Title: Advanced General Relativity - Lecture 10A

Date: Mar 26, 2008 10:30 AM

URL: http://pirsa.org/08030014

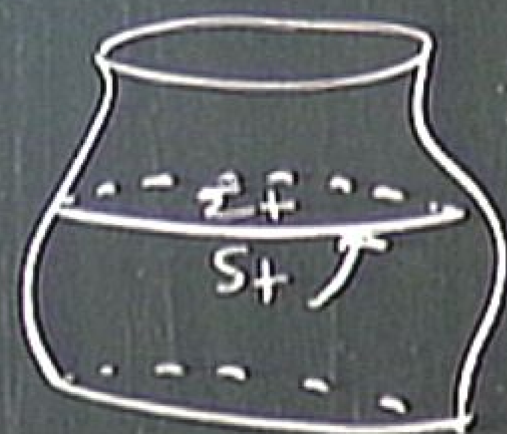
Abstract: Advanced General Relativity

HAMILTONIAN

$$16\pi H_G = \int_{\Sigma_t} \left[N\left(K^{ab}K_{ab} - K^2 - {}^3R\right) - 2N_a D_b\left(K^{ab} - K h^{ab}\right)\right]\sqrt{h}\, d^3y$$

$$- 2\oint_{S_t} \left[N(k - k_0) - N_a\left(K^{ab} - K h^{ab}\right) r_b \right] \sqrt{\sigma}\, d^2\theta$$

$$t^\alpha = N n^\alpha + N^a e^\alpha_a$$



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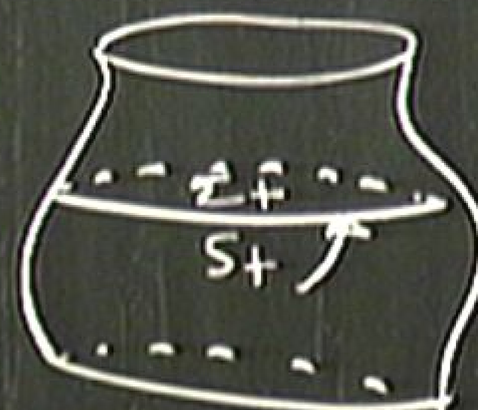
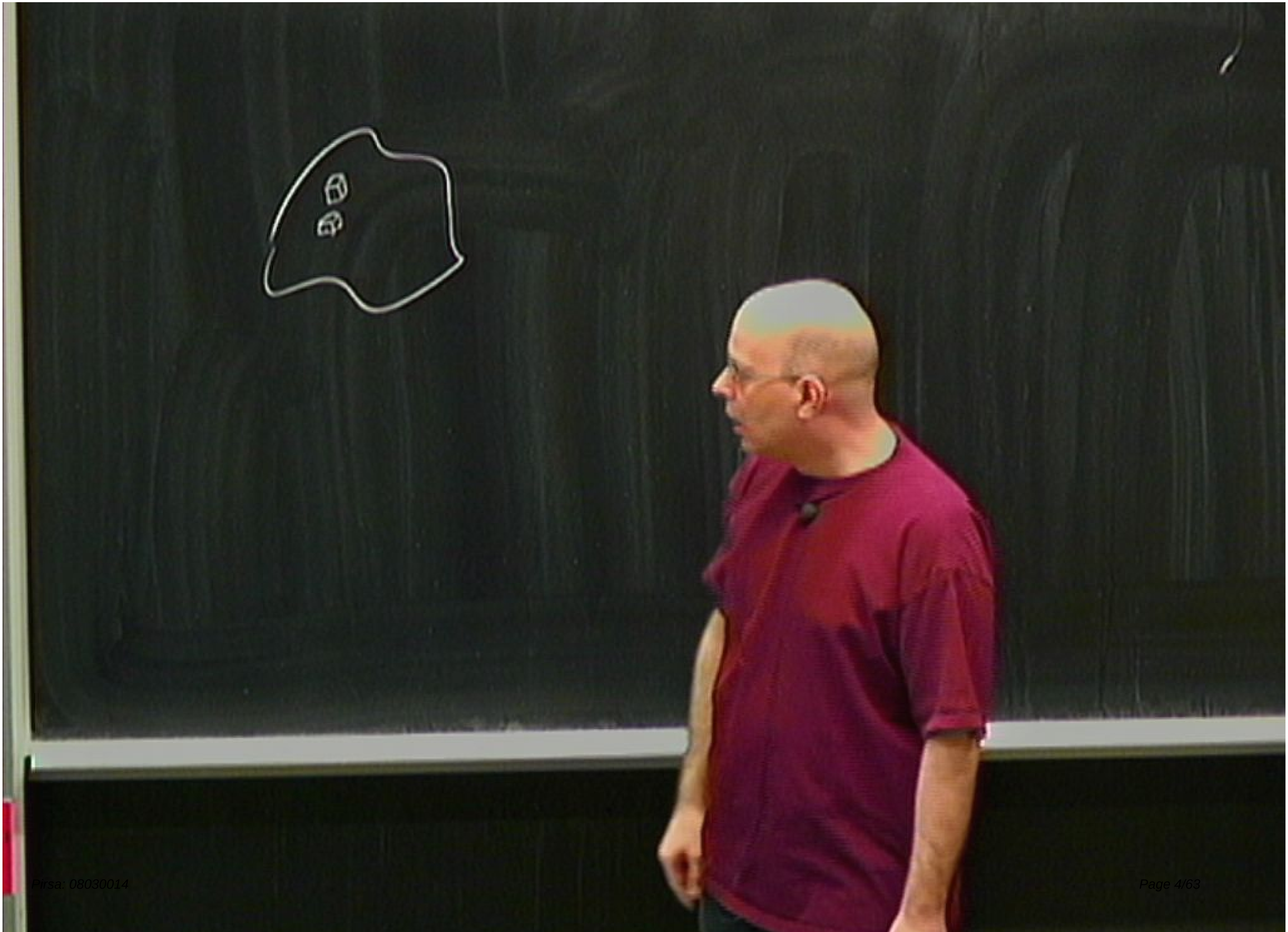
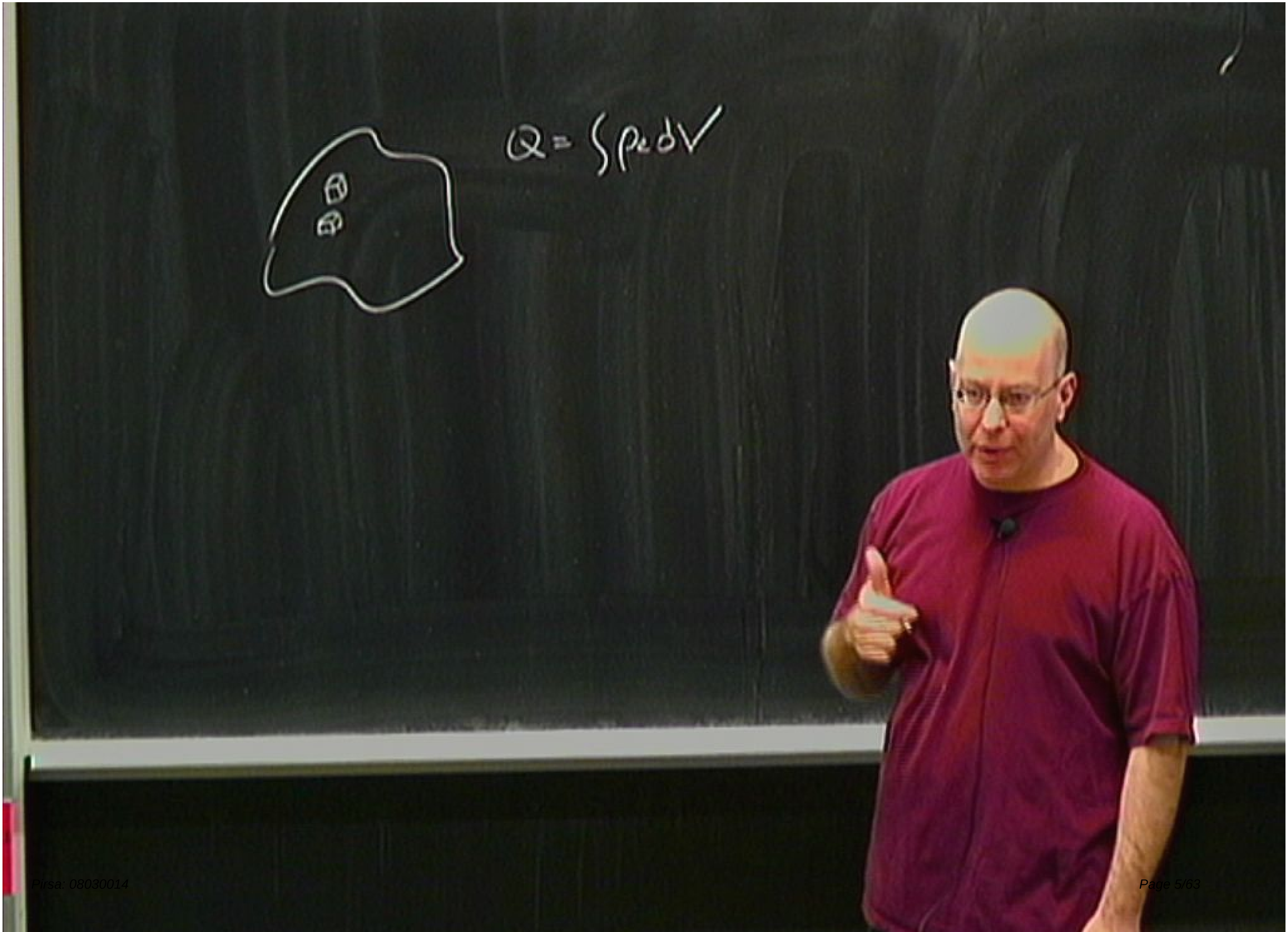
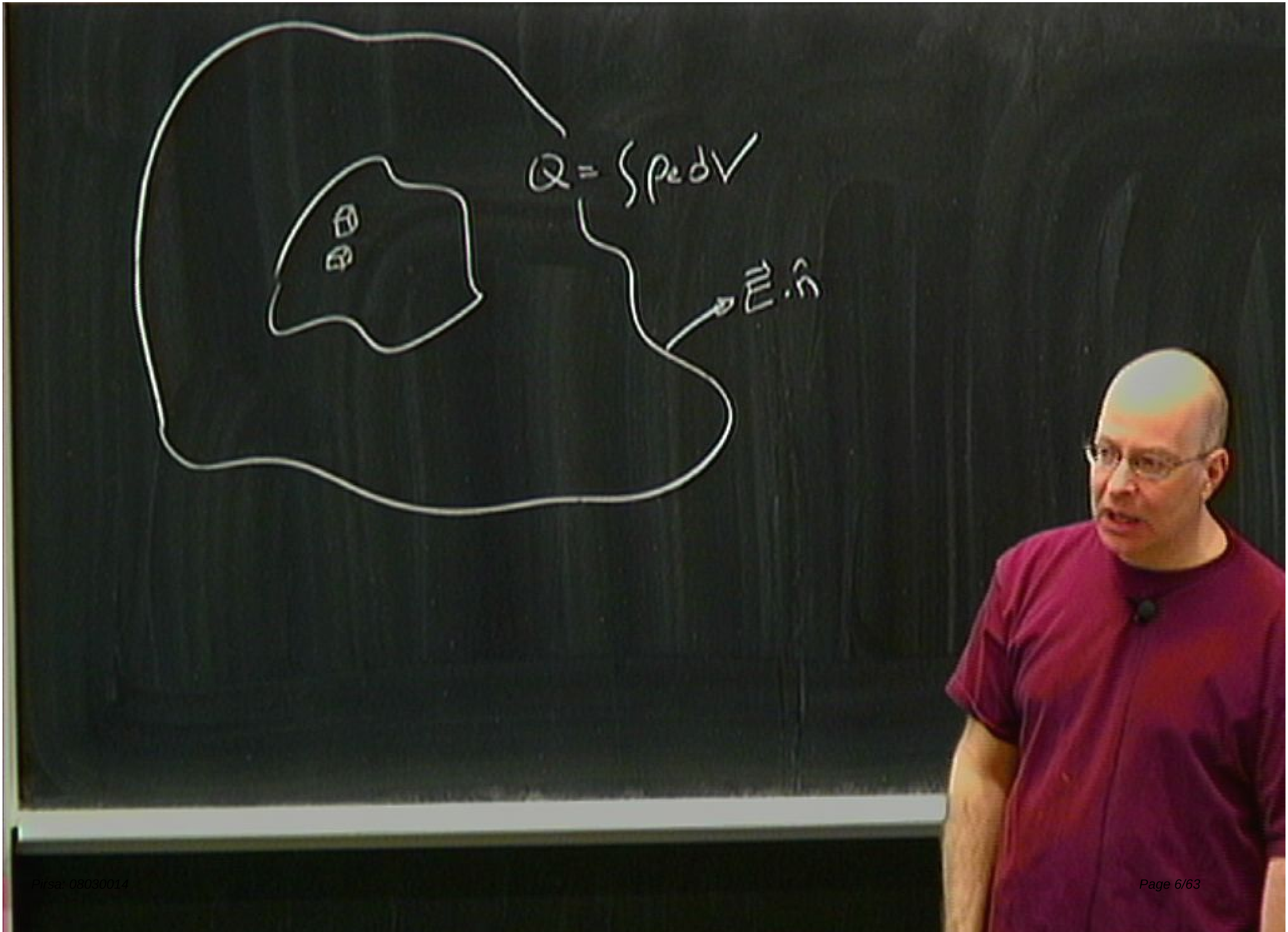
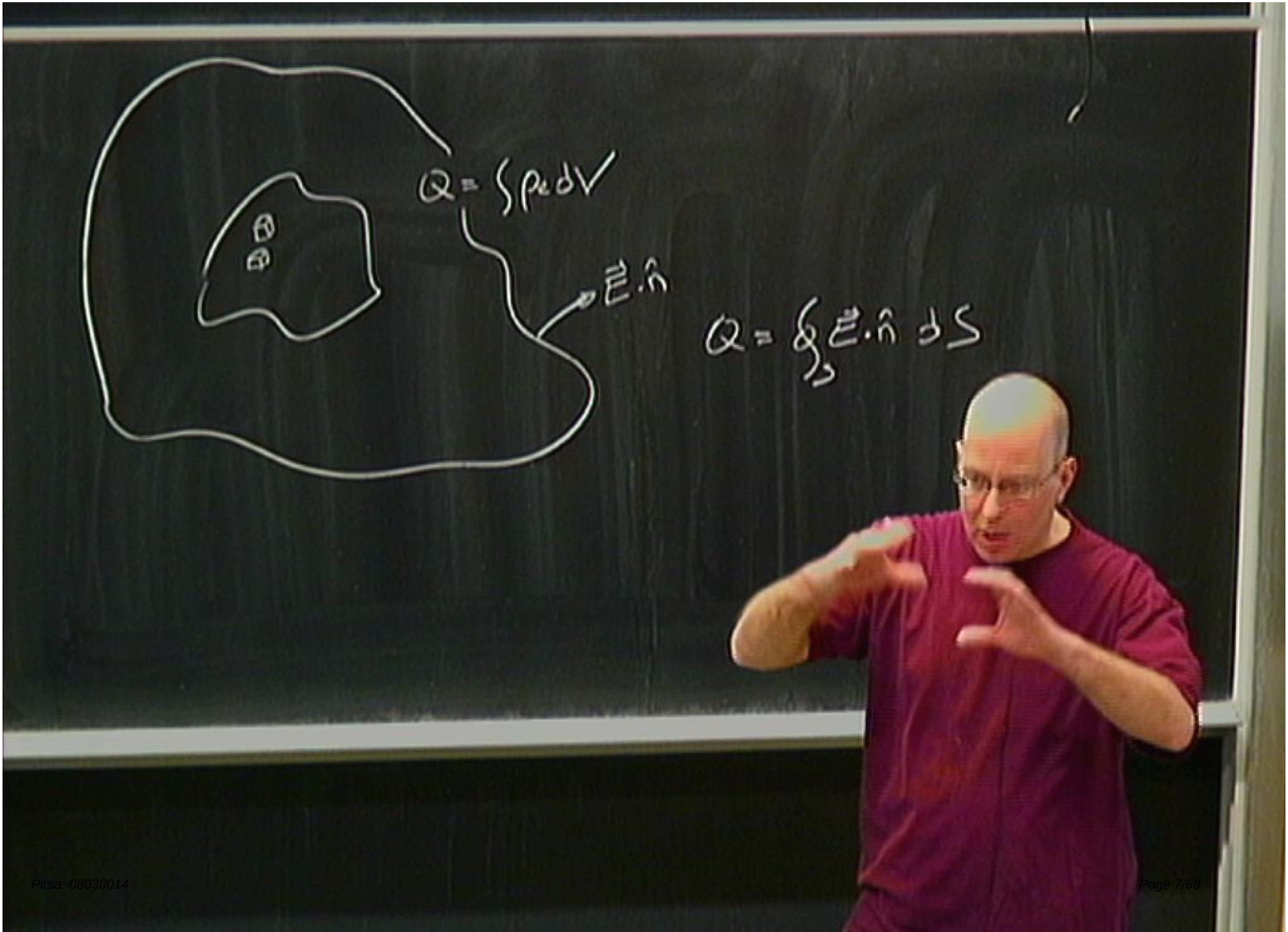


$k_{AB} = r_{a|b}\, e^a_A e^b_B$

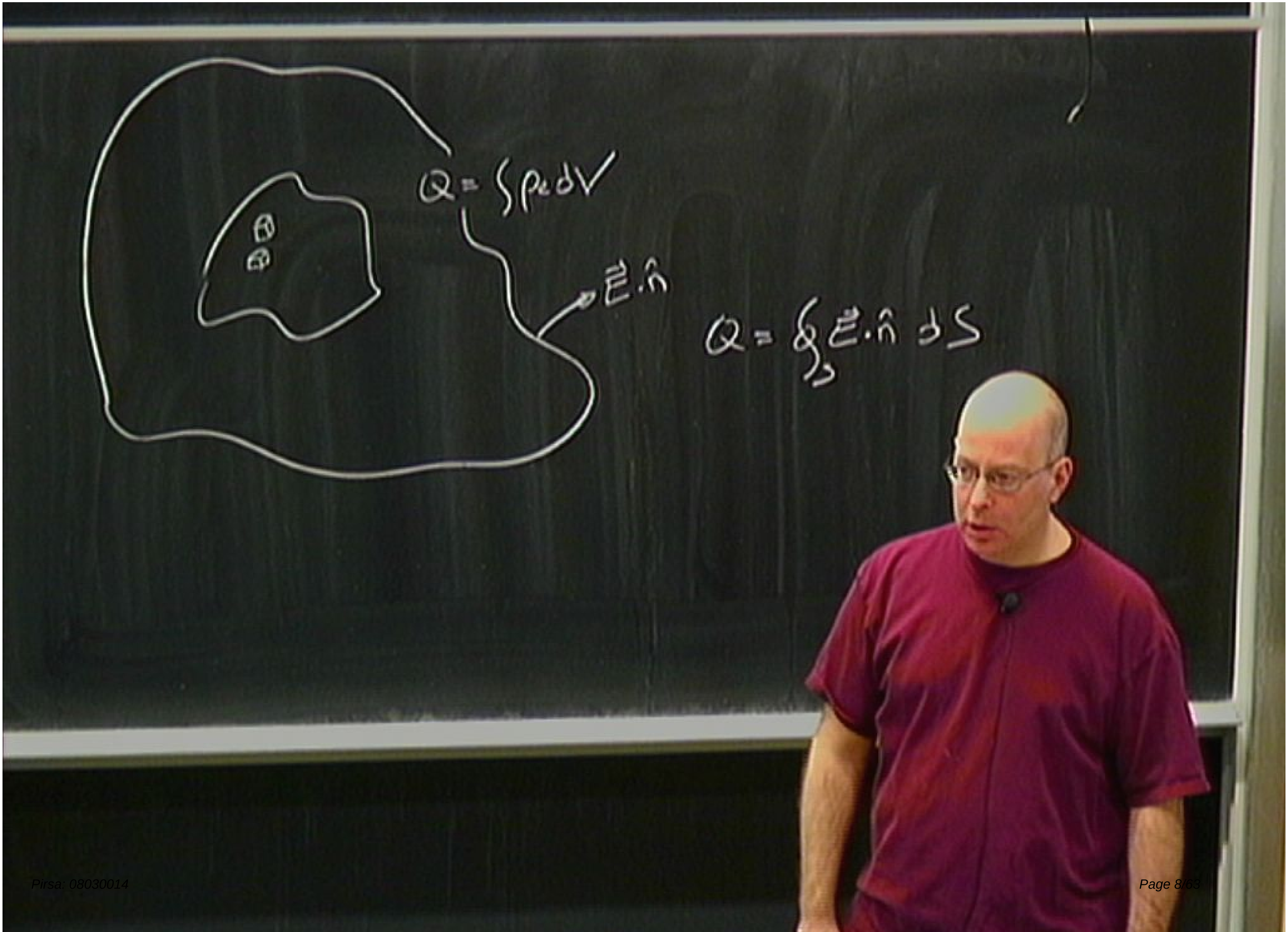


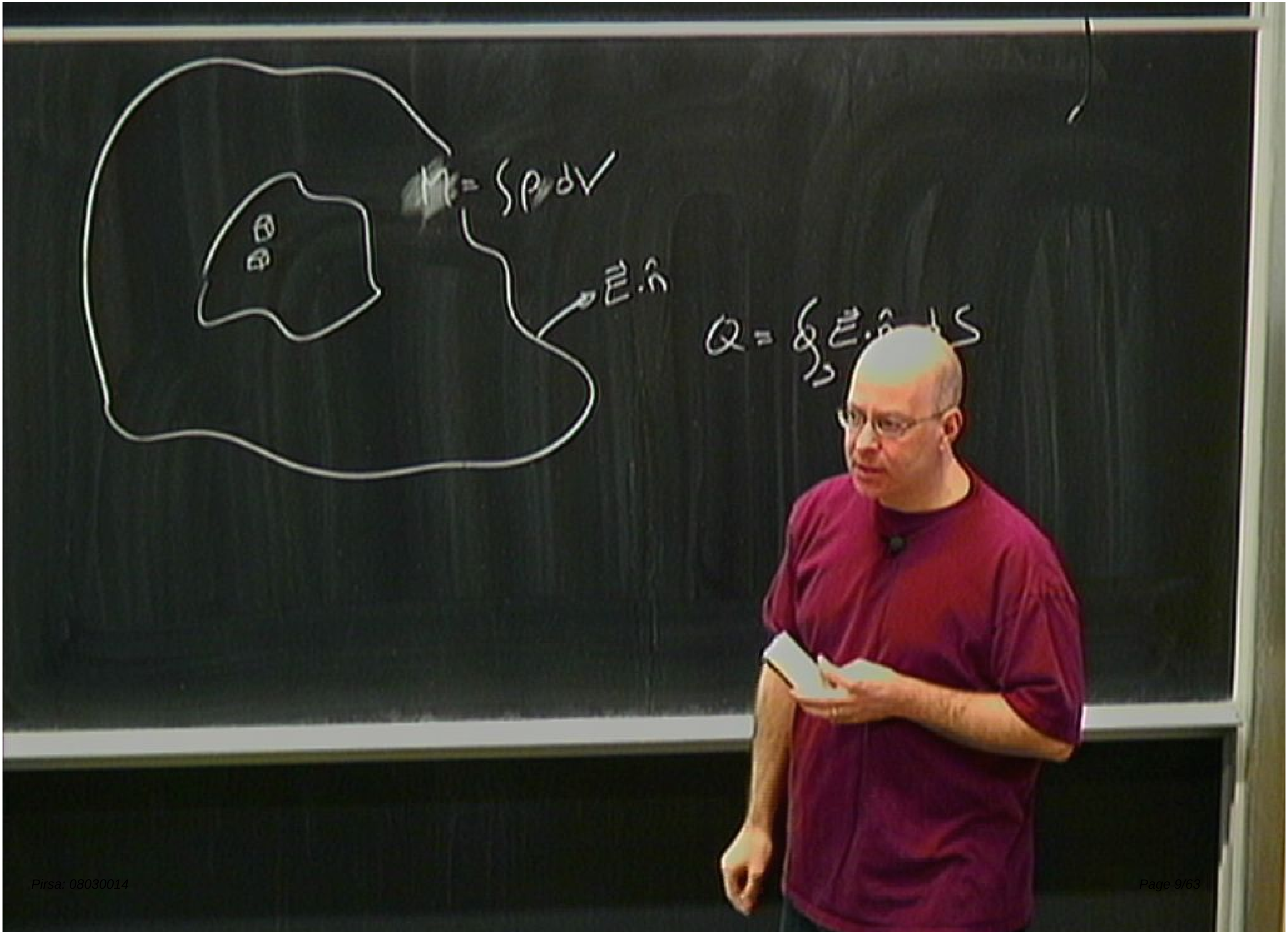

$Q = \int \rho_e \, dV$


$Q = \int \rho_e \, dV$
$\vec{E} \cdot \hat{n}$

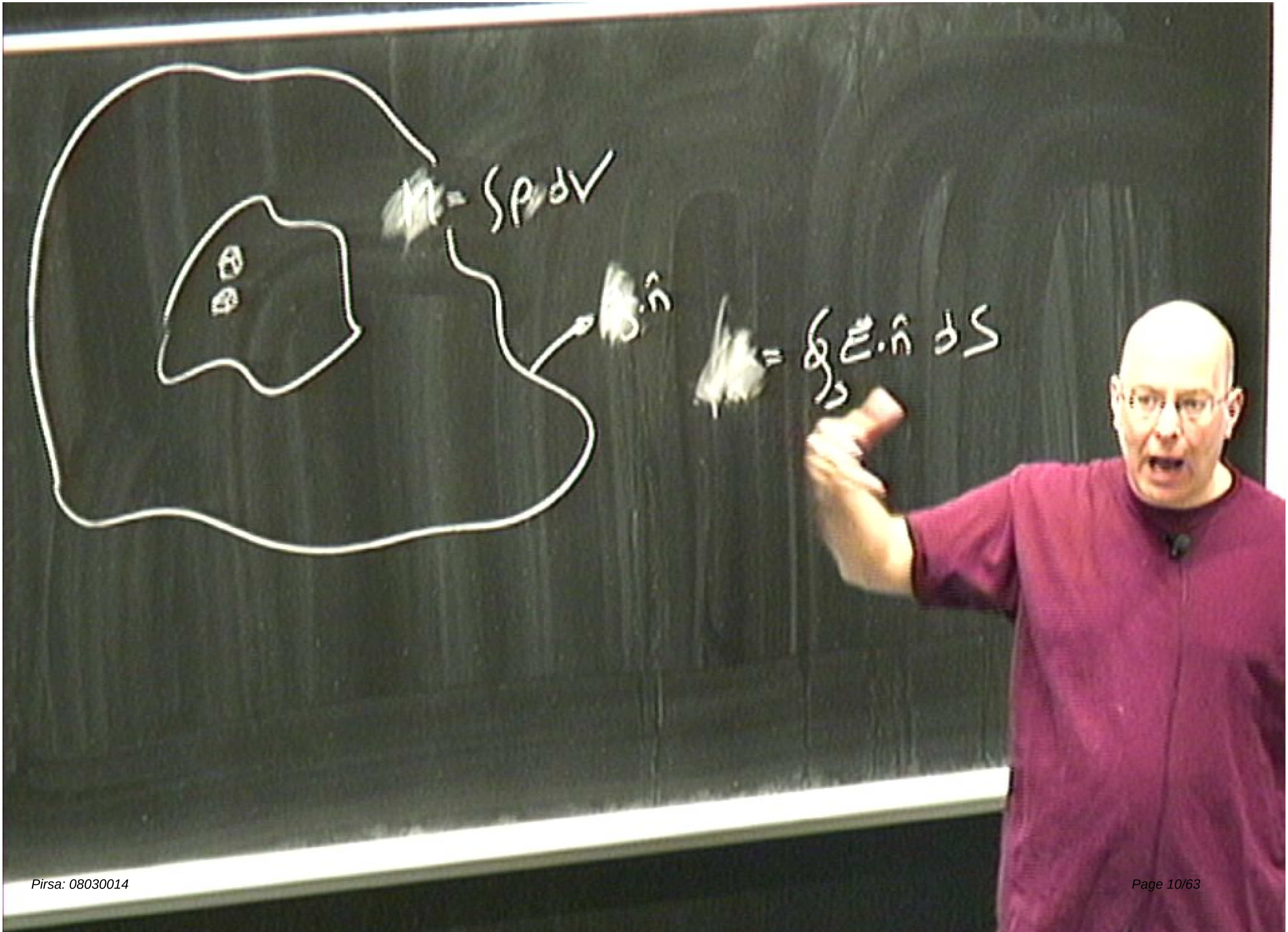


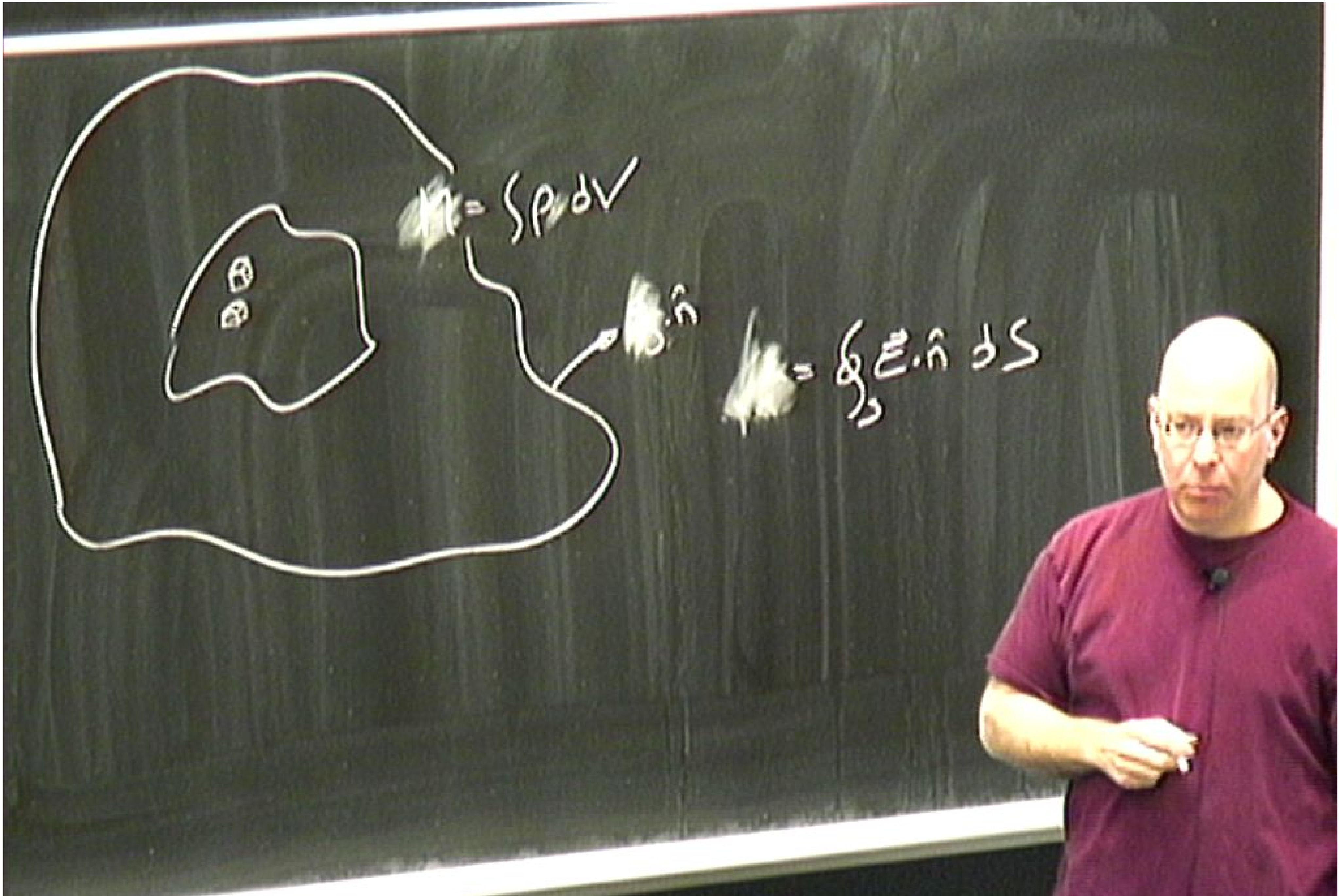
Q = ∫ρe dV
E·n̂
Q = ∮S E·n̂ dS

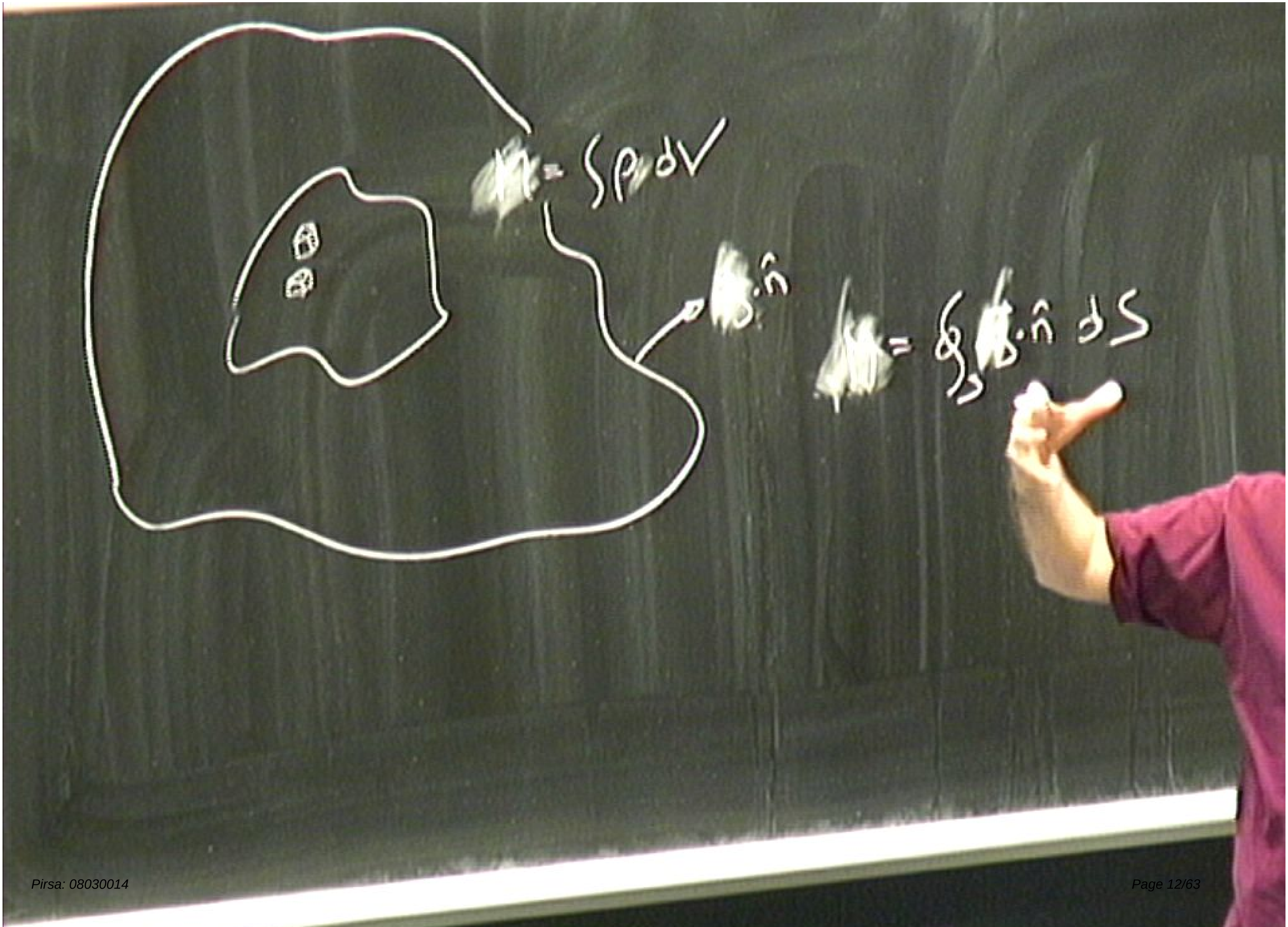

$Q = \int \rho_e \, dV$
$\vec{E} \cdot \hat{n}$
$Q = \oint_S \vec{E} \cdot \hat{n} \, dS$

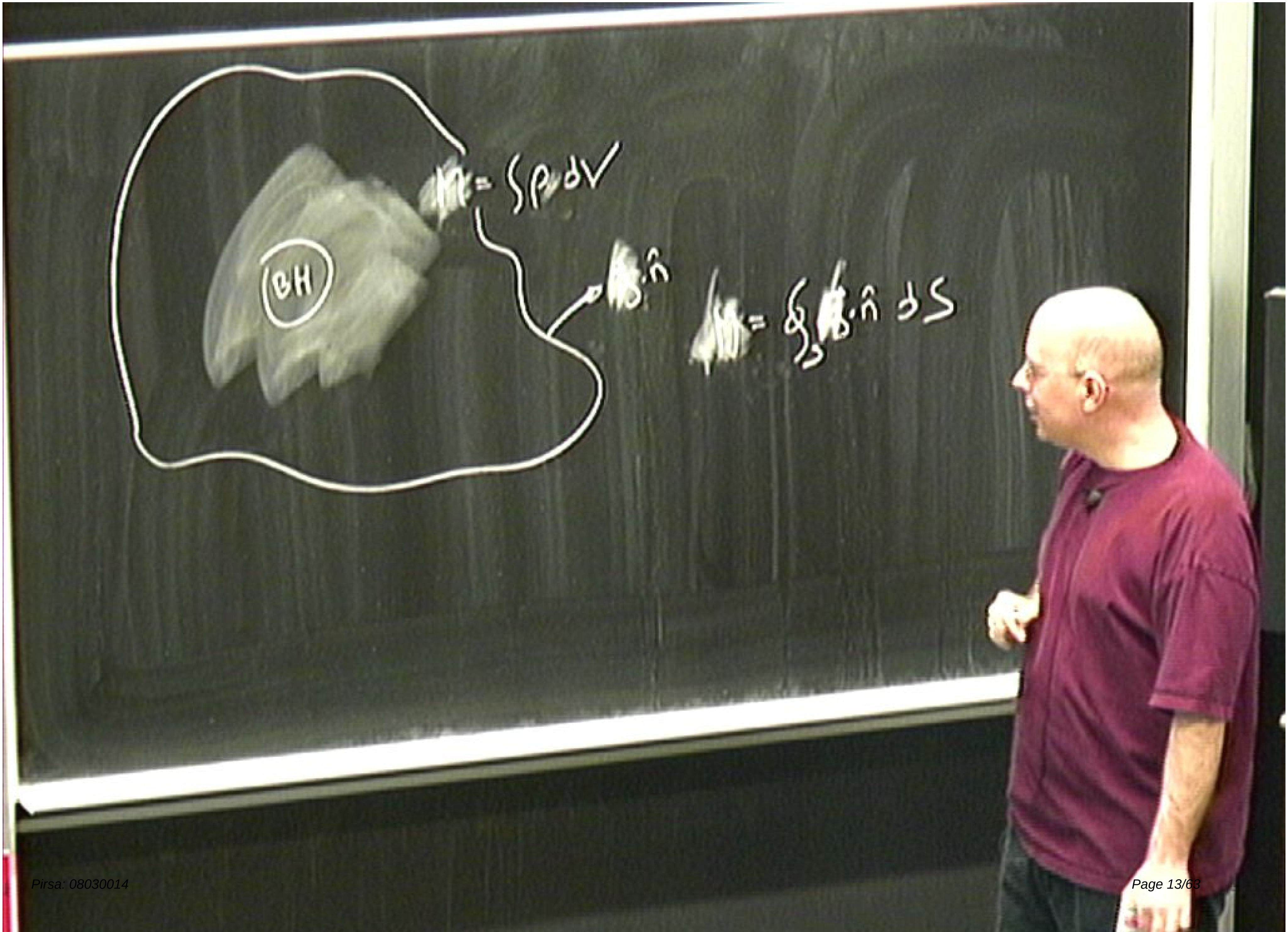


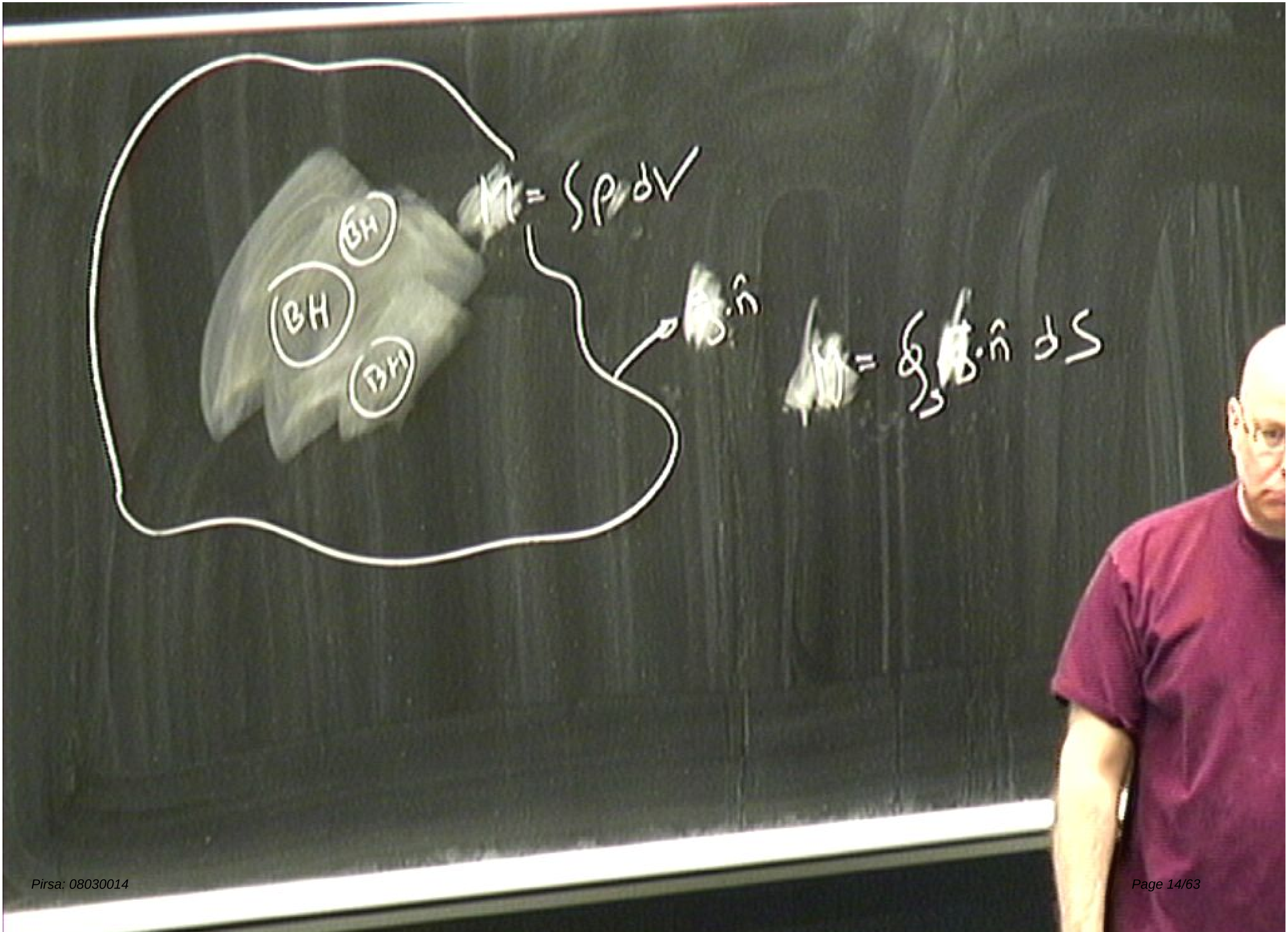
M = ∫ρ dV
E·n̂


$= \int \rho \, dV$
$\cdot \hat{n}$
$= \oint_S E \cdot \hat{n} \, dS$


= ∫ρ dV
= ∮ E·n̂ dS


= ∫ρ dV
·n̂
= ∮S ·n̂ dS


BH
M = ∫ρ dV
g·n̂
M = ∮ g·n̂ dS


BH
BH
BH
ρ dV
n̂ dS

MASS AND ANGULAR MOMENTUM

Near ∞, solve linearized EFE as a perturbation of Minkowski.

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Stationary, axisymmetric spacetime:

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Stationary, axisymmetric spacetime:

$$ds^2 = -\left(1 - \frac{2m}{r} + O(r^{-2})\right)dt^2 + \left(1 + \frac{2m}{r} + O(r^{-2})\right)\left(dr^2 + r^2 d\Omega^2\right)$$

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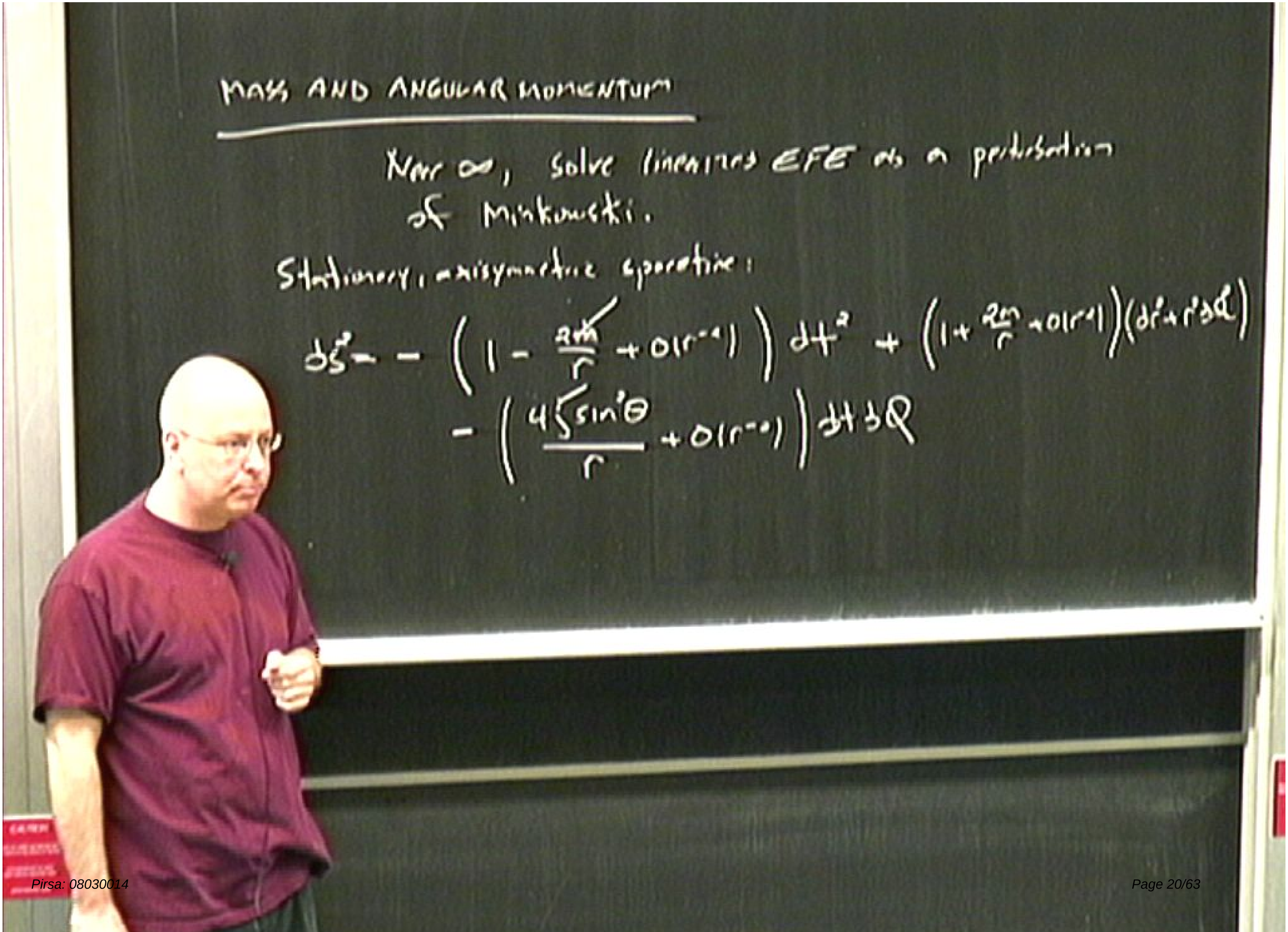
$$- \left(\frac{4J \sin^2\theta}{r} + O(r^{-2})\right) dt\, d\phi$$


MASS AND ANGULAR MOMENTUM
Near ∞, solve linearized EFE as a perturbation of Minkowski.
Stationary, axisymmetric spacetime:
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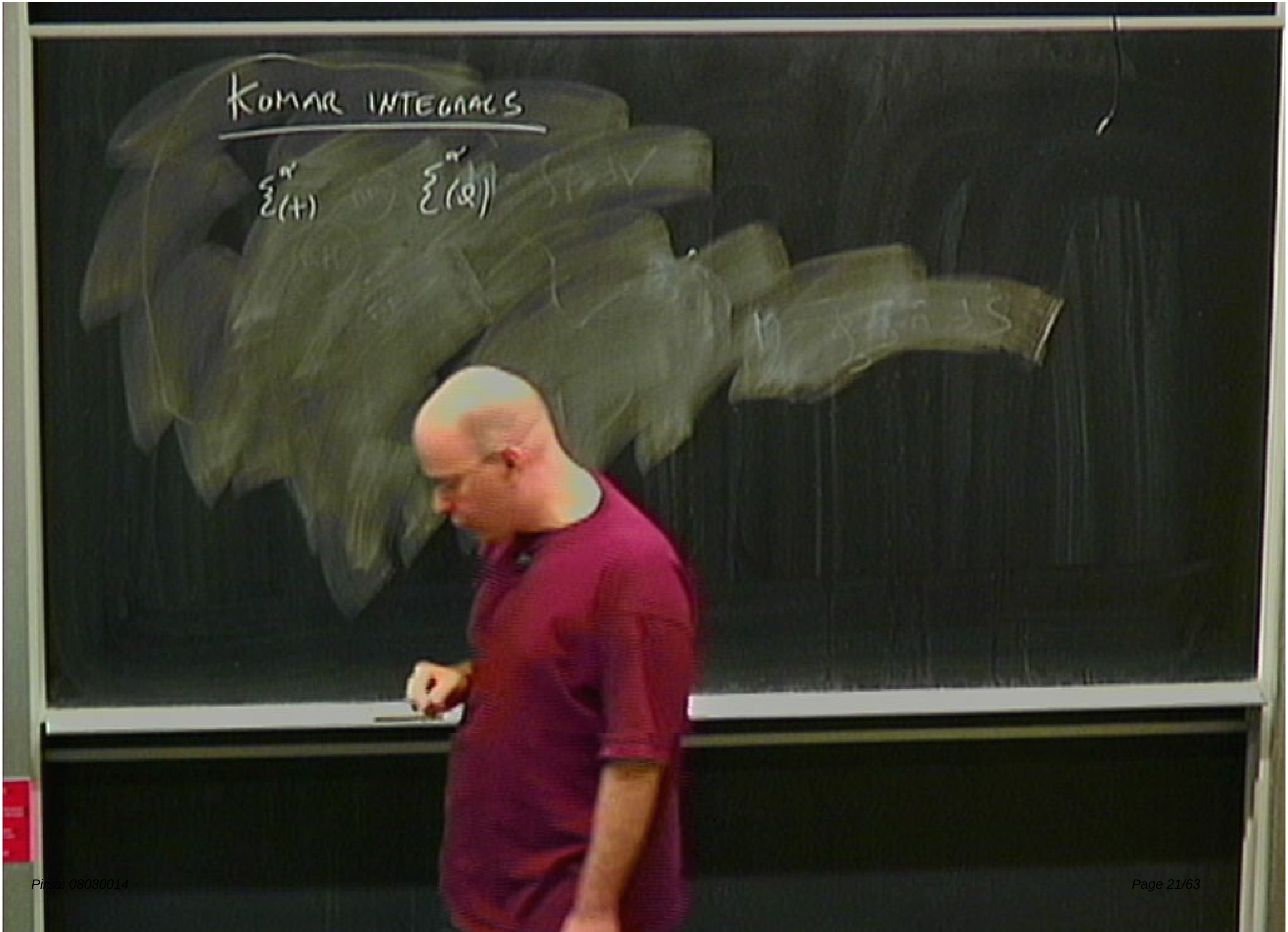
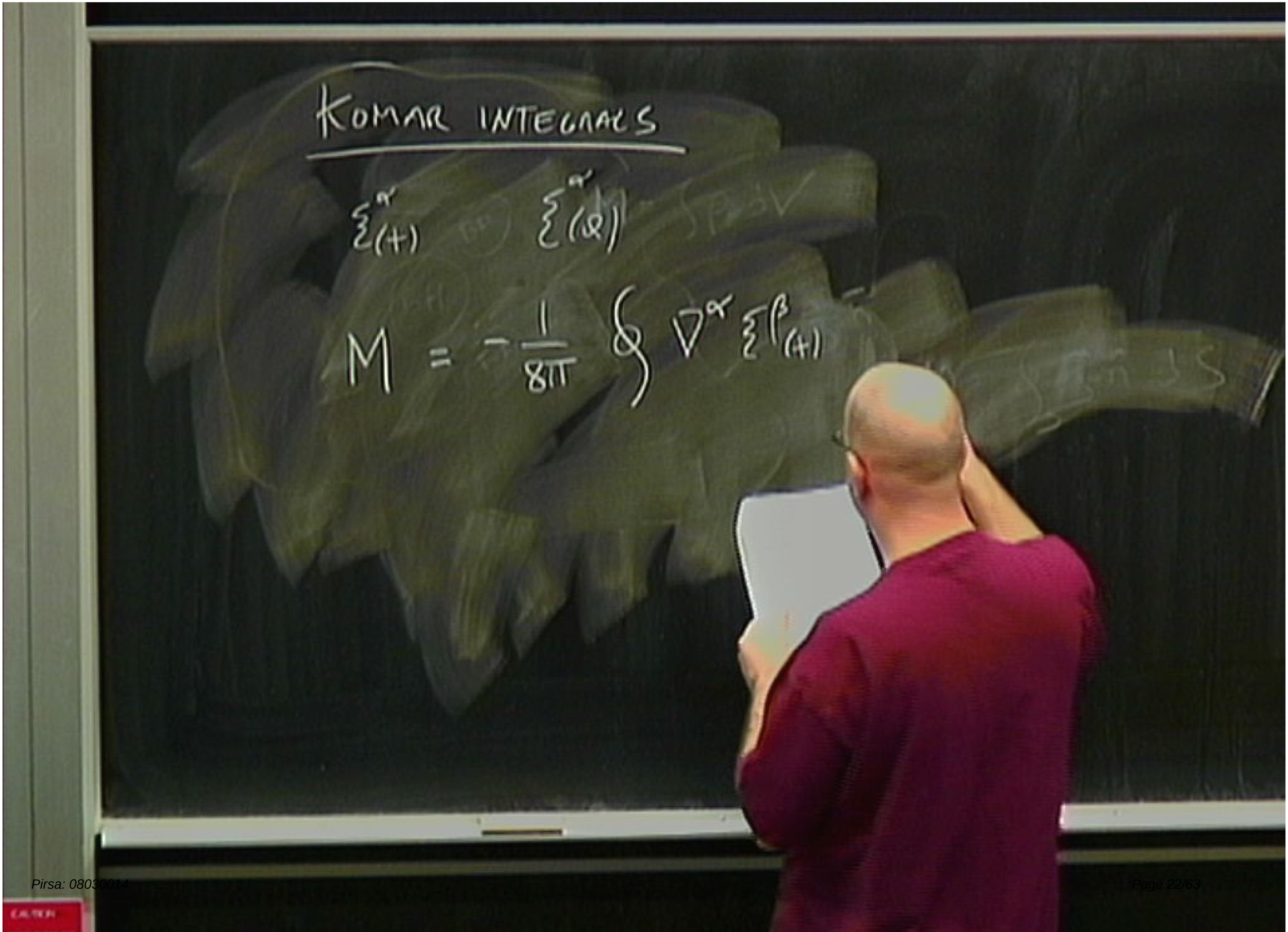
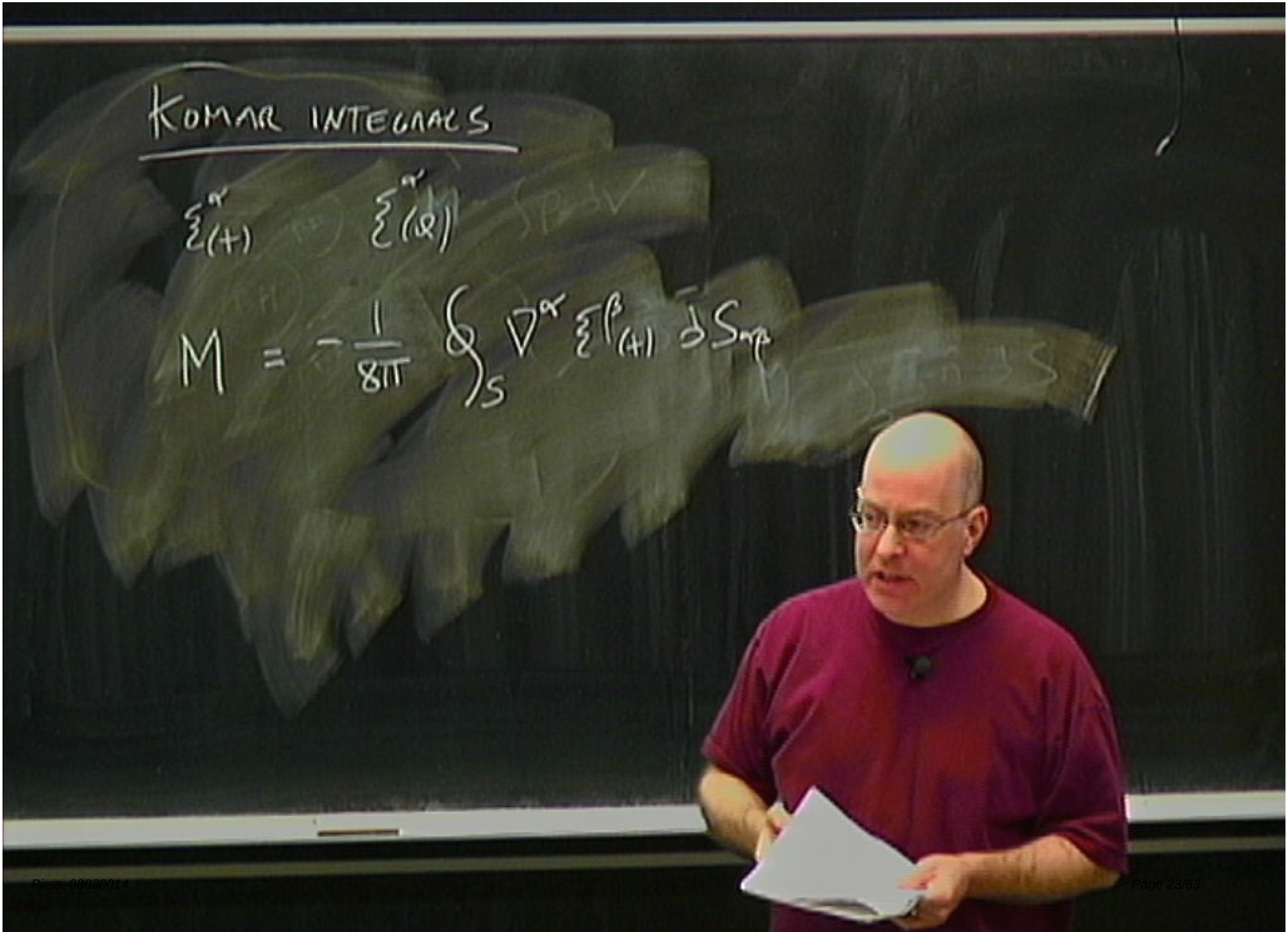

KOMAR INTEGRALS
$\xi^{\alpha}_{(t)}$
$\xi^{\alpha}_{(\phi)}$


KOMAR INTEGRALS
$\xi^{\alpha}_{(t)}$ $\xi^{\alpha}_{(\phi)}$
$M = -\frac{1}{8\pi} \oint \nabla^{\alpha} \xi^{\beta}_{(t)}$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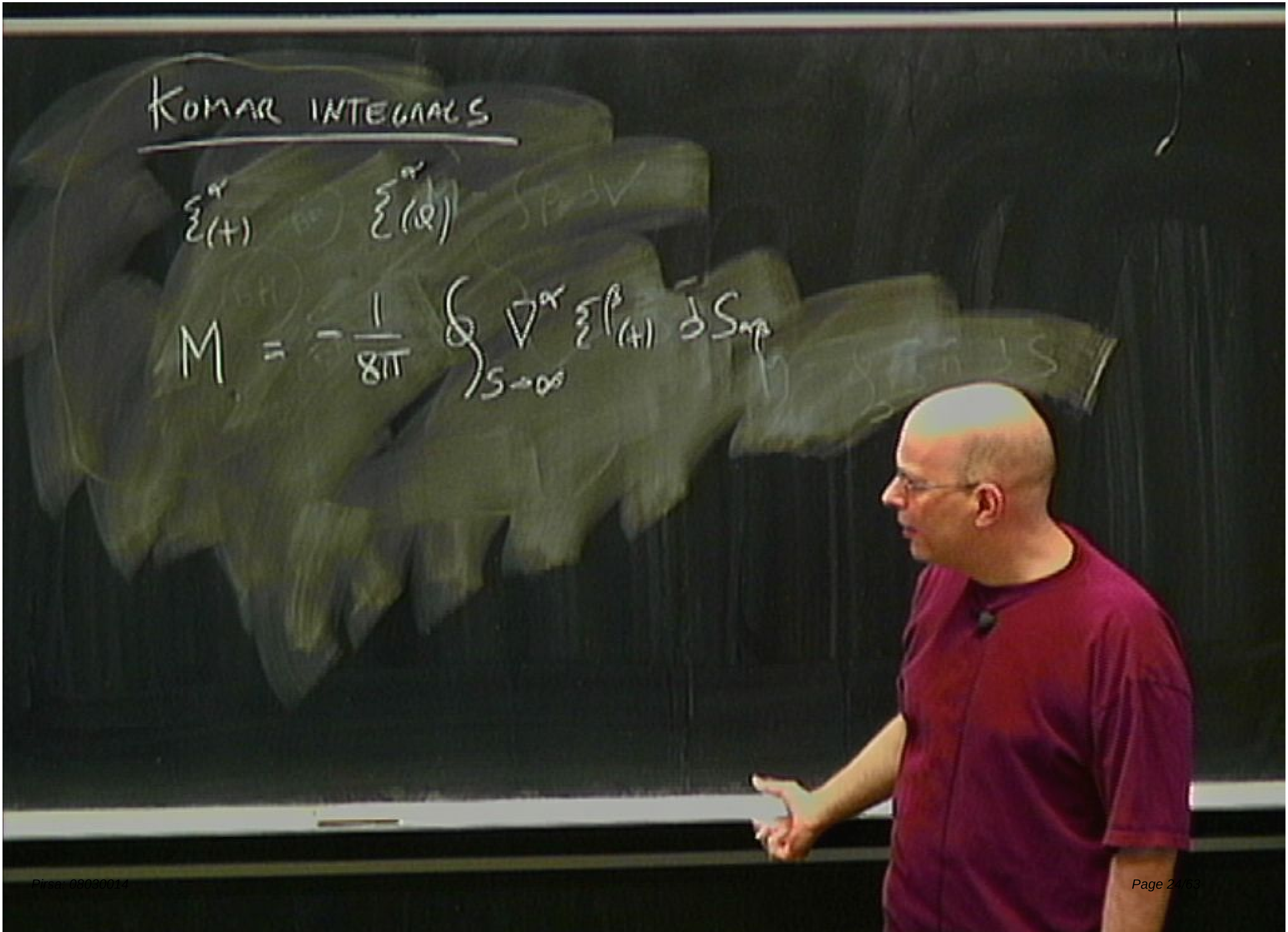
KOMAR INTEGRALS
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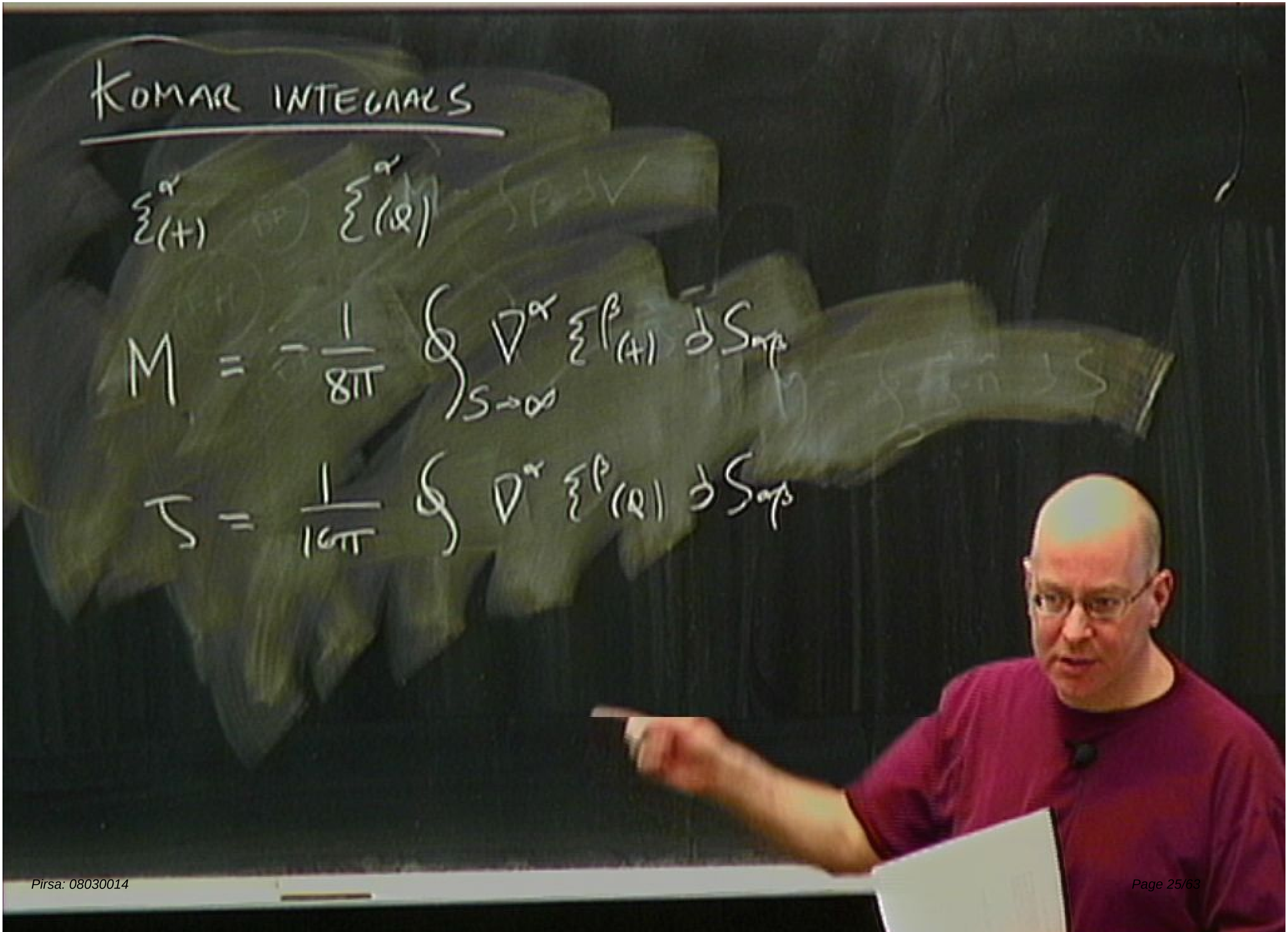
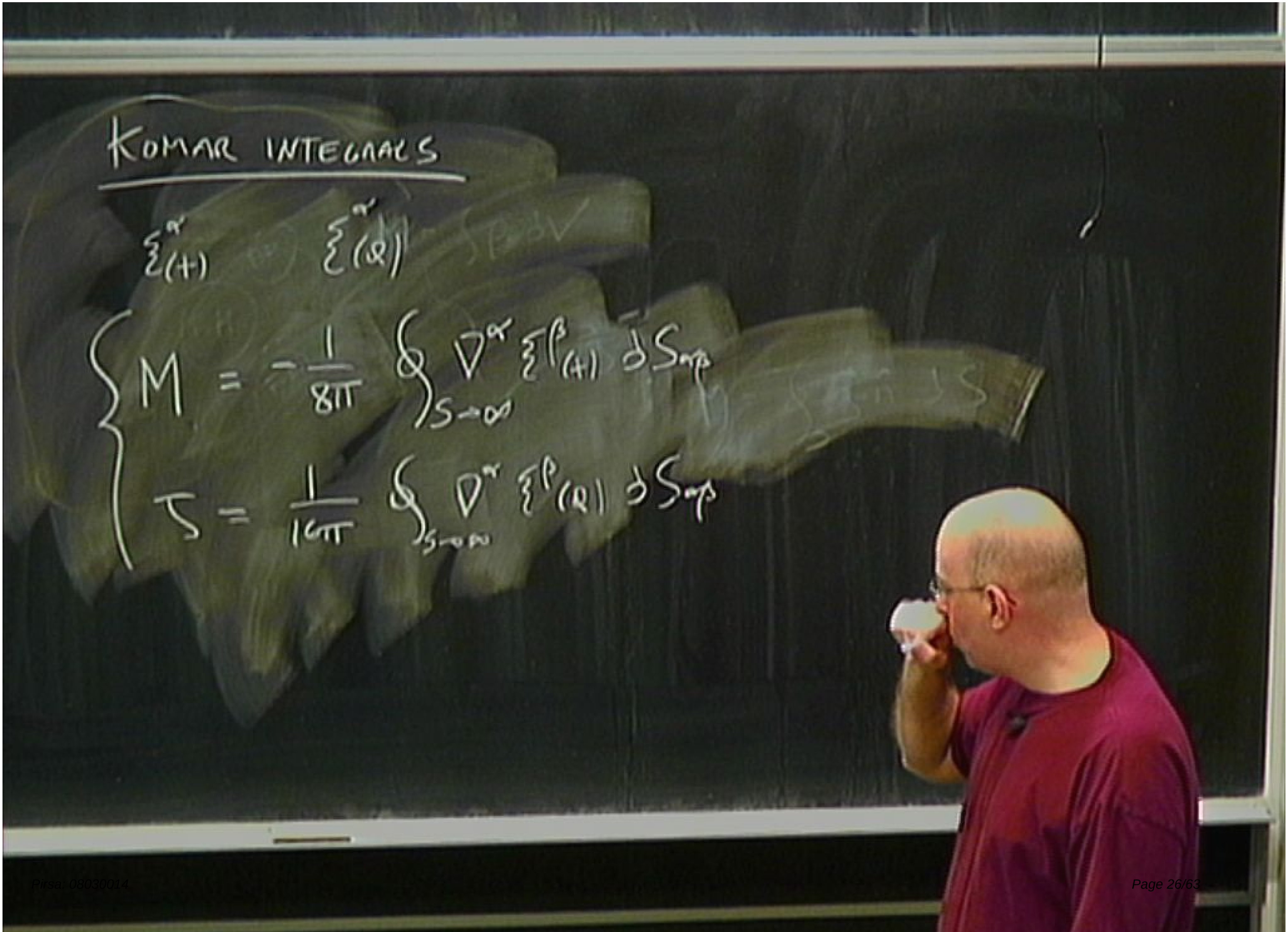
$M = -\frac{1}{8\pi}\oint_{S\to\infty} \nabla^\alpha \xi^\beta_{(t)}\, dS_{\alpha\beta}$


KOMAR INTEGRALS
$\xi^{\alpha}_{(t)}$ $\xi^{\alpha}_{(\phi)}$
$M = -\frac{1}{8\pi} \oint_{S\to\infty} \nabla^{\alpha} \xi^{\beta}_{(t)} \, dS_{\alpha\beta}$
$J = \frac{1}{16\pi} \oint \nabla^{\alpha} \xi^{\beta}_{(\phi)} \, dS_{\alpha\beta}$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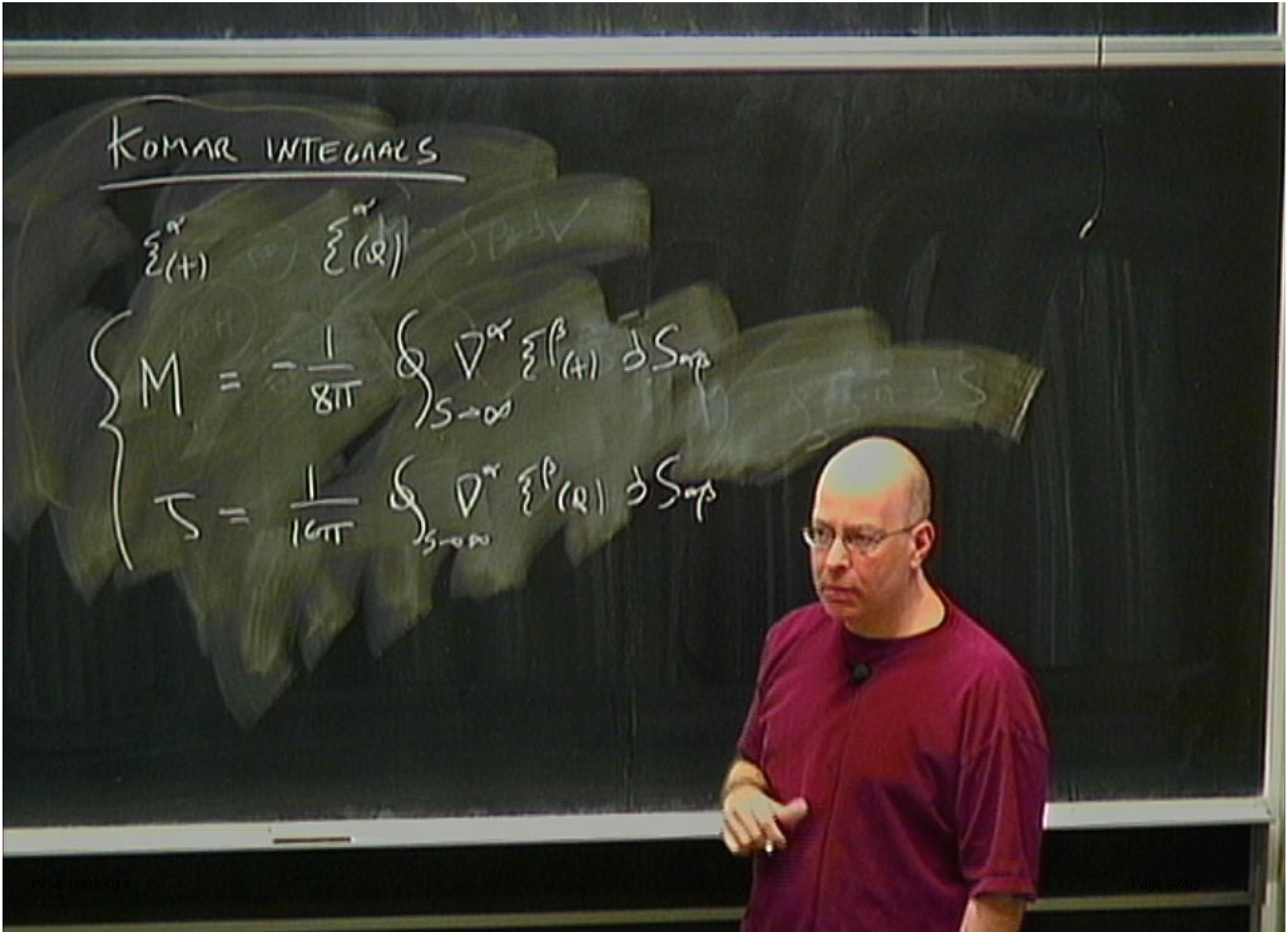
KOMAR INTEGRALS
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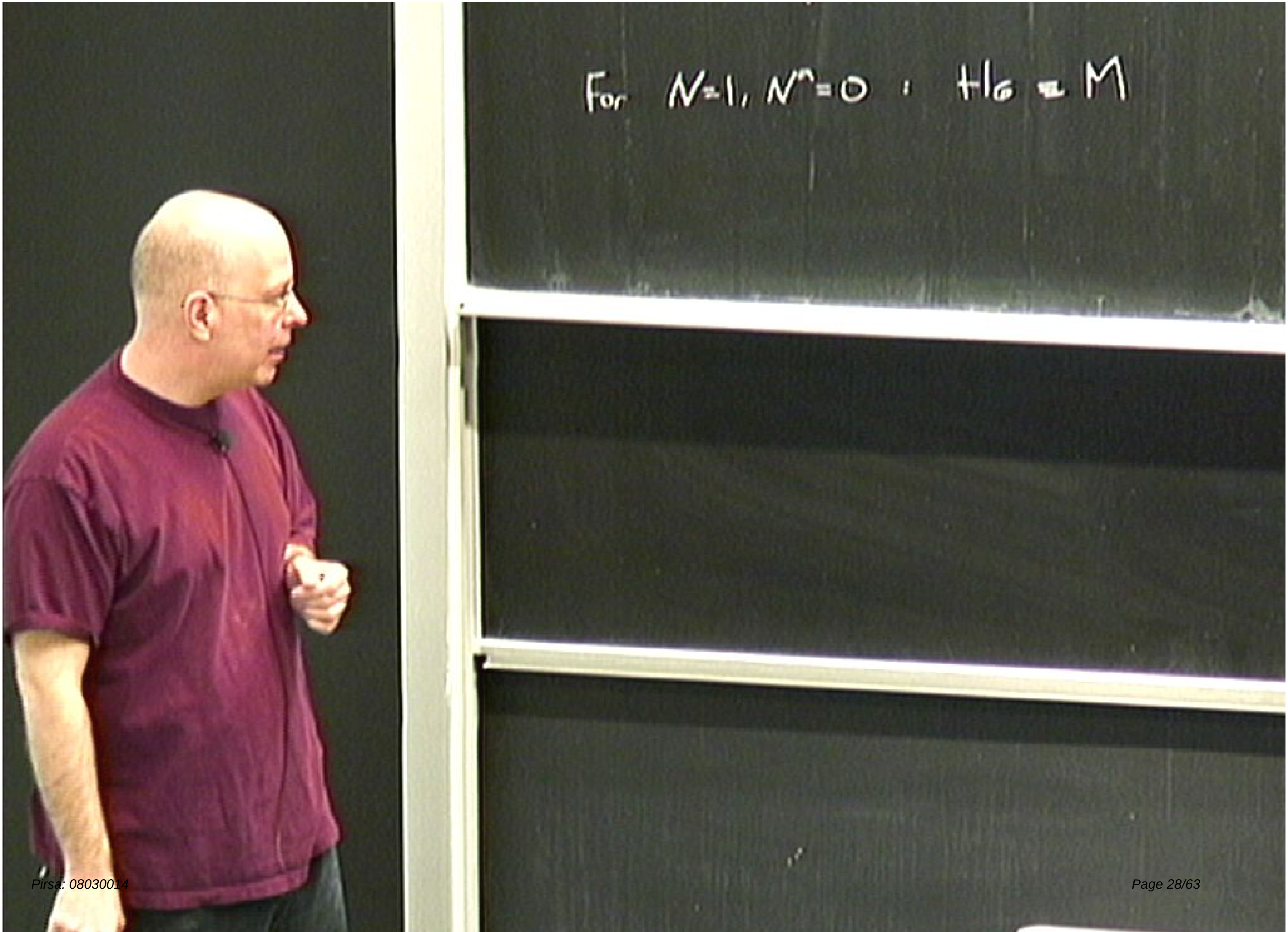
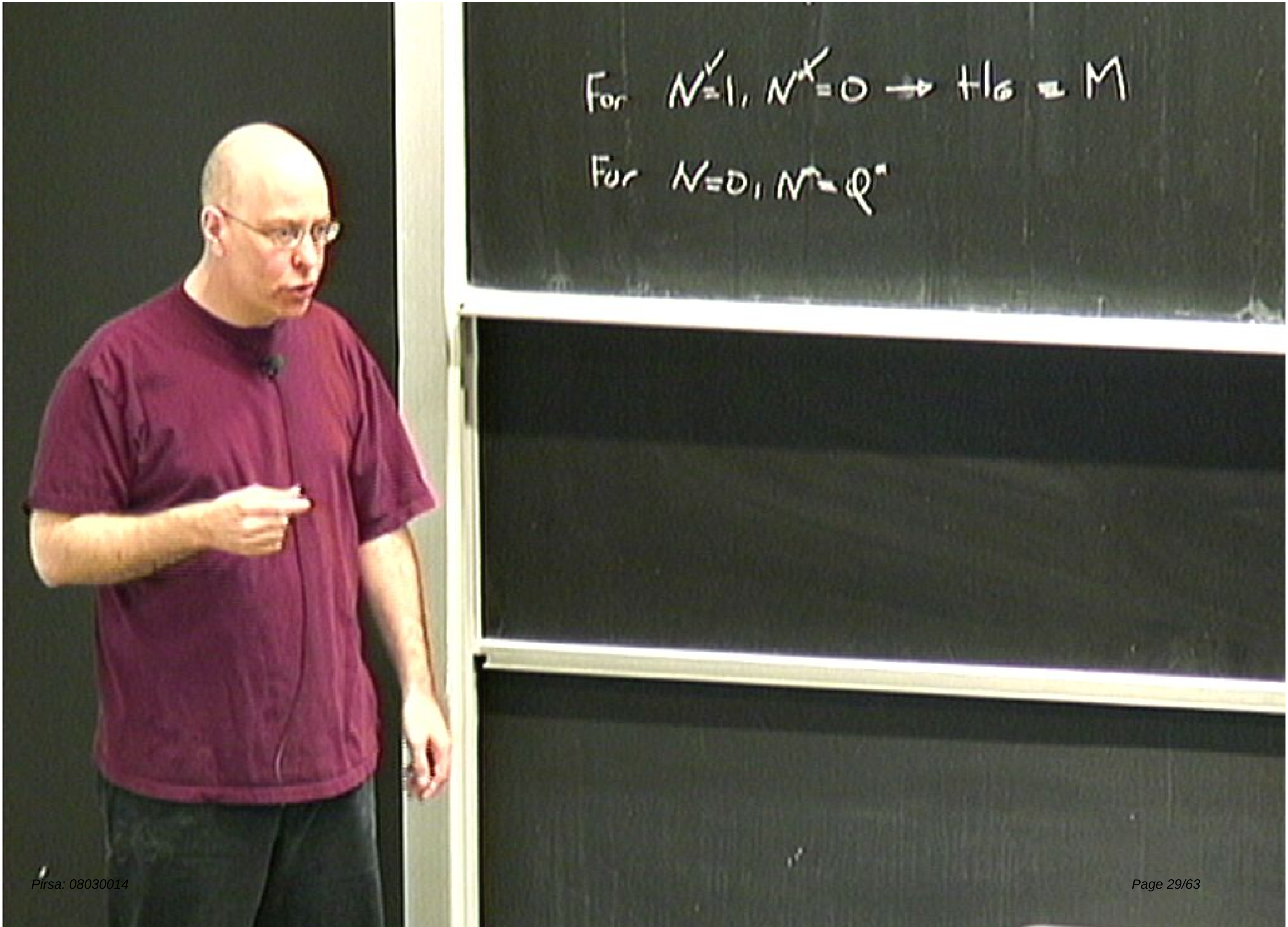
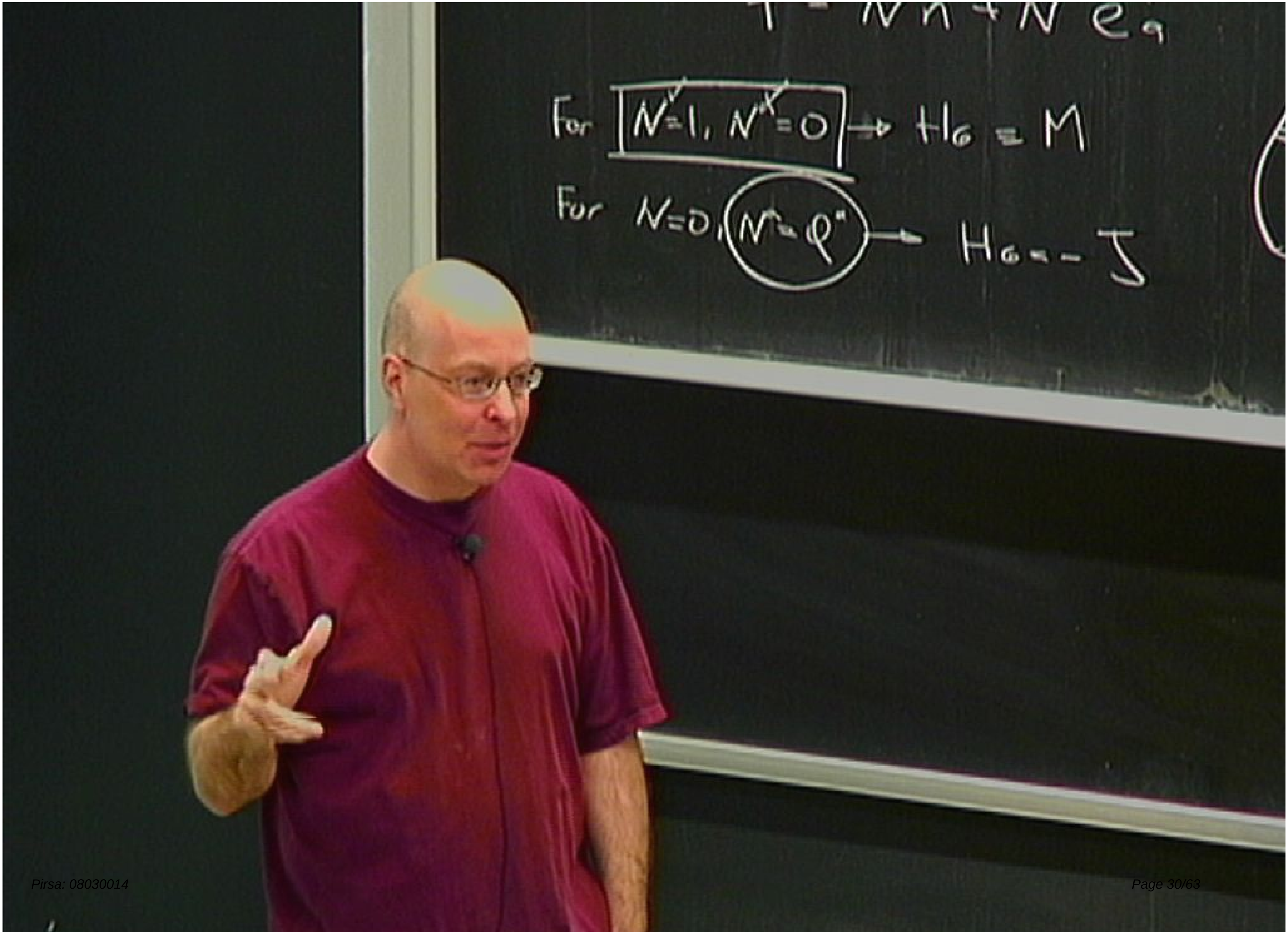
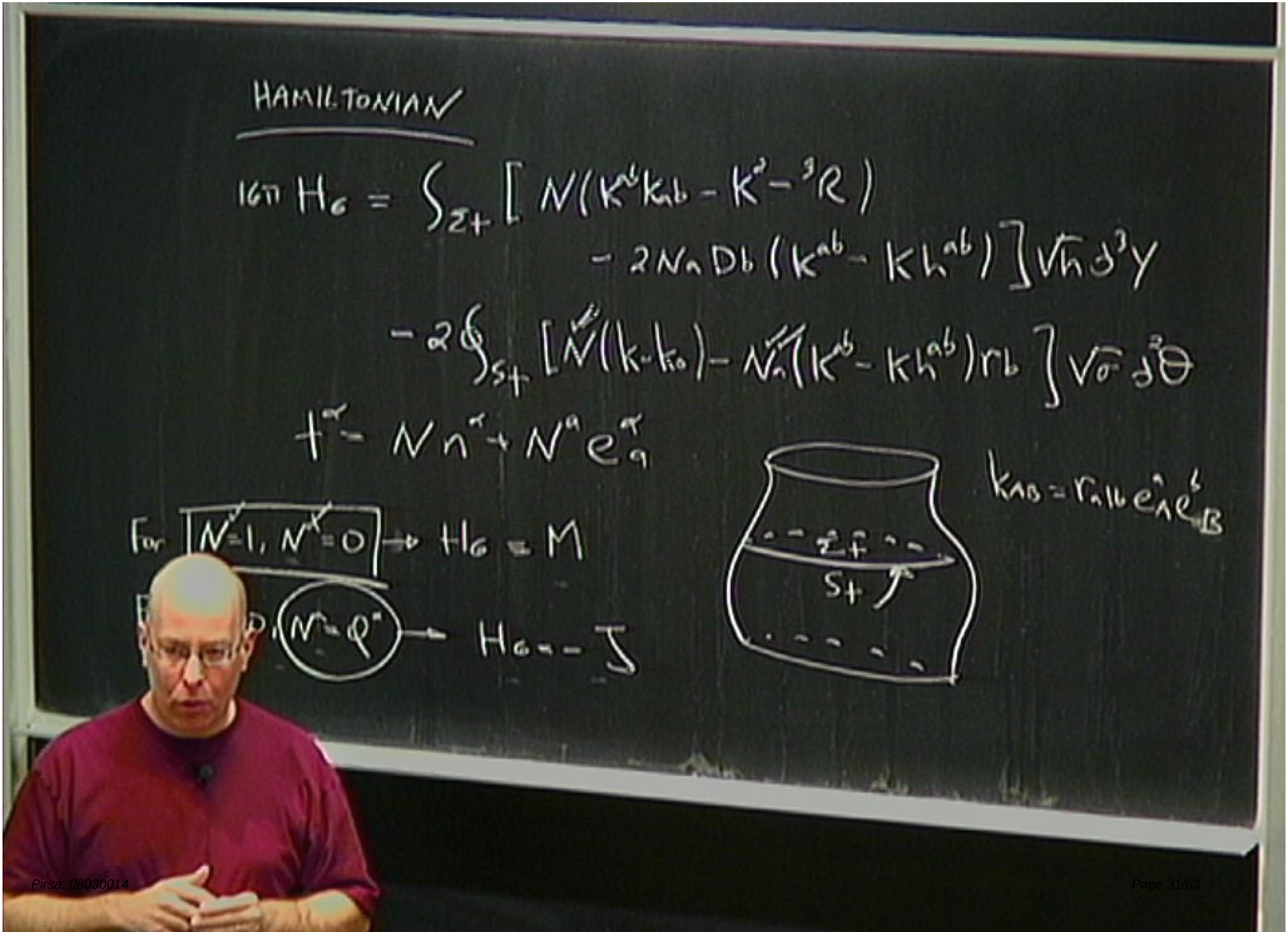
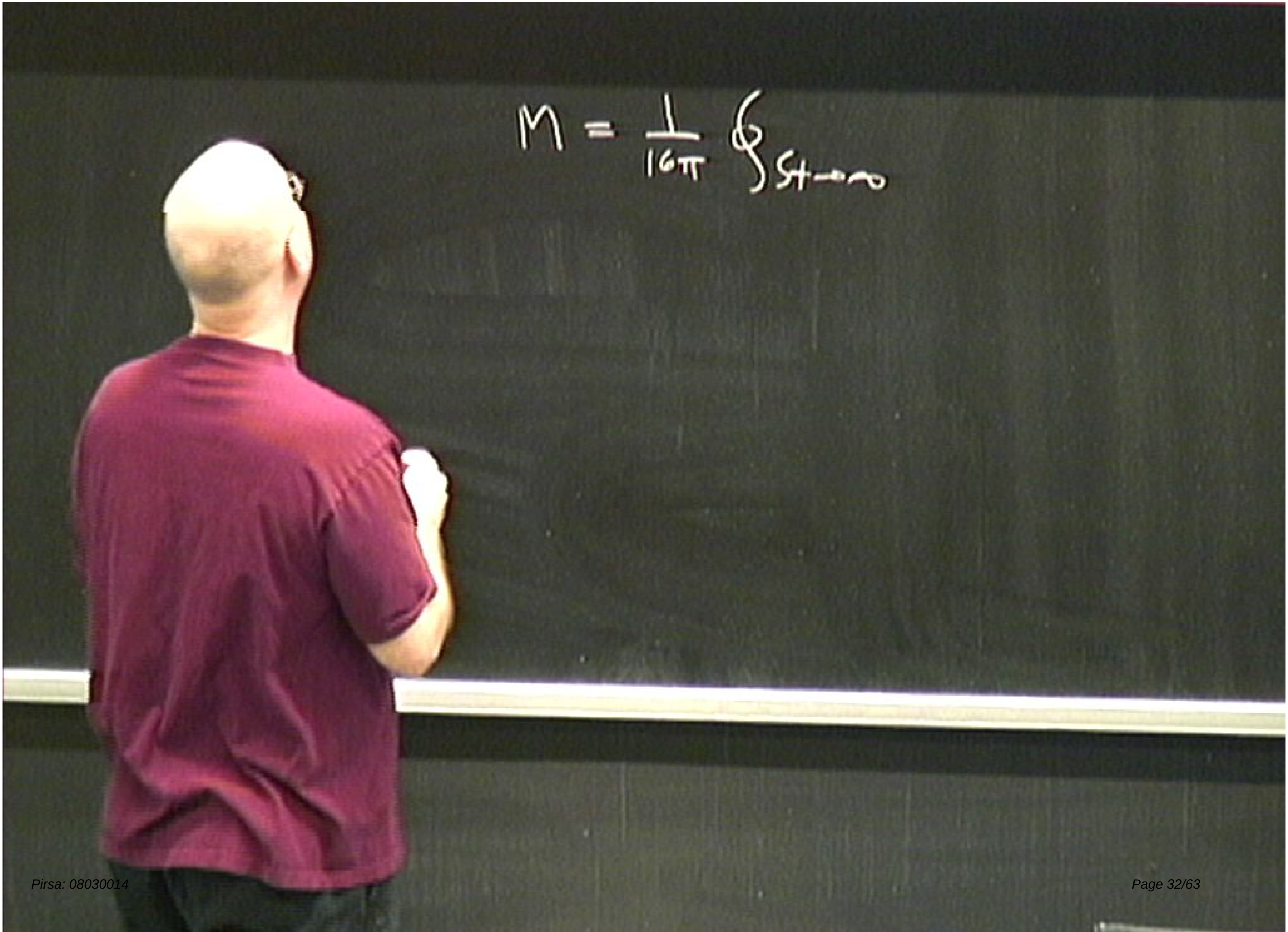

For $\mathcal{N}=1$, $\mathcal{N}^a=0$: $\mathcal{H}_G = M$


For $N^{V}=1,\ N^{K}=0 \to H_G = M$
For $N=0,\ N = \rho$


For N=1, N=0 → H = M
For N=0, N=ρ → H = -J


HAMILTONIAN
$16\pi H_G = \int_{\Sigma_t} [N(K^{ab}K_{ab} - K^2 - {}^3R)$
$- 2N_a D_b(K^{ab} - K h^{ab})] \sqrt{h}\, d^3y$
$-2\oint_{S_t} [N(k - k_0) - N_a(K^{ab} - K h^{ab}) r_b] \sqrt{\sigma}\, d^2\theta$
$t^\alpha = N n^\alpha + N^a e^\alpha_a$
$k_{AB} = r_{a|b} e^a_A e^b_B$
For $N=1, N^a=0 \rightarrow H_G = M$
$N^a = \phi^a \rightarrow H_G = -J$
Σ_t
S_t

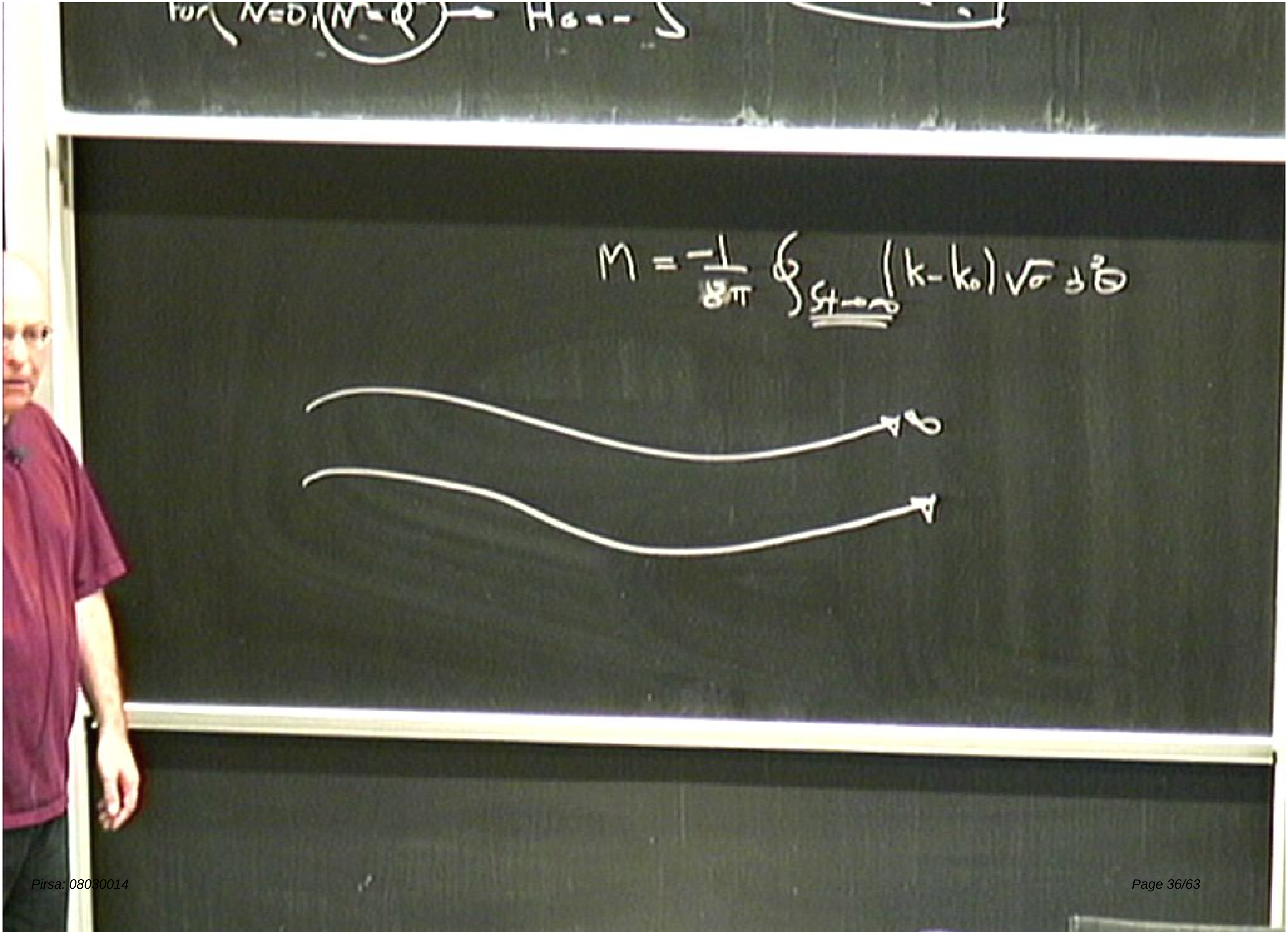

$M = \frac{1}{16\pi} \oint_{S_{t \to \infty}}$

$$M = \frac{1}{16\pi} \oint_{S_{t \to \infty}} (k - k_0) \sqrt{\sigma}\, d^2\theta$$

$$M = \frac{-1}{8\pi} \oint_{S_{t\to\infty}} (k - k_0)\sqrt{\sigma}\, d^2\theta$$

for $(N=0, N=Q) \to H_{GR} = \int$

$$M = -\frac{1}{8\pi}\oint_{S_{t\to\infty}} (k-k_0)\sqrt{\sigma}\, d^2\theta$$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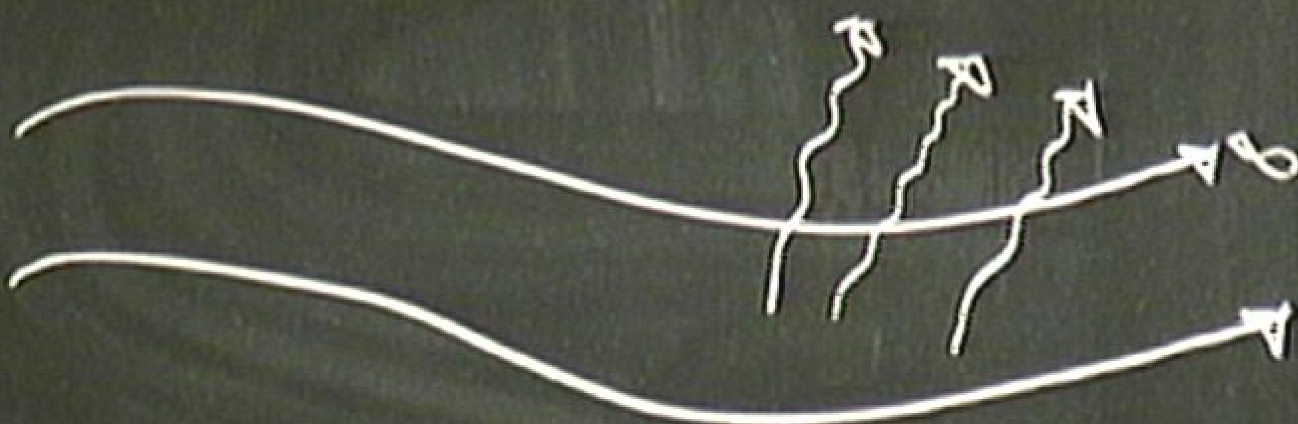
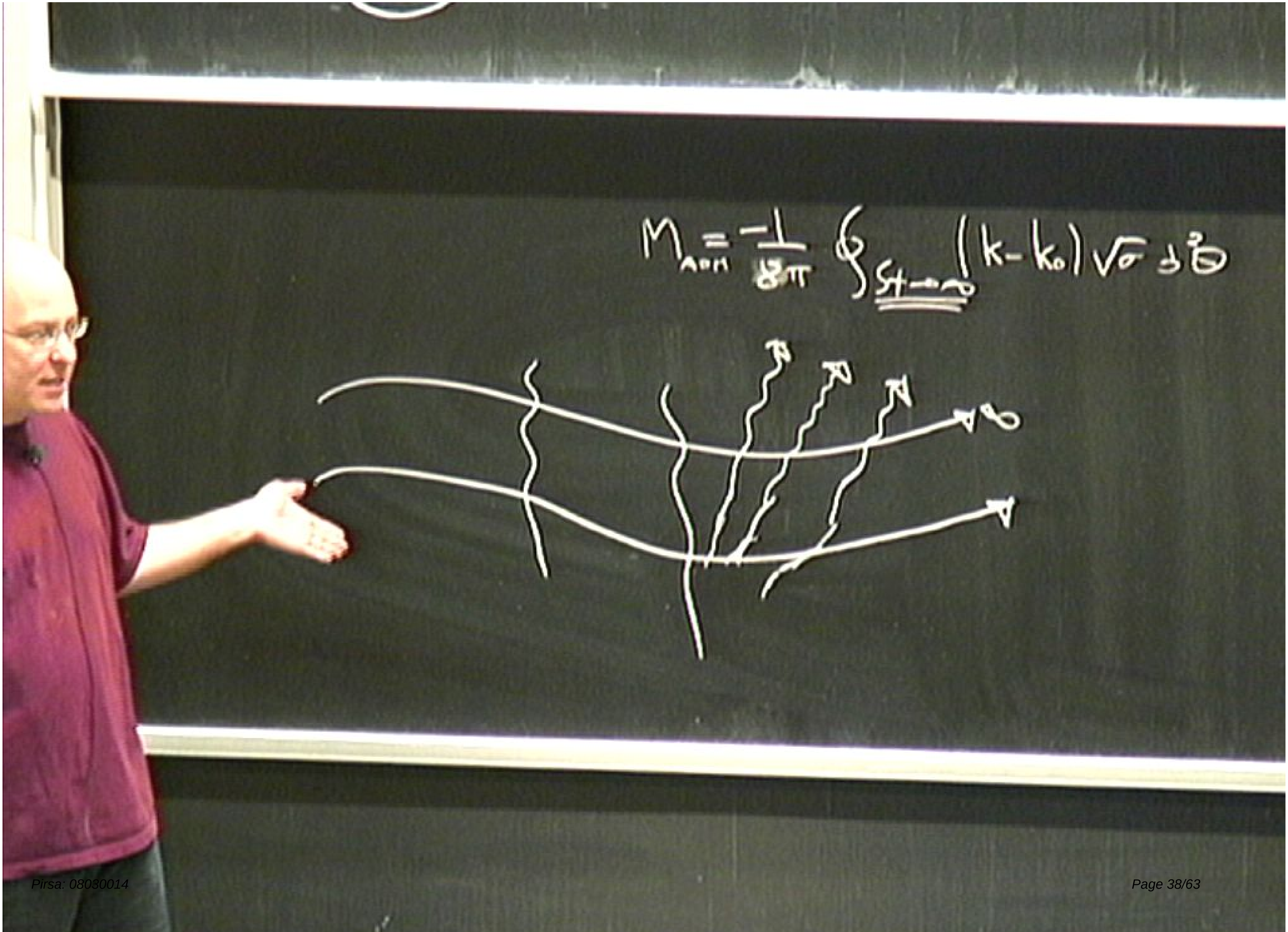
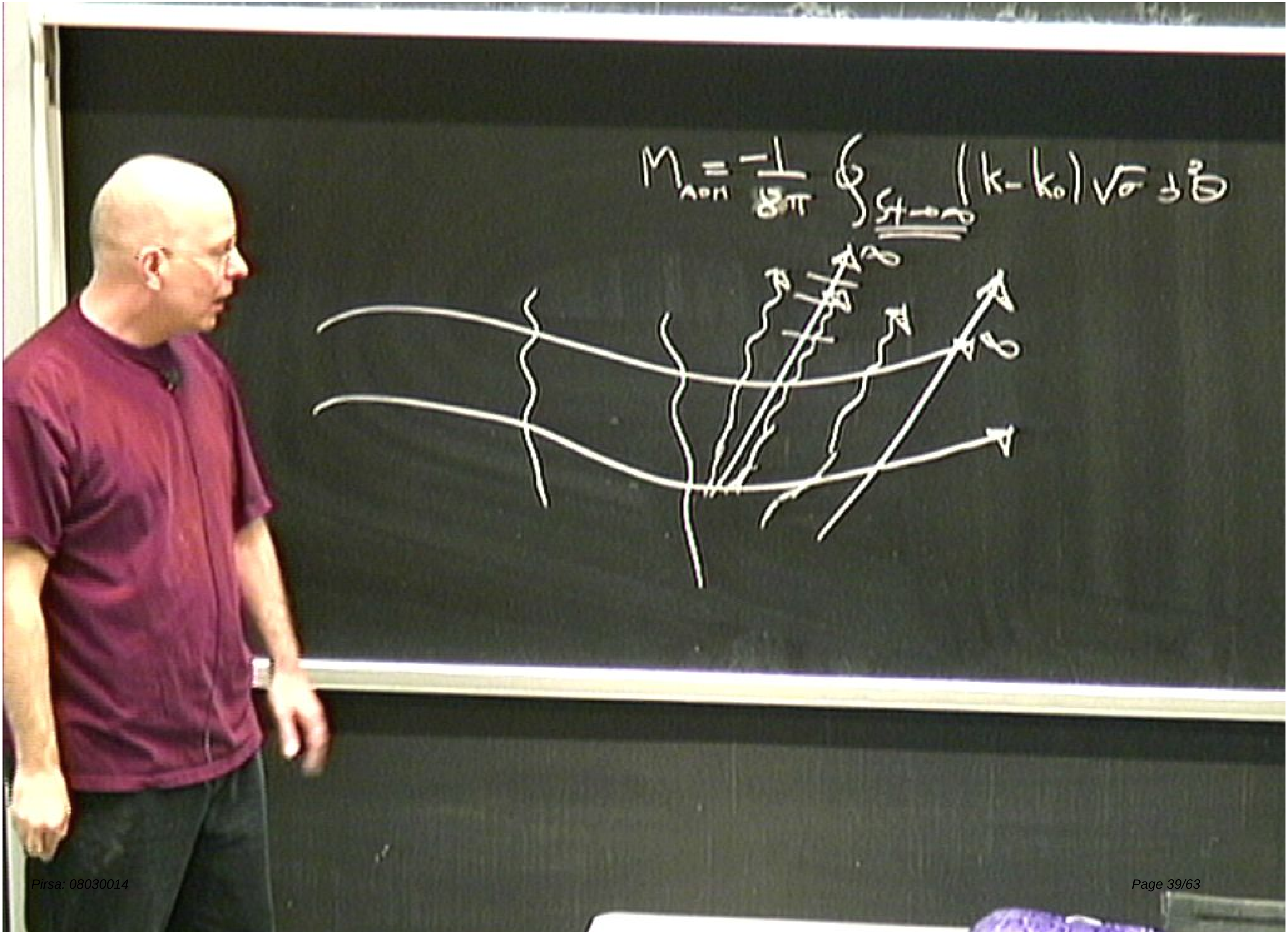



$M_{ADM} = \frac{-1}{8\pi} \oint_{S_{t\to\infty}} (k - k_0)\sqrt{\sigma}\, d^2\theta$
∞


$M_{ADM} = \frac{-1}{8\pi} \oint_{S_{t\to\infty}} (k - k_0)\sqrt{\sigma}\, d^2\theta$

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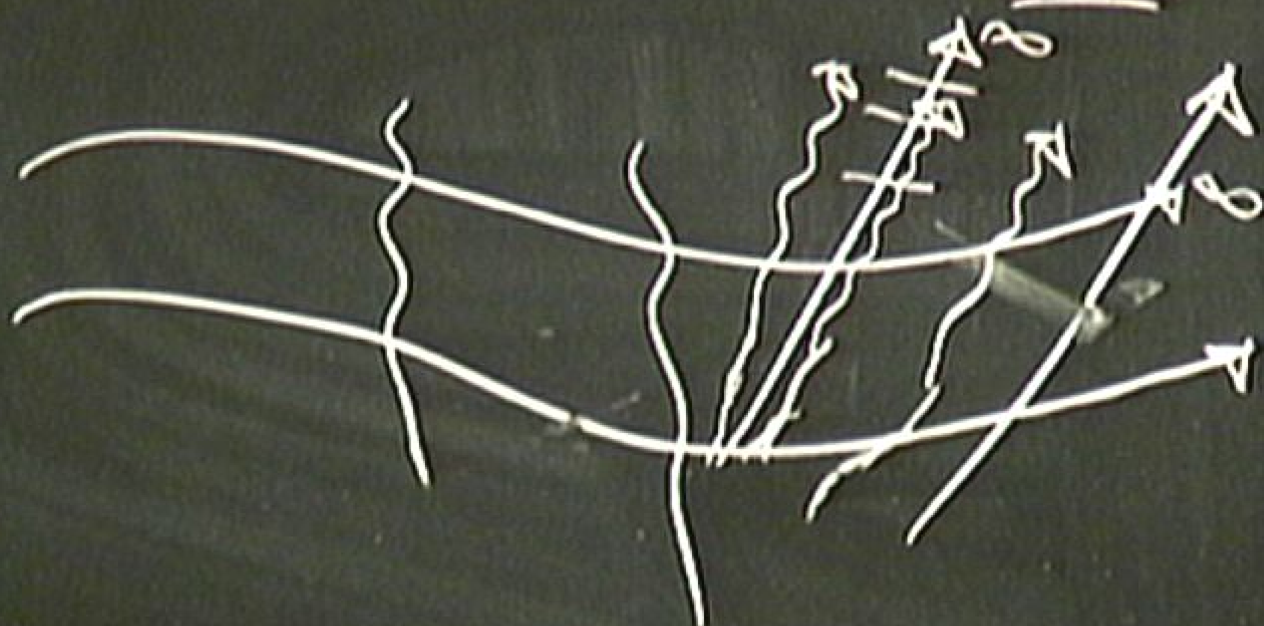
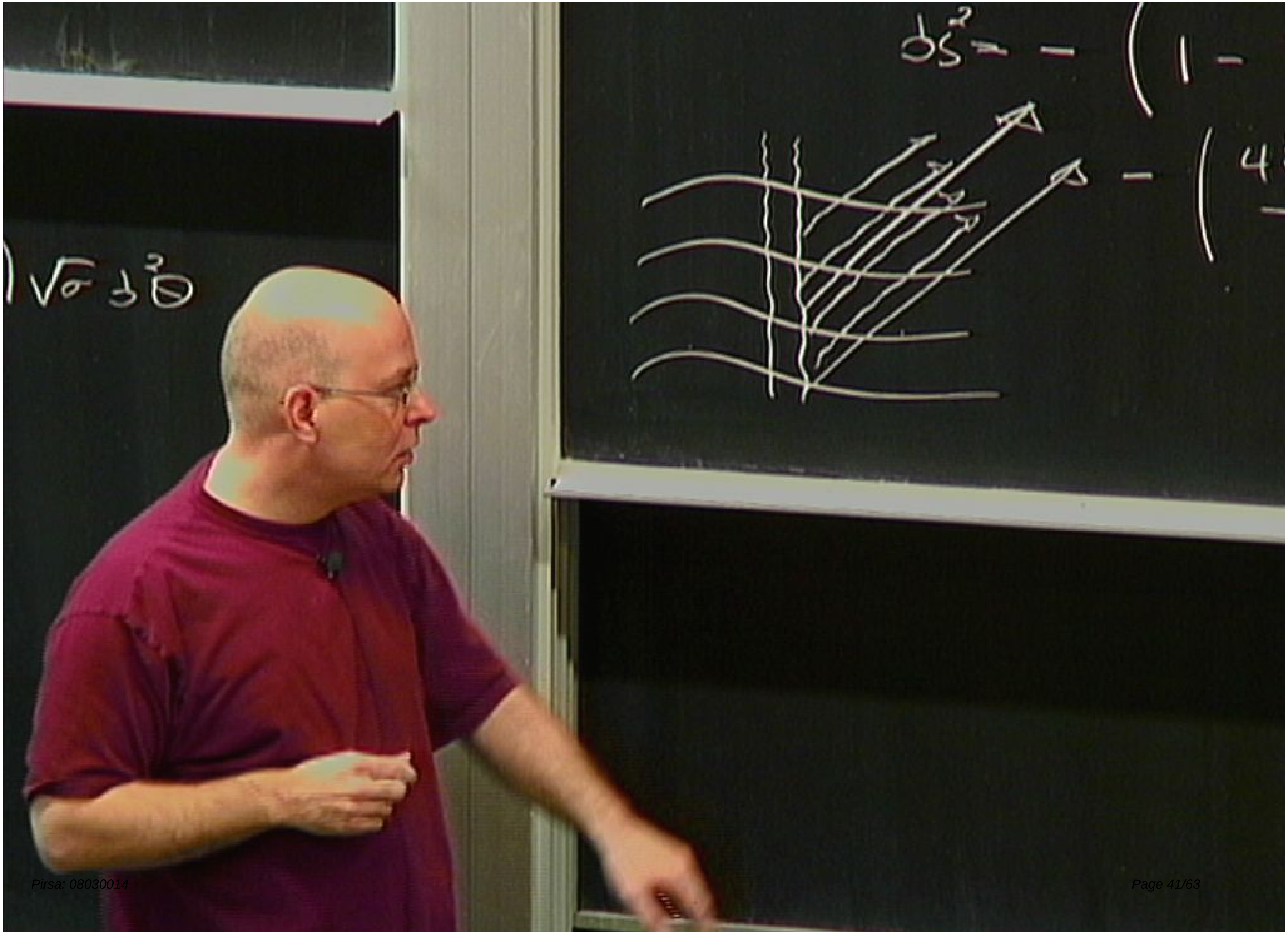
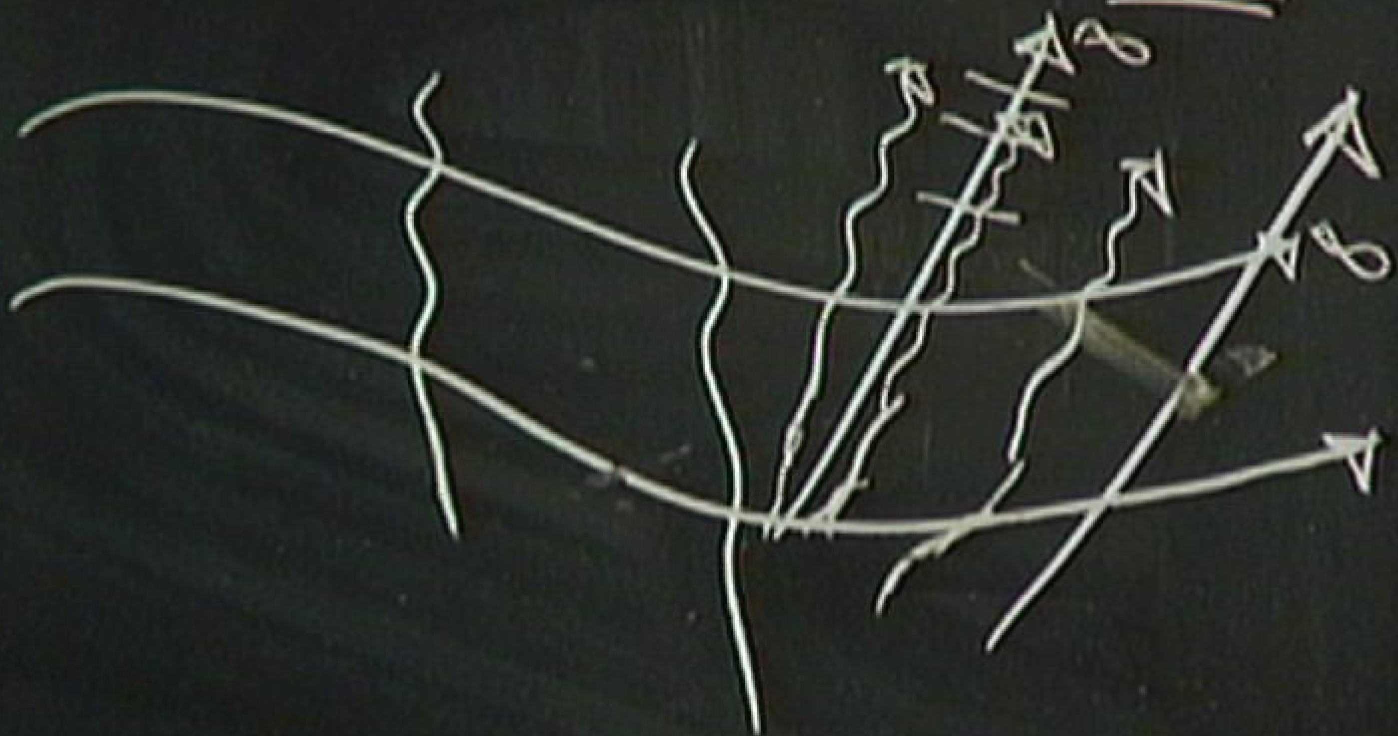
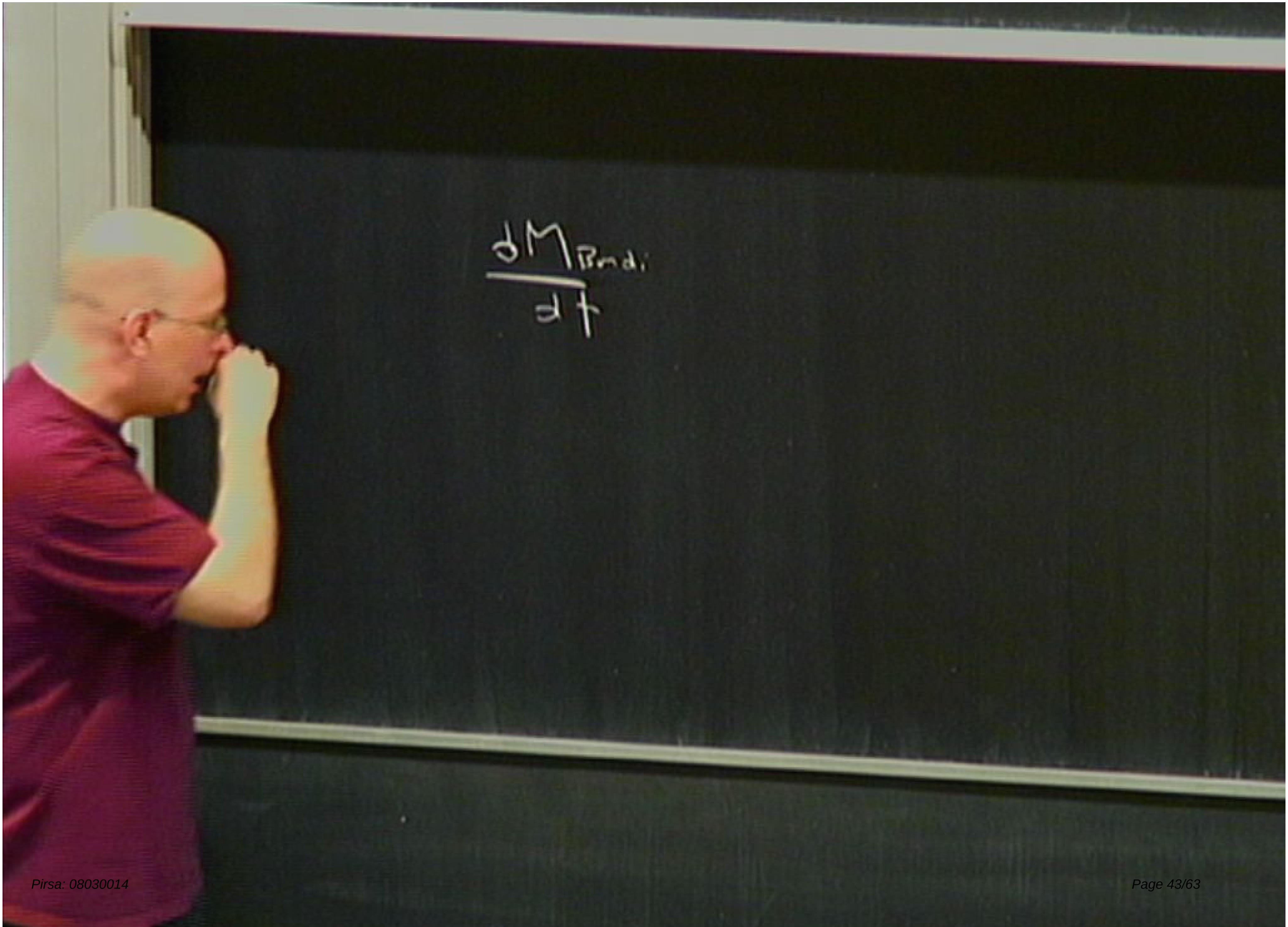


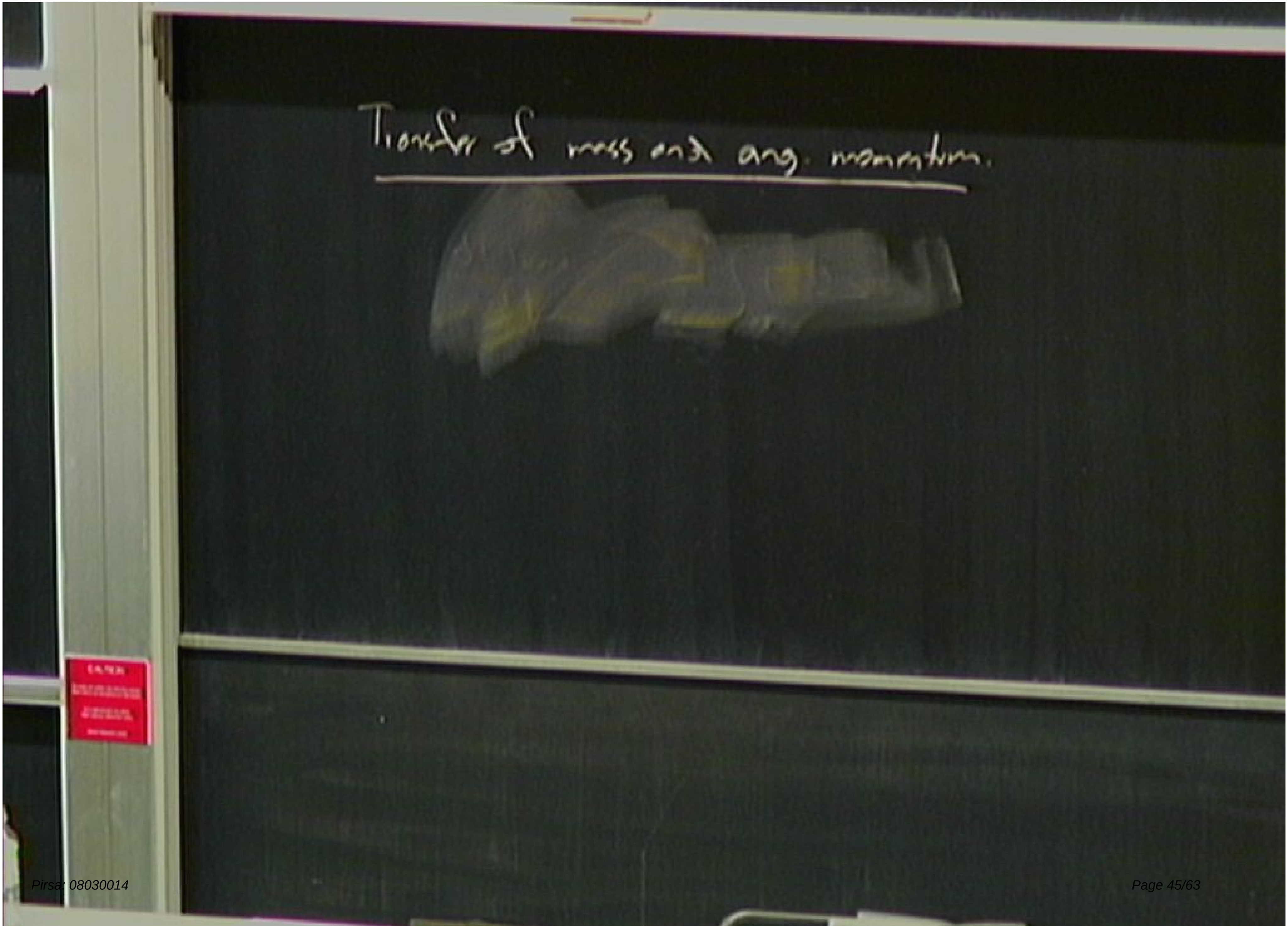


$$M_{\text{Bondi}} = -\frac{1}{8\pi} \oint_{S_{t\to\infty}} (k - k_0)\sqrt{\sigma}\, d^2\theta$$




$\frac{dM_{Bondi}}{dt}$

$$\frac{dM_{Bondi}}{dt} = - \int_{S_t} F \, dS$$


Transfer of mass and ang. momentum.

Transfer of mass and ang. momentum.

tst T_{op}

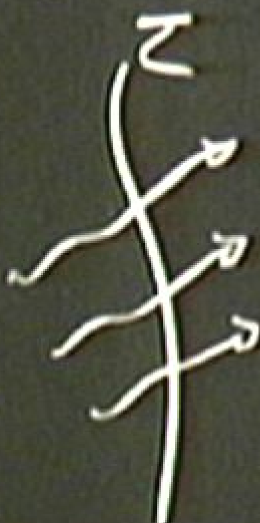
Transfer of mass and ang. momentum.

test T_{ab}

Transfer of mass and ang. momentum.

test T_{ab}

Transfer of mass and ang. momentum.



tot $T_{\alpha\beta}$

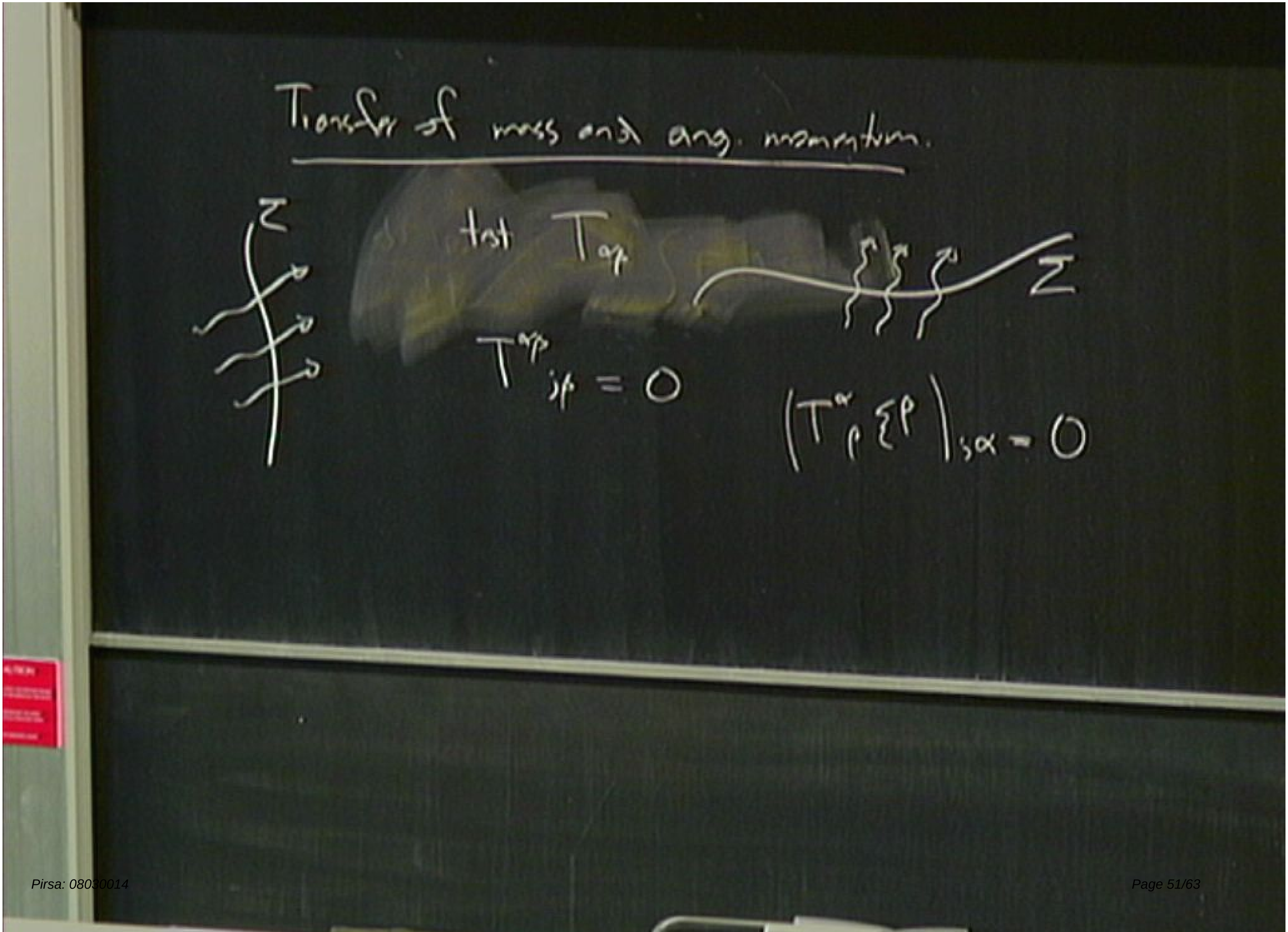
$$T^{\alpha\beta}{}_{;\beta} = 0$$

Transfer of mass and ang. momentum.

test $T_{\alpha\beta}$

$$T^{\alpha\beta}{}_{;\beta} = 0$$

$$T^{\alpha}{}_{\beta}\,\xi^{\beta}$$


Transfer of mass and ang. momentum.
Σ
test $T_{\alpha\beta}$
$T^{\alpha\beta}{}_{;\beta} = 0$
Σ
$(T^{\alpha}{}_{\beta}\xi^{\beta})_{;\alpha} = 0$

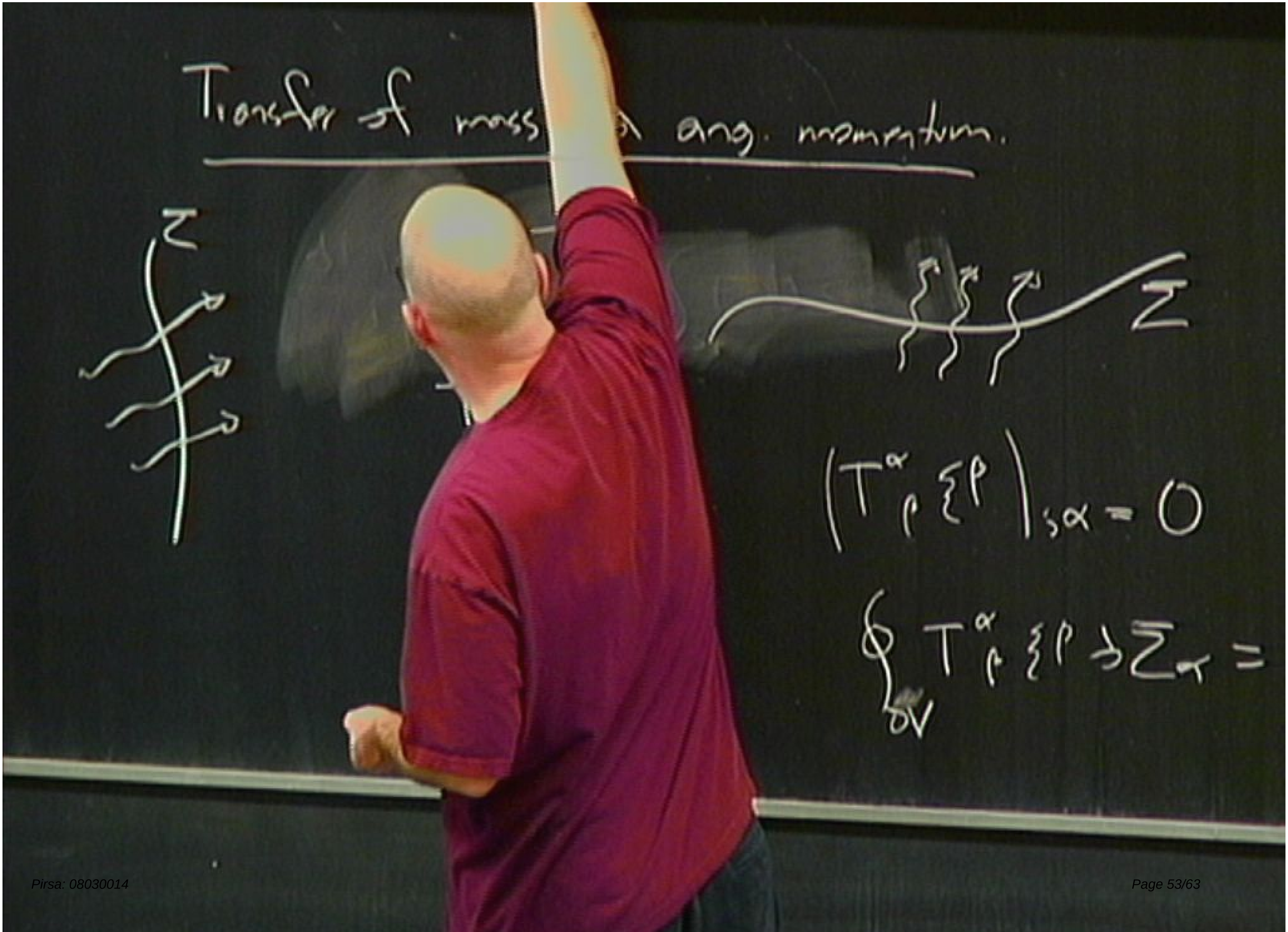
Transfer of mass and ang. momentum.

test $T_{\alpha\beta}$

$$T^{\alpha\beta}{}_{;\beta} = 0$$

$$\left(T^{\alpha}{}_{\beta}\,\xi^{\beta}\right)_{;\alpha} = 0$$

$$\oint_{\partial V} T^{\alpha}{}_{\beta}\,\xi^{\beta}\, d\Sigma_{\alpha}$$


Transfer of mass ang. momentum.
Σ
Σ
$\left(T^{\alpha}{}_{\beta}\,\xi^{\beta}\right)_{;\alpha} = 0$
$\oint_{\partial V} T^{\alpha}{}_{\beta}\,\xi^{\beta}\, d\Sigma_{\alpha} =$

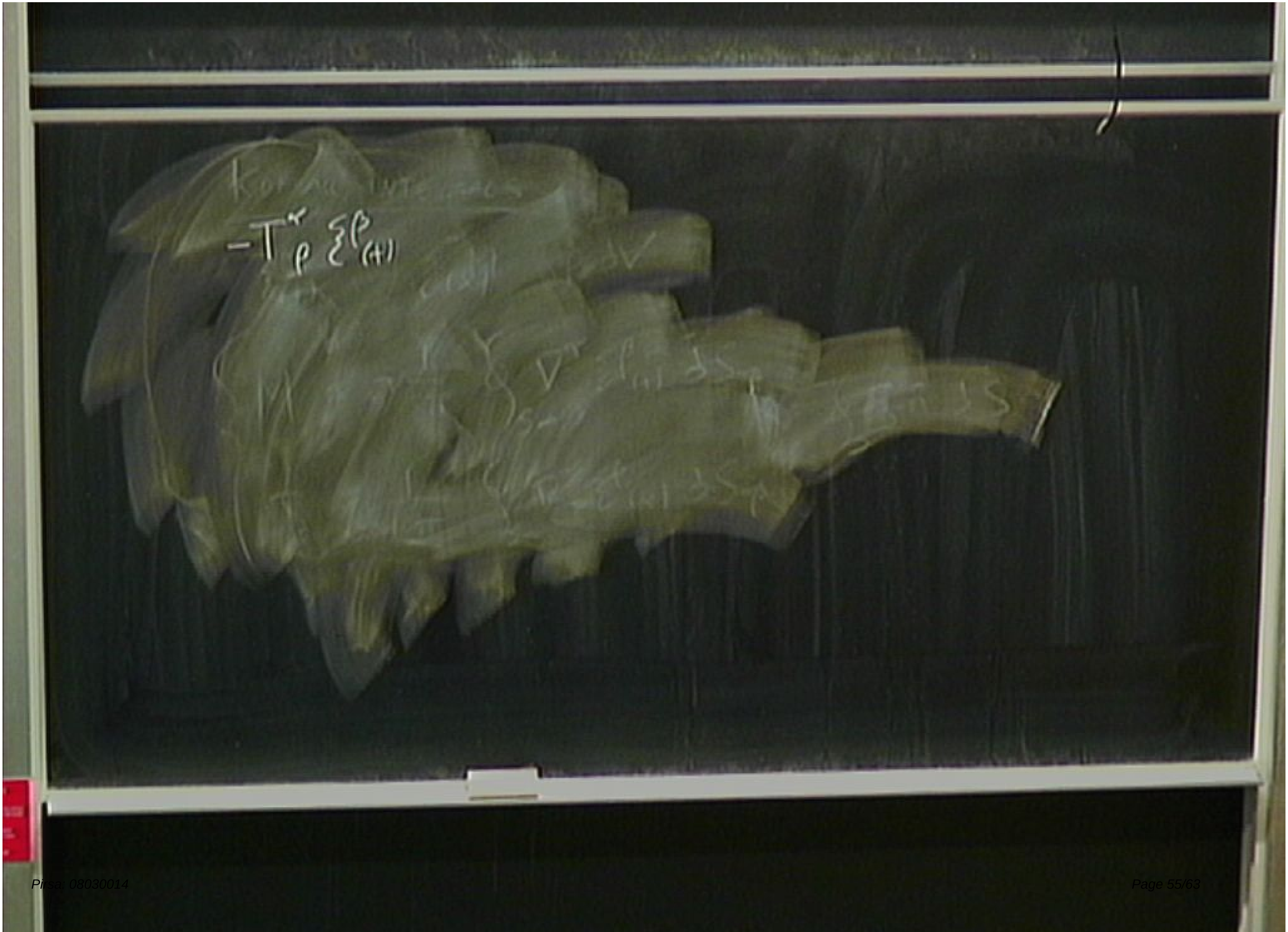
Transfer of mass and ang. momentum.

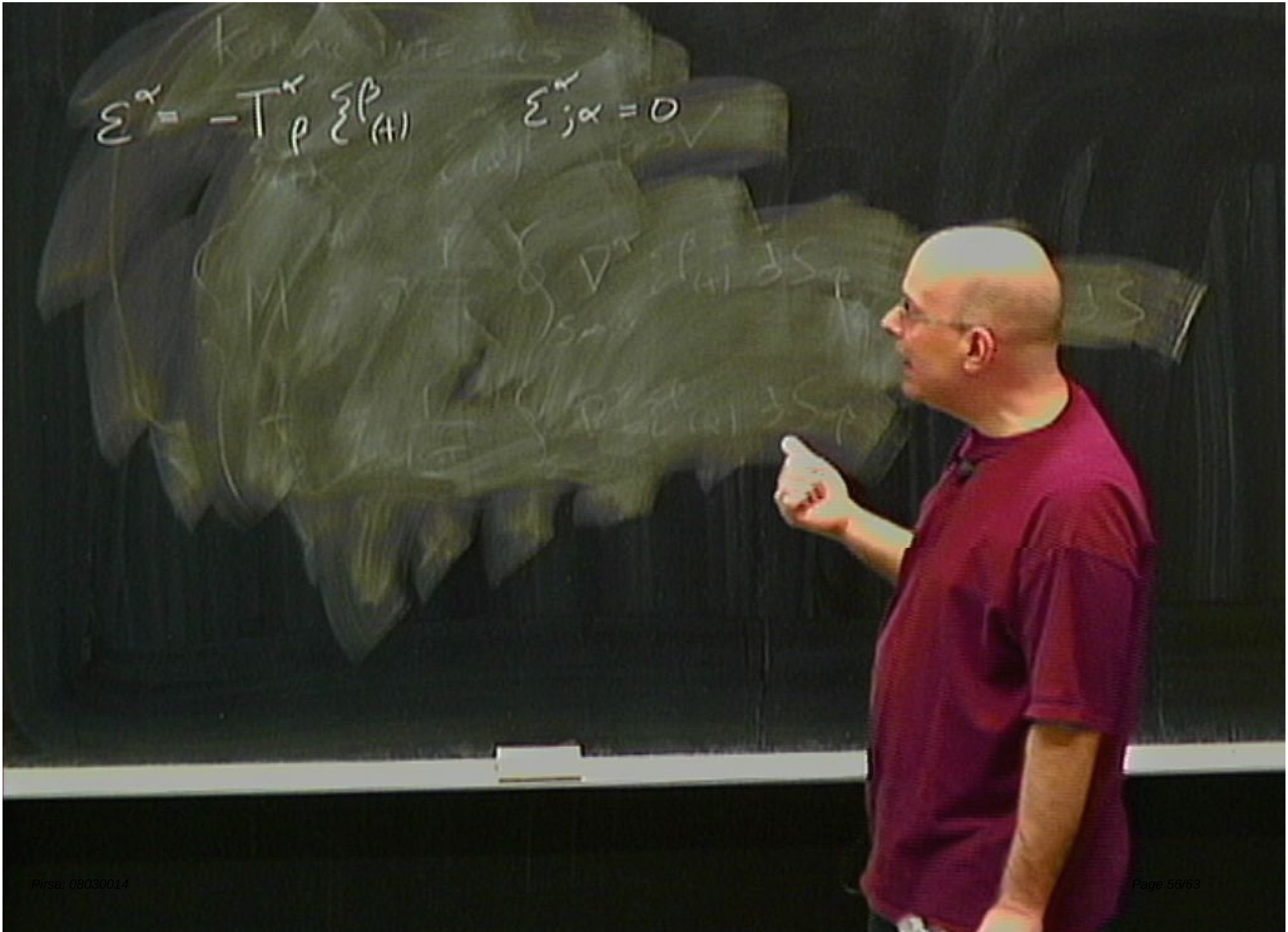
test $T_{\alpha\beta}$

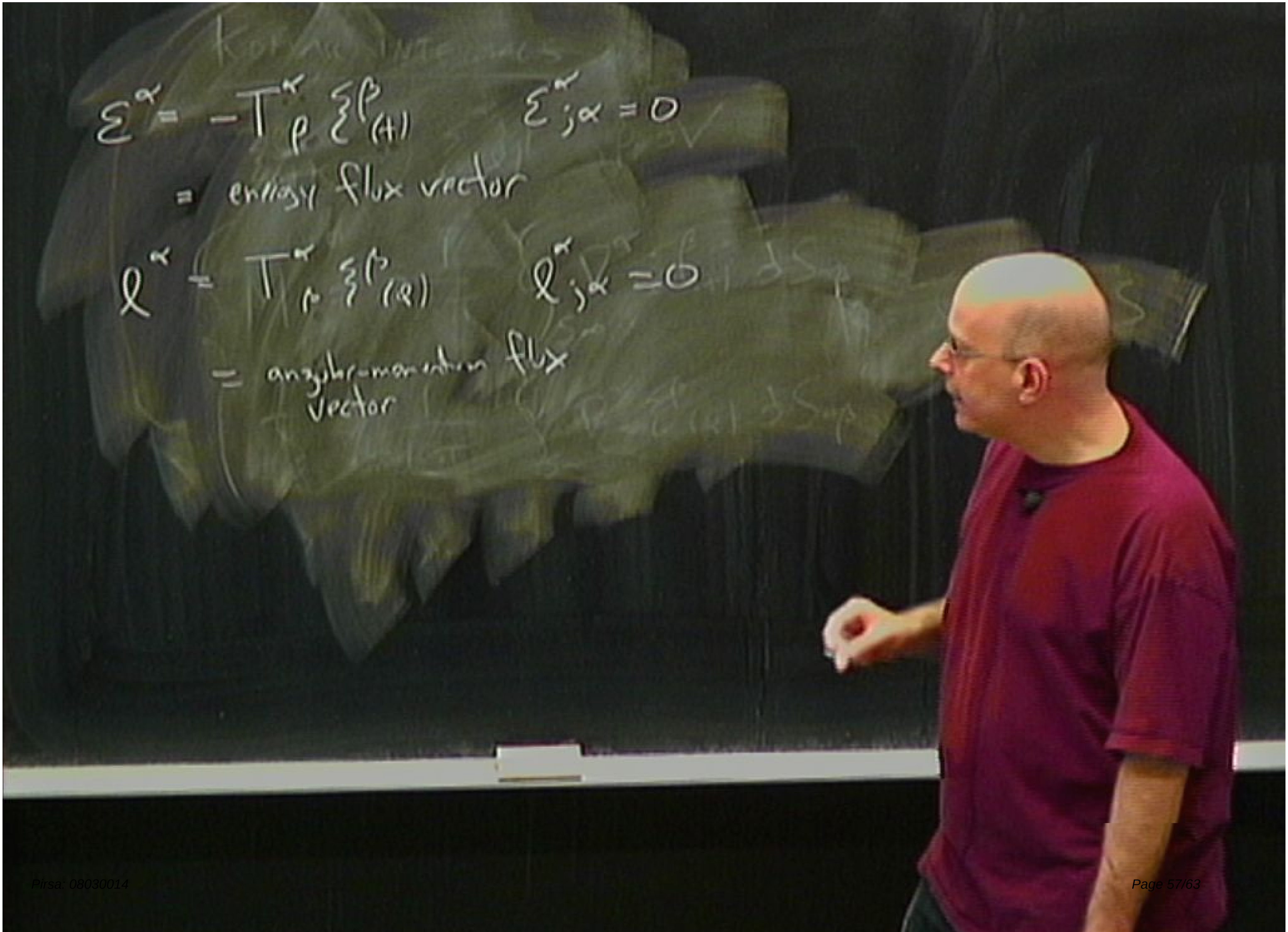
$$T^{\alpha\beta}{}_{;\beta} = 0$$

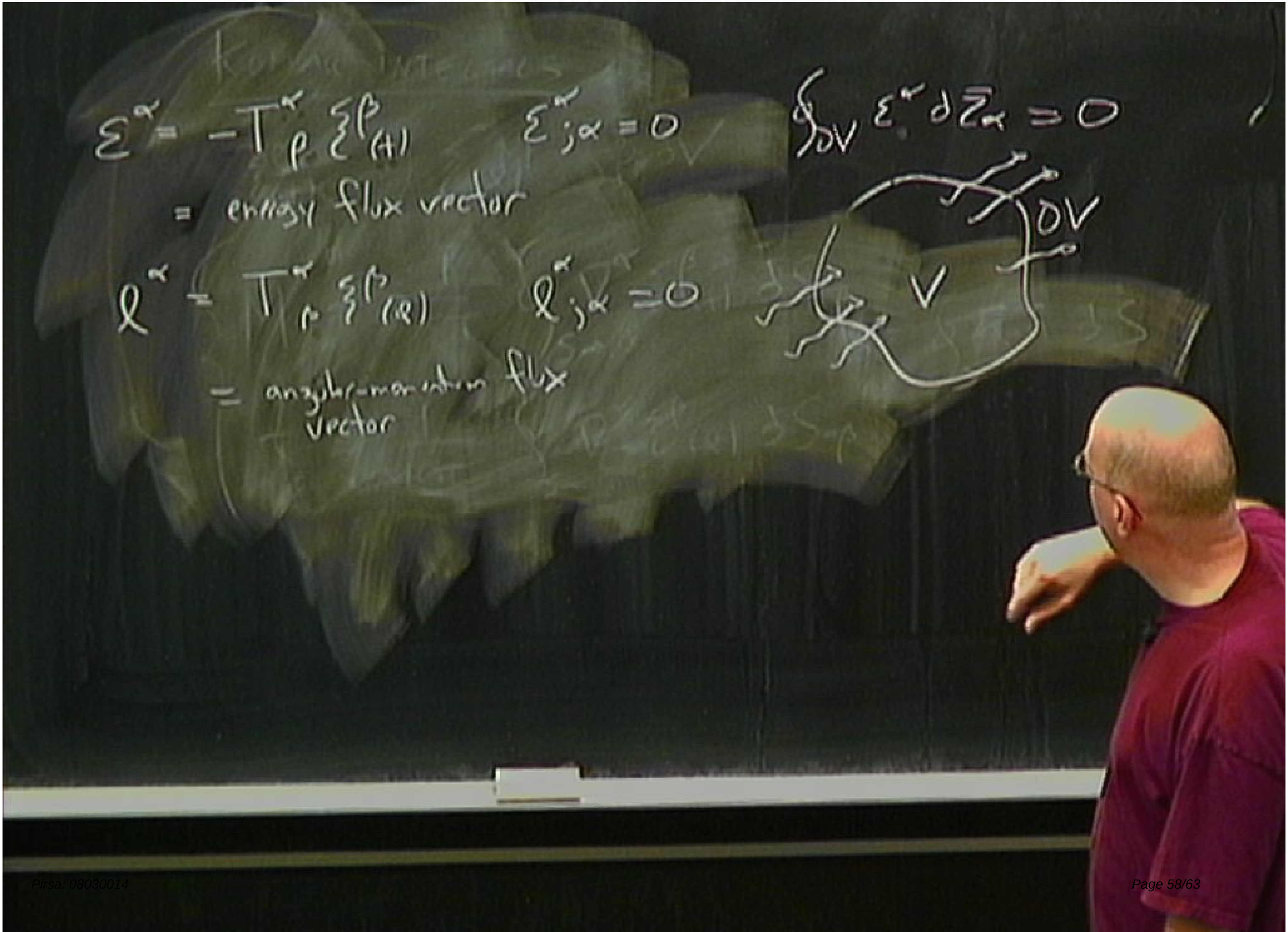
$$\left(T^{\alpha}{}_{\beta}\,\xi^{\beta}\right)_{;\alpha} = 0$$

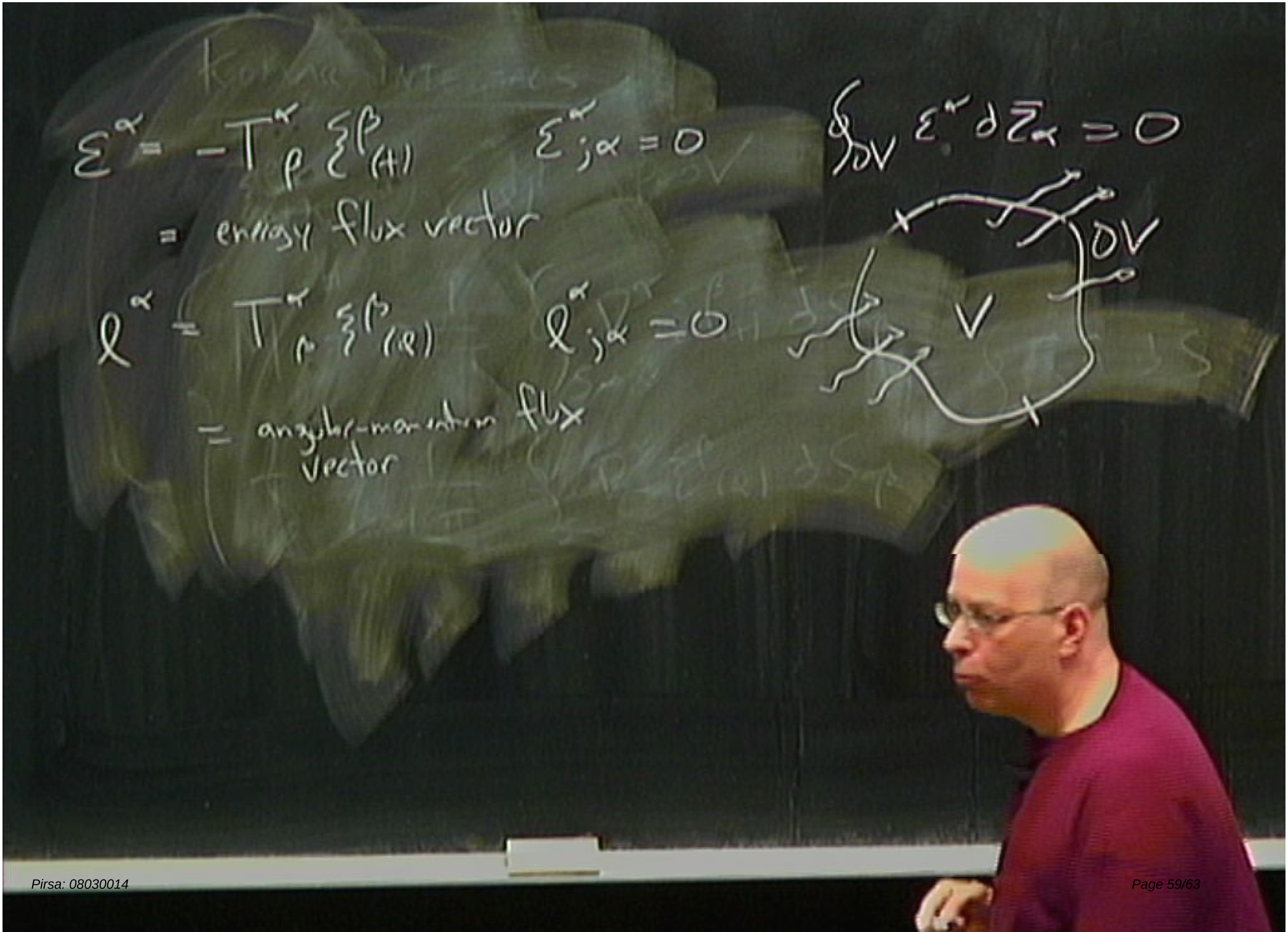
$$\oint_{\partial V} T^{\alpha}{}_{\beta}\,\xi^{\beta}\, d\Sigma_{\alpha} = 0$$

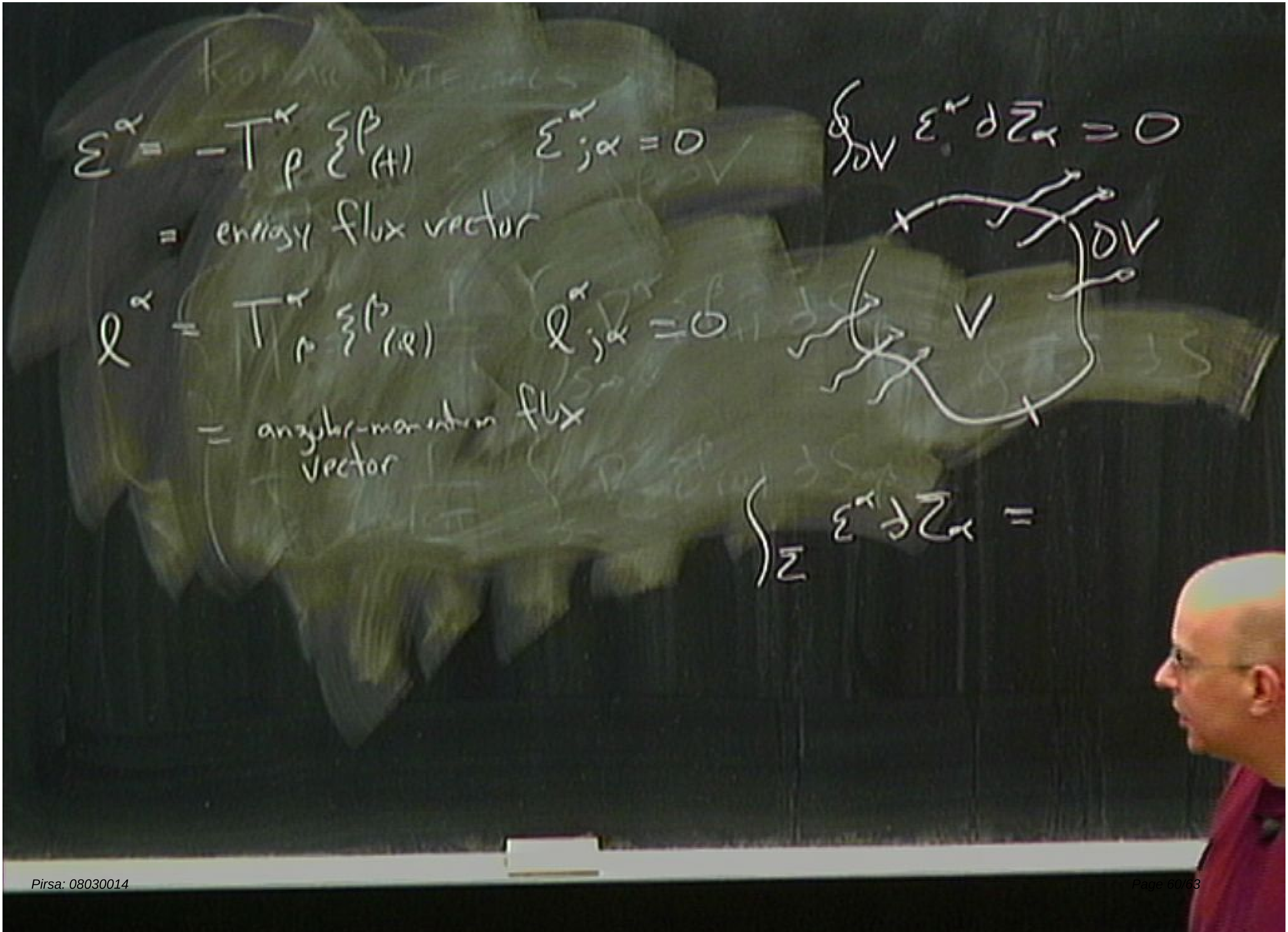

$-T^{\alpha}{}_{\beta}\,\xi^{\beta}_{(t)}$


$\xi^\alpha = -T^\alpha{}_\beta\, \xi^\beta_{(t)}$
$\xi^\alpha{}_{;\alpha} = 0$


$\mathcal{E}^\alpha = -T^\alpha{}_\beta \, \xi^\beta_{(t)}$ $\mathcal{E}^\alpha{}_{;\alpha} = 0$
= energy flux vector
$\ell^\alpha = T^\alpha{}_\beta \, \xi^\beta_{(\phi)}$ $\ell^\alpha{}_{;\alpha} = 0$
= angular-momentum flux vector


$\mathcal{E}^\alpha = -T^\alpha{}_\beta \, \xi^\beta{}_{(t)}$
$\mathcal{E}^\alpha{}_{;\alpha} = 0$
$\oint_{\partial V} \mathcal{E}^\alpha \, d\Sigma_\alpha = 0$
= energy flux vector
∂V
V
$\ell^\alpha = T^\alpha{}_\beta \, \xi^\beta{}_{(\varphi)}$
$\ell^\alpha{}_{;\alpha} = 0$
= angular-momentum flux vector


$\mathcal{E}^\alpha = -T^\alpha{}_\beta \xi^\beta_{(t)}$
$\mathcal{E}^\alpha{}_{;\alpha} = 0$
$\oint_{\partial V} \mathcal{E}^\alpha d\Sigma_\alpha = 0$
= energy flux vector
∂V
V
$\ell^\alpha = T^\alpha{}_\beta \xi^\beta_{(\phi)}$
$\ell^\alpha{}_{;\alpha} = 0$
= angular-momentum flux vector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$\mathcal{E}^\alpha{}_{;\alpha} = 0$
= energy flux vector
$\ell^\alpha = T^\alpha{}_\beta\, \xi^\beta{}_{(\phi)}$
$\ell^\alpha{}_{;\alpha} = 0$
= angular-momentum flux vector
$\oint_{\partial V} \mathcal{E}^\alpha\, d\Sigma_\alpha = 0$
∂V
V
$\int_\Sigma \mathcal{E}^\alpha\, d\Sigma_\alpha =$

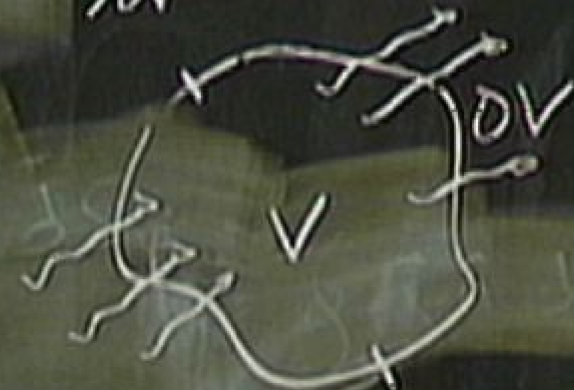
$$\mathcal{E}^\alpha = -T^\alpha{}_\beta\, \xi^\beta_{(t)} \qquad \mathcal{E}^\alpha{}_{;\alpha} = 0$$

= energy flux vector

$$\ell^\alpha = T^\alpha{}_\beta\, \xi^\beta_{(\phi)} \qquad \ell^\alpha{}_{;\alpha} = 0$$

= angular-momentum flux vector

$$\oint_{\partial V} \mathcal{E}^\alpha\, d\Sigma_\alpha = 0$$



$$\int_\Sigma \mathcal{E}^\alpha\, d\Sigma_\alpha = \text{energy crossing } \Sigma$$

$$\int_\Sigma \ell^\alpha\, d\Sigma_\alpha =$$

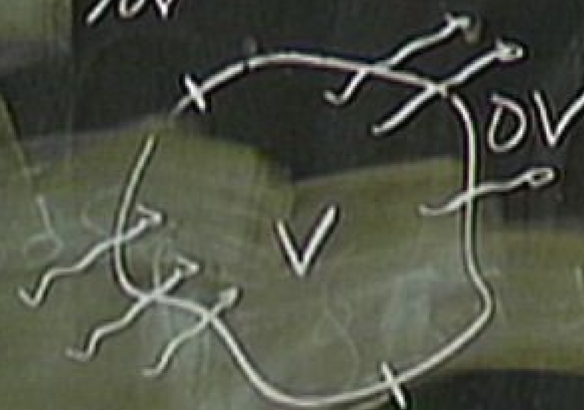
$$\mathcal{E}^\alpha = -T^\alpha{}_\beta\, \xi^\beta_{(t)} \qquad \mathcal{E}^\alpha{}_{;\alpha} = 0$$

= energy flux vector

$$\ell^\alpha = T^\alpha{}_\beta\, \xi^\beta_{(\phi)} \qquad \ell^\alpha{}_{;\alpha} = 0$$

= angular-momentum flux vector

$$\oint_{\partial V} \mathcal{E}^\alpha\, d\Sigma_\alpha = 0$$



$$\int_\Sigma \mathcal{E}^\alpha\, d\Sigma_\alpha = \text{energy crossing } \Sigma$$

$$\int_\Sigma \ell^\alpha\, d\Sigma_\alpha = \text{angular momentum crossing } \Sigma$$

$$\int_\Sigma \epsilon^\alpha \, d\Sigma_\alpha = \text{energy crossing } \Sigma$$

$$\int_\Sigma \ell^\alpha \, d\Sigma_\alpha = \text{angular momentum crossing } \Sigma$$