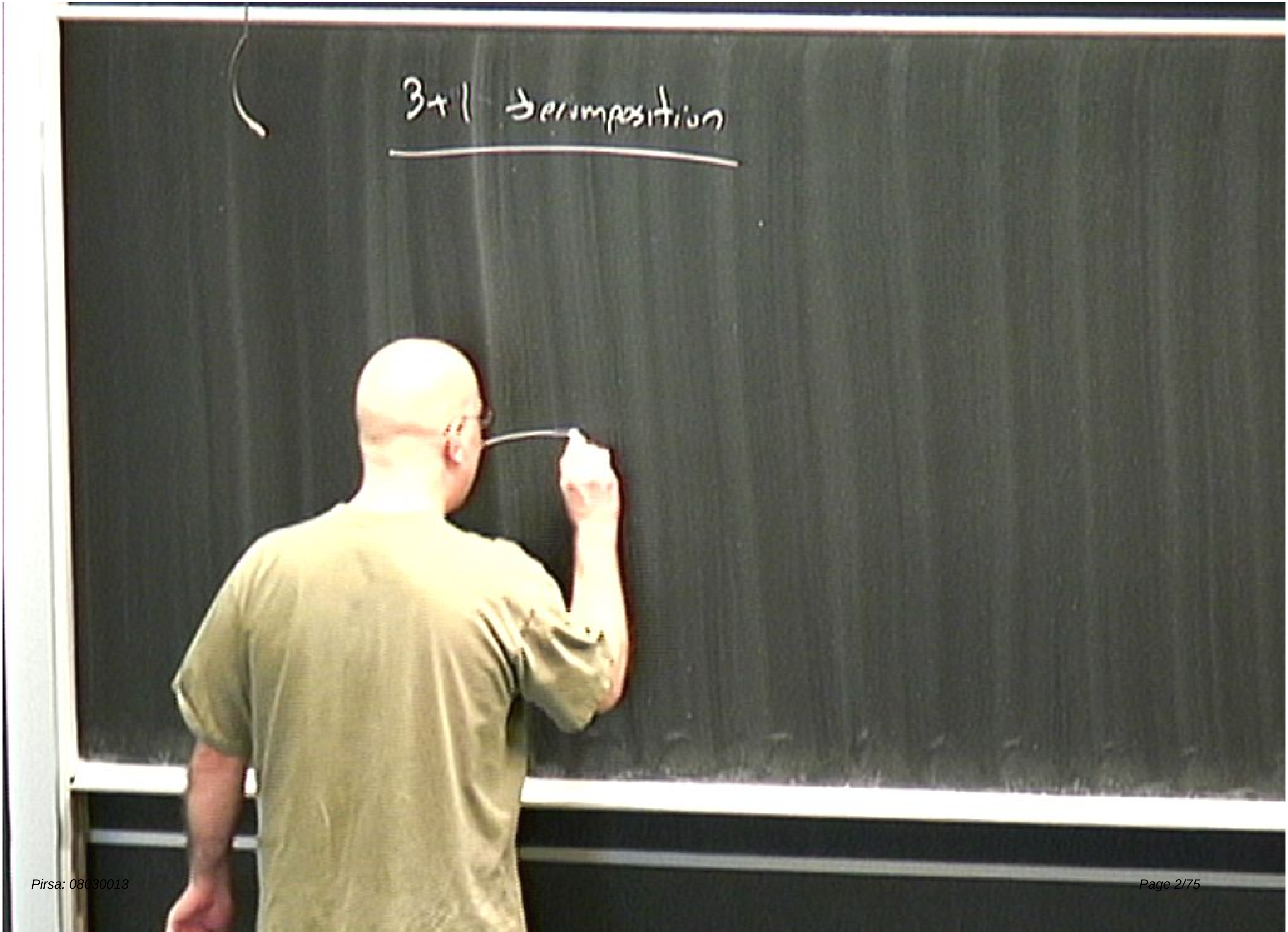
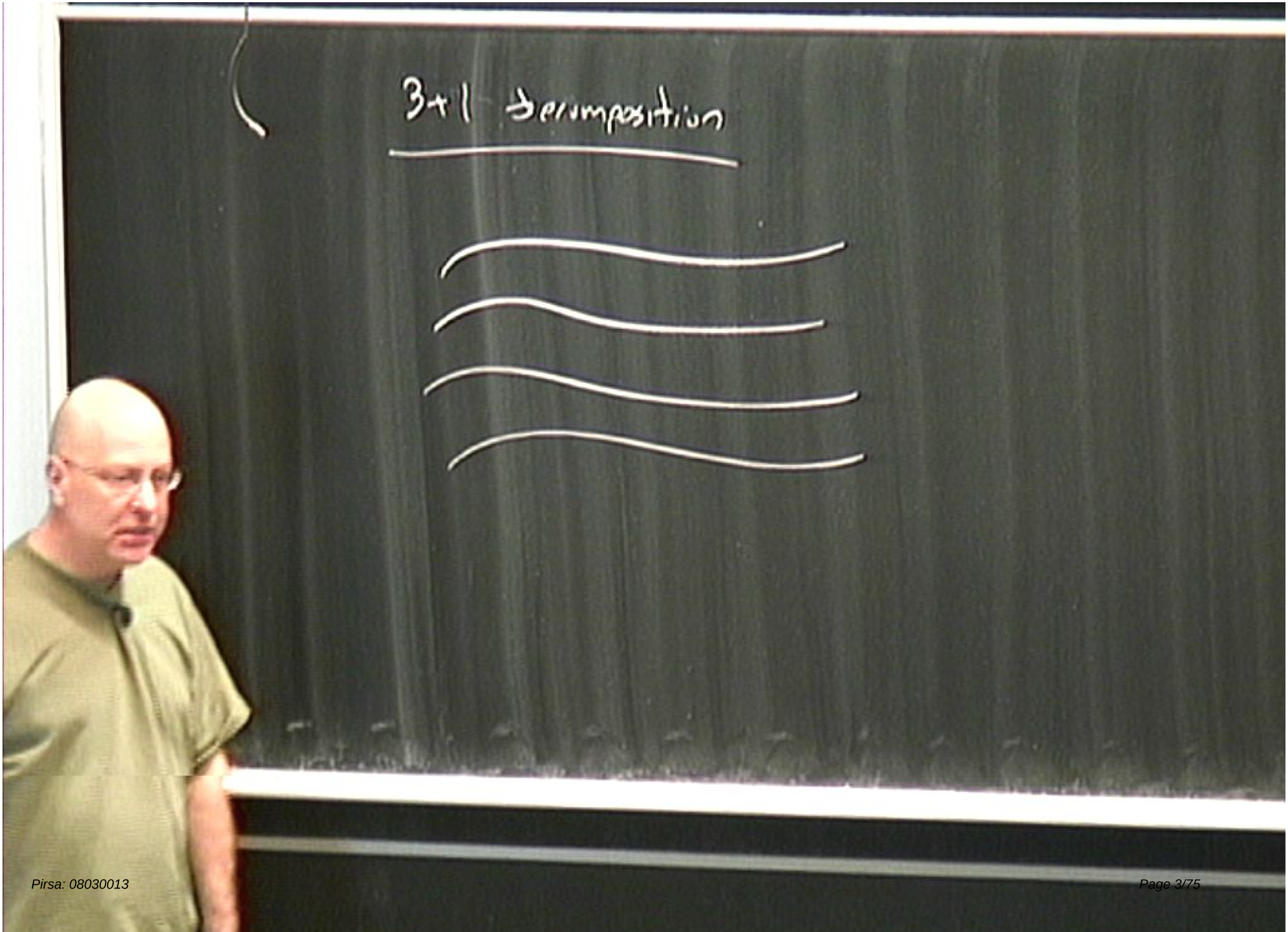
Title: Advanced General Relativity - Lecture 9A

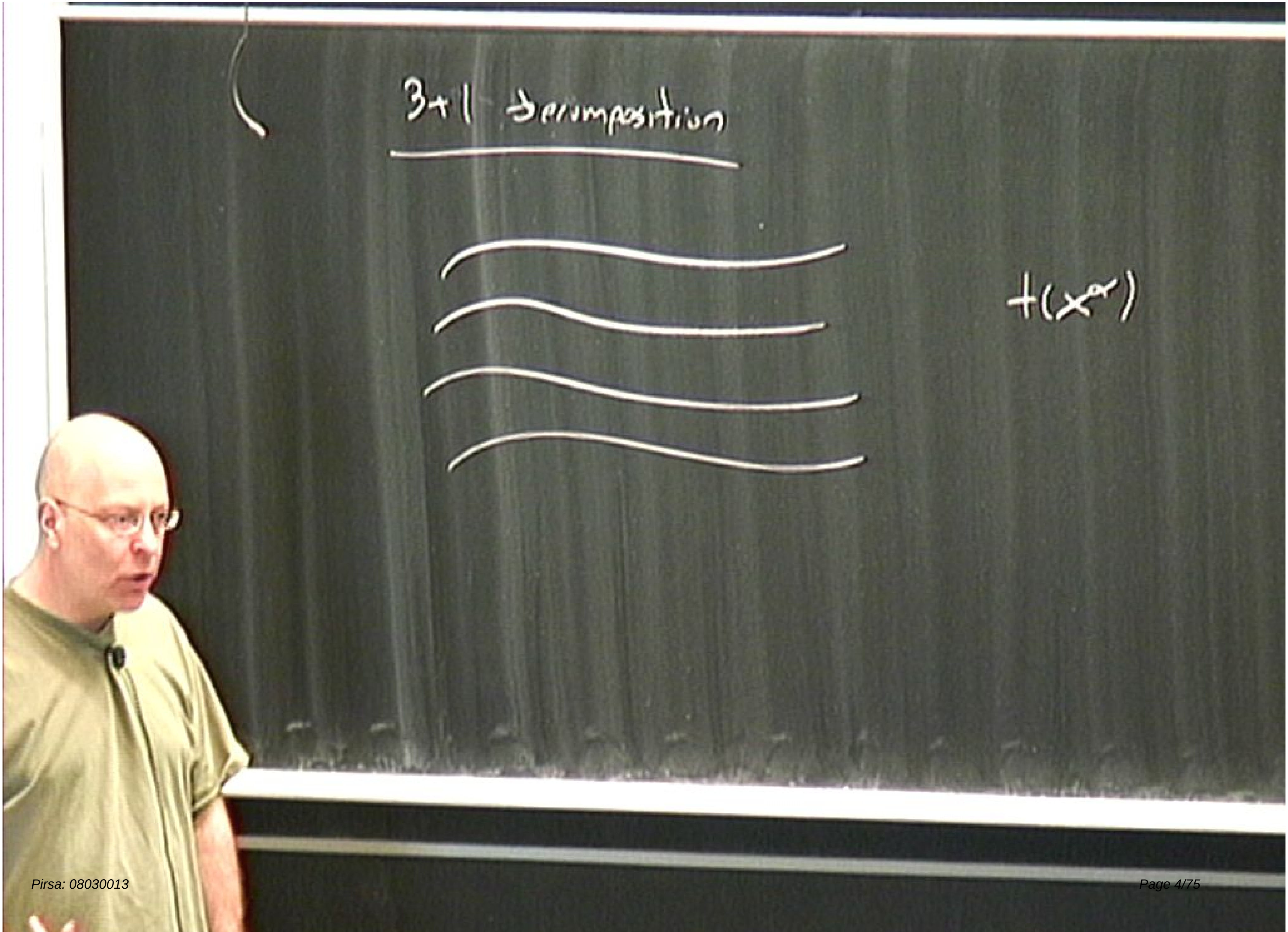
Date: Mar 19, 2008 10:30 AM

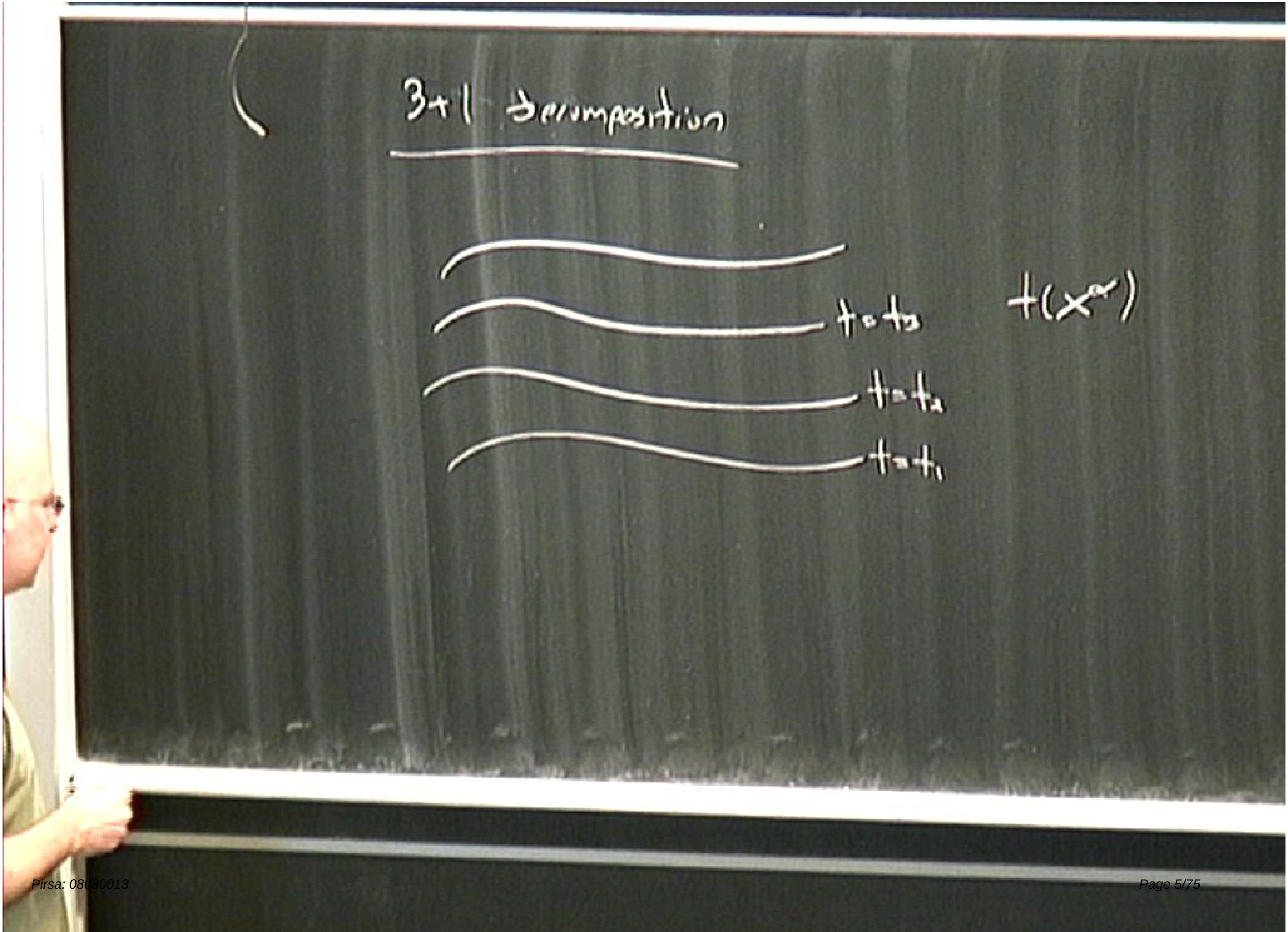
URL: http://pirsa.org/08030013

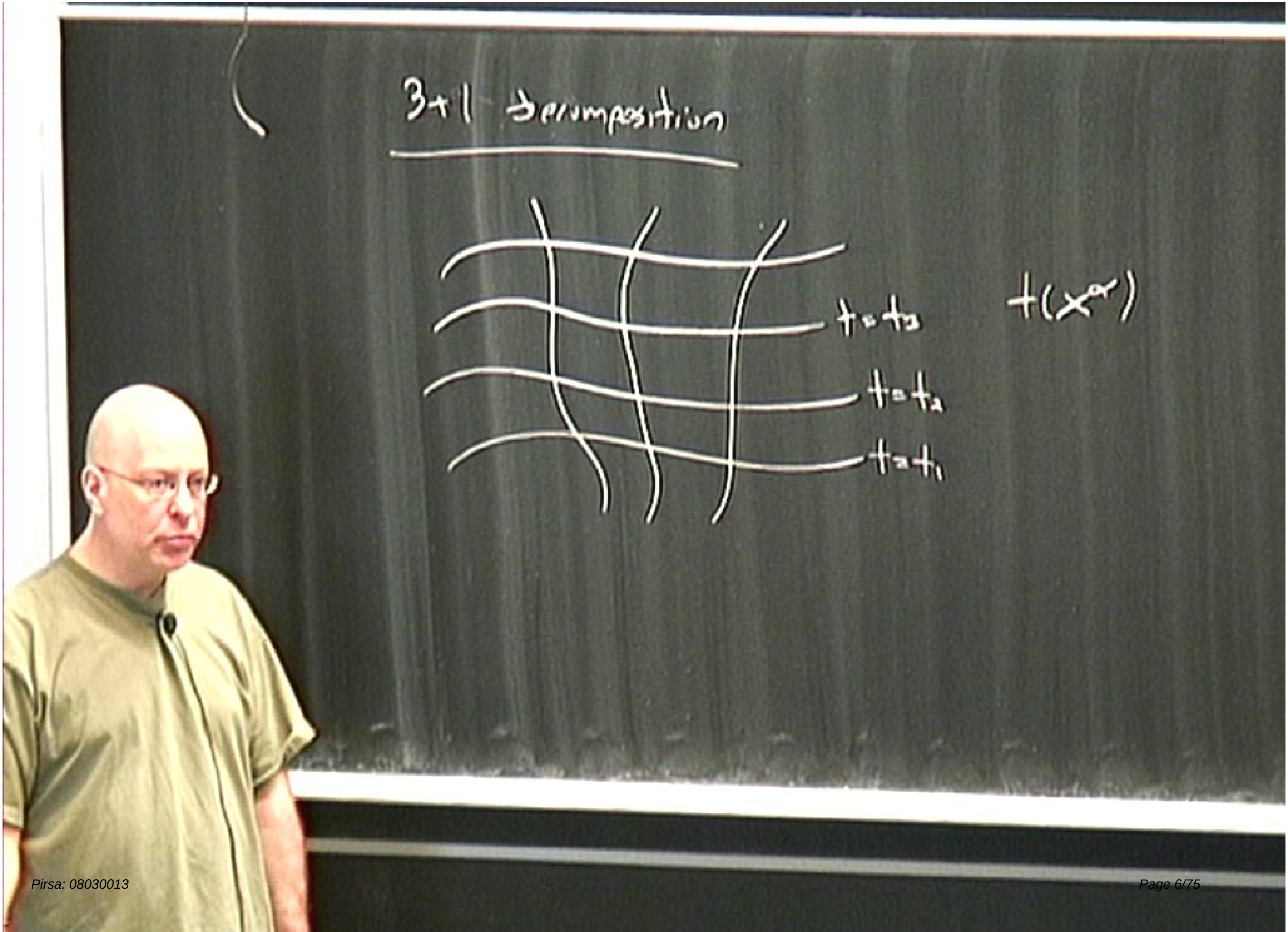
Abstract: Advanced General Relativity

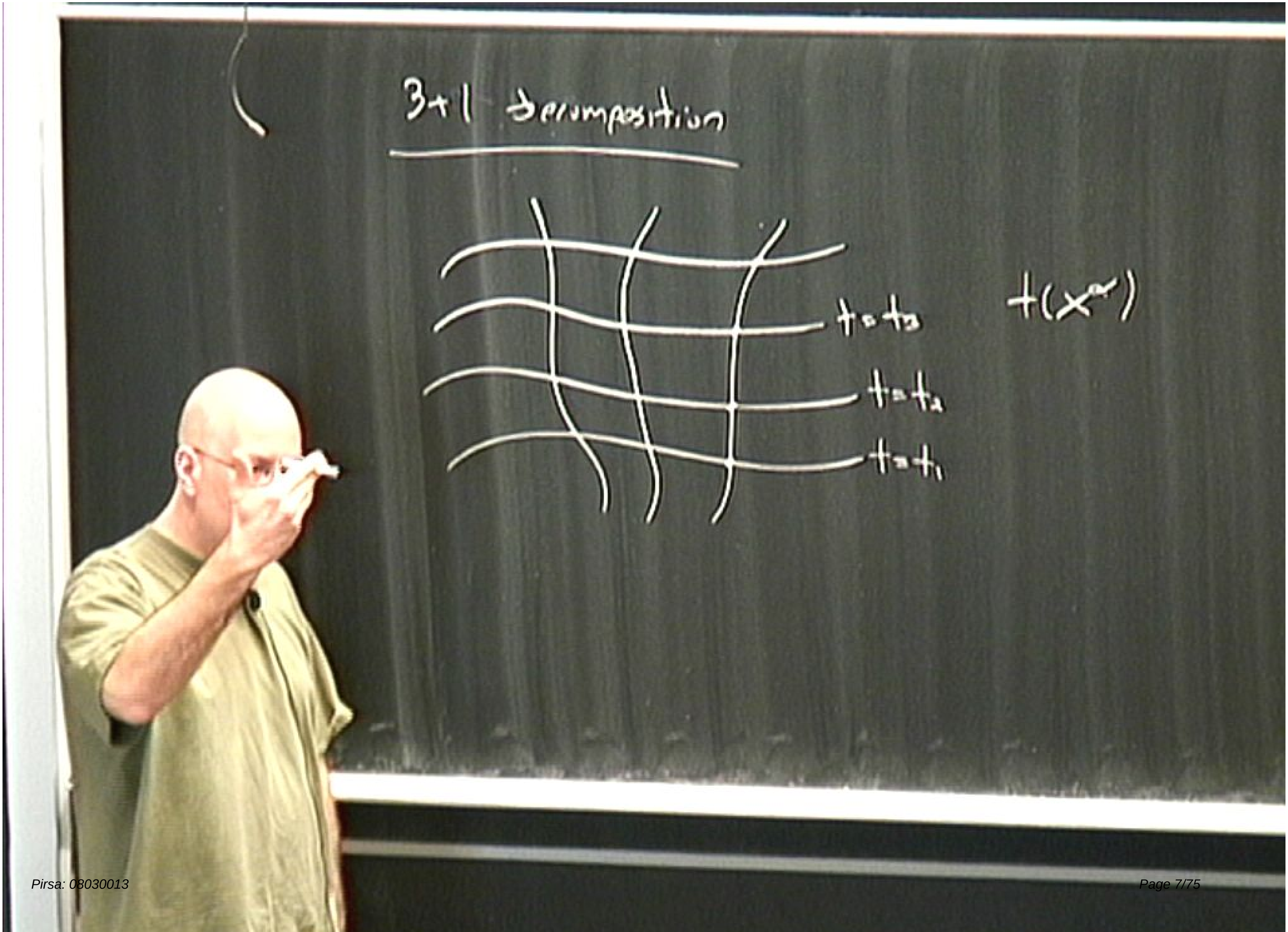

3+1 decomposition

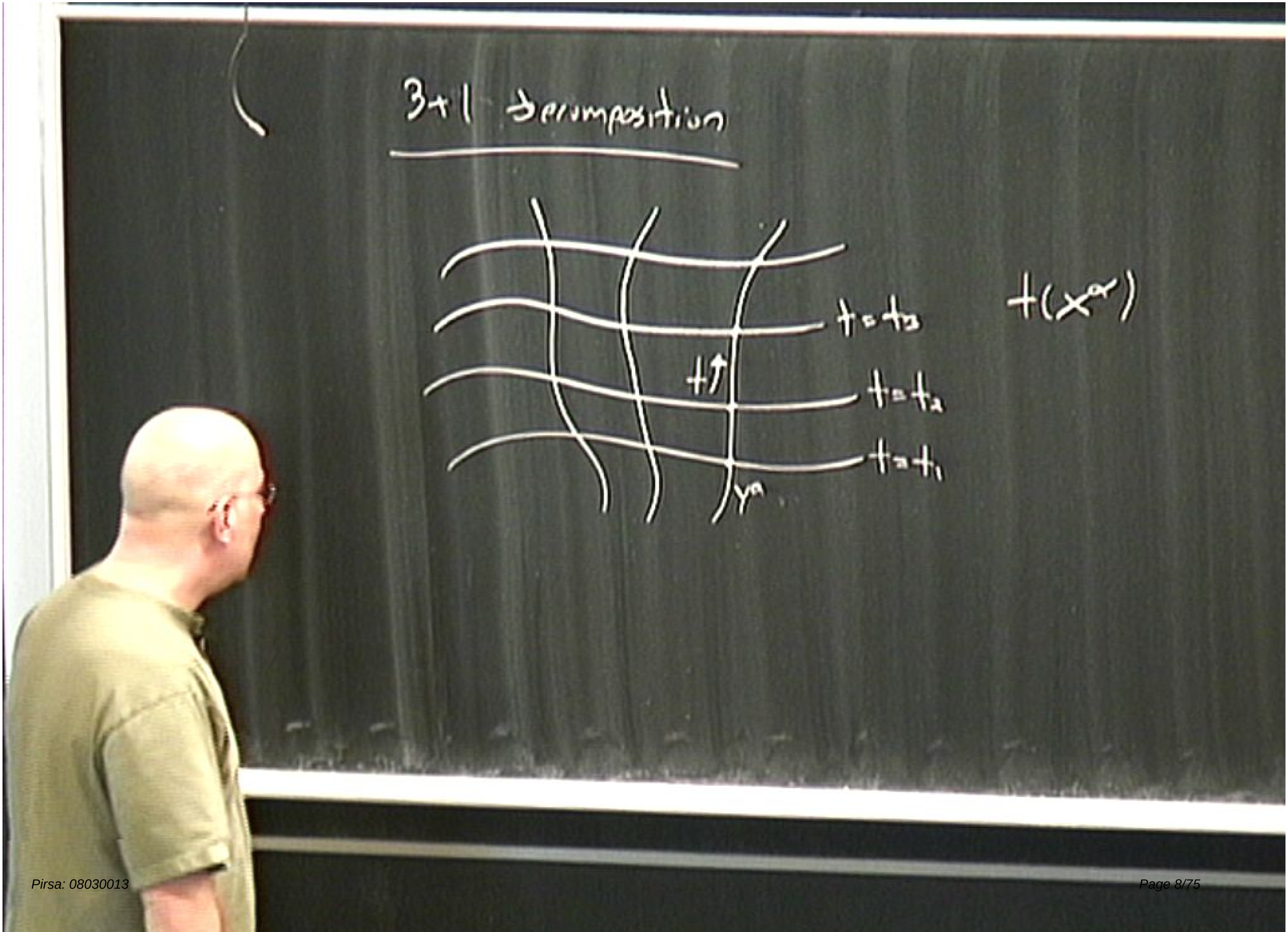

3+1 decomposition

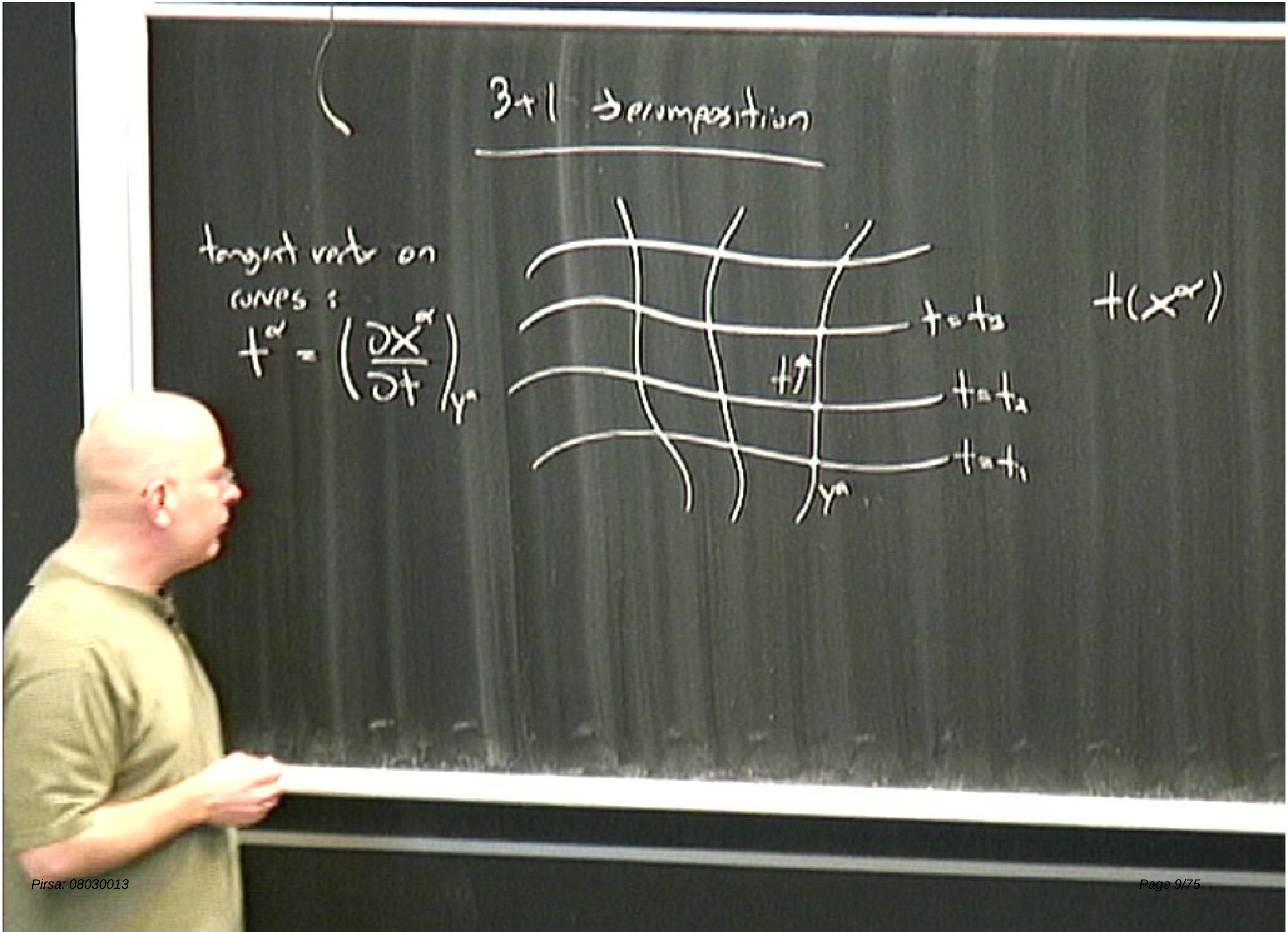

3+1 decomposition
t(x^α)

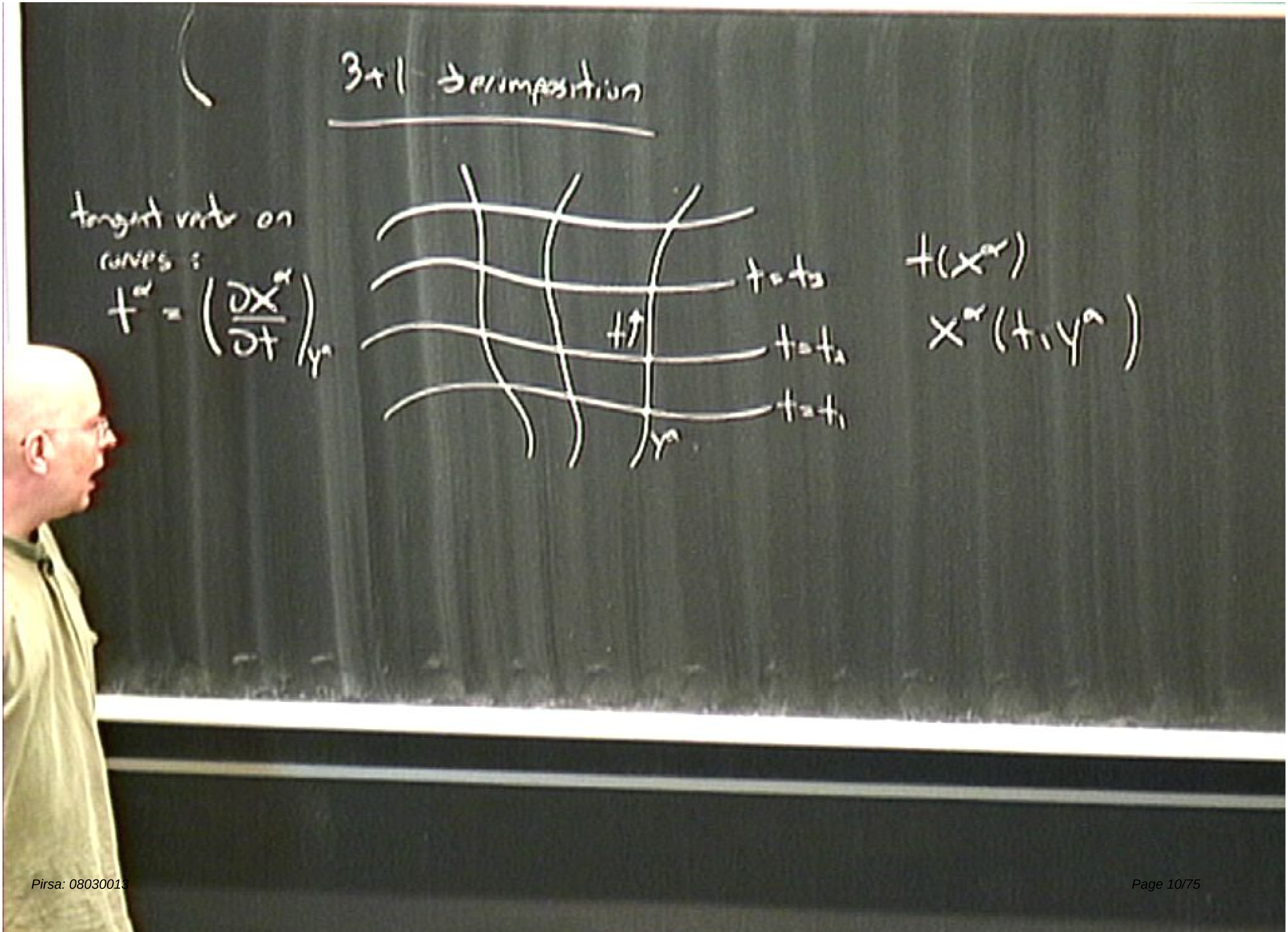

3+1 decomposition
t=t_3
t=t_2
t=t_1
t(x^α)


3+1 decomposition
t=t3
t=t2
t=t1
t(x^α)


3+1 decomposition
t = t3
t = t2
t = t1
t(x^α)


3+1 decomposition
t=t3
t=t2
t=t1
t(xα)
yα


3+1 decomposition
tangent vector on curves:
$t^\alpha = \left(\frac{\partial x^\alpha}{\partial t}\right)_{y^a}$
t^α
$t = t_3$
$t = t_2$
$t = t_1$
y^a
$t(x^\alpha)$

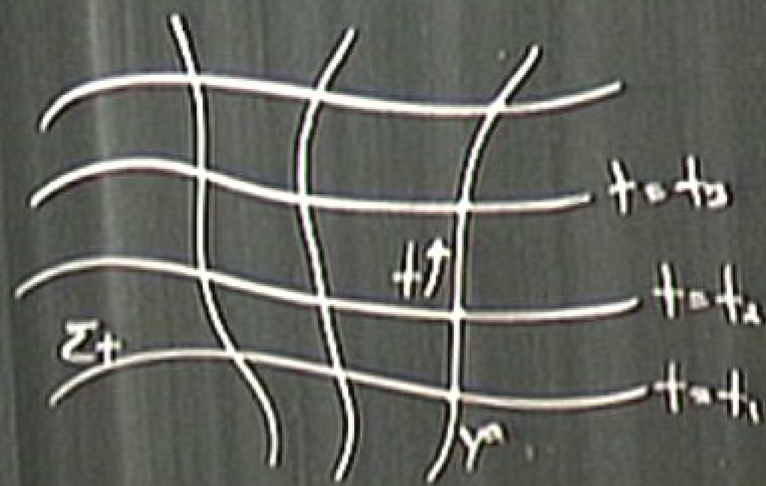

3+1 decomposition
tangent vector on curves:
$t^\alpha = \left(\frac{\partial X^\alpha}{\partial t}\right)_{y^a}$
t^α
$t = t_3$
$t = t_2$
$t = t_1$
y^a
$t(X^\alpha)$
$X^\alpha(t, y^a)$

3+1 Decomposition

tangent vector on curves:

$$t^\alpha = \left(\frac{\partial x^\alpha}{\partial t}\right)_{y^a}$$

tangent vectors on Σ_t



$t(x^\alpha)$

$x^\alpha(t, y^a)$

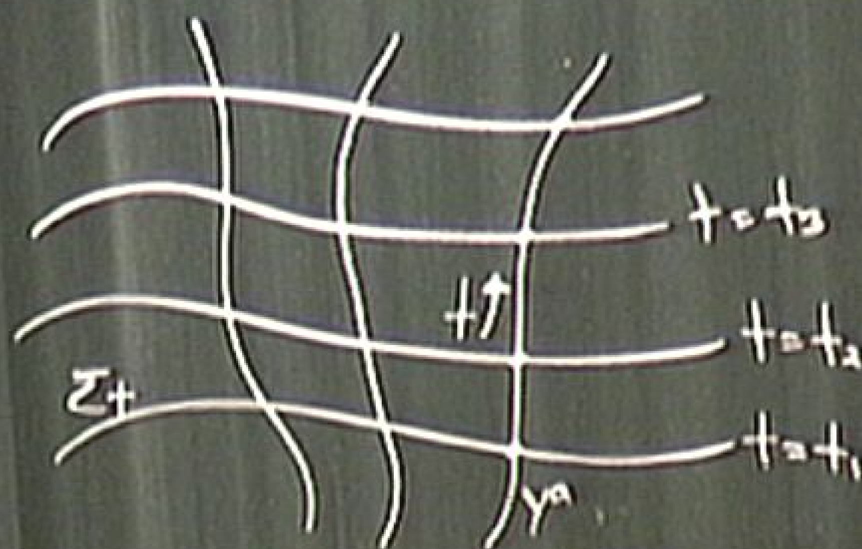
3+1 decomposition

tangent vector on curves:

$$t^\alpha = \left(\frac{\partial x^\alpha}{\partial t}\right)_{y^a}$$

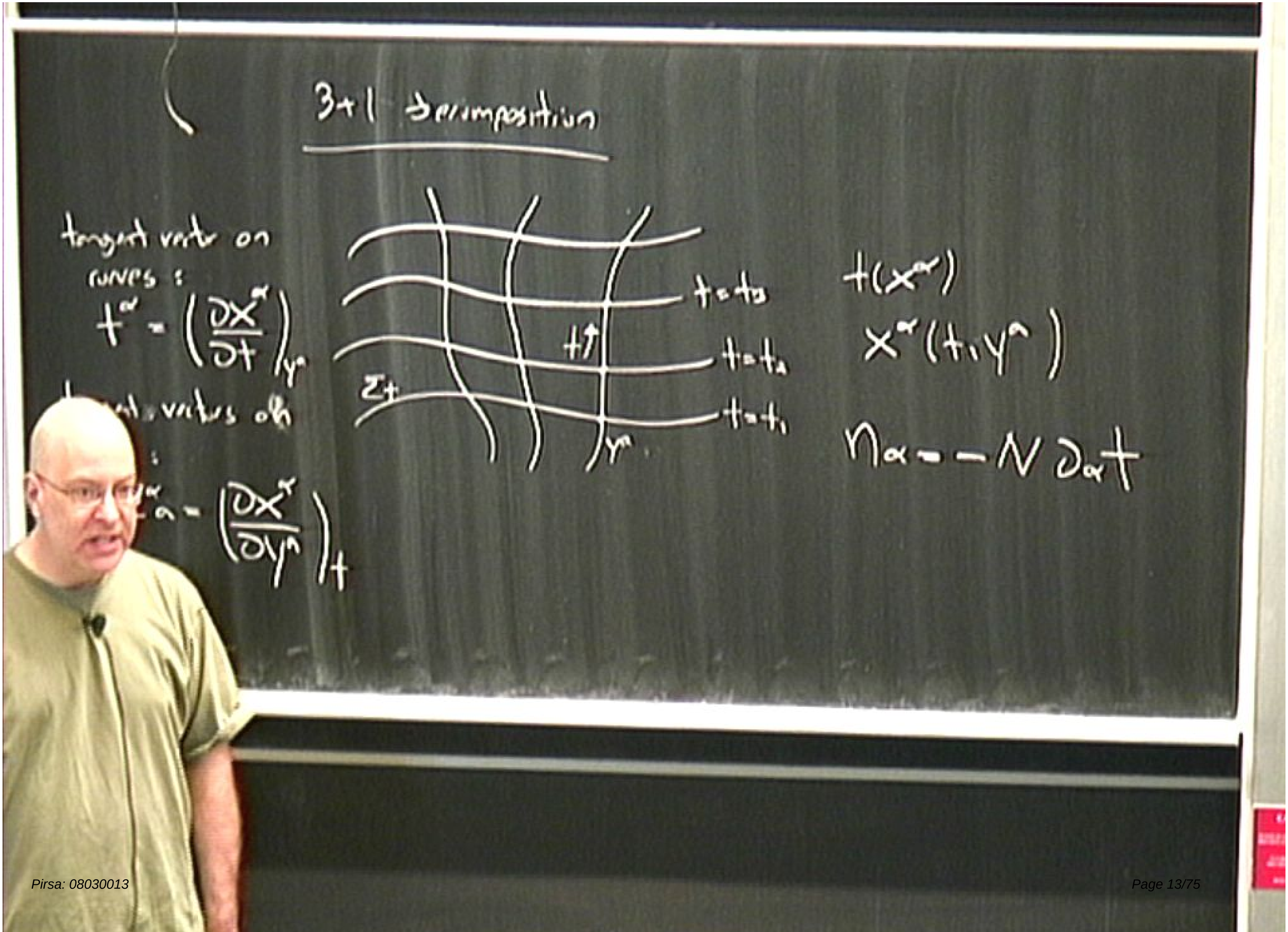
tangent vectors on Σ_t:

$$e^\alpha_a = \left(\frac{\partial x^\alpha}{\partial y^a}\right)_t$$



$t(x^\alpha)$

$x^\alpha(t, y^a)$


3+1 decomposition
tangent vector on curves:
$t^\alpha = \left(\frac{\partial x^\alpha}{\partial t}\right)_{y^a}$
tangent vectors on
$e^\alpha_a = \left(\frac{\partial x^\alpha}{\partial y^a}\right)_t$
Σ_t
$t = t_3$
$t = t_2$
$t = t_1$
t^α
y^a
$t(x^\alpha)$
$x^\alpha(t, y^a)$
$n_\alpha = -N \partial_\alpha t$

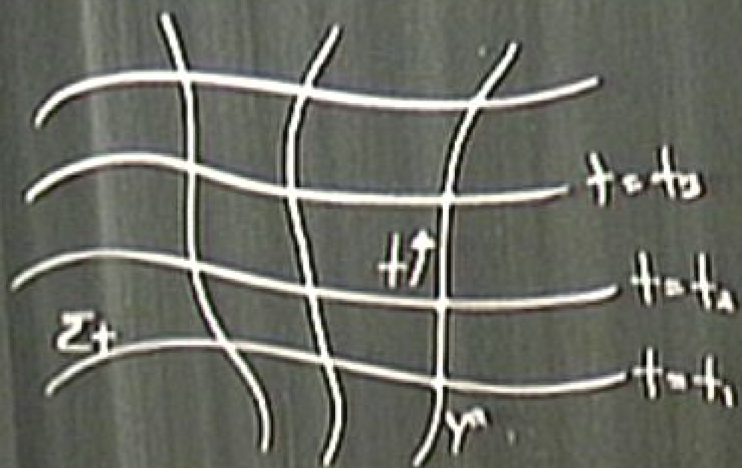
3+1 decomposition

tangent vector on curves:

$$t^\alpha = \left(\frac{\partial x^\alpha}{\partial t}\right)_{y^a}$$

tangent vectors on Σ_t:

$$e^\alpha_a = \left(\frac{\partial x^\alpha}{\partial y^a}\right)_t$$



$t(x^\alpha)$

$x^\alpha(t, y^a)$

$n_\alpha = -N\partial_\alpha t$

3+1 decomposition

tangent vector on curves:

$$t^\alpha = \left(\frac{\partial x^\alpha}{\partial t}\right)_{y^a}$$

tangent vectors of Σ_t:

$$e^\alpha_a = \left(\frac{\partial x^\alpha}{\partial y^a}\right)_t$$

$$t(x^\alpha)$$

$$x^\alpha(t, y^a)$$

$$n_\alpha = -N\,\partial_\alpha t$$

$$t^\alpha = N n^\alpha + N^a e^\alpha_a$$

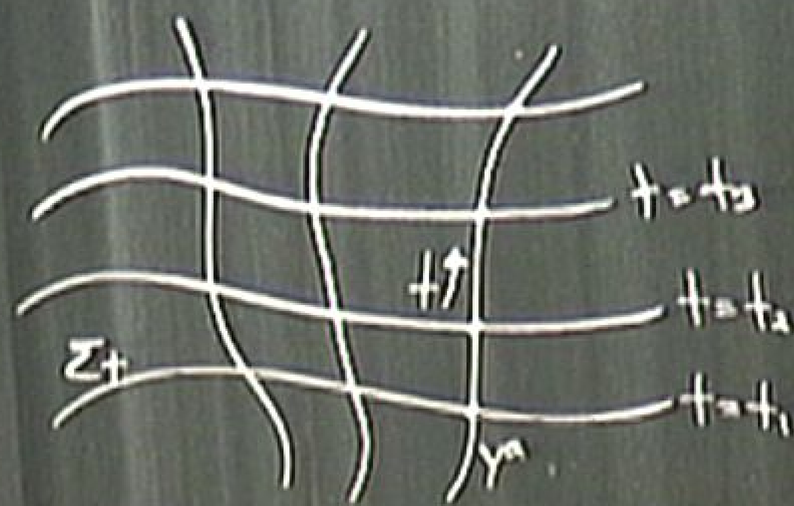
3+1 decomposition

tangent vector on curves:

$$t^\alpha = \left(\frac{\partial x^\alpha}{\partial t}\right)_{y^a}$$

tangent vectors on Σ_t:

$$e^\alpha_a = \left(\frac{\partial x^\alpha}{\partial y^a}\right)_t$$



$$t(x^\alpha)$$

$$x^\alpha(t, y^a)$$

$$n_\alpha = -N\,\partial_\alpha t$$

$$t^\alpha = \underbrace{N}_{\text{lapse}} n^\alpha + \underbrace{N^a}_{\text{shift}} e^\alpha_a$$

metric in new coordinates (t, y^a)

$$dx^\alpha = \frac{\partial x^\alpha}{\partial t}\, dt + \frac{\partial x^\alpha}{\partial y^a}\, dy^a$$

$$= t^\alpha\, dt + e^\alpha_a\, dy^a$$

$$= (N n^\alpha + N^a e^\alpha_a)\, dt + e^\alpha_a\, dy^a$$

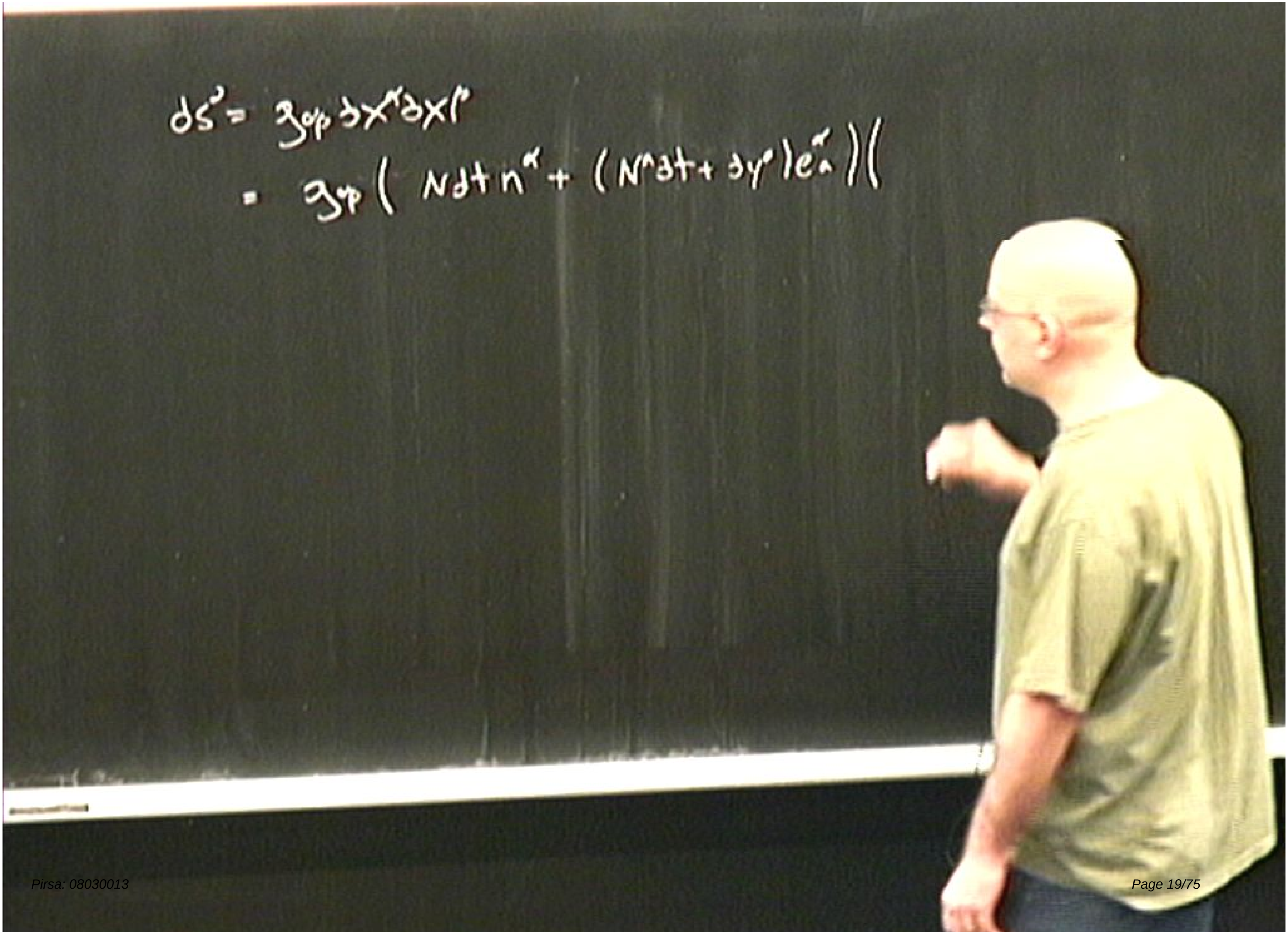
metric in new coordinates (t, y^a)

$$dx^\alpha = \frac{\partial x^\alpha}{\partial t}\, dt + \frac{\partial x^\alpha}{\partial y^a}\, dy^a$$

$$= t^\alpha\, dt + e^\alpha_a\, dy^a$$

$$= (N n^\alpha + N^a e^\alpha_a)\, dt + e^\alpha_a\, dy^a$$

$$= (N dt)\, n^\alpha + (N^a dt + dy^a)\, e^\alpha_a$$


$ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta$
$= g_{\alpha\beta} \left(N dt\, n^\alpha + (N^a dt + dy^a) e^\alpha_a \right)($

$$ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta$$

$$= g_{\alpha\beta}\Big(N dt\, n^\alpha + (N^a dt + dy^a) e^\alpha_a \Big)\Big(N dt\, n^\beta + (N^b dt + dy^b) e^\beta_b \Big)$$

$$ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta$$

$$= g_{\alpha\beta} \left(N dt\, n^\alpha + (N^a dt + dy^a) e^\alpha_a \right)\left(N dt\, n^\beta + (N^b dt + dy^b) e^\beta_b \right)$$

$$= N^2 dt^2$$

$$ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta$$

$$= g_{\alpha\beta}\left(N dt\, n^\alpha + (N^a dt + dy^a) e^\alpha_a \right)\left(N dt\, n^\beta + (N^b dt + dy^b) e^\beta_b \right)$$

$$= N^2 dt^2$$

$$ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta$$

$$= g_{\alpha\beta}\left(N\,dt\, n^\alpha + (N^a dt + dy^a)\, e^\alpha_a \right)\left(N\,dt\, n^\beta + (N^b dt + dy^b)\, e^\beta_b \right)$$

$$\boxed{ds^2 = -N^2 dt^2 + h_{ab}\,(dy^a + N^a dt)(dy^b + N^b dt)}$$

$$ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta$$

$$= g_{\alpha\beta}\left(N dt\, n^\alpha + (N^a dt + dy^a) e^\alpha_a \right)\left(N dt\, n^\beta + (N^b dt + dy^b) e^\beta_b \right)$$

$$\boxed{ds^2 = -N^2 dt^2 + h_{ab}\,(dy^a + N^a dt)(dy^b + N^b dt)}$$

$$ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta$$

$$= g_{\alpha\beta}\left(N\,dt\, n^\alpha + (N^a dt + dy^a) e^\alpha_a \right)\left(N\,dt\, n^\beta + (N^b dt + dy^b) e^\beta_b \right)$$

$$\boxed{ds^2 = -N^2 dt^2 + h_{ab}\,(dy^a + N^a dt)(dy^b + N^b dt)}$$

$$\sqrt{-g}\, d^4x =$$

$$ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta$$

$$= g_{\alpha\beta} \left(N dt\, n^\alpha + (N^a dt + dy^a) e^\alpha_a \right) \left(N dt\, n^\beta + (N^b dt + dy^b) e^\beta_b \right)$$

$$\boxed{ds^2 = -N^2 dt^2 + h_{ab} (dy^a + N^a dt)(dy^b + N^b dt)}$$

$$\sqrt{-g}\, d^4x =$$

$$ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta$$

$$= g_{\alpha\beta}\left(N dt\, n^\alpha + (N^a dt + dy^a) e^\alpha_a \right)\left(N dt\, n^\beta + (N^b dt + dy^b) e^\beta_b \right)$$

$$\boxed{ds^2 = -N^2 dt^2 + h_{ab}\,(dy^a + N^a dt)(dy^b + N^b dt)}$$

$$\sqrt{-g}\, d^4x = N\sqrt{h}$$

$$ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta$$

$$= g_{\alpha\beta}\left(N dt\, n^\alpha + (N^a dt + dy^a) e^\alpha_a \right)\left(N dt\, n^\beta + (N^b dt + dy^b) e^\beta_b \right)$$

$$\boxed{ds^2 = -N^2 dt^2 + h_{ab} (dy^a + N^a dt)(dy^b + N^b dt)}$$

$$\boxed{\sqrt{-g}\, d^4x = N \sqrt{h}\, d^3y\, dt}$$

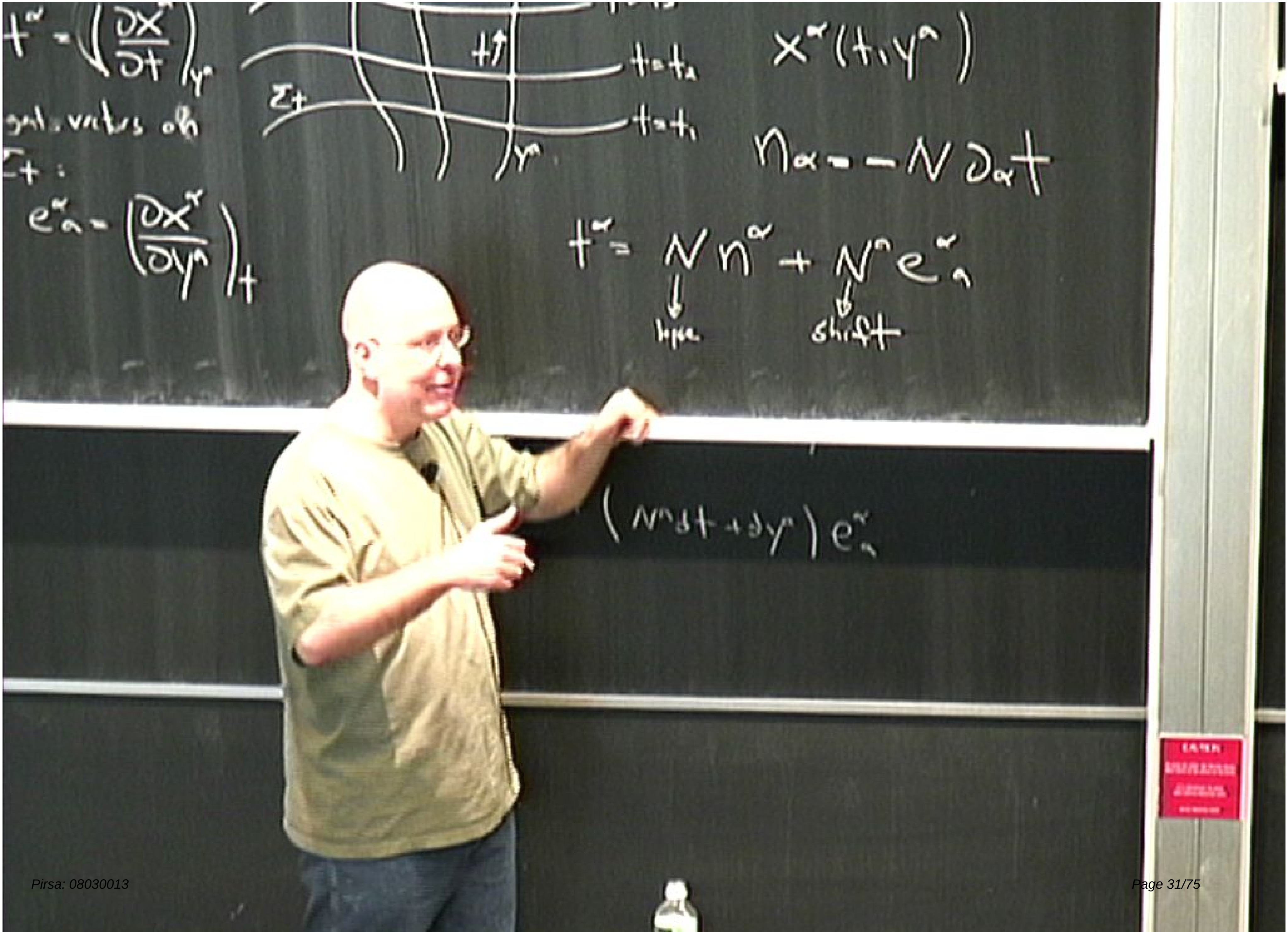
scalar field

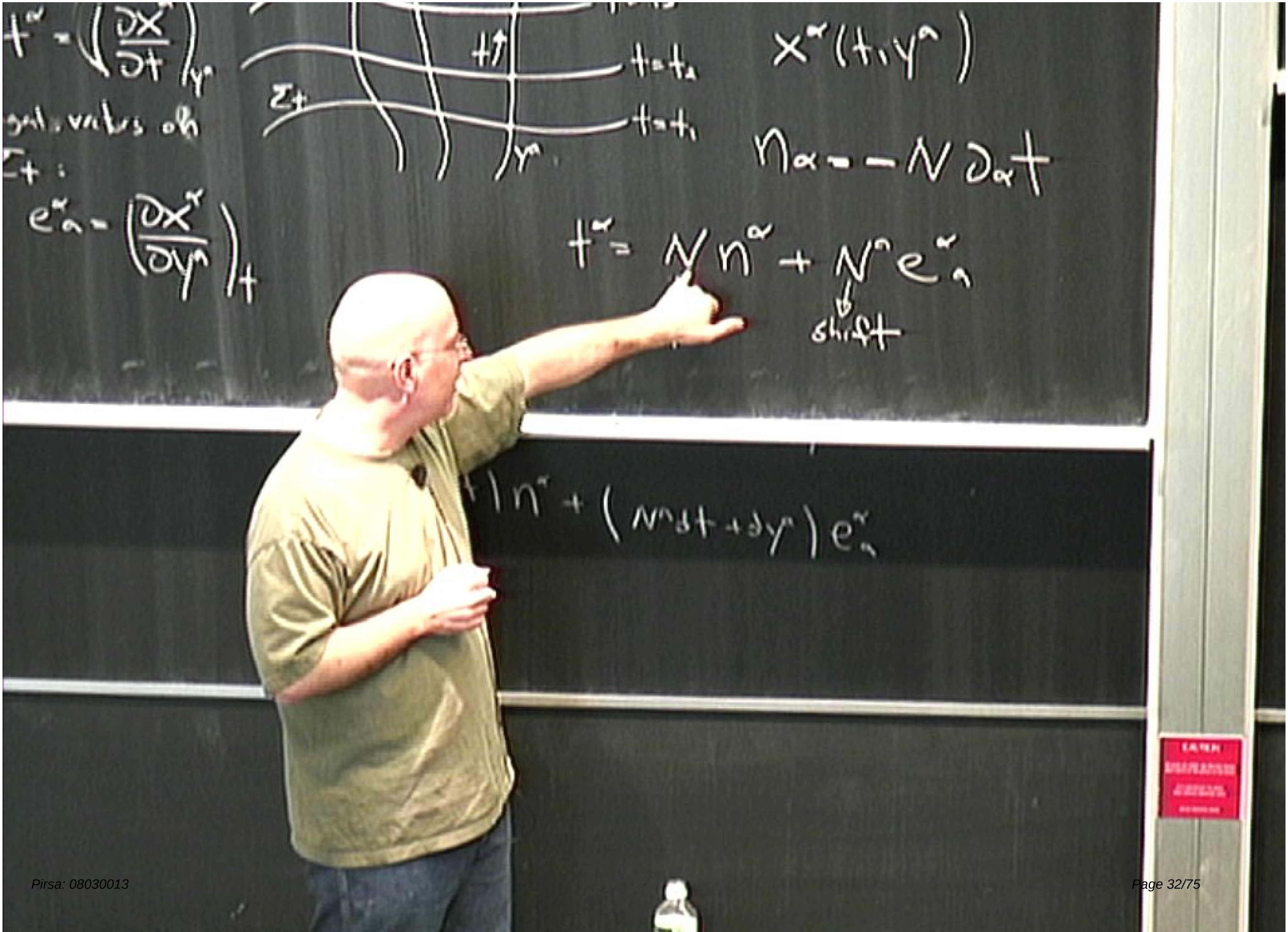
$$\mathcal{L}(q, \partial_\alpha q)$$

scalar field

$$\mathcal{L}(q, \partial_\alpha q)$$

$$\dot{q} \equiv (\partial_\alpha q) t^\alpha$$


t^α = (∂x^α/∂t)_{y^a}
x^α(t, y^a)
Σ_t
t = t_2
t = t_1
n_α = −N ∂_α t
e^α_a = (∂x^α/∂y^a)_t
t^α = N n^α + N^a e^α_a
lapse
shift
(N^a dt + dy^a) e^α_a


$t^\alpha = \left(\frac{\partial x^\alpha}{\partial t}\right)_{y^a}$
Σ_t
$t = t_2$
$t = t_1$
$x^\alpha(t, y^a)$
$n_\alpha = -N \partial_\alpha t$
$e^\alpha_a = \left(\frac{\partial x^\alpha}{\partial y^a}\right)_t$
$t^\alpha = N n^\alpha + N^a e^\alpha_a$
shift
$n^\alpha + (N^a dt + dy^a) e^\alpha_a$

Scalar field

$$\mathcal{L}(q, \partial_\alpha q)$$

$$\dot{q} \equiv (\partial_\alpha q) t^\alpha$$

$$q_n \equiv (\partial_\alpha q) e^\alpha_n$$

Scalar field

$$\mathcal{L}(q, \partial_\alpha q)$$

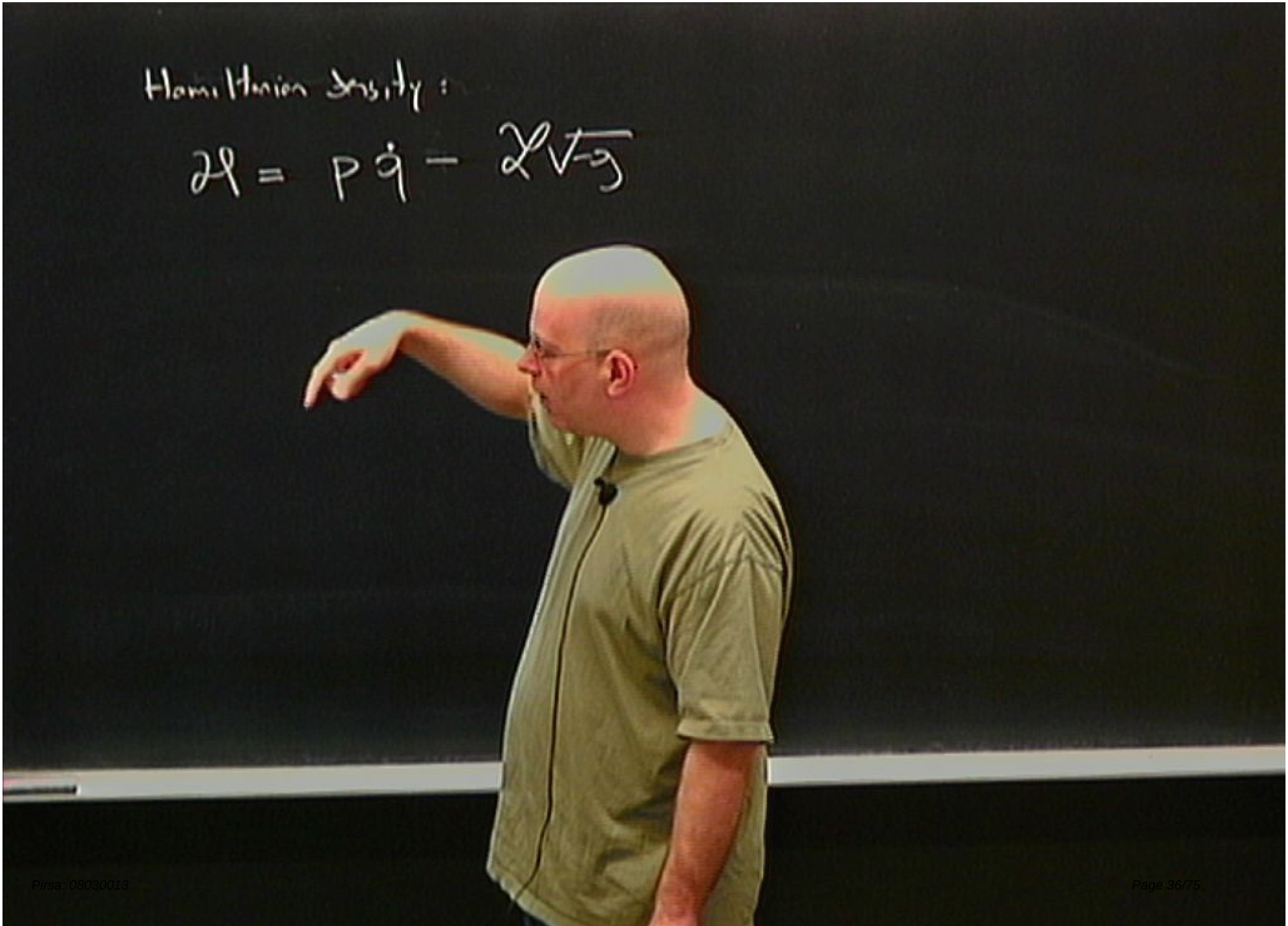
$$\left.\begin{aligned} \dot{q} &\equiv (\partial_\alpha q)\, t^\alpha \\ q_n &\equiv (\partial_\alpha q)\, e^\alpha_n \end{aligned}\right\} \mathcal{L}(q, \dot{q}, q_n)$$

scalar field

$$\mathcal{L}(q, \partial_\alpha q)$$

$$\left.\begin{aligned} \dot{q} &= (\partial_\alpha q) t^\alpha \\ q_n &= (\partial_\alpha q) e^\alpha_n \end{aligned}\right\} \mathcal{L}(q, \dot{q}, q_n)$$

$$p = \frac{\partial}{\partial \dot{q}} \left(\mathcal{L}\sqrt{-g} \right)$$


Hamiltonian density:
$\mathcal{H} = p\dot{q} - \mathcal{L}\sqrt{-g}$

Hamiltonian density :

$$\mathcal{H} = p\dot{q} - \mathcal{L}\sqrt{-g}$$

a function of q, p, q_a

Hamiltonian :

$$H = \int_{\Sigma_+} \mathcal{H} \, d^3y$$

Hamiltonian density:

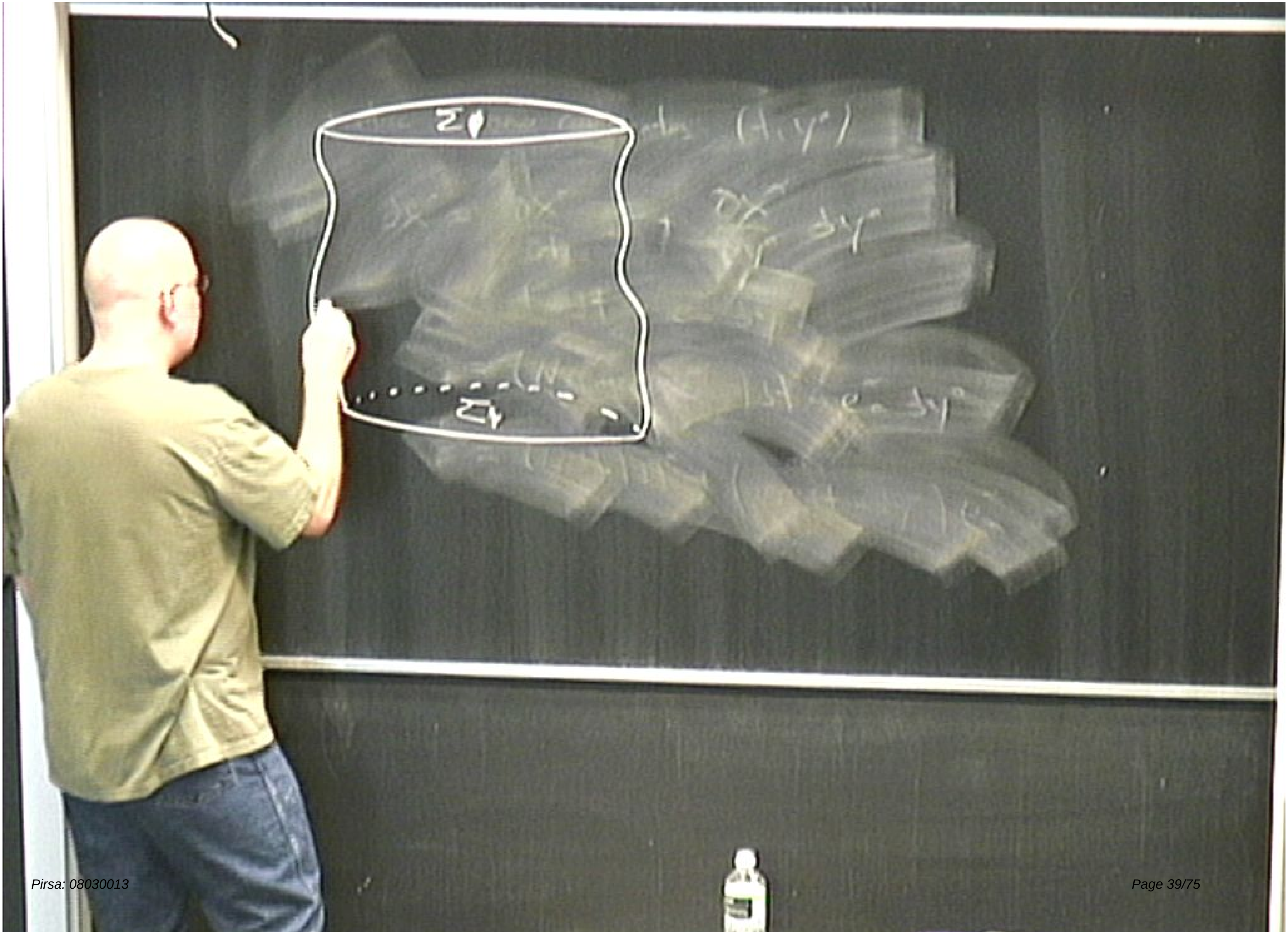
$$\mathcal{H} = p\dot{q} - \mathcal{L}\sqrt{-g}$$

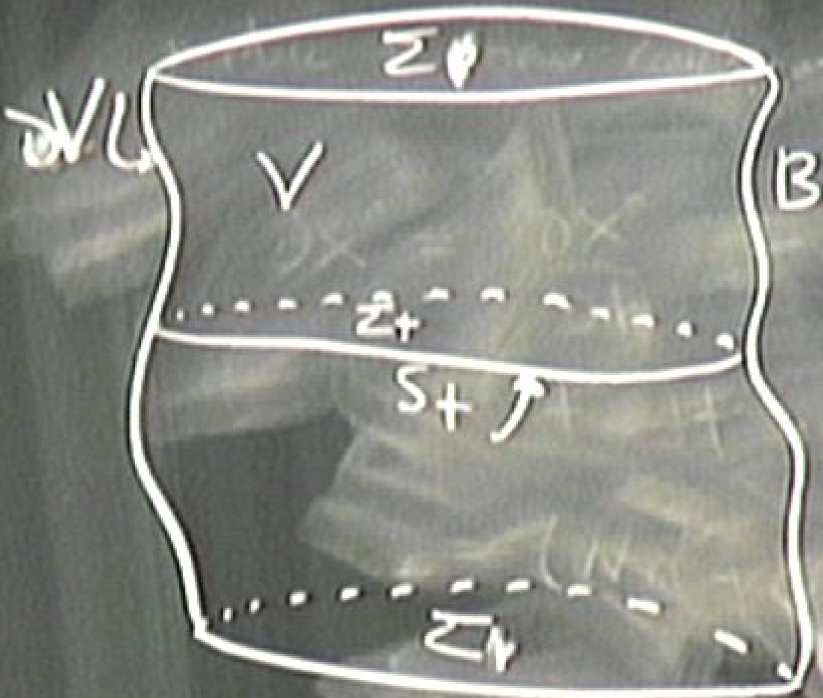
↳ a function of q, p, q_a

Hamiltonian:

$$H = \int_{\Sigma_t} \mathcal{H}\, d^3y$$

↳ a functional of q, p on each Σ_t.





$$\partial V = \Sigma_1 \cup \Sigma_2 \cup B$$

V foliated by Σ_t

B foliated by S_t

$$S_t = \partial \Sigma_t$$

Σ_t embedded in $\mathcal{M}$

$$t(x^\alpha) = \text{const}$$

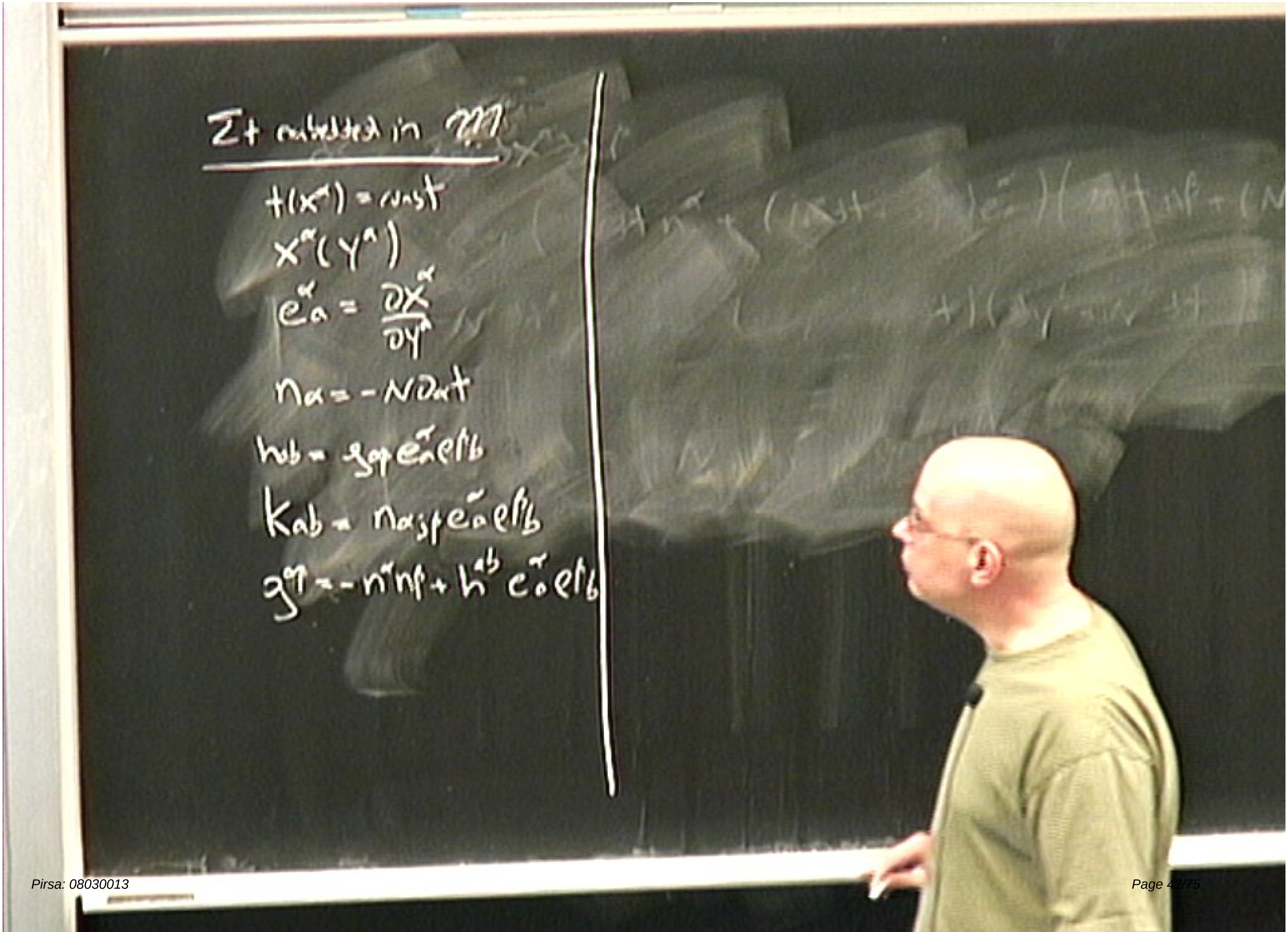
$$x^\alpha(y^a)$$

$$e^\alpha_a = \frac{\partial x^\alpha}{\partial y^a}$$

$$n_\alpha = -N \partial_\alpha t$$

$$h_{ab} = g_{\alpha\beta} e^\alpha_a e^\beta_b$$

$$K_{ab} = n_{\alpha;\beta} e^\alpha_a e^\beta_b$$


Σ_t embedded in $\mathcal{M}$
$t(x^\alpha) = \text{const}$
$x^\alpha(y^a)$
$e^\alpha_a = \frac{\partial x^\alpha}{\partial y^a}$
$n_\alpha = -N \partial_\alpha t$
$h_{ab} = g_{\alpha\beta} e^\alpha_a e^\beta_b$
$K_{ab} = n_{\alpha;\beta} e^\alpha_a e^\beta_b$
$g^{\alpha\beta} = -n^\alpha n^\beta + h^{ab} e^\alpha_a e^\beta_b$

Σ_t embedded in $\mathcal{M}$

$t(x^\alpha) = \text{const}$

$x^\alpha(y^a)$

$e^\alpha_a = \dfrac{\partial x^\alpha}{\partial y^a}$

$n_\alpha = -N \partial_\alpha t$

$h_{ab} = g_{\alpha\beta} e^\alpha_a e^\beta_b$

$K_{ab} = n_{\alpha;\beta} e^\alpha_a e^\beta_b$

$g^{\alpha\beta} = -n^\alpha n^\beta + h^{ab} e^\alpha_a e^\beta_b$

S_t embedded in Σ_t

Σ_t embedded in $\mathcal{M}$

$t(x^\alpha) = \text{const}$

$x^\alpha(y^a)$

$e^\alpha_a = \dfrac{\partial x^\alpha}{\partial y^a}$

$n_\alpha = -N \partial_\alpha t$

$h_{ab} = g_{\alpha\beta} e^\alpha_a e^\beta_b$

$K_{ab} = n_{\alpha;\beta} e^\alpha_a e^\beta_b$

$g^{\alpha\beta} = -n^\alpha n^\beta + h^{ab} e^\alpha_a e^\beta_b$

S_t embedded in Σ_t

$\varphi(y^a) = \text{const}$

$y^a($

Σ_t embedded in $\mathcal{M}$

$t(x^\alpha) = \text{const}$

$x^\alpha(y^a)$

$e^\alpha_a = \dfrac{\partial x^\alpha}{\partial y^a}$

$n_\alpha = -N \partial_\alpha t$

$h_{ab} = g_{\alpha\beta} e^\alpha_a e^\beta_b$

$K_{ab} = n_{\alpha;\beta} e^\alpha_a e^\beta_b$

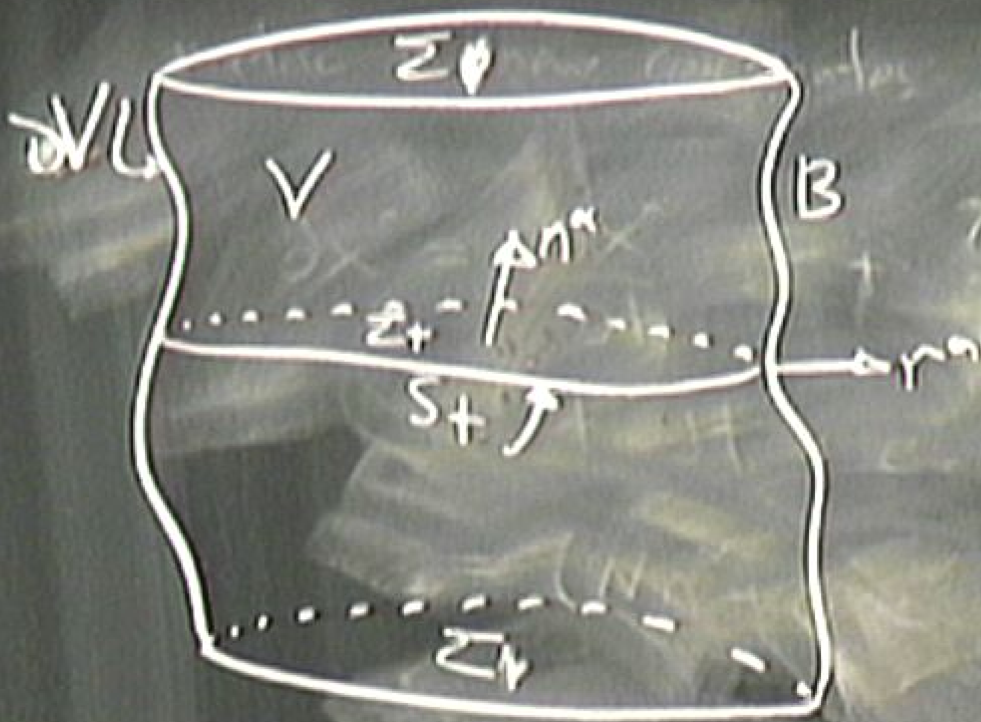
$g^{\alpha\beta} = -n^\alpha n^\beta + h^{ab} e^\alpha_a e^\beta_b$

S_t embedded in Σ_t

$\varphi(y^a) = \text{const}$

$y^a(\theta^A)$

$e^a_A = \dfrac{\partial y^a}{\partial \theta^A}$

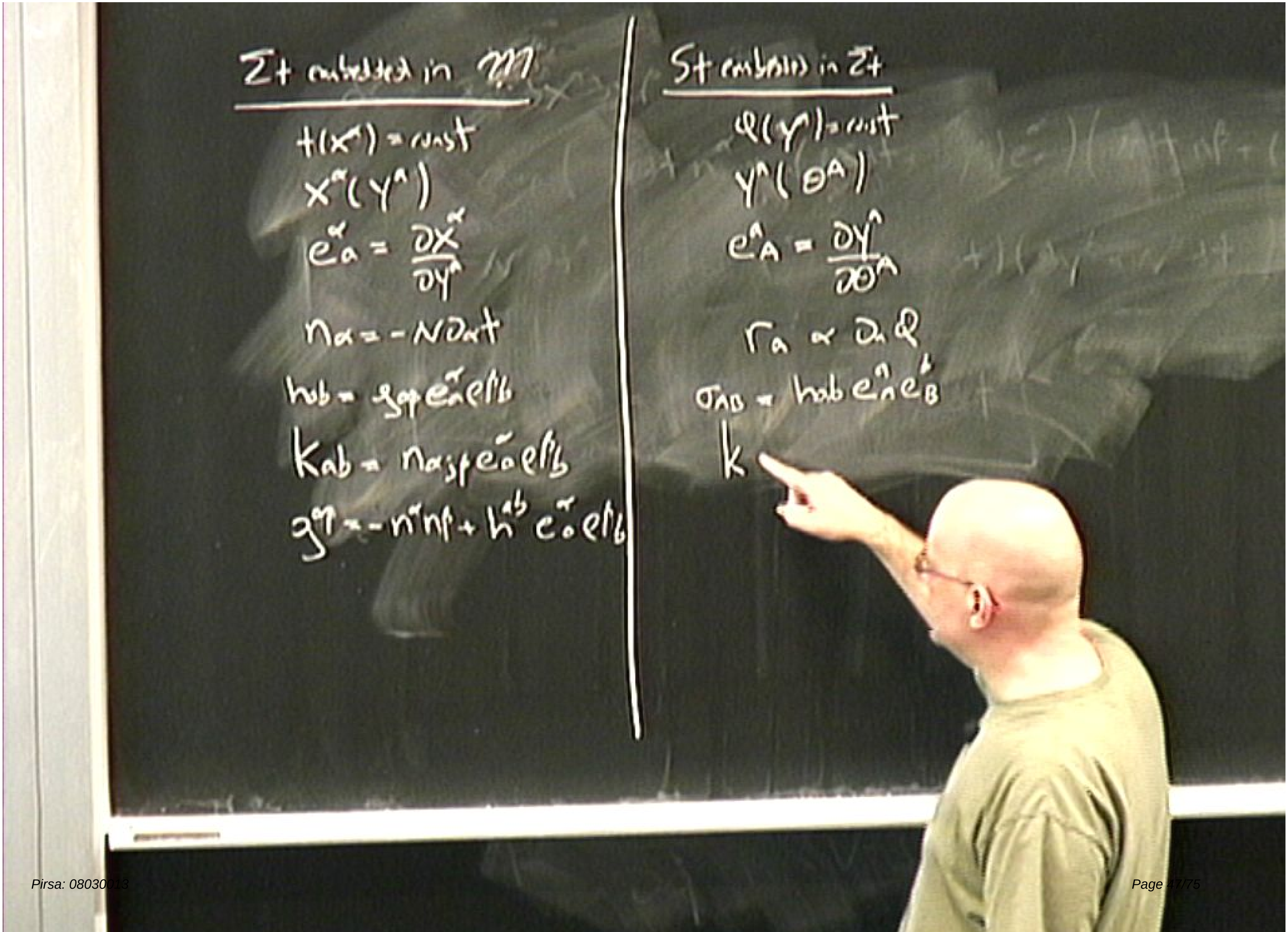


$\partial V = \Sigma_1 \cup \Sigma_2 \cup B$

V foliated by Σ_t

B foliated by S_t

$S_t = \partial \Sigma_t$


Σ_t embedded in $\mathcal{M}$
$t(x^\alpha) = \text{const}$
$x^\alpha(y^a)$
$e^\alpha_a = \frac{\partial x^\alpha}{\partial y^a}$
$n_\alpha = -N \partial_\alpha t$
$h_{ab} = g_{\alpha\beta} e^\alpha_a e^\beta_b$
$K_{ab} = n_{\alpha;\beta} e^\alpha_a e^\beta_b$
$g^{\alpha\beta} = -n^\alpha n^\beta + h^{ab} e^\alpha_a e^\beta_b$
S_t embedded in Σ_t
$\Phi(y^a) = \text{const}$
$y^a(\theta^A)$
$e^a_A = \frac{\partial y^a}{\partial \theta^A}$
$r_a \propto \partial_a \Phi$
$\sigma_{AB} = h_{ab} e^a_A e^b_B$
k

Σ_t embedded in $\mathcal{M}$

$t(x^\alpha) = \text{const}$

$x^\alpha(y^a)$

$e^\alpha_a = \frac{\partial x^\alpha}{\partial y^a}$

$n_\alpha = -N \partial_\alpha t$

$h_{ab} = g_{\alpha\beta} e^\alpha_a e^\beta_b$

$K_{ab} = n_{\alpha;\beta} e^\alpha_a e^\beta_b$

$g^{\alpha\beta} = -n^\alpha n^\beta + h^{ab} e^\alpha_a e^\beta_b$

S_t embedded in Σ_t

$\Phi(y^a) = \text{const}$

$y^a(\theta^A)$

$e^a_A = \frac{\partial y^a}{\partial \theta^A}$

$r_a \propto \partial_a \Phi$

$\sigma_{AB} = h_{ab} e^a_A e^b_B$

$k_{AB} = r_{a|b}$

Σ_t embedded in $\mathcal{M}$

$$t(x^\alpha) = \text{const}$$

$$x^\alpha(y^a)$$

$$e^\alpha_a = \frac{\partial x^\alpha}{\partial y^a}$$

$$n_\alpha = -N \partial_\alpha t$$

$$h_{ab} = g_{\alpha\beta} e^\alpha_a e^\beta_b$$

$$K_{ab} = n_{\alpha;\beta} e^\alpha_a e^\beta_b$$

$$g^{\alpha\beta} = -n^\alpha n^\beta + h^{ab} e^\alpha_a e^\beta_b$$

S_t embedded in Σ_t

$$\Phi(y^a) = \text{const}$$

$$y^a(\theta^A)$$

$$e^a_A = \frac{\partial y^a}{\partial \theta^A}$$

$$r_a \propto \partial_a \Phi$$

$$\sigma_{AB} = h_{ab} e^a_A e^b_B$$

$$k_{AB} = r_{a|b} e^a_A e^b_B$$

$$h^{ab} = r^a r^b + \sigma^{AB} e^a_A e^b_B$$

St embedded in $\mathcal{M}$

$$\psi(x^\alpha) = \text{const}$$

$$x^\alpha(\theta^A) = x^\alpha(y^a(\theta^A))$$

$$e^\alpha_A = \frac{\partial x^\alpha}{\partial \theta^A} = \frac{\partial x^\alpha}{\partial y^a}\frac{\partial y^a}{\partial \theta^A}$$

$$e^a_A e^b_B$$

$$-\ldots e^a_A e^b_B$$

St embedded in $\mathcal{M}$

$$\Psi(x^\alpha) = \text{const}$$

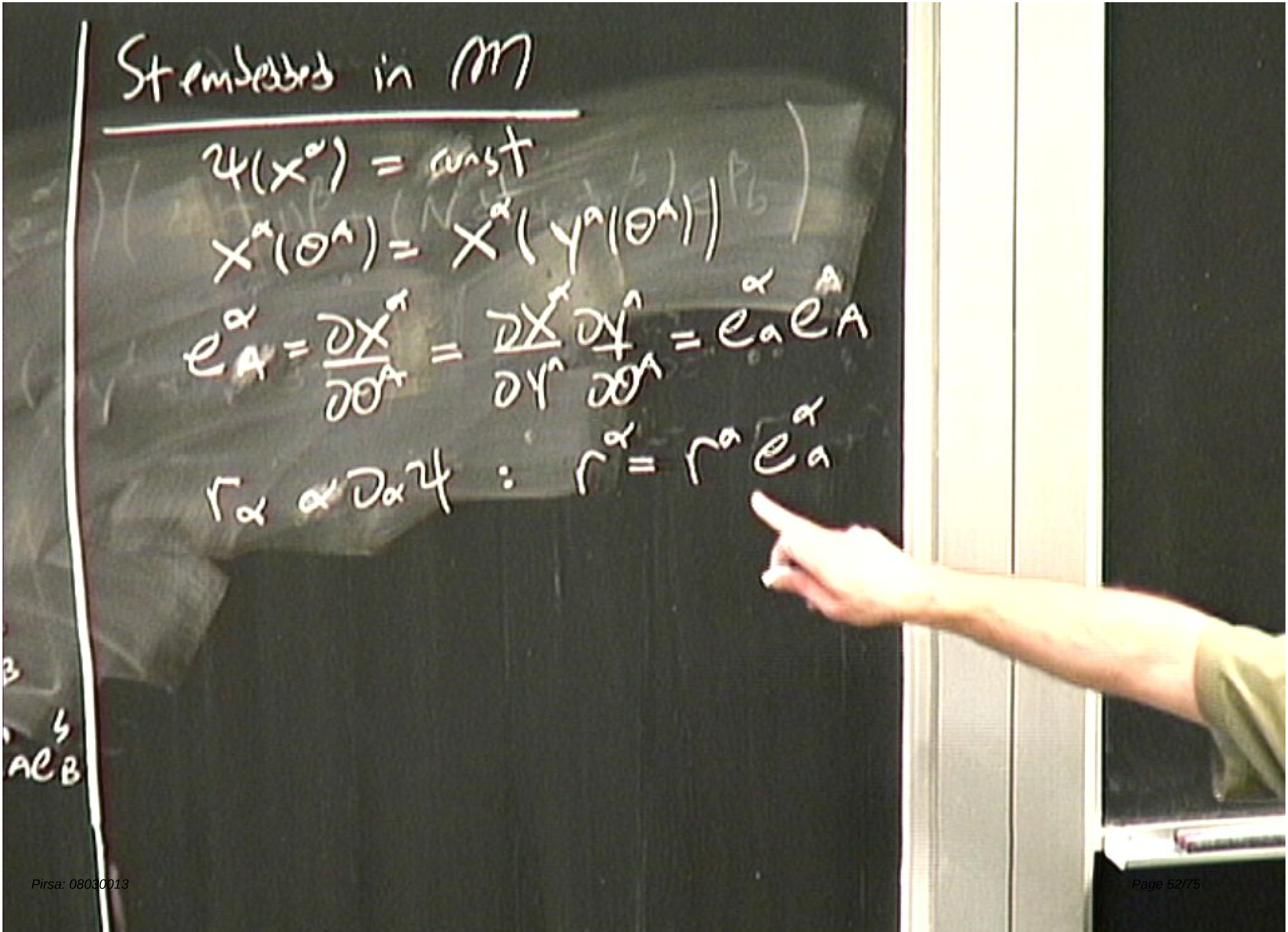
$$x^\alpha(\theta^A) = x^\alpha(y^a(\theta^A))$$

$$e^\alpha_A = \frac{\partial x^\alpha}{\partial \theta^A} = \frac{\partial x^\alpha}{\partial y^a}\frac{\partial y^a}{\partial \theta^A} = e^\alpha_a e^a_A$$

$$r_\alpha$$

$$e^a_A e^b_B$$

$$_{AB}\, e^a_A e^b_B$$



St embedded in M
$\psi(x^\alpha) = \text{const}$
$x^\alpha(\theta^A) = x^\alpha(y^a(\theta^A))$
$e^\alpha_A = \frac{\partial x^\alpha}{\partial \theta^A} = \frac{\partial x^\alpha}{\partial y^a}\frac{\partial y^a}{\partial \theta^A} = e^\alpha_a e^a_A$
$r_\alpha \propto \partial_\alpha \psi \ : \ r^\alpha = r^a e^\alpha_a$

S_t embedded in Σ_t

$$\Phi(y^a) = \text{const}$$

$$y^a(\theta^A)$$

$$e^a_A = \frac{\partial y^a}{\partial \theta^A}$$

$$r_a \propto D_a \Phi$$

$$\sigma_{AB} = h_{ab} e^a_A e^b_B$$

$$k_{AB} = r_{a|b} e^a_A e^b_B$$

$$h^{ab} = r^a r^b + \sigma^{AB} e^a_A e^b_B$$

S_t embedded in $\mathcal{M}$

$$\Psi(x^\alpha) = \text{const}$$

$$x^\alpha(\theta^A) = x^\alpha(y^a(\theta^A))$$

$$e^\alpha_A = \frac{\partial x^\alpha}{\partial \theta^A} = \frac{\partial x^\alpha}{\partial y^a}\frac{\partial y^a}{\partial \theta^A} = e^\alpha_a e^a_A$$

$$r_\alpha \propto D_\alpha \Psi \; : \; r^\alpha = r^a e^\alpha_a$$

$$\sigma_{AB} = g_{\alpha\beta} e^\alpha_A e^\beta_B$$

$$= g_{\alpha\beta}(e^\alpha_a e^a_A)(e^\beta_b e^b_B)$$

Σ_t embedded in $\mathcal{M}$

$t(x^\alpha) = \text{const}$

$x^\alpha(y^a)$

$e^\alpha_a = \frac{\partial x^\alpha}{\partial y^a}$

$n_\alpha = -N \partial_\alpha t$

$h_{ab} = g_{\alpha\beta} e^\alpha_a e^\beta_b$

$K_{ab} = n_{\alpha;\beta} e^\alpha_a e^\beta_b$

$g^{\alpha\beta} = -n^\alpha n^\beta + h^{ab} e^\alpha_a e^\beta_b$

S_t embedded in Σ_t

$\varphi(y^a) = \text{const}$

$y^a(\theta^A)$

$e^a_A = \frac{\partial y^a}{\partial \theta^A}$

$r_a \propto \partial_a \varphi$

$\sigma_{AB} = h_{ab} e^a_A e^b_B$

$k_{AB} = r_{a|b} e^a_A e^b_B$

$h^{ab} = r^a r^b + \sigma^{AB} e^a_A e^b_B$

S_t embedded in $\mathcal{M}$

$\psi(x^\alpha) = \text{const}$

$x^\alpha(\theta^A) = x^\alpha(y^a(\theta^A))$

$e^\alpha_A = \frac{\partial x^\alpha}{\partial \theta^A} = \frac{\partial x^\alpha}{\partial y^a}\frac{\partial y^a}{\partial \theta^A} = e^\alpha_a e^a_A$

$r_\alpha \propto \partial_\alpha \psi \; : \; r^\alpha = r^a e^\alpha_a$

$\sigma_{AB} = g_{\alpha\beta} e^\alpha_A e^\beta_B$

$= g_{\alpha\beta}(e^\alpha_a e^a_A)(e^\beta_b e^b_B)$

$= h_{ab} e^a_A e^b_B$

$g^{\alpha\beta} = -n^\alpha n^\beta + r^\alpha r^\beta + \sigma^{AB} e^\alpha_A e^\beta_B$

Σ_t

$\ldots \mathrm{const}$

$\ldots)$

$\ldots \frac{\partial y^a}{\partial \theta^A}$

$\partial_a \varphi$

$\ldots e^a_A e^b_B$

$r_{a|b}\, e^a_A e^b_B$

$\ldots r^b + \sigma^{AB} e^a_A e^b_B$

S_t embedded in $\mathcal{M}$

$$\Psi(x^\alpha) = \mathrm{const}$$

$$x^\alpha(\theta^A) = x^\alpha(y^a(\theta^A))$$

$$e^\alpha_A = \frac{\partial x^\alpha}{\partial \theta^A} = \frac{\partial x^\alpha}{\partial y^a}\frac{\partial y^a}{\partial \theta^A} = e^\alpha_a e^a_A$$

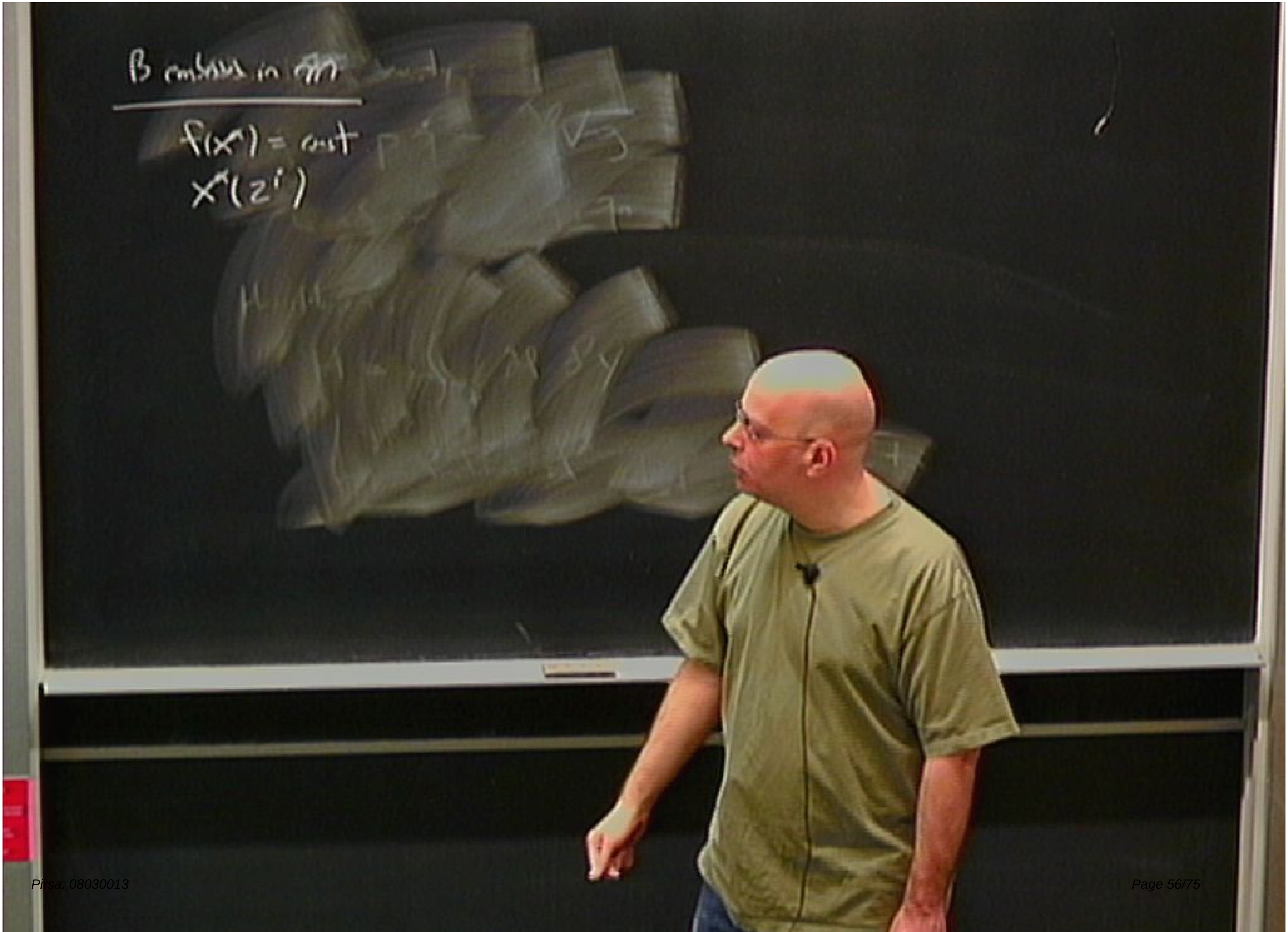
$$r_\alpha \propto \partial_\alpha \Psi \quad : \quad r^\alpha = r^a e^\alpha_a$$

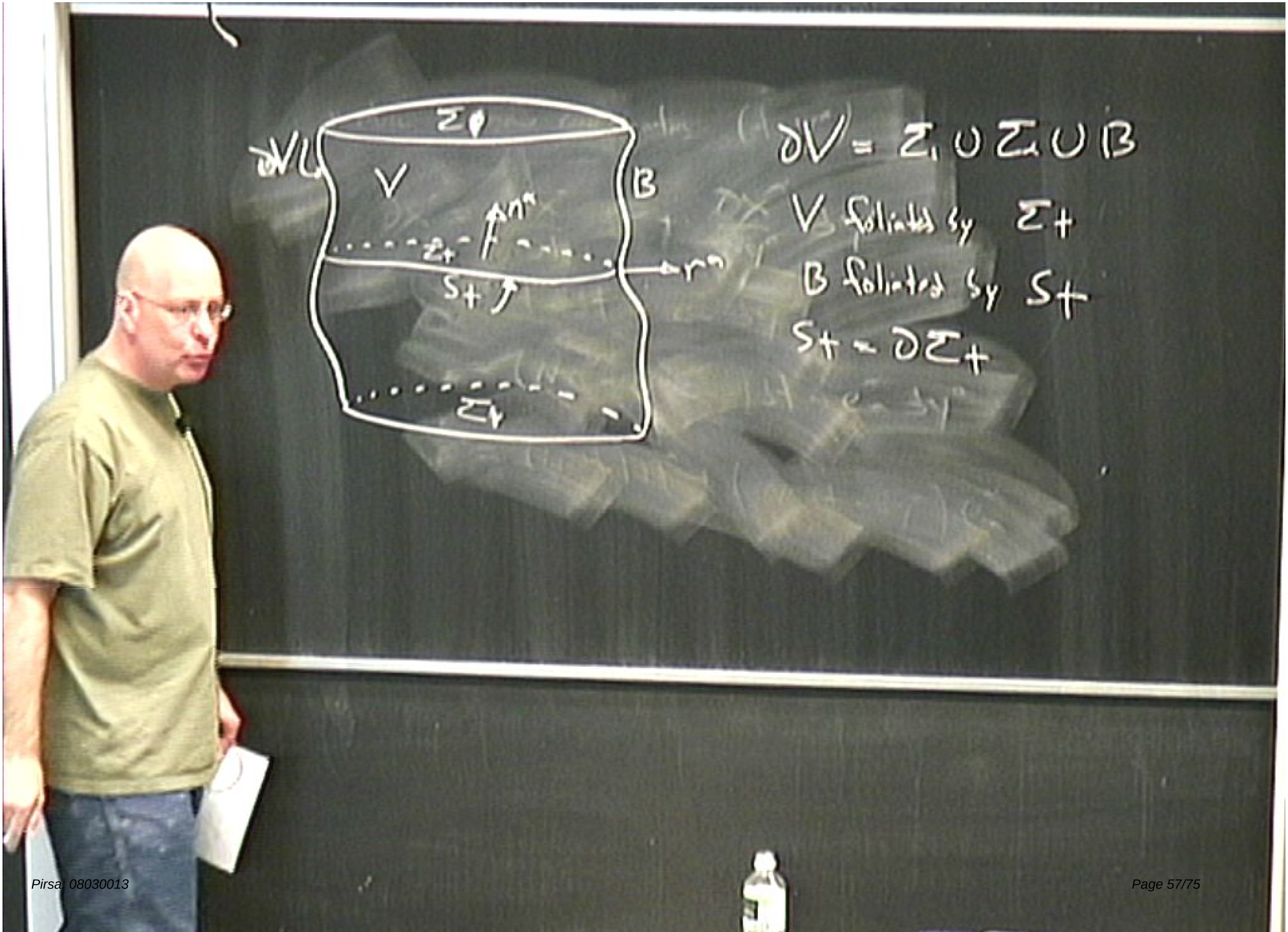
$$\sigma_{AB} = g_{\alpha\beta} e^\alpha_A e^\beta_B$$

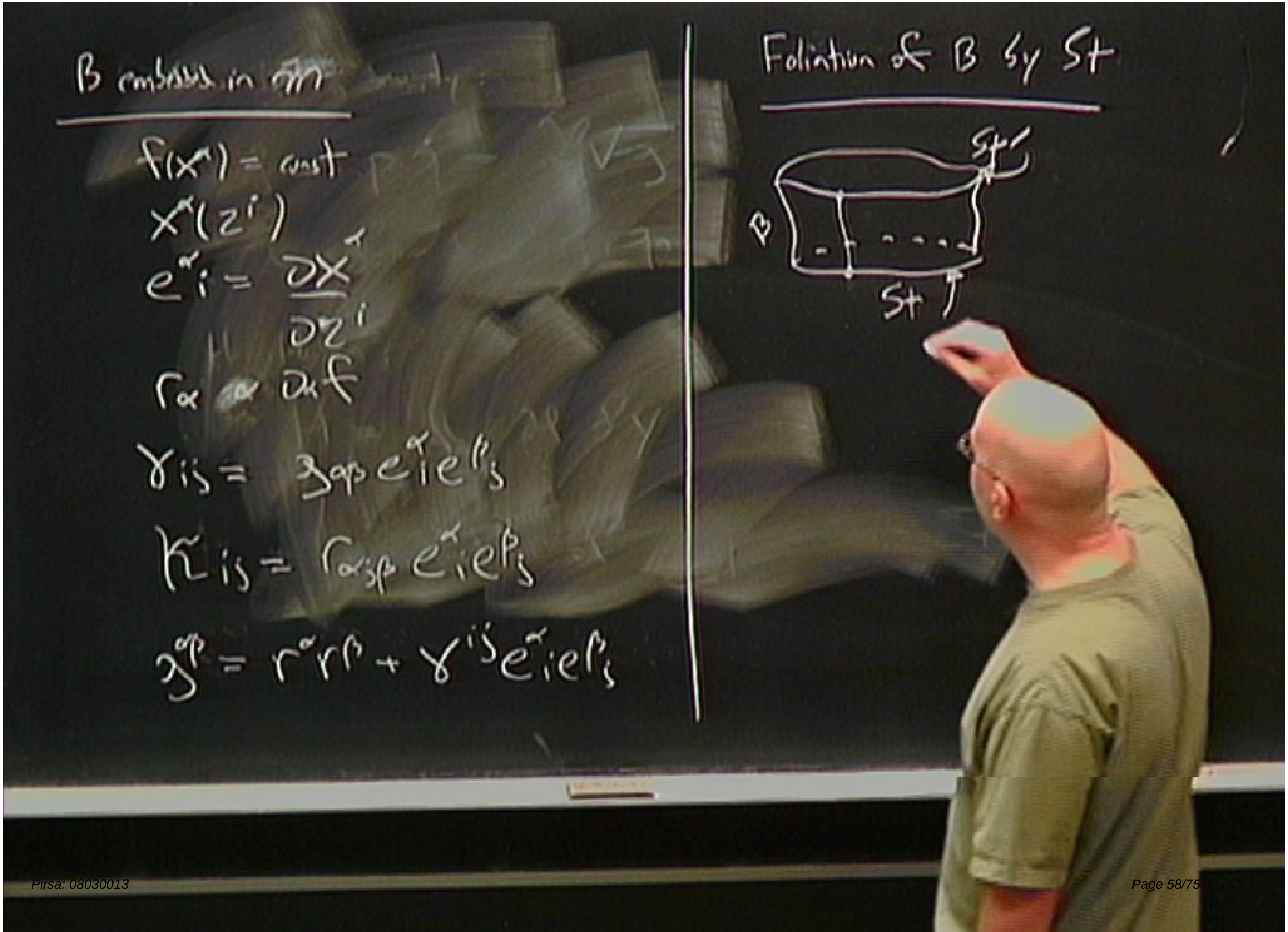
$$= g_{\alpha\beta}(e^\alpha_a e^a_A)(e^\beta_b e^b_B)$$

$$= h_{ab}\, e^a_A e^b_B$$

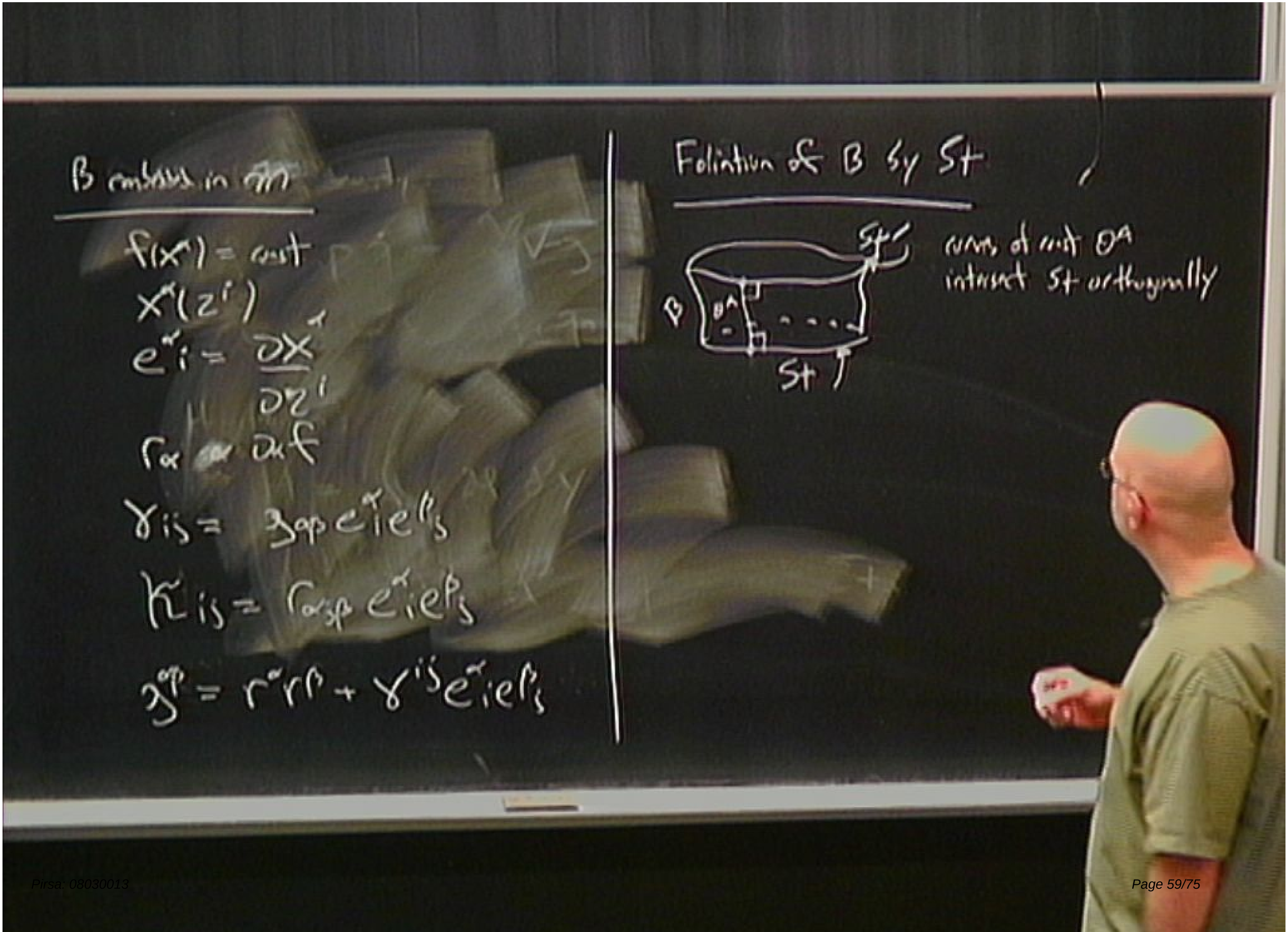
$$g^{\alpha\beta} = -n^\alpha n^\beta + r^\alpha r^\beta + \sigma^{AB} e^\alpha_A e^\beta_B$$

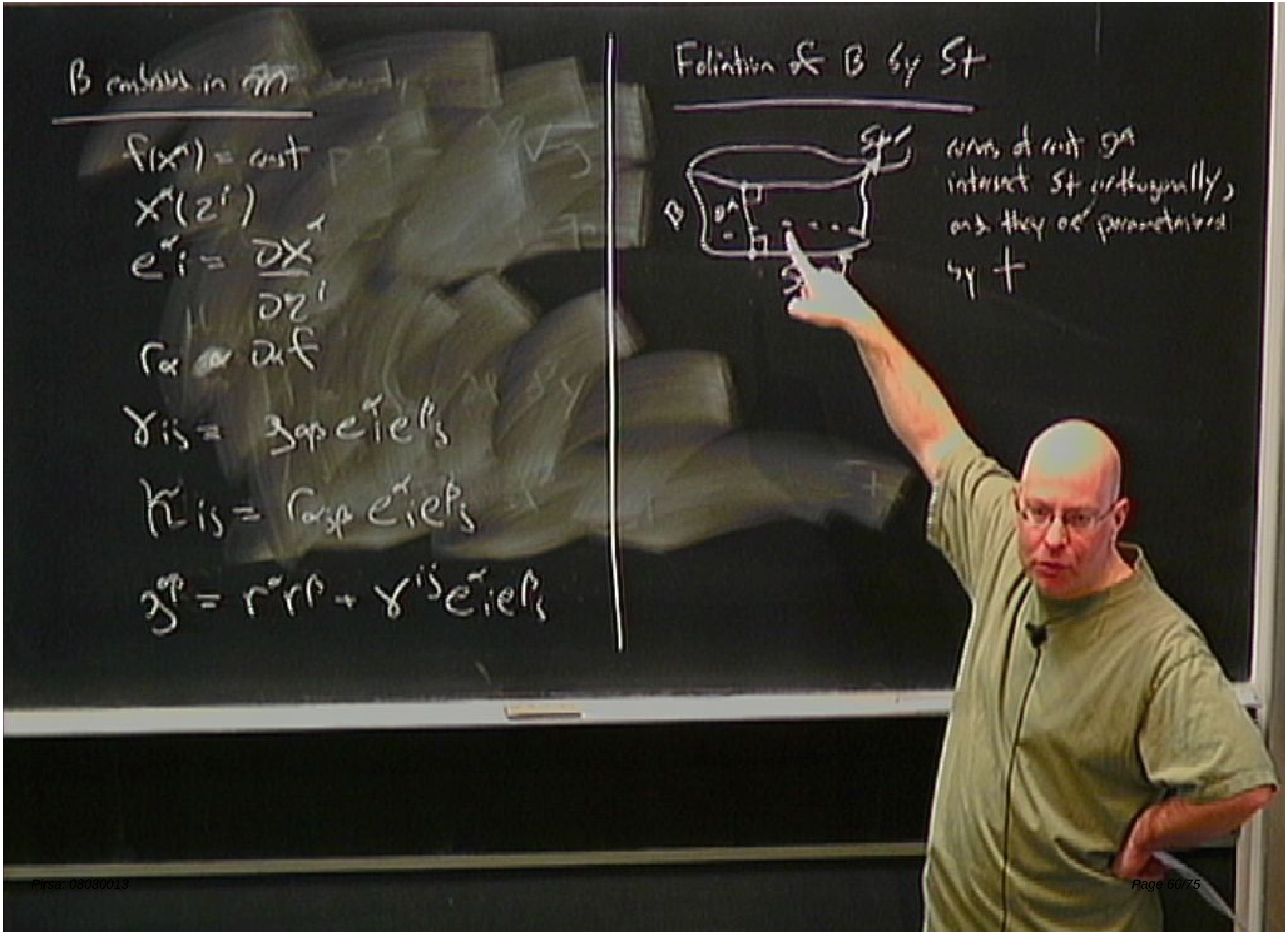

$f(x^\mu) = \text{cst}$
$x^\mu(z^i)$


$\partial V = \Sigma_1 \cup \Sigma_2 \cup B$
V foliated by Σ_t
B foliated by S_t
$S_t = \partial \Sigma_t$
∂V
V
B
n^a
Σ_t
r^a
S_t



B embedded in $\mathcal{M}$
$f(x^\alpha) = \text{const}$
$x^\alpha(z^i)$
$e^\alpha{}_i = \dfrac{\partial x^\alpha}{\partial z^i}$
$r_\alpha \propto \partial_\alpha f$
$\gamma_{ij} = g_{\alpha\beta}\, e^\alpha{}_i e^\beta{}_j$
$K_{ij} = r_{\alpha;\beta}\, e^\alpha{}_i e^\beta{}_j$
$g^{\alpha\beta} = r^\alpha r^\beta + \gamma^{ij} e^\alpha{}_i e^\beta{}_j$
Foliation of B by S_t
$S_{t'}$
B
S_t


B embedded in M
$f(x^\alpha) = \text{const}$
$x^\alpha(z^i)$
$e^\alpha{}_i = \frac{\partial x^\alpha}{\partial z^i}$
$r_\alpha \propto \partial_\alpha f$
$\gamma_{ij} = g_{\alpha\beta} e^\alpha{}_i e^\beta{}_j$
$K_{ij} = r_{\alpha;\beta} e^\alpha{}_i e^\beta{}_j$
$g^{\alpha\beta} = r^\alpha r^\beta + \gamma^{ij} e^\alpha{}_i e^\beta{}_j$
Foliation of B by St
St'
B
θ^A
St
curves of const θ^A intersect St orthogonally



f(x^α) = const
x^α(z^i)
e^α_i = ∂x^α/∂z^i
r_α ∝ ∂_α f
γ_ij = g_αβ e^α_i e^β_j
K_ij = r_α;β e^α_i e^β_j
g^αβ = r^α r^β + γ^ij e^α_i e^β_j
Foliation of B by St
curves of const θ^A
intersect St orthogonally,
and they are parametrized
by t

B embedded in $\mathcal{M}$

$$f(x^\alpha) = \text{const}$$

$$x^\alpha(z^i)$$

$$e^\alpha{}_i = \frac{\partial x^\alpha}{\partial z^i}$$

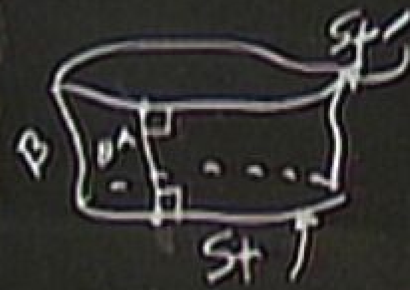
$$r_\alpha \propto \partial_\alpha f$$

$$\gamma_{ij} = g_{\alpha\beta}\, e^\alpha{}_i e^\beta{}_j$$

$$K_{ij} = r_{\alpha;\beta}\, e^\alpha{}_i e^\beta{}_j$$

$$g^{\alpha\beta} = r^\alpha r^\beta + \gamma^{ij} e^\alpha{}_i e^\beta{}_j$$

Foliation of B by S_t



curves of const θ^A intersect S_t orthogonally, and they are parametrized by t

$$\left(\frac{\partial x^\alpha}{\partial t}\right)_{\theta^A} = N n^\alpha \qquad \left(n_\alpha = -N \partial_\alpha t\right)$$

B embedded in $\mathcal{M}$

$$f(x^\alpha) = \text{const}$$

$$x^\alpha(z^i)$$

$$e^\alpha{}_i = \frac{\partial x^\alpha}{\partial z^i}$$

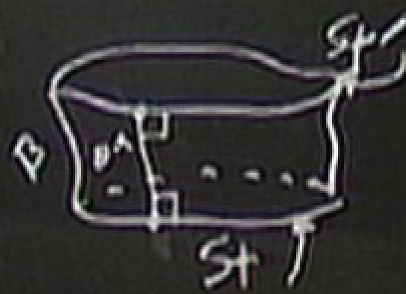
$$r_\alpha \propto \partial_\alpha f$$

$$\gamma_{ij} = g_{\alpha\beta}\, e^\alpha{}_i e^\beta{}_j$$

$$K_{ij} = r_{\alpha;\beta}\, e^\alpha{}_i e^\beta{}_j$$

$$g^{\alpha\beta} = r^\alpha r^\beta + \gamma^{ij} e^\alpha{}_i e^\beta{}_j$$

Foliation of B by S_t



curves of const θ^A intersect S_t orthogonally, and they are parametrized by t

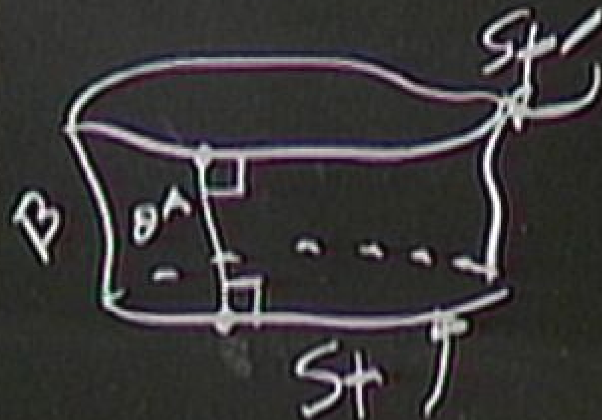
$$\left(\frac{\partial x^\alpha}{\partial t}\right)_{\theta^A} = N n^\alpha \quad \left(n_\alpha = -N \partial_\alpha t\right)$$

Choose $z^i = (t, \theta^A)$

Displacements on B: $dx^\alpha = N n^\alpha dt + e^\alpha{}_A d\theta^A$

$$ds^2\big|_B = -N^2 dt^2 + \sigma_{AB}\, d\theta^A d\theta^B$$

Foliation of B by S_t



curves of const θ^A intersect S_t orthogonally, and they are parametrized by t

$$\left(\frac{\partial x^\alpha}{\partial t}\right)\Big|_{\theta^A} = N n^\alpha \qquad \left(n_\alpha = -N \partial_\alpha t\right)$$

Choose $z^i = (t, \theta^A)$

Displacements on B: $dx^\alpha = N n^\alpha dt + e^\alpha_A d\theta^A$

$$ds^2\Big|_B = -N^2 dt^2 + \sigma_{AB}\, d\theta^A d\theta^B$$

B embedded in M

$$f(x^\alpha) = \text{const}$$

$$x^\alpha(z^i)$$

$$e^\alpha{}_i = \frac{\partial x^\alpha}{\partial z^i}$$

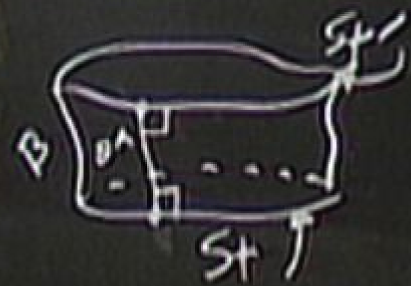
$$r_\alpha \propto \partial_\alpha f$$

$$\gamma_{ij} = g_{\alpha\beta}\, e^\alpha{}_i e^\beta{}_j$$

$$K_{ij} = r_{\alpha;\beta}\, e^\alpha{}_i e^\beta{}_j$$

$$g^{\alpha\beta} = r^\alpha r^\beta + \gamma^{ij} e^\alpha{}_i e^\beta{}_j$$

Foliation of B by S_t



curves of const θ^A intersect S_t orthogonally, and they are parametrized by t

$$\left(\frac{\partial x^\alpha}{\partial t}\right)_{\theta^A} = N n^\alpha \qquad \left(n_\alpha = -N \partial_\alpha t\right)$$

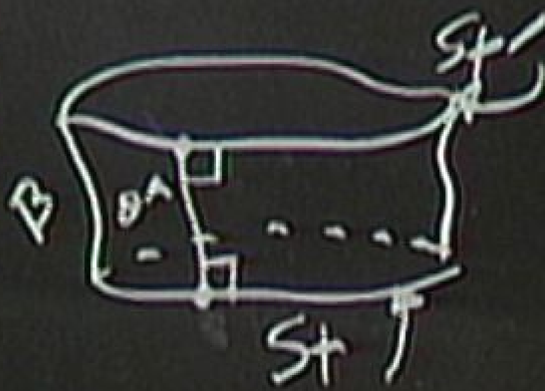
Choose $z^i = (t, \theta^A)$

Displacements on B : $dx^\alpha = N n^\alpha dt + e^\alpha{}_A d\theta^A$

$$ds^2\big|_B = -N^2 dt^2 + \sigma_{AB}\, d\theta^A d\theta^B$$

$$\sqrt{-\gamma} = N\sqrt{\sigma}$$

Foliation of B by S_t



curves of const θ^A intersect S_t orthogonally, and they are parametrized by t

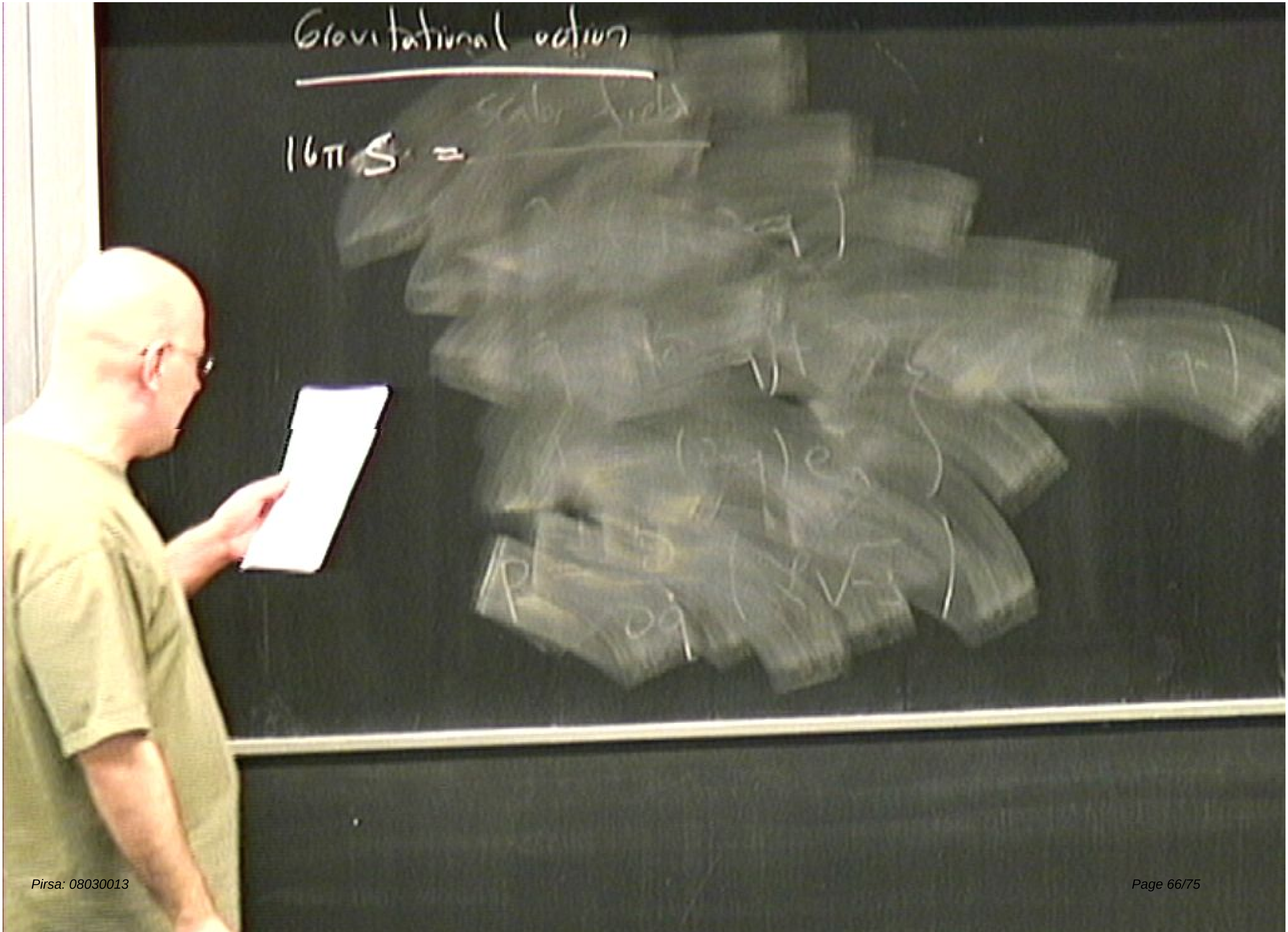
$$\left(\frac{\partial x^\alpha}{\partial t}\right)_{\theta^A} = N n^\alpha \qquad \left(n_\alpha = -N \partial_\alpha t\right)$$

Choose $z^i = (t, \theta^A)$

Displacements on B: $dx^\alpha = N n^\alpha dt + e^\alpha_A d\theta^A$

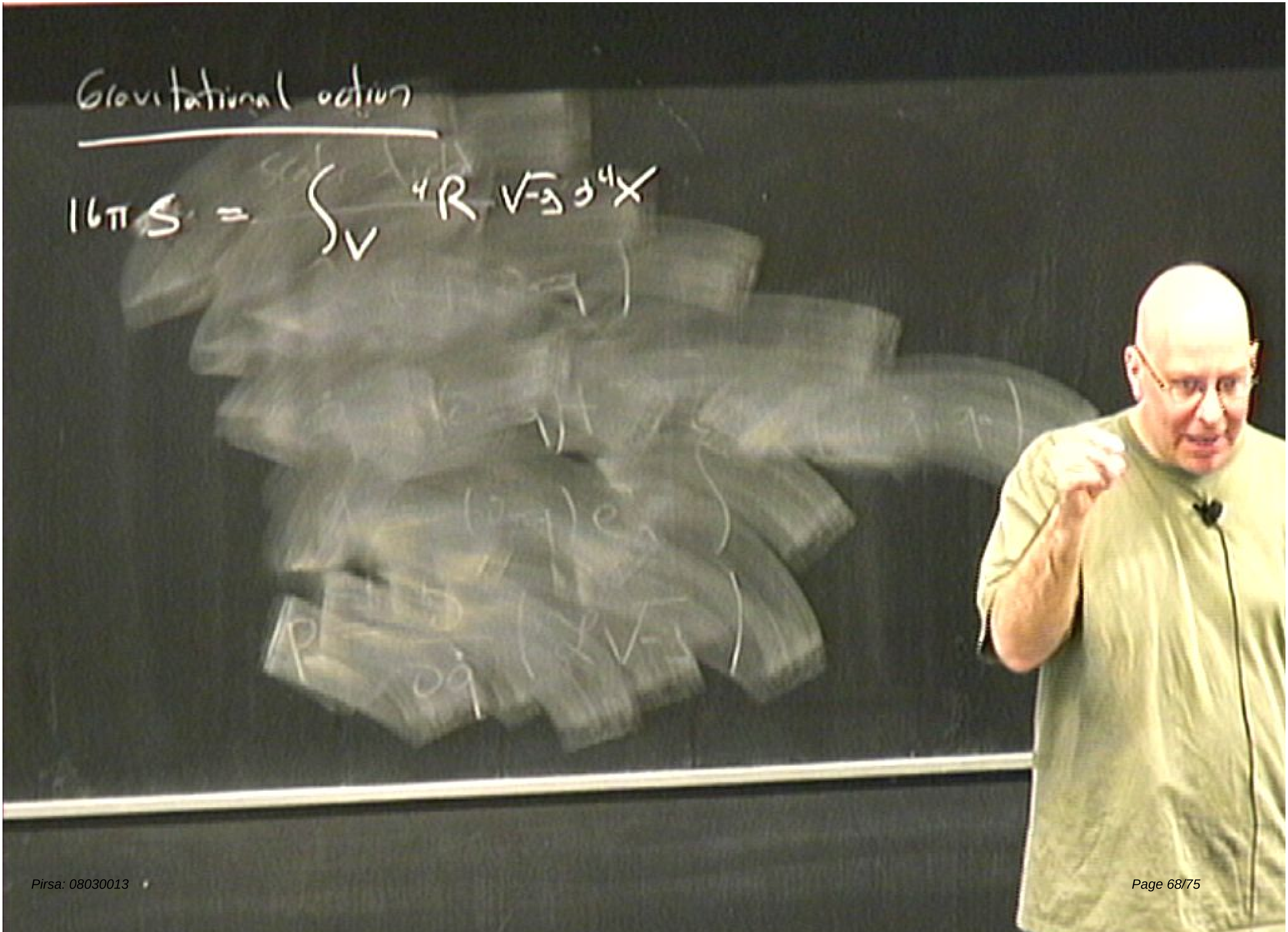
$$ds^2\big|_B = -N^2 dt^2 + \sigma_{AB}\, d\theta^A d\theta^B$$

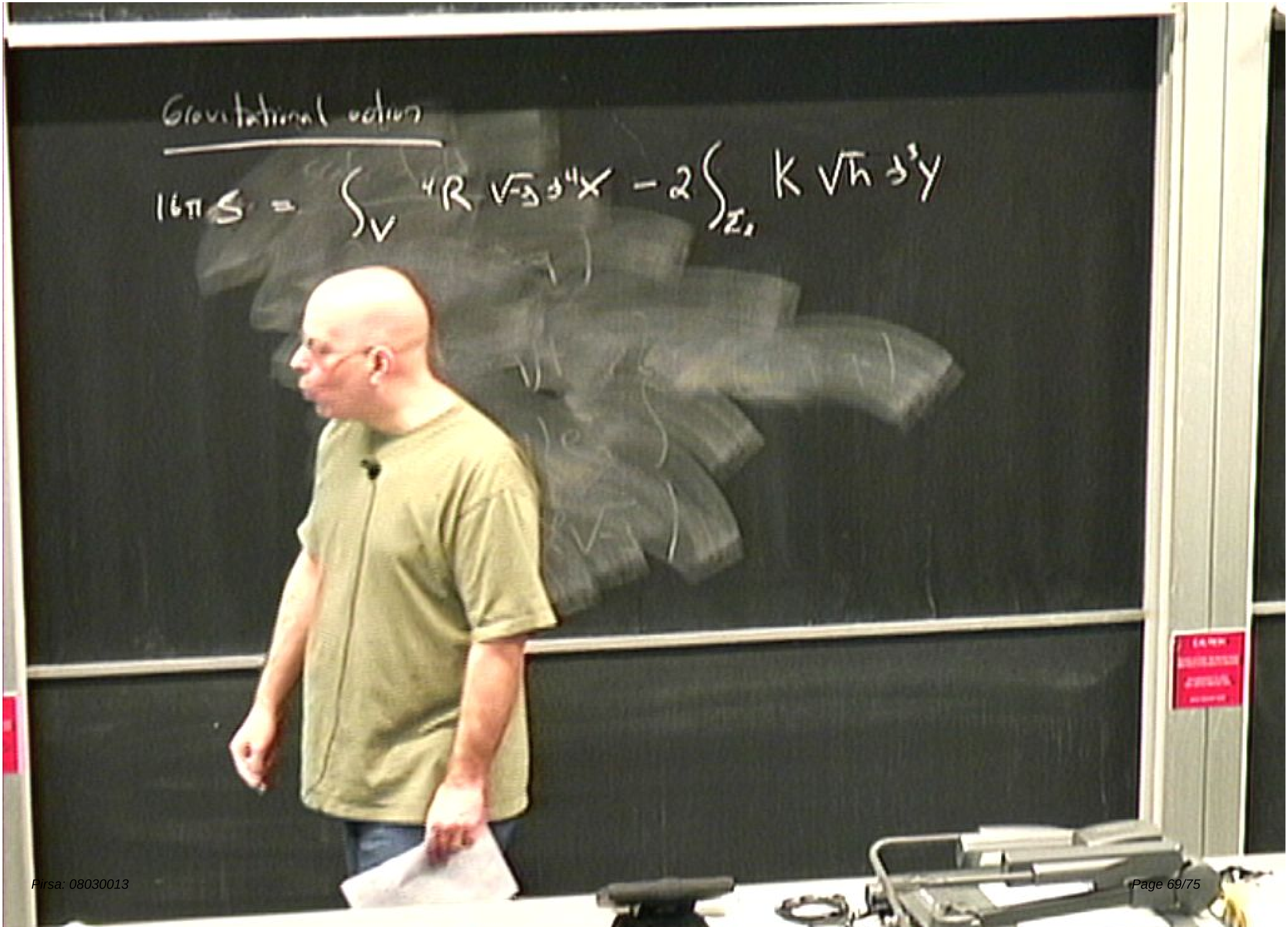
$$\sqrt{-\gamma} = N\sqrt{\sigma}$$


Gravitational action
16π S =

Gravitational action

$$16\pi S = \int_V {}^4R \sqrt{-g}\, d^4x$$


Gravitational action
$16\pi S = \int_V {}^4R \sqrt{-g}\, d^4x$


Gravitational action
$16\pi S = \int_V {}^4R\sqrt{-g}\,d^4x - 2\int_{\Sigma_1} K\sqrt{h}\,d^3y$

Gravitational action

$$16\pi S = \int_V {}^4R \sqrt{-g}\, d^4x - 2\int_{\Sigma_2} K \sqrt{h}\, d^3y$$

$$+ 2\int_{\Sigma_1} K \sqrt{h}\, d^3y + 2\int$$

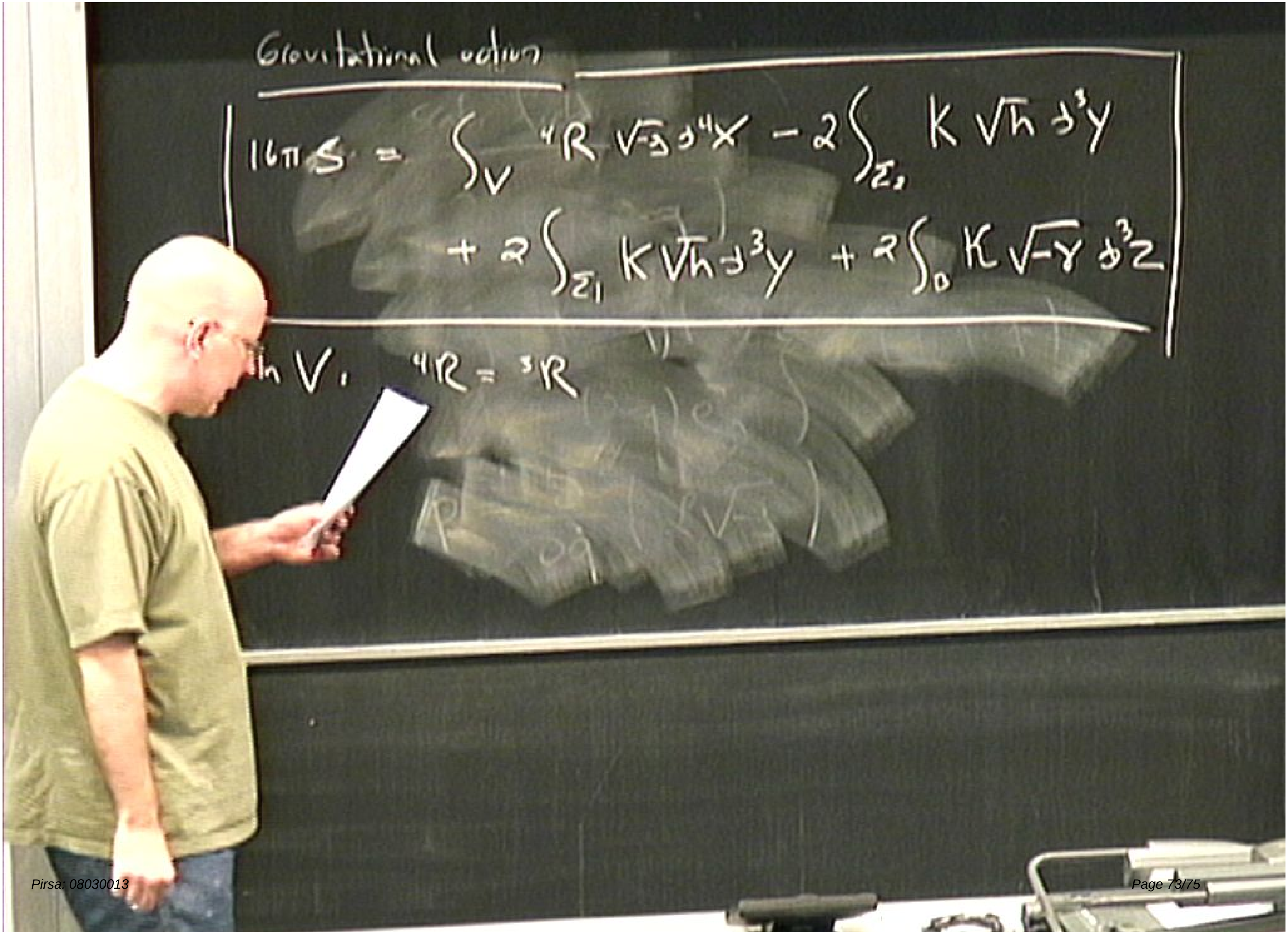
Gravitational action

$$16\pi S = \int_V {}^4R \sqrt{-g}\, d^4x - 2\int_{\Sigma_2} K \sqrt{h}\, d^3y$$

$$+ 2\int_{\Sigma_1} K \sqrt{h}\, d^3y + 2\int_{B} K \sqrt{-\gamma}\, d^3z$$

Gravitational action

$$\boxed{16\pi S = \int_V {}^4R\sqrt{-g}\,d^4x - 2\int_{\Sigma_2} K\sqrt{h}\,d^3y + 2\int_{\Sigma_1} K\sqrt{h}\,d^3y + 2\int_B K\sqrt{-\gamma}\,d^3z}$$


Gravitational action
$16\pi S = \int_V {}^4R \sqrt{-g}\, d^4x - 2\int_{\Sigma_2} K \sqrt{h}\, d^3y + 2\int_{\Sigma_1} K \sqrt{h}\, d^3y + 2\int_{B} K \sqrt{-\gamma}\, d^3z$
In V, ${}^4R = {}^3R$

Gravitational action

$$\boxed{16\pi S = \int_V {}^4R\sqrt{-g}\,d^4x - 2\int_{\Sigma_2} K\sqrt{h}\,d^3y + 2\int_{\Sigma_1} K\sqrt{h}\,d^3y + 2\int_{B} K\sqrt{-\gamma}\,d^3z}$$

In V: $\quad {}^4R = {}^3R + K^{ab}K_{ab} - K^2$

Gravitational action

$$\boxed{16\pi S = \int_V {}^4R\sqrt{-g}\,d^4x - 2\int_{\Sigma_2} K\sqrt{h}\,d^3y + 2\int_{\Sigma_1} K\sqrt{h}\,d^3y + 2\int_{B} K\sqrt{-\gamma}\,d^3z}$$

In V: $\quad {}^4R = {}^3R + K^{ab}K_{ab} - K^2 - 2\left(n^\alpha{}_{;\beta}n^\beta - n^\alpha n^\beta{}_{;\beta}\right)_{;\alpha}$