Title: Living without Birkhoff\'s law

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Abstract: TBA

Living without Birkhoff's law

Speaker:De-Chang Dai

Colleague: Glenn Starkman, Irit Maor, and Reijiro Matsuo.

Accepted: phys. Rev. D

PI:02/08/2008

What is Birkhoff's law

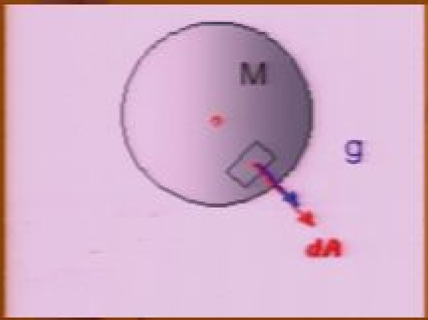
- It is the Gauss's Law in Newtonian gravity
- The gravitational force inside a spherical shell is zero.
- Geometry in spherically symmetric space is a piece of the Schwarzschild geometry.
- Single solution.

Application of Birkhoff's law?

- Gauss's law:

  The mass inside a closed surface can be determined by the gravitational acceleration on the surface.

$$\oint_S \mathbf{g} \cdot \mathrm{d}\mathbf{A} = -4\pi GM$$



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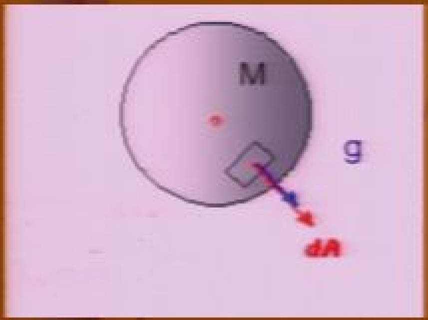

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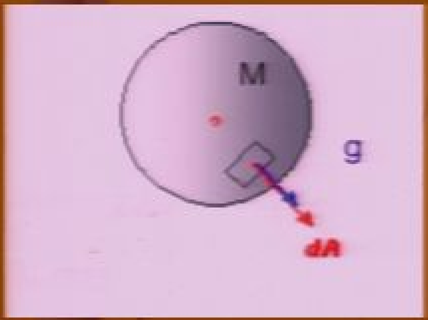

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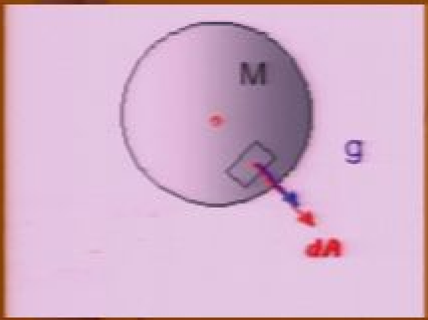


- Local gravitational force can be determined by local matters.

No gravitational acceleration inside a spherical shell

- Newtonian gravity: True
- General relativity:

  True (without considering backreaction)

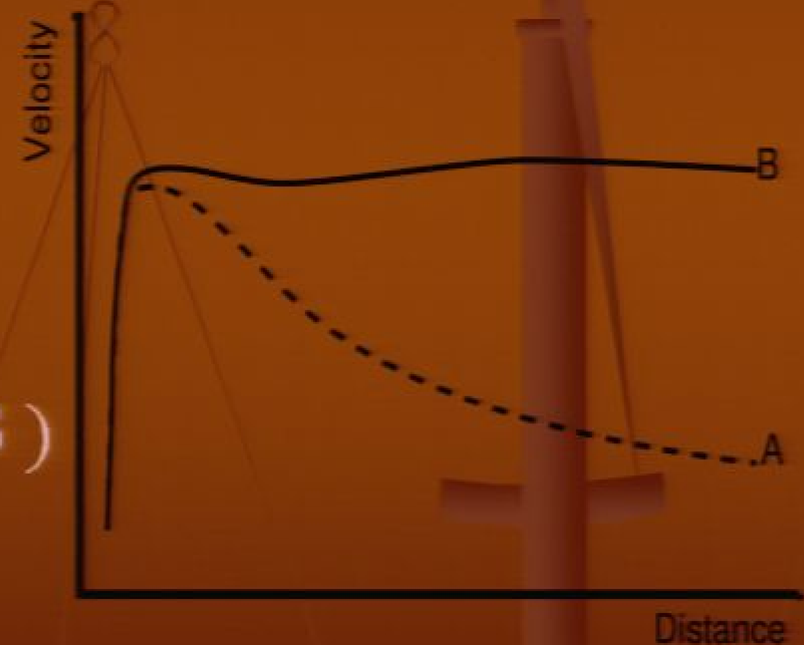
  There is no sphere in an aspherical space.

$$\frac{d^2 x_k}{dt^2} = \sum_{a\neq k} \vec{r}_{ak}\,\frac{M_a}{r_{ak}^3}\Big[1 - 4\sum_{b\neq k}\frac{M_b}{r_{bk}} - \sum_{b\neq a}\frac{M_c}{r_{ca}}\Big(1-\frac{\vec{r}_{ak}\cdot\vec{r}_{ca}}{2r_{ca}^2}\Big)\Big]$$
$$+\frac{7}{2}\sum_{a\neq k}\sum_{c\neq a}\vec{r}_{ca}\,\frac{M_a M_b}{r_{ak}\, r_{ca}^3}$$

$$a_k \propto M_k$$

Problem of Birkhoff's law

There exists unknown dark matter on the galaxy scale.



Alternative solution:

(Mordehai Milgrom, 1983)

Modified Newtonian

Dynamics: MOND

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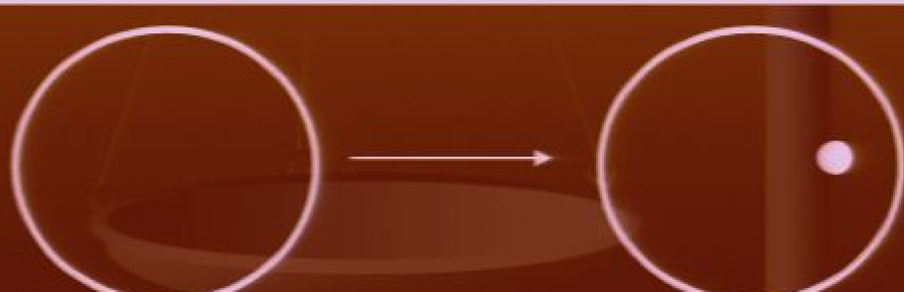

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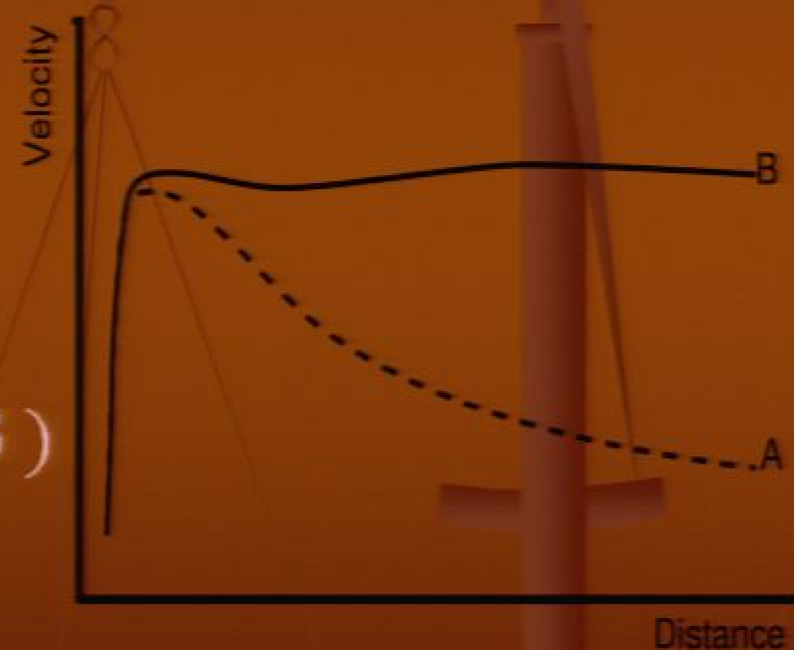

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$$\vec{F} = m\vec{a} \to m\mu(a/a_0)\vec{a}$$

$$a_0 = 1.2\times 10^{-10} m/s^2$$

- Modified Gauss's law

$$\nabla\cdot\nabla\phi = \rho \to \nabla\cdot(\mu(|\nabla\phi|/a_0)\nabla\phi) = \rho$$

$$\mu(x) = \begin{cases} 1, & x \gg 1 \\ x, & x \ll 1 \end{cases}$$

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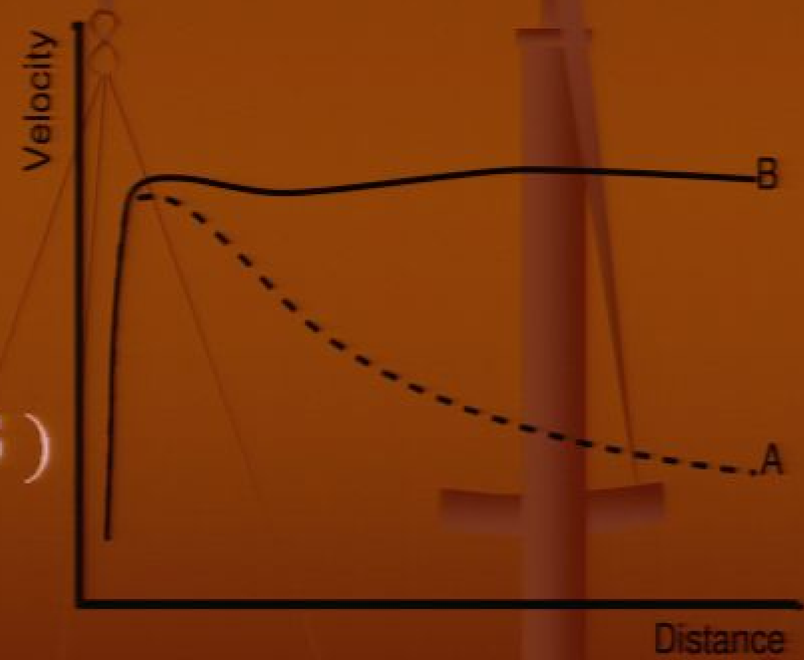

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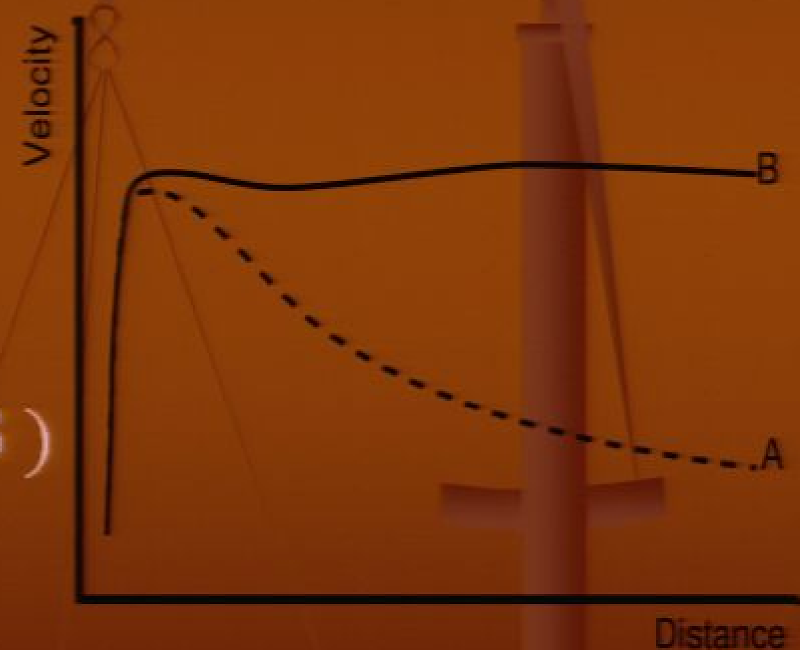

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$$u = \frac{x}{1+x}$$

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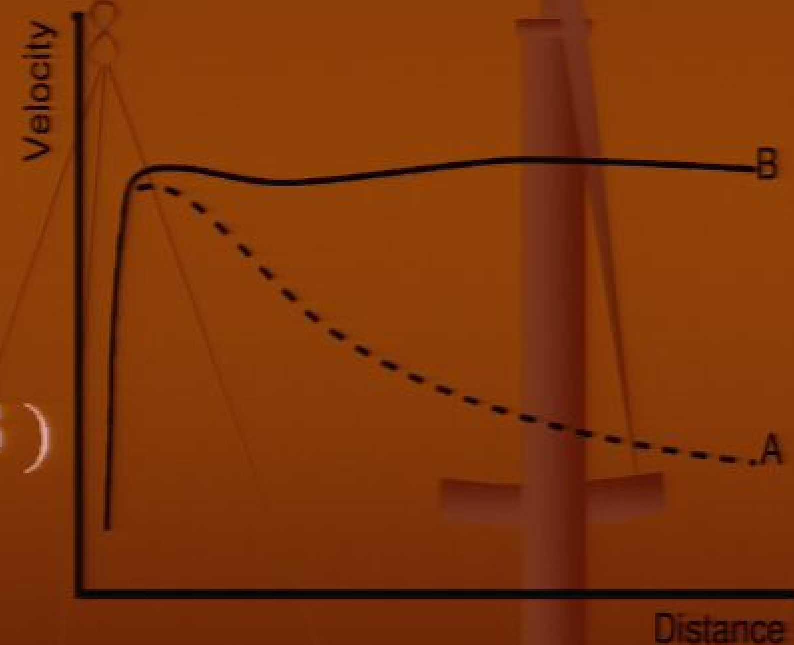

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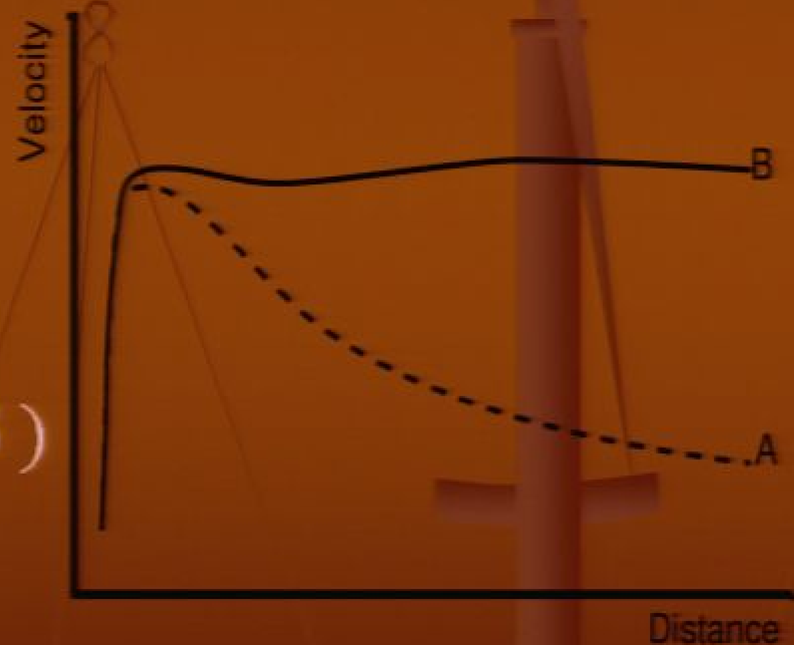

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- A heavy particle and a light shell: false

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$$a \not\propto m$$

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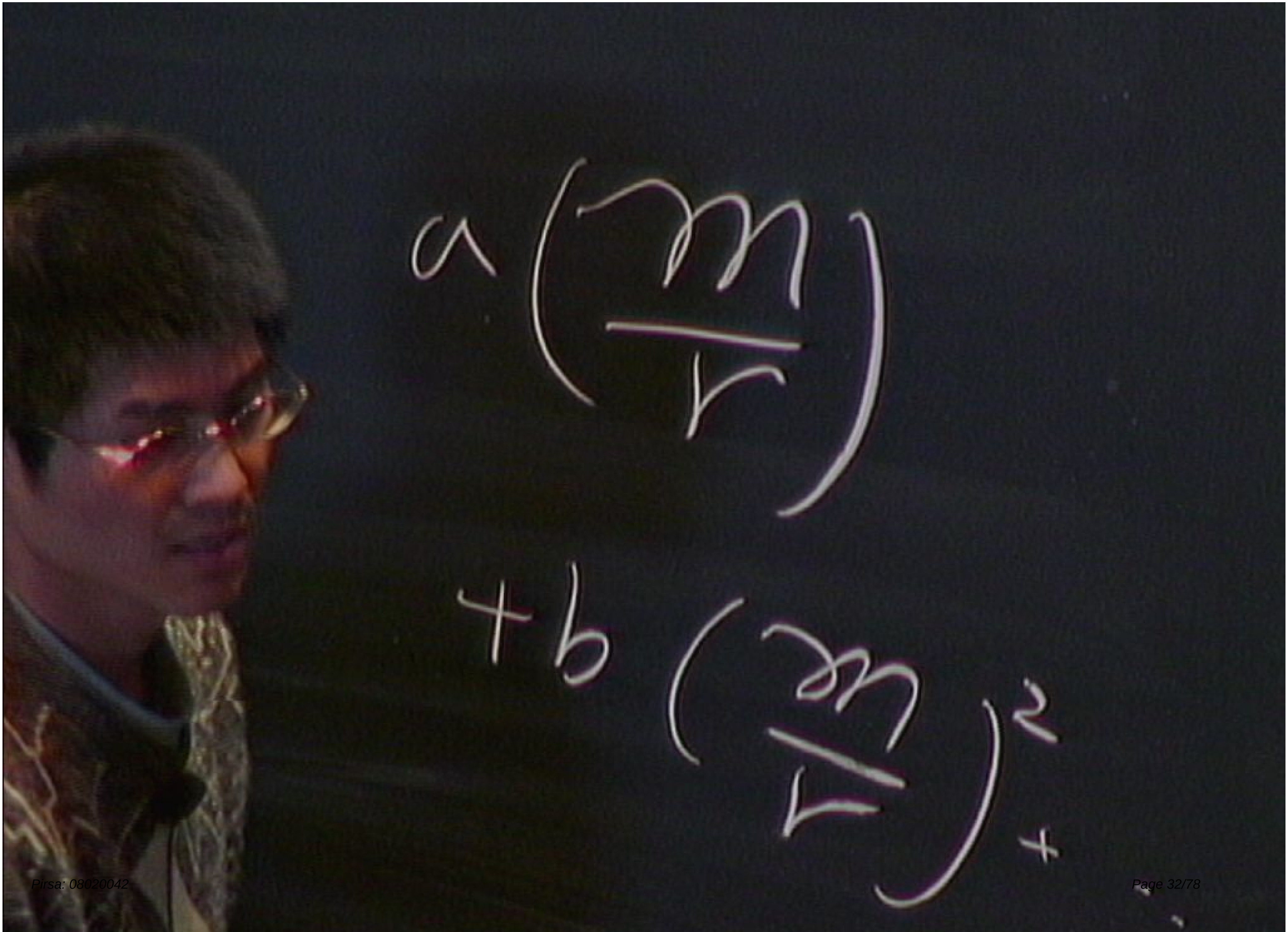
$a\left(\frac{m}{r}\right)$
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How sad, the stars in the sky really affect our life.

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- MOND: Modified Poisson's equation
- DGP: high dimensional theory
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- F(R): $R \longrightarrow F(R)$
- TeVeS: relativistic version of MOND
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- General Einstein Aether: introduce a vector field
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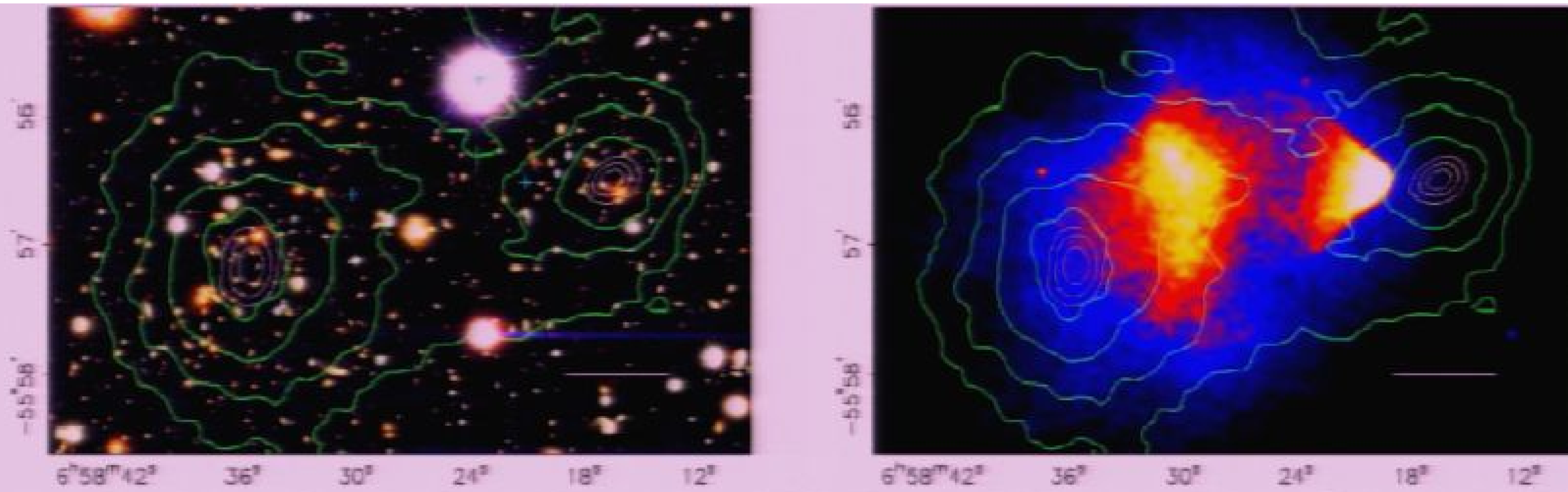
Is ΛCDM distinguishable from modified gravity theories?

- First direct evidence of dark matter ?
  Bullet Cluster(1E0657-56)
- The possible Modified gravity models: Scalar-Tensor-Vector, and TeVeS(?).
- Other models: need ν with m_ν=2eV

- Current: check whether Modified Gravity Theory can explain Bullet Cluster.

NASA



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Astro-ph/0608407

M. Clowe et al.

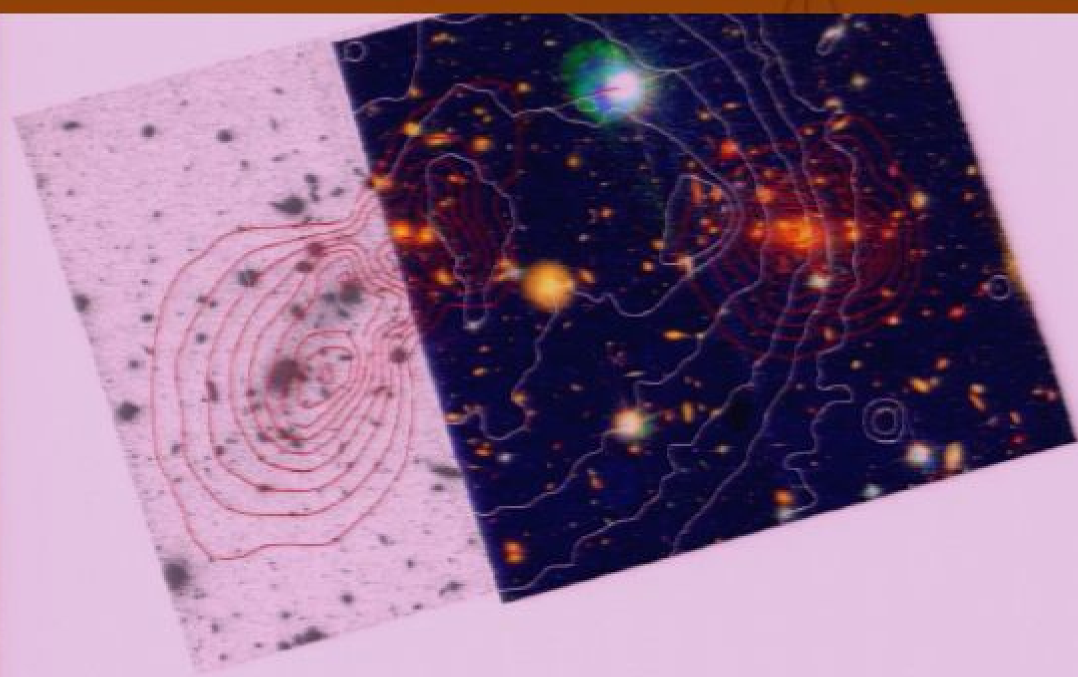

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FIG. 4.— The F435W-F606W-F814 color composite of the cluster 1E0657−56. Overlaid in red contours is the surface mass density κ from the combined weak and strong lensing mass reconstruction (for the purpose of this plot we recalculate the final κ-map from top-left panel of Fig. 6 on a finer grid). The contour levels are linearly spaced with $\Delta\kappa = 0.1$, starting at $\kappa = 0.5$, for a fiducial source at a redshift

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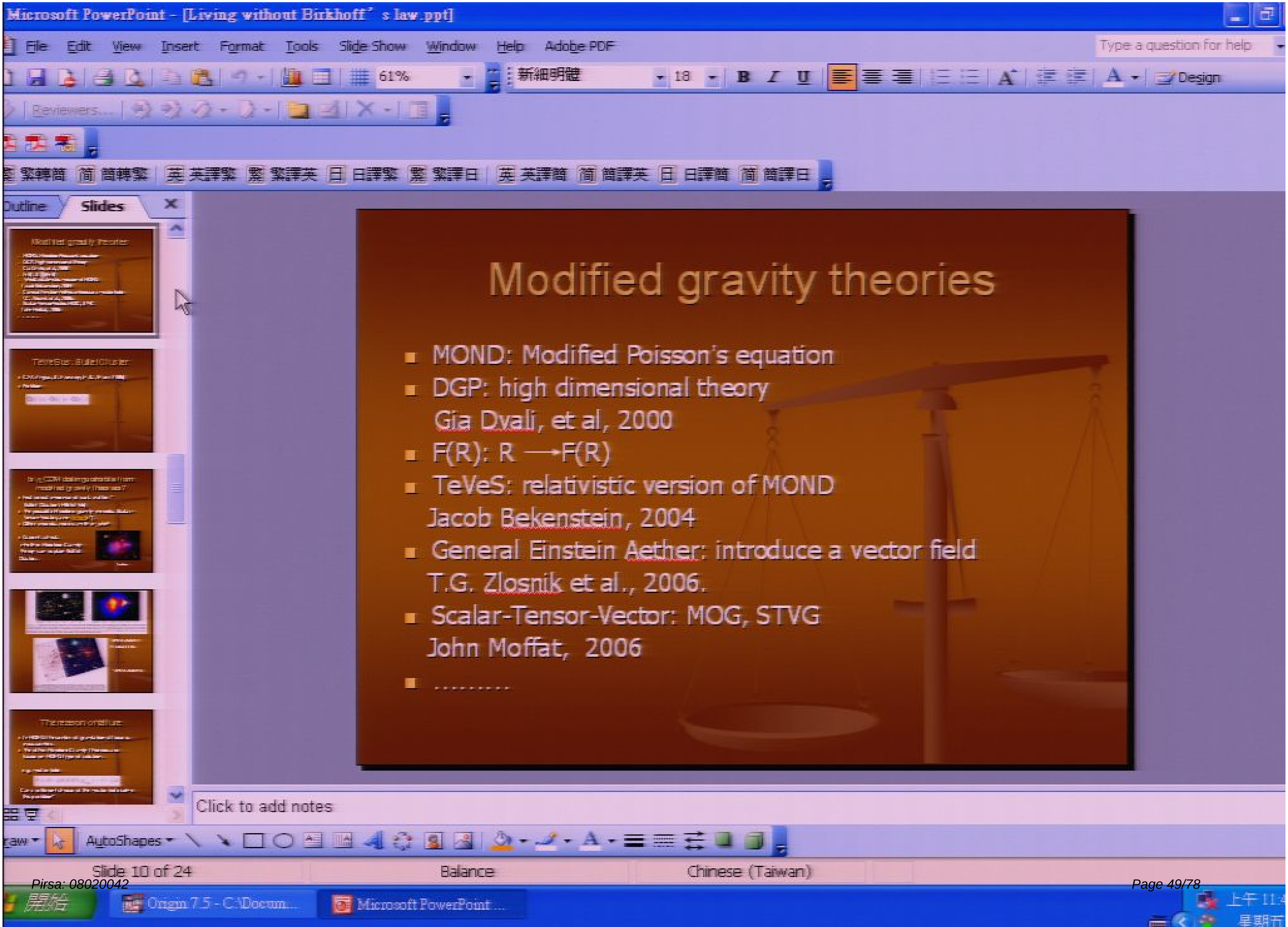
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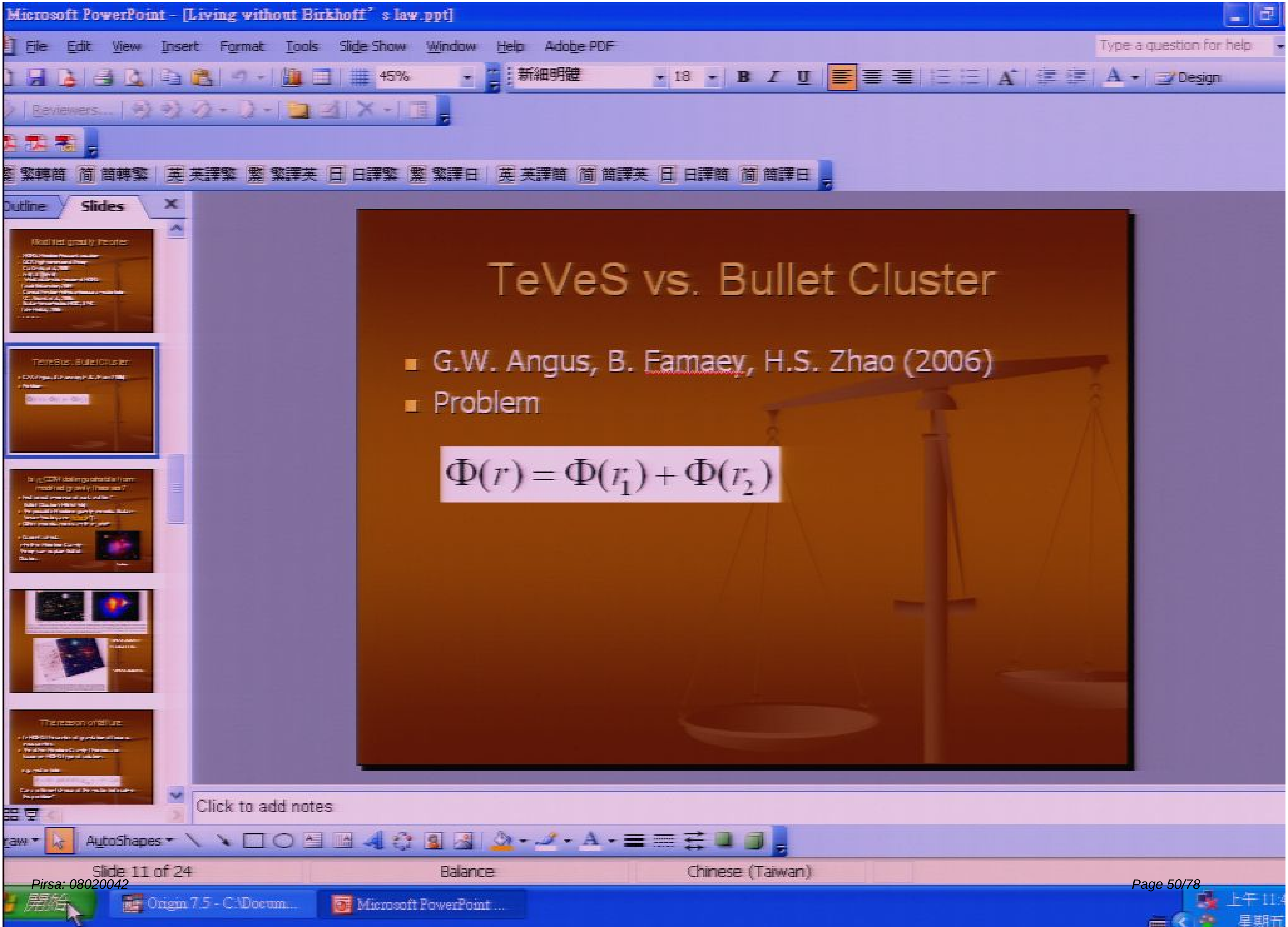
Slide 10 of 24
Balance
Chinese (Taiwan)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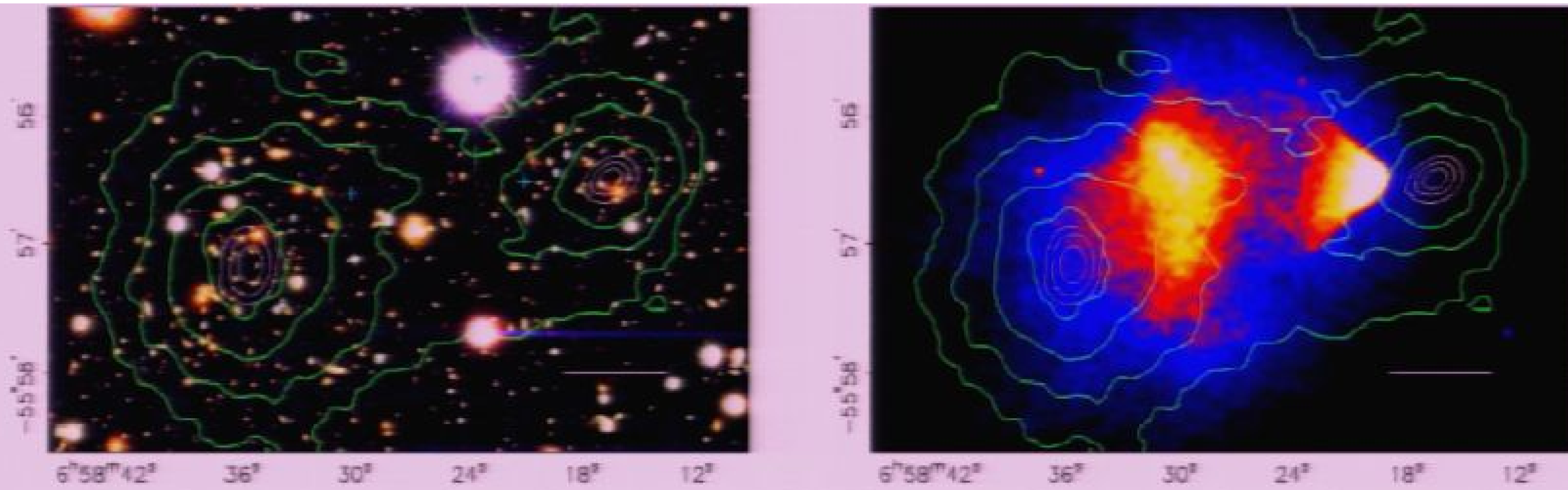

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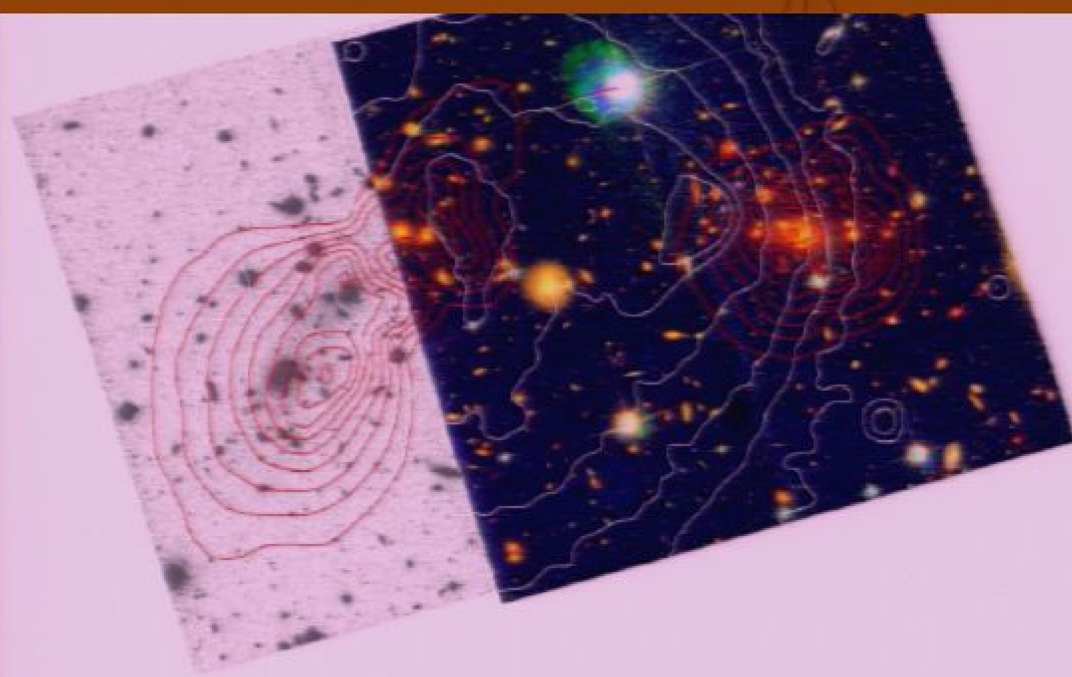

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The reason of failure

- In MOND the center of gravitational force is mass center:
- The other Modified Gravity theories are based on MOND type of solution:

e.g. vector field

$$A = (1-\phi,0,0,0), g_{00} = -1-2\phi$$

Can a different choice of the vector field solve this problem?

A possible choice

- In General Einstein Aether

$$A=(1-\phi,B_1(\phi),B_2(\phi),B_3(\phi)),\ g_{00}=-1-2\phi$$

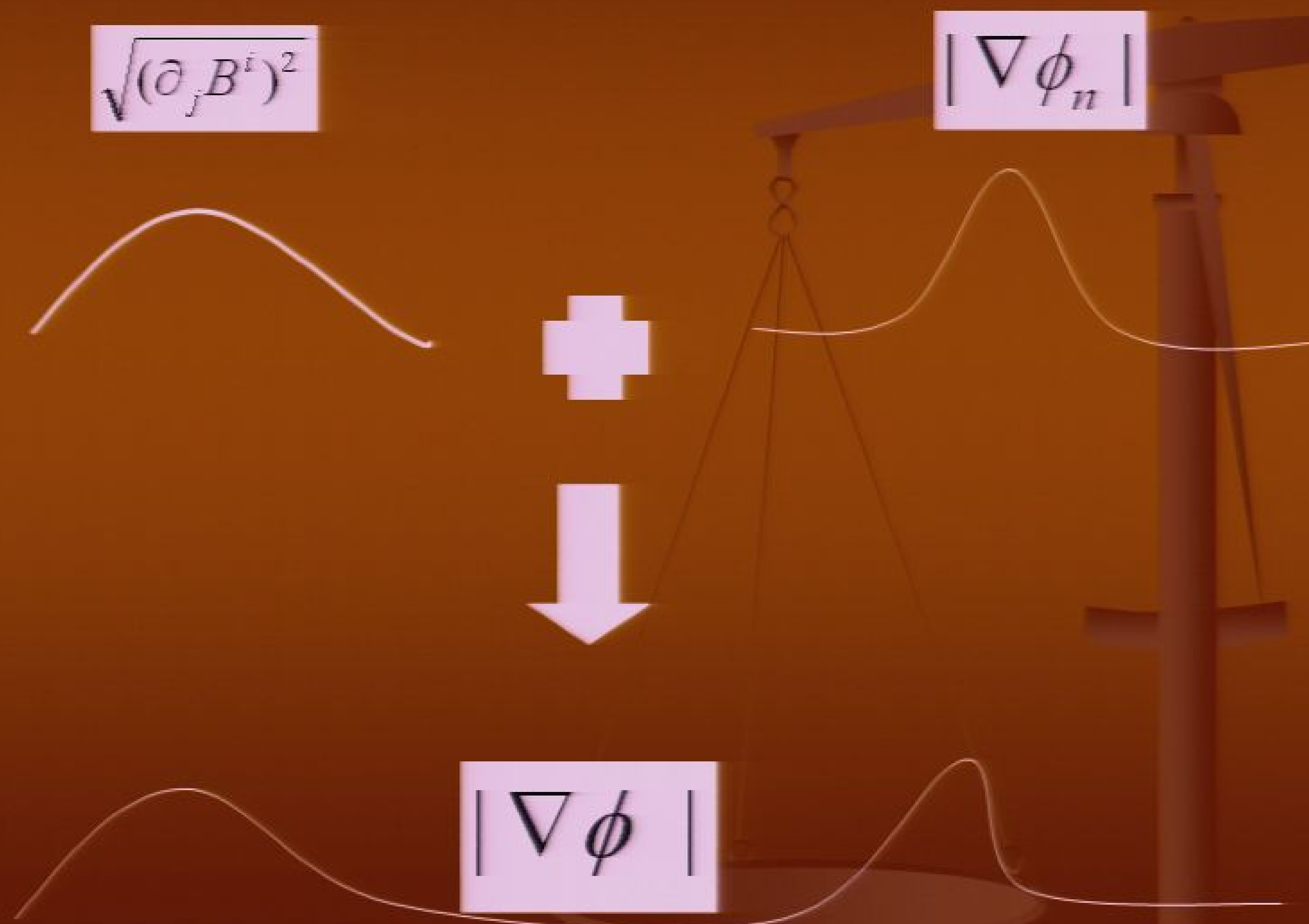
- The first order solution

$$\nabla\cdot\nabla\phi=\rho\rightarrow\nabla\cdot(\mu((\nabla\phi)^2-(\partial_i B_j)^2)\nabla\phi)=\rho$$

- The gravitational acceleration has been enhanced by the vector field.

arXiv:0711.0520 T.G Zlosnik, P.G. Ferreira, G.D. Starkman .

The gravitational acceleration



Violation of Birkhoff's law : DGP

- 4D ⟶ 5D
- Standard Model particles are constrained on a 4D brane.
- A new scalar constant r_0
- Derivative of metrics in extra dimension mimics the ordinary mass.

The geometry

$$S_{(5)} = -\frac{M_5^3}{16\pi}\int dz\, d^4x\sqrt{-g}R - \frac{M_p^2}{16\pi}\int d^4x\sqrt{-g^{(4)}}R^{(4)} + \int d^4x\sqrt{-g^{(4)}}\mathcal{L}_m \ . \quad (1)$$

$$ds^2 = N^2(r,z)dt^2 - A^2(r,z)dr^2 - B^2(r,z) \times [d\theta^2 + sin^2\theta d\phi^2] - dz^2 \ .$$

$$M_5 = 10^{-3} TeV$$

Phys.Rev.D67:064002,2003. Arthur Lue, Glenn Starkman

Equation of motion

$$N(r,z) \equiv 1 + n(r,z)$$
$$A(r,z) \equiv 1 + a(r,z)$$
$$B(r,z) \equiv (1 + b(r,z))r$$

$$\dot{a} + 2\dot{b} = r_0\left(\frac{2a}{r^2} + \frac{2a'}{r}\right) - \frac{r_0}{r^2}R'_g - 3H(r_0 H \pm 1)$$
$$2\dot{b} = r_0\left(\frac{2a}{r^2} - \frac{2n'}{r}\right) - H(3Hr_0 \pm 2) - g \qquad (1$$
$$\dot{a} + \dot{b} = r_0\left(-n'' - \frac{n'}{r} + \frac{a'}{r}\right) - H(3Hr_0 \pm 2) - g.$$

Gravitational acceleration

- $$n(r,0) = n_0 + \int_0^r d\tilde{r}\Big[\frac{f(\tilde{r})}{\tilde{r}} + \frac{1}{2\tilde{r}^2}(R_g(\tilde{r}) - G_g(\tilde{r})) - \frac{g(\tilde{r})\tilde{r}}{4r_0} - (H^2 \pm \frac{H}{2r_0})\tilde{r}\Big] \quad (1$$

$$R_g(r) \equiv \frac{8\pi}{M_p^2}\int_0^r \tilde{r}^2 \rho_g(\tilde{r})d\tilde{r}\,,$$

$$\dot{n}(r,0) = \mp H + g(r)$$

$$G_g(r) \equiv \frac{3}{2}r_0^{-1}\int_0^r \tilde{r}^2 g(\tilde{r})d\tilde{r}\,.$$

$$f(r) = \frac{r}{8r_0^2}\Big[\sqrt{D} - r\left(3 - 2r_0\left(g \mp H\right)\right)\Big]\ ; \quad (1$$

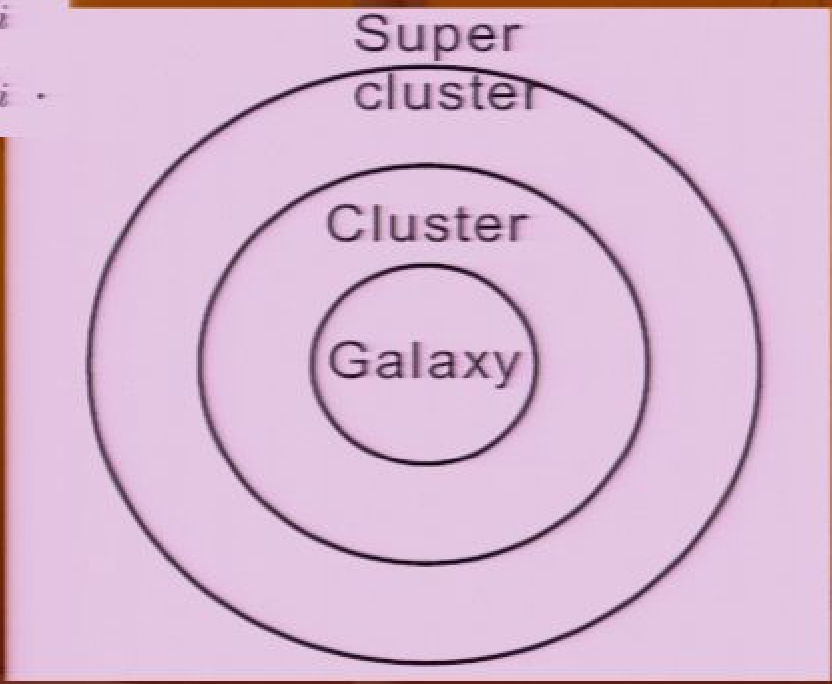
$$D(r) \equiv \left(3 - 2r_0\left(g \mp H\right)\right)^2 r^2 + 12gr^2 r_0 + 16\frac{r_0^2}{r}\Big[Q + \frac{1}{2}(R_g - G_g) + r^3(2H^2 \pm \frac{3H}{2r_0})\Big]$$

$$Q(r) \equiv \mp H(r_0 G_g - \frac{1}{2}gr^3) - \frac{1}{2}\int_0^r g(\tilde{r})g'(\tilde{r})\tilde{r}^3 d\tilde{r}$$

Simulation model

- three simple spherical shells:
- derivative in extra dimension:

$$g(r) = -\sqrt{2GM_i/r_i^3} \times \begin{cases} 1 & \text{for } r \leq r_i \\ (r_i/r)^4 & \text{for } r > r_i \,. \end{cases}$$



Result I

- $M_g=10^{12}M_{sun}$
- $M_c=10^{15}M_{sun}$
- $M_{sc}=10^{17}M_{sun}$

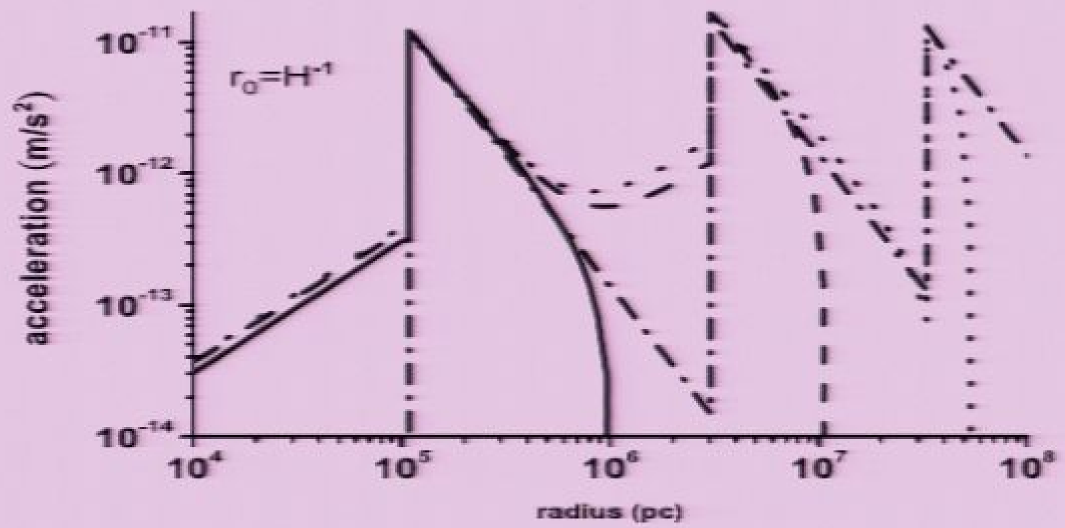
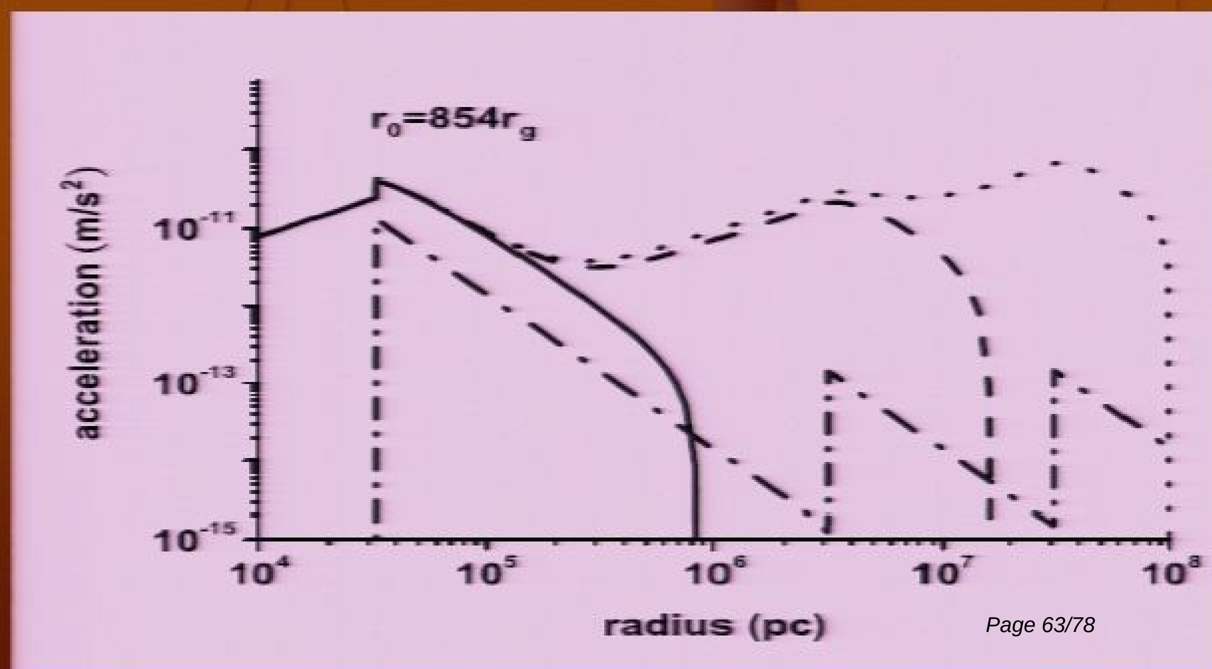


FIG. 2: Gravitational acceleration as a function of radius. g (solid) includes only the galaxy as a source, $g+c$ (dashed) adds the cluster, and $g+c+sc$ (dotted) adds the super-cluster. The Newtonian acceleration is g_n (dot-dashed). The crossover scale is $r_0 = 10^5 r_g = 10^{10}$pc$\approx 1/H_0$.

Result II

- $M_g = 10^{11} M_{sun}$
- $M_c = 10^{13} M_{sun}$
- $M_{sc} = 10^{15} M_{sun}$



Gravitational acceleration

$$n(r,0) = n_0 + \int_0^r d\tilde{r}\Big[\frac{f(\tilde{r})}{\tilde{r}} + \frac{1}{2\tilde{r}^2}(R_g(\tilde{r}) - G_g(\tilde{r})) - \frac{g(\tilde{r})\tilde{r}}{4r_0} - (H^2 \pm \frac{H}{2r_0})\tilde{r}\Big] \quad (1$$

$$R_g(r) \equiv \frac{8\pi}{M_p^2}\int_0^r \tilde{r}^2\rho_g(\tilde{r})d\tilde{r}\,,$$

$$G_g(r) \equiv \frac{3}{2}r_0^{-1}\int_0^r \tilde{r}^2 g(\tilde{r})d\tilde{r}\,.$$

$$\dot{n}(r,0) = \mp H + g(r)$$

$$f(r) = \frac{r}{8r_0^2}\Big[\sqrt{D} - r\left(3 - 2r_0\left(g \mp H\right)\right)\Big]\;; \quad (1$$

$$D(r) \equiv \left(3 - 2r_0\left(g \mp H\right)\right)^2 r^2 + 12gr^2r_0 + 16\frac{r_0^2}{r}\Big[Q + \frac{1}{2}(R_g - G_g) + r^3(2H^2 \pm \frac{3H}{2r_0})\Big]$$

$$Q(r) \equiv \mp H\left(r_0 G_g - \frac{1}{2}gr^3\right) - \frac{1}{2}\int_0^r g(\tilde{r})g'(\tilde{r})\tilde{r}^3 d\tilde{r}$$

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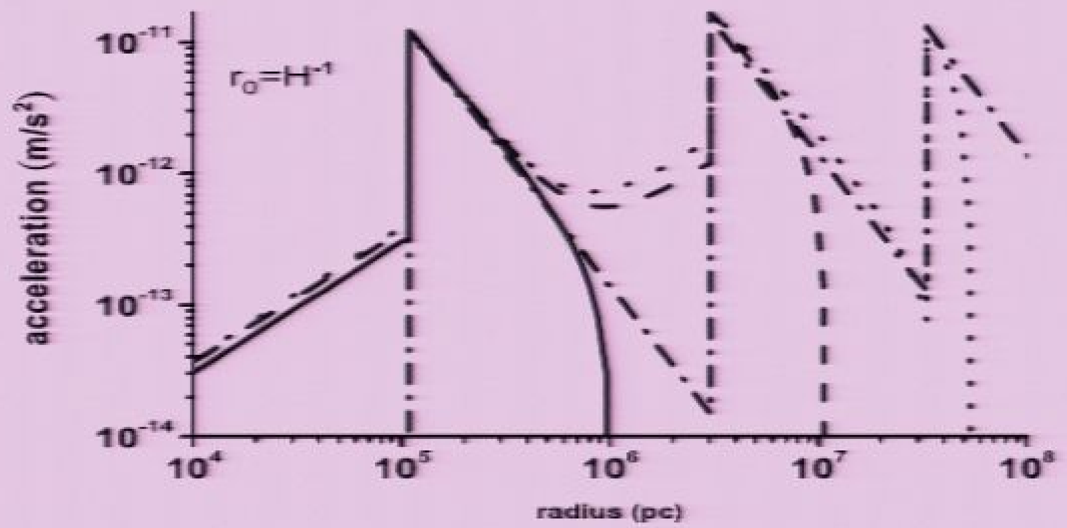

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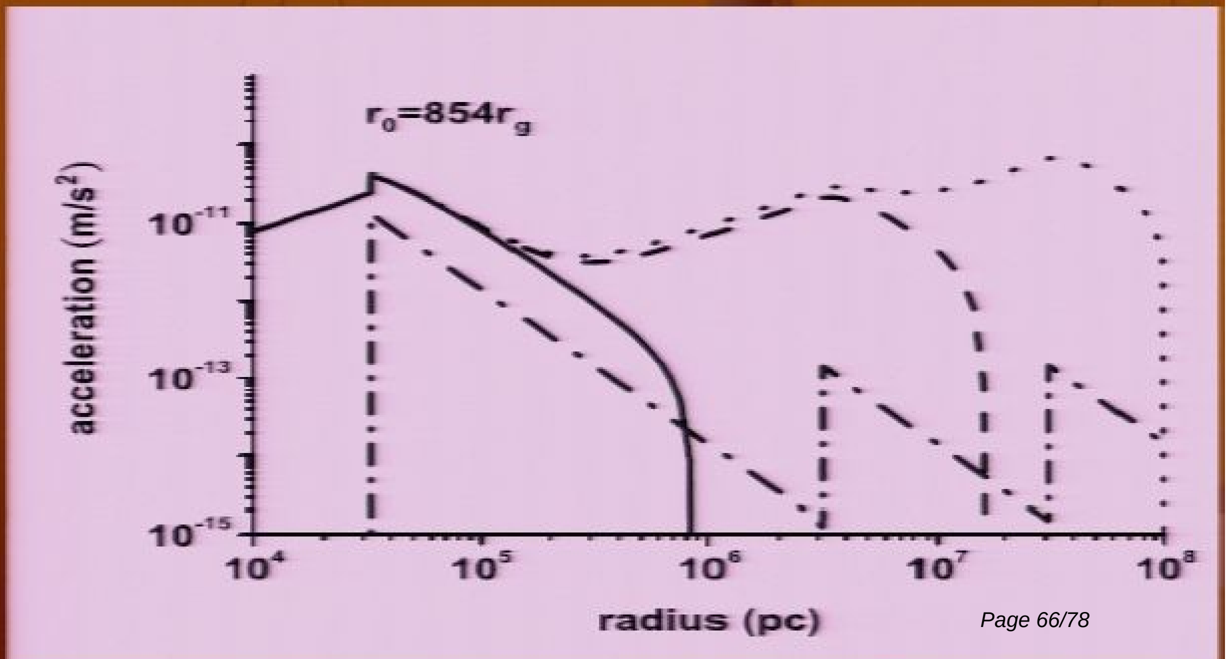

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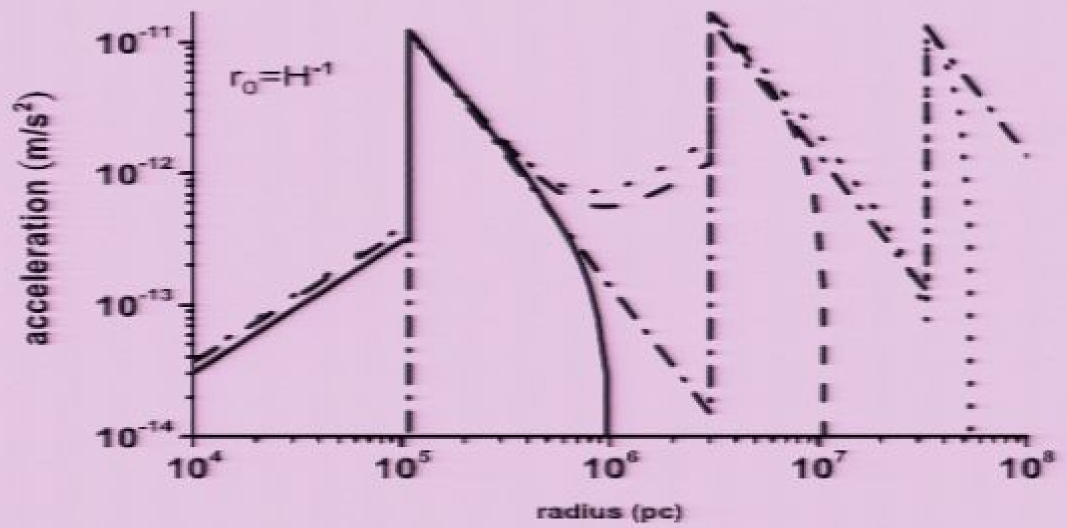

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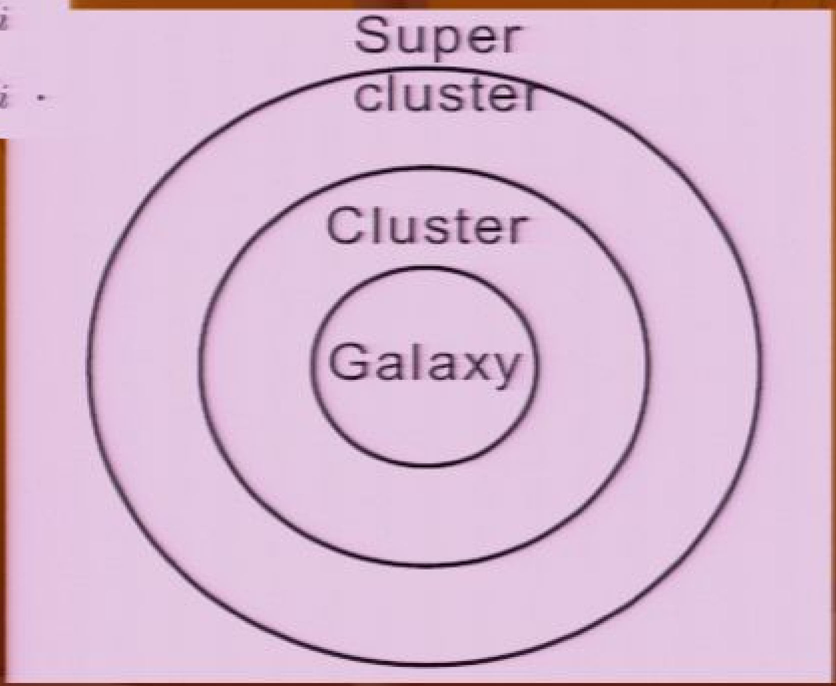
Simulation model

- three simple spherical shells:
- derivative in extra dimension:

$$g(r) = -\sqrt{2GM_i/r_i^3} \times \begin{cases} 1 & \text{for } r \le r_i \\ (r_i/r)^4 & \text{for } r > r_i \,. \end{cases}$$



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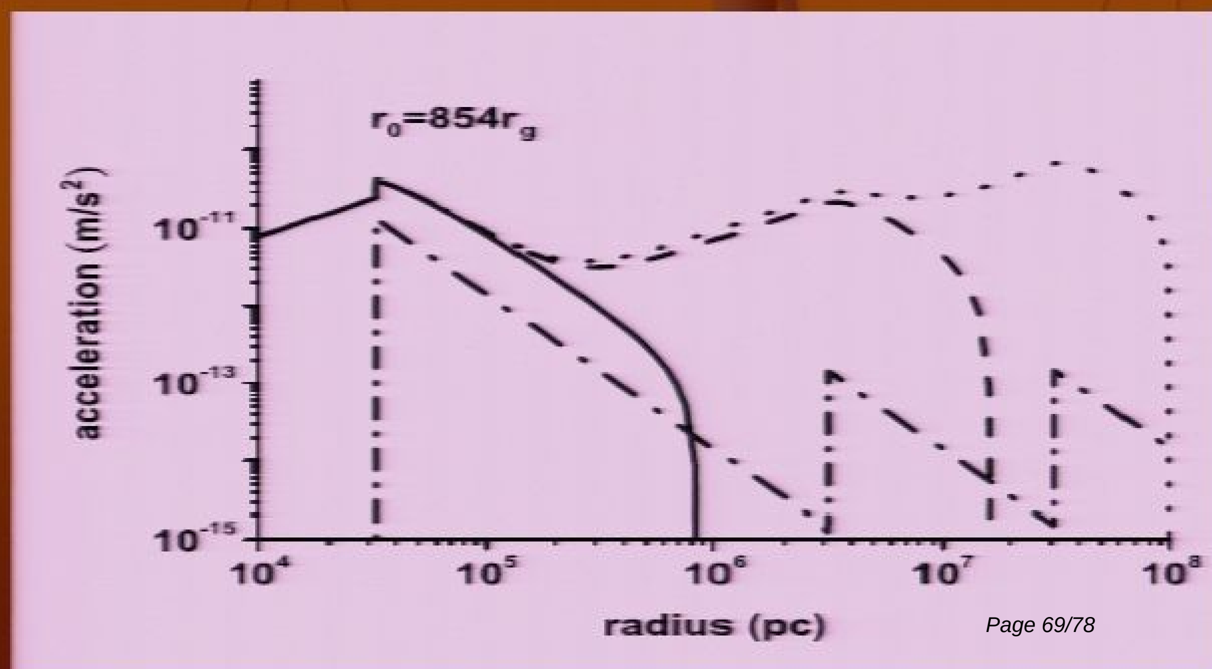

summary

- Gravitational acceleration can come from outside a spherical shell.
- Modified gravity theories are not fully ruled out by the Bullet cluster.
- The free parameters in Modified Gravity theories may be as many as in ΛCDM.

Violation of Birkhoff's law : DGP

- 4D $\longrightarrow$ 5D
- Standard Model particles are constrained on a 4D brane.
- A new scalar constant r_0
- Derivative of metrics in extra dimension mimics the ordinary mass.

A possible choice

- In General Einstein Aether

$$A=(1-\phi, B_1(\phi), B_2(\phi), B_3(\phi)), g_{00}=-1-2\phi$$

- The first order solution

$$\nabla\cdot\nabla\phi=\rho\rightarrow\nabla\cdot(\mu((\nabla\phi)^2-(\partial_i B_j)^2)\nabla\phi)=\rho$$

- The gravitational acceleration has been enhanced by the vector field.

arXiv:0711.0520 T.G Zlosnik, P.G. Ferreira, G.D. Starkman .

End of slide show, click to exit.

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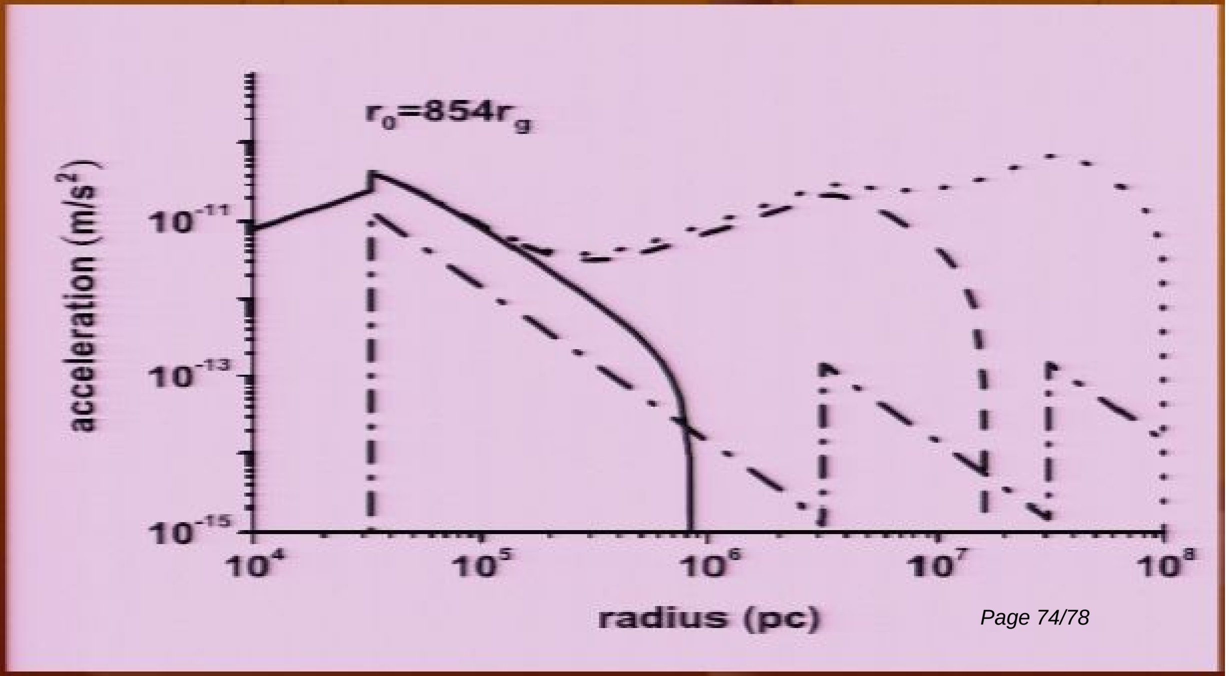

$F \propto \underline{r}$

$)^2 + \ldots$

$\mu = \frac{x}{1+x}$

$\mu = \frac{x}{\sqrt{1+x^2}}$

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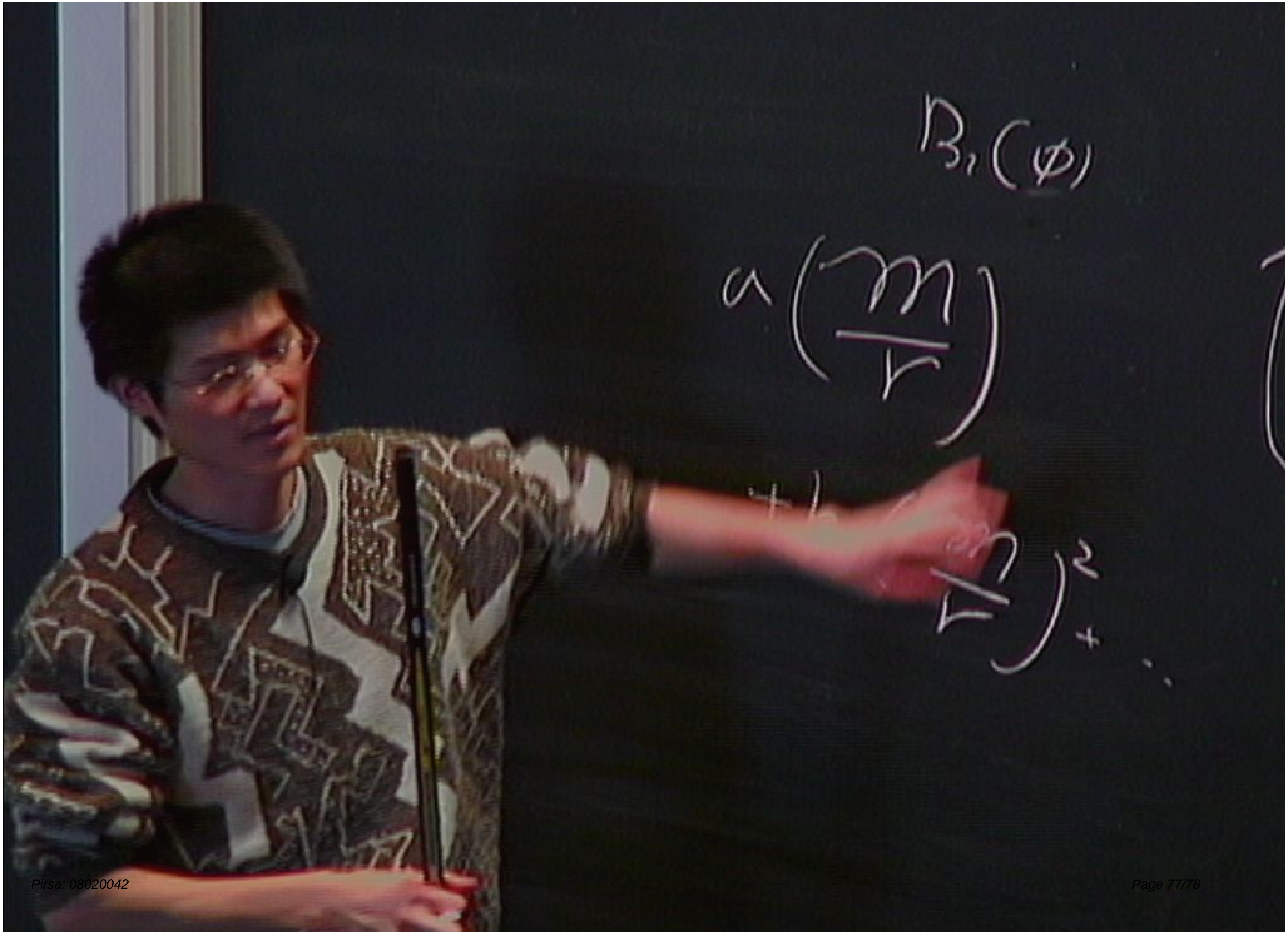
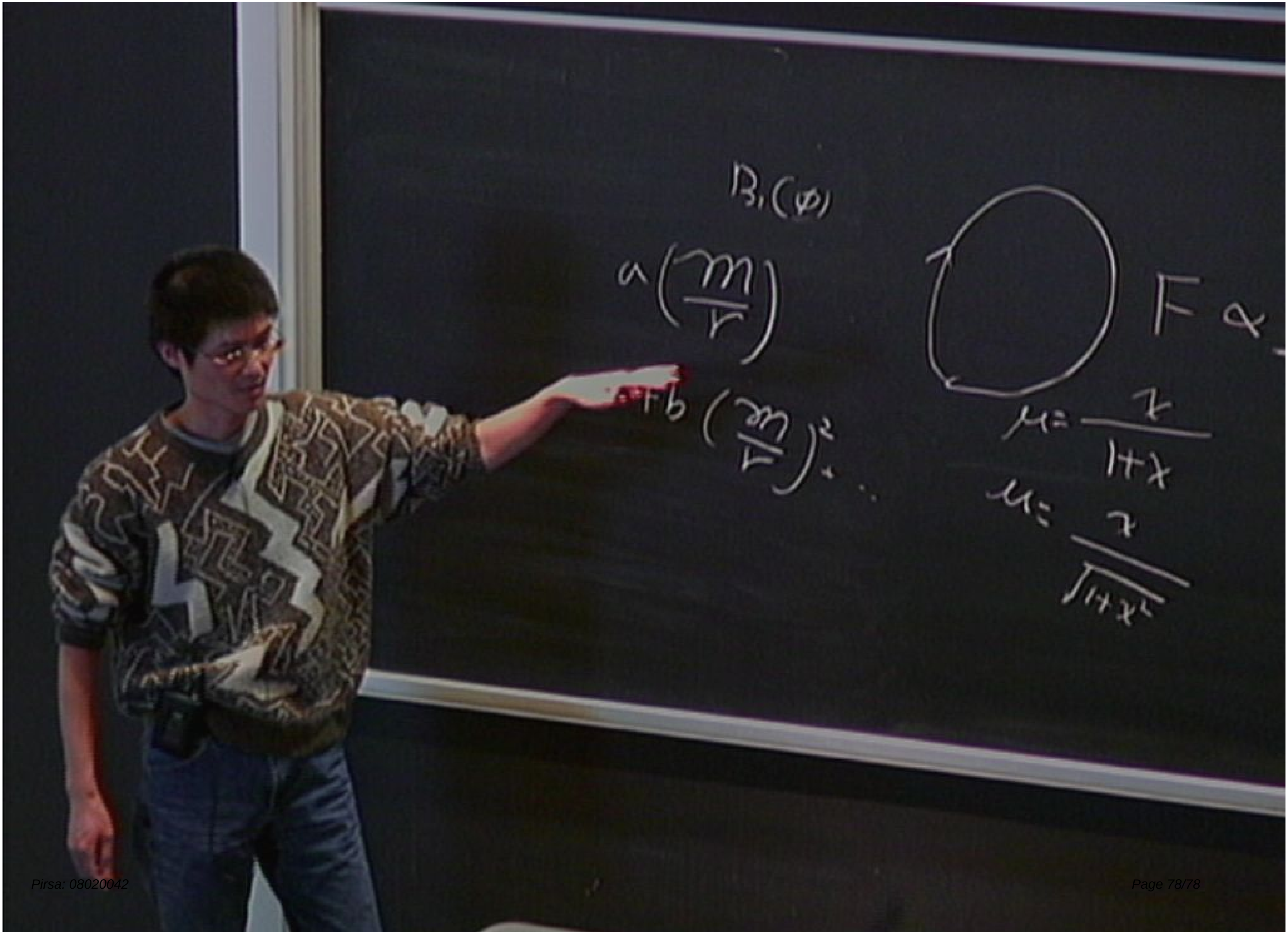

B1(φ)
a(m/r)


B1(φ)
a(m/r)
+b(m/r)^2 + ...
F ∝
μ = x/(1+x)
μ = x/√(1+x^2)