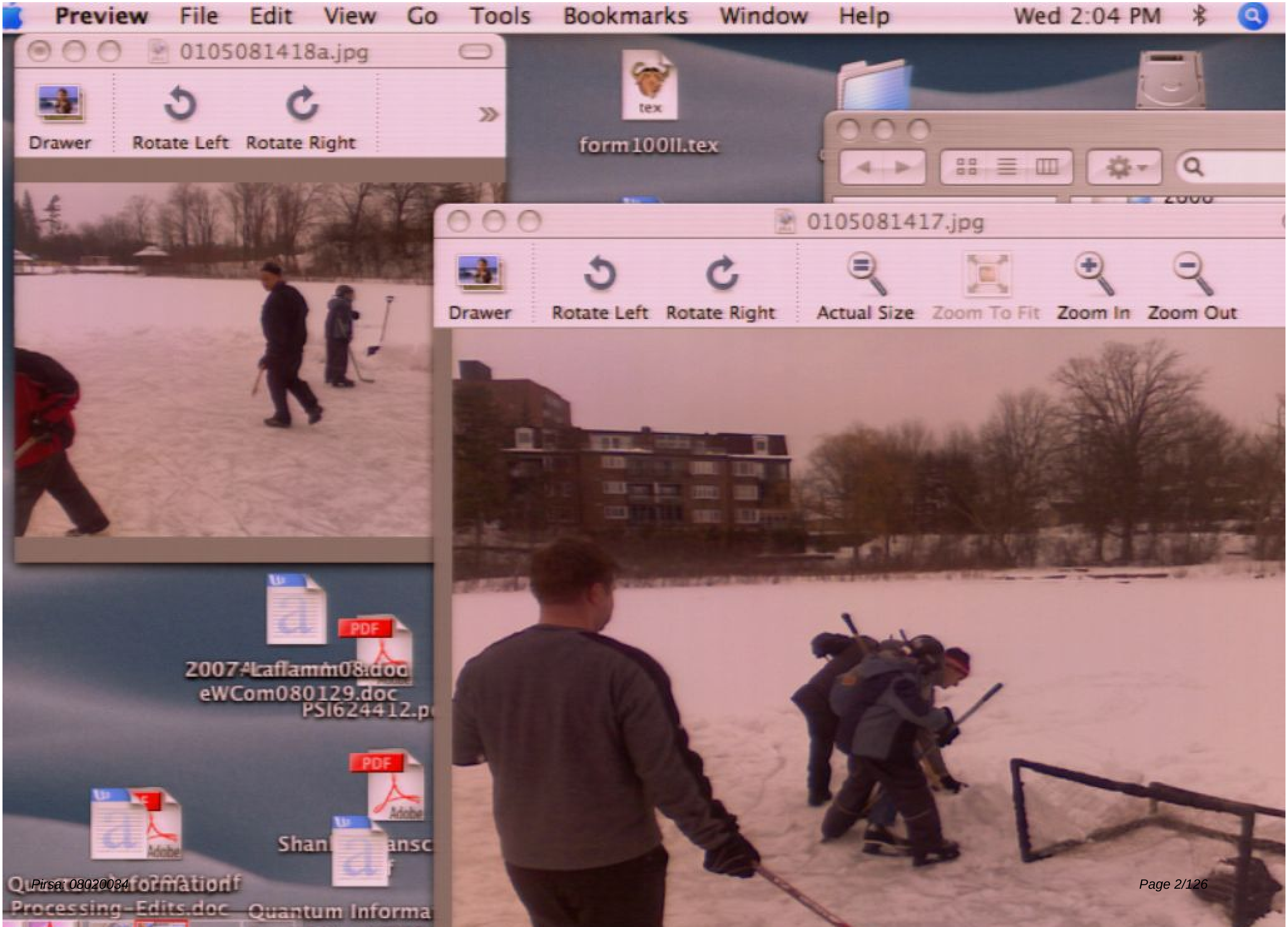
Title: Experimental Quantum Error Correction

Date: Feb 06, 2008 02:00 PM

URL: http://pirsa.org/08020034

Abstract: The Achilles\\\' heel of quantum information processors is the fragility of quantum states and processes. Without a method to control imperfection and imprecision of quantum devices, the probability that a quantum computation succeed will decrease exponentially in the number of gates it requires. In the last ten years, building on the discovery of quantum error correction, accuracy threshold theorems were proved showing that error can be controlled using a reasonable amount of resources as long as the error rate is smaller than a certain threshold. We thus have a scalable theory describing how to control quantum systems. I will briefly review some of the assumptions of the accuracy threshold theorems and comment on experiments that have been done and should be done to turn quantum error correction into an experimental reality.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IQC Institute for Quantum Computing

Experimental Quantum Error Correction

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Help from : Martin Laforest, Osama Moussa other colleagues at IQC

 

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Plan

- The need for quantum error correction and the accuracy threshold theorem
- Characterising noise
- Characterising and demonstrating control
  - Implementing error correcting codes, noiseless subspaces and subsystems
  - Implementing encoded gates
- Extracting entropy, algorithmic cooling
- Conclusion

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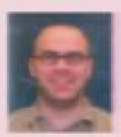 

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Experimental Quantum Error Correction

J. Baugh, J. Emerson, J. Chamillard, T. Cui, A. Hubbard, M. Laforest, C. Madaiah, O. Moussa, G. Pasante, C. Ryan, M. Silva, S. Simmons, U. Sinha, J. Zhang

at MIT D. Cory's group

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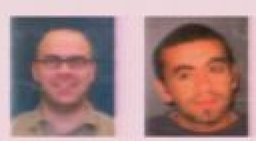

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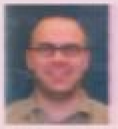

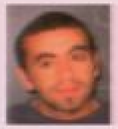

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The death of QComputers (1995)

IEEE TRANSACTIONS ON ELECTRON DEVICES, VOL. 43, NO. 10, OCTOBER 1996

Need for Critical Assessment

Rolf Landauer, *Life Fellow, IEEE*

(Invited Paper)

Abstract—Adventurous technological proposals are subject to inadequate critical assessment. It is the proponents who organize meetings and special issues. Optical logic, mesoscopic switching devices and quantum parallelism are used to illustrate this problem.

Index Terms—Technology assessment, optical logic, mesoscopic devices, quantum parallelism.

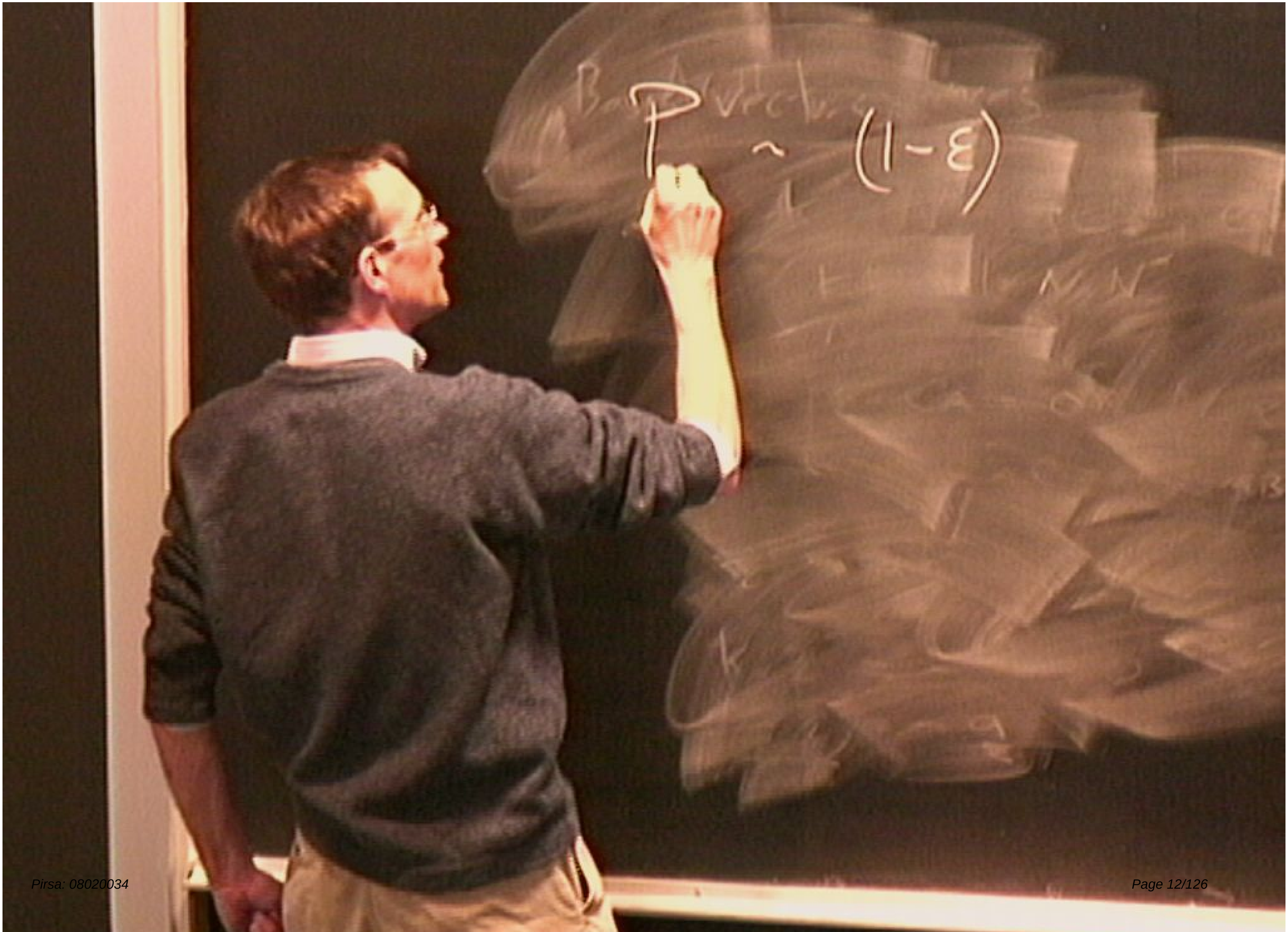
PHYSICAL REVIEW A VOLUME 51, NUMBER 2 FEBRUARY 1995

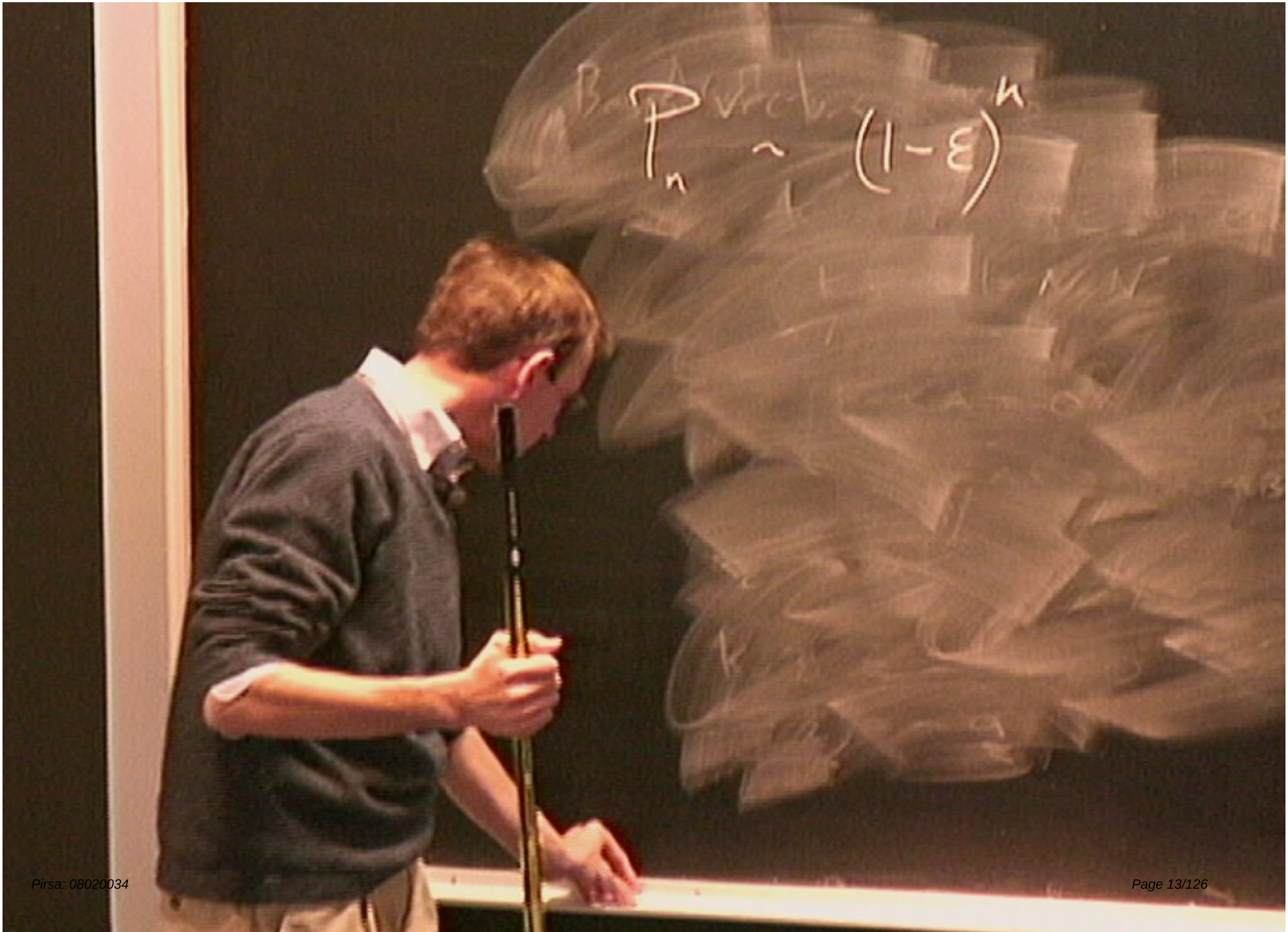
Maintaining coherence in quantum computers

W. G. Unruh*
Canadian Institute for Advanced Research, Cosmology Program, Department of Physics, University of British Columbia, Vancouver, Canada V6T 1Z1

(Received 10 June 1994)

The effects of the inevitable coupling to external degrees of freedom of a quantum computer are examined. It is found that for quantum calculations (in which the maintenance of coherence over a large number of states is important), not only must the coupling be small, but the time taken in the quantum calculation must be less than the thermal time scale $\hbar/k_BT$. For longer times the condition on the strength of the coupling to the external world becomes much more stringent.


P ~ (1-ε)


$P_n \sim (1-\varepsilon)^n$

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not an erroneous 1. We have, essentially, returned to an analog computer as already foreseen by Peres in remarkably perceptive comments made in 1985 [41]. Analog computers can do far more per step than digital computers, but they cannot take very many steps before the accumulated errors have derailed the computation. Of course, almost any device can be labeled *analog computer*, if we want to be generous.

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Threshold theorem

A quantum computation can be as long as required with any desired accuracy as long as the noise level is below a threshold value

$$P < 10^{-6,-5,-4,\ldots,-1?}$$

Knill et al.; Science, 279, 342, 1998

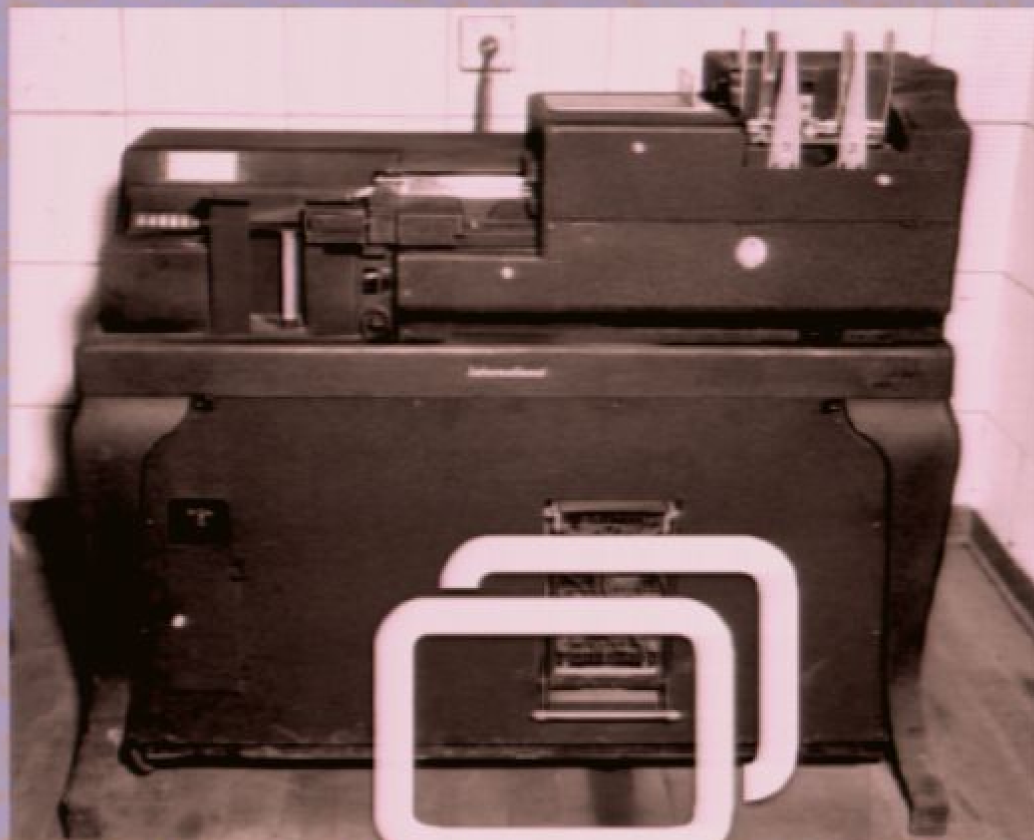
Kitaev, Russ. Math Survey 1997

Aharonov & Ben Or, ACM press

Preskill, PRSL, 454, 257, 1998

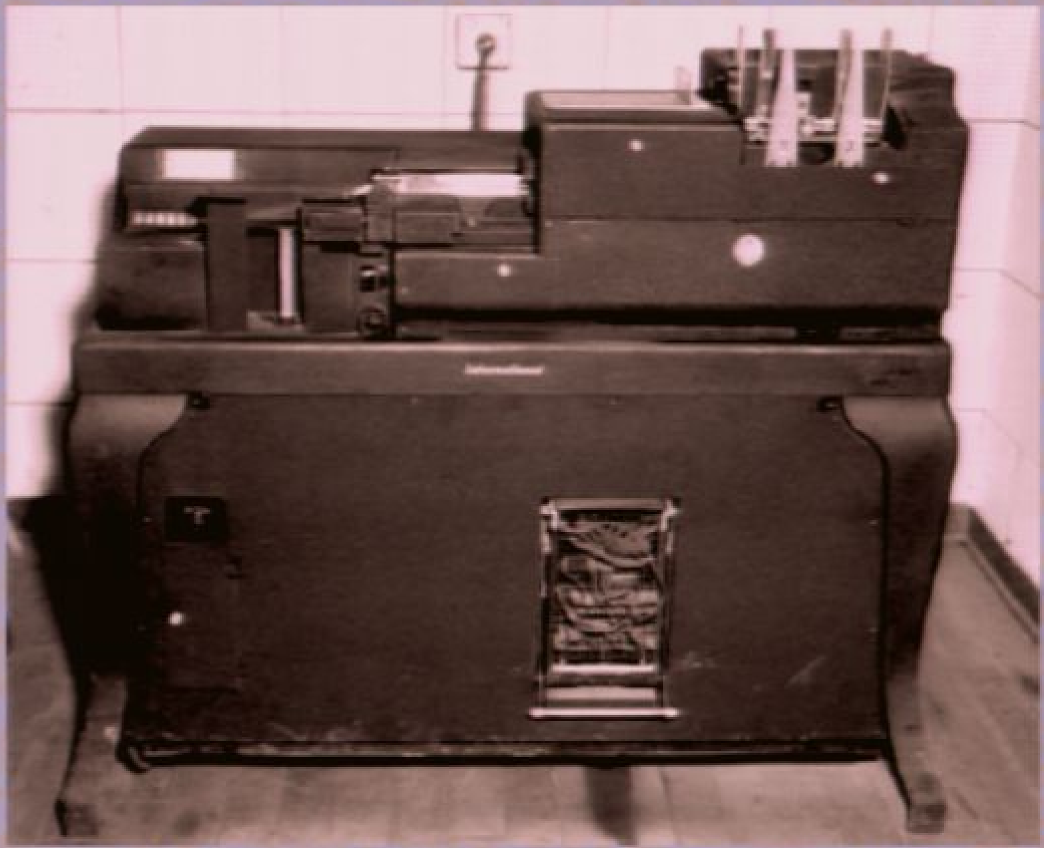
Significance:

- -imperfections and imprecisions are not fundamental objections to quantum computation
- -it gives criteria for scalability
- -its requirements are a guide for experimentalists
- -it is a benchmark to compare different technologies



"Error correction permits to build arbitrarily complex systems of a given reliability from less reliable parts"

R. Hamming



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Classical error correction

	Noise model	Redundancy	Majority voting
Classical	bit flip $0 \overset{\varepsilon}{\longleftrightarrow} 1$	copy information $0 \longrightarrow 000$ $1 \longrightarrow 111$	observe and compare $110 \longrightarrow 111$

Note: if the error rate is ϵ, after error correction, the "effective" error rate is $c\epsilon^2$, and thus the number of reliable gates is $\frac{1}{c\epsilon^2}$

Quantum error correction

	Noise model	Redundancy	Majority voting
Classical	bit flip $0 \leftrightarrow 1$	copy information $0 \longrightarrow 000$ $1 \longrightarrow 111$	observe and compare $110 \longrightarrow 111$
Quantum	bit flip phase flip $\alpha\|0\rangle + \beta\|1\rangle$ $\alpha\|0\rangle - \beta\|1\rangle$		

Evolution of a generic qubit stat $|0\rangle|\epsilon\rangle \rightarrow |0\rangle|\epsilon_0^0\rangle + |1\rangle|\epsilon_0^1\rangle$ $|1\rangle|\epsilon\rangle \rightarrow |0\rangle|\epsilon_1^0\rangle + |1\rangle|\epsilon_1^1\rangle$ environment:

$$
\begin{aligned}
(\alpha|0\rangle + \beta|1\rangle)|\epsilon\rangle \rightarrow \\
& (\alpha|0\rangle + \beta|1\rangle)\frac{1}{2}(|\epsilon_0^0\rangle + |\epsilon_1^1\rangle) \quad (\Rightarrow \mathbb{1}|\Psi\rangle) \\
& +(\alpha|0\rangle - \beta|1\rangle)\frac{1}{2}(|\epsilon_0^0\rangle - |\epsilon_1^1\rangle) \quad (\Rightarrow \mathrm{Z}|\Psi\rangle) \\
& +(\alpha|1\rangle + \beta|0\rangle)\frac{1}{2}(|\epsilon_0^1\rangle + |\epsilon_1^0\rangle) \quad (\Rightarrow \mathrm{X}|\Psi\rangle) \\
& +(\alpha|1\rangle - \beta|0\rangle)\frac{1}{2}(|\epsilon_0^1\rangle - |\epsilon_1^0\rangle) \quad (\Rightarrow \mathrm{iY}|\Psi\rangle)
\end{aligned}
$$

The effect of the noise is to apply the error operators $\mathbb{1}, \mathrm{X}, \mathrm{Y}, \mathrm{Z}$ to the state $|\Psi\rangle$ depending on what the state of the environment is.

Note, the discrete basis implies a dicretization of the error similar to the classical case.

	Noise model	Redundancy	Majority voting
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Quantum	bit flip phase flip $\alpha\|0\rangle + \beta\|1\rangle$ $\alpha\|0\rangle - \beta\|1\rangle$	No cloning theorem (Wootters & Zurek, 1982) Encode $\alpha\|000\rangle + \beta\|111\rangle$	

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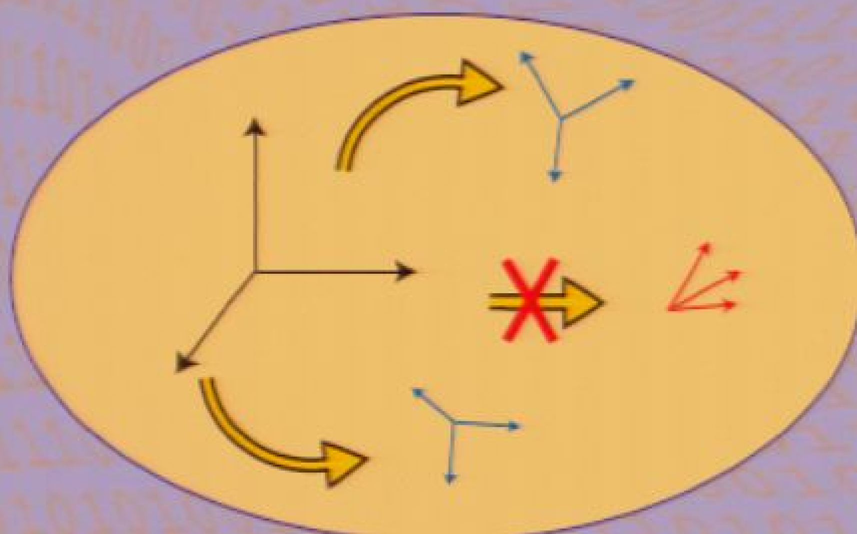
Definition: Quantum error codes

We can generalize the Schrodinger equation to open system:

$$\rho_f = \sum_a A_a \rho_i A_a^\dagger$$

An error correcting code is a code $\mathcal{C}$ defined by basis states $\{\ |i_L\rangle\ \}$ such that

$$\langle i_L | \mathcal{A}_a^\dagger \mathcal{A}_b | j_L \rangle = \delta_{ij} c_{ab}$$

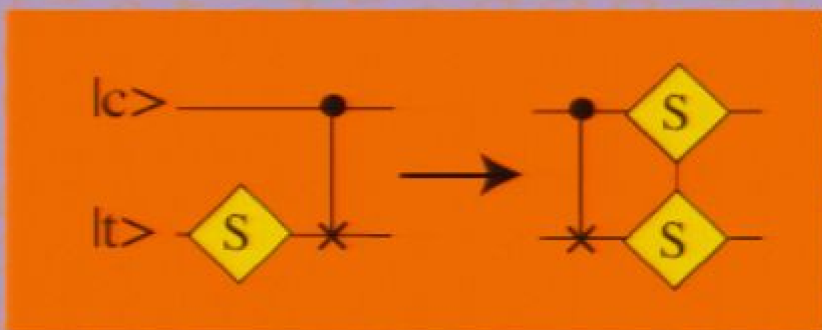


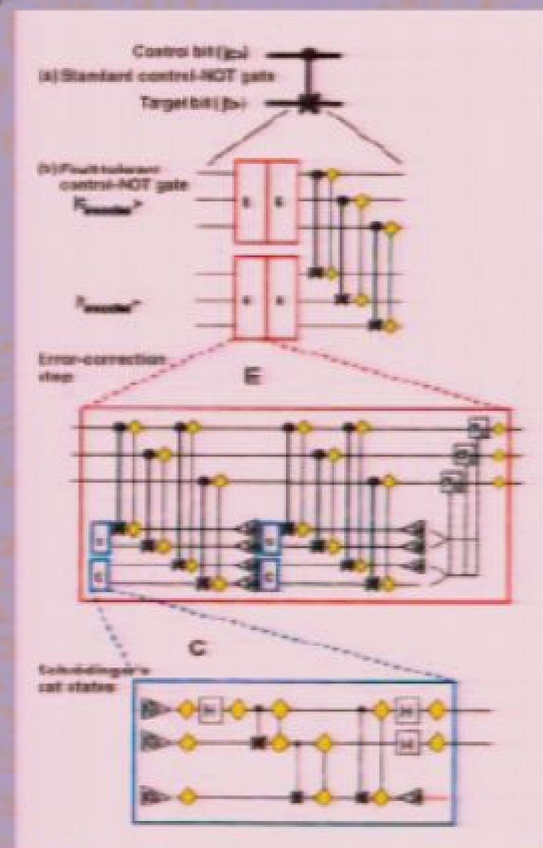
Knill and Laflamme PRA 55, 900, 1997.

Fault Tolerance

Error Propagation

P. Shor, Proc . Symp. Found. Comp. Science, IEEE Press, 56-65, 1996





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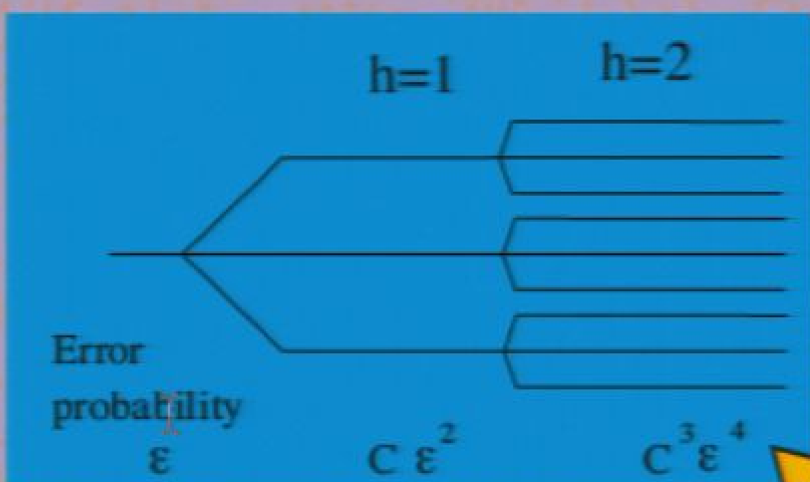

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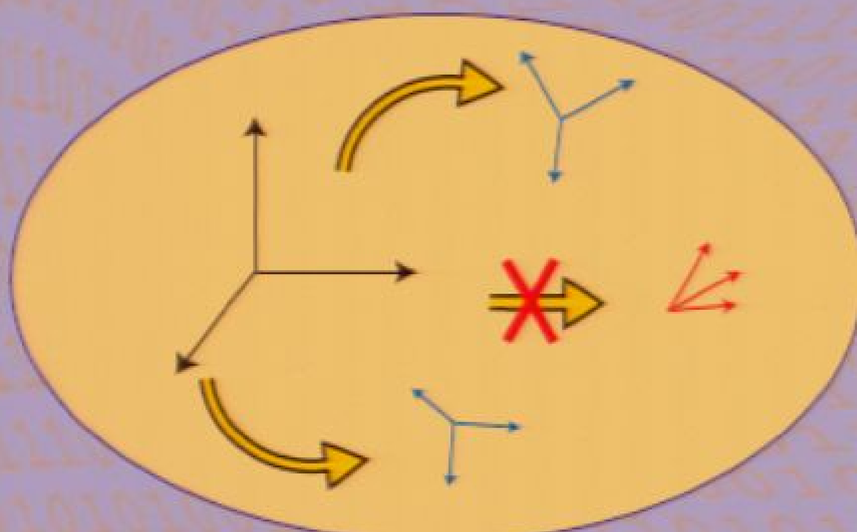

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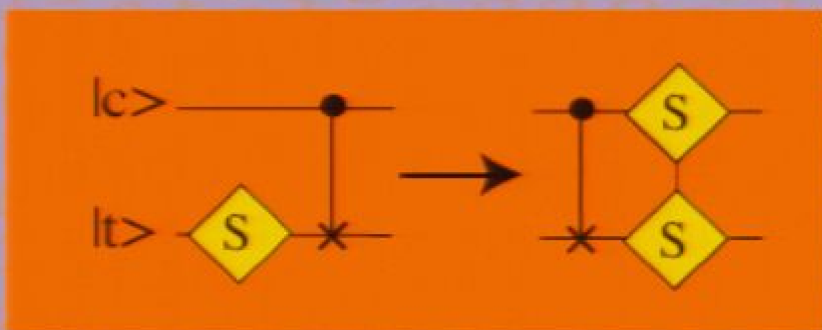

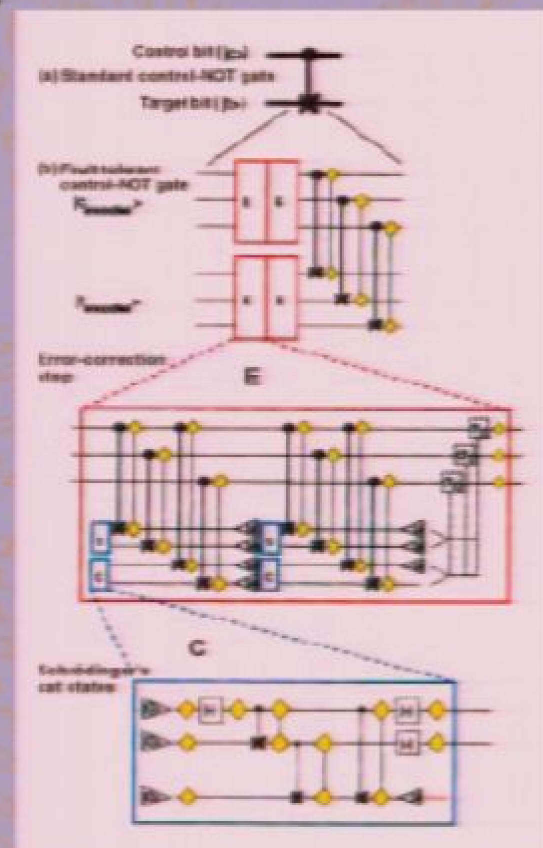

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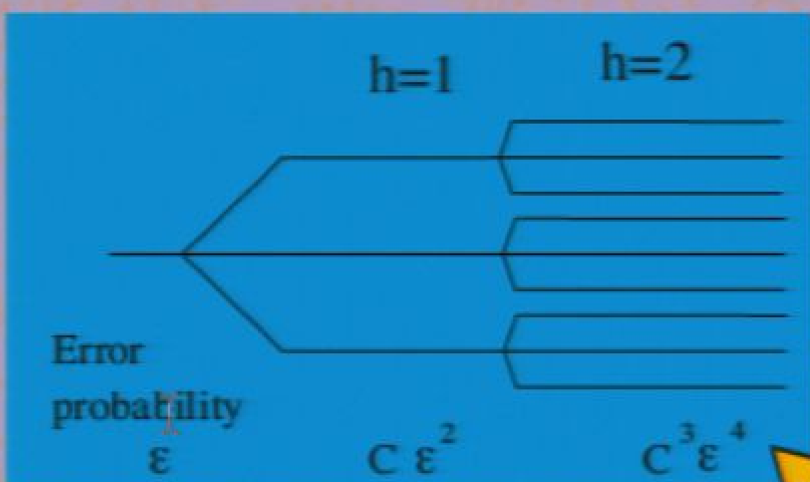

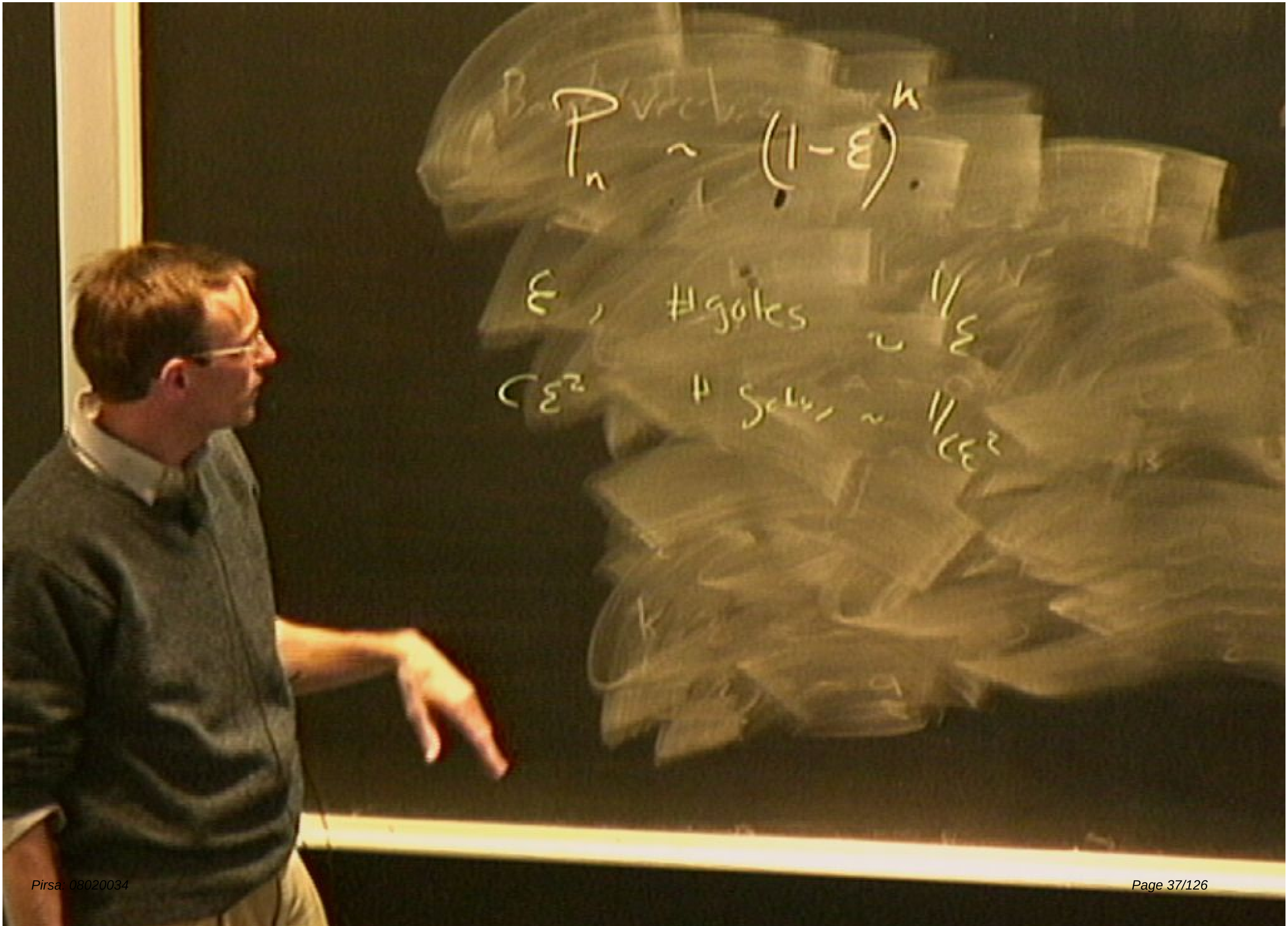
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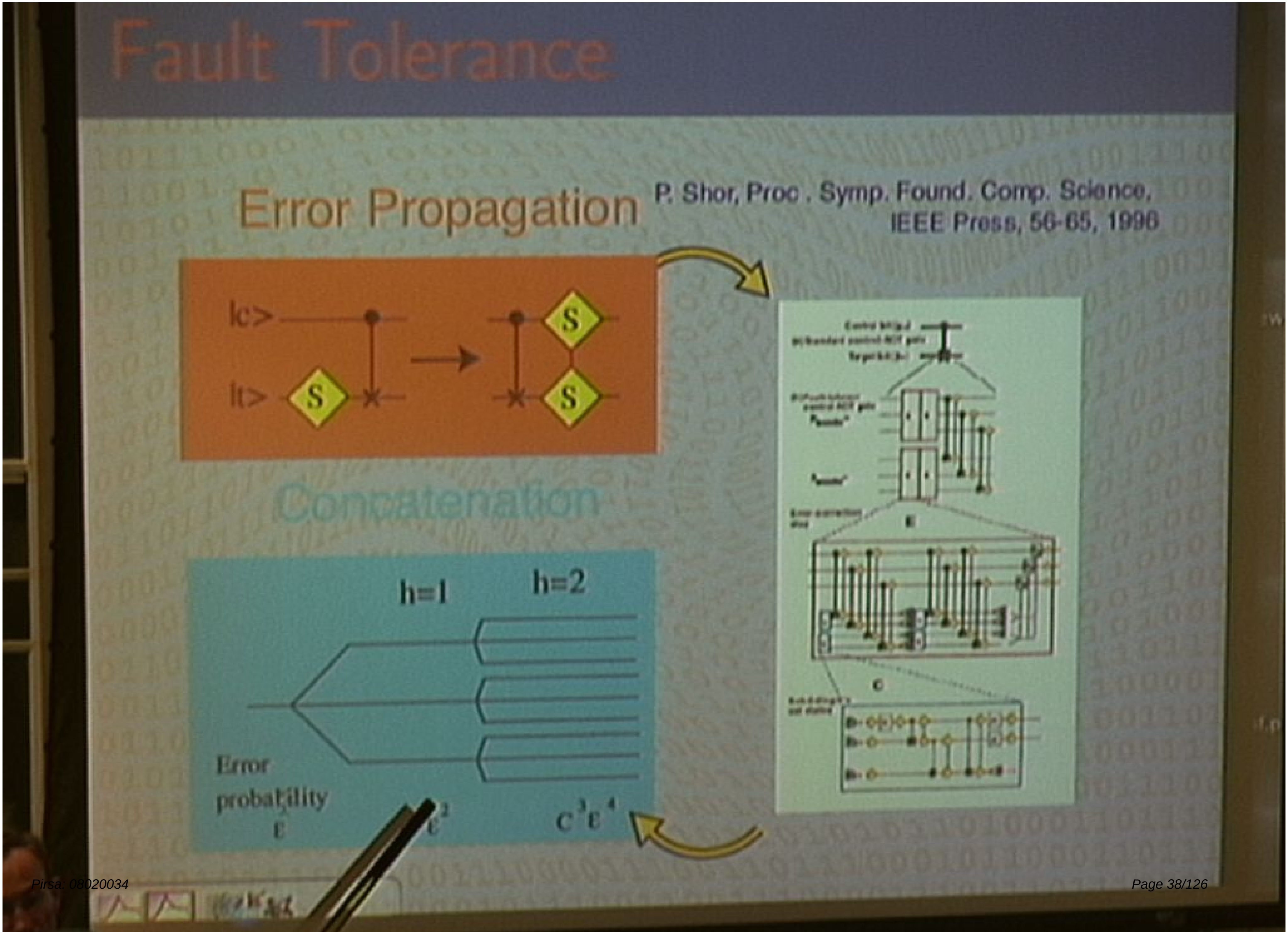
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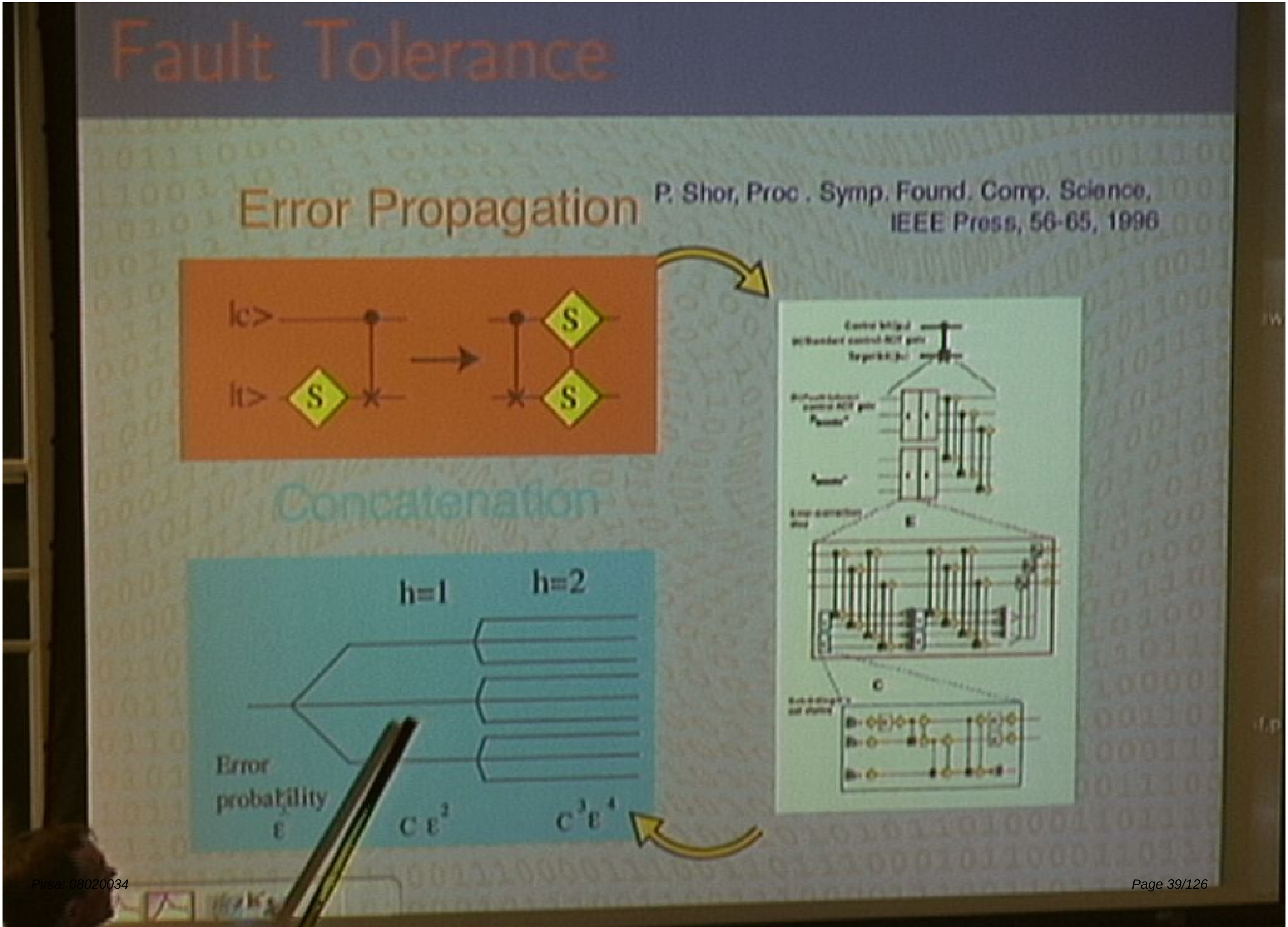

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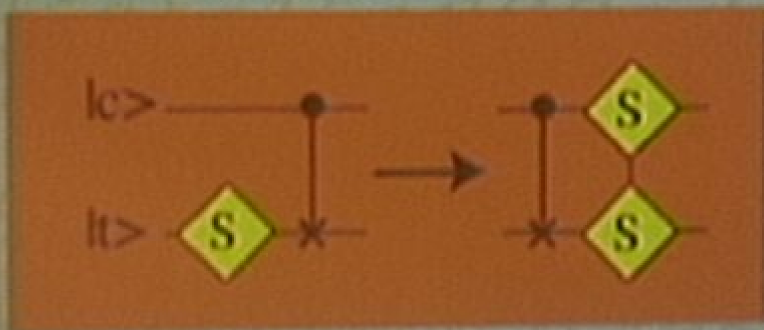

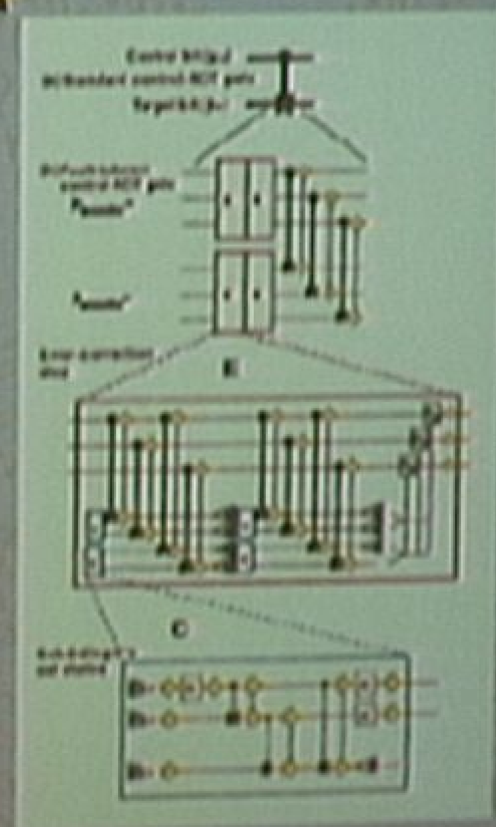

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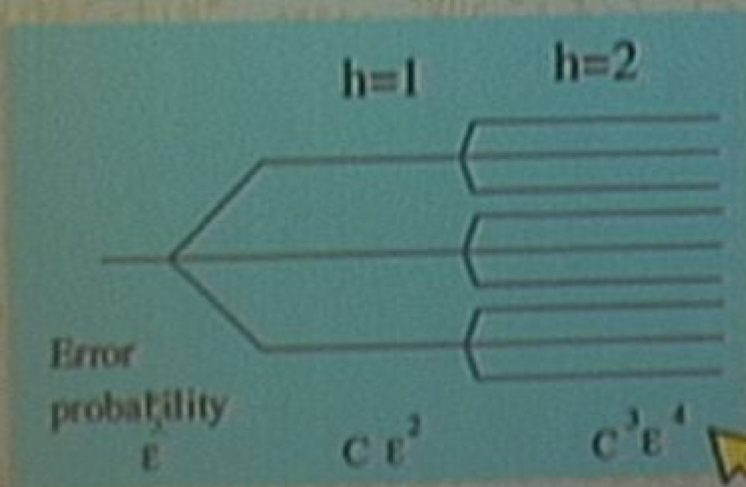

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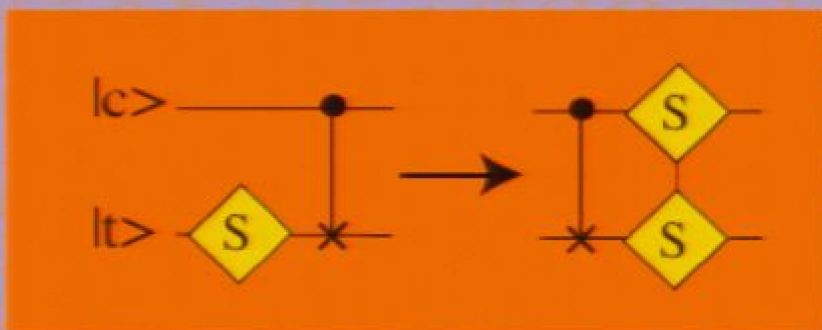
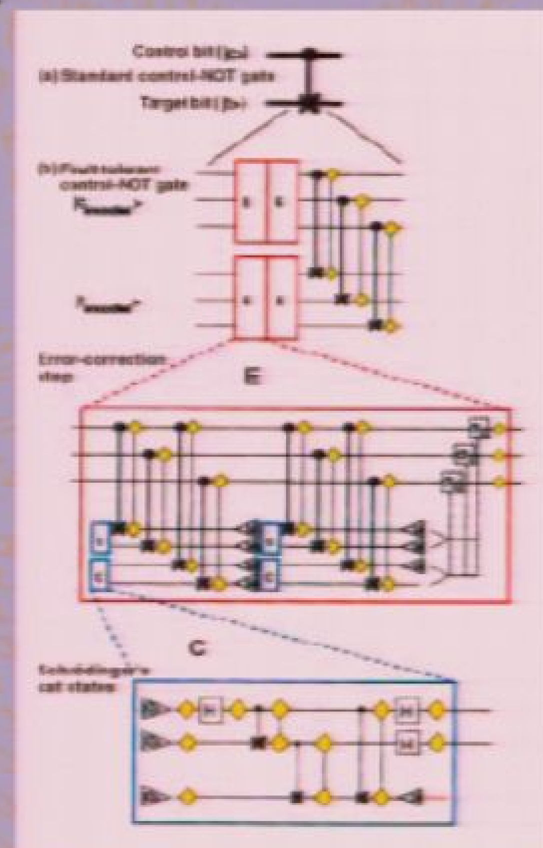
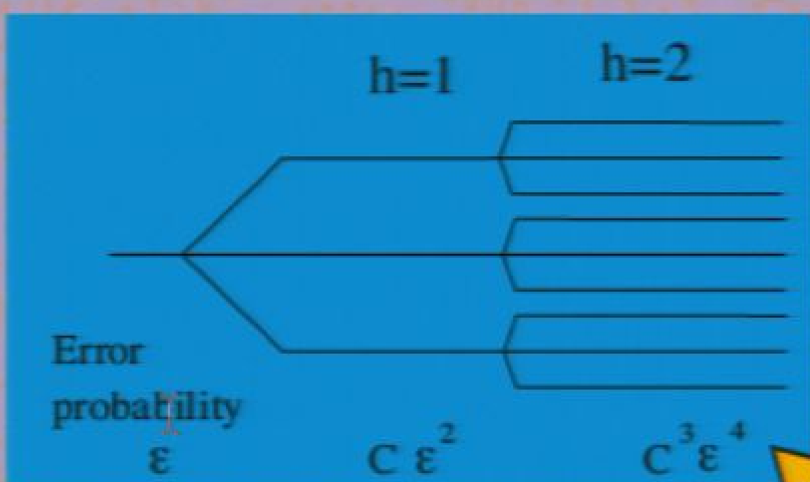
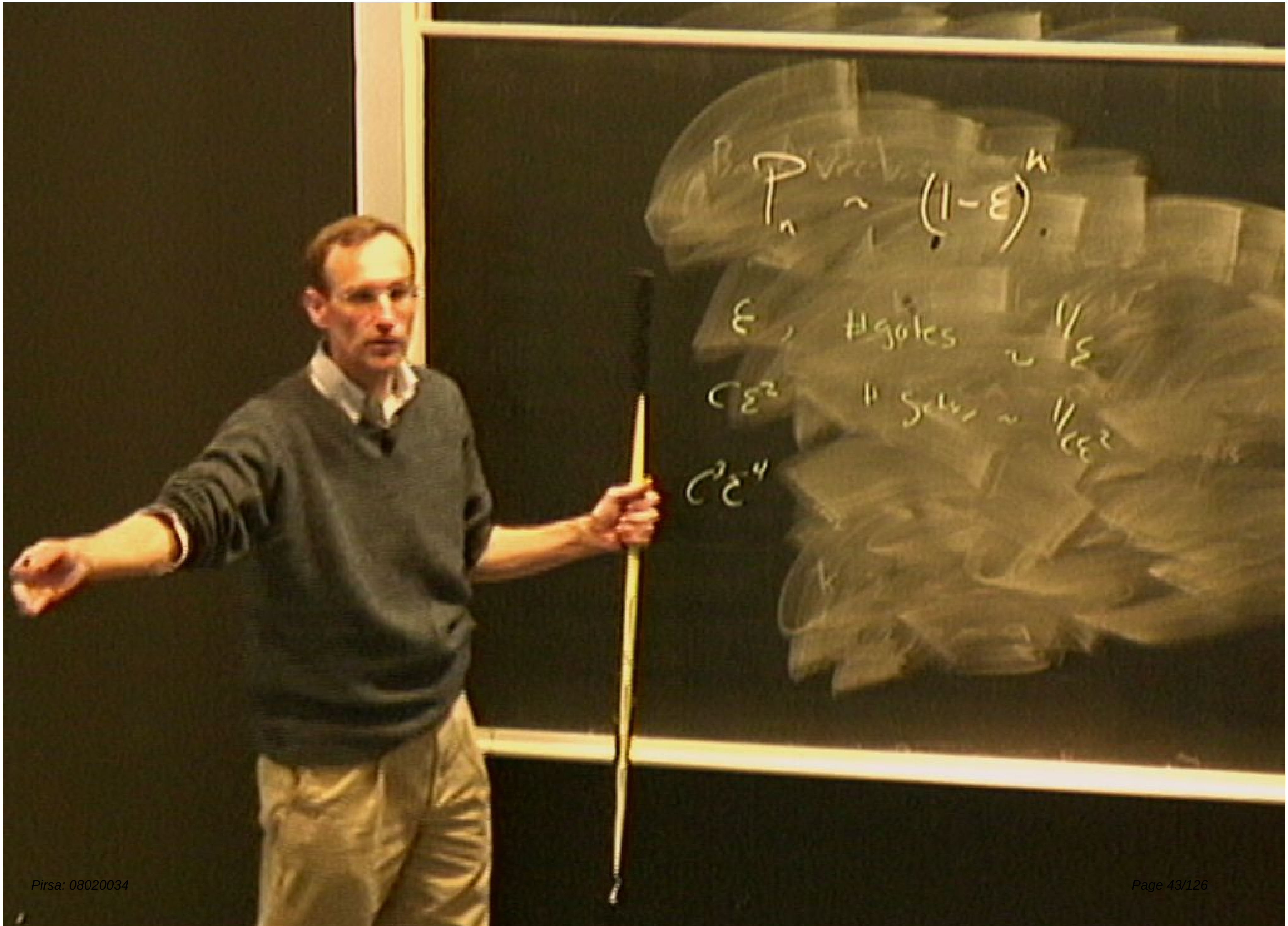




Concatenation



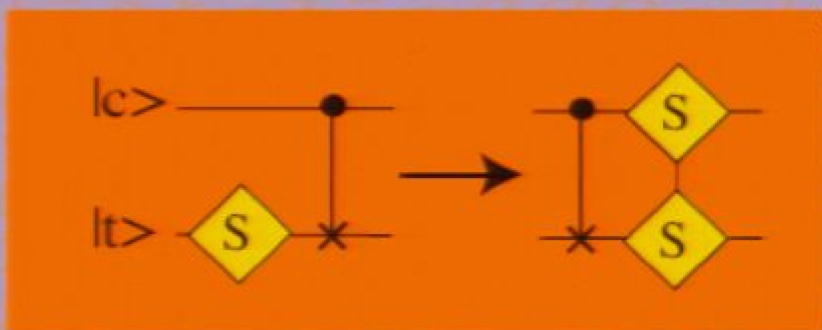


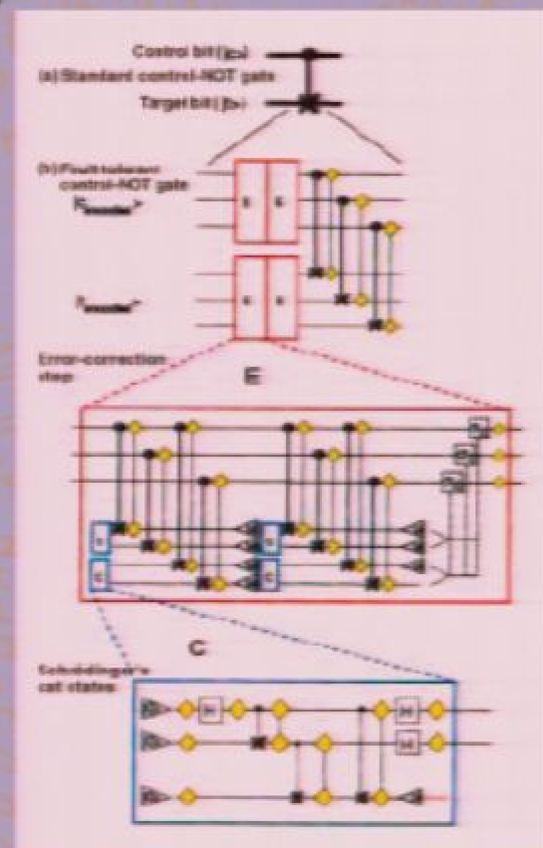
$P_n \sim (1-\varepsilon)^n$
ε, #gates ~ $1/\varepsilon$
$c\varepsilon^2$ # gates ~ $1/c\varepsilon^2$
$c^3\varepsilon^4$

Fault Tolerance

Error Propagation

P. Shor, Proc . Symp. Found. Comp. Science, IEEE Press, 56-65, 1996





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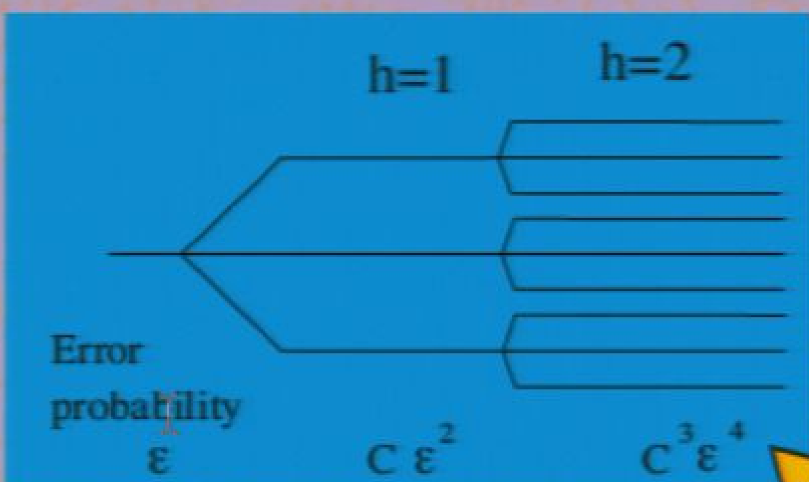

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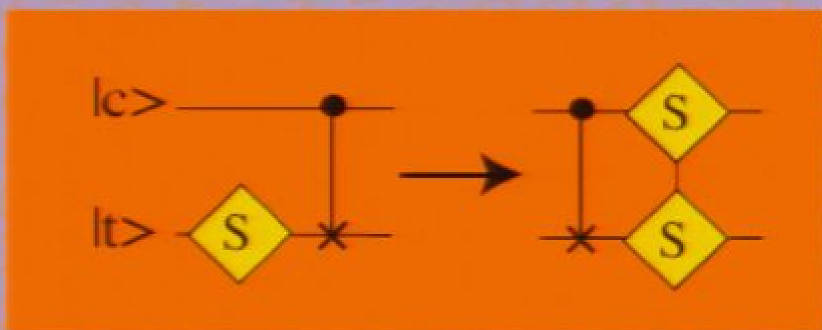

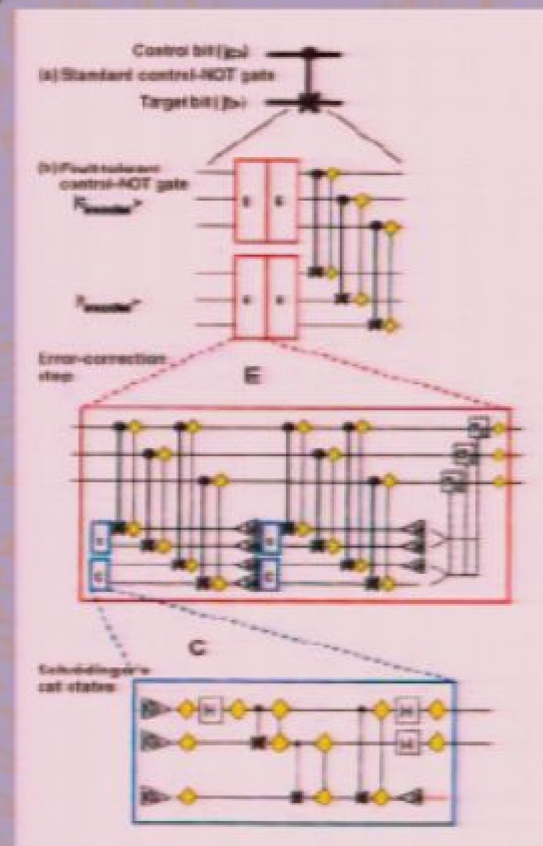

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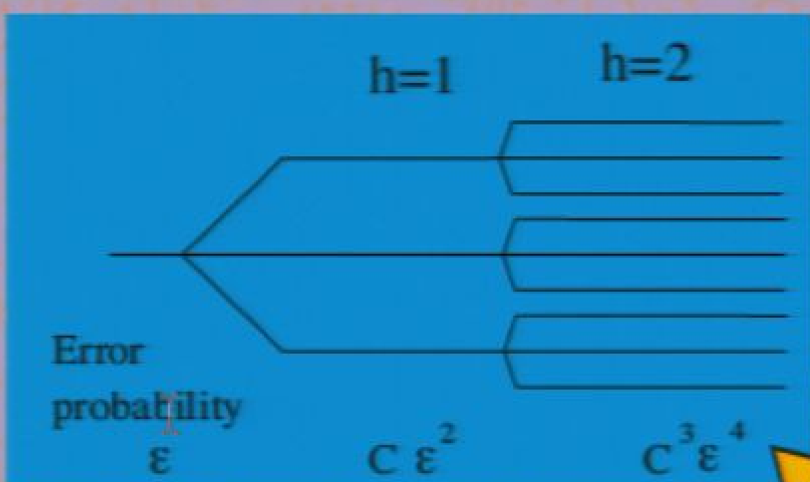


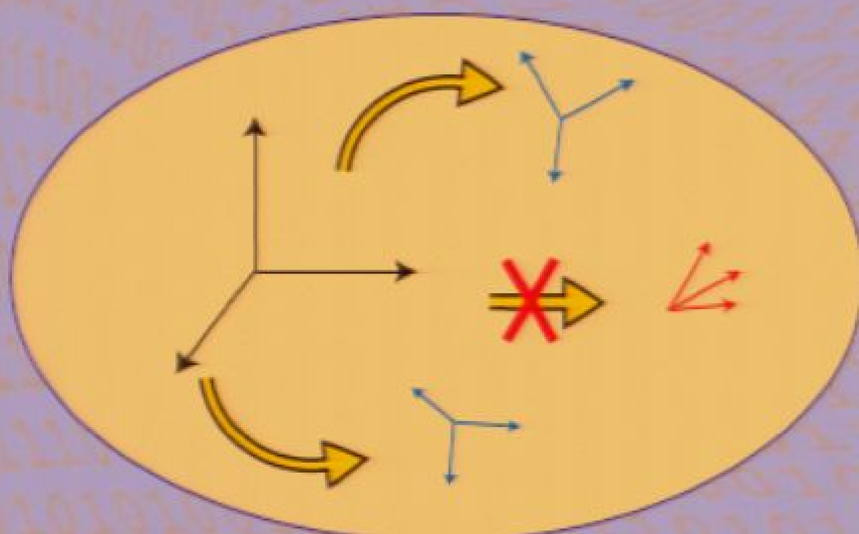
Definition: Quantum error codes

We can generalize the Schrodinger equation to open system:

$$\rho_f = \sum_a A_a \rho_i A_a^\dagger$$

An error correcting code is a code $\mathcal{C}$ defined by basis states $\{ \, |i_L\rangle \, \}$ such that

$$\langle i_L | \mathcal{A}_a^\dagger \mathcal{A}_b | j_L \rangle = \delta_{ij} c_{ab}$$



Knill and Laflamme PRA 55, 900, 1997.

Threshold theorem

A quantum computation can be as long as required with any desired accuracy as long as the noise level is below a threshold value

$$P < 10^{-6,-5,-4,\ldots,-1?}$$

Knill et al.; Science, 279, 342, 1998

Kitaev, Russ. Math Survey 1997

Aharonov & Ben Or, ACM press

Preskill, PRSL, 454, 257, 1998

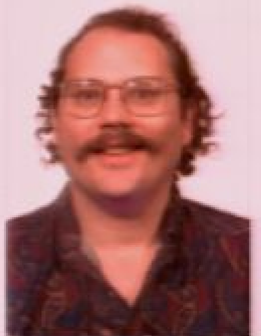
Significance:

- -imperfections and imprecisions are not fundamental objections to quantum computation
- -it gives criteria for scalability
- -its requirements are a guide for experimentalists
- -it is a benchmark to compare different technologies

Ingredients for FTQEC

- **Knowledge of the noise**
- **Good quantum control**
- **Ability to extract entropy**
- **Parallel operations**

Threshold theorem



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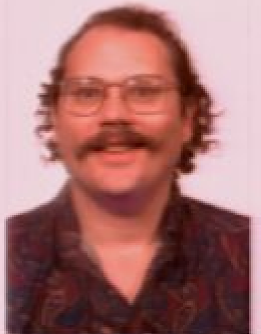

A quantum co
can be as long
with any desir
as long as the
is below a thre
$P < 10$

A Comparative Code Study for Quantum Fault Tolerance

Andrew W. Cross * David P. DiVincenzo † Barbara M. Terhal ‡

February 2, 2008

Abstract

We study a comprehensive list of quantum codes as candidates of codes to be used at the bottom, physical, level in a fault-tolerant code architecture. Using the Aliferis-Gottesman-Preskill (AGP) ex-Rec method we calculate the pseudo-threshold for these codes against depolarizing noise at various levels of overhead. We estimate the logical noise rate as a function of overhead at a physical error rate of $p_0 = 1 \times 10^{-4}$. The Bacon-Shor codes and the Golay code are the best performers in our study.

Knill et al.; Science, 279, 342, 1998
Kitaev, Russ. Math Survey 1997
Aharonov & Ben Or, ACM press
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Ingredients for FTQEC

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- Ability to extract entropy
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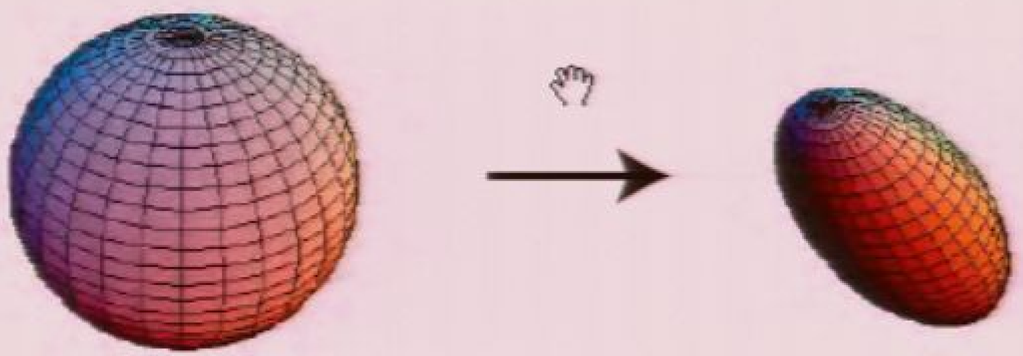
Characterising noise in q. systems

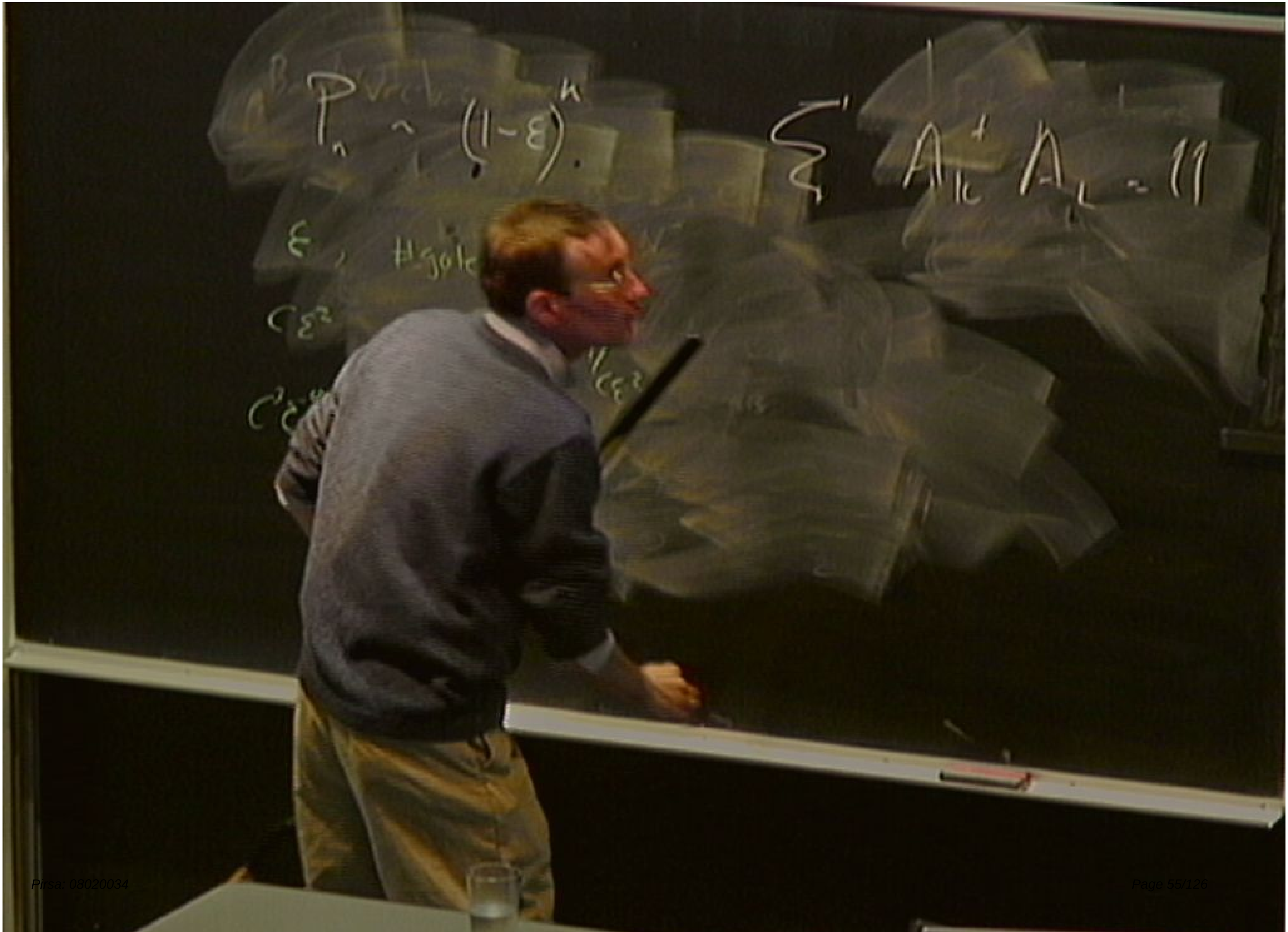
Process tomography:

$$\rho_f = \sum_k A_k \rho_i A_k^\dagger = \sum_{kl} \chi_{kl} P_k \rho_i P_l$$

For one qubit, 12 parameters are required as described by the evolution of the Bloch sphere:



For n qubits, we need to provide $4^{2n} - 4^n$ numbers to do so.



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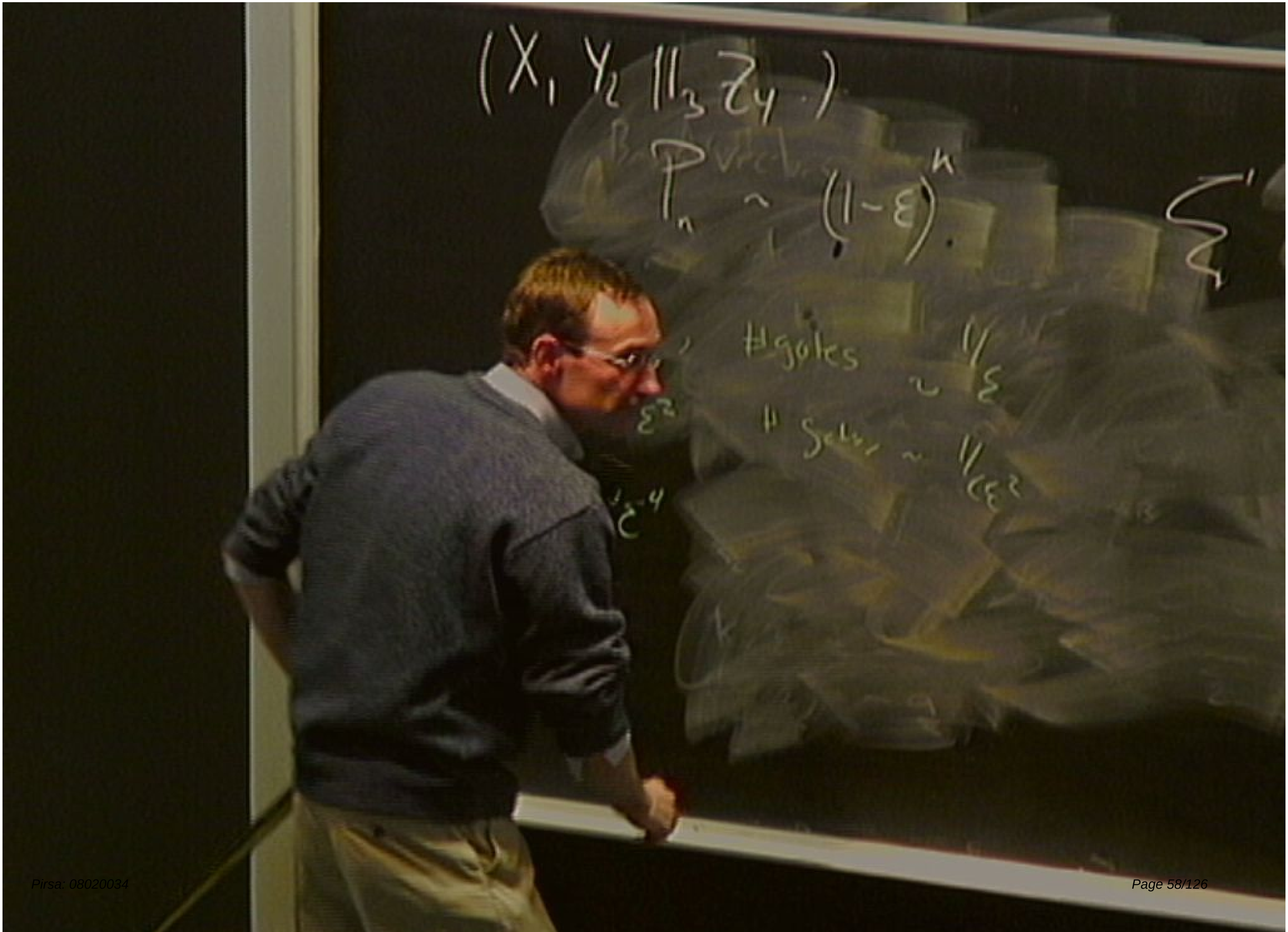
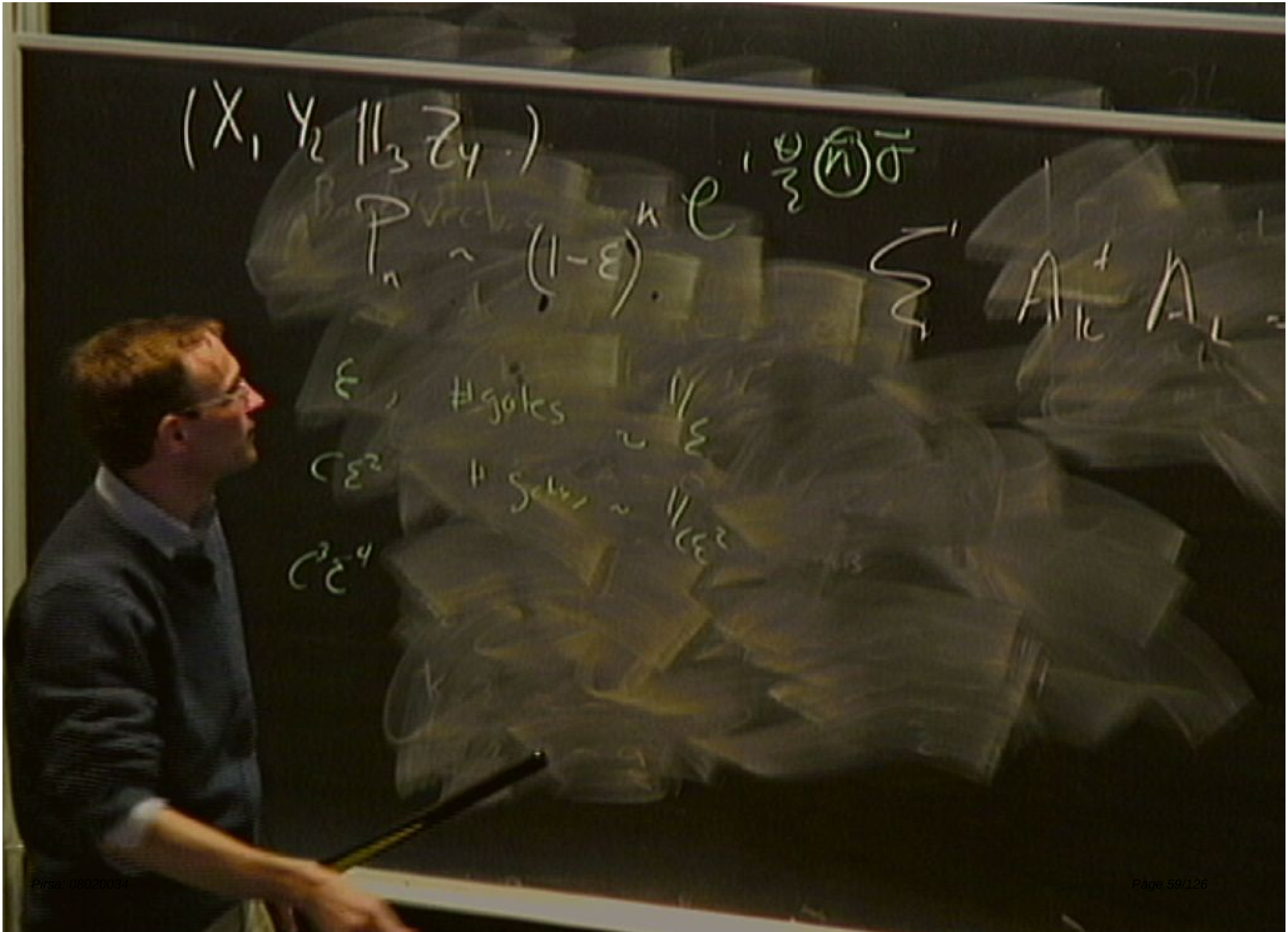
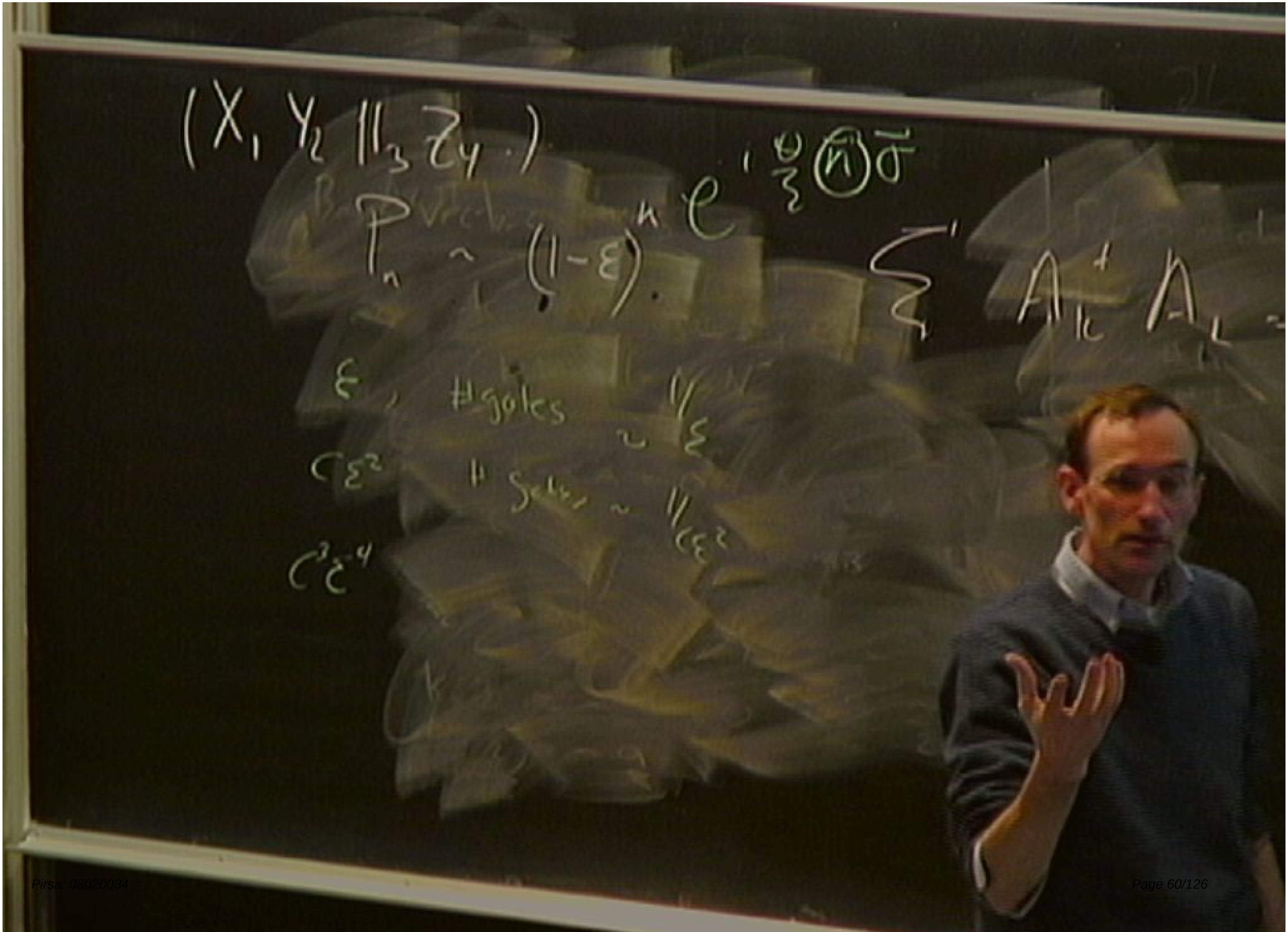



(X1 Y2 11 3 Z4 ...)
Pn ~ (1-ε)
ε
#gates
~ 1/ε
cε²
~ 1/cε²
c³ε⁴



#gates

Characterising noise in q. systems

$\ldots_{kl} P_k \rho_i P_l$

...re required as de-
...loch sphere:

$$\begin{pmatrix} \rho^f_{\mathbb{1}} \\ \rho^f_X \\ \rho^f_Y \\ \rho^f_Z \end{pmatrix} = \begin{pmatrix} \chi_{\mathbb{1},\mathbb{1}} & \chi_{\mathbb{1},X} & \chi_{\mathbb{1},Y} & \chi_{\mathbb{1},Z} \\ \chi_{X,\mathbb{1}} & \chi_{X,X} & \chi_{X,Y} & \chi_{X,Z} \\ \chi_{Y,\mathbb{1}} & \chi_{Y,X} & \chi_{Y,Y} & \chi_{Y,Z} \\ \chi_{Z,\mathbb{1}} & \chi_{Z,X} & \chi_{Z,Y} & \chi_{Z,Z} \end{pmatrix} \begin{pmatrix} \rho^i_{\mathbb{1}} \\ \rho^i_X \\ \rho^i_Y \\ \rho^i_Z \end{pmatrix}$$

$\ldots 4^{2n} - 4^n$ numbers

to do so.

Characterising noise in q. systems

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$$
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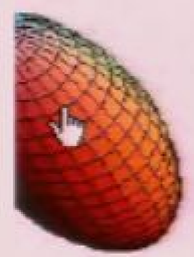
$$



$4^{2n} - 4^n$ **numbers**

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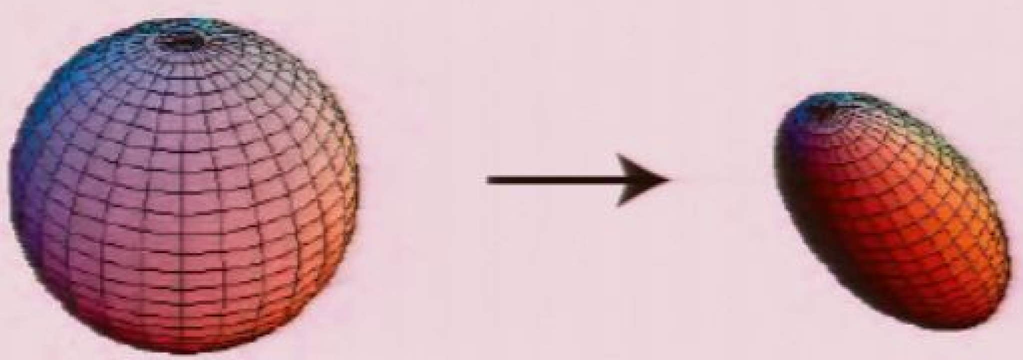

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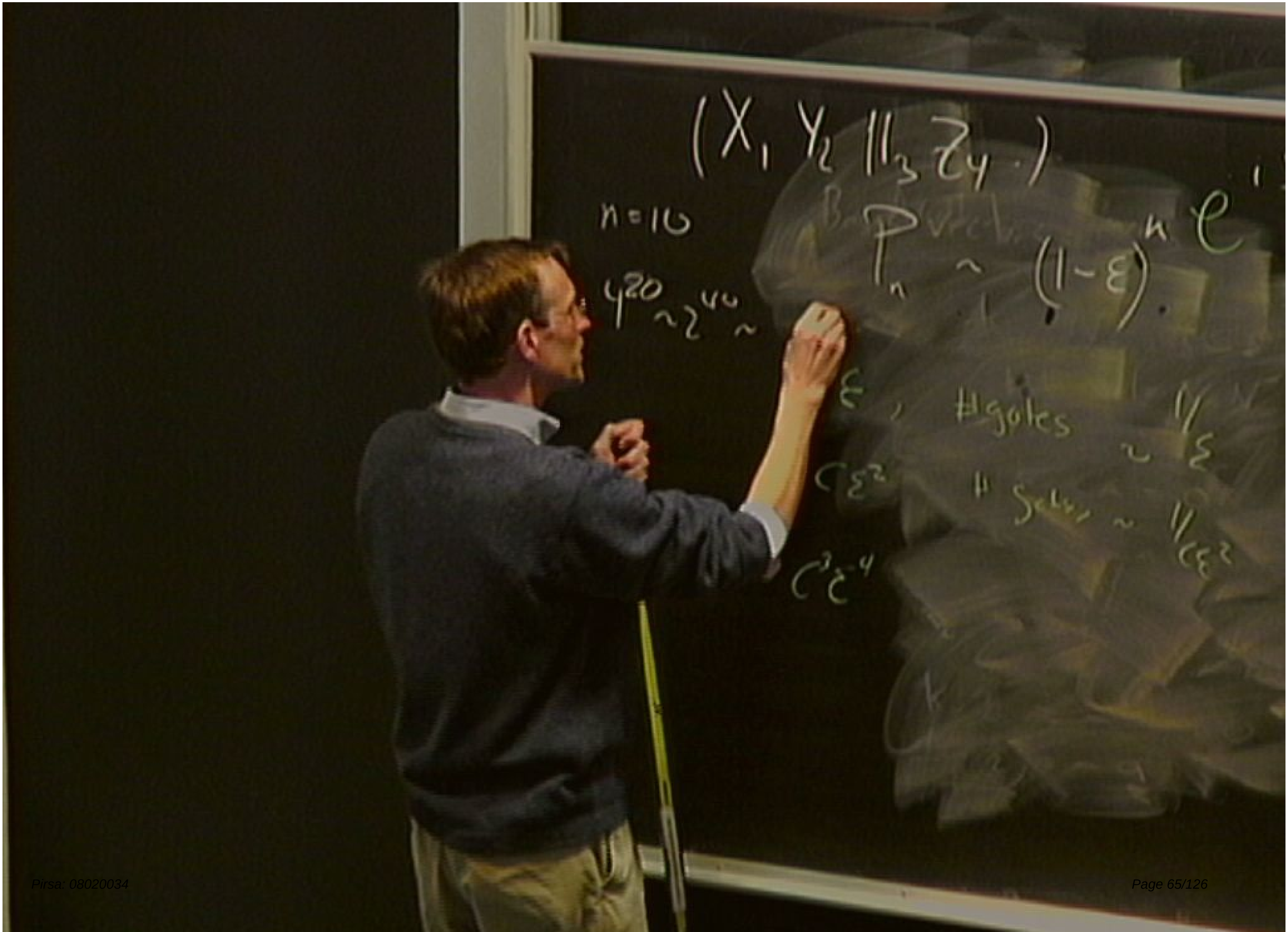
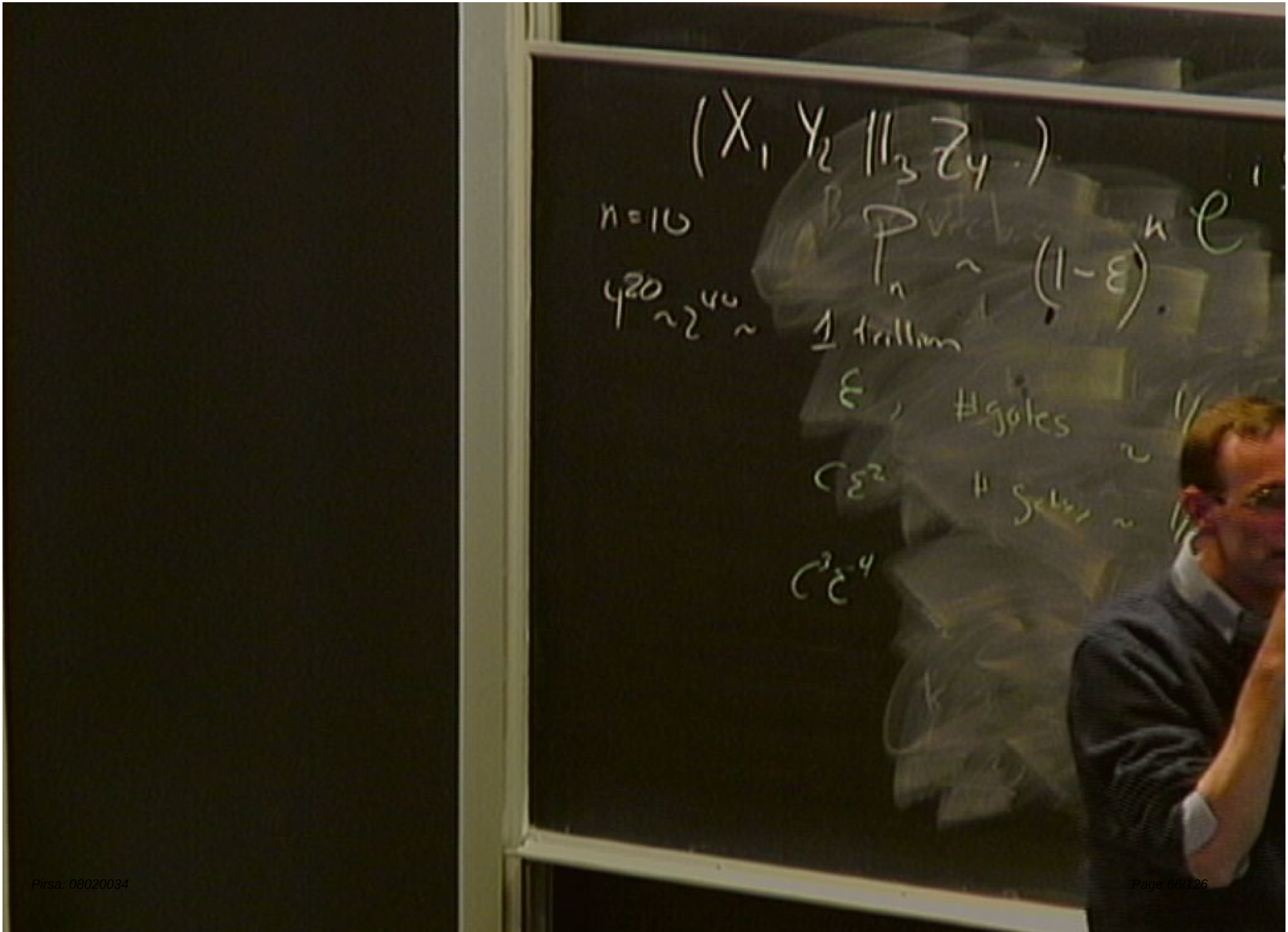
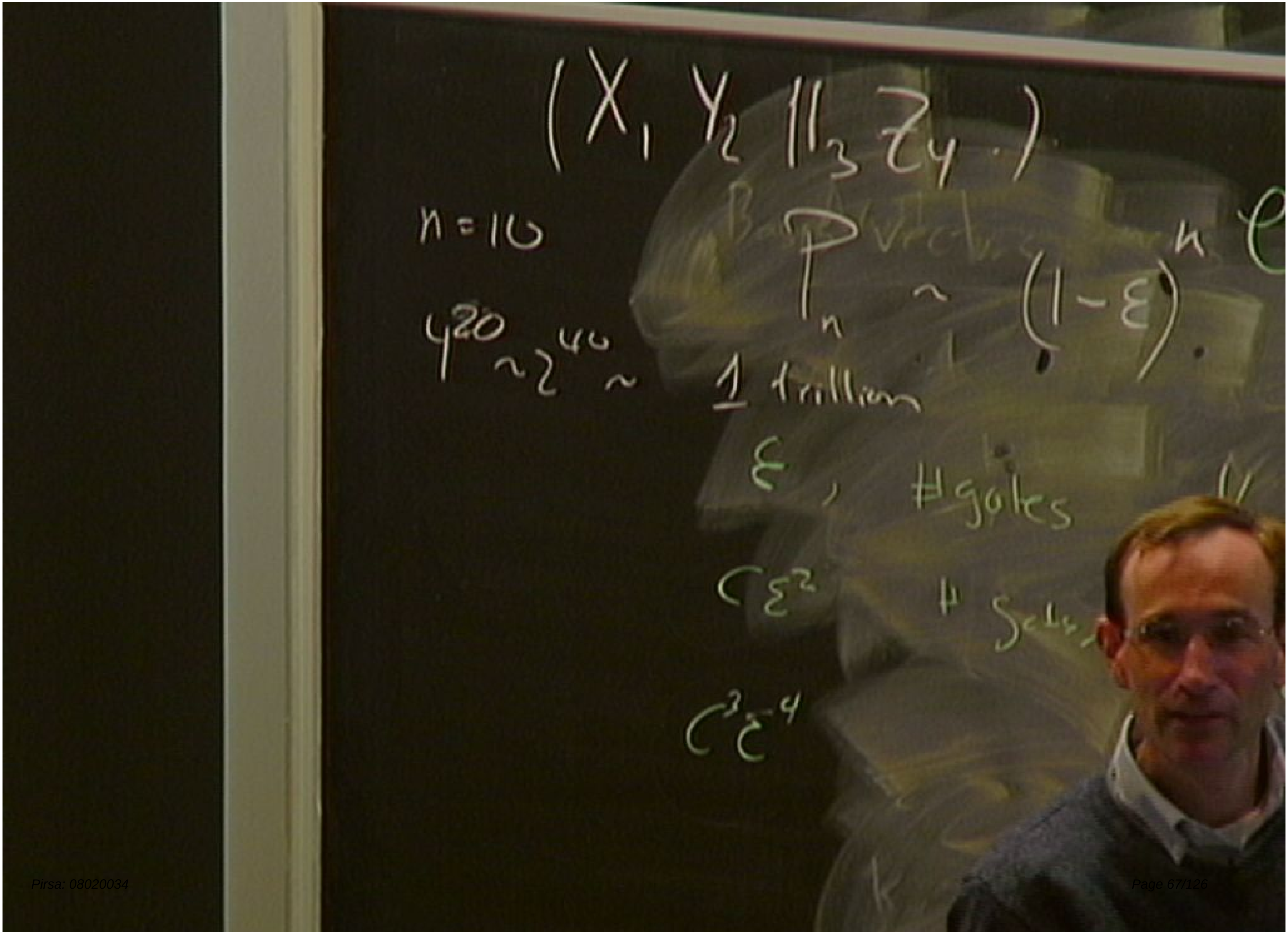
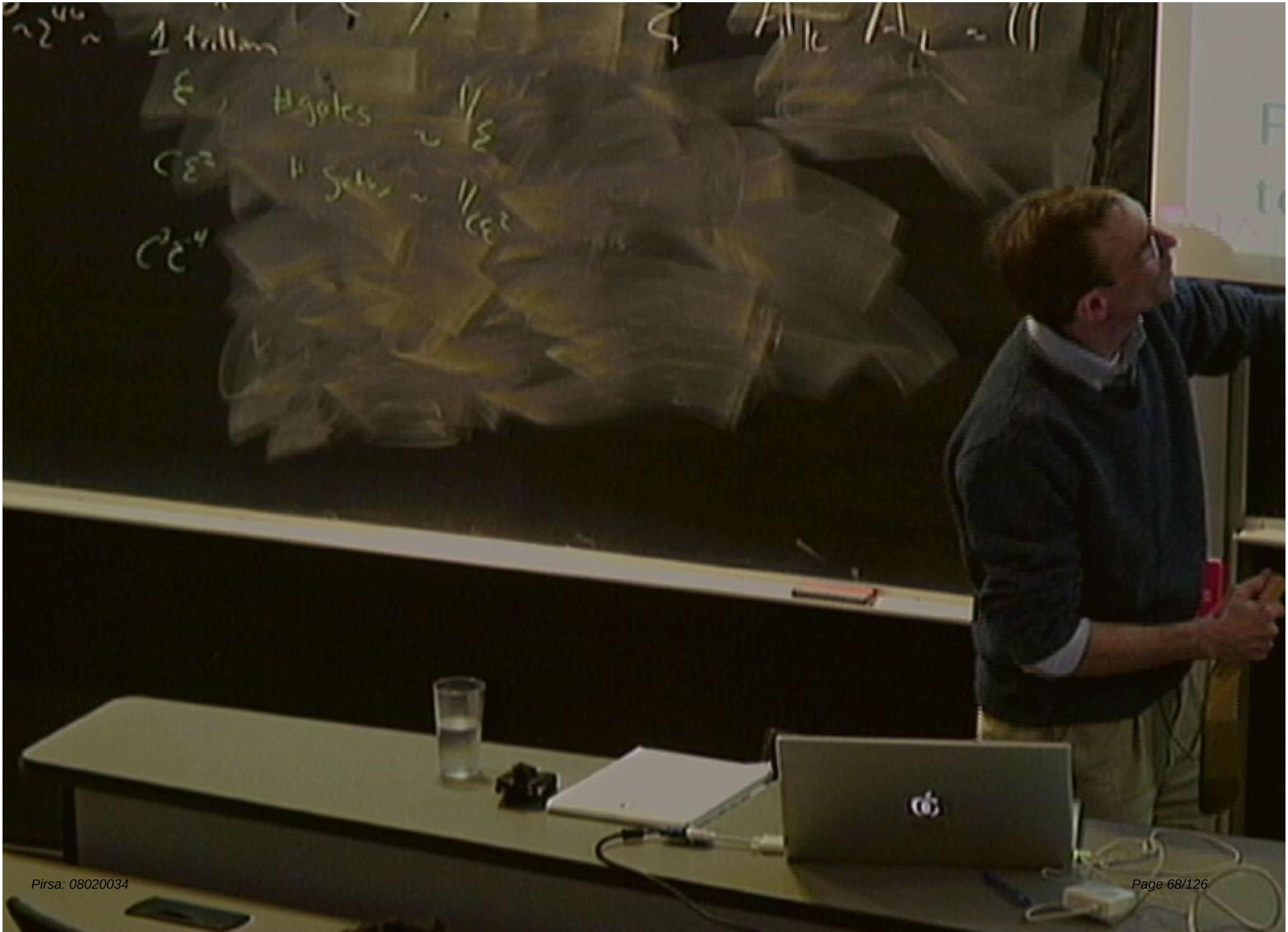
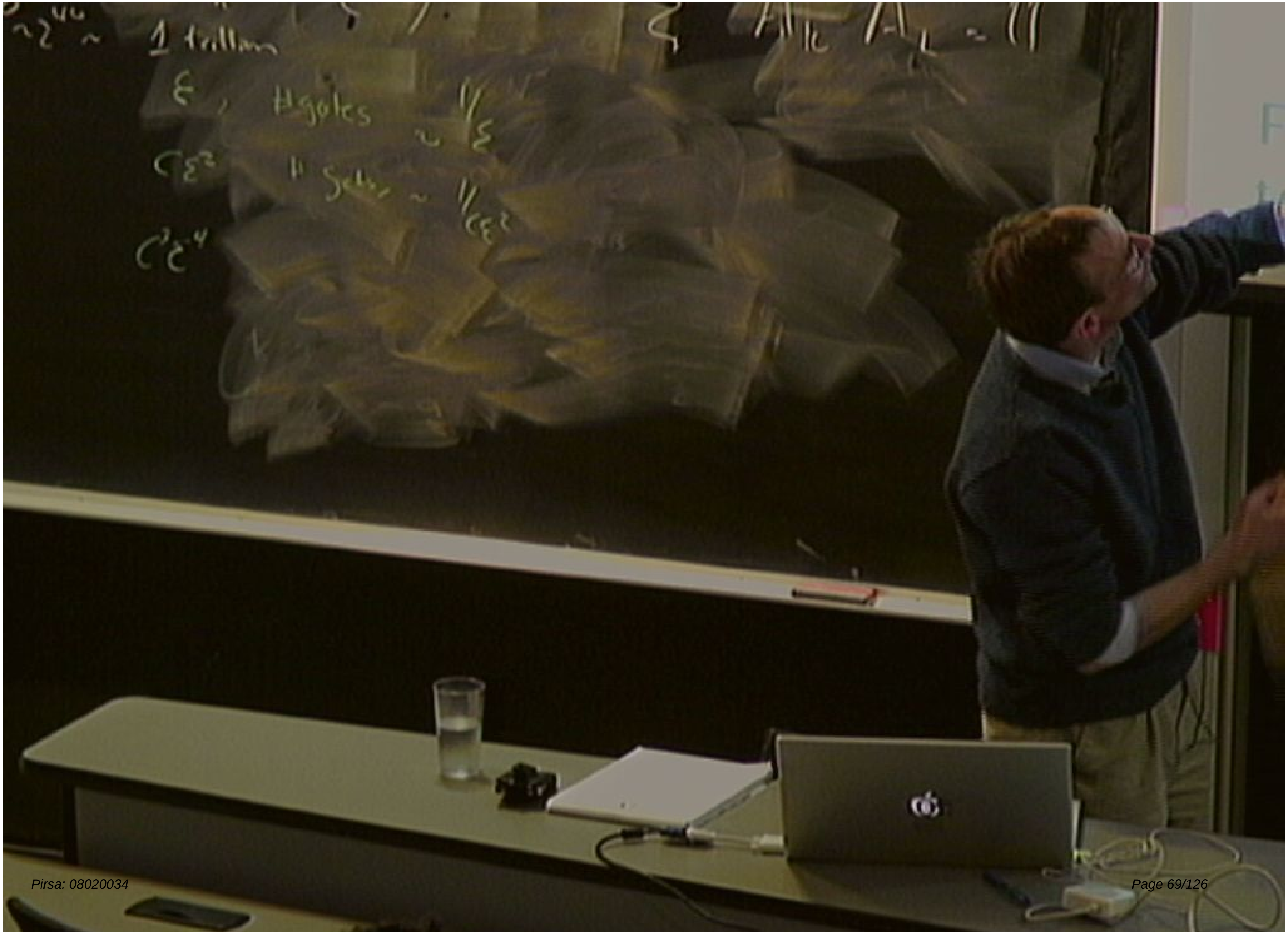
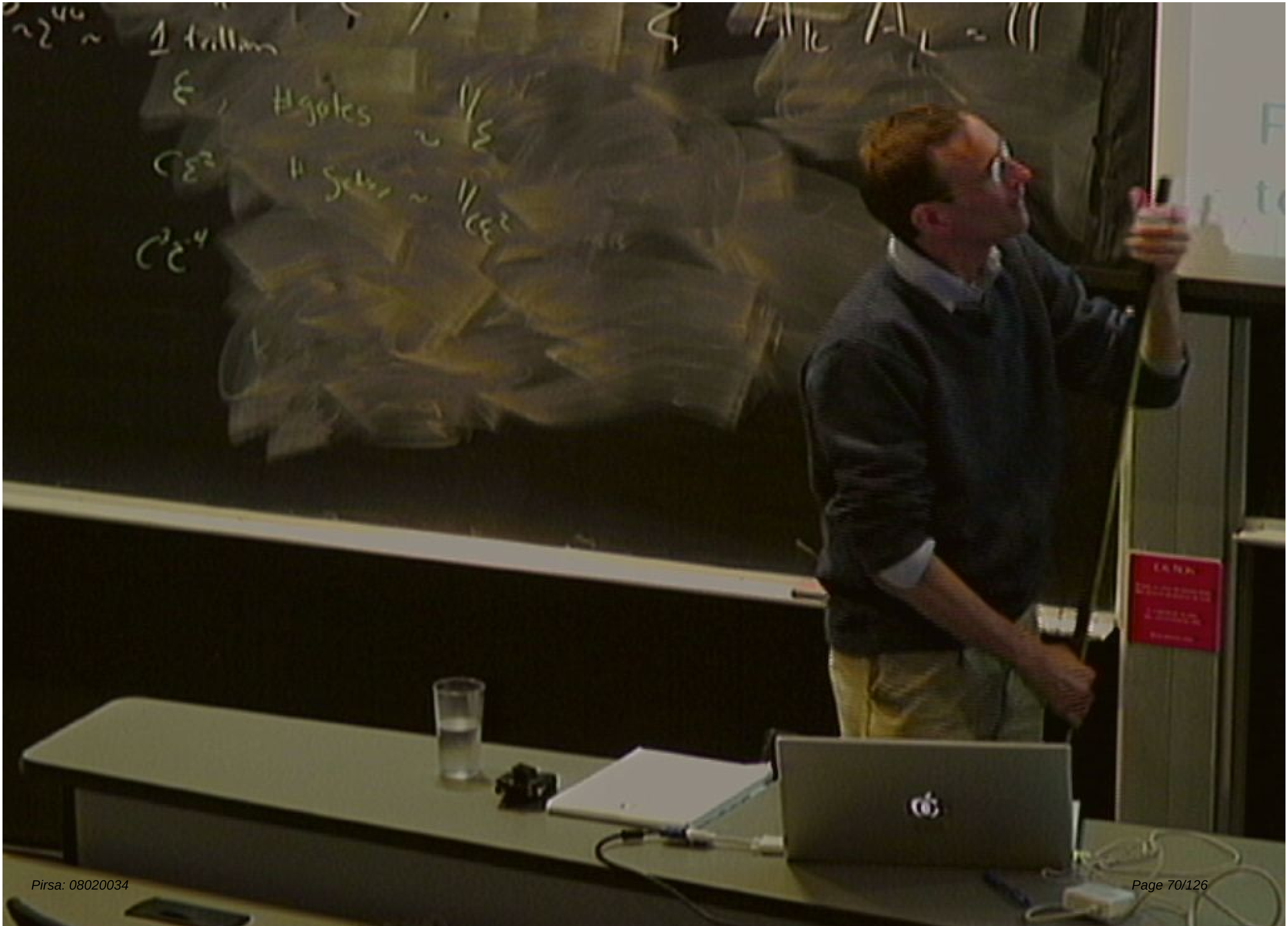

(X, Y Z .)
n=10
#gates ~ 1/ε
Salvos ~ 1/cε²


(X1 Y2 I3 Z4 .)
n = 10
1 trillion
gates


(X1 Y2 I3 Z4 .)
n = 10
4^20 ~ 2^40 ~ 1 trillion
Pn ~ (1−ε)^n
ε , #gates
cε^2
c^3ε^4


1 trillion
#gates


1 trillion
#gates


1 trillion
gates
Gates

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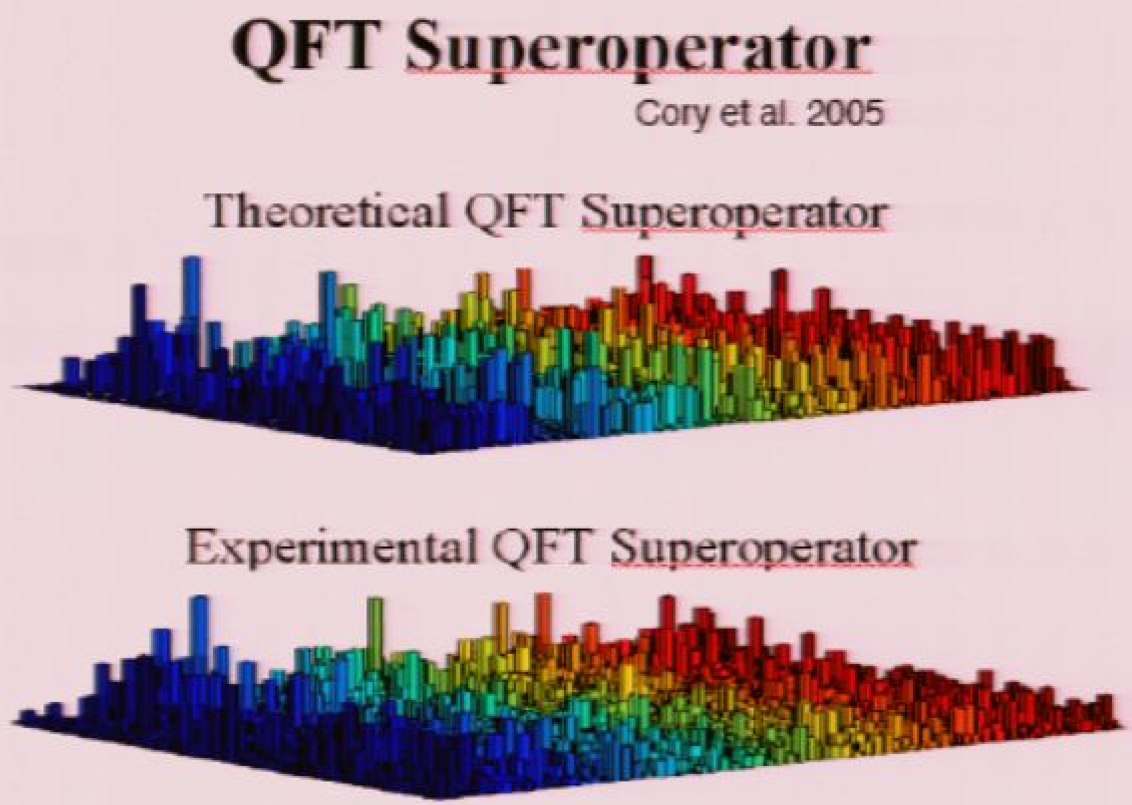

P_l

For one ... **ired as de- scribed** ... **here:**

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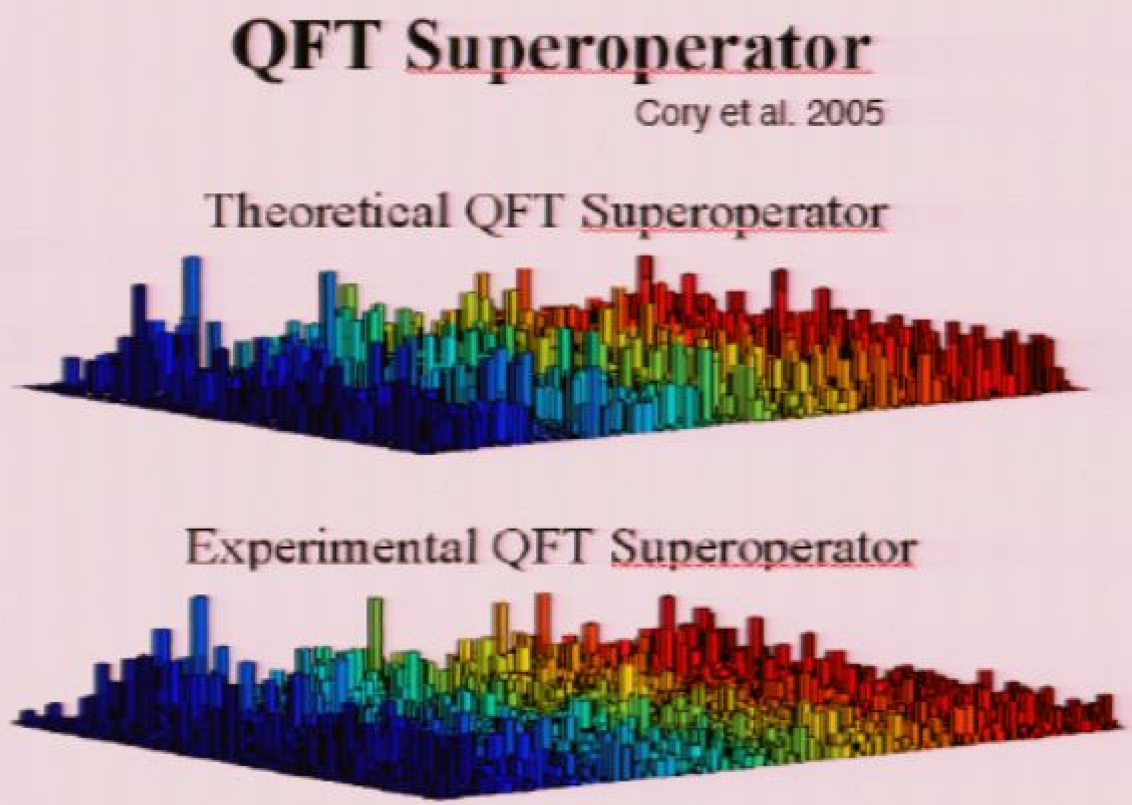
Process tomography:

P_l



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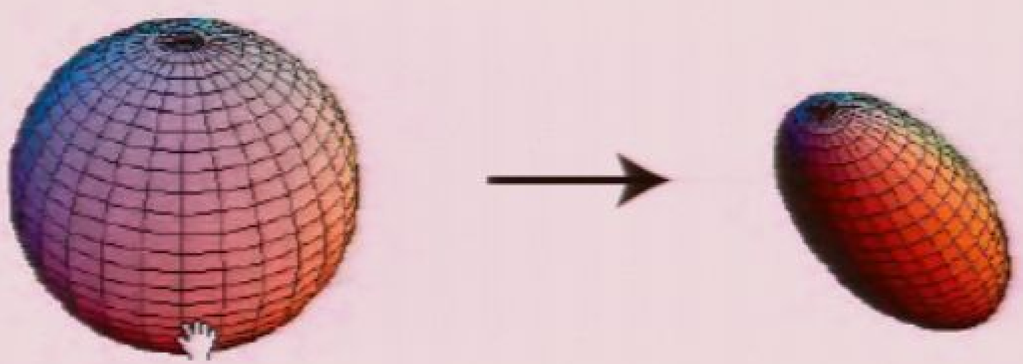

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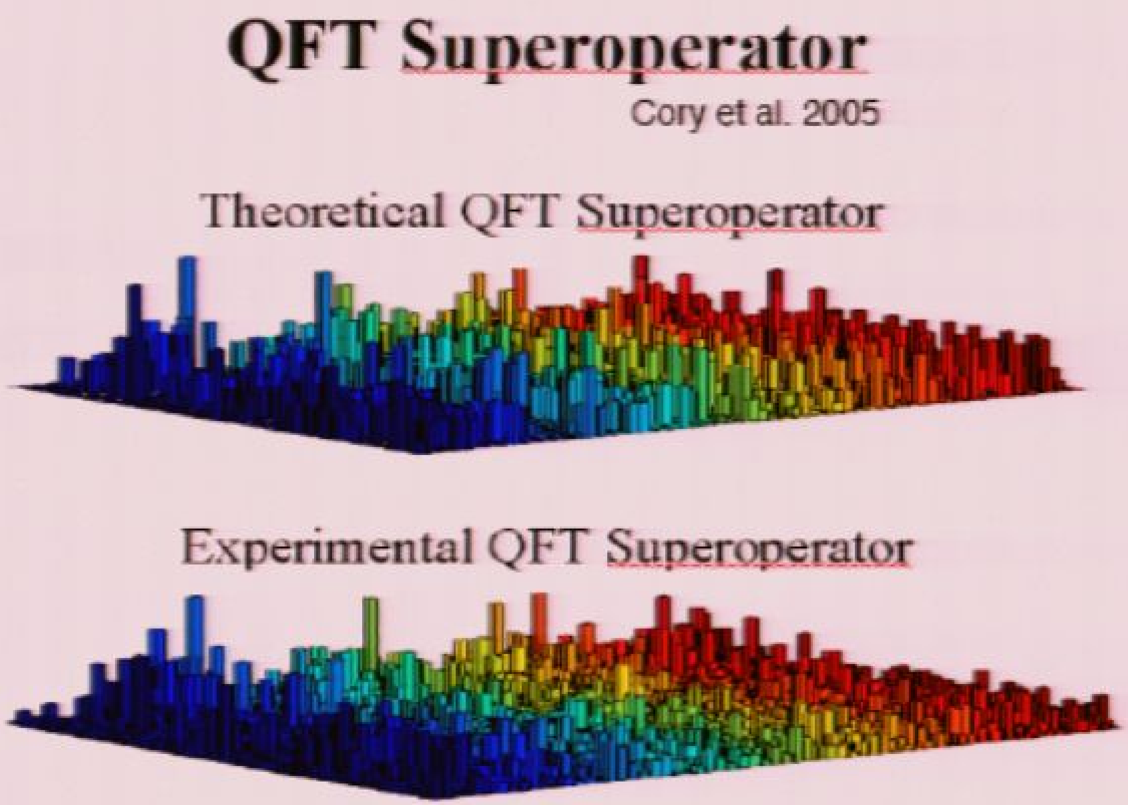

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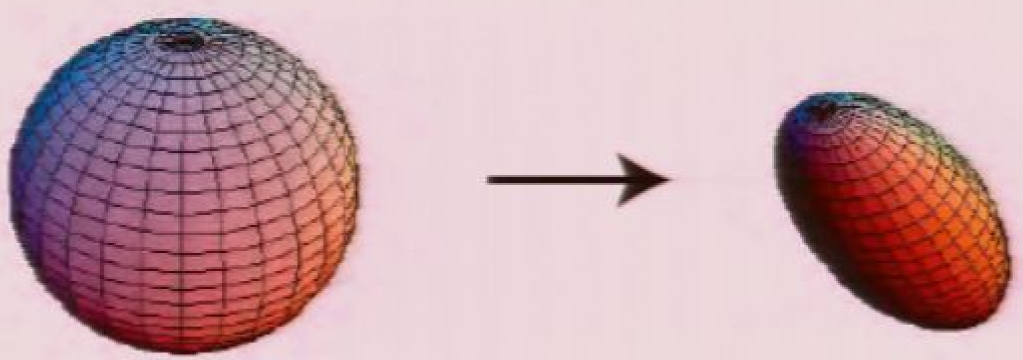


For n qubits, we need to provide $4^{2n} - 4^n$ numbers to do so.

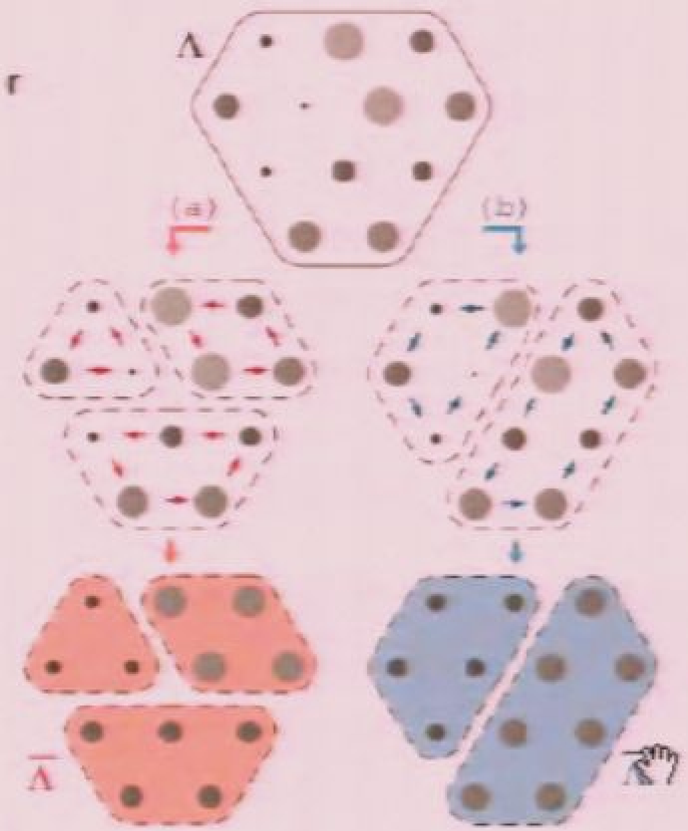
Coarse graining

• We are not interested in all the elements that describe the full noise superopeartor but only a coarse graining of them.

• If we are interested in implementing quantum error corrrection, we can ask what is the probbaility to get one, or two, or k qubit error, independent of the location and independent of teh type of error $\sigma_{x,y,z}$. The question is can we do this efficiently?

• Coarse graining is equivalent to implement a symmetry.

$$\Lambda(\rho) = \sum_{k}^{D^2} A_k \rho A_k^\dagger$$



Schematic illustration of coarse-graining.

Zanardi, Phys.Lett. A258 (1999) 77

Coarse graining

1) to coarse error type average over $SU(2)^{\otimes n}$

$$\rho_f = \sum_k \int d\mu(U) U^\dagger A_k U \rho_i U^\dagger A_k^\dagger U$$

This is an example of a 2-design, and the integral can be replaced by a sum

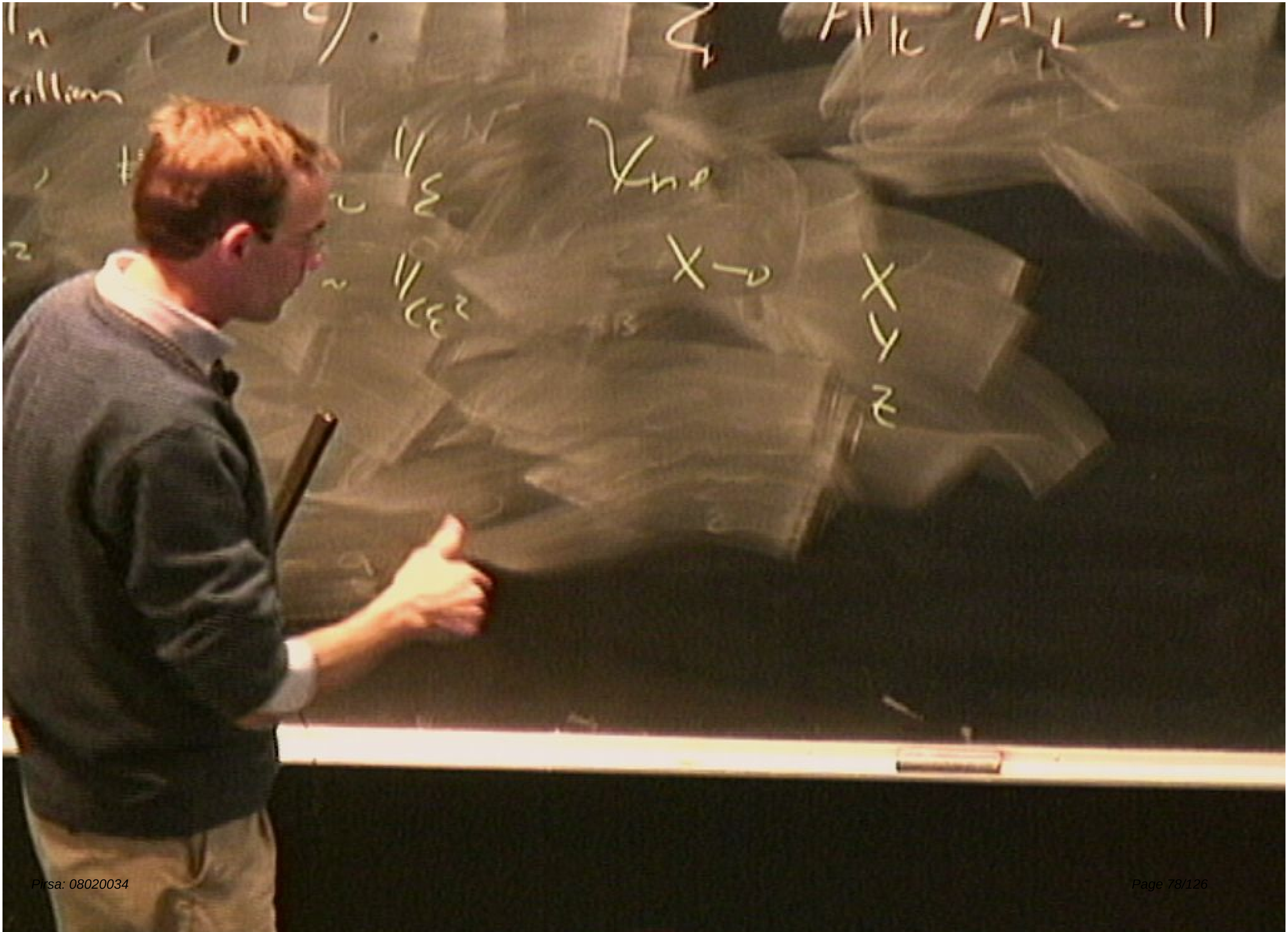
C. Dankert, R. Cleve, J. Emerson, E. Livine, quant-ph/0606161 (2006).

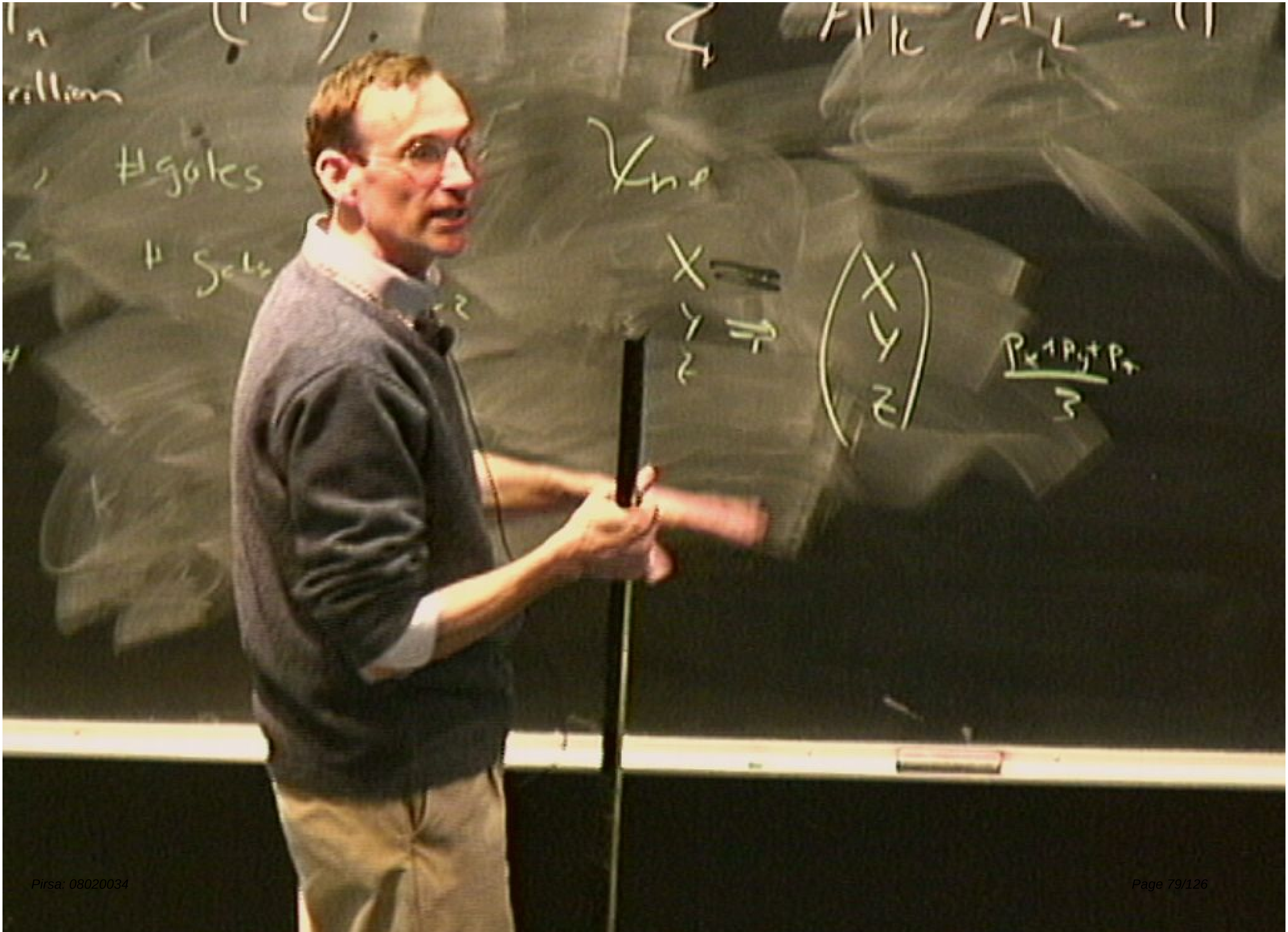
$$\rho_f = \sum_k \sum_\alpha C_\alpha^\dagger A_k C_\alpha \rho_i C_\alpha^\dagger A_k^\dagger C_\alpha$$

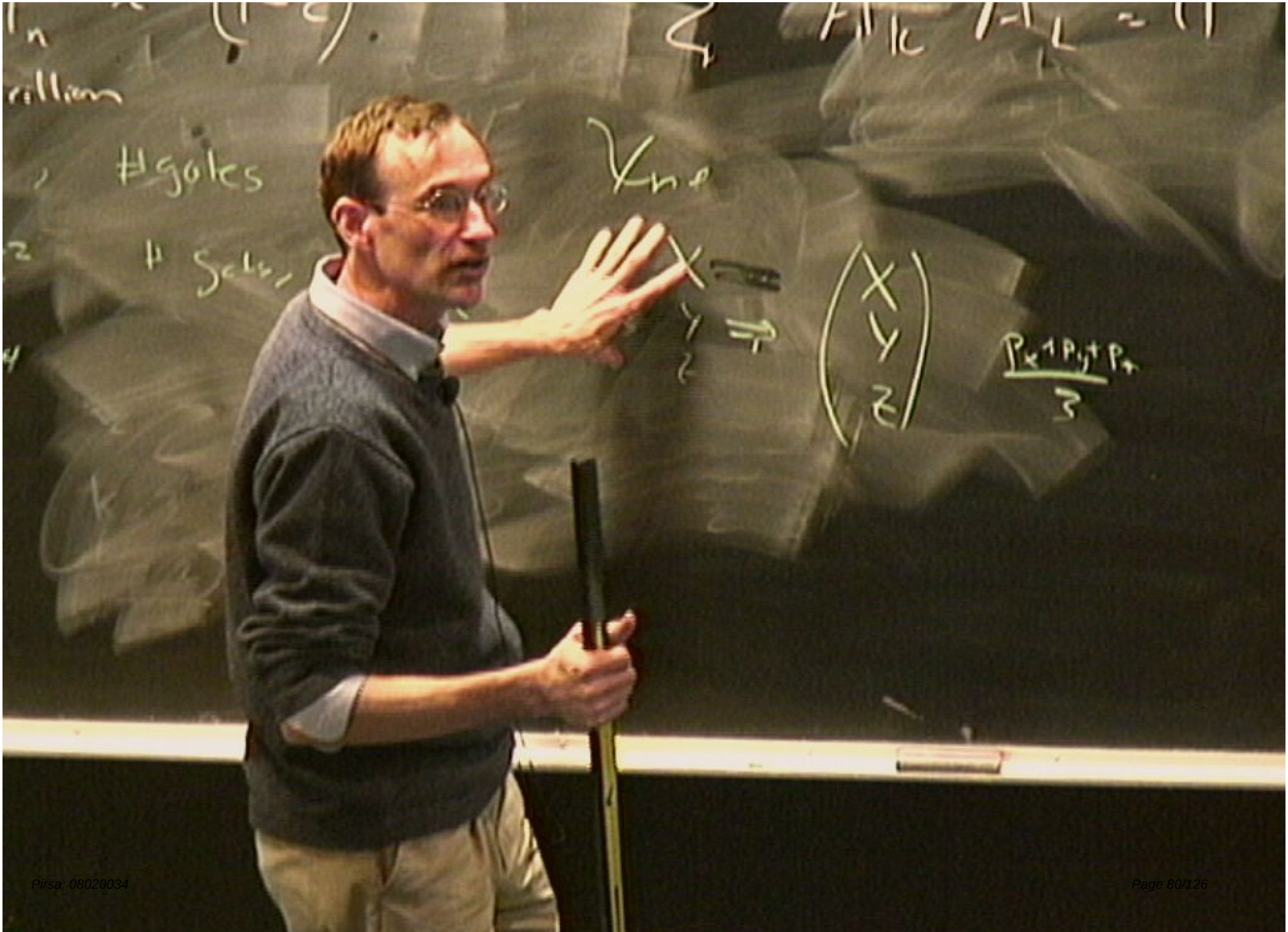
where C_α belongs to the Clifford group $\sim \mathcal{SP}$ with $\mathcal{P} = \{\mathbb{1}, X, Y, Z\}$, $\mathcal{S} = \{e^{-i\frac{\pi}{4}X}, e^{-i\frac{\pi}{4}Y}, e^{-i\frac{\pi}{4}Z}\}$

2) coarse grain the position by symmetrising using permutation π_s

$$\rho_f = \sum_k A_k \rho_i A_k^\dagger = \sum_{kl} \chi_{kl} P_k \rho_i P_l$$

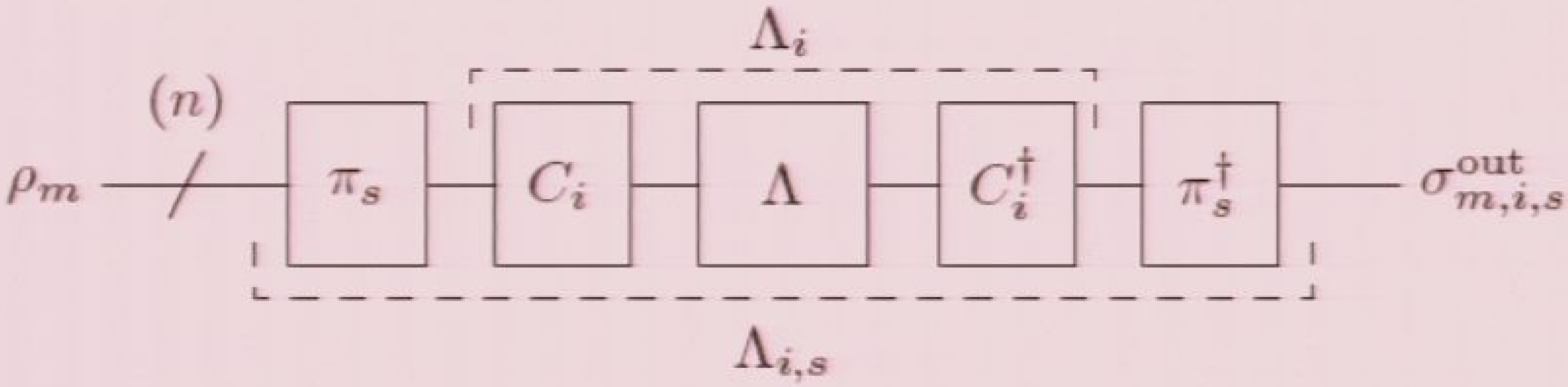



#gates



#goles
X
Y
Z

Coarse graining

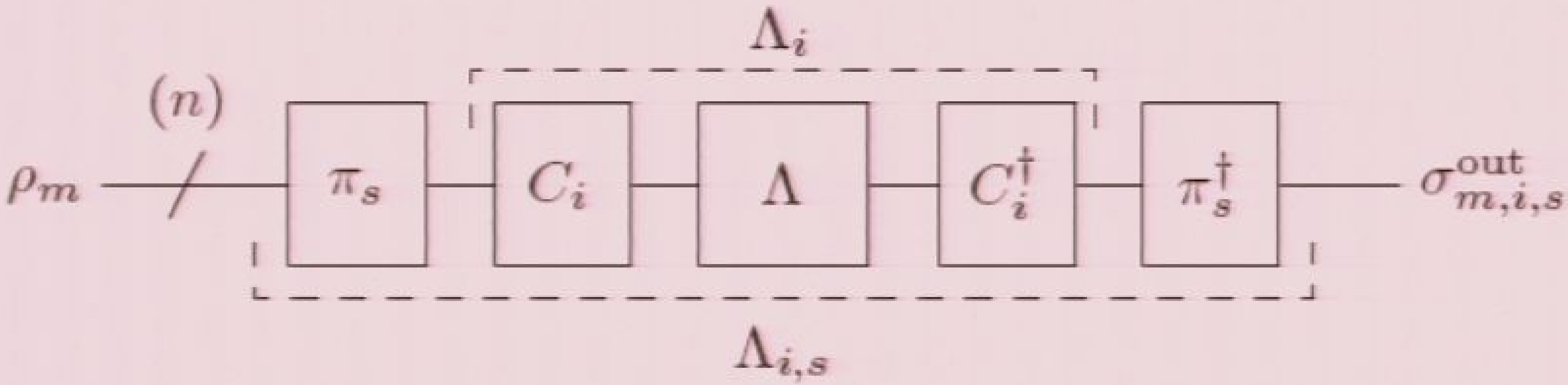


If we implement all the elements in the Clifford group and permutation, we would have an exponential number of terms , but the sum can be estimated by sampling and using the Chernoff bound.

(see Emerson et al. arXiv:0707.0685)

In practice, implementing the symmetrisation can be done by starting with the state $|000\ldots\rangle$ and measure the Hamming size (i.e. the number of 1) in the final state.

Coarse graining



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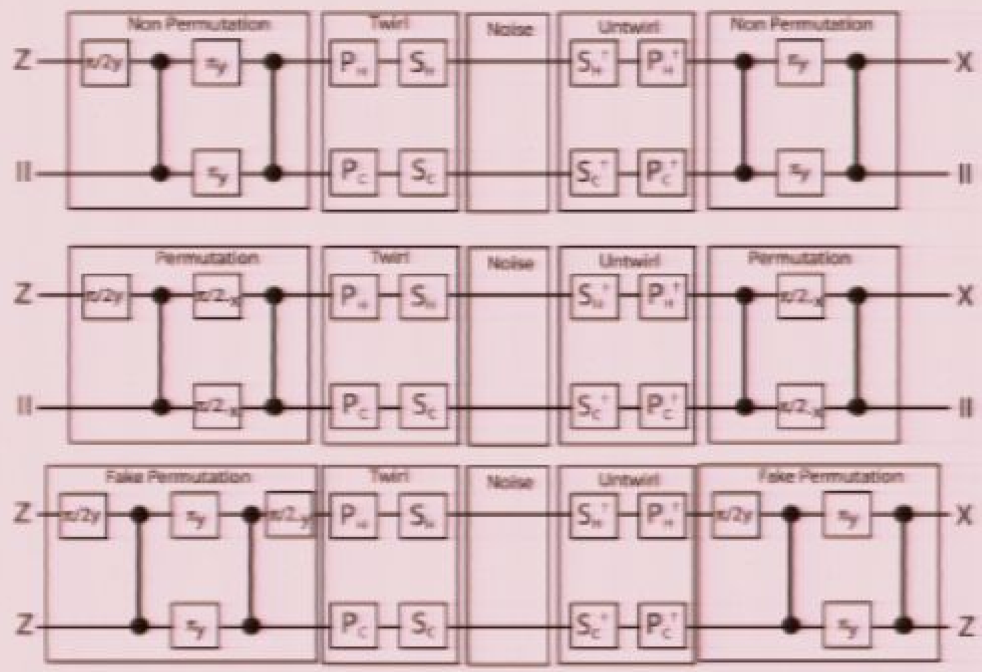
NMR implementation: 2 qubits

Twirling in liquid state NMR

The Implementation

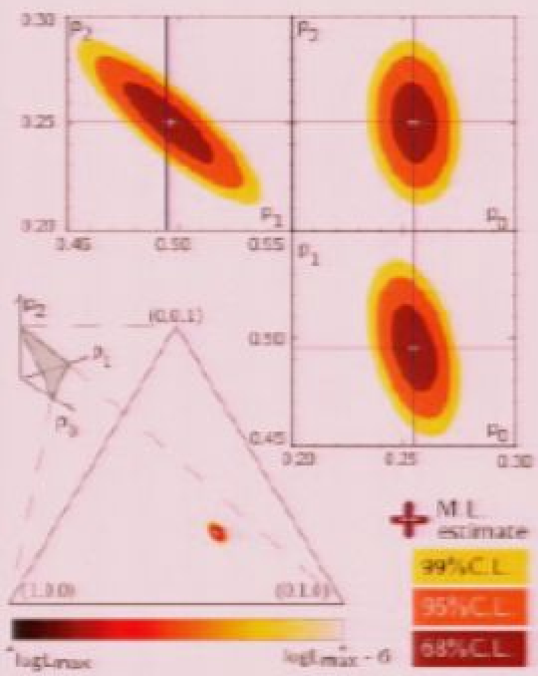


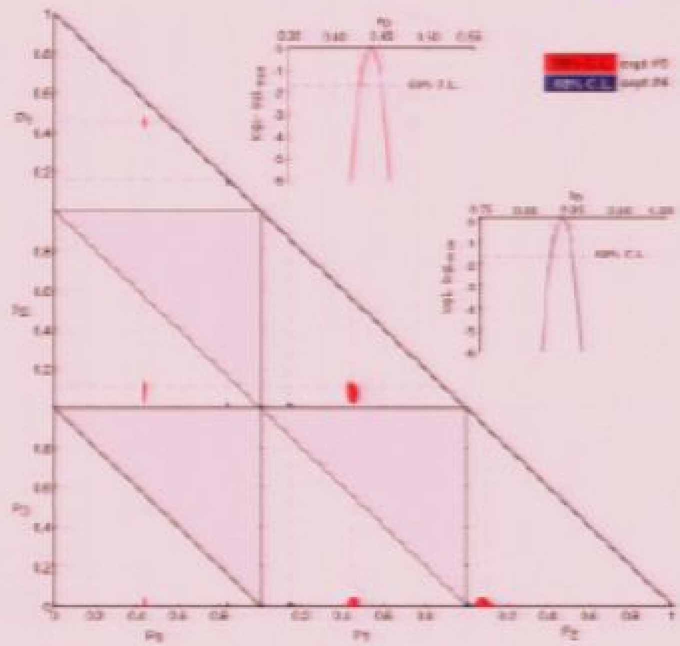
Non-permutation and fake permutation are perform so that all experiment can be compared on the same footing.

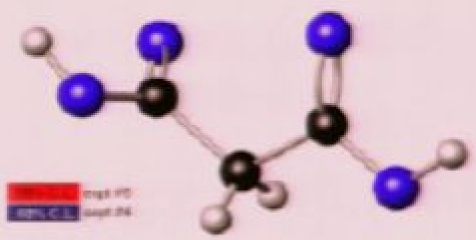
Emerson, Silva, Moussa, Ryan, Laforest, Baugh, Cory, Laflamme, Science 317, 1893, 2007

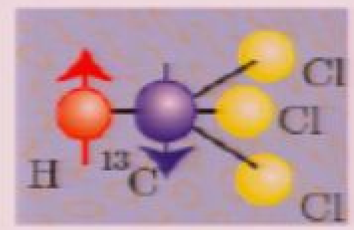
Experimental results

Noise Characterization - NMR results









#	Map Description	Kraus operators (A_k)	k_m	p_0	p_1	p_2	p_3
1	Engineered: p = [0, 1, 0].	$\frac{1}{\sqrt{2}}\{Z_1, Z_2\}$	288	$0.000^{+0.004}$	$0.991^{+0.009}_{-0.015}$	$0.009^{+0.017}_{-0.009}$	–
2	Engineered: p = [0, 0, 1].	$\{Z_1 Z_2\}$	288	$0.001^{+0.006}_{-0.001}$	$0.004^{+0.011}_{-0.004}$	$0.996^{+0.004}_{-0.011}$	–
3	Engineered: p = [1/4, 1/2, 1/4].	$\{exp[i\frac{\pi}{4}(Z_1 + Z_2)]\}$	288	$0.254^{+0.018}_{-0.018}$	$0.495^{+0.021}_{-0.029}$	$0.250^{+0.019}_{-0.019}$	–
4	Engineered: p = [0, 1, 0, 0].	$\frac{1}{\sqrt{3}}\{Z_1, Z_2, Z_3\}$	432	$0.01^{+0.01}_{-0.01}$	$0.99^{+0.01}_{-0.03}$	$0.01^{+0.02}_{-0.01}$	$0.00^{+0.01}$
5	Natural noise (a)	unknown	432	$0.44^{+0.01}_{-0.02}$	$0.45^{+0.03}_{-0.03}$	$0.10^{+0.04}_{-0.08}$	$0.01^{+0.03}_{-0.01}$
6	Natural noise (b)	unknown	432	$0.84^{+0.01}_{-0.01}$	$0.15^{+0.02}_{-0.03}$	$0.01^{+0.03}_{-0.01}$	$0.00^{+0.02}$

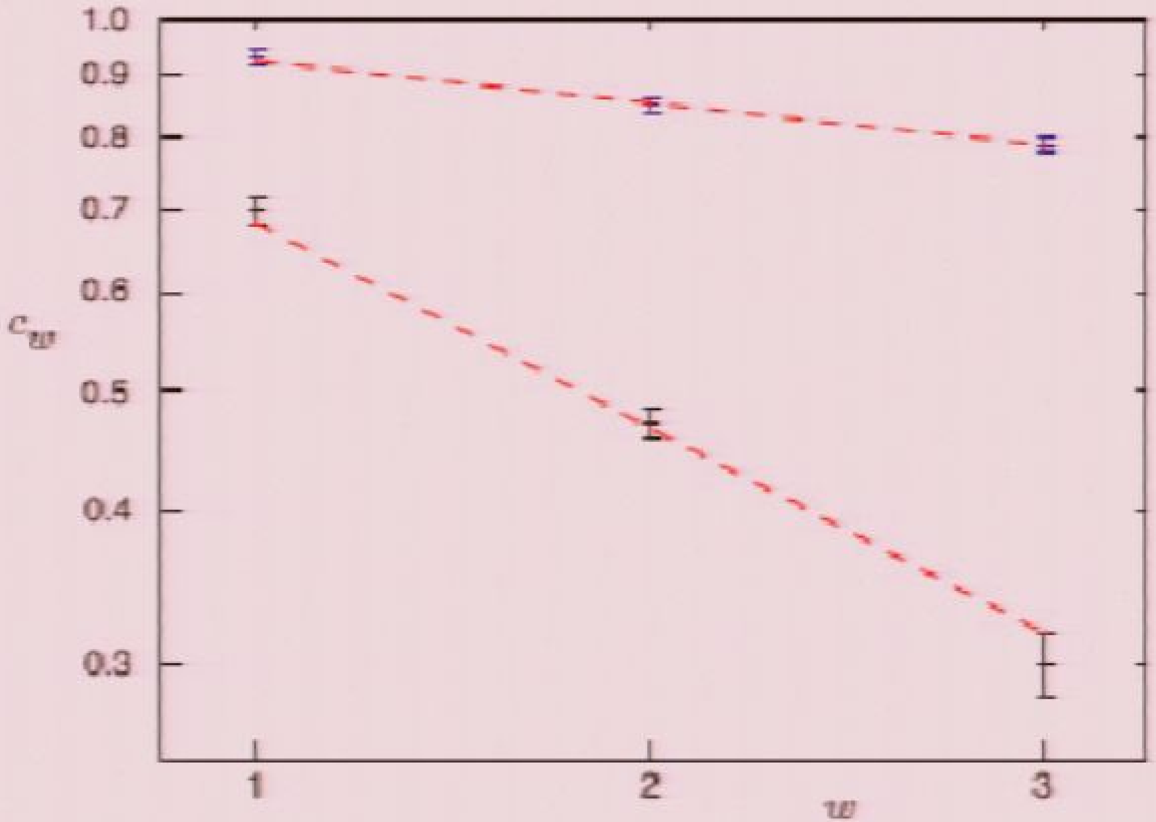
TABLE I: Summary of experimental results.

Emerson, Silva, Moussa, Ryan, Laforest, Baugh, Cory, Laflamme, Science 317, 1893, 2007

Checking for noise independence

If the noise is independent

$$c_\omega = (c_1)^\omega$$

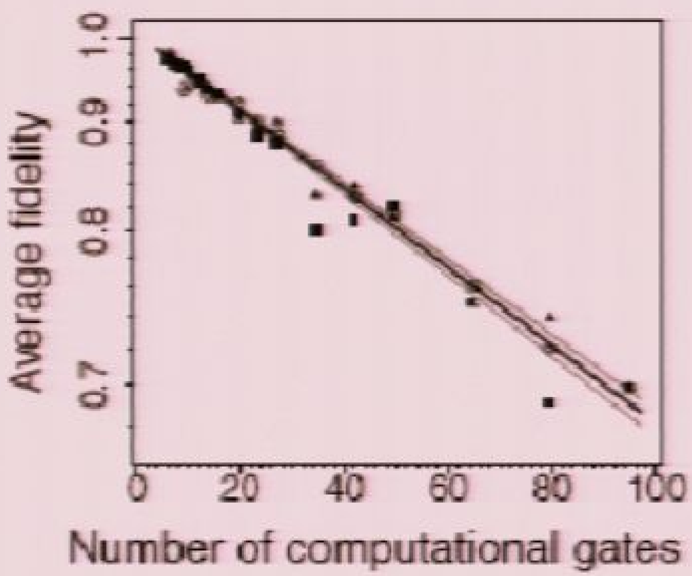


Randomized Benchmarking of Quantum Gates

- Wish to characterize the computationally relevant errors per gate in a general context
- Process tomography gives complete information about a particular instance of a particular gate and is limited by errors in preparation and measurement
- Furthermore, we need a scalable scheme to benchmark a large-scale quantm information processor efficiently
- Scheme is to apply a sequence of random gates and measure the fidelity decay as a function of the number of gates

E. Knill, et al. arXiv:0707.0963

Benchmarking gates in ion traps



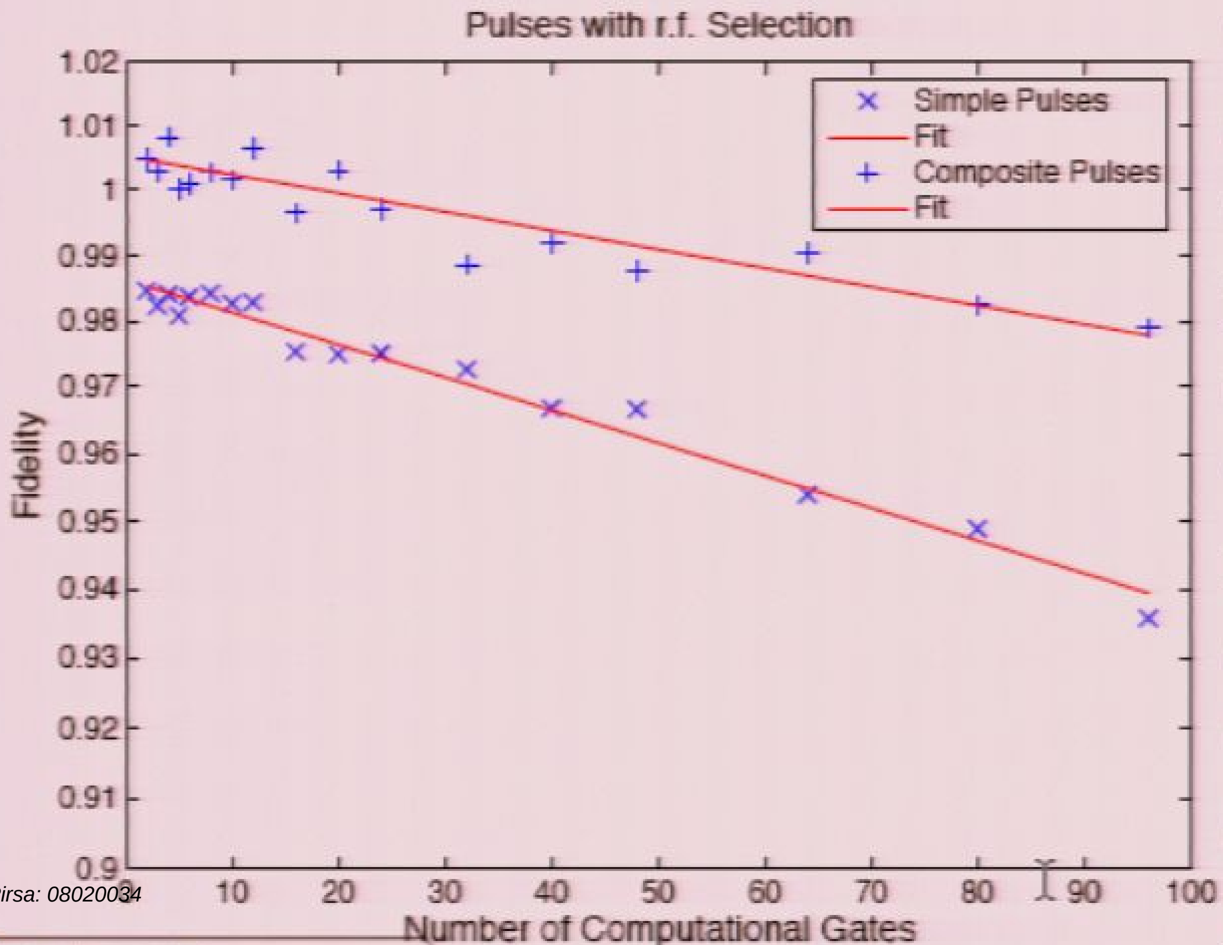
Knill, Liefried, Reichle, Britton, Blakestad, Jost, Lnager, Ozeri, Seidelin, Wineland, arXiv:0707.0963

0.00482 error per randomized gates

FIG. 2: Average fidelity as a function of the number of steps for each computational sequence. The points show the average randomized fidelity for four different computational gate sequences (indicated by the different symbols) as a function of the length. The average fidelity is plotted on a logarithmic scale. The middle line shows the fitted exponential decay. The upper and lower line show the boundaries of the 68% confidence interval for the fit. The standard deviation of each point due to measurement noise ranges from 0.0004 for values near 1 to 0.002 for the lower values, smaller than the size of the symbols. The empirical standard deviation based on the scatter in the points shown in Fig. 1 ranges from 0.0011 to 0.014. The slope implies an error probability of 0.00482(17) per randomized computational gate. The data is consistent with the gate's errors not depending on position in the sequence.

Benchmarking gates in NMR

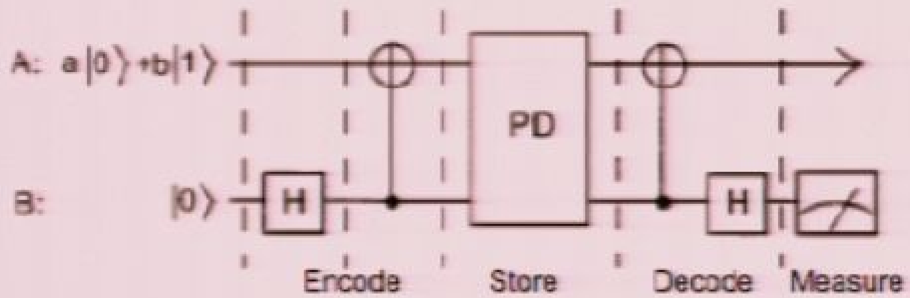
C. Ryan et al. unpublished

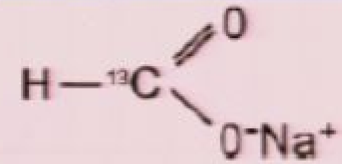


- Exponential decay suggests gate error does not depend on where in sequence it lies
- Suggests an error per randomized computational gate of $5x10^{-4}/3x10^{-4}$ for simple/composite pulses

2 qubit phase damping code

Leung et al., PRA 60, 1924, 1999





- 2 qubits (H and ^{13}C from formate)
- Error model: independent phase damping

$$\begin{bmatrix} a & b^* \\ b & c \end{bmatrix} \to \int p(\theta) e^{-i\theta\hat{z}} \rho e^{i\theta\hat{z}} d\theta \to \begin{bmatrix} a & e^{-\lambda t} b^* \\ e^{-\lambda t} b & c \end{bmatrix}, \lambda = \frac{1}{T_2}$$

- Phase-flip rate:

$$p(t) = (1 - e^{-\lambda t})/2$$

- Stabilizer code, generators ={XX}

$$|0_L\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle), \quad |1_L\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle).$$

- Acceptance of a state depends on the syndrome

$$|\psi_{II}\rangle \overset{dec}{\Rightarrow} (a|0\rangle + b|1\rangle)|0\rangle,$$

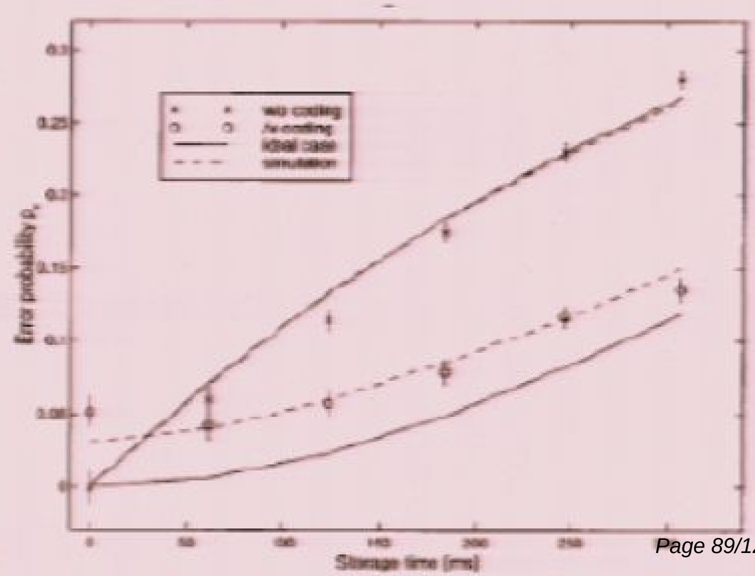
$$|\psi_{ZI}\rangle \Rightarrow (a|0\rangle - b|1\rangle)|1\rangle,$$

$$|\psi_{IZ}\rangle \Rightarrow (a|0\rangle + b|1\rangle)|1\rangle,$$

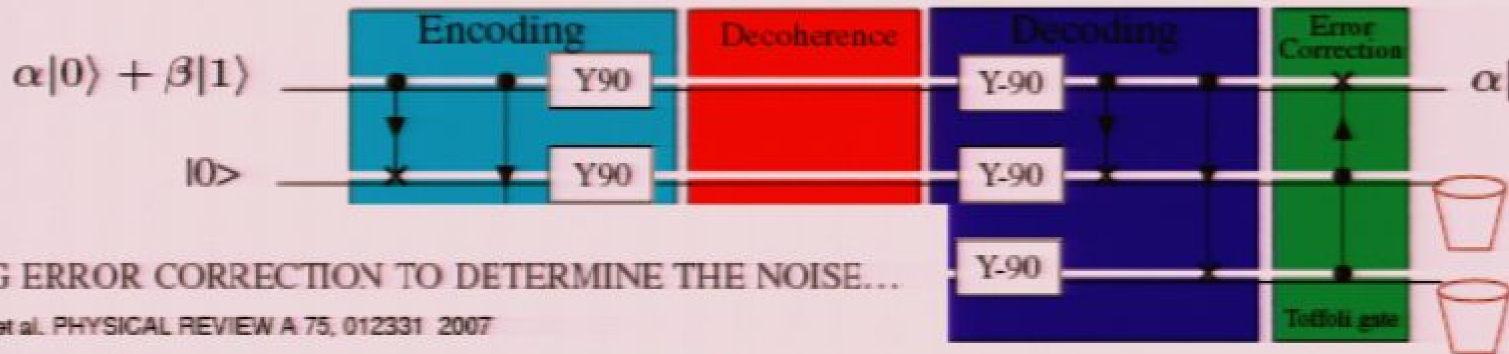
$$|\psi_{ZZ}\rangle \Rightarrow (a|0\rangle - b|1\rangle)|0\rangle.$$

- Conditional probability of error-free storage

$$\frac{(1-p_a)(1-p_b)}{(1-p_a)(1-p_b) + p_a p_b} \sim 1 - p_a p_b$$



Q. Error Correction for Phase



USING ERROR CORRECTION TO DETERMINE THE NOISE...

M. Laforest et al. PHYSICAL REVIEW A 75, 012331 2007

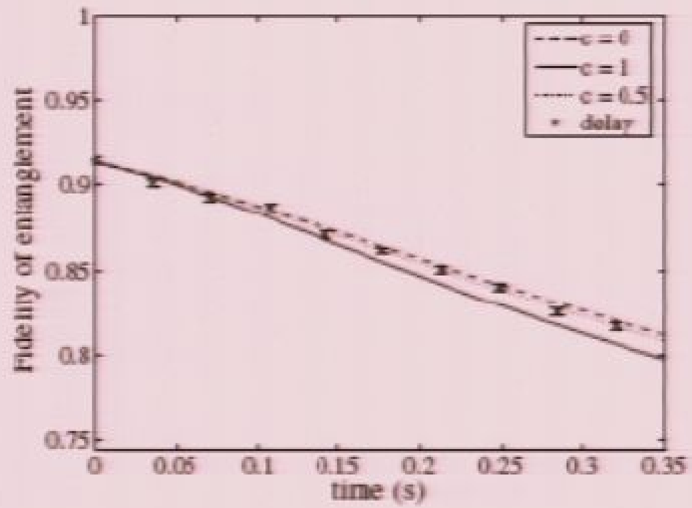


FIG. 3. Experimental results. The lines correspond to the fidelity decay for noise correlation factors of 0, 0.5, and 1 as a function of time simulated from the measured T_2's and the experimental fidelities obtained by implementing engineered noise. The points are the fidelities when the system is affected by natural noise for a various amount of time.

Errors: $+ \rightarrow -$

$- \rightarrow +$

$+ \beta|1\rangle)|00\rangle$

$+ \beta|0\rangle)|11\rangle$

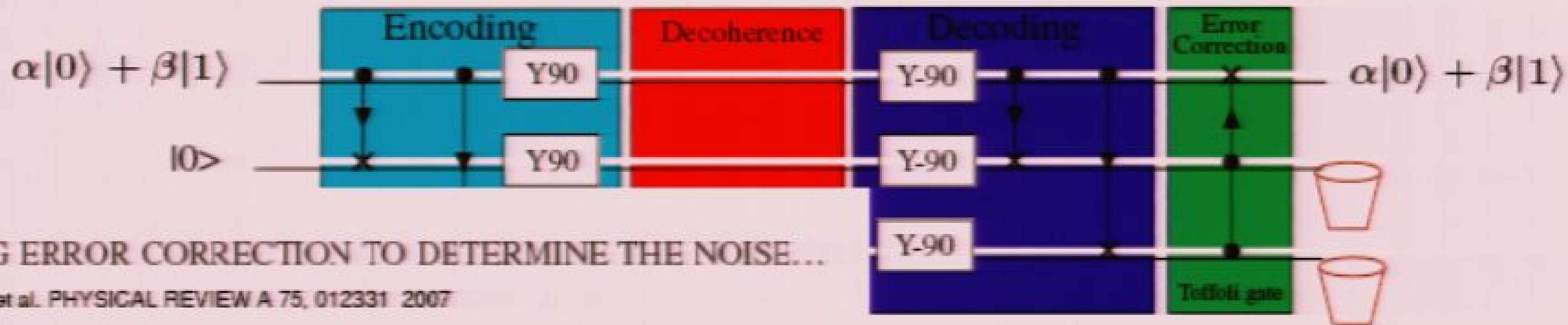
$+ \beta|1\rangle)|01\rangle$

$+ \beta|1\rangle)|10\rangle$

$$(\alpha|0\rangle + \beta|1\rangle) \otimes \begin{cases} 00 \\ 11 \\ 01 \\ 10 \end{cases}$$

$\sim 1 - 3\gamma^2$

Q. Error Correction for Phase



USING ERROR CORRECTION TO DETERMINE THE NOISE…

M. Laforest et al. PHYSICAL REVIEW A 75, 012331 2007

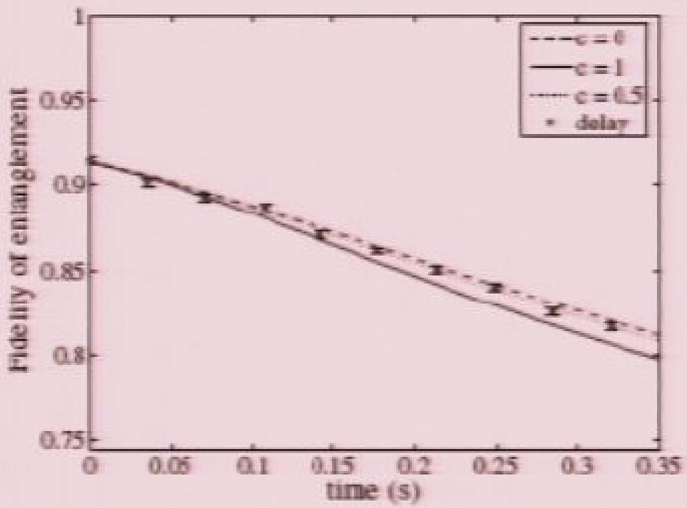


FIG. 3. Experimental results. The lines correspond to the fidelity decay for noise correlation factors of 0, 0.5, and 1 as a function of time simulated from the measured T_2's and the experimental fidelities obtained by implementing engineered noise. The points are the fidelities when the system is affected by natural noise for a various amount of time.

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$+\beta|0\rangle)|11\rangle$

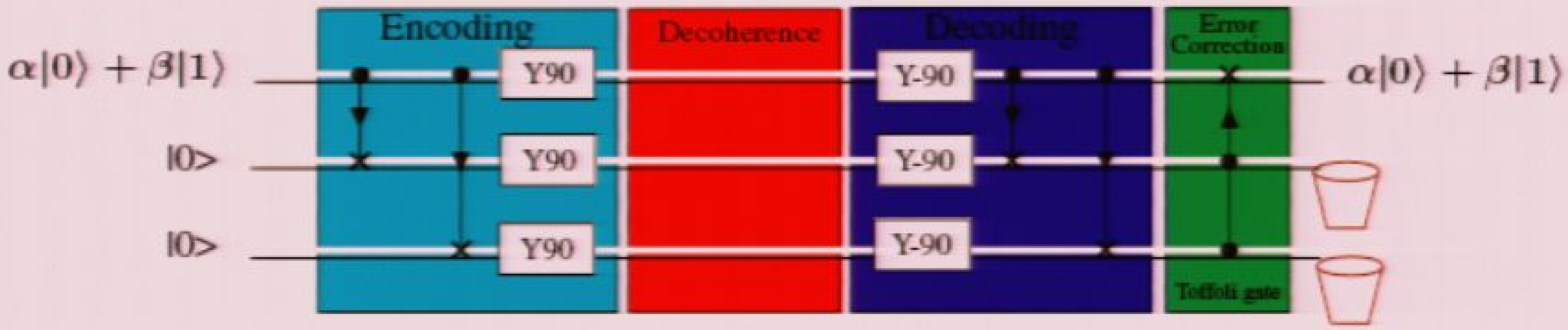
$+\beta|1\rangle)|01\rangle$

$+\beta|1\rangle)|10\rangle$

$$(\alpha|0\rangle + \beta|1\rangle) \otimes \begin{cases} 00 \\ 11 \\ 01 \\ 10 \end{cases}$$

$\sim 1 - 3\gamma^2$

Q. Error Correction for Phase



$\alpha|0\rangle + \beta|1\rangle$

Errors: $+ \to -$

$- \to +$

$\alpha|+++\rangle + \beta|---\rangle$
$\alpha|-++\rangle + \beta|+--\rangle$
$\alpha|+-+\rangle + \beta|-+-\rangle$
$\alpha|++-\rangle + \beta|--+\rangle$

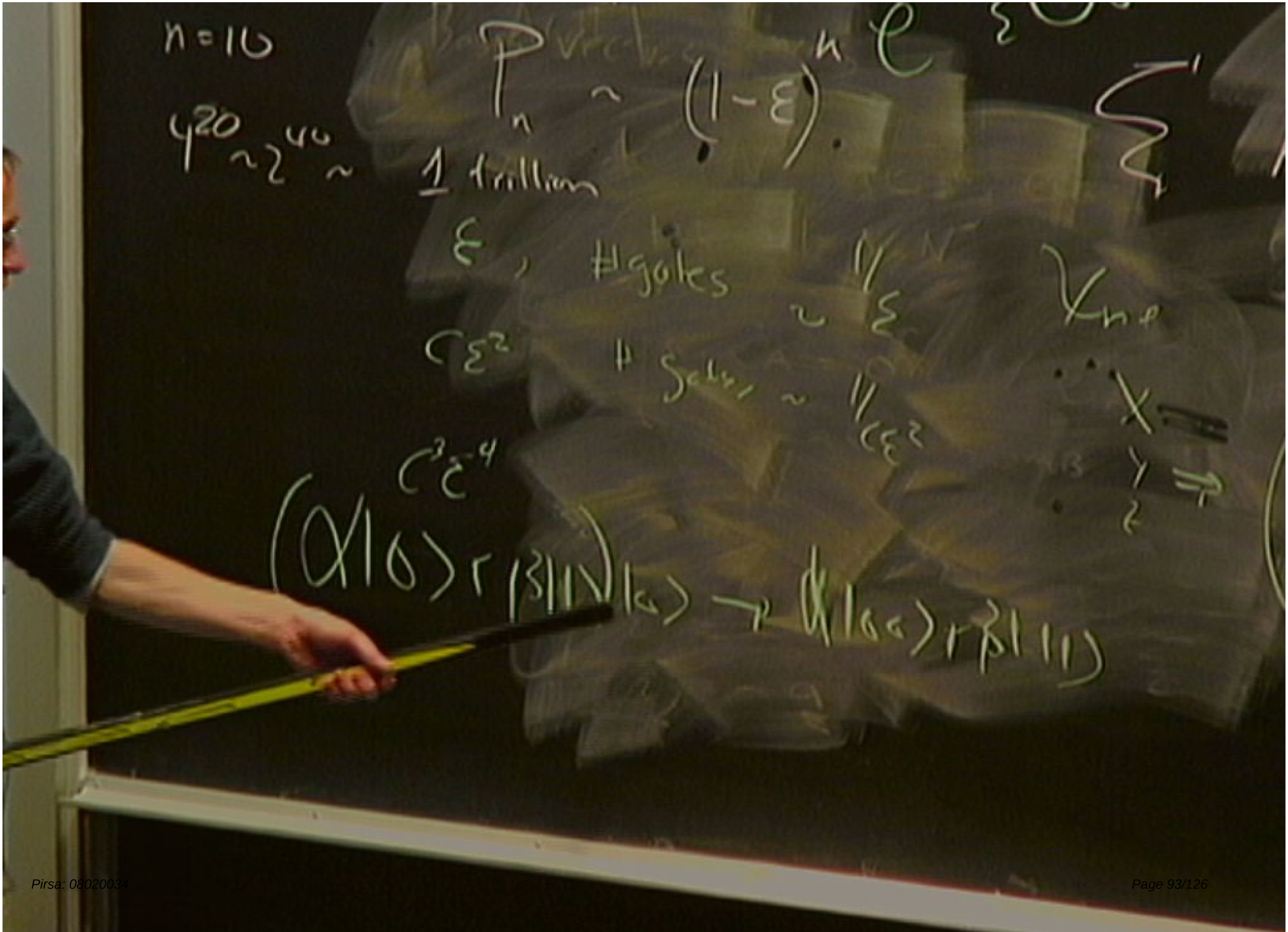
$(\alpha|0\rangle + \beta|1\rangle)|00\rangle$
$(\alpha|1\rangle + \beta|0\rangle)|11\rangle$
$(\alpha|0\rangle + \beta|1\rangle)|01\rangle$
$(\alpha|0\rangle + \beta|1\rangle)|10\rangle$

Control-Not

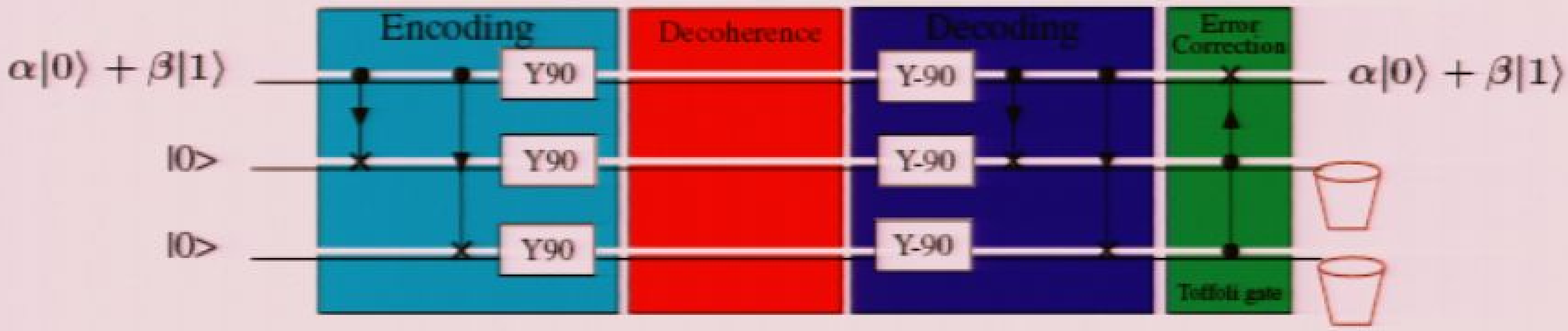
$$= \begin{cases} 00 \to 00 \\ 01 \to 01 \\ 10 \to 11 \\ 11 \to 10 \end{cases}$$

$$(\alpha|0\rangle + \beta|1\rangle) \otimes \begin{cases} 00 \\ 11 \\ 01 \\ 10 \end{cases} \quad \sim 1 - 3\gamma^2$$

$$|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$$


n=10
1 trillion
#gates

Q. Error Correction for Phase



$\alpha|0\rangle + \beta|1\rangle$

Errors: $+ \rightarrow -$

$- \rightarrow +$

$$\alpha|+++\rangle + \beta|---\rangle$$
$$\alpha|-++\rangle + \beta|+--\rangle$$
$$\alpha|+-+\rangle + \beta|-+-\rangle$$
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$$(\alpha|0\rangle + \beta|1\rangle)|00\rangle$$
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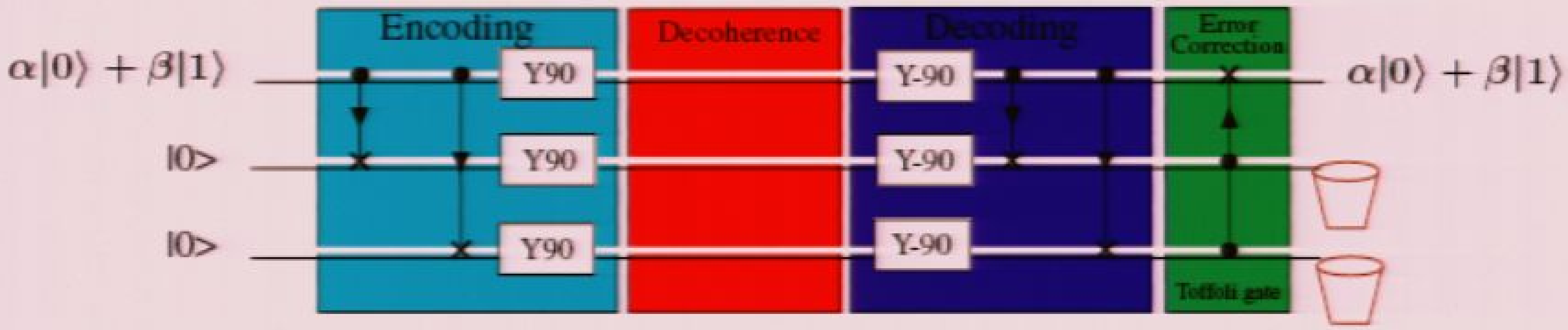
$$(\alpha|0\rangle + \beta|1\rangle) \otimes \begin{cases} 00 \\ 11 \\ 01 \\ 10 \end{cases}$$
$$\sim 1 - 3\gamma^2$$

Control-Not

$$= \begin{cases} 00 \rightarrow 00 \\ 01 \rightarrow 01 \\ 10 \rightarrow 11 \\ 11 \rightarrow 10 \end{cases}$$

$$|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$$

Q. Error Correction for Phase



$\alpha|0\rangle + \beta|1\rangle$

Errors: $+ \rightarrow -$

$- \rightarrow +$

$$\alpha|+++\rangle + \beta|---\rangle$$
$$\alpha|-++\rangle + \beta|+--\rangle$$
$$\alpha|+-+\rangle + \beta|-+-\rangle$$
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$$(\alpha|0\rangle + \beta|1\rangle)|00\rangle$$
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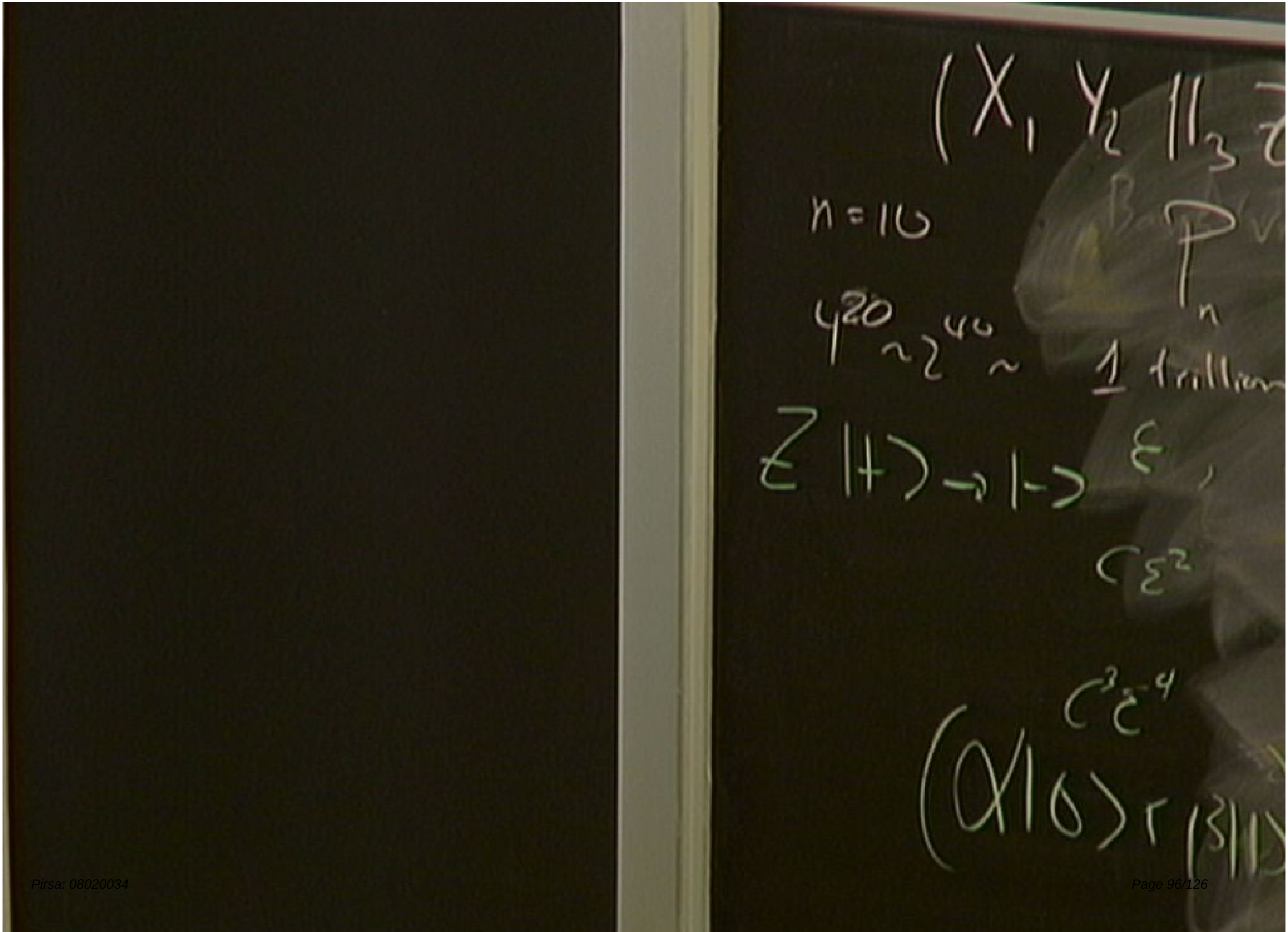
$$(\alpha|0\rangle + \beta|1\rangle)|01\rangle$$
$$(\alpha|0\rangle + \beta|1\rangle)|10\rangle$$

Control-Not

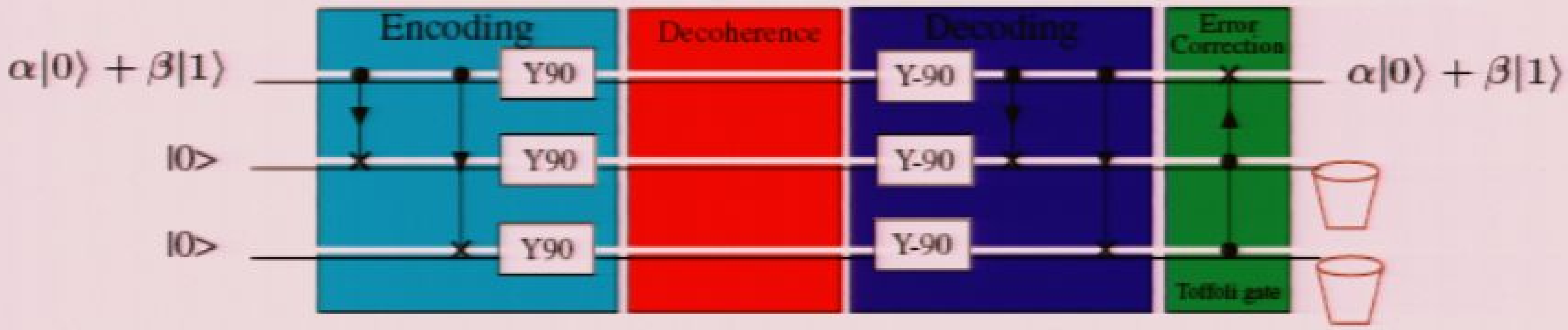
$$= \begin{cases} 00 \rightarrow 00 \\ 01 \rightarrow 01 \\ 10 \rightarrow 11 \\ 11 \rightarrow 10 \end{cases}$$

$$(\alpha|0\rangle + \beta|1\rangle) \otimes \begin{cases} 00 \\ 11 \\ 01 \\ 10 \end{cases} \quad \sim 1 - 3\gamma^2$$

$$|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$$


(X1 Y2 I3
n = 10
4^20 ~ 2^40 ~ 1 trillion
Z |+⟩ → |−⟩
ε
cε^2
c^3ε^-4

Q. Error Correction for Phase



$\alpha|0\rangle + \beta|1\rangle$

Errors: $+ \rightarrow -$

$- \rightarrow +$

$$\alpha|+++\rangle + \beta|---\rangle$$
$$\alpha|-++\rangle + \beta|+--\rangle$$
$$\alpha|+-+\rangle + \beta|-+-\rangle$$
$$\alpha|++-\rangle + \beta|--+\rangle$$

$$(\alpha|0\rangle + \beta|1\rangle)|00\rangle$$
$$(\alpha|1\rangle + \beta|0\rangle)|11\rangle$$
$$(\alpha|0\rangle + \beta|1\rangle)|01\rangle$$
$$(\alpha|0\rangle + \beta|1\rangle)|10\rangle$$

Control-Not

$$= \begin{cases} 00 \rightarrow 00 \\ 01 \rightarrow 01 \\ 10 \rightarrow 11 \\ 11 \rightarrow 10 \end{cases}$$

$$(\alpha|0\rangle + \beta|1\rangle) \otimes \begin{cases} 00 \\ 11 \\ 01 \\ 10 \end{cases} \quad \sim 1 - 3\gamma^2$$

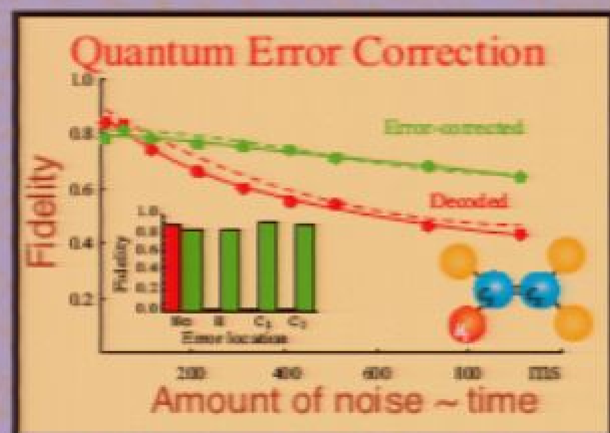
$$|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$$

Q. Error Correction for Phase

Encoding | Decoherence | Decoding | Error Correction

$\alpha|0\rangle + \beta|1\rangle$

Experiments

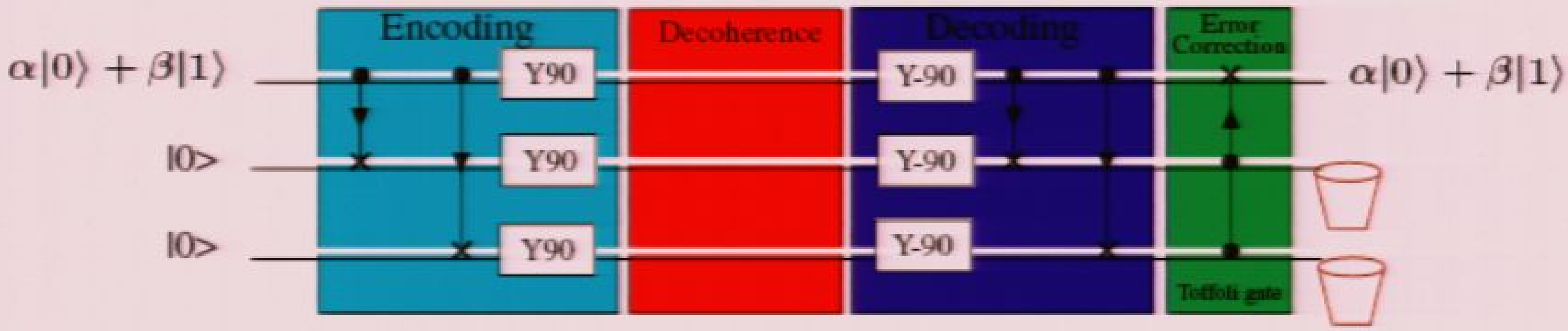


Experimental Quantum Error Correction:
D. G. Cory, M. D. Price, W. Maas, E. Knill, R. Laflamme, W. H. Zurek, T. F. Havel and S. S. Somaroo, PRL 81, 2152, 1998

$1\rangle) \otimes \begin{cases} 00 \\ 11 \\ 01 \\ 10 \end{cases}$

$-3\gamma^2$

Q. Error Correction for Phase



$\alpha|0\rangle + \beta|1\rangle$

Errors: $+ \rightarrow -$

$- \rightarrow +$

$$\alpha|+++\rangle + \beta|---\rangle$$
$$\alpha|-++\rangle + \beta|+--\rangle$$
$$\alpha|+-+\rangle + \beta|-+-\rangle$$
$$\alpha|++-\rangle + \beta|--+\rangle$$

$$(\alpha|0\rangle + \beta|1\rangle)|00\rangle$$
$$(\alpha|1\rangle + \beta|0\rangle)|11\rangle$$
$$(\alpha|0\rangle + \beta|1\rangle)|01\rangle$$
$$(\alpha|0\rangle + \beta|1\rangle)|10\rangle$$

$$(\alpha|0\rangle + \beta|1\rangle) \otimes \begin{cases} 00 \\ 11 \\ 01 \\ 10 \end{cases}$$

$\sim 1 - 3\gamma^2$

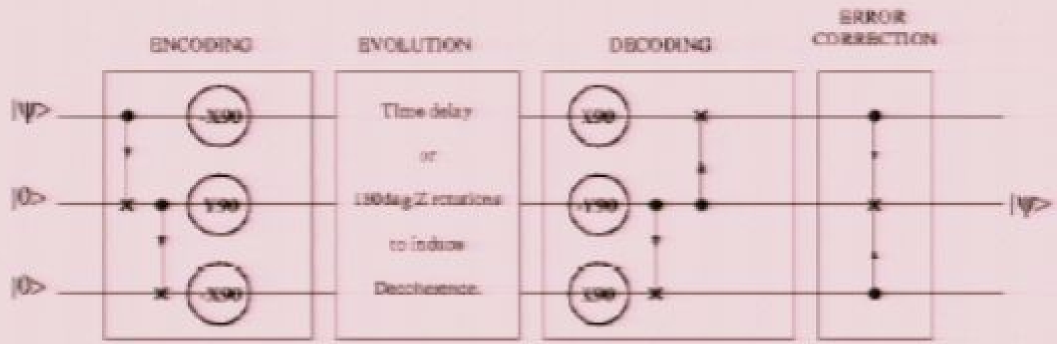
Control-Not

$$= \begin{cases} 00 \rightarrow 00 \\ 01 \rightarrow 01 \\ 10 \rightarrow 11 \\ 11 \rightarrow 10 \end{cases}$$

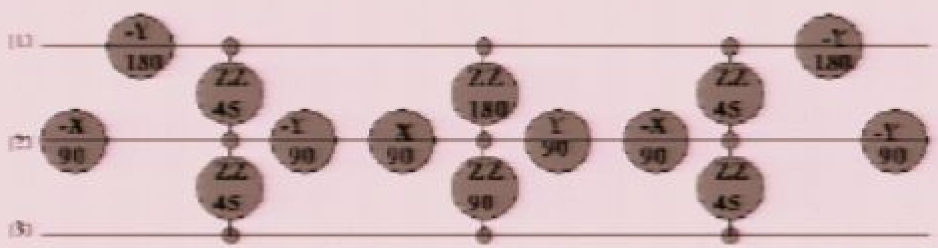
$$|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$$

Phase QEC NMR circuit

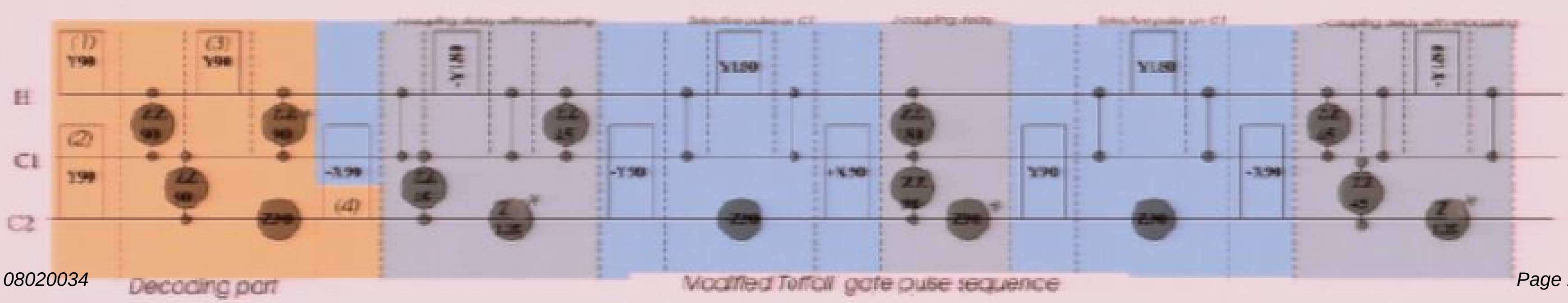
NMR implementation of the decoding and error correction:



Toffoli gate:



and the full decoding and Toffoli, including some optimization

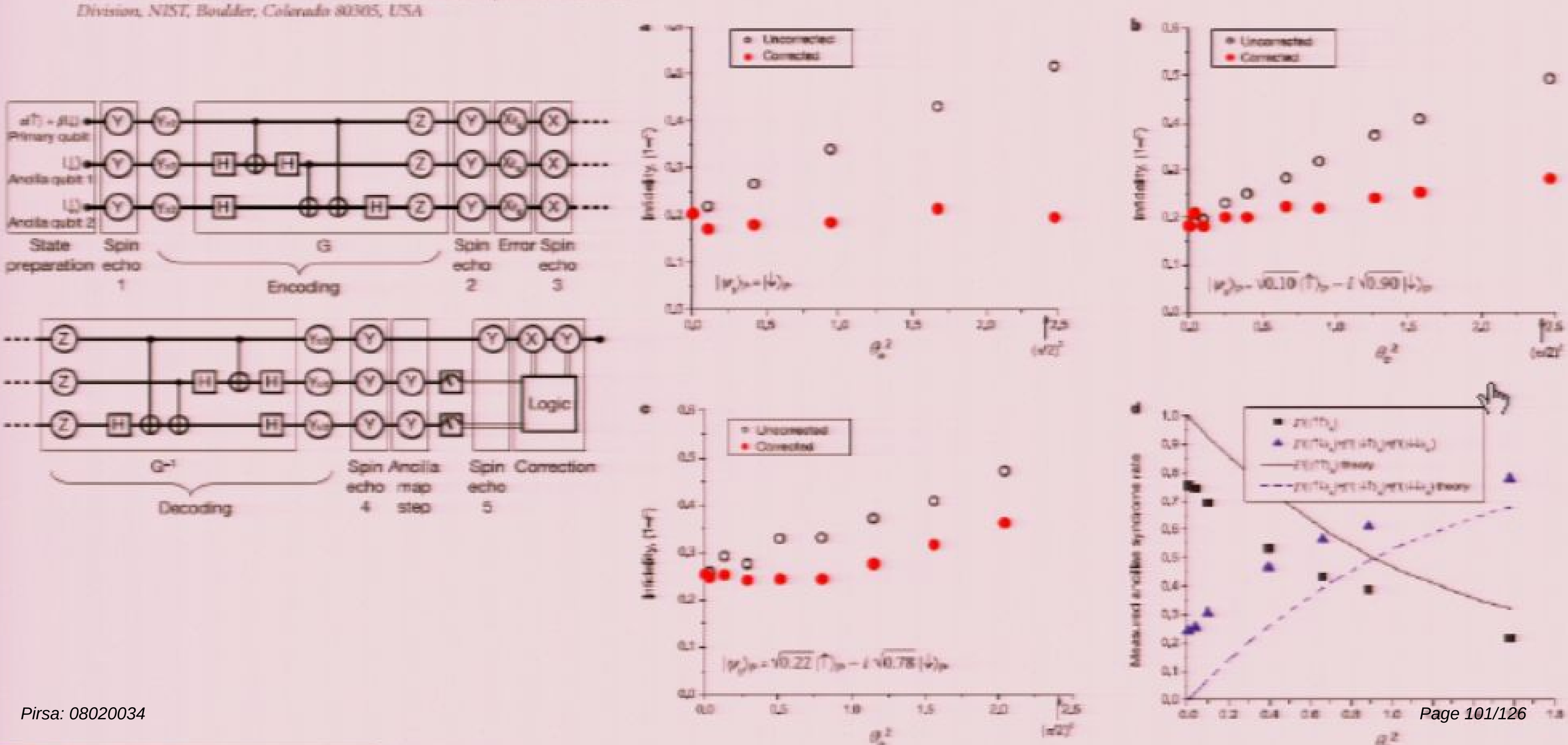


3 bit code in an ion trap

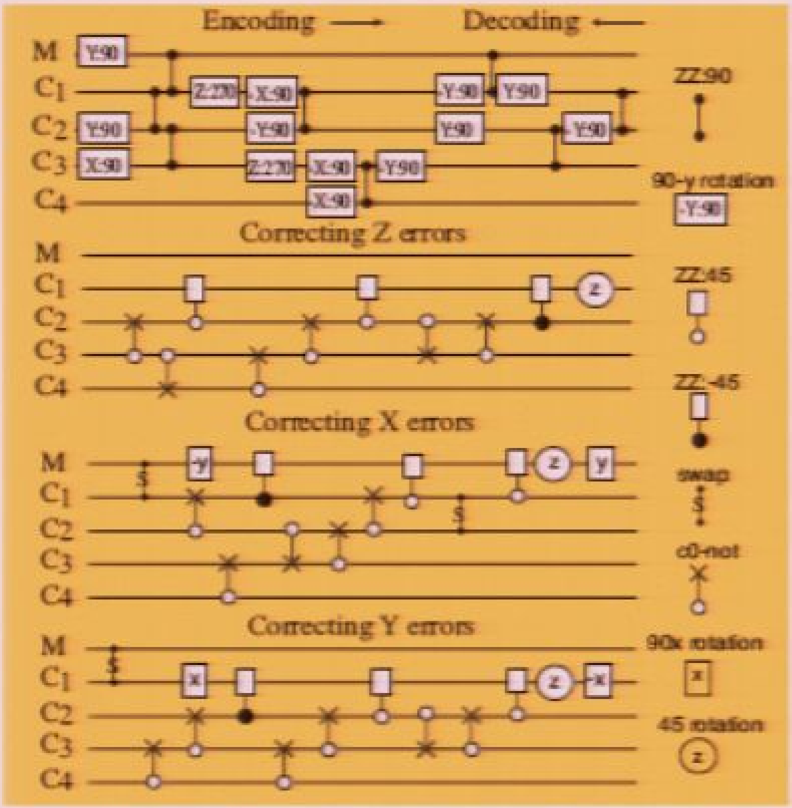
Realization of quantum error correction

J. Chiaverini[1], D. Leibfried[1], T. Schaetz[1,*], M. D. Barrett[1,*], R. B. Blakestad[1], J. Britton[1], W. M. Itano[1], J. D. Jost[1], E. Knill[2], C. Langer[1], R. Ozeri[1] & D. J. Wineland[1]

[1]*Time and Frequency Division,* [2]*Mathematical and Computational Sciences Division, NIST, Boulder, Colorado 80305, USA*

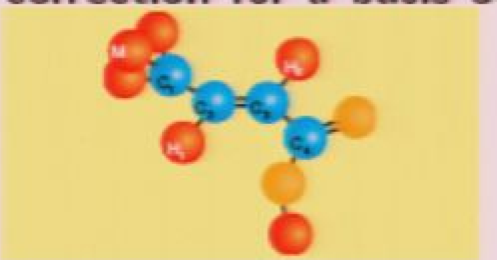


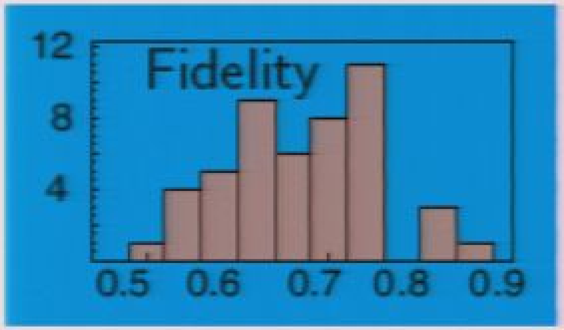
5 bit quantum error correcting code

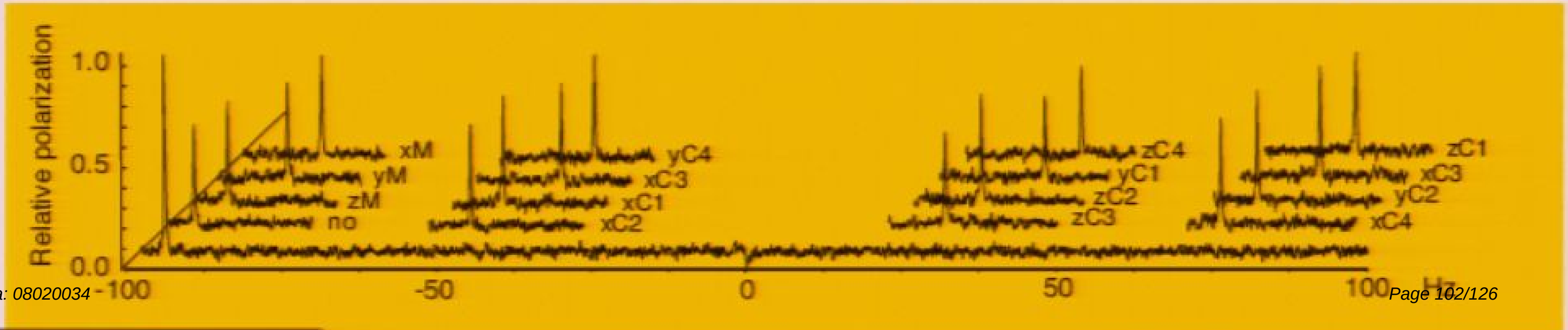


Knill, Laflamme, Martinez, Negrevergne, PRL 404,308,2000

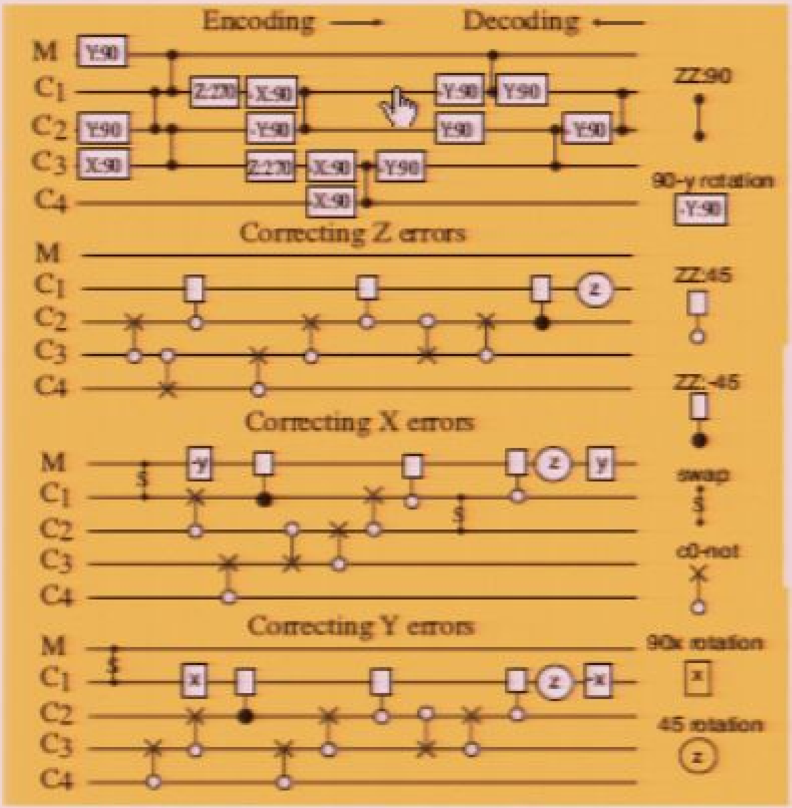
Implementation of the 5 bit code with the stabilizer $Z^2Y^3Y^4X^5$, $Z^1Y^2Y^3X^4$, $Y^2Z^3Z^4Z^5$ and $X^1Z^2X^3Z^4$, including decoding and error correction for a basis of 1 qubit errors.







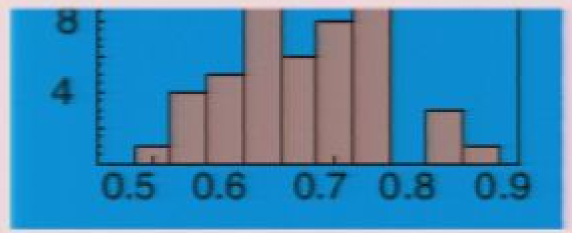
5 bit quantum error correcting code

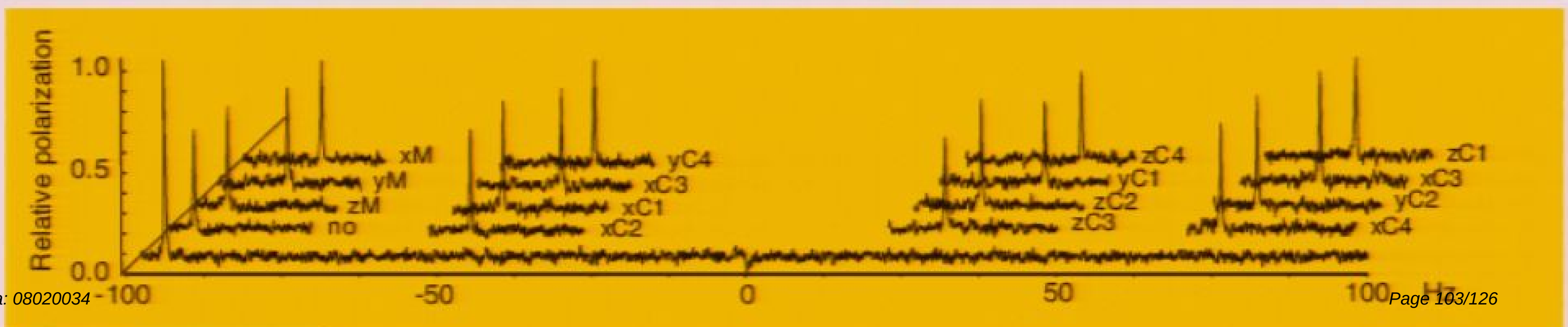


Knill, Laflamme, Martinez, Negrevergne, PRL 404,308,2000

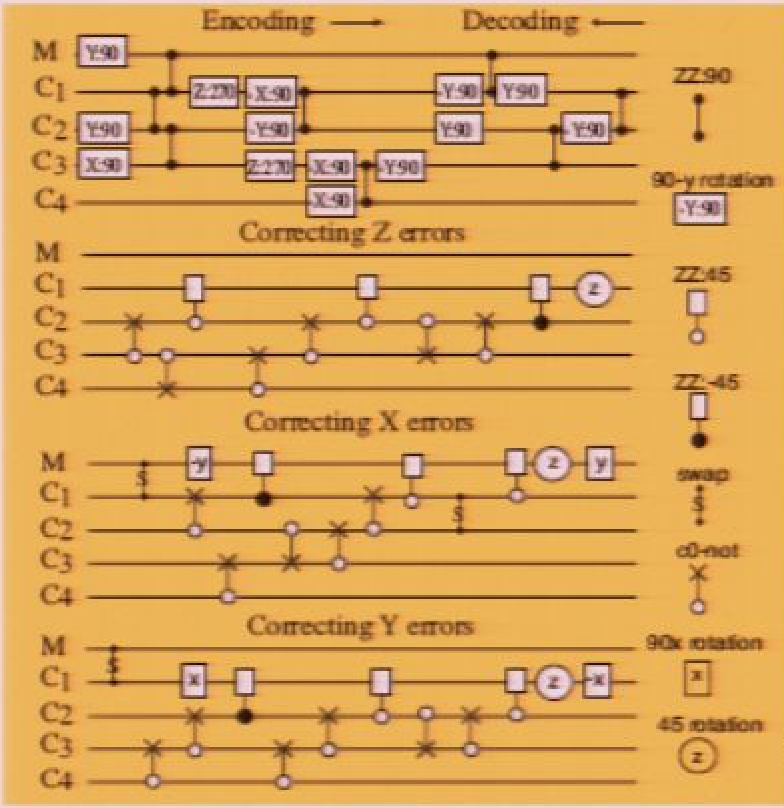
Implementation of the 5 bit code with the stabilizer $Z^2Y^3Y^4X^5$, $Z^1Y^2Y^3X^4$, $Y^2Z^3Z^4Z^5$ and $X^1Z^2X^3Z^4$, including decoding and error correction for a basis of 1 qubit errors.

$$T_{0,1} = \frac{1}{2}(I \pm \frac{1}{\sqrt{3}}(X + Y + Z))$$



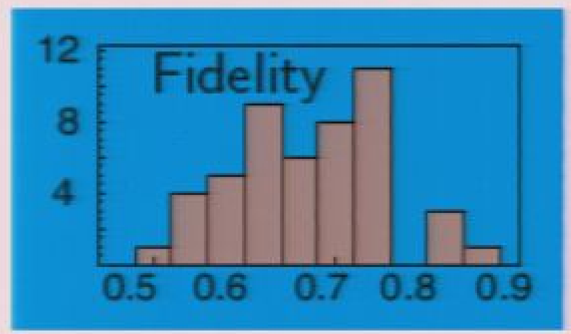


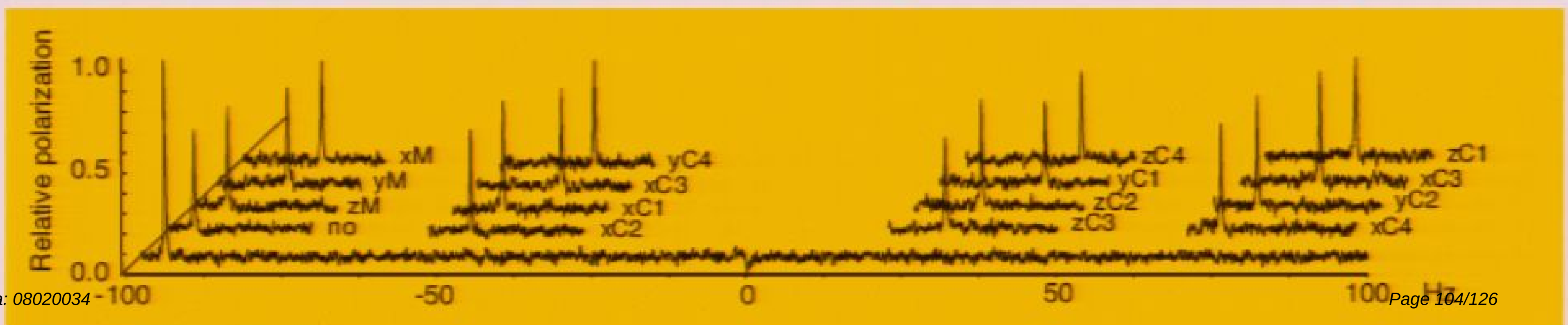
5 bit quantum error correcting code



Knill, Laflamme, Martinez, Negrevergne, PRL 404,308,2000

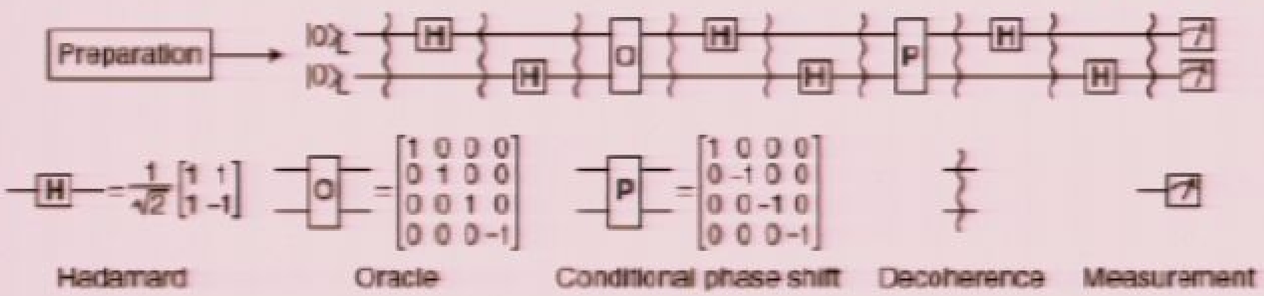
Implementation of the 5 bit code with the stabilizer $Z^2Y^3Y^4X^5$, $Z^1Y^2Y^3X^4$, $Y^2Z^3Z^4Z^5$ and $X^1Z^2X^3Z^4$, including decoding and error correction for a basis of 1 qubit errors.

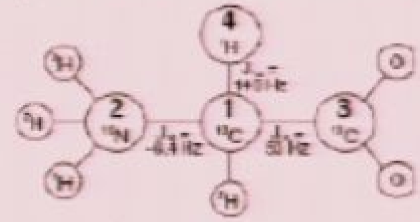




Decoherence free subspaces

Ollerenshaw et al., PRL 91, 217904, 2003





- 4 qubits (H, ^{13}C and ^{15}N from glycine)
- Error model: Sub-system collective bit-flip

$$E_4 = a_{40} I_1 I_2 I_3 I_4 + a_{41} X_1 X_2 I_3 I_4 + a_{42} I_1 I_2 X_3 X_4 + a_{43} X_1 X_2 X_3 X_4,$$

- Strength of noise is controllable (engineered)
- There are 4 orthogonal 4-d simultaneous eigenspace of the noise generators (DFS)

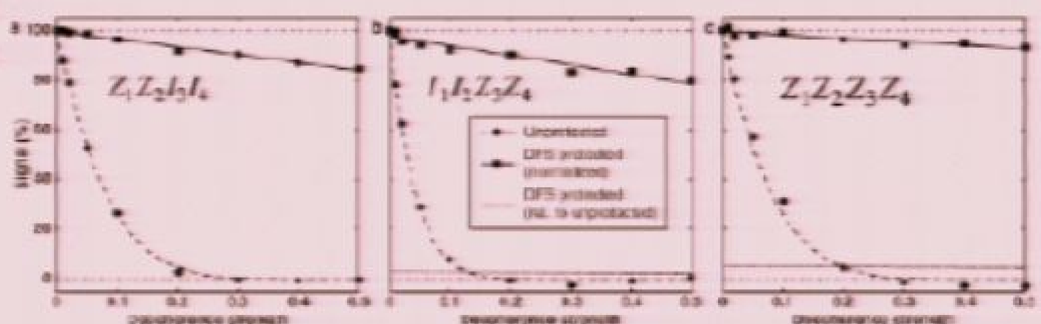
$$|00\rangle_L^1 = (|0000\rangle + |1100\rangle + |0011\rangle + |1111\rangle)/2,$$
$$|01\rangle_L^1 = (|1000\rangle + |0100\rangle + |1011\rangle + |0111\rangle)/2,$$
$$|10\rangle_L^1 = (|0001\rangle + |1101\rangle + |0010\rangle + |1110\rangle)/2,$$
$$|11\rangle_L^1 = (|1001\rangle + |0101\rangle + |1010\rangle + |0110\rangle)/2.$$

- Other DFS only differ by sign flip, e.g.

$$|00\rangle_L^2 = (|0000\rangle + |1100\rangle - |0011\rangle - |1111\rangle)/2,$$
$$|00\rangle_L^3 = (|0000\rangle - |1100\rangle + |0011\rangle - |1111\rangle)/2,$$
$$|00\rangle_L^4 = (|0000\rangle - |1100\rangle - |0011\rangle + |1111\rangle)/2.$$

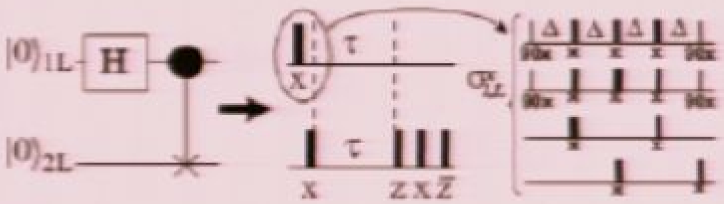
- Encoding was a classical superposition of the 4 DFS

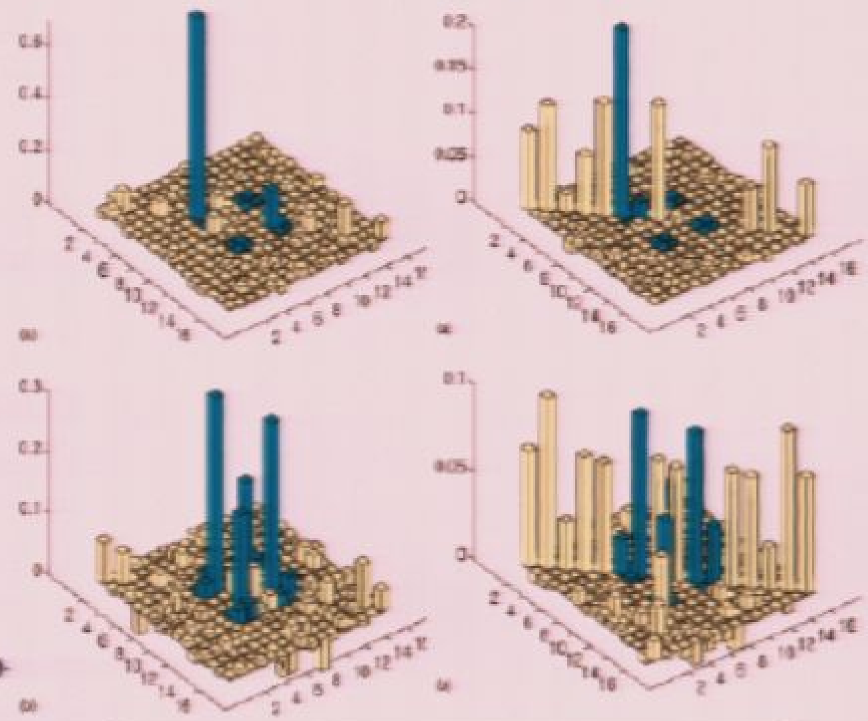
$$\rho_{|\psi\rangle_L} = |\psi\rangle_L\langle\psi|_L = \sum_{i=1}^{4} c_i |\psi\rangle_L^i \langle\psi|_L^i,$$
$$\rho_{|00\rangle_L} = (I_1 I_2 I_3 I_4 + Z_1 Z_2 I_3 I_4 + I_1 I_2 Z_3 Z_4 + Z_1 Z_2 Z_3 Z_4)/16.$$



Encoded gates

J.S. Hodges et al. PRA 75:042320,2007





- 4 qubits ([13]C from crotonic acid)
- Error model: Collective phase-damping

$$\mathcal{H}_{SE} = \gamma\left(\sum_k I_z^{(k)}\right) \otimes B_z$$

- Two qubit DFS spanned by $|01\rangle$ and $|10\rangle$
- Logical Paulis and logical operation:

$$\sigma_z^L \Leftrightarrow \frac{\sigma_z^1 - \sigma_z^2}{2}, \quad \sigma_x^L \Leftrightarrow \frac{\sigma_x^1\sigma_x^2 + \sigma_y^1\sigma_y^2}{2}, \quad U_{C_{NOT}}U_H \Leftrightarrow e^{i\phi}U_4U_3U_2U_1,$$

$$1^L \Leftrightarrow \frac{1^{1,2} - \sigma_z^1\sigma_z^2}{2}, \quad \sigma_y^L \Leftrightarrow \frac{\sigma_y^1\sigma_x^2 - \sigma_x^1\sigma_y^2}{2}, \quad U_1 = e^{-i(\pi/4)\sigma_y^1\sigma_y^2}, \quad U_2 = e^{-i(\pi/4)\sigma_y^3\sigma_y^4}, \quad U_3 = e^{-i(\pi/4)\sigma_y^2\sigma_y^3}, \quad U_4 = e^{-i(\pi/4)\sigma_y^3\sigma_y^4}$$

- Compare performance from system pseudo pure and subsystem pseudo-pure state

$$\mathcal{H} = \mathcal{L} \otimes \mathcal{R}$$

$$\rho_{pp} = |00\rangle\langle 00|_L$$

$$\rho_{spp} = \frac{1}{2}|00\rangle\langle 00|_L + \frac{1}{4}\mathbb{I}_R$$

Quantum state	C_{sim}	C_{expt}	c_{LL}
Full pseudopure state	0.98	0.91	0.99
Full pseudopure Bell state	0.96	0.74	0.95
Subsystem pseudopure state	0.99	0.97	0.99
Subsystem pseudopure Bell state	0.97	0.87	0.91

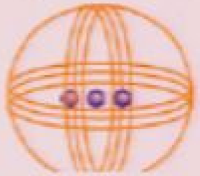
Noiseless subsystem

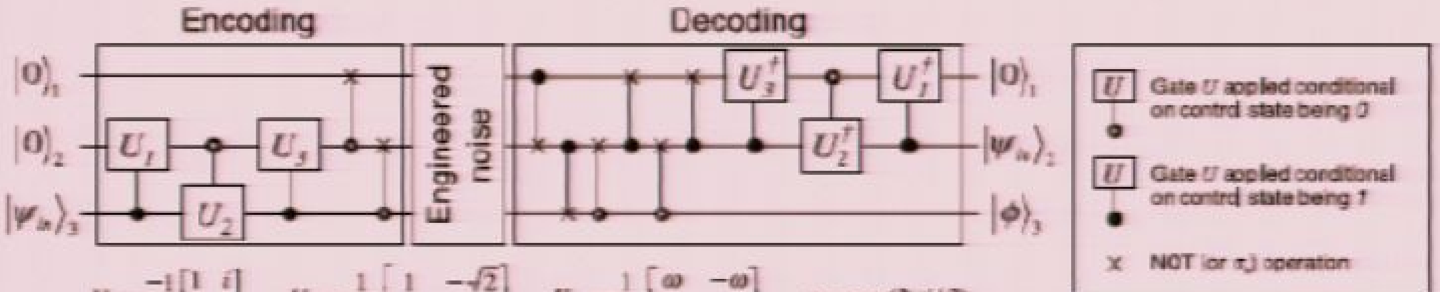
L.Viola, E.M. Fortunato, M.A. Pravia, E. Knill, R. Laflamme, D.G. Cory Science 293, 2059 2001

$$s_{12} = \vec{\sigma}^{(1)} \cdot \vec{\sigma}^{(2)}, \quad s_{23} = \vec{\sigma}^{(2)} \cdot \vec{\sigma}^{(3)}, \quad s_{31} = \vec{\sigma}^{(3)} \cdot \vec{\sigma}^{(1)}$$

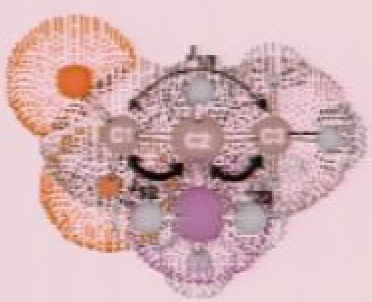
$$\sigma_x^{(L)} = \frac{1}{2}\left(\mathbb{1} + s_{12}\right)P_{1/2} \quad \sigma_y^{(L)} = \frac{\sqrt{3}}{6}\left(s_{23} - s_{31}\right)P_{1/2} \quad \sigma_z^{(L)} = -i\sigma_x^{(L)}\sigma_y^{(L)}$$

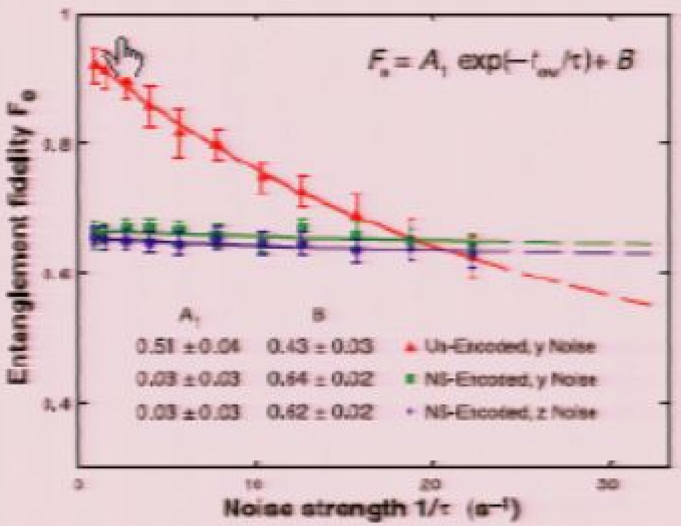
$$P_{1/2} = \frac{1}{2}\left(\mathbb{1} - \frac{s_{12}+s_{23}+s_{31}}{3}\right)$$



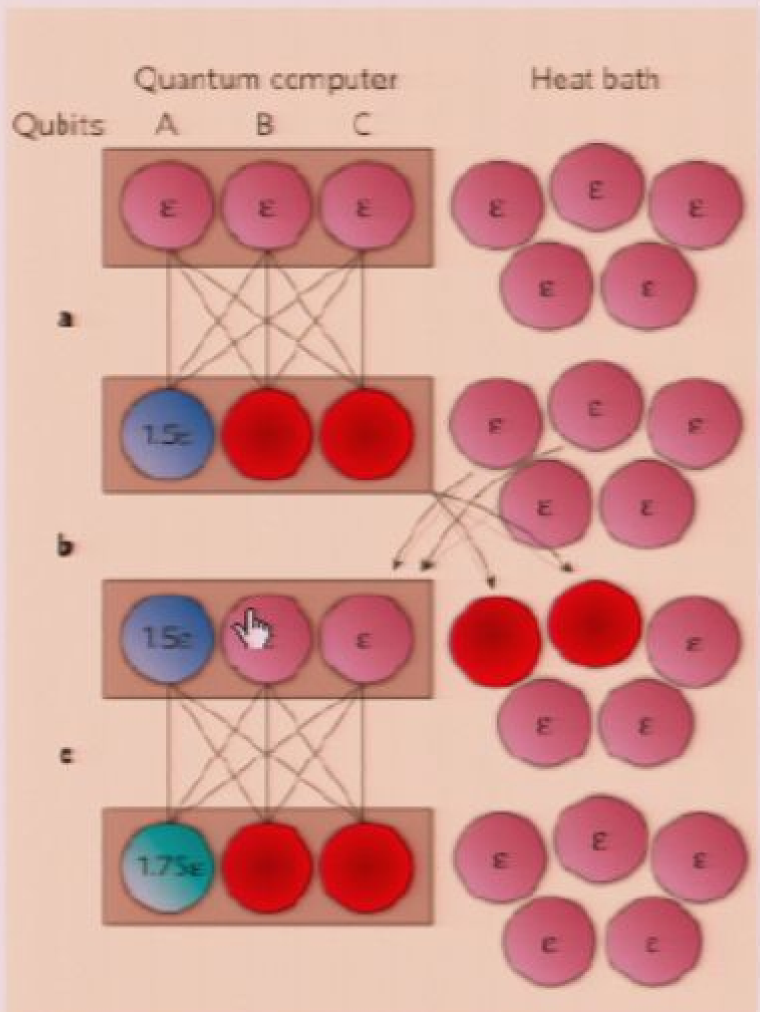


$$U_1 = \frac{-1}{\sqrt{2}}\begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix} \quad U_2 = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & -\sqrt{2} \\ \sqrt{2} & 1 \end{bmatrix} \quad U_3 = \frac{1}{\sqrt{2}}\begin{bmatrix} \omega & -\omega \\ \omega^2 & \omega^2 \end{bmatrix} \quad \omega = \exp(2\pi i/3)$$





Algorithmic cooling with heat bath



Schulman and Vazirani. Proceedings of the 31th Annual ACM Symposium on the Theory of Computation (STOC), pages 322–329, 1998.

Schulman, Mor and Weinstein, PRL94, 2005

Algorithmic cooling

Sorensen [5], Schulman and Vazirani [4]

We have seen that we can cool a subset of spins by swapping states. For excample, with 3 spins, implementing a gate that swaps $|011\rangle \leftrightarrow |100\rangle$ will increase the order of the first spin at the expense of the last two. We could concatenate this process to reach polarization of order 1.

$$\rho \sim e^{-\beta H} \sim \frac{1}{2^n}(\mathbb{1} - \beta\omega(Z_1 + Z_2 + Z_3) + \dots)$$

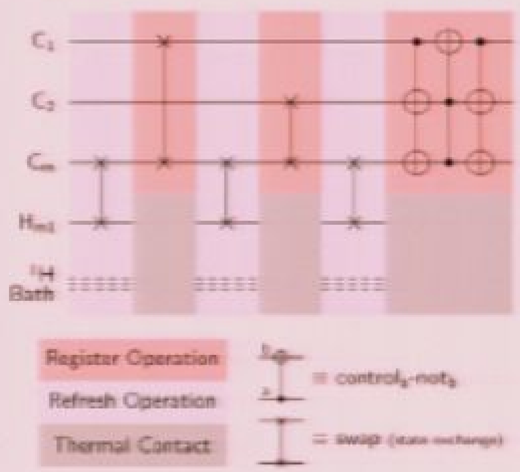
$$\rho^d_{\text{thermal}} \approx \frac{\beta\omega}{8}\begin{pmatrix} 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -3 \end{pmatrix} \Longleftrightarrow \rho^d_{\text{pol}} \approx \frac{\beta\omega}{8}\begin{pmatrix} 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -3 \end{pmatrix}$$

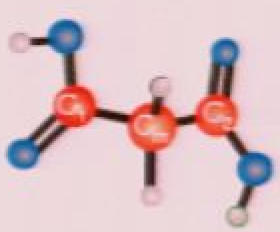
$$\bar{\rho}^d_{\text{pol}} = \text{Tr}_{2,3}\rho^d_{\text{pol}} \approx \frac{3}{4}\beta\omega\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

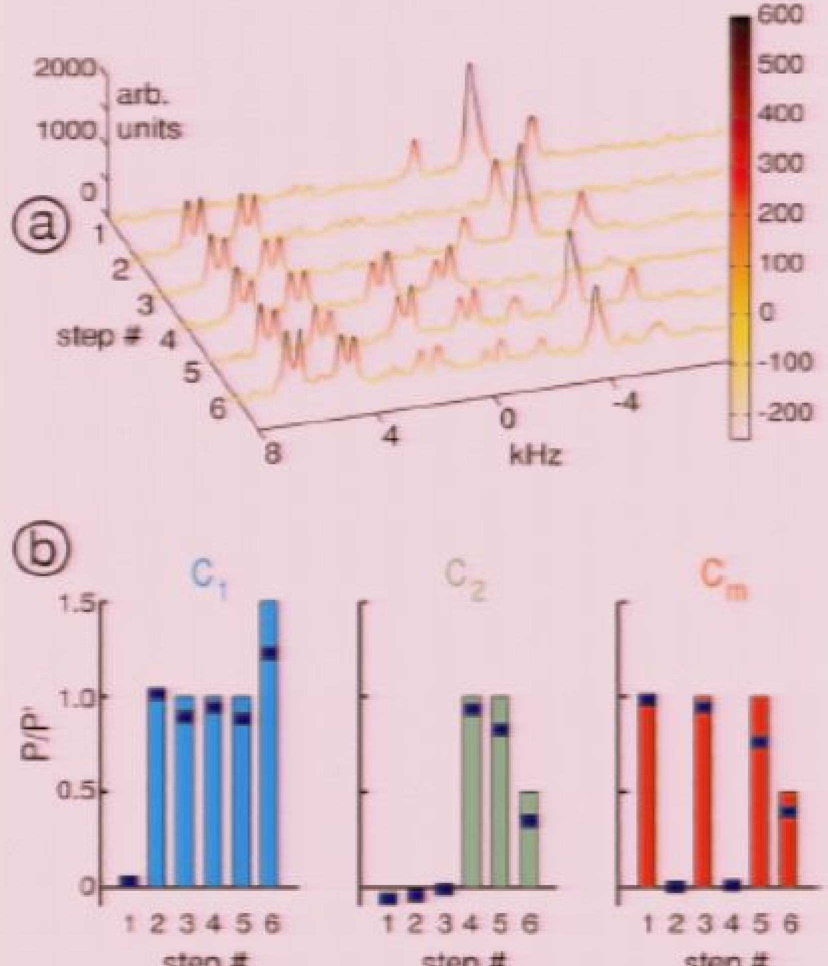
We could concatenate this process to reach polarization of O(1), but this would take a lot of ressources ($\sim 1/\beta^2$).

Experimental results

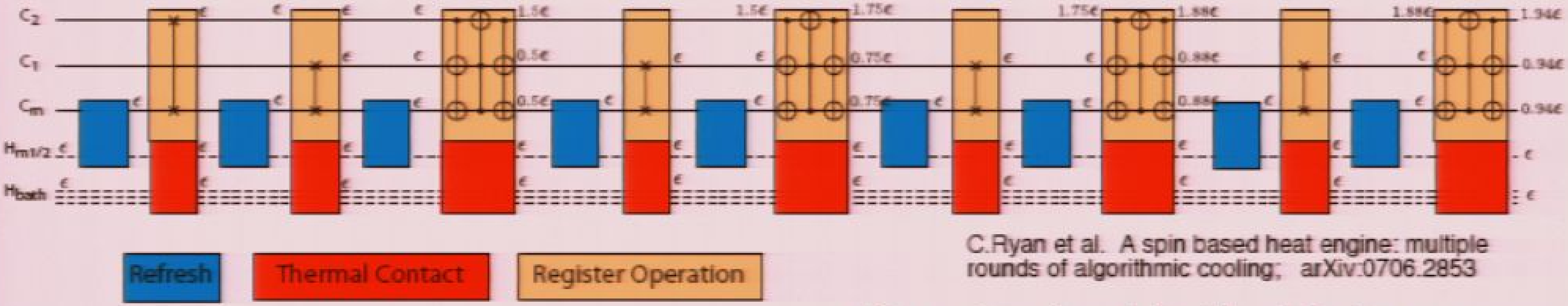
Baugh et al. Nature 438, 470, 2005 [1]





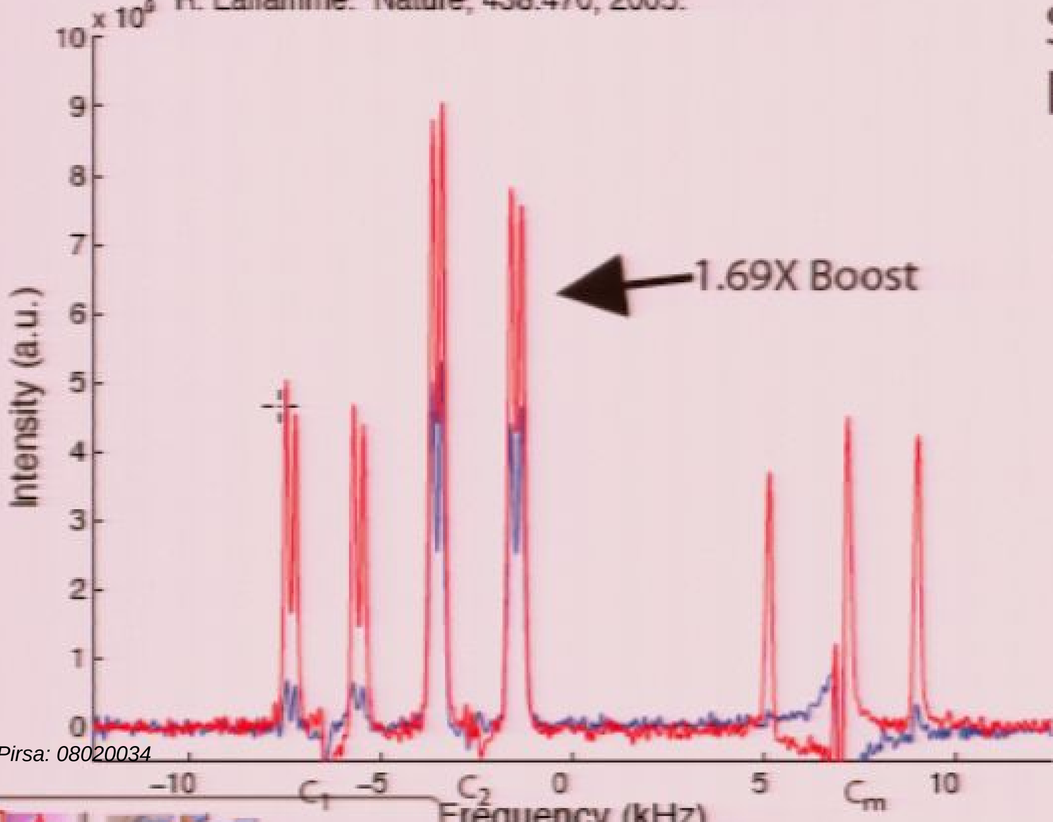


Multiple Rounds of Algorithmic Cooling



C.Ryan et al. A spin based heat engine: multiple rounds of algorithmic cooling; arXiv:0706.2853

J. Baugh, O. Moussa, C. Ryan, A. Nayak, and R. Laflamme. Nature, 438:470, 2005.



- By using heat-bath able to surpass Shannon/Soresnsen bound of 1.5X heat-bath polarization

Polarization Boost w.r.t. heat-bath

Compression Step	C_2	C_1	C_m
1	1.39	0.47	0.49
2	1.56	0.68	0.71
3	1.64	0.76	0.79
4	1.69	0.79	0.84

Conclusion

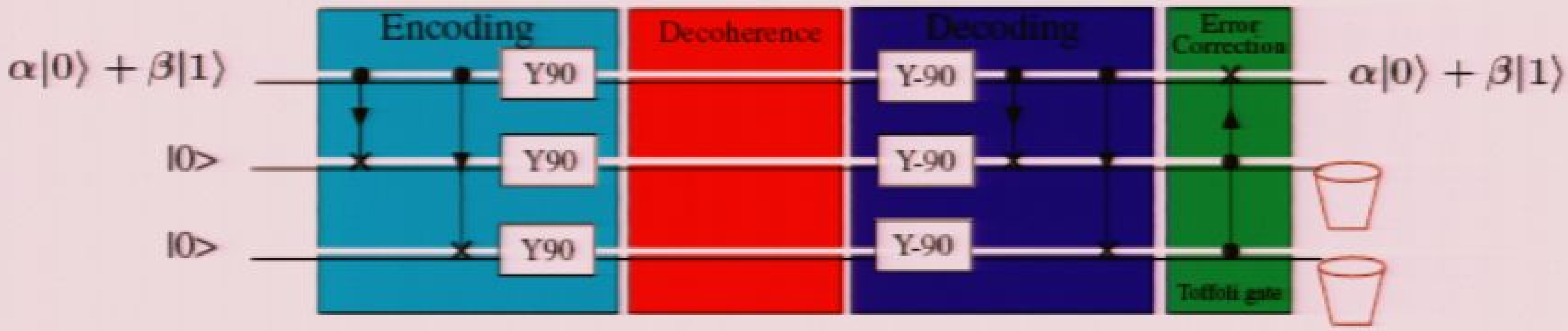
In order to error correct, we need to have

- Knowledge of the noise
- Good quantum control
- Ability to extract entropy
- Parallel operations

Recent experiments have demonstrated these elements individually but we need to pull them together.

It is only the beginning of experimental QEC.

Q. Error Correction for Phase



$\alpha|0\rangle + \beta|1\rangle$

Errors: $+ \rightarrow -$

$- \rightarrow +$

$$\alpha|+++\rangle + \beta|---\rangle$$
$$\alpha|-++\rangle + \beta|+--\rangle$$
$$\alpha|+-+\rangle + \beta|-+-\rangle$$
$$\alpha|++-\rangle + \beta|--+\rangle$$

$$(\alpha|0\rangle + \beta|1\rangle)|00\rangle$$
$$(\alpha|1\rangle + \beta|0\rangle)|11\rangle$$
$$(\alpha|0\rangle + \beta|1\rangle)|01\rangle$$
$$(\alpha|0\rangle + \beta|1\rangle)|10\rangle$$

$$(\alpha|0\rangle + \beta|1\rangle) \otimes \begin{cases} 00 \\ 11 \\ 01 \\ 10 \end{cases}$$

$\sim 1 - 3\gamma^2$

Control-Not

$$= \begin{cases} 00 \rightarrow 00 \\ 01 \rightarrow 01 \\ 10 \rightarrow 11 \\ 11 \rightarrow 10 \end{cases}$$

$$|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$$

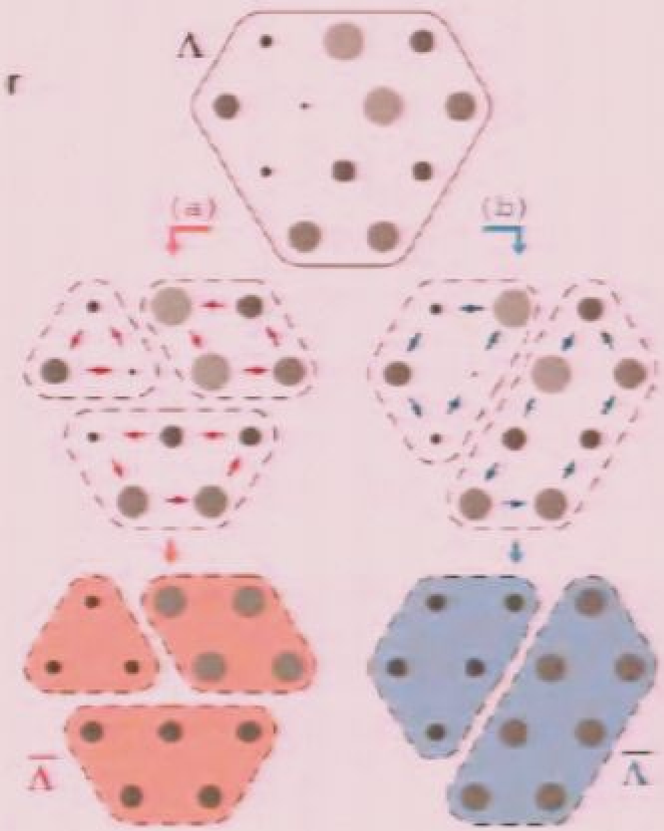
Coarse graining

- We are not interested in all the elements that describe the full noise superopeartor but only a coarse graining of them.

- If we are interested in implementing quantum error corrrection, we can ask what is the probbaility to get one, or two, or k qubit error, independent of the location and independent of teh type of error $\sigma_{x,y,z}$. The question is can we do this efficiently?

- Coarse graining is equivalent to implement a symmetry

$$\Lambda(\rho) = \sum_k^{D^2} A_k \rho A_k^\dagger$$



Schematic illustration of coarse-graining.

Zanardi, Phys.Lett. A258 (1999) 77

The death of QComputers (1995)

IEEE TRANSACTIONS ON ELECTRON DEVICES, VOL. 43, NO. 10, OCTOBER 1996

Need for Critical Assessment

Rolf Landauer, *Life Fellow, IEEE*

(Invited Paper)

Abstract—Adventurous technological proposals are subject to inadequate critical assessment. It is the proponents who organize meetings and special issues. Optical logic, mesoscopic switching devices and quantum parallelism are used to illustrate this problem.

Index Terms—Technology assessment, optical logic, mesoscopic devices, quantum parallelism.

PHYSICAL REVIEW A VOLUME 51, NUMBER 2 FEBRUARY 1995

Maintaining coherence in quantum computers

W. G. Unruh*
Canadian Institute for Advanced Research, Cosmology Program, Department of Physics, University of British Columbia, Vancouver, Canada V6T 1Z1

(Received 10 June 1994)

The effects of the inevitable coupling to external degrees of freedom of a quantum computer are examined. It is found that for quantum calculations (in which the maintenance of coherence over a large number of states is important), not only must the coupling be small, but the time taken in the quantum calculation must be less than the thermal time scale $\hbar/k_BT$. For longer times the condition on the strength of the coupling to the external world becomes much more stringent.

Threshold theorem

A quantum computation can be as long as required with any desired accuracy as long as the noise level is below a threshold value

$$P < 10^{-6,-5,-4,\ldots,-1?}$$

Knill et al.; Science, 279, 342, 1998

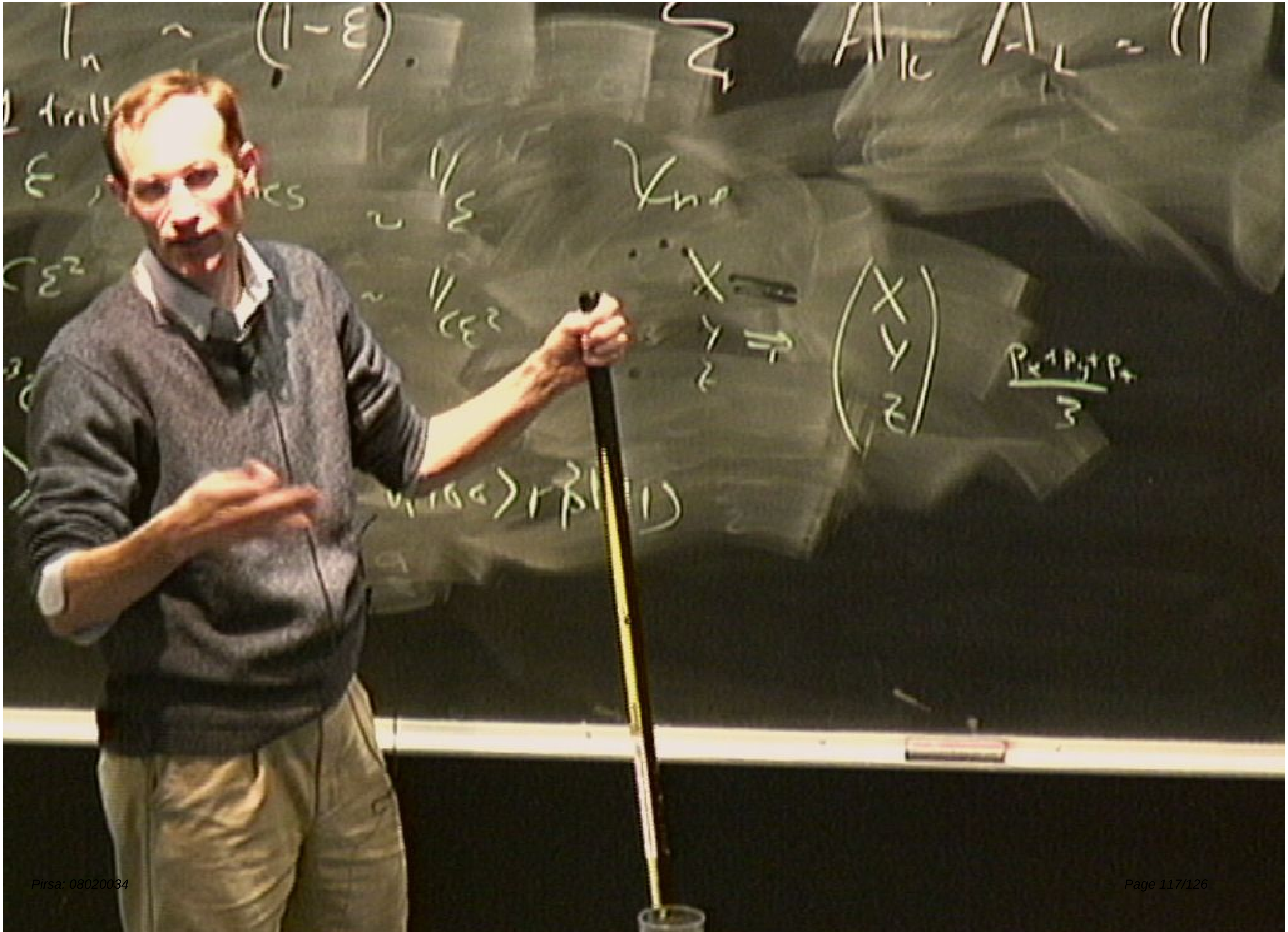
Kitaev, Russ. Math Survey 1997

Aharonov & Ben Or, ACM press

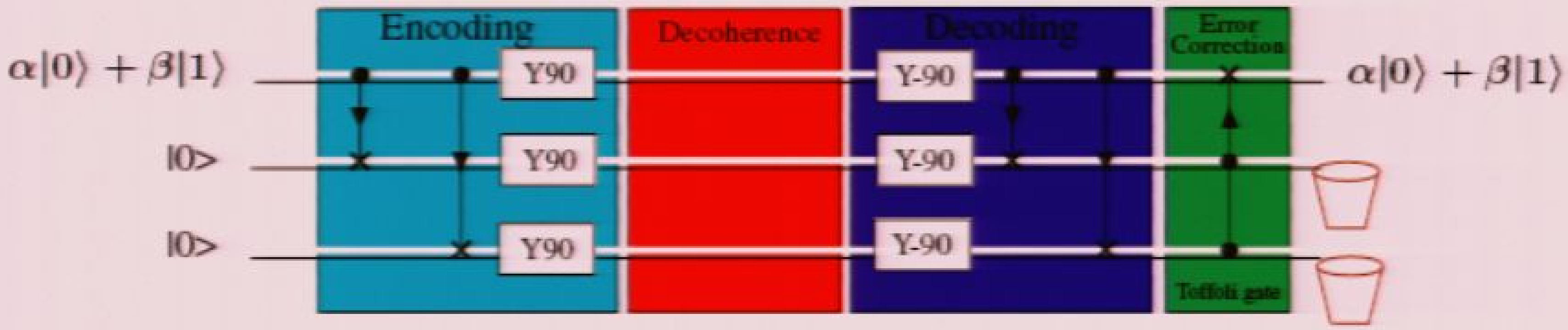
Preskill, PRSL, 454, 257, 1998

Significance:

- -imperfections and imprecisions are not fundamental objections to quantum computation
- -it gives criteria for scalability
- -its requirements are a guide for experimentalists
- -it is a benchmark to compare different technologies


$\frac{p_x+p_y+p_z}{3}$

Q. Error Correction for Phase



$\alpha|0\rangle + \beta|1\rangle$

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$$(\alpha|0\rangle + \beta|1\rangle) \otimes \begin{cases} 00 \\ 11 \\ 01 \\ 10 \end{cases} \quad \sim 1 - 3\gamma^2$$

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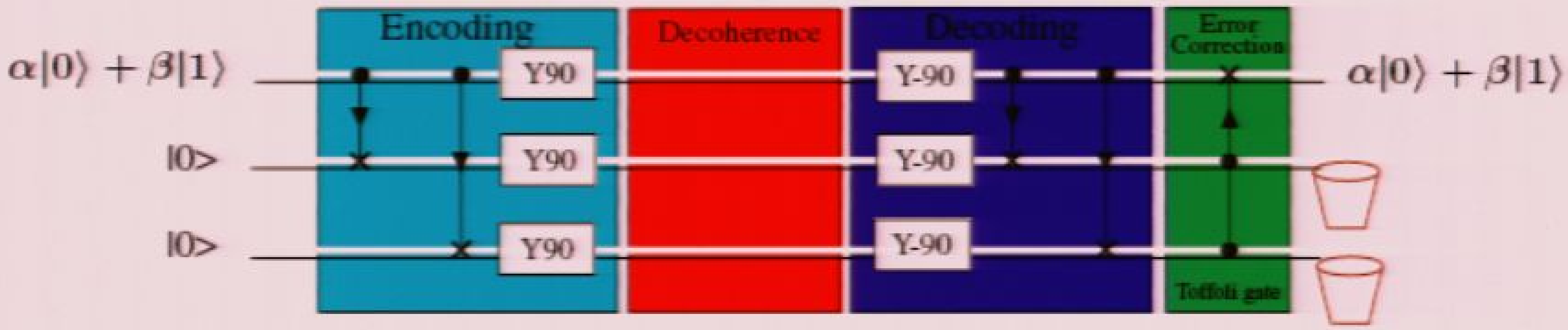

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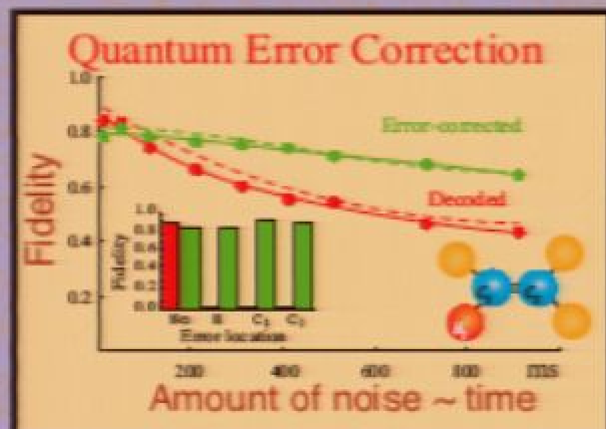
Q. Error Correction for Phase

Encoding | Decoherence | Decoding | Error Correction

$\alpha|0\rangle + \beta|1\rangle$

Experiments

Science
Top 10 breakthroughs of the year, Science 282, 2156, 1998



Experimental Quantum Error Correction:
D. G. Cory, M. D. Price, W. Maas, E. Knill, R. Laflamme, W. H. Zurek, T. F. Havel and S. S. Somaroo, PRL 81, 2152, 1998

$1\rangle) \otimes \begin{cases} 00 \\ 11 \\ 01 \\ 10 \end{cases}$

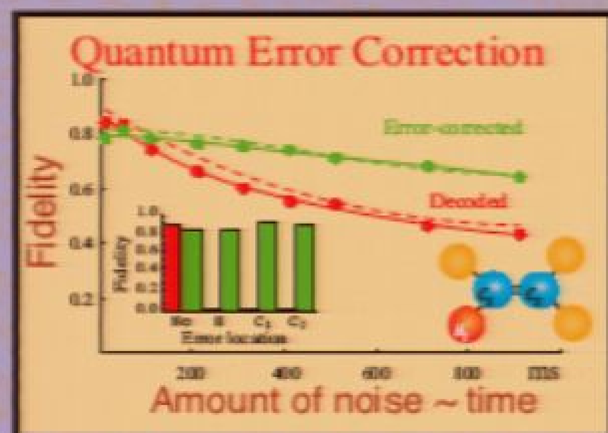
$- 3\gamma^2$

Q. Error Correction for Phase

Encoding | Decoherence | Decoding | Error Correction

$\alpha|0\rangle + \beta|1\rangle$

Experiments



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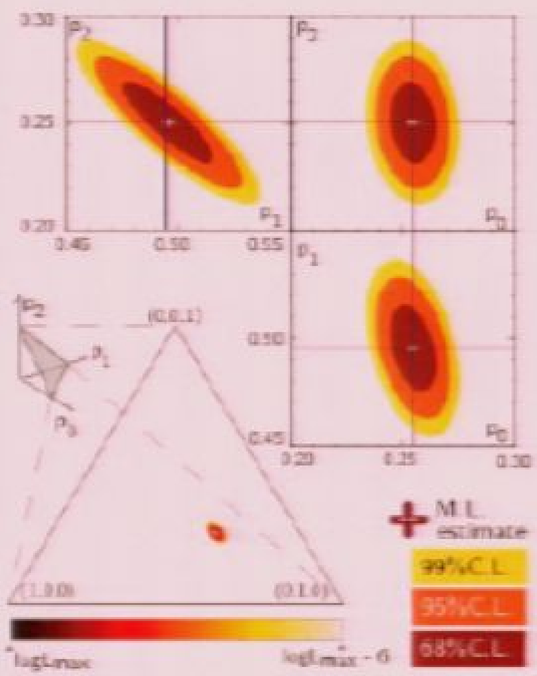
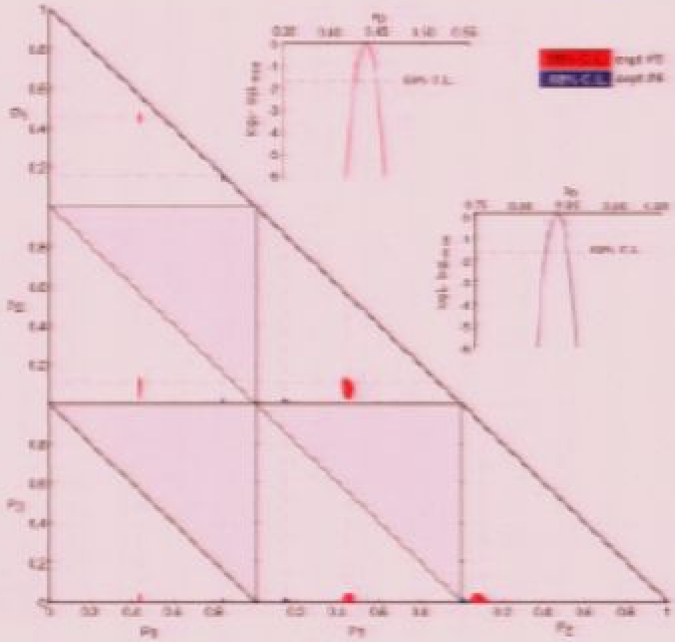
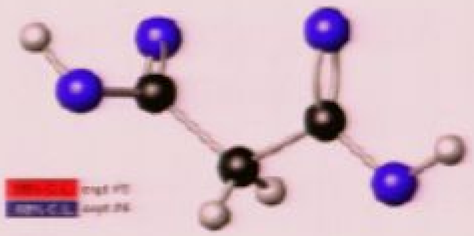
$\otimes \begin{cases} 00 \\ 11 \\ 01 \\ 10 \end{cases}$

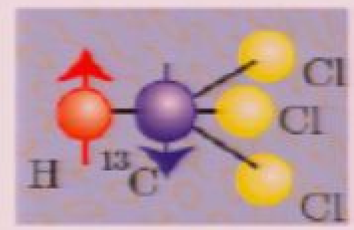
Experimental results

Noise Characterization - NMR results









≠	Map Description	Krauss operators (A_k)	k_m	p_0	p_1	p_2	p_3
1	Engineered: p = [0, 1, 0].	$\frac{1}{\sqrt{2}}\{Z_1, Z_2\}$	288	$0.000^{+0.004}$	$0.991^{+0.009}_{-0.015}$	$0.009^{+0.017}_{-0.009}$	-
2	Engineered: p = [0, 0, 1].	$\{Z_1 Z_2\}$	288	$0.001^{+0.006}_{-0.001}$	$0.004^{+0.011}_{-0.004}$	$0.996^{+0.004}_{-0.011}$	-
3	Engineered: p = [1/4, 1/2, 1/4].	$\{exp[i\frac{\pi}{4}(Z_1 + Z_2)]\}$	288	$0.254^{+0.018}_{-0.018}$	$0.495^{+0.021}_{-0.023}$	$0.250^{+0.019}_{-0.019}$	-
4	Engineered: p = [0, 1, 0, 0].	$\frac{1}{\sqrt{3}}\{Z_1, Z_2, Z_3\}$	432	$0.01^{+0.01}_{-0.01}$	$0.99^{+0.01}_{-0.05}$	$0.01^{+0.02}_{-0.01}$	$0.00^{+0.01}$
5	Natural noise (a)	unknown	432	$0.44^{+0.01}_{-0.02}$	$0.45^{+0.03}_{-0.05}$	$0.10^{+0.04}_{-0.08}$	$0.01^{+0.03}_{-0.01}$
6	Natural noise (b)	unknown	432	$0.84^{+0.01}_{-0.01}$	$0.15^{+0.02}_{-0.03}$	$0.01^{+0.03}_{-0.02}$	$0.00^{+0.02}$

TABLE I: Summary of experimental results.

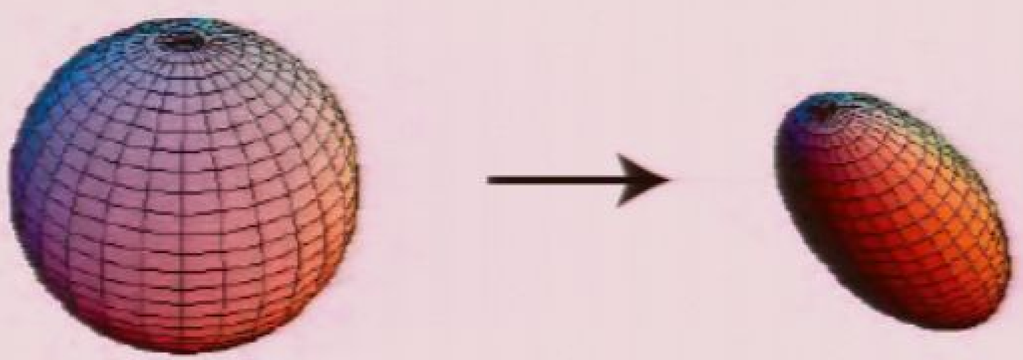
Emerson, Silva, Moussa, Ryan, Laforest, Baugh, Cory, Laflamme, Science 317, 1893, 2007

Characterising noise in q. systems

Process tomography:

$$\rho_f = \sum_k A_k \rho_i A_k^\dagger = \sum_{kl} \chi_{kl} P_k \rho_i P_l$$

For one qubit, 12 parameters are required as described by the evolution of the Bloch sphere:



For n qubits, we need to provide $4^{2n} - 4^n$ numbers to do so.

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$P < 10^{-6,-5,-4,\ldots,-1?}$

Knill et al.; Science, 279, 342, 1998

Kitaev, Russ. Math Survey 1997

Aharonov & Ben Or, ACM press

Preskill, PRSL, 454, 257, 1998

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- -its requirements are a guide for experimentalists
- -it is a benchmark to compare different technologies

Ingredients for FTQEC

- Knowledge of the noise
- Good quantum control
- Ability to extract entropy
- Parallel operations

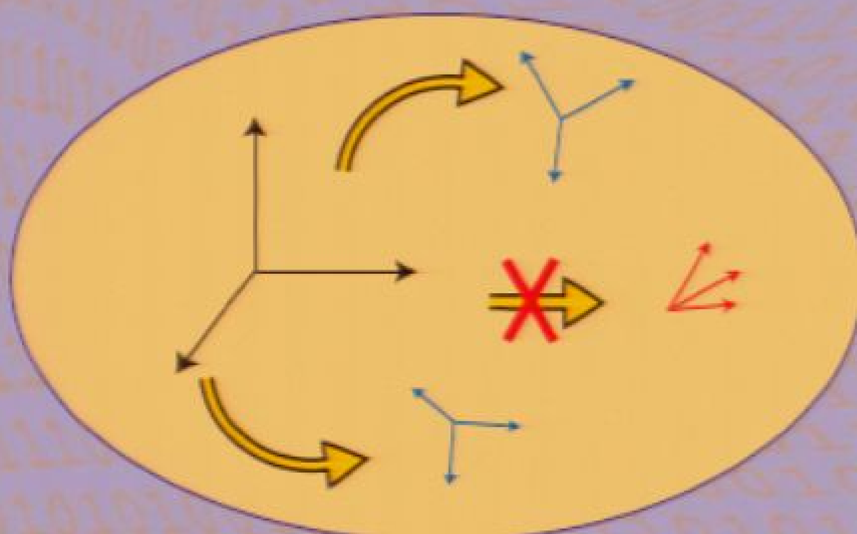
Definition: Quantum error codes

We can generalize the Schrodinger equation to open system:

$$\rho_f = \sum_a A_a \rho_i A_a^\dagger$$

An error correcting code is a code C defined by basis states $\{\ |i_L\rangle\ \}$ such that

$$\langle i_L | \mathcal{A}_a^\dagger \mathcal{A}_b | j_L \rangle = \delta_{ij} c_{ab}$$



Knill and Laflamme PRA 55, 900, 1997.