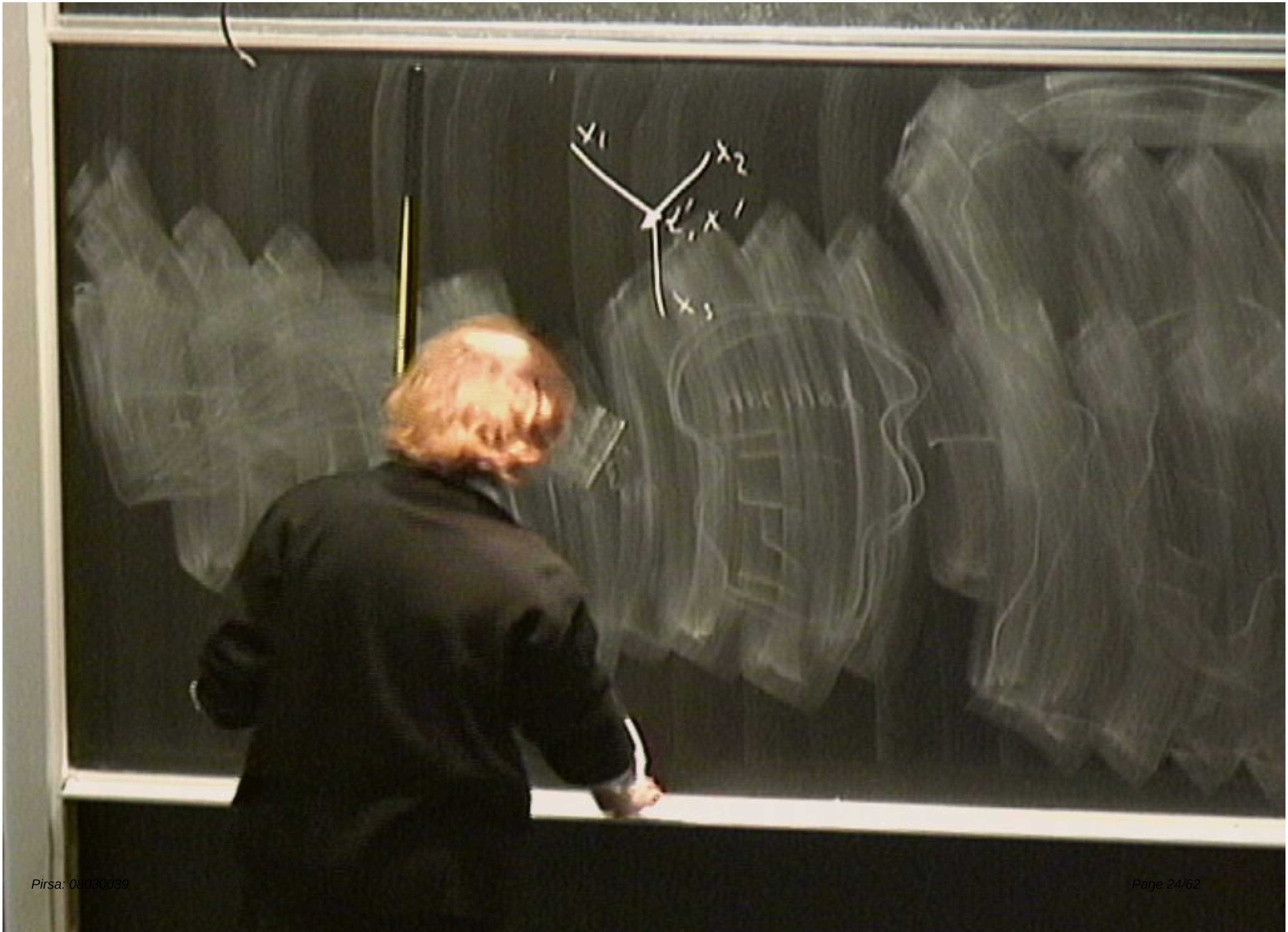
From the three-point vertex of the potential

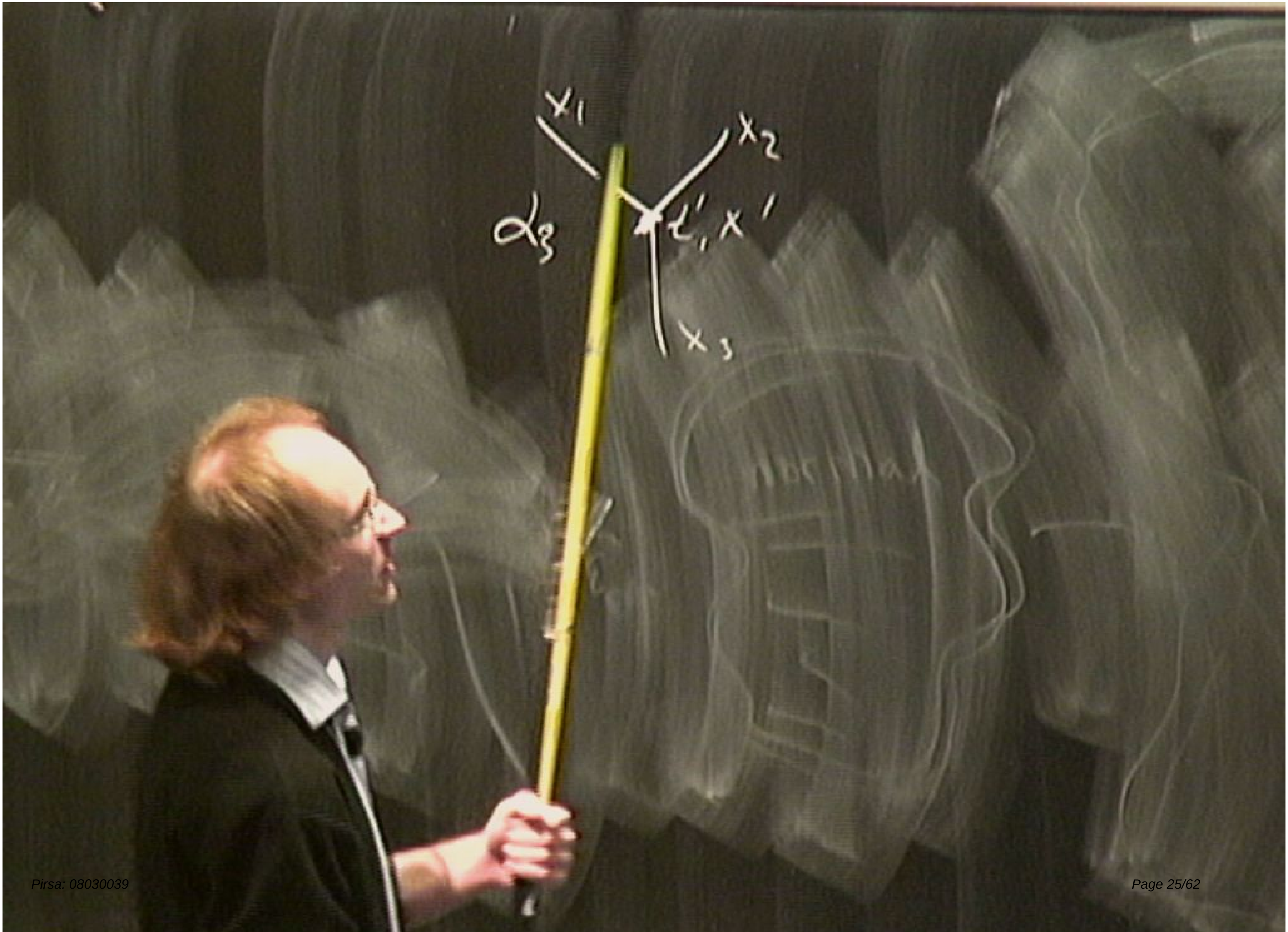
$$G(x_1, x_2, x_3) = -i \int_{-\infty}^{t_{end}} dt' d^3x' [\delta\chi(x_1)\delta\chi(x_2)\delta\chi(x_3), \mathcal{H}_{int}(t', x')]$$

$$\mathcal{H}_{int} = \mathcal{V}(\phi(t)) \frac{\alpha_3}{3!} \frac{\delta\chi^3}{\Lambda^3} = -\frac{2\alpha_3\delta\chi^3}{3! t^2 \Lambda}$$

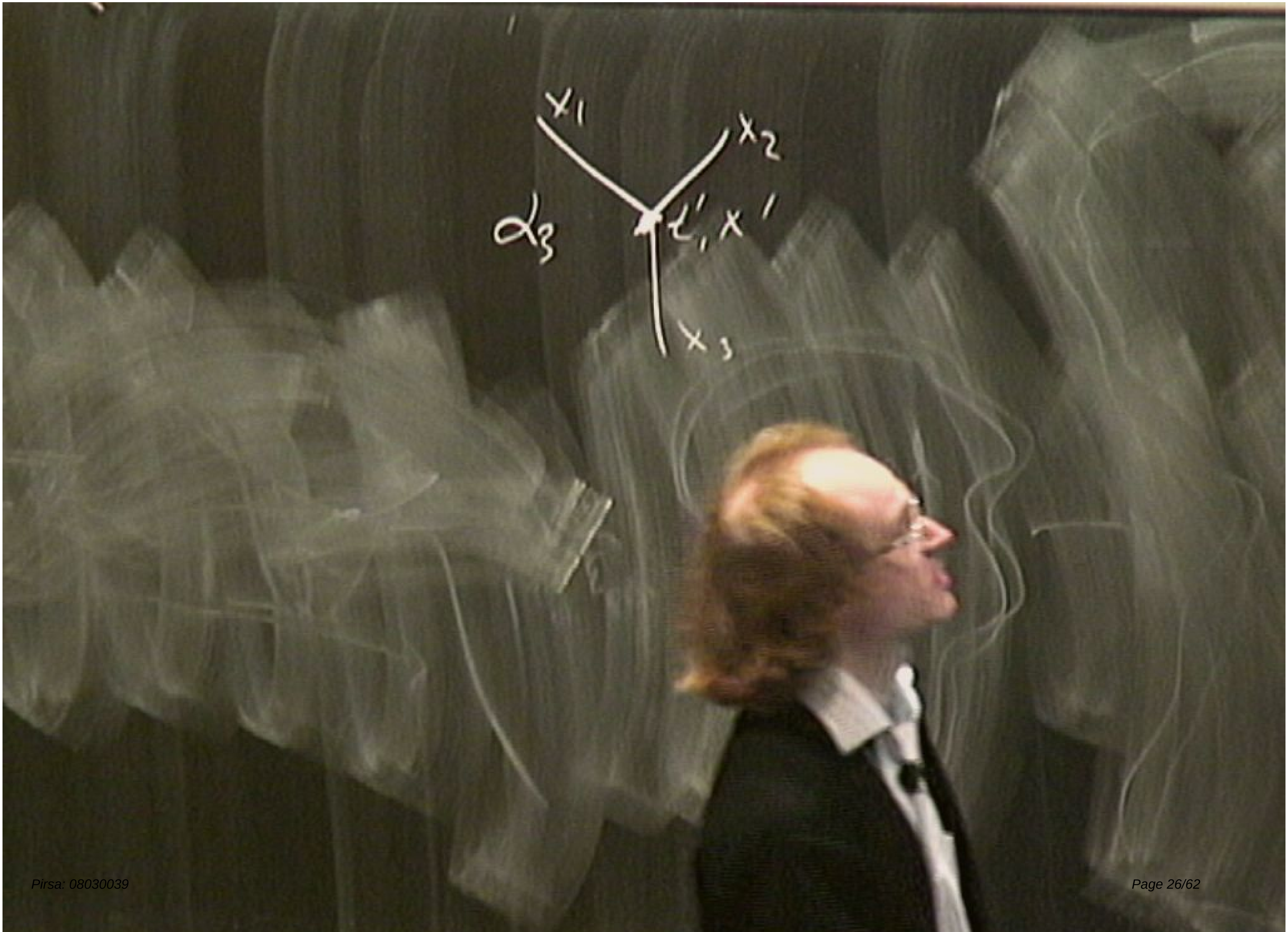
We have to calculate the three-point Green's function with time-dependent coupling.

- Go to momentum space
- Write $\delta\chi_{\vec{p}}$ in terms of the creation and annihilation operators
- $\delta\chi_{\vec{p}} = \delta\chi_p(t) a^{\dagger}_{\vec{p}} + \delta\chi^*_p(t) a_{-\vec{p}}$
  where $\delta\chi_p(t)$ is the scale invariant solution.
- Use the relation between $\delta\chi$ and ζ.
- $f_{NL}^{int} = \mp \frac{5}{18} \frac{\alpha_3}{\beta\epsilon}$


y1
x2
x3



x1
x2
α3
t', x'
x3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- Use the relation between $\delta\chi$ and ζ.
- $f_{NL}^{int} = \mp \frac{5}{18} \frac{\alpha_3}{\beta\epsilon}$

Combining f_{NL}^{conv} and f_{NL}^{int} we get

$$f_{NL} = f_{NL}^{conv} + f_{NL}^{int} = \frac{5}{24\beta^2\epsilon}\left(1 \mp \frac{4}{3}\alpha_3\beta\right)$$

- $f_{NL} \sim 1/\epsilon \Longrightarrow$ tends to be large! For $\beta \sim 1, \epsilon \sim 10^{-2}$

  f_{NL} is consistent with the WMAP data

  $-36 < f_{NL} < 100 \quad \text{(at 95\% C.L.)}$

  for $2.046 > \alpha_3 > -2.85$

  New Ekpyrotic Cosmology predicts large local NG.

- No conflict with the power spectrum

  $k^3 P_\zeta = \beta^2 \frac{H_{end}^2}{2\epsilon M_{Pl}^2} \sim 10^{-10}$

- No conflict with the spectral index

  $n_s - 1 = 4(\epsilon - \delta) \qquad \text{(for simplicity } \eta = 0)$

  One can adjust δ and make f_{NL} and n_s substantially independent.

NG from the Four-Point function

The general form of the answer

$$< \zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}\zeta_{\vec{k}_4} >= (2\pi)^3\delta^3(\vec{k}_1+\vec{k}_2+\vec{k}_3+\vec{k}_4)[T(k_i)+T'(k_i)]$$

$T(k_i)$ and $T'(k_i)$ are two different momentum functions.

$$T(k_i) = \frac{1}{2}\tau_{NL}[\mathcal{P}_\zeta(k_1)\mathcal{P}_\zeta(k_2)\mathcal{P}_\zeta(|\vec{k}_1+\vec{k}_4|) + 23 \text{ permutations}]$$

$$T'(k_i) = \kappa_{NL}[\mathcal{P}_\zeta(k_1)\mathcal{P}_\zeta(k_2)P_\zeta(k_3) + 3 \text{ permutations}]$$

The shape of $T(k_i)$ is consistent with

$$\zeta(x) = \zeta_g(x) + \frac{\sqrt{\tau_{NL}}}{2}\zeta_g^2(x)$$

The shape of $T'(k_i)$ is consistent with

$$\zeta(x) = \zeta_g(x) + \frac{\kappa_{NL}}{6}\zeta_g^3(x)$$

However, we already know that the three point function is consistent with

$$\zeta(x) = \zeta_g(x) + \frac{3}{5} f_{NL} \zeta_g^2(x)$$

Therefore, we don't have to do any calculations of τ_{NL}.

$$\tau_{NL} = 4\left(\frac{3}{5} f_{NL}\right)^2 = \frac{1}{16\beta^4\epsilon^2}\left(1 \mp \frac{4}{3}\alpha_3\beta\right)^2$$

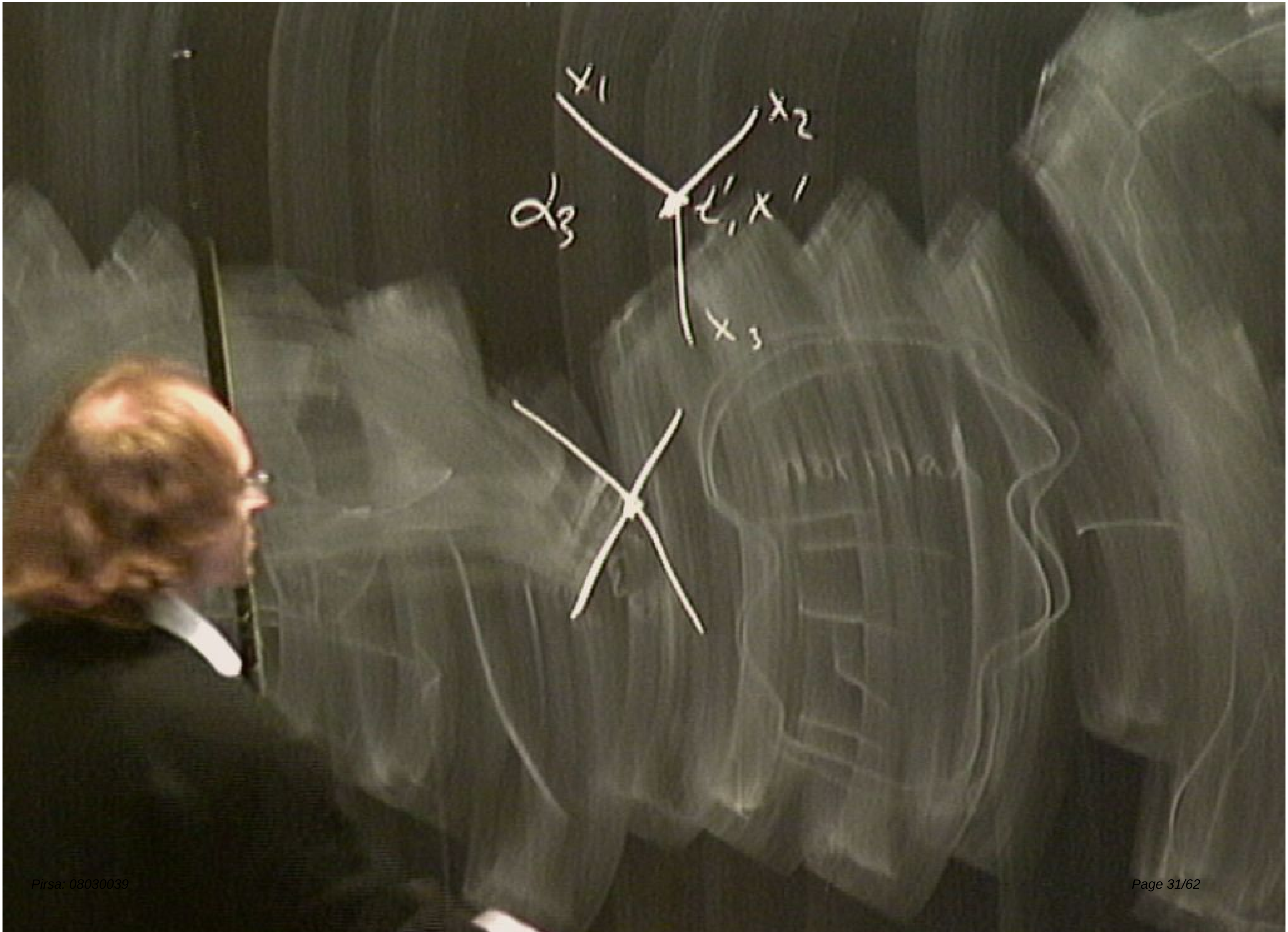
True in any theory. One can confirm this by explicit calculation of the four-point function.

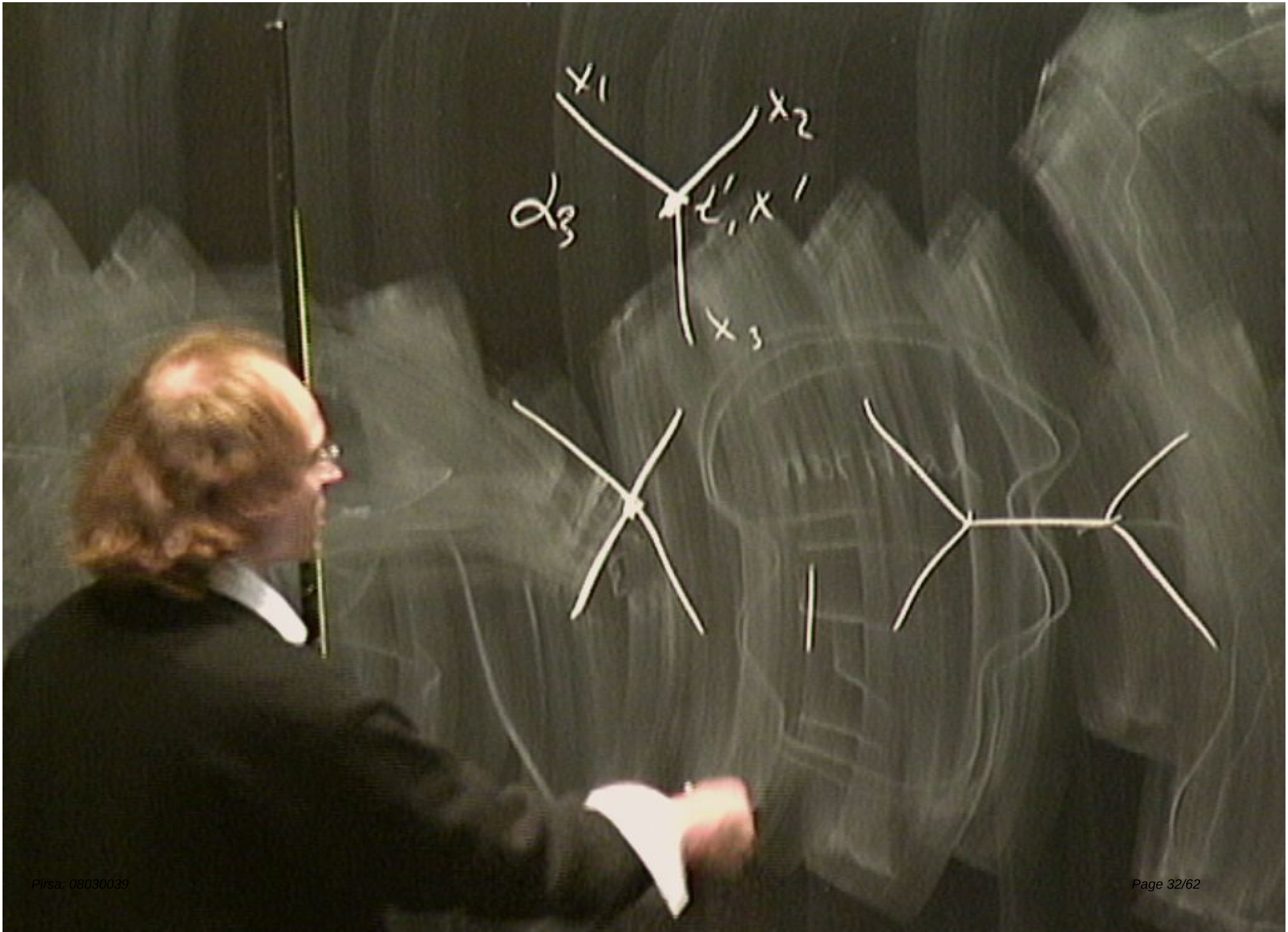
Consider κ_{NL}. From the non-linear relation between ζ and $\delta\chi$ we obtain

$$\kappa_{NL}^{conv} = \mp \frac{\alpha_3}{48\epsilon^2\beta^3}$$

There are additional contributions from the three- and four-point vertices of the potential

$$V(\phi, \chi) = -V_0 e^{-\phi/\Lambda}\left(1 + \frac{\chi^2}{2\Lambda^2} + \frac{\alpha_3}{3!}\frac{\chi^3}{\Lambda^3} + \frac{\alpha_4}{4!}\frac{\chi^4}{\Lambda^4} + \dots\right)$$


x1
x2
α3
t', x'
x3


x1
x2
α3
t', x'
x3

For the four-point vertex it is straightforward to repeat the same steps as in the case of f_{NL} to obtain

$$< \zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}\zeta_{\vec{k}_4} >= (2\pi)^3\delta^3(\vec{k}_1+\vec{k}_2+\vec{k}_3+\vec{k}_4)\frac{\alpha_4}{12\beta^2\epsilon^2}\left(\frac{1}{k_1^3k_2^3k_3^2}+\dots\right)$$

$$\kappa'_{NL} = \frac{\alpha_4}{12\beta^2\epsilon^2}$$

For the three-point vertex we go to the second order in perturbation theory.

$$G(x_1,x_2,x_3,x_4) = \int d^3x' d^3x'' dt' dt'' H_{int}(x',t')\delta\chi^4 H_{int}(x'',t'')$$

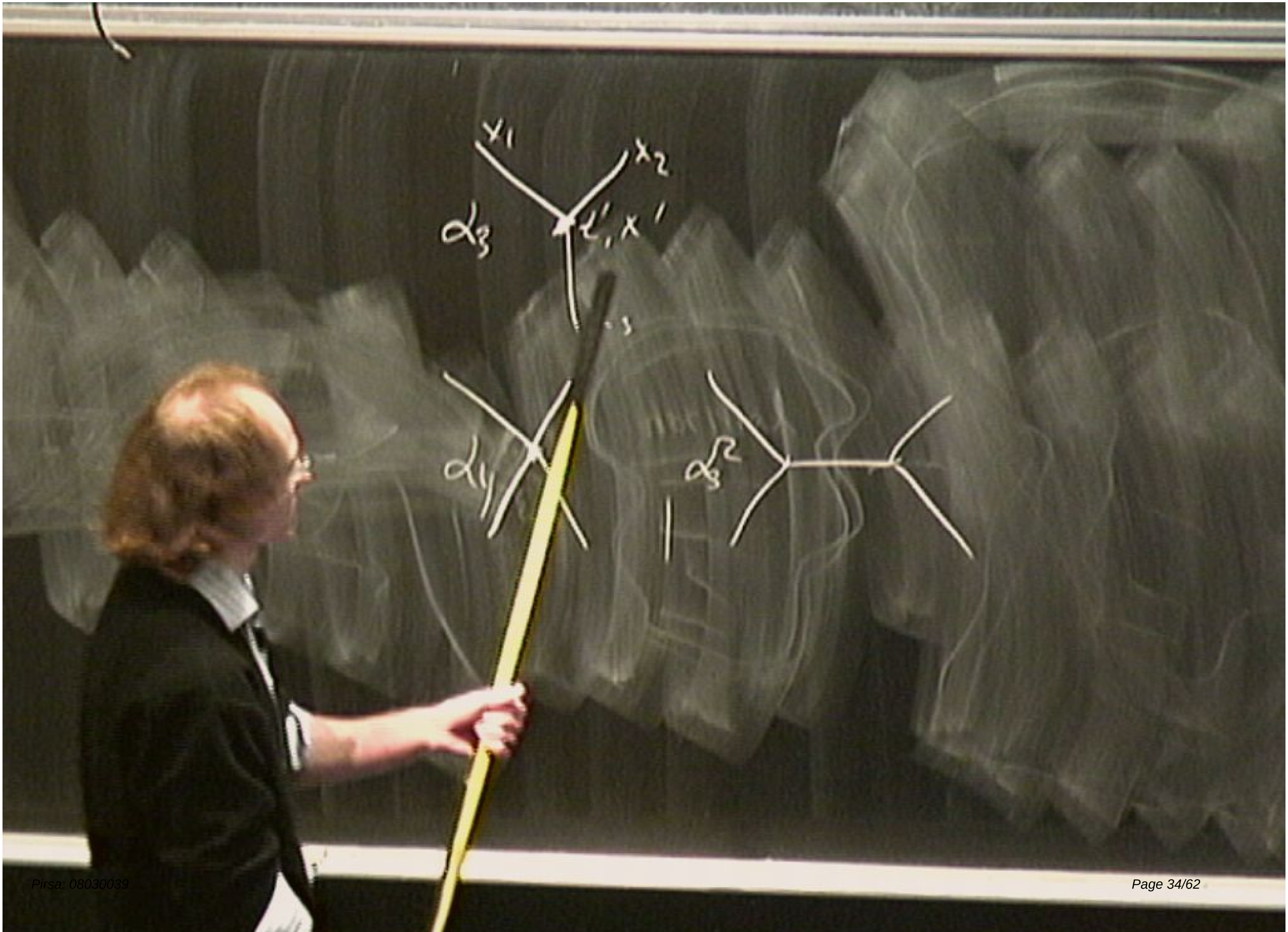
$$-\frac{1}{2}\int d^3x' d^3x'' dt' dt'' H_{int}(x',t') H_{int}(x'',t'')\delta\chi^4$$

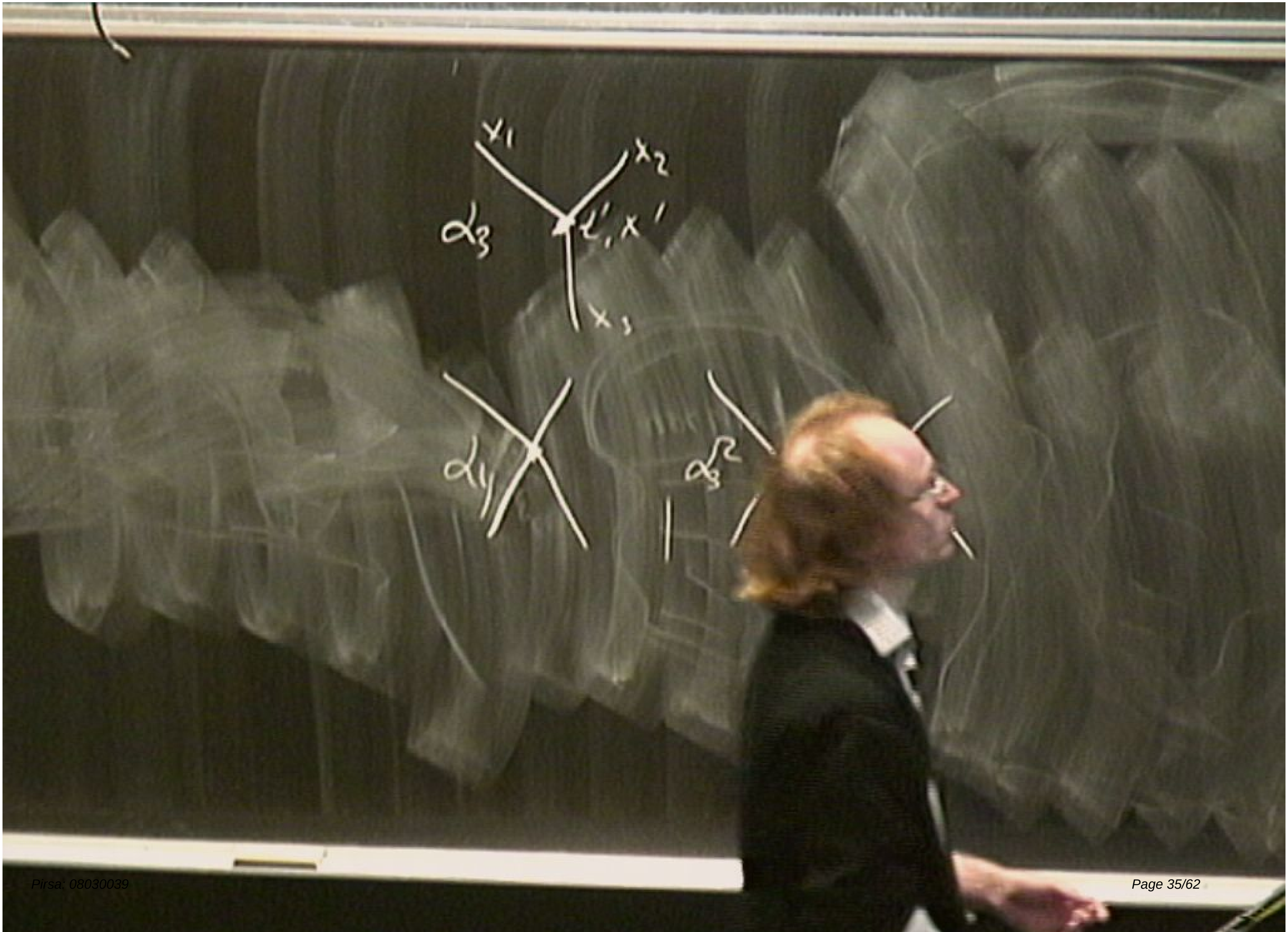
$$-\frac{1}{2}\int d^3x' d^3x'' dt' dt'' \delta\chi^4 H_{int}(x',t') H_{int}(x'',t'')$$

This four-point function contribute to both τ_{NL} and κ_{NL}.

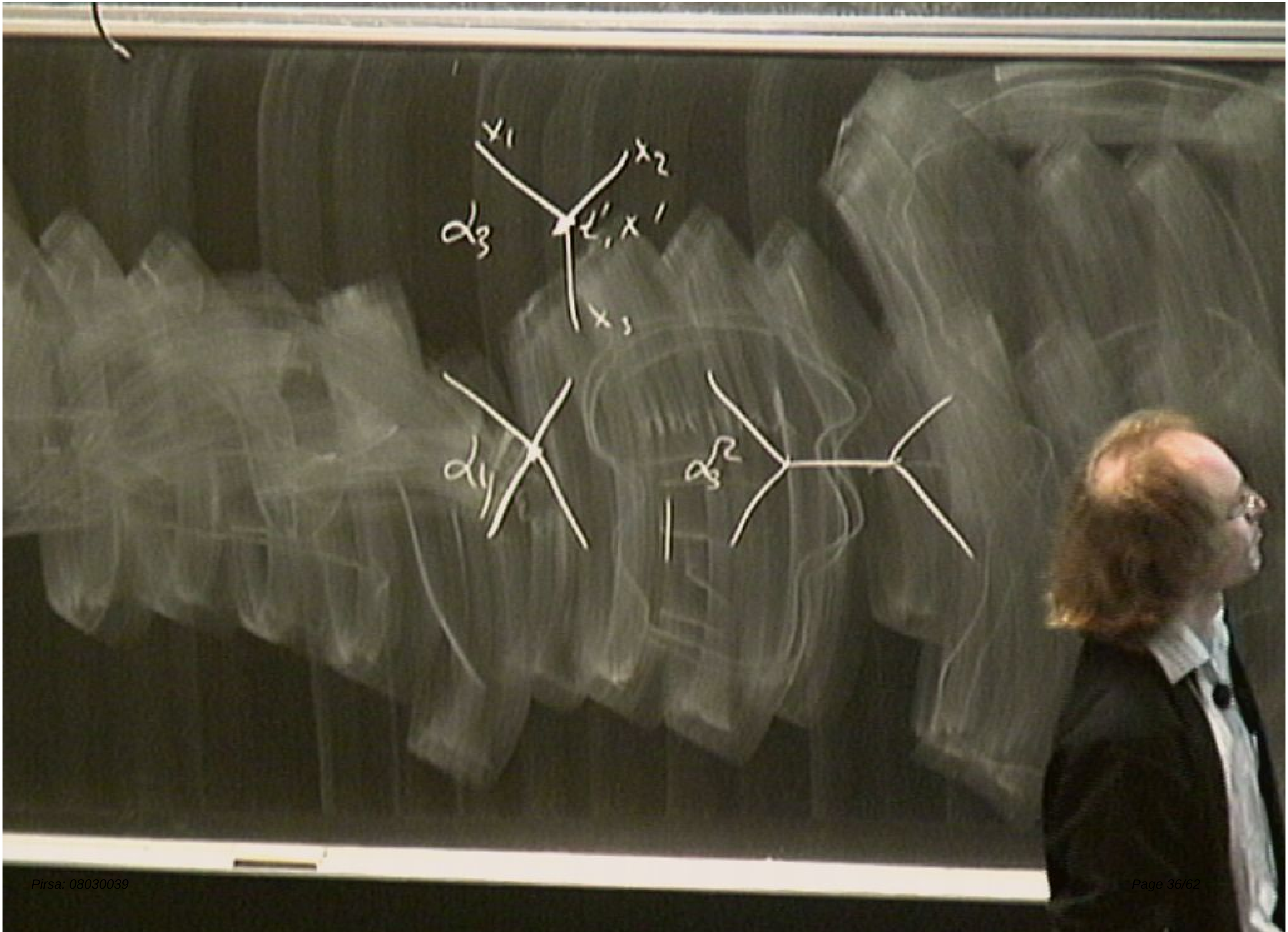
$$\kappa''_{NL} = \frac{\alpha_3^2}{6\beta^2\epsilon^2}$$

The final answer is $\kappa_{NL} = \frac{\alpha_3(8\alpha_3\beta\mp1)+4\beta\alpha_4}{48\beta^3\epsilon^2}$


x1
x2
α3
α4
α3²



x_1
x_2
x_3



x_1
x_2
α_3
t', x'
x_3
α_4
α_3^2

For the four-point vertex it is straightforward to repeat the same steps as in the case of f_{NL} to obtain

$$< \zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}\zeta_{\vec{k}_4} >= (2\pi)^3\delta^3(\vec{k}_1+\vec{k}_2+\vec{k}_3+\vec{k}_4)\frac{\alpha_4}{12\beta^2\epsilon^2}\left(\frac{1}{k_1^3k_2^3k_3^2}+\ldots\right)$$

$$\kappa'_{NL} = \frac{\alpha_4}{12\beta^2\epsilon^2}$$

For the three-point vertex we go to the second order in perturbation theory.

$$G(x_1,x_2,x_3,x_4) = \int d^3x'd^3x''dt'dt''H_{int}(x',t')\delta\chi^4 H_{int}(x'',t'')$$
$$-\frac{1}{2}\int d^3x'd^3x''dt'dt''H_{int}(x',t')H_{int}(x'',t'')\delta\chi^4$$
$$-\frac{1}{2}\int d^3x'd^3x''dt'dt''\delta\chi^4 H_{int}(x',t')H_{int}(x'',t'')$$

This four-point function contribute to both τ_{NL} and κ_{NL}.

$$\kappa''_{NL} = \frac{\alpha_3^2}{6\beta^2\epsilon^2}$$

The final answer is $\kappa_{NL} = \frac{\alpha_3(8\alpha_3\beta\mp 1)+4\beta\alpha_4}{48\beta^3\epsilon^2}$

Conclusion

- Computation of NG in New Ekpyrotic Cosmology

  $f_{NL} \sim \frac{1}{\epsilon}, \quad \tau_{NL} \sim f_{NL}^2, \quad \kappa_{NL} \sim \frac{1}{\epsilon^2}$

- NG are of the **local** form. f_{NL} tends to be large (~ 100).

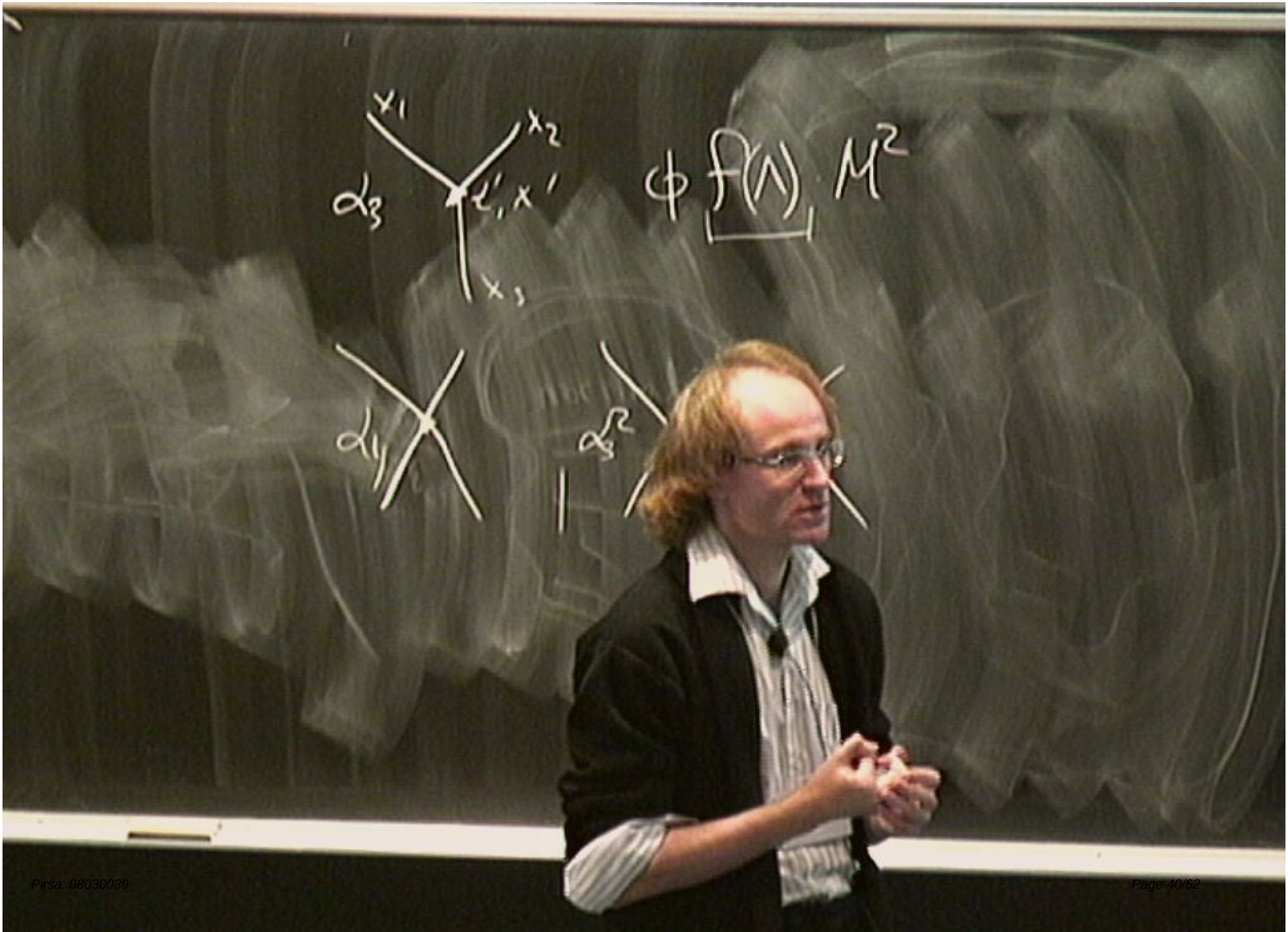
Comparison with inflation

- Different from slow-roll, DBI and any other single field inflation by the shape of the three point function.

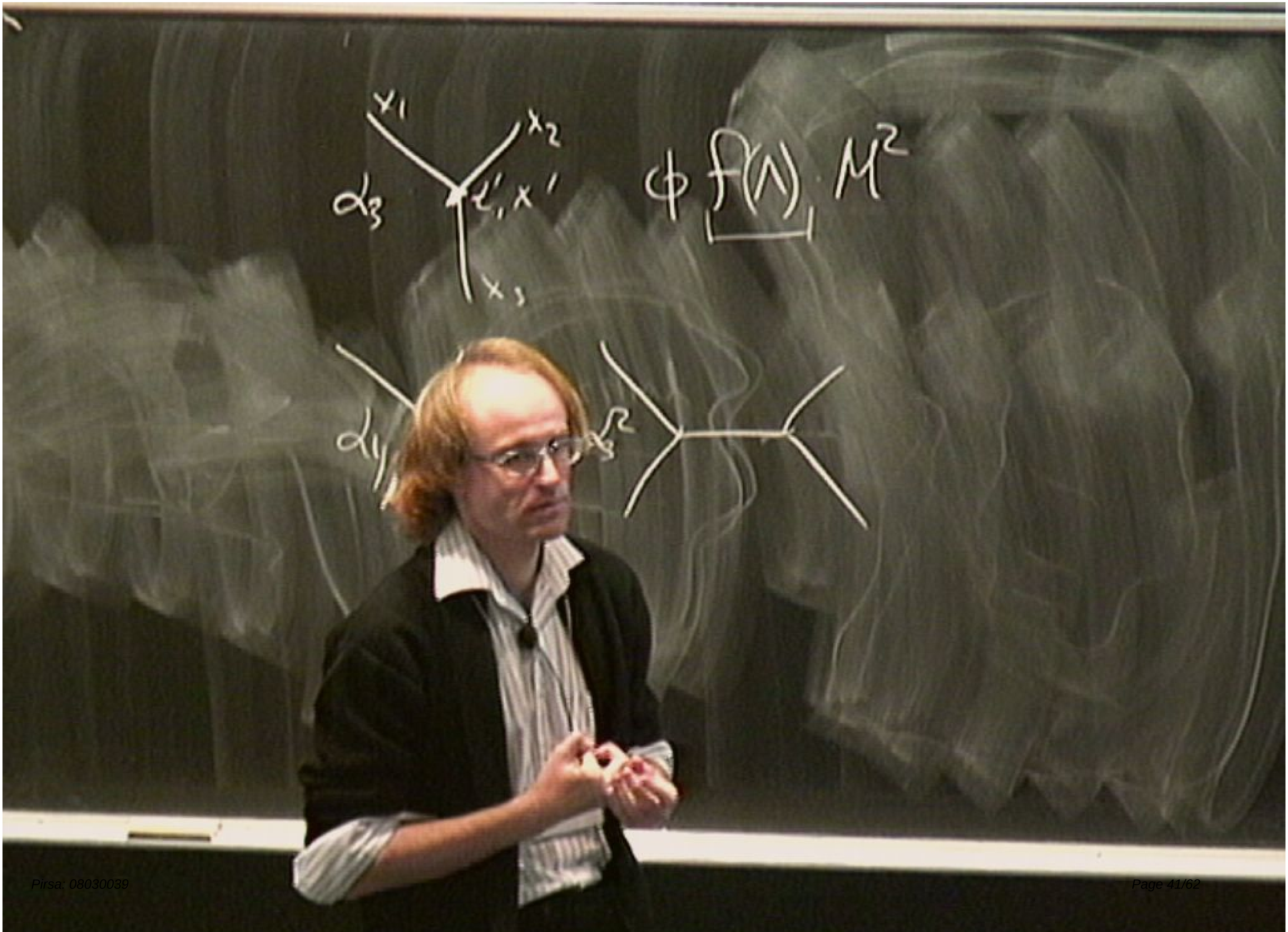
- Different from (the simplest) curvaton, $\kappa_{NL} = 0$.

  Q.-G. Huang, arXiv: 0801.0467: curvaton favors large amplitude of gravity waves (GUT-scale inflation)

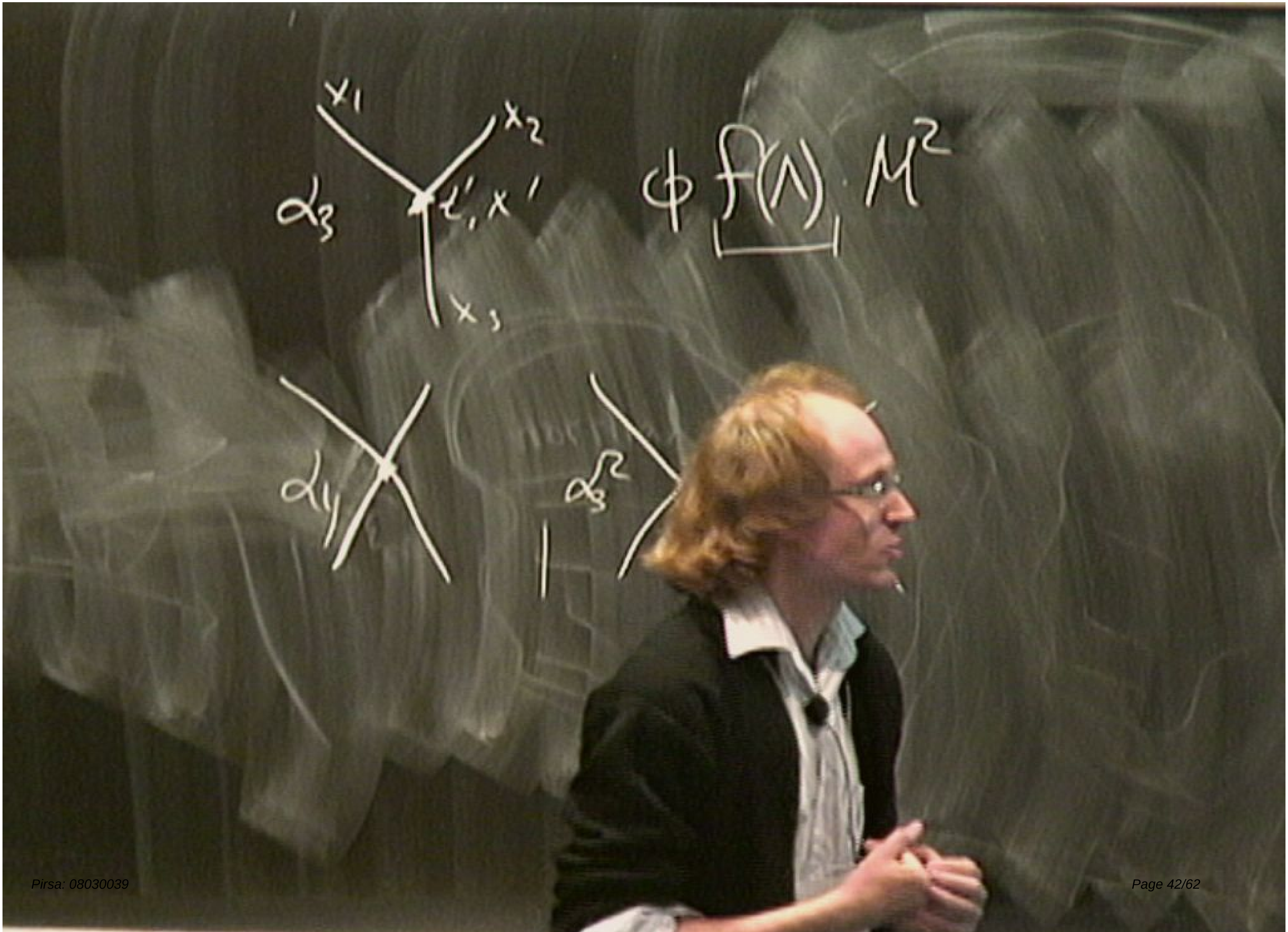
- Modulon inflation - ???



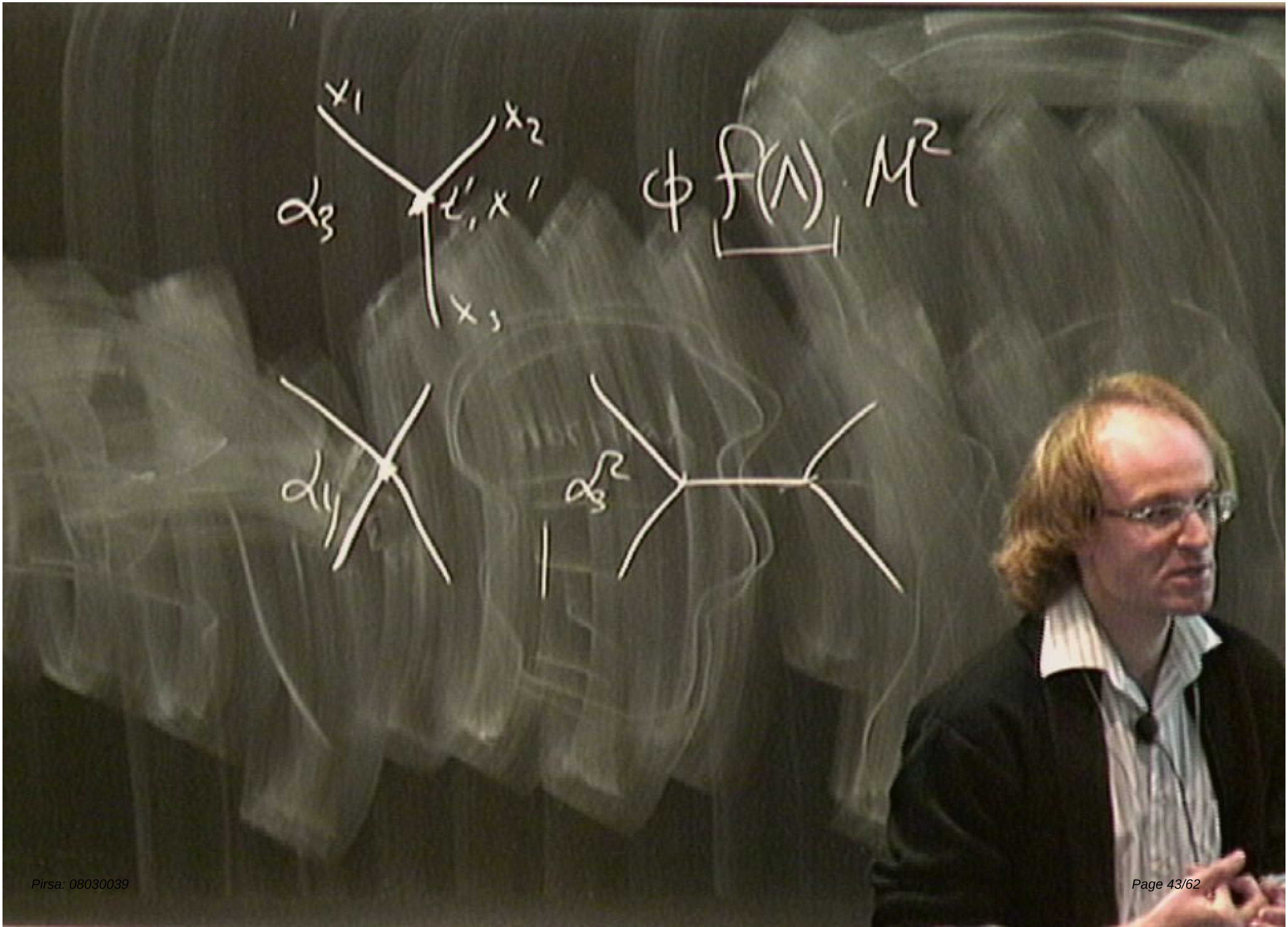
x_1
x_2
α_3
t', x'
$\phi \, f(\Lambda) \; M^2$
x_3
α_1
α_3^2



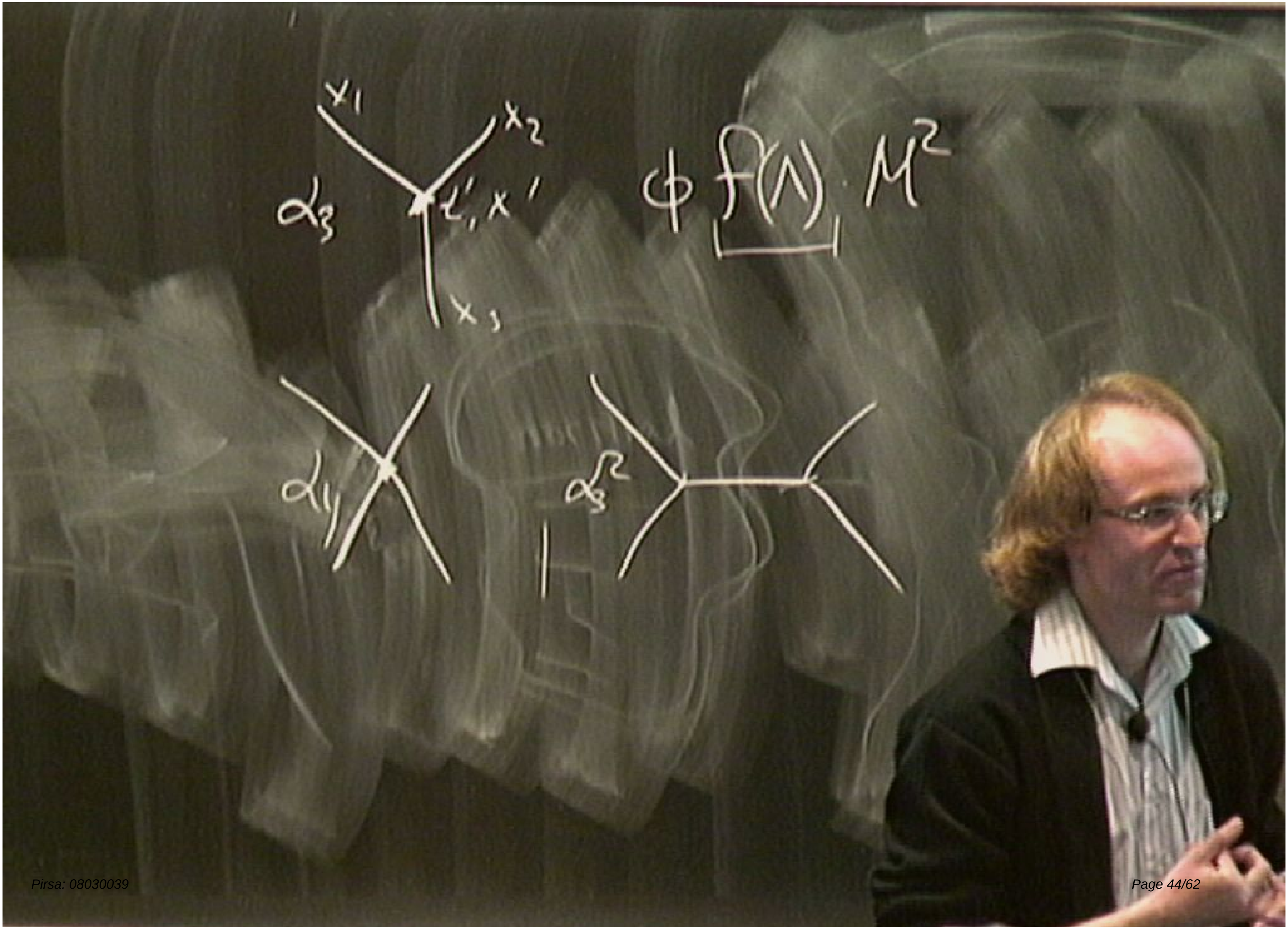
x1
x2
α3
t', x'
x3
φ f(Λ) M²
α1



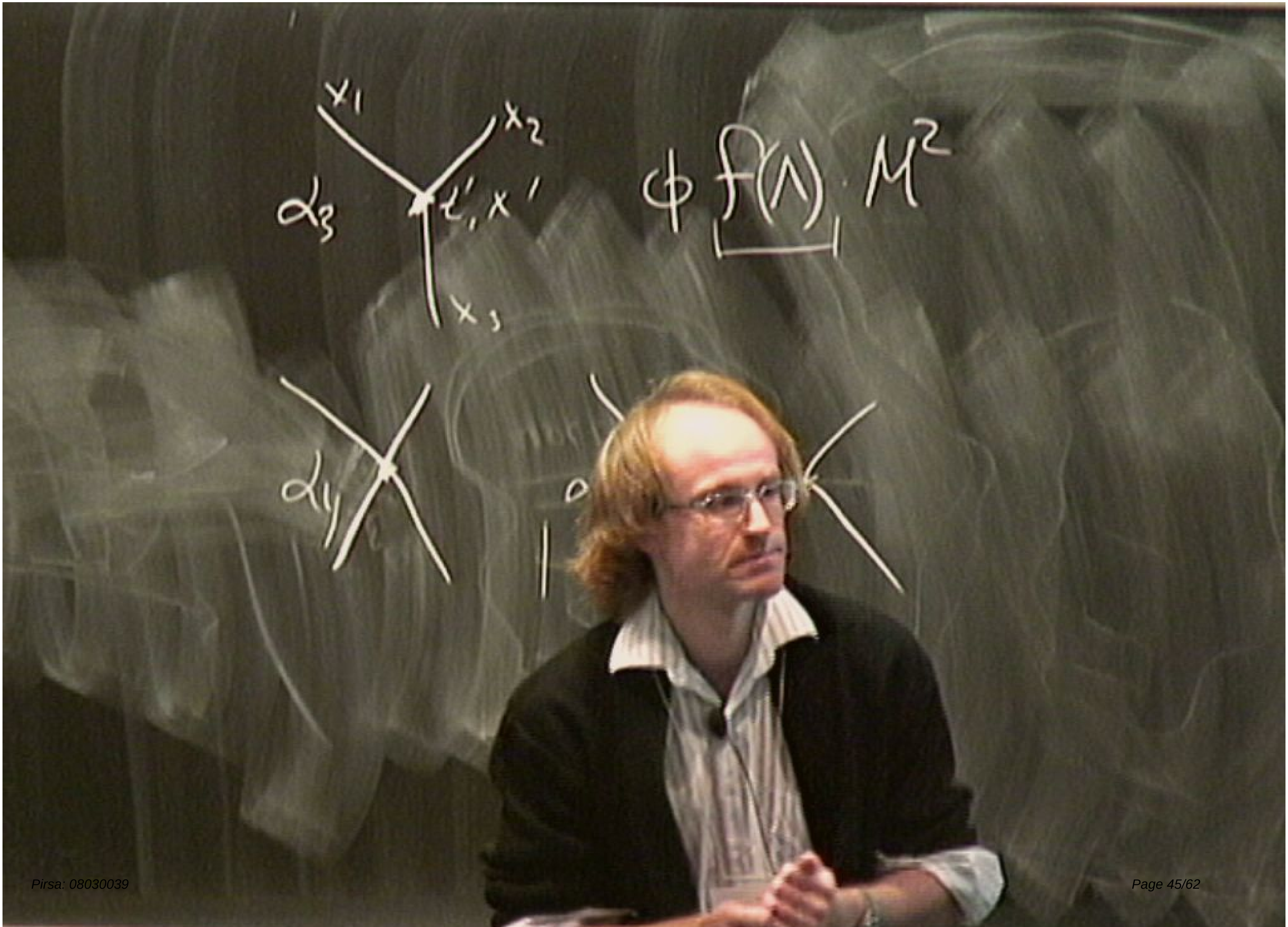
x1
x2
α3
t′, x′
x3
φ f(Λ) M²
α4
α3²

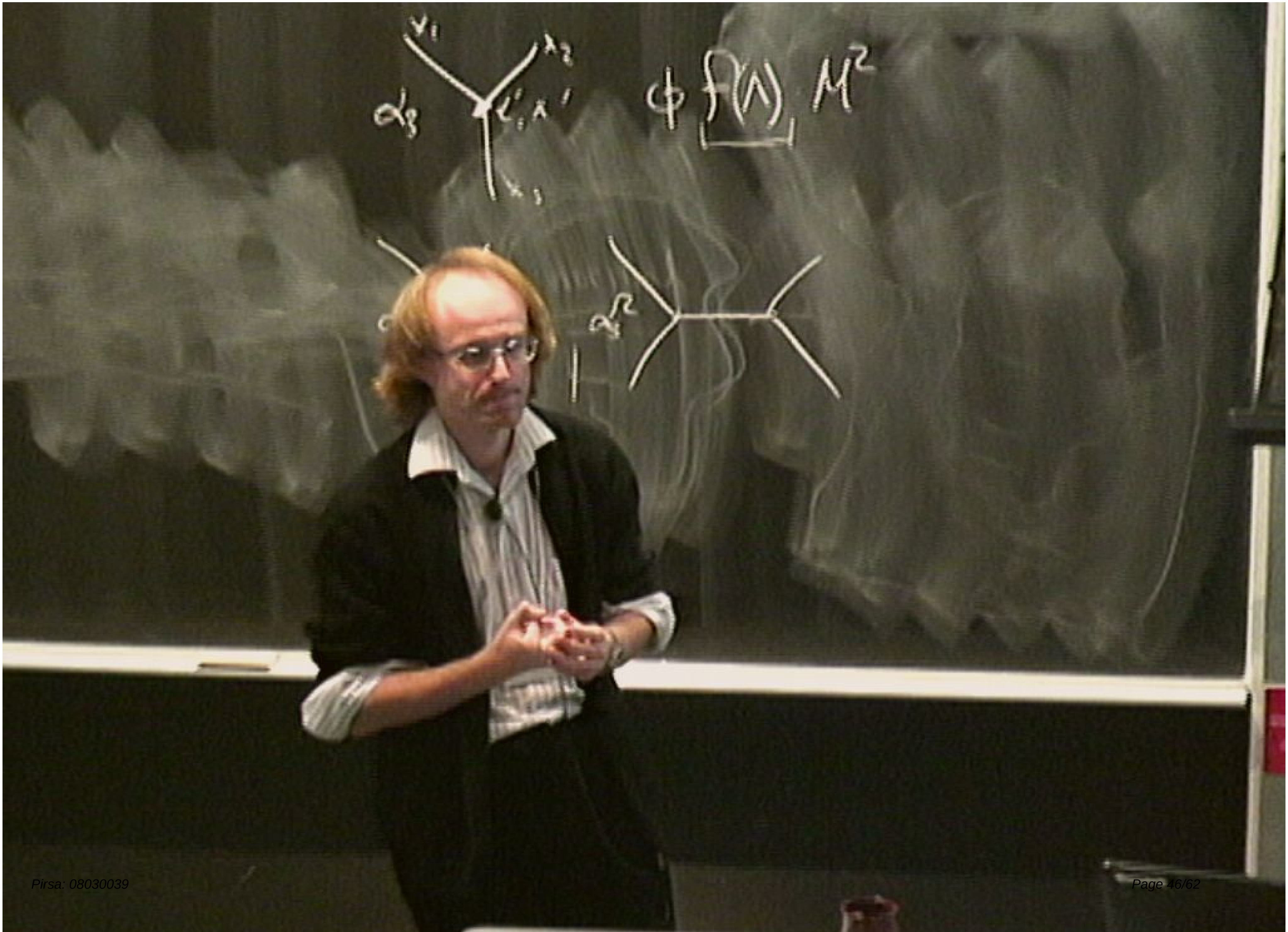


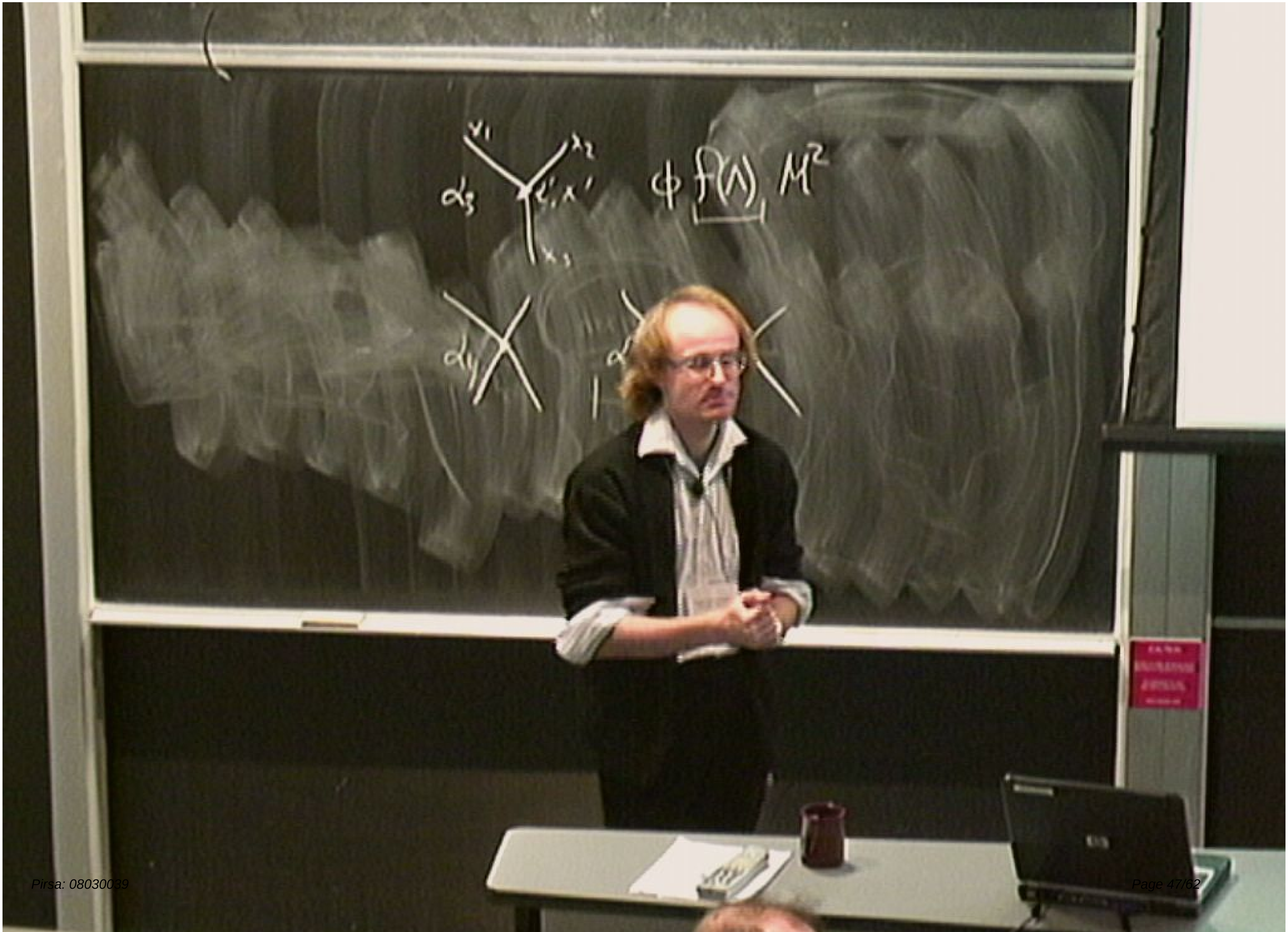
x_1
x_2
α_3
t', x'
x_3
$\phi\ f(\Lambda),\ M^2$
α_4
α_3^2

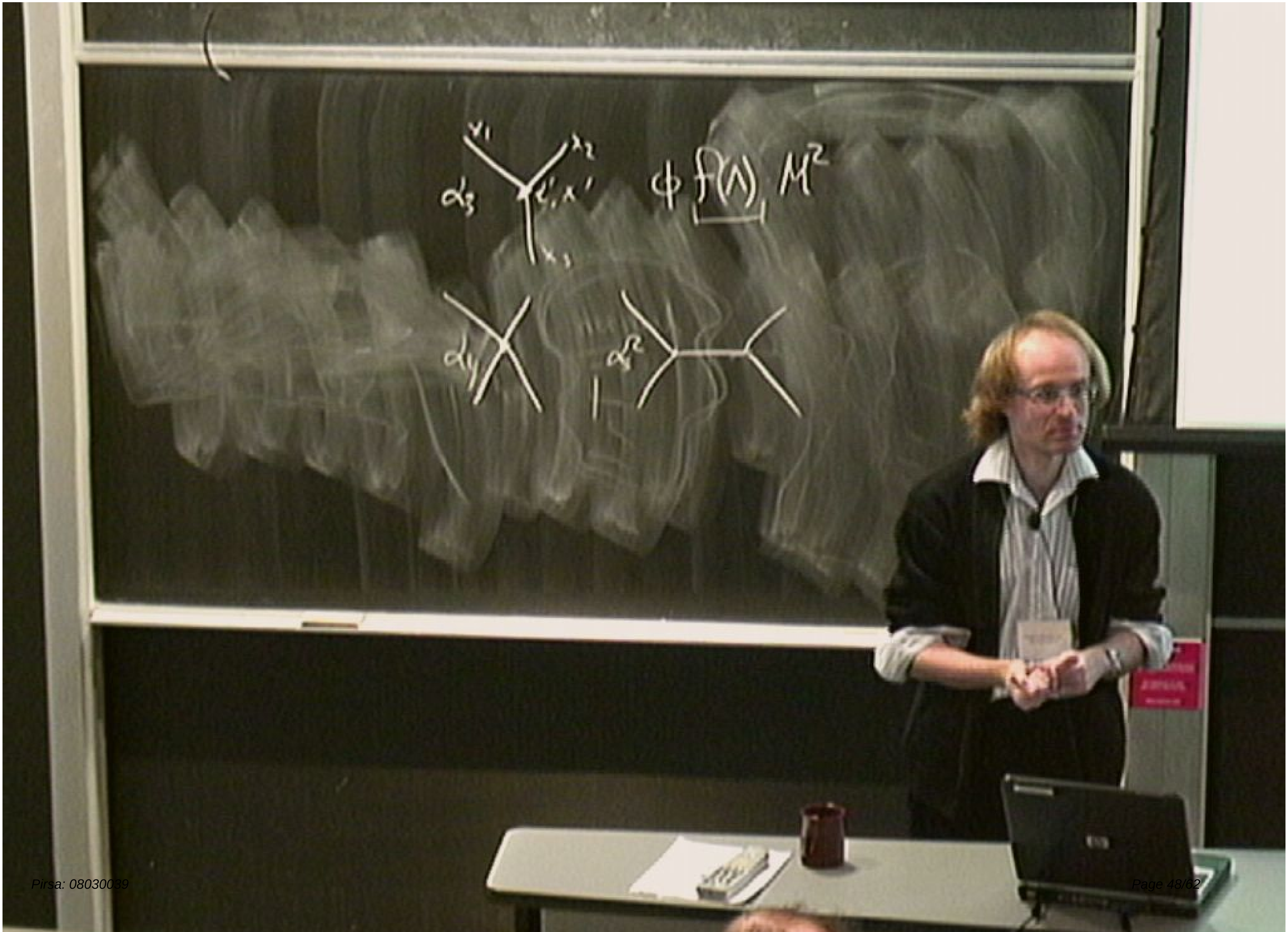


x_1
x_2
α_3
t', x'
x_3
$\phi\, f(\Lambda)\; M^2$
α_4
α_3^2

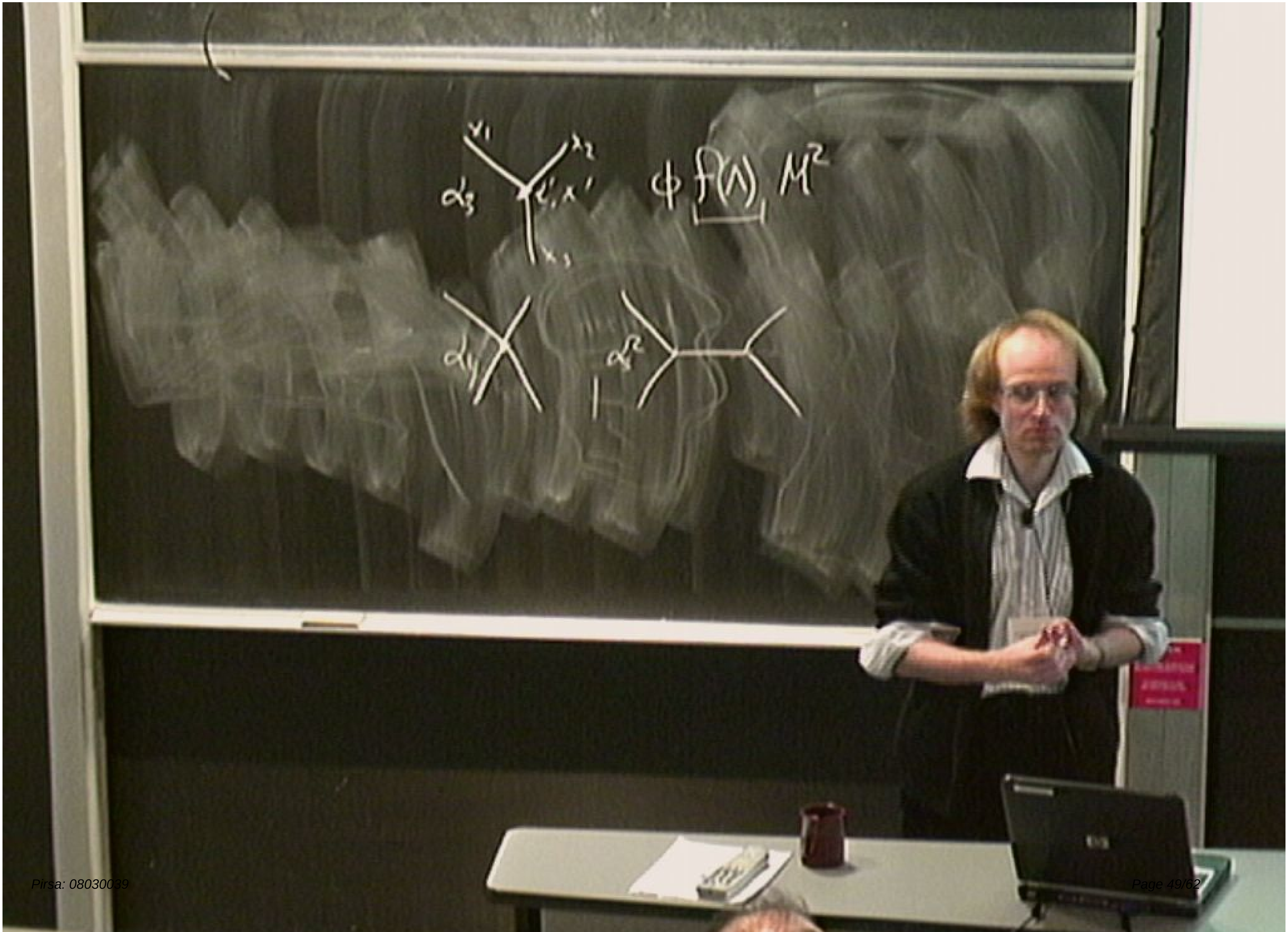

x1
x2
α3
t', x'
x3
φ f(Λ) M²
α4

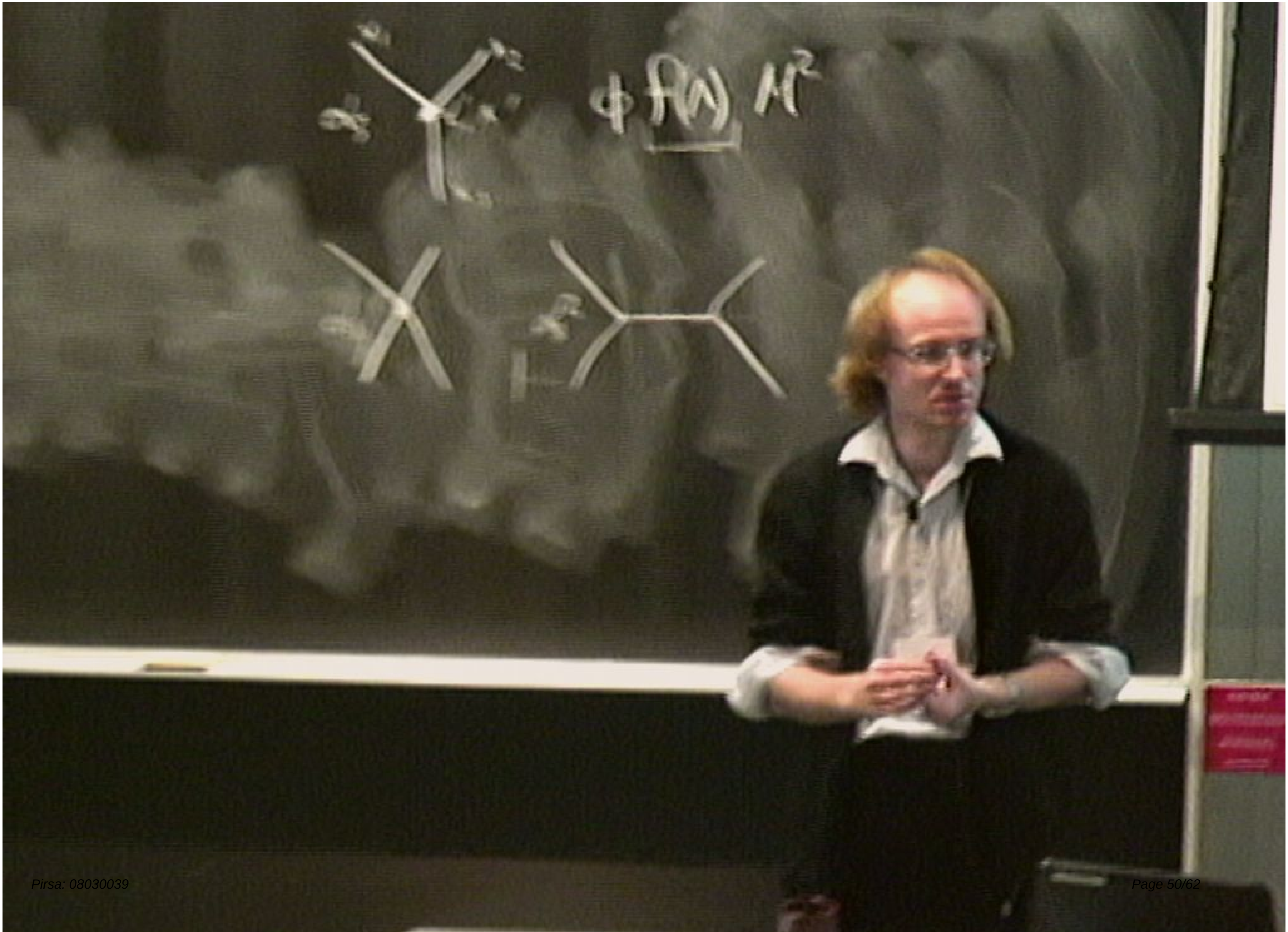


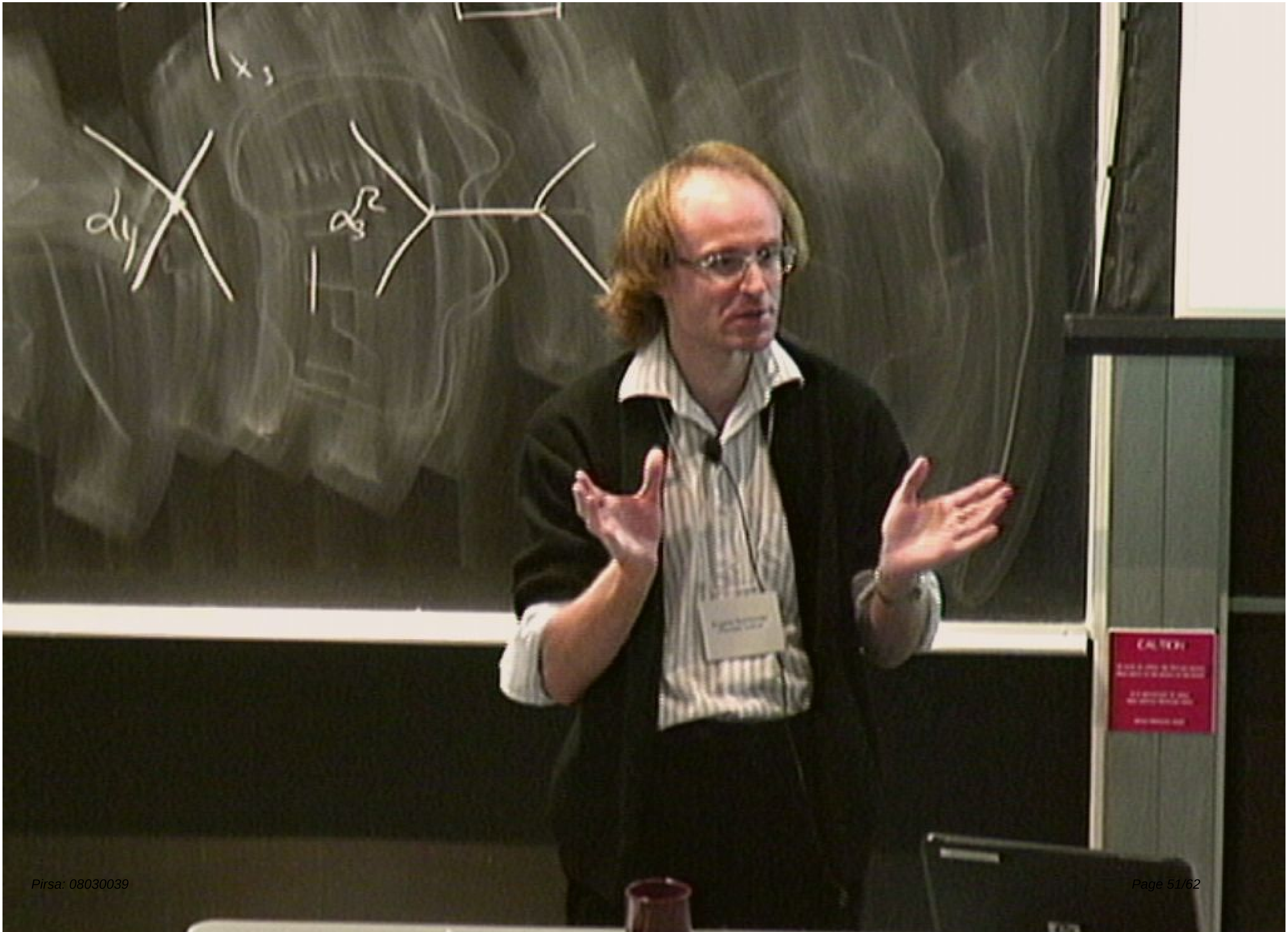

$\phi\, f(\Lambda)\, M^2$

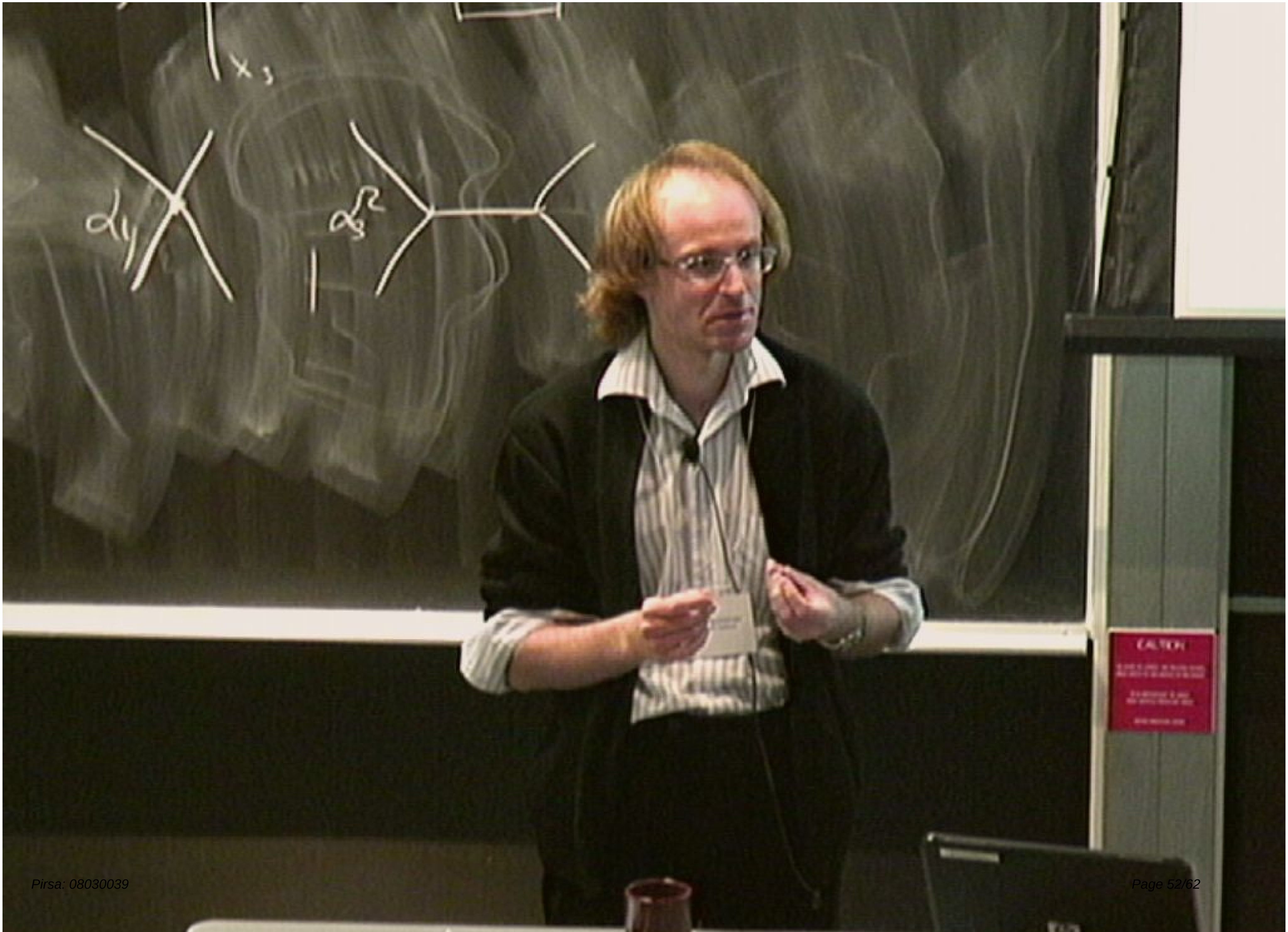


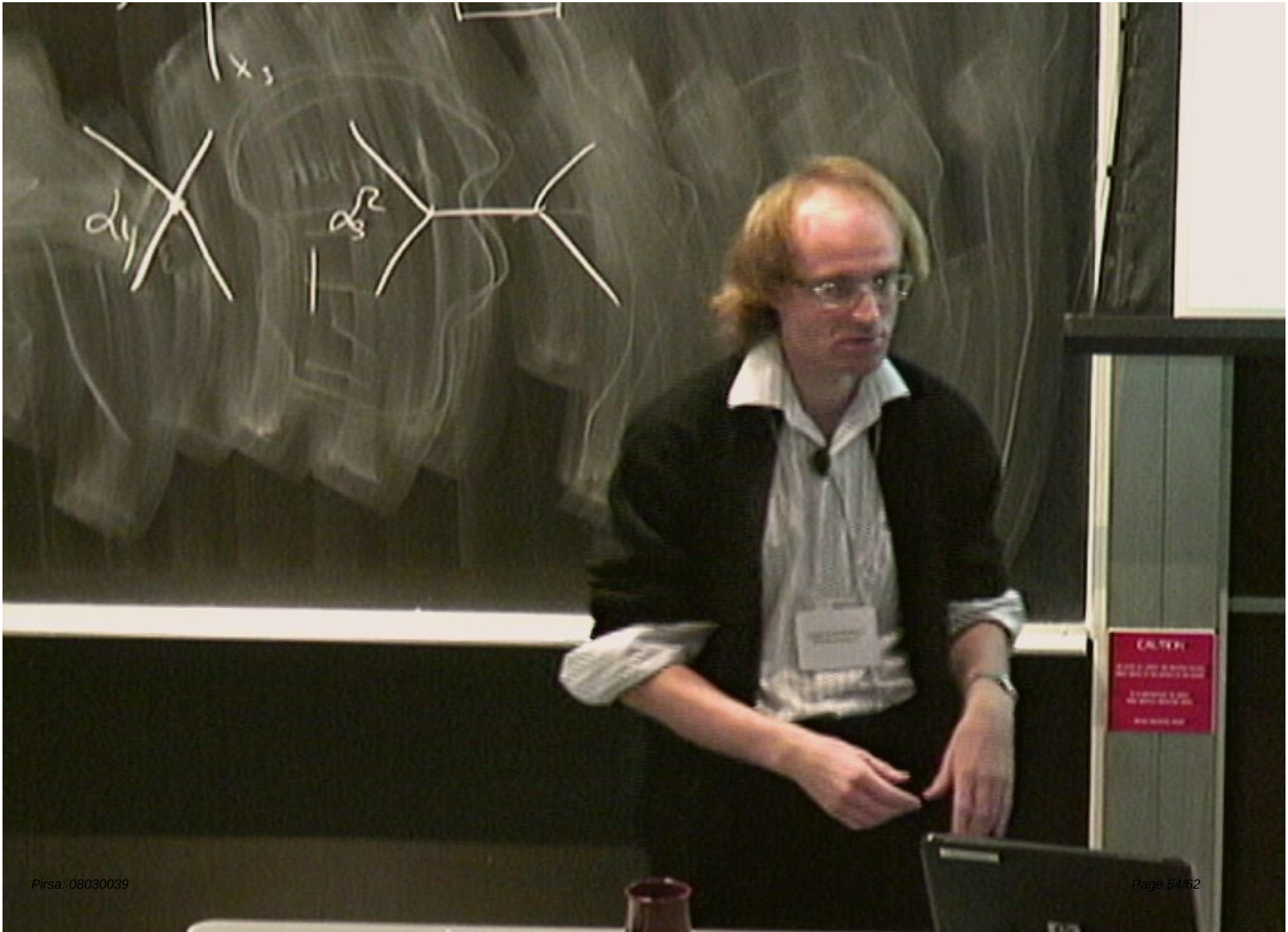
x1
x2
α3
x3
φ f(Λ) M²
α4
α5²

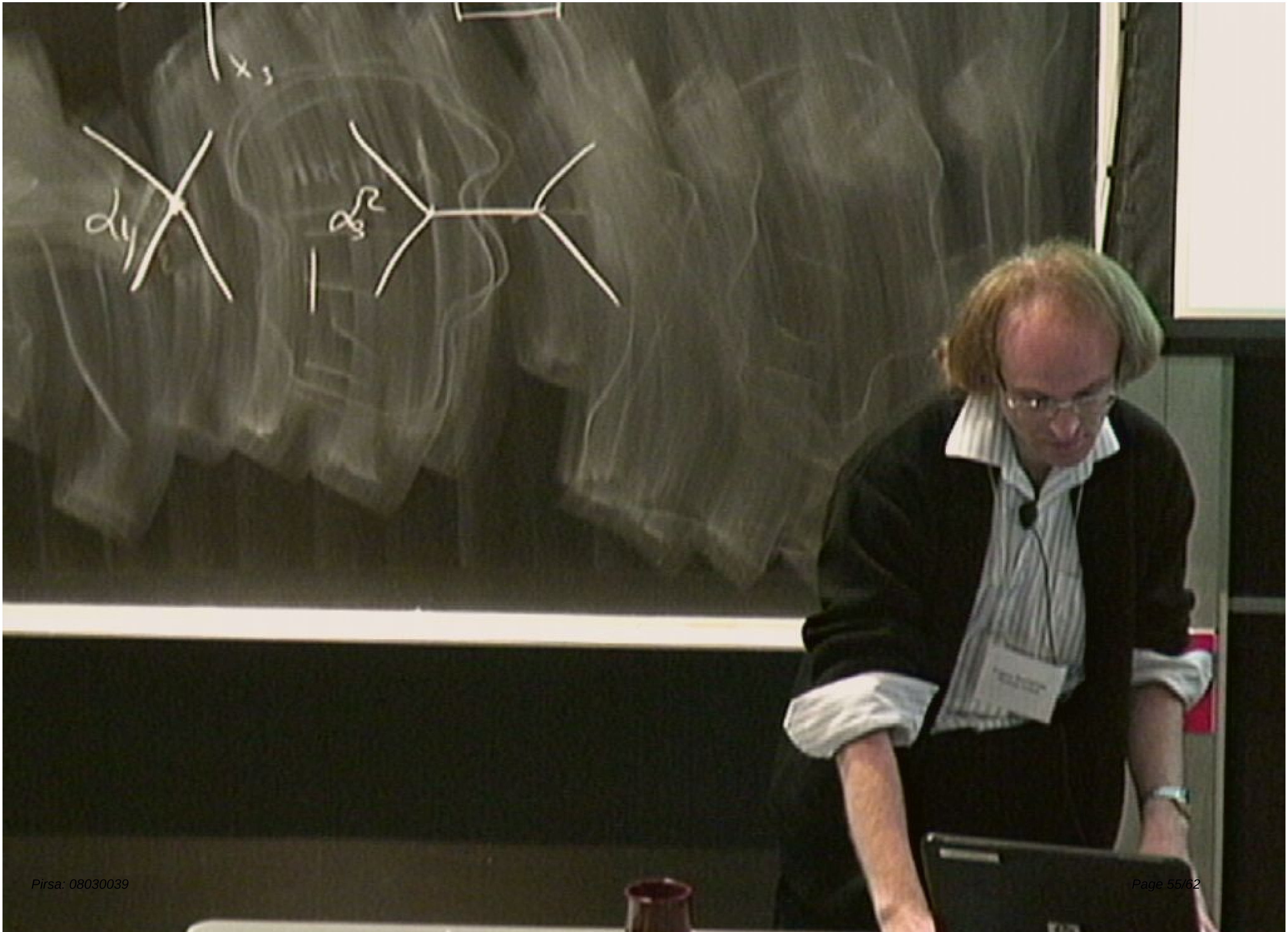


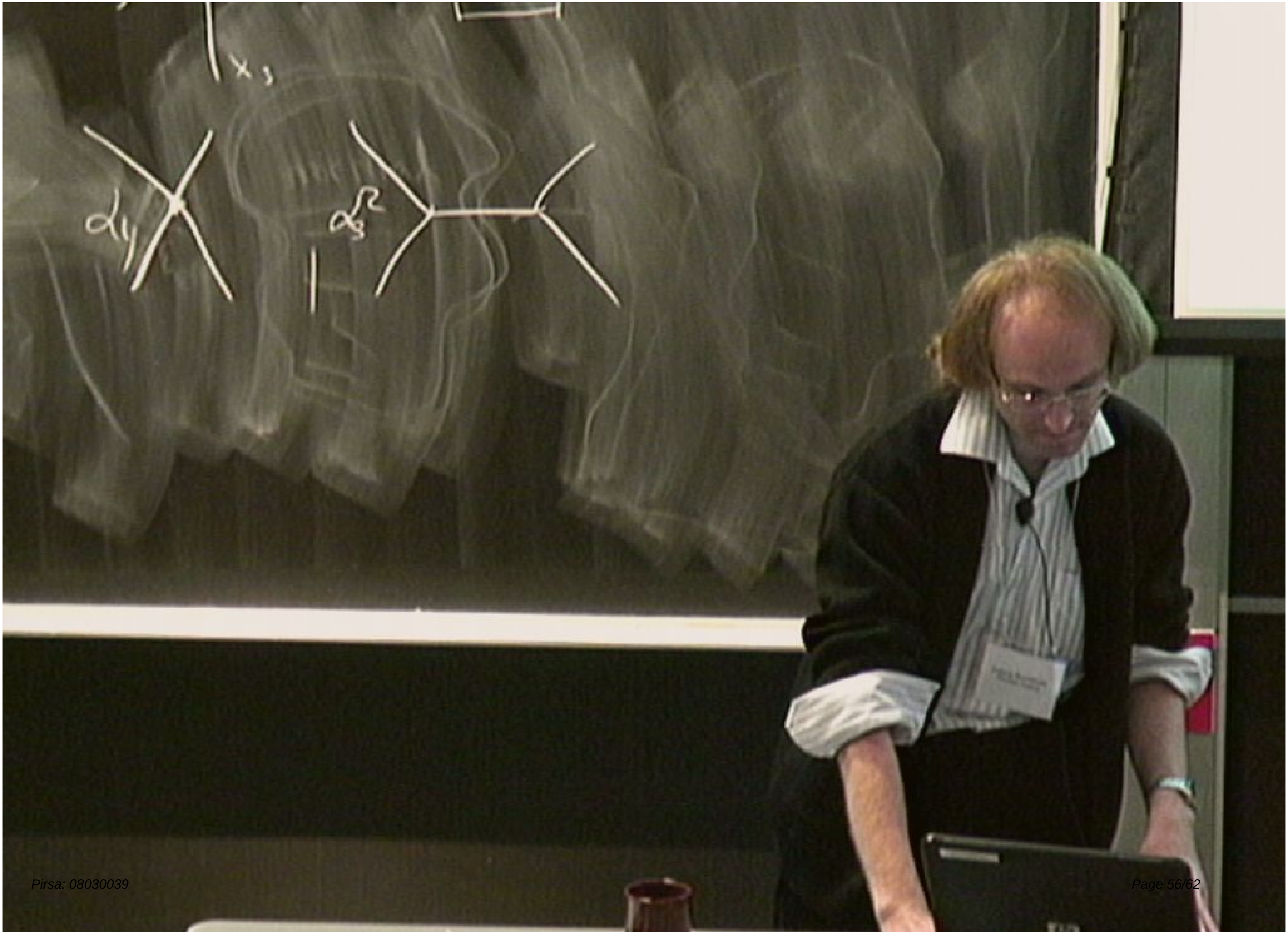


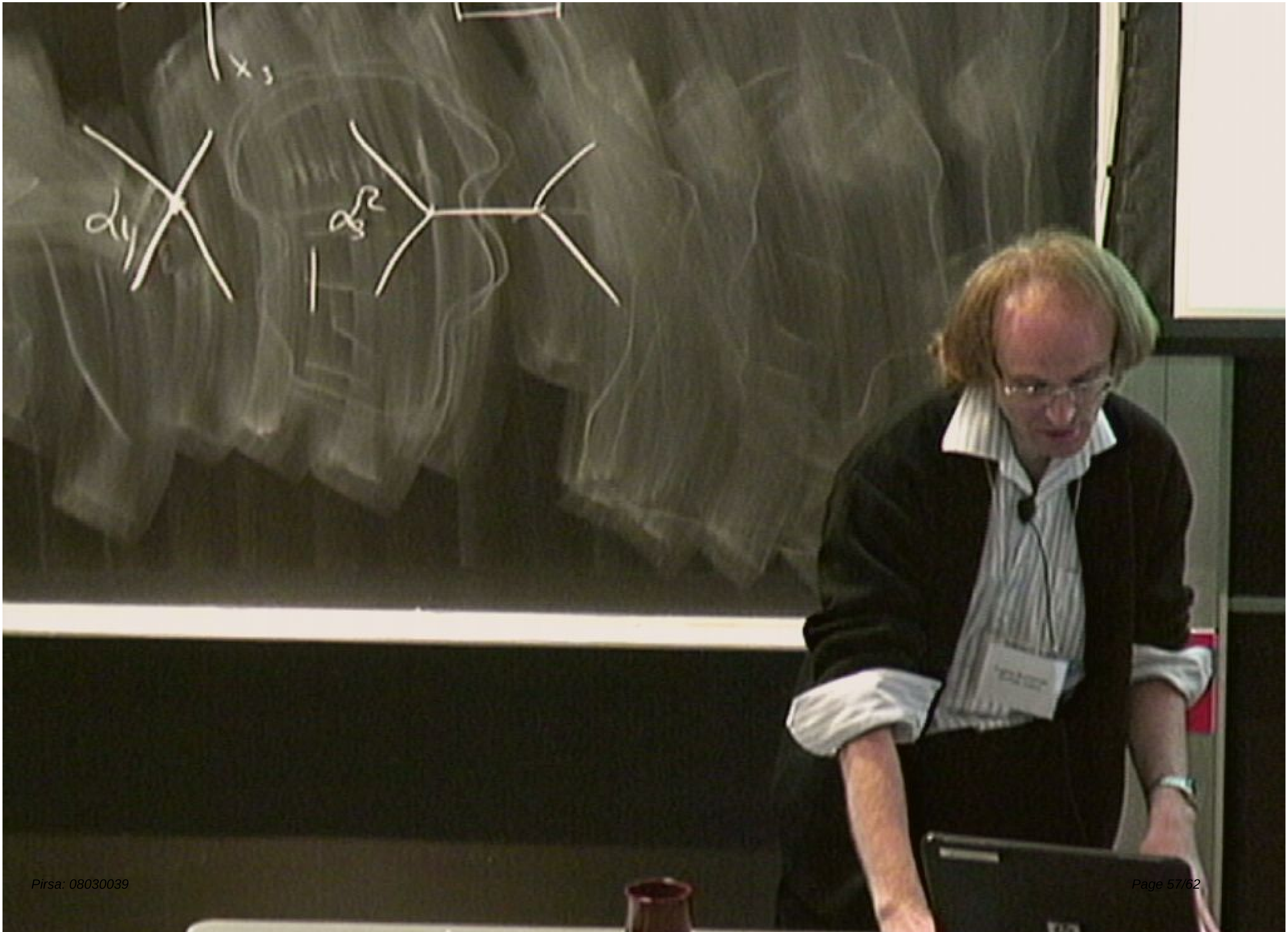

CAUTION

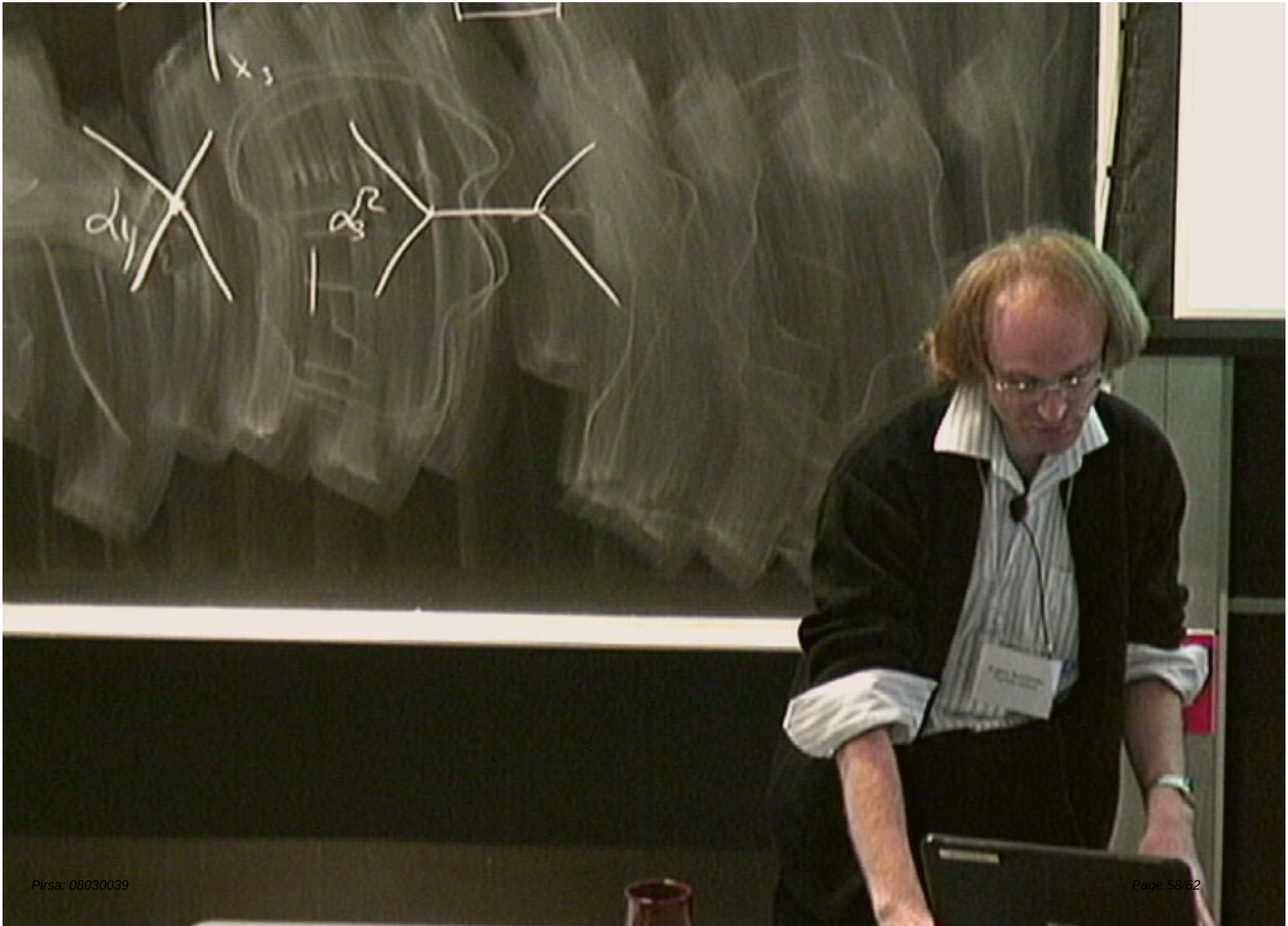


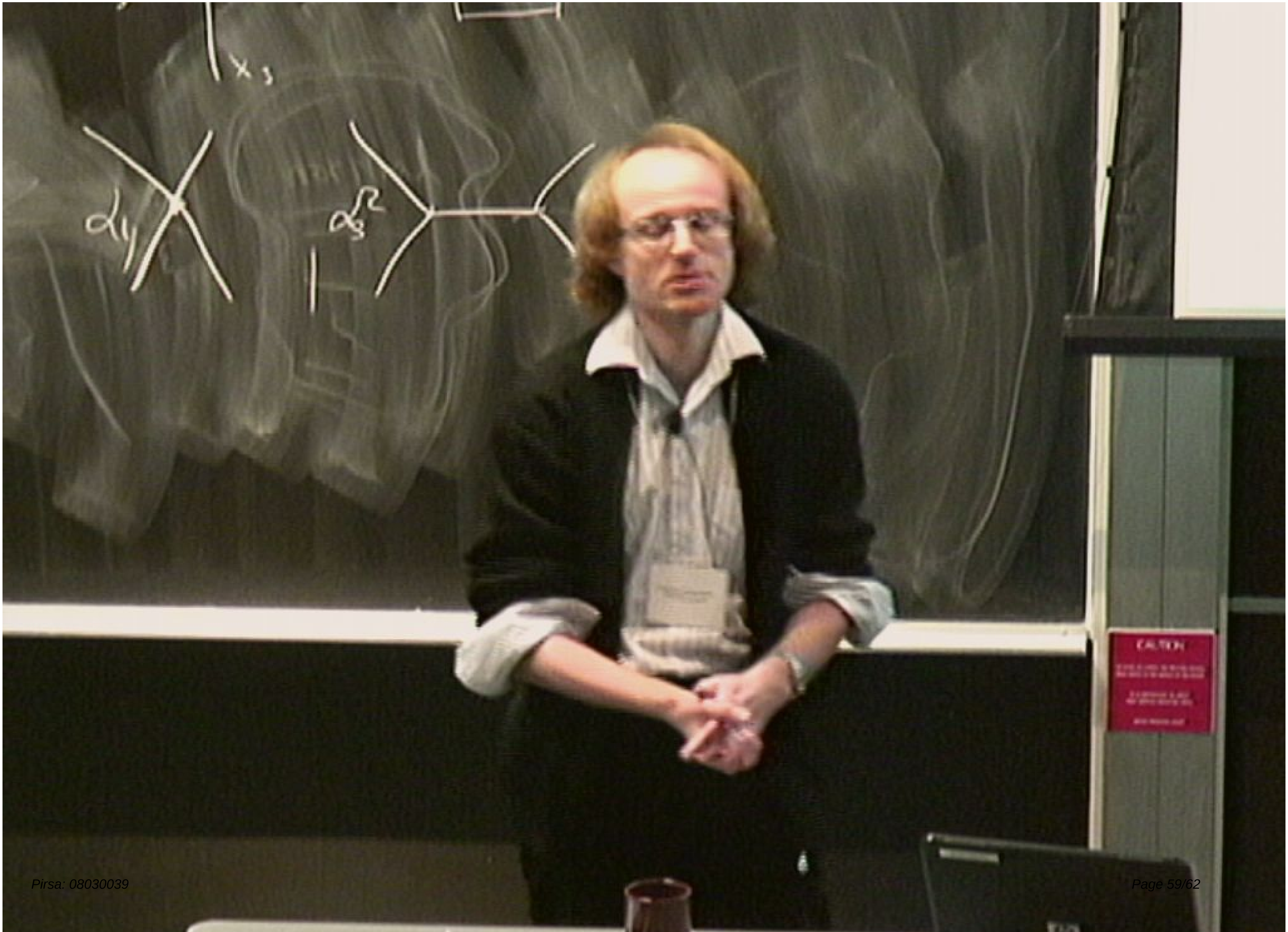











CAUTION