

Title: Playing the quantum harp: from quantum metrology to quantum computing with harmonic oscillators

Date: Feb 06, 2008 04:00 PM

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Abstract: The \"frequency comb\" defined by the eigenmodes of an optical resonator is a naturally large set of exquisitely well defined quantum systems, such as in the broadband mode-locked lasers which have redefined time/frequency metrology and ultra precise measurements in recent years. High coherence can therefore be expected in the quantum version of the frequency comb, in which nonlinear interactions couple different cavity modes, as can be modeled by different forms of graph states. We show that it is possible to thereby generate states of interest to quantum metrology and computing, such as multipartite entangled cluster and Greenberger-Horne-Zeilinger states.

Playing the quantum harp



From quantum metrology to quantum
computing with harmonic oscillators

Olivier Pfister

University of Virginia

<http://faculty.virginia.edu/quantum/>

SUPPORT:



ARO, ONR

Playing the quantum harp



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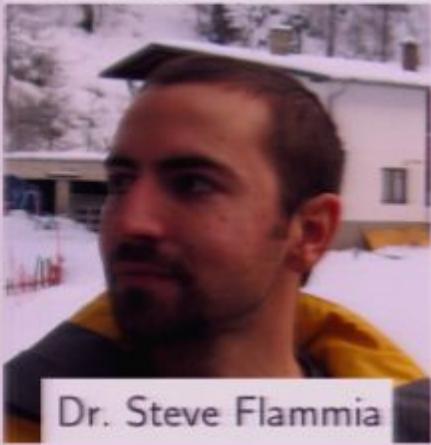
Perimeter Institute for Theoretical Physics, Feb 6, 2008



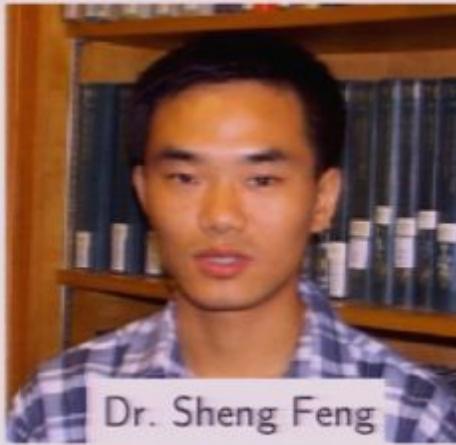
Nick Menicucci



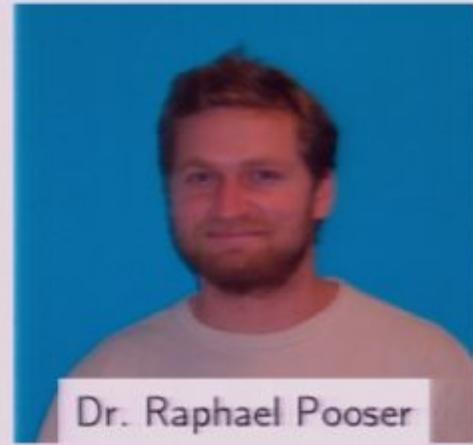
Dr. Jietai Jing



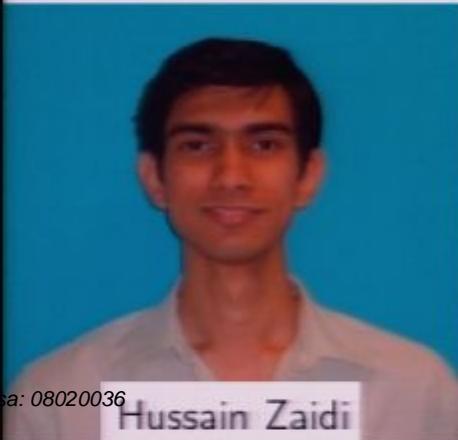
Dr. Steve Flammia



Dr. Sheng Feng



Dr. Raphael Pooser



Pinsa: 08020036

Hussain Zaidi



Penmeyer

Russell Bloomer

2006



Matt Pysher

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The usual
suspects...

*The optical cavity:
A natural set of quantum strings...*



The optical cavity: A natural set of quantum strings...

The eigenmodes of a cavity form a naturally scaled ensemble of classically coherent modes



The optical cavity: A natural set of quantum strings...

The eigenmodes of a cavity form a naturally scaled ensemble of classically coherent modes



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Properly stabilized mode-locked laser = optical frequency comb
(10^6 modes oscillating in phase)

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The eigenmodes of a cavity form a naturally scaled ensemble of classically coherent modes



Properly stabilized mode-locked laser = optical frequency comb
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Jan
Hall



Registered trademark of the Nobel Foundation

Ted
Hänsch



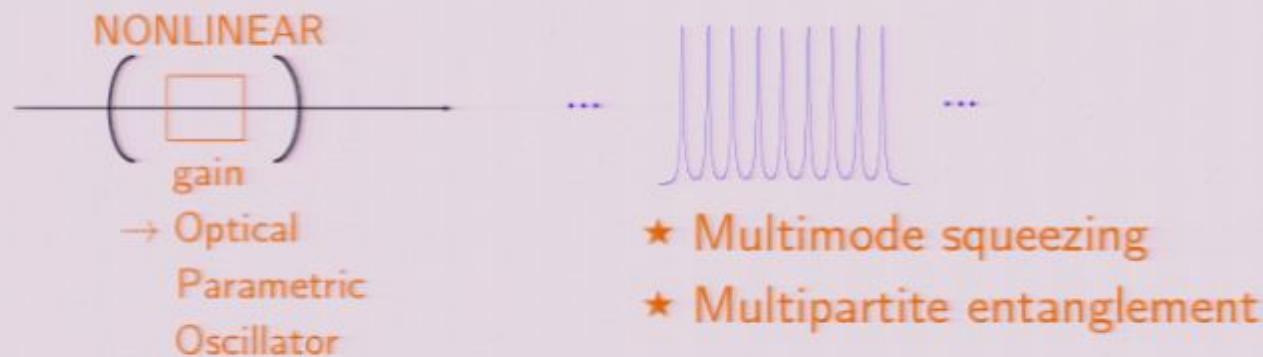
... why not turn it into a quantum register?



... why not turn it into a quantum register?



... why not turn it into a quantum register?



- Continuous-variable states: quadrature operators
- Concrete experimental implementations enabled by nonlinear photonic crystals
- Large scalability potential: theoretical results

Why CV ?

- Quantum harmonic oscillators, hence natural in quantum optics, with first- (linear) and second-order (nonlinear) interactions
- We use ultrastable **phase-locked** OPO above threshold
 - Macroscopic Hong-Ou-Mandel interference (Feng & Pfister, *PRL* 2004)
 - Bipartite CV entanglement (Jing, Feng, Bloomer, & Pfister, *PRA* 2006)
 - Proposal for CV GHZ state in single OPO by multimode squeezing (Pfister Feng, Jennings, Pooser, & Xie, *PRA* 2004)
- **Large-scale integration** battle plan: use
 - optical frequency comb
 - quasiperiodic quasiphase matching (Lifshitz, Arie, Bahabad, *PRL* 2005)

Continuous-variable entanglement

$$\text{EPR state: } \int_{-\infty}^{+\infty} |x\rangle_1 |x\rangle_2 dx = \int_{-\infty}^{+\infty} |p\rangle_1 | -p\rangle_2 dp = \sum_0^{\infty} |n\rangle_1 |n\rangle_2$$

Einstein, Podolsky, and Rosen, PR 47, 777 (1935)

van Enk, PRA 60, 5095 (1999)

(Schmidt basis)

$$X = a + a^\dagger$$

amplitude (position)

$$P = -i(a - a^\dagger)$$

phase (momentum)

$$N = a^\dagger a$$

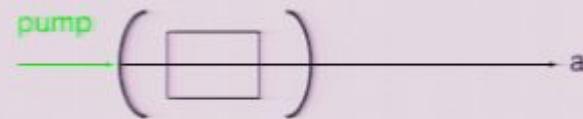
intensity (energy)

$$[X, P] = 2\mathbb{I}$$
$$\Delta X \Delta P \geq 1$$

Natural implementation in quantum optical oscillators
by way of nonlinear (squeezing) interactions

Nonlinear Hamiltonian \rightarrow squeezing

Single degenerate OPO



$$H = i\hbar\chi\beta(a^{\dagger 2} - a^2)$$

$$|out\rangle = \frac{1}{\cosh(\beta\chi t)} \sum_{n=0}^{\infty} \tanh^2(\beta\chi t) |2n\rangle$$

$$\begin{cases} \Delta X = e^{\beta\chi t} \\ \Delta P = e^{-\beta\chi t} \end{cases}$$

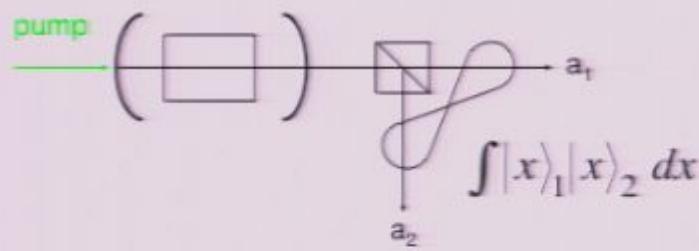
$$\lim_{(\chi\beta t) \rightarrow \infty} |out\rangle = |p\rangle$$

Theory: Walls *Nature* 1983
Expt: Slusher, Hollberg, Yurke, Mertz, Valley, *PRL* 1985
Wu, Kimble, Hall, Wu, *PRL* 1986

Two-mode squeezing

→ bipartite CV entanglement

Single nondegenerate OPO



$$H_{12} = i\hbar\chi\beta(a_1^\dagger a_2^\dagger - H.c.)$$

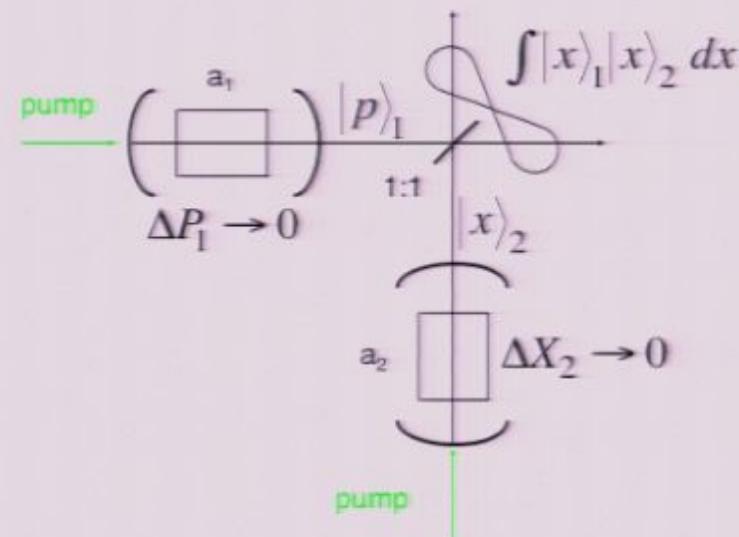
$$\Delta(X_1 - X_2) \rightarrow 0$$

$$\Delta(P_1 + P_2) \rightarrow 0$$

Theory: Reid & Drummond *PRL* 1988,
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OPA Exp: Ou, Pereira, Kimble, Peng, *PRL* 1992
QPO Exp: Jing, Feng, Bloomer, Pfister, *PRA* 2006

Two degenerate OPAs interfering
at a balanced beam splitter



$$H_1 + H_2 = i\hbar\chi\beta(a_1^{\dagger 2} - H.c.) \ominus i\hbar\chi\beta(a_2^{\dagger 2} - H.c.)$$

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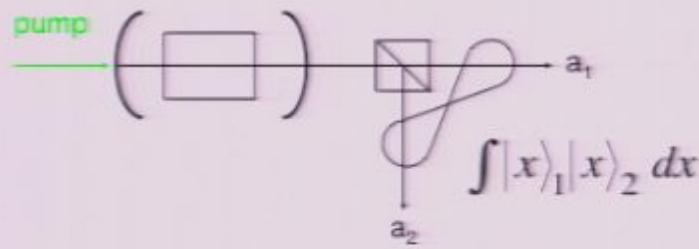
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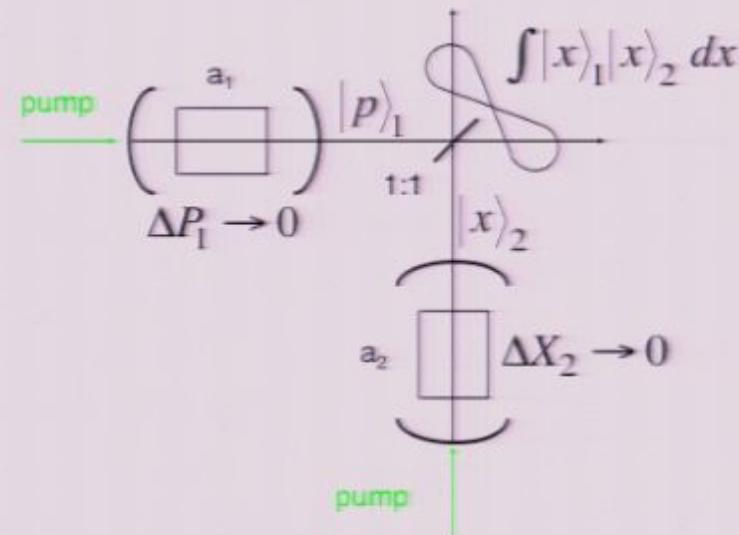
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CV cluster states

(Lloyd & Braunstein, *PRL* 1999)

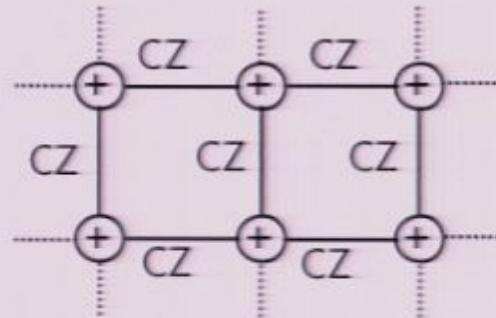
Qbits

$$\{|0\rangle, |1\rangle\}$$

$$|0\rangle|0\rangle + |1\rangle|1\rangle$$

$$\{|+\rangle, |-\rangle\}$$

Pauli group



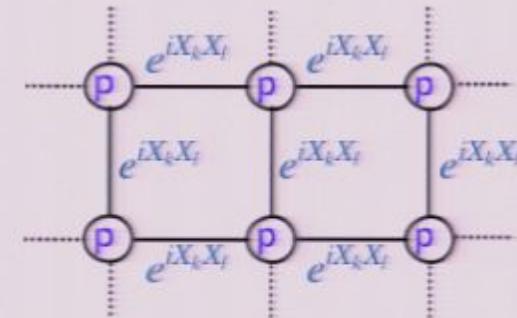
Qnats

$$\{|x\rangle\}_x$$

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Weyl-Heisenberg group



(Zhang & Braunstein, *PRA* 2006)

(Menicucci, van Loock, Gu, Weedbrook, Ralph, & Nielsen, *PRL* 2006)

How to make N -node CV clusters

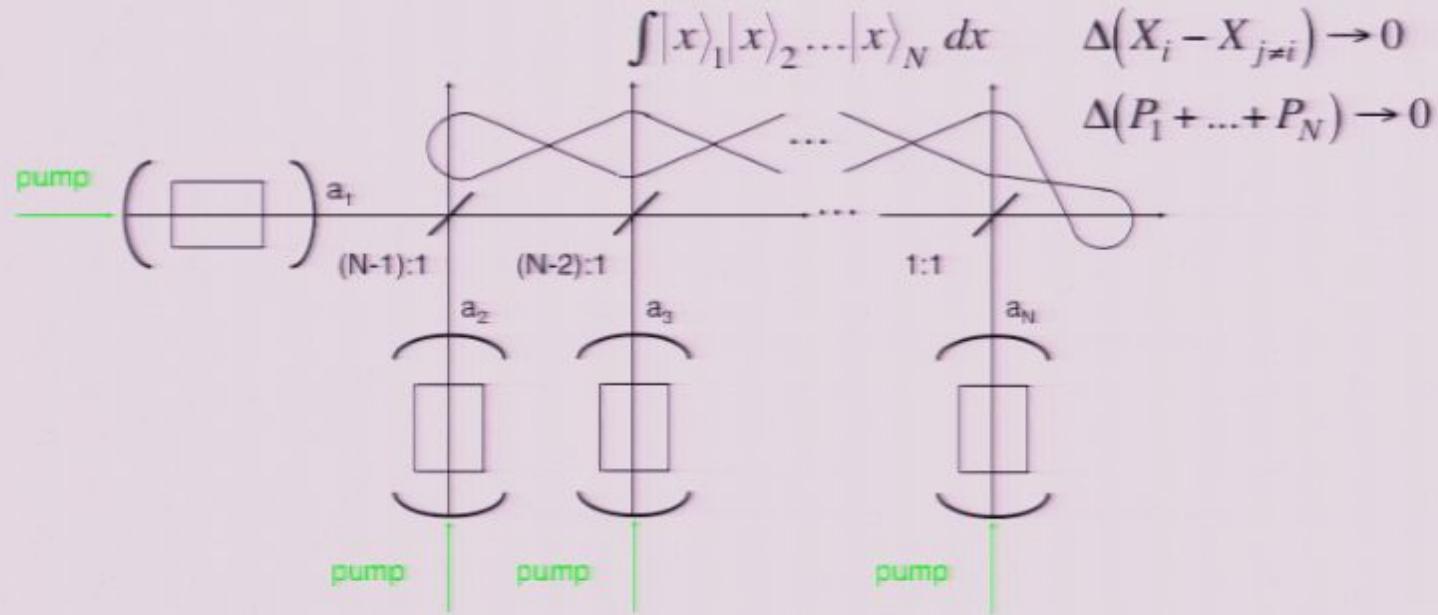
At least 3 methods to produce N -mode CV GHZ and cluster states:

1. Original correspondence:
(Zhang & Braunstein, *PRA* 2006)
 $| N \text{ vacuum squeezers and}$
 $| O(N) \text{ inline squeezers (QND gates)}$
2. Bloch-Messiah reduced:
(van Loock & Braunstein, *PRL* 2000)
(Braunstein, *PRA* 2005)
(van Loock et al., *quant-ph/0610119*)
 $| N \text{ vacuum (offline) squeezers and}$
 $| O(N^p) \text{ beam splitters}$
3. This proposal:
(Pfister et al., *PRA* 2004)
(Menicucci, Flammia, Zaidi, Pfister, *PRA* 2007)
 $| 1 \text{ multimode squeezer with few pumps}$
 $| NO \text{ beamsplitters}$

$$\int dx |x\rangle_1 |x\rangle_2$$

Multimode squeezing → multipartite CV entanglement?

Yes! N degenerate OPA's interfering at an N -beam splitter

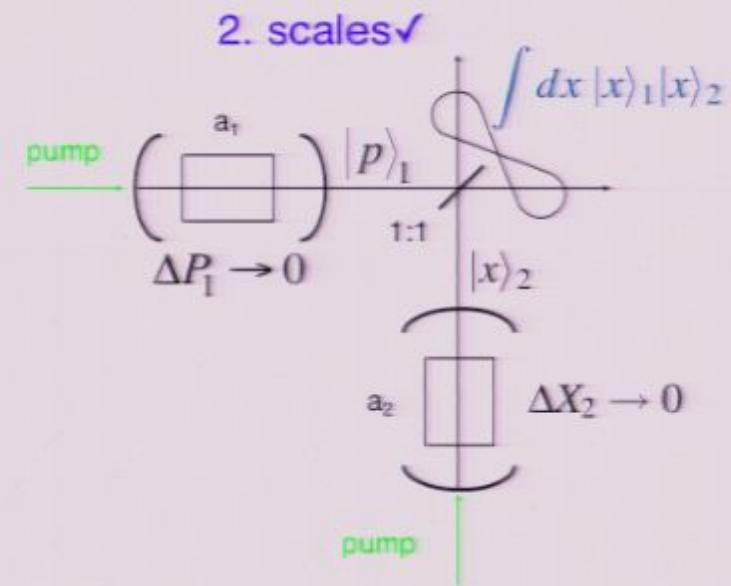
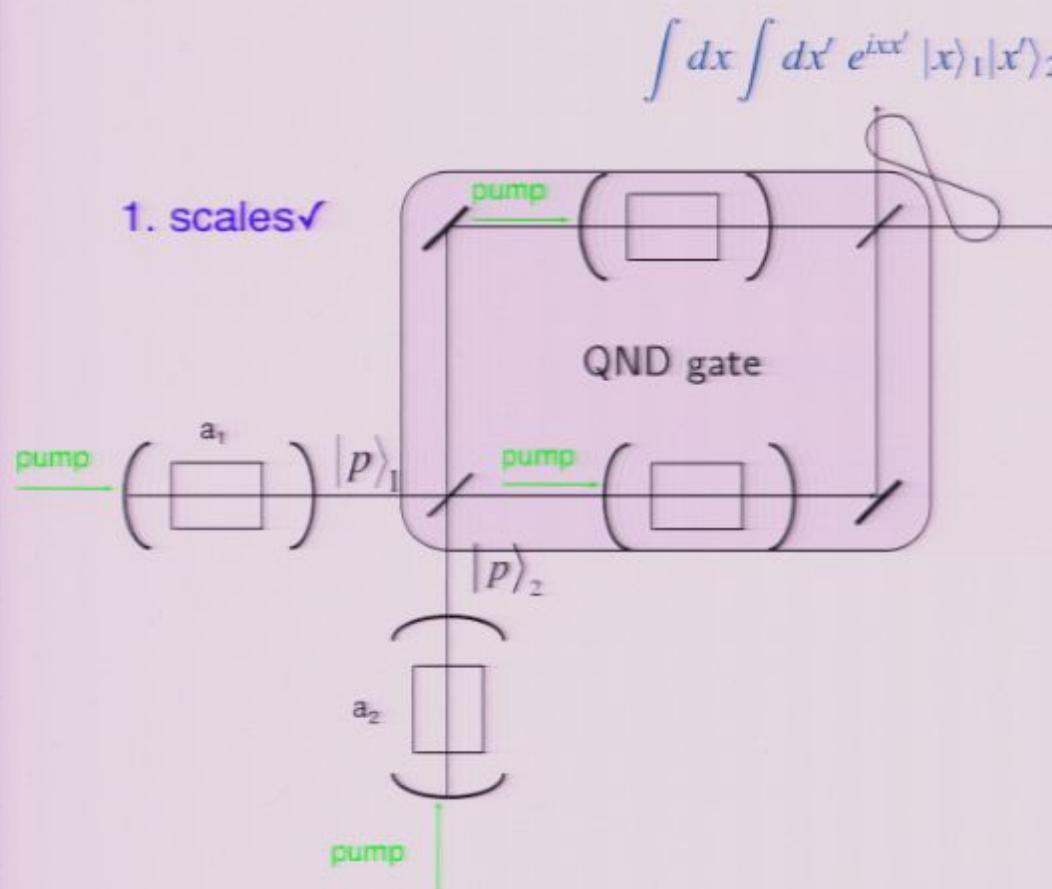


Theory: van Loock & Braunstein *PRL* 84, 3482 (2000)

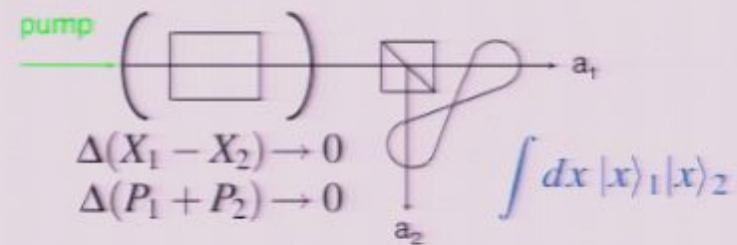
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2-node CV clusters \equiv EPR

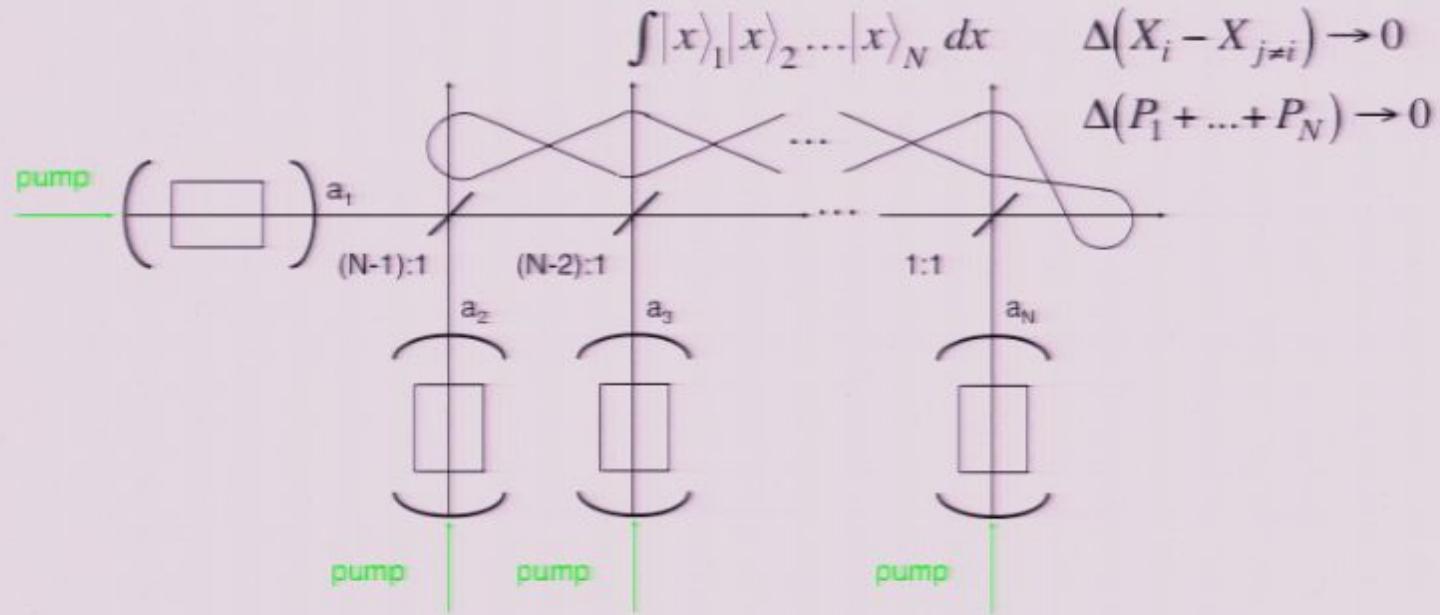


3. scales???



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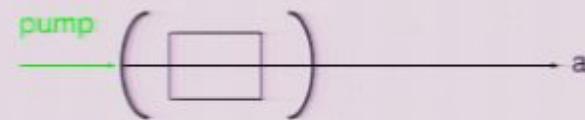


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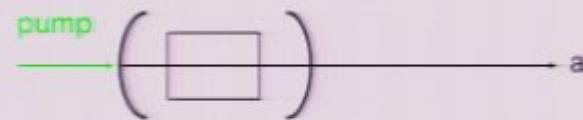
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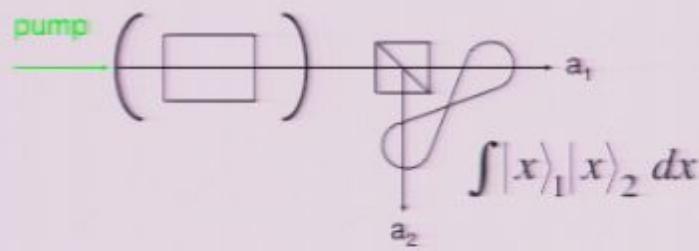
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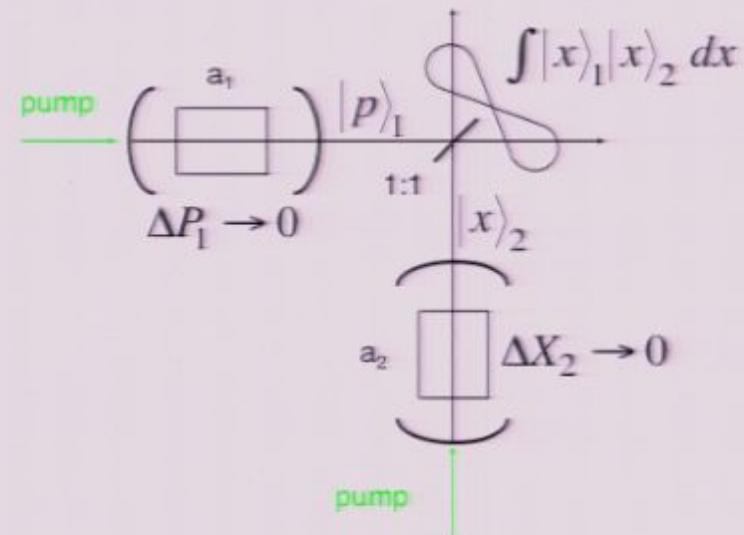
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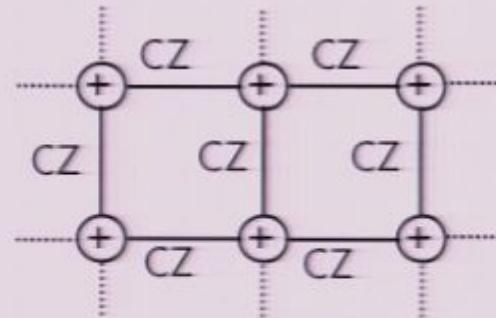
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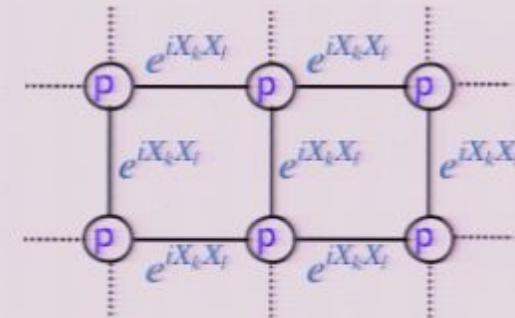
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infinitely squeezed
position eigenstates
finite squeezing OK

CV cluster states

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Qnats

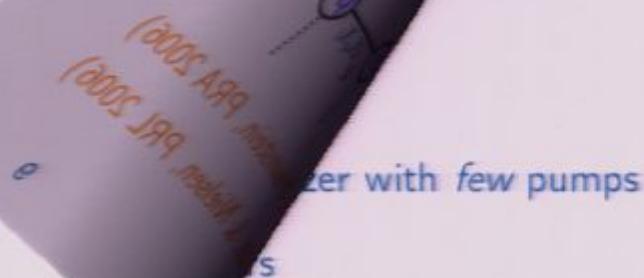
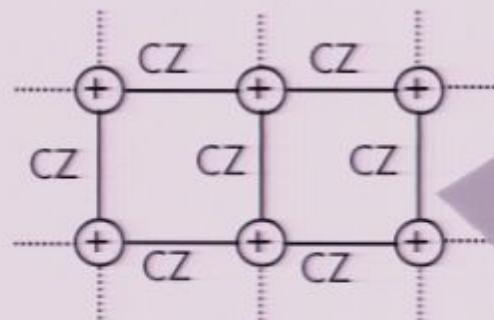
$$\{|x\rangle\}_x$$

infinitely squeezed
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$$\int |x\rangle|x\rangle dx$$

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Pauli group



(Menicucci, van Loock

Single OPO → multipartite CV entanglement?

Single N-beam OPO, no interferometer???
YES!!!

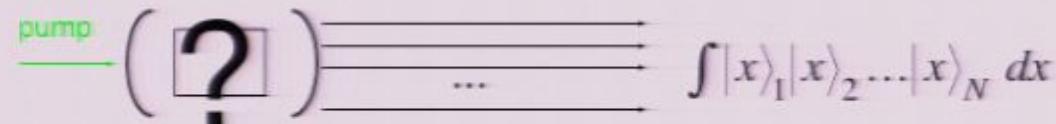
The diagram illustrates a process flow. A green arrow labeled "pump" points to a large black question mark enclosed in a rectangular frame. Three horizontal arrows point from the question mark to the right. To the right of these arrows is the mathematical expression $\int |x\rangle_1 |x\rangle_2 \dots |x\rangle_N dx$.

Theory: Pfister, Feng, Jennings, Pooser, and Xie, *PRA* 2004
Bradley, Olsen, Pfister, and Pooser, *PRA* (in press)

Exp: Pooser and Pfister, *OL* 2005

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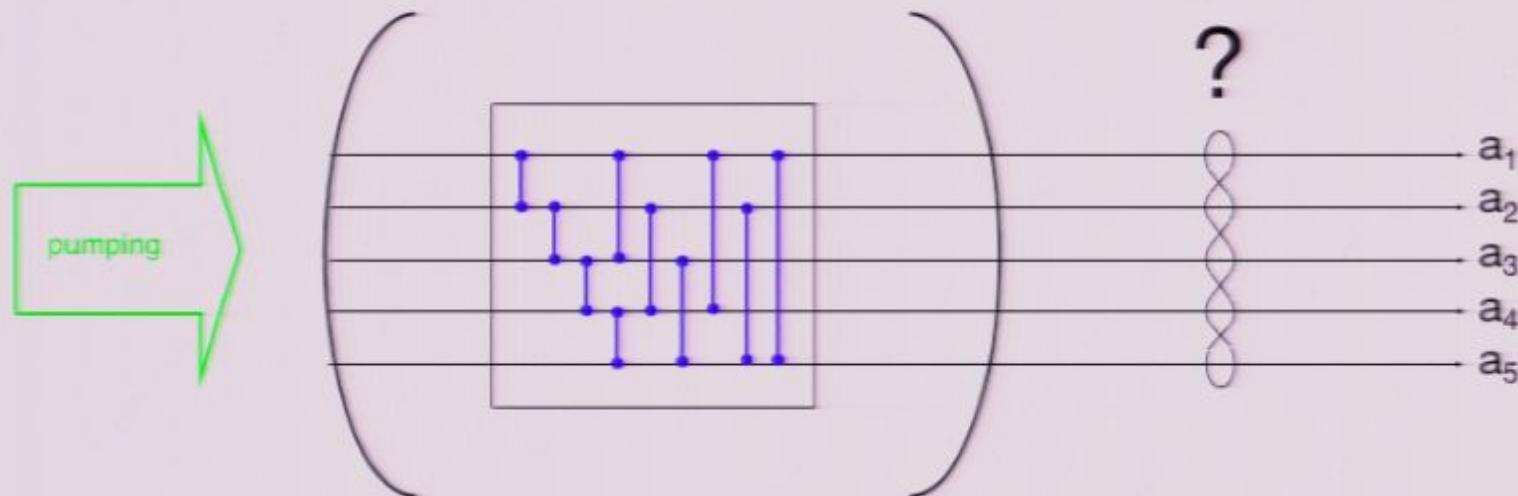


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Single-OPO multifrequency entanglement

- Take advantage of the natural set of quantum modes that is the frequency comb of an optical resonator
- If N modes are nonlinearly coupled to one another, do they evolve into a multipartite entangled state?



Basic equations

Pfister, Feng, Jennings, Pooser, and Xie, *PRA* **70**, 020302 (2004)

Hamiltonian in the interaction picture

$$H_N = \sum_{j < k} H_{jk} = i\hbar\chi\beta \sum_{j < k} (a_j^\dagger a_k^\dagger - H.c.)$$

Heisenberg equations

$$\begin{pmatrix} \dot{a}_1 \\ \vdots \\ \dot{a}_N \end{pmatrix} = \chi\beta \begin{pmatrix} 0 & 1 & \dots & 1 \\ 1 & 0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 1 \\ 1 & \dots & 1 & 0 \end{pmatrix} \begin{pmatrix} a_1 \\ \vdots \\ a_N \end{pmatrix}$$

Solutions

$$\begin{aligned} X_j(t) - X_k(t) &= [X_j(0) - X_k(0)] e^{-\chi\beta t} \\ P_1(t) + \dots + P_N(t) &= [P_1(0) + \dots + P_N(0)] e^{-(N-1)\chi\beta t} \quad \text{e.g., if } \chi \rightarrow \infty, \int |x\rangle_1 |x\rangle_2 \dots |x\rangle_N dx \end{aligned}$$

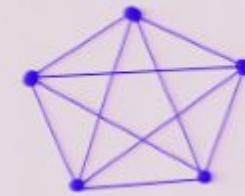
GHZ state

Comparison with van Loock & Braunstein

Us

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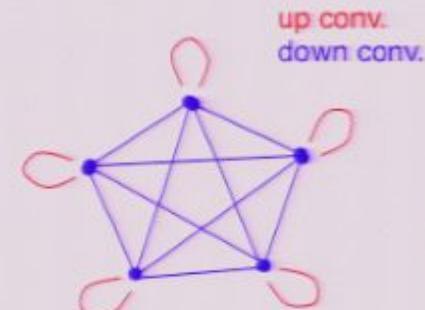


Braunstein *PRA* 2006

van Loock and Braunstein *PRL* 84, 3482 (2000)

$$U_{BS} \left(-H_1 + \sum_{j=2}^N H_j \right) U_{BS}^{-1} = \frac{N-2}{2N} \sum_{j=1}^N H_j \quad \textcircled{-} \quad \frac{2}{N} H_N$$

$$\begin{pmatrix} \dot{a}_1 \\ \vdots \\ \vdots \\ \dot{a}_N \end{pmatrix} = -\frac{\chi\beta}{2N} \begin{pmatrix} -N+2 & 4 & \dots & 4 \\ 4 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 4 \\ 4 & \dots & 4 & -N+2 \end{pmatrix} \begin{pmatrix} a_1 \\ \vdots \\ \vdots \\ a_N \end{pmatrix}$$



vL & B: interference control
Us: down- vs. up-conversion

Bloch-Messiah reduction

Ultracompact N -node CV cluster states ✓

Menicucci, Flammia, Zaidi, Pfister, *PRA* (2007)

Cluster states enable one-way quantum computing. Raussendorf & Briegel, *PRL* (2001)

- For $N=2$, locally equivalent to Bell (EPR) states
- For $N=3$, locally equivalent to GHZ states
- For $N=4$, locally equivalent to neither GHZ nor W states

If we have

$$\left\{ \begin{array}{l} \mathcal{H} = i\hbar \sum_{kl} G_{kl} a_k^\dagger a_l^\dagger + H.c. \quad \text{"H graph"} \\ P_k - \sum_l A_{kl} X_l = 0, \quad \forall k \quad \text{standard cluster} \\ \qquad \qquad \qquad \qquad \qquad \qquad \qquad \text{graph} \end{array} \right.$$

???

Then

where

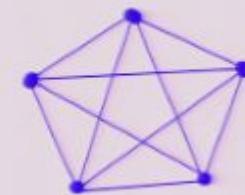
and B, C arbitrary symmetric positive definite matrices

Comparison with van Loock & Braunstein

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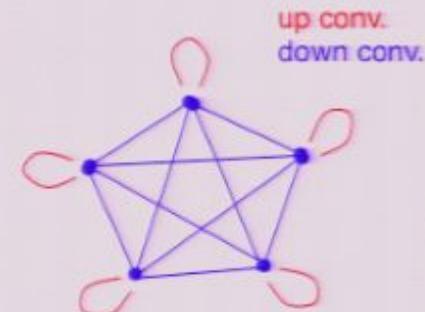


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vL & B: interference control
Us: down- vs. up-conversion

Bloch-Messiah reduction

Ultracompact N -node CV cluster states ✓

Menicucci, Flammia, Zaidi, Pfister, *PRA* (2007)

Cluster states enable one-way quantum computing. Raussendorf & Briegel, *PRL* (2001)

- For $N=2$, locally equivalent to Bell (EPR) states
- For $N=3$, locally equivalent to GHZ states
- For $N=4$, locally equivalent to neither GHZ nor W states

If we have

$$\begin{cases} \mathcal{H} = i\hbar \sum_{kl} G_{kl} a_k^\dagger a_l^\dagger + H.c. & "H \text{ graph}" \\ P_k - \sum_l A_{kl} X_l = 0, \quad \forall k & \text{standard cluster} \\ & \text{graph} \end{cases}$$

???

Then

where

and B, C arbitrary symmetric positive definite matrices

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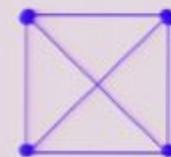
$$G = \begin{pmatrix} I & -A_0 \\ A_0^T & I \end{pmatrix} \begin{pmatrix} B & 0 \\ 0 & -C \end{pmatrix} \begin{pmatrix} I & A_0 \\ -A_0^T & I \end{pmatrix} = \begin{pmatrix} [B - A_0 C A_0^T] [B A_0 + A_0 C] \\ [C A_0^T + A_0^T B] [A_0^T B A_0 - C] \end{pmatrix}$$

where $A = \begin{pmatrix} 0 & A_0 \\ A_0^T & 0 \end{pmatrix}$ and B, C arbitrary symmetric positive definite matrices

Example 1: GHZ state

- Complete TMS graph on 4 modes (Pfister et al., *PRA* 2004)

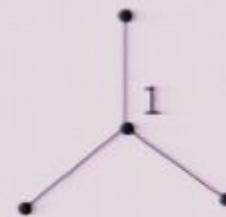
$$\left\{ \begin{array}{l} X_1 + X_2 + X_3 + X_4 \rightarrow 0 \\ P_1 - P_2 \rightarrow 0 \\ P_1 - P_3 \rightarrow 0 \\ P_1 - P_4 \rightarrow 0 \end{array} \right.$$



"H graph"

- Star cluster graph. Phase-shift 1 by $-\pi/2$

$$\left\{ \begin{array}{l} -P_1 + X_2 + X_3 + X_4 \rightarrow 0 \\ X_1 - P_2 \rightarrow 0 \\ X_1 - P_3 \rightarrow 0 \\ X_1 - P_4 \rightarrow 0 \end{array} \right.$$



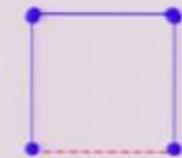
standard graph

- Reminiscent of LC-equivalence of Qbit graphs under local complementation!
(Hein, Dür, Eisert, Raussendorf, Van den Nest, and Briegel, [quant-ph/0602096](https://arxiv.org/abs/quant-ph/0602096))

Example 2: cluster state

- Square TMS weighted graph on 4 modes

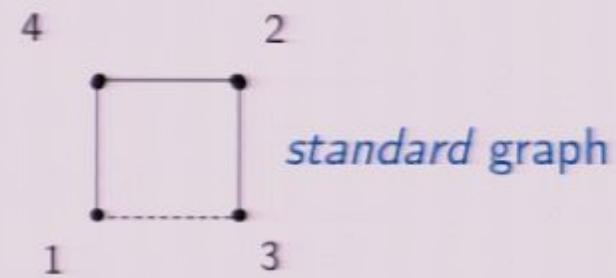
$$\begin{cases} (X_1 + X_2) - \sqrt{2} X_4 \rightarrow 0 \\ (X_1 - X_2) + \sqrt{2} X_3 \rightarrow 0 \\ (P_1 + P_2) + \sqrt{2} P_4 \rightarrow 0 \\ (P_1 - P_2) - \sqrt{2} P_3 \rightarrow 0 \end{cases}$$



"H graph"

- Square cluster graph (One-way QC) Phase-shift 3 & 4 by $\pi/2$

$$\begin{cases} \sqrt{2}P_1 + (X_3 - X_4) \rightarrow 0 \\ \sqrt{2}P_2 - (X_3 + X_4) \rightarrow 0 \\ \sqrt{2}P_3 + (X_1 - X_2) \rightarrow 0 \\ \sqrt{2}P_4 - (X_1 + X_2) \rightarrow 0 \end{cases}$$



standard graph

- Remarkable (nonunique) result $A = G$ in this case!

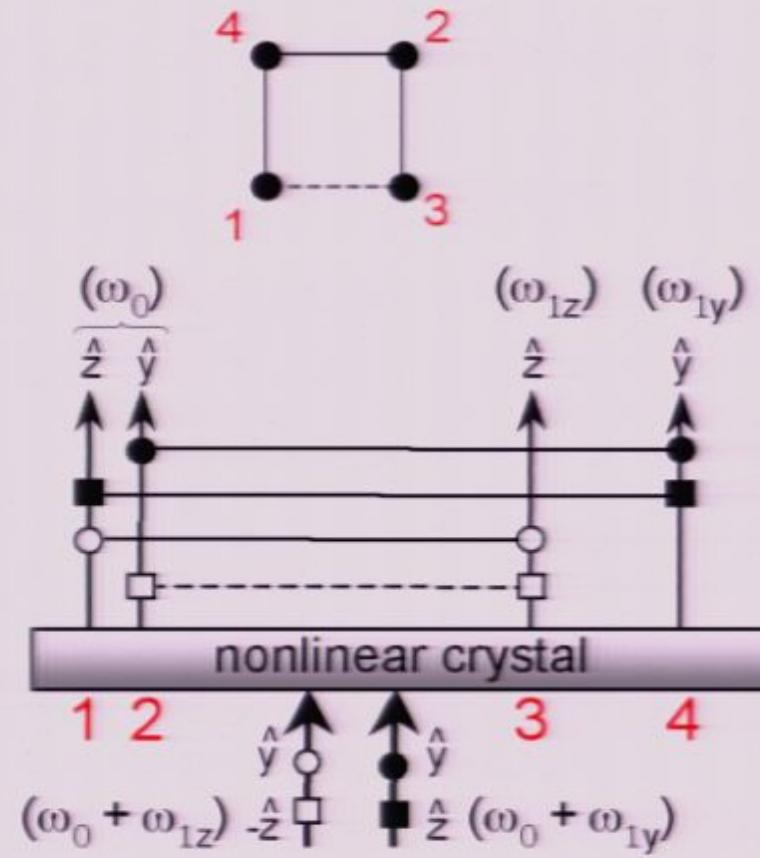
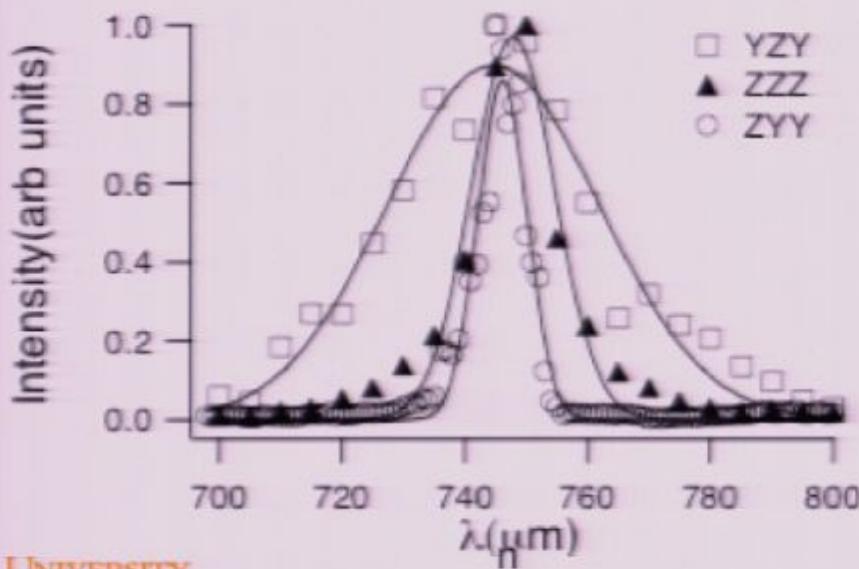
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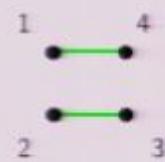
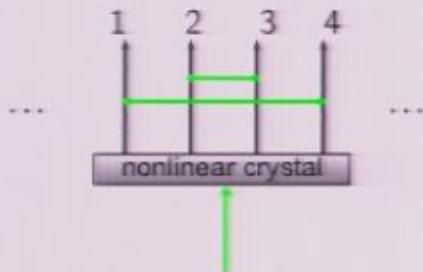
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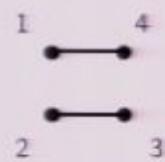
Scaling up towards the square lattice

Zaidi, Menicucci, Flammia, Bloomer, Pysher, and Pfister, quant-ph/0710.4980, Las. Phys. (2008)

Easier than expected!



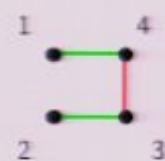
"H graph"



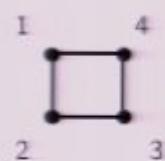
standard graph

entangled pairs

duplicable over the comb



2 x 2 cluster

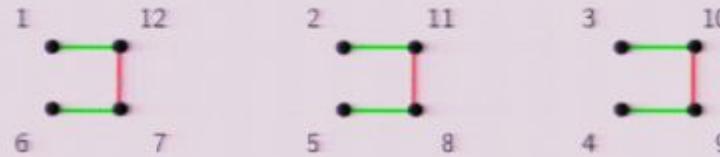


Scaling up towards the square lattice II

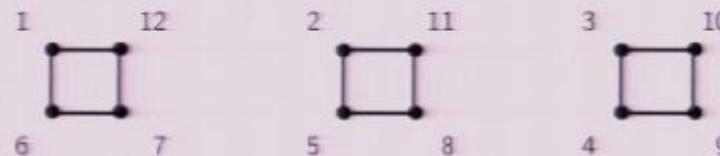
Zaidi, Menicucci, Flammia, Bloomer, Pysher, and Pfister, quant-ph/0710.4980, *Las. Phys.* (2008)



"H graph"



standard graph

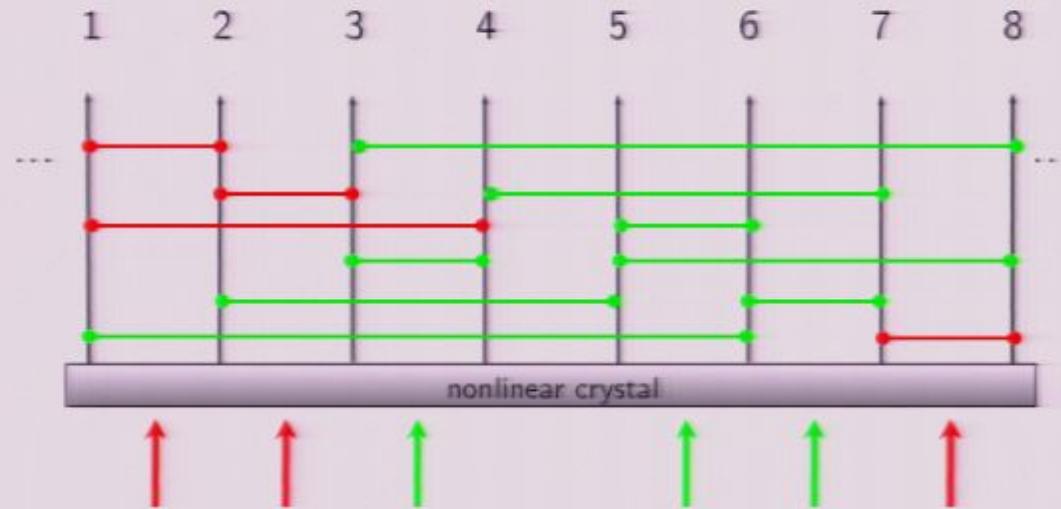


2×2 cluster

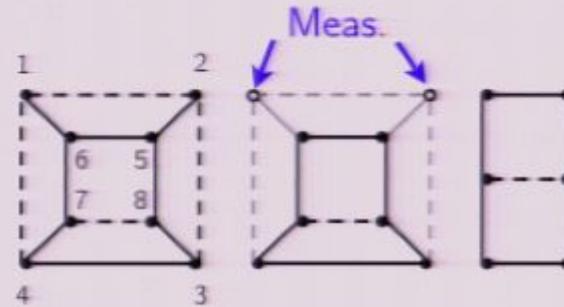
duplicated over the comb

Scaling up towards the square lattice III

Zaidi, Menicucci, Flammia, Bloomer, Pysher, and Pfister, quant-ph/0710.4980, *Las. Phys.* (2008)



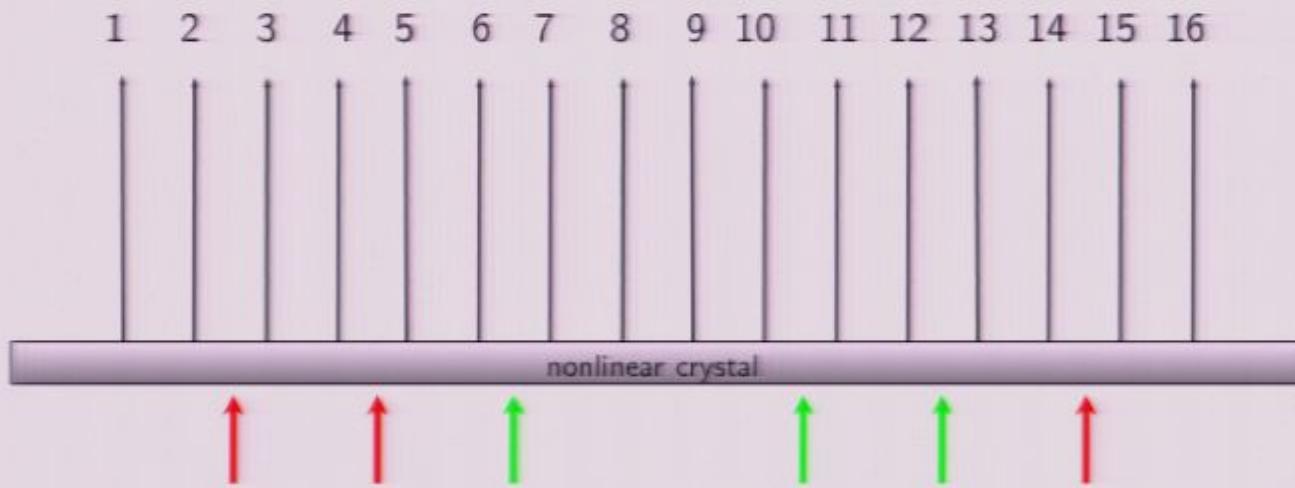
Both graph types are identical here



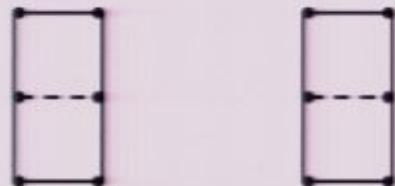
After measurement:
2 x 3 cluster

Scaling up towards the square lattice IV

Zaidi, Menicucci, Flammia, Bloomer, Pysher, and Pfister, quant-ph/0710.4980, *Las. Phys.* (2008)



No additional pump fields
needed to scale.
Just more power



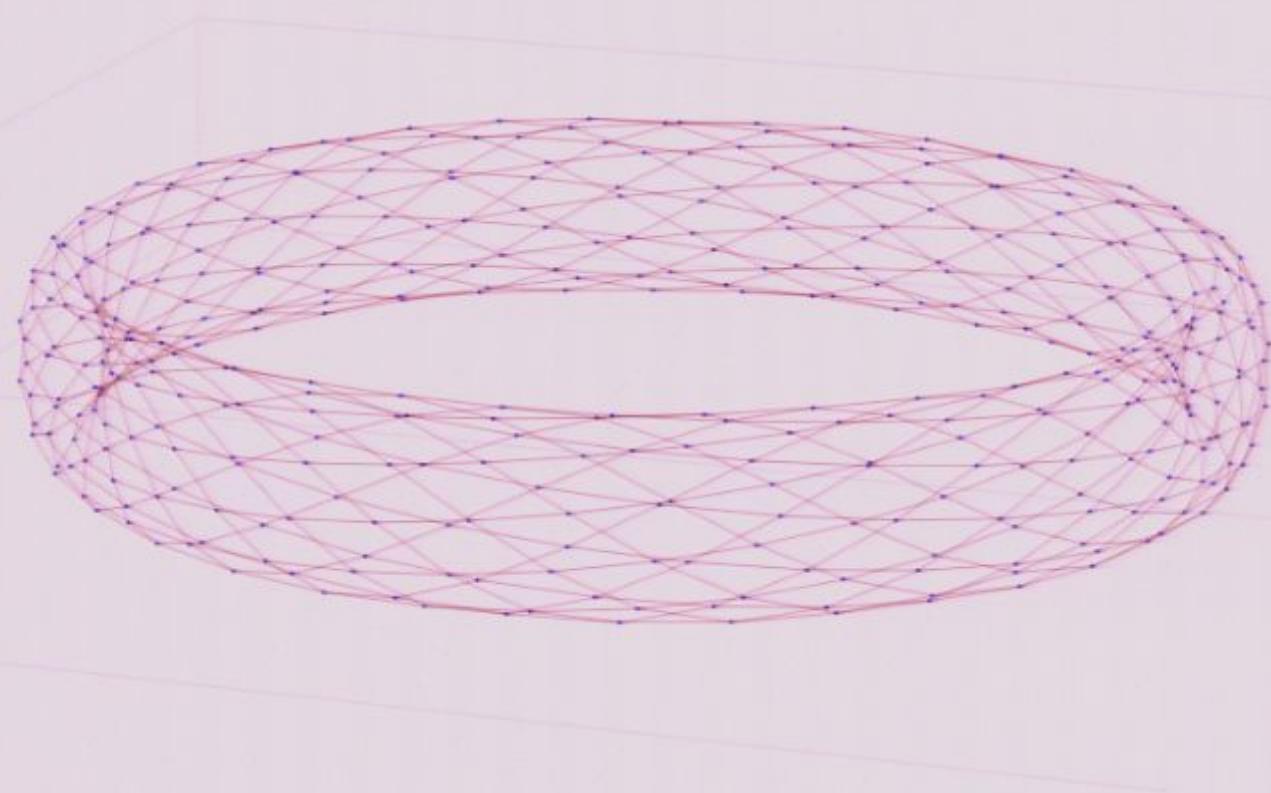
2 x 3 clusters
duplicated over the comb

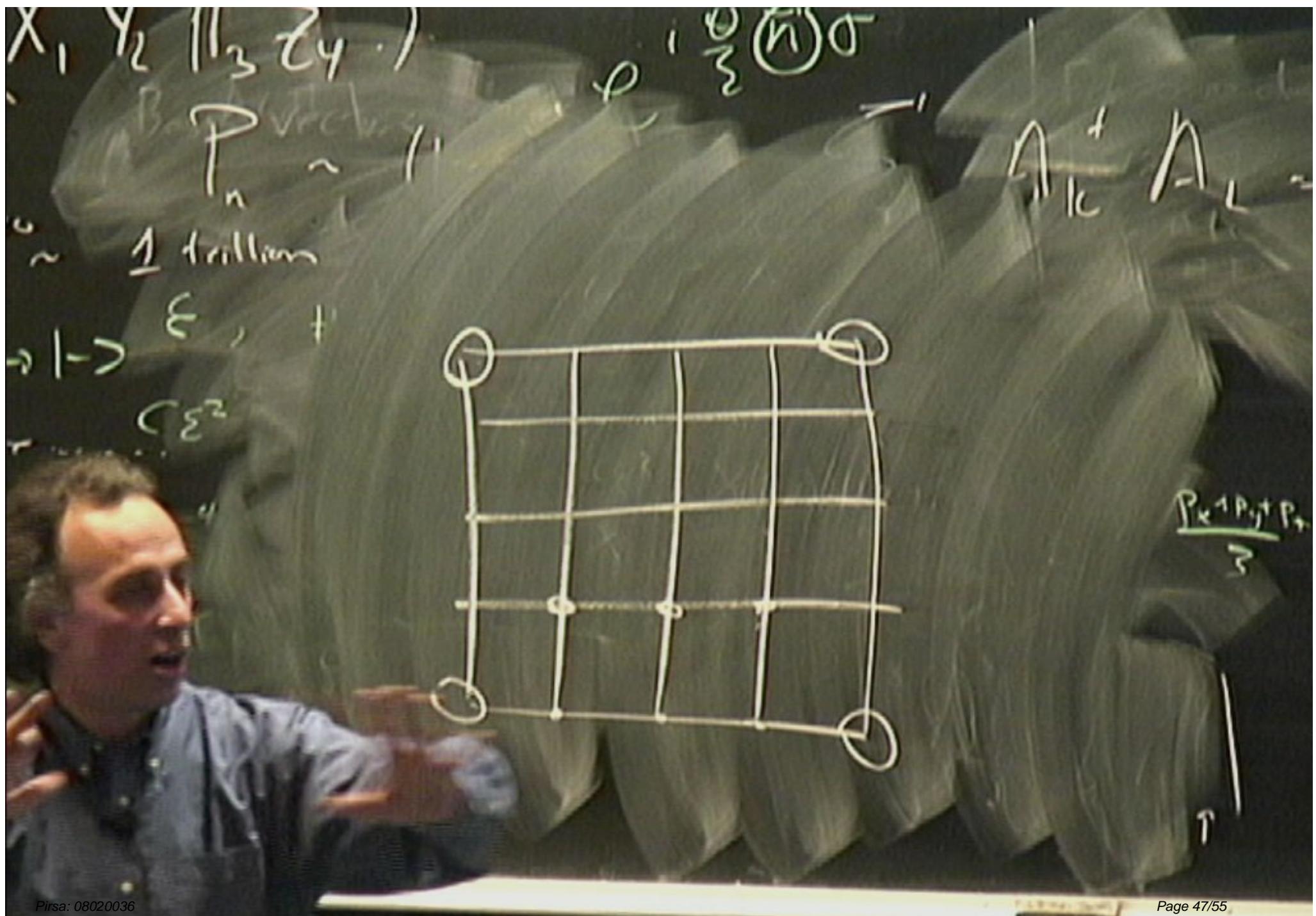
Scaling up to the square lattice, final

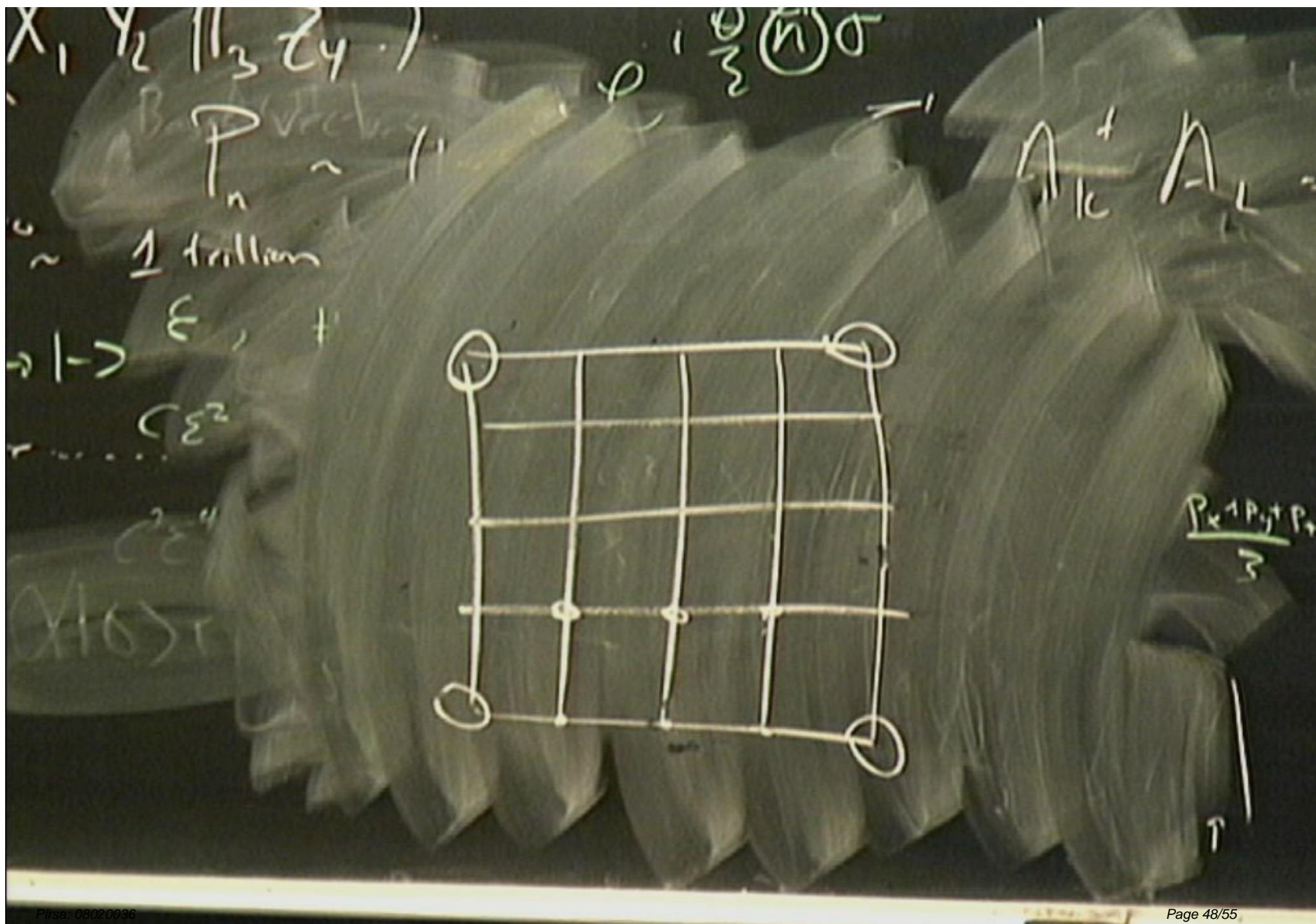
Menicucci, Flammia, Zaidi, and Pfister, *in preparation*

Single OPO. Only 15 pump fields, $\forall N$

... coming up soon!





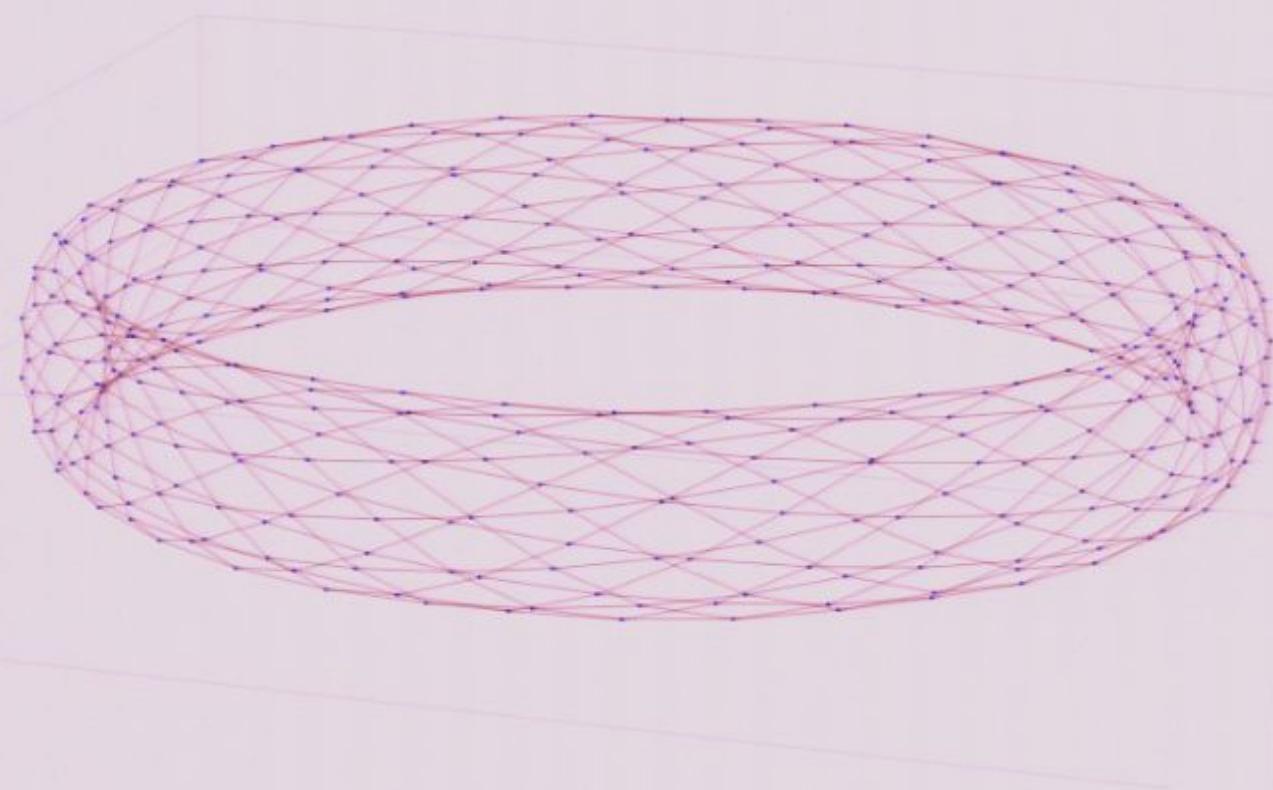


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Conclusion

- Ultrastable quantum optics allows us to work with a **LARGE** set of resonator modes at different frequencies
- Demonstration of bipartite nonclassical light states established experimental methods
 - Bipartite entanglement
 - Heisenberg-limited interferometry
- Ultracompact generation of multipartite entangled states
 - Enabled by photonic crystal technology
 - Scalability to simultaneous generation of 2x3 cluster grids over the quantum harp
 - Generation of arbitrary-size quantum register now figured out, coming up soon!



Nick Menicucci



Dr. Steve Flammia

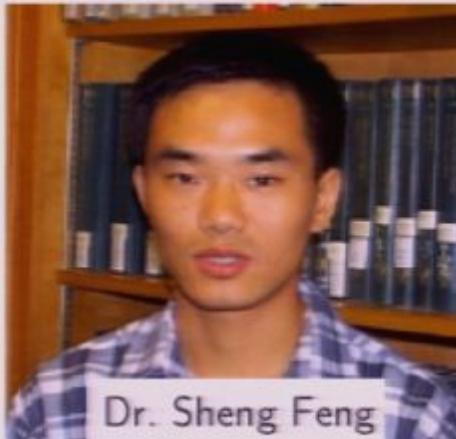


Pinsa: 08020036

Hussain Zaidi



Dr. Jietai Jing



Dr. Sheng Feng



Pennnetech

Russell Bloomer

Thank you for
your attention!



Dr. Raphael Pooser



Matt Pysher

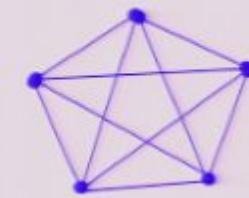
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Comparison with van Loock & Braunstein

Us

$$H_N = \sum_{j < k} H_{jk} = i\hbar\chi\beta \sum_{j < k} (a_j^\dagger a_k^\dagger - H.c.)$$

$$\begin{pmatrix} \dot{a}_1 \\ \vdots \\ \vdots \\ \dot{a}_N \end{pmatrix} = \chi\beta \begin{pmatrix} 0 & 1 & \dots & 1 \\ 1 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 1 \\ 1 & \dots & 1 & 0 \end{pmatrix} \begin{pmatrix} a_1 \\ \vdots \\ \vdots \\ a_N \end{pmatrix}$$

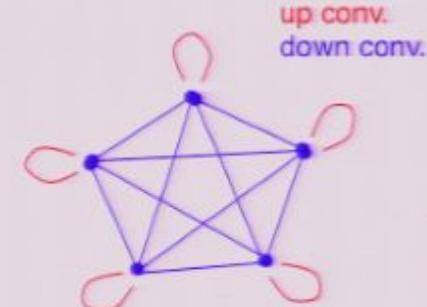


Braunstein PRA 2005

van Loock and Braunstein PRL 84, 3482 (2000)

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$$\begin{pmatrix} \dot{a}_1 \\ \vdots \\ \vdots \\ \dot{a}_N \end{pmatrix} = -\frac{\chi\beta}{2N} \begin{pmatrix} \textcircled{-}N+2 & 4 & \dots & 4 \\ 4 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 4 \\ 4 & \dots & 4 & \textcircled{-}N+2 \end{pmatrix} \begin{pmatrix} a_1 \\ \vdots \\ \vdots \\ a_N \end{pmatrix}$$



Bloch-Messiah reduction

vL & B: interference control
Us: down- vs. up-conversion

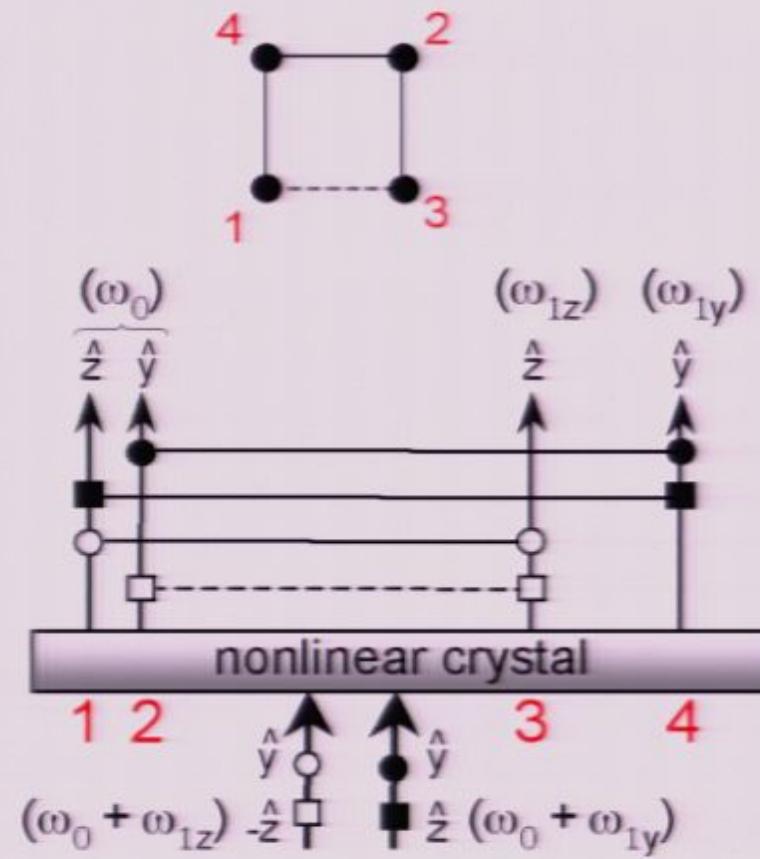
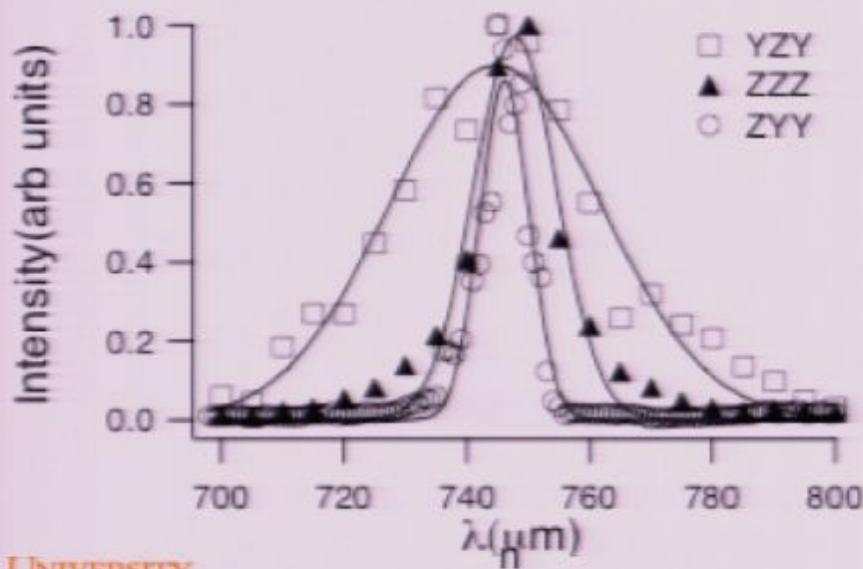
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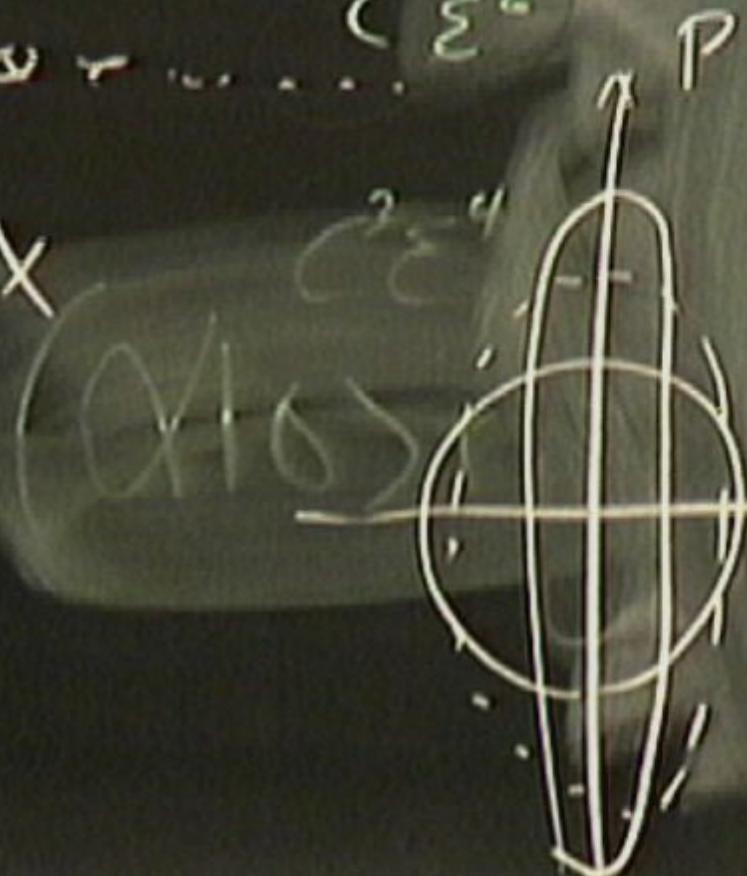
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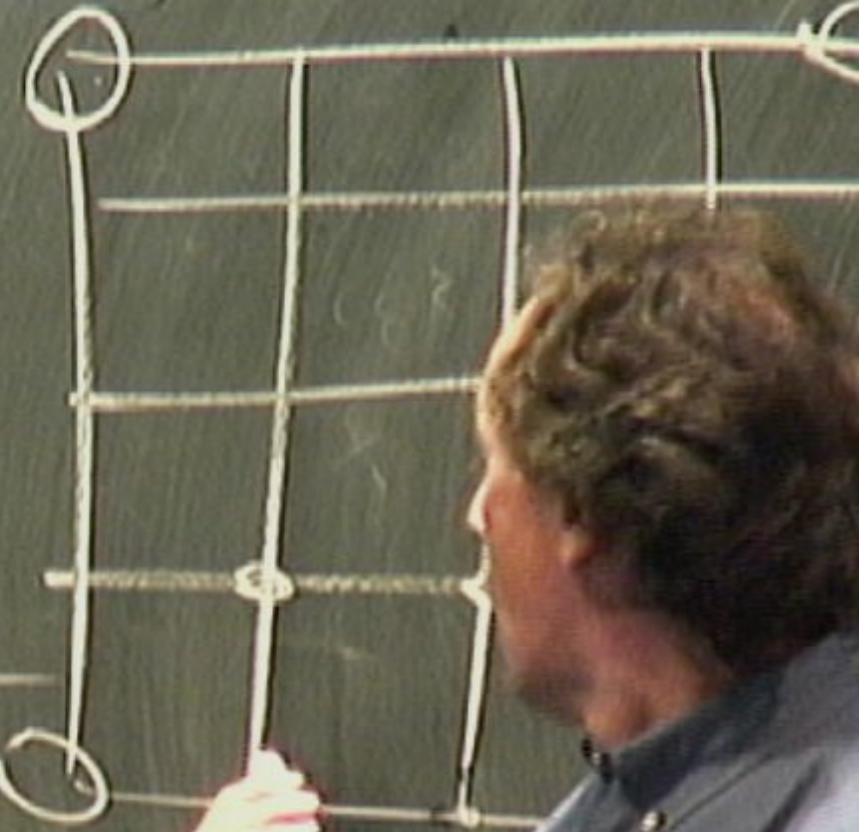
$$|+\rangle \rightarrow |-\rangle \epsilon, +$$

$$\rho \propto r^{-\dots} \propto \epsilon^2$$

$$1+x$$



2



$$|+\rangle \rightarrow |-\rangle$$

$$\rho \omega r \dots \zeta \varepsilon^2$$

$$1+x$$

