Title: Cosmology #2

Date: Mar 27, 2008 06:30 PM

URL: http://pirsa.org/08030010

Abstract: A brief history of our cosmic beginnings, Cosmic Microwave Background. How galaxies form and the existence of dark matter.

Recap

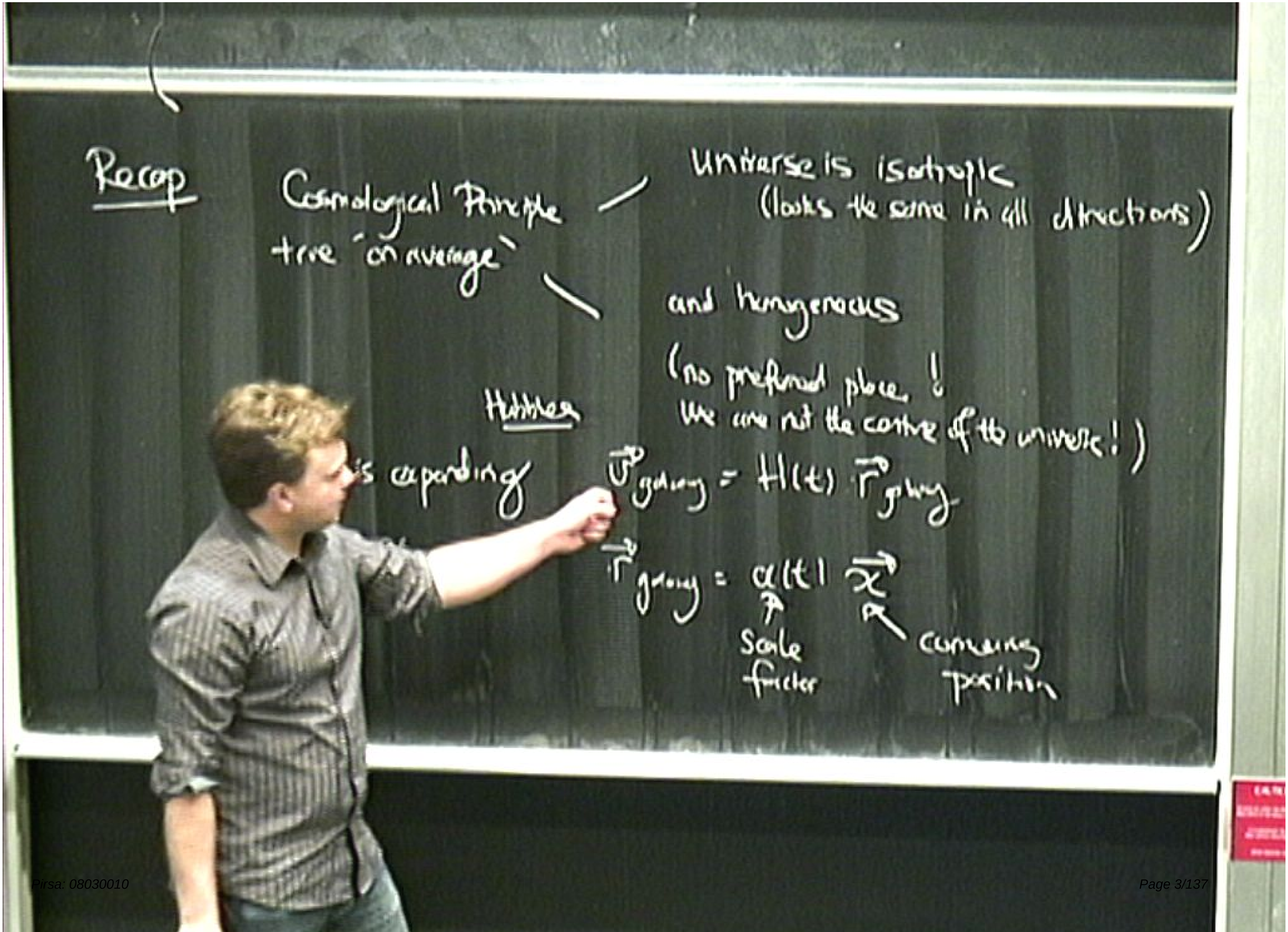
Cosmological Principle true "on average"

- Universe is isotropic (looks the same in all directions)
- and homogeneous (no preferred place! We are not the centre of the universe!)

Universe is expanding

$$\vec{v}_{galaxy} = H(t)\,\vec{r}_{galaxy}$$

$$\vec{r}_{galaxy} = \underbrace{a(t)}_{\text{scale factor}}\ \underbrace{\vec{x}}_{\text{comoving position}}$$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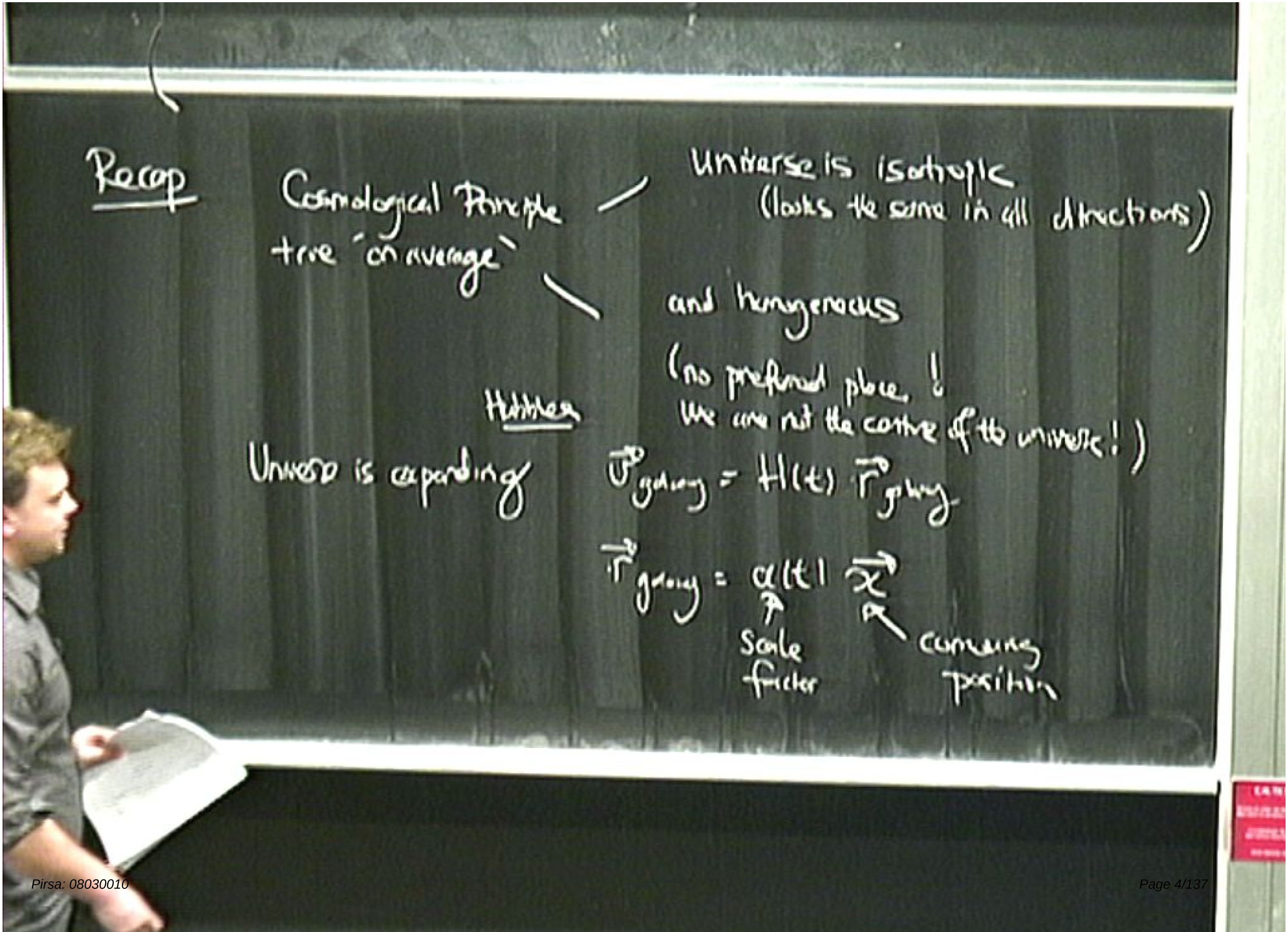
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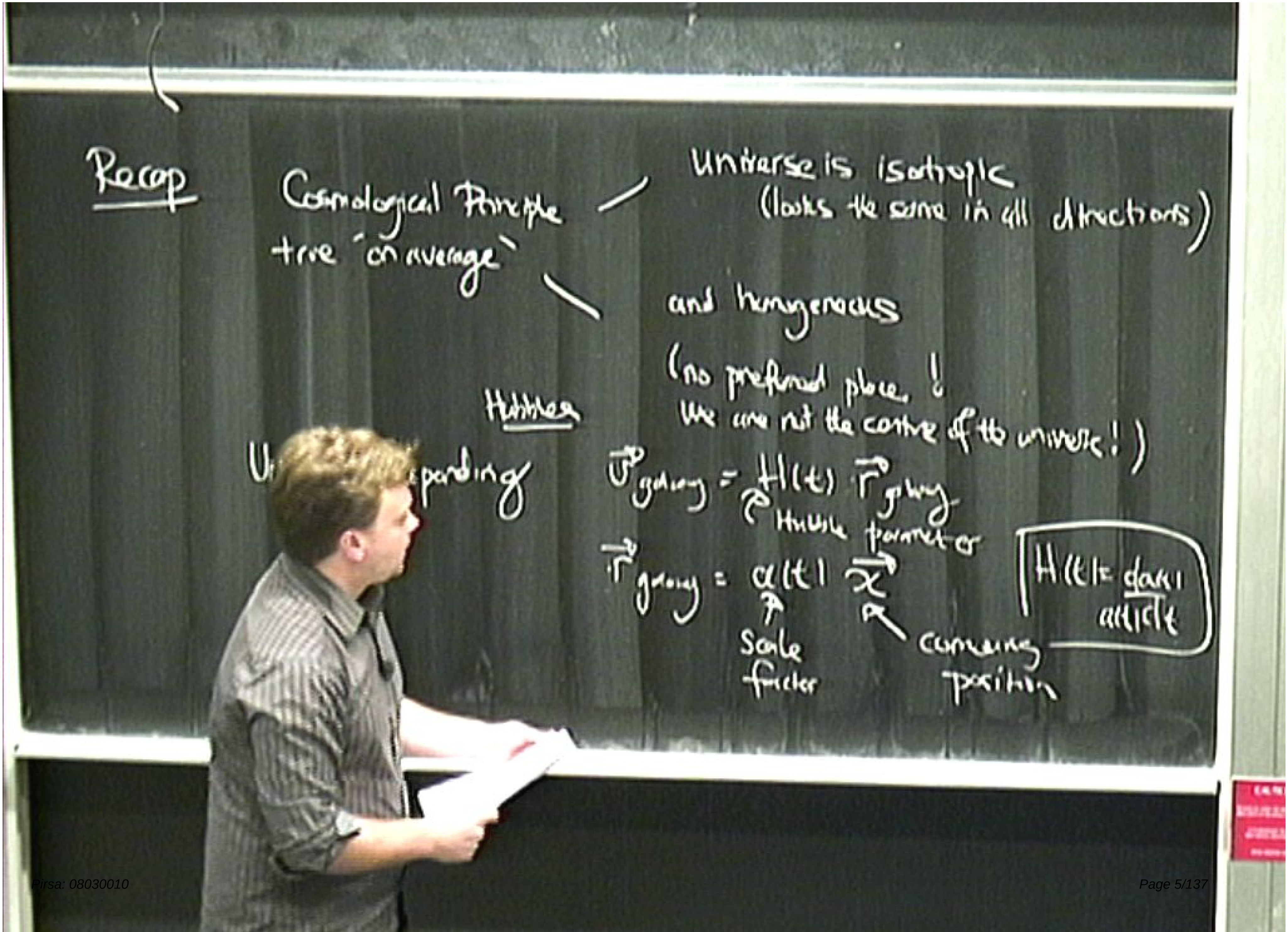
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Hubble parameter
r galaxy = a(t) x
scale factor
comoving position
H(t)

$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$ FRWL metric

Universe filled with energy/mass density ρ with pressure P

'local conservation of energy' $\frac{d\rho}{dt} = -3H(p+\rho)$

Friedmann eq'n $H^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$

$P = w\rho$ with constant w $\rho \sim a^{-3(1+w)}$ $t \sim a^{\frac{2}{3(1+w)}}$

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2 \quad \text{FRW metric}$$

$$x_1 \xrightarrow{a(t)(x_2 - x_1)} x_2$$

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$a(t)(x_2 - x_1)$

$x_1 \longrightarrow x_2$

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$p = w\rho$ with constant $w \qquad \rho \sim a^{-3(1+w)} \qquad t \sim a^{\frac{2}{3(1+w)}}$

$ds = -dt$ ...

Universe filled with energy/mass density ρ with pressure P

'local conservation of energy'

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Friedmann eq'n

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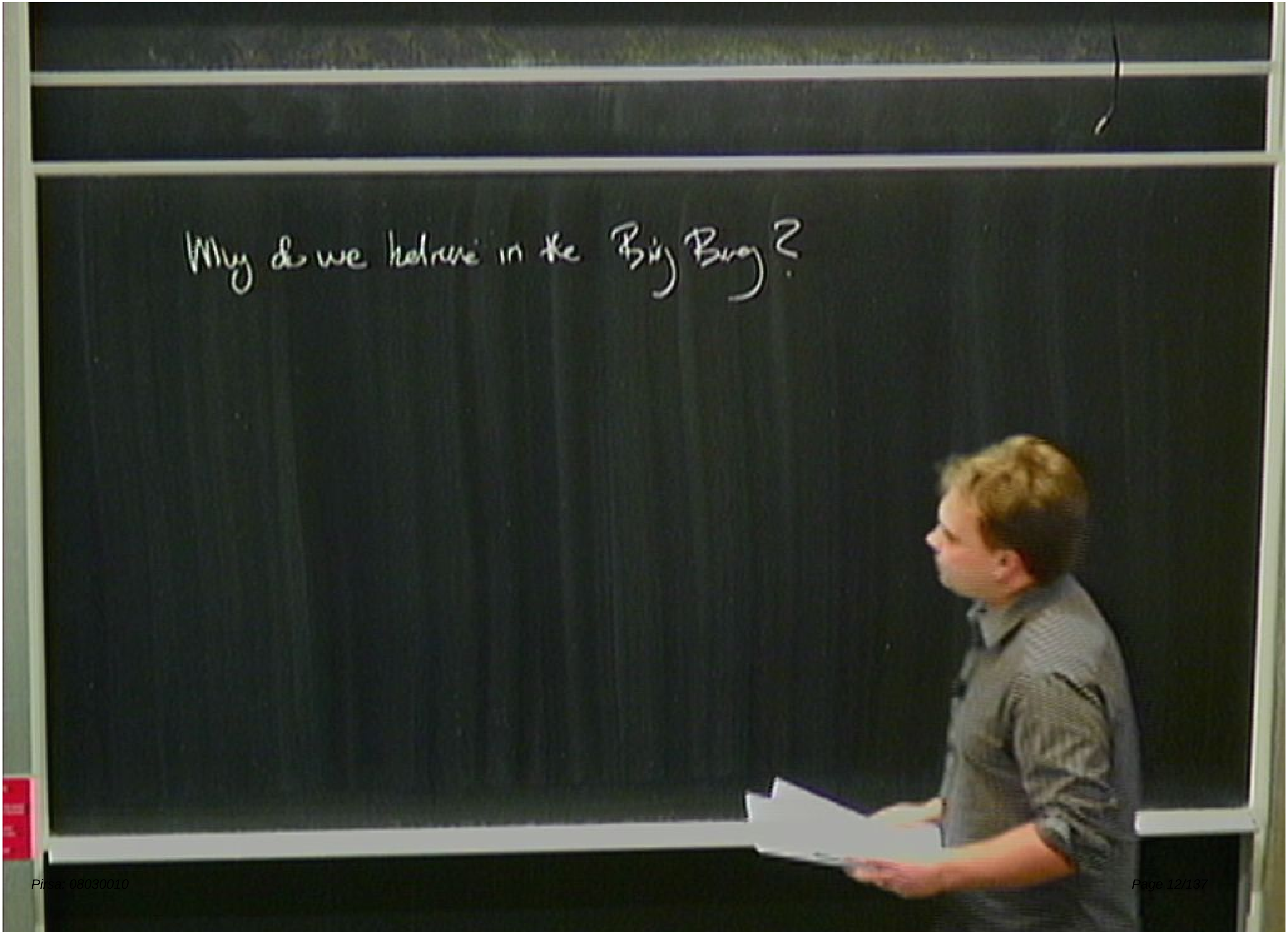
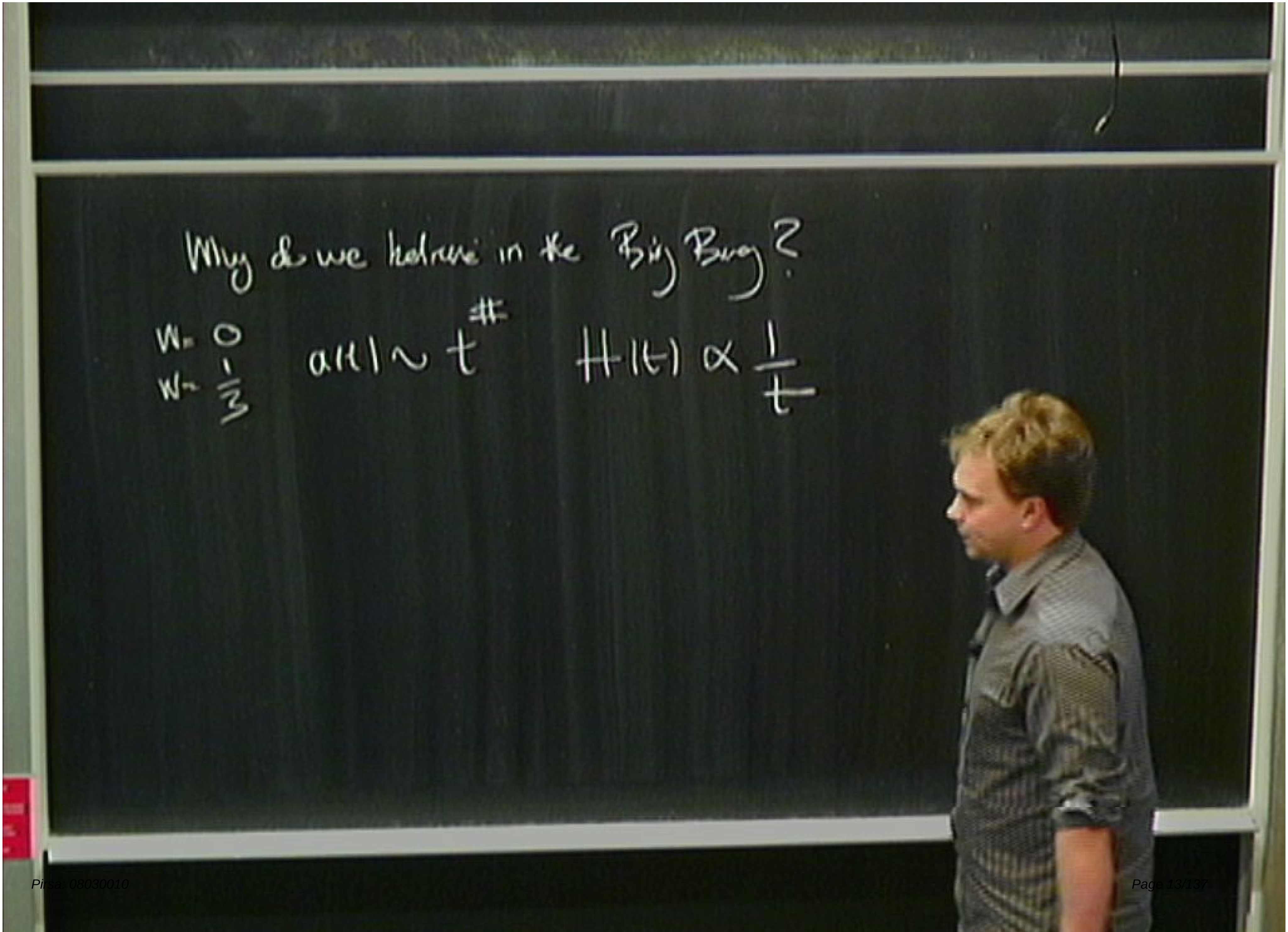
equation of state parameter

$$\rho \sim a^{-3(1+w)}$$

$$\boxed{a(t) \sim t^{\frac{2}{3(1+w)}}}$$


Why do we believe in the Big Bang?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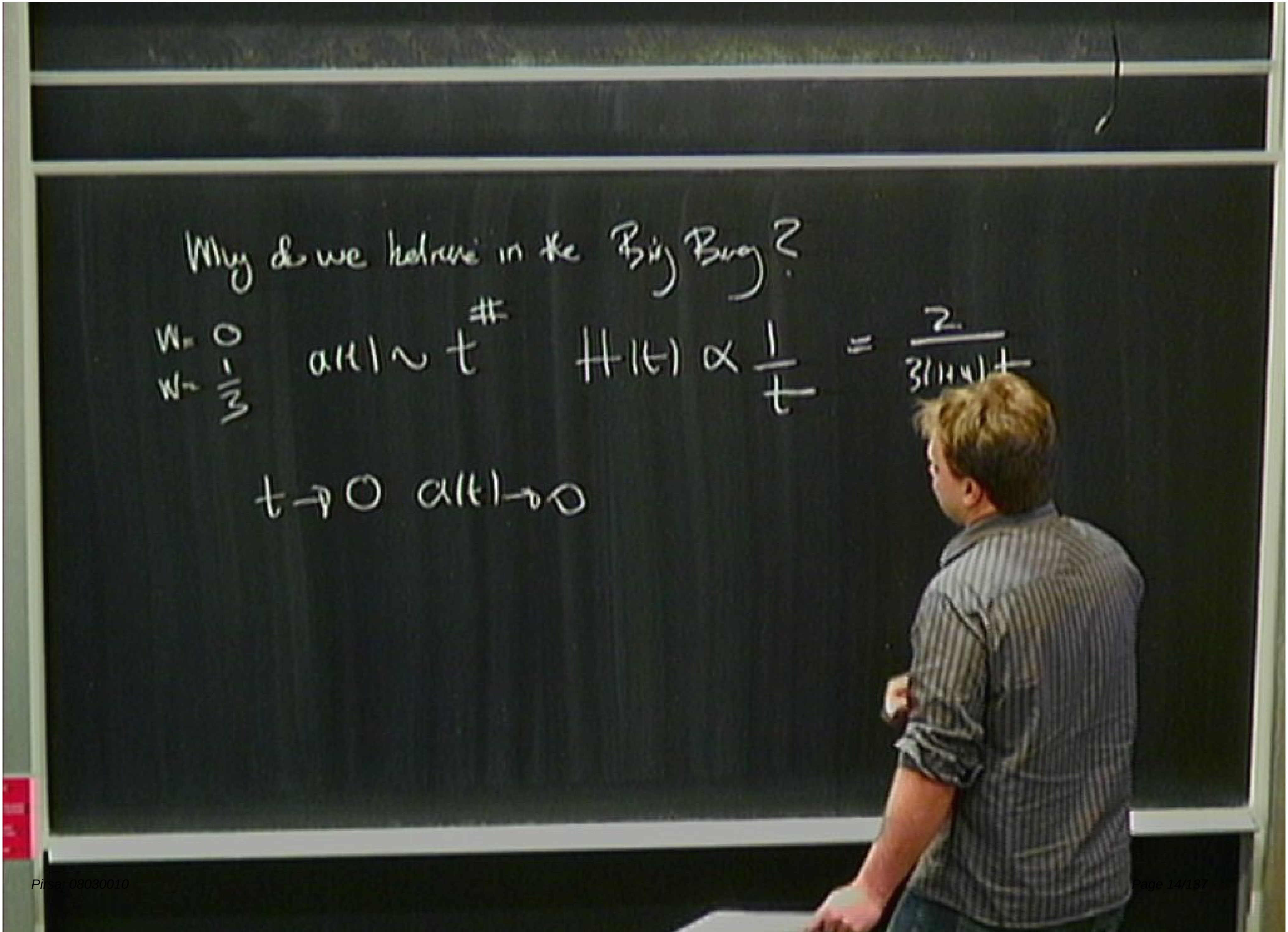
w= 0
w= 1/3
a(t) ~ t^#
H(t) ∝ 1/t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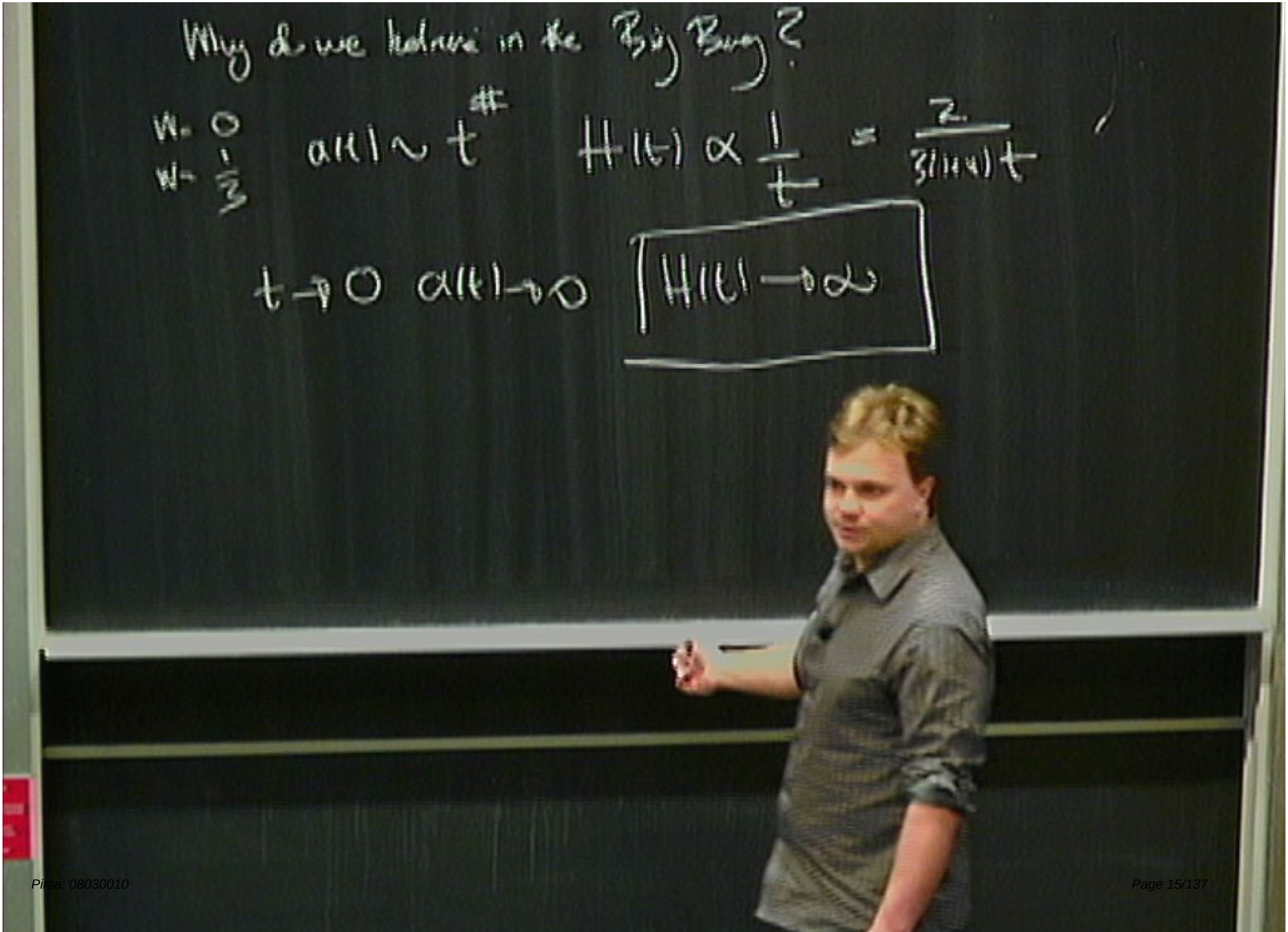
$t \to 0 \quad a(t) \to 0$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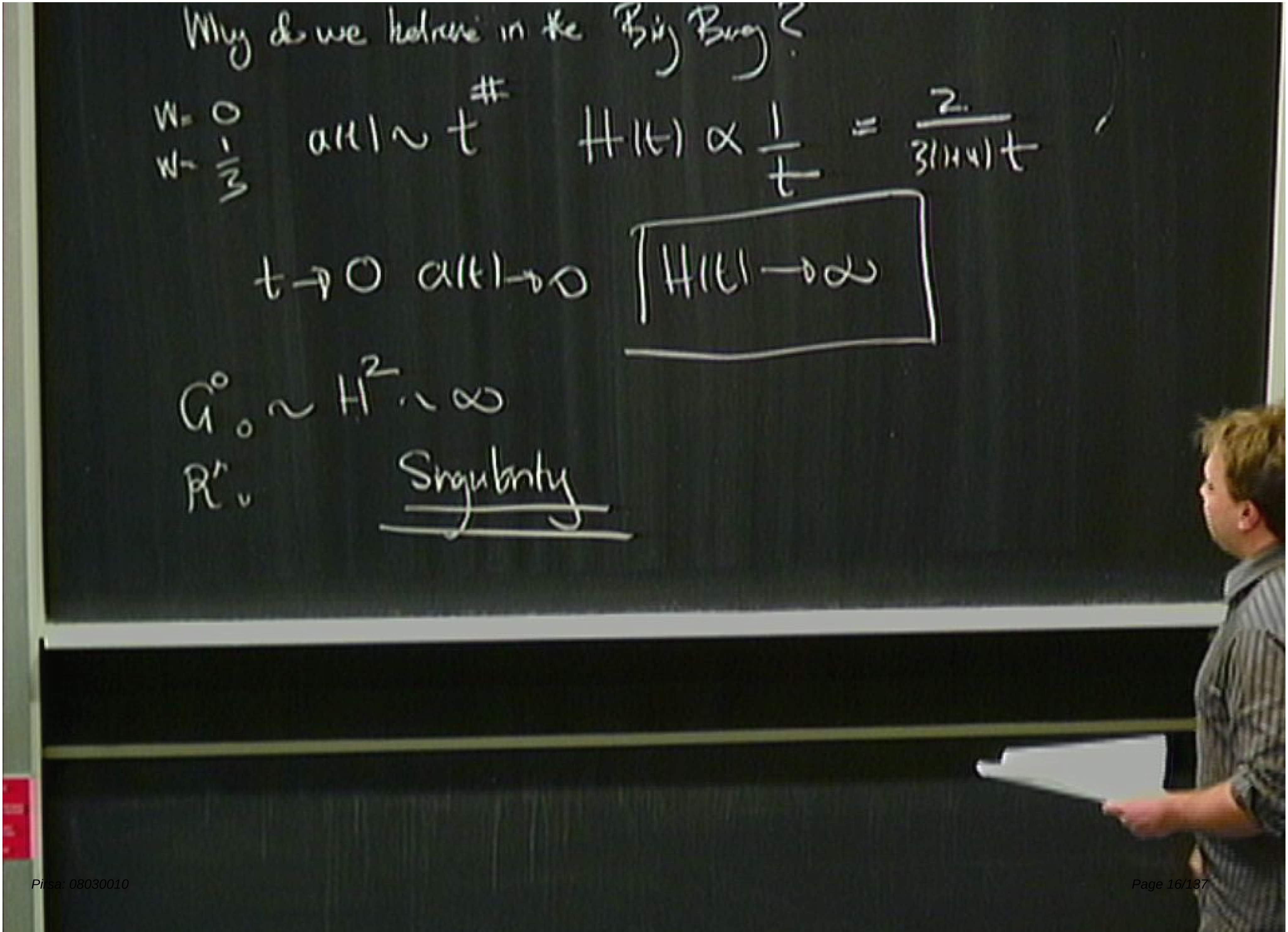
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G^0_0 ~ H^2 ~ ∞
R^μ_ν
Singularity

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$w = \frac{1}{3}$

$t \to 0 \quad a(t) \to 0 \quad \boxed{H(t) \to \infty}$

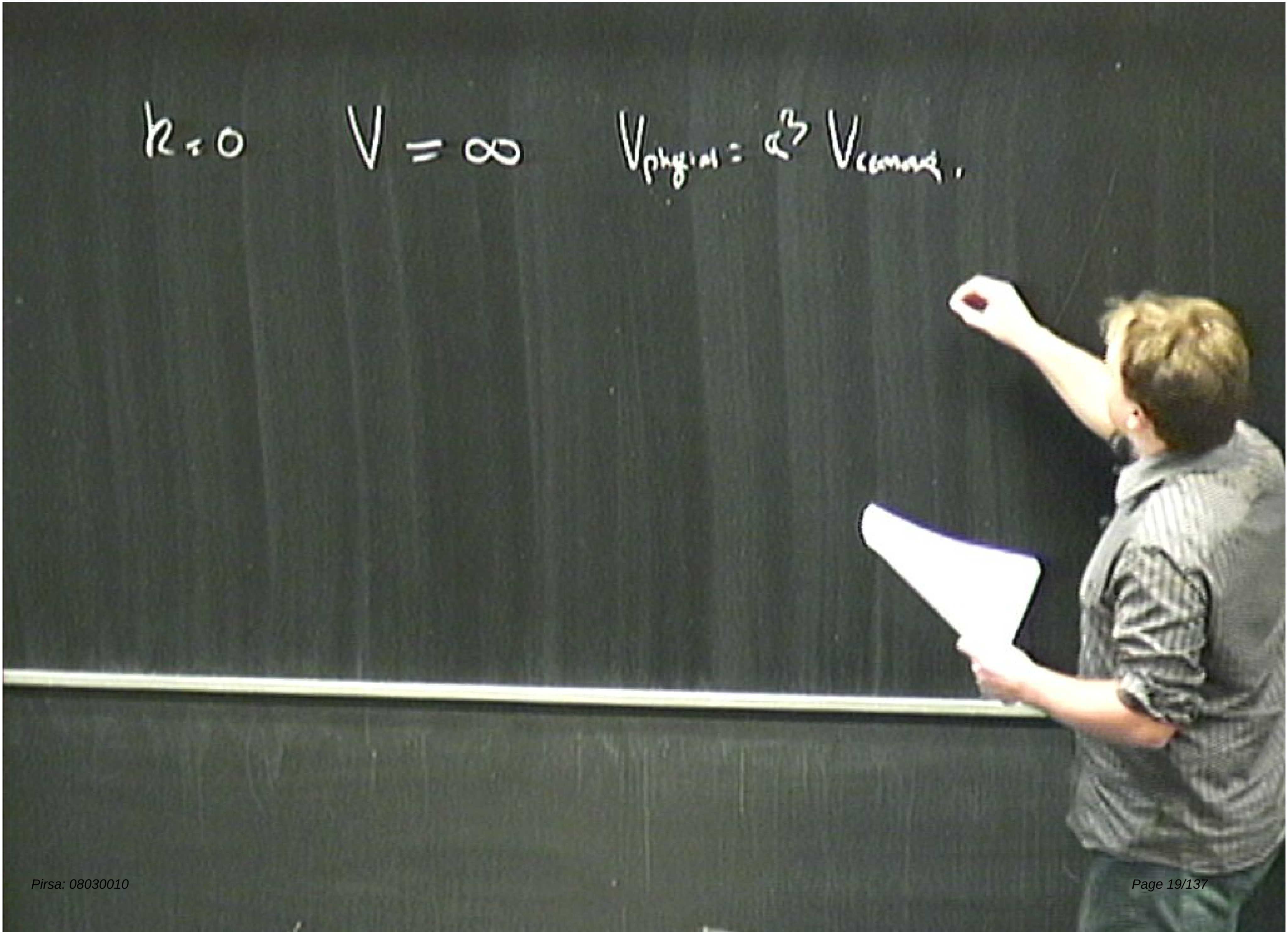
$G^0{}_0 \sim H^2 \sim \infty$

$R^{\mu}{}_{\nu}$

Singularity

As a spacelike singularity

$k = 0 \qquad V = \infty$


$k = 0$
$V = \infty$
$V_{physical} = a^3 V_{comoving}$

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$\int dx_1 dx_2 dx_3$

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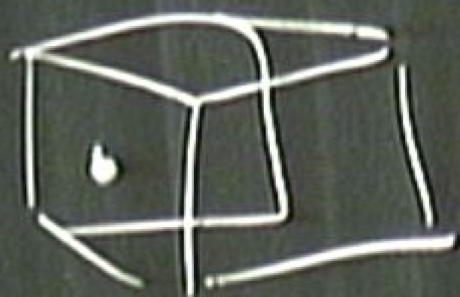

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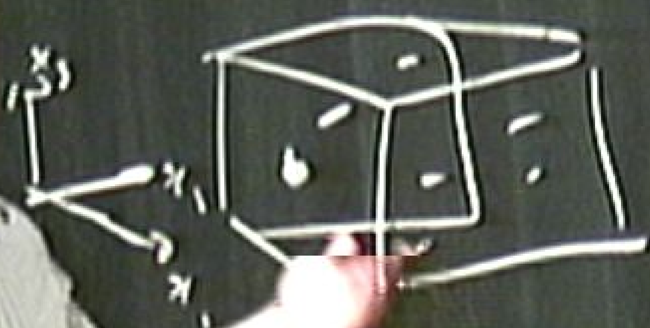
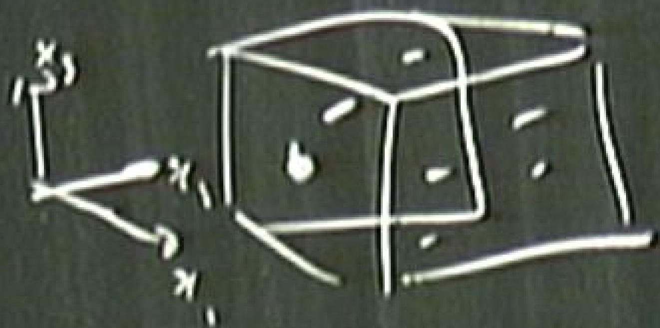


$$k = 0 \qquad V = \infty \qquad V_{\text{physical}} = a^3 \, V_{\text{comoving}}.$$

$$\int dx_1\, dx_2\, dx_3$$



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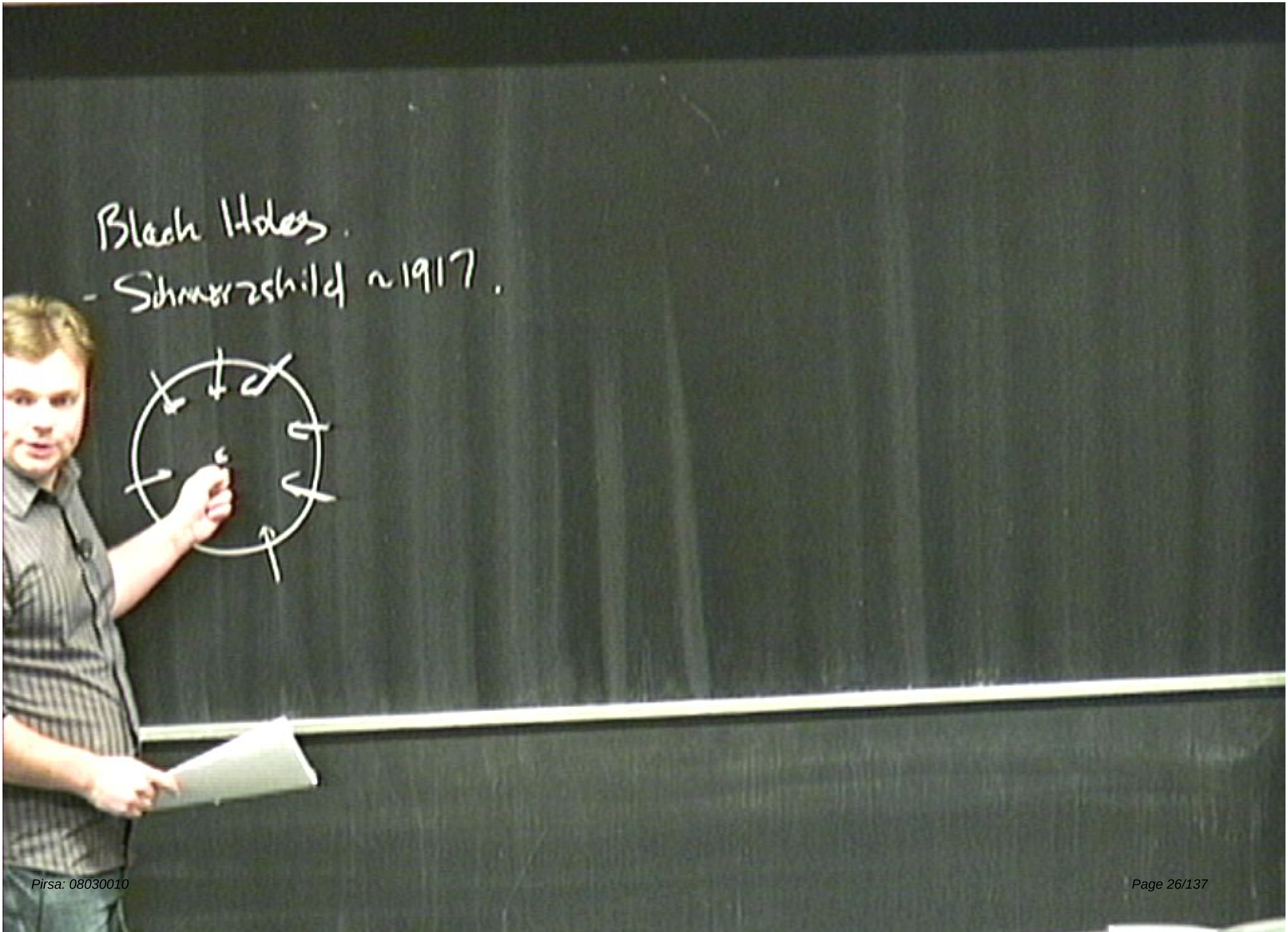
$G^0{}_0 \sim H^2 \sim \infty$ $\qquad$ Its a spacelike singularity

$R^n{}_{\cdots}$ $\qquad$ Singularity

General Relativity breaks down

Black Holes.

- Schwarzschild ~1917.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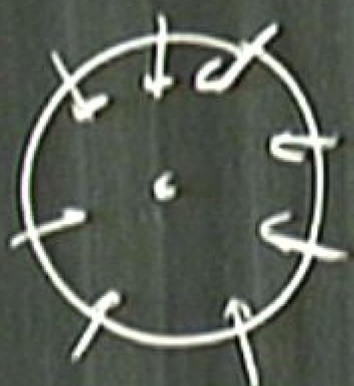

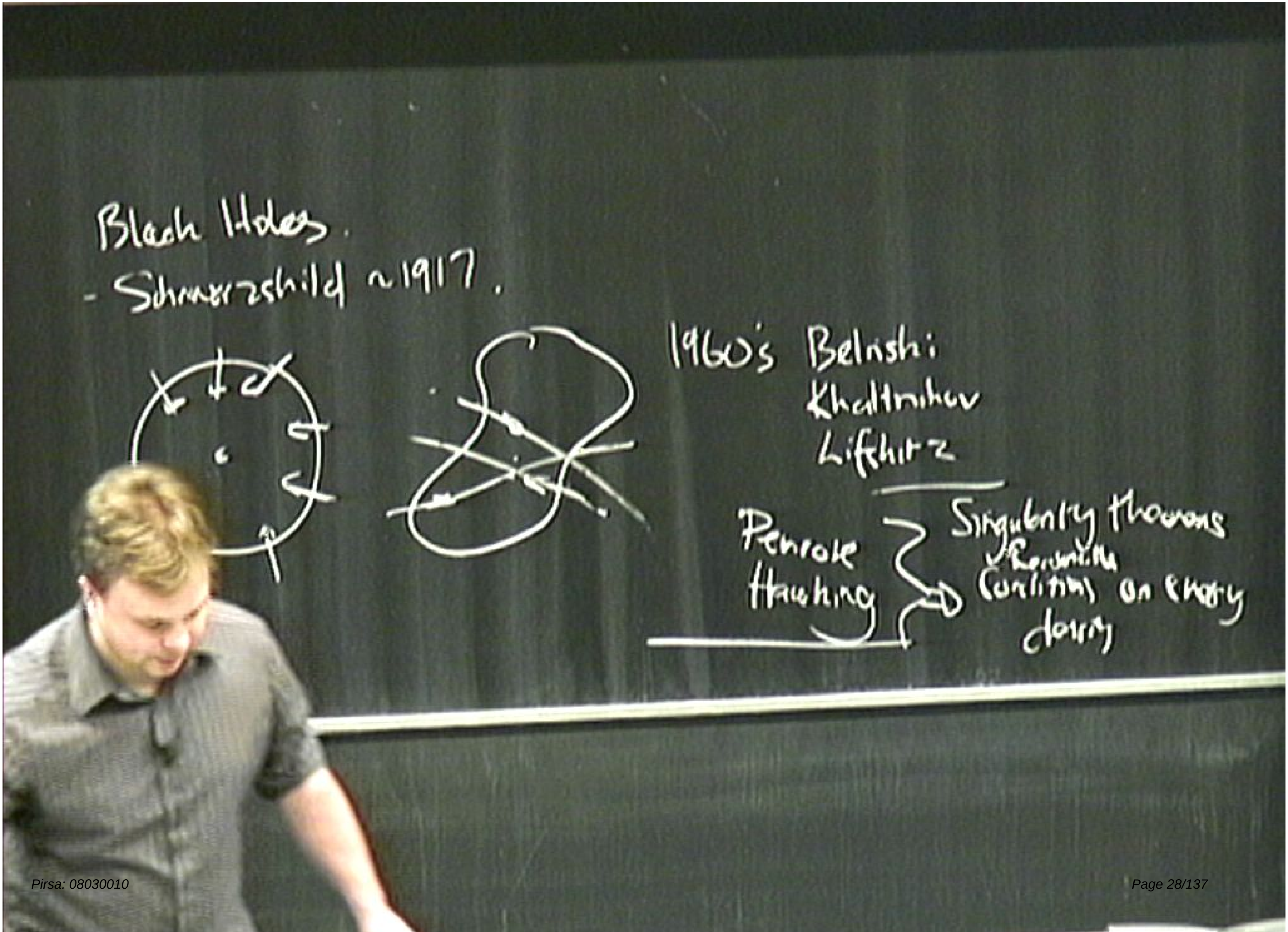
Black Holes.
- Schwarzschild ~1917.
1960's Belinski
Khalatnikov
Lifshitz
Penrose
Hawking
Singularity theorems
Conditions on energy
density

Black Holes.

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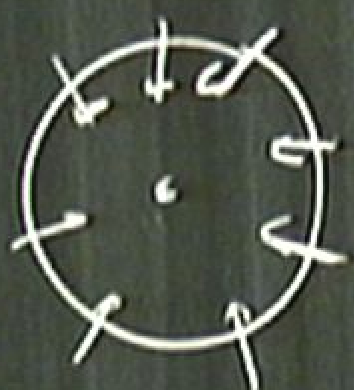

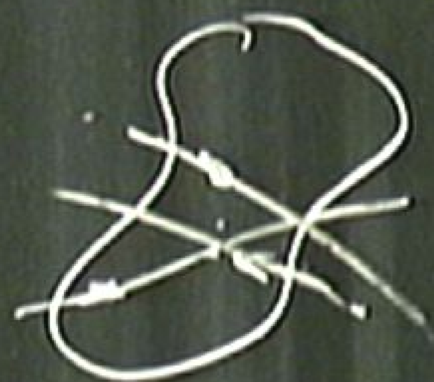

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Singularity theorems

Reasonable conditions on energy density $\Rightarrow$ singularities

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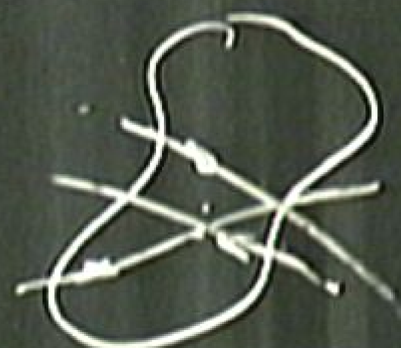


1960's Belinski
Khalatnikov
Lifshitz

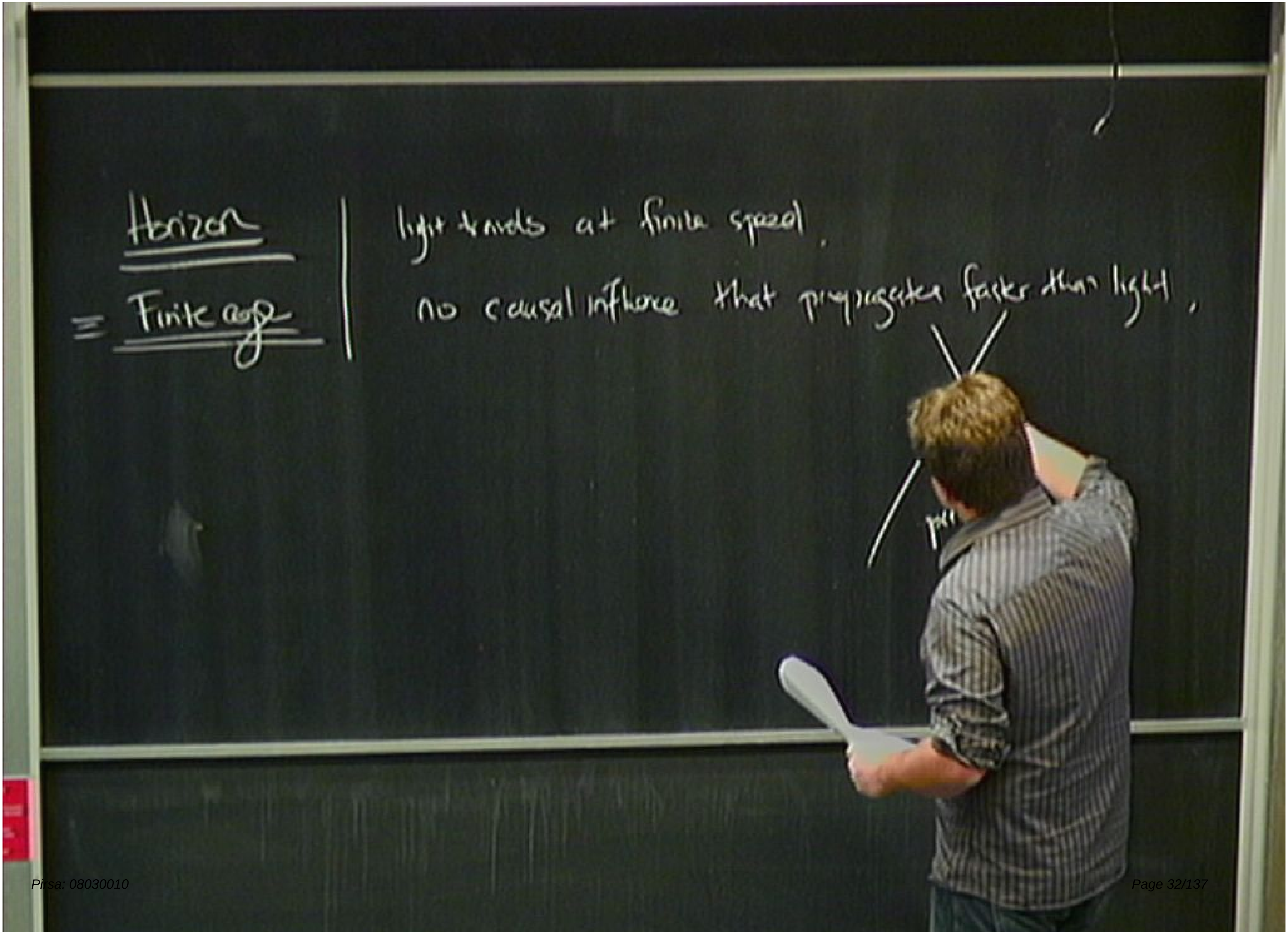
quantum gravity
string theory

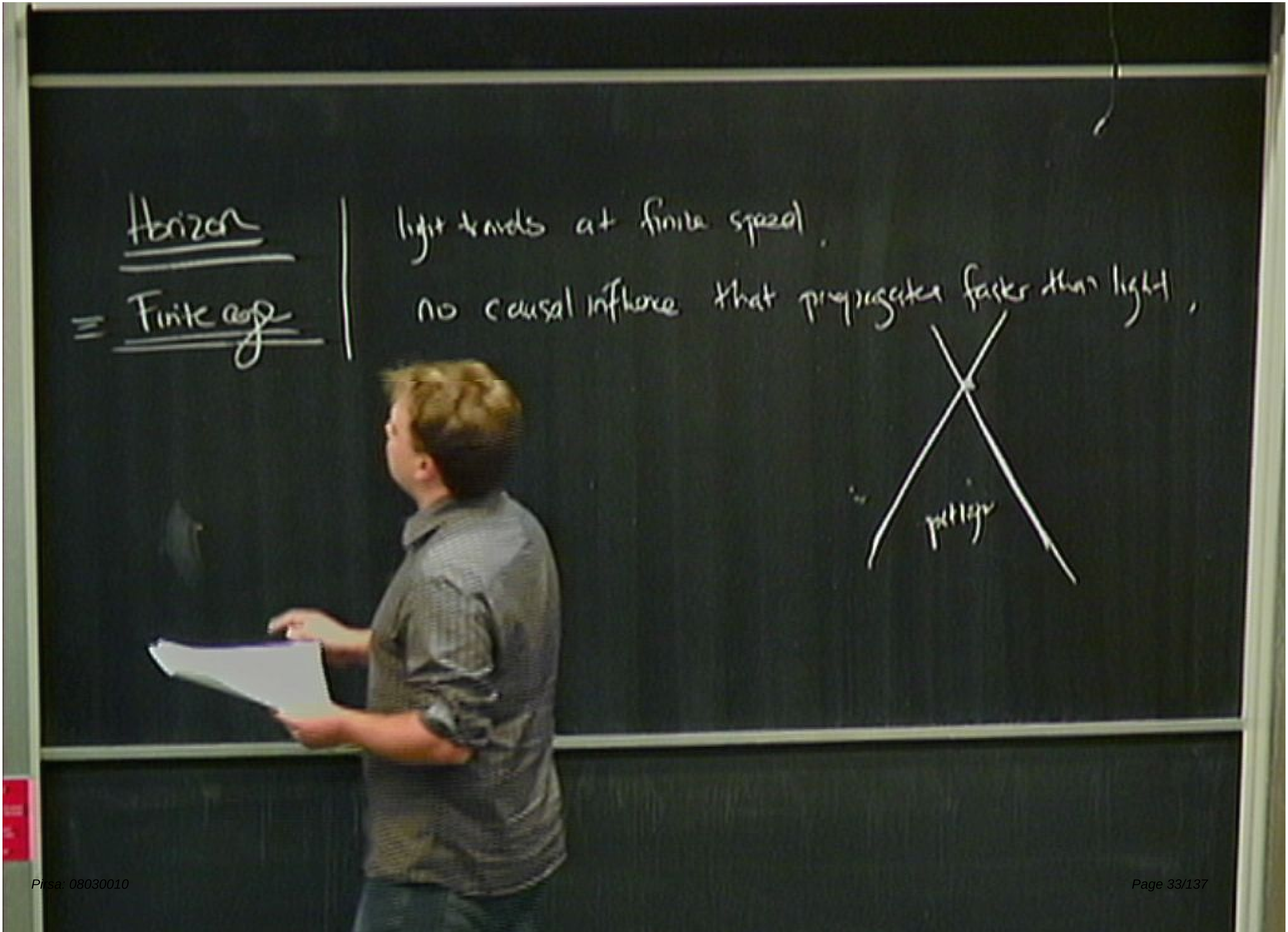
Penrose
Hawking

⇒ Singularity theorems
✓Reasonable conditions on energy density ⇒ singularities

Horizon

$\equiv$ Finite age


Horizon
≡ Finite age
light travels at finite speed.
no causal influence that propagates faster than light.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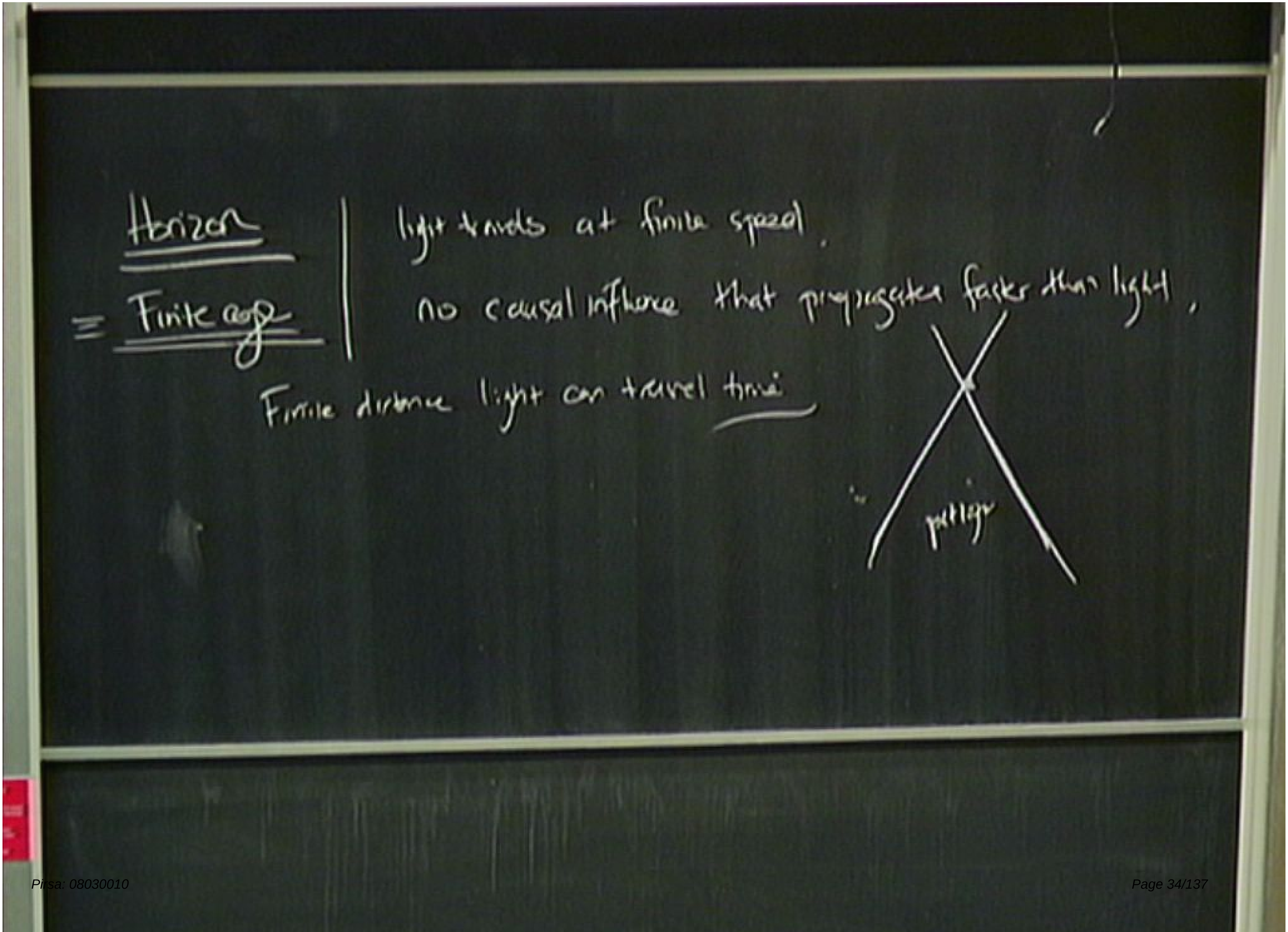
Horizon
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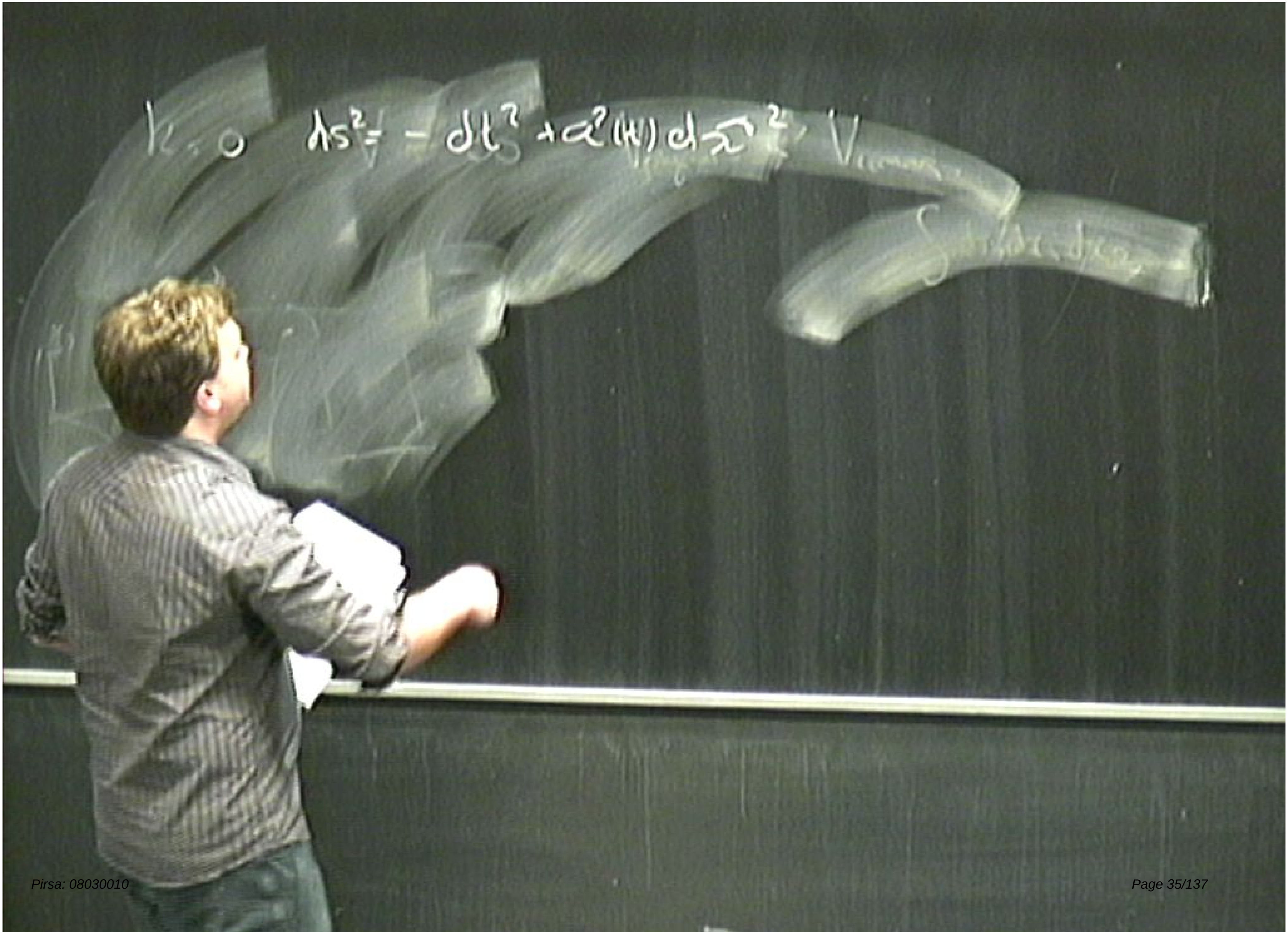
Finite distance light can travel time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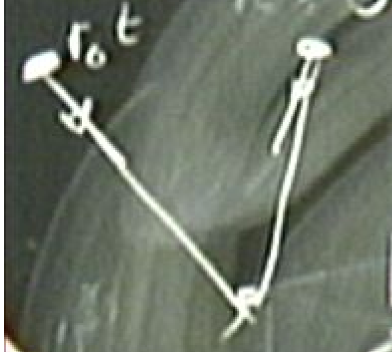
light null rays $\boxed{ds^2 = 0}$



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light null rays

$$\boxed{ds^2 = 0}$$

$\theta = \text{constant}$ $\quad \phi = \text{constant}$

$$dt = a(t)\, dr \quad \text{comoving}$$

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S^2

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$$dt = a(t)\, dr$$ comoving

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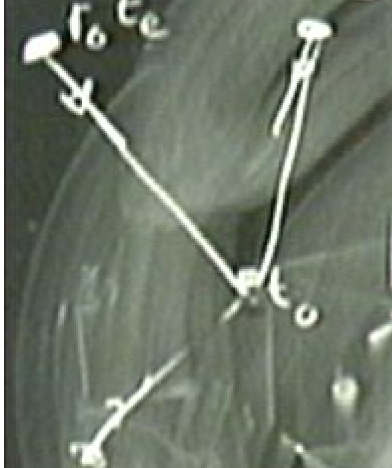

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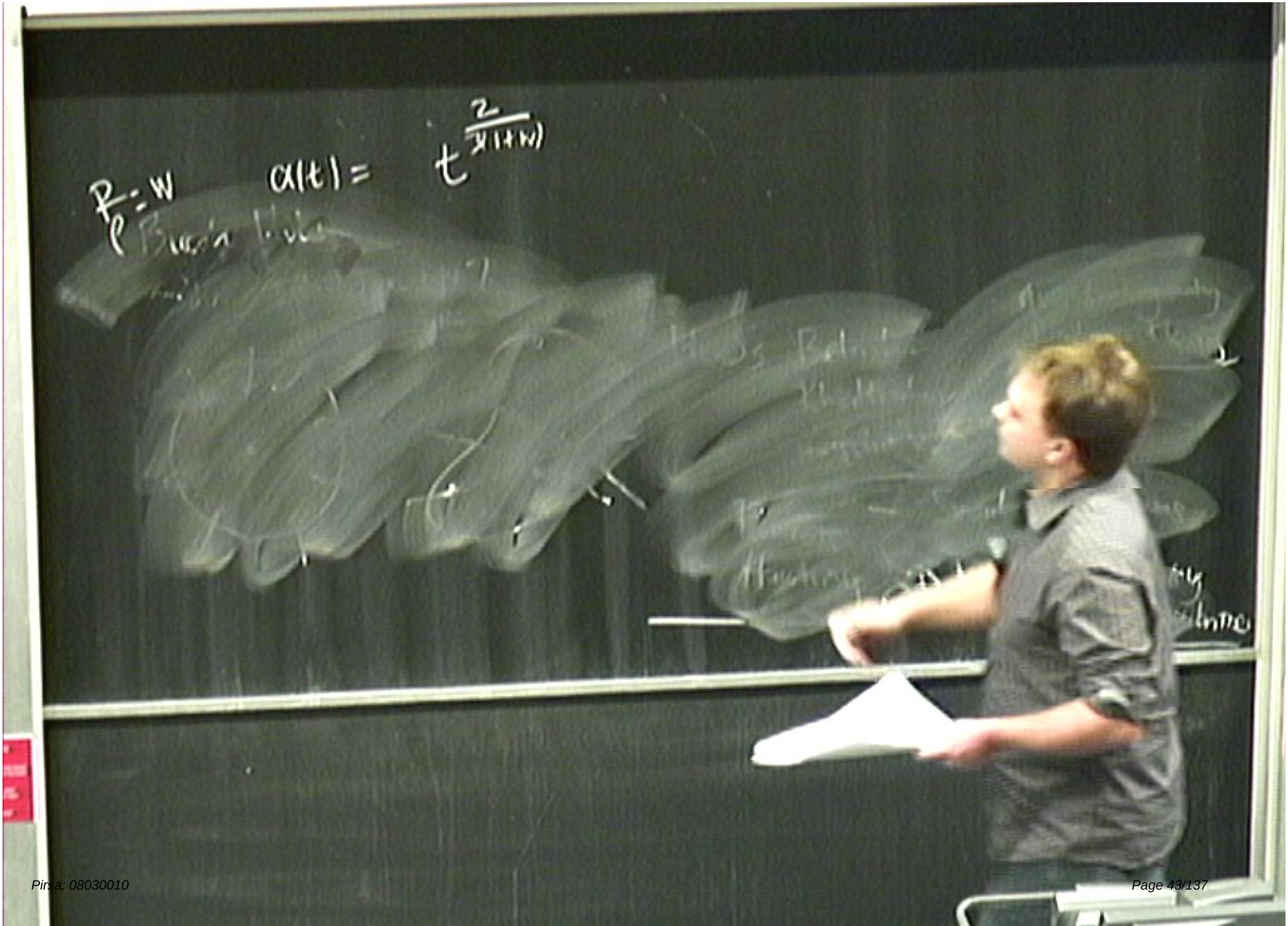
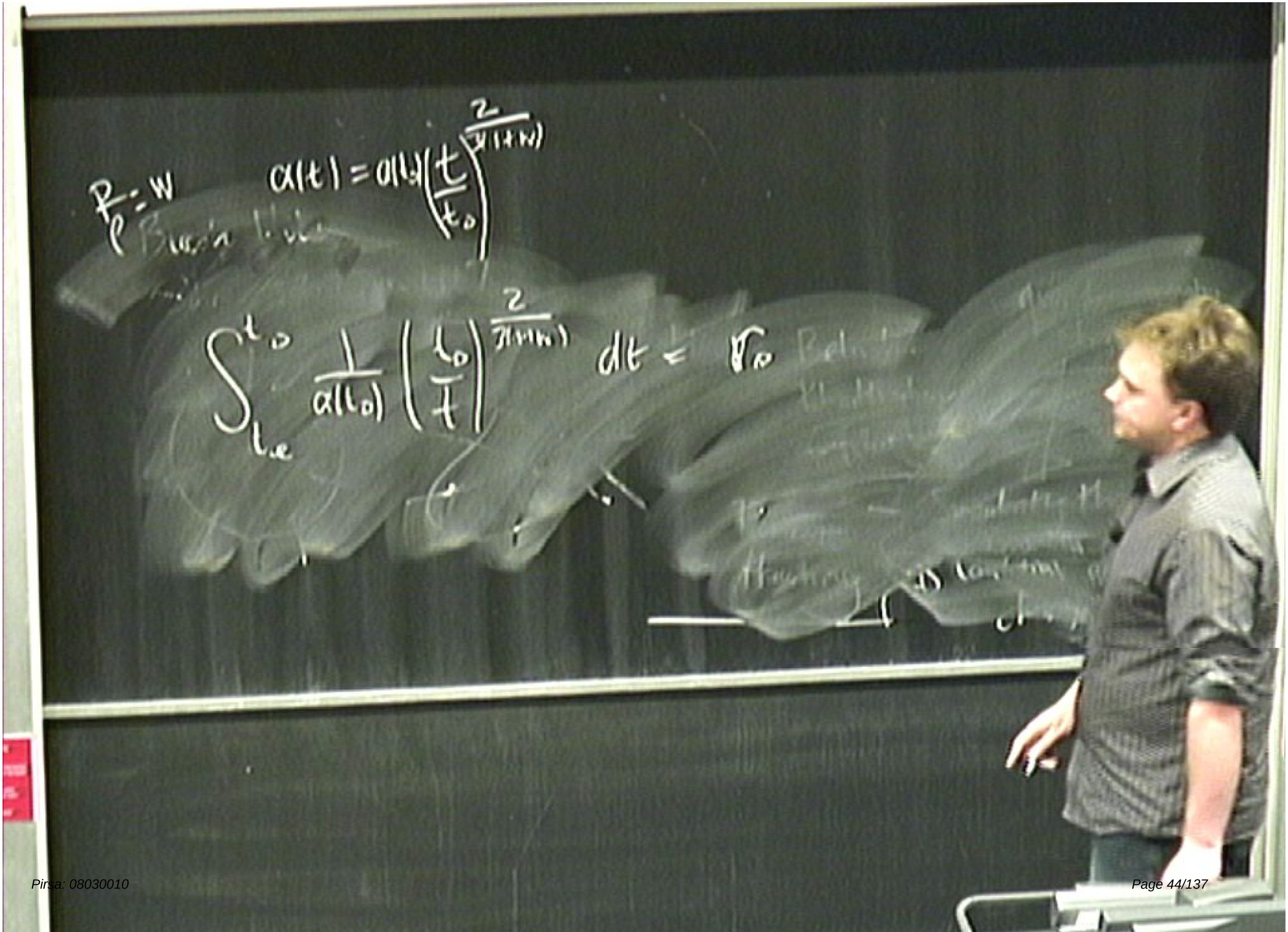
$$\int_{t_e}^{t_0} \frac{1}{a(t)}\, dt = -\int_{r_0}^{0} dr$$


$\frac{P}{\rho} = w$
$a(t) = t^{\frac{2}{3(1+w)}}$


$\frac{p}{\rho} = w$
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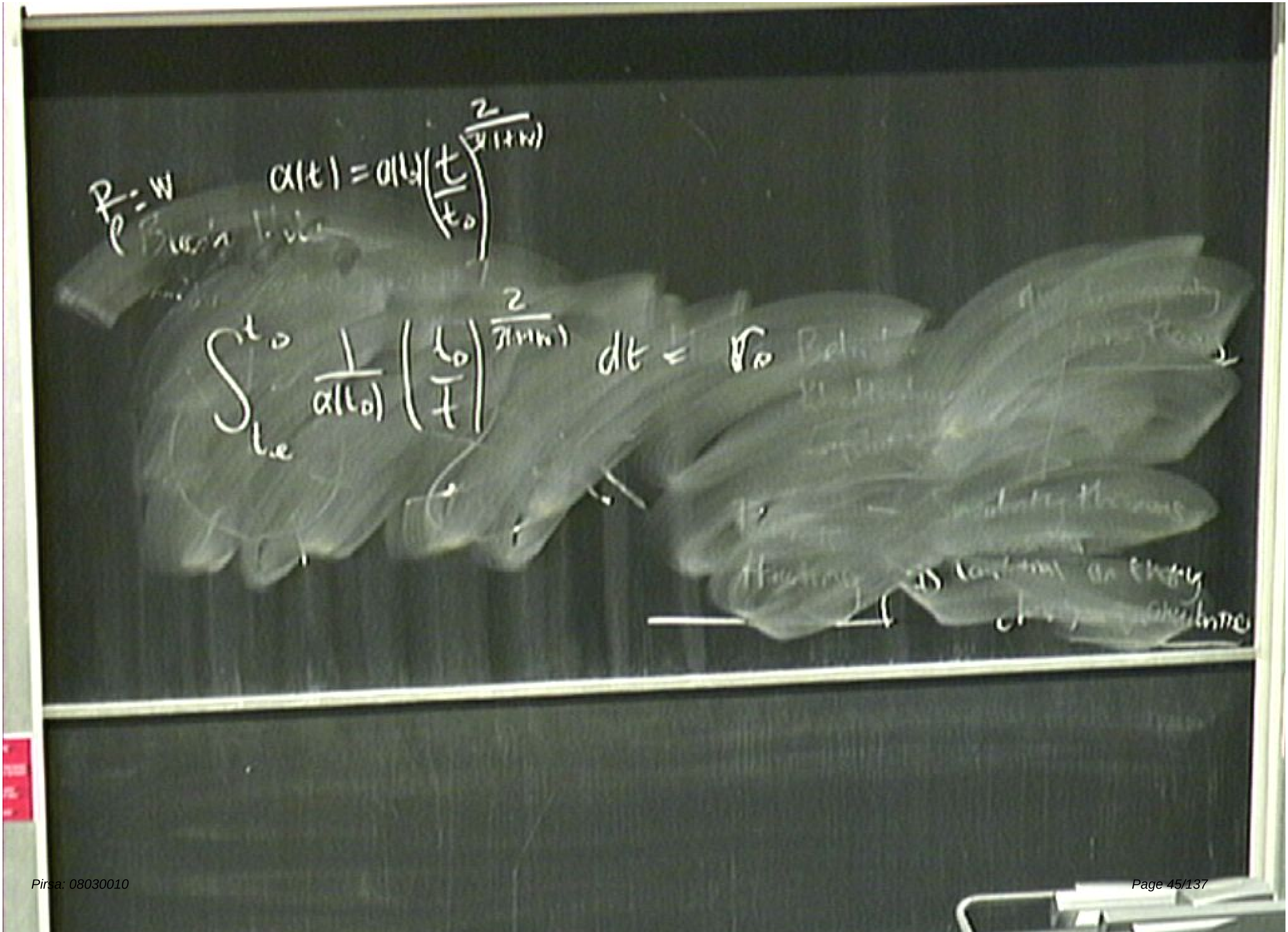
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$$\frac{c}{a(t_0)}\, t_0^{\frac{2}{3(1+w)}} \left[t^{\frac{1+3w}{3(1+w)}} \right]_{t_e}^{t_0}$$

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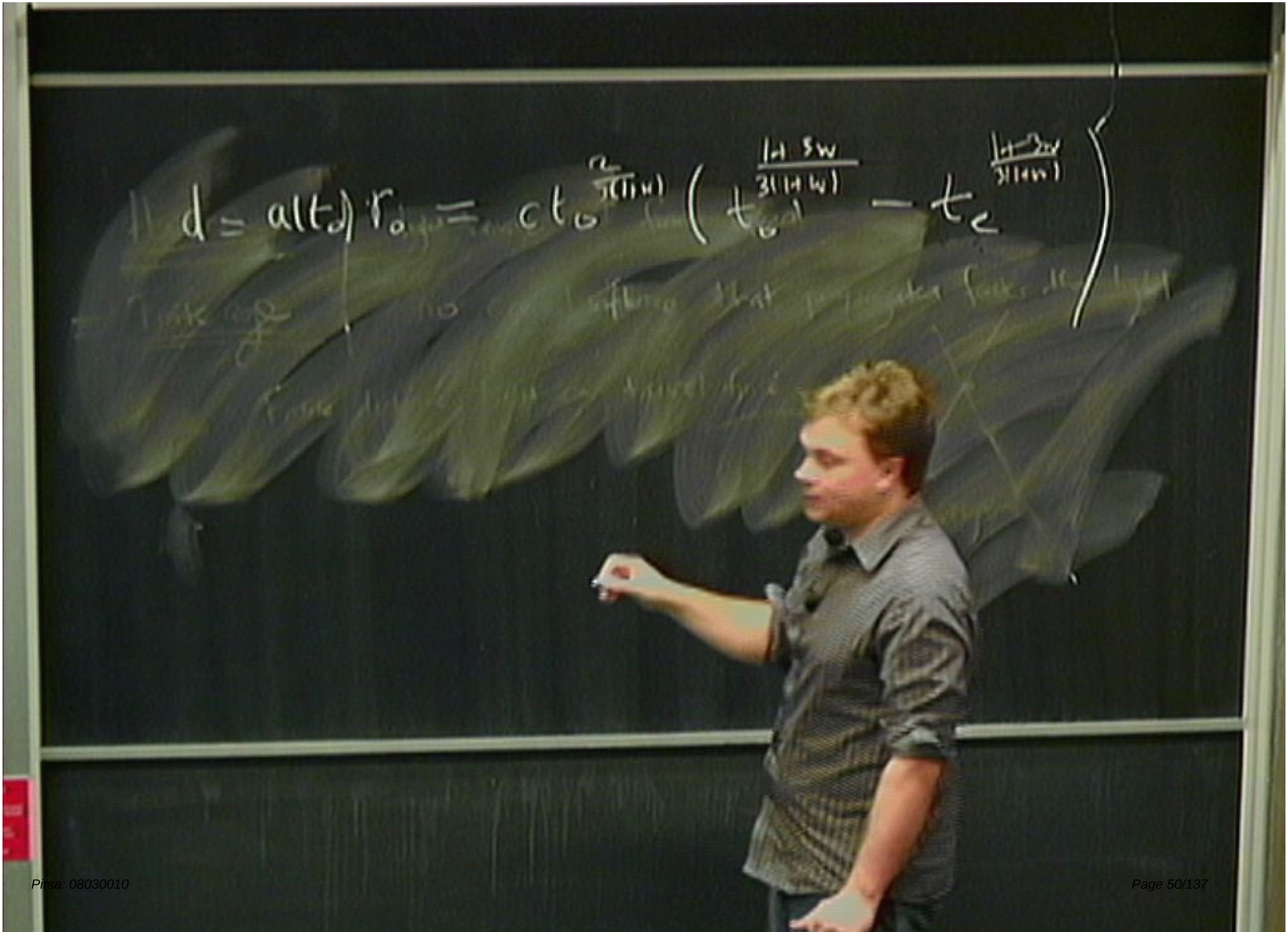
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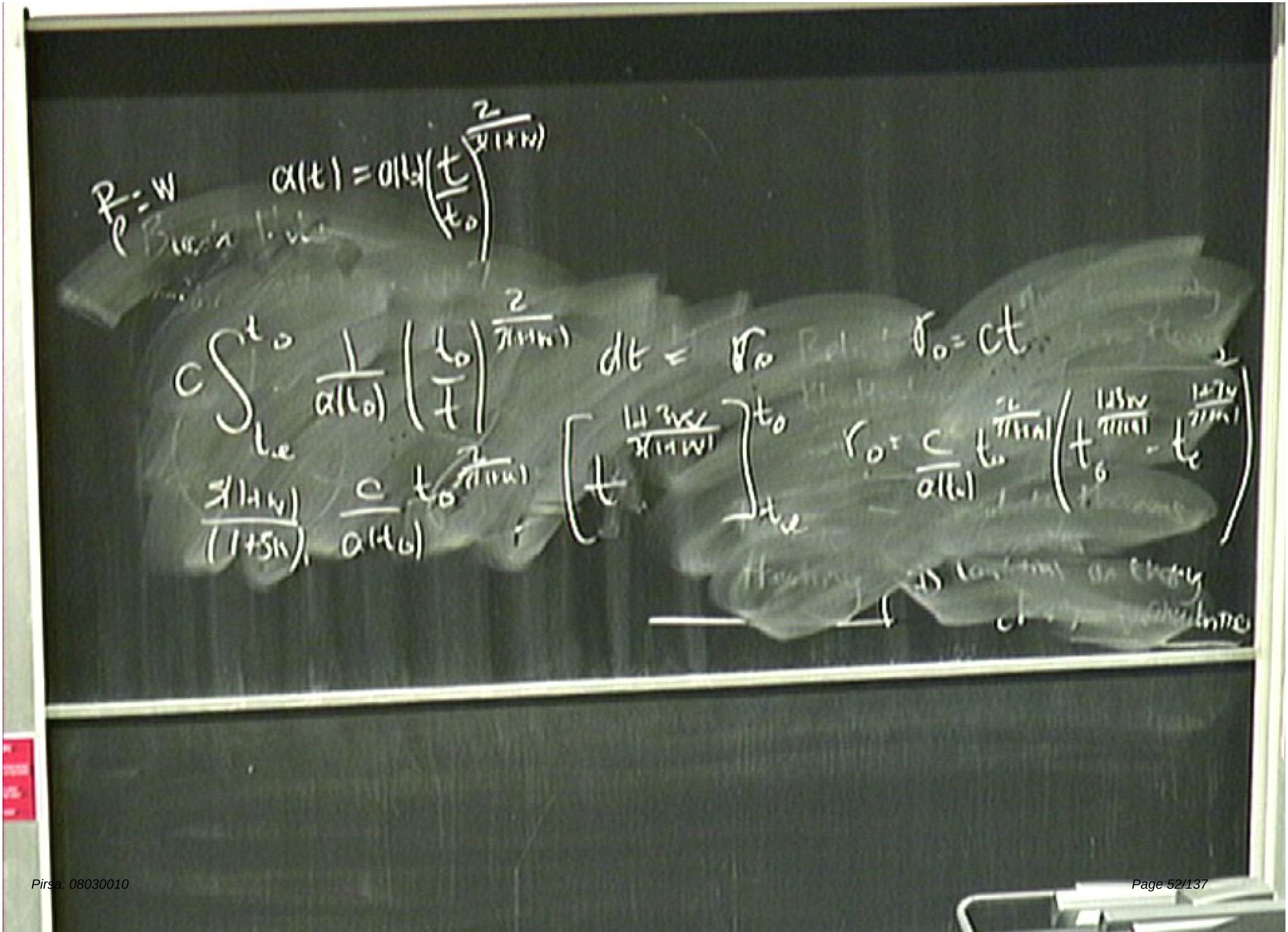
$$d = a(t_0)\, r_0 = c\, t_0^{\frac{2}{3(1+w)}} \left(t_0^{\frac{1+3w}{3(1+w)}} - \cancel{t_e^{\frac{1+3w}{3(1+w)}}} \right)$$

$$\underline{t_e = 0} \qquad =$$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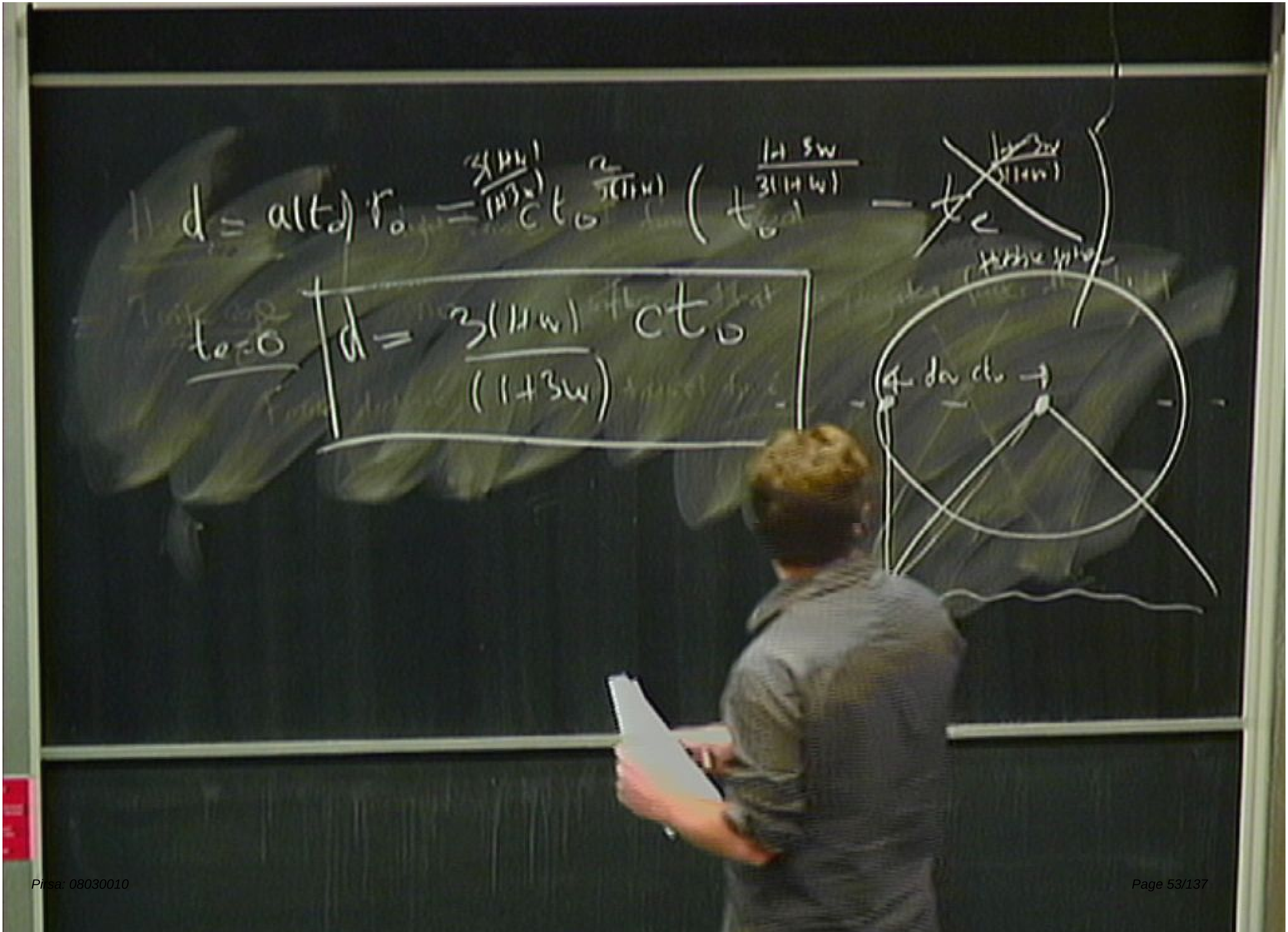
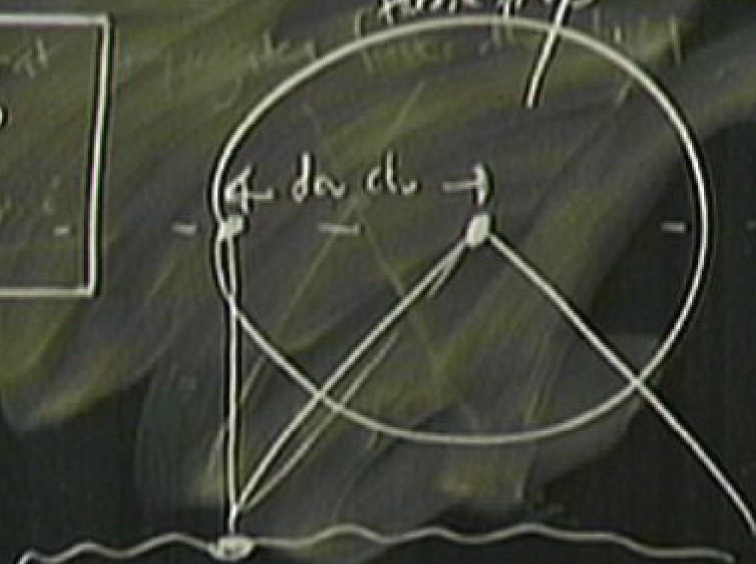
$r_0 = ct$
$\frac{3(1+w)}{(1+3w)} \frac{c}{a(t_0)} t_0^{\frac{2}{3(1+w)}} \left[t^{\frac{1+3w}{3(1+w)}}\right]_{t_e}^{t_0}$
$r_0 = \frac{c}{a(t_0)} t_0^{\frac{2}{3(1+w)}}\left(t_0^{\frac{1+3w}{3(1+w)}} - t_e^{\frac{1+3w}{3(1+w)}}\right)$


d = a(t_0) r_0 = 3(1+w)/(1+3w) c t_0^{2/3(1+w)} (t_0^{(1+3w)/3(1+w)} − t_e^{(1+3w)/3(1+w)})
t_e = 0
d = 3(1+w)/(1+3w) c t_0
d_H c t_0

$$d = a(t_0)\, r_0 = \frac{3(1+w)}{(1+3w)}\, c\, t_0^{\frac{2}{3(1+w)}} \left(t_0^{\frac{1+3w}{3(1+w)}} - \cancel{t_e^{\frac{1+3w}{3(1+w)}}} \right)$$

$t_e = 0$

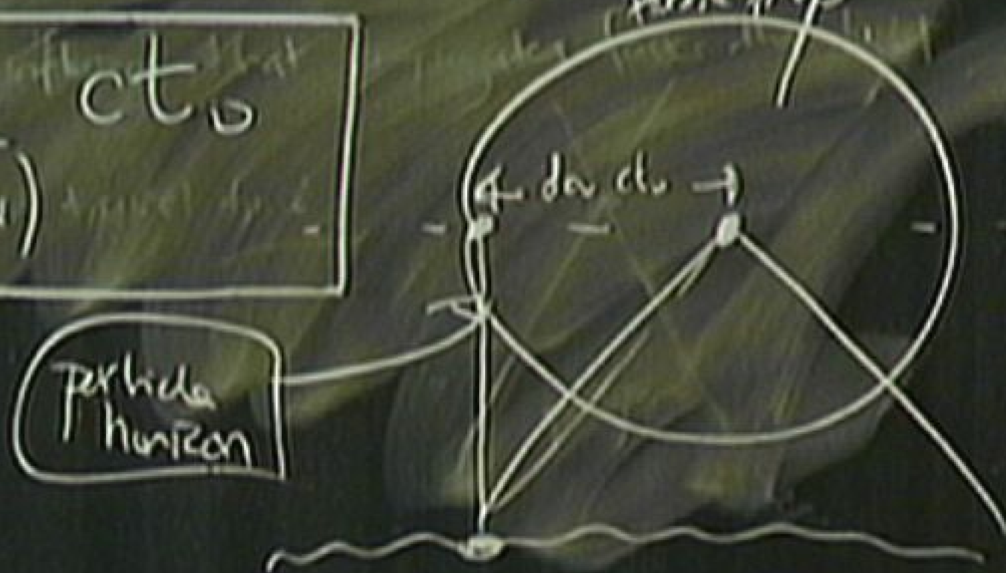
$$\boxed{d = \frac{3(1+w)}{(1+3w)}\, c\, t_0}$$

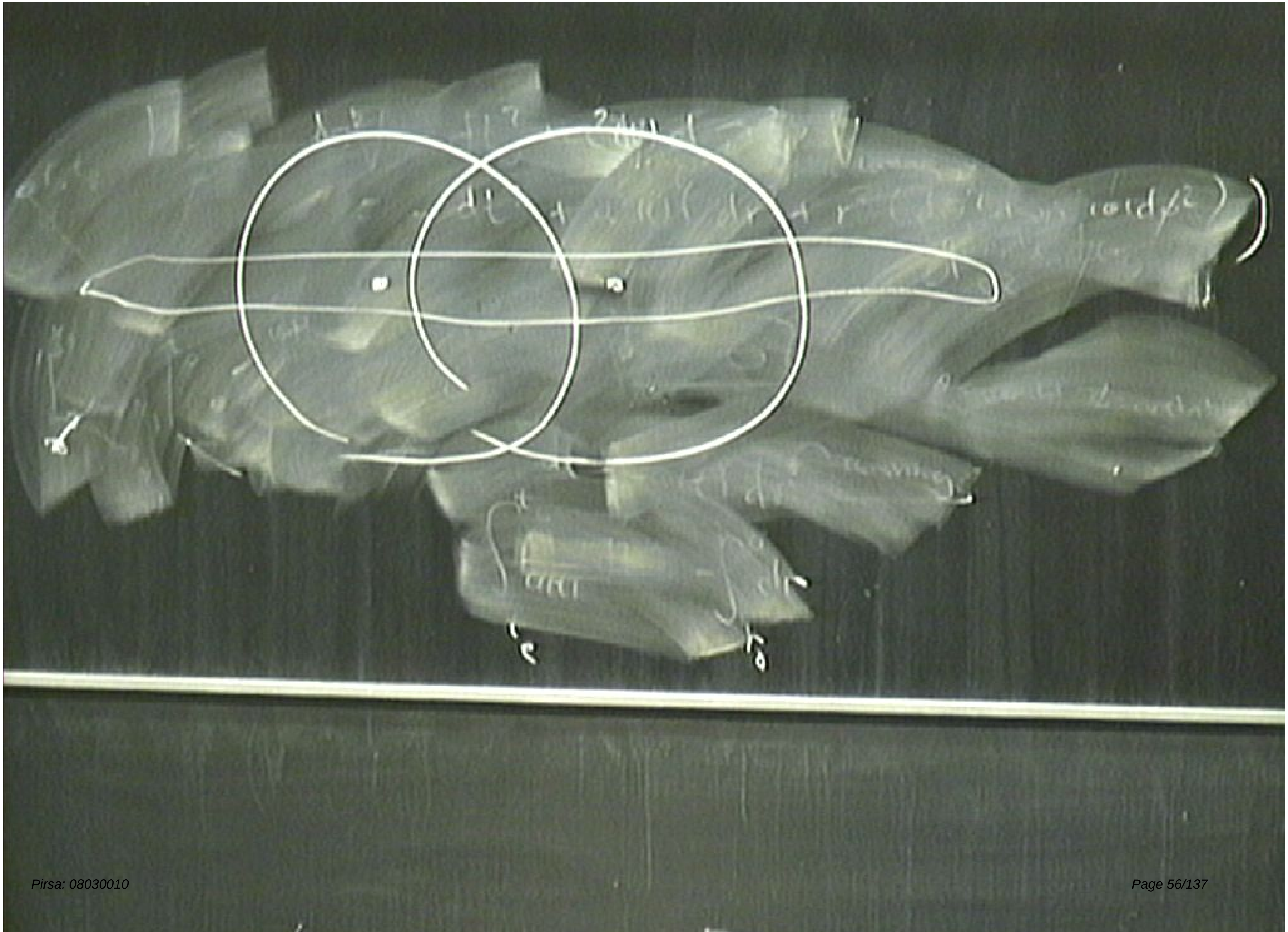


$$d = a(t_0)\, r_0 = \frac{3(1+w)}{(1+3w)}\, c\, t_0^{\frac{2}{3(1+w)}} \left(t_0^{\frac{1+3w}{3(1+w)}} - \cancel{t_e^{\frac{1+3w}{3(1+w)}}} \right)$$

$t_e = 0$

$$\boxed{d = \frac{3(1+w)}{(1+3w)}\, c\, t_0}$$





$$d_H = a(t_0) \int_0^{t_0} \frac{dt}{a(t)}$$

$$d_H = a(t_0) \int_0^{t_0} \frac{dt}{a(t)}$$

$$d_H = a(t_0)\int_0^{t_0} \frac{dt}{a(t)}$$

whole history of universe.

$$d_H = a(t_0)\int_0^{t_0} \frac{dt}{a(t)}$$

whole history of universe.

Integral finite only if it converges

$$a(t) \sim t^2$$

$$d_H = a(t_0) \int_0^{t_0} \frac{dt}{a(t)}$$ whole history of universe.

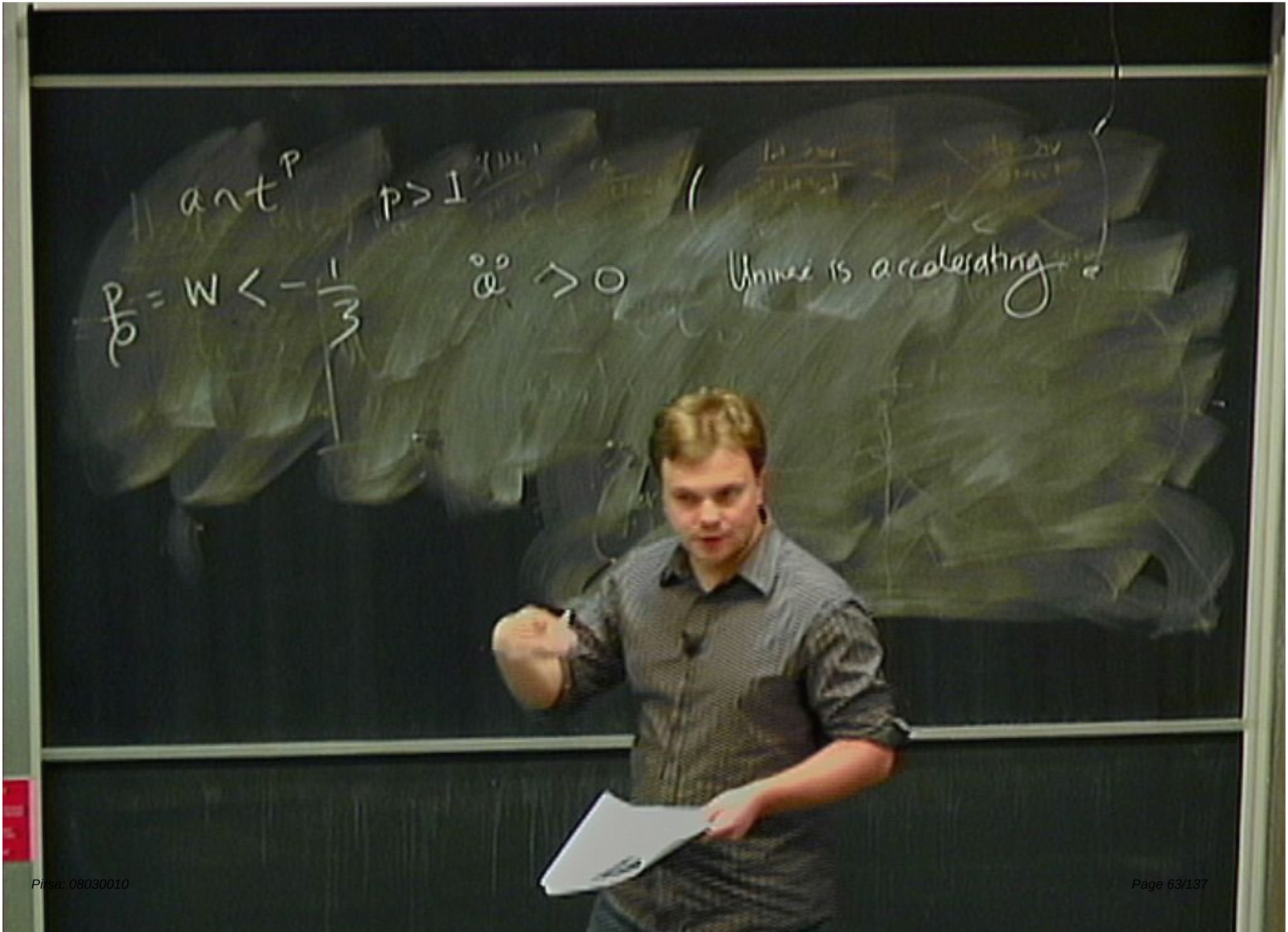
Integral finite only if it converges.

$$a(t) \sim t^2$$

$$d_H = a(t_0)\int_0^{t_0} \frac{dt}{a(t)}$$ whole history of universe.

Integral finite only if it converges

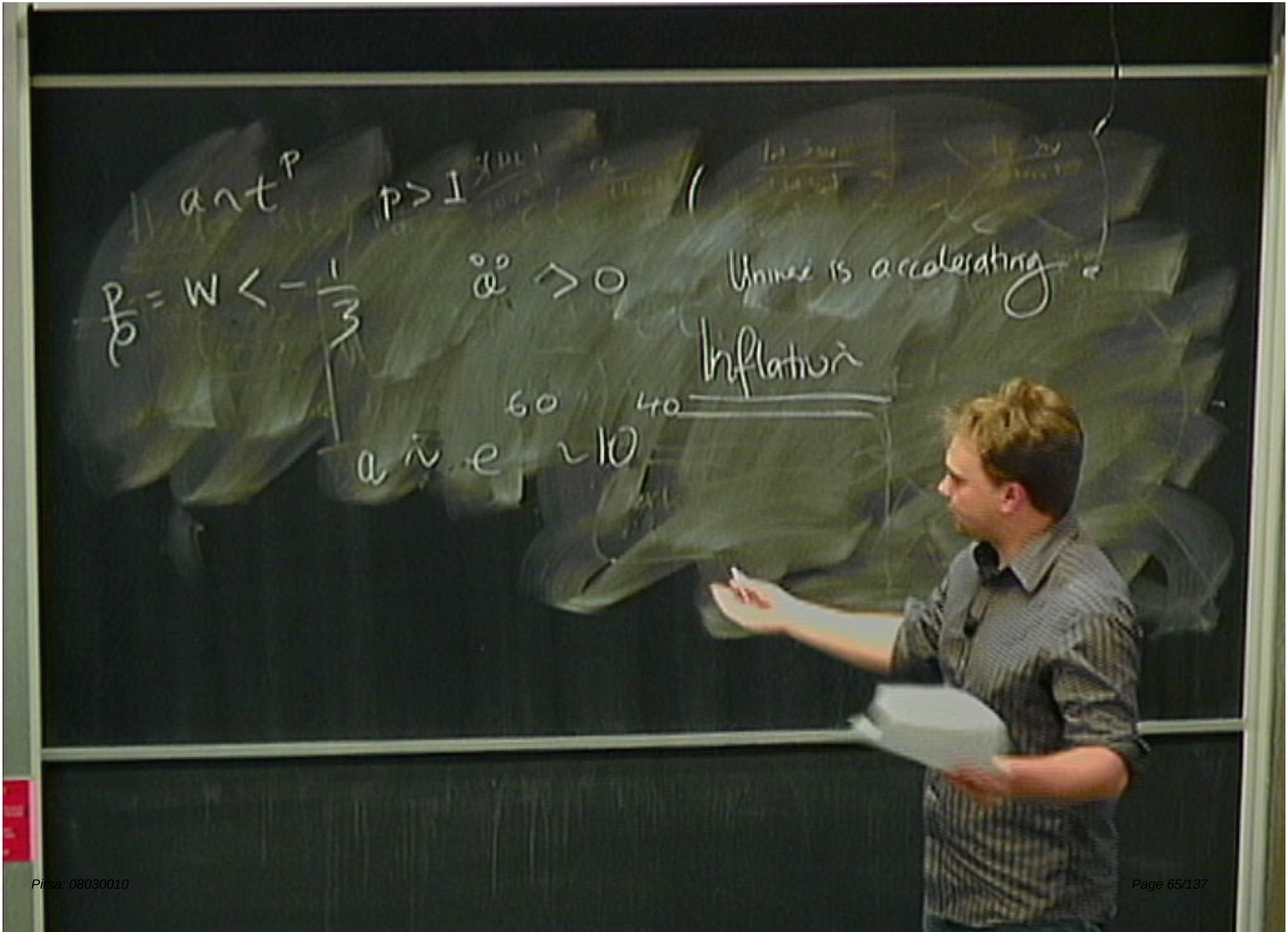
$$a(t) \sim t^2 \qquad \int_0^{t_0} \frac{dt}{t^2} \sim \frac{1}{t} = \infty$$


a ∝ t^p   p > 1
p/ρ = w < −1/3
ä > 0
Universe is accelerating

$a \sim t^p \quad p > 1$

$\frac{p}{\rho} = w < -\frac{1}{3} \quad \ddot{a} > 0$ Universe is accelerating

Inflation


a ~ t^p
p > 1
p/ρ = w < -1/3
ä > 0
Universe is accelerating
Inflation
a ~ e^60 ~ 10^40

$a \sim t^p \quad p > 1$

$\frac{p}{\rho} = w < -\frac{1}{3} \qquad \ddot{a} > 0 \qquad$ Universe is accelerating

Inflation

$a \sim e^{60} \sim 10^{40}$

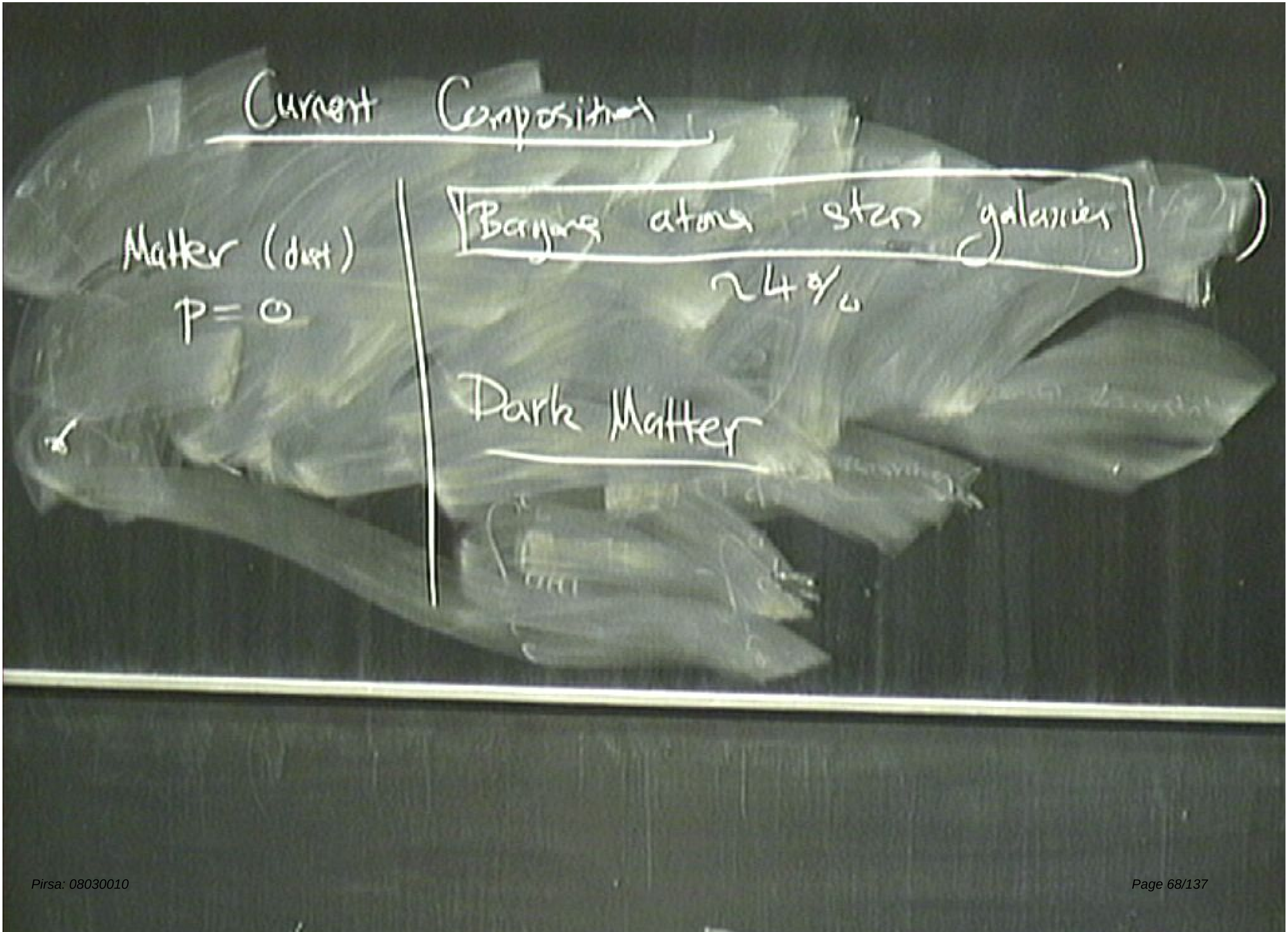
$\boxed{V \sim 10^{100}} \qquad \boxed{t \sim 10^{-30}\, s}$

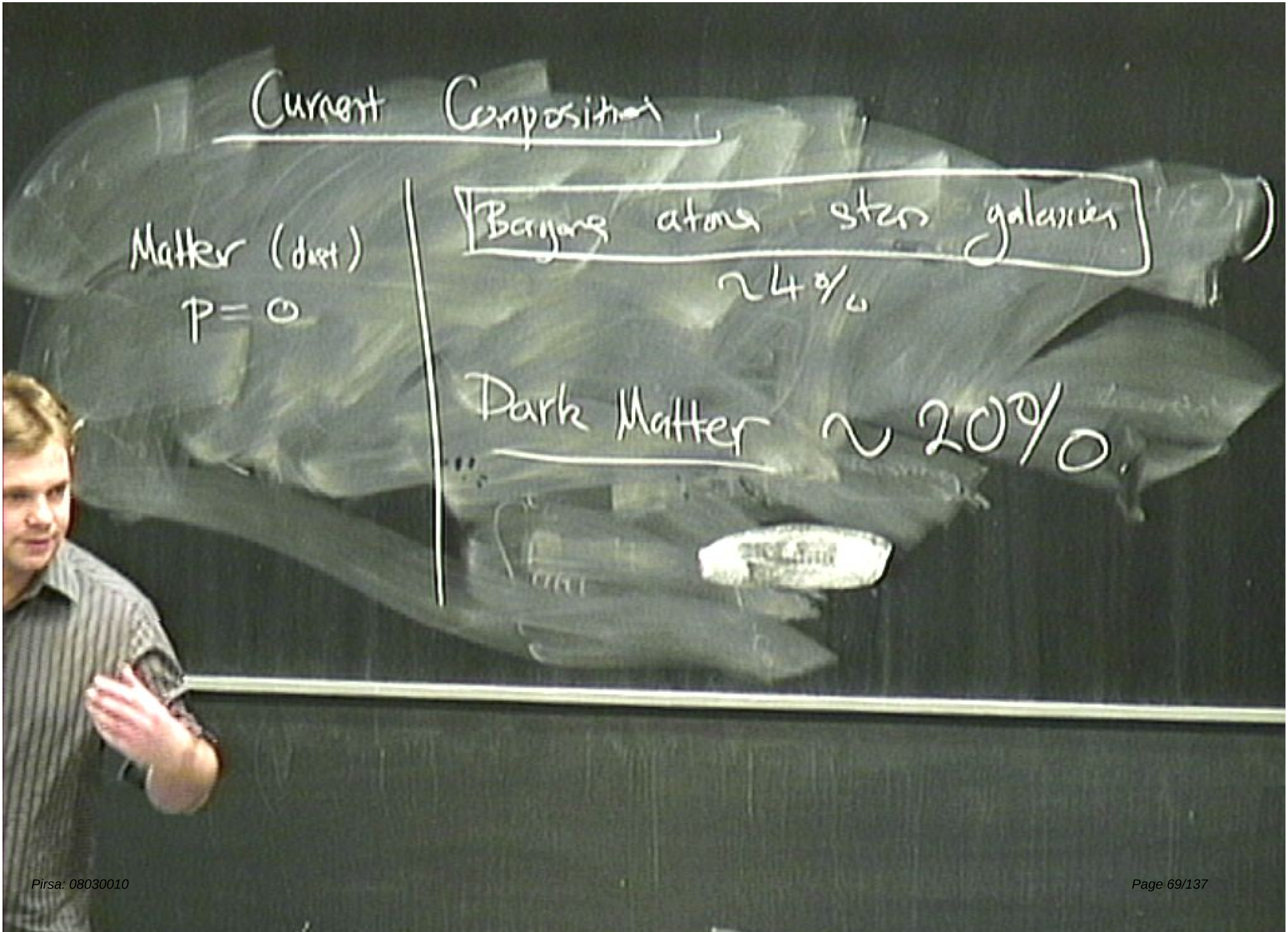
Current Composition

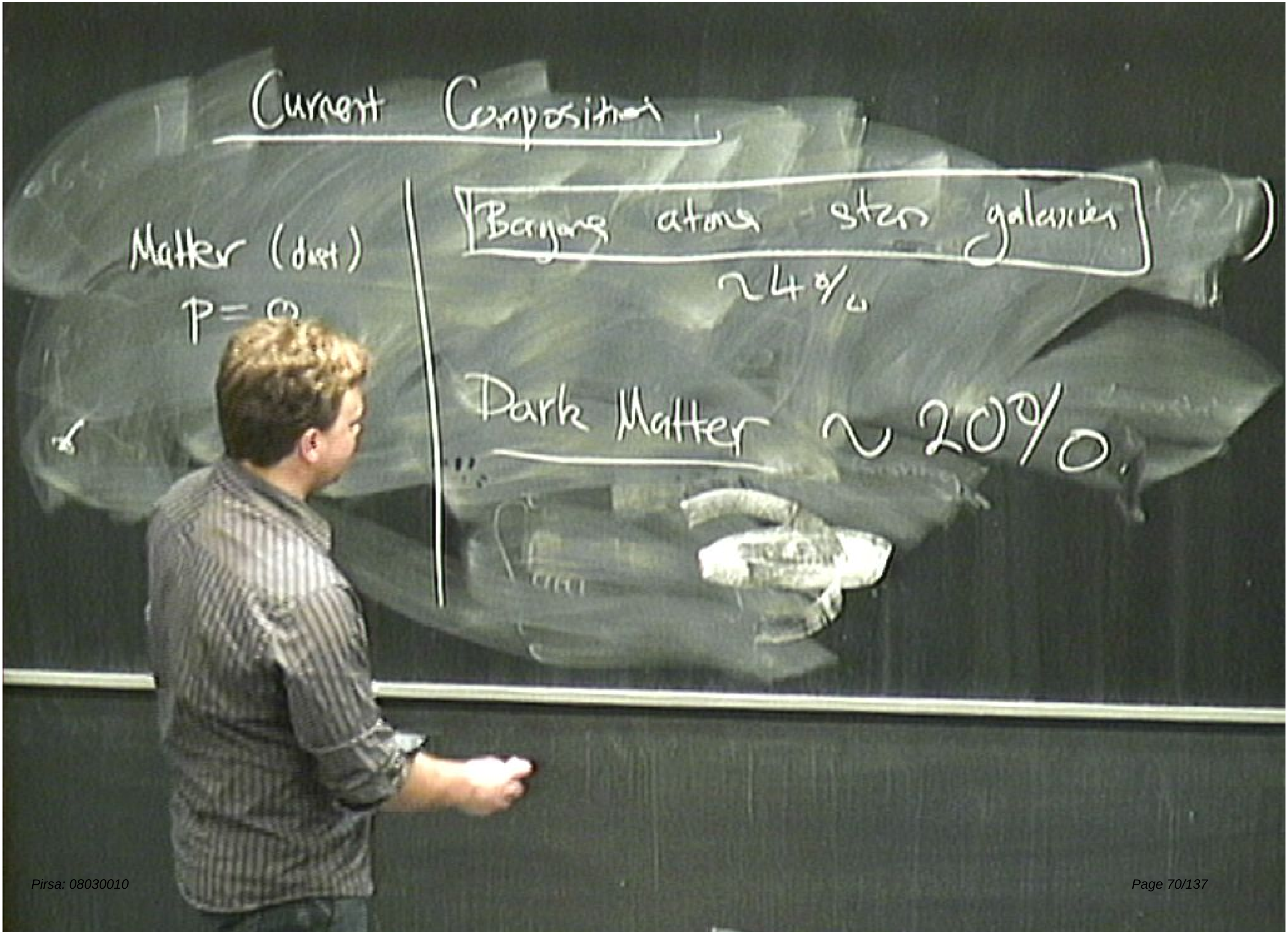
Matter (dust)

$p = 0$

Baryons atoms stars galaxies)


Current Composition
Matter (dust)
P = 0
Baryons atoms stars galaxies
~4%
Dark Matter


Current Composition
Matter (dust)
p = 0
Baryons atoms stars galaxies
~4%
Dark Matter ~ 20%


Current Composition
Matter (dust)
P=
Baryons atoms stars galaxies
~4%
Dark Matter ~ 20%

Current Composition

Matter (dust)

$P = 0$

$\boxed{\text{Baryons atoms stars galaxies}}$

$\sim 4\%$

dark matter halo

Dark Matter $\sim 20\%$

Current Composition

Matter (dust)

$P = 0$

24%

Baryons atoms stars galaxies

~4%

dark matter halo

Dark Matter ~ 20%

Radiation

$p = \frac{1}{3}\rho$

$w = \frac{1}{3}$ $\quad \rho \propto T^4$

$< .1\%$


Radiation
Cosmic Microwave Background

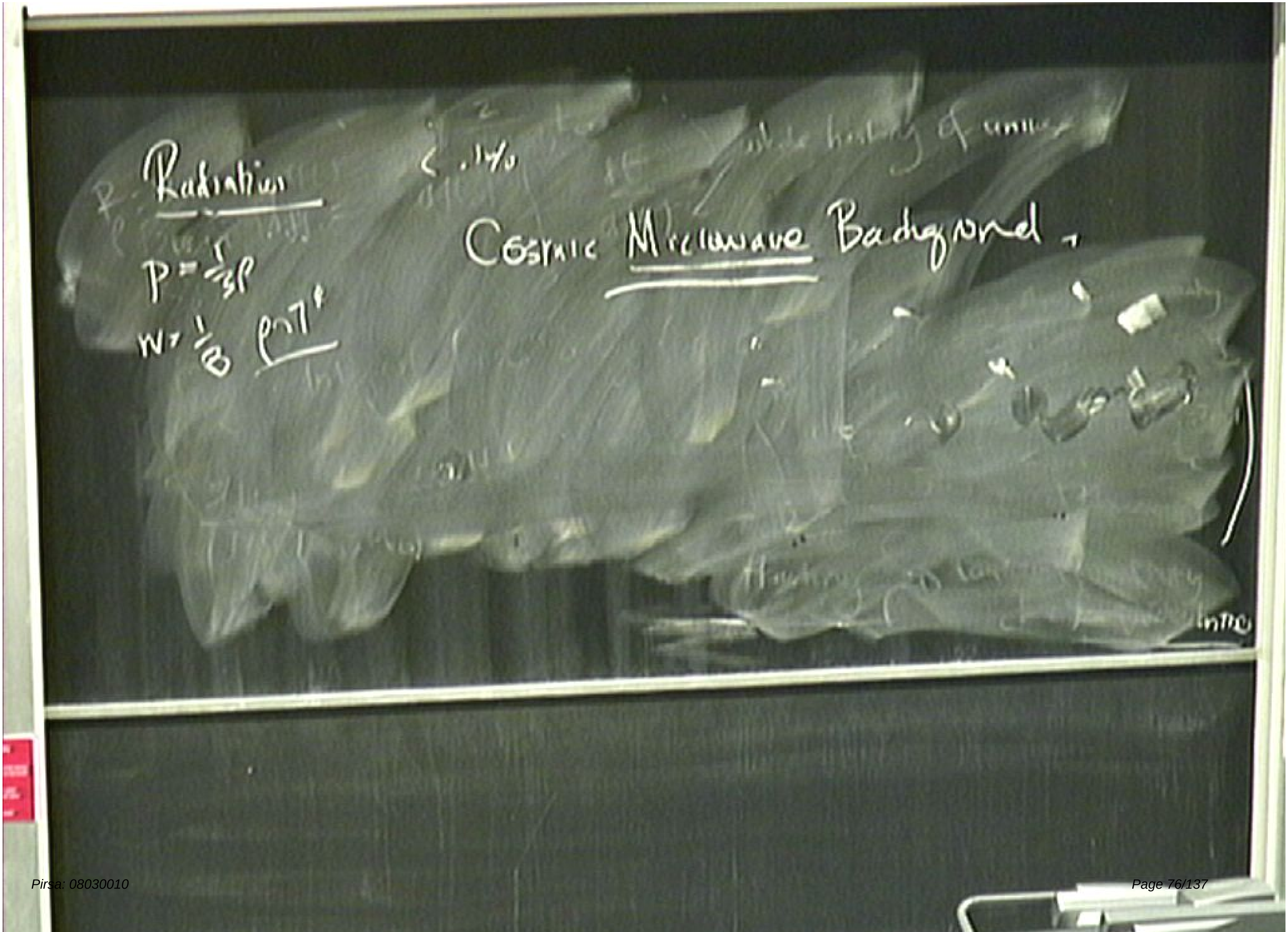
Radiation

$p = \frac{1}{3}\rho$

$w = \frac{1}{3}$ $\rho \propto T^4$

$< .1\%$

Cosmic Microwave Background


Radiation
P = 1/3 ρ
w = 1/3
ρ ∝ T^4
< .1%
Cosmic Microwave Background

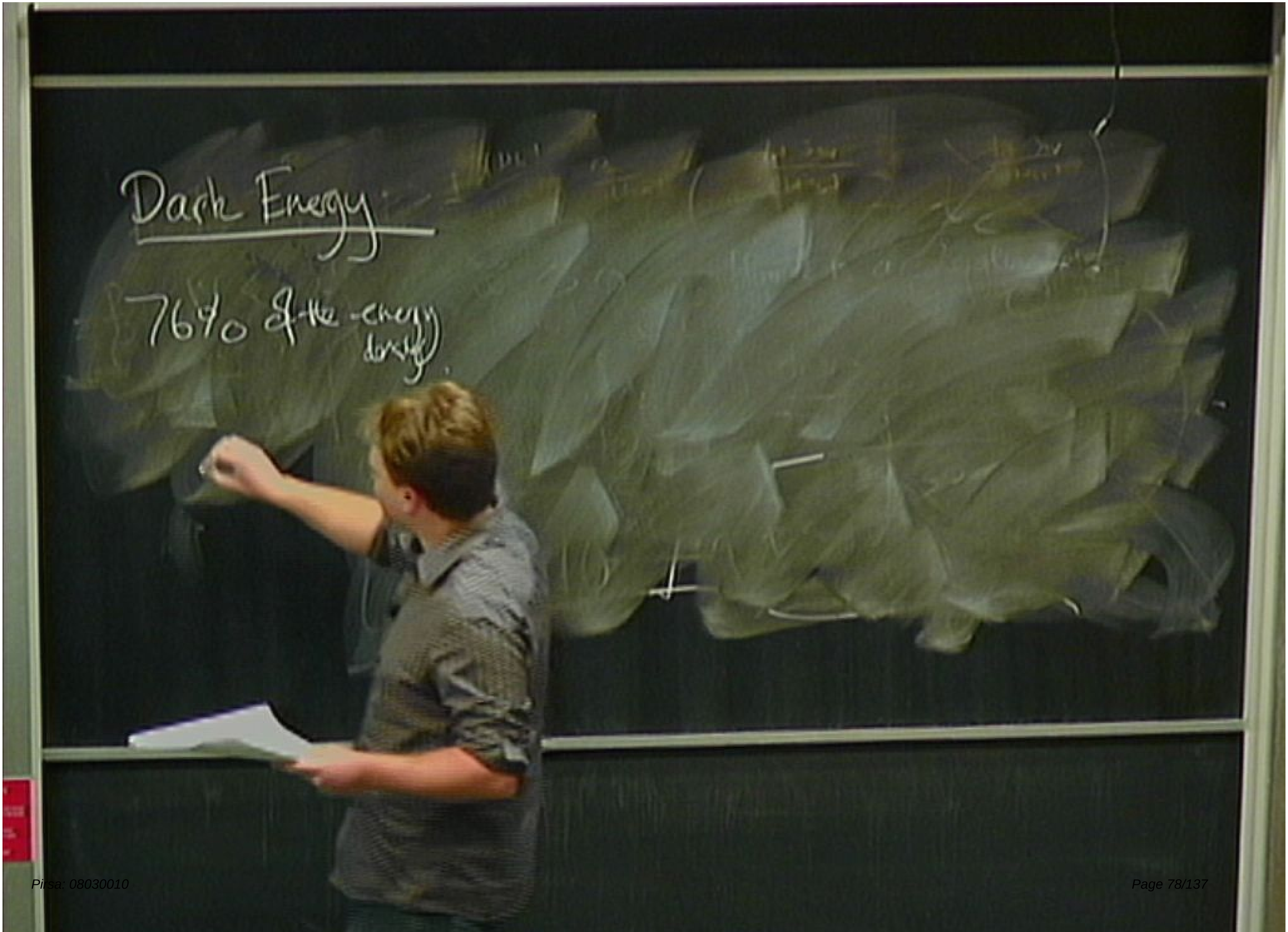
Radiation

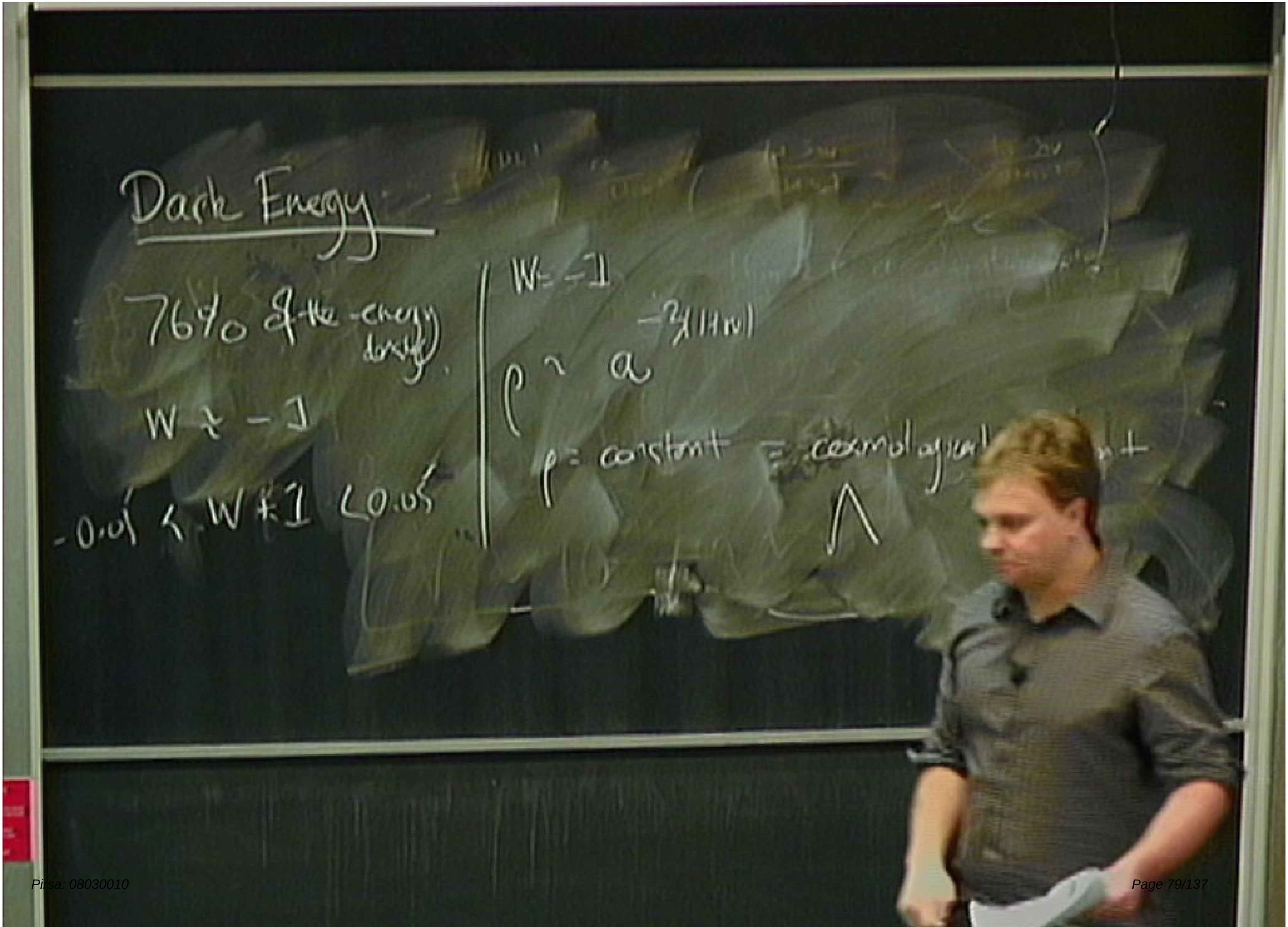
$p = \frac{1}{3}\rho$

$w = \frac{1}{3}$ $\rho \sim T^4$

$< .1\%$

Cosmic Microwave Background


Dark Energy
76% of the energy density


Dark Energy
76% of the energy density.
w ≈ -1
-0.05 < w+1 < 0.05
w = -1
ρ ~ a^-3(1+w)
ρ = constant = cosmological
Λ

Dark Energy

76% of the energy density.

$w \approx -1$

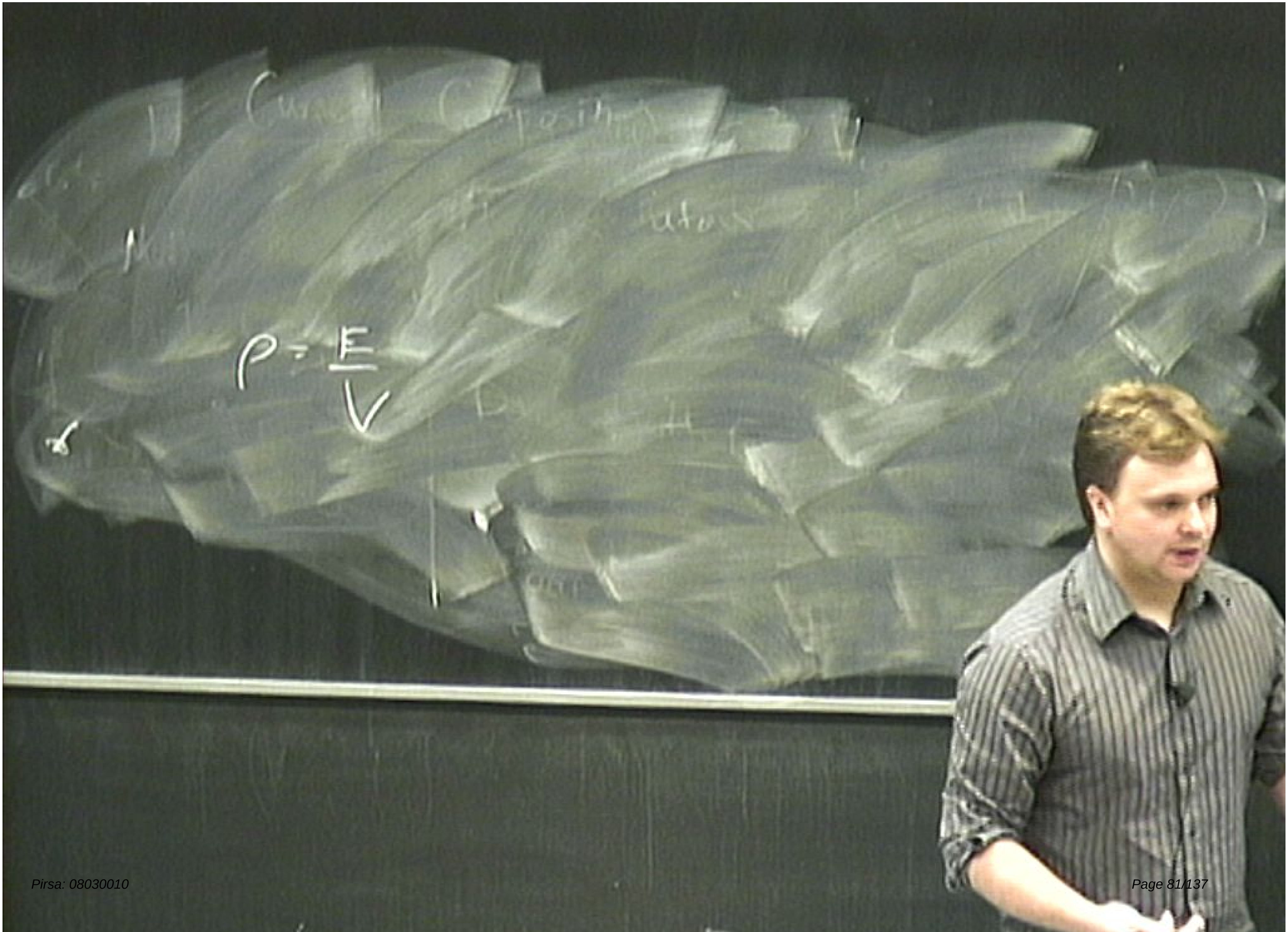
$-0.05 < w + 1 < 0.05$

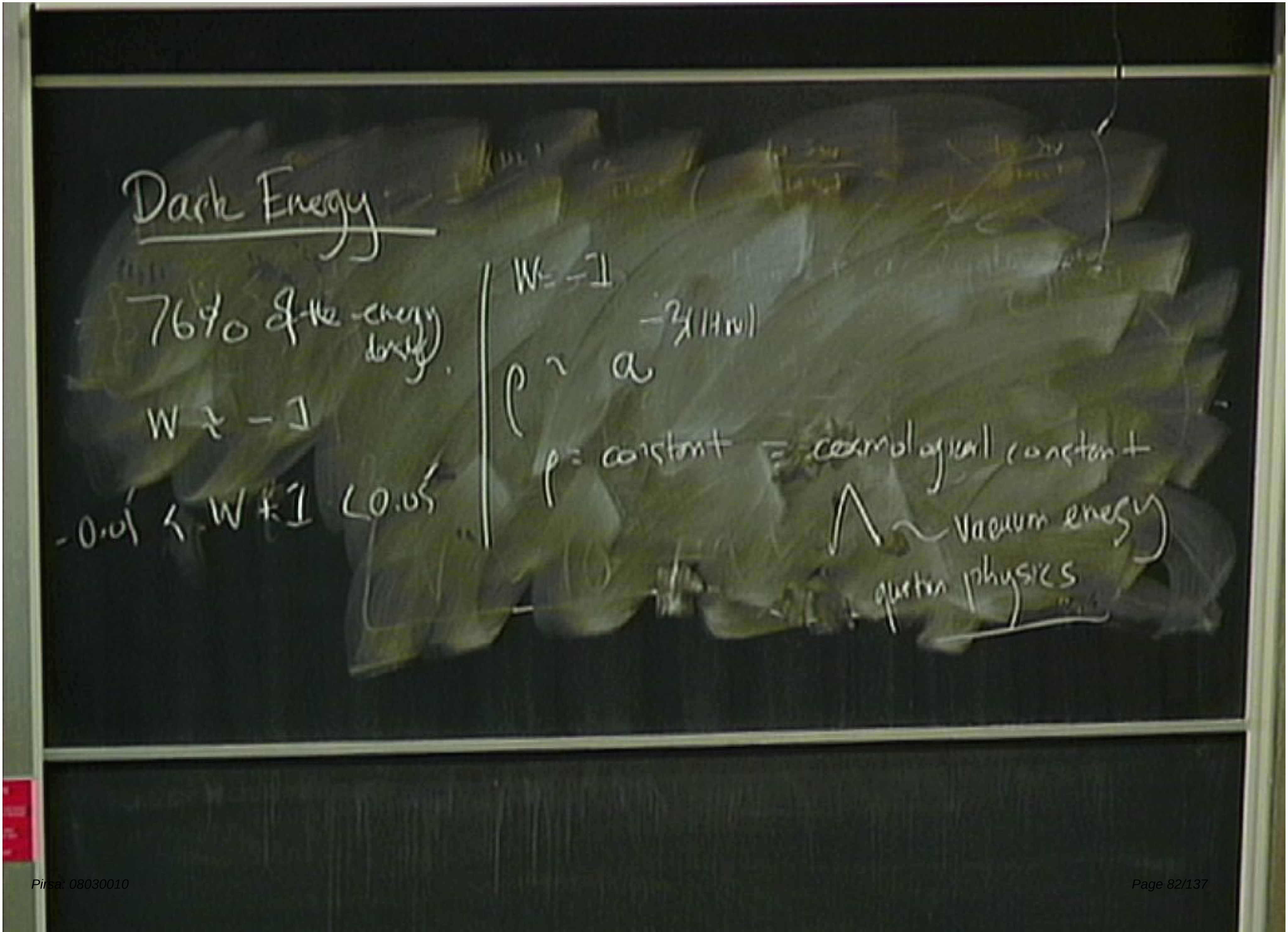
$w = -1$

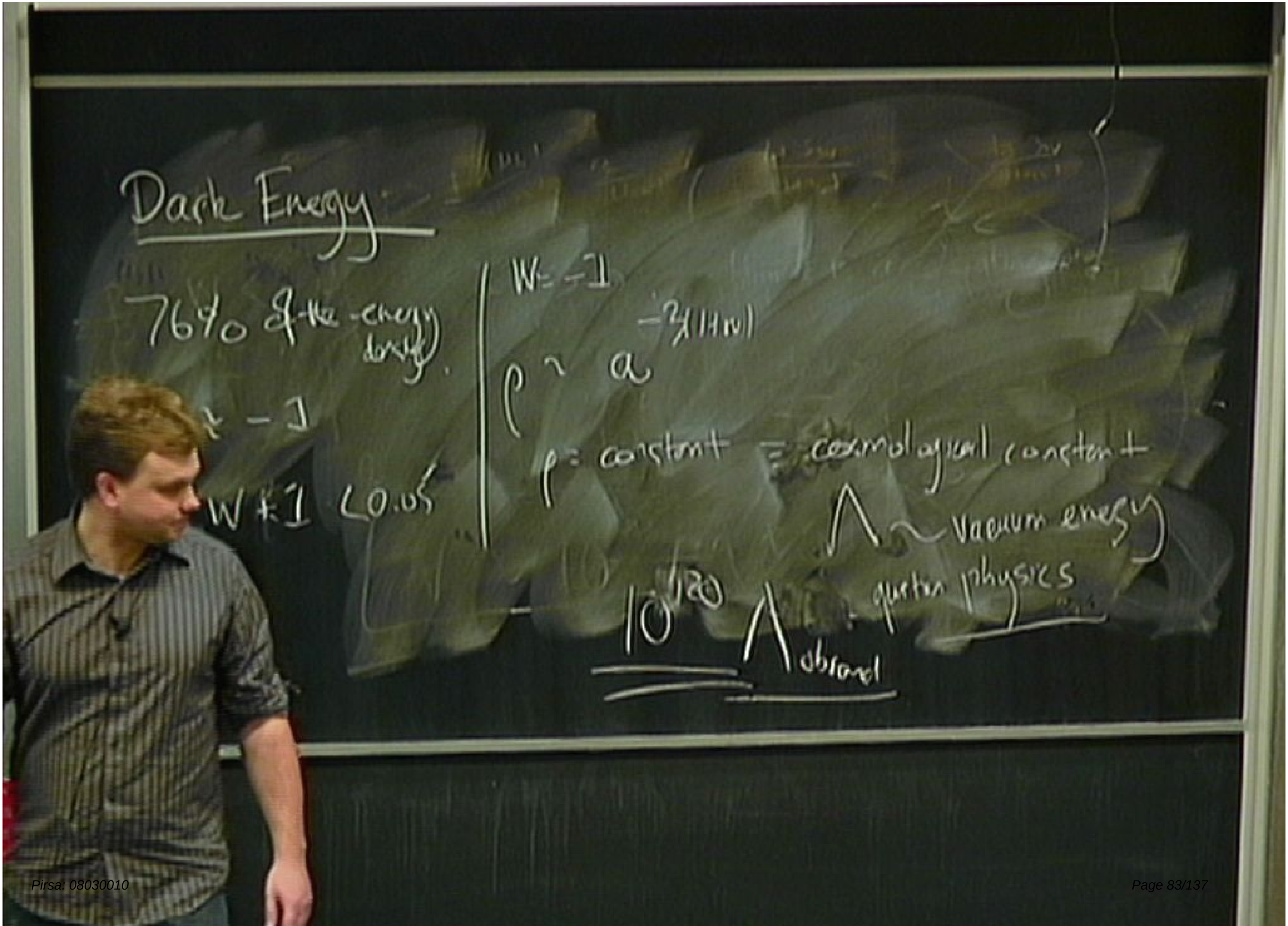
$\rho \sim a^{-3(1+w)}$

$\rho =$ constant $=$ cosmological constant

Λ


ρ = E/V


Dark Energy
76% of the energy density.
$w \approx -1$
$-0.05 < w+1 < 0.05$
$w = -1$
$\rho \sim a^{-3(1+w)}$
ρ = constant = cosmological constant
$\Lambda \sim$ vacuum energy
quantum physics


Dark Energy
76% of the energy density.
w ≈ -1
w+1 < 0.05
w = -1
ρ ~ a^-3(1+w)
ρ = constant = cosmological constant
Λ ~ vacuum energy
quantum physics
10^120 Λ_observed

Dark Energy

76% of the energy density.

$w \approx -1$

$-0.05 < w+1 < 0.05$

Cosmological Constant problem

$w = -1$

$\rho \sim a^{-3(1+w)}$

ρ = constant = cosmological constant

$\Lambda \sim$ vacuum energy

$10^{120} \Lambda_{\text{observed}}$ ~ quantum physics

Radiation

$< .1\%$

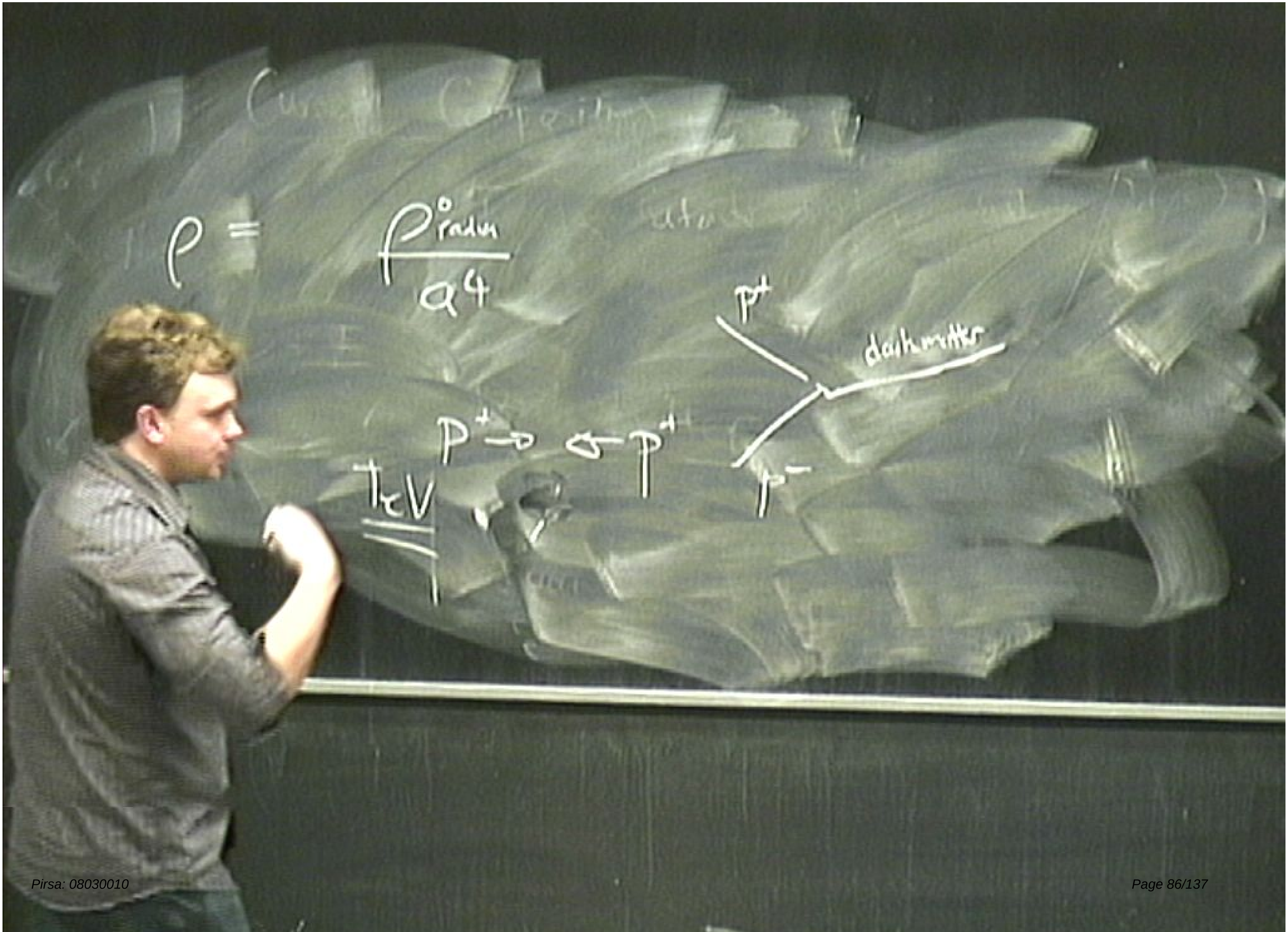
Cosmic Microwave Background

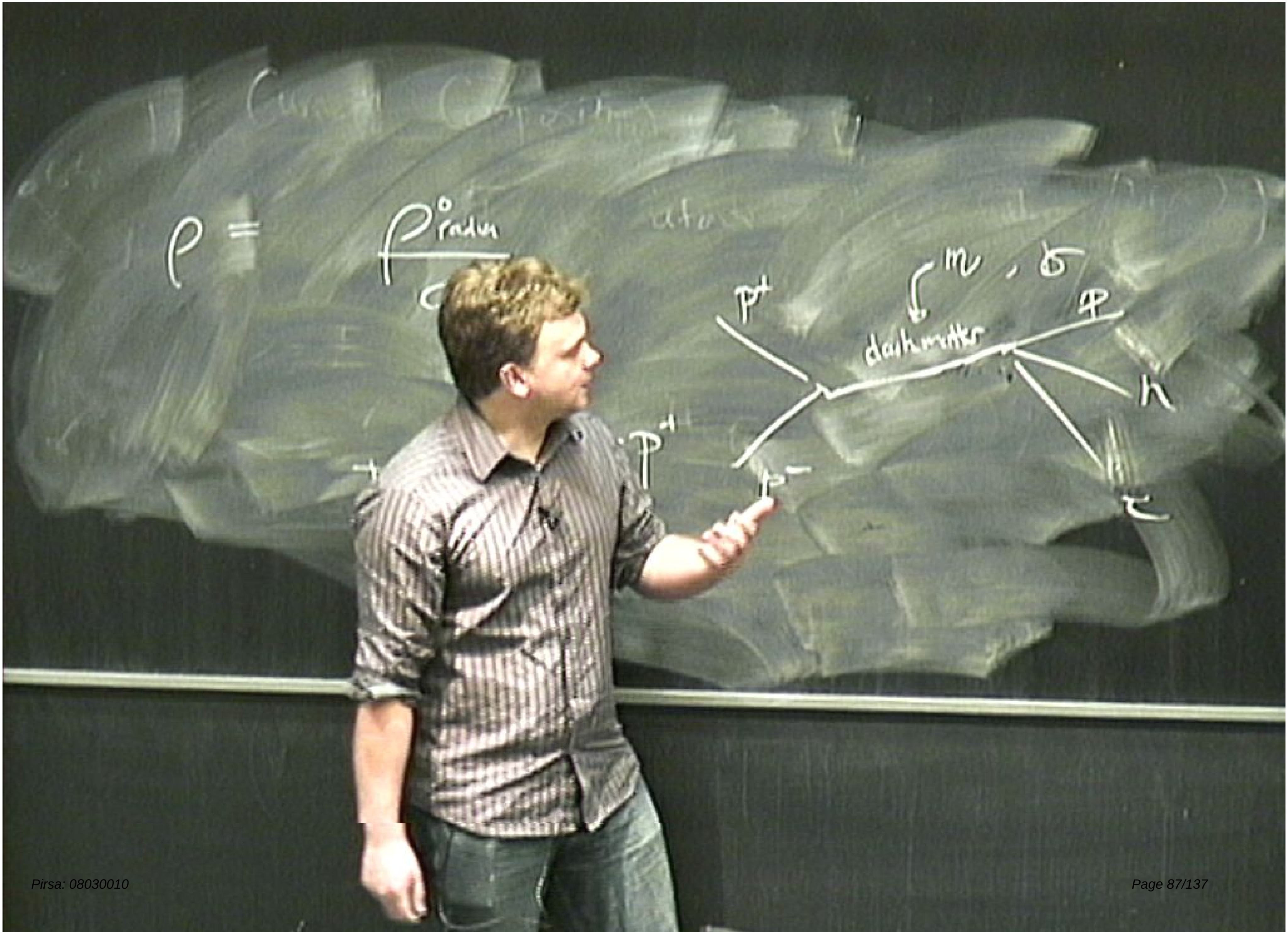
$p = \frac{1}{3}\rho$

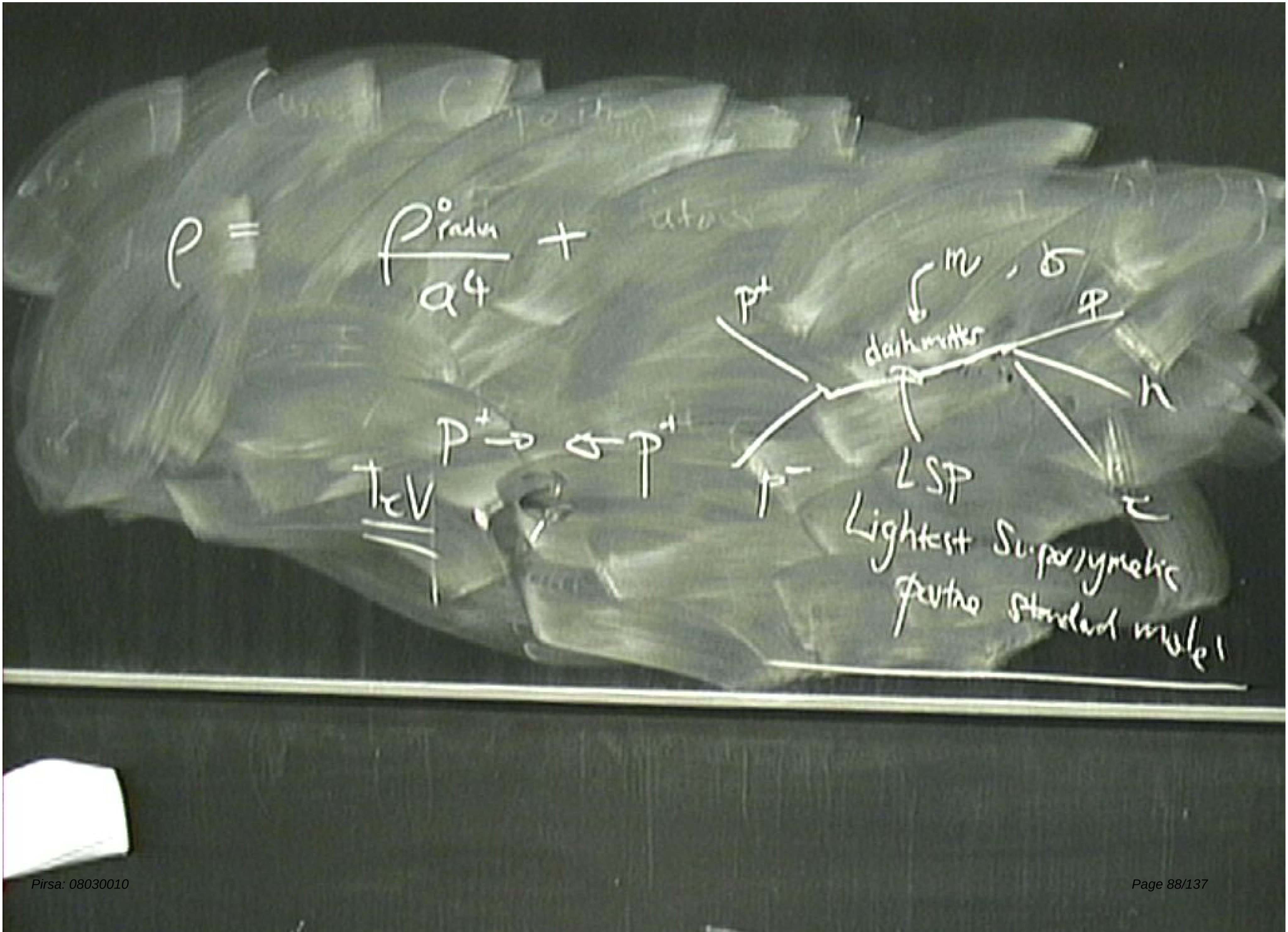
$w = \frac{1}{3}$ $\rho \sim T^4$

Quintessence

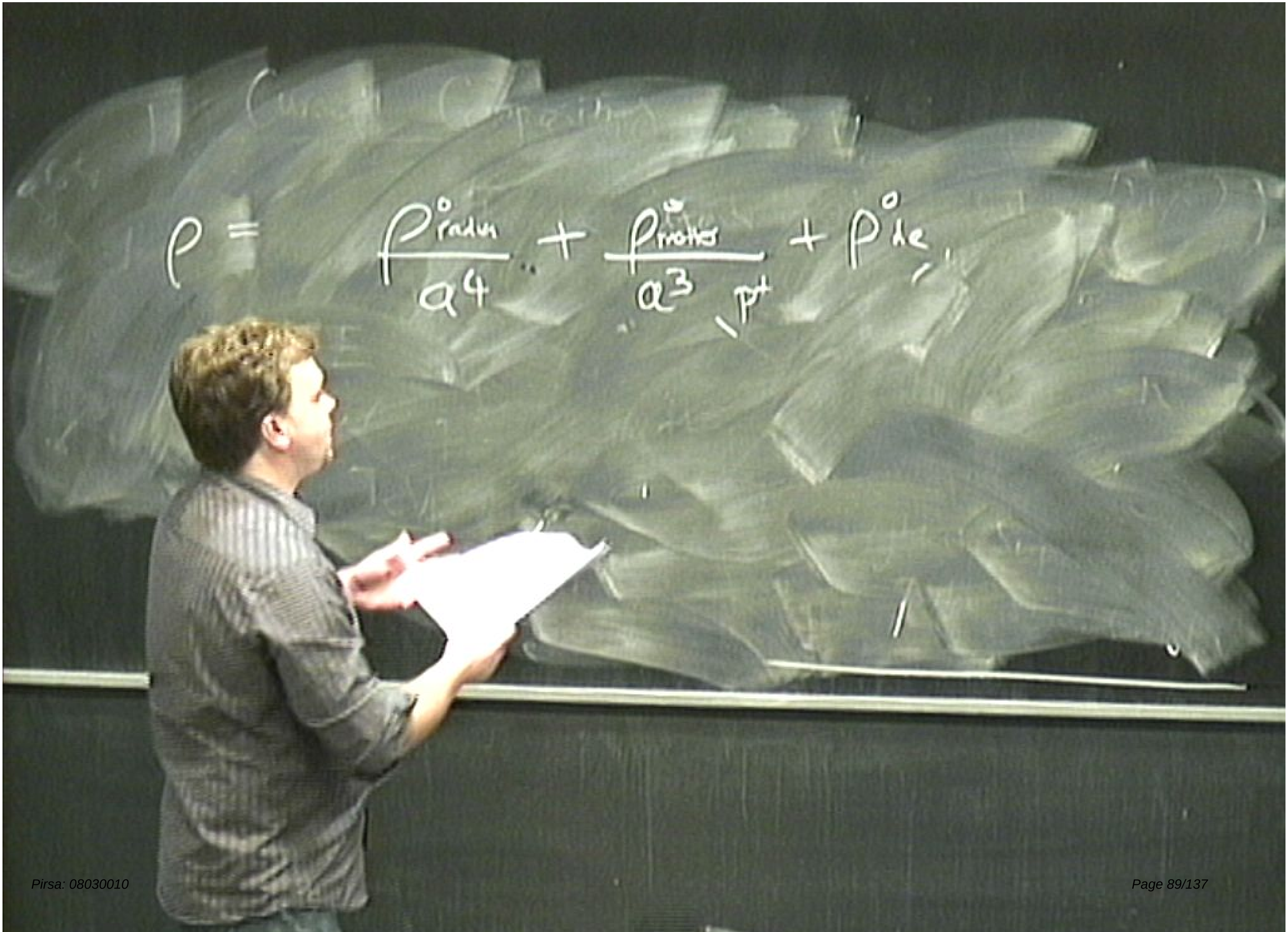
ϕ


ρ = ρ°radn / a4
p+
dark matter
p+ → ← p+
TeV
p−


ρ =
p+
p−
dark matter
h

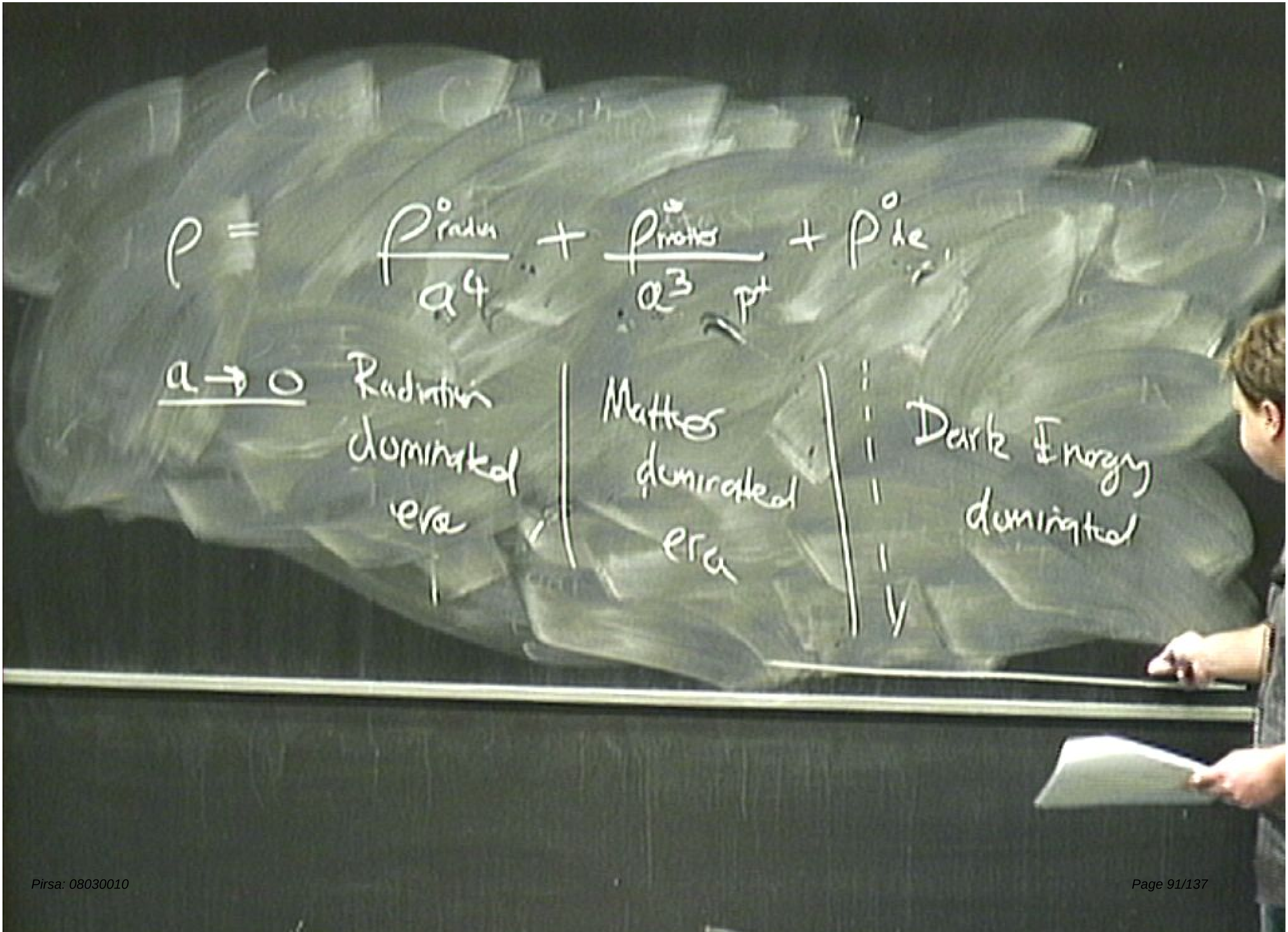


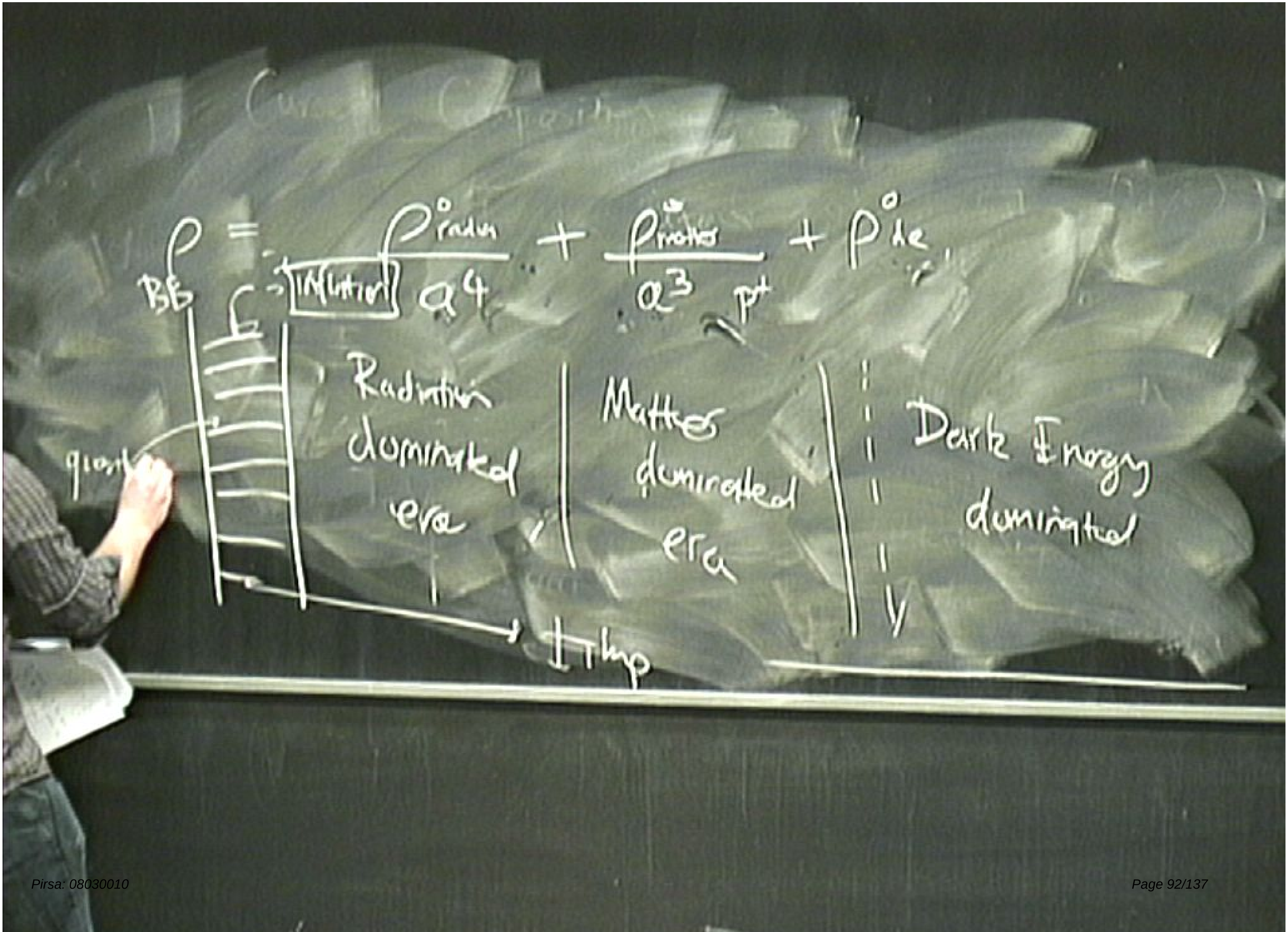
dark matter
LSP
Lightest Supersymmetric
partner standard model
TeV

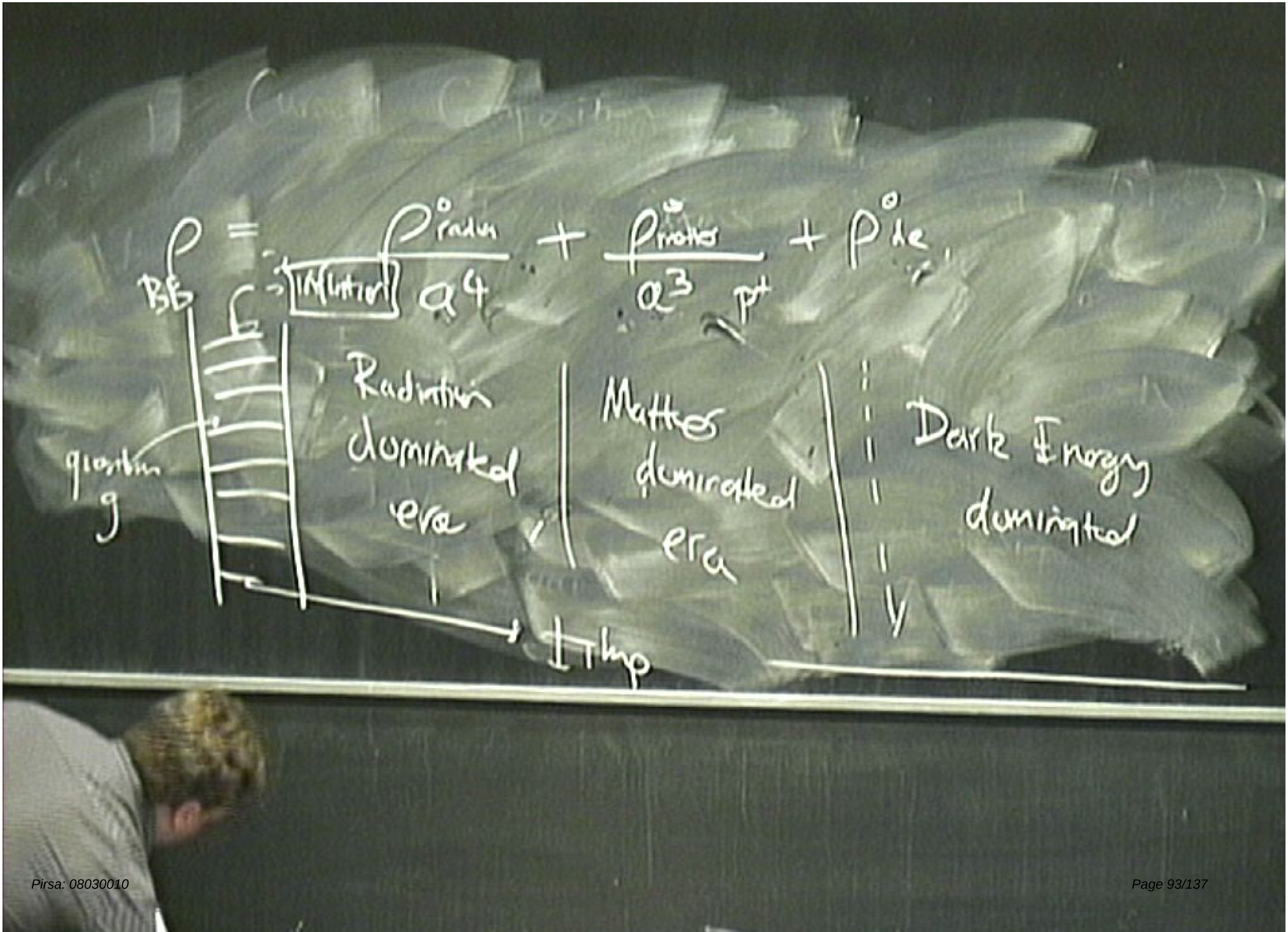

$\rho = \frac{\rho^0_{radn}}{a^4} + \frac{\rho^0_{matter}}{a^3} + \rho^0_{de}$

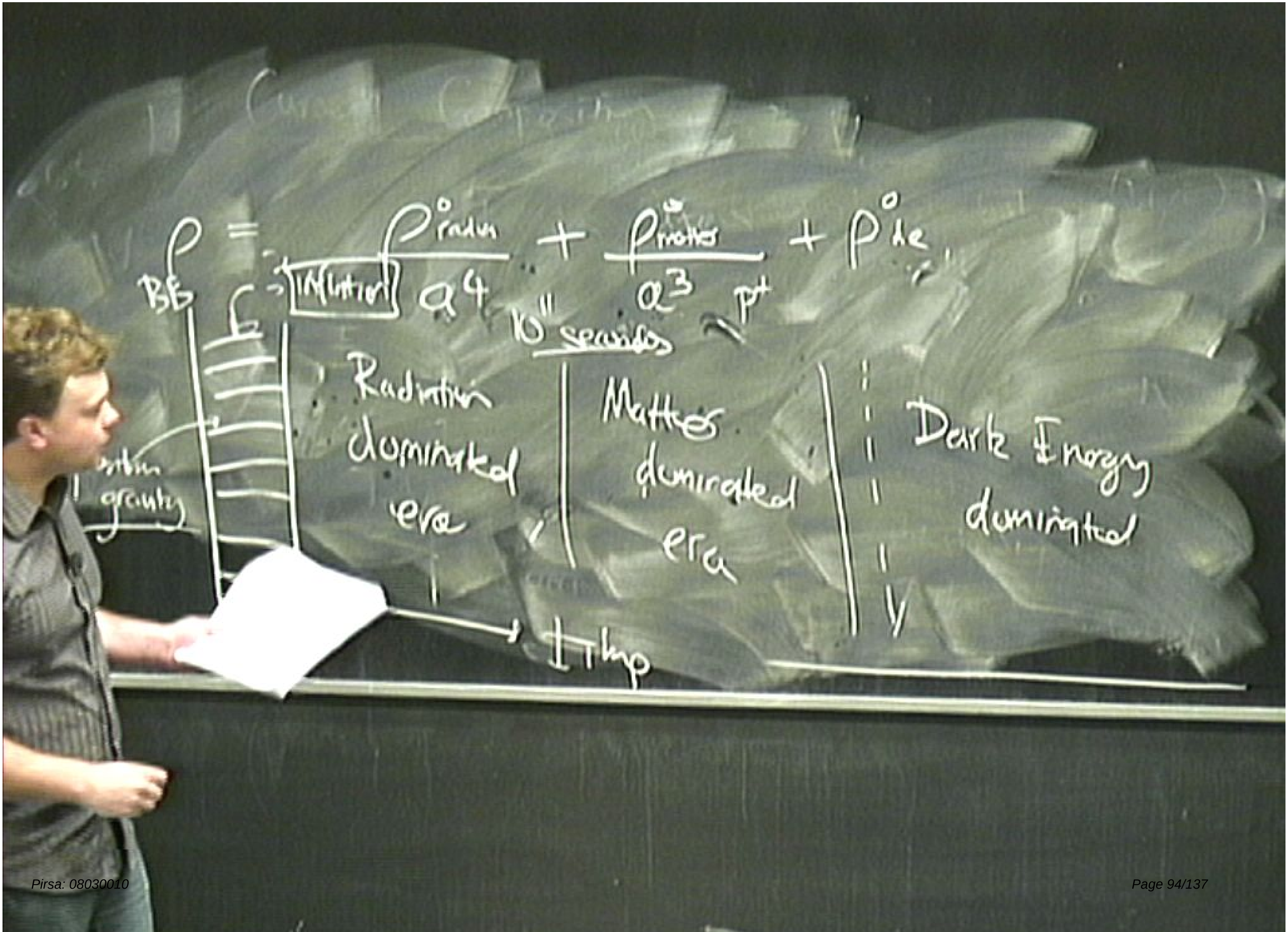
$$\rho = \frac{\rho^0_{radn}}{a^4} + \frac{\rho^0_{matter}}{a^3} + \rho^0_{de}$$

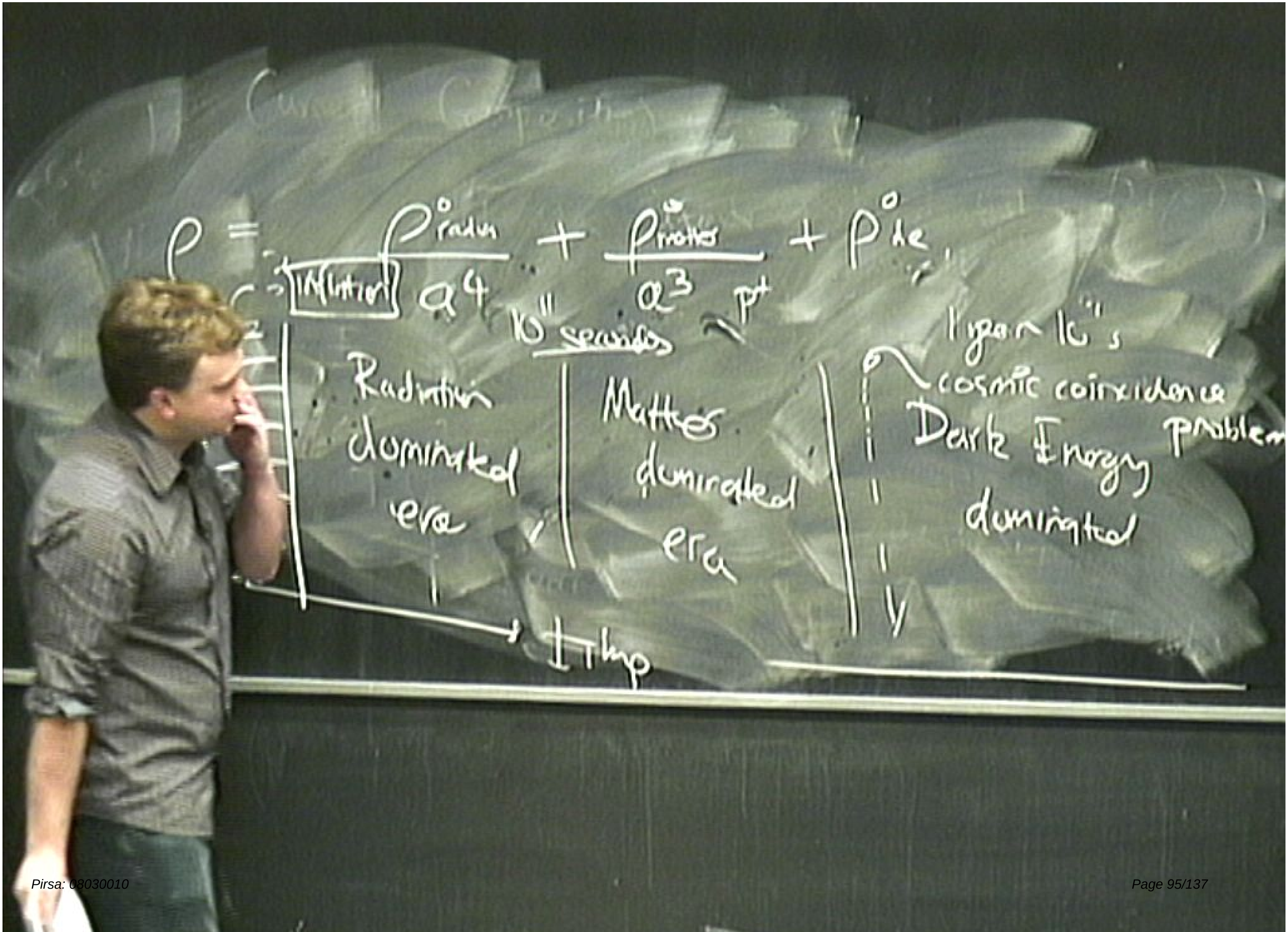
$\underline{a \to 0}$ Radiation dominated era | Matter dominated era | Dark Energy dominated

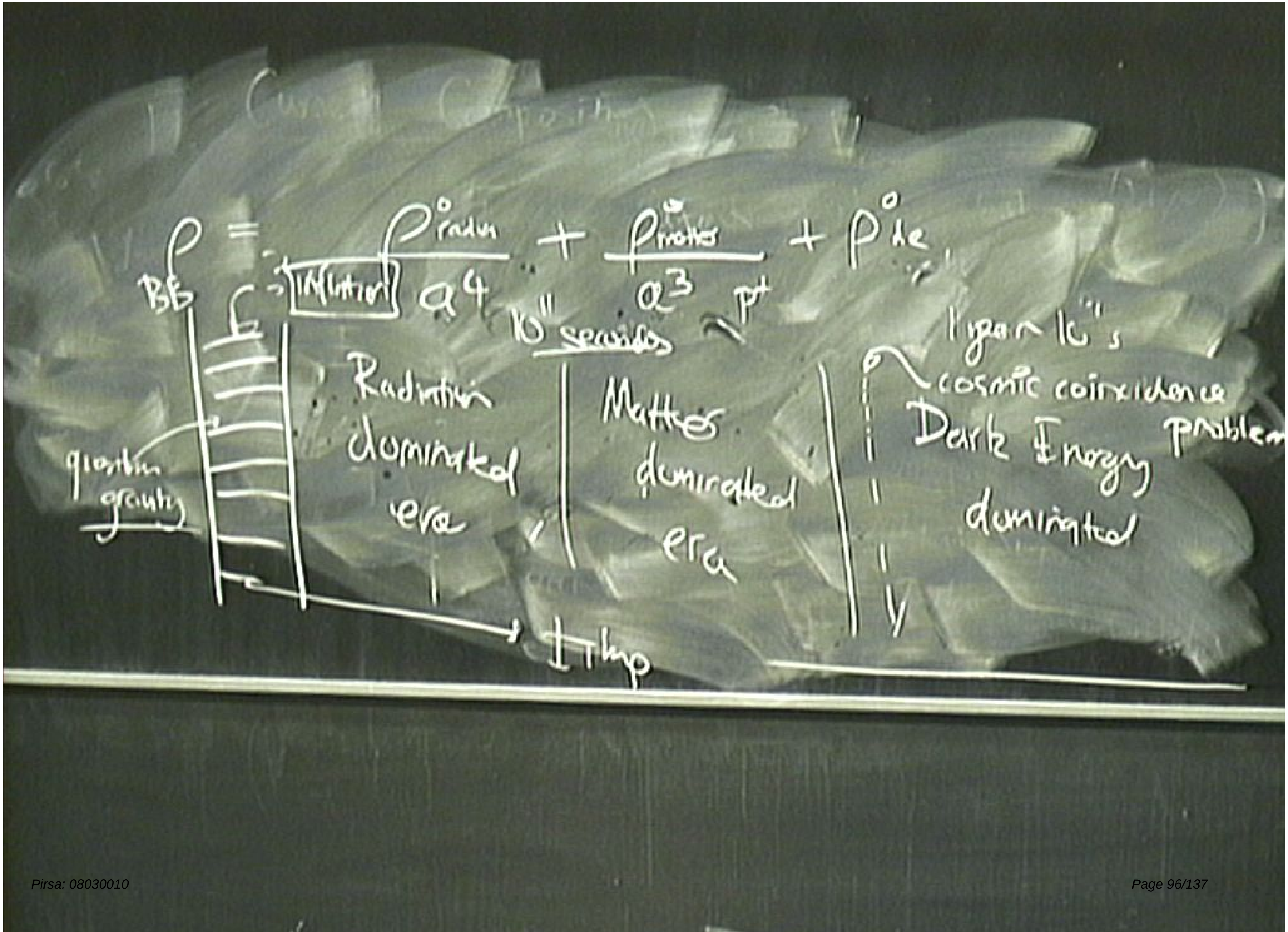

$\rho = \frac{\rho^0_{radn}}{a^4} + \frac{\rho^0_{matter}}{a^3} + \rho^0_{de}$
a → 0
Radiation dominated era
Matter dominated era
Dark Energy dominated


BB
Inflation
Radiation dominated era
Matter dominated era
Dark Energy dominated


ρ = ρ°radn / a⁴ + ρ°matter / a³ + ρ°de
BB
Inflation
Radiation dominated era
Matter dominated era
Dark Energy dominated
quantum g


ρ = ρ°radn / a⁴ + ρ°matter / a³ + ρ°de
BB
Inflation
10¹¹ seconds
Radiation dominated era
Matter dominated era
Dark Energy dominated
gravity


Inflation
10" seconds
Radiation
dominated
era
Matter
dominated
era
cosmic coincidence
problem
Dark Energy
dominated



$\rho = \frac{\rho^0_{radn}}{a^4} + \frac{\rho^0_{matter}}{a^3} + \rho^0_{de}$
BB
Inflation
10^{11} seconds
Radiation dominated era
Matter dominated era
cosmic coincidence
Dark Energy dominated
problem
quantum gravity

$$\rho = \frac{\rho^{\circ}_{radn}}{a^4} + \frac{\rho^{\circ}_{matter}}{a^3} + \rho^{\circ}_{de}$$

BB

$\boxed{\text{Inflation}}$

quantum gravity

10^{11} seconds

Radiation dominated era

Matter dominated era

Dark Energy dominated

cosmic coincidence problem

$$\rho = \frac{\rho^{o}_{radn}}{a^4} + \frac{\rho^{o}_{matter}}{a^3} + \rho^{o}_{de}, \quad a \propto e^{Ht}$$

BB

$\boxed{\text{Inflation}}$

10^{11} seconds

p^{+}

Radiation dominated era

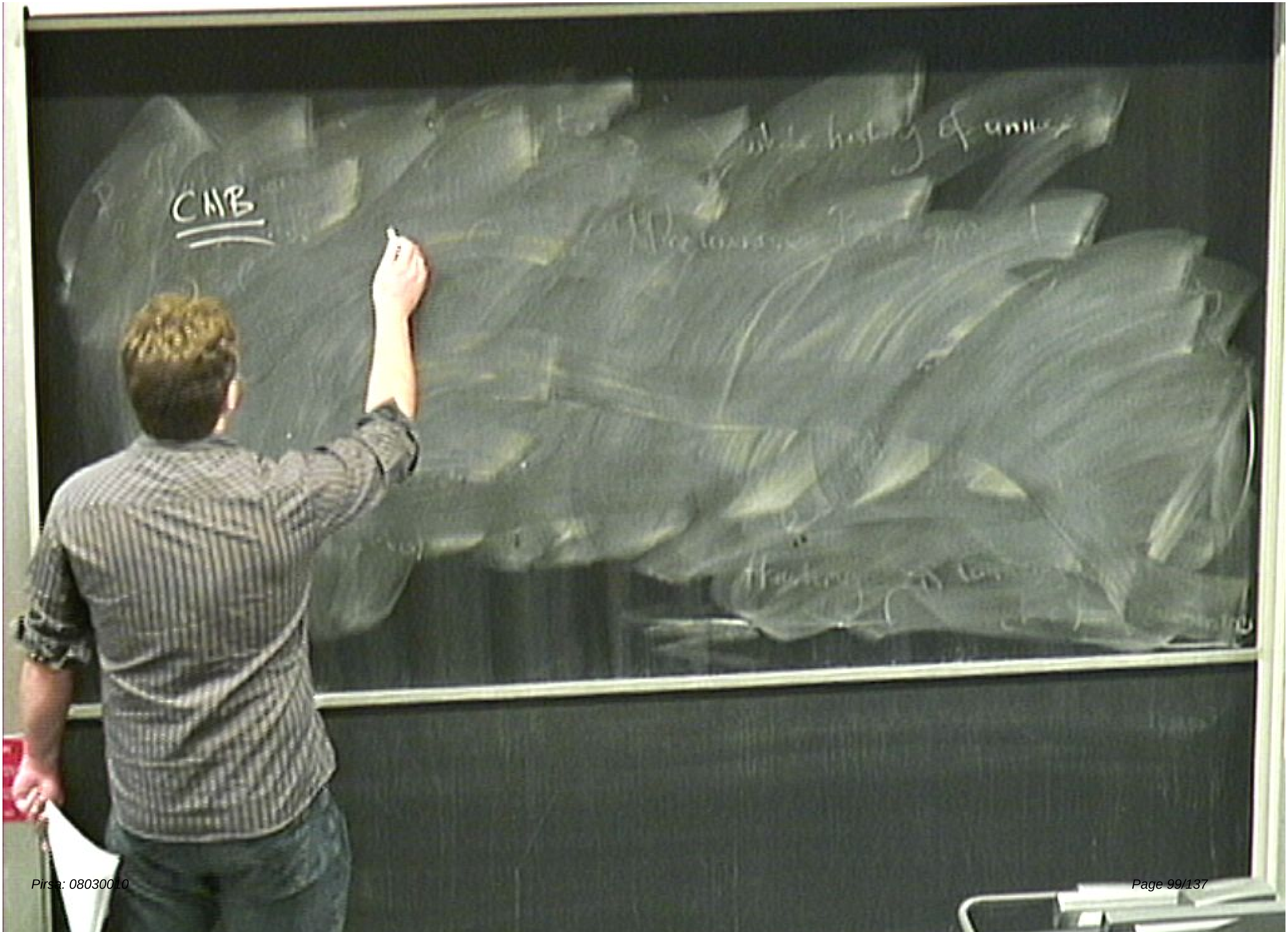
Matter dominated era

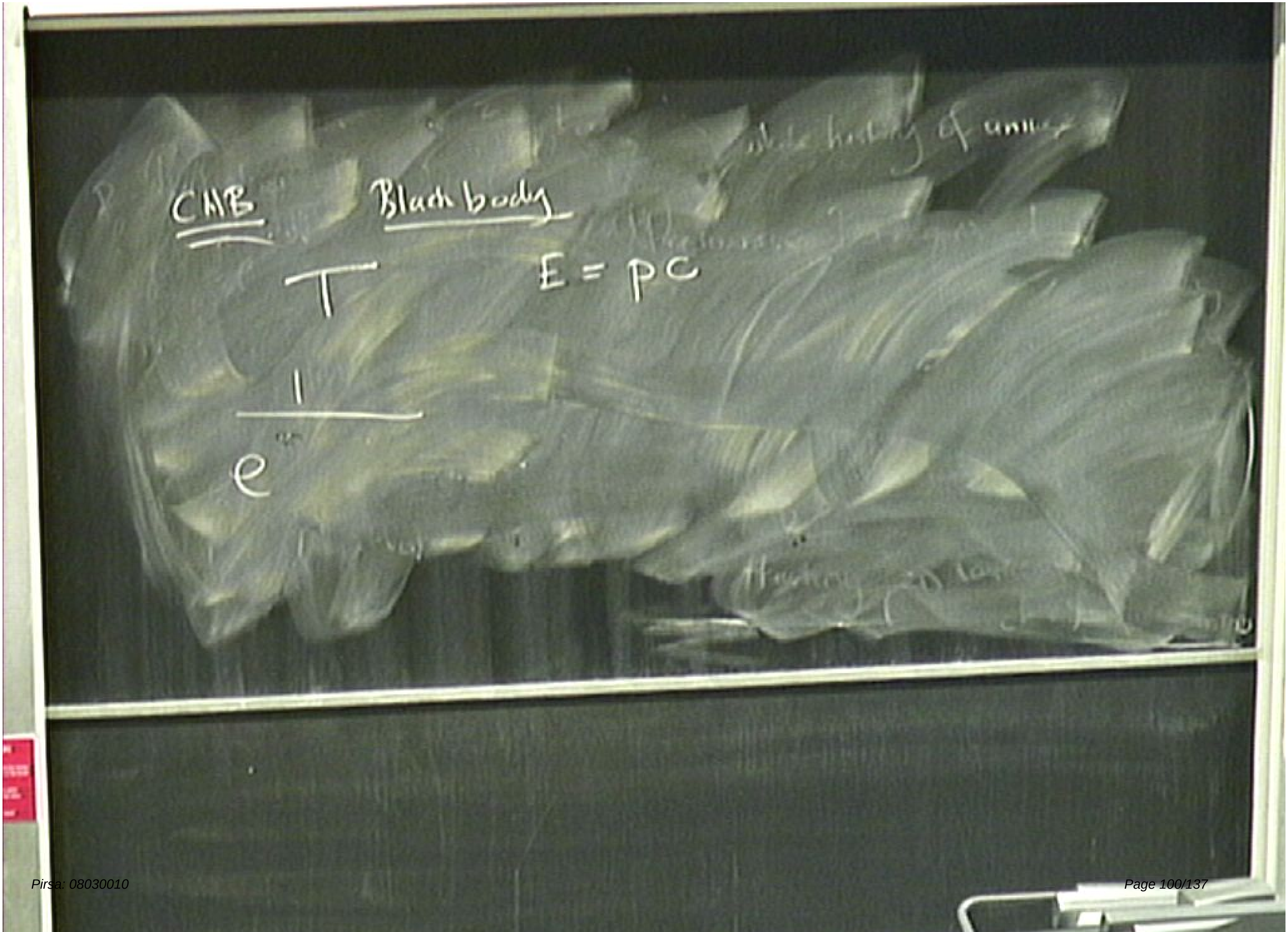
Dark Energy dominated

cosmic coincidence problem

quantum gravity

1 Thp


CMB


CMB
Black body
T
E = pc
1
e

CMB

Black body

$$\boxed{T = 2.73K}$$

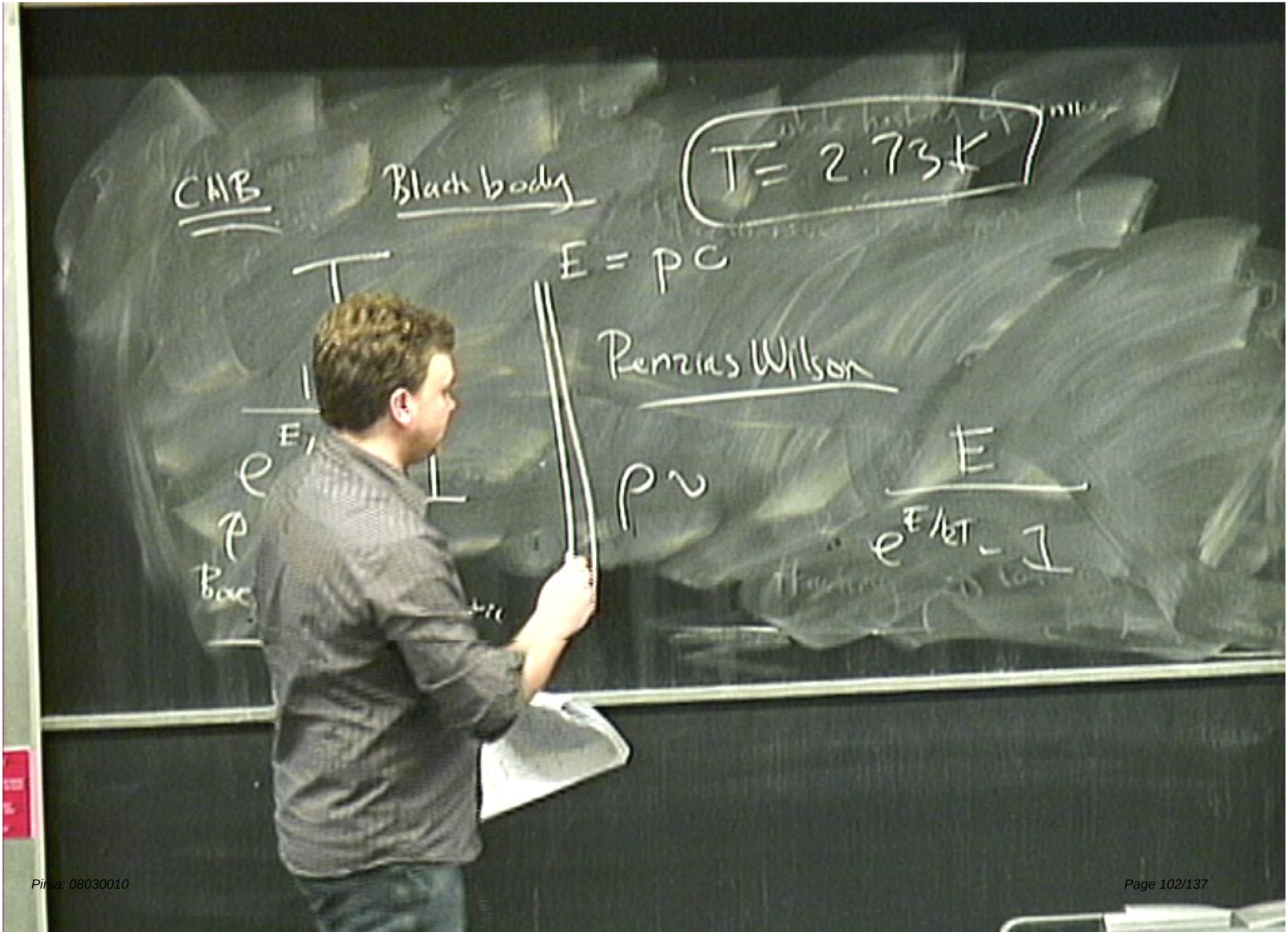
T

$$\frac{1}{e^{E/kT} - 1}$$

$E = pc$

Penzias Wilson

Bose Einstein distribution


CMB
Black body
T = 2.73K
E = pc
Penzias Wilson
ρ ~ E / (e^{E/kT} - 1)

CMB

Black body

$\boxed{T = 2.73K}$

$E = pc$

$$T$$

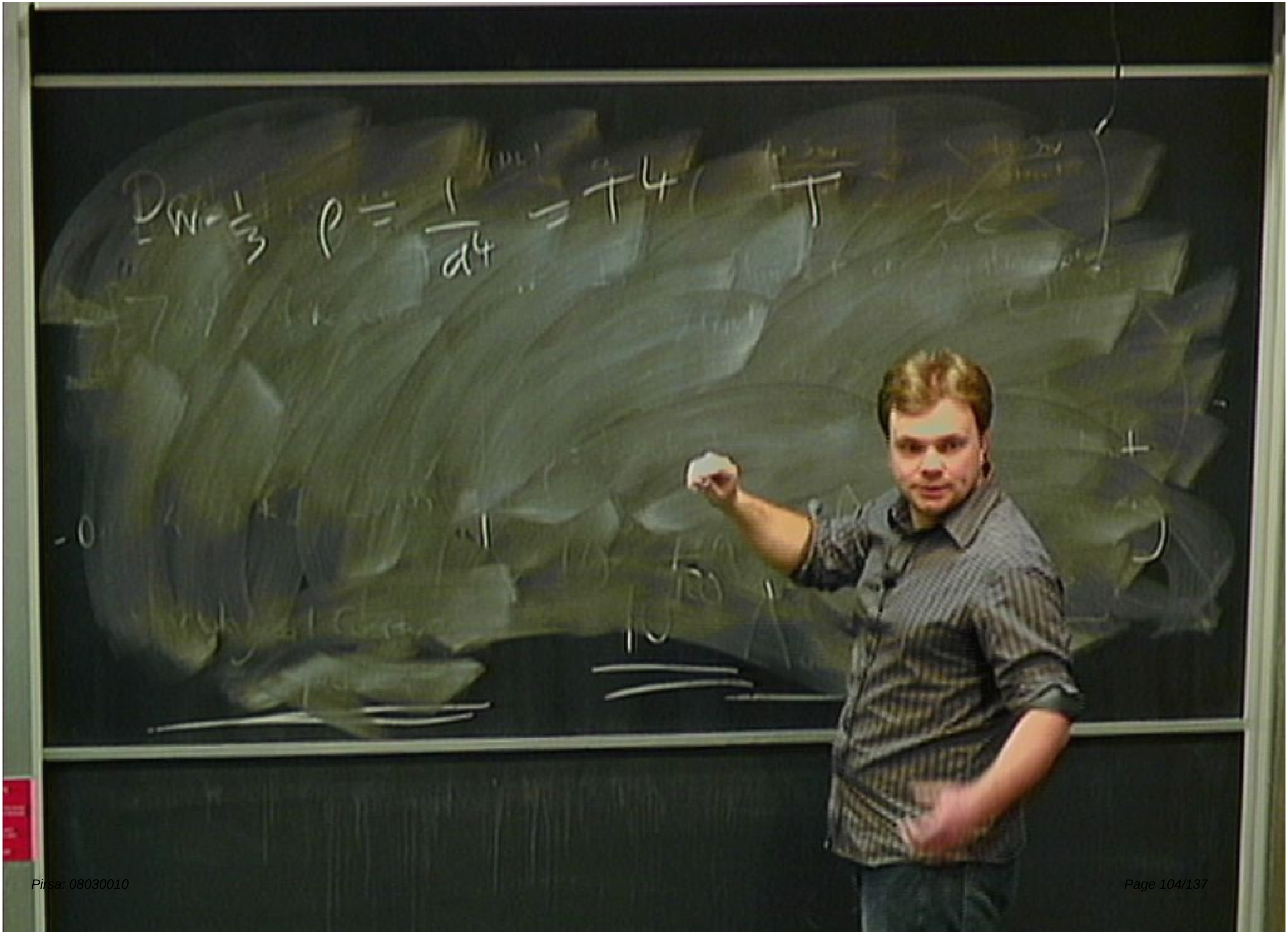
$$\frac{1}{e^{E/kT} - 1}$$

Bose Einstein distribution

Penzias Wilson

$$\rho \approx \int \frac{d^3p}{h^3} \frac{E}{e^{E/kT} - 1} \sim T^4$$

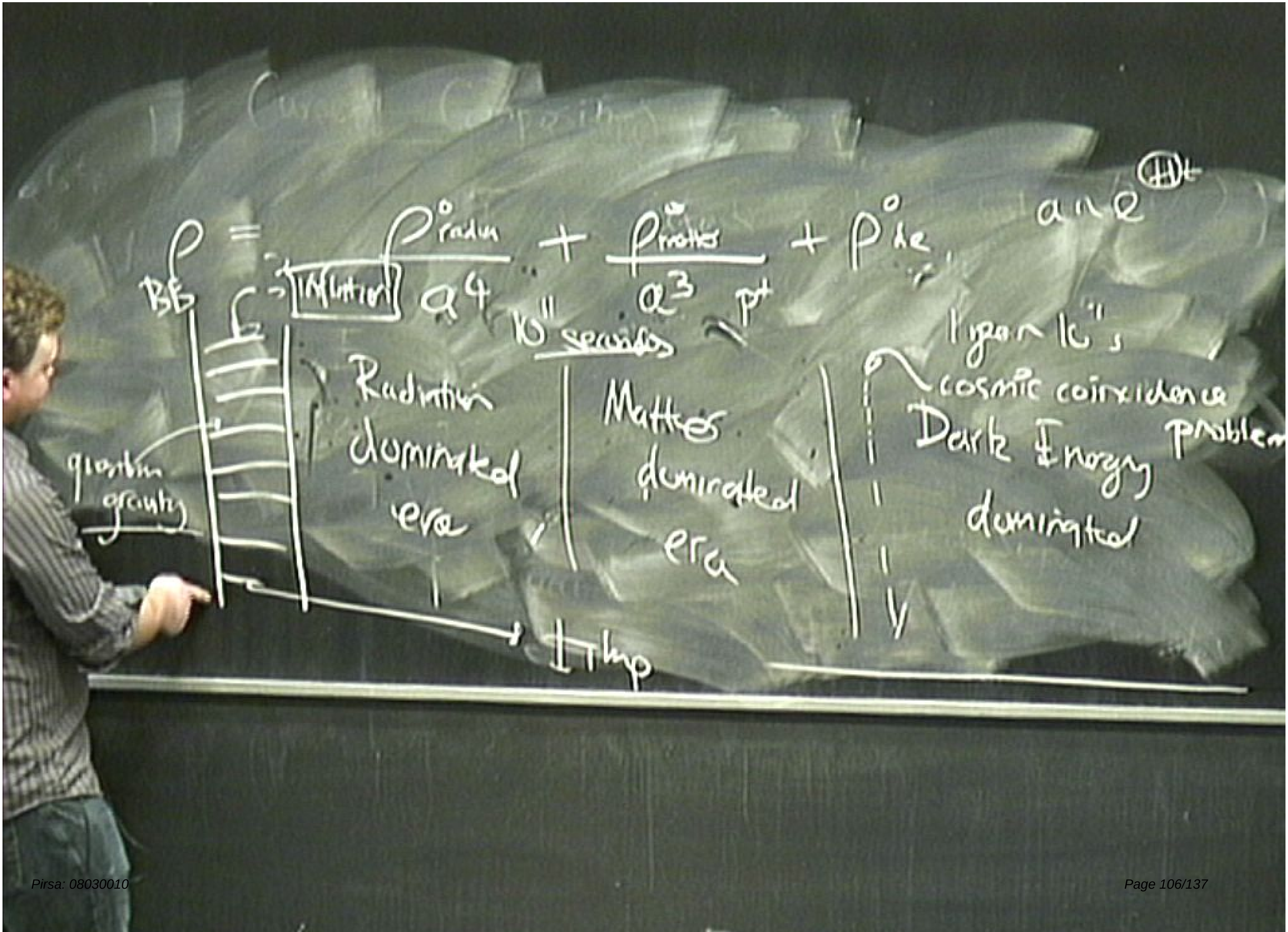
$k=c=1$

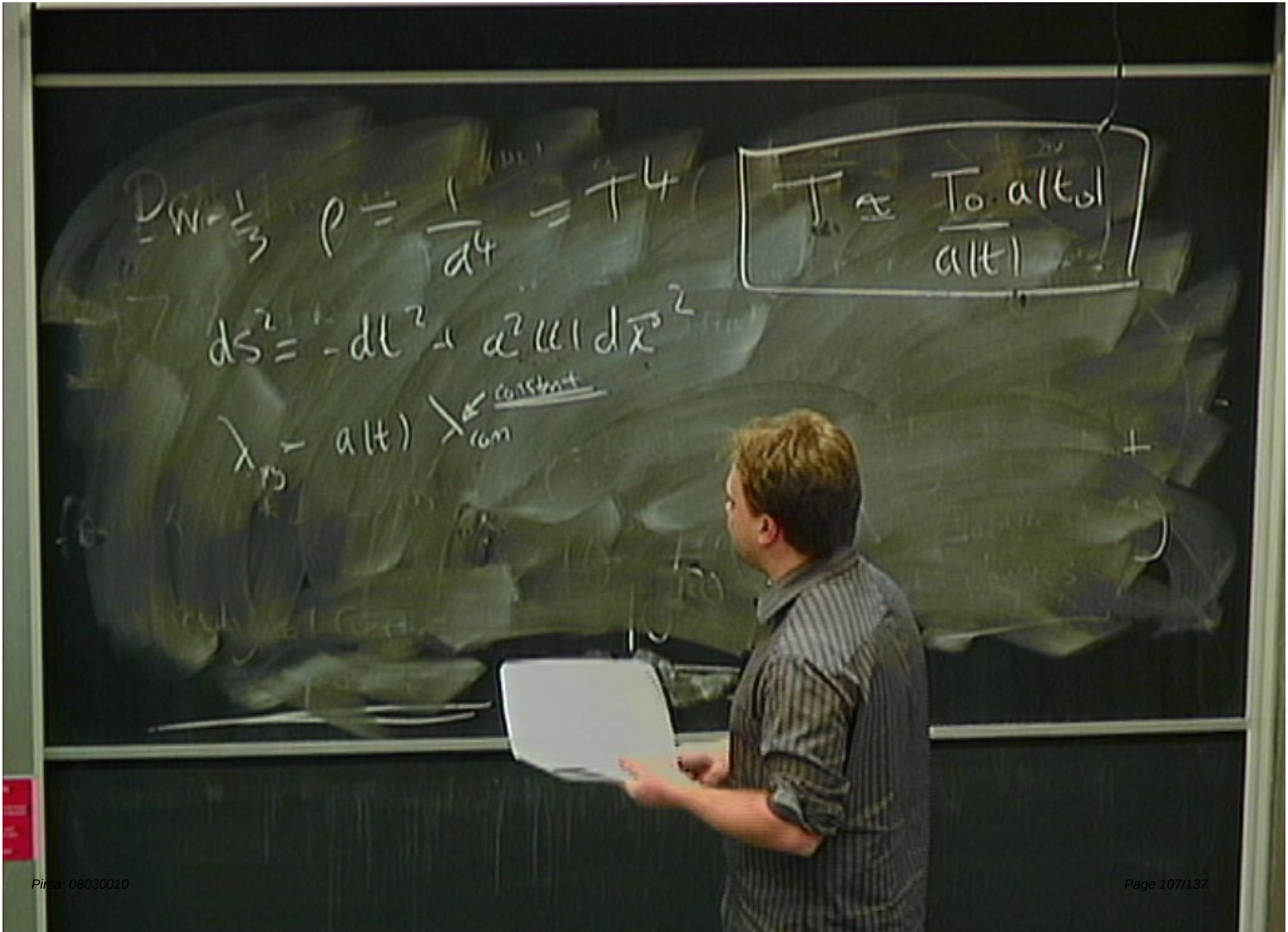


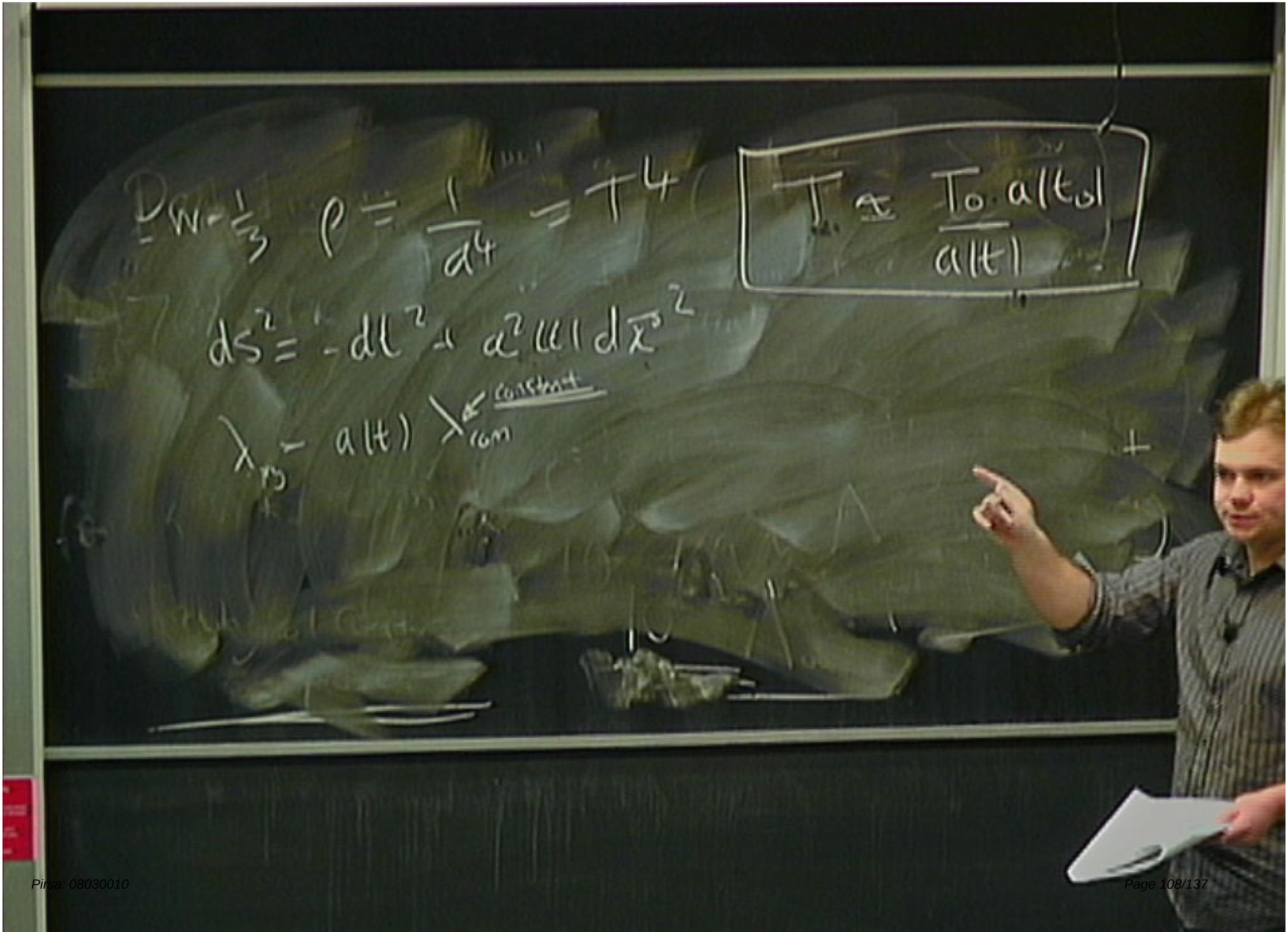
P W = 1/3
ρ ≐ 1/a4 = T4

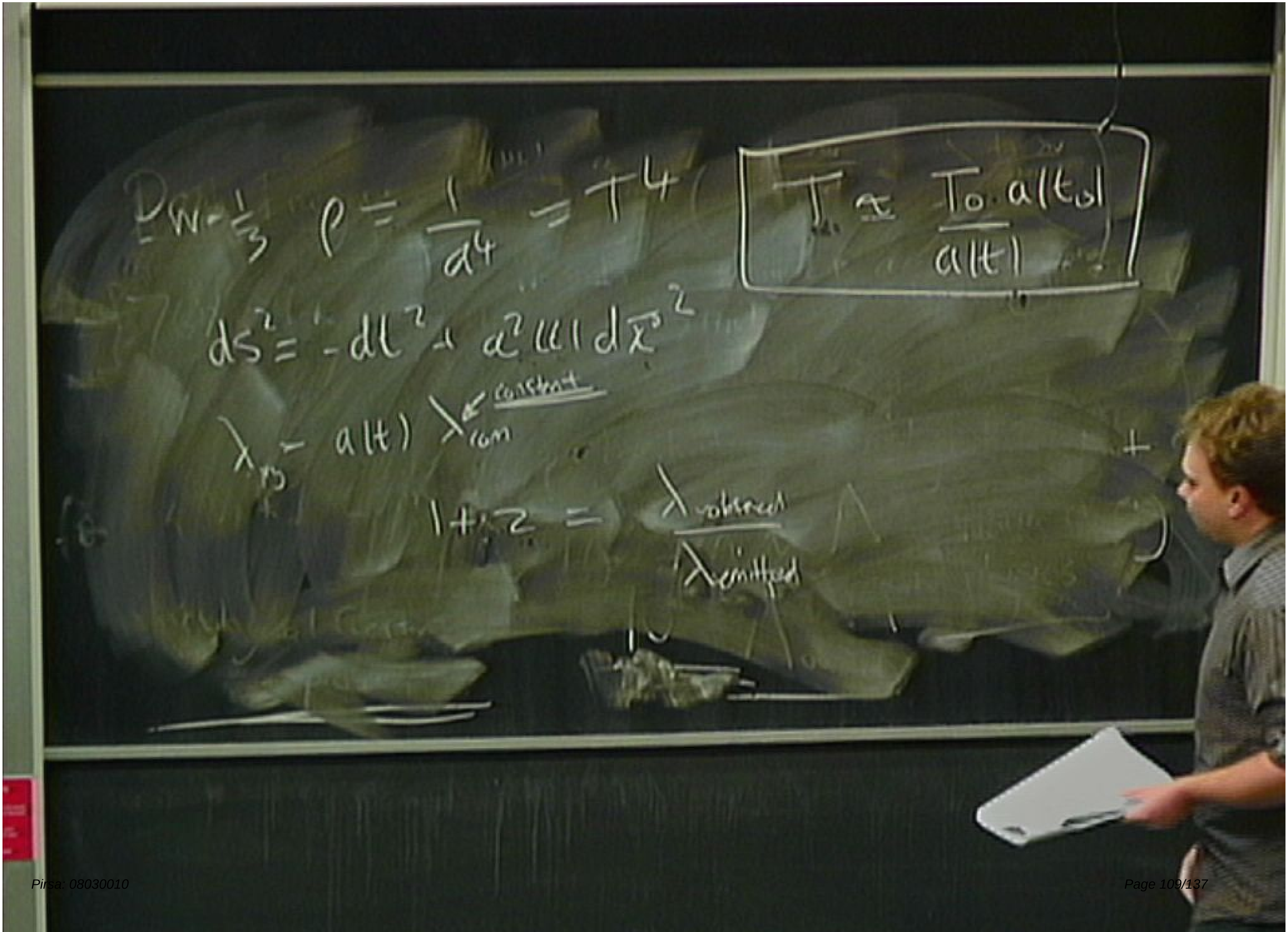
$P w = \frac{1}{3}$ $\quad \rho \doteq \frac{1}{a^4} = T^4$

$$\boxed{T \propto \frac{T_0 \, a(t_0)}{a(t)}}$$


$\rho = \frac{\rho^0_{radn}}{a^4} + \frac{\rho^0_{matter}}{a^3} + \rho^0_{de}$
$a \propto e^{Ht}$
BB
Inflation
10^{11} seconds
Radiation dominated era
Matter dominated era
1 year ICs
cosmic coincidence problem
Dark Energy dominated
quantum gravity
1 Tlyr


$P = w\rho$, $w = \frac{1}{3}$ $\rho \propto \frac{1}{a^4} \propto T^4$
$T \propto \frac{T_0\, a(t_0)}{a(t)}$
$ds^2 = -dt^2 + a^2(t)\, d\vec{x}^2$
$\lambda_{phys} = a(t)\, \lambda_{com}$
constant


P_γ = 1/3 ρ ∝ 1/a^4 ∝ T^4
ds^2 = -dt^2 + a^2(t) dx^2
λ_phys = a(t) λ_com
constant
T ∝ T_0 a(t_0) / a(t)


$P_W = \frac{1}{3}\rho \doteq \frac{1}{a^4} = T^4$
$T \propto \frac{T_0\, a(t_0)}{a(t)}$
$ds^2 = -dt^2 + a^2(t)\, d\vec{x}^2$
constant
$\lambda_{obs} = a(t)\, \lambda_{com}$
$1+z = \frac{\lambda_{observed}}{\lambda_{emitted}}$

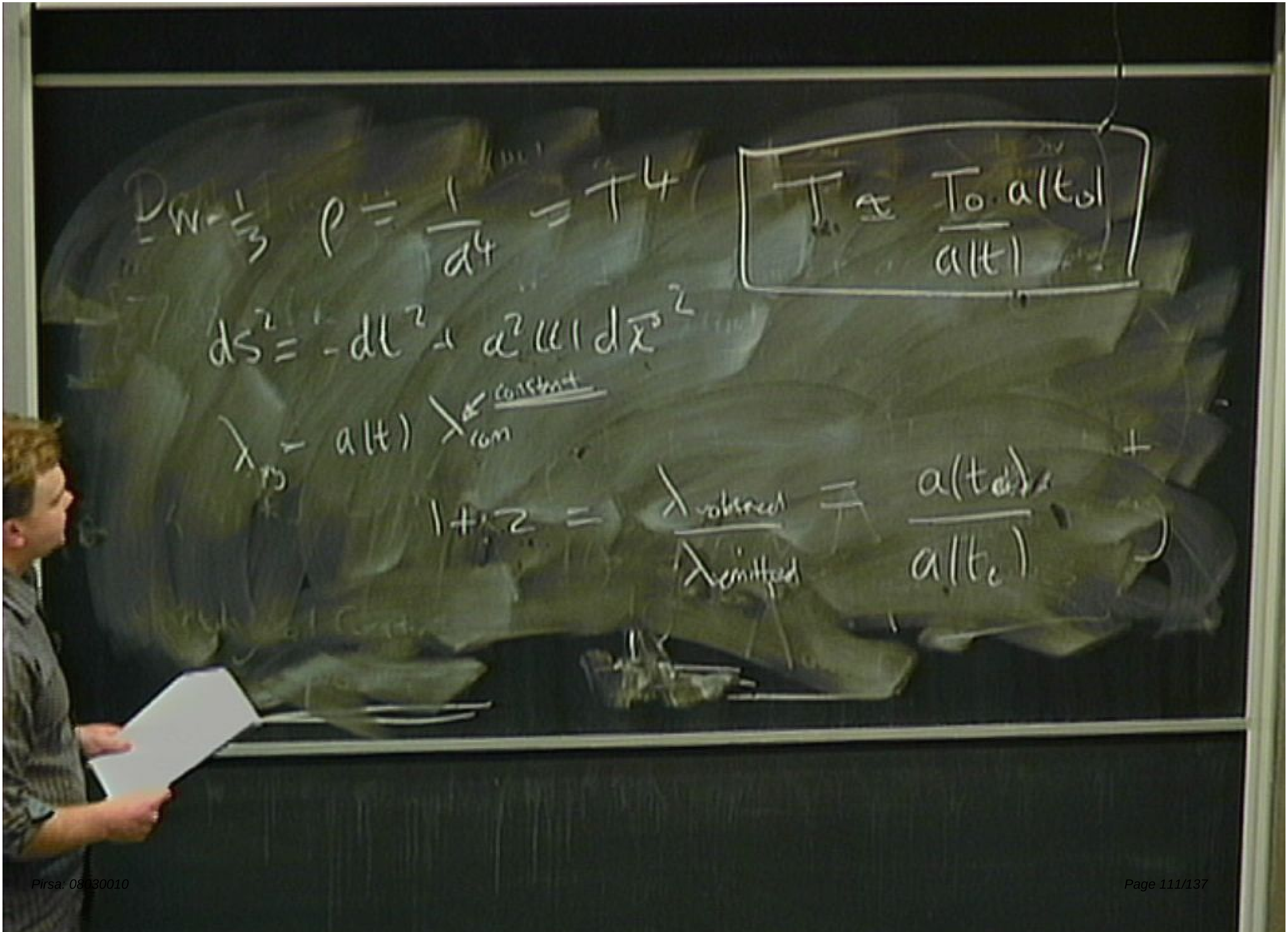
$P_{\gamma} = \frac{1}{3} \rho \doteq \frac{1}{a^4} = T^4$

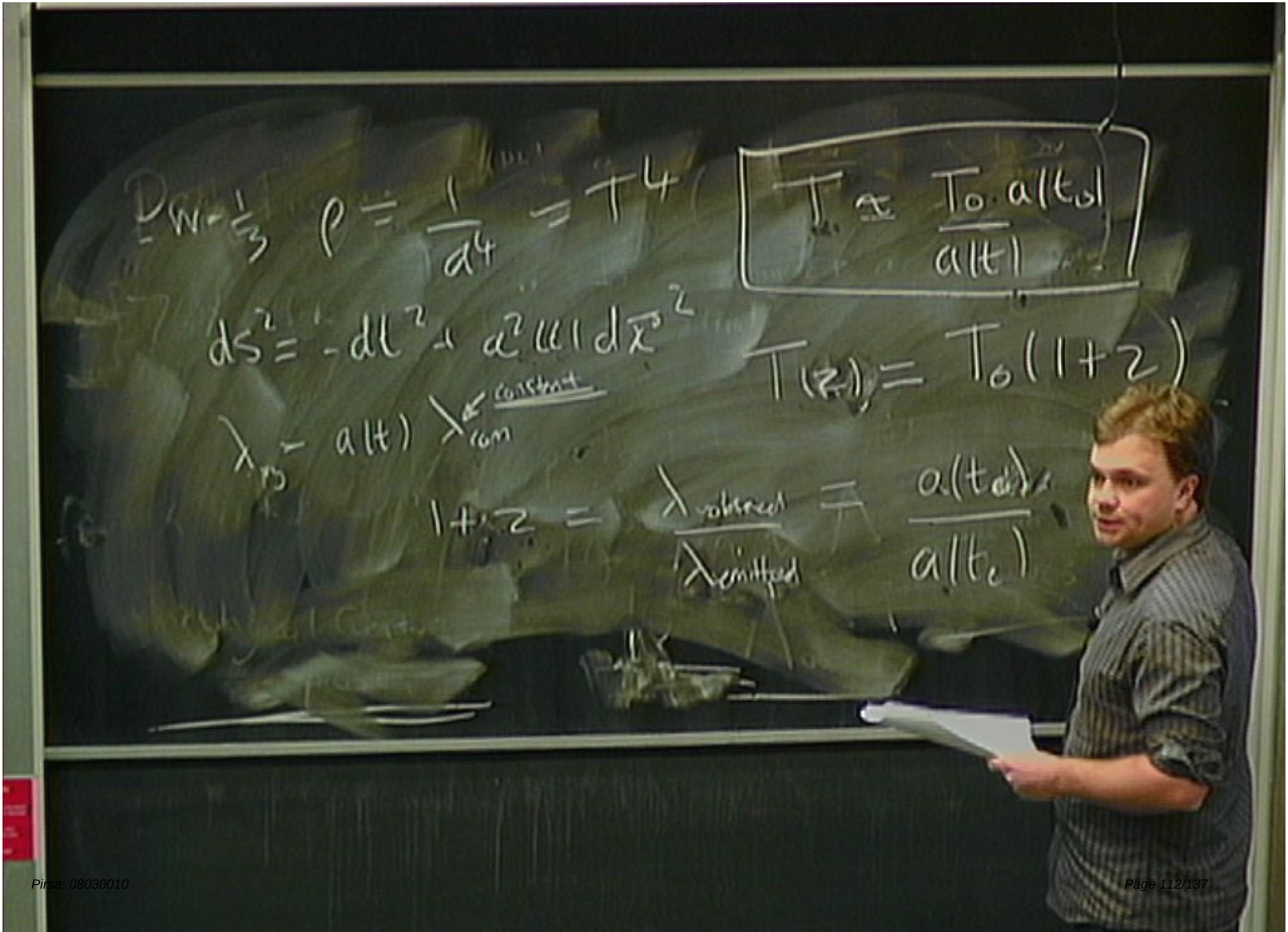
$$\boxed{T = \frac{T_0 \, a(t_0)}{a(t)}}$$

$$ds^2 = -dt^2 + a^2(t)\, d\vec{x}^2$$

$$\lambda_{phys} = a(t)\, \lambda_{com} \leftarrow \text{constant}$$

$$1 + z = \frac{\lambda_{observed}}{\lambda_{emitted}} = \frac{a(t_0)}{a(t_e)}$$


P = w · 1/3 ρ ∝ 1/a⁴ ∝ T⁴
T ∝ T₀ a(t₀) / a(t)
ds² = −dt² + a²(t) dx²
constant
λ_obs = a(t) λ_com
1 + z = λ_observed / λ_emitted = a(t₀) / a(tₑ)



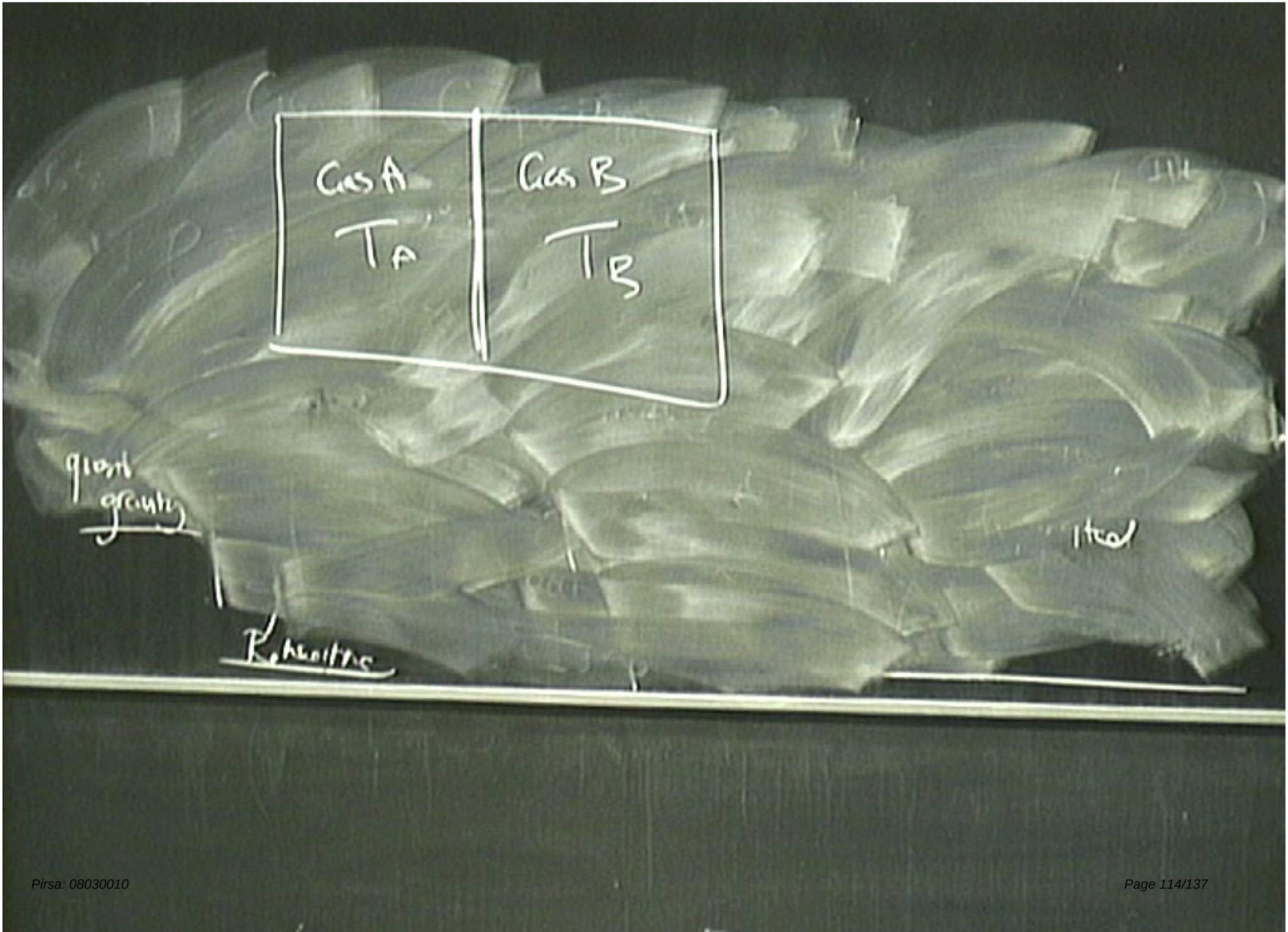
T ∝ T0 a(t0) / a(t)
ds² = -dt² + a²(t) dx²
T(z) = T0 (1+z)
constant
λobs = a(t) λcom
1+z = λobserved / λemitted = a(t0) / a(te)

$$P = w\rho, \quad w = \frac{1}{3} \qquad \rho \propto \frac{1}{a^4} \propto T^4 \qquad \boxed{T \propto \frac{T_0\, a(t_0)}{a(t)}}$$

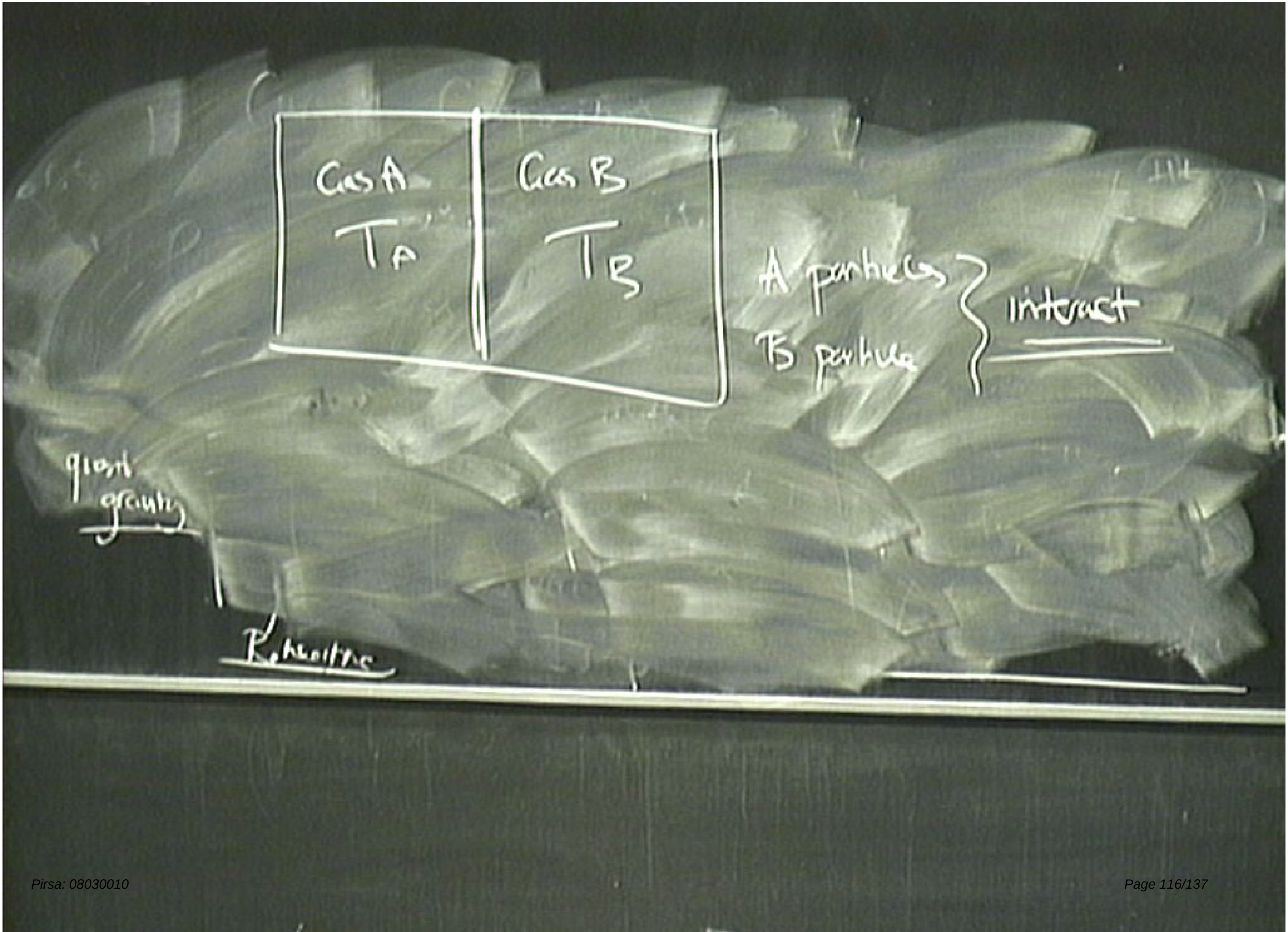
$$ds^2 = -dt^2 + a^2(t)\, d\vec{x}^2 \qquad T(z) = T_0(1+z)$$

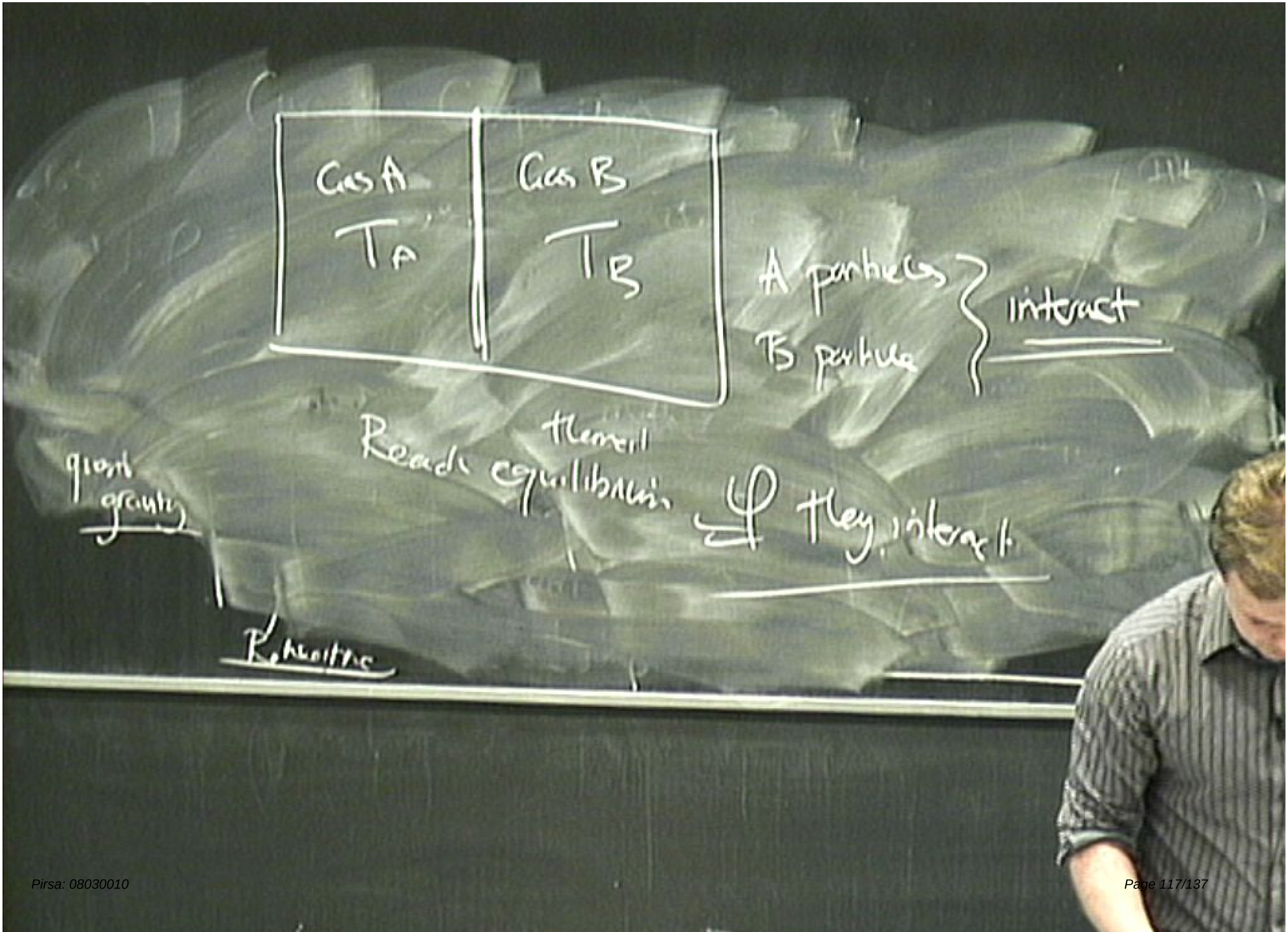
$$\lambda_{ph} \propto a(t)\, \lambda_{com} \quad \leftarrow \underline{\text{constant}}$$

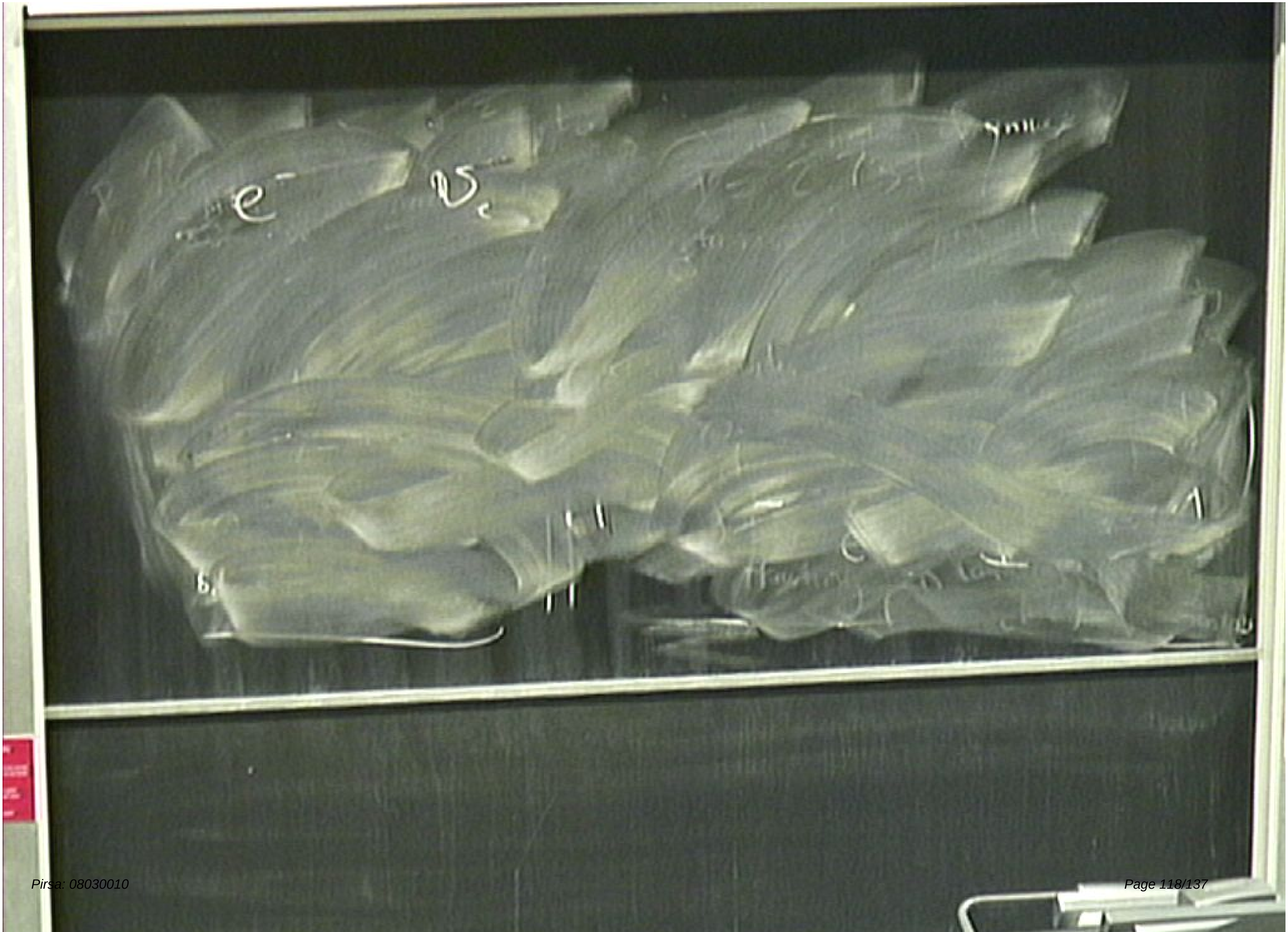
$$1+z = \frac{\lambda_{\text{observed}}}{\lambda_{\text{emitted}}} = \frac{a(t_0)}{a(t_e)}$$


Gas A
T A
Gas B
T B
gravity


Gas A
Gas B
A particles
B particles
interact


Gas A
T_A
Gas B
T_B
A particles
B particles
interact
quant
gravity

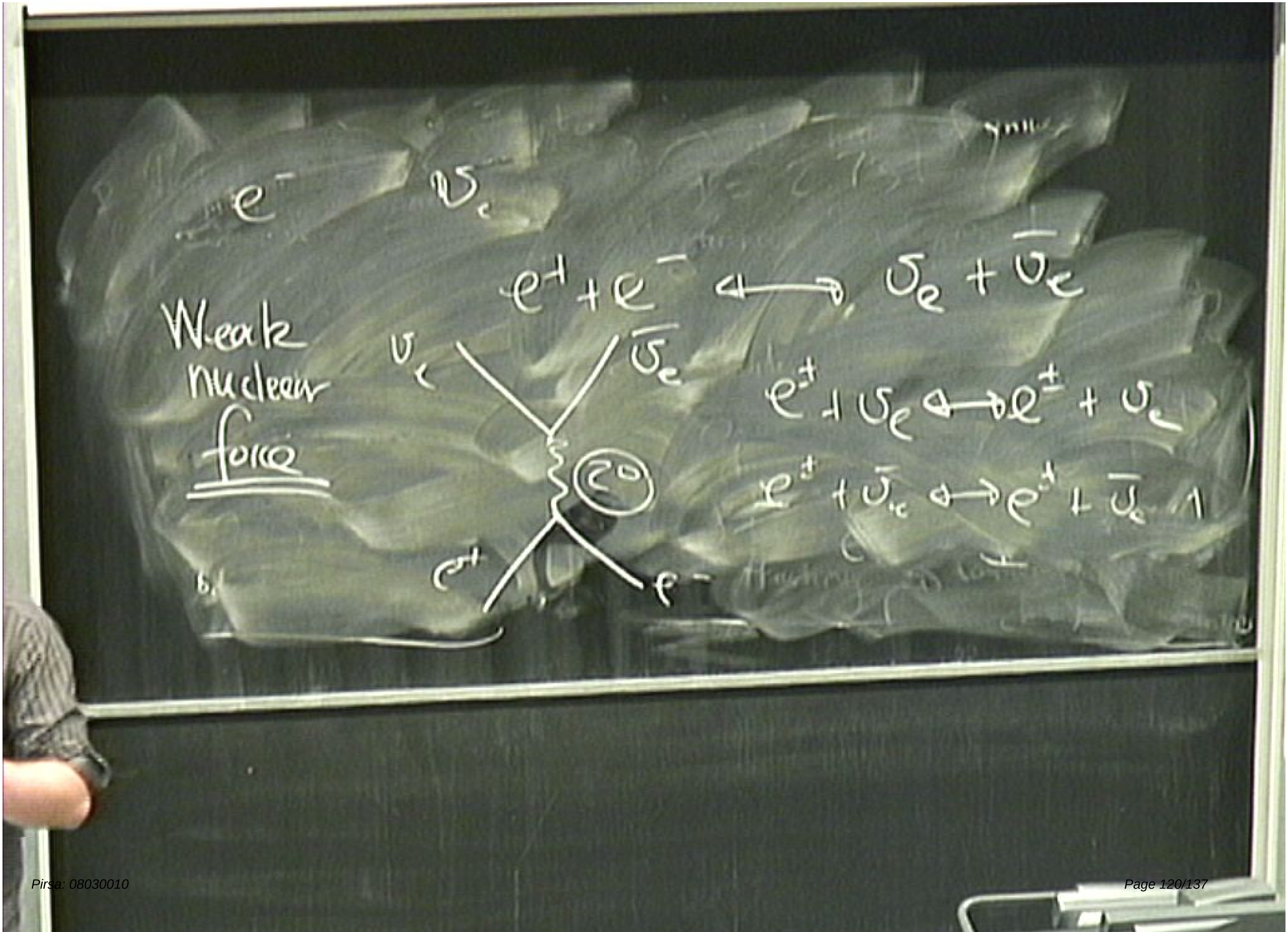

Gas A
T_A
Gas B
T_B
A particles
B particles
interact
thermal
Reach equilibrium iff they interact
quantum gravity
Relativity


e^-
ν_e

e^- ν_e

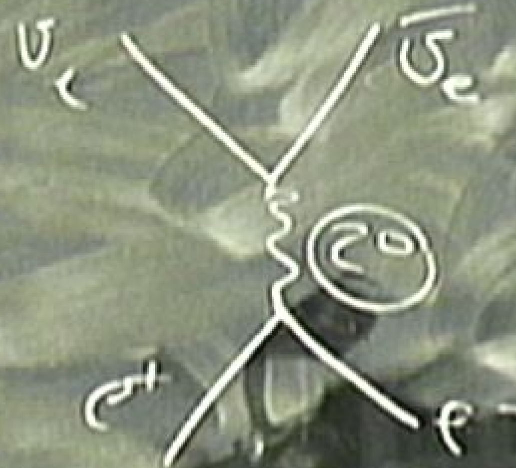
$$e^+ + e^- \leftrightarrow \nu_e + \bar{\nu}_e$$

Weak nuclear force


e⁻
νₑ
e⁺ + e⁻ ⟷ νₑ + ν̄ₑ
Weak nuclear force
Z⁰
e± + νₑ ⟷ e± + νₑ
e± + ν̄ₑ ⟷ e± + ν̄ₑ

Weak nuclear force

$$e^+ + e^- \leftrightarrow \nu_e + \bar{\nu}_e$$



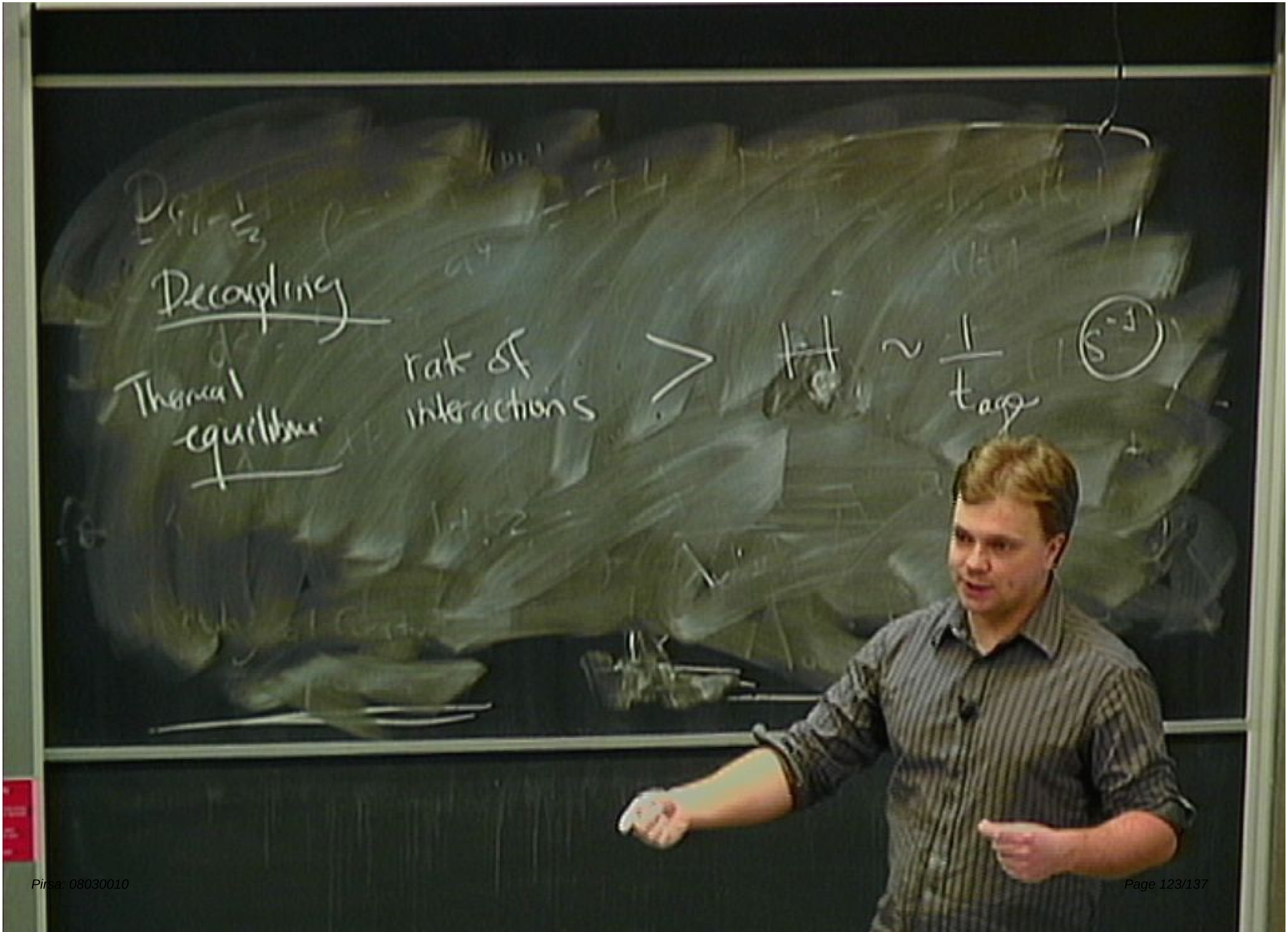
$$e^\pm + \nu_e \leftrightarrow e^\pm + \nu_e$$

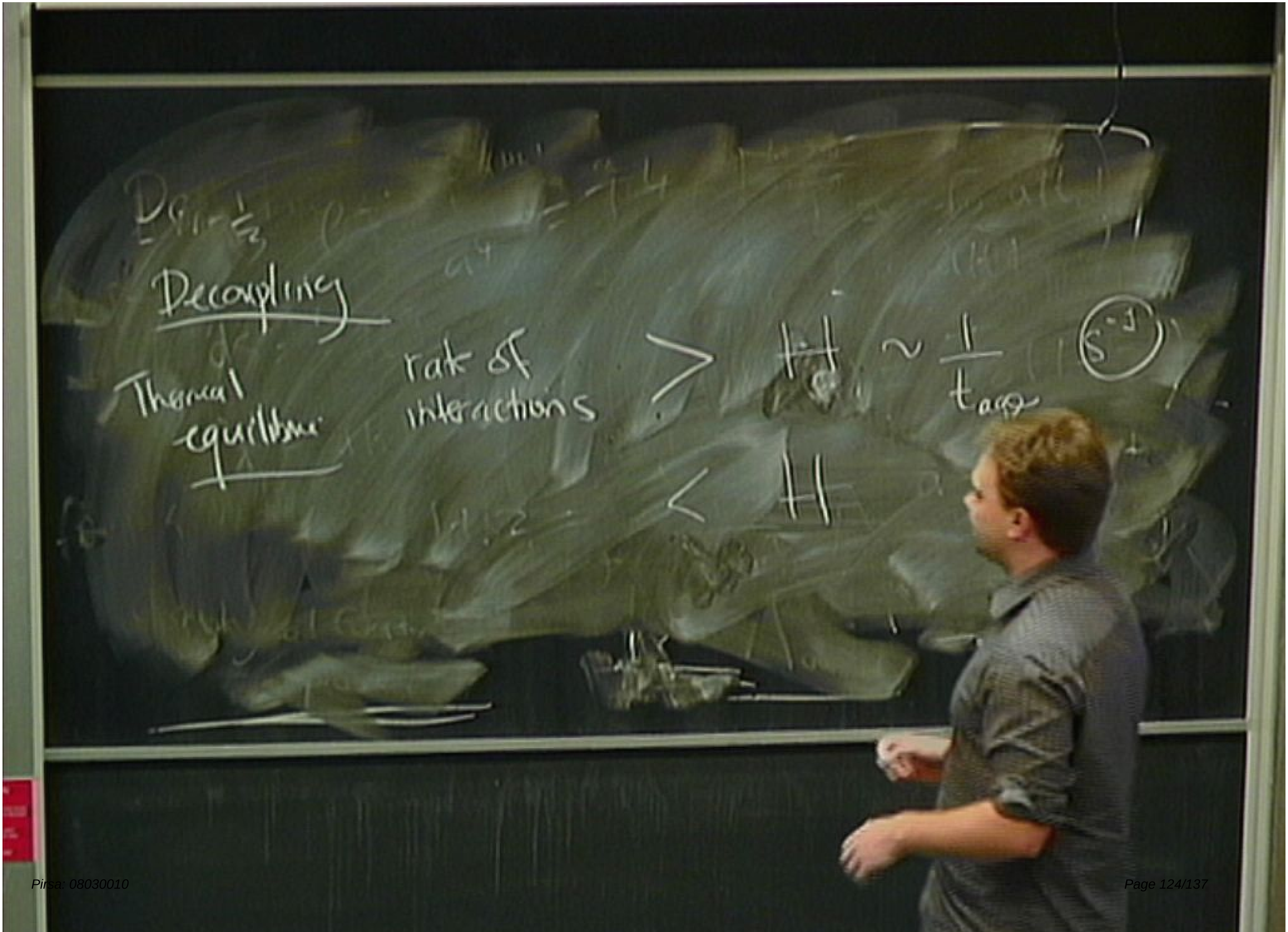
$$e^\pm + \bar{\nu}_e \leftrightarrow e^\pm + \bar{\nu}_e$$

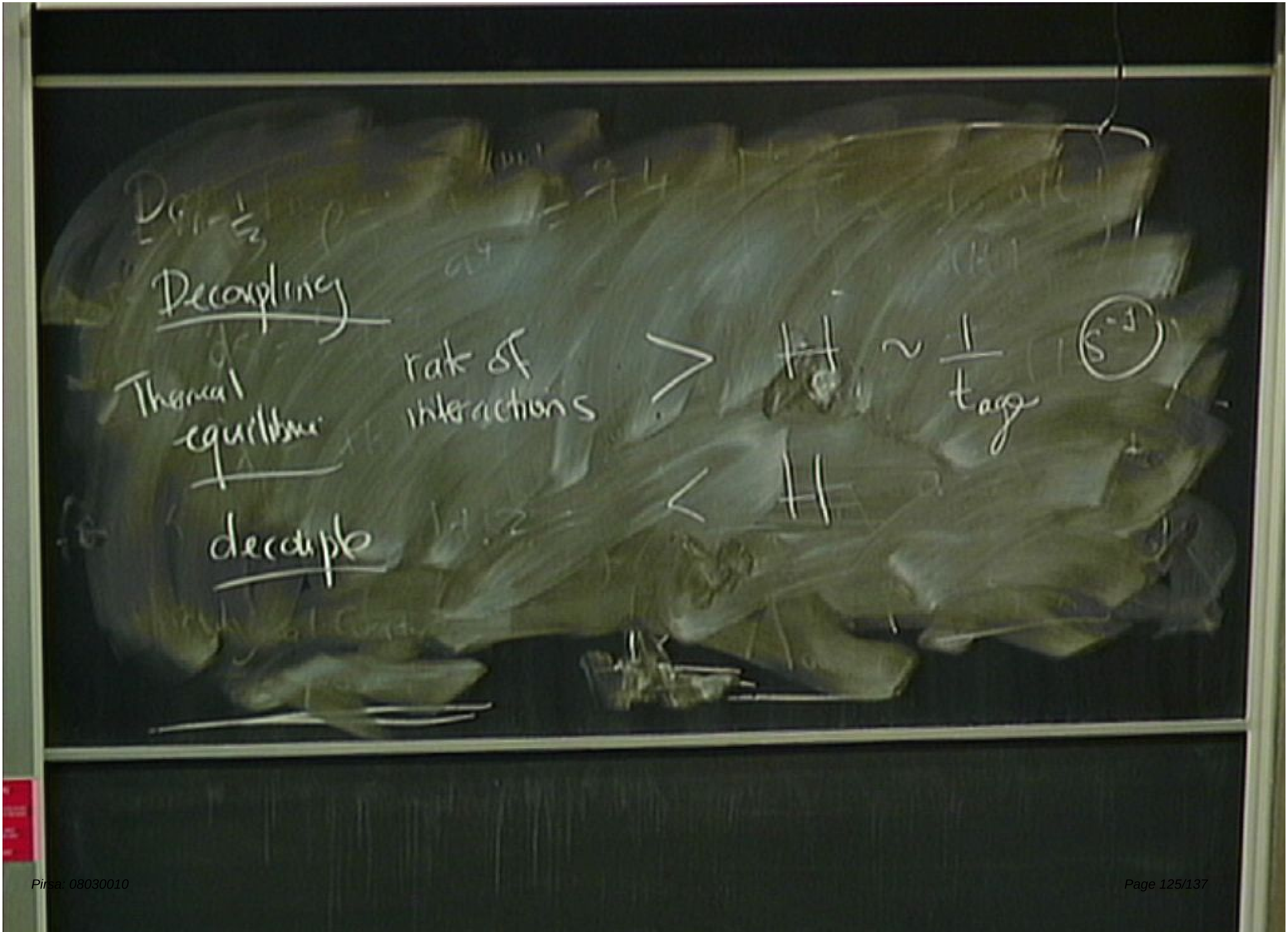
$$T_e = T_\nu$$

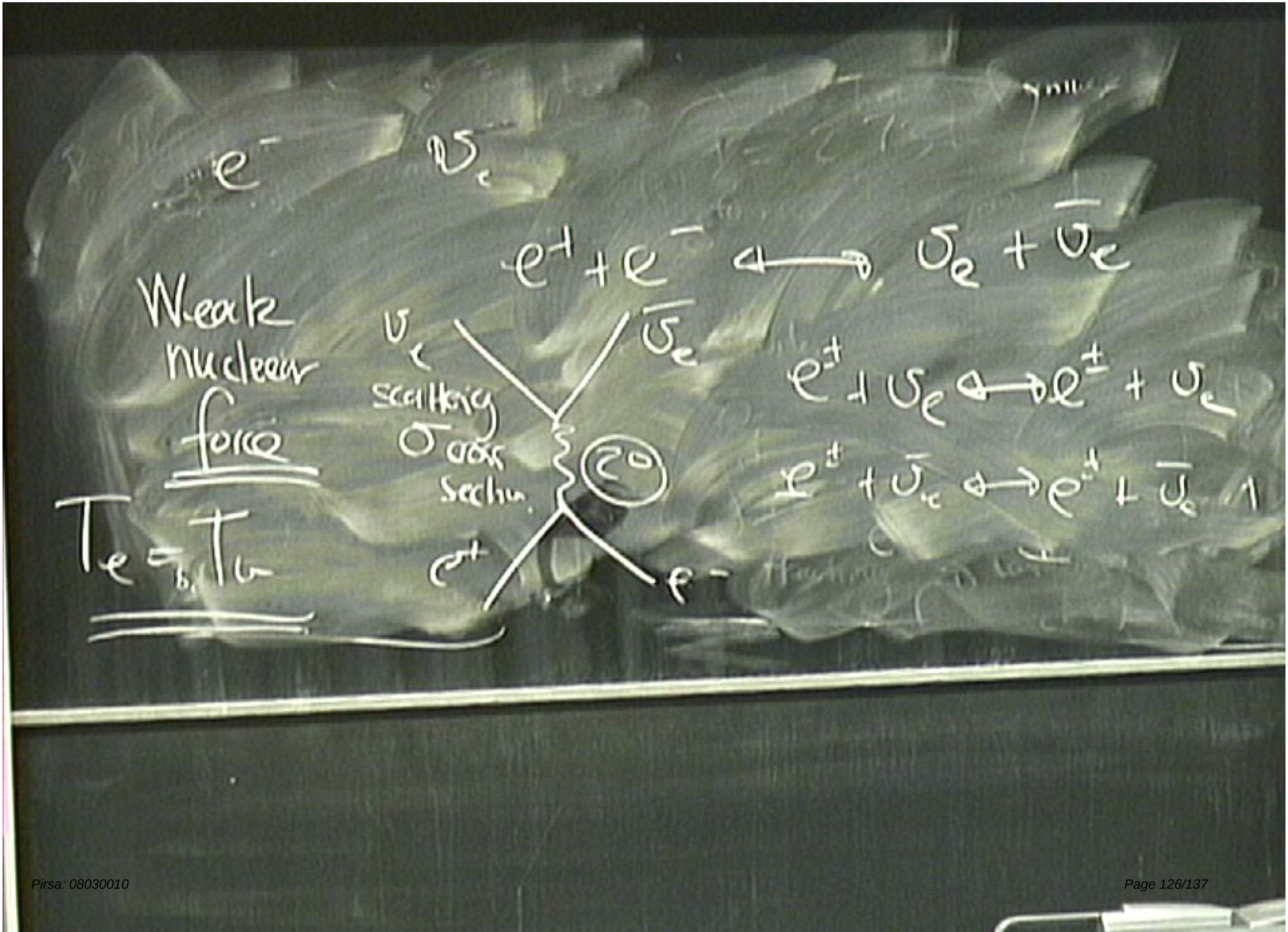
Decoupling

rate of interactions $> H \sim \frac{1}{t_{age}}$

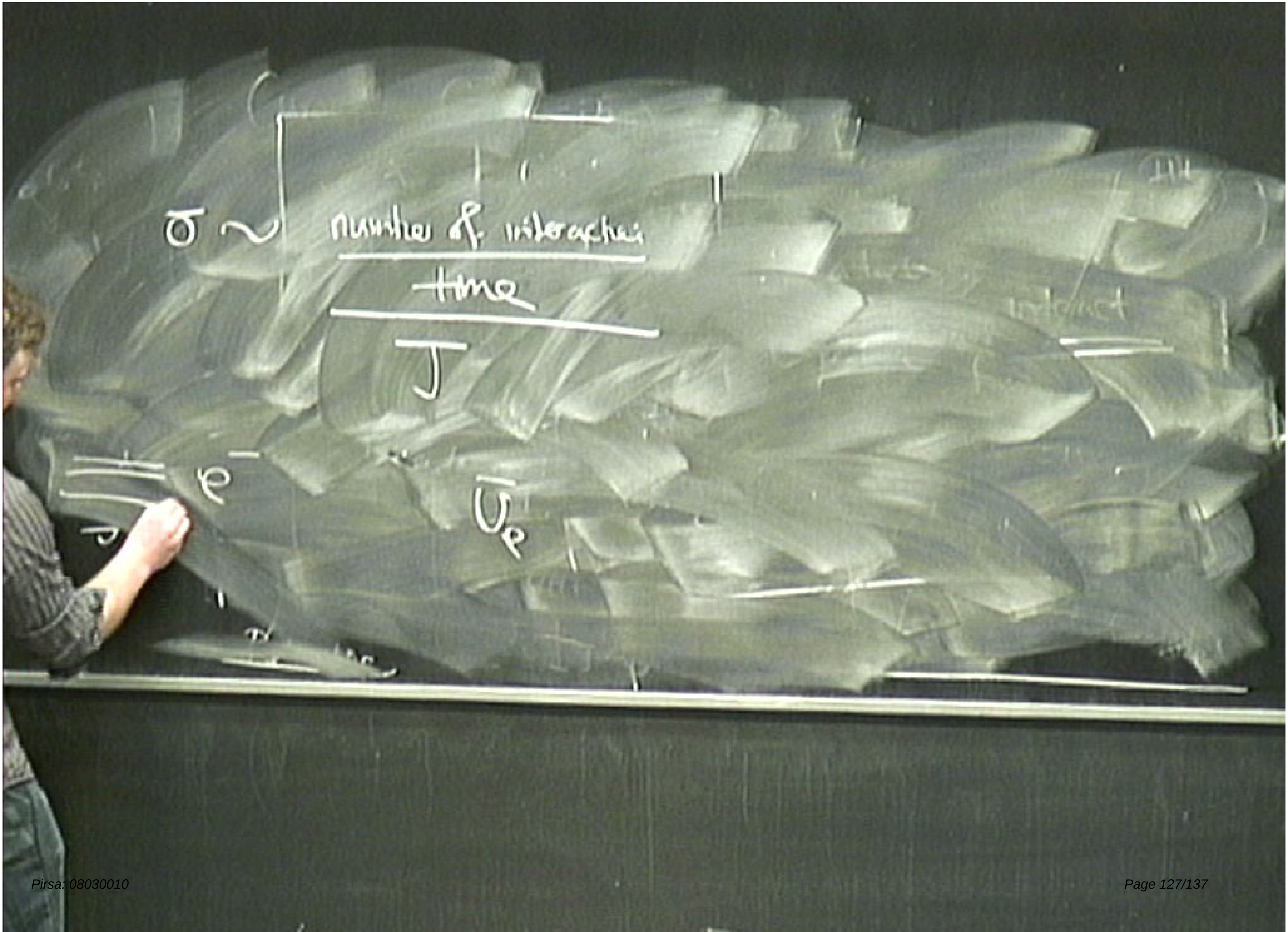

Decoupling
Thermal equilibrium
rate of interactions > H ~ 1/t_age
s^-1

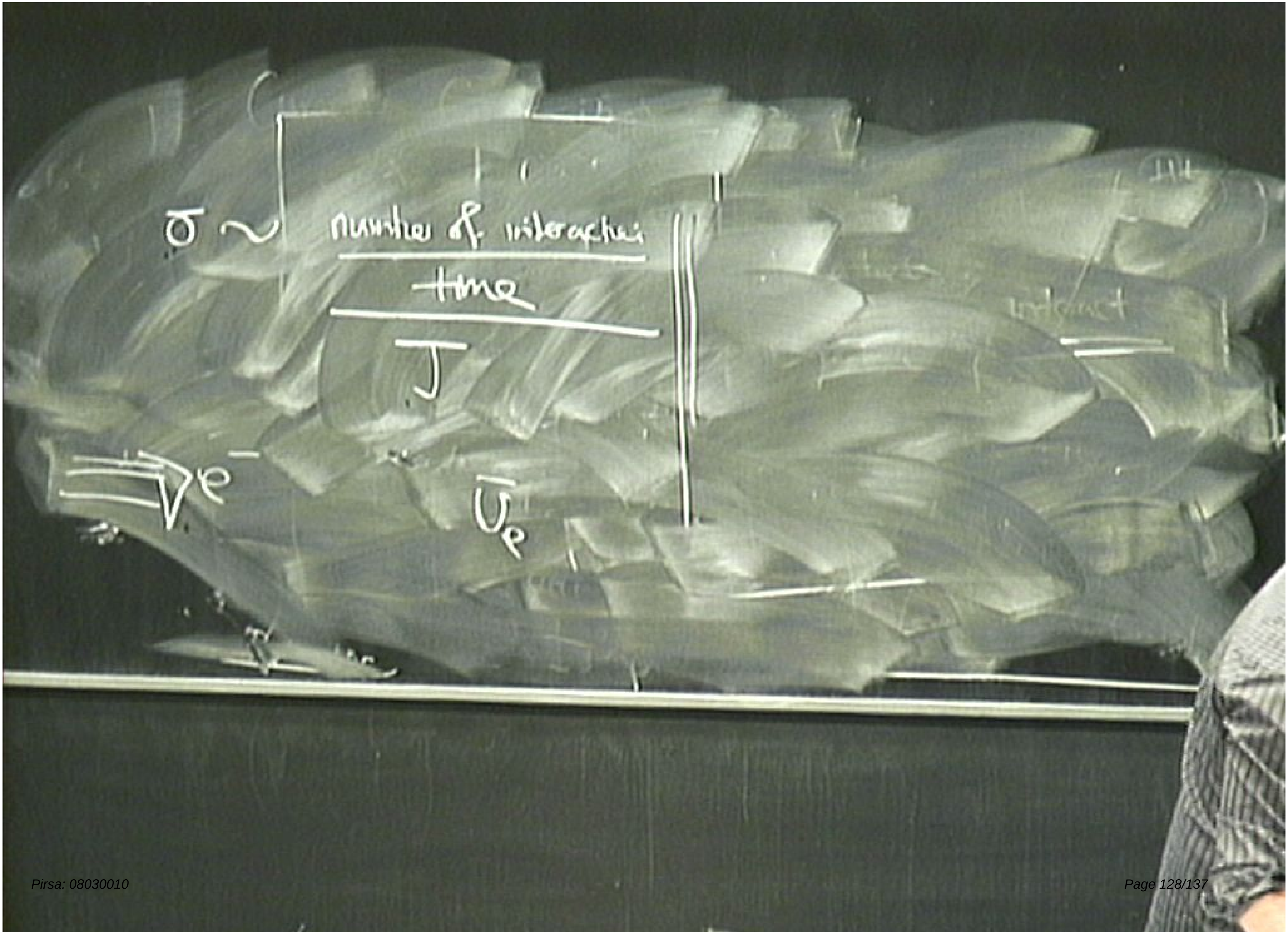

Decoupling
Thermal equilibrium
rate of interactions
> H ~ 1/t_age
s^-1
< H


Decoupling
Thermal equilibrium
rate of interactions > H ~ 1/t_age
s^-1
< H
decouple



e^- ν_e
$e^+ + e^- \longleftrightarrow \nu_e + \bar{\nu}_e$
Weak nuclear force
ν_e $\bar{\nu}_e$
scattering
σ cross section
Z^0
e^+ e^-
$e^\pm + \nu_e \longleftrightarrow e^\pm + \nu_e$
$e^\pm + \bar{\nu}_e \longleftrightarrow e^\pm + \bar{\nu}_e$
$T_e = T_\nu$


σ ~ number of interactions / time / J
e⁻
ν̄e


$\sigma \sim \dfrac{\dfrac{\text{number of interactions}}{\text{time}}}{J}$
νe^-
$\bar{\nu}_e$
interact

$$\sigma \sim \frac{\frac{\text{number of interactions}}{\text{time}}}{J}$$

νe^-

$\bar{\nu}_e$

$$\Gamma = \sigma \times n \times v$$

number of electrons per unit volume

speed

$$\sigma \sim \frac{\left(\frac{\text{number of interactions}}{\text{time}}\right)}{J}$$

$$\langle v^2 \rangle \sim T$$

$$\langle v \rangle \sim \sqrt{T}$$

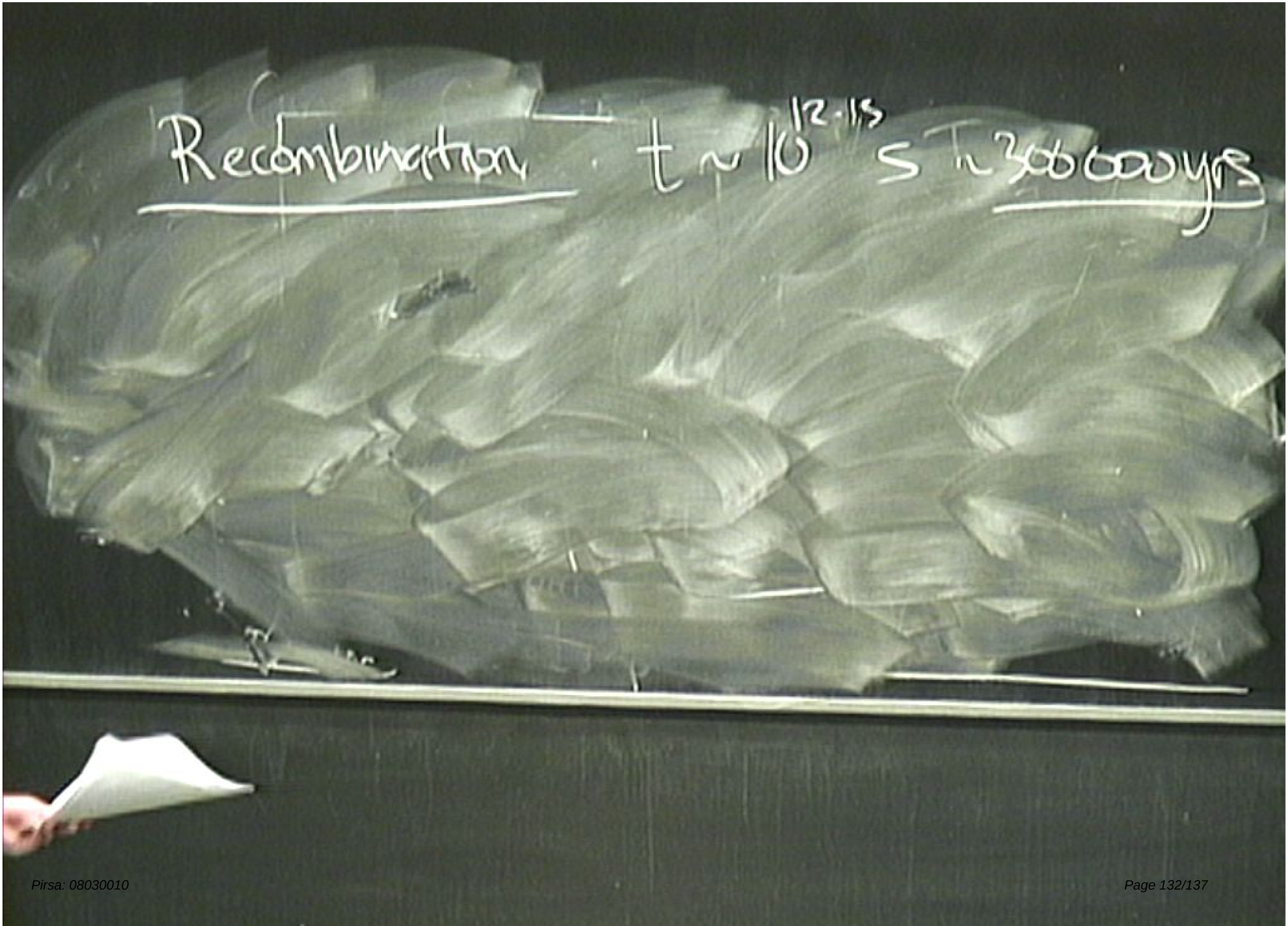
$$\Gamma = \sigma \times \underset{\uparrow}{\overset{\propto \frac{1}{a^3}}{n}} \times \underset{\uparrow}{\overset{\propto}{v}}$$

number of electrons per unit volume

speed

$\bar{\nu} e^-$ $\quad$ $\bar{\nu}_e$


Recombination


Recombination
t ~ 10^12-13 s ≈ 300000 yrs

Recombination $t \sim 10^{12\text{-}13}\,s$ $T \approx 300000$ yrs

13 billion years

Photons decouple

Radiation

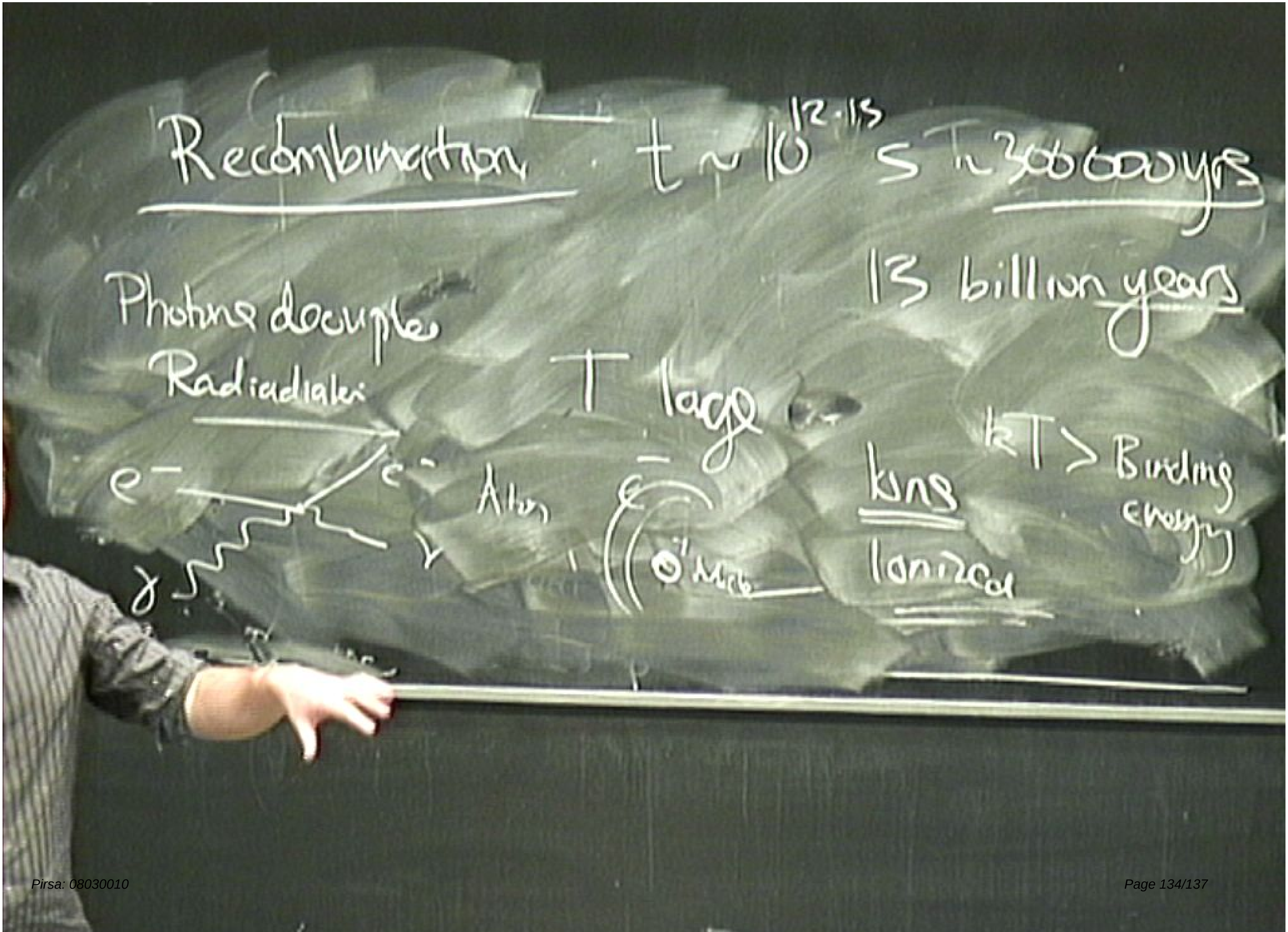
T large

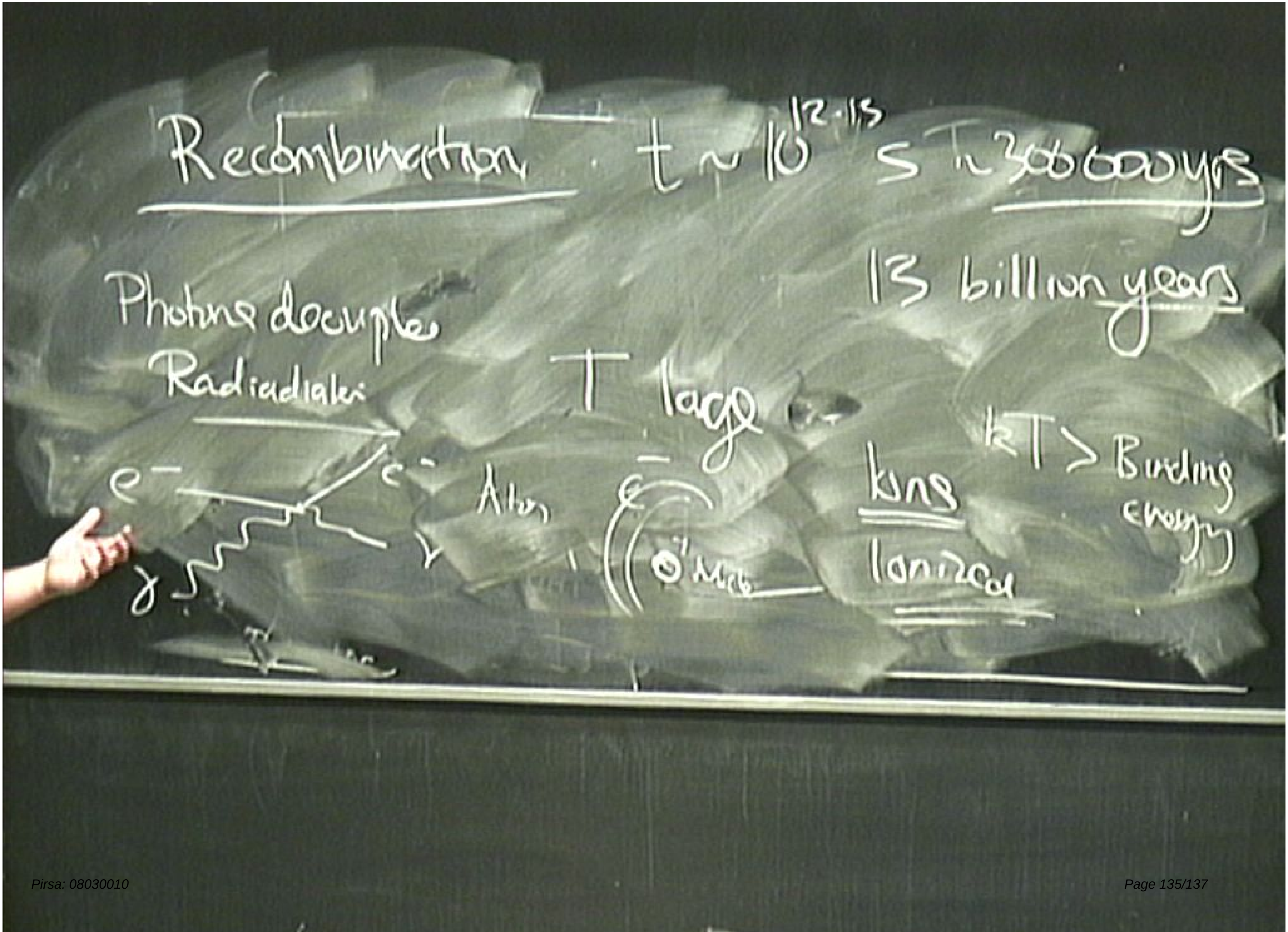
Atom

Ions

Ionized

$kT >$ Binding energy


Recombination
13 billion years
Photons decouple
T large
kT > Binding energy
Ions
Ionized
Atom
e⁻


Recombination
t ~ 10^12-13 s ≈ 300000 yrs
13 billion years
Photons decouple
Radiation
T large
kT > Binding energy
e⁻
e⁻
Atom
Ions
Ionized
γ

Recombination $t \sim 10^{12\text{-}13}$ s $T \approx 300000$yrs

T$\approx$ electrons protons recombine → neutral atoms Hydrogen

13 billion years

Photons decouple

Radiation

T large

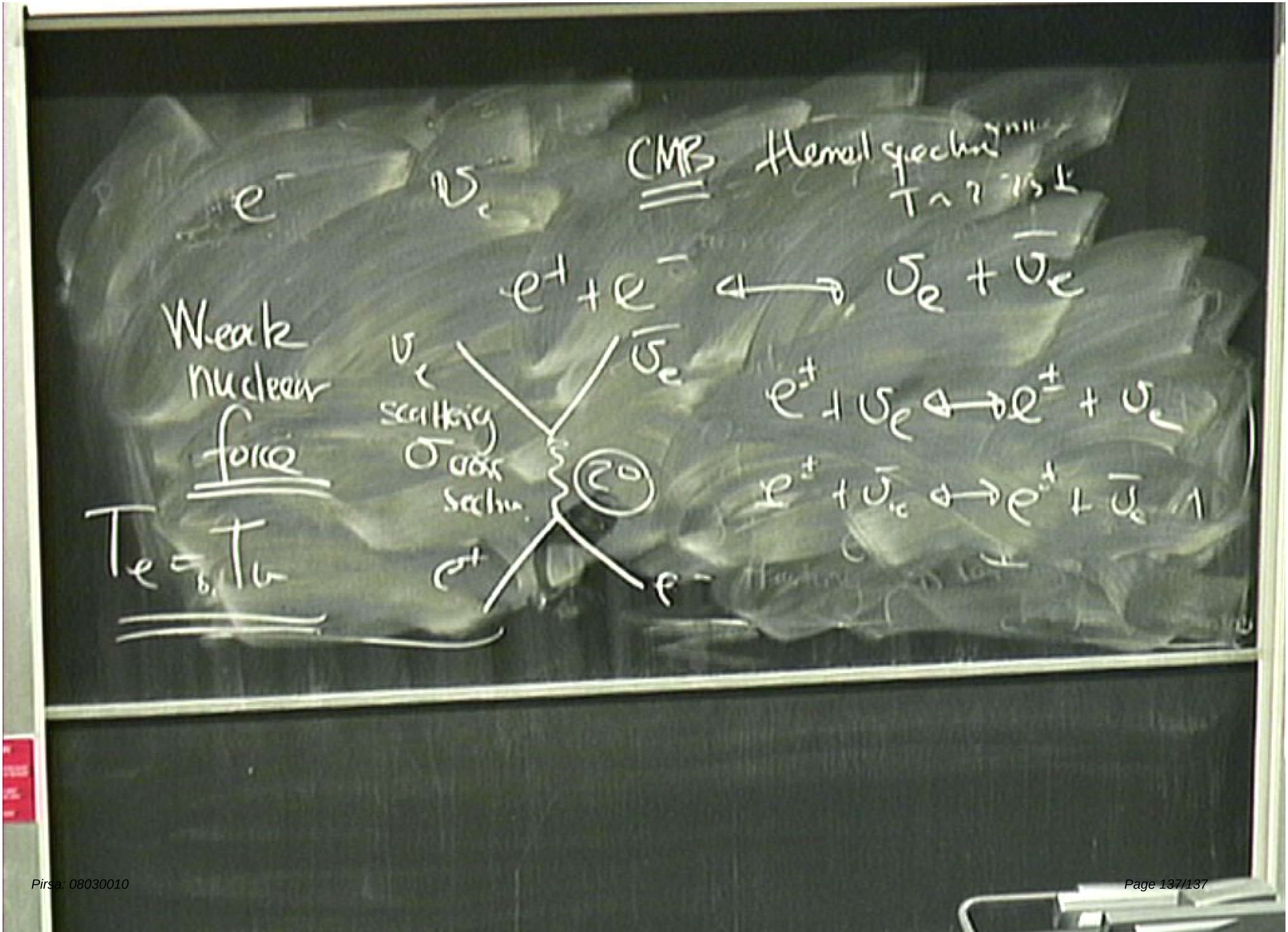
kT > Binding energy

Ions

Ionized

e^- e^- Atom

γ


CMB thermal spectrum
e⁻
ν_e
e⁺ + e⁻ ↔ ν_e + ν̄_e
Weak nuclear force
scattering
σ cross section
Z⁰
e^± + ν_e ↔ e^± + ν_e
e^± + ν̄_e ↔ e^± + ν̄_e
T_e = T_ν