

Title: Advanced General Relativity - Lecture 10A

Date: Mar 26, 2008 10:30 AM

URL: <http://pirsa.org/08030014>

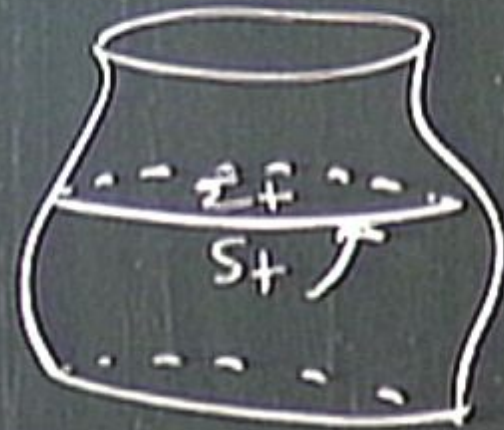
Abstract: Advanced General Relativity

HAMILTONIAN

$$16\pi H_G = \int_{z_+} [N(k^a k_{ab} - K^2 - {}^3R) - 2N_n D_b (k^{ab} - K h^{ab})] \sqrt{h} d^3y$$

$$- 2\oint_{S_+} [N(k - k_0) - N_n (k^b - K h^{ab}) n_b] \sqrt{\sigma} d^2\theta$$

$$T^a = N n^a + N^a e_a^T$$

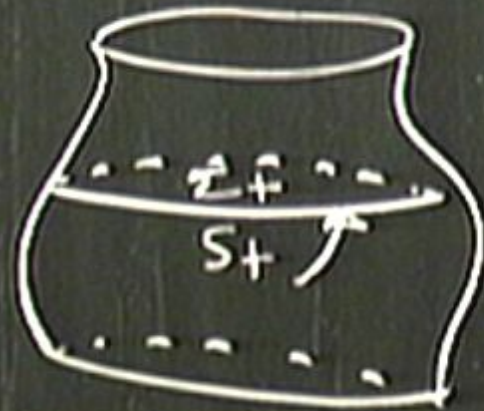


HAMILTONIAN

$$16\pi H_0 = \int_{z_+} [N(k^a k_{ab} - K^2 - {}^3R) - 2N_n D_b (k^{ab} - K h^{ab})] \sqrt{h} d^3y$$

$$- 2 \oint_{S_+} [N(k - k_0) - N_n (k^{ab} - K h^{ab}) n_b] \sqrt{\sigma} d^2\theta$$

$$T^a = N n^a + N^a e_a^a$$



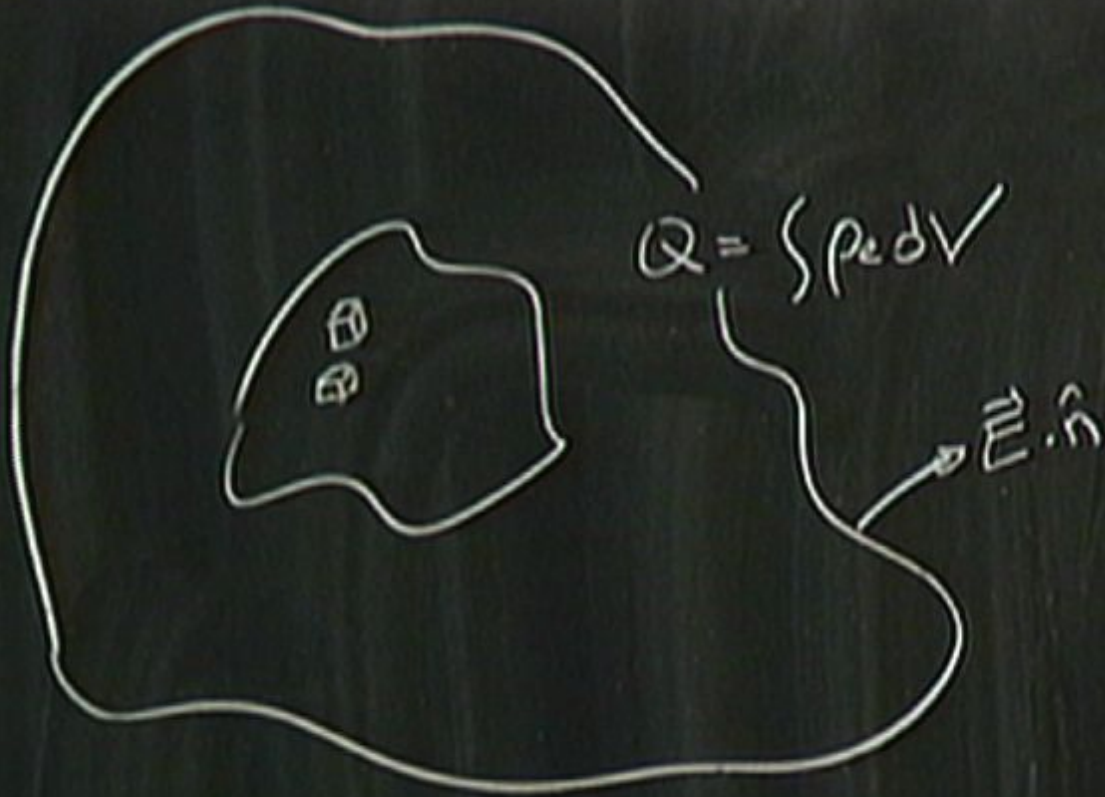
$$k_{ab} = n_{[a} e_{b]}^a e_b^b$$

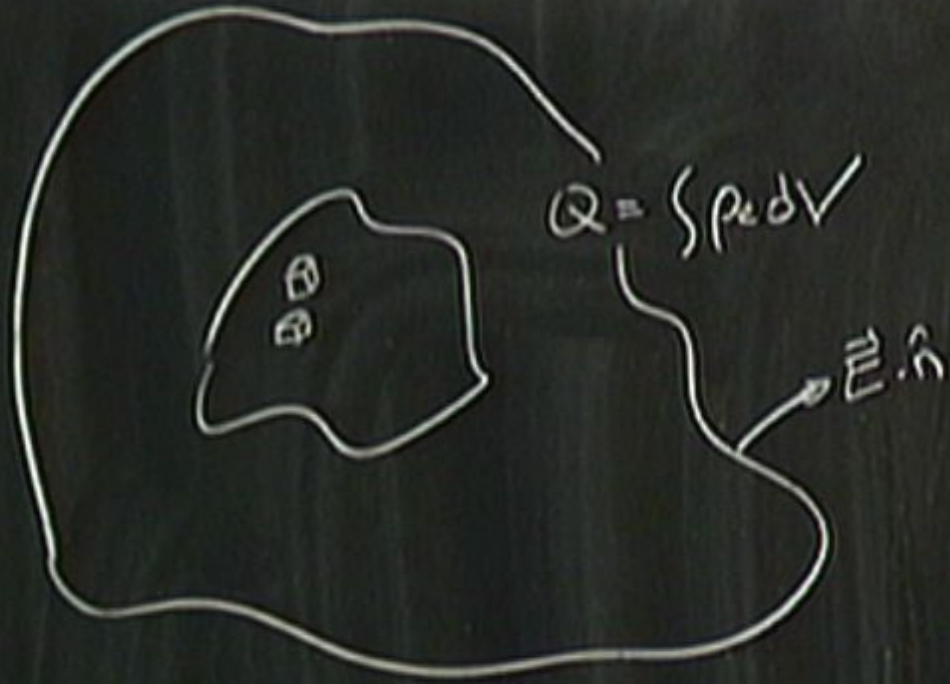




$$Q = \int \rho \mathbf{e} dV$$

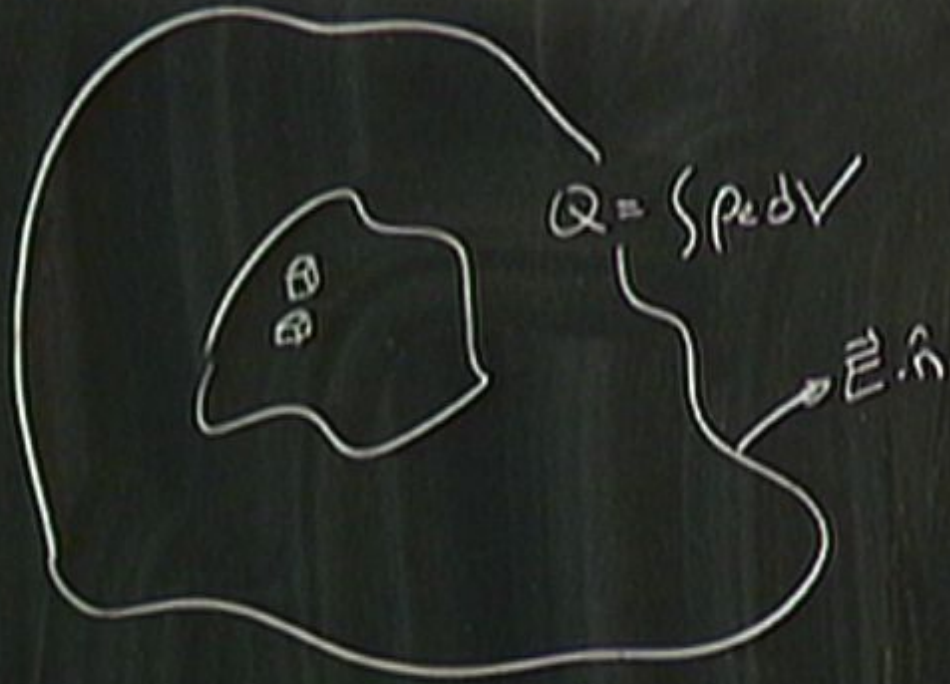






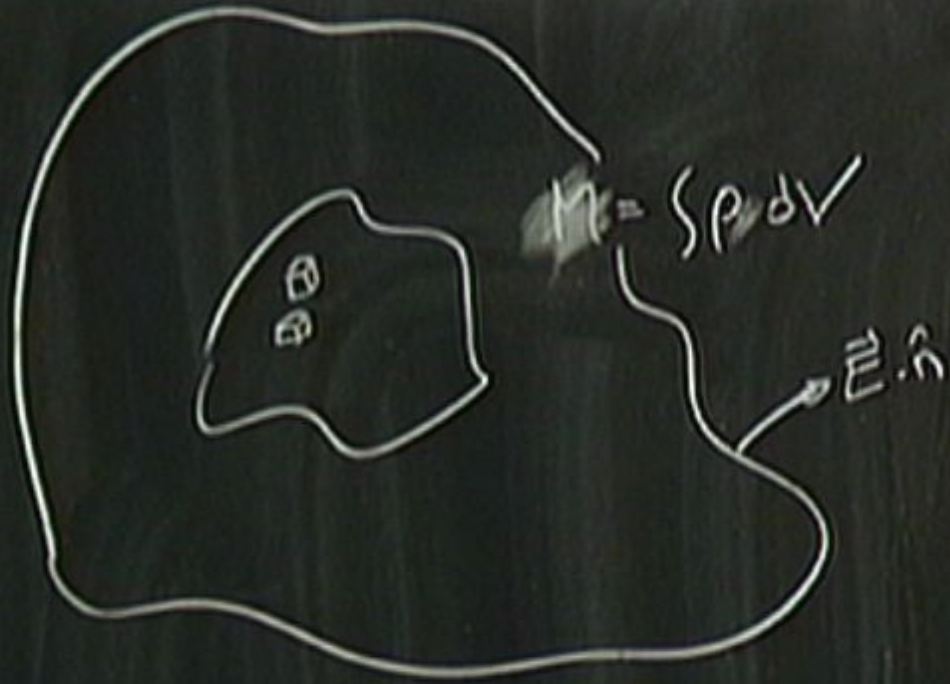
$$Q = \oint_S \vec{E} \cdot \hat{n} dS$$





$$Q = \int \rho_e dV$$

$$Q = \oint_S \vec{E} \cdot \hat{n} dS$$



$$M = \int \rho dV$$

$$\vec{E} \cdot \hat{n}$$

$$Q = \oint \vec{E} \cdot \hat{n} dS$$

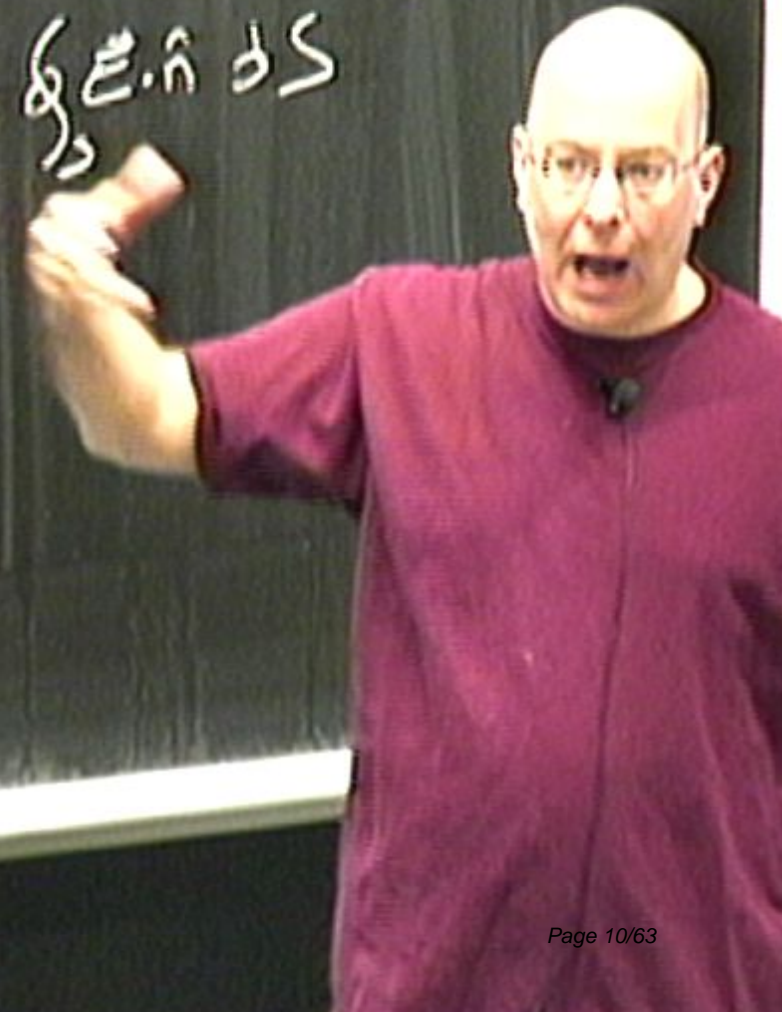




$$M = \int_V \rho \, dV$$

$$F = \oint_S \mathbf{E} \cdot \hat{n} \, dS$$

$$F = \oint_S \mathbf{E} \cdot \hat{n} \, dS$$





$$M = \int \rho \, dV$$

$\oint_S \mathbf{E} \cdot \hat{n} \, dS$

$$Q_{enc} = \int_V \rho \, dV$$

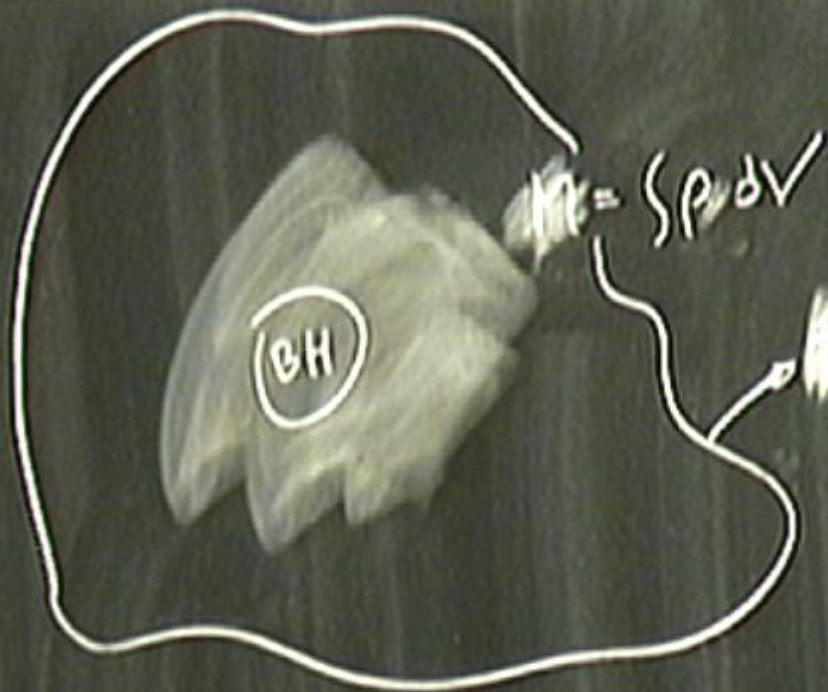




$$\rho = \int \rho dv$$

$$\rho = \int \rho \cdot \hat{n} dS$$

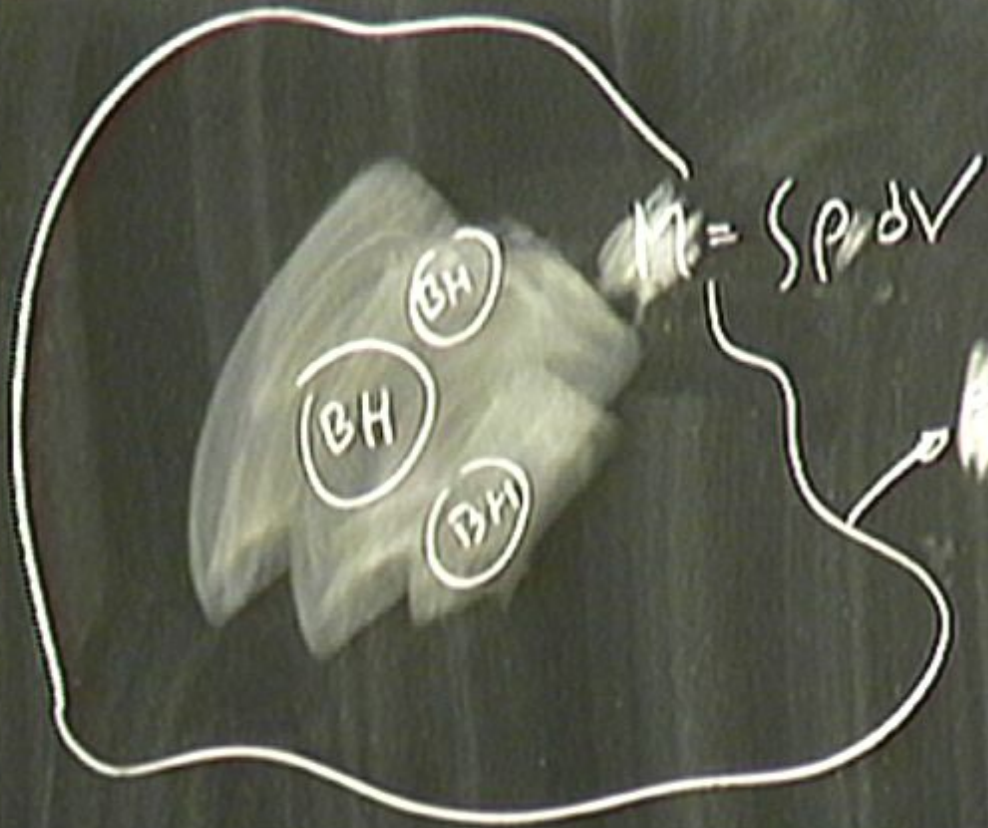
$$\rho = \int \rho \cdot \hat{n} dS$$



$$\vec{g} \cdot \hat{n}$$

$$M = \int_S \vec{g} \cdot \hat{n} dS$$





$$M = \int_S \rho \cdot \hat{n} dS$$

MASS AND ANGULAR MOMENTUM

Near ∞ , solve linearized EFE as a perturbation of Minkowski.

MASS AND ANGULAR MOMENTUM

Near ∞ , solve linearized EFE as a perturbation of Minkowski.

MASS AND ANGULAR MOMENTUM

Near ∞ , solve linearized EFE as a perturbation of Minkowski.

Stationary, axisymmetric, asymptotically flat:

MASS AND ANGULAR MOMENTUM

Near ∞ , solve linearized EFE as a perturbation of Minkowski.

Stationary, axisymmetric spacetime:

$$ds^2 = - \left(1 - \frac{2m}{r} + O(r^{-2}) \right) dt^2 + \left(1 + \frac{2m}{r} + O(r^{-1}) \right) (dr^2 + r^2 d\Omega^2)$$

MASS AND ANGULAR MOMENTUM

Near ∞ , solve linearized EFE as a perturbation of Minkowski.

Stationary, axisymmetric spacetime:

$$ds^2 = - \left(1 - \frac{2m}{r} + O(r^{-2}) \right) dt^2 + \left(1 + \frac{2m}{r} + O(r^{-1}) \right) (dr^2 + r^2 d\Omega^2) - \left(\frac{4j \sin^2 \theta}{r} + O(r^{-2}) \right) dt d\phi$$

MASS AND ANGULAR MOMENTUM

Near ∞ , solve linearized EFE as a perturbation of Minkowski.

Stationary, axisymmetric spacetime:

$$ds^2 = - \left(1 - \frac{2M}{r} + O(r^{-2}) \right) dt^2 + \left(1 + \frac{2M}{r} + O(r^{-1}) \right) (dr^2 + r^2 d\Omega^2) \\ - \left(\frac{4J \sin^2 \theta}{r} + O(r^{-2}) \right) dt d\phi$$



KOMAR INTEGRALS

$$\xi^{\alpha}(\tau) \quad \xi^{\alpha}(\varrho)$$

KOMAR INTEGRALS

$$\xi^\alpha_{(+)} \quad \xi^\alpha_{(-)}$$

$$M = -\frac{1}{8\pi} \oint \nabla^\alpha \xi^\beta_{(+)}$$

KOMAR INTEGRALS

$$\xi^{\alpha}(\mathcal{H}) \quad \xi^{\alpha}(\mathcal{Q})$$

$$M = -\frac{1}{8\pi} \int_S \nabla^{\alpha} \xi^{\beta}(\mathcal{H}) \bar{\delta} S_{\alpha\beta}$$

KOMAR INTEGRALS

$$\xi^{\alpha}(\pm) \quad \xi^{\alpha}(\mathcal{Q})$$

$$M = -\frac{1}{8\pi} \int_{S \rightarrow \infty} \nabla^{\alpha} \xi^{\beta}(\pm) \, dS_{\alpha\beta}$$

KOMAR INTEGRALS

$$\xi^\alpha_{(+)} \quad \xi^\alpha_{(-)}$$

$$M = -\frac{1}{8\pi} \oint_{S \rightarrow \infty} \nabla^\alpha \xi^\beta_{(+)} \partial S_{\alpha\beta}$$

$$J = \frac{1}{16\pi} \oint_{S \rightarrow \infty} \nabla^\alpha \xi^\beta_{(-)} \partial S_{\alpha\beta}$$

KOMAR INTEGRALS

$$\xi^{\alpha}(\mathcal{H}) \quad \xi^{\alpha}(\mathcal{Q}) \quad \mathcal{S} \rightarrow \partial V$$

$$M = -\frac{1}{8\pi} \int_{\mathcal{S} \rightarrow \infty} \nabla^{\alpha} \xi^{\beta}(\mathcal{H}) \partial \mathcal{S}_{\alpha\beta}$$

$$J = \frac{1}{16\pi} \int_{\mathcal{S} \rightarrow \infty} \nabla^{\alpha} \xi^{\beta}(\mathcal{Q}) \partial \mathcal{S}_{\alpha\beta}$$



KOMAR INTEGRALS

$$\xi^{\alpha}(\mathcal{H}) \quad \xi^{\alpha}(\mathcal{R})$$

$$\left\{ \begin{aligned} M &= -\frac{1}{8\pi} \oint_{S \rightarrow \infty} \nabla^{\alpha} \xi^{\beta}(\mathcal{H}) \partial S_{\alpha\beta} \\ J &= \frac{1}{16\pi} \oint_{S \rightarrow \infty} \nabla^{\alpha} \xi^{\beta}(\mathcal{R}) \partial S_{\alpha\beta} \end{aligned} \right.$$



For $N=1, N^{\wedge}=0 : H_0 = M$

For $N^Y=1, N^X=0 \rightarrow H_0 \cong M$

For $N=0, N^X=Q^*$

$$T = N^v n + N^x e_a$$

For $\boxed{N^v=1, N^x=0} \rightarrow H_0 = M$

For $N=0, \textcircled{N^x=q^a} \rightarrow H_0 = -J$



HAMILTONIAN

$$16\pi H_G = \int_{\Sigma_t} \left[N(K^{ab}K_{ab} - K^2 - {}^3R) - 2N_n D_b (K^{ab} - K h^{ab}) \right] \sqrt{h} d^3y$$

$$- 2 \oint_{S_t} \left[\vec{N} \cdot (k - k_0) - N_n (K^{ab} - K h^{ab}) r_b \right] \sqrt{\sigma} d^2\theta$$

$$t^\alpha = N n^\alpha + N^a e_a^\alpha$$

For $\boxed{N=1, N^a=0} \rightarrow H_G = M$

For $\boxed{N=0, N^a=q^a} \rightarrow H_G = -J$



$$k_{ab} = r_{ab} e_a^\alpha e_b^\beta$$

$$M = \frac{1}{16\pi} \int_{-\infty}^{\infty} \dots$$

$$M = \frac{1}{16\pi} \oint_{\mathcal{H} \rightarrow \infty} (k - k_0) \sqrt{\sigma} \, d^3\theta$$

$$M = -\frac{1}{8\pi} \oint_{S \rightarrow \infty} (k - k_0) \sqrt{\sigma} \, d^3\mathbf{O}$$

for $N=0, N=Q \rightarrow H_0 \dots$

$$M = -\frac{1}{8\pi} \int_{\underline{S}_4 \rightarrow \underline{S}_3} (k - k_0) \sqrt{\sigma} d^3\theta$$

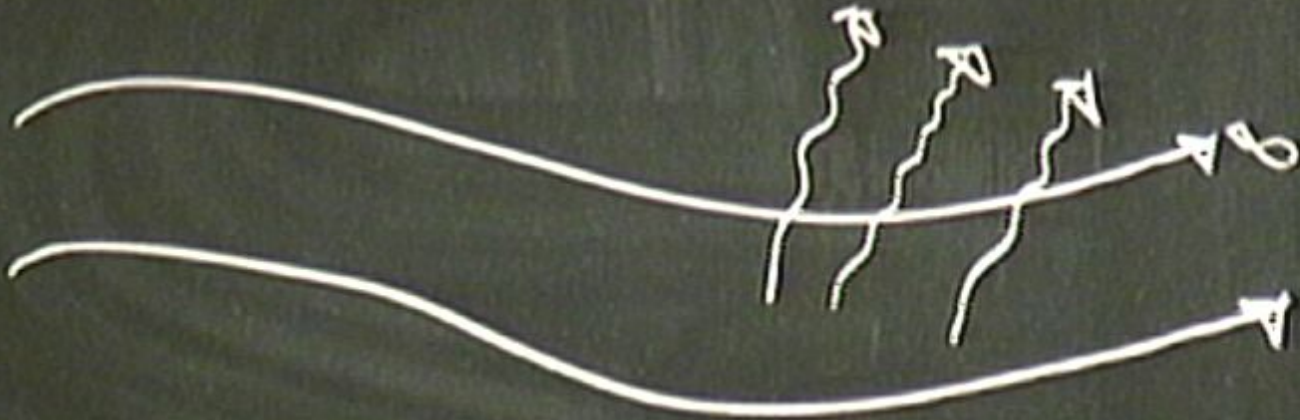


for $N=0, N=Q \rightarrow H_0 \dots$

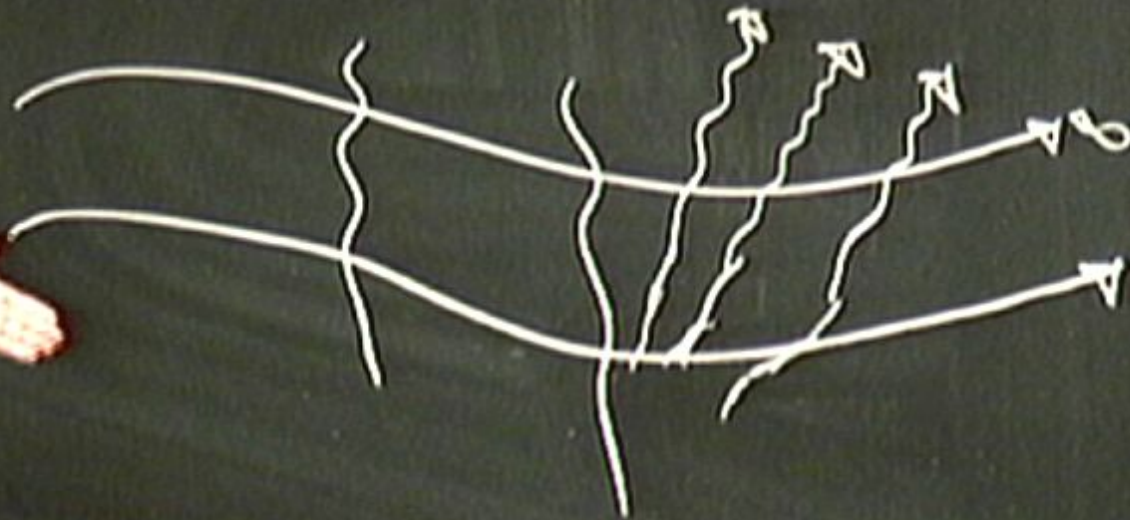
$$M = -\frac{1}{8\pi} \int_{\underline{S}_4 \rightarrow \mathbb{R}^3} (k - k_0) \sqrt{\sigma} d^3\theta$$



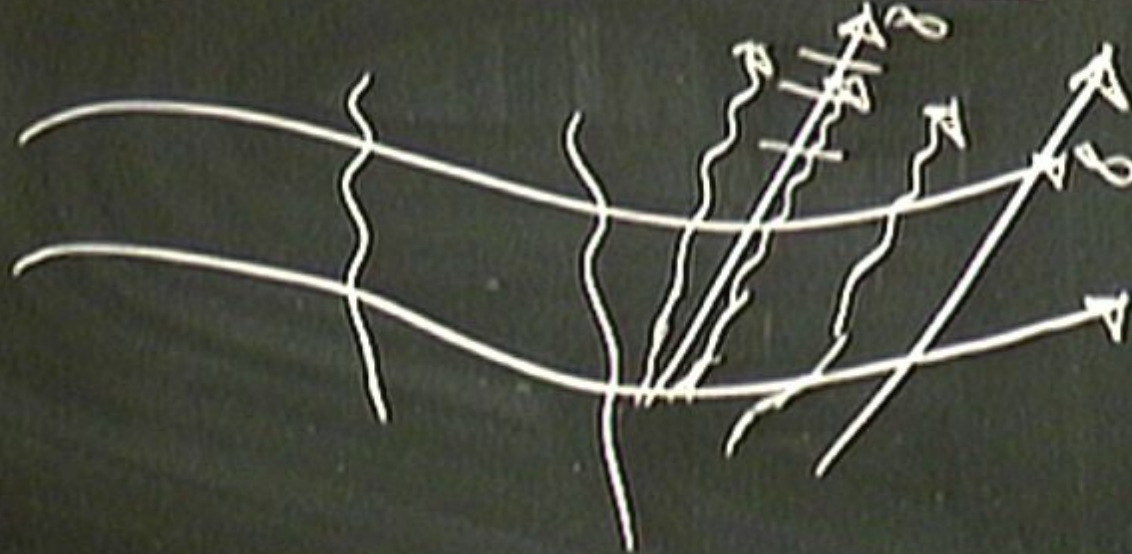
$$M = -\frac{1}{\omega} \int_{\mathcal{H}_0} (k - k_0) \sqrt{\sigma} \, d^2\theta$$



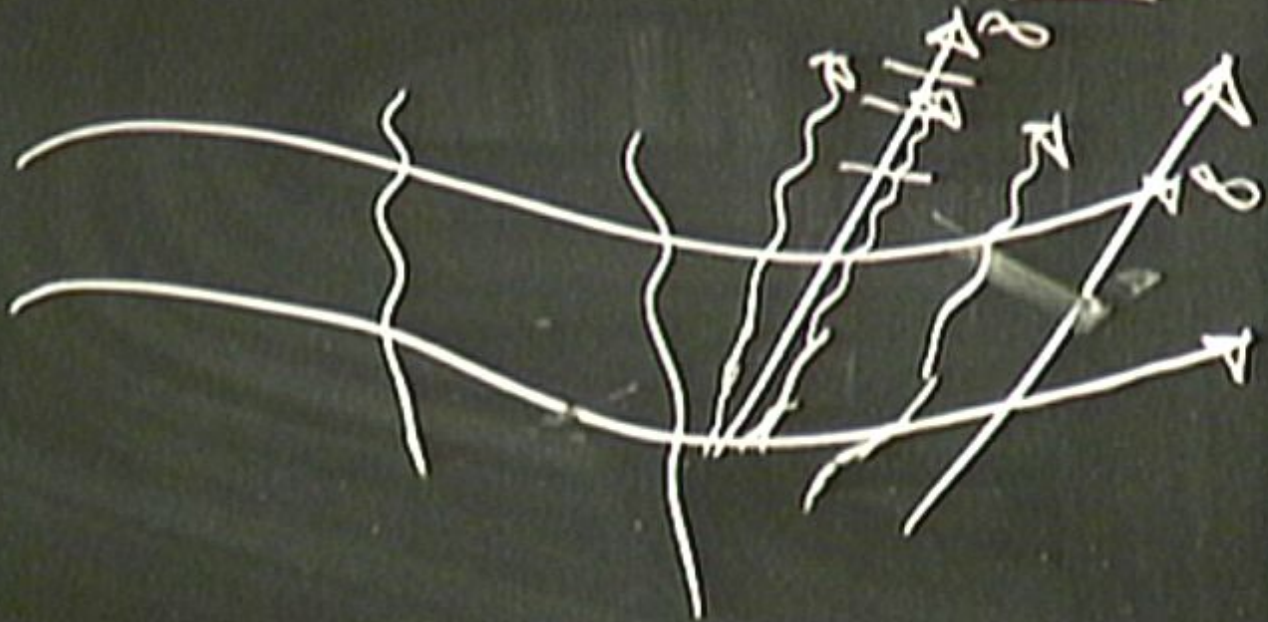
$$M_{\text{Apt}} = \frac{-1}{8\pi} \int_{\mathcal{H} \rightarrow \mathcal{B}} (k - k_0) \sqrt{\sigma} \, d^2\theta$$

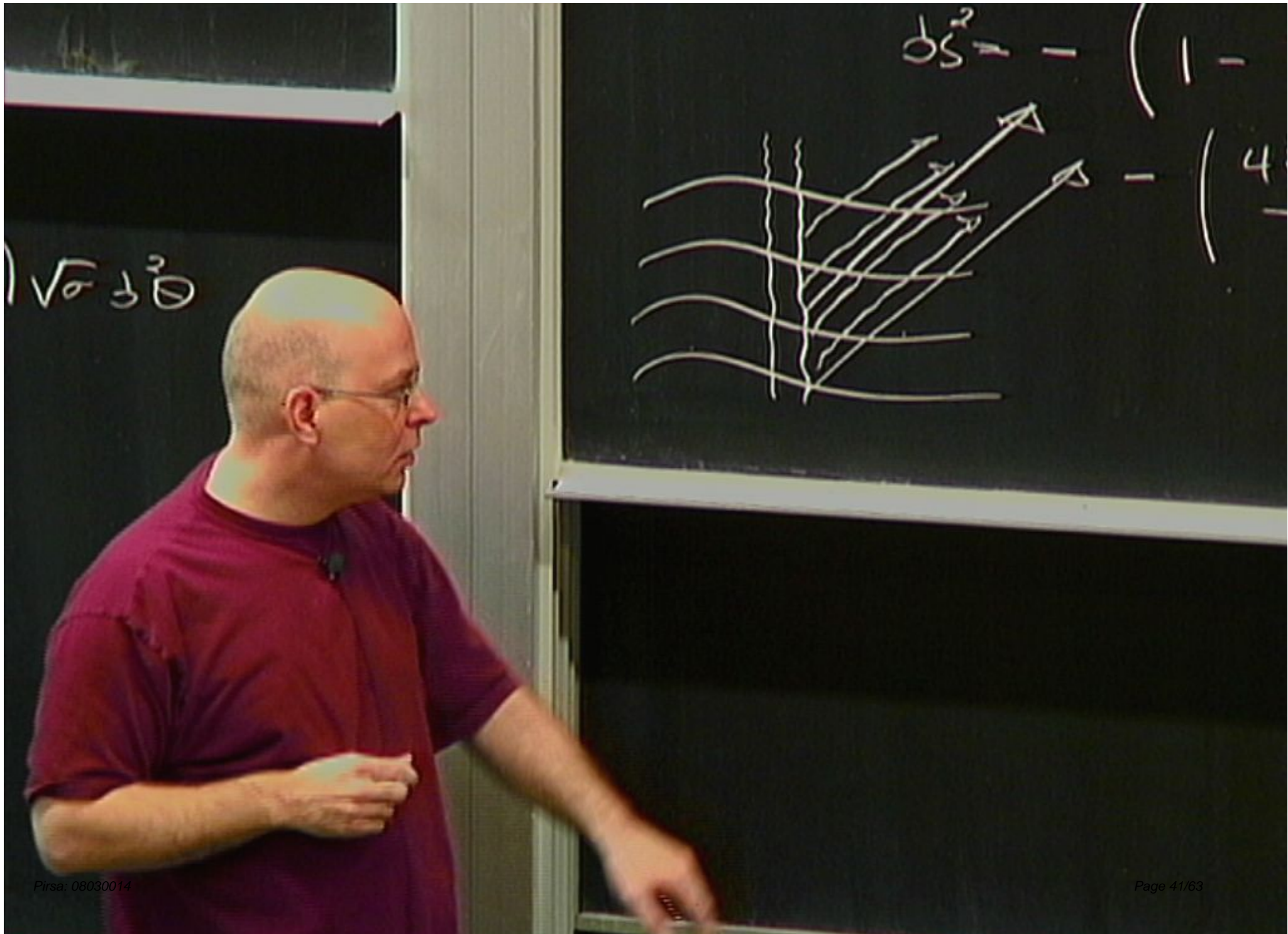


$$M_{\text{eff}} = -\frac{1}{8\pi} \int_{\mathcal{H} \rightarrow \mathcal{B}} (k - k_0) \sqrt{\sigma} \, d^2\theta$$



$$M_{A0n} = -\frac{1}{8\pi} \int_{\Sigma_{t \rightarrow \infty}} (k - k_0) \sqrt{\sigma} d^3\theta$$

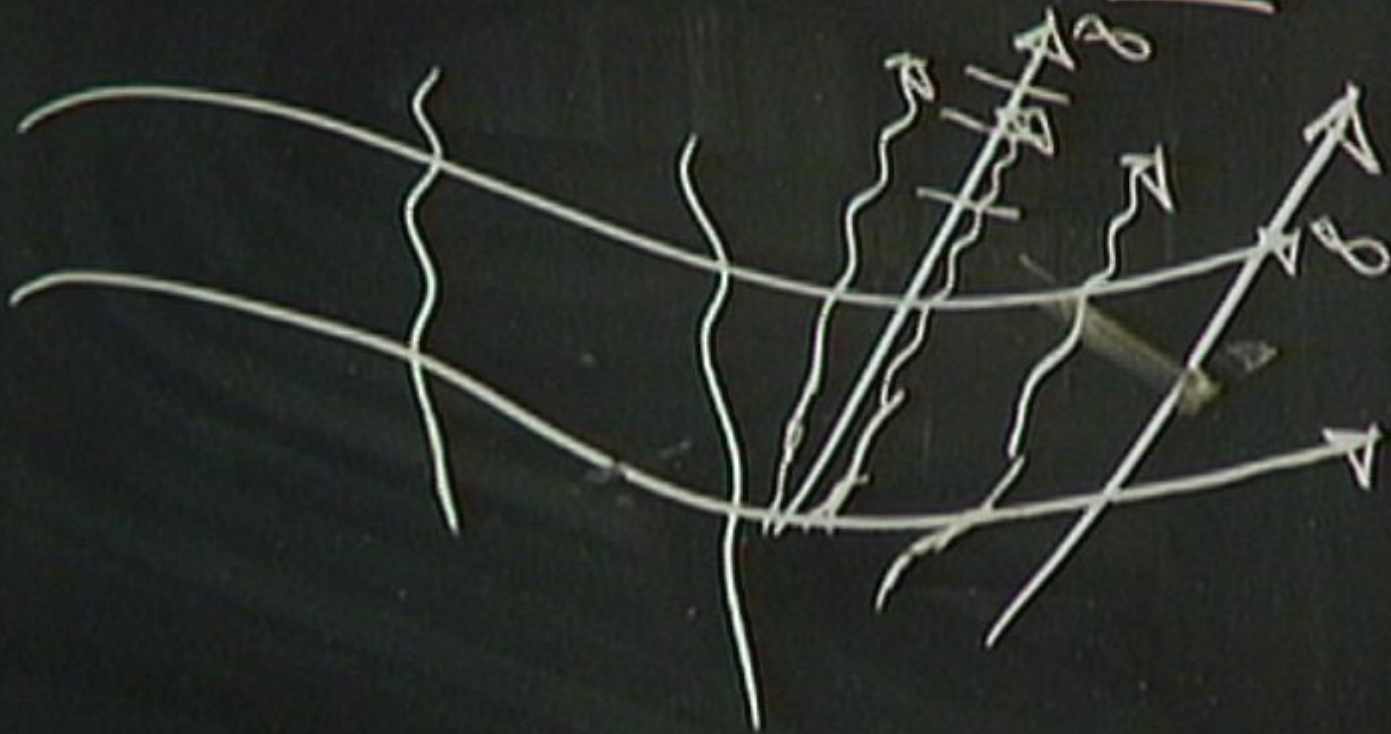




$$\sqrt{\sigma} \partial^3 \theta$$

$$\partial S^2 = \dots \begin{pmatrix} 1 & \dots \\ \dots & 4 \end{pmatrix}$$

$$M = -\frac{1}{8\pi} \int_{\mathcal{H}_0} (k - k_0) \sqrt{\sigma} \, d^3\mathcal{O}$$



$$\frac{\partial M_{Pndi}}{\partial f}$$

$$\frac{\delta M_{Bnd.}}{\delta t} = - \int_{S_t} F \delta S$$

CAUTION
Do not touch the board
Do not use the board
Do not write on the board

Transfer of mass and ang. momentum.



CAUTION
ELECTRIC
EQUIPMENT
DO NOT TOUCH

Transfer of mass and ang. momentum.

t_{st} T_{ap}

Transfer of mass and ang. momentum.

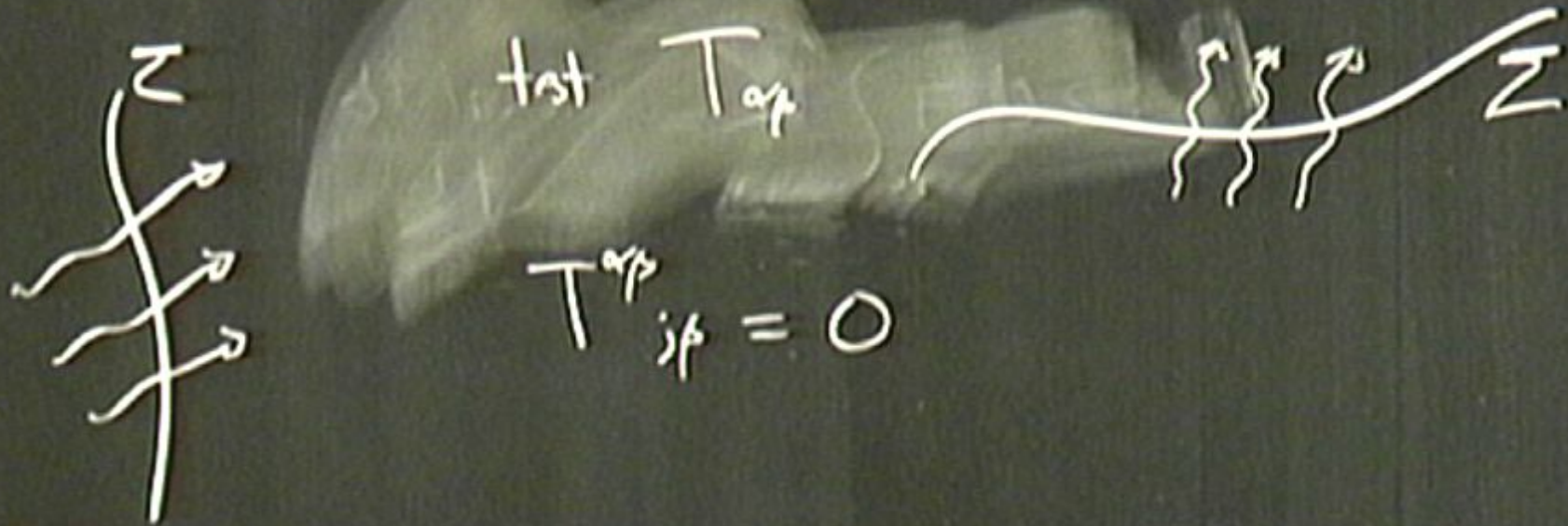


Transfer of mass and ang. momentum.



CAUTION
Do not touch the board
without the permission of the
lecturer.

Transfer of mass and ang. momentum.



Transfer of mass and ang. momentum.



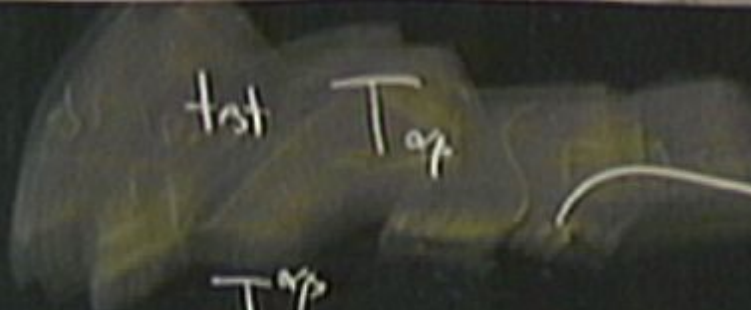
tst $T_{\alpha\beta}$

$$T^{\alpha\beta}_{ip} = 0$$



$$T^{\alpha}_{\rho} \xi^{\rho}$$

Transfer of mass and ang. momentum.



test T_{op}

$$T_{op}^{ij} = 0$$

$$(T^{\alpha}_{\rho} \xi^{\rho})_{, \alpha} = 0$$

Transfer of mass and ang. momentum.



test $T_{\alpha\beta}$

$$T^{\alpha\beta}_{; \beta} = 0$$

$$(T^{\alpha}_{\rho} \xi^{\rho})_{; \alpha} = 0$$

$$\oint_{\partial V} T^{\alpha}_{\rho} \xi^{\rho} \rightarrow \Sigma_{\alpha}$$

Transfer of mass & ang. momentum.



$$\left(T^{\alpha}_{\rho} \xi^{\rho} \right)_{;\alpha} = 0$$

$$\oint_{\partial V} T^{\alpha}_{\rho} \xi^{\rho} \rightarrow \Sigma_{\alpha} =$$

Transfer of mass and ang. momentum.



$$T^{\alpha\beta} \text{ if } = 0$$

$$(T^{\alpha}_{\rho} \xi^{\rho})_{,\alpha} = 0$$

$$\oint_{\partial V} T^{\alpha}_{\rho} \xi^{\rho} \rightarrow \bar{\Sigma}_{\alpha} = 0$$

Konformitätsabbildung

$$-T \rho \xi^{\rho}(\eta)$$

KOHNEN INTEGRALS

$$\Sigma^\alpha = -T^\alpha_\rho \Sigma^\rho \quad \Sigma^\alpha_{;\alpha} = 0$$

[The rest of the chalkboard is heavily obscured by multiple overlapping layers of chalk scribbles and erasures, making the text illegible.]



Kontinuitätsgleichungen

$$\Sigma^\alpha = -T^\alpha{}_\rho \xi^\rho(t) \quad \Sigma^\alpha{}_{;\alpha} = 0$$

= energy flux vector

$$\mathcal{L}^\alpha = T^\alpha{}_\rho \xi^\rho(x) \quad \mathcal{L}^\alpha{}_{;\alpha} = 0$$

= angular-momentum flux
vector

KONJUGATE INTEGRALS

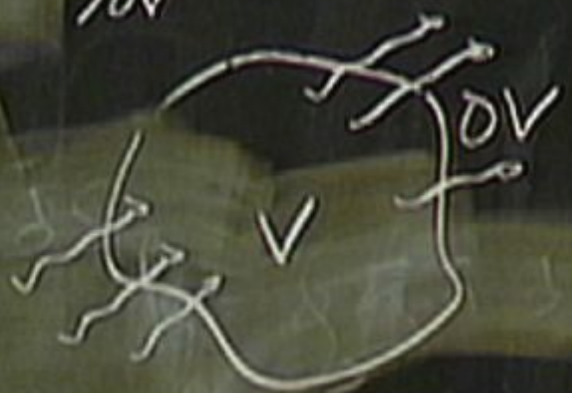
$$\Sigma^\alpha = -T^\alpha_\rho \xi^\rho(t) \quad \Sigma^\alpha_{;\alpha} = 0$$

= energy flux vector

$$\mathcal{L}^\alpha = T^\alpha_\rho \xi^\rho(r) \quad \mathcal{L}^\alpha_{;\alpha} = 0$$

= angular-momentum flux vector

$$\int_{\partial V} \Sigma^\alpha d\bar{z}_\alpha = 0$$



Kohnen integrals

$$\Sigma^\alpha = -T^\alpha_\rho \xi^\rho \quad (\text{H})$$

= energy flux vector

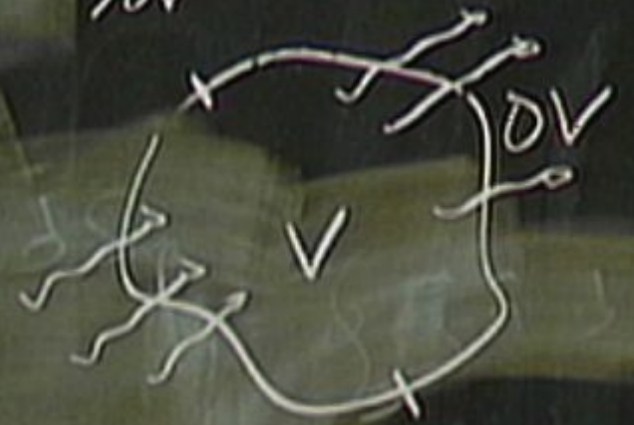
$$\Sigma^\alpha_{;\alpha} = 0$$

$$\oint_{\partial V} \epsilon^\alpha d\bar{z}_\alpha = 0$$

$$\mathcal{L}^\alpha = T^\alpha_\rho \xi^\rho \quad (\text{R})$$

= angular-momentum flux vector

$$\mathcal{L}^\alpha_{;\alpha} = 0$$



KOHAN INTEGRALS

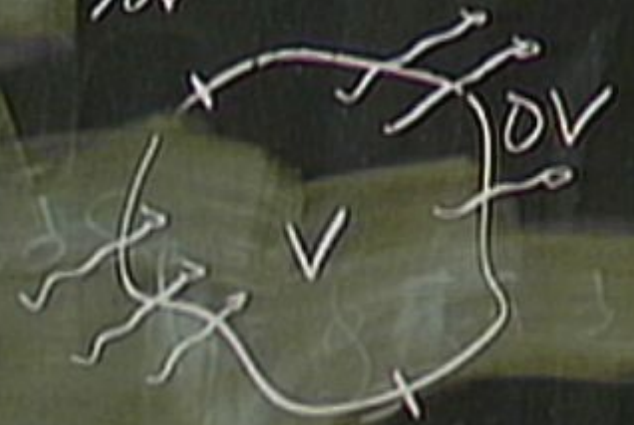
$$\Sigma^\alpha = -T^\alpha{}_\rho \xi^\rho \quad \Sigma^\alpha{}_{;\alpha} = 0$$

= energy flux vector

$$\mathcal{L}^\alpha = T^\alpha{}_\rho \xi^\rho \quad \mathcal{L}^\alpha{}_{;\alpha} = 0$$

= angular-momentum flux vector

$$\int_{\partial V} \epsilon^\alpha d\bar{z}_\alpha = 0$$



$$\int_S \epsilon^\alpha d\bar{z}_\alpha =$$

Klein's Integrals

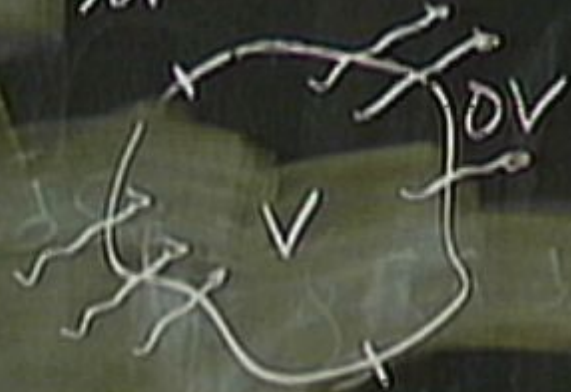
$$\Sigma^\alpha = -T^\alpha_\rho \xi^\rho(t) \quad \Sigma^\alpha_{;\alpha} = 0$$

= energy flux vector

$$\mathcal{L}^\alpha = T^\alpha_\rho \xi^\rho(r) \quad \mathcal{L}^\alpha_{;\alpha} = 0$$

= angular-momentum flux vector

$$\int_{\partial V} \Sigma^\alpha d\bar{z}_\alpha = 0$$



$$\int_{\Sigma} \Sigma^\alpha d\bar{z}_\alpha = \text{Energy flowing } \Sigma$$

$$\int_{\Sigma} \mathcal{L}^\alpha d\bar{z}_\alpha =$$



Kontinuitätsgleichungen

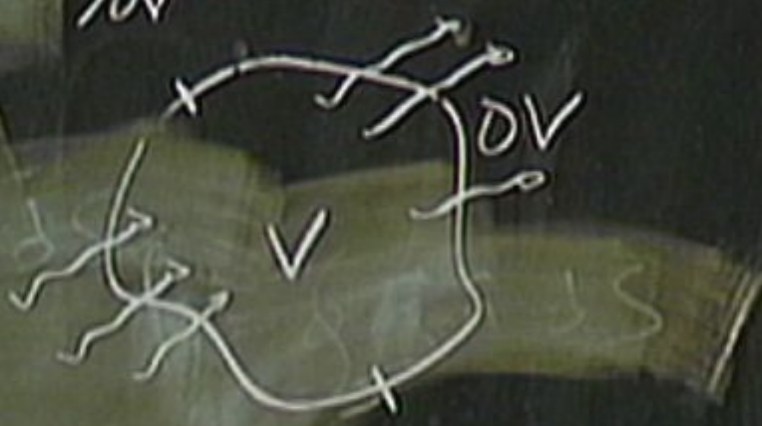
$$\Sigma^\alpha = -T^\alpha_\rho \xi^\rho(t) \quad \Sigma^\alpha_{;\alpha} = 0$$

= energy flux vector

$$\mathcal{L}^\alpha = T^\alpha_\rho \xi^\rho(x) \quad \mathcal{L}^\alpha_{;\alpha} = 0$$

= angular-momentum flux vector

$$\int_{\partial V} \Sigma^\alpha d\bar{z}_\alpha = 0$$



$$\int_{\Sigma} \Sigma^\alpha d\bar{z}_\alpha = \text{energy crossing } \Sigma$$

$$\int_{\Sigma} \mathcal{L}^\alpha d\bar{z}_\alpha = \text{angular momentum crossing } \Sigma$$

$$\int_{\Sigma} \varepsilon^{\alpha} \delta \vec{Z}_{\alpha} = \text{Energy crossing } \Sigma$$

$$\int_{\Sigma} \varrho^{\alpha} \delta \vec{Z}_{\alpha} = \text{angular momentum crossing } \Sigma$$