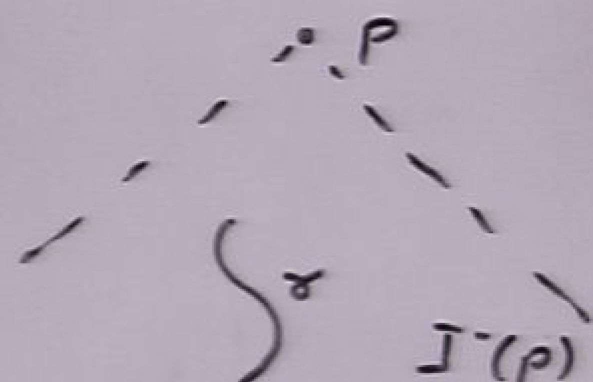
Title: Computation and Physics

Date: Mar 12, 2008 02:00 PM

URL: http://pirsa.org/08030001

Abstract: There are two notions that play a central role in the mathematical theory of computation. One is that of a computable problem, i.e., of a problem that can, in principle, be solved by an (idealized) computer. It is known that there exist problems that \'have answers\', but for which those answers are not computable. The other is that of the difficulty of a computation, i.e. of the number of (idealized) steps required actually to carry out that computation. It is known that, given any appropriate \'degree of difficulty\', there exists a problem that, while computable, is at least that difficult. These two notions, while purely mathematical, are designed to reflect, in some broad sense, the physics of the computation process. But there are indications that physics may have something further to say about them. Indeed, it has been suggested that, by using general relativity, some problems that are (mathematically) non-computable may become computable; and that, by using quantum mechanics, some problems that are (mathematically) difficult may become less so. Are there, in principle, any limitations on what physics can do for us in this area?

A space-time is Malament-Hogarth if it contains a timelike curve γ, of infinite length into the future, such that γ lies entirely in the past of some point p.



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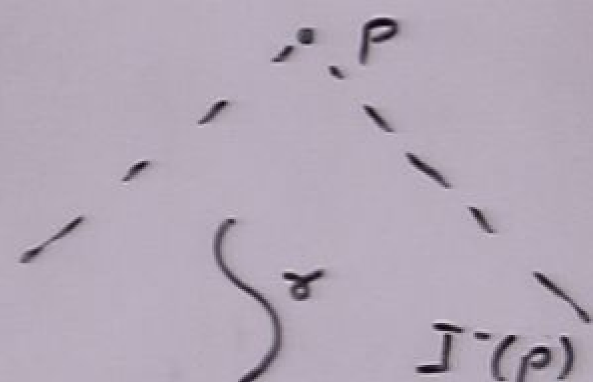
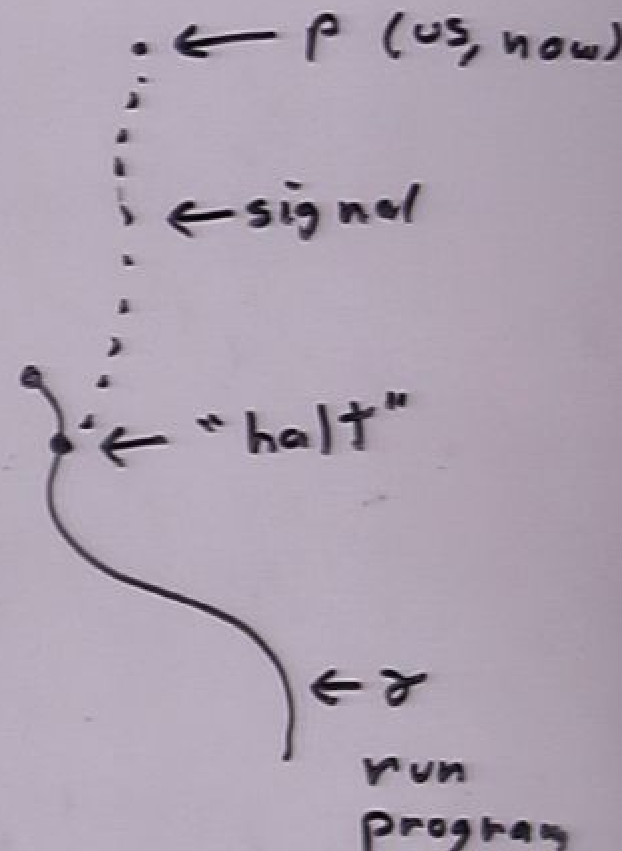


Solving the Halting Problem in a M-H Spacetime



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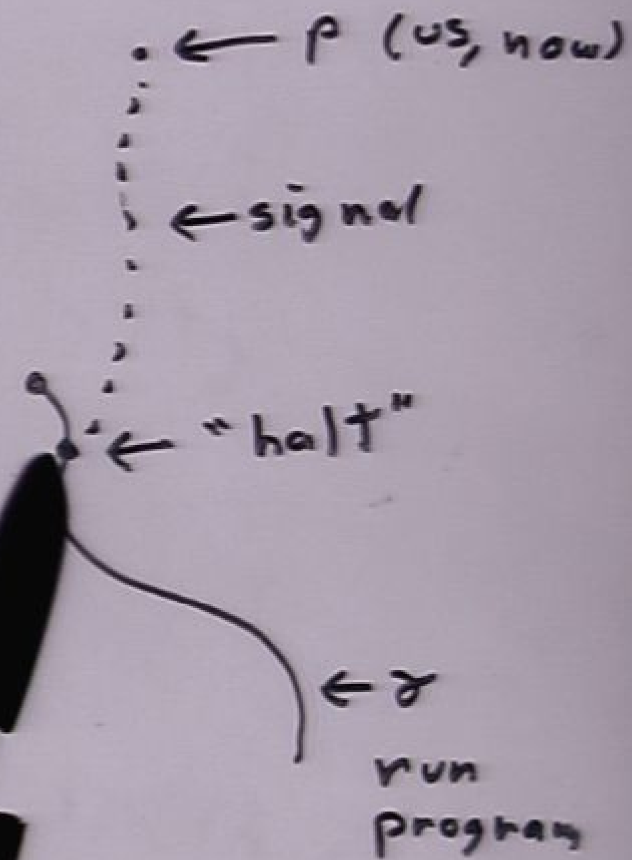

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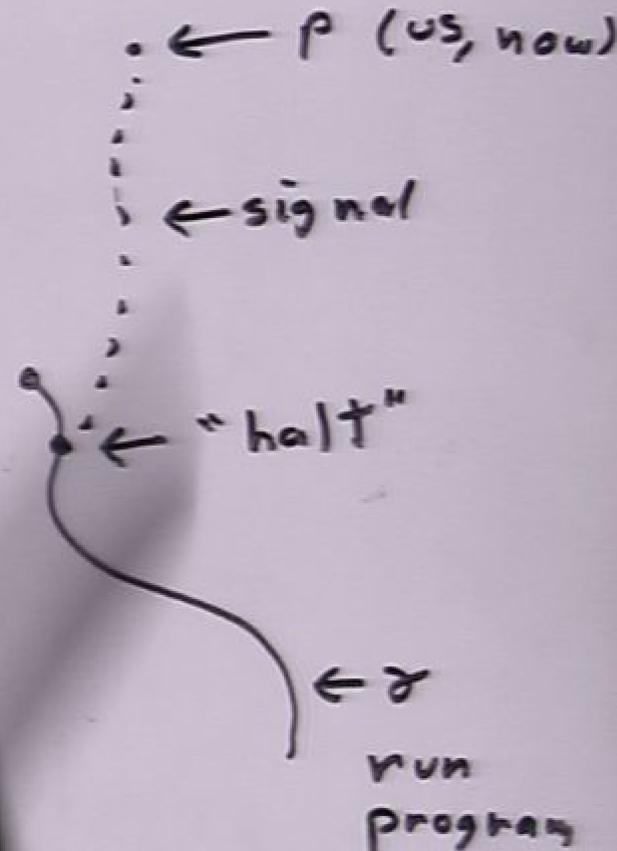
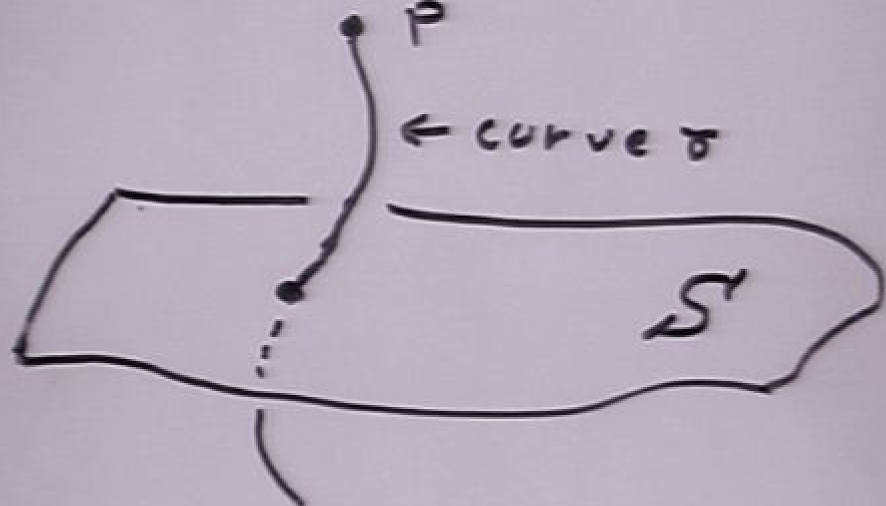


Cauchy Surface

Spacelike surface S such that every timelike or null curve ("every signal") meets S.



The spacetimes one can "build" are the spacetimes with Cauchy surfaces.

Theorem: In a space-time with a Cauchy surface, no infinite-length time-like curve lies entirely in the past of a point p.

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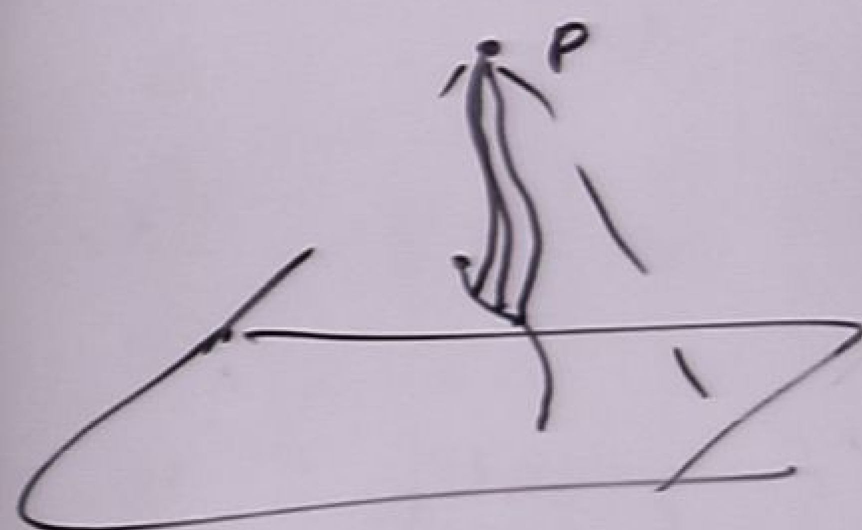

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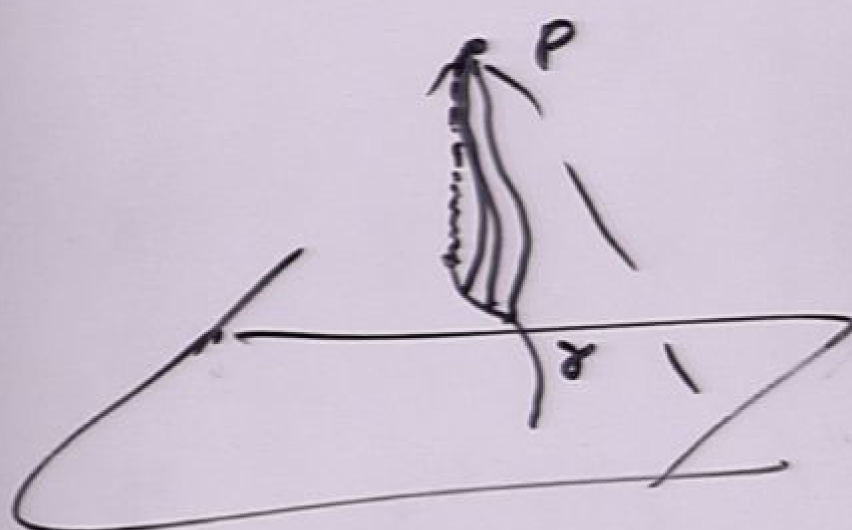

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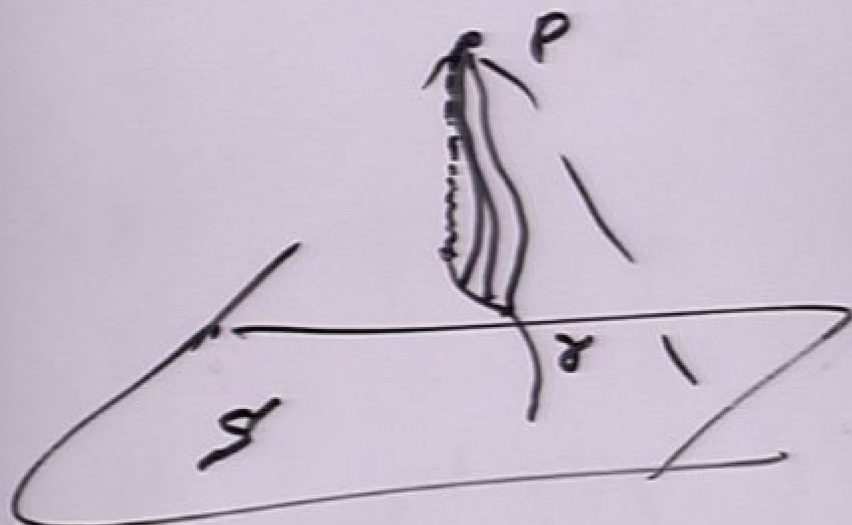

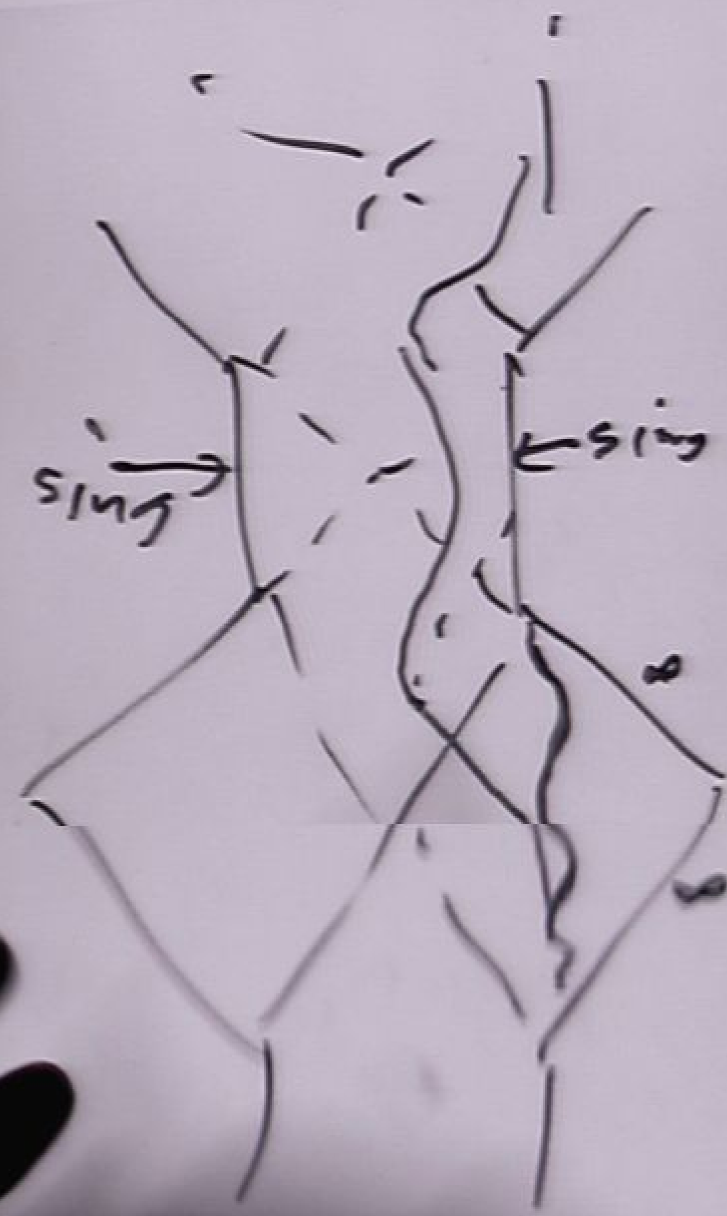
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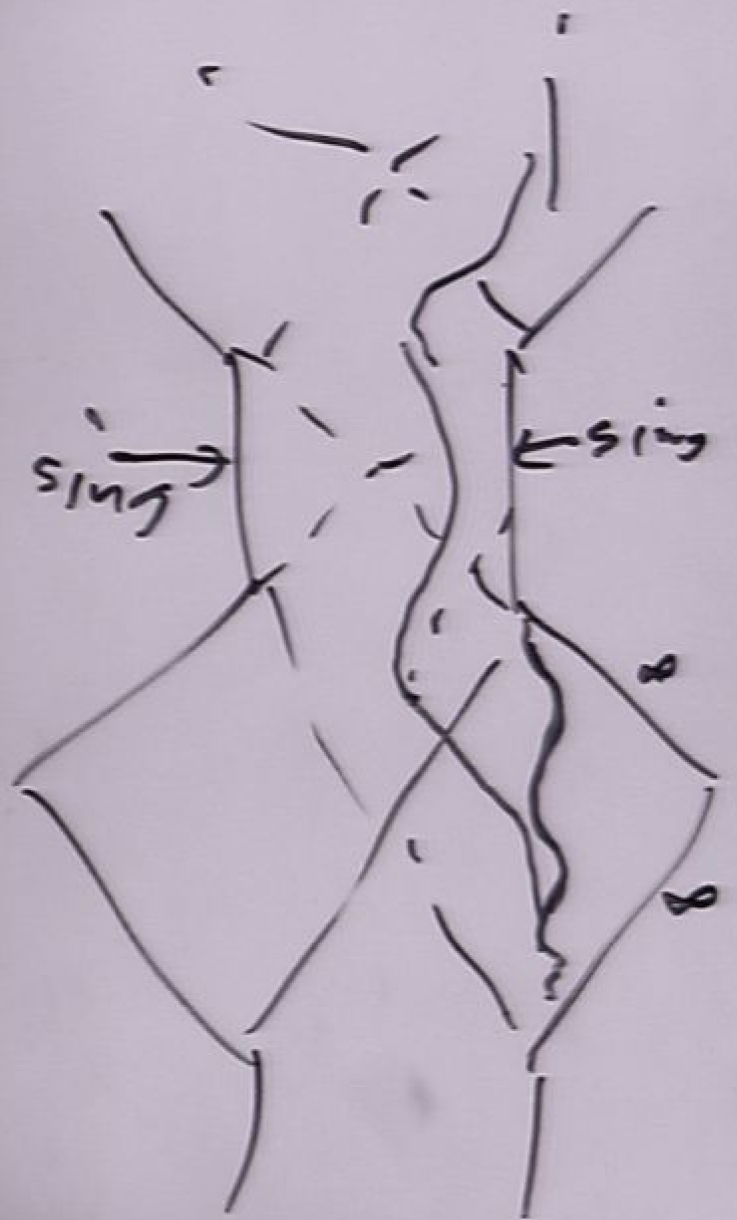
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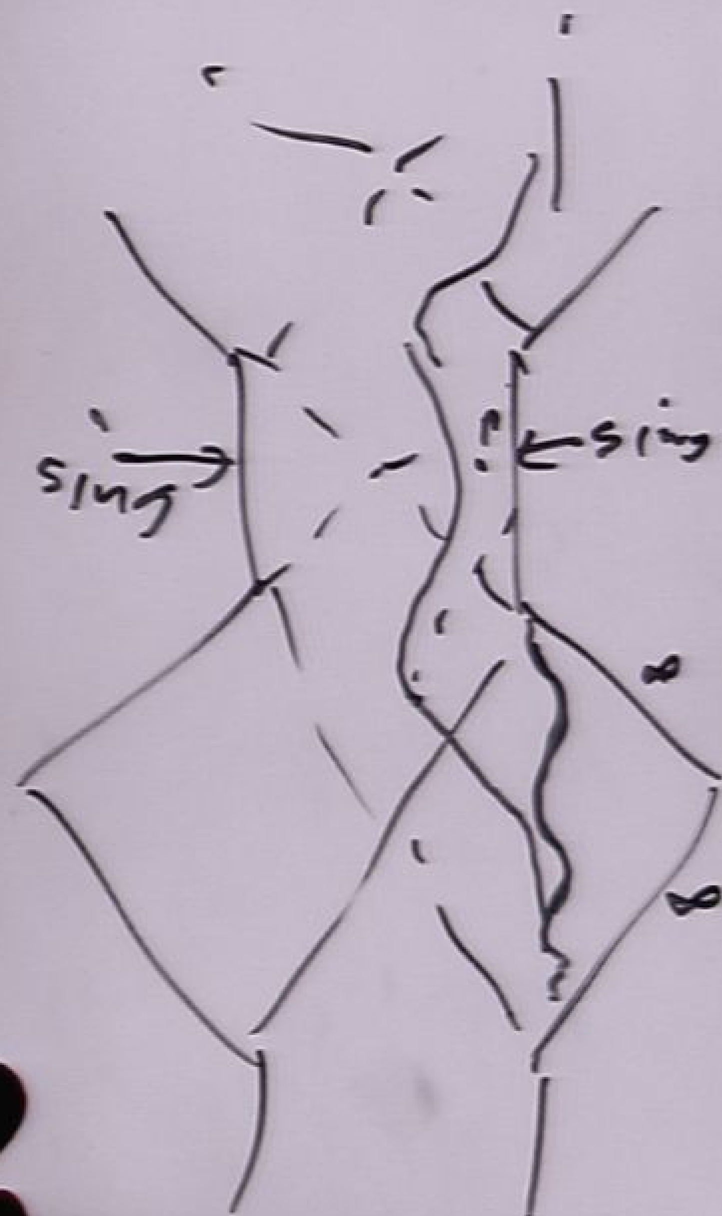
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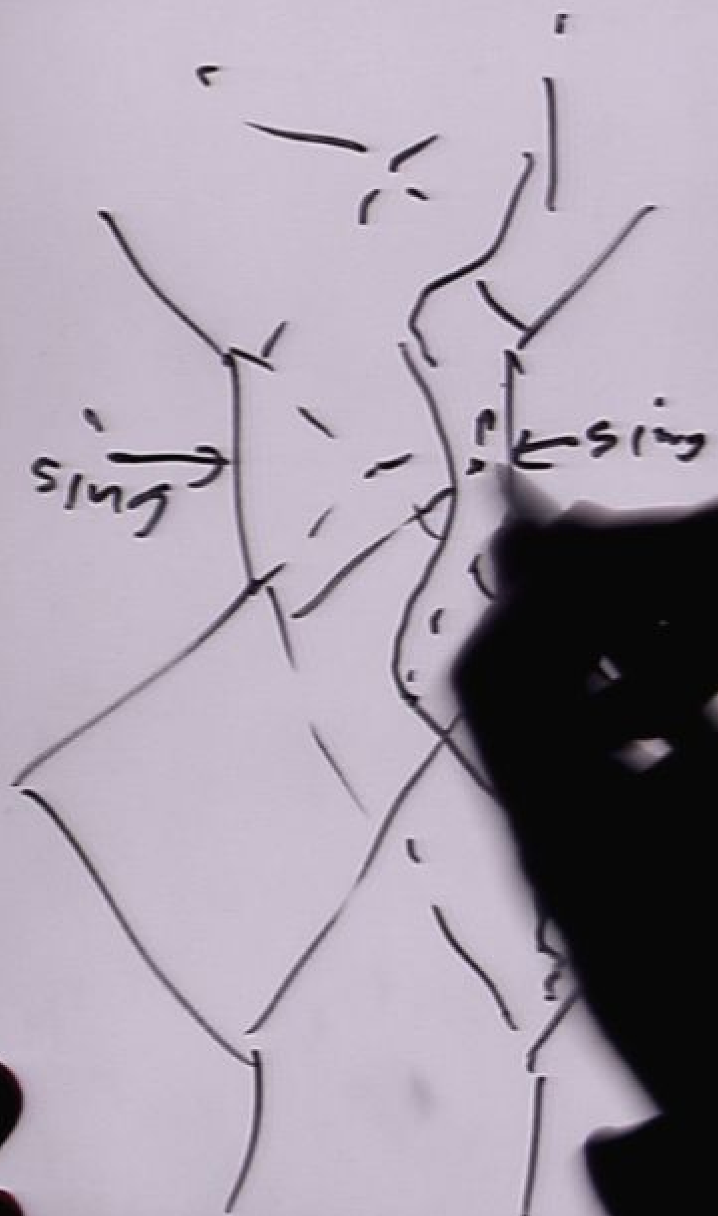
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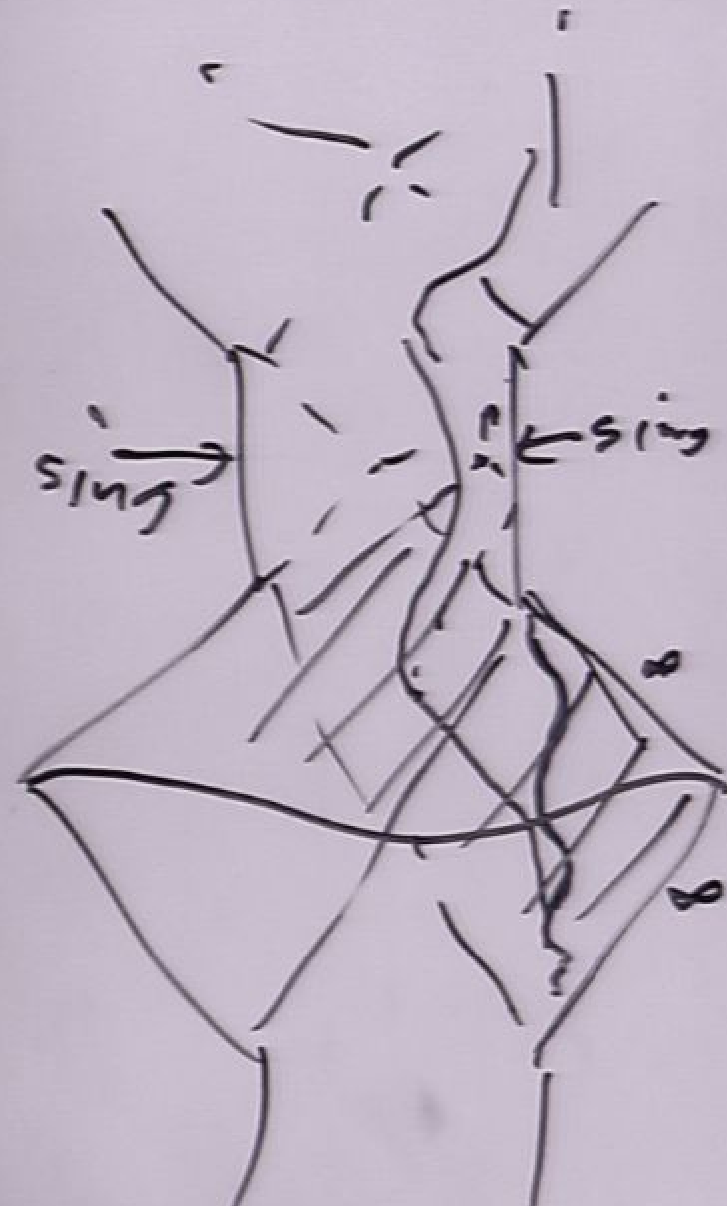
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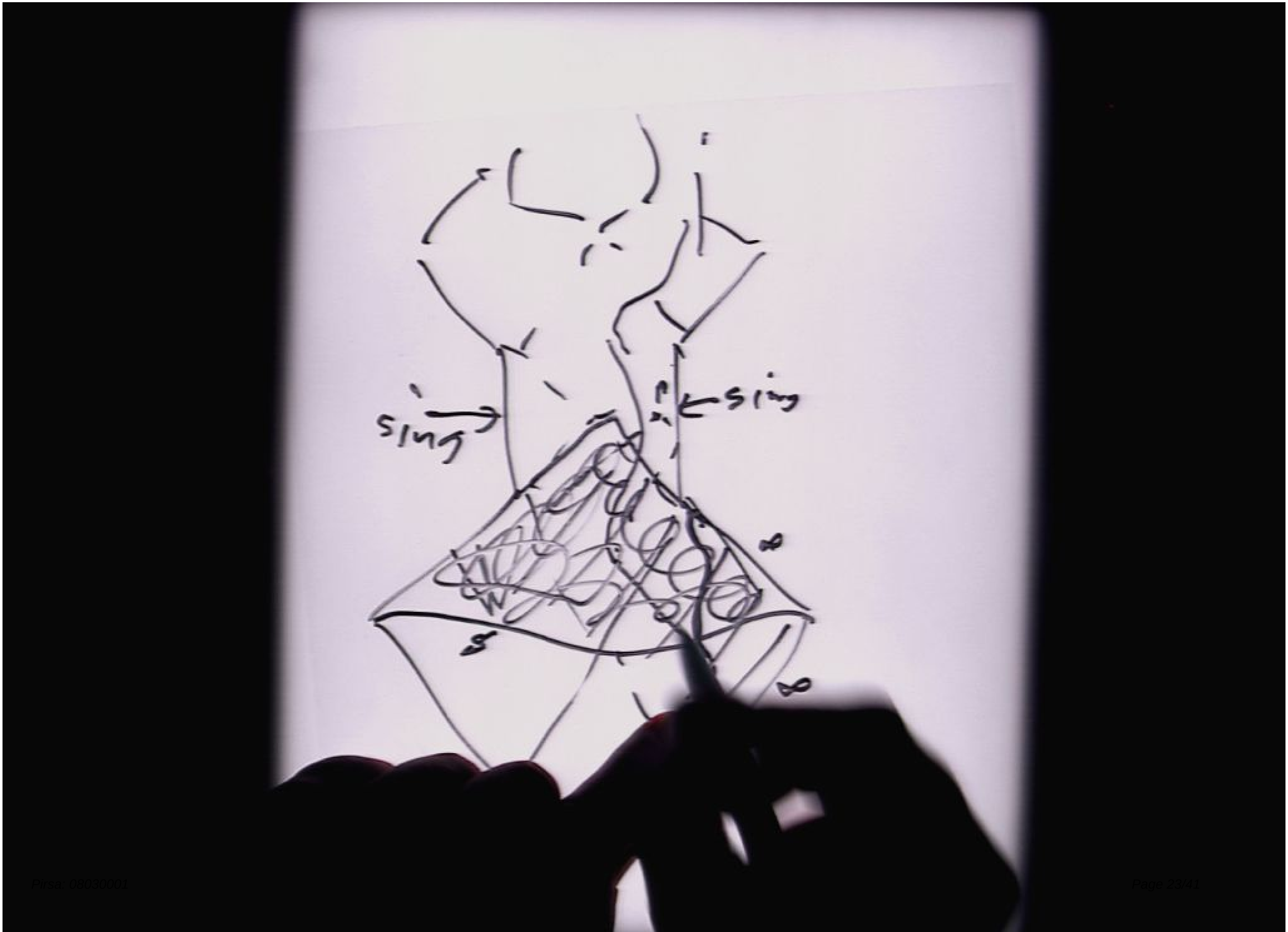
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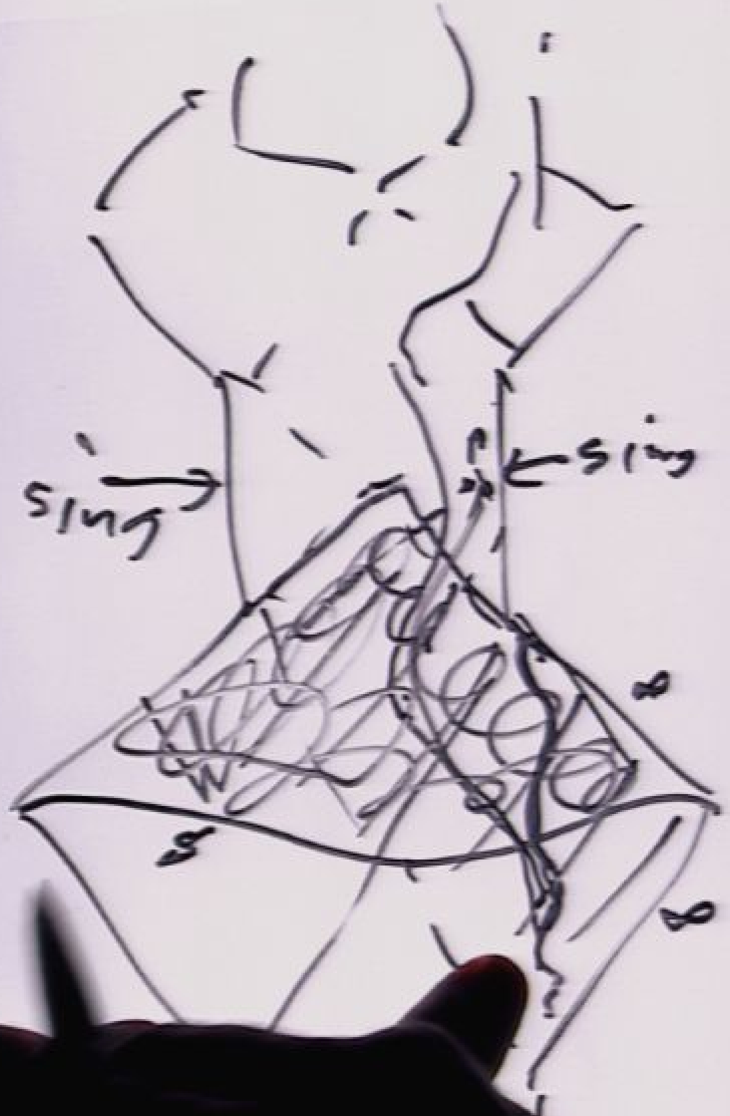
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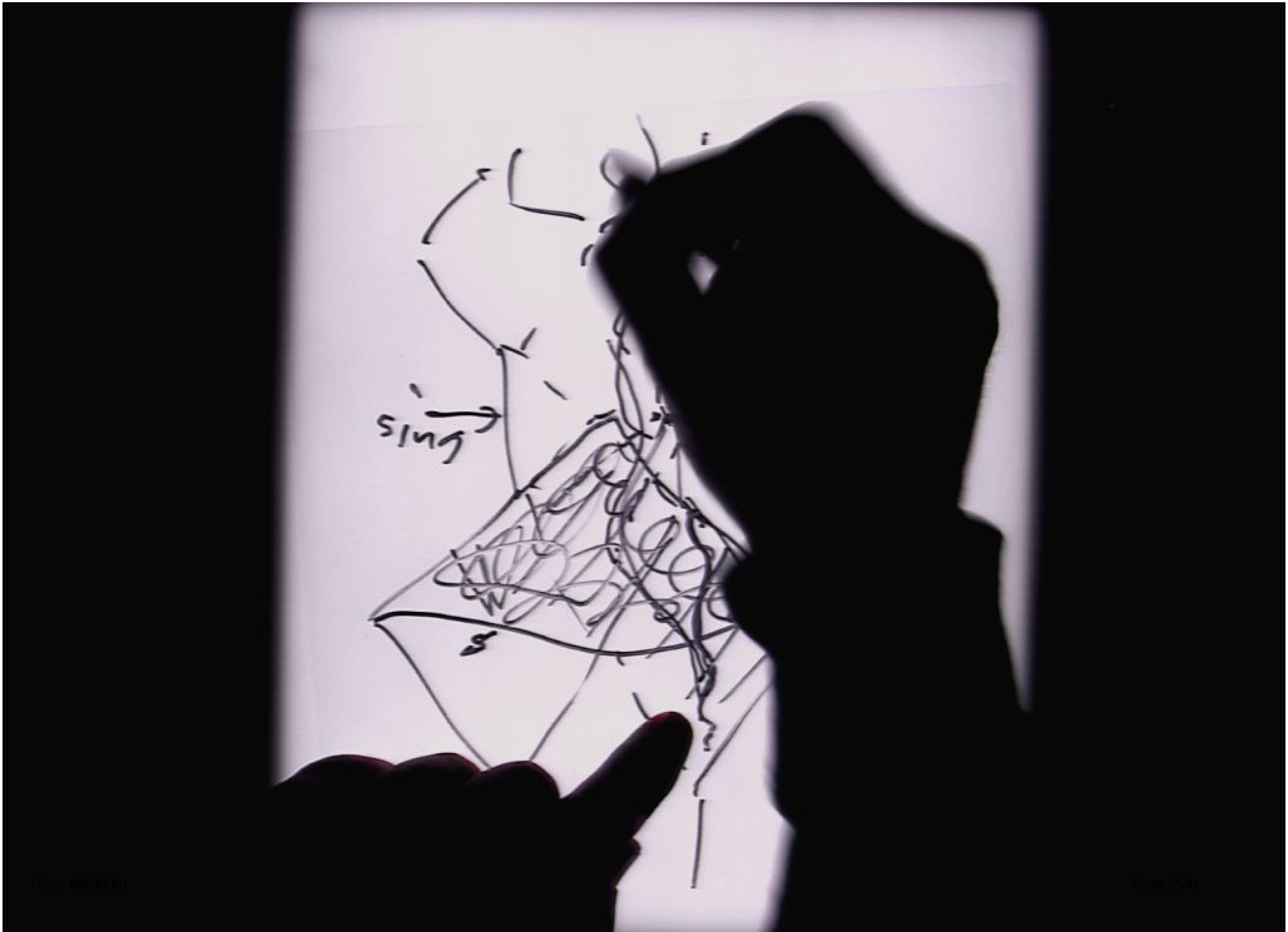
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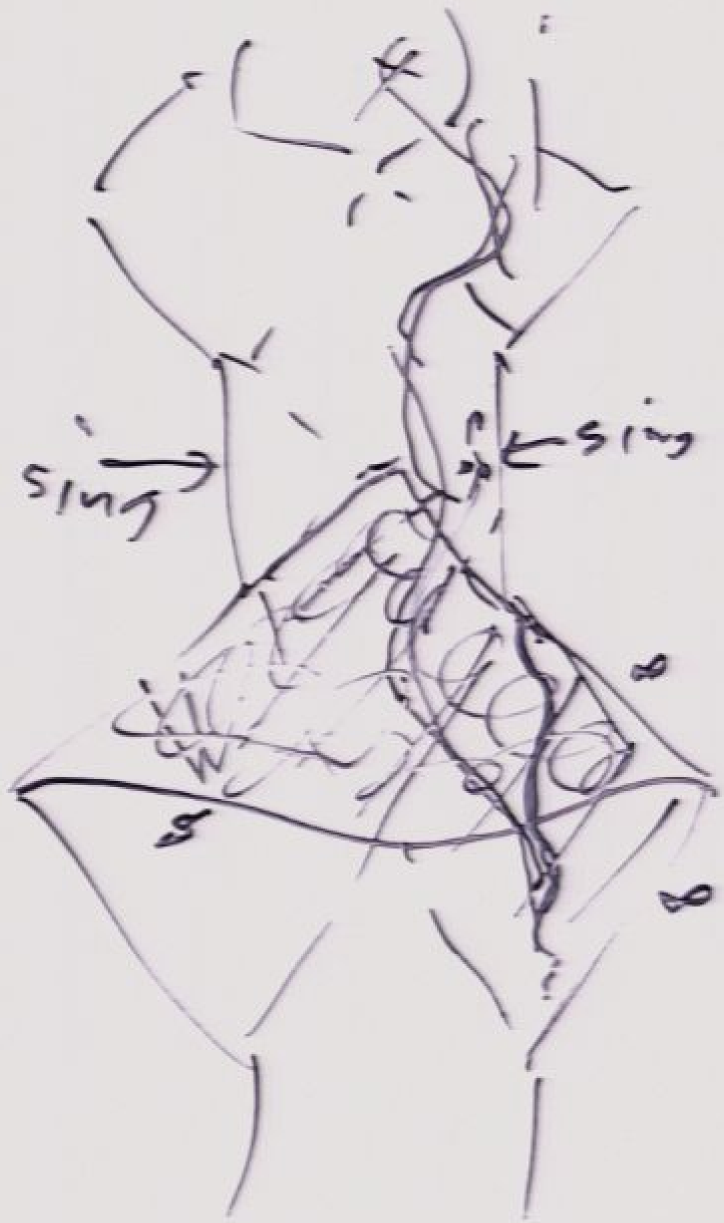
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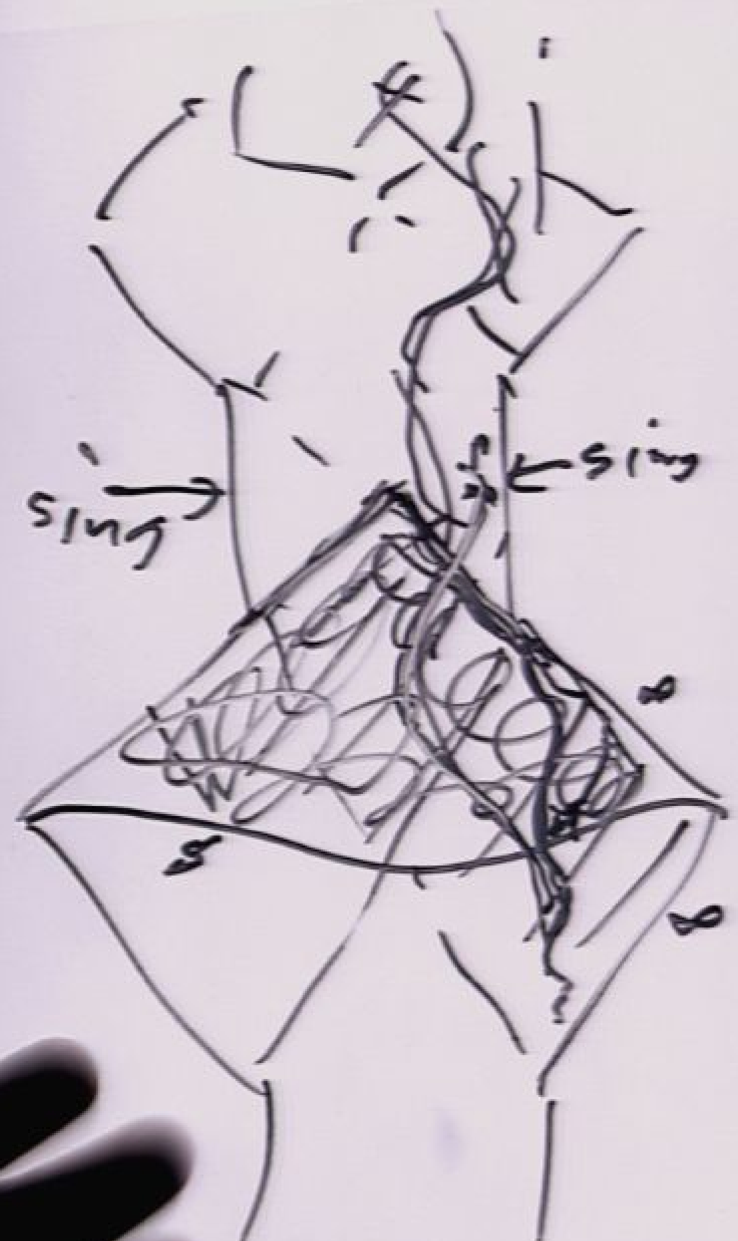
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A real number x is <u>computable</u> if there exists a "Fortran" program such that: when any rational number $\epsilon > 0$ is used as input, the program ultimately halts, with output a rational number r satisfying:

$$|x - r| \le \epsilon.$$

Ex: $1/3$, $\sqrt{2}$, π,

$\log[\sqrt{5}\sin(\pi + 3.4/e^{.71}) - \cosh 8]$,

Anything else you can think of offhand.

Example of a non-computable number:

$$C = \sum_{n=1}^{\infty} a_n / 3^n ,$$

where

$$a_n = \begin{cases} 0 & \text{if } n^{\text{th}} \text{ program halts} \\ 2 & \text{if } n^{\text{th}} \text{ program not halt} \end{cases}$$

E.g.,

a_7 $a_7 = 0$ if Goldbach conjecture is false; 2 if true

$a_{10} = 0$ if Poincare conjecture is true; 2 if false

etc . . .

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Computation

1. Introd

One of the most intrigu
Beautiful results - Gö
Seemingly simple [open] - F
Further ↔ physics als
I'll: 3 Ex - "so clos

2. H-M Spacetimes

* Def
* Solve halting
Exist H-M solns? Can y
** Cauchy Surf - Theorem

3. Q Grav

** Computable # - Ex
Is phys # comp? [meaningle
* Q grav
[Hard to compute

4. Computation

* Problem
* Assertion

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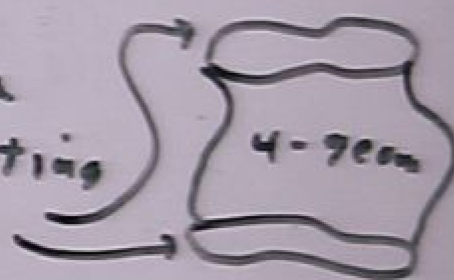
predict computable

"Quantum Gravity" by Feynman sum. [Hartle]

[Transition amplitude between two 3-geometries ("configurations of the gravitational field")]

$$= \sum_{\text{histories}} e^{iS(\text{history})/\hbar}$$

A "history" is a 4-geometry connecting the 3-geometries



Represent a history 4-manifold as a simplicial complex; then eliminate duplications.
But "do two simplicial complexes give rise to the same 4-manifold?" is not computable!

__Theorem__ (Blum). Let f be any recursive function. Then there exists a computable number x with the following property: Any "Fortran" program that returns, for each integer n, a rational within $1/n$ of x requires at least $f(n)$ "steps" to do this calculation.

Ex: $f(n) = 2^{2^{\cdot^{\cdot^{\cdot^{2}}}}}$ (n times)

then $f(5) \approx 10^{15,000}$ (more resources than there are in the universe).

Needle in Haystack Problem

N (+ integer) "haystack"

x ($= 1, 2, \ldots, N$) "needle"

Program P

input: N, x

output: "yes" or "no"

P gives, for each N, "yes" for one, only one, x [x_0]

P runs in $h(N)$ steps

Problem: Given N, find x_0

Requires $N\,h(N)$ steps (naive checking)

Use Quantum Mech

Requires $\sqrt{N}\, h(N)$ steps
(more or less)

Assertion:

Let Π map $(+\ \overset{N}{\text{int}}) \to$ (integers $1, \dots, N$)

Let P test (N, x) in $h(N)$ steps.

Then there exists a program that, given N, produces x_0, in $\sqrt{N}\, h(N)$ steps.

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