

Title: New Ekpyrotic Cosmology

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Abstract:

NEW EKPYROTIC COSMOLOGY

Evgeny Buchbinder, Justin Khoury, Burt Ovrut

- * Creminelli, Senatore
- * Lehners, McFadden, Turok, Steinhardt

- Pre-Big Bang - Gasperini, Veneziano
- Ekpyrotic Cosmology - Khoury, Ovrut, Steinhardt, Turok
- Big Crunch/Big-Bang - K,O,S,T+Seiberg
- Cyclic Models - Steinhardt, Turok

Ekpyrotic Phase

real scalar field - ϕ

Lagrangian density -

$$\frac{\mathcal{L}}{\sqrt{-g}} = -\frac{1}{2}(\partial_\mu \phi)^2 - V(\phi)$$

here the KE is canonical, Text

$$V(\phi) < 0$$

$$\epsilon \equiv M_P^{-2} \left(\frac{V}{V_{,\phi}} \right)^2, \quad \eta \equiv 1 - \frac{V_{,\phi\phi} V}{V_{,\phi}^2}$$

satisfy the “fast-roll” conditions

$$\epsilon \ll 1, \quad |\eta| \ll 1$$

> $V(\phi)$ is steep and nearly exponential

assume spatially flat, FRW spacetime

$$ds^2 = -dt^2 + a(t)^2 \vec{dx}^2$$

► Friedmann equation

$$3H^2 M_P^2 = \frac{1}{2} \dot{\phi}^2 + V(\phi)$$

scalar equation

$$\ddot{\phi} + 3H\dot{\phi} = -V_{,\phi}$$

solving gives the exact scaling solution

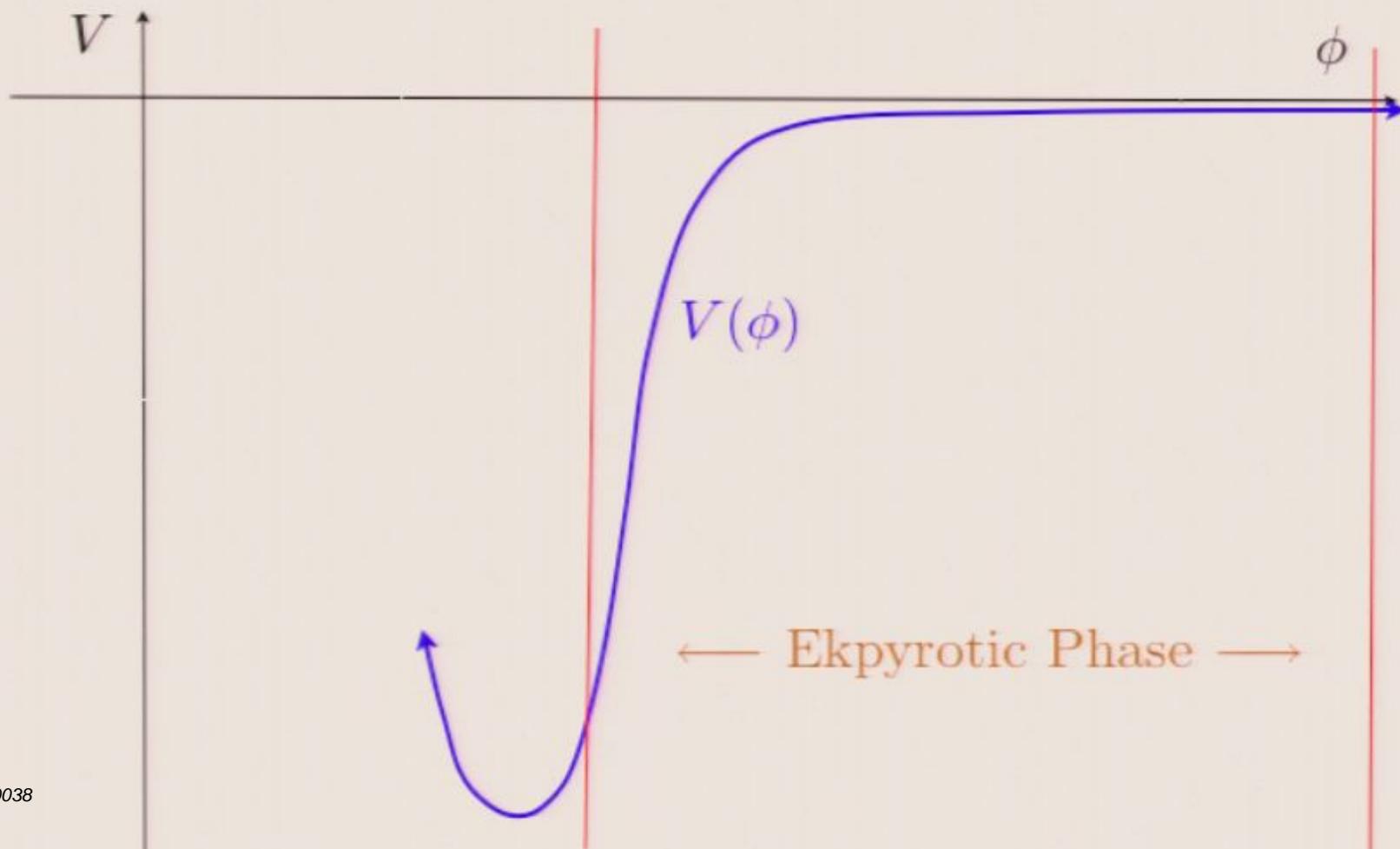
$$a(t) \sim (-t)^p, \quad H = \frac{p}{t}$$

$$\phi(t) = \sqrt{2p} M_P \ln \left(-\sqrt{\frac{V_0}{M_p^2 p (1-3p)}} t \right)$$

For example,

$$V(\phi) = -V_0 e^{-\sqrt{\frac{2}{p}} \frac{\phi}{M_P}}, \quad p \ll 1$$

➤ $\epsilon = \frac{p}{2} \ll 1, \quad \eta = 0$



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here



properties:

$$a(t) \sim (-t)^p \xrightarrow{t \rightarrow 0} 0$$

slowly contracting spacetime
~ constant

$p \ll 1 \implies$ slowly

$$R_H \equiv |H^{-1}| = \frac{-t}{p} \xrightarrow{t \rightarrow 0} 0 \quad \text{decreasing Hubble radius}$$

⇒ “dual” to inflation!

equation of state

$$\omega \equiv \frac{\mathcal{P}}{\rho} = \frac{2}{3p} - 1 \gg 1$$

$\rho_\phi \sim a^{-2/p}$ “blueshifts” faster than curvature ($\sim a^{-2}$),
matter ($\sim a^{-3}$), radiation ($\sim a^{-4}$), anisotropy ($\sim a^{-6}$)

Density Perturbations

Consider the gauge invariant variable

$$u \equiv \frac{a\Phi}{\phi'}$$

is the Newtonian potential and $' \equiv \frac{d}{d\tau}$ with conformal time. Fourier modes satisfy

$$u_k'' + \left(k^2 - \frac{p}{(1-p)^2 \tau^2} \right) u_k = 0$$

The solution is

$$u_k = \frac{\sqrt{p}}{(2k)^{3/2} M_P} \sqrt{\frac{\pi}{2}} \sqrt{-k\tau} H_\nu^{(1)}(-k\tau)$$

here $H_\nu^{(1)}$ is a Hankel function with

$$\nu = \frac{(1+p)}{2(1-p)}$$

n large scales

$$k^3 u k^2 \sim k^{-2p/(1-p)}$$

nce $p \ll 1$, the spectrum is nearly scale invariant with

$$n_s - 1 \approx -2p \quad (= -4\epsilon)$$

slight **red** tilt. For more general $V(\phi)$

$$n_s - 1 = -4(\epsilon + \eta)$$

characterizes deviations from the pure exponential and
n be positive or negative. \Rightarrow spectrum can have a small
blue tilt if η is negative

Problem!!

The relevant variable to track through three bounce is the
curvature perturbation

$$\zeta \propto \left(\frac{\Phi}{a'/a^3} \right)'$$

or the scaling solution

$$\Phi_k \longrightarrow \frac{a'}{a^3} \frac{1}{k^{3/2}}$$

or large modes \Rightarrow

$$\zeta_k \longrightarrow 0 \frac{1}{k^{3/2}} + \dots$$

> a strong blue tilt!

Solution!!

Part A:

Consider two (or more) real scalar fields- ϕ_1, ϕ_2

Lagrangian density-

$$\frac{\mathcal{L}}{\sqrt{-g}} = -\frac{1}{2}(\partial_\mu \phi_1)^2 - \frac{1}{2}(\partial_\mu \phi_2)^2 - V(\phi_1 \phi_2)$$

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or example

$$V(\phi_1, \phi_2) = -V_1 e^{-\sqrt{\frac{2}{p_1}} \frac{\phi_1}{M_P}} - V_2 e^{-\sqrt{\frac{2}{p_2}} \frac{\phi_2}{M_P}}$$

with $p_1 \ll 1, p_2 \ll 1$. Defining new variables

$$\phi = \frac{\sqrt{p_1} \phi_1 + \sqrt{p_2} \phi_2}{\sqrt{p_1 + p_2}}, \quad \chi = \frac{\sqrt{p_2} \phi_1 - \sqrt{p_1} \phi_2}{\sqrt{p_1 + p_2}}$$

$$V(\phi, \chi) = -V_0 e^{-\sqrt{\frac{2}{p}} \frac{\phi}{M_P}} \left(1 + \frac{1}{p M_P^2} (\chi - \chi_t)^2 + \dots \right)$$

Koyama,
Wands

here $p \equiv p_1 + p_2$ ($\ll 1$),

$$\chi_t \equiv \sqrt{\frac{p_1 p_2}{2p}} \ln \left(\frac{p_2 V_1}{p_1 V_2} \right)$$

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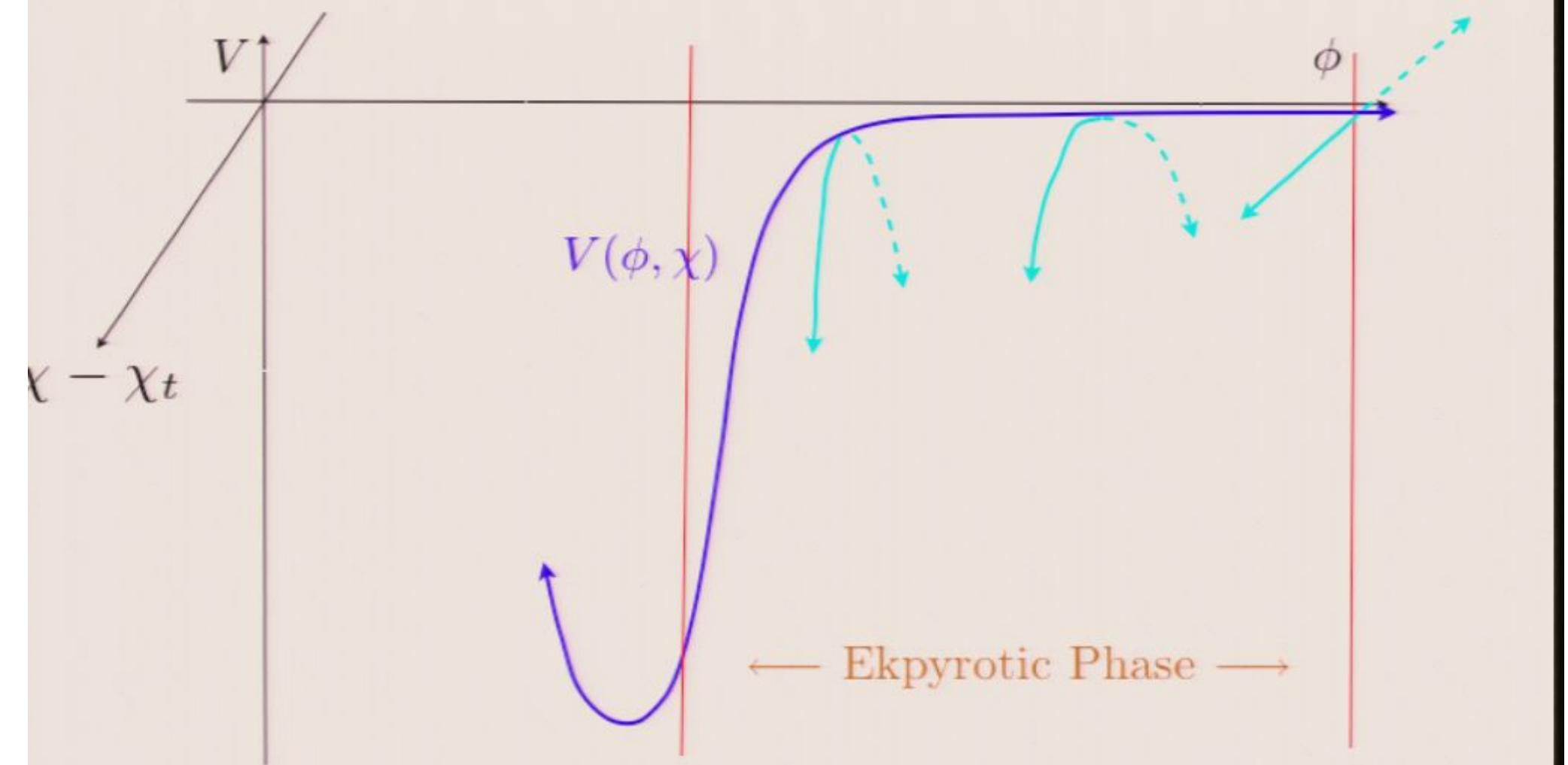
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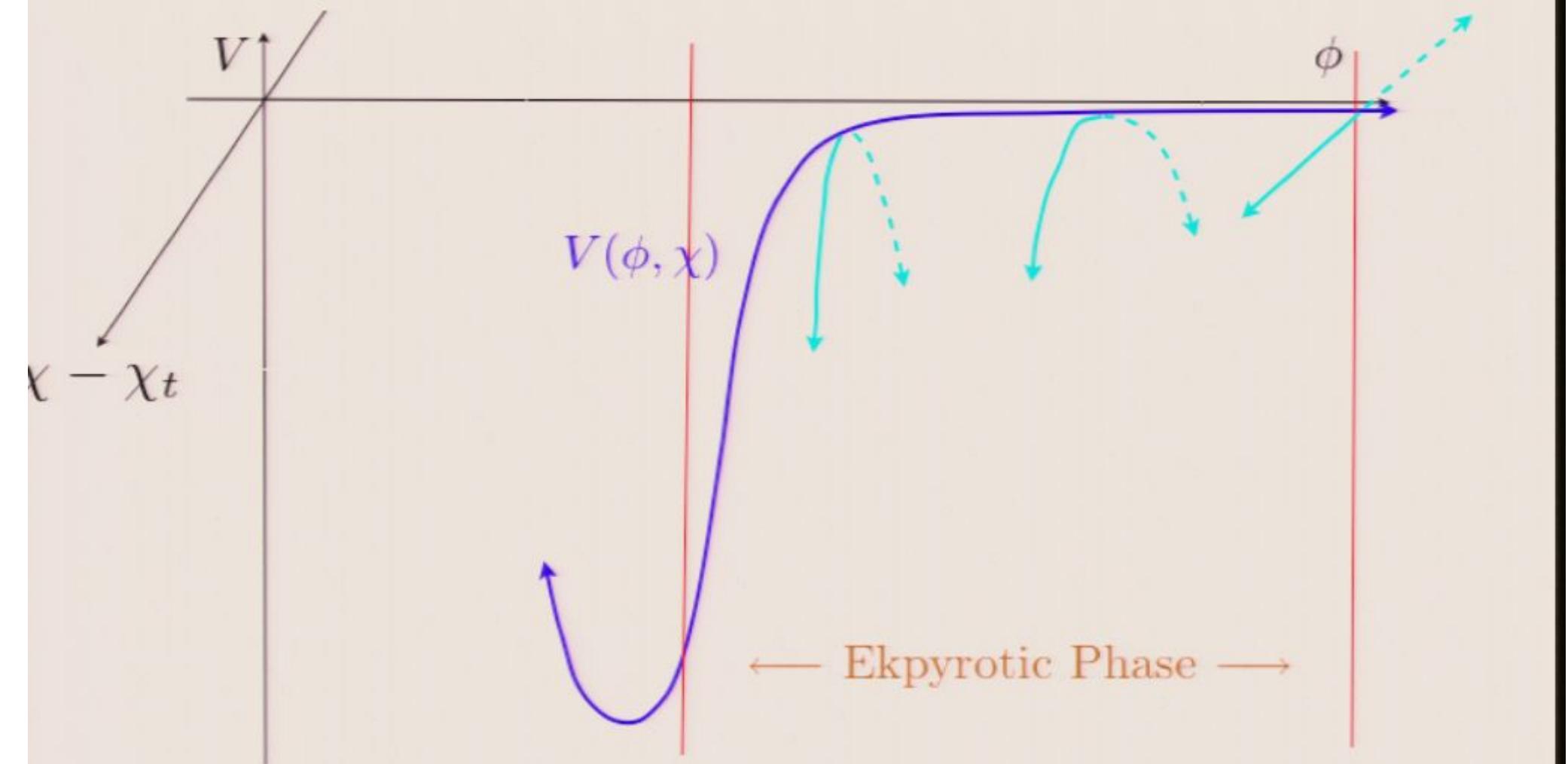
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scalar equations

$$\ddot{\phi} + 3H\dot{\phi} = -V_{,\phi}, \quad \ddot{\chi} + 3H\dot{\chi} = -V_{,\chi}$$

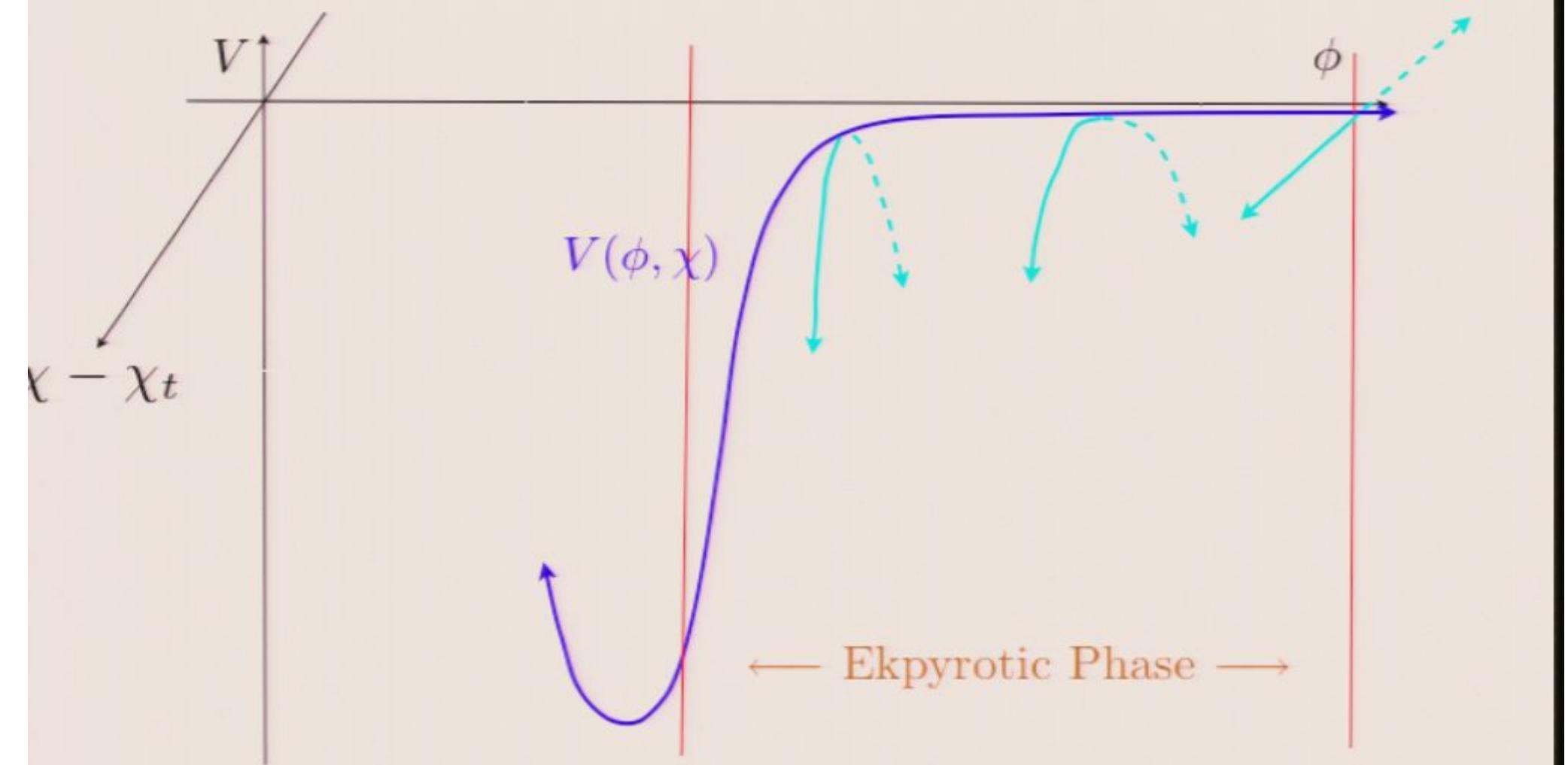
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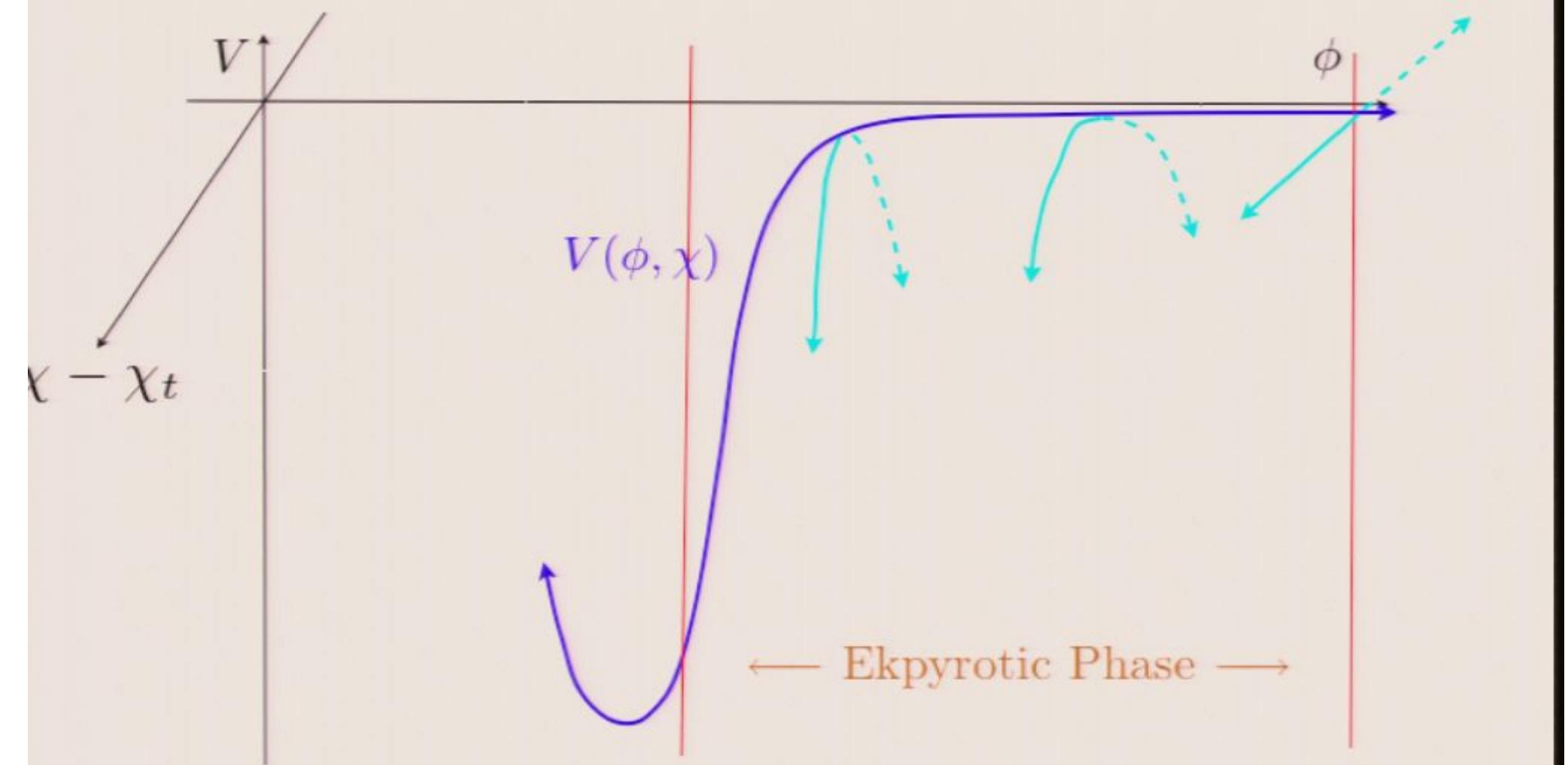
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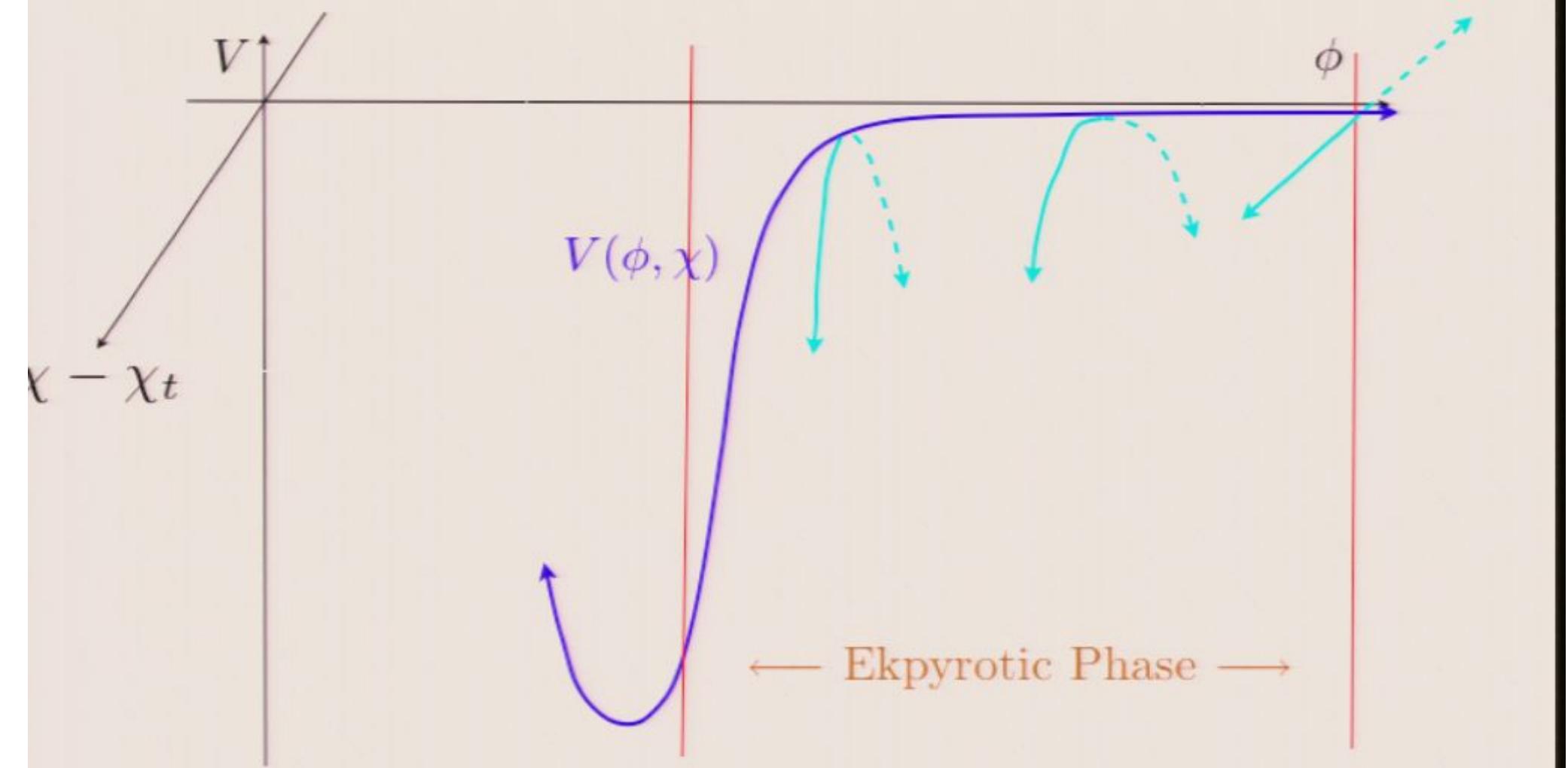
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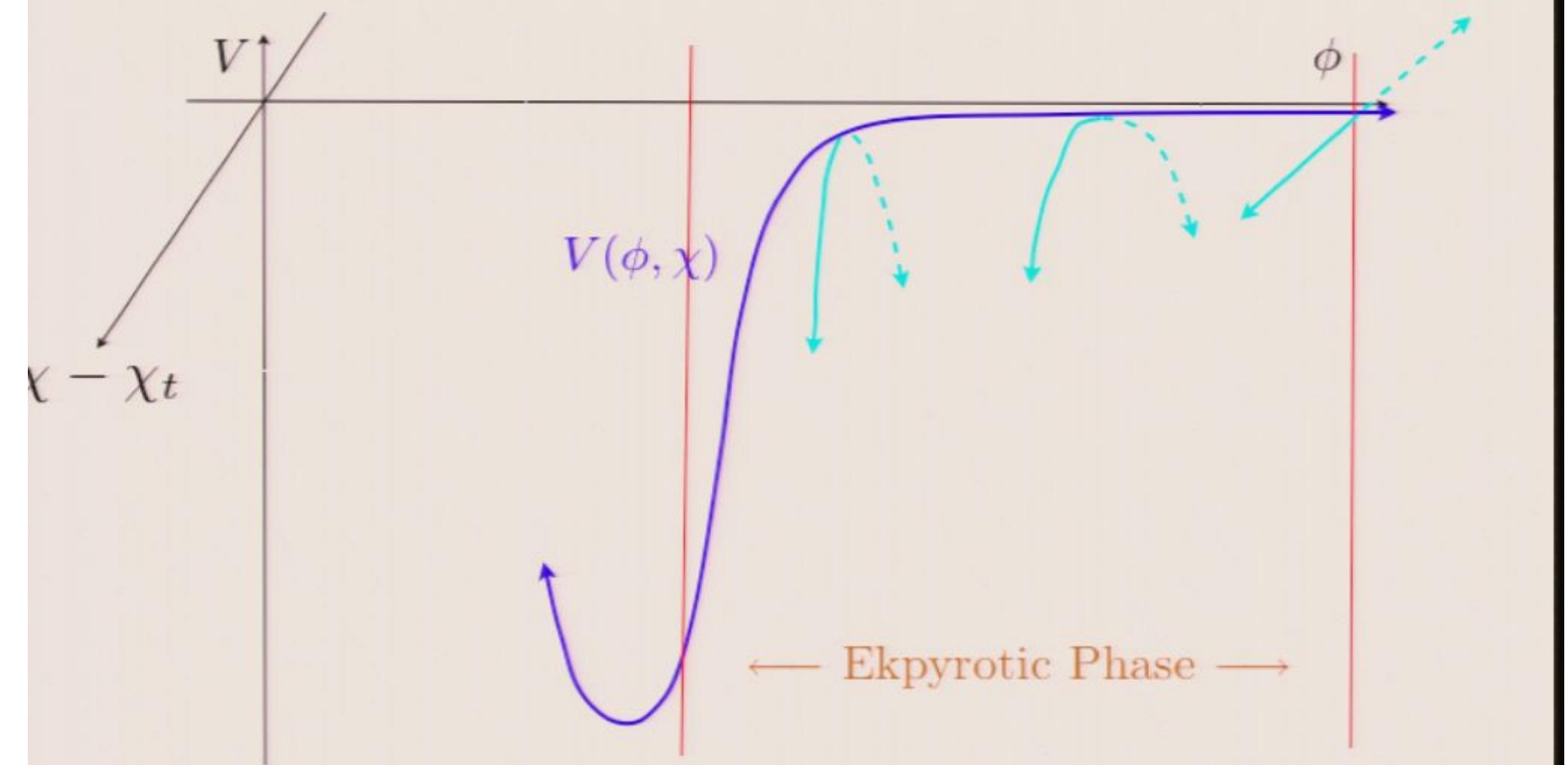
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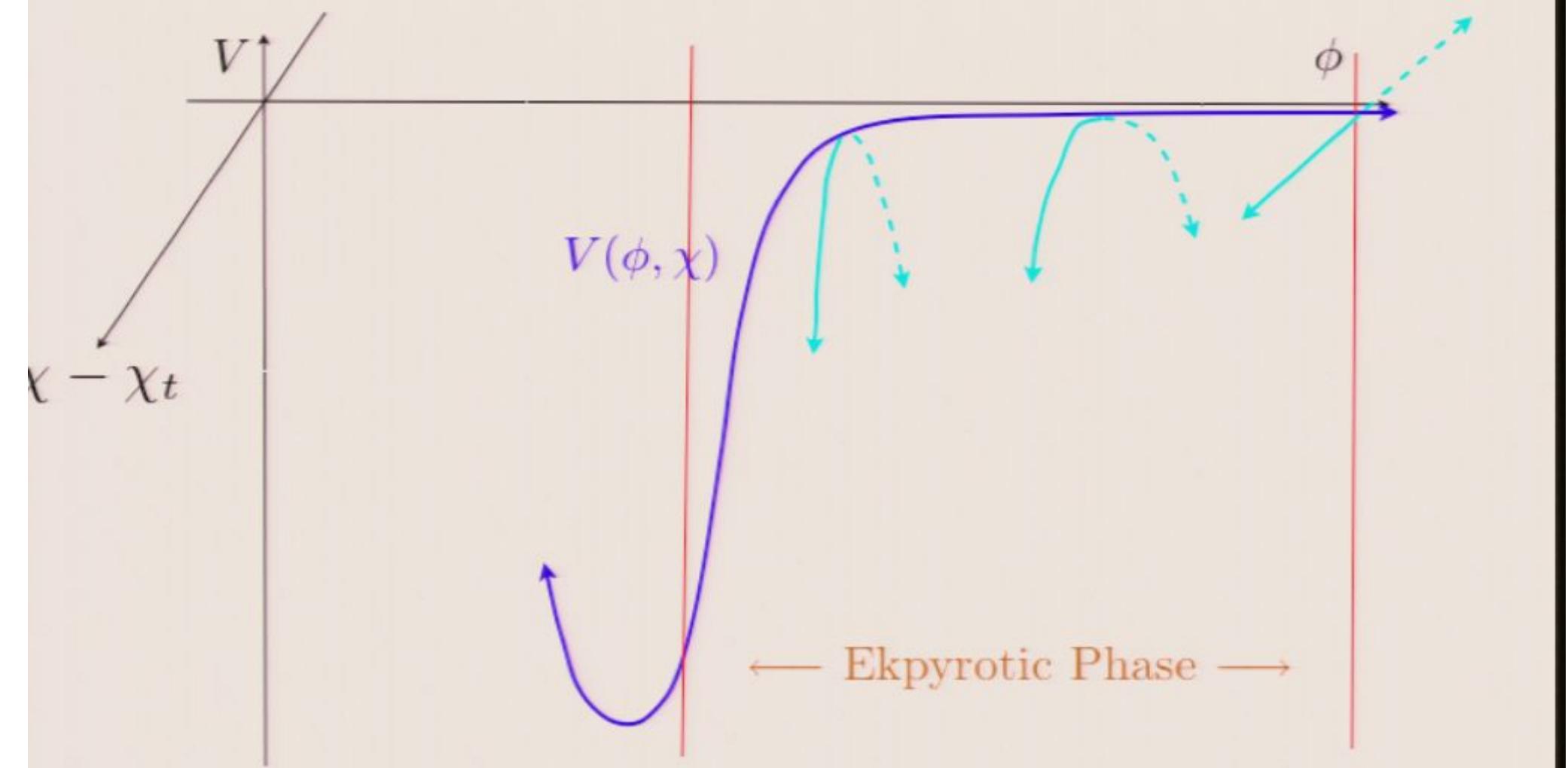
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$$\chi = \chi_t$$

Properties:

Slowly contracting spacetime

decreasing Hubble radius

$$\omega = \frac{2}{3p} - 1 \quad \gg 1$$

$$V_{,\phi\phi}|_{\chi=\chi_t} = V_{,\chi\chi}|_{\chi=\chi_t}, \quad V_{,\phi\phi}|_{\chi=\chi_t} \approx -\frac{2}{t^2}$$

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Density Perturbations

Consider the entropy fluctuation - $\delta\chi$
Fourier modes satisfy

$$\delta\ddot{\chi}_k + 3H\delta\dot{\chi}_k + \left(\frac{k^2}{a^2} + V_{,\phi\phi} \right) \delta\chi_k = 0$$

Using property 4. \Rightarrow

$$k^3\delta\chi_k \sim k^{2p}$$

spectrum is nearly scale invariant with

$$n_s - 1 = 2p (= 4\epsilon)$$

\Rightarrow slight blue tilt

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General form for the two field potential is

$$V(\phi, \chi) = \mathcal{V} \left(1 + \frac{1}{2} \frac{\mathcal{V}_{,\phi\phi}}{\mathcal{V}} f(\phi) (\chi - \chi_t)^2 + F(\phi, \chi - \chi_t) \right)$$

or any

$$\chi_t = \text{constant}, f(\phi) \approx 1$$

function $F(\phi, \chi - \chi_t)$ such that

$$F_{,\chi\chi}|_{\chi=\chi_t} = 0$$

and $\mathcal{V}(\phi)$ where

$$\epsilon \equiv M_P^{-2} \left(\frac{\mathcal{V}}{V_{,\phi}} \right)^2 \ll 1 \quad |\eta| \equiv \left| 1 - \frac{\mathcal{V}_{,\phi\phi} \mathcal{V}}{\mathcal{V}_\phi^2} \right| \ll 1$$

Note that

$$V_{,\phi\phi}|_{\chi=\chi_t} \approx V_{,\chi\chi}|_{\chi=\chi_t} \approx -\frac{2}{t^2}$$

Choosing

$$f(\phi) \equiv 1 + 3\delta, |\delta| \ll 1$$

General form for the two field potential is

$$V(\phi, \chi) = \mathcal{V} \left(1 + \frac{1}{2} \frac{\mathcal{V}_{,\phi\phi}}{\mathcal{V}} f(\phi) (\chi - \chi_t)^2 + F(\phi, \chi - \chi_t) \right)$$

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the spectral index generalizes to

$$n_s - 1 = 4(\epsilon - \eta - \delta)$$

⇒ can have red tilt if η and/or δ are positive

Part B:

or a rapid turn in field space from $\phi \rightarrow \chi$
the entropy perturbation $\delta\chi_k$ “sources” the curvature
perturbation ζ_k as

$$|\zeta_k| \approx \frac{2\epsilon}{M_P} \Delta\theta \delta\chi_k$$

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where

$$\Delta\theta \sim \mathcal{O}(1)$$

the spectral index generalizes to

$$n_s - 1 = 4(\epsilon - \eta - \delta)$$

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$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + \mathcal{V}(\phi), \quad \rho_\chi = \frac{1}{2}\dot{\chi}^2 - \frac{1}{2}m^2_{tachyon}\chi^2 + \mathcal{O}(\chi^3)$$

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$$\Delta\chi_{ek\text{-}end} \lesssim pM_P$$

and, hence

$$\Delta\chi_{ek\text{-}beg} \lesssim e^{-N_{ek}} pM_P \quad (**)$$

• exponentially fine-tuned initial conditions!

must have the constraint

$$|\rho_\chi| \lesssim |\rho_\phi|$$

this \Rightarrow

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here N_{ek} is the number of e-folds. For example,

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Add to the potential energy

$$V(\phi, \chi) = \dots + \frac{1}{2} m_\chi^2(\phi) \chi^2$$

the χ mass is

$$m_{eff}^2(\phi) = -m_{tachyon}^2(\phi) + m_\chi^2(\phi)$$

choose $m_\chi(\phi)$ so that for large ϕ

$m_{eff}^2(\phi) > 0 \Rightarrow$ pre-Ekpyrotic phase

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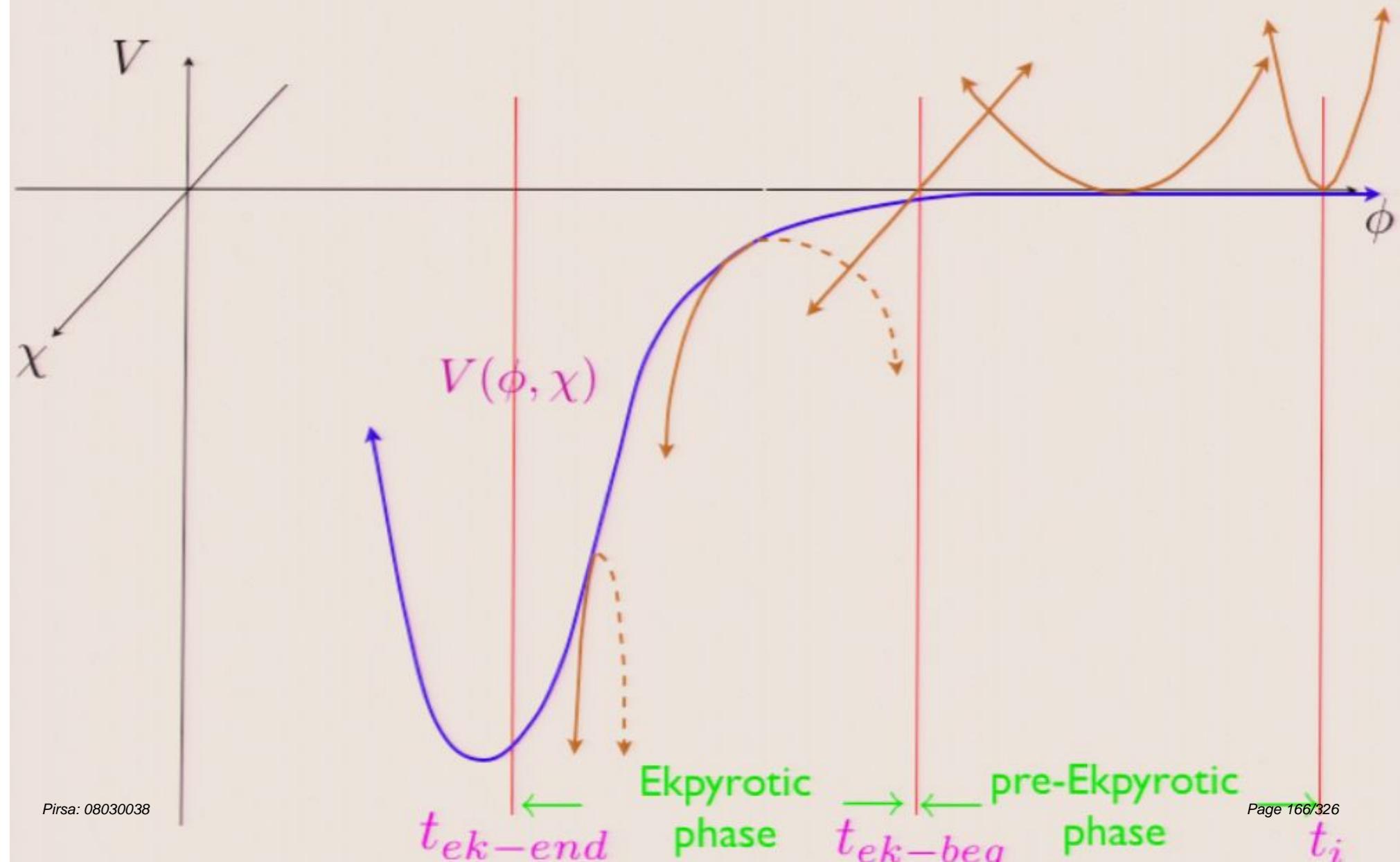
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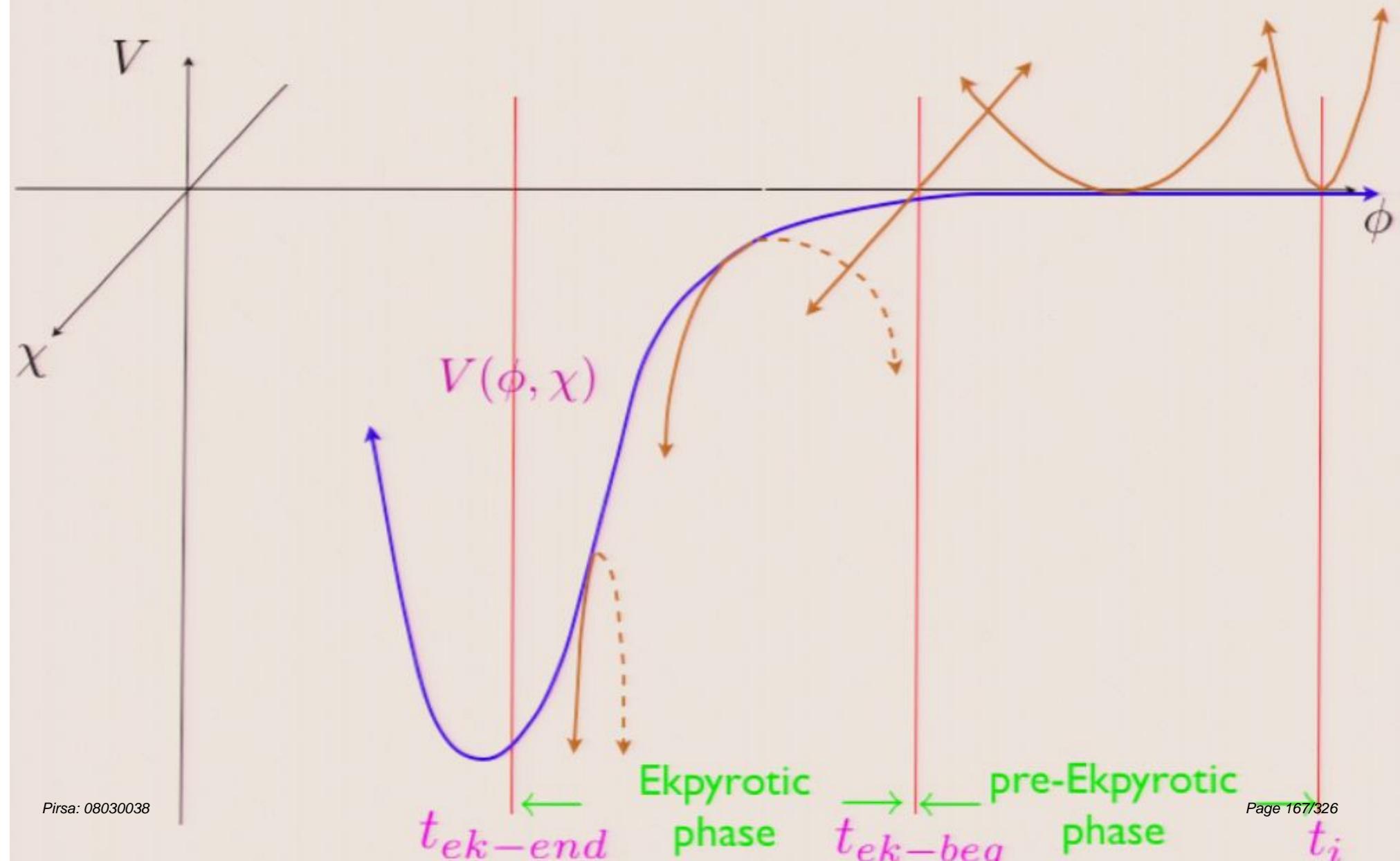
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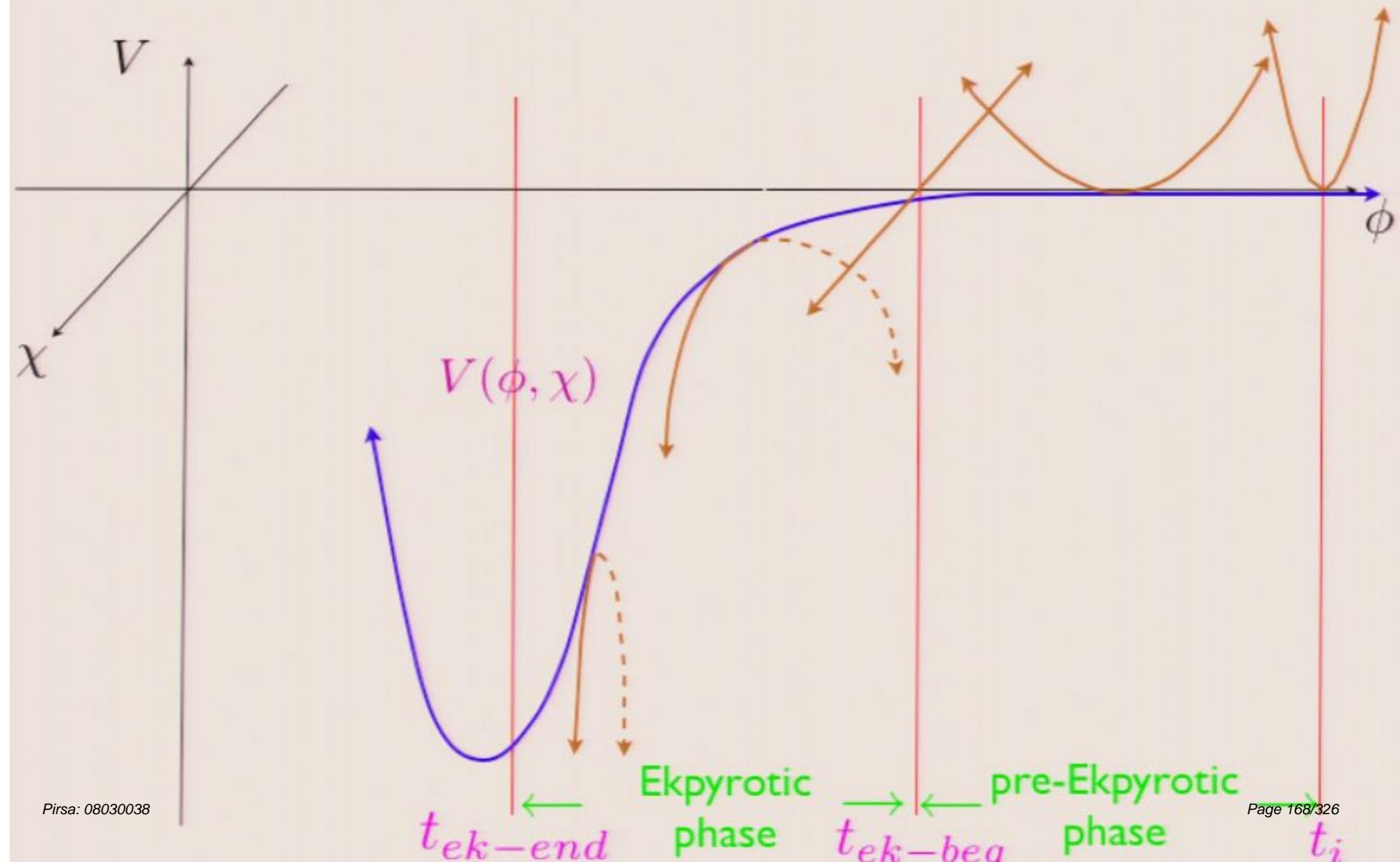
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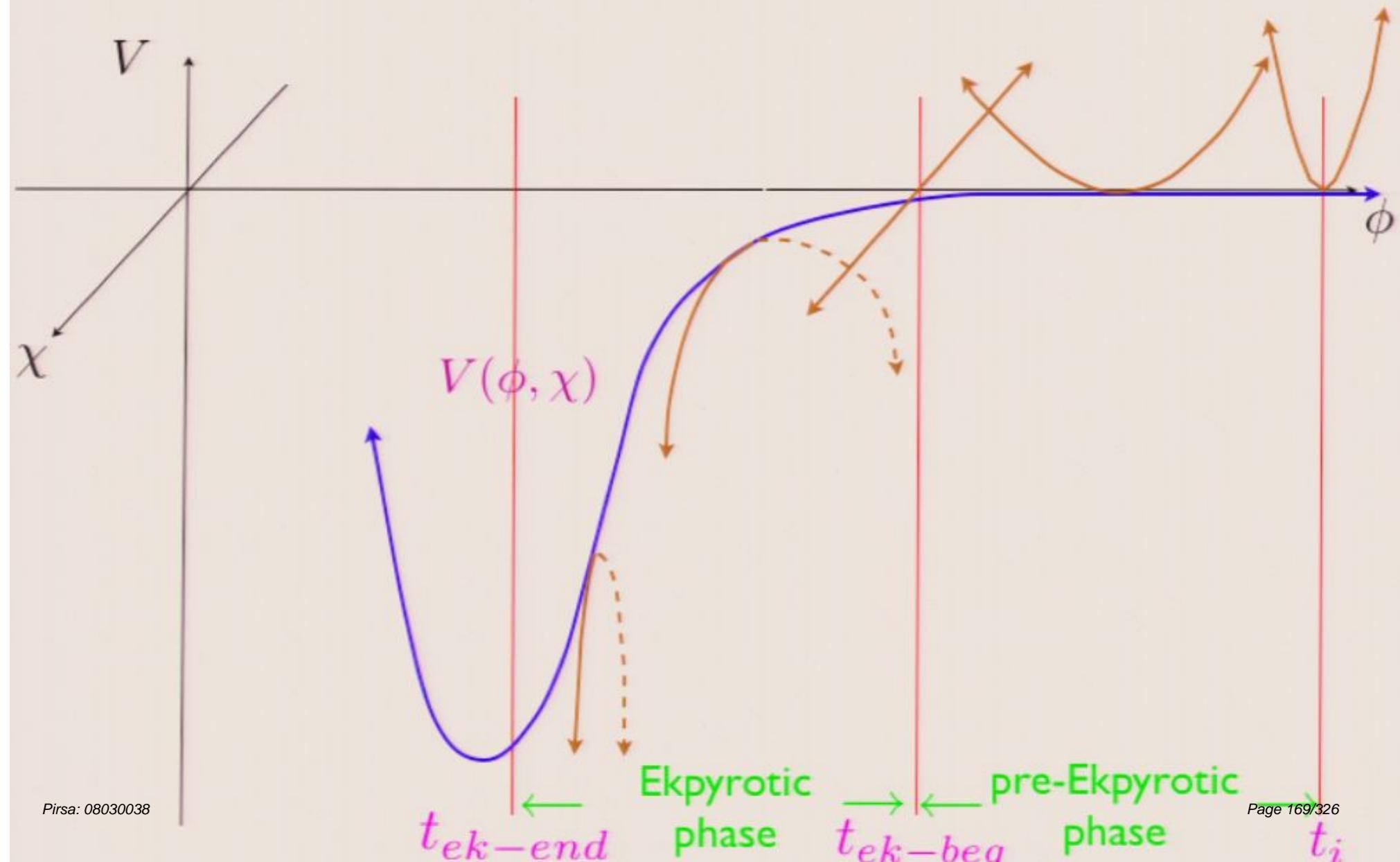
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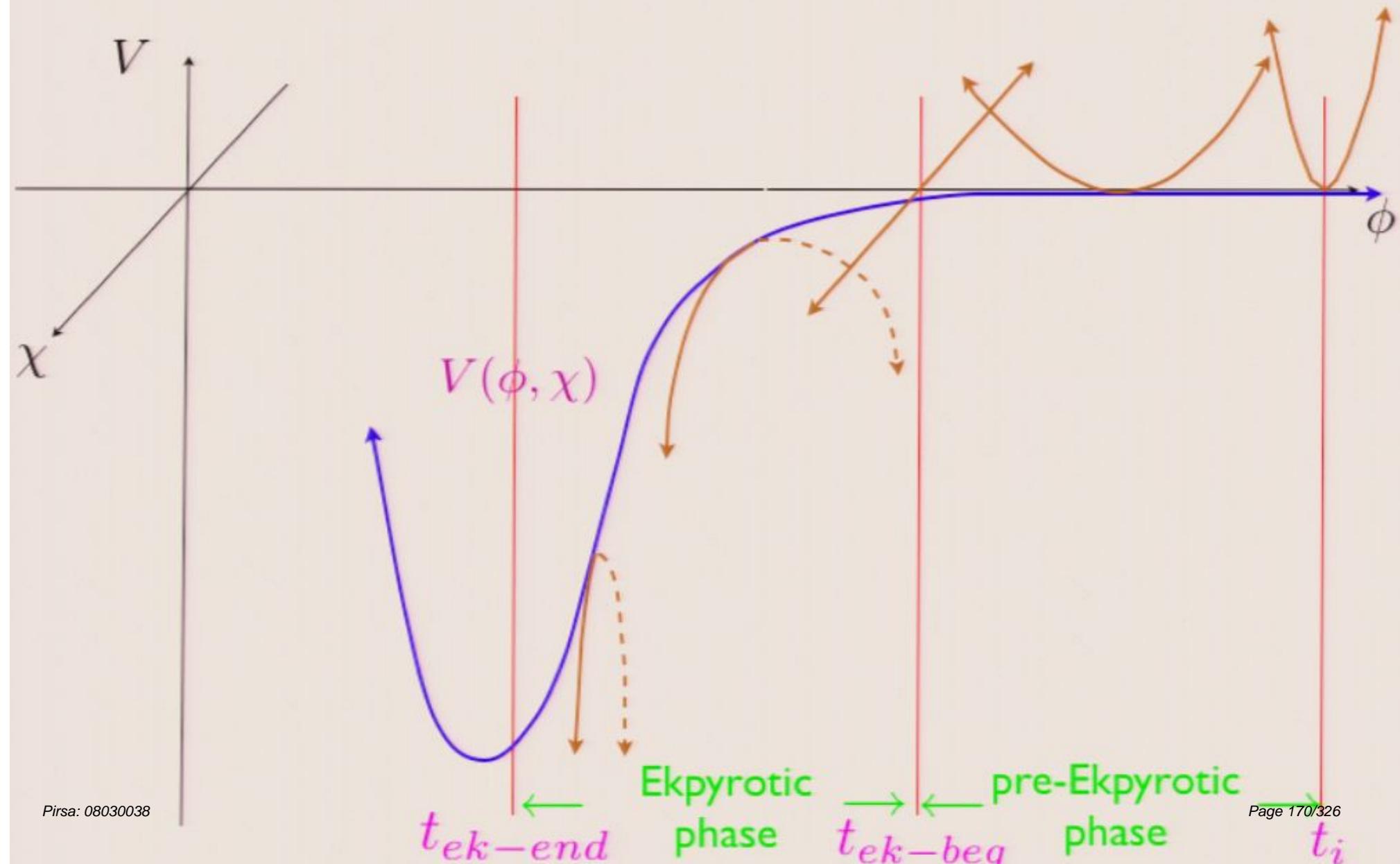
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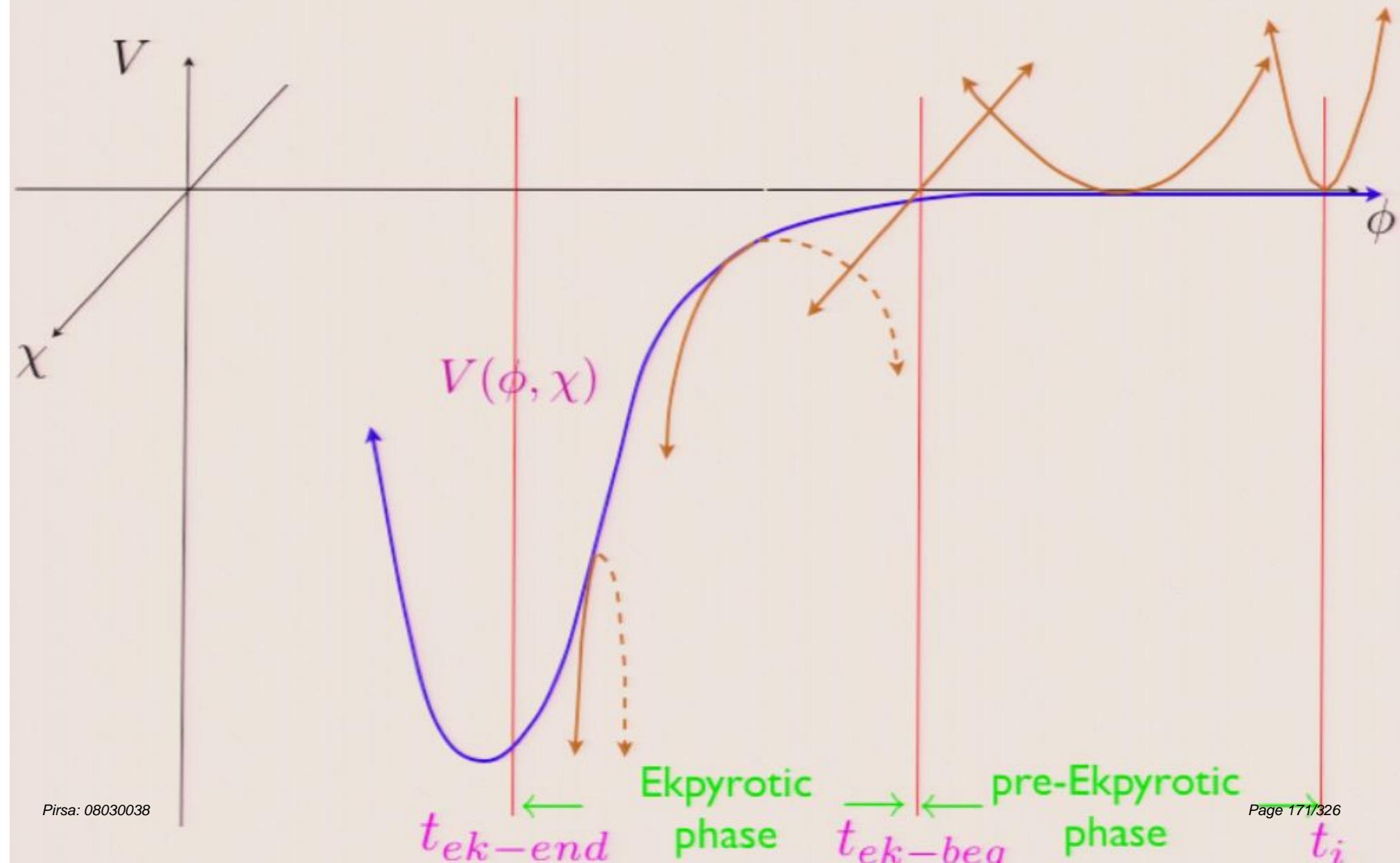
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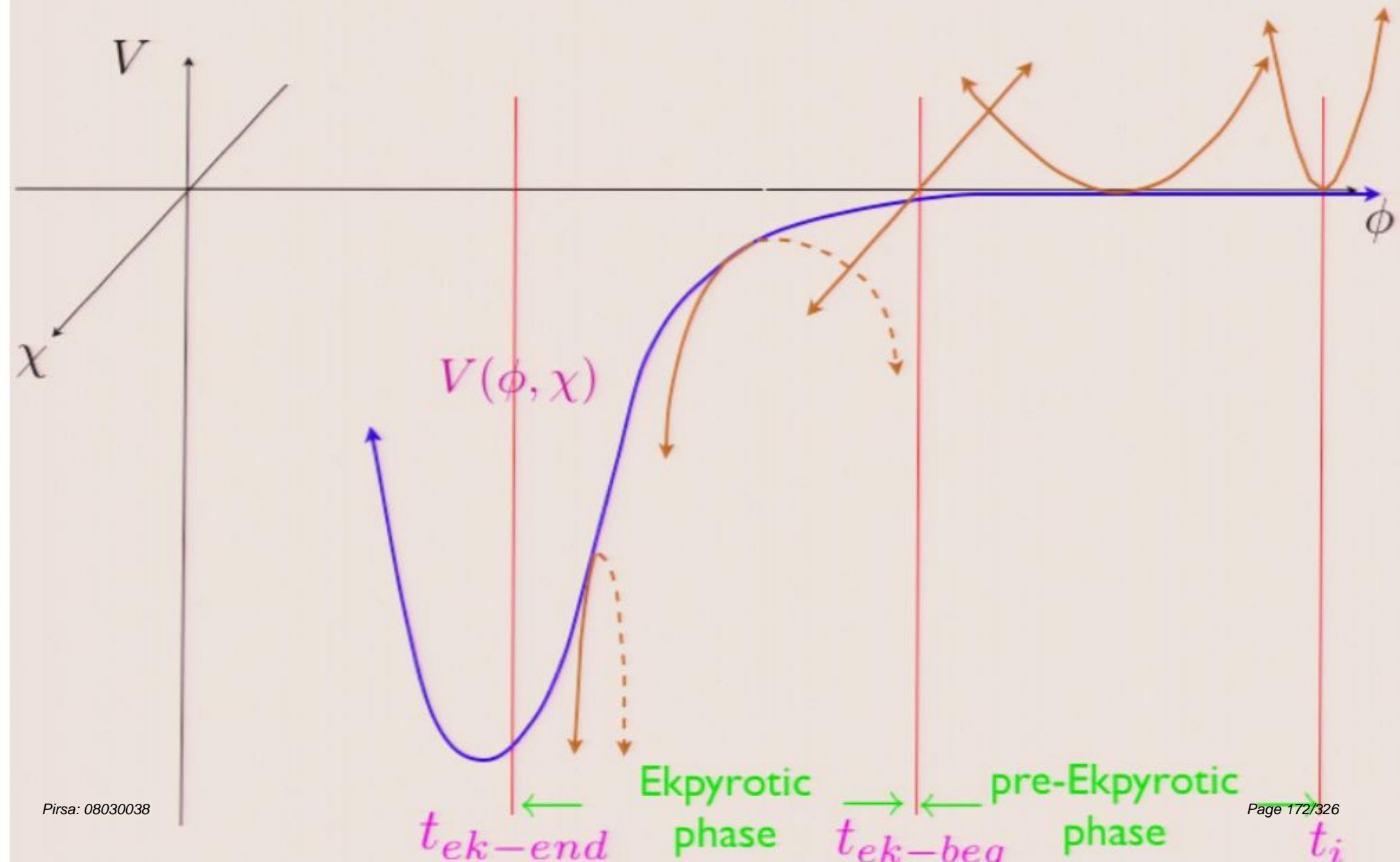
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- to achieve a bounce must violate the NEC. Requires phase where

$$\omega < -1$$

Ghost Condensate

real scalar field- ϕ

Lagrangian density-

$$\frac{\mathcal{L}}{\sqrt{-g}} = M^4 P(X)$$

where $P(X)$ is a function of

$$X = -\frac{1}{2m^4}(\partial_\mu \phi)^2$$

and m, M are arbitrary mass scales

scalar equation

$$\frac{d}{dt}(a^3 P_{,X} \dot{\phi}) = 0$$

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$$\phi(t) = -m^2 t \quad \text{ghost condensate}$$

Corresponds to

$$\omega = -1$$

To obtain $\omega < -1$ add potential $V(\phi)$ where

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$$\ddot{\pi}_0 + 3H\dot{\pi}_0 \simeq -\frac{V_{,\phi}}{K} \left(\frac{m}{M}\right)^4$$

where $K = P_{,XX}|_{X_0=\frac{1}{2}}$. For any solution

$$\rho \simeq \frac{-KM^2\dot{\pi}_0}{m^2} + V, \quad \mathcal{P} \simeq -V$$

\Rightarrow

$$\rho + \mathcal{P} = -\frac{KM^2\dot{\pi}_0}{m^2}$$

Can choose $V(\phi)$ so that $\dot{\pi}_0 > 0 \Rightarrow$

$$\rho + \mathcal{P} < 0, \quad \omega < -1$$

\Rightarrow ghost condensate violates the NEC! For example, take

$$V(\phi) = \Lambda^4 \left(1 - \frac{\Lambda^2}{m^2} \frac{\phi}{M_P}\right), \quad \Lambda^2 \ll M^2 K$$

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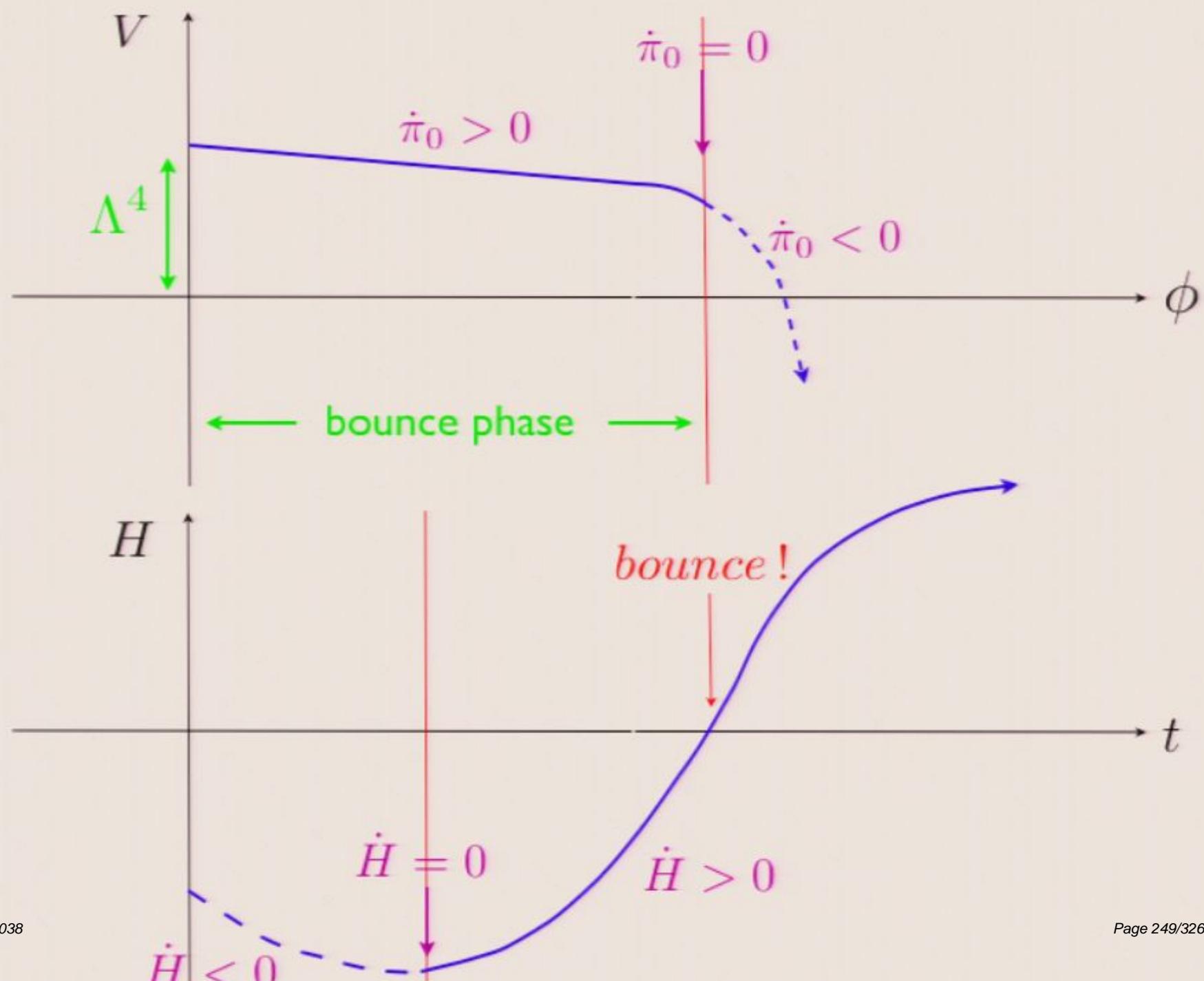
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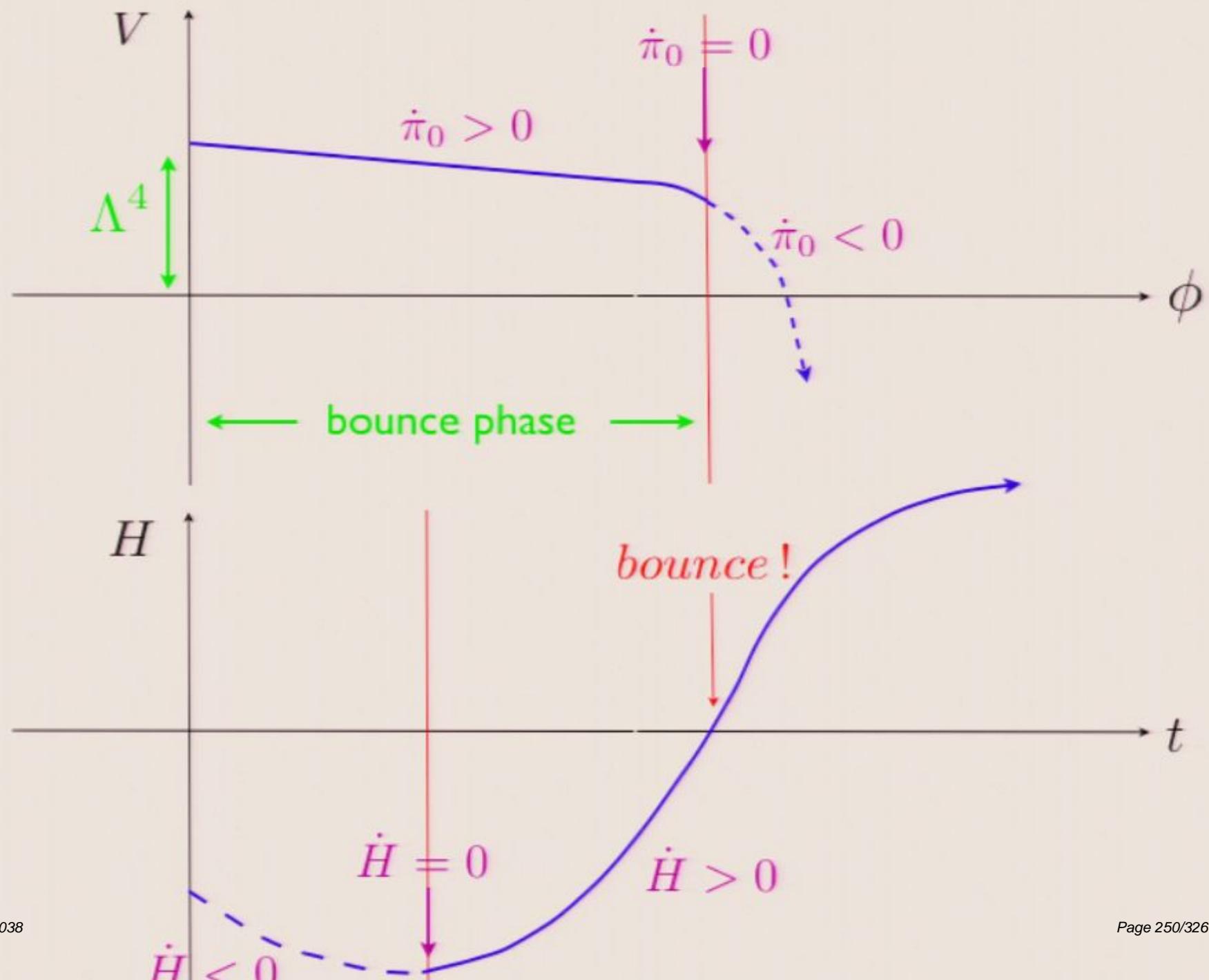
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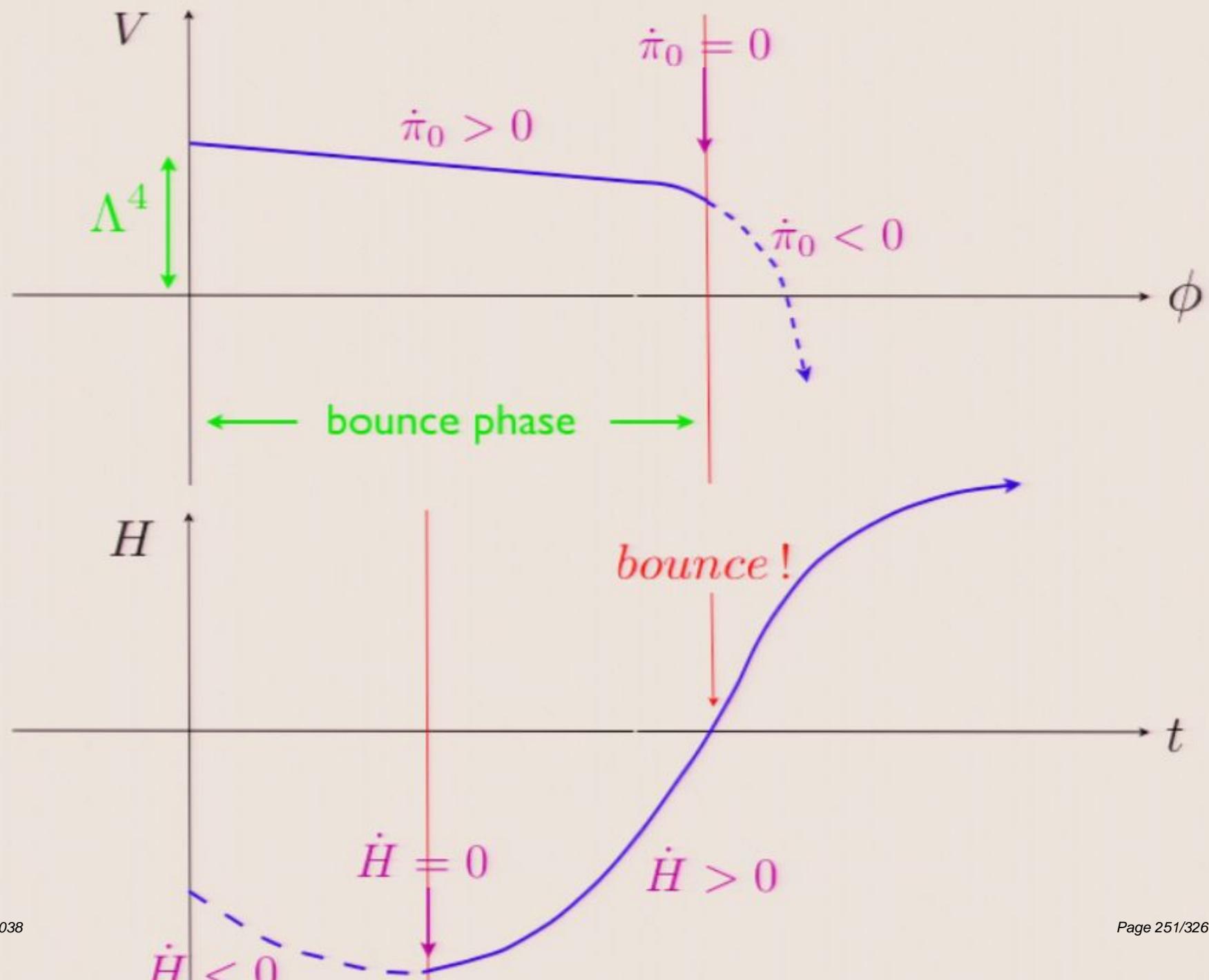
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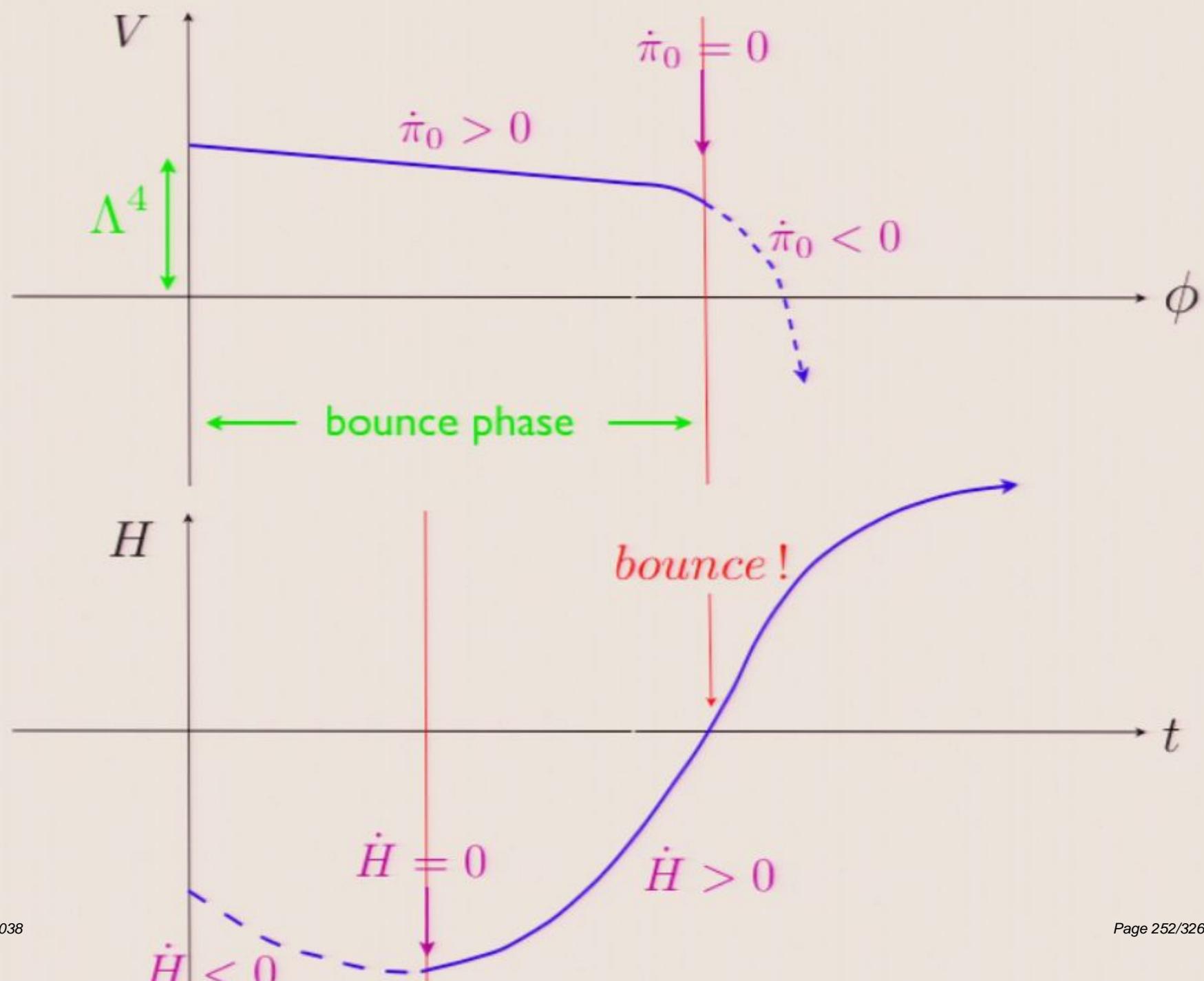
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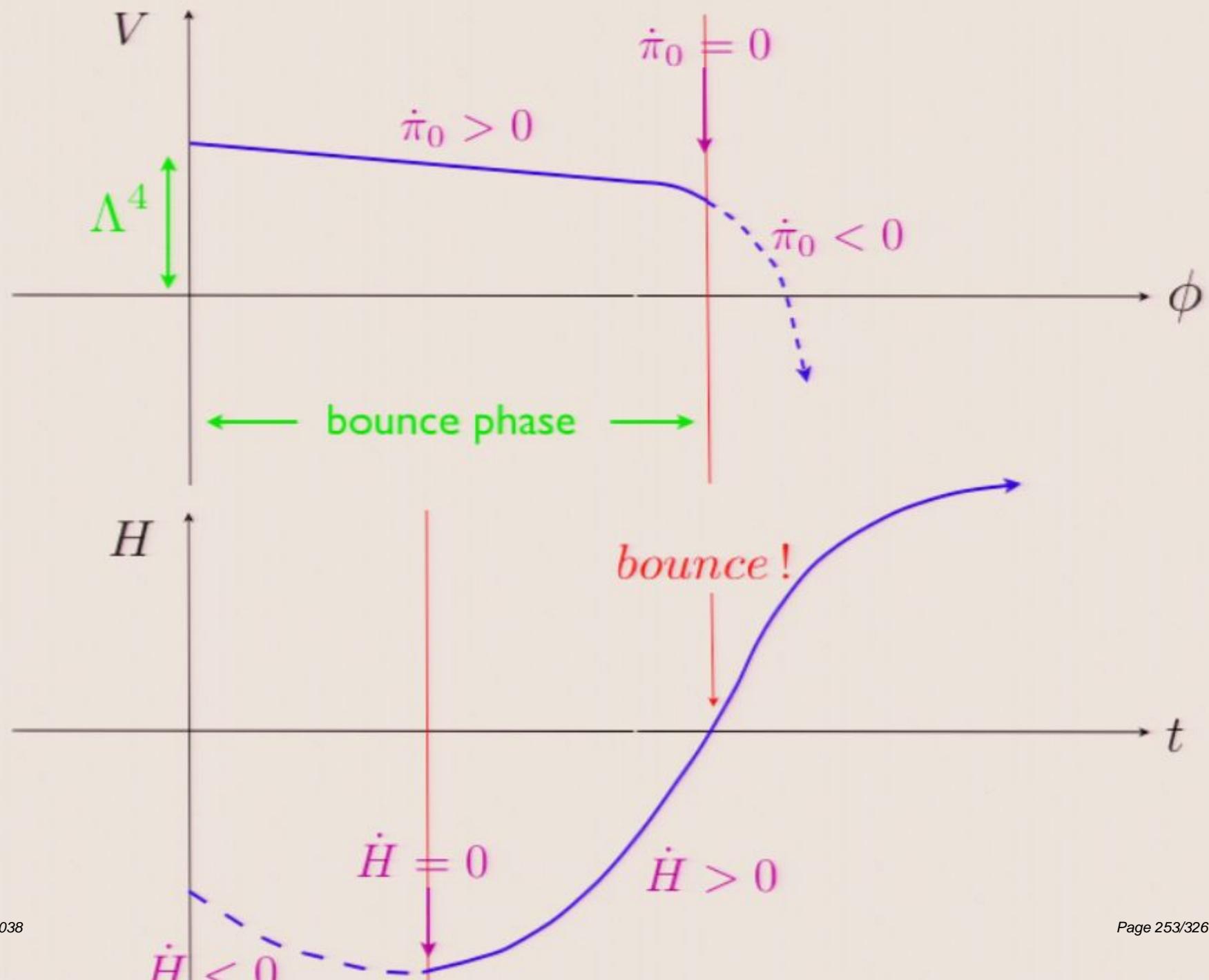
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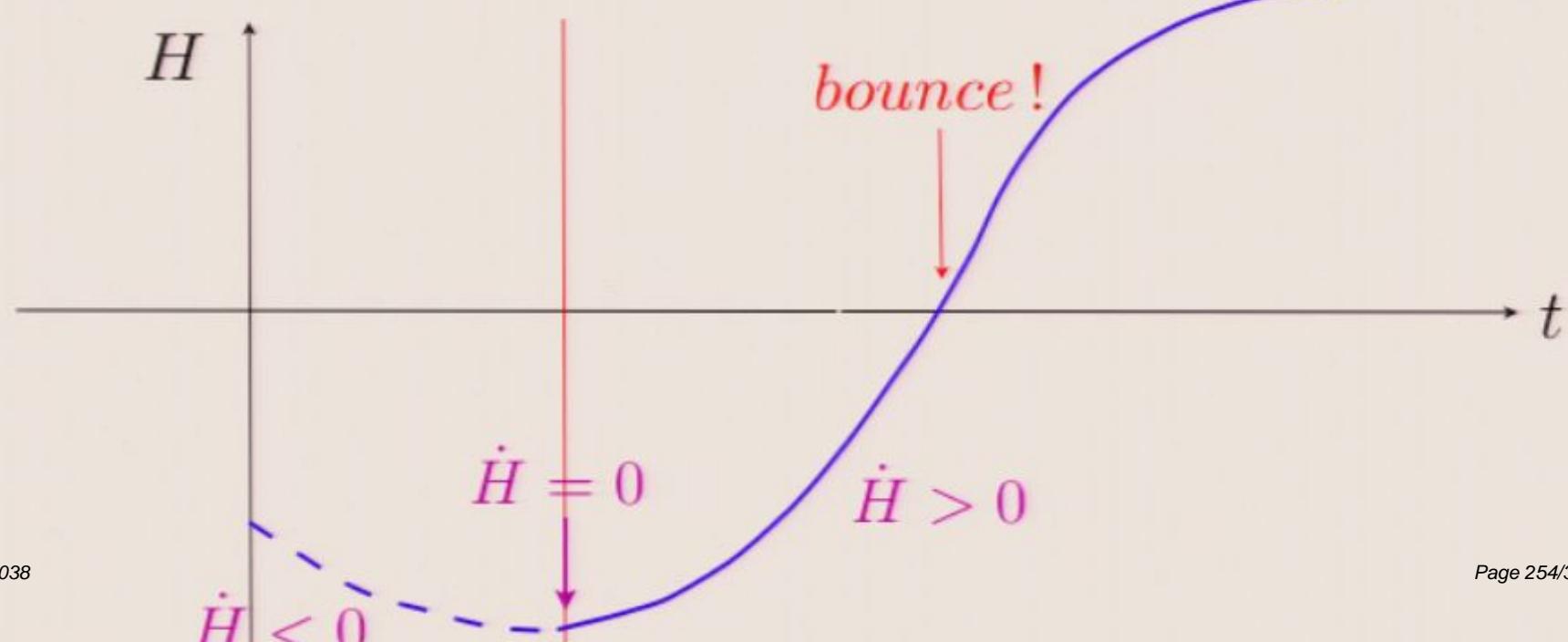
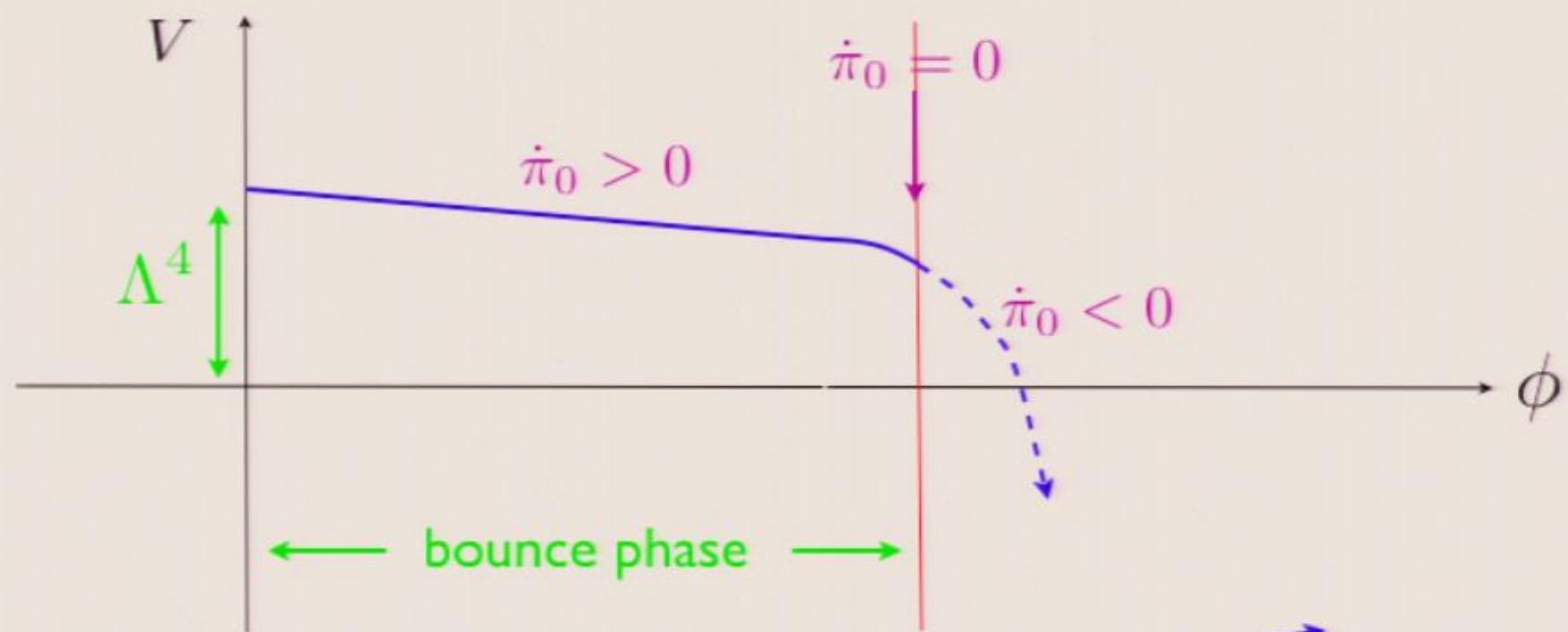


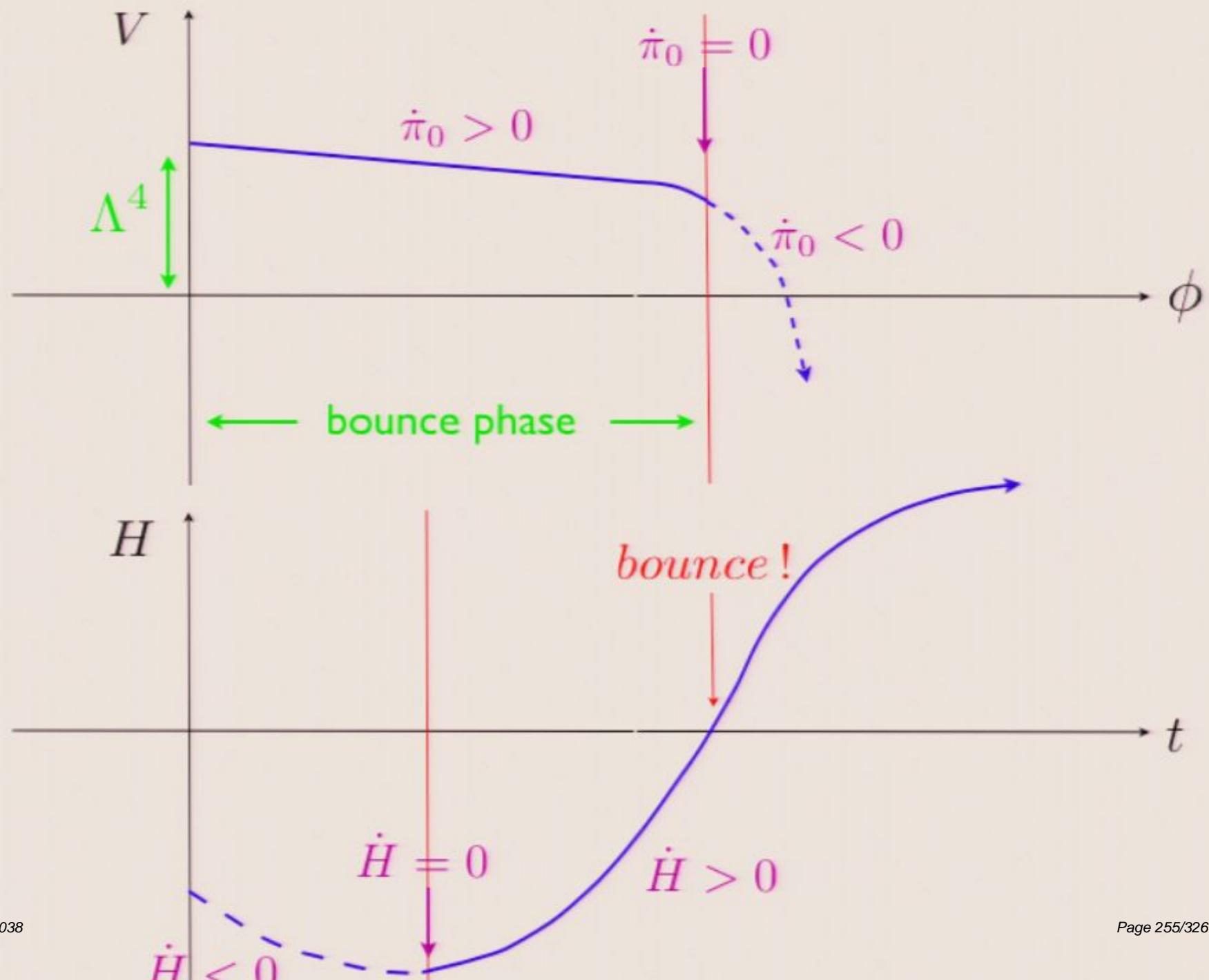


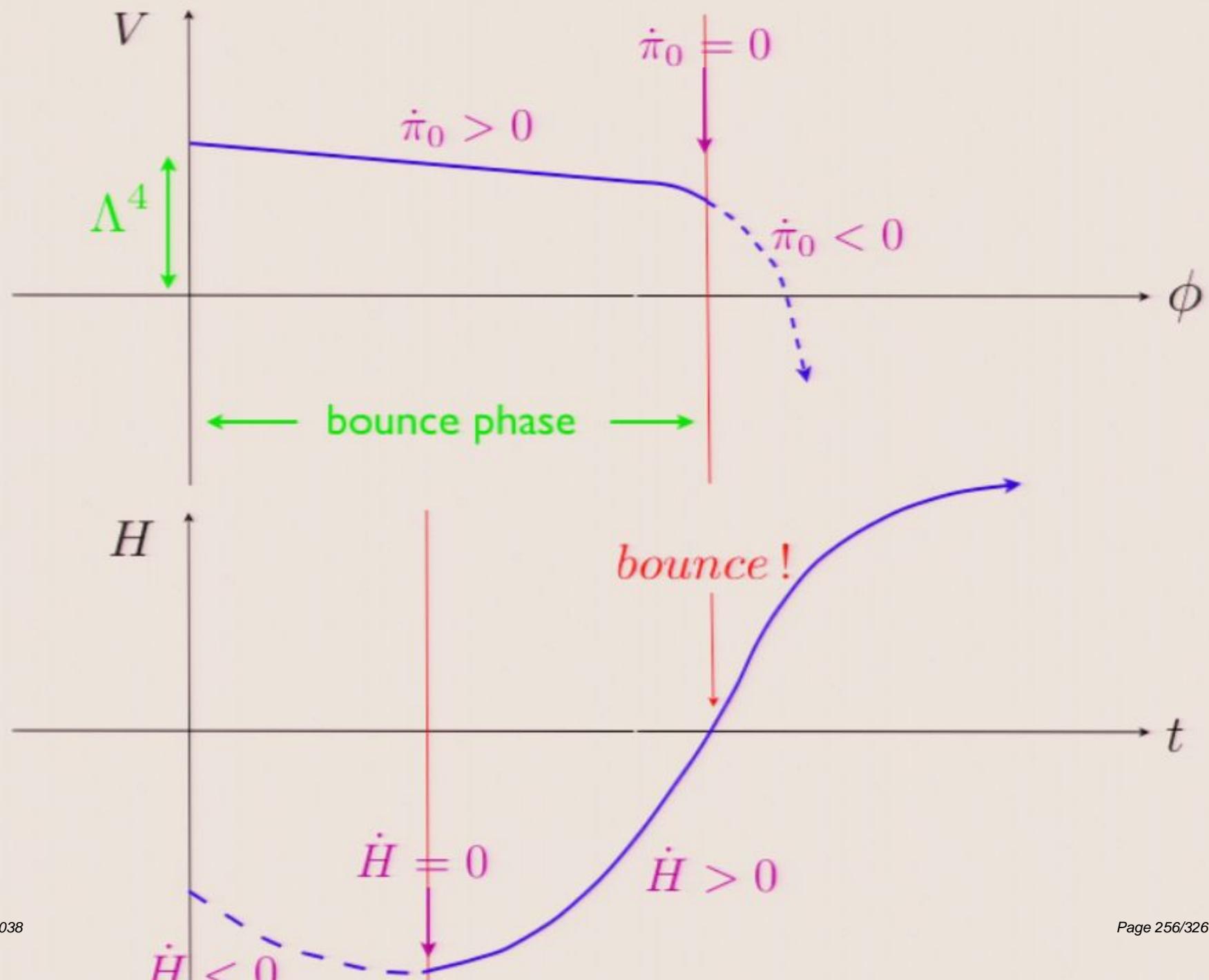


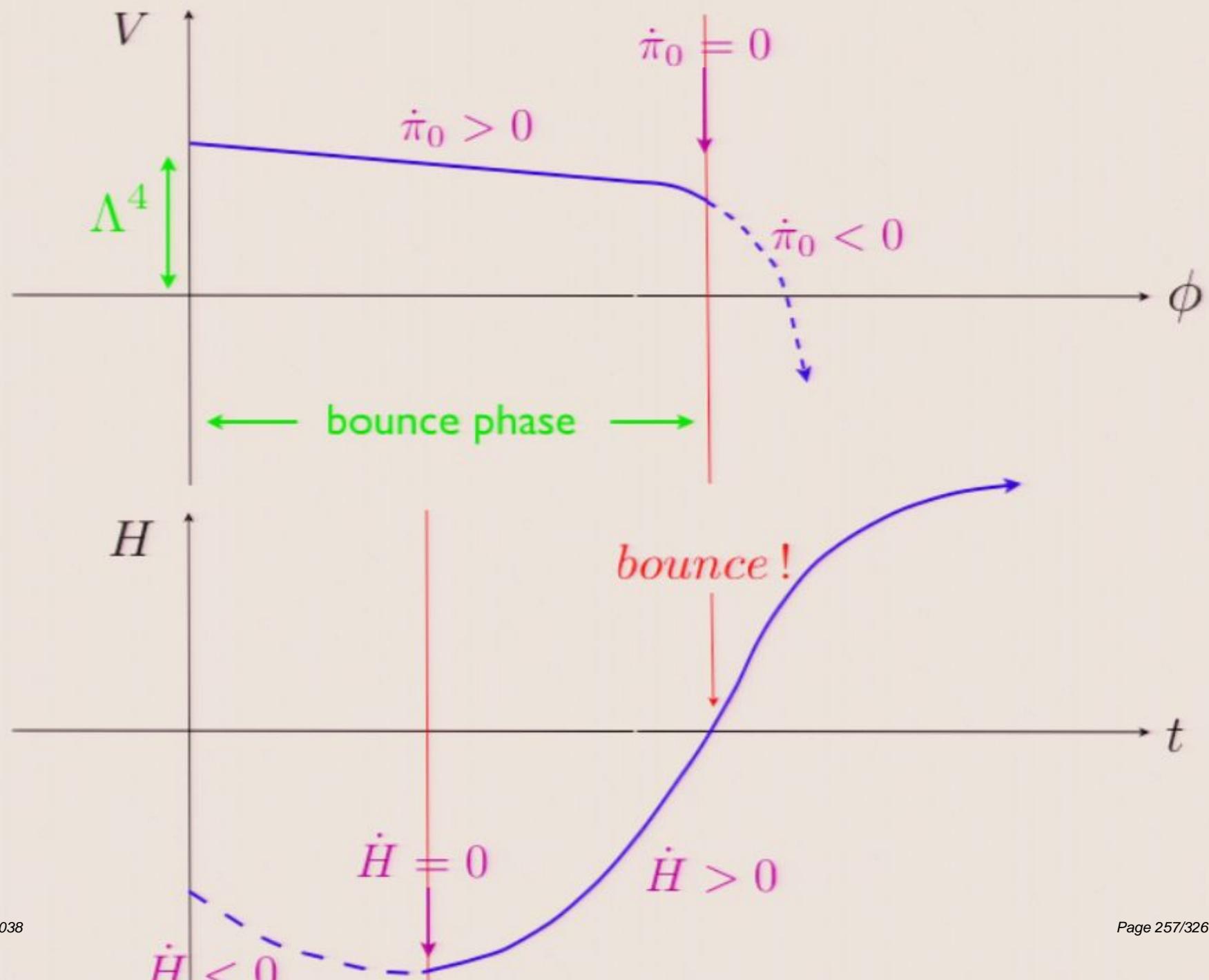


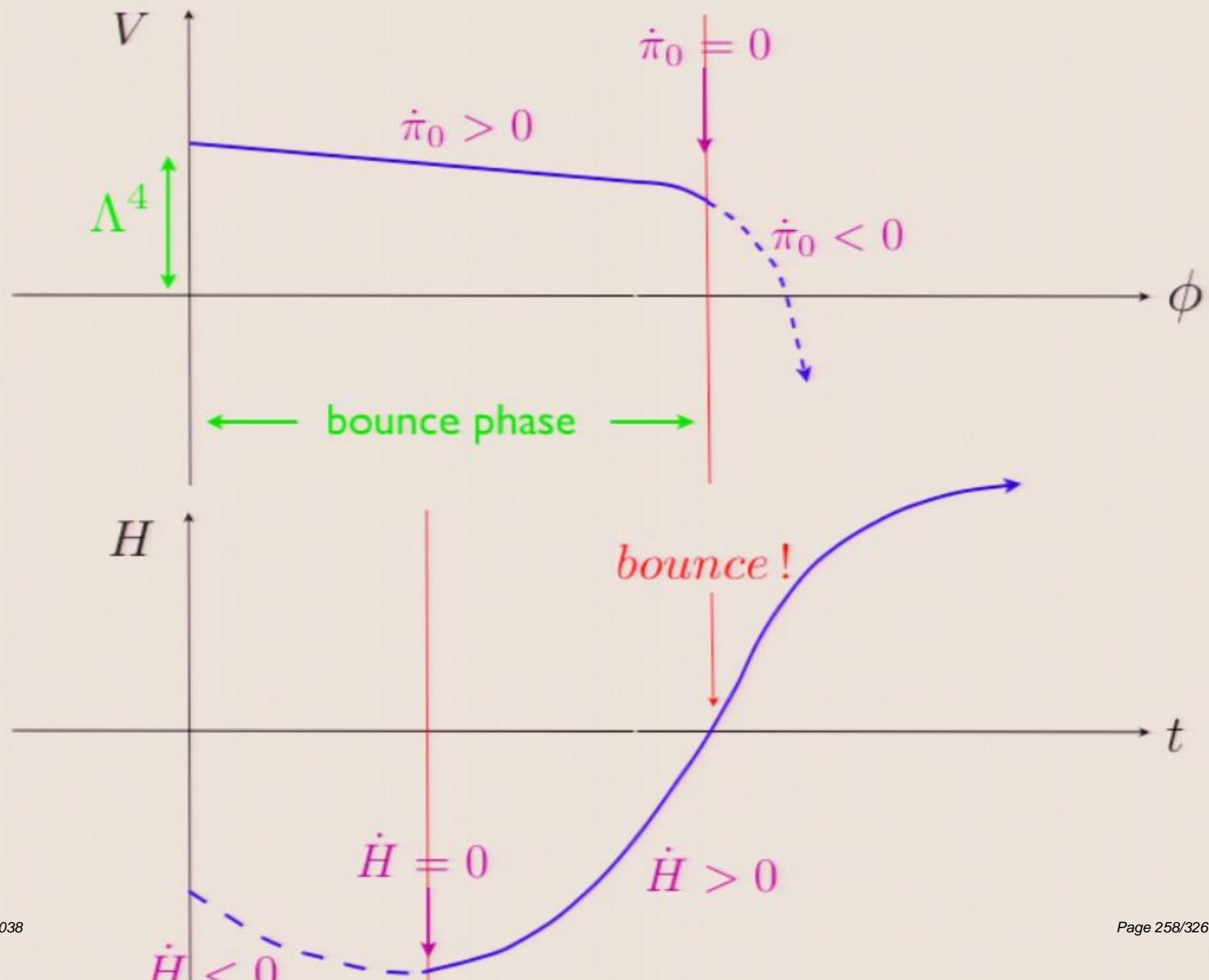


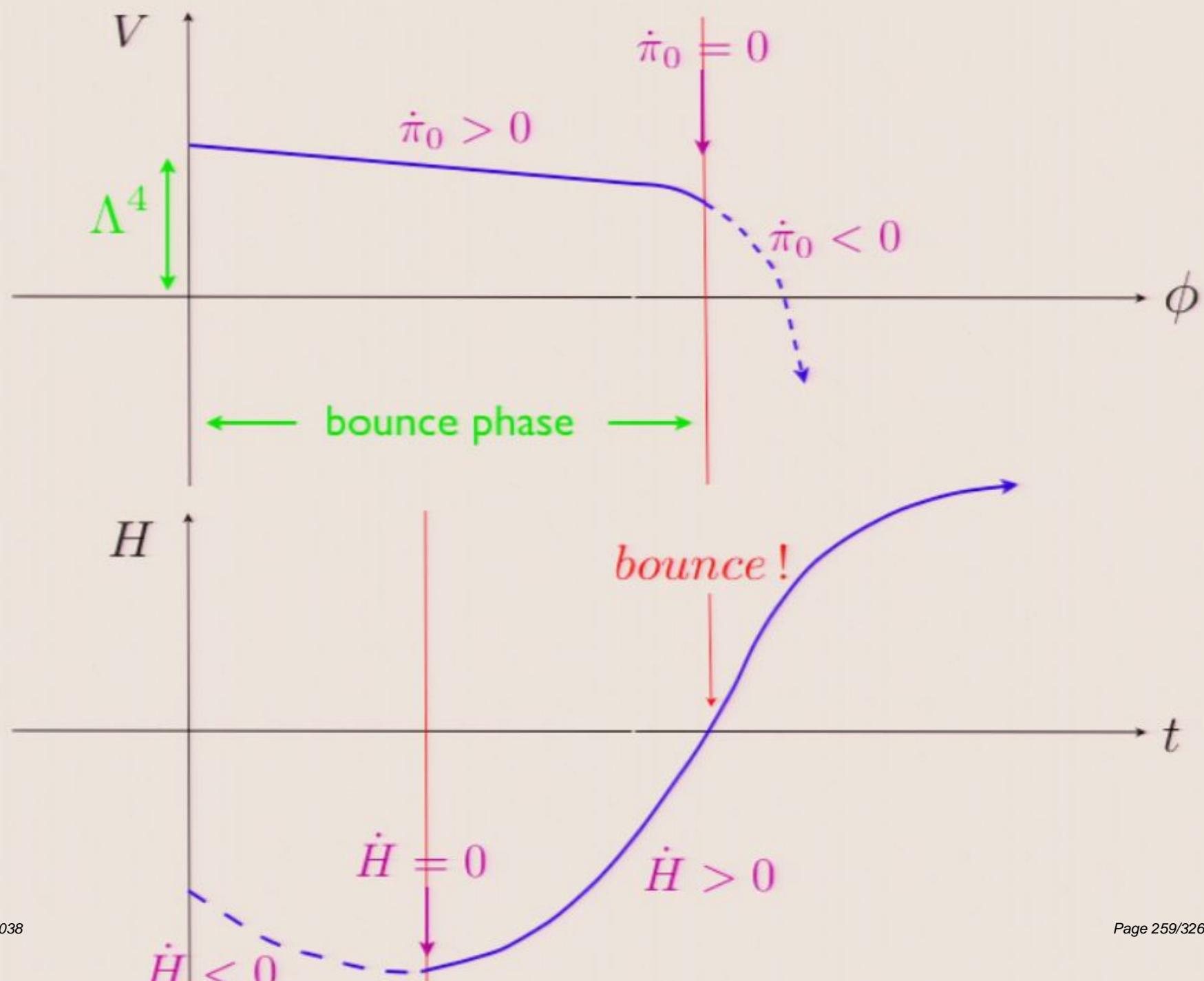


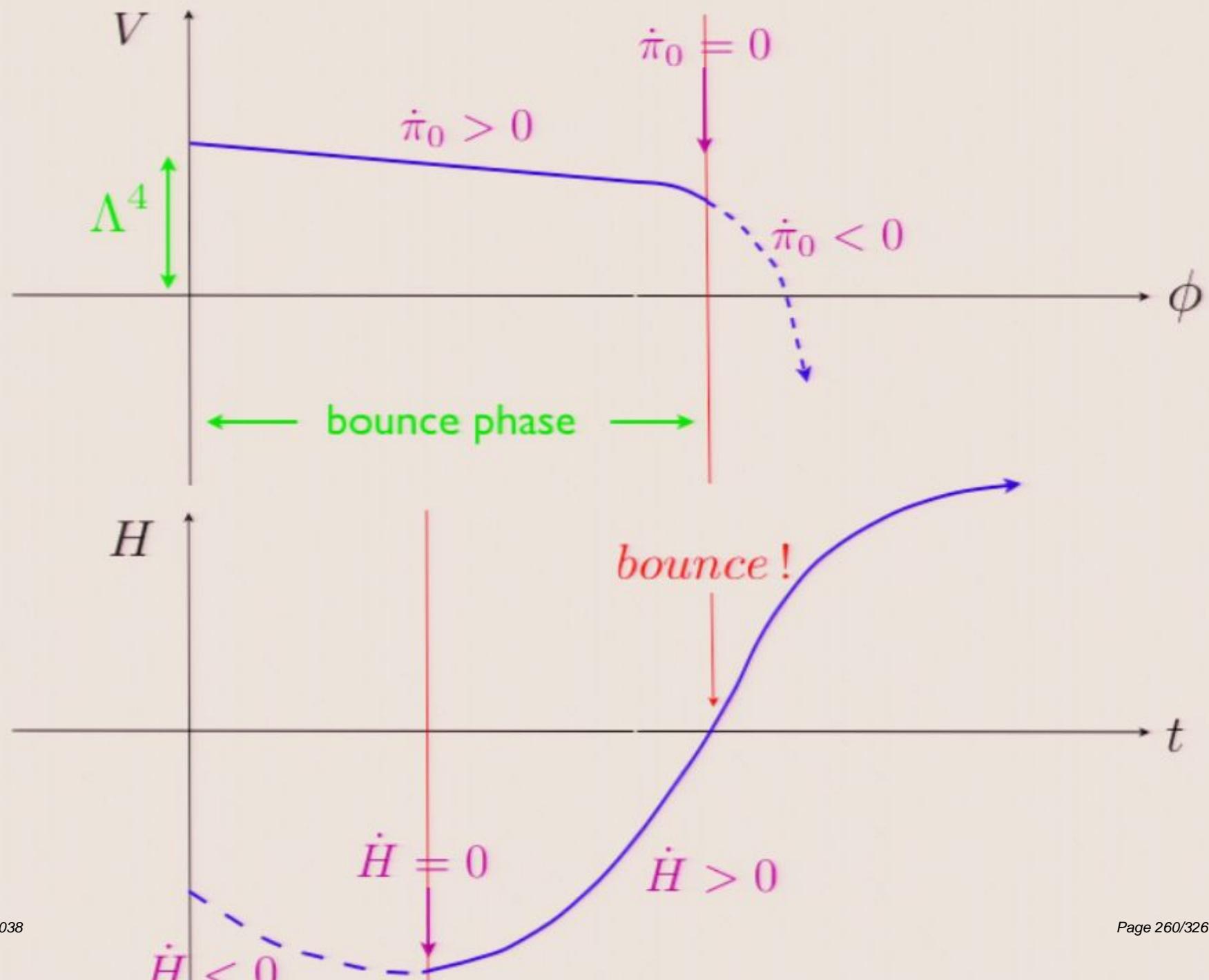


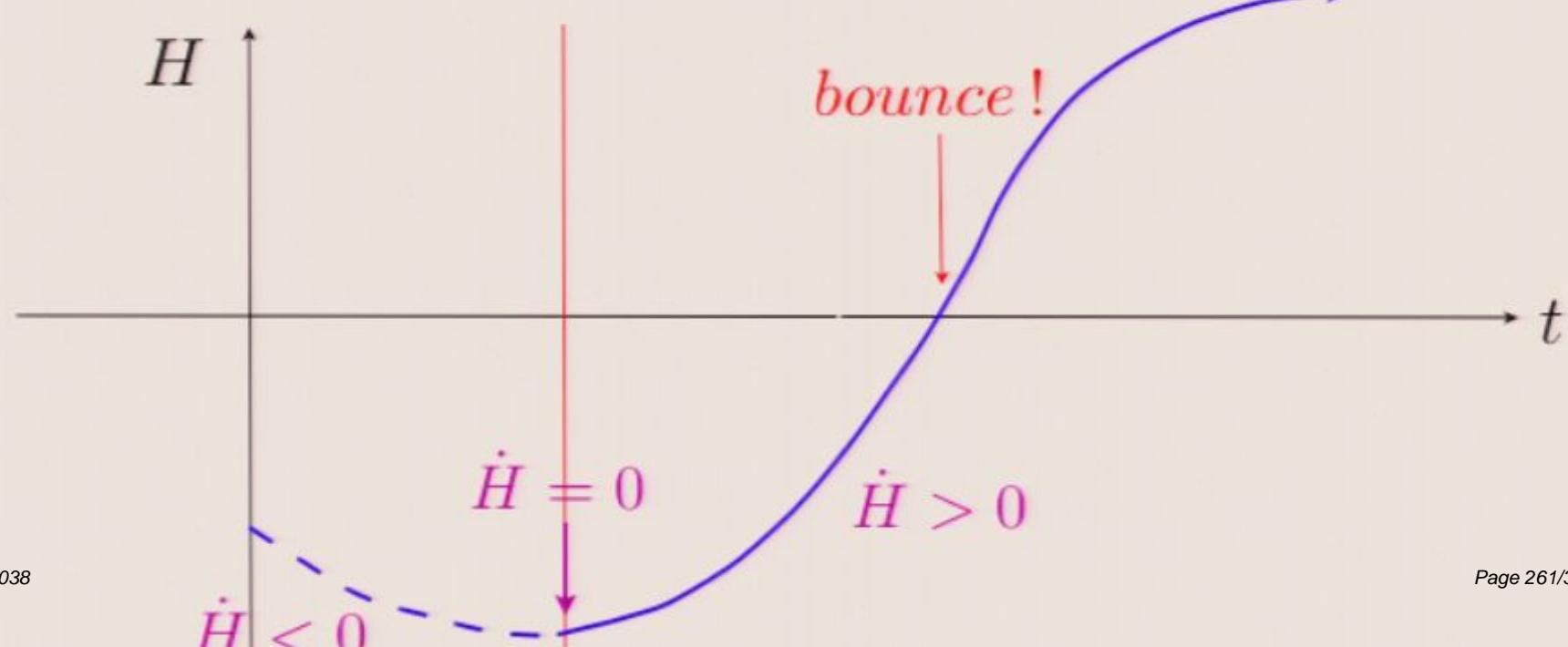
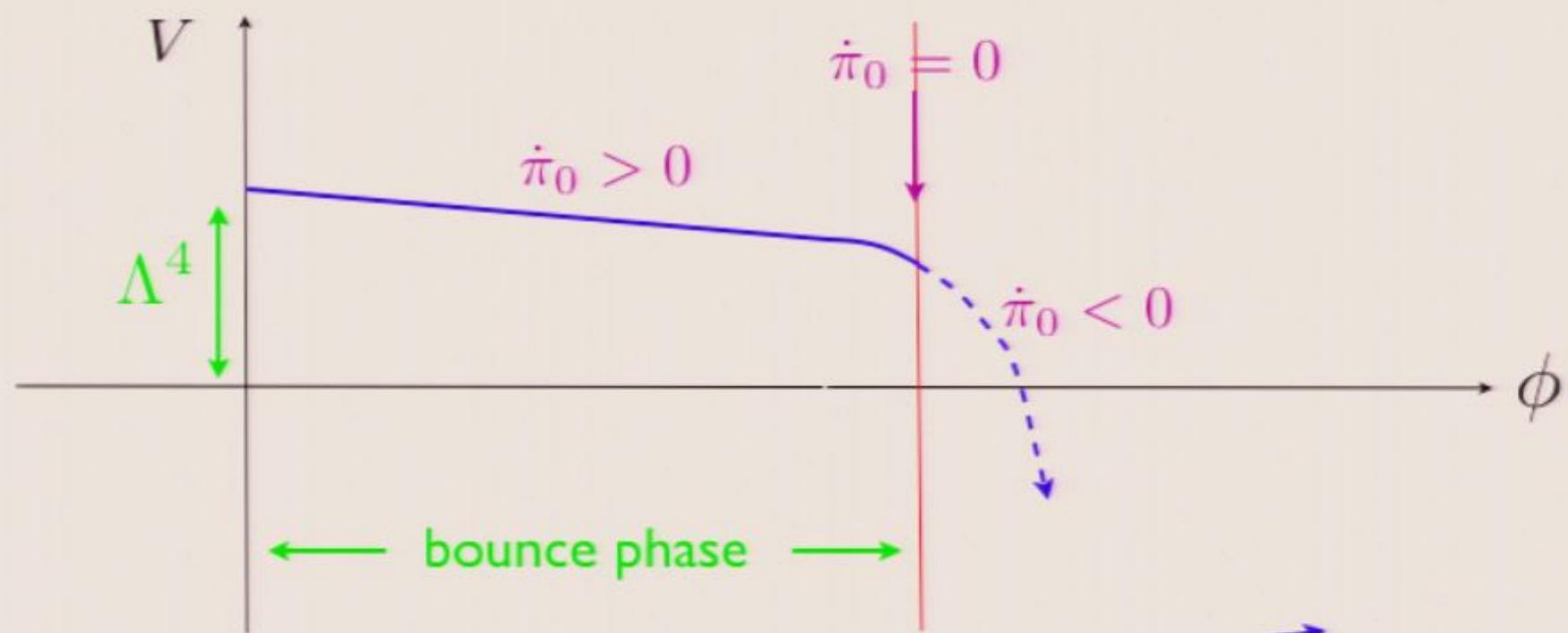


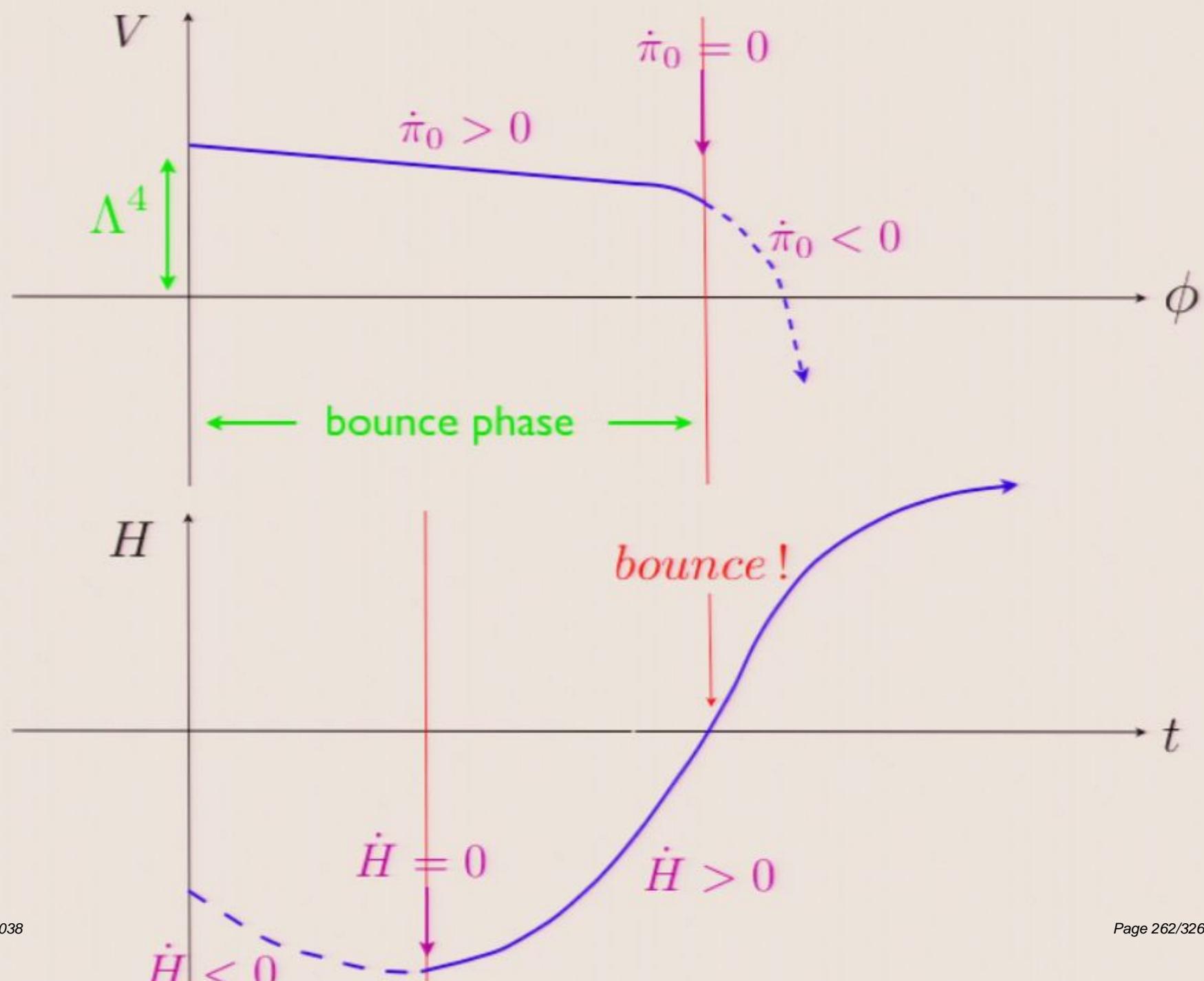


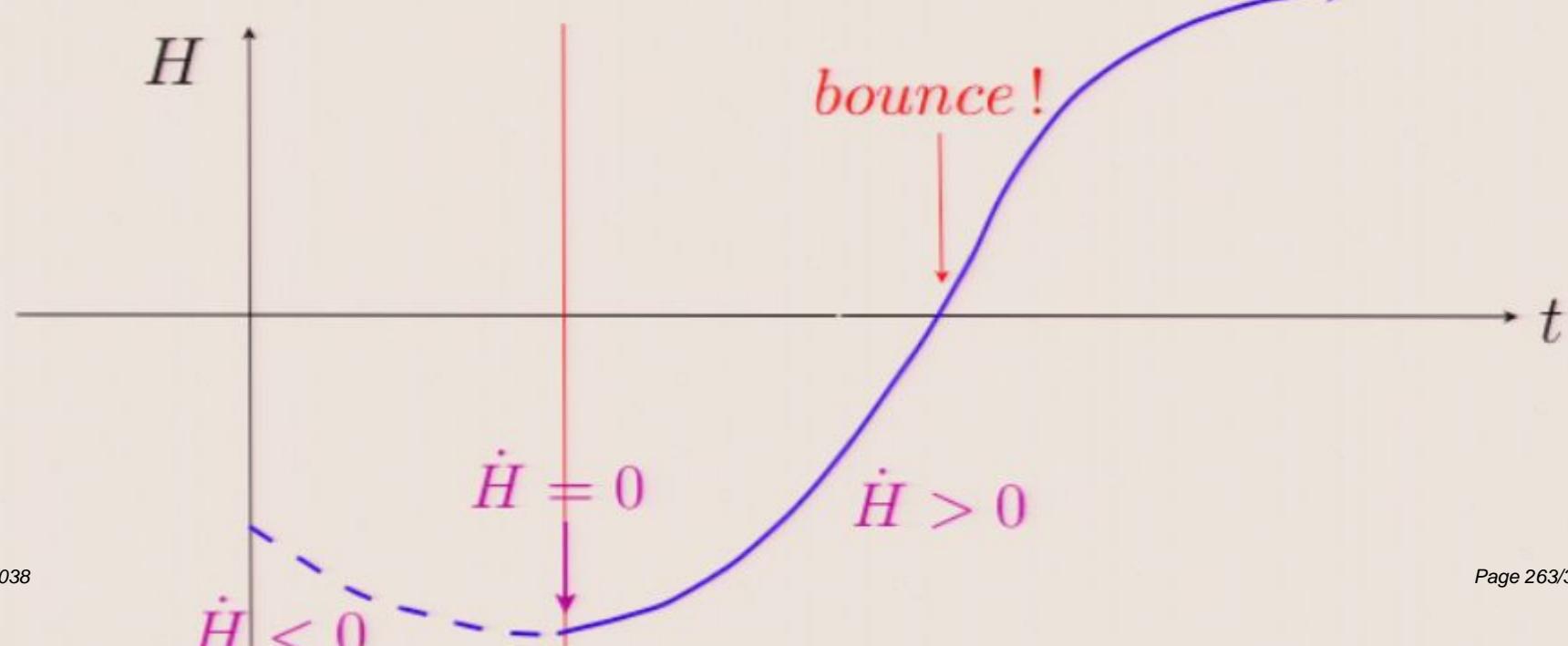
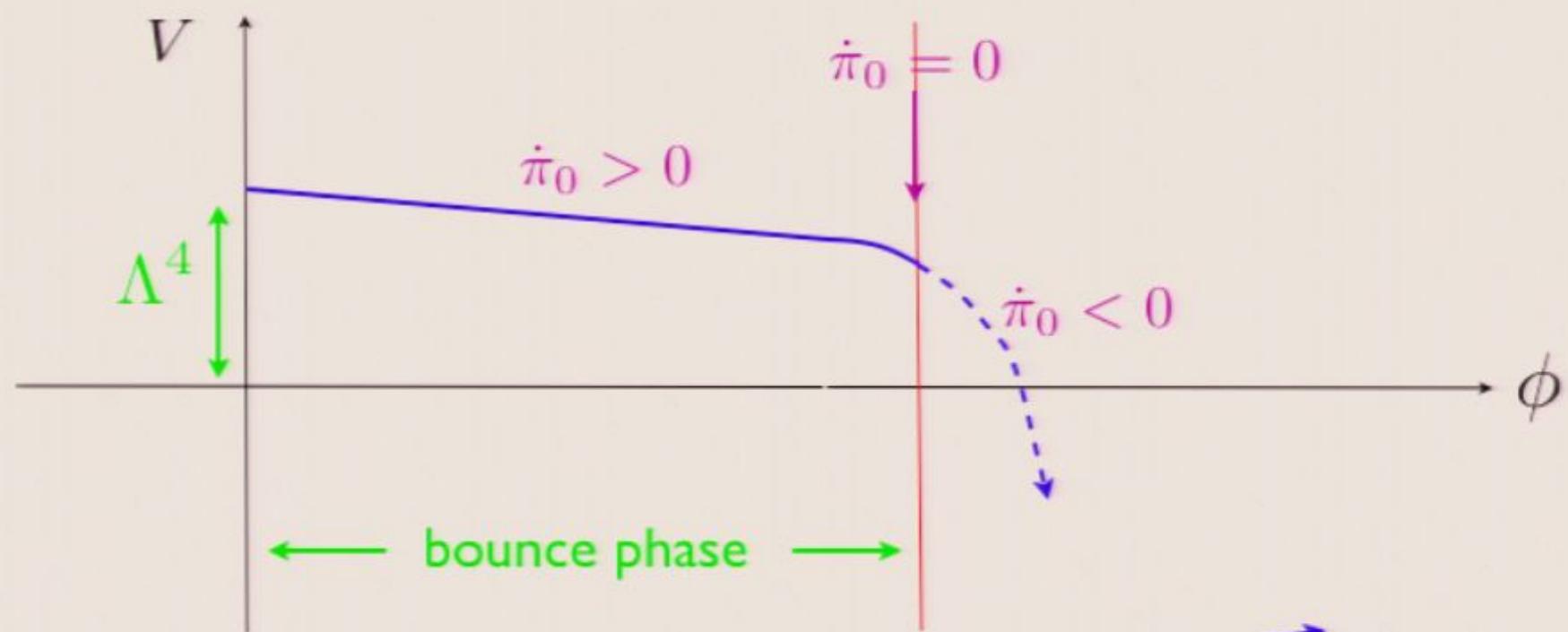


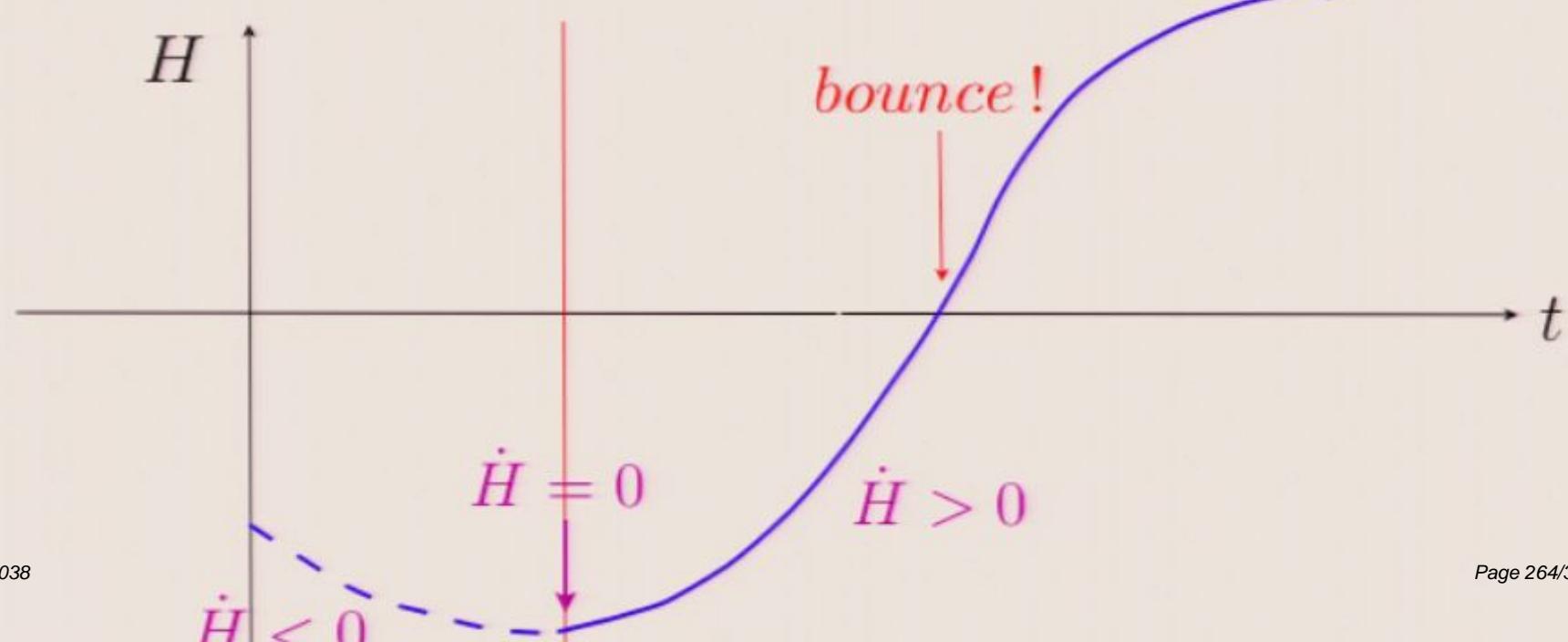
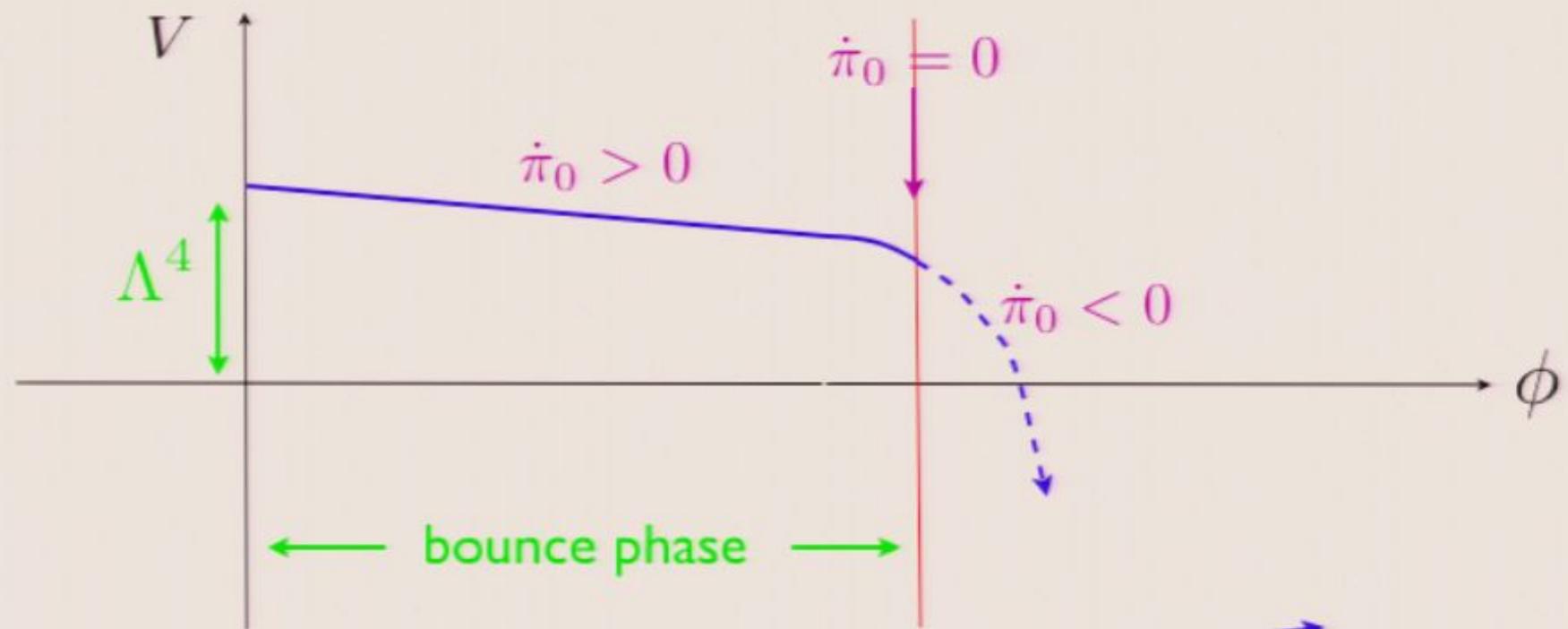


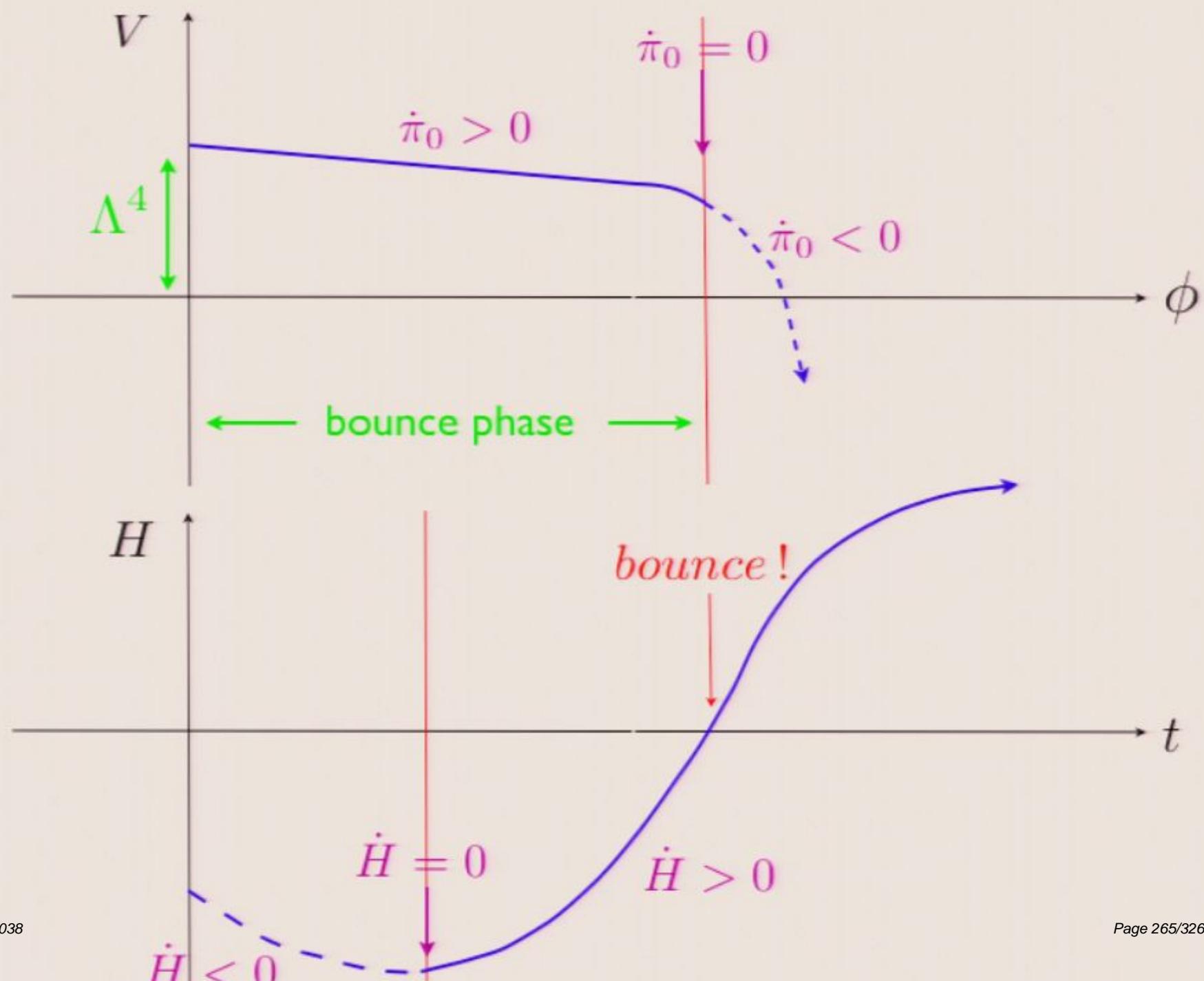


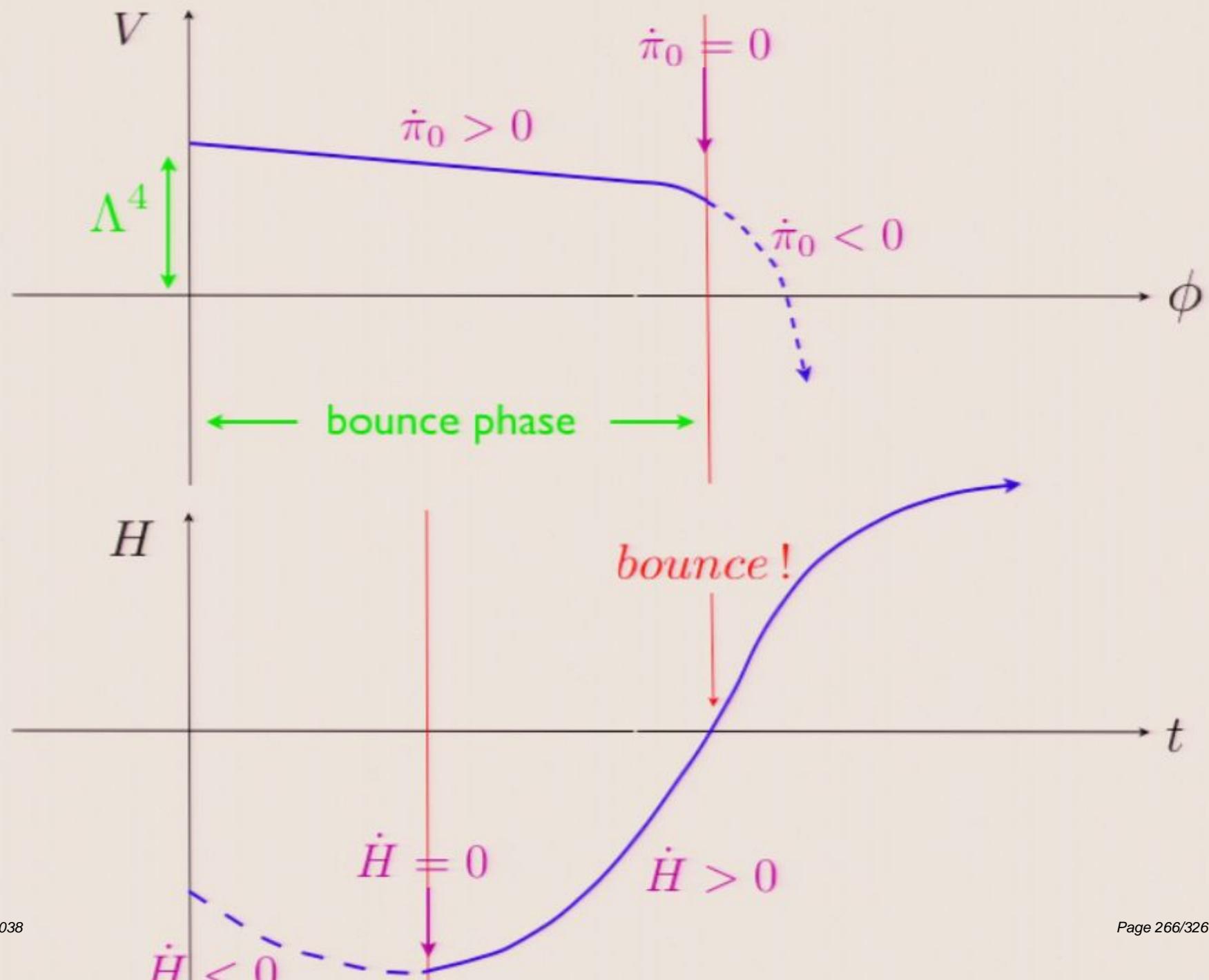


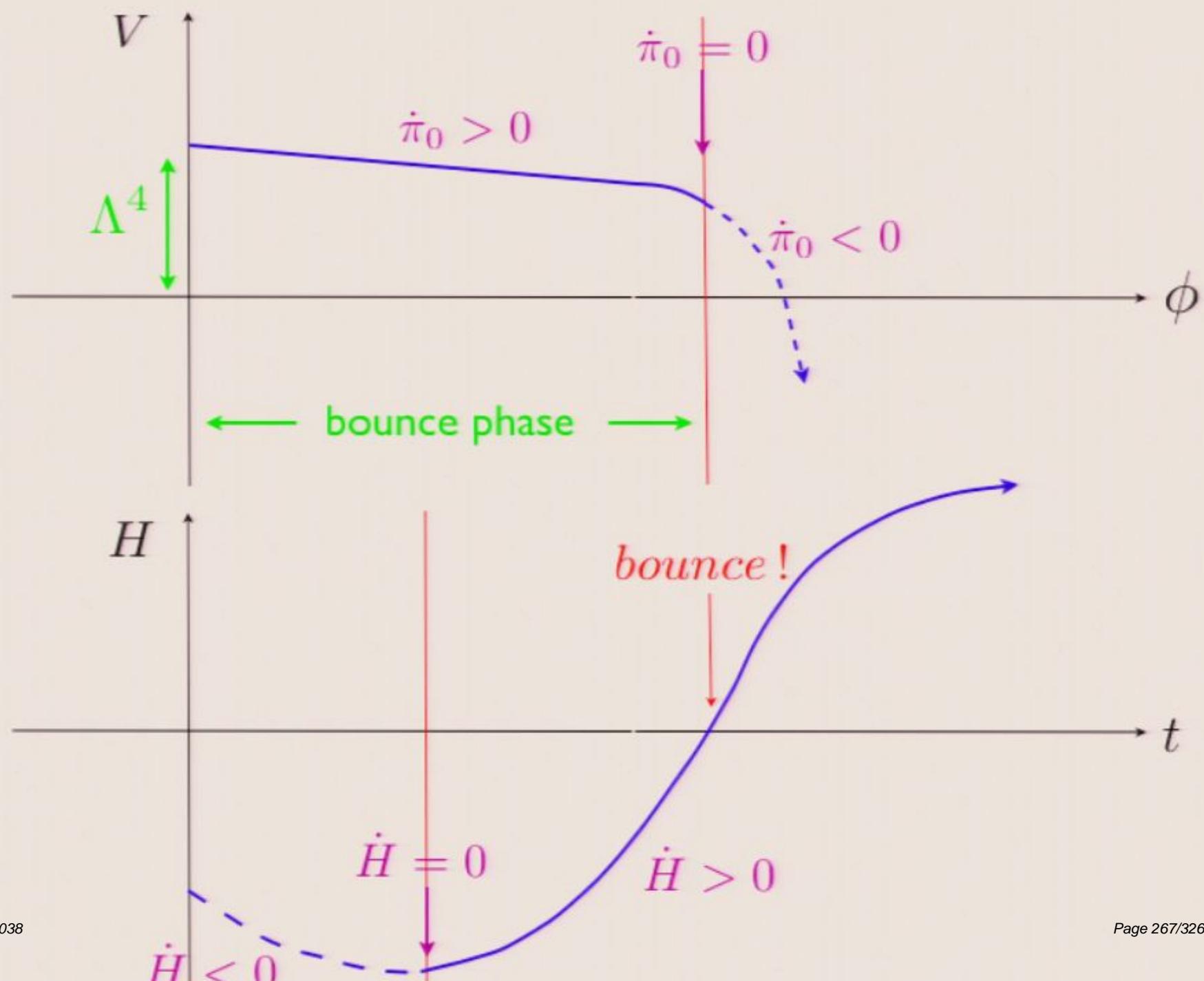


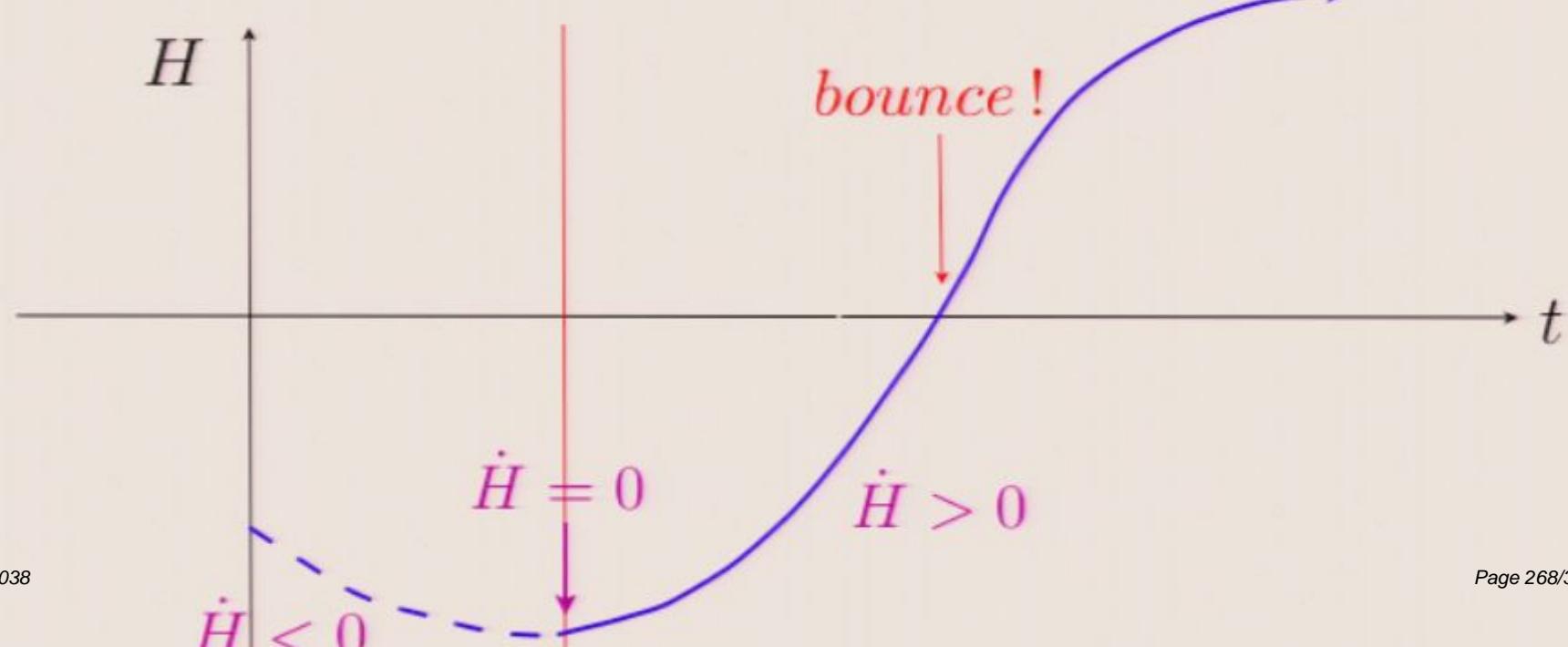
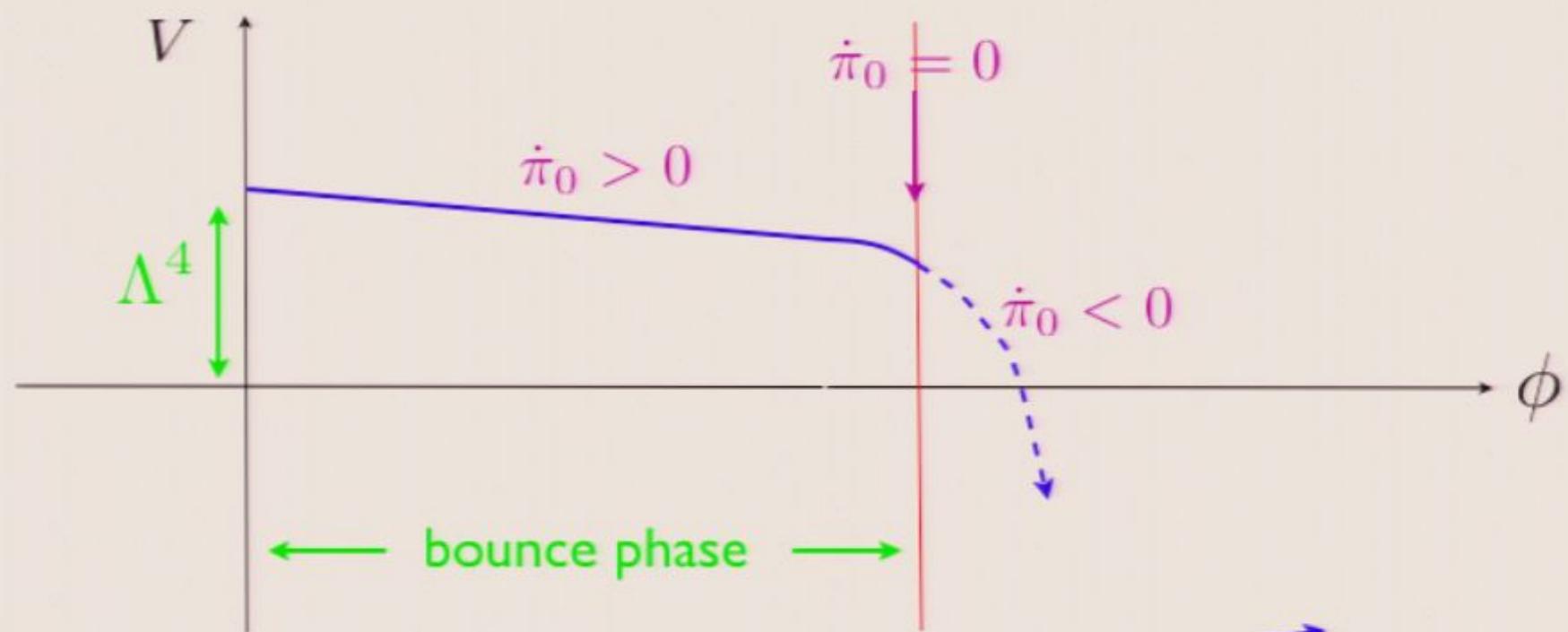


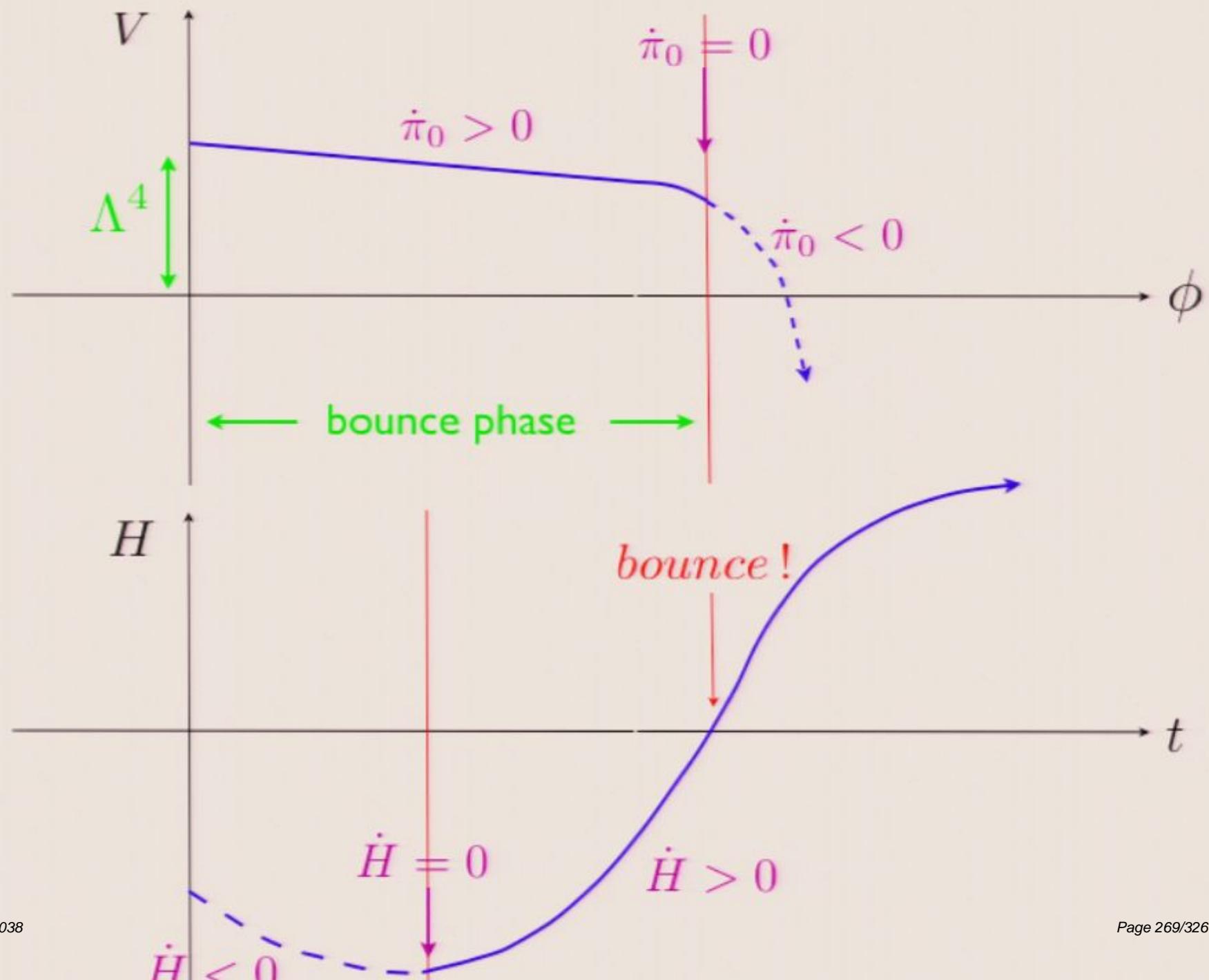


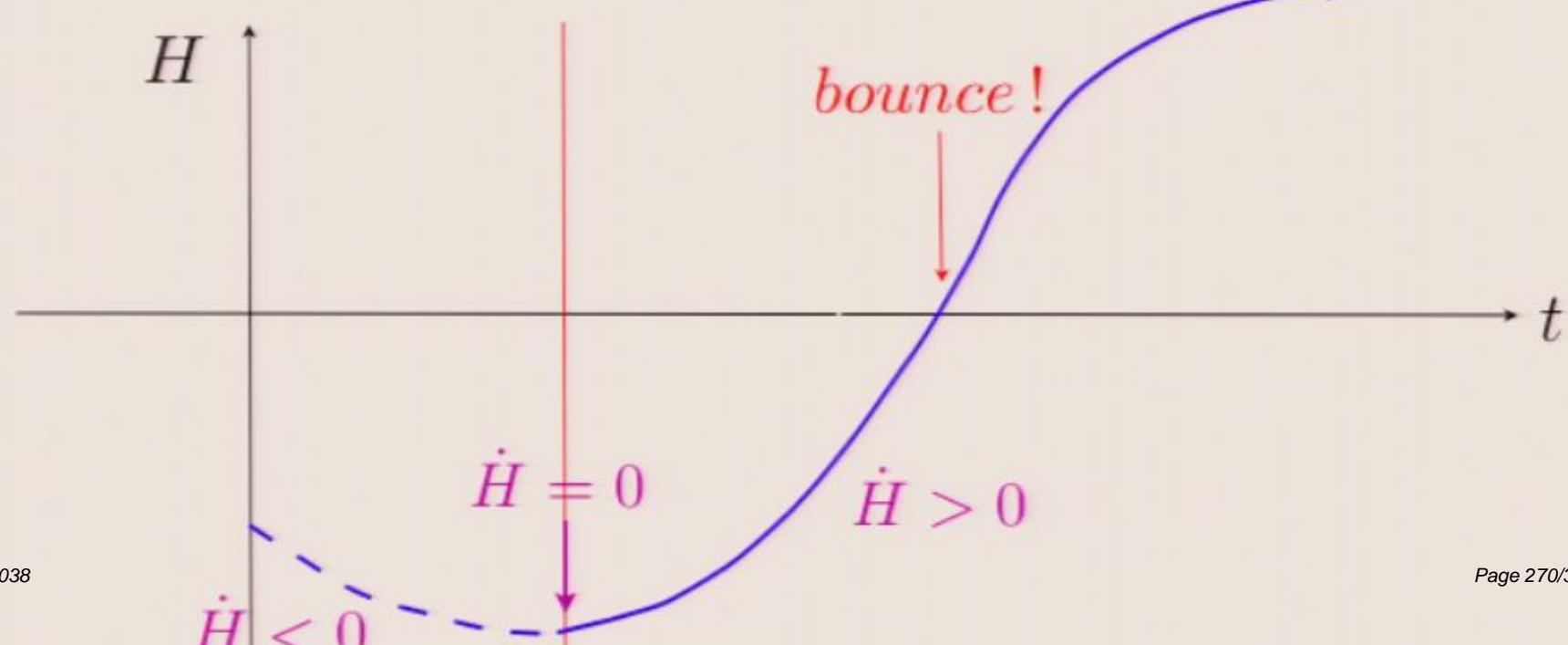
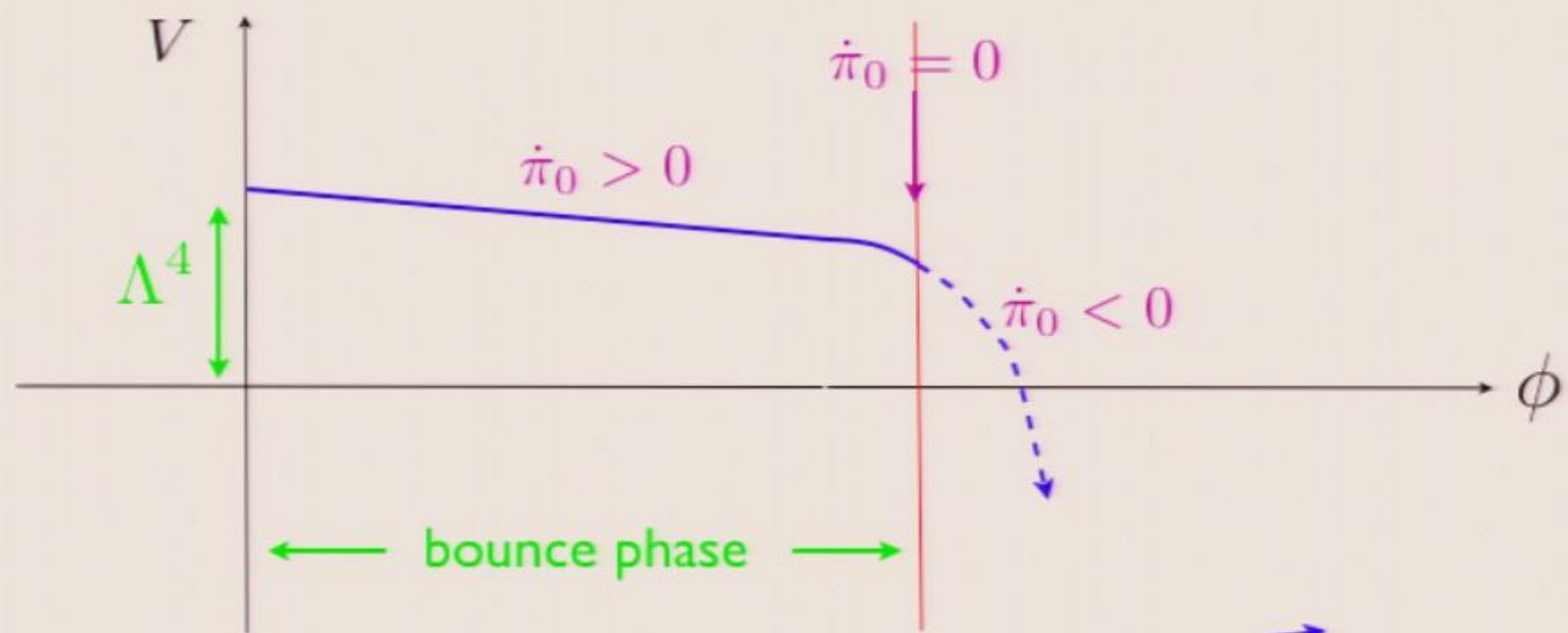


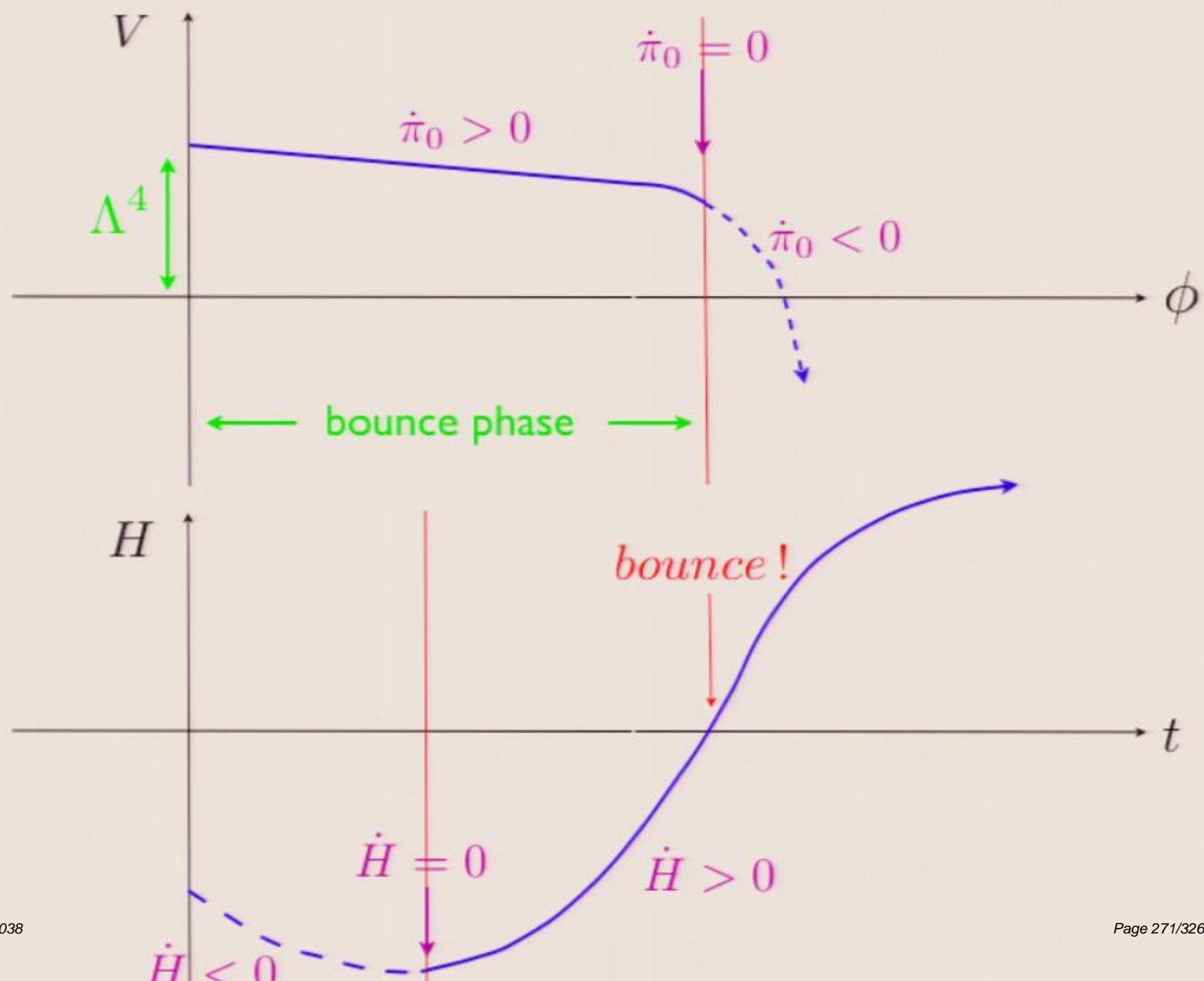


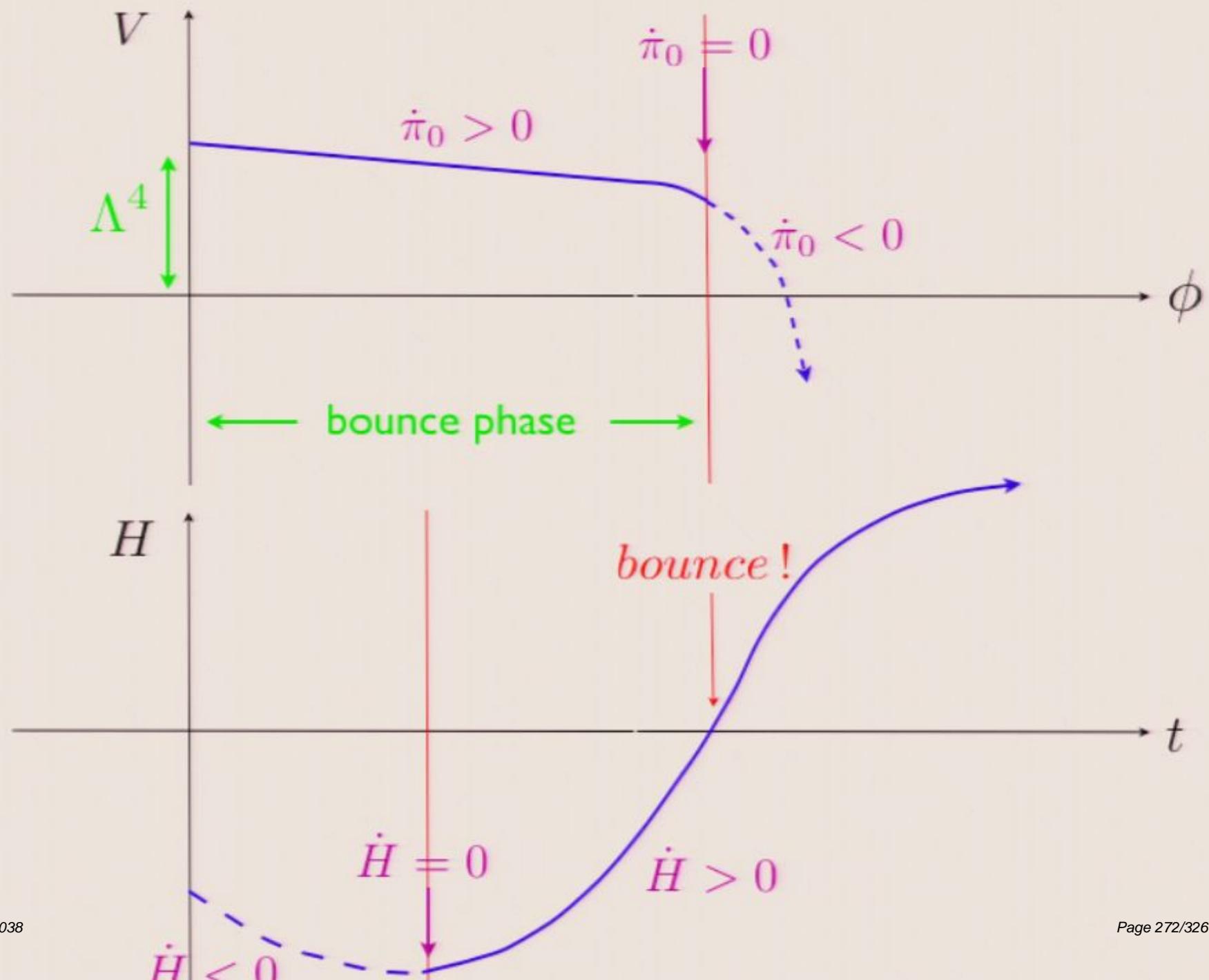


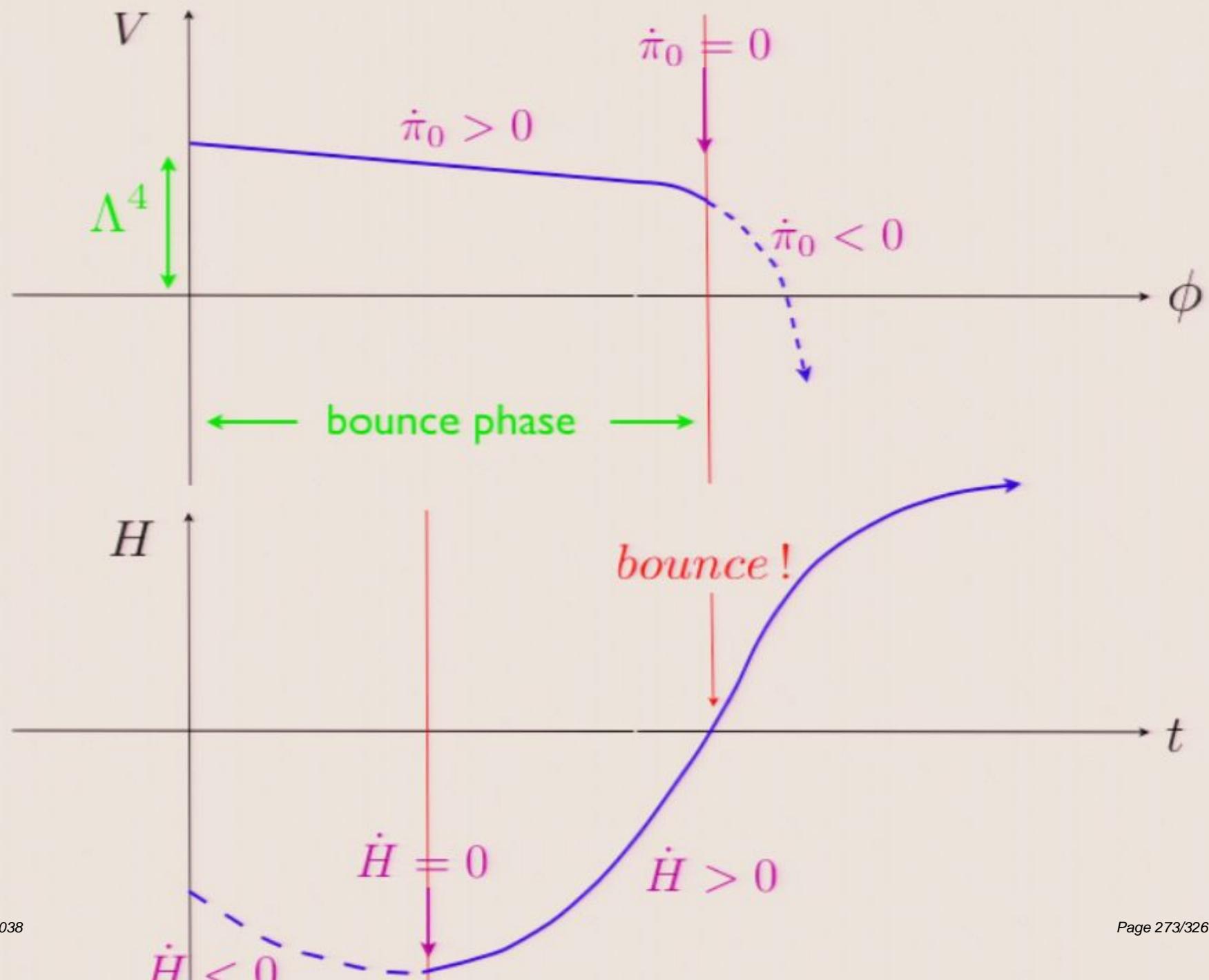


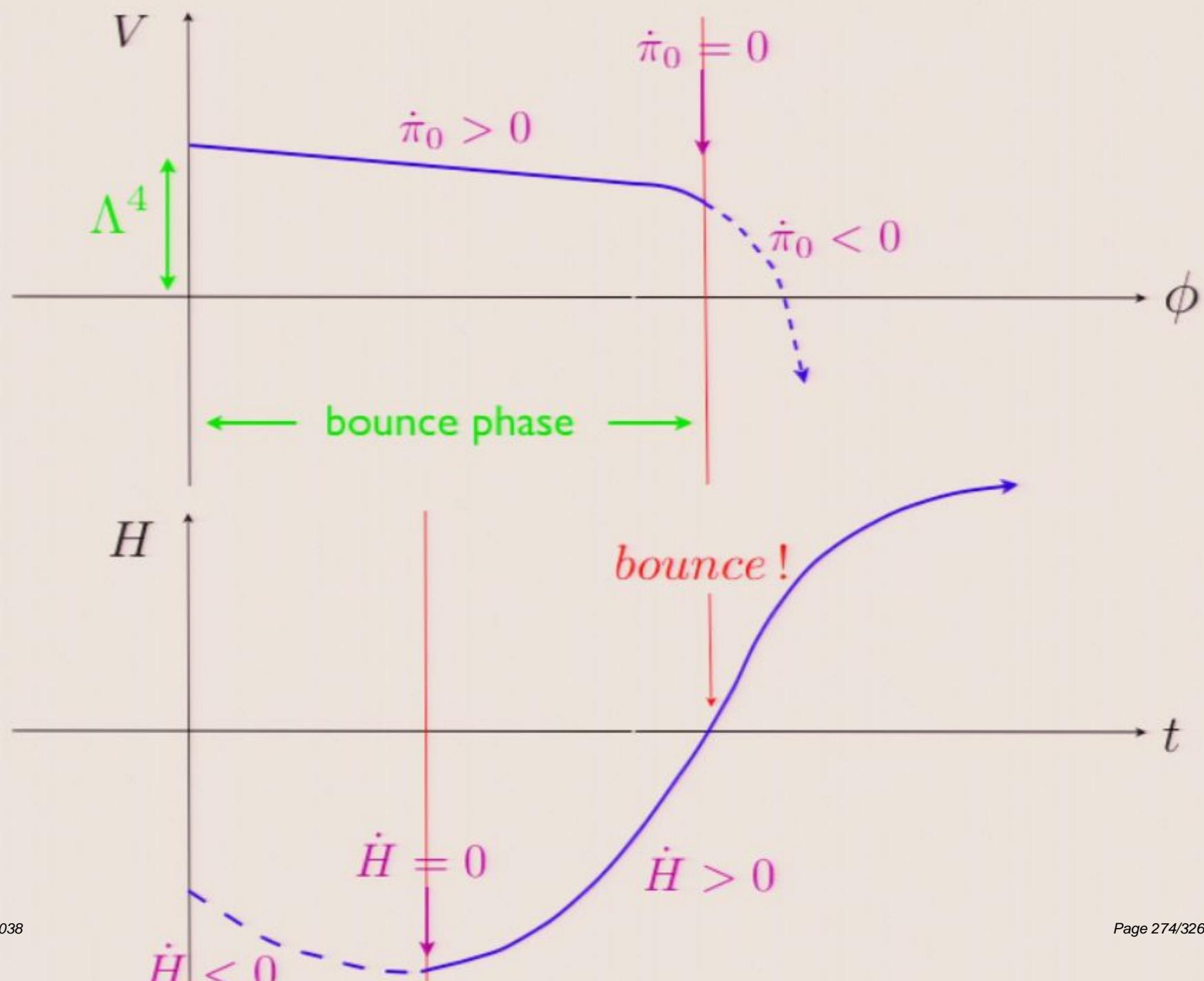


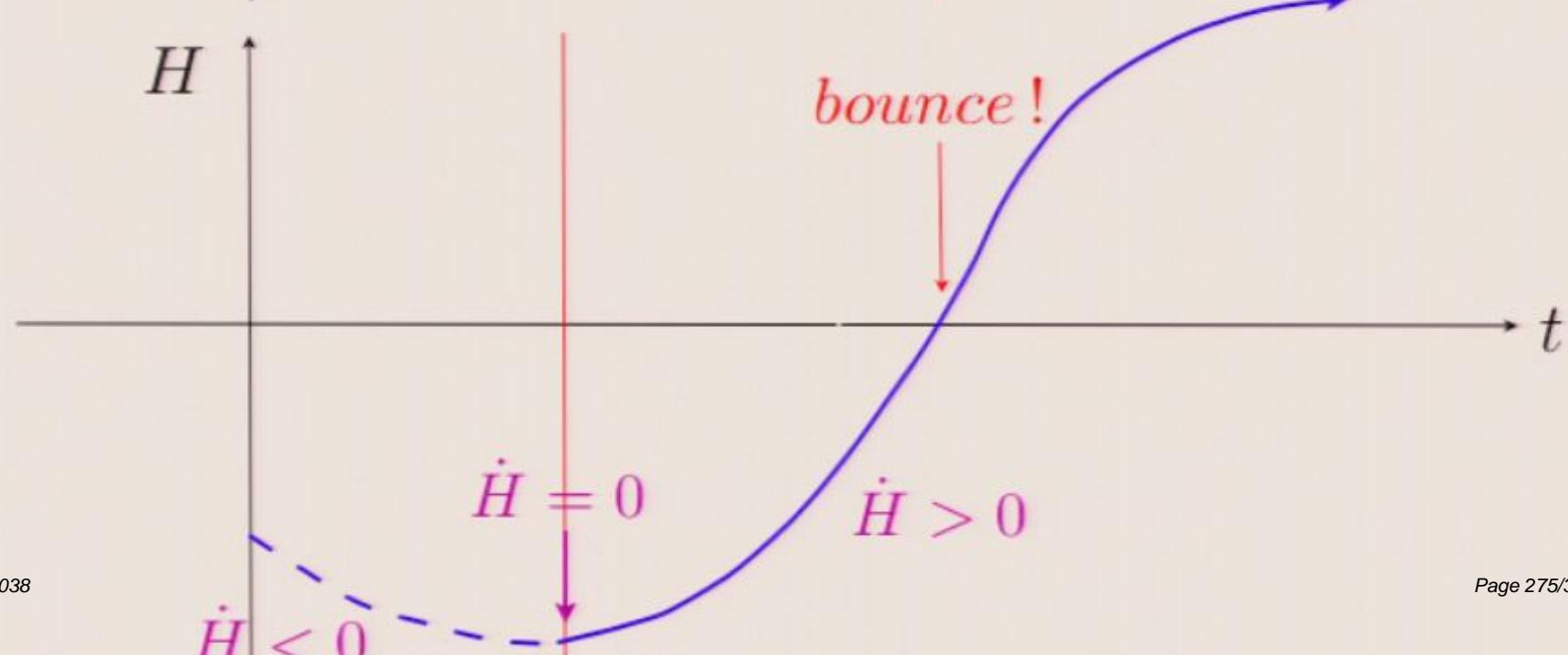
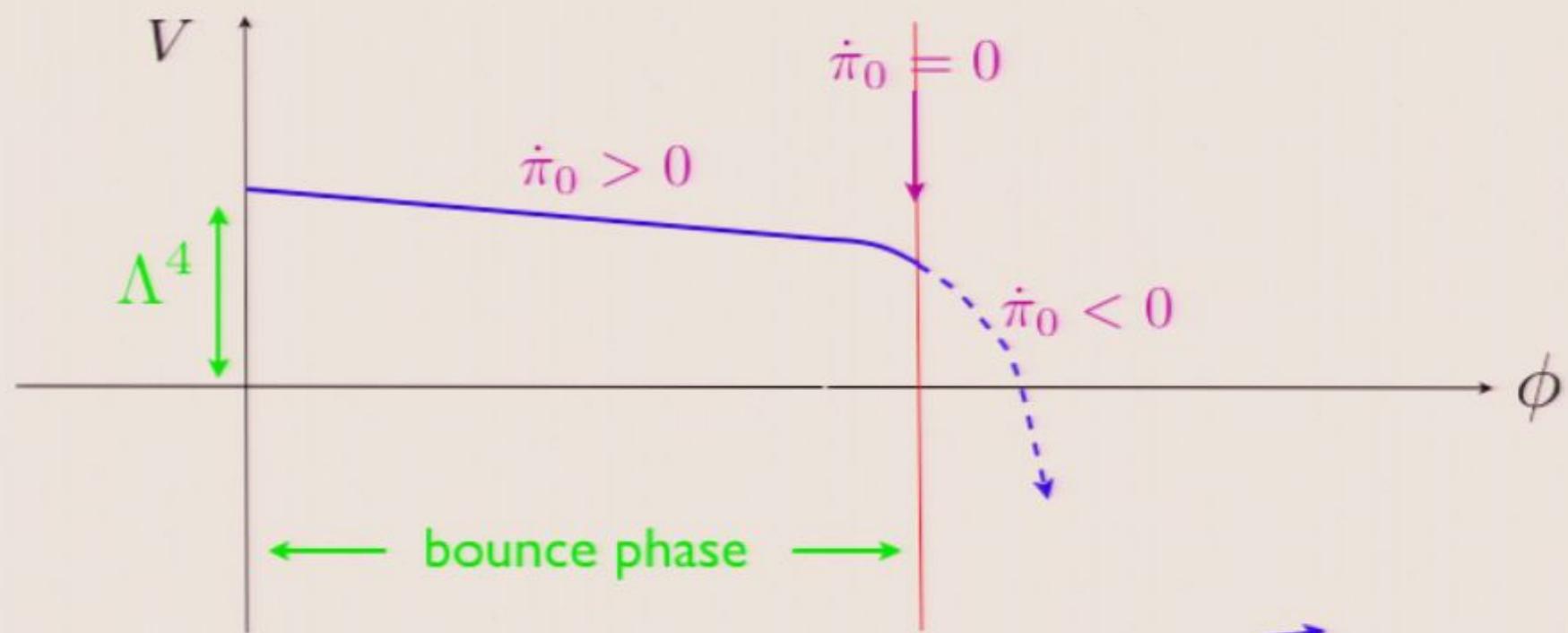


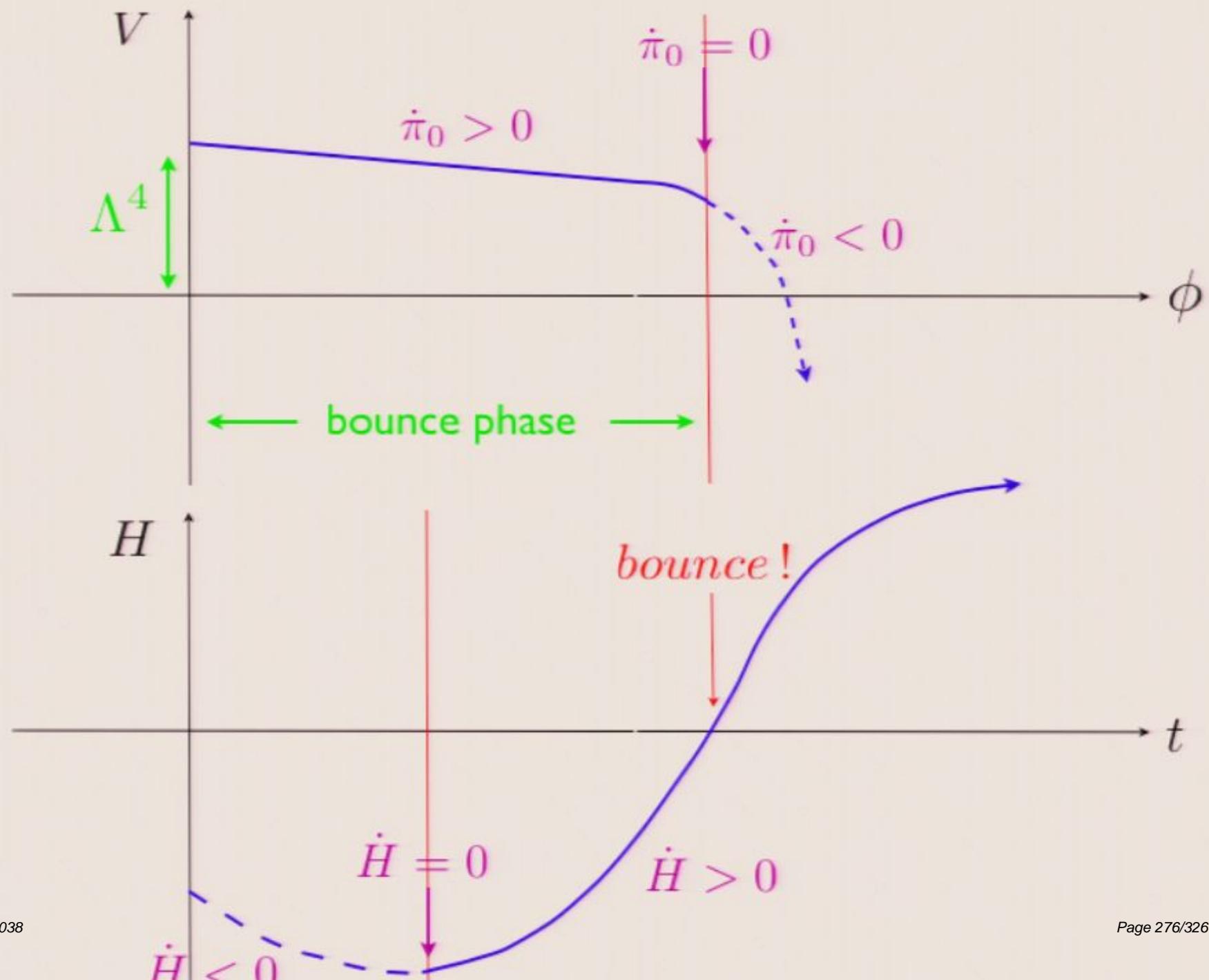


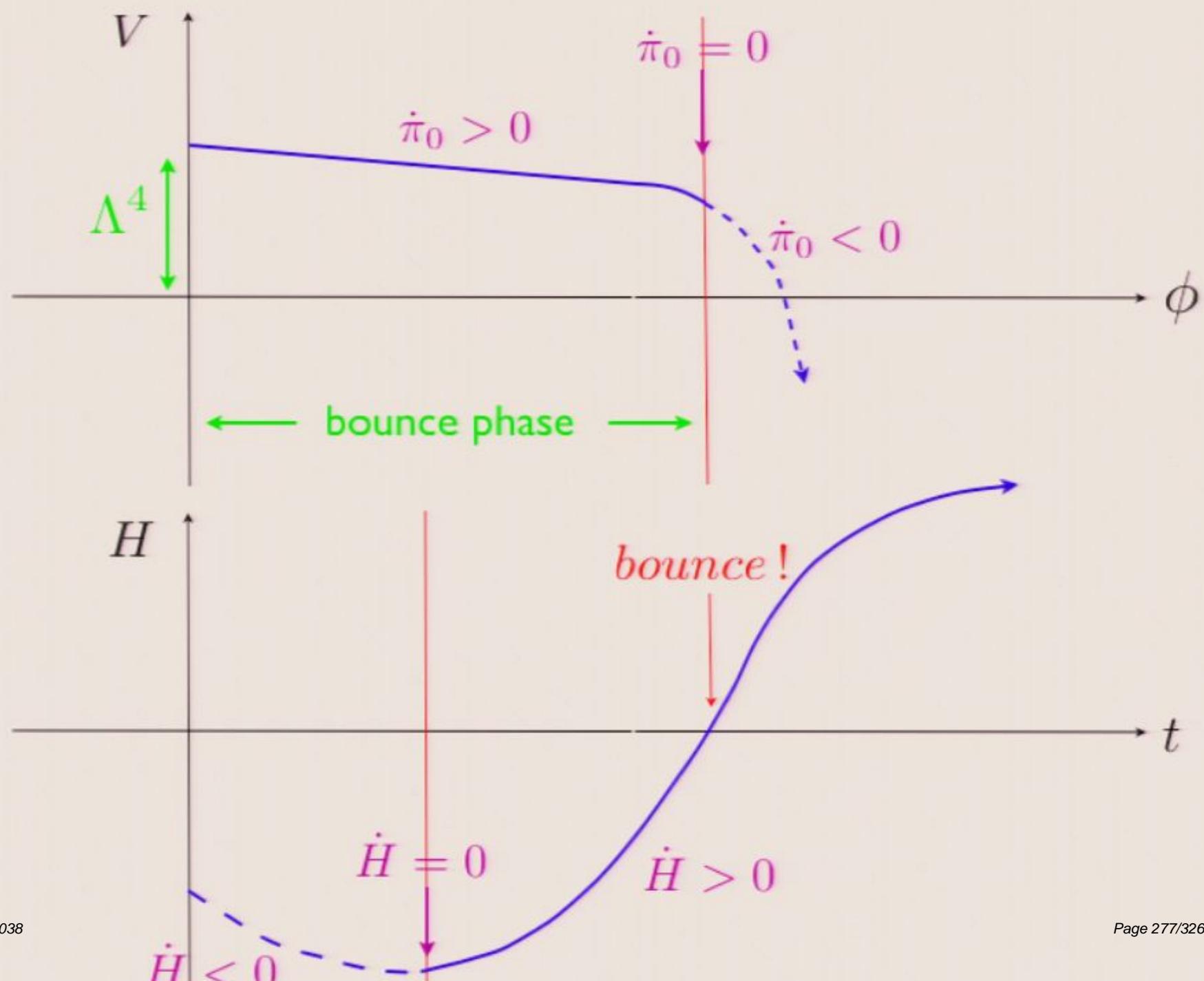


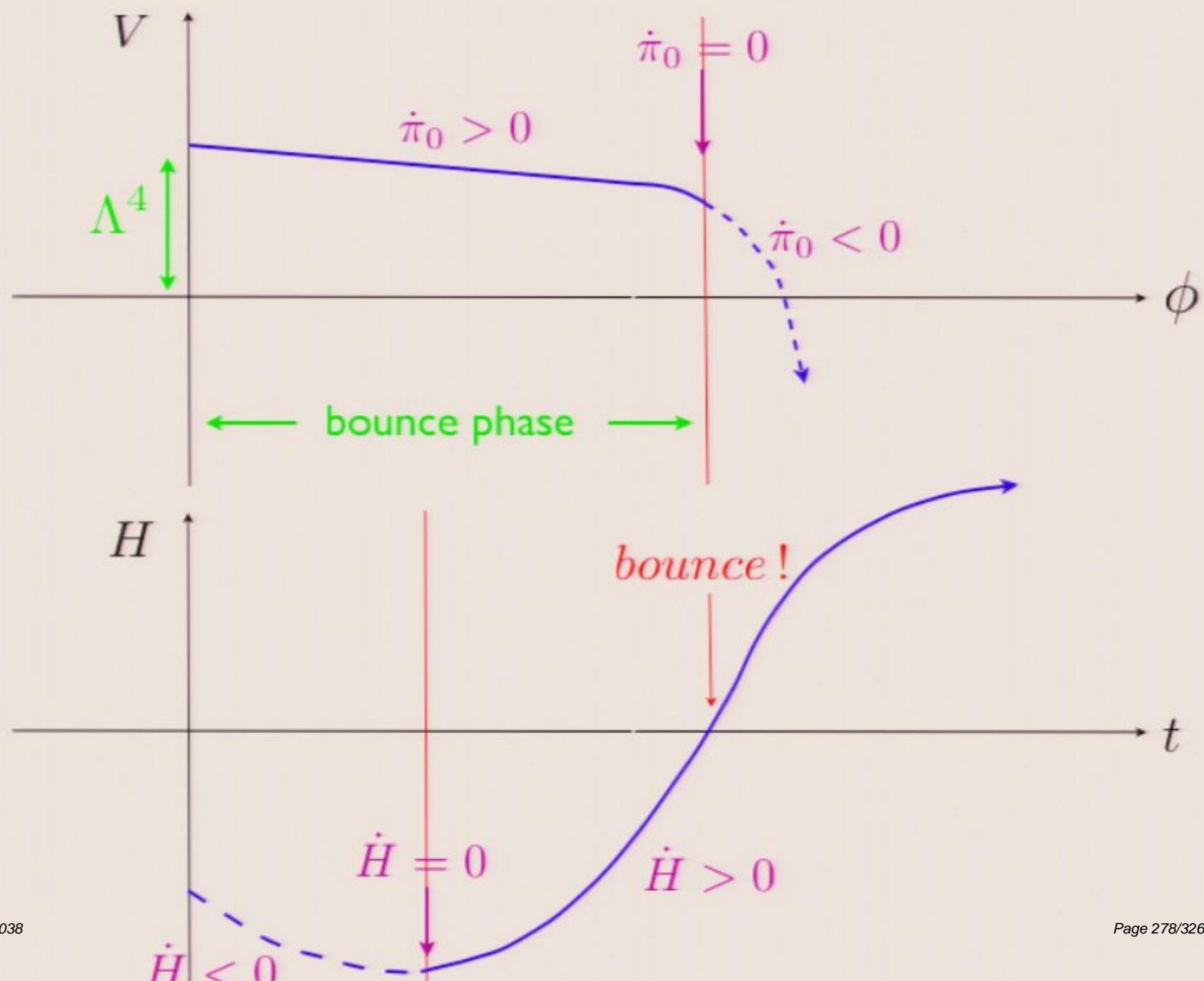


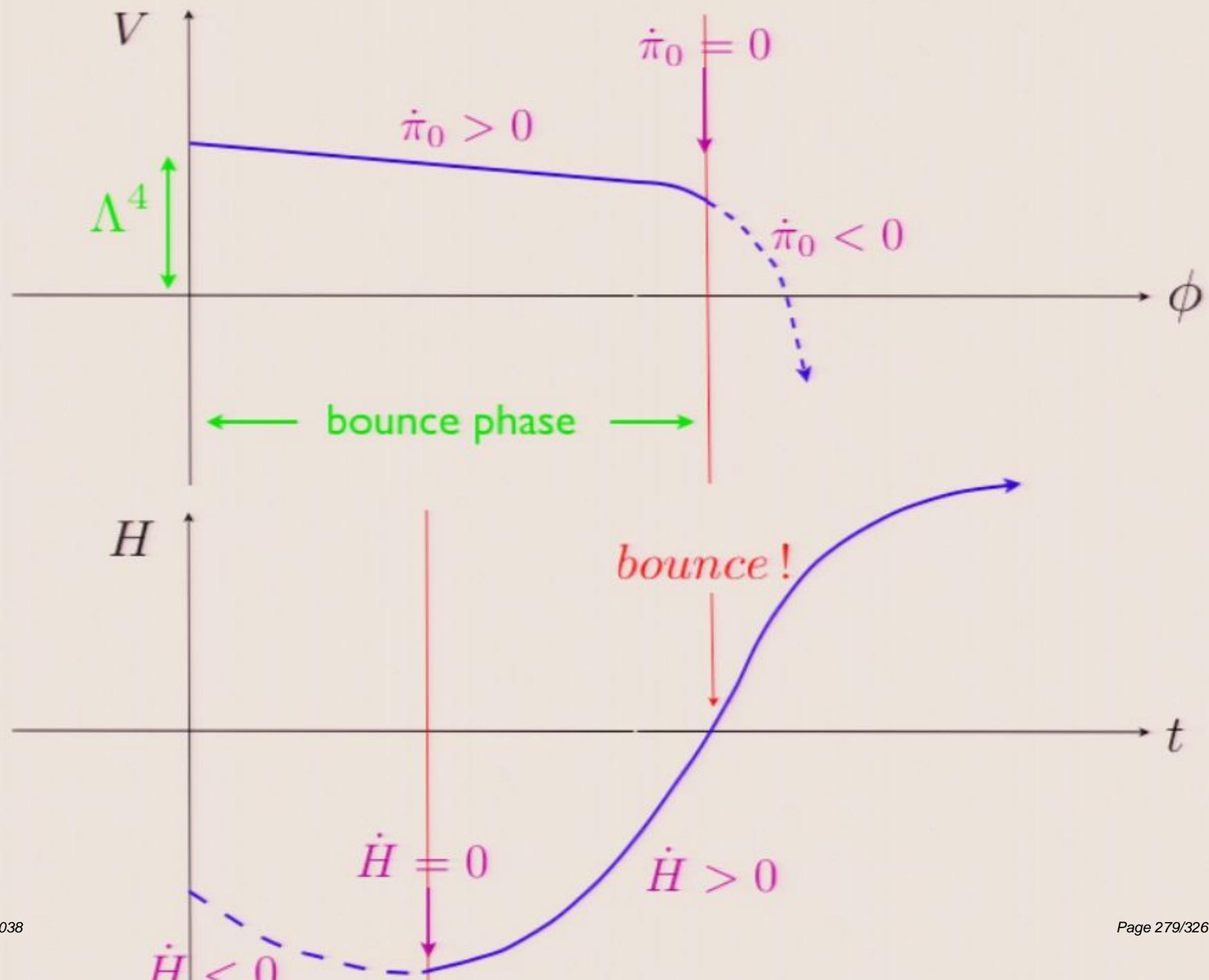


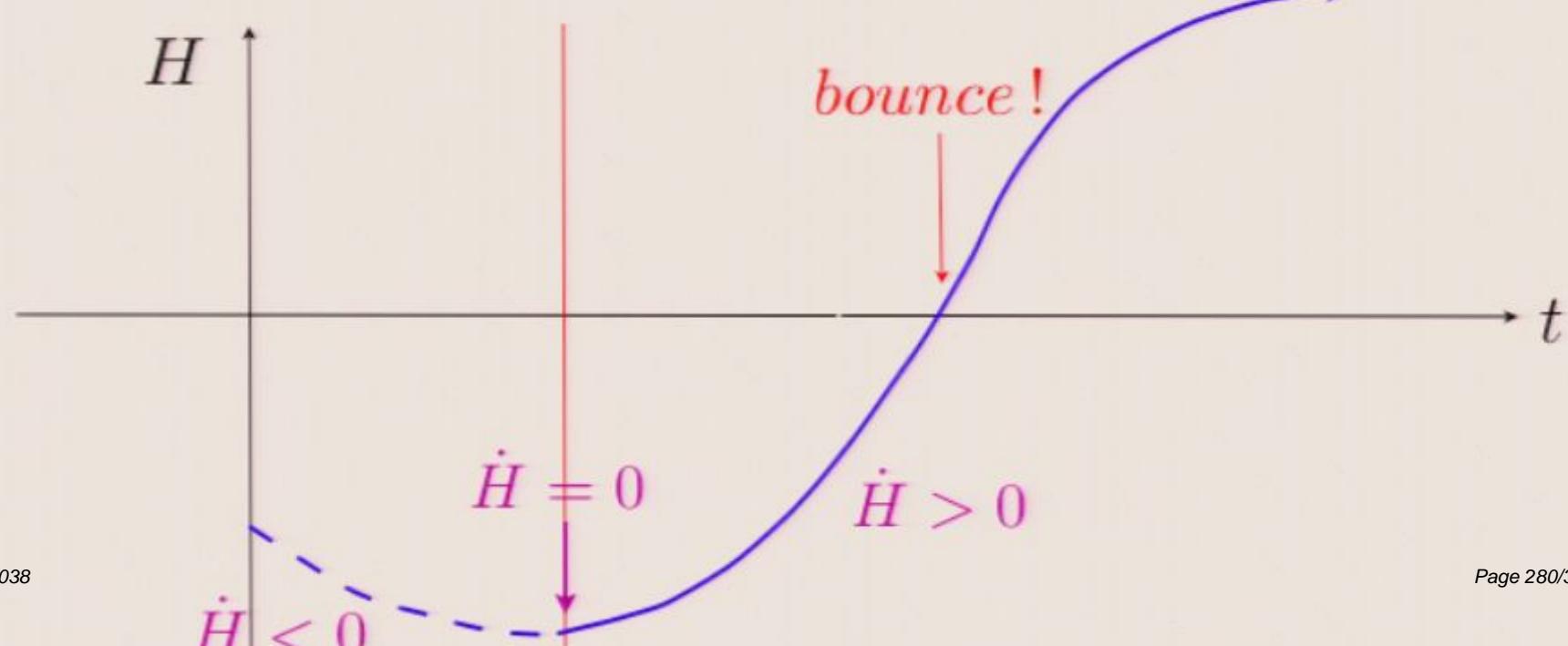
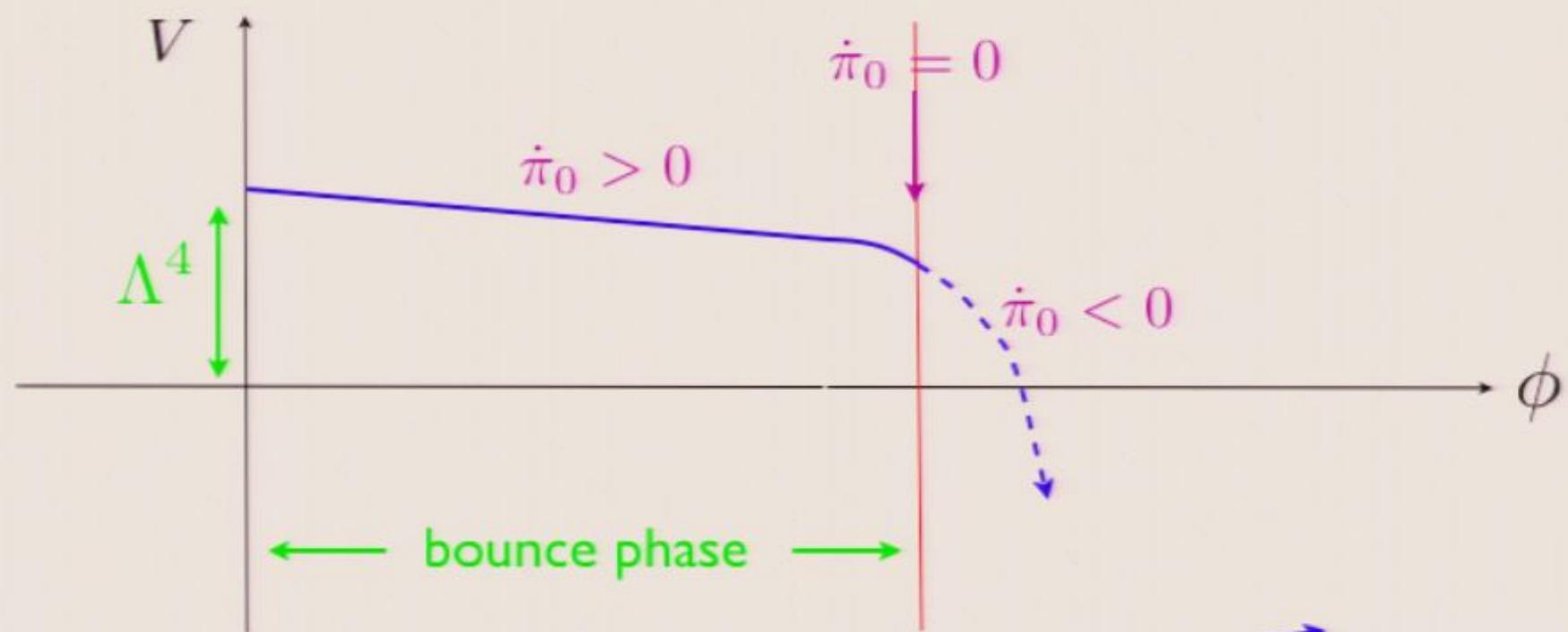


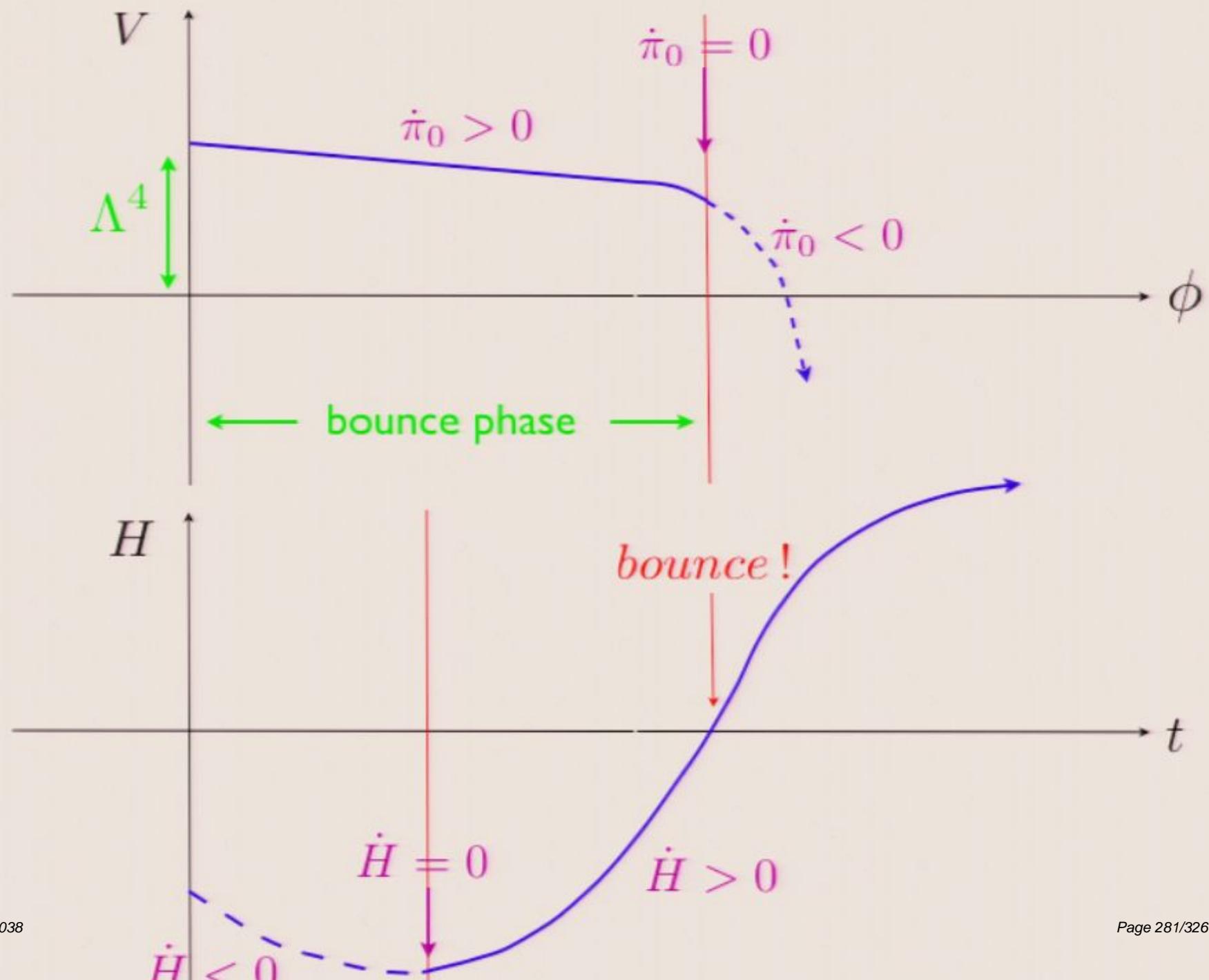


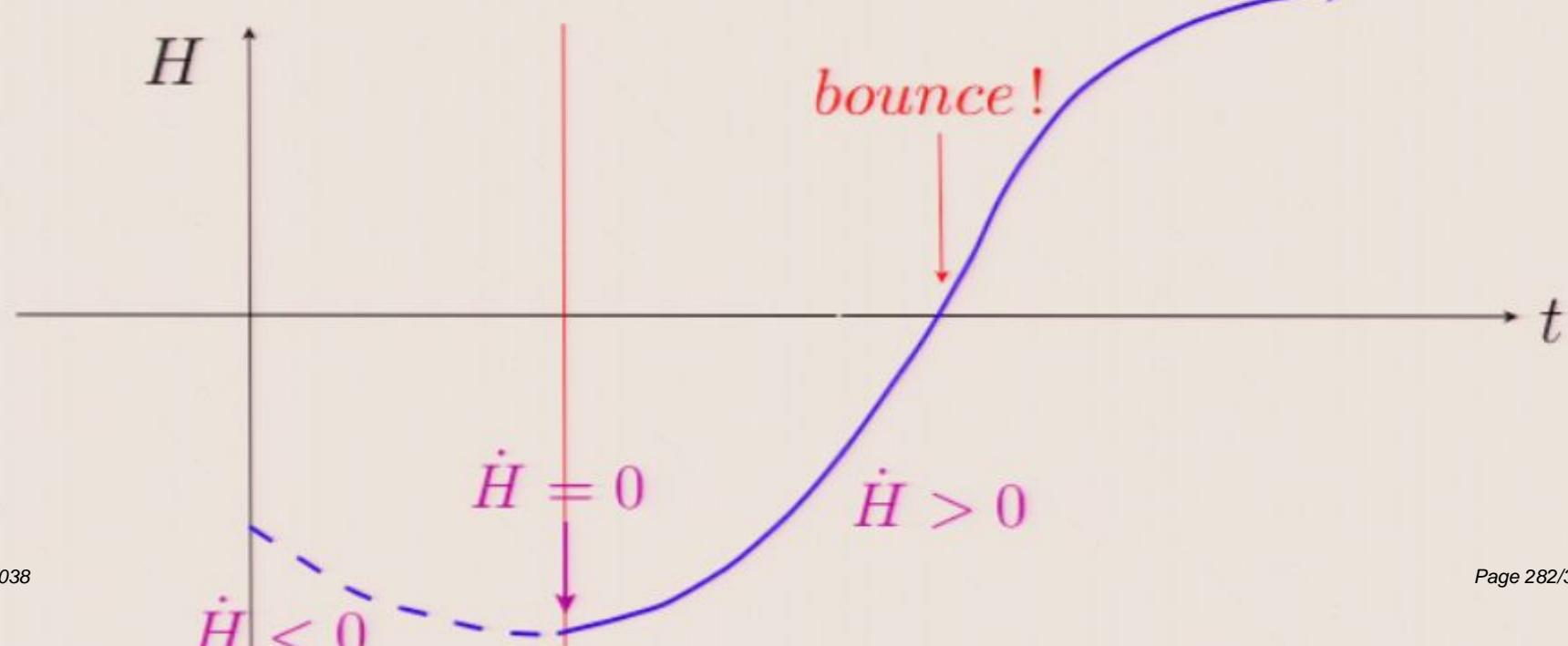
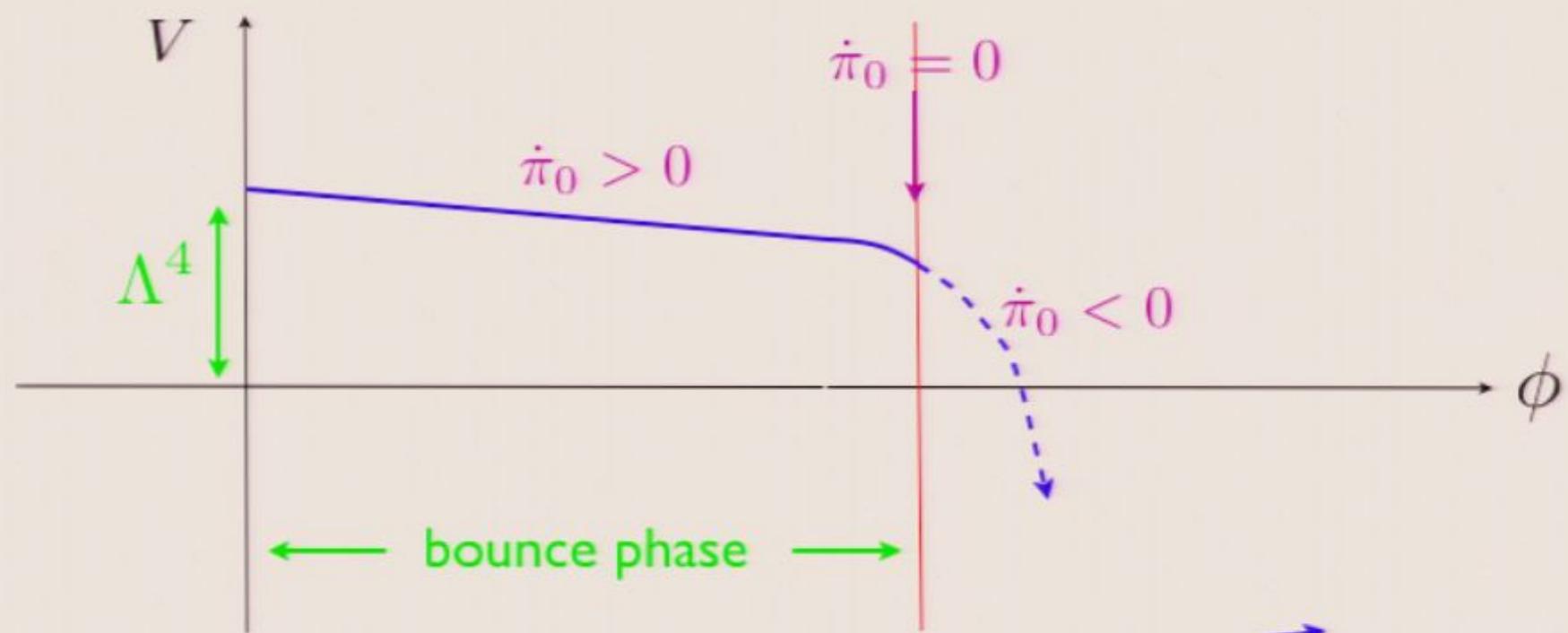


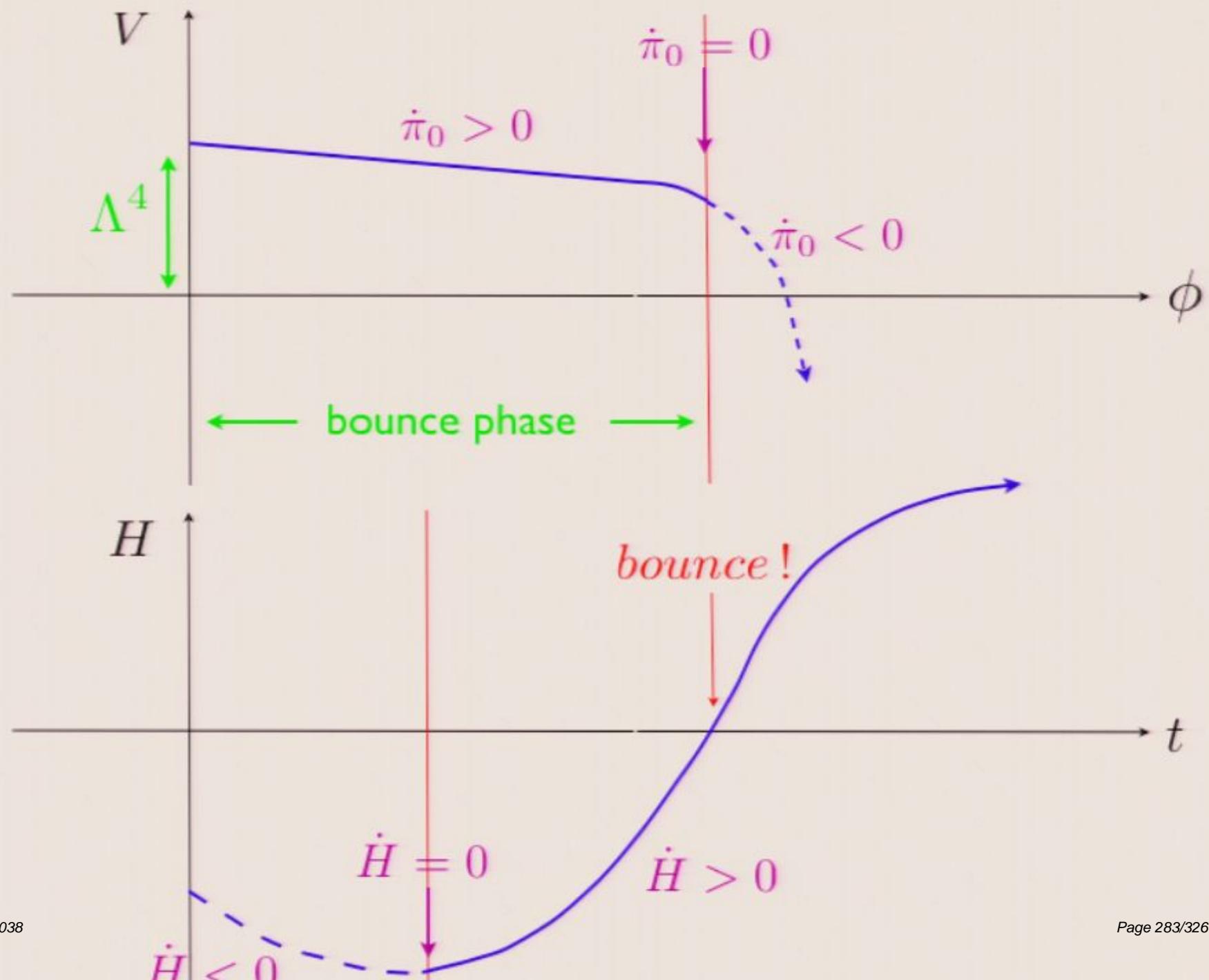


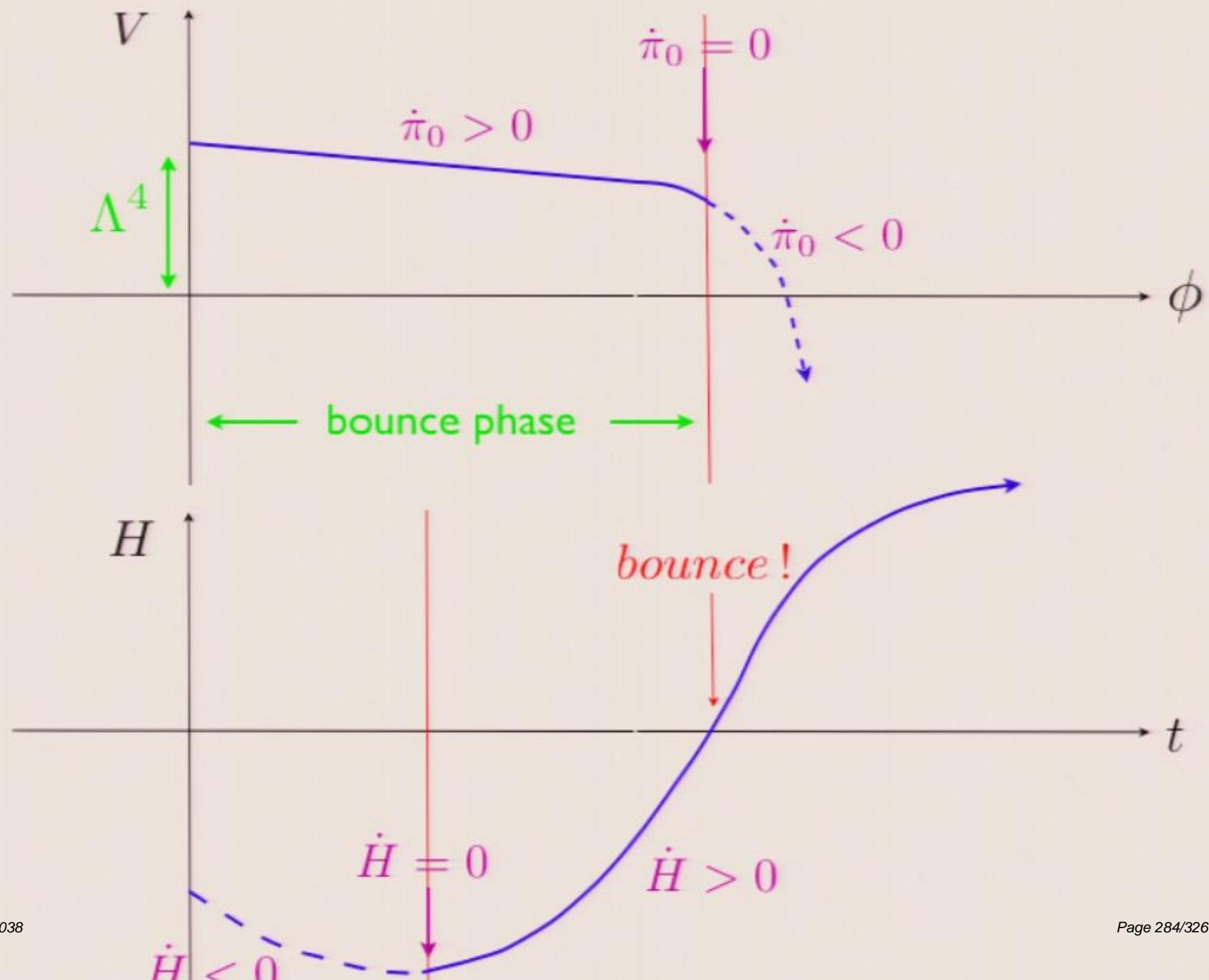


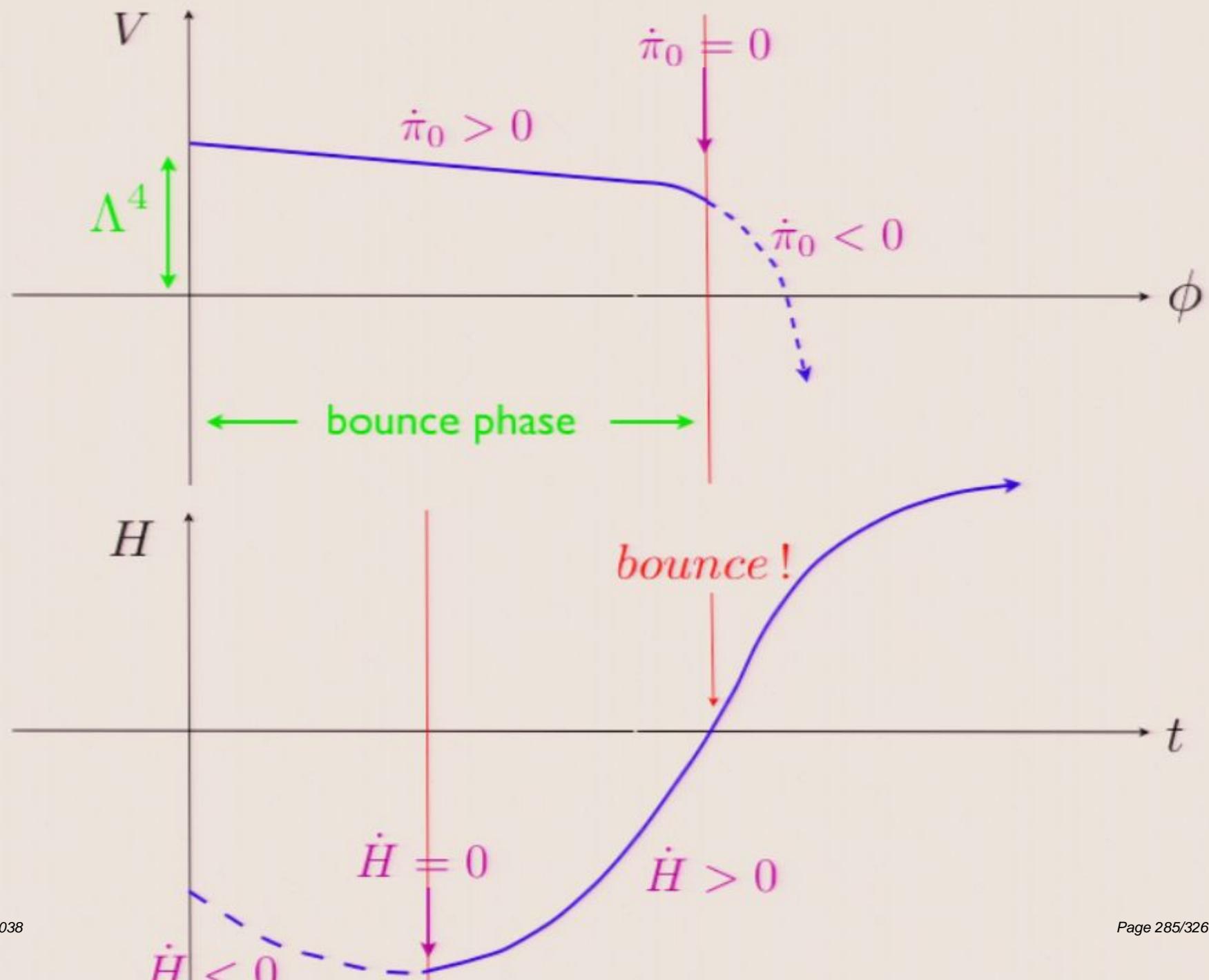


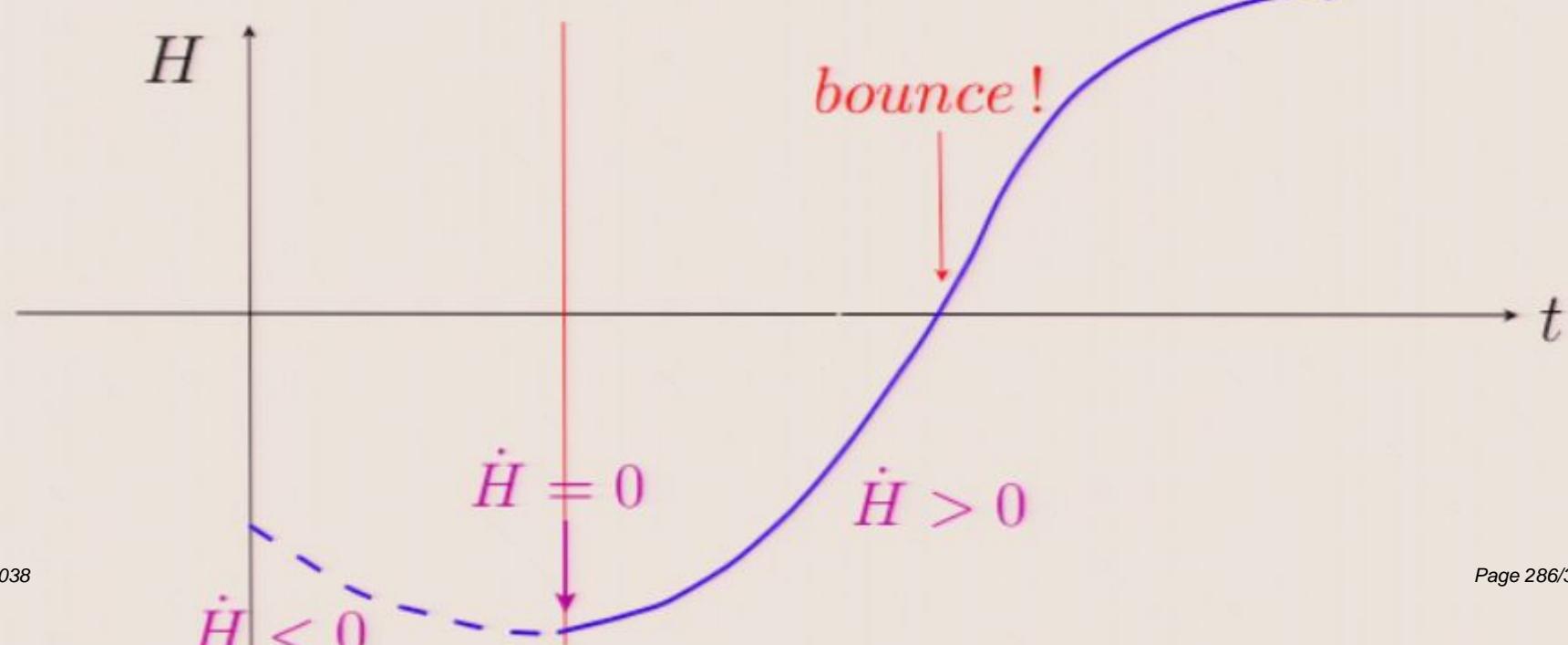
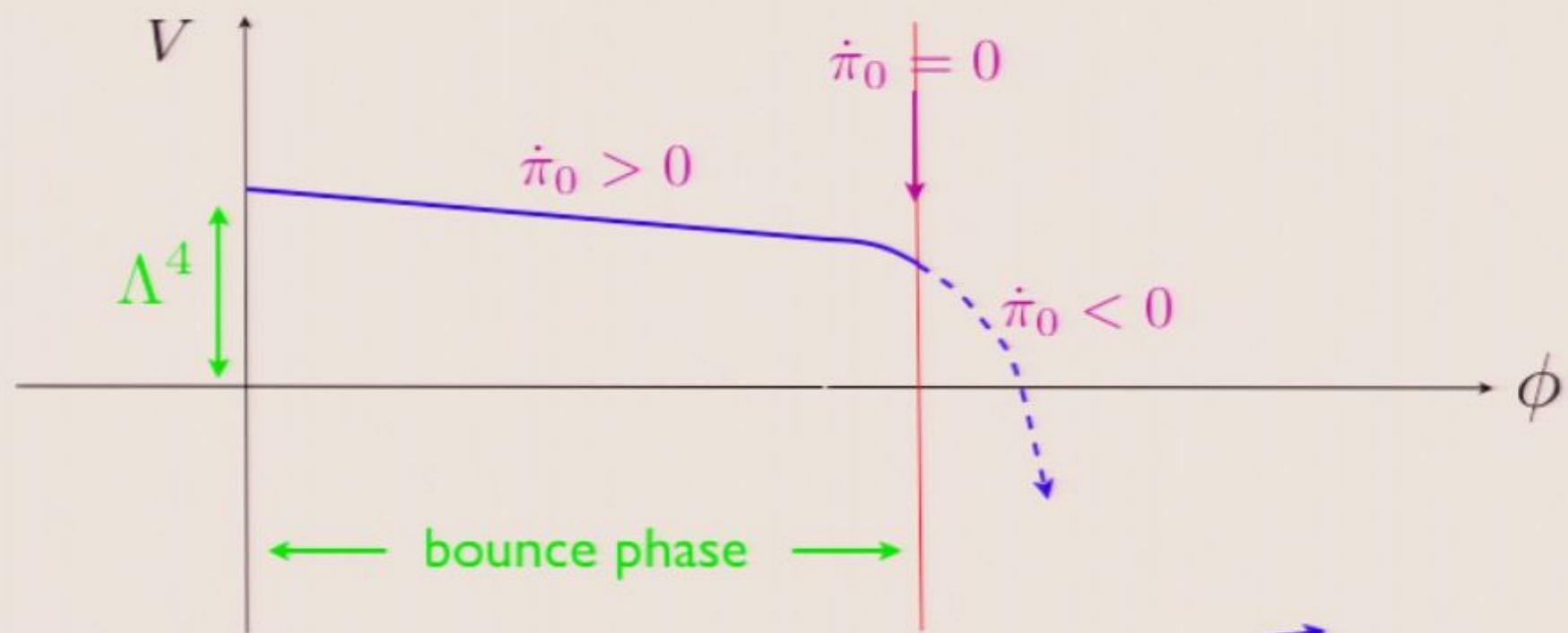




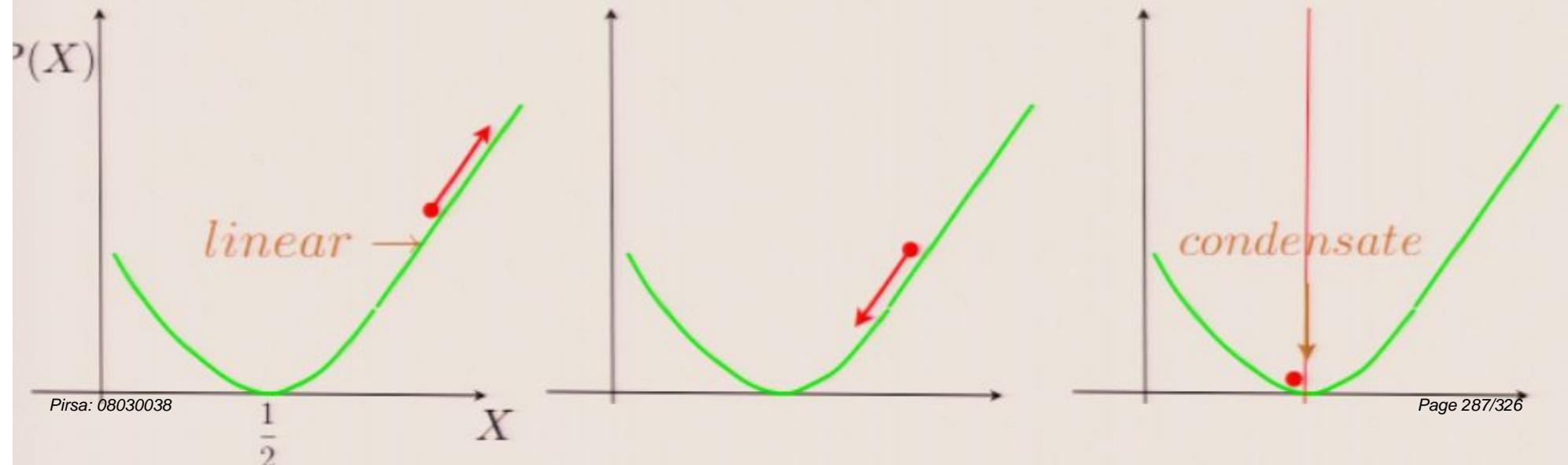
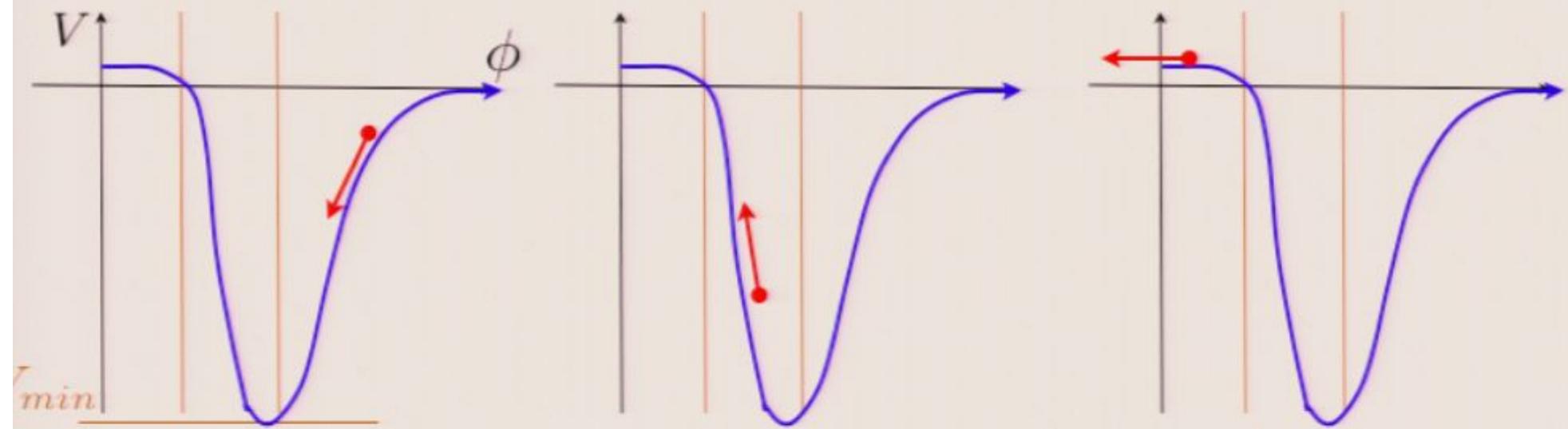




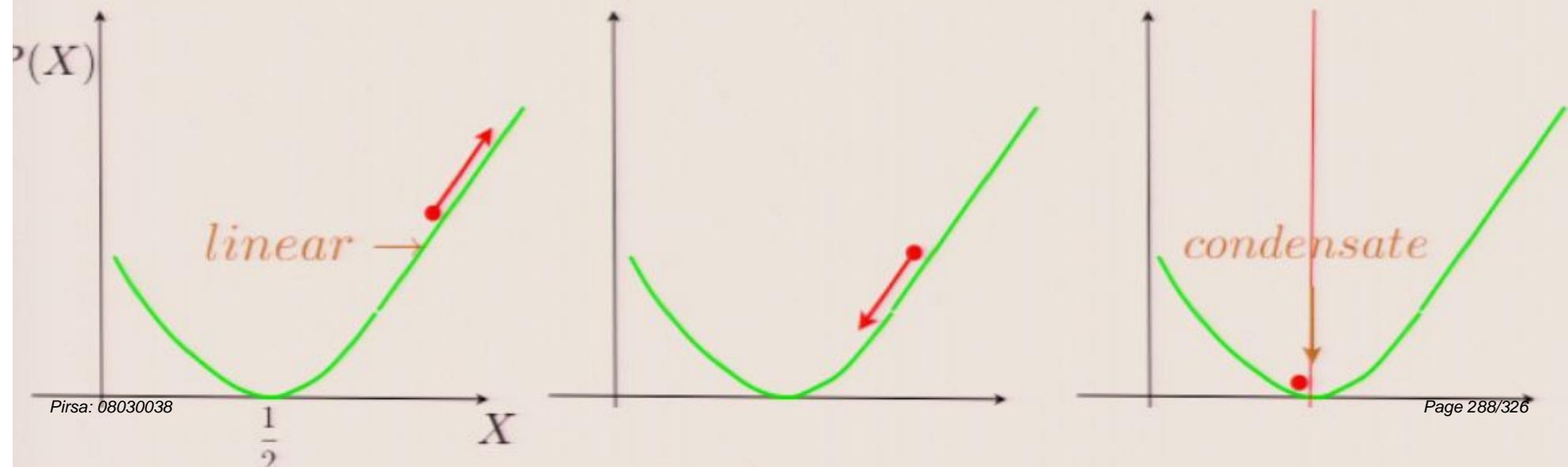
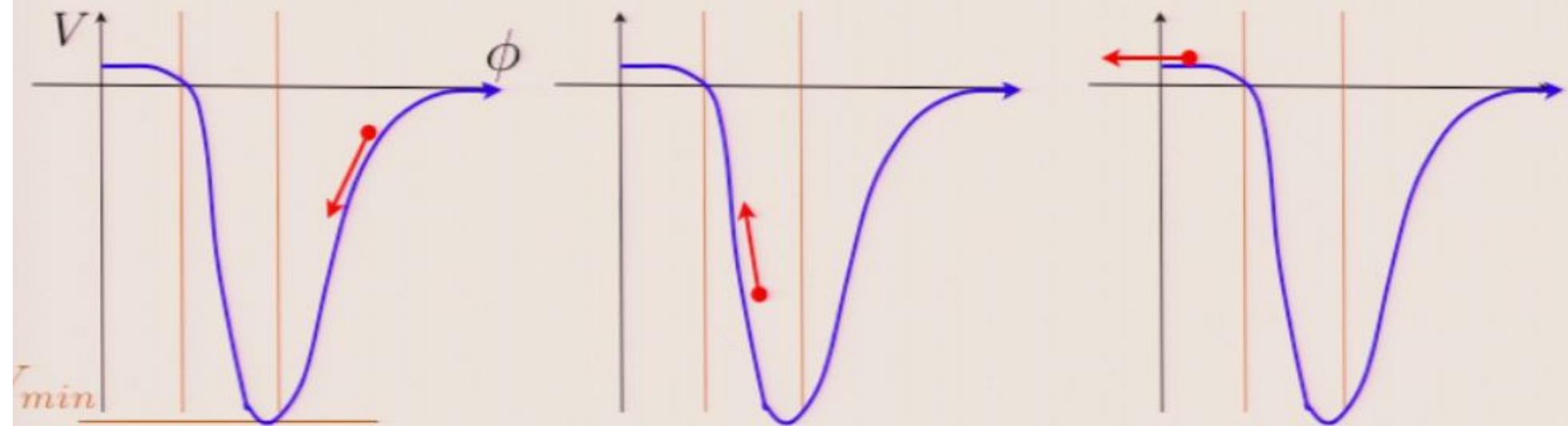




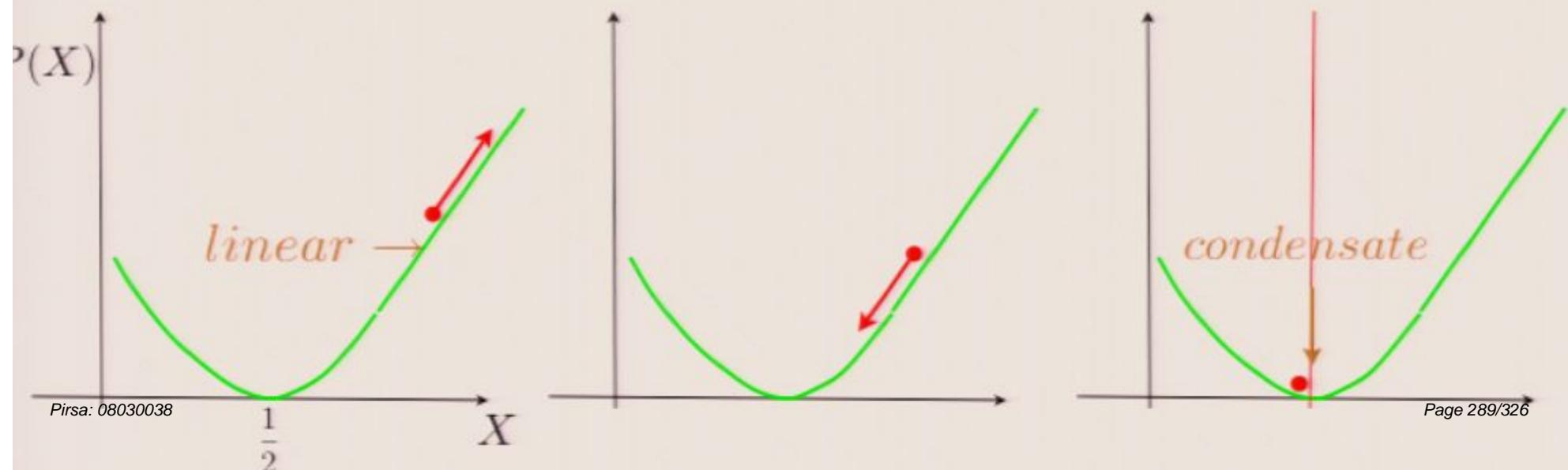
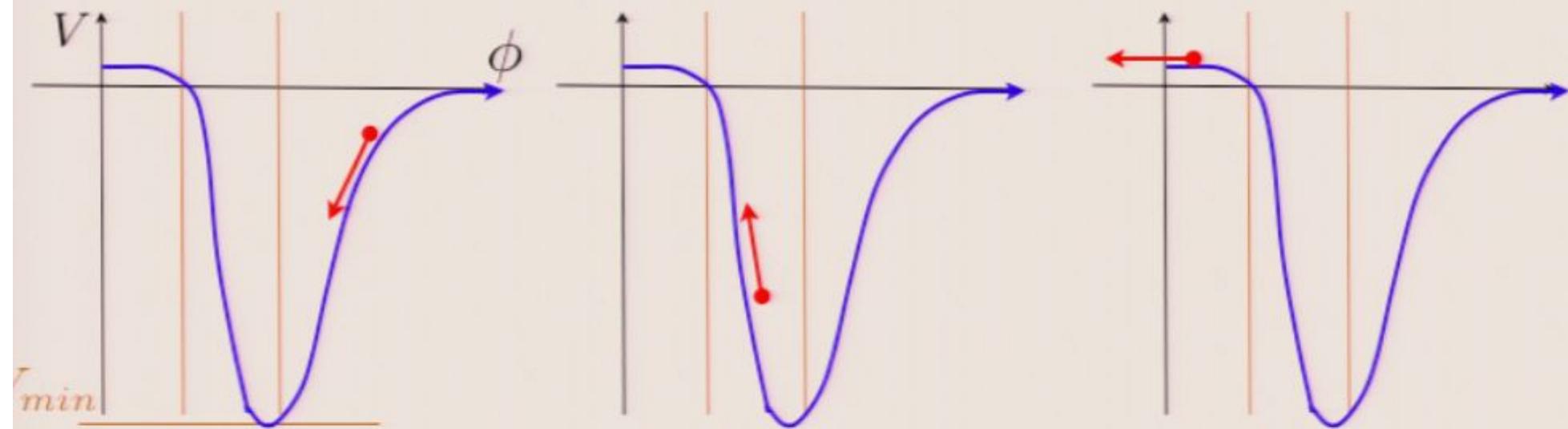
b) Merger with Ekpyrosis



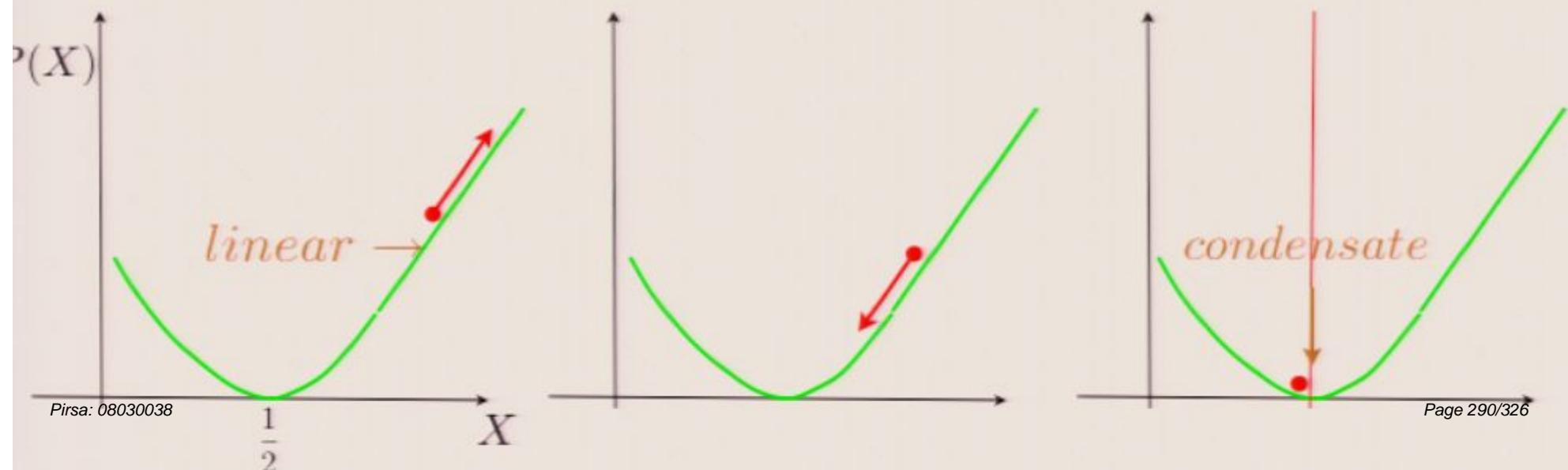
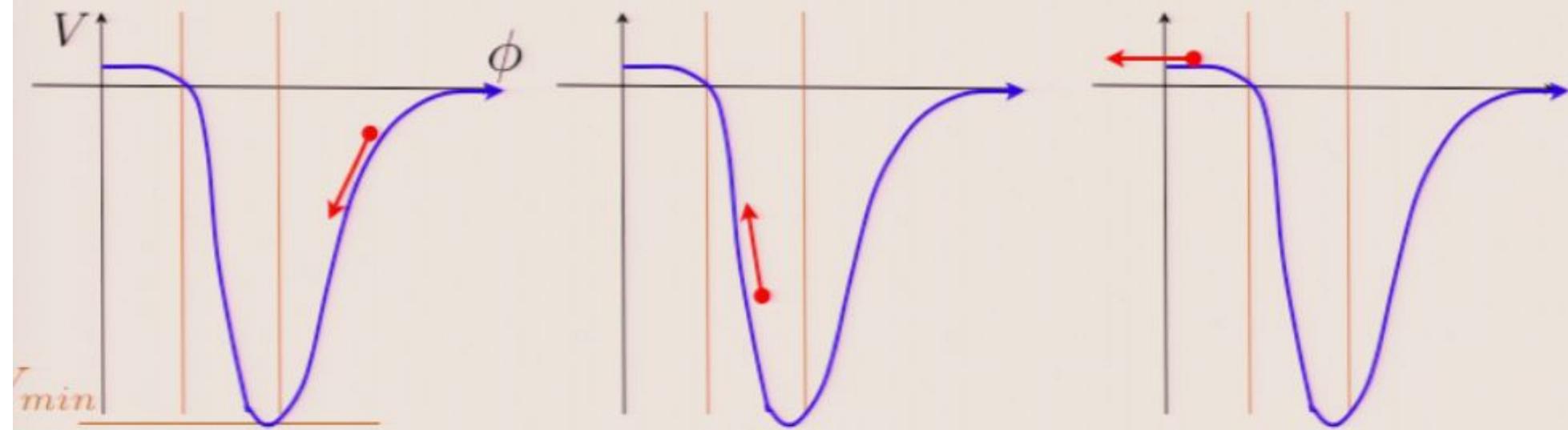
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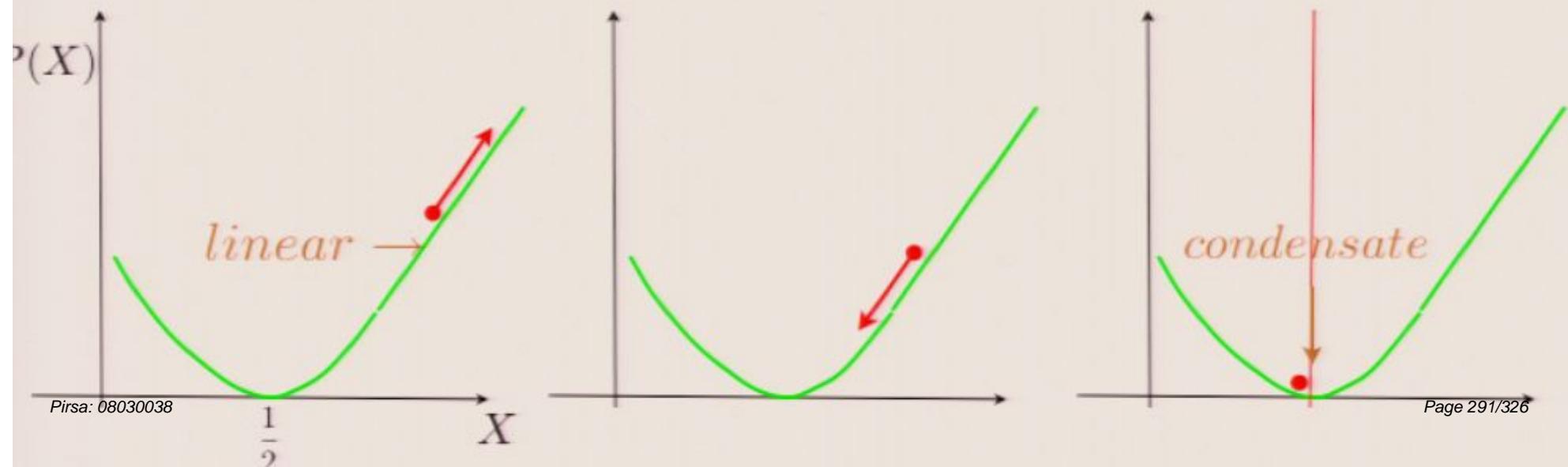
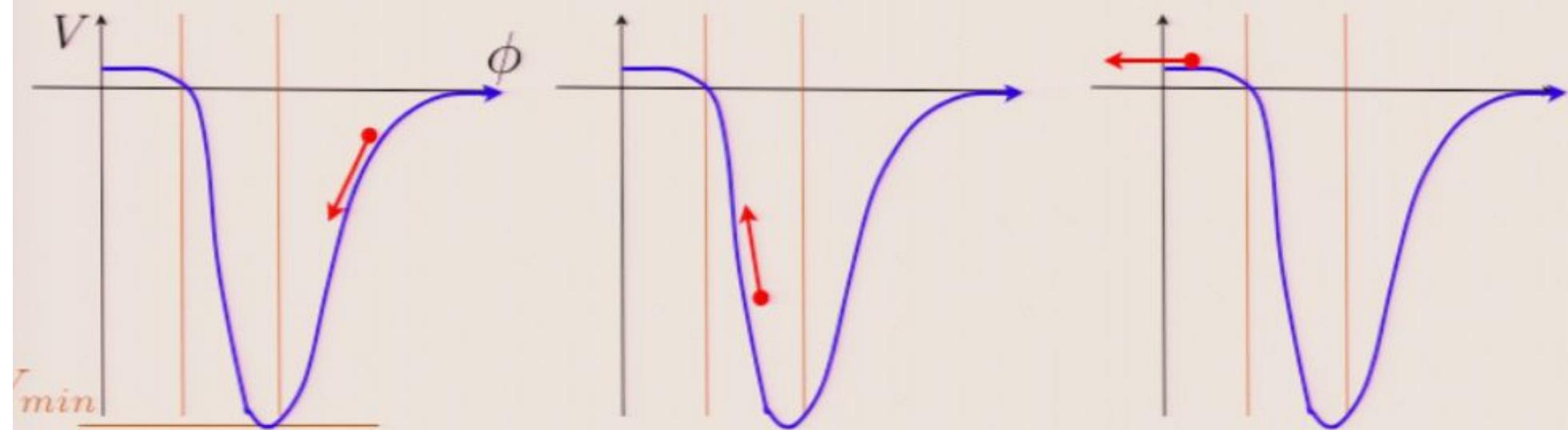
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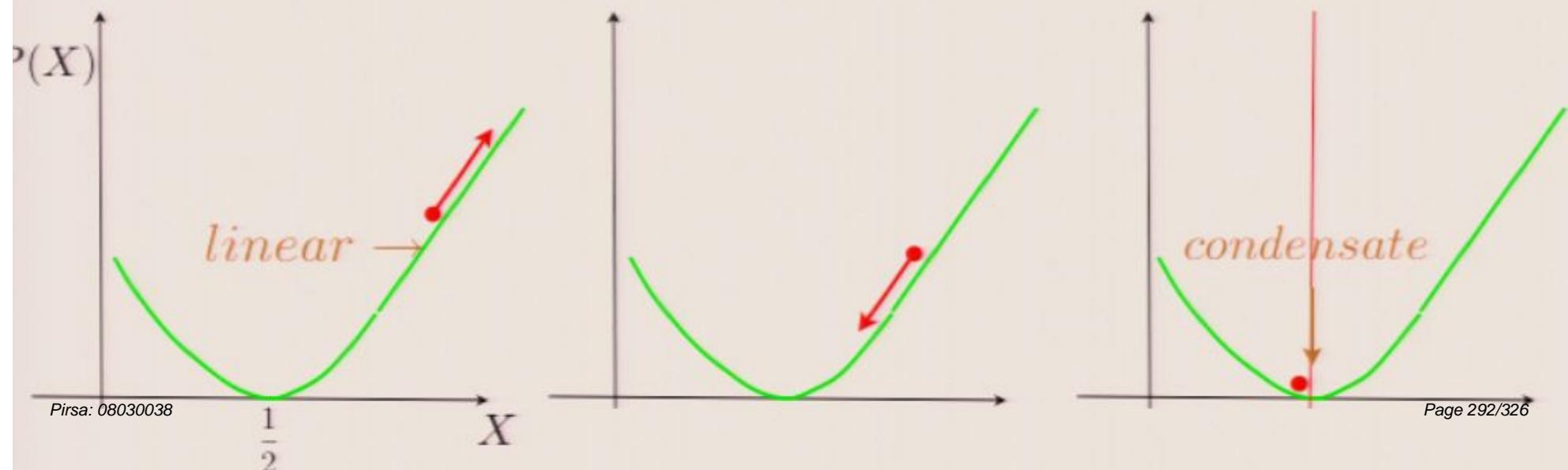
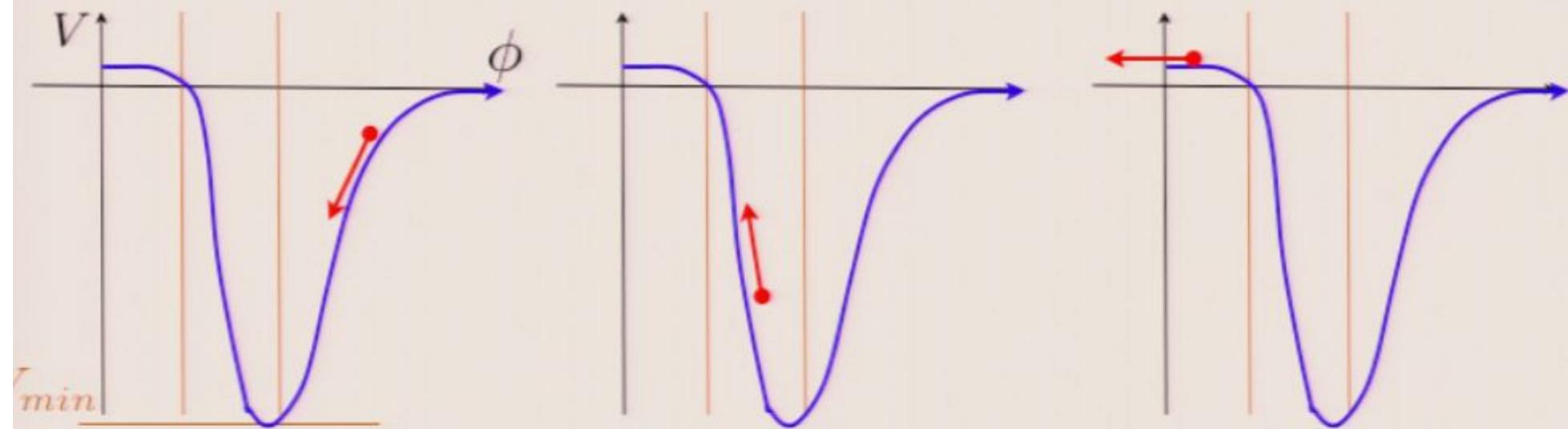
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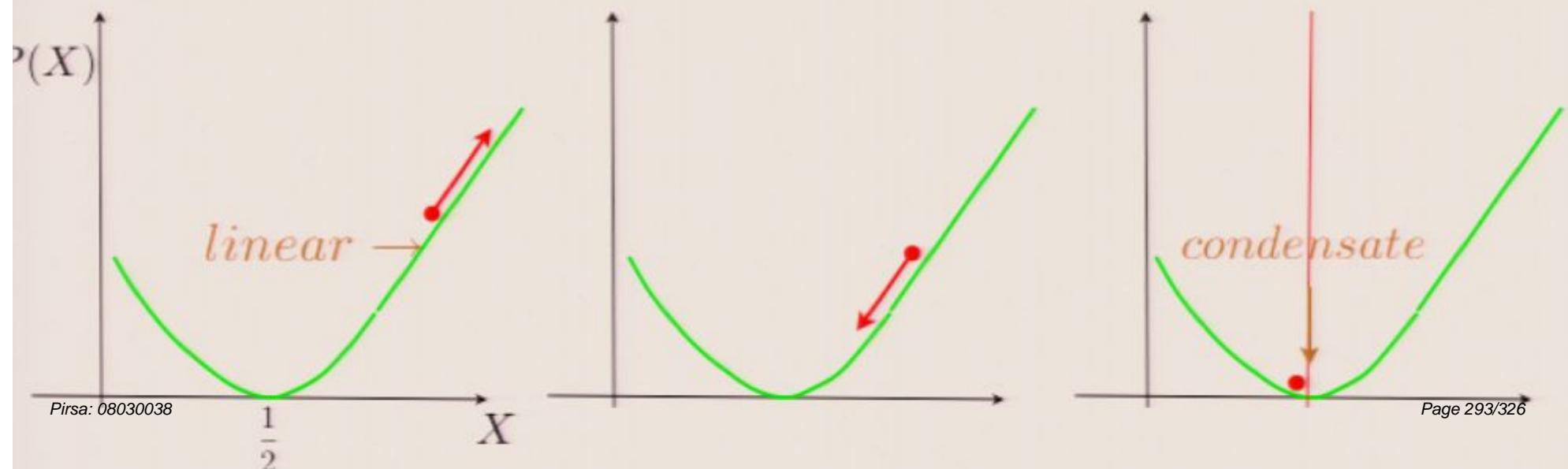
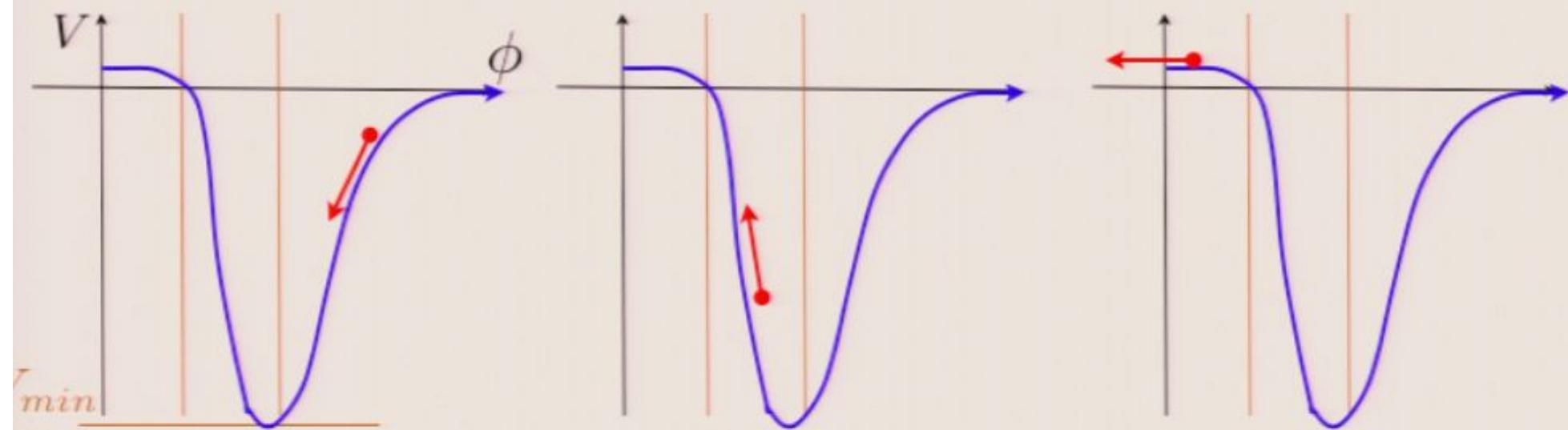
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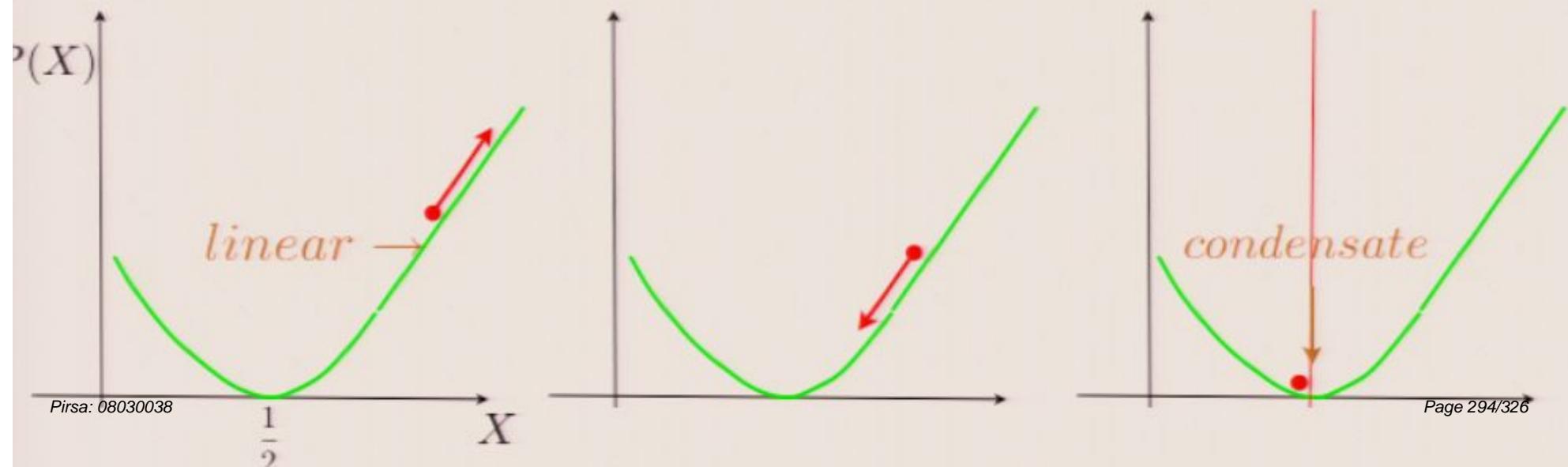
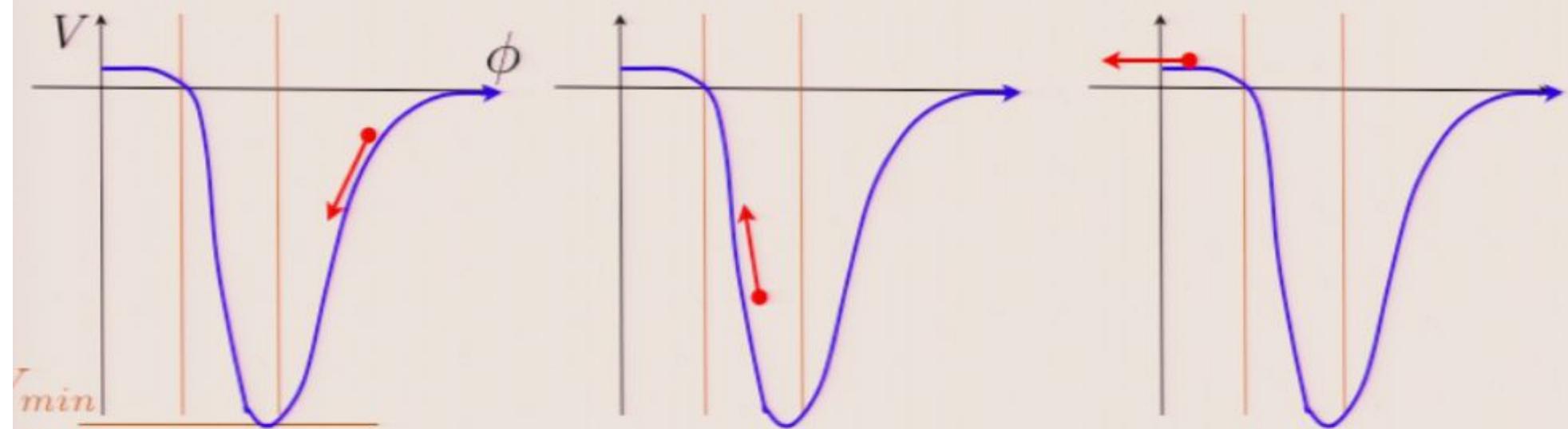
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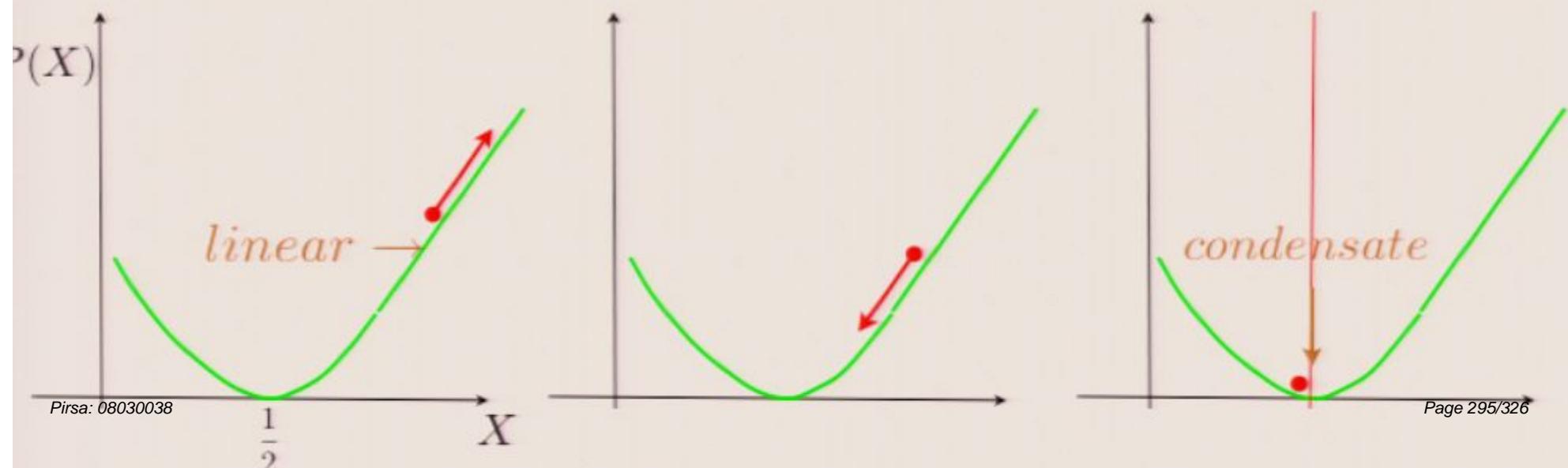
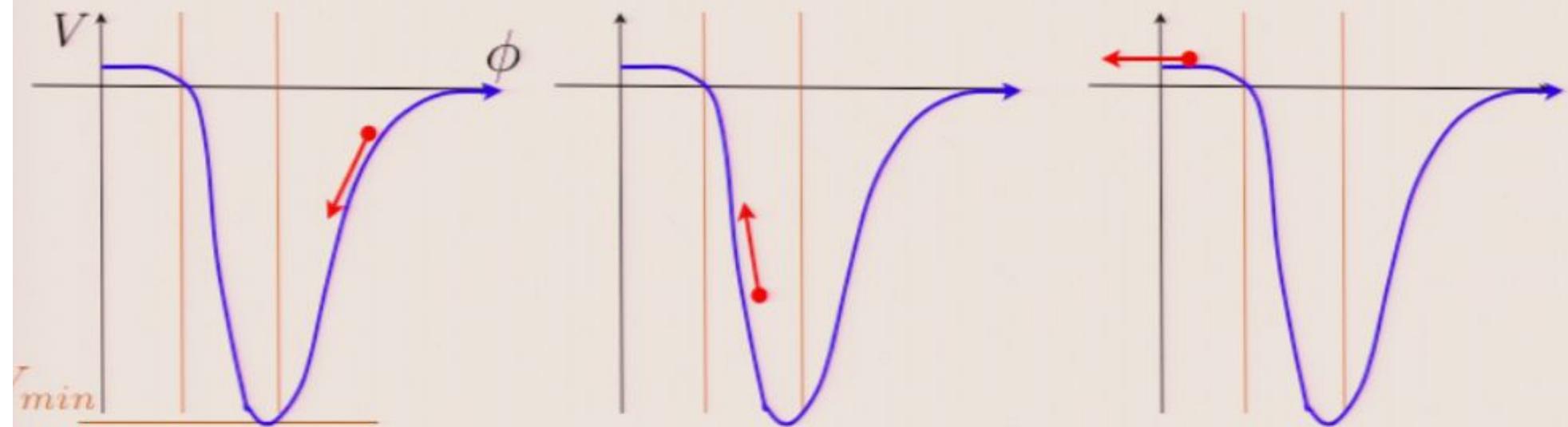
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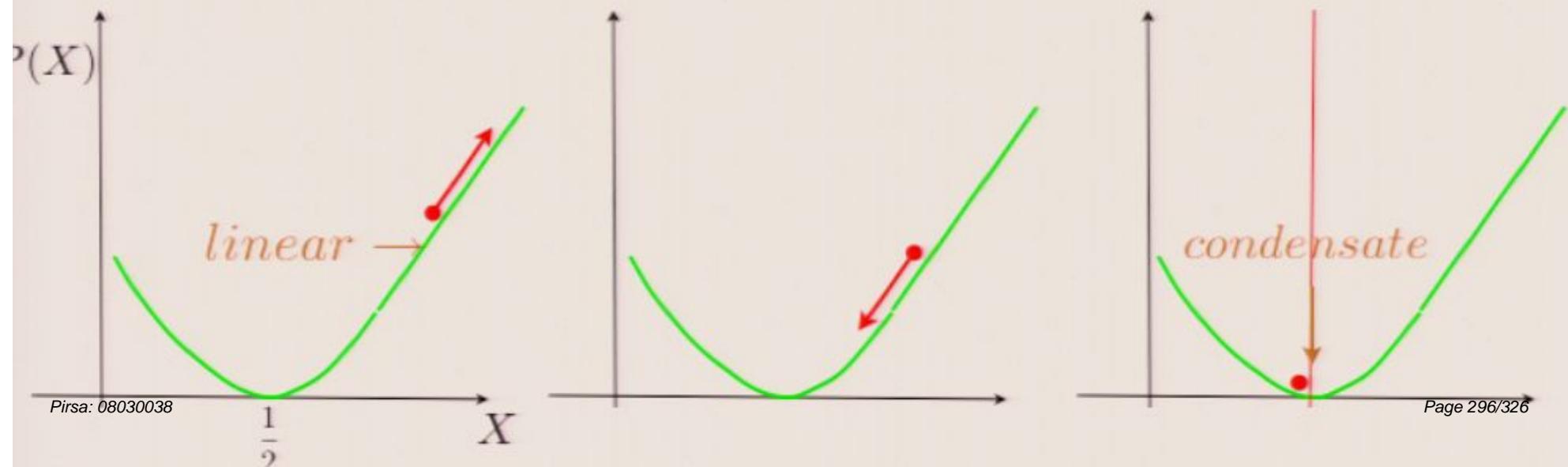
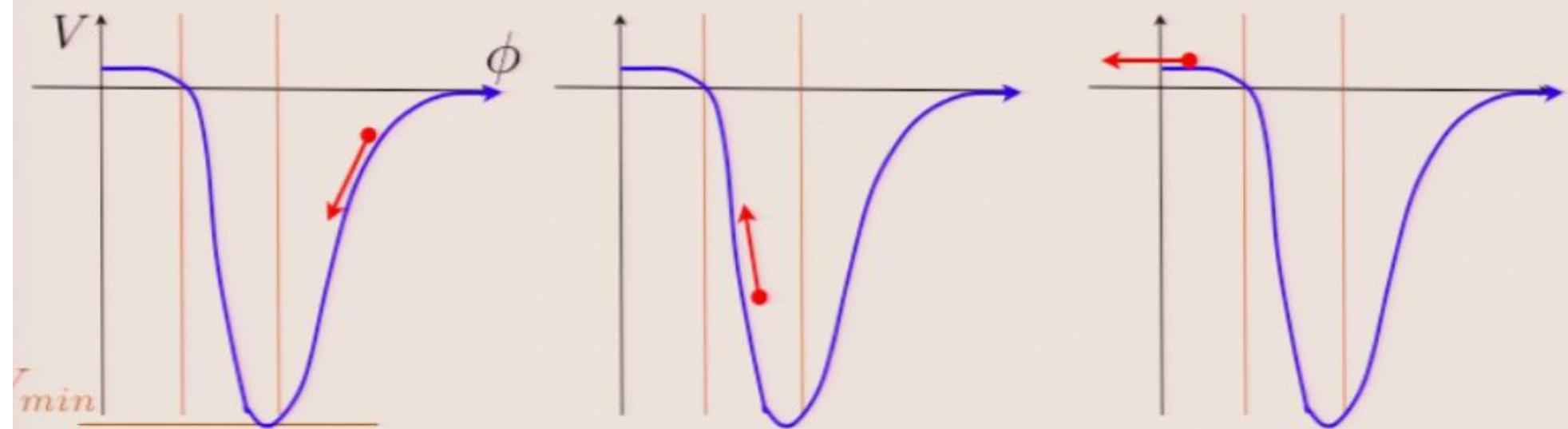
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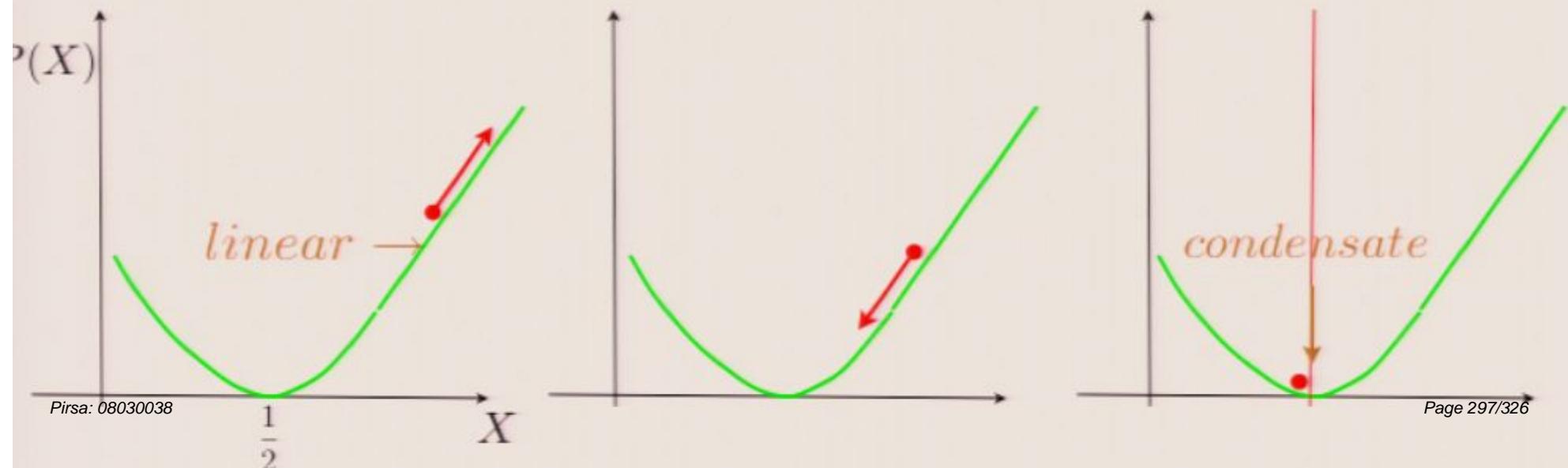
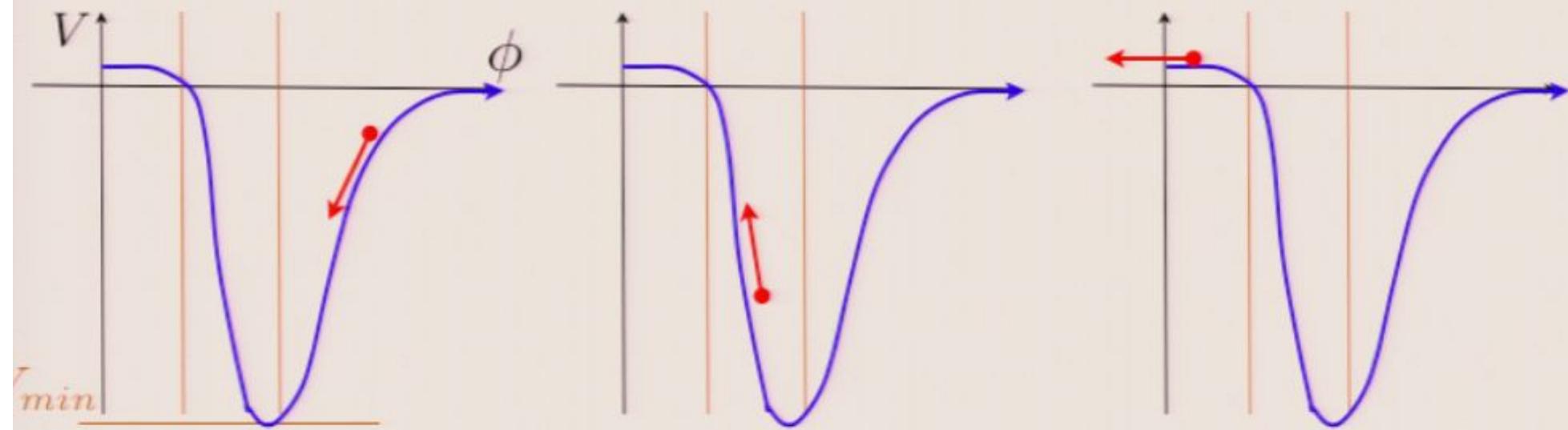
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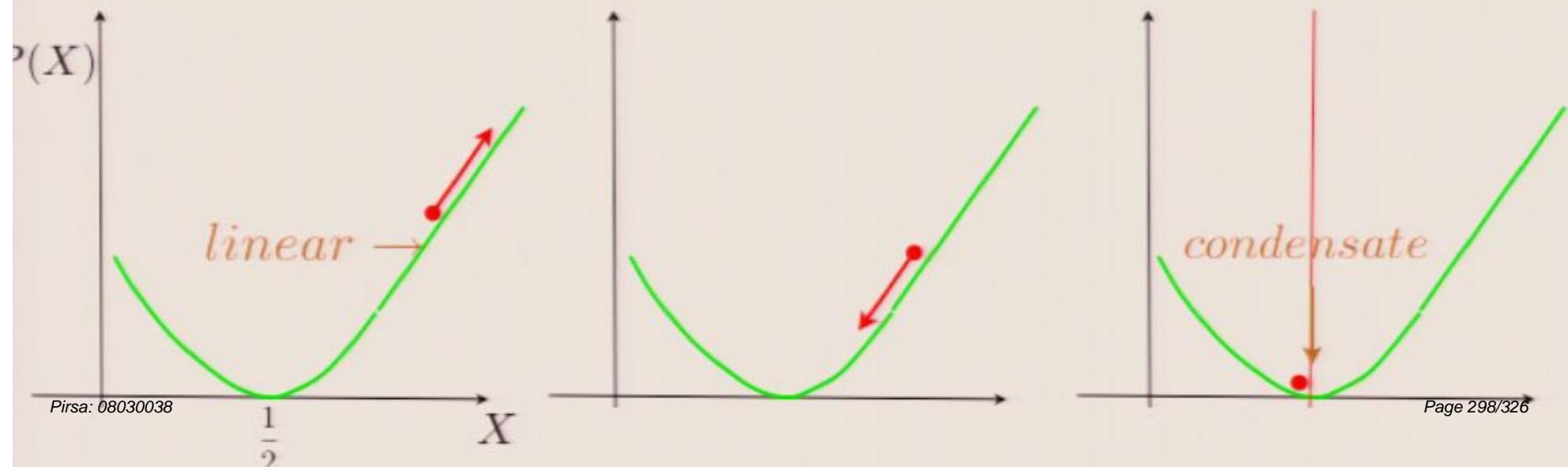
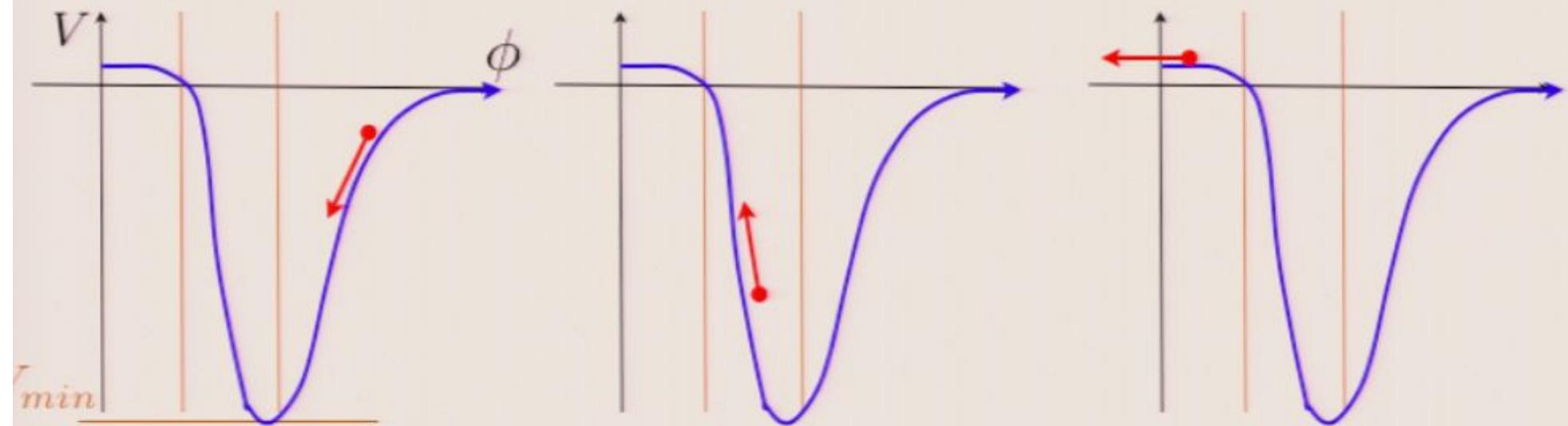
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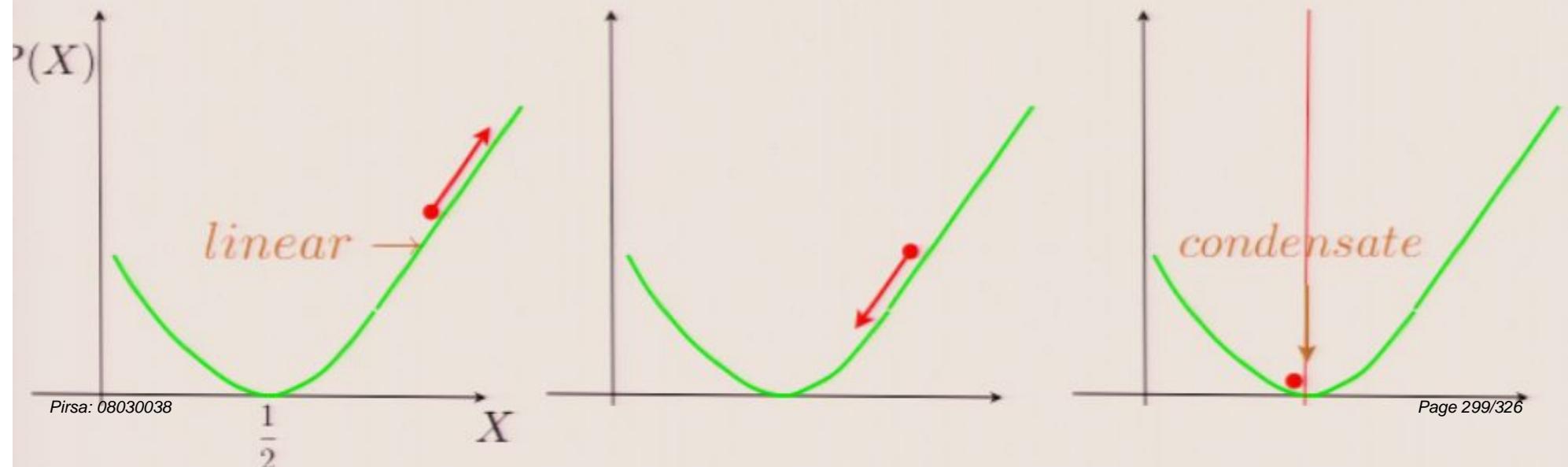
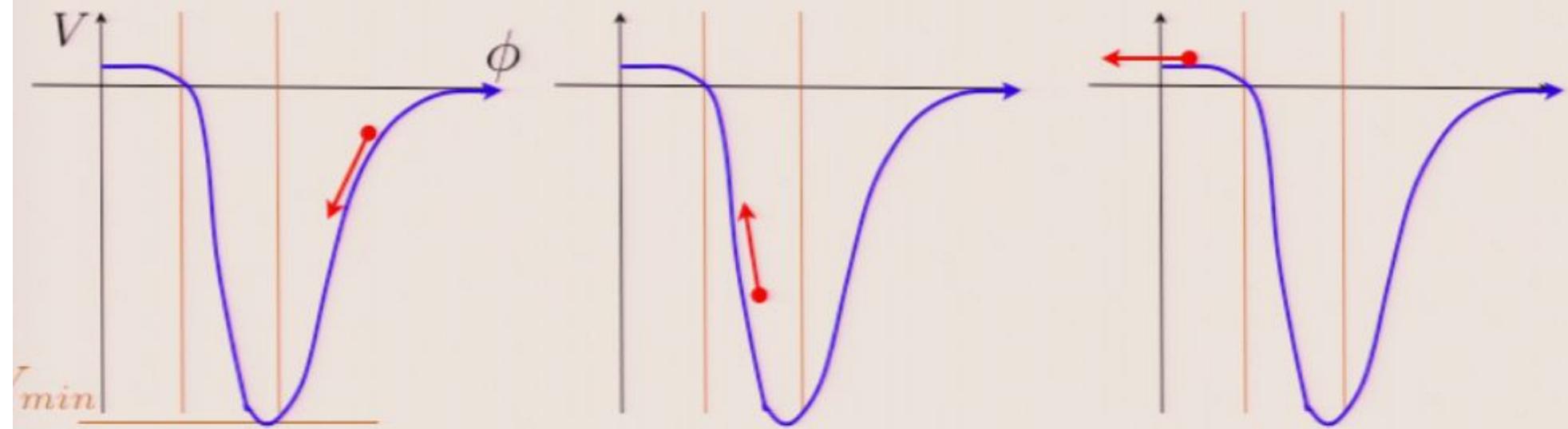
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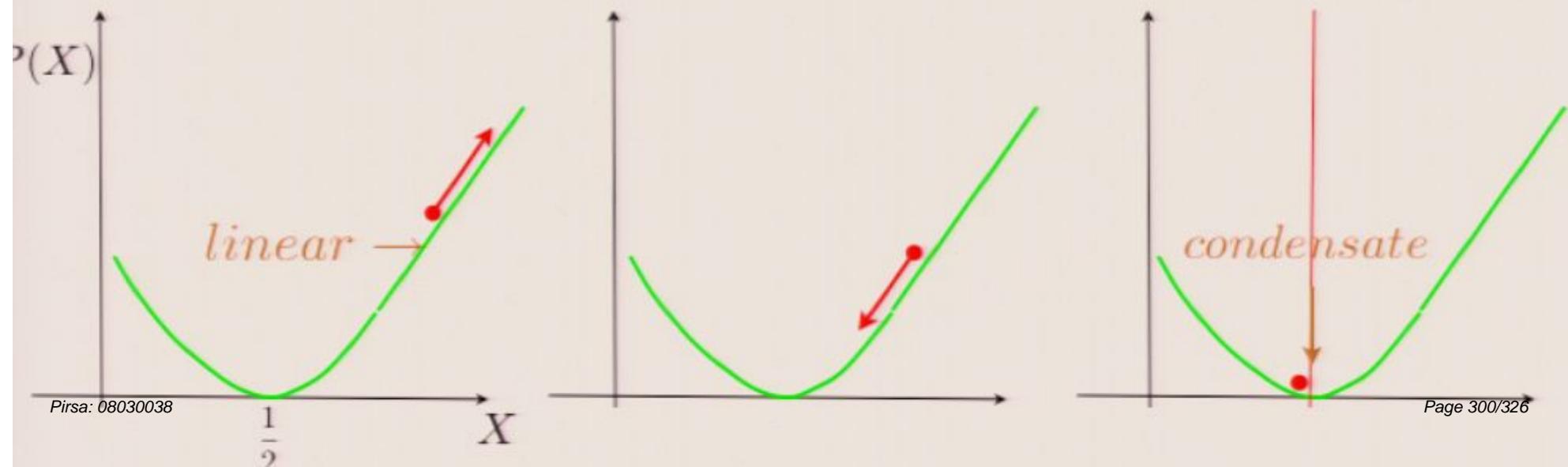
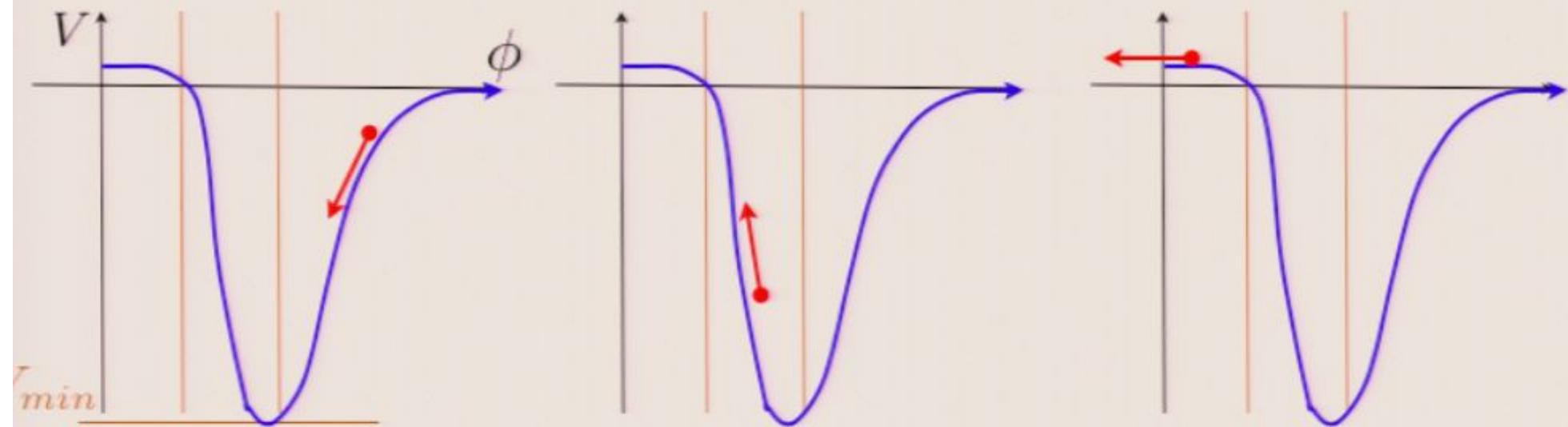
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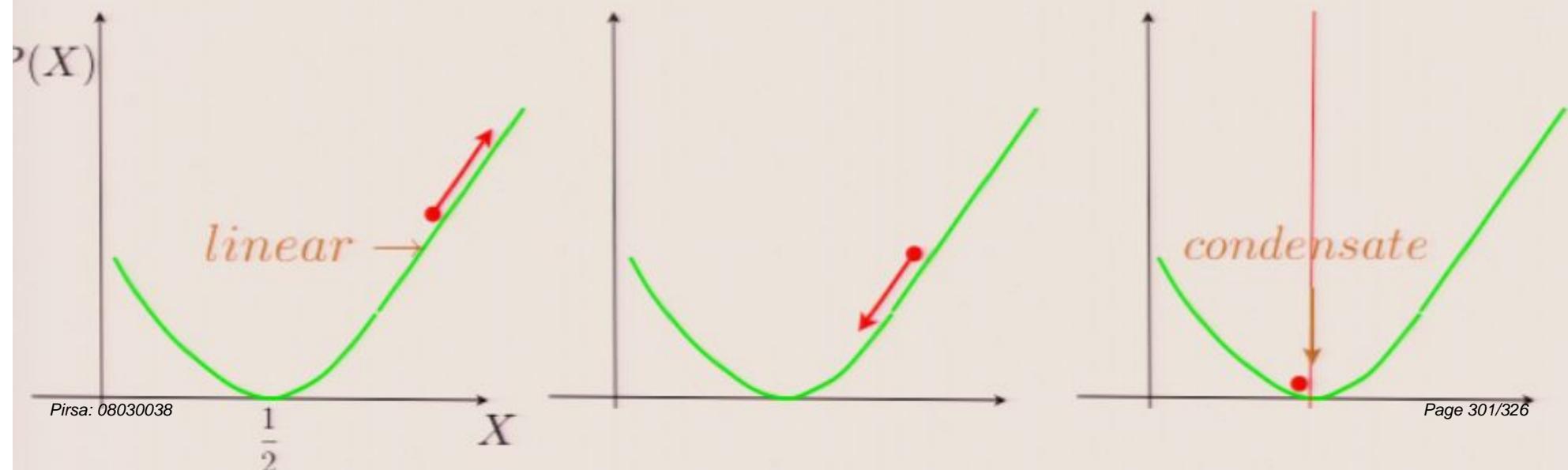
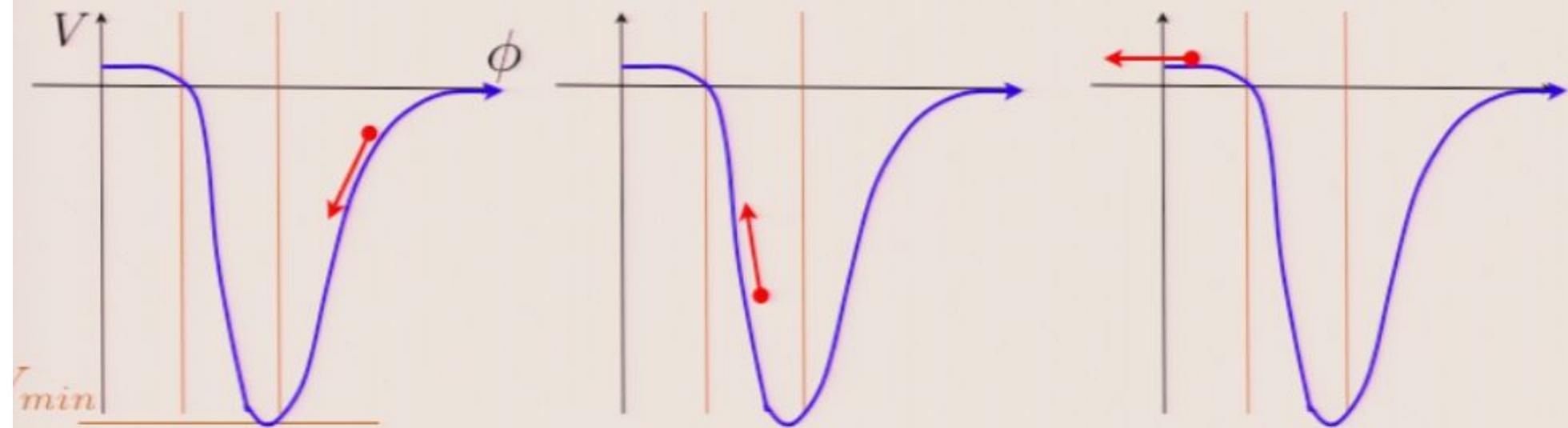
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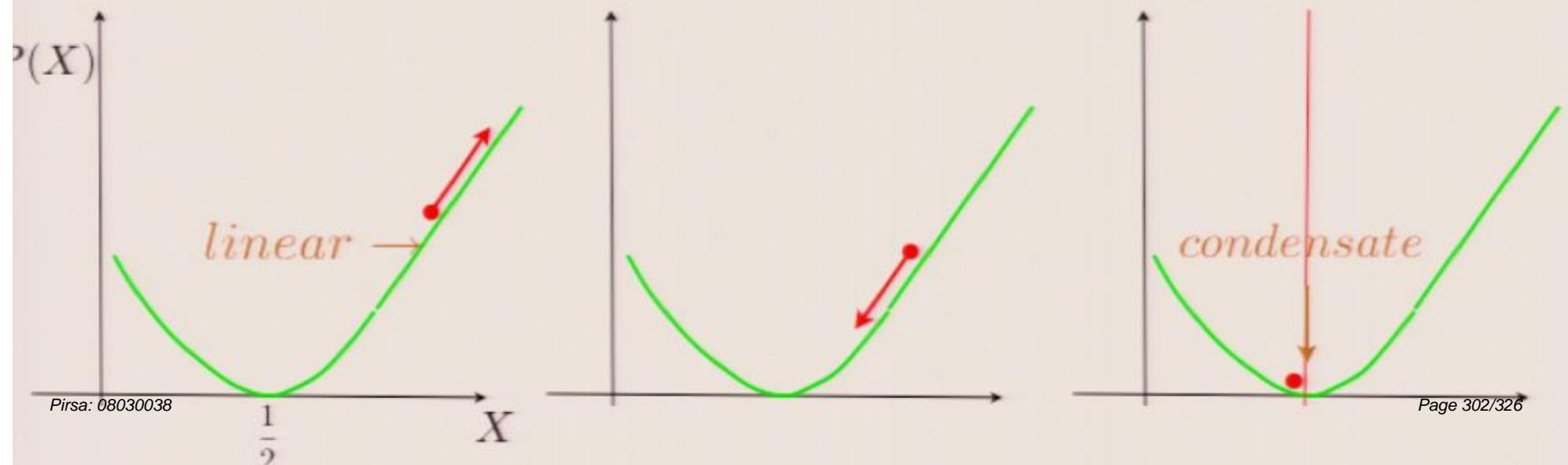
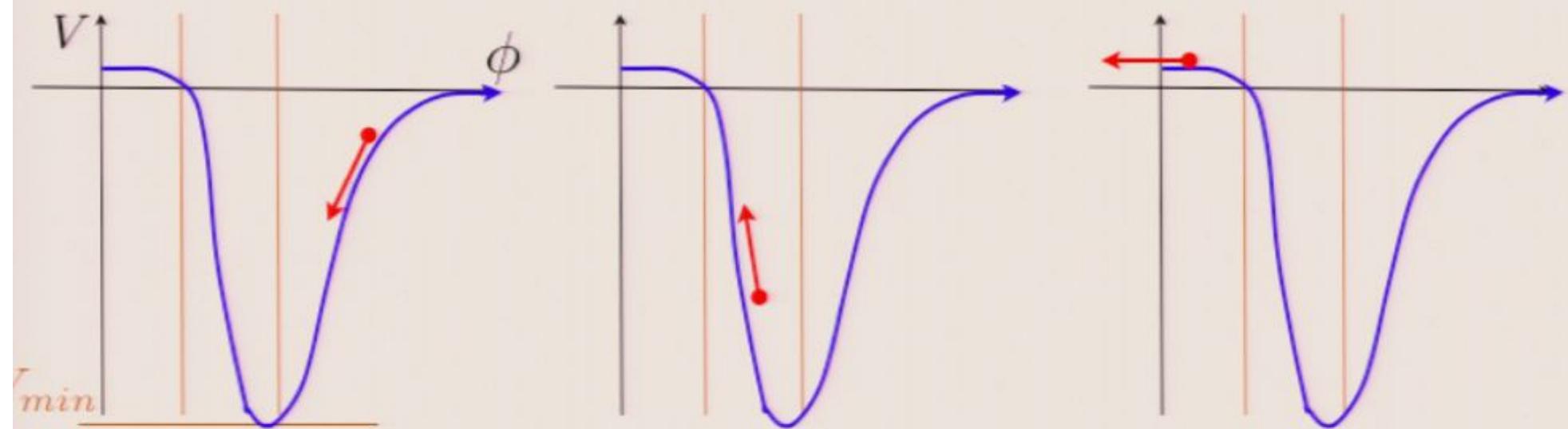
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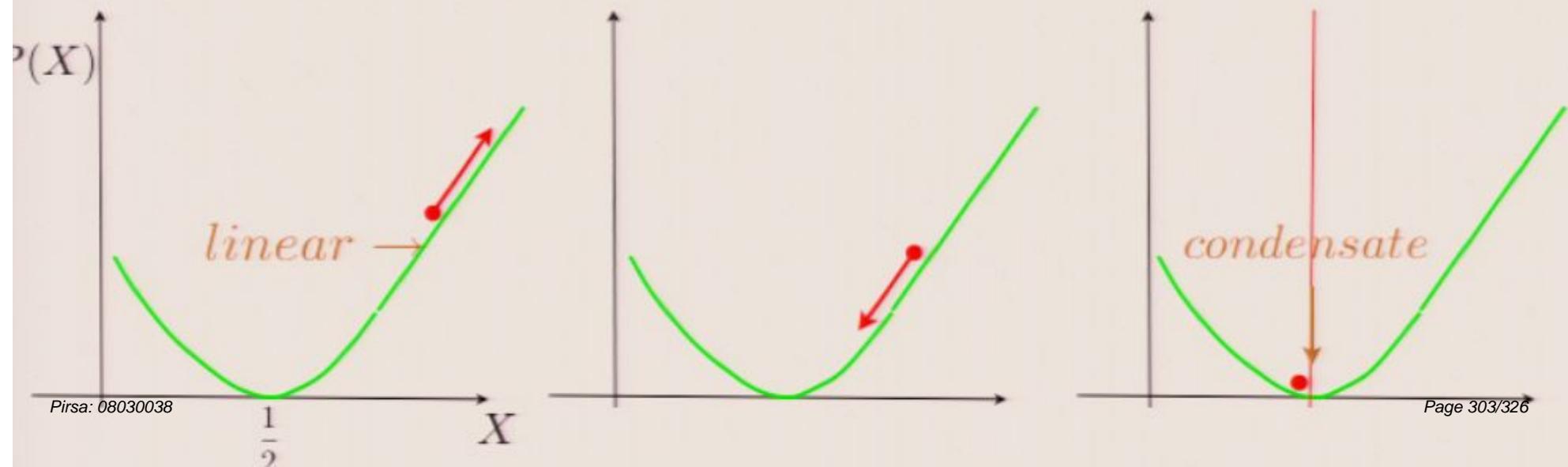
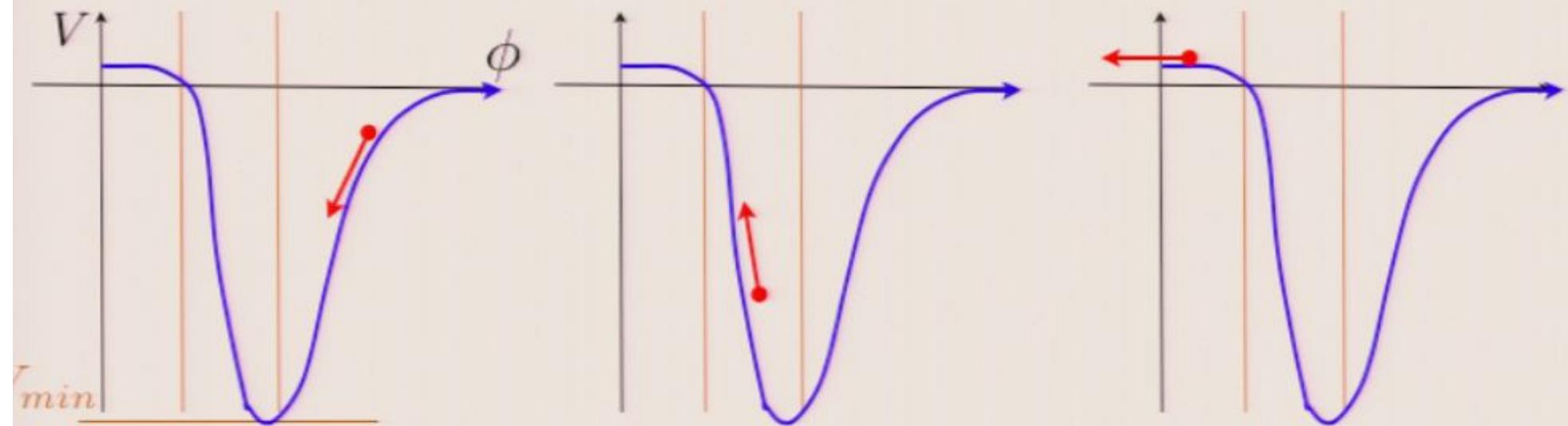
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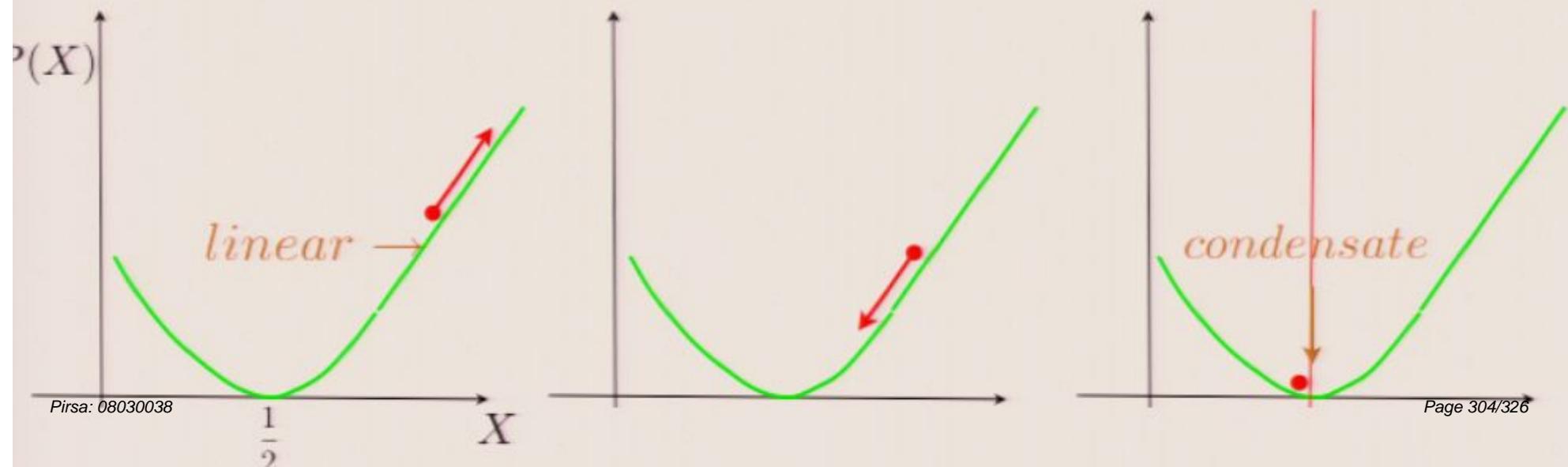
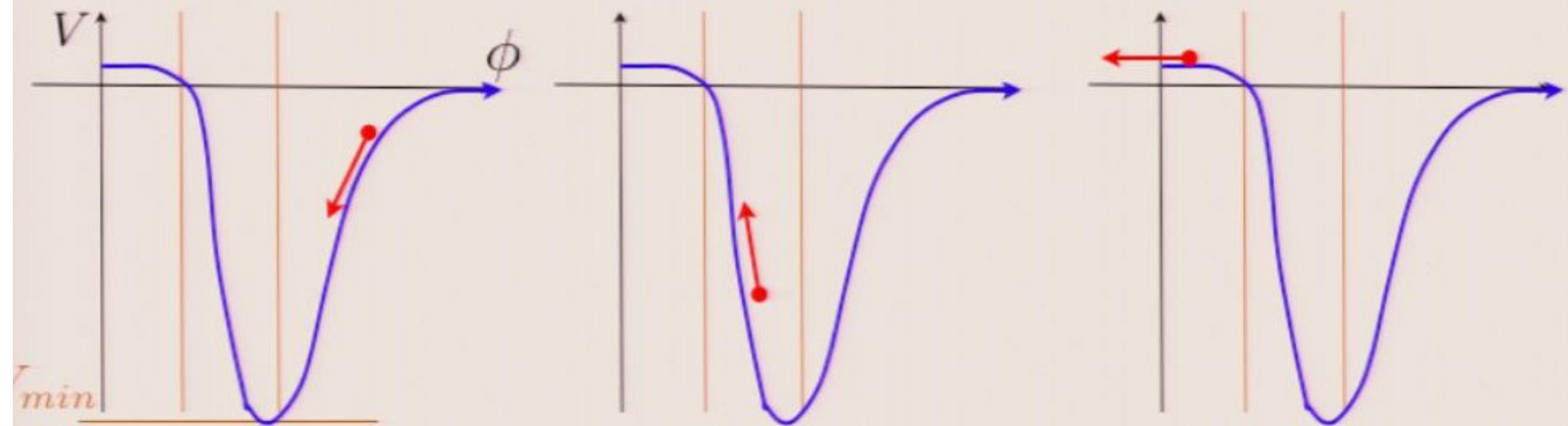
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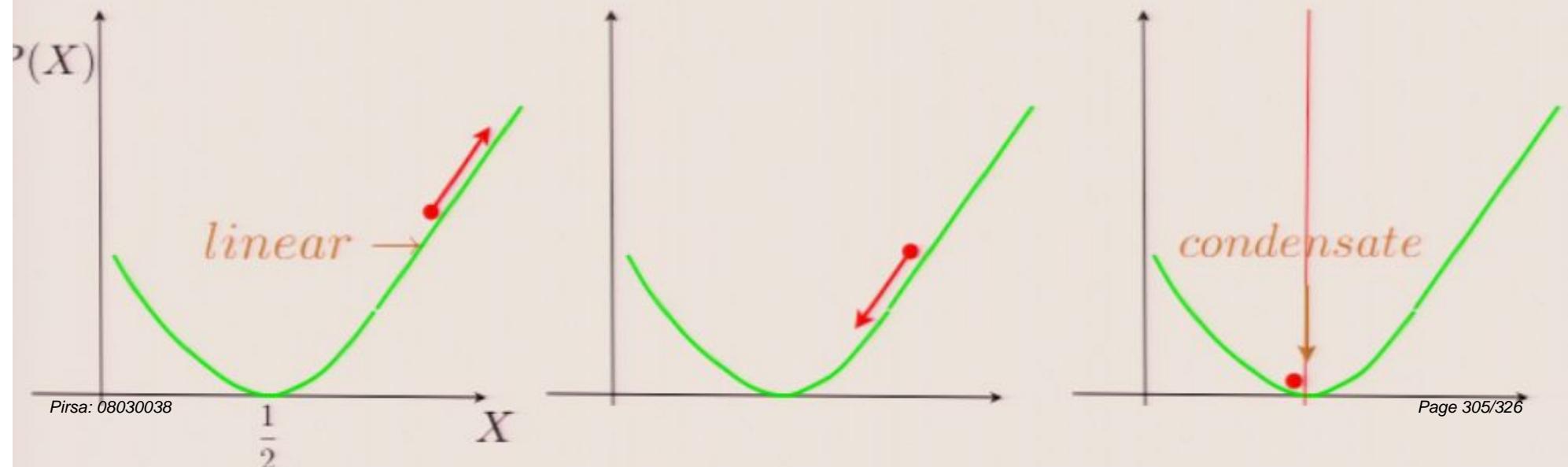
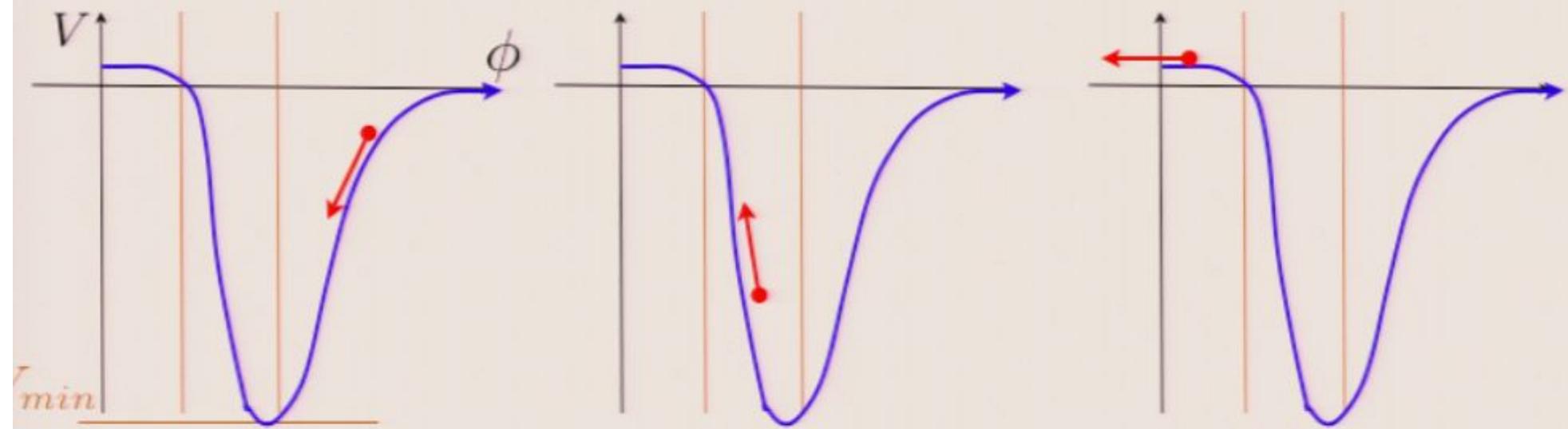
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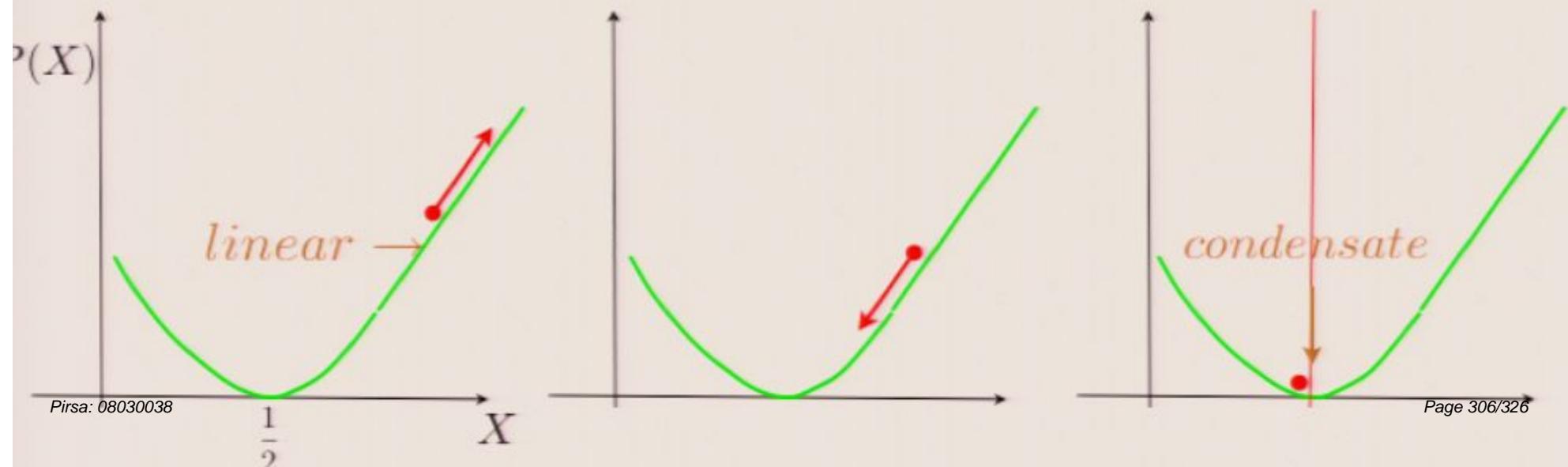
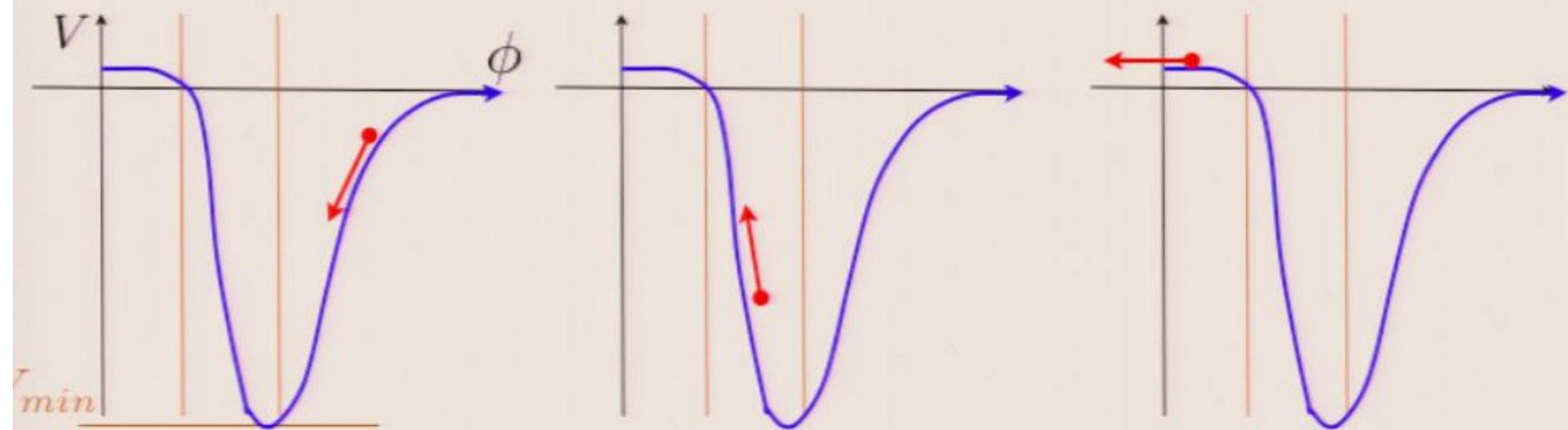
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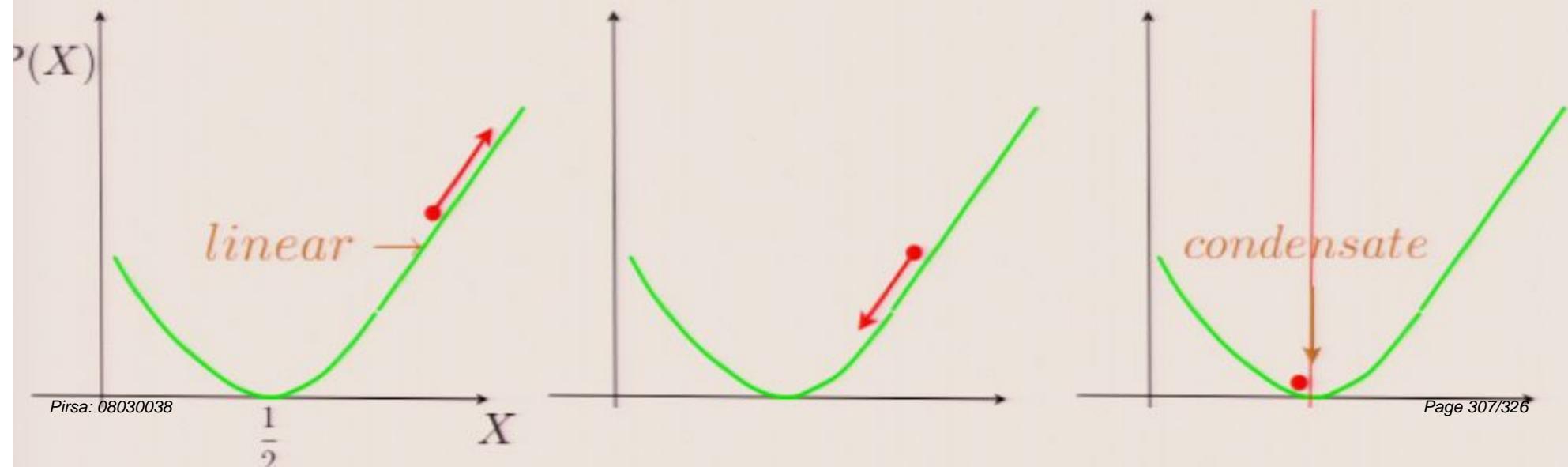
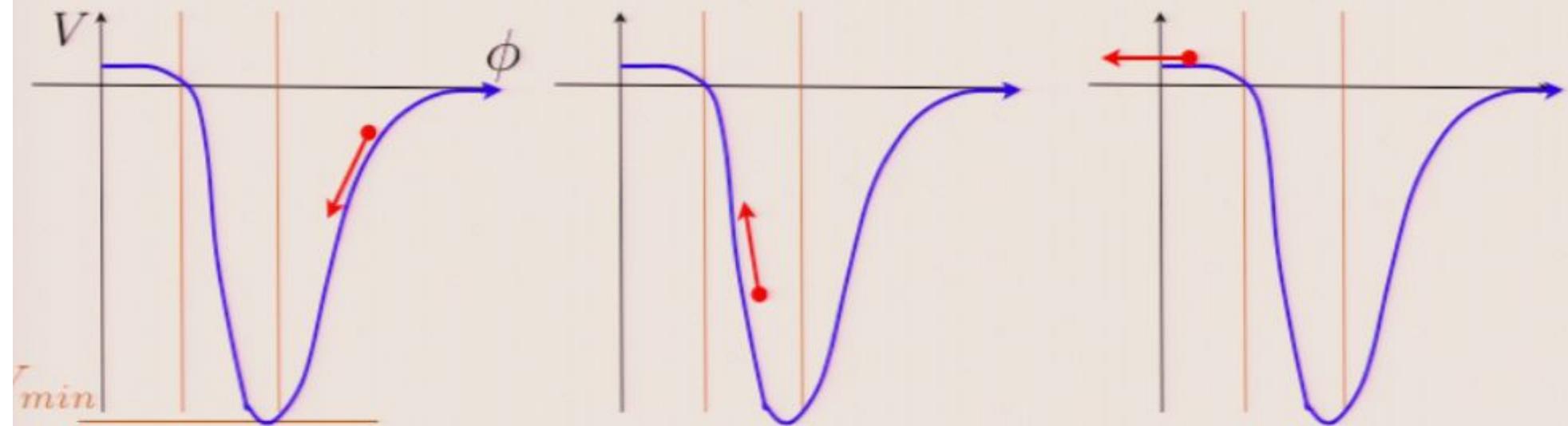
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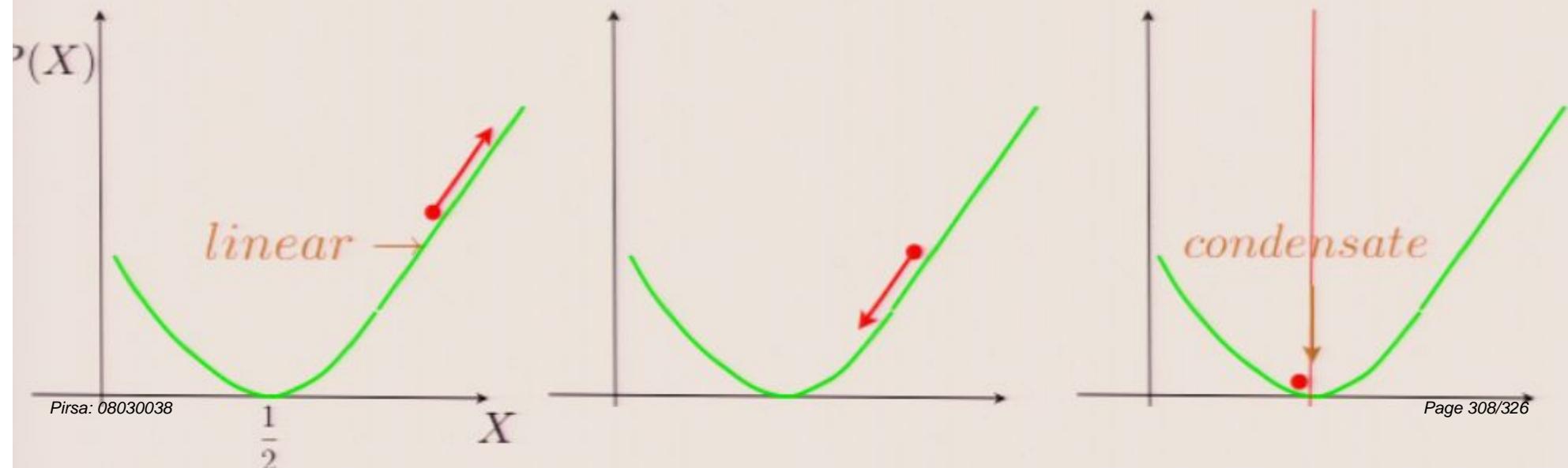
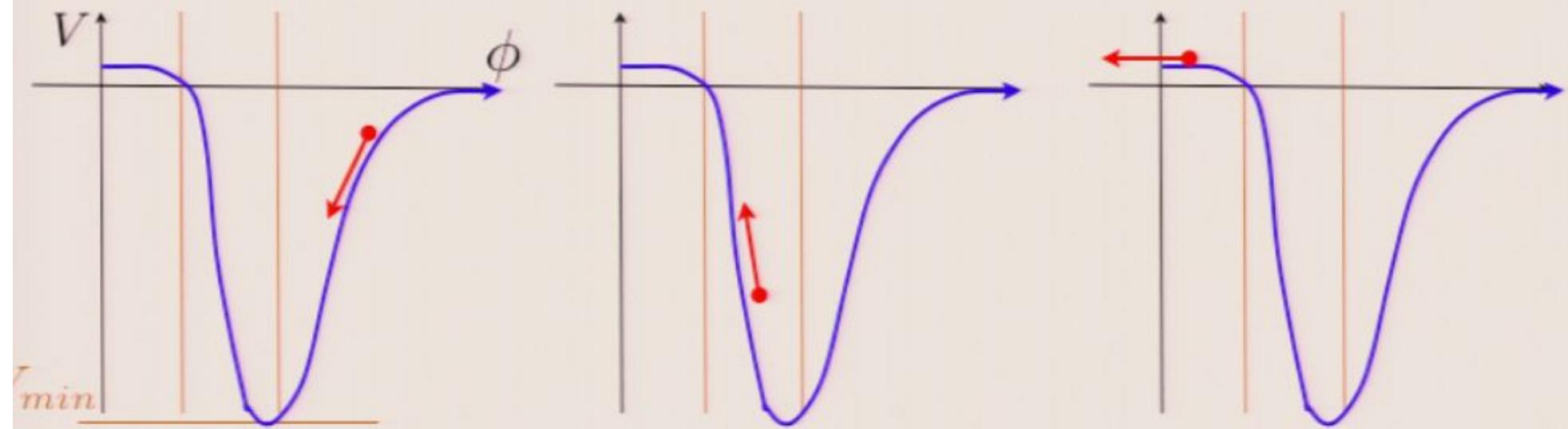
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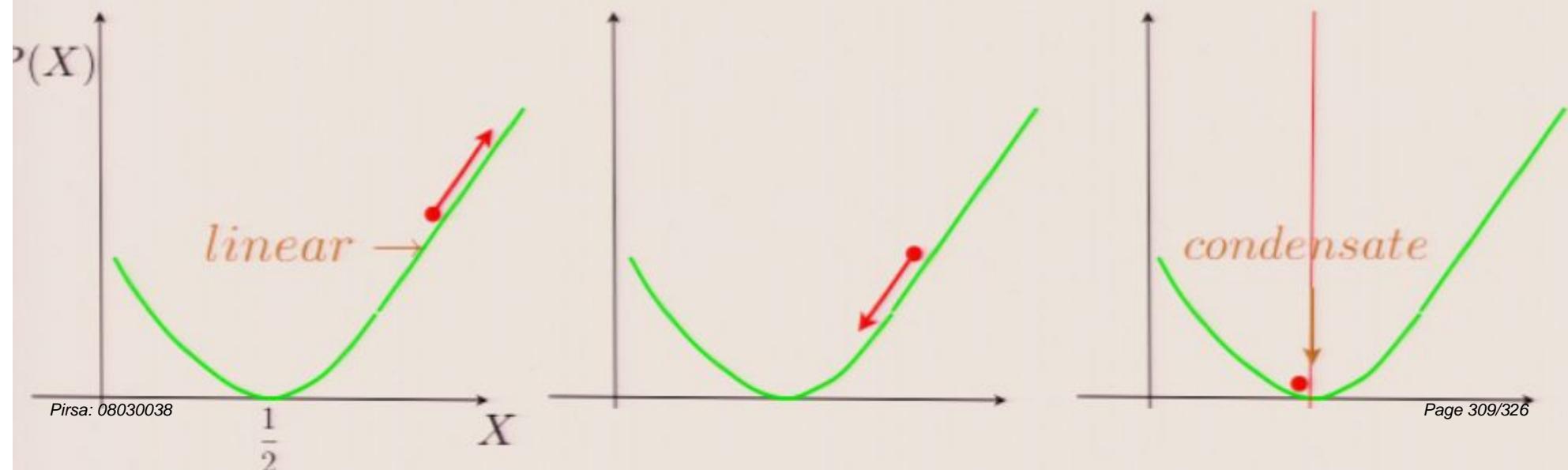
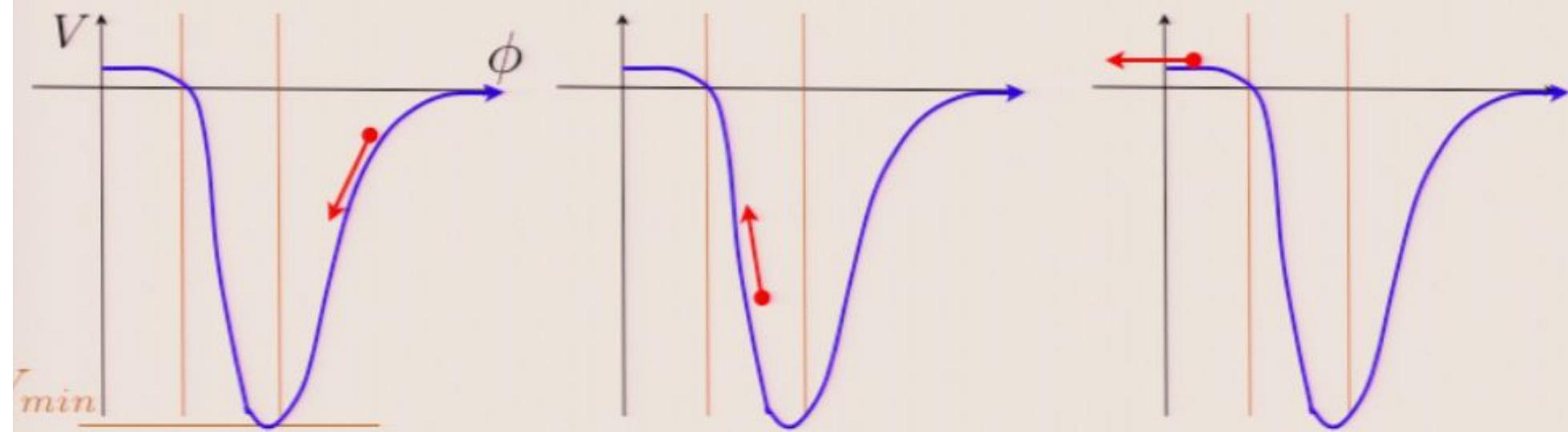
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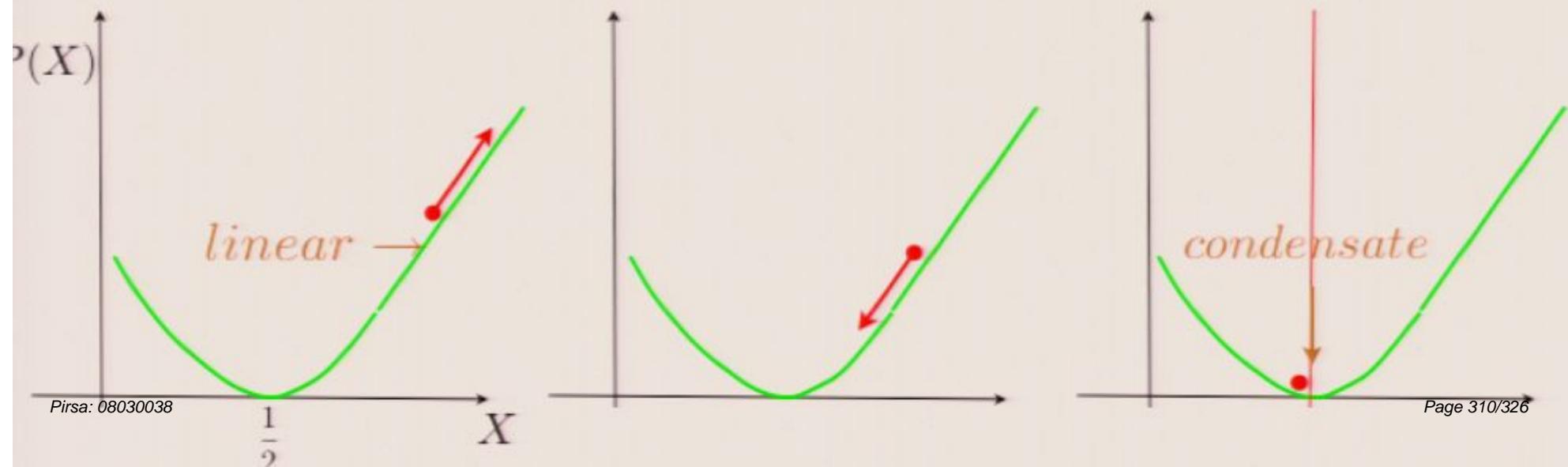
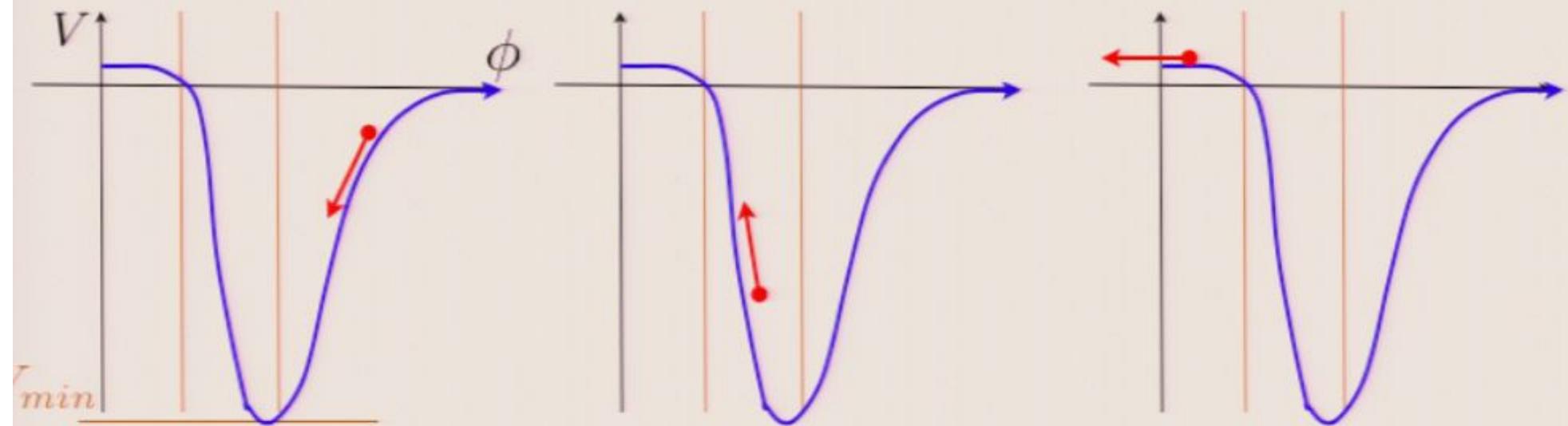
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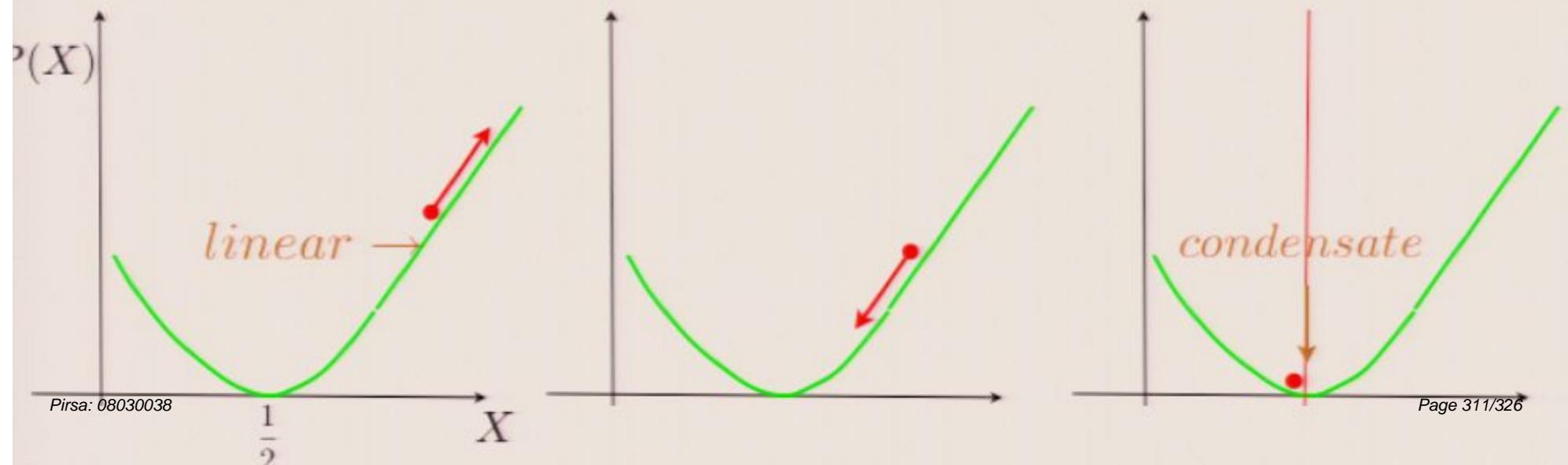
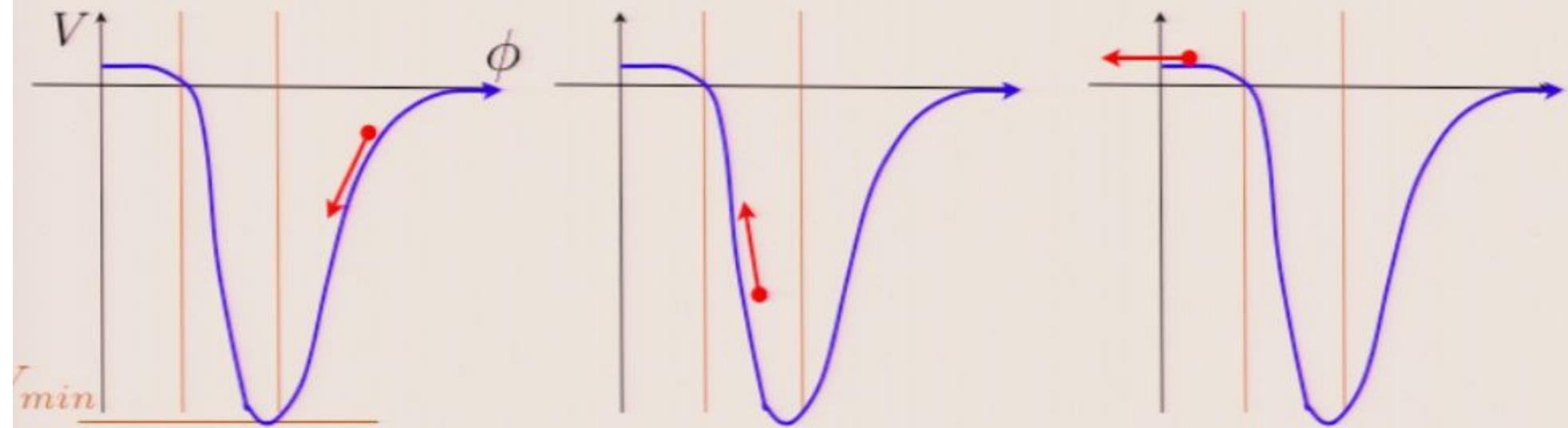
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Merger Condition

$X \gg \frac{1}{2}$ during the Ekpyrotic phase plus $X \approx \frac{1}{2}$

during bounce phase \Rightarrow

$$m^4 e^{2N_{ek}} \ll |V_{min}| \ll \frac{M^4 K}{p}$$

for example, $T_{reheat} = 10^{15} GeV$, $N_{ek} \simeq 60$, $p \simeq 10^{-2} \Rightarrow$

$$m \lesssim 10^3 GeV, M > 10^{16} GeV$$

Ghosts?

Recall

$$\frac{\mathcal{L}}{\sqrt{-g}} = M^4 P(X) - V(\phi), \quad X = -\frac{1}{2m^4} (\partial_\mu \phi)^2$$

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Expand

$$\phi(t, \vec{x}) = -m^2 t + \pi_0(t) + \pi(t, \vec{x})$$

Using $P_{,X}|_{\frac{1}{2}} = 0, P_{,XX}|_{\frac{1}{2}} = K \Rightarrow$

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{M^4 K}{m^4} \left(\frac{\dot{\pi}^2}{2} + \frac{\dot{\pi}_0}{2m^2} (\vec{\nabla} \phi)^2 + \dots \right)$$

Recall

$$M_P^2 \dot{H} = -\frac{1}{2}(\rho + \mathcal{P}) = \frac{KM^4 \dot{\pi}_0}{2m^2} \quad \Rightarrow \quad \frac{\dot{\pi}_0}{2m^2} = \frac{\dot{H} M_P^2}{KM^4}$$

Hence

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{M^4 K}{m^4} \left(\frac{\dot{\pi}^2}{2} + \frac{\dot{H} M_P^2}{KM^4} (\vec{\nabla} \pi)^2 \right)$$

⇒ the dispersion relation

$$\omega^2 = -2 \frac{\dot{H} M_P^2}{K M^4} |\vec{k}|^2$$

in the NEC violating phase

$\dot{H} > 0 \Rightarrow$ “gradient” instability

Add

$$\frac{\mathcal{L}}{\sqrt{-g}} = \dots - \frac{M^4 K}{m^4} \frac{(\Box\phi)^2}{2M'^2} + \text{higher dimension operators}$$

Can ignore the higher dimension operators for

$$k^2 \ll M'$$

⇒ the dispersion relation is modified to

$$\omega^2 = -\frac{2\dot{H} M_P^2}{K M^4} |\vec{k}|^2 + \frac{|\vec{k}|^4}{M'^2}$$

Has instability only at long wavelengths. The rate of instability peaks at

$$\omega_{grad}^2 = - \left(\frac{\dot{H} M_P^2 M'}{K M^4} \right)^2$$

Harmless if

$$|\omega_{grad}| \lesssim |H|$$

⇒ the bound

$$\frac{\dot{H}}{H} \lesssim \frac{K M^4}{M_P^2 M'}$$

which we implement.

Now, even though its the **wrong thing to do**, let us analyze the above dispersion relation for **arbitrary k^2** but **dropping the higher dimension operators**. For simplicity, ignore the term proportional to $\dot{H} |\vec{k}|^2$ and take $a = 1$

\Rightarrow

$$\omega^2 = \frac{(-\omega^2 + |\vec{k}|^2)^2}{M'^2}$$

Four solutions $\omega = \pm\omega_i, i = 1, 2$

$$\omega_1 = \frac{1}{2} \left(\sqrt{M'^2 + 4|\vec{k}|^2} - M' \right), \quad \omega_2 = \frac{1}{2} \left(\sqrt{M'^2 + 4|\vec{k}|^2} + M' \right)$$

For $|\vec{k}|^2 \ll M'^2 \Rightarrow$

$$\omega \approx \pm \frac{|\vec{k}|^2}{M'}, \text{ low frequency}$$

but also

$$\omega \approx \pm M', \text{ high frequency}$$

Claim (Linde et. al.):

High frequency solution corresponds to “ghosts” with negative energy

$$\omega_{ghost} \approx -M'$$

Answer: Note that $k = (\omega, \vec{k}) \Rightarrow$

$$k_{ghost}^2 \approx M'^2$$

But effective theory is only valid for $k^2 \ll M'^2$. As $k^2 \rightarrow M'^2$ must include all higher dimension operators!

Explicit example:

Consider

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu \phi')^2 - \frac{1}{2}(\partial_\mu \chi')^2 - \frac{\Lambda^2}{2}\chi'^2$$

Both fields have canonical KE with zero and positive (*mass*)² respectively. Let

$$\phi' = \phi + \chi', \quad \chi' = \frac{1}{\sqrt{2}}\chi$$

\Rightarrow

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu \phi)^2 - (\partial_\mu \phi)(\partial^\mu \chi) - \frac{1}{2}(\partial_\mu \chi)^2 - \frac{1}{2}\left(\frac{\Lambda^2}{2}\right)\chi^2$$

‘Integrating out’ the heavy field $\chi \Rightarrow$

$$\begin{aligned}\mathcal{L} &= -\frac{1}{2}(\partial_\mu \phi)^2 + \frac{1}{2\Lambda^2}(\square \phi) \frac{1}{(1 - 2\frac{\square}{\Lambda^2})}(\square \phi) \\ &= -\frac{1}{2}(\partial_\mu \phi)^2 + \frac{1}{2\Lambda^2}(\square \phi)^2 + \frac{1}{\Lambda^4}(\square \phi)(\square^2 \phi) + \dots\end{aligned}$$

Truncate the Lagrangian to

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu \phi)^2 + \frac{1}{2\Lambda^2}(\square \phi)^2 \quad (+0)$$

This is equivalent to

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu \phi)^2 - (\partial_\mu \psi)(\partial^\mu \phi) - \frac{1}{2}\Lambda^2 \psi^2$$

where ψ is an auxiliary field.

Now let

$$\phi = \rho - \psi$$

\Rightarrow

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu \rho)^2 + \frac{1}{2}(\partial_\mu \chi)^2 - \frac{1}{2}\Lambda^2 \psi^2$$

Ghost with **negative** energy

$$\omega_{ghost} = -\Lambda$$

But, its **not** really there. An **artifact** of truncating the
higher dimension operators!

Non-Gaussianity

The **3-point function** in New Ekpyrotic cosmology is

$$\langle \zeta_{k_1}, \zeta_{k_2}, \zeta_{k_3} \rangle = (2\pi)^3 \delta^3(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) B(k_1, k_2, k_3)$$

where

$$B(k_1, k_2, k_3) = \frac{6}{5} f_{NL} \{ P_\zeta(k_1) P_\zeta(k_2) + \text{perm} \}$$

is of the “local” form and

$$k^3 P_\zeta(k) = \beta^2 \frac{H_{end}^2}{2\epsilon M_P^2}$$

⇒ the 3-point function fully specified by f_{NL} .

Two contributions:

$$\mathcal{L} = \dots - \mathcal{V} \frac{\alpha_3}{3! \epsilon^{\frac{3}{2}} M_P^3} \chi^3 \Rightarrow f_{NL}^{int} = \mp \frac{5}{18} \frac{\alpha_3}{\beta \epsilon}$$

b) Non-linear relation between $\zeta, \delta\chi \Rightarrow$

$$f_{NL}^{conv} = \frac{5}{24} \frac{1}{\beta^2 \epsilon}$$

Adding these gives

$$f_{NL} = \frac{5}{24\beta^2\epsilon} \left(1 \mp \frac{4}{3}\alpha_3\beta \right)$$

Conclusions:

I) $f_{NL} \sim \frac{1}{\epsilon}, \epsilon \ll 1 \Rightarrow$ large non-Gaussianity !

Contrast to slow-roll inflation where $f_{NL} \sim \epsilon \Rightarrow$ very small non-Gaussianity.

II) Choosing $\beta = 1, \epsilon = 10^{-2}, (-)sign, 2.046 > \alpha_3 > -2.85 \Rightarrow$

$$-36 < f_{NL} < 100 \quad \text{“old” WMAP bound}$$