Title: Loop Quantum Gravity and Deformation Quantization

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Abstract: Loop Quantum Gravity and Deformation Quantization
Abstract: We propose a unified approach to loop quantum gravity and Fedosov quantization of gravity following the geometry of double spacetime fibrations and their quantum deformations. There are considered pseudo--Riemannian manifolds enabled with 1) a nonholonomic 2+2 distribution defining a nonlinear connection (N--connection) structure and 2) an ADM 3+1 decomposition. The Ashtekar-Barbero variables are generalized and adapted to the N-connection structure which allows us to write the general relativity theory equivalently in terms of Lagrange-Finsler variables and related canonical almost symplectic forms and connections. The Fedosov results are re-defined for gravitational gauge like connections and there are analyzed the conditions when the star product for deformation quantization is computed in terms of geometric objects in loop quantum gravity. We speculate on equivalence of quantum gravity theories with 3+1 and 2+2 splitting and quantum analogs of the Einstein equations.

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Loop Quantum Gravity and Deformation Quantization

Sergiu I. Vacaru

visitor at *Fields Institute*,
Rep. Moldova and Romania

Perimeter Institute
Visit: February 6–8, 2008
Seminar: QUANTUM GRAVITY, February 7, 2008

Purposes

∃ two non-perturbative (background free) formalisms of quantization GR: LQG and nonholonomic Fedosov DQ.

A "bridge" between LQG and DQ

A nontrivial task for nonlinear classical/ quantum theories: Dictionary for two communities in quantum gravity working separately with 3+1 ADM/Ashtekar–Barbero variables and 2+2 splitting of almost Kähler structures.

1. Compare results & methods of LQG and DQ of GR.
2. An unified geometric approach to LQG and Fedosov quantization, nonholonomic Ashtekar variables.
3. Formulate a model of Fedosov–Ashtekar DQ of GR.
4. Applications of LQG and DQ to nonlinear systems, geometric mechanics and analogous gravity.

Language: talk about DQ and Lagrange–Finsler methods in GR

using the language of moving frames and double 3+1 and 2+2 splitting in GR and LQG.

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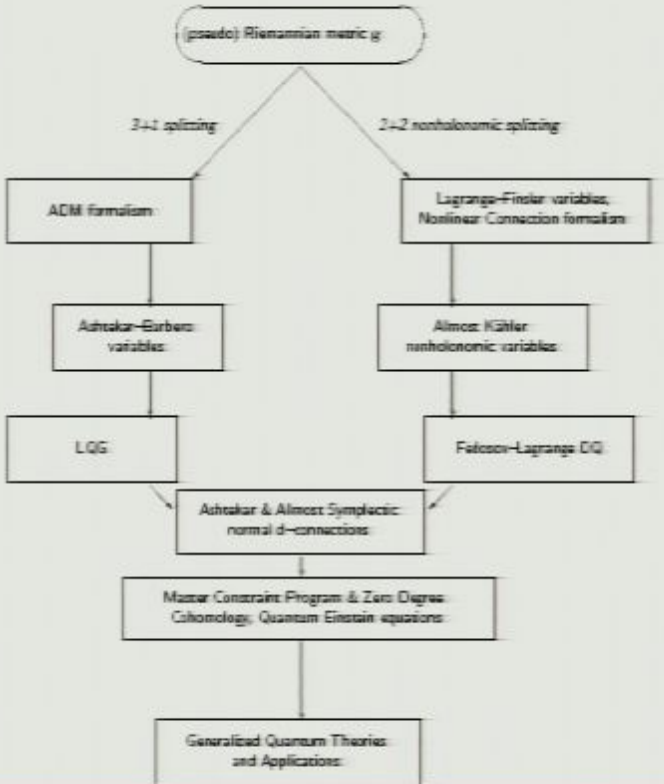

Introduction: Fedosov Quantization

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2. S. Vacaru Ashtekar–Barbero Constraints for Lagrange–Finsler Variables in General Relativity, Geometric Mechanics and Analogous Quantum Gravity (under prep.)
Key steps in DQ: M. Flato, A. Lichnerowicz, D. Sternheimer, ... DQ (1978)
B. Fedosov (symplectic str., Index theory, 1988), M. Kontsevich (Poisson manifolds, 1988)
A. Karabegov, M. Schlichenmaier (almost Kähler, 2001)

DQ of Almost Kähler Structures

almost symplectic manifold $(M, J, \theta) : \forall X, Y \in TM$
almost complex structure J, $\theta(JX, JY) = \theta(X, Y)$
symplectic 2–form and metric $g(X, Y) = \theta(JX, Y)$
Neijenhuis tensor:
$^{J}\Omega(X, Y) = -[X, Y] + [JX, JY] - J[JX, Y] - J[JX, JY]$

Locally $\{x^k\}$ on M, $\partial_j = \partial/\partial x^j, \theta_{ij} = -\theta_{ji}, g_{ij} = g_{ji}$, and Ω^k_{ij} is defined by $J^k_j = g_{ja}\theta^{ak} = g^{ka}\theta_{aj}$, $J^2 = -I$.

Results: 1) Let $^{K}D = \{\,^{K}\Gamma^i_{jk}\}$, torsion $^{K}T^i_{jk}$. Then $^{K}Dg = {}^{K}D\theta = {}^{K}DJ = 0 \Longleftrightarrow {}^{K}T^i_{jk} = (-1/4)\Omega^i_{jk}$.
2) Fedosov quantization for $d\theta = 0$; Kähler: $\theta = d\varphi$.

Questions: How to apply DQ to GR, not modifying the classical theory (generalizations: gauge gravity,

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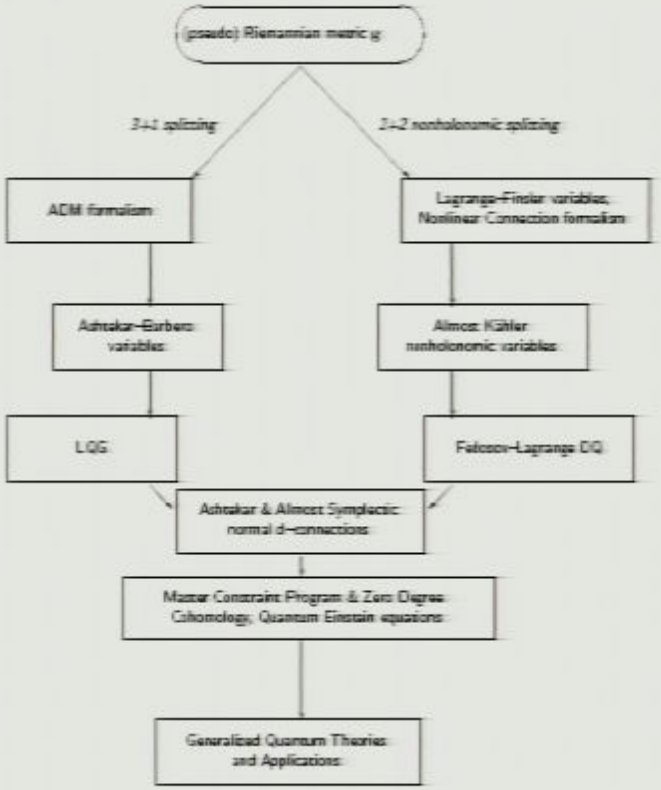

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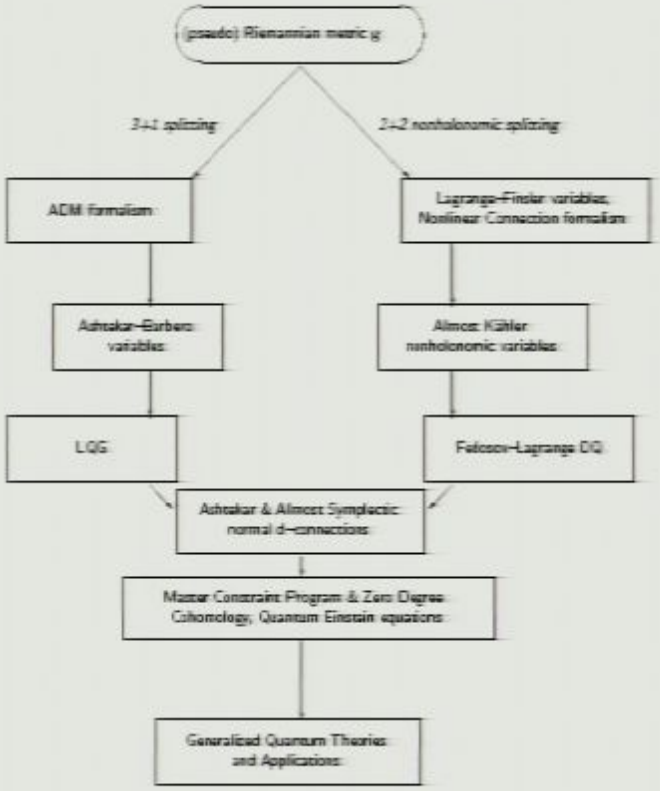

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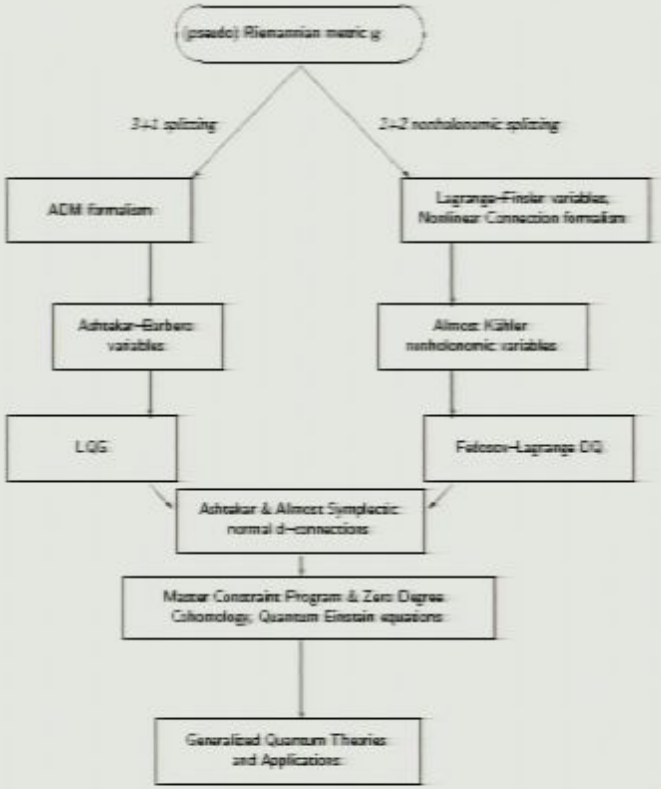

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Generalized Quantum Theories and Applications

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Lagrange–Finsler variables in GR

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New Results [1]:

I. Any Finsler (and Lagrange) geometry, for the Cartan connection[3,4] (NOT Chern connection; NOT metric compatible [5]) can be formulated as an almost Kähler structure and quantized following Fedosov DQ.

II. Introduce Finsler/Lagrange variables in GR, construct

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Geometrization of nonholonomic mechanics $\Longrightarrow$ concept of nonholonomic manifold $\mathbf{V} = (M, \mathcal{D})$ smooth & orientable M, non–integrable distribution $\mathcal{D}$
Local coordinates $u^\alpha = (x^i, y^a)$, for $i, j, ... = 1, 2$; $a, b, ... = 3, 4$.

N–anholonomic manifold is provided with N–connection structure $\mathbf{N} = N_i^a(u)\, dx^i \otimes \frac{\partial}{\partial y^a}$
Particular case: $N_i^a(u) = \Gamma^a_{bi}(x) y^b$

$\exists$ N–adapted (co) frames (vielbeins)

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II. Introduce Finsler/Lagrange variables in GR, construct an equivalent almost Kähler model: apply DQ [2].

Main Idea: Usual Finsler geometry is constructed on TM with a nonholonomic splitting (N–connection)

$$N: \; TTM = hTM \oplus vTM$$

Why not consider similar 2+2, $(n+n)$, decompositions in GR? For a manifold $\mathbf{V}$ with local fibered structure

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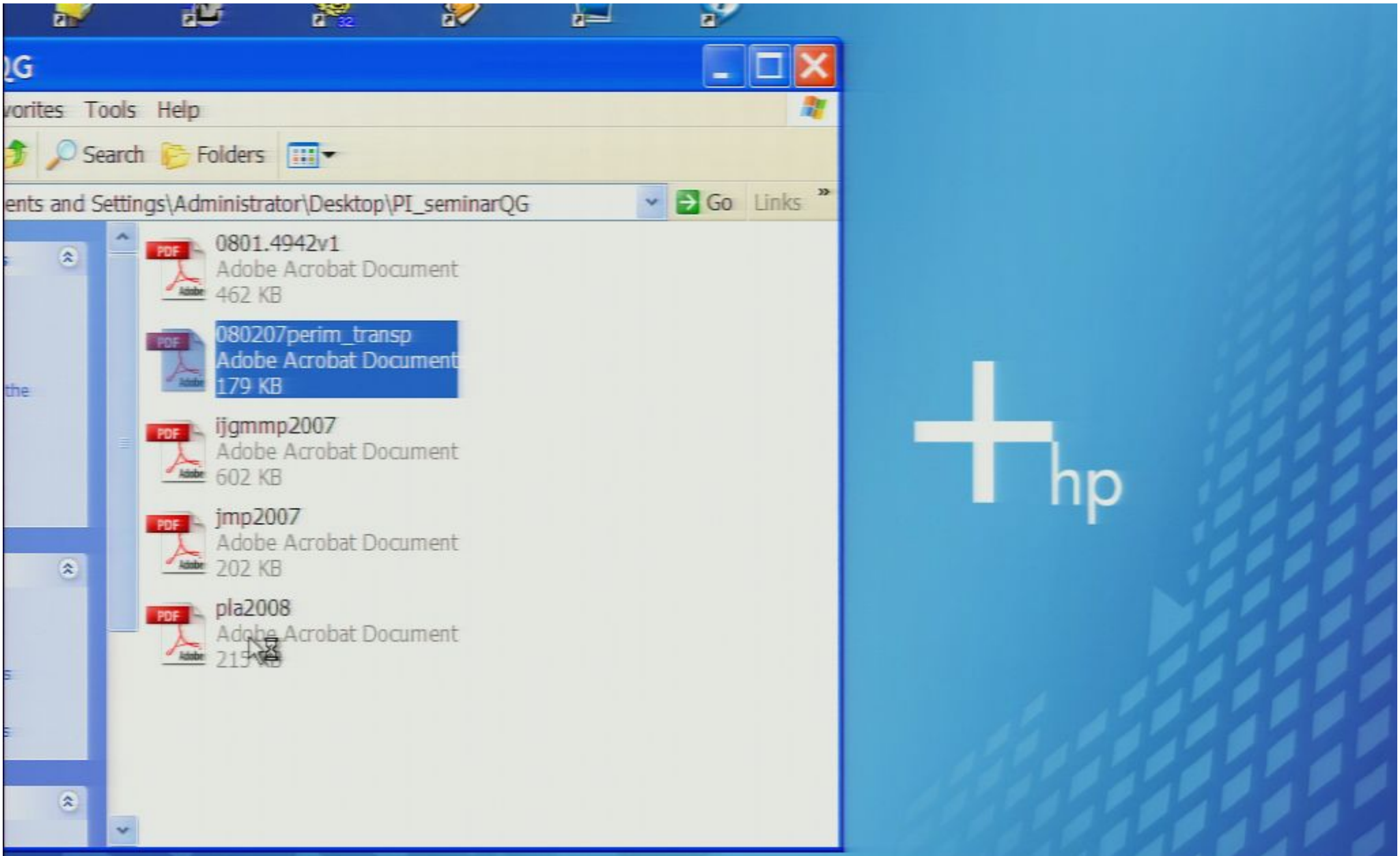
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Deformation quantization of nonholonomic almost Kähler models and Einstein gravity

Sergiu I. Vacaru

The Fields Institute for Research in Mathematical Science 222 College Street, 2d Floor, Toronto M5T 3J1, Canada

Received 15 August 2007; received in revised form 24 December 2007; accepted 10 January 2008

Communicated by P.R. Holland

Abstract

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MSC: 83C45; 81S10; 53D55; 53B40; 53B35; 53D50

Keywords: Deformation quantization; Quantum gravity; Einstein spaces; Finsler and Lagrange geometry; Almost Kähler geometry

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1. G. Vranceanu, Sure les espaces non holonomes. C. R. Acad. Paris 103 (1926) 852–854
2. A. Bejancu and H. R. Farran, Foliations and Geometric Structures (Springer, 2005)
3. Clifford and Riemann- Finsler Structures in Geometric Mechanics and Gravity, Selected Works, by S. Vacaru and all. Differential Geometry – Dynamical Systems, Monograph 7 (Geometry Balkan Press, 2006); www.mathem.pub.ro/dgds/mono/va-t.pdf and gr-qc/0508023

Nonholonomic manifolds

Geometrization of nonholonomic mechanics $\Longrightarrow$ concept of nonholonomic manifold $\mathbf{V} = (M, \mathcal{D})$ smooth & orientable M, non–integrable distribution $\mathcal{D}$

Local coordinates $u^\alpha = (x^i, y^a)$, for $i, j, ... = 1, 2;\; a, b, ... = 3, 4$.

N–anholonomic manifold is provided with N–connection structure $\mathbf{N} = N_i^a(u)\, dx^i \otimes \frac{\partial}{\partial y^a}$

Particular case: $N_i^a(u) = \Gamma^a_{bi}(x) y^b$

$\exists$ N–adapted (co) frames (vielbeins)

$$\mathbf{e}_\nu = \left(\mathbf{e}_i = \frac{\partial}{\partial x^i} - N_i^a(u) \frac{\partial}{\partial y^a}, e_a = \frac{\partial}{\partial y^a} \right),$$

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Lagrange–Finsler variables in GR

Related works: 1. S. Vacaru, Deformation Quantization of Almost Kähler Models and Lagrange-Finsler Spaces, J. Math. Phys. 48 (2007) 123509, arXiv: 0707.1519
2. S. Vacaru, Deformation Quantization of Nonholonomic Almost Kähler Models and Einstein Gravity [accepted Phys. Lett. A (2008)], arXiv: 0707.1667
3. E. Cartan, Les Espaces de Finsler (Paris, Hermann, 1935)
4. R. Miron and M. Anastasiei, The Geometry of Lagrange Spaces: Theory and Applications, FTPH no. 59 (Kluwer, 1994)
5. D. Bao, S. -S. Chern, and Z. Shen, An Introduction to Riemann–Finsler Geometry, Graduate Texts in Math., 200 (Springer–Verlag, 2000)
6. Vacaru, Finsler and Lagrange Geometries in Einstein and String Gravity [accepted Int. J. Geom. Methods. Mod. Phys. 5 (2008)], arXiv: 0801.4958
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N–adapted/distinguished, d–metrics

Any metric on $\mathbf{V}$, $\mathbf{g} = \underline{g}_{\alpha\beta}(u)\, du^\alpha \otimes du^\beta$

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$${}^L N_j^i(x,y) = \frac{\partial\, {}^L G^i}{\partial y^{2+j}}, \qquad {}^L G^i = \frac{1}{4}\, {}^L g^{ij}\left(\frac{\partial^2 L}{\partial y^j \partial x^k} y^k - \frac{\partial L}{\partial x^j}\right)$$

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Curvature: $\mathbf{R}(\mathbf{X},\mathbf{Y}) \doteqdot \mathbf{D}_\mathbf{X}\mathbf{D}_\mathbf{Y} - \mathbf{D}_\mathbf{Y}\mathbf{D}_\mathbf{X} - \mathbf{D}_{[\mathbf{X},\mathbf{Y}]}$, with N–adapted decomposition

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In general, $\mathbf{R}_{\beta\gamma} \neq \mathbf{R}_{\gamma\beta}$.

Set of d–connections compatible to a d–metric

A very important construction in Lagrange geometry, not considered in GR, is: Let be given $\mathbf{g}_{\alpha\beta} = {}^L\mathbf{g}_{\alpha\beta}$, ${}^L N^b_j \Rightarrow$ work equivalently $\forall \mathbf{\Gamma}^\gamma{}_{\alpha\beta}(\mathbf{g}) = \mathbf{\Gamma}^\gamma{}_{\alpha\beta}(\mathbf{g}) + {}_Z\mathbf{\Gamma}^\gamma{}_{\alpha\beta}(\mathbf{g})$

Almost Kähler formulation of GR

Proposal: construct from $\mathbf{g}$ metric/symplectic–compatible almost Kähler connections, necessary for DG, $\Leftrightarrow \nabla$.

$\exists$ Canonical d–connect. ${}^c\mathbf{D}(\mathbf{g}) = \{{}^c\mathbf{\Gamma}^\gamma{}_{\alpha\beta}(g_{ij}, g_{ab}, N^a_i)\}$ ${}^c\mathbf{D}\mathbf{g} = 0$ and $h\,{}^c\mathbf{T}(hX, hY) = 0$, $v\,{}^c\mathbf{T}(vX, vY) = 0$.

$$_c\mathbf{\Gamma}^\gamma{}_{\alpha\beta} = {}^c\mathbf{\Gamma}^\gamma{}_{\alpha\beta}(g_{ij}, g_{ab}, N^a_i) + {}_Z\mathbf{\Gamma}^\gamma{}_{\alpha\beta}(g_{ij}, g_{ab}, N^a_i).$$

For even dimensions, $\exists$ a unique normal d–connection $\widehat{\mathbf{D}}$, $\{\widehat{\mathbf{\Gamma}}^\alpha_{\beta\gamma} = (\widehat{L}^i_{jk}, {}^v\widehat{L}^{2+i}_{2+j\ 2+k} = \widehat{L}^i_{jk}, \widehat{C}^i_{jc} = {}^v\widehat{C}^{2+i}_{2+j\ c}, {}^v\widehat{C}^a_{bc} = \widehat{C}^a_{bc})\}$, defined $\widehat{\mathbf{D}}_\alpha = (\widehat{D}_k, \widehat{D}_c)$, $\widehat{\mathbf{\Gamma}}^\alpha_{\beta\gamma} = (\widehat{L}^i_{jk}, {}^v\widehat{C}^a_{bc})$,

$$\widehat{L}^i_{jk} = \frac{1}{2}\,{}^L g^{ih}\left(\mathbf{e}_k\,{}^L g_{jh} + \mathbf{e}_j\,{}^L g_{hk} - \mathbf{e}_h\,{}^L g_{jk}\right).$$

$$\widehat{C}^a_{jk} = \frac{1}{2}\,{}^L g^{ih}\left(\frac{\partial\,{}^L g_{jh}}{\partial y^k} + \frac{\partial\,{}^L g_{hk}}{\partial y^j} - \frac{\partial\,{}^L g_{jk}}{\partial y^h}\right).$$

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9 / 17 52.5% Find

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Signal Strength: Very Good

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7 / 17 52.5% Find

can be represented as a d–metric $\mathbf{g} = {}^h g \oplus_N {}^v g = [{}^h g, {}^v g]$

$$\mathbf{g} = g_{ij}(x,y)\, e^i \otimes e^j + g_{ab}(x,y)\, \mathbf{e}^a \otimes \mathbf{e}^b$$

Example 1: *Lagrange spaces.* $\mathbf{V} = \mathbf{TM}$, $L(x,y)$, ${}^L g_{ab} = \frac{1}{2}\frac{\partial^2 L}{\partial y^a \partial y^b}$, $det|g_{ab}| \neq 0$. Canonical N–connection ${}^L N^i_j(x,y) = \frac{\partial\, {}^L G^i}{\partial y^{2+j}}$, ${}^L G^i = \frac{1}{4}\, {}^L g^{ij}(\frac{\partial^2 L}{\partial y^j \partial x^k} y^k - \frac{\partial L}{\partial x^j})$

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Example 2: *Finsler spaces.* $F(x, \lambda y) = |\lambda| F(x,y)$, for $0 \neq \lambda \in \mathbb{R}$, particular case $L = F^2$.

Theorem: *Any Lagrange* (Finsler) geometry can be modelled effectively on a N–anholonomic Riemann manifold $\mathbf{V}$, or equivalently on $\mathbf{TM}$, with canonically induced by L (F) d–metric structure

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Torsion, Curvature and Ricci d–tensors

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LQG and DQ

Curvature: $\mathbf{R}(\mathbf{X},\mathbf{Y}) \doteqdot \mathbf{D}_\mathbf{Y}$ ... tion of GR

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N–adapted/distinguished, d–metrics

Any metric on $\mathbf{V}$, $\mathbf{g} = \underline{g}_{\alpha\beta}(u)\, du^\alpha \otimes du^\beta$

$$\underline{g}_{\alpha\beta} = \begin{bmatrix} g_{ij} + N_i^a N_j^b g_{ab} & N_j^e g_{ae} \\ N_i^e g_{be} & g_{ab} \end{bmatrix}, \text{ for } N_j^e(u) = N_j^e(x,y),$$

can be represented as a d–metric $\mathbf{g} = h\mathbf{g} \oplus_{\mathbf{N}} v\mathbf{g} = [\,{}^h g,\ {}^v g]$

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Existence of Lagrange–Finsler Variables in GR

follows from the quadratic algebraic system for $e^{\alpha'}_{\ \alpha}$
$\mathbf{g}_{\alpha'\beta'} e^{\alpha'}_{\ \alpha} e^{\beta'}_{\ \beta} = {}^L\mathbf{g}_{\alpha\beta}$, for given $\mathbf{g}_{\alpha'\beta'}$, ${}^L\mathbf{g}_{\alpha\beta}$.
See the preprint for computations for many realistic cases.

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5 / 17 52.5% Find

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New Results [1]:

I. Any Finsler (and Lagrange) geometry, for the Cartan connection[3,4] (NOT Chern connection; NOT metric compatible [5]) can be formulated as an almost Kähler structure and quantized following Fedosov DQ.

II. Introduce Finsler/Lagrange variables in GR, construct an equivalent almost Kähler model: apply DQ [2].

Main Idea: Usual Finsler geometry is constructed on TM with a nonholonomic splitting (N–connection)

$$N: \; TTM = hTM \oplus vTM$$

Why not consider similar 2+2, $(n+n)$, decompositions in GR? For a manifold $\mathbf{V}$ with local fibered structure

$$\mathbf{N}: T\mathbf{V} = h\mathbf{V} \oplus v\mathbf{V}$$

Nonholonomic manifolds

Geometrization of nonholonomic mechanics $\Longrightarrow$ concept of nonholonomic manifold $\mathbf{V} = (M, \mathcal{D})$ smooth & orientable M, non–integrable distribution $\mathcal{D}$

Local coordinates $u^{\alpha} = (x^i, y^a)$, for $i, j, ... = 1, 2$; $a, b, ... = 3, 4$.

N–anholonomic manifold is provided with N–connection structure $\mathbf{N} = N_i^a(u)\, dx^i \otimes \frac{\partial}{\partial y^a}$

Particular case: $N_i^a(u) = \Gamma^a_{bi}(x) y^b$

$\exists$ N–adapted (co) frames (vielbeins)

$$\mathbf{e}_{\nu} = \left(\mathbf{e}_i = \frac{\partial}{\partial x^i} - N_i^a(u) \frac{\partial}{\partial y^a}, e_a = \frac{\partial}{\partial y^a} \right),$$

$$\mathbf{e}^{\mu} = \left(e^i = dx^i, \mathbf{e}^a = dy^a + N_i^a(u) dx^i \right).$$

Nonholonomy: $[\mathbf{e}_{\alpha}, \mathbf{e}_{\beta}] = \mathbf{e}_{\alpha}\mathbf{e}_{\beta} - \mathbf{e}_{\beta}\mathbf{e}_{\alpha} = W^{\gamma}_{\alpha\beta}\mathbf{e}_{\gamma}$

anholonomy coefficients $W^b_{ia} = \partial_a N_i^b$ and $W^a_{ji} = \Omega^a_{ij}$,

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holonomic/ integrable case $W^{\gamma}_{\alpha\beta} = 0$.

N–adapted/distinguished, ... -Finsler Variables in GR

Any metric on $\mathbf{V}$, ... metric algebraic system for $g^{\alpha'}$

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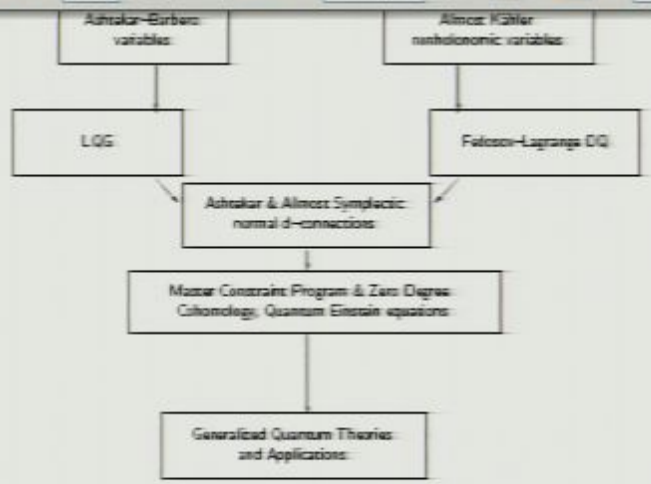

Figure 1: LQG and DQ

almost complex structure J, $\theta(JX,JY)=\theta(X,Y)$
symplectic 2–form and metric $g(X,Y)=\theta(JX,Y)$
Neijenhuis tensor:
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Lagrange–Finsler variables in GR

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Nonholonomic manifolds

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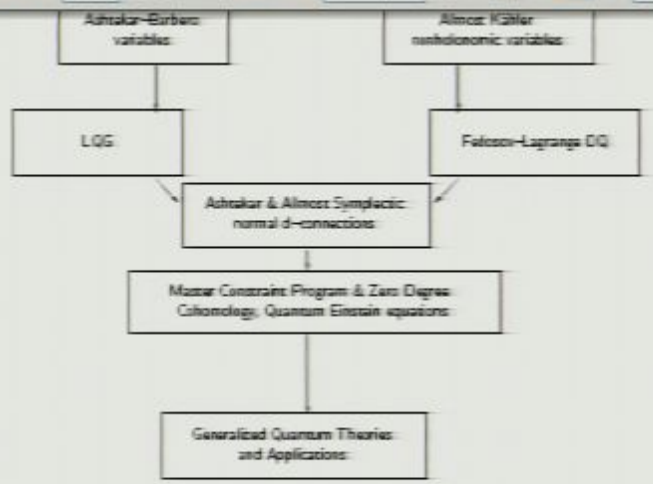

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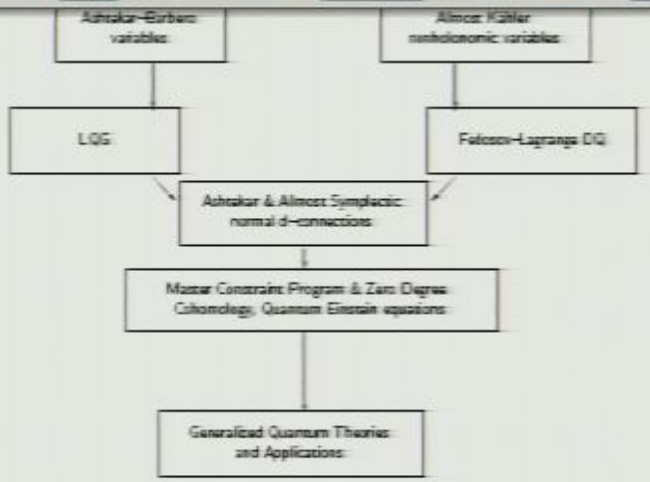

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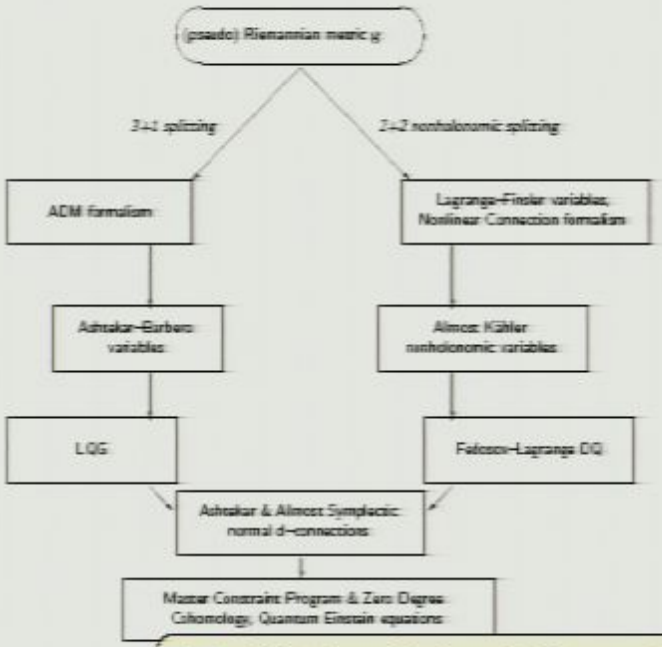

Introduction: Fedosov Quantization

Recent works for this seminar: 1. S. Vacaru, Loop Quantum Gravity in Ashtekar and Lagrange–Finsler Variables and Fedosov Quantization of General Relativity, arXiv: 0801.4992
2. S. Vacaru Ashtekar–Barbero Constraints for Lagrange–Finsler Variables in General Relativity, Geometric Mechanics and Analogous Quantum Gravity (under prep.)
Key steps in DQ: M. Flato, A. Lichnerowicz, D. Sternheimer, ... DQ (1978)
B. Fedosov (symplectic str., Index theory, 1988), M. Kontsevich (Poisson manifolds, 1988)
A. Karabegov, M. Schlichenmaier (almost Kähler, 2001)

DQ of Almost Kähler Structures

almost symplectic manifold $(M, J, \theta) : \forall X, Y \in TM$
almost complex structure J, $\theta(JX, JY) = \theta(X, Y)$
symplectic 2–form and metric $g(X, Y) = \theta(JX, Y)$
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7. S. Vacaru, Parametric Nonholonomic Frame Transforms and Exact Solutions in Gravity, Int. J. Geom. Methods. Mod. Phys. 4 (2007) 1285-1334, arXiv: 0704.3986

New Results [1]:

I. Any Finsler (and Lagrange) geometry, for the Cartan connection[3,4] (NOT Chern connection; NOT metric compatible [5]) can be formulated as an almost Kähler structure and quantized following Fedosov DQ.

II. Introduce Finsler/Lagrange variables in GR, construct an equivalent almost Kähler model: apply DQ [2].

Main Idea: Usual Finsler geometry is constructed on TM with a nonholonomic splitting (N–connection)

$$N: \; TTM = hTM \oplus vTM$$

Why not consider similar 2+2, $(n+n)$, decompositions in GR? For a manifold $\mathbf{V}$ with local fibered structure

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Local coordinates $u^\alpha = (x^i, y^a)$, for $i, j, ... = 1, 2$; $a, b, ... = 3, 4$.

N–anholonomic manifold is provided with N–connection structure $\mathbf{N} = N_i^a(u)\, dx^i \otimes \frac{\partial}{\partial y^a}$

Particular case: $N_i^a(u) = \Gamma^a_{bi}(x) y^b$

$\exists$ N–adapted (co) frames (vielbeins)

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N–adapted/distinguished, d–metrics

Any metric on $\mathbf{V}$, $\mathbf{g} = \underline{g}_{\alpha\beta}(u) du^\alpha \otimes du^\beta$

$$\underline{g}_{\alpha\beta} = \begin{bmatrix} g_{ij} + N_i^a N_j^b g_{ab} & N_j^e g_{ae} \\ N_i^e g_{be} & g_{ab} \end{bmatrix}, \text{ for } N_j^e(u) = N_j^e(x,y),$$

can be represented as a d–metric $\mathbf{g} = hg \oplus_{\mathbf{N}} vg = [{}^h g, {}^v g]$

$$\mathbf{g} = g_{ij}(x,y)\, e^i \otimes e^j + g_{ab}(x,y)\, \mathbf{e}^a \otimes \mathbf{e}^b$$

Example 1: *Lagrange spaces.* $\mathbf{V} = \mathbf{TM}$, $L(x,y)$,

${}^L g_{ab} = \frac{1}{2} \frac{\partial^2 L}{\partial y^a \partial y^b}$, d

${}^L N_j^i(x,y) = \frac{\partial}{\partial y^{2+}} {}^L G$

when nonlinear g

Existence of Lagrange–Finsler Variables in GR

follows from the quadratic algebraic system for $e^{\alpha'}_{\alpha}$

$\mathbf{g}_{\alpha'\beta'} e^{\alpha'}_{\alpha} e^{\beta'}_{\beta} = {}^L\mathbf{g}_{\alpha\beta}$, for given $\mathbf{g}_{\alpha'\beta'}$, ${}^L\mathbf{g}_{\alpha\beta}$.

See the preprint for computations for many realistic cases.

A **d–connection** $\mathbf{D}$ on $\mathbf{V}$ is a linear connection preserving under parallelism the N–connection splitting.

Locally, $\mathbf{D} \Rightarrow \Gamma^\gamma_{\alpha\beta} = \left(L^i_{jk}, L^a_{bk}, C^i_{jc}, C^a_{bc} \right)$.

${}^h D = (L^i_{jk}, L^a_{bk})$ and ${}^v D = (C^i_{jc}, C^a_{bc})$

$= h\mathbf{X} + v\mathbf{X} = {}^h\mathbf{X} + {}^v\mathbf{X}, \mathbf{Y} = h\mathbf{Y} + v\mathbf{Y}.$

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Example 1: *Lagrange spaces.* $\mathbf{V} = \mathbf{TM}$, $L(x,y)$, ${}^L g_{ab} = \frac{1}{2}\frac{\partial^2 L}{\partial y^a \partial y^b}$, $det|g_{ab}| \neq 0$. Canonical N–connection ${}^L N_j^i(x,y) = \frac{\partial\ {}^L G^i}{\partial y^{2+j}}$, ${}^L G^i = \frac{1}{4}\ {}^L g^{ij}(\frac{\partial^2 L}{\partial y^j \partial x^k} y^k - \frac{\partial L}{\partial x^i})$

when nonlinear geodesic equations for $x^i(\tau)$, $y^i = \frac{dx^i}{d\tau}$

$$\frac{d^2 x^i}{d\tau^2} + 2\ {}^L G^i(x^k, \frac{dx^j}{d\tau}) = 0$$

are equivalent to Euler–Lagrange eqs $\frac{d}{d\tau}(\frac{\partial L}{\partial y^i}) - \frac{\partial L}{\partial x^i} = 0$

Example 2: *Finsler spaces.* $F(x, \lambda y) = |\lambda| F(x,y)$, for $0 \neq \lambda \in \mathbb{R}$, particular case $L = F^2$.

Theorem: *Any Lagrange* (Finsler) geometry can be modelled effectively on a N–anholonomic Riemann manifold $\mathbf{V}$, or equivalently on $\mathbf{TM}$, with canonically induced by L (F) d–metric structure

$${}^L\mathbf{g} = {}^L g_{ij}(u)\ e^i \otimes e^j + {}^L g_{ab}(u)\ {}^L\mathbf{e}^a \otimes {}^L\mathbf{e}^b$$
$$e^i = dx^i,\ {}^L\mathbf{e}^b = dy^b + {}^L N_j^b(u) dx^j.$$

$\mathbf{g}_{\alpha'\beta'} e^{\alpha'}{}_{\alpha} e^{\beta'}{}_{\beta} = {}^{\sim}\mathbf{g}_{\alpha\beta}$, for given $\mathbf{g}_{\alpha'\beta'}$, ${}^{\sim}\mathbf{g}_{\alpha\beta}$.
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${}^h D = (L^i_{jk}, L^a_{bk})$ and ${}^v D = (C^i_{jc}, C^a_{bc})$

d–vectors $\mathbf{X} = h\mathbf{X} + v\mathbf{X} = {}^h\mathbf{X} + {}^v\mathbf{X}$, $\mathbf{Y} = h\mathbf{Y} + v\mathbf{Y}$.

Torsion, Curvature and Ricci d–tensors

Torsion of $\mathbf{D} = ({}^h D, {}^v D)$, field

$$\mathbf{T}(\mathbf{X},\mathbf{Y}) \doteqdot \mathbf{D}_{\mathbf{X}}\mathbf{Y} - \mathbf{D}_{\mathbf{Y}}\mathbf{X} - [\mathbf{X},\mathbf{Y}],$$

N–adapted decomposition into

$$\mathbf{T}(\mathbf{X},\mathbf{Y}) = \{h\mathbf{T}(h\mathbf{X},h\mathbf{Y}), h\mathbf{T}(h\mathbf{X},v\mathbf{Y}), h\mathbf{T}(v\mathbf{X},h\mathbf{Y}), h\mathbf{T}(v\mathbf{X},v\mathbf{Y}), v\mathbf{T}(h\mathbf{X},h\mathbf{Y}), v\mathbf{T}(h\mathbf{X},v\mathbf{Y}), v\mathbf{T}(v\mathbf{X},h\mathbf{Y}), v\mathbf{T}v\mathbf{X},v\mathbf{Y})\}.$$

N–adapted coefficients

$$\mathbf{T} = \{\mathbf{T}^{\alpha}{}_{\beta\gamma} = -\mathbf{T}^{\alpha}{}_{\gamma\beta} = (T^i{}_{jk}, T^i{}_{ja}, T^a{}_{jk}, T^b{}_{ja}, T^b{}_{ca})\}$$

Curvature: $\mathbf{R}(\mathbf{X}$ ... formulation of GR

with N–adapted ...

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Nonholonomic manifolds

Geometrization of nonholonomic mechanics $\Longrightarrow$
concept of nonholonomic manifold $\mathbf{V} = (M, \mathcal{D})$
smooth & orientable M, non–integrable distribution $\mathcal{D}$
Local coordinates $u^\alpha = (x^i, y^a)$, for
$i, j, ... = 1, 2;\; a, b, ... = 3, 4.$

N–anholonomic manifold is provided with N–connection structure $\mathbf{N} = N_i^a(u)\, dx^i \otimes \frac{\partial}{\partial y^a}$
Particular case: $N_i^a(u) = \Gamma^a_{bi}(x) y^b$

$\exists$ N–adapted (co) frames (vielbeins)

$$\mathbf{e}_\nu = \left(\mathbf{e}_i = \frac{\partial}{\partial x^i} - N_i^a(u) \frac{\partial}{\partial y^a}, e_a = \frac{\partial}{\partial y^a} \right),$$

$$\mathbf{e}^\mu = \left(e^i = dx^i, \mathbf{e}^a = dy^a + N_i^a(u) dx^i \right).$$

Nonholonomy: $[\mathbf{e}_\alpha, \mathbf{e}_\beta] = \mathbf{e}_\alpha \mathbf{e}_\beta - \mathbf{e}_\beta \mathbf{e}_\alpha = W^\gamma_{\alpha\beta} \mathbf{e}_\gamma$

anholonomy coefficients $W^b_{ia} = \partial_a N_i^b$ and $W^a_{ji} = \Omega^a_{ij}$,

N–connection curvature $\Omega^a_{ij} = \mathbf{e}_j (N_i^a) - \mathbf{e}_i (N_j^a)$

holonomic/ integrable case $W^\gamma_{\alpha\beta} = 0$.

Wireless Network Connection is now connected

Connected to: GuestPass
Signal Strength: Low

N–adapted/distinguished, d–metrics

Any metric on $\mathbf{V}$, $\mathbf{g} = \underline{g}_{\alpha\beta}(u)\, du^\alpha \otimes du^\beta$

$$\underline{g}_{\alpha\beta} = \begin{bmatrix} g_{ij} + N_i^a N_j^b g_{ab} & N_j^e g_{ae} \\ N_i^e g_{be} & g_{ab} \end{bmatrix}, \text{ for } N_j^e(u) = N_j^e(x,y),$$

can be represented as a d–metric $\mathbf{g} = hg \oplus_{\mathbf{N}} vg = [\,{}^h g,\ {}^v g]$

$$\mathbf{g} = g_{ij}(x,y)\, e^i \otimes e^j + g_{ab}(x,y)\, \mathbf{e}^a \otimes \mathbf{e}^b$$

Example 1: *Lagrange spaces.* $\mathbf{V} = \mathbf{TM}$, $L(x,y)$, ${}^L g_{ab} = \frac{1}{2}\frac{\partial^2 L}{\partial y^a \partial y^b}$, $det|g_{ab}| \neq 0$. Canonical N–connection ${}^L N_j^i(x,y) = \frac{\partial\, {}^L G^i}{\partial y^{2+j}}$, ${}^L G^i = \frac{1}{4}\, {}^L g^{ij}(\frac{\partial^2 L}{\partial y^j \partial x^k} y^k - \frac{\partial L}{\partial x^i})$

when nonlinear geodesic equations for $x^i(\tau), y^i = \frac{dx^i}{d\tau}$

$$\frac{d^2 x^i}{d\tau^2} + 2\, {}^L G^i(x^k, \frac{dx^j}{d\tau}) = 0$$

are equivalent to Euler–Lagrange eqs $\frac{d}{d\tau}(\frac{\partial L}{\partial y^i}) - \frac{\partial L}{\partial x^i} = 0$

Example 2: *Finsler spaces.* $F(x, \lambda y) = |\lambda| F(x,y)$, for $0 \neq \lambda \in \mathbb{R}$, particular case $L = F^2$.

Theorem: *Any Lagrange* (Finsler) geometry can be modelled effectively on a N–anholonomic Riemann manifold $\mathbf{V}$, or e
induced by L (F
${}^L\mathbf{g} = {}^L g$
$e^i = dx^i$

Existence of Lagrange–Finsler Variables in GR

follows from the quadratic algebraic system for $e^{\alpha'}_{\ \alpha}$
$\mathbf{g}_{\alpha'\beta'} e^{\alpha'}_{\ \alpha} e^{\beta'}_{\ \beta} = {}^L\mathbf{g}_{\alpha\beta}$, for given $\mathbf{g}_{\alpha'\beta'}$, ${}^L\mathbf{g}_{\alpha\beta}$.
See the preprint for computations for many realistic cases.

A **d–connection** $\mathbf{D}$ on $\mathbf{V}$ is a linear connection preserving under parallelism the N–connection splitting.
Locally, $\mathbf{D} \Rightarrow \mathbf{\Gamma}^\gamma_{\ \alpha\beta} = (L^i_{jk}, L^a_{bk}, C^i_{jc}, C^a_{bc})$,
${}^h D = (L^i_{jk}, L^a_{bk})$ and ${}^v D = (C^i_{jc}, C^a_{bc})$

d–vectors $\mathbf{X} = h\mathbf{X} + v\mathbf{X} = {}^h\mathbf{X} + {}^v\mathbf{X}$, $\mathbf{Y} = h\mathbf{Y} + v\mathbf{Y}$,

Torsion, Curvature and Ricci d–tensors

Torsion of $\mathbf{D} = ({}^h D, {}^v D)$, field

$$\mathbf{T}(\mathbf{X}, \mathbf{Y}) \doteqdot \mathbf{D}_{\mathbf{X}}\mathbf{Y} - \mathbf{D}_{\mathbf{Y}}\mathbf{X} - [\mathbf{X}, \mathbf{Y}],$$

N–adapted decomposition into

$$\mathbf{T}(\mathbf{X}, \mathbf{Y}) = \{h\mathbf{T}(h\mathbf{X}, h\mathbf{Y}), h\mathbf{T}(h\mathbf{X}, v\mathbf{Y}),$$
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Theorem: *Any Lagrange* (Finsler) geometry can be modelled effectively on a N–anholonomic Riemann manifold $\mathbf{V}$, or equivalently on $\mathbf{TM}$, with canonically induced by L (F) d–metric structure

$${}^L\mathbf{g} = {}^L g$$
$$e^i = dx^i$$

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$$\ldots_{\gamma} = -\mathbf{T}^\alpha_{\ \gamma\beta} = \left(T^i_{\ jk}, T^i_{\ ja}, T^a_{\ jk}, T^b_{\ ja}, T^b_{\ ca}\right)\}$$

Curvature: $\mathbf{R}(\mathbf{X},\mathbf{Y}) \doteqdot \mathbf{D}_{\mathbf{X}}\mathbf{D}_{\mathbf{Y}} - \mathbf{D}_{\mathbf{Y}}\mathbf{D}_{\mathbf{X}} - \mathbf{D}_{[\mathbf{X},\mathbf{Y}]}$,
with N–adapted decomposition

$$\mathbf{R}(\mathbf{X},\mathbf{Y})\mathbf{Z} = \{\mathbf{R}(h\mathbf{X},h\mathbf{Y})h\mathbf{Z}, \mathbf{R}(h\mathbf{X},v\mathbf{Y})h\mathbf{Z}, \mathbf{R}(v\mathbf{X},h\mathbf{Y})h\mathbf{Z}, \mathbf{R}(v\mathbf{X},v\mathbf{Y})h\mathbf{Z}, \mathbf{R}(h\mathbf{X},h\mathbf{Y})v\mathbf{Z}, \mathbf{R}(h\mathbf{X},v\mathbf{Y})v\mathbf{Z}, \mathbf{R}(v\mathbf{X},h\mathbf{Y})v\mathbf{Z}, \mathbf{R}(v\mathbf{X},v\mathbf{Y})v\mathbf{Z}\}.$$

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Ricci: $\mathbf{Ric} \doteqdot \{\mathbf{R}_{\beta\gamma} = \mathbf{R}^\alpha{}_{\beta\gamma\alpha} = (R_{ij}, R_{ia}, R_{ai}, R_{ab})\}$.

In general, $\mathbf{R}_{\beta\gamma} \neq \mathbf{R}_{\gamma\beta}$.

Set of d–connections compatible to a d–metric

A very important construction in Lagrange geometry, not considered in GR, is: Let be given $\mathbf{g}_{\alpha\beta} = {}^L\mathbf{g}_{\alpha\beta}$, ${}^L N^b_j \Rightarrow$ work equivalently $\forall \mathbf{\Gamma}^\gamma{}_{\alpha\beta}$, ${}_|\Gamma^\gamma{}_{\alpha\beta}(\mathbf{g}) = \mathbf{\Gamma}^\gamma{}_{\alpha\beta}(\mathbf{g}) + {}_|Z^\gamma{}_{\alpha\beta}(\mathbf{g})$,

$$\mathbf{D}\mathbf{g} = 0,$$

where $\exists$ Levi Civita (not good for DQ)
$\nabla(\mathbf{g}) = \{{}_|\Gamma^\gamma{}_{\alpha\beta}(g_{ij}, g_{ab}, N^a_i)\}$, ${}_|T^\alpha{}_{\beta\gamma} = 0$ and $\nabla\mathbf{g} = 0$.

This is not an us
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off–diagonal/N–c

Almost Kähler formulation of GR

Proposal: construct from $\mathbf{g}$ metric/symplectic–compatible almost Kähler connections, necessary for DG, $\Leftrightarrow \nabla$.

$\exists$ Canonical d–connect. ${}^c\mathbf{D}(\mathbf{g}) = \{{}^c\mathbf{\Gamma}^\gamma{}_{\alpha\beta}(g_{ij}, g_{ab}, N^a_i)\}$
${}^c\mathbf{D}\mathbf{g} = 0$ and $h\,{}^c\mathbf{T}(hX, hY) = 0$, $v\,{}^c\mathbf{T}(vX, vY) = 0$.

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For even dimensions, $\exists$ a unique normal d–connection $\widehat{\mathbf{D}}$, $\{\widehat{\mathbf{\Gamma}}^\alpha_{\beta\gamma} = (\widehat{L}^i_{jk}, {}^v\widehat{L}^{2+i}_{2+j\ 2+k} = \widehat{L}^i_{jk}, \widehat{C}^i_{jc} = {}^v\widehat{C}^{2+i}_{2+j\ c}, {}^v\widehat{C}^a_{bc} = \widehat{C}^a_{bc})\}$, defined $\widehat{\mathbf{D}}_\alpha = (\widehat{D}_k, \widehat{D}_c)$, $\widehat{\mathbf{\Gamma}}^\alpha_{\beta\gamma} = (\widehat{L}^i_{jk}, {}^v\widehat{C}^a_{bc})$,

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Almost complex operator, $\mathbf{J}(\mathbf{e}_i) = -e_{2+i}$, $\mathbf{J}(e_{2+i}) = \mathbf{e}_i$,
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${}^L\theta(\mathbf{X},\mathbf{Y}) \doteqdot {}^L\mathbf{g}(\mathbf{J}\mathbf{X},\mathbf{Y})$, $\widehat{\mathbf{D}}\mathbf{J} = \widehat{\mathbf{D}}\,{}^L\theta = \widehat{\mathbf{D}}\,{}^L\mathbf{g} = 0$

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Conclusion: Prescribing a generating effective Lagrangian $L(x, y)$, we induce canonically a $(2+2)_L$ splitting and construct an almost Kähler model of GR.

N–adapted 3+1 splitting = "double" $(2+2)_L$ **& 3+1**

S. Mercuri, From the Einstein–Cartan to the Ashtekar–Barbero canonical constraints, passing through the Nieh–Yah functional, 0708.0037

with N–adapted decomposition

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Set of d–connections compatible to a d–metric

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$$\mathbf{D}\mathbf{g} = 0,$$

where $\exists$ Levi Civita (not good for DQ)
$\nabla(\mathbf{g}) = \{{}_|\Gamma^\gamma{}_{\alpha\beta}(g_{ij}, g_{ab}, N^a_i)\}$, ${}_|T^\alpha{}_{\beta\gamma} = 0$ and $\nabla\mathbf{g} = 0$.

distorsion (induced torsion) ${}_|Z^\gamma{}_{\alpha\beta}(\mathbf{g})$ is not an additional field but completely defined by the metric $\mathbf{g}$, off–diagonal/N–connection/anholonomy coefficients N^a_i.

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${}^L\theta(\mathbf{X},\mathbf{Y}) \doteqdot {}^L\mathbf{g}(\mathbf{J}\mathbf{X},\mathbf{Y})$, $\widehat{\mathbf{D}}\mathbf{J} = \widehat{\mathbf{D}}\,{}^L\theta = \widehat{\mathbf{D}}\,{}^L\mathbf{g} = 0$

${}^L\theta = {}^Lg_{ij}(x, y)(dy^{2+i} + {}^LN^{2+i}_k dx^k)\wedge dx^j$.

${}^L_{(i)} = \frac{1}{2}\frac{\partial L}{\partial x^i}$, ${}^L\theta = d\,{}^L_{(i)} \Rightarrow d\,{}^L\theta = dd\,{}^L_{(i)} = 0$

Conclusion: Prescribing a generating effective Lagrangian $L(x, y)$, we induce canonically a $(2+2)_L$ splitting and construct an almost Kähler model of GR.

N–adapted 3+1 splitting = "double" $(2+2)_L$ & 3+1

S. Mercuri, From the Einstein–Cartan to the Ashtekar–Barbero canonical constraints, passing through the Nieh–Yan functional, 0708.0037

$$\mathbf{R}(hX, vY)vZ, \mathbf{R}(vX, hY)vZ, \mathbf{R}(vX, vY)vZ\}.$$

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Conclusion: Prescribing a generating effective Lagrangian $L(x, y)$, we induce canonically a $(2+2)_L$ splitting and construct an almost Kähler model of GR.

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V is topologically ${}^3\Sigma \times \mathbb{R}$, $\mathbf{t}^\alpha = {}_l\mathbf{N}\mathbf{n}^\alpha + {}_s\mathbf{N}^\alpha$,
lapse function ${}_lN$, shift d–vector $\mathbf{N}^\alpha$

S. Mercuri, From the Einstein–Cartan to the Ashtekar–Barbero canonical constraints, passing through the Nieh–Yah functional, 0708.0037

Ashtekar–Barbero d–connections

modelled effectively on a N–anholonomic Riemann manifold V, or equivalently on TM, with canonically induced by L (F) d–metric structure

$$^L\mathbf{g} = {}^L g_{ij}(u)\, e^i \otimes e^j + {}^L g_{ab}(u)\, {}^L\mathbf{e}^a \otimes {}^L\mathbf{e}^b$$
$$e^i = dx^i,\ {}^L\mathbf{e}^b = dy^b + {}^L N^b_j(u) dx^j.$$

$$h\mathbf{T}(v\mathbf{X}, h\mathbf{Y}), h\mathbf{T}(v\mathbf{X}, v\mathbf{Y}), v\mathbf{T}(h\mathbf{X}, h\mathbf{Y}),$$
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$$\mathbf{T} = \{\mathbf{T}^\alpha{}_{\beta\gamma} = -\mathbf{T}^\alpha{}_{\gamma\beta} = (T^i{}_{jk}, T^i{}_{ja}, T^a{}_{jk}, T^b{}_{ja}, T^b{}_{ca})\}$$

Curvature: $\mathbf{R}(\mathbf{X}, \mathbf{Y}) \doteqdot \mathbf{D}_\mathbf{X}\mathbf{D}_\mathbf{Y} - \mathbf{D}_\mathbf{Y}\mathbf{D}_\mathbf{X} - \mathbf{D}_{[\mathbf{X},\mathbf{Y}]}$, with N–adapted decomposition

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Almost Kähler formulation of GR

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N–adapted/distinguished, d–metrics

Any metric on $\mathbf{V}$, $\mathbf{g} = \underline{g}_{\alpha\beta}(u)\, du^\alpha \otimes du^\beta$

$$\underline{g}_{\alpha\beta} = \begin{bmatrix} g_{ij} + N_i^a N_j^b g_{ab} & N_j^e g_{ae} \\ N_i^e g_{be} & g_{ab} \end{bmatrix}, \text{ for } N_j^e(u) = N_j^e(x,y),$$

can be represented as a d–metric $\mathbf{g} = hg \oplus_{\mathbf{N}} vg = [\,{}^h g,\ {}^v g]$

$$\mathbf{g} = g_{ij}(x,y)\, e^i \otimes e^j + g_{ab}(x,y)\, \mathbf{e}^a \otimes \mathbf{e}^b$$

Example 1: *Lagrange spaces.* $\mathbf{V} = \mathbf{TM}$, $L(x,y)$, ${}^L g_{ab} = \frac{1}{2}\frac{\partial^2 L}{\partial y^a \partial y^b}$, $det|g_{ab}| \neq 0$. Canonical N–connection ${}^L N_j^i(x,y) = \frac{\partial\, {}^L G^i}{\partial y^{2+j}}$, ${}^L G^i = \frac{1}{4}\, {}^L g^{ij}(\frac{\partial^2 L}{\partial y^j \partial x^k} y^k - \frac{\partial L}{\partial x^i})$

when nonlinear geodesic equations for $x^i(\tau), y^i = \frac{dx^i}{d\tau}$

$$\frac{d^2 x^i}{d\tau^2} + 2\, {}^L G^i(x^k, \frac{dx^j}{d\tau}) = 0$$

are equivalent to Euler–Lagrange eqs $\frac{d}{d\tau}(\frac{\partial L}{\partial y^i}) - \frac{\partial L}{\partial x^i} = 0$

Example 2: *Finsler spaces.* $F(x, \lambda y) = |\lambda| F(x,y)$, for $0 \neq \lambda \in \mathbb{R}$, particular case $L = F^2$.

Theorem: *Any Lagrange* (Finsler) geometry can be modelled effectively on a N–anholonomic Riemann manifold $\mathbf{V}$, or equivalently on $\mathbf{TM}$, with canonically induced by L (F) d–metric structure

$${}^L\mathbf{g} = {}^L g_{ij}(u)\, e^i \otimes e^j + {}^L g_{ab}(u)\, {}^L\mathbf{e}^a \otimes {}^L\mathbf{e}^b$$
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Existence of Lagrange–Finsler Variables in GR

follows from the quadratic algebraic system for $e^{\alpha'}{}_\alpha$ $\mathbf{g}_{\alpha'\beta'} e^{\alpha'}{}_\alpha e^{\beta'}{}_\beta = {}^L\mathbf{g}_{\alpha\beta}$, for given $\mathbf{g}_{\alpha'\beta'}$, ${}^L\mathbf{g}_{\alpha\beta}$.
See the preprint for computations for many realistic cases.

A **d–connection** $\mathbf{D}$ on $\mathbf{V}$ is a linear connection preserving under parallelism the N–connection splitting.
Locally, $\mathbf{D} \Rightarrow \Gamma^\gamma{}_{\alpha\beta} = (L^i_{jk}, L^a_{bk}, C^i_{jc}, C^a_{bc})$.
${}^h D = (L^i_{jk}, L^a_{bk})$ and ${}^v D = (C^i_{jc}, C^a_{bc})$

d–vectors $\mathbf{X} = h\mathbf{X} + v\mathbf{X} = {}^h\mathbf{X} + {}^v\mathbf{X}$, $\mathbf{Y} = h\mathbf{Y} + v\mathbf{Y}$.

Torsion, Curvature and Ricci d–tensors

Torsion of $\mathbf{D} = ({}^h D, {}^v D)$, field

$$\mathbf{T}(\mathbf{X}, \mathbf{Y}) \doteqdot \mathbf{D}_\mathbf{X}\mathbf{Y} - \mathbf{D}_\mathbf{Y}\mathbf{X} - [\mathbf{X}, \mathbf{Y}].$$

N–adapted decomposition into

$$\mathbf{T}(\mathbf{X}, \mathbf{Y}) = \{h\mathbf{T}(h\mathbf{X}, h\mathbf{Y}), h\mathbf{T}(h\mathbf{X}, v\mathbf{Y}),$$
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can be represented as a d–metric $\mathbf{g} = h\mathbf{g} \oplus_{\mathbf{N}} v\mathbf{g} = [\,{}^h\mathbf{g},\ {}^v\mathbf{g}]$

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Curvature: $\mathbf{R}(\mathbf{X},\mathbf{Y}) \doteqdot \mathbf{D}_{\mathbf{X}}\mathbf{D}_{\mathbf{Y}} - \mathbf{D}_{\mathbf{Y}}\mathbf{D}_{\mathbf{X}} - \mathbf{D}_{[\mathbf{X},\mathbf{Y}]}$,
with N–adapted decomposition

$$\mathbf{R}(\mathbf{X},\mathbf{Y})\mathbf{Z} = \{\mathbf{R}(h\mathbf{X},h\mathbf{Y})h\mathbf{Z}, \mathbf{R}(h\mathbf{X},v\mathbf{Y})h\mathbf{Z}, \mathbf{R}(v\mathbf{X},h\mathbf{Y})h\mathbf{Z}, \mathbf{R}(v\mathbf{X},v\mathbf{Y})h\mathbf{Z}, \mathbf{R}(h\mathbf{X},h\mathbf{Y})v\mathbf{Z}, \mathbf{R}(h\mathbf{X},v\mathbf{Y})v\mathbf{Z}, \mathbf{R}(v\mathbf{X},h\mathbf{Y})v\mathbf{Z}, \mathbf{R}(v\mathbf{X},v\mathbf{Y})v\mathbf{Z}\}.$$

N–adapted components $\mathbf{X} = \mathbf{e}_\alpha$, $\mathbf{Y} = \mathbf{e}_\beta$ and $\mathbf{Z} = \mathbf{e}_\gamma$,

$$\mathbf{R} = \{\mathbf{R}^\alpha{}_{\beta\gamma\delta} = (R^i{}_{hjk}, R^a{}_{bjk}, R^i{}_{hja}, R^c{}_{bja}, R^i{}_{hba}, R^c{}_{bea})\}$$

Ricci: $\mathbf{Ric} \doteqdot \{\mathbf{R}_{\beta\gamma} = \mathbf{R}^\alpha{}_{\beta\gamma\alpha} = (R_{ij}, R_{ia}, R_{ai}, R_{ab})\}$.

In general, $\mathbf{R}_{\beta\gamma} \neq \mathbf{R}_{\gamma\beta}$.

Set of d–connections compatible to a d–metric

A very important construction in Lagrange geometry, not considered in GR, is: Let be given $\mathbf{g}_{\alpha\beta} = {}^L\mathbf{g}_{\alpha\beta}$, ${}^L N^b_j \Rightarrow$ work equivalently $\forall \mathbf{\Gamma}^\gamma{}_{\alpha\beta}$, ${}_|\Gamma^\gamma{}_{\alpha\beta}(\mathbf{g}) = \mathbf{\Gamma}^\gamma{}_{\alpha\beta}(\mathbf{g}) + {}_|Z^\gamma{}_{\alpha\beta}(\mathbf{g})$,

$$\mathbf{D}\mathbf{g} = 0,$$

where $\exists$ Levi Civita (not good for DQ)
$\nabla(\mathbf{g}) = \{{}_|\Gamma^\gamma{}_{\alpha\beta}(g_{ij}, g_{ab}, N^a_i)\}$, ${}_|T^\alpha{}_{\beta\gamma} = 0$ and $\nabla\mathbf{g} = 0$.

This is not an usual Einstein–Cartan space because distorsion (induced torsion) ${}_|Z^\gamma{}_{\alpha\beta}(\mathbf{g})$ is not an additional field but completely defined by the metric $\mathbf{g}$,
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Almost Kähler formulation of GR

Proposal: construct from $\mathbf{g}$ metric/symplectic–compatible almost Kähler connections, necessary for DG, $\Leftrightarrow \nabla$.

$\exists$ Canonical d–connect. ${}^c\mathbf{D}(\mathbf{g}) = \{{}^c\mathbf{\Gamma}^\gamma{}_{\alpha\beta}(g_{ij}, g_{ab}, N^a_i)\}$
${}^c\mathbf{D}\mathbf{g} = 0$ and $h\,{}^c\mathbf{T}(hX, hY) = 0$, $v\,{}^c\mathbf{T}(vX, vY) = 0$.

$${}_|\Gamma^\gamma{}_{\alpha\beta} = {}^c\mathbf{\Gamma}^\gamma{}_{\alpha\beta}(g_{ij}, g_{ab}, N^a_i) + {}_|Z^\gamma{}_{\alpha\beta}(g_{ij}, g_{ab}, N^a_i).$$

For even dimensions, $\exists$ a unique normal d–connection $\widehat{\mathbf{D}}$, $\{\widehat{\mathbf{\Gamma}}^\alpha_{\beta\gamma} = (\widehat{L}^i_{jk}, {}^v\widehat{L}^{2+i}_{2+j\ 2+k} = \widehat{L}^i_{jk}; \widehat{C}^i_{jc} = {}^v\widehat{C}^{2+i}_{2+j\ c}, {}^v\widehat{C}^a_{bc} = \widehat{C}^a_{bc})\}$, defined $\widehat{\mathbf{D}}_\alpha = (\widehat{D}_k, \widehat{D}_c)$, $\widehat{\mathbf{\Gamma}}^\alpha_{\beta\gamma} = (\widehat{L}^i_{jk}, {}^v\widehat{C}^a_{bc})$,

$$\widehat{L}^i_{jk} = \frac{1}{2}\,{}^Lg^{ih}\left(\mathbf{e}_k\,{}^Lg_{jh} + \mathbf{e}_j\,{}^Lg_{hk} - \mathbf{e}_h\,{}^Lg_{jk}\right).$$

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Almost complex operator, $\mathbf{J}(\mathbf{e}_i) = -e_{2+i}$ $\mathbf{J}(e_{2+i}) = \mathbf{e}_i$,
$\mathbf{J} = -\frac{\partial}{\partial y^i}\otimes dx^i + (\frac{\partial}{\partial x^i} - {}^LN^{2+j}_i\frac{\partial}{\partial y^j})\otimes(dy^i + {}^LN^{2+i}_k dx^k)$

$${}^L\theta(\mathbf{X},\mathbf{Y}) \doteqdot {}^L\mathbf{g}(\mathbf{J}\mathbf{X},\mathbf{Y}),\ \widehat{\mathbf{D}}\mathbf{J} = \widehat{\mathbf{D}}\,{}^L\theta = \widehat{\mathbf{D}}\,{}^L\mathbf{g} = 0$$

$${}^L\theta = {}^Lg_{ij}(x,y)(dy^{2+i} + {}^LN^{2+i}_k dx^k)\wedge dx^j.$$

$${}^L\omega = \tfrac{1}{2}\tfrac{\partial L}{\partial y^i}dx^i,\ {}^L\theta = d\,{}^L\omega, \Rightarrow d\,{}^L\theta = dd\,{}^L\omega = 0.$$

Conclusion: Prescribing a generating effective Lagrangian $L(x,y)$, we induce canonically a $(2+2)$–splitting and construct an almost Kähler model of GR.
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$${}^{L}\omega = \frac{1}{2}\frac{\partial L}{\partial y^{i}}dx^{i},\ {}^{L}\theta = d\,{}^{L}\omega, \Rightarrow d\,{}^{L}\theta = dd\,{}^{L}\omega = 0.$$

Conclusion: Prescribing a generating effective Lagrangian $L(x, y)$, we induce canonically a $(2+2)_{L}$ splitting and construct an almost Kähler model of GR.

N–adapted 3+1 splitting = "double" $(2+2)_{L}$ & 3+1

$\mathbf{V}$ is topologically ${}^{3}\Sigma \times \mathbb{R}$, $\mathbf{t}^{\alpha} = {}_{l}\mathbf{N}\mathbf{n}^{\alpha} + {}_{s}\mathbf{N}^{\alpha}$,
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This is not an usual Einstein–Cartan space because distorsion (induced torsion) ${}_{|}Z^{\gamma}{}_{\alpha\beta}(\mathbf{g})$ is not an additional field but completely defined by the metric $\mathbf{g}$, off–diagonal/N–connection/anholonomy coefficients N_i^a.

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Ashtekar–Barbero d–connections

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N–adapted 3+1 splitting = "double" $(2+2)_L$ & 3+1

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S. Mercuri, From the Einstein–Cartan to the Ashtekar–Barbero canonical constraints, passing through the Nieh–Yan functional, 0708.0037

Ashtekar–Barbero d–connections

Classical phase space and Hamilton formalism for
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Ashtekar–Barbero d–connections

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DQ and Fedosov–Ashtekar Manifolds

Fedosov–Ashtekar d–operators on ${}^{3}\Sigma$

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the internal and spacetime projection maps, q_I^α and $q_{\alpha'}^{I'}$,
$SO(1,3) \to SO(3)$, invariant $\mathbf{n}^{\alpha'}$.

$so(1,3)$–valued $\Gamma^{\alpha'}_{\ \gamma'} = \Gamma^{\alpha'}_{\ \gamma'\beta'} \mathbf{e}^{\beta'} \Rightarrow$ two $so(3)$–valued
$\Gamma^{I'}_{\ I} \doteqdot \frac{1}{2} q_I^\alpha q_{\alpha'}^{I'} \epsilon^{\alpha'\beta'}_{\ \ \gamma'\tau'} \mathbf{n}_{\beta'} \Gamma^{\gamma'\tau'}_{\ \ \alpha}$ and $K^{I'}_{\ I} \doteqdot q_{\alpha'}^{I'} q_I^\alpha \mathbf{n}_{\beta'} \Gamma^{\alpha'\beta'}_{\ \ \alpha}$,
the spin connection and extrinsic curvature on shell.

LQG is formulated for $({}_{\ell}\Gamma^{I'}_{\ I}, {}_{\ell}K^{I'}_{\ I})$, induced by
$\Gamma^\gamma_{\ \alpha\beta} = {}_{\ell}\Gamma^\gamma_{\ \alpha\beta}(\mathbf{g})$.

${}_{\ell}A^{I'}_{\ I} \doteqdot {}_{\ell}\Gamma^{I'}_{\ I} + \beta\, {}_{\ell}K^{I'}_{\ I}$, $E_{I'}^{\ \ I} = (\kappa\beta)^{-1} \sqrt{q} \mathbf{e}^I_{\ I'}$,

$q \doteqdot \det|\mathbf{q}_{IJ}|$, "tilde" for densities.

Suggestion: generalize to a d–connection ${}^A\mathcal{D} = \{\mathbf{A}^{I'}_{\ I}\}$,

${}_{\ell}A^{I'}_{\ I}({}^L\mathbf{g}) \to \mathbf{A}^{I'}_{\ I}({}^L\mathbf{g}) = {}_{\ell}A^{I'}_{\ I}({}^L\mathbf{g}) + {}_{\ell}^{\ A}\mathbf{Z}^{I'}_{\ I}({}^L\mathbf{g})$,

${}_{\ell}^{\ A}\mathbf{Z}^{I'}_{\ I} \doteqdot q_I^\alpha q_{\alpha'}^{I'} \mathbf{n}_{\beta'} \left(\frac{1}{2} \epsilon^{\alpha'\beta'}_{\ \ \gamma'\tau'}\ {}_{\ell}^{\ A}\mathbf{Z}^{\gamma'\tau'}_{\ \ \alpha} + \beta\ {}_{\ell}^{\ A}\mathbf{Z}^{\alpha'\beta'}_{\ \ \alpha} \right)$

${}_{\ell}^{\ A}\mathbf{Z}^\alpha_{\ \beta\gamma} = \frac{1}{8}(2\ {}^J\Omega^\alpha_{\ \beta\gamma} - {}^L\mathbf{g}^{\alpha\tau}\, {}^L\mathbf{g}_{\beta\xi}\ {}^J\Omega^\xi_{\ \tau\gamma})$,

${}^J\Omega^\gamma_{\ \alpha\beta} = (\mathbf{e}_\tau \mathbf{J}^\gamma_{\ \alpha}) \mathbf{J}^\tau_{\ \beta} - (\mathbf{e}_\tau \mathbf{J}^\gamma_{\ \beta}) \mathbf{J}^\tau_{\ \alpha} + (\mathbf{e}_\alpha \mathbf{J}^\tau_{\ \beta} - \mathbf{e}_\beta \mathbf{J}^\tau_{\ \alpha}) \mathbf{J}^\gamma_{\ \tau}$

Ashtekar d–connection $\mathbf{A}^{I'}_{\ I}$, or the nonholonomic
deformation of the Ashtekar–Barbero connection ${}_{\ell}A^{I'}_{\ I}$

$4\ {}^A\mathbf{T}^{\alpha'}_{\ \alpha\beta} = -\ {}^J\Omega^{\alpha'}_{\ \alpha\beta}$ for ${}^A\mathcal{D}_{[\alpha} \mathbf{e}_{\beta]}^{\ \alpha'} = {}^A\mathbf{T}^{\alpha'}_{\ \alpha\beta}$,

$4\ {}^A\mathbf{T}^{I'}_{\ I} = -\ {}^J\Omega^{I'}_{\ I}$, torsion of $\mathbf{A}^{I'}_{\ I}$.

$\exists\ {}^A\mathbf{\Gamma}^\alpha_{\ \beta\gamma}({}^L\mathbf{g})$, or ${}^A\mathbf{\Gamma}^{I'}_{\ I} \doteqdot \frac{1}{2} q_I^\alpha q_{\alpha'}^{I'} \epsilon^{\alpha'\beta'}_{\ \ \gamma'\tau'} \mathbf{n}_{\beta'}\ {}^A\mathbf{\Gamma}^{\gamma'\tau'}_{\ \ \alpha}$,
${}^A\mathbf{K}^{I'}_{\ I} \doteqdot q_{\alpha'}^{I'} q_I^\alpha \mathbf{n}_{\beta'}\ {}^A\mathbf{\Gamma}^{\alpha'\beta'}_{\ \ \alpha}$.

May perform DQ of GR in nonholonomic Ashtekar
variables, or LQG with induced torsion.

DQ and Fedosov–Ashtekar Manifolds

Star products

Fedosov–Ashtekar d–operators on ${}^3\Sigma$

Formal Wick product:

${}^\Sigma\circ\ (z) \doteqdot \exp\left(i\frac{\hbar}{2} \mathbf{\Lambda}^{IJ} \frac{\partial^2}{\partial z^I \partial z'^J} \right) a(z) b(z_{[1]}) \,|_{z=z_{[1]}}$.

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$\Gamma^{I'}{}_{I} \doteqdot \frac{1}{2} q^{\alpha}_{I} q^{I'}_{\alpha'} \epsilon^{\alpha'\beta'}{}_{\gamma'\tau'} \mathbf{n}_{\beta'} \Gamma^{\gamma'\tau'}{}_{\alpha}$ and $K^{I'}{}_{I} \doteqdot q^{I'}_{\alpha'} q^{\alpha}_{I} \mathbf{n}_{\beta'} \Gamma^{\alpha'\beta'}{}_{\alpha'}$, the spin connection and extrinsic curvature on shell.

LQG is formulated for $({}_{l}\Gamma^{I'}{}_{I}, {}_{l}K^{I'}{}_{I})$, induced by $\Gamma^{\gamma}{}_{\alpha\beta} = {}_{l}\Gamma^{\gamma}{}_{\alpha\beta}(\mathbf{g})$.

$\exists\ {}^{A}\mathbf{\Gamma}^{\alpha}_{\beta\gamma}({}^{L}\mathbf{g})$, or ${}^{A}\mathbf{\Gamma}^{I'}{}_{I} \doteqdot \frac{1}{2} q^{\alpha}_{I} q^{I'}_{\alpha'} \epsilon^{\alpha'\beta'}{}_{\gamma'\tau'} \mathbf{n}_{\beta'}\ {}^{A}\mathbf{\Gamma}^{\gamma'\tau'}{}_{\alpha}$, ${}^{A}\mathbf{K}^{I'}{}_{I} \doteqdot q^{I'}_{\alpha'} q^{\alpha}_{I} \mathbf{n}_{\beta'}\ {}^{A}\mathbf{\Gamma}^{\alpha'\beta'}{}_{\alpha'}$.

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DQ and Fedosov–Ashtekar Manifolds

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Consider ${}^{L}\mathbf{\Lambda}^{\alpha\beta} \doteqdot {}^{L}\theta^{\alpha\beta} - i\ {}^{L}\mathbf{g}^{\alpha\beta}$ and $\mathbf{\Lambda}^{IJ} = q^{I}_{\alpha} q^{J}_{\beta} \mathbf{\Lambda}^{\alpha\beta}$, $u = \{u^{\alpha}\}$ on $\mathbf{V}$ and $(u, z) = (u^{\alpha}, z^{\beta})$ on $T_u\mathbf{V}$, fiber z^{β}

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${}^{1}f * {}^{2}f = \sum_{r=0}^{\infty} {}_{r}C({}^{1}f, {}^{2}f)\, v^{r}$; ${}_{r}C, r \geq 0$ are bilinear operators on $C^{\infty}(\mathbf{V})$, ${}_{0}C({}^{1}f, {}^{2}f) = {}^{1}f\, {}^{2}f$ and ${}_{1}C({}^{1}f, {}^{2}f) - {}_{1}C({}^{2}f, {}^{1}f) = i\{{}^{1}f, {}^{2}f\}$; $i^2 = -1$.

formal Wick product for elements a and b,

$$a \circ b\,(z) \doteqdot \exp\left(i\frac{v}{2}\mathbf{\Lambda}^{\alpha\beta}\frac{\partial^2}{\partial z^{\alpha}\partial z^{\beta}_{[1]}}\right) a(z) b(z_{[1]})\,|_{z=z_{[1]}},$$

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Fedosov–Ashtekar d–operators on ${}^{3}\Sigma$

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the spin connection and extrinsic curvature on shell.

LQG is formulated for $({}_{1}\Gamma^{I'}{}_{I}, {}_{1}K^{I'}{}_{I})$, induced by $\Gamma^{\gamma}{}_{\alpha\beta} = {}_{1}\Gamma^{\gamma}{}_{\alpha\beta}(g)$.

$${}^{A}K^{I'}{}_{I} \doteqdot q^{I'}_{\alpha'} q^{\beta}_{I} n_{\beta'}\, {}^{A}\Gamma^{\alpha'\beta'}{}_{\alpha'}.$$

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DQ and Fedosov–Ashtekar Manifolds

Star products

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bilinear operators on $C^{\infty}(\mathbf{V})$, ${}_{0}C({}^{1}f, {}^{2}f) = {}^{1}f\, {}^{2}f$ and ${}_{1}C({}^{1}f, {}^{2}f) - {}_{1}C({}^{2}f, {}^{1}f) = i\{{}^{1}f, {}^{2}f\}$; $i^{2} = -1$.

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Fedosov–Ashtekar d–operators on ${}^{3}\Sigma$

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Commutators on $({}^{L}\mathcal{W} \otimes {}^{\Sigma}\Lambda, {}^{\Sigma}\circ)$, ad_{Wick} defined as a

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Star products

Consider ${}^L\Lambda^{\alpha\beta} \doteqdot {}^L\theta^{\alpha\beta} - i\ {}^L g^{\alpha\beta}$ and $\Lambda^{IJ} = q^I_\alpha q^J_\beta \Lambda^{\alpha\beta}$, $u = \{u^\alpha\}$ on $\mathbf{V}$ and $(u, z) = (u^\alpha, z^\beta)$ on $T_u\mathbf{V}$, fiber z^β

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$$[{}^\Sigma\mathbf{D}, {}^\Sigma\delta] = \frac{i}{v} ad_{Wick}({}^A_z\mathcal{T}),$$

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Star products

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Flat Fedosov–Ashtekar d–connection

$\forall\, \mathbf{g} = {}^{L}\mathbf{g}_{\alpha\beta}$ $\exists$ a flat Fedosov–Ashtekar d–connection

$${}^{\Sigma}\mathcal{D} \doteqdot -{}^{\Sigma}\delta + {}^{A}\mathcal{D} - \frac{i}{v} ad_{Wick}({}^{\Sigma}r)$$

${}^{\Sigma}\mathcal{D}^2 = 0$, the unique element ${}^{\Sigma}r \in {}^{L}\mathcal{W} \otimes {}^{\Sigma}\mathbf{\Lambda}$, $\deg_u({}^{\Sigma}r) = 1$, ${}^{\Sigma}\delta^{-1}\,{}^{\Sigma}r = 0$, solves (recurrently)

$${}^{\Sigma}\delta r = {}^{A}_{\Sigma}T + {}^{A}\mathcal{F} + {}^{A}\mathcal{D}\,{}^{\Sigma}r - \frac{i}{v}\,{}^{\Sigma}r \circ {}^{\Sigma}r.$$

2–forms: ${}^{A}_{\Sigma}T$, curvature ${}^{A}\mathcal{F}$ for ${}^{A}\mathcal{D} = \{\mathbf{A}^{I'}_{I}\}$.

The ${}^{\Sigma}*$–product is defined on $C^\infty(\mathbf{V})[[v]]$:

$${}^{1}f\;{}^{\Sigma}*\;{}^{2}f \doteqdot {}^{\Sigma}\sigma({}^{\Sigma}\tau({}^{1}f)) \circ {}^{\Sigma}\sigma({}^{\Sigma}\tau({}^{2}f)),$$

compute recurrently projectors ${}^{\Sigma}\sigma : {}^{L}\mathcal{W} \to C^\infty(\mathbf{V})[[v]]$

The zero–degree cohomology for Einstein spaces

$\exists$ another canonical class ${}^{\Sigma}\varepsilon$ for ${}^{N}T\,{}^{3}\Sigma = h\,{}^{3}\Sigma \oplus v\,{}^{3}\Sigma$ related to Chern–Weyl form ${}^{\Sigma}\gamma = -\frac{1}{4}\mathbf{J}^{I}_{K'}\,\mathbf{F}^{K'}_{IJ} \wedge \mathbf{e}^J$,

$$c_0({}^{\Sigma}*) = -(1/2i)\,{}^{\Sigma}\varepsilon, \quad {}^{\Sigma}\varepsilon \doteqdot [\,{}^{\Sigma}\gamma].$$

4d, closed Chern–Weyl form $\varepsilon \doteqdot [\gamma]$, $4\gamma = -\mathbf{J}_{\tau}{}^{\alpha'}\,{}^{A}\mathcal{R}^{\tau}{}_{\alpha'}$

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3–form Einstein equations

Deformations of Einstein tensor for ${}^{A}\Gamma^{\alpha}_{\beta\gamma}({}^{L}\mathbf{g}) = \widehat{\Gamma}^{\alpha}_{\beta\gamma}({}^{L}\mathbf{g}) + \widehat{\mathbf{Z}}^{\alpha}_{\beta\gamma}({}^{L}\mathbf{g}) = {}_{1}\Gamma^{\alpha}_{\beta\gamma}({}^{L}\mathbf{g}) + {}^{A}_{1}\mathbf{Z}^{\alpha}_{\beta\gamma}({}^{L}\mathbf{g})$,

defining deformations of curvatures

$\exists$ standard grading $\deg_s$ on Λ and other gradings, $\circ$ on ${}^L\mathcal{W}\otimes\Lambda$ is bigrated; d–algebra $({}^L\mathcal{W}\otimes\Lambda,\circ)$; also consider ${}^L\mathcal{W}\otimes{}^\Sigma\Lambda$ for Λ^{IJ}.

$$[{}^\Sigma\mathbf{D},{}^\Sigma\delta] = \frac{i}{v}ad_{Wick}({}^A_\Sigma T),$$
$$ {}^\Sigma\mathbf{D}^2 = -\frac{i}{v}ad_{Wick}({}^A_\Sigma\mathcal{R}).$$

Flat Fedosov–Ashtekar d–connection

$\forall\ \mathbf{g}={}^L\mathbf{g}_{\alpha\beta}\ \exists$ a flat Fedosov–Ashtekar d–connection

$$ {}^\Sigma\mathcal{D}\doteqdot -{}^\Sigma\delta+{}^A\mathcal{D}-\frac{i}{v}ad_{Wick}({}^\Sigma r)$$

${}^\Sigma\mathcal{D}^2=0$, the unique element ${}^\Sigma r\in{}^L\mathcal{W}\otimes{}^\Sigma\Lambda$, $\deg_s({}^\Sigma r)=1$, ${}^\Sigma\delta^{-1}\,{}^\Sigma r=0$, solves (recurrently)

$$ {}^\Sigma\delta r={}^A_\Sigma T+{}^A\mathcal{F}+{}^A\mathcal{D}\,{}^\Sigma r-\frac{i}{v}\,{}^\Sigma r\circ{}^\Sigma r,$$

2–forms: ${}^A_\Sigma T$, curvature ${}^A\mathcal{F}$ for ${}^A\mathcal{D}=\{\mathbf{A}^{I'}_I\}$.

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$\forall$ symplectic $(M,\theta)\ \exists$ a formal cohomology class

$$ cl({}^\Sigma*)\in(1/iv)[\theta]+H^2(M,\mathbb{C})[[v]]$$
$$ =(1/iv)[\theta]+c_0({}^\Sigma*)+\ldots$$

Most intriguing is $c_0({}^\Sigma*)$ characterizing DQ

The zero–degree cohomology for Einstein spaces

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$$ {}_1\mathcal{R}^\tau{}_\gamma={}_1R^\tau{}_{\gamma\alpha\beta}\,\mathbf{e}^\alpha\wedge\mathbf{e}^\beta,$$
$$ \widehat{\mathcal{R}}^\tau{}_\gamma=\widehat{\mathbf{R}}^\tau{}_{\gamma\alpha\beta}\,\mathbf{e}^\alpha\wedge\mathbf{e}^\beta,\quad {}^A\mathcal{R}^\tau{}_\gamma={}^A\mathbf{R}^\tau{}_{\gamma\alpha\beta}\,\mathbf{e}^\alpha\wedge\mathbf{e}^\beta,$$
$$ {}^A\mathcal{R}^\tau{}_\gamma={}_1\mathcal{R}^\tau{}_\gamma+{}_1^A\mathcal{Z}^\tau{}_\gamma,\ \widehat{\mathcal{R}}^\tau{}_\gamma={}_1\mathcal{R}^\tau{}_\gamma+{}_1\widehat{\mathcal{Z}}^\tau{}_\gamma,$$

$$\epsilon_{\alpha\beta\gamma\tau}\left(\mathbf{e}^\alpha\wedge{}^A\mathcal{R}^{\beta\gamma}+\lambda\mathbf{e}^\alpha\wedge\mathbf{e}^\beta\wedge\mathbf{e}^\gamma\right)=8\pi G\,{}^A_\Sigma T_\tau$$
$$ {}^A_\Sigma T_\tau={}^m_\Sigma T_\tau+{}^A_Z T_\tau,$$

consider ${}^{L}\mathcal{W}\otimes{}^{\Sigma}\mathbf{\Lambda}$ for $\mathbf{\Lambda}^{IJ}$.

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$$\begin{aligned} &{}_{|}\mathcal{R}^{\tau}{}_{\gamma} = {}_{|}R^{\tau}{}_{\gamma\alpha\beta}\,\mathbf{e}^{\alpha}\wedge\mathbf{e}^{\beta}, \\ &\widehat{\mathcal{R}}^{\tau}{}_{\gamma} = \widehat{\mathbf{R}}^{\tau}{}_{\gamma\alpha\beta}\,\mathbf{e}^{\alpha}\wedge\mathbf{e}^{\beta},\quad {}^{A}\mathcal{R}^{\tau}{}_{\gamma} = {}^{A}\mathbf{R}^{\tau}{}_{\gamma\alpha\beta}\,\mathbf{e}^{\alpha}\wedge\mathbf{e}^{\beta}, \\ &{}^{A}\mathcal{R}^{\tau}{}_{\gamma} = {}_{|}\mathcal{R}^{\tau}{}_{\gamma} + {}^{A}_{|}\mathcal{Z}^{\tau}{}_{\gamma},\ \widehat{\mathcal{R}}^{\tau}{}_{\gamma} = {}_{|}\mathcal{R}^{\tau}{}_{\gamma} + {}_{|}\widehat{\mathcal{Z}}^{\tau}{}_{\gamma}, \end{aligned}$$

$$\epsilon_{\alpha\beta\gamma\tau}\left(\mathbf{e}^{\alpha}\wedge{}^{A}\mathcal{R}^{\beta\gamma} + \lambda\mathbf{e}^{\alpha}\wedge\mathbf{e}^{\beta}\wedge\mathbf{e}^{\gamma}\right) = 8\pi G\,{}^{A}T_{\tau},$$

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$${}^{m}T_{\tau} = {}^{m}\mathbf{T}^{\alpha}_{\tau}\epsilon_{\underline{\alpha\beta\gamma\delta}}du^{\underline{\beta}}\wedge du^{\underline{\gamma}}\wedge du^{\underline{\delta}},$$

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$$ \widehat{\mathcal{R}}^{\tau}{}_{\gamma} = \widehat{\mathbf{R}}^{\tau}{}_{\gamma\alpha\beta}\,\mathbf{e}^{\alpha} \wedge \mathbf{e}^{\beta}, \quad {}^{A}\mathcal{R}^{\tau}{}_{\gamma} = {}^{A}\mathbf{R}^{\tau}{}_{\gamma\alpha\beta}\,\mathbf{e}^{\alpha} \wedge \mathbf{e}^{\beta}, $$
$$ {}^{A}\mathcal{R}^{\tau}{}_{\gamma} = {}_{|}\widehat{\mathcal{R}}^{\tau}{}_{\gamma} + {}_{|}^{A}\mathcal{Z}^{\tau}{}_{\gamma}, \quad \widehat{\mathcal{R}}^{\tau}{}_{\gamma} = {}_{|}\mathcal{R}^{\tau}{}_{\gamma} + {}_{|}\widehat{\mathcal{Z}}^{\tau}{}_{\gamma}, $$

$$ \epsilon_{\alpha\beta\gamma\tau}\left(\mathbf{e}^{\alpha} \wedge {}^{A}\mathcal{R}^{\beta\gamma} + \lambda\mathbf{e}^{\alpha} \wedge \mathbf{e}^{\beta} \wedge \mathbf{e}^{\gamma}\right) = 8\pi G\,{}^{A}T_{\tau}, $$
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$$ {}^{m}T_{\tau} = {}^{m}\mathbf{T}^{\alpha}{}_{\tau}\epsilon_{\underline{\alpha\beta\gamma\delta}}du^{\underline{\beta}} \wedge du^{\underline{\gamma}} \wedge du^{\underline{\delta}}, $$
$$ {}^{A}_{Z}T_{\tau} = (8\pi G)^{-1}\,{}_{|}^{A}\mathcal{Z}^{\alpha}{}_{\tau}\epsilon_{\underline{\alpha\beta\gamma\delta}}du^{\underline{\beta}} \wedge du^{\underline{\gamma}} \wedge du^{\underline{\delta}}, $$

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Discussion and Conclusions

1. **Difference and Similarity:** LQG and DQ of GR are performed following different geometric methods, common splitting "philosophy". In both cases, "conservative" particle physicists not yet satisfied by approaches but math schemes are rigorous.
2. **Equivalence:** For generic nonlinear theories, it is not a trivial task to state the conditions when two different methods of quantization are equivalent: we provided a strong mutual support for both schemes.
3. **Poincare geometry–theory dualism:** The real physical world of classical and quantum gravity can be differently encoded into "catalogs" by different "librarians" / mathematical languages. Dictionary?
4. **Complimentary character:** A set of important physical problems treated differently: quantum gravitational eqs.
5. **Applications** (speculation)
   LQG methods for nonlinear systems: model verifications
   LQG & DQ methods for analogous gravity
   LQG, noncommutative geometry, nonsymmetric metrics, nonholonomic Ricci flows, even nonperturbative methods for string theory.

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N–adapted 3+1 splitting = "double" $(2+2)_L$ & 3+1

V is topologically ${}^3\Sigma\times\mathbb{R}$, $\mathbf{t}^\alpha = {}_l\mathbf{N}\mathbf{n}^\alpha + {}_s\mathbf{N}^\alpha$,
lapse function ${}_l\mathbf{N}$, shift d–vector ${}_s\mathbf{N}^\alpha$,
time–evolution d–vector $\mathbf{t}^\alpha$, $\mathbf{t}^\alpha(d\mathbf{t})_\alpha = 1$; the unit normal d–vector $\mathbf{n}^\alpha$ of ${}^3\Sigma$, $\eta_{\alpha'\beta'}\mathbf{n}^{\alpha'}\mathbf{n}^{\beta'} = -1$ for $\mathbf{n}_{\alpha'} \equiv \mathbf{n}_\alpha \mathbf{e}^\alpha{}_{\alpha'}$.
Indices $\alpha', \beta', ...$ are internal, $\alpha' = (0', I')$, $\beta' = (0', J')$, ...
$I', J', ... = 1, 2, 3$ in the internal space on ${}^3\Sigma$.

Transf. $\mathbf{e}_\alpha = \mathbf{e}_\alpha^{\underline{\alpha}}\partial_{\underline{\alpha}}$, $\mathbf{e}_\alpha^{\underline{\alpha}}(u) = \begin{bmatrix} e_i^{\underline{i}}(u) & {}^L N_i^b(u) e_b^{\underline{a}}(u) \\ 0 & e_a^{\underline{a}}(u) \end{bmatrix}$,

${}^L\underline{\mathbf{g}}_{\alpha\beta} = \mathbf{e}_\alpha^{\alpha'}\mathbf{e}_\beta^{\beta'}\eta_{\alpha'\beta'} = \mathbf{e}_\alpha^{\underline{\alpha}}\mathbf{e}_\beta^{\underline{\beta}}\underline{\mathbf{g}}_{\underline{\alpha}\underline{\beta}}$, for $\mathbf{e}_\alpha = (\mathbf{e}_0, \mathbf{e}_I)$
$\eta_{\alpha'\beta'} = diag[-1, 1, 1, 1]$, ${}^L\mathbf{g} = {}^L\underline{\mathbf{g}}_{\alpha\beta}(u)\, du^\alpha \otimes du^\beta$,

${}^L\underline{\mathbf{g}}_{\alpha\beta} = \begin{bmatrix} {}^L g_{ij} + {}^L N_i^a\, {}^L N_j^b\, {}^L g_{ab} & {}^L N_j^e\, {}^L g_{ae} \\ {}^L N_i^e\, {}^L g_{be} & {}^L g_{ab} \end{bmatrix}$.

3d. metric ${}^L\mathbf{q}_{IJ} = \mathbf{e}_I^{I'}\mathbf{e}_J^{J'}\delta_{I'J'}$, $\mathbf{e}_I^{I'} \doteqdot \mathbf{e}_\alpha^{\alpha'} q_{\alpha'}^{I'} q_I^\alpha$,
the internal and spacetime projection maps, q_I^α and $q_{\alpha'}^{I'}$,
$SO(1,3) \to SO(3)$, invariant $\mathbf{n}^{\alpha'}$.

$so(1,3)$–valued $\Gamma^{\alpha'}{}_{\gamma'} = \Gamma^{\alpha'}{}_{\gamma'\beta'}\mathbf{e}^{\beta'} \Rightarrow$ two $so(3)$–valued
$\Gamma^{I'}{}_I \doteqdot \frac{1}{2} q_I^\alpha q_{\alpha'}^{I'} \epsilon^{\alpha'\beta'}{}_{\gamma'\tau'}\mathbf{n}_{\beta'}\Gamma^{\gamma'\tau'}{}_\alpha$ and $K^{I'}{}_I \doteqdot q_{\alpha'}^{I'} q_I^\alpha \mathbf{n}_{\beta'}\Gamma^{\alpha'\beta'}{}_\alpha$,
the spin connection and extrinsic curvature on shell.

LQG is formulated for $({}_l\Gamma^{I'}{}_I, {}_l K^{I'}{}_I)$, induced by
$\Gamma^\gamma{}_{\alpha\beta} = {}_l\Gamma^\gamma{}_{\alpha\beta}(\mathbf{g})$.

S. Mercuri, From the Einstein–Cartan to the Ashtekar–Barbero canonical constraints, passing through the Nieh–Yan functional, 0708.0037

Ashtekar–Barbero d–connections

Classical phase space and Hamilton formalism for
$({}_l A^{I'}{}_I, \widetilde{E}_{I'}{}^I)$, the configuration and conjugate momentum

$$ {}_l A^{I'}{}_I \doteqdot {}_l\Gamma^{I'}{}_I + \beta\, {}_l K^{I'}{}_I, \quad \widetilde{E}_{I'}{}^I = (\kappa\beta)^{-1}\sqrt{q}\mathbf{e}^I{}_{I'}, $$

$q \doteqdot \det|\mathbf{q}_{IJ}|$, "tilde" for densities.

Suggestion: generalize to a d–connection ${}^A\mathcal{D} = \{\mathbf{A}^{I'}{}_I\}$,

$$ {}_l A^{I'}{}_I({}^L\mathbf{g}) \to \mathbf{A}^{I'}{}_I({}^L\mathbf{g}) = {}_l A^{I'}{}_I({}^L\mathbf{g}) + {}_l^A\mathbf{Z}^{I'}{}_I({}^L\mathbf{g}), $$

$$ {}_l^A\mathbf{Z}^{I'}{}_I \doteqdot q_I^\alpha q_{\alpha'}^{I'}\mathbf{n}_{\beta'}\left(\frac{1}{2}\epsilon^{\alpha'\beta'}{}_{\gamma'\tau'}\ {}_l^A\mathbf{Z}^{\gamma'\tau'}{}_\alpha + \beta\ {}_l^A\mathbf{Z}^{\alpha'\beta'}{}_\alpha\right) $$

$$ {}_l^A\mathbf{Z}^\alpha_{\beta\gamma} = \frac{1}{8}({}^J\Omega^\alpha_{\beta\gamma} - {}^L\mathbf{g}^{\alpha\tau}{}^L\mathbf{g}_{\beta\varepsilon}\ {}^J\Omega^\varepsilon_{\tau\gamma}), $$

$$ {}^J\Omega^\gamma_{\alpha\beta} = (\mathbf{e}_\tau \mathbf{J}^\gamma{}_\alpha)\mathbf{J}^\tau{}_\beta - (\mathbf{e}_\tau \mathbf{J}^\gamma{}_\beta)\mathbf{J}^\tau{}_\alpha + (\mathbf{e}_\alpha \mathbf{J}^\tau{}_\beta - \mathbf{e}_\beta \mathbf{J}^\tau{}_\alpha)\mathbf{J}^\gamma{}_\tau $$

Ashtekar d–connection $\mathbf{A}^{I'}{}_I$, or the nonholonomic deformation of the Ashtekar–Barbero connection ${}_l A^{I'}{}_I$

$$ 4\ {}^A\mathbf{T}^{\alpha'}_{\alpha\beta} = -\ {}^J\Omega^{\alpha'}_{\alpha\beta} \quad \text{for} \quad {}^A\mathcal{D}_{[\alpha}\mathbf{e}_{\beta]}^{\alpha'} = {}^A\mathbf{T}^{\alpha'}_{\alpha\beta}. $$

$$ 4\ {}^A\mathbf{T}^{I'}{}_I = -\ {}^J\Omega^{I'}{}_I, \ \text{torsion of } \mathbf{A}^{I'}{}_I. $$

$\exists\ {}^A\mathbf{\Gamma}^\alpha_{\beta\gamma}({}^L\mathbf{g})$, or ${}^A\mathbf{\Gamma}^{I'}{}_I \doteqdot \frac{1}{2} q_I^\alpha q_{\alpha'}^{I'}\epsilon^{\alpha'\beta'}{}_{\gamma'\tau'}\mathbf{n}_{\beta'}\ {}^A\mathbf{\Gamma}^{\gamma'\tau'}{}_\alpha$,
${}^A\mathbf{K}^{I'}{}_I \doteqdot q_{\alpha'}^{I'} q_I^\alpha \mathbf{n}_{\beta'}\ {}^A\mathbf{\Gamma}^{\alpha'\beta'}{}_\alpha$.

May perform DQ of GR in nonholonomic Ashtekar variables, or LQG with induced torsion.

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The zero–degree cohomology for Einstein spaces

$\exists$ another canonical class ${}^{\Sigma}\varepsilon$ for ${}^{N}T\,{}^{3}\Sigma = h\,{}^{3}\Sigma \oplus v\,{}^{3}\Sigma$ related to Chern–Weyl form ${}^{\Sigma}\gamma = -\frac{1}{4}\mathbf{J}^{I}_{K'}\,\mathbf{F}^{K'}_{IJ} \wedge \mathbf{e}^{J}$,

$$ c_0({}^{\Sigma}*) = -(1/2i)\,{}^{\Sigma}\varepsilon, \quad {}^{\Sigma}\varepsilon \doteqdot [\,{}^{\Sigma}\gamma]. $$

4d, closed Chern–Weyl form $\varepsilon \doteqdot [\gamma]$, $4\gamma = -\mathbf{J}_{\tau}^{\alpha'}\,{}^{A}\mathcal{R}^{\tau}{}_{\alpha'}$

$$ \mathbf{e}^{\alpha} \wedge \gamma = \frac{1}{4}\mathbf{J}_{\tau\beta}\mathbf{e}^{\alpha} \wedge {}^{A}\mathcal{R}^{\beta\tau} $$

3–form Einstein equations

Deformations of Einstein tensor for ${}^{A}\mathbf{\Gamma}^{\alpha}_{\beta\gamma}({}^{L}\mathbf{g}) = \widehat{\Gamma}^{\alpha}_{\beta\gamma}({}^{L}\mathbf{g}) + \widehat{\mathbf{Z}}^{\alpha}_{\beta\gamma}({}^{L}\mathbf{g}) = {}_{|}\Gamma^{\alpha}_{\beta\gamma}({}^{L}\mathbf{g}) + {}_{|}^{A}\mathbf{Z}^{\alpha}_{\beta\gamma}({}^{L}\mathbf{g})$,

defining deformations of curvatures

$$ {}_{|}\mathcal{R}^{\tau}{}_{\gamma} = {}_{|}R^{\tau}{}_{\gamma\alpha\beta}\,\mathbf{e}^{\alpha} \wedge \mathbf{e}^{\beta}, $$
$$ \widehat{\mathcal{R}}^{\tau}{}_{\gamma} = \widehat{\mathbf{R}}^{\tau}{}_{\gamma\alpha\beta}\,\mathbf{e}^{\alpha} \wedge \mathbf{e}^{\beta}, \quad {}^{A}\mathcal{R}^{\tau}{}_{\gamma} = {}^{A}\mathbf{R}^{\tau}{}_{\gamma\alpha\beta}\,\mathbf{e}^{\alpha} \wedge \mathbf{e}^{\beta}, $$
$$ {}^{A}\mathcal{R}^{\tau}{}_{\gamma} = {}_{|}\mathcal{R}^{\tau}{}_{\gamma} + {}_{|}^{A}\mathcal{Z}^{\tau}{}_{\gamma}, \quad \widehat{\mathcal{R}}^{\tau}{}_{\gamma} = {}_{|}\mathcal{R}^{\tau}{}_{\gamma} + {}_{|}\widehat{\mathcal{Z}}^{\tau}{}_{\gamma}, $$

$$ \epsilon_{\alpha\beta\gamma\tau}\left(\mathbf{e}^{\alpha} \wedge {}^{A}\mathcal{R}^{\beta\gamma} + \lambda\mathbf{e}^{\alpha} \wedge \mathbf{e}^{\beta} \wedge \mathbf{e}^{\gamma}\right) = 8\pi G\,{}^{A}T_{\tau}, $$
$$ {}^{A}T_{\tau} = {}^{m}T_{\tau} + {}^{A}_{Z}T_{\tau}, $$
$$ {}^{m}T_{\tau} = {}^{m}\mathbf{T}^{\alpha}_{\tau}\epsilon_{\underline{\alpha\beta\gamma\delta}}du^{\underline{\beta}} \wedge du^{\underline{\gamma}} \wedge du^{\underline{\delta}}, $$
$$ {}^{A}_{Z}T_{\tau} = (8\pi G)^{-1}\,{}_{|}^{A}\mathcal{Z}^{\alpha}_{\tau}\epsilon_{\underline{\alpha\beta\gamma\delta}}du^{\underline{\beta}} \wedge du^{\underline{\gamma}} \wedge du^{\underline{\delta}}, $$