Title: New Era for Cosmic Inflation

Date: Feb 26, 2008 02:00 PM

URL: http://pirsa.org/08020045

Abstract: Cosmological observations will soon distinguish between the standard slow roll inflationary paradigm and some of its recently developed alternatives. Driven by developments in string theory, many new models include features such as non-minimal kinetic terms, leading to large non-gaussianities, making them observationally testable in the CMB. Models of slow roll inflation can also give rise to large non- gaussianities if the initial inflationary state was sufficiently excited, with a shape dependence that will be clearly distinguishable. I will review these different possibilities and discuss how they provide new theoretical challenges in understanding the initial conditions problem and the global structure of the inflationary universe.

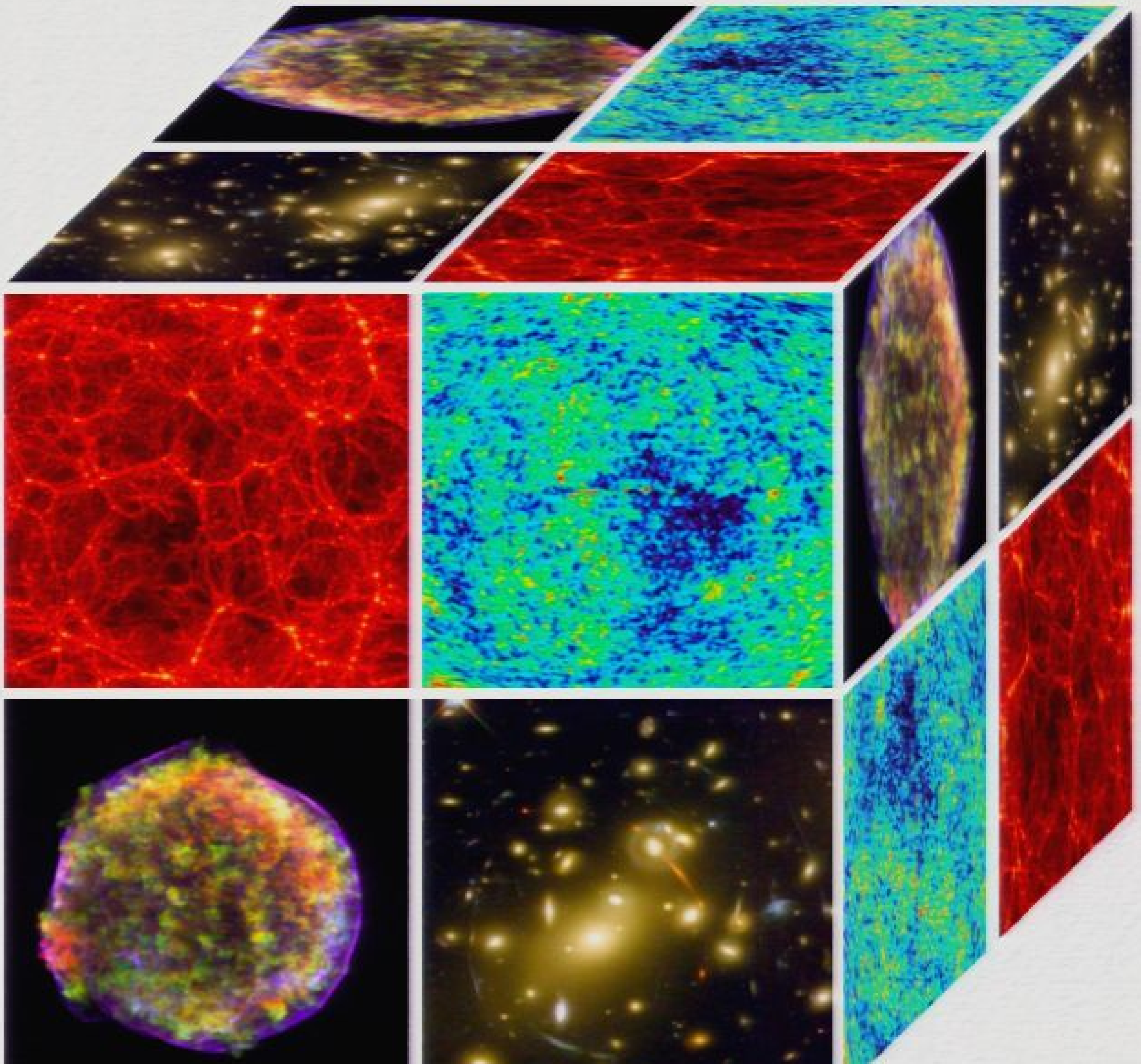
New Era for Cosmic Inflation

Andrew J. Tolley
Perimeter Institute

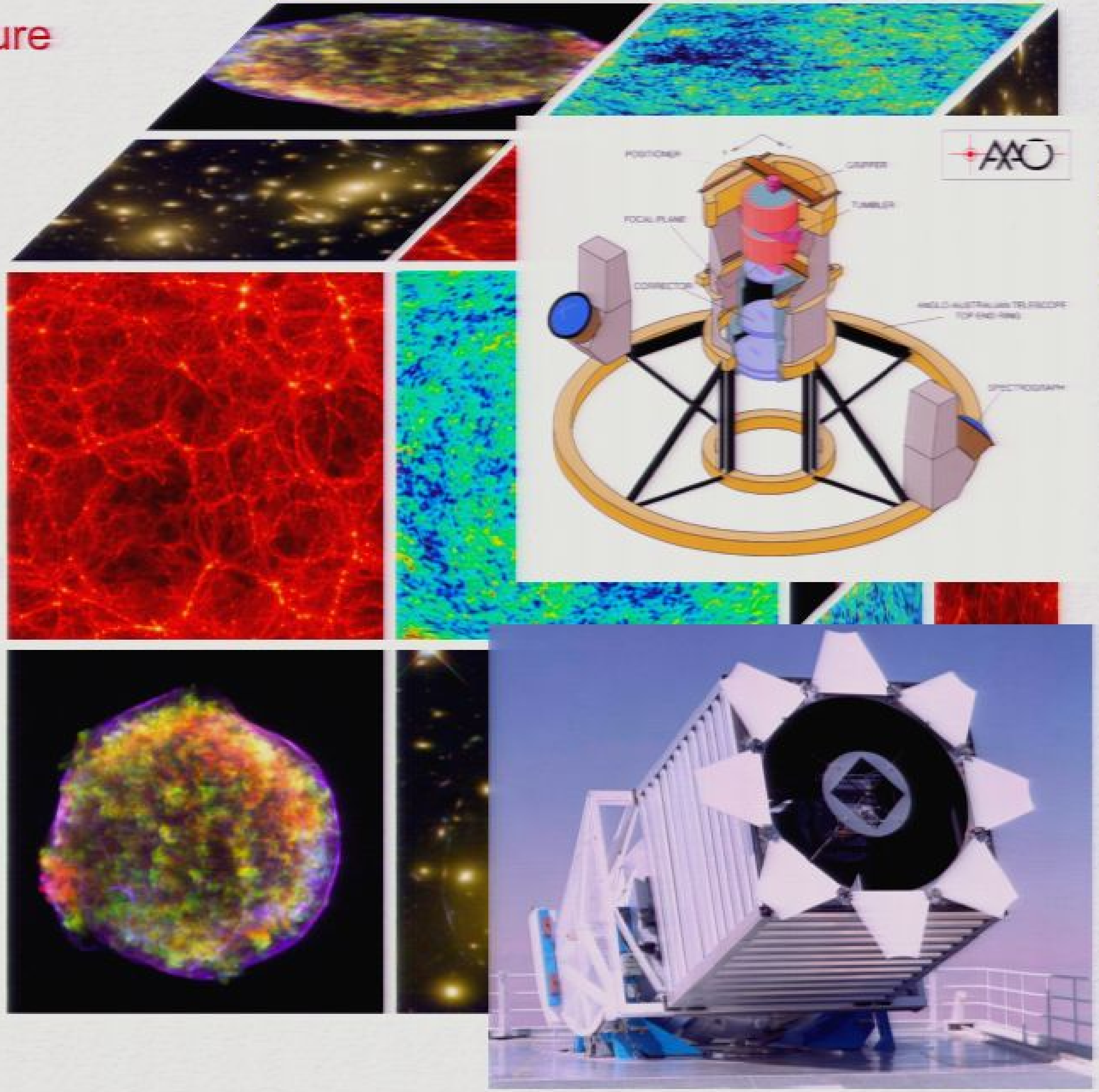
Feb 26, 2008 @ Perimeter Institute

Based on work with
D. Wesley, hep-th/0703101
M. Wyman, 0801.1854
R. Holman, 0710.1302
+ work in progress

A wealth of cosmological data...



large scale structure



Measures:

- abundance of galaxy clusters
- size ….

Tests:

Cosmic expansion history

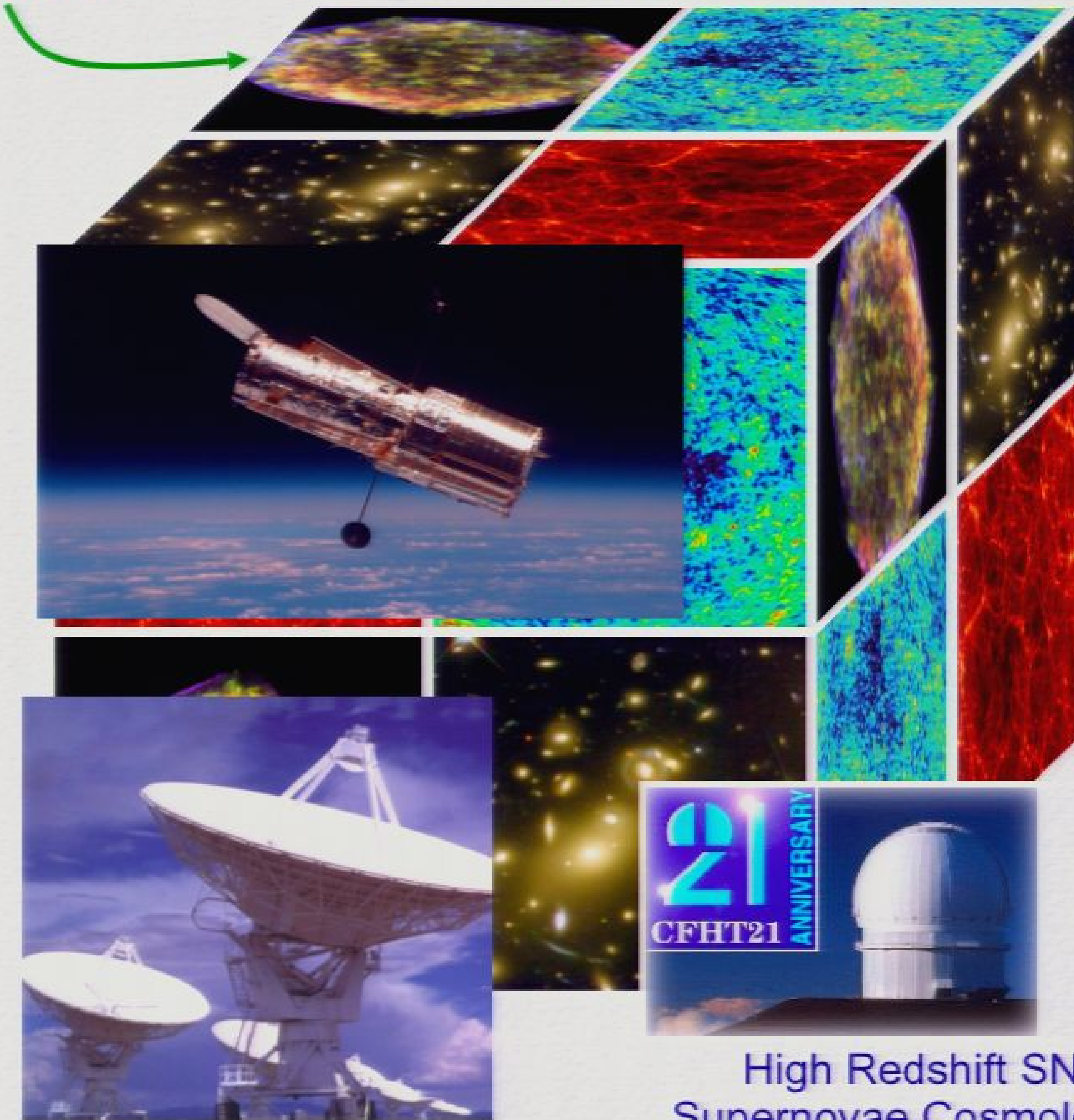
growth of matter clustering

Probes:

- 2dFGRS
- SDSS

Type Ia Supernovae

Tycho's Nova remnant



<u>Probes</u>:

- CTIO
- CFHT
- ESO
- HST
- VLT
- Keck
- UKIRT
- WIYN
- APO

<u>Measures</u>:
- SN redshift

<u>Tests</u>:
- SN are standard candles
- Cosmic expansion
- Universe is accelerating

High Redshift SN Search

Supernovae Cosmology Project

Gravitational lensing

Measures:
- matter density through its gravitational pull

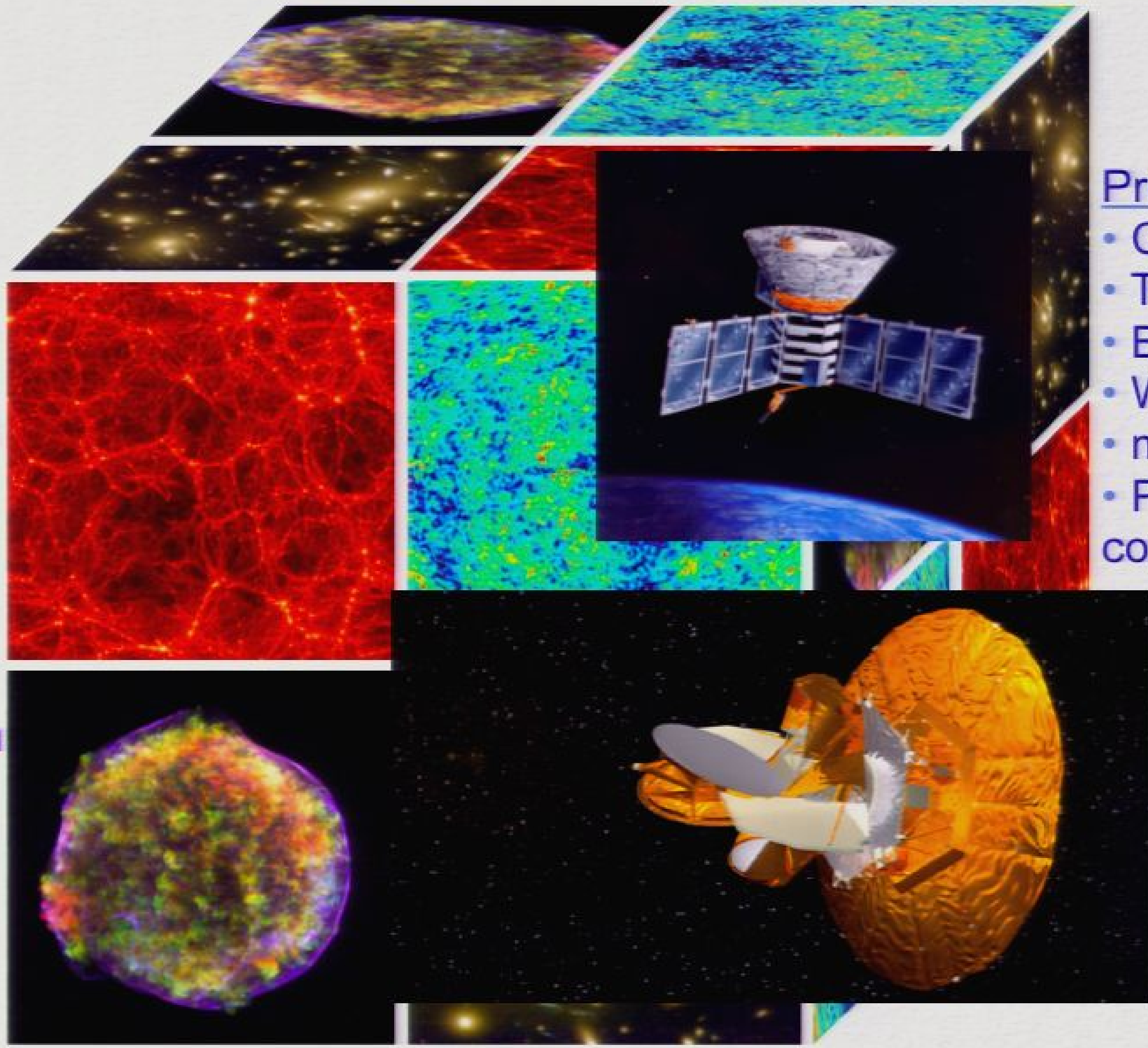
Tests:
- Composition of the Universe

Probes:
- CFHT
- William Hersche telescope
- Hubble Space Telescope

Cosmic Microwave Background

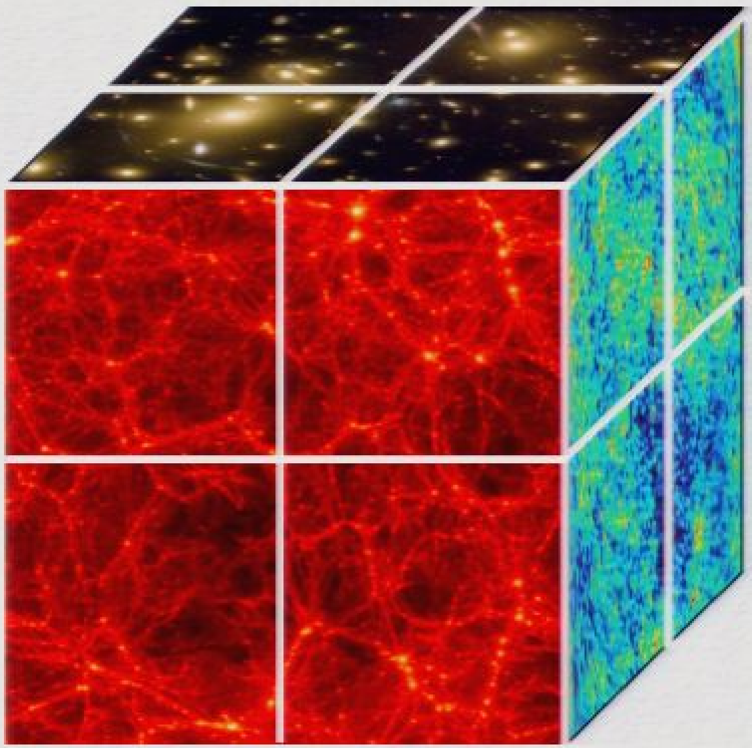
Measures:
- radiation from last scattering surface 400,000yrs

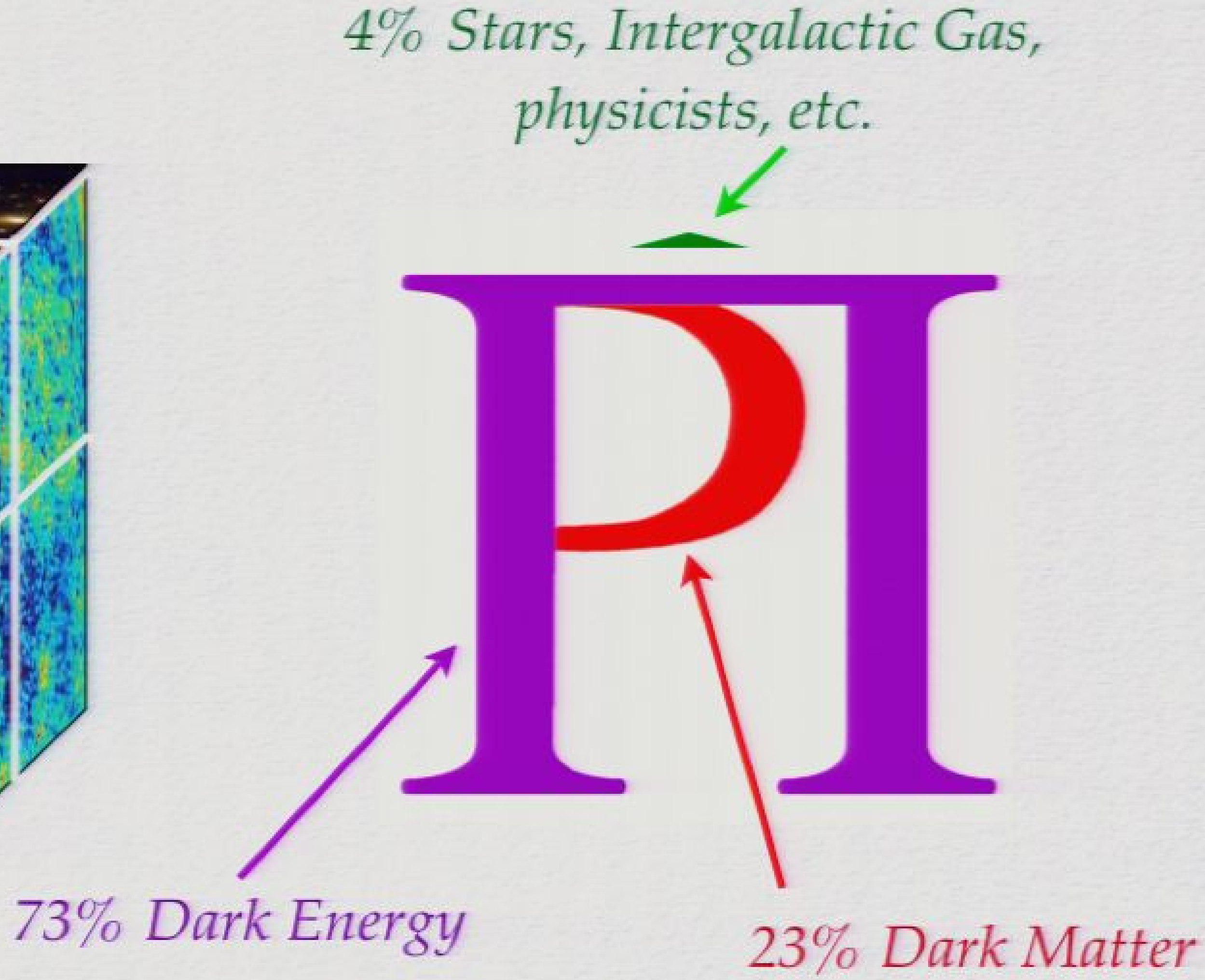
Tests:
- Cosmological parameters of previous period
- Cosmic expansion history
- Composition of the Universe
- Primordial power spectrum



Probes:
- COBE
- Toco
- Boomerang
- WMAP
- many others!
- PLANCK to come

... leading to the Concordance Model





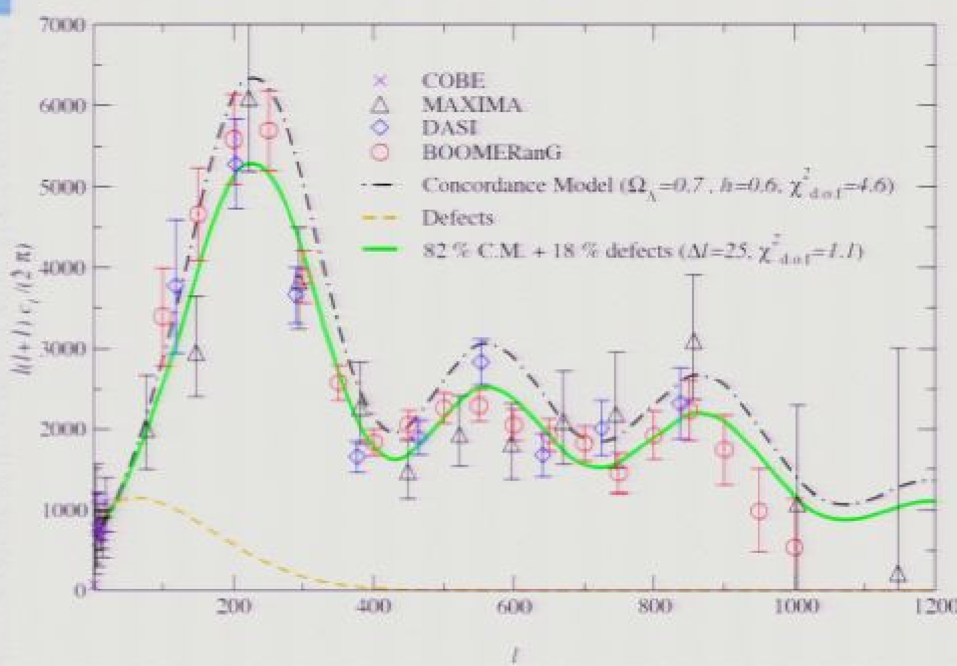
What does this tell us about the early universe?

- Large scale structure grew out of a primordial spectrum of density fluctuations that were outside the horizon at recombination
- Fluctuations were *nearly:*
    - Adiabatic
    - Scale Invariant
    - Gaussian Statistics

How to generate nearly scale invariant fluctuations?

- Scale invariance is a very simple concept!

- Cosmic strings and textures can give rise to scale invariant fluctuations, but they are not coherent!
  (Do not give rise to acoustic peaks!)



- Coherence implies **scalar field**, be it fundamental or effective degree of freedom (e.g. condensate)

- Given the familiar low energy effective theories we are used to, how many ways can we generate scale invariance fluctuations?

AJT+Wesley '07

- Observations tell us that to good approximation the spectral index is constant (small running)

$$\int \frac{d^3k}{(2\pi)^3} P_\zeta(k) = \int \frac{d^3k}{(2\pi)^3} k^{-3} k^{n_s-1} \mathcal{A}^2 \qquad \frac{dn_s}{d\ln k} \ll 1$$

If rescale $k \to e^\lambda k$ then $P_\zeta(k) \to e^{(n_s-4)\lambda} P_\zeta(k)$

- Only arise from vacuum fluct's if action has scaling symmetry

$$g_{\mu\nu} \to e^{-2\lambda} g_{\mu\nu} \qquad V \to e^{2\lambda} V$$

$$\frac{d\phi^A}{d\lambda} = \xi^A(\phi) \qquad G_{AB} \to G_{AB} \qquad S \to e^{-2\lambda} S$$

killing vector on

- In special case $n_s = 1$ scaling symmetry degenerates into a shift symmetry: Adiabatic mode is a pseudo-Goldstone boson with corrections suppressed by $n_s - 1$

Only two options: Inflation or ....

AJT+Wesley '07

- In an expanding universe, a general analysis of the scaling symmetric actions shows that nearly scale invariance is only achieved in either the adiabatic or isocurvature modes if

$$w = \frac{p}{\rho} \approx -1$$

since the universe will be accelerating $w < -\frac{1}{3}$:

INFLATION

the proximity to this solution is governed by two slow roll parameters

$$\epsilon = 2M_P^2 \left(\frac{1}{H} \frac{dH}{d\phi} \right)^2 \qquad \eta = 2M_P^2 \left(\frac{1}{H} \frac{d^2 H}{d\phi^2} \right)$$

... or Ekpyrosis

AJT+Wesley '07

- In collapsing universe with one field, scale invariance is only possible if $w = 0$ but highly unstable!

Finelli + Brandenberger astro-ph/0211276
Wands gr-qc/9809062

- In collapsing universe with more than one field possible to generate scale invariant isocurvature modes (also unstable) and subsequently convert them to adiabatic modes

Typically requires $w \gg 1$ this mechanism is known as:

EKPYROSIS

OLD: Khoury et al. hep-th/0103239
NEW: Lehners et al. hep-th/0702153
Buchbinder et al. hep-th/0702154
Creminelli et al. hep-th/0702165

Other alternatives exist but are not vacuum fluctuations (eg. thermal initial states) or not from a local field theory! Cosmic strings and textures can give scale invariance, but fluctuations are incoherent and hence ruled out

Inflation and its successes

- Exponential expansion of inflation also gives `solution' to Flatness, Monopole and Horizon problems
- Typically requires ~60 e-folds of expansion to solve the above
- In simplest models prediction of near scale invariance is justified
- Inflation gives a natural means to reheat the universe required for starting the Hot Big Bang (provided couples to SM particles).
- Adding more fields allows greater freedom, but physics essentially unchanged
- Very robust as an idea!

... or Ekpyrosis

AJT+Wesley '07

- In collapsing universe with one field, scale invariance is only possible if $w = 0$ but highly unstable!

Finelli + Brandenberger astro-ph/0211276
Wands gr-qc/9809062

- In collapsing universe with more than one field possible to generate scale invariant isocurvature modes (also unstable) and subsequently convert them to adiabatic modes

Typically requires $w \gg 1$ this mechanism is known as:

EKPYROSIS

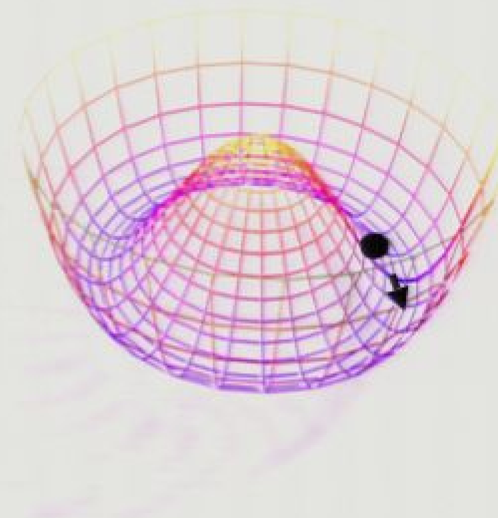
OLD: Khoury et al. hep-th/0103239
NEW: Lehners et al. hep-th/0702153
Buchbinder et al. hep-th/0702154
Creminelli et al. hep-th/0702165

Other alternatives exist but are not vacuum fluctuations (eg. thermal initial states) or not from a local field theory! Cosmic strings and textures can give scale invariance, but fluctuations are incoherent and hence ruled out

Inflation and its successes

- Exponential expansion of inflation also gives `solution' to Flatness, Monopole and Horizon problems
- Typically requires ~60 e-folds of expansion to solve the above
- In simplest models prediction of near scale invariance is justified
- Inflation gives a natural means to reheat the universe required for starting the Hot Big Bang (provided couples to SM particles).
- Adding more fields allows greater freedom, but physics essentially unchanged
- Very robust as an idea!

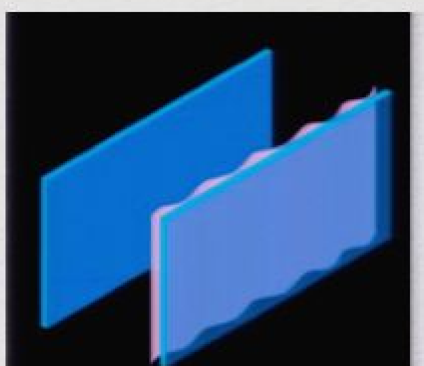
PROBLEMS OF INFLATION



- Zoo of inflationary models, how do we distinguish? What is the inflaton?
- What sets the initial conditions for inflation? A large fine-tuning is involved.
- Inflaton must start at the top of the hill, with small velocity, is this natural?
- Higgs is the only necessary scalar field in the Standard Model, but even that has not been observed and couldn't be the inflaton!!

Problems of ekpyrosis

- There is rapidly becoming a *menagerie* of ekpyrotic models
- Since the generation mechanism i unstable they have their own initi conditions problem
- Need to bounce from collapsing universe to expanding one. Much

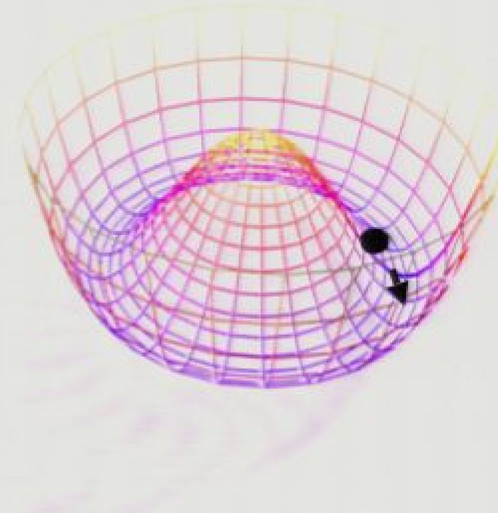


Inflation and its successes

- Exponential expansion of inflation also gives `solution' to Flatness, Monopole and Horizon problems
- Typically requires ~60 e-folds of expansion to solve the above
- In simplest models prediction of near scale invariance is justified
- Inflation gives a natural means to reheat the universe required for starting the Hot Big Bang (provided couples to SM particles).
- Adding more fields allows greater freedom, but physics essentially unchanged
- Very robust as an idea!

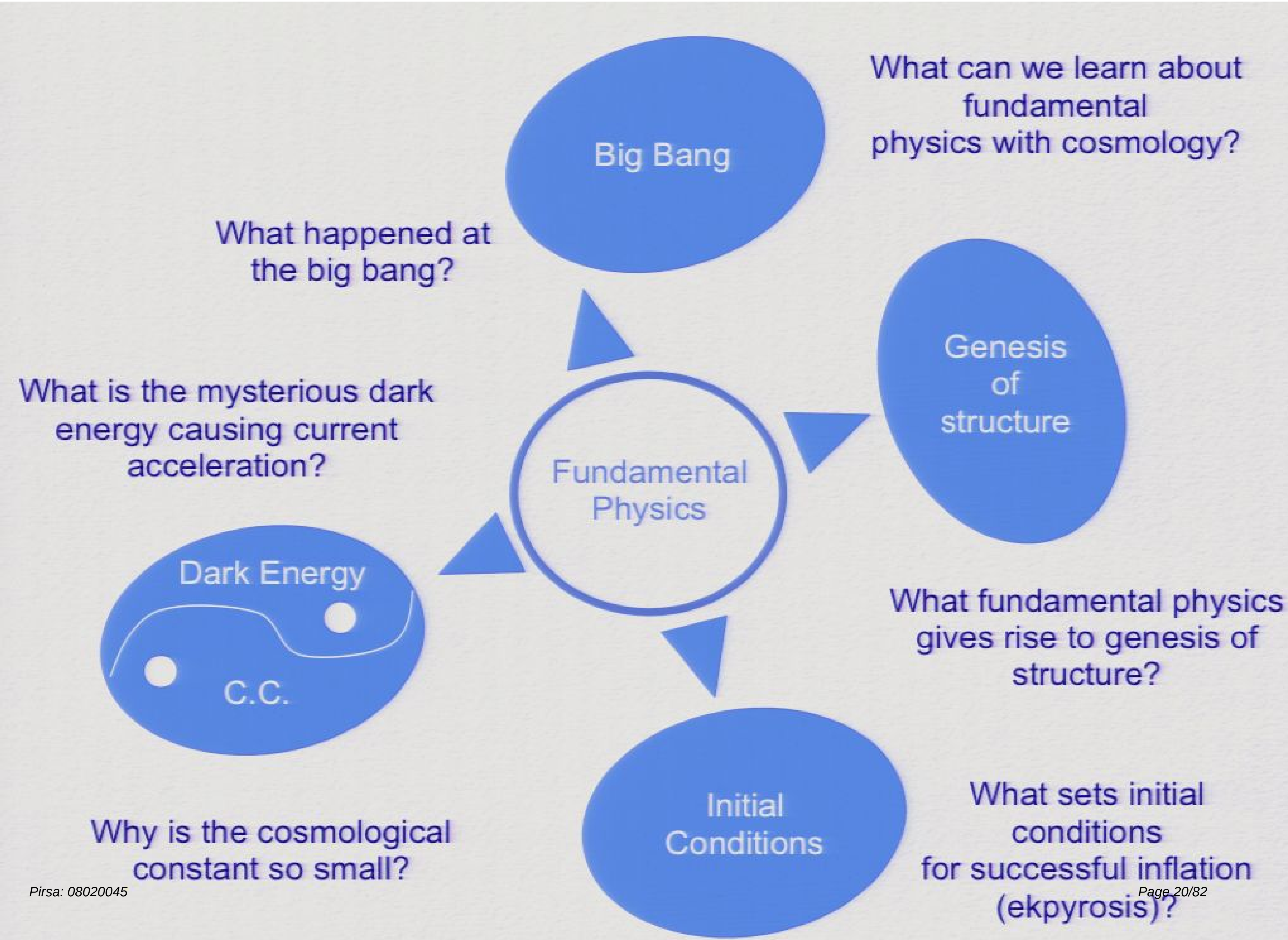
PROBLEMS OF INFLATION

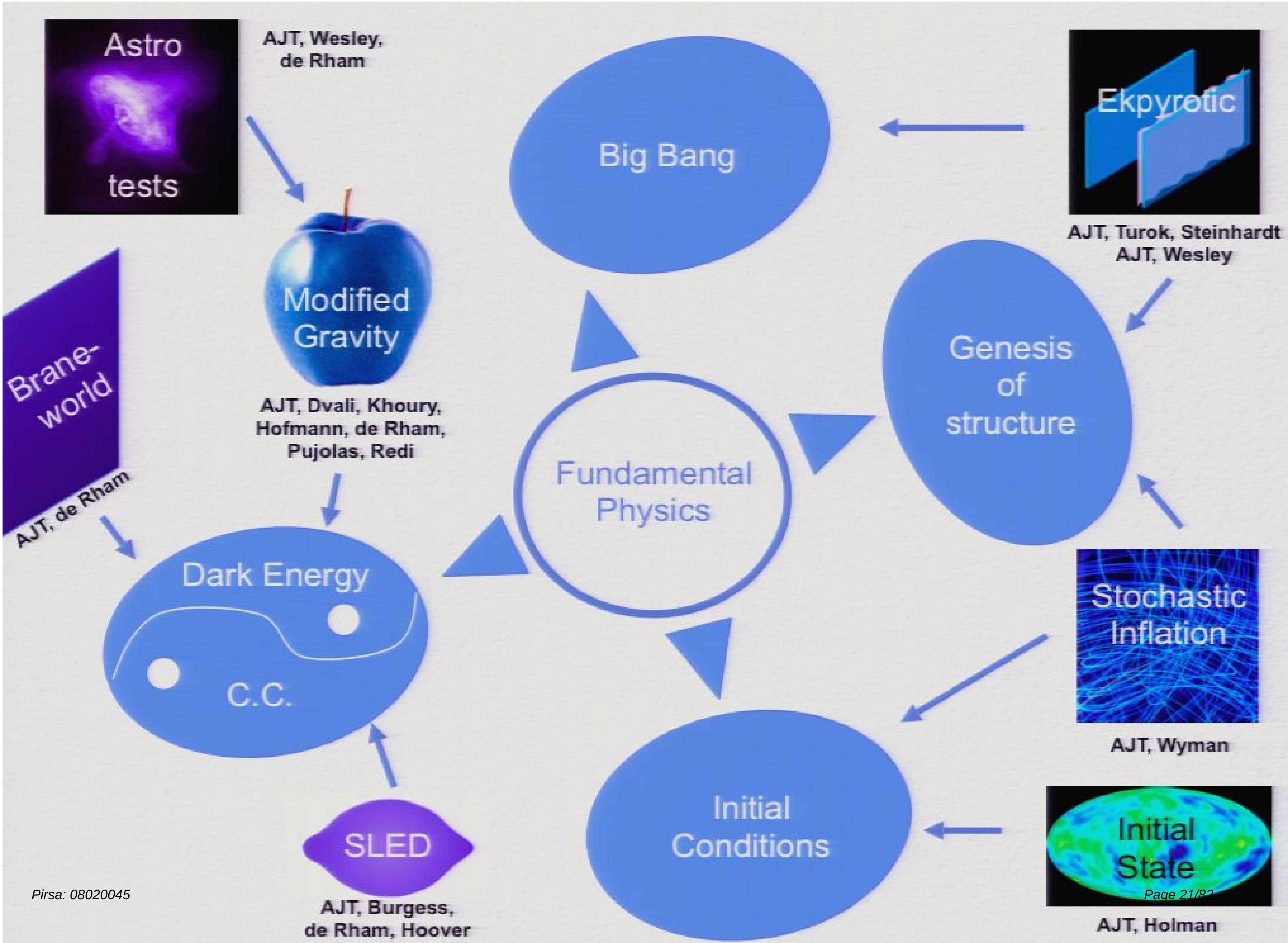
- Zoo of inflationary models, how do we distinguish? What is the inflaton?
- What sets the initial conditions for inflation? A large fine-tuning is involved.
- Inflaton must start at the top of the hill, with small velocity, is this natural?
- Higgs is the only necessary scalar field in the Standard Model, but even that has not been observed and couldn't be the inflaton!!



Problems of ekpyrosis

- There is rapidly becoming a *menagerie* of ekpyrotic models
- Since the generation mechanism i unstable they have their own initi conditions problem
- Need to bounce from collapsing universe to expanding one. Much

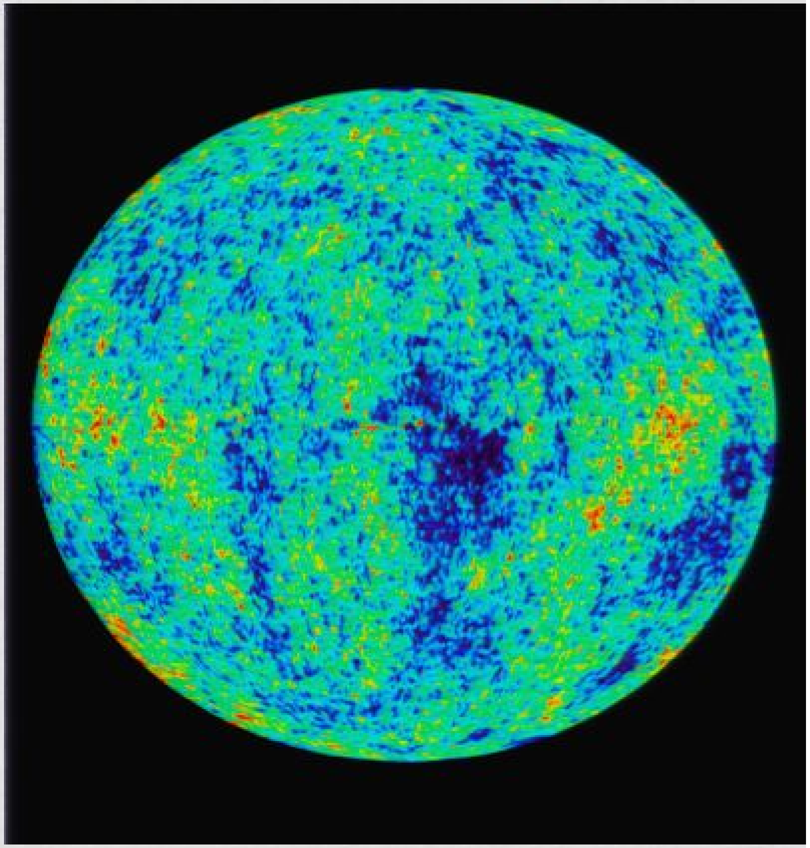

Big Bang
What can we learn about fundamental physics with cosmology?
What happened at the big bang?
Genesis of structure
What is the mysterious dark energy causing current acceleration?
Fundamental Physics
Dark Energy
C.C.
What fundamental physics gives rise to genesis of structure?
Initial Conditions
What sets initial conditions for successful inflation (ekpyrosis)?
Why is the cosmological constant so small?


Astro
tests
AJT, Wesley,
de Rham
Big Bang
Ekpyrotic
AJT, Turok, Steinhardt
AJT, Wesley
Modified
Gravity
AJT, Dvali, Khoury,
Hofmann, de Rham,
Pujolas, Redi
Brane-
world
AJT, de Rham
Genesis
of
structure
Fundamental
Physics
Dark Energy
C.C.
Stochastic
Inflation
AJT, Wyman
Initial
Conditions
SLED
AJT, Burgess,
de Rham, Hoover
Initial
State
AJT, Holman

INFLATION THE NEW ERA

Cosmology as a probe of fundamental physics

Cosmic Microwave Background



- Cosmology is one of our best probes of fundamental physics

1. Departure from scale invariance

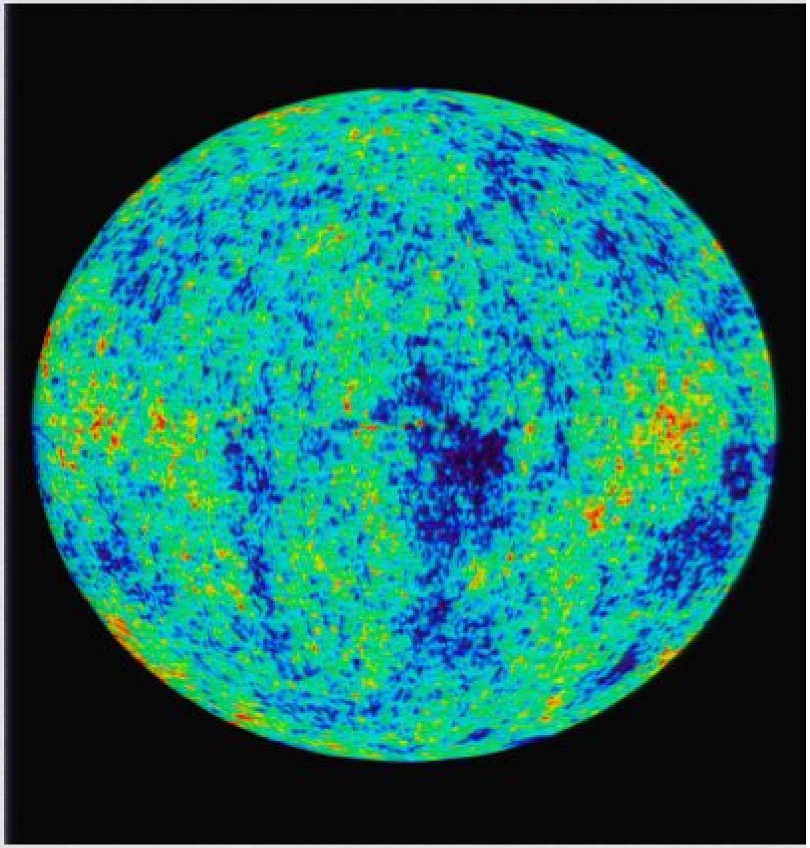
2. Gravity waves

3. Departure from Gaussianity

Testing all 3 is necessary to determine fundamental physics behind origin of large scale structure

Cosmology as a probe of fundamental physics

Cosmic Microwave Background



1. Departure from scale invariance

2. Gravity waves

* Contribution directly related to value of H in inflation.

* Their observation → big vacuum energy

* Usually negligible in ekpyrotic etc....

3. Departure from Gaussianity

Testing all 3 is necessary to determine fundamental physics behind origin of large scale structure

The first hints!

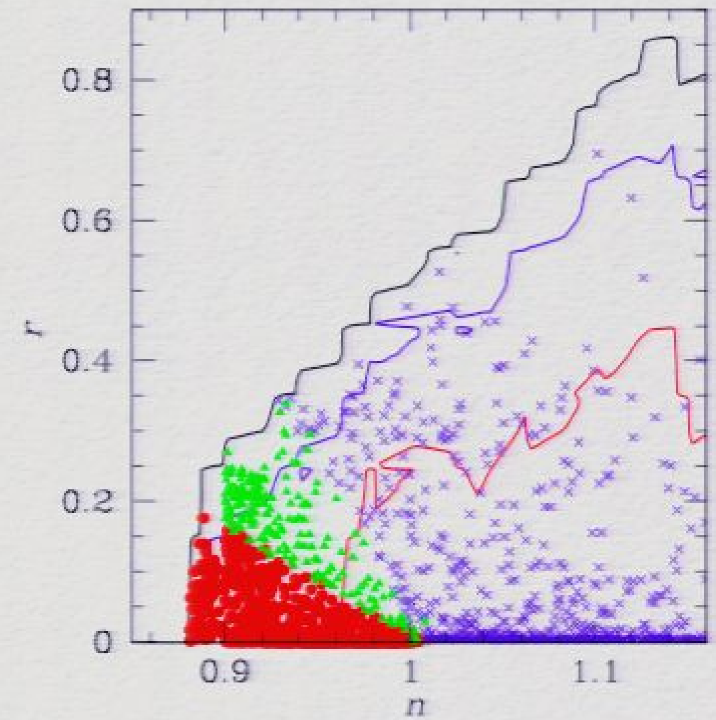
- Current WMAP data already favor departure from scale invariance.

$$n_s = 0.95 \pm 0.015$$ Spergel et al '07

- A red spectrum

- Most models of inflation predict

$$|n_s - 1| \sim \frac{1}{N_e}$$



Kinney et al `07

- Departure from scale invariance required by slow-roll paradigm but also by its alternatives!

The first hints!

- Reanalysis of WMAP data by Yadav + Wandelt has suggested that nG's have already been discovered in the WMAP data!

$$+27 < f_{NL}(\text{local}) < +147\,(2\sigma)$$

not included in range at 99.5% CL!

Yadav and Wandelt 0712.1148

$$\Phi(\vec{x}) \rightarrow \Phi_G(\vec{x}) + f_{NL}\Phi_G^2(\vec{x})$$

Newtonian potential

The future!

- In any case nG's will be measured with some accuracy in future data eg PLANCK potentially getting down to (including E polarization)

$$f_{NL} \approx 2.9$$

- very close to ideal experimental limit (including E polarization)

$$f_{NL} \approx 1.6$$

Babich et al. astro-ph/0408455

(limited by cosmic variance and gravitational lensing)

- Potential existence of nG's is far more significant discriminator of fundamental physics than departure from scale invariance!

A short course in cosmological perturbation theory

First we determine the background FRW geometry

$$ds^2 = a^2(\tau)(-d\tau^2 + d\vec{x}^2) \qquad H^2(\tau) = \left(\frac{a'}{a^2}\right)^2 = \frac{1}{3M_P^2}\rho - \frac{k}{a^2}$$

R^3 , S^3 , H^3

where the energy density $\rho(\tau)$ is given by

$$\rho = \frac{1}{2a^2} G^{AB}(\phi)\phi'_A\phi'_B + V(\phi)$$

and the scalar field equations of motion are

$$\left(a^2 G^{AB}(\phi)\phi'_A\right)' - a^4 \frac{\partial V}{\partial \phi_B} = 0$$

At linear level perturbations decompose into the representations of the spatial isometry group, E(3), SO(4), SO(1,3)

N SCALARS   0 ~~VECTORS~~   2 TENSORS

$$\phi^A(\tau, \vec{x}) = \phi_0^A(\tau) + \delta\phi^A(\tau, \vec{x})$$

Beyond linear order scalars and tensors couple,
coupling suppressed by inverse powers of M_{Planck}

Inflation well described by nonlinear scalar field theory + free gravitational waves (tensors) living on fixed FRW background

Focusing on the single field case: $\phi = \phi_0 + \delta\phi$

scalar tensor

$$ds^2 = -a^2 e^{2\Phi} d\tau^2 + a^2 e^{-2\psi} h_{ij} (dx^i - N^i d\tau)(dx^j - N^j d\tau)$$

$$Det(h) = 1$$

Φ and N_i are non-dynamical, fixed by constraints

Things considerably simplify if we work with....

$$\zeta = \psi + aH \frac{\delta\phi}{\phi_0'}$$

ζ is gauge invariant + conserved at long wavelengths

$$\zeta' \approx 0$$

crucially, can be defined nonlinearly, still conserved

Why is zeta conserved at long wavelengths?

AJT+Wyman '08

Textbook arguments attribute to conservation of energy

but much simpler to see that from the existence of a **scaling symmetry**, at long wavelengths: rescale the 3-space by

$\vec{x} \rightarrow e^c \vec{x}$ metric is invariant if rescale $\psi \rightarrow \psi + c$

Implies E.o.M. for ζ must be invariant under

$$\zeta \rightarrow \zeta + c$$

In turn implies one mode of ζ is always constant in time. Provided $w < 1$ this mode is always the `growing' mode: ζ conserved!

To quadratic order action for scalar fluctuations looks = massless scalar on spacetime with scale factor z

$$S = \frac{1}{2}\int d\tau d^3x z^2\left(\zeta'^2 - (\nabla\zeta)^2\right)$$

Exhibits promised symmetry!

$$\zeta \to \zeta + c$$

For constant equation of state w: $z \propto a$

If background is close to de Sitter, then power spectrum at long wavelengths is close to scale invariant:

Determines 2-pt function of temperature fluct's in CMB

$\langle \delta T(x)\delta T(y)\rangle$

+ matter power spectrum

$\langle \delta\rho(x)\delta\rho(y)\rangle$

$$\langle\zeta(x)\zeta(y)\rangle = \int \frac{d^3k}{(2\pi)^3}\frac{1}{k^3}\frac{H^2(k)}{4\epsilon^2 M_P^2}e^{ik.(x-y)}$$

evaluate at horizon crossing

scale invariant

Building up the Interactions: the slow roll case

$$S = \int d^4x e^{3\zeta}\mathcal{L}_2 + \left(\frac{\dot{\phi}^2}{H^2}\right) e^{3\zeta}(\epsilon,\eta)^s H^{n_1}\dot{\zeta}^{n_2}(e^{-\zeta}\nabla\zeta)^{n_3}$$

- In slow roll case we can build up interactions by using following principles:

1. ζ is conserved

$$\zeta \to \zeta + c$$

2. ϕ is approximately a goldstone boson

$$\phi \to \phi + c$$

3. Action contains either zero or two derivatives

- where

$$n_1 + n_2 + n_3 = (0,2)$$

- Looking at the details of Maldacena's calculation gives terms of this form

Maldacena
astro-ph/0210603

To quadratic order action for scalar fluctuations looks = massless scalar on spacetime with scale factor z

$$S = \frac{1}{2}\int d\tau d^3x z^2 \left(\zeta'^2 - (\nabla\zeta)^2\right)$$

Exhibits promised symmetry!

$$\zeta \to \zeta + c$$

For constant equation of state w: $z \propto a$

If background is close to de Sitter, then power spectrum at long wavelengths is close to scale invariant:

Determines 2-pt function of temperature fluct's in CMB

$\langle \delta T(x) \delta T(y) \rangle$

+ matter power spectrum

$\langle \delta\rho(x) \delta\rho(y) \rangle$

evaluate at horizon crossing

$$\langle \zeta(x)\zeta(y) \rangle = \int \frac{d^3k}{(2\pi)^3}\frac{1}{k^3}\frac{H^2(k)}{4\epsilon^2 M_P^2} e^{ik.(x-y)}$$

scale invariant

Building up the Interactions: the slow roll case

$$S = \int d^4x e^{3\zeta} \mathcal{L}_2 + \left(\frac{\dot{\phi}^2}{H^2}\right) e^{3\zeta} (\epsilon, \eta)^s H^{n_1} \dot{\zeta}^{n_2} (e^{-\zeta} \nabla \zeta)^{n_3}$$

- In slow roll case we can build up interactions by using following principles:

1. ζ is conserved
$$\zeta \to \zeta + c$$
2. ϕ is approximately a goldstone boson
$$\phi \to \phi + c$$
3. Action contains either zero or two derivatives

- where
$$n_1 + n_2 + n_3 = (0, 2)$$
- Looking at the details of Maldacena's calculation gives terms of this form

Maldacena
astro-ph/0210603

Consequence for slow roll models: non-gaussianity is suppressed by slow roll parameters

stripped of delta function

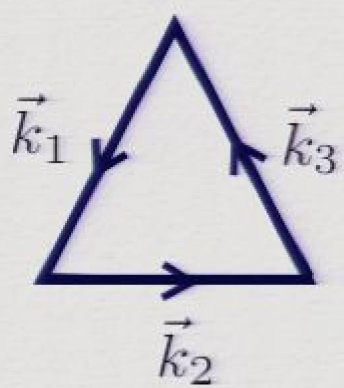
$$\frac{6}{5} f_{NL} = \frac{\langle \zeta(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3)\rangle}{P_\zeta(k_1)P_\zeta(k_2) + P_\zeta(k_2)P_\zeta(k_3) + P_\zeta(k_3)P_\zeta(k_1)}$$

where $\langle \zeta(\vec{k}_1)\zeta(\vec{k}_2)\rangle = (2\pi)^3 \delta^3(\vec{k}_1 + \vec{k}_2) P_\zeta(k_1)$

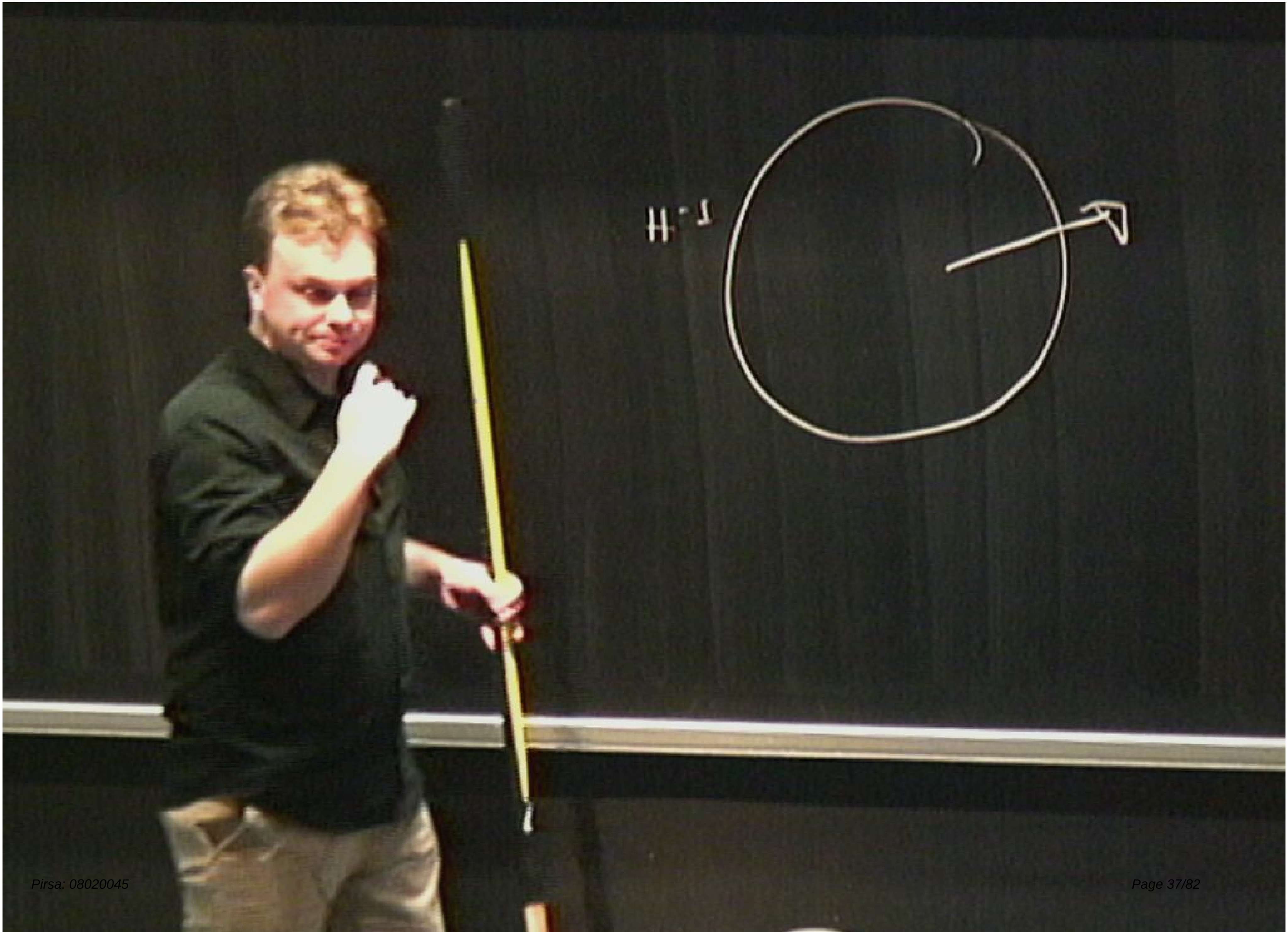
For single field slow roll: $f_{NL} \sim \epsilon + \eta$

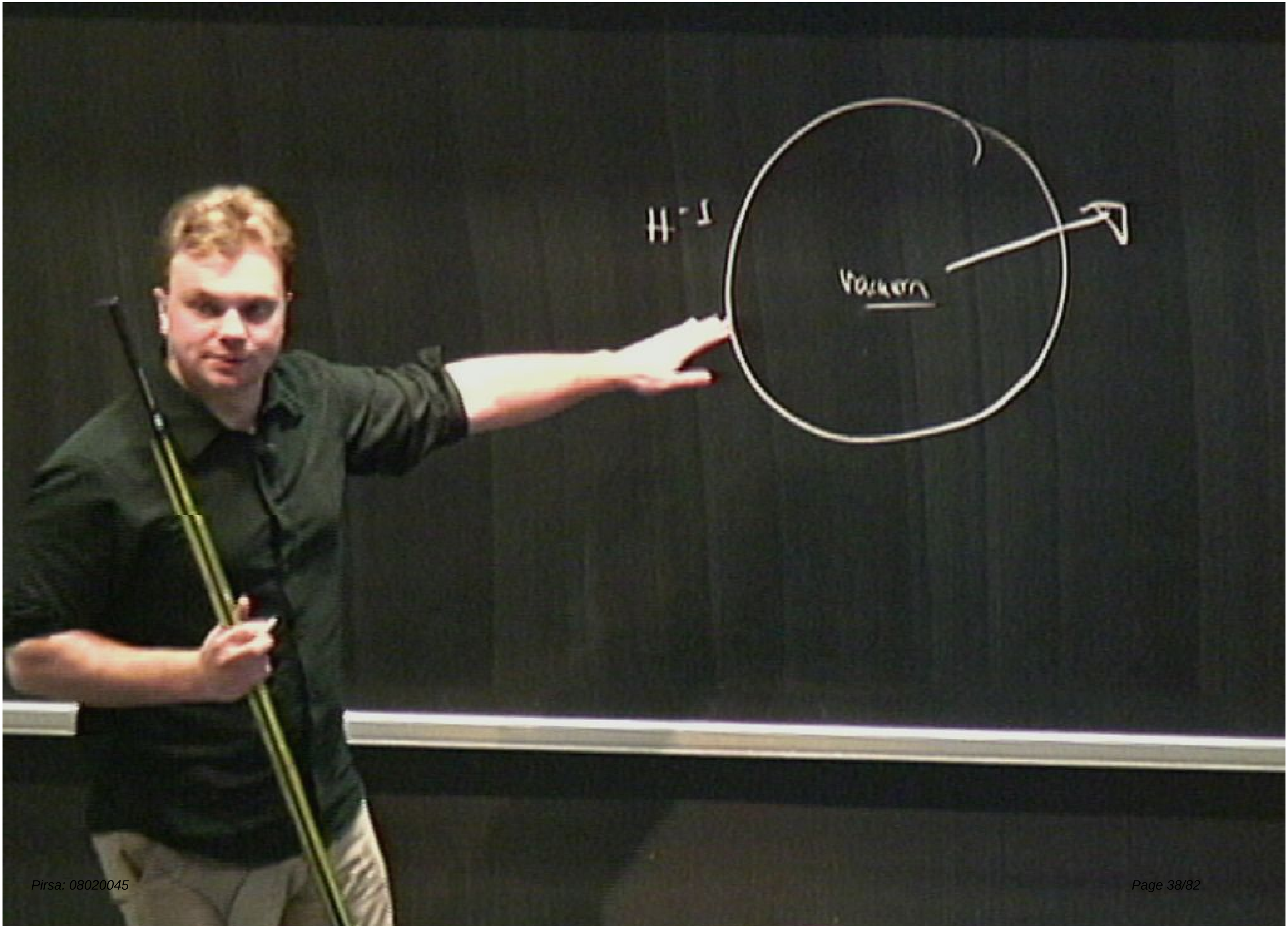
Which is dominated by equilateral triangles

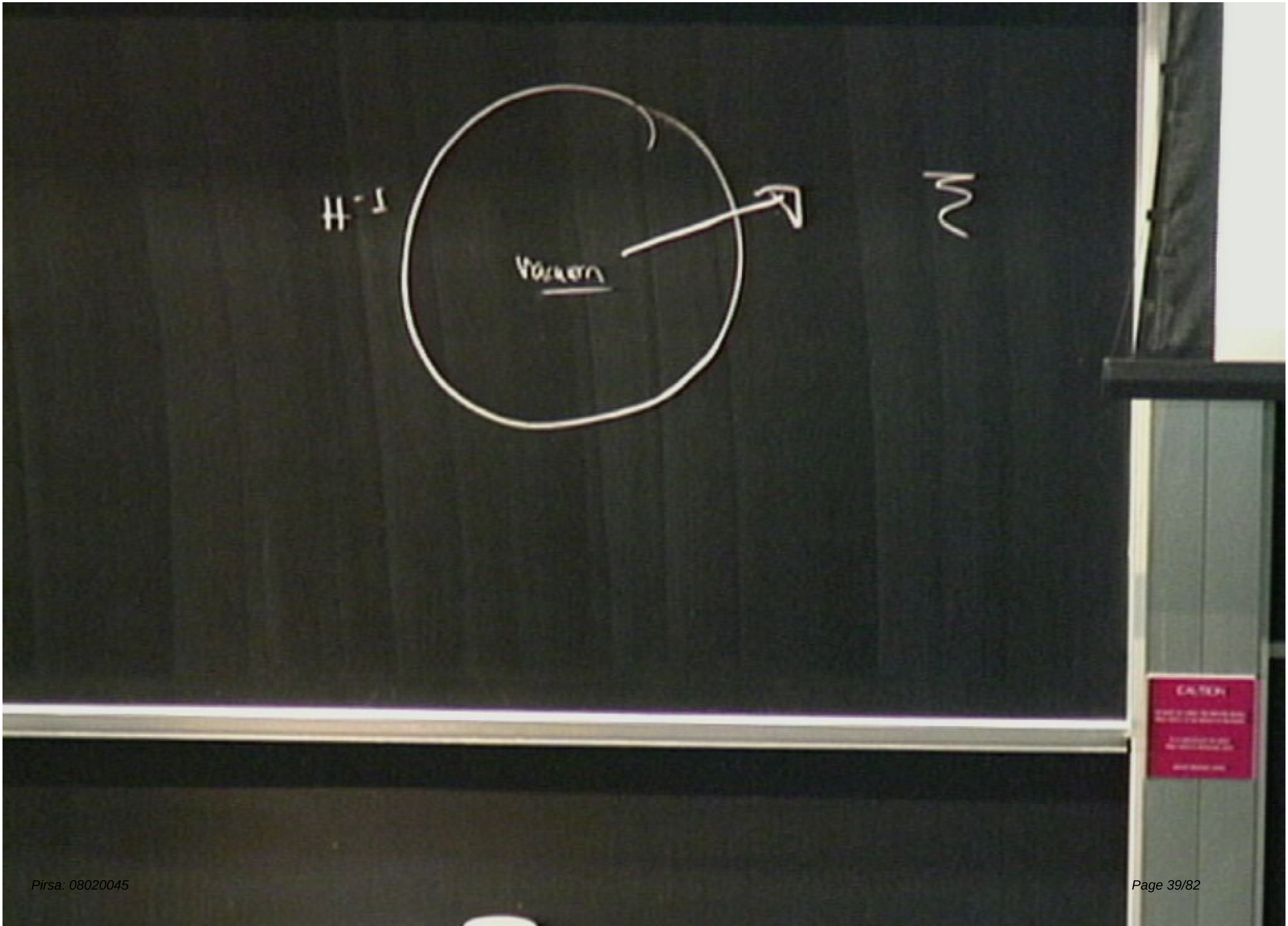
$\vec{k}_1 + \vec{k}_2 + \vec{k}_3 = 0$



dominant contribute when each mode crosses horizon




Vacuum

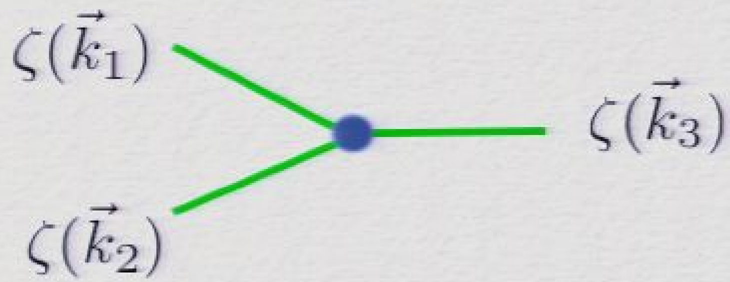

Vacuum

Non-Gaussianities measure inflaton interactions

nG's are to cosmologists what scattering experiments are to particle physicists: Measure strength of interactions!

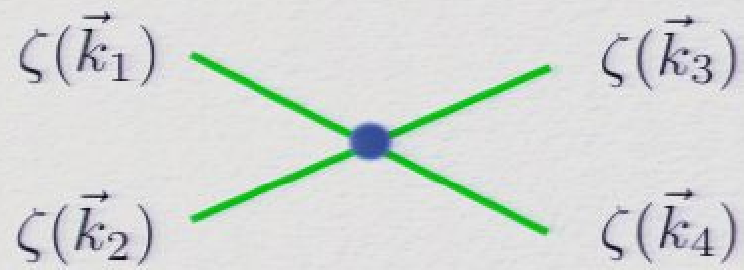
3pt point interactions:

$$S_I = \int d^4x \left(\frac{\dot{\phi}^2}{H^2}\right) f_{NL} H^{-1} \dot{\zeta}^3$$



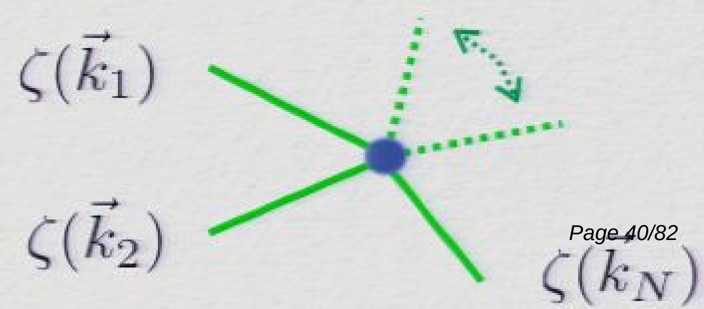
4pt point interactions:

$$S_I = \int d^4x \left(\frac{\dot{\phi}^2}{H^2}\right) \tau_{NL} H^{-2} \dot{\zeta}^4 + \dots$$



Npt point interactions:

$$S_I = \int d^4x \left(\frac{\dot{\phi}^2}{H^2}\right) f_{NL}^{(N)} H^{2-N} \dot{\zeta}^N + \dots$$

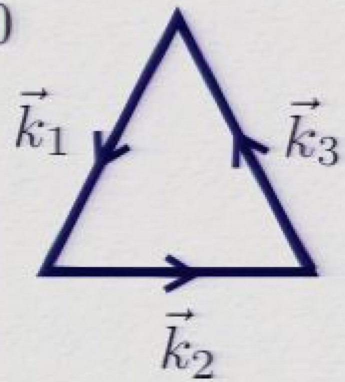


shapes from interactions:

most attention focused on two types of triangles

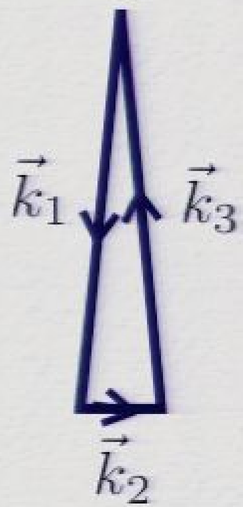
$\vec{k}_1 + \vec{k}_2 + \vec{k}_3 = 0$

horizon crossing:
equilateral



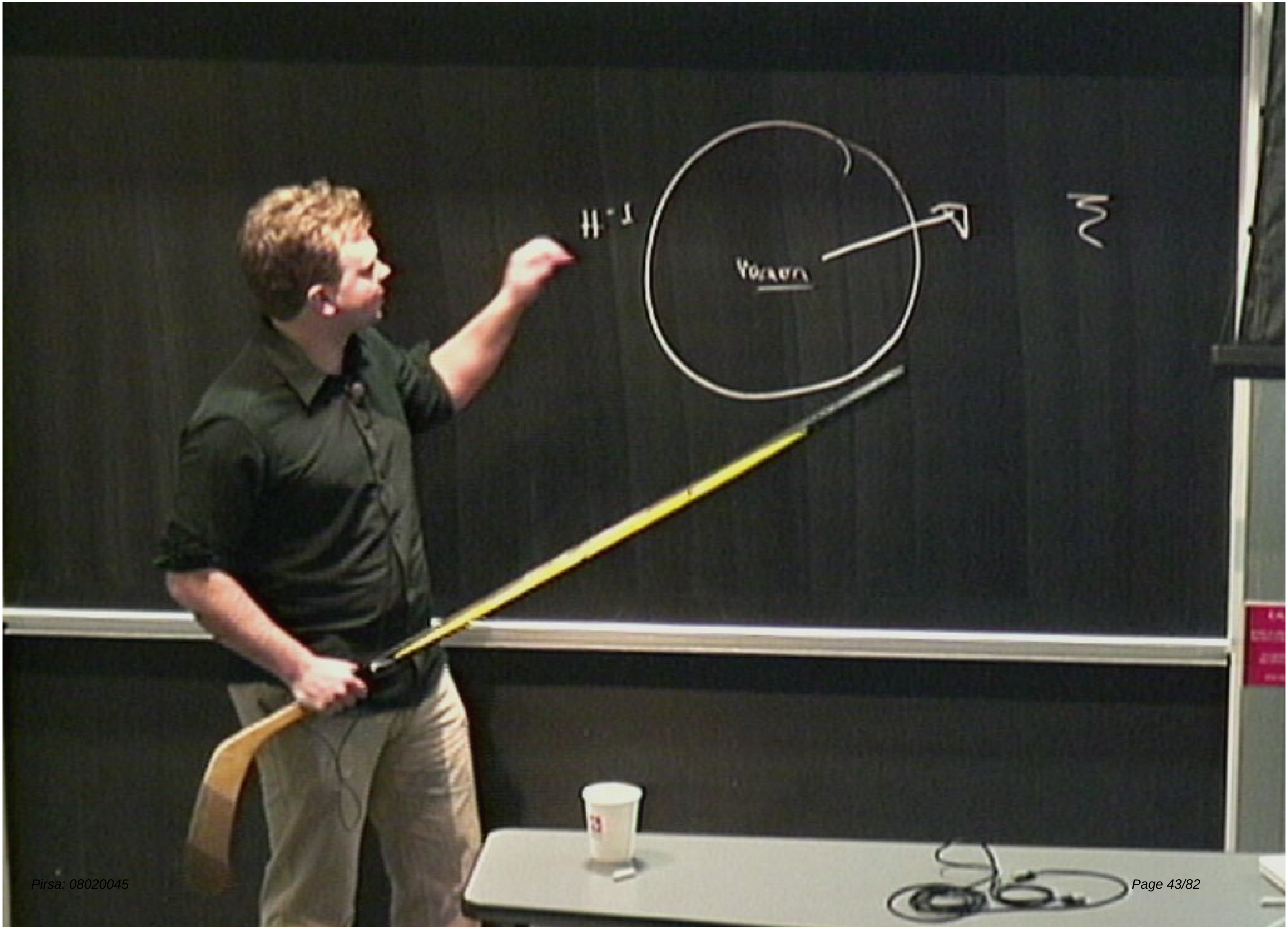
$|\vec{k}_1| \sim |\vec{k}_2| \sim |\vec{k}_3|$ Single field models

superhorizon:
local /squeezed



Curvaton models
Ekpyrotic models

$|k_i| \sim 0$ *conversion mechanism occurring after horizon crossing!*



but they also measure initial state!

AJT+Holman `07

Even a free theory can have nG's!

$$\psi(\zeta(\vec{x}),\tau) = e^{-\int d^3x d^3y \zeta(\vec{x}) K(\vec{x},\vec{y},\tau)\zeta(\vec{y})}$$
$$\times \; e^{-\int d^3x d^3y \int d^3z \zeta(\vec{x}) H(\vec{x},\vec{y},\vec{z},\tau)\zeta(\vec{y})\zeta(\vec{z}) + \ldots}$$

Directly contributes to f_{NL}

This effect can be pronounced since at subhorizon scales: $\zeta \sim \frac{1}{a} e^{\pm ik\tau}$

$$f_{NL} \sim \frac{\langle \zeta^3 \rangle}{\langle \zeta^2 \rangle^2} \sim \frac{a^{-3}}{(a^{-2})^2} \sim a$$

but they also measure initial state!

AJT+Holman `07

Even a free theory can have nG's!

$$\psi(\zeta(\vec{x}),\tau) = e^{-\int d^3x d^3y \zeta(\vec{x})K(\vec{x},\vec{y},\tau)\zeta(\vec{y})}$$
$$\times\ e^{-\int d^3x d^3y \int d^3z \zeta(\vec{x})H(\vec{x},\vec{y},\vec{z},\tau)\zeta(\vec{y})\zeta(\vec{z})+\dots}$$

Directly contributes to f_{NL}

This effect can be pronounced since at subhorizon scales: $\zeta \sim \frac{1}{a}e^{\pm ik\tau}$

$$f_{NL} \sim \frac{\langle\zeta^3\rangle}{\langle\zeta^2\rangle^2} \sim \frac{a^{-3}}{(a^{-2})^2} \sim a$$

Exciting the Initial State

- In recent years, considerable interest in testing initial state of inflaton via the CMB
- Most work has naturally focused on 2pt power spectrum
- Many crude models describing these effects
- However, possible to estimate the magnitude of effects based on general principles to be explained
- nG's could prove to be much more interesting probe of inflationary initial state

Backreaction bound on 2-pt function

Most direct bound comes from backreaction:

Energy density of the excited state cannot be bigger than inflaton energy density at beginning of inflation

Define excited initial state by Bogoliubov transformation

$$\left[\alpha_k a_k + \beta_k a_k^\dagger\right] |\psi\rangle = 0 \qquad |\alpha_k|^2 - |\beta_k|^2 = 1$$

annihilation/creation operators of adiabatic/Bunch-Davies vacuum

$$\langle\psi|T^0{}_0|\psi\rangle - \langle 0|T^0{}_0|0\rangle = \int \frac{d^3k}{(2\pi)^3} k|\beta_k|^2 \ll H^2 M_P^2$$

implies $\quad |\beta_k| \leq \frac{H M_P}{M_*^2}$

physical length scale of CMB modes at beginning of inflation

Transplanckian problem

Backreaction condition $$|\beta_k| \leq \frac{H M_P}{M_*^2}$$

implies if CMB modes were transplanckian at beginning of inflation $$M_* \geq M_P$$

$$\frac{\delta P_\zeta(k)}{P_\zeta(k)} \approx |\beta_k| \leq \frac{H}{M_P} \sim 10^{-6}$$

adding slow roll consistency

$$|\beta_k| \leq 10^{-8}$$

i.e. No Transplanckian Problem!
(unless modify energy density)

Best chance to see initial state effects is if inflation was sufficiently short (little more than required 60 efolds) + reheat temperature low so that CMB modes always cisplanckian

$$M_* \ll M_P$$

Backreaction bound on 2-pt function

Most direct bound comes from backreaction:

Energy density of the excited state cannot be bigger than inflaton energy density at beginning of inflation

Define excited initial state by Bogoliubov transformation

$$\left[\alpha_k a_k + \beta_k a_k^\dagger\right]|\psi\rangle = 0 \qquad |\alpha_k|^2 - |\beta_k|^2 = 1$$

annihilation/creation operators of adiabatic/Bunch-Davies vacuum

$$\langle\psi|T^0{}_0|\psi\rangle - \langle 0|T^0{}_0|0\rangle = \int \frac{d^3k}{(2\pi)^3} k|\beta_k|^2 \ll H^2 M_P^2$$

implies $$|\beta_k| \leq \frac{H M_P}{M_*^2}$$

physical length scale of CMB modes at beginning of inflation

Transplanckian problem

Backreaction condition $|\beta_k| \leq \frac{H M_P}{M_*^2}$

implies if CMB modes were transplanckian at beginning of inflation $M_* \geq M_P$

$$\frac{\delta P_\zeta(k)}{P_\zeta(k)} \approx |\beta_k| \leq \frac{H}{M_P} \sim 10^{-6}$$

adding slow roll consistency

$$|\beta_k| \leq 10^{-8}$$

i.e. No Transplanckian Problem!
(unless modify energy density)

Best chance to see initial state effects is if inflation was sufficiently short (little more than required 60 efolds) + reheat temperature low so that CMB modes always cisplanckian

$$M_* \ll M_P$$

Backreaction bound on 2-pt function

Most direct bound comes from backreaction:

Energy density of the excited state cannot be bigger than inflaton energy density at beginning of inflation

Define excited initial state by Bogoliubov transformation

$$\left[\alpha_k a_k + \beta_k a_k^\dagger\right] |\psi\rangle = 0 \qquad\qquad |\alpha_k|^2 - |\beta_k|^2 = 1$$

annihilation/creation operators of adiabatic/Bunch-Davies vacuum

$$\langle\psi|T^0{}_0|\psi\rangle - \langle 0|T^0{}_0|0\rangle = \int \frac{d^3k}{(2\pi)^3} k|\beta_k|^2 \ll H^2 M_P^2$$

implies $\quad |\beta_k| \leq \dfrac{H M_P}{M_*^2}$

physical length scale of CMB modes at beginning of inflation

Transplanckian problem

Backreaction condition $|\beta_k| \leq \frac{H M_P}{M_*^2}$

implies if CMB modes were transplanckian at beginning of inflation $M_* \geq M_P$

$$\frac{\delta P_\zeta(k)}{P_\zeta(k)} \approx |\beta_k| \leq \frac{H}{M_P} \sim 10^{-6}$$

adding slow roll consistency

$$|\beta_k| \leq 10^{-8}$$

i.e. No Transplanckian Problem!
(unless modify energy density)

Best chance to see initial state effects is if inflation was sufficiently short (little more than required 60 efolds) + reheat temperature low so that CMB modes always cisplanckian

$$M_* \ll M_P$$

Backreaction bound on 3-pt function

AJT+Holman
In progress

Similar bound exists for 3-pt function

$$\langle\psi|T^0{}_0|\psi\rangle - \langle 0|T^0{}_0|0\rangle \ll H^2 M_P^2$$

where l.h.s. picks up contributions from cubic terms in action

$$T^0{}_0 \sim \zeta'^3$$

This condition amounts to statement $\langle\dot{\zeta}^3(\vec{x})\rangle \lesssim H^3$ at beginning of inflation

since $\zeta \sim \frac{1}{a}$ $\quad \langle\zeta_H^3\rangle \lesssim \left(\frac{H}{M_*}\right)^6$

which implies $\quad f_{NL} \sim \frac{\langle\zeta_H^3\rangle}{\langle\zeta_H^2\rangle^2} \lesssim \epsilon^2 \frac{H^2 M_P^4}{M_*^6}$

nG's generated from initial state interactions

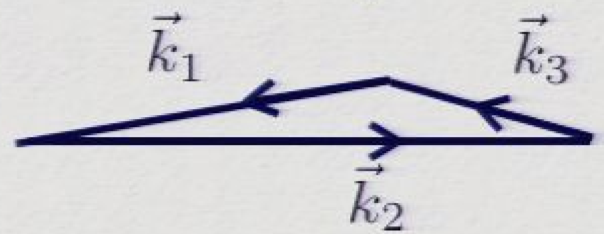
AJT+Holman `07

Even if initial state is gaussian but not Bunch-Davies, interactions in early stages of inflation will generate nG's

$$\langle \zeta_{k_1}\zeta_{k_2}\zeta_{k_3}(\tau)\rangle = -2\mathcal{R}e\int_{\tau_i}^{\tau} d\tau' i\langle \zeta_{k_1}\zeta_{k_2}\zeta_{k_3}(\tau)H_I(\tau')\rangle$$

$$\sim \beta_{k_1}\frac{A(k)}{k_2+k_3-k_1} + \text{permutations}$$

Resonance type interaction shows up in flattened triangles (energy conservation)



$$\delta\theta \sim \frac{H}{M_*}$$

For minimal slow-roll inflation effect is measure suppressed

$$\frac{\delta f_{NL}}{f_{NL}} \sim \beta_k \frac{M_*}{H} \times \frac{H}{M_*} \sim \beta_k \ll 1$$

Building up the Interactions: non-minimal case

Inflaton is effective degree of freedom in effective field theory

Expect `higher derivative correction' suppressed by cutoff

$$S = \int d^4x\sqrt{-g}\left(-\frac{1}{2}(\partial\phi)^2 - V - \frac{\lambda}{M^4}(\partial\phi)^4 + \dots\right)$$

These terms respect Goldstone shift symmetry $\phi \to \phi + c$

and so unsuppressed by slow roll parameters

$$S_I = \int d\tau d^3x a\frac{\lambda\dot{\phi}^4}{2H^3M^4}\zeta'(\zeta'^2 - (\nabla\zeta)^2) + \dots$$

Creminelli
astro-ph/0306122

Computing non-gaussianity: $f_{NL} \sim \lambda\frac{\dot{\phi}^2}{M^4}$

can easily be larger than ϵ, η if M is sufficiently low

Initial state effects in non-minimal case

AJT+Holman `07

For higher derivative interactions, resonance enhancement is greater

$$\langle \zeta_{k_1}\zeta_{k_2}\zeta_{k_3}(\tau)\rangle = -2\mathcal{R}e\int_{\tau_i}^{\tau} d\tau' i\langle \zeta_{k_1}\zeta_{k_2}\zeta_{k_3}(\tau)H_I(\tau')\rangle$$

$$\sim \beta_{k_1}\frac{A(k)}{(k_2+k_3-k_1)^2} + \text{permutations}$$

Chen et al.
astro-ph/0605045

After taking account of measure factor

$$\frac{\delta f_{NL}}{f_{NL}} \sim \beta_k\left(\frac{M_*}{H}\right)^2 \times \frac{H}{M_*} \sim \beta_k\frac{M_*}{H}$$

not a large effect, can easily turn an unobservable nG
into an observable one.

Non-minimal models give $f_{NL} \gtrsim 1$

This idea can be pushed to its extreme, consider general action of form

$$S = \int d^4x \sqrt{-g}\, p(\phi, (\partial\phi)^2)$$

Chen et al.
astro-ph/0605045

Gives rise to several new features:

1. Linear perturbations have nontrivial speed of sound

$$S_{(2)} = \int d\tau d^3x \frac{z^2}{2} \left(\zeta'^2 - c_s^2 (\nabla\zeta)^2\right) \qquad c_s^2 = \frac{p_{,X}}{\rho_{,X}}$$

2. Allowing $c_s \ll 1$ equivalent to higher derivative terms comparable to leading kinetic term

3. Can achieve near scale invariance without ϕ being a pseudo-Goldstone boson: no slow roll suppression in interactions

Initial state effects in non-minimal case

AJT+Holman `07

For higher derivative interactions, resonance enhancement is greater

$$\langle \zeta_{k_1}\zeta_{k_2}\zeta_{k_3}(\tau)\rangle = -2\mathcal{R}e\int_{\tau_i}^{\tau} d\tau' i\langle \zeta_{k_1}\zeta_{k_2}\zeta_{k_3}(\tau)H_I(\tau')\rangle$$

$$\sim \beta_{k_1}\frac{A(k)}{(k_2+k_3-k_1)^2} + \text{permutations}$$

Chen et al.
astro-ph/0605045

After taking account of measure factor

$$\frac{\delta f_{NL}}{f_{NL}} \sim \beta_k\left(\frac{M_*}{H}\right)^2 \times \frac{H}{M_*} \sim \beta_k\frac{M_*}{H}$$

not a large effect, can easily turn an unobservable nG
into an observable one.

Building up the Interactions: non-minimal case

Inflaton is effective degree of freedom in effective field theory

Expect `higher derivative correction' suppressed by cutoff

$$S = \int d^4x\sqrt{-g}\left(-\frac{1}{2}(\partial\phi)^2 - V - \frac{\lambda}{M^4}(\partial\phi)^4 + \dots\right)$$

These terms respect Goldstone shift symmetry $\phi \to \phi + c$

and so unsuppressed by slow roll parameters

$$S_I = \int d\tau d^3x a\frac{\lambda\dot{\phi}^4}{2H^3M^4}\zeta'(\zeta'^2 - (\nabla\zeta)^2) + \dots$$

Creminelli
astro-ph/0306122

Computing non-gaussianity: $f_{NL} \sim \lambda\frac{\dot{\phi}^2}{M^4}$

can easily be larger than ϵ, η if M is sufficiently low

Non-minimal models give $f_{NL} \gtrsim 1$

This idea can be pushed to its extreme, consider general action of form

$$S = \int d^4x \sqrt{-g}\, p(\phi, (\partial\phi)^2)$$

Gives rise to several new features:

Chen et al.
astro-ph/0605045

1. Linear perturbations have nontrivial speed of sound

$$S_{(2)} = \int d\tau d^3x \frac{z^2}{2} \left(\zeta'^2 - c_s^2 (\nabla\zeta)^2\right) \qquad c_s^2 = \frac{p_{,X}}{\rho_{,X}}$$

2. Allowing $c_s \ll 1$ equivalent to higher derivative terms comparable to leading kinetic term

3. Can achieve near scale invariance without ϕ being a pseudo-Goldstone boson: no slow roll suppression in interactions

An explicit example: DBI inflation

DBI inflation is string model of inflation consisting of D-brane moving in warped throat geometry

Silverstein et al. hep-th/0310221
Alishahiha et al. het-th/0404084

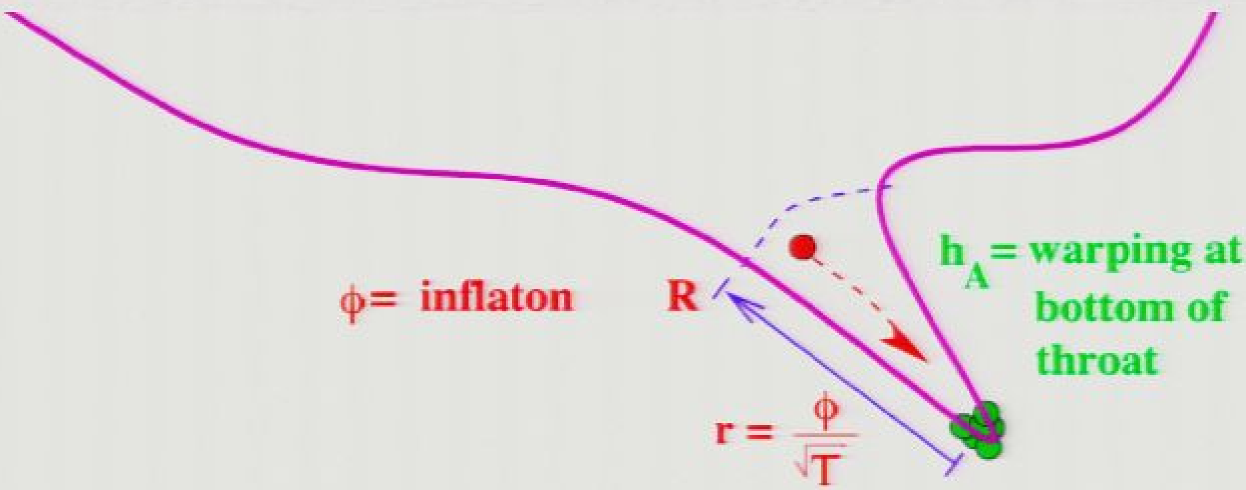
In brane inflation models, inflaton has a natural interpretation: the field corresponds to the distance between 2 branes, or moduli

Models:

* Brane-brane inflation
* Brane-Antibrane inflation
* D3/D7 brane inflation
* DBI Inflation

Dvali+Tye hep-th/9812482
+ considerable work

picture from hep-th/0702107, Bean et.al.



- The Lorentz factor, γ tracks the motion of the brane

$$\gamma(\phi) = \frac{1}{\sqrt{1-\dot{\phi}^2/T(\phi)}}$$

- The inflaton speed is limited by the warped tension $T(\phi)$,

$$\dot{\phi}^2 < T(\phi)$$

- T decreases towards the bottom of the throat

⇒ the limiting speed decreases rapidly
inflaton can be moving slowly but ultra-relativistically
predictions quite different from usual slow-roll.

nG's in DBI inflation

Effective 4 dimensional theory is determined by Dirac-Born-Infeld action. Essentially the action for a relativistic particle with a space dependent mass

$$S_{DBI} = \int d^4x\sqrt{-g}\left(-T(\phi)\sqrt{1+\frac{(\partial\phi)^2}{T(\phi)}} + T(\phi) - V(\phi)\right)$$

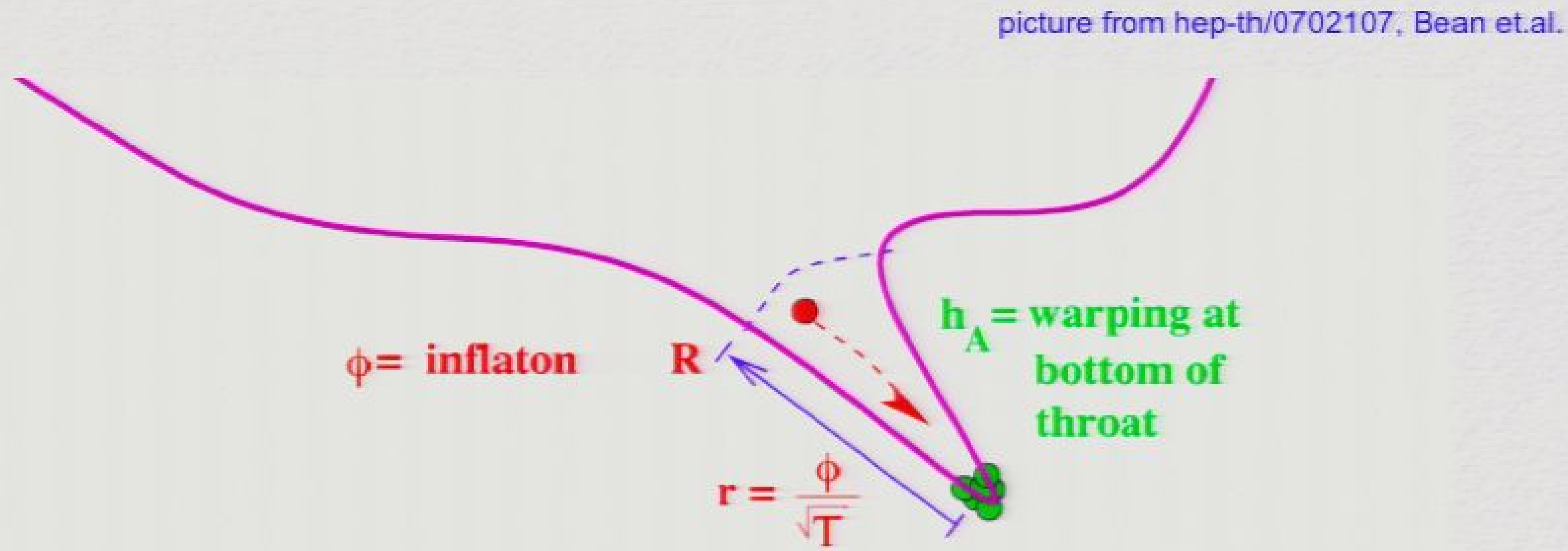
where $T(\phi)$ is the tension of the brane $V(\phi) = \frac{1}{2}m^2\phi^2 + V_o\left(1 - \frac{V_o}{4\pi v\phi^4}\right)$

in simplest models $T(\phi) = \frac{(\phi^2+b^2)^2}{\Lambda^2}$

$$c_s^2 = \frac{1}{\sqrt{1+\frac{(\partial\phi)^2}{T(\phi)}}}$$

giving following non-gaussian term

$$f_{NL} \approx \frac{1}{c_s^2} - 1 + \text{slow roll contributions}$$



- The Lorentz factor, γ tracks the motion of the brane

$$\gamma(\phi) = \frac{1}{\sqrt{1 - \dot{\phi}^2/T(\phi)}}$$

- The inflaton speed is limited by the warped tension $T(\phi)$,

$$\dot{\phi}^2 < T(\phi)$$

- T decreases towards the bottom of the throat

→ the limiting speed decreases rapidly
inflaton can be moving slowly but ultra-relativistically
predictions quite different from usual slow-roll.

nG's in DBI inflation

Effective 4 dimensional theory is determined by Dirac-Born-Infeld action. Essentially the action for a relativistic particle with a space dependent mass

$$S_{DBI} = \int d^4x\sqrt{-g}\left(-T(\phi)\sqrt{1+\frac{(\partial\phi)^2}{T(\phi)}} + T(\phi) - V(\phi)\right)$$

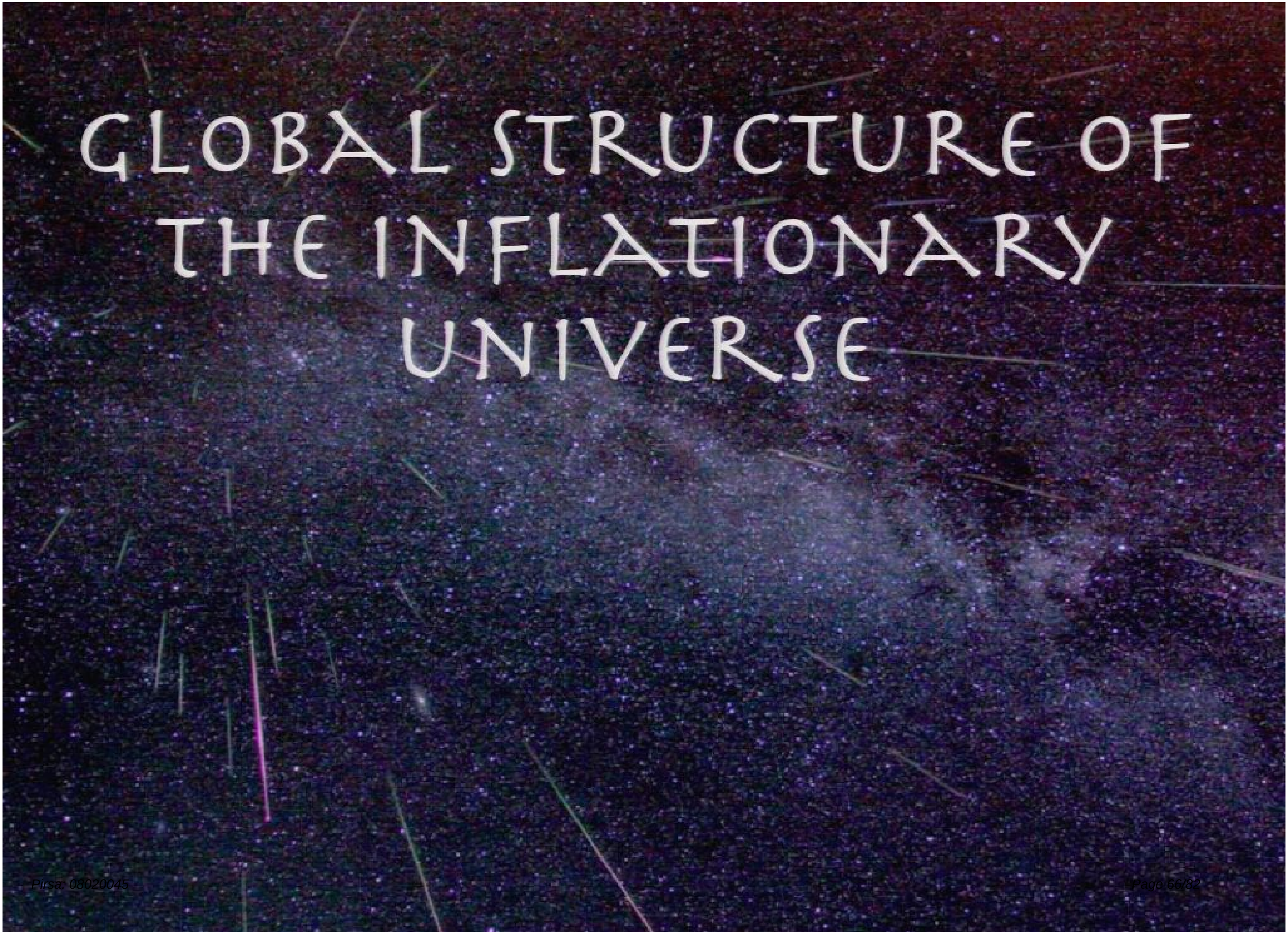
where $T(\phi)$ is the tension of the brane $V(\phi) = \frac{1}{2}m^2\phi^2 + V_o\left(1 - \frac{V_o}{4\pi v\phi^4}\right)$

in simplest models $T(\phi) = \frac{(\phi^2+b^2)^2}{\Lambda^2}$

$$c_s^2 = \frac{1}{\sqrt{1+\frac{(\partial\phi)^2}{T(\phi)}}}$$

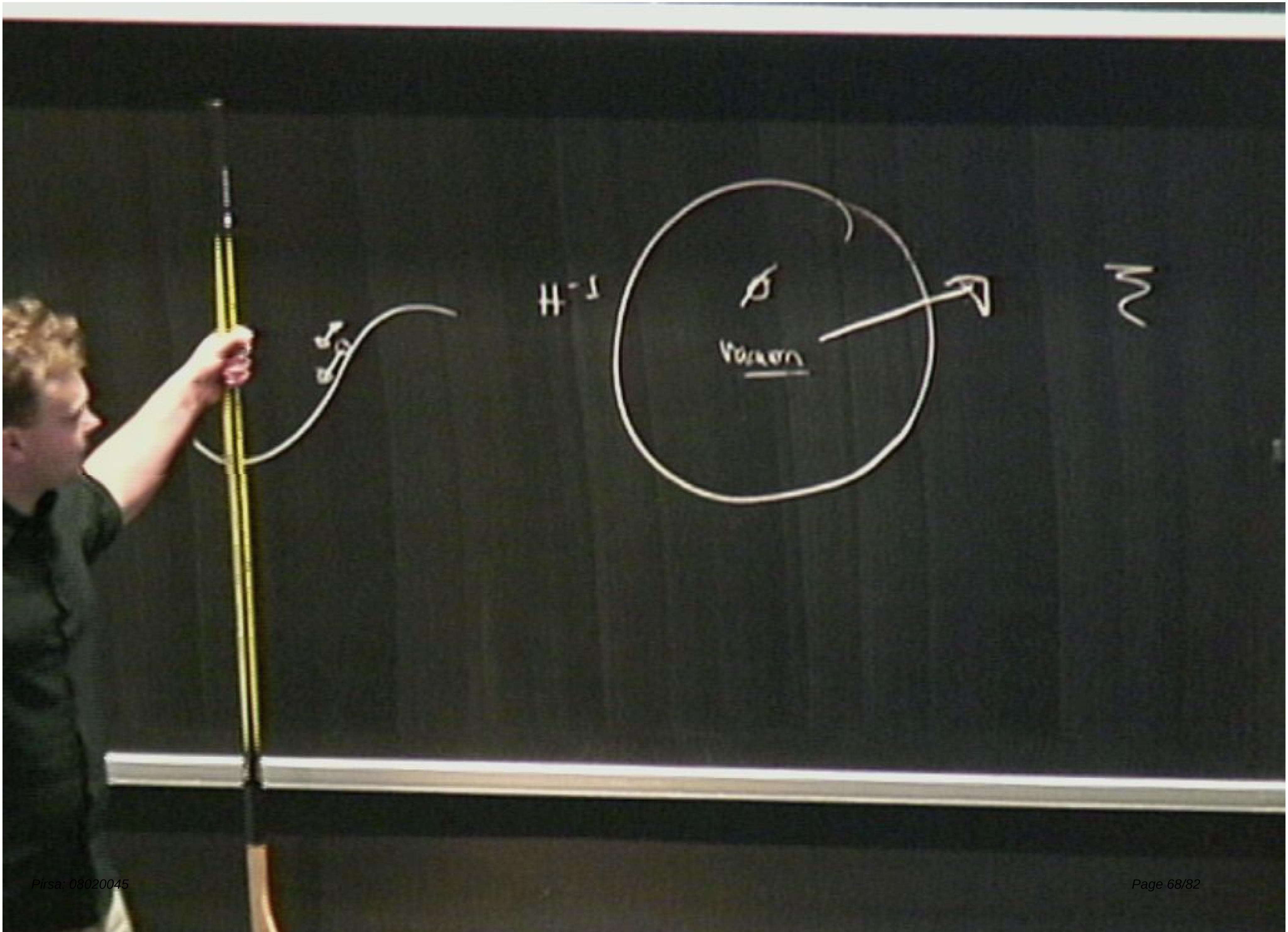
giving following non-gaussian term

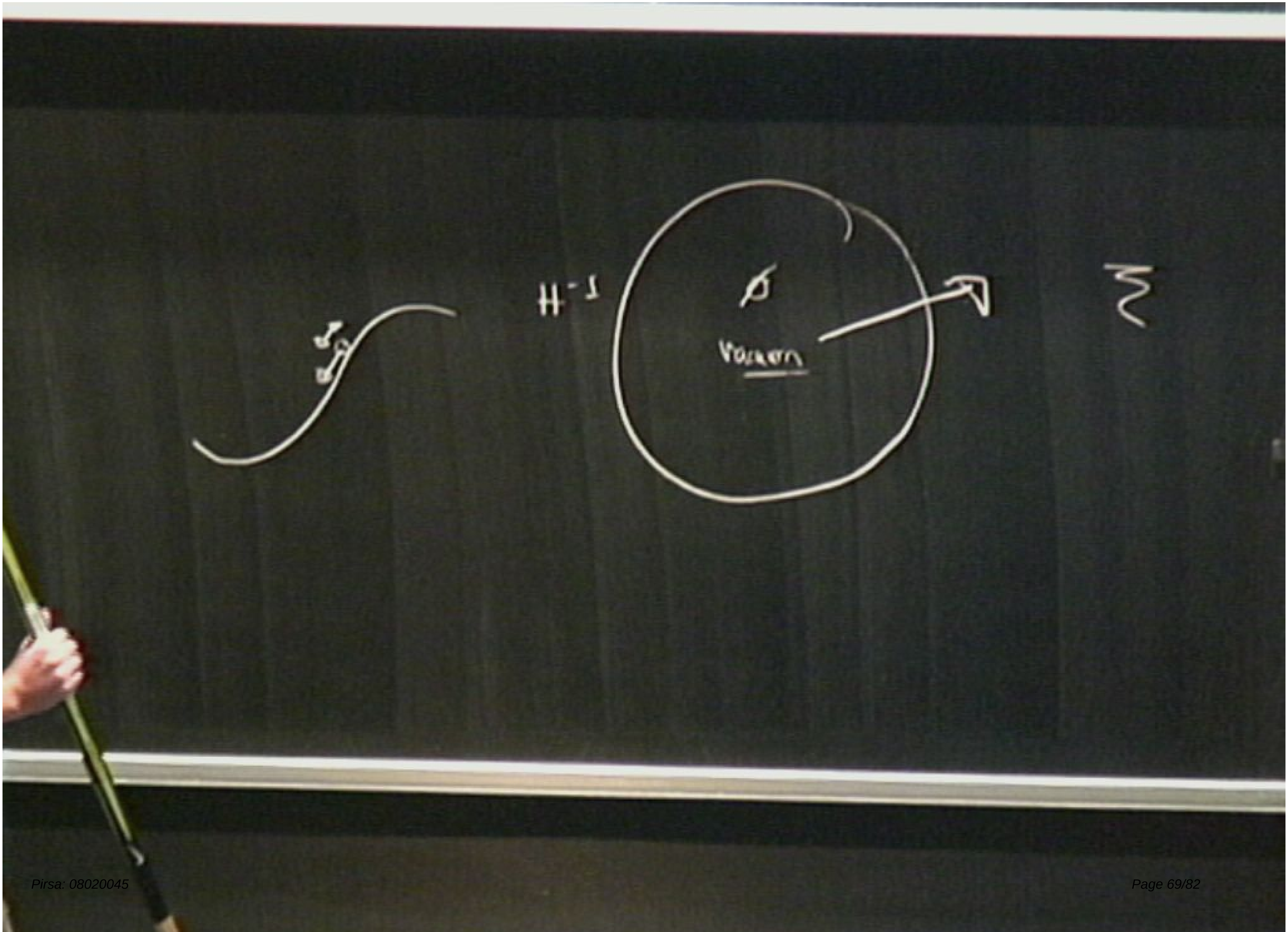
$$f_{NL} \approx \frac{1}{c_s^2} - 1 + \text{slow roll contributions}$$

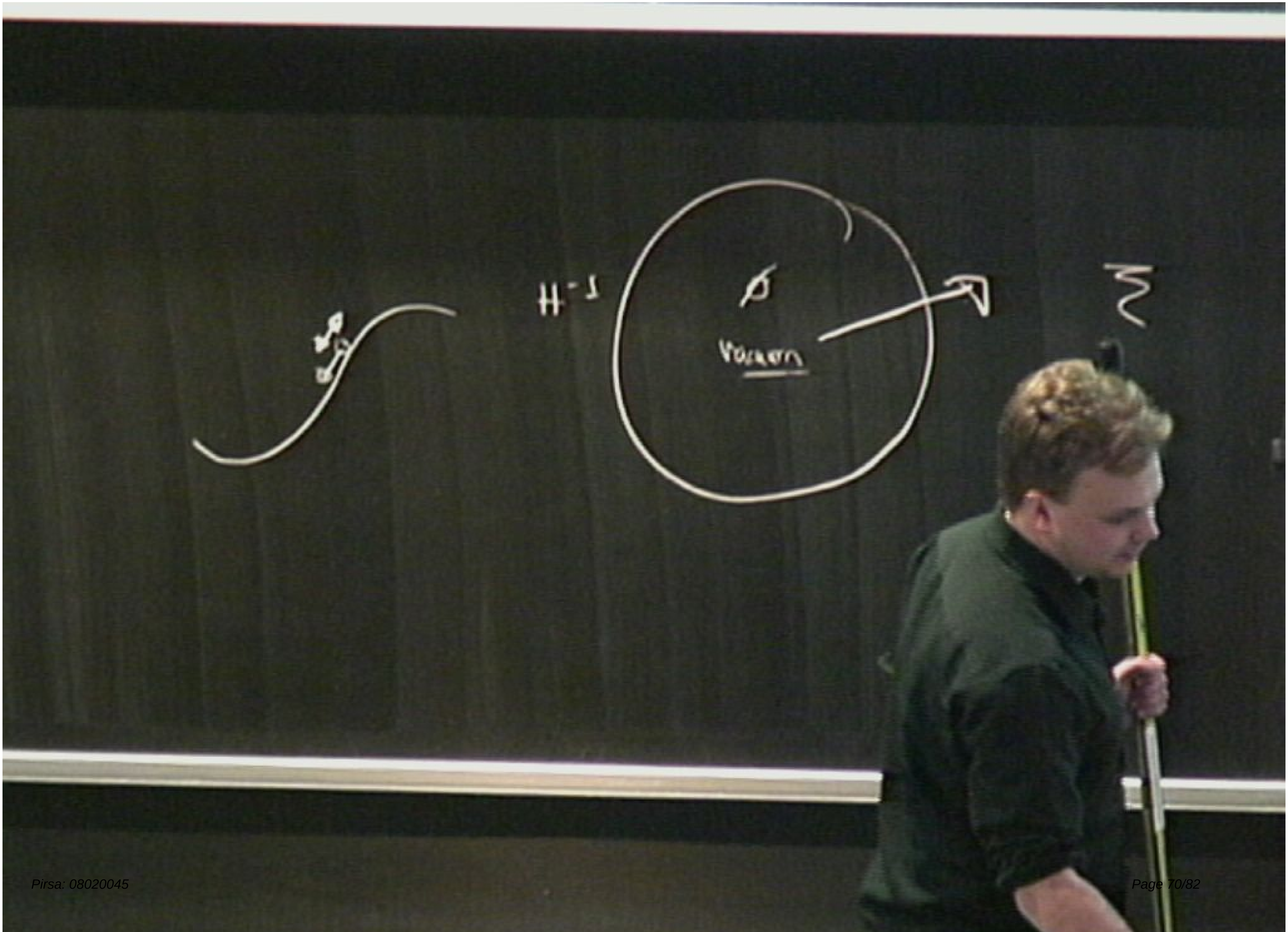

GLOBAL STRUCTURE OF
THE INFLATIONARY
UNIVERSE

Probabilities in the multiverse

- Proper treatment of the inflationary universe requires understanding probability on space of solutions
- Does eternal inflation happen in DBI models?
- How do we describe tunneling in DBI models?
- String landscape predicts multitude of vacua and multiple fields, how likely is it that a universe like ours can arise?
- One framework: Stochastic Inflation
- Classical Evolution combined with quantum fluctuations!


vacuum


H^{-1}
Vacuum


Vacuum

Coarse Graining Inflation

Key idea that governs cosmology is existence of a curvature scale $cH^{-1} \approx cT$ which separates long and short wavelengths

Superhorizon + subhorizon modes decouple i.e. weakly interacting

We can divide system up into
SYSTEM = Long wavelengths
ENVIRONMENT = Short wavelengths

Naturally leads us to think about coarse graining,
i.e. integrating out short wavelength modes

$$\hat{\rho} = Tr_{\mathrm{k>aH}}|\psi\rangle\langle\psi|$$

This is `dual' of de Sitter complementarity picture

Coarse Graining Inflation

Key idea that governs cosmology is existence of a curvature scale
$cH^{-1} \approx cT$ which separates long and short wavelengths

Superhorizon + subhorizon modes decouple i.e. weakly interacting

We can divide system up into
SYSTEM = Long wavelengths
ENVIRONMENT = Short wavelengths

Naturally leads us to think about coarse graining,
i.e. integrating out short wavelength modes

$$\hat{\rho} = Tr_{\mathrm{k>aH}}|\psi\rangle\langle\psi|$$

This is `dual' of de Sitter complementarity picture

Stochastic Inflation: beyond slow roll

AJT+Wyman `08

Coarse graining is achieved by splitting the field as

$$\phi(x) = \phi_L(x) + \phi_S(x) = \sum \int \frac{d^3k}{(2\pi)^3}\theta(\pm(k - aH))\phi(k,\tau)$$

technically simplest to work in phase space, i.e. Wigner distributions!

$$\rho(\phi, \pi)$$

Neglecting subhorizon fluctuations:

Hubble damping

$$\frac{\partial \rho}{\partial \lambda} = \frac{\partial}{\partial \phi}\left(-\frac{\partial H_{\rm eq}}{\partial \pi}\rho\right) + \frac{\partial}{\partial \pi}\left(3\pi\rho + \frac{\partial H_{\rm eq}}{\partial \phi}\rho\right)$$

e-folds

Including fluctuations

Fokker-Planck equation

AJT+Wyman `08

$$\frac{\partial \rho}{\partial \lambda} = \frac{\partial}{\partial \phi}\left(-\frac{\partial H_{\rm eq}}{\partial \pi}\rho\right) + \frac{\partial}{\partial \pi}\left(3\pi\rho + \frac{\partial H_{\rm eq}}{\partial \phi}\rho + D\frac{\partial \rho}{\partial \pi}\right)$$

Subhorizon fluctuations
induce a diffusion term

Describes a stochastic system in which long wavelength dynamics evolves classically, losing 'energy' through damping

energy is input by noise/diffusion from short wavelength modes

These two processes eventually balance leading to an `equilibrium'

equilibrium = eternal inflation

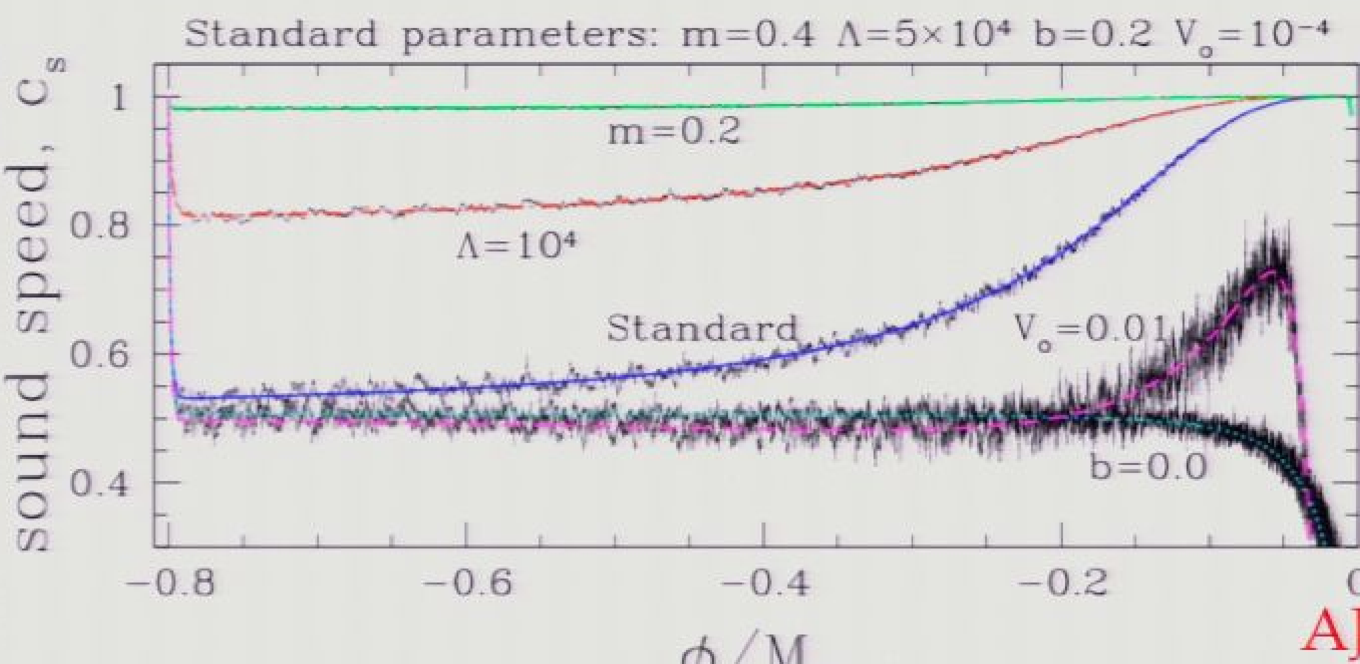
Application: DBI inflation

In the case of DBI inflation described by the effective action

$$S_{DBI} = \int d^4x\sqrt{-g}\left(-T(\phi)\sqrt{1+\frac{(\partial\phi)^2}{T(\phi)}}+T(\phi)-V(\phi)\right)$$

the Hamiltonian is given by

$$H^2_{\rm eq} = 12M_P^2\left(T\sqrt{1+T^{-1}\pi^2}-T+V\right)$$



AJT+Wyman `08

Eternal Inflation and the strong coupling regime

Fokker-Planck equation is a good description of evolution from classical regime to onset of eternal inflation

Eternal inflation is defined as period when quantum fluctuations become comparable to classical $\langle\zeta^2\rangle \sim 1$

This in turn implies $\langle\zeta^3\rangle \sim f_{NL}$

Leblond+Shandera
0802.2290

Eternally inflating region is weakly coupled if $f_{NL} \lesssim 1$

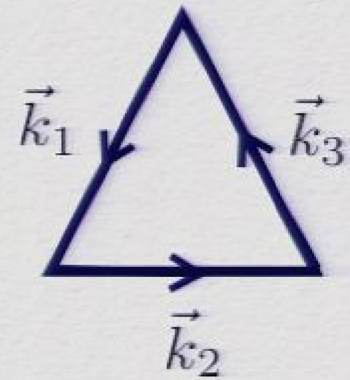
and strongly coupled if $f_{NL} \gtrsim 1$

New physics at work!

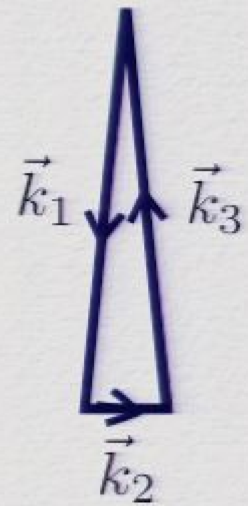
Conclusions

- Near future (+ possibly existing!) data will tell us whether theory that describes evolution of structure was weakly coupled or not.
- Essentially any alternative to slow roll inflation gives $f_{NL} \gtrsim 1$
- Shape dependence tells us whether from superhorizon physics, horizon crossing physics or initial state interactions

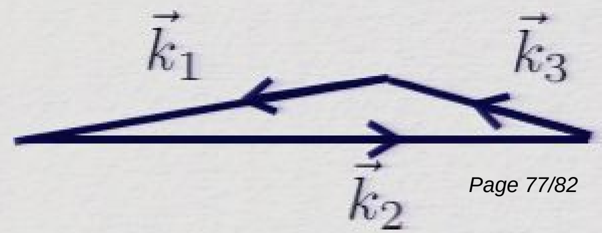
horizon crossing: equilateral



superhorizon: local /squeezed



initial state: flattened



Conclusions

- None-weakly coupled physics will provide a marvellous theoretical challenge, and more accurate window into fundamental theory.
- nG's in flattened triangles would give us new information on the inflationary initial state
- In either case it will force a rethink in the conceptual underpinnings of inflation and its alternatives eg. eternal inflation