

Title: From Large N Double Scaling Limits to Non-Critical Superstrings

Date: Feb 19, 2008 11:00 AM

URL: <http://pirsa.org/08020033>

Abstract: We consider the large N limit of a class of fourdimensional supersymmetric theories in conjunction with a limit in their parameter space towards singular points where extra baryonic states become light, which causes the low-energy description to break down. However, this can be cured by defining a large N double scaling limit where one approaches the singularity by keeping the mass M of these states fixed. This limit has several interesting features. For example, the conventional 't Hooft limit leads to a free theory of colour singlet states where all interactions are suppressed by powers of $1/N$. In this case, the large N Hilbert space splits into two decoupled sectors, and one of them keeps residual interactions whose strength is inversely proportional to the mass M. We argue that for a class of these models the dynamics of this sector is dual to a non-critical superstring background, which is exactly solvable.

From Large N Double Scaling Limits to Non-Critical Superstrings

Gaetano Bertoldi
University of Toronto

- Work done in collaboration with [Nick Dorey](#) (DAMTP, Cambridge University), [Tim Hollowood](#) (Swansea University) and [J. Luis Miramontes](#) (University of Santiago de Compostela)
- Talk based on [hep-th/0507075](#), [0603075](#), [0603122](#), [0611016](#)

Outline

- Motivation and results
- 4d $N=1$ SUSY & Dijkgraaf-Vafa Matrix Model
- The large N Double scaling limit (DSL)
- Double scaled Little String Theory
- Duality proposal
- From Large N DSL to Matrix Model DSL
- Open problems

- 't Hooft: large N limit of Yang-Mills $g_{YM}^2 N$ fixed \rightarrow
String Theory: $g_s \sim 1/N$
stable glueballs \rightarrow excitations of the string
- AdS/CFT: examples of this scenario for **confining** models. Dual string theory is compactification of 10d critical superstring theory
- **Problems:** Ramond-Ramond flux, obstacle for quantization of the string;
- In the supergravity approximation only lightest glueball states survive
- Unwanted Kaluza-Klein states may not decouple

Results in brief

- We will propose large N limits of partially confining theories where the dual string theory has:
- No RR flux
- Exactly solvable worldsheet description
- String coupling can be made small
- This is achieved by a **Large N Double scaling limit (DSL)**
- This Large N DSL maps to a **Matrix Model DSL**
- Matrix Model DSL is analogous to ones considered in the study of $c \leq 1$ non-critical bosonic strings
- There is also a class of Large N DSL where “simple” dual string is not known

The Models

- 4d N=1 SUSY Yang-Mills $U(N)$ + chiral adjoint field Φ + polynomial superpotential

$$W(\Phi) = \varepsilon \text{Tr}_N \left[\frac{\Phi^{m+1}}{m+1} - \sum_{l=1}^m g^l \frac{\Phi^l}{l} \right]$$

- Vacua:

$$U(N) \rightarrow \prod_{i=1}^m U(N_i) \rightarrow \hat{G} = U(1)^m$$

no mass gap but partial confinement

- Large N spectrum:
- conventional weakly interacting glueballs
- Magnetic/electric **dibaryons** with mass $\sim N$

- There are special vacua where the mass M of some dibaryons vanishes:
 1. Conventional $1/N$ expansion breaks down
 2. Glueball interactions are NOT suppressed
- In the DSL where $N \rightarrow \infty$, superpotential couplings $g_l \rightarrow g_{l,crit}$ but M is kept fixed a sector of the theory remains interacting

$$g_{eff} \sim 1/N_{eff} = \frac{\sqrt{T}}{M}$$

- **Proposal:** dynamics of this sector has a dual non-critical string description

$$R^{3,1} \times \left(\frac{SL(2)_k}{U(1)} \times LG(W) \right) / Z_m$$

Dijkgraaf-Vafa Matrix Model

- N=1 SUSY YM U(N) with adjoint Φ

$$W_{tree}(\Phi) \Rightarrow W_{eff} = \sum_l N_l \frac{\partial F_0}{\partial S_l} + 2\pi i(\tau_0 + b_l) S_l$$

- F_0 is free energy of an auxiliary complex one-matrix model in the planar limit

$$\exp \sum_{g=0}^{\infty} F_g g_s^{2g-2} = \int d\hat{\Phi} \exp \left(-g_s^{-1} \text{Tr} W(\hat{\Phi}) \right).$$

- Spectral curve: $y^2 = W'(x)^2 + f_{m-1}(x)$

$$S_l = \oint_{A_l} y dx \quad \frac{\partial F_0}{\partial S_l} = \oint_{B_l} y dx$$

- String dual picture: Calabi-Yau geometry

$$W'(x)^2 + f_{m-1}(x) + y^2 + z^2 + w^2 = 0$$

- There are special vacua where the mass M of some dibaryons vanishes:
 1. Conventional $1/N$ expansion breaks down
 2. Glueball interactions are NOT suppressed
- In the DSL where $N \rightarrow \infty$, superpotential couplings $g_l \rightarrow g_{l,crit}$ but M is kept fixed a sector of the theory remains interacting

$$g_{eff} \sim 1/N_{eff} = \frac{\sqrt{T}}{M}$$

- **Proposal:** dynamics of this sector has a dual non-critical string description

$$R^{3,1} \times \left(\frac{SL(2)_k}{U(1)} \times LG(W) \right) / Z_m$$

Dijkgraaf-Vafa Matrix Model

- N=1 SUSY YM U(N) with adjoint Φ

$$W_{tree}(\Phi) \Rightarrow W_{eff} = \sum_l N_l \frac{\partial F_0}{\partial S_l} + 2\pi i(\tau_0 + b_l) S_l$$

- F_0 is free energy of an auxiliary complex one-matrix model in the planar limit

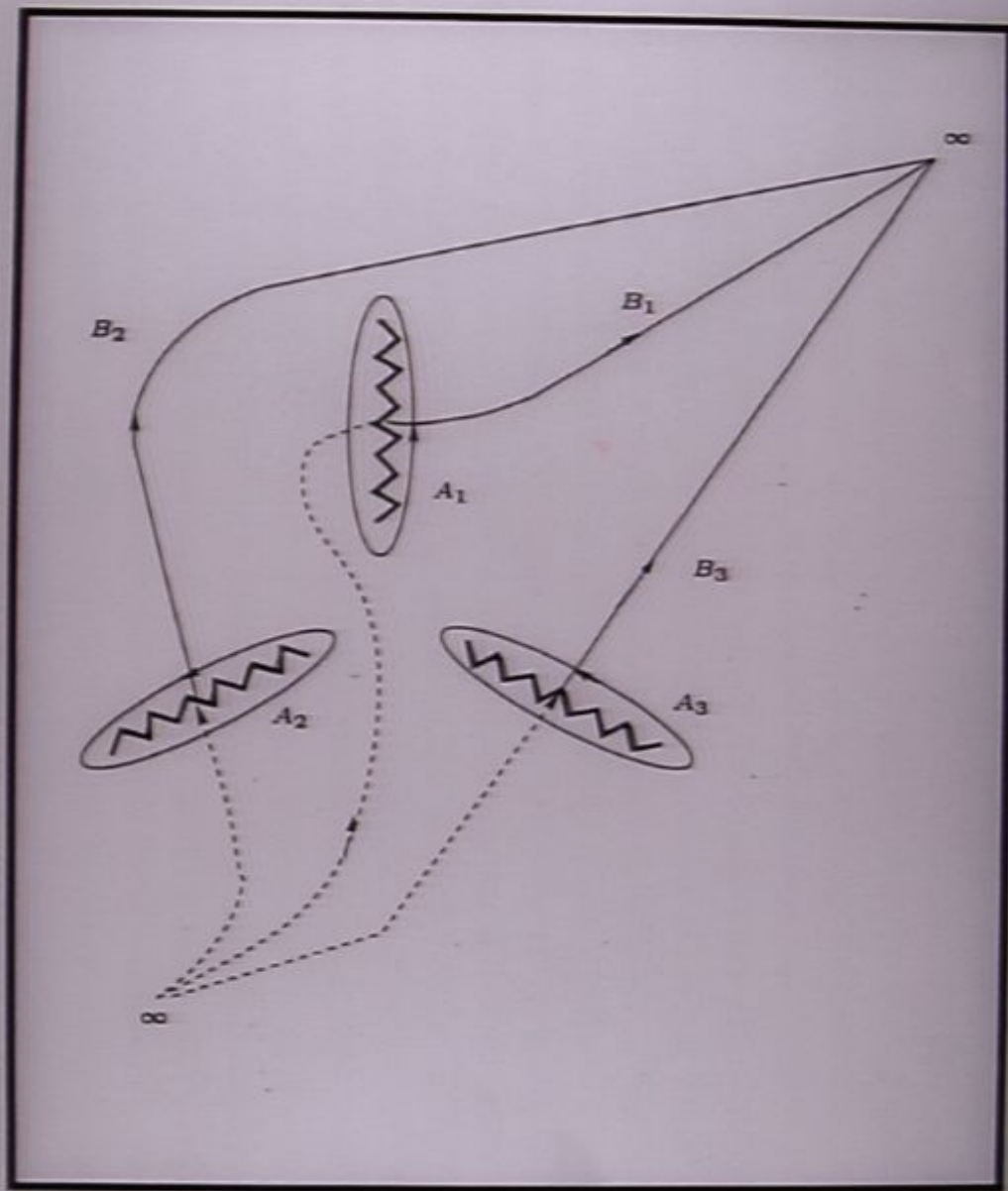
$$\exp \sum_{g=0}^{\infty} F_g g_s^{2g-2} = \int d\hat{\Phi} \exp \left(-g_s^{-1} \text{Tr} W(\hat{\Phi}) \right).$$

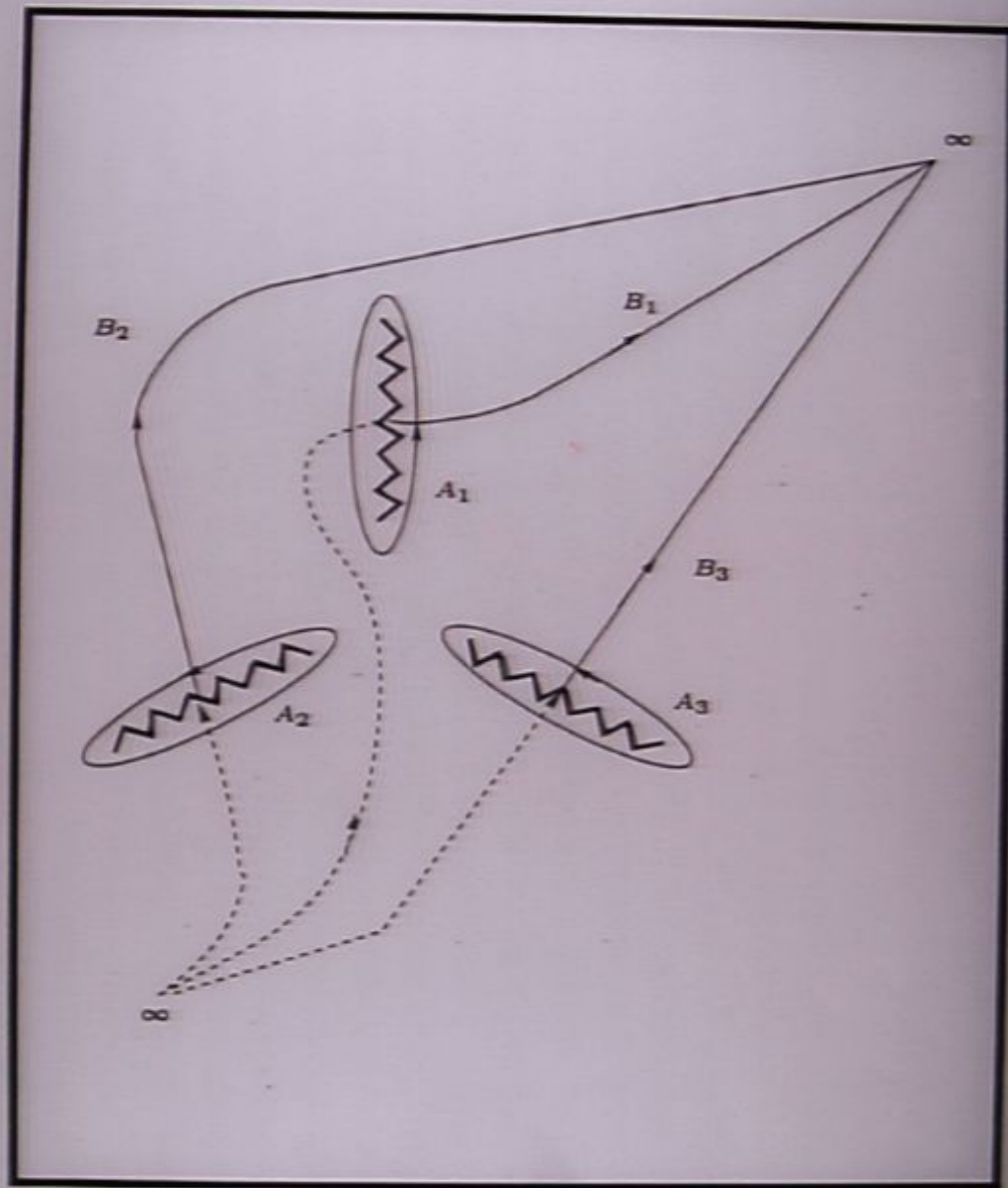
- Spectral curve: $y^2 = W'(x)^2 + f_{m-1}(x)$

$$S_l = \oint_{A_l} y dx \quad \frac{\partial F_0}{\partial S_l} = \oint_{B_l} y dx$$

- String dual picture: Calabi-Yau geometry

$$W'(x)^2 + f_{m-1}(x) + y^2 + z^2 + w^2 = 0$$





Dijkgraaf-Vafa Matrix Model

- N=1 SUSY YM U(N) with adjoint Φ

$$W_{tree}(\Phi) \Rightarrow W_{eff} = \sum_l N_l \frac{\partial F_0}{\partial S_l} + 2\pi i(\tau_0 + b_l) S_l$$

- F_0 is free energy of an auxiliary complex one-matrix model in the planar limit

$$\exp \sum_{g=0}^{\infty} F_g g_s^{2g-2} = \int d\hat{\Phi} \exp \left(-g_s^{-1} \text{Tr} W(\hat{\Phi}) \right).$$

- Spectral curve: $y^2 = W'(x)^2 + f_{m-1}(x)$

$$S_l = \oint_{A_l} y dx \quad \frac{\partial F_0}{\partial S_l} = \oint_{B_l} y dx$$

- String dual picture: Calabi-Yau geometry

$$W'(x)^2 + f_{m-1}(x) + y^2 + z^2 + w^2 = 0$$

- RR and NS fluxes

$$W_{eff} = \int_{CY} (H_{RR} + \tau H_{NS}) \wedge \Omega$$

$$\Omega \rightarrow ydx \quad (H_{RR} + \tau H_{NS}) \rightarrow Tdx$$

$$N_i = \oint_{A_i} Tdx \quad b_i = \oint_{B_i} Tdx$$

- Massless Dibaryons \rightarrow singular spectral curve, vanishing cycles

$$M \sim \int_{3\text{-cycle}} \Omega \rightarrow \int_{1\text{-cycle}} ydx$$

- Example of singularity: m branch points colliding

$$y^2 \approx x^m$$

The Exact F-term effective action

- F-term effective action

$$L_F = \int d^2\theta \left[W_{eff}^{(0)} + W_{eff}^{(2)} \right]$$

$$W_{eff}^{(2)} = \frac{1}{2} \sum_l \frac{\partial F_0}{\partial S_l \partial S_k} w_{\alpha,l} w_k^\alpha$$

$$W_{eff}^{(0)} = \sum_l N_l \frac{\partial F_0}{\partial S_l} + 2\pi i (\tau_0 + b_l) S_l$$

- In terms of the glueball superfields

$$S_l = -\frac{1}{32\pi^2} \text{Tr}_{N_l} [W_{\alpha,l} W_l^\alpha] = s_l + \theta_\alpha \chi_l^\alpha + \dots,$$

$$w_{\alpha,l} = \frac{1}{4\pi} \text{Tr}_{N_l} [W_{\alpha,l}] = \lambda_{\alpha l} + \theta_\beta f_{\alpha l}^\beta + \dots$$

$$L_F \supset V_{ij}^{(2)} f_{\alpha\beta}^i f^{\alpha\beta j} + V_{ijk}^{(3)} \chi_\alpha^i f^{\alpha\beta j} \lambda_\beta^k + \dots$$

$$L_F \supset H_{ij} \chi_\alpha^i \chi^{\alpha j}$$

L-point vertex

$$V_{i_1 \dots i_L}^{(L)} = \left\langle \frac{\partial^L F_0}{\partial S_1 \dots \partial S_L} \right\rangle$$

“Mass” matrix

$$H_{ij} = \left\langle \frac{\partial^2 W^{(0)}}{\partial S_i \partial S_j} \right\rangle$$

Breakdown of the 1/N expansion

- In the 't Hooft large N limit $\epsilon \sim N, g_l \sim N^0$

$$\langle S_l \rangle \sim N \quad V^{(L)} \sim N^{2-L} \quad H_{ij} \sim N^0$$

- At singularity $y^2 = x^m - \delta, \delta \rightarrow 0$

$$V_{ijk}^{(3)} \sim \frac{1}{S - S_{crit}} \sim \frac{1}{N \delta^{\frac{m+2}{2m}}}$$

- Define the **double scaling limit**

$$N \rightarrow \infty, \epsilon \sim N, \Lambda \sim \text{const}, \quad \delta \rightarrow 0$$

where $\Delta \equiv \epsilon \delta^{\frac{m+2}{2m}}$ is kept fixed

The Large N DSL

- Hilbert space splits into 2 decoupled sectors
- One sector becomes free in the DSL, while the other has finite interactions weighted by

$$1/N_{eff} \sim 1/\Delta \sim \sqrt{T}/M$$

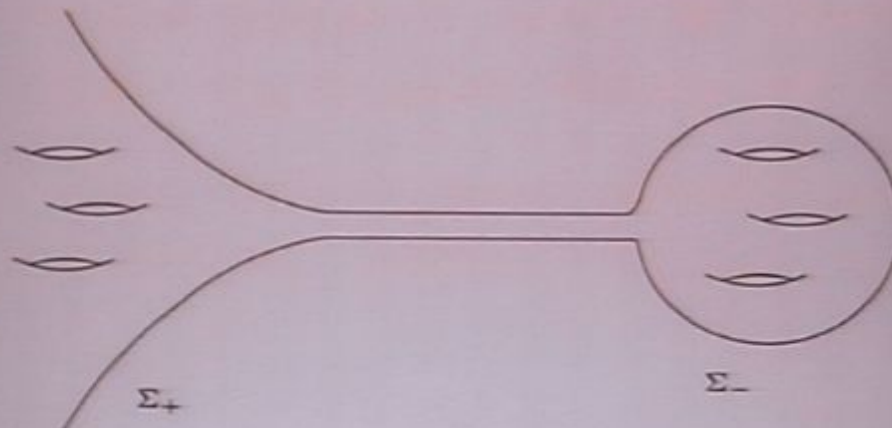
- Spectrum and interactions of the latter sector compatible with N=2 SUSY
- How is this possible given the presence of RR flux in initial geometry?
- **More general singularities:** the above picture partially holds: 1) Large N DSL is well-defined 2) no SUSY enhancement in general

$$y^2 \rightarrow Z_m(x)^2 B_n(x)$$

The field theory DSL

- there are two sectors \mathcal{H}_{\pm} in the Hilbert space which decouple in the large N DSL.
- \mathcal{H}_{-} contains $p = \frac{m-1}{2} U(1) \mathcal{N} = 1$ vector multiplets, \tilde{w}_{val} together with p neutral chiral multiplets S_i . These chiral multiplets become massless in the DSL.
- interactions between colour-singlet states in \mathcal{H}_{-} are controlled by

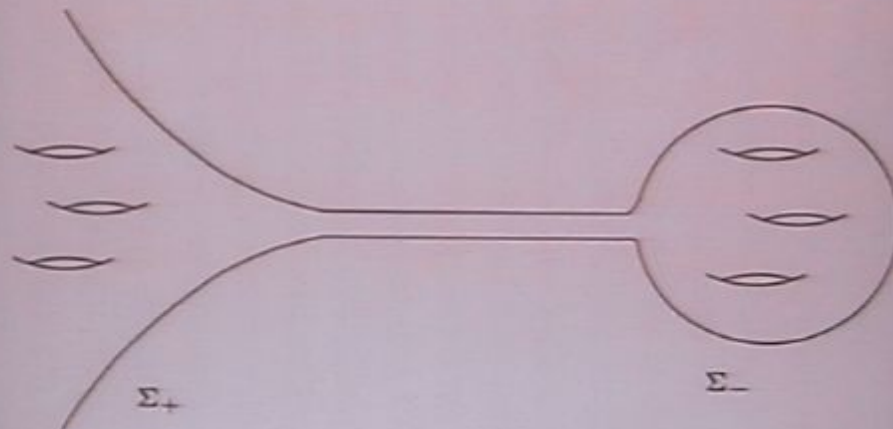
$$1/N_{\text{eff}} \sim 1/\Delta$$



The field theory DSL

- there are two sectors \mathcal{H}_{\pm} in the Hilbert space which decouple in the large N DSL.
- \mathcal{H}_{-} contains $p = \frac{m-1}{2} U(1) \mathcal{N} = 1$ vector multiplets, $\bar{w}_{\alpha i}$ together with p neutral chiral multiplets S_i . These chiral multiplets become massless in the DSL.
- interactions between colour-singlet states in \mathcal{H}_{-} are controlled by

$$1/N_{eff} \sim 1/\Delta$$



The Large N DSL

- Hilbert space splits into 2 decoupled sectors
- One sector becomes free in the DSL, while the other has finite interactions weighted by

$$1/N_{eff} \sim 1/\Delta \sim \sqrt{T}/M$$

- Spectrum and interactions of the latter sector compatible with N=2 SUSY
- How is this possible given the presence of RR flux in initial geometry?
- **More general singularities**: the above picture partially holds: 1) Large N DSL is well-defined 2) no SUSY enhancement in general

$$y^2 \rightarrow Z_m(x)^2 B_n(x)$$

The Large N DSL

- Hilbert space splits into 2 decoupled sectors
- One sector becomes free in the DSL, while the other has finite interactions weighted by

$$1/N_{eff} \sim 1/\Delta \sim \sqrt{T}/M$$

- Spectrum and interactions of the latter sector compatible with N=2 SUSY
- How is this possible given the presence of RR flux in initial geometry?
- **More general singularities:** the above picture partially holds: 1) Large N DSL is well-defined 2) no SUSY enhancement in general

$$y^2 \rightarrow Z_m(x)^2 B_n(x)$$

Little String Theory

- How does the non-critical superstring emerge? String Theory in the proximity of certain CY singularities
- Consider CY in IIB with no fluxes

$$z^2 + w^2 + y^2 + x^m = \mu$$

- At $\mu = 0$ singularity \rightarrow D3-branes wrapping shrinking cycles \rightarrow massless states \rightarrow singular string amplitudes
- Target space \rightarrow Throat with linear dilaton

$$g_s = g_0 \exp\langle\phi\rangle \rightarrow \infty$$

- In the limit $g_0 \rightarrow 0$ states localized at singularity decouple from bulk 10d modes BUT maintain non-trivial interactions
- \rightarrow non-critical 4d string theory without gravity dubbed Little String Theory
- LST has N=2 SUSY in 4d. Below the string scale it reduces to an Argyres-Douglas SCFT

Double Scaled LST

- LST has a holographic dual description in terms of a linear dilaton background corresponding to the infinite throat region

Problem: the string coupling diverges

- Giveon and Kutasov considered the **double scaling limit**

$$g_0 \rightarrow 0, \mu \rightarrow 0, \quad \kappa^{-1} = \frac{\mu^{\frac{m+2}{2m}}}{g_0} \quad \text{fixed}$$

The singularity is cured by giving a mass to the wrapped D3-brane states. The string dual to DSLST is now given by

$$R^{3,1} \times \left(\frac{SL(2)_k}{U(1)} \times LG(W) \right) / Z_m$$

$$\kappa \sim 1/M_{D3} \sqrt{\alpha'}$$

- The semi-infinite “cigar” geometry of $SL(2)/U(1)$ replaces the linear dilaton throat. **The string coupling has an upper bound**

Proposal and Checks

- The Double Scaling Limit of LST maps to the field theory large N DSL. The above models in the large N DSL have a dual string description given by non-critical susy backgrounds

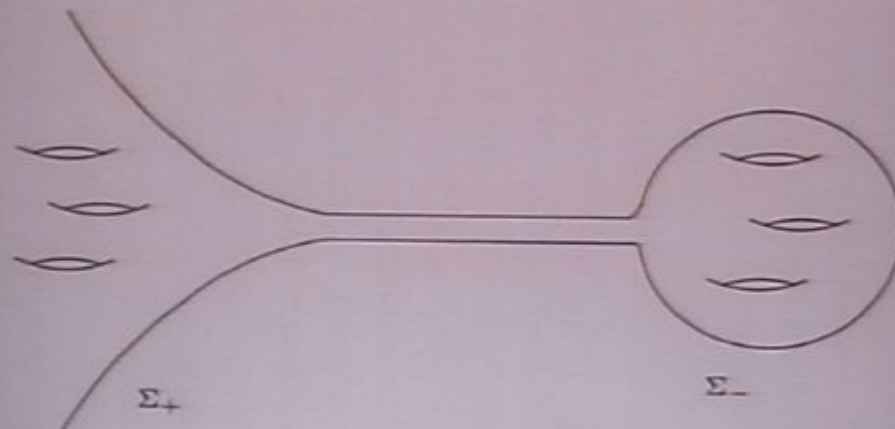
$$\kappa = \frac{\Lambda^{3-m/2}}{\Delta} F_1\left(\frac{\epsilon}{N}, \Lambda, M_{UV}\right) \quad \alpha' = \frac{1}{\Lambda^2} F_2\left(\frac{\epsilon}{N}, \Lambda, M_{UV}\right)$$

- Puzzle: DSLST has N=2 SUSY (no RR flux) vs N=1 (RR flux not zero)
- Massless spectrum and F-term interactions are the same as DSLST
- Fact that degrees of freedom of interacting sector in field theory become exactly massless in the large N DSL and that their superpotential vanishes is strong indication of SUSY enhancement from N=1 to N=2

The field theory DSL

- there are two sectors \mathcal{H}_{\pm} in the Hilbert space which decouple in the large N DSL.
- \mathcal{H}_- contains $p = \frac{m-1}{2} U(1) \mathcal{N} = 1$ vector multiplets, $\bar{w}_{\alpha I}$ together with p neutral chiral multiplets S_I . These chiral multiplets become massless in the DSL.
- interactions between colour-singlet states in \mathcal{H}_- are controlled by

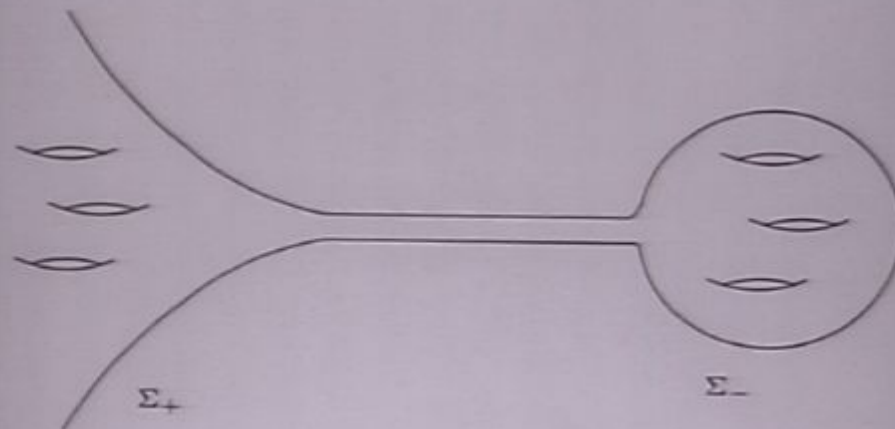
$$1/N_{eff} \sim 1/\Delta$$



The field theory DSL

- there are two sectors \mathcal{H}_{\pm} in the Hilbert space which decouple in the large N DSL.
- \mathcal{H}_- contains $p = \frac{m-1}{2} U(1) \mathcal{N} = 1$ vector multiplets, $\bar{w}_{\alpha i}$ together with p neutral chiral multiplets S_i . These chiral multiplets become massless in the DSL
- interactions between colour-singlet states in \mathcal{H}_- are controlled by

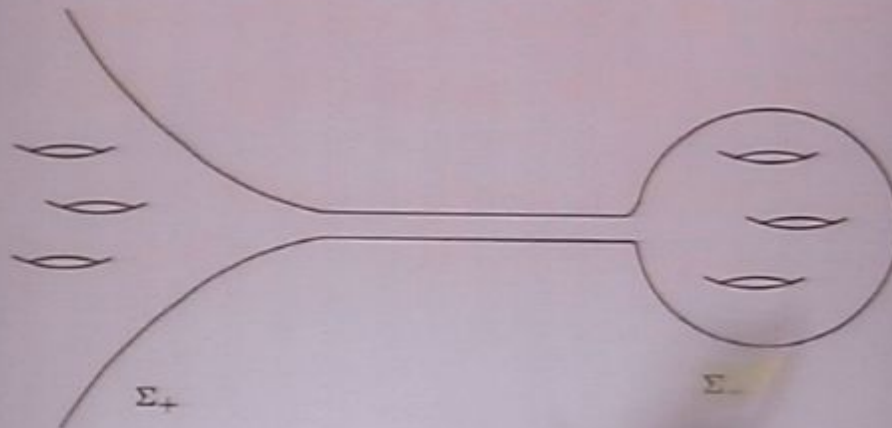
$$1/N_{eff} \sim 1/\Delta$$



The field theory DSL

- there are two sectors \mathcal{H}_{\pm} in the Hilbert space which decouple in the large N DSL.
- \mathcal{H}_- contains $p = \frac{m-1}{2} U(1) \mathcal{N} = 1$ vector multiplets, $\tilde{w}_{\alpha l}$ together with p neutral chiral multiplets S_l . These chiral multiplets become massless in the DSL
- interactions between colour-singlet states in \mathcal{H}_- are controlled by

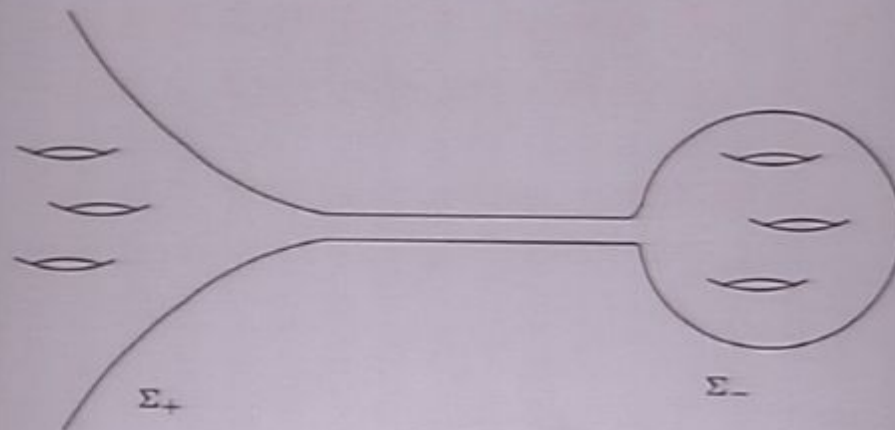
$$1/N_{eff} \sim 1/\Delta$$



The field theory DSL

- there are two sectors \mathcal{H}_{\pm} in the Hilbert space which decouple in the large N DSL.
- \mathcal{H}_{-} contains $p = \frac{m-1}{2} U(1) \mathcal{N} = 1$ vector multiplets, $\bar{w}_{\alpha I}$ together with p neutral chiral multiplets S_I . These chiral multiplets become massless in the DSL
- interactions between colour-singlet states in \mathcal{H}_{-} are controlled by

$$1/N_{eff} \sim 1/\Delta$$



Summary

- Novel example of duality between a SUSY field theory in a large N limit and a string background.
- Field Theory is in a **partially confined phase**. String background is **non-critical** and **exactly solvable** in α' . We can go beyond the supergravity approximation and there is no RR flux.
- The large N DSLs of these 4d SUSY theories map to DSLs of the auxiliary Dijkgraaf-Vafa Matrix Model. In particular they are well-defined in higher genus as well.
- The non-critical $c \leq 1$ bosonic string associated to MM DSL should correspond to a topological twist of the dual non-critical superstring background.
- Can define large N DSLs where there is no SUSY enhancement. In these cases dual string background is not known.

Summary

- Novel example of duality between a SUSY field theory in a large N limit and a string background.
- Field Theory is in a **partially confined phase**. String background is **non-critical** and **exactly solvable** in α' . We can go beyond the supergravity approximation and there is no RR flux.
- The large N DSLs of these 4d SUSY theories map to DSLs of the auxiliary Dijkgraaf-Vafa Matrix Model. In particular they are well-defined in higher genus as well.
- The non-critical $c \leq 1$ bosonic string associated to MM DSL should correspond to a topological twist of the dual non-critical superstring background.
- Can define large N DSLs where there is no SUSY enhancement. In these cases dual string background is not known.

Open problems

- Determine the non-critical bosonic string dual to MM DSL and compare with the topologically twisted superstring background. This would be a non-trivial “topological” test of the above duality proposal
- Complicated problem: near-critical spectral curve has in general genus ≥ 1 .
- Is the dual to generalized large N DSL with N=1 SUSY still simple?
- Explicit evaluation of higher genus free energy F_g and its asymptotic behaviour for large g . Perturbative series is not expected to be convergent \rightarrow non-perturbative D-brane effects?

DSLs from Matrix Model point of view

- The Large N DSLs map to Matrix Model DSLs
- “New” universality classes \rightarrow “near-critical” spectral curve

$$\Sigma : y^2 = Z_m(x)^2 \sigma_{2s}(x) \rightarrow C Z_m(x)^2 B_n(x)$$

$$z_j, b_i \rightarrow x_0, \quad x = a\tilde{x}, \quad z_i = a\tilde{z}_i, \quad b_j = a\tilde{b}_j \quad a \rightarrow 0$$

$$\Sigma_- : y_-^2 = \tilde{Z}_m(\tilde{x})^2 \tilde{B}_n(\tilde{x}) .$$

- Scaling of Matrix Model free energy

$$F_g(\Sigma) \rightarrow (Na^{m+n/2+1})^{2-2g} F_g(\Sigma_-)$$

- Non-critical bosonic string dual to MM DSL? Topological twist of Non-critical superstring background
- Explicit check for simplest singularity (n=2, conifold) related to c=1 non-critical bosonic string

$$y^2 = x^2 - \delta$$

Double Scaled LST

- LST has a holographic dual description in terms of a linear dilaton background corresponding to the infinite throat region

Problem: the string coupling diverges

- Giveon and Kutasov considered the **double scaling limit**

$$g_0 \rightarrow 0, \mu \rightarrow 0, \quad \kappa^{-1} = \frac{\mu^{\frac{m+2}{2m}}}{g_0} \quad \text{fixed}$$

The singularity is cured by giving a mass to the wrapped D3-brane states. The string dual to DSLST is now given by

$$R^{3,1} \times \left(\frac{SL(2)_k}{U(1)} \times LG(W) \right) / Z_m$$

$$\kappa \sim 1/M_{D3} \sqrt{\alpha'}$$

- The semi-infinite “cigar” geometry of $SL(2)/U(1)$ replaces the linear dilaton throat. **The string coupling has an upper bound**

Open problems

- Determine the non-critical bosonic string dual to MM DSL and compare with the topologically twisted superstring background. This would be a non-trivial “topological” test of the above duality proposal
- Complicated problem: near-critical spectral curve has in general genus ≥ 1 .
- Is the dual to generalized large N DSL with N=1 SUSY still simple?
- Explicit evaluation of higher genus free energy F_g and its asymptotic behaviour for large g . Perturbative series is not expected to be convergent \rightarrow non-perturbative D-brane effects?

Summary

- Novel example of duality between a SUSY field theory in a large N limit and a string background.
- Field Theory is in a **partially confined phase**. String background is **non-critical** and **exactly solvable** in α' . We can go beyond the supergravity approximation and there is no RR flux.
- The large N DSLs of these 4d SUSY theories map to DSLs of the auxiliary Dijkgraaf-Vafa Matrix Model. In particular they are well-defined in higher genus as well.
- The non-critical $c \leq 1$ bosonic string associated to MM DSL should correspond to a topological twist of the dual non-critical superstring background.
- Can define large N DSLs where there is no SUSY enhancement. In these cases dual string background is not known.

Open problems

- Determine the non-critical bosonic string dual to MM DSL and compare with the topologically twisted superstring background. This would be a non-trivial “topological” test of the above duality proposal
- Complicated problem: near-critical spectral curve has in general genus ≥ 1 .
- Is the dual to generalized large N DSL with N=1 SUSY still simple?
- Explicit evaluation of higher genus free energy F_g and its asymptotic behaviour for large g . Perturbative series is not expected to be convergent \rightarrow non-perturbative D-brane effects?



LargeNds1.ppt

- From Large N Double Scaling Limits to Non-Critical Superstrings
Gaetano Bertoldi
University of Toronto
- Work done in collaboration with Nick Dorey (DAMTP, Cambridge University), Tim Hollowood (Swansea University) and J. Luis Miramontes (University of Santiago de Compostela)
- Talk based on hep-th/0507075, 0603075, 0603122, 0611016
- Outline
 - Motivation and results
 - 4d N=1 SUSY & Dijkgraaf-Vafa Matrix Model
 - The large N Double scaling limit (DSL)
 - Double scaled Little String Theory
 - Duality proposal
 - From Large N DSL to Matrix Model DSL
 - Open problems
- 't Hooft: large N limit of Yang-Mills fixed \rightarrow String Theory:
stable glueballs \rightarrow excitations of the string
- AdS/CFT: examples of this scenario for confining models. Dual string theory is compactification of 10d critical superstring theory
- Problems: Ramond-Ramond flux, obstacle for quantization of the string;
- In the supergravity approximation only lightest glueball states survive
- Unwanted Kaluza-Klein states may not decouple
- Results in brief
 - We will propose large N limits of partially confining theories where the dual string theory has:
 - No RR flux
 - Exactly solvable worldsheet description

From Large N Double Scaling Limits to Non-Critical Superstrings

Gaetano Bertoldi
University of Toronto

Click to add notes

acam.0.9.0.dmg

HousesCars

Macintosh HD

UkBank

PHY1510Ff.doc



DSLtalk+

Network
Macintosh HD
Desktop
gaetano
Applications
Documents
Music
Pictures

LargeNdsl.ppt
Comment on consistency of identifica...gularity.doc
Title.doc
0507075bertoldidorey

1 of 4 selected, 36.29 GB available

Bergmann

PSmath

PSdirichlet

DSLtalk+

Action: Perform tasks with the selected item.

- Macintosh HD
- Desktop
- gaetano
- Applications
- Documents
- Music
- Pictures

LargeNdsl.ppt

Comment on consistency of identifica...gularity.doc

Title.doc

0507075bertoldidorey

1 of 4 selected, 36.29 GB available

acam.0.9.0.dmg

HousesCars

Macintosh HD

UkBank

PHY1510Ff.doc

Bergmann

PSmath

PSdirichlet

