Title: Introduction of bosonic fields into causal set theory

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Abstract: The purpose of this talk is to describe bosonic fields and their Lagrangians in the causal set context. Spin-0 fields are defined to be real-valued functions on a causal set. Gauge fields are viewed as SU(n)-valued functions on the set of pairs of elements of a causal set, and gravity is viewed as the causal relation itself. The purpose of this talk is to come up with expressions for the Lagrangian densities of these fields in such a way that they approximate the Lagrangian densities expected from regular Quantum Field Theory on a differentiable manifold in the special case where the causal set is a random sprinkling of points in the manifold. I will then conjecture that that same expression is appropriate for an arbitrary causal set.

BOSONIC FIELDS AND THEIR LAGRANGIANS IN CAUSAL SET THEORY

Roman Sverdlov, University of Michigan

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QFT in causal sets = extrapolation

Example of extrapolation: trigonometric functions of non- reals

Geometry → Taylor series a theorem → Taylor series a definition → no geometry

Coordinates → Theorem: L = . . . → Definition: L = . . . → No coordinates

Trigonometry:

a) Problem: compute the sin of an angle without a device to measure angles and/or a ruller to measure lengths. NOTE: angles and lengths DO exist but are unknown.

b) Solution: Taylor Series

c) Definition: neither angles nor lengths exist. Taylor series is definition of trigonometric function

Key: Unknown → Non-existant

GETTING RID OF COORDINATES IN QFT:

a)Given:

(i) Lorentzian manifold, which both has coordinates and is a continuum
(ii) $p_1, \ldots, p_n$ are Poisson distribution of points in bounded region of that space
(iii) φ, $A^{a\mu}$, and $g_{\mu\nu}$ are differentiable functions on Minkowski space that vary very slowly compared to the density of scattered points
(iv) Causal relations between the points
(v) $\varphi(p_1)=\ldots, \varphi(p_2)=\ldots, \ldots, \varphi(p_n)=\ldots$
(vi) $\exp \int_{l(p_i p_j)} T^a A^a{}_\mu dx^\mu = U(p_i, p_j)$ where $l(p_i p_j)$ is a geodesic connecting p_i and p_j

Unknowns: p_i^μ, $A^{a\mu}(p_i)$ and $g_{\mu\nu}(p_i)$

Find: Lagrangian densities associated with each of the above fields around $p_1, \ldots, p_n$

b) Solution: some expressions that don't explicitly involve coordinates

c)Definition: expressions in part b are actually definitions of Lagrangian densities for causal sets

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SCALAR FIELD

SIMPLER PROBLEM: EXTRA ASSUMPTIONS

(vii) We are in a FLAT Minkowski space
(vii) φ is linear

$S = \{P_1, \ldots, P_n\}$

We are interested in evaluating $L(\varphi ; p)$ where $p \in S$

Find $q \in S$ such that $\#(S \cap \alpha(p,q)) > M$ where $\alpha(p,q) = \{r | p < r < q\}$ is Alexandroff set based on p and q.

Select coordinate system where t axis passes through p and q with origin lying between these two points.

$$\varphi(x) = a^{\mu} x_{\mu} + b$$

$$\alpha(p,q) = \{r | p \prec r \prec q\}$$

$$A_d (q^0 - p^0)^d = \frac{1}{\rho} \#(S \cap \alpha(p,q))$$

$$a_0 = \frac{\varphi(q) - \varphi(p)}{q^0 - p^0} = \frac{(\varphi(q) - \varphi(p))(\rho A)^{1/d}}{(\#(S \cap \alpha(p,q)))^{1/d}} \qquad (1)$$

$$\frac{1}{\rho^2} \sum_{r,s \in S \cap \alpha(p,q)} (\varphi(r) - \varphi(s))^2 = \int_{r,s \in \alpha(p,q)} d^d r \, d^d s \, a^{\mu} a^{\nu} (r_{\mu} - s_{\mu})(r_{\nu} - s_{\nu})$$

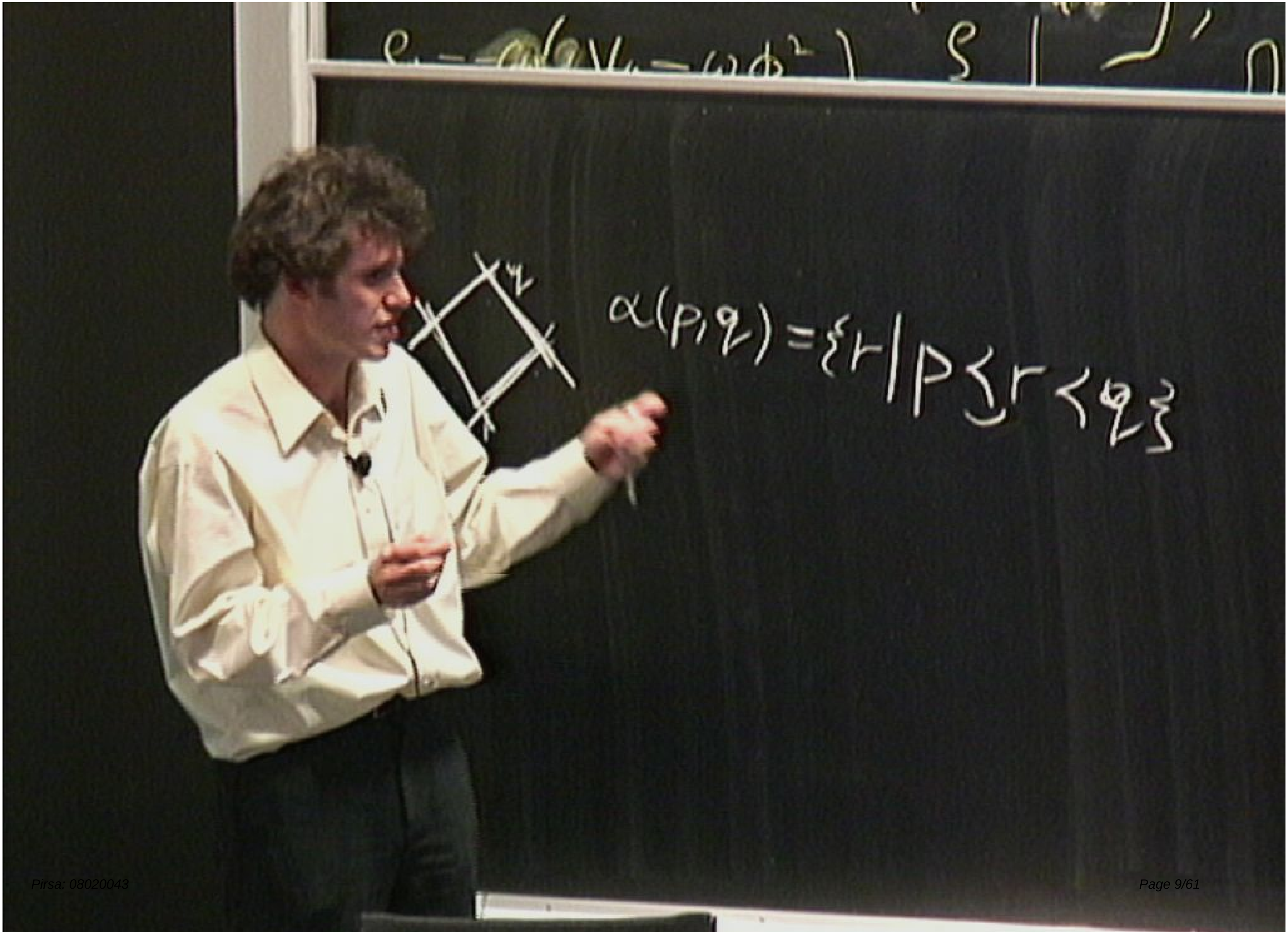
$$= \frac{(\#(S \cap \alpha(p,q)))^{2+2/d}}{\rho^{2+2/d}} (B_d (a_0)^2 + C_d \sum_{k=1}^{d-1} (a_k)^2)$$

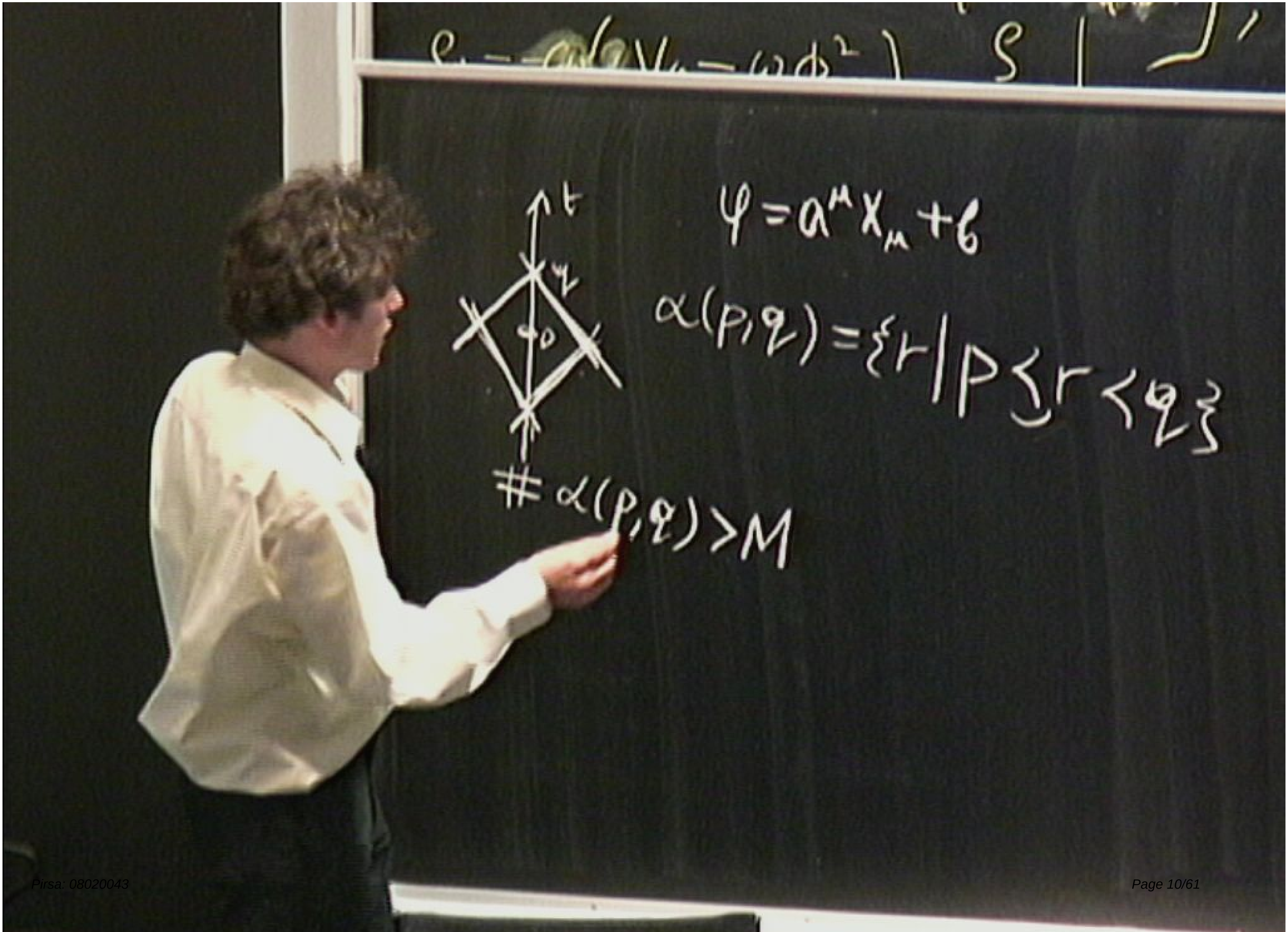
$$B_d (a_0)^2 + C_d \sum (a_k)^2 = \frac{\rho^{2/d}}{(\#(S \cap \alpha(p,q)))^{2+2/d}} \sum_{[illegible]} (\varphi(r) - \varphi(s))^2 \qquad (2)$$

$$(1 + \frac{B}{C})(1)^2 - \frac{1}{C}(2) \Rightarrow$$

$$\Rightarrow a^{\mu} a_{\mu} = (1 + \frac{B_d}{C_d}) \frac{(\varphi(q) - \varphi(p))^2 (\rho A)^{2/d}}{(\#(S \cap \alpha(p,q)))^{2+2/d}} - \frac{1}{C_d} \frac{\rho^{2/d}}{(\#(S \cap \alpha(p,q)))^{2+2/d}} \sum_{r,s \in \alpha(p,q)} (\varphi(r) - \varphi(s))^2$$

$$L(\varphi; p) = \frac{1}{2} (1 + \frac{B_d}{C_d}) \frac{(\varphi(q) - \varphi(p))^2 (\rho A)^{2/d}}{(\#(S \cap \alpha(p,q)))^{2+2/d}} - \frac{1}{2C_d} \frac{\rho^{2/d}}{(\#(S \cap \alpha(p,q)))^{2+2/d}} \sum_{r,s \in \alpha(p,q)} (\varphi(r) - \varphi(s))^2 - \frac{m^2 \varphi^2(p)}{2}$$


α(p,q) = {r | p ≤ r < q}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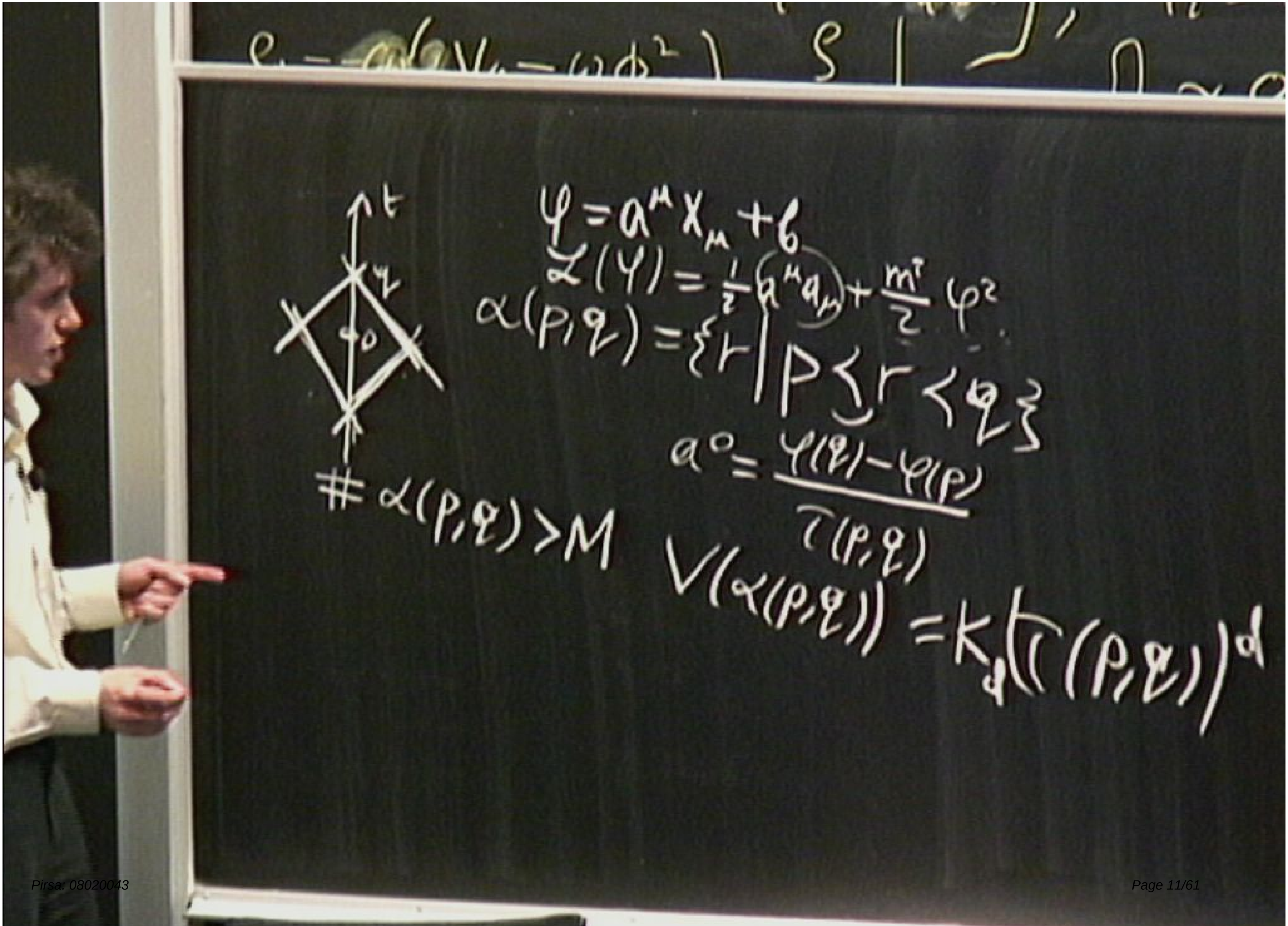
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ψ
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α(p,q) = {r | p ≤ r < q}
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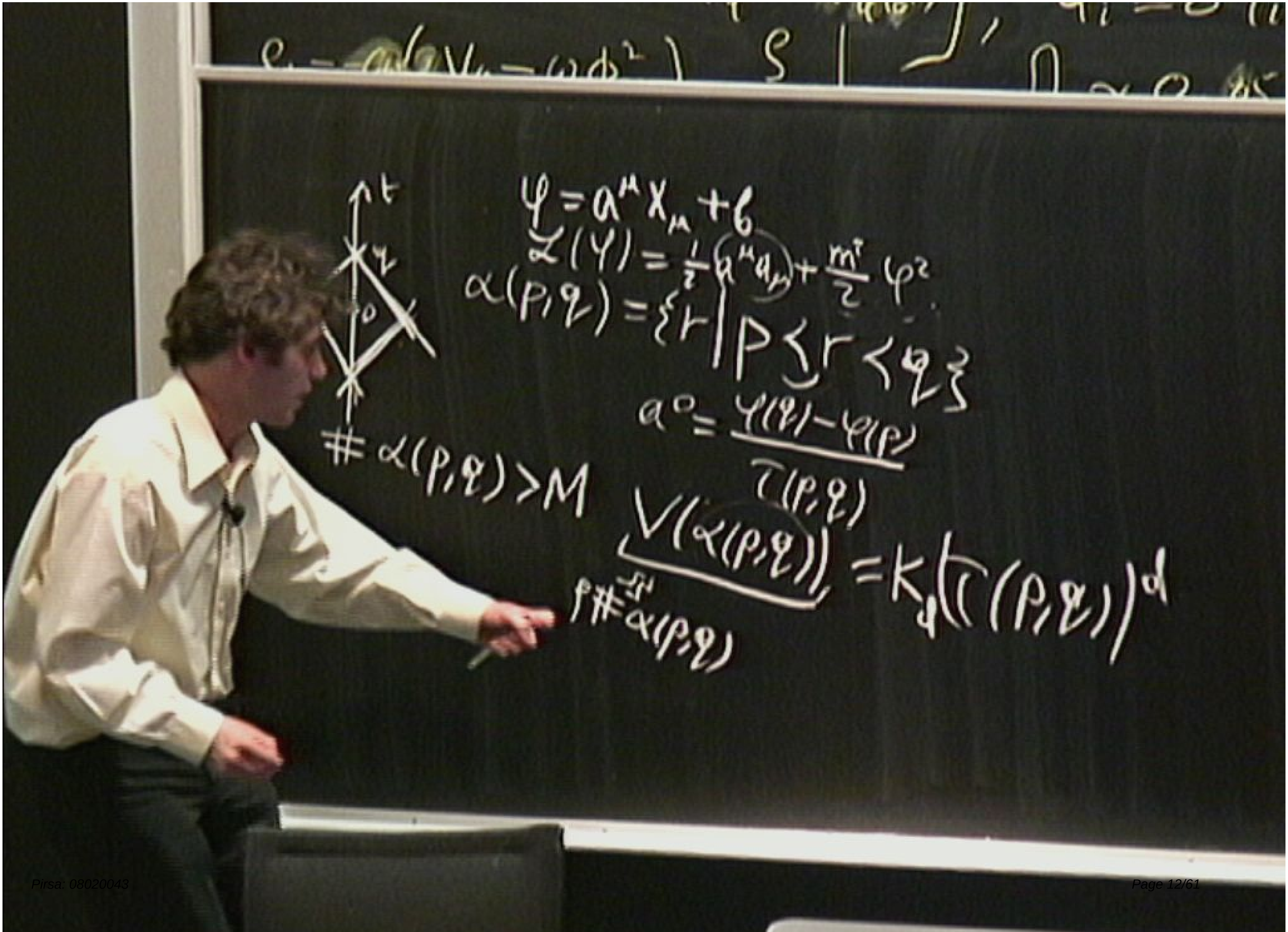
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$$\frac{1}{\rho^2} \sum_{r,s \in S \cap \alpha(p,q)} (\varphi(r) - \varphi(s))^2 = \int [illegible] \, a^{\mu} a^{\nu} (r_{\mu} - s_{\mu})(r_{\nu} - s_{\nu})$$

$$= \frac{(\#(S \cap \alpha(p,q)))[illegible]}{\rho^{2+2/d}} [illegible] (B_d (a_0)^2 + C_d \sum_{k=1}^{d-1} (a_k)^2)$$

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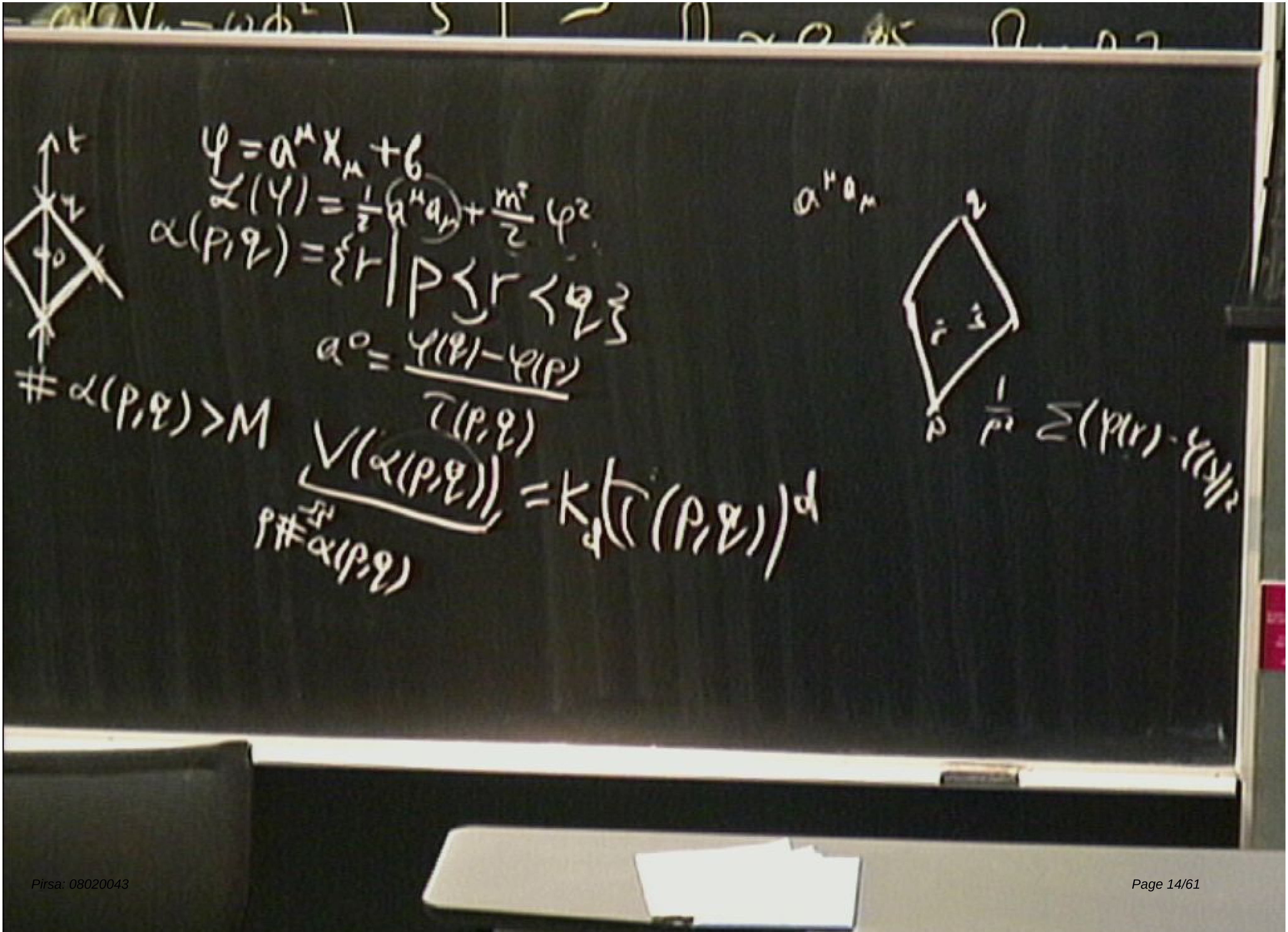
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$$\frac{1}{\rho^2} \sum_{r,s \in S \cap \alpha(p,q)} (\varphi(r)-\varphi(s))^2 \approx \int_{r,s \in \alpha(p,q)} d^d r\, d^d s\, a^\mu a^\nu (r_\mu - s_\mu)(r_\nu - s_\nu)$$

$$= \frac{(\#(S \cap \alpha(p,q)))^{2+2/d}}{\rho^{2+2/d}} (B_d (a_0)^2 + C_d \sum_{k=1}^{d-1} (a_k)^2)$$

$$B_d (a_0)^2 + C_d \sum (a_k)^2 \approx \frac{\rho^{2/d}}{(\#(S \cap \alpha(p,q)))^{2+2/d}} \sum_{p \prec r \prec q,\, p \prec s \prec q,\, r,s \in S} (\varphi(r)-\varphi(s))^2 \qquad (2)$$

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$$\Rightarrow a^\mu a_\mu \approx (1 + \frac{B_d}{C_d}) \frac{(\varphi(q)-\varphi(p))^2 (\rho A)^{2/d}}{(\#(S \cap \alpha(p,q)))^{2+2/d}} - \frac{1}{C_d} \frac{\rho^{2/d}}{(\#(S \cap \alpha(p,q)))^{2+2/d}} \sum_{r,s \in \alpha(p,q)} (\varphi(r)-\varphi(s))^2$$

$$L(\varphi, p) \approx \frac{1}{2}(1 + \frac{B_d}{C_d}) \frac{(\varphi(q)-\varphi(p))^2 (\rho A)^{2/d}}{(\#(S \cap \alpha(p,q)))^{2+2/d}} - \frac{1}{2C_d} \frac{\rho^{2/d}}{(\#(S \cap \alpha(p,q)))^{2+2/d}} \sum_{r,s \in \alpha(p,q)} (\varphi(r)-\varphi(s))^2 - \frac{m^2 \varphi^2(p)}{2}$$

LINEAR FIELD IN FLAT SPACE → GENERAL CASE

Field F is linear and space is flat $\Rightarrow L(F;p) = L^*(F;p,q)$ whenever $\#\alpha(p,q) > M$ where L^* is some expression that doesn't involve Lorentz indices

Relax assumption of flat space and linearity and replace them with assumptions of slowly varying curvature and slowly varying derivatives of fields.

GOAL: find point q such that F is approximately linear and space is approximately flat on $\alpha(p,q)$.

NOTE: small Lorentzian distance between p and q (measured through $\#\alpha(p,q)$ or other means) doesn't help us because the Lorentzian distance is arbitrarily small in vicinity of a light cone

SOLUTION

Choose N>>M. Find point q>p such that

(i) $\#(S \cap \alpha(p,q)) > N$

(ii) $L(F,\alpha(r_1,s_1)) = L(F,\alpha(r_2,s_2))$ whenever $p \prec r_1 \prec s_1 \prec q$, $p \prec r_2 \prec s_2 \prec q$, $\#\alpha(r_1,s_1) > M$ and $\#\alpha(r_2,s_2) > M$

$$err_{M,N}(\alpha(p,q))$$
$$= \frac{1}{2}(\max\{L(F;r,s) | p \prec r \prec s \prec q \,\&\, \#\alpha(r,s) > M\} - \min\{L(F;r,s) | p \prec r \prec s \prec q \,\&\, \#\alpha(r,s) > M\})$$

Let $q_{N,M}(p,F)$ be a point such that

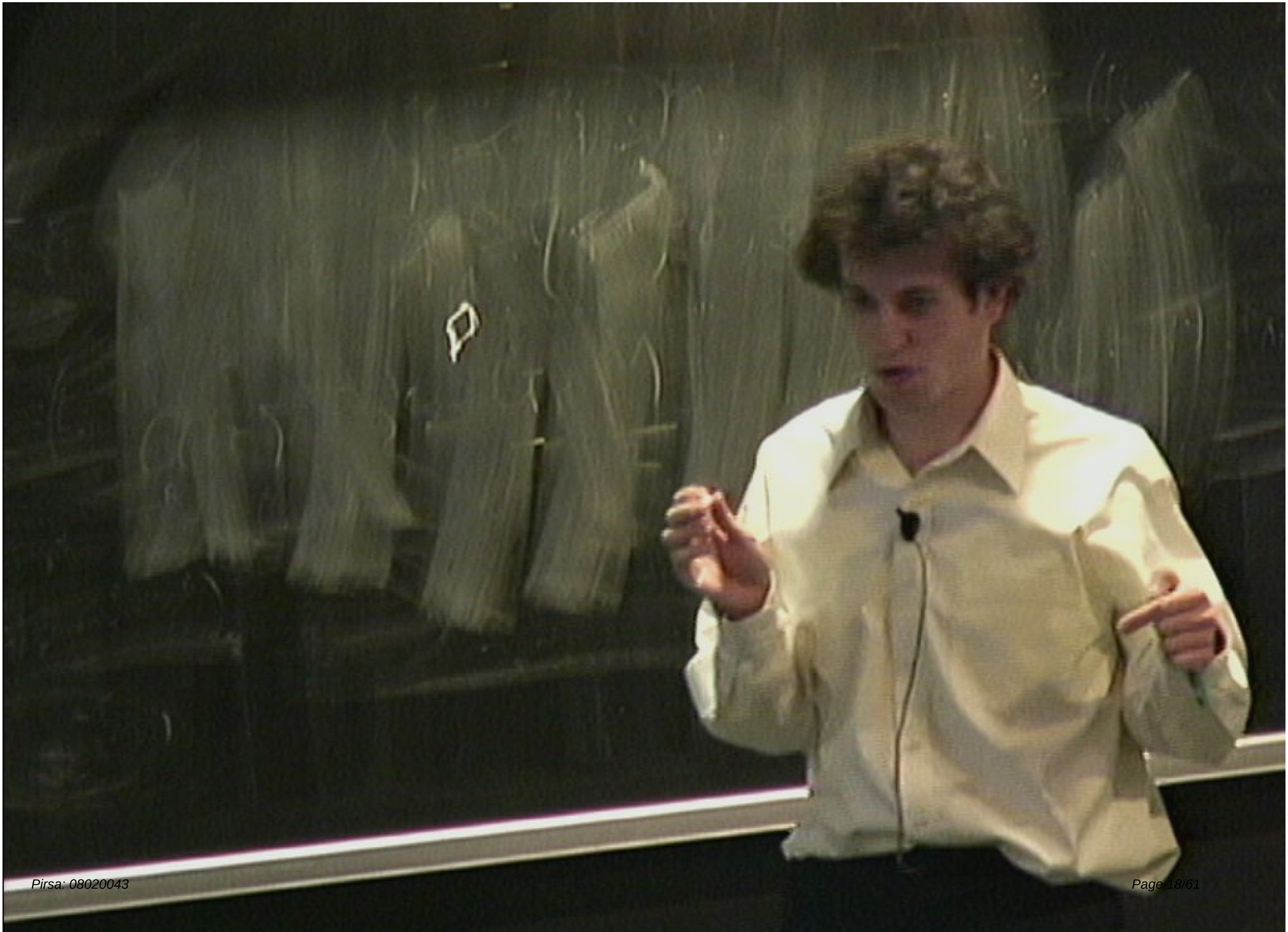
(i) $\#(S \cap \alpha(p,q_{M,N}(F,p))) > N$

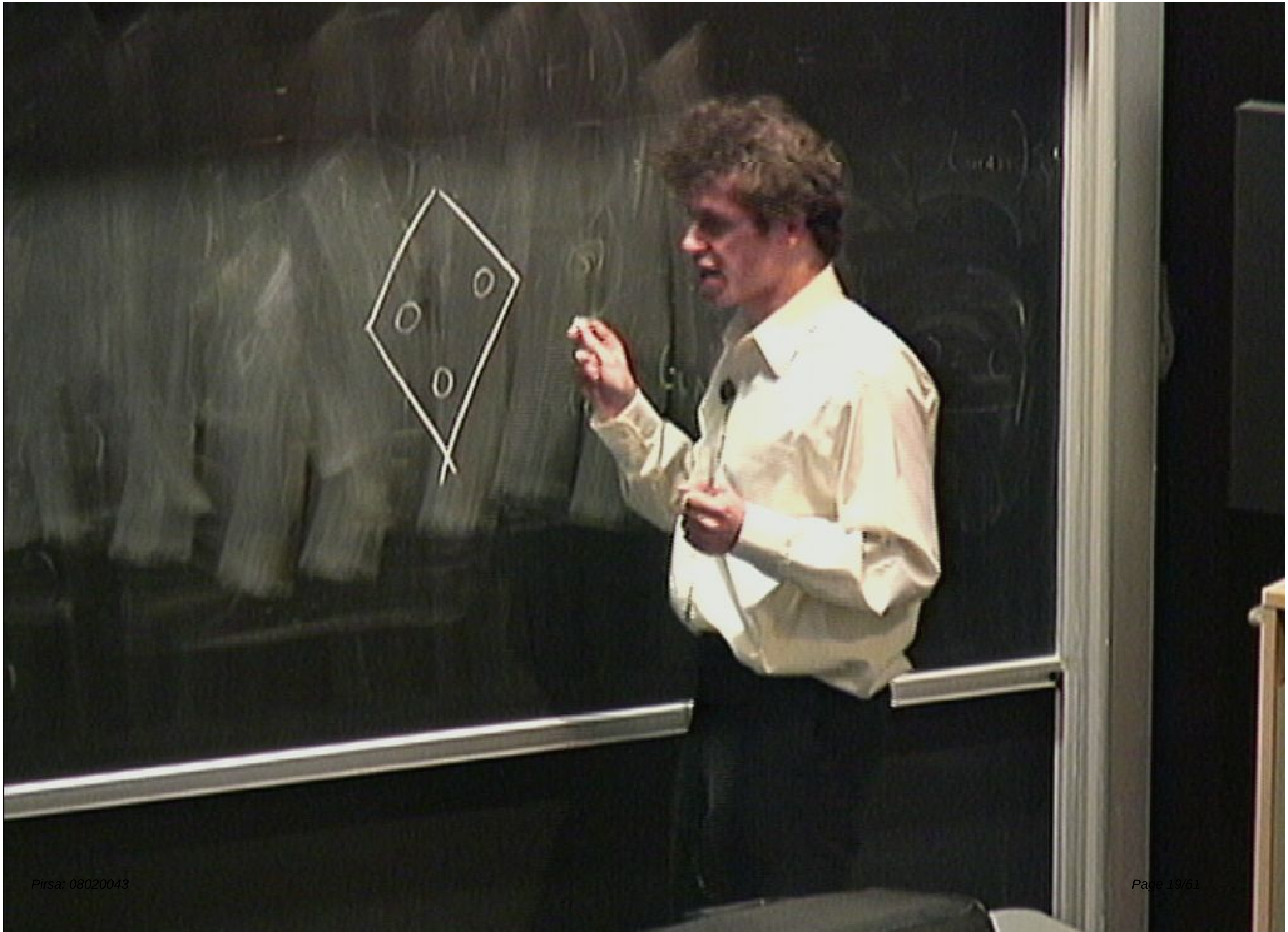
(ii) $err_{M,N}(p,q) \geq err_{M,N}(p,q(F,p))$ for any other q satisfying $\#\alpha(p,q) > N$

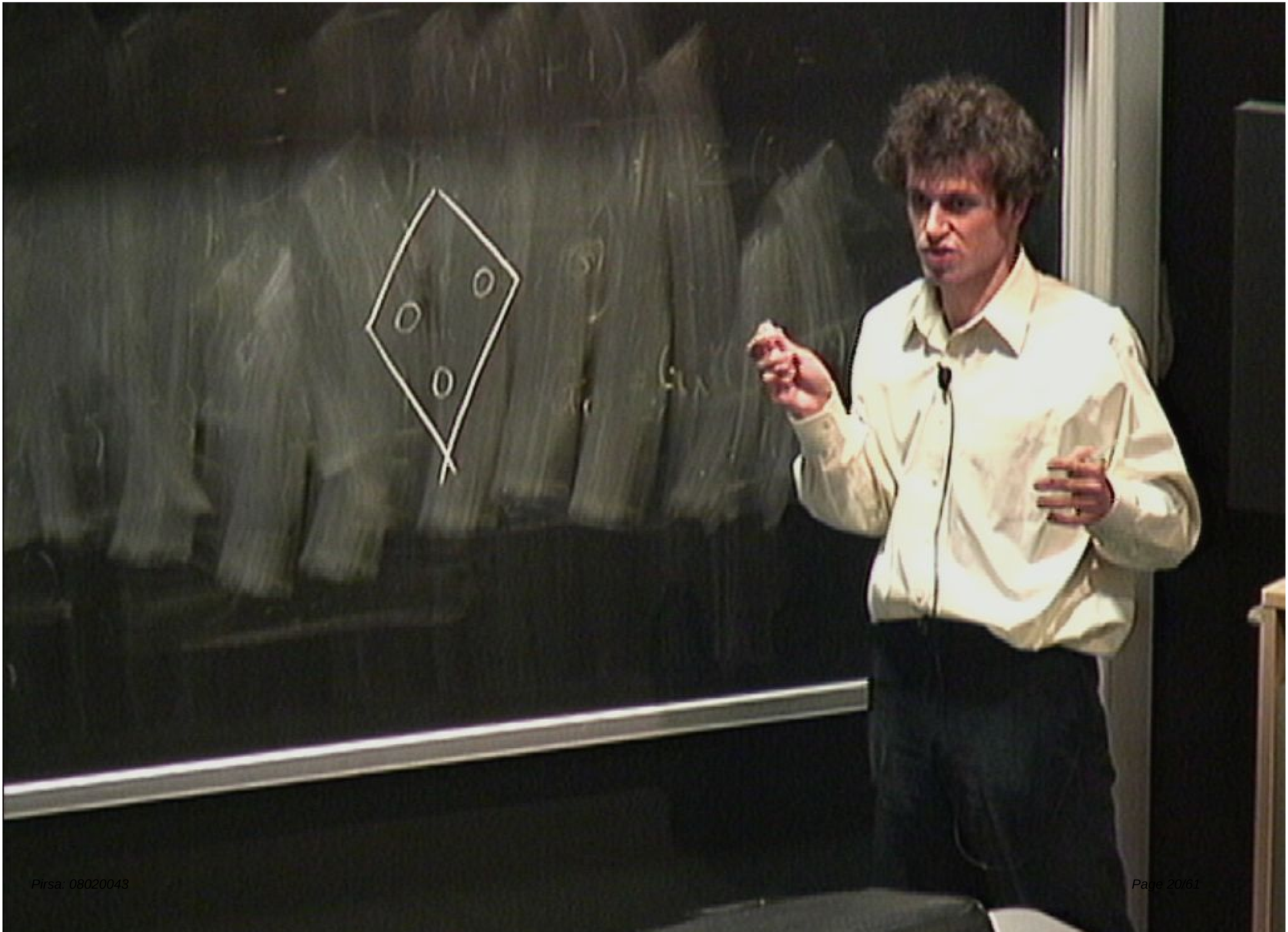
If there is more than one choice for the above, then it is undefined, so we throw such cases out of path integration, but this would be a set of measure 0.

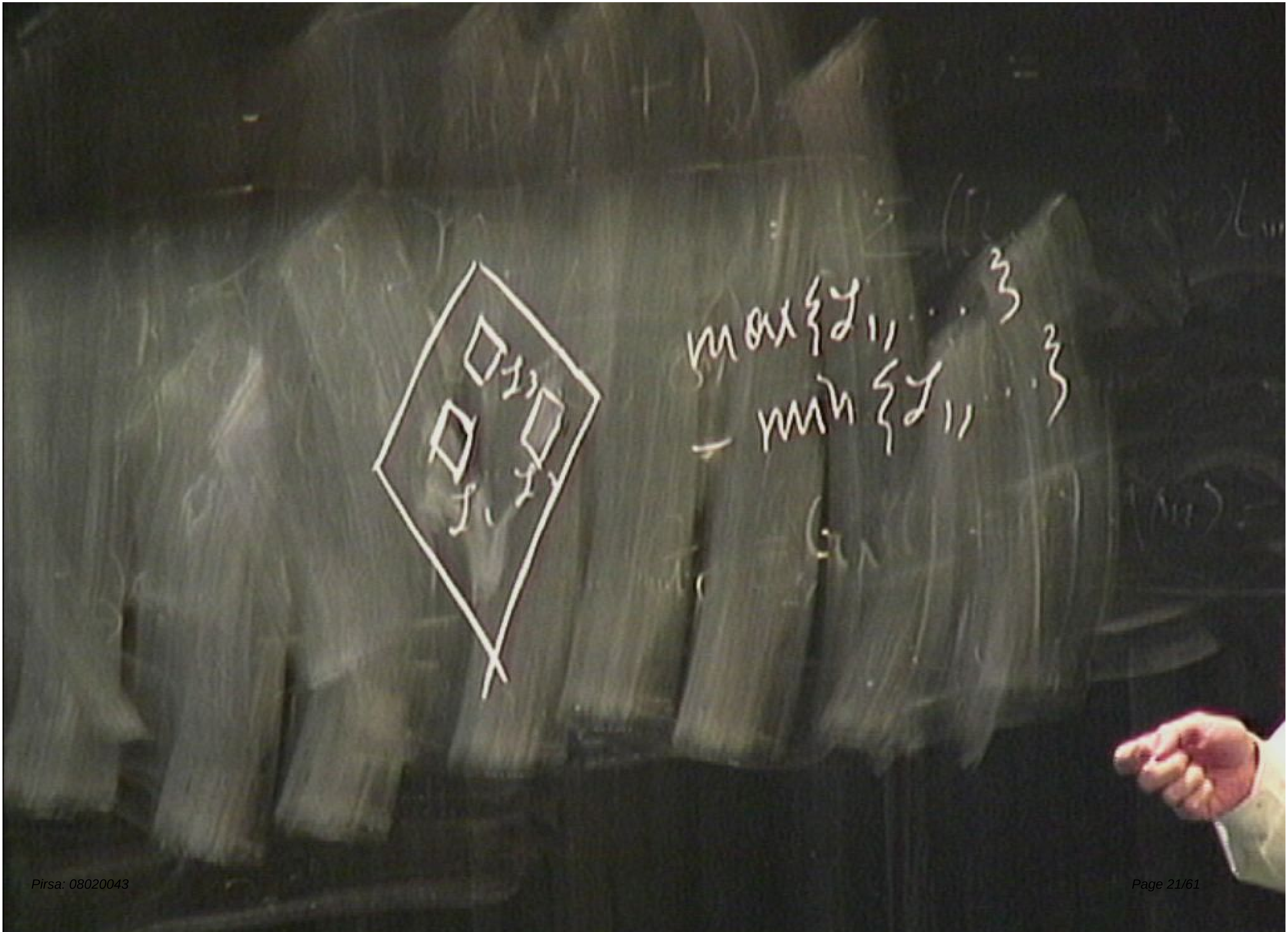
FINAL ANSWER: $L_{M,N}(F,p) = L_{M,N}^*(F,p,q(F,p))$ given that curvature of spacetime as well as fields and their derivatives vary slowly enough

DEFINITION: $L^*_{M,N}(F,p) = L^*_{M,N}(F,p,q(F,p)) = \lim^*_{M,N,q \to p} L(F,p,q)$; For discrete case equate S with entire set of points.

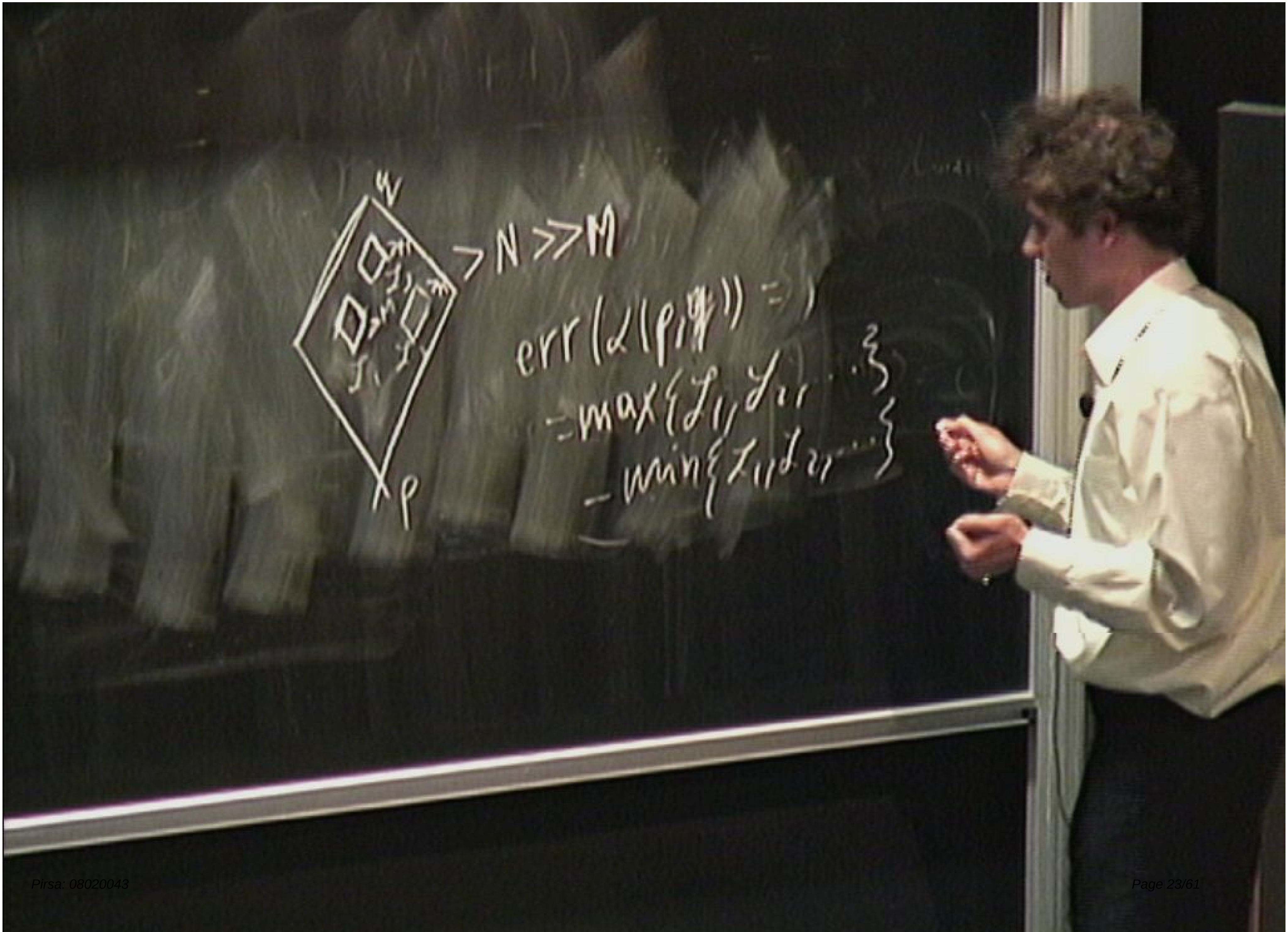









> N >> M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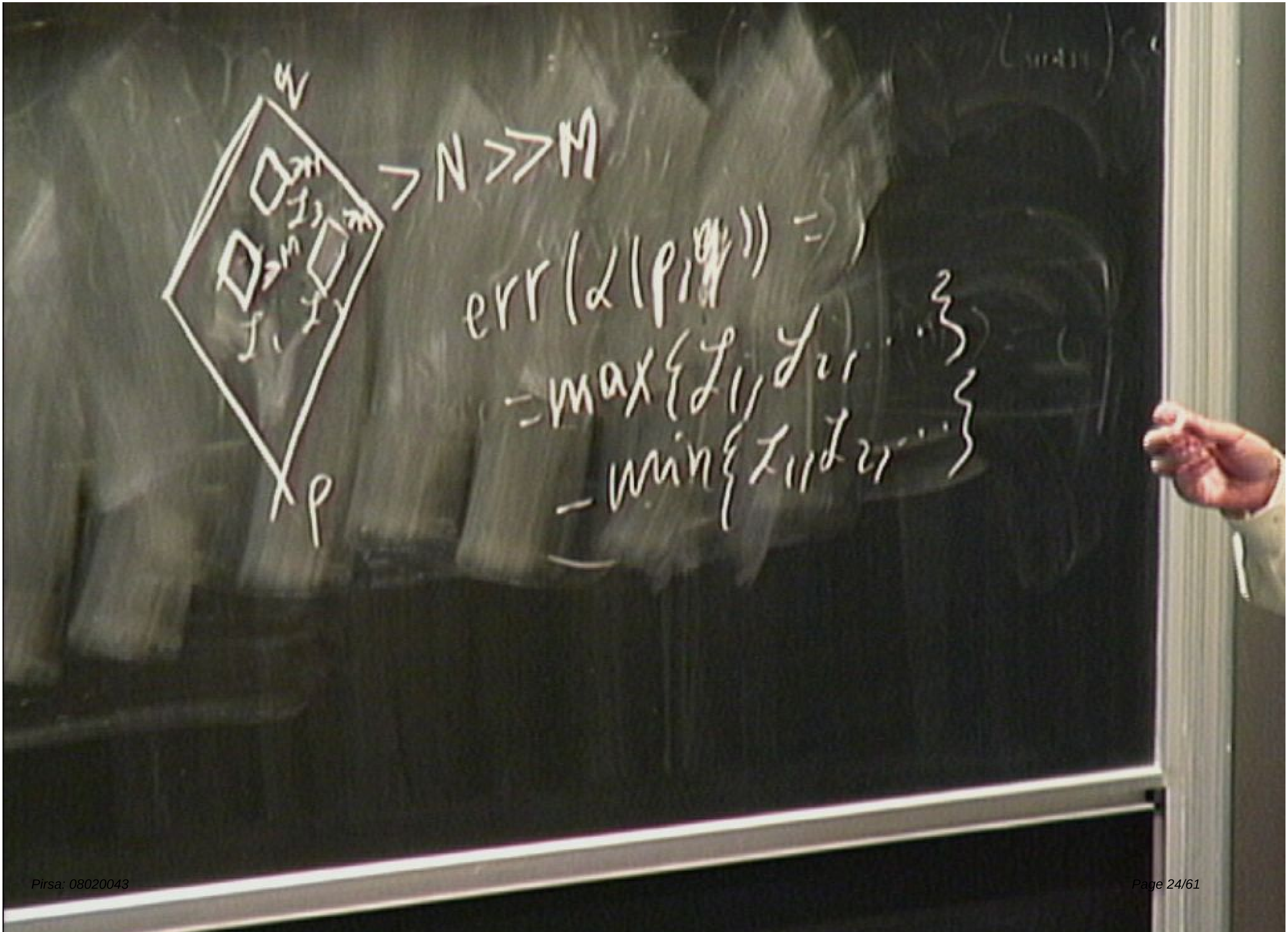
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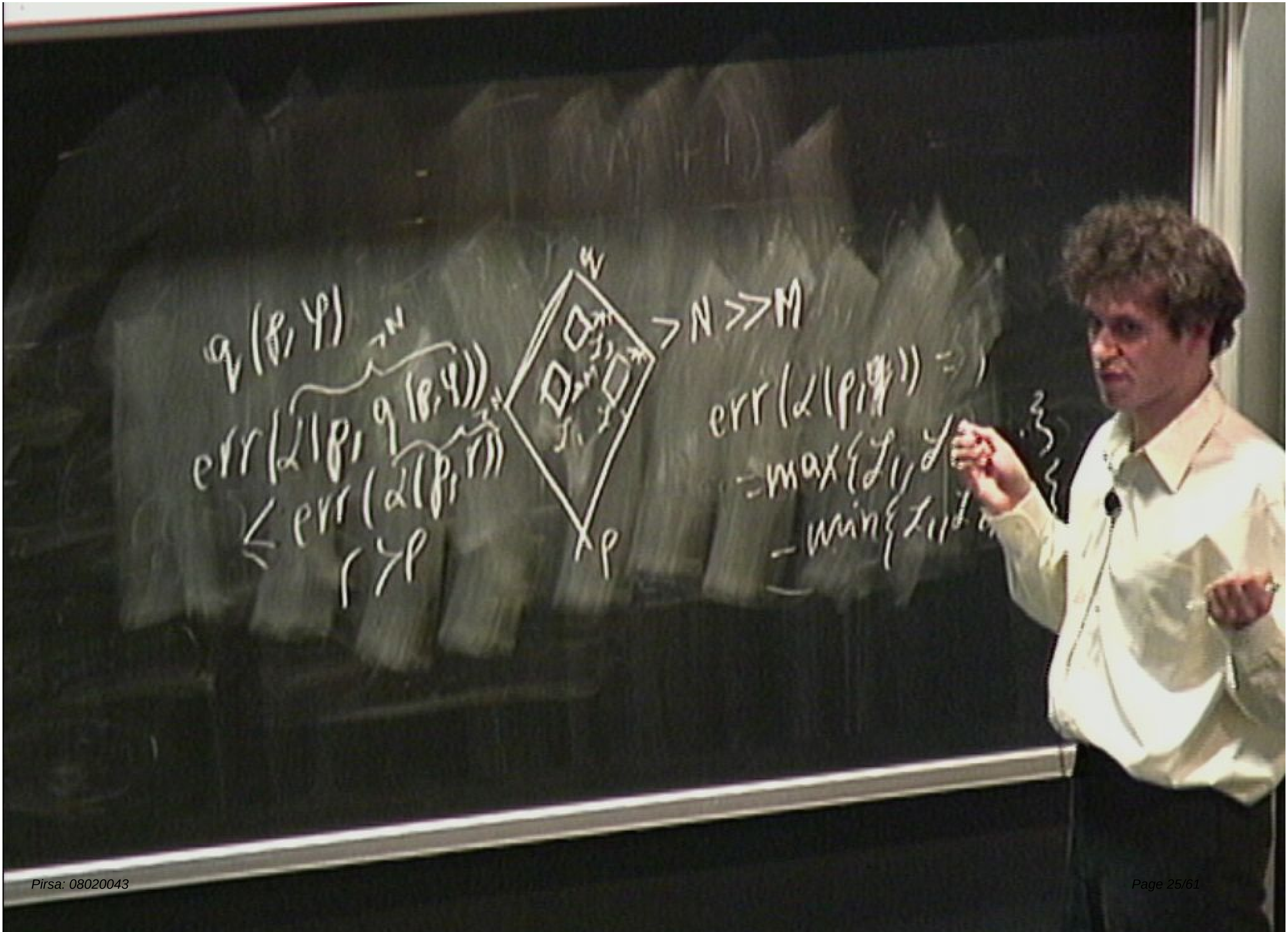
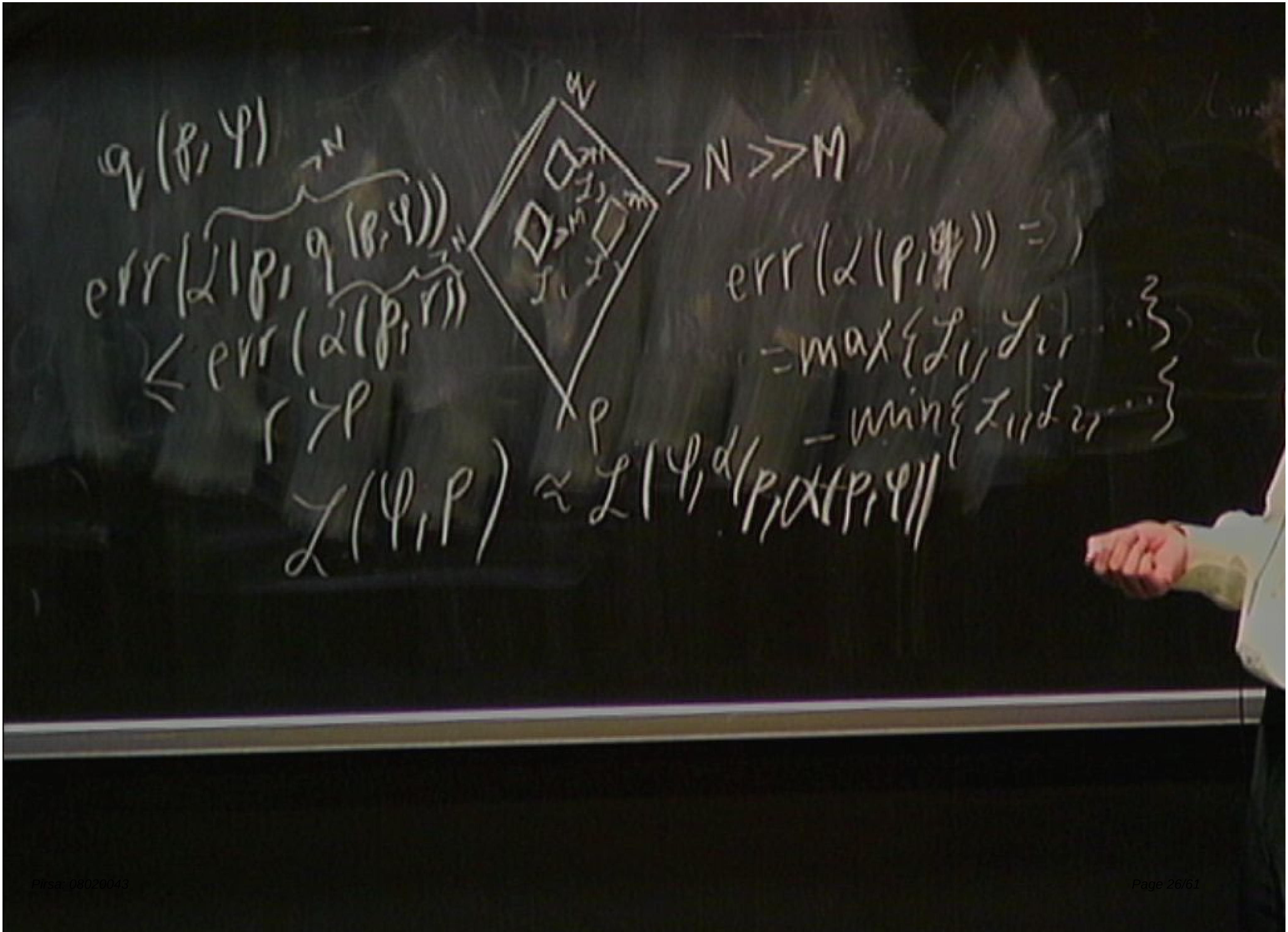



N >> M

GAUGE FIELD

Given: $\exp \int_{\gamma(p_i p_j)} T^a A^a{}_\mu dx^\mu = U(p_i, p_j)$

Unknowns: p^μ and $A^{a\mu}$

Suppose we know that for some GIVEN q the following is true:

a) $\#(S \cap \alpha(p,q)) > M$

b) Both $A^{a\mu}$ and B(r,s) are linear on $\alpha(p,q)$

$$U(a,b)U(b,c)U(c,a) = 1 + \frac{1}{2} T^a F^a{}_{\mu\nu}(a^\mu - c^\mu)(b^\nu - c^\nu)$$

$$Tr((U(a,b)U(b,c)U(c,a) - 1)^2) = \frac{C_2}{4} F^a{}_{\mu\nu} F^a{}_{\rho\sigma}(a^\mu - c^\mu)(b^\nu - c^\nu)(a^\rho - c^\rho)(b^\sigma - c^\sigma)$$

Choose coordinate system so that t axis passes through p and q and origin lies in the middle between two points

$$\frac{1}{\rho} \sum_{r \in S \cap \alpha(p,q)} Tr((U(p,q)U(q,r)U(r,p) - 1)^2)$$

$$= \frac{C_2}{4} \int_{\alpha(p,q)} d^d r F^a{}_{\mu\nu} F^a{}_{\rho\sigma}(p^\mu - r^\mu)(q^\nu - r^\nu)(p^\rho - r^\rho)(q^\sigma - r^\sigma) = \frac{k_1}{\rho^{1+4/d}} \sum (F^a{}_{0i}(p))^2$$

$$\sum (F^a{}_{0i}(p))^2 = \frac{\rho^{4/d}}{k_1} \sum_{r \in S \cap \alpha(p,q)} Tr((U(p,q)U(q,r)U(r,p) - 1)^2) (1)$$

$$\frac{1}{\rho^3} \sum_{a,b,c \in S \cap \alpha(p,q)} Tr((U(a,b)U(b,c)U(c,a) - 1)^2)$$

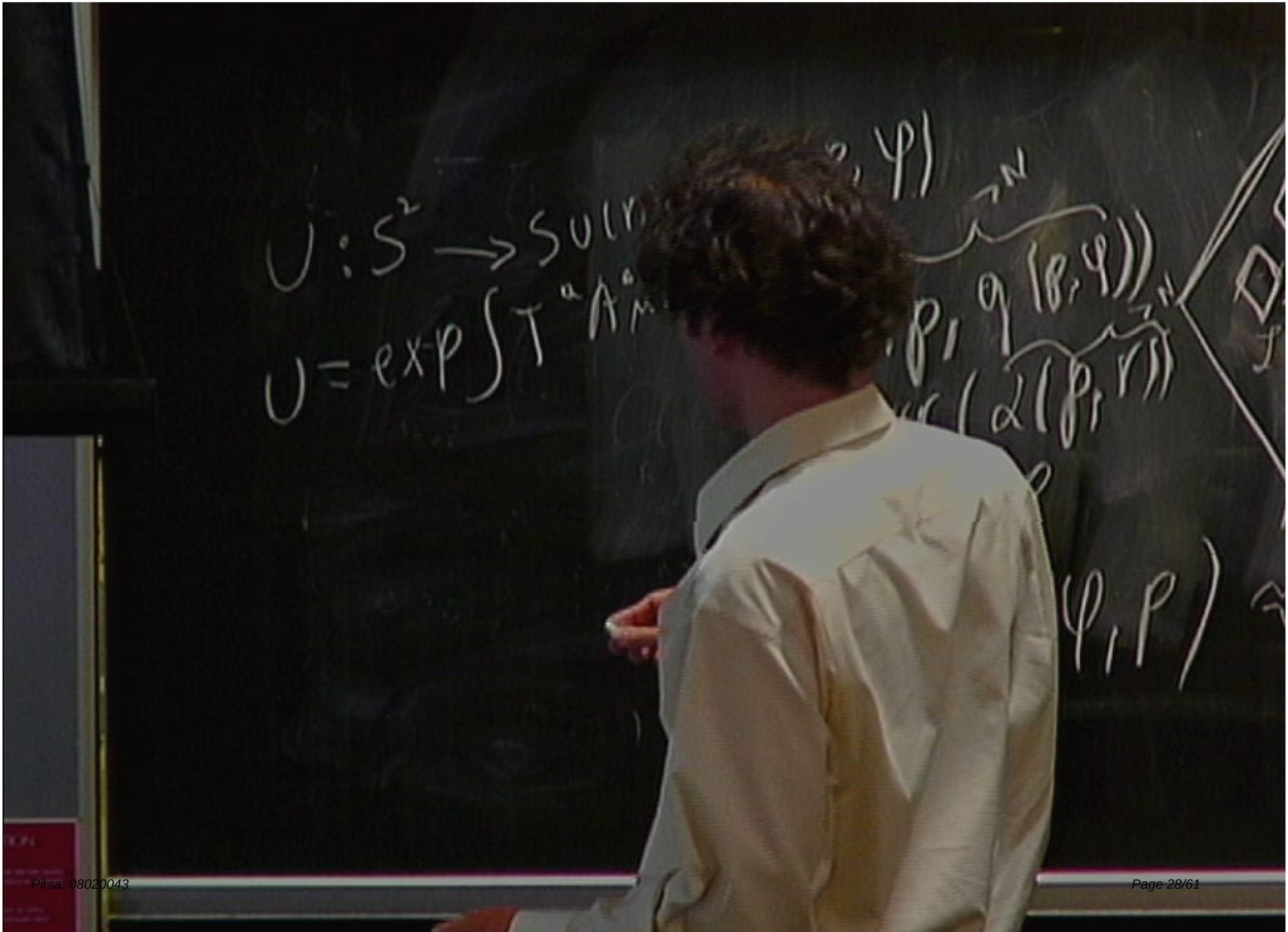
$$= \frac{C_2}{4} \int_{a,b,c \in \alpha(p,q)} d^d a\, d^d b\, d^d c\, F^a{}_{\mu\nu} F^a{}_{\rho\sigma}(p^\mu - r^\mu)(q^\nu - r^\nu)(p^\rho - r^\rho)(q^\sigma - r^\sigma)$$

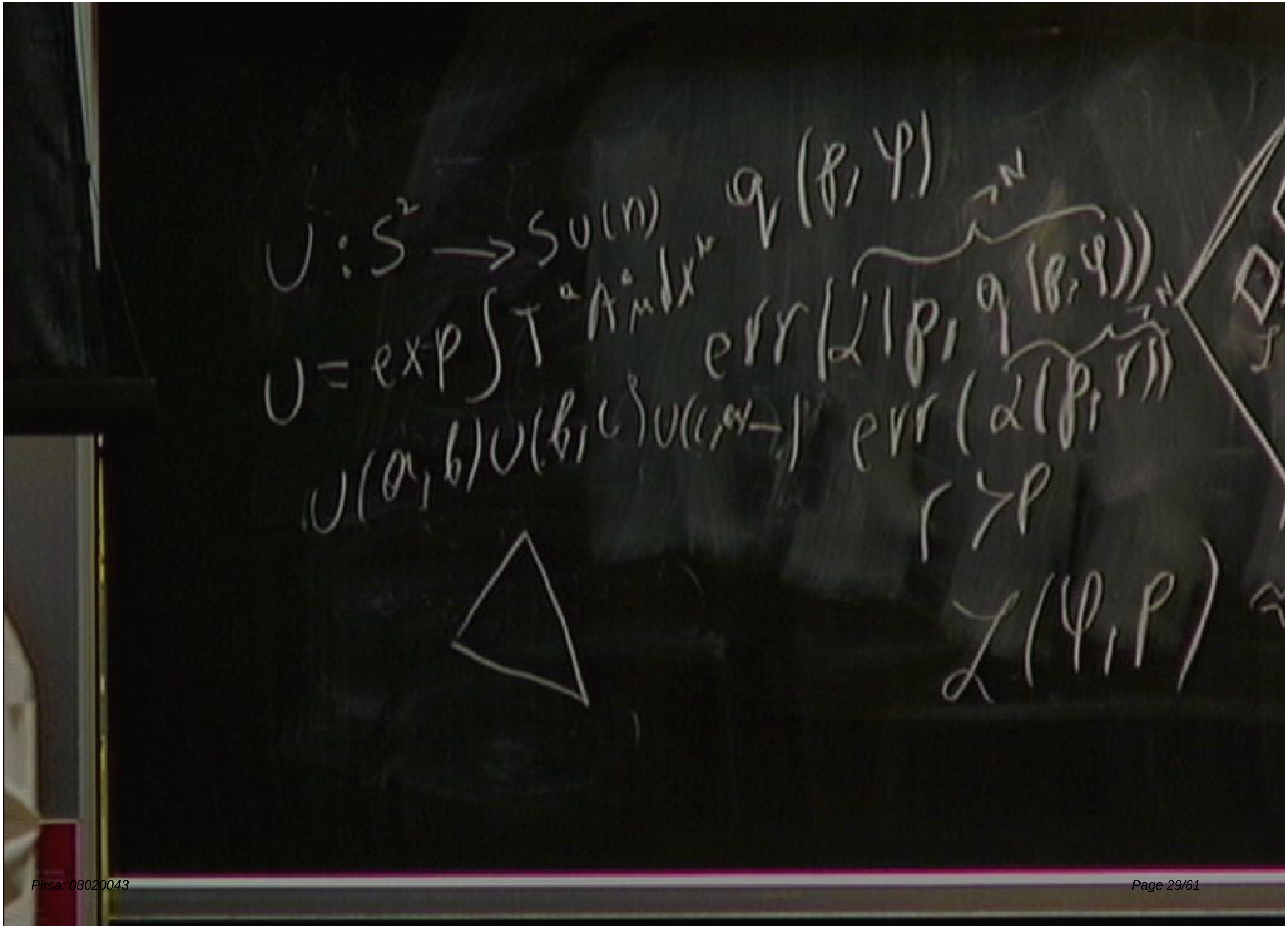
$$= \frac{1}{\rho^{3+4/d}} \left(k_2 \sum_{i=1}^{d-1} (F^a{}_{0i}(p))^2 + k_3 \sum_{i,j} (F^a{}_{ij}(p))^2\right)$$

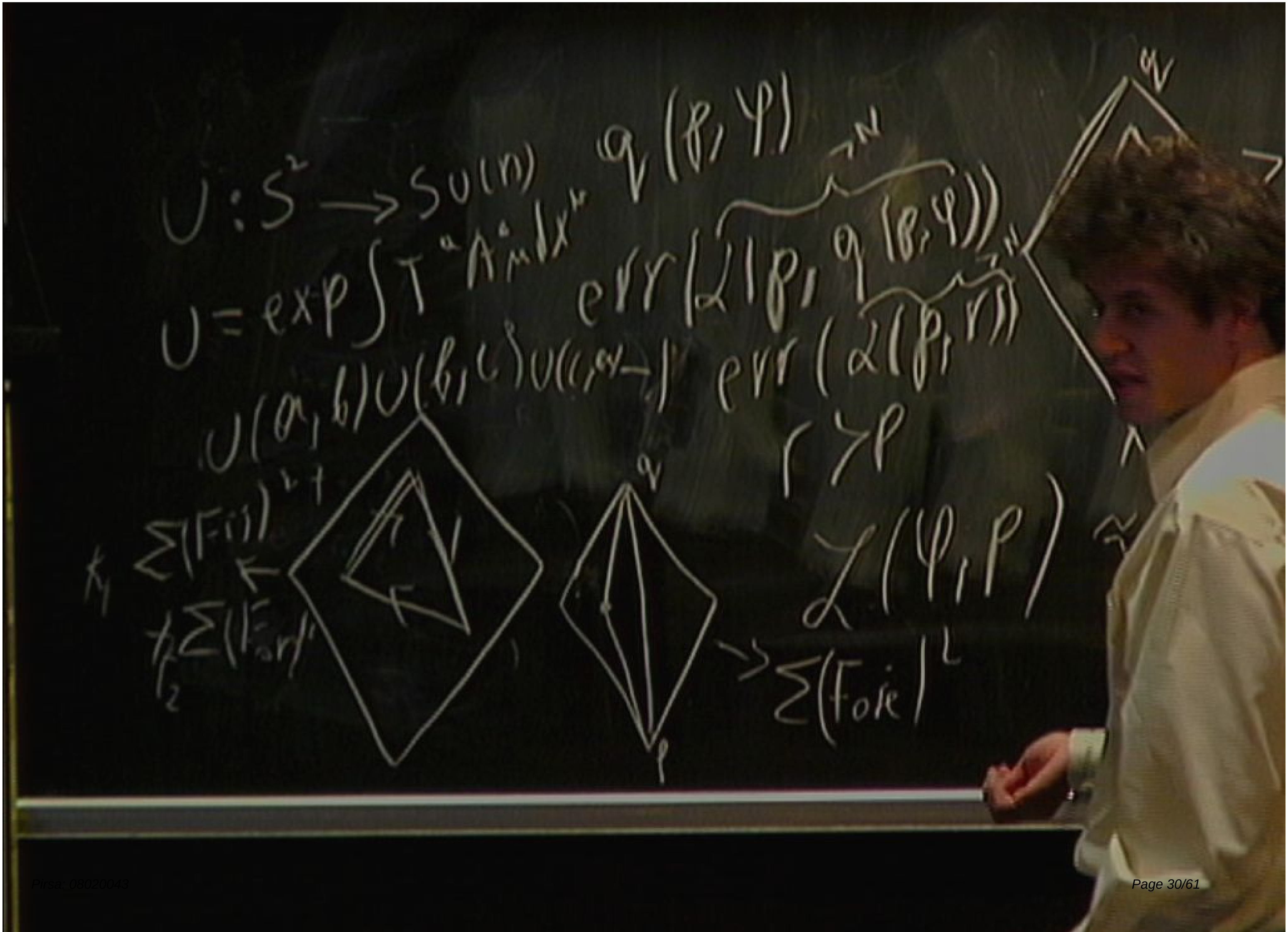
$$k_2 \sum_{i \neq 0} (F^a{}_{0i}(p))^2 + k_3 \sum_{i,j \neq 0} (F^a{}_{ij}(p))^2) = \rho^{4/d} \sum_{a,b,c \in S \cap \alpha(p,q)} Tr((U(p,q)U(q,r)U(r,p) - 1)^2) (2)$$

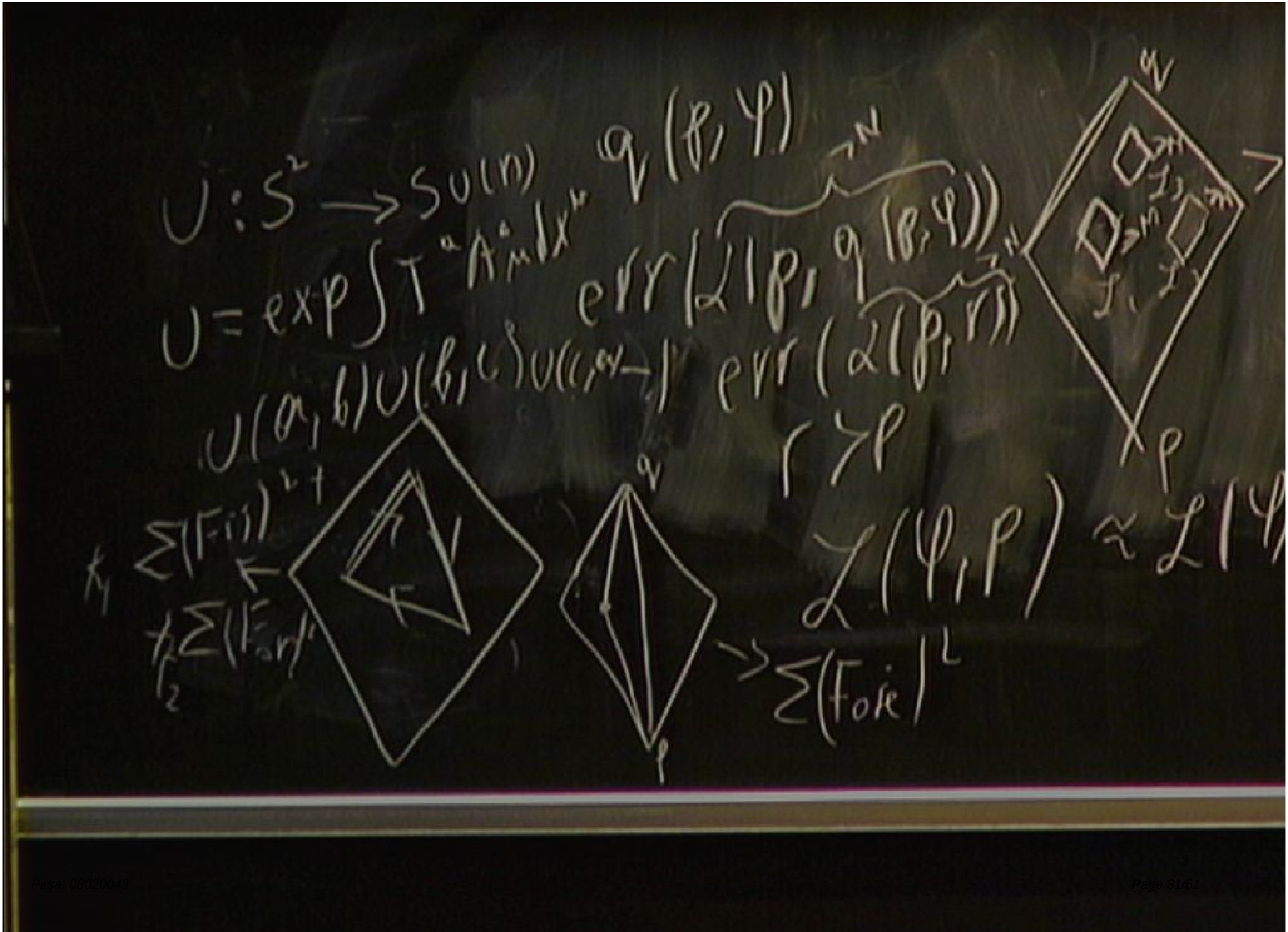
$$\frac{1}{k_3}(2) - (1 + \frac{k_2}{k_1})(1) \Rightarrow F^{a\mu\nu} F^a{}_{\mu\nu} = \frac{\rho^{4/d}}{k_3} \sum_{a,b,c \in S \cap \alpha(p,q)} Tr((U(p,q)U(q,r)U(r,p) - 1)^2)$$

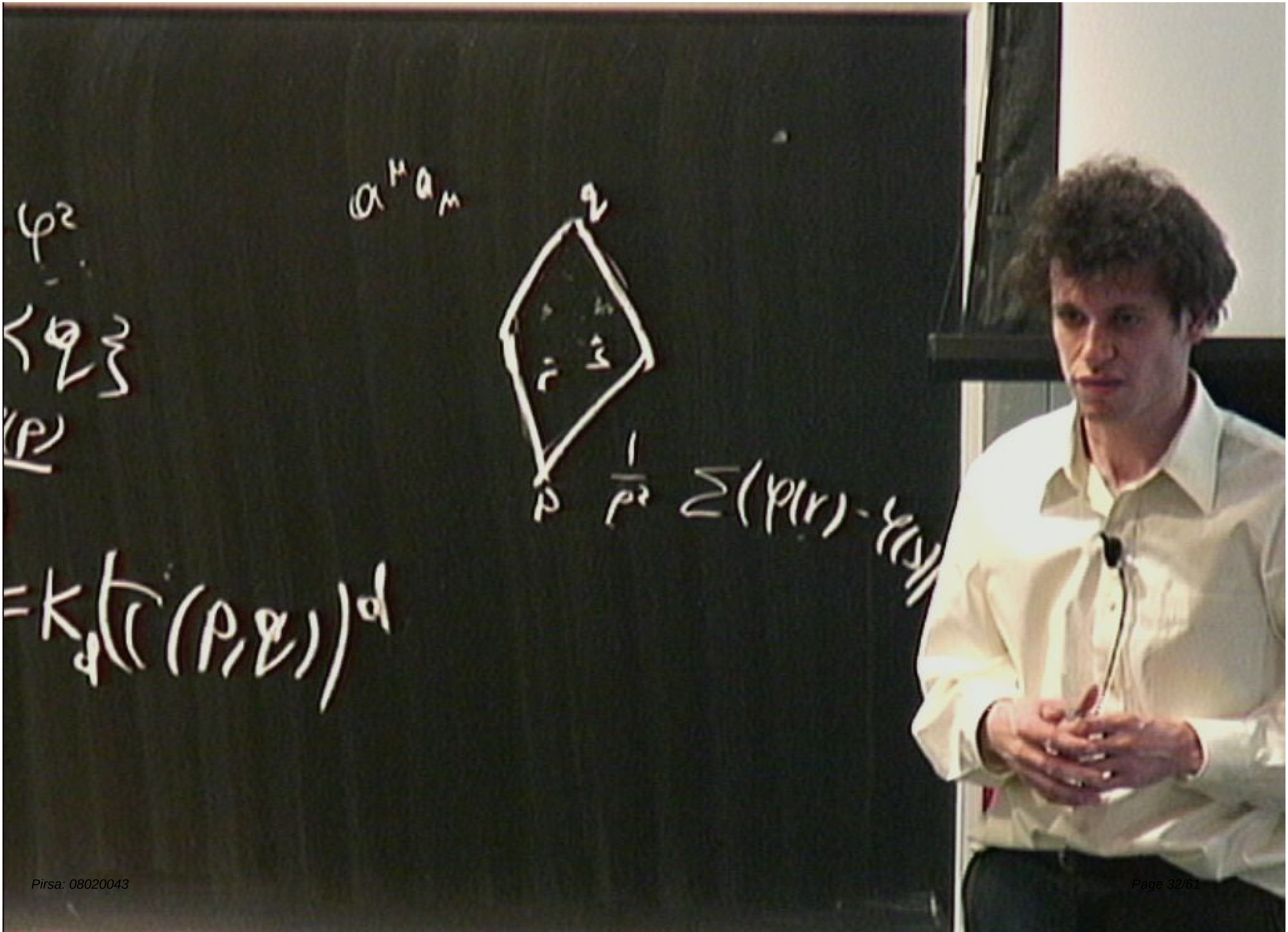
$$-(1 + \frac{k_2}{k_3}) \frac{\rho^{4/d}}{k_1} \sum_{r \in S \cap \alpha(p,q)} Tr((U(p,q)U(q,r)U(r,p) - 1)$$

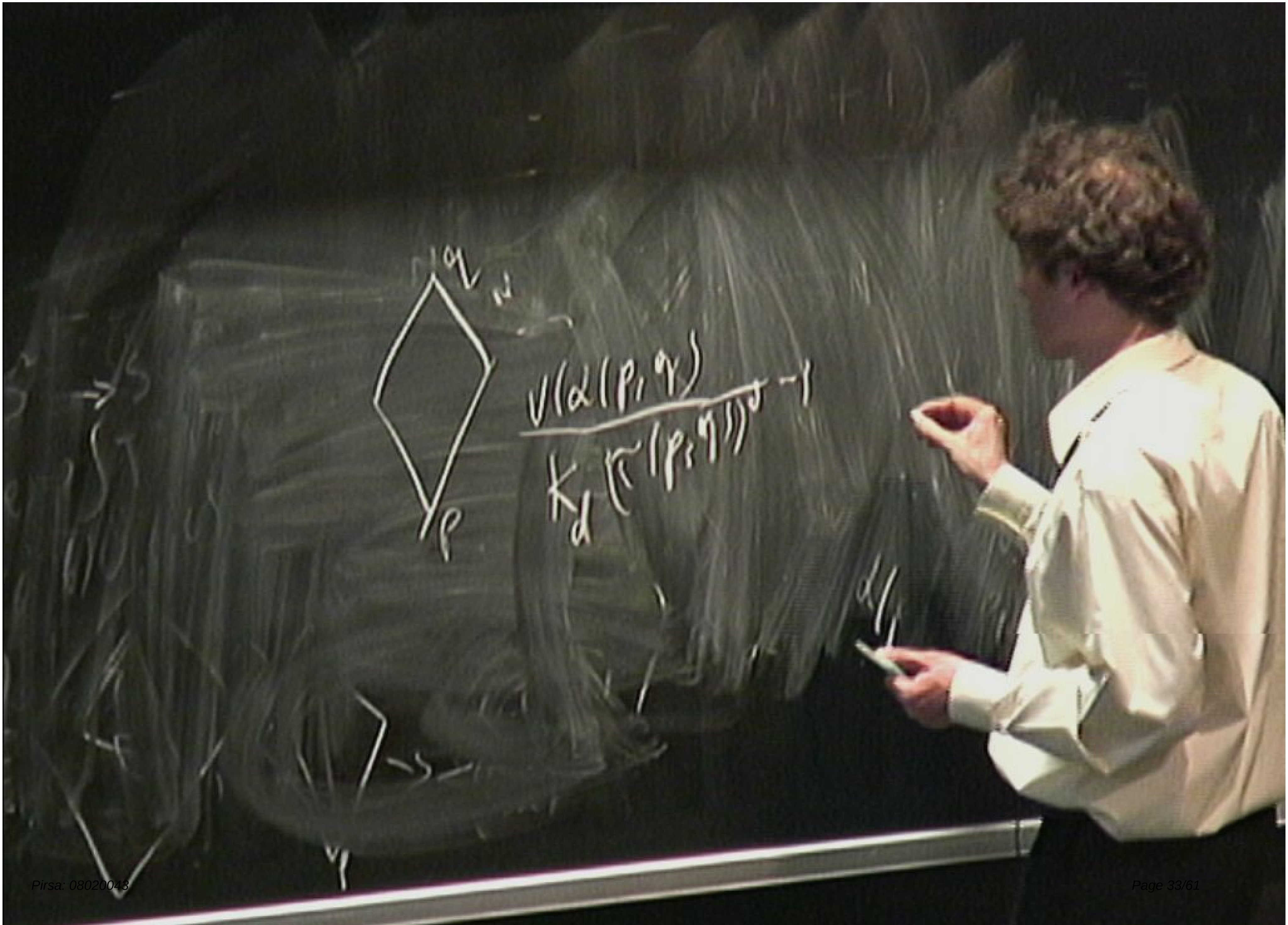


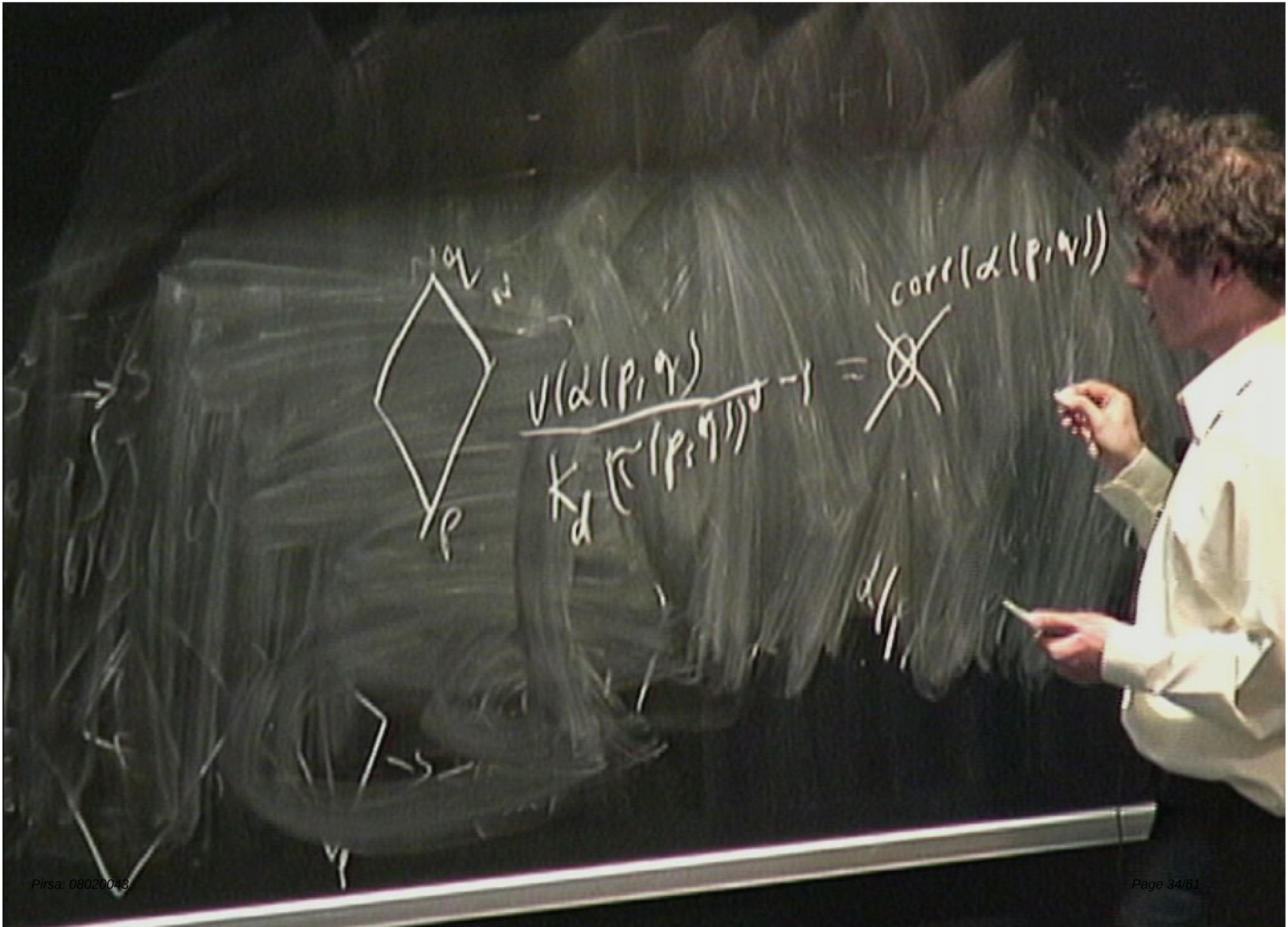

U : S² → SU(n)
U = exp ∫ T^a A^a_μ dx^μ
q(ρ, φ)
err(α | ρ, q(ρ, φ))
err(α(ρ, r))
α(φ, ρ)

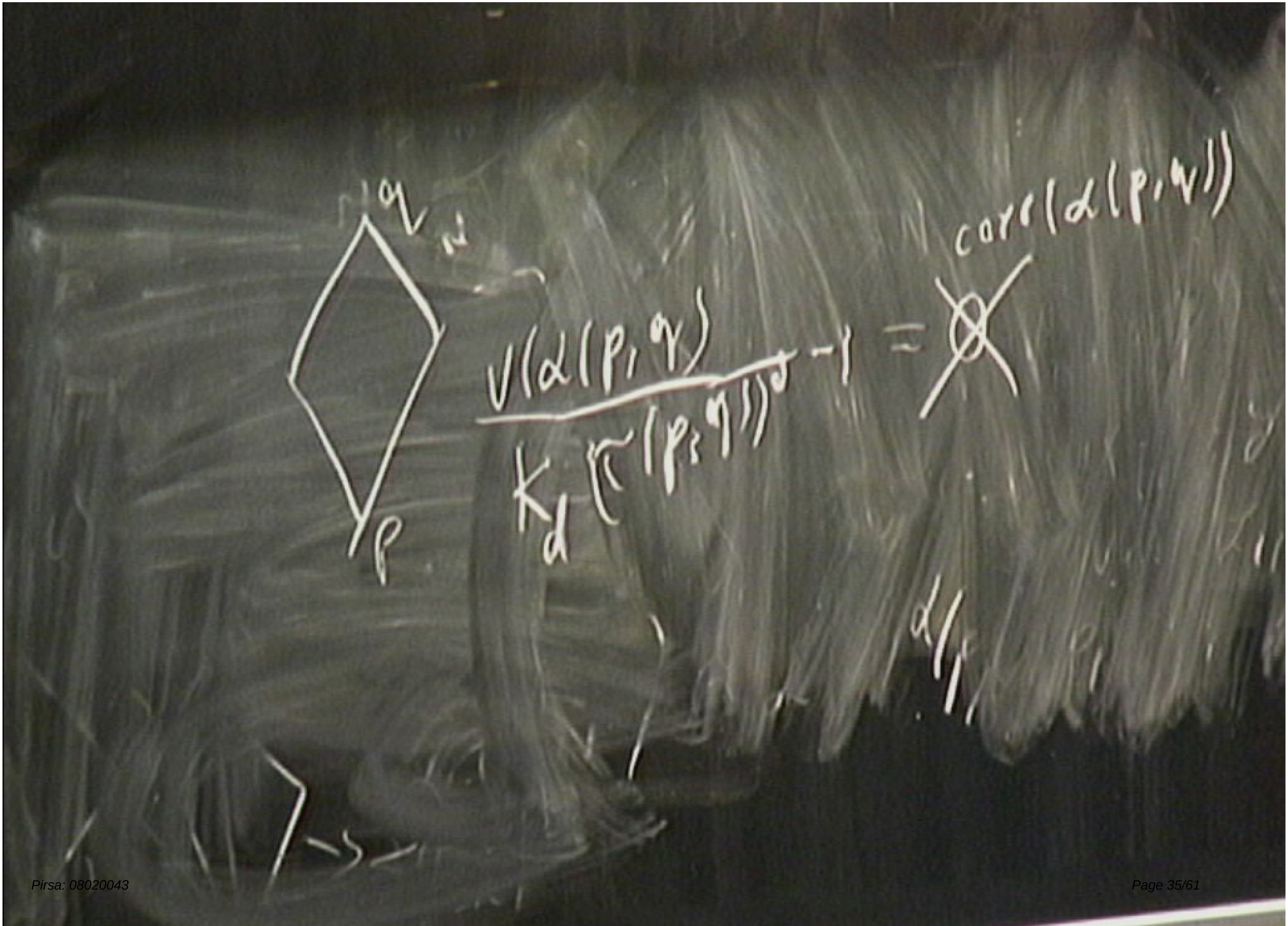

U : S² → SU(n)
U = exp ∫ T^a A^a_μ dx^μ

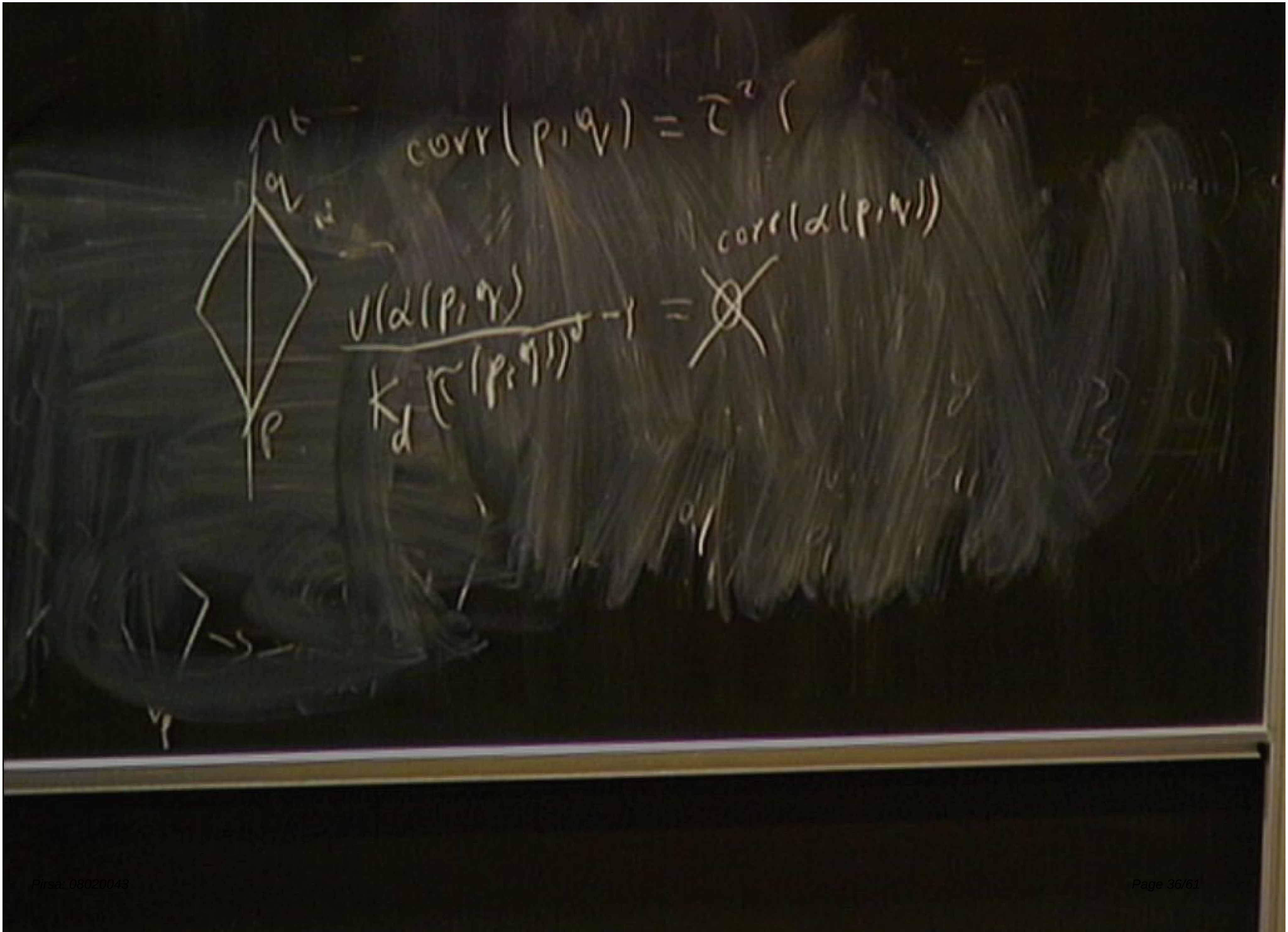

U : S² → SU(n)
U = exp ∫ T^a A^a_μ dx^μ

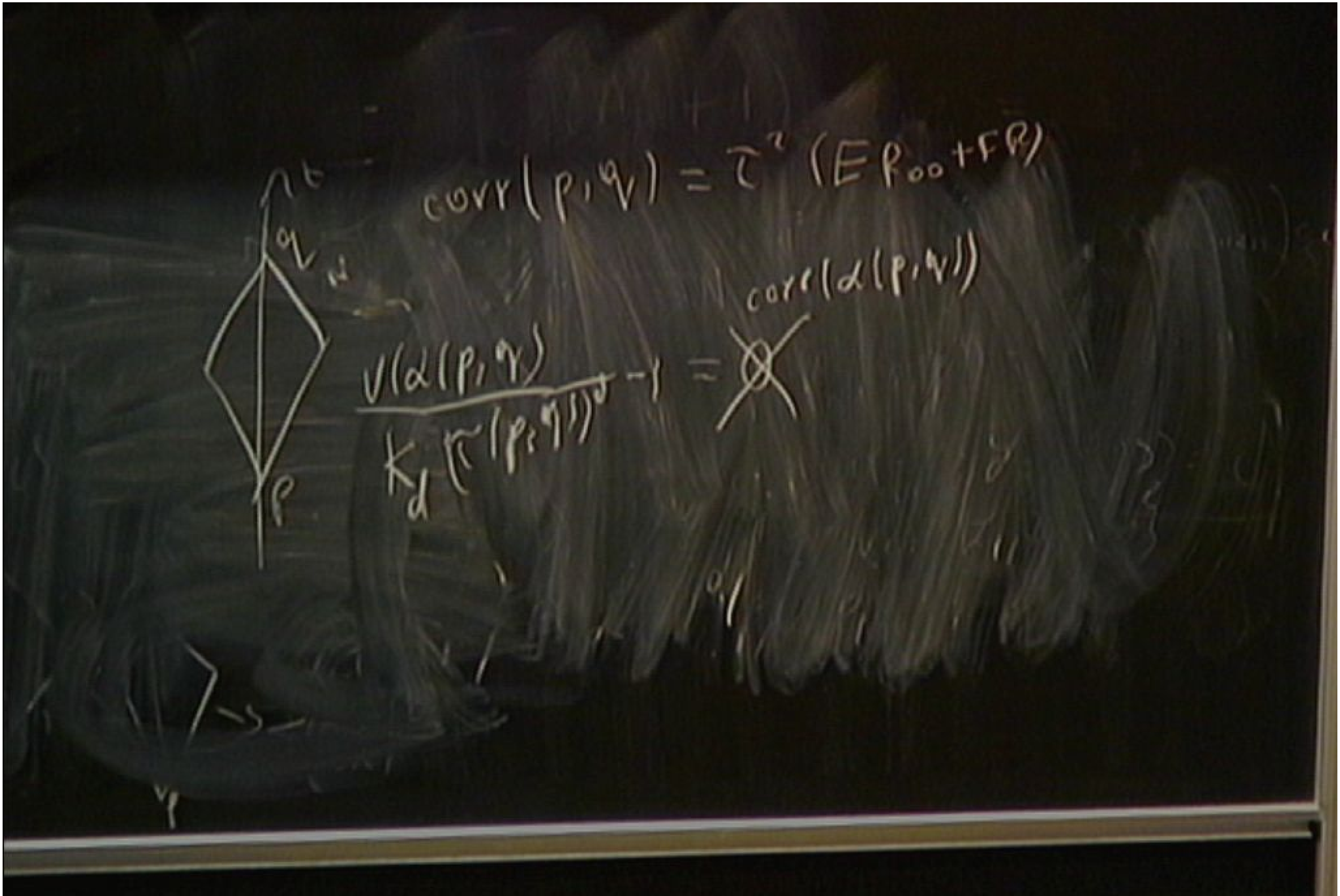


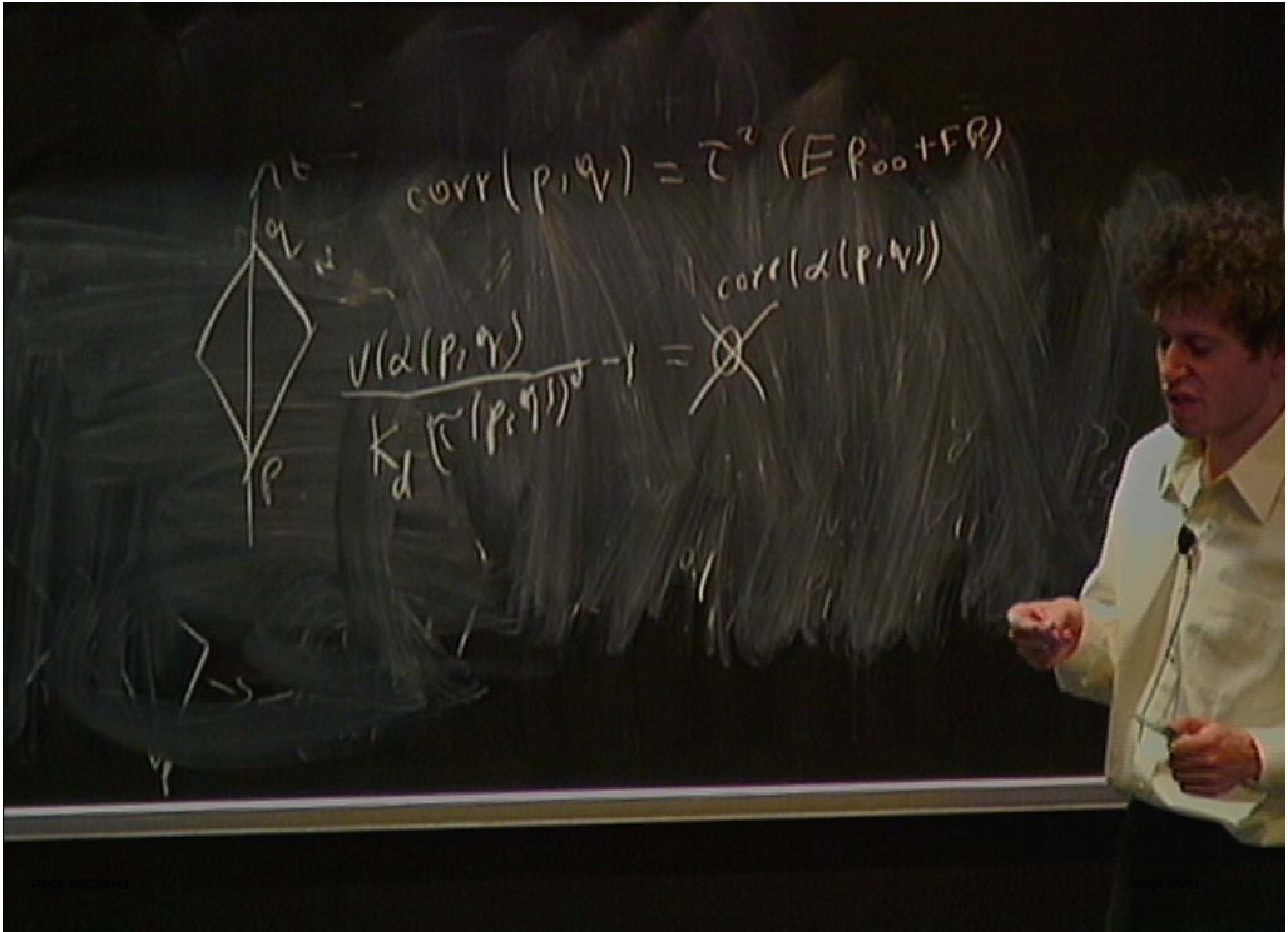


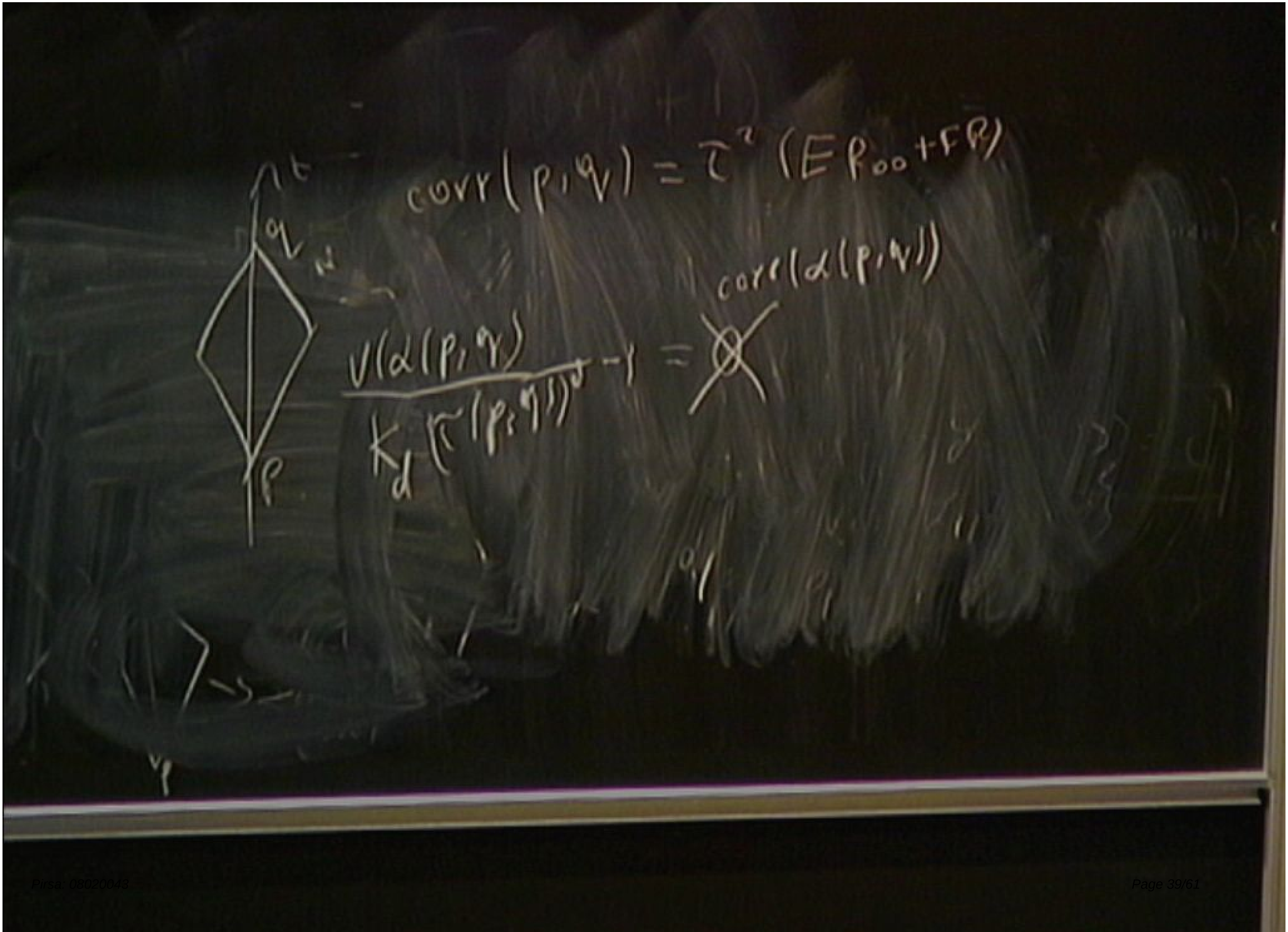

corr(α(p,q))

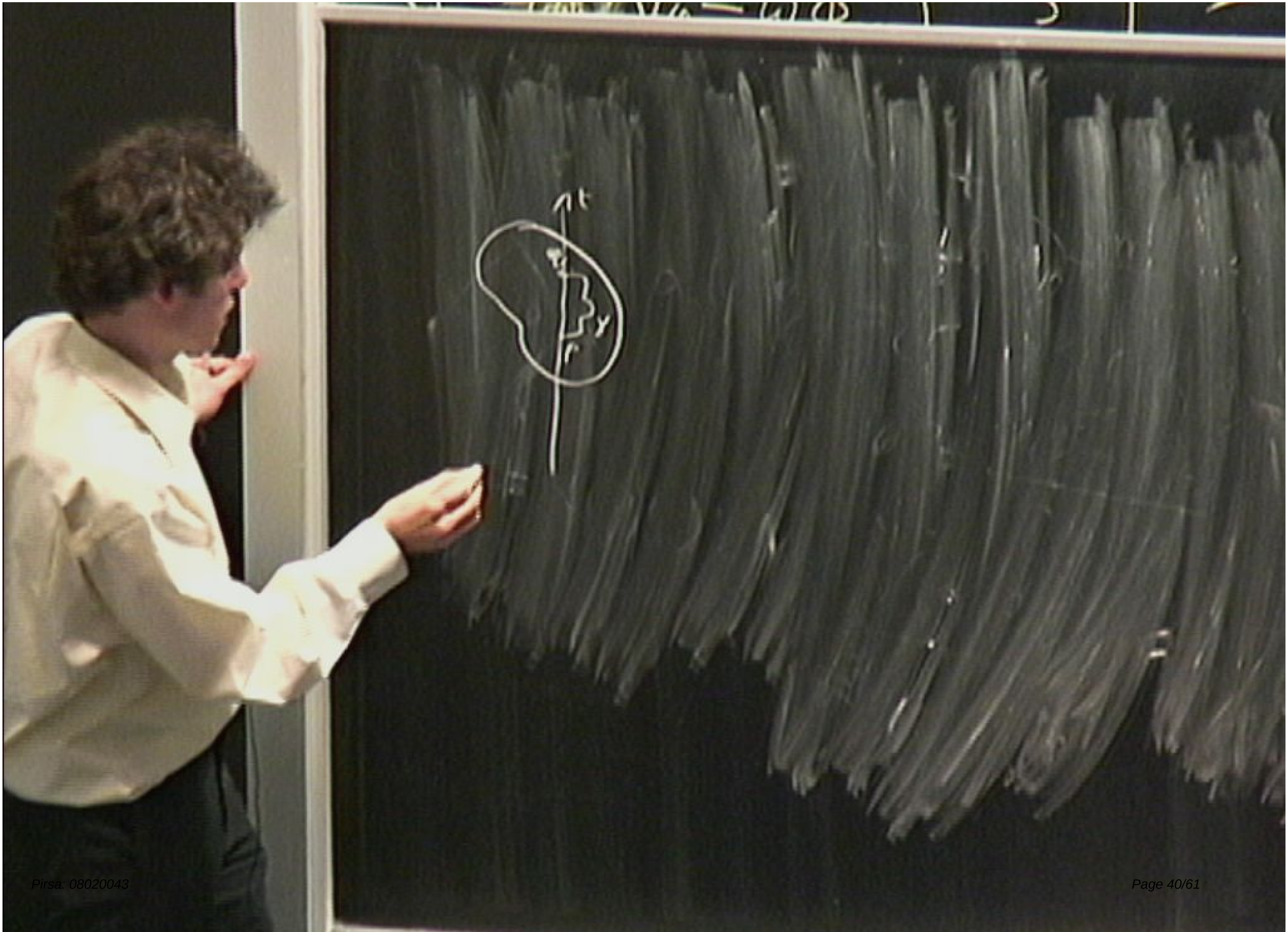

q
p
corr(α(p,q))

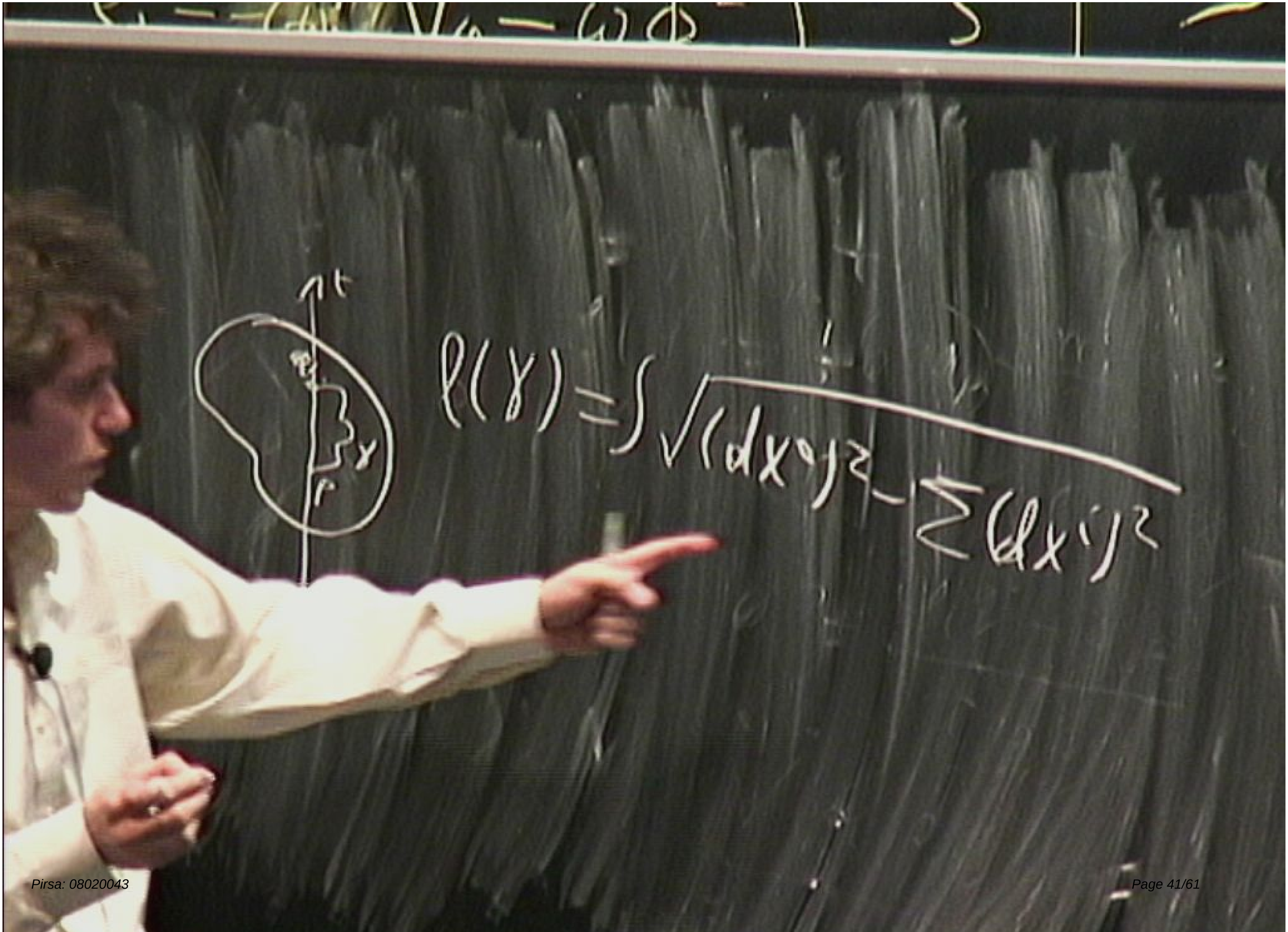

corr(p,q) = τ² (
corr(α(p,q))
V(α(p,q))
K_d (τ(p,q))^d

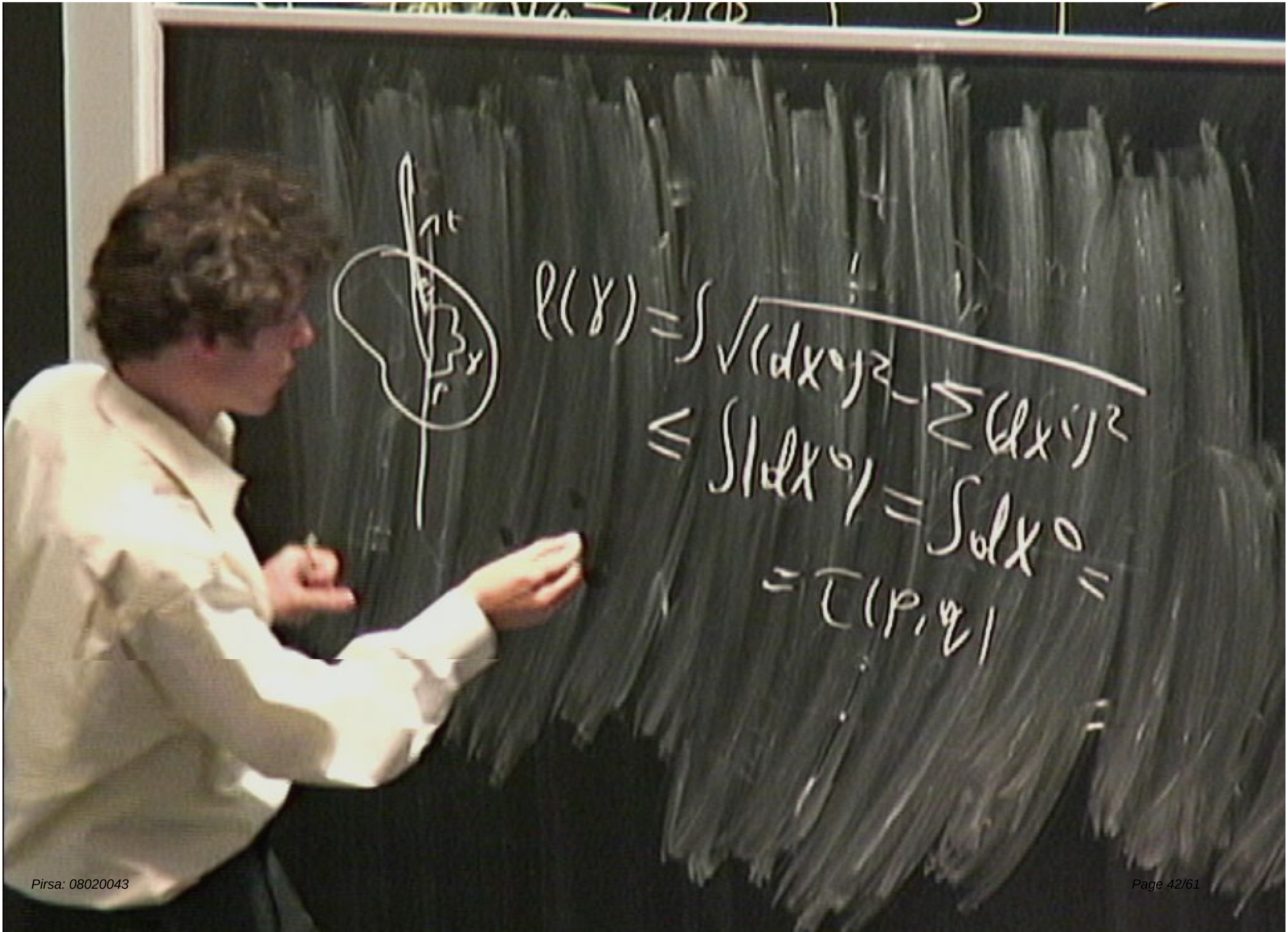

corr(p,q) = τ² (E R00 + F R)
corr(d(p,q))
V(d(p,q)) / Kd (τ(p,q))^d − 1 =
t
q
p

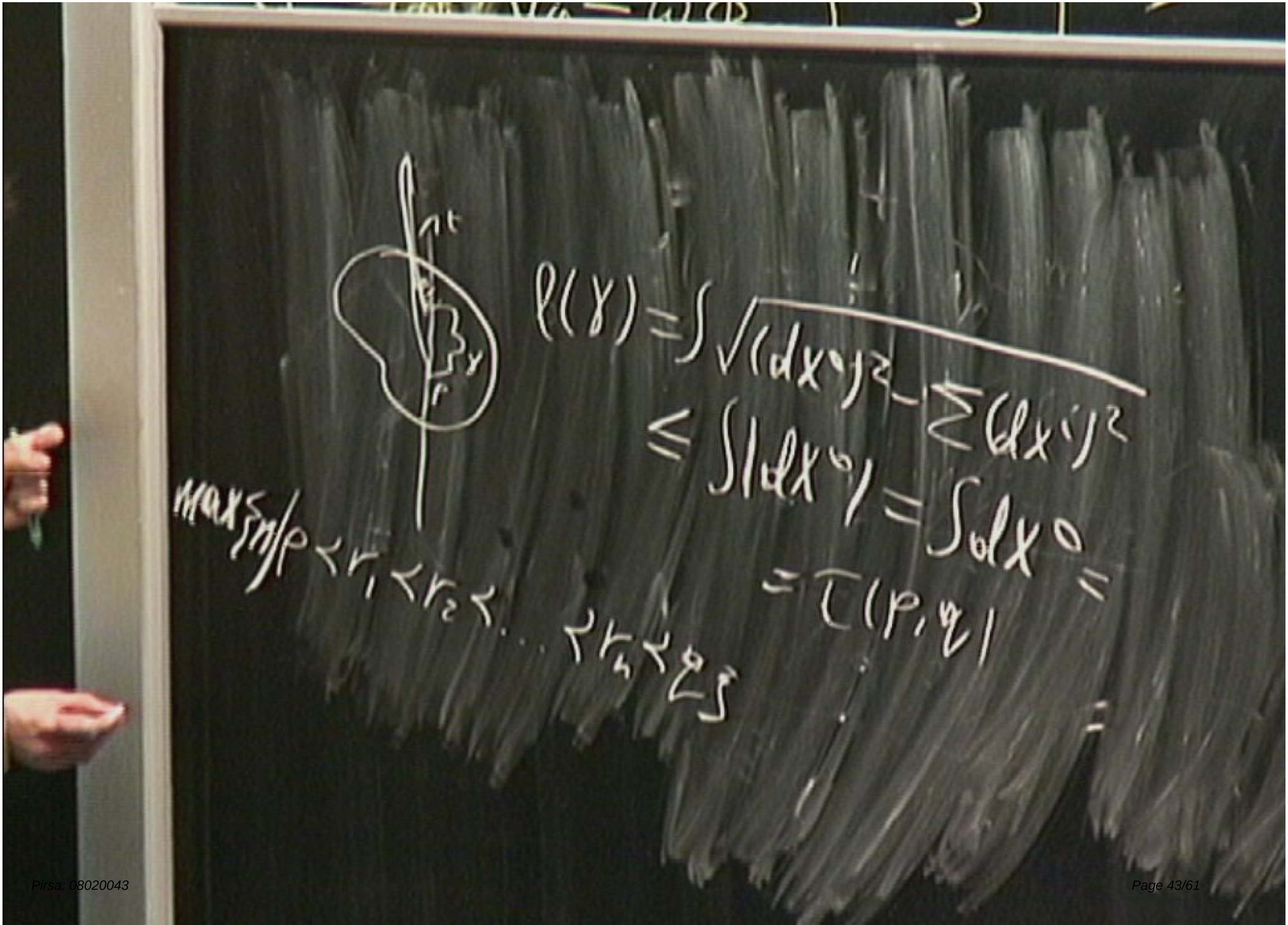

corr(p,q)
corr(α(p,q))

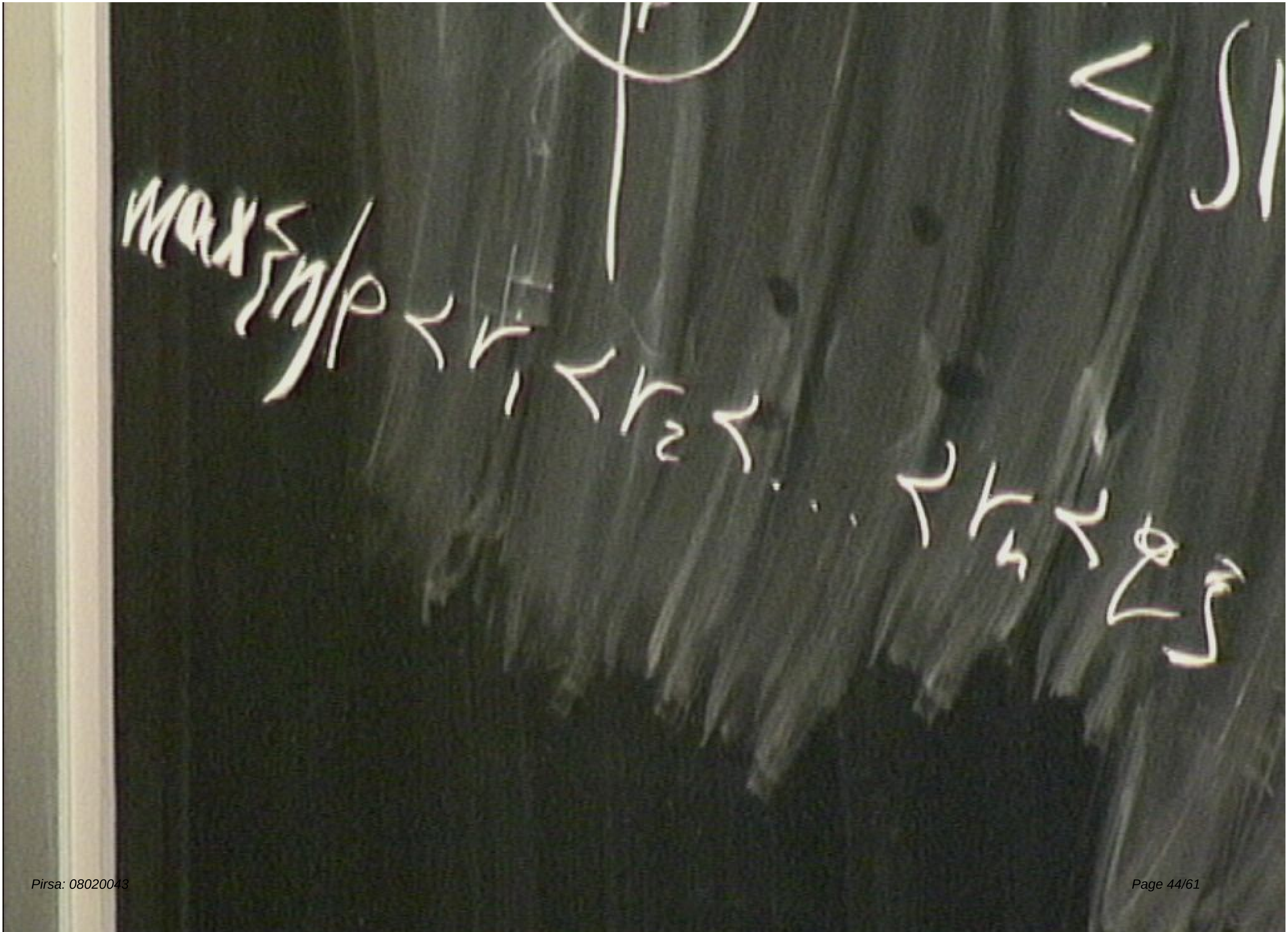

corr(p,q) = τ² (E R₀₀ + F R)
corr(α(p,q))
V(α(p,q))
K_d (τ(p,q))^d
t
q
p

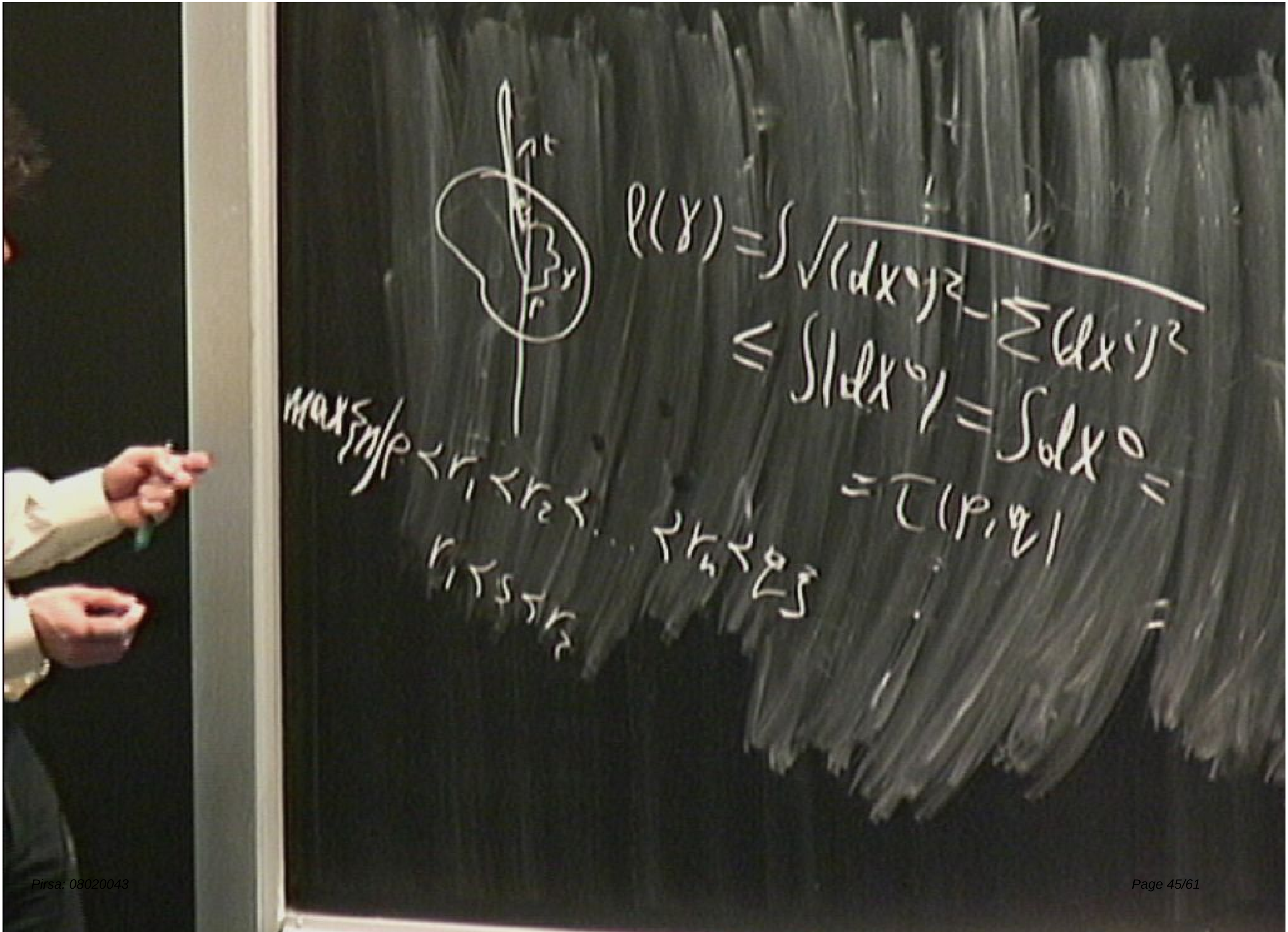


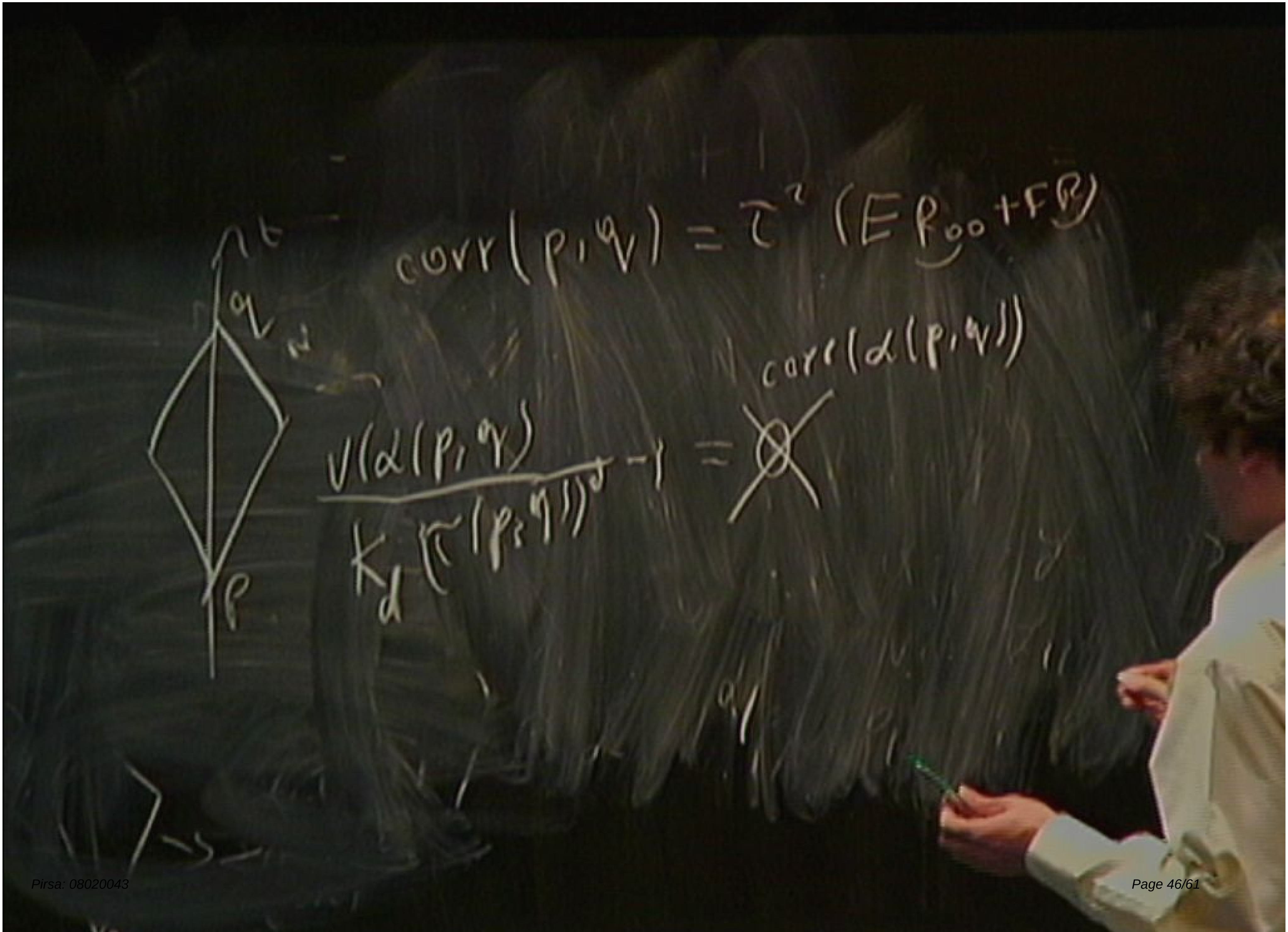

t
γ
ℓ(γ) = ∫ √((dx⁰)² − Σ(dxⁱ)²)

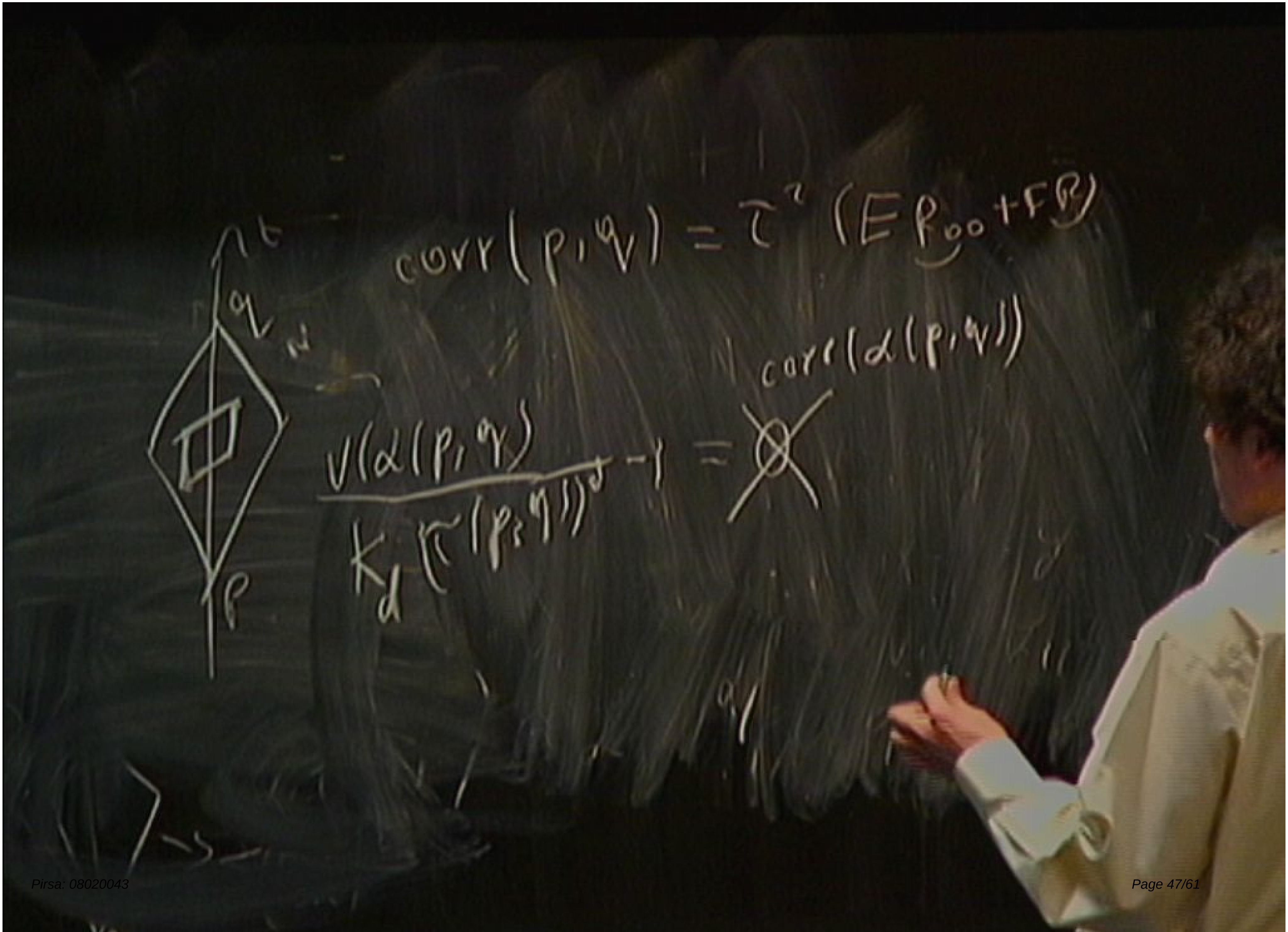

ℓ(γ) = ∫ √((dx⁰)² − Σ(dxⁱ)²)
≤ ∫ |dx⁰| = ∫ dx⁰
= τ(p, q)

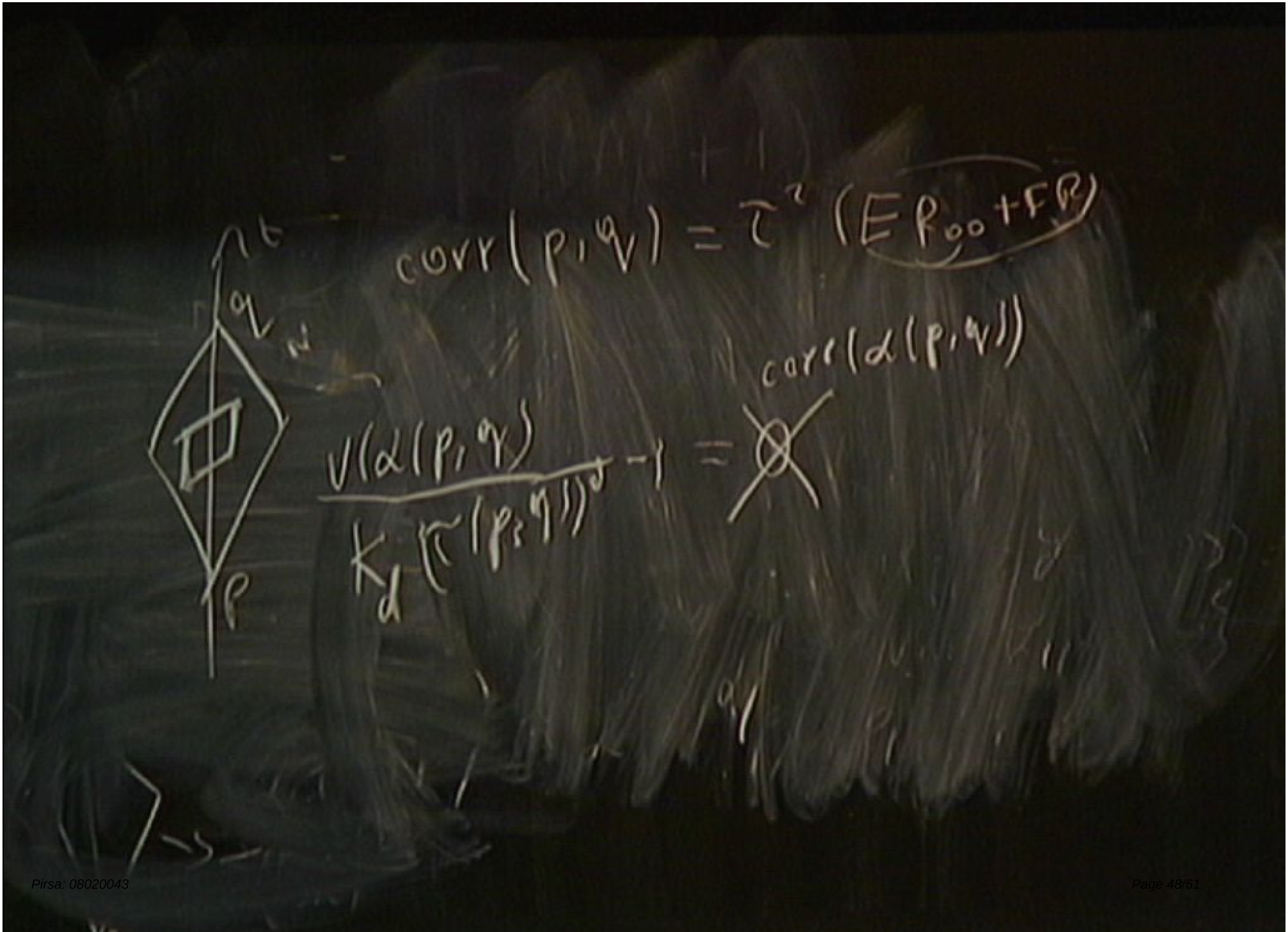

ℓ(γ) = ∫ √((dx⁰)² − Σ(dxⁱ)²)
≤ ∫ |dx⁰| = ∫ dx⁰ =
= τ(p, q)
max{n / p < r₁ < r₂ < … < rₙ < q}


$\max_{\xi} n/\rho < r_1 < r_2 < \dots < r_n < \phi_{\xi}$


$\ell(\gamma) = \int \sqrt{(dx^0)^2 - \sum (dx^i)^2}$
$\le \int |dx^0| = \int dx^0 =$
$= \tau(p,q)$
$\max \{ n / p < r_1 < r_2 < \dots < r_n < q \}$
$r_1 < s < r_2$


corr(p,q) = τ² (E R00 + F R)
corr(α(p,q))
V(α(p,q)) / Kd (τ(p,q))^d − 1 =
t
q
p


t
q
p
corr(p,q) = τ² (E R00 + F R)
corr(α(p,q))
V(α(p,q))
Kd (τ(p,q))


corr(p,q) = τ² (E R₀₀ + F R)
corr(α(p,q))
V(α(p,q)) / K_d (τ(p,q))^d − 1 = 0

ρ

$$corr(p,q) = \frac{\#\alpha(p,q)}{k_d(\max\{r \mid \exists r_1,\ldots,r_n(p<r_1<\ldots<r_n<q)\})^d} - 1 = (ER_{\mu\nu} + FRg_{\mu\nu})(p^\mu - q^\mu)(p^\nu - q^\nu)$$

whenever $\#\alpha(p,q) \geq M_1$

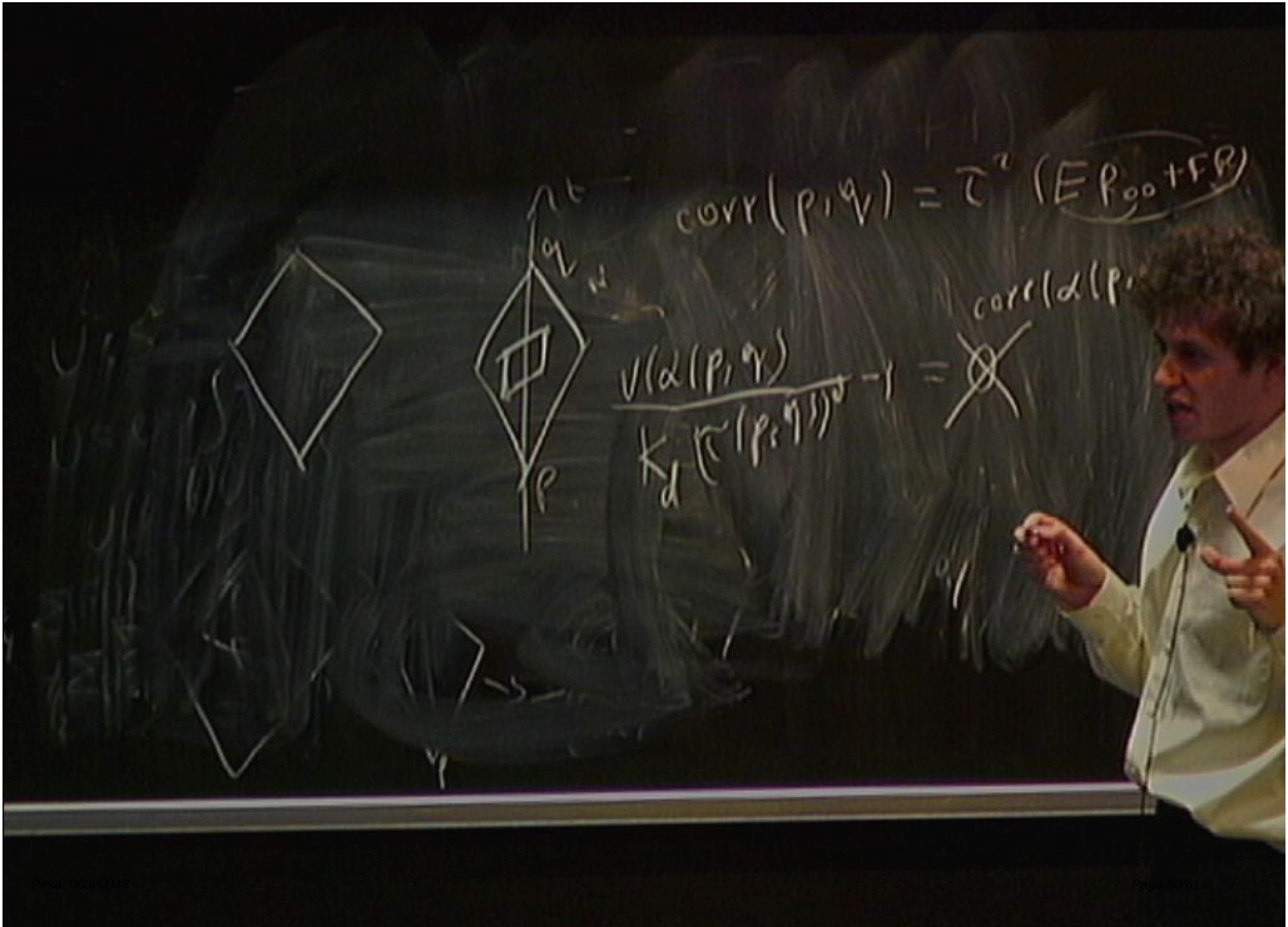
Suppose $\#\alpha(p,q) \geq M_2 >> M_1$ and again assume that t axis passes through p and q with origin in the middle: $p = (-\frac{\tau}{2},000)$ and $q = (\frac{\tau}{2},0,0,0)$

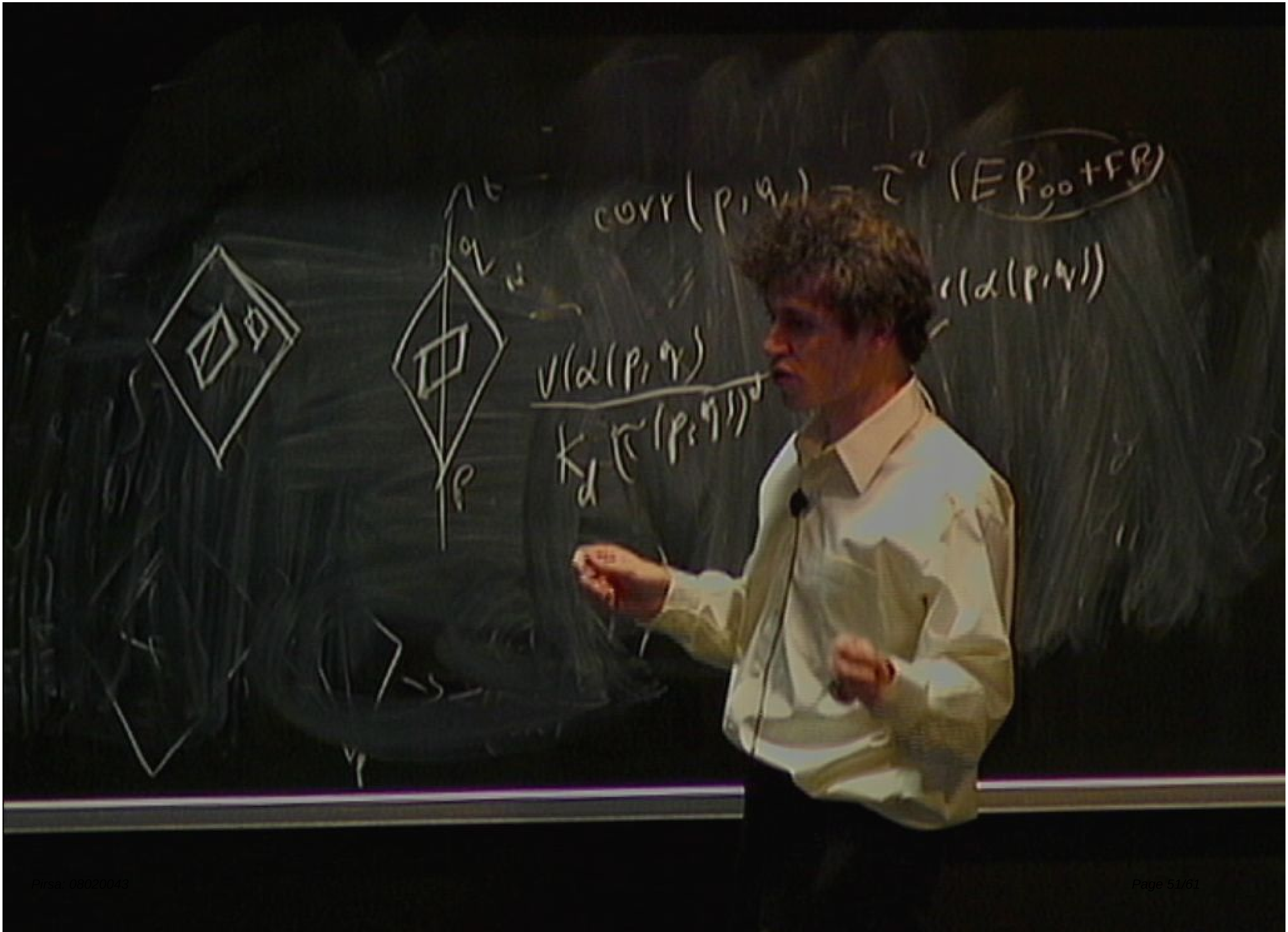
$$corr(p,q) = (ER_{00} + FR)\tau^2$$

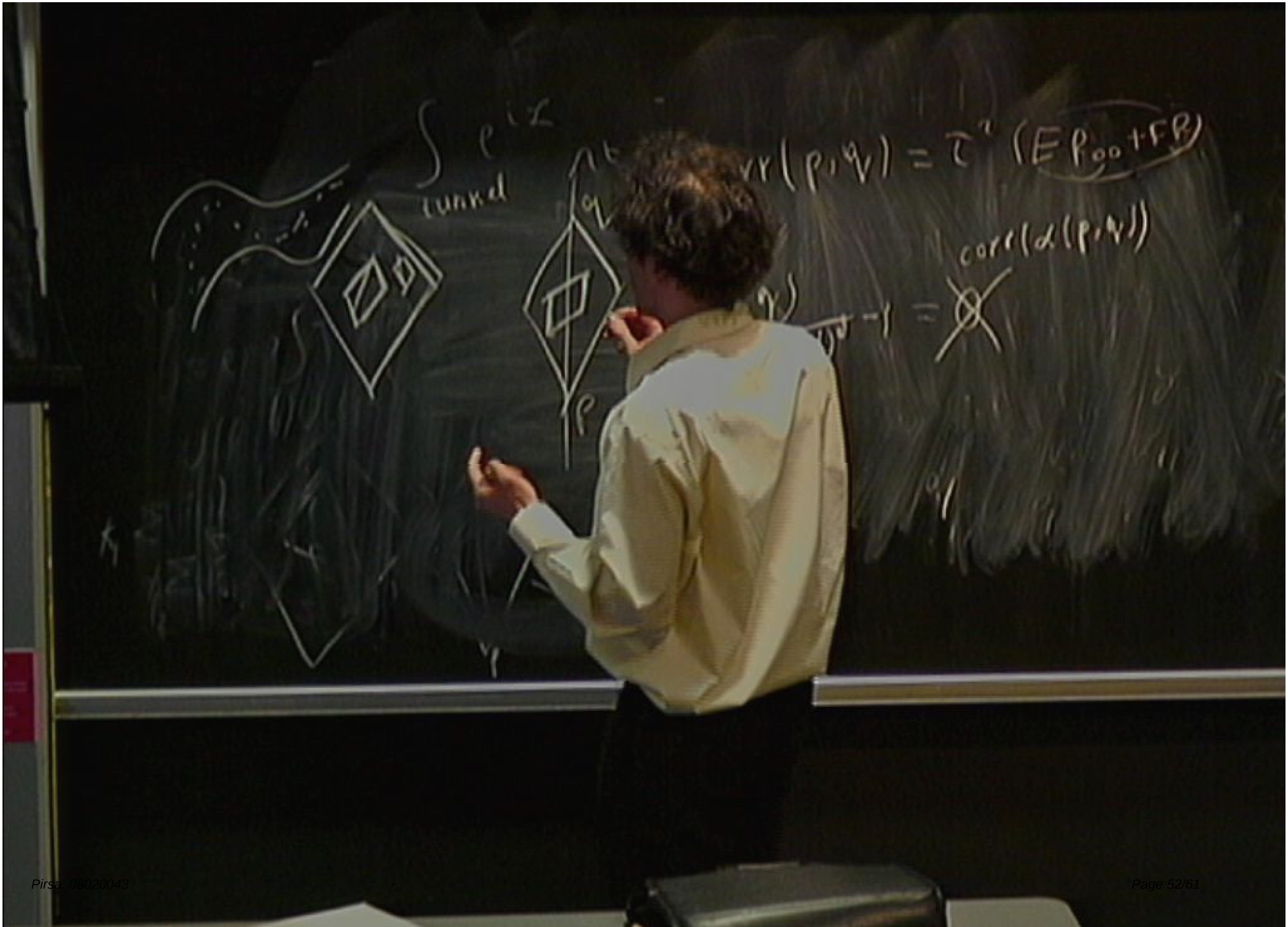
$$\frac{1}{\rho^2} \sum_{p<r<s<q,\ \#\alpha(r,s)\geq M_1} corr(\alpha(r,s)) = \int_{p<r<s<q} d^d r\, d^d s (ER_{\mu\nu} + FRg_{\mu\nu})(r^\mu - s^\mu)(r^\nu - s^\nu) = \tau^{2d+2}(GR_{00} + HR)$$

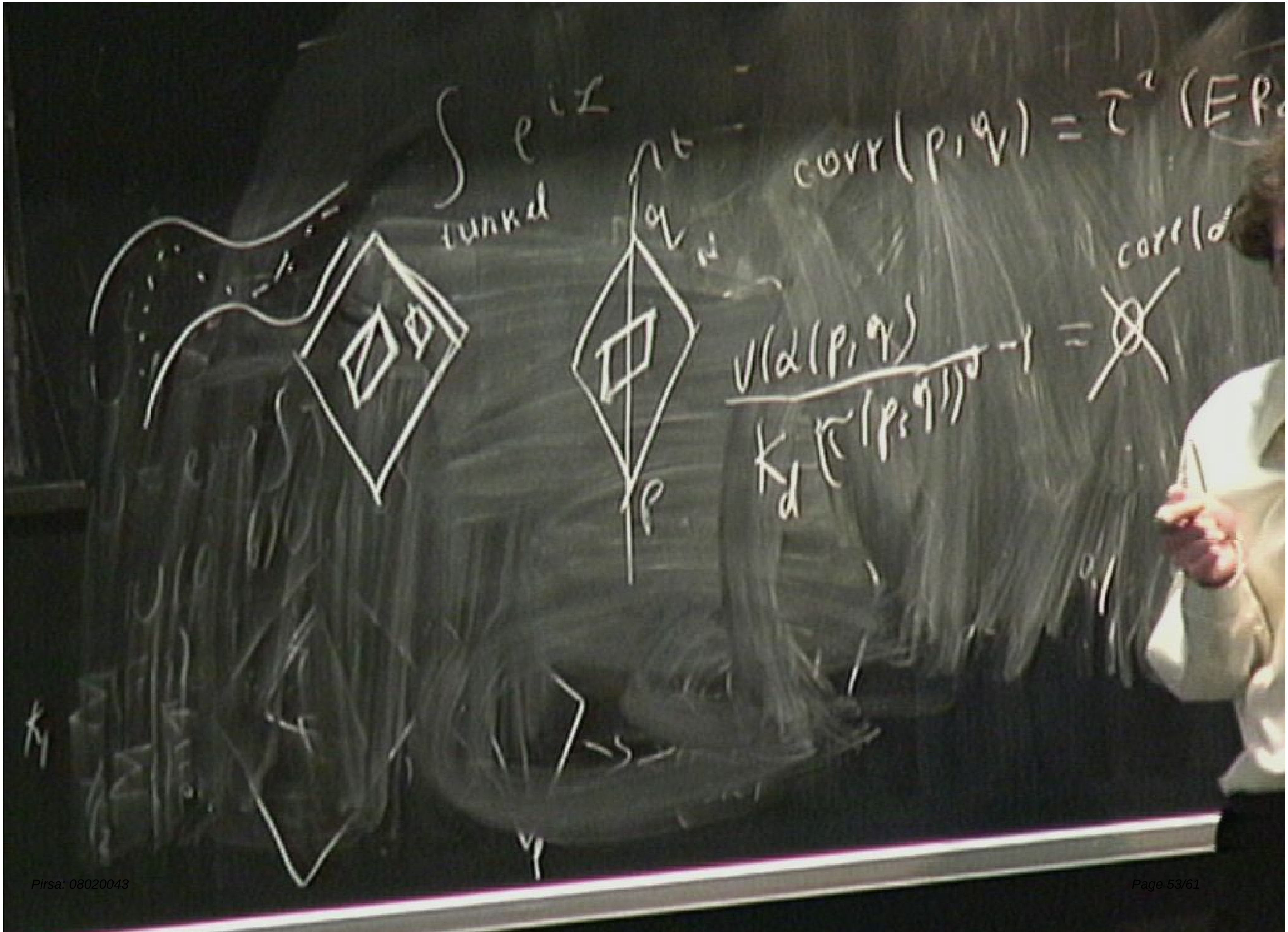
$$R = \frac{1}{\frac{E}{F} - \frac{G}{H}}\left(\frac{corr(\alpha(p,q))}{E\tau^2} - \frac{\sum_{p<r<s<q,\ \#\alpha(r,s)\geq M_1} corr(\alpha(r,s))}{F\tau^{2d+2}}\right)$$

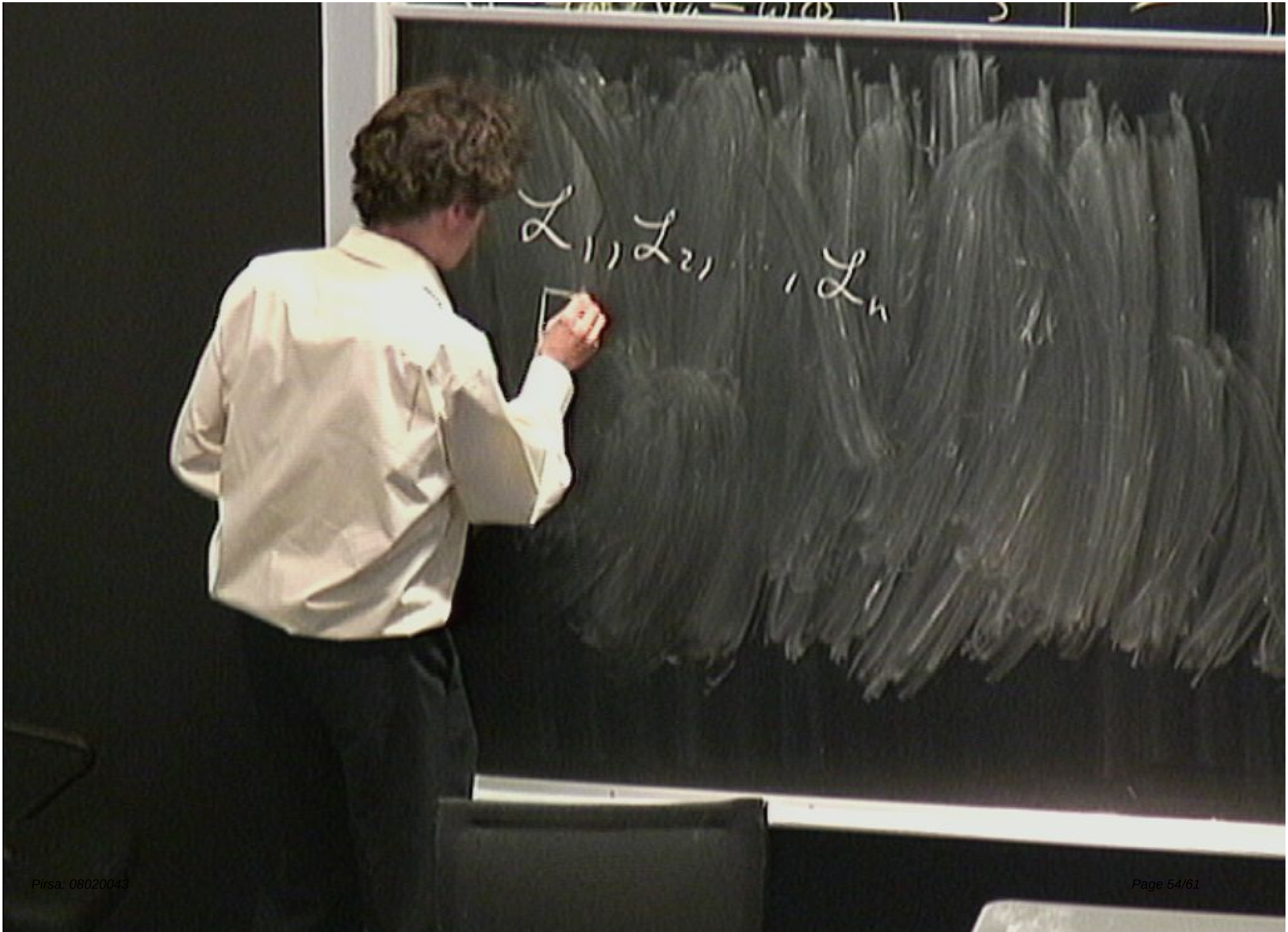
$$= \frac{1}{\frac{E}{F} - \frac{G}{H}}\left(\frac{\rho^{2/d} corr(\alpha(p,q))}{E(\max\{r \mid \exists r_1,\ldots,r_n(p<r_1<\ldots<r_n<q)\})^2} - \frac{\rho^{2+2/d} \sum_{p<r<s<q,\ \#\alpha(r,s)\geq M_1} corr(\alpha(r,s))}{F(\max\{r \mid \exists r_1,\ldots,r_n(p<r_1<\ldots<r_n<q)\})^{2d+2}}\right)$$

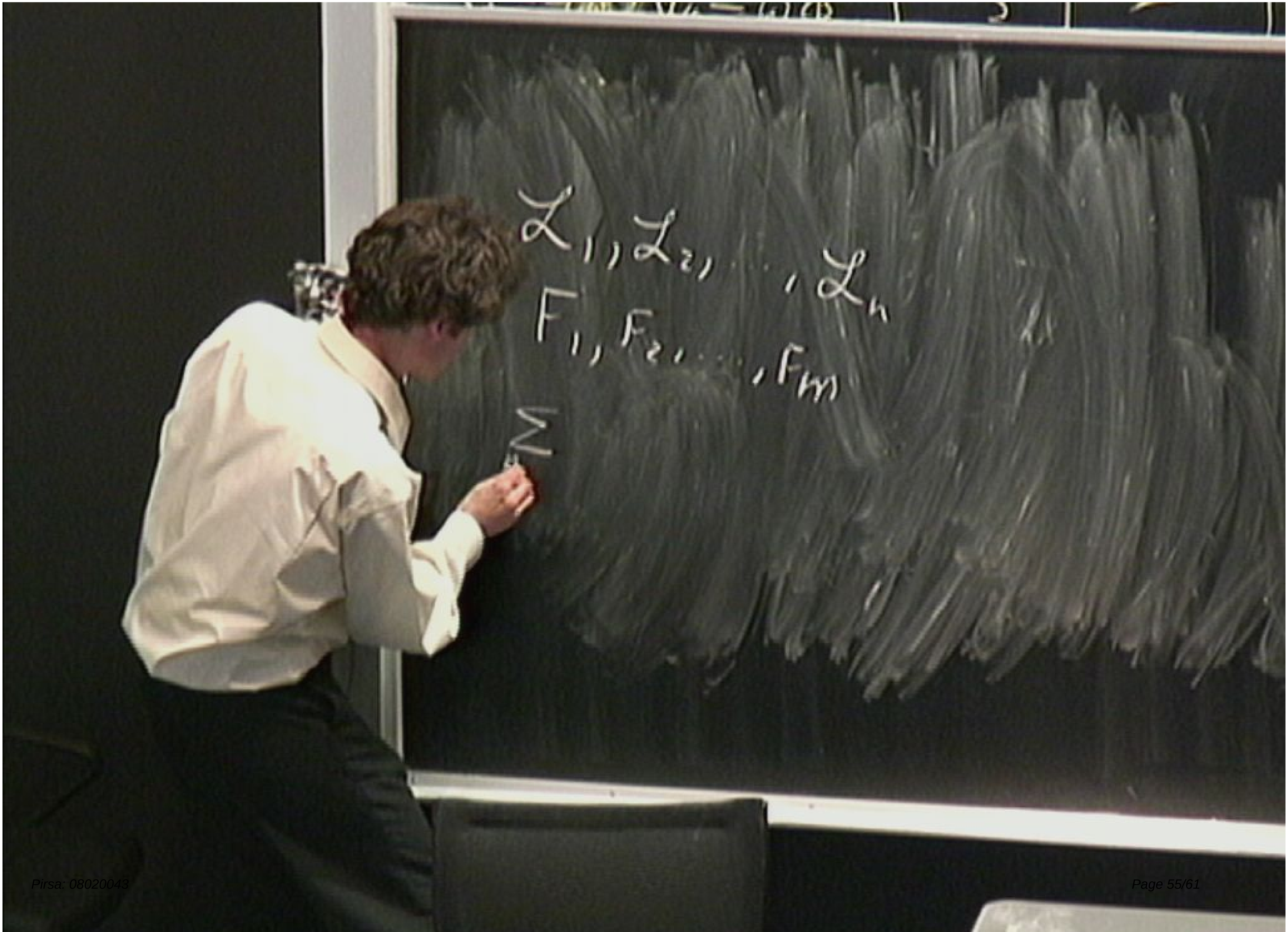

corr(p,q) = τ² (ER00 + FR)
corr(α(p,

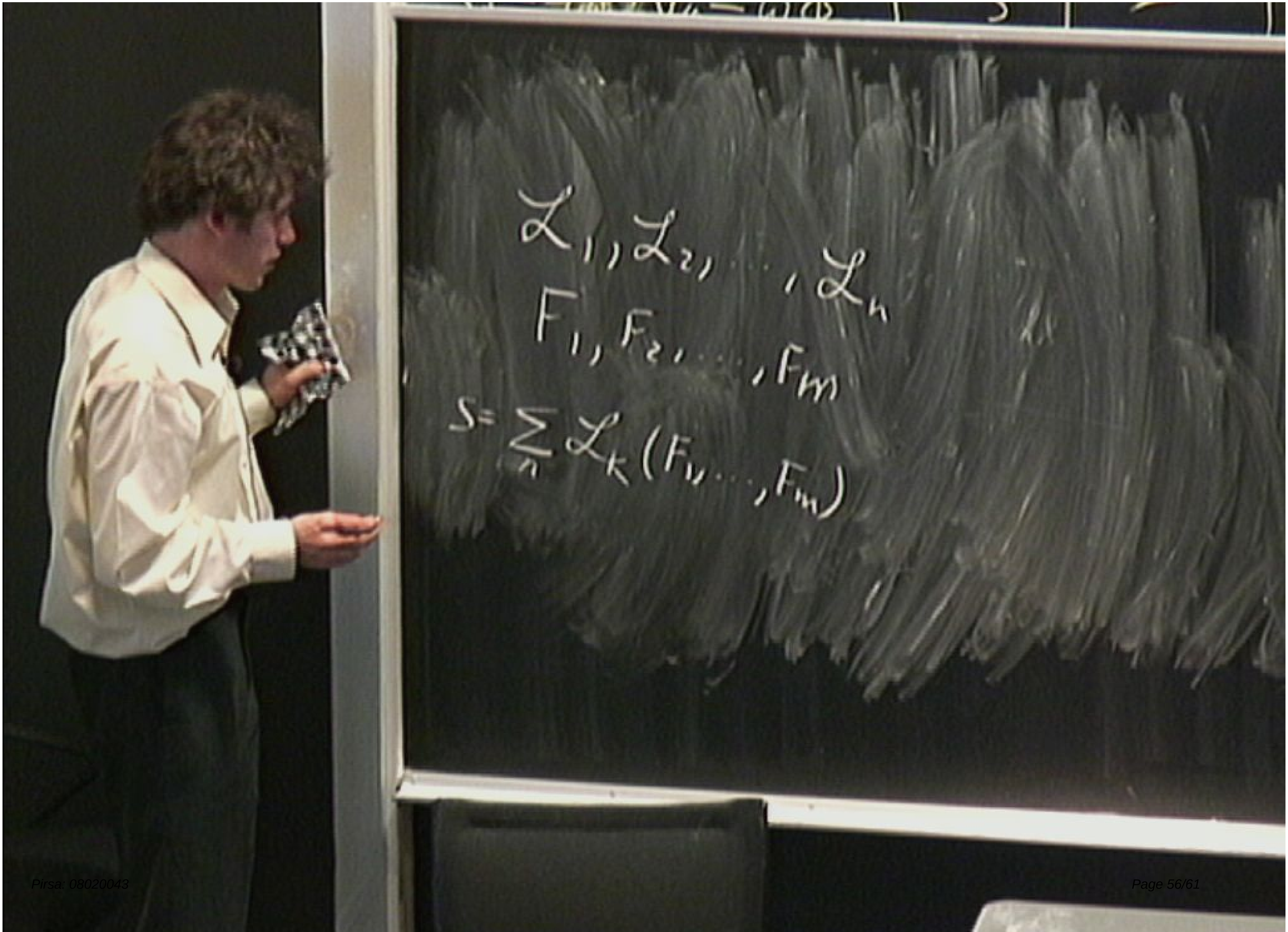

corr(p,q)
V(α(p,q))

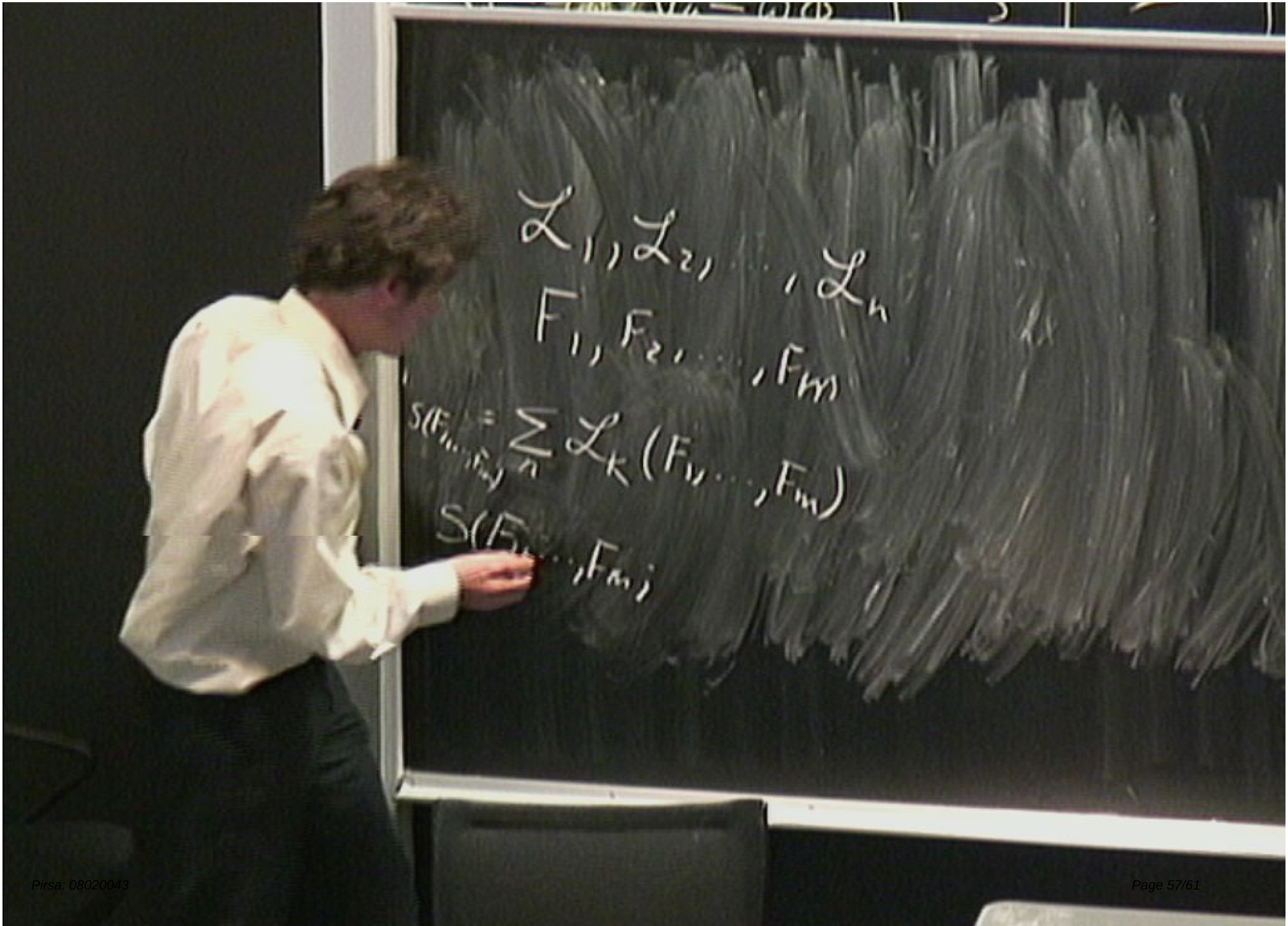

tunnel

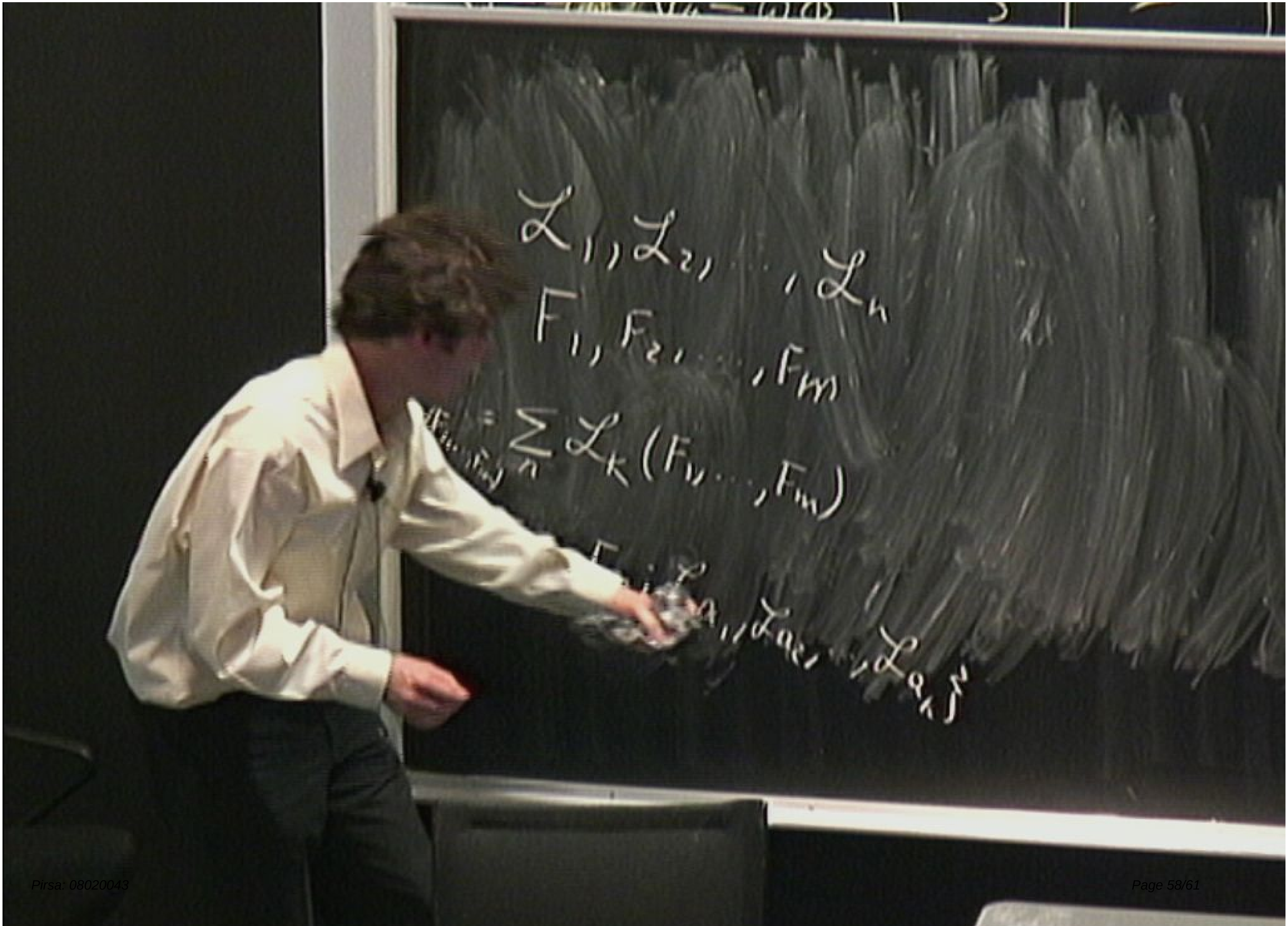

corr(p,q) =
tunnel


$\mathcal{L}_1, \mathcal{L}_2, \ldots, \mathcal{L}_n$

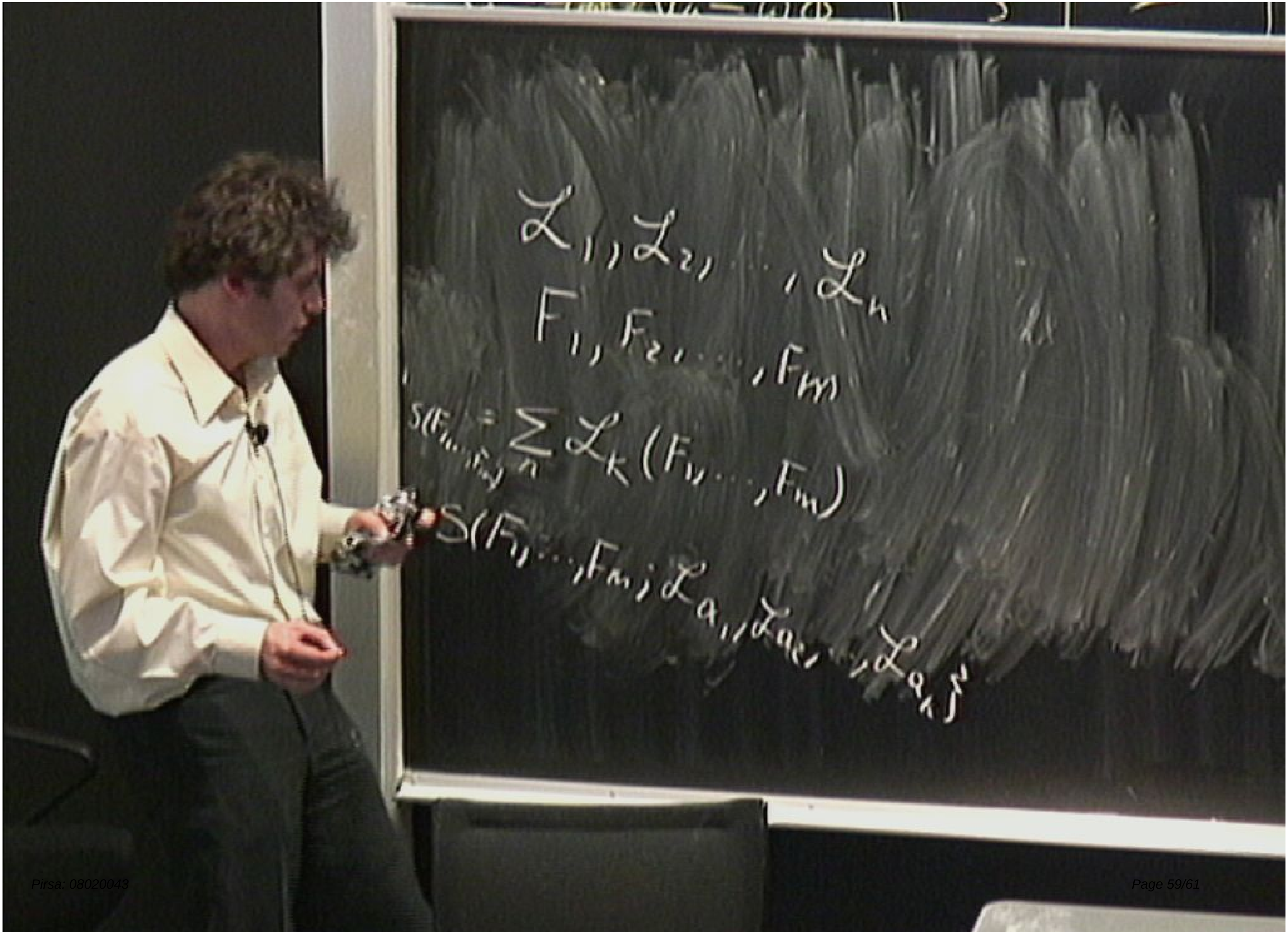

$\mathcal{L}_1, \mathcal{L}_2, \ldots, \mathcal{L}_n$
$F_1, F_2, \ldots, F_m$
Σ

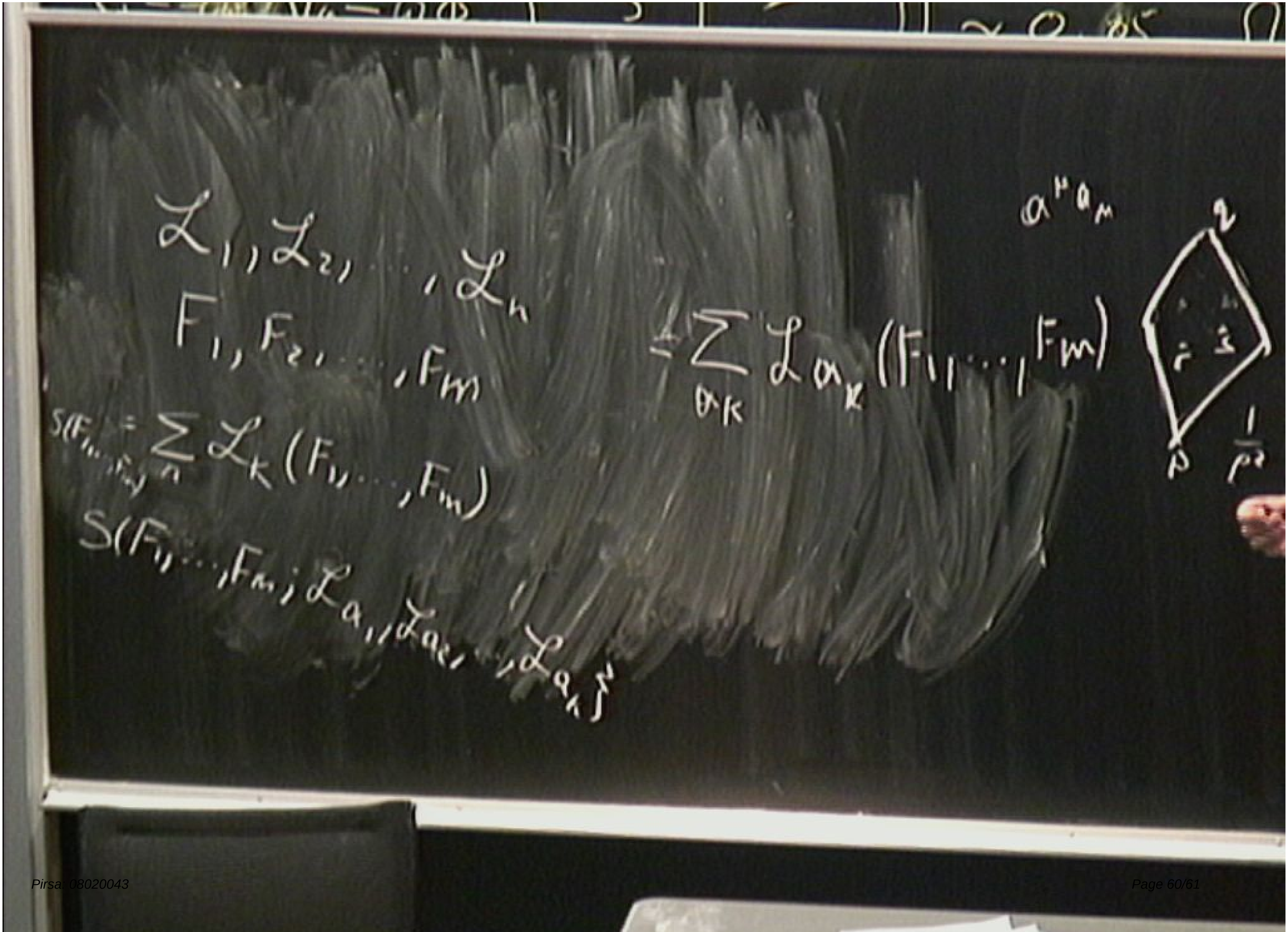

$\mathcal{L}_1, \mathcal{L}_2, \ldots, \mathcal{L}_n$
$F_1, F_2, \ldots, F_m$
$S = \sum_n \mathcal{L}_k(F_1, \ldots, F_m)$


$\mathcal{L}_1, \mathcal{L}_2, \ldots, \mathcal{L}_n$
$F_1, F_2, \ldots, F_m$
$S(F_1, \ldots, F_m) = \sum_n \mathcal{L}_k(F_1, \ldots, F_m)$
$S(F_1, \ldots, F_m;$



$\mathcal{L}_1, \mathcal{L}_2, \ldots, \mathcal{L}_n$
$F_1, F_2, \ldots, F_m$
$\sum_n \mathcal{L}_k(F_1, \ldots, F_m)$


𝓛1, 𝓛2, …, 𝓛n
F1, F2, …, Fm
S(F1,…,Fm) = Σn 𝓛k(F1,…,Fm)
S(F1,…,Fm; 𝓛a1, 𝓛a2, …, 𝓛an)


$\mathcal{L}_1, \mathcal{L}_2, \ldots, \mathcal{L}_n$
$F_1, F_2, \ldots, F_m$
$S(F_1, \ldots, F_m) = \sum_n \mathcal{L}_k(F_1, \ldots, F_m)$
$S(F_1, \ldots, F_m; \mathcal{L}_{a_1}, \mathcal{L}_{a_2}, \ldots, \mathcal{L}_{a_n})$
$= \sum_{a_k} \mathcal{L}_{a_k}(F_1, \ldots, F_m)$
$a^\mu a_\mu$
$\frac{1}{p^2}$

$\mathcal{L}_1, \mathcal{L}_2, \ldots, \mathcal{L}_n$

$F_1, F_2, \ldots, F_m$

$S(F_1,\ldots,F_m) = \sum_n \mathcal{L}_k(F_1,\ldots,F_m)$

$S(F_1,\ldots,F_m; \mathcal{L}_{a_1}, \mathcal{L}_{a_2}, \ldots, \mathcal{L}_{a_n})$

$= \sum_{a_k} \mathcal{L}_{a_k}(F_1,\ldots,F_m)$

$a^\mu a_\mu$