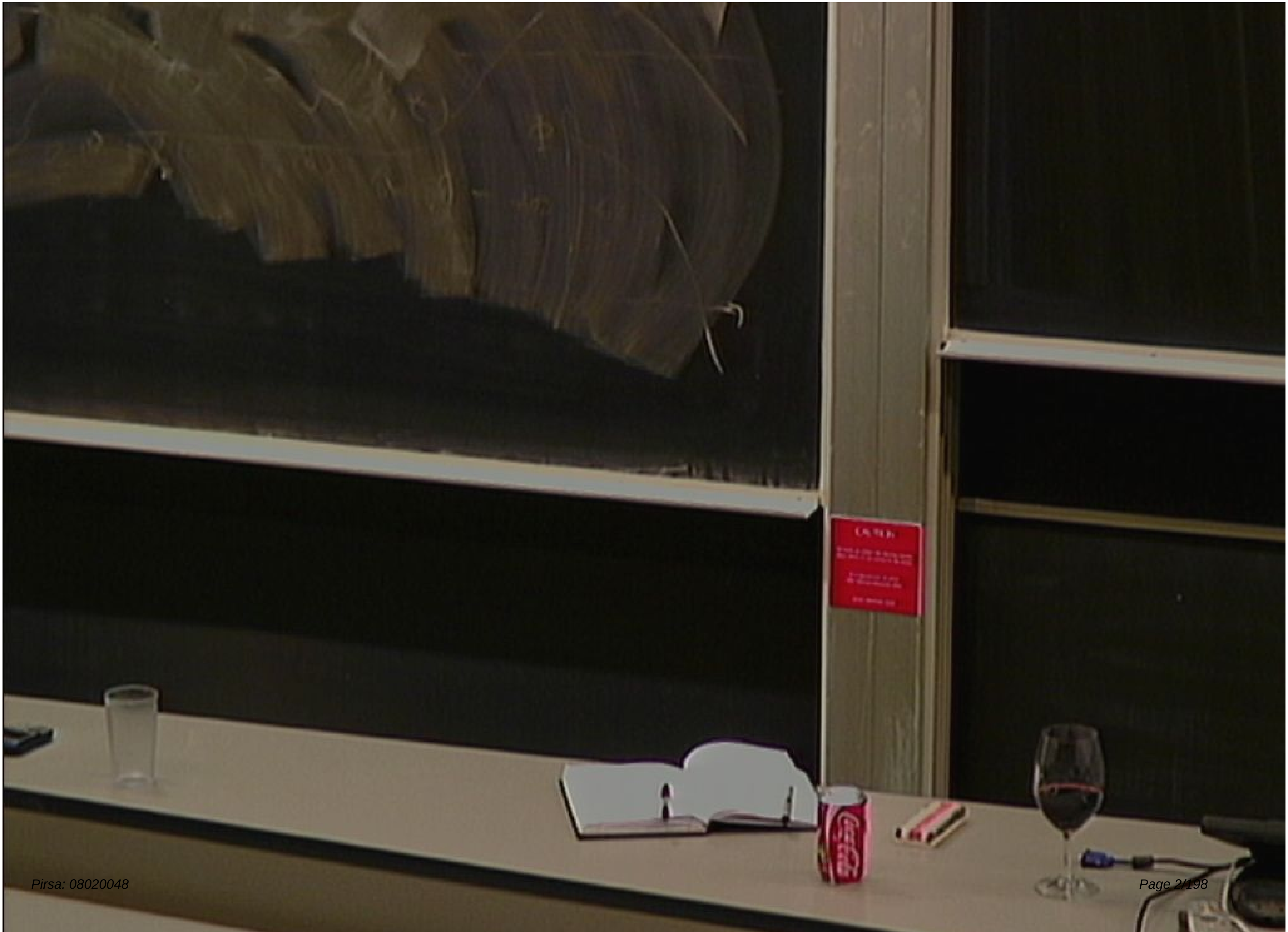
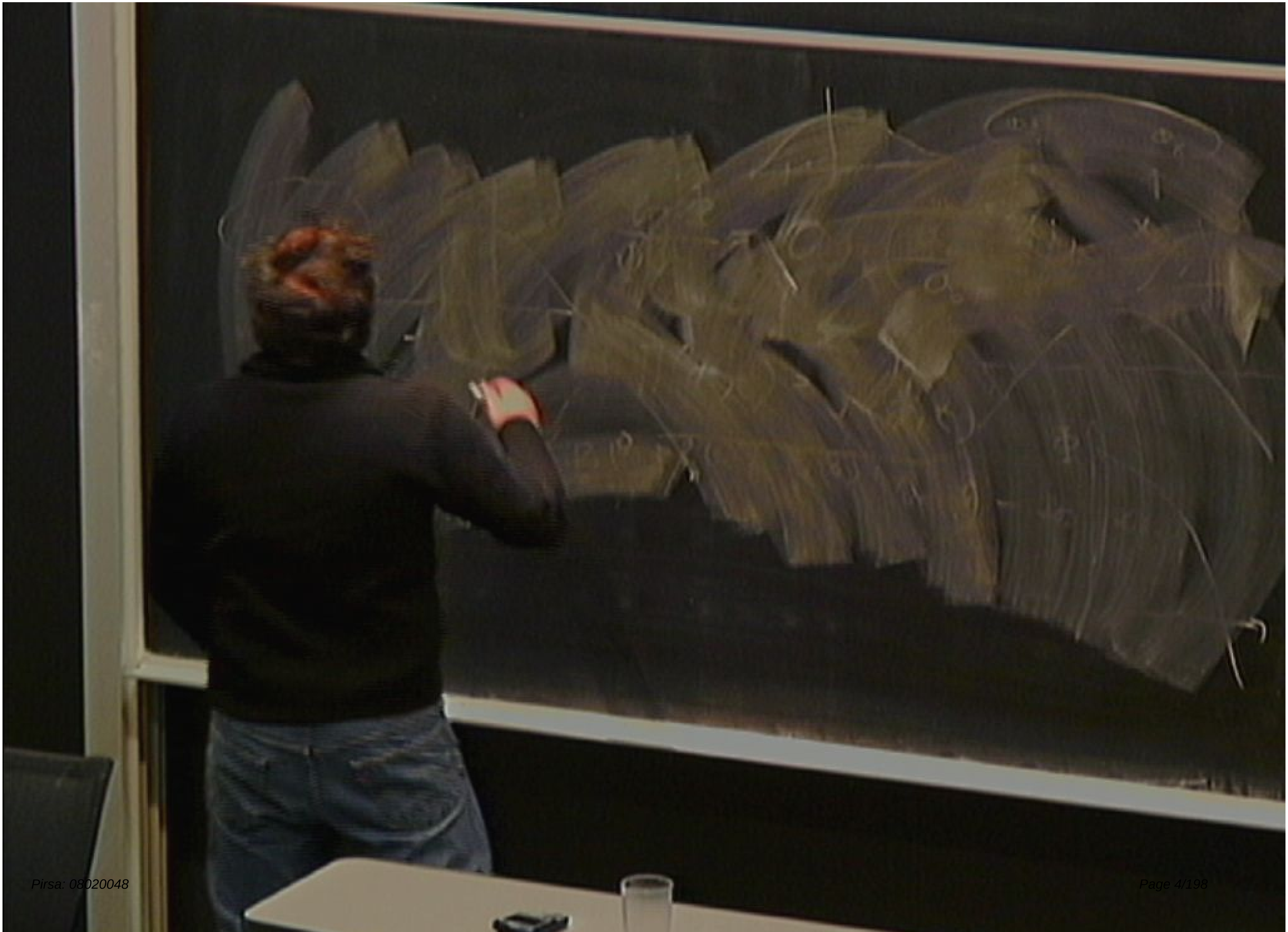
Title: Special Topics in Physics - Lecture 6C

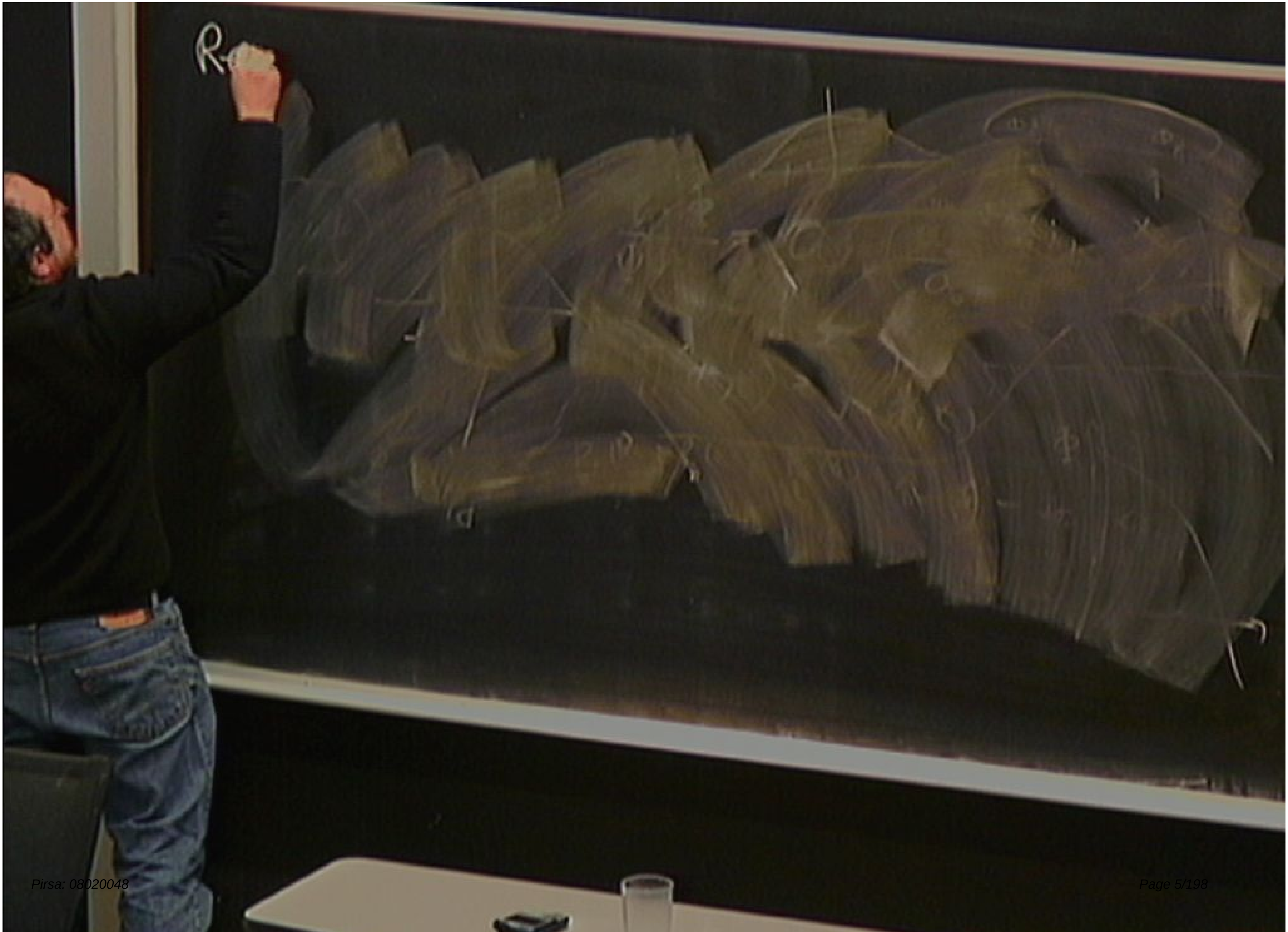
Date: Feb 20, 2008 07:00 PM

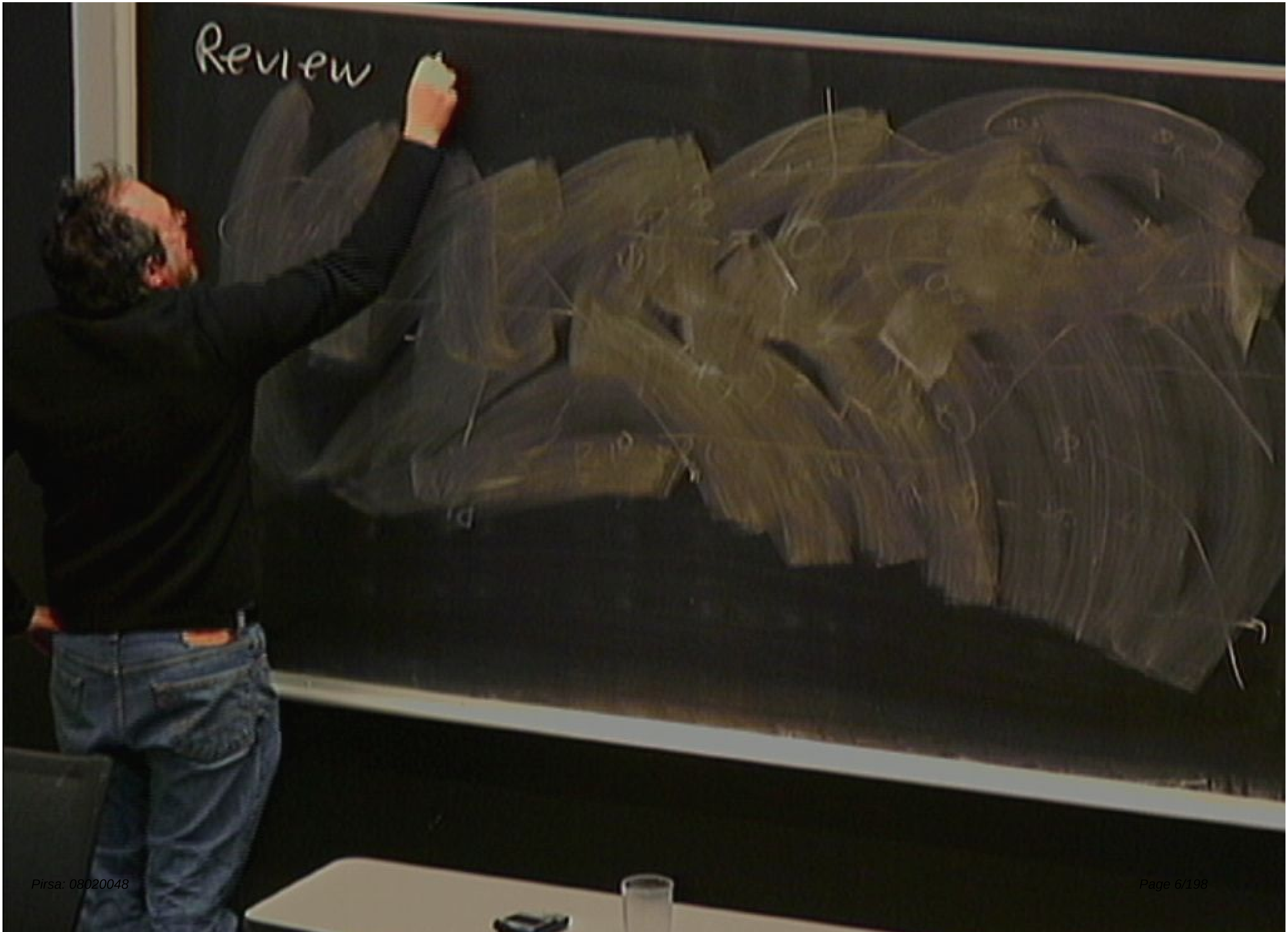
URL: http://pirsa.org/08020048

Abstract: The Problem of Time in Quantum Gravity and Cosmology





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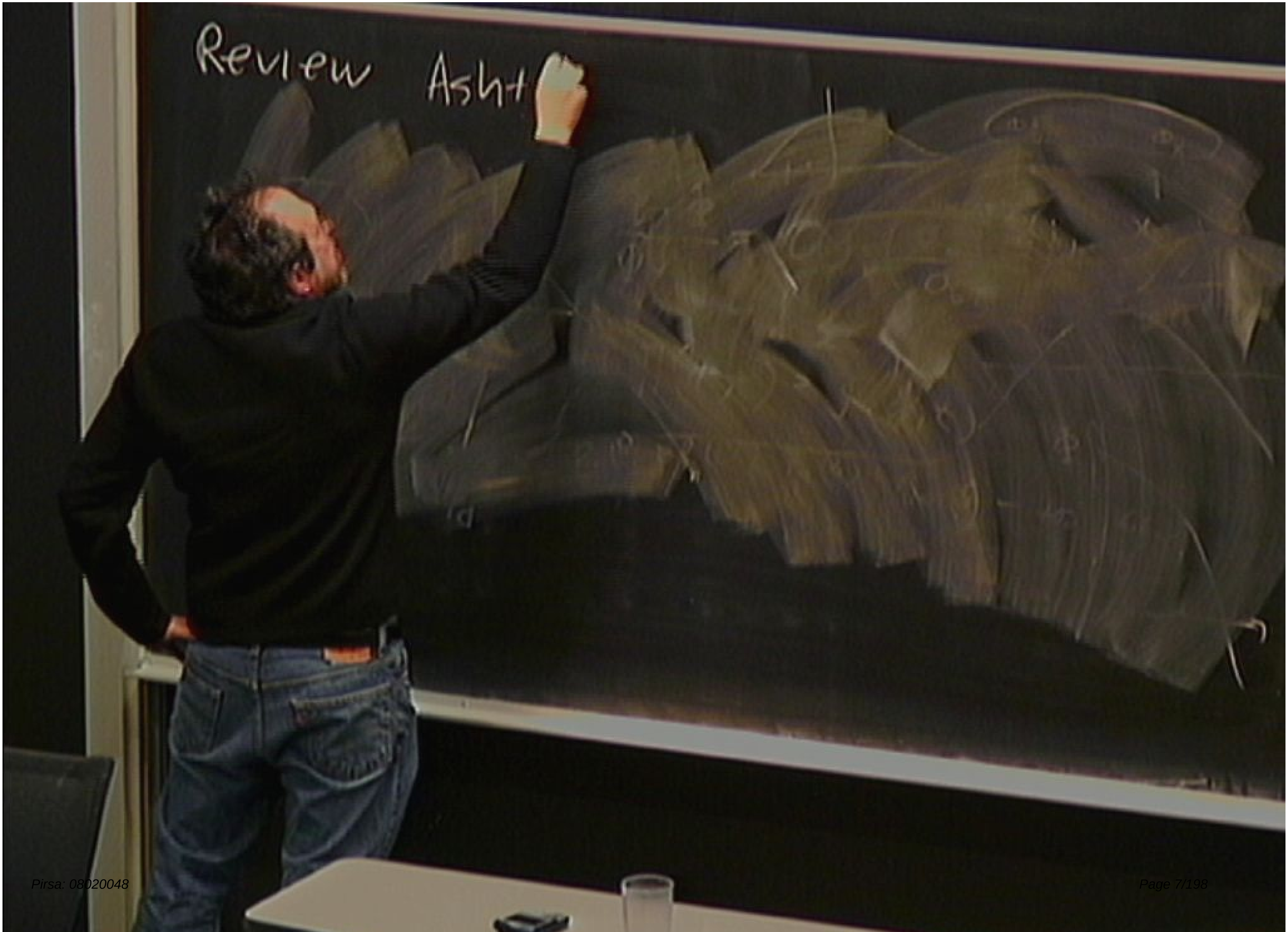
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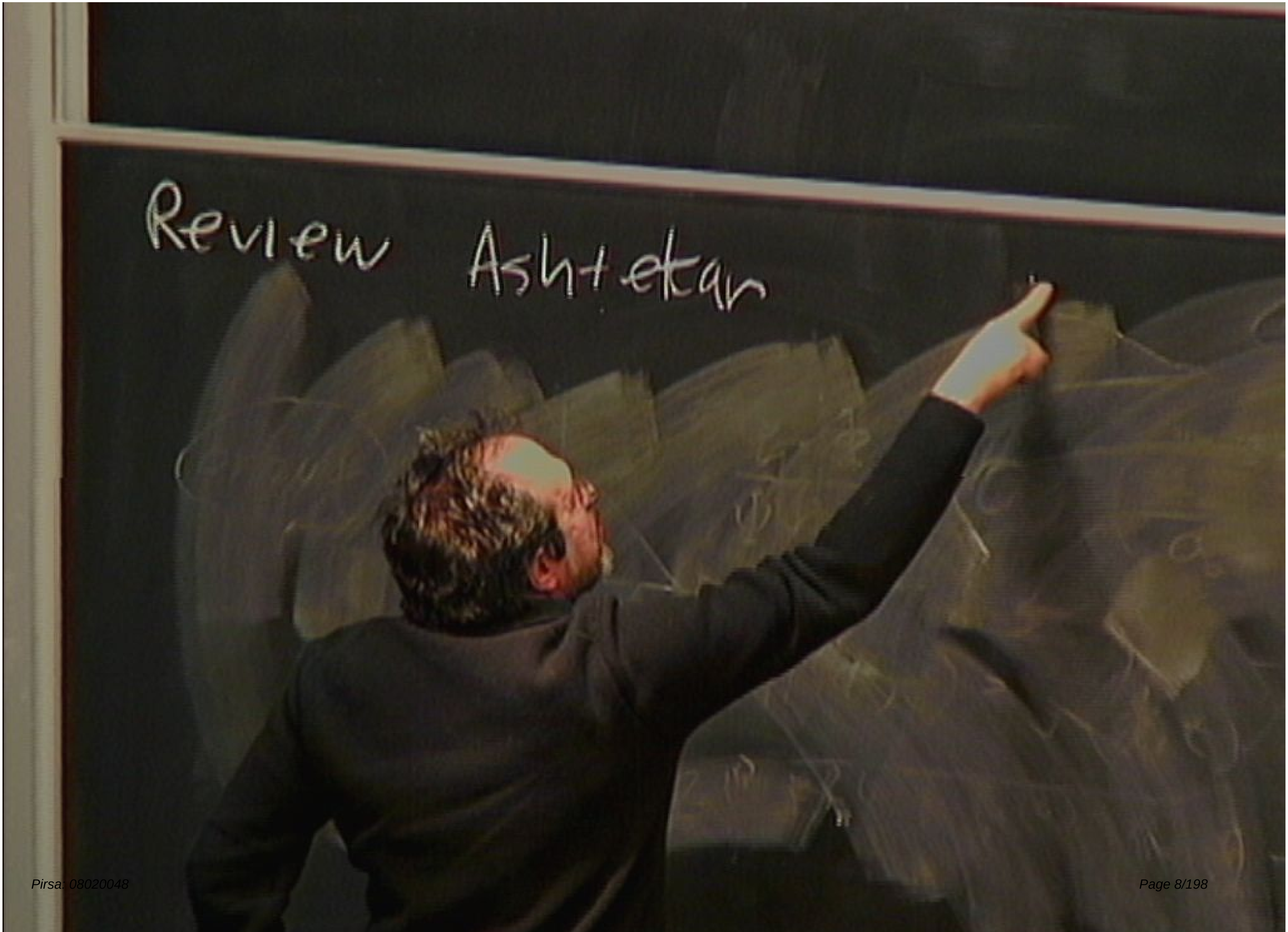
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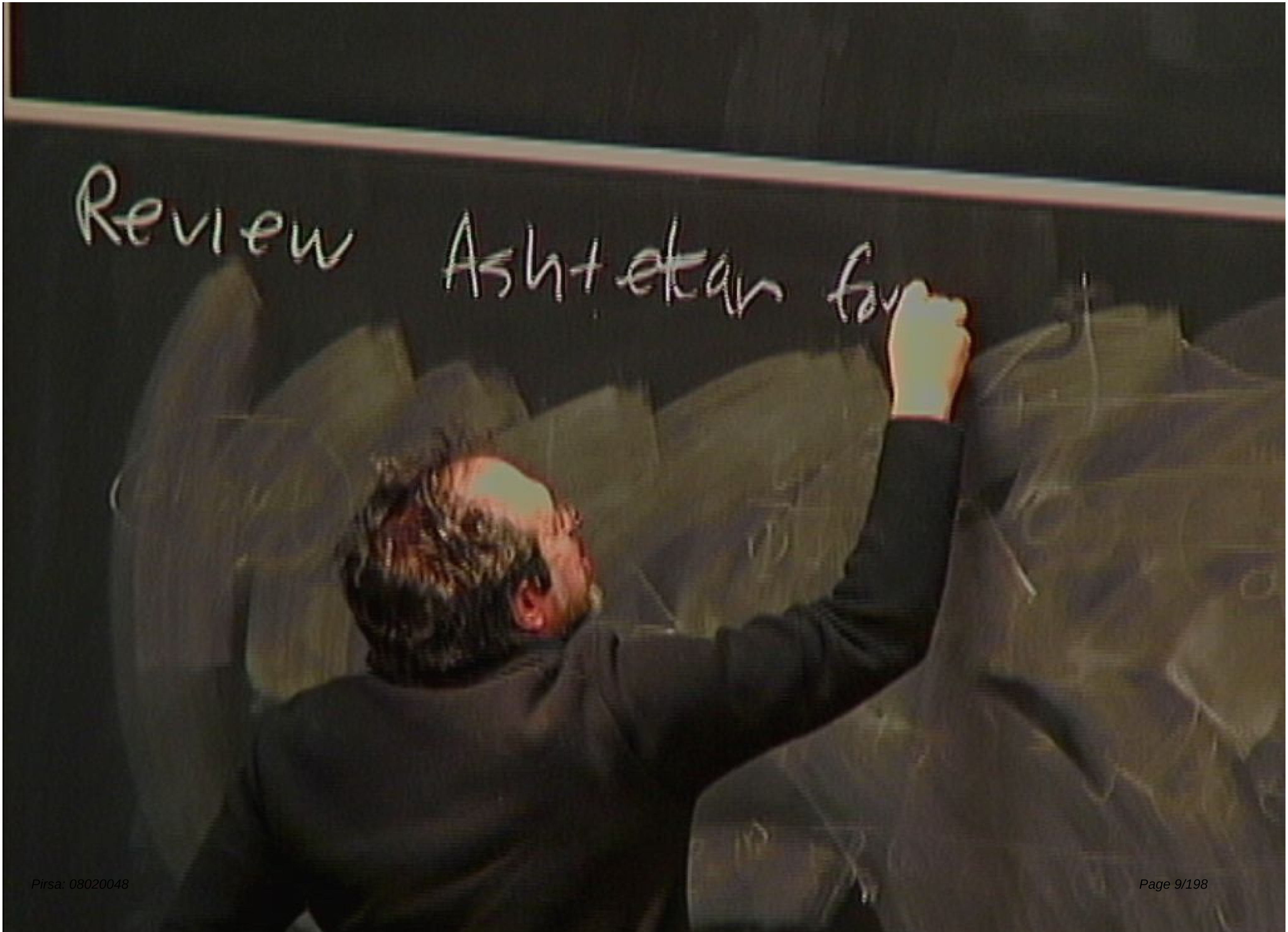
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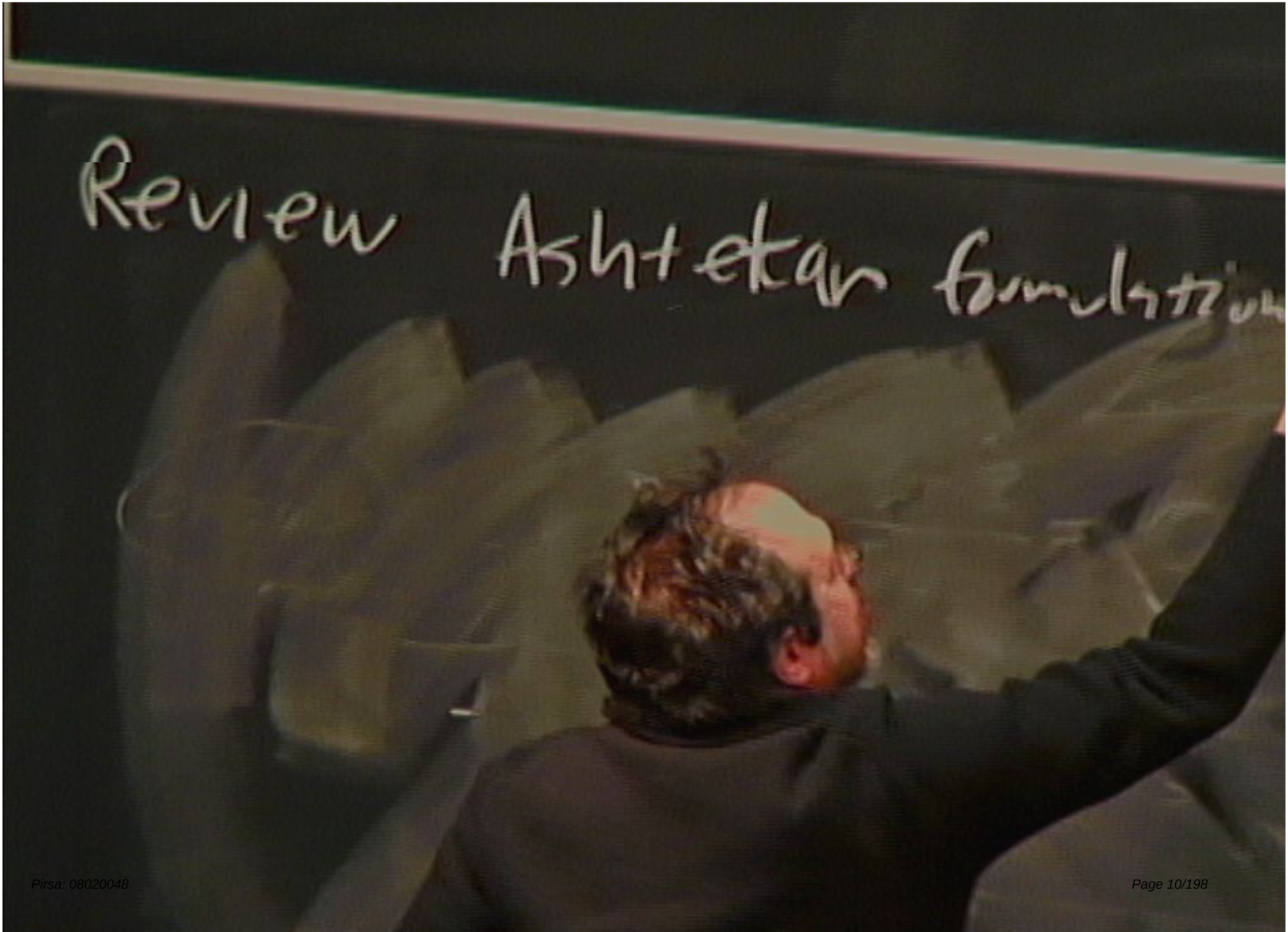
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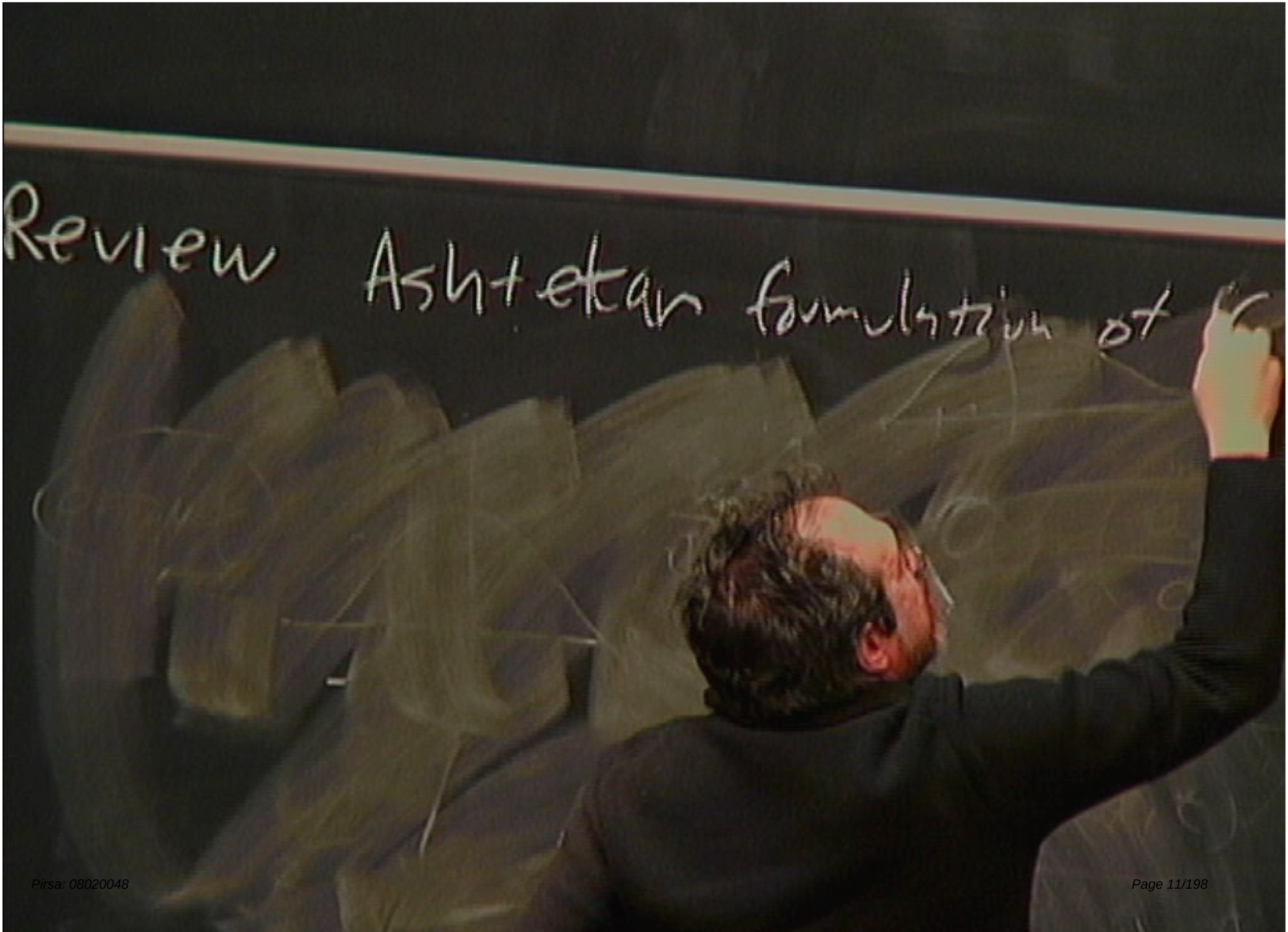
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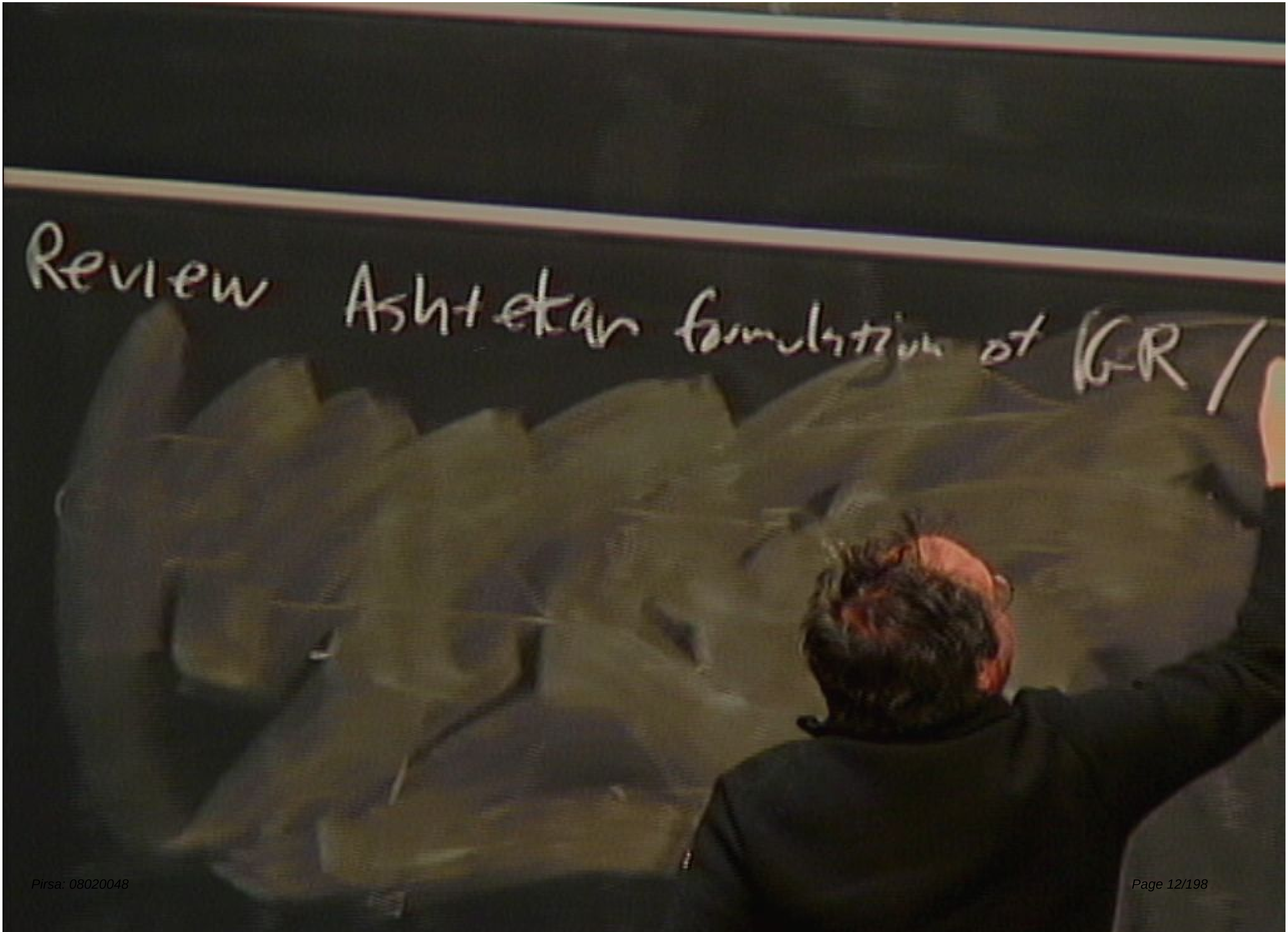
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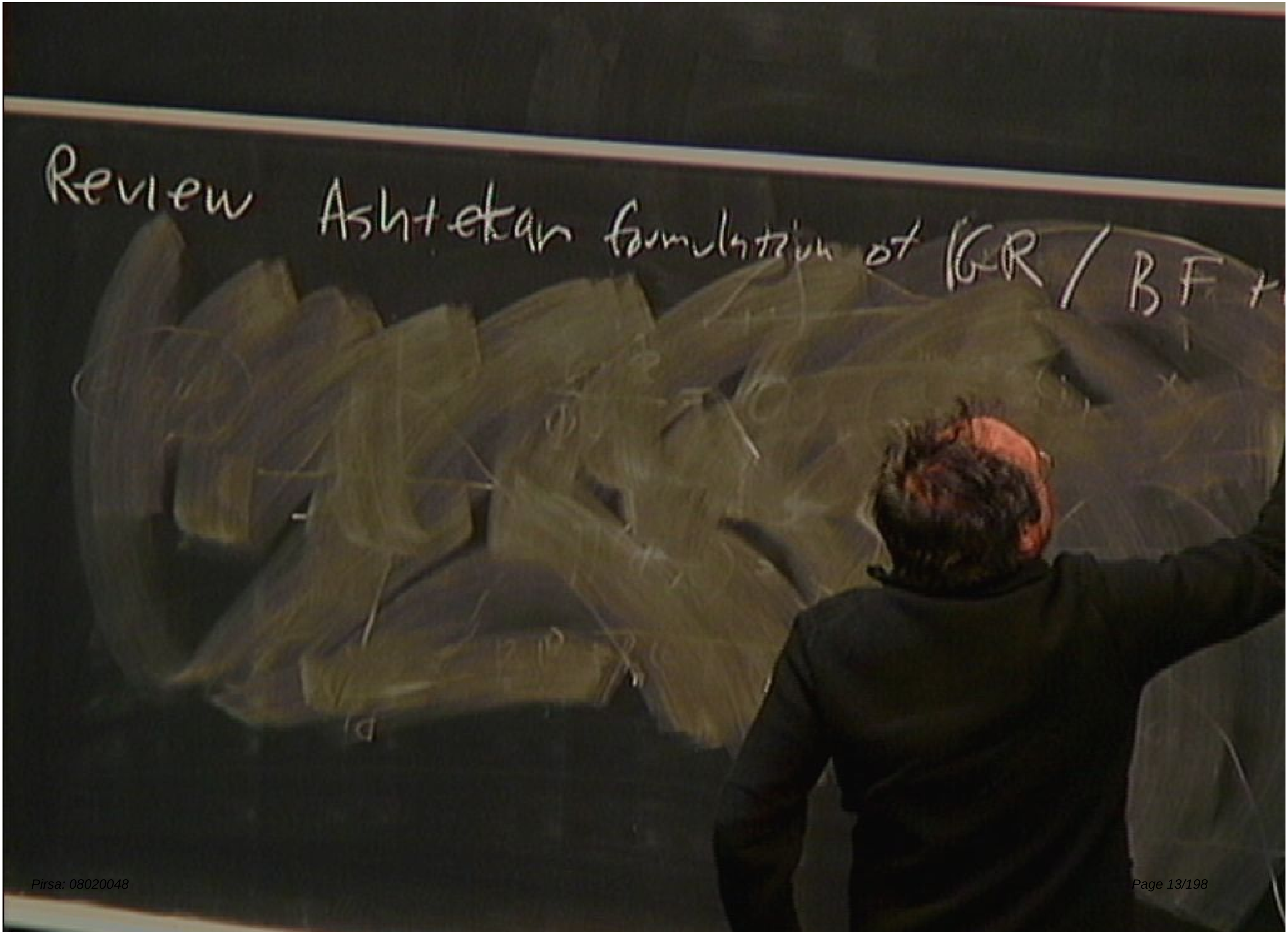
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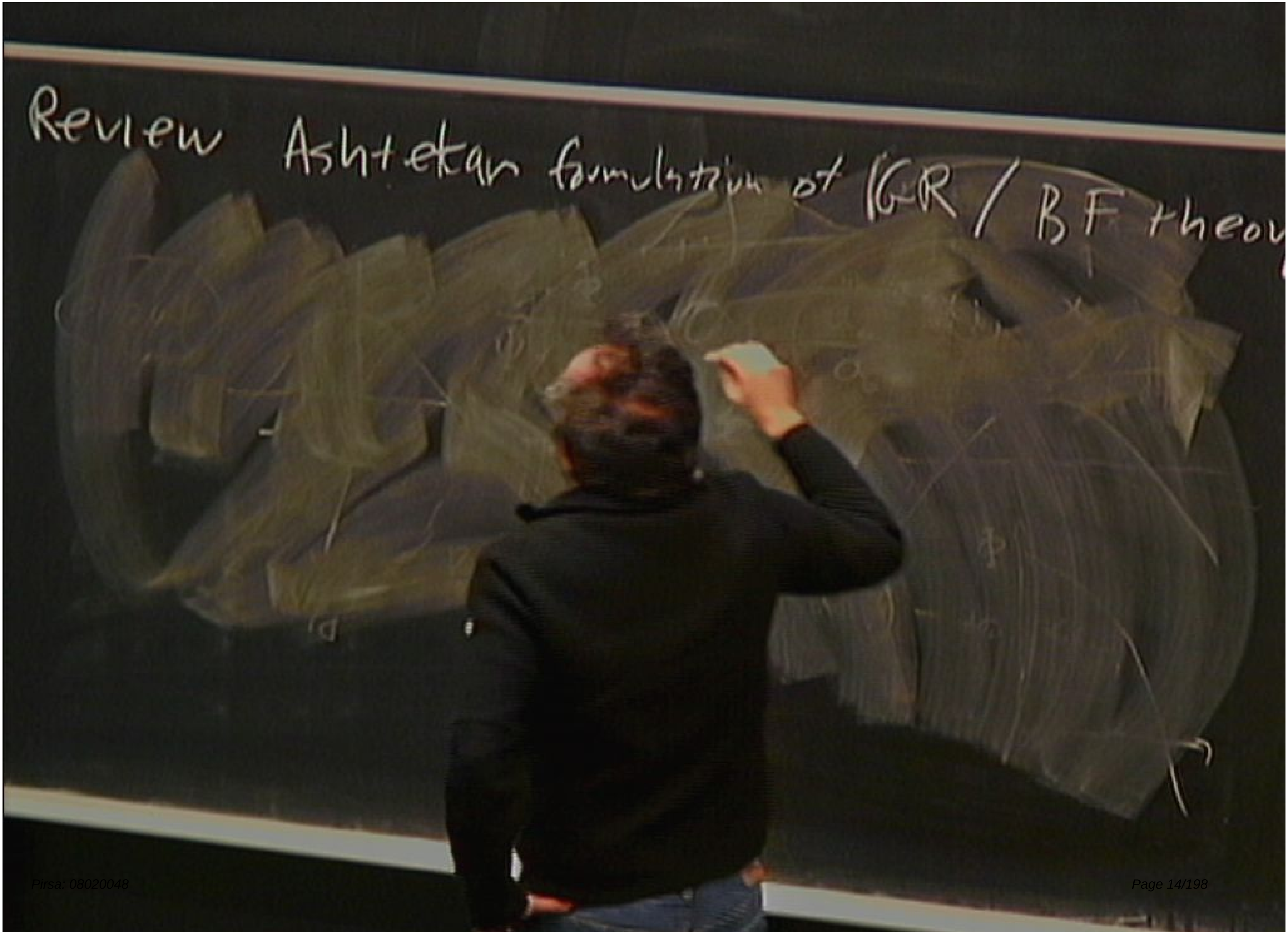
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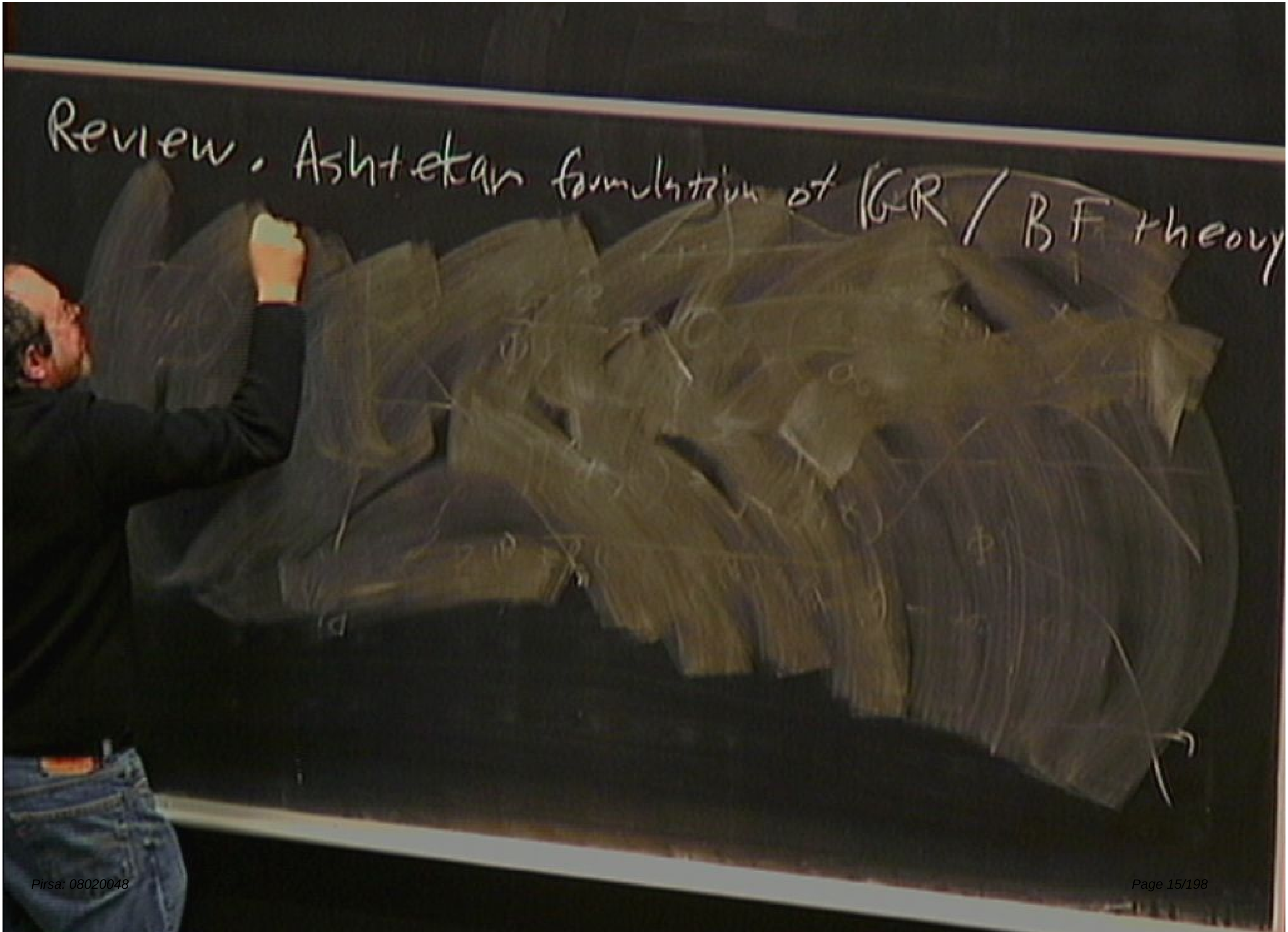
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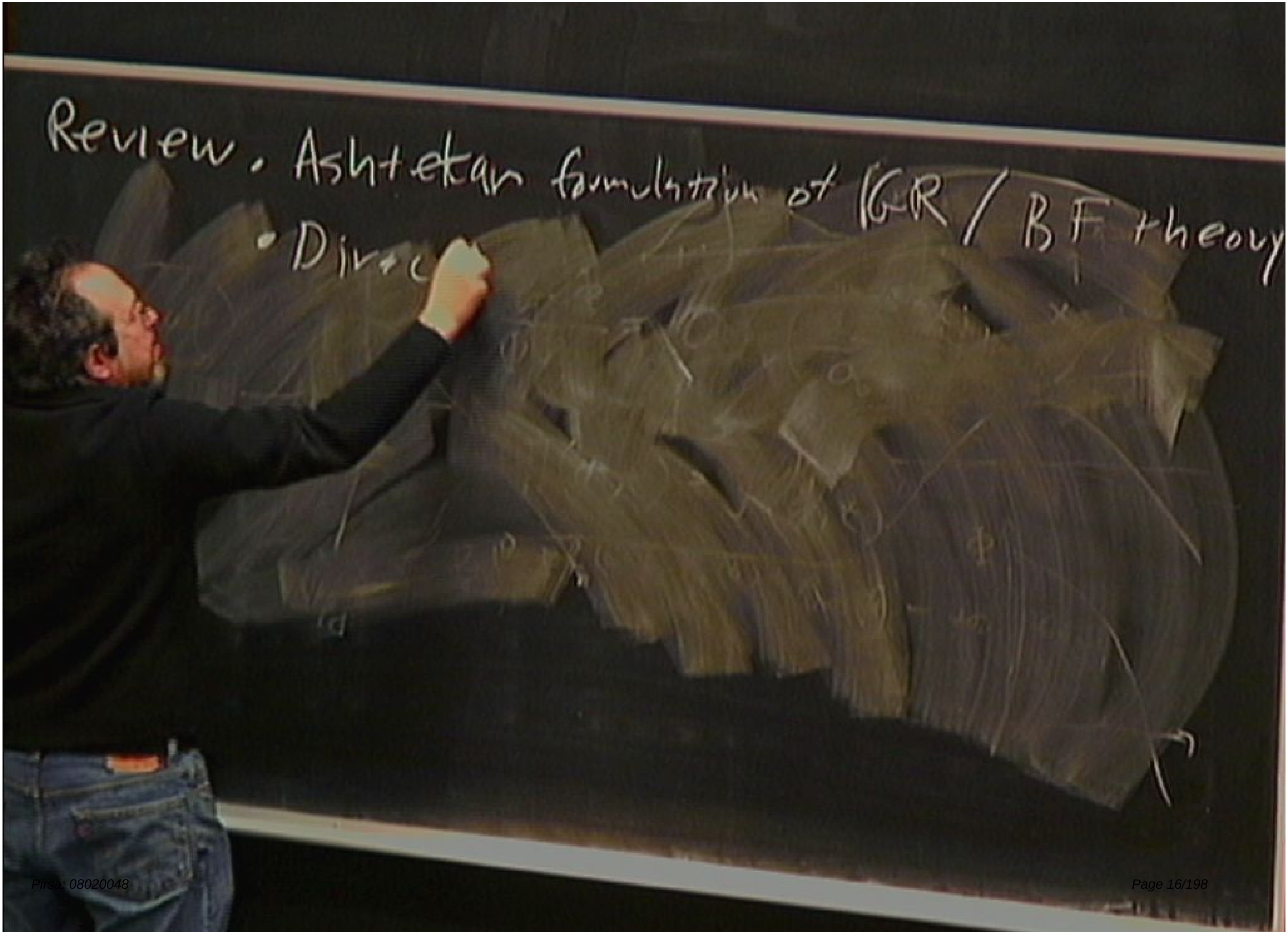
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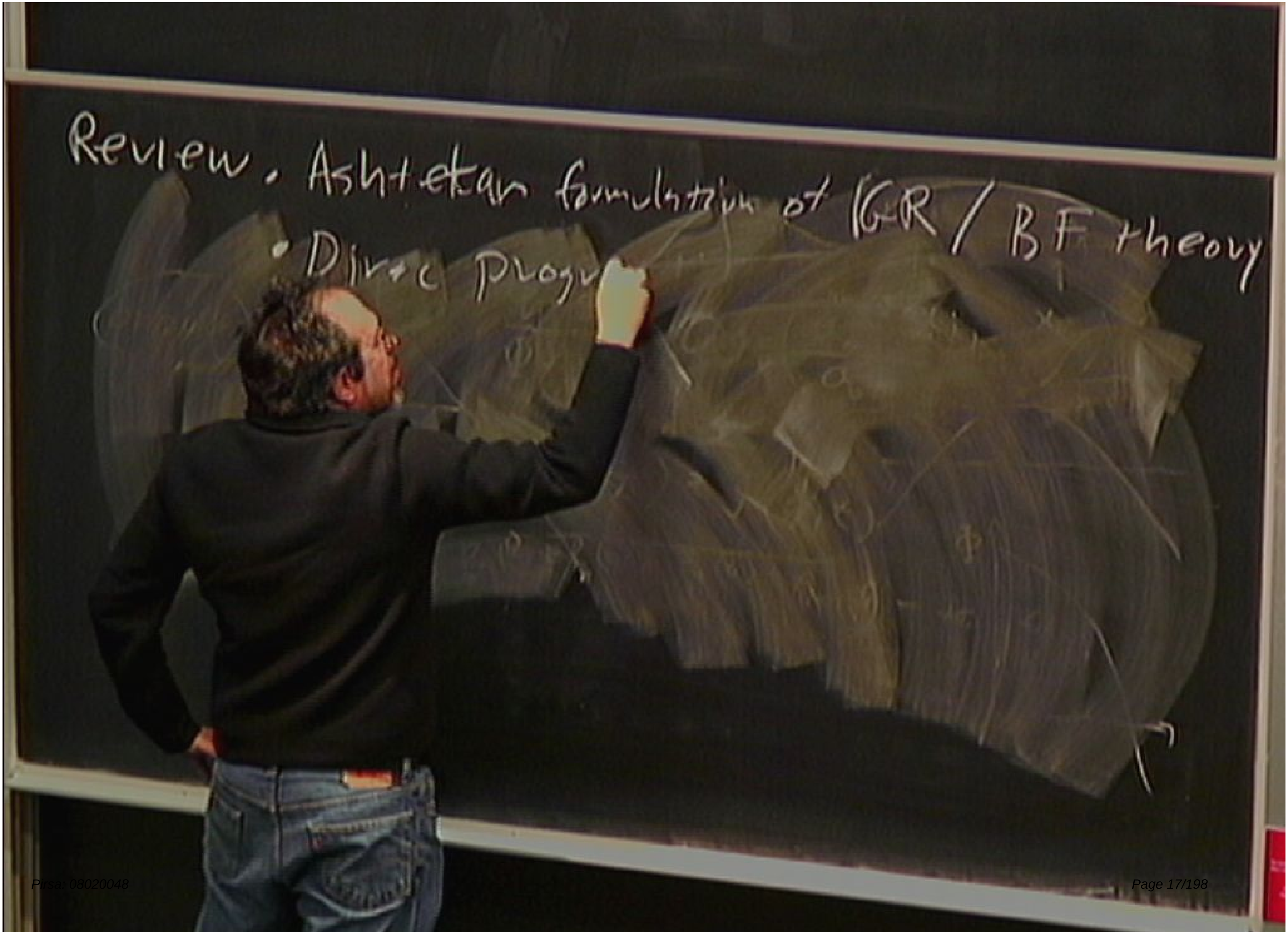
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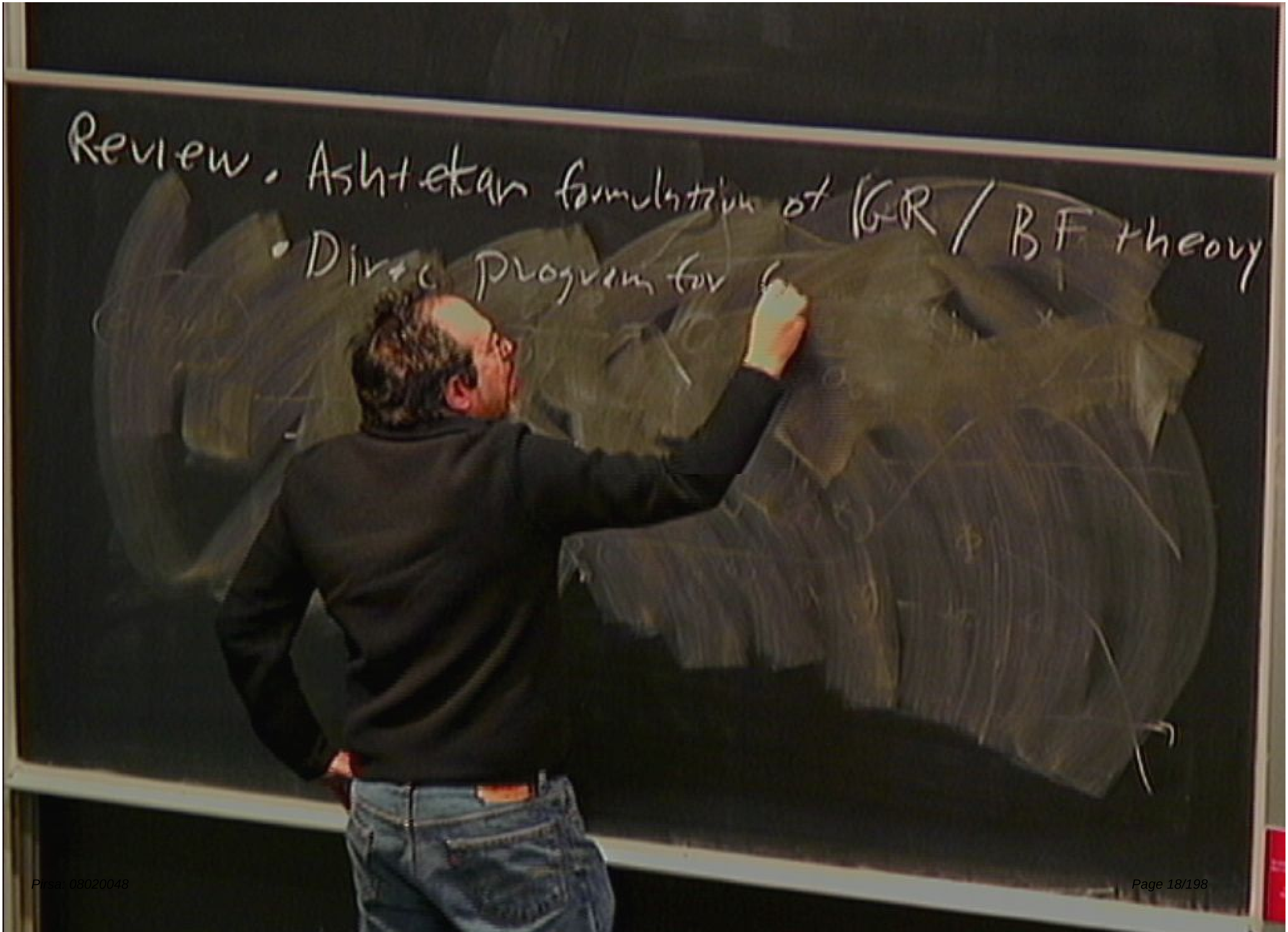
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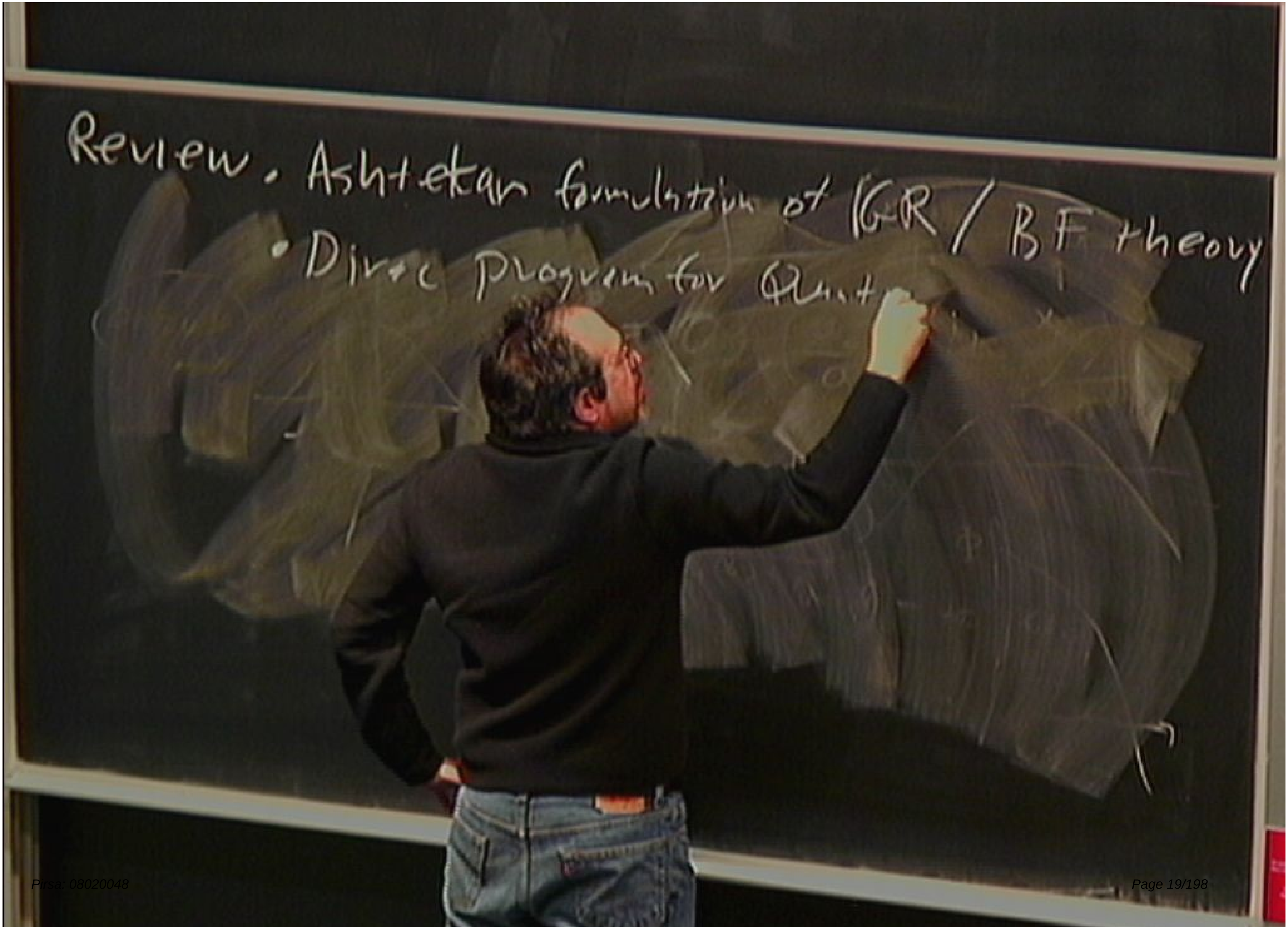
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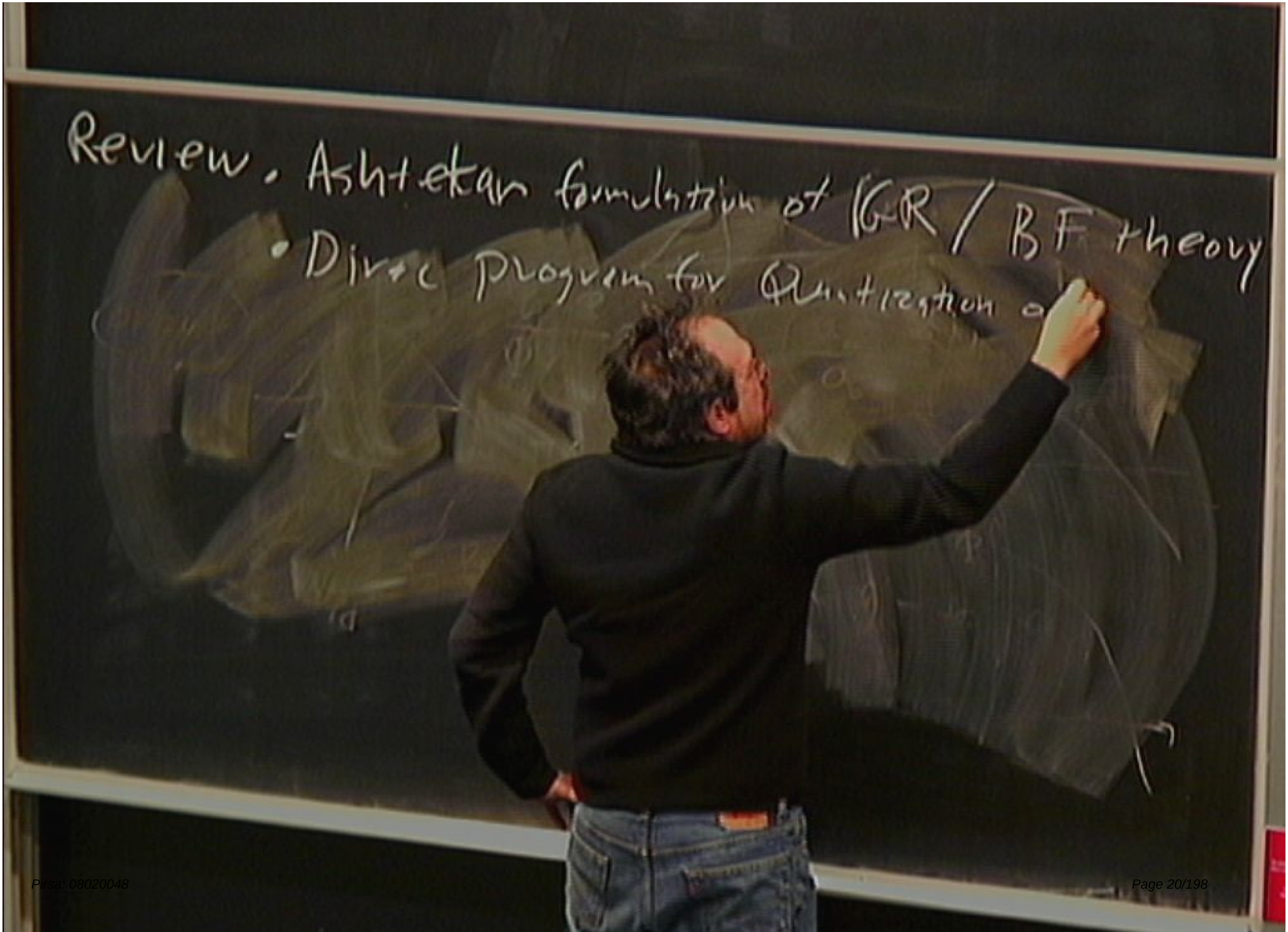
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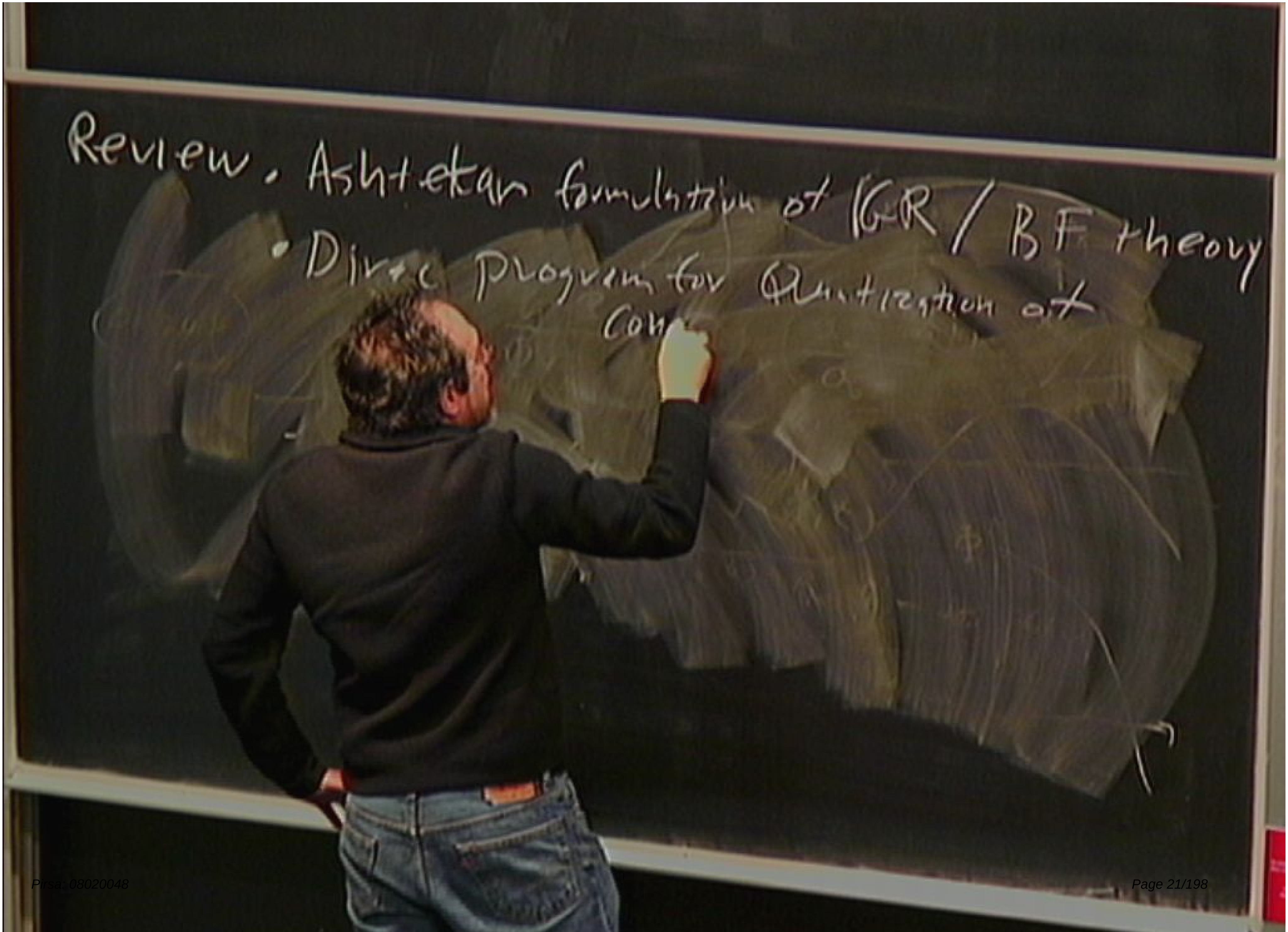
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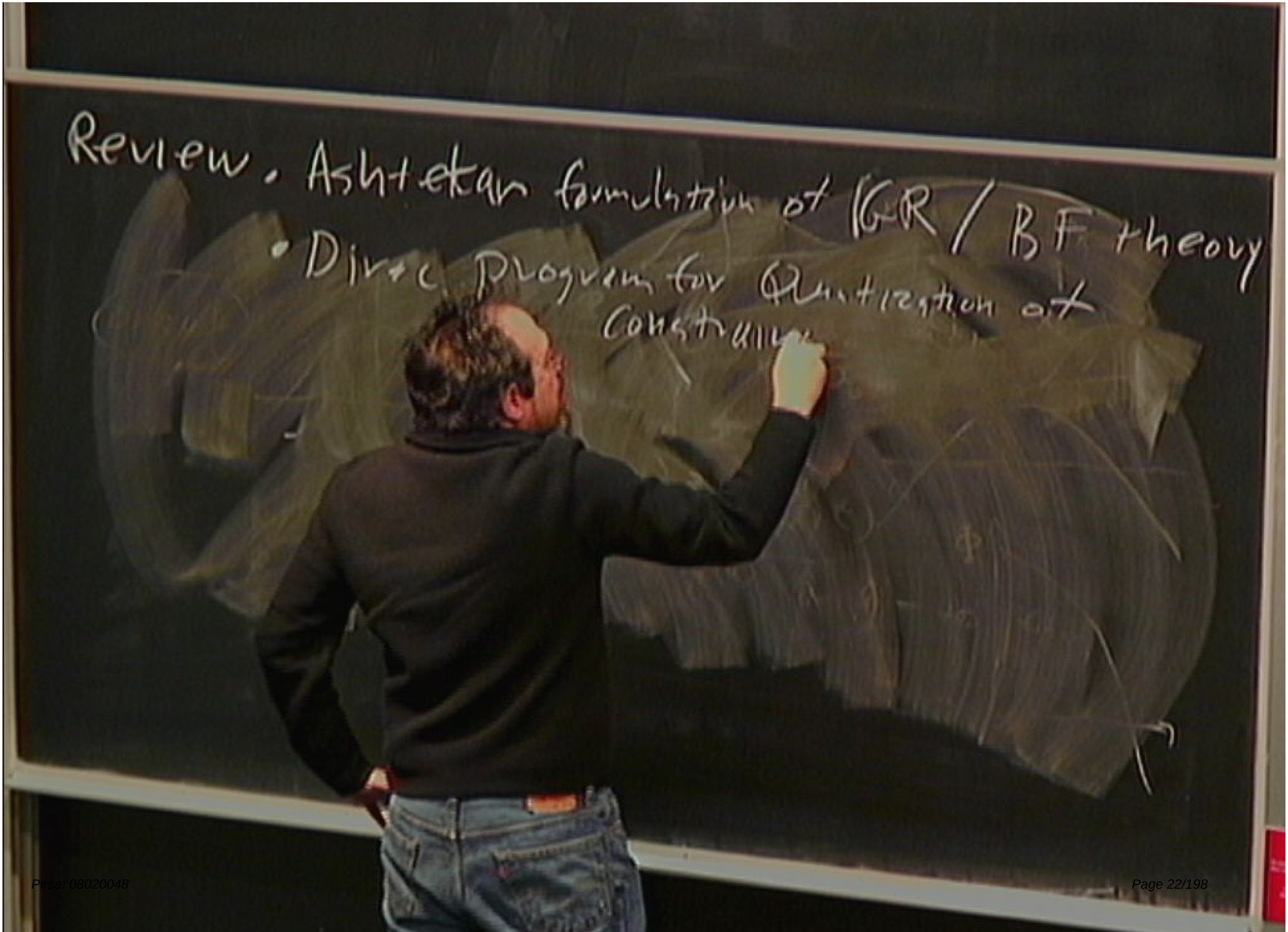
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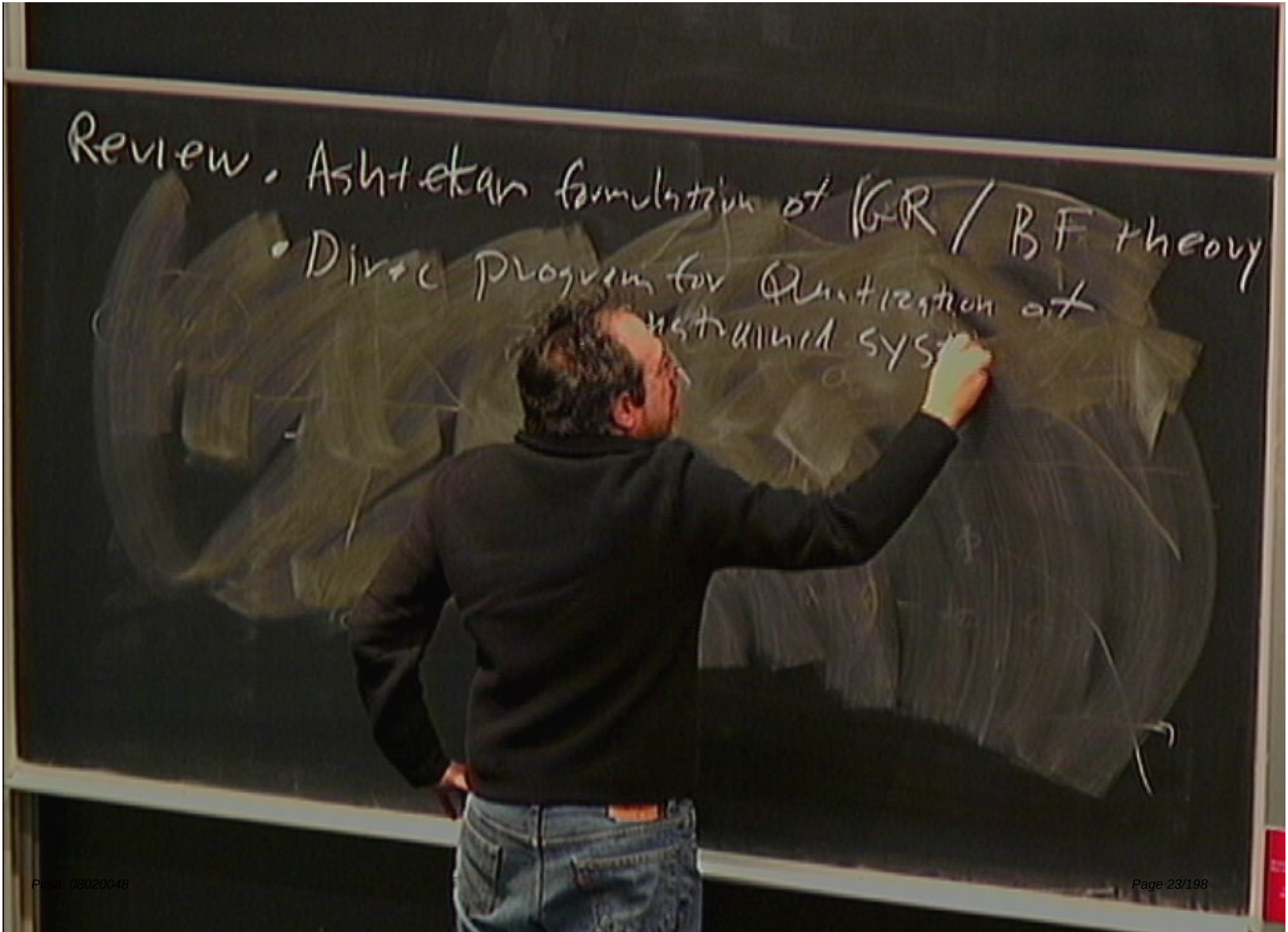
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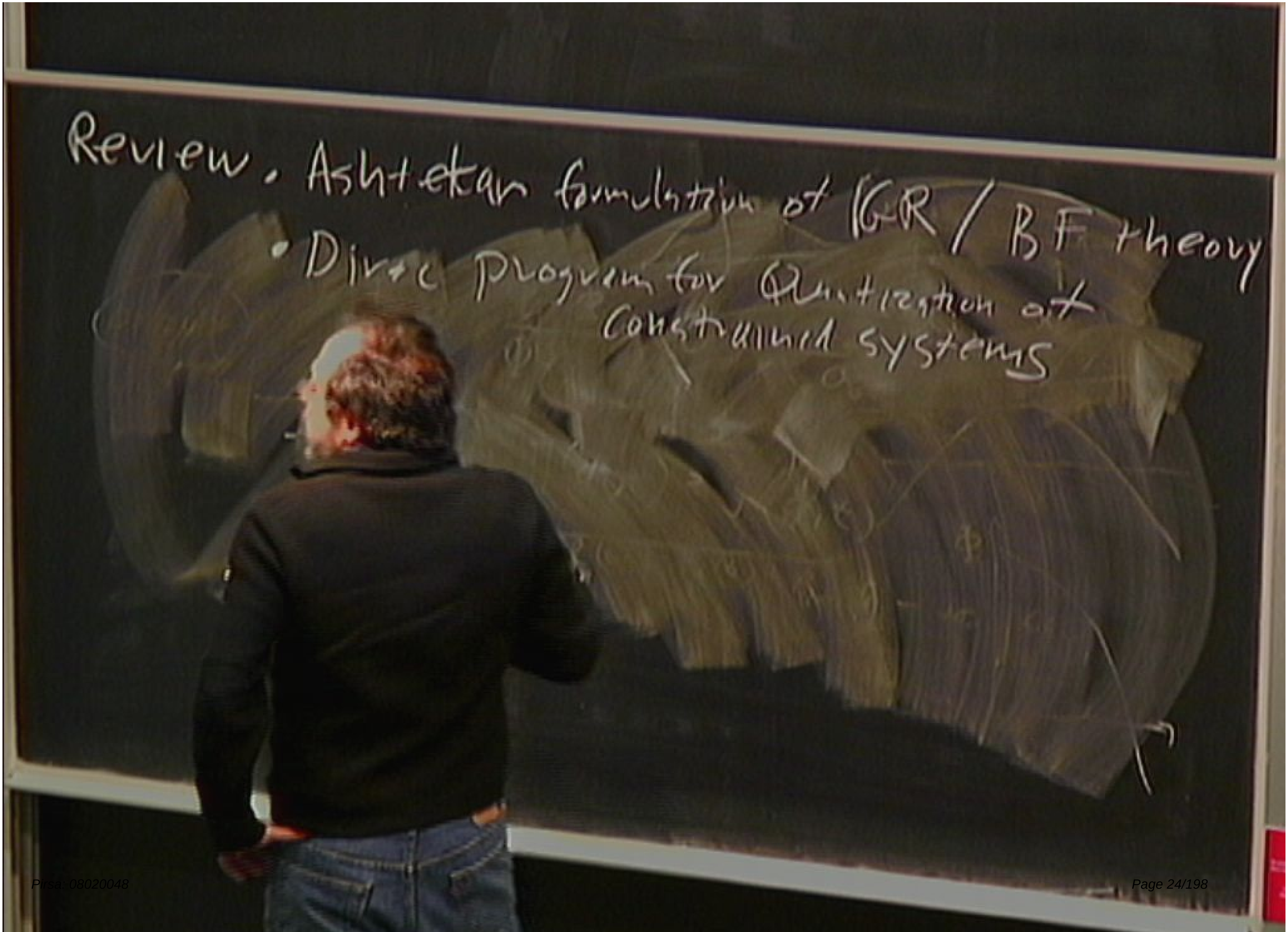
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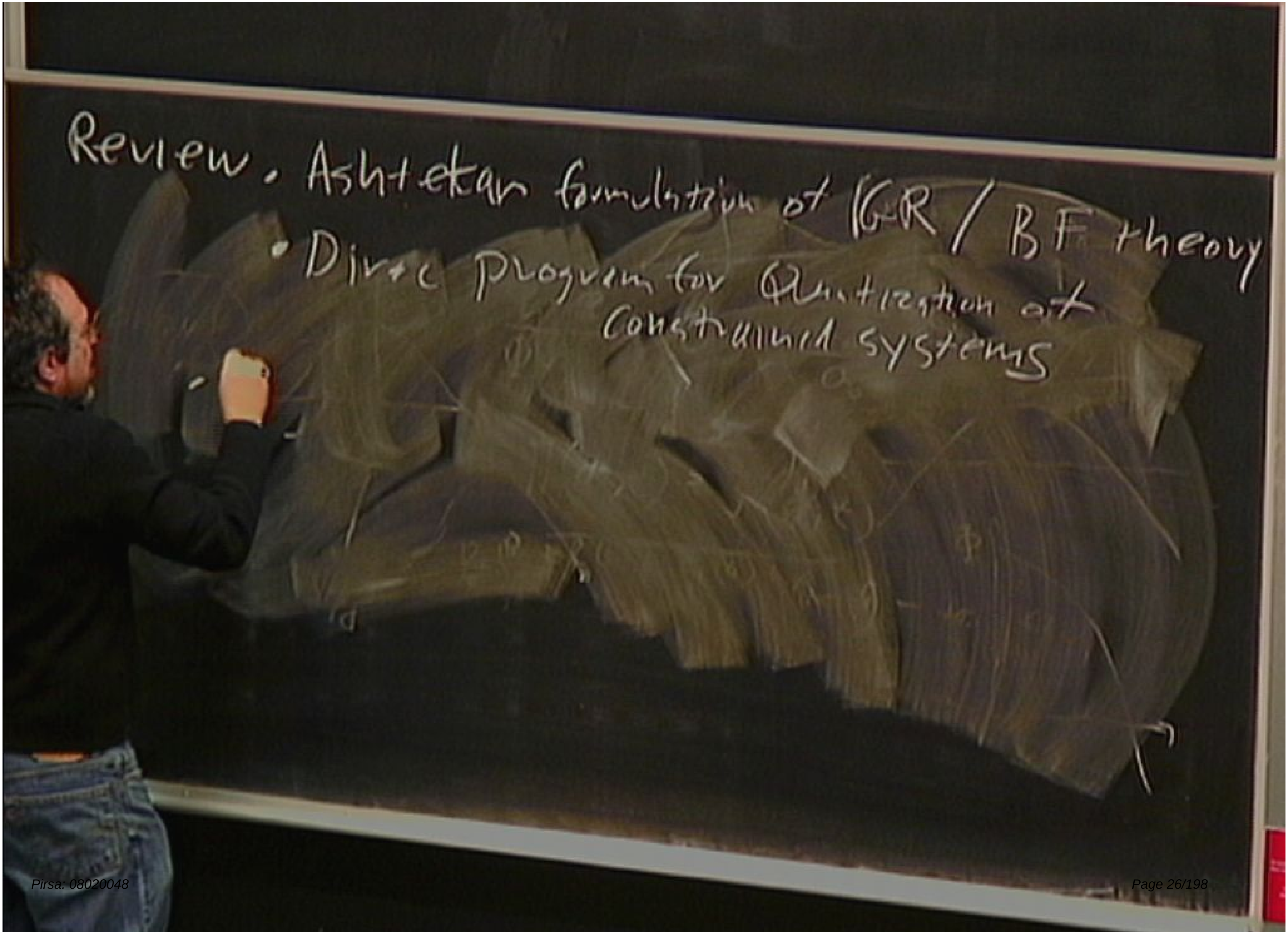
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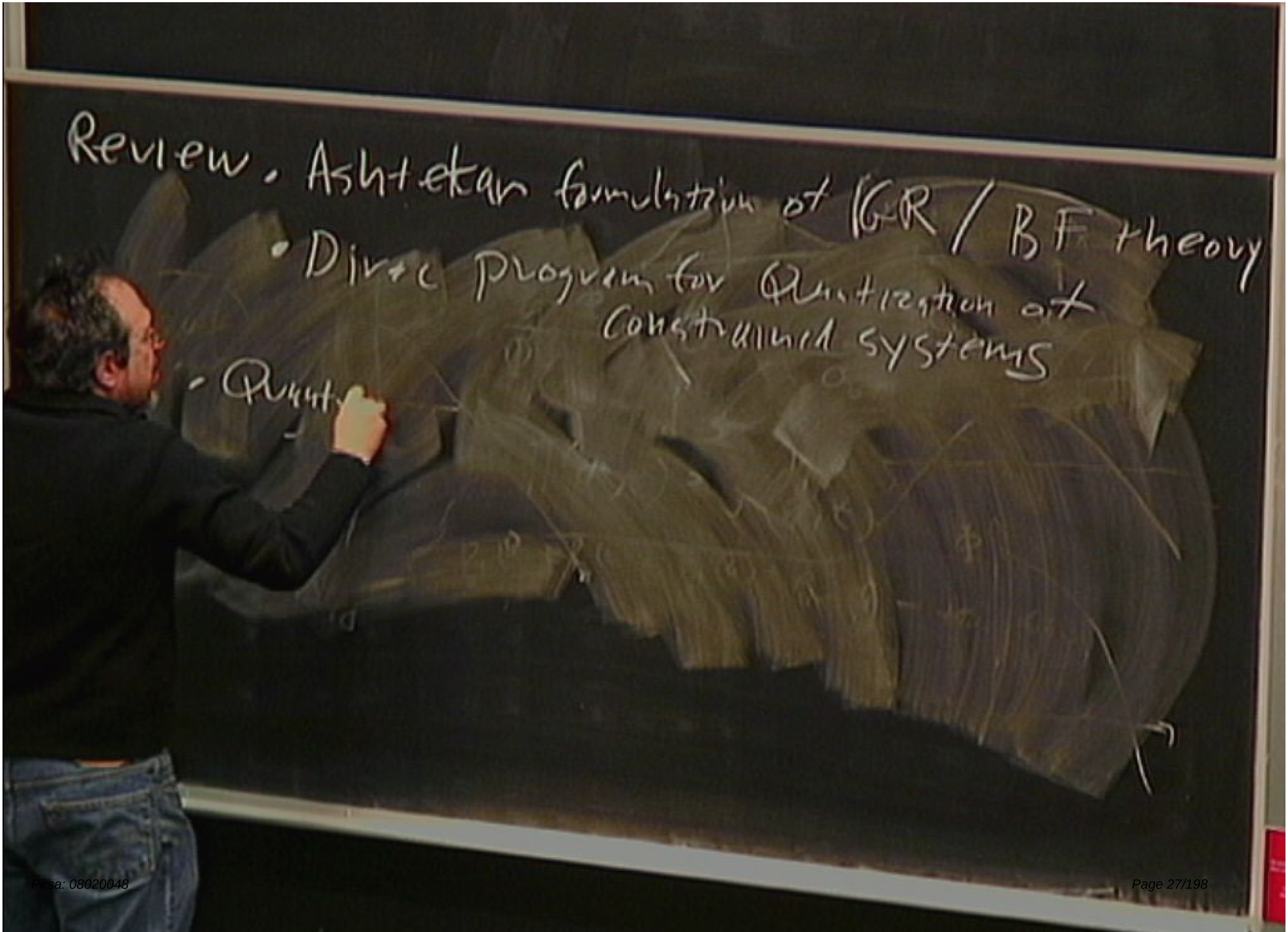
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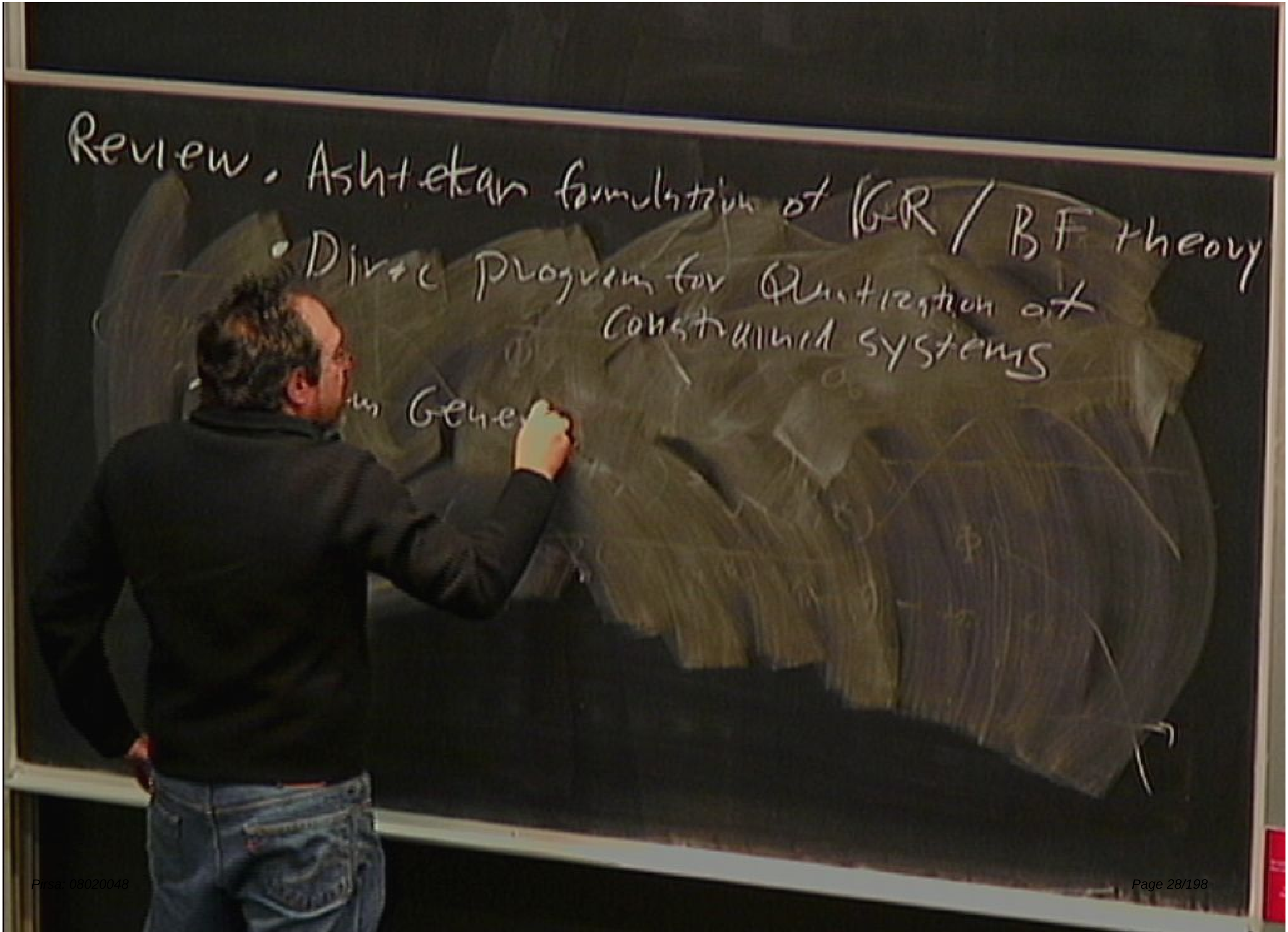
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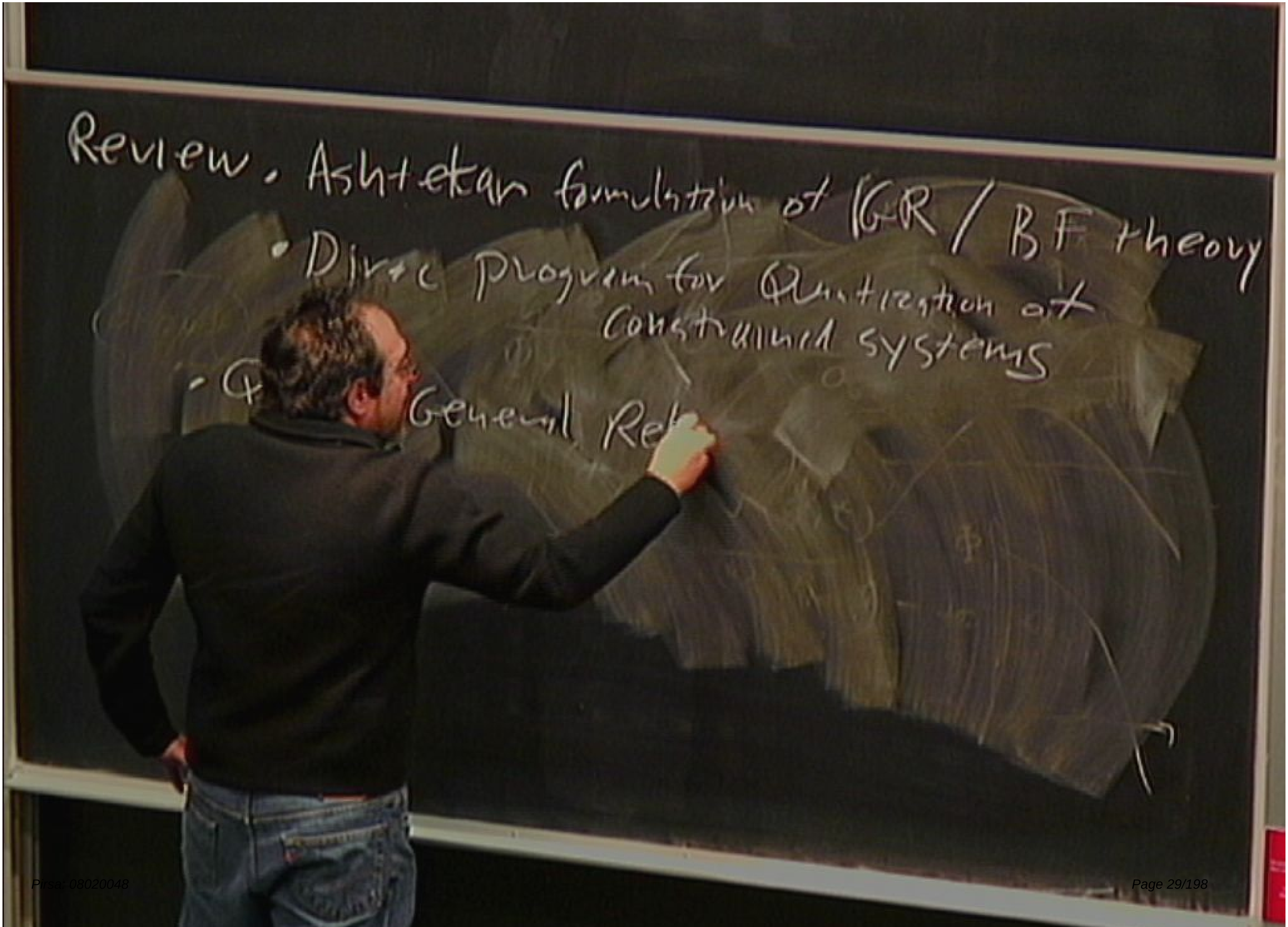
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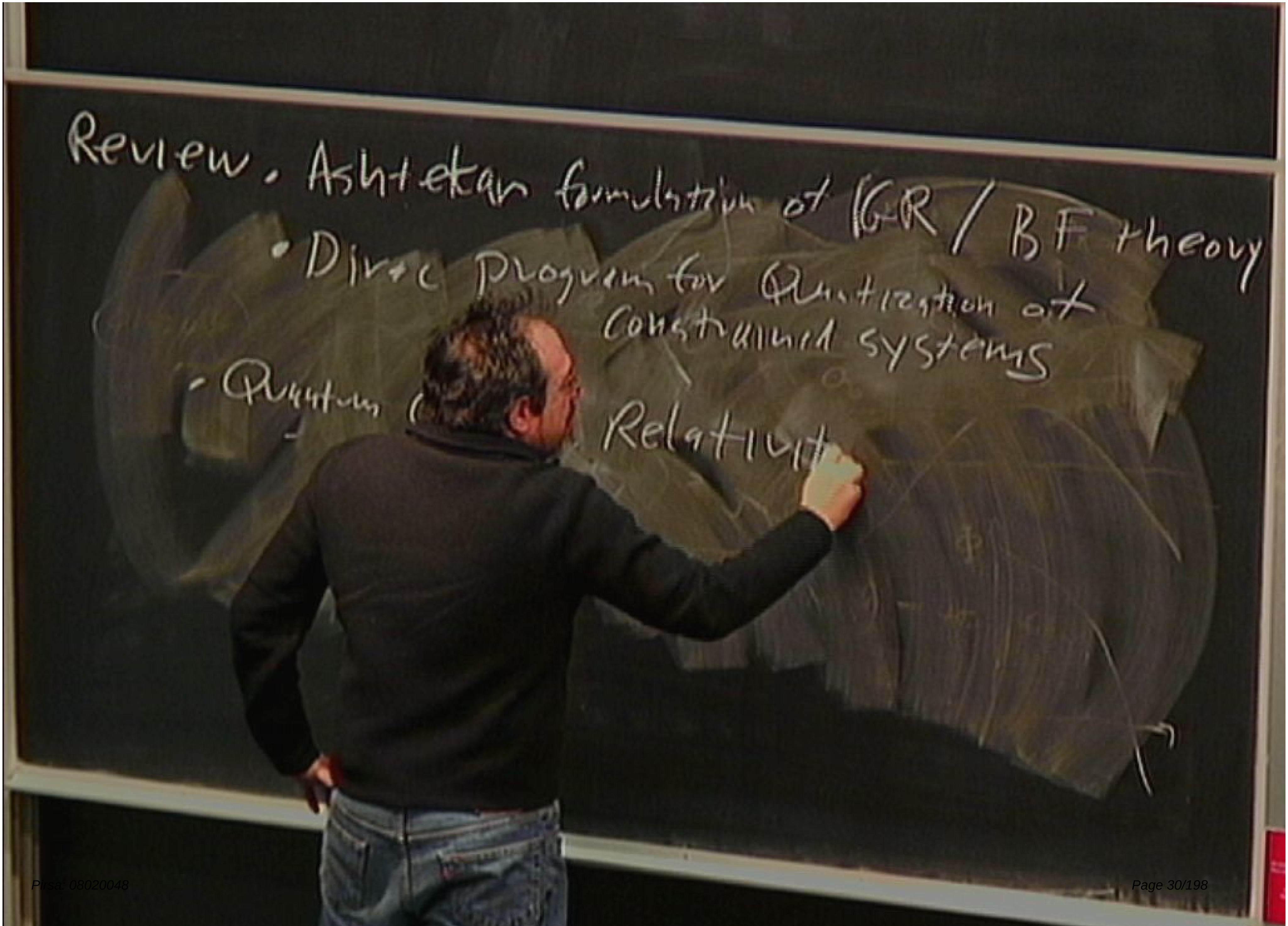
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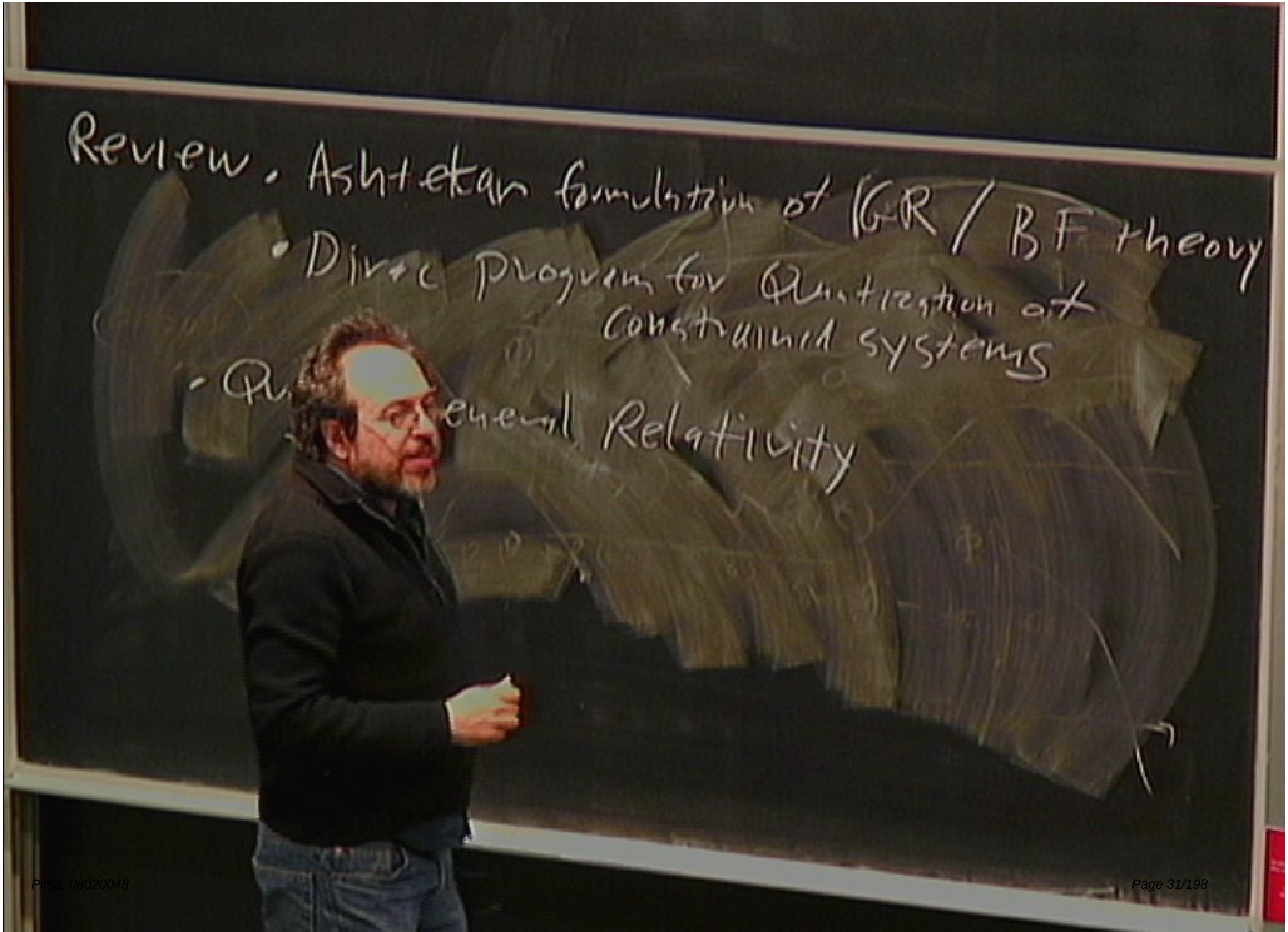
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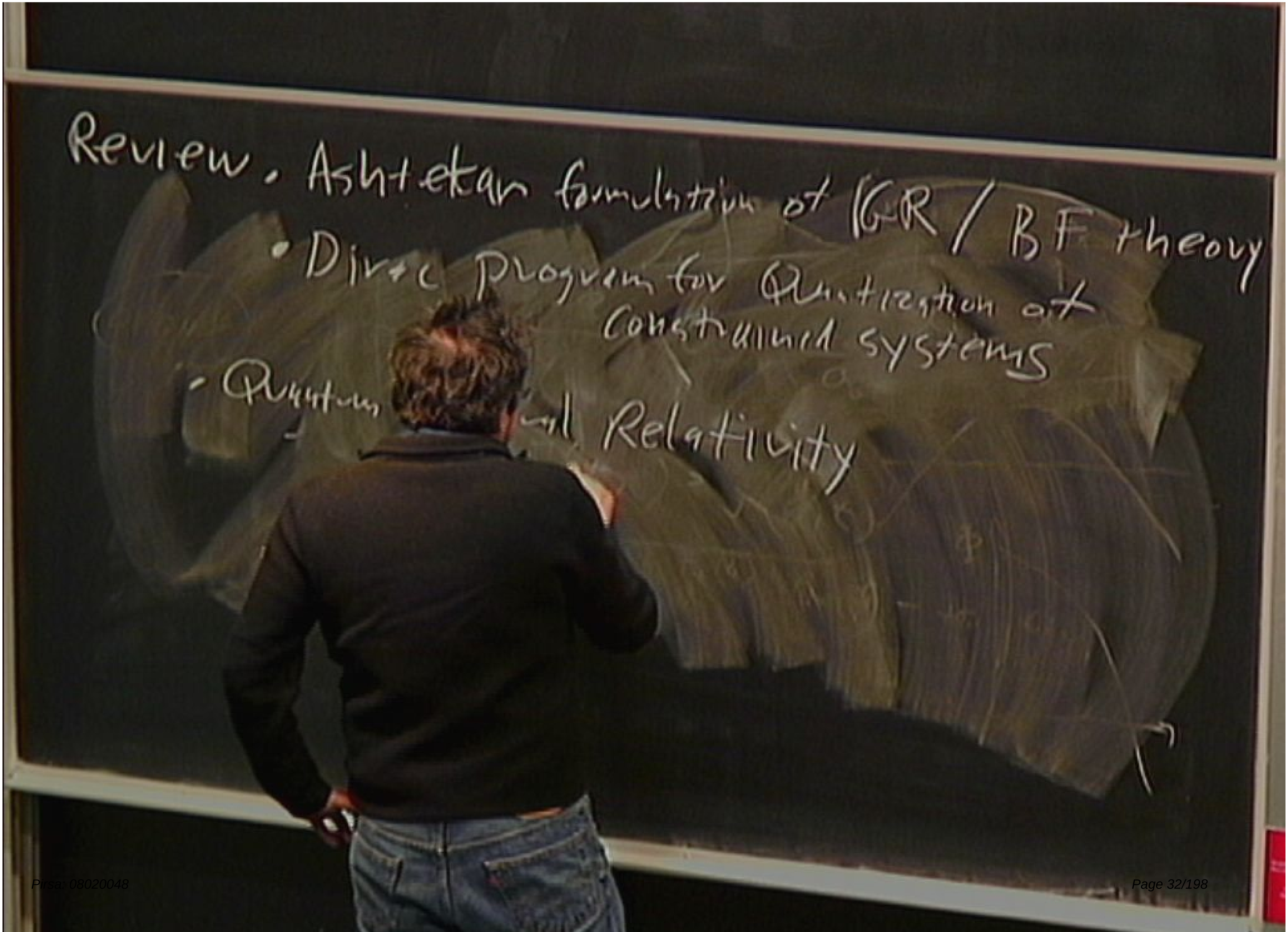
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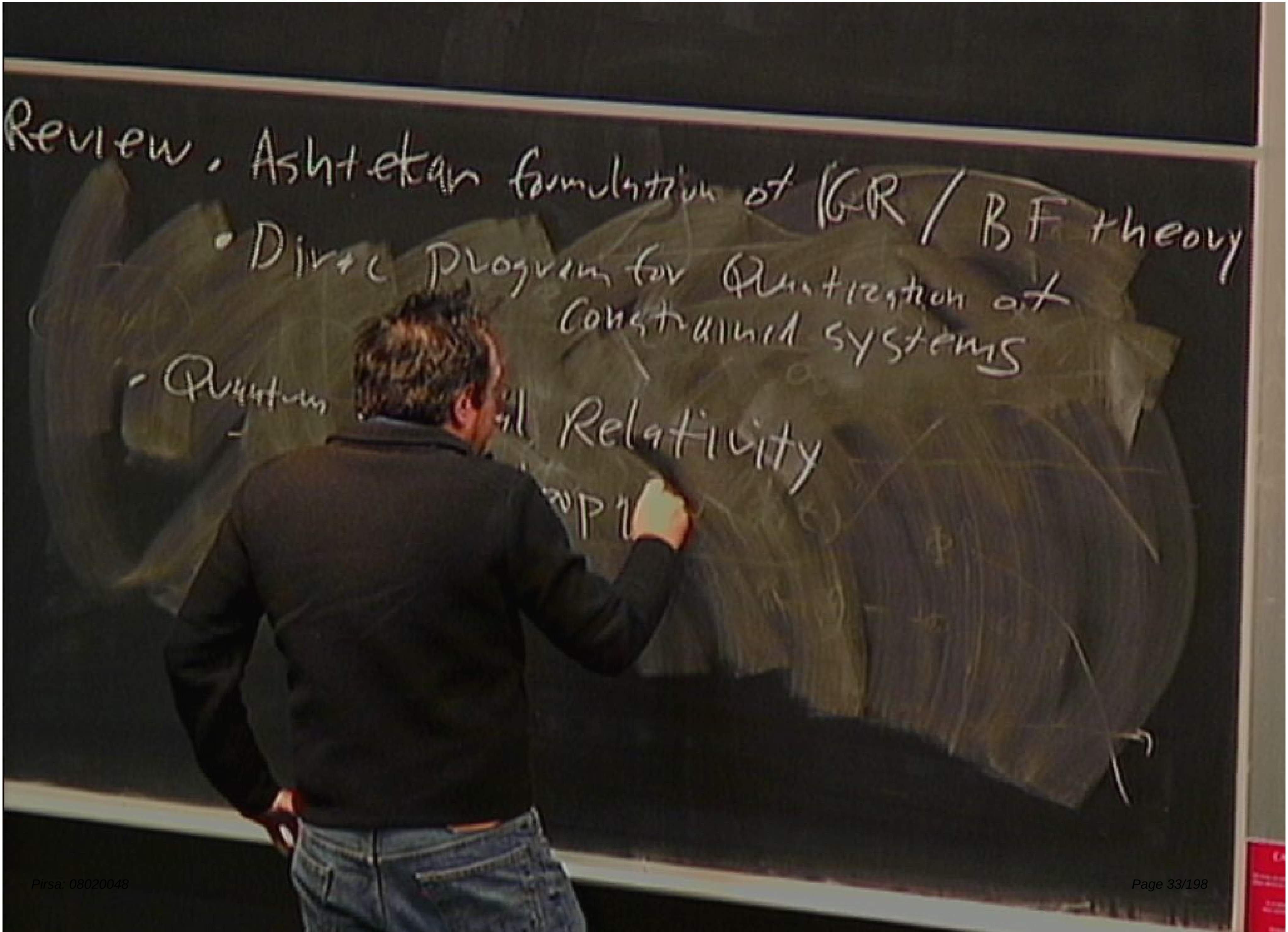
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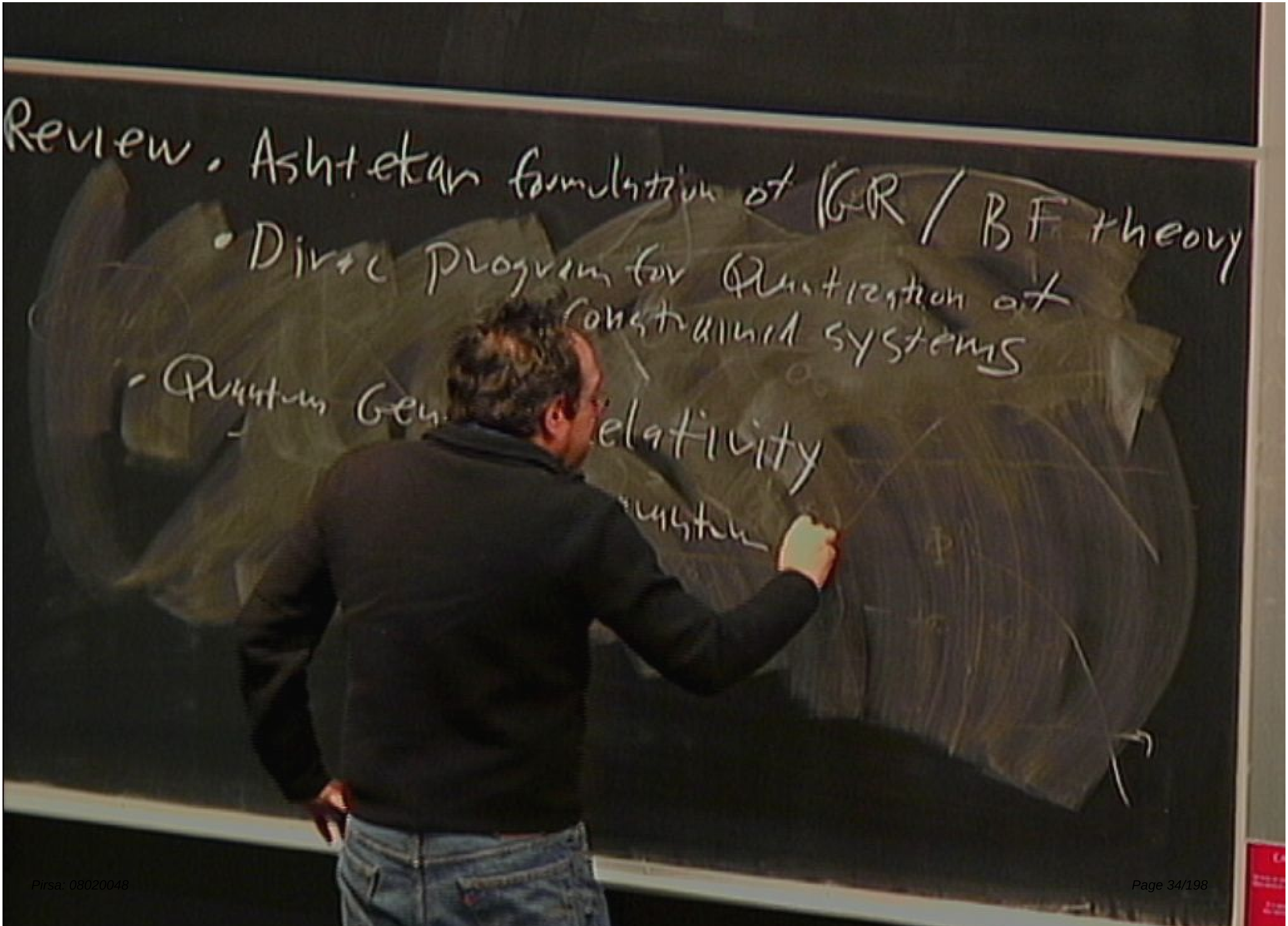
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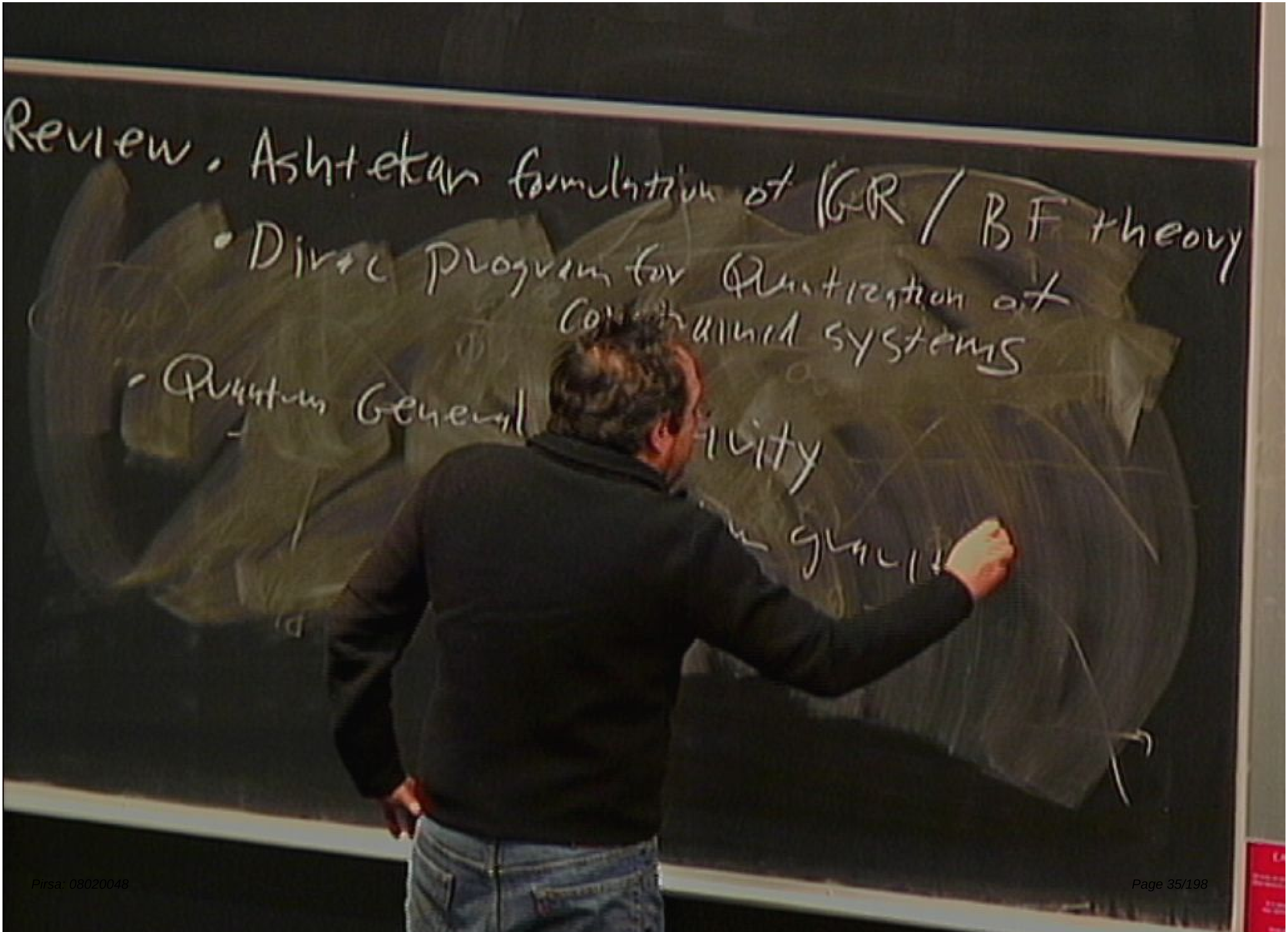
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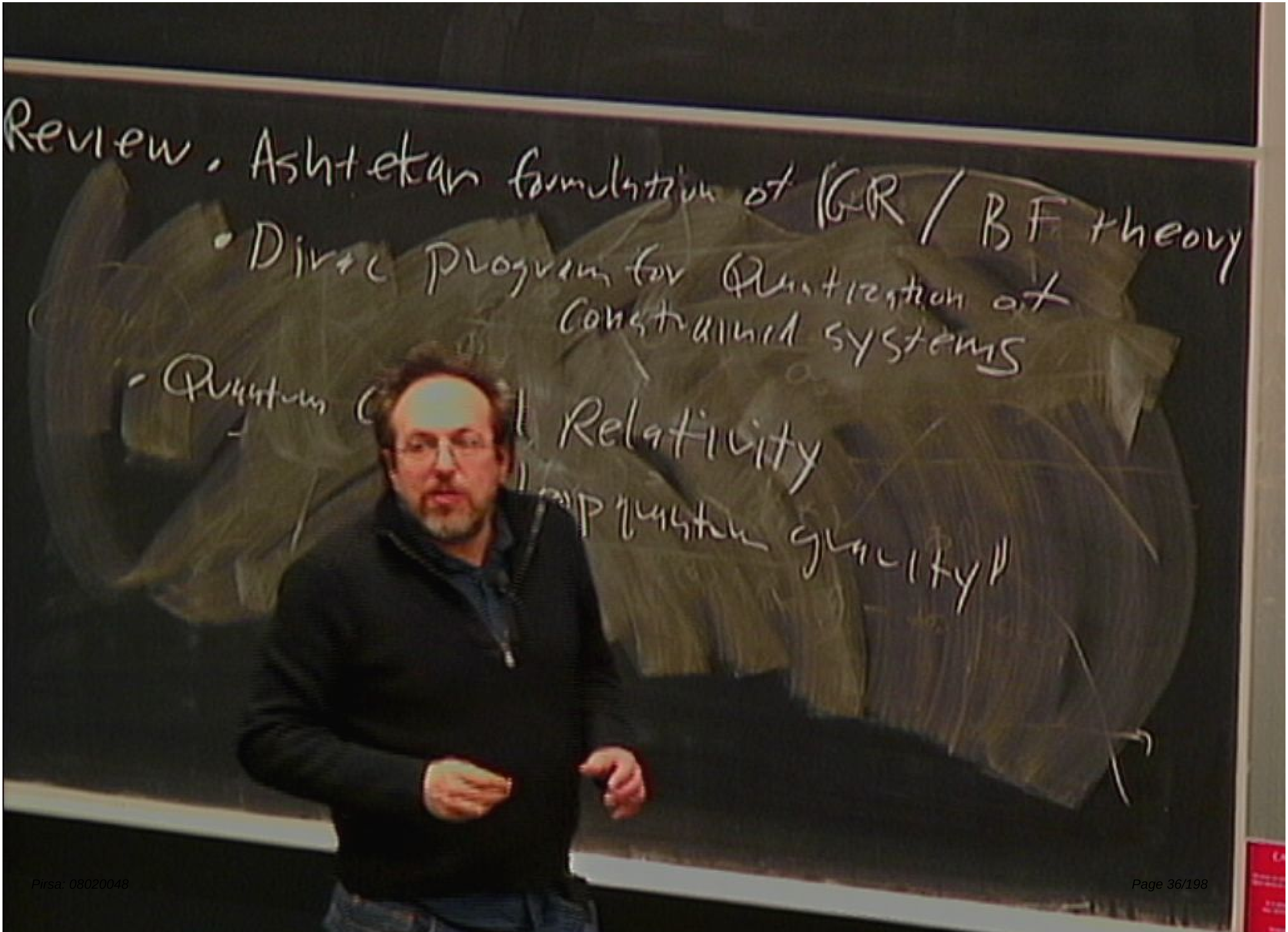

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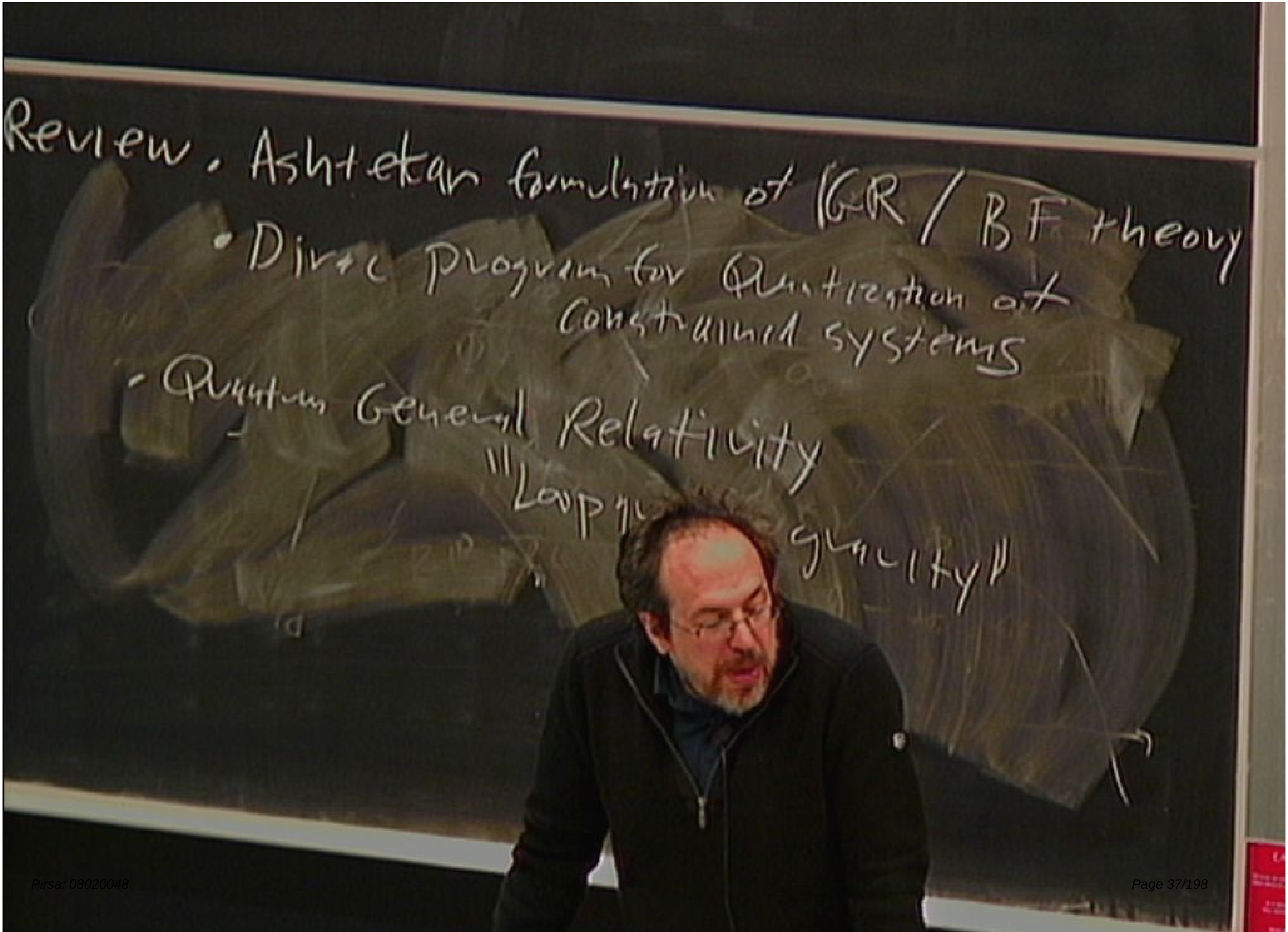
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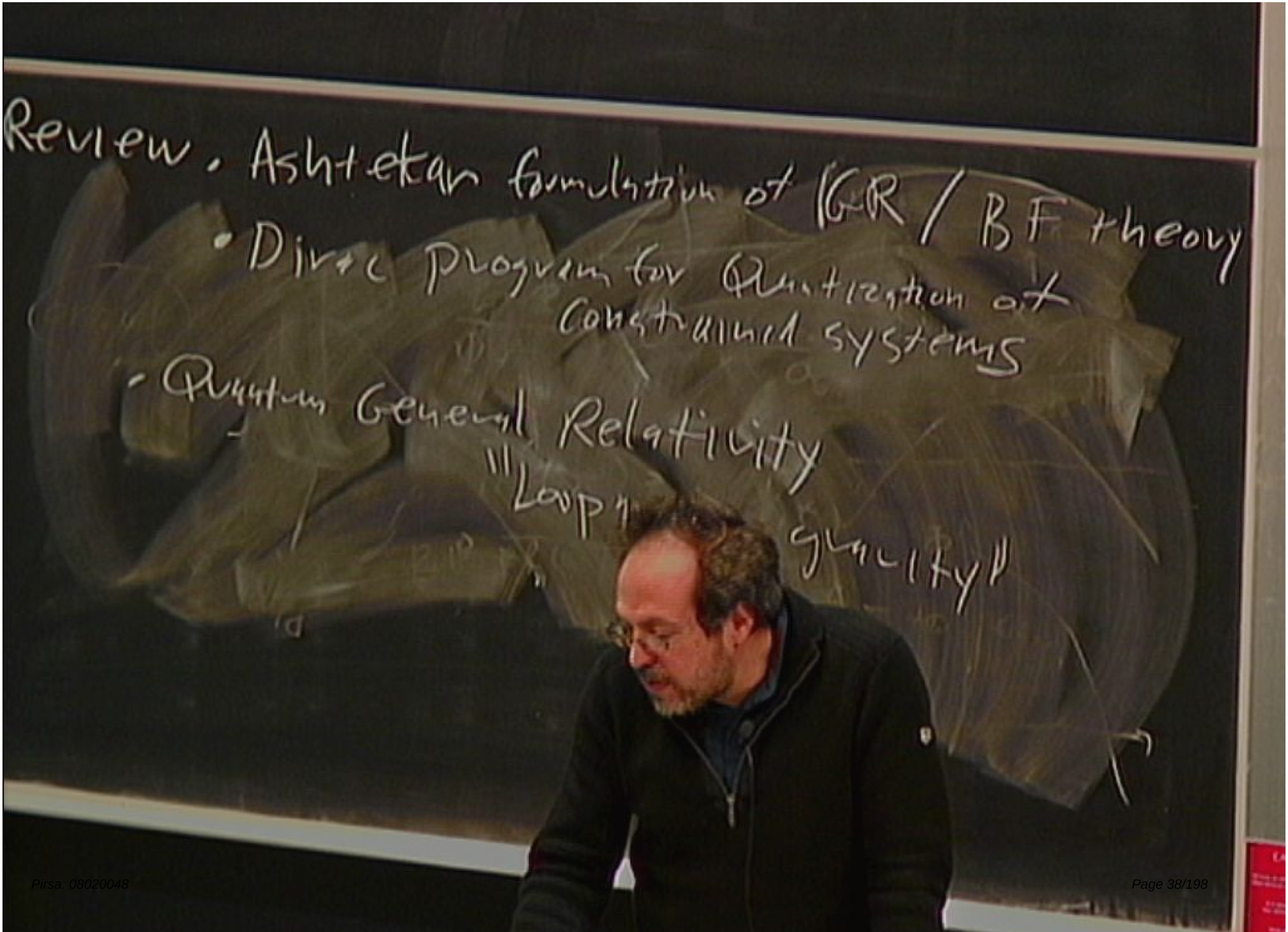
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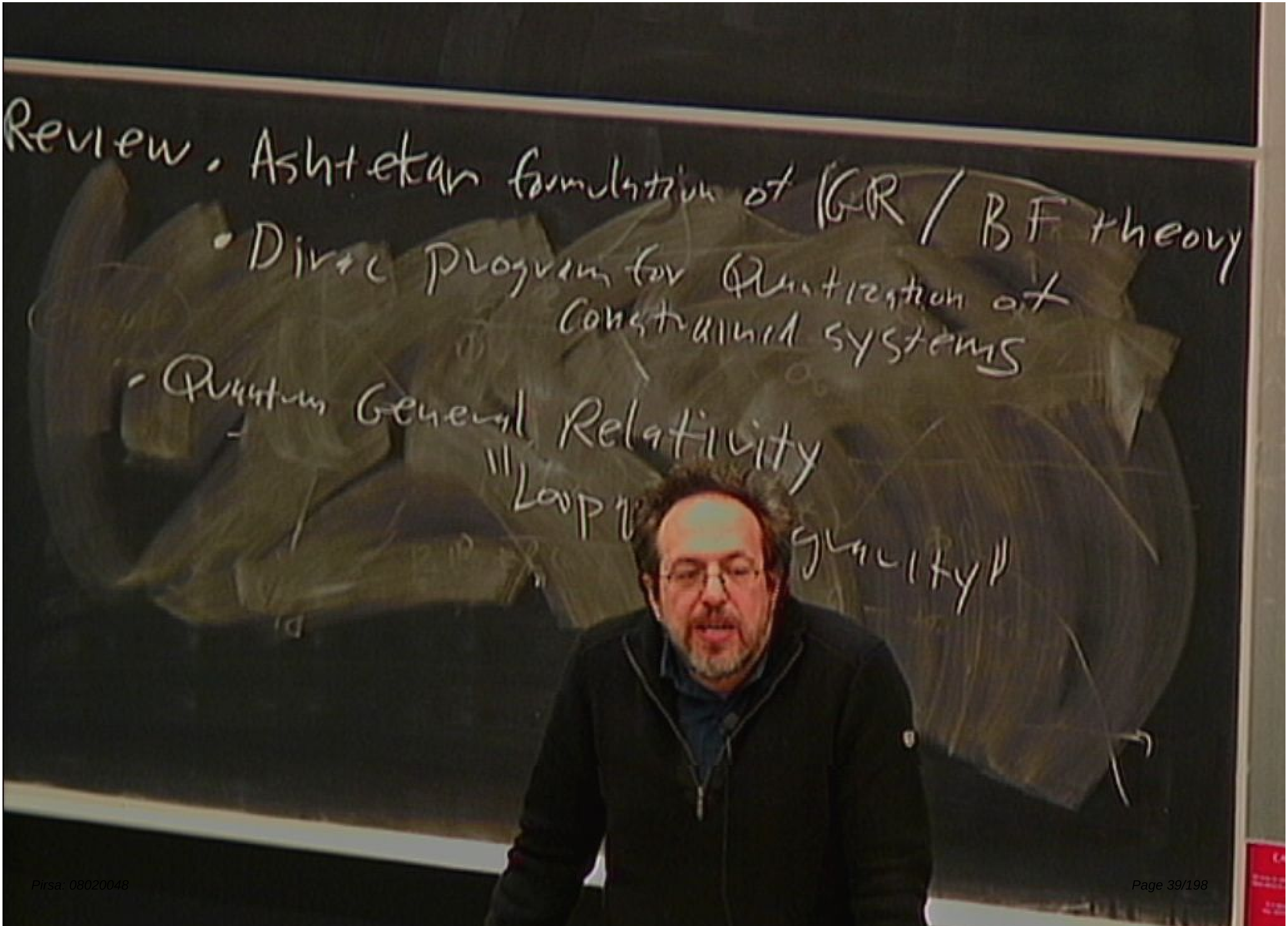
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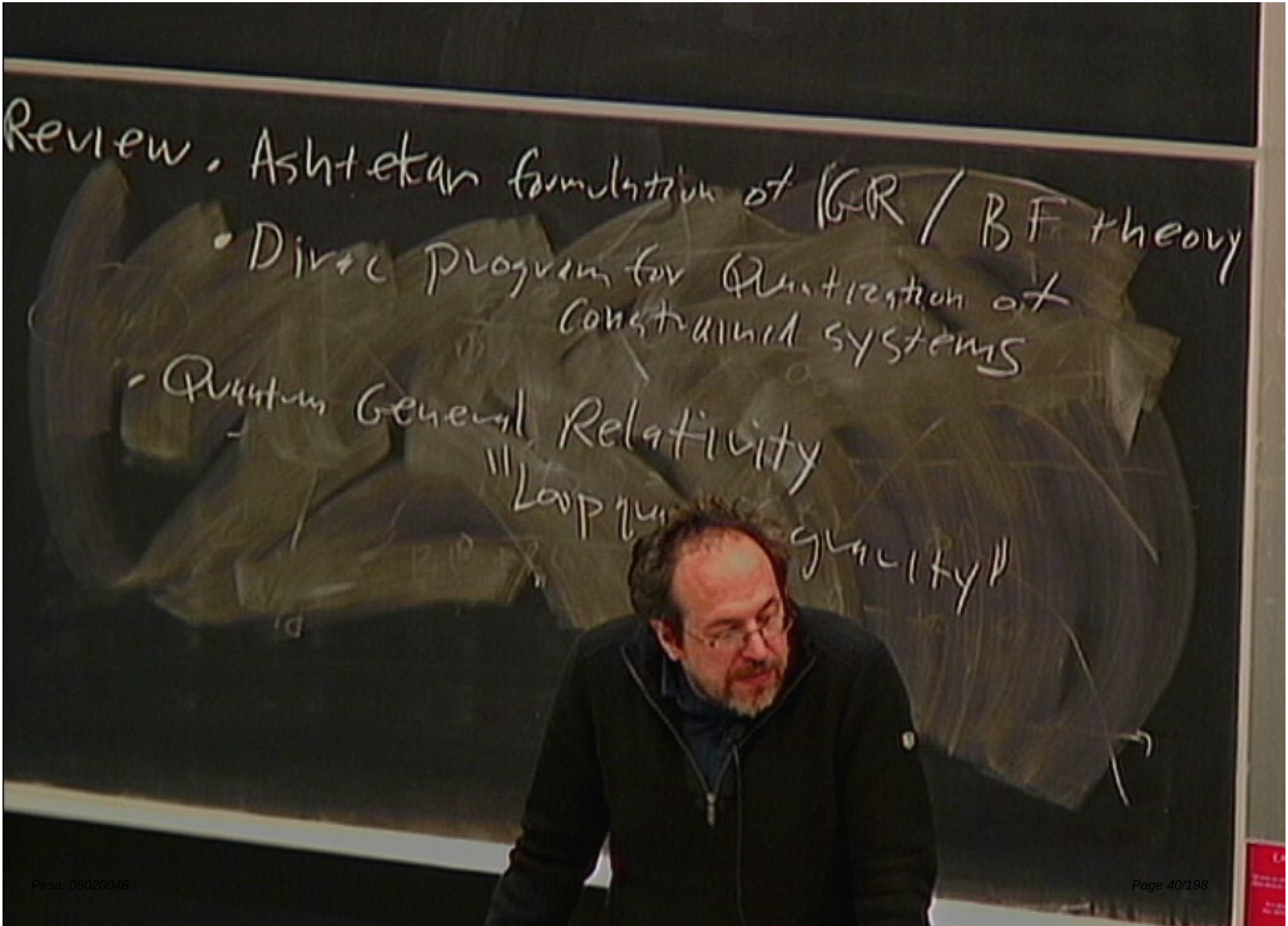
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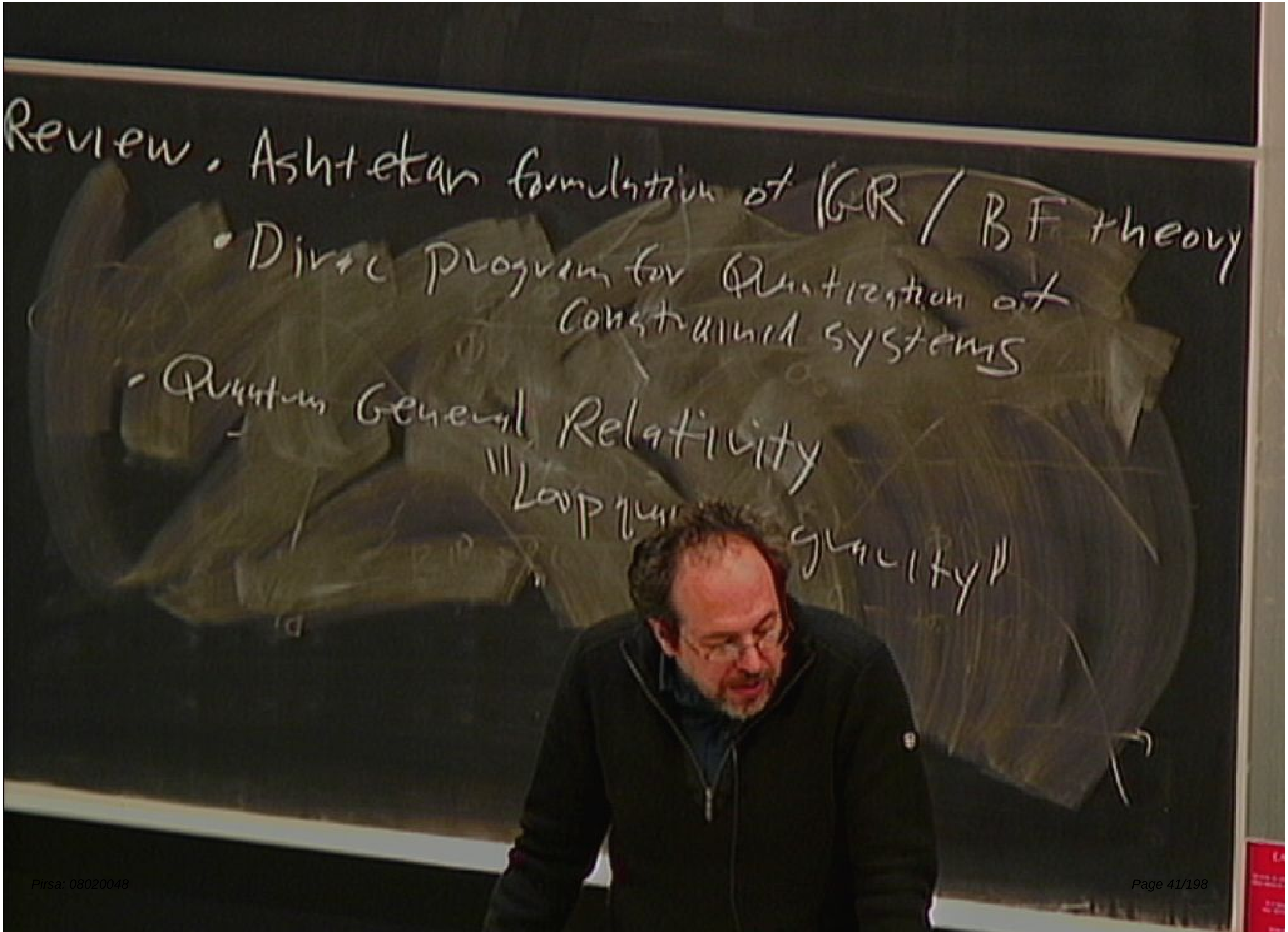
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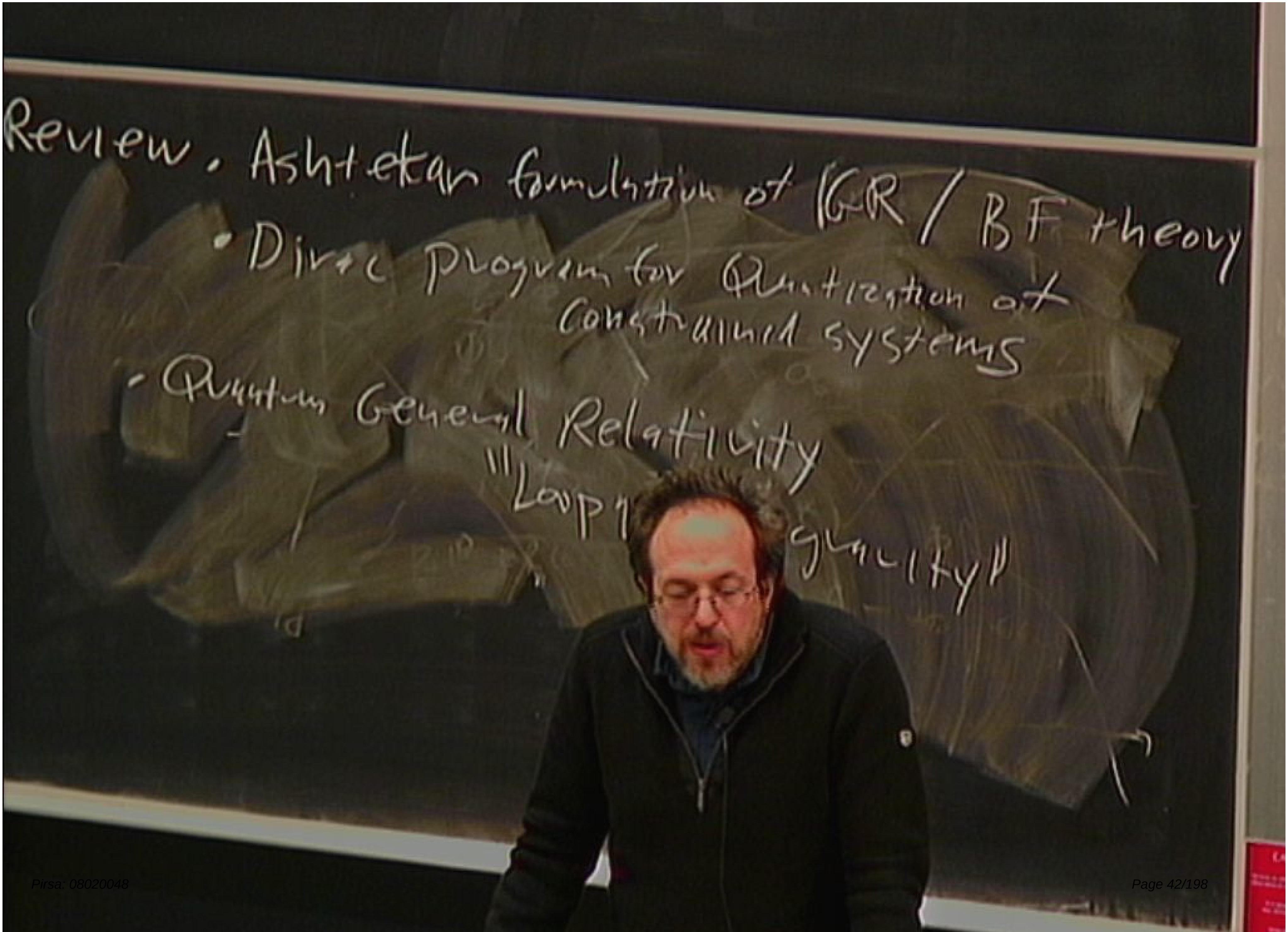
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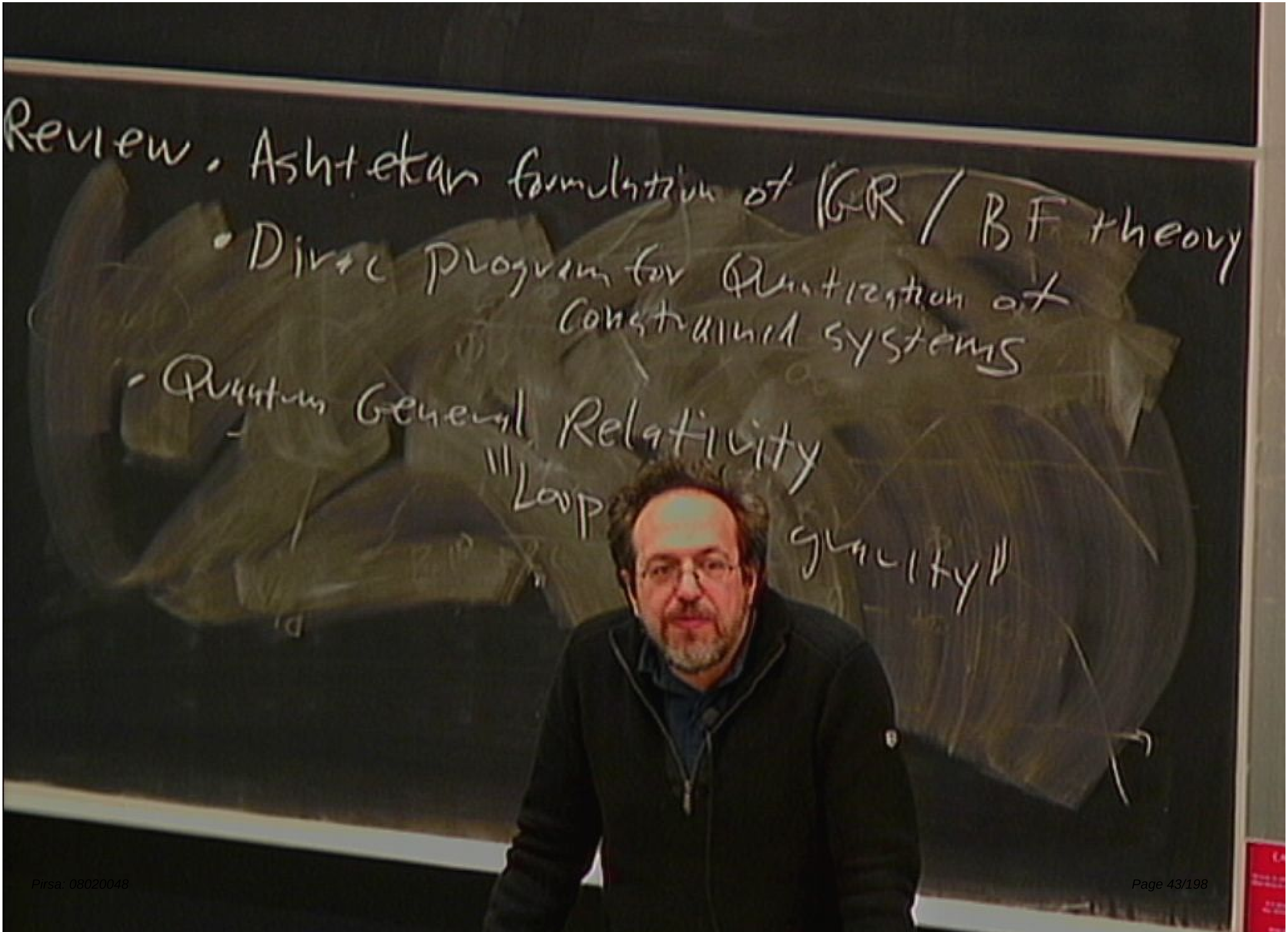
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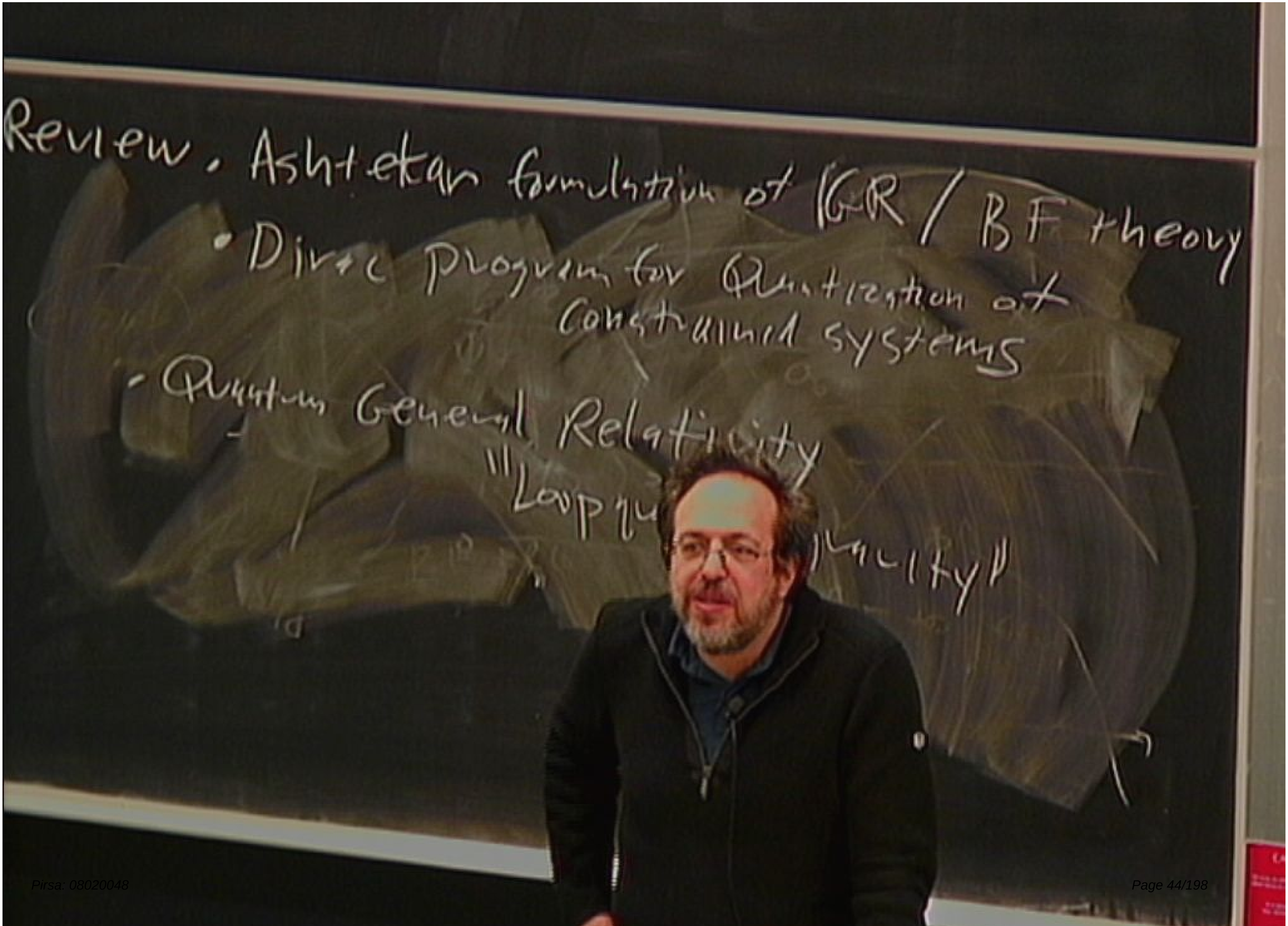
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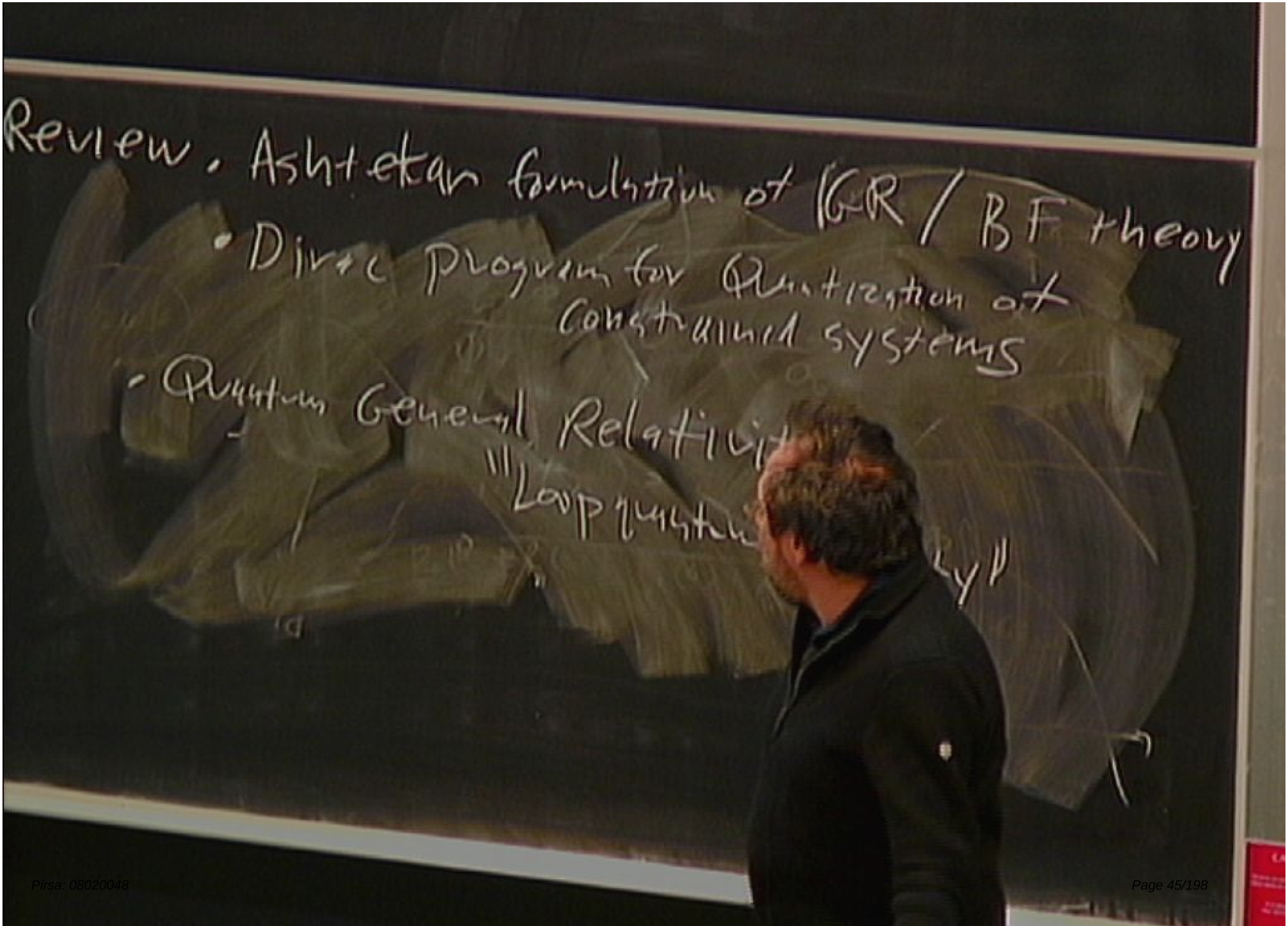
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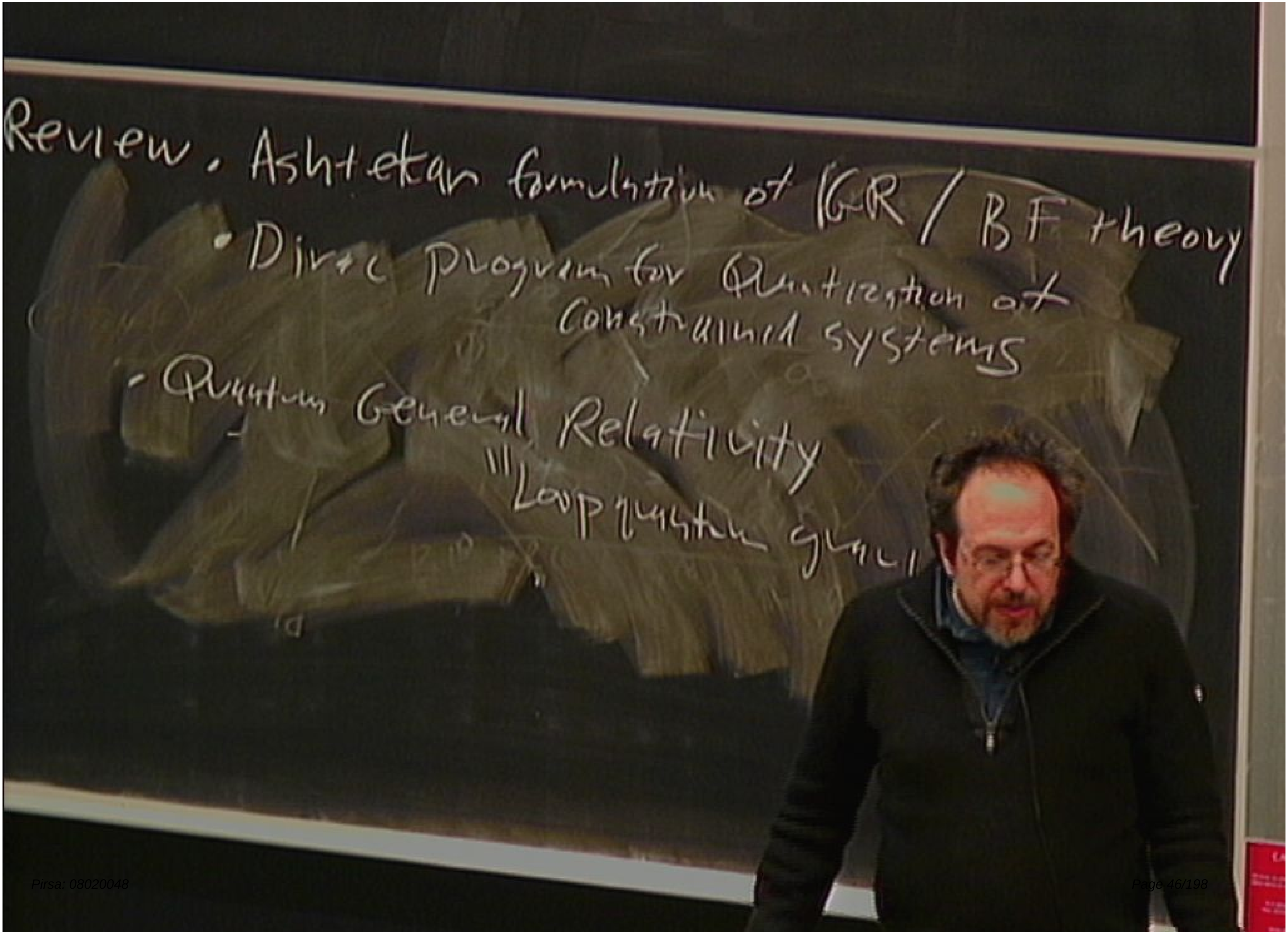
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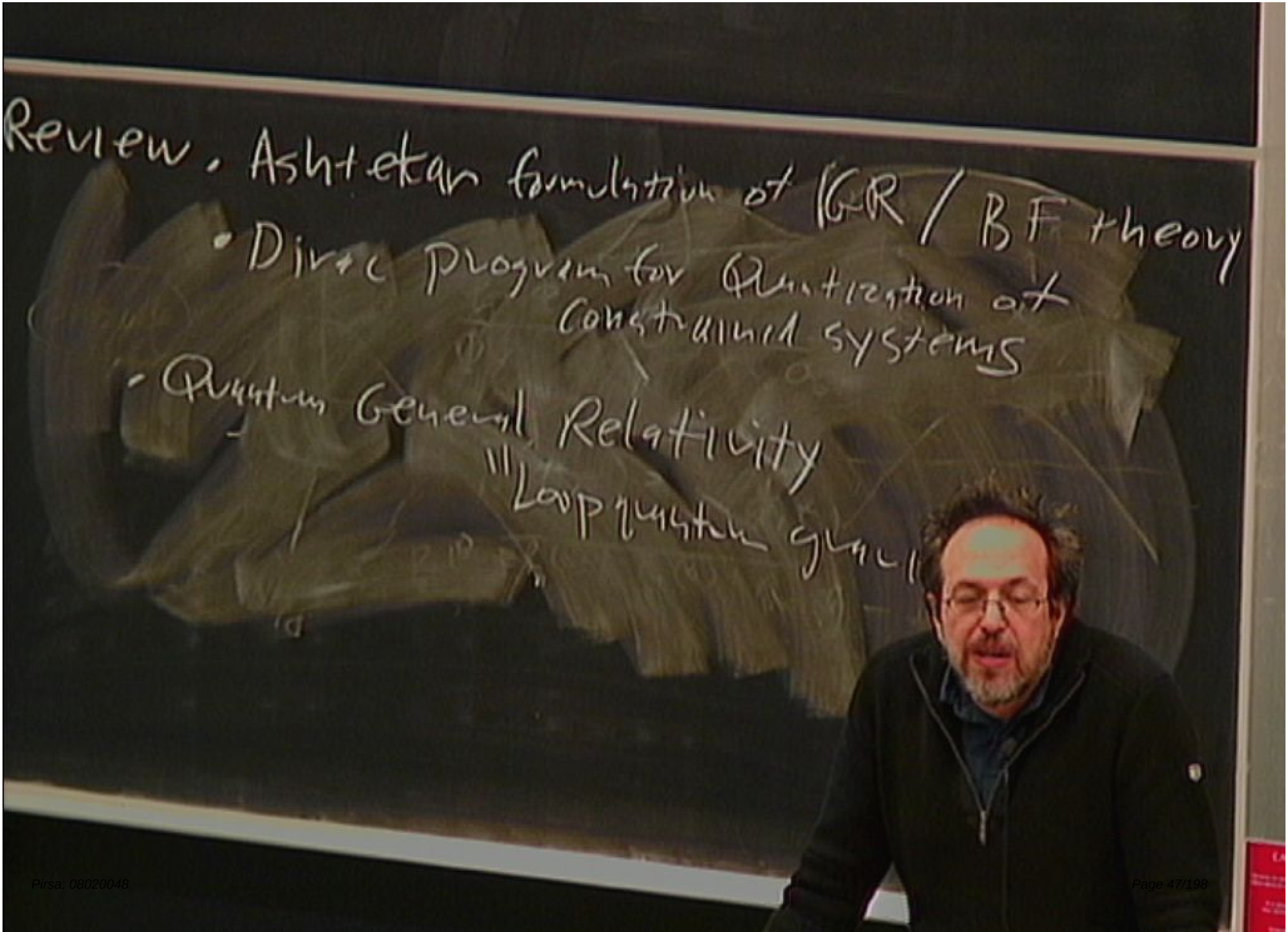
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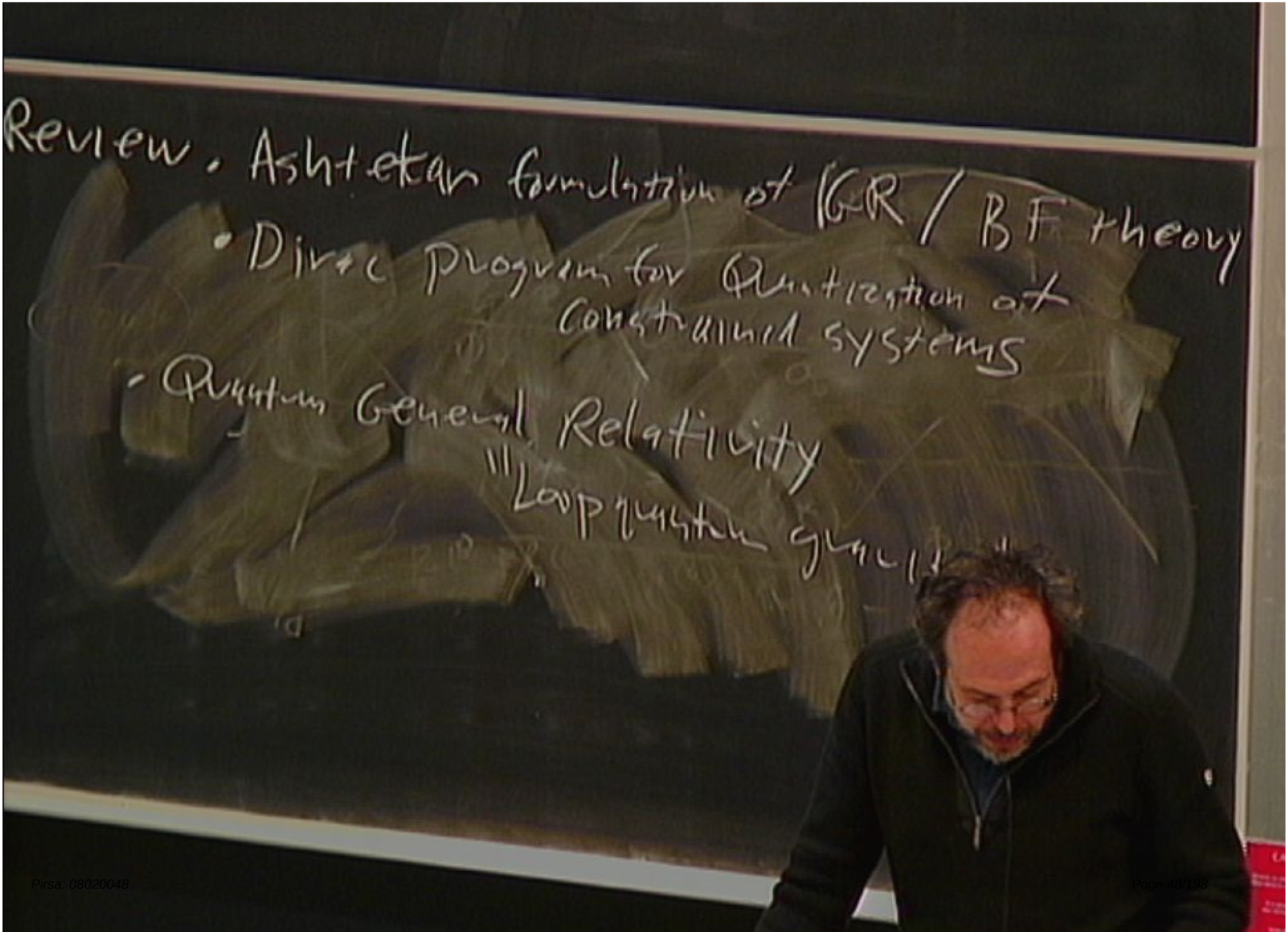
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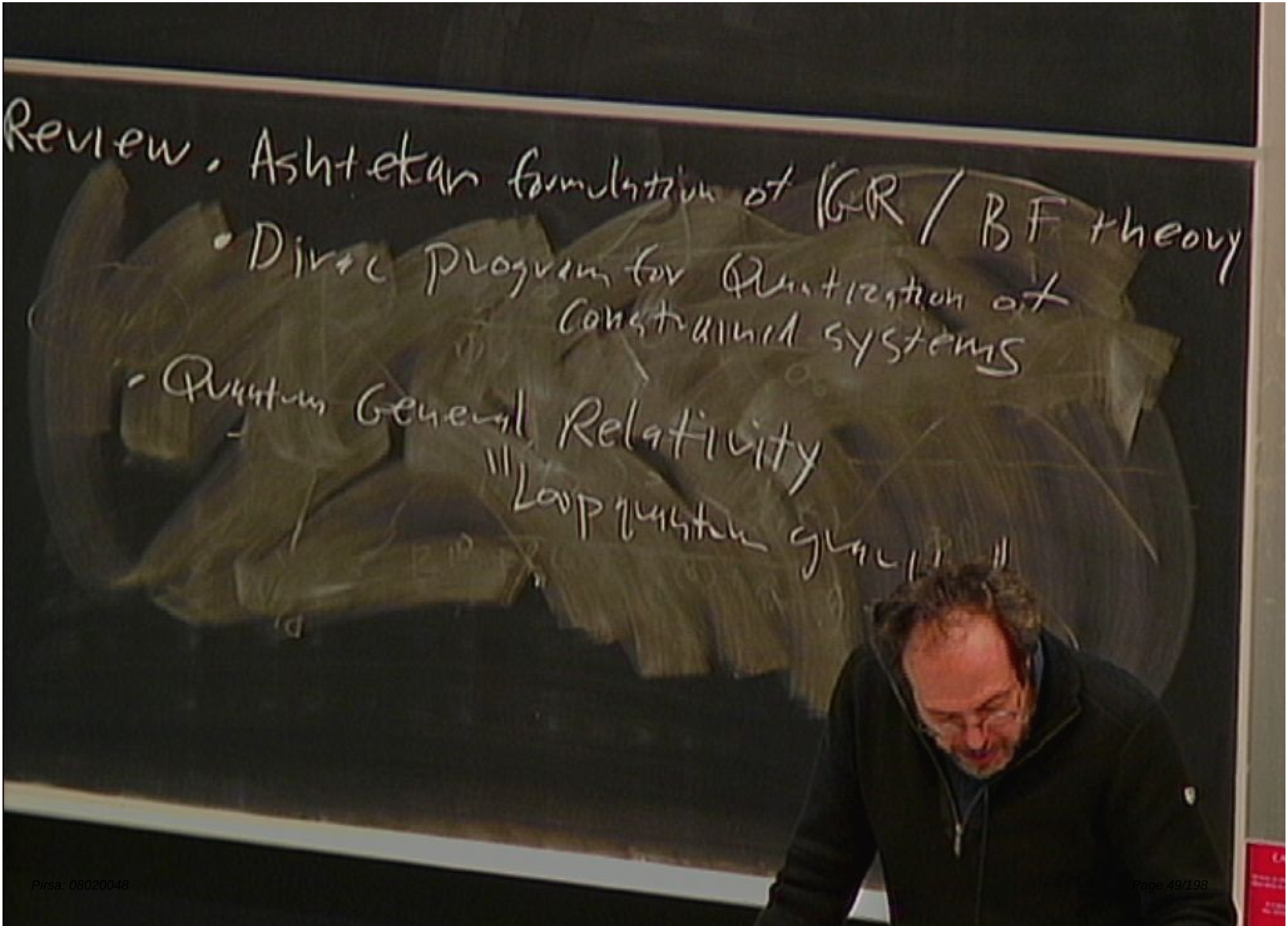
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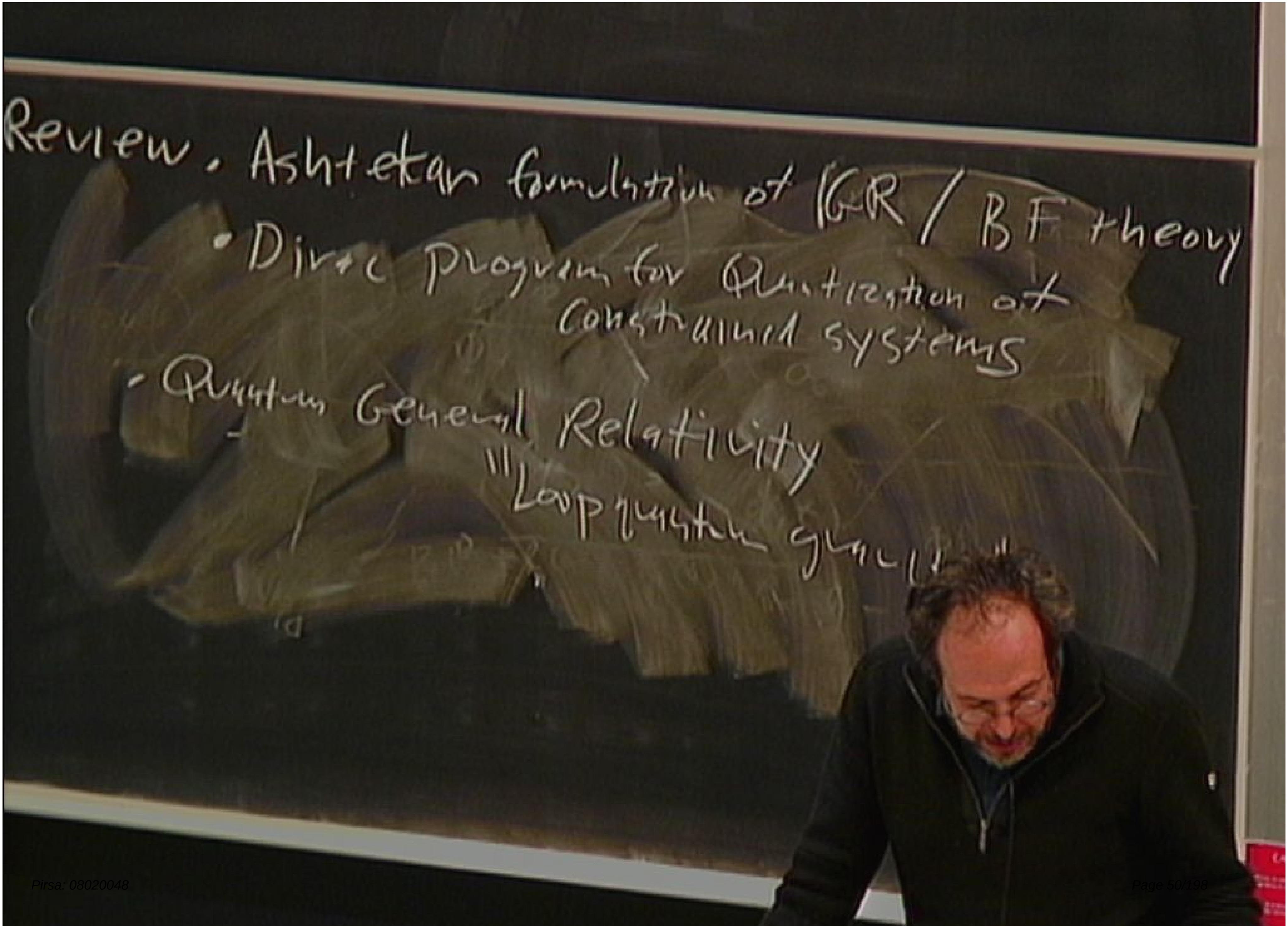
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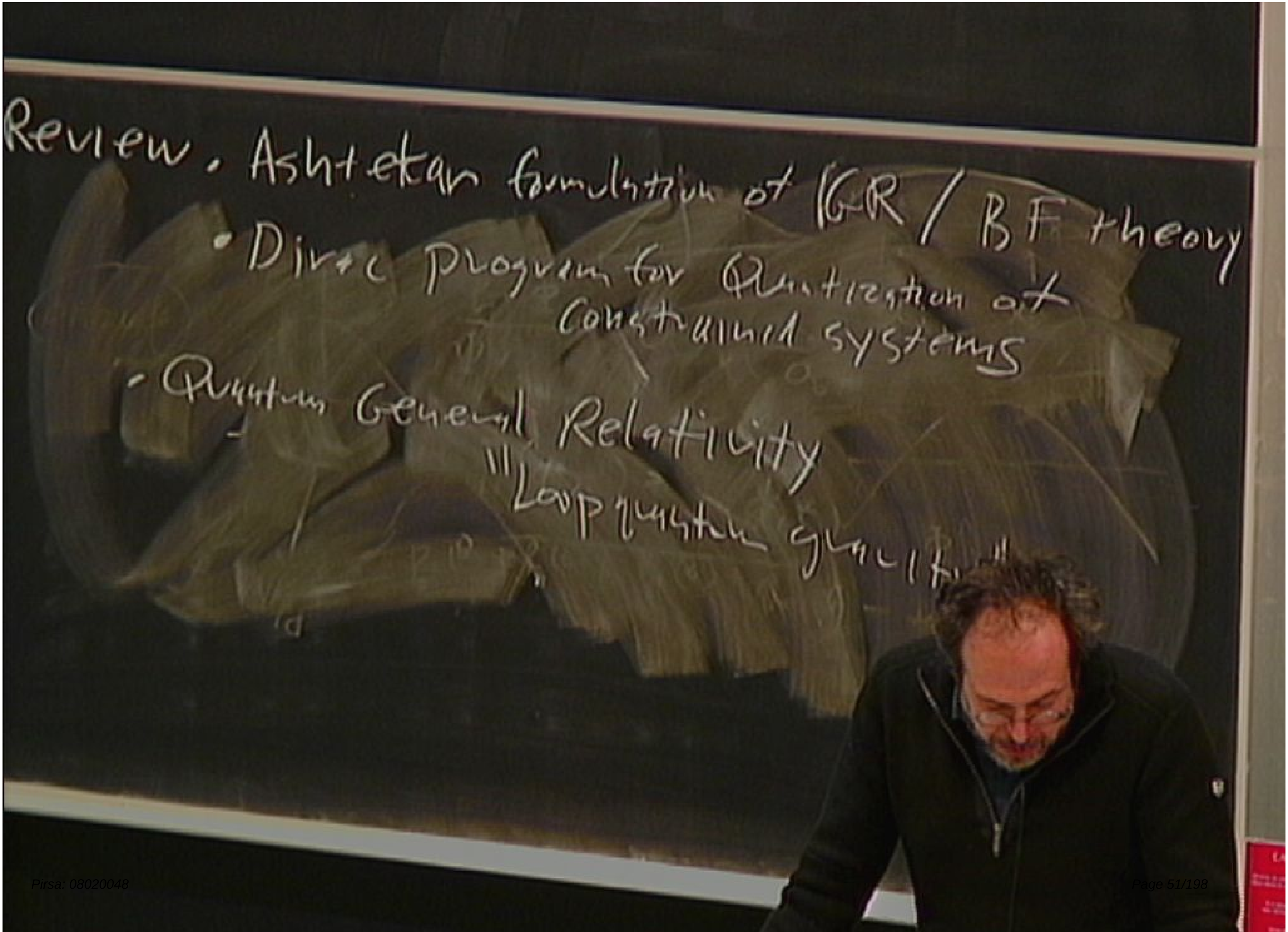
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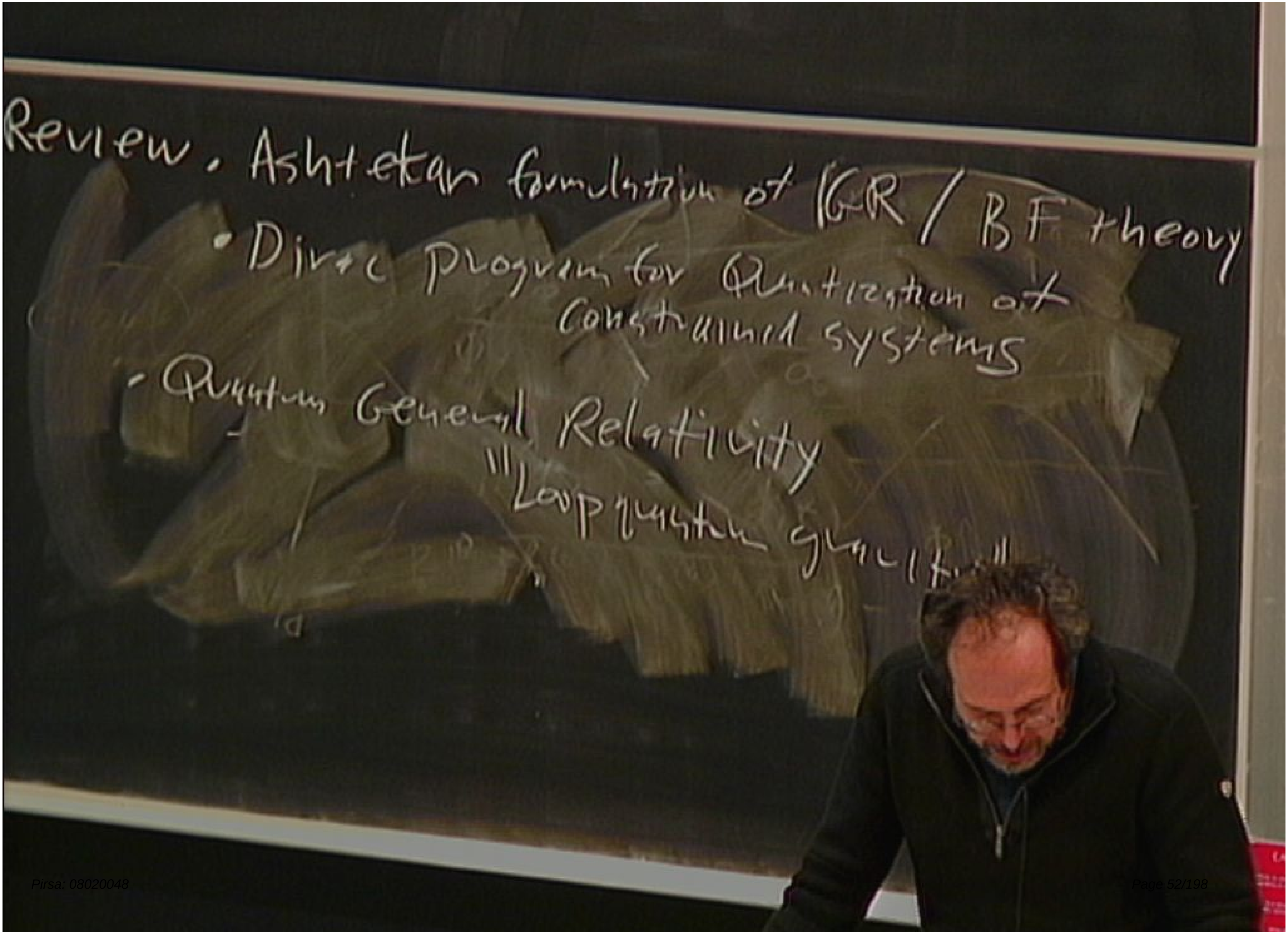
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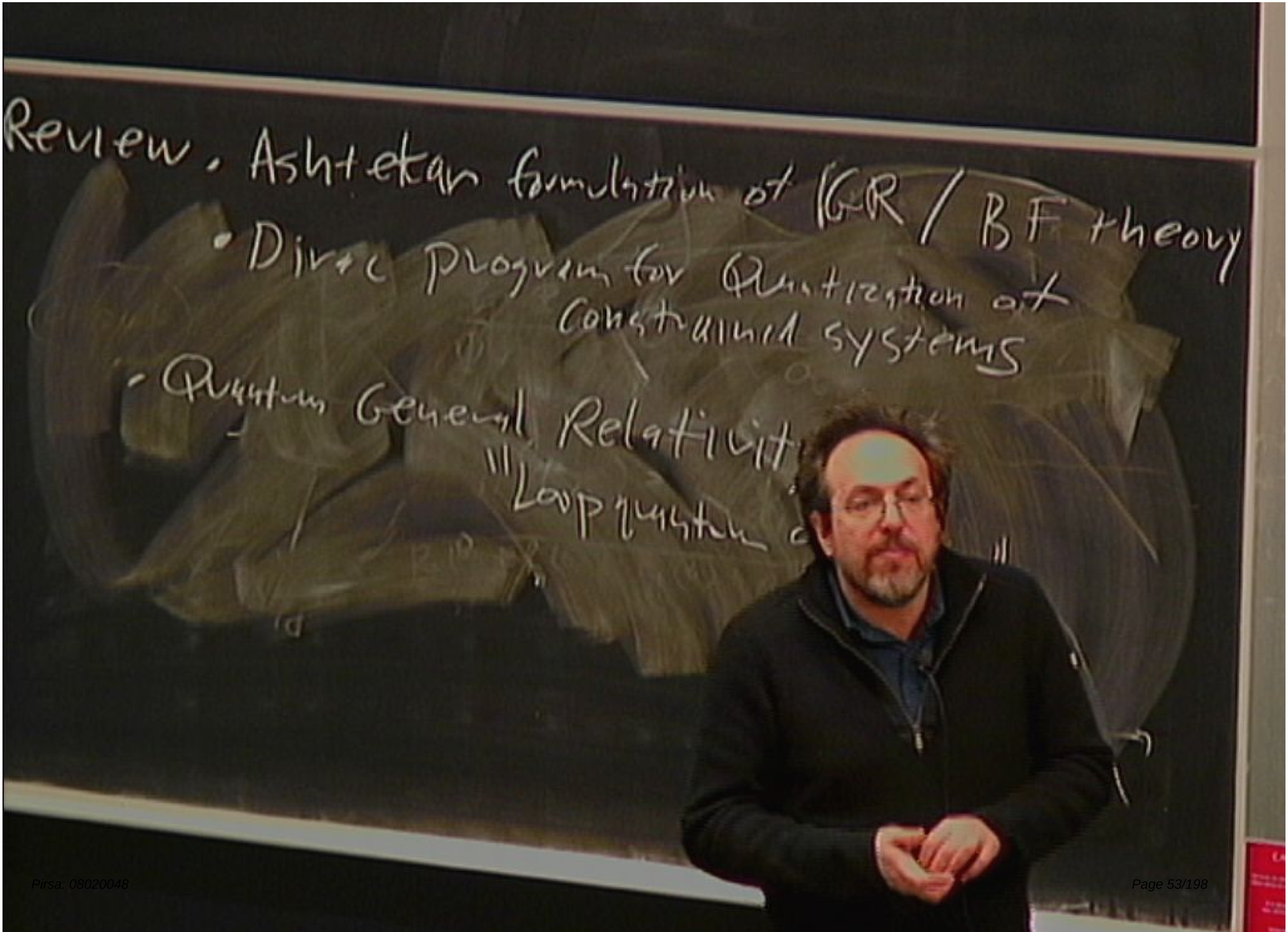
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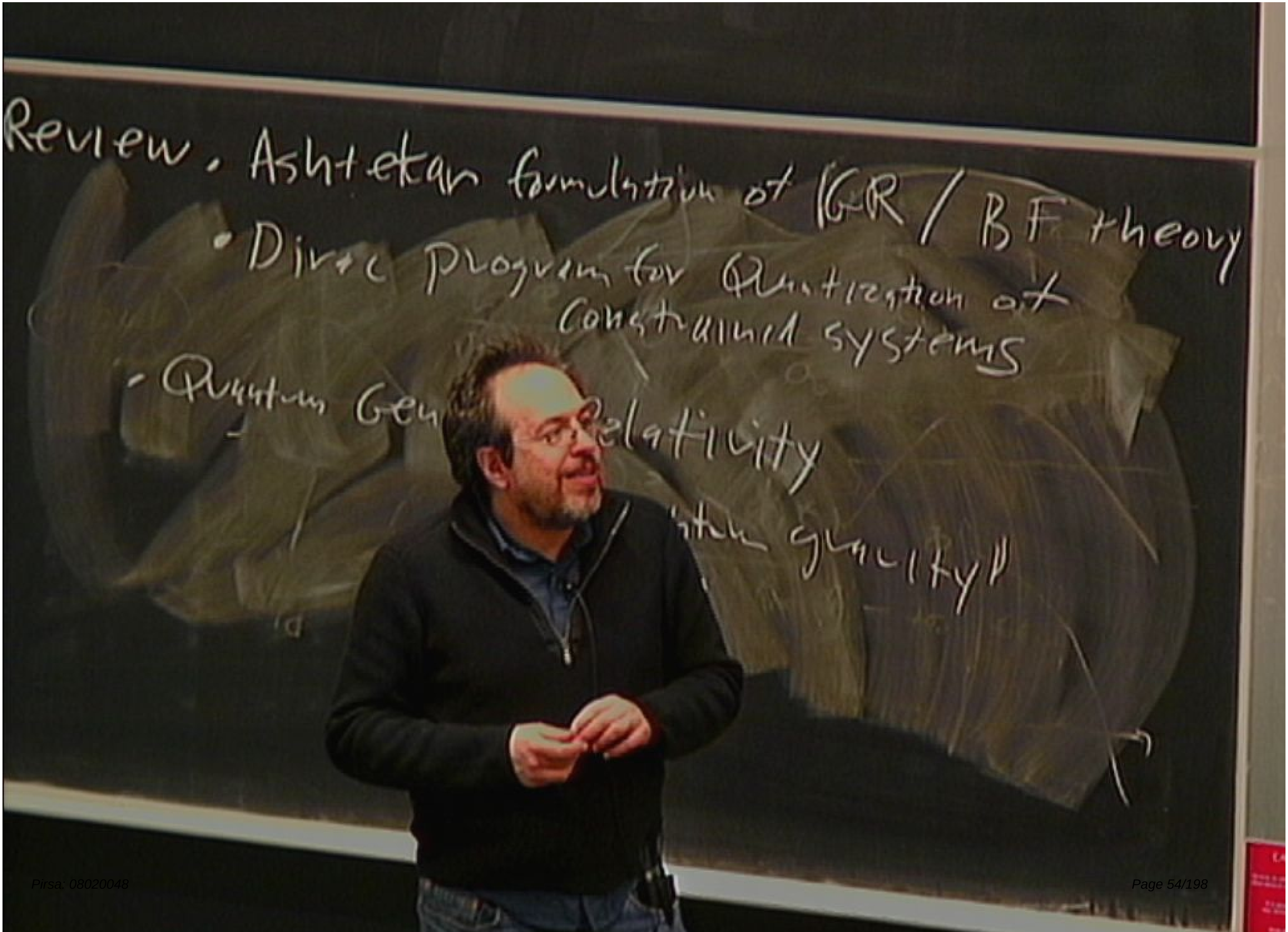
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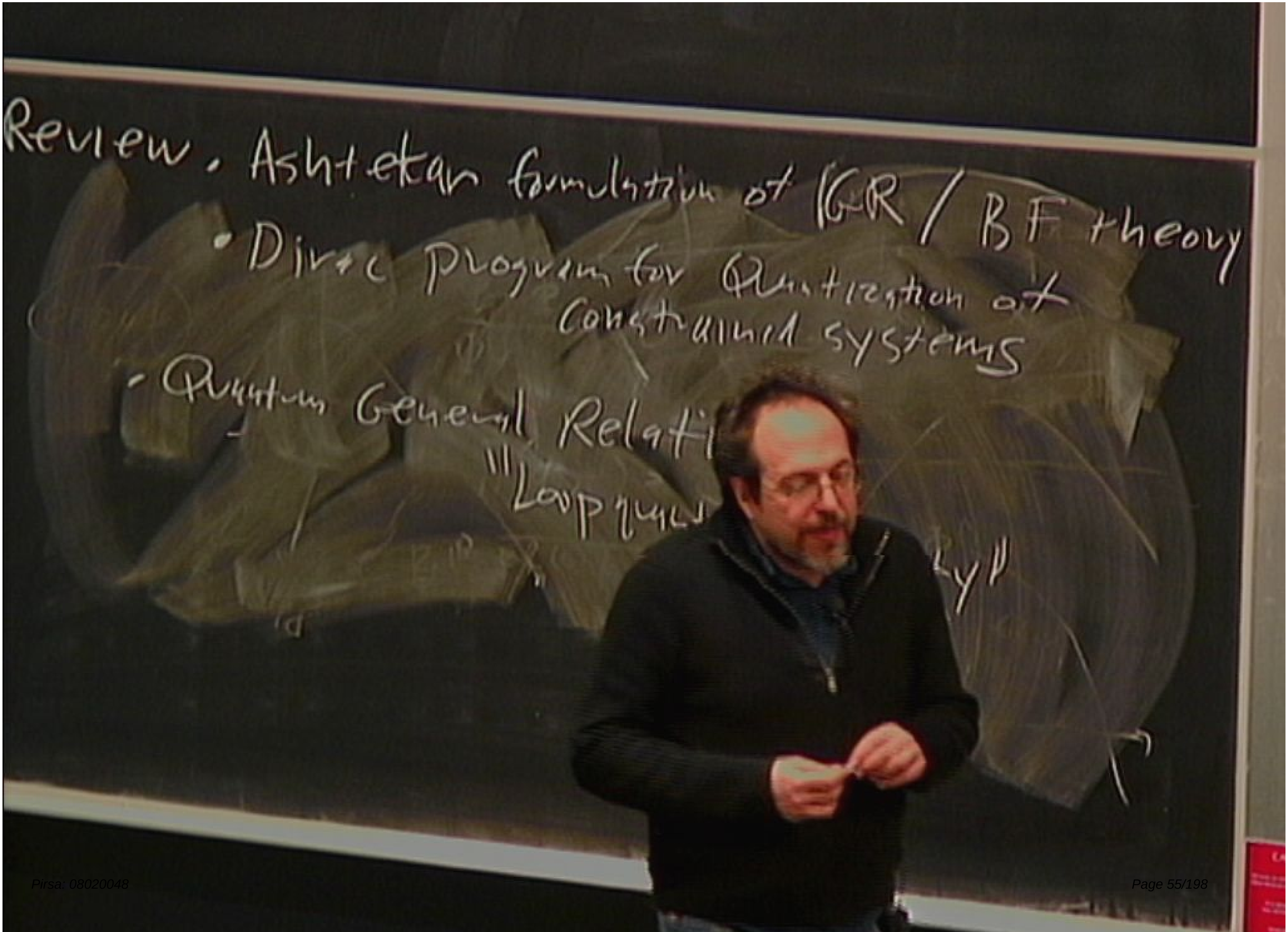
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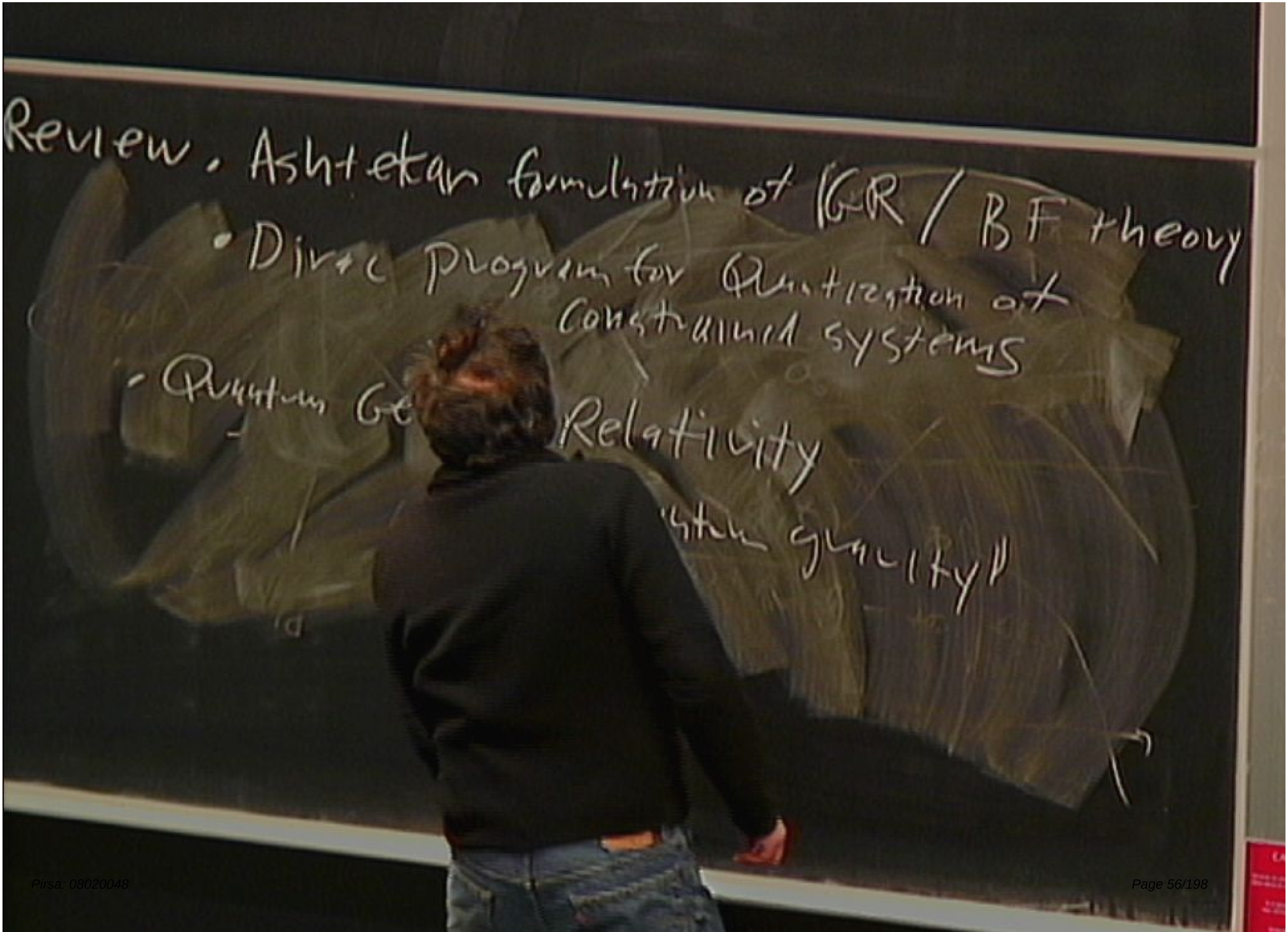
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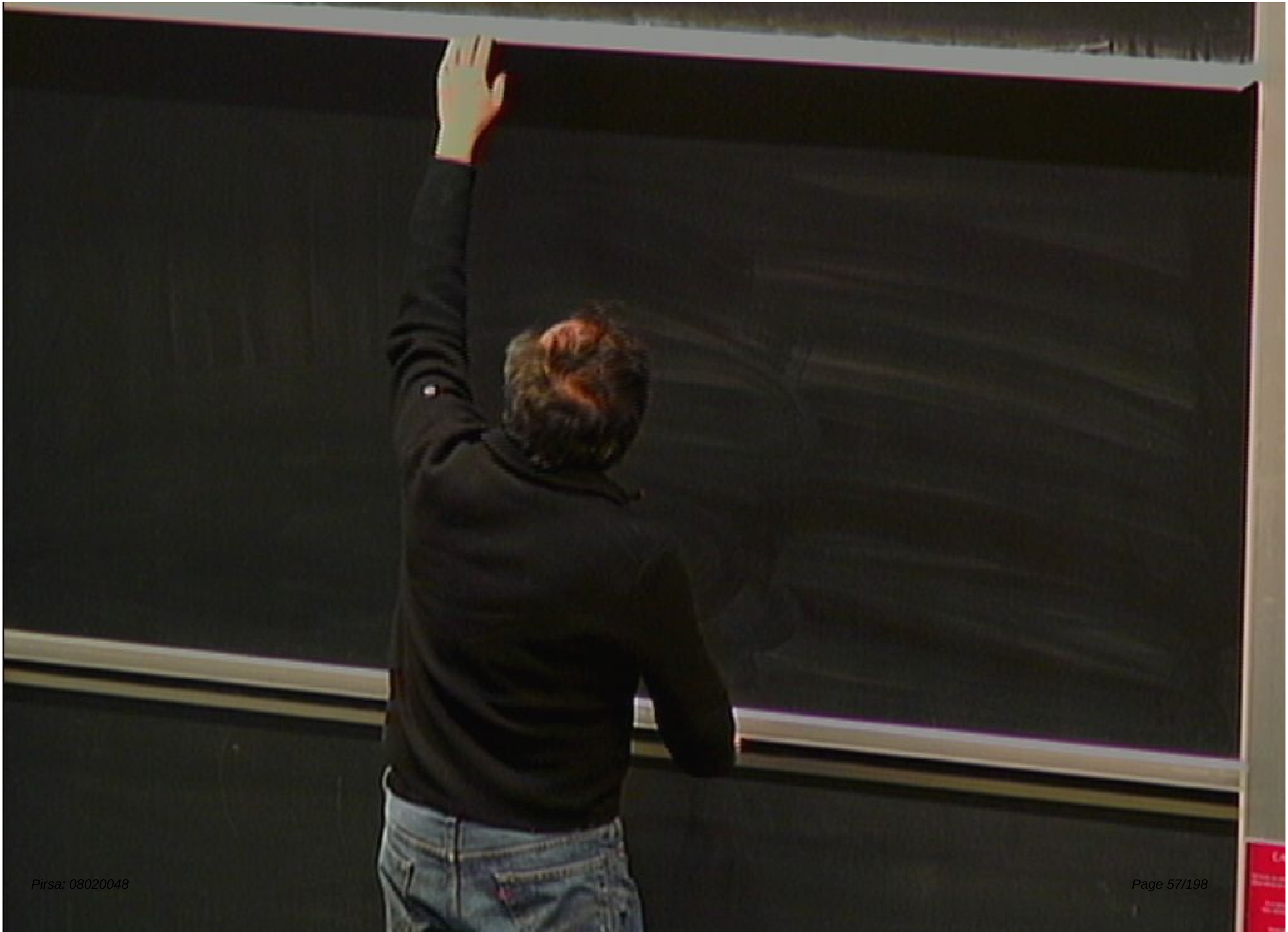

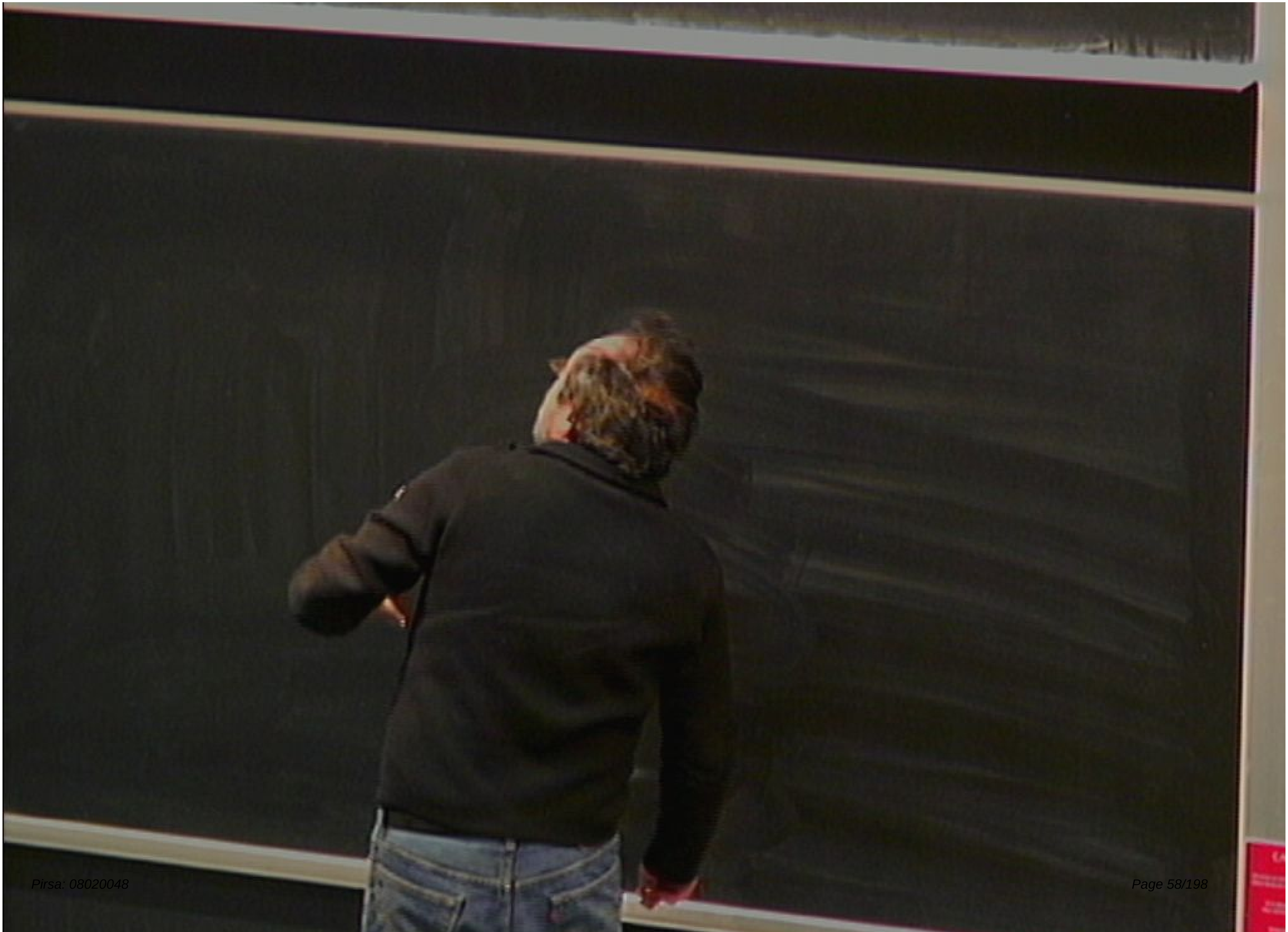

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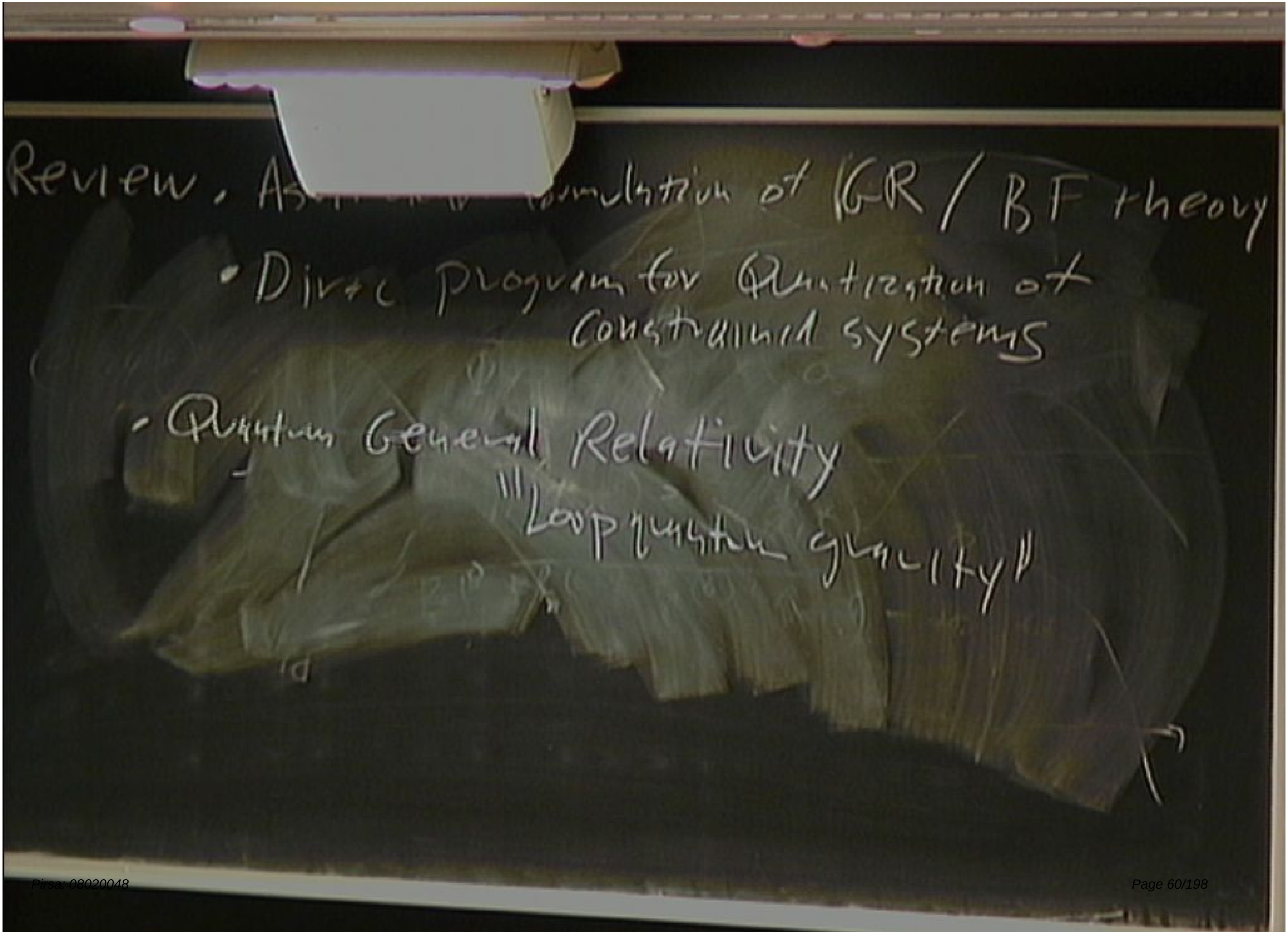
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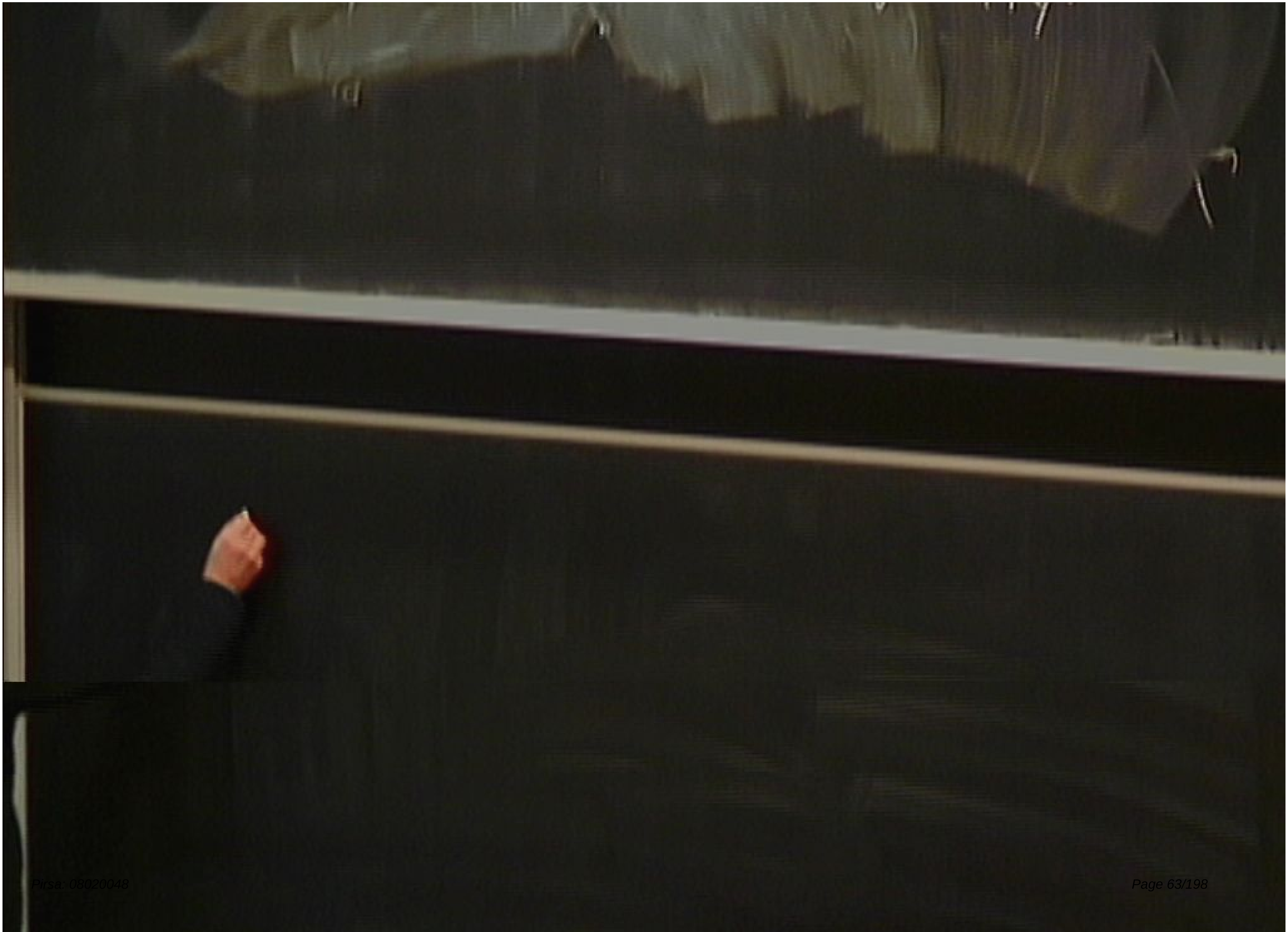

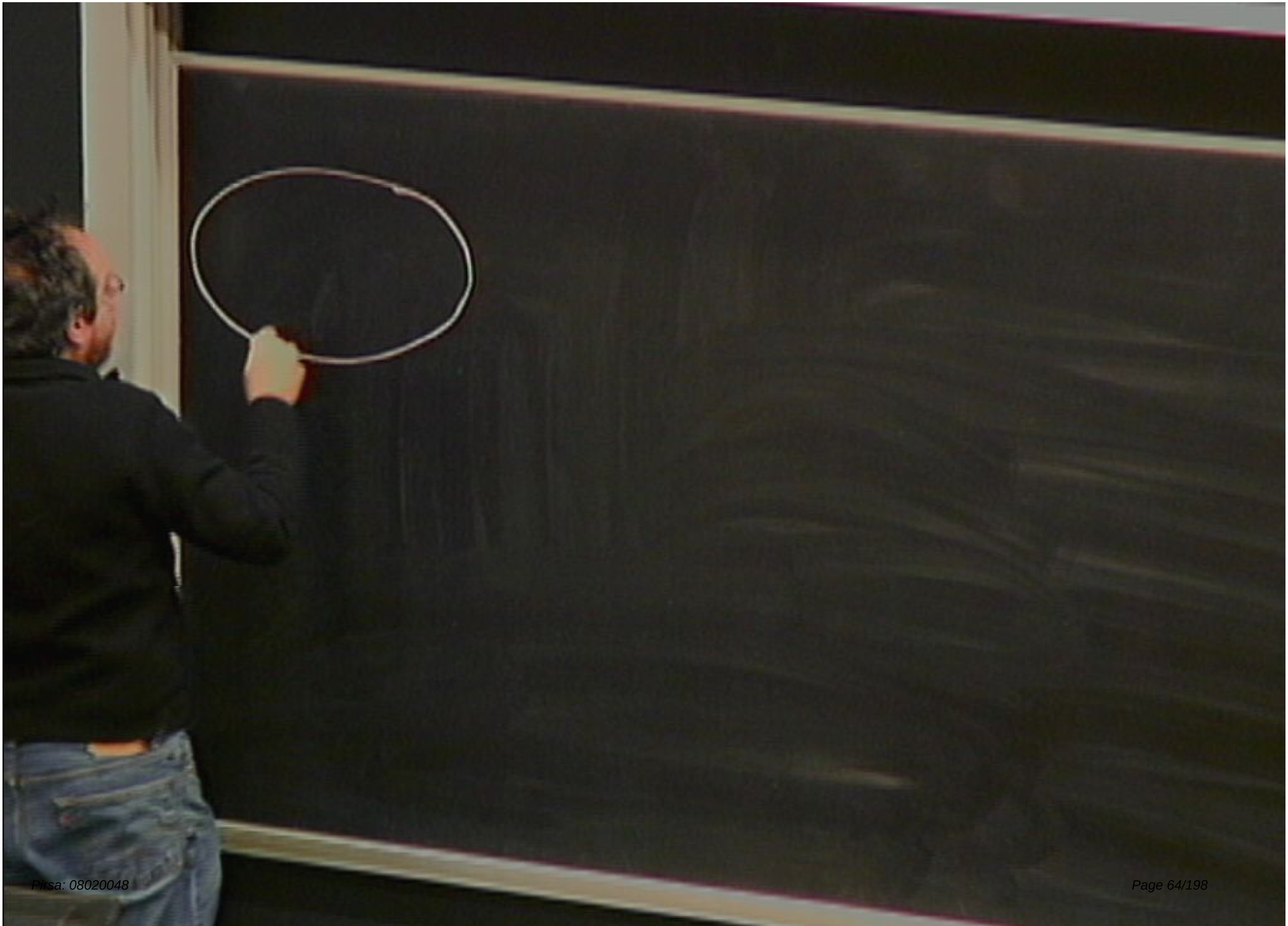

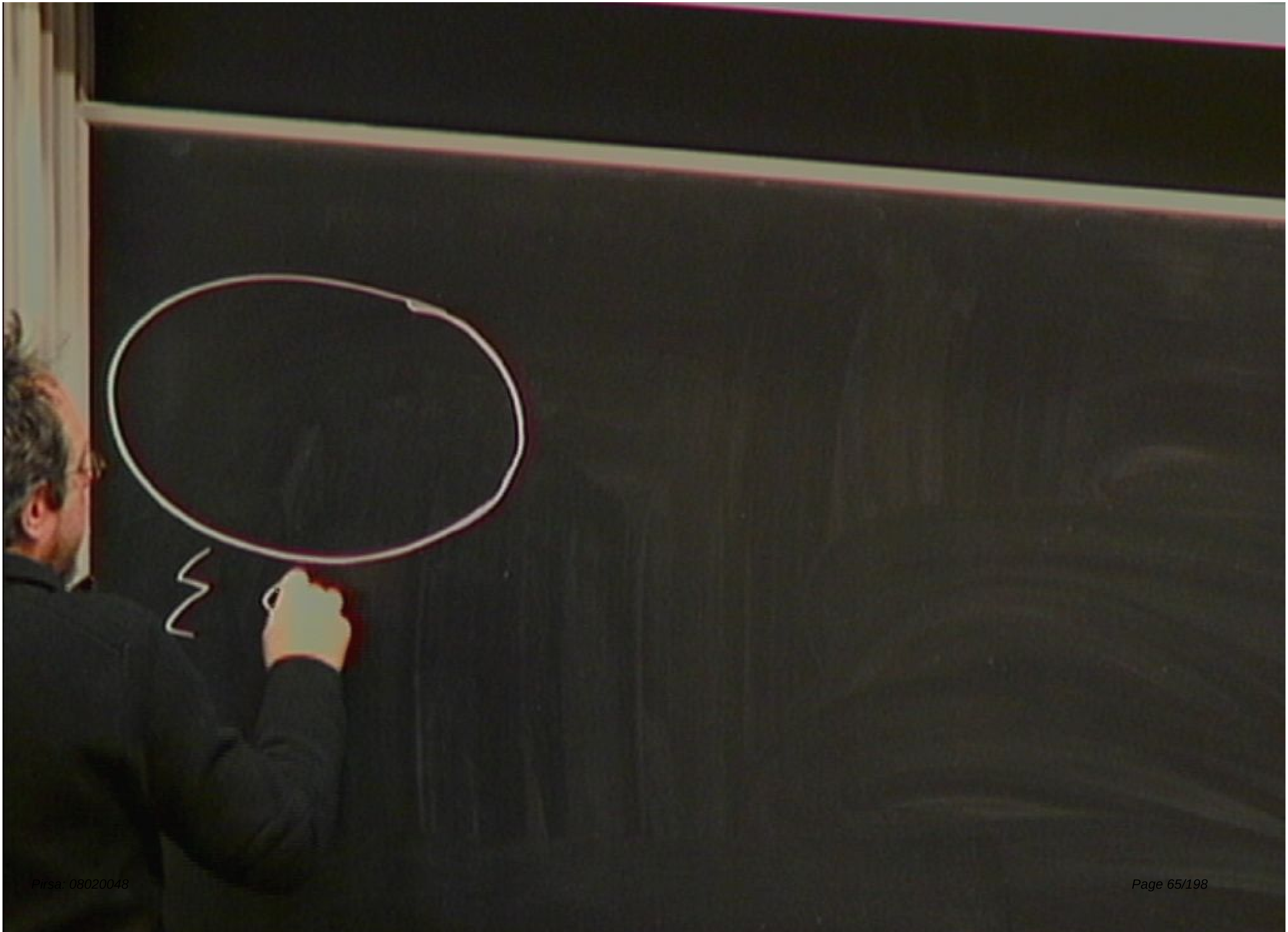

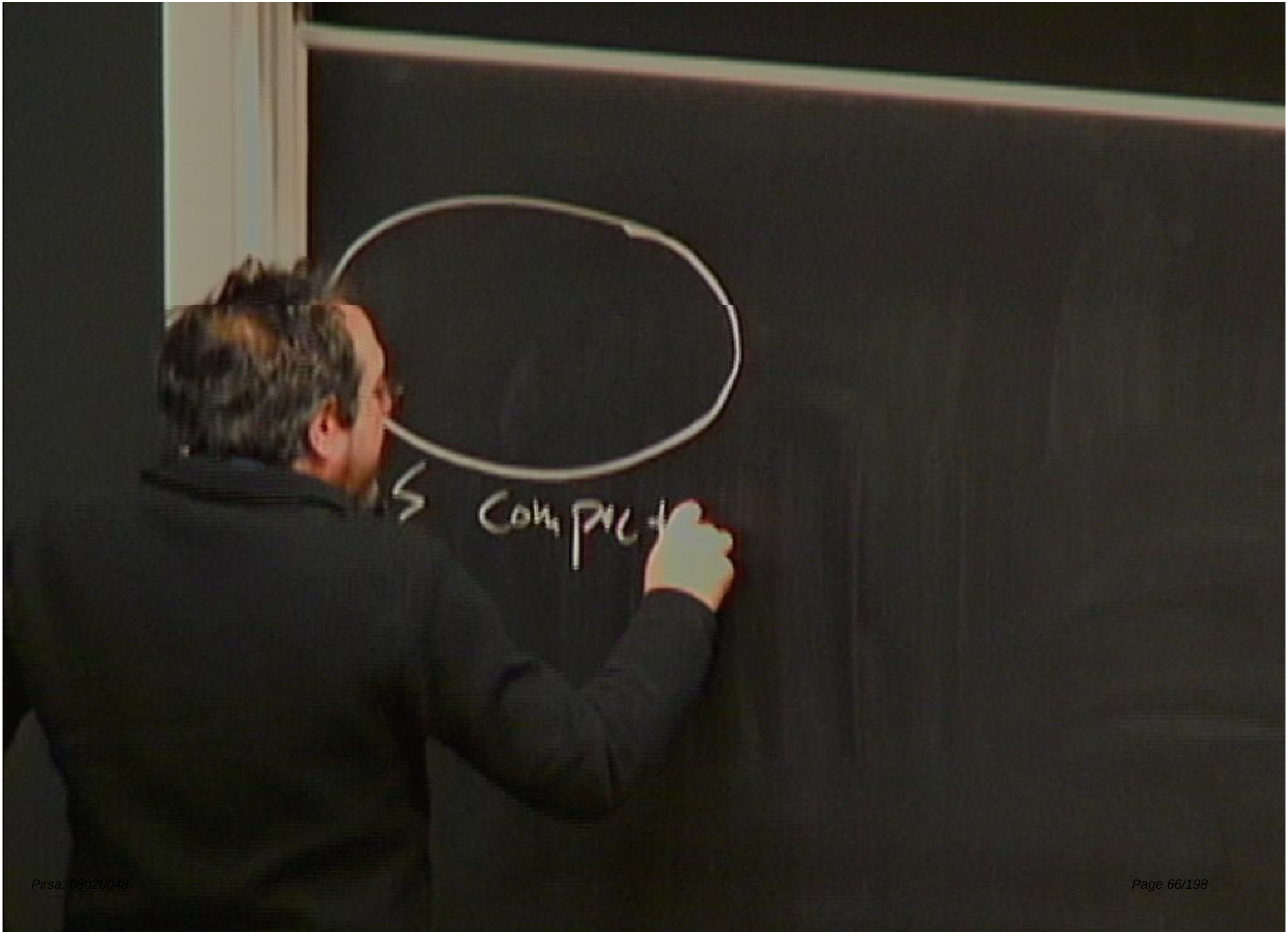

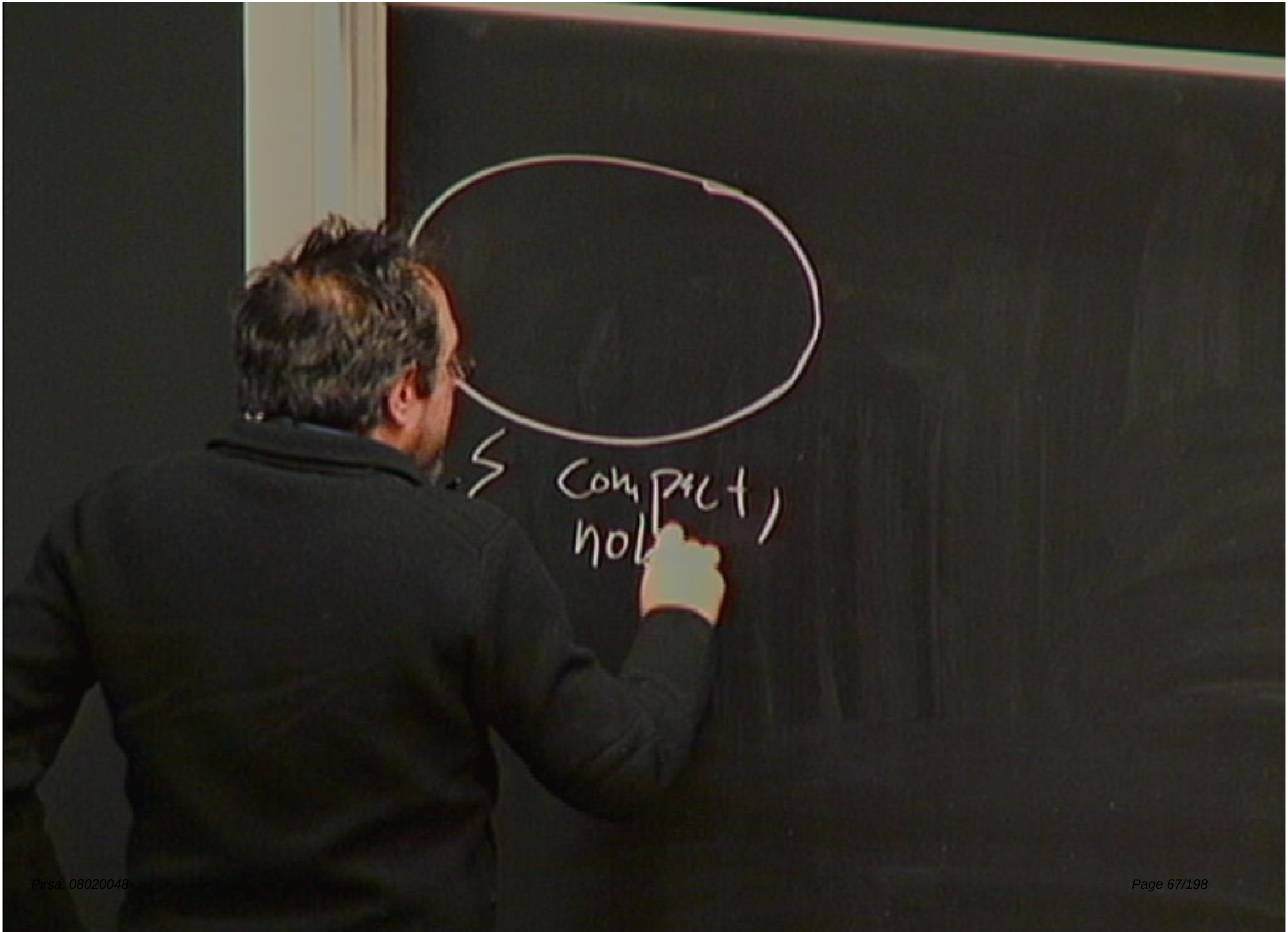
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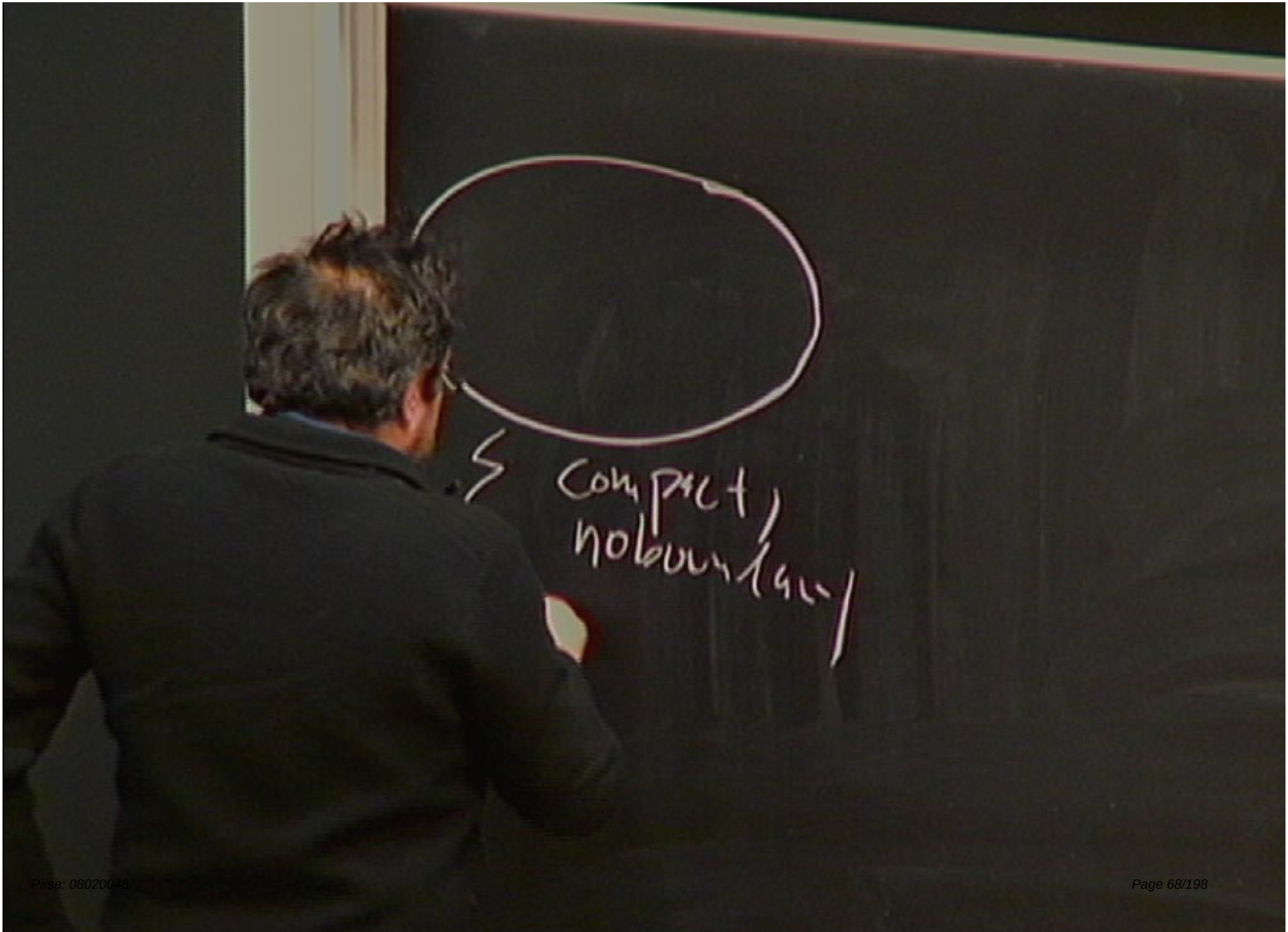
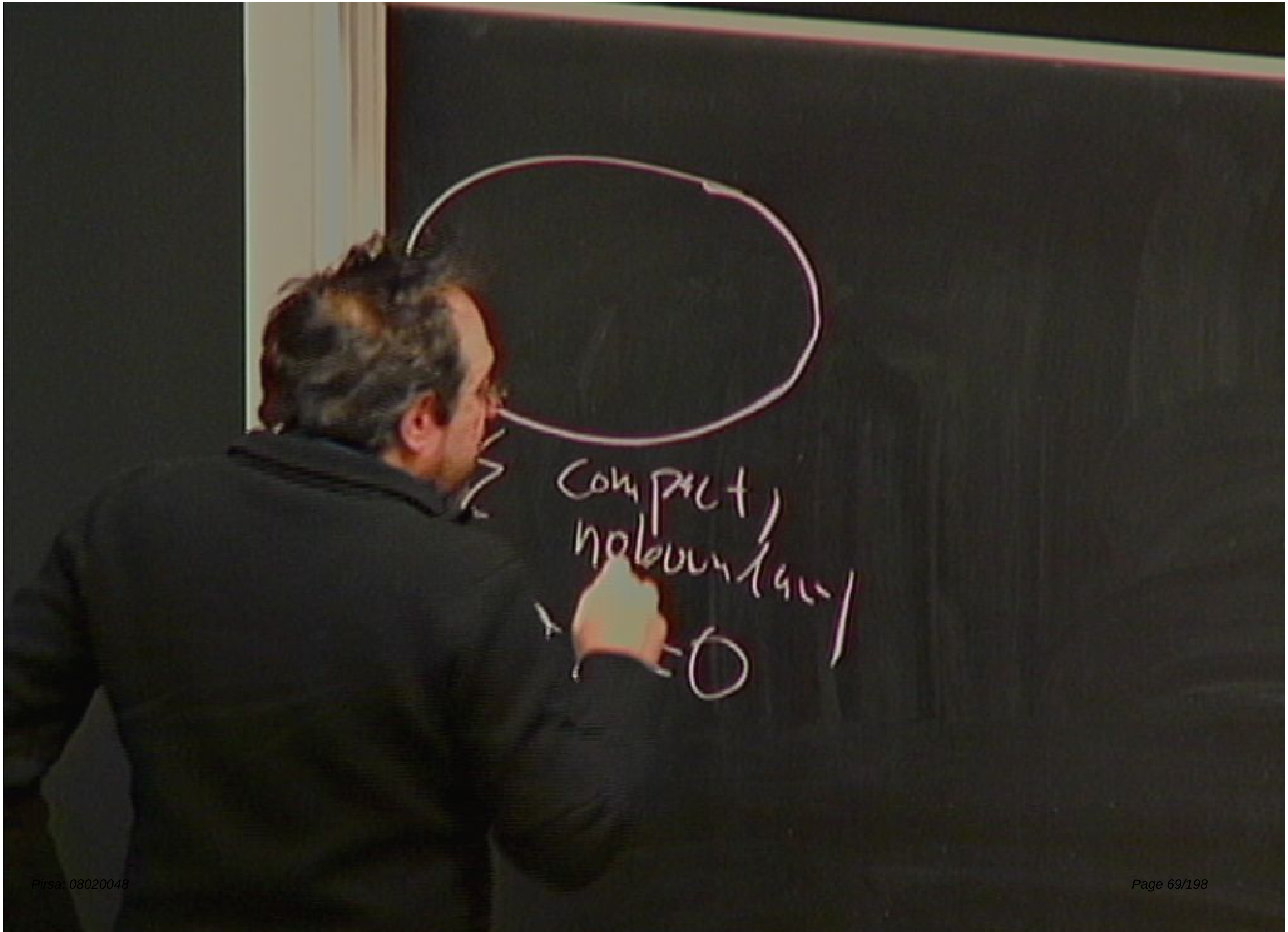
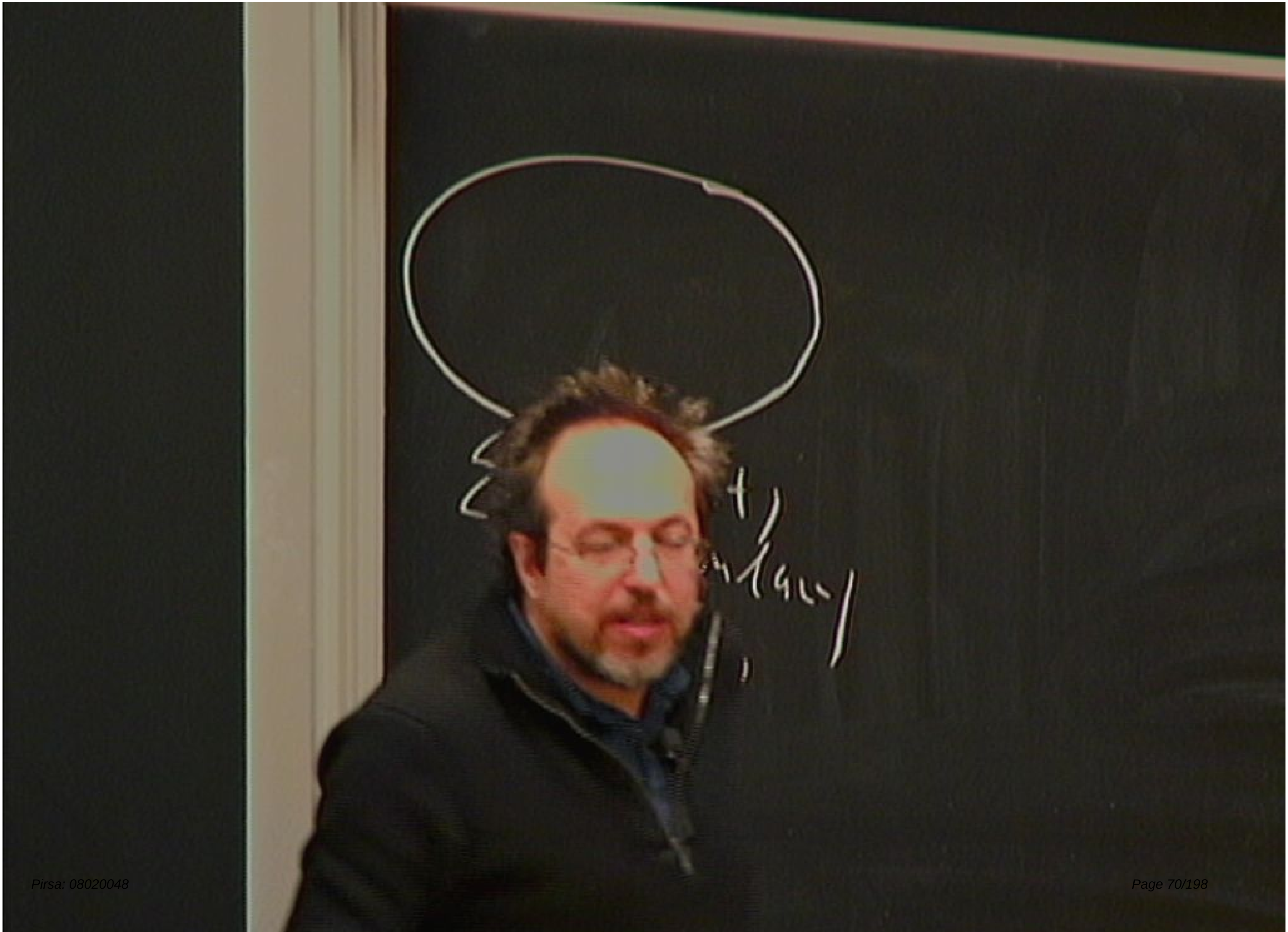
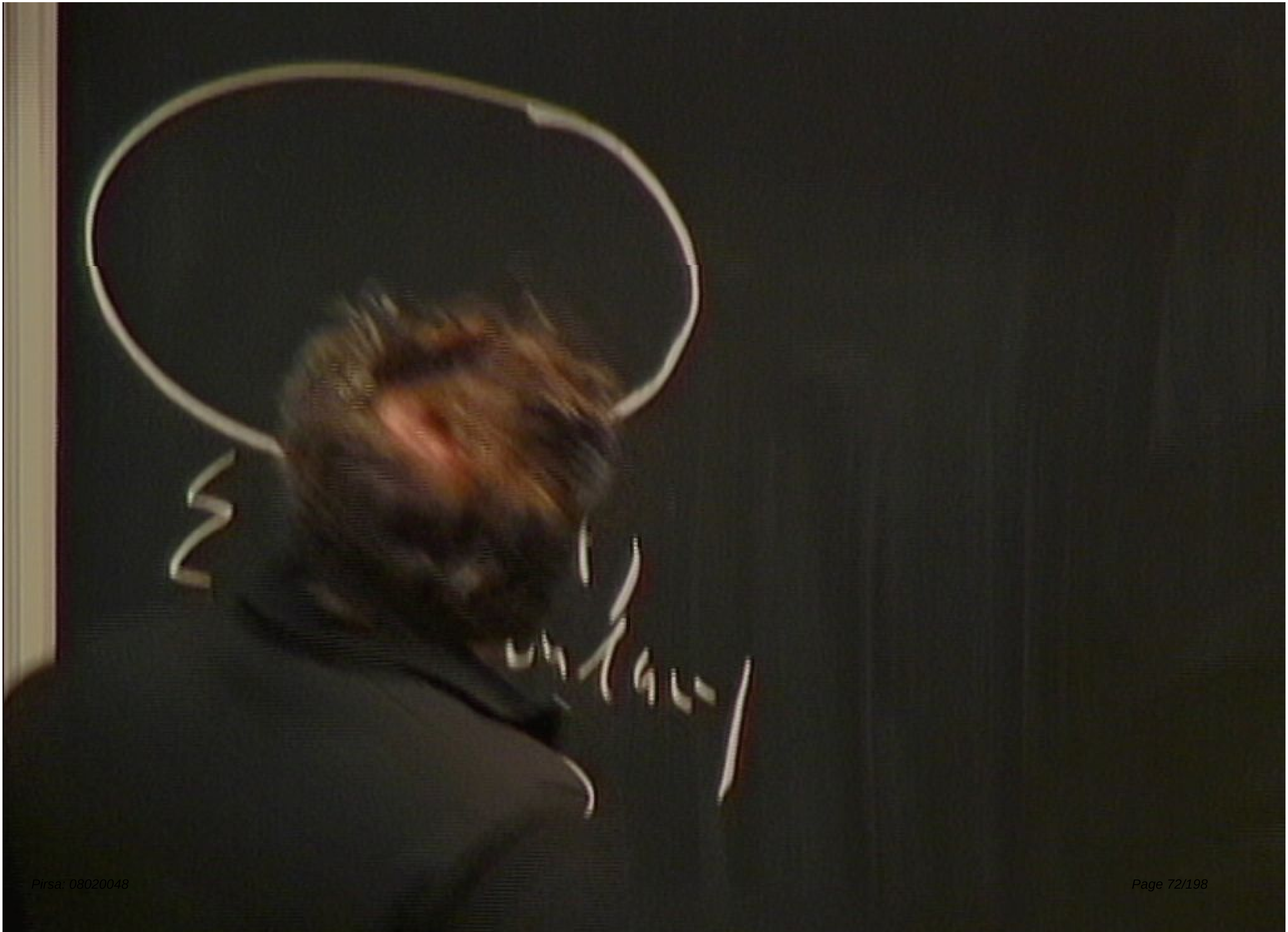

Σ compact,
no boundary

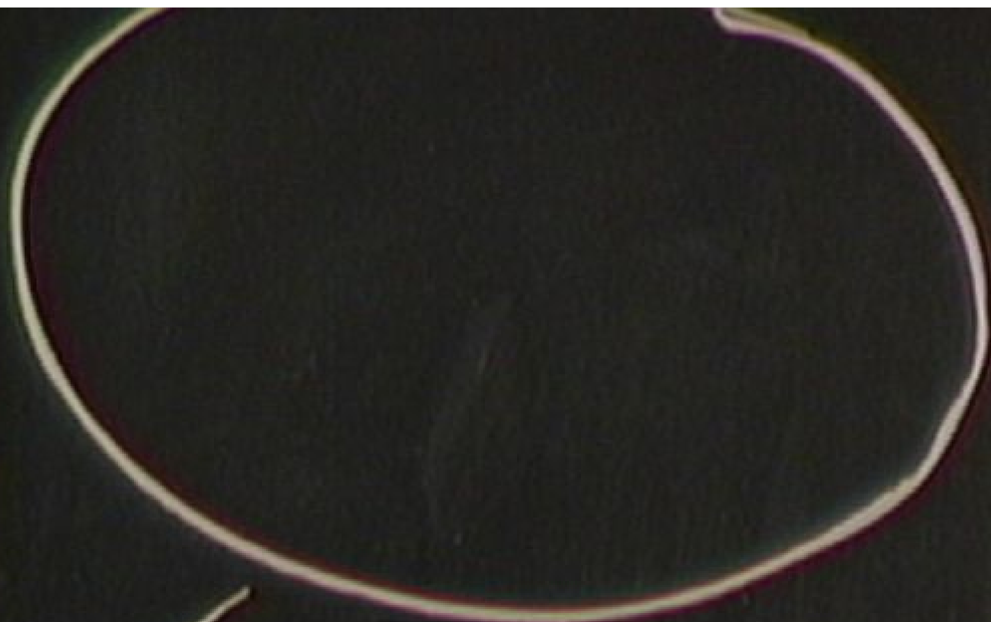

compact,
no boundary



Σ compact, no boundary

$\partial\Sigma = 0$





Σ compact, no boundary

$\partial\Sigma = 0$

$d = \text{space} = 3$

$G = SU(2)$

E $\quad A_a \in G$

Σ compact, no boundary, $\partial\Sigma = 0$

$d = \text{space} = 3$

$G = SU(2)$

$E^a \quad A_a \in G$

Σ compact, no boundary

$\partial\Sigma = 0$

$$\Pi_i^a \rightarrow E_i^a$$

$G = SU(2)$

$\Pi^a_i \rightarrow \tilde{E}$

$$\{A^i_a(x), \tilde{E}^b_j(y)\} = \delta^b_a \, \delta^i_j \, \delta^3(x, y)$$

$d = \text{space} = 3$

$E^a \quad A_a \in G$

Σ compact, no boundary

$\partial\Sigma = 0$

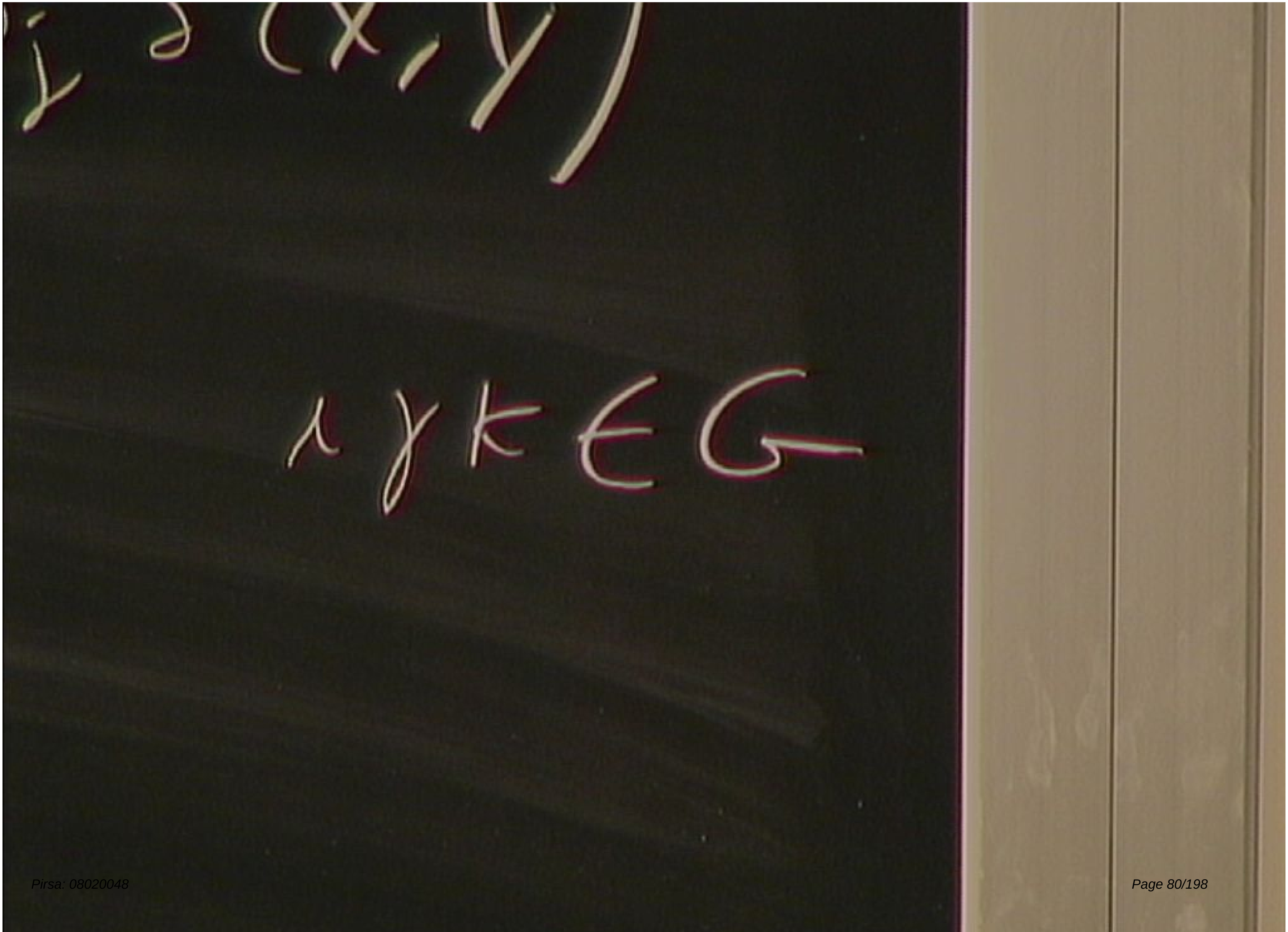
no metric

$G = SU(2)$

$\{A_a^i(x), \tilde{E}^b_j(y)\} = \delta_a^b$

$H = \sum$ constraints

$$\boxed{H = \sum \text{constraints}}$$


λγκ ∈ G

$$\boxed{H = \sum \text{constraints}}$$

B-F Theory Constraints

$$G^i = \left(\mathcal{D}_a \tilde{E}^a\right)^i = 0$$

$$\boxed{H = \sum \text{constraints}}$$

$\lambda\, \gamma\, k \in G$

B-F Theory Constraints

$$G^i = (\mathcal{D}_a \hat{E}^a)^i = 0 = \partial_a \hat{E}^{ai} + \epsilon^{ijk} A_{ja} \hat{E}^{ak}$$

$$\boxed{\mathcal{H} = \sum \text{constraints}}$$

$i\, j\, k \in G$

$\widetilde{E}^a = \epsilon^{abc} E_{bc}$

B-F Theory constraints

$(dE)_{abc}\, \epsilon^{abc}$

$$G^i = (\mathcal{D}_a \widetilde{E}^a)^i = 0 = \partial_a \widetilde{E}^{ai} + \epsilon^{ijk} A_{ja} \widetilde{E}^{ak}$$

B-F Theory constraints

$$G^i = (\mathcal{D}_a \tilde{E}^a)^i = 0 = \partial_a \tilde{E}^{ai} + \epsilon^{ij\ldots}$$

$$\tilde{E}^a =$$

$$\Sigma^i_{ab} = F^i_{ab}(A) - \Lambda \epsilon_{abc} \tilde{E}^{ci} = 0$$

Ashteka formulation constraints

$$G^i = 0$$

Ashtekar formulation constraints

$G^i = 0$ $\qquad$ $D_a = F_{ab}^{i} \tilde{E}^b_i$ "diffeomorphism constraint"

$$C = \epsilon^{ijk} \tilde{E}^a_i \tilde{E}^b_j \left[F_{abk} + \Lambda \, \epsilon_{abc} \tilde{E}^c_k \right] = 0$$ Hamiltonian constraint

$$C = \epsilon^{ijk} \tilde{E}^a_i \tilde{E}^b_j \left[F_{abk} + \Lambda\, \epsilon_{abc} \tilde{E} \right.$$

connection to ADM variables

$H \,] = 0$ Hamiltonian constraint

3+1

g_{ab}

connection to ADM

$${}^{4}g_{ab} \to q_{ab}$$

$$C = \epsilon^{ijk} E_i E_j \left[F_{ab\,k} + \Lambda\, \epsilon \right.$$

onnection to ADM variable

$g^4_{ab} \Rightarrow q_{ab}$ spatial slices

$$C = \epsilon^{ijk} \tilde{E}^a_i \tilde{E}^b_j \left[F_{abk} + \Lambda \, \epsilon_{abc} \tilde{E}^c_k \right]$$

nnection to ADM variables

$g^4_{ab} \longrightarrow q_{ab}$ spatial slices

$$\Pi^{ab} \sim \sqrt{\det q} \ldots q_{..} \, q^{..} \Gamma^{..} + \ldots$$

$\ldots \Sigma_i \Sigma_j \left[F_{abk} + \Lambda \ldots \right.$

connection to ADM variabl...

$g^4_{ab} \Rightarrow q_{ab}$ spatial slices

$\tilde{\Pi}^{ab} \sim \sqrt{\det q} \ldots \dot{q}_{ab} \ldots q^{\ldots}$

$\{q_{ab}(x), \tilde{\Pi}^{cd}(y)\} = \delta^c_a \delta^d_b \delta^3(x, \tilde{y})$

Translation:

$$\tilde{E}^a_i \hat{E}^b_i$$

Translation:

$$\tilde{E}^a_i \hat{E}^b_i = \qquad q^{ab}$$

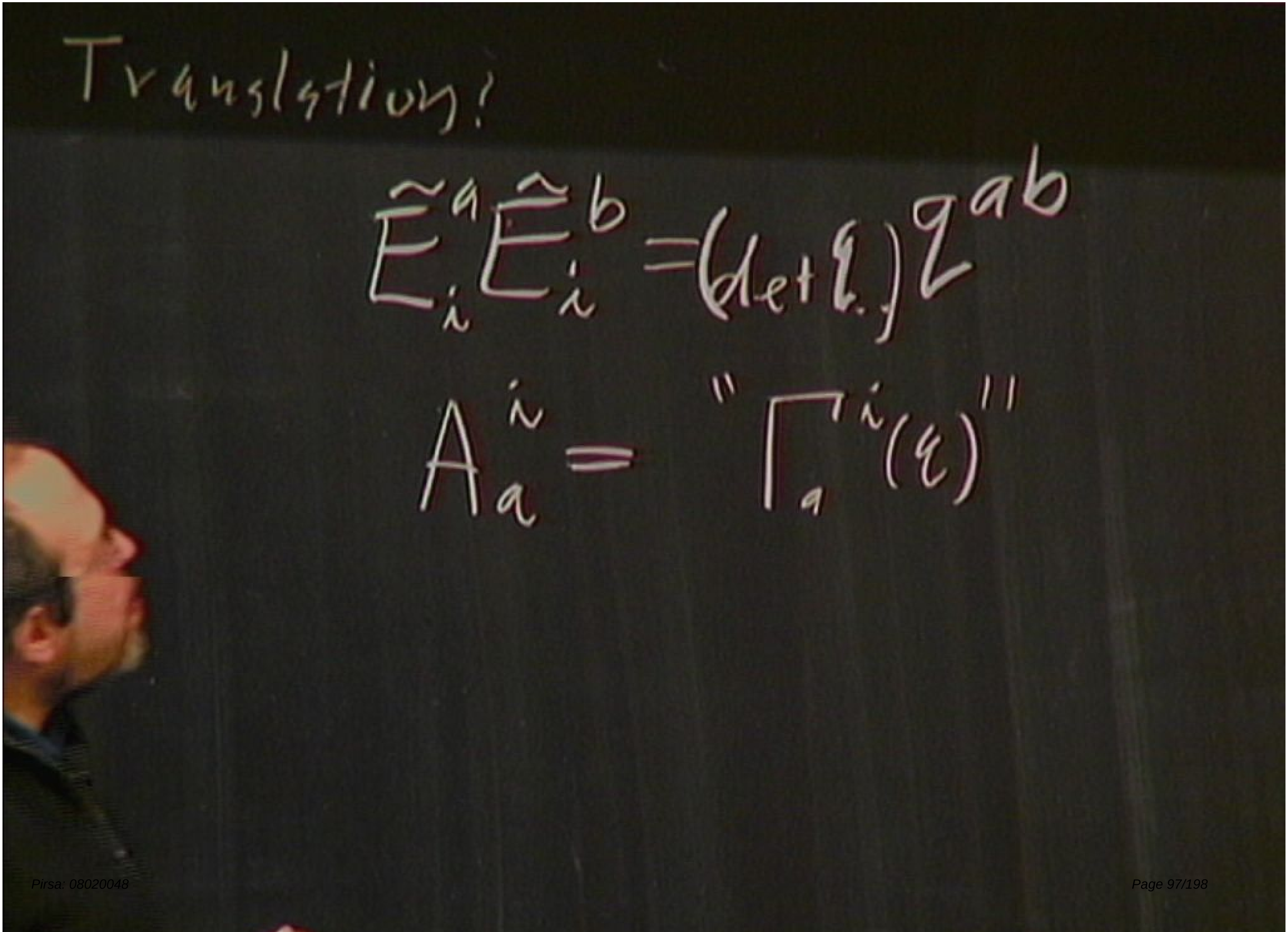
Translation:

$$\tilde{E}^a_i \hat{E}^b_i = (\det q_{..})\, q^{ab}$$

Translation:

$$\tilde{E}^a_i \hat{E}^b_i = (\det q_{..})\, q^{ab}$$

$$A^i_a =$$


Translation:
$\tilde{E}^a_i \hat{E}^b_i = (\det q)\, q^{ab}$
$A^i_a =$ "$\Gamma^i_a(q)$"

Translation:

$$\tilde{E}^a_i \hat{E}^b_i = (\det q)\, q^{ab}$$

$$A^i_a = \text{"}\Gamma^i_a(q)\text{"} + i\, \frac{\Pi_{ab}\, \tilde{E}^{bi}}{\det q}$$

$(y)\} = \delta_a \delta_b\, \delta^3(x, \tilde{y})$

tion!

$$\tilde{E}^a_i \tilde{E}^b_i = (\det q)\, q^{ab}$$

$$G A^i_a = \text{“}\Gamma^i_a(q)\text{”} + \frac{i\, \Pi_{ab} \tilde{E}^{bi}}{G^2 \det q} \qquad [G A] \sim \frac{1}{\text{length}}$$

$$\ldots(y)\} = \delta_a^a \delta_b^a \, \delta^3(x, \tilde{y})$$

...tion!

$$\tilde{E}^a_i \tilde{E}^b_i = (\det q)\, q^{ab}$$

$$A^i_a = \text{“}\Gamma^i_a(E)\text{''} + \frac{i\, \Pi_{ab} \tilde{E}^{bi}}{G \det q}$$

$$[GA] = \frac{1}{\text{length}}$$

$$\}(y)\} = \delta_a \delta_b^a \delta^3(x, \tilde{y})$$

-tion!

$$\tilde{E}^a_i \tilde{E}^b_i = (\det q)\, q^{ab}$$

$$A^i_a = \text{``}\Gamma^i_a(q)\text{''} + \frac{i\, \Pi_{ab} \tilde{E}^{bi}}{G \det q}$$

$$[A] = \frac{1}{\text{length}}$$

$G = SU(2)$

$\Pi_i^a \to \tilde{E}_i^a$

$\{A_a^i(x), \tilde{E}_j^b(y)\} = G \delta_a^b \delta_j^i \delta^3(x, \tilde{y})$

$= \{$ constraints

$\lambda\, \gamma\, k \in G$

$\tilde{E}^a = \epsilon^{abc} E$

Theory constraints

tion?

$$\tilde{E}^a_i \hat{E}^b_i = (\det \tilde{E}.)\, q^{ab}$$

$$A^i_a = \text{"}\Gamma^i_a(\tilde{E})\text{"} + \frac{i}{G} \frac{\Pi_{ab} \tilde{E}^{bi}}{\det q}$$

Σ compact, no boundary

$\partial\Sigma = 0$

no metric

$$S = \frac{1}{G}\int B \wedge F$$

$$\boxed{\mathcal{H} = \sum \text{constr}}$$

B-F Theory

$$G^i = (\mathcal{D}_a \tilde{E}$$

$$\tilde{\Sigma}^i_{ab} = F^i_{ab}(A) -$$

$G = SU(2)$

$\Pi^a_i \rightarrow \tilde{E}^a_i$

$$\{A^i_a(x), \tilde{E}^b_j(y)\} = G\delta^b_a \delta^i_j \delta^3(x,y)$$

$\boxed{= \{\text{constraints}}$

$\lambda\gamma k \in C$

$\tilde{E}^a = \epsilon^{abc}$

Theory constraints

$(d E)_{ab}$

$(D\tilde{E}^a)^i$

$$A_a^i = \text{"}\Gamma_a^i(e)\text{"} + i\,\frac{\Pi\ldots}{\sigma\, d\ldots}$$

Euclidean signature $i = 1$

$$\tilde{E}^a_i \hat{E}^b_i = (\det q)\, q^{ab}$$

$$A^i_a = \text{"}\Gamma^i_a(q)\text{"} + \frac{i\, \Pi_{ab} \tilde{E}^{bi}}{G \det q}$$

Euclidean signature $i = 1$ $\quad G = SU(2)$ real

Euclidean signature $\lambda = 1$ G

Lorentzian $A \in \mathbb{C}\, SU(2)$

Lorentzian reality conditions

$$q_{ab} = \overline{q_{ab}} \qquad E^{ai*} = E^{a\bar{i}}$$

Lorentzian reality conditions

$$q_{ab} = \overline{q_{ab}} \qquad E_i^{a\,i\,*} = E^{a\,i}$$

$$A + \overline{A} =$$

Lorentzian reality conditions

$$q_{ab} = \overline{q_{ab}} \qquad E^{ai\,*} = E^{a\bar{i}}$$

$$A + \overline{A} = \Gamma(q)$$

Lorentzian reality conditions

$$\boxed{\begin{gathered} q_{ab} = \bar{q}_{ab} \qquad E^{ai*}_{i} = E^{ai} \\ A + \bar{A} = \Gamma(q) \end{gathered}}$$

$$(\epsilon)'' + \frac{i\,\Pi_{ab}\,E^{bi}}{G\ \det q} \qquad [\ A] \Rightarrow \frac{1}{\text{length}}$$

$= 1 \qquad G = SU(2)$ real $\qquad A, E$ real

$SU(2)$

$$\tilde{E}^a_i \tilde{E}^b_i = (\det q_{..}) q^{ab}$$

$$A^i_a = \text{"}\Gamma^i_a(q)\text{"} + \sigma \, i \, \frac{\Pi_{ab}}{\det q}$$

Euclidean signature $i = 1$ $\quad \sigma =$

Lorentzian $A \in \mathbb{C}\, SU(2)$

Lorentzian reality conditions

$$\boxed{\begin{gathered} q_{ab} = \bar{q}_{ab} \qquad E^{ai*} = E^{ai} \\ A + \bar{A} = \Gamma(q) \end{gathered}}$$

Ashtekar – Barbero

$$A = \Gamma + G\,\Pi E$$

Lorentzian reality conditions

$$\boxed{\begin{array}{c} q_{ab} = \bar{q}_{ab} \qquad E_i^{ai\,*} = E^{ai} \\ A + \bar{A} = \Gamma(q) \end{array}}$$

Ashtekar–Barbero

$$A = \Gamma + G \Pi E \qquad A_a^{i\,*} = A_a^i \qquad G = SU(2)$$

$$\{A_a^i(x), A_b^j(y)\} = 0$$

$A_a^{i*} = A_a^i \quad G = SU(2)$

$$q_{ab} = \bar{q}_{ab} \qquad E_i^{ai} = E^{ai}$$

$$\boxed{A + \bar{A} = \Gamma(q)}$$

Ashtekar – Barbero

$$A = \Gamma + G\,\Pi\,E \qquad A_a^{i*} = A$$

$$C = C^{\text{Ashtekar}} + \text{new term}$$

$$\{A_a^i(x),\ A_b^j(y)\}_P = 0$$

$$A_a^{i*} = A_a^i \qquad G = SU(2) \qquad \gamma \in \mathbb{C}$$

term

$$\{A_a^i(x), A_b^j(y)\}_? = 0$$

γ Immirzi

$$A_a^{i*} = A_a^i \quad G = SU(2) \quad \gamma \in \mathbb{C}$$

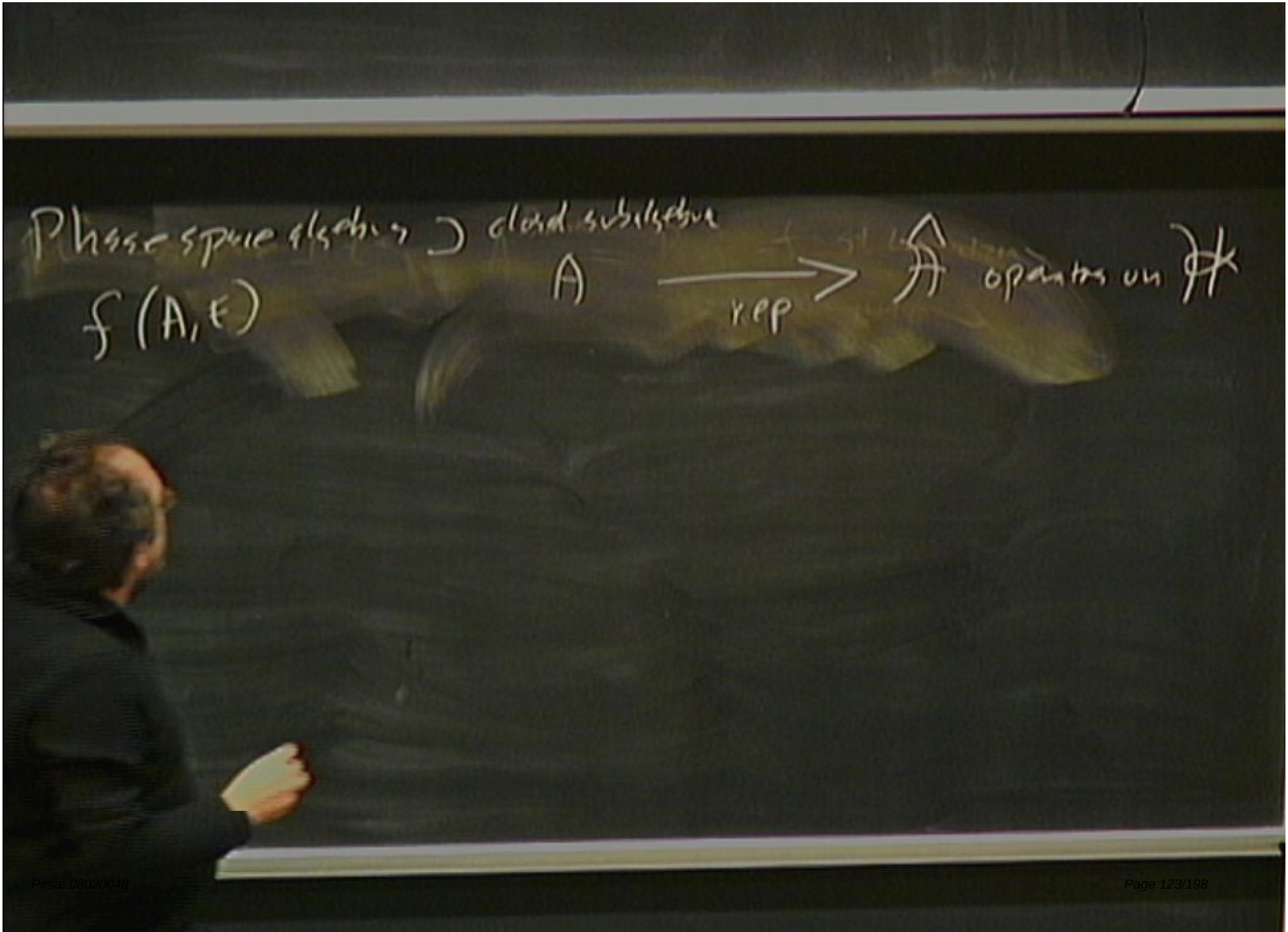
term

$\uparrow t^a$ $\rightarrow$ 3 space $\perp t^a$
S

A^{Barbero} = Proj of 4d Lorentzian connection into S

Phase space algebra $\supset$ closed subalgebra

$f(A, E)$ $\qquad$ A


Phase space algebra ⊃ closed subalgebra
A —rep→ Â operators on H
f(A, E)

Phase space algebra ⊃ closed subalgebra

$f(A, E)$ $\quad \mathcal{A}^{kin} \xrightarrow{\text{rep}} \hat{\mathcal{A}}^{kin}$ operators on $\mathcal{H}^{kin}$

sed subalgebra

$$A^{kin} \xrightarrow[\text{rep}]{} \hat{A}^{kin} \text{ operators on } \mathcal{H}^{kin}$$

$$= (i\hbar)\,\delta^b_a\,\delta^i_j\,\delta^3(x, y)$$

Canonical algebra

ts

nstraints

$$\lambda\, \gamma\, k \in G$$

$$\tilde{E}^a = \epsilon^{abc} E_{bc}$$

$$\sim (dE)_{abc}\,\epsilon^{abc}$$

$$= 0 = \partial_a \tilde{E}^{ai} + \epsilon^{ijk} A_{ja} \tilde{E}^{ak}$$

Phase space algebra $\supset$ closed subalgebra

$f(A, E)$ $\qquad A^{kin} \xrightarrow{\text{rep}} \hat{A}^{kin}$ op

$A^{canonical} \xrightarrow[\text{rep}]{\text{Fock space}}$ background metric

choice of time coordinate

$e^{-i\omega t} \not\sim e^{i\omega t}$

rep

choic

$\det q > 0$

$\mathcal{F}(A,t)$

$A^{\text{canonical}} \xrightarrow[\text{rep}]{\text{Fock space}}$ bac… choice…

Isham $\det q > 0$

Polyakov:

$A^{canonical}$ $\xrightarrow[\text{rep}]{\text{Fock space}}$ ba... choic...

Isham $\det q > 0$

Polyatov: right observables

fields/coordinates

...ackground metric

...nce of time coordinate

$$e^{-i\omega t} \quad \times \quad e^{i\omega t}$$

$$T[A,\gamma] = \mathrm{Tr}\, P\, e^{\oint_\gamma A\cdot dx}$$

$A^{\text{canonical}}$ $\xrightarrow[\text{rep}]{\text{Fock space}}$ bac choice

Ishum $\det q > 0$

Polyatov: right observables fields/coordinates

...oice of time coordinates

$e^{-i\omega t} \quad x \quad e^{i\omega t}$

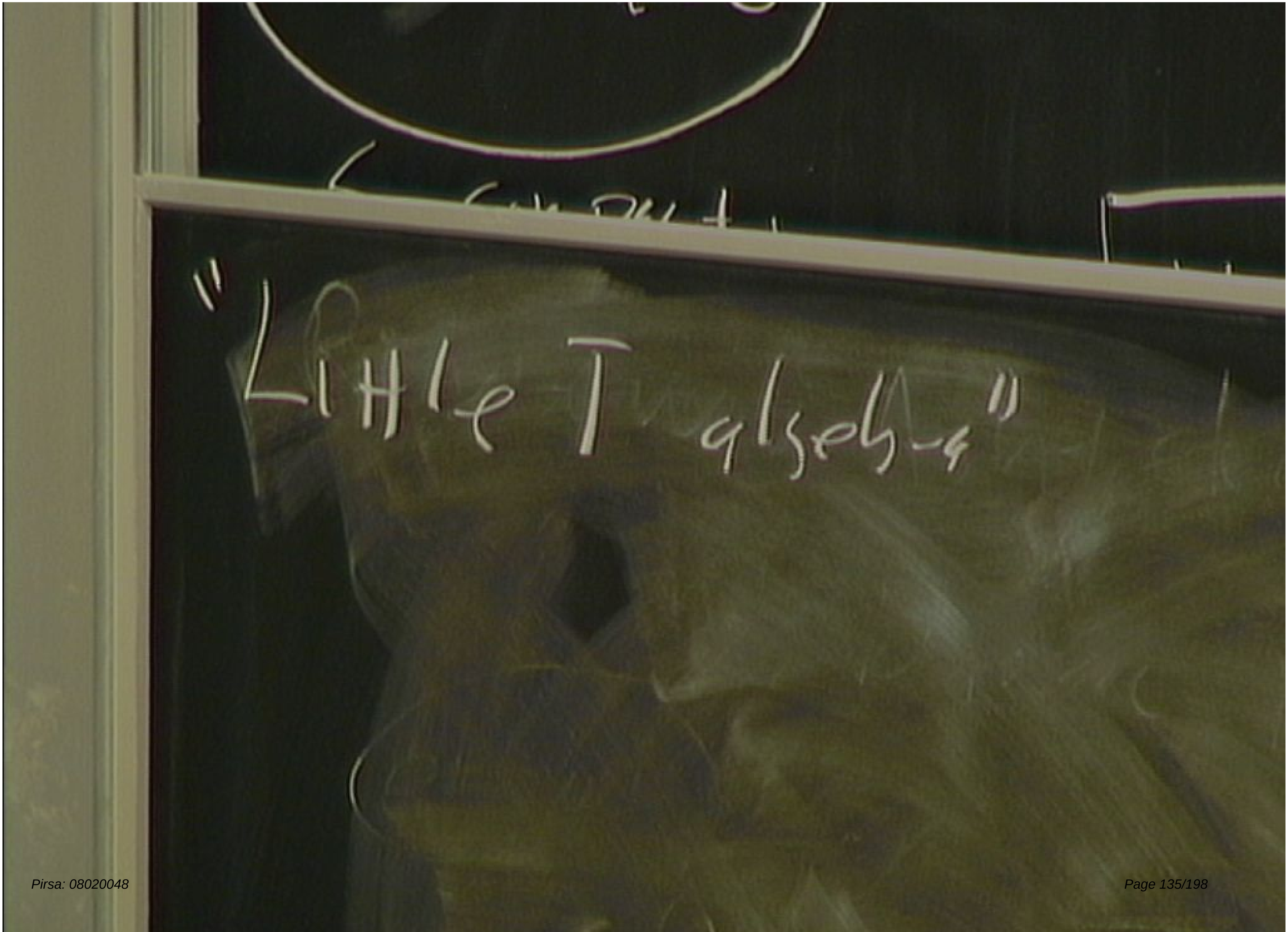
$$T[A, \alpha] = \mathrm{Tr}\, P\, e^{\oint_\alpha A \cdot dx}$$

...nckground metric

...nce of time coordinates

$e^{-i\omega t} \quad x \; e^{i\omega t}$

$$\underbrace{T[A,\gamma] = \mathrm{Tr}\, P\, e^{\oint_\gamma A\cdot dx}}$$


"Little T algebra"

Loop = Strip algebra

"Little T algebra"

$\gamma(0) = p$

$$\tilde{E}^a(p)\, U_\gamma$$

"Little T algebra"

$\gamma(0) = p$

$$\mathrm{Tr}\left[\tilde{E}^a(p)\, U_\gamma\right] \equiv T^a[\gamma]$$

Loop

$$A = \Gamma + \gamma G \, \Pi \, E \qquad A_a = A \qquad G =$$

$$C = C^{\text{Ashtekar}} + \text{new term}$$

under $\delta_f A_a = (D_a f)^i = \{A_a, G(f)\}$

$$\delta_f E^{a}_{i} = \epsilon^{ijk} E^{ka} f_j$$

Phase space algebra $\supset$ closed subalgebra

$$f(A, E) \qquad \mathcal{A}^{\text{kin}} \xrightarrow{\text{rep}} \hat{\mathcal{A}}^{\text{kin}} \text{ op}$$

$$\mathcal{A}^{\text{canonical}} \xrightarrow[\text{rep}]{\text{Fock space}}$$ background metric

choice of time condensa

$$e^{-i\omega t} \quad \times \quad e^{i\omega t}$$

let $a \to 0$

"Little T algebra"

"T-one"

$\gamma(0)=p$

$$\mathrm{Tr}\left[E^a(p)\,U_\gamma\right] \equiv T^{a}[\gamma]$$

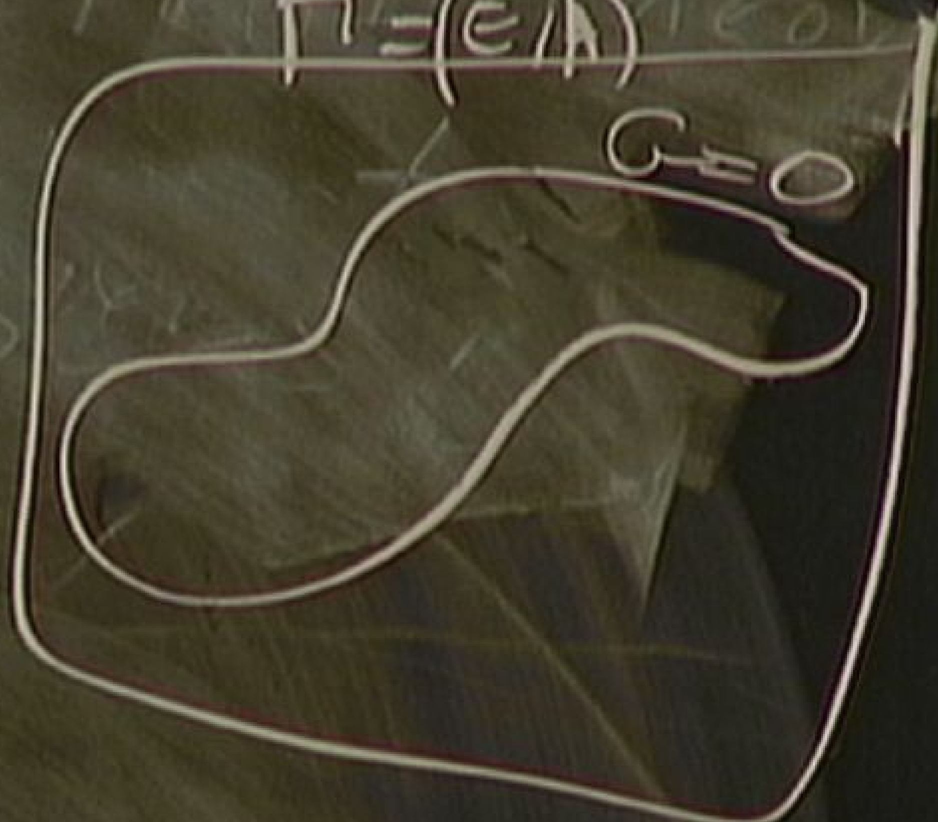
$\lambda \gamma k \in G$

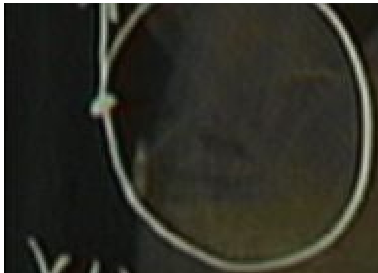
one"

Loop-Strip algebra

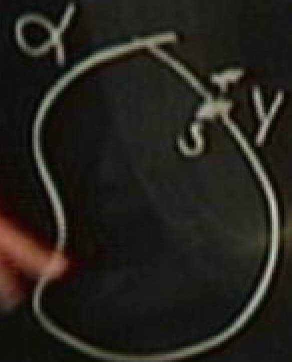
$\Pi = (e, A)$

$T^a[\gamma]$





$\gamma(0)=p$



$$\mathrm{Tr}\left[\tilde{E}^a(p)\, U_\gamma\right] \equiv T^a[\gamma]$$

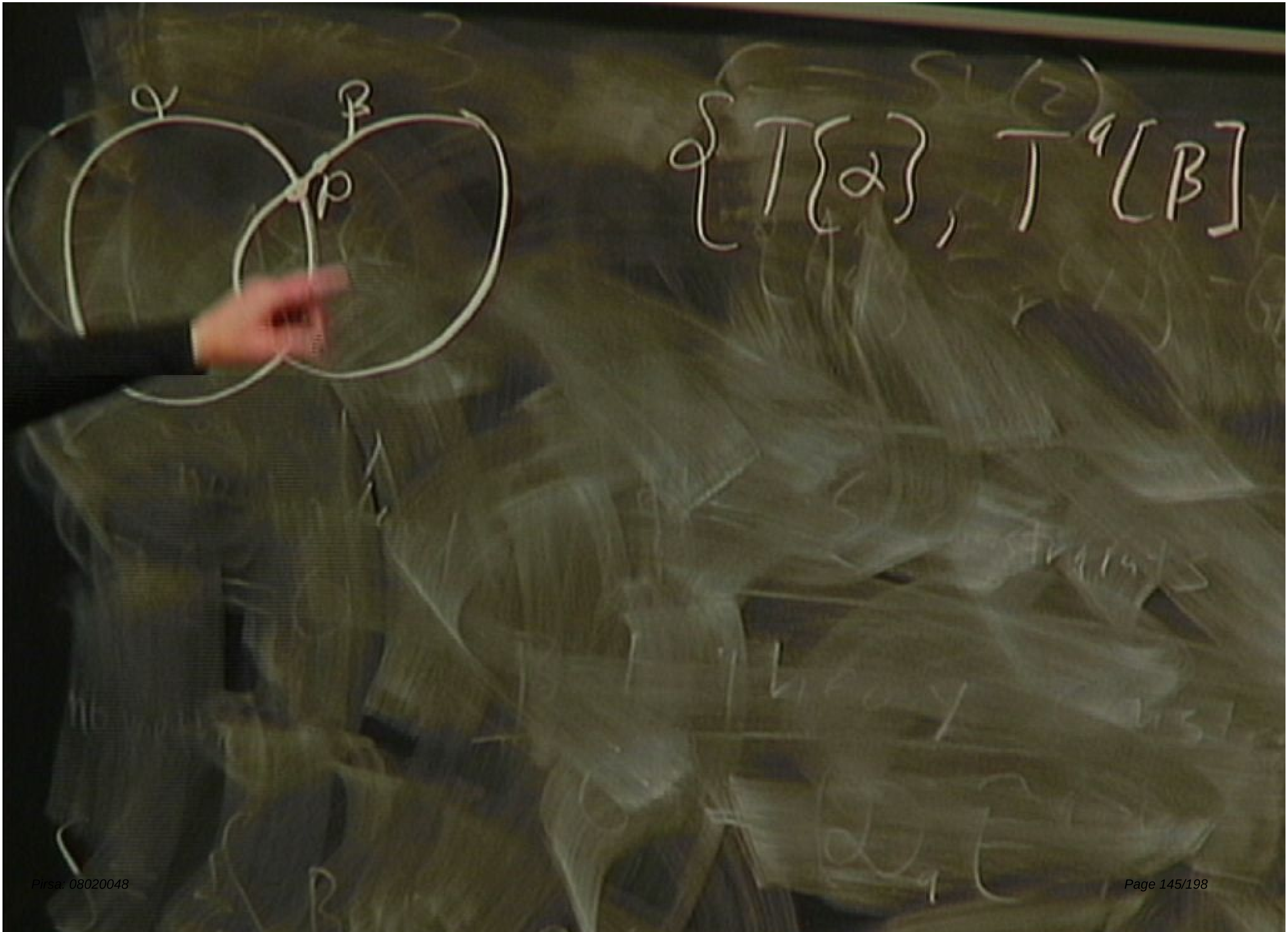
$$\{T[\alpha], \tilde{E}^{ai}(y)\} = \int ds\, \dot{\alpha}^i(s)\, \delta^3(\alpha(s), y)\ \mathrm{Tr}\, U_{\gamma(s)}$$

$$\{T[\alpha], E^{ai}(y)\} = \int ds\, \dot{\alpha}^a(s)\, \delta^3(\alpha(s), y)\, \mathrm{Tr}\, U_\gamma(s) | \tau^i$$

$$\int ds\, a(s)\, \delta(\alpha(s), y)]$$

$$\mathrm{Tr}\, U(s)\, \tau^i$$

$$= \frac{\delta\, T[\alpha]}{\delta A_a^i(x)}$$


α
β
p
q{T[α], T^q[β]]

$$\mathrm{Tr}\left[\tilde{E}^a(p)\, U_\gamma\right] \equiv T^a[\gamma]$$

$$\left\{ T[\alpha], \tilde{E}^{ai}(y) \right\} = \int ds\, \dot{\alpha}^i(s)\, \delta^3(\alpha(s), y)\; \mathrm{Tr}\, U_\gamma(s)\, \tau^i$$

$$= \int T[\alpha$$

$$q\{T[\alpha], T^a[\beta]\} = \int ds\, \dot{\alpha}^a(s)\, \delta^3(\alpha(s), p)$$

$$\{T[\alpha], T^a[\beta]\} = \int ds\, \dot{\alpha}^a(s)\, \delta^3(\alpha(s), p)\, \mathrm{Tr}(U_\alpha(s)\gamma^i)\, \mathrm{Tr}(U_\beta \gamma^i)$$

$\mathrm{Tr}(U_\alpha$

$$\mathrm{Tr}(A\,T^i)\,\mathrm{Tr}(B\,T^i)$$

$$= \mathrm{Tr}(AB) - \mathrm{Tr}(AB')$$

Check

$$\mathrm{Tr}(A\tau_i)\,\mathrm{Tr}\,B\tau_i$$

$$= \mathrm{Tr}(AB) - \mathrm{Tr}(AB')$$

(check

$$\mathrm{Tr}\left(U_\alpha(\omega)\,\gamma^a\right)\,\mathrm{Tr}\left(U_\beta\,\gamma^a\ldots\right.$$

$$= \int ds\,\dot{\alpha}^a(s)\,\delta^3(\alpha(s), p)$$

$$\oint T\left[\alpha \circ \beta\right]$$

$$\{T[\alpha], T^a[\beta]\} = i G \gamma \int ds\, \dot{\alpha}^a(s)\, \delta^3(\alpha(s), \ldots$$

$$\mathrm{Tr}(U_\alpha(s)\,\tau^i)\,\mathrm{Tr}(U_\beta\,\tau^i)$$

$$\tau^i$$

$$= i G \gamma \int ds\, \dot{\alpha}^a(s)\, \delta^3(\alpha(s), p)\, (d$$

$$\mathrm{Tr}(A\beta')$$

$$\{T[\alpha \circ \beta] - T[\alpha \circ \beta']\}$$

$$=$$

constraints

$$\{T^{a} \dots T^{b}\} = \sum T^{a}{}'s$$

ADM

$\{T^a \dots T^y\} = \sum T^a$'s check

$$\{E^a(y)\} = i\kappa \oint_\gamma ds\, \dot{\alpha}^a(s)\, \delta^3(\alpha(s), y)$$

$$\mathrm{Tr}\, U_\gamma(s)\, \tau^i$$

exercise

$$= \frac{\delta T[\alpha]}{\delta A^i_a(x)}$$

$$\{T[\alpha], T[\beta]\} = 0$$

$$= i G \gamma \int ds\, \dot{\alpha}^a(s)\, \delta^3(\alpha(s), p)$$

$$\ldots E^a(y)\} = i\kappa \int_\gamma ds\, \dot\alpha^a(s)\, \delta^3(\alpha(s), y)$$

$$\mathrm{Tr}\, U_\gamma(s) \, \tau^i$$

exercise

$$= \frac{\delta T[\alpha]}{\delta A_a^i(x)}$$

$$\{T[\alpha], T[\beta]\} = 0$$

$$= i G \gamma \int ds\, \dot\alpha^a(s)\, \delta^3(\alpha(s), p)$$

$$\int T[\ldots$$

$$\left\{ T \ldots T \right\}_b = \sum T$$

SU(2) real

SU(2) real

$\Psi[A] =$

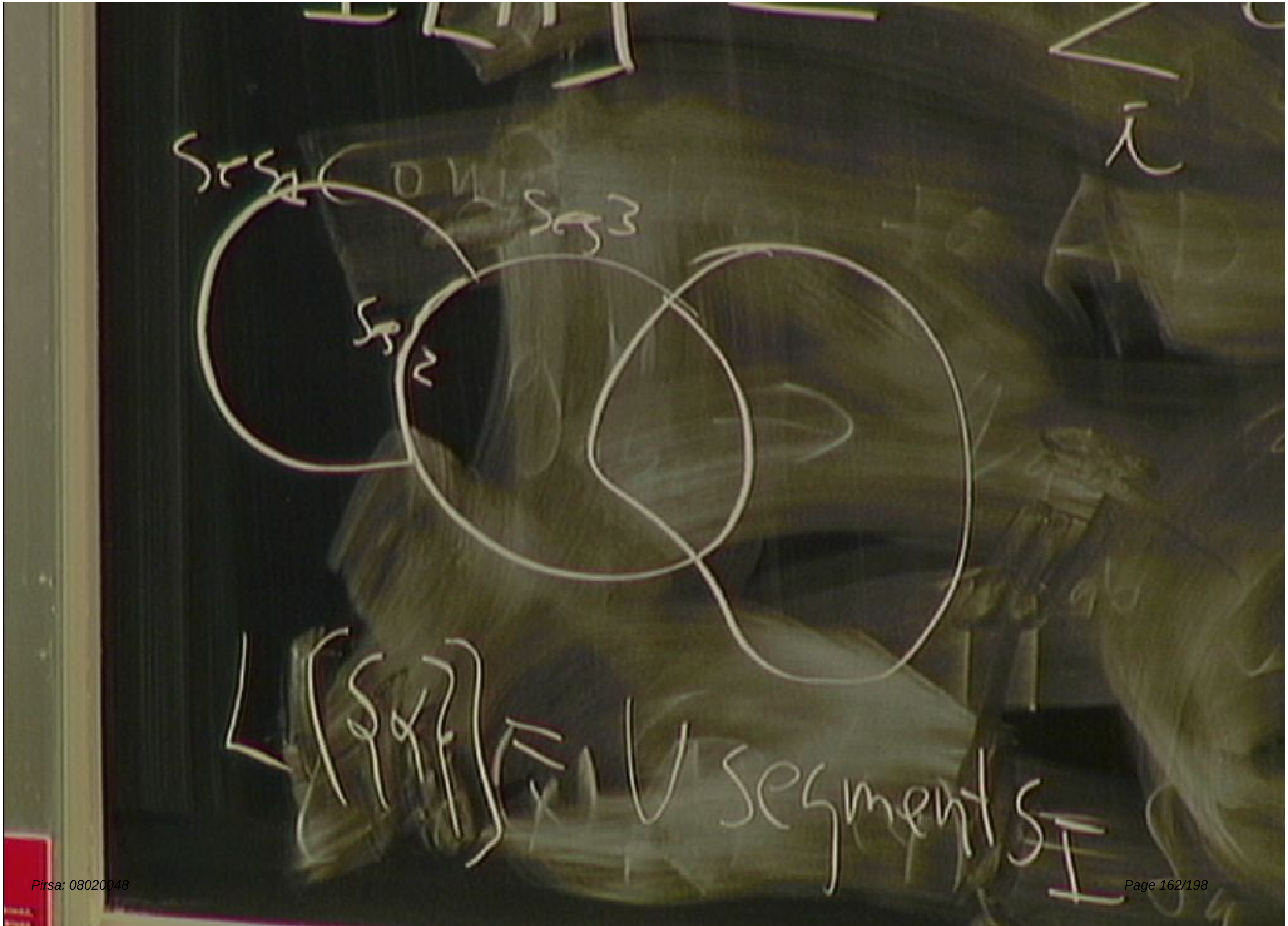
SU(2) real

$$\Psi[A] = \sum_i a_{i \ldots} \prod_\alpha T[\alpha]$$

variables

countable sum

Hamiltonian
constraint


Seg 1
Seg 3
Seg 2
L[Seg] = ∪ segments

countable sum

$$\alpha_i \prod_{\delta\alpha\xi} T[\alpha]$$

SU(2) lattice gauge theory on L

$\alpha_i \quad \prod_{\{\alpha\}} T[\alpha]$ countable sum

SU(2) lattice gauge theory on L

$C = \{U_I\}$ SU(2) element to each segment

$$\mathcal{C} = \{ U_\Gamma \} \quad su\ldots$$

$$U_T[\alpha] \in \mathcal{C}$$

$$\{\alpha\}$$

$$L \supset C_L$$

$$L \Rightarrow C_L \quad f(u_I \ldots)^{ab}$$

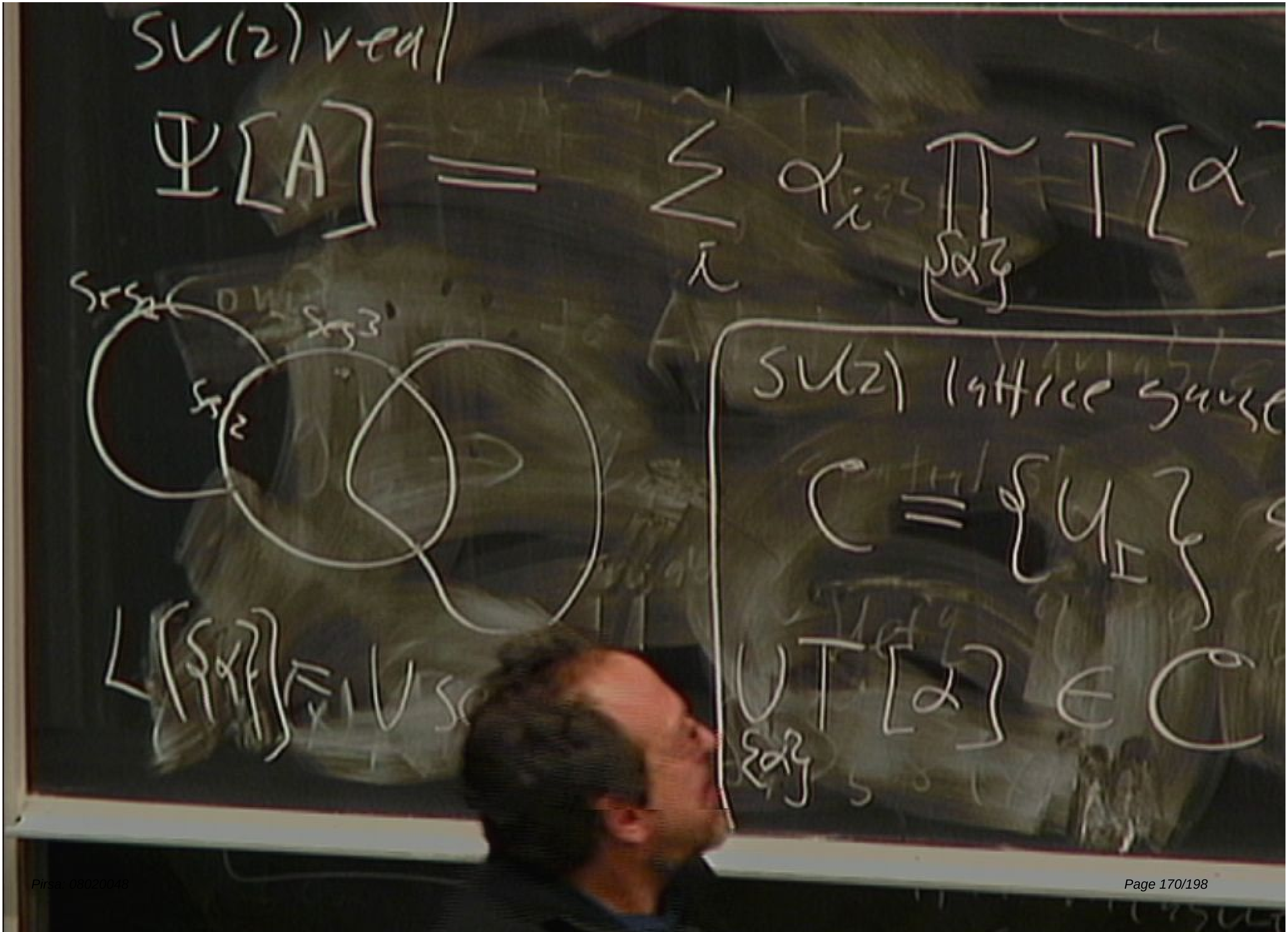
$$L \Rightarrow C_L \quad f(u_I \ldots)^{ab}$$

$$|f| = \prod_I \int d\mu(u)$$

$$\Rightarrow C_L \quad f(u_I \ldots)$$

$$|f| = \prod_I \int d\mu(u_I)\, f(u_1 \ldots u_2)$$

Haar measure
grouping


SU(2) real
Ψ[A] = Σ α_i ∏ T[α
SU(2) lattice gauge
C = {U_I}
∪ T[α] ∈ C

$$\boxed{|f| = \prod_I \int d\mu(U_I)\, f \ldots}$$

Haar measure

$$|\Psi[\Gamma]| = \sum_i |f(L_i)|$$

check

countable sum

$$\sum_i |\alpha|^2 < \infty$$

theory on L

$$\hat{T}[\gamma]\,\Psi(T[\alpha]) \equiv T[A,\gamma]\,\Psi[T[\alpha\gamma]]$$

$$T[g]\,\Psi(\cdots\,|e_g\rangle) = T[h,$$

$$\hat{T}^a[\alpha]\,\Psi$$

$$T[\gamma]\,\Psi(\ell_\gamma) = T[n, \ldots$$

$$\hat{T}^a[n]\,\Psi =$$

$$\hat{T}[\gamma]\,\Psi(T[\gamma]) \equiv T[A,\gamma]$$

$$\hat{T}^a[n]\,\Psi = \sum_i \Pi$$

$$\hat{T}[\gamma]\,\Psi(T[A,\gamma]) \equiv T[A,\gamma]\,\Psi[T[\alpha,\gamma]]$$

$$\hat{T}^a[\eta]\,\Psi = \sum_i \prod_\alpha \int ds\, \dot{\alpha}_i^a(s)\, \delta^3(\alpha_i(s), p)\, \left[T[\eta p \alpha_i]\right]$$

$$\hat{T}^a[n]\,\Psi = \sum_{i}$$

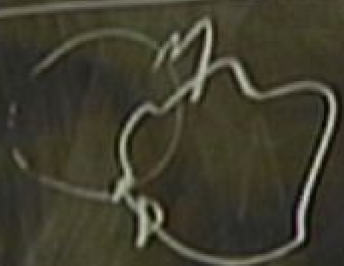
$$\equiv T[A,\gamma]\,\Phi[\,\eta_{\alpha_\varphi}]$$

$$\prod_{\alpha}\int ds\,\dot{\alpha}_i^a(s)\,\delta^3(\alpha_i(s),p)$$

$$\left[T[\eta\rho\alpha_i]-T[\eta'\rho\alpha_i]\right]$$

$$\hat{T}[\gamma]\,\Psi(T[A,\gamma]) \equiv T[A,\gamma]\,\Psi[T[\alpha,\gamma]]$$

$$\hat{T}^a[m]\,\Psi = G\gamma \sum_i \prod_\alpha \int ds\, \dot{\alpha}_i^a(s)\, \delta^3(\alpha_i(s), p)\, \Big[T[m p \alpha_i] - T[m' p \alpha_i]\Big]$$



$$\hat{T}[\gamma]\,\Psi(T[\alpha]) \equiv T[A$$

$$\hat{T}^{a}[n]\,\Psi = \frac{i\hbar}{G}\gamma \sum_{i} \prod_{\alpha} \int ds$$

$$\Big[\, T[n$$

$= \sum_i a_i \prod_{\{\alpha\}} T[\alpha]$ $\quad \sum_i |\alpha|^2 < \infty$

SU(2) lattice gauge theory on L

$C = \{U_I\}$ SU(2) element to each segment

segments s_I

$\prod_{\{\alpha\}} T[\alpha] \in C$

$\|\psi\|^2 = |\psi^* \psi|$

$= \sum_i |f(L_i)|$

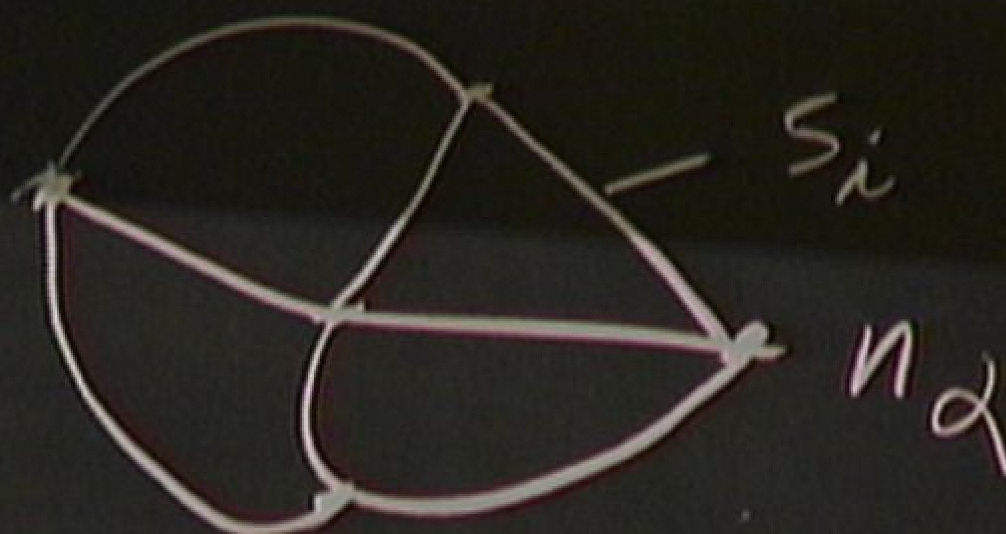
theory on L

$U(2)$ element to each segment

$$\langle \Psi | \chi \rangle = \int \Psi^* \chi[A]$$

Orthonormal basis

C_L^0

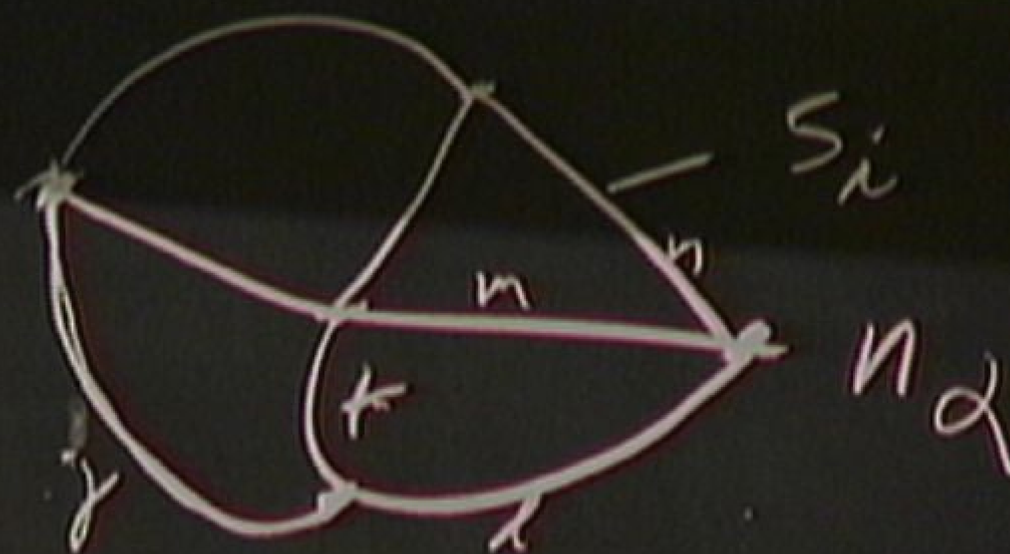


orthonormal basis

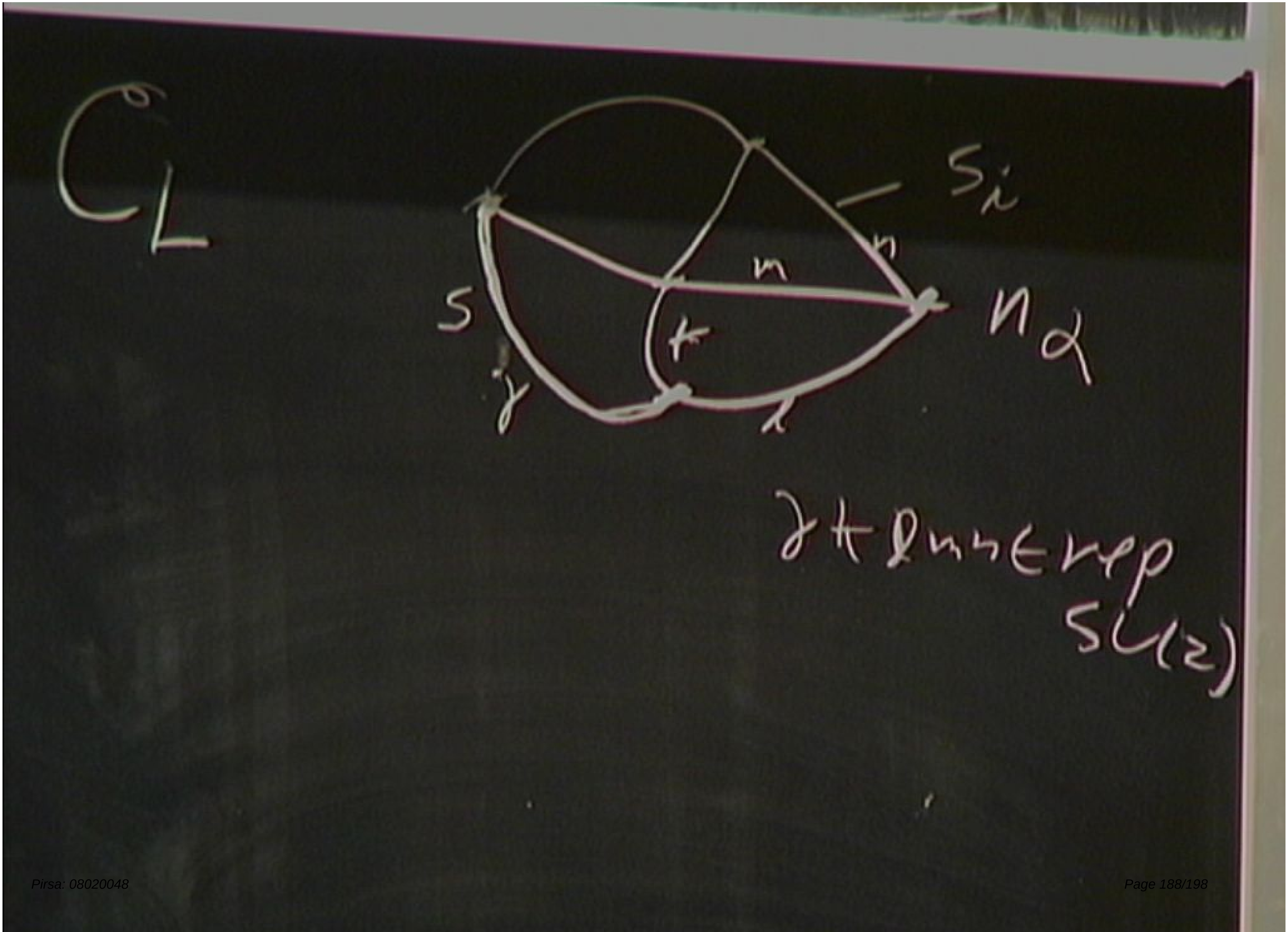
$$\mathcal{C}_L$$

$$\mathcal{C} = \{ u_i \}$$

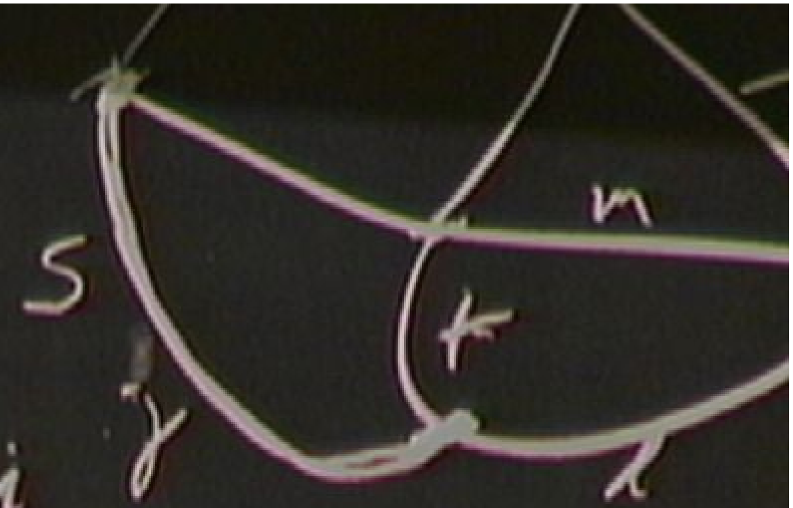
C_L



∂ + l inner rep
$SU(2)$


C_L
s_i
n
n
s
n_α
t
γ
l
γ t l m n ∈ rep
SU(2)

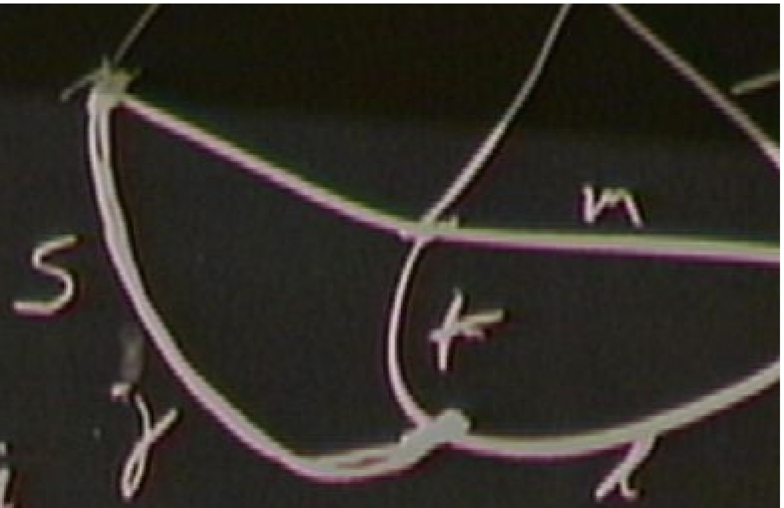
CL



$$U_s^j = P e^{\int ds A^i T^i_j}$$

← jth rep of th

CL



$$U_s^j = P e^{\int ds A^i T^i_j}$$

j th rep

$$C = \{ u_i \}$$

$$u_s^{\gamma} = P e$$

$$T[u]^{\gamma \cdots} = \prod_s u_s^{\gamma_s}$$

$$U_s^{\gamma} = P_e \, S$$

$$T[U\triangle]^{\gamma\ldots} = \prod_s \left(U_s^{\gamma_s}\right)_{i_{\gamma_s}}^{\;j_{\gamma_s}}$$

$= P\, e^{\int ds\, A^i T^i_j}$

jth rep

$j\,k\,l$

$)^{i}_{j_5}$

$)_{i j_5}$

$$\int ds\, A^i\, T^i_j$$

j ← j th rep

$j+\ell$ inner rep SU

$$\mathcal{N}[r_j \otimes r_k \otimes r_\ell \otimes r_m] \to \mathrm{Inv}$$

$$= P e^{\int ds A^i T^i_j}$$

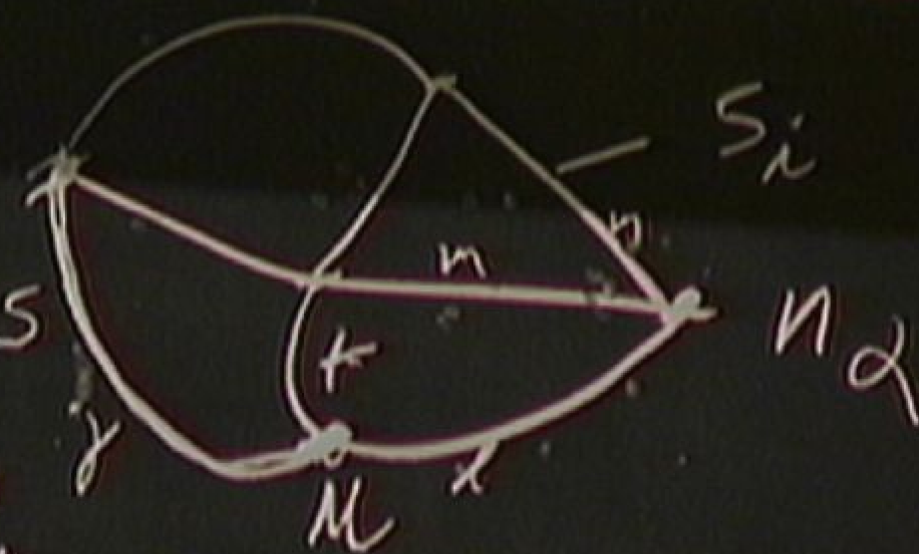
$j \leftarrow$ jth rep

jth line

$$\Big)^{j_{\gamma_s}}_{i_{\gamma_s}} \prod_{\text{nodes}} \left(\mathrm{Inv} \right)$$

$$\mu\,[r_j \otimes r_k \otimes r_l \otimes r_m] \longrightarrow \mathrm{Inv}$$

$\overset{\circ}{C}_L$



$= P e^{\int ds A^i T^i_j}$ ← jth rep

$2j+1$ dimen rep $SU(2)$

$)^{*j_s}_{i j_s} \prod_{\text{nodes}} (\mathrm{Inv})^{\dots j \dots}_{\dots}$

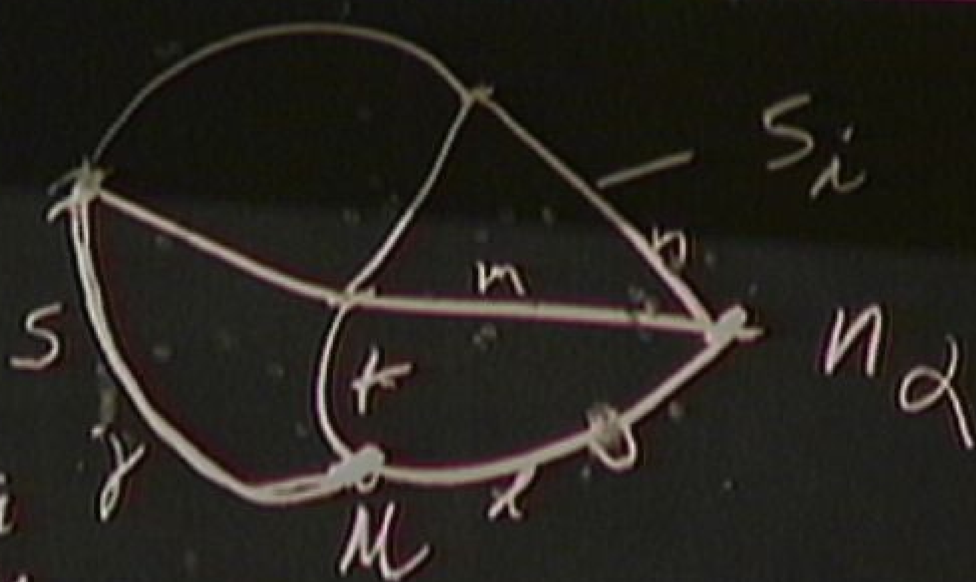
$$U_s^{\gamma} = P e$$

$$T[U]^{\gamma \ldots}_{\Delta u} = \prod_s \left(U_s{}^{\gamma_s}\right)^{j_{\gamma_s}}_{i_{\gamma_s}} \prod_{\text{nodes}} ($$

spin-networks basis

$\delta A^a_i(x)$ $\qquad (T[\alpha], T[\beta])_g = 0$

C_L



$= P e^{\int ds A^i T^i_j}$ ← jth rep, $2j+1$ dim rep $SU(2)$

$)^{i}_{j}\ \gamma_s \prod (\mathrm{Inv})^{\dots j \dots}$ k