Title: Why the quantum? Insights from classical theories with a statistical restriction

Date: Feb 27, 2008 02:00 PM

URL: http://pirsa.org/08020051

Abstract: It is common to assert that the discovery of quantum theory overthrew our classical conception of nature. But what, precisely, was overthrown? Providing a rigorous answer to this question is of practical concern, as it helps to identify quantum technologies that outperform their classical counterparts, and of significance for modern physics, where progress may be slowed by poor physical intuitions and where the ability to apply quantum theory in a new realm or to move beyond quantum theory necessitates a deep understanding of the principles upon which it is based.
In this talk, I demonstrate that a large part of quantum theory can be obtained from a single innovation relative to classical theories, namely, that there is a fundamental restriction on the sorts of statistical distributions over classical states that can be prepared. This restriction implies a fundamental limit on the amount of knowledge that any observer can have about the classical state. I will also discuss the quantum phenomena that are not captured by this principle, and I will end with a few speculations on what conceptual innovations might underlie the latter set and what might be the origin of the statistical restriction.

Why the Quantum?
Insights from Classical Theories with a Statistical Restriction

Robert Spekkens
DAMTP, University of Cambridge

Feb, 27, 2008, PI Colloquium

Funding: The Royal Society

Classical statistical theory

+

fundamental restriction on statistical distributions

$\Downarrow$

A large part of quantum theory

In the sense of reproducing the operational predictions

Classical statistical theory

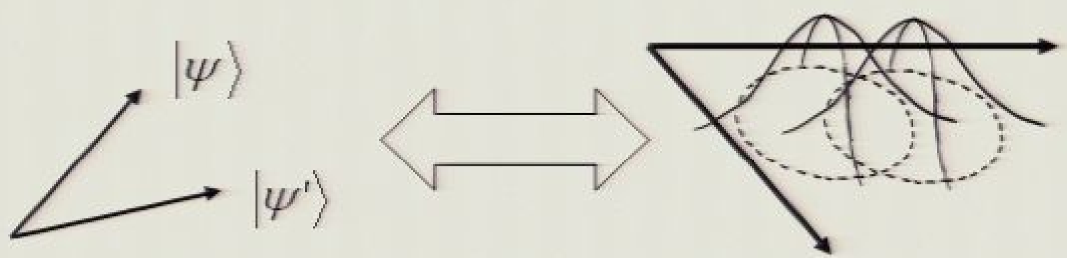
+

fundamental restriction on statistical distributions

$\Downarrow$

A large part of quantum theory

In the sense of reproducing the operational predictions



Strong evidence for the view that

quantum states are states of knowledge,

not states of reality


The fire
Prisoners
shadows cast
on wall
Roadway where
puppeteers perform

Classical theory	Statistical theory for the classical theory	Restricted Statistical theory for the classical theory
Mechanics	Liouville mechanics	Restricted Liouville mechanics = Gaussian quantum mechanics

Gaussian QM includes:

- **Most basic quantum phenomena**

e.g. noncommutativity, Interference, coherent superposition, collapse, complementary bases, no-cloning, …

- **Most quantum information-processing tasks**

e.g. teleportation, key distribution, quantum error correction, improvements in metrology, …

- **A large part of entanglement theory**

e.g. monogamy, distillation, deterministic and probabilistic single copy entanglement transformation, catalysis, …

- **A large part of the formalism of quantum theory**

e.g. Choi-Jamiolkowski isomorphism, Naimark extension, Stinespring dilation, multiple convex decompositions of states, …

A great deal of what we usually take to be mysteriously nonclassical!

Classical theory	Statistical theory for the classical theory	Restricted Statistical theory for the classical theory
Mechanics	Liouville mechanics	Restricted Liouville mechanics = Gaussian quantum mechanics

Classical theory	Statistical theory for the classical theory	Restricted Statistical theory for the classical theory
Mechanics	Liouville mechanics	Restricted Liouville mechanics = Gaussian quantum mechanics
Bits	Statistical theory of bits	Restricted statistical theory of bits $\simeq$ Stabilizer theory for qubits

Classical theory	Statistical theory for the classical theory	Restricted Statistical theory for the classical theory
Mechanics	Liouville mechanics	Restricted Liouville mechanics = Gaussian quantum mechanics
Bits	Statistical theory of bits	Restricted statistical theory of bits $\simeq$ Stabilizer theory for qubits
Trits	Statistical theory of trits	Restricted statistical theory of trits = Stabilizer theory for qutrits

Classical theory	Statistical theory for the classical theory	Restricted Statistical theory for the classical theory
Mechanics	Liouville mechanics	Restricted Liouville mechanics = Gaussian quantum mechanics
Bits	Statistical theory of bits	Restricted statistical theory of bits $\simeq$ Stabilizer theory for qubits
Trits	Statistical theory of trits	Restricted statistical theory of trits = Stabilizer theory for qutrits
Optics	Statistical optics	Restricted statistical optics = linear quantum optics

Classical theory	Statistical theory for the classical theory	Restricted Statistical theory for the classical theory
Mechanics	Liouville mechanics	Restricted Liouville mechanics = Gaussian quantum mechanics
Bits	Statistical theory of bits	Restricted statistical theory of bits $\simeq$ Stabilizer theory for qubits
Trits	Statistical theory of trits	Restricted statistical theory of trits = Stabilizer theory for qutrits
Optics	Statistical optics	Restricted statistical optics = linear quantum optics
Electrodynamics	Statistical electrodynamics	Restricted statistical electrodynamics =/$\simeq$ part of QED?

Classical theory	Statistical theory for the classical theory	Restricted Statistical theory for the classical theory
Mechanics	Liouville mechanics	Restricted Liouville mechanics = Gaussian quantum mechanics
Bits	Statistical theory of bits	Restricted statistical theory of bits $\simeq$ Stabilizer theory for qubits
Trits	Statistical theory of trits	Restricted statistical theory of trits = Stabilizer theory for qutrits
Optics	Statistical optics	Restricted statistical optics = linear quantum optics
Electrodynamics	Statistical electrodynamics	Restricted statistical electrodynamics =/$\simeq$ part of QED?
General relativity	Statistical GR	Restricted statistical GR =/$\simeq$ part of quantum gravity?

Practical applications of this characterization of part of quantum theory

Practical applications of this characterization of part of quantum theory

- Developing Quantum technologies
  - computation
  - cryptography
  - metrology

Practical applications of this characterization of part of quantum theory

- Developing Quantum technologies
  - computation
  - cryptography
  - metrology
- Extending the range of applicability of quantum theory
- Modifying quantum theory, if necessary

Categorizing quantum phenomena

Those arising in a restricted statistical classical theory	Those not arising in a restricted statistical classical theory

Categorizing quantum phenomena

Those arising in a restricted statistical classical theory

Those not arising in a restricted statistical classical theory

Wave-particle duality

collapse

Interference

noncommutativity

Teleportation

entanglement

No cloning

Quantized spectra

Coherent superposition

Quantum eraser

Key distribution

Bell inequality violations + no-signalling

Bell-Kochen-Specker theorem

Improvements in metrology

Pre and post-selection "paradoxes"

Computational speed-up

Particle statistics

Categorizing quantum phenomena

Those arising in a restricted statistical classical theory	Those not arising in a restricted statistical classical theory
Interference	**Bell inequality violations + no-signalling**
Noncommutativity	**Computational speed-up (if it exists)**
Entanglement	Bell-Kochen-Specker theorem
Collapse	Certain aspects of items on the left
Wave-particle duality	Others…
Teleportation	
No cloning	
Key distribution	
Improvements in metrology	
Quantum eraser	
Coherent superposition	
Pre and post-selection	
"paradoxes"	
Others…	

Quantized spectra?
Particle statistics?
Others…

Categorizing quantum phenomena

Those arising in a restricted statistical classical theory	Those not arising in a restricted statistical classical theory
Interference	**Bell inequality violations + no-signalling**
Noncommutativity	**Computational speed-up (if it exists)**
Entanglement	Bell-Kochen-Specker theorem
Collapse	Certain aspects of items on the left
Wave-particle duality	Others…
Teleportation	
No cloning	
Improvements in metrology	
Quantum eraser	
Coherent superposition	
Pre and post-selection "paradoxes"	
Others…	

Not so strange after all!

Still surprising!
Find more!
Focus on these

Quantized spectra?
Particle statistics?
Others…

A research program

The success of restricted statistical theories suggests that:

Quantum theory is best understood as a kind of probability theory
and that
Quantum states are incomplete descriptions of a deeper reality

Speculative possibility for an axiomatization of quantum theory

Principle 1: There is a fundamental restriction on observers capacities to know and control the systems around them

Principle 2: **???** (Some change to the classical picture of the world)

We need to explore possibilities for principle 2, even if only in toy theories
Ultimately, we need to derive principle 1

Statistical classical theories with a restriction on knowledge

Restricted Liouville mechanics

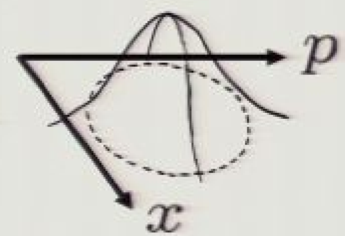
Joint work with Stephen Bartlett and Terry Rudolph

Liouville mechanics

Liouville distribution on phase space

$$\mu(x_1, p_1, x_2, p_2, \dots)$$

e.g.



What is a good statistical restriction to apply ?

-- look to quantum theory

Quantum particle mechanics

Covariance matrix

$$\gamma(\hat{\rho}) = 2\begin{pmatrix} \Delta^2 x_1 & C_{x_1,p_1} & C_{x_1,x_2} & C_{x_1,p_2} & \cdots \\ C_{p_1,x_1} & \Delta^2 p_1 & C_{p_1,x_2} & C_{p_1,p_2} & \\ C_{x_2,x_1} & C_{x_2,p_1} & \Delta^2 x_2 & C_{x_2,p_2} & \\ C_{p_2,x_1} & C_{p_2,p_1} & C_{p_2,x_2} & \Delta^2 p_2 & \\ \vdots & & & & \ddots \end{pmatrix}$$

where
$$\Delta^2 x \equiv \langle \hat{x}^2 \rangle - \langle \hat{x} \rangle^2$$
$$C_{x,p} \equiv \frac{1}{2}\langle \hat{x}\hat{p} + \hat{p}\hat{x} \rangle - \langle \hat{x} \rangle \langle \hat{p} \rangle$$

General form of the uncertainty principle is:

$$\boxed{\gamma(\hat{\rho}) + i\hbar\Sigma \geq 0}$$

where

$$\Sigma = \begin{pmatrix} 0 & -1 & & & \cdots \\ 1 & 0 & & & \\ & & 0 & -1 & \\ & & 1 & 0 & \\ \vdots & & & & \ddots \end{pmatrix}$$

For a single canonical degree of freedom

$$2\begin{pmatrix}\Delta^2 x & C_{x,p}\\ C_{p,x} & \Delta^2 p\end{pmatrix} + i\hbar\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix} \geq 0$$

$$2\begin{pmatrix}\Delta^2 x & C_{x,p} - \frac{1}{2}i\hbar\\ C_{p,x} + \frac{1}{2}i\hbar & \Delta^2 p\end{pmatrix} \geq 0$$

$$4\Delta^2 x \Delta^2 p \geq (\langle \hat{x}\hat{p} + \hat{p}\hat{x}\rangle - 2\langle \hat{x}\rangle\langle \hat{p}\rangle)^2 + \hbar^2$$

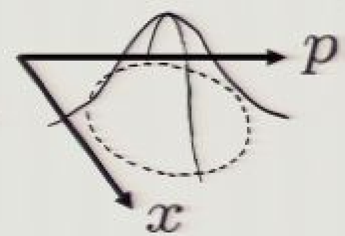
$$\Delta x \Delta p \geq \hbar/2$$

Liouville mechanics

Liouville distribution on phase space

$$\mu(x_1, p_1, x_2, p_2, \ldots)$$

e.g.



Covariance matrix

$$\gamma(\mu) = 2 \begin{pmatrix} \Delta^2 x_1 & C_{x_1,p_1} & C_{x_1,x_2} & C_{x_1,p_2} & \cdots \\ C_{p_1,x_1} & \Delta^2 p_1 & C_{p_1,x_2} & C_{p_1,p_2} & \\ C_{x_2,x_1} & C_{x_2,p_1} & \Delta^2 x_2 & C_{x_2,p_2} & \\ C_{p_2,x_1} & C_{p_2,p_1} & C_{p_2,x_2} & \Delta^2 p_2 & \\ \vdots & & & & \ddots \end{pmatrix}$$

where $\Delta^2 x \equiv \langle x^2 \rangle - \langle x \rangle^2$

$C_{x,p} \equiv \langle xp \rangle - \langle x \rangle \langle p \rangle$

$\langle f(x,p) \rangle \equiv \int dxdp f(x,p) \mu(x,p)$

Liouville mechanics with a statistical restriction

Assume:

The classical uncertainty principle:

The only Liouville distributions that can be prepared are those that satisfy

$$\gamma(\mu) + i\hbar\Sigma \geq 0$$

and that have maximal entropy for a given set of second-order moments.

Liouville mechanics with a statistical restriction

Assume:

The classical uncertainty principle:

The only Liouville distributions that can be prepared are those that satisfy

$$\gamma(\mu) + i\hbar\Sigma \geq 0$$

and that have maximal entropy for a given set of second-order moments.

Among valid μ with a given γ, multi-variate Gaussians maximize the entropy

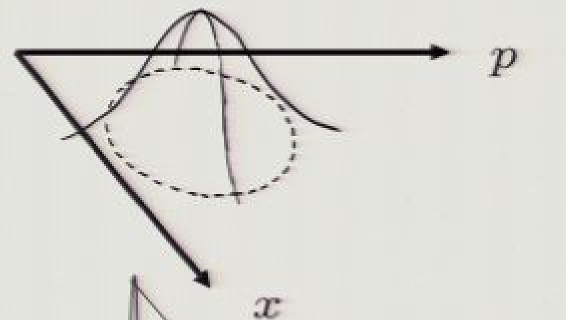
$$\mu(\mathbf{z}) = \frac{1}{(2\pi)^{n/2}|\gamma|^{1/2}} \exp\left(-\frac{1}{2}(\mathbf{z} - \langle\mathbf{z}\rangle)^T \gamma^{-1} (\mathbf{z} - \langle\mathbf{z}\rangle)\right)$$

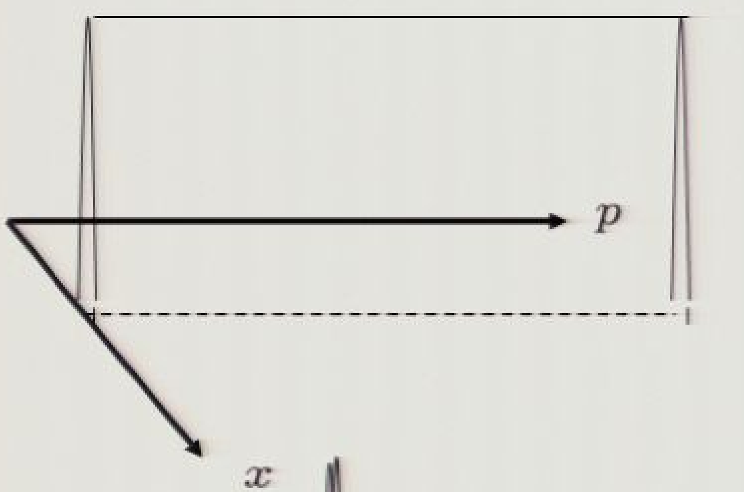
where $\mathbf{z} \equiv (x_1, p_1, x_2, p_2, \ldots)$

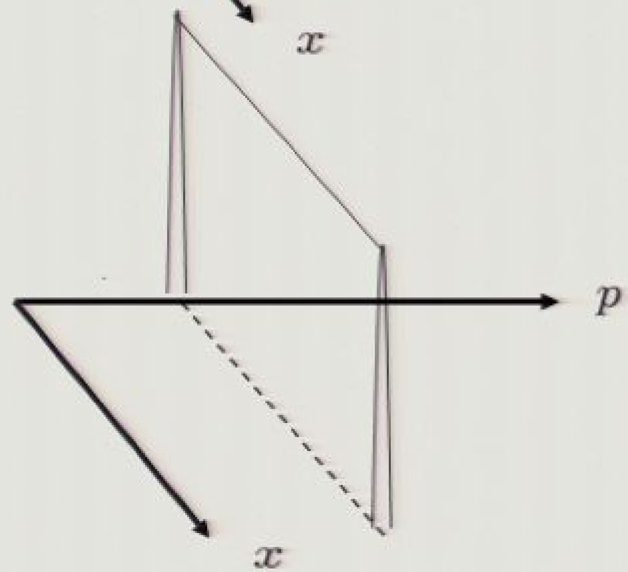
Valid distributions for one canonical system

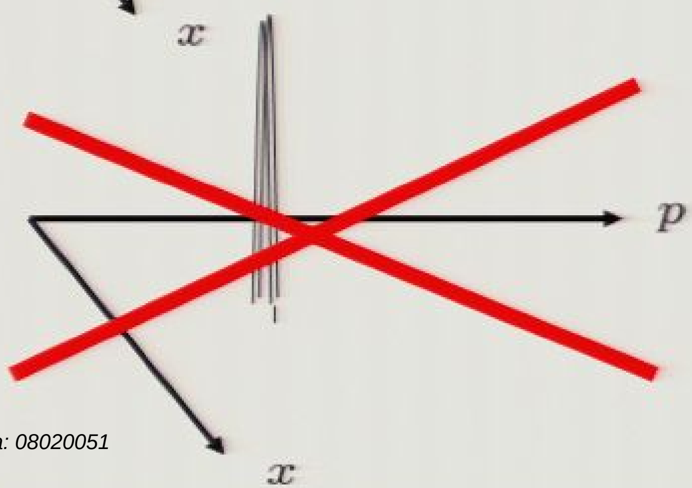
$$\mu(x,p) \geq 0$$

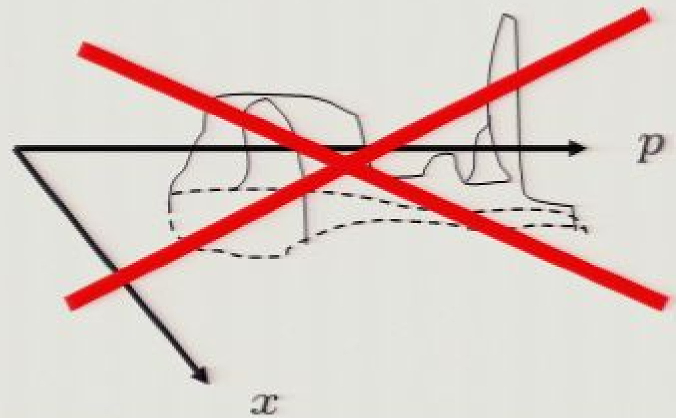
$$\int \mu(x,p)\,dx\,dp = 1$$











Valid distributions for a pair of canonical systems

Uncorrelated distributions

$$\mu(x_1, p_1, x_2, p_2) = \mu(x_1, p_1)\ \mu(x_2, p_2)$$

Correlated distributions

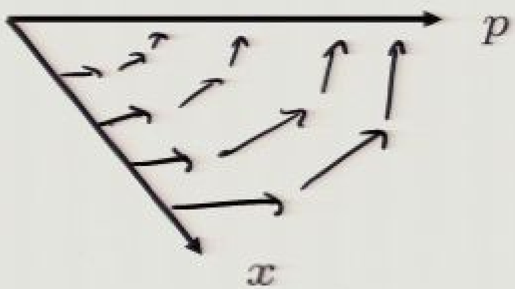
e.g. $$\mu(x_1, p_1, x_2, p_2) = \frac{1}{N}\delta(x_1 - x_2)\delta(p_1 + p_2)$$

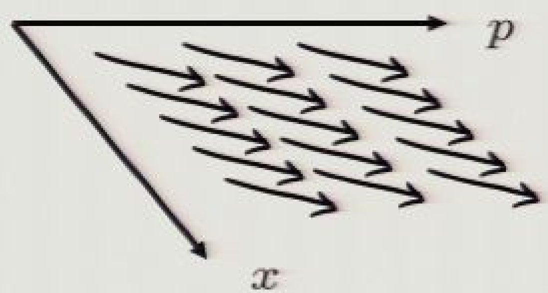
This corresponds to the entangled state of Einstein, Podolsky and Rosen

Valid deterministic transformations

The group of canonical transformations with quadratic Hamiltonian

Only canonical transformations preserve the CUP
Only quadratic Hamiltonians preserve the gaussianity





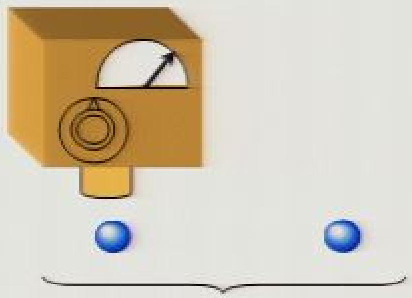
One can also determine the probabilistic transformations
→ Found to be isomorphic to the bipartite distributions

Valid measurements

Sets of indicator functions $\{\xi_k(x,p)\}$

$\xi_k(x,p)$ = probability of k given (x,p)

$\xi_k(x_1,p_1)$



$\mu(x_1,p_1,x_2,p_2)$

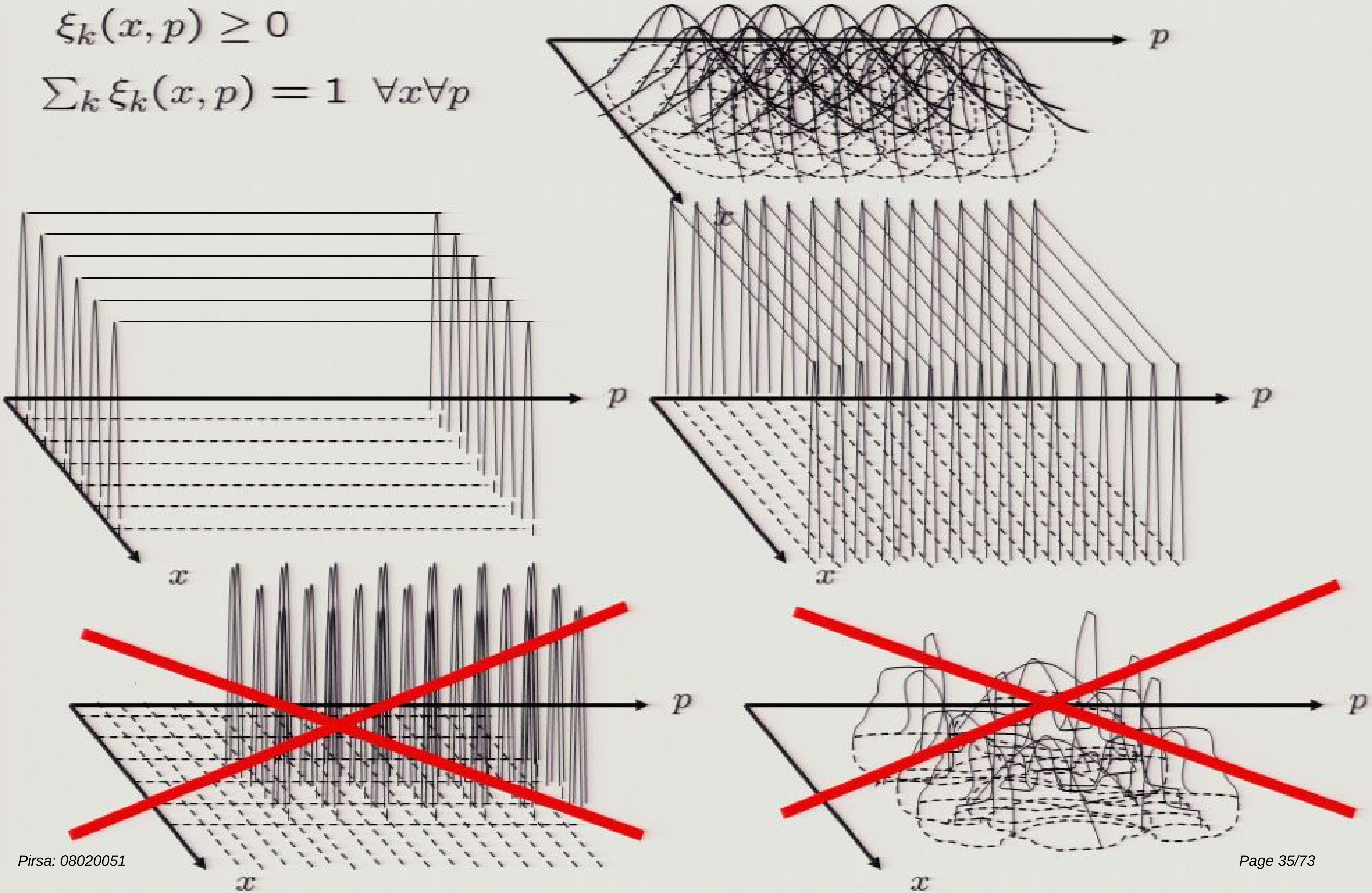
$$\mu(x_2,p_2) \propto \int dx_1 dp_1\, \xi(x_1,p_1)\, \delta(x_1 - x_2)\delta(p_1 + p_2)$$
$$= \xi(x_2,-p_2)$$

$$\mu(x_2,p_2) \propto \int dx_1 dp_1\, \xi(x_1,p_1)\, \mu(x_1,p_1,x_2,p_2)$$

Valid measurements for one canonical system

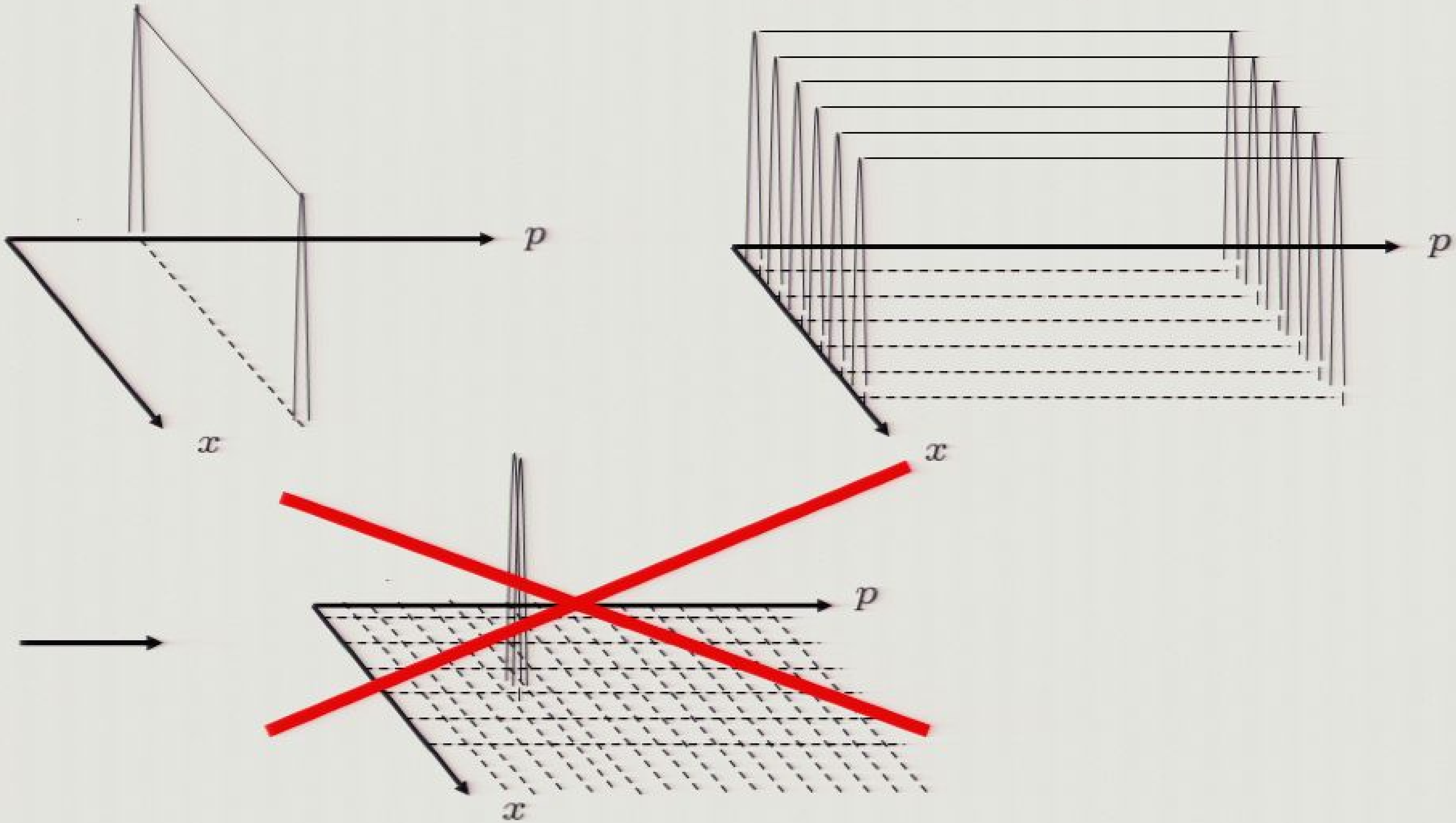
$\xi_k(x,p) \geq 0$

$\sum_k \xi_k(x,p) = 1 \;\; \forall x \forall p$



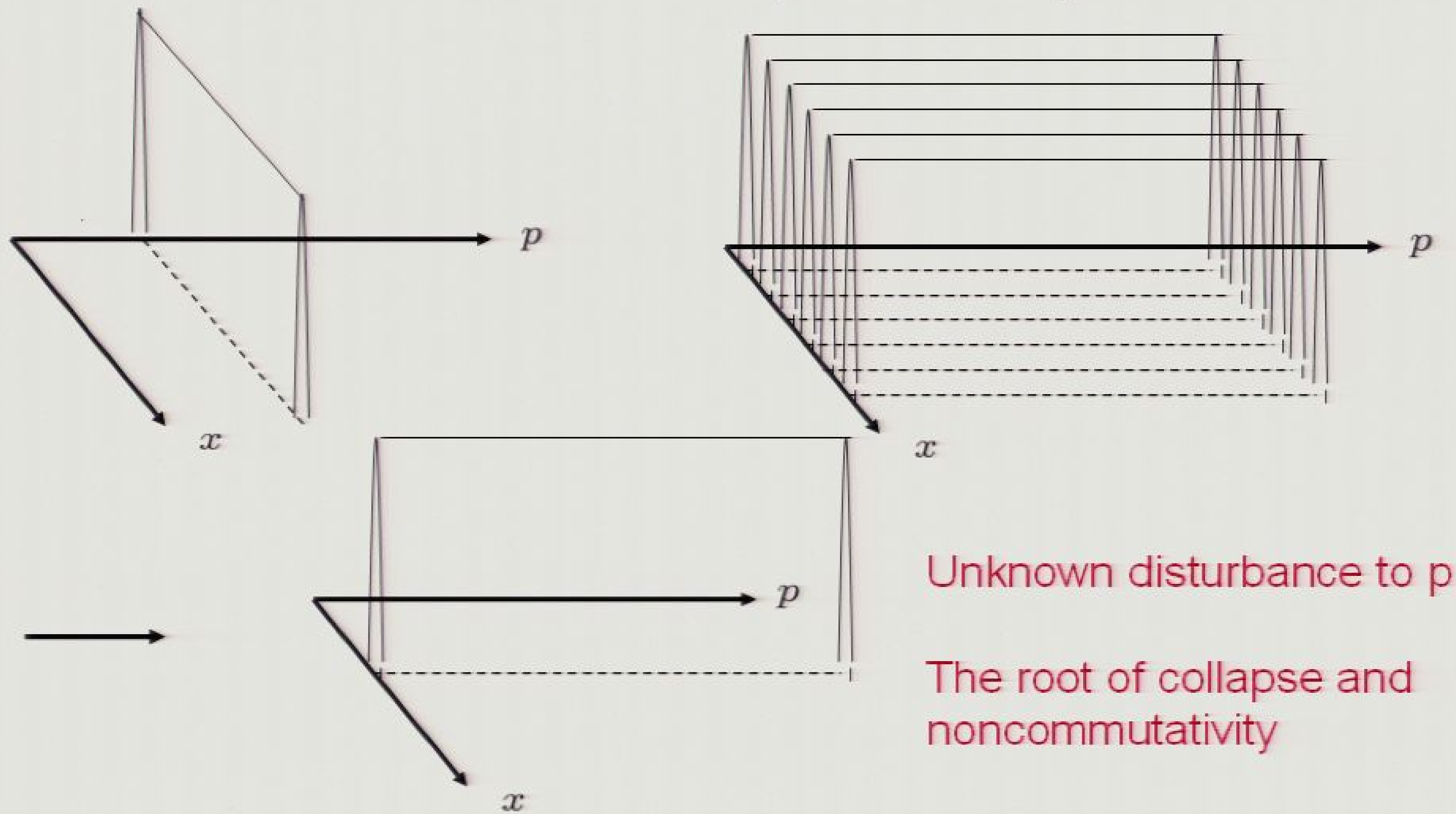
Measurement-induced transformations

Measure x in a reproducible way



Measurement-induced transformations

Measure x in a reproducible way



Unknown disturbance to p

The root of collapse and noncommutativity

Note: the evolution is *deterministic* if the apparatus is treated internally

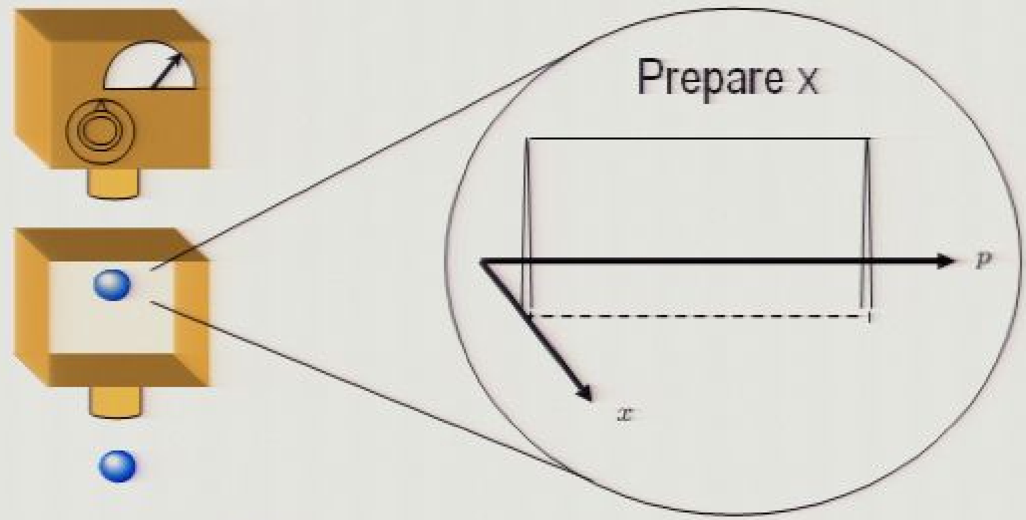
External apparatus

Measure x

Unknown disturbance to p

Internal apparatus

Measure x



Interact by $H_{int} = x_{sys}\, p_{app}$

Final x of apparatus reflects initial x of system
Final p of system reflects initial p of apparatus

*The position of the internal-external **cut** doesn't matter*

Gaussian quantum mechanics

Define $\hat{A}_{\mathbf{z}} = \bigotimes_{i=1}^{n} \hat{A}_{x_i,p_i}$ $\quad \mathbf{z} \equiv (x_1, p_1, x_2, p_2, \ldots)$

$$\hat{A}_{x_i,p_i} = \frac{1}{\pi\hbar}\int e^{-ip_i y/\hbar} \left| x_i - \tfrac{1}{2}y \right\rangle \left\langle x_i + \tfrac{1}{2}y \right| dy$$

Wigner rep'n of ρ. $\quad W_\rho(\mathbf{z}) = \mathrm{Tr}(\rho \hat{A}_{\mathbf{z}})$

Gaussian state ρ: one that has a Gaussian Wigner rep'n

$$W_\rho(\mathbf{z}) = \frac{1}{(2\pi)^{n/2}|\gamma|^{1/2}} \exp\left(-\tfrac{1}{2}(\mathbf{z} - \langle\mathbf{z}\rangle)^T \gamma^{-1} (\mathbf{z} - \langle\mathbf{z}\rangle)\right)$$

Note: $\langle \hat{x}^k \hat{p}^l \rangle_{\hat{\rho}} = \langle x^k p^l \rangle_{W_{\hat{\rho}}}$ therefore $\gamma(\hat{\rho}) = \gamma(W_{\hat{\rho}})$

Gaussian measurements and transformations: preserve Gaussianity

Gaussian quantum mechanics

Define $\hat{A}_{\mathbf{z}} = \otimes_{i=1}^{n} \hat{A}_{x_i,p_i}$ $\quad \mathbf{z} \equiv (x_1, p_1, x_2, p_2, \ldots)$

$$\hat{A}_{x_i,p_i} = \frac{1}{\pi\hbar} \int e^{-ip_i y/\hbar} \left| x_i - \tfrac{1}{2}y \right\rangle \left\langle x_i + \tfrac{1}{2}y \right| dy$$

Wigner rep'n of ρ. $\quad W_\rho(\mathbf{z}) = \mathrm{Tr}(\rho \hat{A}_{\mathbf{z}})$

Gaussian state ρ: one that has a Gaussian Wigner rep'n

$$W_\rho(\mathbf{z}) = \frac{1}{(2\pi)^{n/2}|\gamma|^{1/2}} \exp\left(-\tfrac{1}{2}(\mathbf{z} - \langle \mathbf{z} \rangle)^T \gamma^{-1} (\mathbf{z} - \langle \mathbf{z} \rangle)\right)$$

Note: $\langle \hat{x}^k \hat{p}^l \rangle_{\hat{\rho}} = \langle x^k p^l \rangle_{W_{\hat{\rho}}}$ therefore $\gamma(\hat{\rho}) = \gamma(W_{\hat{\rho}})$

Gaussian measurements and transformations: preserve Gaussianity

One can prove

Theorem: Restricted Liouville mechanics is empirically equivalent to Gaussian quantum mechanics

Gaussian QM includes:

- **Most basic quantum phenomena**

e.g. noncommutativity, Interference, coherent superposition, collapse, complementary bases, no-cloning, …

- **Most quantum information-processing tasks**

e.g. teleportation, key distribution, quantum error correction, improvements in metrology, …

- **A large part of entanglement theory**

e.g. monogamy, distillation, deterministic and probabilistic single copy entanglement transformation, catalysis, …

- **A large part of the formalism of quantum theory**

e.g. Choi-Jamiolkowski isomorphism, Naimark extension, Stinespring dilation, multiple convex decompositions of states, …

A great deal of what we usually take to be mysteriously nonclassical!

Restricted statistical theory of trits

Joint work with Olaf Schreiber

Building upon:

Spekkens, quant-ph/0401052 [Phys. Rev. A 75, 032110 (2007)]

S. van Enk, arxiv:0705.2742 [Found. Phys. 37, 1447 (2007)]

D. Gross, quant-ph/0602001 [J. Math. Phys. 47, 122107 (2006)]

Does quantum theory suggest a good statistical restriction?

For finite-d system, no good analogue of

$$\Delta x \Delta p \geq \hbar/2$$

Instead, note simply that:
Jointly-measurable observables = a commuting set of observables

This suggests
Jointly-knowable variables = a set of canonically **nonconjugate** variables

Continuous case: variables with vanishing Poisson bracket

$$\{f,g\}_{PB} = 0$$

where

$$\{f,g\}_{PB} \equiv \sum_{i=1}^{n} \left(\frac{\partial f}{\partial x_i}\frac{\partial g}{\partial p_i} - \frac{\partial f}{\partial p_i}\frac{\partial g}{\partial x_i}\right)$$

Discrete configuration space

$(X_1, ..., X_n) \in (\mathbb{Z}_3)^n$ $\qquad \mathbb{Z}_3 = \{0, 1, 2\}$

Discrete phase space

$(X_1, P_1, ..., X_n, P_n) \in (\mathbb{Z}_3)^{2n}$

General canonical variable

$f = a_1 X_1 + b_1 P_1 + \cdots + a_n X_n + b_n P_n$

$a_1, b_1, \ldots, a_n, b_n \in \mathbb{Z}_3$

Vector in symplectic vector space associated with variable

$v_f = (a_1, b_1, \ldots, a_n, b_n) \in (\mathbb{Z}_3)^{2n}$

Discrete case: **nonconjugate variables** are those such that

$$[v_f, v_g] = 0$$

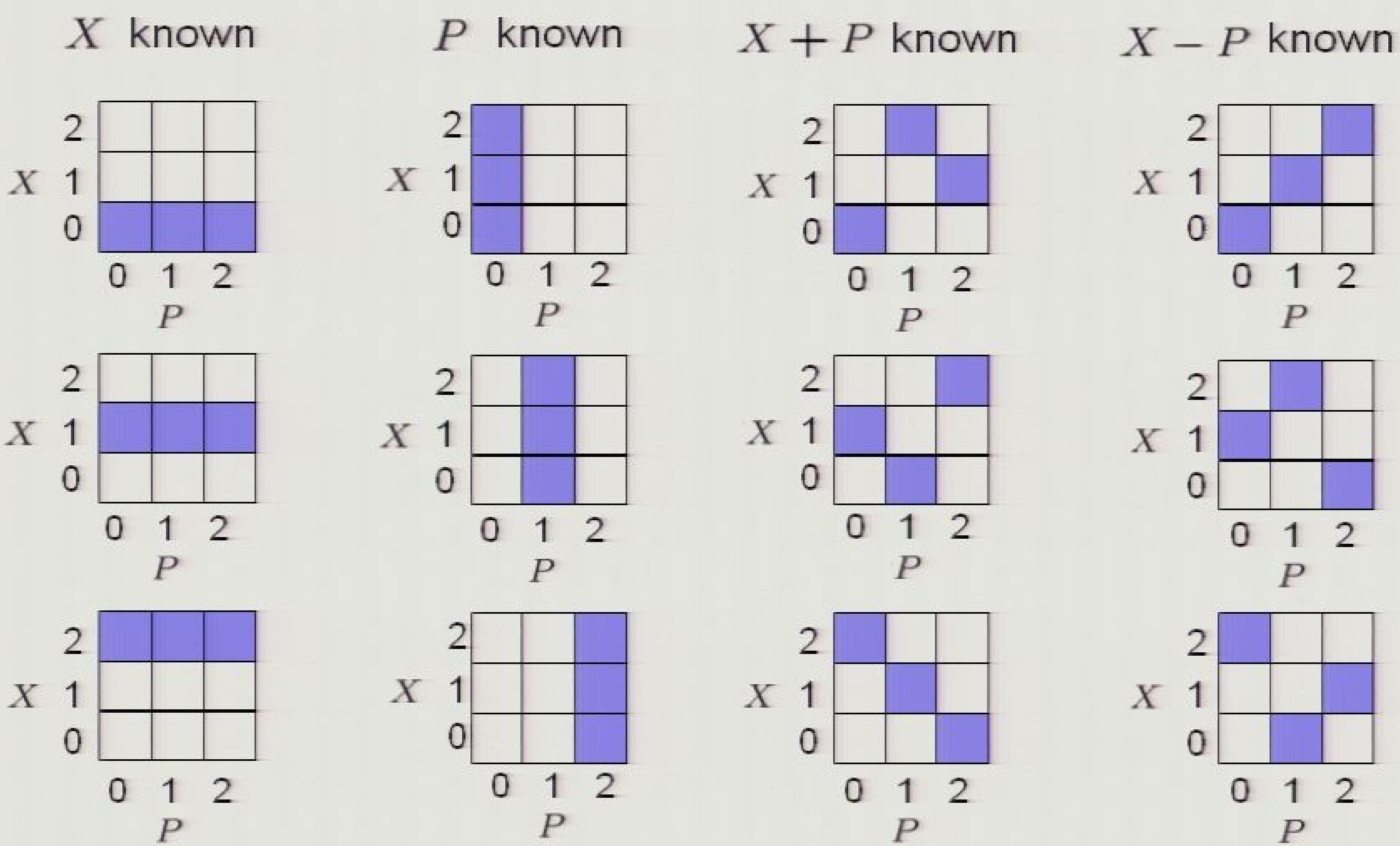
where

$$[u, v] = u^T \Sigma v$$

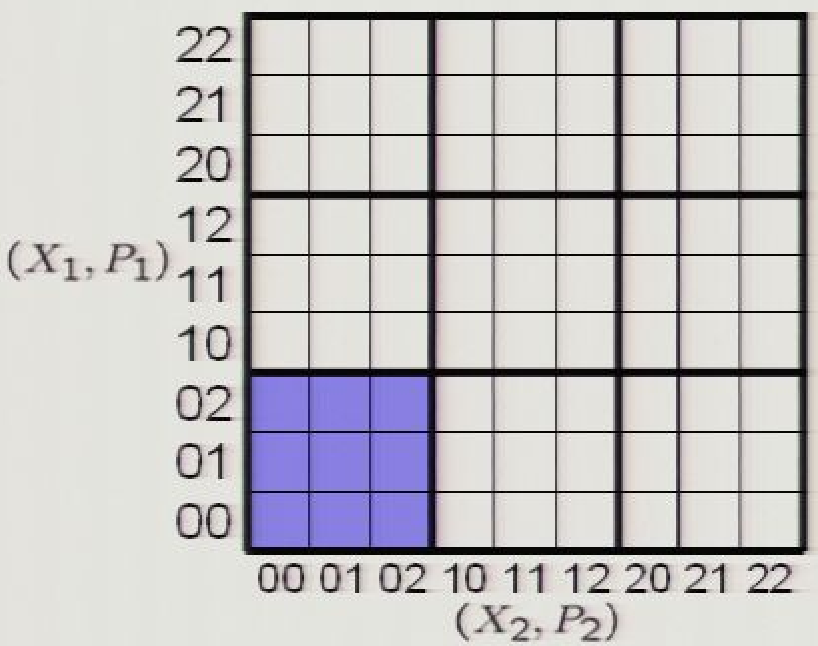
The discrete classical uncertainty principle:

The only distributions that can be prepared are those that correspond to fixing the values of a set of canonical and **nonconjugate** variables

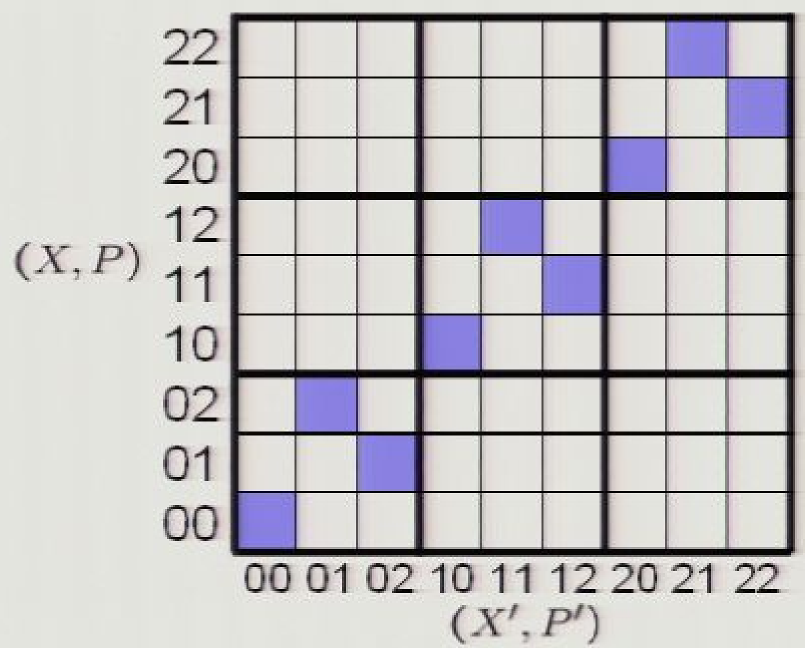
An observer can know at most half of the full set of variables (and only certain halves)


X known
P known
X + P known
X − P known
X
P
0 1 2

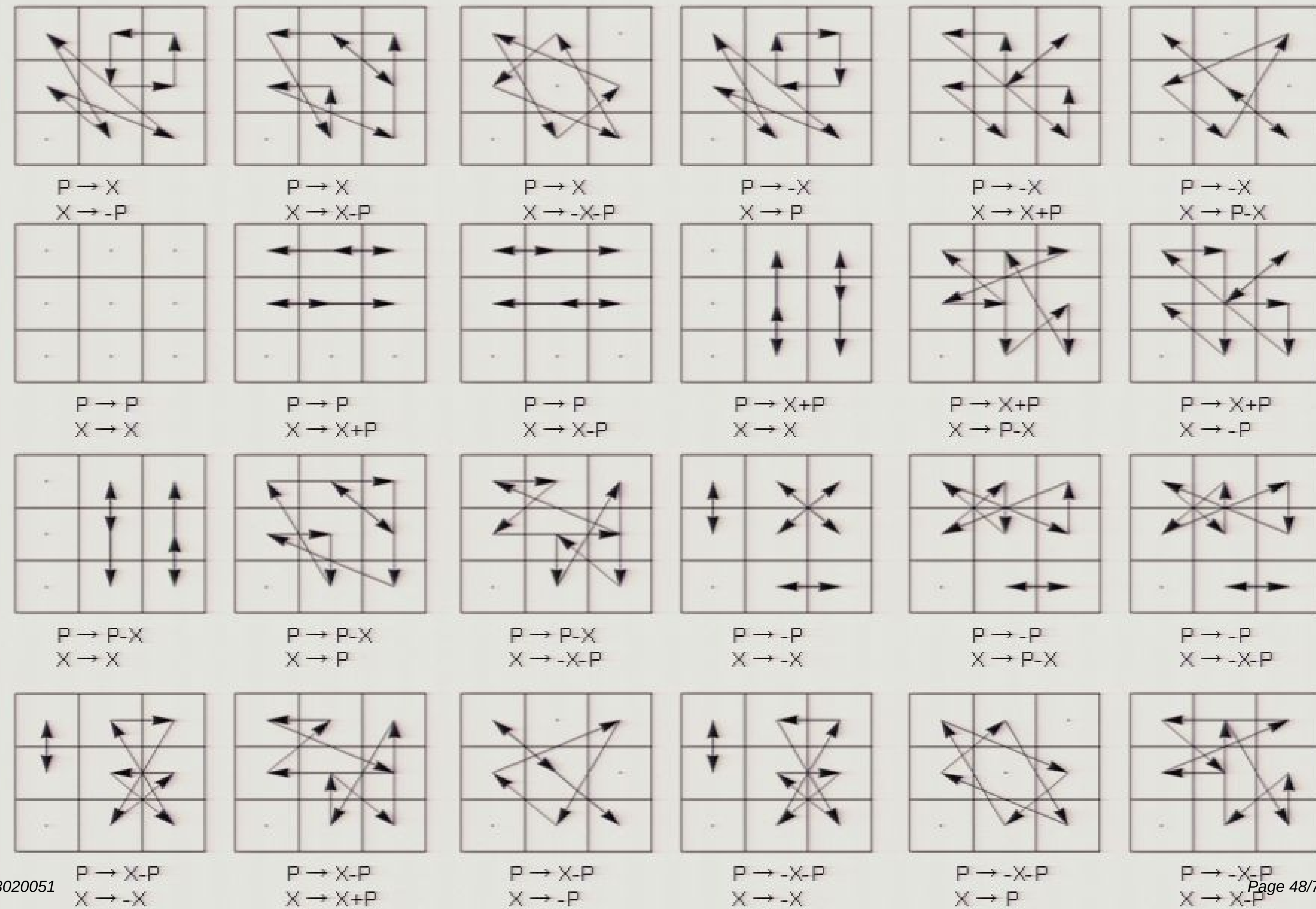
X_1 and X_2 known



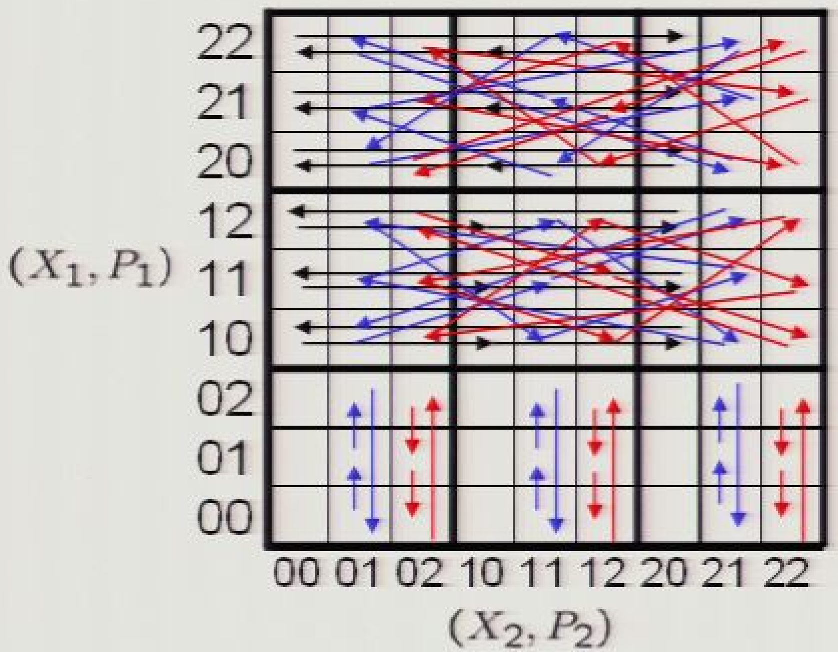
$X_1 - X_2$ and $P_1 + P_2$ known

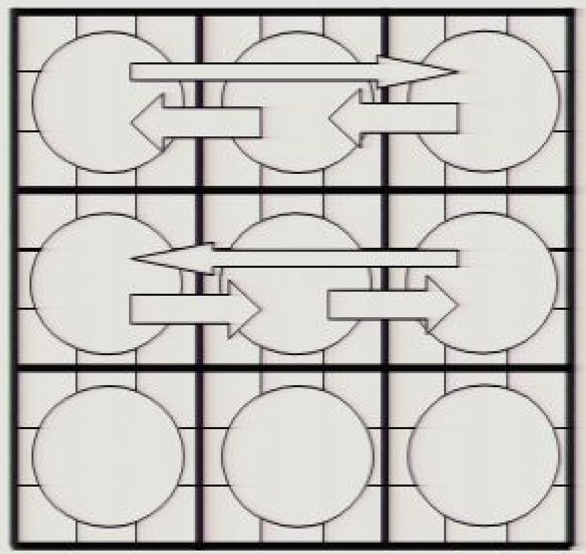


Symplectic transformations for a single trit



$$X_1 \mapsto X_1$$
$$P_1 \mapsto P_1 + P_2$$
$$X_2 \mapsto X_1 + X_2$$
$$P_2 \mapsto P_2$$





Qutrit Stabilizer theory

Stabilizer state: an eigenstate of a commuting set of canonical observables defined by symplectic transformations of

$\hat{X} = \sum_{x\in\mathbb{Z}_3} |x\rangle\langle x|$

$\hat{P} = \sum_{p\in\mathbb{Z}_3} |p\rangle\langle p|$ where $\langle x|p\rangle = \frac{1}{\sqrt{3}} e^{i\frac{2\pi}{3}xp}$

Equivalently, a state stabilized by an associated set of discrete phase-space displacement operators (generalized Pauli operators)

Qutrit Stabilizer theory

Stabilizer state: an eigenstate of a commuting set of canonical observables defined by symplectic transformations of

$$\hat{X} = \sum_{x\in\mathbb{Z}_3} |x\rangle\langle x|$$

$$\hat{P} = \sum_{p\in\mathbb{Z}_3} |p\rangle\langle p| \quad \text{where } \langle x|p\rangle = \frac{1}{\sqrt{3}} e^{i\frac{2\pi}{3}xp}$$

Equivalently, a state stabilized by an associated set of discrete phase-space displacement operators (generalized Pauli operators)

For the stabilizer states, the discrete Wigner rep'n of Gross (quant-ph/0602001) is equal to the valid distributions of the restricted statistical trit theory

Qutrit Stabilizer theory

Stabilizer state: an eigenstate of a commuting set of canonical observables defined by symplectic transformations of

$\hat{X} = \sum_{x \in \mathbb{Z}_3} |x\rangle\langle x|$

$\hat{P} = \sum_{p \in \mathbb{Z}_3} |p\rangle\langle p|$ where $\langle x|p\rangle = \frac{1}{\sqrt{3}} e^{i\frac{2\pi}{3}xp}$

Equivalently, a state stabilized by an associated set of discrete phase-space displacement operators (generalized Pauli operators)

For the stabilizer states, the discrete Wigner rep'n of Gross (quant-ph/0602001) is equal to the valid distributions of the restricted statistical trit theory

Therefore

Theorem: The restricted statistical theory of trits is empirically equivalent to the Stabilizer theory for qutrits

Stabilizer qutrit theory includes:

- **Most basic quantum phenomena**

e.g. noncommutativity, Interference, coherent superposition, collapse, complementary bases, no-cloning, …

- **Most quantum information-processing tasks**

e.g. teleportation, key distribution, quantum error correction, improvements in metrology, dense coding, …

- **A large part of entanglement theory**

e.g. monogamy, distillation, deterministic and probabilistic single copy entanglement transformation, catalysis, …

- **A large part of the formalism of quantum theory**

e.g. Choi-Jamiolkowski isomorphism, Naimark extension, Stinespring dilation, multiple convex decompositions of states, …

A great deal of what we usually take to be mysteriously nonclassical!

Restricted statistical theory of bits
(a.k.a. "the toy theory"©)

Spekkens, quant-ph/0401052 [Phys. Rev. A 75, 032110 (2007)]

The knowledge-balance principle:

The only distributions that can be prepared are those that correspond to knowing at most **half** of a set of canonical variables sufficient to specify the state

The constraint of nonconjugacy isn't workable in this context
However, a constraint of locality does most of its work

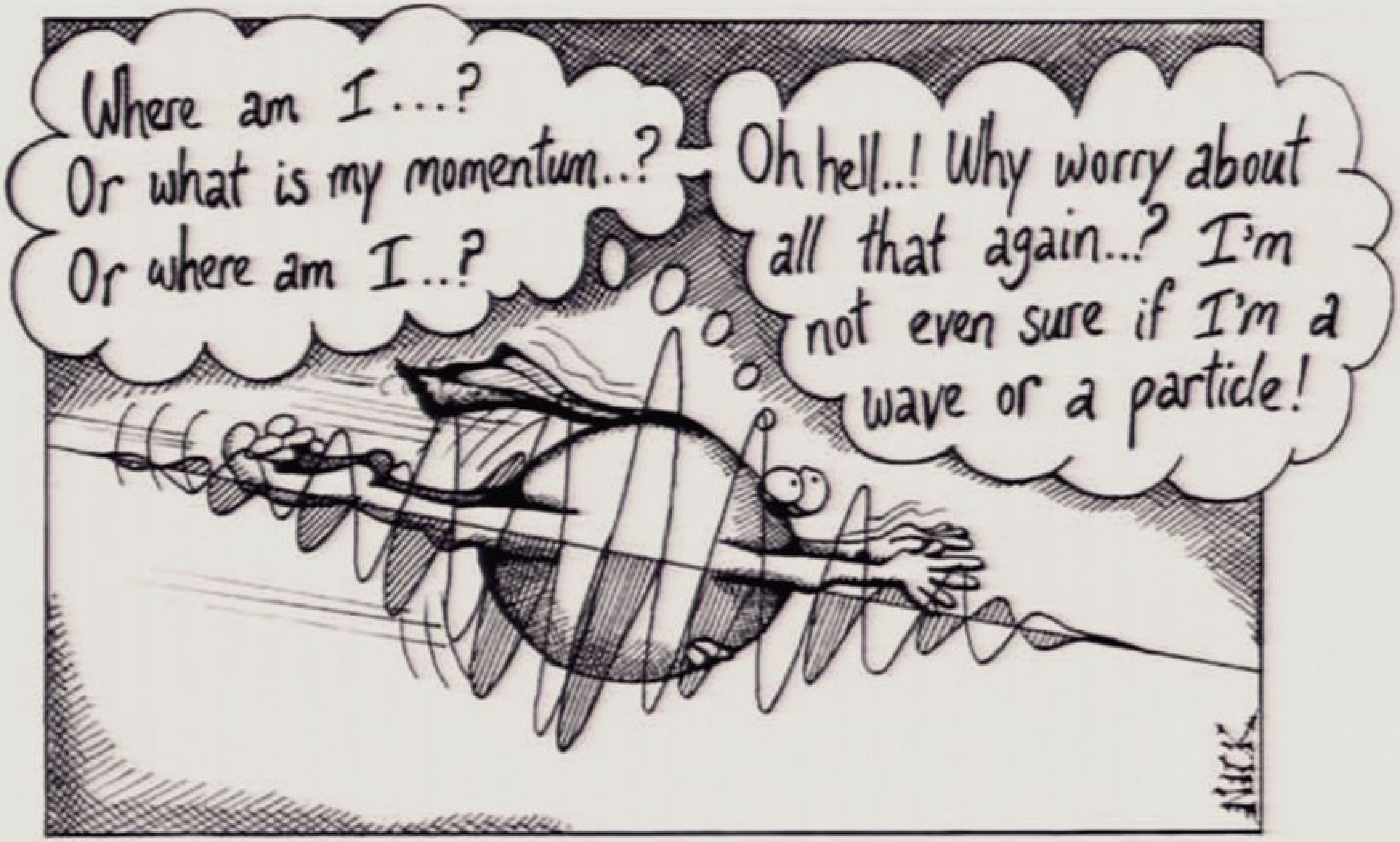
The theory is close to but not precisely the Stabilzer theory for qubits

In restricted statistical theories for d-level systems
d even and d odd are fundamentally different

Restricted statistical optics

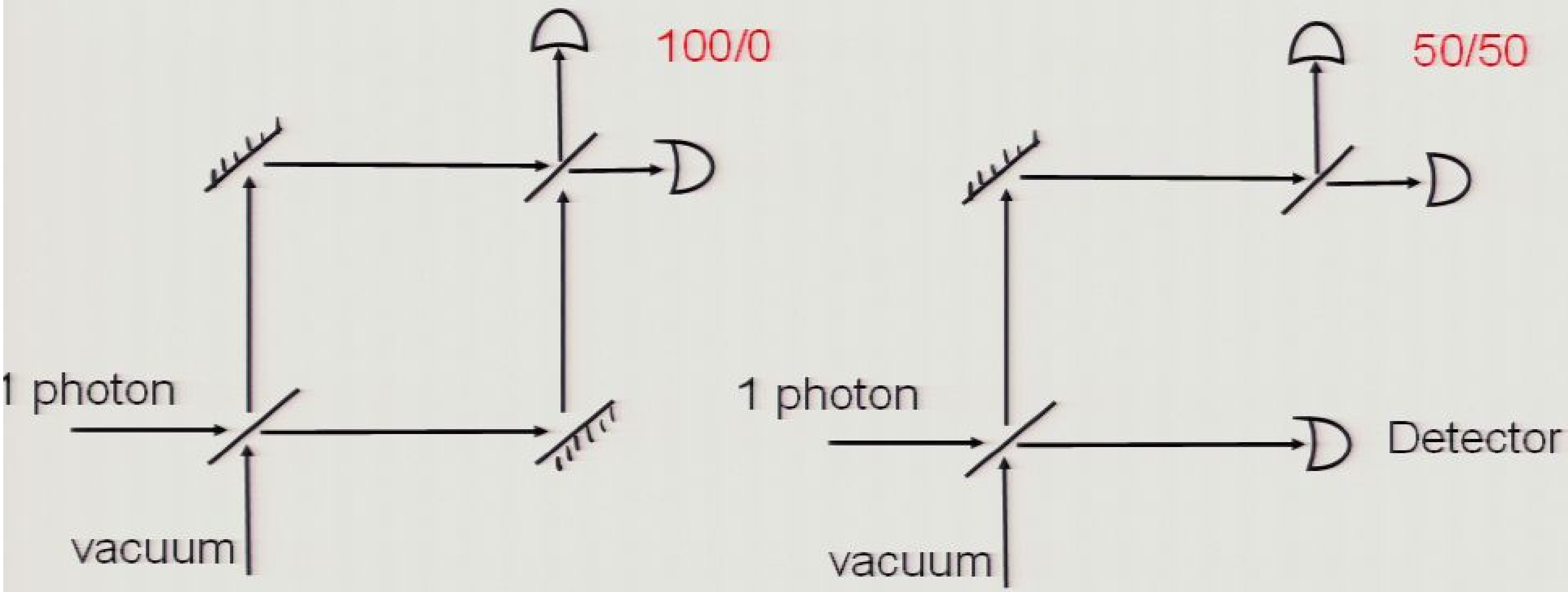
Simply take x,p → E,B in restricted Liouville mechanics

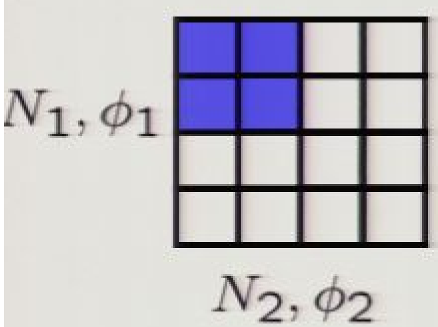
Equivalent to linear quantum optics


Where am I....?
Or what is my momentum..?
Or where am I..?
Oh hell..! Why worry about all that again..? I'm not even sure if I'm a wave or a particle!
NICK

Restricted statistical theory of "bit-valued fields"

Joint work with Elliott Martin





Occupation number and discrete phase both well-defined

One obtains:

- Wave-like interference with particle-like detection
- Interaction-free measurement (the safer x-ray)
- quantum eraser

Restricted statistical theory of electrodynamics

Future research

"Stochastic electrodynamics"

See: Boyer, Marshall & Santos, de la Peno & Cetto, Cole
(Note: unlike most proponents, I do not expect a derivation of all of quantum theory)

Low energy physics that proponents claim to reproduce:

- Stability of ground state of Hydrogen
- Radius of ground state of Hydrogen
- Planck blackbody spectrum
- Einstein A and B coefficients
- Lamb shift
- Casimir effect
- Unruh effect
- Aharonov-Bohm effect
- Energy quantization

Restricted statistical theory of <insert classical theory>

More future research

Phenomena **not** arising from a statistical restriction

Categorizing quantum phenomena

Those arising in a restricted statistical classical theory	Those not arising in a restricted statistical classical theory
Interference	**Bell inequality violations + no-signalling**
Noncommutativity	**Computational speed-up (if it exists)**
Entanglement	Bell-Kochen-Specker theorem
Collapse	Certain aspects of items on the left
Wave-particle duality	Others…
Teleportation	
No cloning	
Key distribution	
Improvements in metrology	
Quantum eraser	
Coherent superpositions	
Pre and post-selection "paradoxes"	
Others…	

Key for identifying the missing conceptual innovations of quantum theory

Quantized spectra?
Particle statistics?
Others…

Contextuality as an umbrella for the remaining nonclassicality

(Spekkens, PRA **71**, 052108)

Notions of nonclassicality found to be instances of contextuality:

• all variants of the **Bell-Kochen-Specker theorem** -- algebraic, state-specific, statistical, continuous, discrete

• all variants of **Bell's theorem**

• all **no-go theorems for noncontextuality of preparations, transformations and unsharp measurements**, including the 2d ones (Spekkens, PRA **71**, 052108, 2005)

• All variants of **von Neumann's no-go theorem**, including the negative-energy tunneling "paradox" and independence of angular momentum spectrum on reference point (Spekkens, PIRSA:07060037)

• Necessity of **negativity in quasi-probability representations** such as the Wigner representation (Spekkens, arXiv:0710.5549; Emerson and Ferrie, arXiv:0711.2658)

• Quantum **improvements in certain IP tasks** e.g. "parity-oblivious random access codes" (Spekkens and Toner, in preparation)

• Aspects of **pre- and post-selected "paradoxes"** (Leifer and Spekkens, PRL **95**, 200405, 2005)

One of the key remaining mysteries

Bell inequality Violations (i.e. superluminal causation at the subquantum level)	**tension** ⟷	No superluminal signalling (i.e. no way to make use of the superluminal causation)

One of the key remaining mysteries

Bell inequality Violations (i.e. superluminal causation at the subquantum level)		No superluminal signalling (i.e. no way to make use of the superluminal causation)
		

1. Deny Bell inequality violations
(experimental loopholes, superdeterminism)

One of the key remaining mysteries

Bell inequality Violations
(i.e. superluminal causation at the subquantum level)

tension

No superluminal signalling
(i.e. no way to make use of the superluminal causation)

1. ***Deny Bell inequality violations***
(experimental loopholes, superdeterminism)

2. ***Admit the possibility of superluminal signalling***
Theorem (Valentini):
"subquantum" distributions + Bell inequality violations + determinism → signalling

One of the key remaining mysteries

Bell inequality Violations
(i.e. superluminal causation at the subquantum level)

tension

No superluminal signalling
(i.e. no way to make use of the superluminal causation)

1. ***Deny Bell inequality violations***
(experimental loopholes, superdeterminism)

2. ***Admit the possibility of superluminal signalling***
Theorem (Valentini):
"subquantum" distributions + Bell inequality violations + determinism → signalling

3. ***Find a worldview for which the tension disappears***

One of the key remaining mysteries

Bell inequality Violations (i.e. superluminal causation at the subquantum level)	⟺ tension	No superluminal signalling (i.e. no way to make use of the superluminal causation)

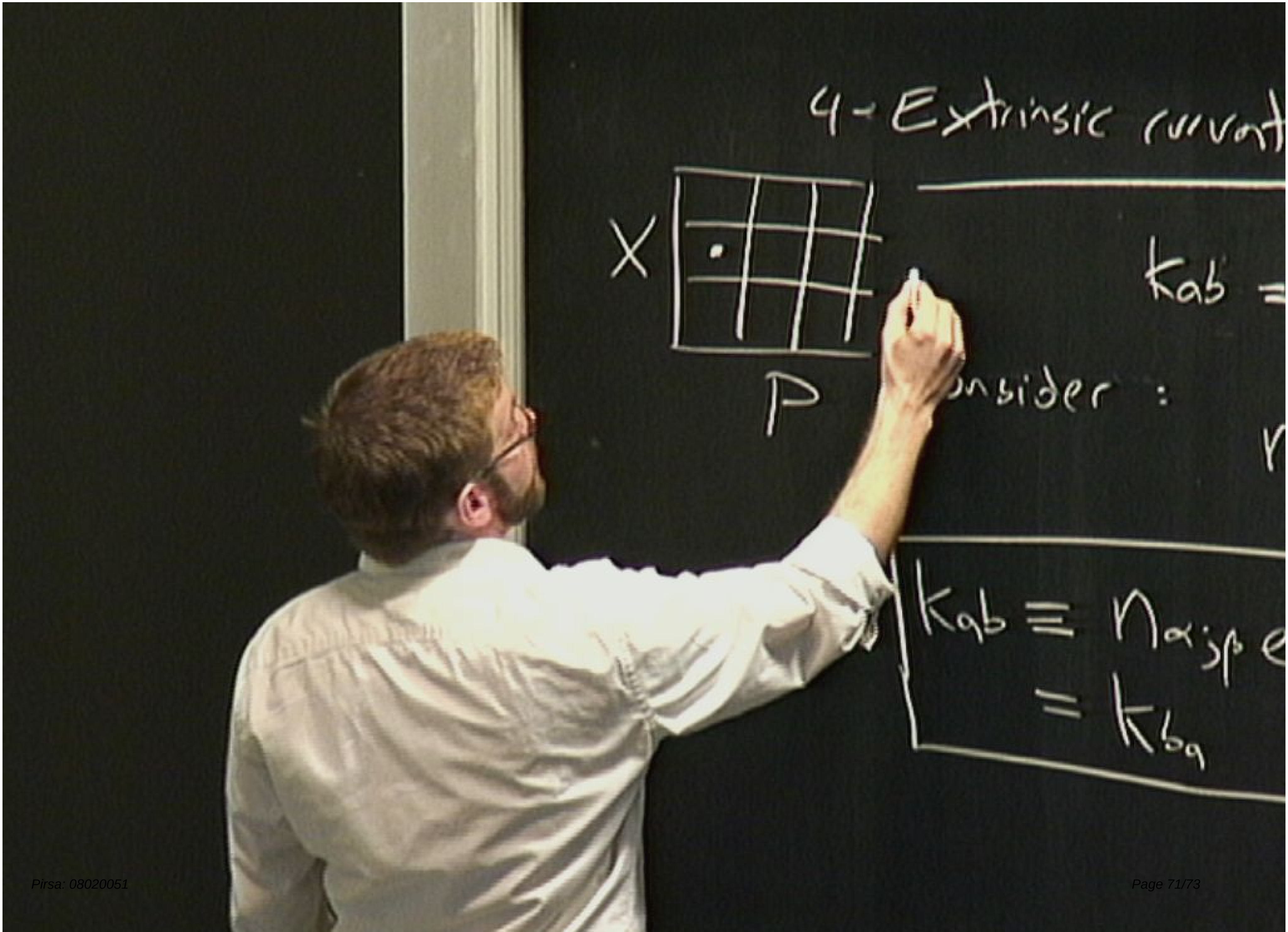
1. ***Deny Bell inequality violations***
(experimental loopholes, superdeterminism)

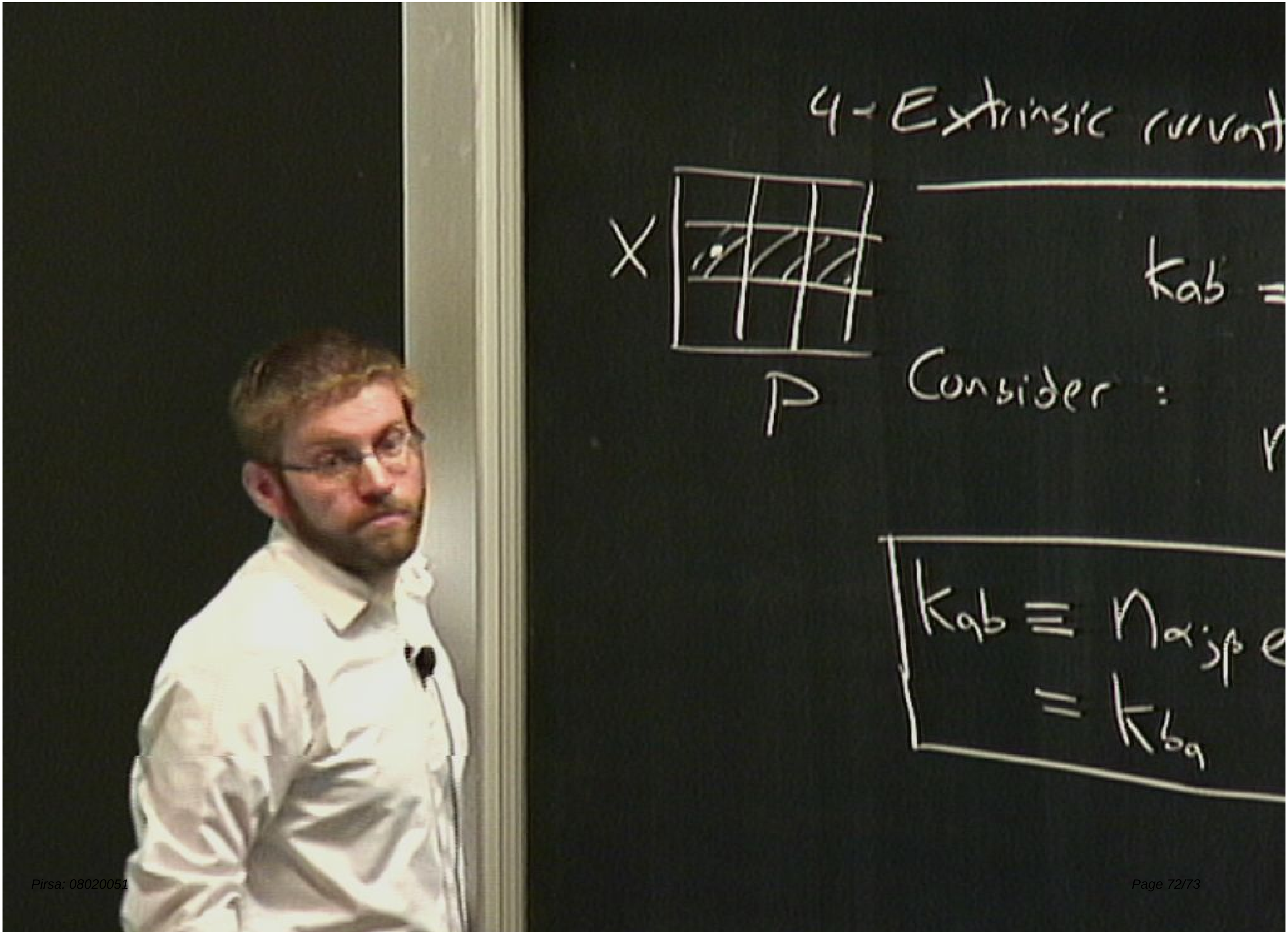
2. ***Admit the possibility of superluminal signalling***
Theorem (Valentini):
"subquantum" distributions + Bell inequality violations + determinism → signalling

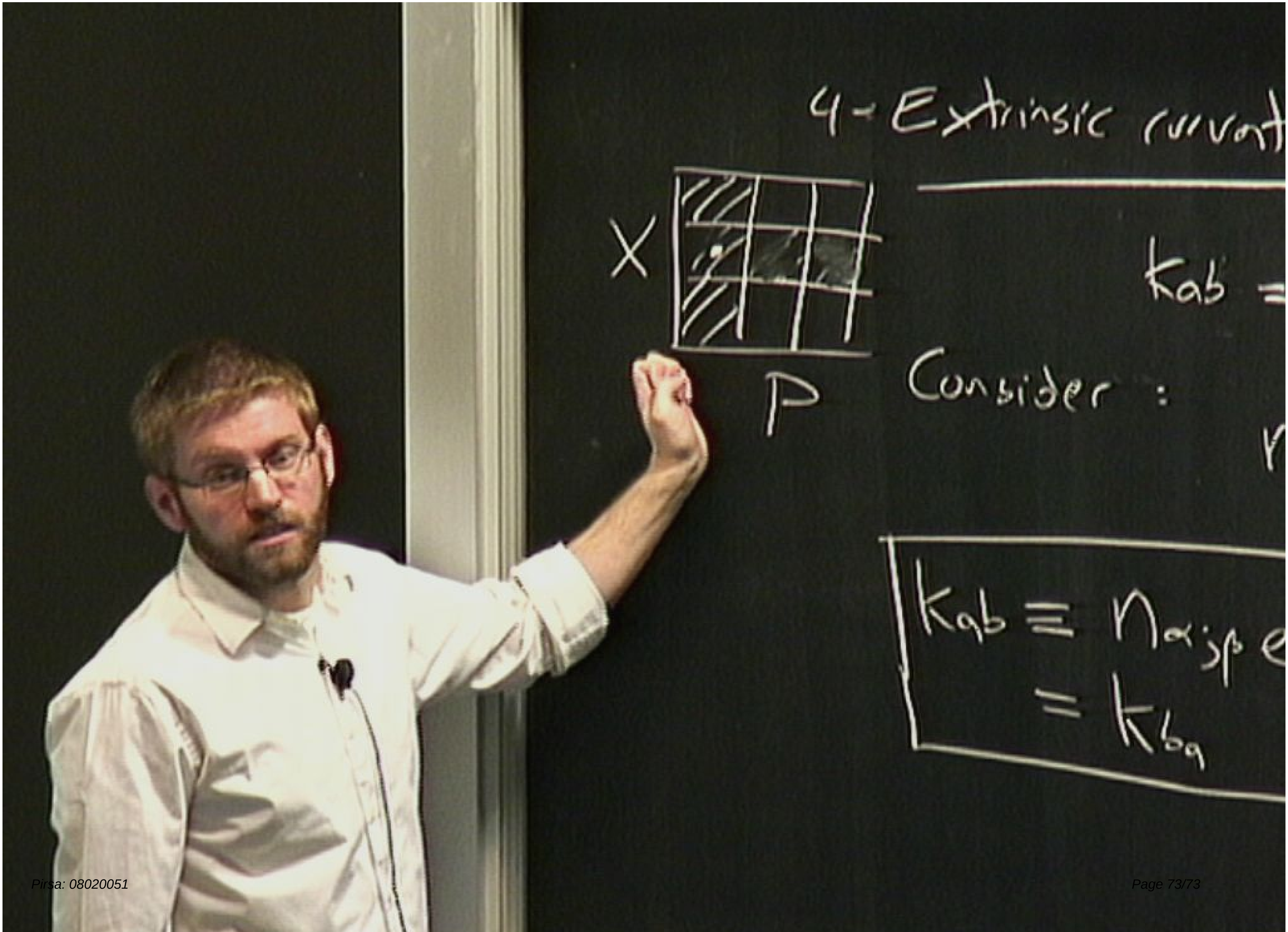
3. ***Find a worldview for which the tension disappears***

no-signalling + Bell inequality violations + determinism → statistical restriction

What assumptions will get us precisely the statistical restriction of QT?


4 - Extrinsic curvat
X
P
Kab =
onsider :
Kab ≡ nα;β e
= Kba


4 - Extrinsic curvat
X
P
Kab =
Consider :
Kab ≡ nα;β e
= Kba


4 - Extrinsic curvat
X
P
Kab =
Consider :
Kab ≡ nα;β e
= Kba