Title: Foundations of Quantum Mechanics #6

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Abstract: Interferometry, measurement and interpretation. Beyond the quanta.

New Horizons Lectures.

Lecture 14: Reconstructing Quantum Theory

Philip Goyal

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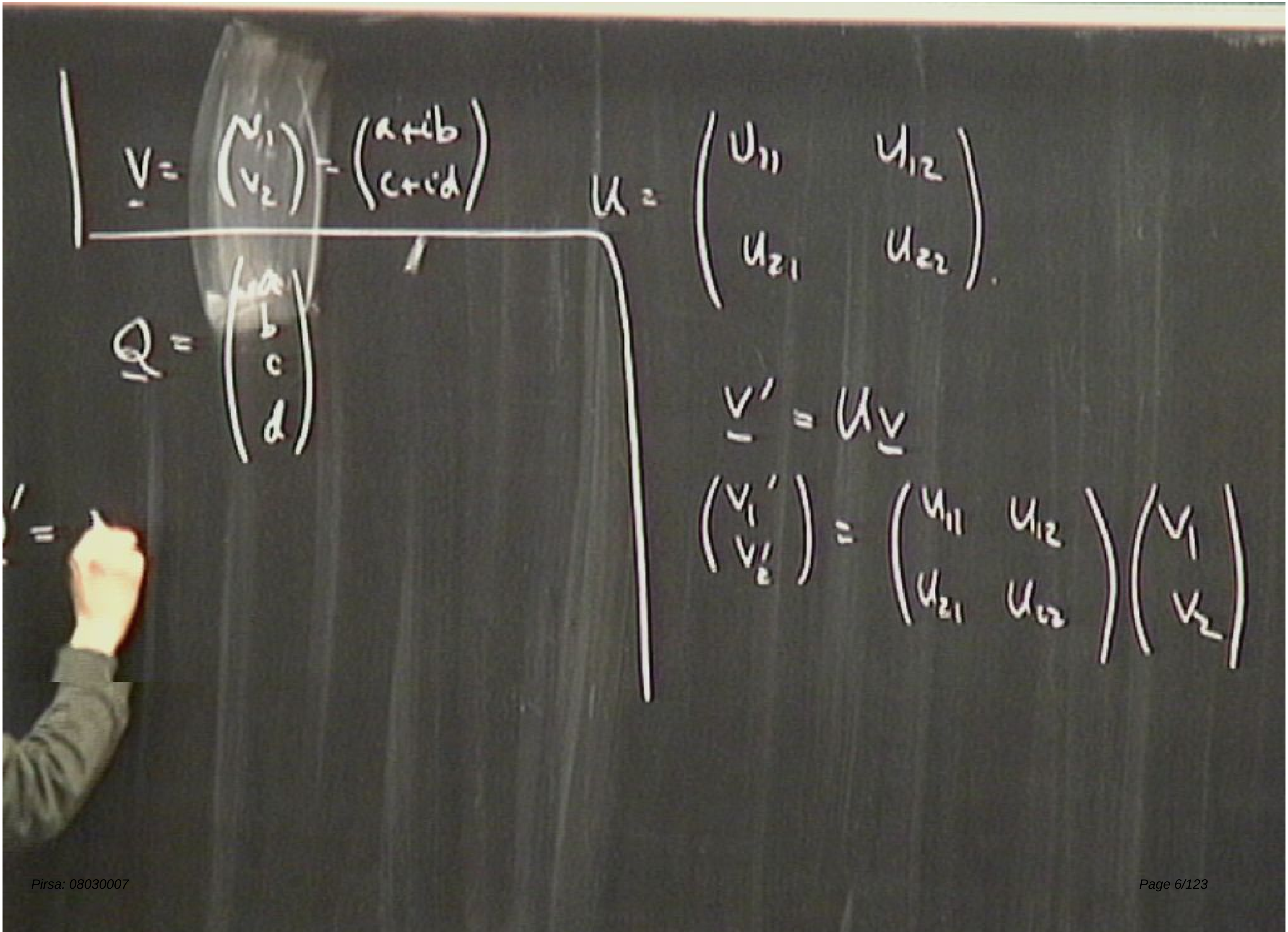
Philip Goyal

Recap

- An understanding of quantum theory (at the level of classical physics) requires construction of a conception of reality that underpins the quantum modelling framework.
- One obstacle: the quantum modelling framework has many mathematical features whose physical origin is obscure.
- Reconstruction of quantum theory: formulate a set of physical assumptions from which the quantum modelling framework can be derived.

What needs to be explained?

- States: why are states represented by complex vectors, and not simply a vector of real numbers?
- Transformations and Dynamics: why are these represented by unitary transformations of the vector space, and not simply by length-preserving one-to-one maps?
- Measurements: why are measurement outcomes subject to the Born rule?


$\underline{v} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} a+ib \\ c+id \end{pmatrix}$
$\underline{Q} = \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$
$U = \begin{pmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{pmatrix}$.
$\underline{v}' = U\underline{v}$
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↑ rotation matrix

very special type of rotation matrix.

Objectives of this lecture

- Convey a sense of the diversity of approaches to reconstruction that currently exist.
- Investigate the type of strategy one can adopt in attempting to reconstruct quantum theory.
- Give examples of the kinds of insights have been obtained.

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Reconstruction: The Goal

- “The task is not to make sense of the quantum axioms by heaping more structure, more definitions, more science-fiction imagery on top of them, but to throw them away wholesale and start afresh. We should be relentless in asking ourselves: From what deep physical principles might we derive this exquisite mathematical structure?” — **Chris Fuchs**

Special Relativity: A Model of Reconstruction

- The Lorentz transformations can be written down by inspection of the symmetry group of Maxwell's equations. But they conflict with Galileo's transformations. What is the physical origin of this discrepancy?
- Einstein showed that the Lorentz transformations can be deduced from two main postulates and a careful operational definition of time.

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Experimental Basis of Probabilistic Assumption

- When we perform simple experiments with quantum systems (such as spin-1/2 systems), we can look to see if the data is probabilistic or not.
- We find that, indeed, the data is best modelled by a probabilistic source.
- So, the assumption of probabilistic outcomes is well-supported by experiment.

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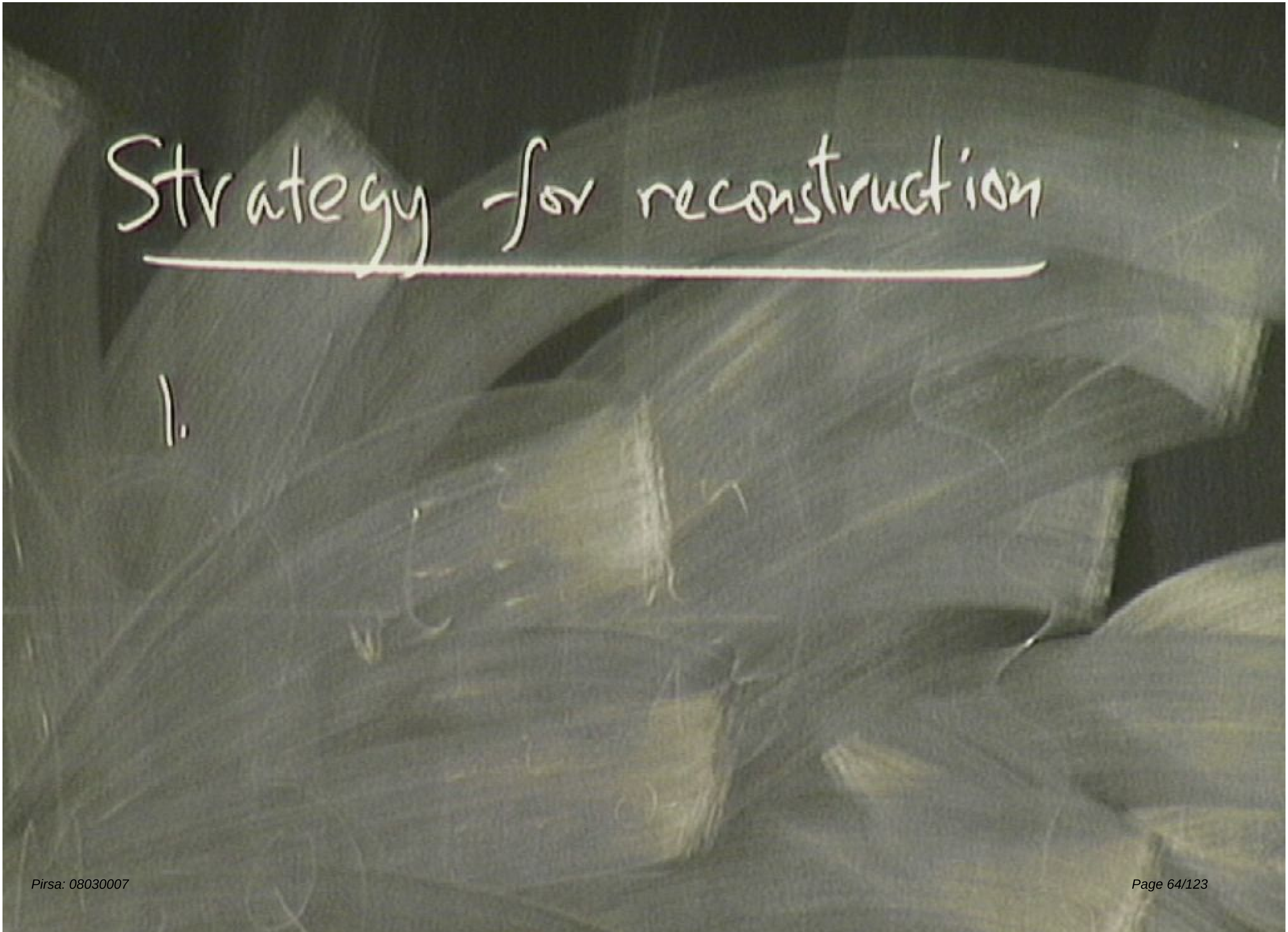
Strategy for reconstruction
1.

Strategy for reconstruction

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   state, $S =$

- system, background environment, measuring device, rest of the universe

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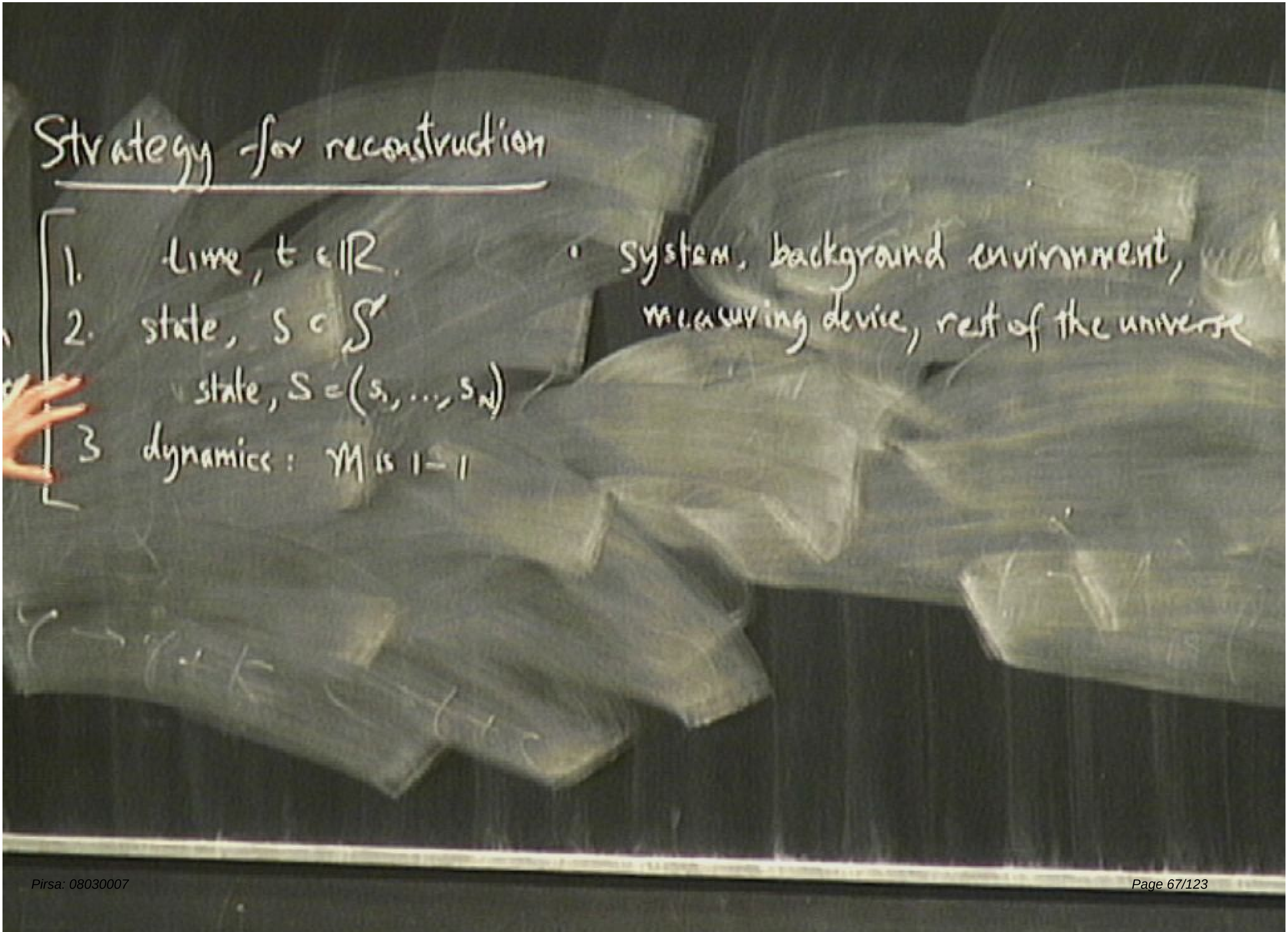
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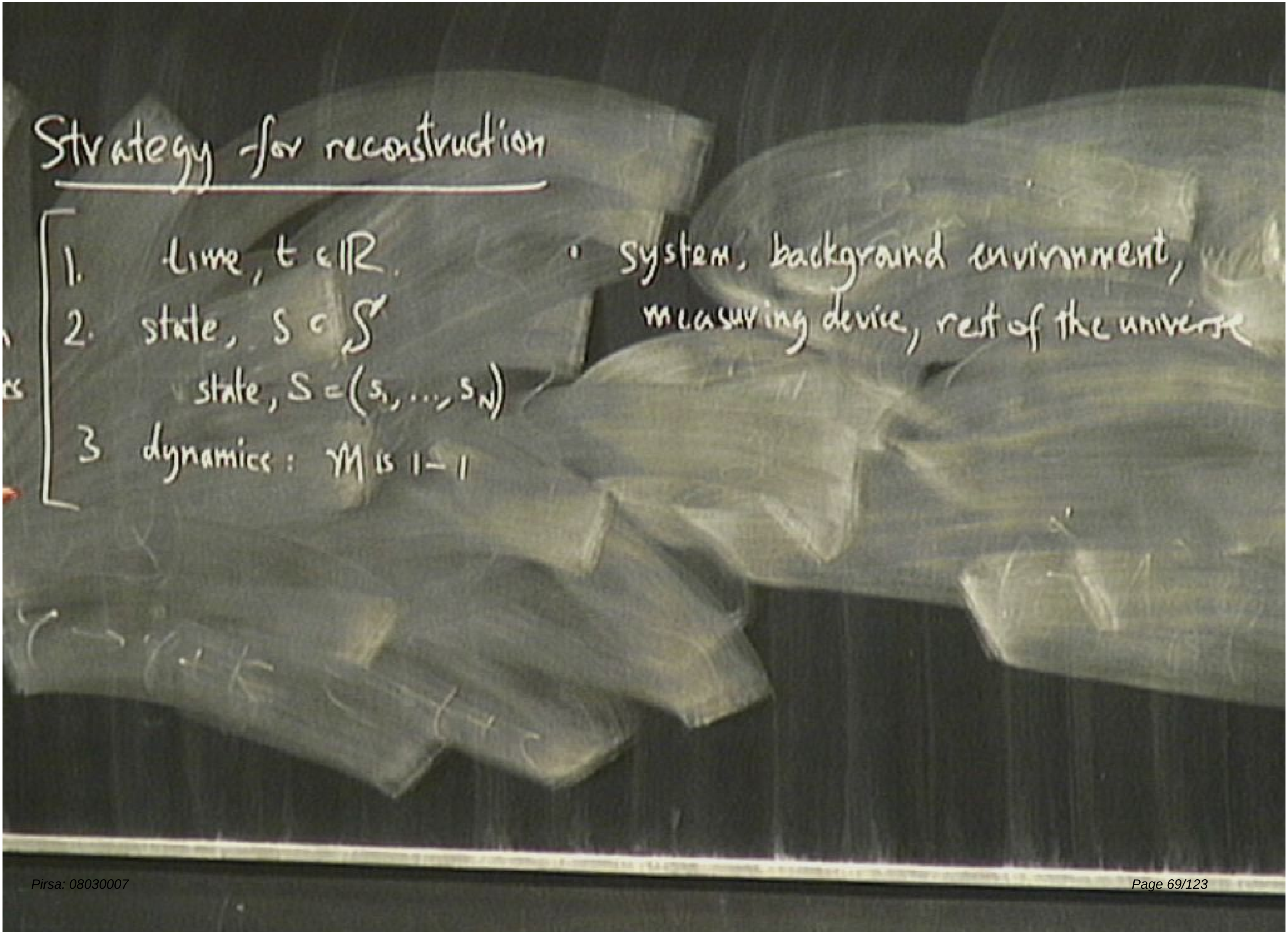
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differences

1. statistical outcomes of measurement
2. finiteness of # of possible measurement outcomes.
3. complementarity
4. non-locality
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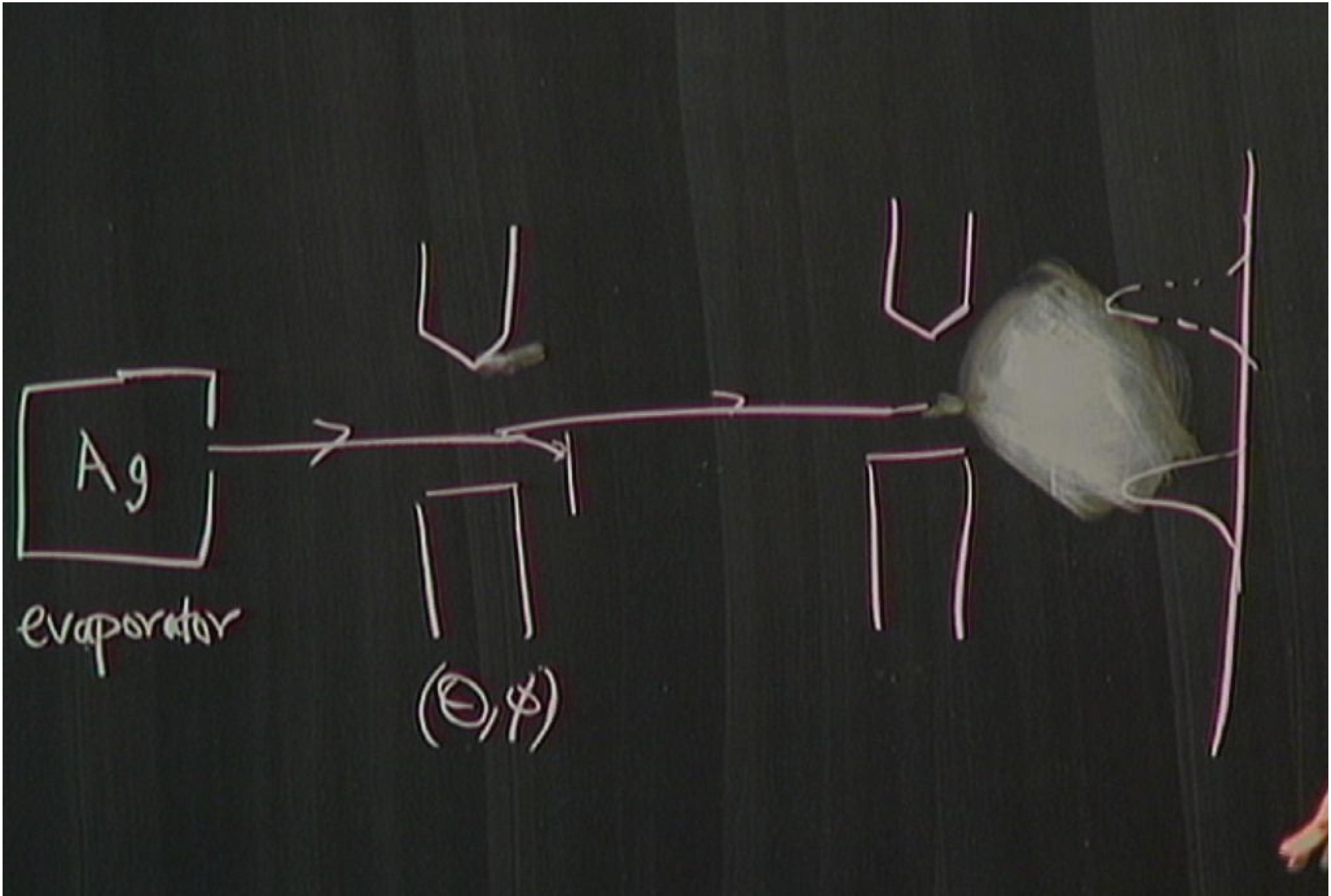
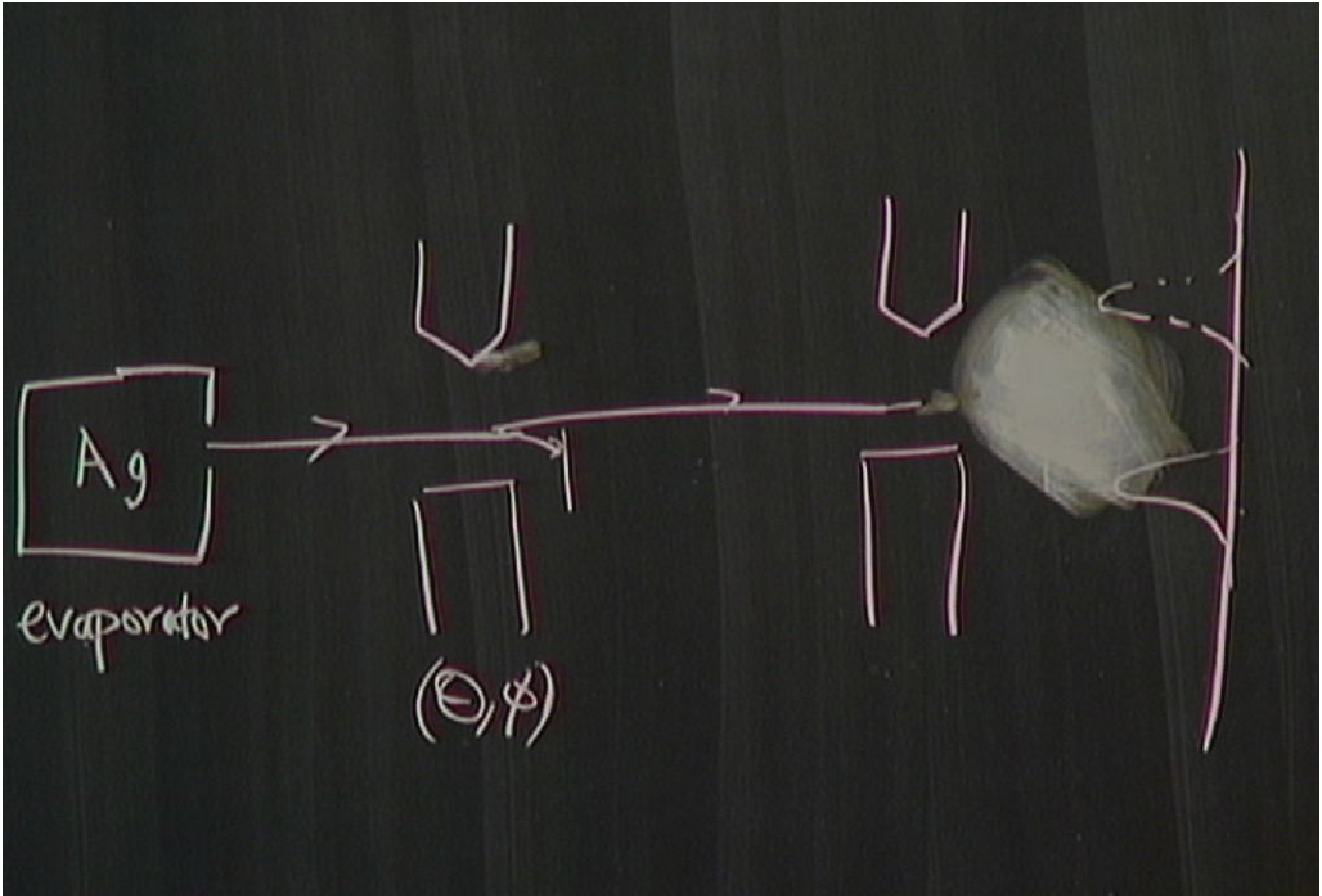
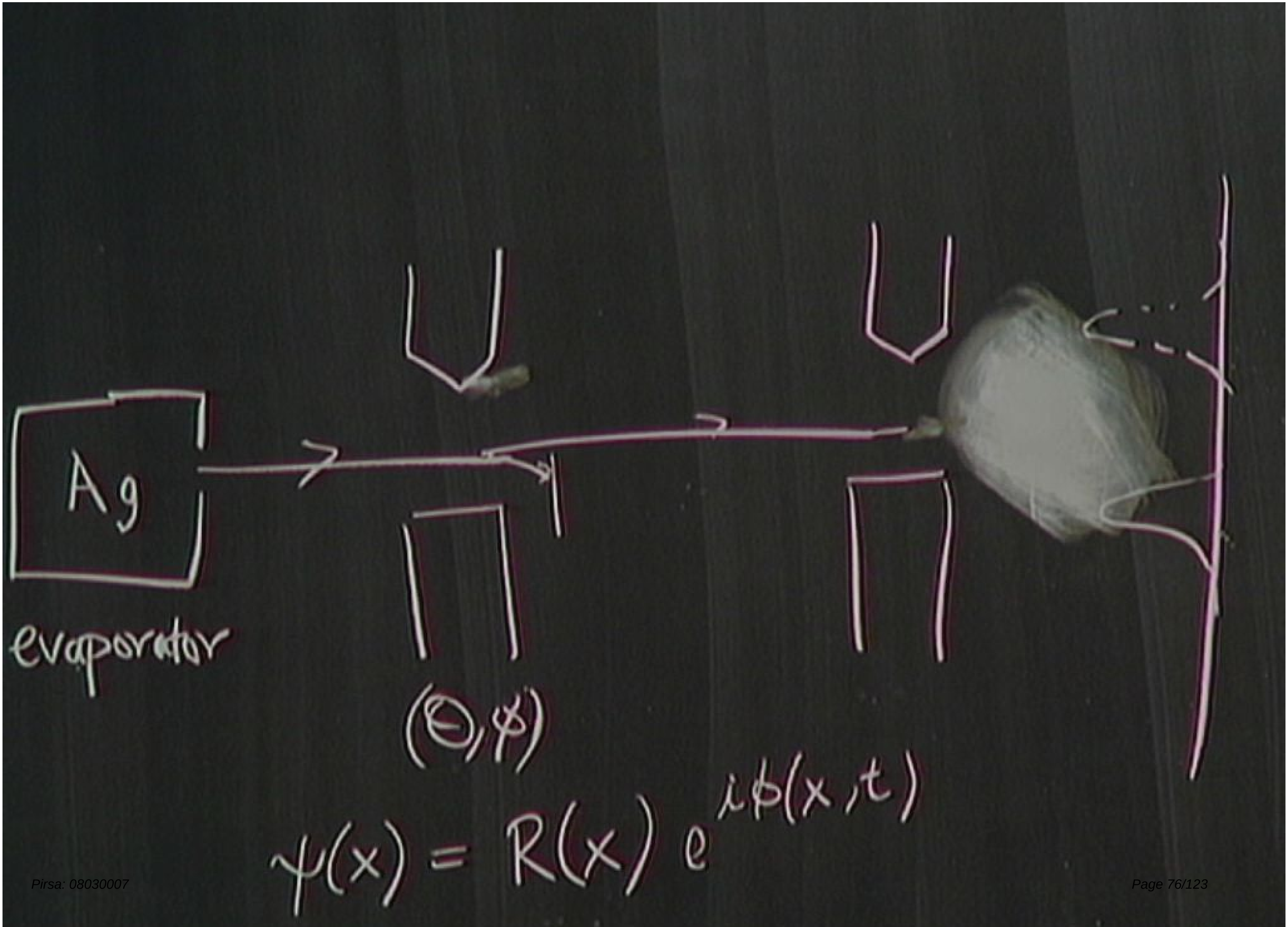
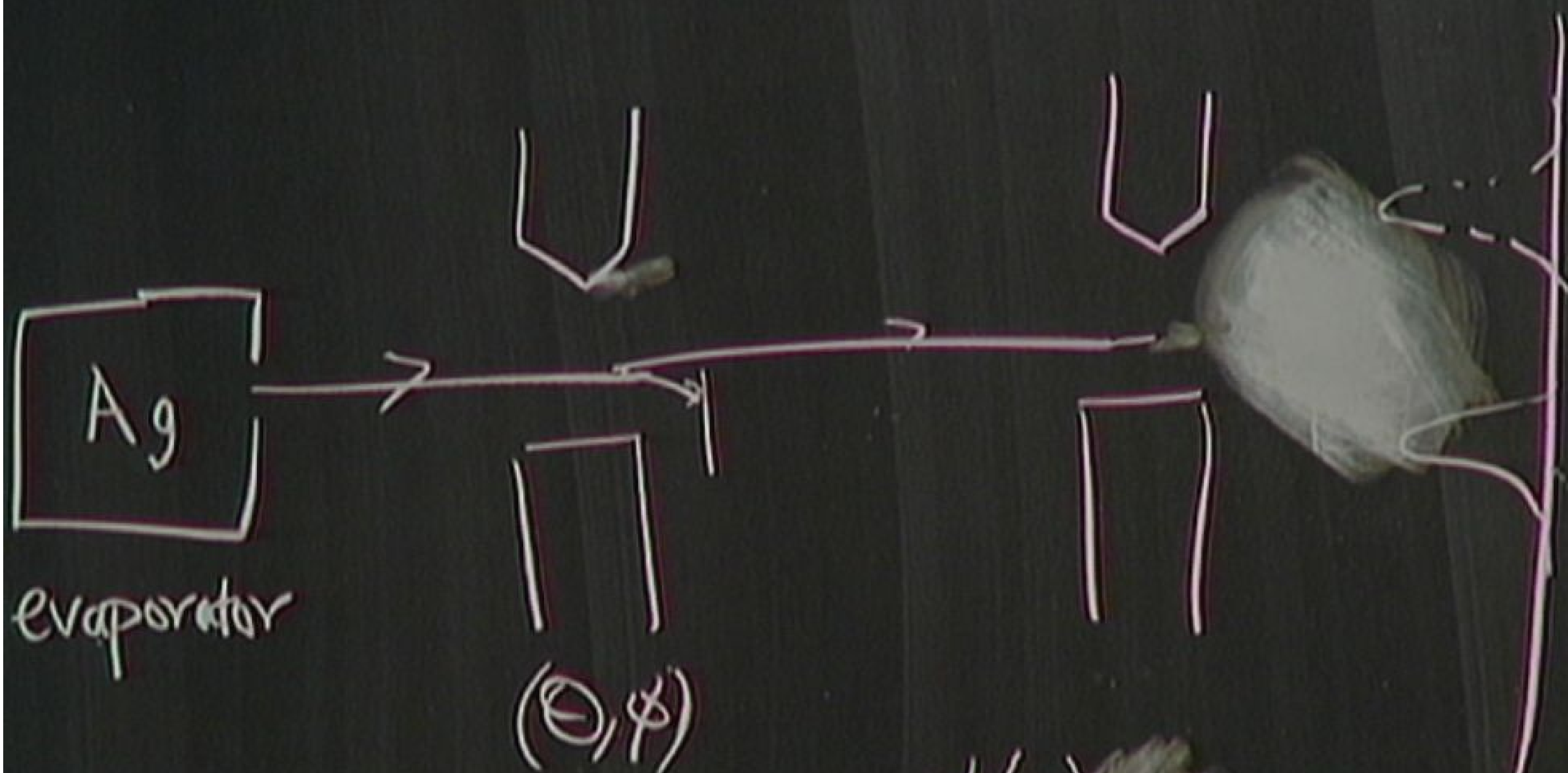
4. non-locality
5. non-contextuality.

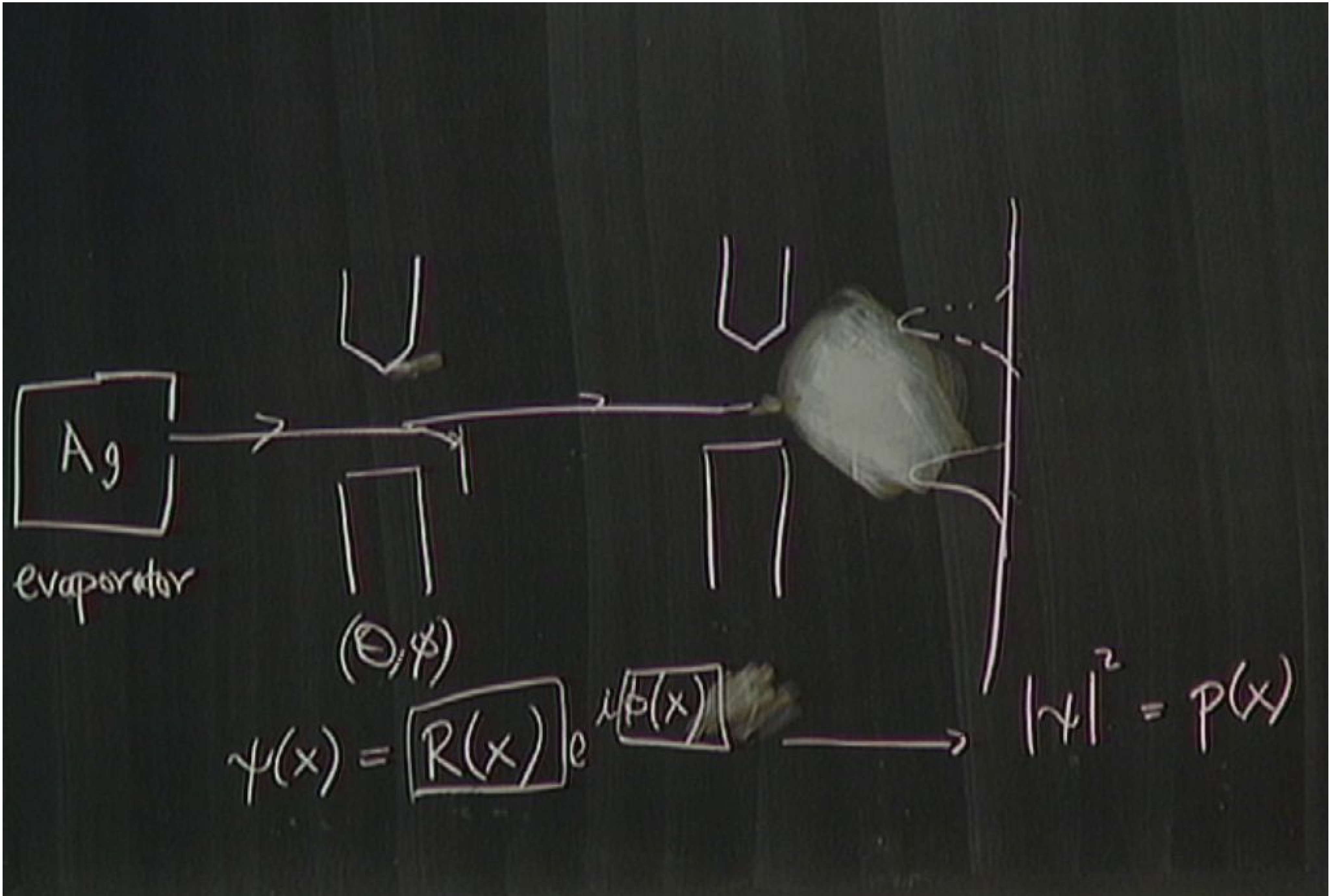

Ag
evaporator
(θ,φ)

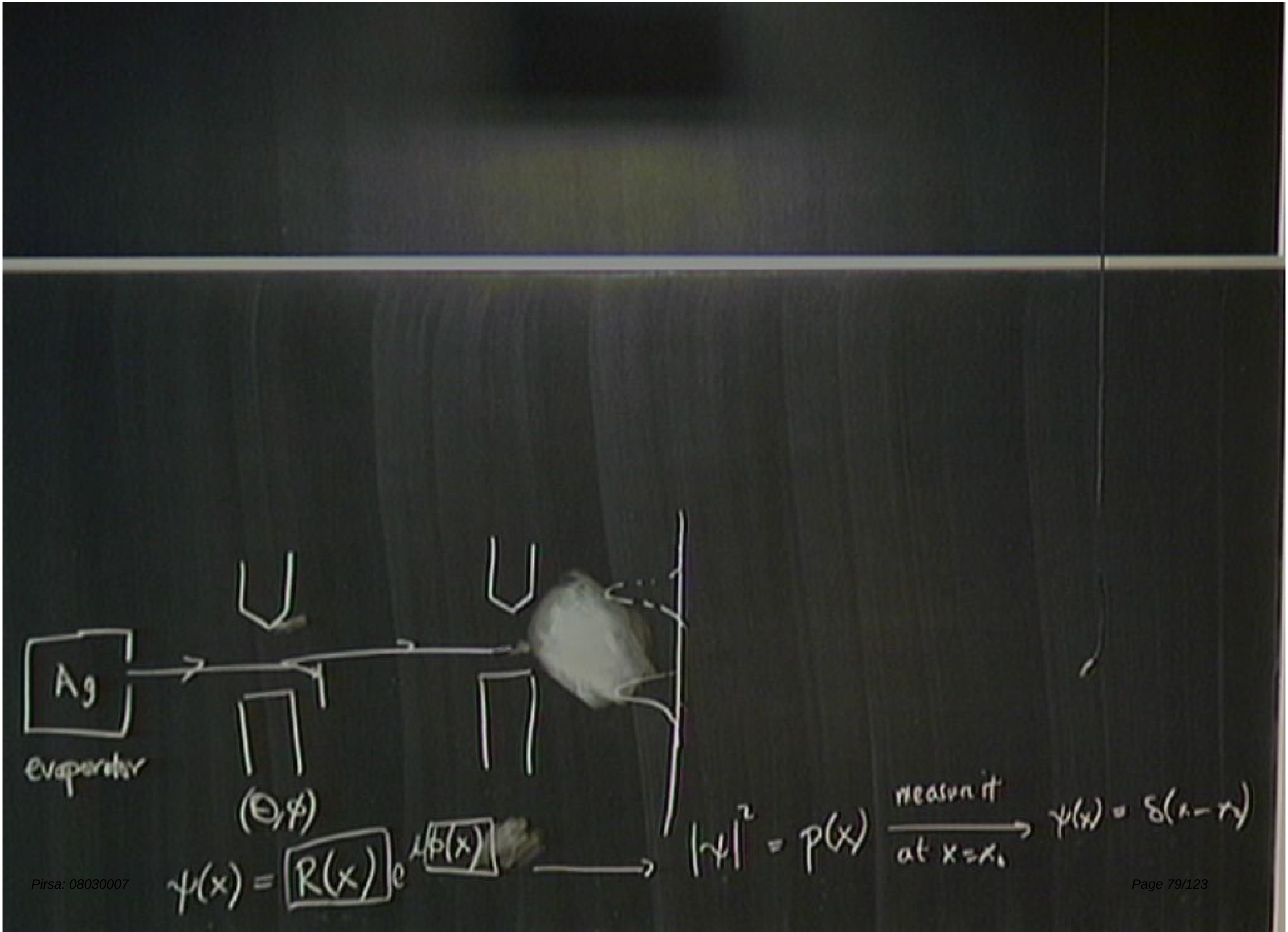

Ag
evaporator
(Θ,φ)


Ag
evaporator
(θ,φ)
ψ(x) = R(x) e^{iφ(x,t)}



$$\psi(x) = R(x)\, e^{i\phi(x)}$$


Ag
evaporator
(θ,φ)
ψ(x) = R(x) e^{iφ(x)}
|ψ|² = P(x)


Ag
evaporator
(θ,φ)
ψ(x) = R(x) e^{iφ(x)}
|ψ|² = P(x)
measure it
at x=x₀
ψ(x) = δ(x−x₀)

Strategy for reconstruction

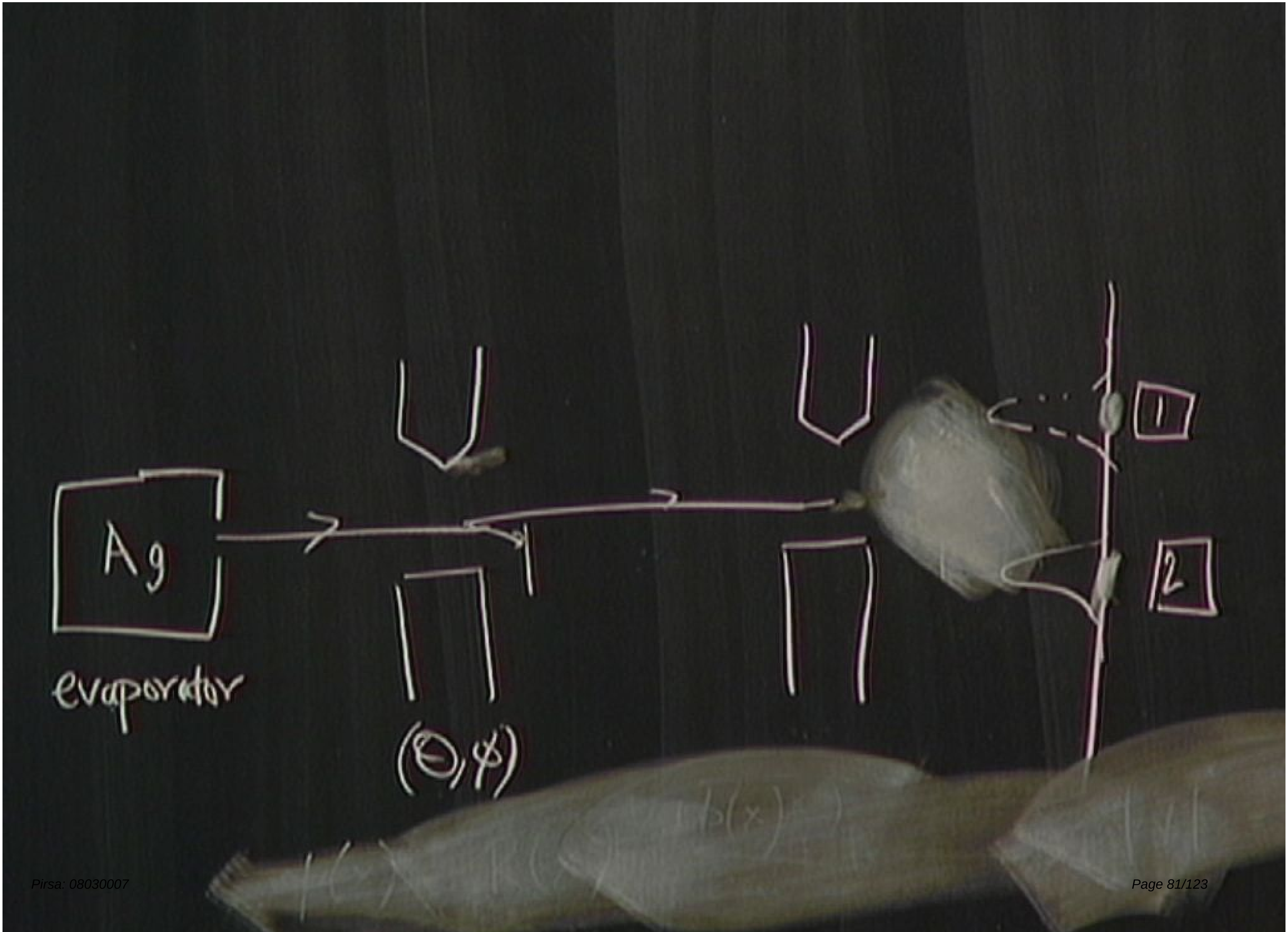
Common features

1. time, $t \in \mathbb{R}$.
2. state, $s \in \mathcal{S}$
   state, $S = (s_1, \ldots, s_N)$
3. dynamics: $\mathcal{M}$ is 1–1
4. measurements are reproducible.

• system, background environment, measuring device, rest of the universe

differences

1. statistical outcomes of measurement ; $|\psi_x|^2 = p(x)$ ($\psi_x(x)$)
2. finiteness of # of possible measurement outcomes.
3. complementarity
4. non-locality
5. non-contextuality.


Ag
evaporator
(θ,φ)
1
2

Strategy for reconstruction

Common features

1. time, $t \in \mathbb{R}$.
2. state, $S \in \mathcal{S}$
   state, $S = (s_1, \ldots, s_N)$
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differences

*1 statistical outcomes of measurement ; $\psi(x)$ → $|\psi|^2 = P(x)$

*2. finiteness of # of possible measurement outcomes.

*3. complementarity

* 4. non-locality

* 5. non-contextuality.

State $\mathbb{P} = (p_1, p_2, \ldots, p_N)$

Measurement: yields outcome i with probability p_i

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Measurement outcome = 1.

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Reproducibility $\Rightarrow$ state afterwards is $\underline{p}' = (1, 0, 0, \ldots, 0)$

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$\textcircled{\underline{p}} \longrightarrow$ "1"

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Measurement : yelds outcome i with probability p_i

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Reproducibility $\Rightarrow$ state afterwards is $\underline{p}' = (1, 0, 0, \ldots, 0)$

$\textcircled{\underline{p}} \longrightarrow$ "1"

State $\underline{p} = (p_1, p_2, \dots, p_N)$

Measurement: yields outcome i with probability p_i

Measurement outcome = 1.

Reproducibility $\Rightarrow$ state afterwards is $\underline{p}' = (1, 0, 0, \dots, 0)$

$(\underline{p}) \longrightarrow$ "1"

$(\underline{p}) \longrightarrow$ "2"

$(\underline{p}) \longrightarrow$ "1"

$\vdots$

$\underline{p} = (p_1, p_2)$; measurement gives

m_1 "1's"

m_2 "2's"

if done n times.

How much information does the data provide about the state ?

How much information does the data provide about the state?

$$\Pr(p_1 \mid f_1, n, I)$$

$\underline{p}' = (1, 0, 0, \ldots, 0)$

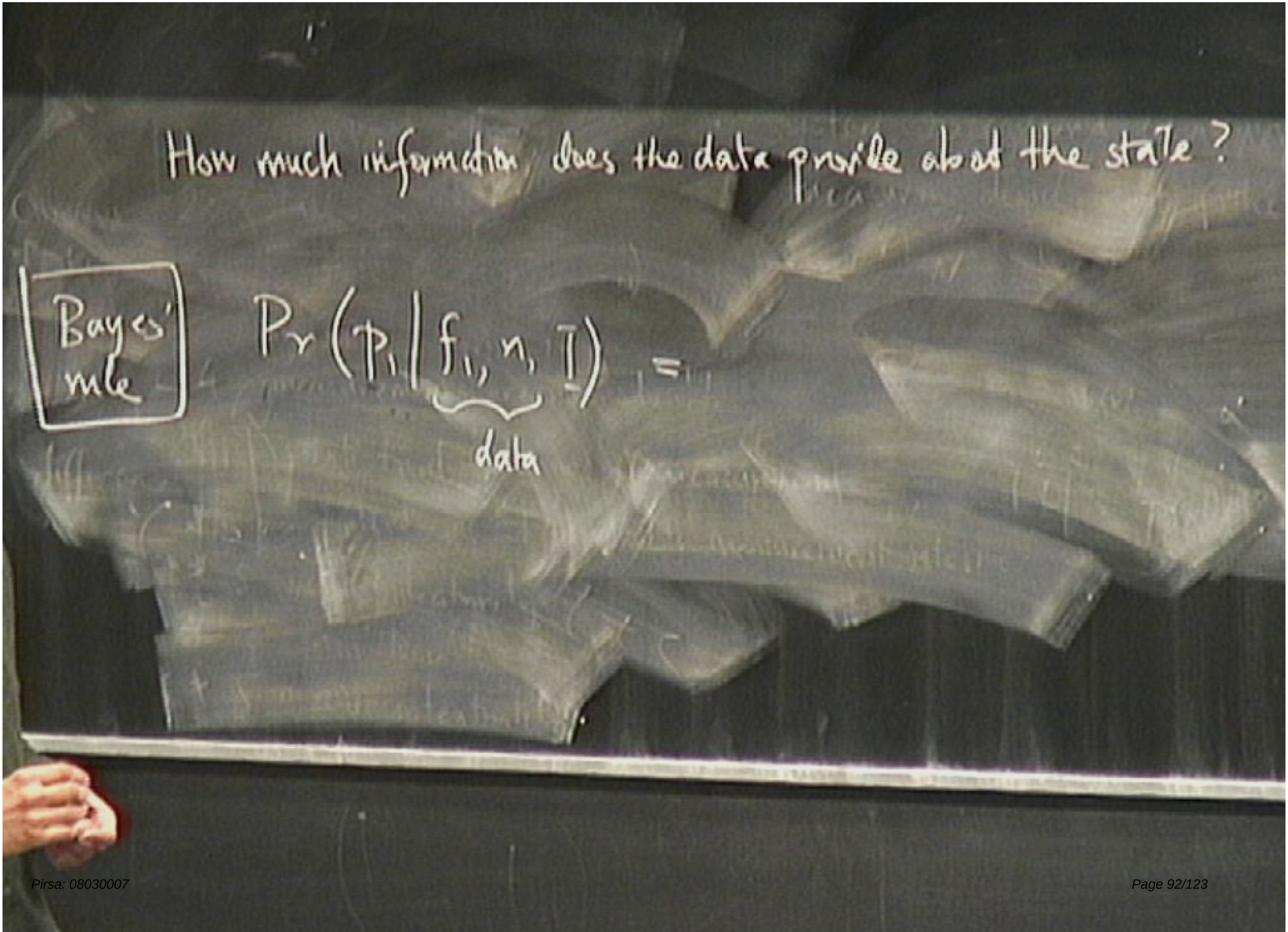
$P_2)$; measurement gives

m_1 "1's"

m_2 "2's"

if done n times.

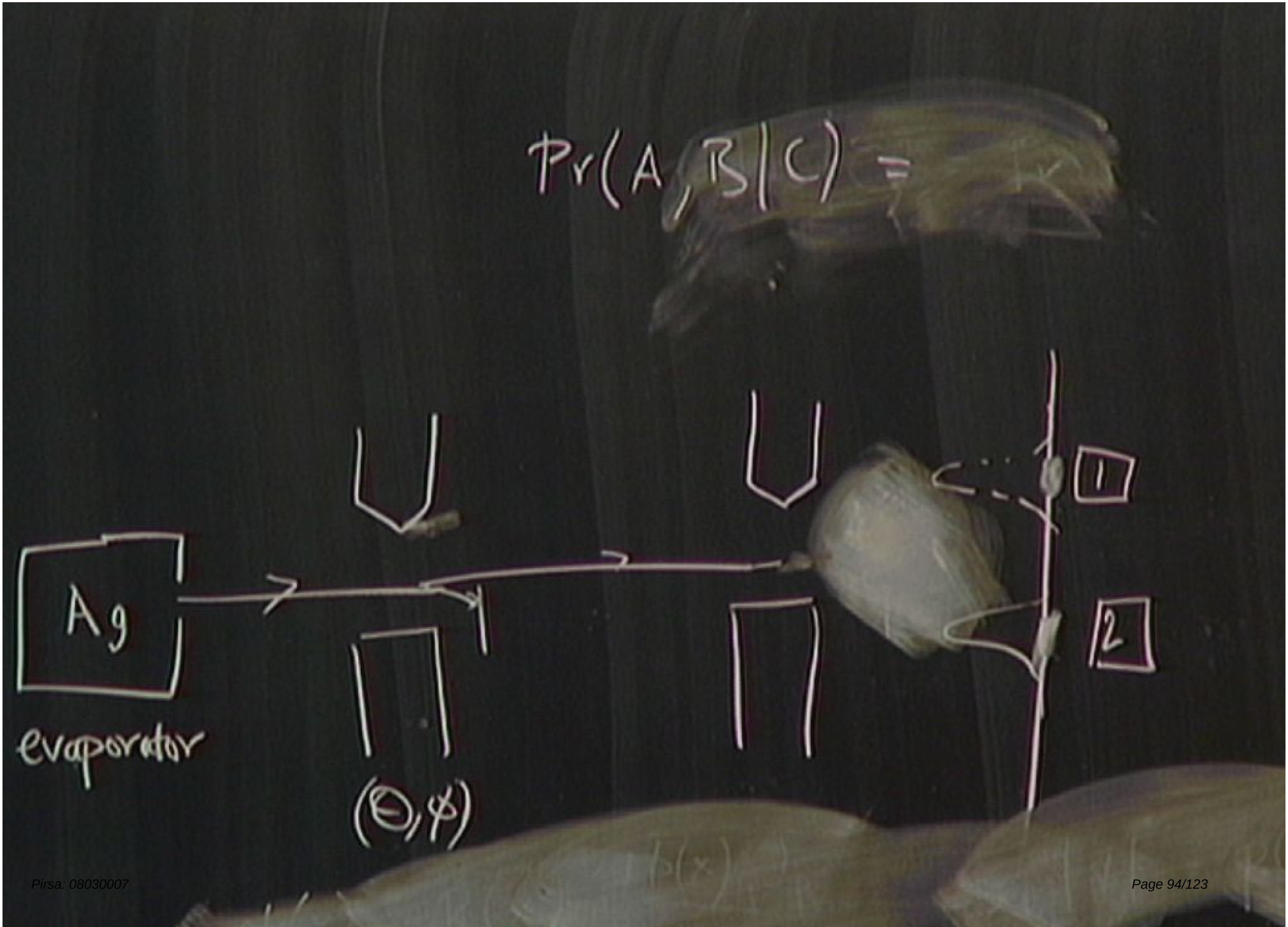
$$f_1 = \frac{m_1}{n}$$

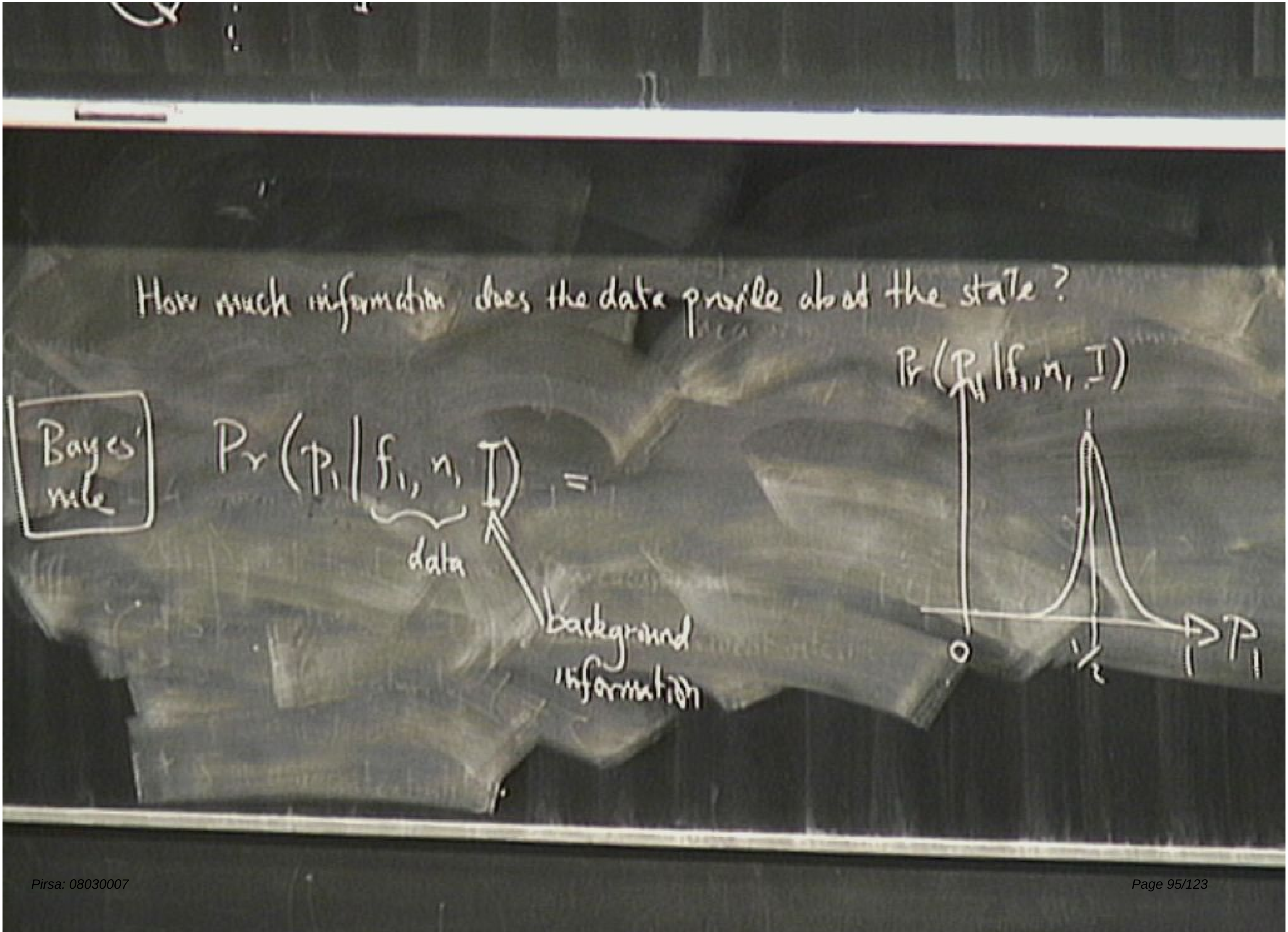

How much information does the data provide about the state ?
Bayes' rule
$\Pr(p_1 | f_1, n, I) =$
data

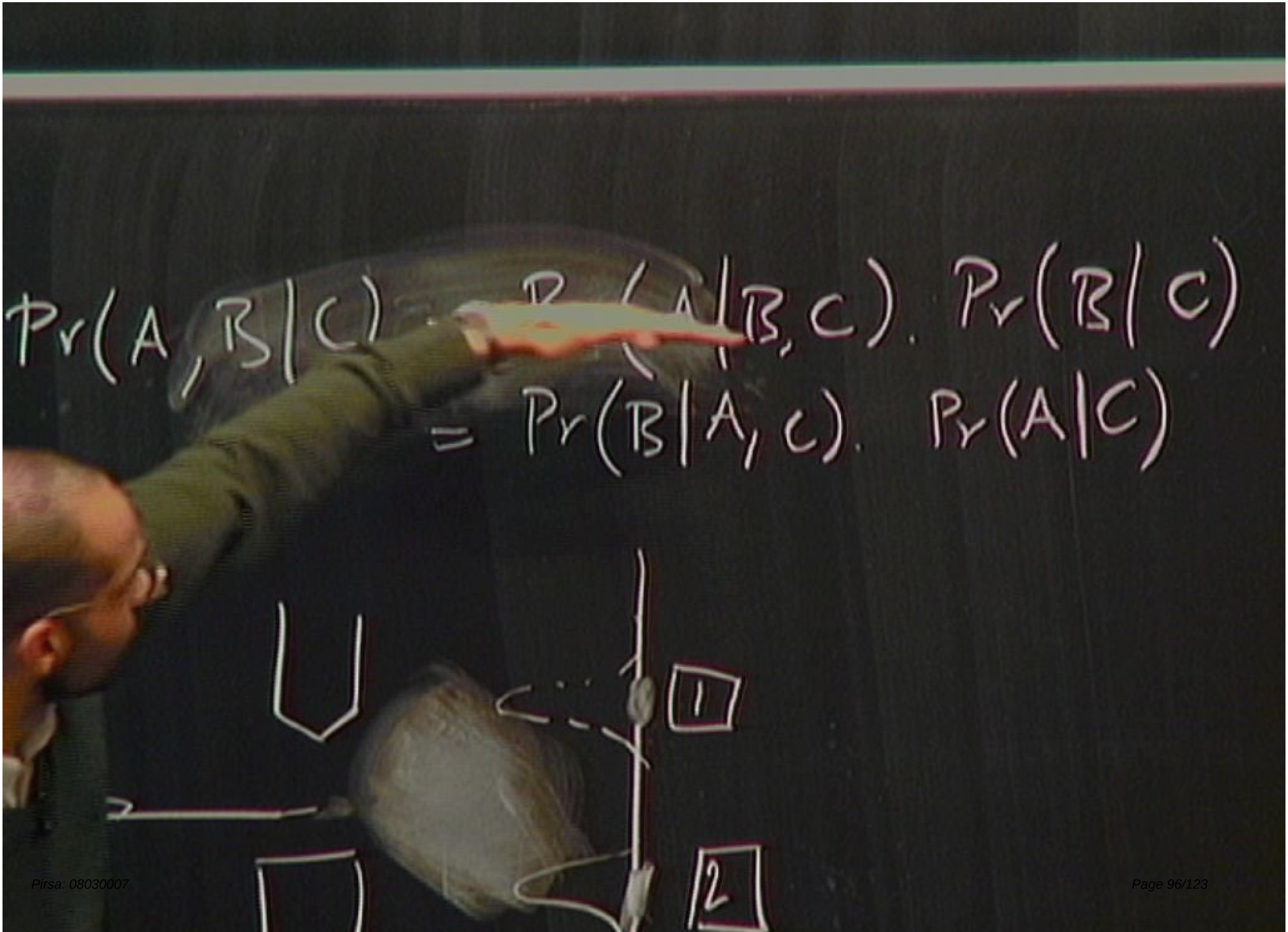
How much information does the data provide about the state ?

Bayes' rule

$$\Pr(p_1 \,|\, \underbrace{f_1, n,}_{\text{data}} I) =$$

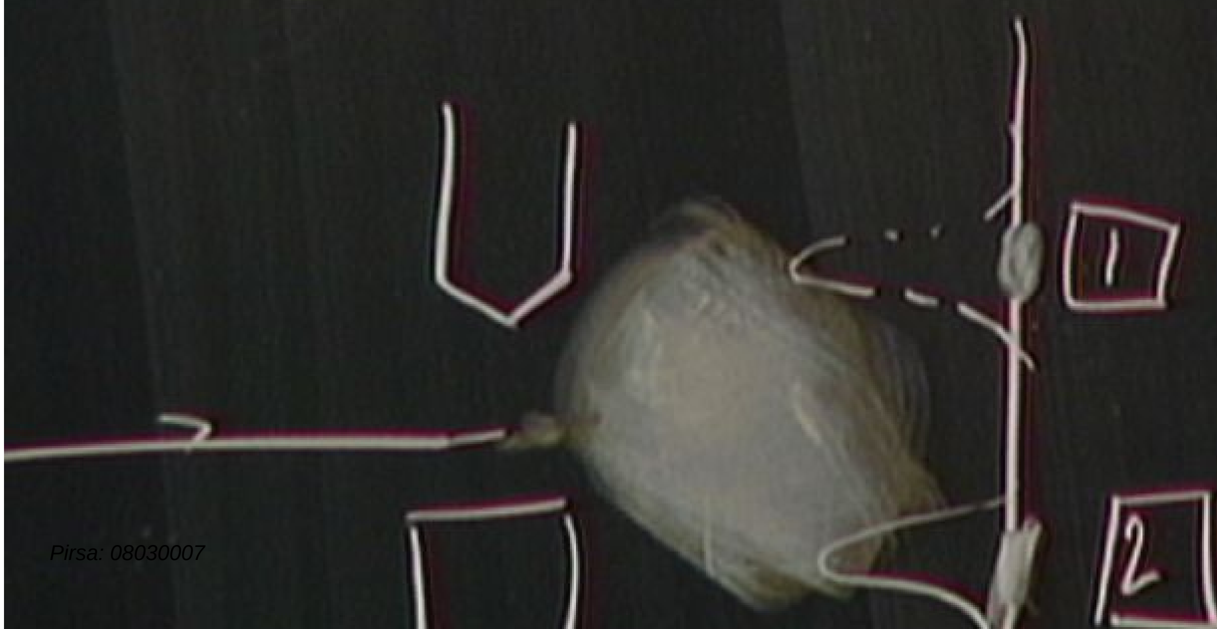

Pr(A, B|C) =
Ag
evaporator
(θ, φ)
1
2


How much information does the data provide about the state?
Bayes' rule
Pr(p1 | f1, n, I) =
data
background information
Pr(p1 | f1, n, I)
0
1/2
1
p1



$Pr(A,B|C) = Pr(A|B,C) \cdot Pr(B|C)$
$= Pr(B|A,C) \cdot Pr(A|C)$
1
2

$$\Pr(A,B|C) = \Pr(A|B,C)\cdot\Pr(B|C)$$
$$= \Pr(B|A,C)\cdot\Pr(A|C)$$



$$\ldots B, C) \cdot Pr(B|C)$$

$$\ldots C) \cdot Pr(A|C)$$

$$Pr(A|B,C) = \frac{Pr(B|A,C) \cdot Pr(A|C)}{Pr(B|C)}$$

Bayes' Rule.

How much information does the data provide about the state?

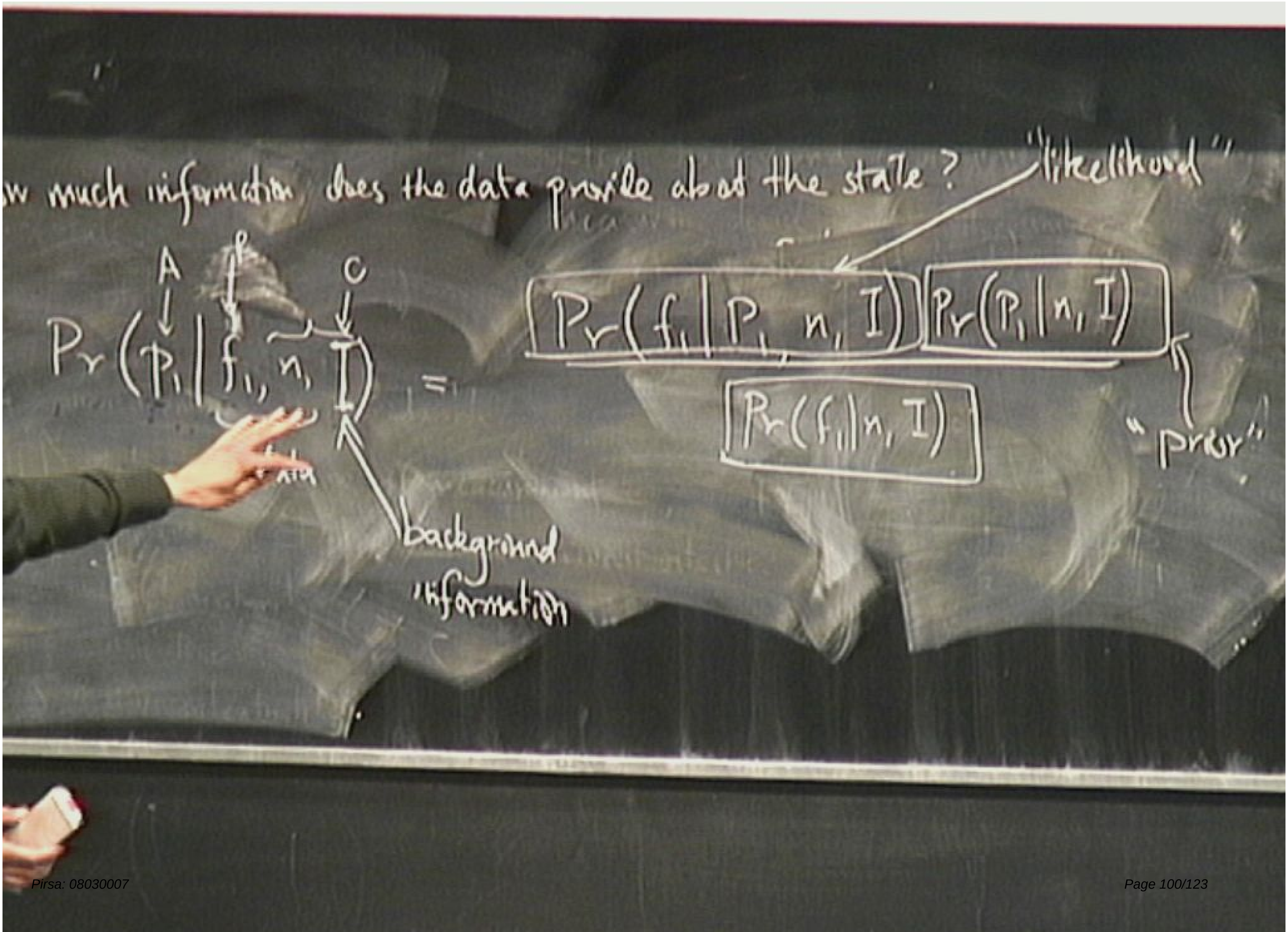
Bayes' rule

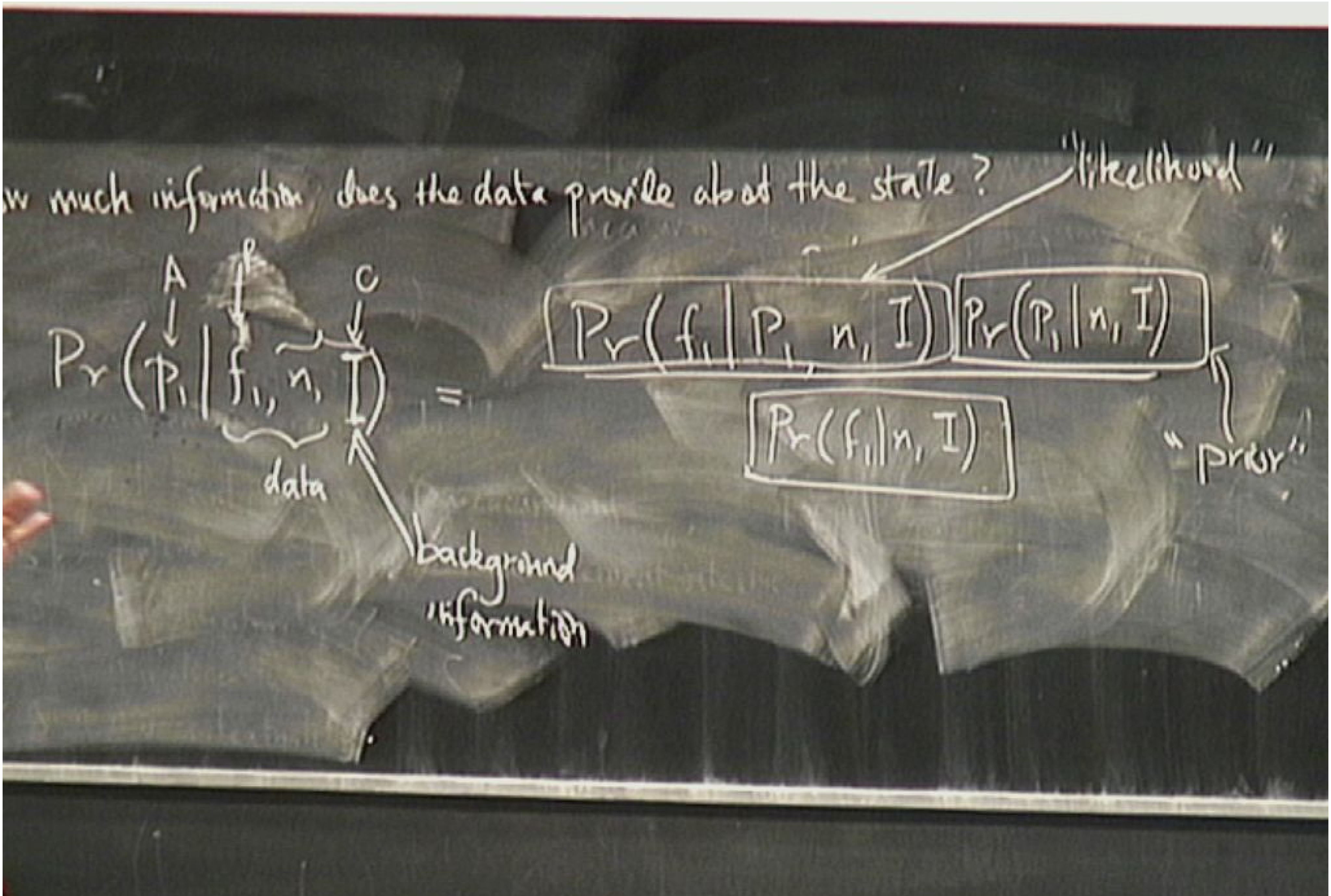
A → p_1, B → f_1, n, C → I

$$\Pr(p_1 \mid f_1, n, I) = \Pr$$

f_1, n: data

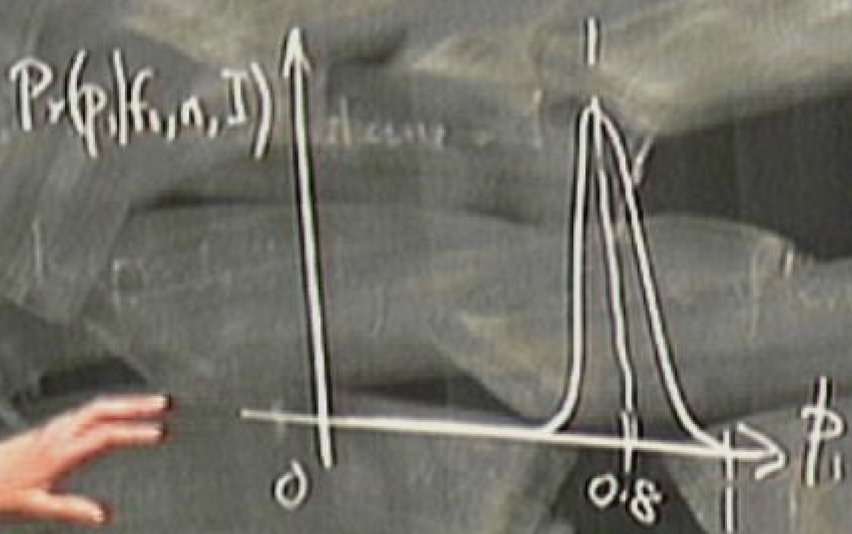
I: background information

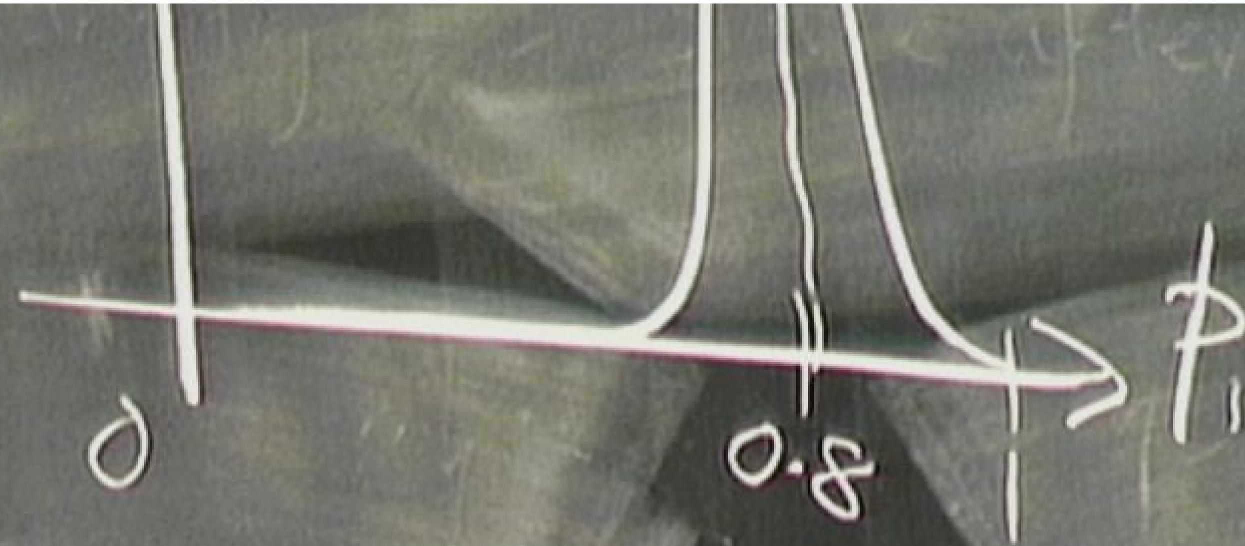

w much information does the data provide about the state?
"likelihood"
A
B
C
Pr(p1 | f1, n, I) =
Pr(f1 | p1, n, I) Pr(p1 | n, I)
Pr(f1 | n, I)
"prior"
data
background information


...w much information does the data provide about the state?
"likelihood"
A
B
C
$Pr(P_1 | f_1, n, I) = \frac{Pr(f_1 | P_1, n, I)\, Pr(P_1 | n, I)}{Pr(f_1 | n, I)}$
data
background information
"prior"

n v. large $\Rightarrow$ posterior is Gaussian, centre $\mu = f_1$

standard deviation $\sigma = \sqrt{\dfrac{f_1(1-f_1)}{n}}$

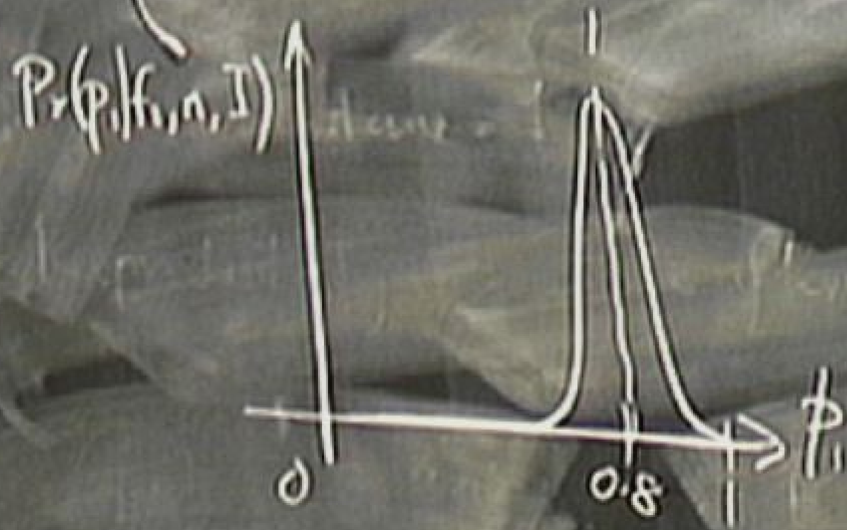




Shannon:

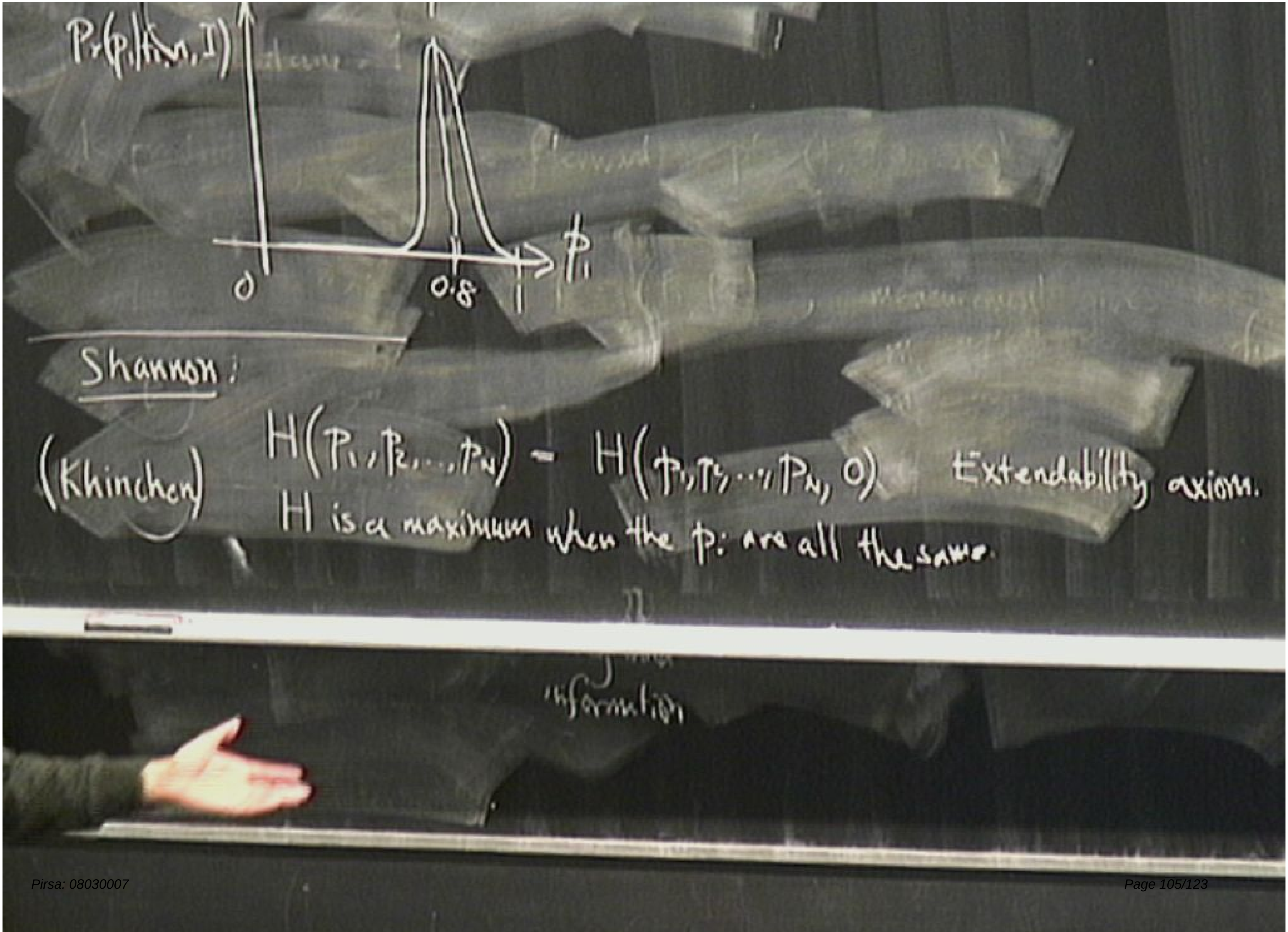
$$H(p_1, p_2, \ldots, p_N)$$

Standard deviation $\sigma = \sqrt{\frac{\ldots}{n}}$



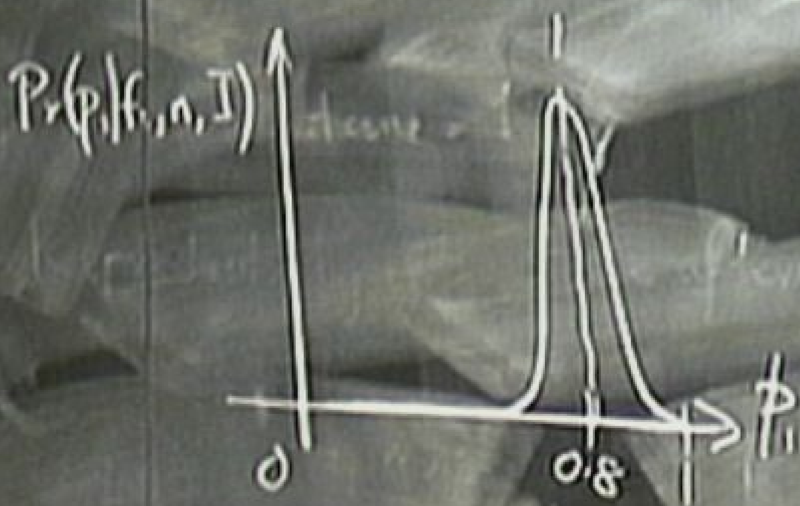
Shannon:

(Khinchen) $H(p_1, p_2, \ldots, p_N) = H(p_1, p_2, \ldots, p_N, 0)$ Extendability axiom.


Pr(p1|H,n,I)
0
0.8
1
p1
Shannon:
(Khinchen)
H(p1, p2, ..., pN) = H(p1, p2, ..., pN, 0) Extendability axiom.
H is a maximum when the pi are all the same.
information

n v. large ⇒ posterior is Gaussian, centre $\mu = f_1$

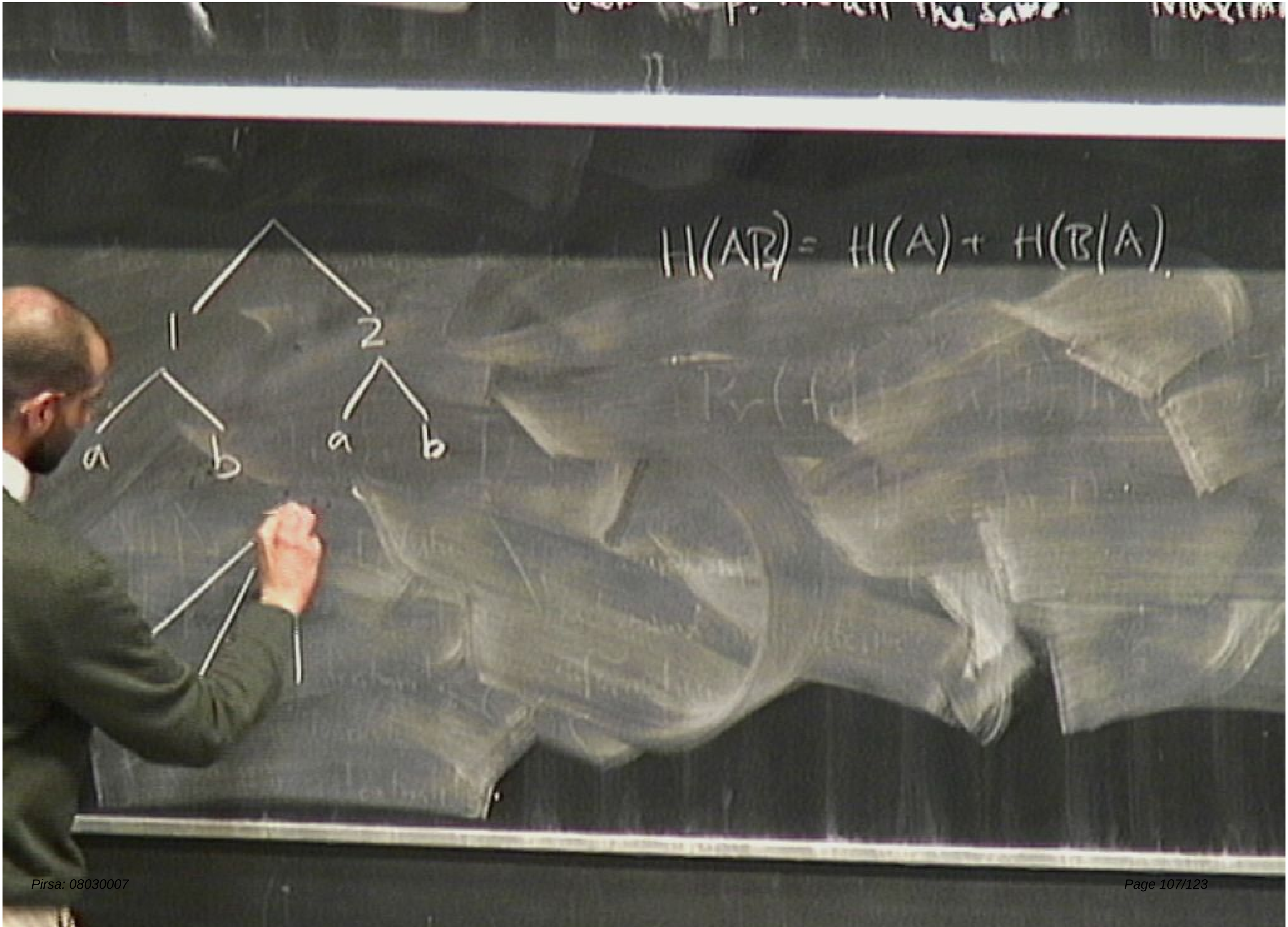
standard deviation $\sigma = \sqrt{\dfrac{f_1(1-f_1)}{n}}$

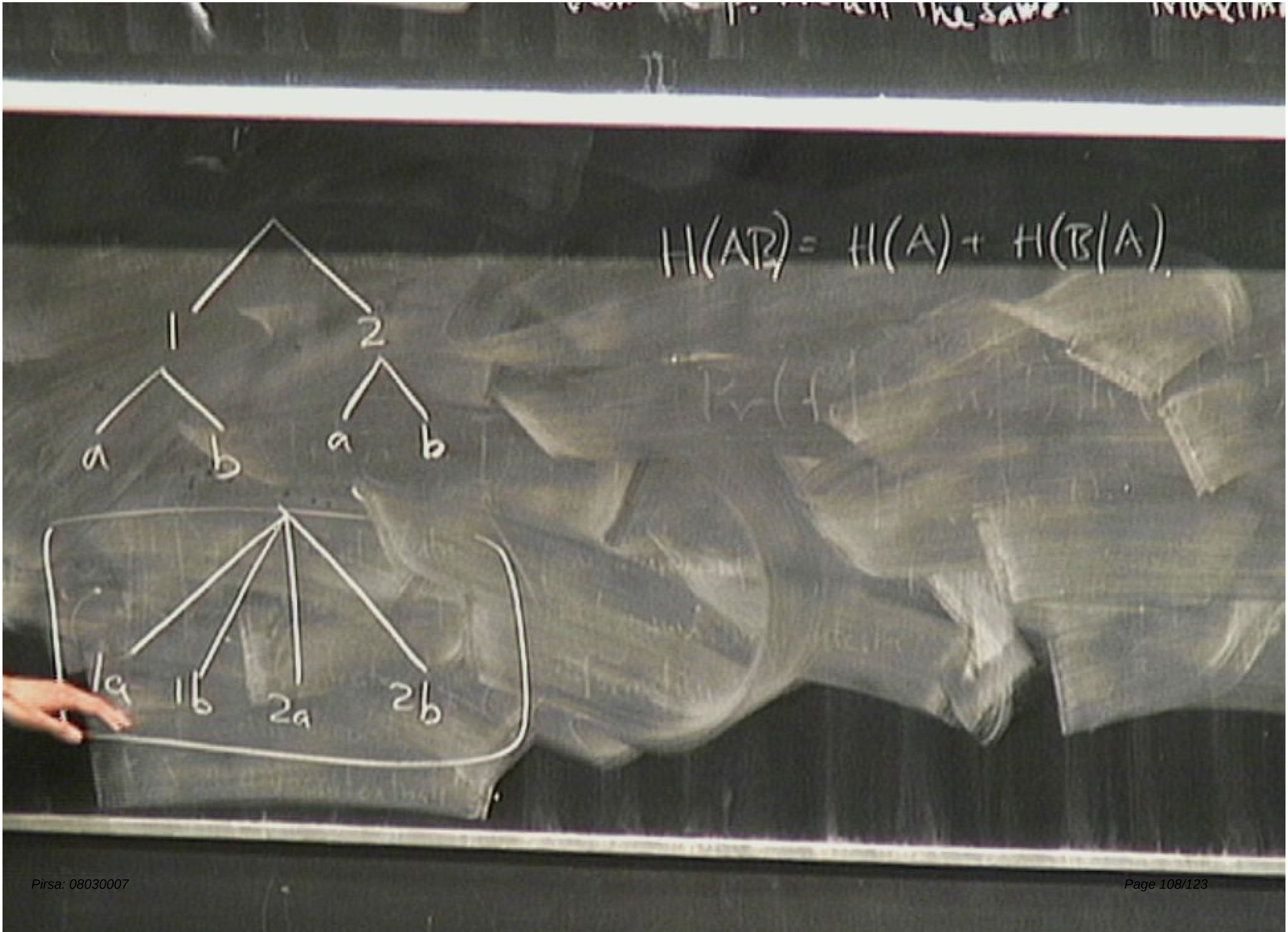


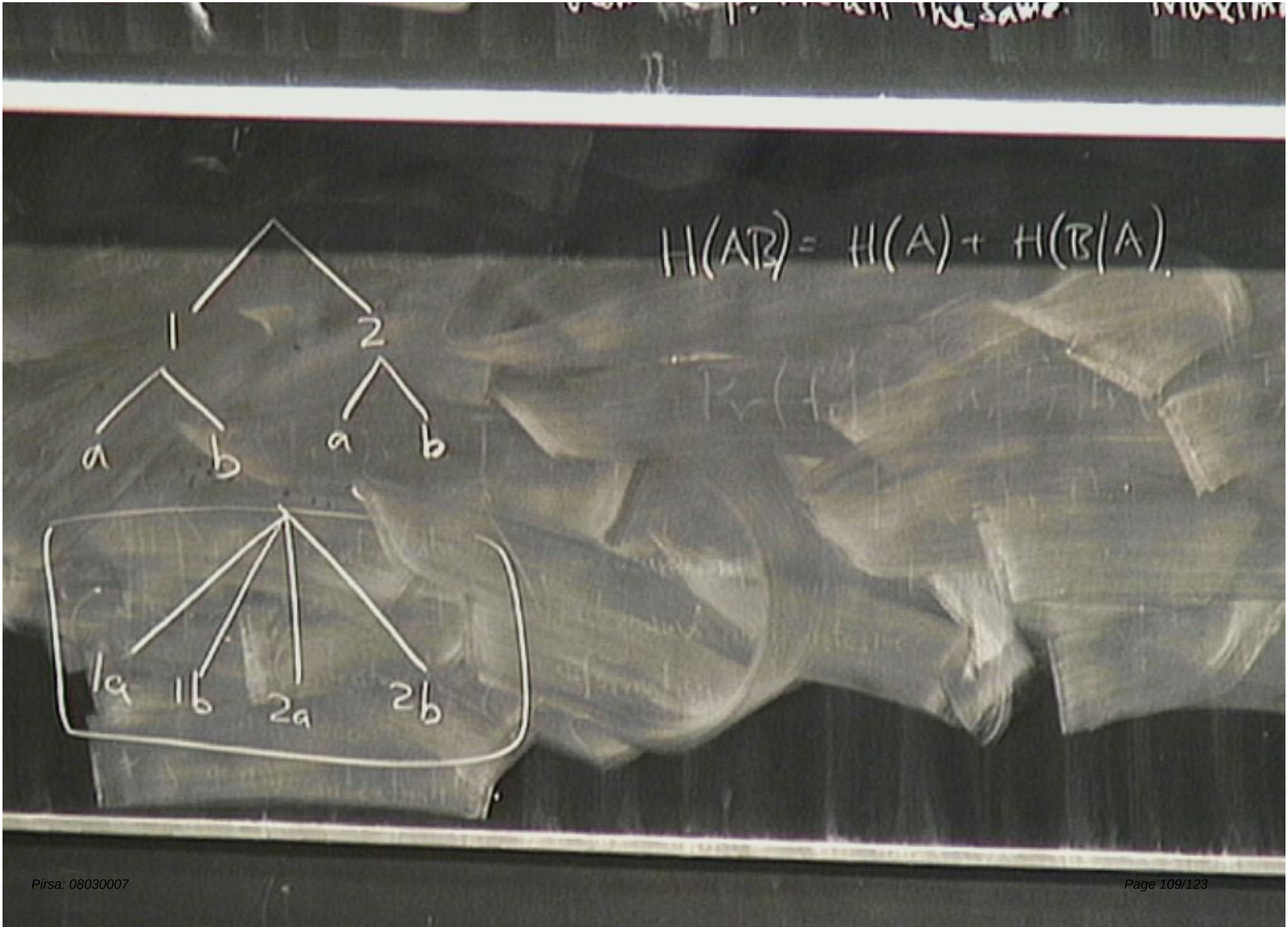
Shannon:

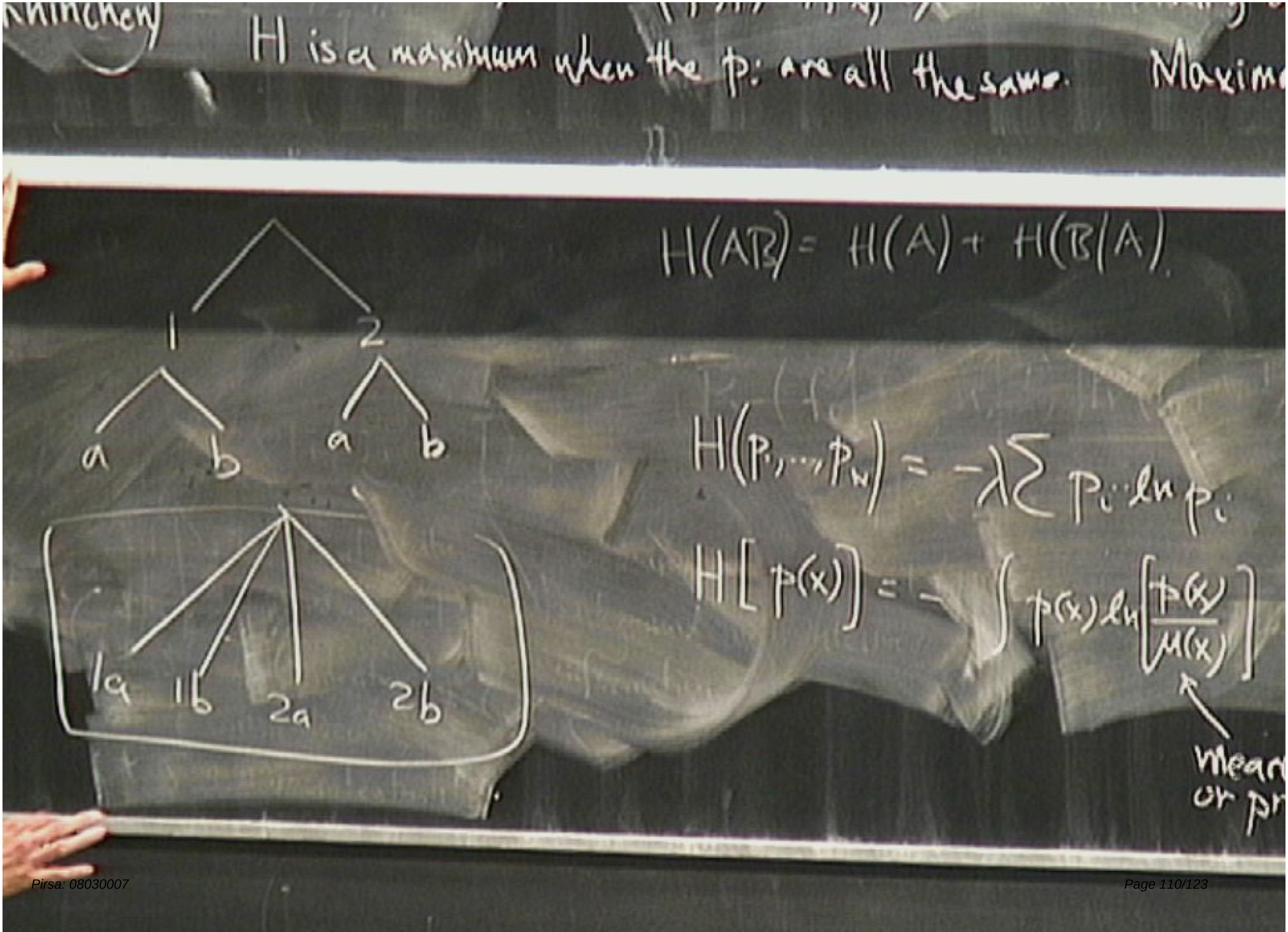
(Khinchin) $H(p_1, p_2, \ldots, p_N) = H(p_1, p_2, \ldots, p_N, 0)$ Extendability axiom.

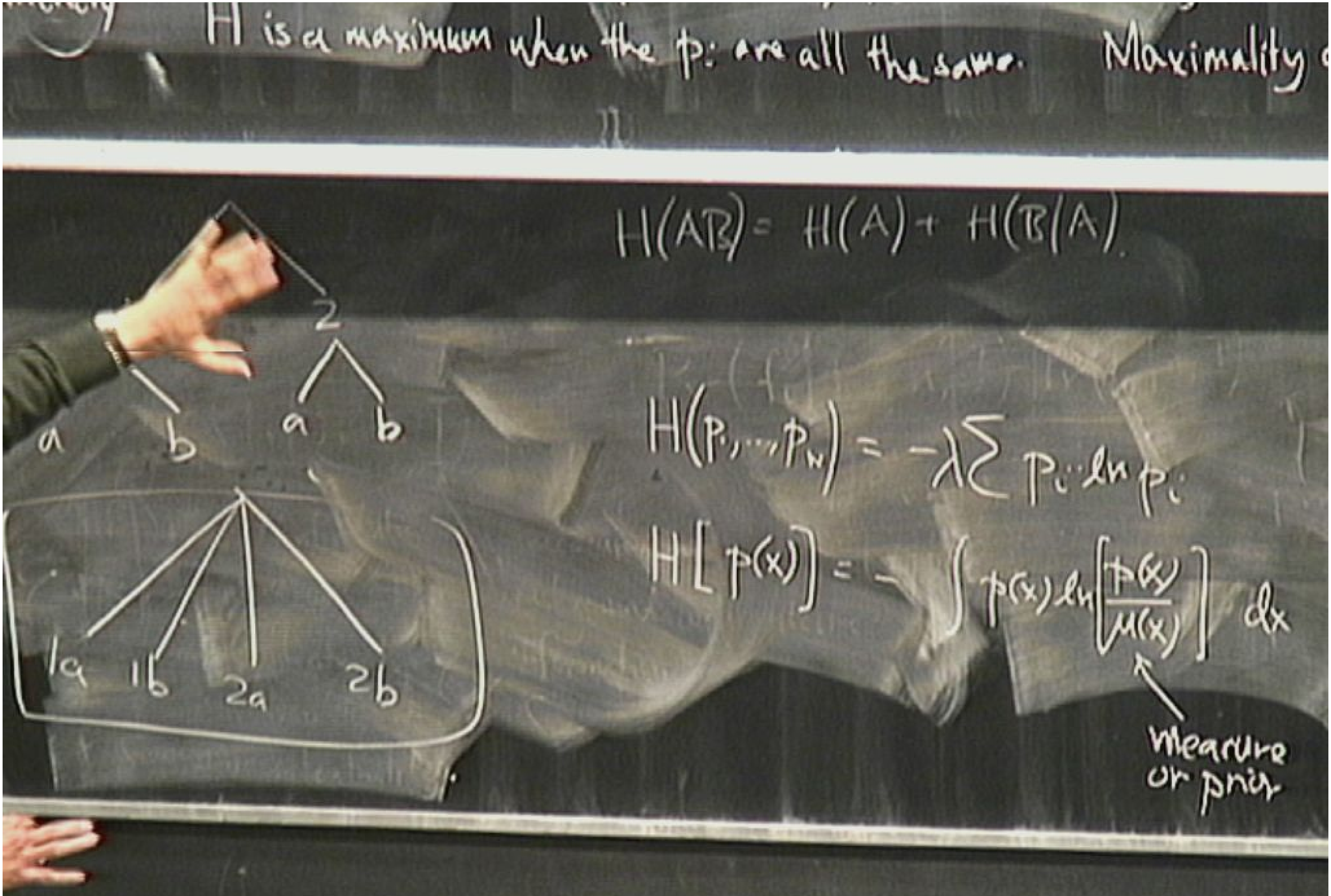
H is a maximum when the p_i are all the same. Maximality axiom.

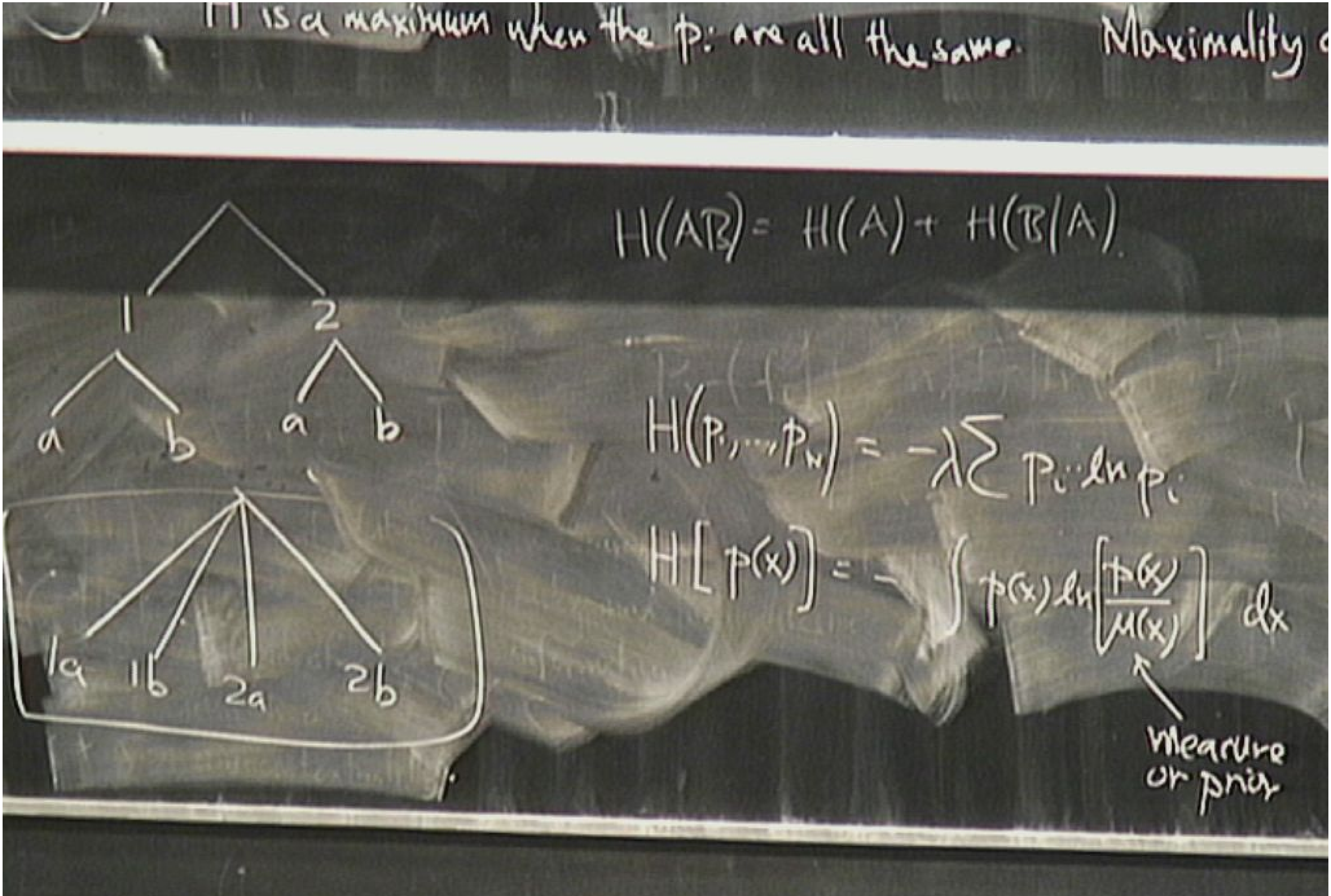

H(AB) = H(A) + H(B|A)
1
2
a
b
a
b

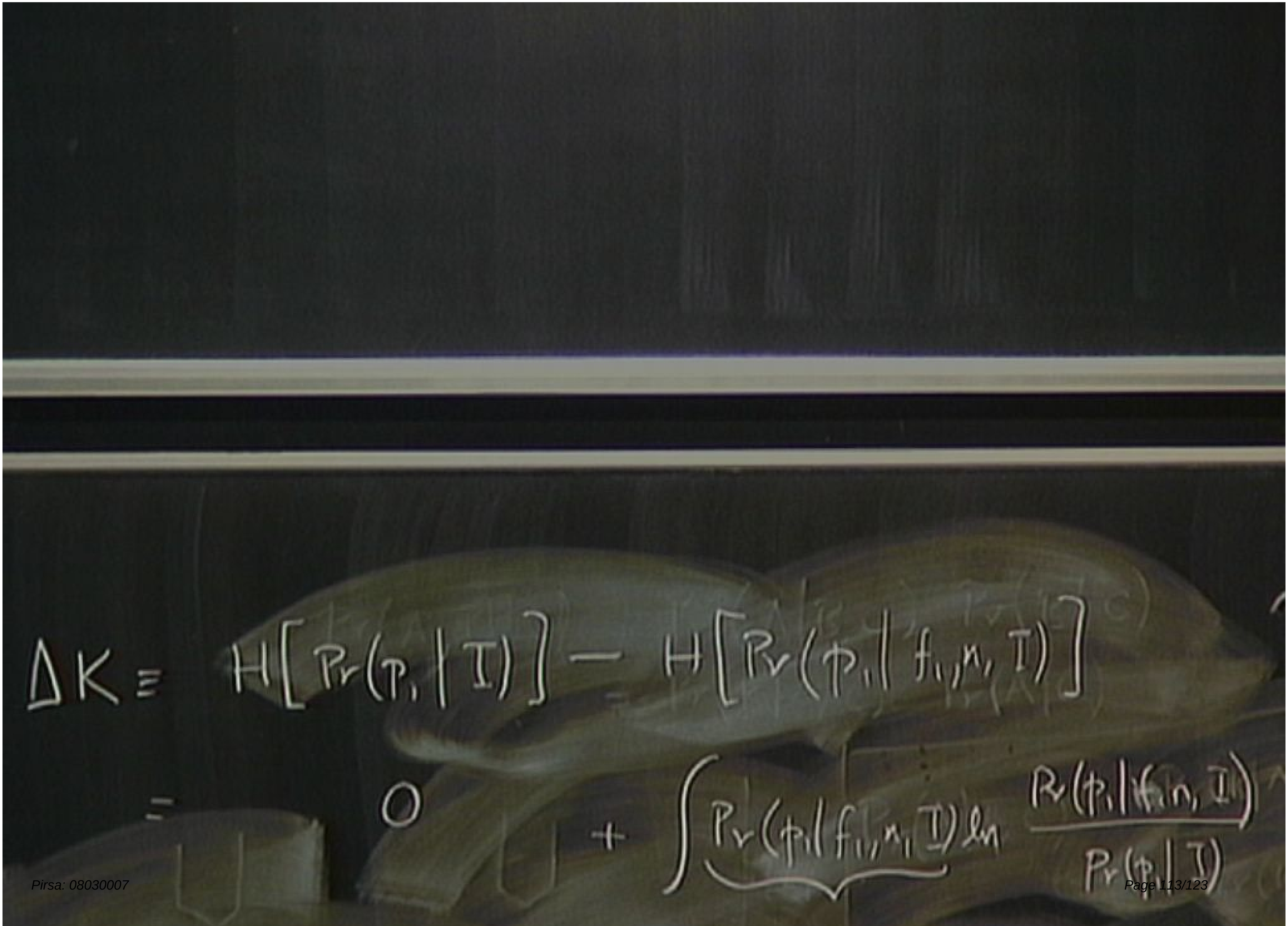

H(AB) = H(A) + H(B|A)
1
2
a
b
a
b
1a
1b
2a
2b


H(AB) = H(A) + H(B|A).
1
2
a
b
a
b
1a
1b
2a
2b


H is a maximum when the p_i are all the same.
Maxim
$H(AB) = H(A) + H(B|A)$
1
2
a
b
a
b
1a
1b
2a
2b
$H(p_1, \dots, p_N) = -\lambda \sum p_i \ln p_i$
$H[p(x)] = -\int p(x) \ln\left[\frac{p(x)}{\mu(x)}\right]$
mean
or pr


H is a maximum when the p_i are all the same.
Maximality
$H(AB) = H(A) + H(B|A)$
2
a
b
a
b
1a
1b
2a
2b
$H(p_1, \ldots, p_N) = -\lambda \sum p_i \ln p_i$
$H[p(x)] = -\int p(x) \ln \left[\frac{p(x)}{\mu(x)} \right] dx$
measure or prior


H is a maximum when the p_i are all the same. Maximality
$H(AB) = H(A) + H(B|A)$
1 2
a b a b
1a 1b 2a 2b
$H(p_1, \ldots, p_n) = -\lambda \sum p_i \ln p_i$
$H[p(x)] = -\int p(x) \ln\left[\frac{p(x)}{\mu(x)}\right] dx$
Measure or prior


$\Delta K \equiv H[\Pr(p_1 | I)] - H[\Pr(p_1 | f_1, n, I)]$
$= 0 + \int \Pr(p_1 | f_1, n, I) \ln \frac{\Pr(p_1 | f_1, n, I)}{\Pr(p_1 | I)}$

$$\Delta K \equiv H\left[\Pr(p_1 \mid I)\right] - H\left[\Pr(p_1 \mid f_1, n, I)\right]$$

$$= \quad 0 \quad + \int \underbrace{\Pr(p_1 \mid f_1, n, I)}\, \ln \frac{\Pr(p_1 \mid f_1, n, I)}{\boxed{\Pr(p_1 \mid I)}}$$

← uniform

$$= \frac{1}{2}\ln n + \ldots + \ldots \ln \sqrt{\bar{p}_1(1-\bar{p}_1)}$$

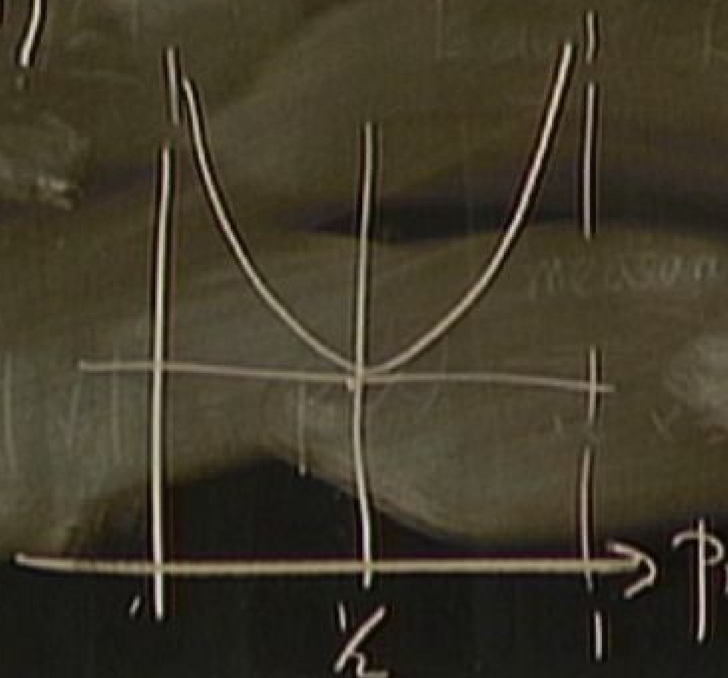
$$= \frac{1}{2}\ln n + \dots + \dots \ln\sqrt{p_1(1-p_1)}$$

$$\Pr(p_1 \mid I) = \frac{1}{\pi} \cdot \frac{1}{\sqrt{p_1(1-p_1)}}$$

$$0 \quad + \int \underbrace{\Pr(p_1|f_1,n_1,I)}\ln \frac{\Pr(p_1|f_1,n_1, \ldots}{\boxed{\Pr(p_1|I \ldots}}$$

$$= \frac{1}{2}\ln n + \ldots + \ldots \ln\sqrt{p_1(1-p_1)}$$

$$\Pr(p_1|I) = \frac{1}{\pi} \cdot \frac{1}{\sqrt{p_1(1-p_1)}}$$

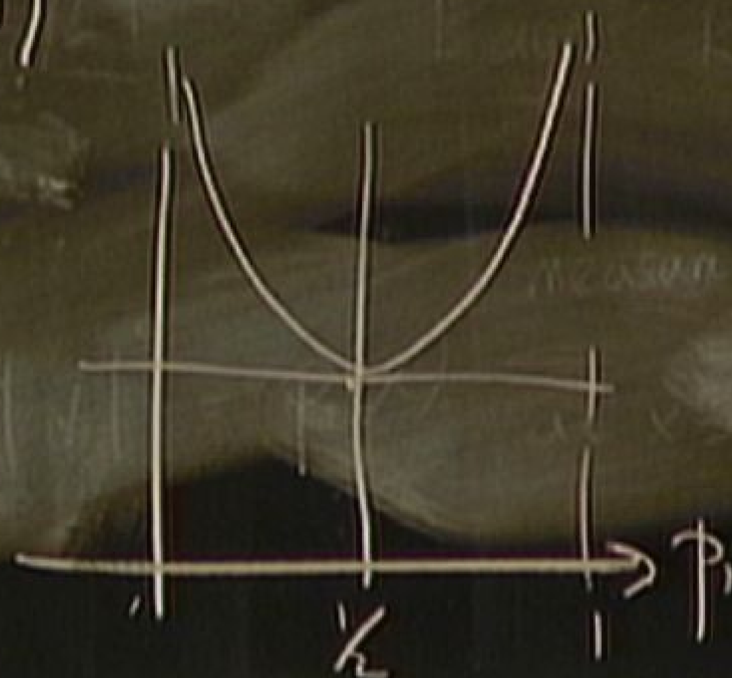


$$0 \quad + \int \Pr(p_1 | f_1, n_1 I) \ln \frac{\Pr(p_1 | f_1 n_1 \ldots}{\Pr(p_1 | I \ldots}$$

$$= \frac{1}{2} \ln n + \ldots + \ldots \ln \sqrt{p_1(1-p_1)}$$

$$\Pr(p_1 | I) = \frac{1}{\pi} \cdot \frac{1}{\sqrt{p_1(1-p_1)}}$$

$$\Pr(p_1, p_2 | I) \propto \frac{1}{\sqrt{p_1 p_2}}$$



$$\Pr(p_1, \ldots, p_N \mid I) \propto \frac{1}{\sqrt{p_1 p_2 \cdots \cdots p_N}}$$

$$\Pr(p_1, \ldots, p_N \mid I) \propto \frac{1}{\sqrt{p_1 p_2 \cdots \cdots p_N}}$$

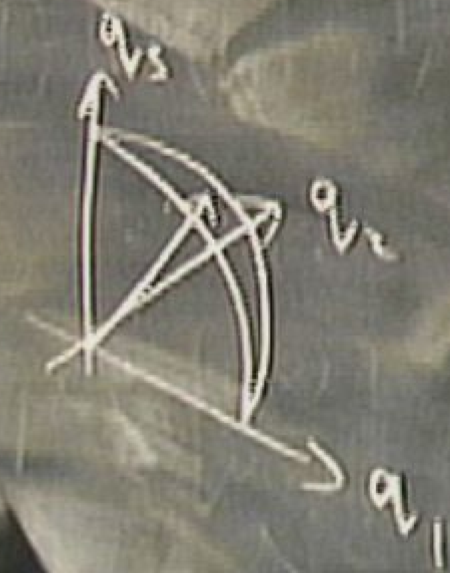
Define $q_1 = \sqrt{p_1}, \ldots, q_N = \sqrt{p_N}$

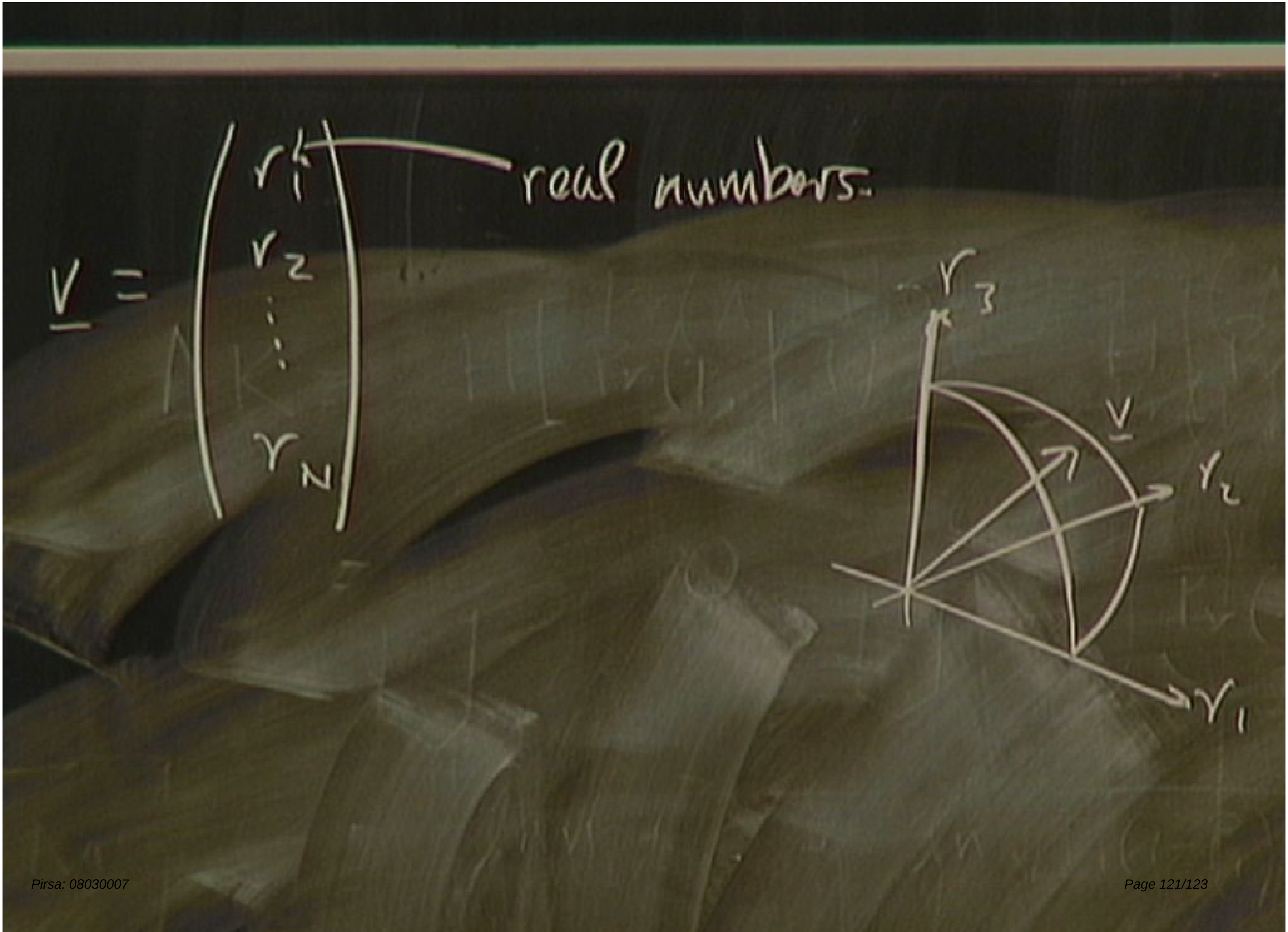
$$\Pr(q_1, \ldots, q_N) = \text{uniform on } \sum q_i^2 = 1.$$

$$\Pr(p_1, \dots, p_N \mid I) \propto \frac{1}{\sqrt{p_1 p_2 \cdots p_N}}$$

Define $q_1 = \sqrt{p_1}, \dots, q_N = \sqrt{p_N}$

$\Pr(q_1, \dots, q_N)$ = uniform on $\sum q_i^2 = 1$

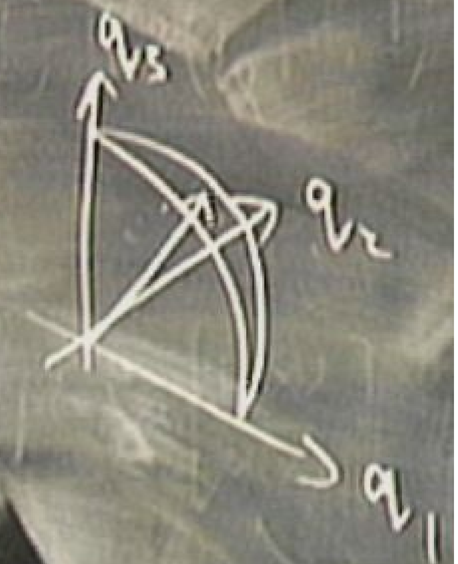


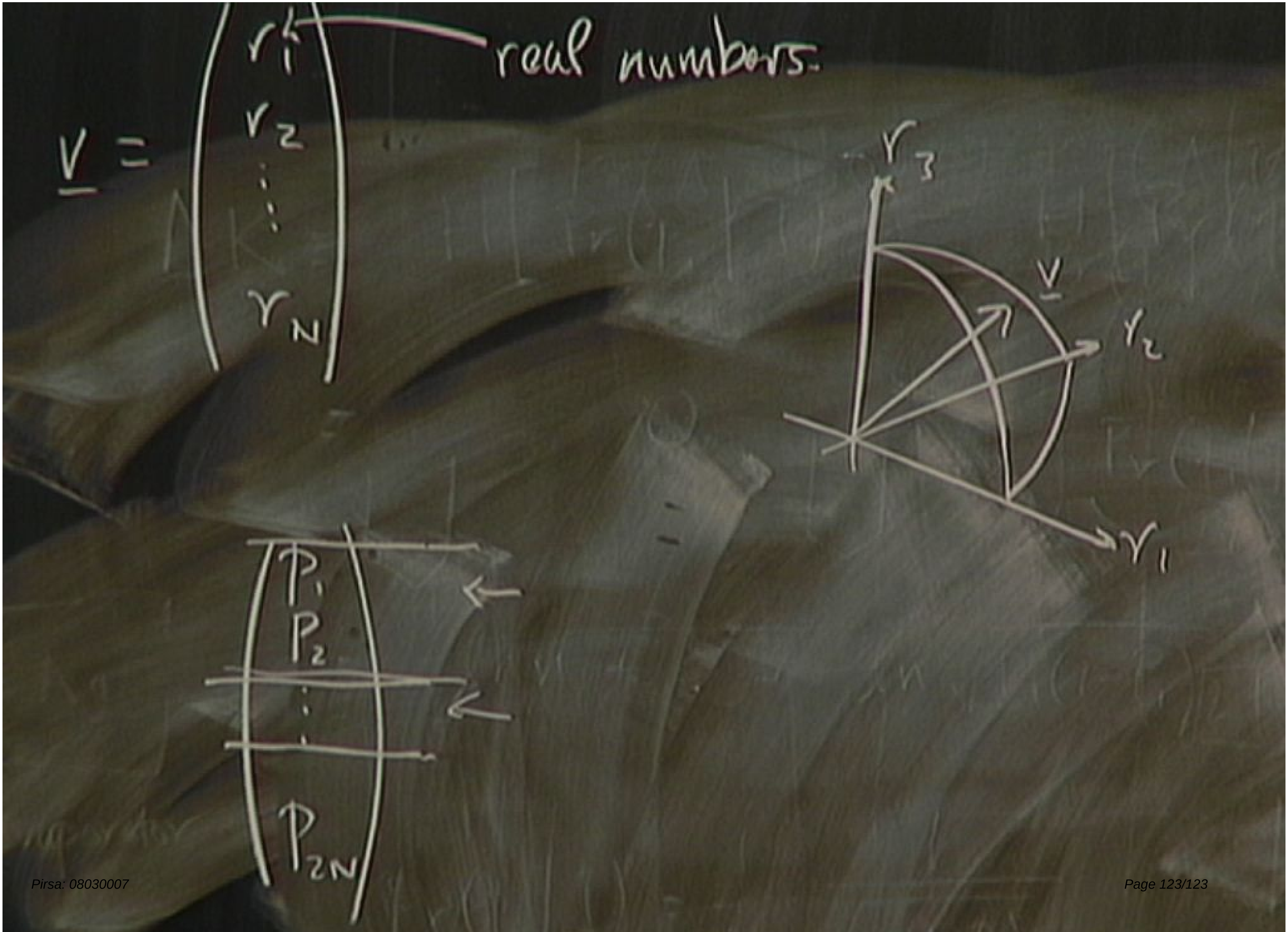

$\underline{v} = \begin{pmatrix} r_1 \\ r_2 \\ \vdots \\ r_N \end{pmatrix}$
real numbers
r_3
$\underline{v}$
r_2
r_1

$$\Pr(p_1, \ldots, p_N \mid I) \propto \frac{1}{\sqrt{p_1 p_2 \cdots \cdots p_N}}$$

Define $q_1 = \sqrt{p_1}, \ldots, q_N = \sqrt{p_N}$

$$\Pr(q_1, \ldots, q_N) = \text{uniform on } \sum q_i^2 = 1.$$




V =
r1
r2
rN
real numbers.
r3
V
r2
r1
P1
P2
P2N