

Title: Advanced General Relativity - Lecture 9A

Date: Mar 19, 2008 10:30 AM

URL: <http://pirsa.org/08030013>

Abstract: Advanced General Relativity

3+1 decomposition



3+1 decomposition

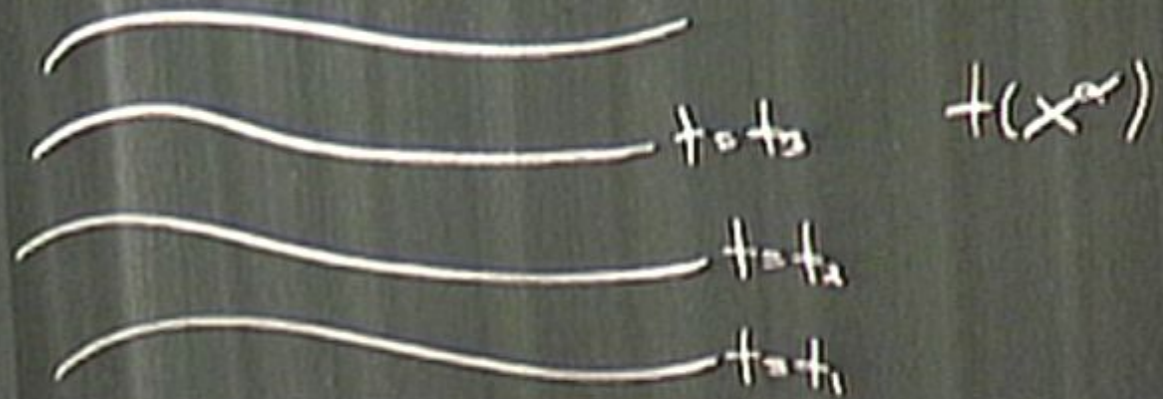


3+1 decomposition

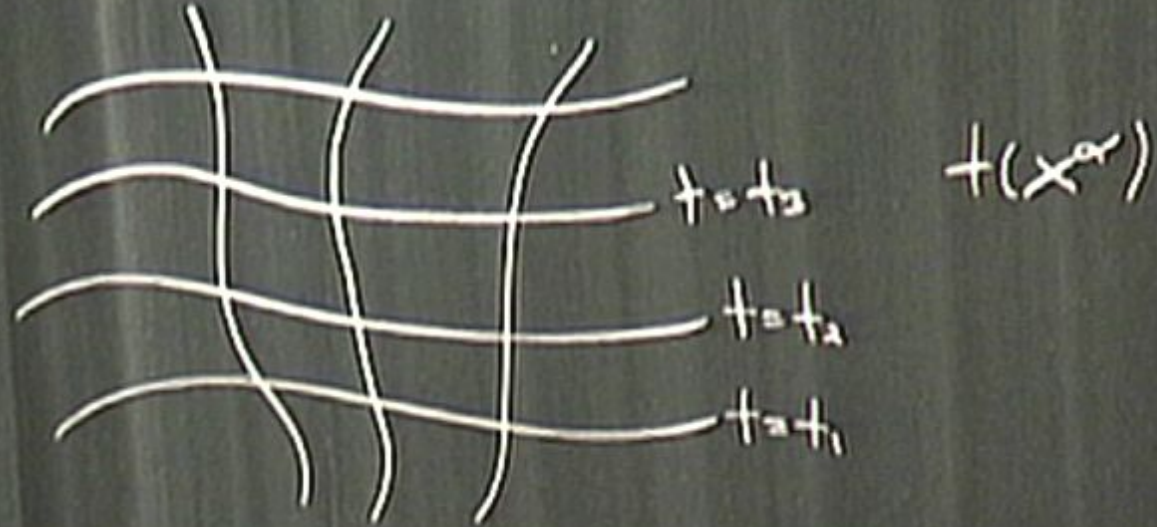


$t(x^i)$

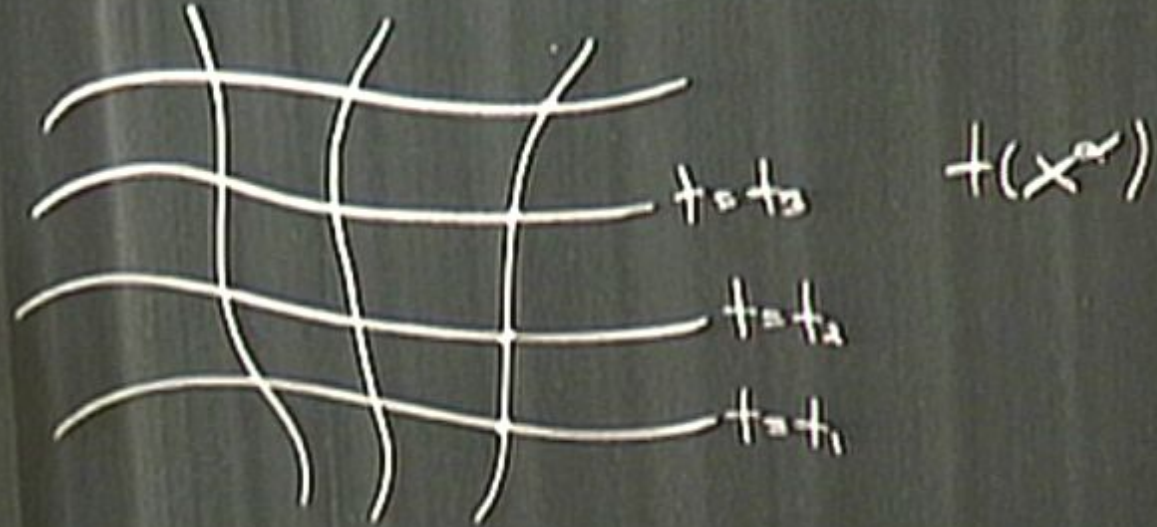
3+1 decomposition



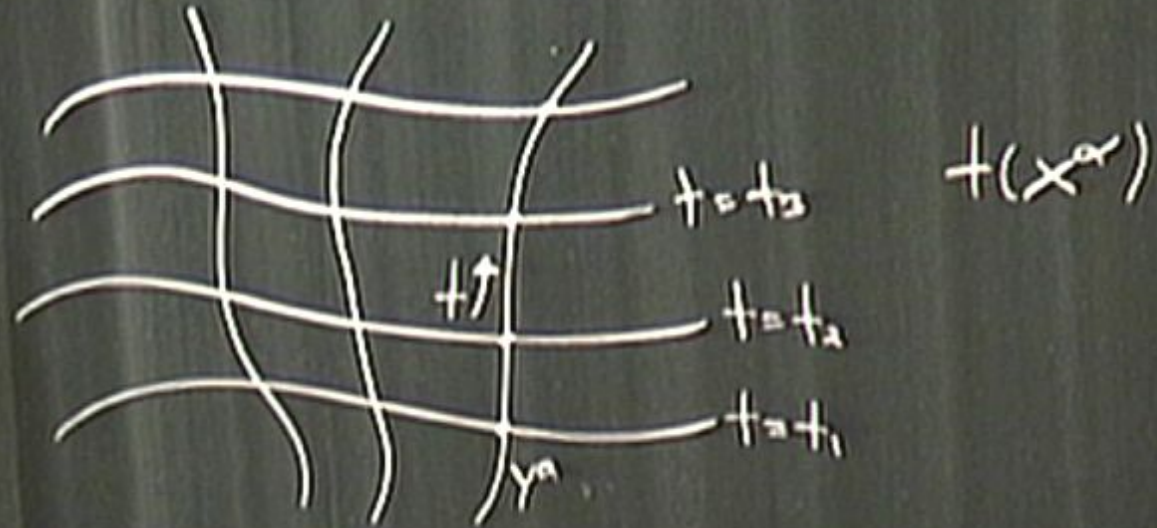
3+1 decomposition



3+1 decomposition



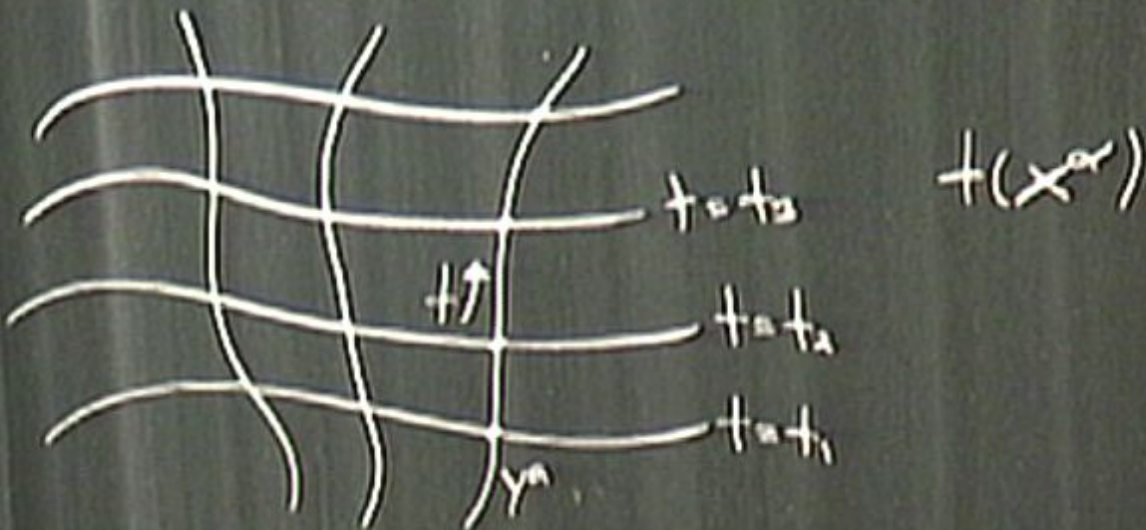
3+1 decomposition



3+1 decomposition

tangent vector on
 worldlines:

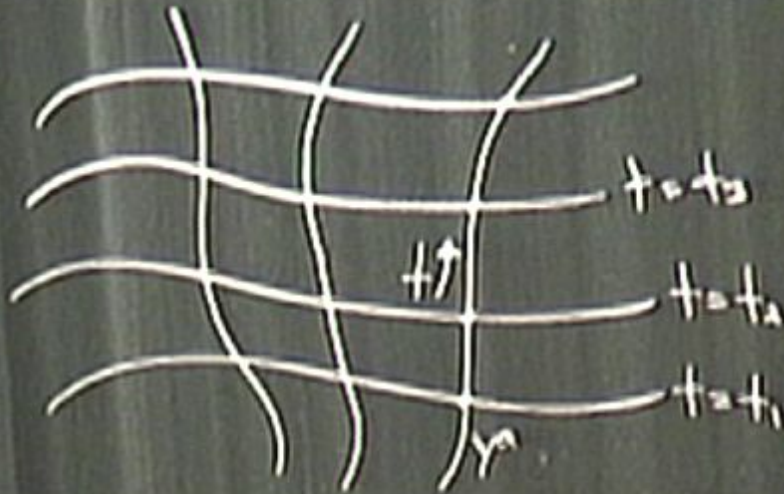
$$t^\alpha = \left(\frac{\partial x^\alpha}{\partial t} \right)_{y^a}$$



3+1 decomposition

tangent vector on
curves:

$$t^\alpha = \left(\frac{\partial X^\alpha}{\partial t} \right)_{Y^a}$$



$$t(X^\alpha) \\ X^\alpha(t, Y^a)$$

3+1 decomposition

tangent vector on
curves:

$$\dot{x}^\alpha = \left(\frac{\partial x^\alpha}{\partial t} \right)_{y^a}$$

tangent vectors of
 Z_t



$$t(x^\alpha) \\ x^\alpha(t, y^a)$$

3+1 decomposition

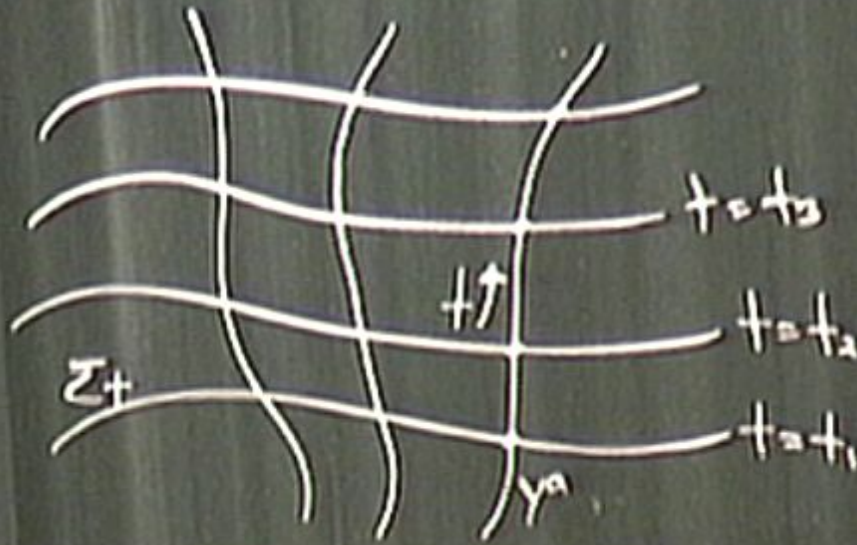
tangent vector on
curves:

$$t^\alpha = \left(\frac{\partial x^\alpha}{\partial t} \right)_{y^a}$$

tangent vectors on

Σ_t :

$$e_a^\alpha = \left(\frac{\partial x^\alpha}{\partial y^a} \right)_t$$



$t(x^\alpha)$

$x^\alpha(t, y^a)$

3+1 decomposition

tangent vector on
curves:

$$t^\alpha = \left(\frac{\partial x^\alpha}{\partial t} \right)_{y^a}$$

normal vectors of

$$n_\alpha = -N \left(\frac{\partial x^\alpha}{\partial y^a} \right)_t$$



$$t(x^\alpha)$$

$$x^\alpha(t, y^a)$$

$$n_\alpha = -N \partial_\alpha t$$

3+1 decomposition

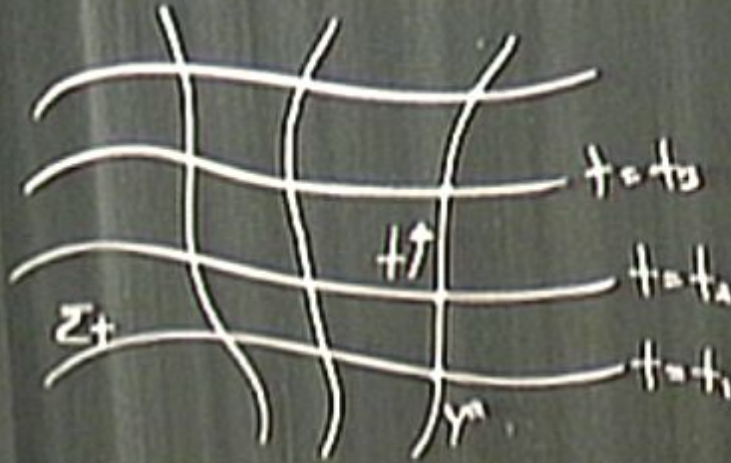
tangent vector on
curves:

$$\dot{t}^\alpha = \left(\frac{\partial x^\alpha}{\partial t} \right)_y$$

tangent vectors of

Σ_t :

$$e_a^\alpha = \left(\frac{\partial x^\alpha}{\partial y^a} \right)_t$$



$$t(x^\alpha)$$

$$x^\alpha(t, y^a)$$

$$\eta_\alpha = -N \partial_\alpha t$$

3+1 decomposition

tangent vector on
curves:

$$t^\alpha = \left(\frac{\partial x^\alpha}{\partial t} \right)_y$$

tangent vectors of

Σ_t :

$$e_a^\alpha = \left(\frac{\partial x^\alpha}{\partial y^a} \right)_t$$



$t(x^\alpha)$

$x^\alpha(t, y^a)$

$$n_\alpha = -N \partial_\alpha t$$

$$t^\alpha = N n^\alpha + N^a e_a^\alpha$$

3+1 decomposition

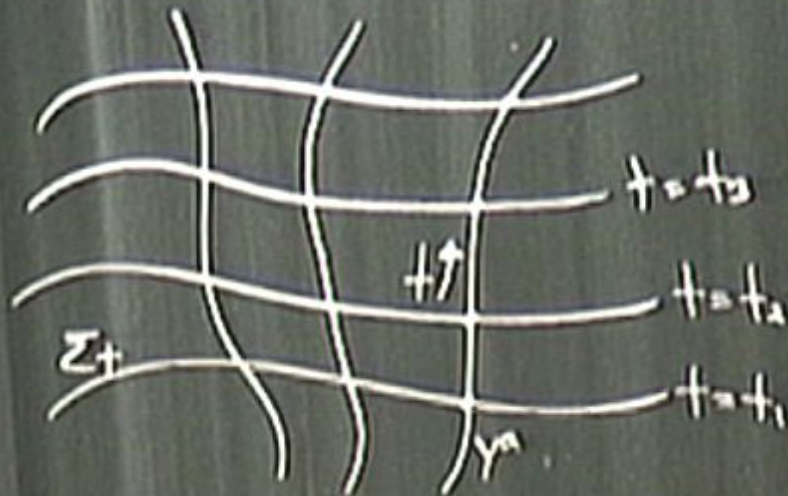
tangent vector on
curves:

$$t^\alpha = \left(\frac{\partial x^\alpha}{\partial t} \right)_{y^a}$$

tangent vectors of

Σ_t :

$$e_a^\alpha = \left(\frac{\partial x^\alpha}{\partial y^a} \right)_t$$



$$t(x^\alpha)$$

$$x^\alpha(t, y^a)$$

$$n_\alpha = -N \partial_\alpha t$$

$$t^\alpha = \underbrace{N}_{\text{lapse}} n^\alpha + \underbrace{N^a}_{\text{shift}} e_a^\alpha$$

metric in new coordinates (t, y^a)

$$dx^\alpha = \frac{\partial x^\alpha}{\partial t} dt + \frac{\partial x^\alpha}{\partial y^a} dy^a$$

$$= t^\alpha dt + e_a^\alpha dy^a$$

$$= (Nn^\alpha + N^a e_a^\alpha) dt + e_a^\alpha dy^a$$

metric in new coordinates (t, y^a)

$$\begin{aligned} dx^\alpha &= \frac{\partial x^\alpha}{\partial t} dt + \frac{\partial x^\alpha}{\partial y^a} dy^a \\ &= t^\alpha dt + e_a^\alpha dy^a \\ &= (Nn^\alpha + N^a e_a^\alpha) dt + e_a^\alpha dy^a \\ &= (Ndt) n^\alpha + (N^a dt + dy^a) e_a^\alpha \end{aligned}$$

$$\begin{aligned}
 ds^2 &= g_{\mu\nu} dx^\mu dx^\nu \\
 &= g_{\mu\nu} \left(N dt + (N^a dt + dy^a) e_a^\mu \right) \left(N dt + (N^b dt + dy^b) e_b^\nu \right)
 \end{aligned}$$



$$\begin{aligned}
 ds^2 &= g_{\mu\nu} dx^\mu dx^\nu \\
 &= g_{\mu\nu} \left(N dt n^\mu + (N^\alpha dt + dy^\alpha) e^\mu_\alpha \right) \left(N dt n^\nu + (N^\beta dt + dy^\beta) e^\nu_\beta \right)
 \end{aligned}$$

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

$$= g_{\mu\nu} \left(N dt n^\mu + (N^\alpha dt + dy^\alpha) e^\mu_\alpha \right) \left(N dt n^\nu + (N^\beta dt + dy^\beta) e^\nu_\beta \right)$$

$$= N^2 dt^2$$

$$\begin{aligned}
 ds^2 &= g_{\mu\nu} dx^\mu dx^\nu \\
 &= g_{\mu\nu} \left(N dt n^\mu + (N^\alpha dt + dy^\alpha) e_\alpha^\mu \right) \left(N dt n^\nu + (N^\beta dt + dy^\beta) e_\beta^\nu \right) \\
 &= N^2 dt^2
 \end{aligned}$$

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

$$= g_{\mu\nu} (N dt + n^\alpha (N^\alpha dt + dy^\alpha) e^\alpha_\mu) (N dt + n^\beta (N^\beta dt + dy^\beta) e^\beta_\nu)$$

$$\boxed{ds^2 = -N^2 dt^2 + h_{ab} (dy^a + N^a dt) (dy^b + N^b dt)}$$

$$ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta$$

$$= g_{\alpha\beta} (N dt n^\alpha + (N^\alpha dt + dy^\alpha) e_\alpha^i) (N dt n^\beta + (N^\beta dt + dy^\beta) e_\beta^j)$$

$$\boxed{ds^2 = -N^2 dt^2 + h_{ab} (dy^a + N^a dt) (dy^b + N^b dt)}$$

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

$$= g_{\mu\nu} (N dt + N^\alpha e_\alpha^\mu) (N dt + N^\beta e_\beta^\nu)$$

$$\boxed{ds^2 = -N^2 dt^2 + h_{ab} (\partial y^a + N^a dt) (\partial y^b + N^b dt)}$$

$$\sqrt{-g} d^4 X =$$

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

$$= g_{\mu\nu} (N dt + n^\alpha + (N^\alpha dt + \gamma^\alpha) e^\alpha) (N dt + n^\beta + (N^\beta dt + \gamma^\beta) e^\beta)$$

$$ds^2 = -N^2 dt^2 + h_{ab} (\gamma^a + N^a dt) (\gamma^b + N^b dt)$$

$$\sqrt{-g} d^4x =$$

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

$$= g_{\mu\nu} (N dt + n^\alpha + (N^\alpha dt + \partial y^\alpha) e^\alpha) (N dt + n^\beta + (N^\beta dt + \partial y^\beta) e^\beta)$$

$$ds^2 = -N^2 dt^2 + h_{ab} (\partial y^a + N^a dt) (\partial y^b + N^b dt)$$

$$\sqrt{-g} d^4x = N \sqrt{h}$$

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

$$= g_{\mu\nu} (N dt n^\mu + (N^\alpha dt + dy^\alpha) e_\alpha^\mu) (N dt n^\nu + (N^\beta dt + dy^\beta) e_\beta^\nu)$$

$$ds^2 = -N^2 dt^2 + h_{ab} (dy^a + N^a dt) (dy^b + N^b dt)$$

$$\sqrt{-g} d^4x = N \sqrt{h} d^3y dt$$

Scalar field

$$\mathcal{L}(q, \dot{q}, t)$$

Scalar field

$$\mathcal{L}(\varphi, \partial_\mu \varphi)$$

$$\dot{\varphi} \equiv (\partial_\mu \varphi) H^\mu$$

$t^\alpha = \left(\frac{\partial x}{\partial t} \right)_y$
 gen. values of t :
 $e_\alpha^\alpha = \left(\frac{\partial x^\alpha}{\partial y^\alpha} \right)_t$

$x^\alpha(t, y^\alpha)$
 $\eta_\alpha = -N \partial_\alpha t$

$t^\alpha = \underbrace{N}_\text{time} \eta^\alpha + \underbrace{N^\alpha}_\text{shift} e_\alpha^\alpha$



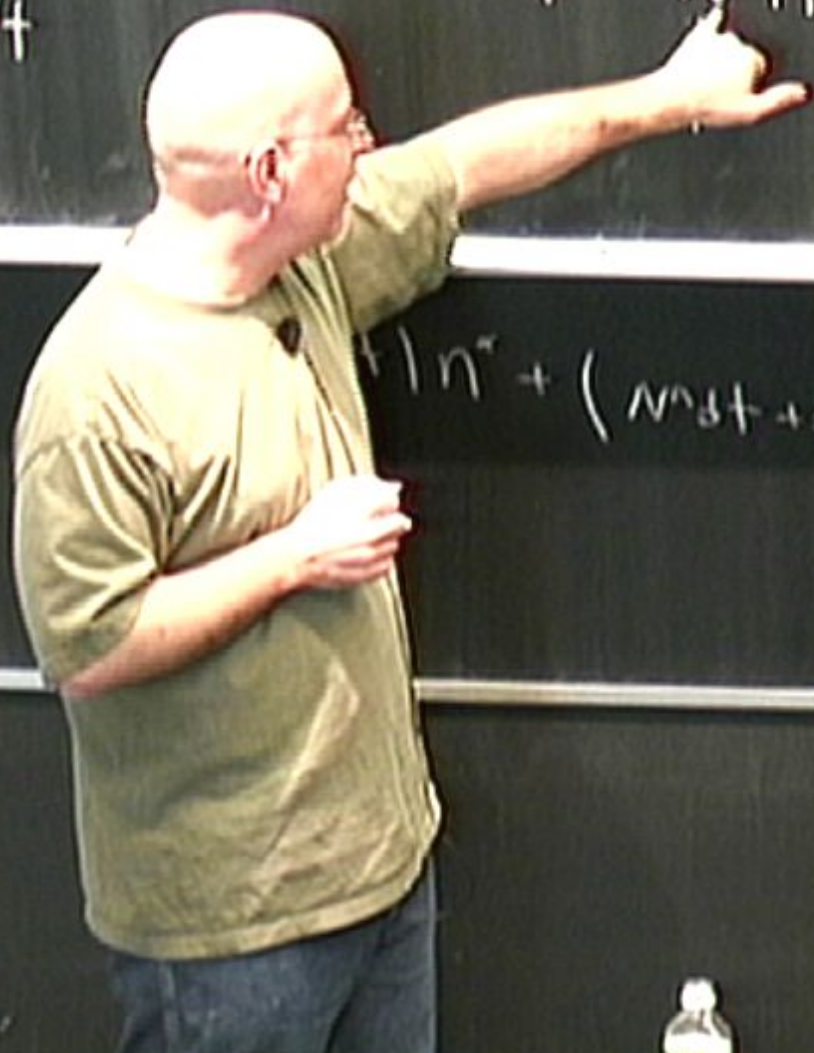
$(N^\alpha dt + dy^\alpha) e_\alpha^\alpha$

EATEN
 REPERATED
 JUSTICE

$t^x = \left(\frac{\partial x}{\partial t} \right)_y$
 gen. values of t
 t_+
 $e^x = \left(\frac{\partial x^x}{\partial y^y} \right)_t$

$x^x(t, y^y)$
 $\eta_\alpha = -N \partial_\alpha t$

$t^x = N \eta^\alpha + N^\alpha e^x_\alpha$
 shift



$t^x = N \eta^\alpha + (N^\alpha t + dy^\alpha) e^x_\alpha$

EATEN
 DISAPPEARED
 REPAIRED
 2000-00-00

Scalar field

$$\mathcal{L}(q, \partial_\mu q)$$

$$\dot{q} \equiv (\partial_\mu q) t^\mu$$

$$q_n \equiv (\partial_\mu q) e_n^\mu$$

Scalar field

$$\mathcal{L}(\varphi, \partial_\mu \varphi)$$

$$\left. \begin{aligned} \dot{\varphi} &= (\partial_\mu \varphi) \dot{t}^\mu \\ \varphi_n &= (\partial_\mu \varphi) e_n^\mu \end{aligned} \right\} \mathcal{L}(\varphi, \dot{\varphi}, \varphi_n)$$

Scalar field

$$\mathcal{L}(q, \dot{q})$$

$$\left. \begin{aligned} \dot{q} &= (\dot{q}_r) \mathbf{H}^r \\ q_n &= (q_n) \mathbf{e}_n \end{aligned} \right\} \mathcal{L}(q, \dot{q}, q_n)$$

$$P = \frac{\partial}{\partial \dot{q}} (\mathcal{L} \sqrt{g})$$

Hamiltonian density:

$$\mathcal{H} = p\dot{q} - \mathcal{L}(\sqrt{g})$$



Hamiltonian density:

$$\mathcal{H} = p\dot{q} - \mathcal{L}$$

↳ a function of q, p, \dot{q}

Hamiltonian:

$$H = \int_{\Sigma_t} \mathcal{H} d^3y$$

Hamiltonian density:

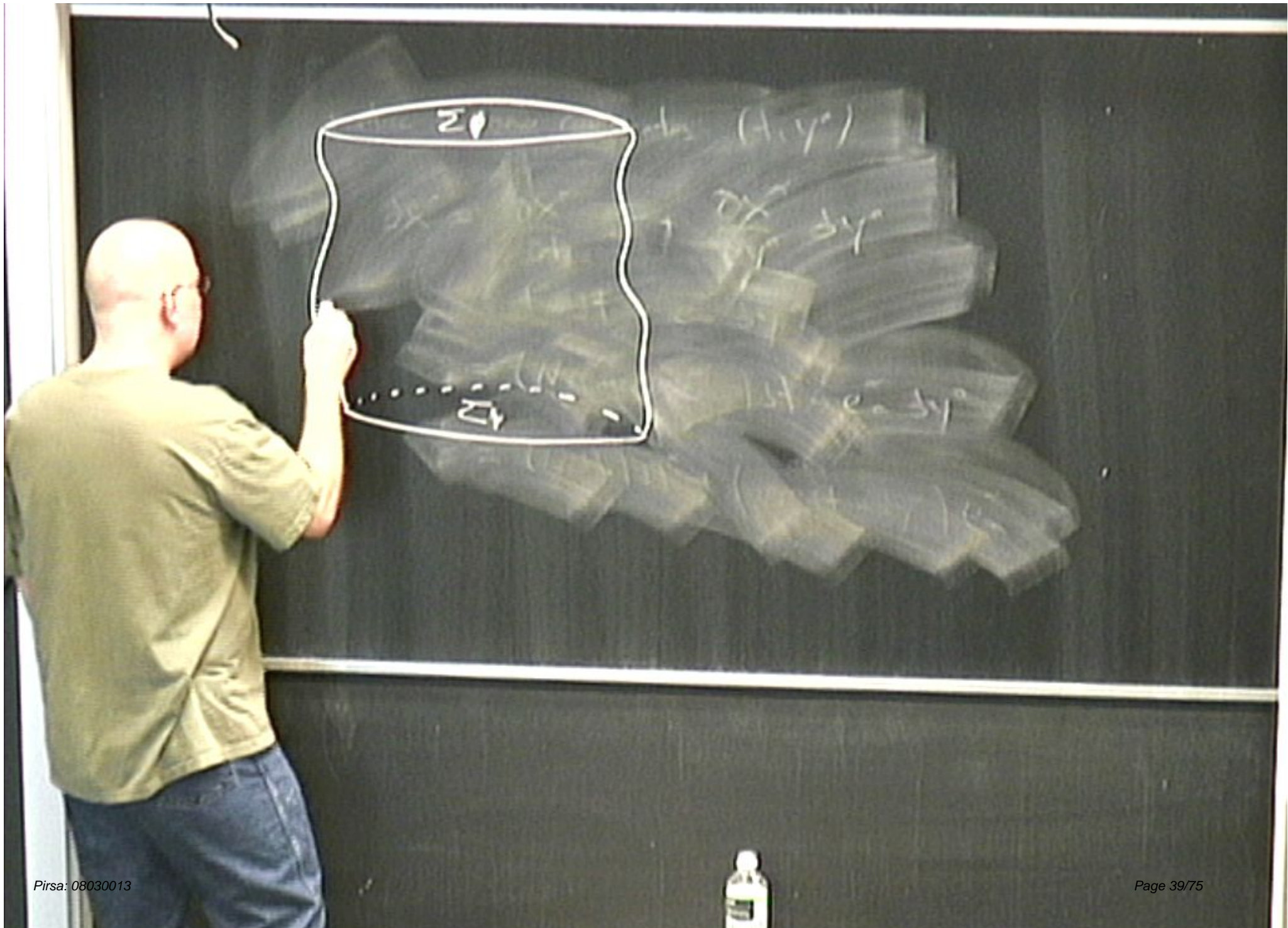
$$\mathcal{H} = p\dot{q} - \mathcal{L}\sqrt{-g}$$

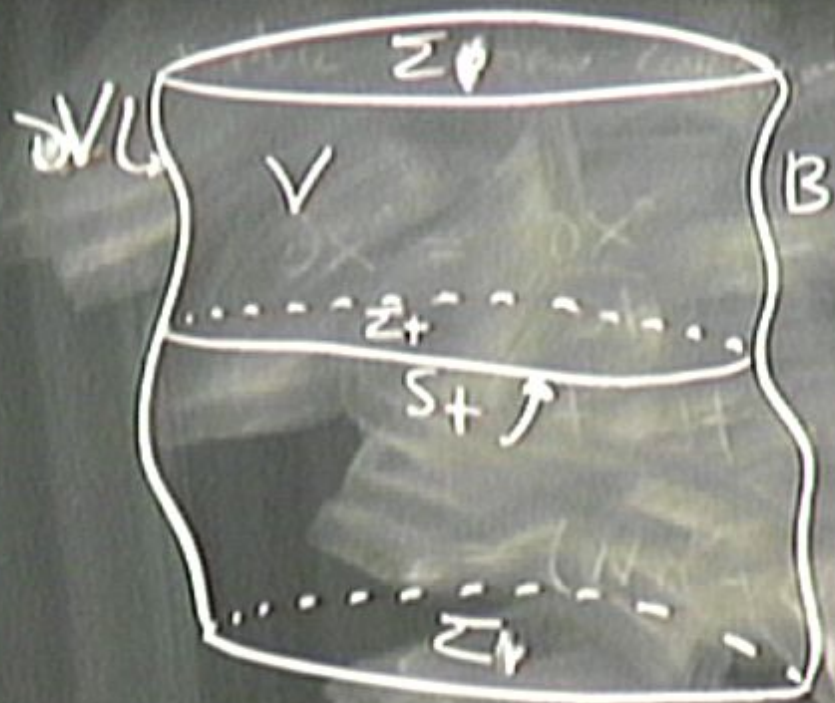
↳ a function of q, p, \dot{q}

Hamiltonian:

$$H = \int_{\Sigma_t} \mathcal{H} d^3y$$

↳ a functional of q, p on each Σ_t .





$$\partial V = \Sigma_0 \cup \Sigma_+ \cup \Sigma_- \cup B$$

V foliated by Σ_+

B foliated by S_+

$$S_+ = \partial \Sigma_+$$

Z^+ embedded in M

$$t(x^a) = \text{const}$$

$$x^a(y^a)$$

$$e_a^{\check{x}} = \frac{\partial x^{\check{x}}}{\partial y^a}$$

$$n_a = -N \partial_x t$$

$$h_{ab} = g_{\check{x}\check{x}} e_a^{\check{x}} e_b^{\check{x}}$$

$$K_{ab} = n_{\check{x}} e_a^{\check{x}} e_b^{\check{x}}$$

Z_t embedded in M

$$t(x^a) = \text{const}$$

$$x^a(y^a)$$

$$e^a = \frac{\partial x^a}{\partial y^a}$$

$$n_a = -N \partial_t t$$

$$h_{ab} = g_{\alpha\beta} e^{\alpha} e^{\beta}$$

$$K_{ab} = n_{\alpha} s^{\alpha} e^{\alpha} e^{\beta}$$

$$g_{\alpha\beta} = -n^{\alpha} n^{\beta} + h^{ab} e^{\alpha} e^{\beta}$$

Σ_t embedded in M

$$t(x^\alpha) = \text{const}$$

$$x^\alpha(y^\alpha)$$

$$e_\alpha^\sigma = \frac{\partial x^\sigma}{\partial y^\alpha}$$

$$n_\alpha = -N \partial_\alpha t$$

$$h_{ab} = g_{\sigma\tau} e^\sigma_a e^\tau_b$$

$$K_{ab} = n_\sigma e^\sigma_a e^\tau_b$$

$$g_{\sigma\tau} = -n^\sigma n^\tau + h^{ab} e^\sigma_a e^\tau_b$$

S_t embedded in Σ_t

Σ_t embedded in \mathcal{M}

$$t(x^\alpha) = \text{const}$$

$$x^\alpha(y^\alpha)$$

$$e_\alpha^\alpha = \frac{\partial x^\alpha}{\partial y^\alpha}$$

$$n_\alpha = -N \partial_\alpha t$$

$$h_{ab} = g_{\alpha\beta} e^\alpha_a e^\beta_b$$

$$K_{ab} = n_\alpha e^\alpha_a e^\beta_b$$

$$g_{\alpha\beta} = -n_\alpha n_\beta + h^{\alpha\beta} e_\alpha e_\beta$$

Σ_t embedded in Σ_t

$$Q(y^\alpha) = \text{const}$$

$$y^\alpha$$

Σ_t embedded in M

$$t(x^a) = \text{const}$$

$$x^a(y^A)$$

$$e^{\vec{a}} = \frac{\partial x^{\vec{a}}}{\partial y^A}$$

$$n_a = -N \partial_t$$

$$h_{ab} = g_{\mu\nu} e^{\mu}_{\vec{a}} e^{\nu}_{\vec{b}}$$

$$K_{ab} = n_{\mu} \partial_{\nu} e^{\mu}_{\vec{a}} e^{\nu}_{\vec{b}}$$

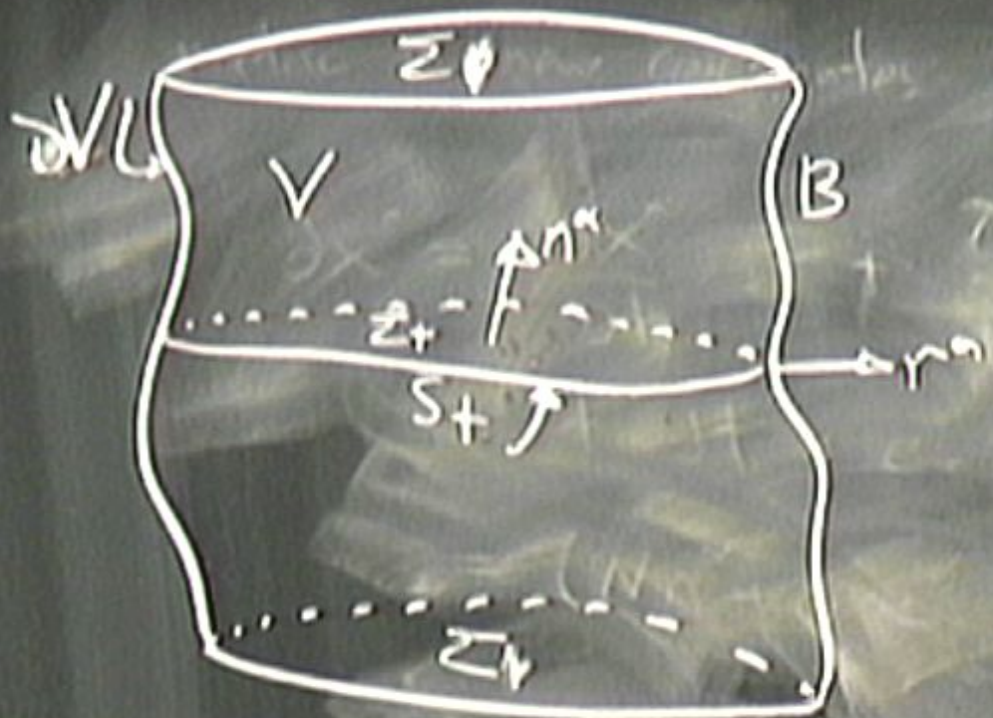
$$g_{\mu\nu} = -n_{\mu} n_{\nu} + h^{\vec{a}\vec{b}} e^{\mu}_{\vec{a}} e^{\nu}_{\vec{b}}$$

S_t embedded in Σ_t

$$\varphi(y^A) = \text{const}$$

$$y^A(\theta^A)$$

$$e^{\vec{A}} = \frac{\partial y^{\vec{A}}}{\partial \theta^A}$$



$$\partial V = \Sigma_1 \cup \Sigma_2 \cup B$$

V foliated by Σ_+

B foliated by S_+

$$S_+ = \partial \Sigma_+$$

Σ_t embedded in \mathcal{M}

$$t(x^\alpha) = \text{const}$$

$$x^\alpha(y^A)$$

$$e_a^\alpha = \frac{\partial x^\alpha}{\partial y^a}$$

$$n_a = -N \partial_{x^t}$$

$$h_{ab} = g_{\alpha\beta} e_a^\alpha e_b^\beta$$

$$K_{ab} = n_\alpha \partial_{x^\alpha} e_b^\alpha$$

$$g_{\alpha\beta} = -n_\alpha n_\beta + h^{ab} e_a^\alpha e_b^\beta$$

S_t embedded in Σ_t

$$Q(y^A) = \text{const}$$

$$y^A(\theta^A)$$

$$e_A^a = \frac{\partial y^A}{\partial x^a}$$

$$\Gamma_a \propto \partial_{x^a} Q$$

$$\sigma_{AB} = h_{ab} e_a^A e_b^B$$

k



Σ_t embedded in \mathcal{M}

$$t(x^\alpha) = \text{const}$$

$$x^\alpha(y^A)$$

$$e^{\vec{\alpha}} = \frac{\partial x^\alpha}{\partial y^A}$$

$$n_\alpha = -N \partial_\alpha t$$

$$h_{ab} = g_{\alpha\beta} e^{\vec{\alpha}} e^{\vec{\beta}}$$

$$K_{ab} = n_\alpha \partial_a e^{\vec{\alpha}} e^{\vec{\beta}} b$$

$$g_{\alpha\beta} = -n_\alpha n_\beta + h^{ab} e^{\vec{\alpha}} e^{\vec{\beta}}$$

S_t embedded in Σ_t

$$Q(y^A) = \text{const}$$

$$y^A(\theta^A)$$

$$e^{\vec{A}} = \frac{\partial y^A}{\partial \theta^A}$$

$$\Gamma_A \propto \partial_A Q$$

$$\sigma_{AB} = h_{ab} e^{\vec{a}} e^{\vec{b}}$$

$$K_{AB} = \Gamma_{AB}$$

Σ_t embedded in \mathcal{M}

$$t(x^\alpha) = \text{const}$$

$$x^\alpha(y^A)$$

$$e^\alpha_a = \frac{\partial x^\alpha}{\partial y^a}$$

$$n_\alpha = -N \partial_\alpha t$$

$$h_{ab} = g_{\alpha\beta} e^\alpha_a e^\beta_b$$

$$K_{ab} = n_\alpha \partial_a e^\alpha_b$$

$$g_{\alpha\beta} = -n^\alpha n_\beta + h^{ab} e^\alpha_a e^\beta_b$$

S_t embedded in $\bar{\Sigma}_t$

$$Q(y^A) = \text{const}$$

$$y^A(\theta^A)$$

$$e^A_A = \frac{\partial y^A}{\partial \theta^A}$$

$$\Gamma_a \propto \partial_a Q$$

$$\sigma_{AB} = h_{ab} e^a_A e^b_B$$

$$K_{AB} = \Gamma_{a|b} e^a_A e^b_B$$

$$h^{ab} = \Gamma^a \Gamma^b + \sigma^{AB} e^a_A e^b_B$$

St embedded in M

$$\psi(x^\alpha) = \text{const}$$

$$x^\alpha(\theta^A) = x^\alpha(y^a(\theta^A))$$

$$e^a_A = \frac{\partial x^\alpha}{\partial \theta^A} = \frac{\partial x^\alpha}{\partial y^a} \frac{\partial y^a}{\partial \theta^A}$$

$$e^a_A e^b_B$$

$$e^a_A e^b_B$$

St embedded in M

$$\psi(x^\alpha) = \text{const}$$

$$x^\alpha(\theta^A) = x^\alpha(y^A(\theta^A))$$

$$e_A^\alpha = \frac{\partial x^\alpha}{\partial \theta^A} = \frac{\partial x^\alpha}{\partial y^A} \frac{\partial y^A}{\partial \theta^A} = e_a^\alpha e^A$$

\sqrt{g}

3
 $e^a e^b$

4
 $e^a e^b$

St embedded in M

$$\psi(x^\alpha) = \text{const}$$

$$x^\alpha(\theta^A) = X^\alpha(\psi^A(\theta^A))$$

$$e_a^\alpha = \frac{\partial X^\alpha}{\partial \theta^A} = \frac{\partial X^\alpha}{\partial \psi^A} \frac{\partial \psi^A}{\partial \theta^A} = e_a^A e_A^\alpha$$

$$\Gamma_\alpha \propto \partial_\alpha \psi : \Gamma^\alpha = \Gamma^a e_a^\alpha$$



3
4
A ∈ B

St embeddings in \mathbb{R}^4

$$\varphi(\gamma) = \text{const}$$

$$\gamma^\alpha(\theta^A)$$

$$e^A = \frac{\partial \gamma^\alpha}{\partial \theta^A}$$

$$\Gamma_\alpha \propto \partial_\alpha \varphi$$

$$\sigma_{AB} = \text{hab } e^a e^b$$

$$k_{AB} = \Gamma_{ab} e^a e^b$$

$$h^{ab} = \Gamma^a \Gamma^b + \sigma^{AB} e^A e^B$$

St embeddings in M

$$\varphi(x^\alpha) = \text{const}$$

$$X^\alpha(\theta^A) = X^\alpha(\varphi^\alpha(\theta^A))$$

$$e^A = \frac{\partial X^\alpha}{\partial \theta^A} = \frac{\partial X^\alpha}{\partial \varphi^\alpha} \frac{\partial \varphi^\alpha}{\partial \theta^A} = e^a e^A$$

$$\Gamma_\alpha \propto \partial_\alpha \varphi : \Gamma^\alpha = \Gamma^a e^a$$

$$\begin{aligned} \sigma_{AB} &= \text{zap } e^a e^b \\ &= \text{zap}(e^a e^A)(e^B e^B) \end{aligned}$$

Σ embedded in M

$$t(x) = \text{const}$$

$$x^\alpha(y^\alpha)$$

$$e_a^\alpha = \frac{\partial x^\alpha}{\partial y^a}$$

$$n_\alpha = -N \partial_t$$

$$h_{ab} = g_{\alpha\beta} e_a^\alpha e_b^\beta$$

$$K_{ab} = n_\alpha \partial_a e_b^\alpha$$

$$g^{\alpha\beta} = -n^\alpha n^\beta + h^{ab} e_a^\alpha e_b^\beta$$

Σ embedded in Σ_t

$$Q(y) = \text{const}$$

$$y^\alpha(\theta^\alpha)$$

$$e_A^\alpha = \frac{\partial y^\alpha}{\partial \theta^A}$$

$$\Gamma_A \propto \partial_A Q$$

$$\sigma_{AB} = h_{ab} e_A^a e_B^b$$

$$K_{AB} = \Gamma_{ab} e_A^a e_B^b$$

$$h^{ab} = r^a r^b + \sigma^{AB} e_A^a e_B^b$$

Σ embedded in M

$$\psi(x^\alpha) = \text{const}$$

$$x^\alpha(\theta^\alpha) = X^\alpha(Y^\alpha(\theta^\alpha))$$

$$e_A^\alpha = \frac{\partial x^\alpha}{\partial \theta^A} = \frac{\partial x^\alpha}{\partial Y^\beta} \frac{\partial Y^\beta}{\partial \theta^A} = e_A^\beta e_B^\alpha$$

$$\Gamma_A \propto \partial_A \psi : r^\alpha = r^a e_A^\alpha$$

$$\begin{aligned} \sigma_{AB} &= g_{\alpha\beta} e_A^\alpha e_B^\beta \\ &= g_{\alpha\beta} (e_A^a e_B^b) (e_C^c e_D^d) \\ &= h_{ab} e_A^a e_B^b \end{aligned}$$

$$g^{\alpha\beta} = -n^\alpha n^\beta + r^a r^b + \sigma^{AB} e_A^a e_B^b$$

St embedded in M

$$\psi(x^\alpha) = \text{const}$$

$$x^\alpha(\theta^\alpha) = X^\alpha(\psi^\alpha(\theta^\alpha))$$

$$e^\alpha_A = \frac{\partial X^\alpha}{\partial \theta^\alpha} = \frac{\partial X^\alpha}{\partial \psi^\alpha} \frac{\partial \psi^\alpha}{\partial \theta^\alpha} = e^\alpha_a e^a_A$$

$$\Gamma_\alpha \propto \partial_\alpha \psi : \Gamma^\alpha = \Gamma^a e^a_\alpha$$

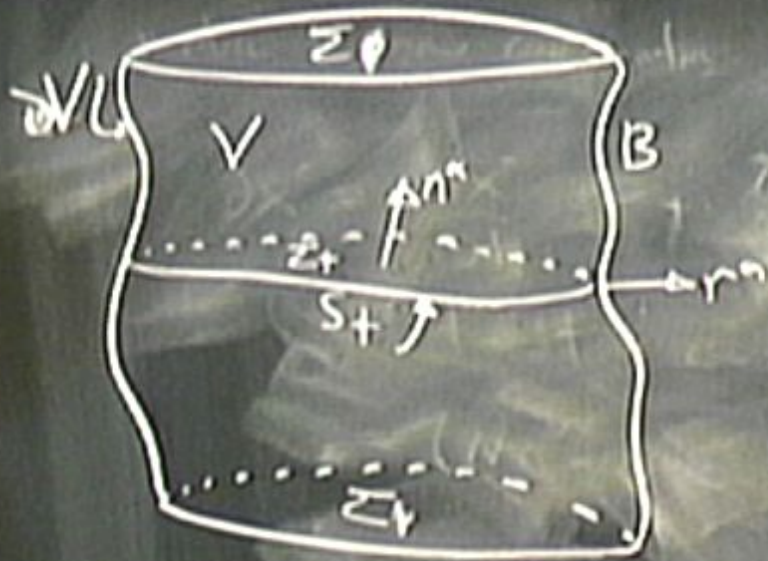
$$\begin{aligned} \sigma_{AB} &= \gamma_{\alpha\beta} e^\alpha_A e^\beta_B \\ &= \gamma_{\alpha\beta} (e^\alpha_a e^a_A) (e^\beta_b e^b_B) \\ &= h_{ab} e^a_A e^b_B \end{aligned}$$

$$\begin{aligned} \gamma^{\alpha\beta} &= -n^\alpha n^\beta + r^\alpha r^\beta \\ &\quad + \sigma^{AB} e^a_A e^b_B \end{aligned}$$

B embed in \mathbb{R}^m

$$f(x) = \text{cost}$$

$$x^*(z^i)$$



$$\partial V = \Sigma_+ \cup \Sigma_- \cup B$$

V foliated by Σ_+

B foliated by S_+

$$S_+ = \partial \Sigma_+$$

B embeds in \mathbb{R}^n

$$f(x) = \text{const}$$

$$x(z^i)$$

$$e^{\alpha}_i = \frac{\partial x^{\alpha}}{\partial z^i}$$

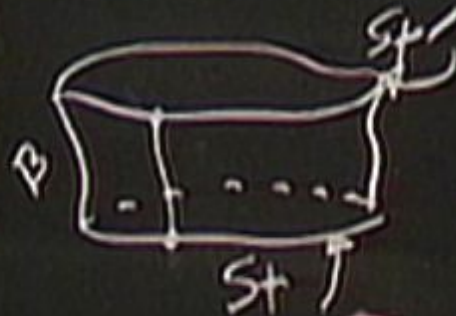
$$r_{\alpha} = \frac{\partial f}{\partial x^{\alpha}}$$

$$\gamma_{ij} = g_{\alpha\beta} e^{\alpha}_i e^{\beta}_j$$

$$K_{ij} = r_{\alpha\beta} e^{\alpha}_i e^{\beta}_j$$

$$g^{\alpha\beta} = r^{\alpha} r^{\beta} + \gamma^{ij} e^{\alpha}_i e^{\beta}_j$$

Foliation of B by S^1



B contains in \mathbb{R}^n

$$f(x) = \text{const}$$

$$x^i(z^i)$$

$$e^{\alpha}_i = \frac{\partial x^{\alpha}}{\partial z^i}$$

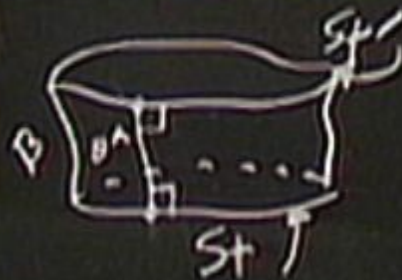
$$r_{\alpha} = \frac{\partial f}{\partial x^{\alpha}}$$

$$\gamma_{ij} = g_{\alpha\beta} e^{\alpha}_i e^{\beta}_j$$

$$K_{ij} = r_{\alpha\beta} e^{\alpha}_i e^{\beta}_j$$

$$g^{\alpha\beta} = r^{\alpha} r^{\beta} + \gamma^{ij} e^{\alpha}_i e^{\beta}_j$$

Foliation of B by S^1



curves of const θ^{α}
intersect S^1 orthogonally

B embeds in \mathbb{R}^n

$$f(x^i) = \text{out}$$

$$x^i(z^i)$$

$$e^i = \frac{\partial x^i}{\partial z^i}$$

$$r_{\alpha} = \frac{\partial f}{\partial x^i}$$

$$\gamma_{is} = z_{\alpha p} e^i e^s$$

$$k_{is} = r_{\alpha p} e^i e^s$$

$$g^p = r^{\alpha} r^{\beta} + \gamma^{is} e^i e^s$$

Foliation of B by S^1



curves of unit S^1
intersect S^1 orthogonally,
and they are parametrized
by t

B embed in \mathbb{R}^n

$$f(x) = \text{const}$$

$$x(z^i)$$

$$e^{\alpha}_i = \frac{\partial x^{\alpha}}{\partial z^i}$$

$$r_{\alpha} = \frac{\partial f}{\partial x^{\alpha}}$$

$$\gamma_{ij} = g_{\alpha\beta} e^{\alpha}_i e^{\beta}_j$$

$$K_{ij} = r_{\alpha\beta} e^{\alpha}_i e^{\beta}_j$$

$$g^{\mu\nu} = r^{\alpha} r^{\beta} + \gamma^{ij} e^{\alpha}_i e^{\beta}_j$$

Foliation of B by S_t



curves of const θ^{α}
intersect S_t orthogonally,
and they are parametrized
by t

$$\left(\frac{\partial x^{\alpha}}{\partial t} \right)_{\theta^{\alpha}} = N n^{\alpha} \quad (n_{\alpha} = -N \partial_{\alpha} t)$$

B embed in \mathbb{R}^n

$$f(x) = \text{const}$$

$$x^i(z^i)$$

$$e^i = \frac{\partial x^i}{\partial z^i}$$

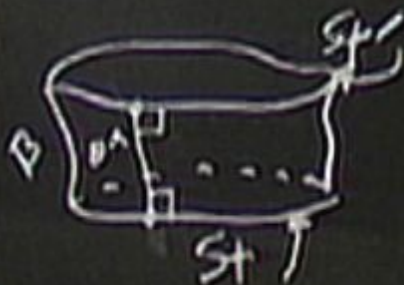
$$r_\alpha = \frac{\partial f}{\partial x^\alpha}$$

$$\gamma_{ij} = g_{\alpha\beta} e^i e^j$$

$$K_{ij} = r_{\alpha\beta} e^i e^j$$

$$g^{\alpha\beta} = r^\alpha r^\beta + \gamma^{ij} e^i e^j$$

Foliation of B by S^1



curves of int ∂A
intersect S^1 orthogonally,
and they are parametrized
by t

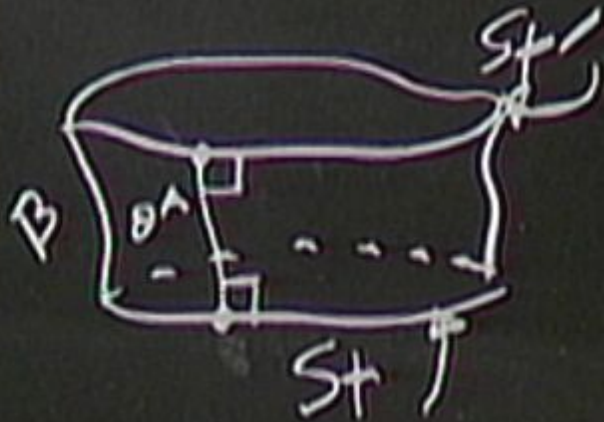
$$\left(\frac{\partial x^i}{\partial t} \right)_{\partial A} = N n^i \quad (n_{0i} = -N \partial_{\alpha t})$$

$$\text{choose } z^i = (t, \theta^A)$$

$$\text{Differentials on B: } dx^i = N n^i dt + e^A_{\alpha} d\theta^A$$

$$ds^2|_B = -N^2 dt^2 + g_{AB} d\theta^A d\theta^B$$

Foliation of B by St



curves of const θ^A
intersect St orthogonally,
and they are parametrized
by t

$$\left(\frac{dx^\alpha}{dt} \right)_{\theta^A} = N n^\alpha \quad (n_{\alpha t} = -N \partial_\alpha t)$$

Choose $z^i = (t, \theta^A)$

Displacements on B: $dx^\alpha = N n^\alpha dt + e^{\alpha A} d\theta^A$

$$ds^2|_B = -N^2 dt^2 + \sigma_{AB} d\theta^A d\theta^B$$

B embeds in \mathbb{R}^n

$$f(x^\alpha) = \text{const}$$

$$x^\alpha(z^i)$$

$$e^\alpha_i = \frac{\partial x^\alpha}{\partial z^i}$$

$$r_\alpha = \alpha f$$

$$\gamma_{ij} = \gamma_{\alpha\beta} e^\alpha_i e^\beta_j$$

$$K_{ij} = r_{\alpha\beta} e^\alpha_i e^\beta_j$$

$$g^{\mu\nu} = r^\alpha r^\beta + \gamma^{ij} e^\alpha_i e^\beta_j$$

Foliation of B by S^1



curves of int. ∂A
intersect S^1 orthogonally,
and they are parametrized
by t

$$\left(\frac{\partial x^\alpha}{\partial t} \right)_{\partial A} = N n^\alpha \quad (n_\alpha = -N \alpha_t)$$

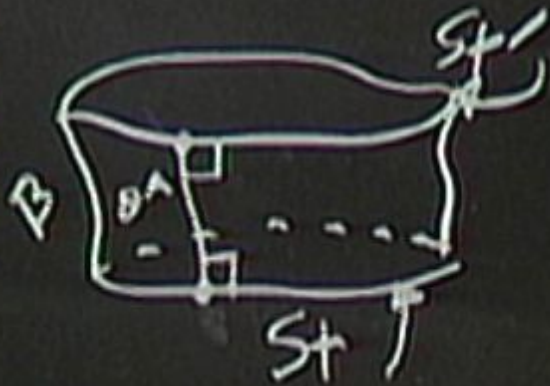
$$\text{choose } z^i = (t, \theta^A)$$

$$\text{Differentials on B: } dx^\alpha = N n^\alpha dt + e^\alpha_A d\theta^A$$

$$ds^2|_B = -N^2 dt^2 + \sigma_{AB} d\theta^A d\theta^B$$

$$\sqrt{-\gamma} = N \sqrt{\sigma}$$

Foliation of B by S_t



curves of const θ^A
intersect S_t orthogonally,
and they are parametrized
by t

$$\left(\frac{dx^\alpha}{dt} \right)_{\theta^A} = N n^\alpha \quad (n_{\alpha t} = -N \partial_\alpha t)$$

Choose $z^i = (t, \theta^A)$

Displacements on B : $dx^\alpha = N n^\alpha dt + e^{\alpha A} d\theta^A$

$$ds^2|_B = -N^2 dt^2 + \sigma_{AB} d\theta^A d\theta^B$$

$$\sqrt{-\gamma} = N \sqrt{\sigma}$$

Gravitational action

$16\pi S =$ scalar field

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Gravitational action

$$16\pi S = \int_V {}^4R \sqrt{-g} d^4x$$

Gravitational action

$$16\pi S = \int_V {}^4R \sqrt{-g} d^4X$$

Gravitational action

$$16\pi S = \int_V \sqrt{-g} \mathcal{L} - 2 \int_{\Sigma} K \sqrt{h} d^3y$$

Gravitational action

$$16\pi S = \int_V \sqrt{-g} R - 2 \int_{\Sigma_1} K \sqrt{h} d^3y$$
$$+ 2 \int_{\Sigma_2} K \sqrt{h} d^3y + 2 \int$$

Gravitational action

$$16\pi S = \int_V {}^4R \sqrt{-g} d^4x - 2 \int_{\Sigma_1} K \sqrt{h} d^3y$$
$$+ 2 \int_{\Sigma_2} K \sqrt{h} d^3y + 2 \int_{\Sigma_0} K \sqrt{-\gamma} d^3z$$

Gravitational action

$$16\pi S = \int_V {}^4R \sqrt{-g} d^4x - 2 \int_{\Sigma_2} K \sqrt{h} d^3y \\ + 2 \int_{\Sigma_1} K \sqrt{h} d^3y + 2 \int_{\partial} K \sqrt{-\gamma} d^3z$$

Gravitational action

$$16\pi S = \int_V {}^4R \sqrt{-g} d^4x - 2 \int_{\Sigma_1} K \sqrt{h} d^3y \\ + 2 \int_{\Sigma_2} K \sqrt{h} d^3y + 2 \int_{\Sigma_0} K \sqrt{-g} d^3z$$

in V , ${}^4R = {}^3R$

Gravitational action

$$16\pi S = \int_V {}^4R \sqrt{-g} d^4x - 2 \int_{\Sigma_2} K \sqrt{h} d^3y \\ + 2 \int_{\Sigma_1} K \sqrt{h} d^3y + 2 \int_{\partial} K \sqrt{-g} d^3z$$

$$\ln V: {}^4R = {}^3R + K^{ab} K_{ab} - K^2$$

Gravitational action

$$16\pi S = \int_V {}^4R \sqrt{-g} d^4x - 2 \int_{\Sigma_1} K \sqrt{h} d^3y$$
$$+ 2 \int_{\Sigma_2} K \sqrt{h} d^3y + 2 \int_{\partial} K \sqrt{-\gamma} d^3z$$

$$\ln V, \quad {}^4R = {}^3R + K^{ab} K_{ab} - K^2 - 2(n^{\alpha}{}_{; \beta} n^{\beta} - n^{\alpha} n^{\beta}{}_{; \beta})_{; \alpha}$$