

Title: Binary black hole merger: symmetry and the spin expansion

Date: Mar 04, 2008 02:00 PM

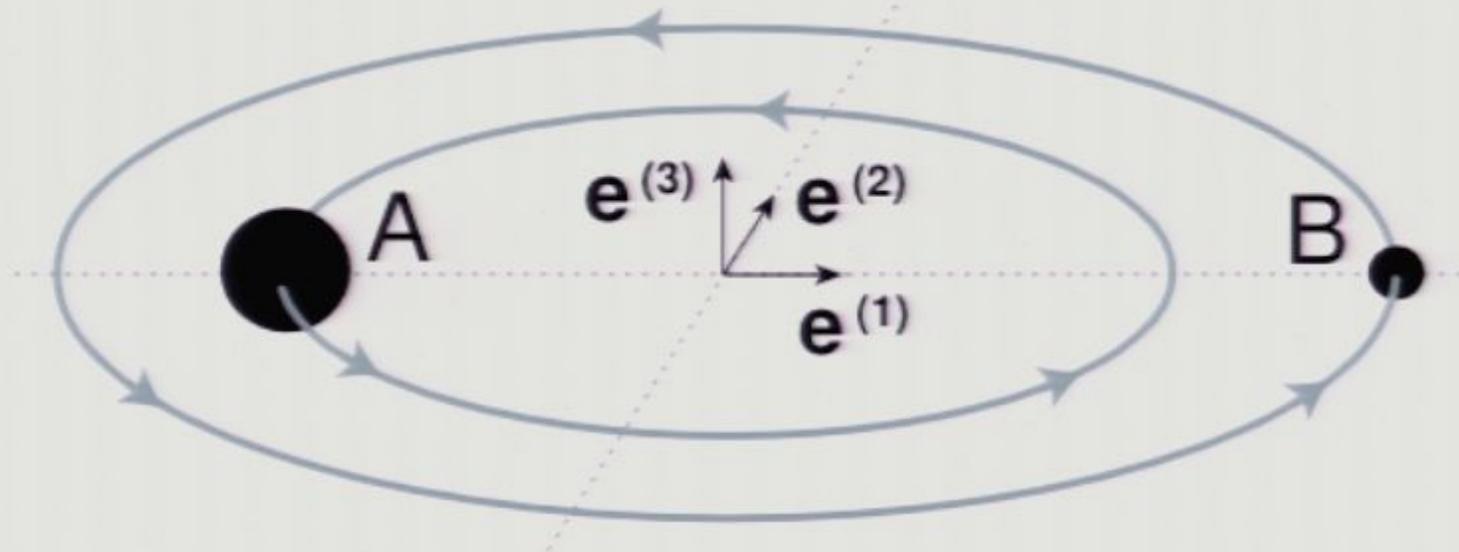
URL: <http://pirsa.org/08030032>

Abstract: Two spinning black holes emit gravitational waves as they orbit, and eventually merge to form a single black hole. How do the properties of the final black hole depend on those of the initial black holes? This is a classic problem in general relativity, with implications for astrophysics, cosmology, and gravitational wave detection. I will describe the rapid numerical and theoretical progress over the past two years, and discuss some open questions and future directions.

Latham Boyle

Binary Black Hole Merger:  
Symmetry and the Spin Expansion

0709.0299 (LB, M. Kesden, S. Nissanke),  
0712.2819 (LB, M. Kesden)



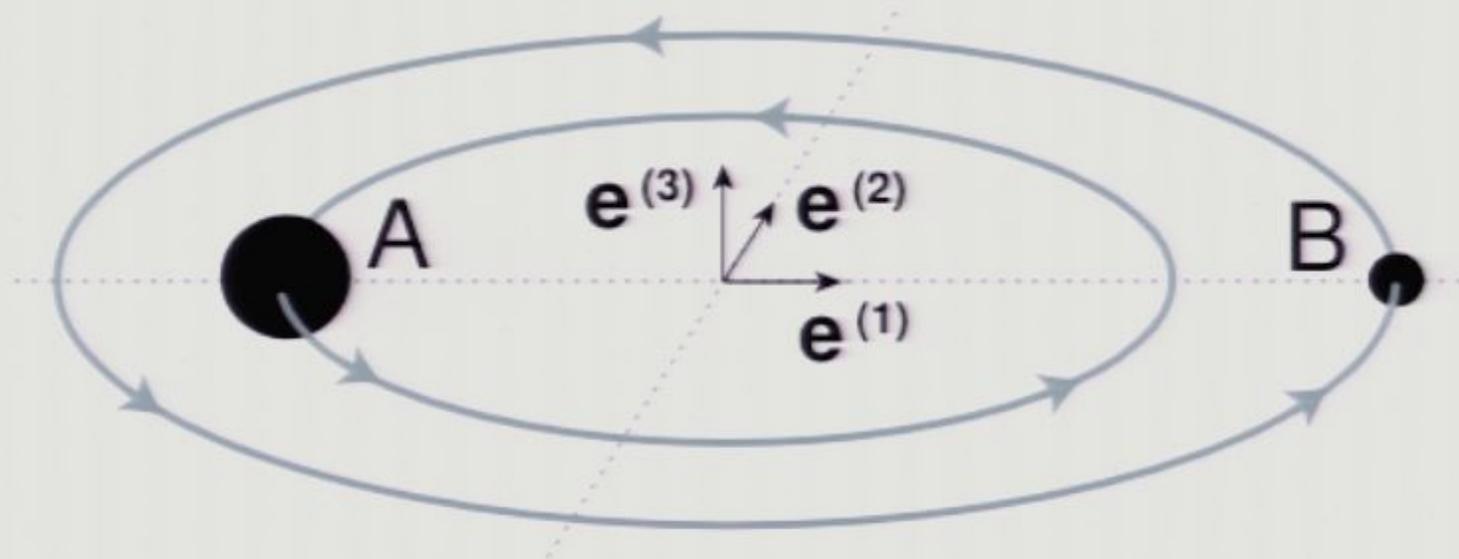
## Kerr metric (1963)

$$g_{\mu\nu} = \eta_{\mu\nu} + 2hk_{\mu}k_{\nu}$$

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# Gravitational Radiation from Colliding Black Holes

S. W. Hawking

*Institute of Theoretical Astronomy, University of Cambridge, Cambridge, England*

(Received 11 March 1971)

It is shown that there is an upper bound to the energy of the gravitational radiation emitted when one collapsed object captures another. In the case of two objects with equal masses  $m$  and zero intrinsic angular momenta, this upper bound is  $(2-\sqrt{2})m$ .

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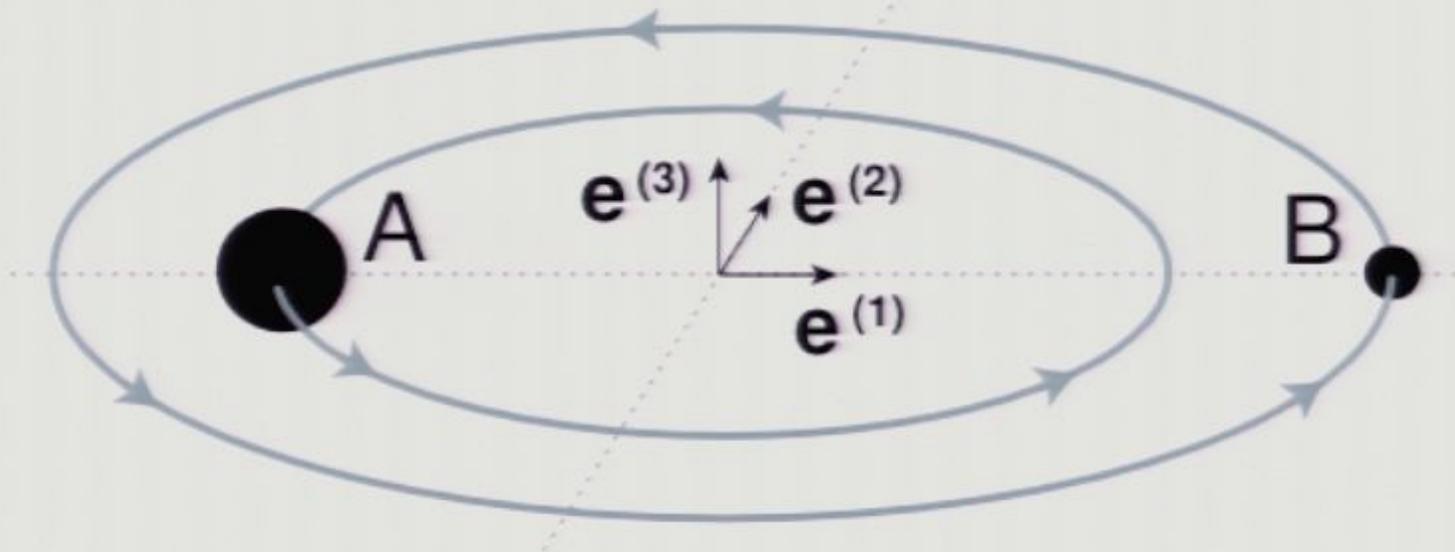
$$A(M_f, S_f) \geq A(M_1, S_1) + A(M_2, S_2)$$

$$k_1 = k_1(q, a_1, a_2, a_3, b_1, b_2, b_3)$$

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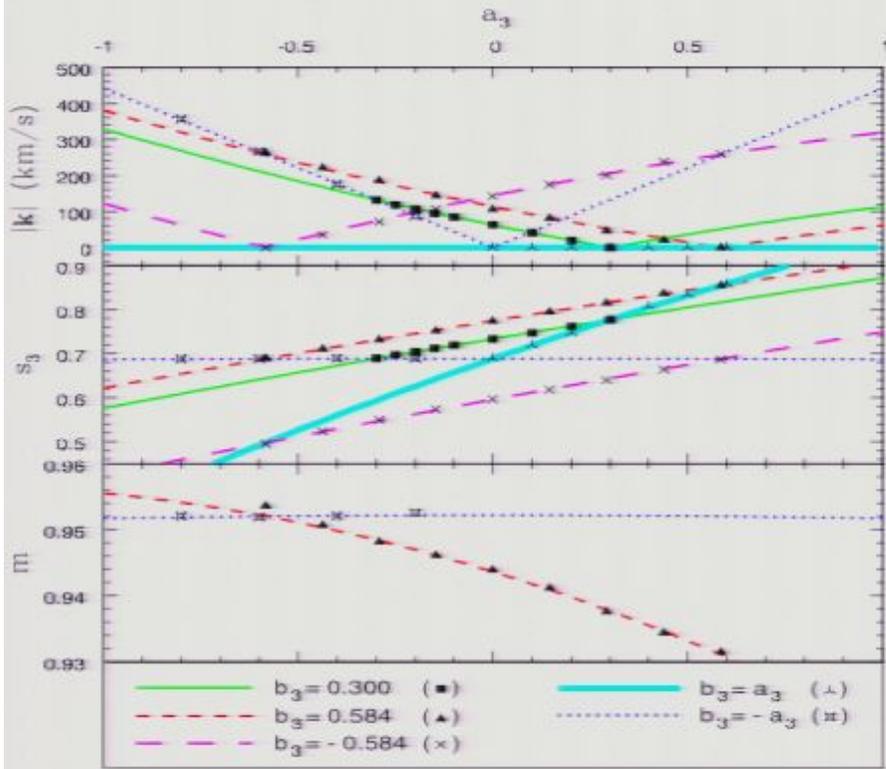
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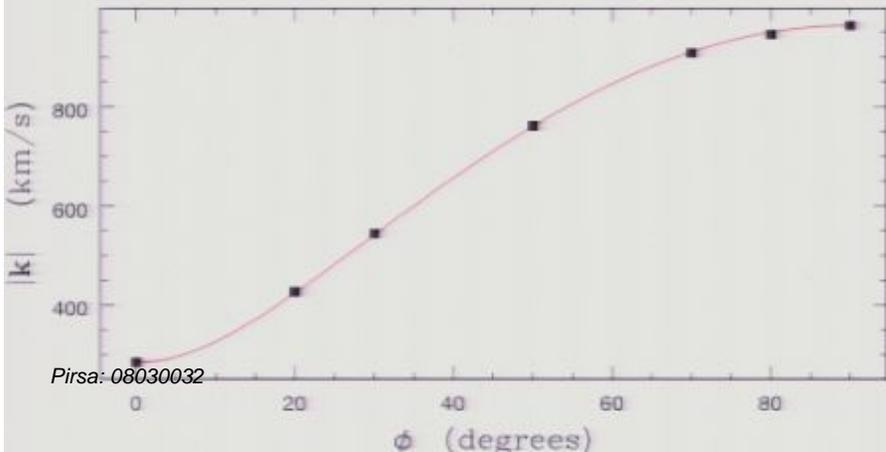
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$$\begin{aligned} k_1 &= k_1^{001|000} (a_3 - b_3) \\ &+ k_1^{200|000} (a_1^2 - b_1^2) + k_1^{020|000} (a_2^2 - b_2^2) + k_1^{002|000} (a_3^2 - b_3^2) \\ &+ k_1^{110|000} (a_1 a_2 - b_1 b_2) + k_1^{100|010} (a_1 b_2 - b_1 a_2) \end{aligned}$$



L. Rezzolla et al, arXiv:0708.3999

F. Herrmann et al, arXiv:0706.2541



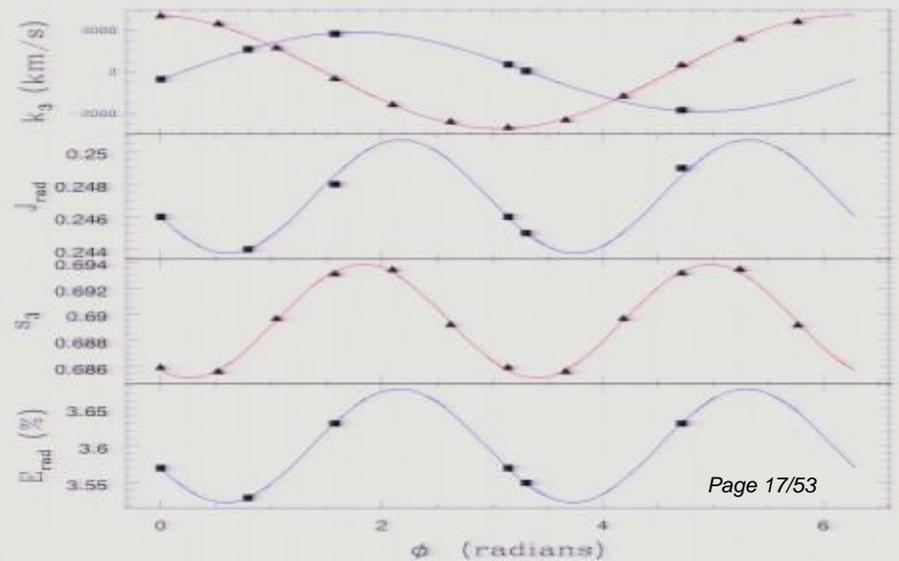
Pirsa: 08030032



W. Tichy et al, gr-qc/0703075

M. Campanelli et al, PRL 98, 231102 (2007)

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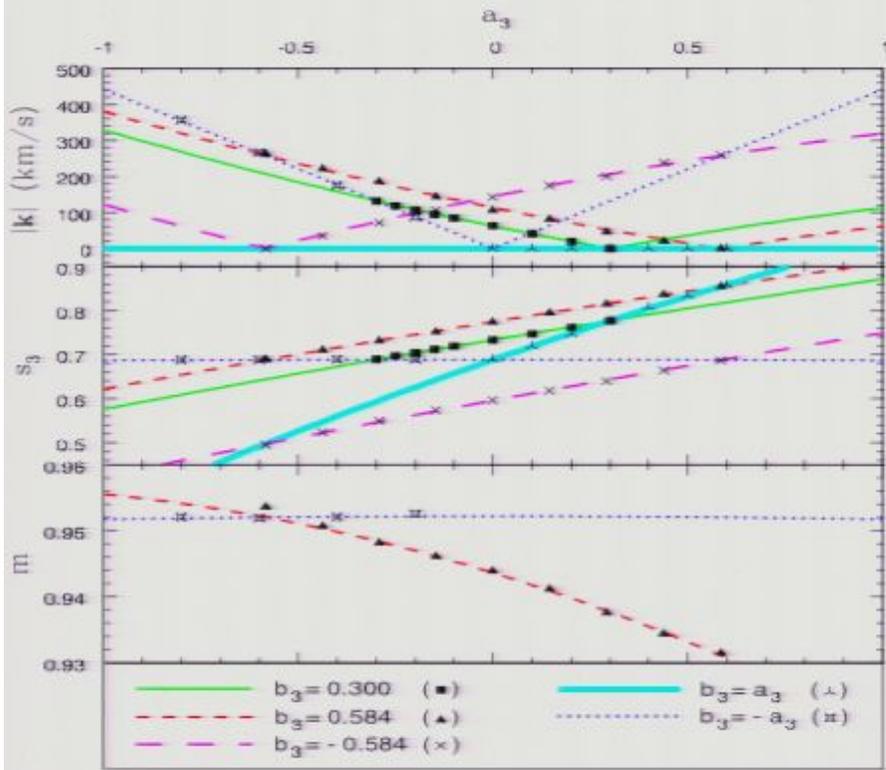


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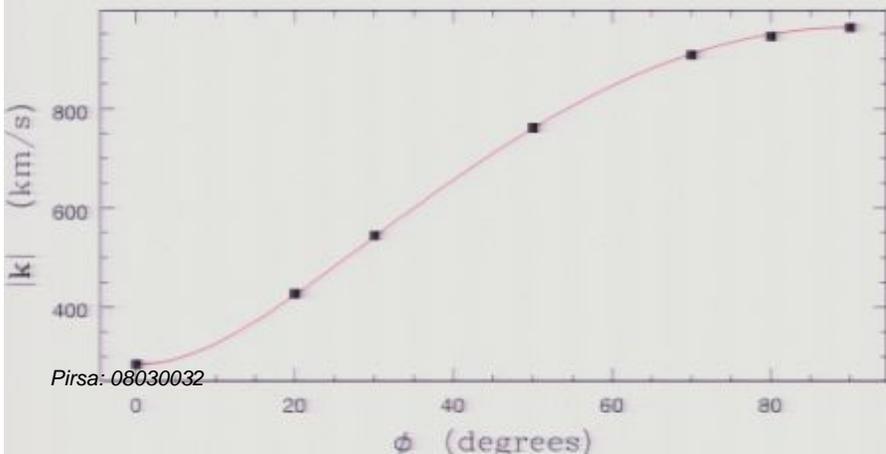
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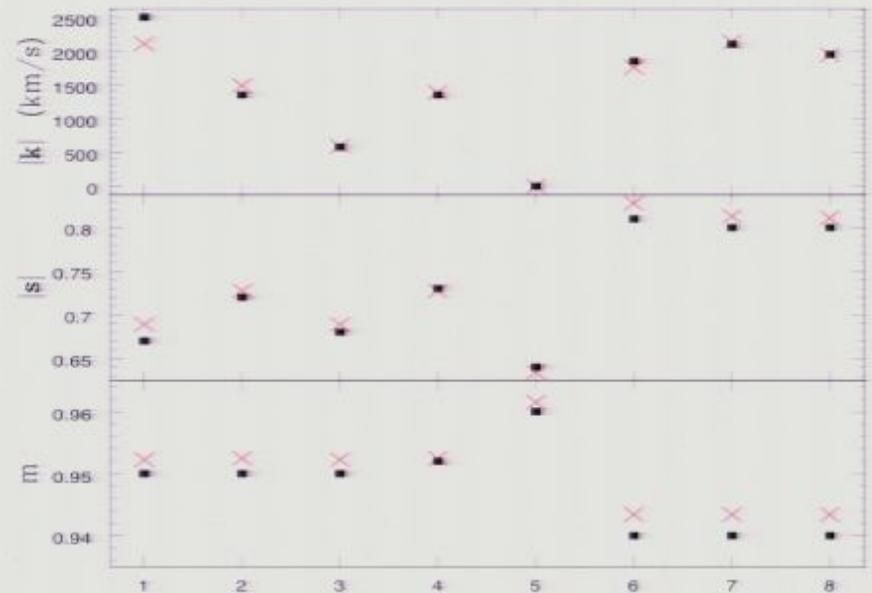


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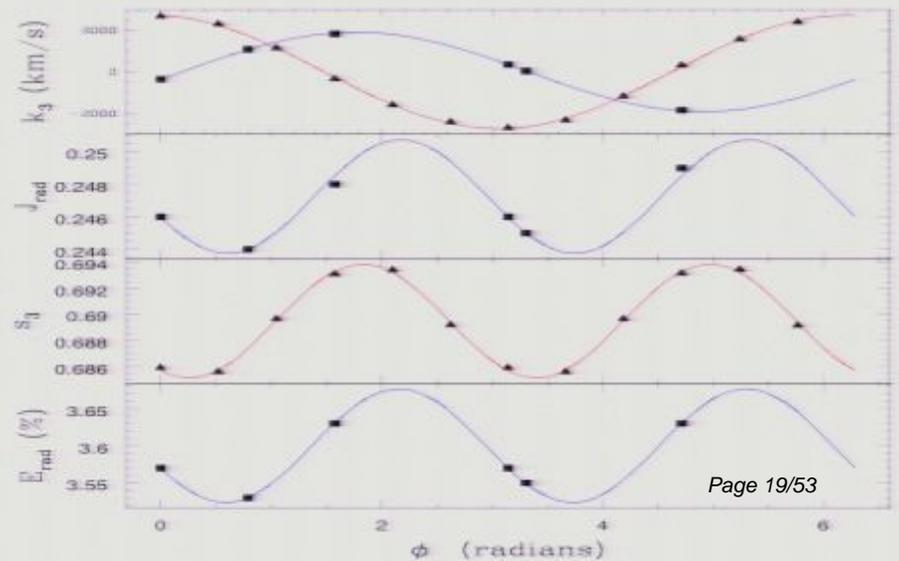
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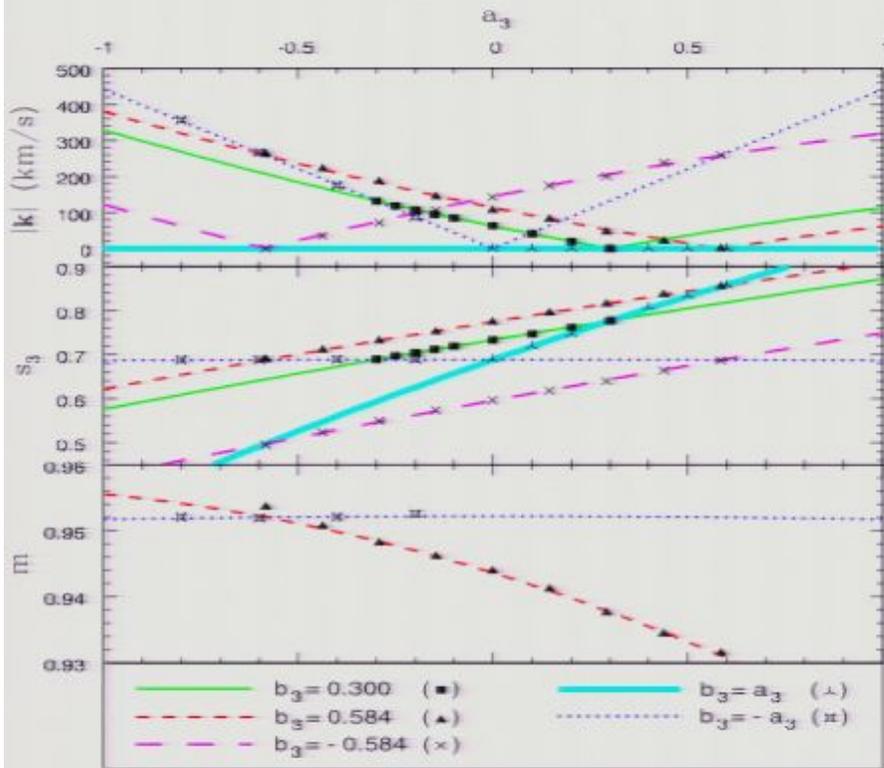


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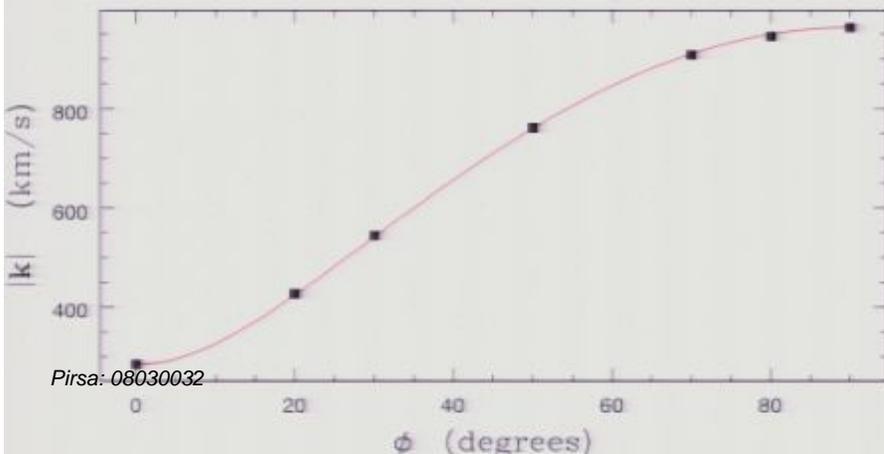
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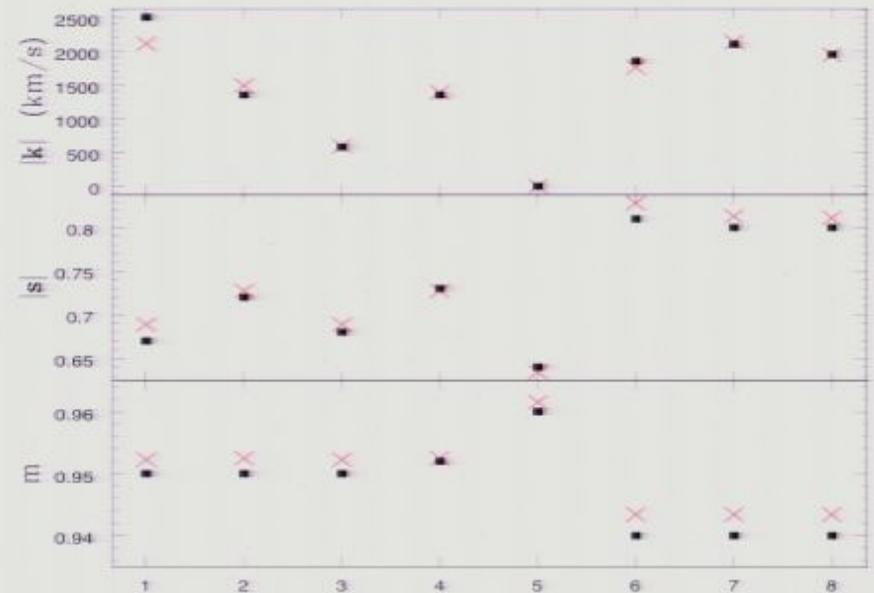


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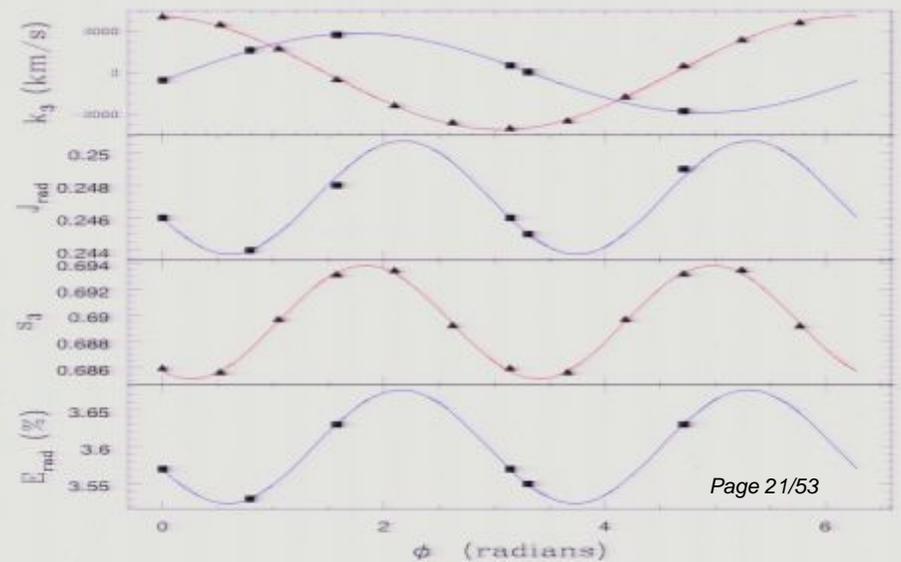
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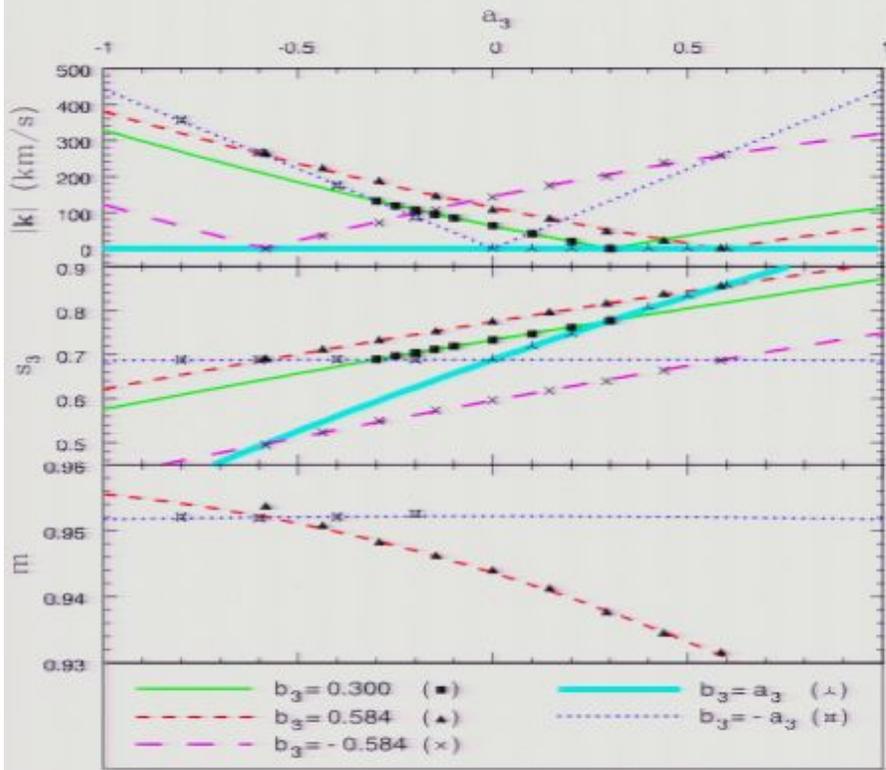
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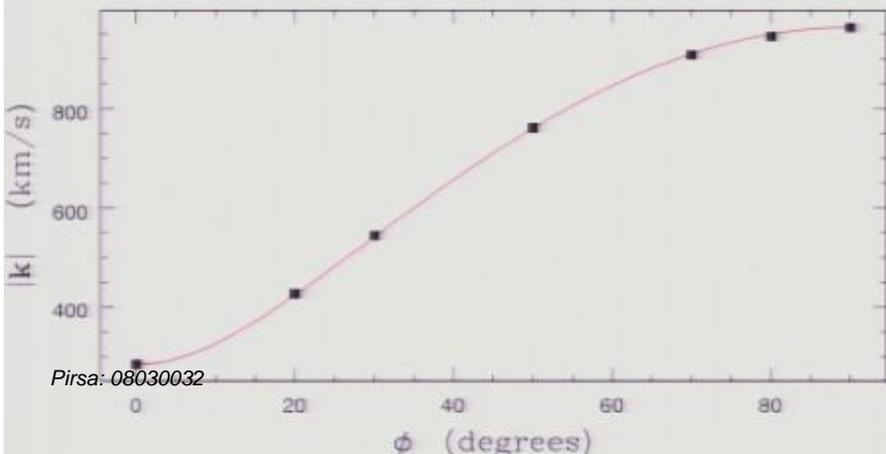
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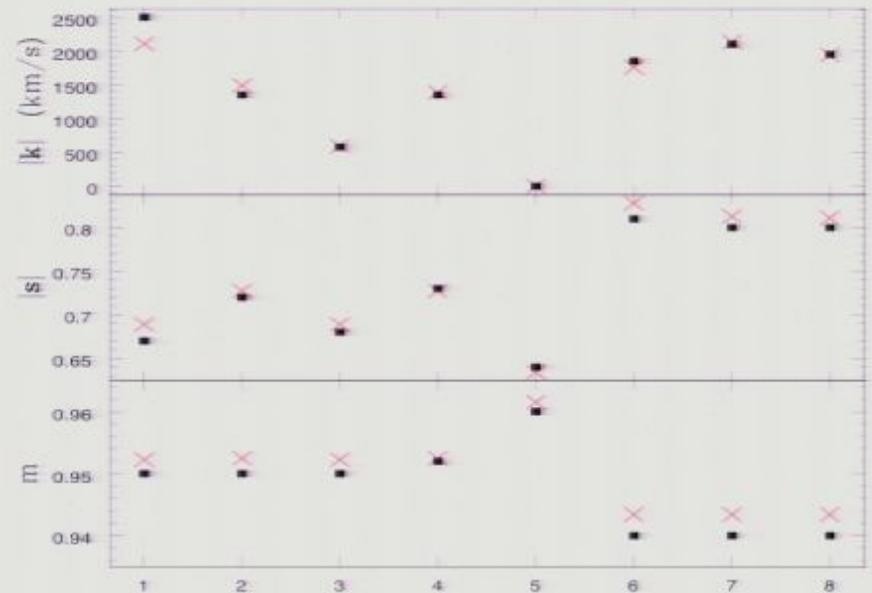


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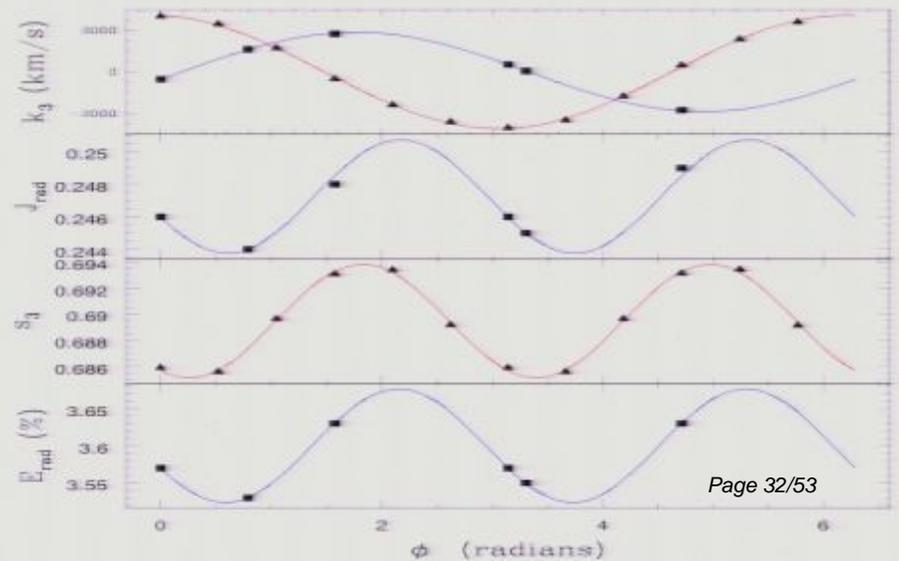
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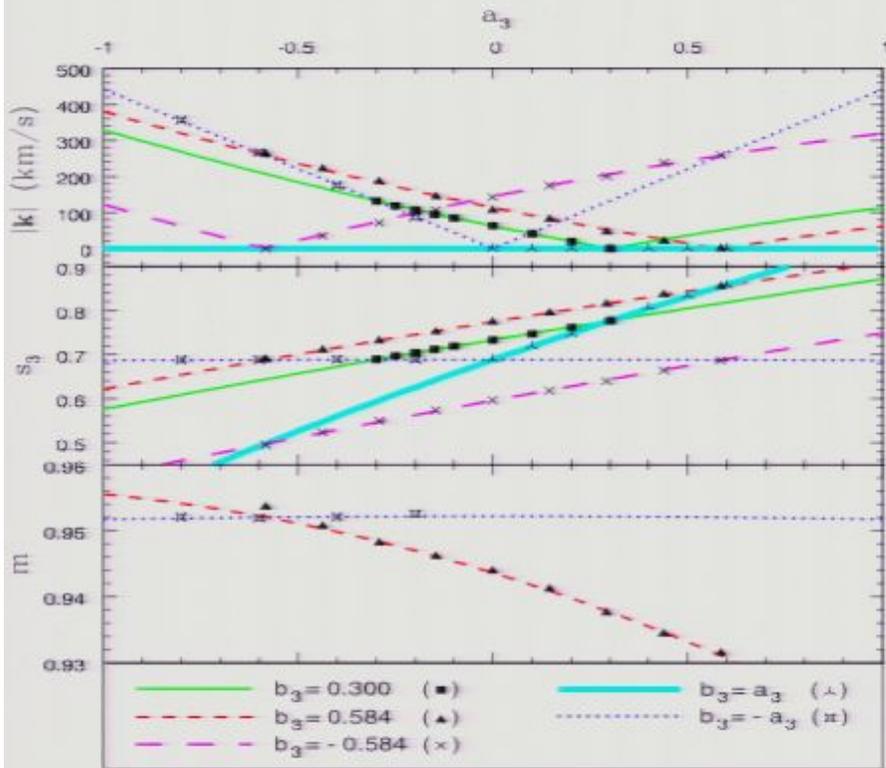
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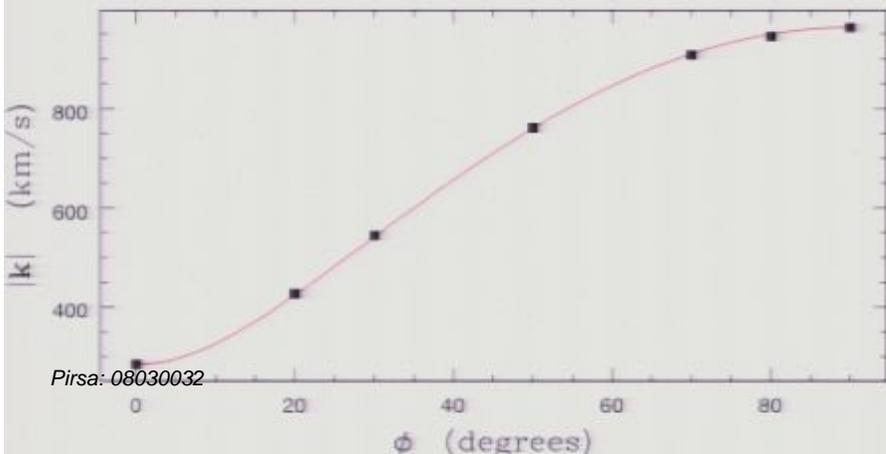


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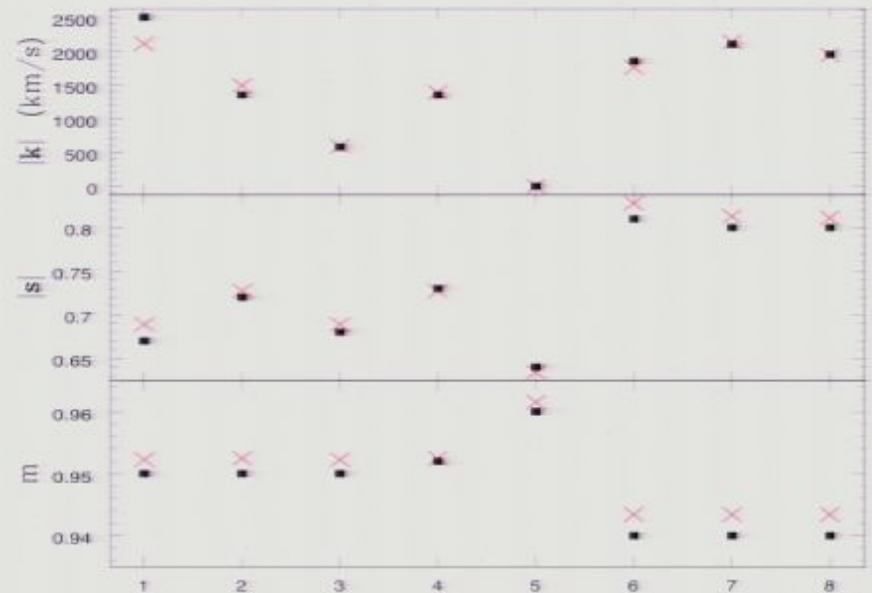


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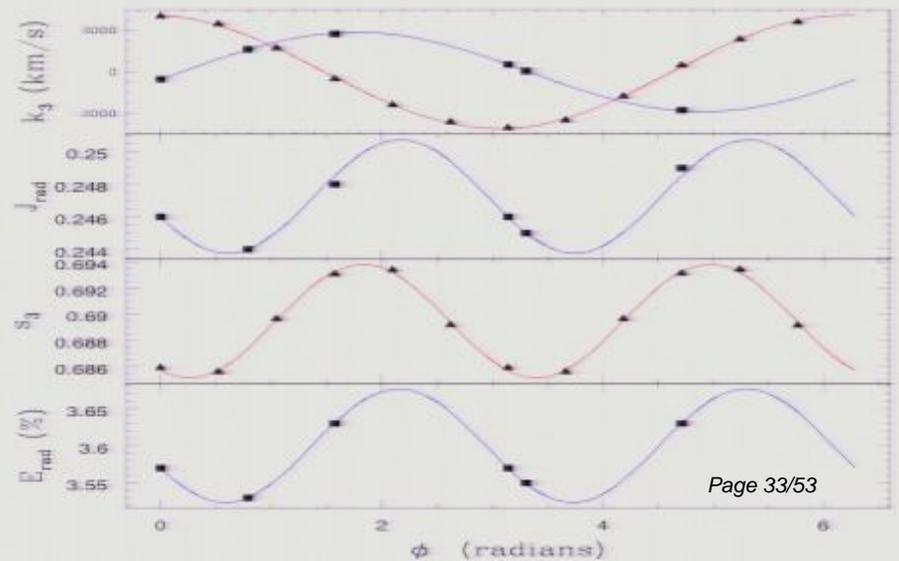
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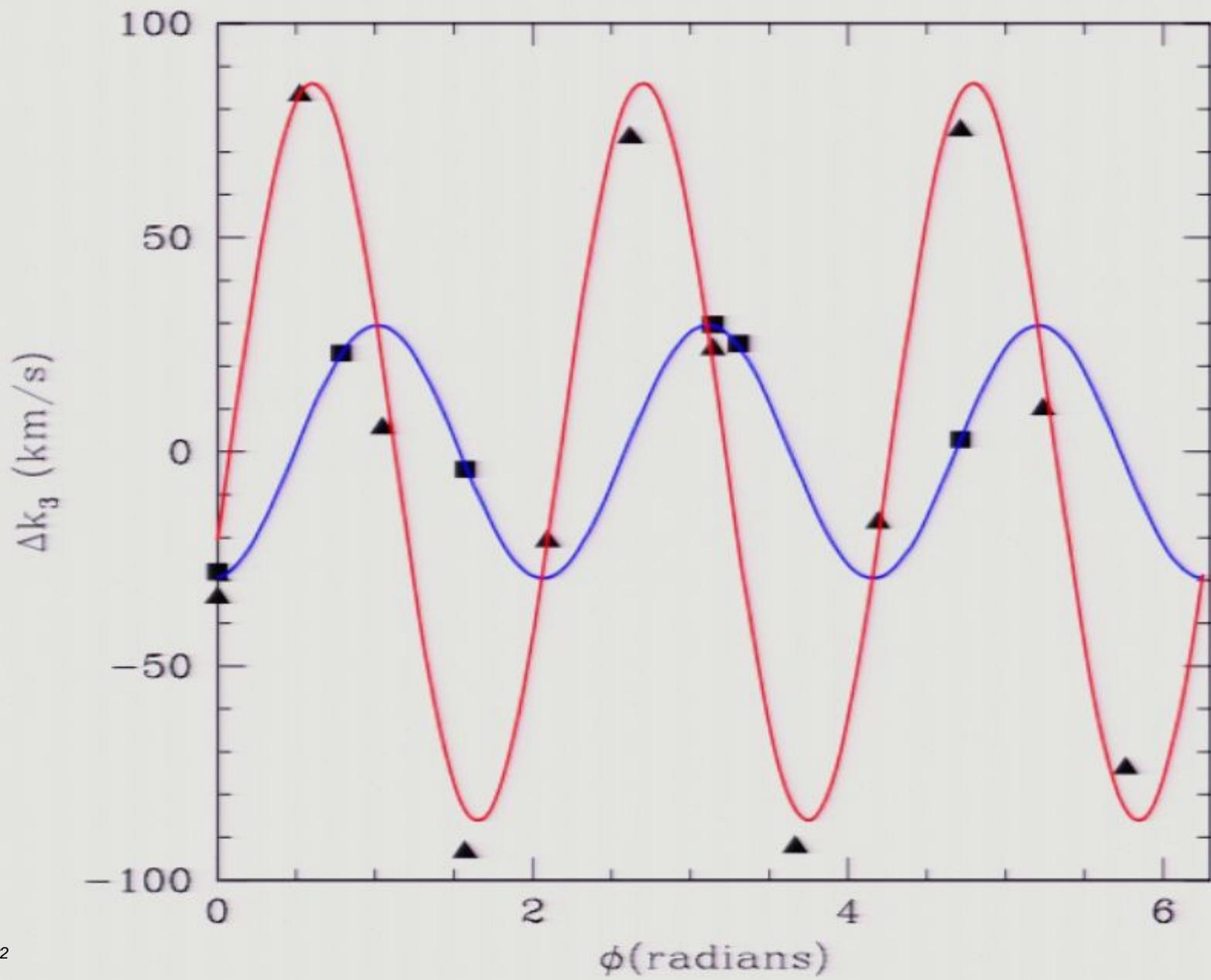
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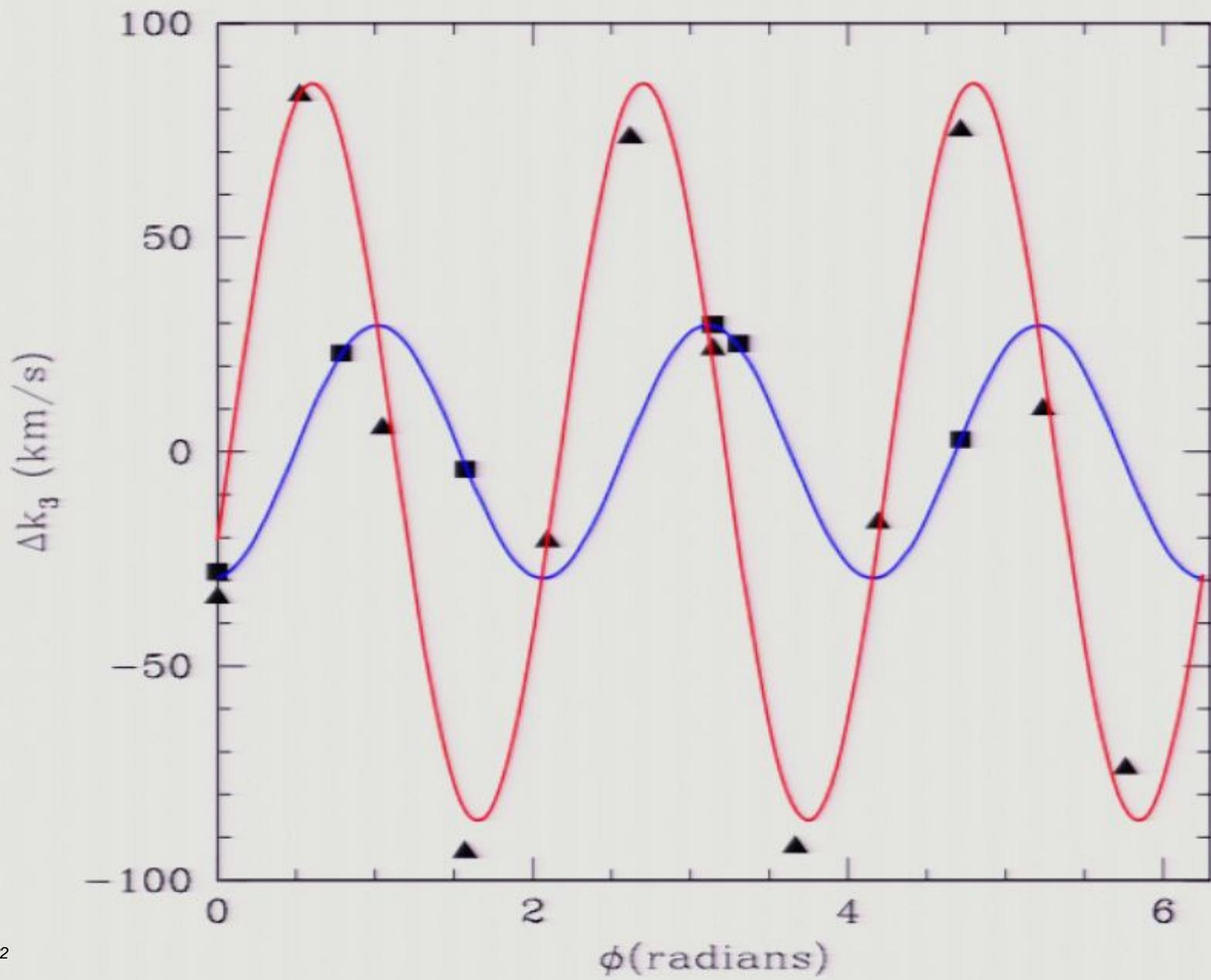
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$$q = 1 \quad \Leftrightarrow \quad \alpha = 0$$

$$(q \rightarrow 1/q) \quad \Leftrightarrow \quad (\alpha \rightarrow -\alpha)$$

$$k_1^{000|000} = A_1\alpha^1 + A_3\alpha^3 + A_5\alpha^5 + \dots$$

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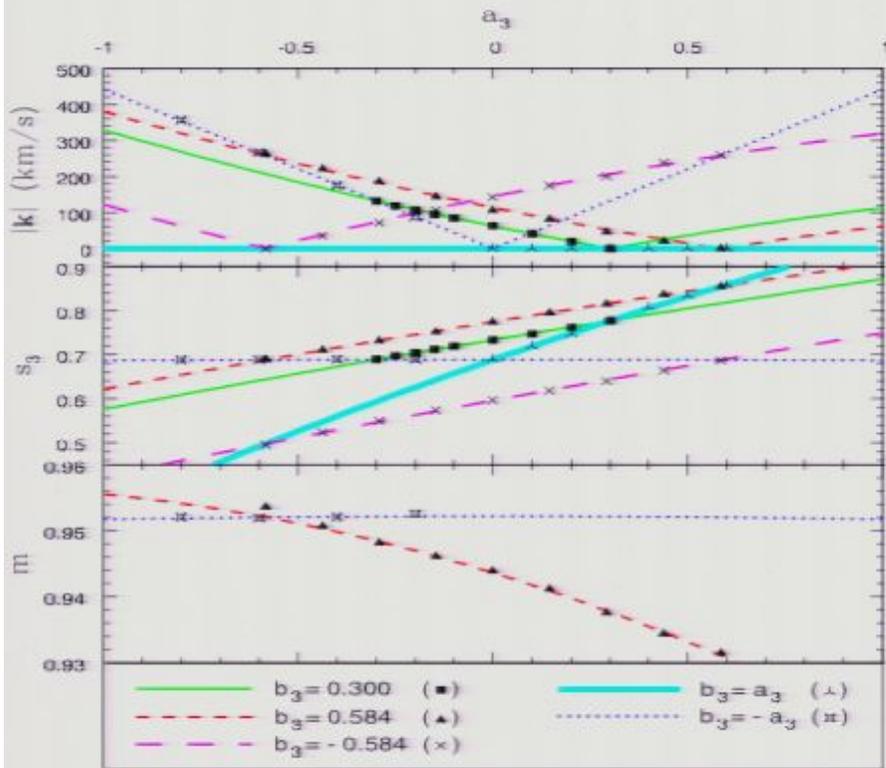
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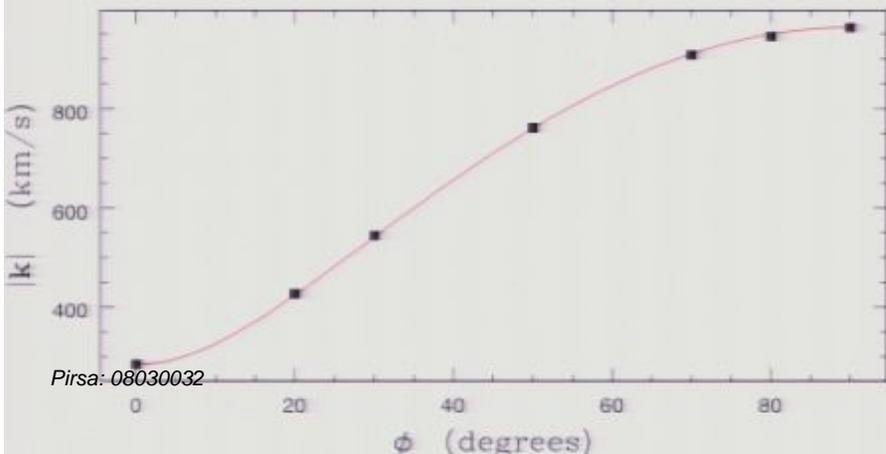
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- 3) Separates non-linearities from elementary considerations.
- 4) Efficient.
- 5) But what's it good for?

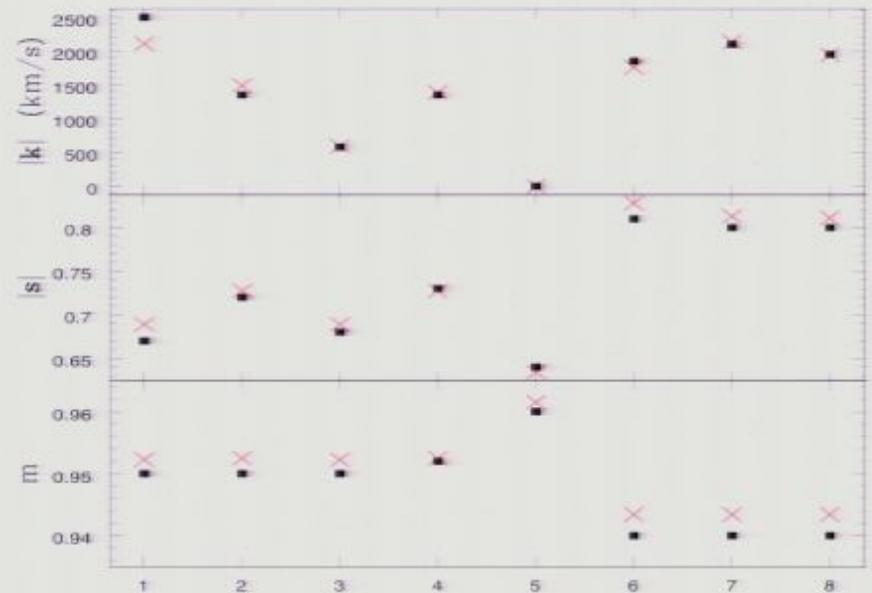


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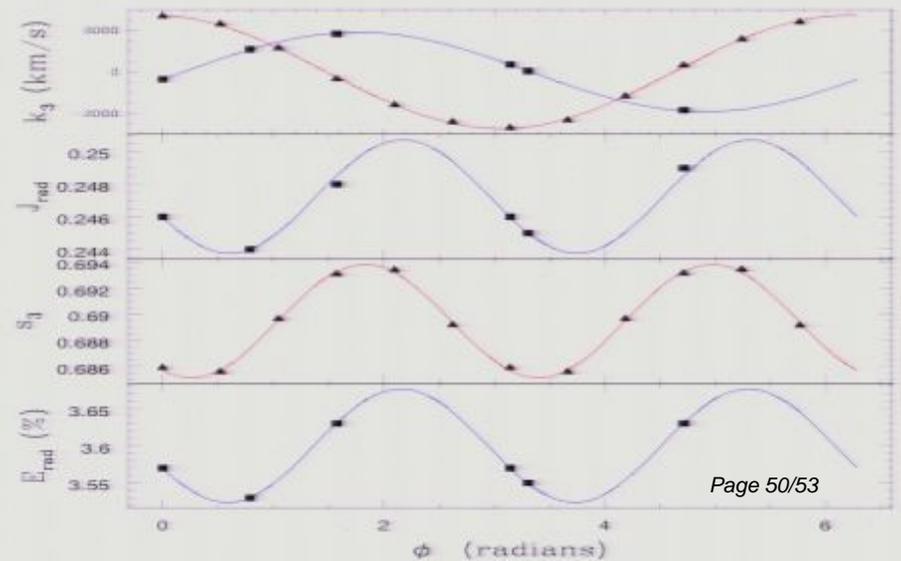
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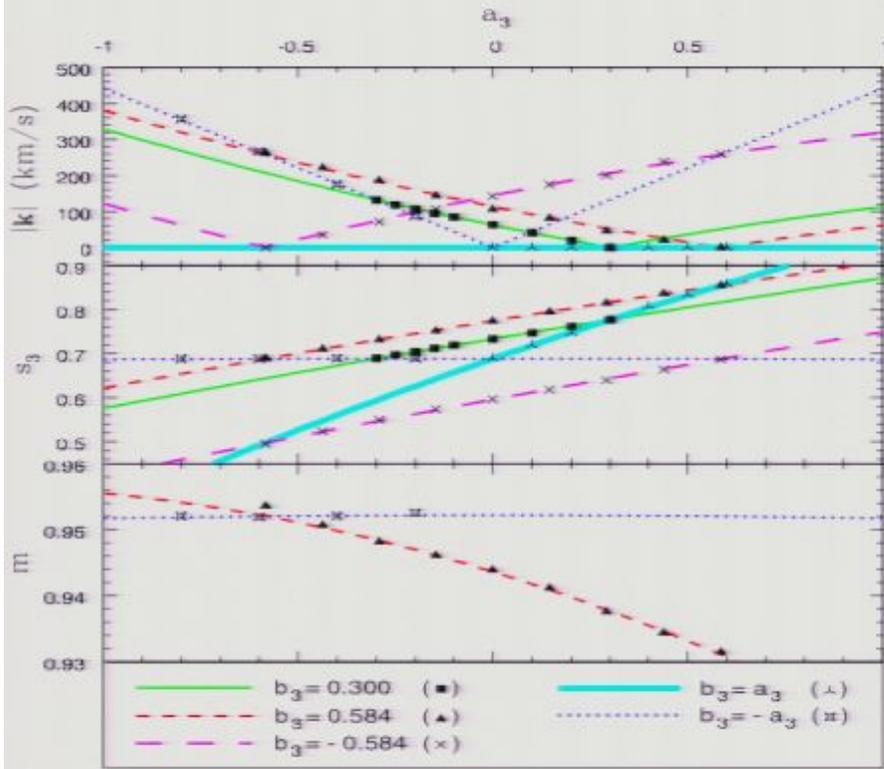


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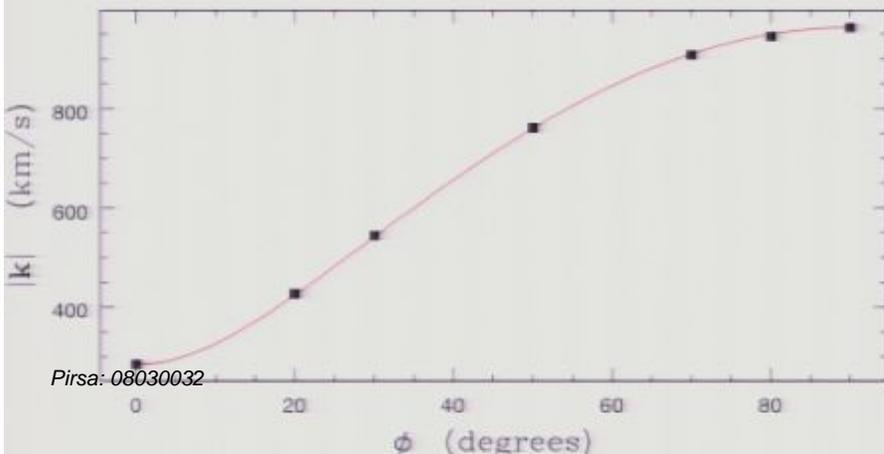
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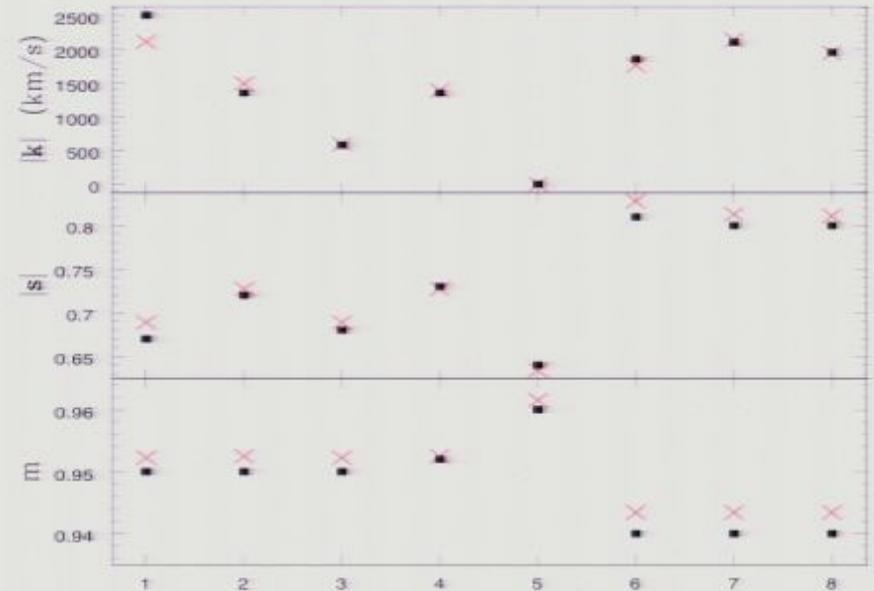


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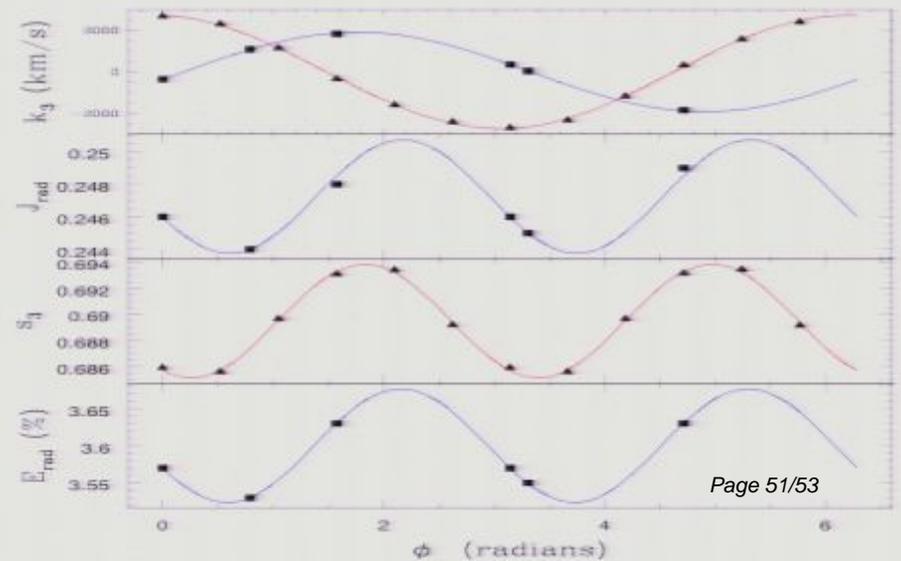
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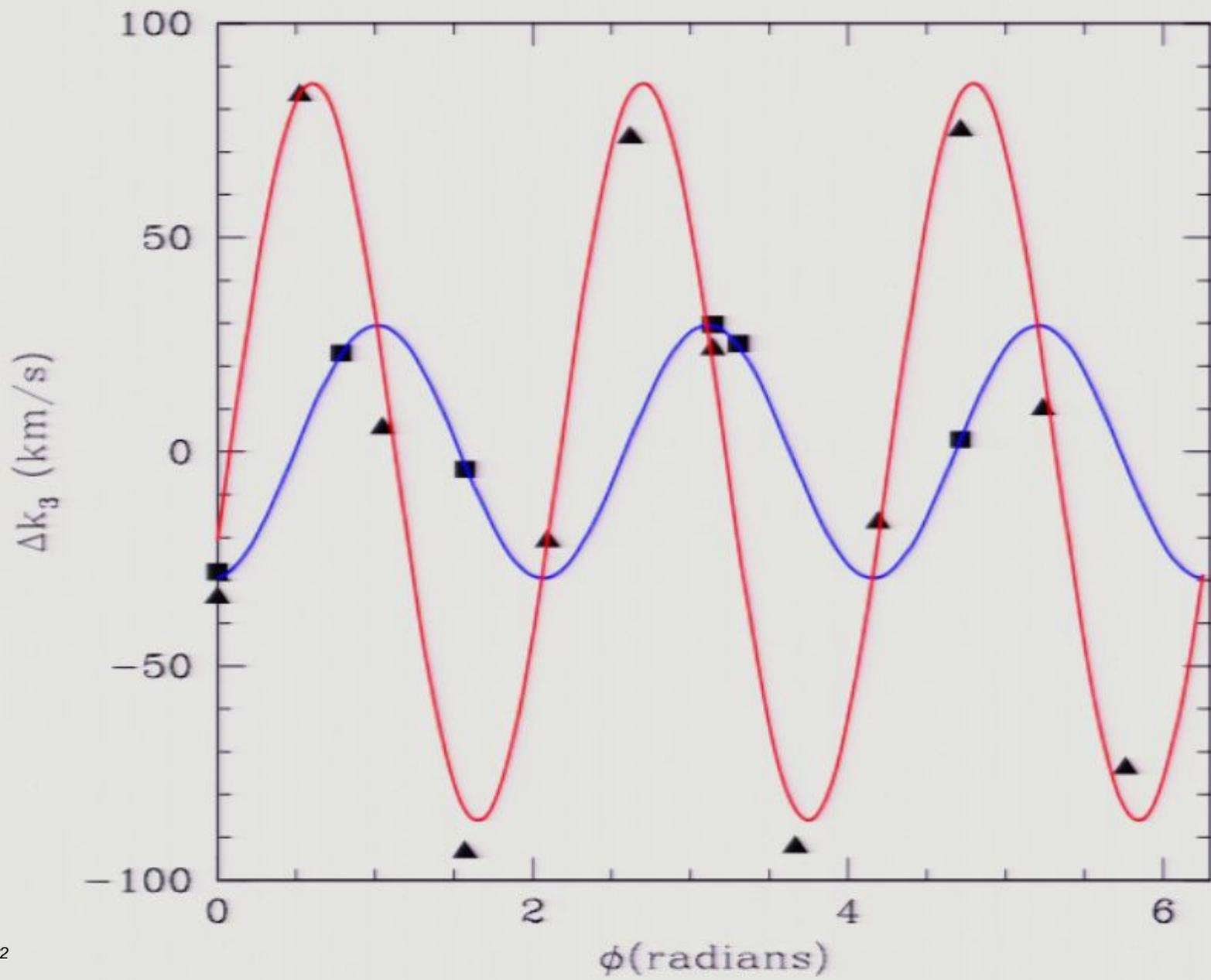
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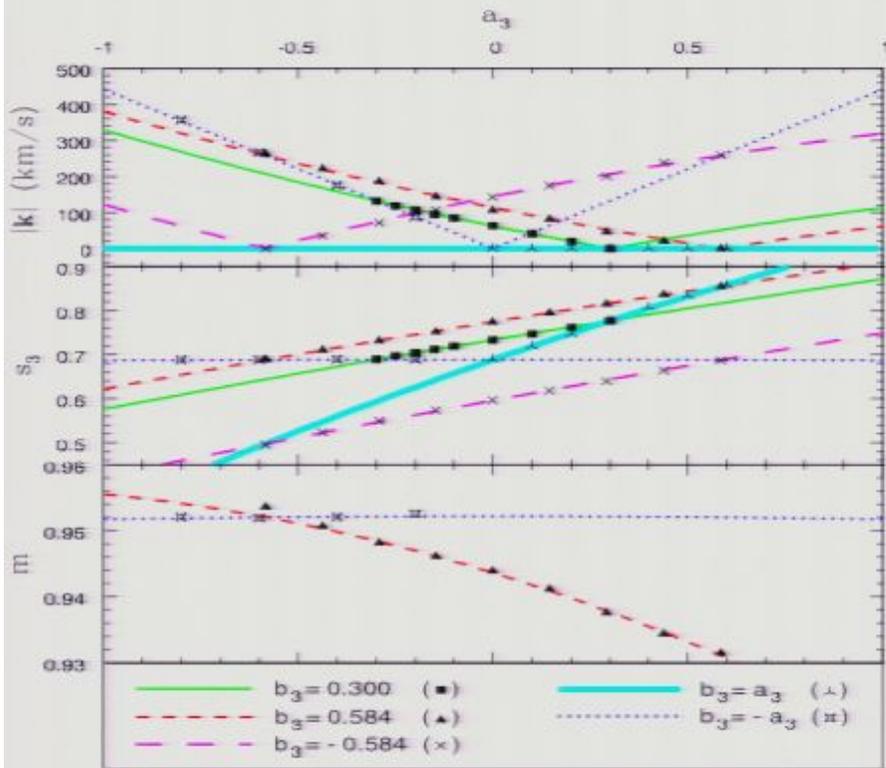
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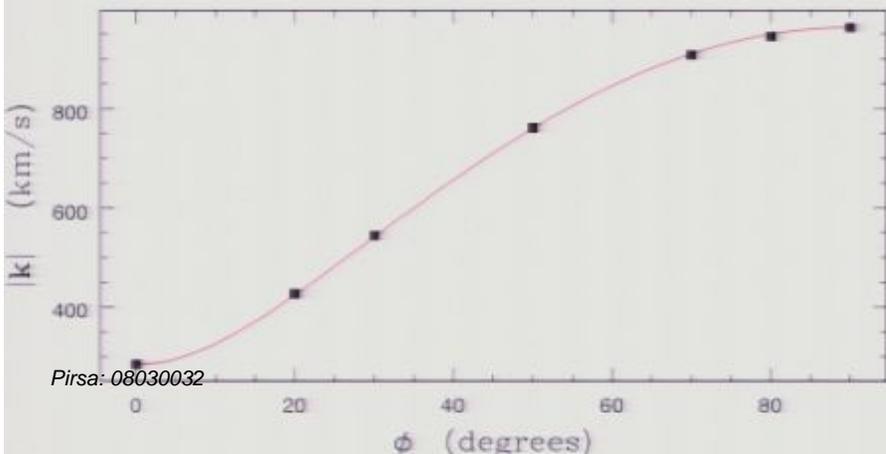
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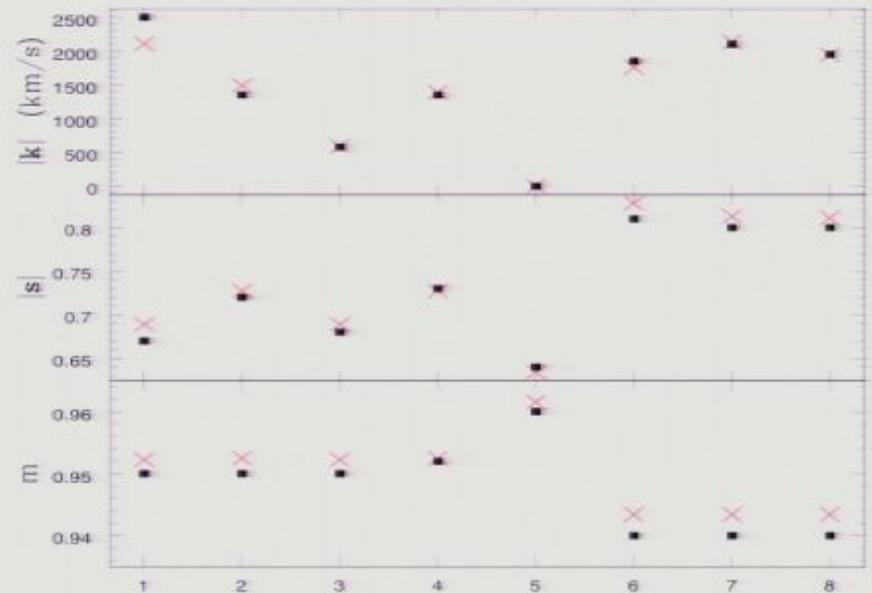


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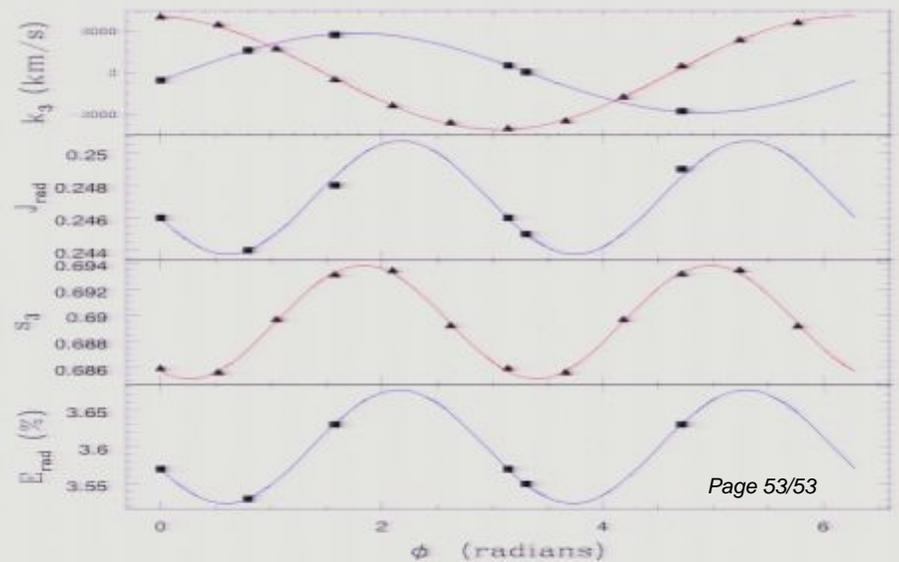
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