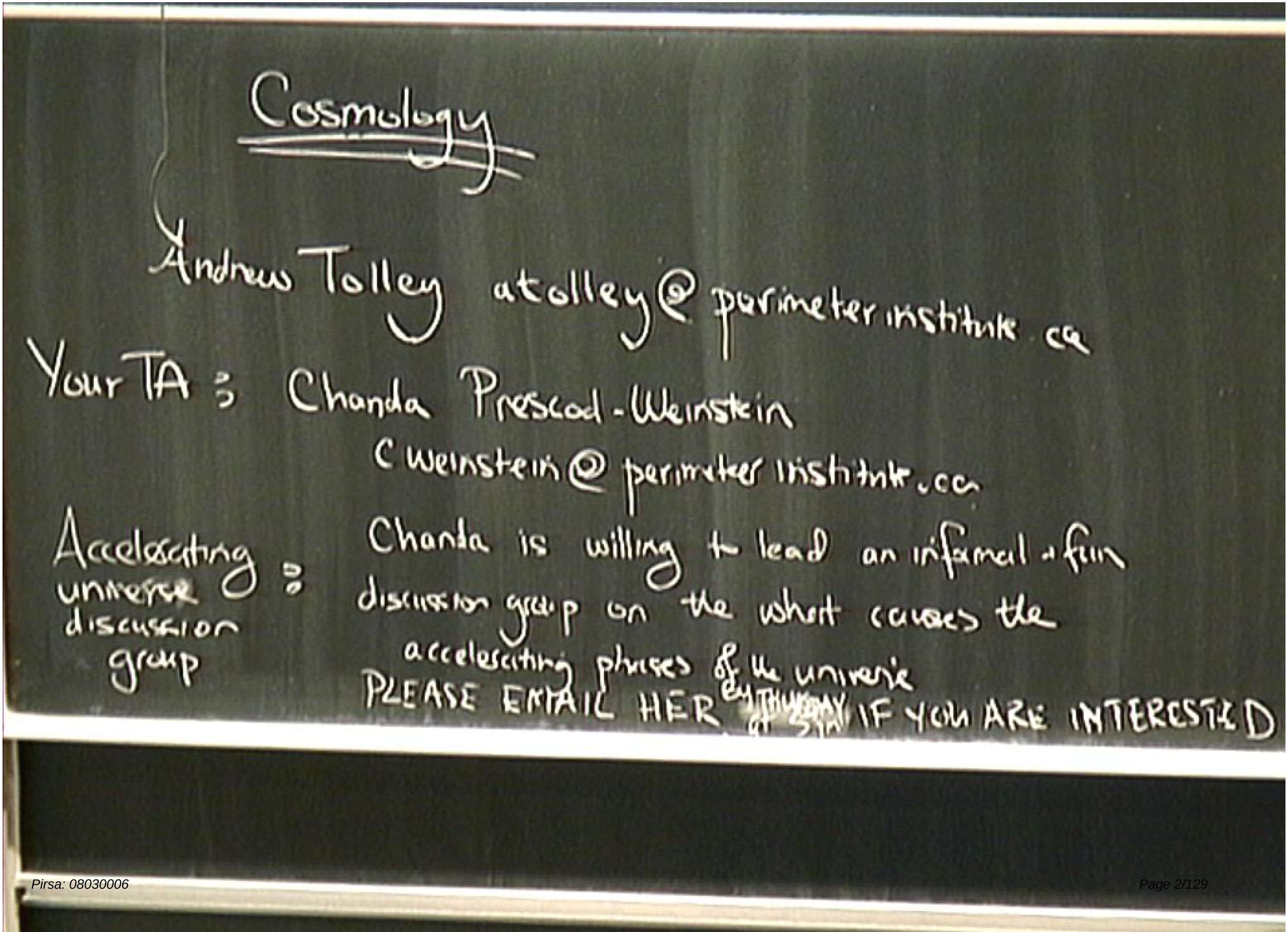
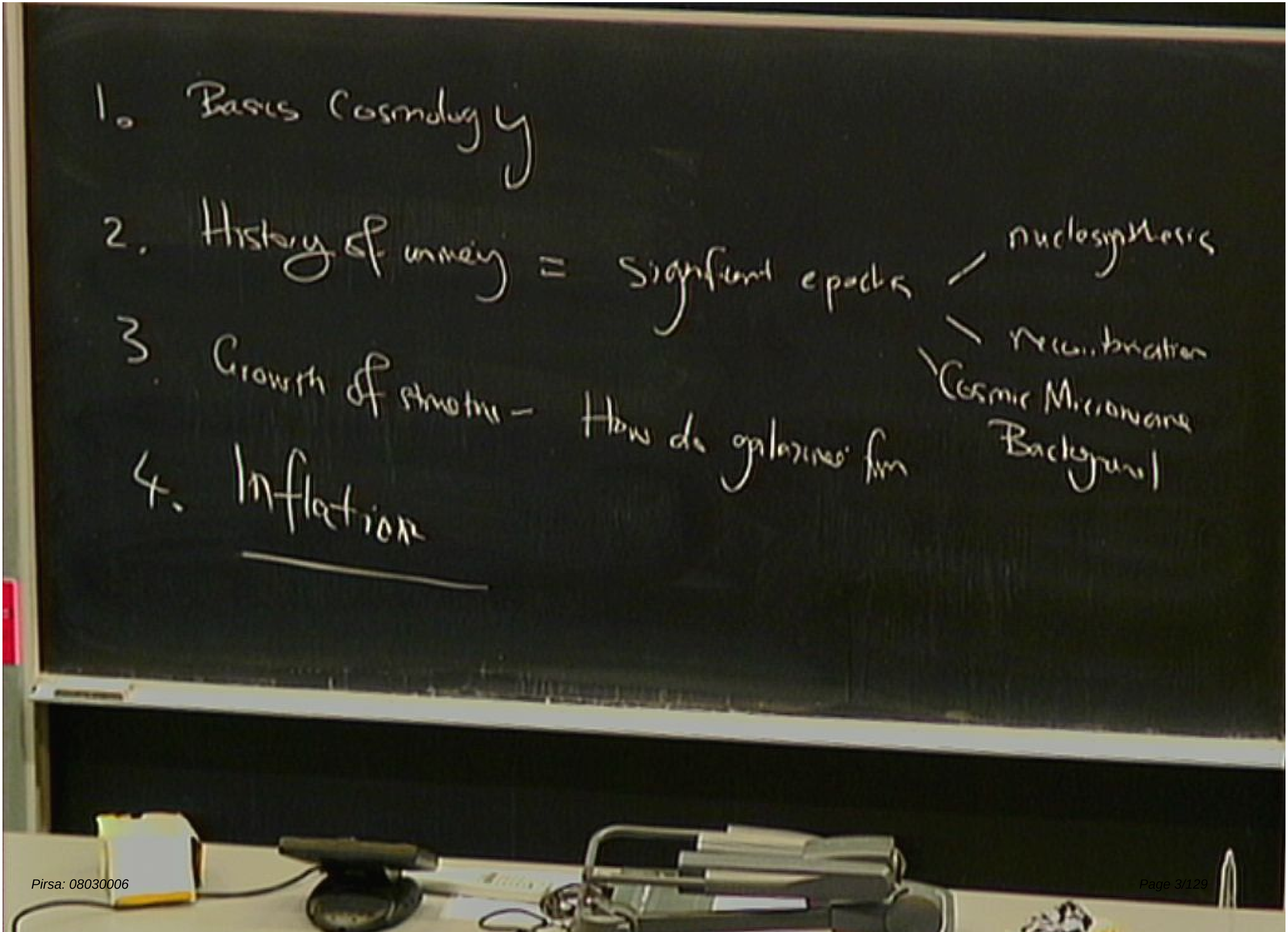
Title: Cosmology #1

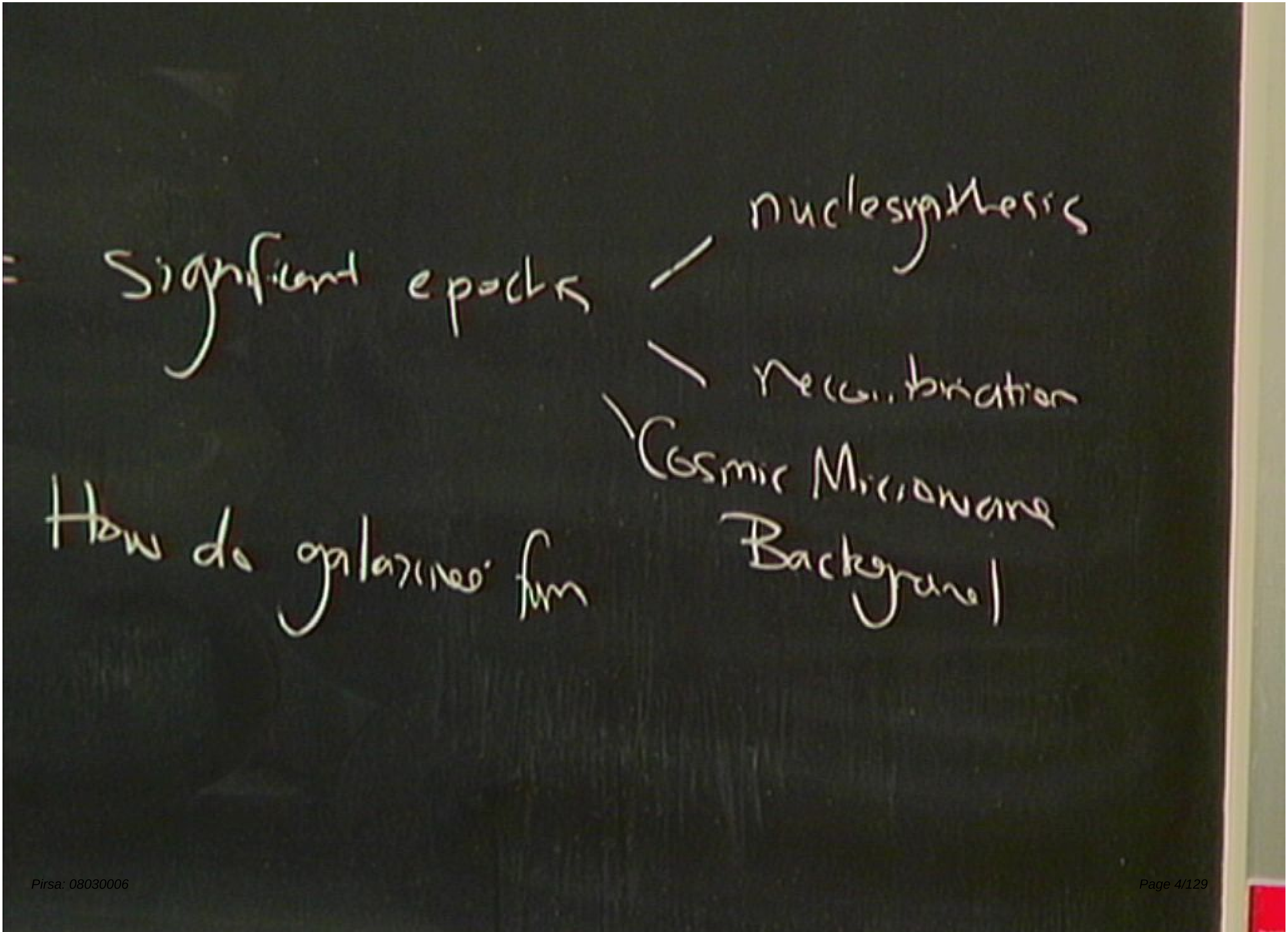
Date: Mar 25, 2008 06:30 PM

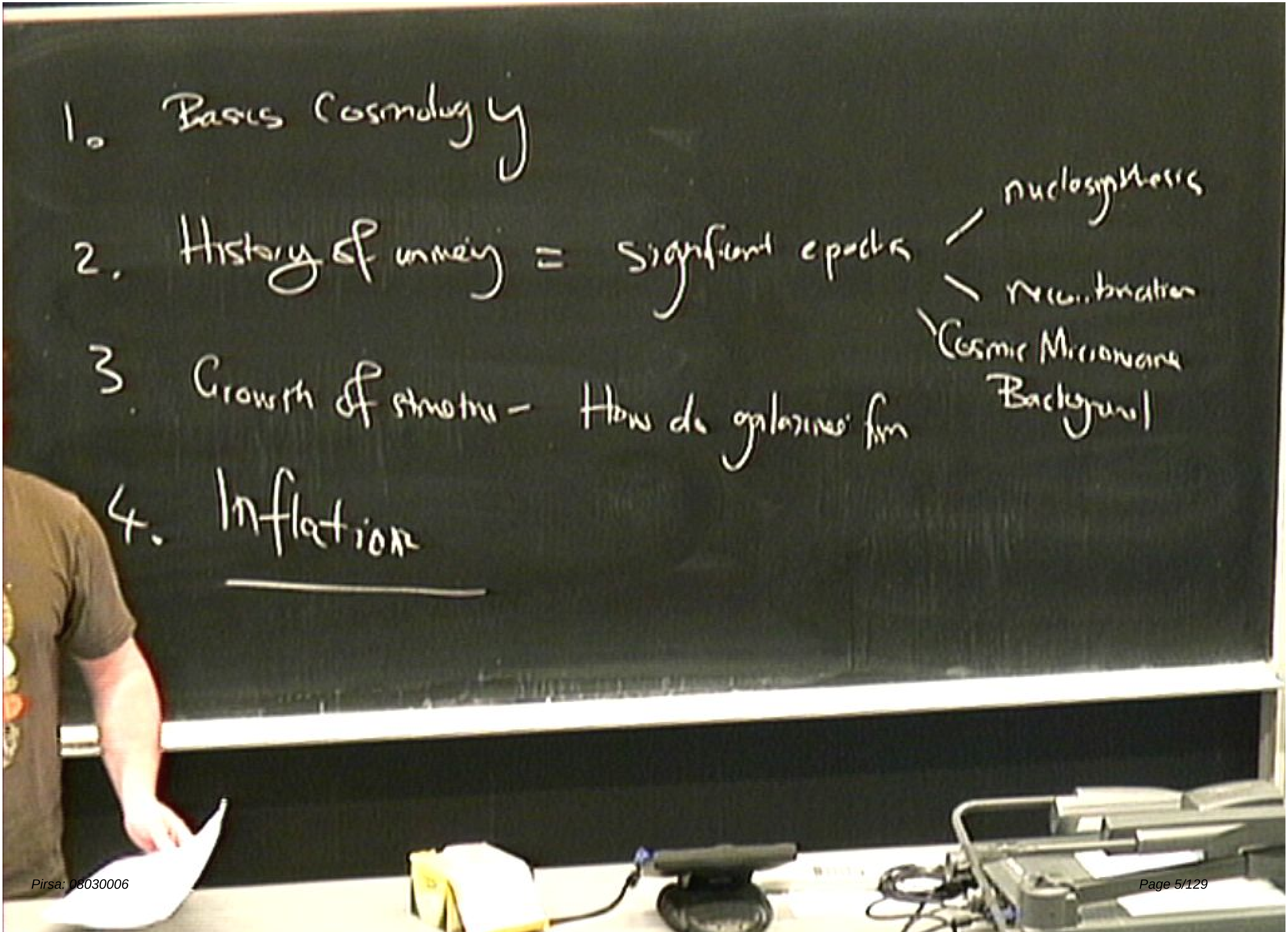
URL: http://pirsa.org/08030006

Abstract: A brief history of our cosmic beginnings, Cosmic Microwave Background. How galaxies form and the existence of dark matter.


Cosmology
Andrew Tolley atolley@perimeterinstitute.ca
Your TA: Chanda Prescod-Weinstein
cweinstein@perimeterinstitute.ca
Accelerating universe discussion group:
Chanda is willing to lead an informal & fun discussion group on the what causes the accelerating phases of the universe
PLEASE EMAIL HER IF YOU ARE INTERESTED


1. Basics Cosmology
2. History of universe = Significant epochs
nucleosynthesis
recombination
Cosmic Microwave Background
3. Growth of structure - How do galaxies form
4. Inflation


Significant epochs
nucleosynthesis
recombination
Cosmic Microwave Background
How do galaxies form


1. Basics Cosmology
2. History of universe = significant epochs
nucleosynthesis
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Cosmic Microwave Background
3. Growth of structure - How do galaxies form
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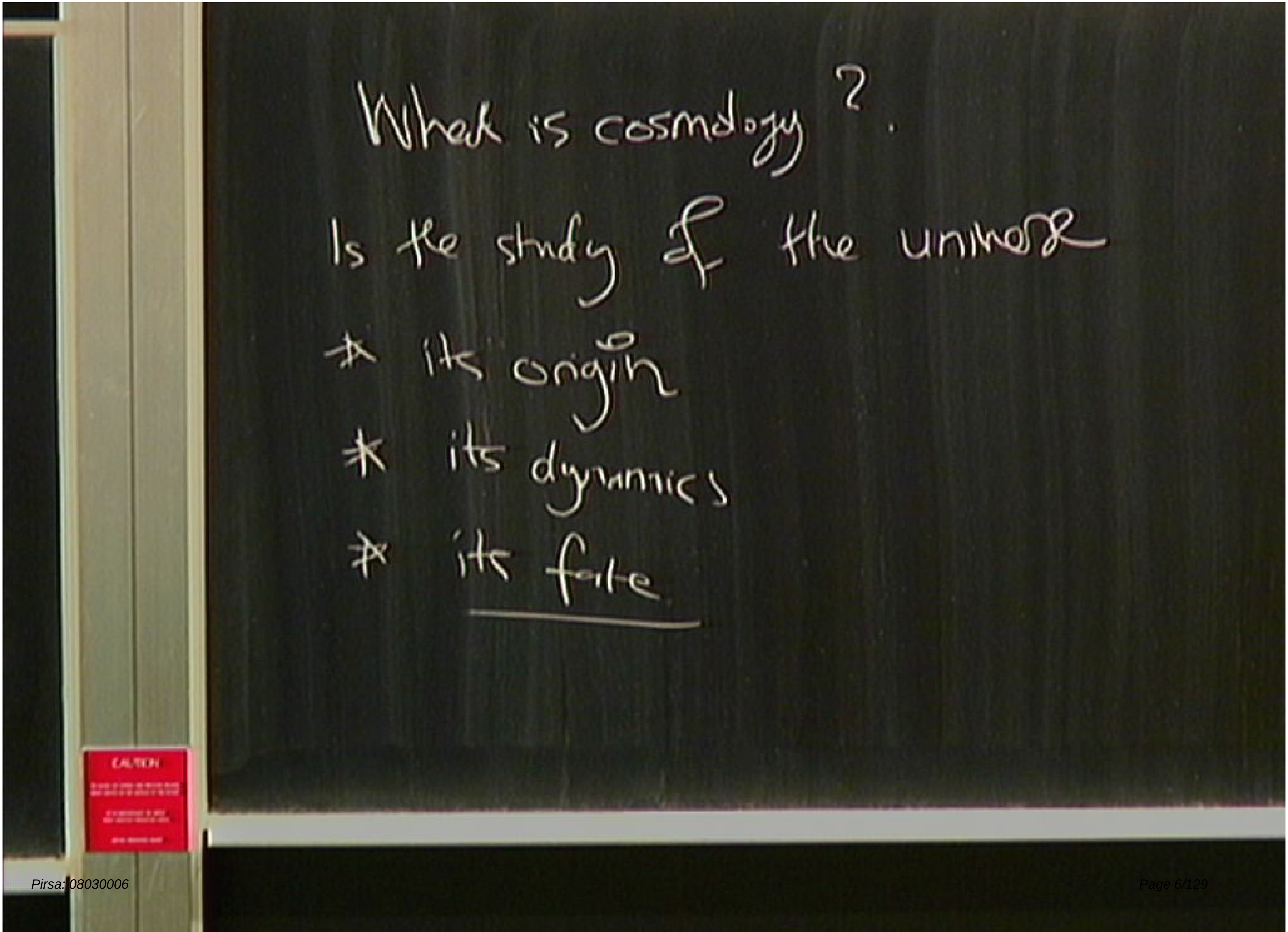
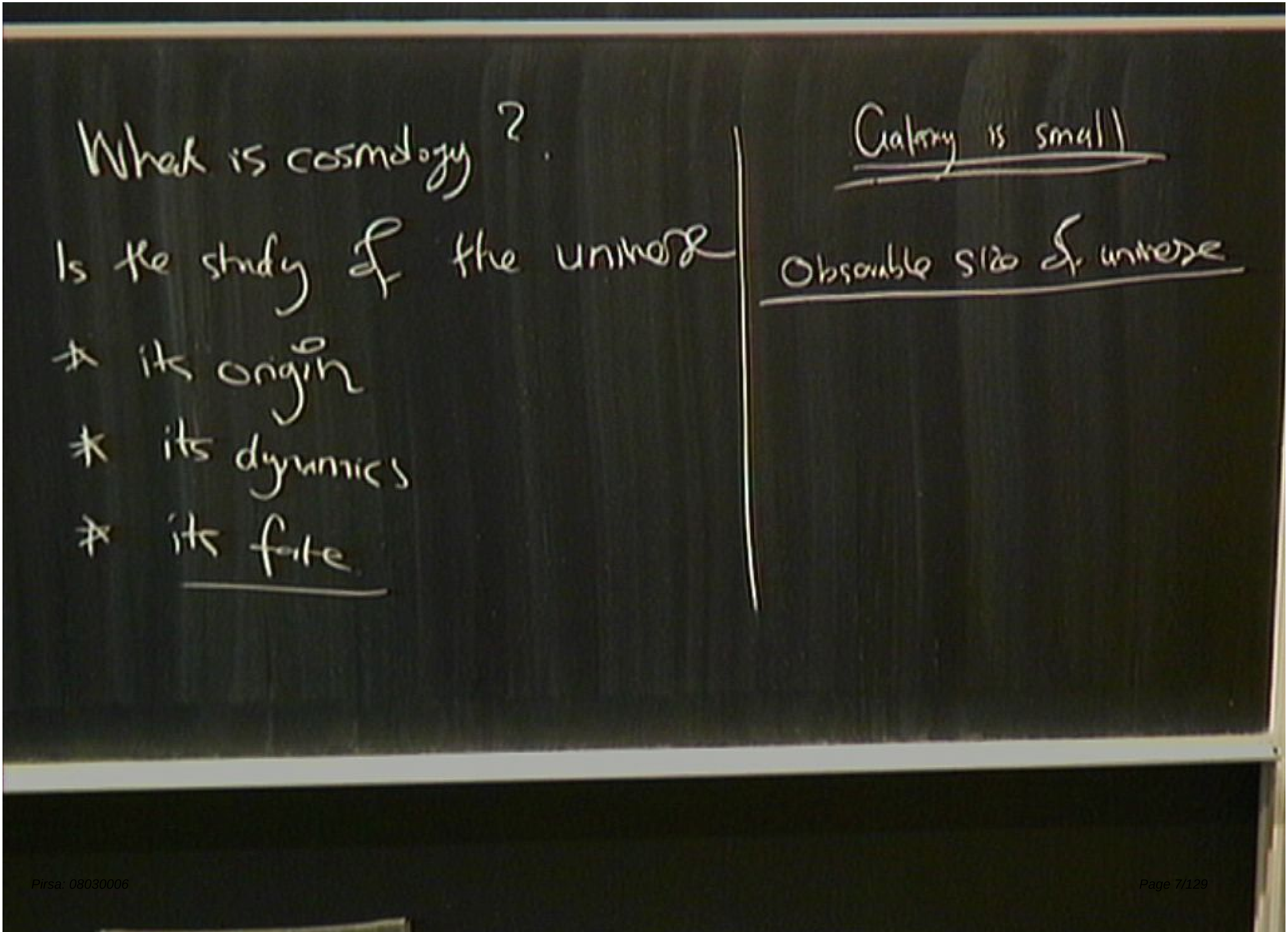

What is cosmology ?
Is the study of the universe
* its origin
* its dynamics
* its fate


What is cosmology?
Is the study of the universe
* its origin
* its dynamics
* its fate
Galaxy is small
Observable size of universe

What is cosmology?

Is the study of the universe

* its origin
* its dynamics
* its fate

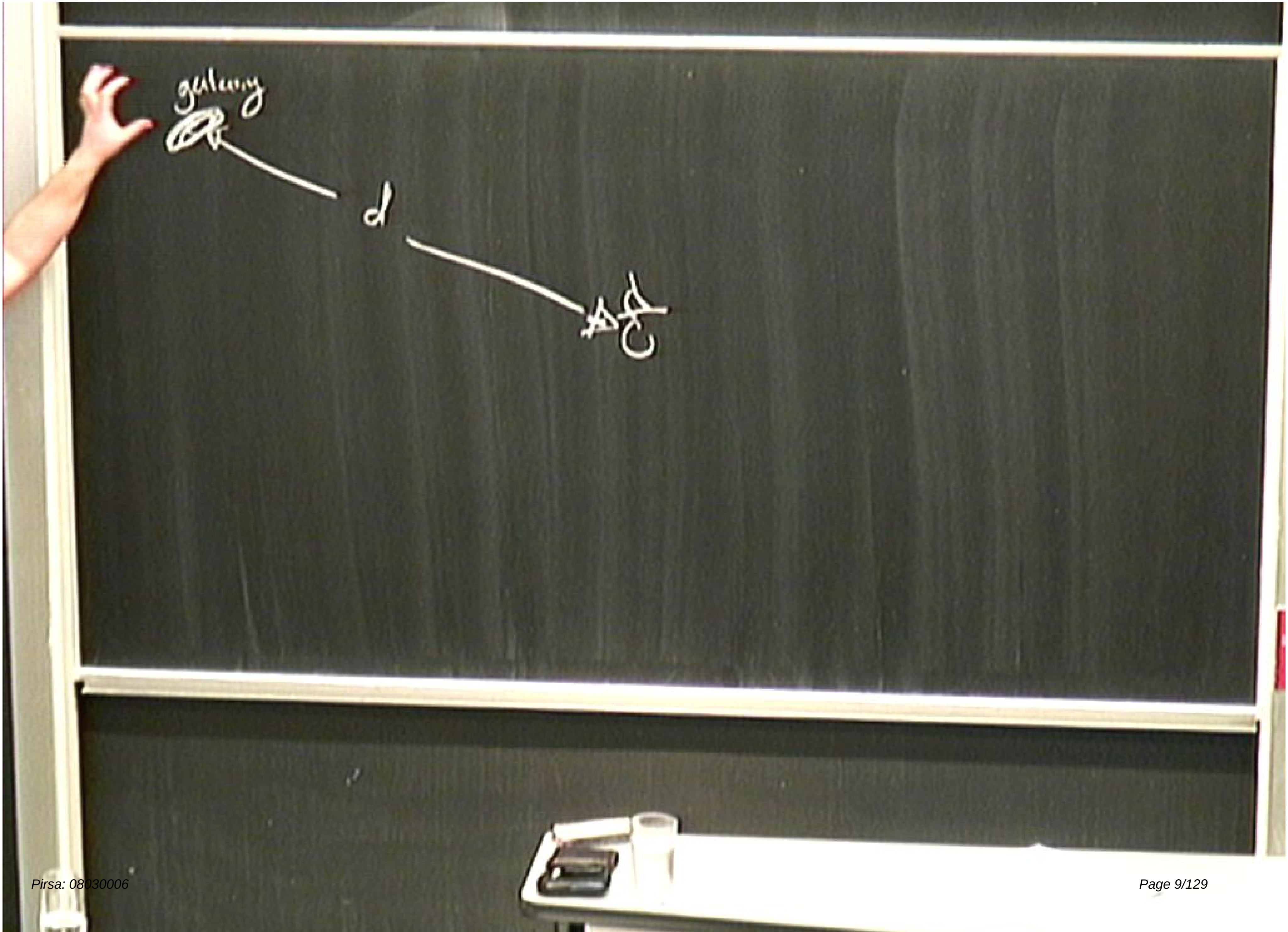
Galaxy is small

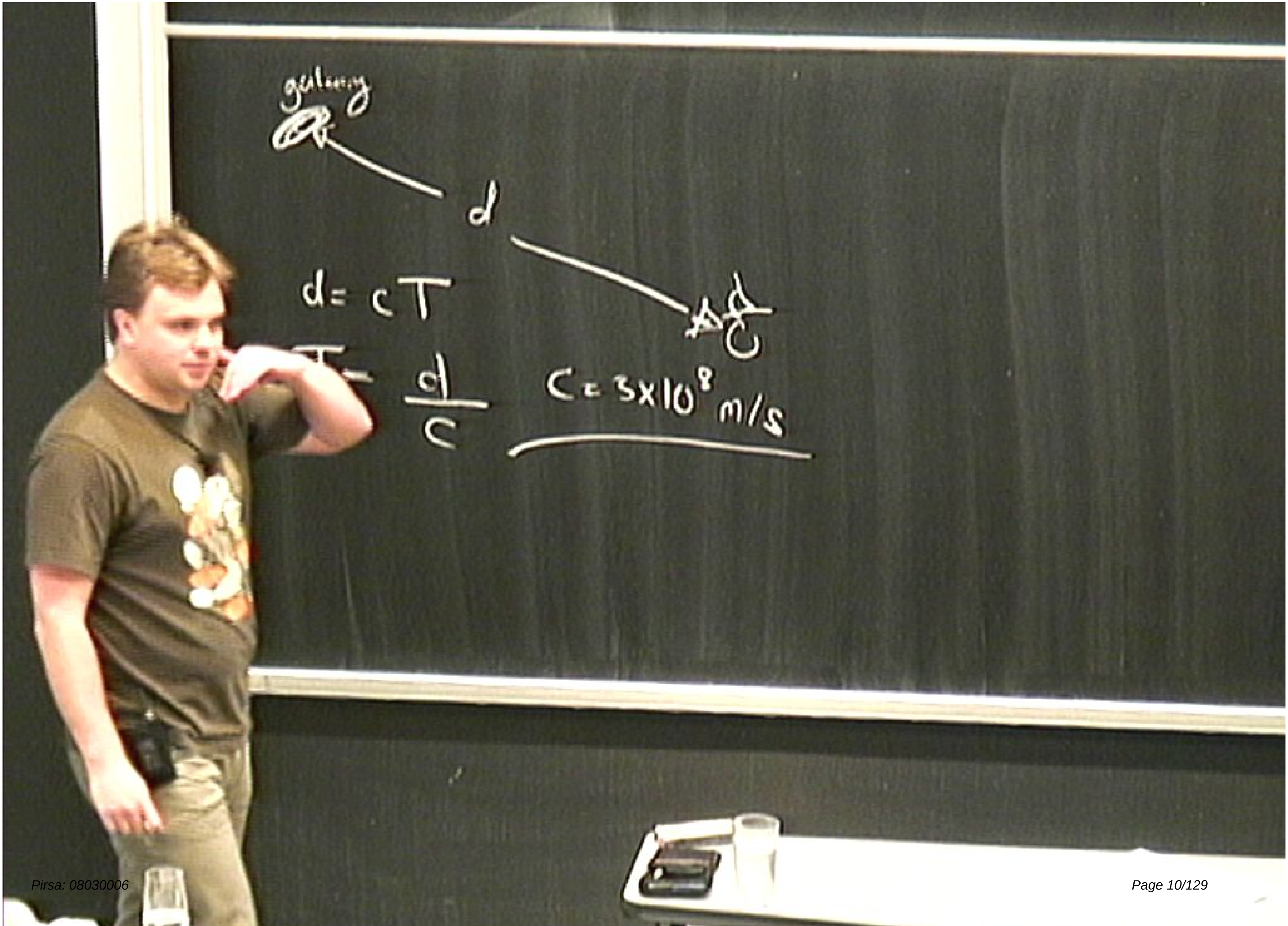
Observable size of universe

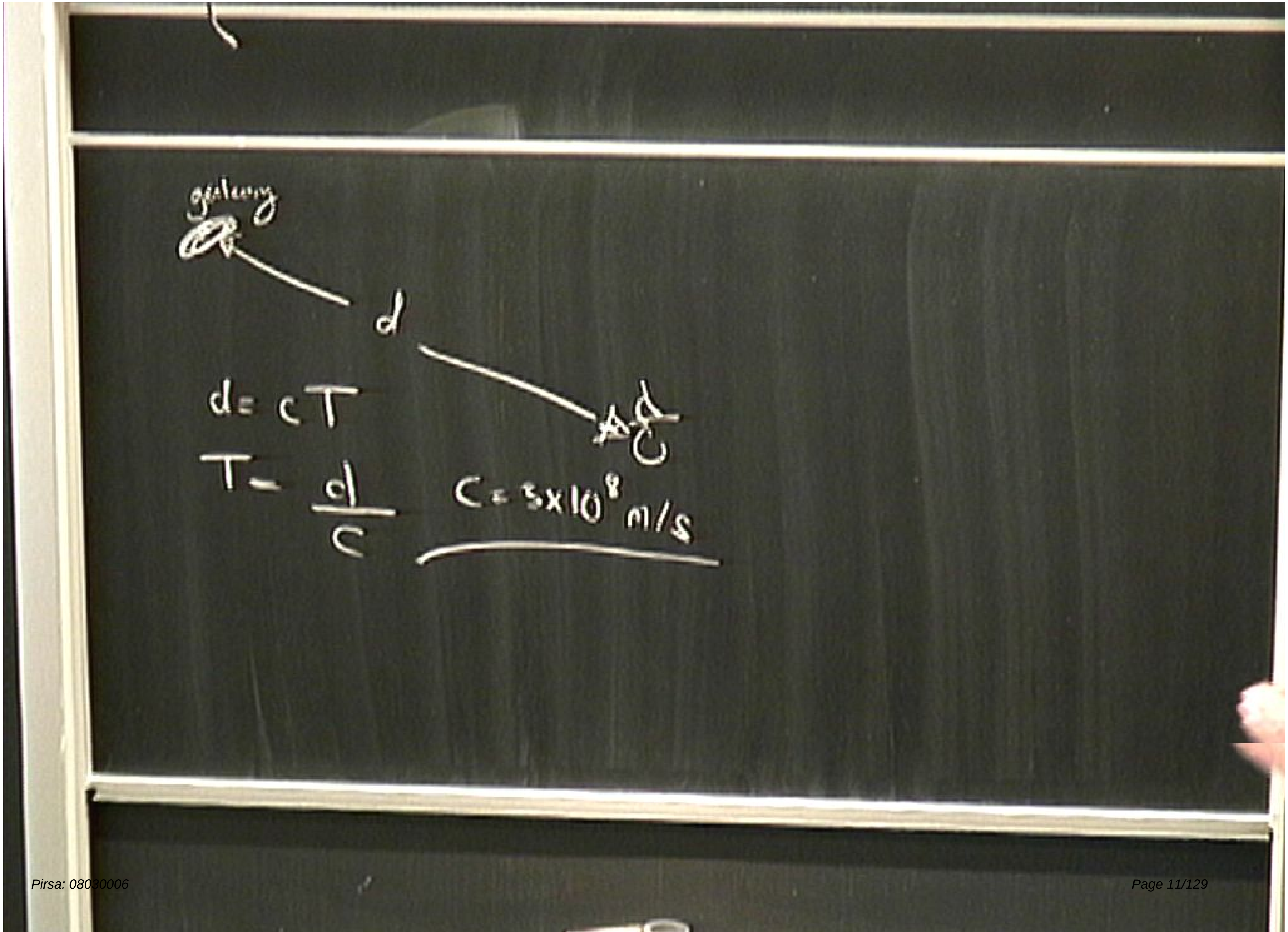
$d = 4000 \text{ Mpc}$

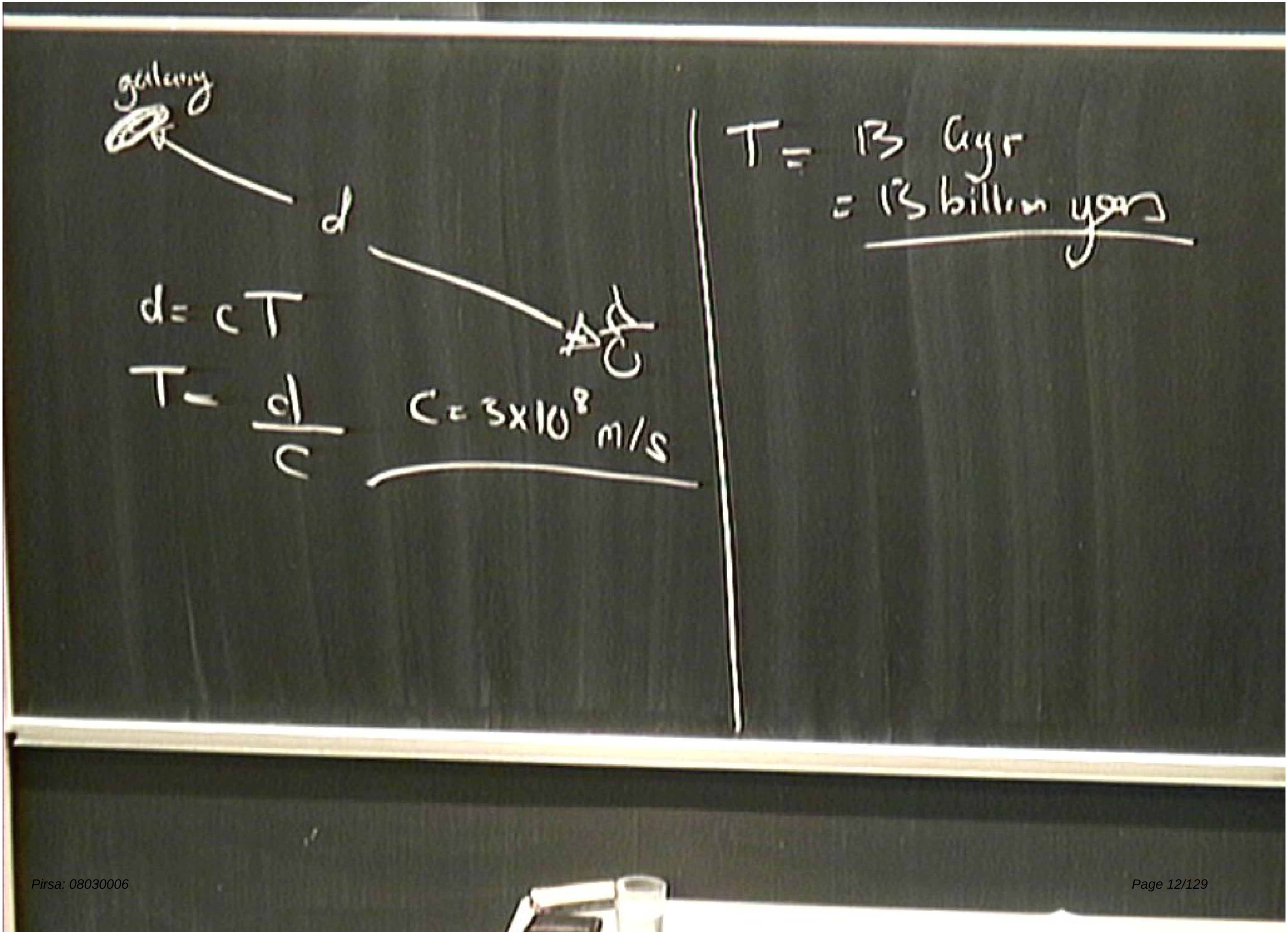
$1 \text{ pc} \approx 3.26$ light years

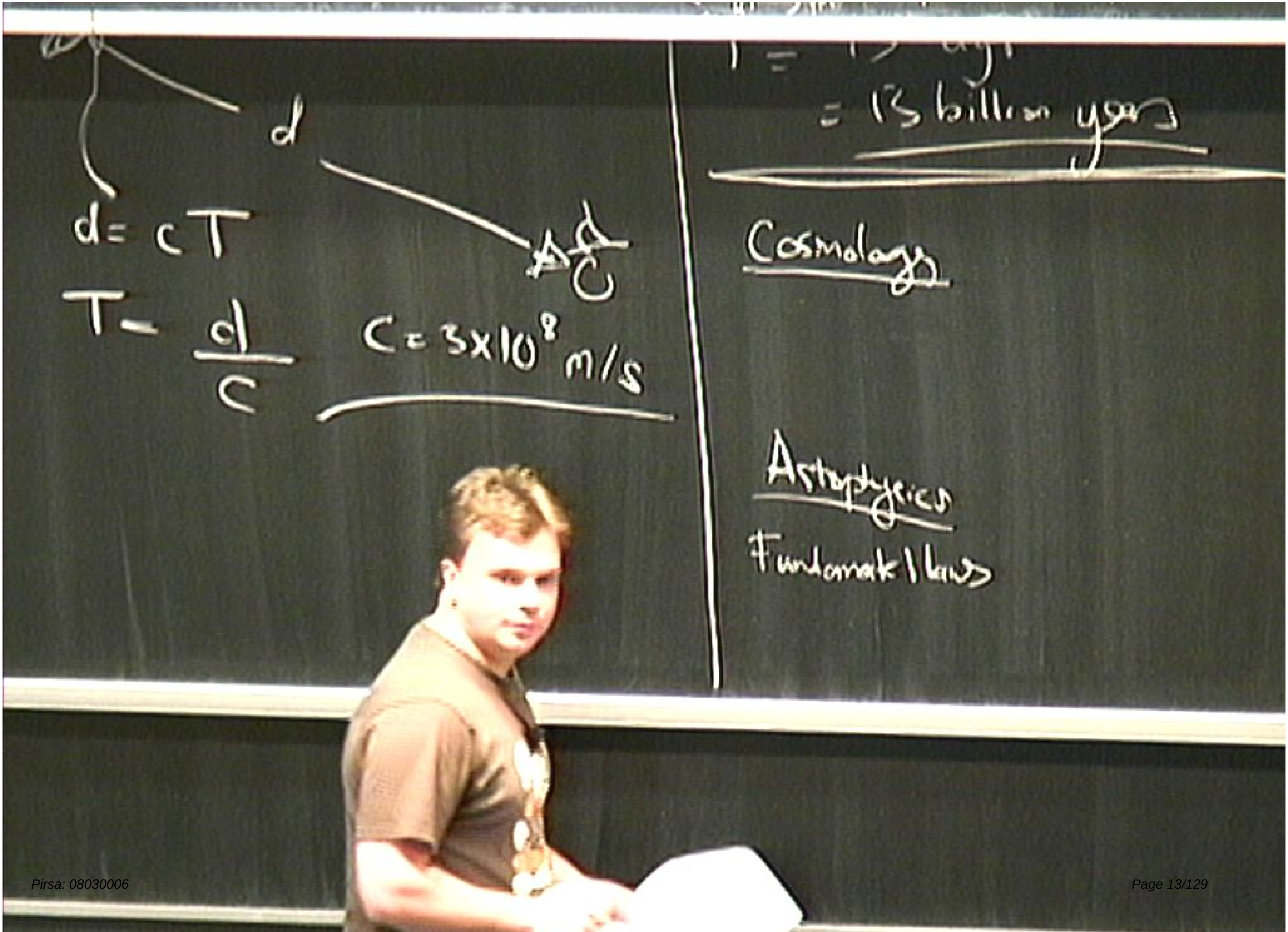
$= 3 \times 10^{16}$ m

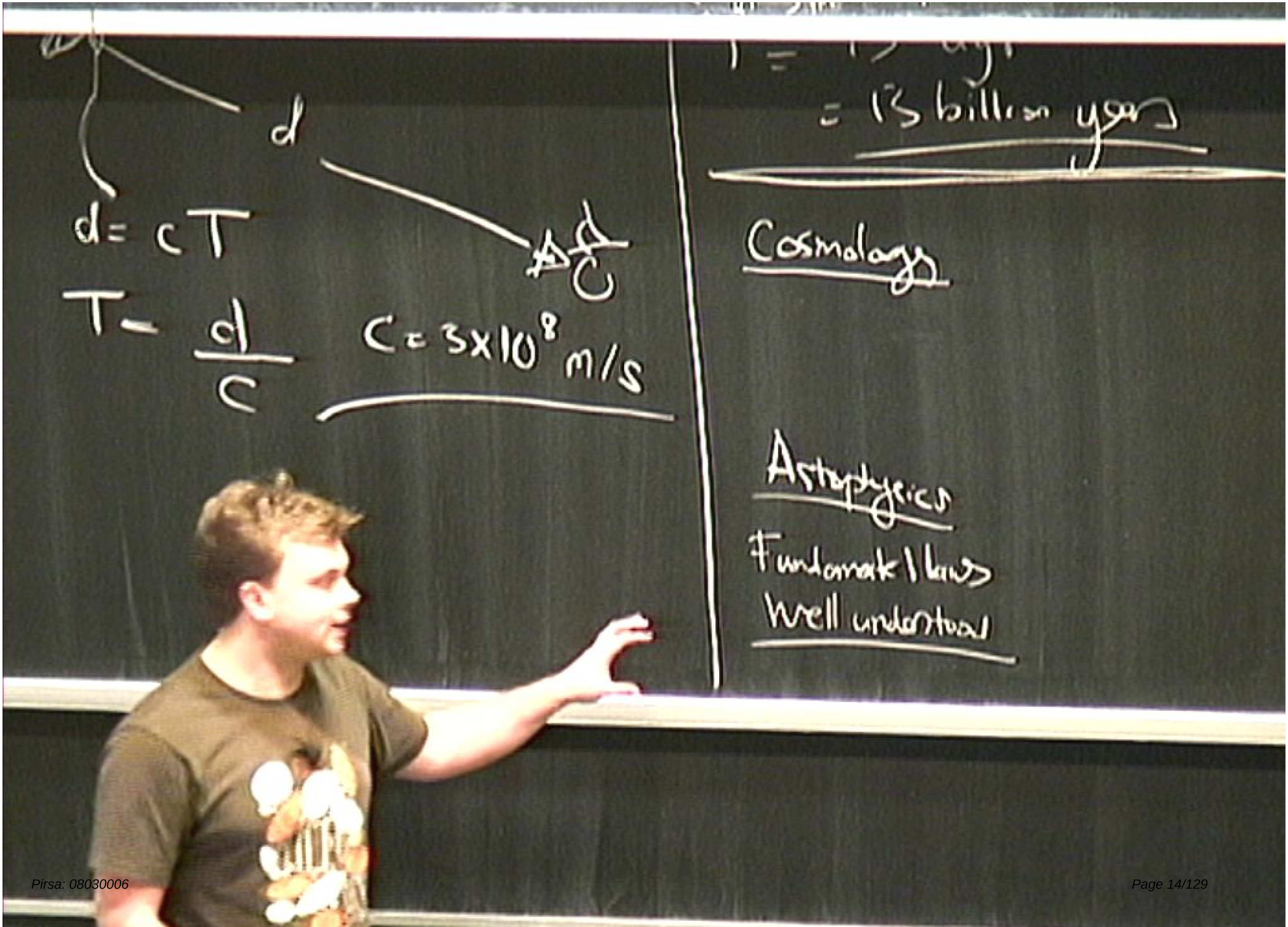

galaxy
d

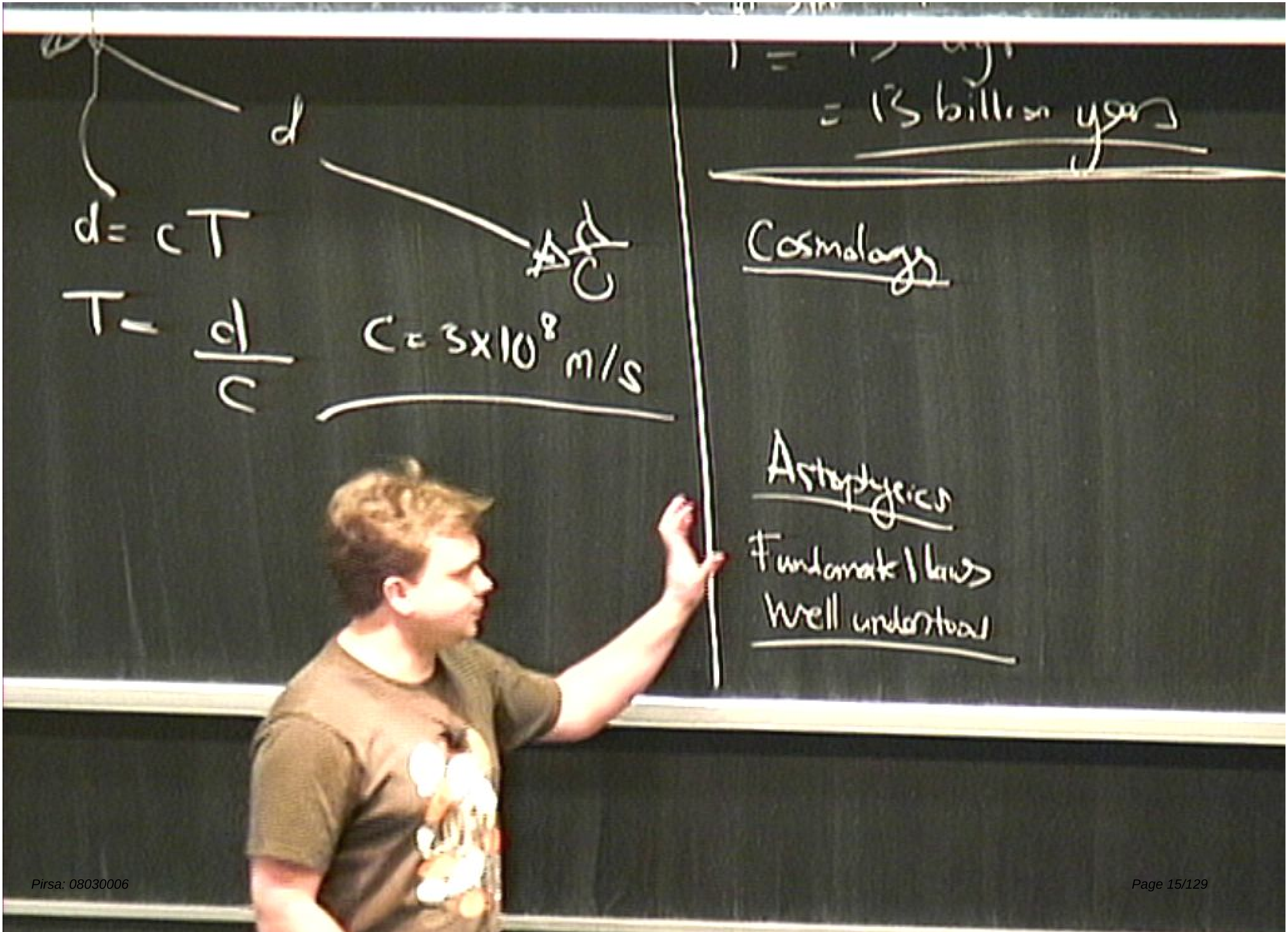

galaxy
d
d= cT
T= d/c
c = 3x10^8 m/s

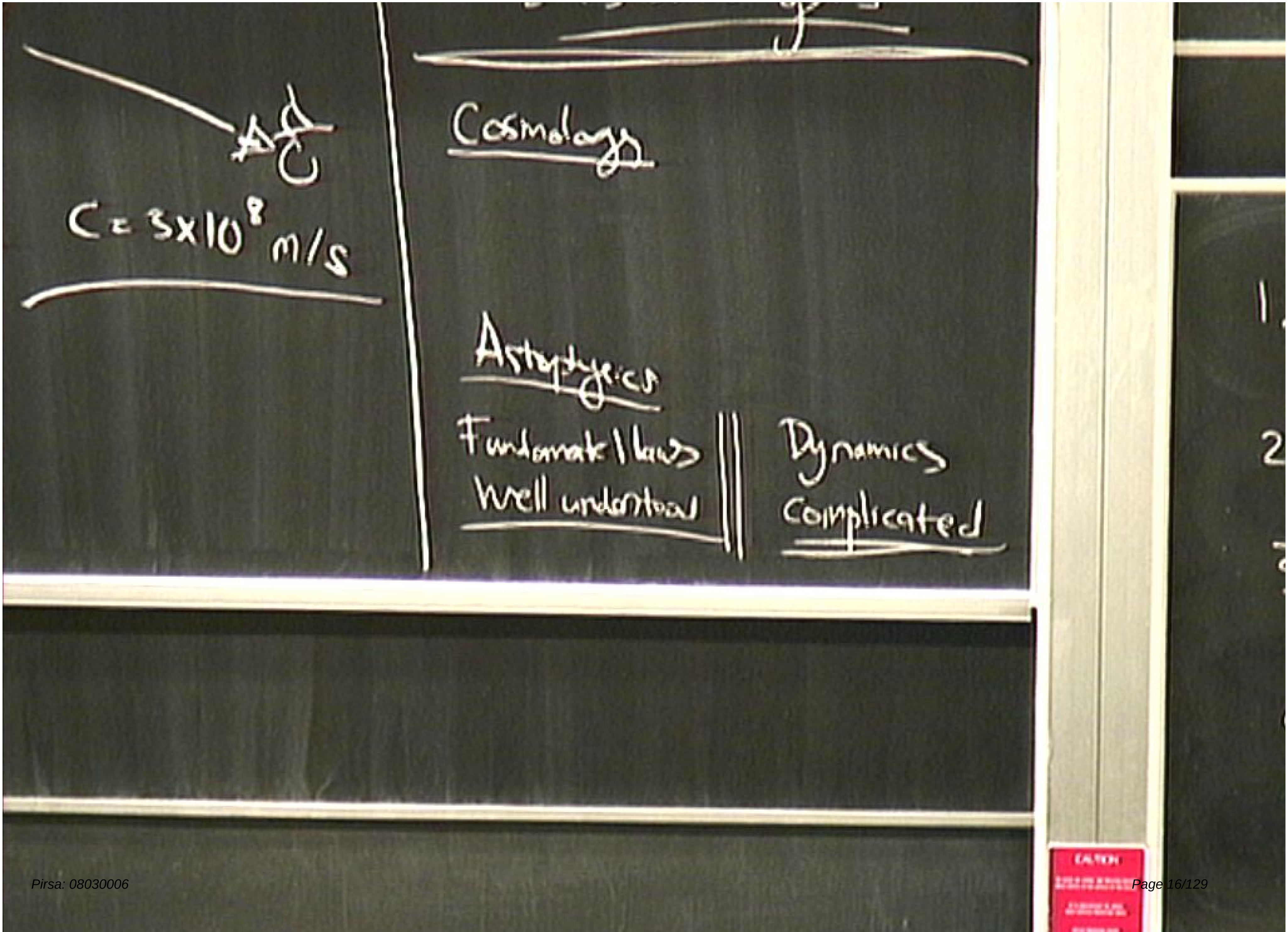

galaxy
d
d = cT
T = d/c
c = 3x10^8 m/s


galaxy
d
d = cT
T = d/c
c = 3x10^8 m/s
T = 13 Gyr
= 13 billion years

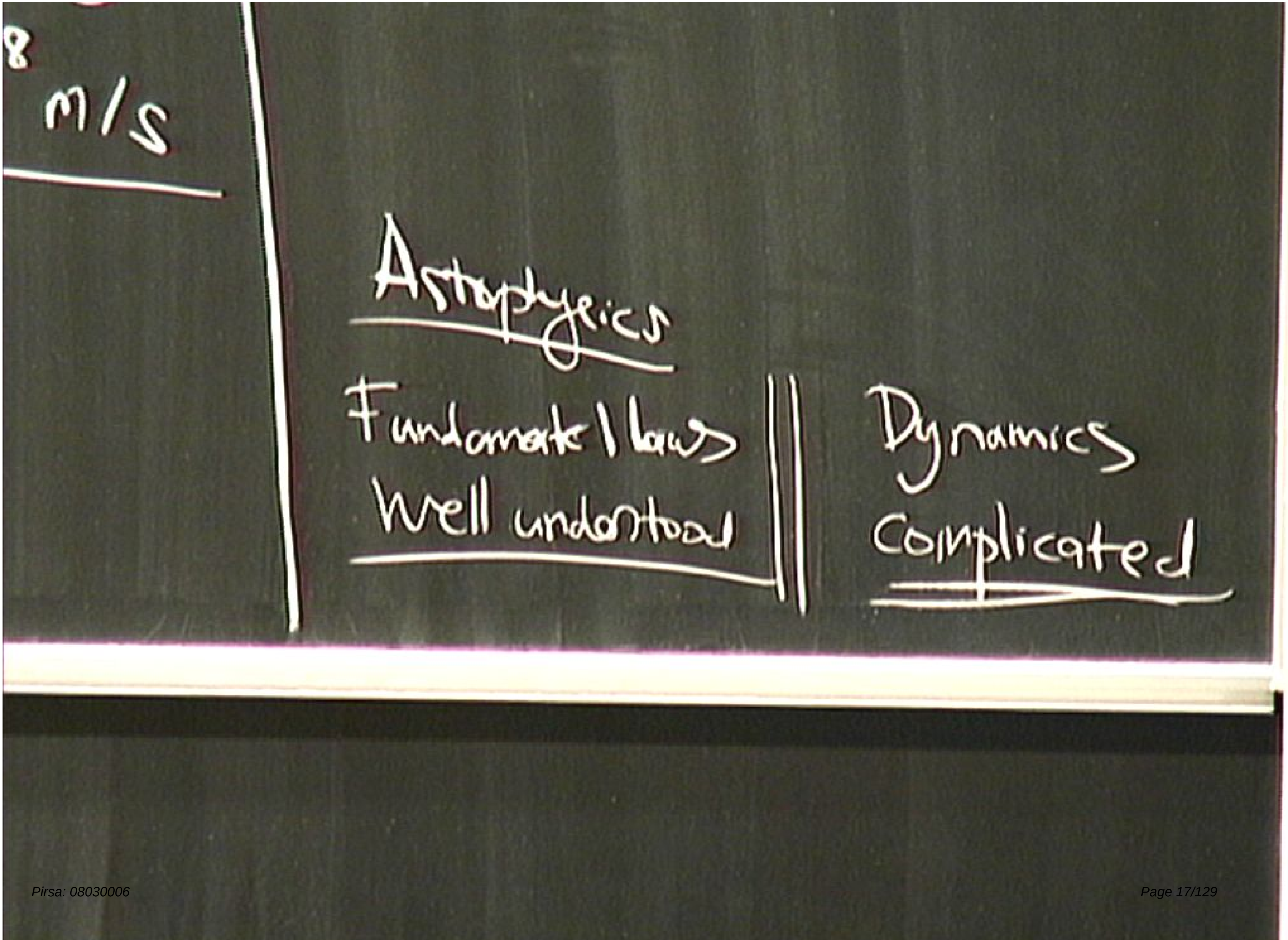

d
d= cT
T= d/c
c = 3x10^8 m/s
= 13 billion years
Cosmology
Astrophysics
Fundamental laws

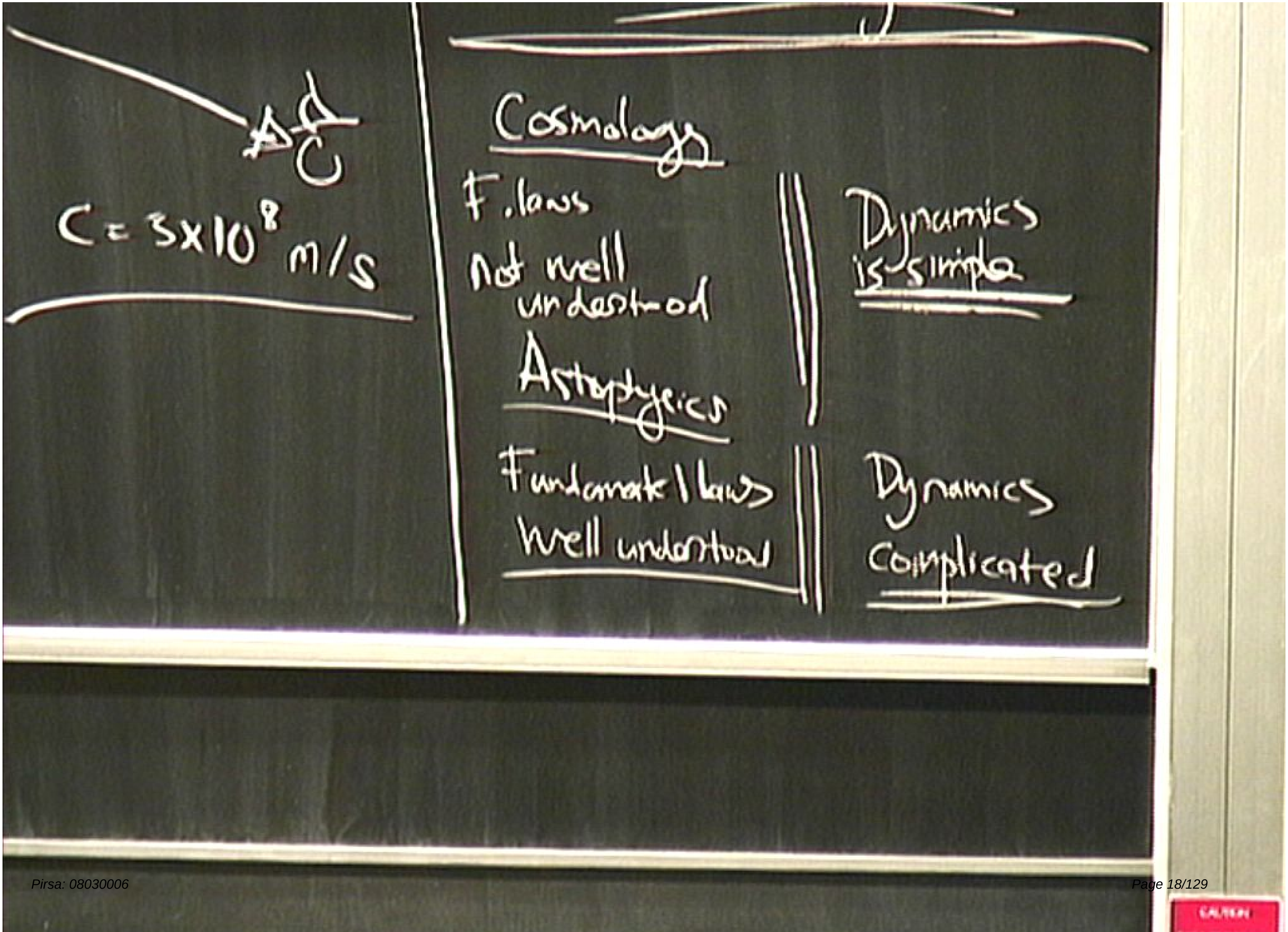

d
d= cT
T= d/c
c = 3x10^8 m/s
= 13 billion years
Cosmology
Astrophysics
Fundamental laws
Well understood

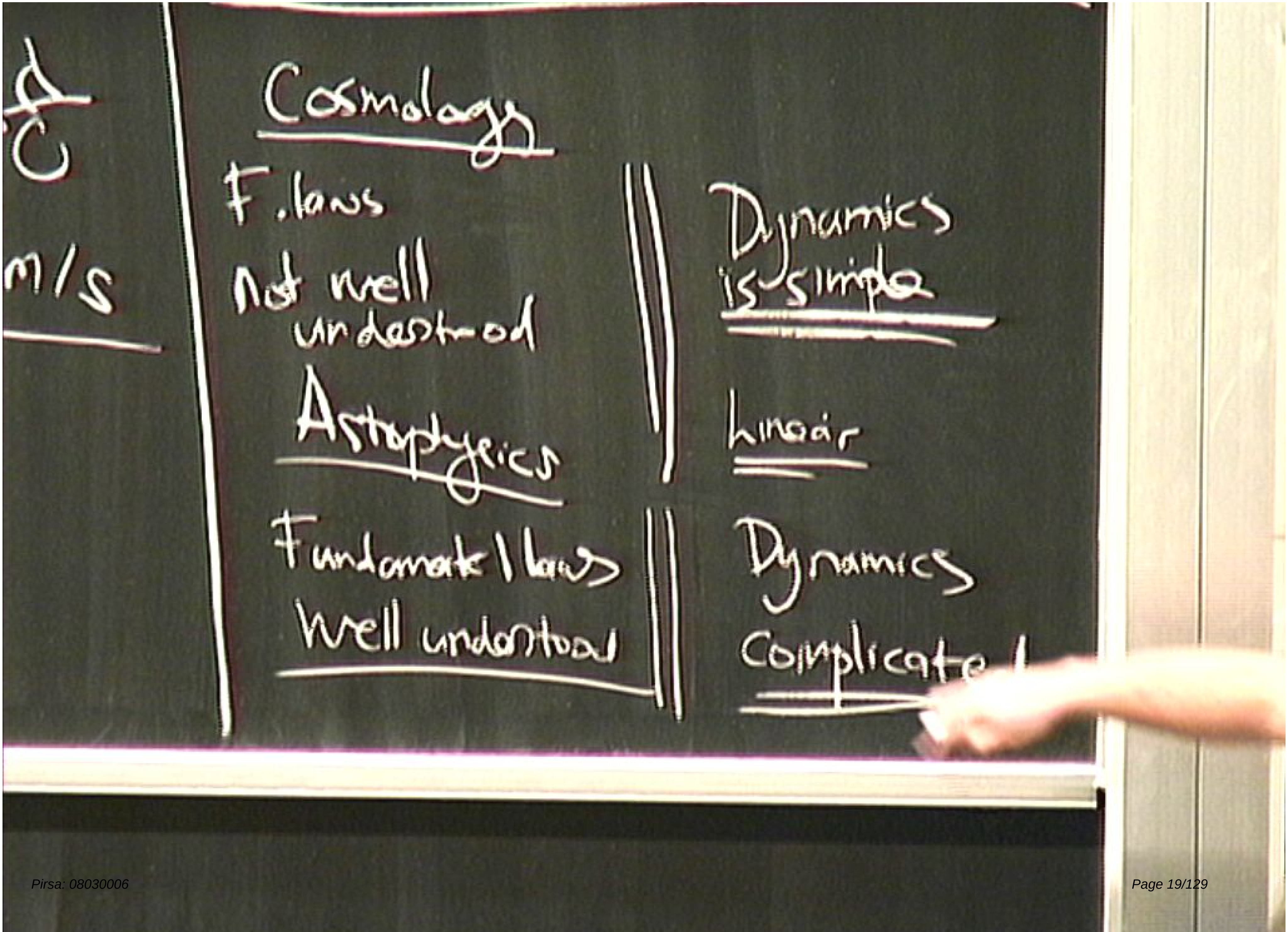

= 13 billion years
d
d= cT
T= d/c
c= 3x10^8 m/s
Cosmology
Astrophysics
Fundamental laws
Well understood

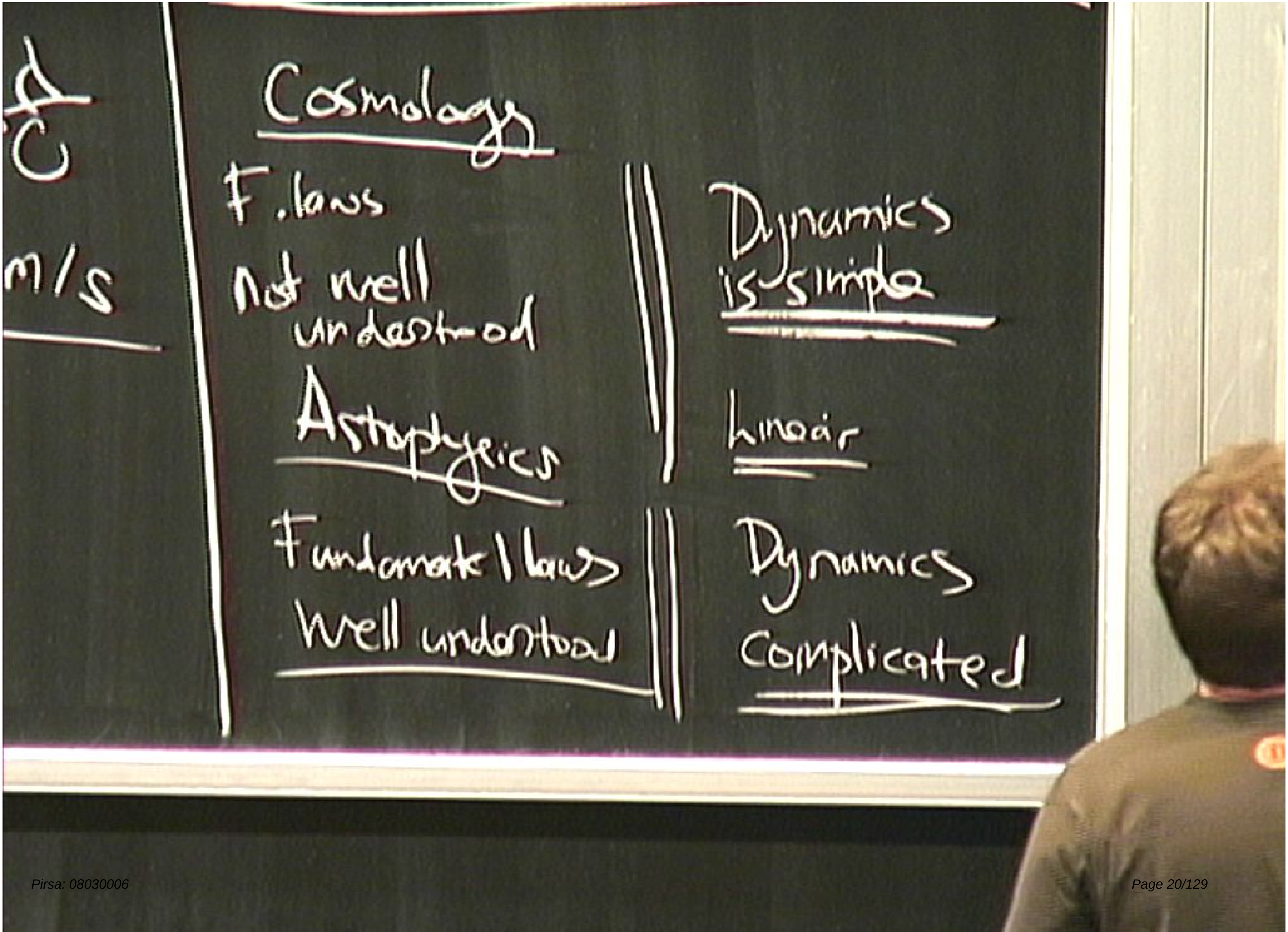


C = 3x10^8 m/s
Cosmology
Astrophysics
Fundamental laws
Well understood
Dynamics
Complicated


8
m/s
Astrophysics
Fundamental laws
Well understood
Dynamics
Complicated


c = 3x10^8 m/s
Cosmology
F. laws
not well understood
Dynamics is simple
Astrophysics
Fundamental laws
Well understood
Dynamics
Complicated


m/s
Cosmology
F. laws
not well understood
Astrophysics
Fundamental laws
Well understood
Dynamics
is simple
linear
Dynamics
Complicate


m/s
Cosmology
F. laws
not well
understood
Astrophysics
Fundamental laws
Well understood
Dynamics
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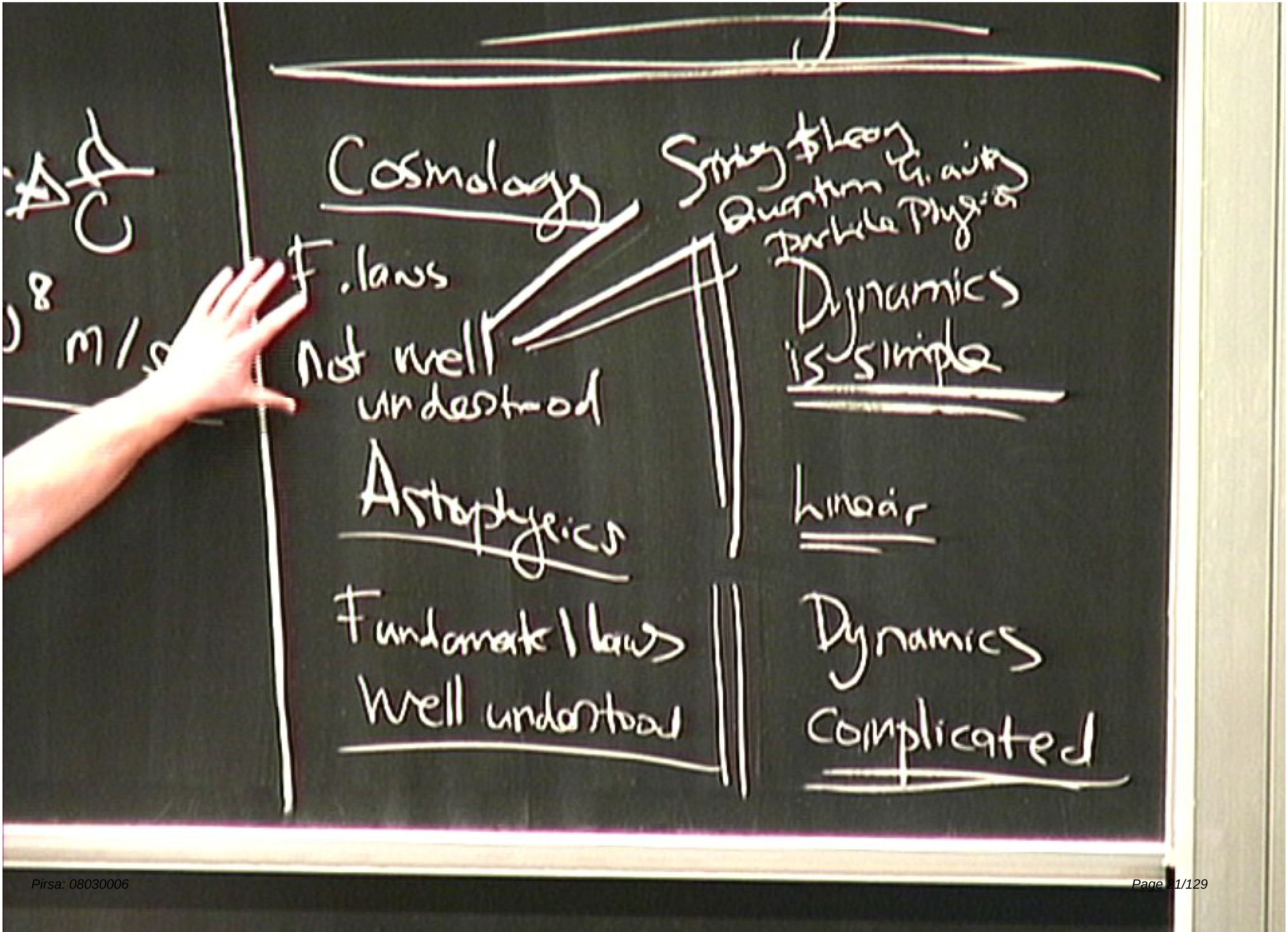
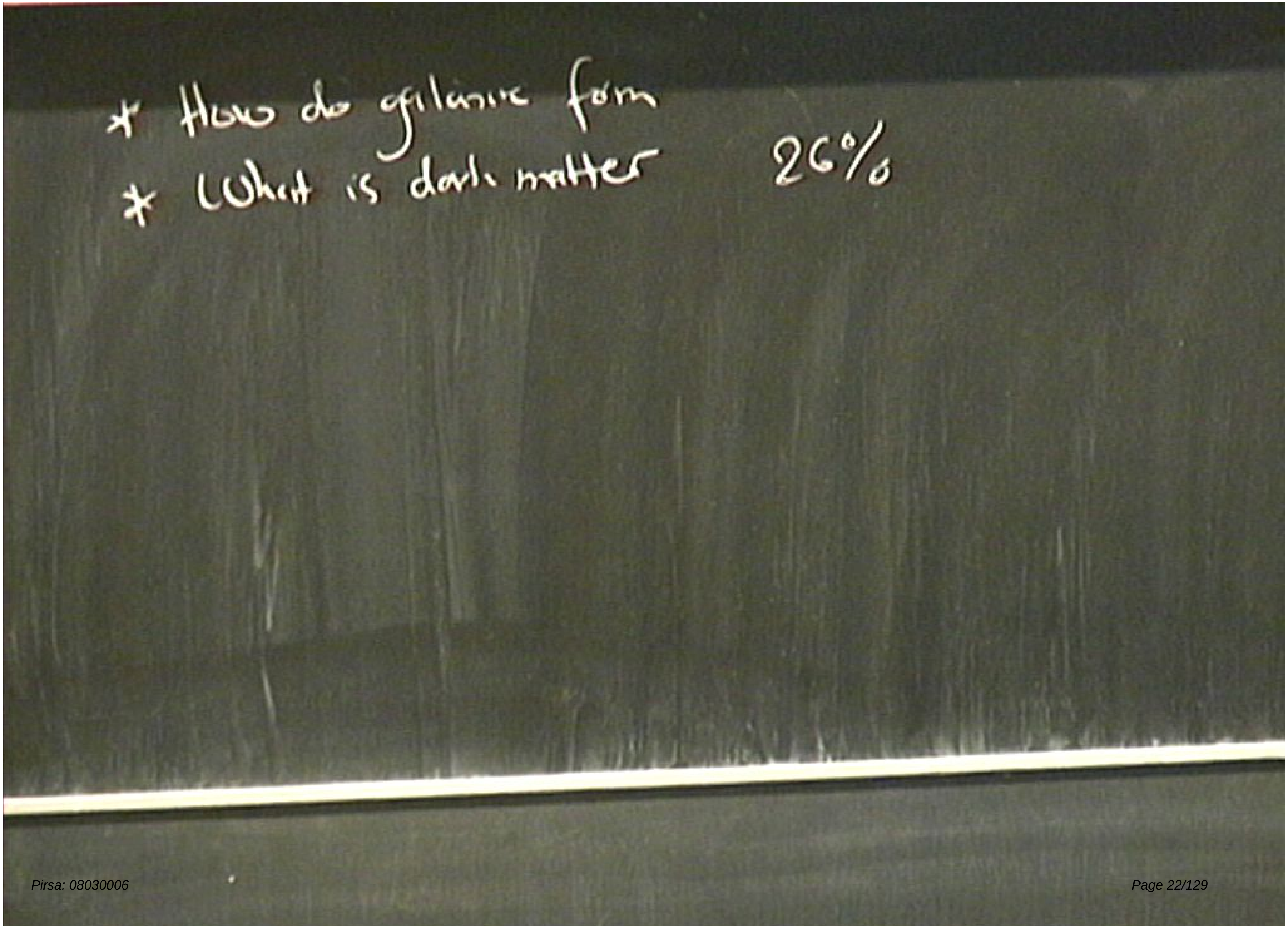
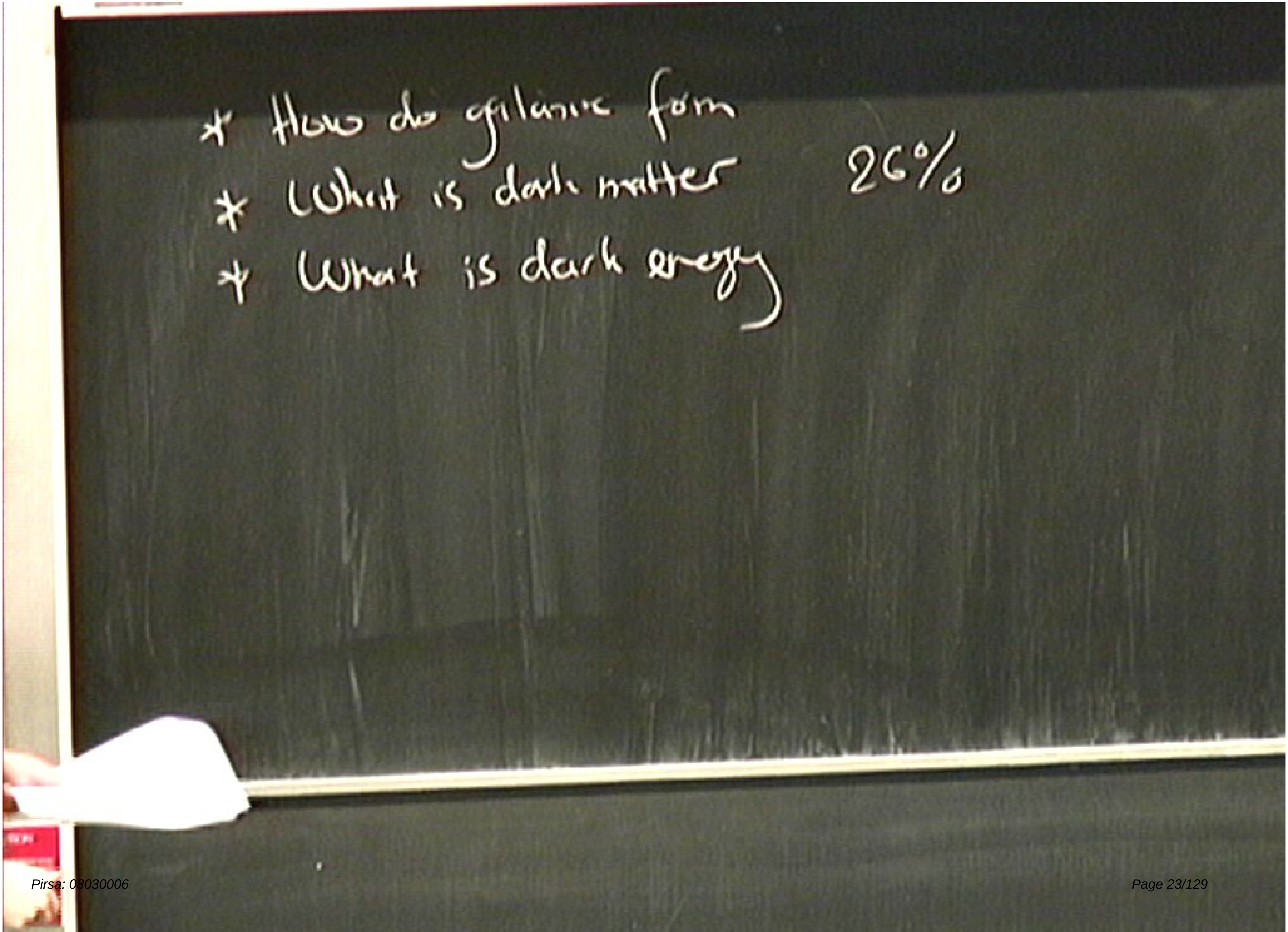
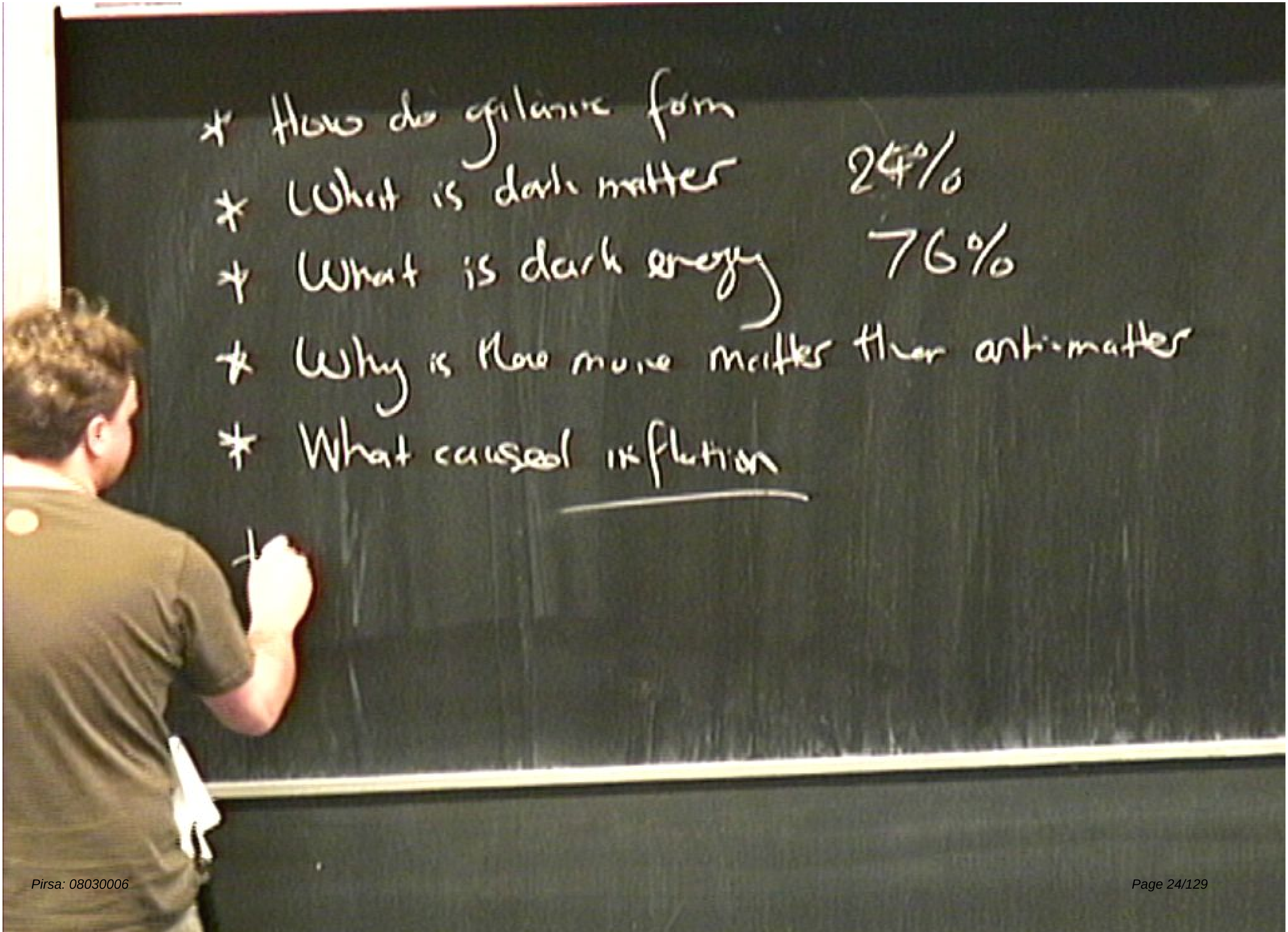
linear
Dynamics
Complicated

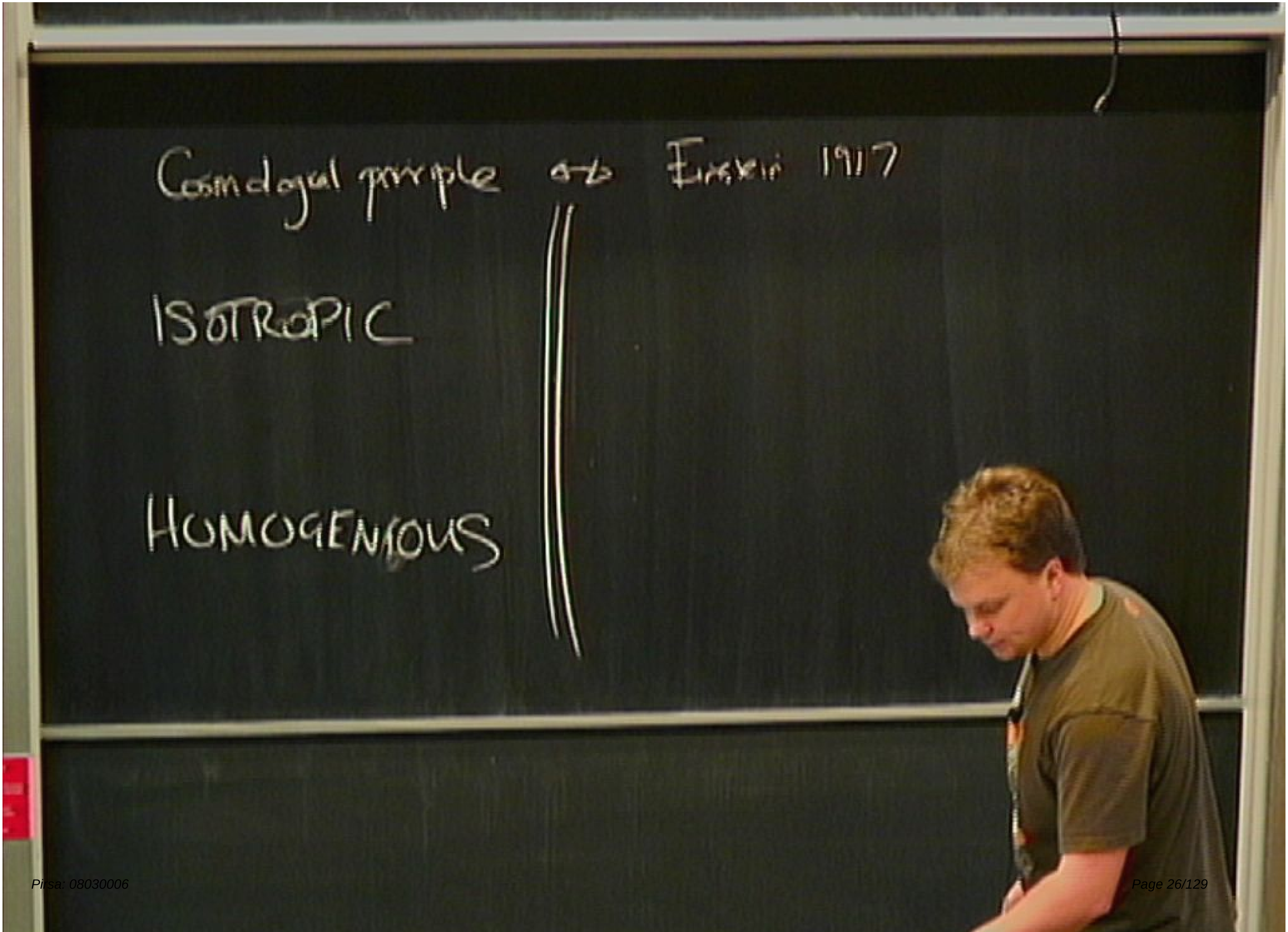

Cosmology
String theory
Quantum Gravity
Particle Physics
F. laws
not well understood
Dynamics is simple
Astrophysics
linear
Fundamental laws
Well understood
Dynamics
Complicated


* How do galaxies form
* What is dark matter
26%

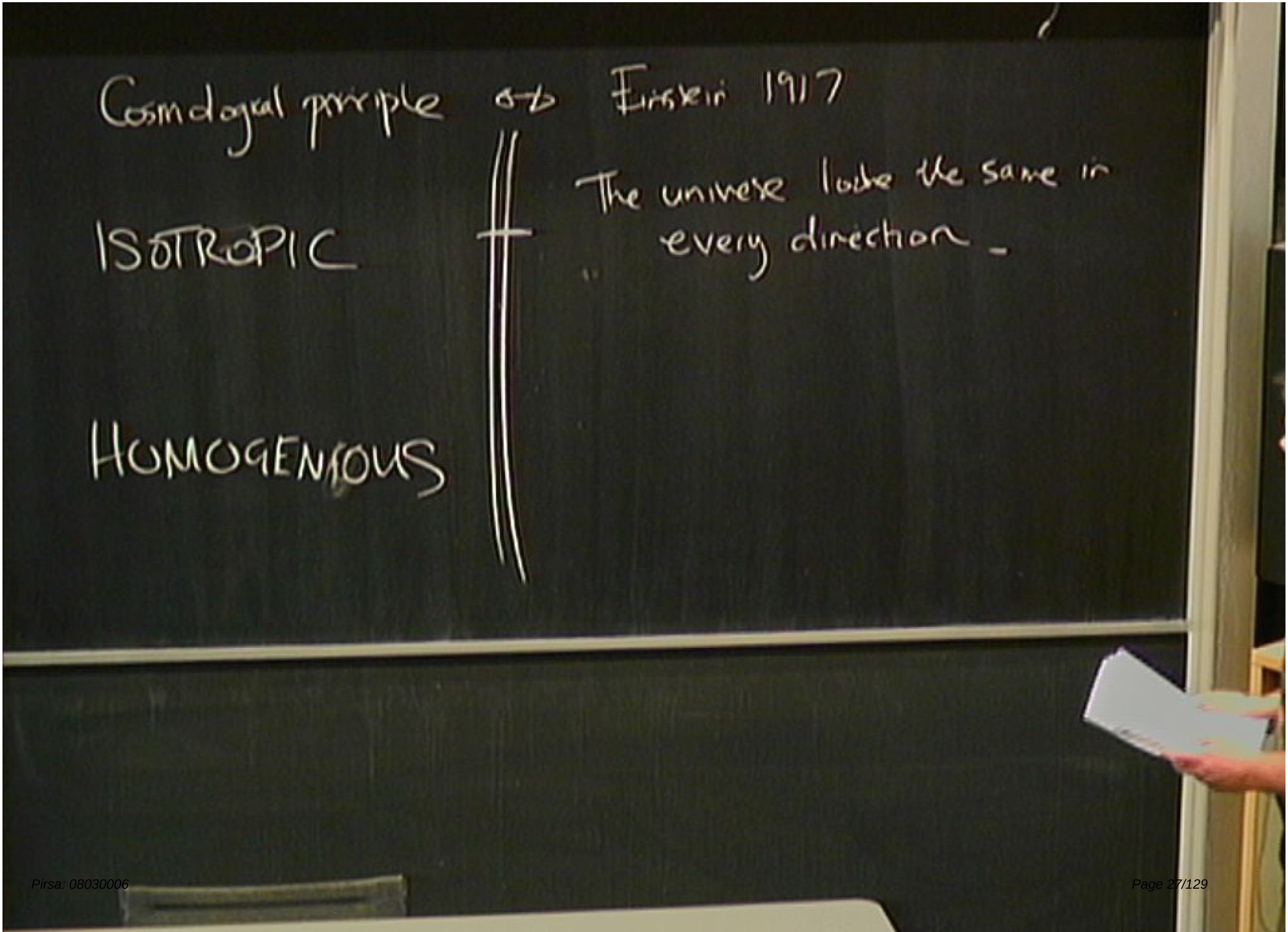

* How do galaxies form
* What is dark matter
26%
* What is dark energy

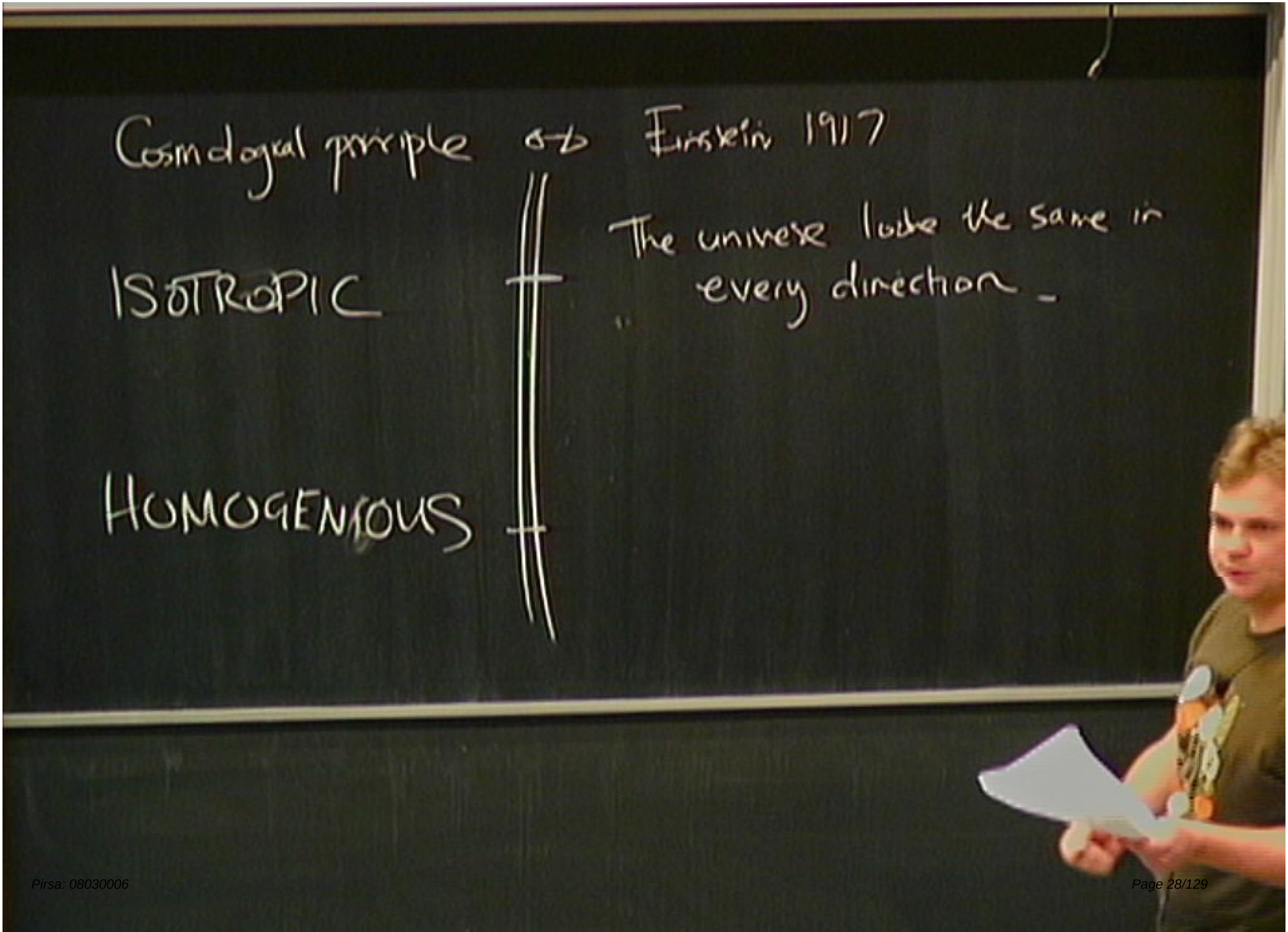

* How do galaxies form
* What is dark matter 24%
* What is dark energy 76%
* Why is there more matter than anti-matter
* What caused inflation

* How do galaxies form
* What is dark matter 24%
* What is dark energy 76%
* Why is there more matter than anti-matter
* What caused inflation
* Where do the density fluctuations that are responsible for galaxy formation come from



Cosmological principle
Einstein 1917
ISOTROPIC
HOMOGENOUS


Cosmological principle
Einstein 1917
ISOTROPIC
The universe looks the same in every direction.
HOMOGENEOUS


Cosmological principle ↔ Einstein 1917
ISOTROPIC
The universe looks the same in every direction.
HOMOGENEOUS

Cosmological principle $\leftrightarrow$ Einstein 1917

ISOTROPIC — The universe looks the same in every direction.

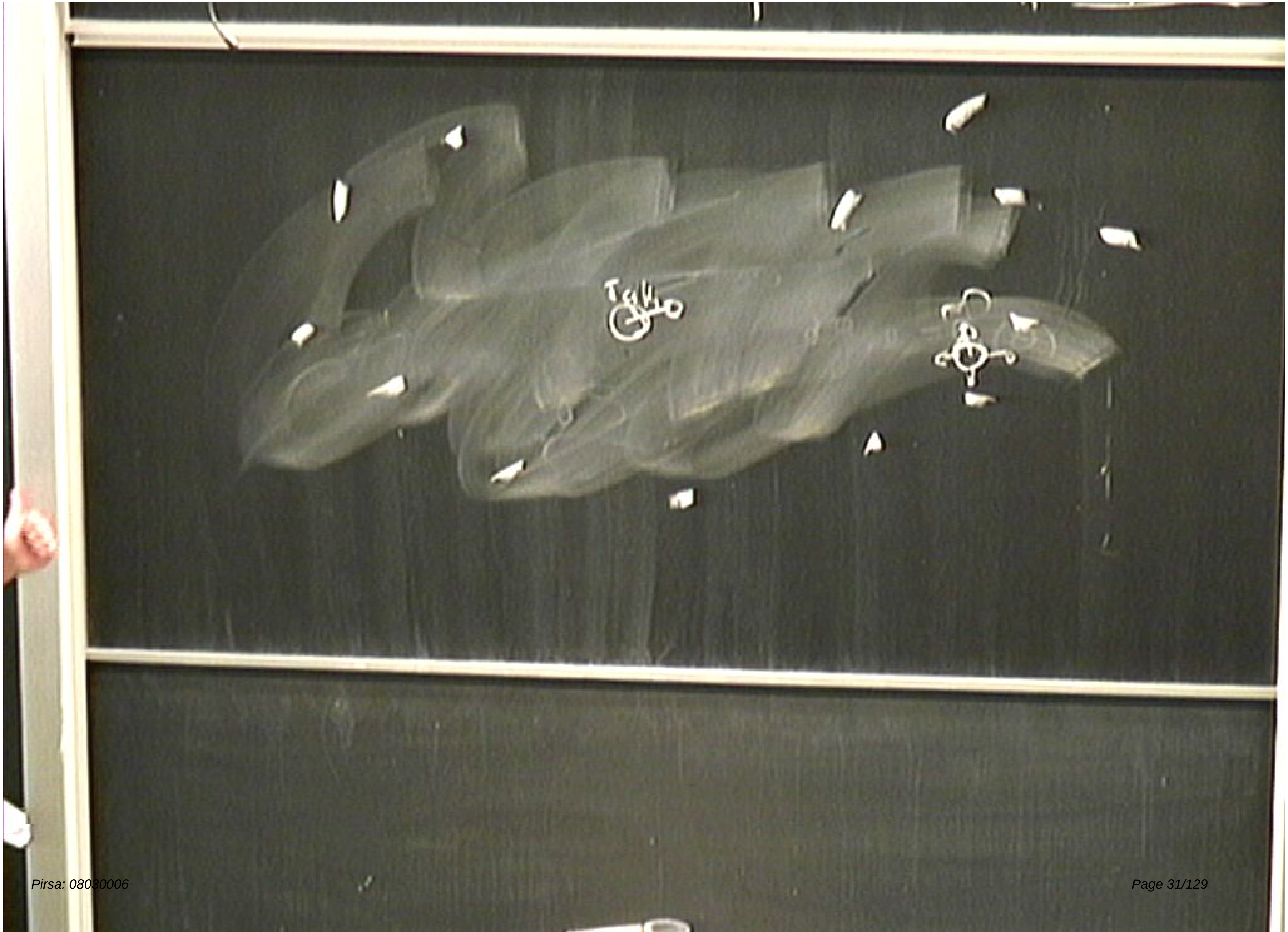
HOMOGENEOUS

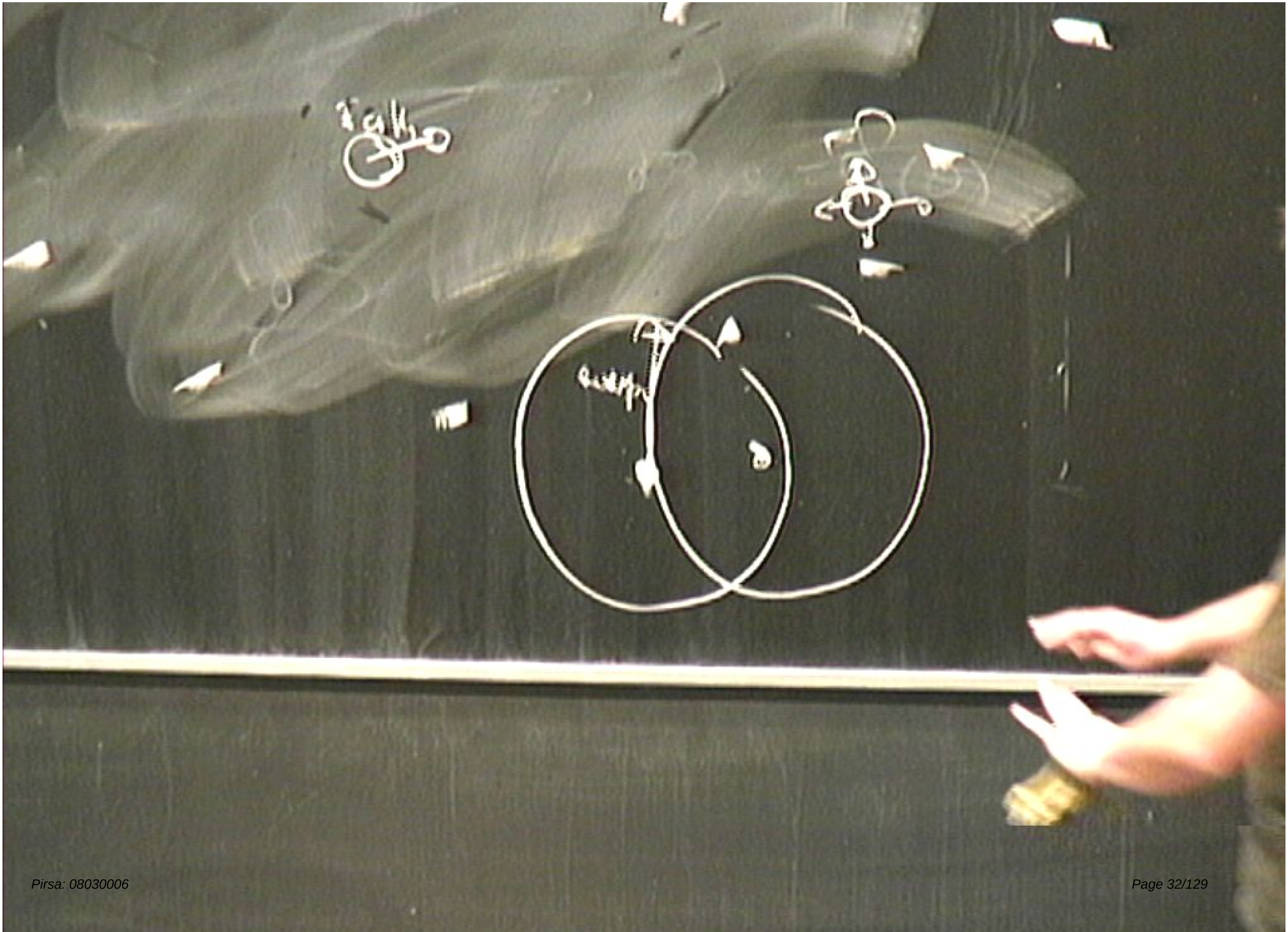
Cosmological principle $\leftrightarrow$ Einstein 1917

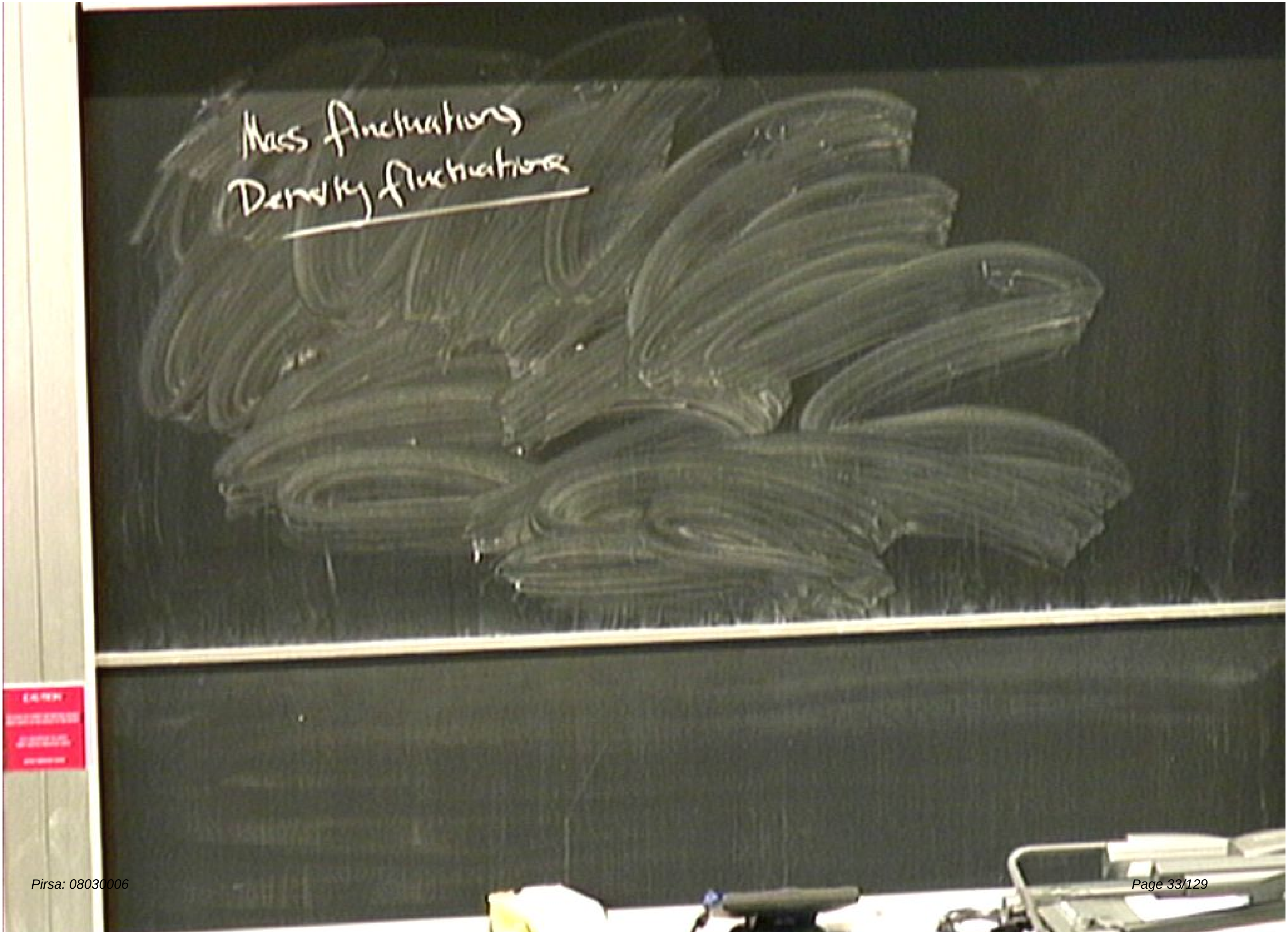
ISOTROPIC — The universe looks the same in every direction.

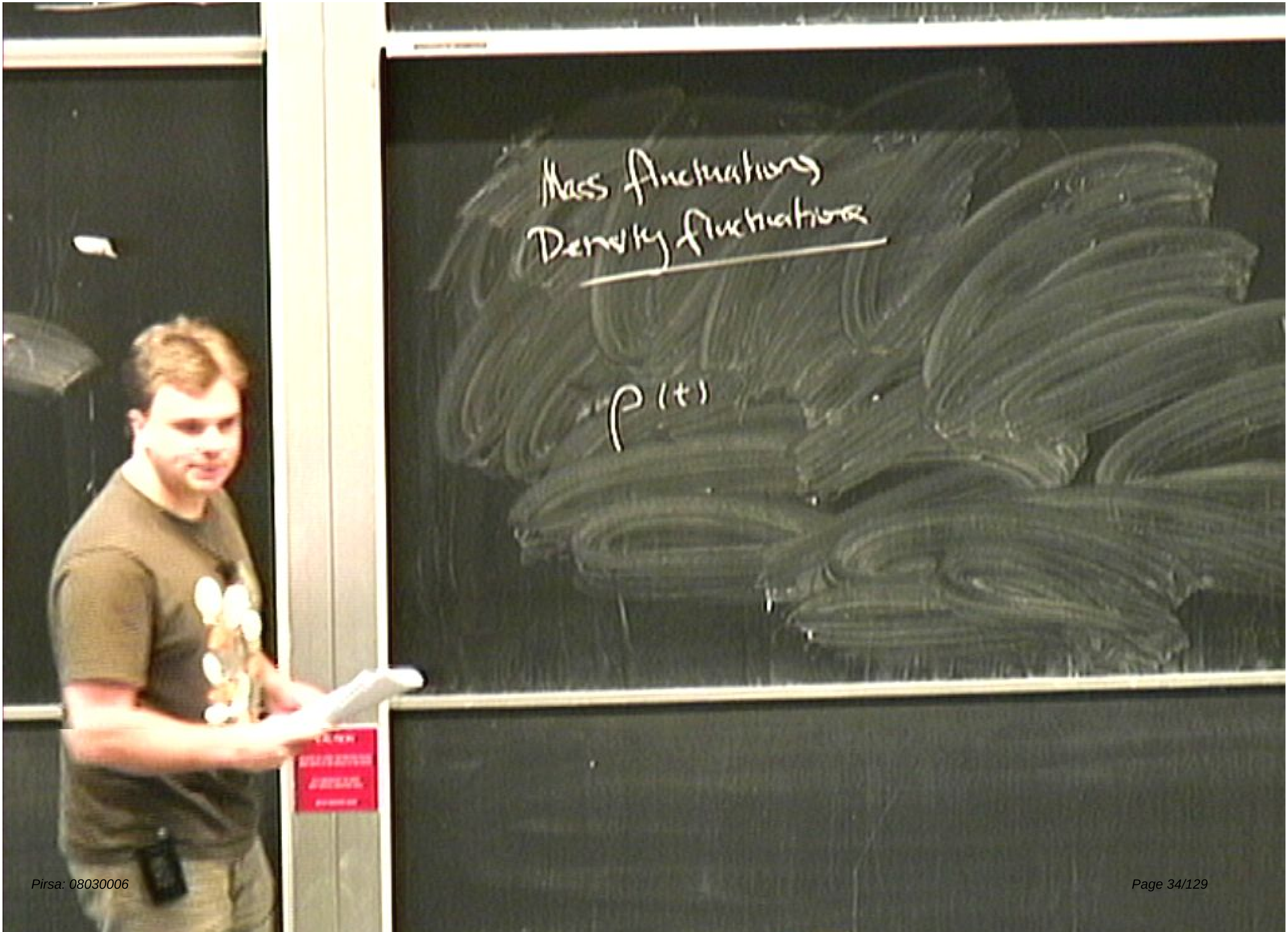
HOMOGENEOUS — The universe looks the same from every point in space

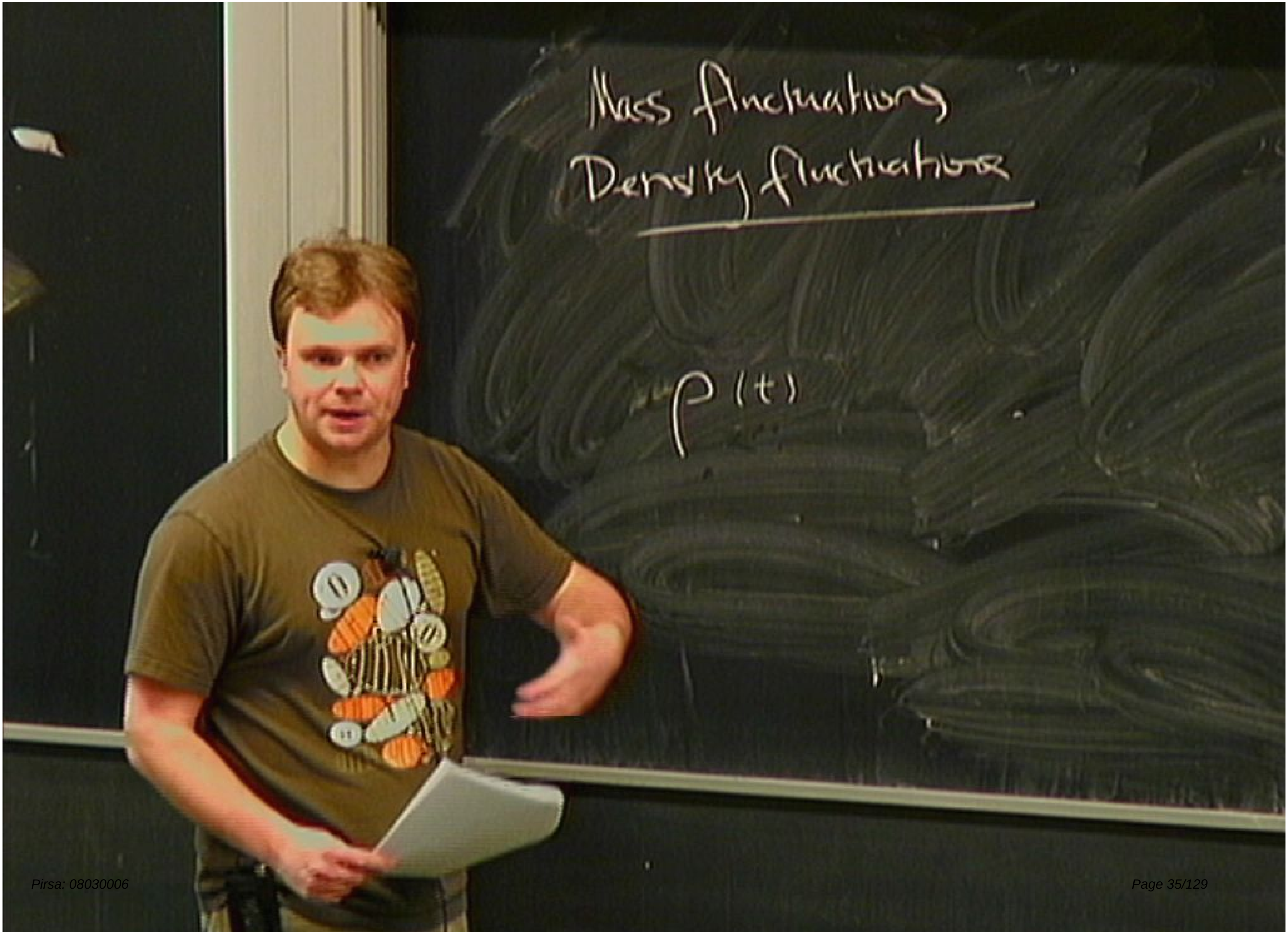
$\Rightarrow$ No preferred point in space

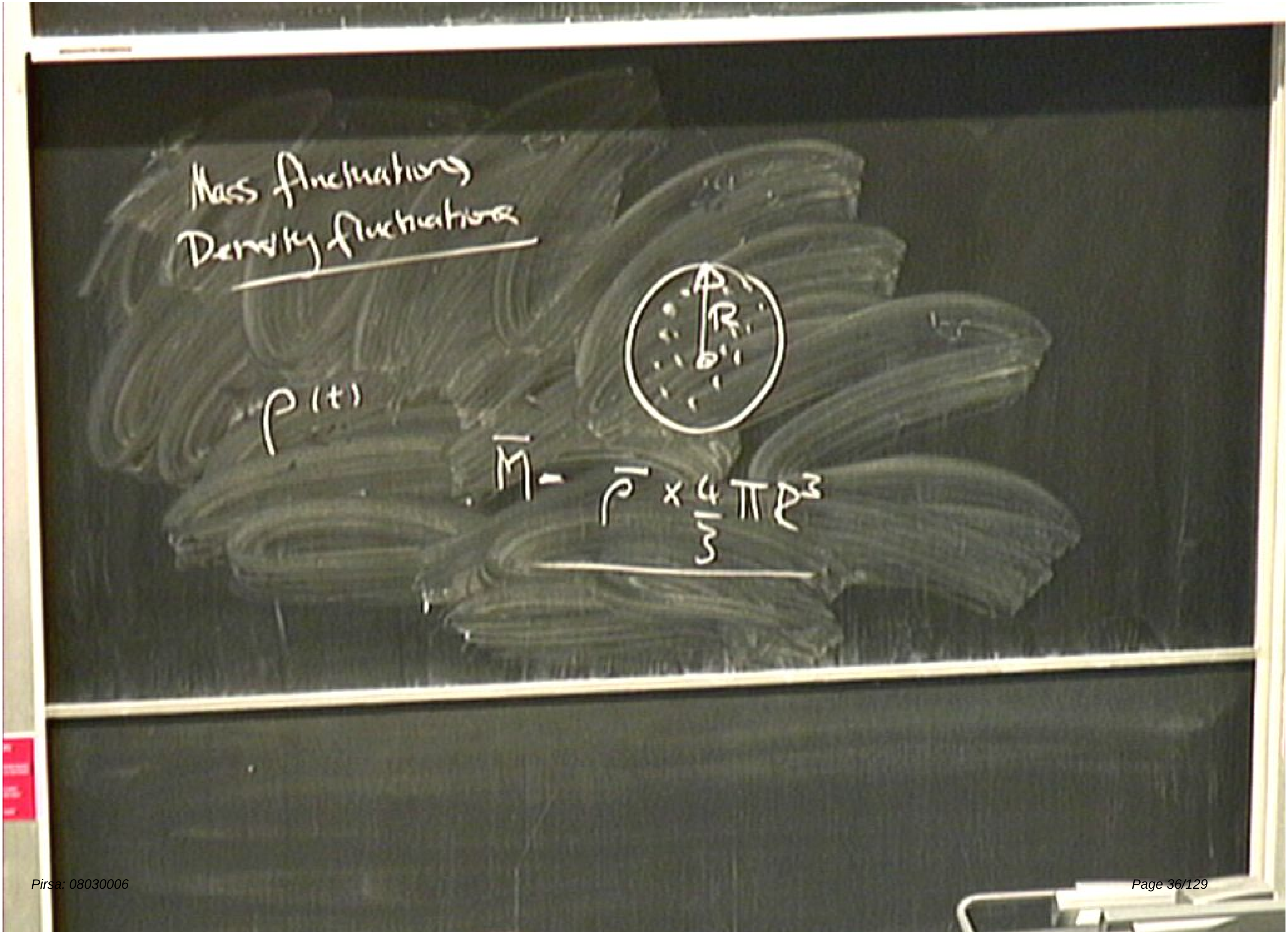





Mass fluctuations
Density fluctuations


Mass fluctuations
Density fluctuations
ρ(t)


Mass fluctuations
Density fluctuations
ρ(t)

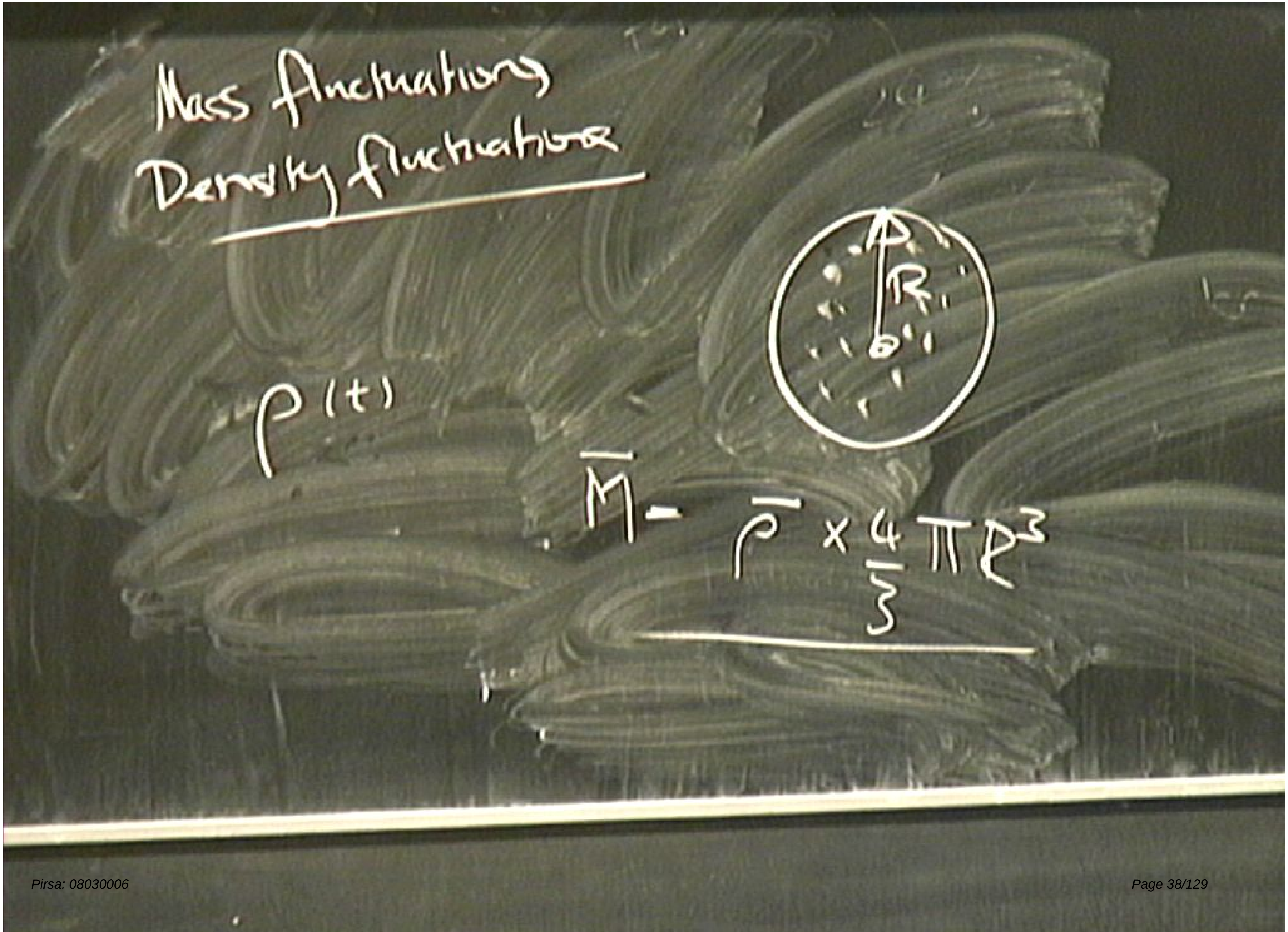

Mass fluctuations
Density fluctuations
ρ(t)
R
M̄ = ρ̄ × 4/3 π R³

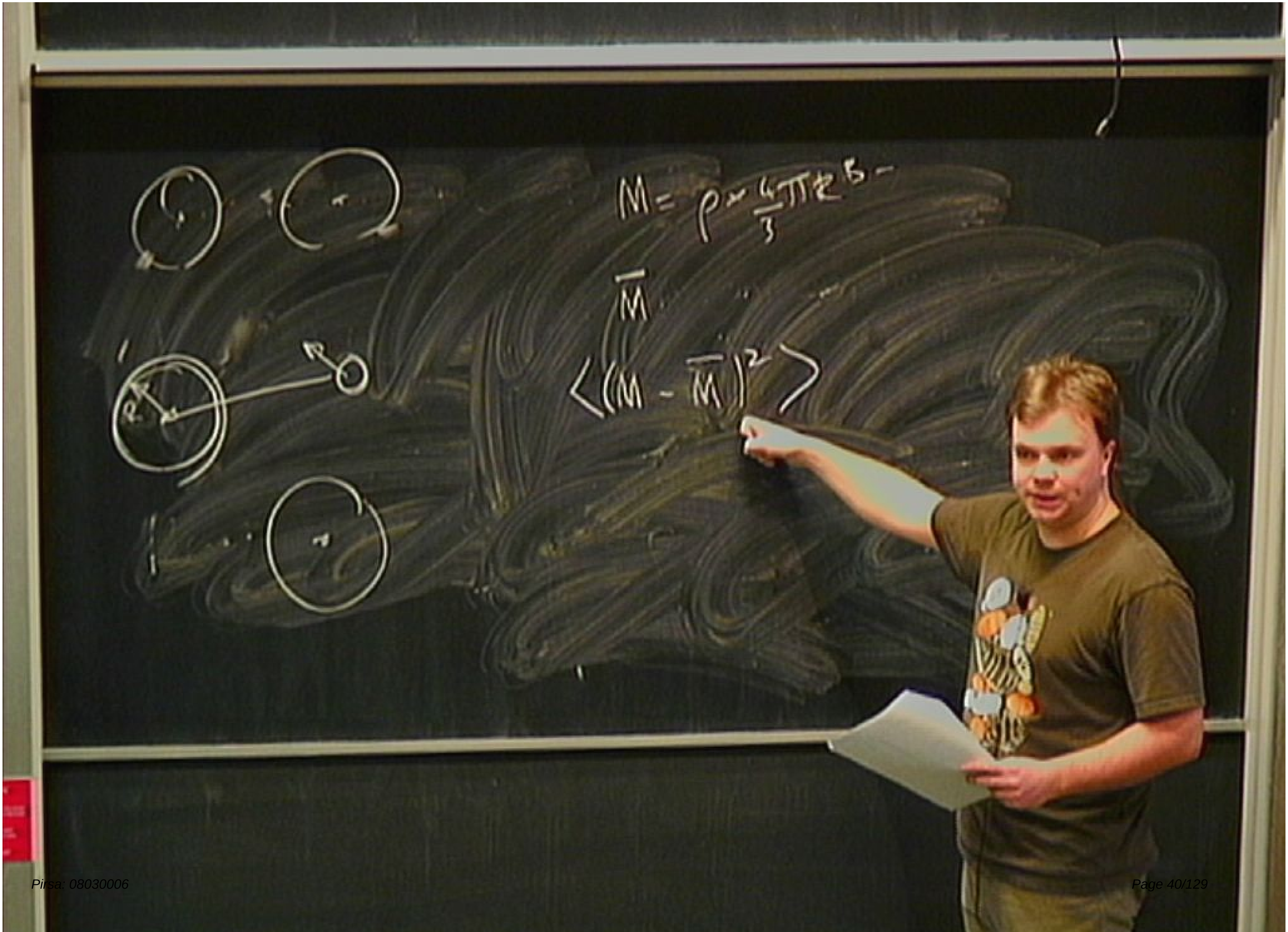
Mass fluctuations

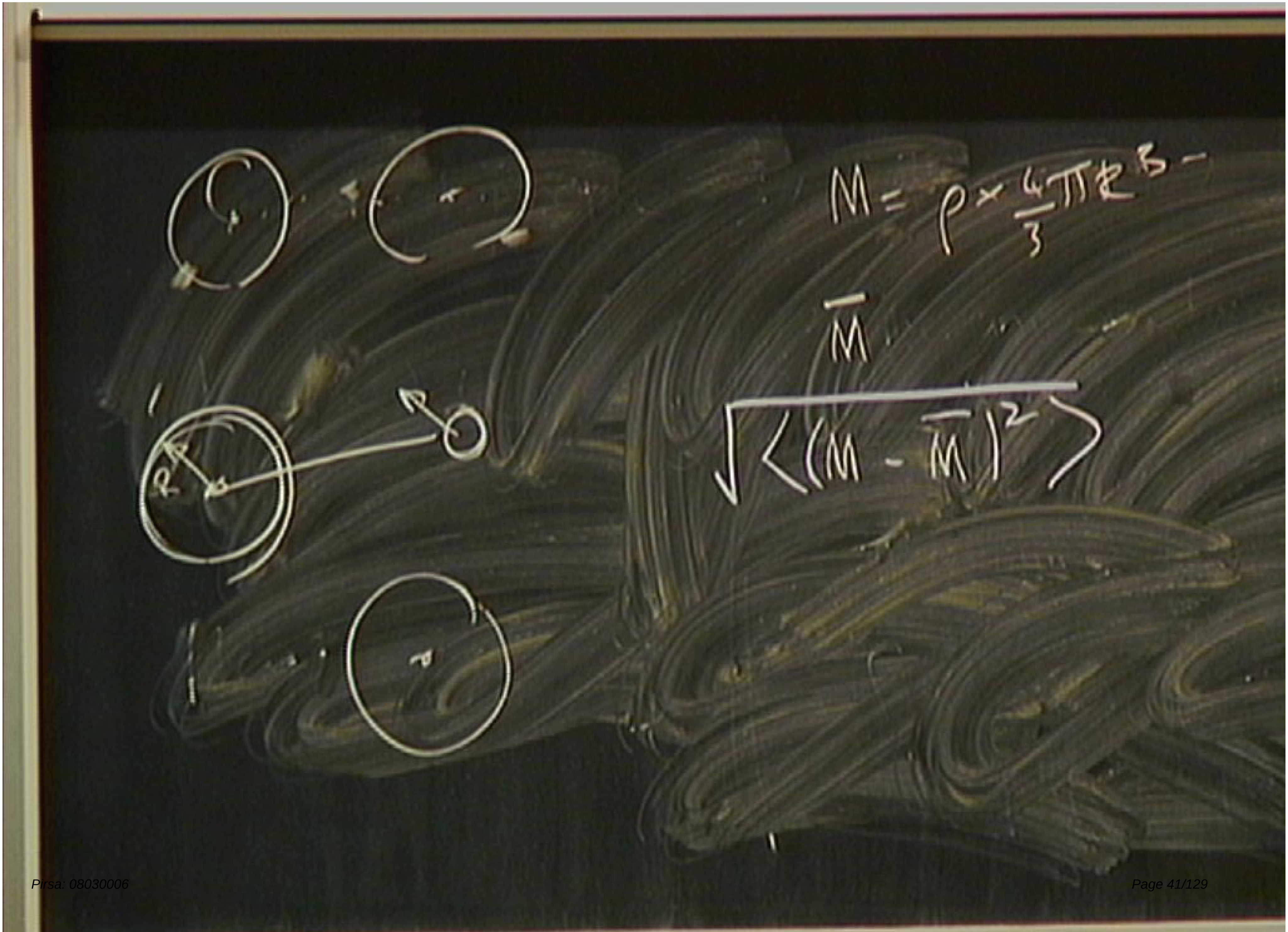
Density fluctuations

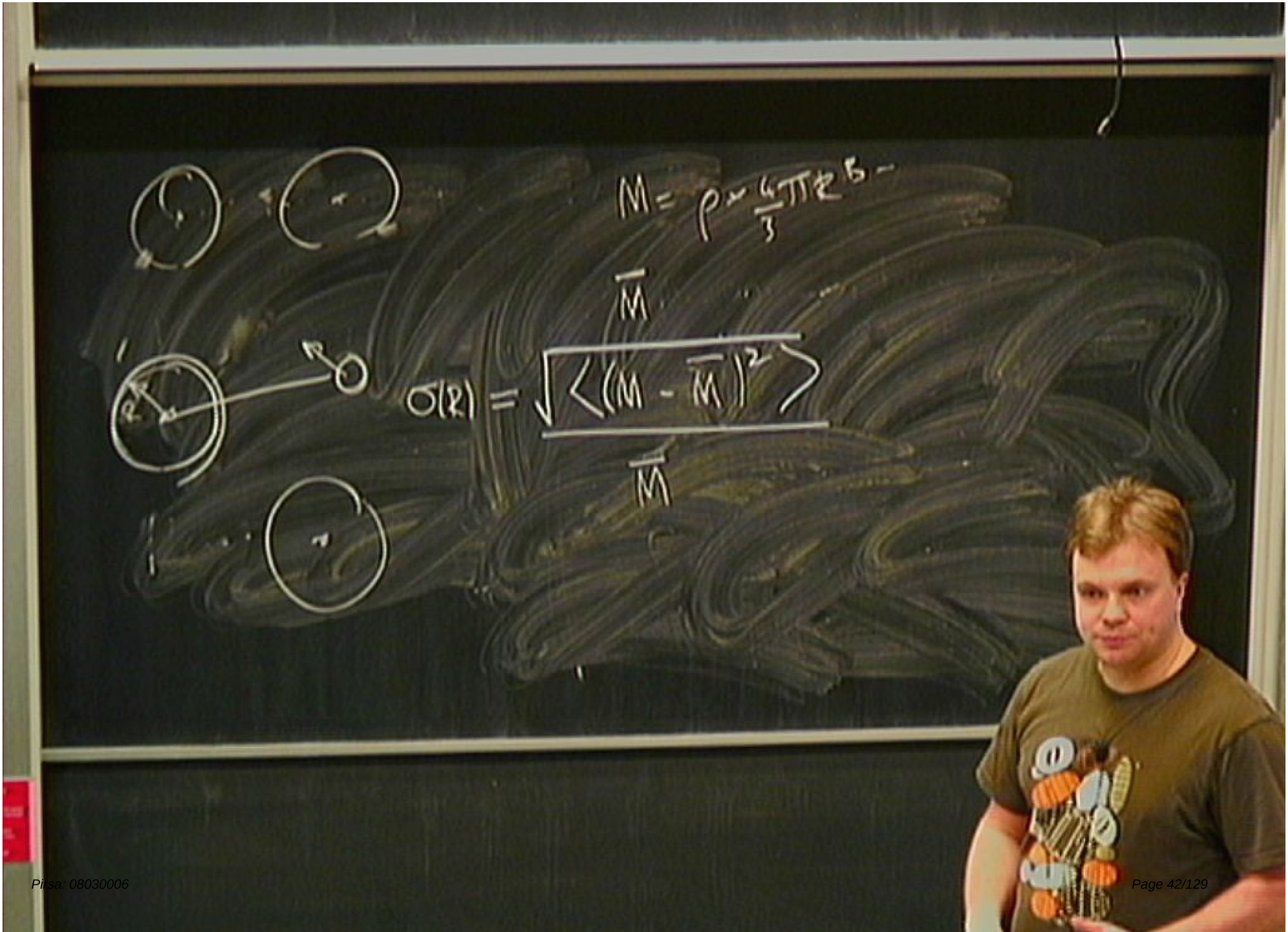
$\rho(t)$

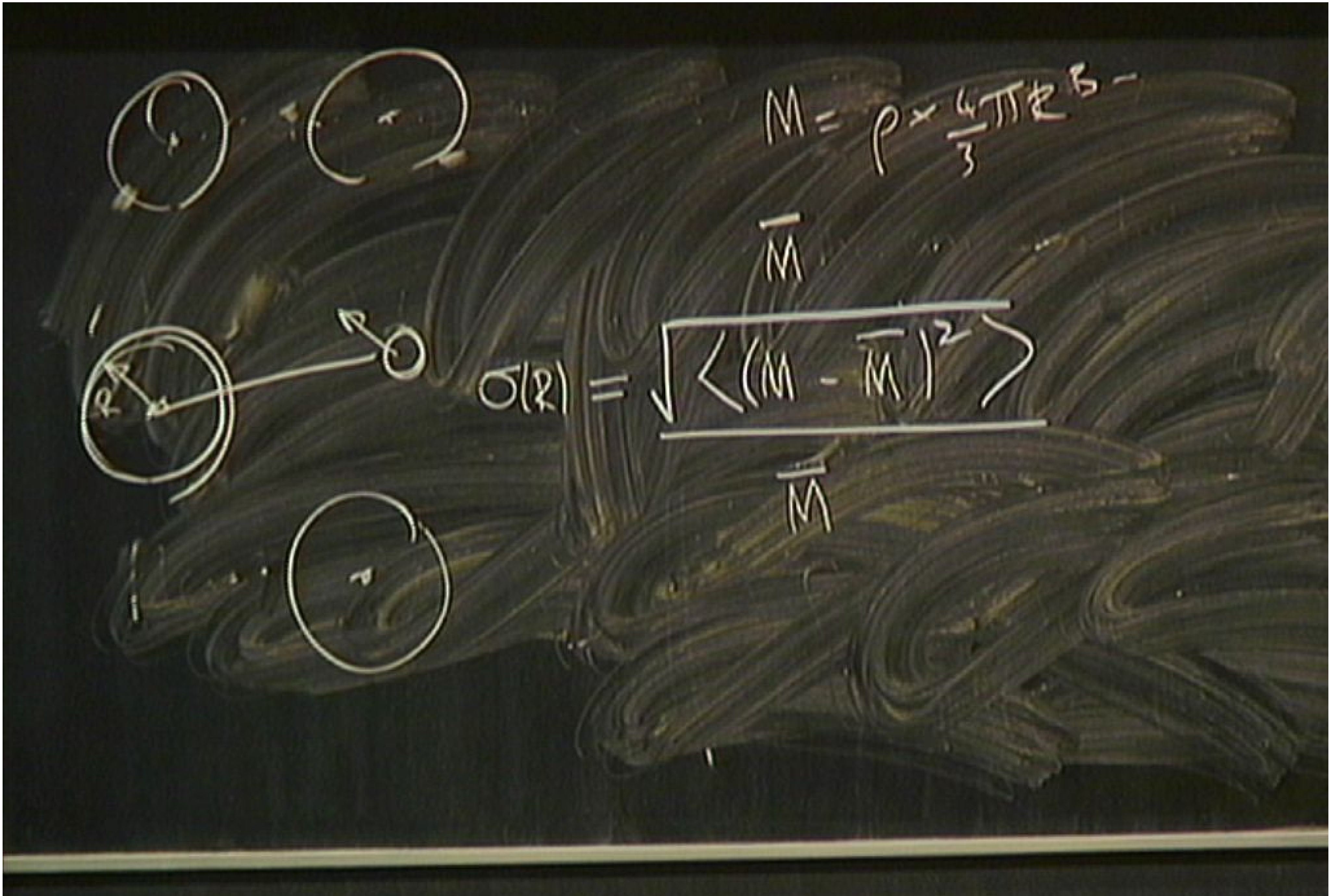
$$\bar{M} = \bar{\rho} \times \frac{4}{3}\pi R^3$$

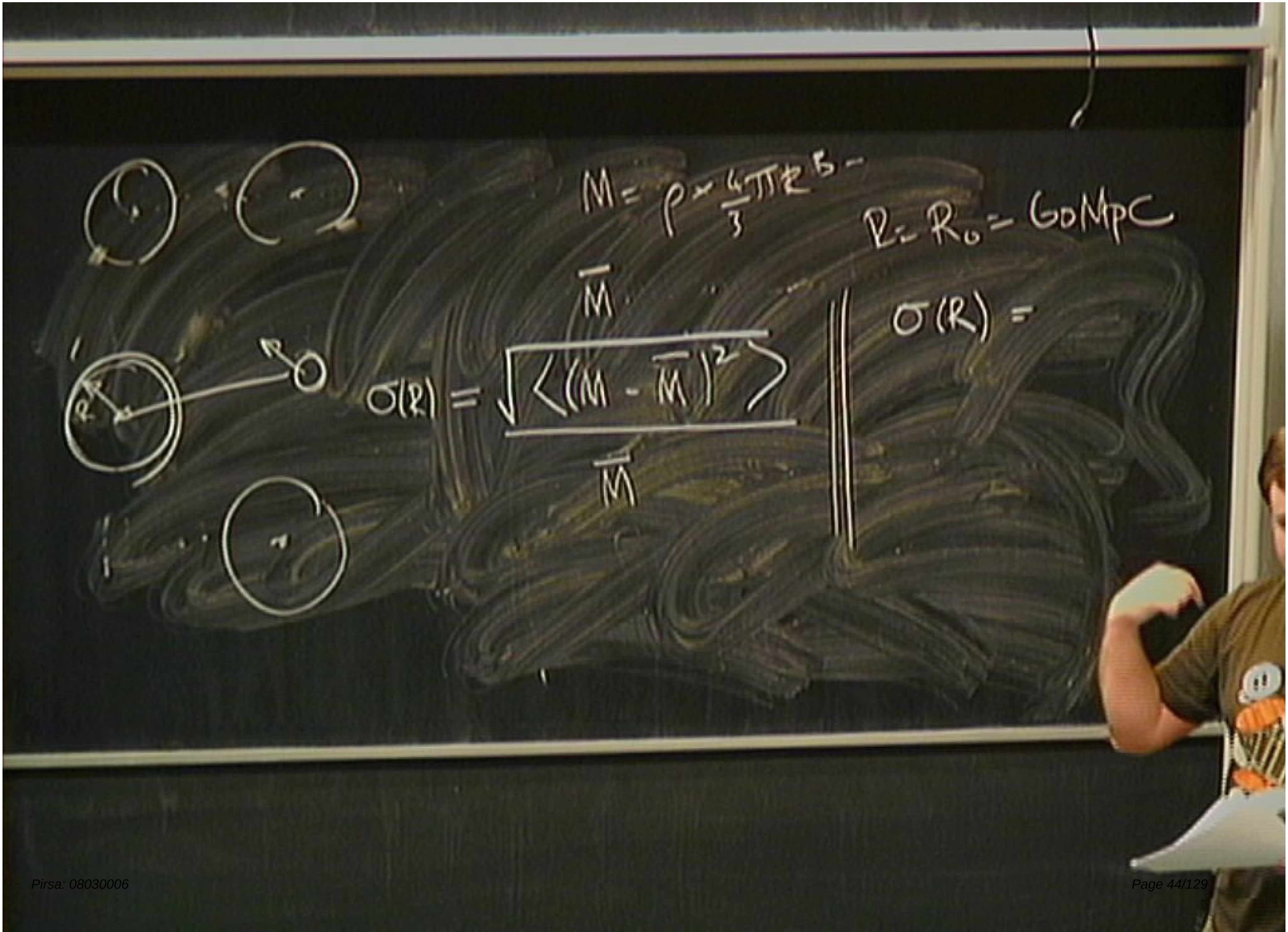

Mass fluctuations
Density fluctuations
ρ(t)
R
M̄ = ρ̄ × 4/3 π R³

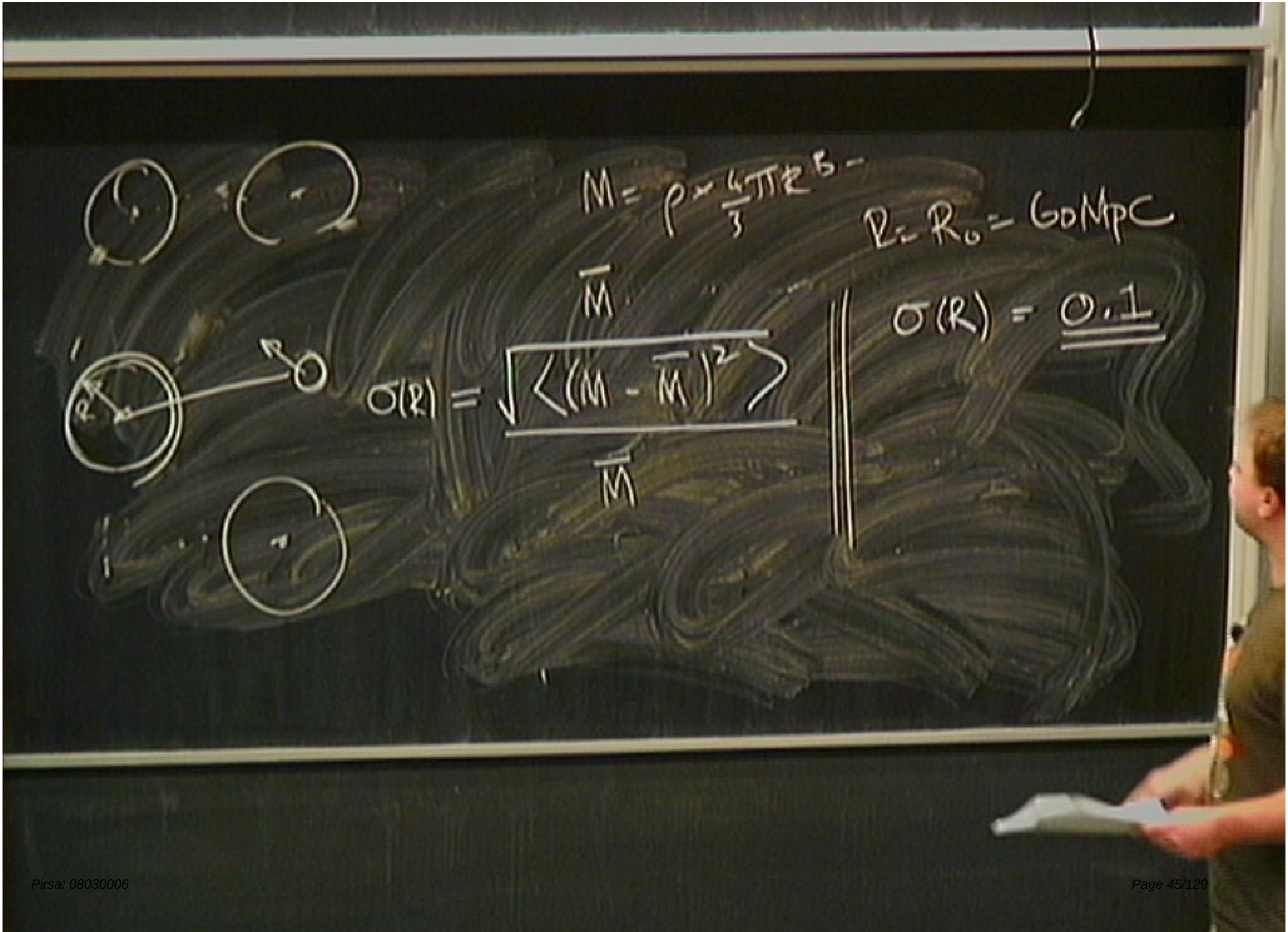

M = ρ × 4/3 πR3 -
M̄
⟨(M - M̄)²⟩

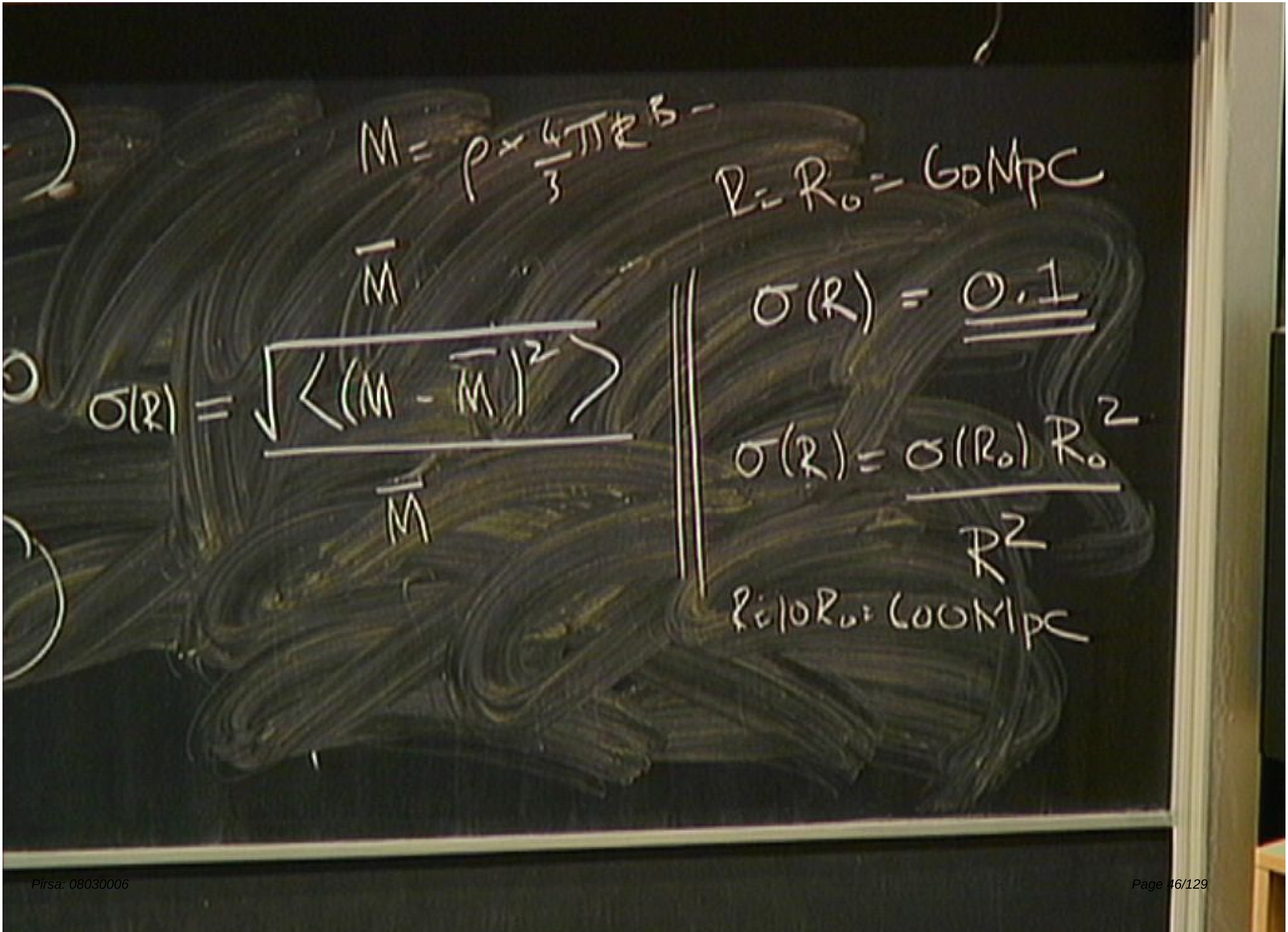

$M = \rho \times \frac{4}{3}\pi R^3 -$
$\bar{M}$
$\sqrt{\langle (M - \bar{M})^2 \rangle}$
R


M = ρ × 4/3 π R^3 ...
M̄
σ(R) = √⟨(M − M̄)²⟩ / M̄
R


M = ρ × 4/3 π R³ −
M̄
R
σ(R) = √⟨(M − M̄)²⟩ / M̄


M = ρ × 4/3 π R^3 ~
R = R0 = 60 Mpc
M̄
σ(R) = √⟨(M − M̄)^2⟩ / M̄
σ(R) =
R


$M = \rho \times \frac{4}{3}\pi R^3$
$R = R_0 = 60\,\text{MpC}$
$\bar{M}$
$\sigma(R) = 0.1$
$\sigma(R) = \frac{\sqrt{\langle (M - \bar{M})^2 \rangle}}{\bar{M}}$
R


M = ρ × 4/3 π R^3 -
R = R_0 = 60 Mpc
M̄
σ(R) = √⟨(M - M̄)^2⟩ / M̄
σ(R) = 0.1
σ(R) = σ(R_0) R_0^2 / R^2
R = 10 R_0 = 600 Mpc

$M = \rho \times \frac{4}{3}\pi R^3$

$R = R_0 = 60\,\mathrm{Mpc}$

$\bar{M}$

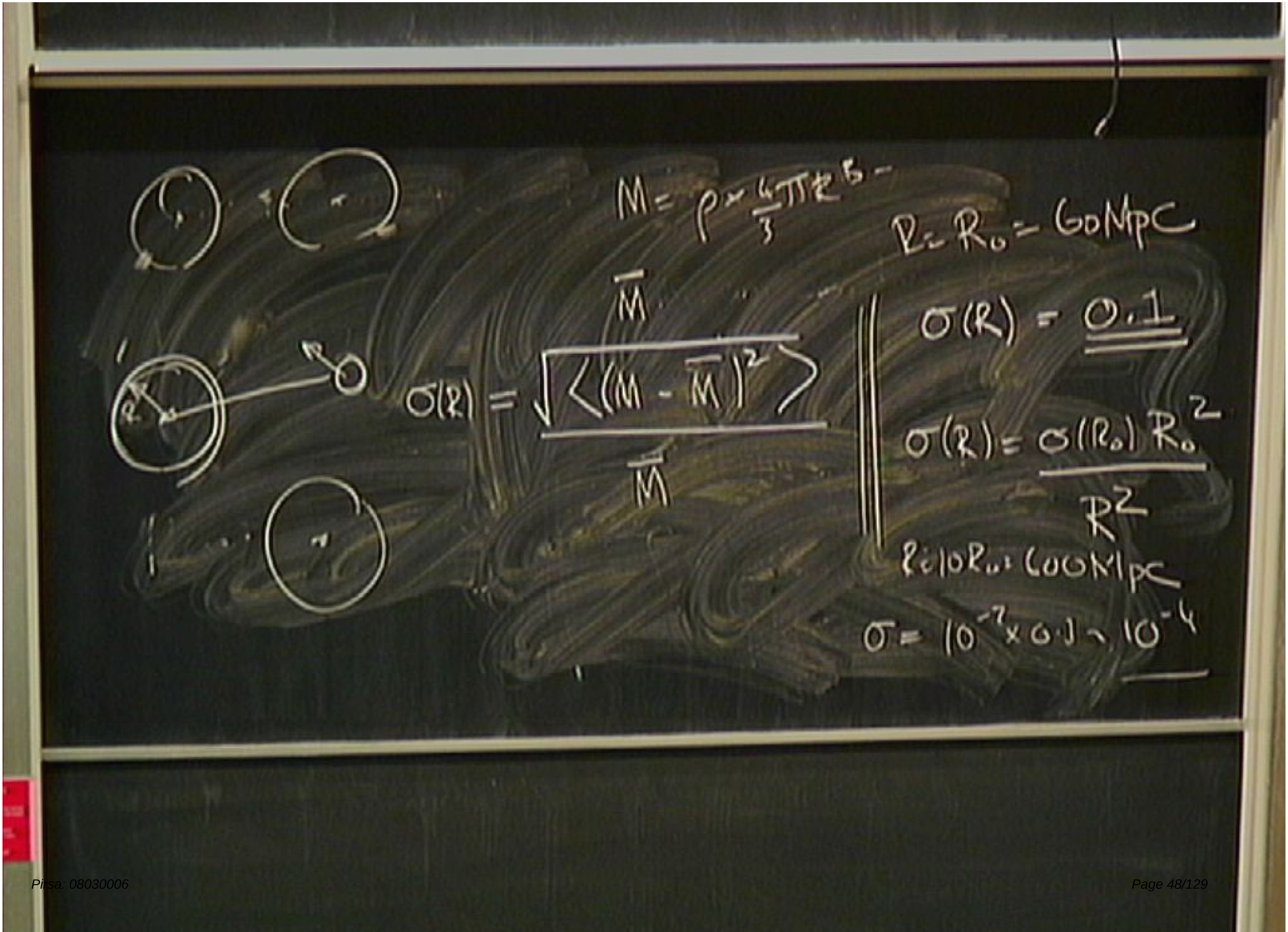
$$\sigma(R) = \frac{\sqrt{\langle (M - \bar{M})^2 \rangle}}{\bar{M}}$$

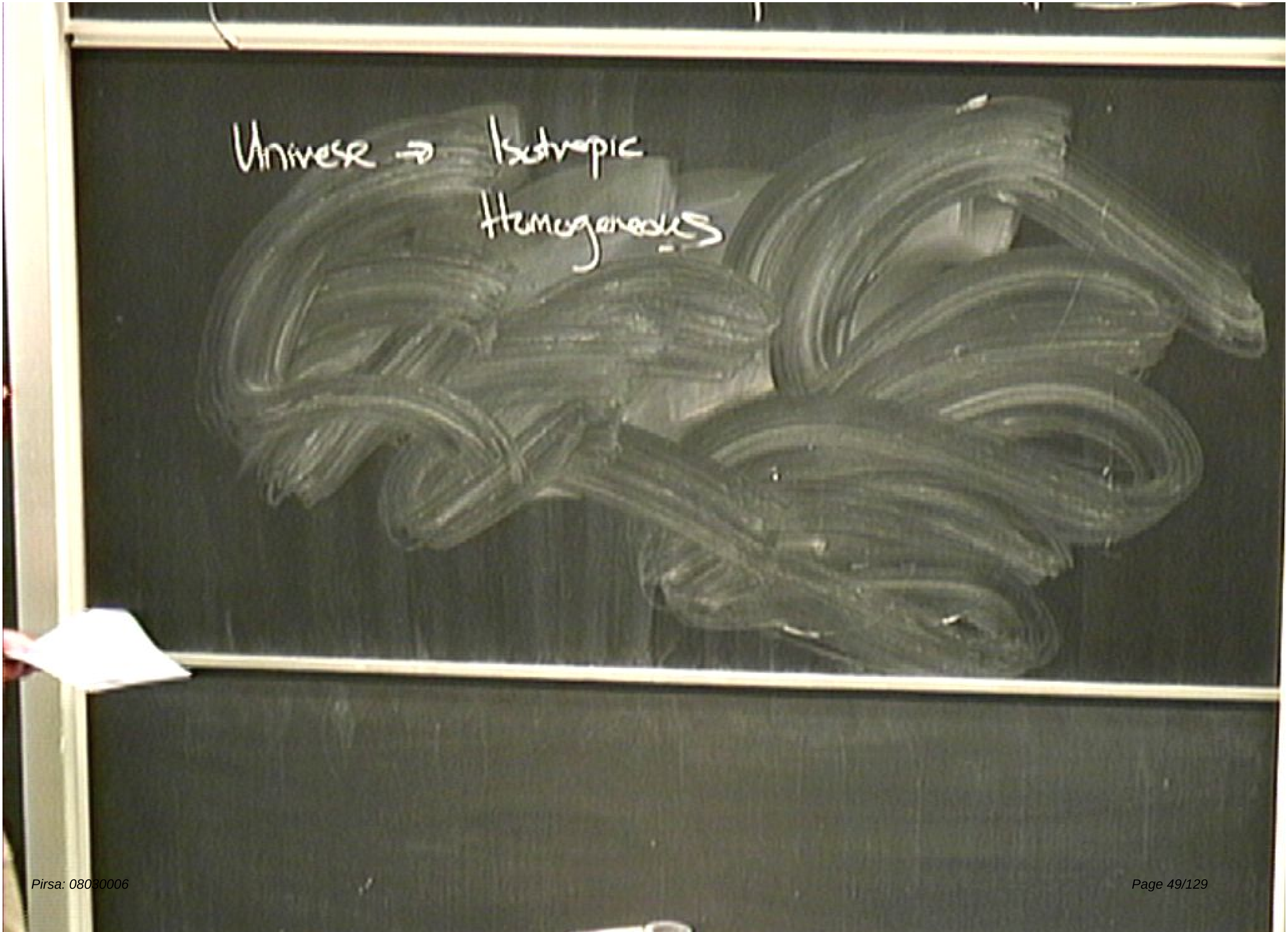
$\sigma(R) = \underline{\underline{0.1}}$

$$\sigma(R) = \frac{\sigma(R_0)\, R_0^2}{R^2}$$

$R = 10 R_0 = 600\,\mathrm{Mpc}$

$\sigma = 10^{-2} \times 0.1 \sim 10^{-4}$


M = ρ × 4/3 π R³ -
R ~ R₀ ~ 60 MpC
σ(R) = 0.1
σ(R) = √⟨(M − M̄)²⟩ / M̄
σ(R) = σ(R₀) R₀² / R²
R ~ 10 R₀ ~ 600 Mpc
σ = 10⁻² × 0.1 ~ 10⁻⁴

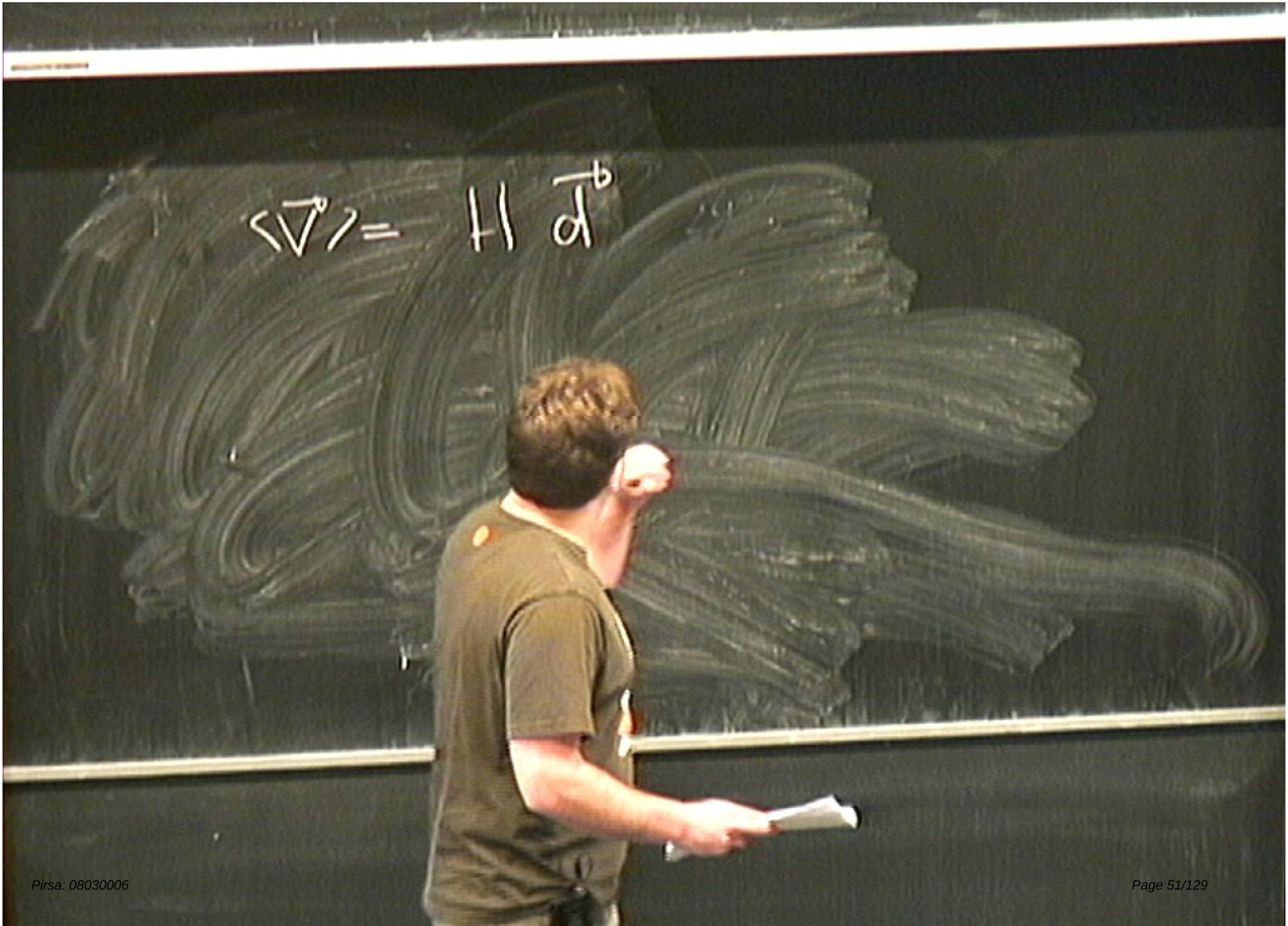

Universe ⇒ Isotropic
Homogeneous

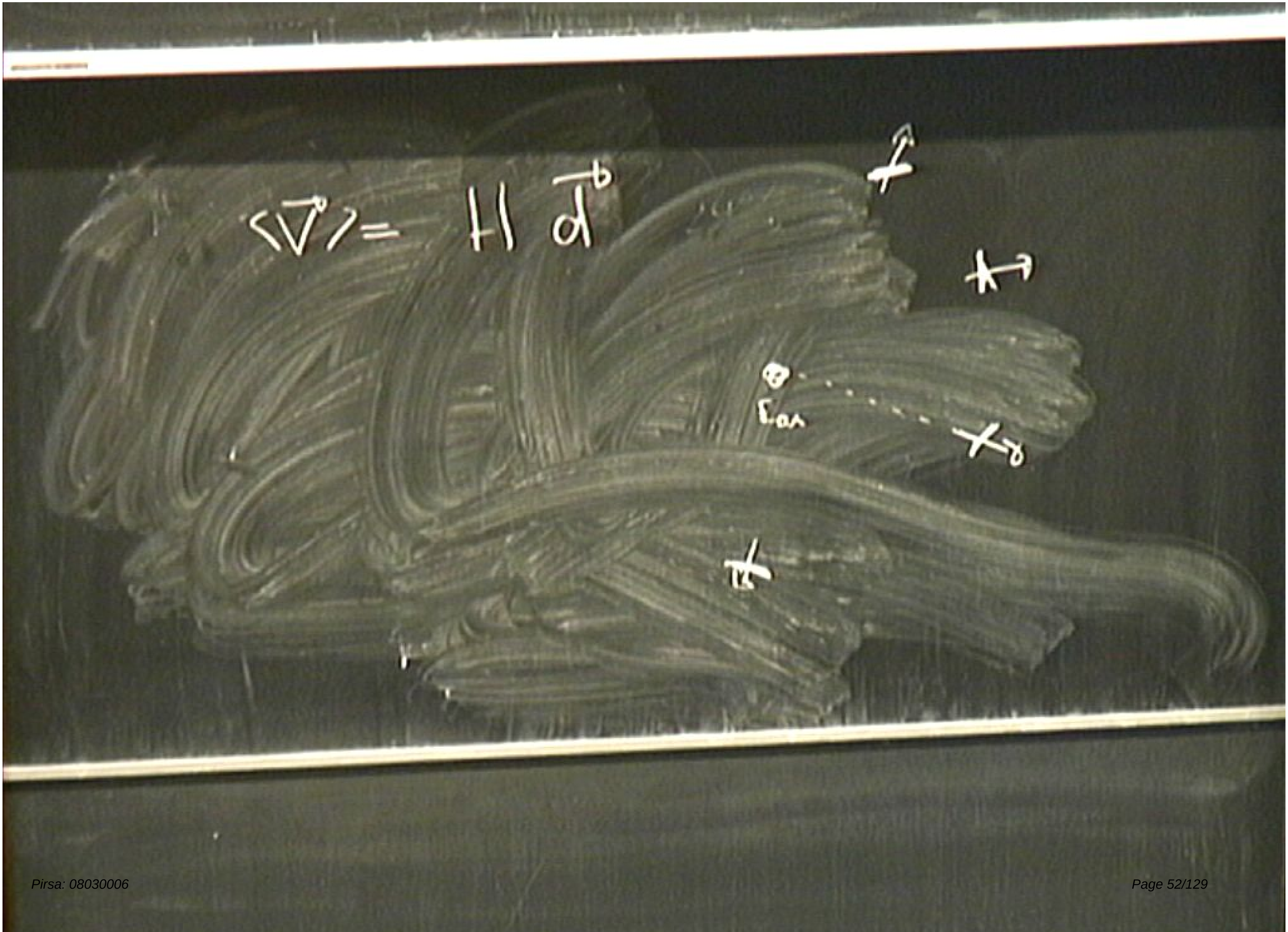
Universe → Isotropic

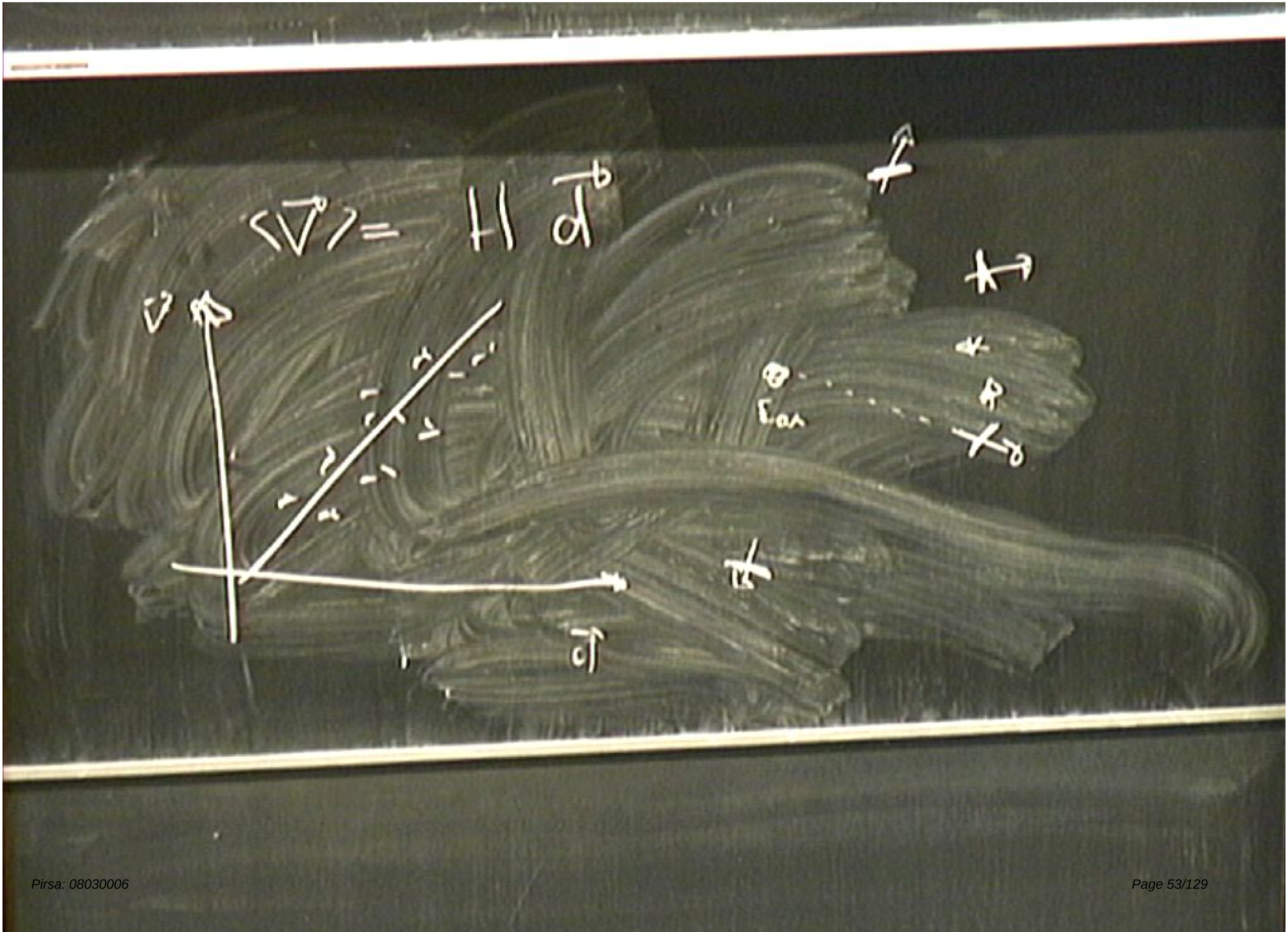
Homogeneous

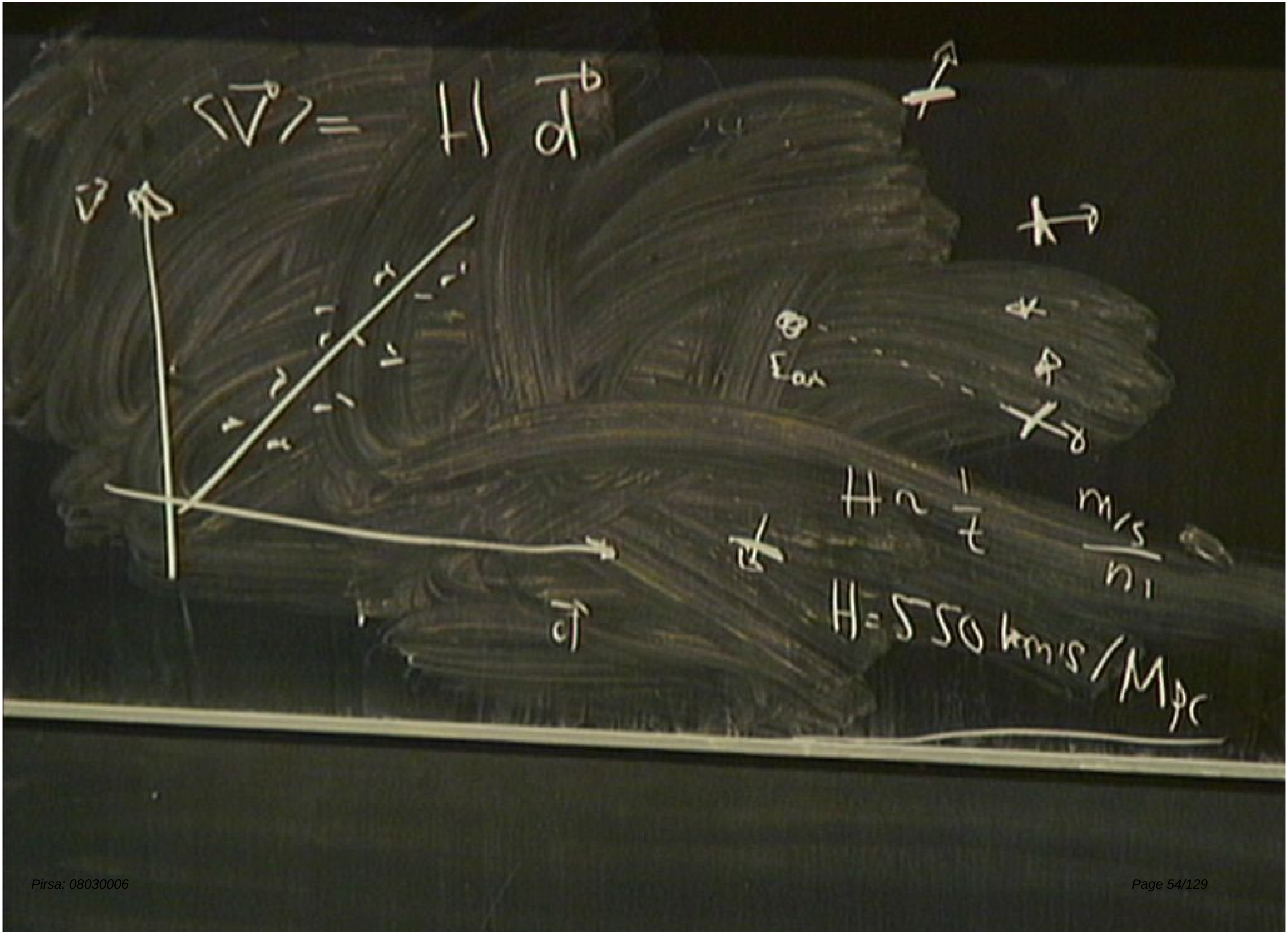
Expanding

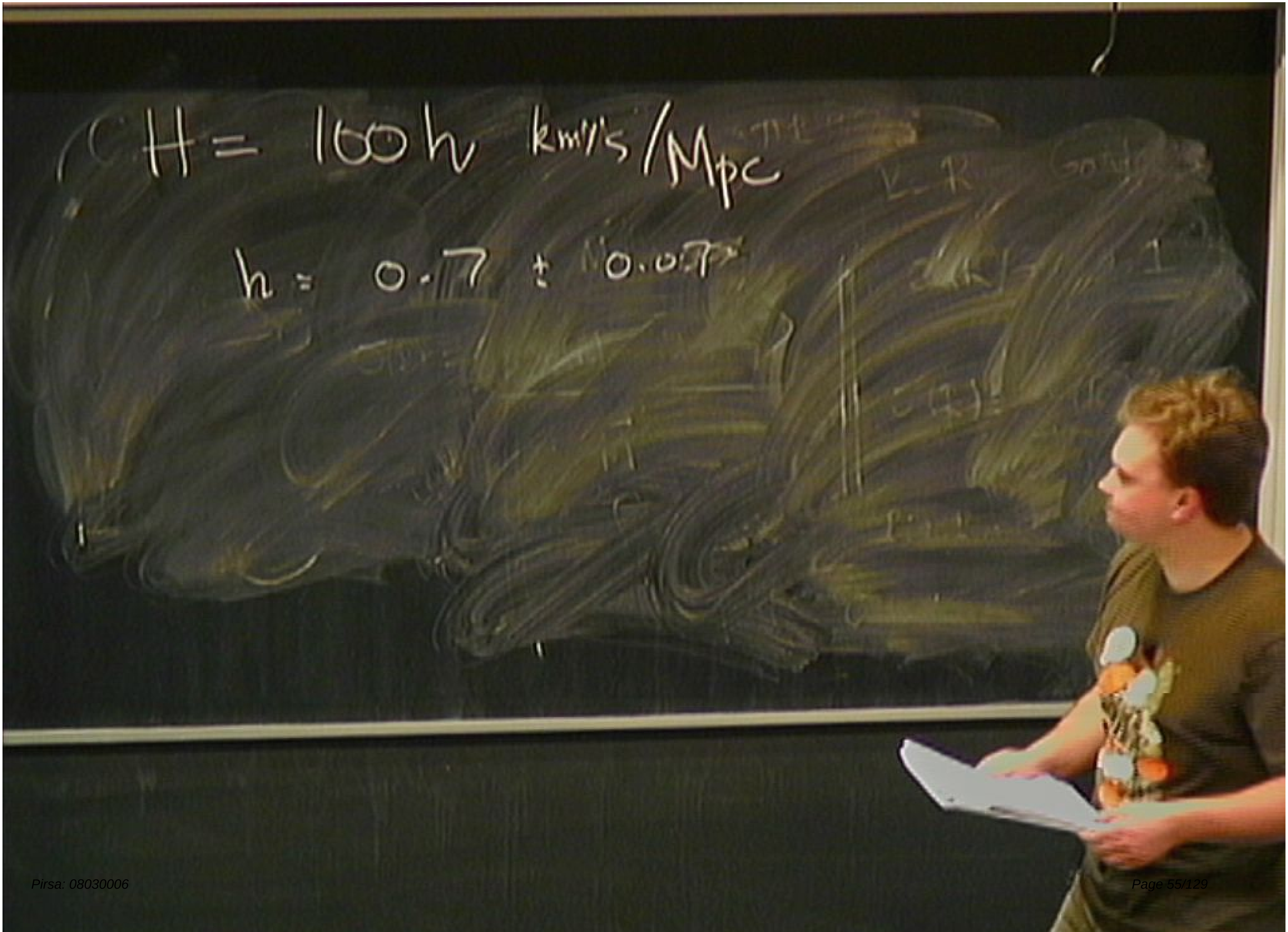
Edwin Hubble 1920-30

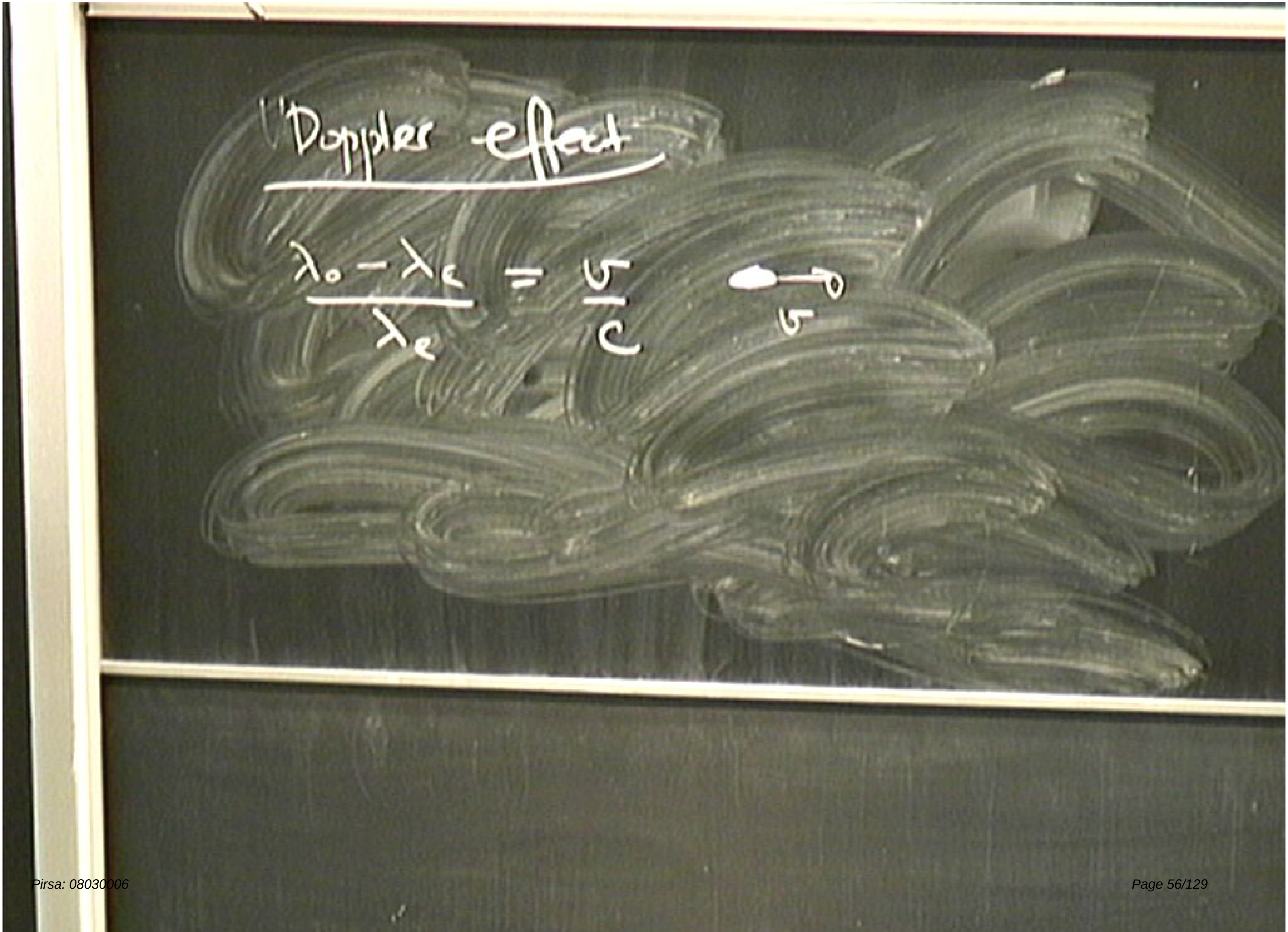

⟨∇⟩= H d

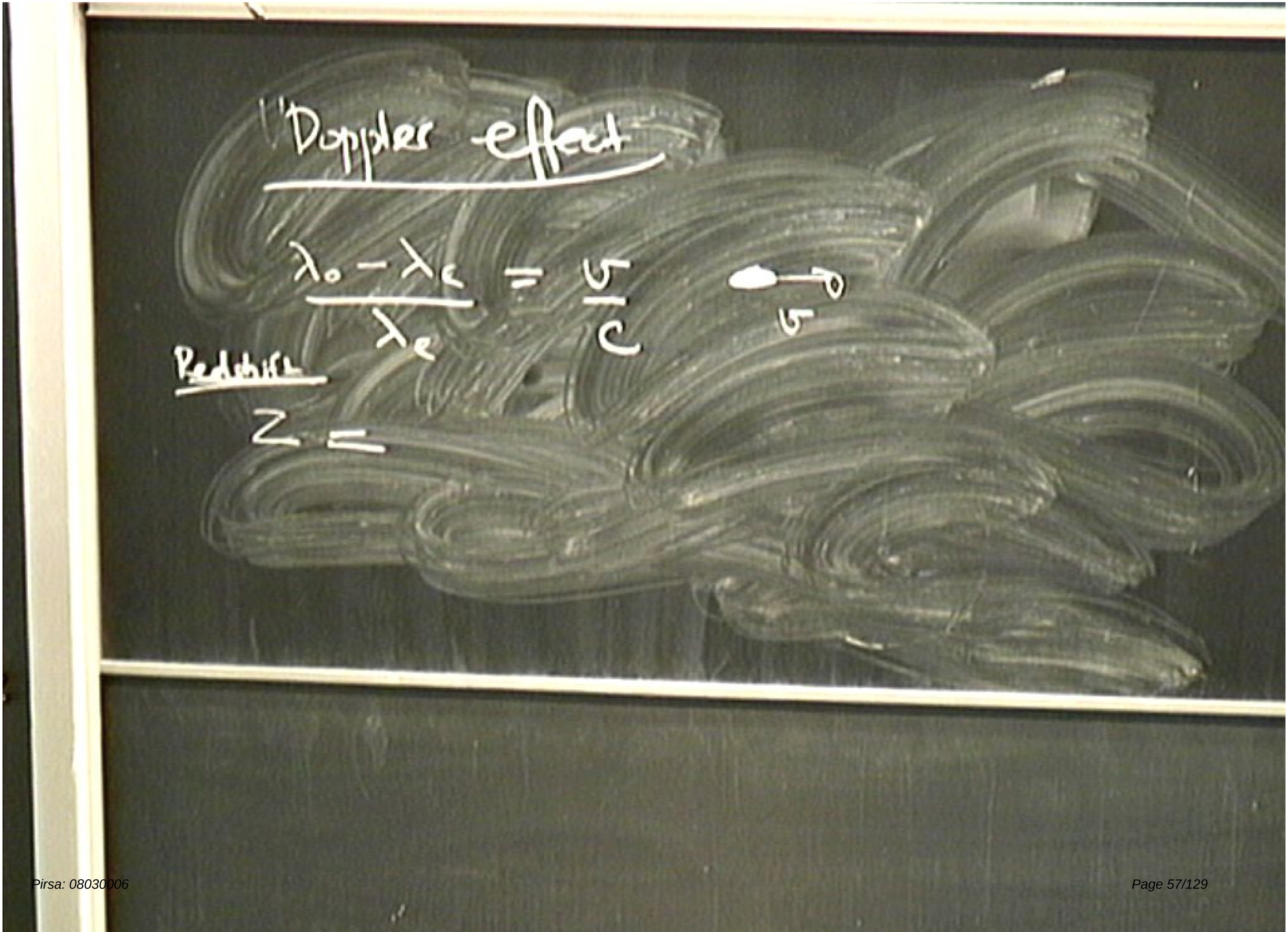

$\langle \vec{\nabla} \rangle = H \vec{d}$


⟨v⟩ = H d


<V> = H d
H ~ 1/t
m/s / m
H = 550 km/s/Mpc


H = 100 h km/s / Mpc
h = 0.7 ±


"Doppler effect"
$\frac{\lambda_o - \lambda_e}{\lambda_e} = \frac{v}{c}$
v


"Doppler effect
λo − λe / λe = v/c
v
Redshift
z =

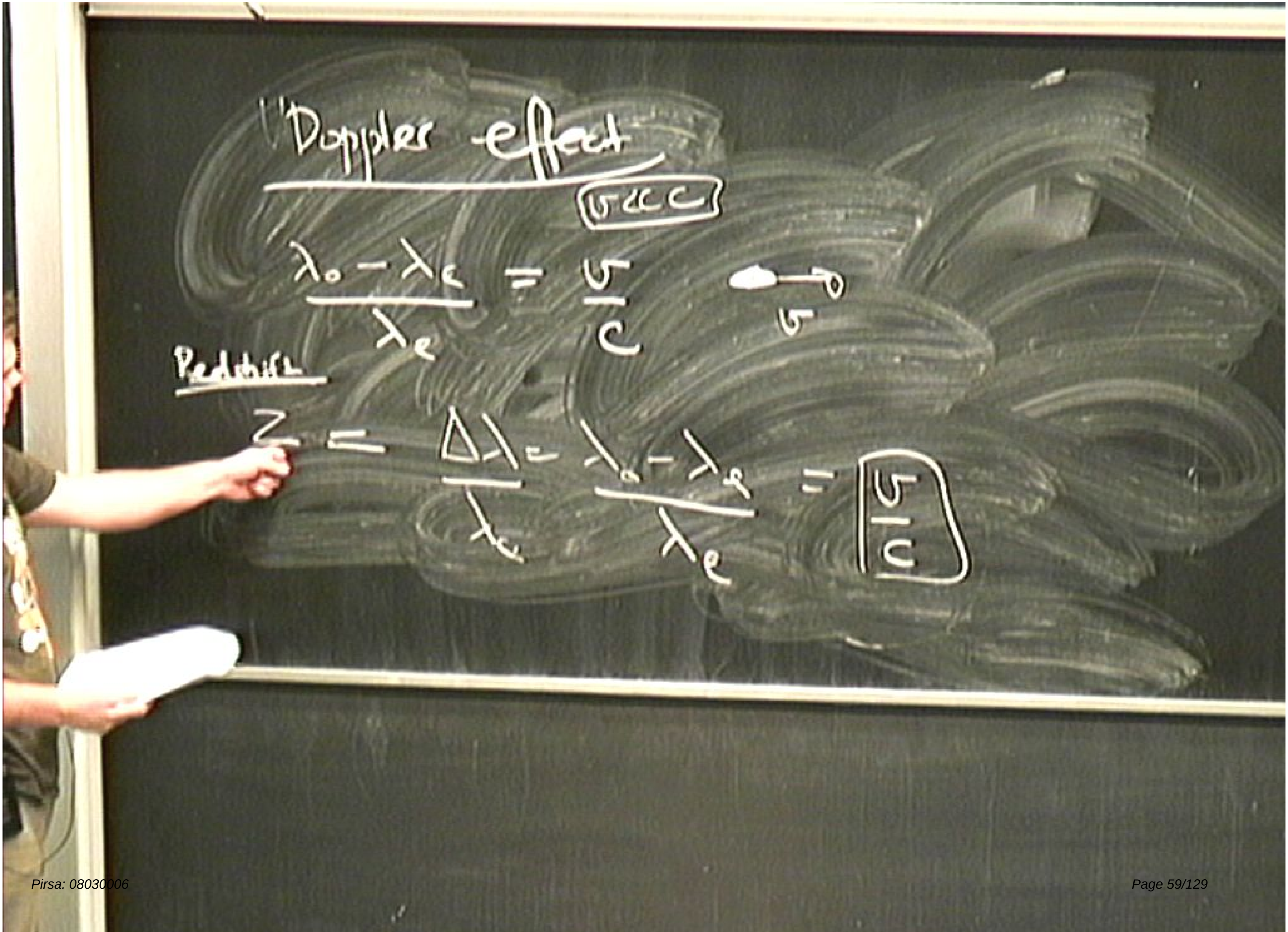
"Doppler effect"

$v \ll c$

$$\frac{\lambda_o - \lambda_e}{\lambda_e} = \frac{v}{c}$$

Redshift

$$z = \frac{\Delta\lambda}{\lambda_e} = \frac{\lambda_o - \lambda_e}{\lambda_e}$$


"Doppler effect"
v<<c
$\frac{\lambda_o - \lambda_e}{\lambda_e} = \frac{v}{c}$
v
Redshift
$z = \frac{\Delta\lambda}{\lambda_e} = \frac{\lambda_o - \lambda_e}{\lambda_e} = \frac{v}{c}$

"Doppler effect"

$v \ll c$

$$\frac{\lambda_o - \lambda_e}{\lambda_e} = \frac{v}{c} \qquad \bullet \xrightarrow{v}$$

Redshift

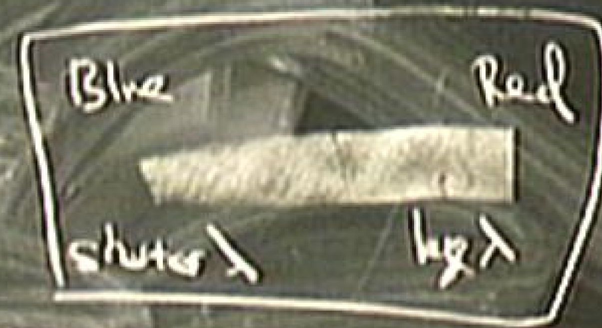
$$z = \frac{\Delta\lambda}{\lambda_e} = \frac{\lambda_o - \lambda_e}{\lambda_e} = \boxed{\frac{v}{c}}$$

$v > 0 \quad z > 0 \quad \lambda_o > \lambda_e$

Doppler effect

$v \ll c$

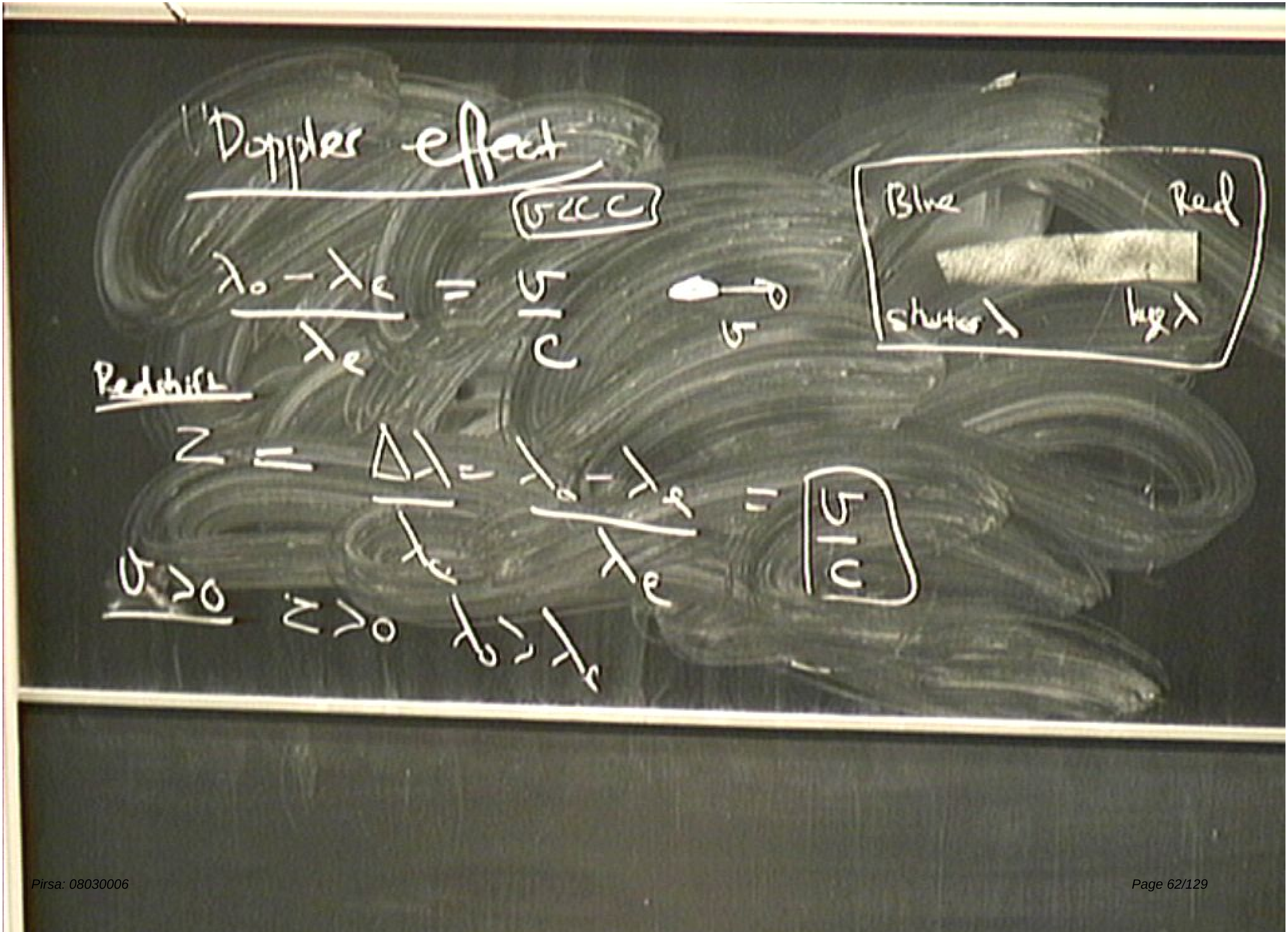
$$\frac{\lambda_o - \lambda_e}{\lambda_e} = \frac{v}{c}$$

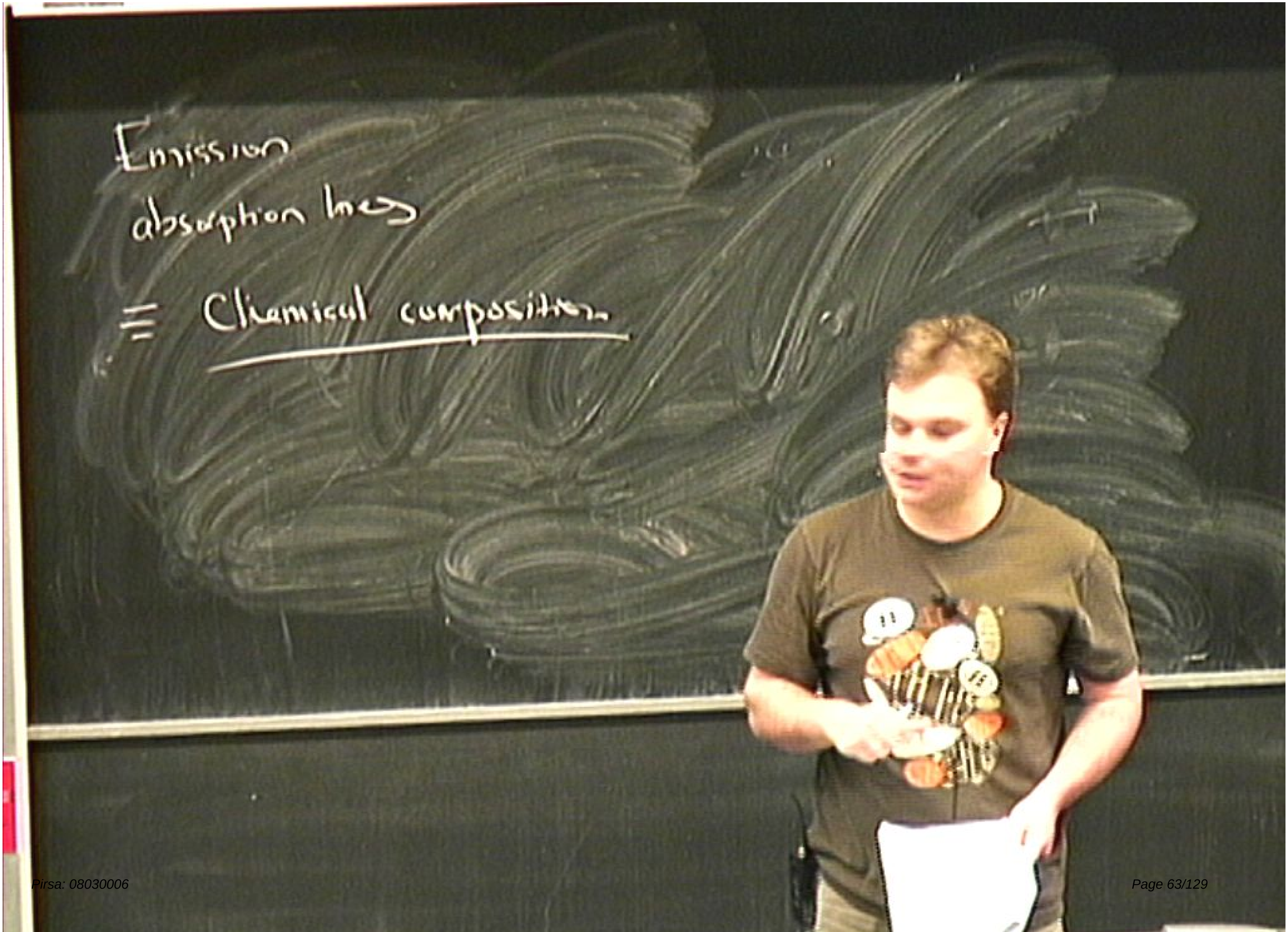


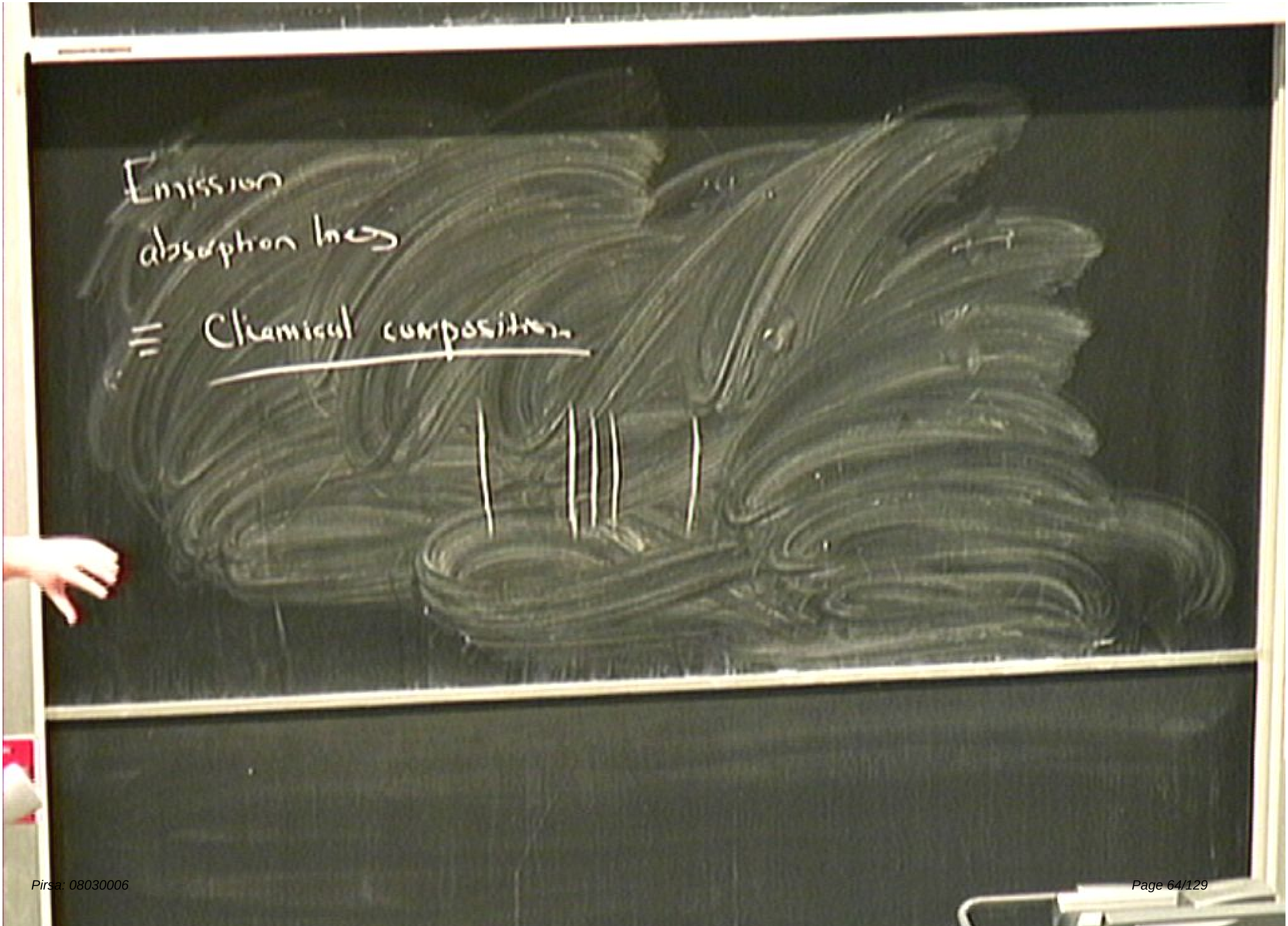
Redshift

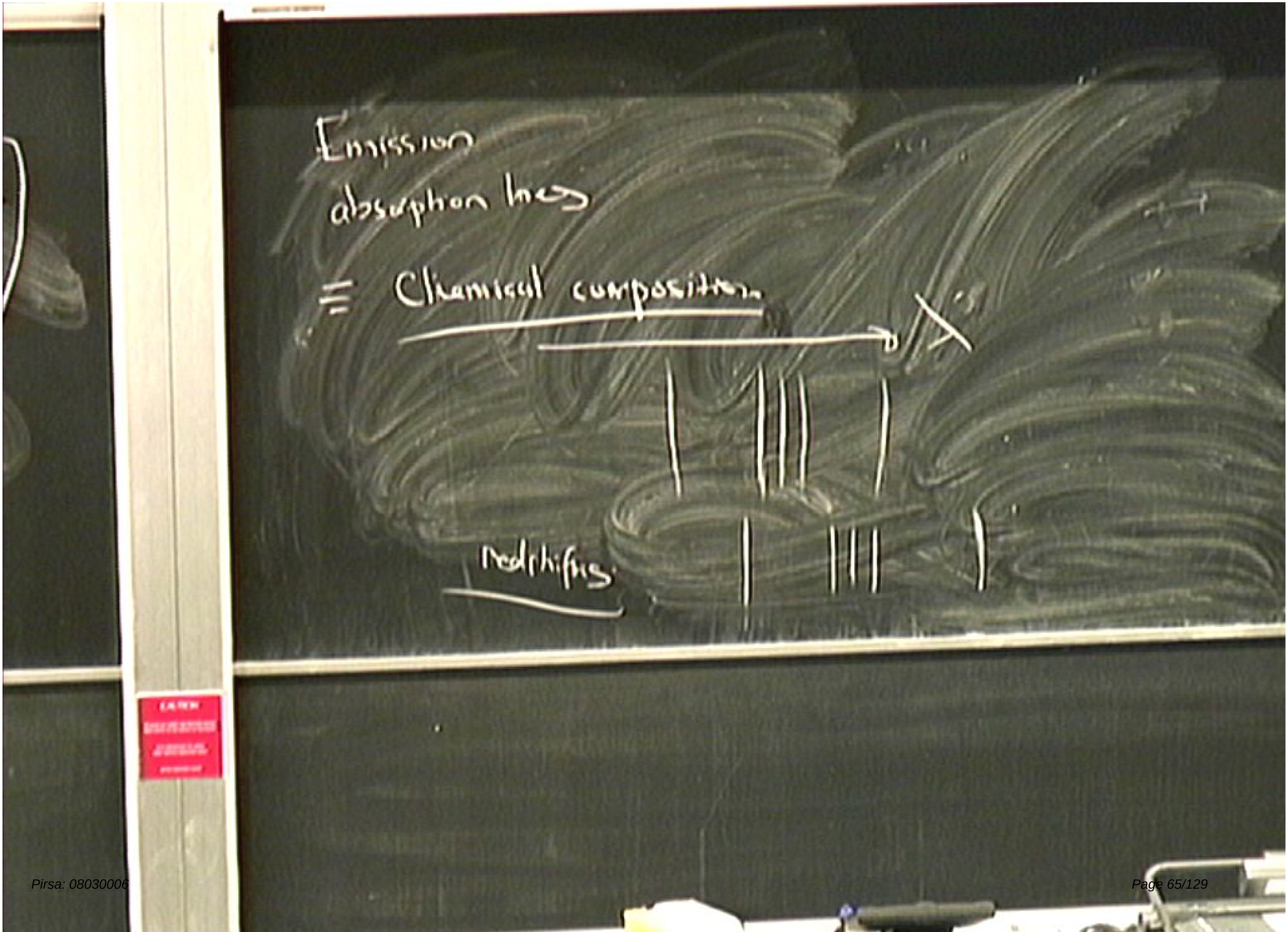
$$z = \frac{\Delta\lambda}{\lambda_e} = \frac{\lambda_o - \lambda_e}{\lambda_e} = \frac{v}{c}$$

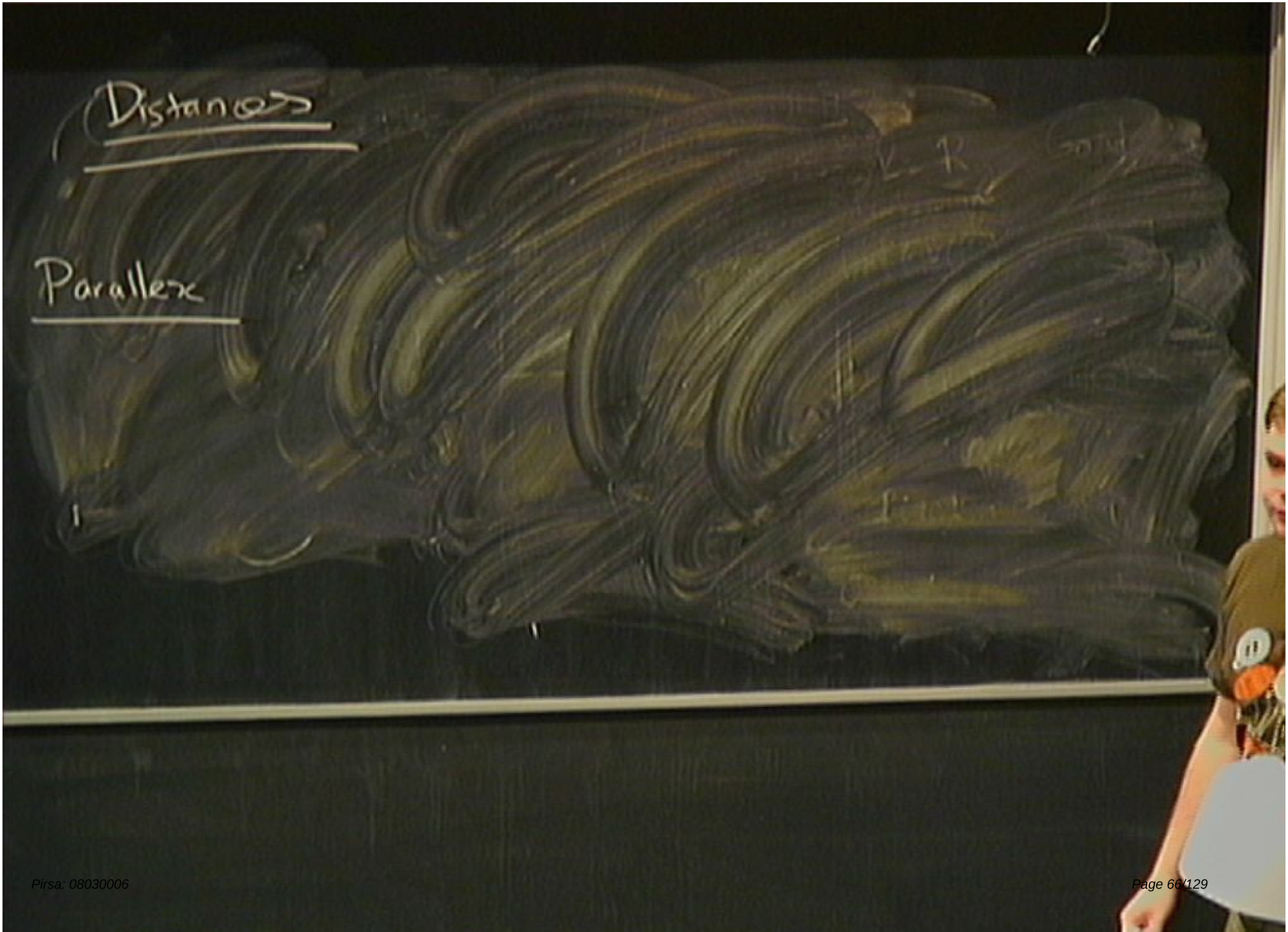
$v > 0$ $\quad z > 0 \quad \lambda_o > \lambda_e$

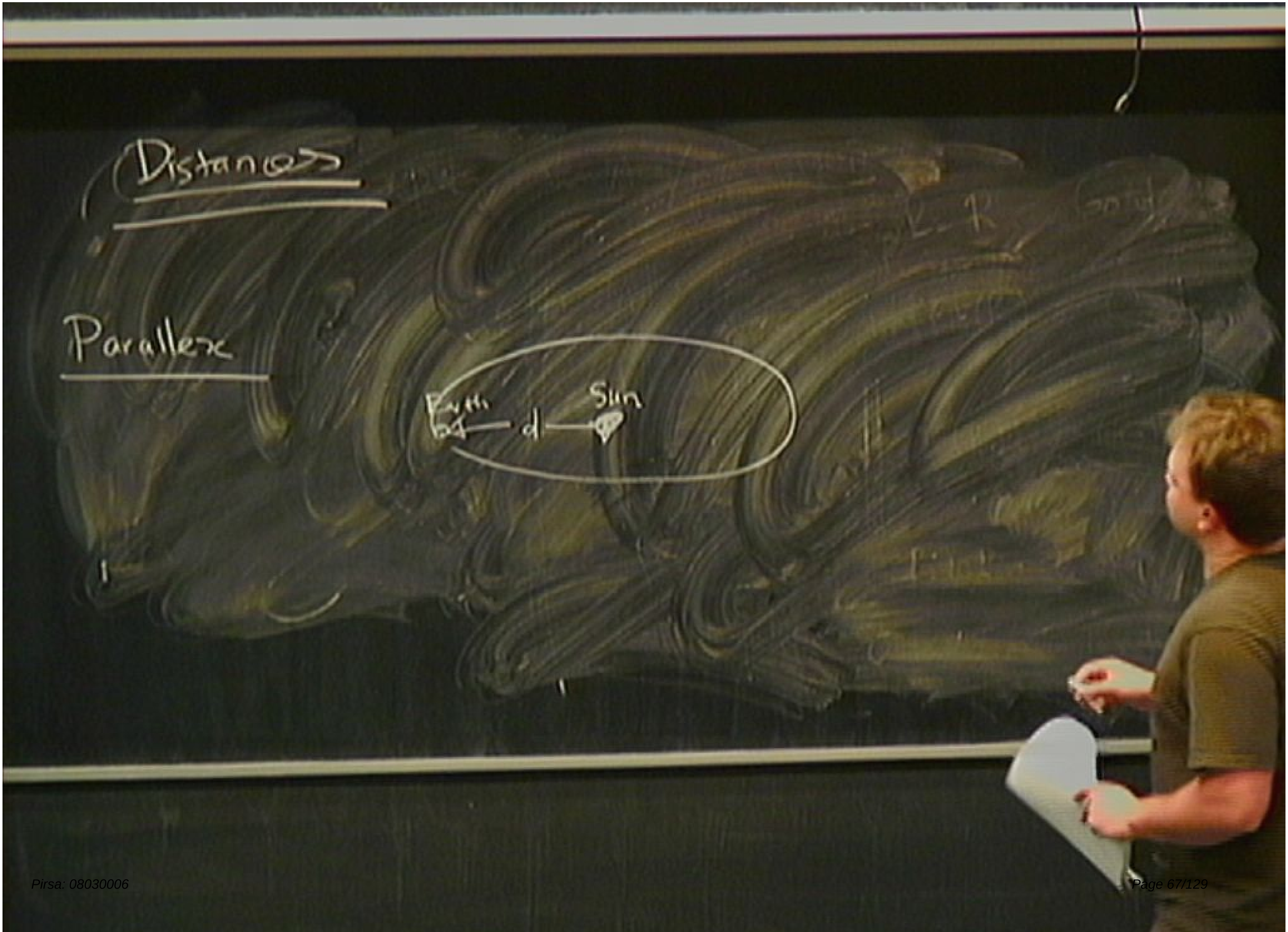

Doppler effect
v << c
$\frac{\lambda_o - \lambda_e}{\lambda_e} = \frac{v}{c}$
v
Blue
Red
shorter λ
longer λ
Redshift
$z = \frac{\Delta\lambda}{\lambda_e} = \frac{\lambda_o - \lambda_e}{\lambda_e} = \frac{v}{c}$
v > 0
$z > 0$
$\lambda_o > \lambda_e$

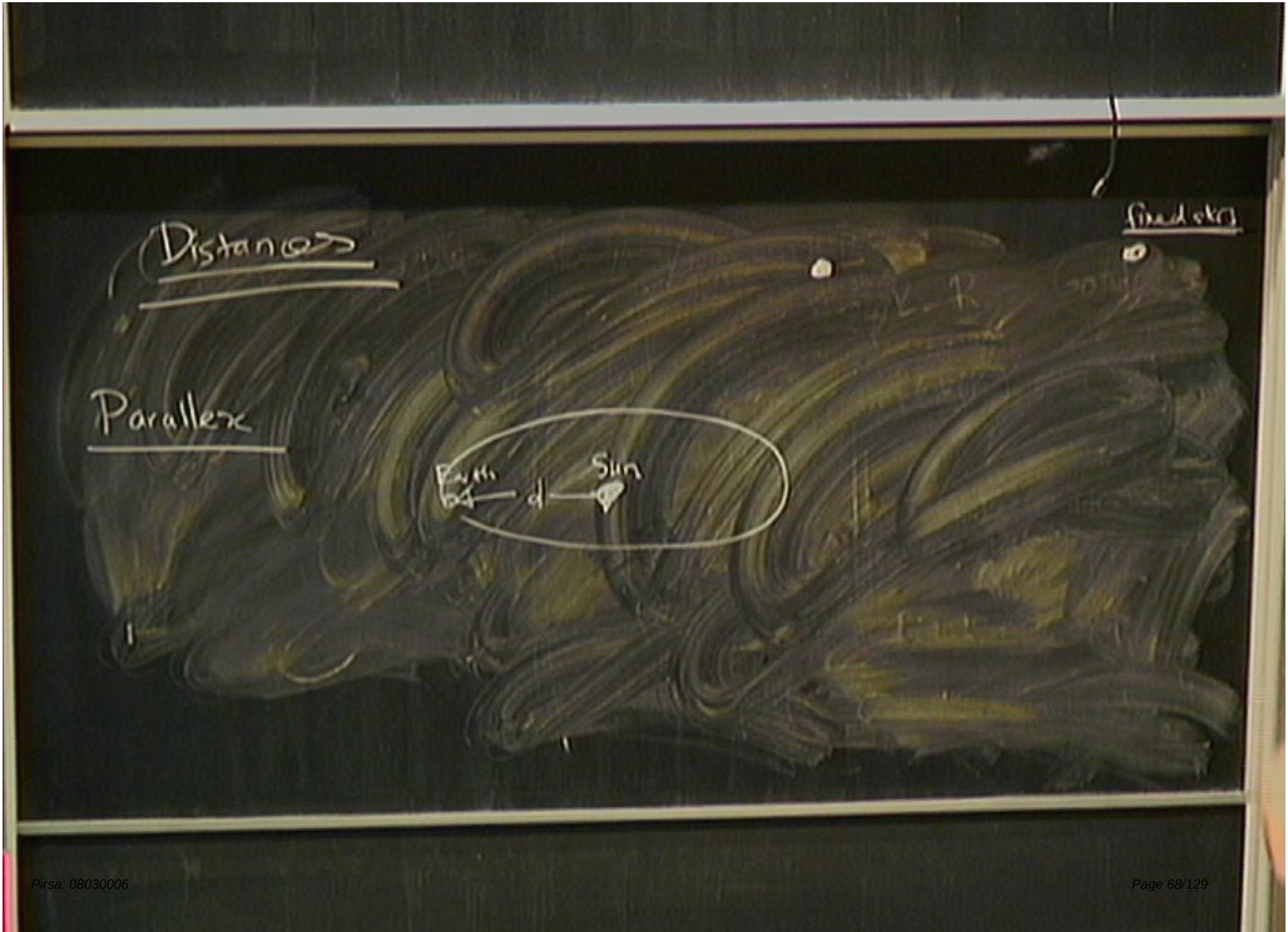

Emission
absorption lines
≡ Chemical composition

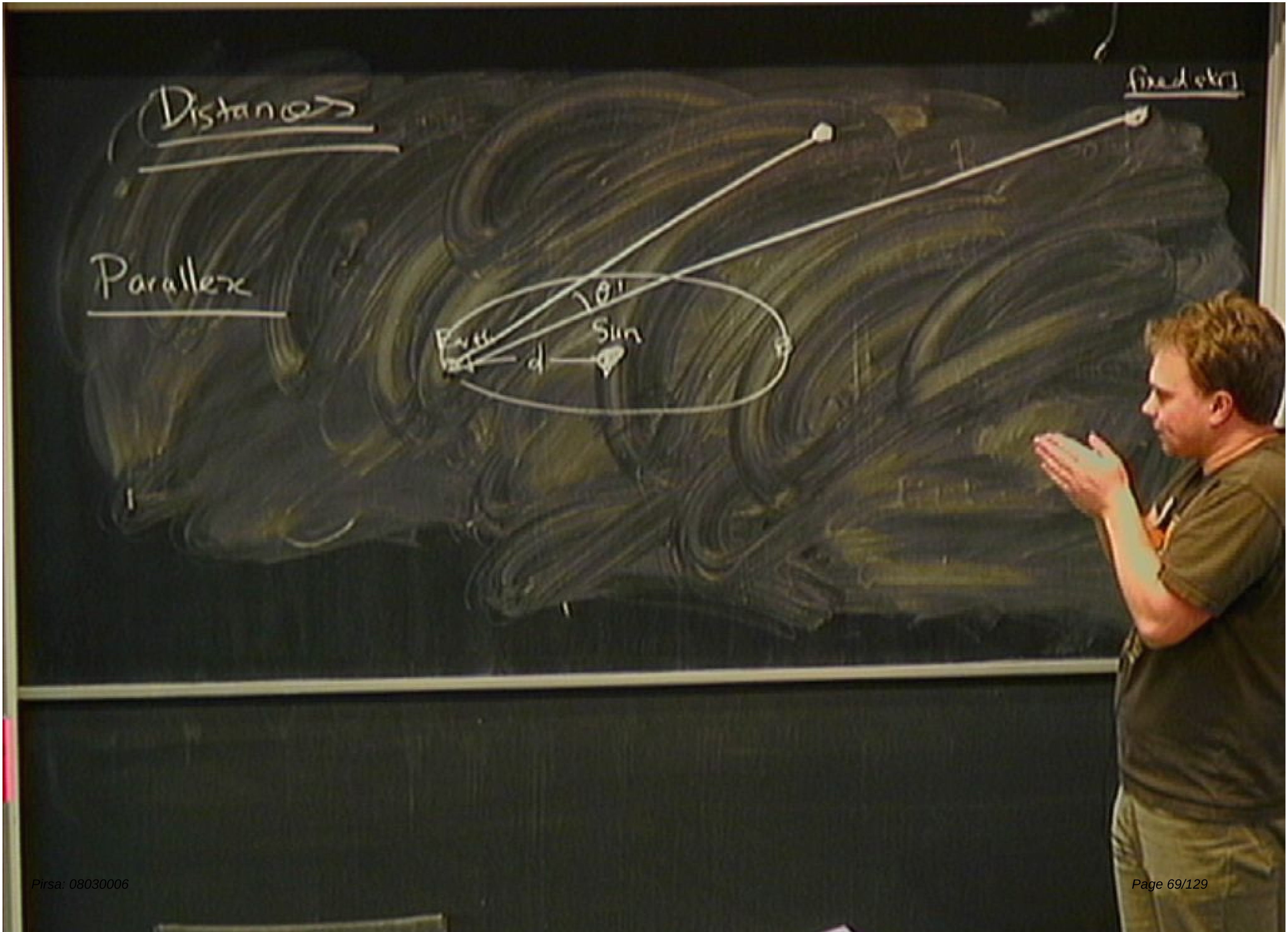

Emission
absorption lines
≡ Chemical composition

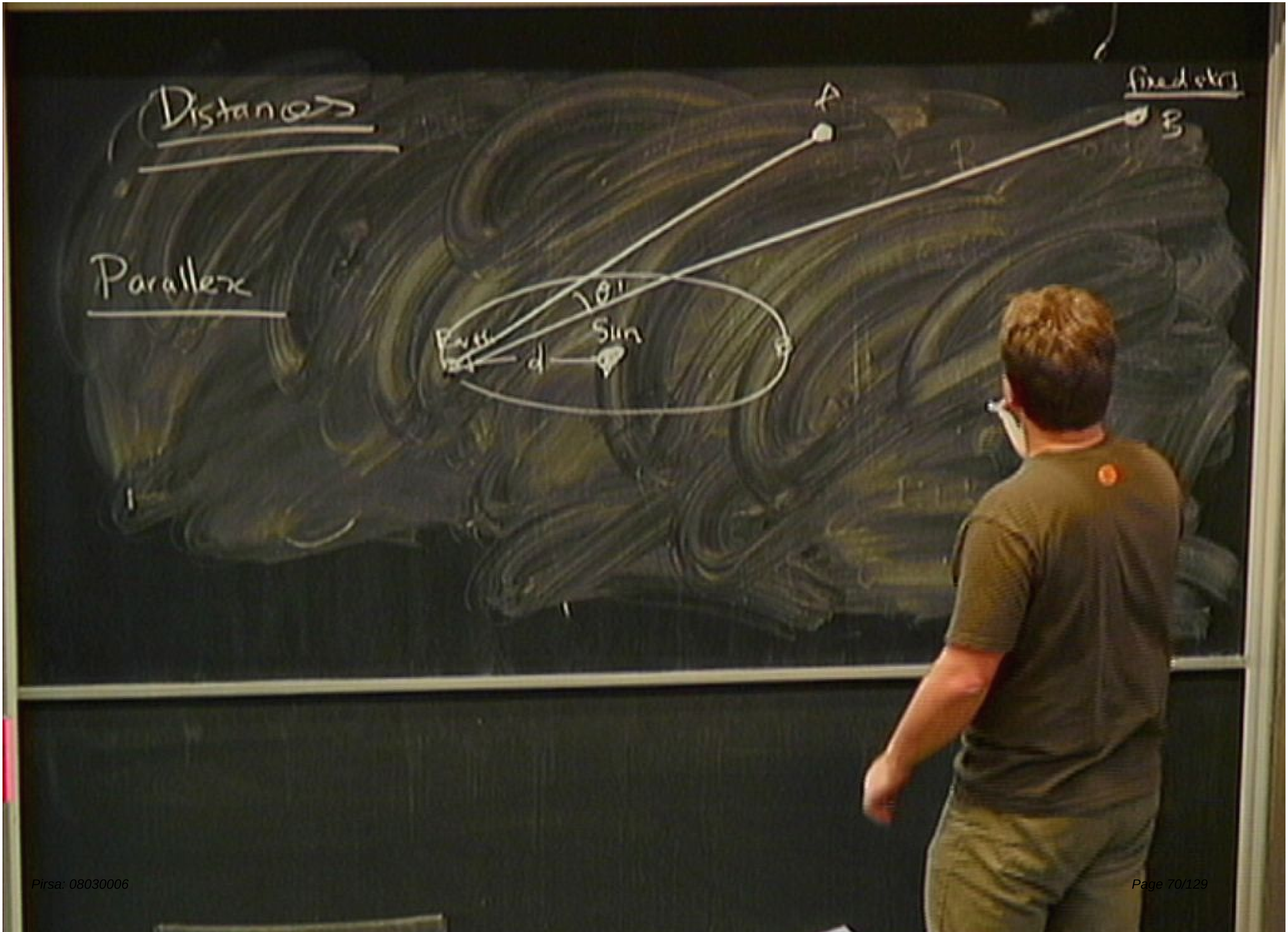

Emission
absorption lines
≡ Chemical composition
redshifts

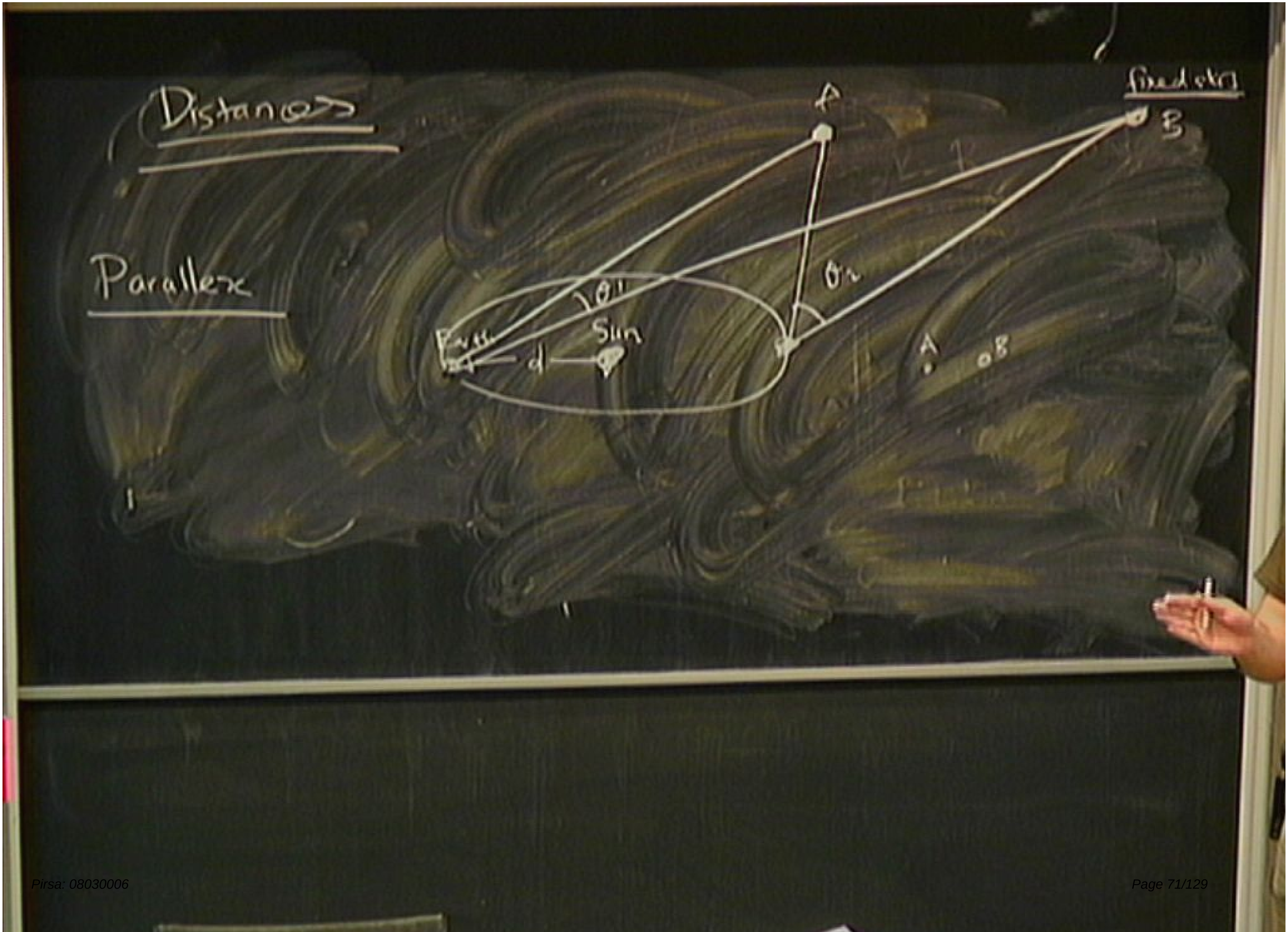

Distances
Parallex


Distances
Parallex
Earth
d
Sun

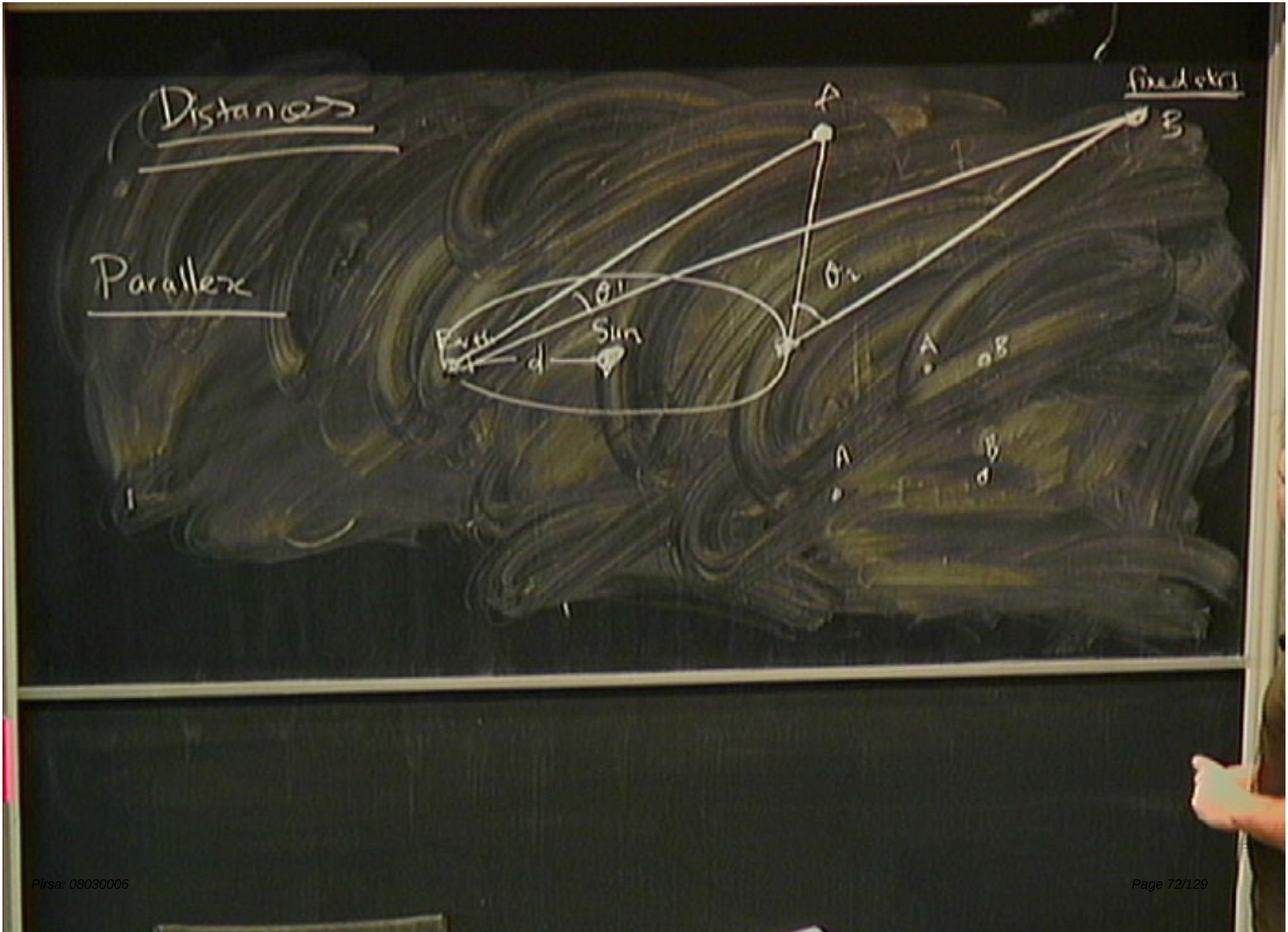

Distances
Parallex
Earth
d
Sun

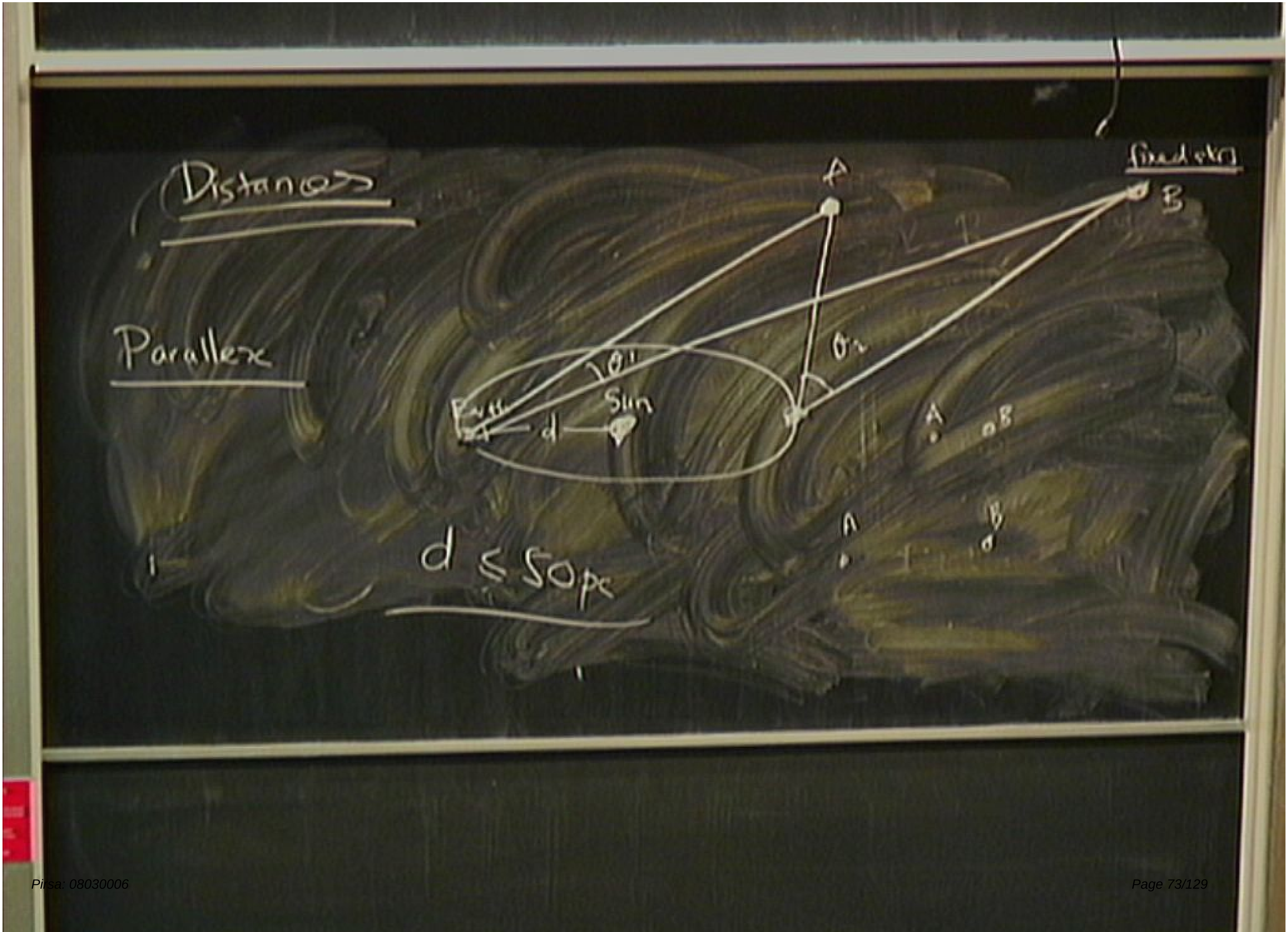

Distances
Parallax
Sun
d

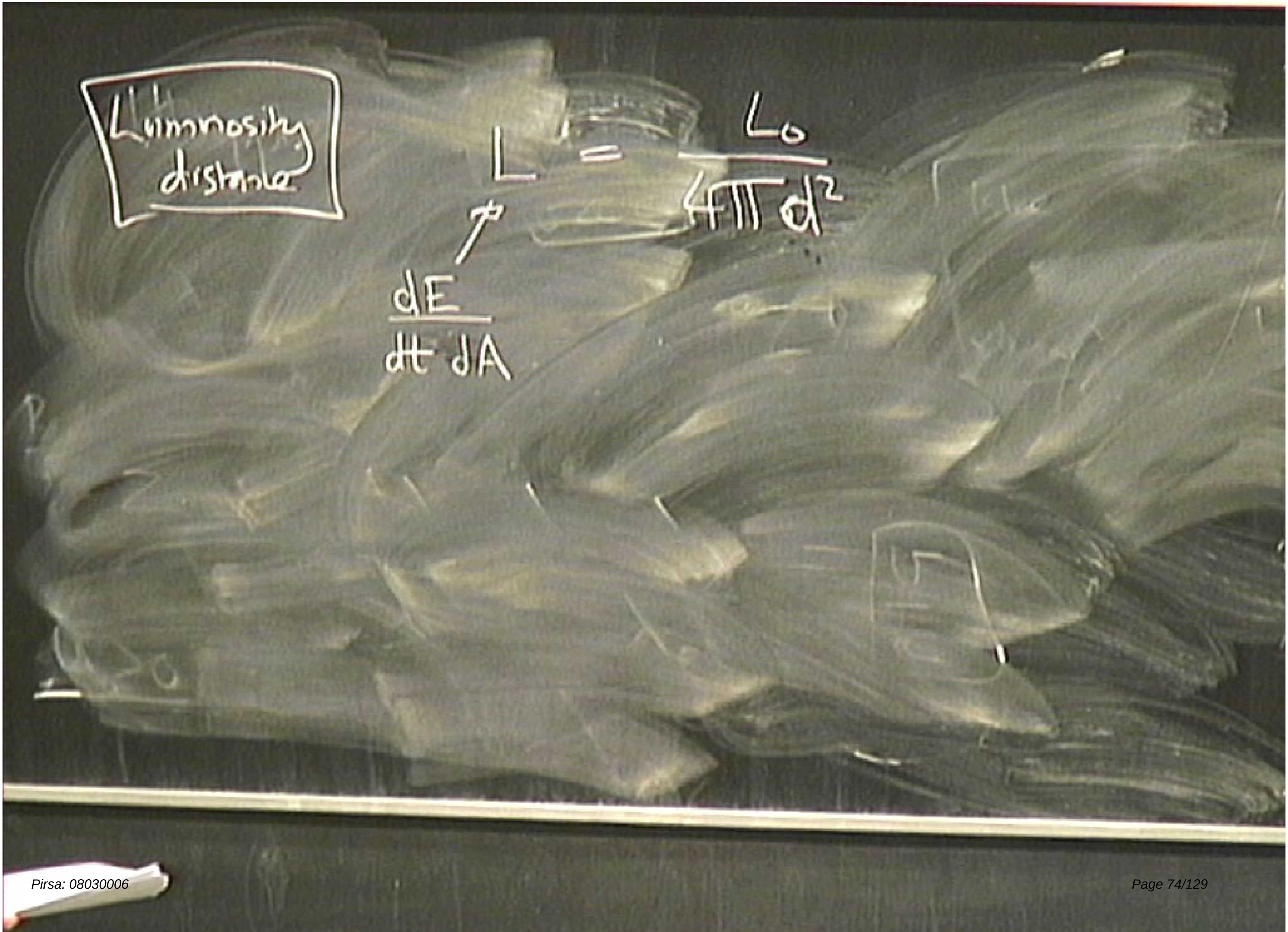

Distances
Parallax
A
B
fixed stars
Sun
d

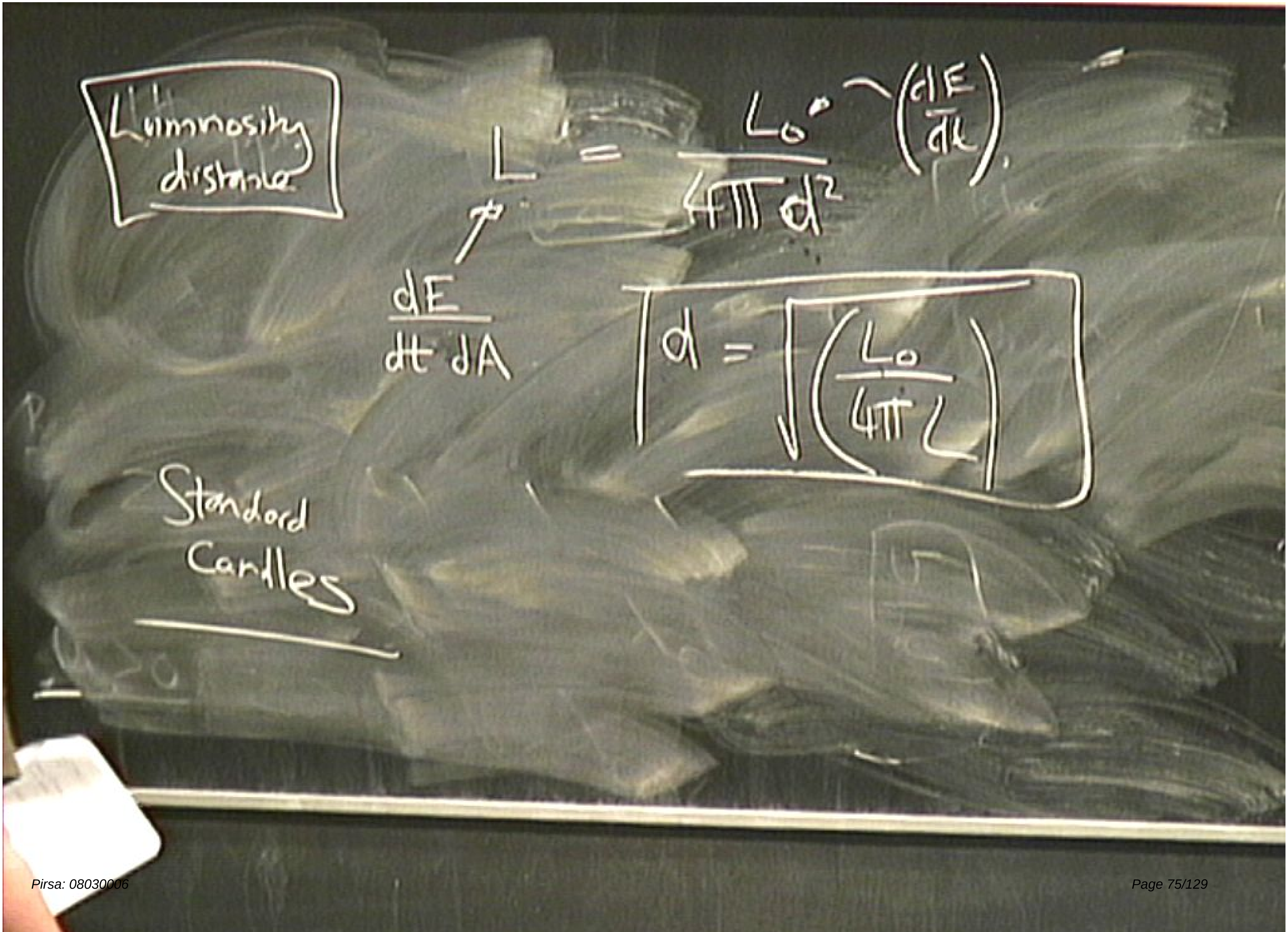


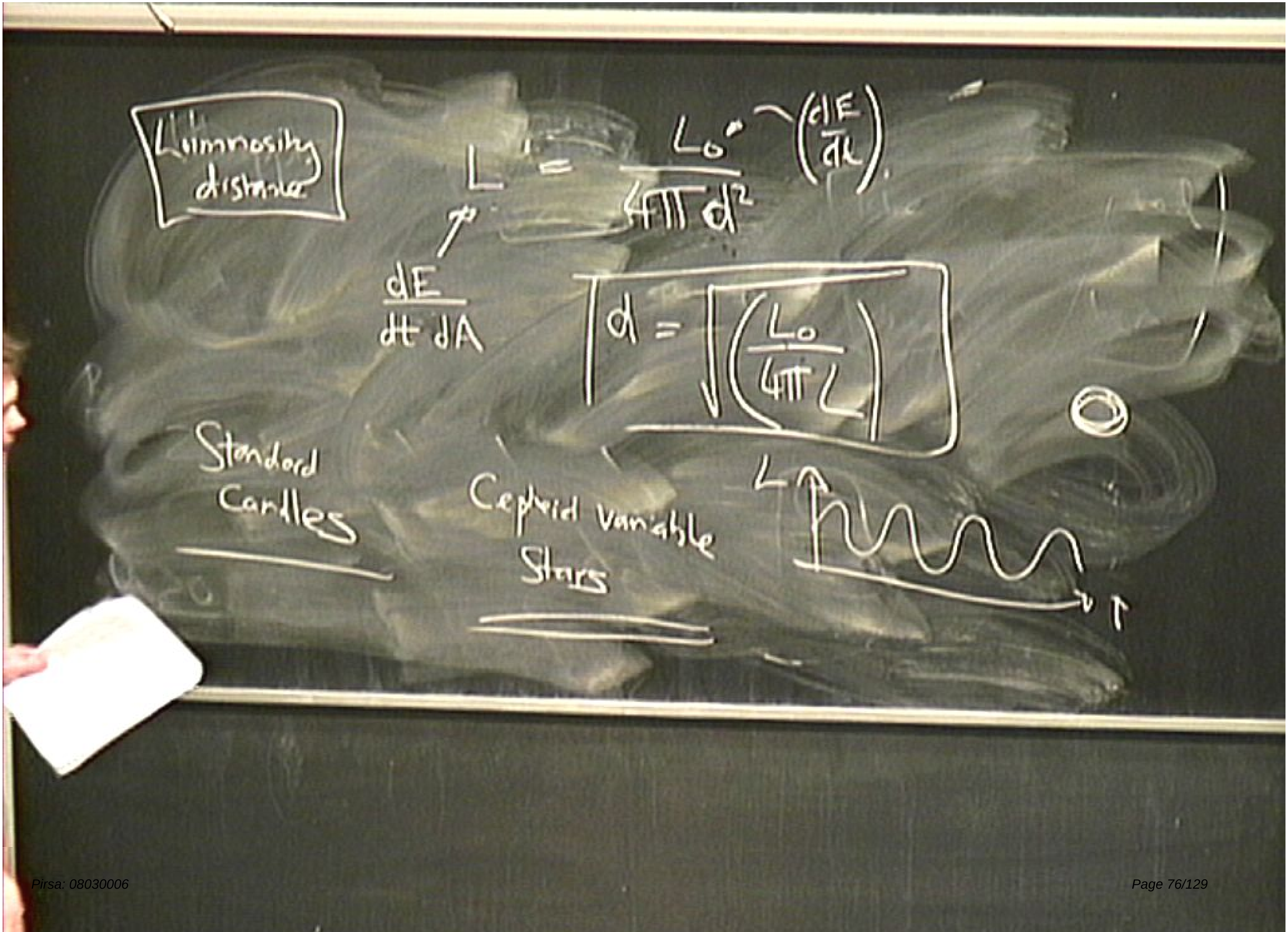
Distances
Parallex
fixed stars
A
B
θ1
θ2
Sun
d
A
B

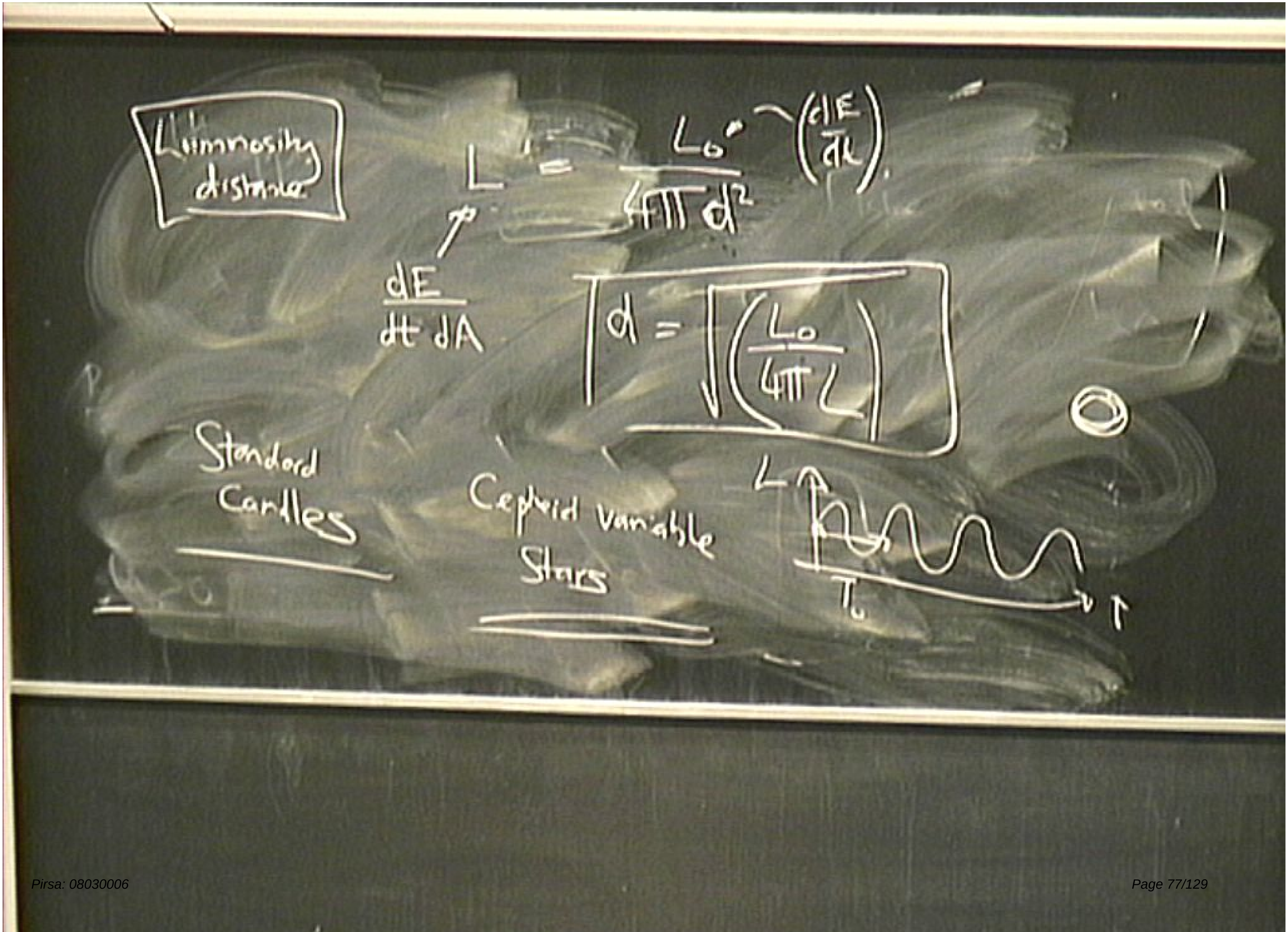

Distances
Parallex
A
fixed stars
B
θ1
θ2
Sun
d

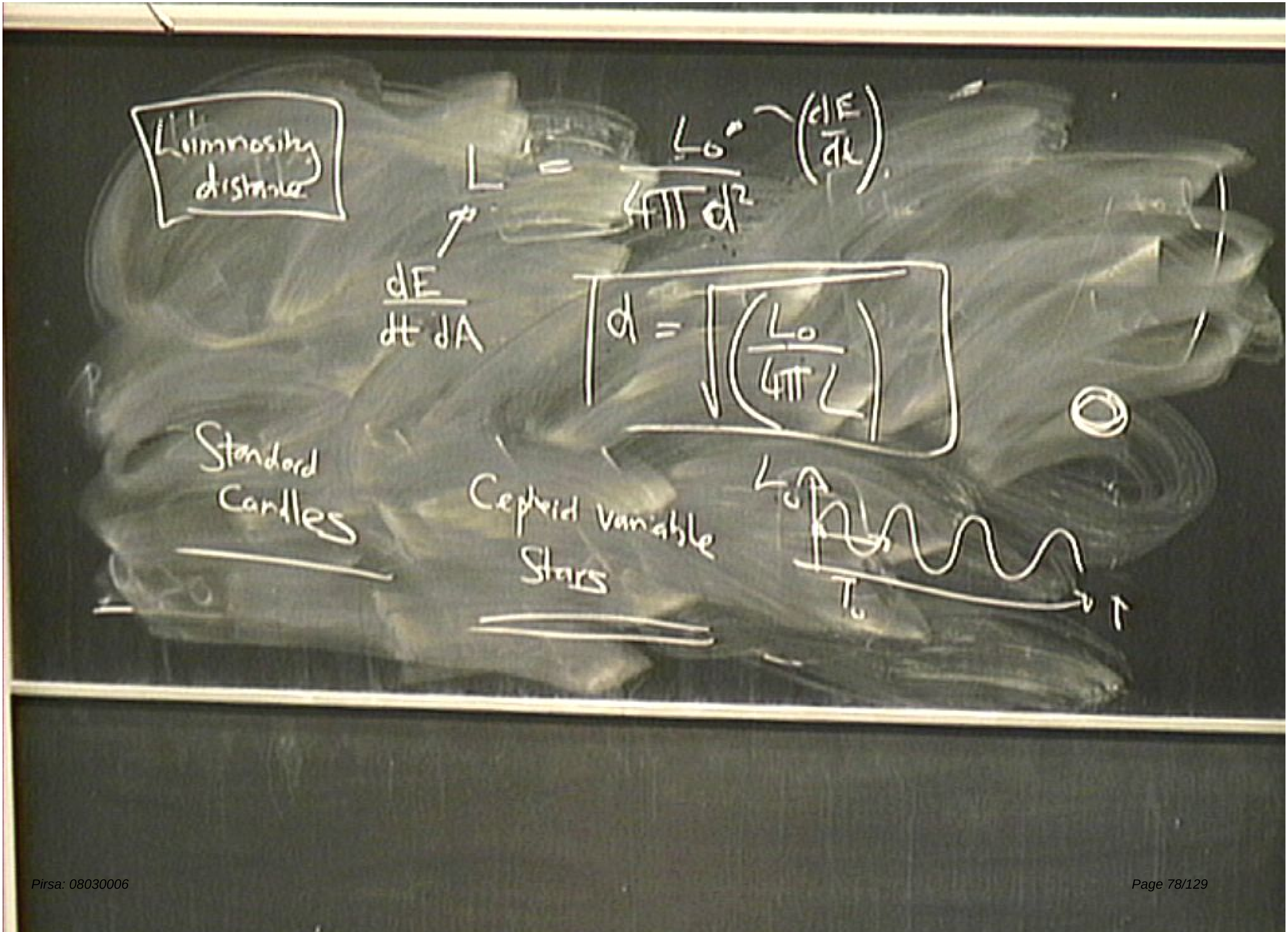

Distances
Parallex
Sun
d
A
B
A
B
d ≤ 50pc

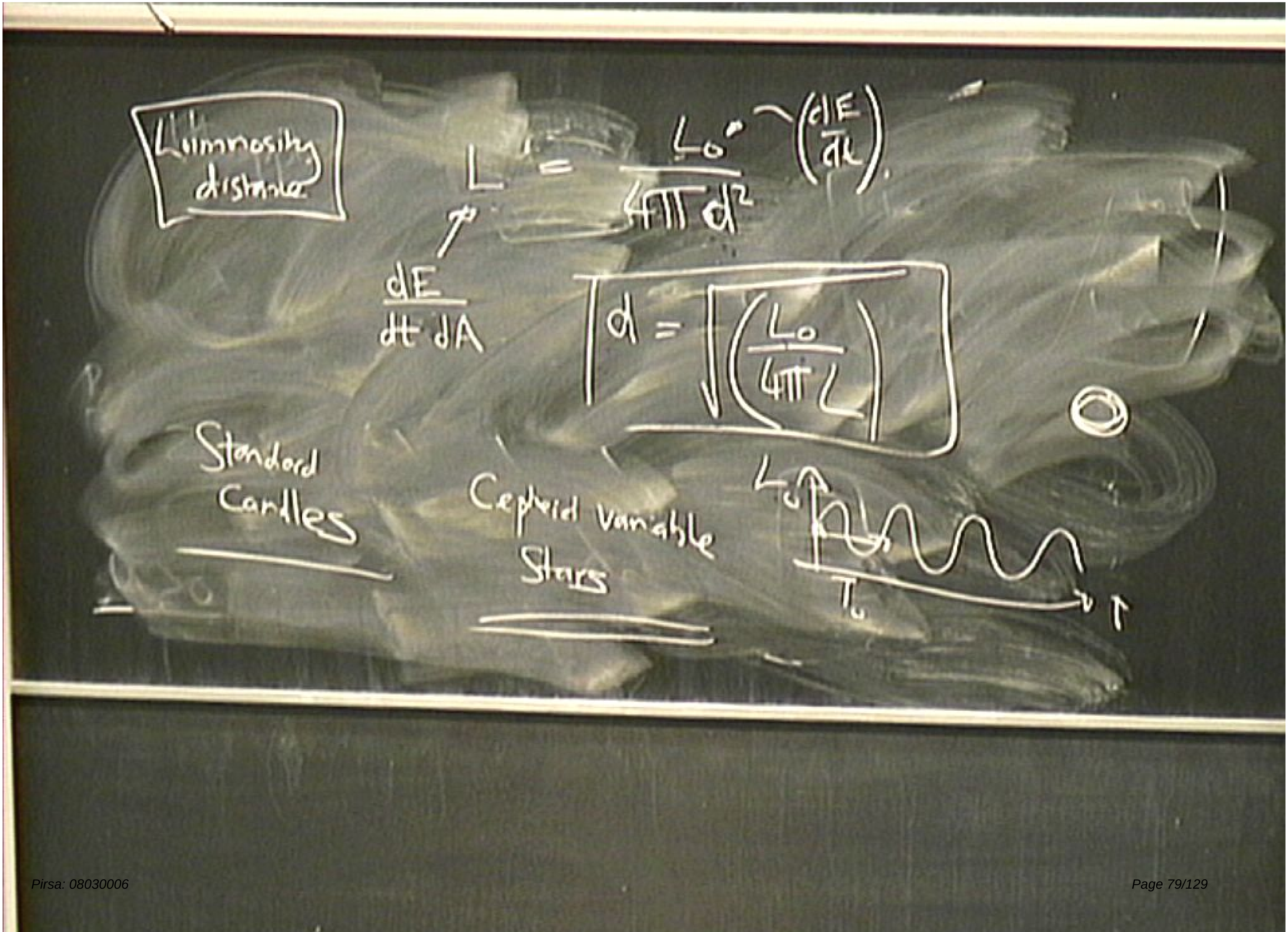

Luminosity distance
L = Lo / 4π d²
dE / dt dA

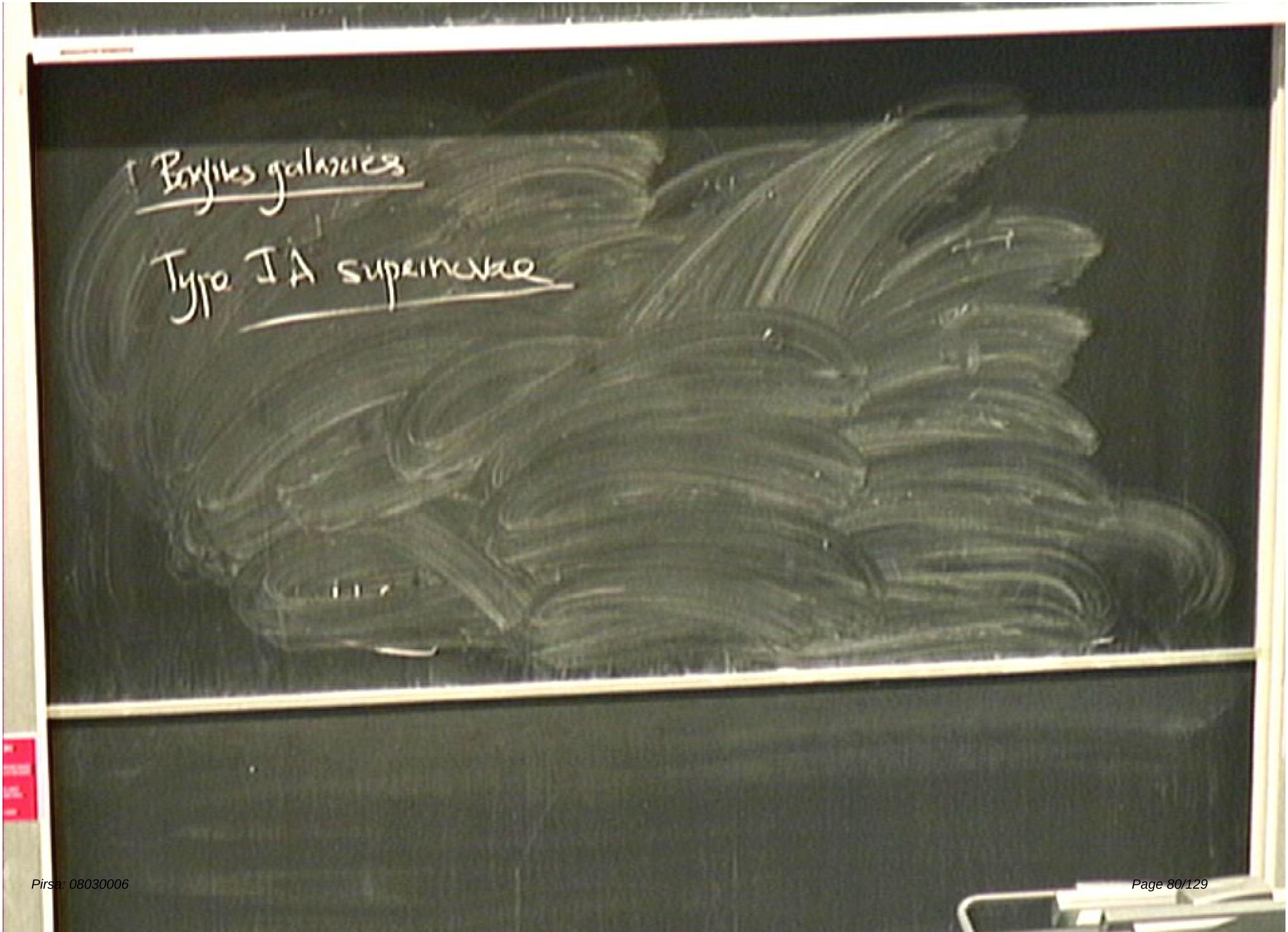

Luminosity distance
L = L_0 / 4π d^2 ~ (dE/dt)
dE / dt dA
d = √(L_0 / 4π L)
Standard Candles

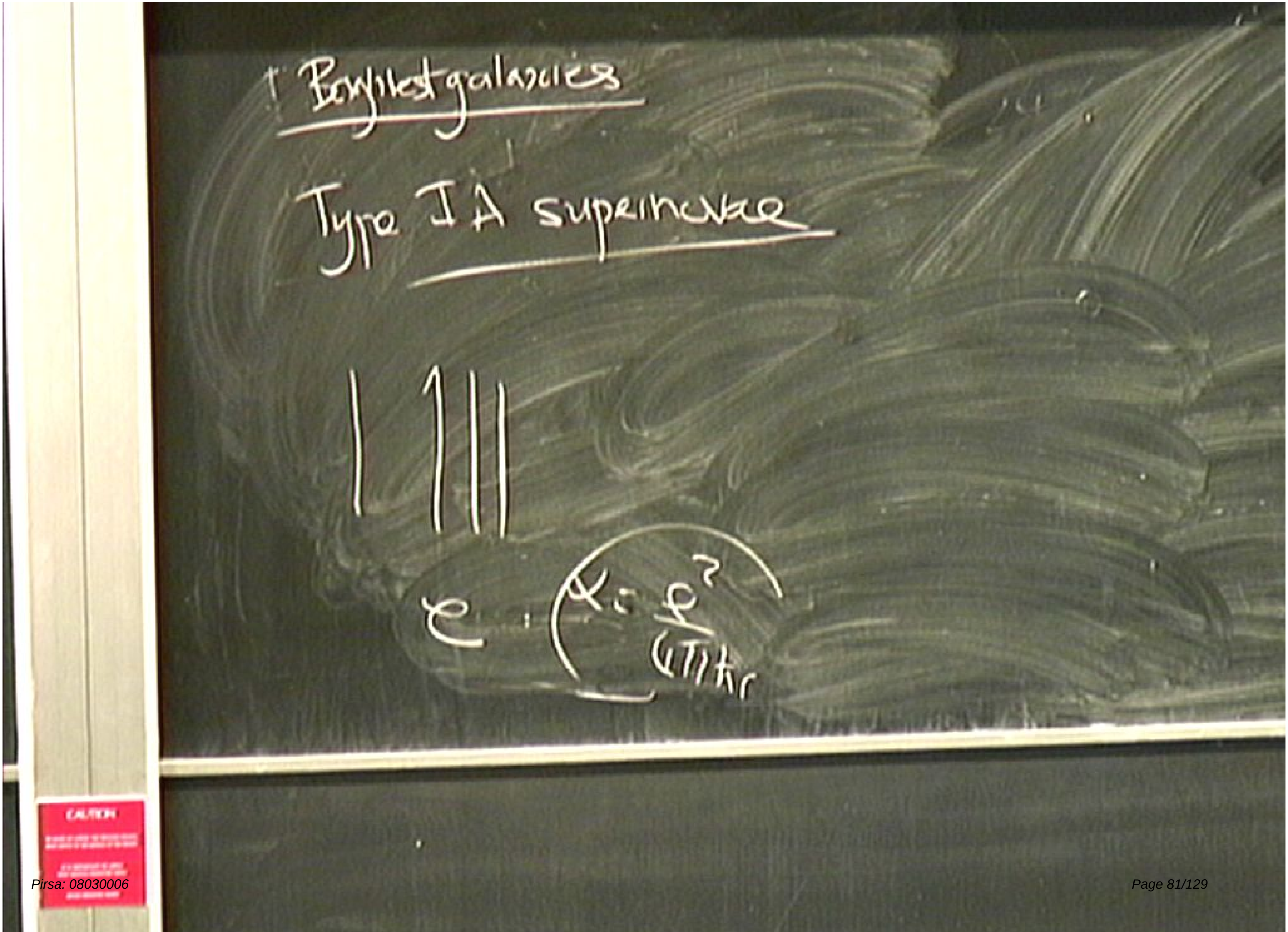

Luminosity distance
L = L₀ / 4π d² ~ (dE/dt)
dE / dt dA
d = √(L₀ / 4π L)
Standard Candles
Cepheid variable Stars
L
t

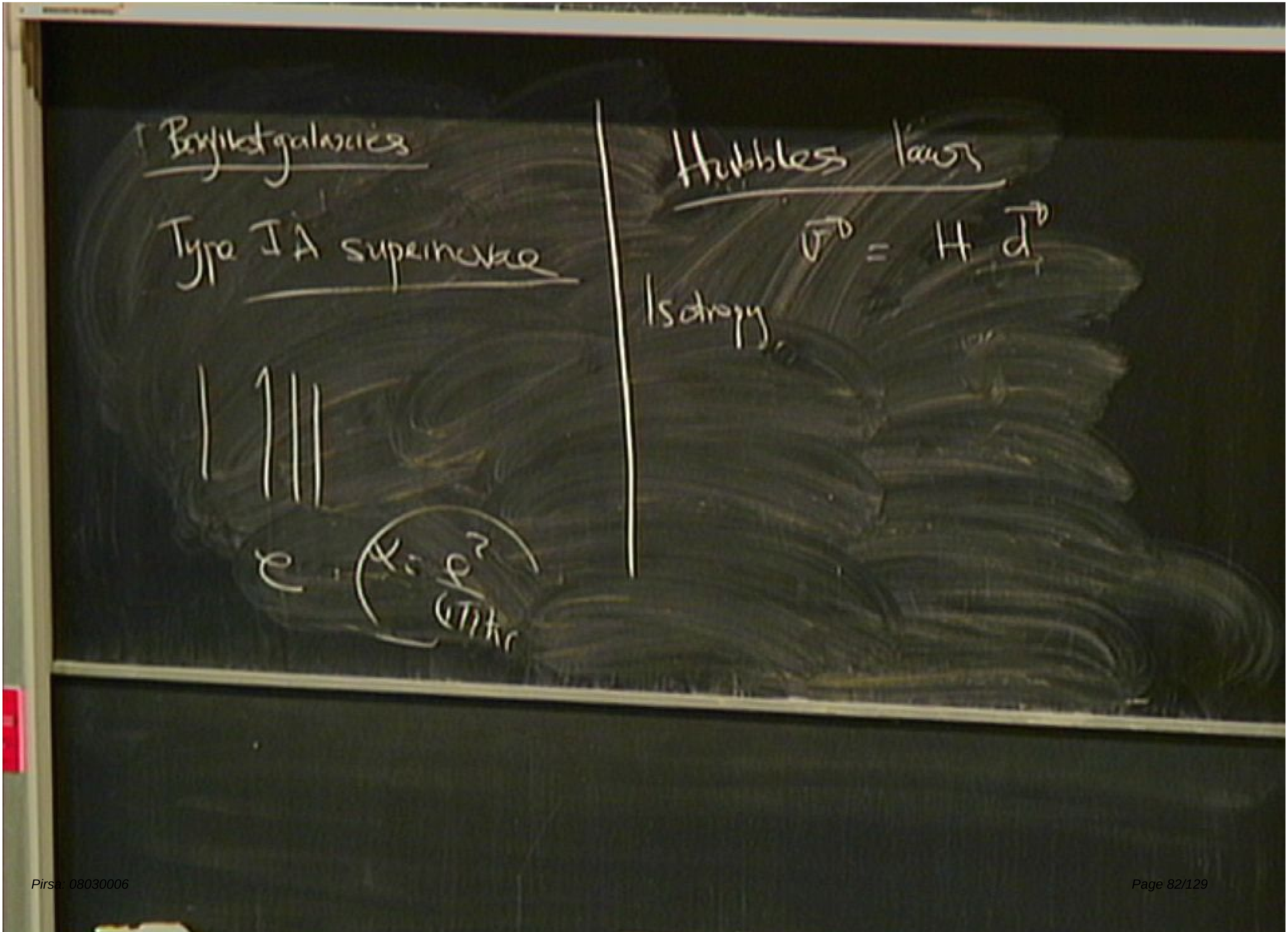

Luminosity distance
L = L₀ / 4π d² ~ (dE/dt)
dE / dt dA
d = √(L₀ / 4πL)
Standard Candles
Cepheid variable Stars
L
T₀

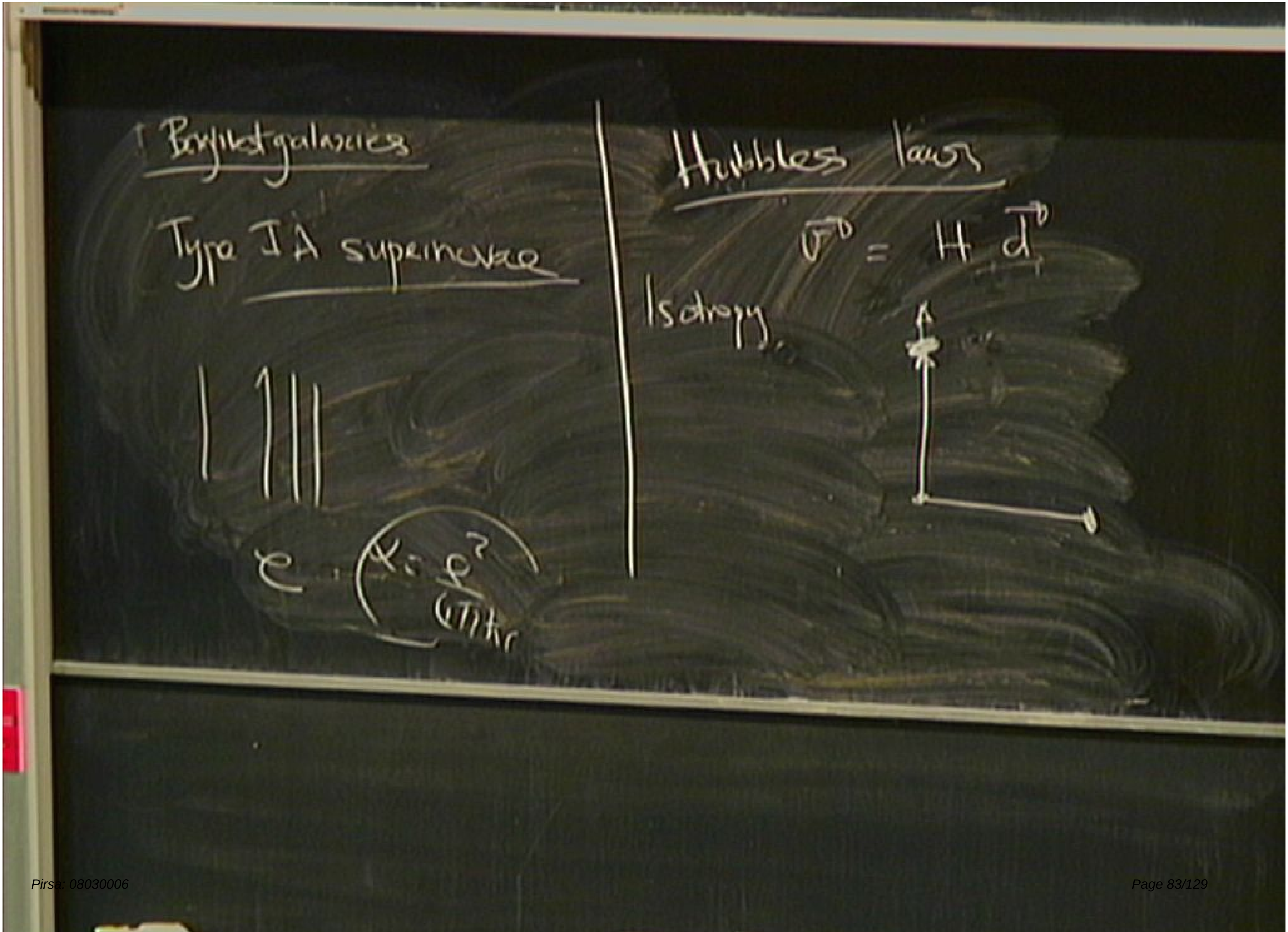

Luminosity distance
L = L0 / 4π d² (dE/dt)
dE / dt dA
d = √(L0 / 4π L)
Standard Candles
Cepheid variable Stars
L0
T0
t

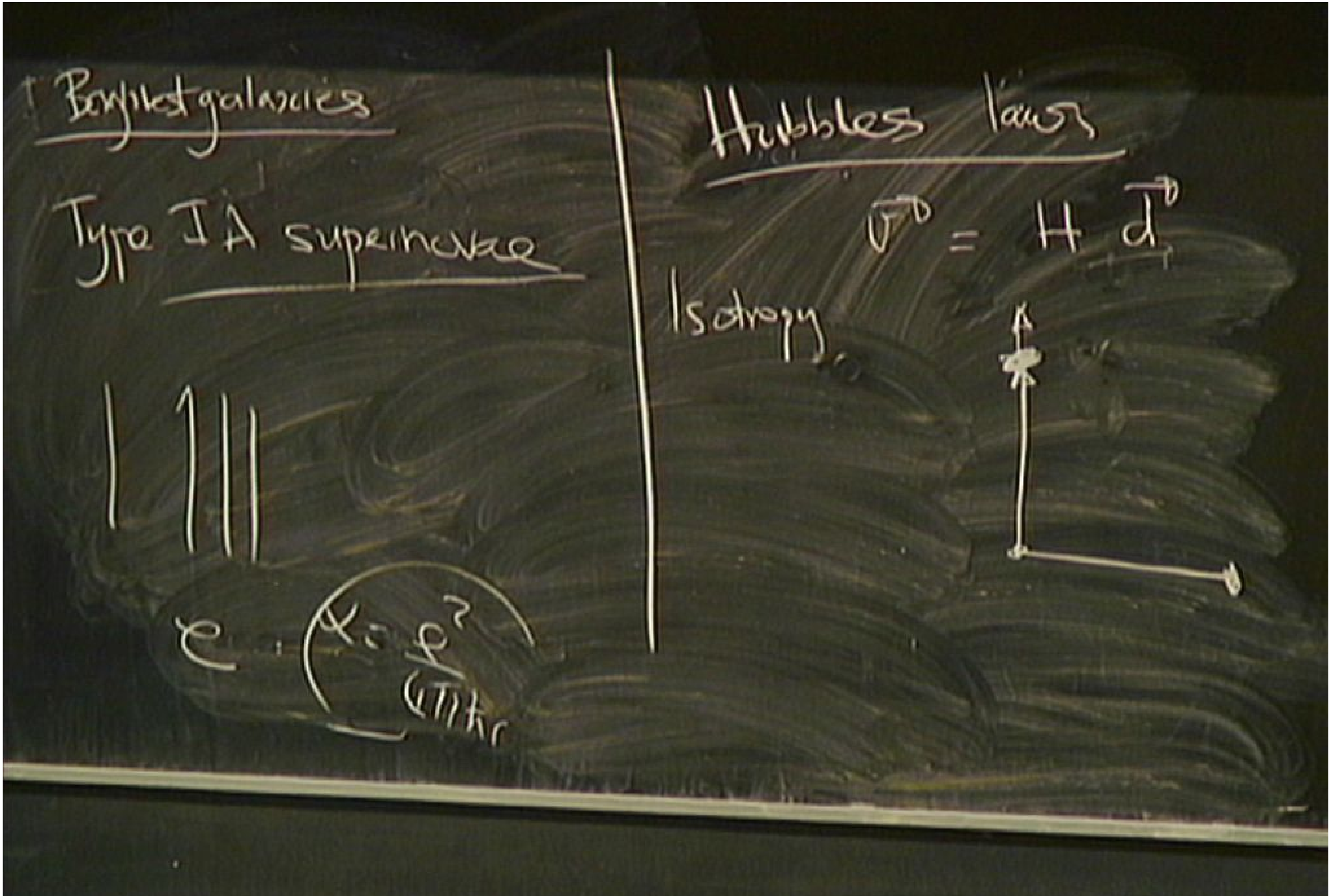

Luminosity distance
L = L0 / 4π d² ~ (dE/dt)
dE / dt dA
d = √(L0 / 4π L)
Standard Candles
Cepheid variable Stars
L0
T0
t

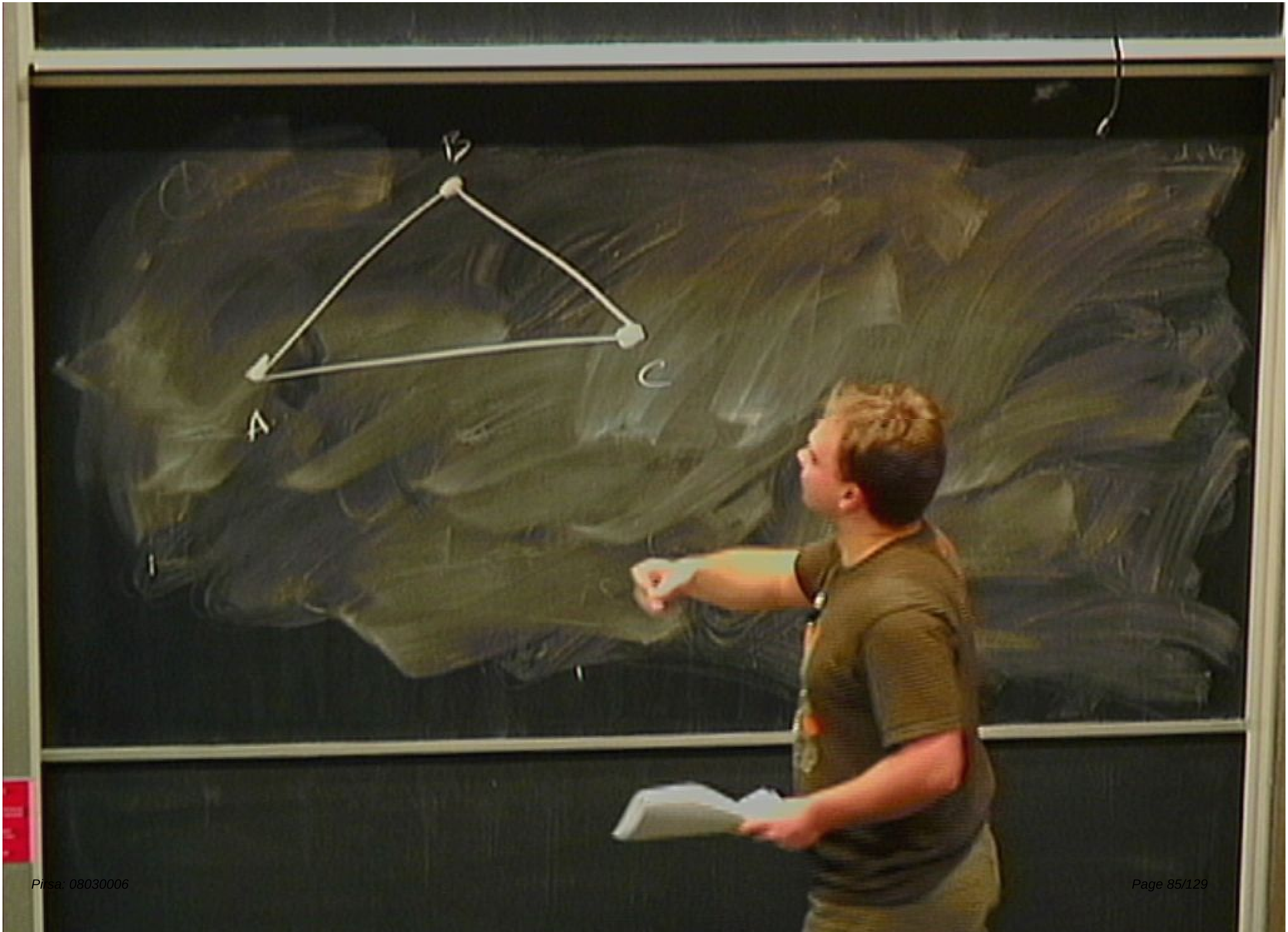

Type IA supernovae

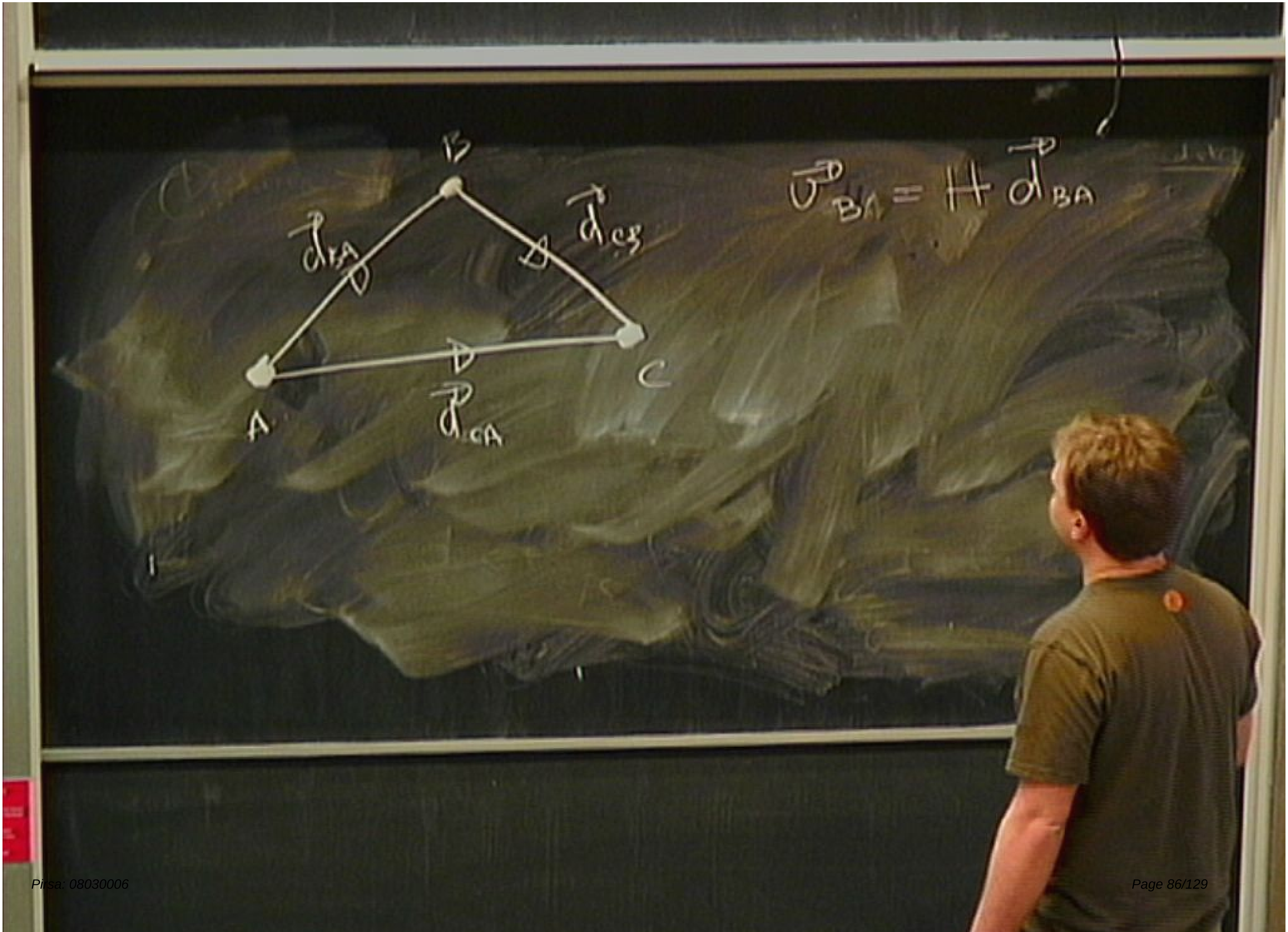

Brightest galaxies
Type IA supernovae


Brightest galaxies
Type IA supernovae
Hubbles law
v = H d
Isotropy

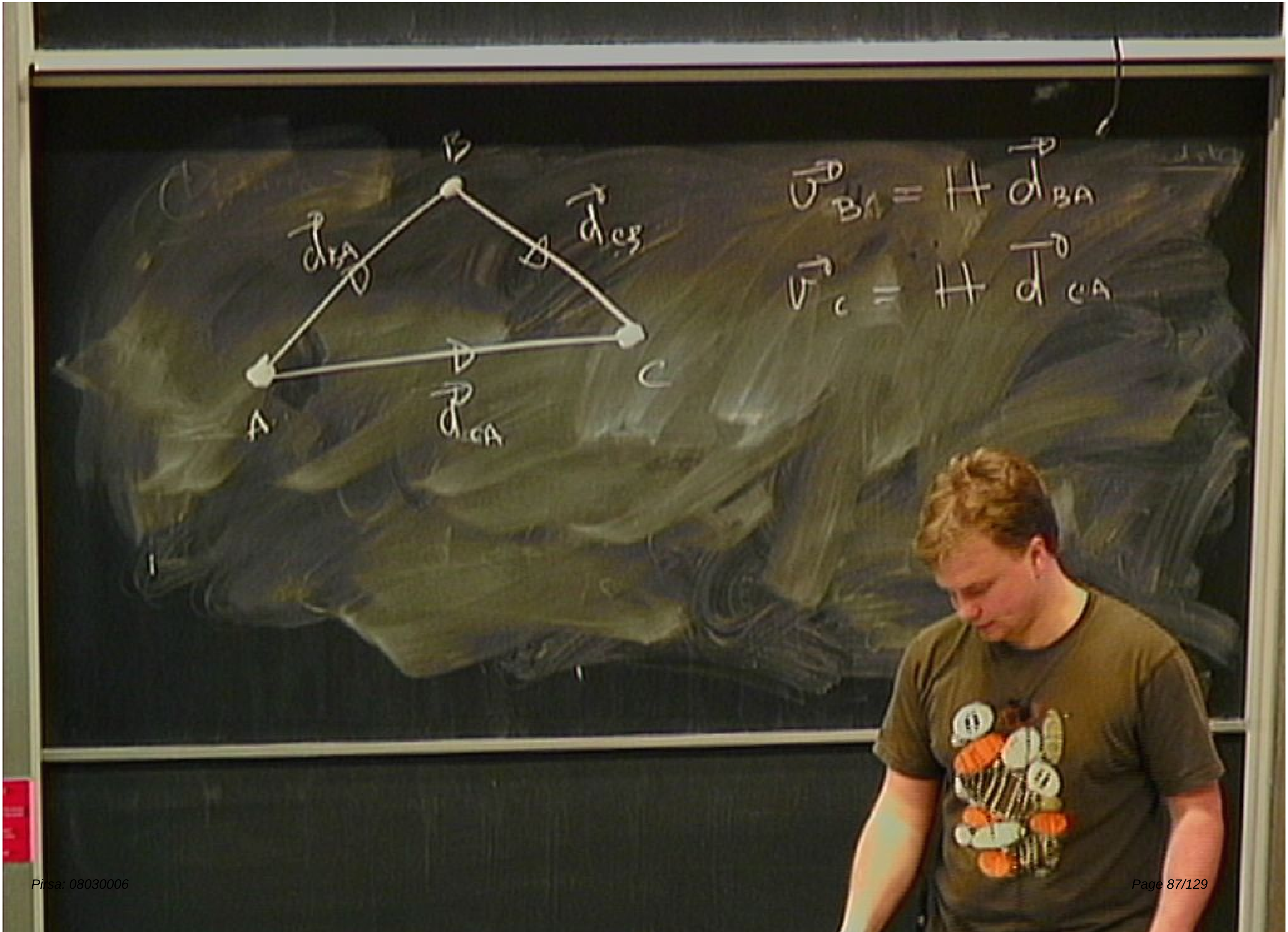

Brightest galaxies
Type IA supernovae
Hubbles law
v = H d
Isotropy

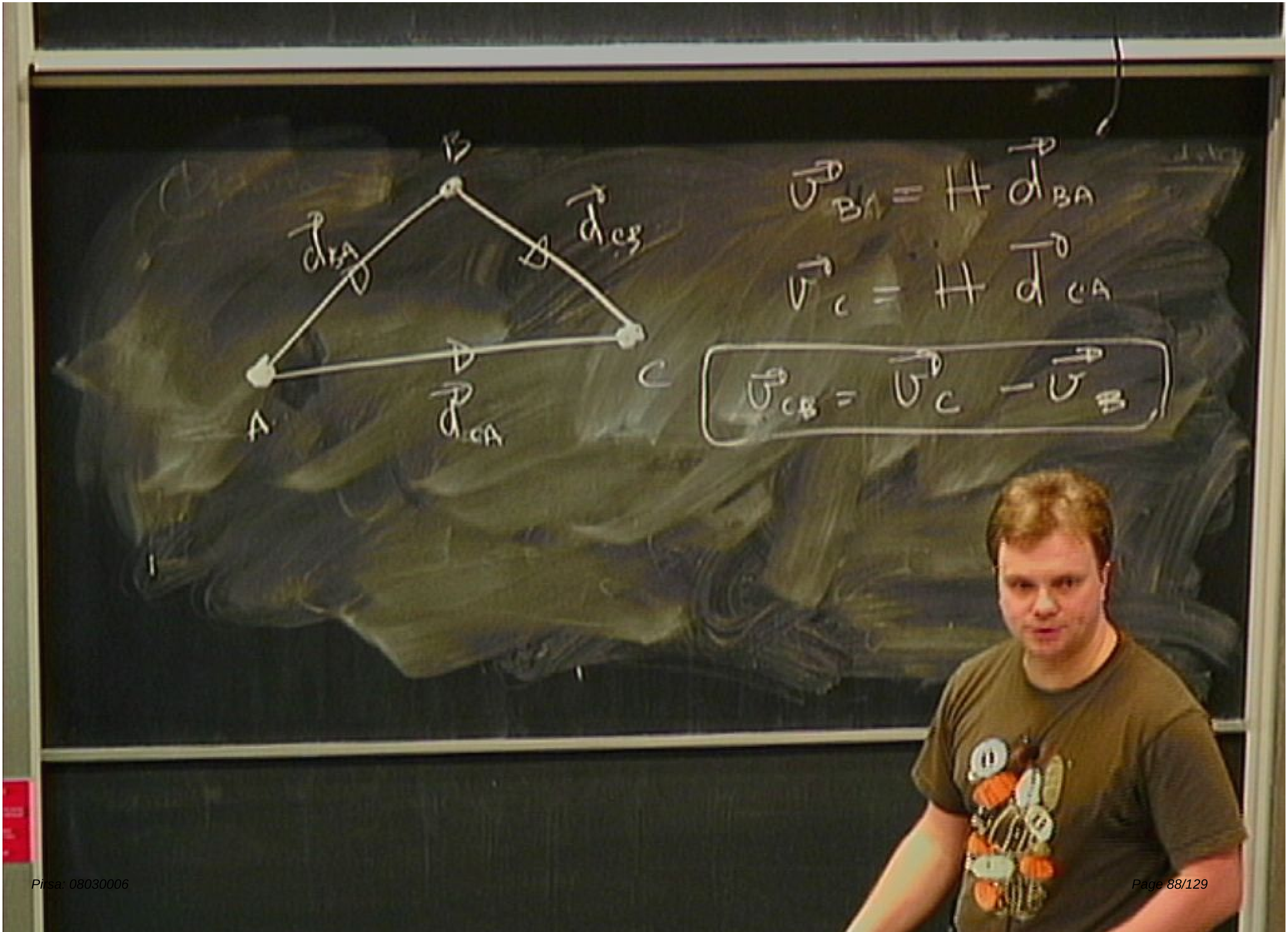

Brightest galaxies
Type IA supernovae
Hubbles law
$\vec{v} = H \vec{d}$
Isotropy

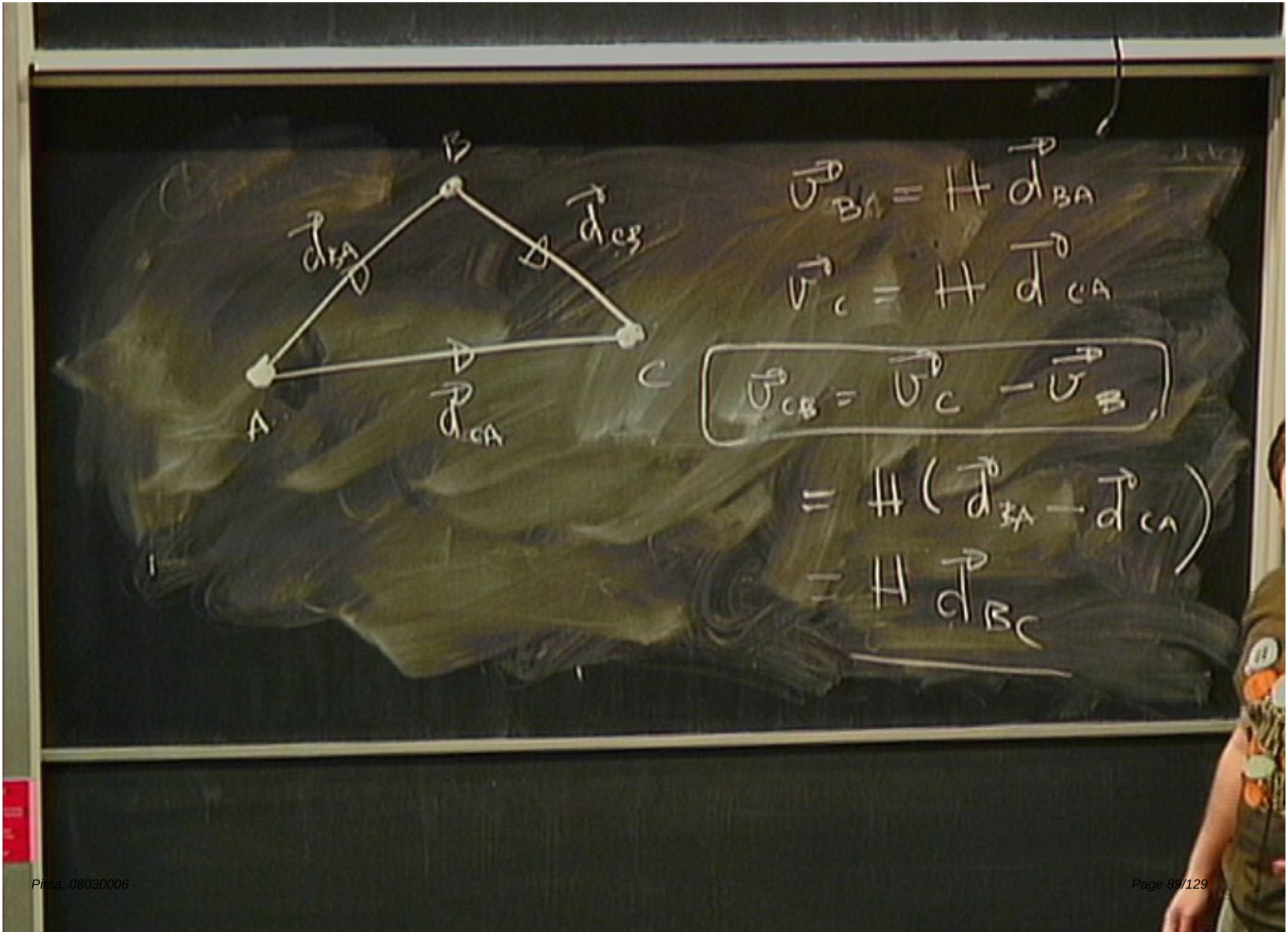

B
A
C

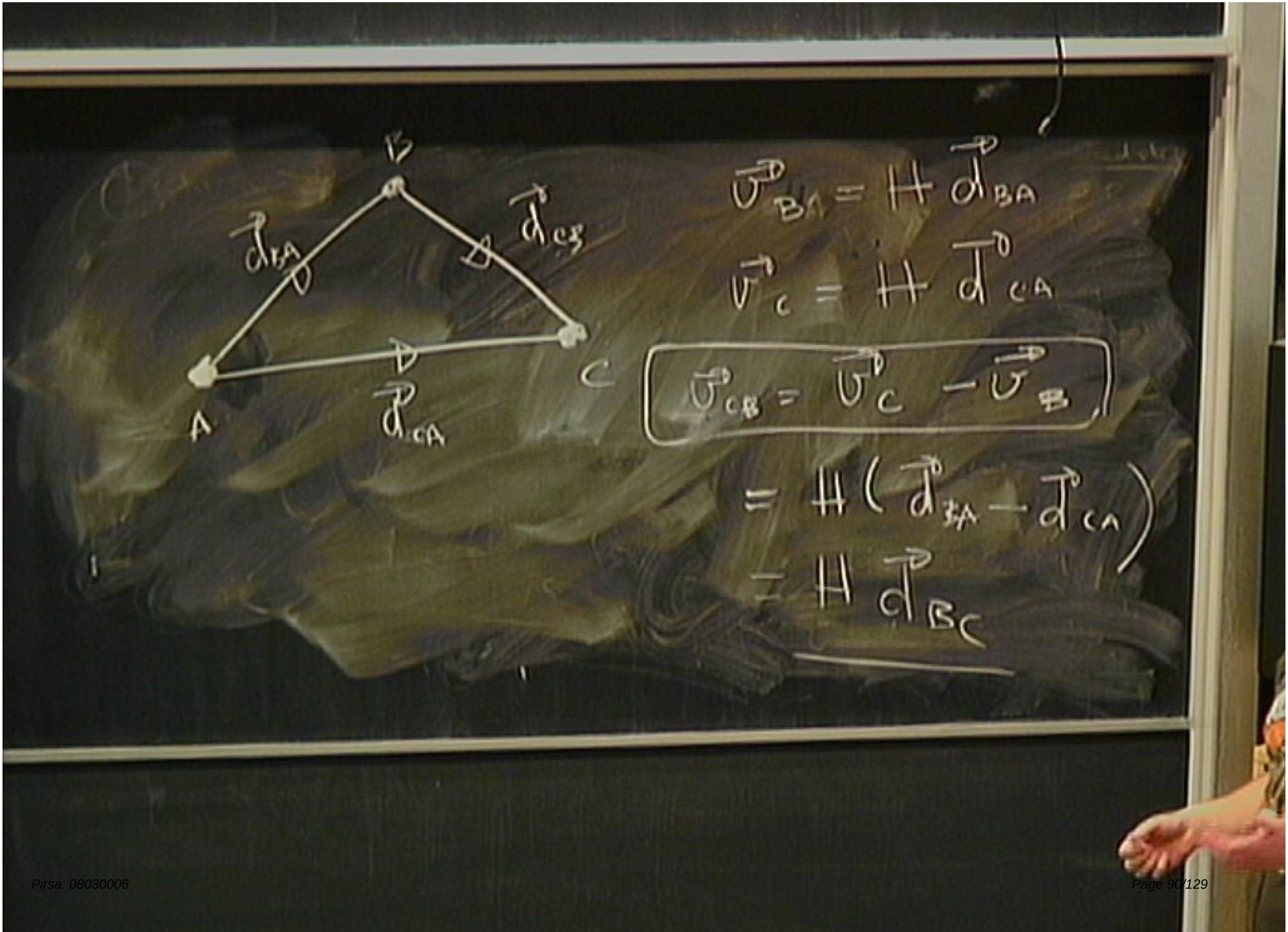


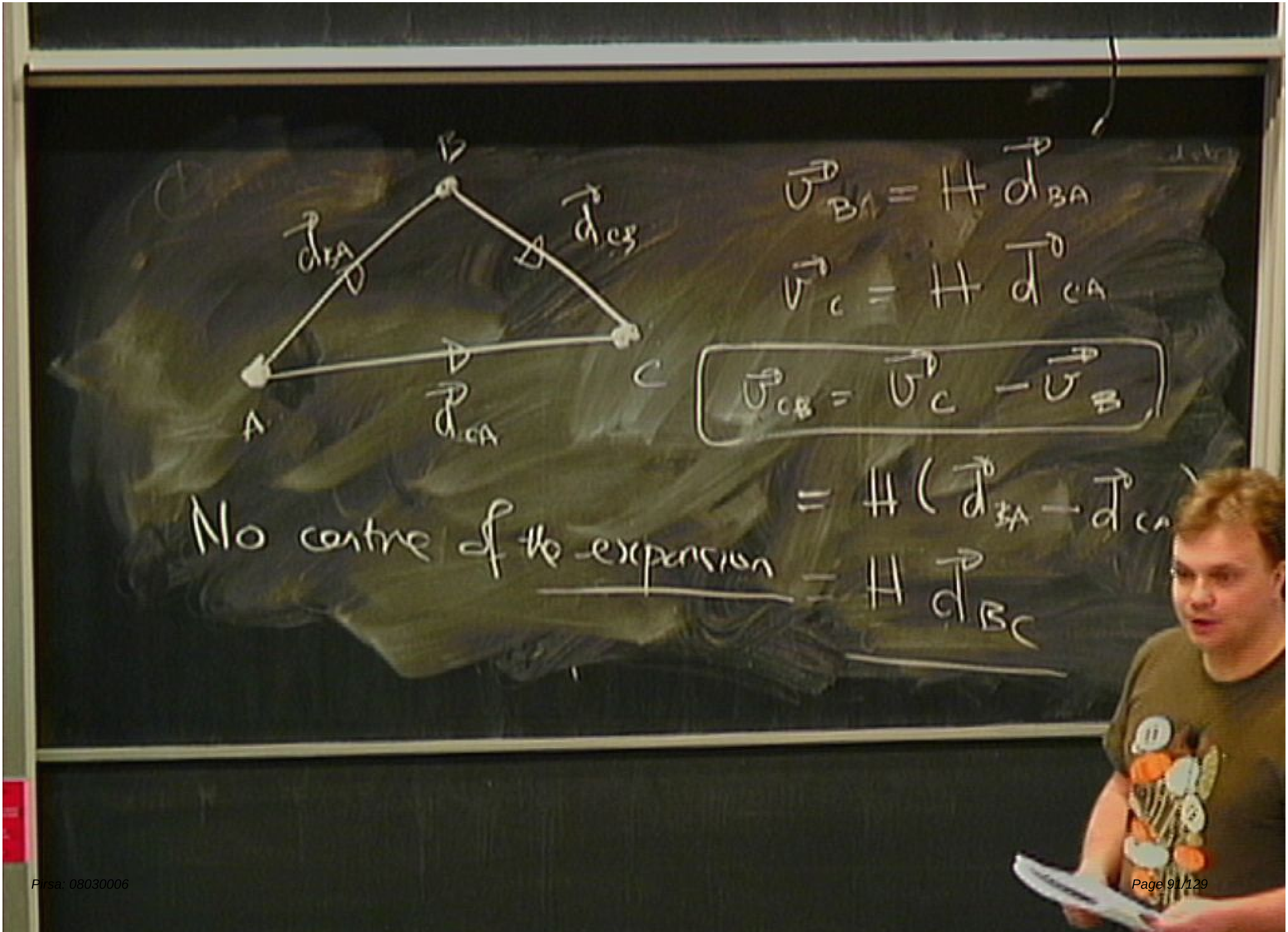
B
A
C
$\vec{d}_{BA}$
$\vec{d}_{CB}$
$\vec{d}_{CA}$
$\vec{v}_{BA} = H \vec{d}_{BA}$

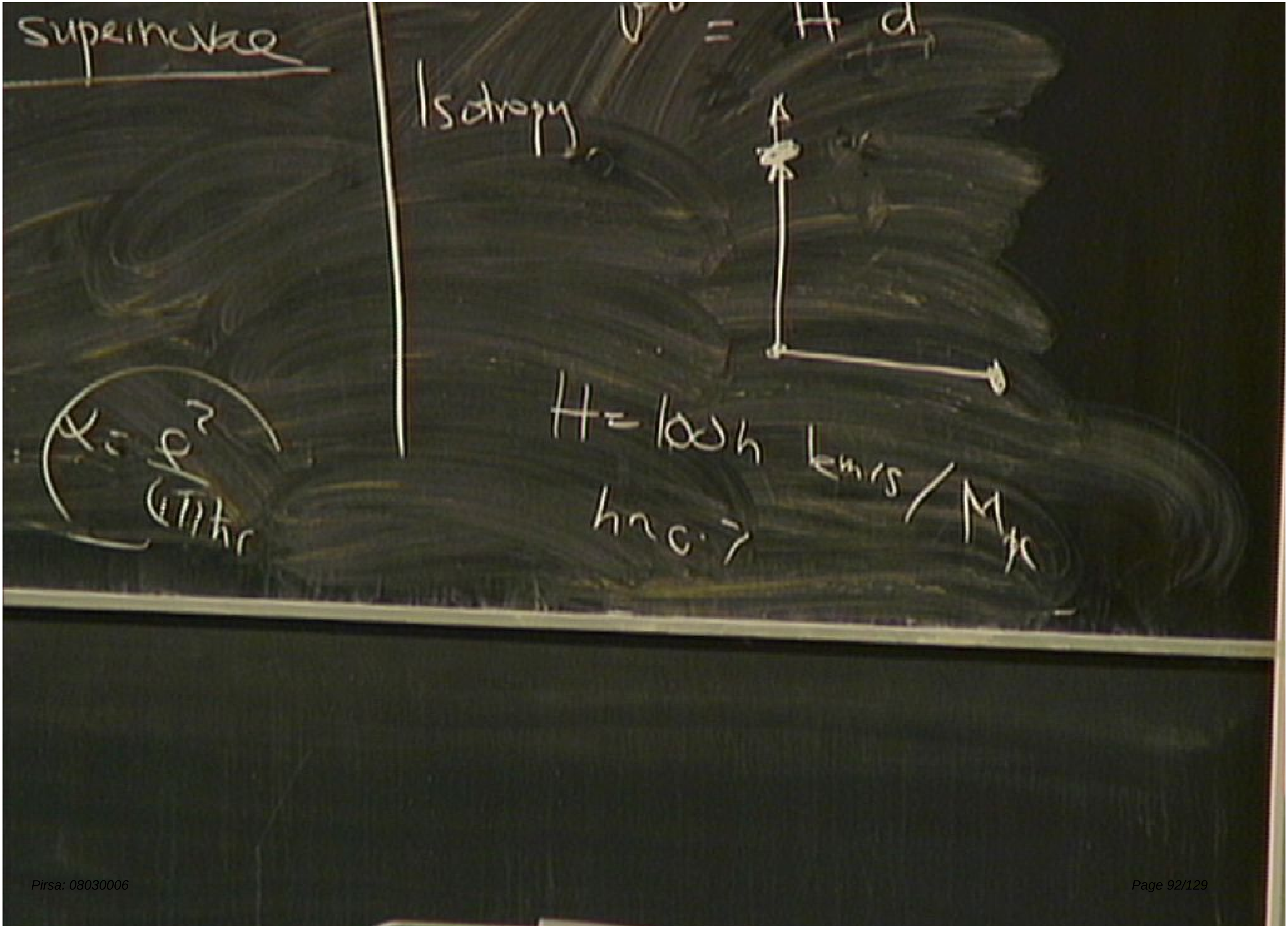

B
A
C
$\vec{d}_{BA}$
$\vec{d}_{CB}$
$\vec{d}_{CA}$
$\vec{v}_{BA} = H\,\vec{d}_{BA}$
$\vec{v}_{C} = H\,\vec{d}_{CA}$

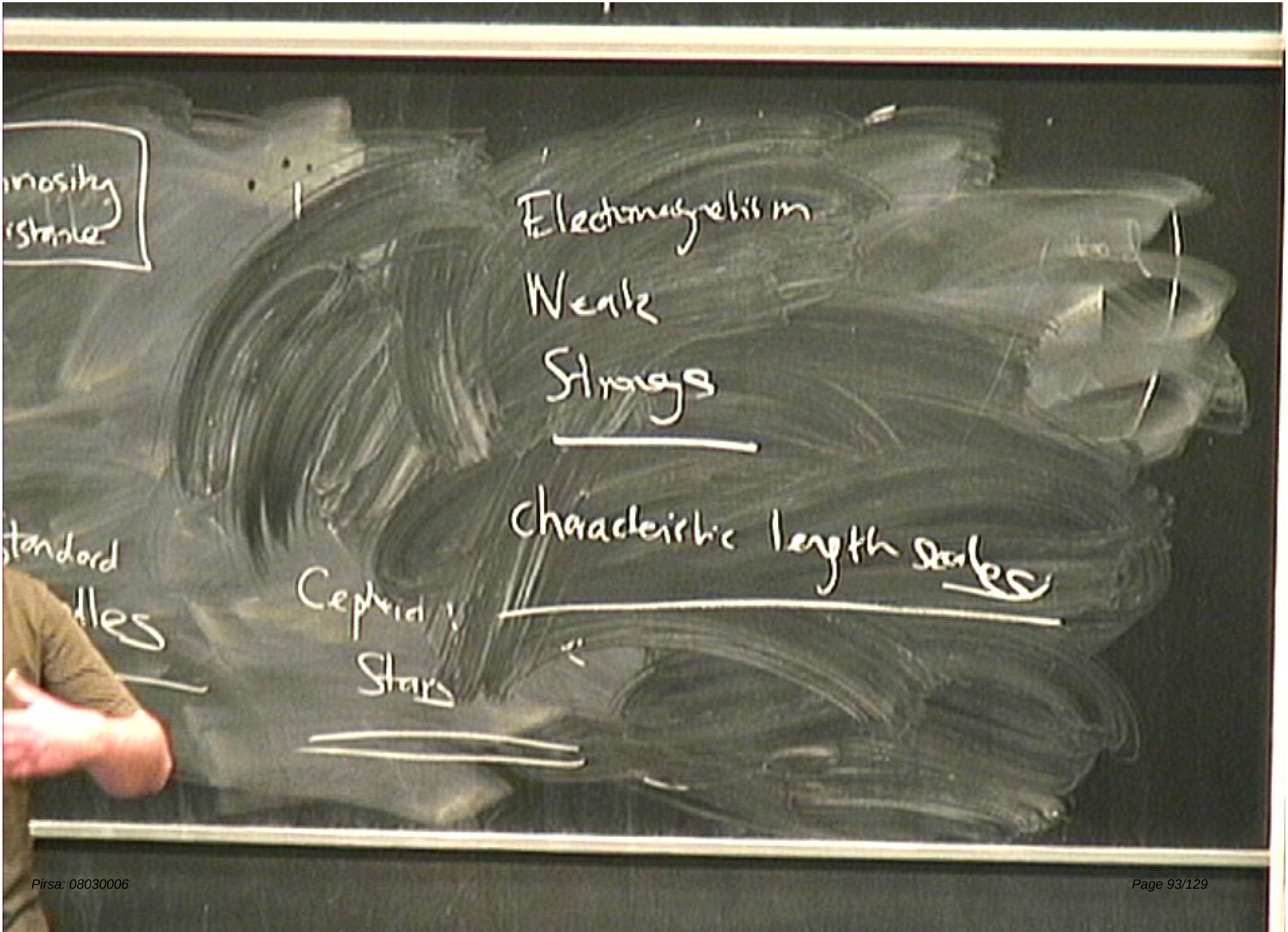

B
$\vec{d}_{BA}$
$\vec{d}_{CB}$
A
C
$\vec{d}_{CA}$
$\vec{v}_{BA} = H\,\vec{d}_{BA}$
$\vec{v}_{C} = H\,\vec{d}_{CA}$
$\vec{v}_{CB} = \vec{v}_{C} - \vec{v}_{B}$

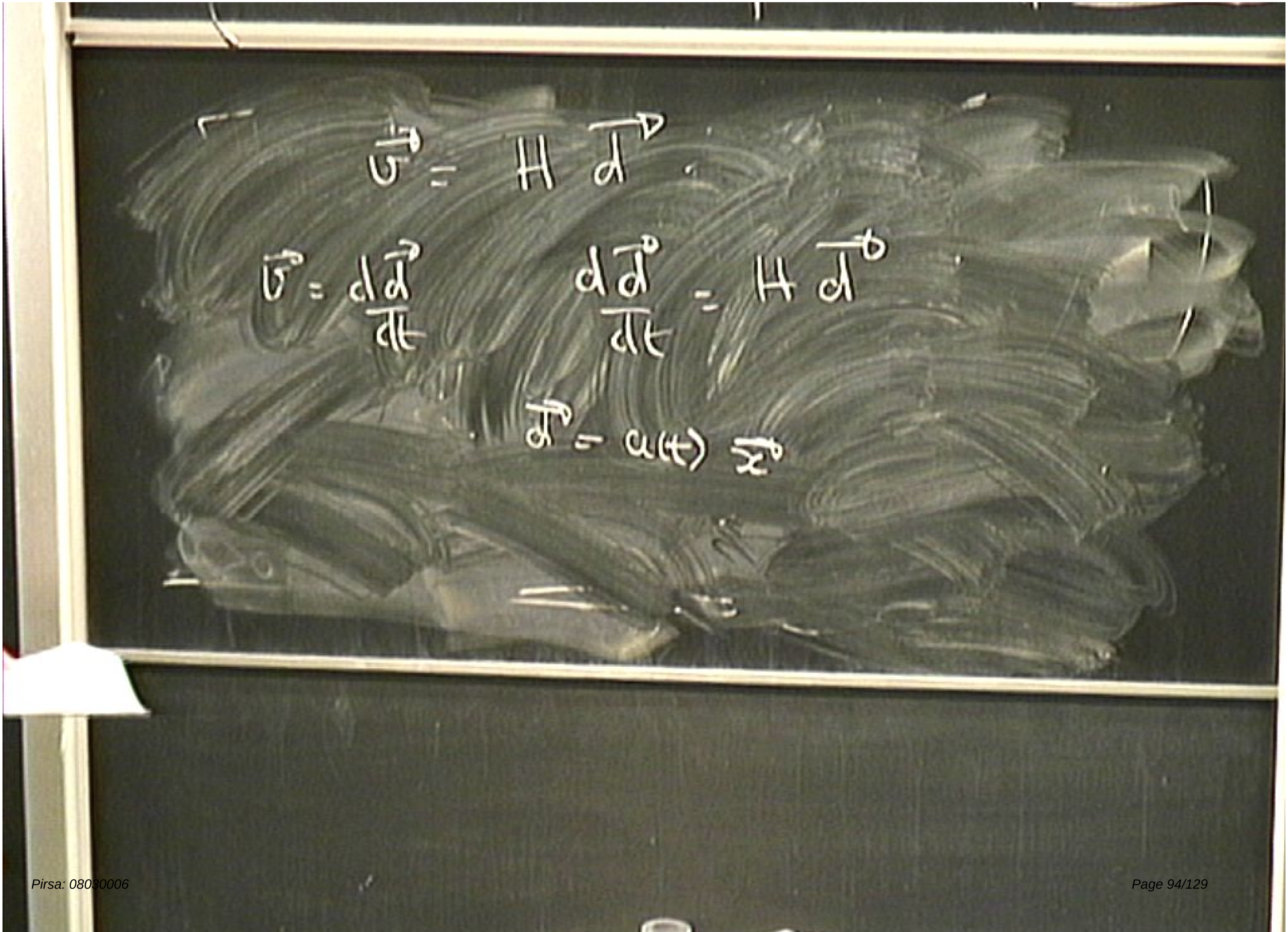

B
A
C
$\vec{d}_{BA}$
$\vec{d}_{CB}$
$\vec{d}_{CA}$
$\vec{v}_{BA} = H\,\vec{d}_{BA}$
$\vec{v}_{C} = H\,\vec{d}_{CA}$
$\vec{v}_{CB} = \vec{v}_{C} - \vec{v}_{B}$
$= H(\vec{d}_{BA} - \vec{d}_{CA})$
$= H\,\vec{d}_{BC}$


B
A
C
$\vec{d}_{BA}$
$\vec{d}_{CB}$
$\vec{d}_{CA}$
$\vec{v}_{BA} = H\,\vec{d}_{BA}$
$\vec{v}_{C} = H\,\vec{d}_{CA}$
$\vec{v}_{CB} = \vec{v}_{C} - \vec{v}_{B}$
$= H(\vec{d}_{BA} - \vec{d}_{CA})$
$= H\,\vec{d}_{BC}$


B
A
C
$\vec{d}_{BA}$
$\vec{d}_{CB}$
$\vec{d}_{CA}$
$\vec{v}_{BA} = H \vec{d}_{BA}$
$\vec{v}_{C} = H \vec{d}_{CA}$
$\vec{v}_{CB} = \vec{v}_{C} - \vec{v}_{B}$
$= H(\vec{d}_{BA} - \vec{d}_{CA})$
$= H \vec{d}_{BC}$
No centre of the expansion


supernovae
Isotropy
H = 100h km/s / Mpc
h ≈ 0.7


Electromagnetism
Weak
Strong
Characteristic length scales
Cepheid
Stars


$\vec{v} = H \vec{d}$
$\vec{v} = \frac{d\vec{d}}{dt}$
$\frac{d\vec{d}}{dt} = H \vec{d}$
$\vec{d} = a(t)\, \vec{x}$

$$\vec{v} = H\vec{d}$$

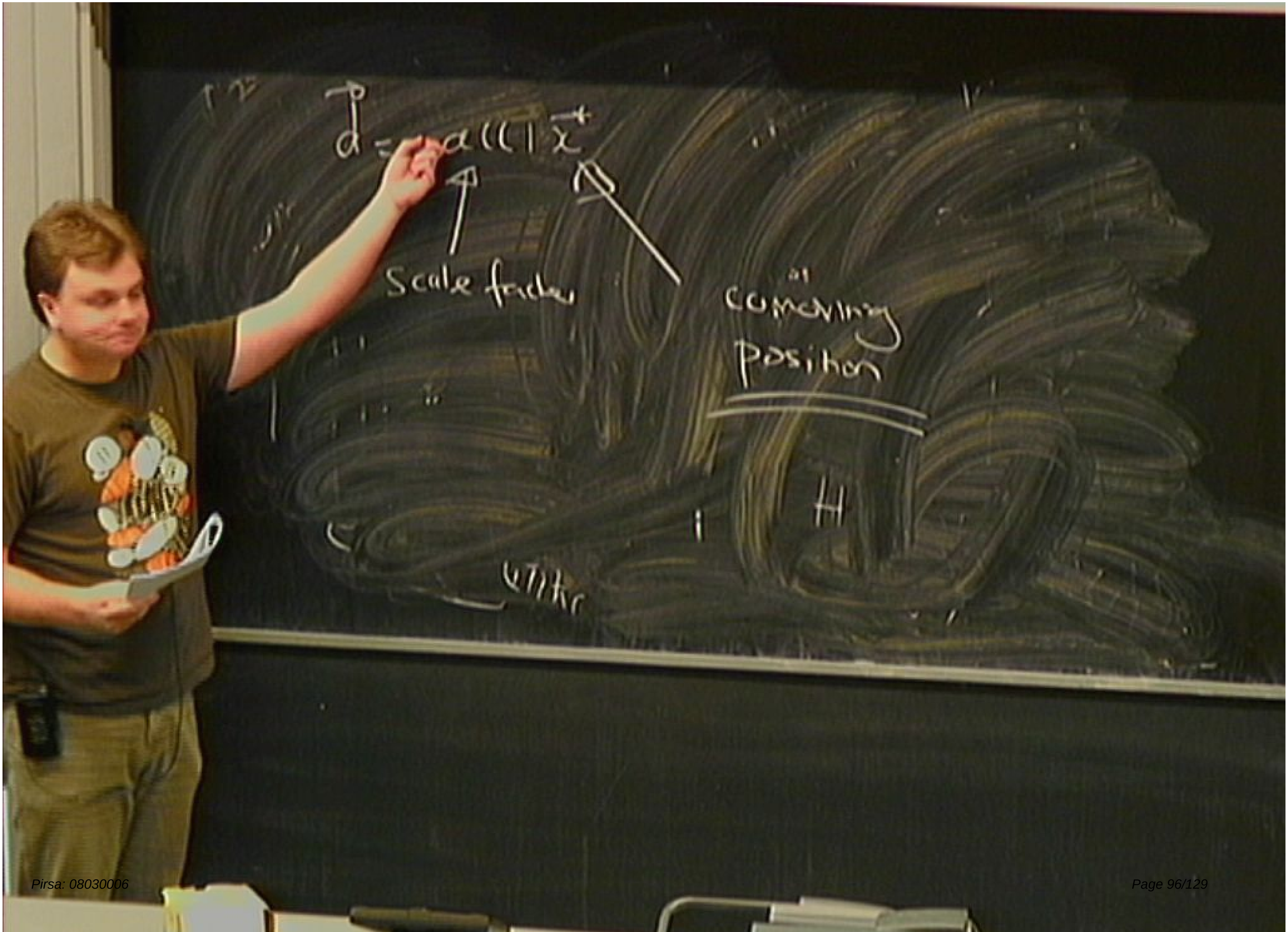
$$\vec{v} = \frac{d\vec{d}}{dt} \qquad \frac{d\vec{d}}{dt} = H\vec{d}$$

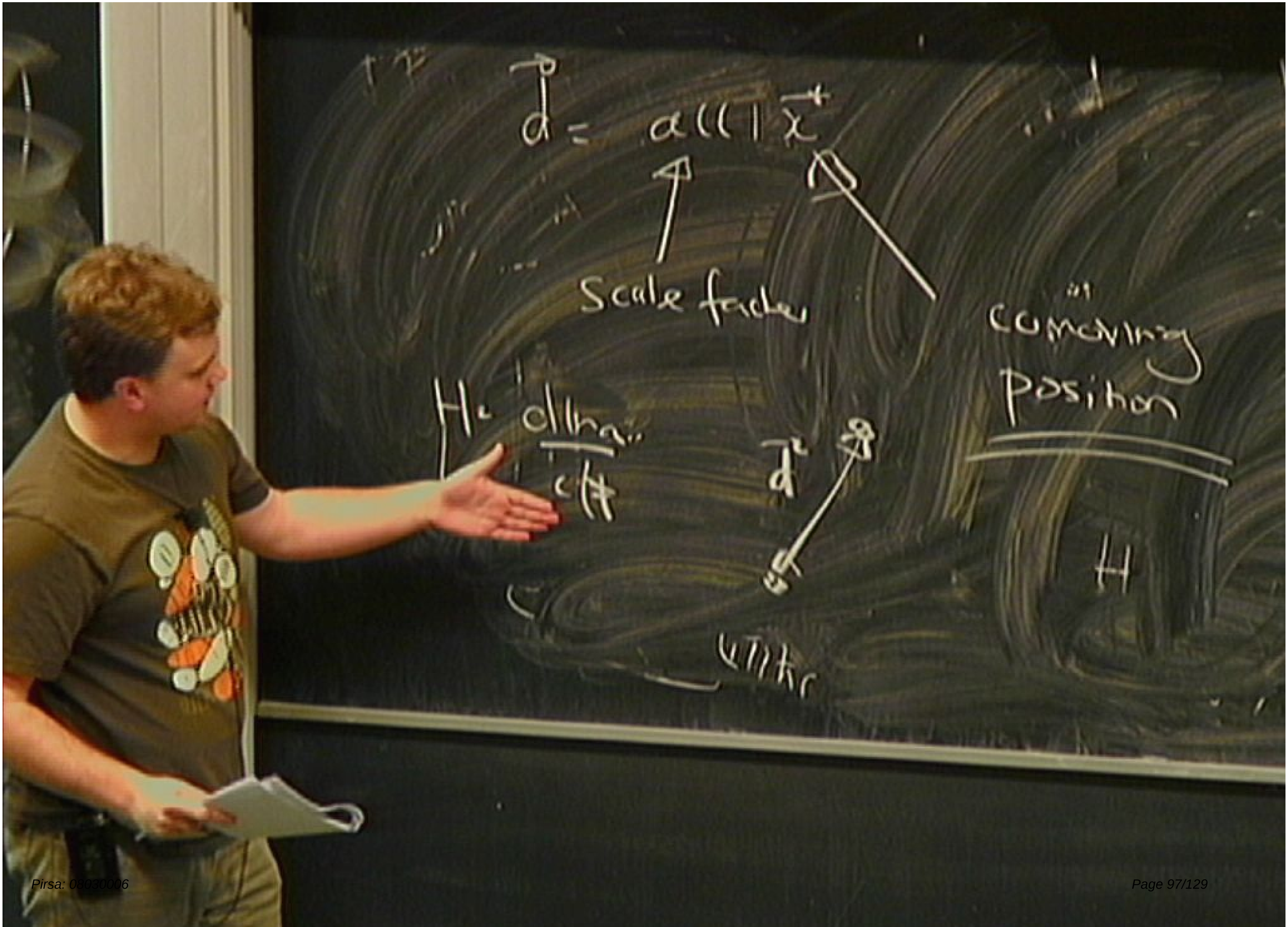
$$\vec{d} = a(t)\,\vec{x}$$

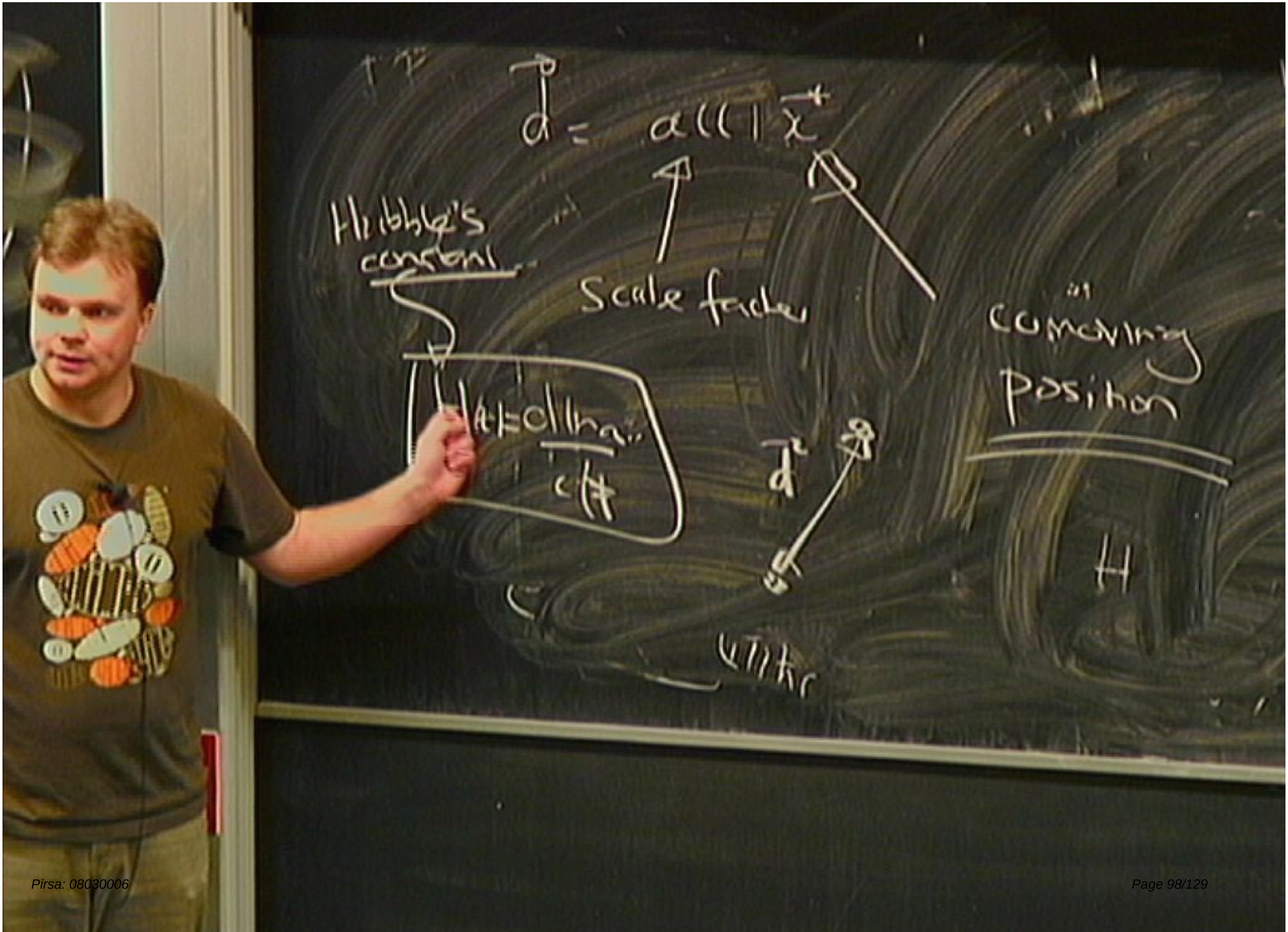
$$\frac{da}{dt}\vec{x} = Ha\vec{x}$$

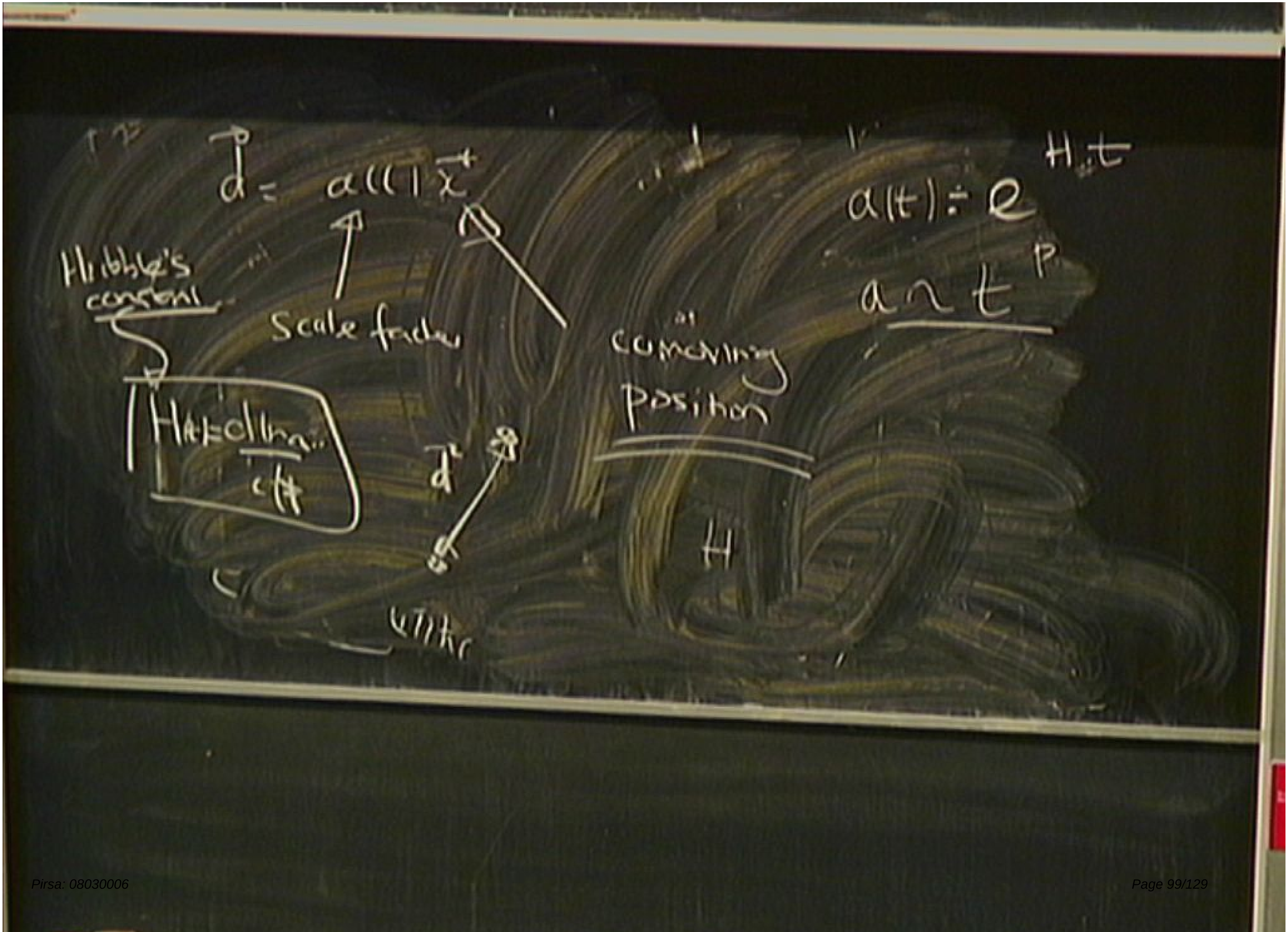
$$\frac{da}{a\,dt} = H$$

$$H = \frac{d\ln a}{dt}$$

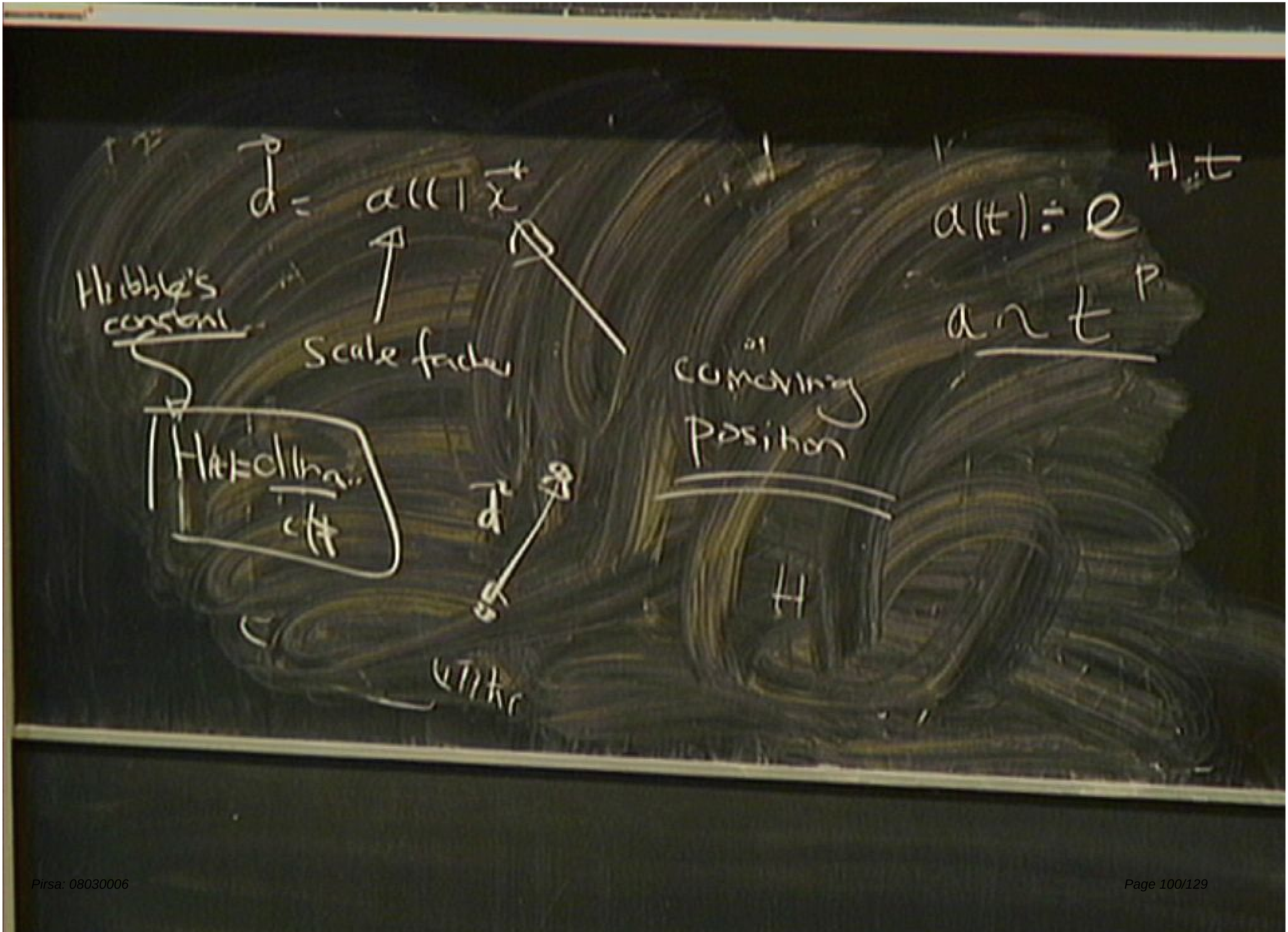

Scale factor
comoving
position
H

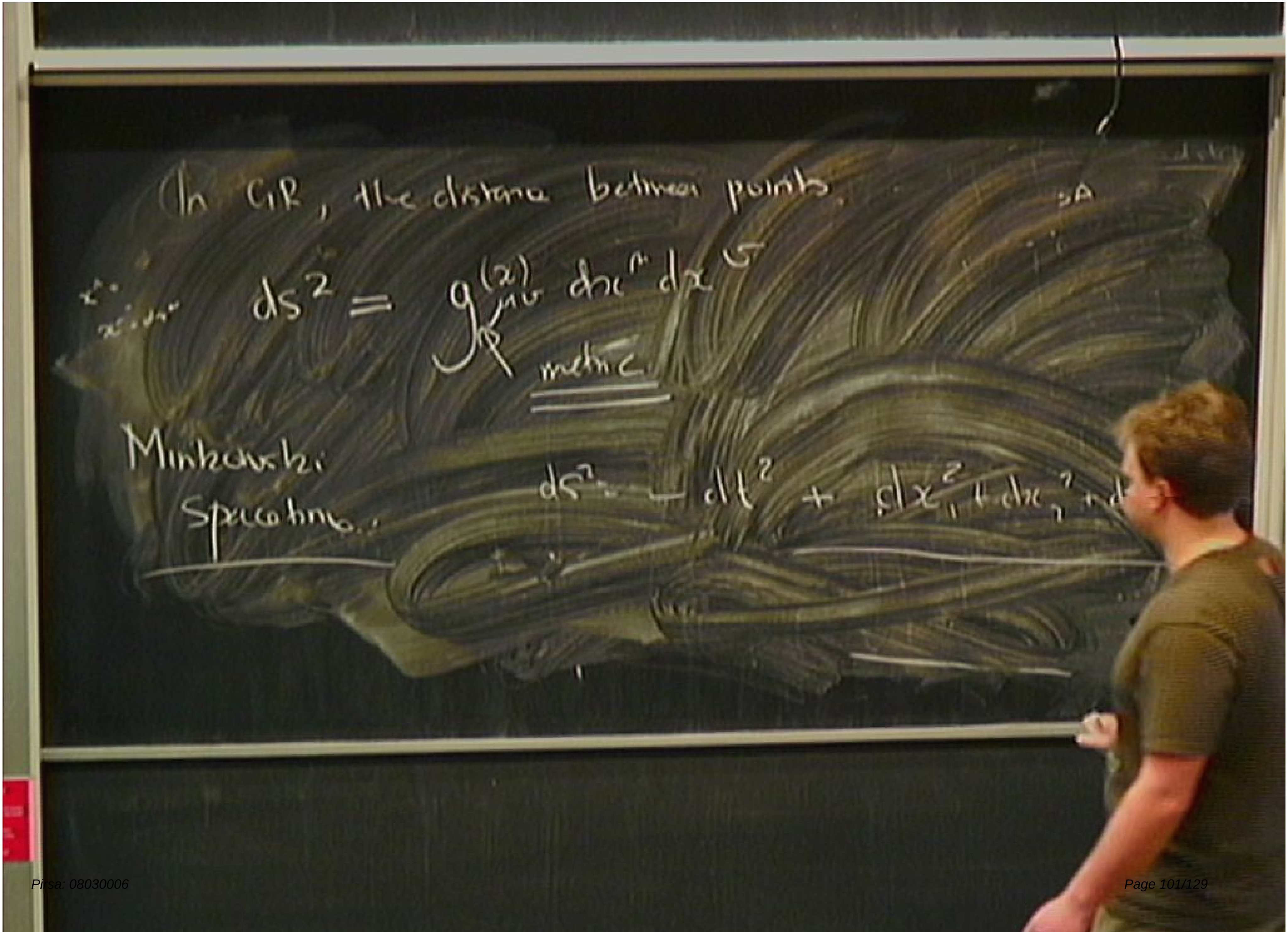

d = a(t) x
Scale factor
comoving
position
H = dlna/dt
H


Hubble's constant
Scale factor
comoving position
H



d = a(t) x
Hubble's constant
Scale factor
comoving position
a(t) = e^{H₀t}
a ~ t^p


d = a(t) x
Hubble's constant
Scale factor
comoving position
a(t) = e^{H t}
a ~ t^p
H


In GR, the distance between points
ds² = g_μν(x) dx^μ dx^ν
metric
Minkowski Spacetime:
ds² = − dt² + dx₁² + dx₂² + d

In GR, the distance between points

$$ds^2 = g_{\mu\nu}(x)\, dx^\mu dx^\nu$$

metric

Minkowski Spacetime:

$$ds^2 = -dt^2 + dx_1^2 + dx_2^2 + dx_3^2$$

In GR, the distance between points

$$ds^2 = g_{\mu\nu}(x)\, dx^\mu dx^\nu$$

metric

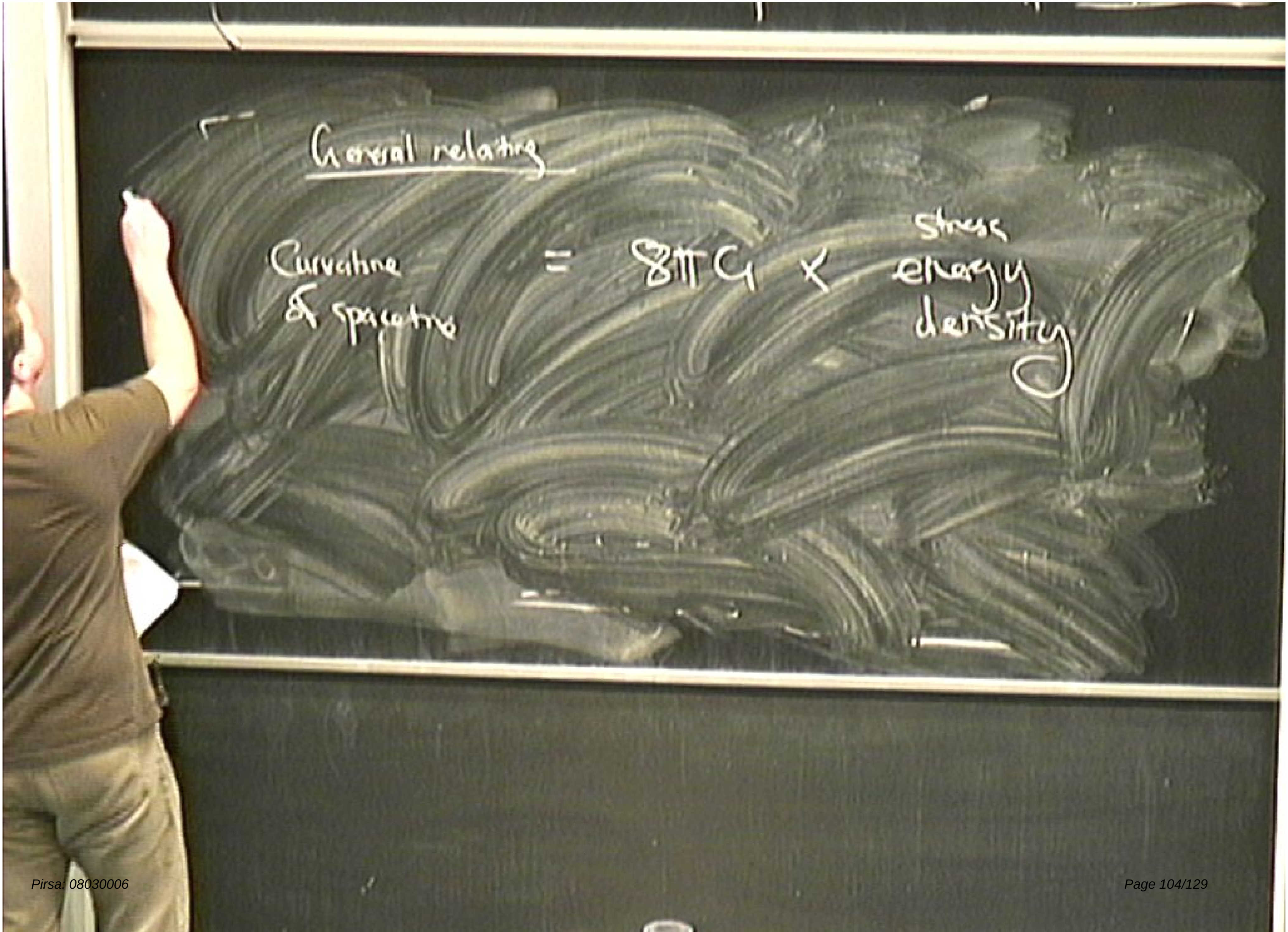
Minkowski spacetime:

$$ds^2 = -dt^2 + dx_1^2 + dx_2^2 + dx_3^2$$

$$ds^2 = -dt^2 + a^2(t)\,(dx_1^2 + dx_2^2 + dx_3^2)$$

$S^3,\ H^3$

FRWL Friedmann, Robertson, Walker, Lemaitre


General relativity
Curvature of spacetime = 8πG × stress energy density

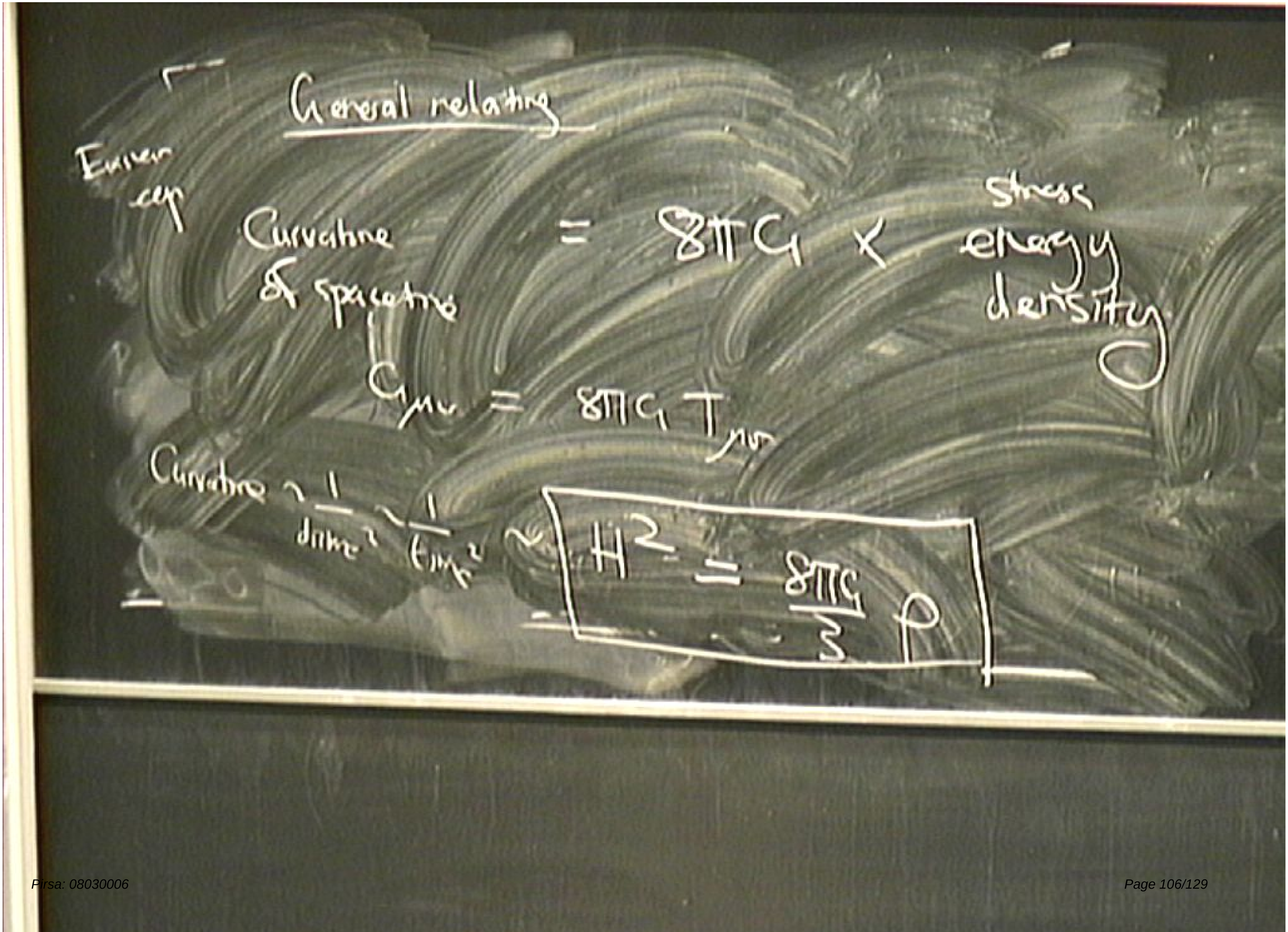
General relativity

Einstein eqn

Curvature of spacetime = $8\pi G$ × stress energy density

$$G_{\mu\nu} = 8\pi G \, T_{\mu\nu}$$

Curvature $\sim \frac{1}{\text{dist}^2} \sim \frac{1}{\text{time}^2} \sim H^2$


General relativity
Curvature of spacetime = 8πG × stress energy density
$G_{\mu\nu} = 8\pi G T_{\mu\nu}$
Curvature ~ $\frac{1}{dist^2}$ ~ $\frac{1}{time^2}$ ~ $H^2 = \frac{8\pi G}{3}\rho$

General relativity

Einstein eqn

Curvature of spacetime $=$ $8\pi G$ $\times$ stress energy density

$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

Friedmann equation

Curvature $\sim \frac{1}{\text{dist}^2} \sim \frac{1}{\text{time}^2} \sim$

$$H^2 = \frac{8\pi G}{3}\bar{\rho} - \frac{k}{a^2}$$

$k = -1, 0, 1$

General relativity

Einstein eqn

Curvature of spacetime $= 8\pi G \times$ stress energy density

$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

$$\text{Curvature} \sim \frac{1}{\text{dist}^2} \sim \frac{1}{\text{time}^2} \sim$$

Friedmann equation

$$H^2 = \frac{8\pi G}{3}\bar{\rho} + \frac{k}{R^2} \qquad k = -1, 0, 1$$

General relativity

Einstein eqn

Curvature of spacetime = $8\pi G$ × stress energy density

$H = \frac{da}{a\,dt}$

$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

Friedmann equation

Curvature $\sim \frac{1}{\text{dime}^2} \sim \frac{1}{\text{time}^2} \sim$

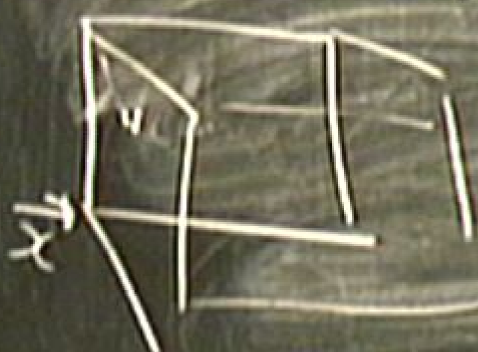
$$H^2 = \frac{8\pi G}{3}\bar{\rho}(t) - \frac{k}{a^2}$$

$k = -1, 0, 1$

Conservation of energy

$$\nabla^\nu T_{\mu\nu} = 0$$

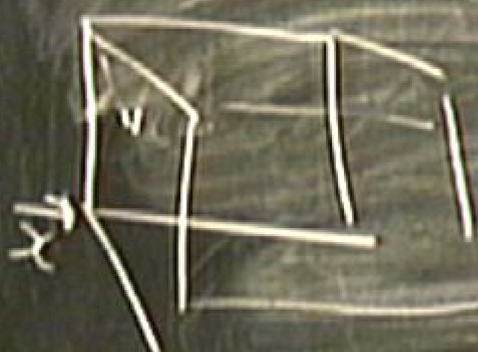
$$\frac{d}{dt}\Big($$



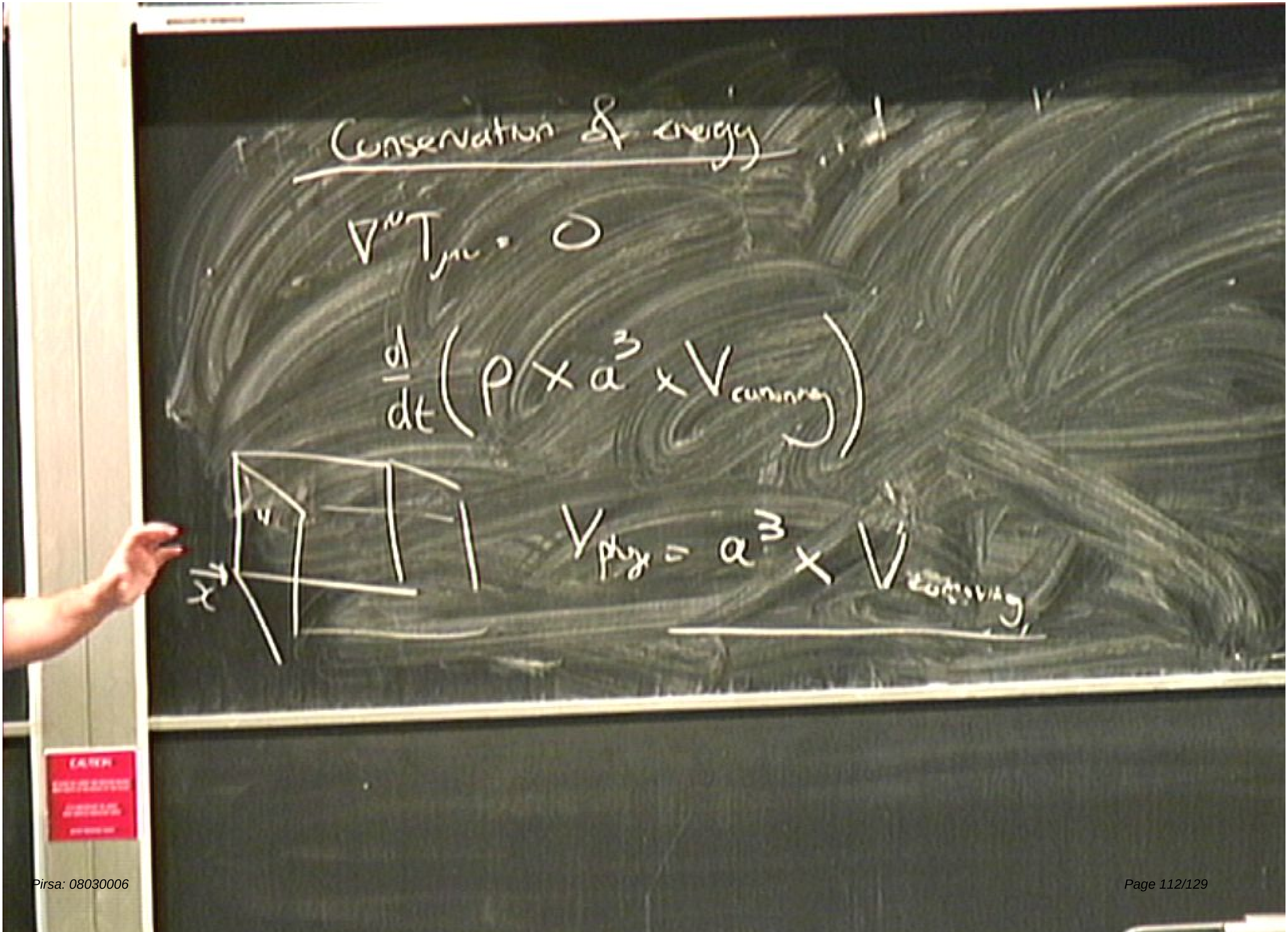
Conservation of energy

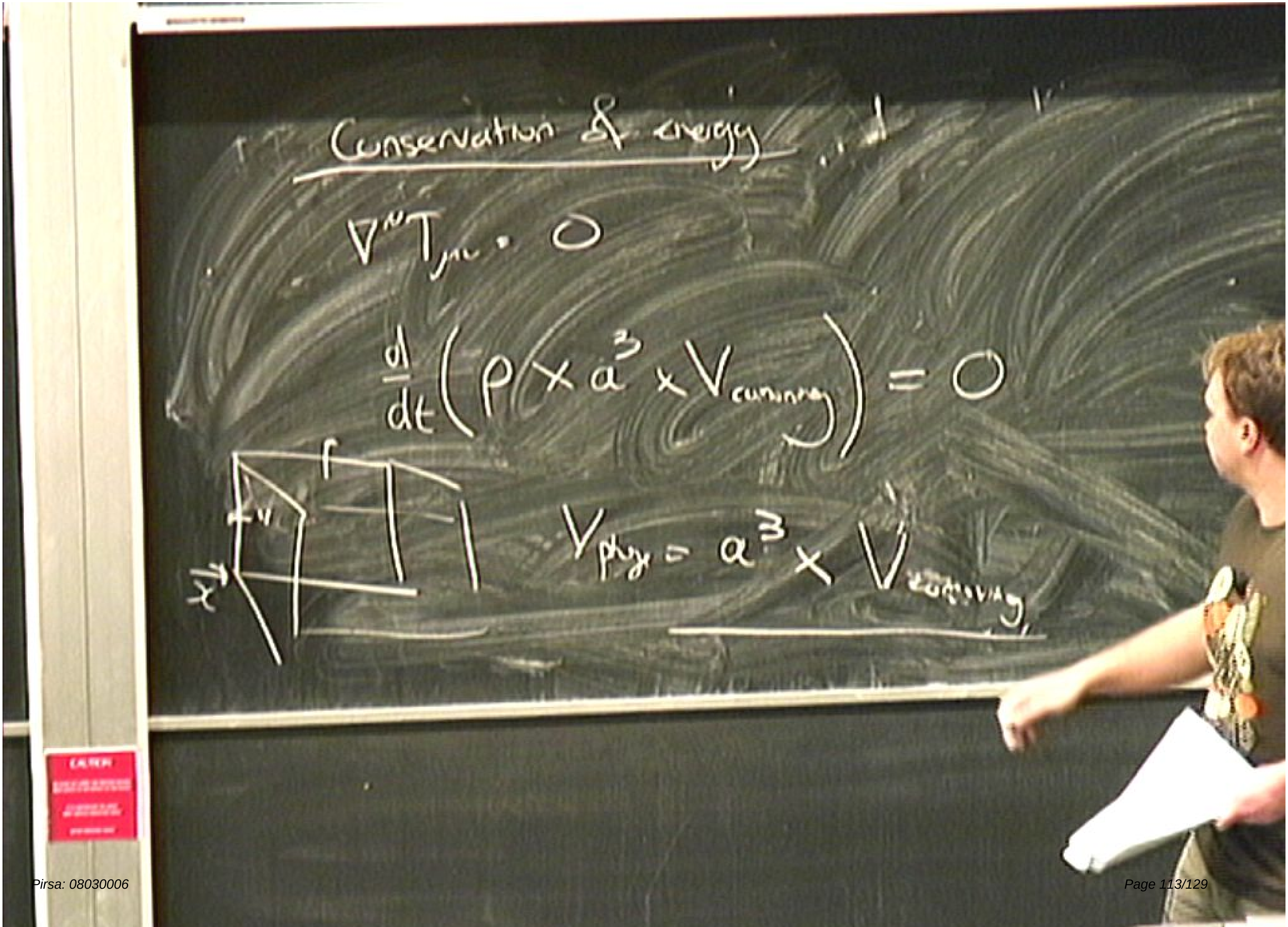
$$\nabla^\nu T_{\mu\nu} = 0$$

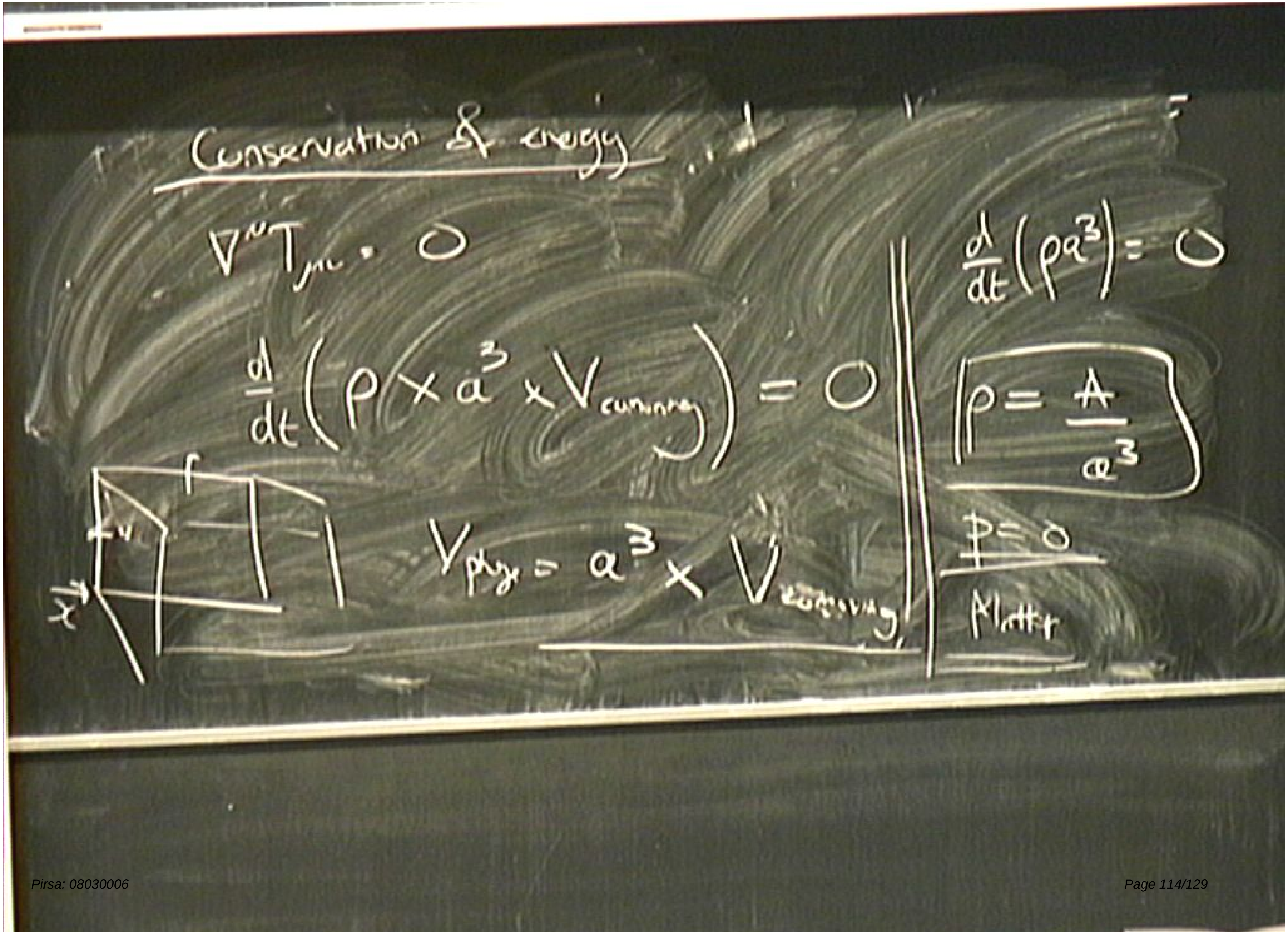
$$\frac{d}{dt}\Big($$

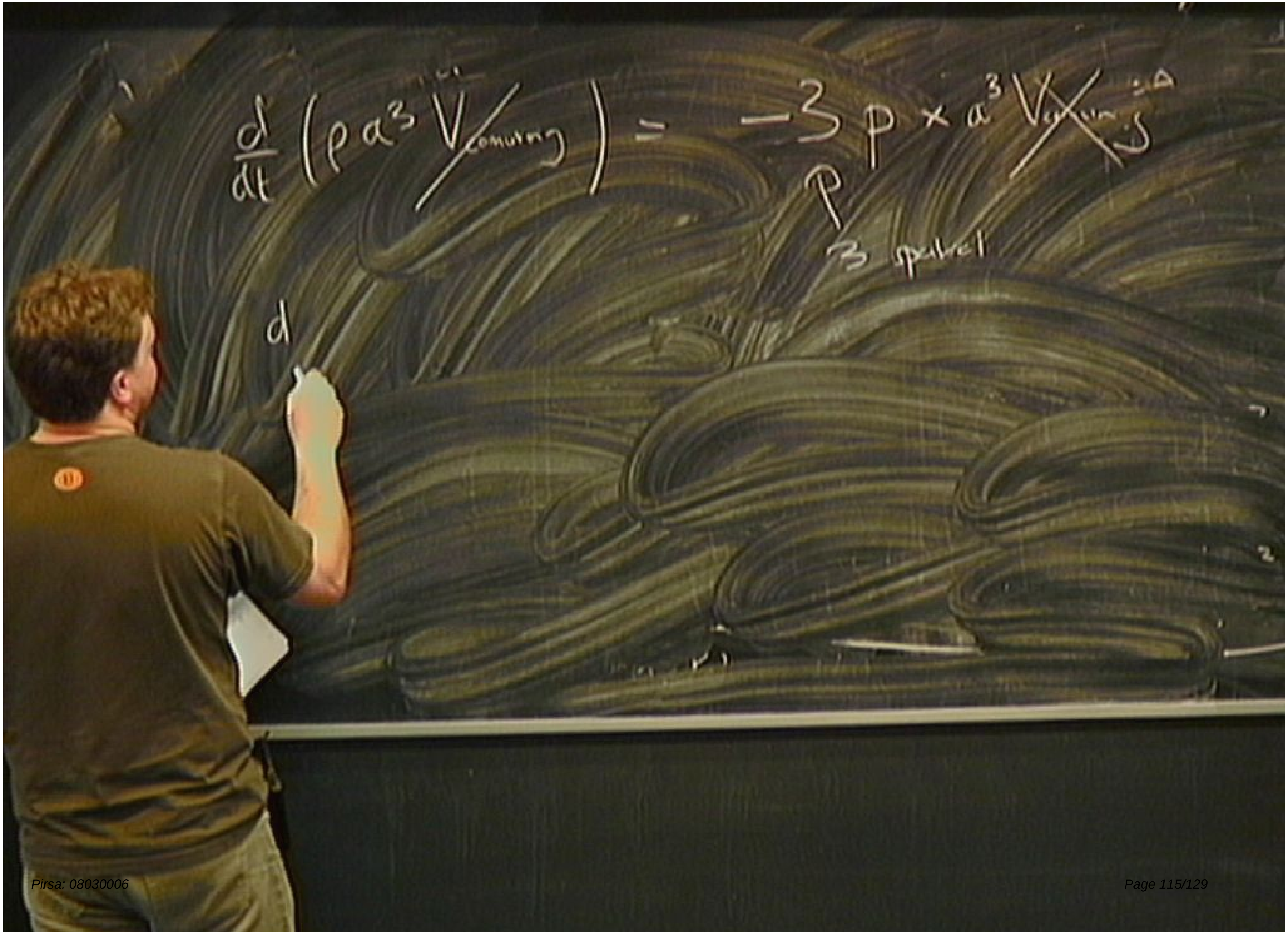


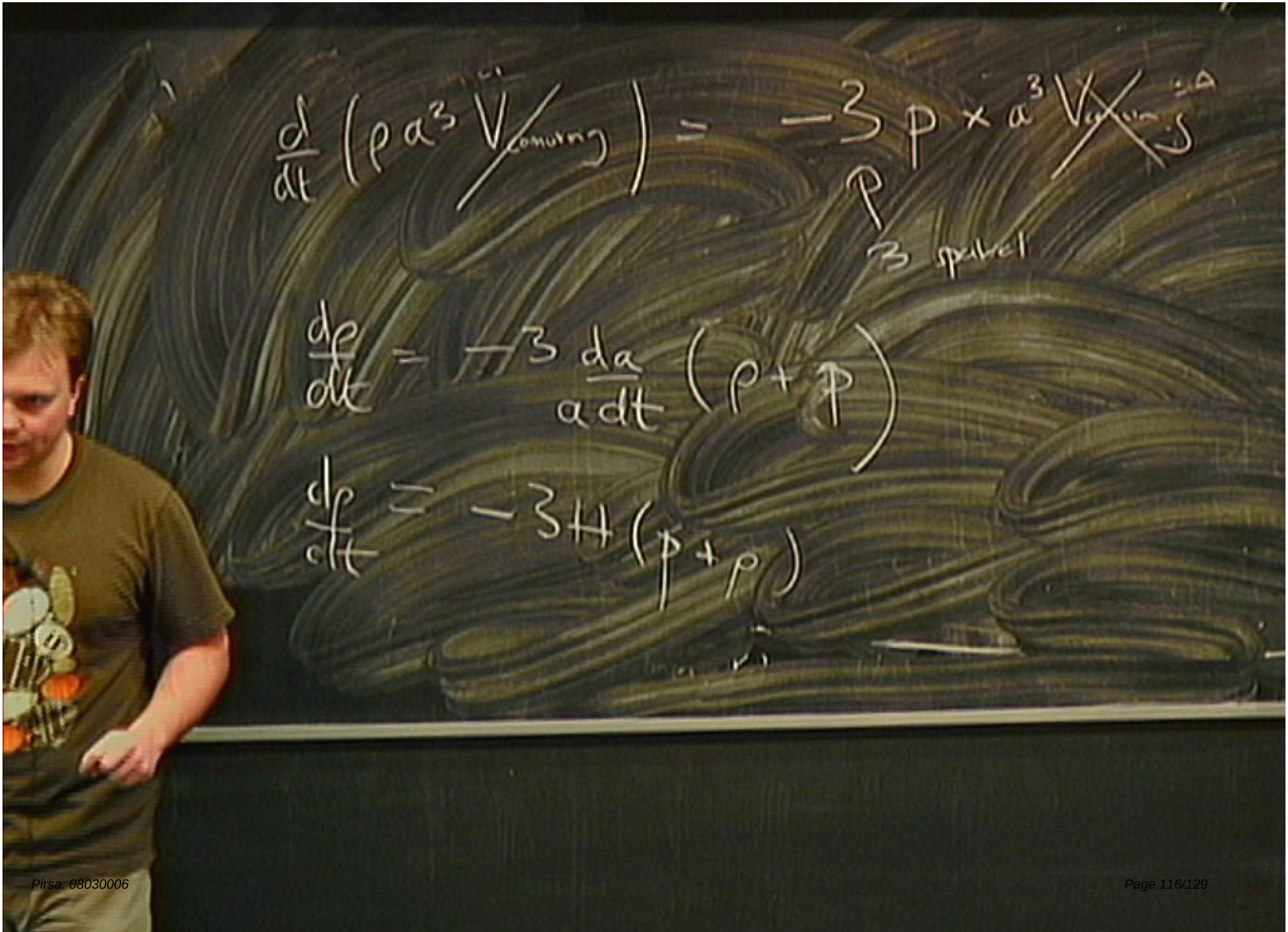
$$V_{phys} = a^3 \times V_{comoving}$$


Conservation of energy
$\nabla^\nu T_{\mu\nu} = 0$
$\frac{d}{dt}(\rho \times a^3 \times V_{comoving})$
$V_{phys} = a^3 \times V_{comoving}$


Conservation of energy
∇^μ T_μν = 0
d/dt (ρ × a^3 × V_comoving) = 0
V_phys = a^3 × V_comoving


Conservation of energy
$\nabla^\nu T_{\mu\nu} = 0$
$\frac{d}{dt}\left(\rho \times a^3 \times V_{comoving}\right) = 0$
$V_{phys} = a^3 \times V_{comoving}$
$\frac{d}{dt}(\rho a^3) = 0$
$\rho = \frac{A}{a^3}$
$P = 0$
Matter


comoving


d/dt (ρ a³ V_comoving) = −3 p × a³ V_comoving
dρ/dt = −3 da/(a dt) (ρ + p)
dρ/dt = −3H (p + ρ)

$$\frac{d}{dt}\left(\rho a^3 V_{comoving}\right) = -3 p \times a^3 V_{comoving}$$

p

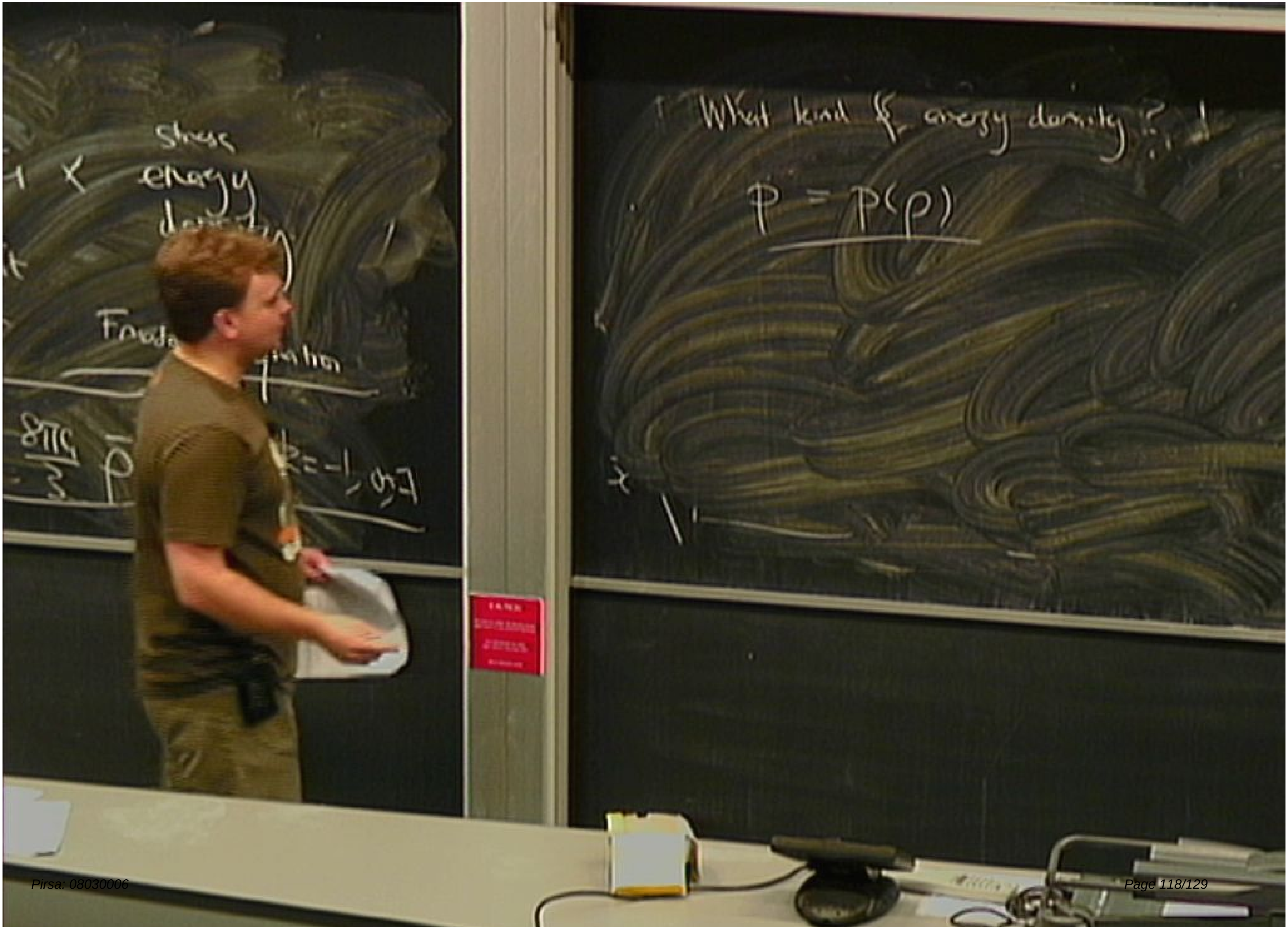
3 pulsel

$$\frac{d\rho}{dt} = -3\frac{da}{a\,dt}\left(\rho + p\right)$$

$$\frac{d\rho}{dt} = -3H\left(\rho + p\right)$$

$\nabla^\mu T_{\mu\nu} = 0$

conservation energy

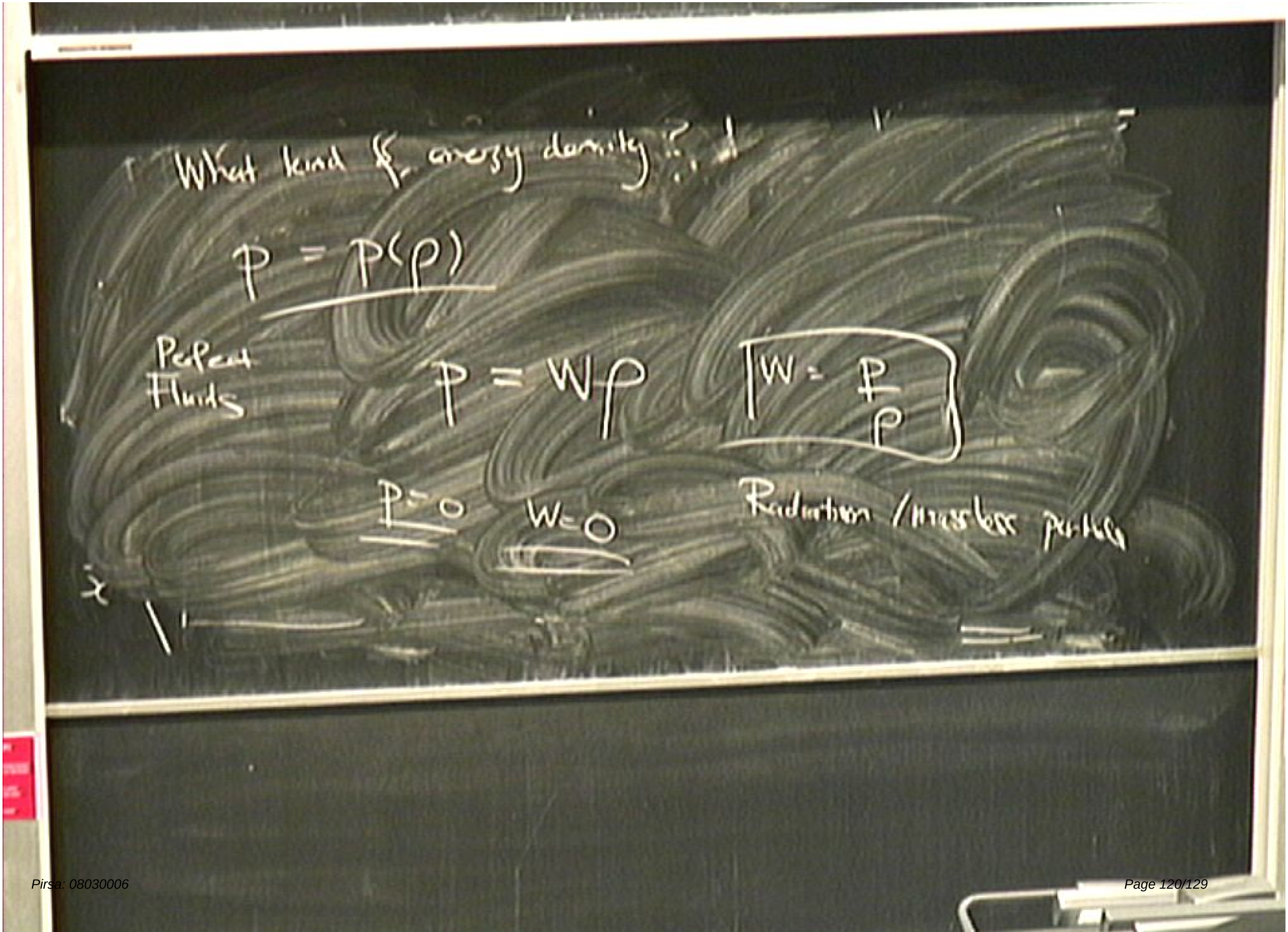

stress
energy
What kind of energy density?
P = P(ρ)

What kind of energy density?

$$P = P(\rho)$$

Perfect Fluids

$$P = w\rho \qquad \boxed{w = \frac{P}{\rho}}$$


What kind of energy density?
P = P(ρ)
Perfect Fluids
P = wρ
w = P/ρ
P=0
w=0
Radiation / massless particles

What kind of energy density?

$$p = p(\rho)$$

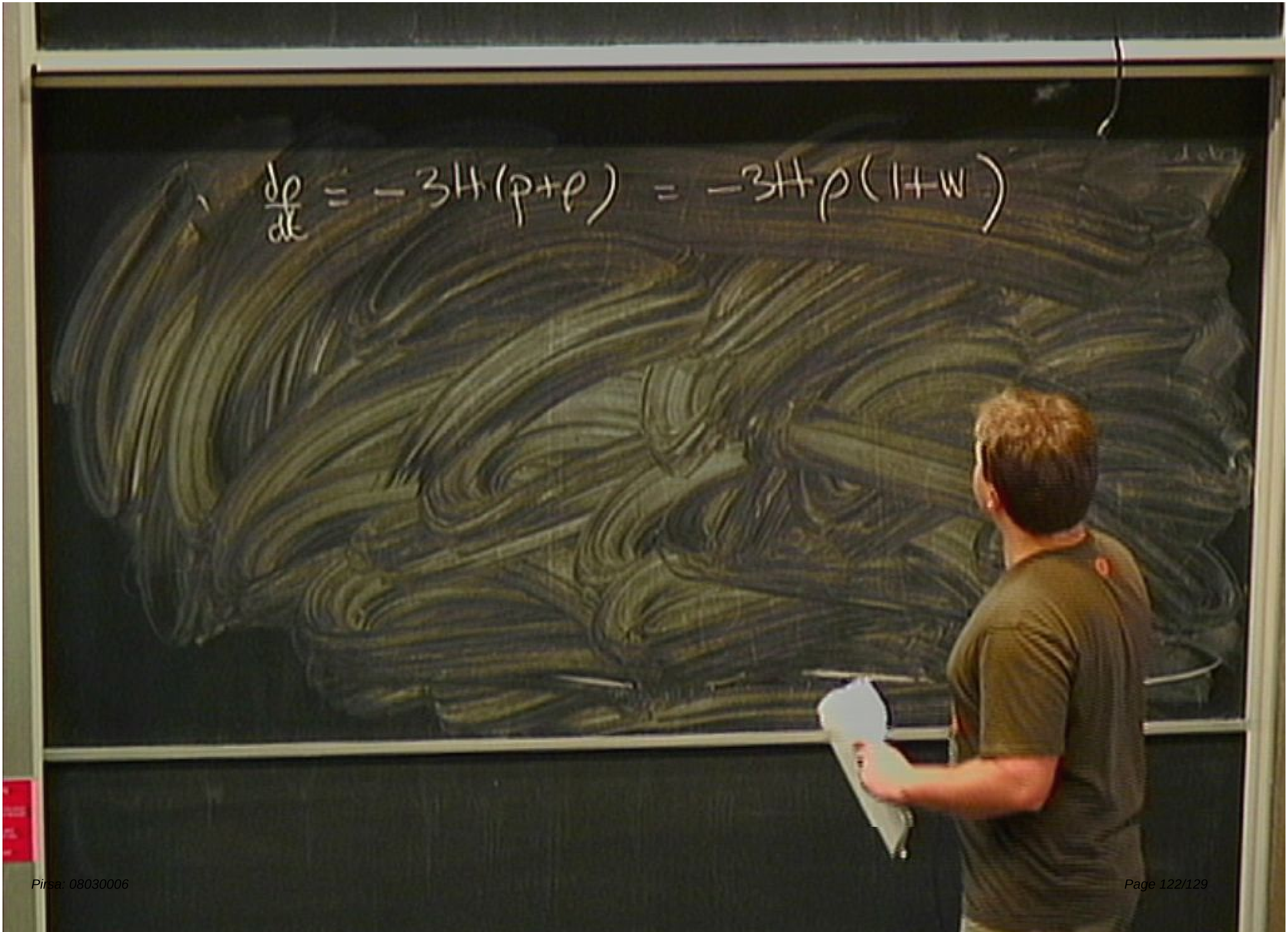
Perfect Fluids

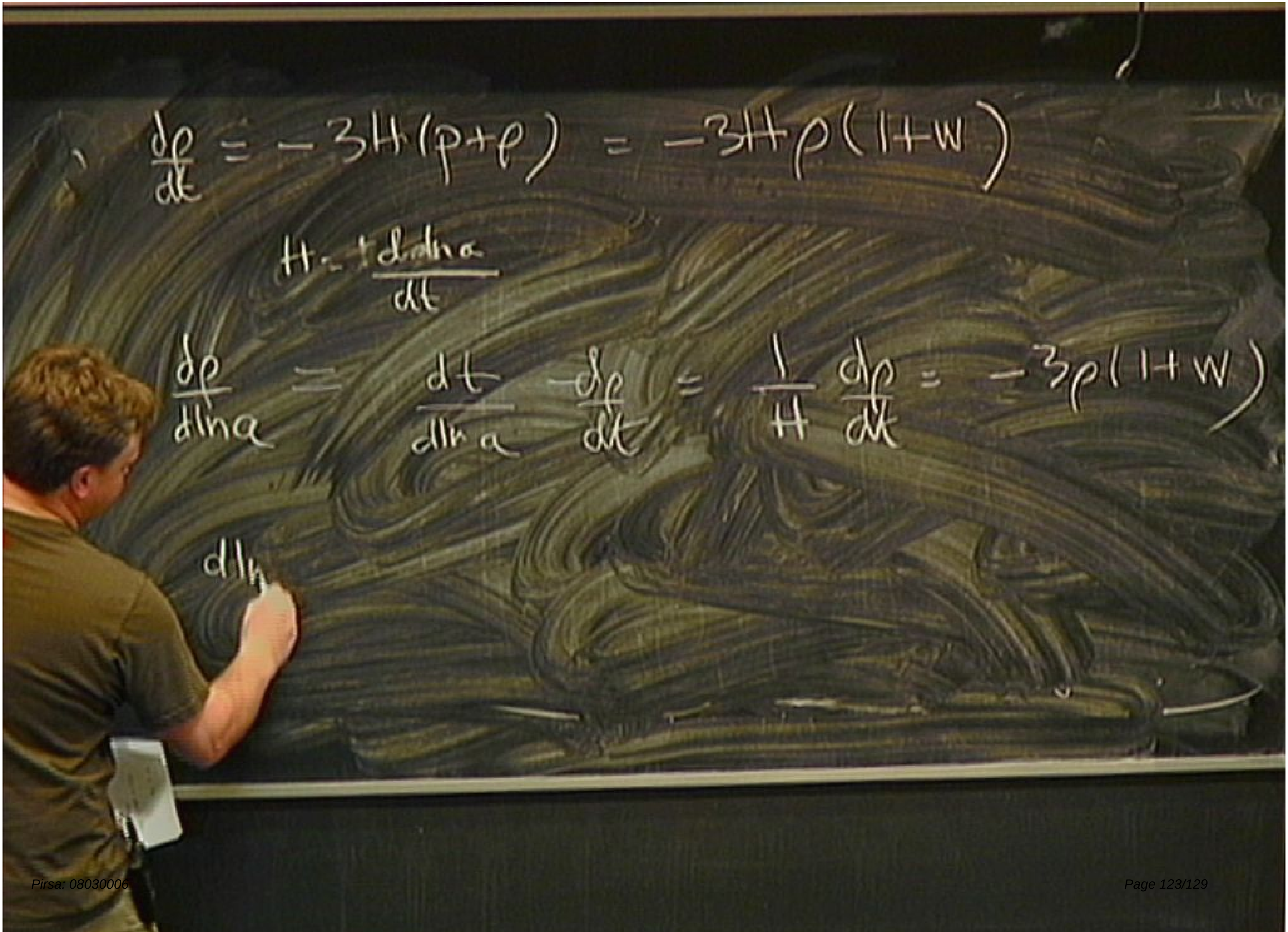
$$p = w\rho \qquad w = \frac{p}{\rho}$$

$$p = 0 \qquad w = 0$$

Radiation / massless particles

$$p = \frac{1}{3}\rho$$


dρ/dt = −3H(p+ρ) = −3Hρ(1+w)


$\frac{d\rho}{dt} = -3H(p+\rho) = -3H\rho(1+w)$
$H = \frac{d\ln a}{dt}$
$\frac{d\rho}{d\ln a} = \frac{dt}{d\ln a}\frac{d\rho}{dt} = \frac{1}{H}\frac{d\rho}{dt} = -3\rho(1+w)$
d ln

$$\frac{d\rho}{dt} = -3H(p+\rho) = -3H\rho(1+w)$$

$$H = \frac{d\ln a}{dt}$$

$$\frac{d\rho}{d\ln a} = \frac{dt}{d\ln a}\frac{d\rho}{dt} = \frac{1}{H}\frac{d\rho}{dt} = -3\rho(1+w)$$

$$\frac{d\ln\rho}{d\ln a} = -3(1+w) \Rightarrow \rho = \rho_0\left(\frac{a_0}{a(t)}\right)^{3(1+w)}$$

$$\rho \sim a^{-3(1+w)}$$

$$H^2 = \frac{8\pi G}{3} \cdot \frac{\rho_0 a_0^{3(1+w)}}{a^{3(1+w)}}$$

$$\frac{1}{a}\frac{da}{dt} = \sqrt{\frac{8\pi G}{3} \frac{\rho_0 a_0^{3(1+w)}}{a^{3(1+w)}}}$$

$\ddot{x}$

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$$\frac{da}{dt} \sim B\, a^{1 - \frac{3(1+w)}{2}}$$

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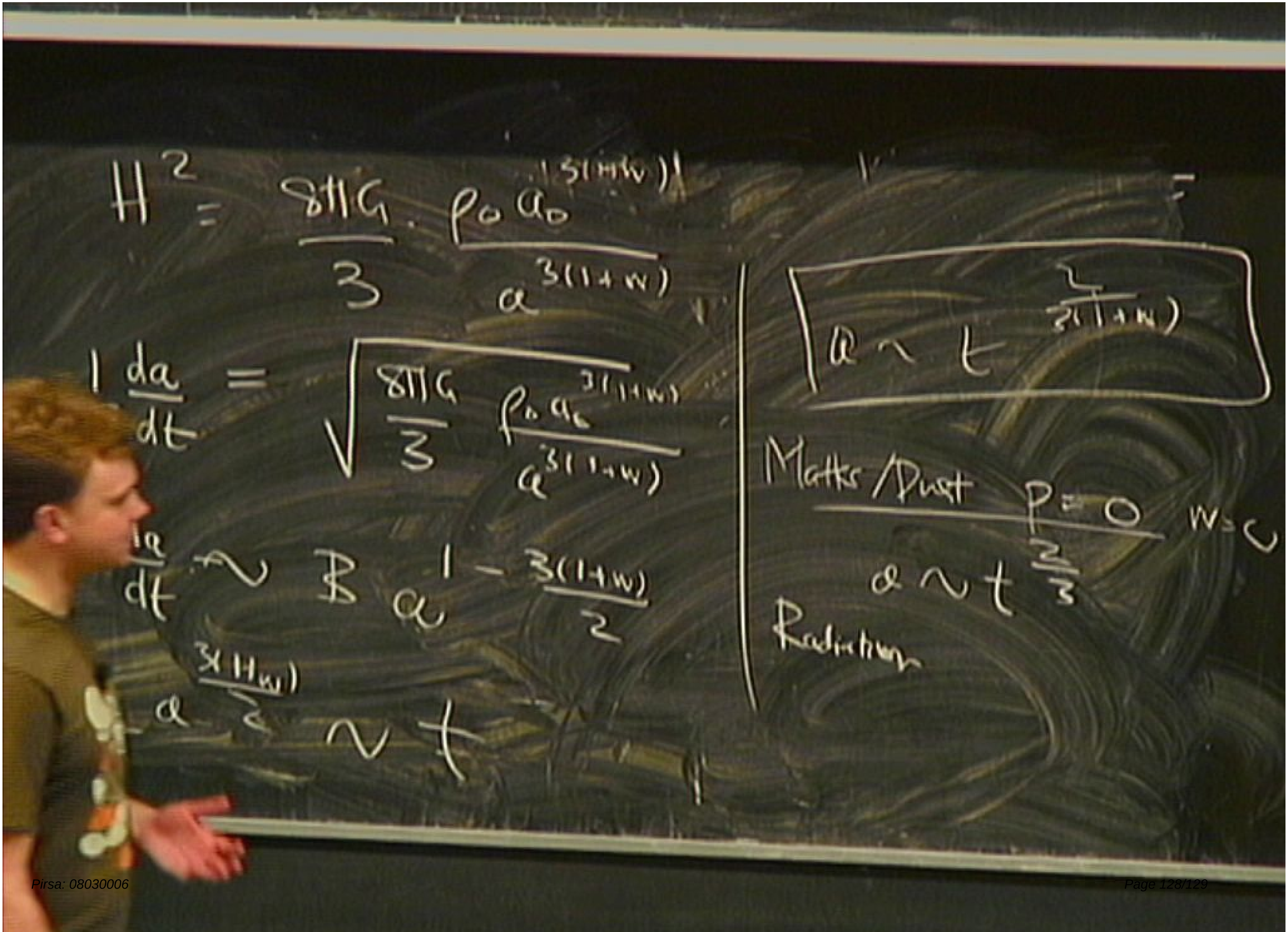
$$a^{\frac{3(1+w)}{2}} \sim t$$

$$a \sim t^{\frac{2}{3(1+w)}}$$


Matter / Dust
P = 0
w = 0
Radiation

$$H^2 = \frac{8\pi G}{3} \frac{\rho_0 a_0^{3(1+w)}}{a^{3(1+w)}}$$

$$\frac{1}{a}\frac{da}{dt} = \sqrt{\frac{8\pi G}{3} \frac{\rho_0 a_0^{3(1+w)}}{a^{3(1+w)}}}$$

$$\frac{da}{dt} \sim B\, a^{1 - \frac{3(1+w)}{2}}$$

$$a^{\frac{3(1+w)}{2}} \sim t$$

$$\boxed{a \sim t^{\frac{2}{3(1+w)}}}$$

Matter/Dust $P = 0$ $w = 0$

$$a \sim t^{\frac{2}{3}}$$

Radiation $P = \frac{1}{3}\rho$ $w = \frac{1}{3}$

$$a \sim t^{\frac{1}{2}}$$