Title: Modified Gravity and Its Consequences for Astronomy, Astrophysics and Cosmology

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Abstract: The consequences of a modified gravity (MOG) are explored.
I demonstrate how the solutions of the field equations obtained from the action principle of the MOG lead to a theory without any free, adjustable parameters or ad-hoc empirical formulae. The theory successfully explains solar system observations, the dispersion velocities of globular clusters, the rotation curves of galaxies, the mass profiles of X-ray clusters, the dispersion velocities of satellite galaxies, the Bullet Cluster and cosmological observations without exotic dark matter. The peculiar features of the recent data obtained for the merging cluster Abell 520 are discussed. MOG predicts agreement with data from the scale of the solar system to cosmological scales without dark matter. With no undetermined free parameters, the theory can be used to make firm predictions that may be verifiable in the foreseeable future.

Modified Gravity and its Consequences for Astronomy, Astrophysics and Cosmology

Perimeter Institute
and
University of Waterloo

Talk given at Perimeter Institute, 19 February, 2008

Contents

1. Introduction

- The preferred standard model of cosmology today is the LambdaCDM model, which fits well the available cosmological data, but at a substantial cost: according to this model 96% of the matter and energy of the universe is either invisible or undetectable, or possibly both.

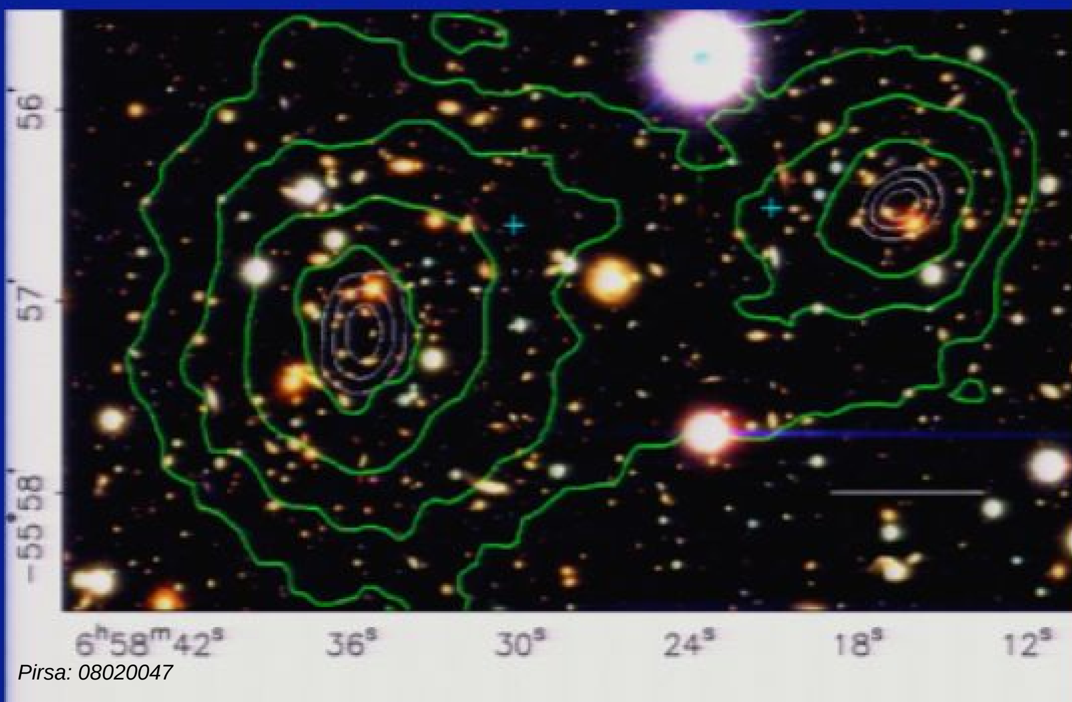
- This fact provides a strong incentive to seek alternative explanations that can account for cosmological observations, without resorting to dark matter or Einstein's cosmological constant.

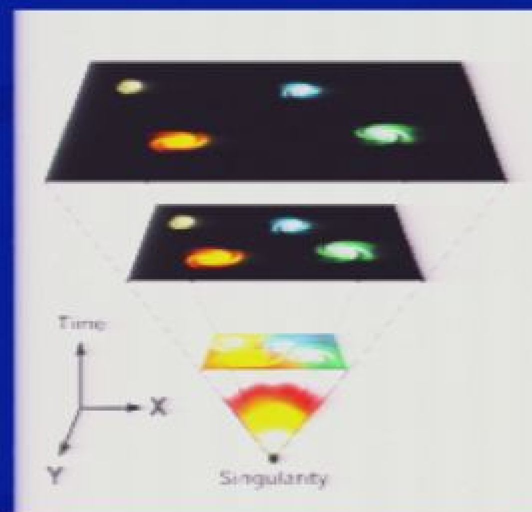
Contents

• The dark matter and dark energy are the most puzzling parts of the standard cosmology.

Dark matter is cold and collisionless, ~ 25 %

Dark energy ~ 70 % is smooth and appears about 9 billion years after the Big Bang (supernovae measurements)





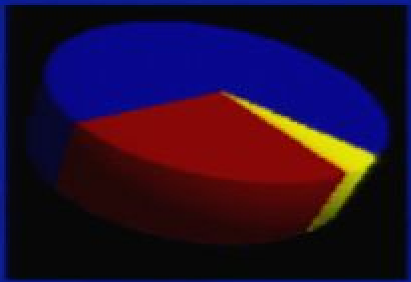
• Dark matter and dark energy are inferred from the motions of visible matter in a gravitational field. It is possible that dark matter doesn't exist and that Einstein's General Relativity (GR) has to be modified. Is this possible? Yes.

• The Equivalence Principle implies a metric and GR is determined uniquely in a 4-dimensional spacetime. To modify GR we either change the Einstein-Hilbert action, pseudo-Riemannian geometry or add new fields.

• The new fields may be independently dynamical – "dark matter" - or they may be sourced by ordinary matter – "modified gravity" (MOG).

Ingredients of the standard cosmology:

- General Relativity
- Large-scale homogeneity and isotropy
- 5% ordinary matter (baryons and electrons)
- 25% dark matter
- 70% dark energy
- Uniform CMB radiation, T ~ 2.73 degrees
- Scale-free adiabatic fluctuations $\Delta T/T \sim 10^{-5}$



A MOG should explain the following:

- Solar system observations
- Galaxy rotation curves
- Mass profiles of X-ray clusters of galaxies
- The CMB data including the power spectrum data
- The formation of proto-galaxies in the early universe and the growth of galaxies
- Gravitational lensing data for galaxies and clusters of galaxies
- The supernovae data and the accelerating universe
- The Bullet Cluster 1E0-657-56 and the merging cluster Abell 520

• MOG has been used successfully to account for galaxy cluster masses (Brownstein & JM, 2006) the rotation curves of galaxies (Brownstein & JM, 2006), velocity dispersions of satellite galaxies (JM & Toth, 2007), and globular clusters (JM & Toth, 2007) without exotic dark matter. It also offers an explanation for the Bullet Cluster (Brownstein & JM, 2007) without resorting to dark matter. Remarkably MOG also meets the challenge posed by cosmological observations (JM & Toth, 2007). It produces an acoustical power spectrum, a matter power spectrum, and a luminosity distance relationship that are in good agreement with observations, and require no dark matter.

2. Modified gravity action

- Modified gravity (MOG) is a fully relativistic theory of gravitation. Its simplest version is Scalar-Tensor-Vector-Gravity (STVG) (JM 2006) involving scalar, tensor, and vector fields. The action is given by

$$S = \int (\mathcal{L}_G + \mathcal{L}_\phi + \mathcal{L}_S)\sqrt{-g}\, d^4x + S_M$$

$$S_G = \frac{1}{16\pi}\int \frac{1}{G}(R + 2\Lambda)\sqrt{-g}\, d^4x,$$

$$S_\phi = -\int \omega \left[\frac{1}{4}B^{\mu\nu}B_{\mu\nu} - \frac{1}{2}\mu^2\phi_\mu\phi^\mu + V_\phi(\phi)\right]\sqrt{-g}\, d^4x$$

$$S_S = -\int \frac{1}{G}\left[\frac{1}{2}g^{\mu\nu}\left(\frac{\nabla_\mu G\nabla_\nu G}{G^2} + \frac{\nabla_\mu\mu\nabla_\nu\mu}{\mu^2} - \nabla_\mu\omega\nabla_\nu\omega\right) - \frac{V_G(G)}{G^2} - \frac{V_\mu(\mu)}{\mu^2} - V_\omega(\omega)\right]\sqrt{-g}\, d^4x$$

S_M is the matter action, and $B_{\mu\nu} = \partial_{[\mu}\phi_{\nu]}$ while $V_\phi(\phi)$, $V_G(G)$, $V_\omega(\omega)$, and $V_\mu(\mu)$ denote the self-interaction potentials associated with the vector field and the three scalar fields G, μ, and ω. We assume that Einstein's cosmological constant $\Lambda = 0.$

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S_M}{\delta g^{\mu\nu}}. \qquad J^\nu = -\frac{1}{\sqrt{-g}}\frac{\delta S_M}{\delta \phi_\nu}.$$

- We assume that the variation of the matter action w.r.t. to the scalar fields vanishes:

$$\frac{\delta S_M}{\delta X} = 0 \quad (X = G, \mu, \omega).$$

• A Dirac-Hamiltonian constraint analysis can be used to show that STVG is physically consistent. It is free of ghosts and instabilities.

• All the formulas and predictions of MOG follow from the action principle. There are no *ad hoc* formulas and no free adjustable parameters.

3. Field equations

• If we hold the scalar fields constant $\partial_\mu X = 0 \quad (X = G, \mu, \omega)$ we get the field equations

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} - 8\pi G\omega\left(B_{\mu\kappa}B_\nu{}^\kappa - \frac{1}{4}g_{\mu\nu}B_{\kappa\lambda}B^{\kappa\lambda}\right)$$

$$+ 8\pi G\omega\mu^2\phi_\mu\phi_\nu = 8\pi GT_{\mu\nu}.$$

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$$\phi_0'' + \frac{2}{r}\phi_0' - \mu^2\phi_0 = 0, \qquad \phi_0(r) = -\beta\frac{e^{-\mu r}}{r},$$

- Variation with respect to the scalar fields with $V_X(X) = 0$, gives

$$\begin{aligned}\nabla^\nu\nabla_\nu G - \frac{3}{2G}\nabla^\nu G\nabla_\nu G \\ +\frac{G}{2}\left(\nabla^\nu\omega\nabla_\nu\omega - \frac{\nabla^\nu\mu\nabla_\nu\mu}{\mu^2}\right) &= 0, \\ \nabla^\nu\nabla_\nu\mu - \frac{1}{\mu}\nabla^\nu\mu\nabla_\nu\mu - \frac{1}{G}\nabla^\nu G\nabla_\nu\mu \\ -G\mu^3\omega\phi^\nu\phi_\nu &= 0,\end{aligned}$$

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4. Modified acceleration law

- The equation of motion of a particle is given by

$$m\left(\frac{d^2x^\mu}{d\tau^2}+\Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau}\right)=\kappa m\omega B^\mu{}_\nu\frac{dx^\nu}{d\tau}.$$

- The radial acceleration is

$$\boxed{\frac{d^2r}{dt^2}=-\frac{GM}{r^2}+\beta\kappa\omega(1+\mu r)\frac{e^{-\mu r}}{r^2}.}$$

- Setting $\beta\kappa\omega = \alpha GM/(1+\alpha),$ we get

$$\frac{d^2r}{dt^2}=-\frac{GM}{r^2}\left[1-\frac{\alpha}{1+\alpha}(1+\mu r)e^{-\mu r}\right]$$

$$\alpha_Y=-\frac{\alpha}{1+\alpha}=-\frac{\beta\kappa\omega}{GM},$$

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- Obtaining values for α_Y and λ_Y requires solving for the scalar fields G, ω and μ .

- The modified Newtonian law of acceleration is given by

$$\alpha = \frac{M}{(\sqrt{M}+C_1')^2}\left(\frac{G_\infty}{G_N}-1\right)$$

$$\mu = \frac{C_2'}{\sqrt{M}},$$

$$a_{\rm MOG} = -\frac{G_N M}{r^2}\left[1+\frac{19M}{(\sqrt{M}+B)^2}\left(1-\exp\left(-\frac{Cr}{\sqrt{M}}\right)\left(1+\frac{Cr}{\sqrt{M}}\right)\right)\right]$$

$$B = 2.5\times 10^4\, M_\odot^{1/2}, \quad C = 6.25\times 10^3\, M_\odot^{1/2}\,{\rm kpc}^{-1}$$

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5. Cosmology

- To compute the scalar field G we set the curvature k=0 and keep $\Lambda = 0$ and solve the field equation (JM & VToth 2007):

$$\ddot{G} + \frac{3\eta}{t}\dot{G} + \frac{6\eta^2 - 3\eta}{8\pi t^2}G - \frac{3}{2}\frac{\dot{G}^2}{G} + \frac{G}{2}\left[\dot{\omega}^2 - \frac{\dot{\mu}^2}{\mu^2}\right] = 0.$$

- We get for large t the solution ($a = t^\eta$ and $H = \eta/t$):

$$G \simeq G_0 t^{3\eta - 1 - \sqrt{(3\eta-1)^2 + 3\eta(2\eta-1)/4\pi}}.$$

Between $1/2 < \eta < 1$ we have $G \simeq \text{const.}$ and we get for $\nabla_\nu \phi^\nu = 0$ that ϕ_0 is constant and

$$\ddot{\mu} + \frac{3\eta}{t}\dot{\mu} - \frac{\dot{\mu}^2}{\mu} - G\mu^3\omega\phi_0^2 = 0,$$

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$$G(r) = G_N\left[1+\frac{19M}{(\sqrt{M}+B)^2}\left(1-\exp\left(-\frac{Cr}{\sqrt{M}}\right)\left(1+\frac{Cr}{\sqrt{M}}\right)\right)\right]$$

- This acceleration law can be used to predict the dynamical data for any virialized, gravitationally bound system in the universe without dark matter and free adjustable parameters (JM & VToth 2007).

5. Cosmology

- To compute the scalar field G we set the curvature k=0 and keep $\Lambda = 0$ and solve the field equation (JM & VToth 2007):

$$\ddot{G} + \frac{3\eta}{t}\dot{G} + \frac{6\eta^2 - 3\eta}{8\pi t^2}G - \frac{3}{2}\frac{\dot{G}^2}{G} + \frac{G}{2}\left[\dot{\omega}^2 - \frac{\dot{\mu}^2}{\mu^2}\right] = 0.$$

- We get for large t the solution ($a = t^\eta$ and $H = \eta/t$):

$$G \simeq G_0 t^{3\eta - 1 - \sqrt{(3\eta-1)^2 + 3\eta(2\eta-1)/4\pi}}.$$

Between $1/2 < \eta < 1$ we have $G \simeq \text{const.}$ and we get for $\nabla_\nu \phi^\nu = 0$ that ϕ_0 is constant and

$$\ddot{\mu} + \frac{3\eta}{t}\dot{\mu} - \frac{\dot{\mu}^2}{\mu} - G\mu^3\omega\phi_0^2 = 0,$$

$$\ddot{\omega} + \frac{3\eta}{t}\dot{\omega} + \frac{1}{2}G\mu^2\phi_0^2 = 0.$$

- The modified Newtonian law of acceleration is given by

$$\alpha = \frac{M}{(\sqrt{M}+C_1')^2}\left(\frac{G_\infty}{G_N}-1\right) \qquad \mu = \frac{C_2'}{\sqrt{M}}.$$

$$a_{\rm MOG} = -\frac{G_N M}{r^2}\left[1+\frac{19M}{(\sqrt{M}+B)^2}\left(1-\exp\left(-\frac{Cr}{\sqrt{M}}\right)\left(1+\frac{Cr}{\sqrt{M}}\right)\right)\right]$$

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$$\ddot{\omega} + \frac{3\eta}{t}\dot{\omega} + \frac{1}{2}G\mu^2\phi_0^2 = 0.$$

- Using the FLRW line element

$$ds^2 = dt^2 - a^2(t)[(1 - kr^2)^{-1}dr^2 + r^2 d\Omega^2],$$

$$\phi_i = 0 \; (i = 1, 2, 3)$$

we get the Friedmann equations for $\dot{G} \simeq 0$:

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho,$$
$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$

$$\dot{\rho} + 3\frac{d\ln a}{dt}(\rho + p) = 0$$

- We determine from the field equations $\omega = K_2 t^2$ and $\mu = K_1/t$ where

$$K_1 = \sqrt{(3\eta - 1)/G\omega\phi_0^2}, \quad K_2 = G(2V_\phi(\phi) - \mu^2\phi_0^2)/(4 + 12\eta)$$

- We choose k=0 for a spatially flat universe.

- The massive vector field ϕ_μ produces the density and pressure:

$$\rho_\phi = -\omega\left[\frac{1}{2}\mu^2\phi_0^2 + V_\phi(\phi)\right]$$
$$p_\phi = -\omega\left[\frac{1}{2}\mu^2\phi_0^2 - V_\phi(\phi)\right]$$

$$V_\phi(\phi) < 0$$

- When $V_\phi(\phi)$ dominates, then we obtain the equation of state $w_\phi = -1$ corresponding to a vacuum dark energy:

$$\Omega_\phi = \frac{\rho_\phi}{\rho_{\rm crit}} \simeq 0.7 \qquad \rho_{\rm crit} = \frac{3H^2}{8\pi G} \qquad \Lambda = 0.$$

- MOG explains the supernovae data and the accelerating universe without dark matter within a unified model.

- The MOG acceleration law is associated with the potential:

$$\Phi = -\frac{G_\infty M}{r}\left[1 - \frac{\alpha}{1+\alpha}e^{-\mu r}\right] = \Phi_N + \Phi_Y$$

$$\Phi_N = -\frac{G_\infty M}{r} \qquad \Phi_Y = \frac{\alpha}{1+\alpha} G_\infty M \frac{e^{-\mu r}}{r}$$

- The Poisson equation is (for cosmology $1/\mu \simeq 14 \times 10^9$ light years):

$$\begin{aligned} \nabla^2 \Phi &= -4\pi G_\infty \rho(\mathbf{r}) + \mu^2 \Phi_Y(\mathbf{r}) \\ &= -4\pi G_\infty \rho(\mathbf{r}) + \alpha \mu^2 G_N \int \frac{e^{-\mu|\mathbf{r}-\tilde{\mathbf{r}}|}\rho(\tilde{\mathbf{r}})}{|\mathbf{r}-\tilde{\mathbf{r}}|}\, d^3\tilde{\mathbf{r}} \end{aligned}$$

$$G_{\text{eff}} \simeq 0.3 G_\infty \qquad G_\infty \simeq 20 G_N \qquad \alpha \simeq 19$$

• To estimate G we note that for baryonic matter $(\rho_M = \rho_b)$ and from BBN at z ~ 1100 (G -> G_N for r -> 0):

$$\Omega_M = \frac{\rho_M}{\rho_{\rm crit}} \simeq 0.04\Big|_{G=G_N}$$

where G_N is Newton's constant. If we use instead $G \simeq 7G_N$, we get

$$\Omega_M \simeq 0.3 \qquad \Omega_\phi = \frac{\rho_\phi}{\rho_{\rm crit}} \simeq 0.7$$

$$\Omega = \Omega_M + \Omega_\phi = 1 \qquad (k=0)$$

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$$\Omega_M \simeq 0.3 \qquad \Omega_\phi = \frac{\rho_\phi}{\rho_{\rm crit}} \simeq 0.7$$

$$\Omega = \Omega_M + \Omega_\phi = 1 \qquad (k=0)$$

- The equation determining the density profile δ is

$$\ddot{\delta} + 2H\dot{\delta} - \frac{c_s^2}{a^2}\nabla^2\delta - 4\pi G_N \left[1 + \alpha\left(1 + \frac{\mu a}{k}\right) e^{-\mu a/k}\right] \rho\delta = 0$$

- The transfer function T is given by (Eisenstein and Hu, 1998):

$$T(k) = \frac{\Omega_b}{\Omega_m} T_b(k) + \frac{\Omega_c}{\Omega_m} T_c(k)$$ $$P(k) = T^2(k) P_0(k)$$ $$P_0(k) \propto k^n$$ $$n \simeq 1$$

- The baryonic transfer function is

$$T_b(k) = \left[\frac{\tilde{T}_0(k,1,1)}{1 + (ks/5.2)^2} + \frac{\alpha_b e^{-(k/k_{\rm Silk})^{1.4}}}{1 + (\beta_b/ks)^3}\right] \frac{\sin k\tilde{s}}{k\tilde{s}}$$

- We use Mukhanov's semi-analytical, cmbslow formalism to calculate the correlation function $C(l)$, which gives the angular (acoustical) power spectrum (Mukhanov 2005).

- For a flat universe $(k=0)$ we have $\Omega = \Omega_M + \Omega_\phi = 1$, $\Omega_b \simeq 0.035$ and $\Omega_M \simeq 0.3$ because $G \simeq 7G_N$.

- We note that the baryon fraction Ω_b does not depend on the effective G, because Ω_b depends on the speed of sound, which depends on the baryonic matter density, regardless of gravitation.

- Silk damping is also reduced because of the increase in G: $k_{\rm Silk} \propto G^{3/4}$. .

6. MOG predictions

- MOG has no free adjustable parameters. Given the mass of a physical system such as a galaxy, we can fit the data. This holds for the solar system, globular clusters, dwarf galaxies, spiral galaxies, clusters of galaxies and the large-scale structure of the universe without exotic dark matter.

- All the predictions of MOG follow from the action principle (field theory).

- MOG tells us that Einstein and Newtonian gravity agree with solar system data to a high degree of accuracy, including all Earth based experiments.

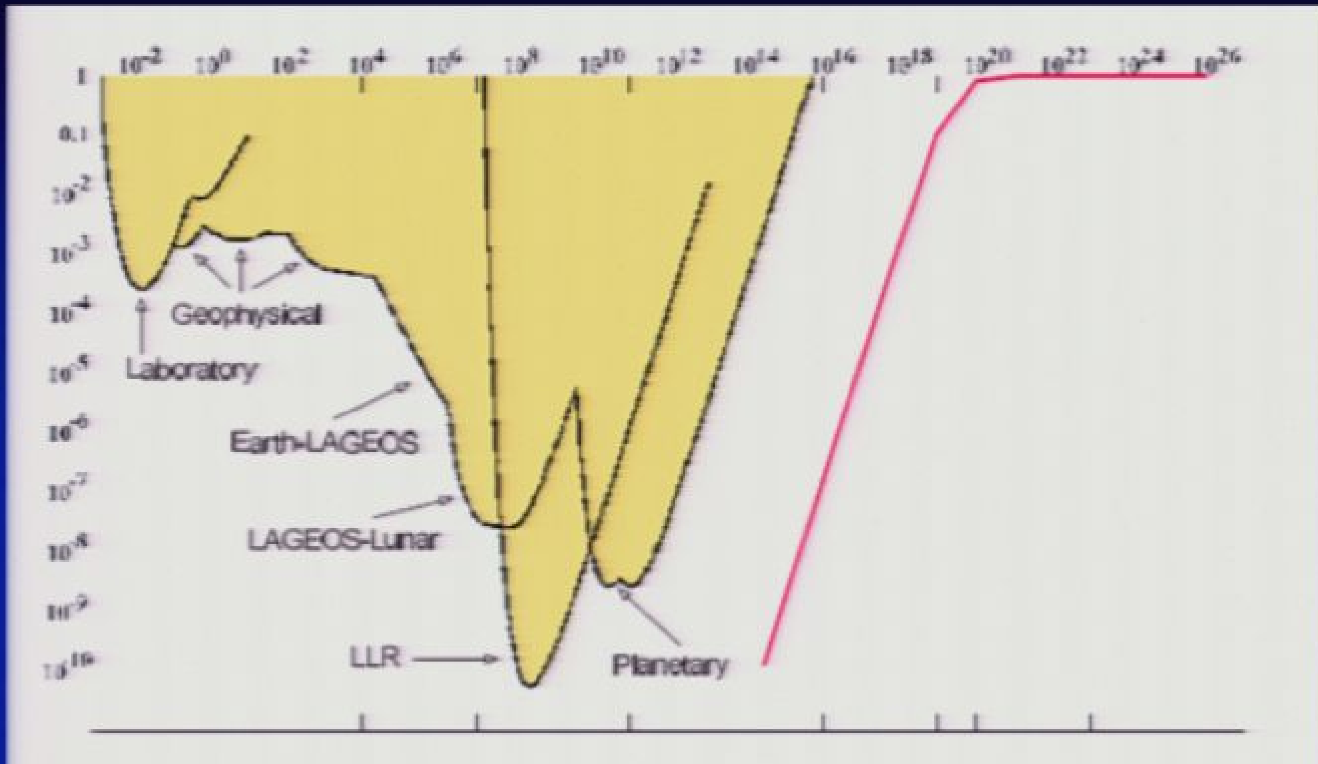


FIG. 5: Predictions of the Yukawa-parameters from the MOG field equations are not in violation of solar system and laboratory constraints. Predicted values of λ_Y (horizontal axis, in m) vs. $|\alpha_Y|$ are indicated by the solid red line. Plot adapted from [14].

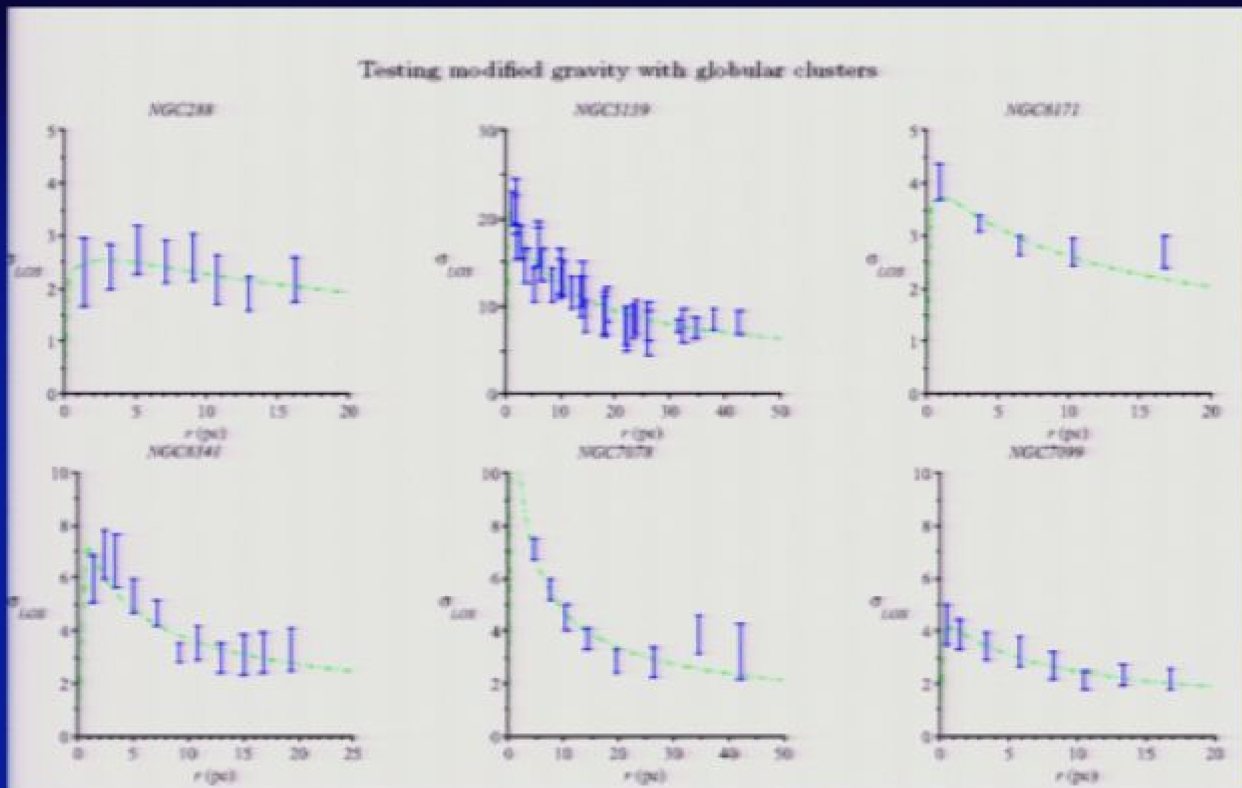


FIG. 1.— Fitting velocity dispersions obtained from the Jeans equation to globular cluster data, using the Hernquist model and MOG or Newtonian gravity.

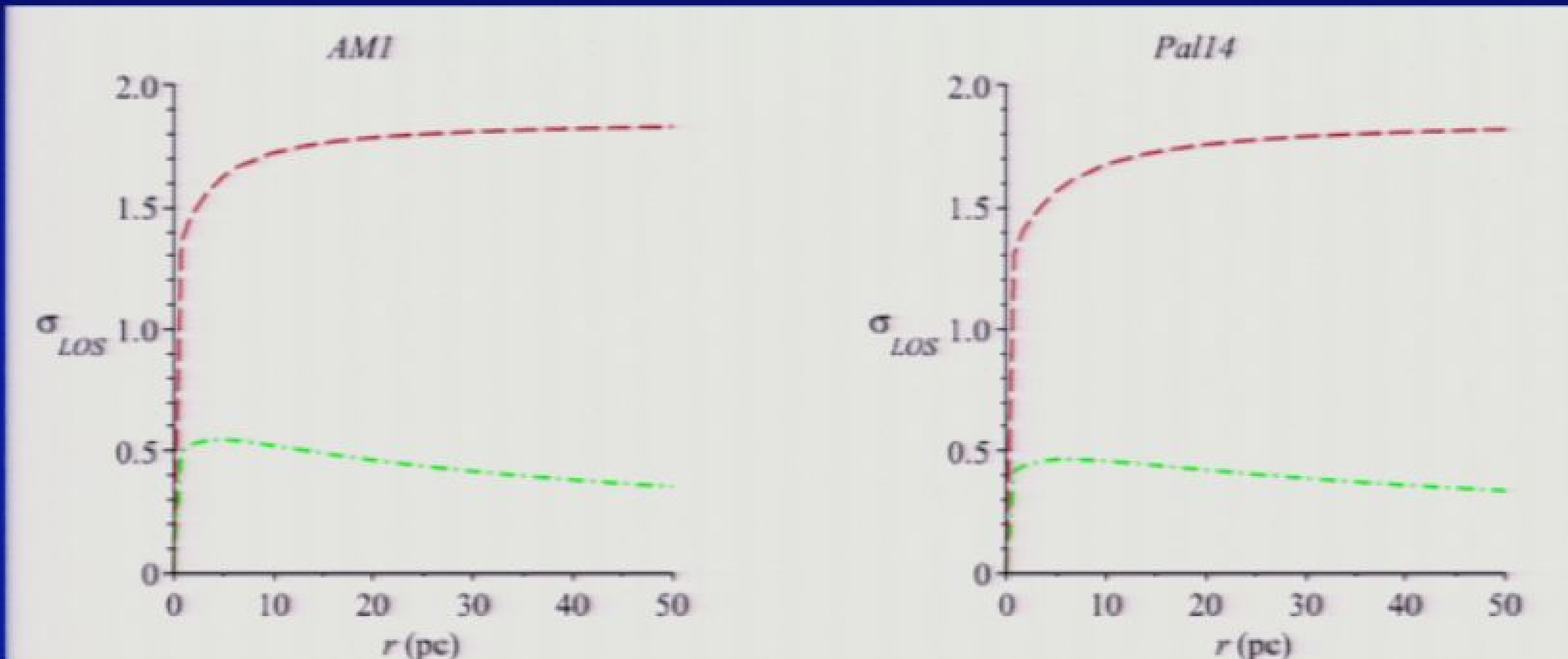
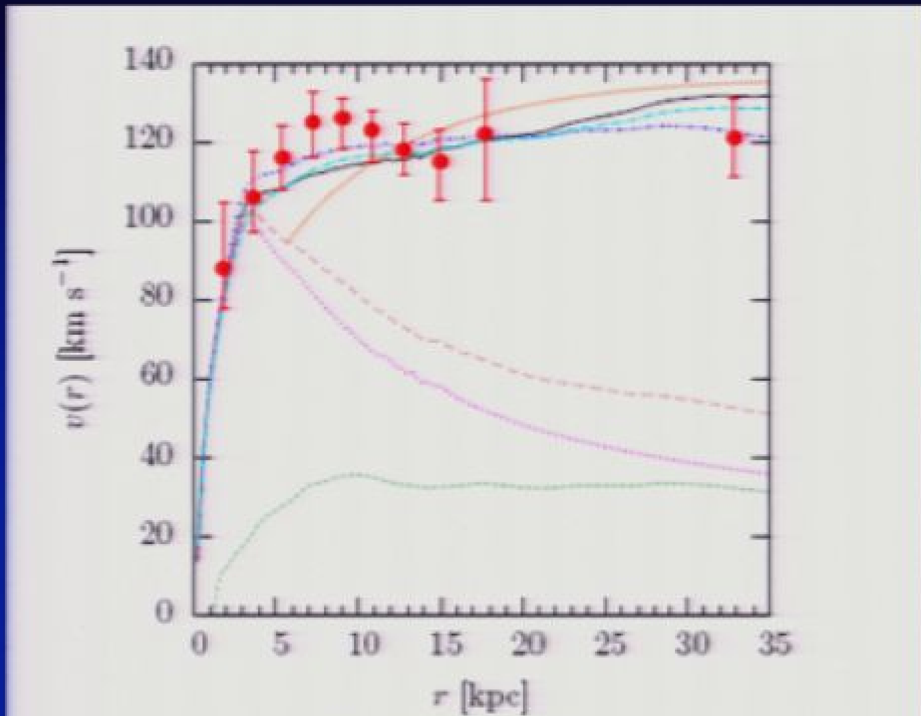
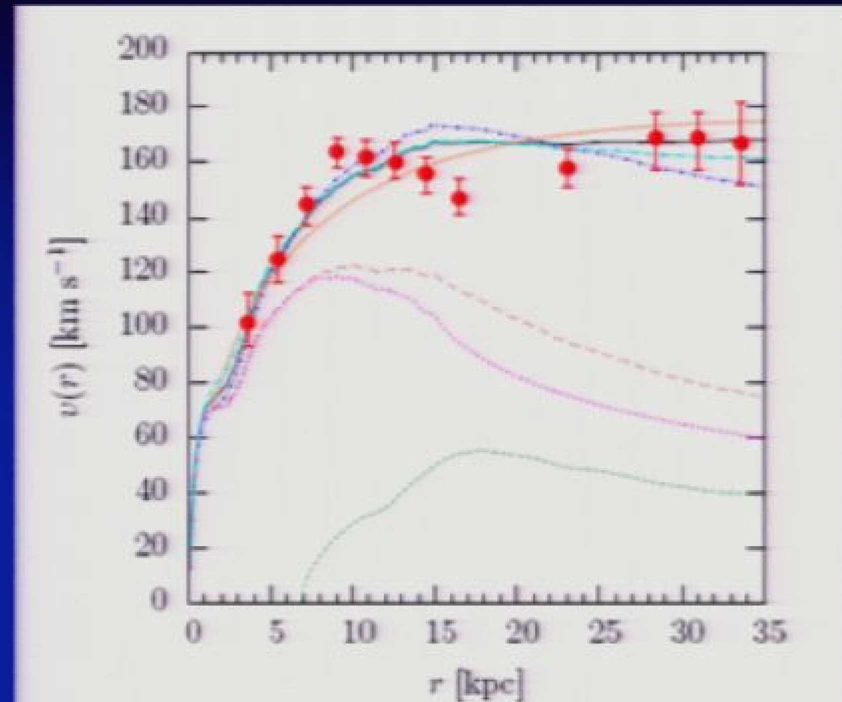


FIG. 2.— Predicted velocity dispersion curves for two distant GCs. Dash-dot line (green) is the prediction obtained using MOG or Newtonian gravity; dashed (brown) curve is the MOND prediction. In both cases, we used $M/L = 2$ and we equated the parameter r_0 of the Hernquist model with the half-light radius.

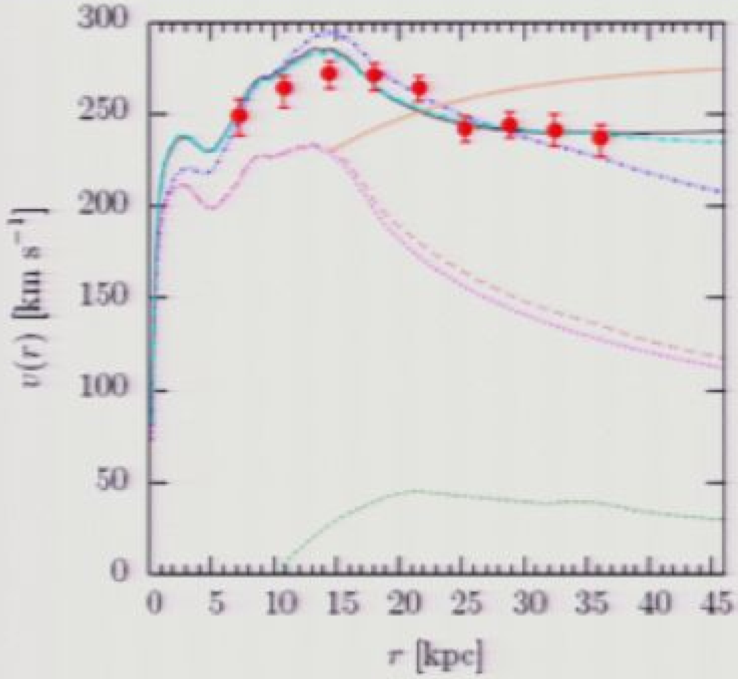


(b) NGC 3769

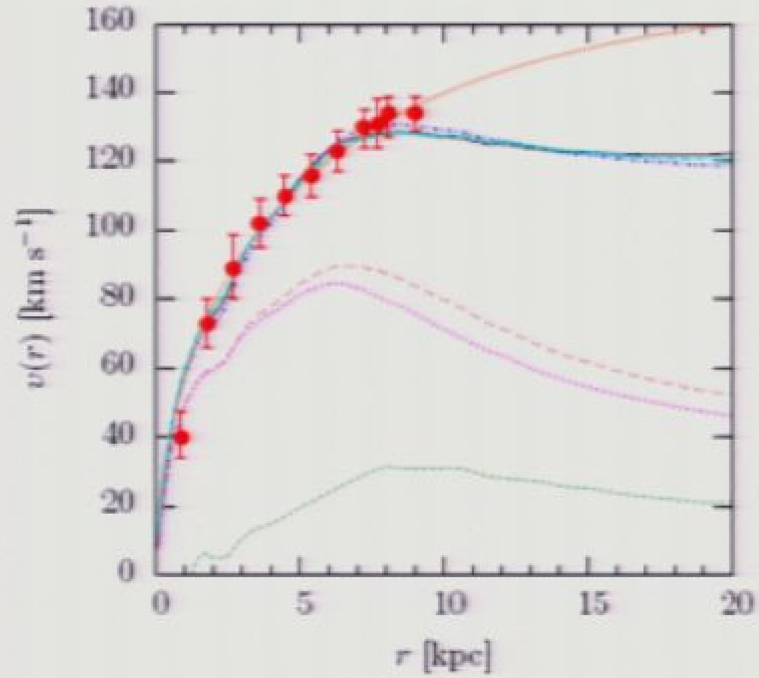


(a) NGC 3726

v(r)	red filled circles with error bars	STVG	black solid line
HI gas	green dashed line	MSTG	blue short dash-dotted line
Stellar disk	magenta dotted line	MOND	cyan long dash-dotted line
Newton (visible)	brown dot-dotted line	best-fit NFW	orange fine-dotted line

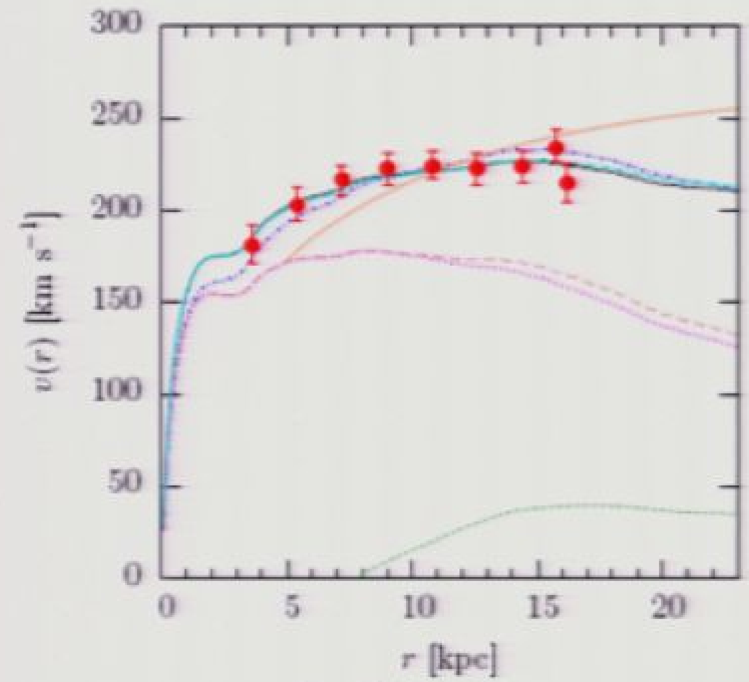


(h) NGC 3992

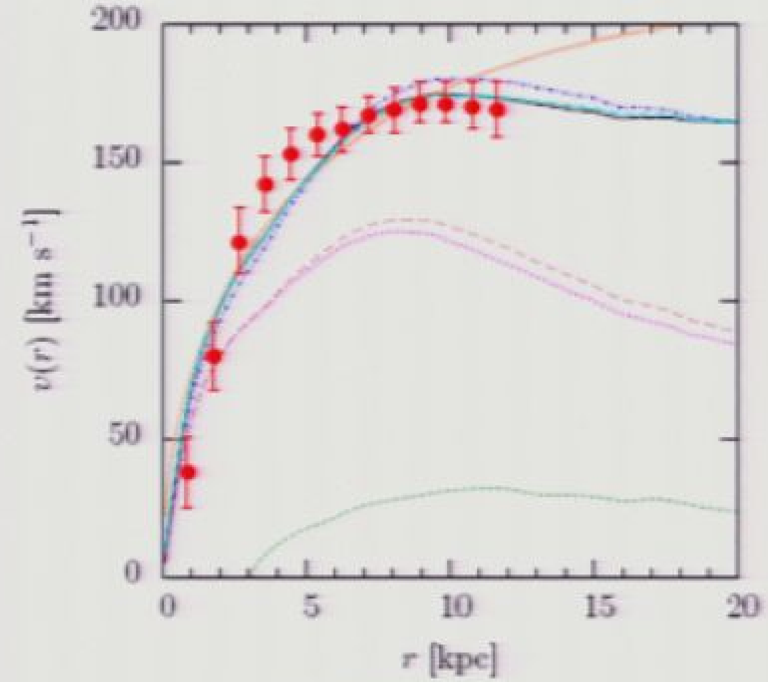


(g) NGC 3972

v(r)	red filled circles with error bars	STVG	black solid line
HI gas	green dashed line	MSTG	blue short dash-dotted line
Stellar disk	magenta dotted line	MOND	cyan long dash-dotted line
Newton (visible)	brown dot-dotted line	best-fit NFW	orange fine-dotted line

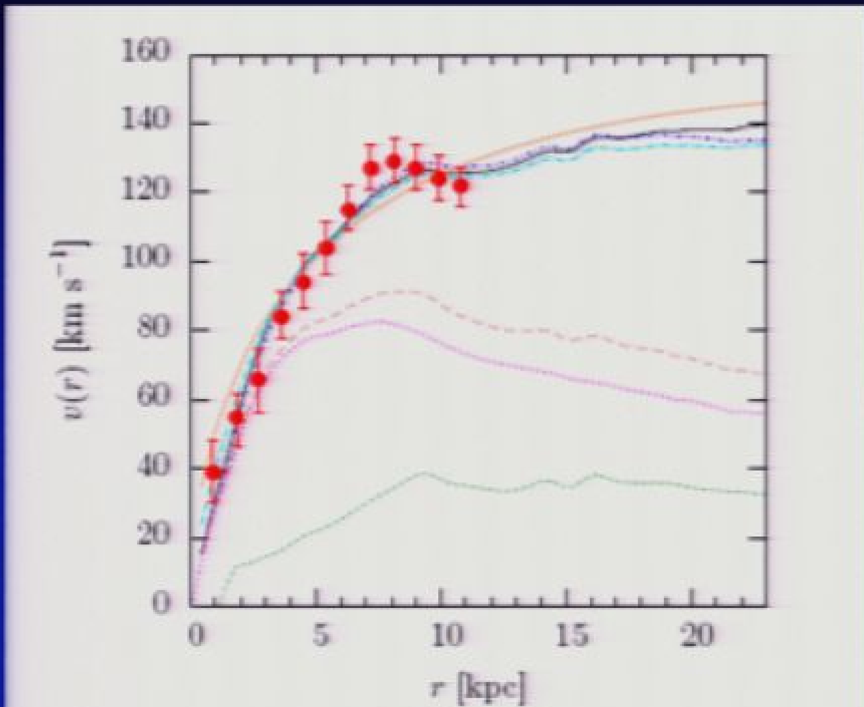


(f) NGC 3953

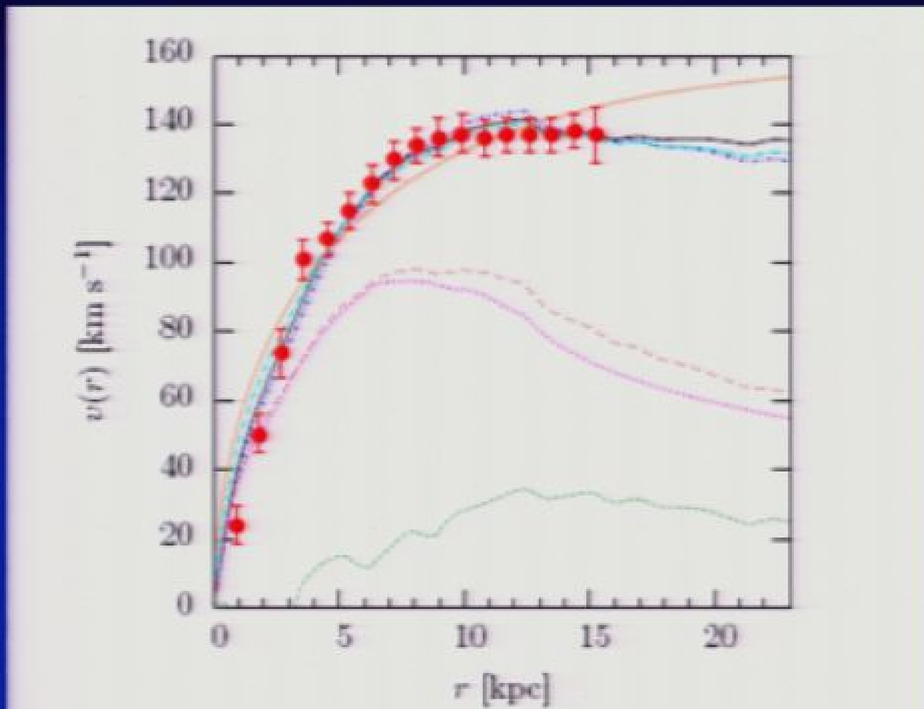


(c) NGC 3877

v(r)	red filled circles with error bars	STVG	black solid line
HI gas	green dashed line	MSTG	blue short dash-dotted line
Stellar disk	magenta dotted line	MOND	cyan long dash-dotted line
Newton (visible)	brown dot-dotted line	best-fit NFW	orange fine-dotted line



(b) NGC 4010



(a) NGC 3917

v(r)	red filled circles with error bars	STVG	black solid line
HI gas	green dashed line	MSTG	blue short dash-dotted line
Stellar disk	magenta dotted line	MOND	cyan long dash-dotted line
Newton (visible)	brown dot-dotted line	best-fit NFW	orange fine-dotted line

- Kepler's laws of orbital motion yielded a relationship between circular velocity at radius r from a mass M in the form:

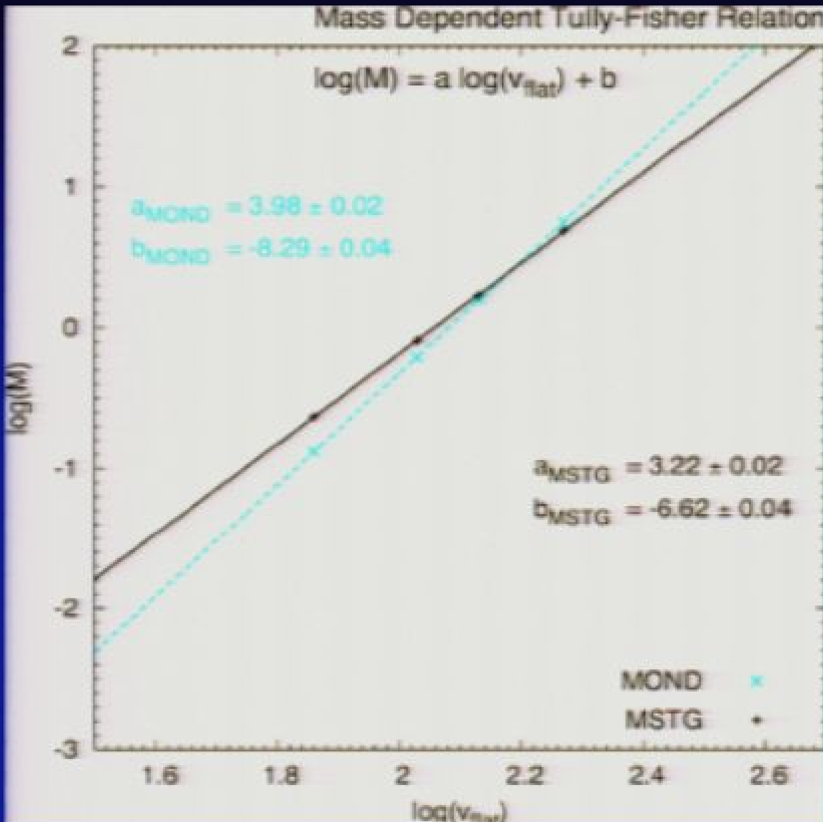
$$\frac{v_c^2}{r} = \frac{GM}{r^2}.$$

In contrast, Tully -Fisher determined that for galaxies, assuming that the brightness of a galaxy and its mass are correlated, the flat part of the rotation curve obeys the empirical relationship:

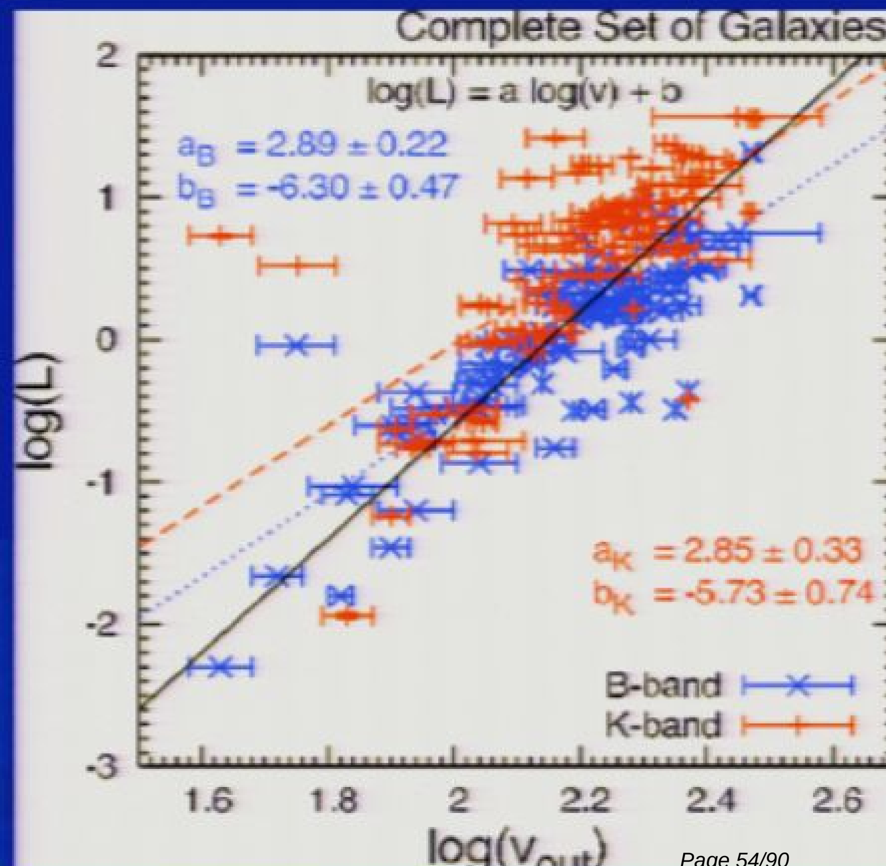
$$v_c^n \propto M,$$

- From the solutions of the MOG field equations, we predict that for a characteristic radius $r \sim \bar{r}$:

$$v_c^2 \propto \frac{M}{\sqrt{M}} = \sqrt{M}.$$



Mass dependent Tully-Fisher relation for the photometry of the 4 benchmark of galaxies. The **vertical axis** is the (base 10) logarithm of the total mass of the galaxy (in $10^{10} M_{\odot}$) resulting from the respective fits. The **horizontal axis** is the (base 10) logarithm of the flat rotational velocity (in km/s) as determined from the fits. The cyan crosses are the MOND results, and the black plus signs are the MSTG results. The Tully-Fisher relation is parametrized by $\log(M) = a\log(v)+b$, and the best-fit results using a nonlinear least-squares fitting routine including estimated errors are shown for MSTG and MOND. The dashed cyan line is the best-fit solution for the MOND results, and the solid black line is the best-fit solution for MSTG results.



Observed B-band K-band Tully-Fisher relation for the Complete Set of galaxies of J. R. Brownstein & J. W. Moffat (2006) Astrophys. J. **636** 721. The **vertical axis** is the (base 10) logarithm of the observed galaxy luminosity (in $10^{10} L_{\odot}$), and the **horizontal axis** is the (base 10) logarithm of the observed rotational velocity (in km/s) at the maximum observed radius. The blue crosses are the observed B-band luminosity data and the red plus signs are the observed K-band luminosity data. The Tully-Fisher relation is parametrized by $\log(L) = a\log(v) + b$. The blue dotted line is the best-fit B-band Tully-Fisher relation and the red dashed line is the best-fit K-band Tully-Fisher relation. The solid black line is the MOND prediction with $\langle M/L \rangle = 1$.

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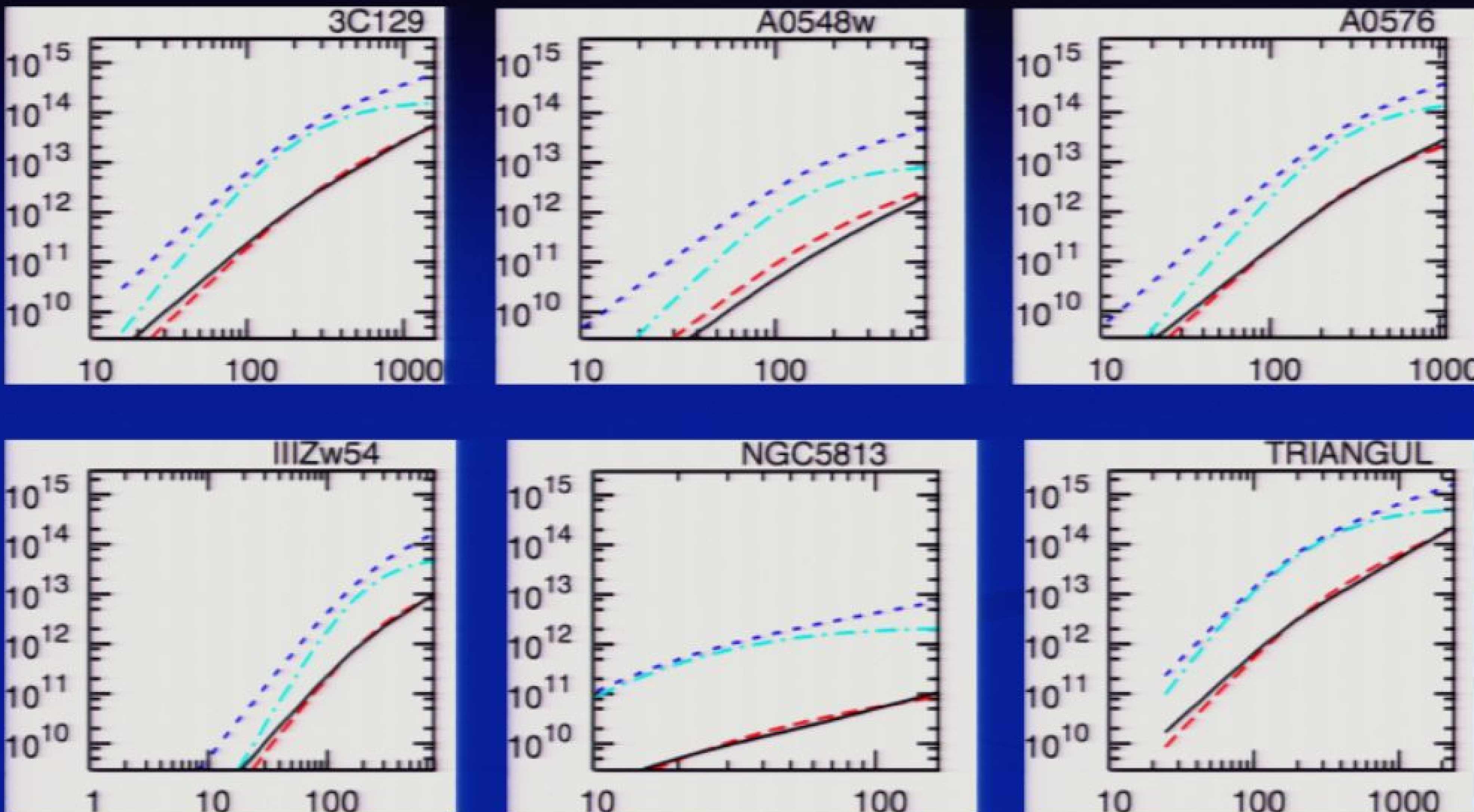


astro-ph/0507222 From 106 Cluster Fits: In all cases, the horizontal axis is the radius in kpc and the vertical axis is mass in units of $M_{\odot}$. The red long dashed curve is the ICM gas mass inferred from X-ray observations; the short dashed blue curve is the Newtonian dynamic mass; the dashed-dotted cyan curve is the MOND dynamic mass; and the solid black curve is the MSTG dynamic mass. The Newtonian, MOND and MSTG dynamic masses are calculated within the context of the β−model isothermal, isotropic sphere.

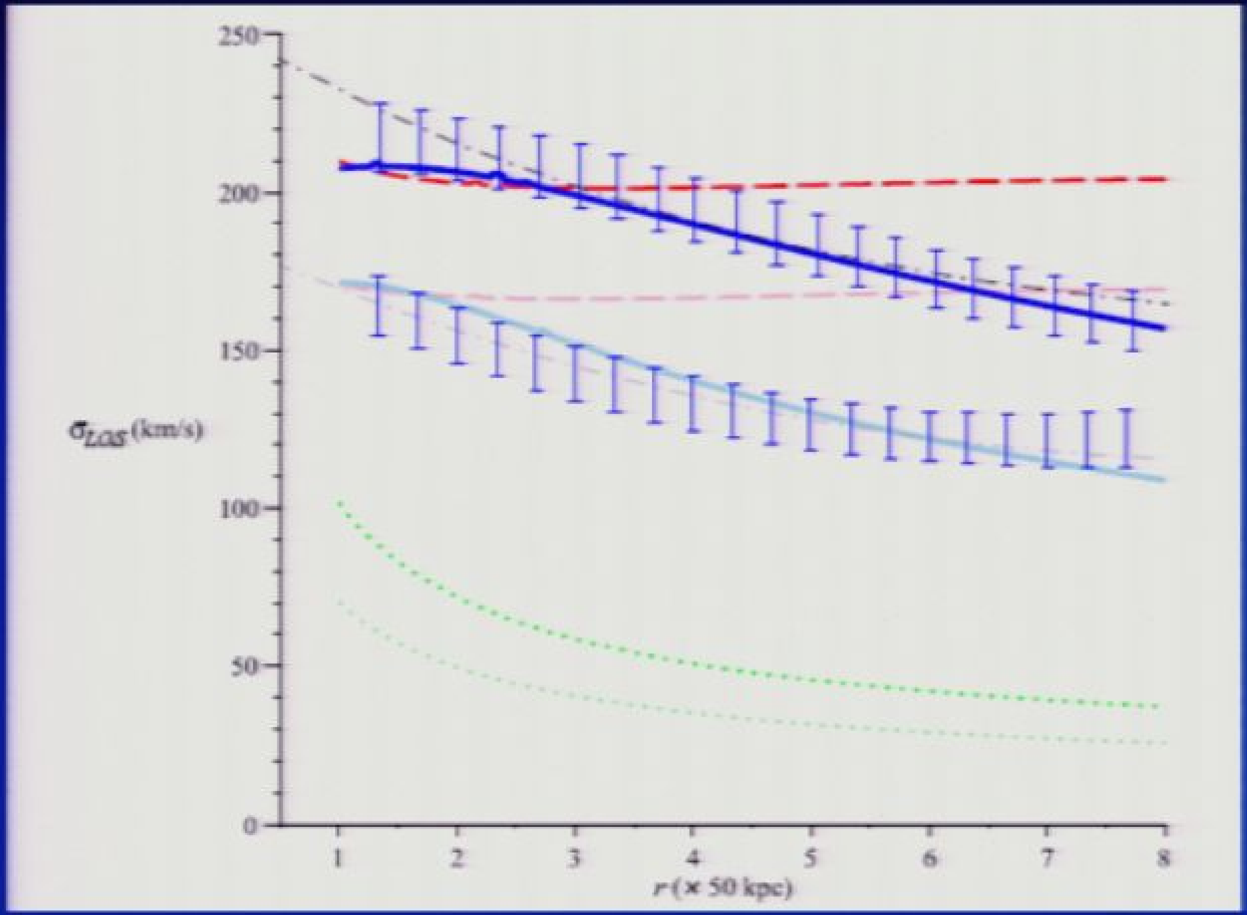


FIG. 1.— Predicted line-of-sight velocity dispersions for MOG (solid blue line), MOND (dotted green line) and Newtonian gravity (dotted green line) in comparison with the ΛCDM predictions (dash-dot black line) and data read from Fig. 2 in (Klypin & Prada 2007). Error bars represent 68% confidence levels as reported by Klypin & Prada (2007). The lower curves represent host galaxies in the $-20.5 > M_g > -21.1$ luminosity range. The upper curves represent host galaxies of $-21.1 > M_g > -21.6$.

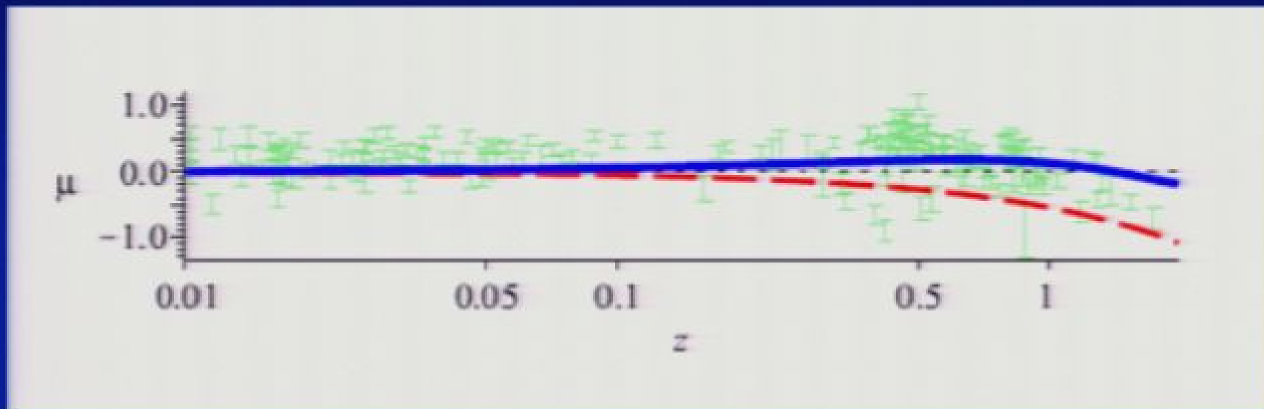
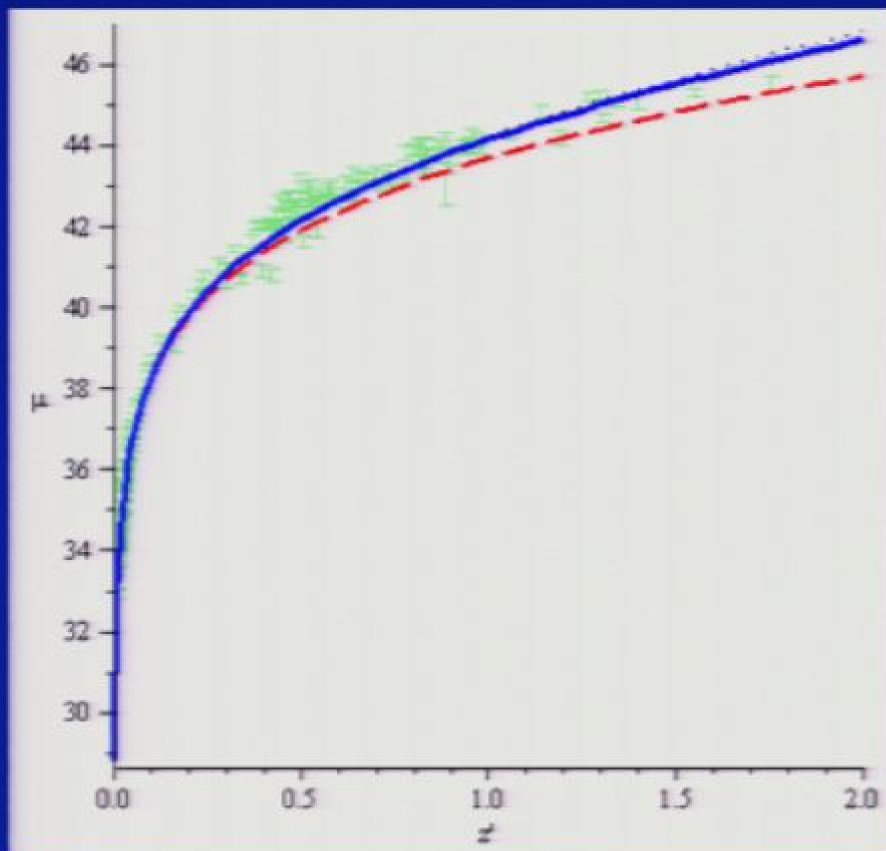


Figure 1. Type Ia supernova luminosity-redshift data (Riess et al. 2004) and the MOG/ΛCDM predictions. No astrophysical dimming was applied. The horizontal axis corresponds with the $q = 0$ empty universe. Dashed (red) line is a matter-dominated Einstein de-Sitter universe with $\Omega_M = 1$, $q = 0.5$. Thick (blue) line is the MOG/ΛCDM prediction.

- Magnitude versus redshift showing difference between q=0.05 and LambdaCDM prediction.



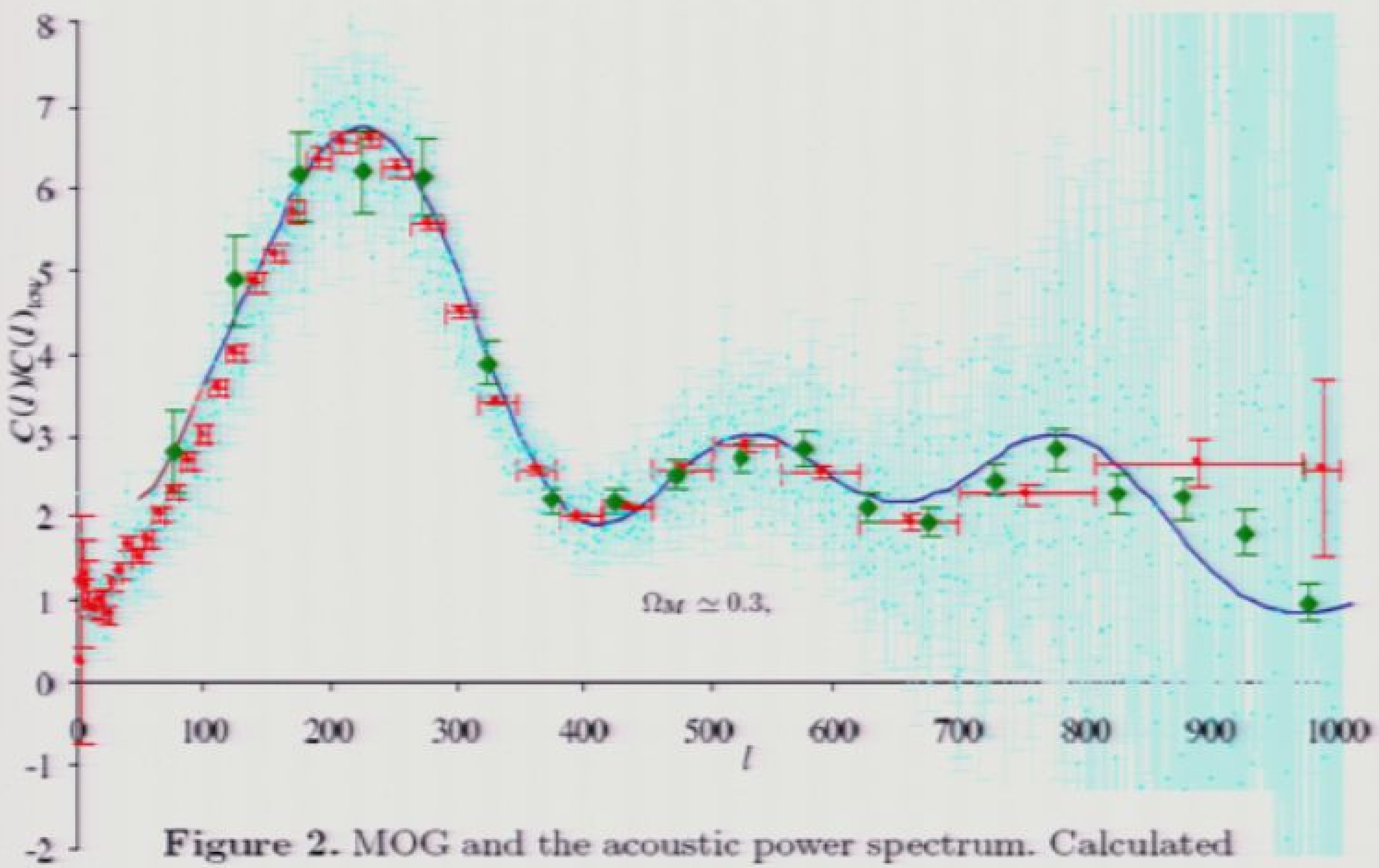


Figure 2. MOG and the acoustic power spectrum. Calculated using $\Omega_M = 0.3$, $\Omega_b = 0.035$, $H_0 = 71$ km/s/Mpc. Also shown are the raw WMAP 3-year data set (light blue), binned averages with horizontal and vertical error bars provided by the WMAP project (red), and data from the Boomerang experiment (green).

$$\Omega_M = \frac{\rho_M}{\rho_{\rm crit}} \simeq 0.04\Big|_{G=G_N}$$

$(\rho_M = \rho_b)$

$G \simeq 7G_N$

$\Omega_M \simeq 0.3$.

$\Omega = \Omega_M + \Omega_\phi = 1$

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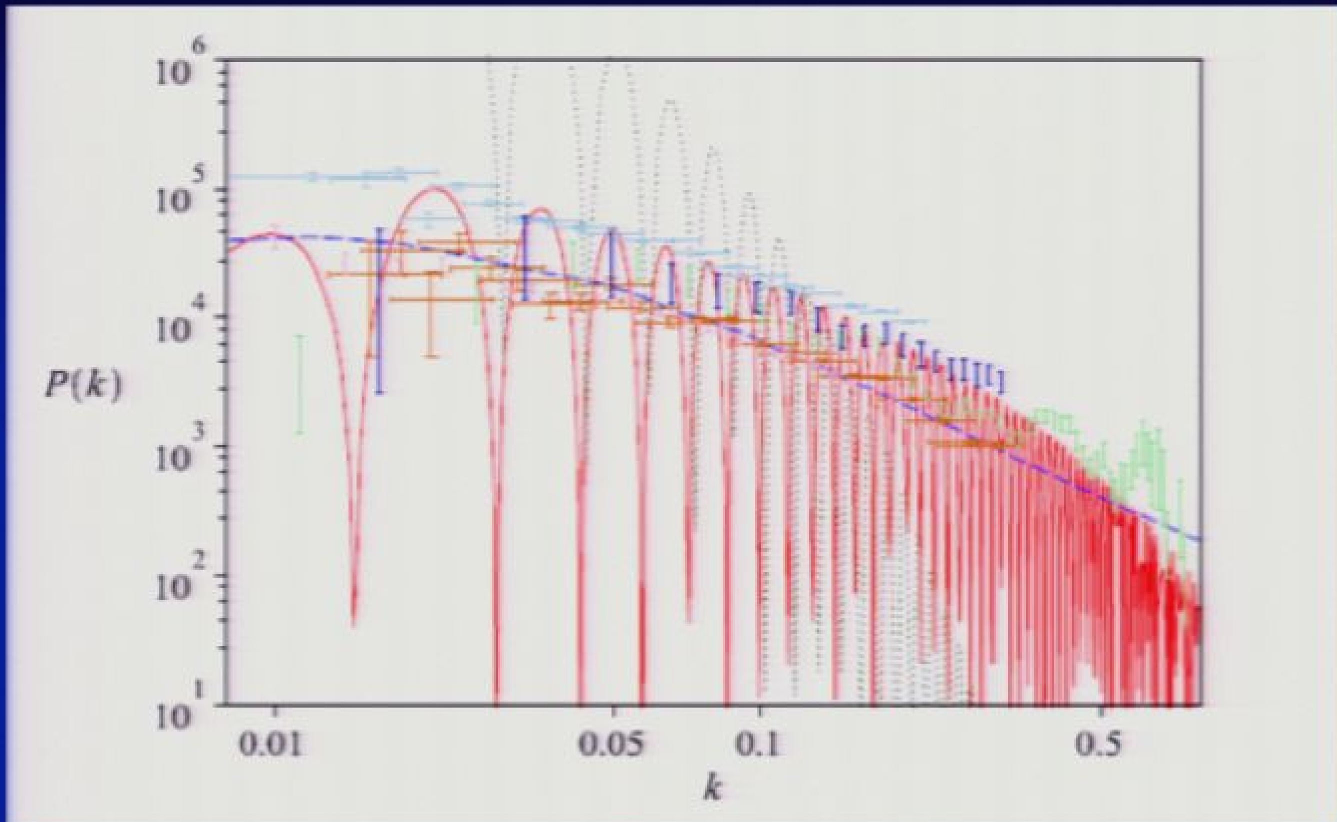
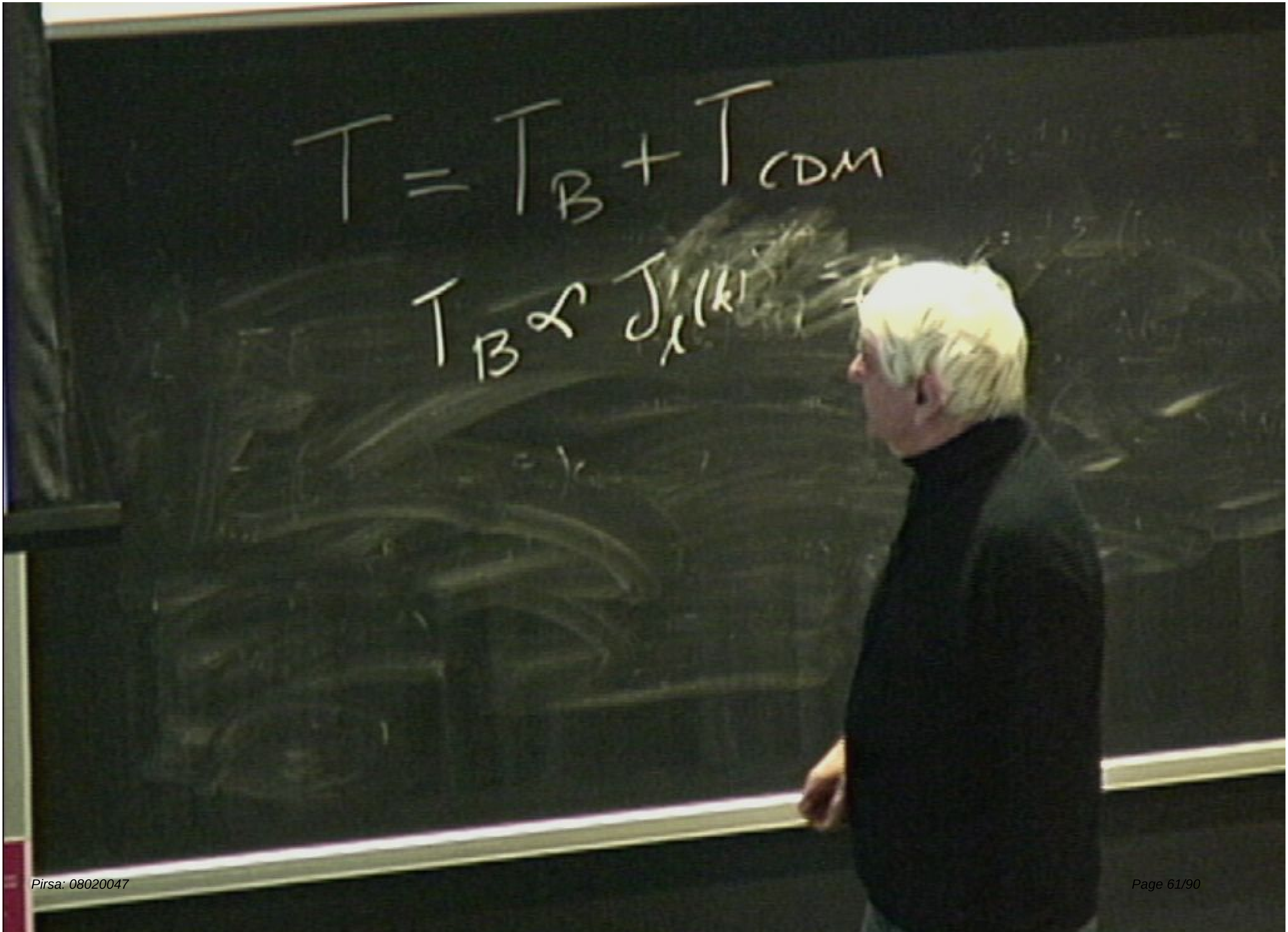
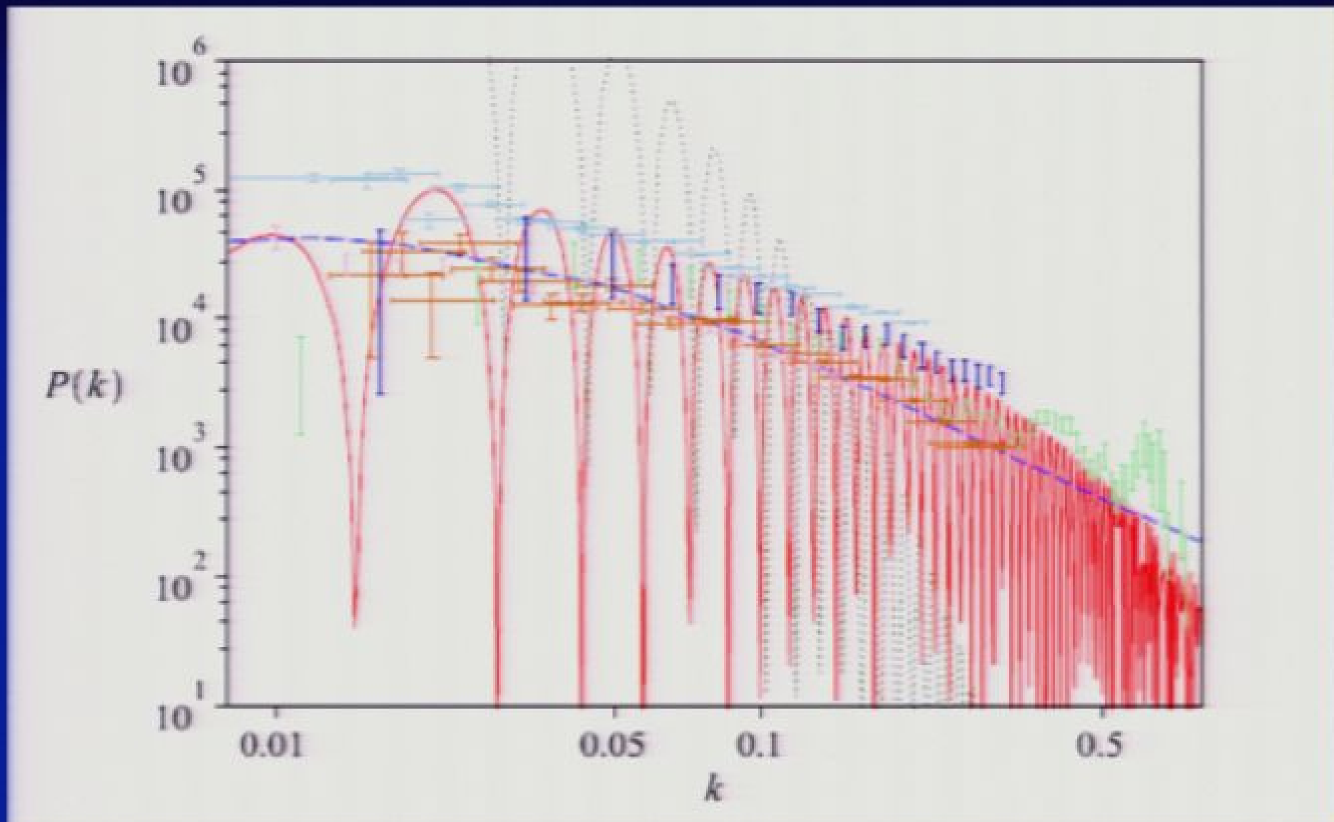


Figure 3. The matter power spectrum. Three models are compared against five data sets (see text): ΛCDM (dashed blue line, $\Omega_b = 0.035$, $\Omega_c = 0.245$, $\Omega_\Lambda = 0.72$, $H = 71$ km/s/Mpc), a baryon-only model (dotted green line, $\Omega_b = 0.035$, $H = 71$ km/s/Mpc), and MOG (solid red line, $\alpha = 19$, $\mu =$ inverse radius of the visible universe, $\Omega_b = 0.035$, $H = 71$ km/s/Mpc.) Data points are colored light blue (SDSS 2006), gold (SDSS 2004), pink (2dF), light green (UKST), and dark blue (CfA).


$T = T_B + T_{CDM}$
$T_B \propto j'_l(k)$



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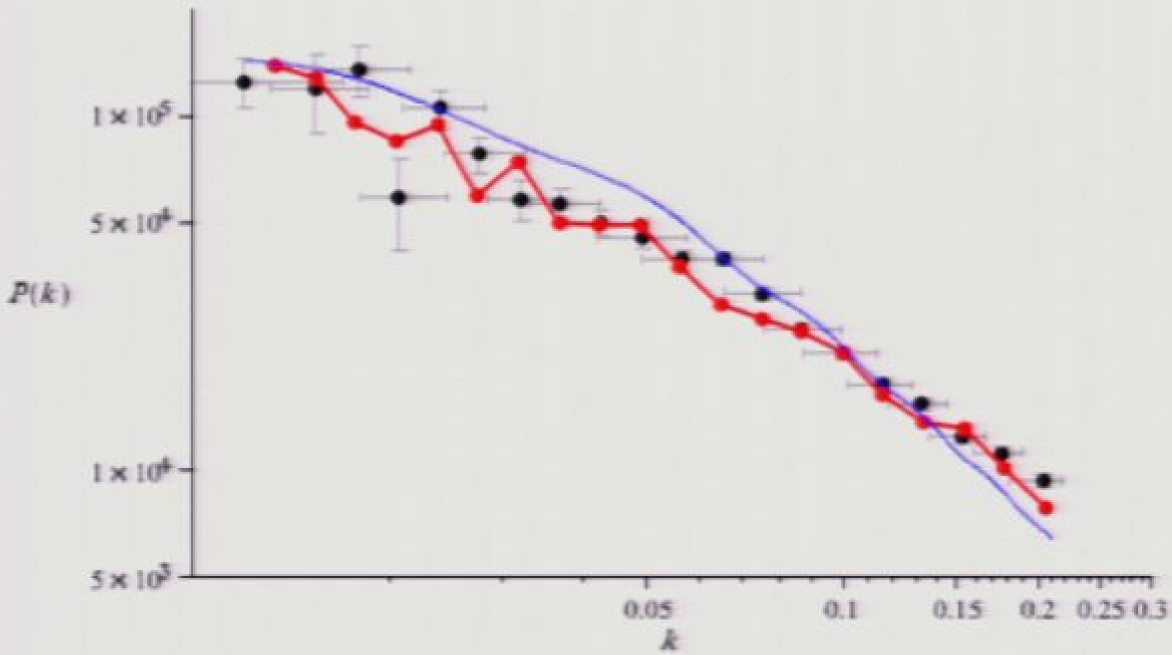


Figure 4. The effect of window functions on the power spectrum is demonstrated by applying the SDSS luminous red galaxy survey window functions to the MOG prediction. Baryonic oscillations are greatly dampened in the resulting curve (solid red line), yielding excellent agreement with the data after normalization. A normalized linear ΛCDM estimate is also shown (thin blue line) for comparison.

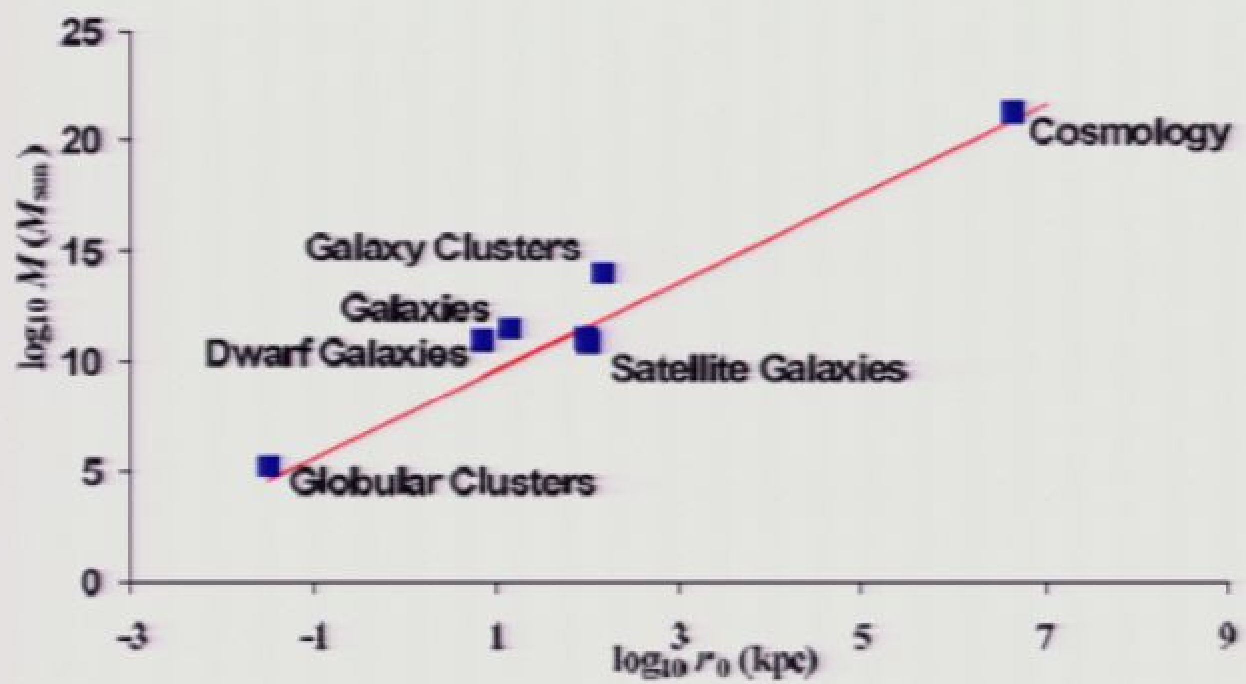


FIG. 4: The relationship $\mu^2 M$ = const. between mass M and the Yukawa-parameter $r_0 = \mu^{-1}$ across many orders of magnitude remains valid. The solid red line represents our theoretical prediction in accordance with (79). We are using cosmological data from [3]; galaxy cluster fits from [6]; galaxy and dwarf galaxy fits from [9, 10]; satellite galaxy fits from [7]; and globular cluster fits from [8]. Note that the dwarf galaxy, galaxy, and galaxy cluster outliers are removed when these objects are recalculated using the results presented in this paper.

7. The Bullet Cluster

- The merging clusters 1E0657-56 (z= 0.296) (discovered by Tucker et al. 1995) is claimed to prove empirically the existence of dark matter (Clowe et al. 2003-2006, Bradac et al. 2006). Due to the collision of two clusters, the dissipationless stellar component and the X-ray emitting plasma are spatially segregated. The claim is that the gravitationally lensing maps show that that the gravitational potential does not trace the plasma distribution – the dominant baryonic mass component – but rather approximately traces the distribution of galaxies.

- *It is necessary in MOG to explain the 8σ significant spatial offset of the center of the total mass from the center of the baryonic mass peaks (JWM, 2006, J. R. Brownstein and JWM, MNRAS, 2007.)*

- The main cluster of E10657-56 is close to being isothermal, whereas the smaller bullet cluster is not isothermal and out of equilibrium (shock wave).

- We treat the subcluster as a perturbation, and neglect it as a zeroth order approximation.

- Include the subcluster as a perturbation, and shift the origin of the varying gravitational coupling G(r) toward the subcluster (toward the center-of-mass of the system).

- Use the concentric cylinder mass M(R) as an approximation for M(R), and shift that toward the subcluster (toward the MOG center).

- Treat the subcluster as a perturbation, and utilize the isothermal model to approximate M(R) and shift that towards the subcluster (towards the center-of-mass of the system – where the origin of G(r) is located).

- The goal of the strong and weak lensing survey of Bradac et al. (2006), Clowe et al. (2006) was to obtain a convergence κ-map by measuring the distortion images of background galaxies by the deflection of light as it passes the bullet cluster (lens). The distortions in image ellipticity are only measurable statistically with a large number of sources. The reduced shear $g=\gamma/(1-\kappa)$ is measured where γ is the anisotropic stretching of the galaxy image, and the convergence κ is the shape-independent change in the size of the image. By recovering the κ-map from the measured rduced shear field, a measure of the local curvature is obtained. In GR and in MOG, the local curvature is related to the distribution of mass/energy.

$$\kappa(x,y) = \int \frac{4\pi G(r)}{c^2}\frac{D_l D_{ls}}{D_s}\rho(x,y,z)dz \equiv \frac{\Sigma(x,y)}{\Sigma_c(r)}$$

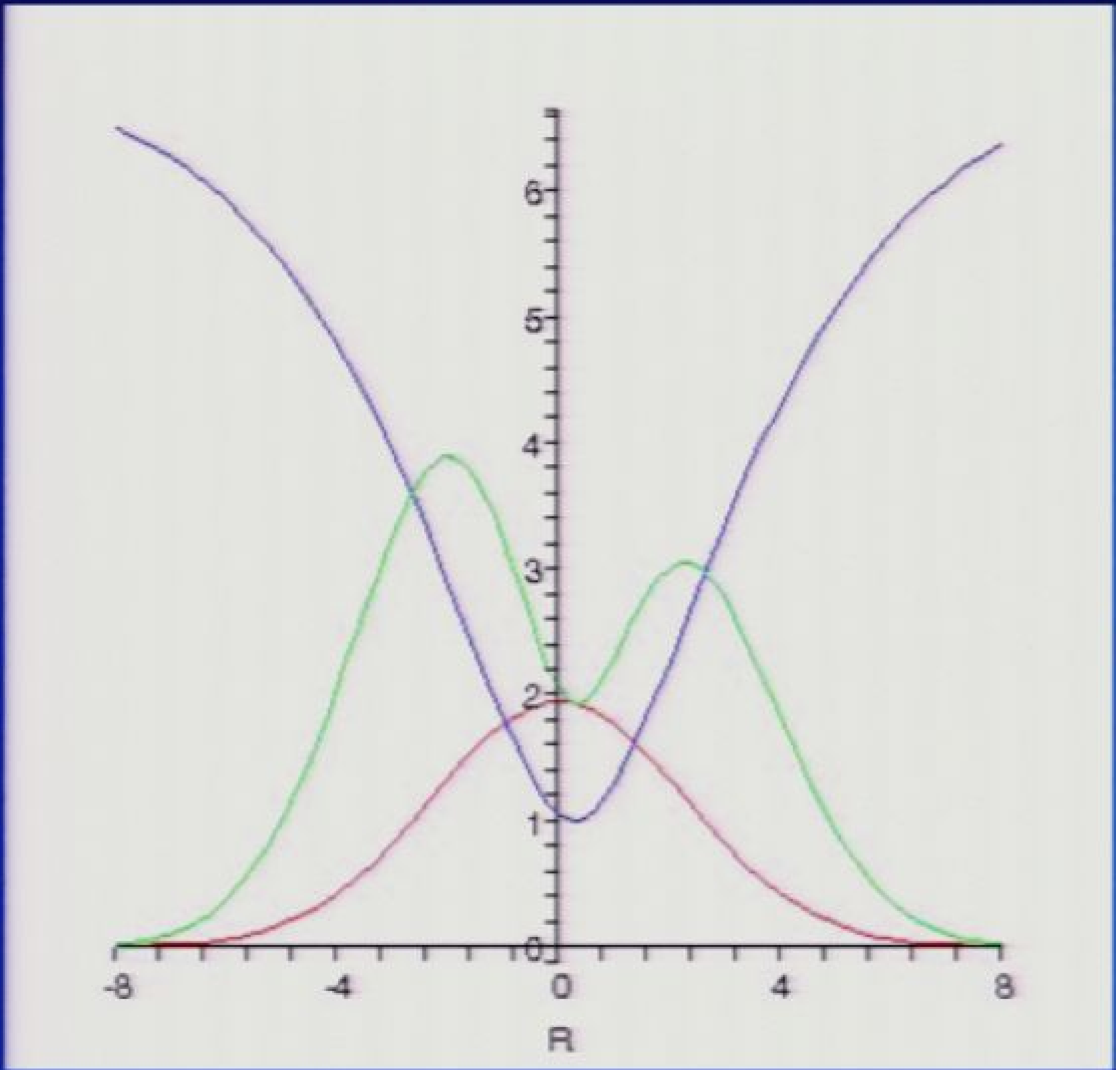
$$\Sigma_c = \frac{c^2}{4\pi G_N}\frac{D_s}{D_l D_{ls}} \approx 3.1\times 10^9\ M_\odot/\mathrm{kpc}^2$$

$$\kappa(x,y) = \int \frac{4\pi G(r)}{c^2}\frac{D_l D_{ls}}{D_s}\rho(x,y,z)dz \equiv \frac{\bar\Sigma(x,y)}{\Sigma_c}$$

$$\bar\Sigma(x,y) = \int \mathcal{G}(r)\rho(x,y,z)dz.$$

$$\frac{D_l D_{ls}}{D_s} \approx 540\ \mathrm{kp}$$

One-dimensional cartoon sketch of MOG lensing prediction



Red curve is X-ray gas surface density Σ(R)

Blue curve is $G(R)/G_N$

Green curve is predicted κ(R) lensing

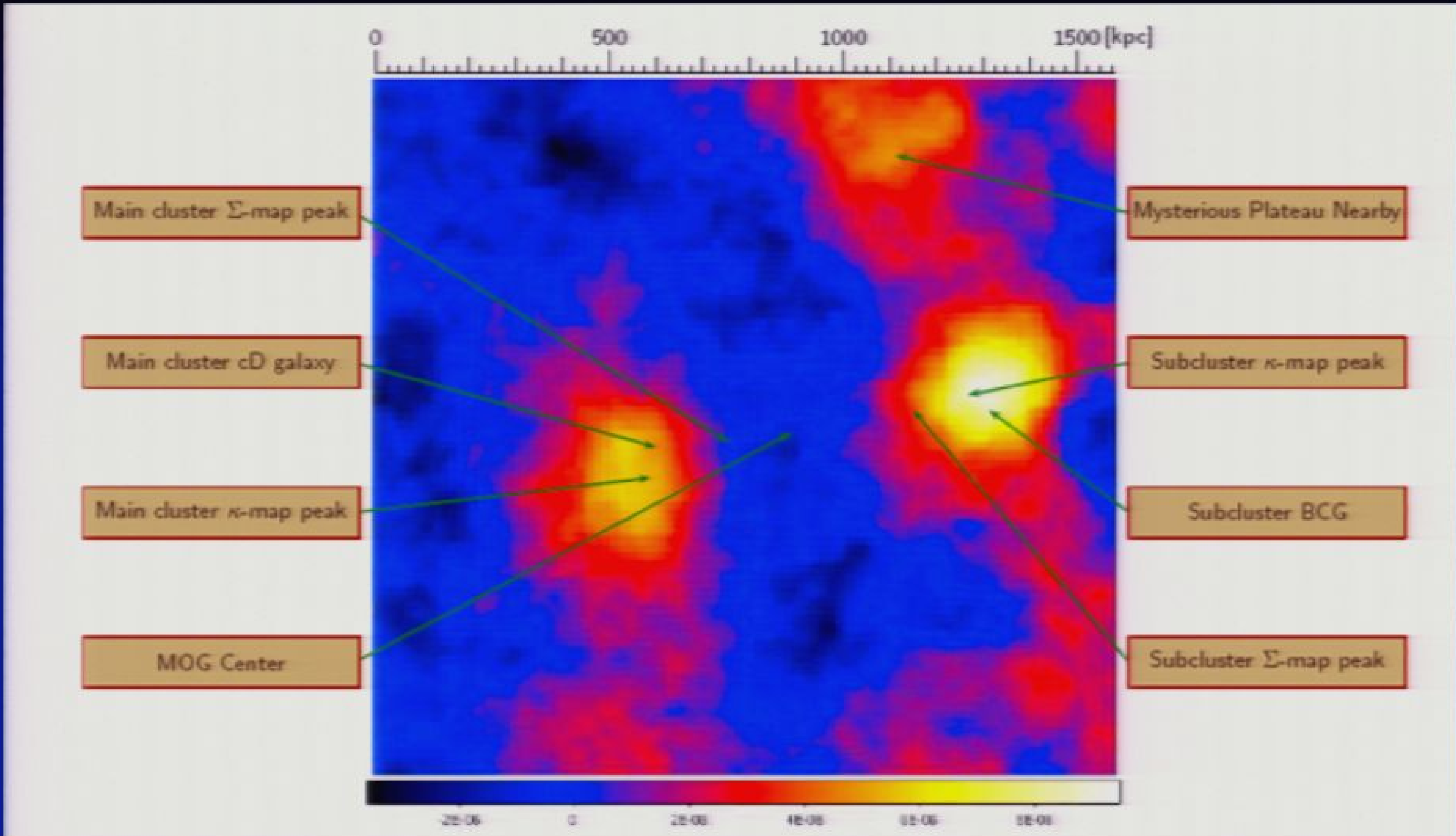
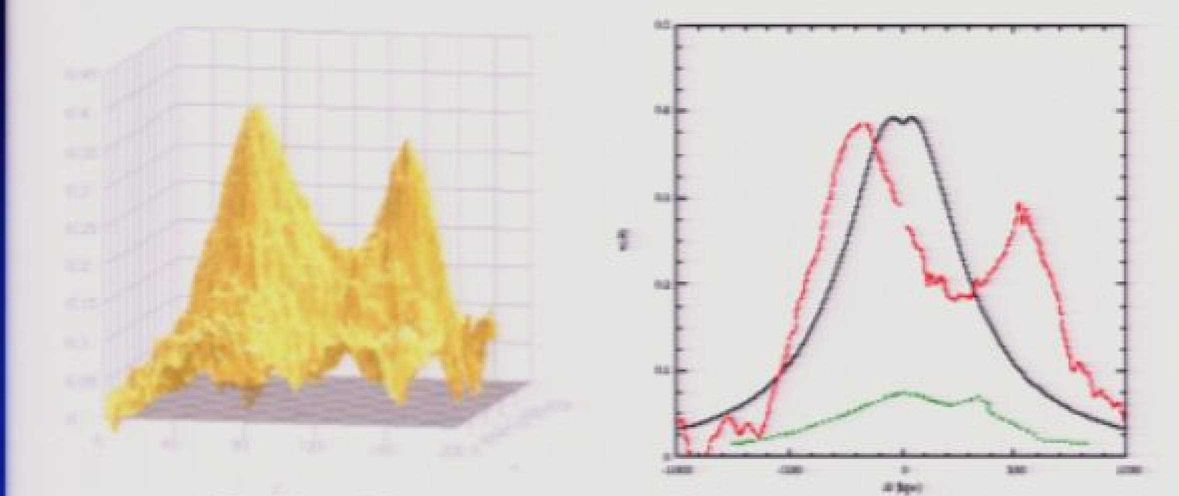


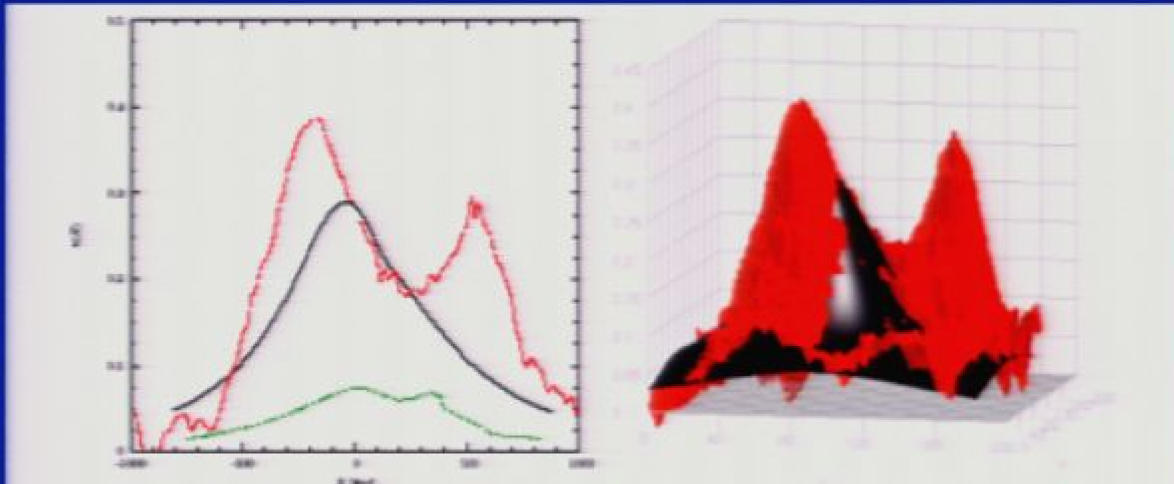
FIG. 13: The galaxy surface density Σ-map prediction.
The prediction of the Σ-map due to the galaxies as computed by the difference between the κ-map and our MOG κ-model, scaled as surface mass density according to Equation (78). Σ-map and κ-map observed peaks are shown for comparison. The central dominant (cD) galaxy of the main cluster, the brightest cluster galaxy (BCG) of the subcluster, and the MOG predicted gravitational center are shown. J2000 and map (x,y) coordinates are listed in Table 1. Component masses (integrated within a 100 kpc radius aperture) for the main and subcluster, the MOG center and the total predicted baryonic mass, $M_{\rm bary}$, for the Bullet Cluster 1E0657-558 are shown in Table 3.

45



(a) 3D visualization of the κ-map November 15, 2006 data release (Clowe *et al.*, 2006c)

(b) 0^{th}-order approximation – neglecting the subcluster



(c) Best-fit κ-model

(d) 3D visualization of the best-fit κ-model

FIG. 12: The convergence κ-map November 15, 2006 data release (Clowe *et al.*, 2006c) and our κ-models. The best-fit MOG κ-model is shown in solid black in Figures 12b, 12c and 12d. The convergence κ-map November 15, 2006 data release (Clowe *et al.*, 2006c) is shown as Figure 12a and in red in Figures 12b, 12c and 12d. The scaled Σ-map, $\Sigma(x,y)/\Sigma_c$ data is shown in short-dashed green, also shown in Figure 3.

Abell 520

- Abel 520 is a cosmic "train wreck". Data from Chandra X-ray and France-Canada-Hawaii and Subaru optical telescope observations.

- Observational results contradict those for the Bullet Cluster.

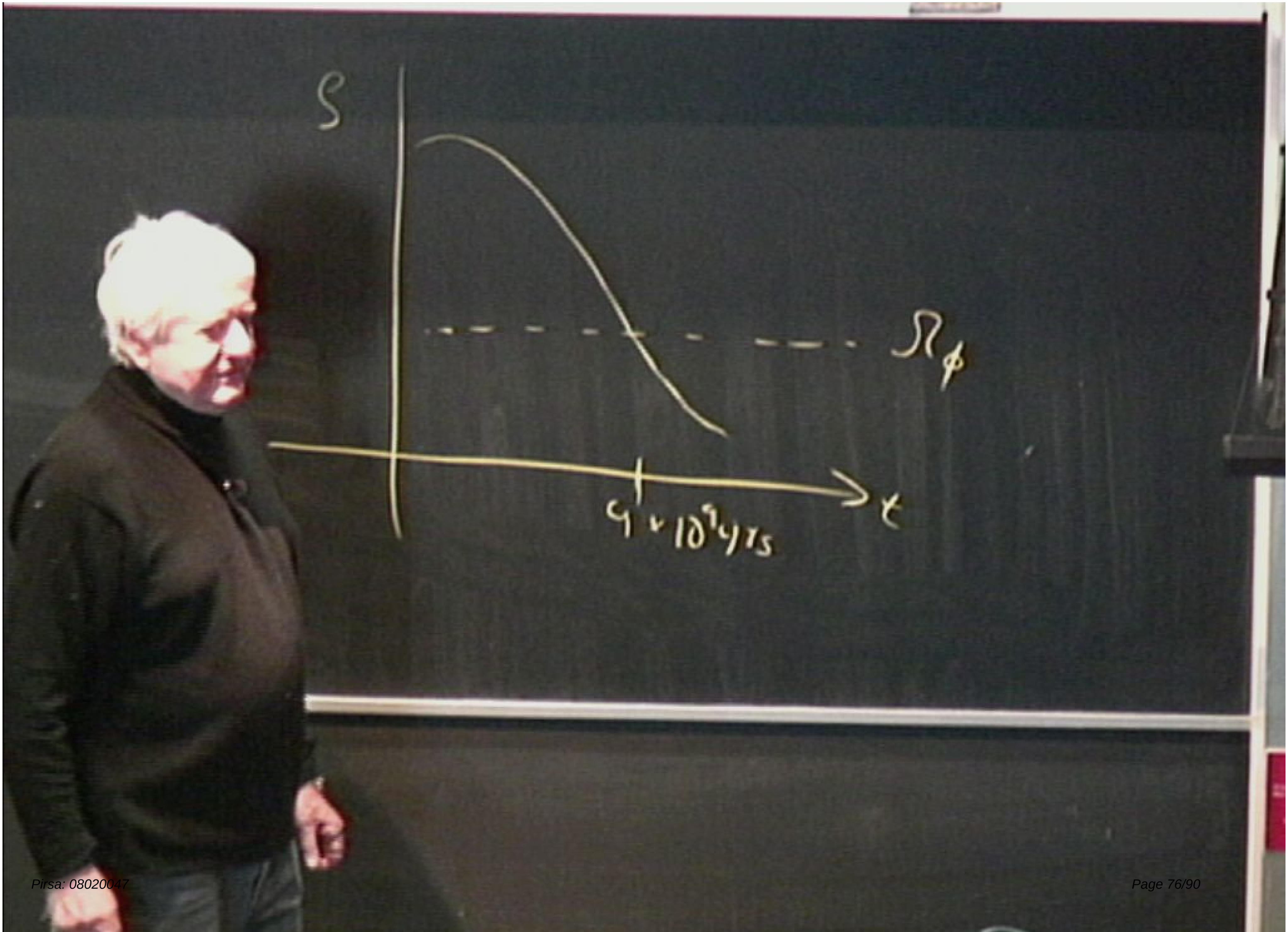
- "Dark matter" has not separated from the hot plasma. Galaxies are observed without dark matter (Mahdavi et al. 2007).

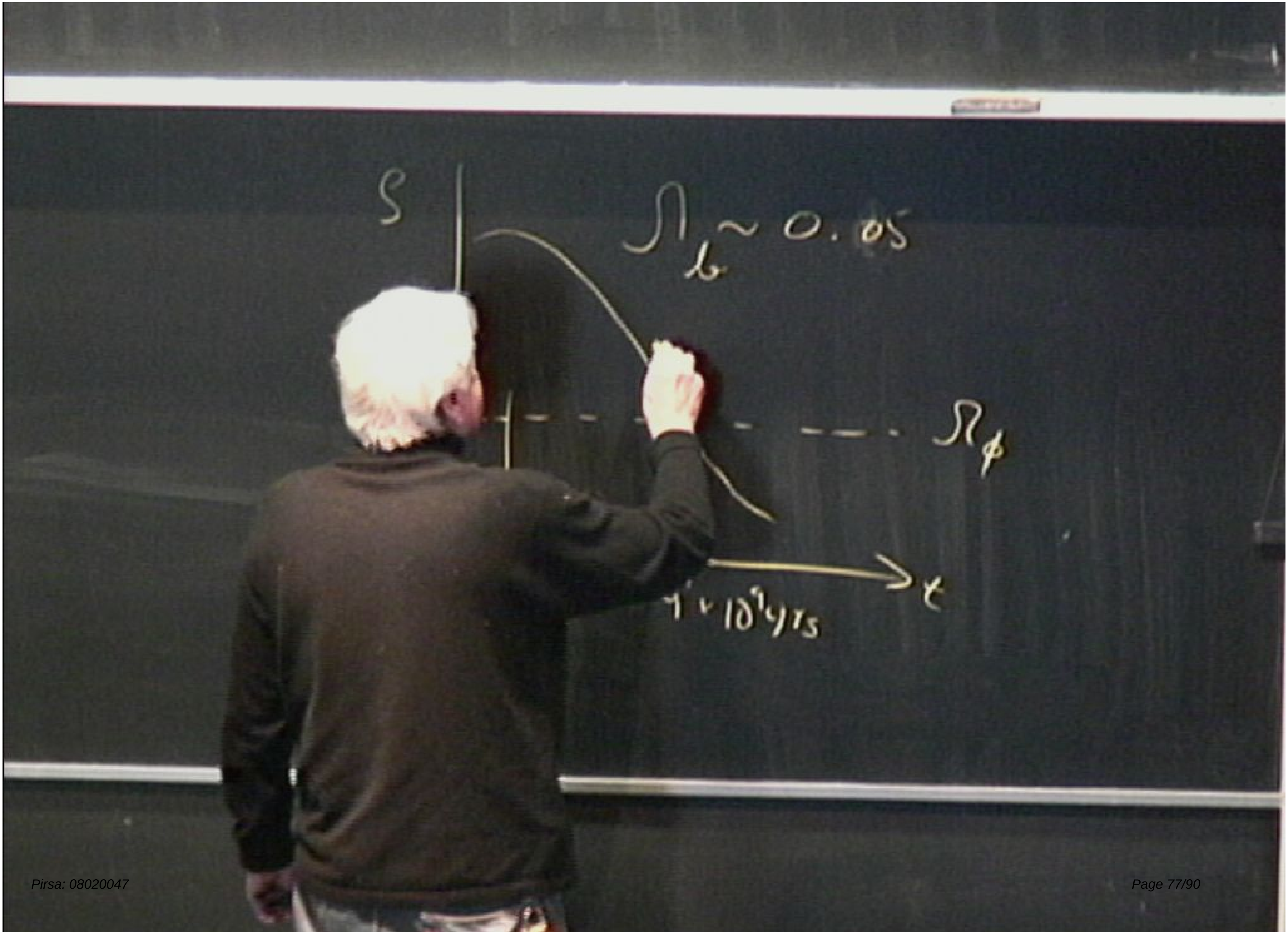

1
2
3
4
5

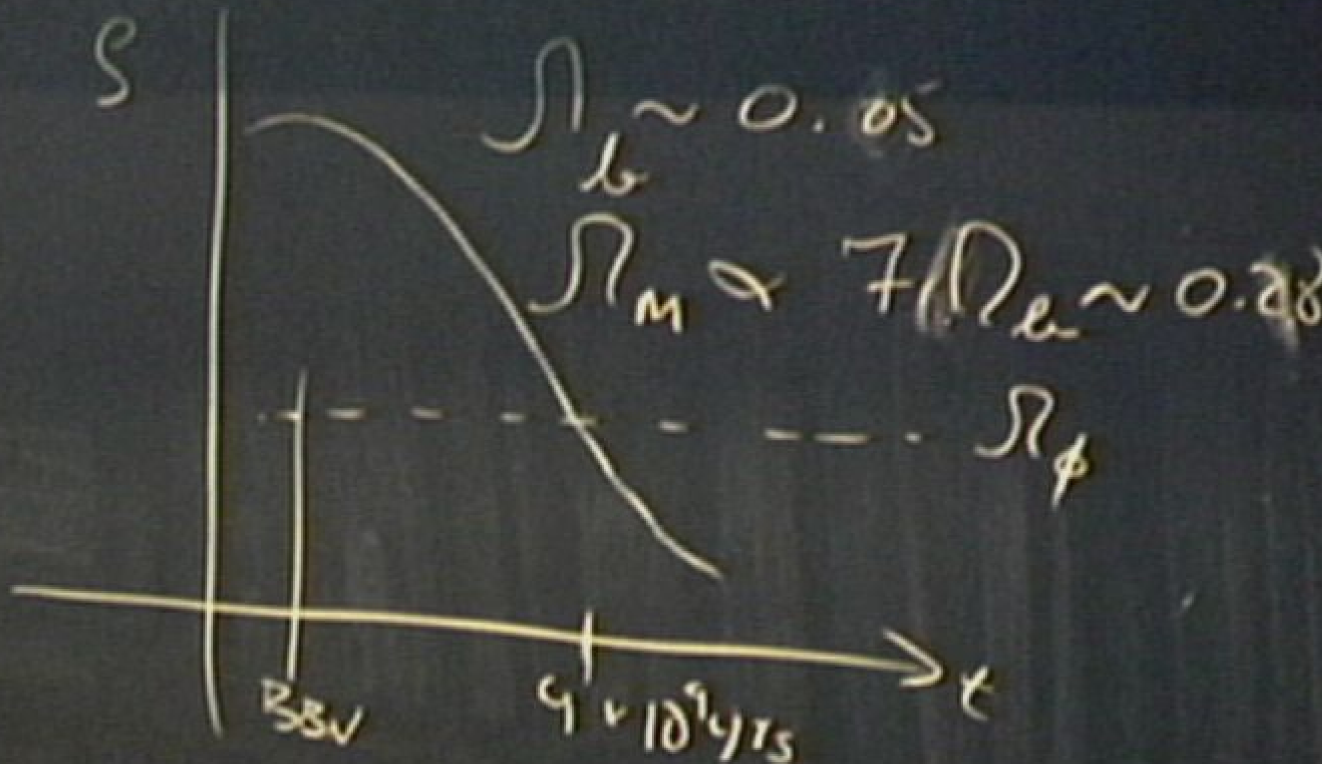
8. Conclusions

• We have demonstrated how results and predictions can be derived directly from the modified gravity (MOG) action principle, without resorting to *ad hoc* assumptions. After we fix two universal integration constants from observation, no free adjustable parameters remain and the theory can make definite predictions.

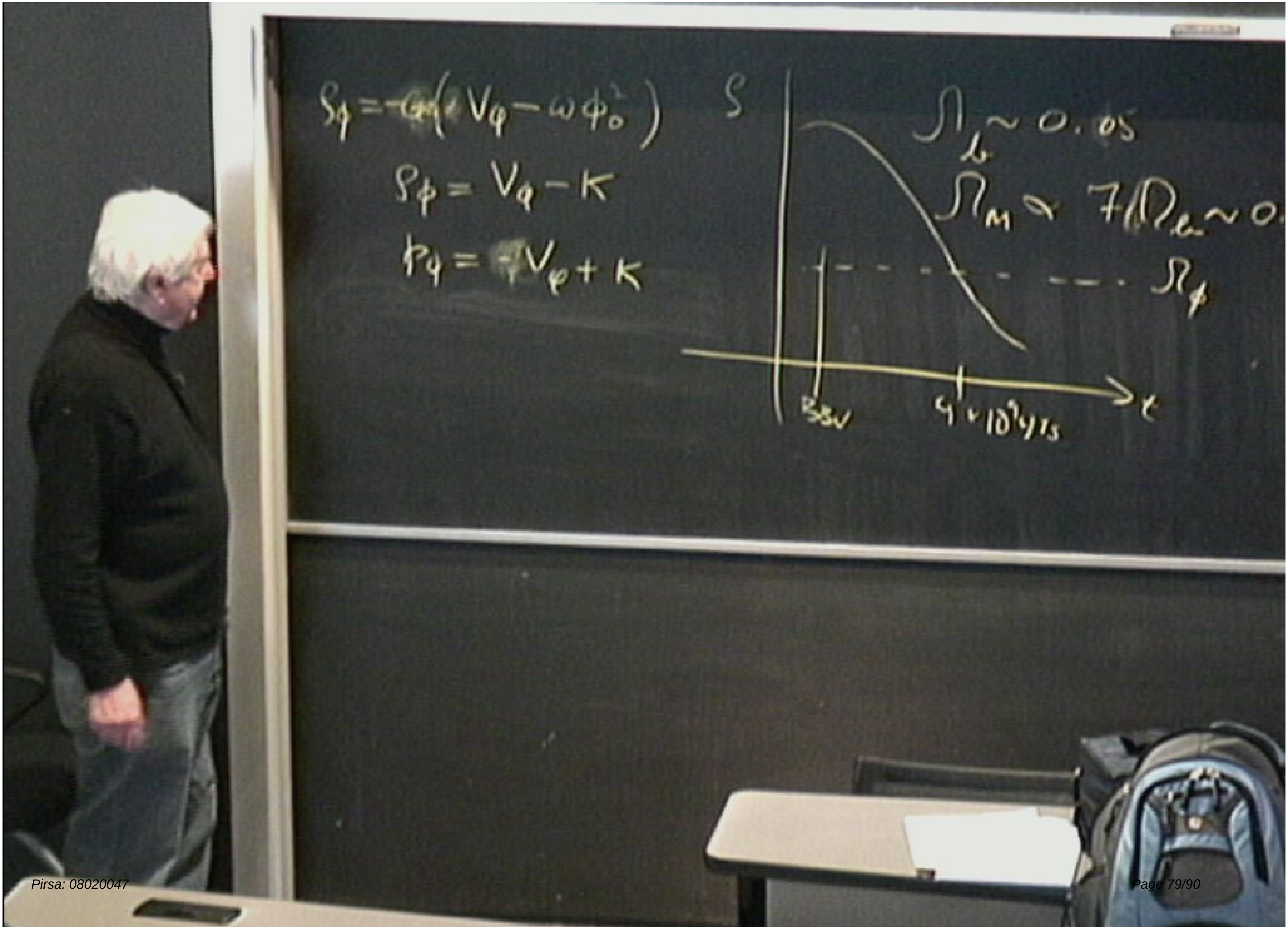
• The predictions fit the data over 22 orders of magnitude in mass for globular clusters, dwarf galaxies, spiral galaxies, clusters of galaxies, the bullet cluster and cosmology.


ρ
Ω_ϕ
t


ρ
$\Omega_b \sim 0.05$
Ω_ϕ
t
10^{9} yrs



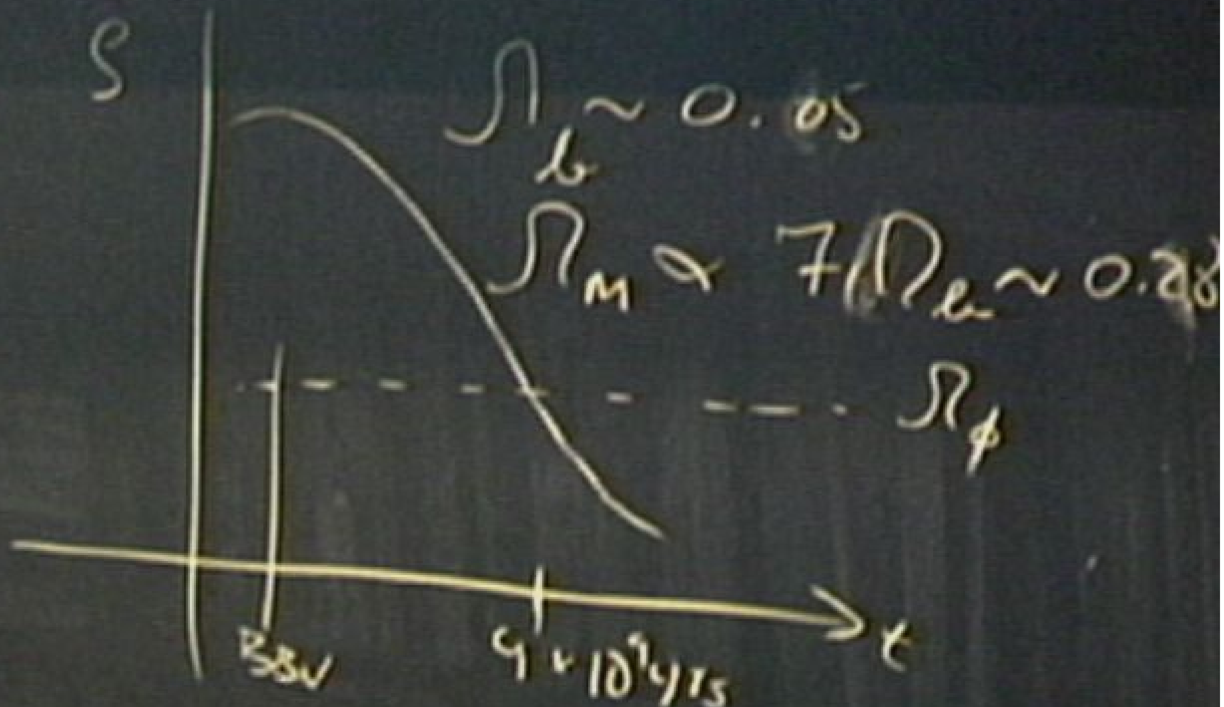
ρ
$\Omega_b \sim 0.05$
$\Omega_M \propto 7\Omega_b \sim 0.28$
Ω_ϕ
t
BBN

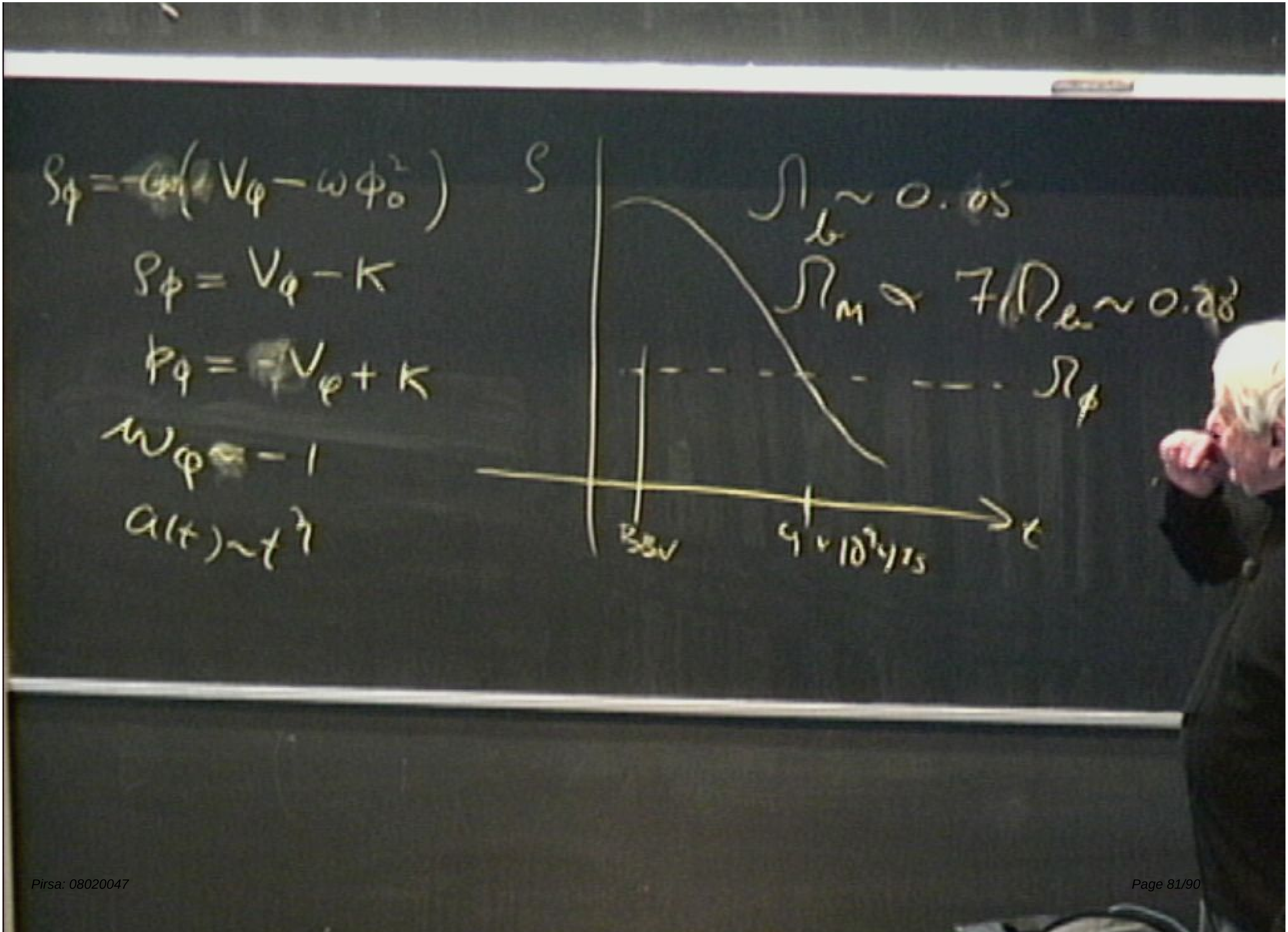

ρφ = Vφ − K
pφ = Vφ + K
ρ
Ωb ~ 0.05
ΩM
Ωφ
BBN
t

$$\rho_\phi = -\omega(V_\phi - \omega\phi_0^2)$$

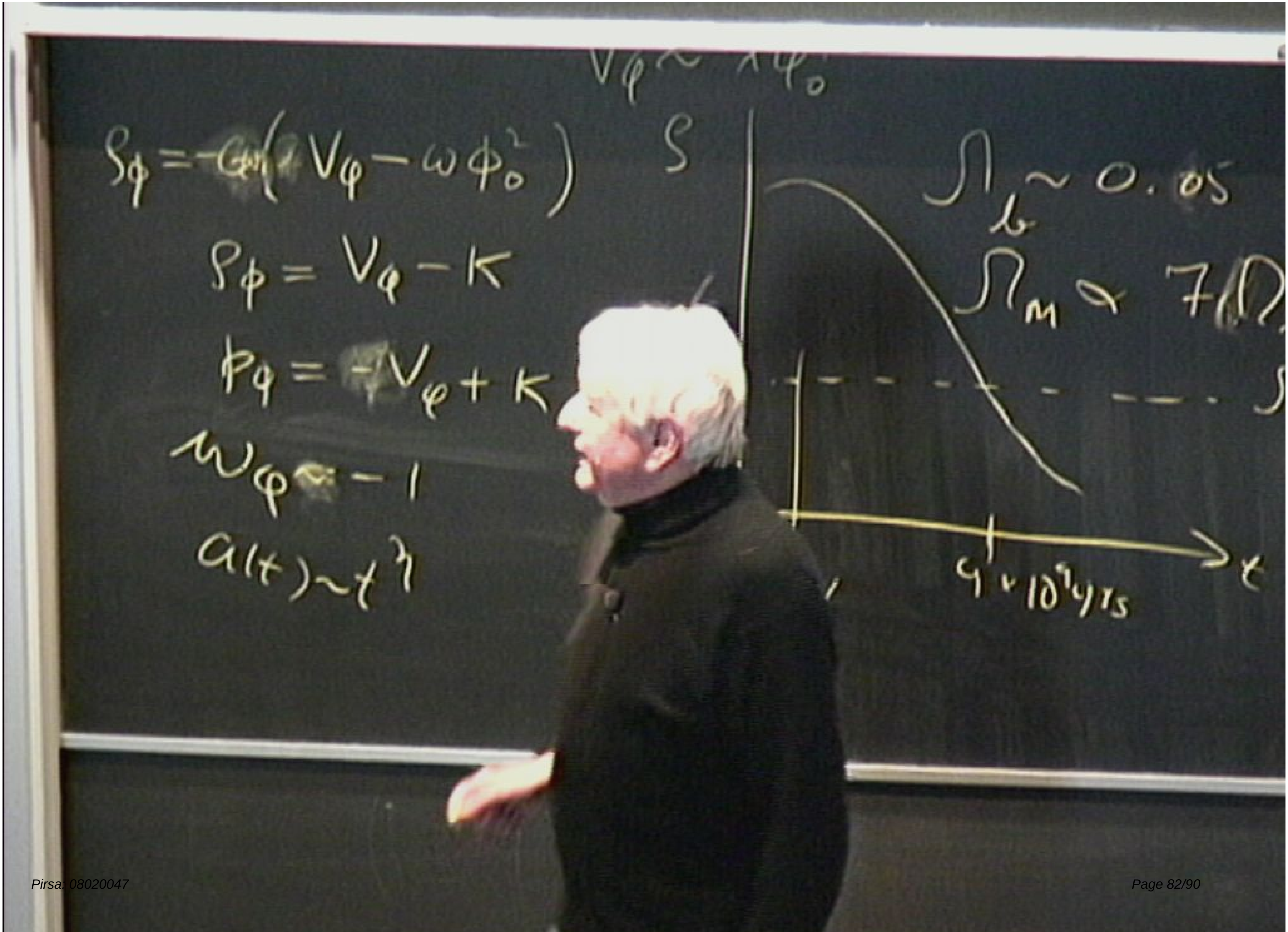
$$\rho_\phi = V_\phi - K$$

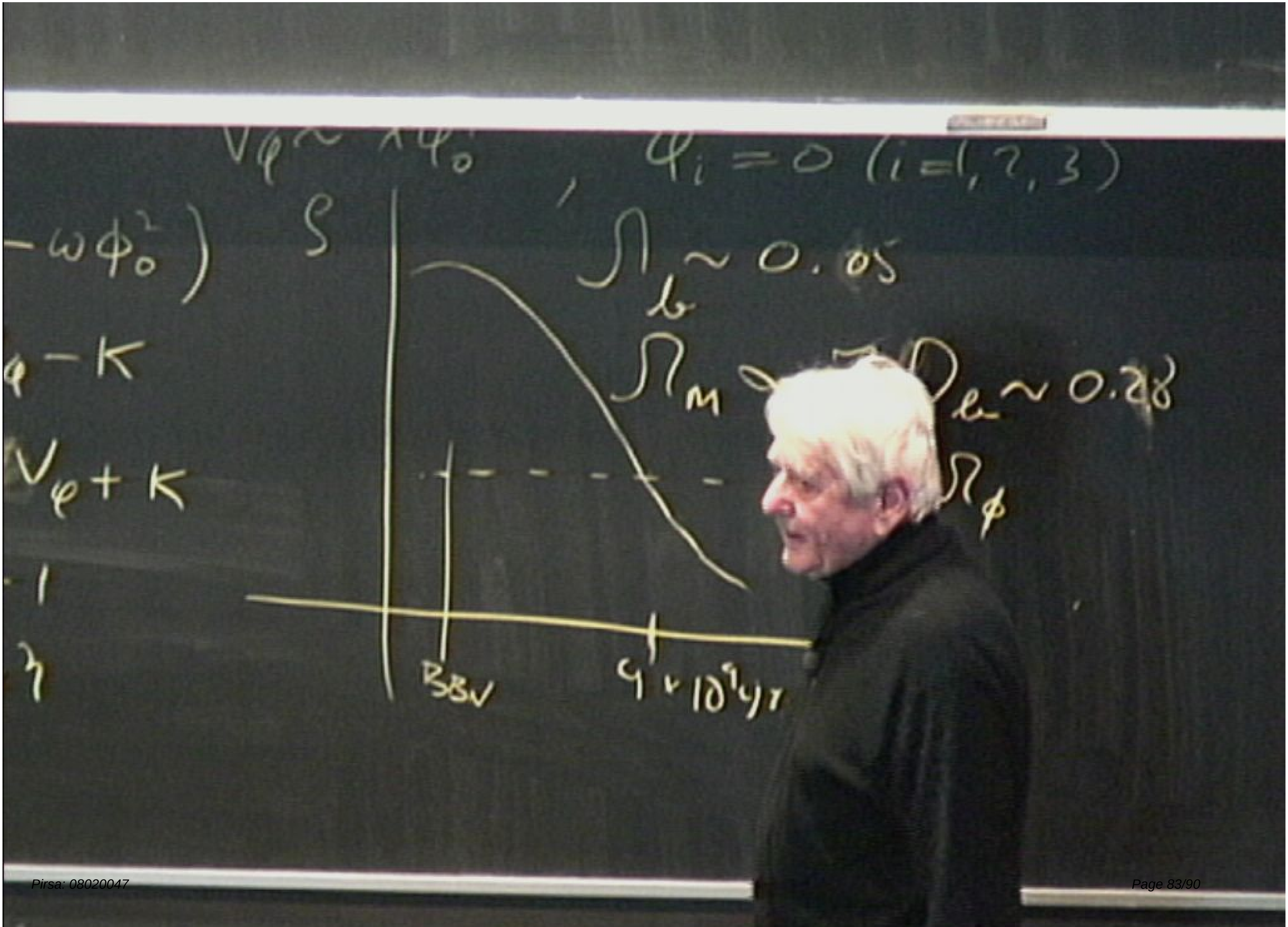
$$p_\phi = -V_\phi + K$$

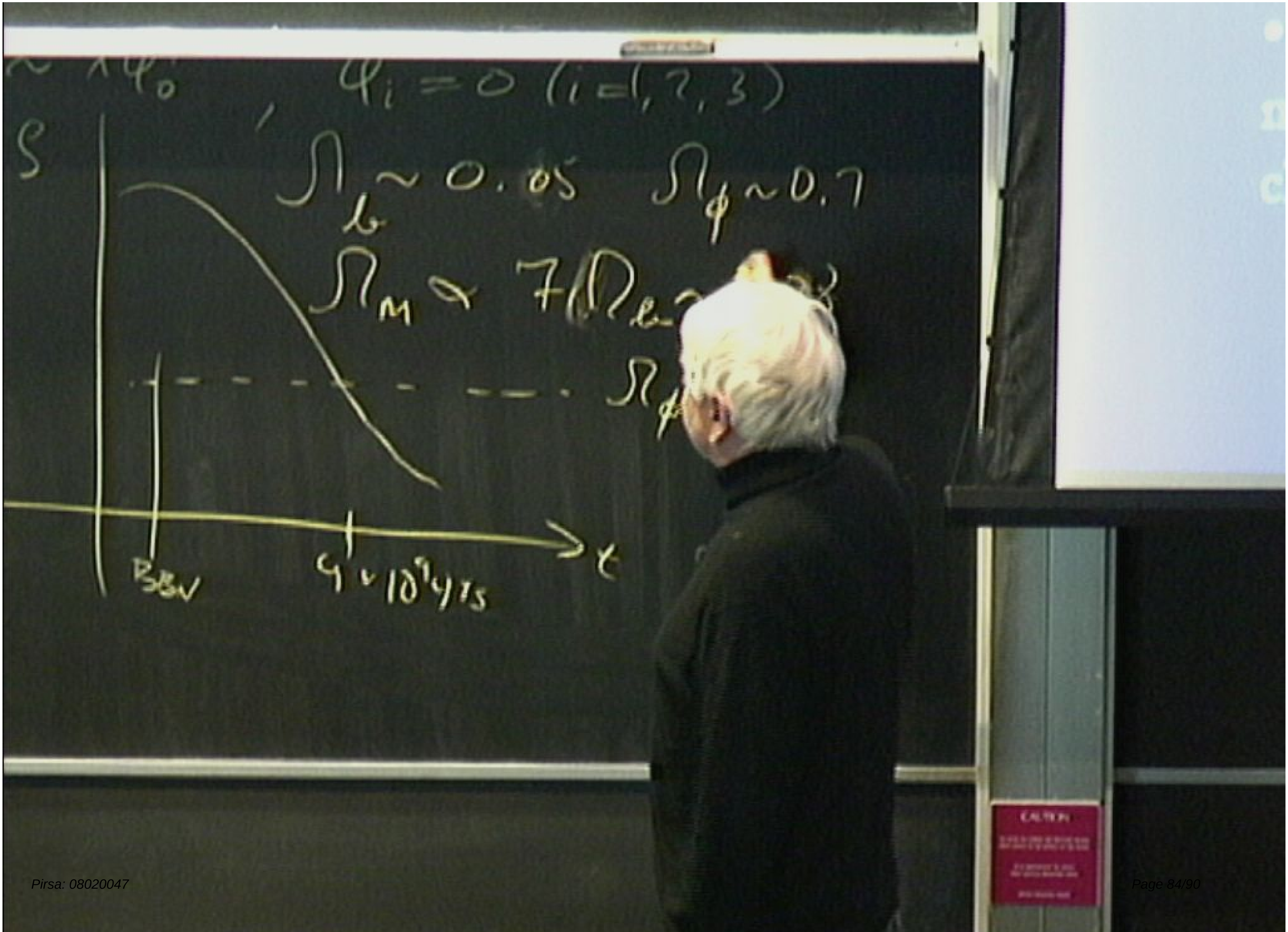


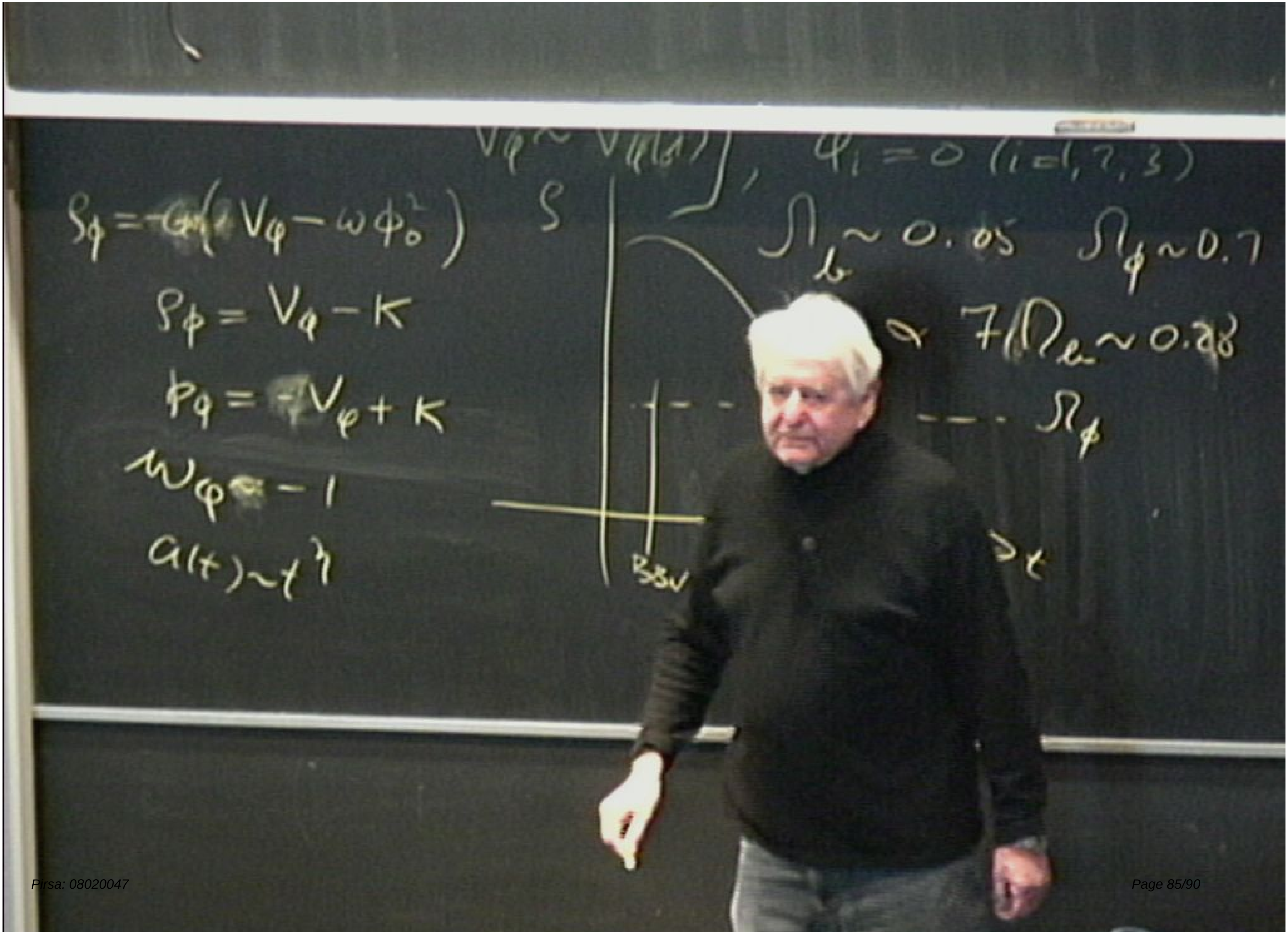


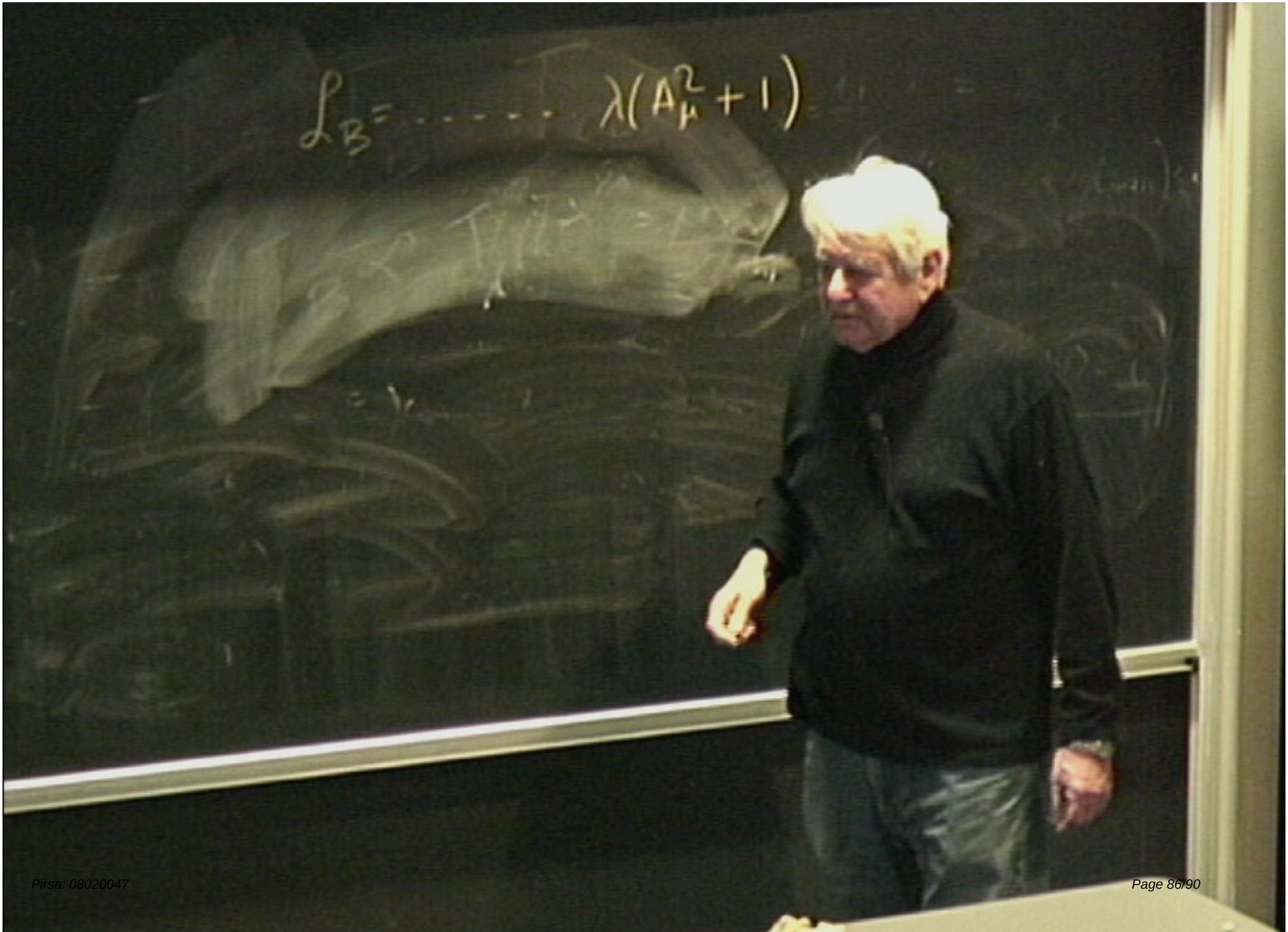
$\rho_\phi = -(V_\phi - \omega\phi_0^2)$
$\rho_\phi = V_\phi - K$
$p_\phi = -V_\phi + K$
$w_\phi = -1$
$a(t) \sim t^{?}$
$\Omega_b \sim 0.05$
$\Omega_M \approx 7\Omega_b \sim 0.28$
Ω_ϕ
BBN
t


$\rho_\phi = V_\phi - K$
$p_\phi = -V_\phi + K$
$w_\phi \simeq -1$
$a(t) \sim t^{?}$
$\Omega_b \sim 0.05$
t


φ_i = 0 (i=1,2,3)
Ω_b ~ 0.05
Ω_φ
BBN


φ_i = 0 (i=1,2,3)
Ω_b ~ 0.05   Ω_φ ~ 0.7
Ω_M
t
BBN
~10^9 yrs

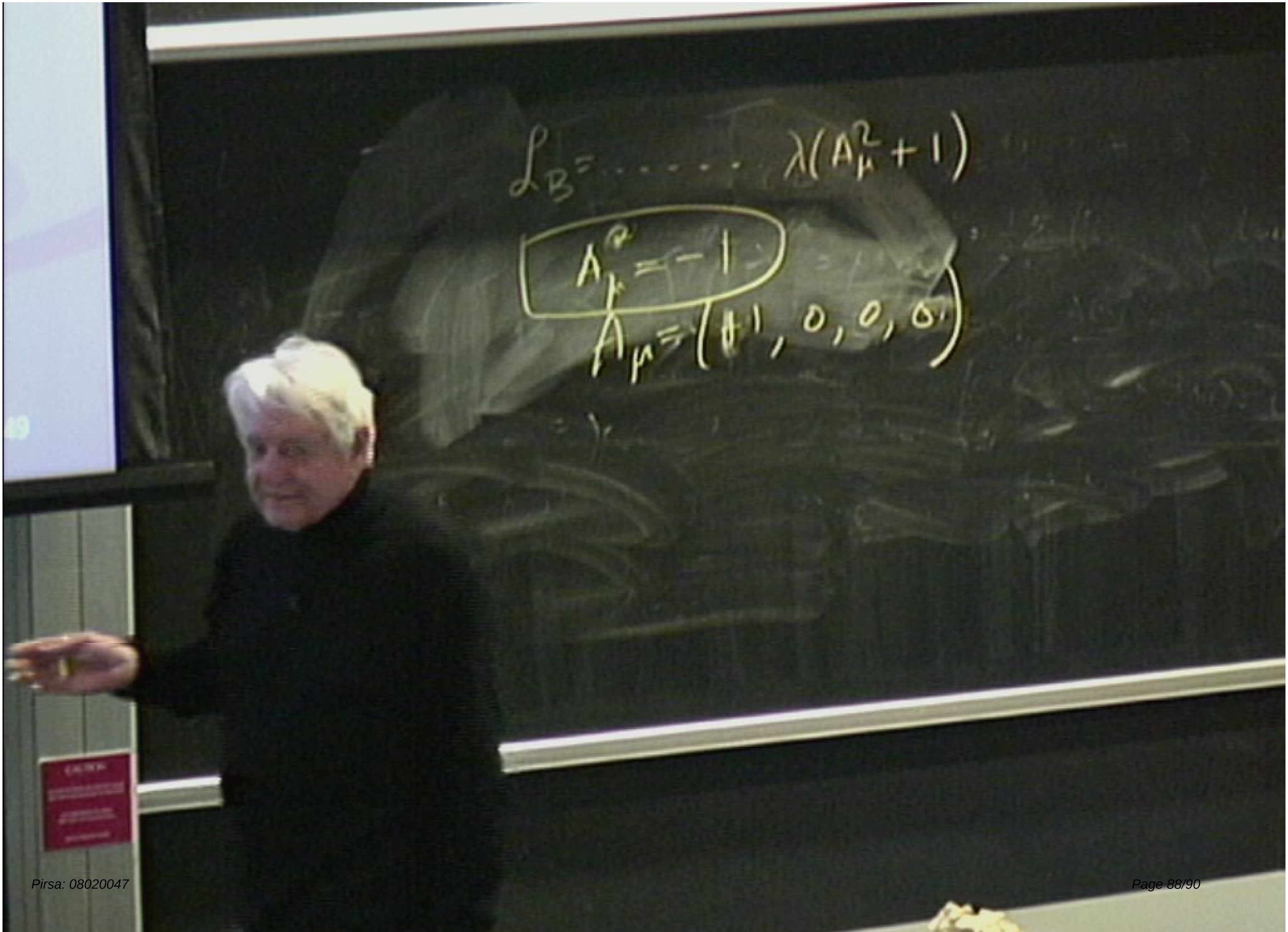

ρφ = Vφ − K
pφ = −Vφ + K
wφ ≈ −1
a(t) ~ t^η
Ωb ~ 0.05
Ωφ ~ 0.7
Ωφ


$\mathcal{L}_B = \ldots\ldots \lambda(A_\mu^2 + 1)$

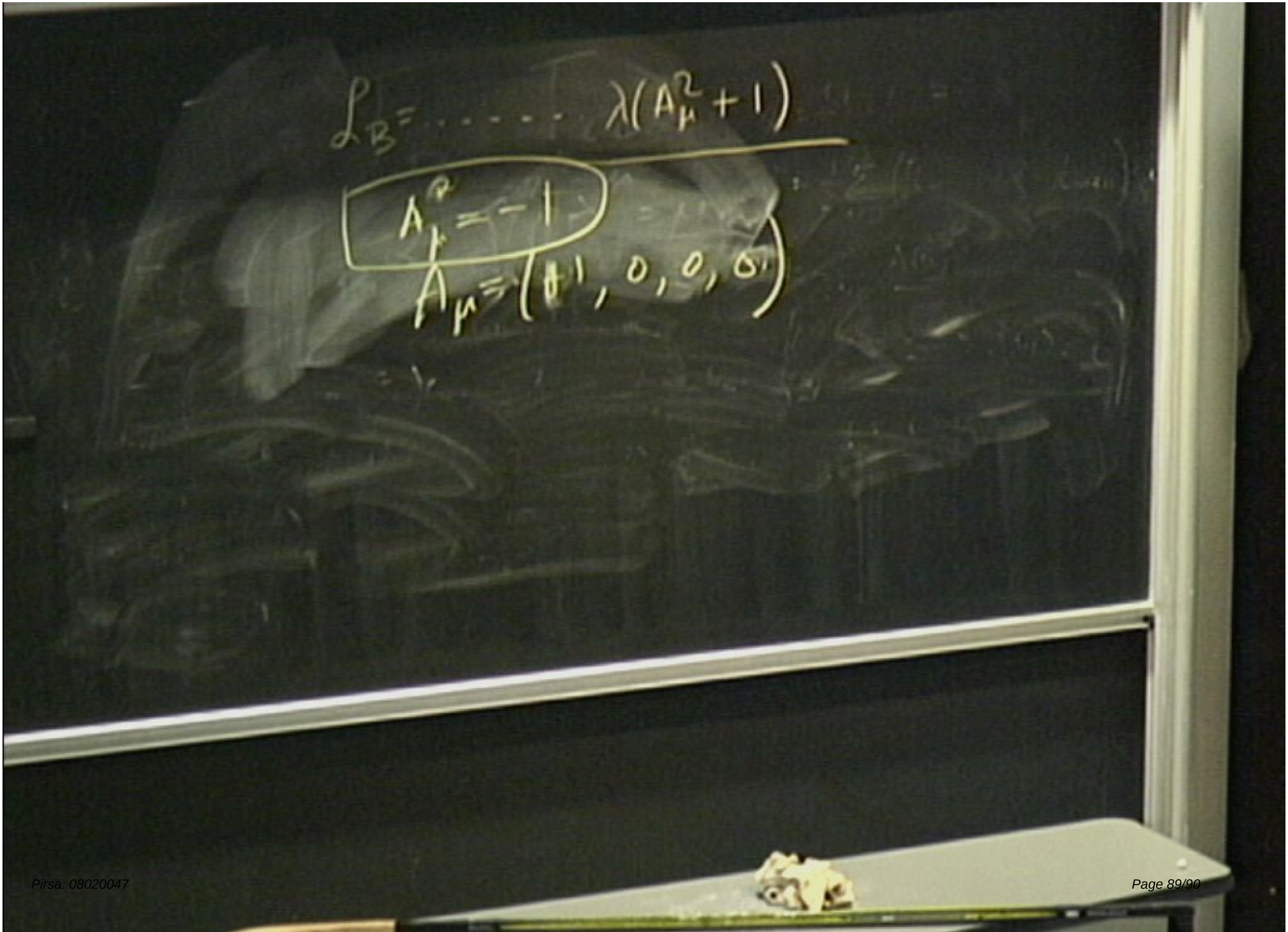
$$\mathcal{L}_B = \ldots\ldots \lambda(A_\mu^2 + 1)$$

$$A_\mu^2 = -1$$

$$A_\mu = (\pm 1, 0, 0, 0)$$



L_B = ... λ(A_μ^2 + 1)
A_μ^2 = -1
A_μ = (±1, 0, 0, 0)


$\mathcal{L}_B = \ldots\ldots \lambda(A_\mu^2 + 1)$
$A_\mu^2 = -1$
$A_\mu = (1, 0, 0, 0)$

- The MOG acceleration law is associated with the potential:

$$\Phi = -\frac{G_\infty M}{r}\left[1 - \frac{\alpha}{1+\alpha}e^{-\mu r}\right] = \Phi_N + \Phi_Y$$

$$\Phi_N = -\frac{G_\infty M}{r}$$

$$\Phi_Y = \frac{\alpha}{1+\alpha}G_\infty M\frac{e^{-\mu r}}{r}$$

- The Poisson equation is (for cosmology $1/\mu \simeq 14 \times 10^9$ light years):

$$\begin{aligned}\nabla^2\Phi &= -4\pi G_\infty\rho(\mathbf{r}) + \mu^2\Phi_Y(\mathbf{r}) \\ &= -4\pi G_\infty\rho(\mathbf{r}) + \alpha\mu^2 G_N \int \frac{e^{-\mu|\mathbf{r}-\tilde{\mathbf{r}}|}\rho(\tilde{\mathbf{r}})}{|\mathbf{r}-\tilde{\mathbf{r}}|}\, d^3\tilde{\mathbf{r}}\end{aligned}$$

$$G_{\rm eff} \simeq 0.3 G_\infty$$

$$G_\infty \simeq 20 G_N$$

$$\alpha \simeq 19$$