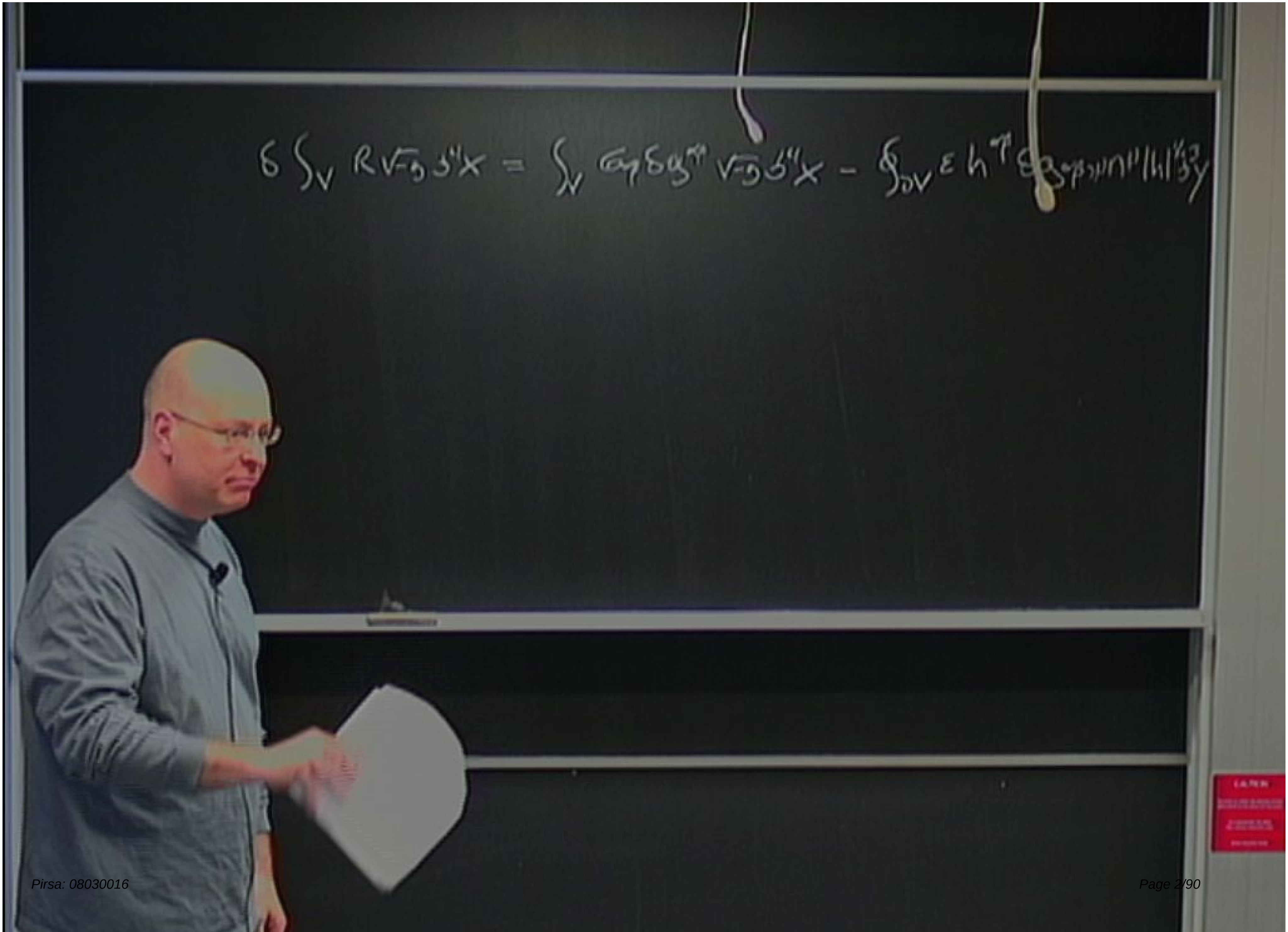
Title: Advanced General Relativity - Lecture 8B

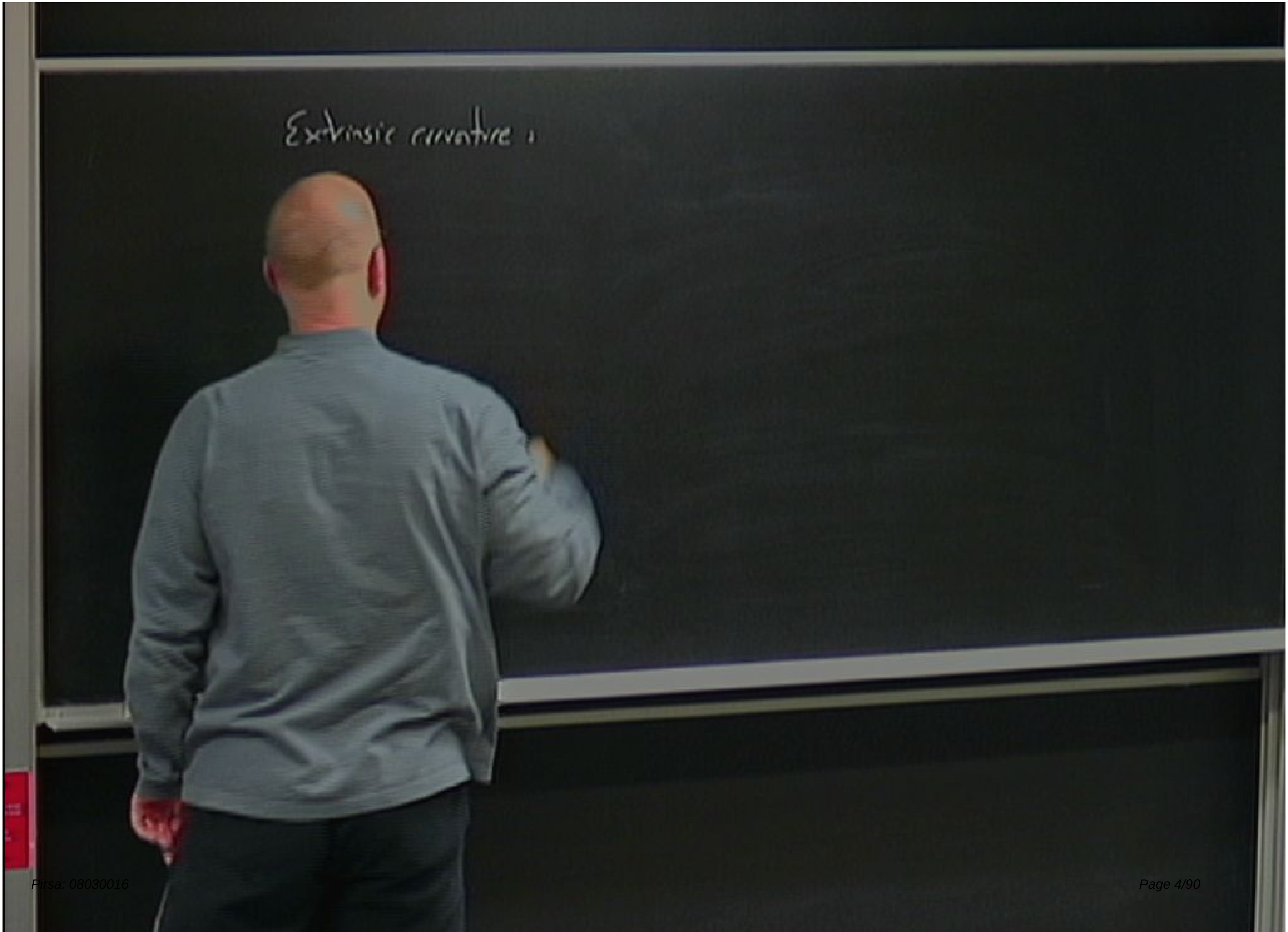
Date: Mar 12, 2008 04:00 PM

URL: http://pirsa.org/08030016

Abstract: Advanced General Relativity


$\delta \int_V R\sqrt{-g}\, d^4x = \int_V G_{\alpha\beta}\, \delta g^{\alpha\beta} \sqrt{-g}\, d^4x - \oint_{\partial V} \varepsilon\, h^{\alpha\beta}\, \delta g_{\alpha\beta,\mu} n^\mu |h|^{1/2} d^3y$

$$\delta \int_V R\sqrt{-g}\, d^4x = \int_V G_{\alpha\beta}\, \delta g^{\alpha\beta} \sqrt{-g}\, d^4x - \oint_{\partial V} \varepsilon h^{\alpha\beta}\, \delta g_{\alpha\beta ; \mu} n^\mu |h|^{1/2} d^3y$$


Extrinsic curvature :

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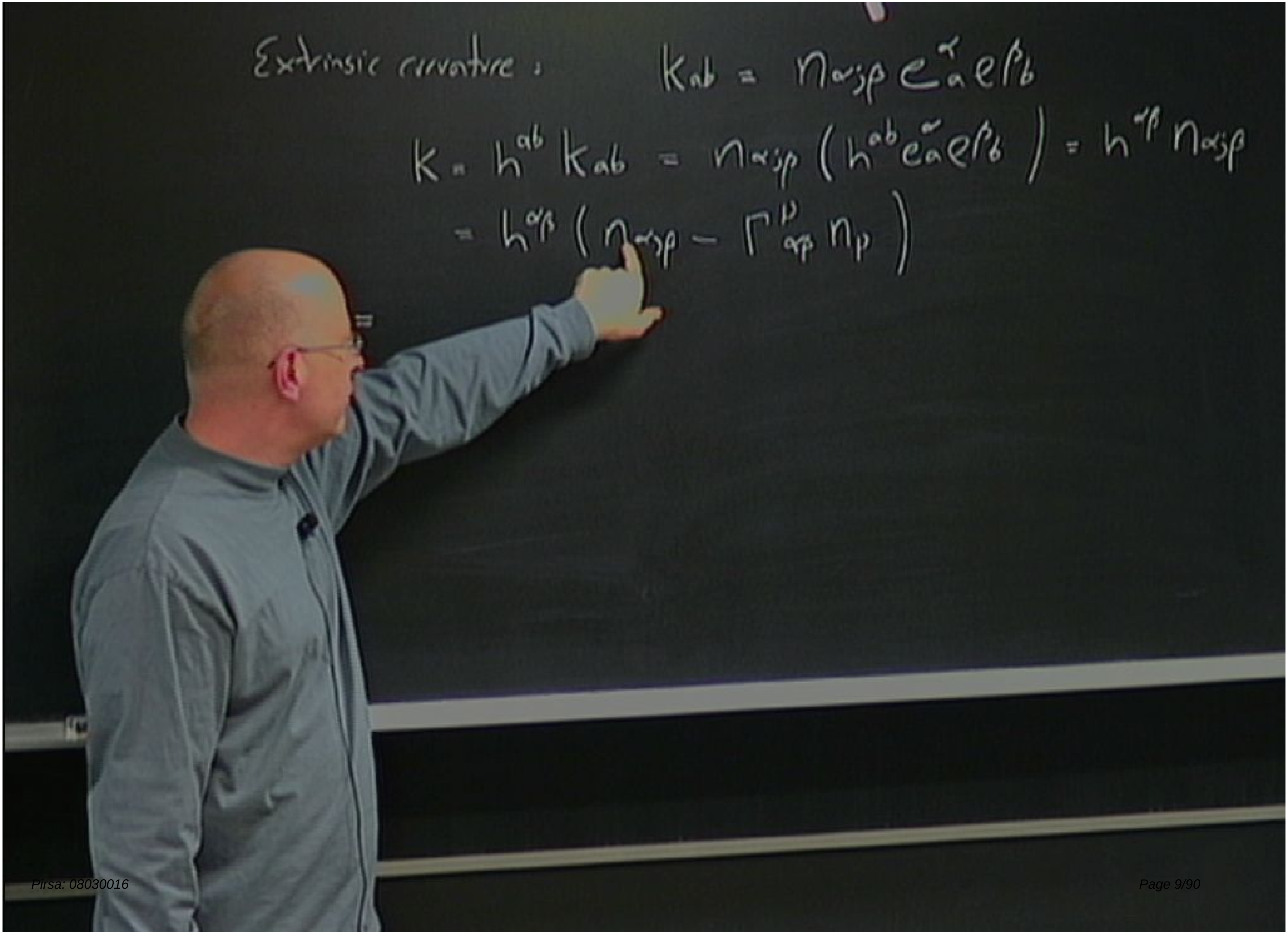
$$K = h^{ab} K_{ab} = n_{\alpha;\beta} \left(h^{ab} e^{\alpha}_{a} e^{\beta}_{b} \right)$$

Extrinsic curvature: $K_{ab} = n_{\alpha;\beta}\, e^{\alpha}_{a} e^{\beta}_{b}$

$$K = h^{ab} K_{ab} = n_{\alpha;\beta}\left(h^{ab} e^{\alpha}_{a} e^{\beta}_{b} \right) = h^{\alpha\beta} n_{\alpha;\beta}$$


Extrinsic curvature: $K_{ab} = n_{\alpha;\beta} e^{\alpha}_{a} e^{\beta}_{b}$
$K = h^{ab} K_{ab} = n_{\alpha;\beta} (h^{ab} e^{\alpha}_{a} e^{\beta}_{b}) = h^{\alpha\beta} n_{\alpha;\beta}$
$= h^{\alpha\beta} (n_{\alpha,\beta} - \Gamma^{\mu}_{\alpha\beta} n_{\mu})$

Extrinsic curvature: $K_{ab} = n_{\alpha;\beta} e^{\alpha}_{a} e^{\beta}_{b}$

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$$= h^{\alpha\beta} \left(n_{\alpha,\beta} - \Gamma^{\mu}_{\alpha\beta} n_{\mu} \right)$$

$$\delta K = - h^{\alpha\beta} n_{\mu} \delta\Gamma^{\mu}_{\alpha\beta}$$

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$$\delta K = - h^{\alpha\beta} n_{\mu} \delta\Gamma^{\mu}_{\alpha\beta}$$

$$= -\tfrac{1}{2} h^{\alpha\beta} n_{\mu} g^{\mu\nu} \left(\delta g_{\nu\alpha,\beta} + \delta g_{\nu\beta,\alpha} - \delta g_{\alpha\beta,\nu} \right)$$

$$= -\tfrac{1}{2} h^{\alpha\beta} \left(\delta g_{\nu\alpha,\beta} + \delta g_{\nu\beta,\alpha} - \delta g_{\alpha\beta,\nu} \right) n^{\nu}$$

Extrinsic curvature: $K_{ab} = n_{\alpha;\beta}\, e^{\alpha}_{a} e^{\beta}_{b}$

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$$= h^{\alpha\beta} \left(n_{\alpha,\beta} - \Gamma^{\mu}_{\alpha\beta} n_{\mu} \right)$$

$$\delta K = - h^{\alpha\beta} n_{\mu} \delta \Gamma^{\mu}_{\alpha\beta}$$

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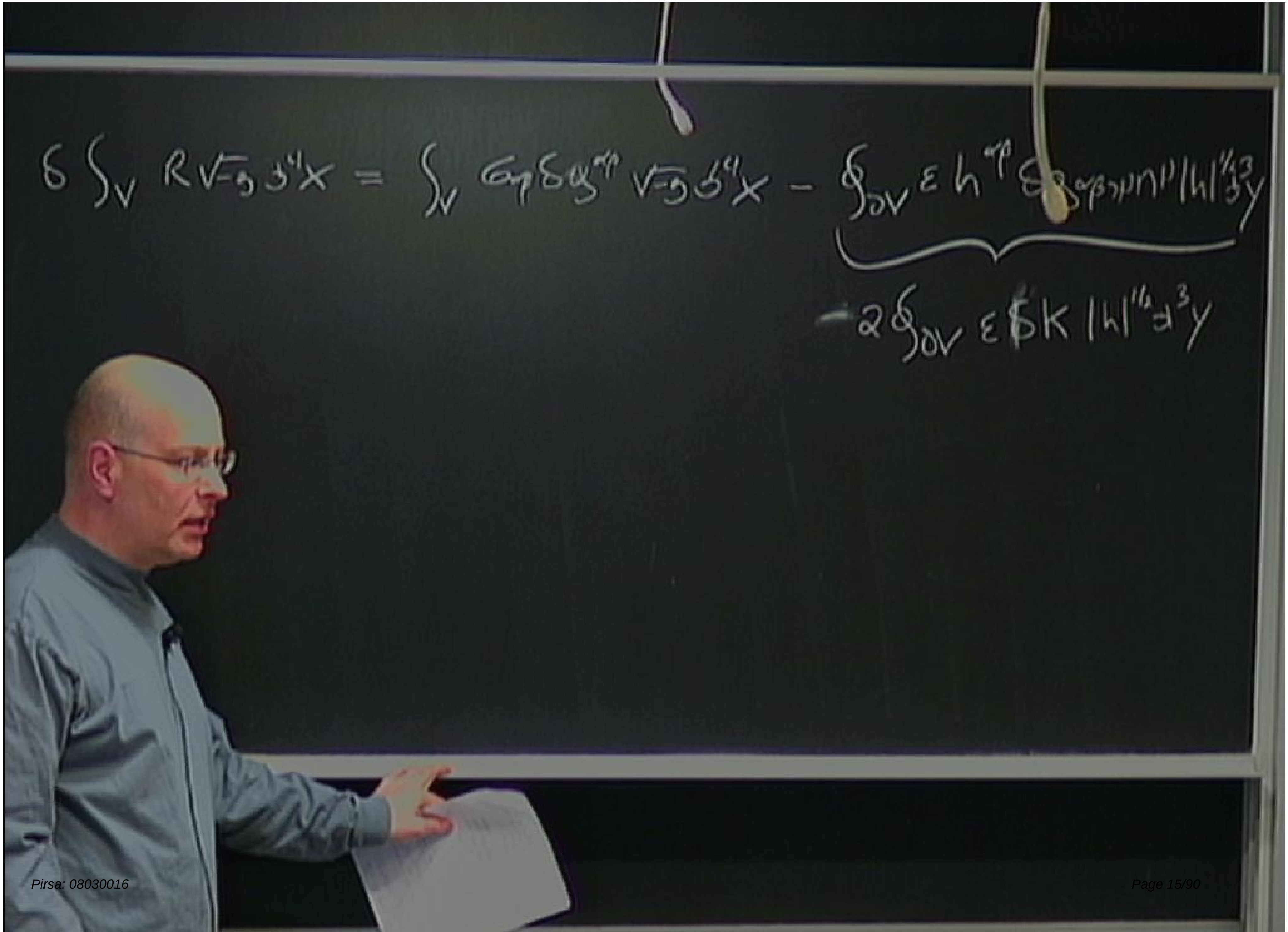
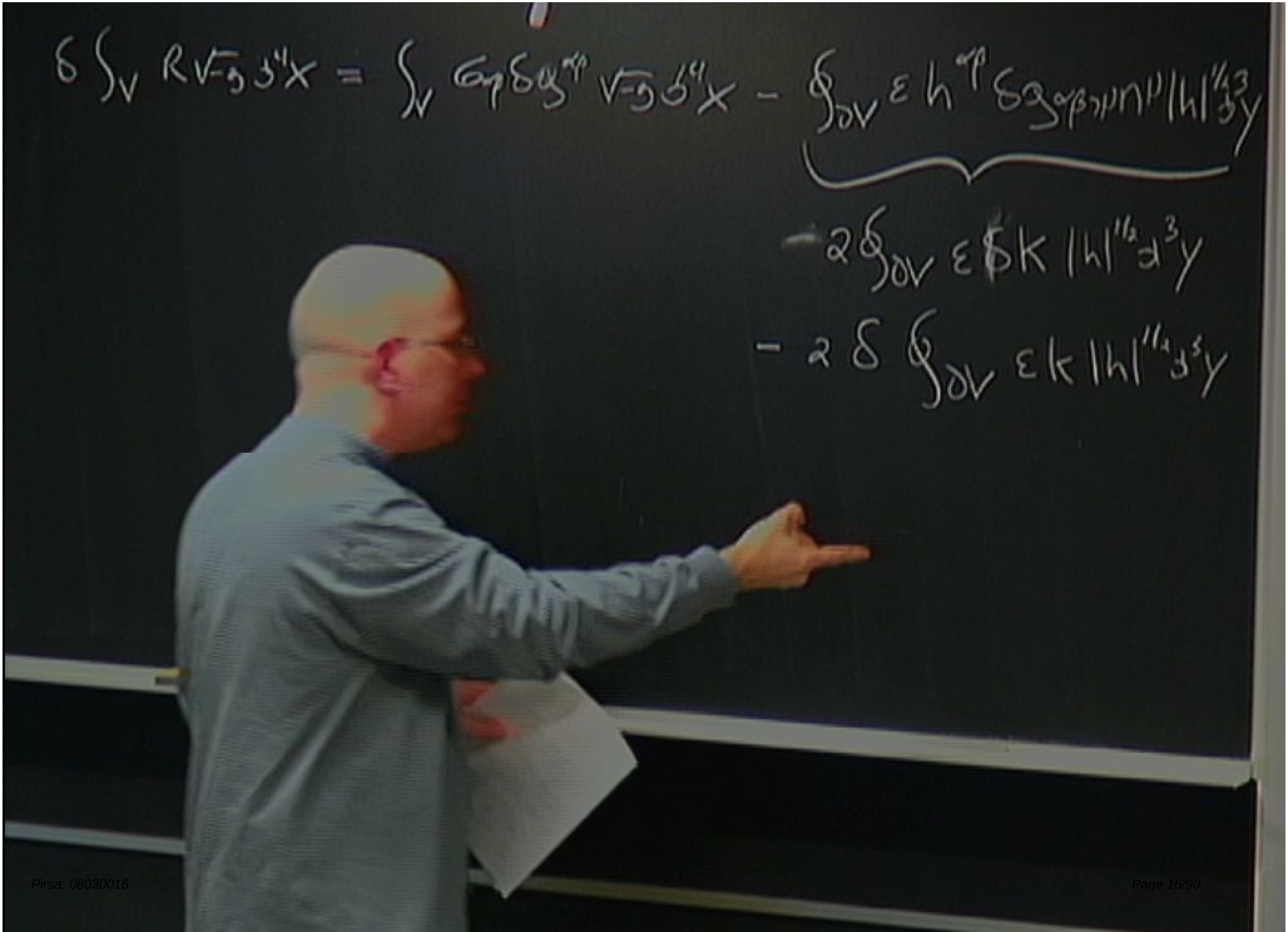
$$= -\tfrac{1}{2} h^{\alpha\beta}\, \delta g_{\alpha\beta,\nu}\, n^{\nu}$$

$$\delta \int_V R\sqrt{-g}\, d^4x = \int_V G_{\alpha\beta}\, \delta g^{\alpha\beta} \sqrt{-g}\, d^4x - \underbrace{\oint_{\partial V} \varepsilon\, h^{\alpha\beta}\, \delta g_{\alpha\beta,\mu} n^{\mu} |h|^{1/2} d^3y}$$


$\delta \int_V R\sqrt{-g}\, d^4x = \int_V G_{\alpha\beta}\,\delta g^{\alpha\beta} \sqrt{-g}\, d^4x - \underbrace{\oint_{\partial V} \varepsilon\, h^{\alpha\beta}\, \delta g_{\alpha\beta,\gamma}\, n^\gamma |h|^{1/2} d^3y}_{-2\oint_{\partial V} \varepsilon\, \delta K\, |h|^{1/2} d^3y}$


$\delta \int_V R\sqrt{-g}\, d^4x = \int_V G_{\alpha\beta}\, \delta g^{\alpha\beta} \sqrt{-g}\, d^4x - \oint_{\partial V} \varepsilon h^{\alpha\beta} \delta g_{\alpha\beta,\gamma} n^\gamma |h|^{1/2} d^3y$
$-2 \oint_{\partial V} \varepsilon\, \delta K\, |h|^{1/2} d^3y$
$-2\, \delta \oint_{\partial V} \varepsilon K |h|^{1/2} d^3y$

$$\sqrt{-g}\, d^4x - \underbrace{\oint_{\partial V} \varepsilon\, h^{\alpha\beta}\, \delta g_{\alpha\beta,\mu}\, n^{\mu}\, |h|^{1/2}\, d^3y}_{-2\oint_{\partial V} \varepsilon\, \delta K\, |h|^{1/2}\, d^3y}$$

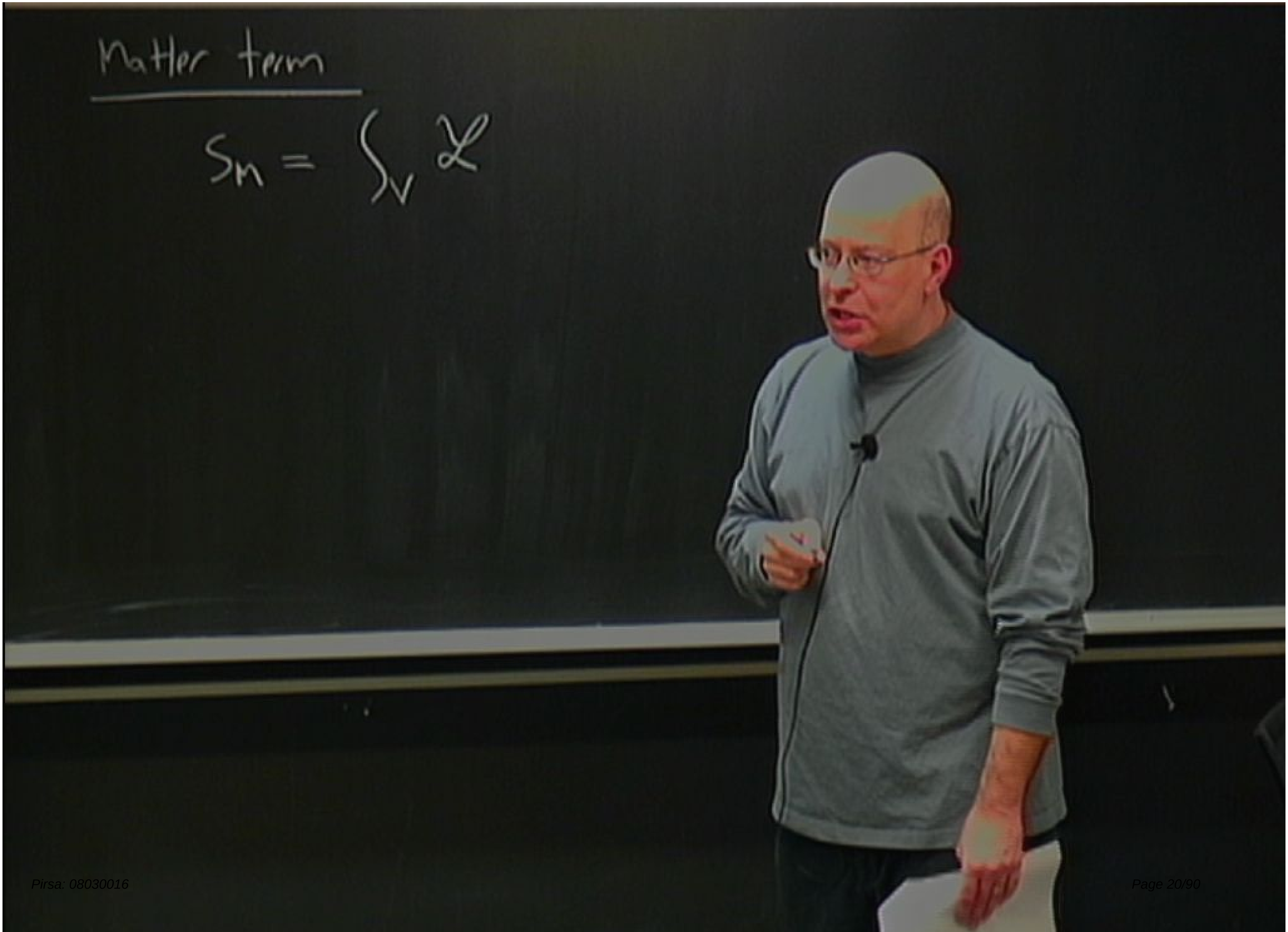
$$-2\,\delta \oint_{\partial V} \varepsilon\, K\, |h|^{1/2}\, d^3y$$

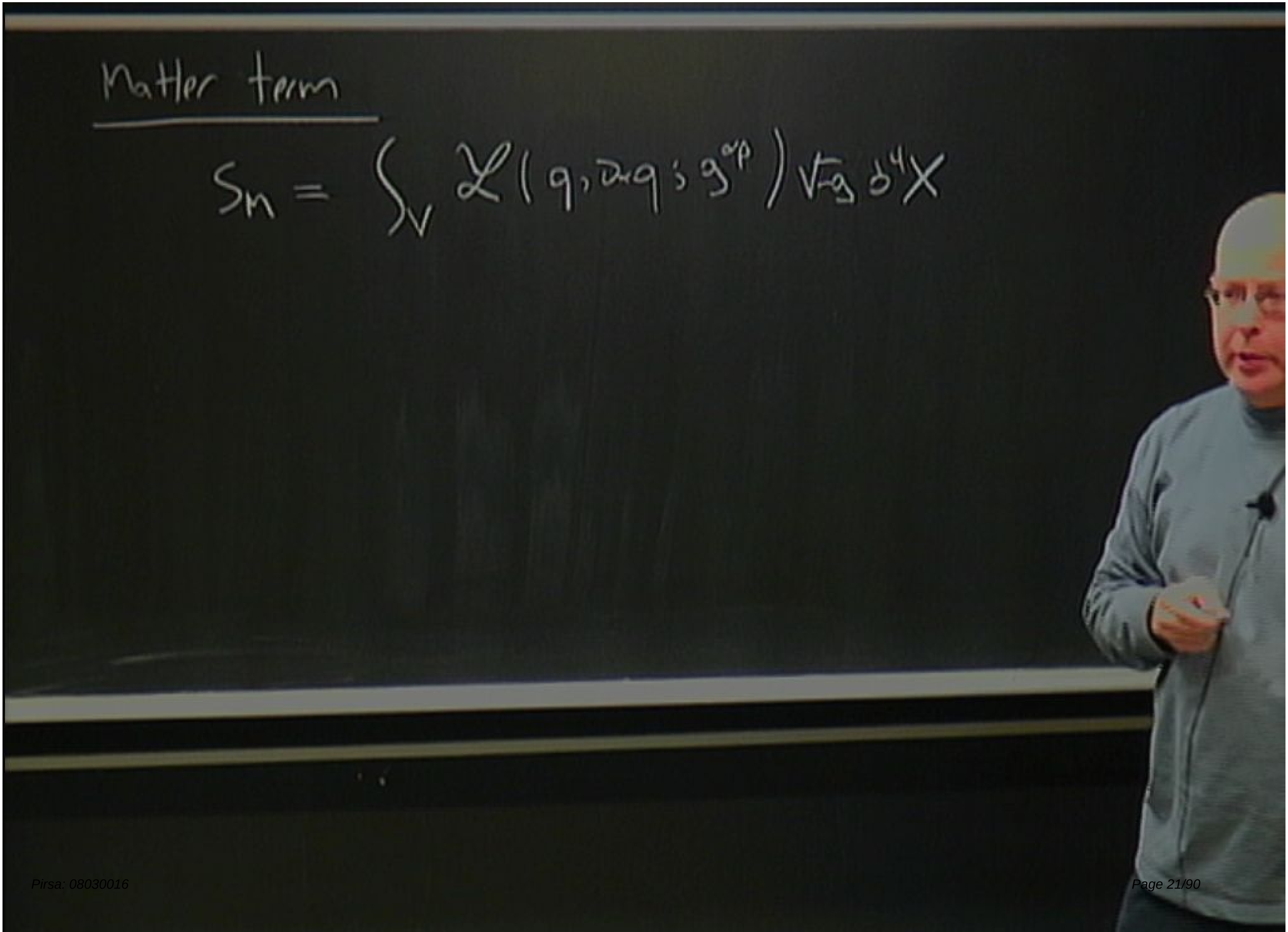
$$\delta\int_V R\sqrt{-g}\,d^4x = \int_V G_{\alpha\beta}\,\delta g^{\alpha\beta}\sqrt{-g}\,d^4x - \underbrace{\oint_{\partial V}\varepsilon h^{\alpha\beta}\,\delta g_{\alpha\beta,\gamma}n^\gamma |h|^{1/2} d^3y}_{-2\oint_{\partial V}\varepsilon\,\delta k\,|h|^{1/2}d^3y}$$

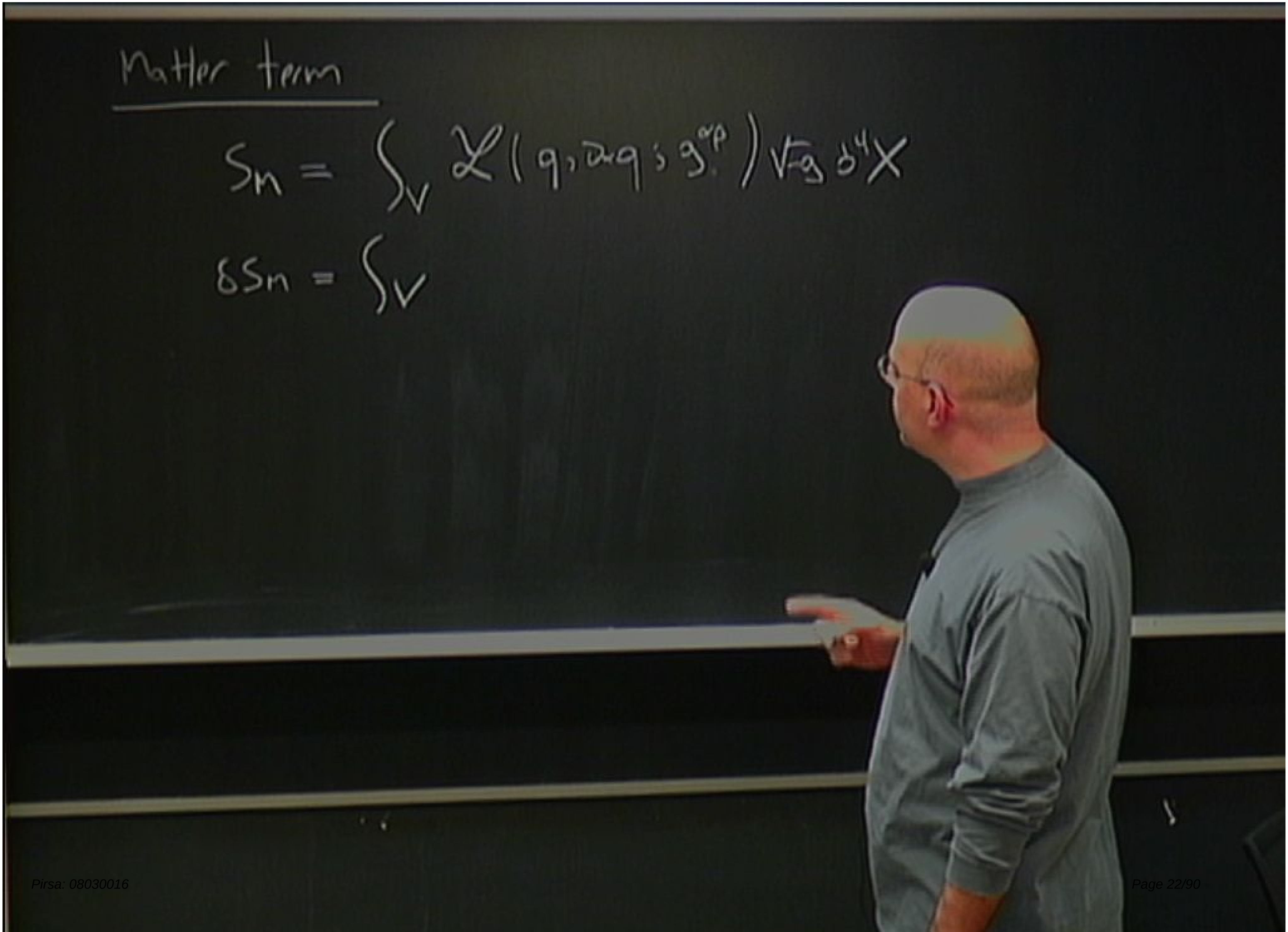
$$\boxed{\delta\left(\int_V R\sqrt{-g}\,d^4x + 2\oint_{\partial V}\varepsilon k\,|h|^{1/2}d^3y\right) = \int_V G_{\alpha\beta}\,\delta g^{\alpha\beta}\sqrt{-g}\,d^4x} - 2\,\delta\oint_{\partial V}\varepsilon k\,|h|^{1/2}d^3y$$

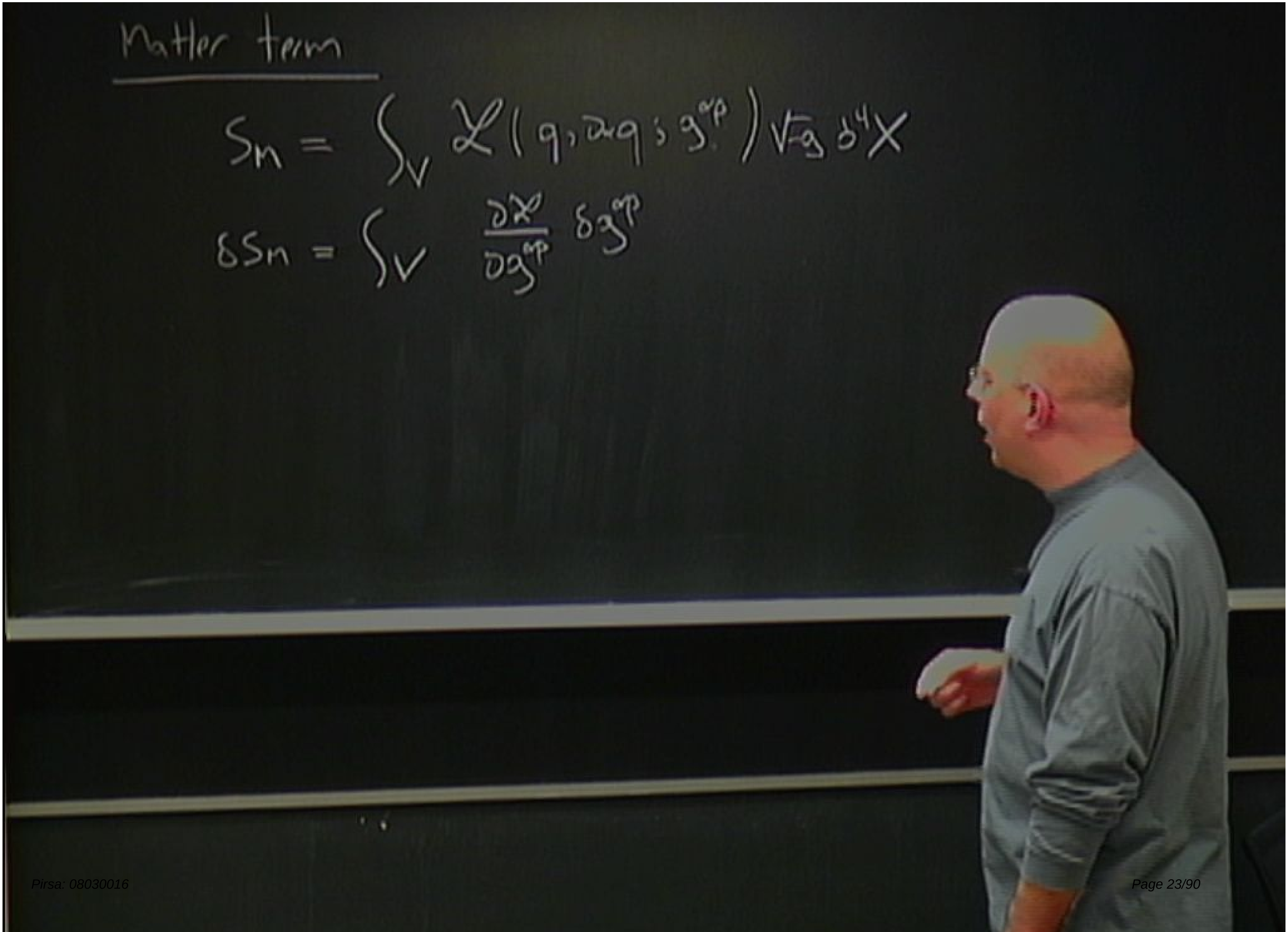
$$\delta \int_V R\sqrt{-g}\,d^4x = \int_V G_{\alpha\beta}\,\delta g^{\alpha\beta}\sqrt{-g}\,d^4x - \underbrace{\oint_{\partial V} \varepsilon\, h^{\alpha\beta}\,\delta g_{\alpha\beta,\mu}\, n^{\mu}\, |h|^{1/2}\, d^3y}_{-2\oint_{\partial V}\varepsilon\,\delta K\,|h|^{1/2}\,d^3y}$$

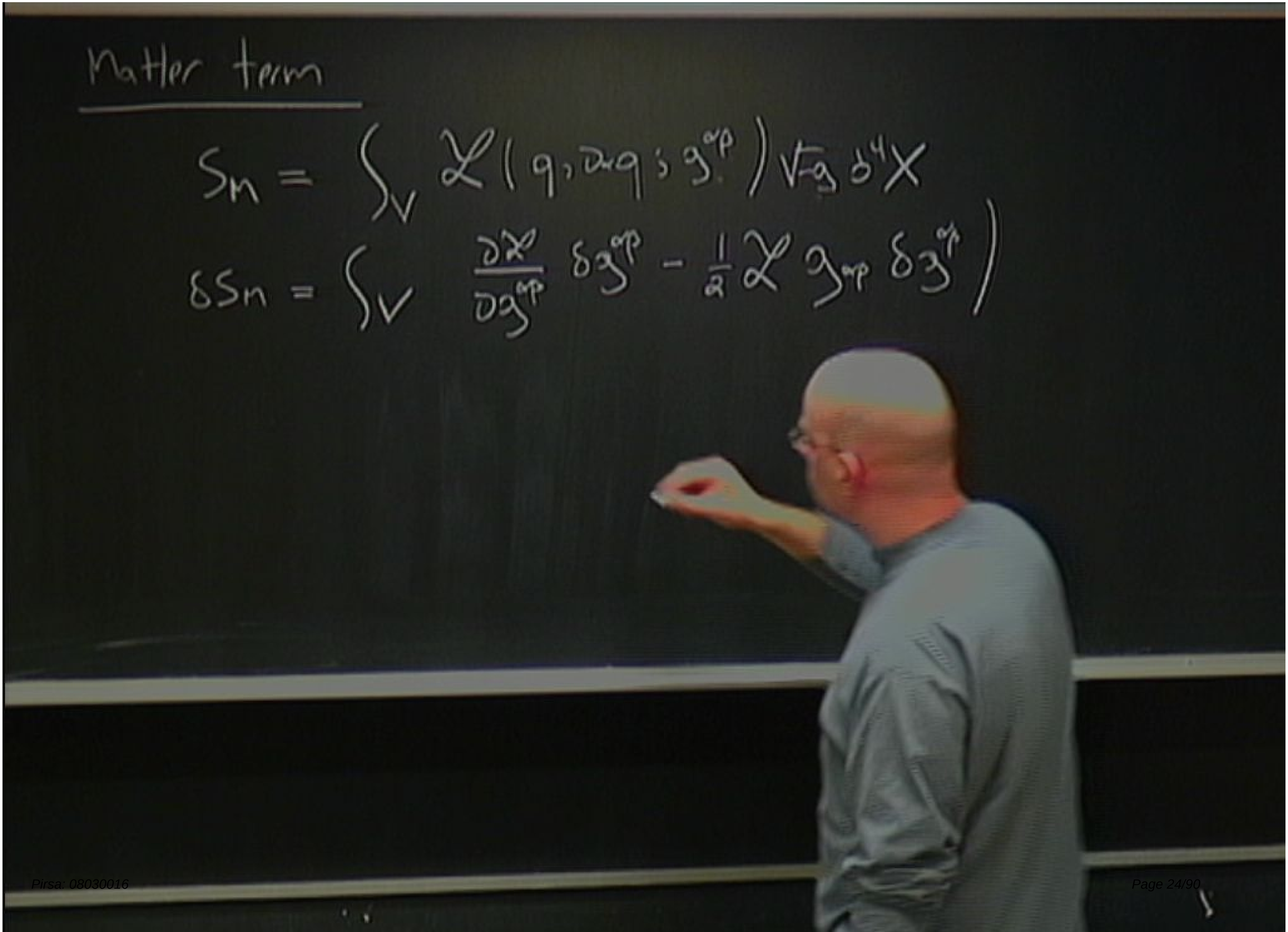
$$\boxed{\delta\left(\int_V R\sqrt{-g}\,d^4x + 2\oint_{\partial V}\varepsilon K\,|h|^{1/2}\,d^3y\right) = \int_V G_{\alpha\beta}\,\delta g^{\alpha\beta}\sqrt{-g}\,d^4x} \qquad - 2\,\delta\oint_{\partial V}\varepsilon K\,|h|^{1/2}\,d^3y$$


Matter term
$S_M = \int_V \mathcal{L}$


Matter term
$S_m = \int_V \mathcal{L}(q, \partial_\alpha q ; g^{\alpha\beta}) \sqrt{-g}\, d^4x$


Matter term
$S_m = \int_V \mathcal{L}(q, \partial_\alpha q; g^{\alpha\beta}) \sqrt{-g}\, d^4x$
$\delta S_m = \int_V$

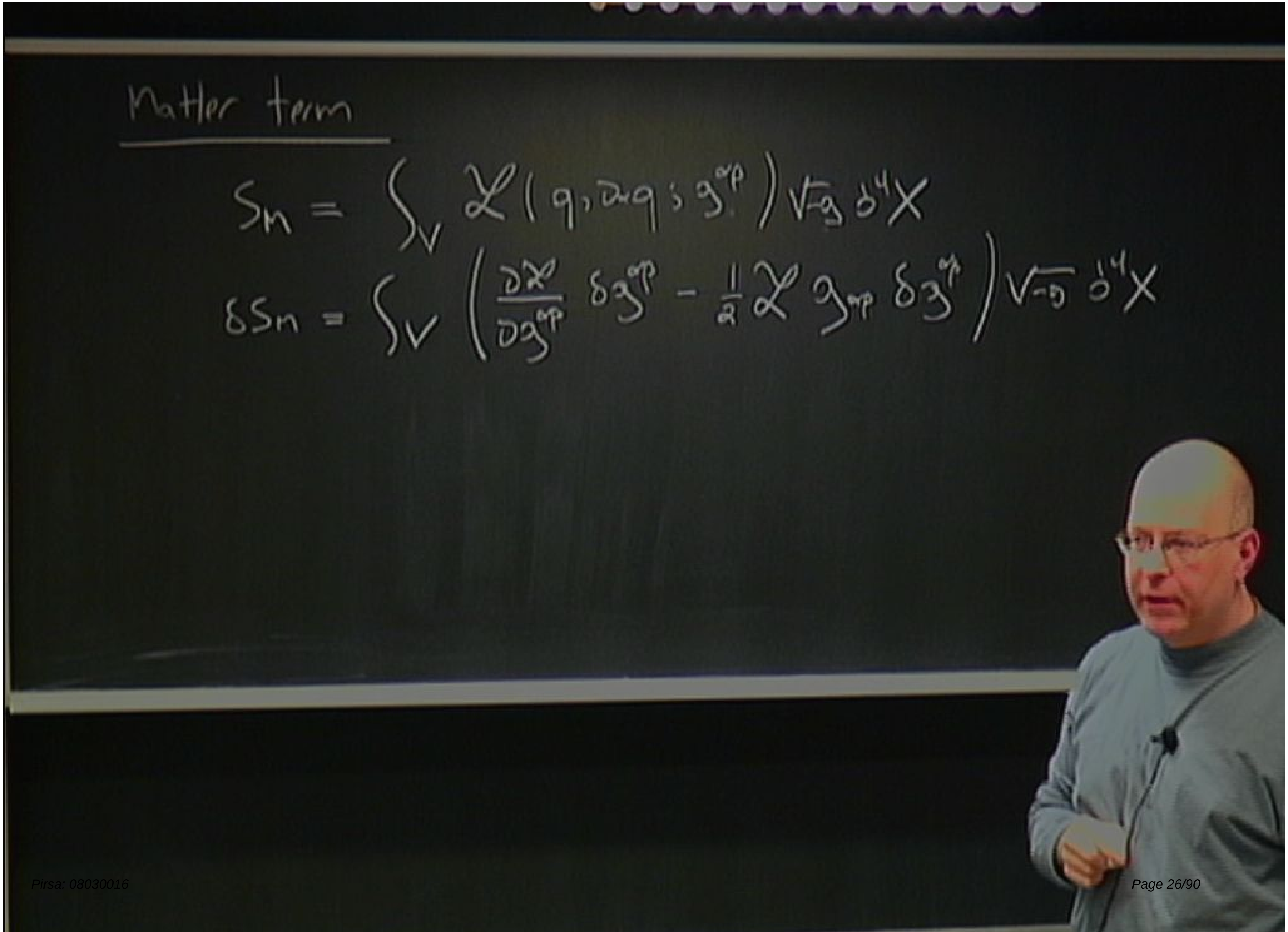

Matter term
$S_m = \int_V \mathcal{L}(q, \partial_\alpha q; g^{\alpha\beta}) \sqrt{-g}\, d^4x$
$\delta S_m = \int_V \frac{\partial \mathcal{L}}{\partial g^{\alpha\beta}} \delta g^{\alpha\beta}$


Matter term
$S_m = \int_V \mathcal{L}(q, \partial_\alpha q ; g^{\alpha\beta}) \sqrt{-g}\, d^4x$
$\delta S_m = \int_V \frac{\partial \mathcal{L}}{\partial g^{\alpha\beta}} \delta g^{\alpha\beta} - \frac{1}{2} \mathcal{L} g_{\alpha\beta} \delta g^{\alpha\beta})$

Matter term

$$S_m = \int_V \mathcal{L}(q, \partial_\alpha q; g^{\alpha\beta}) \sqrt{-g}\, d^4x$$

$$\delta S_m = \int_V \left(\frac{\partial \mathcal{L}}{\partial g^{\alpha\beta}} \delta g^{\alpha\beta} - \frac{1}{2} \mathcal{L} g_{\alpha\beta} \delta g^{\alpha\beta} \right) \sqrt{-g}\, d^4x$$


Matter term
$S_m = \int_V \mathcal{L}(q, \partial_\mu q ; g^{\alpha\beta}) \sqrt{-g}\, d^4x$
$\delta S_m = \int_V \left(\frac{\partial \mathcal{L}}{\partial g^{\alpha\beta}} \delta g^{\alpha\beta} - \frac{1}{2} \mathcal{L} g_{\alpha\beta} \delta g^{\alpha\beta} \right) \sqrt{-g}\, d^4x$

Matter term

$$S_M = \int_V \mathcal{L}(q, \partial_\alpha q ; g^{\alpha\beta}) \sqrt{-g}\, d^4x$$

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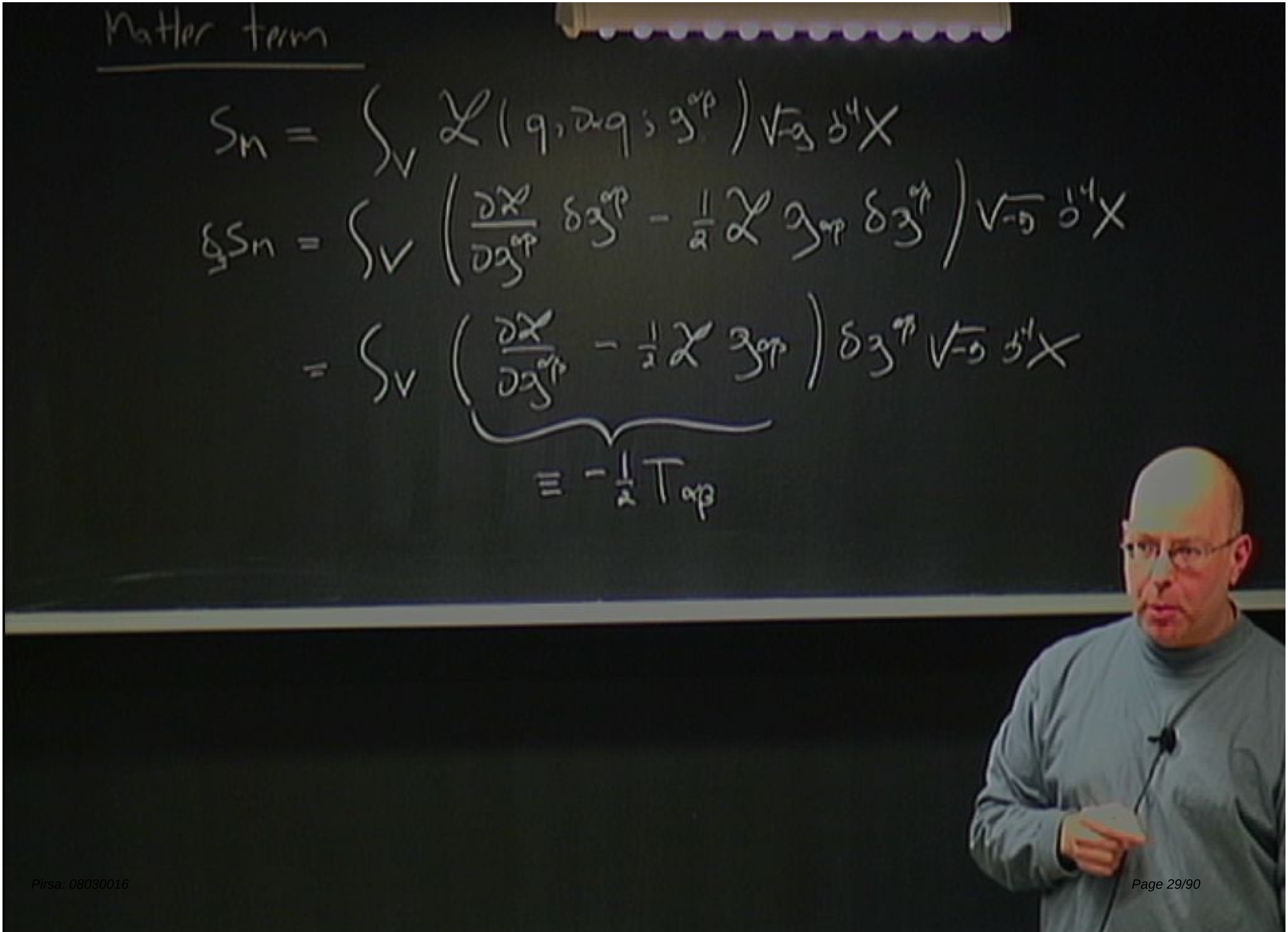
$$= \int_V \left(\frac{\partial \mathcal{L}}{\partial g^{\alpha\beta}} - \frac{1}{2} \mathcal{L} g_{\alpha\beta} \right) \delta g^{\alpha\beta} \sqrt{-g}\, d^4x$$

Matter term

$$S_m = \int_V \mathcal{L}(q, \partial_\mu q; g^{\alpha\beta}) \sqrt{-g}\, d^4x$$

$$\delta_g S_m = \int_V \left(\frac{\partial \mathcal{L}}{\partial g^{\alpha\beta}} \delta g^{\alpha\beta} - \frac{1}{2} \mathcal{L} g_{\alpha\beta} \delta g^{\alpha\beta} \right) \sqrt{-g}\, d^4x$$

$$= \int_V \underbrace{\left(\frac{\partial \mathcal{L}}{\partial g^{\alpha\beta}} - \frac{1}{2} \mathcal{L} g_{\alpha\beta} \right)}_{\equiv -\frac{1}{2} T_{\alpha\beta}} \delta g^{\alpha\beta} \sqrt{-g}\, d^4x$$


Matter term
$S_m = \int_V \mathcal{L}(q, \partial_\alpha q ; g^{\alpha\beta}) \sqrt{-g}\, d^4x$
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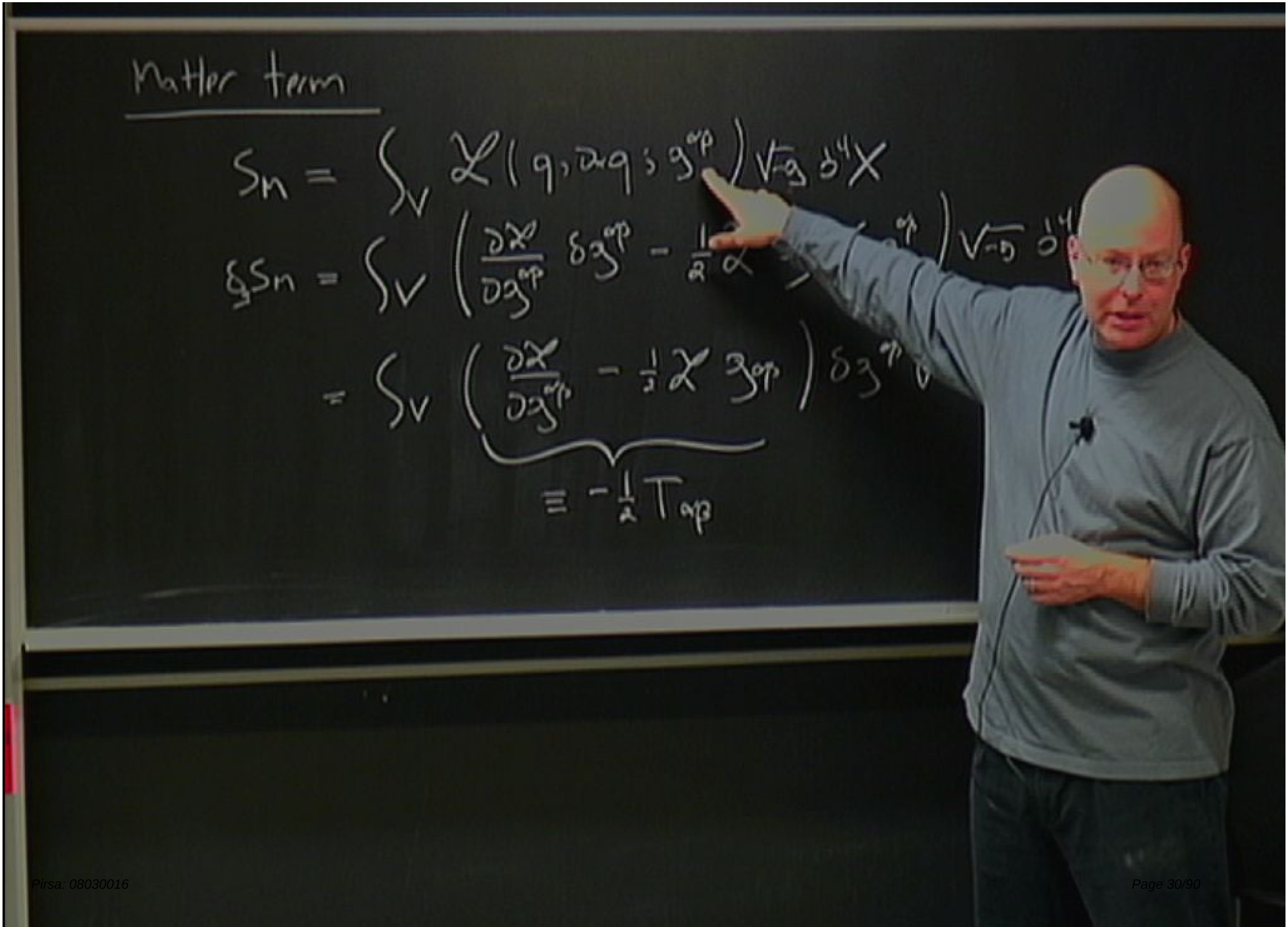
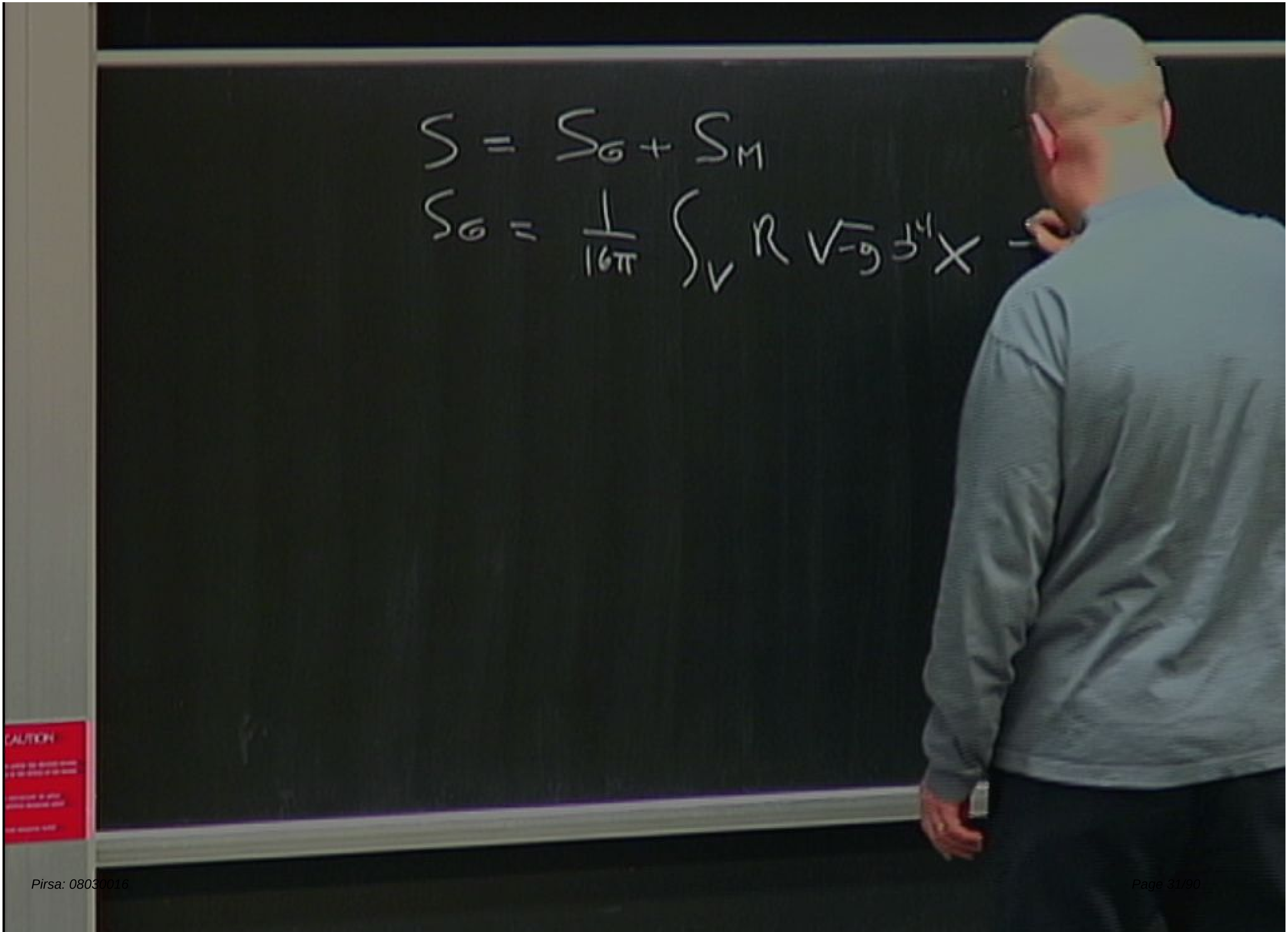
$= \int_V \underbrace{\left(\frac{\partial \mathcal{L}}{\partial g^{\alpha\beta}} - \frac{1}{2} \mathcal{L} g_{\alpha\beta} \right)}_{\equiv -\frac{1}{2} T_{\alpha\beta}} \delta g^{\alpha\beta} \sqrt{-g}\, d^4x$


Matter term
$S_M = \int_V \mathcal{L}(q, \partial_\alpha q; g^{\alpha\beta}) \sqrt{-g}\, d^4x$
$\delta_g S_M = \int_V \left(\frac{\partial \mathcal{L}}{\partial g^{\alpha\beta}} \delta g^{\alpha\beta} - \frac{1}{2} \mathcal{L}\right)$
$= \int_V \underbrace{\left(\frac{\partial \mathcal{L}}{\partial g^{\alpha\beta}} - \frac{1}{2} \mathcal{L}\, g_{\alpha\beta} \right)}_{\equiv -\frac{1}{2} T_{\alpha\beta}} \delta g^{\alpha\beta}$


$S = S_G + S_M$
$S_G = \frac{1}{16\pi} \int_V R \sqrt{-g}\, d^4x$ -

$$S = S_G + S_M$$

$$S_G = \frac{1}{16\pi}\int_V R\sqrt{-g}\,d^4x + \frac{1}{8\pi}\oint_{\partial V}\varepsilon K|h|^{1/2}\,d^3y$$

$$S_M = \int_V \mathcal{L}\sqrt{-g}\,d^4x$$

$$\delta S = \int_V \Big(\underbrace{\frac{1}{16\pi}G_{\alpha\beta} - \frac{1}{2}T_{\alpha\beta}}_{0}\Big)\,\delta g^{\alpha\beta}\sqrt{-g}\,d^4x$$

$$\Rightarrow G_{\alpha\beta} = 8\pi T_{\alpha\beta}$$

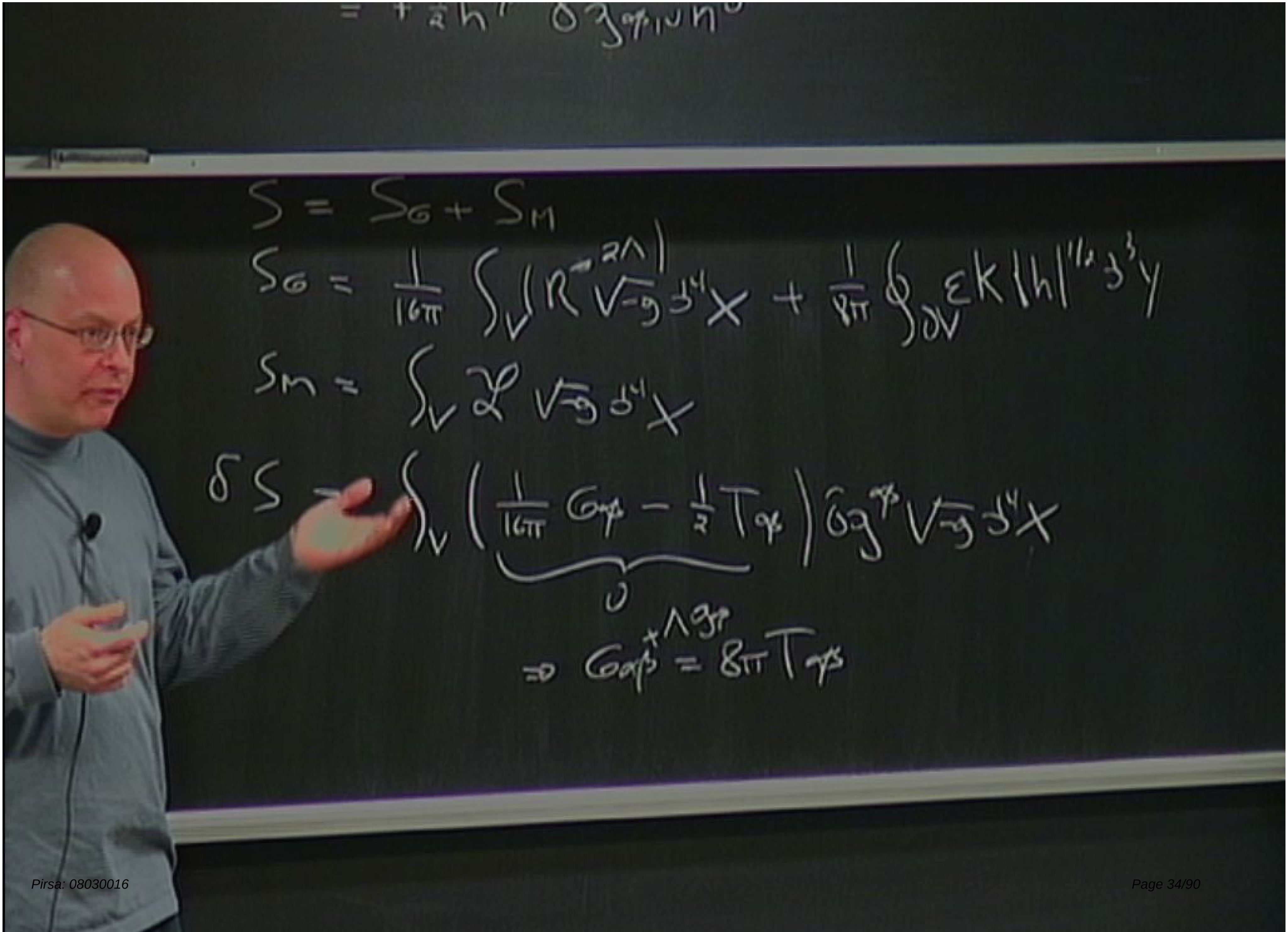
$$S = S_G + S_M$$

$$S_G = \frac{1}{16\pi}\int_V (R - 2\Lambda)\sqrt{-g}\, d^4x + \frac{1}{8\pi}\oint_{\partial V} \varepsilon K |h|^{1/2} d^3y$$

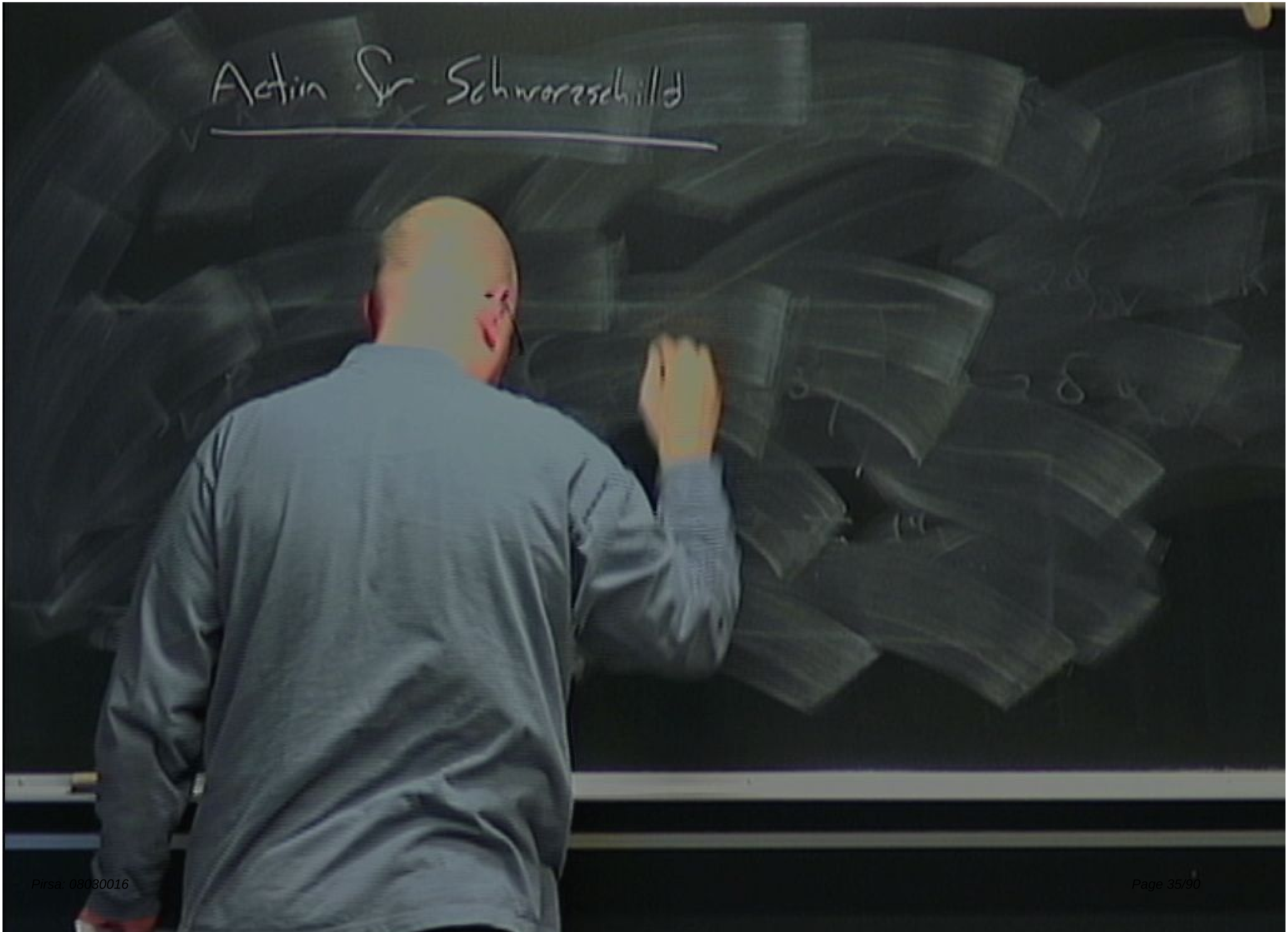
$$S_M = \int_V \mathcal{L}\sqrt{-g}\, d^4x$$

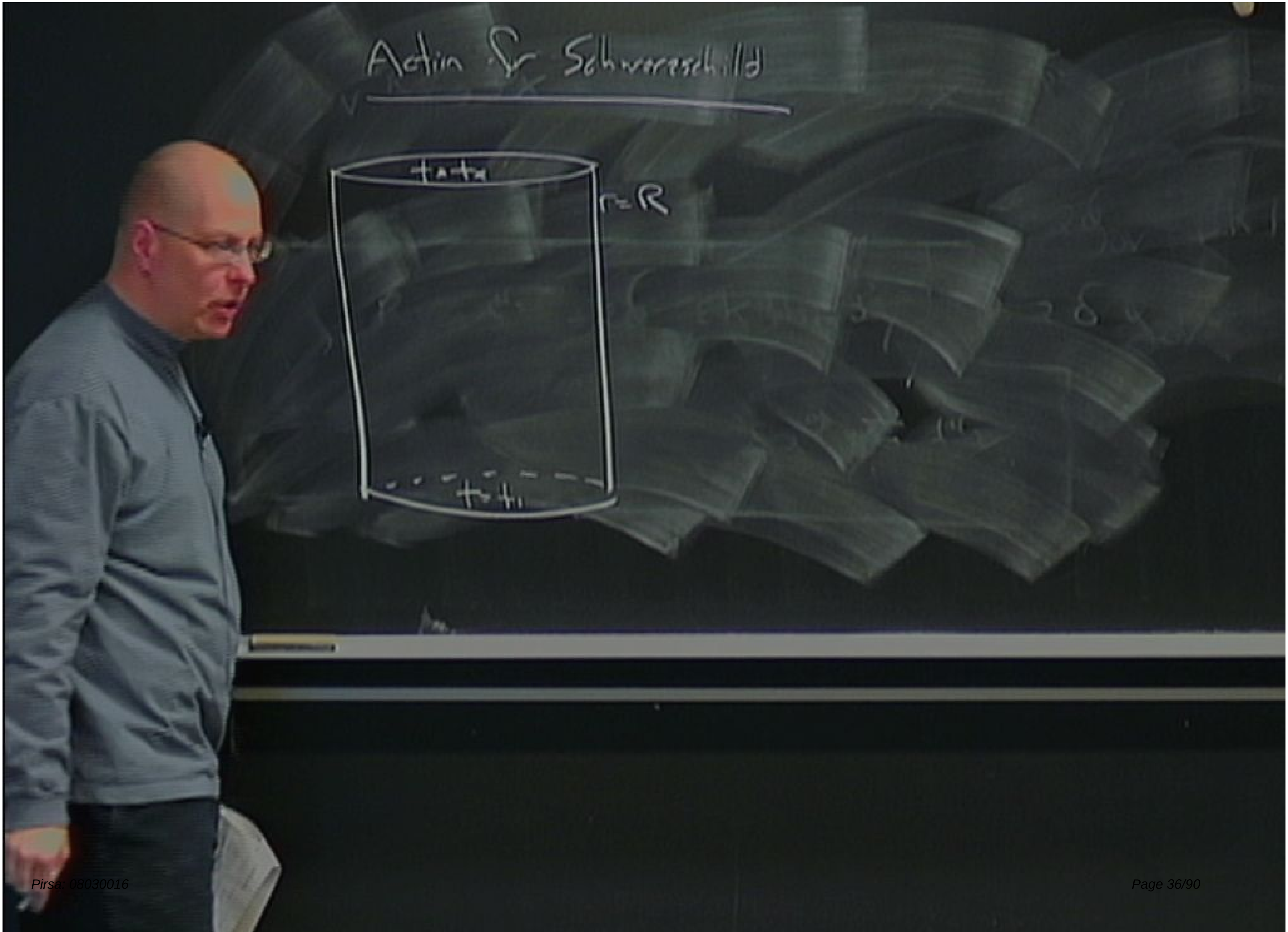
$$\delta S = \int_V \Big(\underbrace{\frac{1}{16\pi} G_{\alpha\beta} - \frac{1}{2} T_{\alpha\beta}}_{0}\Big)\delta g^{\alpha\beta}\sqrt{-g}\, d^4x$$

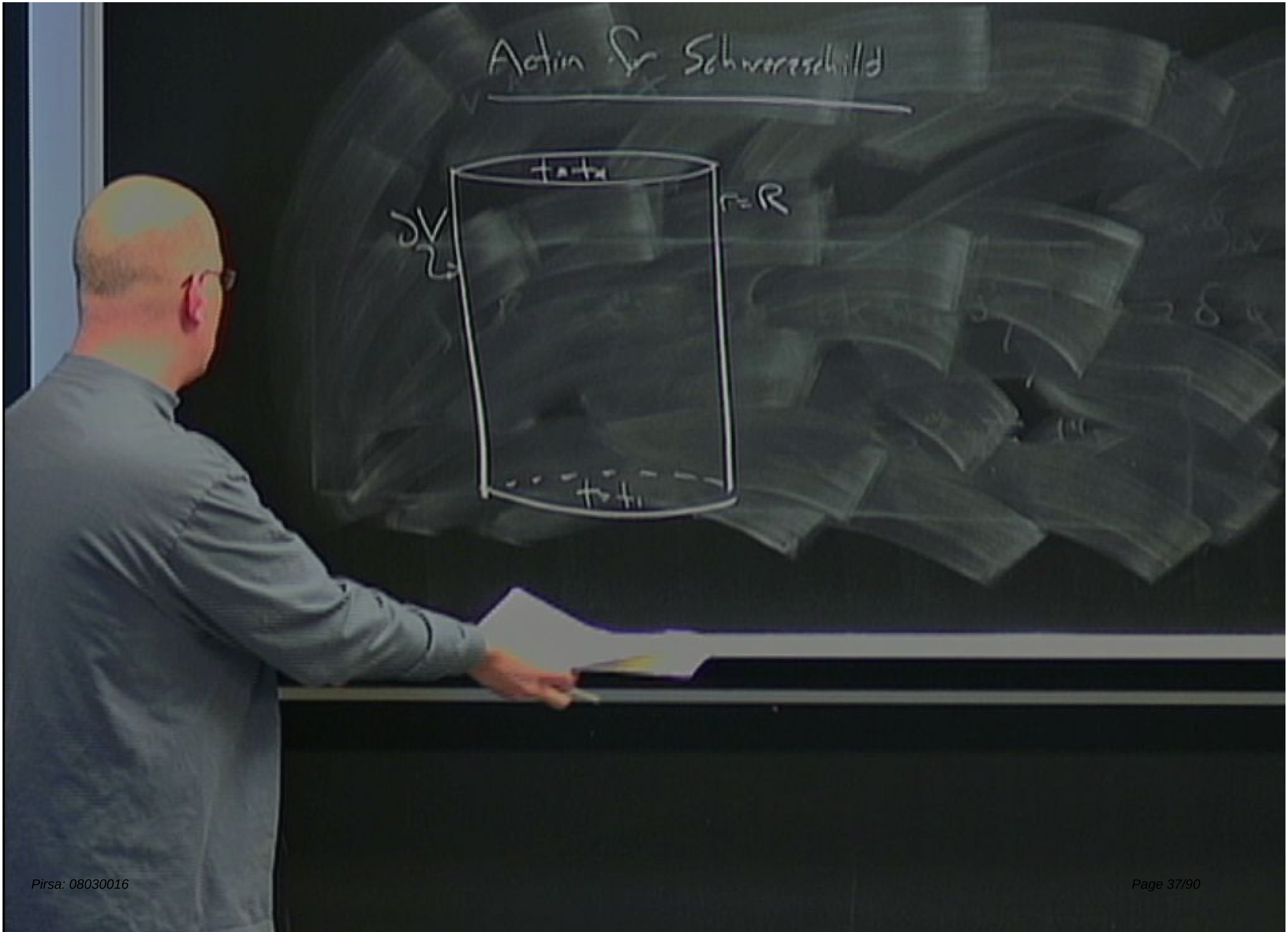
$$\Rightarrow G_{\alpha\beta} + \Lambda g_{\alpha\beta} = 8\pi T_{\alpha\beta}$$



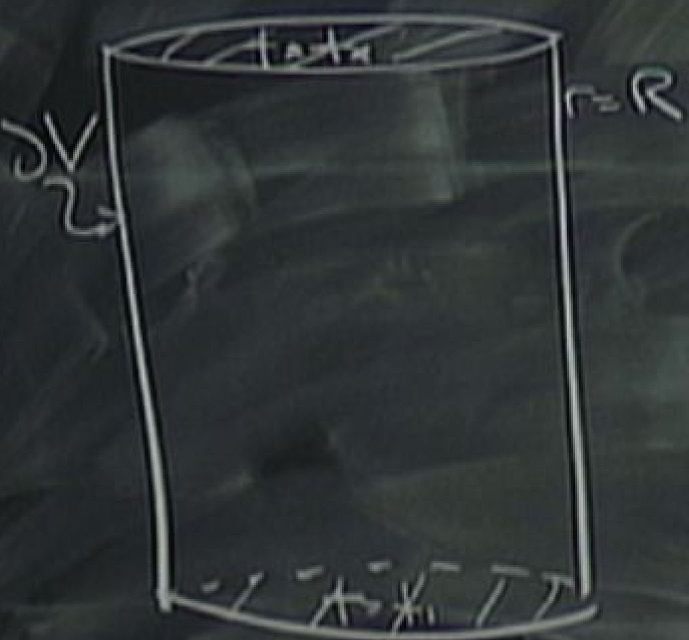
$S = S_G + S_M$
$S_G = \frac{1}{16\pi}\int_V (R - 2\Lambda)\sqrt{-g}\, d^4x + \frac{1}{8\pi}\oint_{\partial V} \varepsilon K |h|^{1/2} d^3y$
$S_M = \int_V \mathcal{L}\sqrt{-g}\, d^4x$
$\delta S = \int_V \underbrace{\left(\frac{1}{16\pi} G_{\alpha\beta} - \frac{1}{2} T_{\alpha\beta}\right)}_{0} \delta g^{\alpha\beta}\sqrt{-g}\, d^4x$
$\Rightarrow G_{\alpha\beta} + \Lambda g_{\alpha\beta} = 8\pi T_{\alpha\beta}$


Action for Schwarzschild


Action for Schwarzschild
r=R
t=t2
t=t1

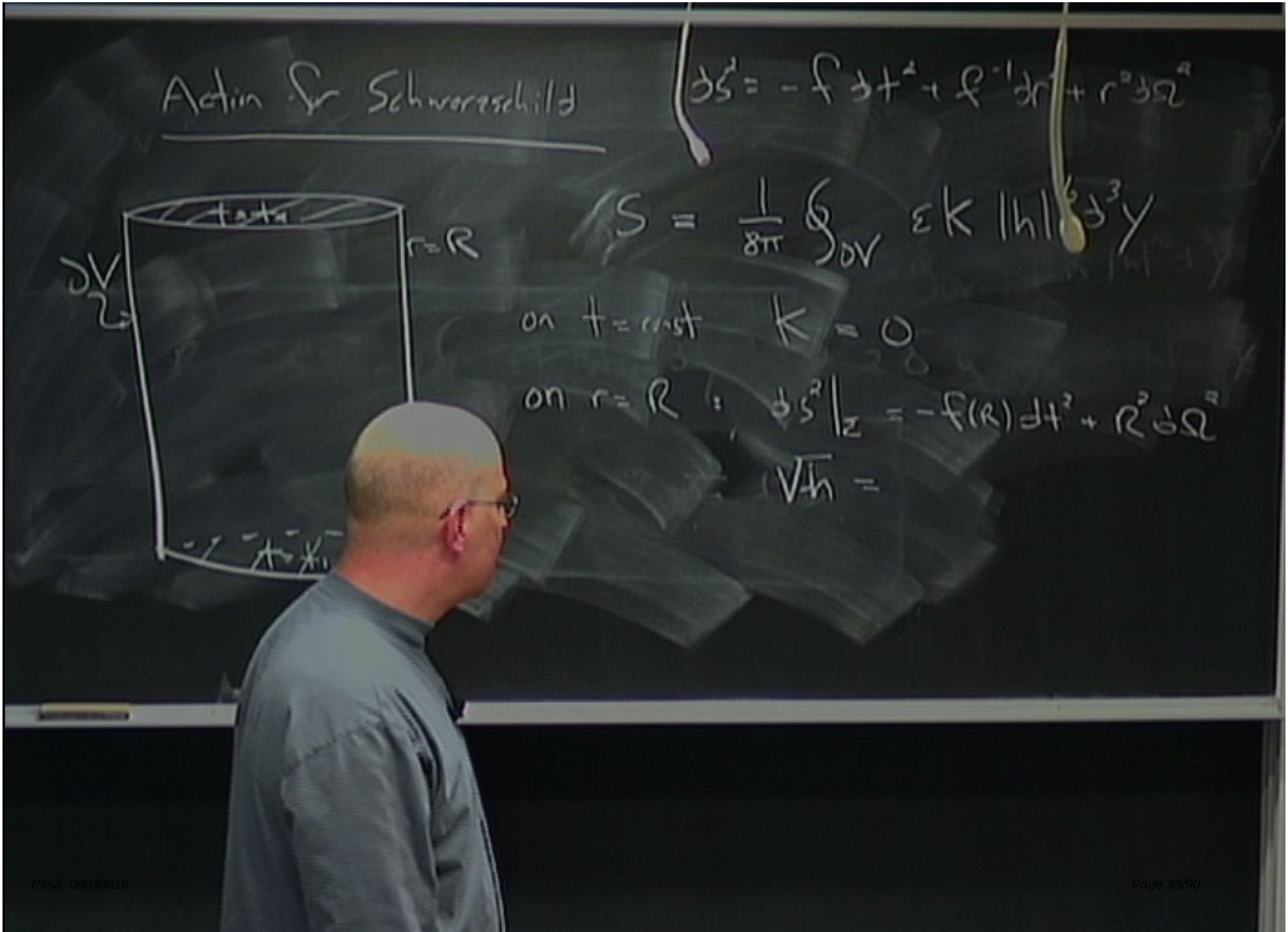

Action for Schwarzschild
t=t2
∂V
r=R
t=t1

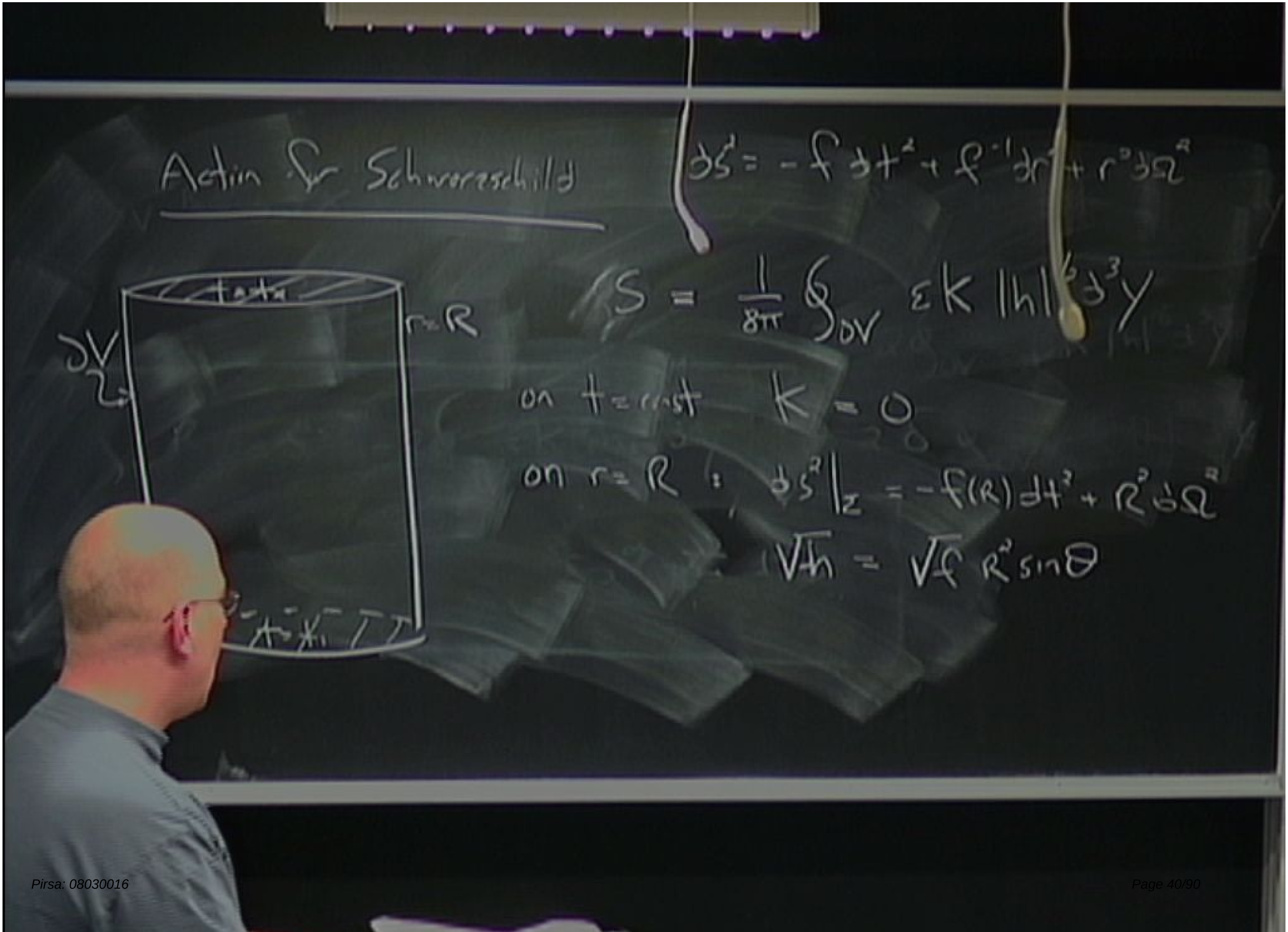
Action for Schwarzschild

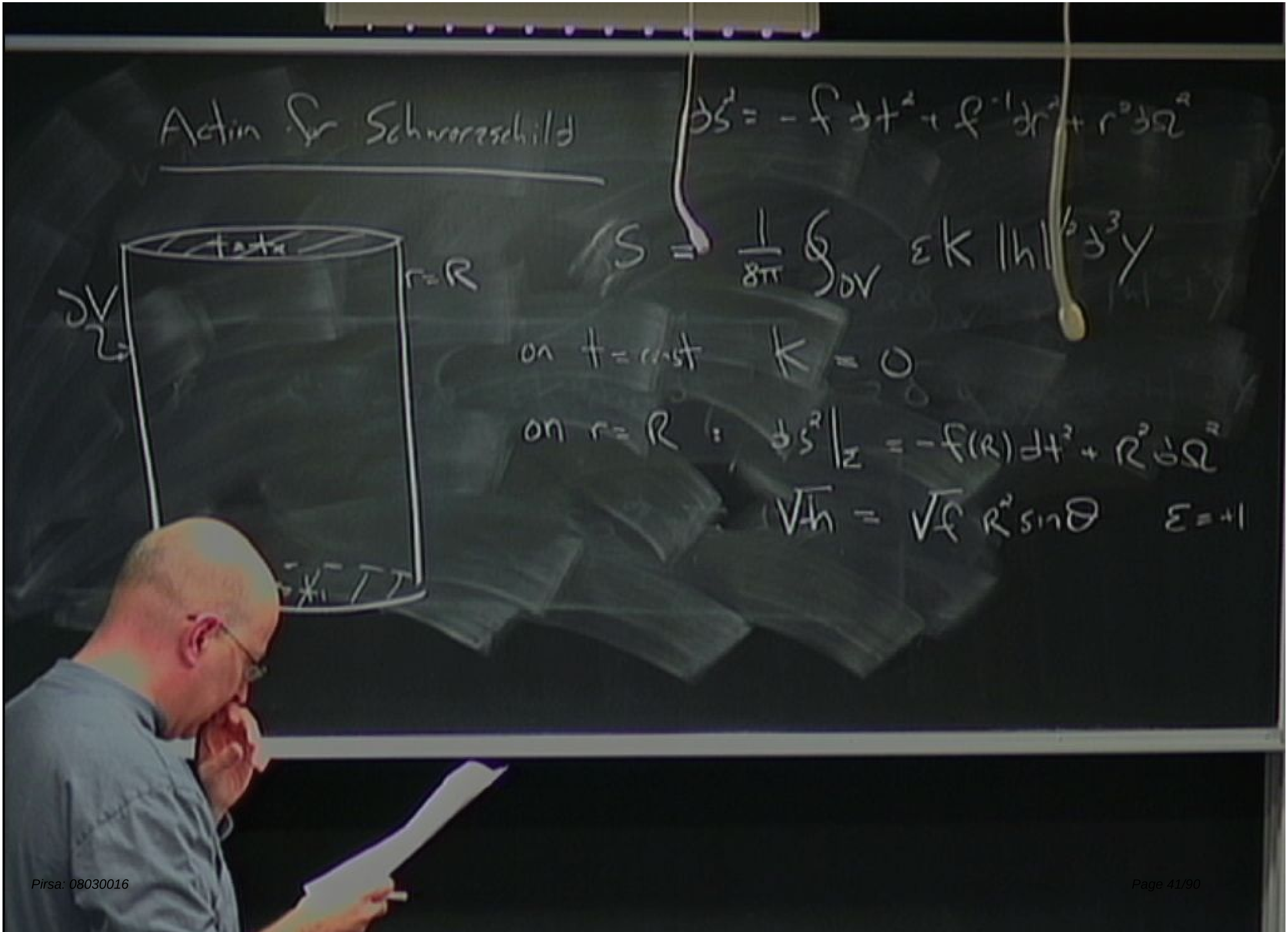


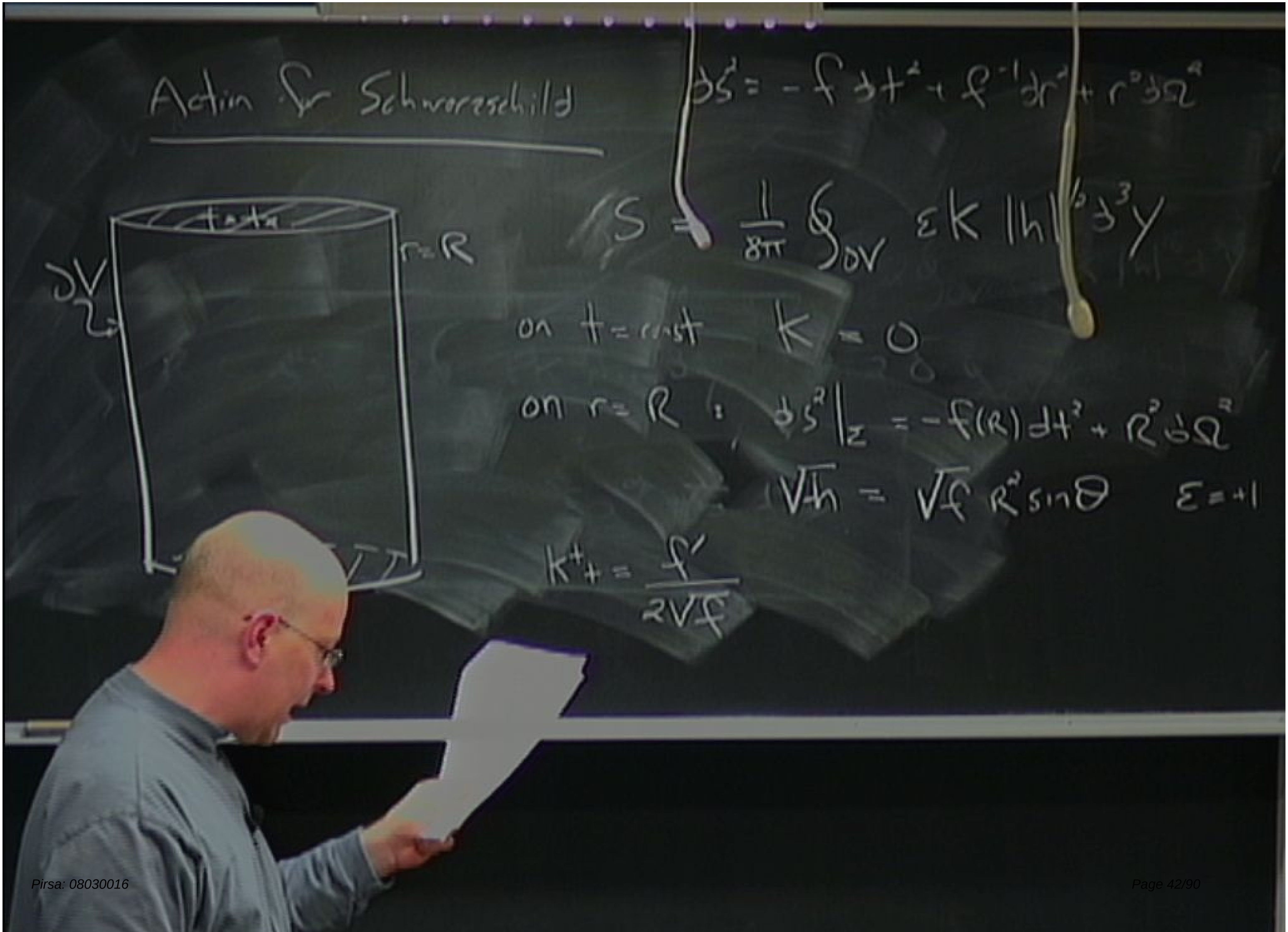
$$S = \frac{1}{8\pi} \oint_{\partial V} \varepsilon K \, |h|^{1/2} d^3y$$

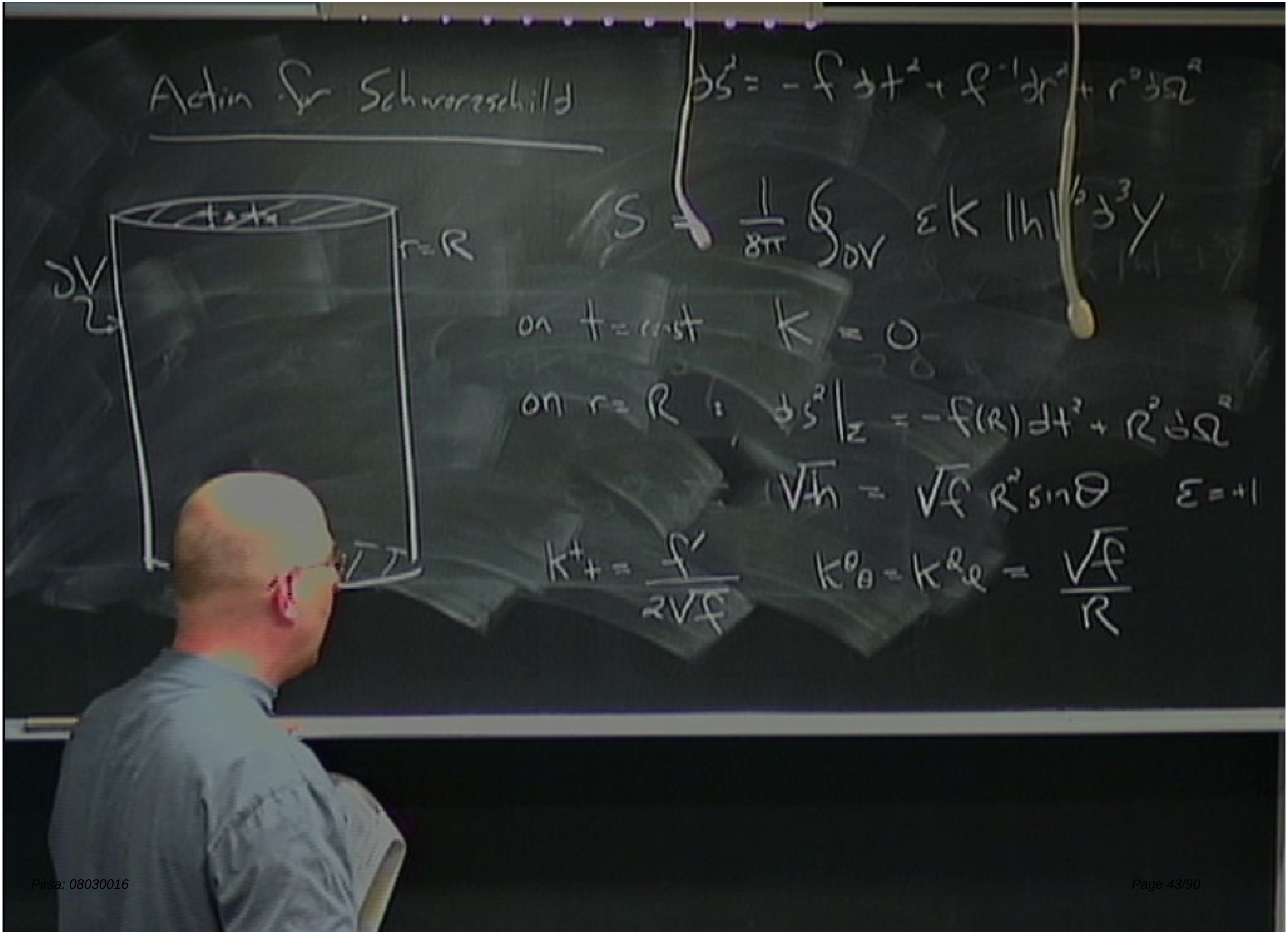
on $t = \text{const}$ $\quad K = 0$


Action for Schwarzschild
$ds^2 = -f\,dt^2 + f^{-1}dr^2 + r^2 d\Omega^2$
$S = \frac{1}{8\pi}\oint_{\partial V} \varepsilon K |h|^{1/2} d^3y$
on $t = $ const $K = 0$
on $r = R$: $ds^2|_\Sigma = -f(R)\,dt^2 + R^2 d\Omega^2$
$\sqrt{-h} =$
∂V
$r=R$
$t=t_2$
$t=t_1$


Action for Schwarzschild
$ds^2 = -f\,dt^2 + f^{-1}dr^2 + r^2 d\Omega^2$
$S = \frac{1}{8\pi}\oint_{\partial V} \varepsilon K\, |h|^{1/2} d^3y$
∂V
$r=R$
on $t = \text{const}$ $K = 0$
on $r = R$: $ds^2|_\Sigma = -f(R)\,dt^2 + R^2 d\Omega^2$
$\sqrt{-h} = \sqrt{f}\, R^2 \sin\theta$


Action for Schwarzschild
$ds^2 = -f\,dt^2 + f^{-1}dr^2 + r^2 d\Omega^2$
$S = \frac{1}{8\pi}\oint_{\partial V} \varepsilon K |h|^{1/2} d^3y$
on $t = \text{const}$ $K = 0$
on $r = R$: $ds^2\big|_\Sigma = -f(R)\,dt^2 + R^2 d\Omega^2$
$\sqrt{-h} = \sqrt{f}\, R^2 \sin\theta$ $\varepsilon = +1$
$r = R$
∂V


Action for Schwarzschild
$ds^2 = -f\,dt^2 + f^{-1}dr^2 + r^2 d\Omega^2$
∂V
$r=R$
$S = \frac{1}{8\pi}\oint_{\partial V} \varepsilon K |h|^{1/2} d^3y$
on t = const $K = 0$
on $r = R$: $ds^2|_\Sigma = -f(R)\,dt^2 + R^2 d\Omega^2$
$\sqrt{-h} = \sqrt{f}\, R^2 \sin\theta$ $\varepsilon = +1$
$K^t{}_t = \frac{f'}{2\sqrt{f}}$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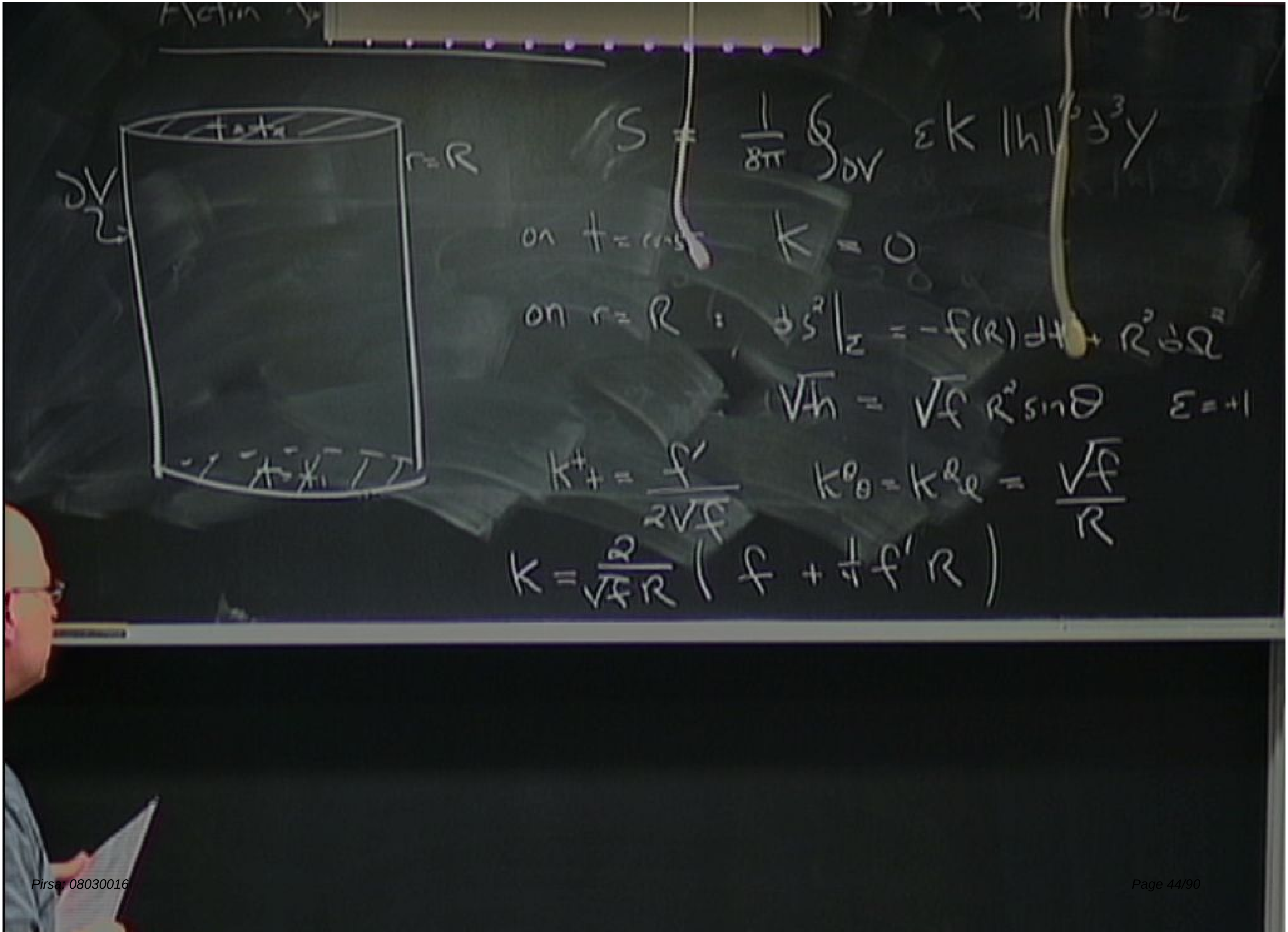
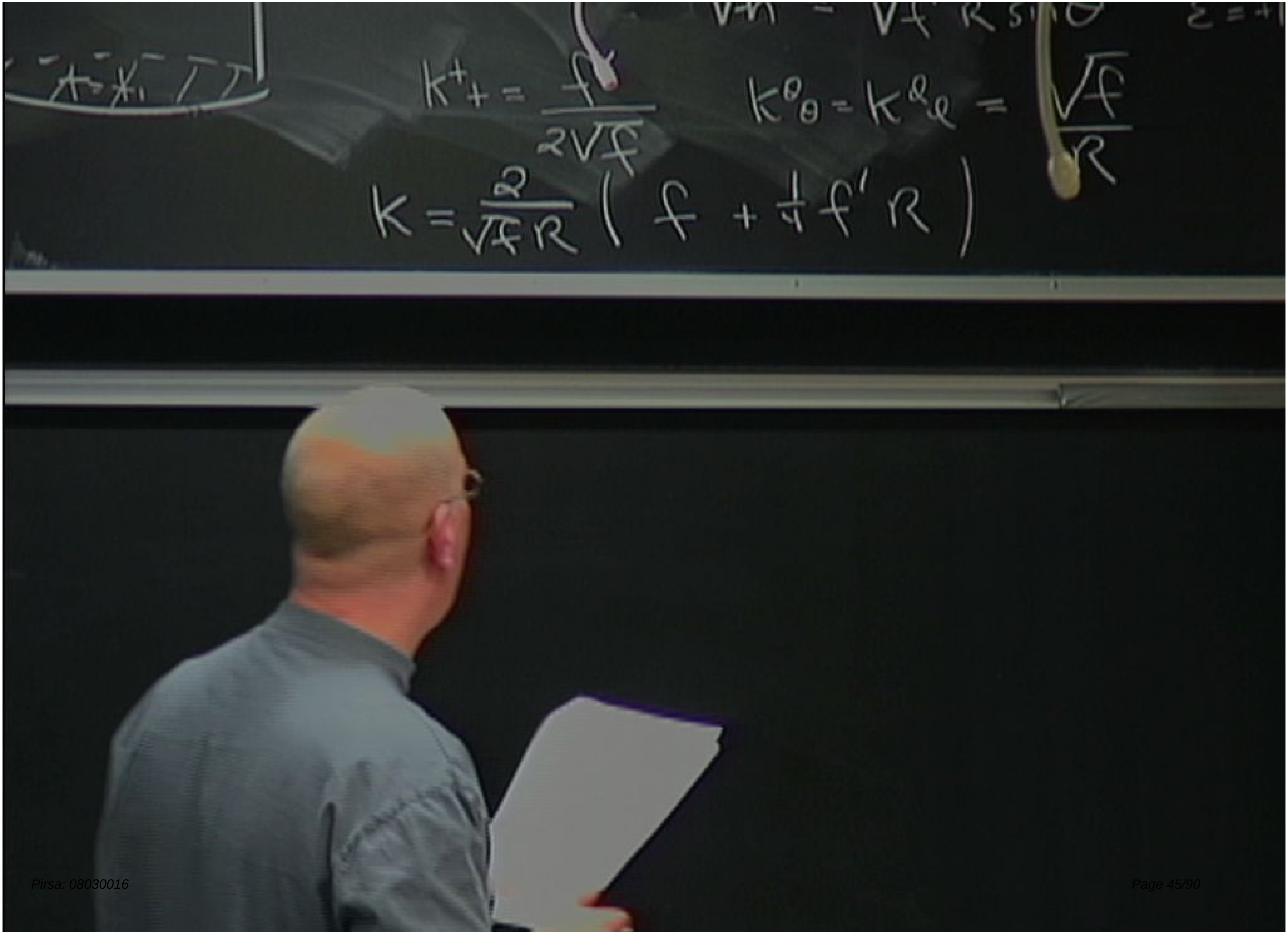
$ds^2 = -f\,dt^2 + f^{-1}dr^2 + r^2 d\Omega^2$
$S = \frac{1}{8\pi}\oint_{\partial V} \varepsilon K |h|^{1/2} d^3y$
∂V
$r = R$
on $t = \text{const}$ $K = 0$
on $r = R$: $ds^2\big|_\Sigma = -f(R)\,dt^2 + R^2 d\Omega^2$
$\sqrt{-h} = \sqrt{f}\, R^2 \sin\theta$ $\varepsilon = +1$
$K^t{}_t = \frac{f'}{2\sqrt{f}}$ $K^\theta{}_\theta = K^\phi{}_\phi = \frac{\sqrt{f}}{R}$



$t = t_2$
$r = R$
∂V
$t = t_1$
$S = \frac{1}{8\pi} \oint_{\partial V} \varepsilon K |h|^{1/2} d^3y$
on $t = \text{const}$ $K = 0$
on $r = R$: $ds^2|_\Sigma = -f(R)\,dt^2 + R^2 d\Omega^2$
$\sqrt{-h} = \sqrt{f}\, R^2 \sin\theta$ $\varepsilon = +1$
$K^t{}_t = \frac{f'}{2\sqrt{f}}$ $K^\theta{}_\theta = K^\phi{}_\phi = \frac{\sqrt{f}}{R}$
$K = \frac{2}{\sqrt{f} R}\left(f + \frac{1}{4} f' R \right)$


$k^+{}_+ = \frac{f'}{2\sqrt{f}}$
$k^\theta{}_\theta = k^\varphi{}_\varphi = \frac{\sqrt{f}}{R}$
$K = \frac{2}{\sqrt{f}R}\left(f + \frac{1}{4} f' R \right)$

on $r=R$: $ds^2\big|_\Sigma = -f(R)\,dt^2 + R^2\,d\Omega^2$

$\sqrt{-h} = \sqrt{f}\,R^2\sin\theta \qquad \varepsilon = +1$

$K^t{}_t = \dfrac{f'}{2\sqrt{f}} \qquad K^\theta{}_\theta = K^\varphi{}_\varphi = \dfrac{\sqrt{f}}{R}$

$$K = \frac{2}{\sqrt{f}R}\left(f + \tfrac{1}{4} f' R \right)$$

$$S = \frac{1}{8\pi}\int \frac{2}{\sqrt{f}R}\left(f + \tfrac{1}{4} f' R\right)\sqrt{f}\,R^2\,d\Omega\,dt$$

$$S = \frac{1}{8\pi} \int \frac{2}{\cancel{\sqrt{f}}\, R} \left(f + \frac{1}{4} f' R \right) \cancel{\sqrt{f}}\, R^2 \, d\Omega \, dt$$

$$= \frac{1}{4\pi} \int R \left(1 - \frac{3m}{2R} \right) d\Omega \, dt$$

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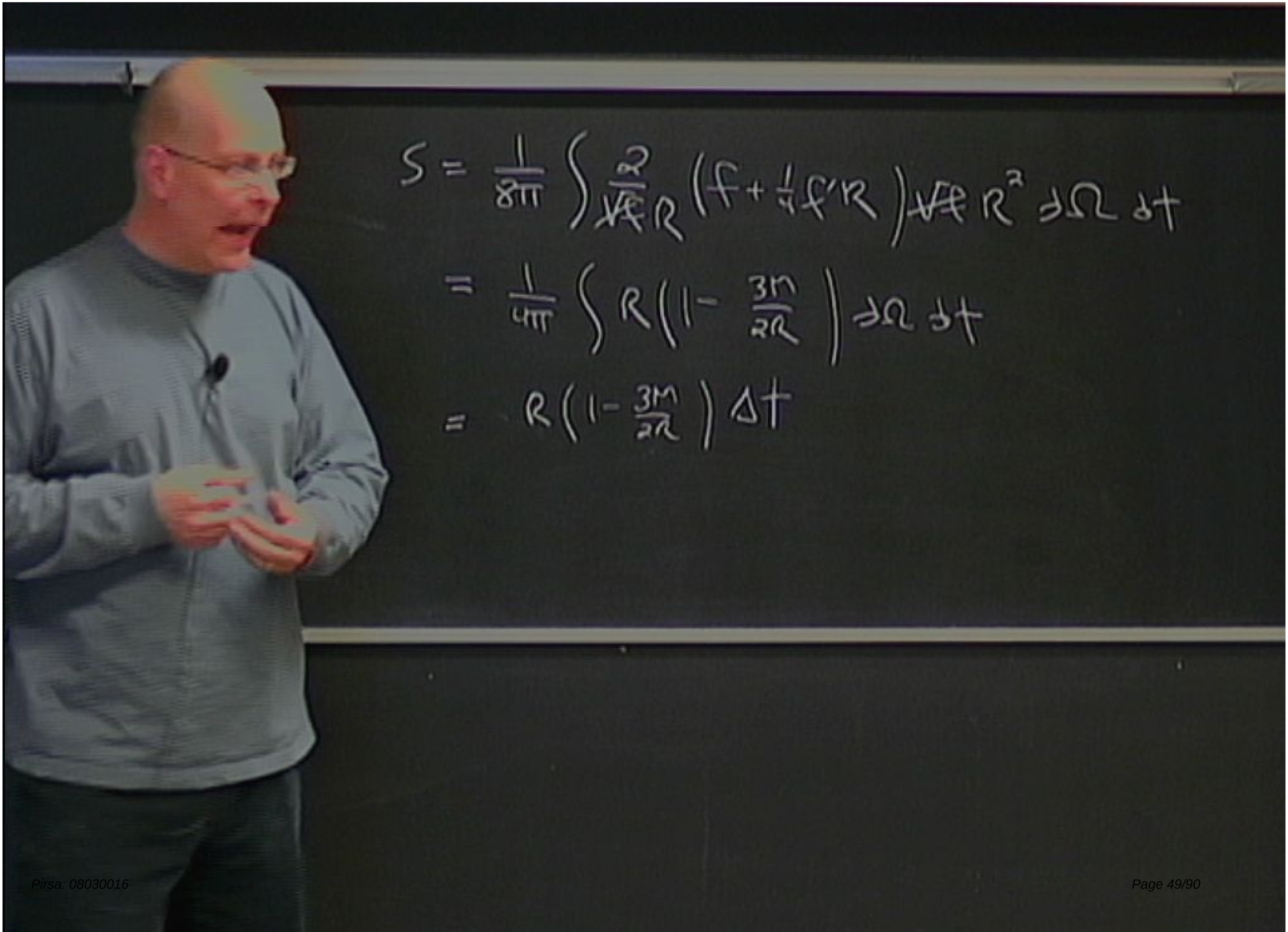
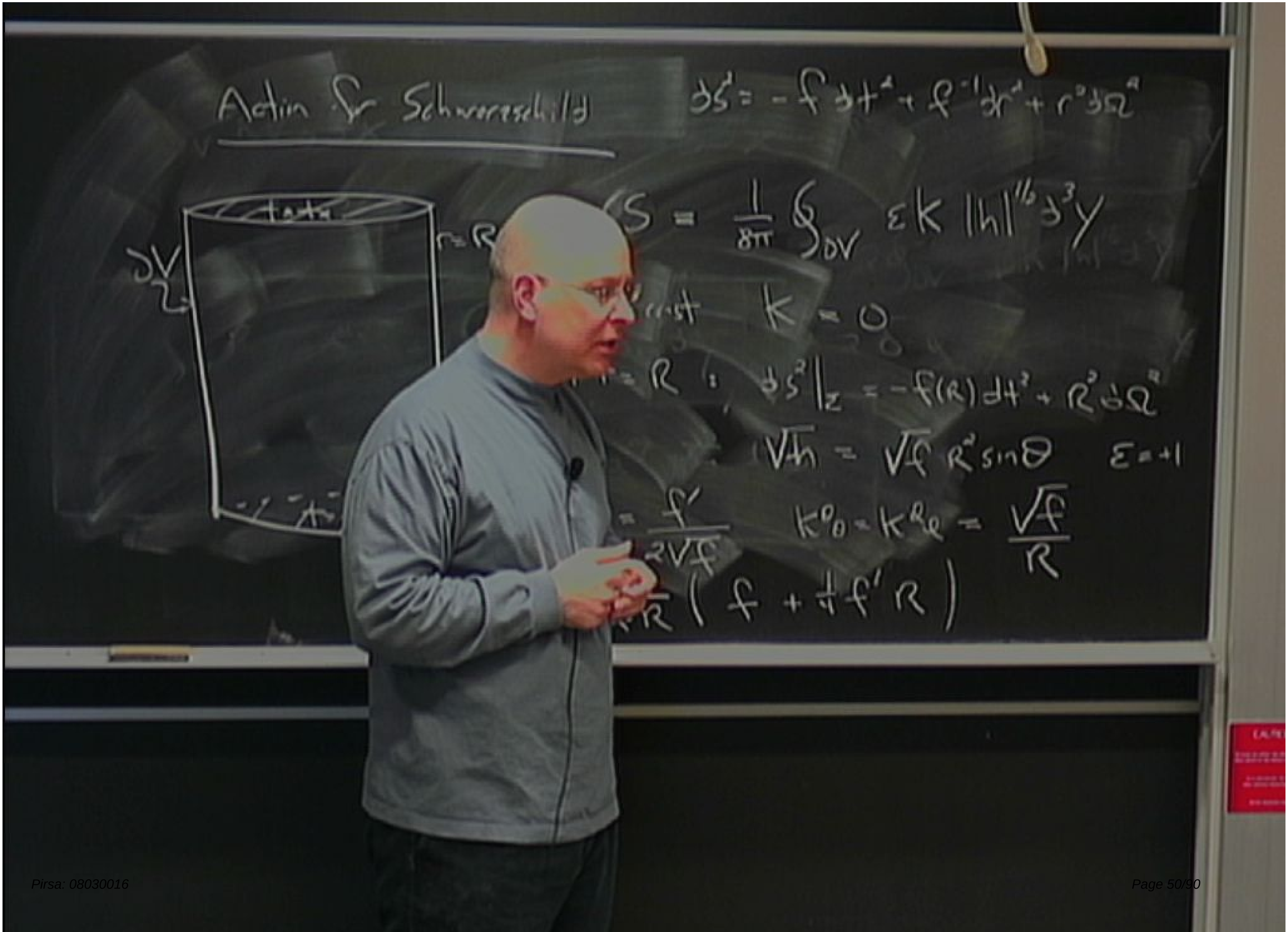
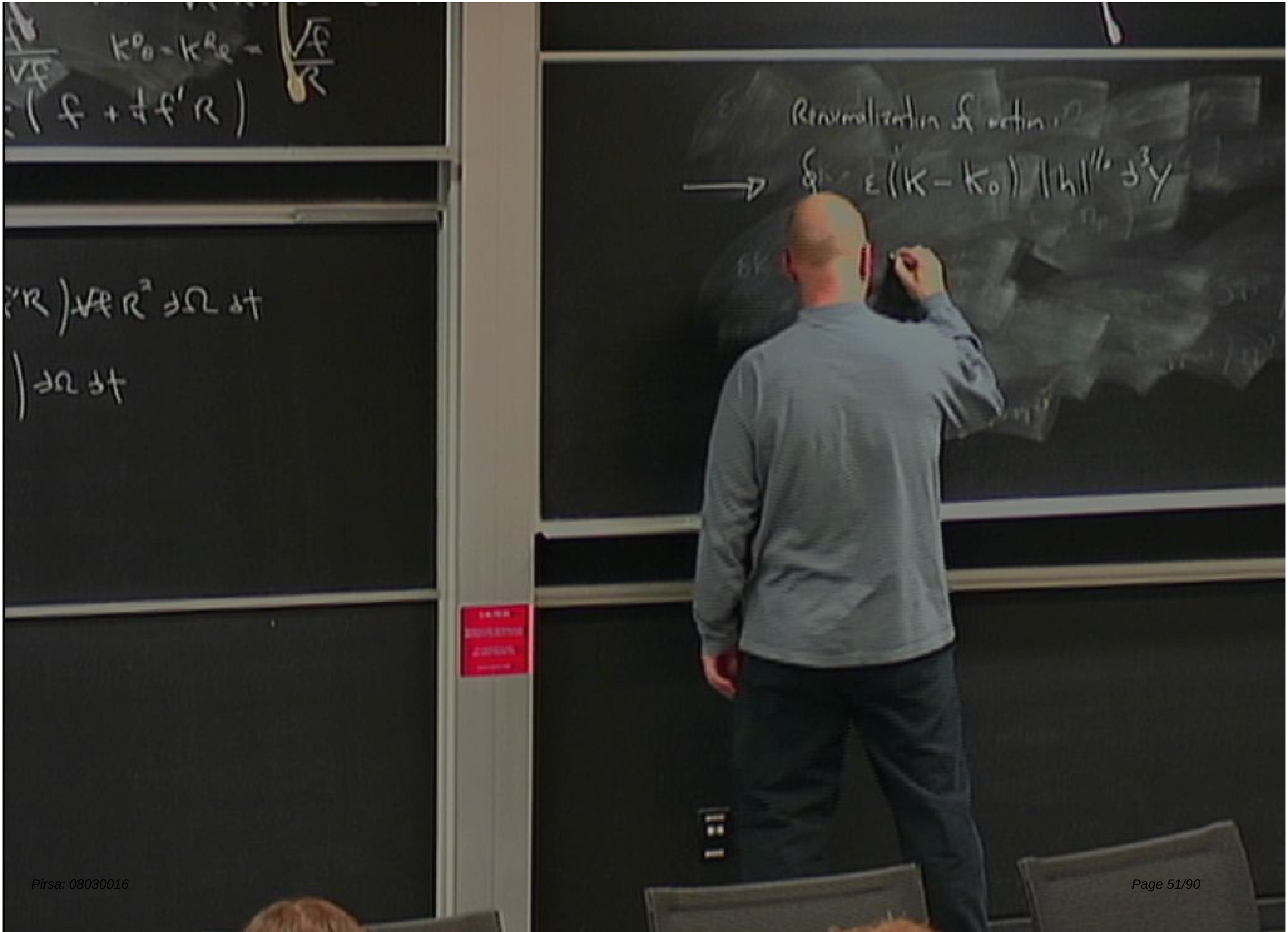
$$= \frac{1}{4\pi} \int R \left(1 - \frac{3m}{2R} \right) d\Omega \, dt$$

$$R \left(1 - \frac{3m}{2R} \right) \Delta t$$


$S = \frac{1}{8\pi} \int \frac{2}{\cancel{\sqrt{f}} R} \left(f + \frac{1}{4} f' R \right) \cancel{\sqrt{f}} R^2 \, d\Omega \, dt$
$= \frac{1}{4\pi} \int R \left(1 - \frac{3m}{2R} \right) d\Omega \, dt$
$= R \left(1 - \frac{3m}{2R} \right) \Delta t$


Action fr Schwarzschild
$ds^2 = -f\,dt^2 + f^{-1}dr^2 + r^2 d\Omega^2$
∂V
$r = R$
$S = \frac{1}{8\pi}\oint_{\partial V} \varepsilon K |h|^{1/2} d^3y$
$K = 0$
$ds^2|_\Sigma = -f(R)\,dt^2 + R^2 d\Omega^2$
$\sqrt{-h} = \sqrt{f}\,R^2 \sin\theta$
$\varepsilon = +1$
$K^\theta{}_\theta = K^\phi{}_\phi = \frac{\sqrt{f}}{R}$


Renormalization of action:

$$S = \frac{1}{8\pi}\int \frac{2}{\cancel{\sqrt{f}}\,R}\left(f + \frac{1}{4}f'R\right)\cancel{\sqrt{f}}\,R^2\, d\Omega\, dt$$

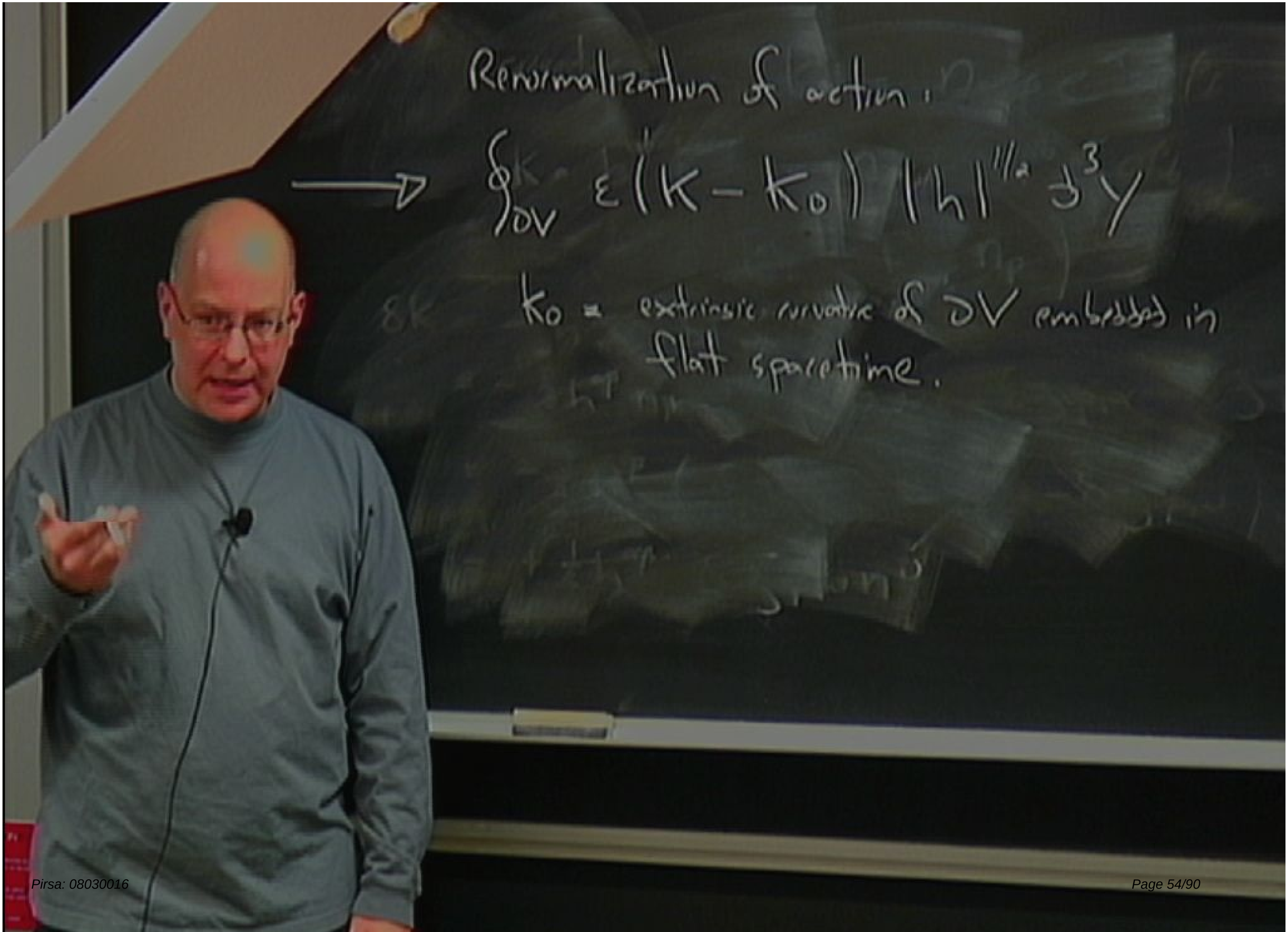
$$= \frac{1}{4\pi}\int R\left(1 - \frac{3m}{2R}\right) d\Omega\, dt$$

$$= R\left(1 - \frac{3m}{2R}\right)\Delta t$$

Renormalization of action:

$$\longrightarrow \int_{\partial V} \varepsilon (K - K_0) |h|^{1/2} d^3y$$

K_0 = extrinsic curvature of ∂V embedded in flat spacetime.


Renormalization of action:
$\rightarrow \oint_{\partial V} \varepsilon (K - K_0) |h|^{1/2} d^3y$
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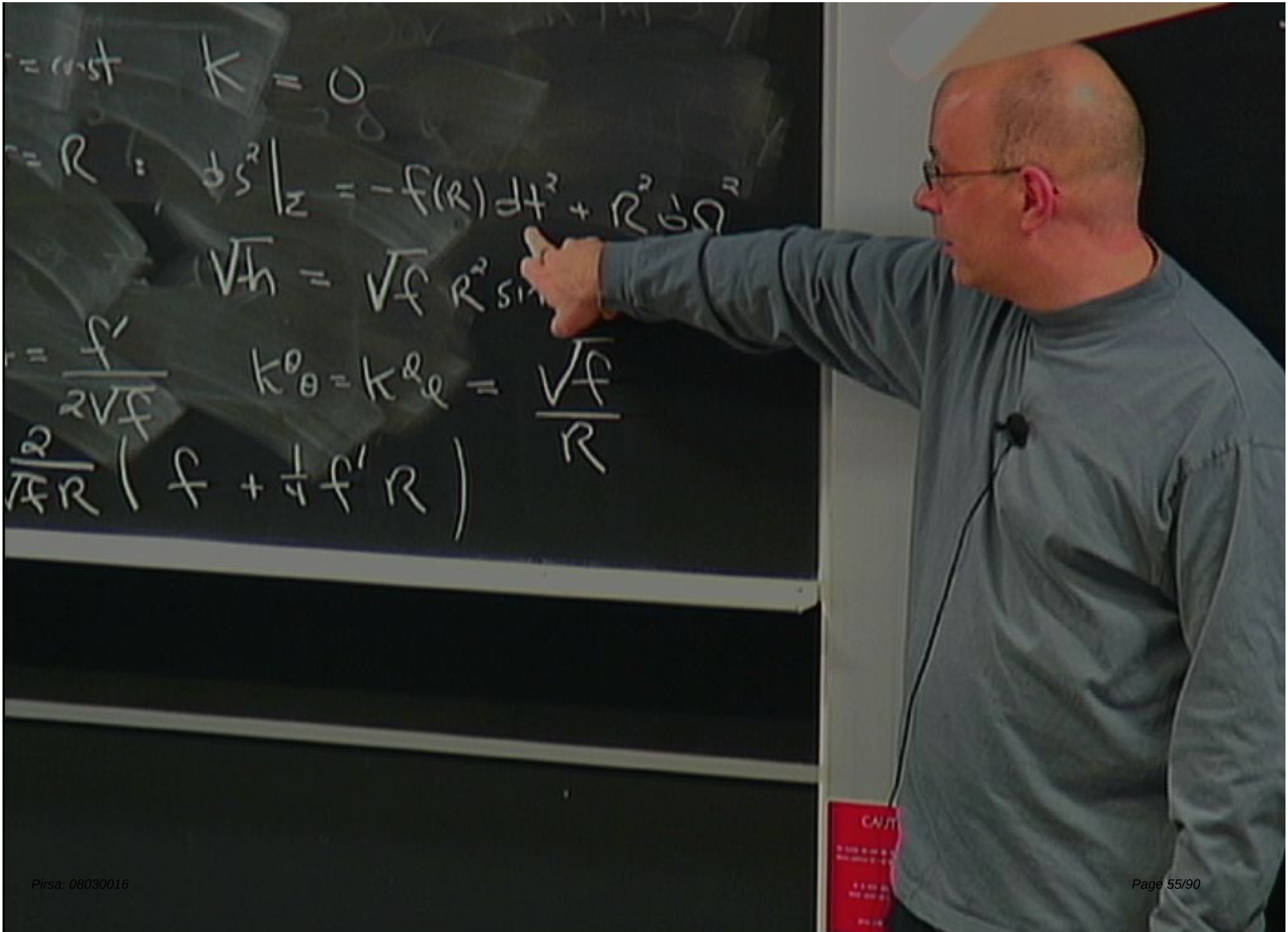
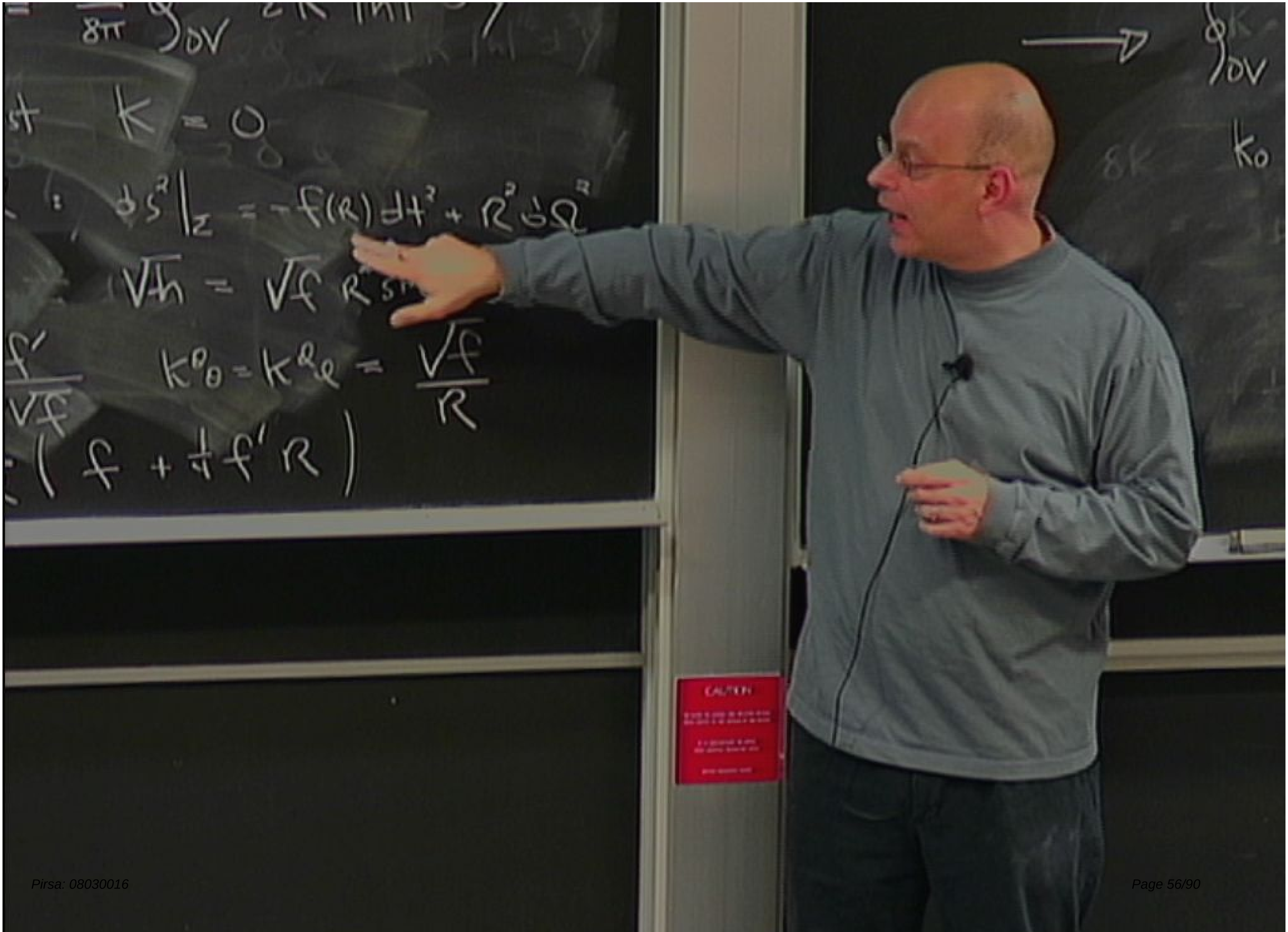
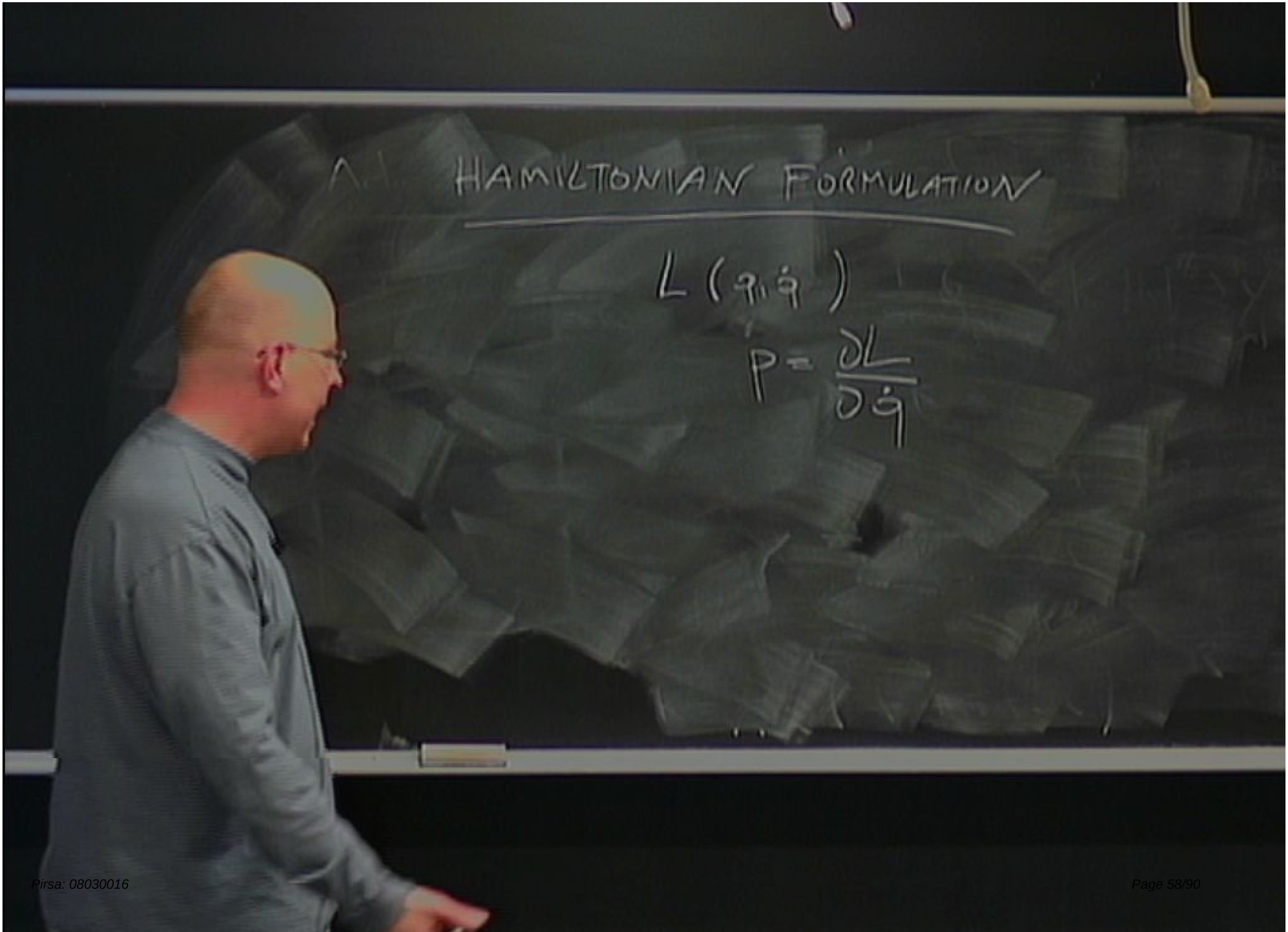

= const K = 0
= R : ds²|Σ = −f(R) dt² + R² dΩ²
√h = √f R² sin
kθθ = kφφ = √f / R

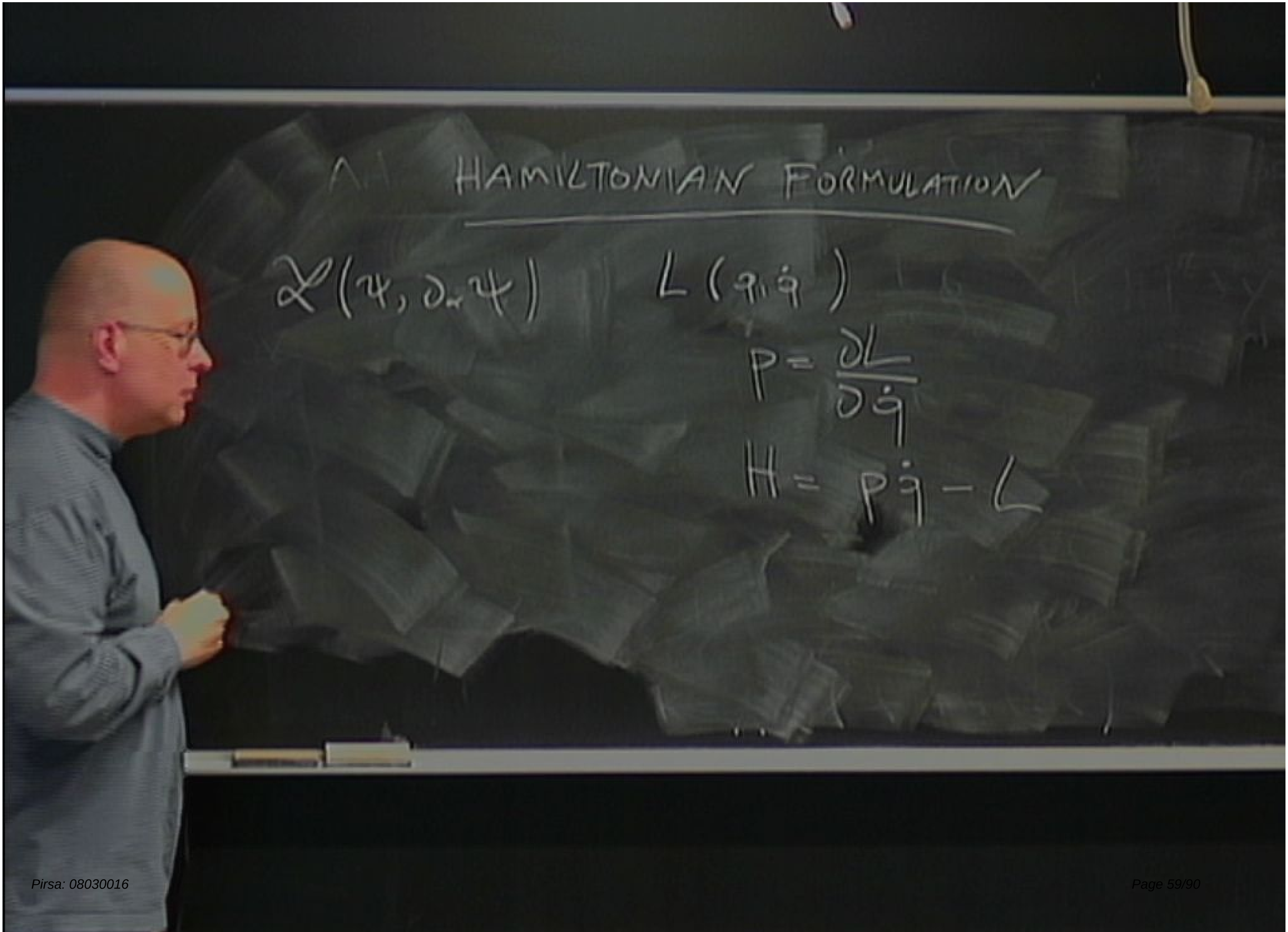

K = 0
ds²|Σ = −f(R) dt² + R² dΩ²
√h = √f R
kθθ = kφφ = √f / R
(f + ½ f′ R)
k₀
CAUTION

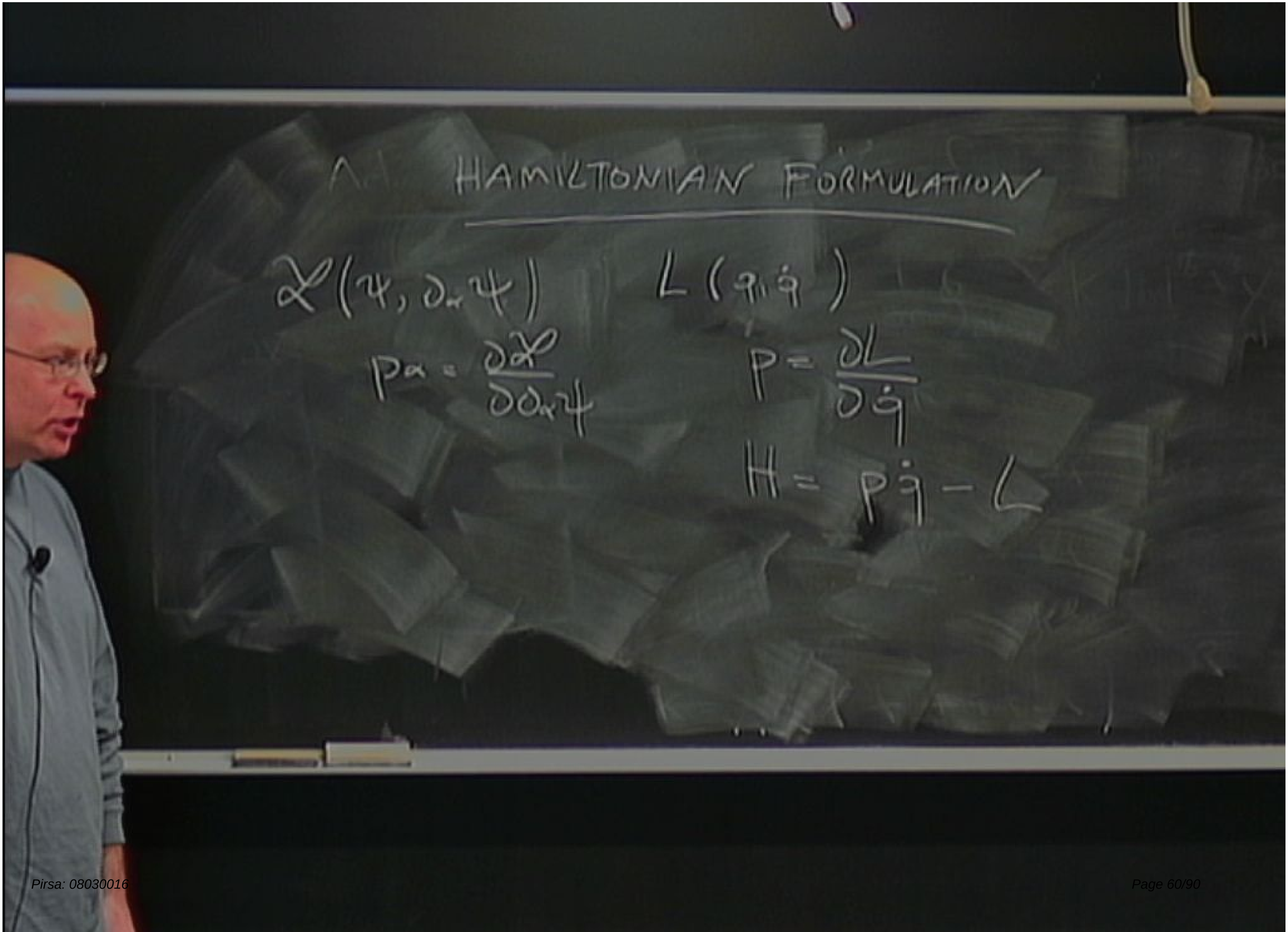
Renormalization of action:

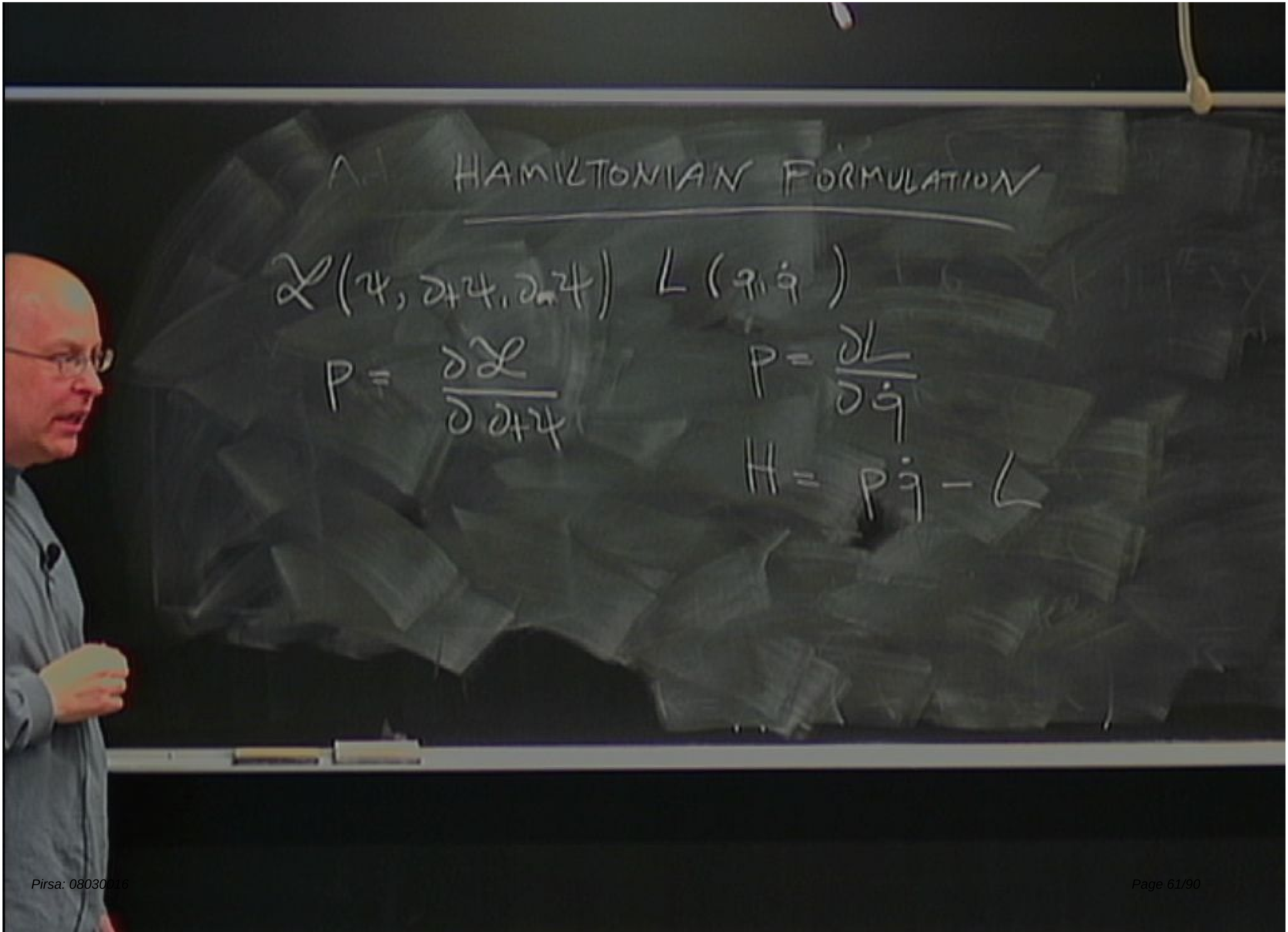
$$\longrightarrow \oint_{\partial V} \varepsilon (K - K_0)\, |h|^{1/2}\, d^3y$$

K_0 = extrinsic curvature of ∂V embedded in flat spacetime.


HAMILTONIAN FORMULATION
$L(q, \dot{q})$
$p = \frac{\partial L}{\partial \dot{q}}$


HAMILTONIAN FORMULATION
$\mathcal{L}(\psi, \partial_\mu \psi)$
$L(q, \dot{q})$
$p = \frac{\partial L}{\partial \dot{q}}$
$H = p\dot{q} - L$


A. HAMILTONIAN FORMULATION
$\mathcal{L}(\psi, \partial_\alpha \psi)$
$L(q, \dot{q})$
$p_\alpha = \frac{\partial \mathcal{L}}{\partial \partial_\alpha \psi}$
$p = \frac{\partial L}{\partial \dot{q}}$
$H = p\dot{q} - L$


A. HAMILTONIAN FORMULATION
$\mathcal{L}(\psi, \partial_t \psi, \partial_m \psi)$ $L(q, \dot{q})$
$p = \frac{\partial \mathcal{L}}{\partial \partial_t \psi}$
$p = \frac{\partial L}{\partial \dot{q}}$
$H = p\dot{q} - L$

A.1 HAMILTONIAN FORMULATION

$$\mathcal{L}(\psi, \partial_t\psi, \partial_m\psi) \qquad L(q, \dot{q})$$

$$p = \frac{\partial \mathcal{L}}{\partial\, \partial_t\psi} \qquad p = \frac{\partial L}{\partial \dot{q}}$$

$$\mathcal{H} = p\, \partial_t\psi - \mathcal{L} \qquad H = p\dot{q} - L$$

A. HAMILTONIAN FORMULATION

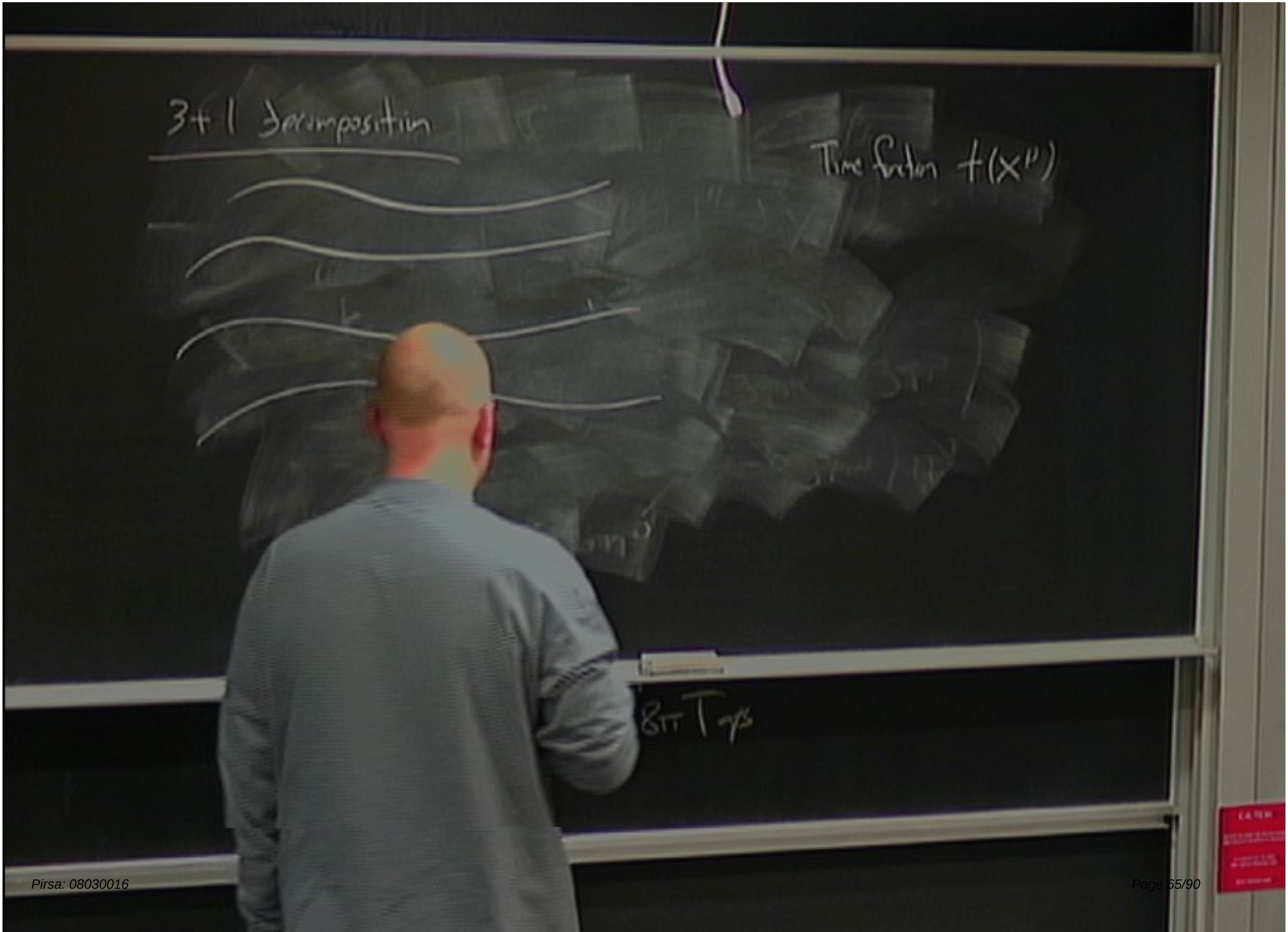
$\mathcal{L}(\psi, \partial_t\psi, \partial_x\psi)$ $\quad L(q, \dot{q})$

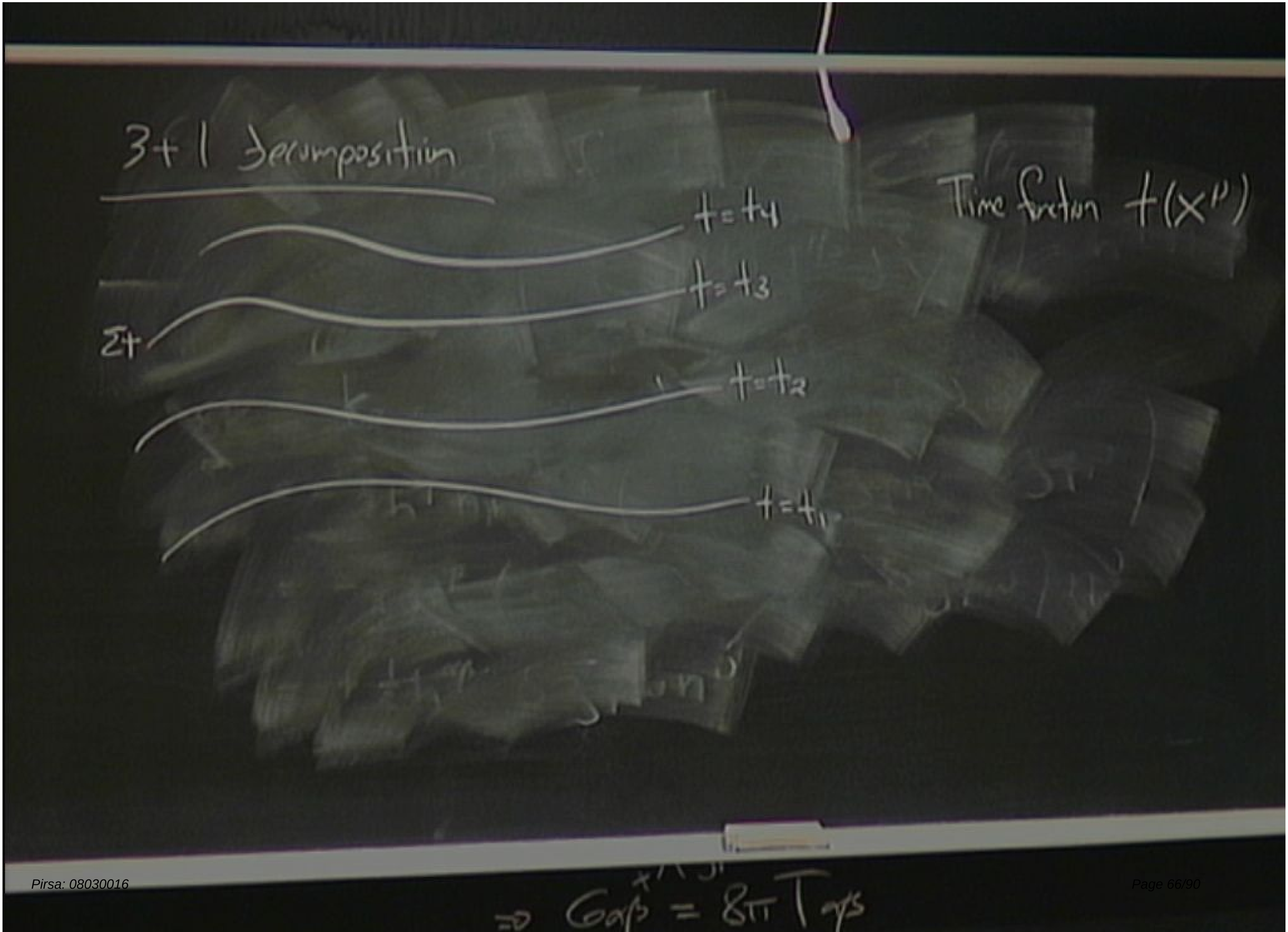
$P = \dfrac{\partial \mathcal{L}}{\partial\, \partial_t\psi}$ $\quad p = \dfrac{\partial L}{\partial \dot{q}}$

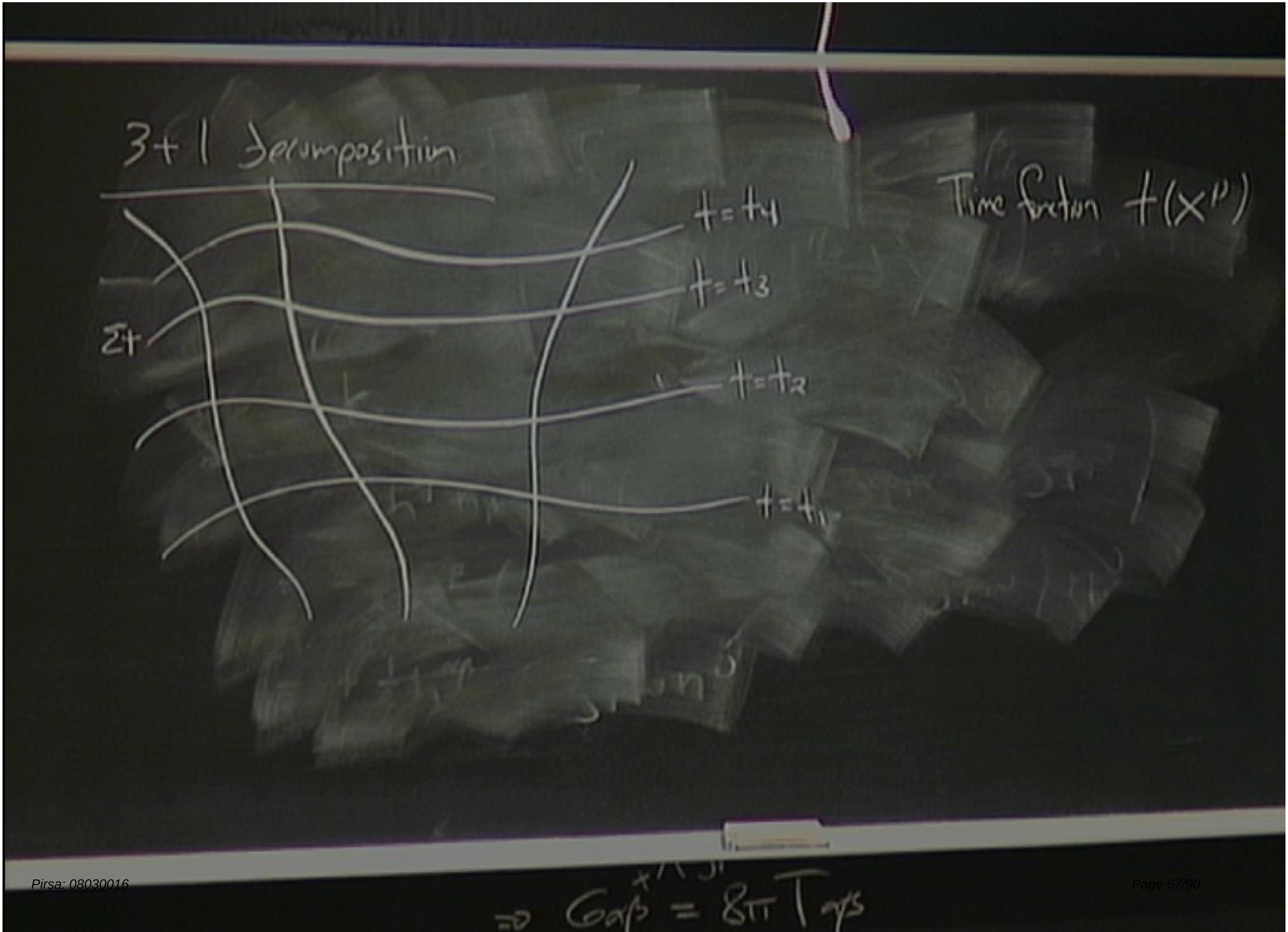
$\mathcal{H} = P\, \partial_t\psi - \mathcal{L}$ $\quad H = p\dot{q} - L$

3+1 decomposition

$$\Rightarrow G_{\alpha\beta} = 8\pi T_{\alpha\beta}$$


3+1 Decomposition
Time function t(x^μ)


3+1 decomposition
t = t4
t = t3
Σt
t = t2
t = t1
Time function t(xμ)
⇒ Gαβ = 8π Tαβ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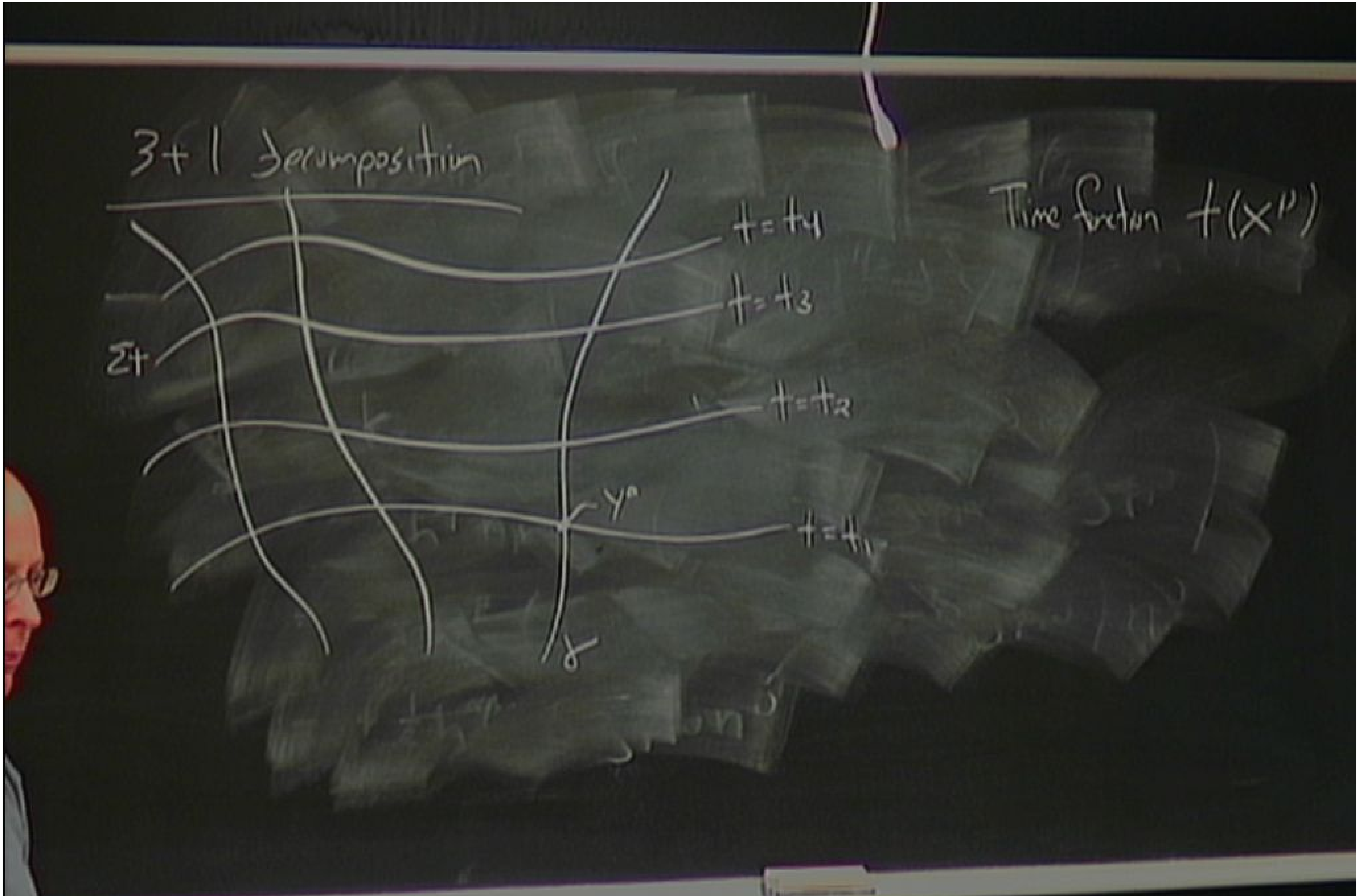
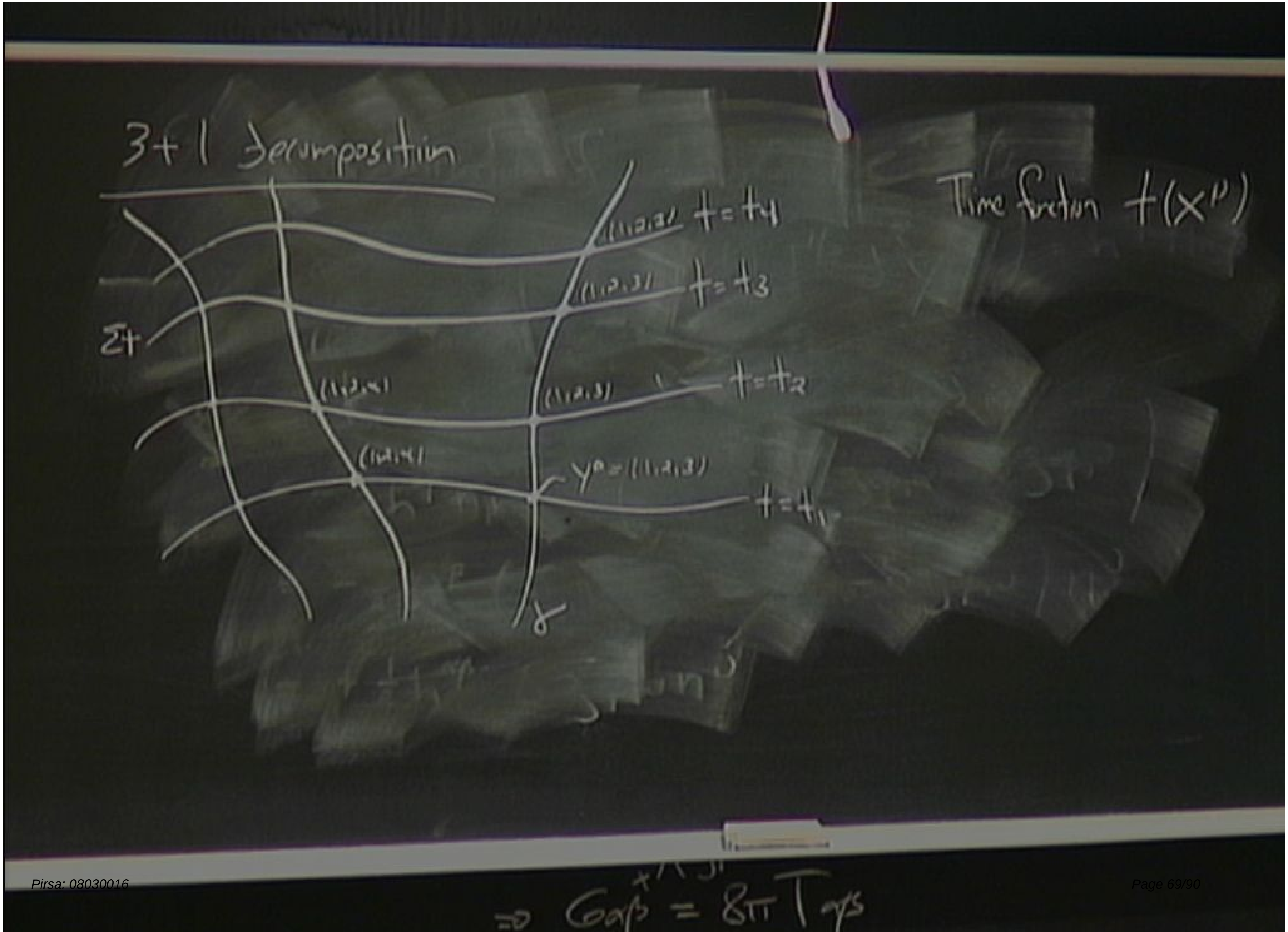
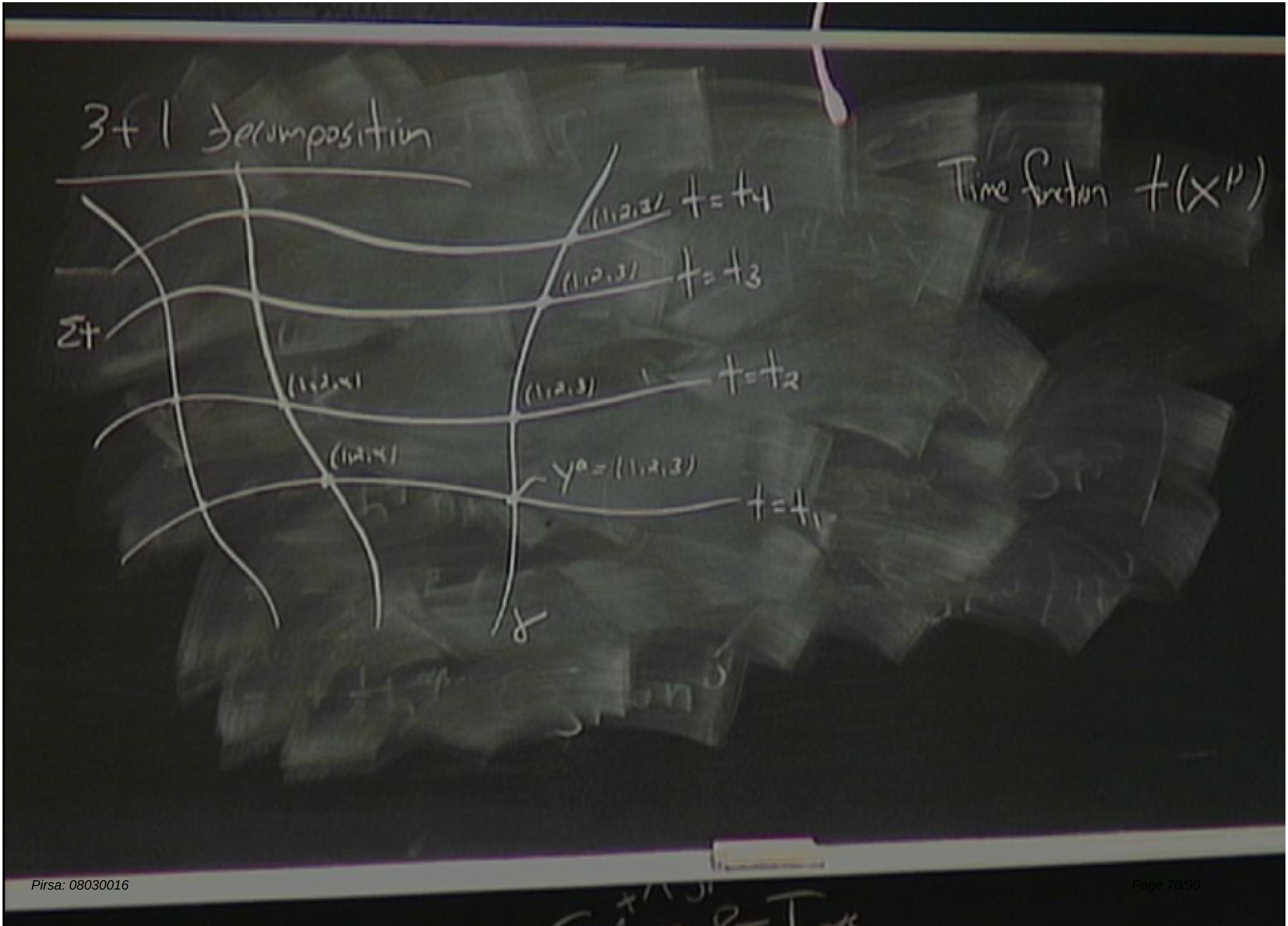
3+1 decomposition

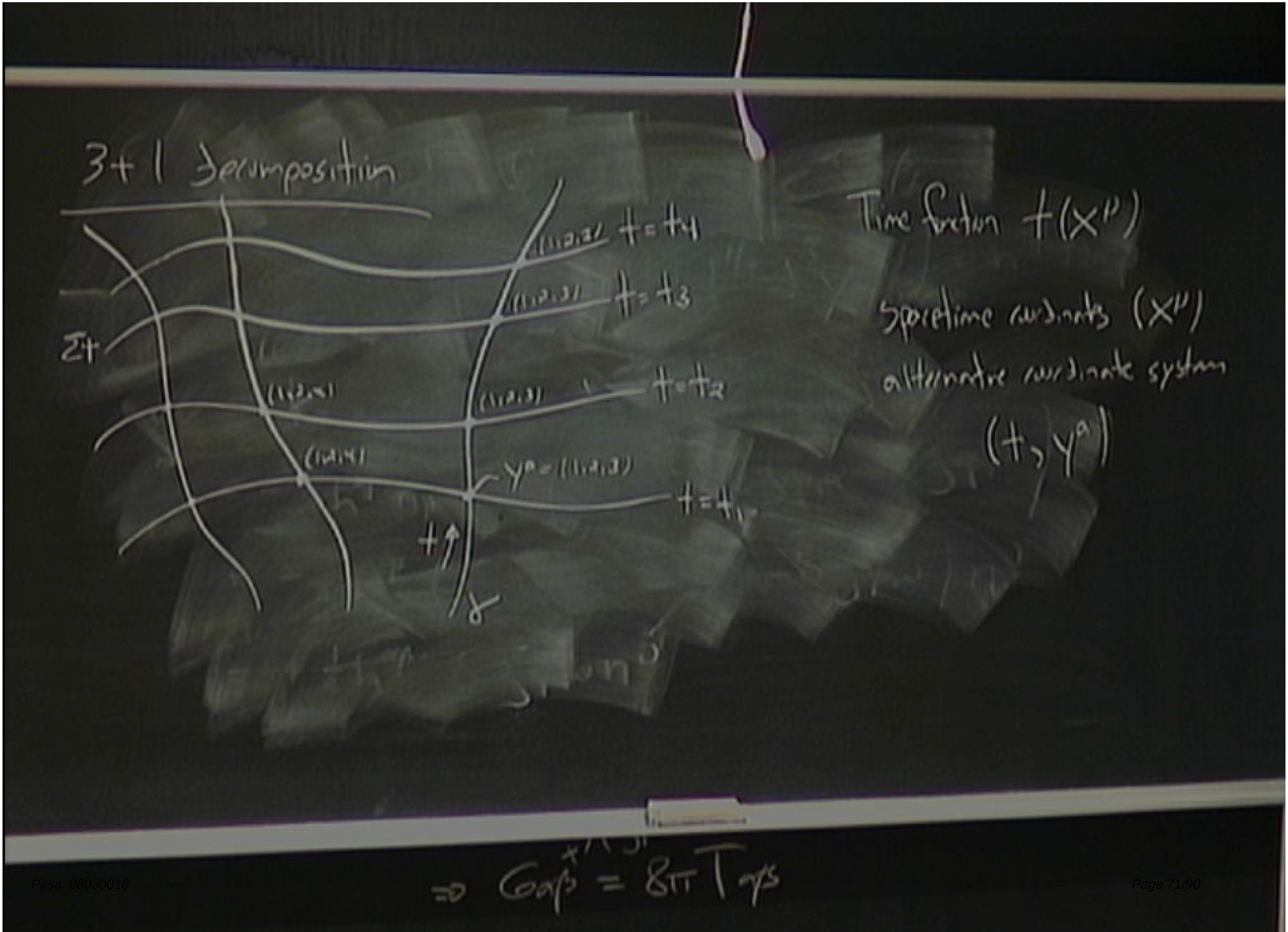


$\Rightarrow G_{\alpha\beta} = 8\pi T_{\alpha\beta}$



3+1 decomposition
Time function $t(x^\mu)$
$t = t_4$
$t = t_3$
$t = t_2$
$t = t_1$
Σ_t
(1,2,3)
(1,2,3)
(1,2,3)
$y^a = (1,2,3)$
γ
$\Rightarrow G_{\alpha\beta} = 8\pi T_{\alpha\beta}$


3+1 decomposition
Time function t(x^μ)
(1,2,3) t = t4
(1,2,3) t = t3
Σt
(1,2,3) (1,2,3) t = t2
(1,2,3)
y^a = (1,2,3)
t = t1
γ


3+1 decomposition
t = t4
t = t3
t = t2
t = t1
Σt
(1,2,3)
ya = (1,2,3)
t
γ
Time function t(xμ)
Spacetime coordinates (xμ)
alternative coordinate system
(t, ya)
⇒ Gαβ = 8π Tαβ

Time function $t(x^\mu)$

Spacetime coordinates (x^μ)

alternative coordinate system

$$(t, y^a)$$

$= t_4$

$= t_3$

$t = t_2$

$t = t_1$

$t = t_4$

$t = t_3$

$t = t_2$

$t = t_1$

Time function $t(x^\mu)$

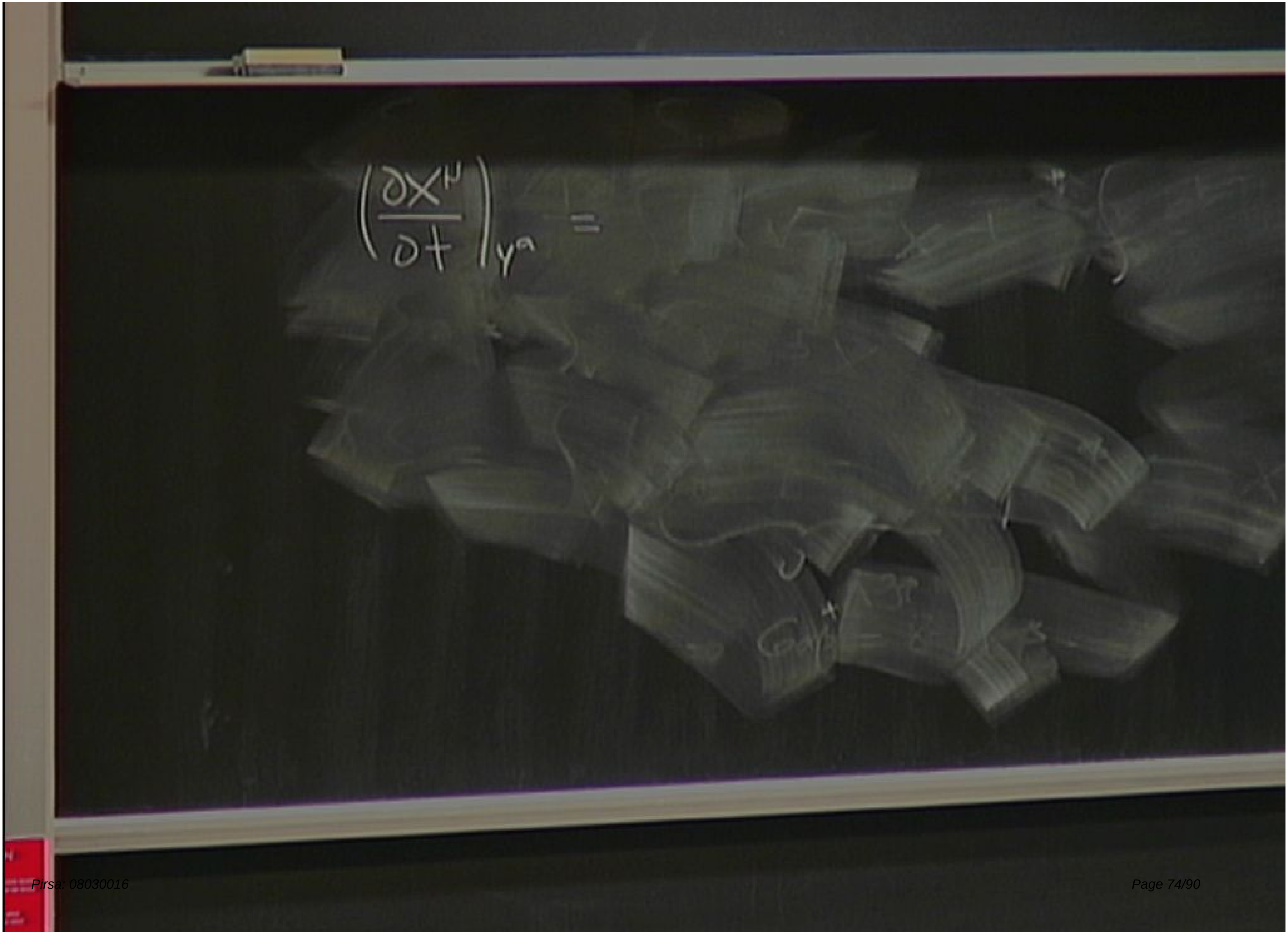
Spacetime coordinates (x^μ)

alternative coordinate system

$$(t, y^a)$$

$\exists$ coordinate transf:

$$x^\mu = x^\mu(t, y^a)$$


$\left(\frac{\partial x^\mu}{\partial t}\right)_{y^a} =$

$$\left(\frac{\partial x^\mu}{\partial t}\right)_{y^a} = \text{tangent vector field on congruence}$$

$$\equiv t^\alpha$$

$$\left(\frac{\partial x^\mu}{\partial t}\right)_{y^a} = \text{tangent vector field on congruence}$$

$$\equiv t^\alpha$$

$$\left(\frac{\partial x^\mu}{\partial y^a}\right)_t = \text{tangential vectors on each } \Sigma_t$$

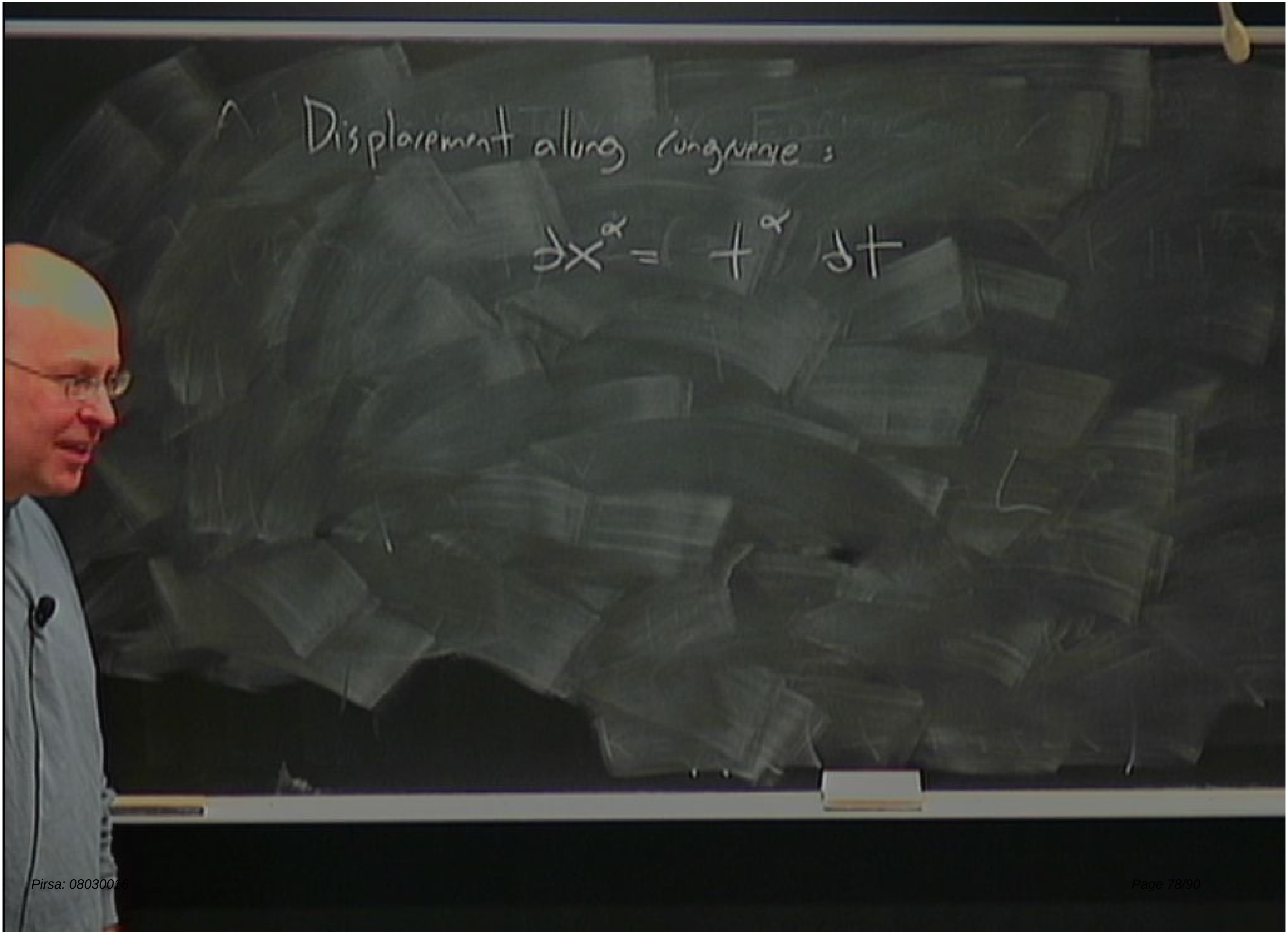
$$\equiv e^\alpha_a$$

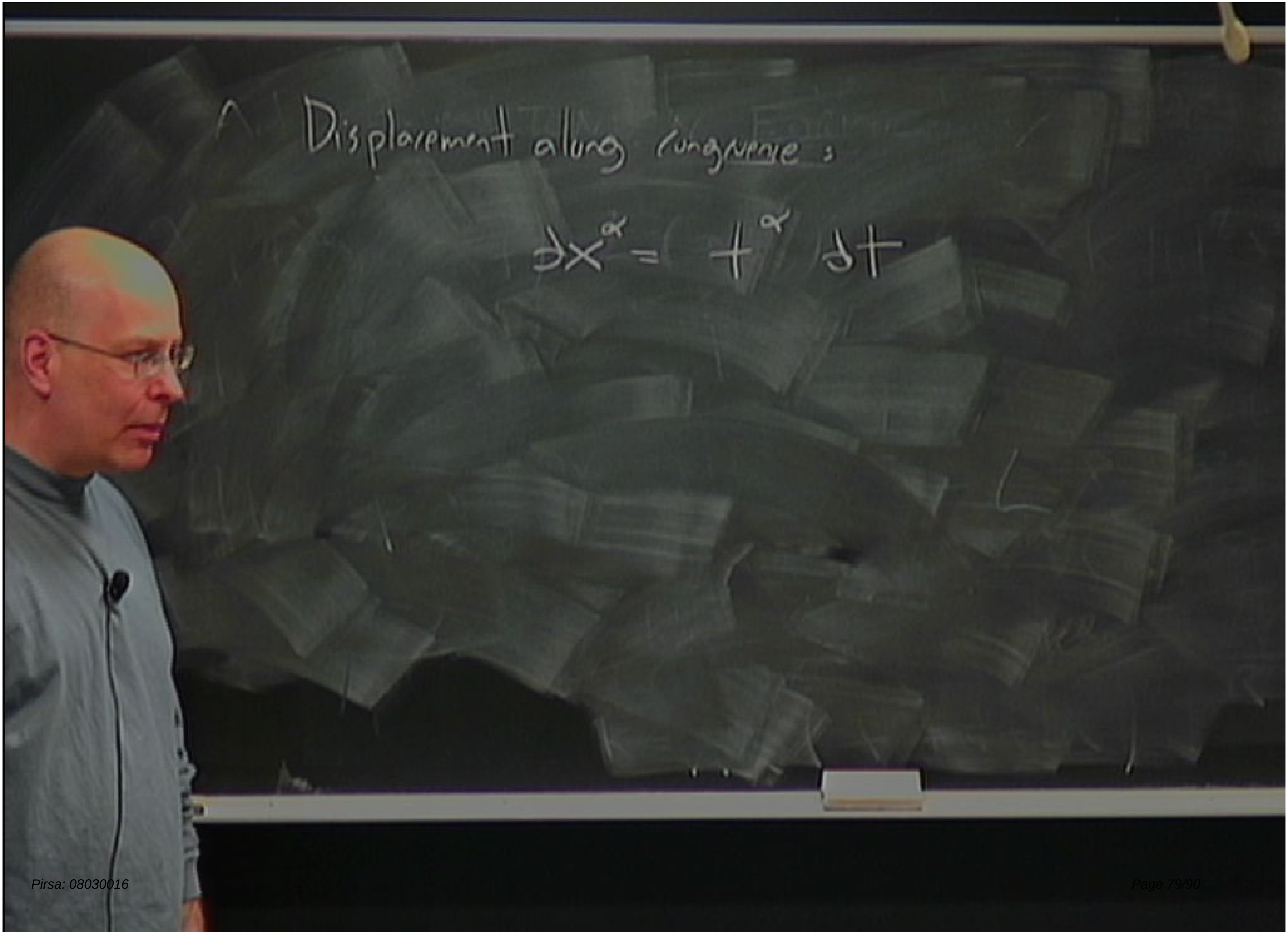
$$\left(\frac{\partial x^\mu}{\partial t}\right)_{y^a} = \text{tangent vector field on congruence}$$

$$\equiv t^\alpha$$

$$\left(\frac{\partial x^\mu}{\partial y^a}\right)_t = \text{tangential vectors on each } \Sigma_t$$

$$\equiv e^\alpha_a$$


Displacement along congruence :
$dx^\alpha = t^\alpha dt$


Displacement along congruence :
$dx^\alpha = t^\alpha \, dt$

Displacement along congruence :

$$dx^\alpha = t^\alpha \, dt$$

Change of $t(x^\mu)$ from such a displacement :

$$dt$$

Displacement along congruence:

$$dx^\alpha = t^\alpha \, dt$$

Change of $t(x^\mu)$ from such a displacement:

$$dt = \frac{\partial t}{\partial x^\mu} \, dx^\mu$$

Displacement along congruence:

$$dx^\alpha = t^\alpha \, dt$$

Change of $t(x^\mu)$ from such a displacement:

$$dt = \frac{\partial t}{\partial x^\mu} \, dx^\mu = \frac{\partial t}{\partial x^\mu} \, t^\mu \, dt$$

Displacement along congruence:

$$\partial x^{\alpha} = t^{\alpha} \, \partial t$$

Change of $t(x^{\mu})$ from such a displacement:

$$\partial t = \frac{\partial t}{\partial x^{\mu}} \, \partial x^{\mu} = \frac{\partial t}{\partial x^{\mu}} \, t^{\mu} \, \partial t$$

$$t^{\alpha} \partial_{\alpha} t = 1$$

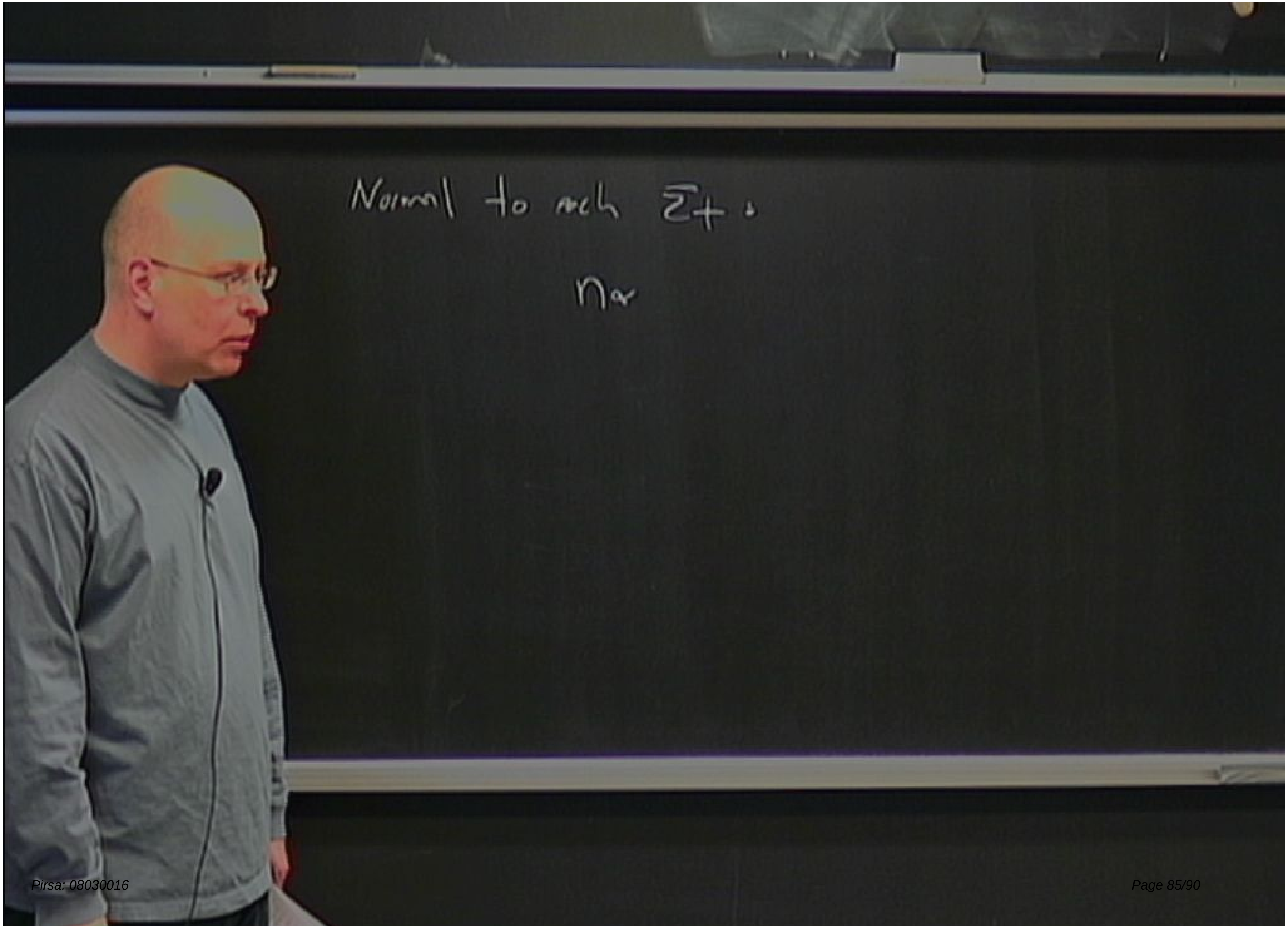
Displacement along congruence:

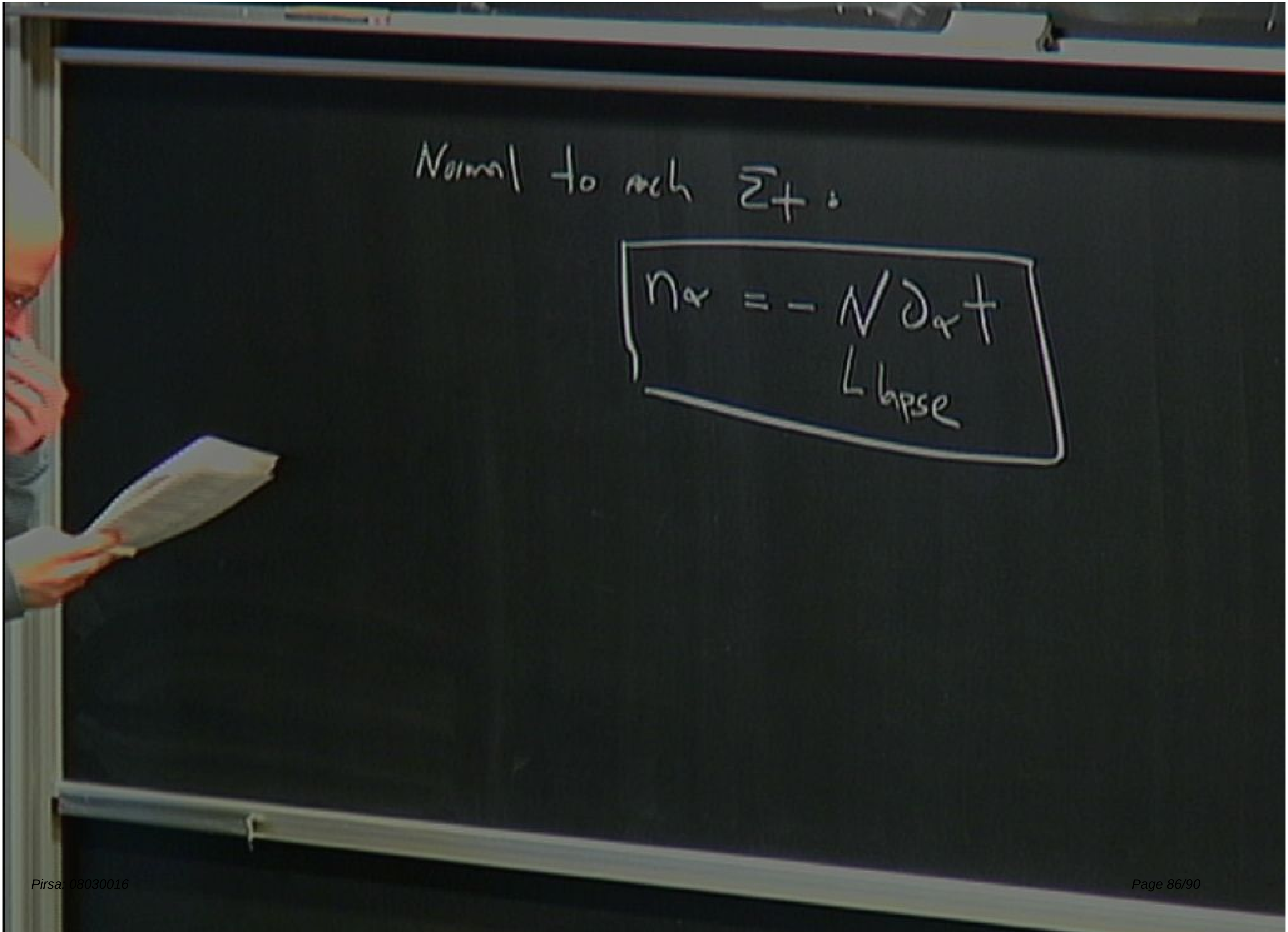
$$\partial x^\alpha = t^\alpha \, \partial t$$

Change of $t(x^\mu)$ from such a displacement:

$$\partial t = \frac{\partial t}{\partial x^\mu} \partial x^\mu = \frac{\partial t}{\partial x^\mu} t^\mu \, \partial t$$

$$\boxed{t^\alpha \partial_\alpha t = 1}$$


Normal to each Σt :
na



Normal to each Σ_t :
$n_\alpha = - N \partial_\alpha t$
Lapse

Normal to each Σ_t:

$$\boxed{n_\alpha = -N\partial_\alpha t}$$

Lapse

$$n_\alpha e^\alpha_a = 0$$

Normal to each Σ_t:

$$\boxed{\underset{\text{Lapse}}{n_\alpha = -N \partial_\alpha t}} \qquad n_\alpha e^\alpha_a = 0$$

$$t^\alpha = N n^\alpha$$

Normal to each Σ_t:

$$\boxed{n_\alpha = -\underbrace{N}_{\text{Lapse}} \partial_\alpha t} \qquad n_\alpha e^\alpha_a = 0$$

$$\boxed{t^\alpha = N n^\alpha + N^a e^\alpha_a}$$

Normal to each Σ_t:

$$\boxed{n_\alpha = -\underbrace{N}_{\text{lapse}}\partial_\alpha t} \qquad n_\alpha e^\alpha_a = 0$$

$$\boxed{t^\alpha = N n^\alpha + N^a e^\alpha_a}$$

N^a ↳ shift vector.