Title: Special Topics in Physics - Lecture 6A

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Abstract: The Problem of Time in Quantum Gravity and Cosmology

General Relativity as a gauge theory

General Relativity as a gauge theory

Einstein-Hilbert $\rightarrow$ Plebanski review

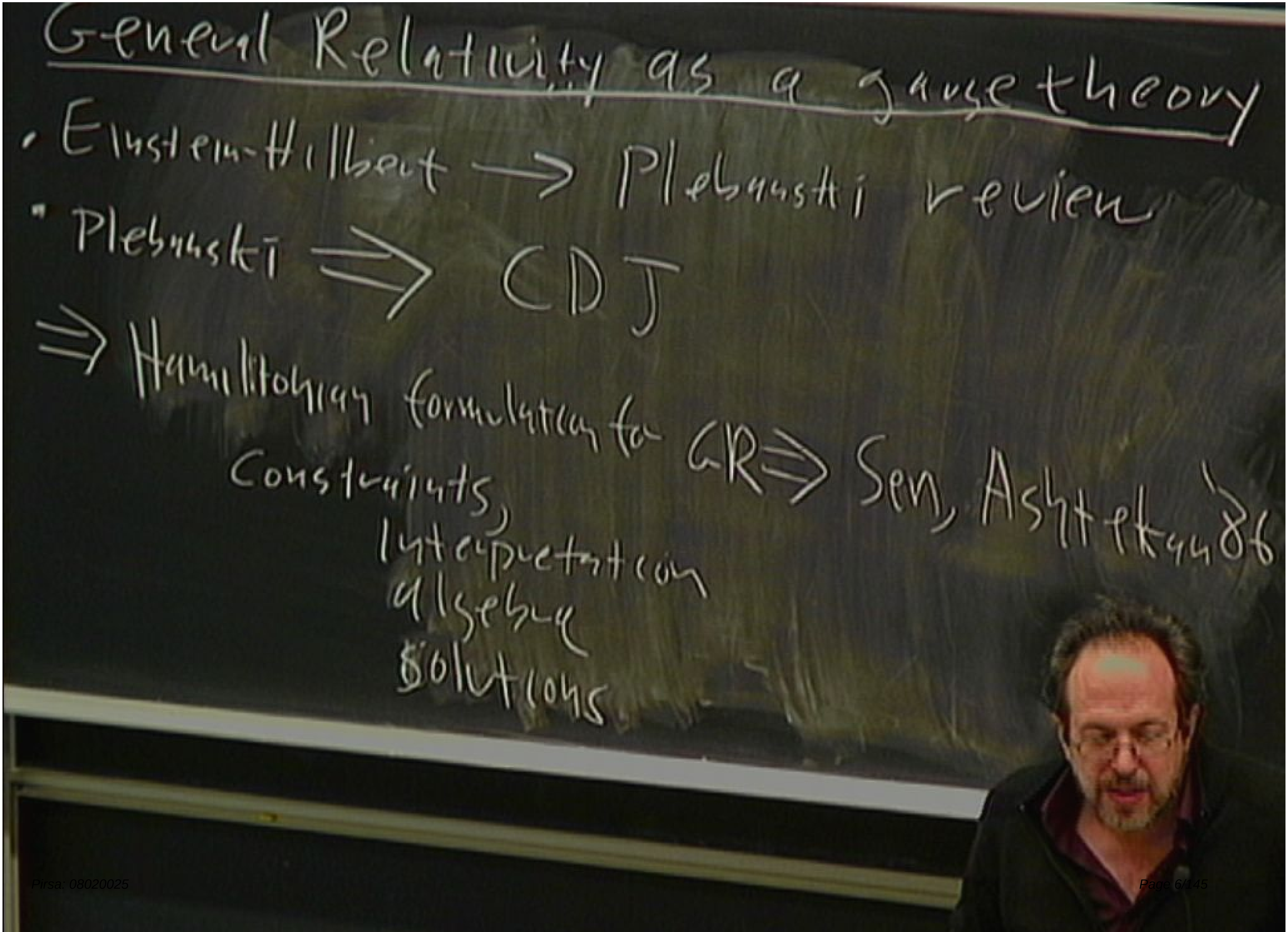
General Relativity as a gauge theory

- Einstein-Hilbert $\longrightarrow$ Plebanski review
- Plebanski $\Longrightarrow$ CDJ

General Relativity as a gauge theory

- Einstein-Hilbert $\longrightarrow$ Plebanski review
- Plebanski $\Longrightarrow$ CDJ

$\Longrightarrow$ Hamiltonian formulation for GR $\Longrightarrow$ Sen, Ashtekar '86


General Relativity as a gauge theory
• Einstein-Hilbert → Plebanski review
• Plebanski ⇒ CDJ
⇒ Hamiltonian formulation for GR ⇒ Sen, Ashtekar '86
Constraints,
Interpretation
algebra
Solutions

...iltonian formulation for GR $\Rightarrow$ Sen, Ashtekar 86

Constraints,

Interpretation

algebra

Solutions

Dirac quantum w/ constraints

usual $g_{\mu\nu}$ metric

usual $g_{\mu\nu}$ metric $\Longrightarrow$ connection

$$\Gamma_{\mathrm{Chr}\,\mu\nu}{}^{\alpha} = \frac{1}{2} g^{\alpha\beta}\left(\partial_\mu g_{\beta\nu} + \partial_\nu g_{\beta\mu} - \partial_\beta g_{\mu\nu}\right)$$

$\Rightarrow$ Connection

$$\Gamma_{\text{Chr.}\,\mu\nu}^{\ \ \alpha} = \frac{1}{2} g^{\alpha\beta} \left(\partial_\mu g_{\beta\nu} + \partial_\nu g_{\beta\mu} - \partial_\beta g_{\mu\nu} \right)$$

$$= R = \partial\Gamma + \Gamma\Gamma$$

$$S = \int \sqrt{-g}\, R \qquad = R$$

$$S = \int \sqrt{-g}\,(R - 2\Lambda) \qquad = \ldots$$

$$S = \int \sqrt{-g}\,(R - 2\Lambda)$$

Palatini – Cartan

$g_{\mu\nu} \to$ frame fields $e_\mu^{\ a}$

$$S = \int \sqrt{-g}\,(R - 2\Lambda)$$

Palatini — Cartan

$$g_{\mu\nu} \to \text{frame fields } e_\mu^{\ a}$$

$\Gamma \to A^{b}_{mq}$ free

algebraic solutions } w/ constraints

$\Gamma^{\lambda}_{\mu\nu} = \frac{1}{2} g^{\lambda\beta}(\partial_\mu g_{\beta\nu} + \partial_\nu g_{\beta\mu}$

$$S(g) = \int \sqrt{g}\,(R - 2\Lambda) \qquad R = \partial\Gamma + \Gamma\Gamma \qquad -\partial_\beta g_{\mu\nu}$$

Palatini – Cartan

$g_{\mu\nu} \to$ frame fields $e_\mu^{\ a}$; $\Gamma \to A_{\mu a}^{\ \ b}$ free

$$S^{Palat}(e, A) = \int (e^a \wedge e^b \wedge F^{cd} + 2\Lambda\, e^a \wedge e^b \wedge e^c \wedge e^d)\,\epsilon_{abcd}$$

$$S(g) = \int \sqrt{-g}\,(R - 2\Lambda)$$

Palatini – Cartan

$g_{\mu\nu} \to$ frame fields e_μ^a

$$S^{Palat}(e, A) = \int (e^a \wedge e^b \wedge F^{cd}(A)$$

$\Gamma_{\mathrm{Ch}, \mu\nu} \ldots \frac{1}{2} g \, (\partial_\mu g_{\beta\nu} + \partial_\nu g_{\beta\mu}$

$(R - 2\Lambda) \quad = R = \partial\Gamma + \Gamma\Gamma \quad - \partial_\beta g_{\mu\nu})$

= Cartan

frame fields $e_\mu^{\ a}$ $\quad \Gamma \to A_{\mu a}^{\ \ b}$ free

$) = \int (e^a \wedge e^b \wedge F^{cd}(A) + 2\Lambda \, e^a \wedge e^b \wedge e^c \wedge e^d) \, \epsilon_{abcd}$

Plebanski (A_a

$$\Gamma \to A_{\mu a}{}^{b} \quad (\text{free} \in SO(1,3))$$

$$\wedge e^b \wedge F^{cd}(A) + \Lambda\, e^a \wedge e^b \wedge e^c \wedge e^d \big)\, \epsilon_{abcd}$$

Plebanski $\left\{ \begin{array}{l} A_\mu^i \in su(2) \quad (A_\mu) = A_\mu^i [\tau_i] \\ B_{\mu\nu}^i \in su(2) \end{array} \right.$

Plebanski $A^{i}_{\mu} \in su(2)$ $(A_\mu) = A^{i}_{\mu}[T_i]$

$B^{i}_{\mu\nu} \in su(2)$

$$S = \int \{ B^i \wedge F_i - \frac{\Lambda}{2} B^i \wedge B_i$$

Plebanski $A_\mu^i \in su(2)$ $(A_\mu) = A_\mu^i [\tau_i]$

$B_{\mu\nu}^i \in su(2)$

$$S[B,A,\phi] = \int \{ B^i \wedge F_i - \frac{\Lambda}{2} B^i \wedge B_i - \frac{1}{2} \phi_{ij} B^i \wedge B^j \qquad \phi^{ii} = 0$$

$$= \int \{ B^i \wedge F_i - \frac{\Lambda}{2} B^i \wedge B_i - \frac{1}{2} \phi_{ij}$$

$$= \int B^i \wedge F_i - \phi_{ij} B^i \wedge B^j$$

Plebanski $\quad A_\mu^i \in su(2) \quad (A_\mu) = A_\mu^i \{\tau$

$B^i_{\mu\nu} \in su(2)$

$$S[B,A,\phi] = \int \left\{ B^i \wedge F_i - \frac{\Lambda}{2} B^i \wedge B_i + \frac{1}{2} \phi_{ij} B^i \wedge B^j \right.$$

$$\boxed{S^{Pl,\Lambda} = \int B^i \wedge F_i + \phi_{ij} B^i \wedge B^j} \qquad \boxed{\phi^i_i = -\Lambda}$$

EOM:

$$\boxed{F^i = \Phi^i{}_j\, B^j}$$

$$B^i \wedge B^j - \frac{1}{3}\delta^{ij} B^k \wedge B_k$$

EOM:

$$\boxed{F^i = \Phi^i{}_j B^j}$$

$$\hat{\mathcal{D}} \wedge B^i = 0$$

$$B^i \wedge B^j - \frac{1}{3}\delta^{ij} B^k \wedge B_k = 0$$

$$\boxed{F^{i} = \Phi^{i}{}_{j} B^{j}} \qquad \hat{D} \wedge B^{i} = 0$$

$$B^{i} \wedge B^{j} - \frac{1}{3}\delta^{ij} B^{k} \wedge B_{k} = 0$$

$$\Rightarrow \exists\, e^{a}_{\mu} \text{ s.t. } \boxed{B^{i} = e^{[a} \wedge e^{b]}\big|_{SD} = e^{a} \wedge e^{b} + \tfrac{1}{2}\epsilon^{ab}{}_{cd}\, e^{c} \wedge e^{d}}$$

$$= e^{[a} \wedge e^{b]\,SD} = \boxed{e^a \wedge e^b + i\epsilon^{ab}{}_{cd}\, e^c \wedge e^d}$$

$$= \Sigma^i$$

self-dual 2-forms of metric

$$g_{\mu\nu} = e_\mu{}^a \otimes e_\nu{}^b \eta_{ab}$$

$$\boxed{F^{i}(A) = \Phi^{i}{}_{\bar{j}}\, B^{\bar{j}}}$$

$$B^{i} \wedge B^{\bar{j}} - \frac{1}{3}\delta^{i\bar{j}} B^{k} \wedge B_{k}$$

Σ^{a}_{μ} s.t. $B^{i} = e^{a} \wedge e^{b}$

$e_\mu \; \xi^i$ $\boxed{B^i = e^a \wedge e^b}$

$= \Sigma^i$ Sel

$$F^i(A) = \psi^i_{ab}\, e^a \wedge e^b$$

$B^k \wedge B_k = 0$ $\qquad \Sigma^i = (\quad -i \quad)$

$$\boxed{(e^{[a} \wedge e^{b]})^{SD} = e^a \wedge e^b + i\epsilon^{ab}{}_{cd}\, e^c \wedge e^d}$$

Self-dual 2-forms of metric

$$g_{\mu\nu} = e_\mu^a \otimes e_\nu^b \eta_{ab}$$

$e^a \wedge e^b$

$\Sigma^i +$

$$\boxed{B = e \wedge e = e \wedge e}$$

$$= \Sigma^i \quad \text{self-dual 2-forms}$$

$$F^i(A) = \psi^i_{ab}\, e^a \wedge e^b$$

$$\boxed{F^i(A) = \phi^i_j\, \Sigma^j + \bar{\Phi}^i_j\, \bar{\Sigma}^j}$$

$B = e \wedge e = e$

$= \Sigma^i$ Self-dual 2

$$F^i(A) = \Psi^i_{ab}\, e^a \wedge e^b$$

$$\boxed{F^i(A) = \phi^i_j\, \Sigma^i +}$$

$$B = e \wedge e \quad = e$$

$$= \Sigma^i \qquad \text{Self-dual 2}$$

$$F^i(A) = \Psi^i_{ab}\, e^a \wedge e^b$$

$$\boxed{F^i(A) = \phi^i_j\, \Sigma^j}$$

$$\boxed{F^i(A) = \phi^i_j \Sigma^j \ldots}$$

$$F^i(\bar{A})$$

$$= \Sigma^{i} /$$

Self

$$F^{i}(A) = \Psi^{i}_{ab}\, e^{a} \wedge e^{b}$$

$$\boxed{F^{i}(A) = \phi^{i}_{j}\, \Sigma^{j}}$$

$$F^{i}(\bar{A}) = \bar{\phi}^{i}_{j}\, \bar{\Sigma}^{j}$$

simple solution $\Phi_{ij} = \frac{1}{3}\delta_{ij}$

simple solution $\Phi_{ij} = \frac{1}{3}\delta_{ij}$

$$\boxed{F^{\bar{i}} = \frac{1}{3}\xi^{\bar{i}}}$$

simple solution $\boxed{\Phi_{ij} = \frac{1}{3}\delta_{ij}}$ ✓

$$\boxed{F^{\bar{i}} = \frac{1}{3}\Sigma^{\bar{i}}}$$

(A)De Sitter

$$B_{\mu\nu}$$

$$S[B,A,\phi] = \int \left\{ B^i \wedge F_i - \frac{\Lambda}{2} B^i \wedge B_i + \frac{1}{2}\phi \right.$$

$$\boxed{S^{Pl} = \int B^i \wedge F_i - \frac{1}{2}\phi_{ij} B^i \wedge B^j}$$

EOM: $\boxed{F^i(A) = \Phi^i{}_j B^j}$

$B^i \wedge B^j - \frac{1}{3}\delta^{ij} B^k \wedge B_k =$

$\Rightarrow \exists\, e_\mu^a$ s.t. $\boxed{B^i = e^{[a} \wedge e^{b]} \ldots}$

$= \Sigma^i$

$F^i(A) \ldots$

$$B^i = (\Phi^{-1})^i{}_j F^j$$

$$B^i \wedge B^j - \frac{1}{3}\delta^{ij} B^k \wedge B_k =$$

$$\Rightarrow \exists\; e_\mu^a \text{ s.t. } \boxed{B^i = e^{[a} \wedge e^{b]}}$$

$$= \Sigma^i$$

$F^i(A)$

$$\boxed{S = \int B^i \wedge F_i - \frac{1}{2}\phi_{ij} B^i \wedge B^j}$$

$$B^i = (\Phi^{-1})^i{}_j F^j$$

$$S = \frac{1}{2}\int F^i \wedge F^j (\phi^{-1})_{ij}$$

$$\Rightarrow \exists\, e_\mu^a \text{ s.t. } \boxed{B^i = e^{[a} \wedge e^{b]SD}}$$

$$= \Sigma^i$$

$$\boxed{S^{pl} = \int B^i \wedge F_i - \frac{1}{2}\Phi_{ij} B^i \wedge B^j}$$

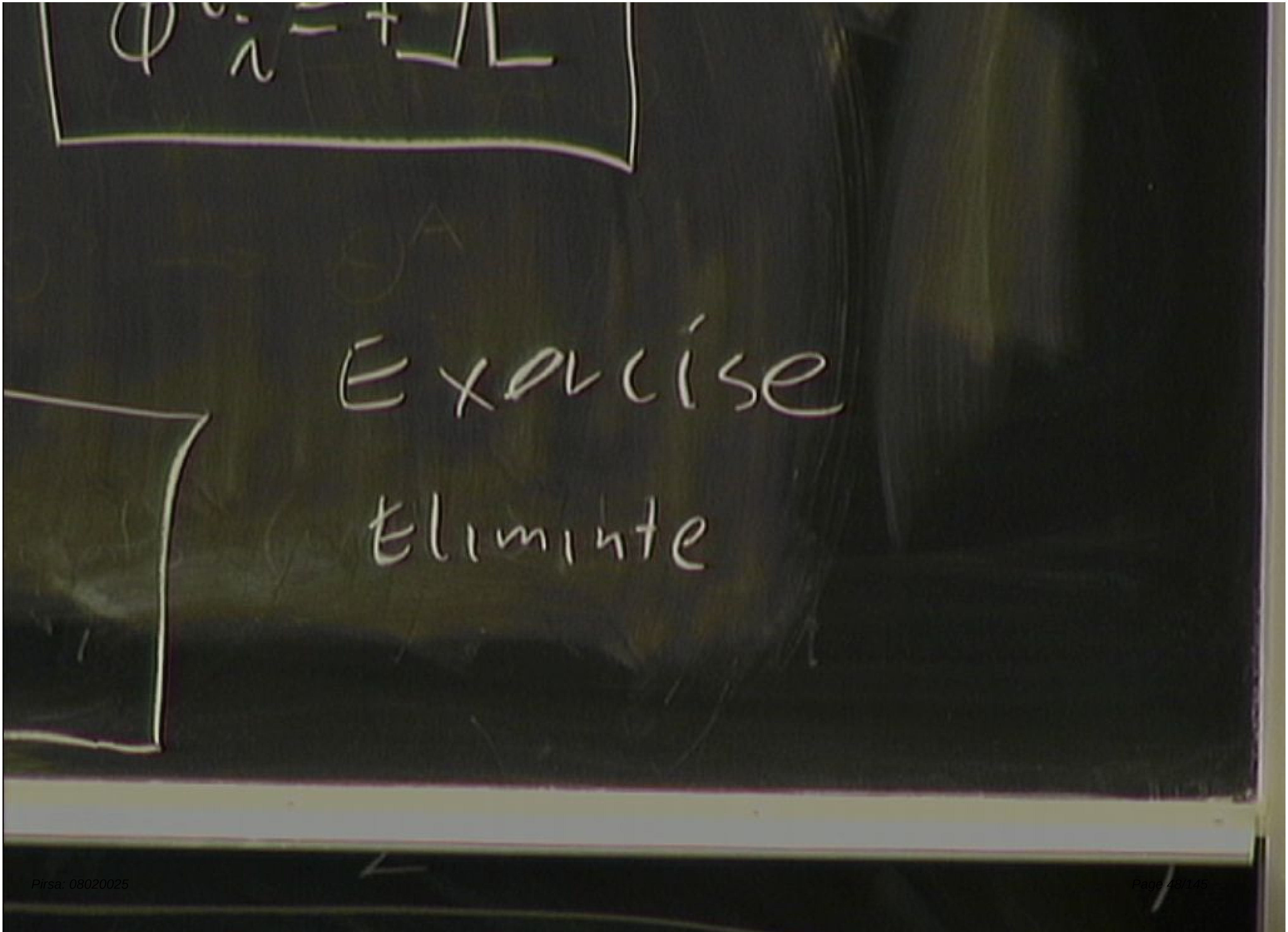
$$B^i = (\Phi^{-1})^i{}_j F^j$$

$$\boxed{S^{[A,\Phi]} = \frac{1}{2}\int F^i \wedge F^j (\Phi^{-1})_{ij}}$$

$\Rightarrow \exists\, e_\mu^a$ s.t. $\boxed{B^i = e^{[a} \wedge e^{b]SD} =}$

simple solution $\boxed{\Phi_{ij} = \frac{1}{3}\delta_{ij}}$ ✓ (A) De Sitter

$$\boxed{F^{i} = \frac{1}{3}\,\xi^{i}}$$


Exercise
Eliminte

$\phi^{cl}_{n} = \ldots$

Exercise

Eliminate Ψ out of S

$S[F, \Psi]$

$$\boxed{S^{pl[A]} = \int B^i \wedge F_i - \frac{1}{2}\phi_{ij} B^i \wedge B^j} \qquad \boxed{\phi^i_{\;i}\ldots}$$

$$B^i = (\Phi^{-1})^i{}_j F^j \qquad \boxed{\text{CDJ}}$$

$$\boxed{S^{[A,\phi]} = \frac{1}{2}\int F^i \wedge F^j (\phi^{-1})_{ij}}$$

$\Rightarrow \exists\; e_\mu^a$ s.t. $\boxed{B^i = e^{[a} \wedge e^{b]SD} = e^{\ldots}}$

$= \Sigma^i$ selfdual 2…

$F^i(A) \ldots$

$$\phi^{i\gamma}\phi^{-1}_{\gamma\ell}=\delta^{i}_{\ell}$$

$$\delta\phi^{i\gamma}\phi^{-1}_{\gamma\ell}=-\phi^{i\gamma}(\delta\phi^{-1})_{\gamma\ell}$$

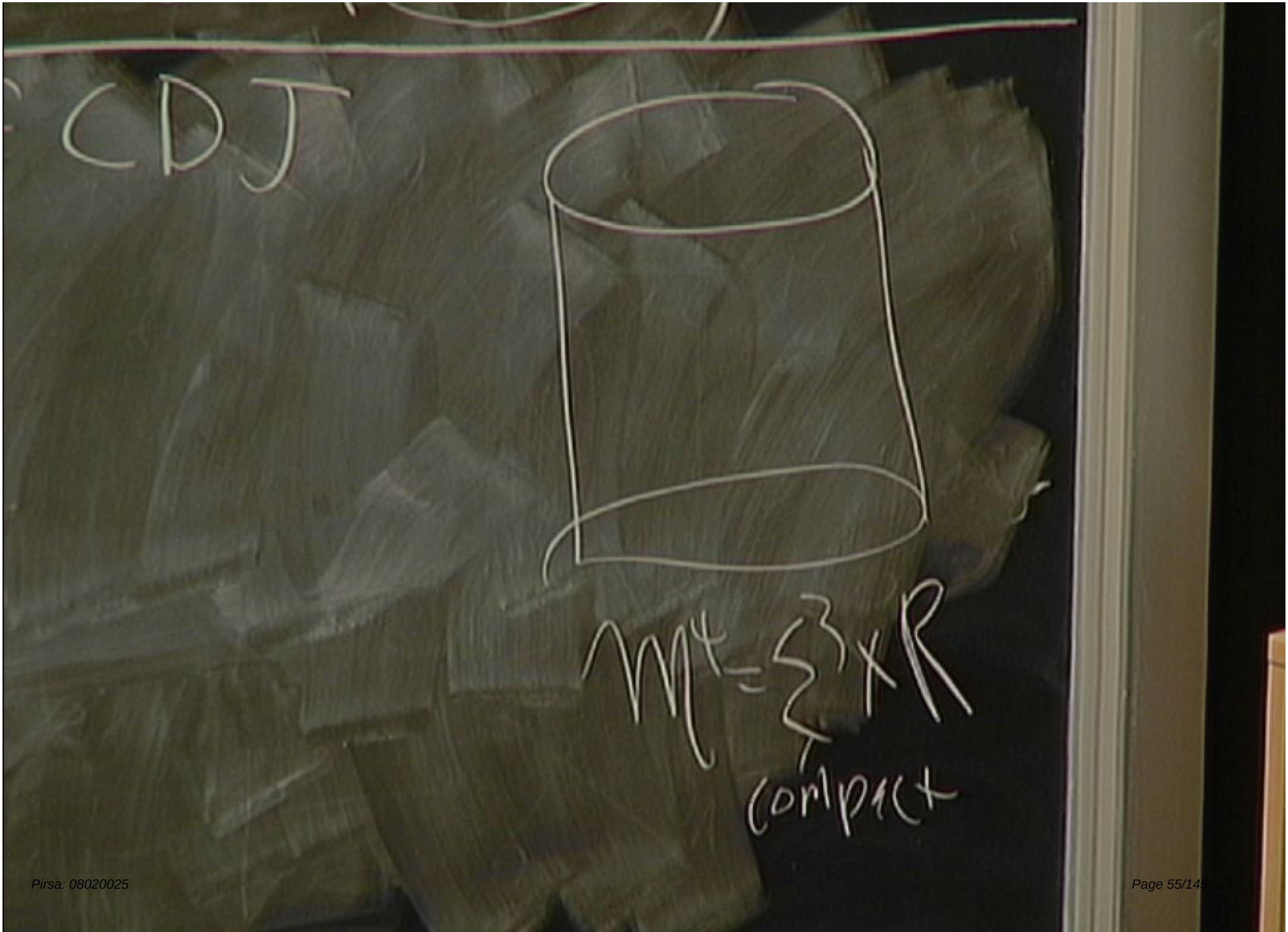
$$(\delta\phi^{-1})_{\ell m}=(\delta\phi)_{i\gamma}(\phi^{-1})_{\ell}{}^{i}(\phi^{-1})_{m}{}^{\gamma}$$

$$F^{i} \wedge F^{j} \phi^{-1}_{i\ell} \phi^{-1}_{jm} = 0$$

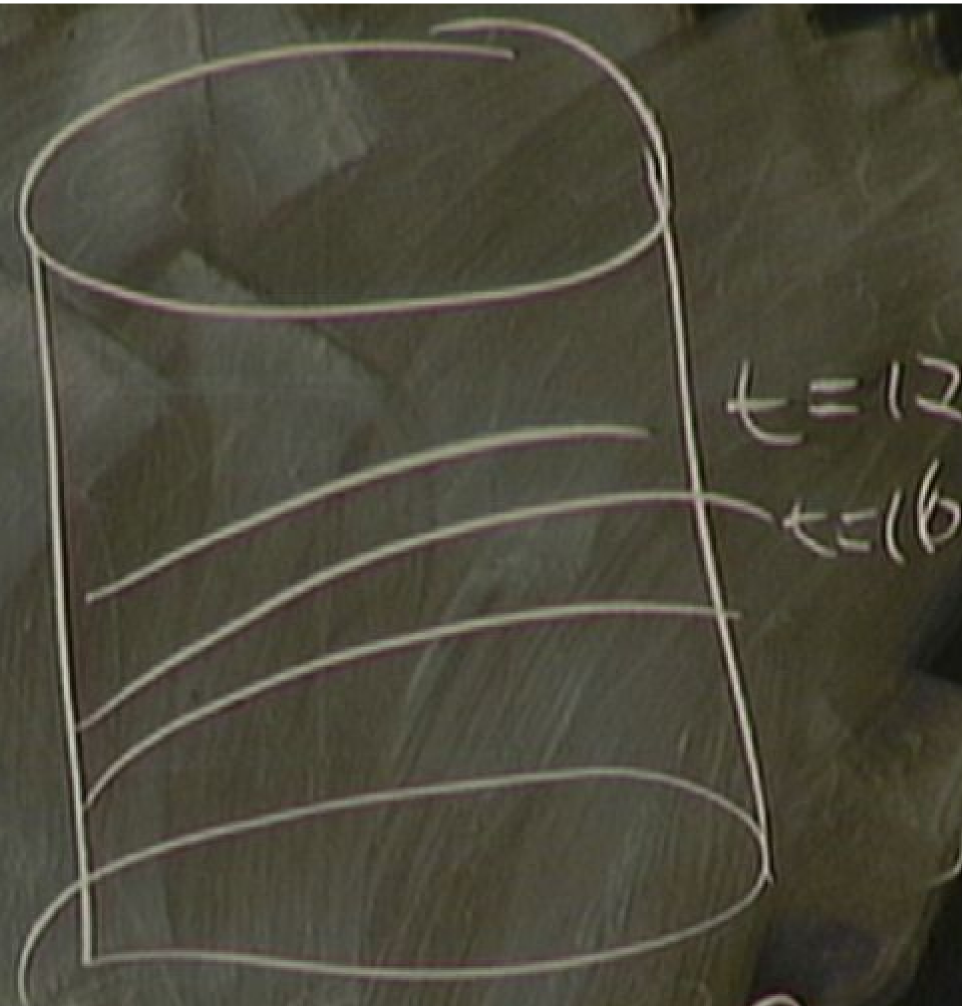
$$\mathcal{D} \wedge \left(\bar{\phi}_i \, \gamma \, F^i \right) = 0$$

$$F^i \wedge F^j \, \phi^{-1}_{il} \, \phi^{-1}_{jm} = 0 \qquad \mathcal{D} \wedge ($$

Hamiltonian Formulation of CDJ


CDJ
M^4 = Σ^3 × R
compact

DJ



$$M^4 = \Sigma^3 \times R$$

compact

Hamiltonian Formulation

$$S = \frac{1}{2}\int dt \int_{\Sigma}$$

Hamiltonian Formulation

$$S = \frac{1}{2} \int dt \int_{\Sigma} \epsilon^{0abc}$$

Hamiltonian Formulation of CDJ

$$S = \frac{1}{2}\int dt \int_{\Sigma} \epsilon^{0abc} F^{i}_{0a} F^{j}_{bc} (\Phi^{-1})_{ij}$$

$$= \frac{1}{2}\int dt$$

...miltonian Formulation of CDJ

$$= \frac{1}{2}\int dt \int_{\Sigma} \epsilon^{0abc} F^{i}_{0a} F^{j}_{bc} (\phi^{-1})_{ij}$$

$$= \frac{1}{2}\int dt \int_{\Sigma} \epsilon^{abc} \left[\partial_0 A^{i}_{a} - \mathcal{D}_a A^{i}_{0}\right] F^{j}_{bc} (\phi^{-1})_{ij}$$

$$\Pi^a_i = \frac{\delta S}{\delta \bar{A}^i_a} = \epsilon^{abc} F_{bc}{}^{j} (\bar{\phi}^{-1})_{ij}$$

$$\alpha = \pm g^{\alpha\beta} /$$

$$\Pi^0_i = 0$$

$$\Pi_{\phi_{i\gamma}} = \frac{\delta S}{\delta \dot{\phi}_{i\gamma}} = 0$$

$$F_{bc}{}^{a}(\phi)_{ij} \qquad \Pi_i^a = 0$$

$$E_i^a = \Pi_i^a - \frac{1}{2}\epsilon^{abc} F_{bc\,j}\,\phi_i{}^{j}$$

$${}^{c}F_{b}{}_{c}^{\,j}(\phi^{-1})_{ij} \qquad \Pi^{0}_{i} = 0$$

$$E^{a}_{i} = \Pi^{a}_{i} - \tfrac{1}{2}\epsilon^{abc}\, F_{bc\,j}\, \phi^{-1\,j}_{\;i} = 0$$

$$\ldots \Pi_i \ldots f_{bc}{}^{j}\, \phi_i{}^{g} = \ldots$$

$$G^i = D_a \Pi^{ai}$$

$$\Pi_{\phi^{i}_{z}} = \frac{\delta S}{\delta \dot{\phi}_{iz}} = 0 \qquad E_i =$$

$$H = \int \Pi \dot{A} + \ldots \qquad \mathcal{L} = \int A_0^i G_i$$

G

$$\ldots c \; f_{bc}{}^{j}\, \Phi_i{}^{g} = 0$$

$$D_a \Pi^{ai} \quad \text{"}G\text{"} = \mathcal{D}(\mathcal{E} F \bar{\phi}')$$

$$\Pi_{\bar\phi_{i\gamma}} \; \frac{\delta}{\delta \bar\phi_{iz}} = 0 \qquad E_i = \Pi_i$$

$$G^i -$$

$$H = \int \Pi \bar{A} + \ldots$$

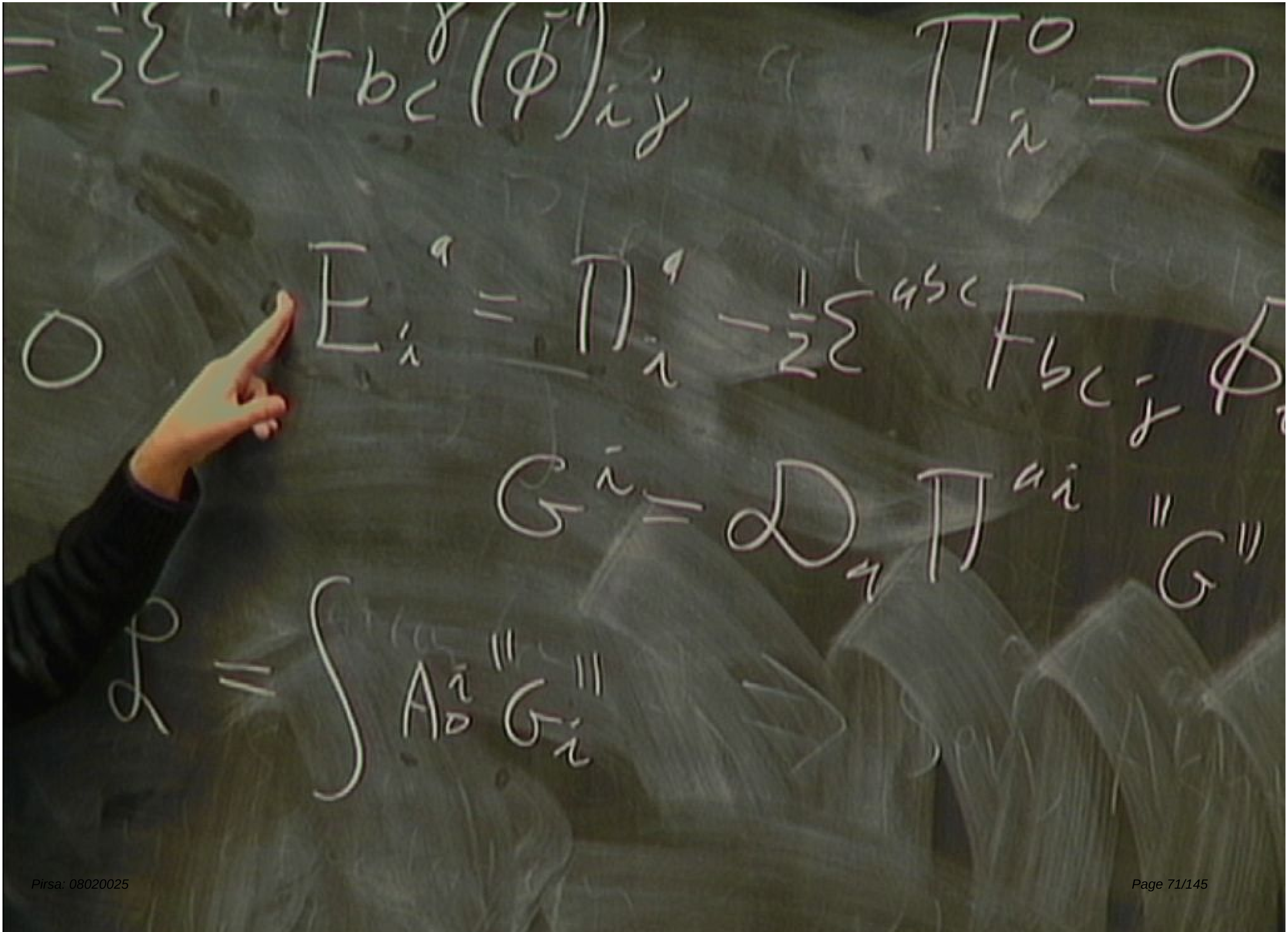
$$\mathcal{L} = \int A_0^i \; {}^{\shortparallel}G_i{}^{\shortparallel}$$

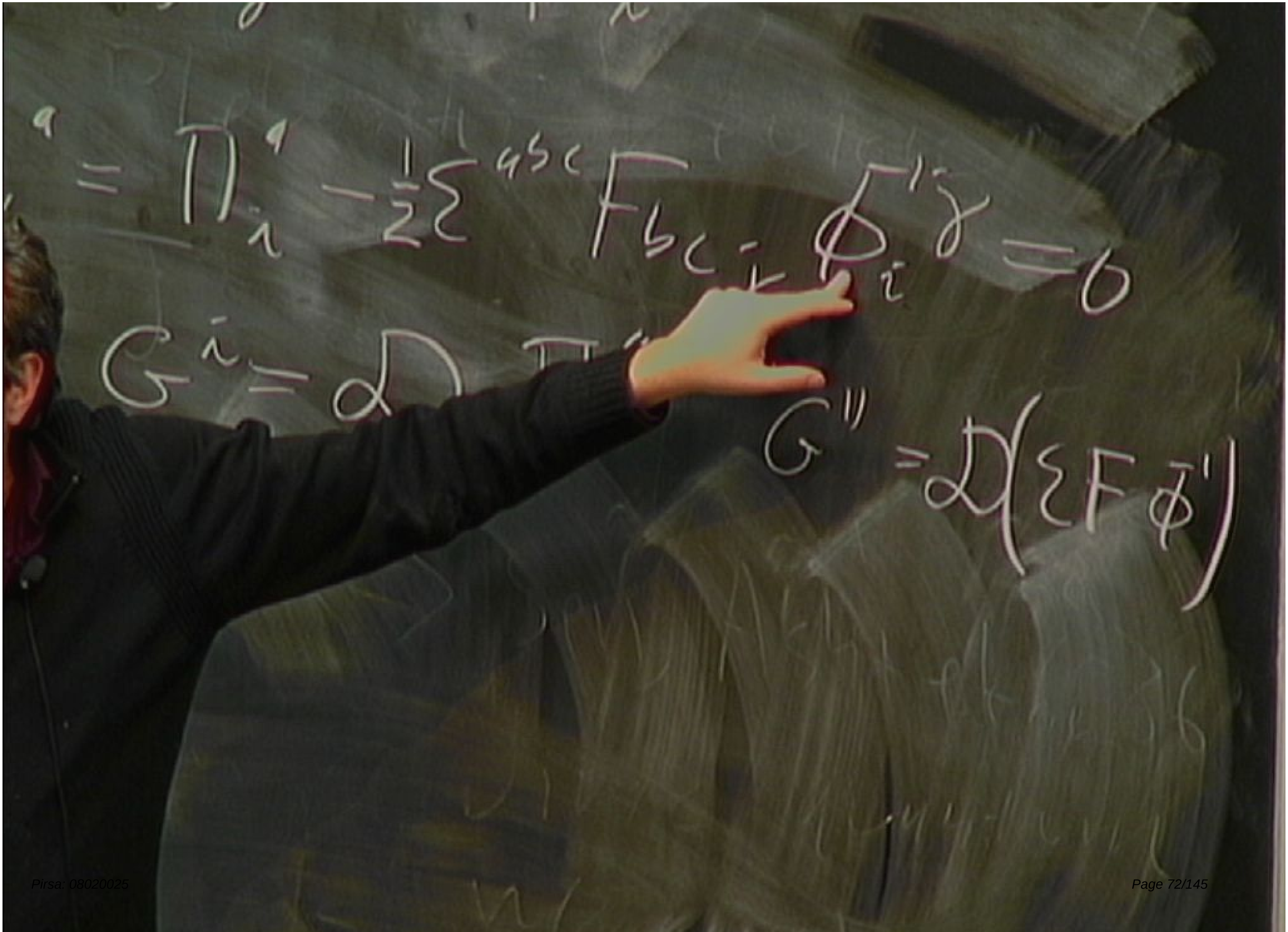
$$\Pi_i = \frac{\delta S}{\delta \dot{A}_i^i} = \frac{1}{2}\epsilon \quad F_{bc}\,\gamma(\phi)$$

$$\Pi_{\phi^i_j} = \frac{\delta S}{\delta \dot{\phi}_{ij}} = 0 \qquad E^a_i = \Pi$$

$$G$$

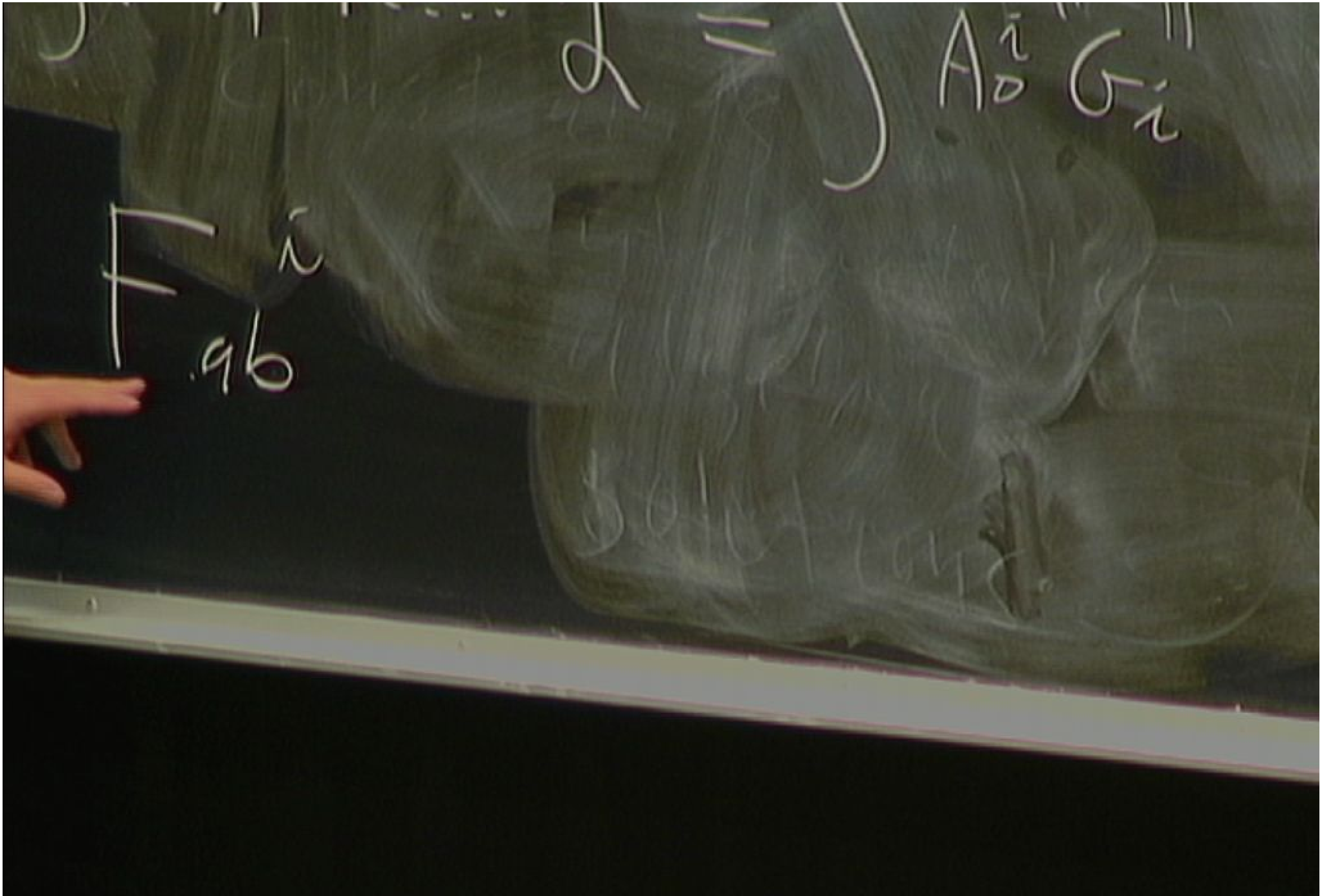
$$H = \int \Pi \dot{A} + \dots \qquad \mathcal{L} = \int A_0^i\, G_i$$


$= \frac{1}{2}\epsilon^{abc} F_{bc}(\bar{\phi})_{ij}$
$\Pi^0_i = 0$
$0 \quad E^a_i = \Pi^a_i - \frac{1}{2}\epsilon^{abc} F_{bc\,j} \phi$
$G^i = D_a \Pi^{ai}$ "G"
$\mathcal{L} = \int A^i_0$ "G_i"



$$G^{ij} = \mathcal{D}(\epsilon F \bar{\phi}')$$

$$\partial_a^{\,i} F_{ab}^{\,i} - \Lambda \epsilon_{abc} \Pi_i^c = 0$$


$\mathcal{L} = \int A_0^i G_i$
F^i_{ab}

$$\lambda = \int A_0^i G_i$$

$$F^i_{ab} \Pi^b_i$$

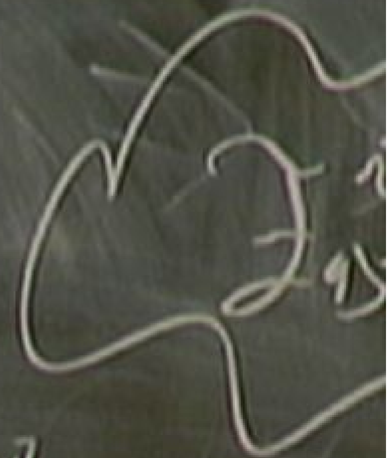
$$\mathcal{D}^i_{ab} = F^i_{ab} - \Lambda \epsilon_{abc} \Pi^c_i = 0$$

$$E \Rightarrow F^j_{bc} = \epsilon_{abc} \Pi^a_i \phi^{ij}$$

$$\Pi A + \ldots \quad \mathcal{L} = \int A_0^i \; '' G_i ''$$

$$F^i_{ab}\,\Pi^b_i = \epsilon_{abc}\,\Pi^b_i\,\Pi^c_j\,\phi^{ij}$$

$$\int \Pi \dot{A} + \ldots \quad \mathcal{L} = \int A_0^i \, "G_i"$$



$$F_{ab}^{\;i} \, \Pi_i^{\;b} = \epsilon_{abc} \, \Pi_i^b \, \Pi_j^c \, \phi^{[ij]} = 0 \; E$$

$$G^i = \mathcal{D} \ldots$$

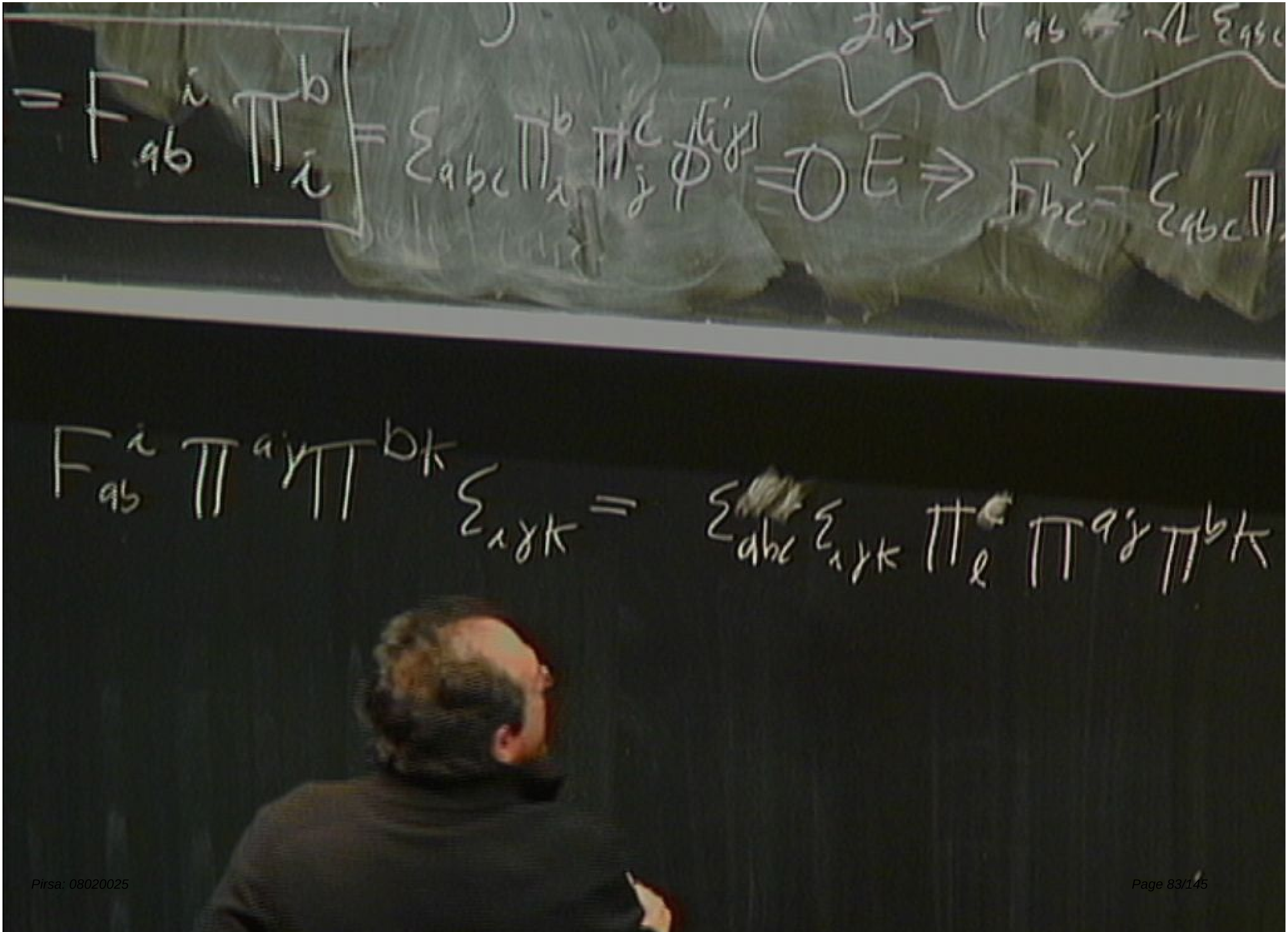
$$H = \int \Pi \dot{A} + \ldots \qquad \mathcal{L} = \int A_0^i G_i$$

$$\boxed{D_a = F_{ab}^i \Pi_i^b} = \epsilon_{abc} \Pi_i^b \Pi_j^c \phi^{[ij]} = 0$$

$$F^{i}_{ab}\,\Pi^{a}\,\Pi$$

$$F_{as}^{\;i}\,\Pi^{aj}\,\Pi^{bk}$$

$$F_{ab}^{\ \ i}\,\Pi^{aj}\,\Pi^{bk}\,\epsilon_{ijk}$$


$= F_{ab}^{i} \Pi^{b}_{i}$
$= \epsilon_{abc} \Pi^{b}_{i} \Pi^{c}_{j} \phi^{ij} = 0 \quad E \Rightarrow$
$F^{i}_{ab} \Pi^{ai} \Pi^{bk} \epsilon_{ijk} = \epsilon_{abc} \epsilon_{ijk} \Pi^{c}_{l} \Pi^{aj} \Pi^{bk}$

$$\boxed{= F_{ab}^{\;\;i}\,\Pi^b_i} = \epsilon_{abc}\,\Pi^b_i\,\Pi^c_j\,\phi^{ij} = 0 \quad E \Rightarrow F^{j}_{bc} = \epsilon_{abc}\,\Pi \ldots$$

$$F_{ab}^{\;\;i}\,\Pi^{aj}\,\Pi^{bk}\,\epsilon_{ijk} = \epsilon_{abc}\,\epsilon_{ijk}\,\Pi_l\,\Pi^{aj}\,\Pi^{bk}$$

$$F_{ab}^{i}\,\Pi^{aj\gamma}\,\Pi^{bk}\,\epsilon_{\lambda\gamma k} = \epsilon_{abc}\,\epsilon_{\lambda\gamma k}\,\Pi_{l}^{c}\,\Pi^{aj\gamma}\,\Pi^{bk}\,\phi^{il}$$

$$\ldots{}_{jk} = \Big[\underbrace{\epsilon_{abc}\,\epsilon_{ijk}\,\Pi^{c}_{l}\,\Pi^{aj}\,\Pi^{bk}}_{W_{il}}\Big] \ldots$$

$$\ldots_{ab}\ \Pi\ \Pi \qquad \epsilon_{ijk} = \Big[\ldots$$

$$W_{il} = \epsilon_{ijk}\, \epsilon_{abc}\, \Pi^{a}_{l}\, \Pi^{bj}\, \Pi^{ck}$$

$$F_{ab}^{\ i}\, \Pi^{aj} \Pi^{bk} \epsilon_{ijk} = \Big[\epsilon_{abc}\, \epsilon_{ijk}\, \Pi_l^{\ \cdot}\, \Pi^{a}$$

$$W_{il} = \epsilon_{ijk}\, \epsilon_{abc}\, \Pi_l^{\ a}\, \Pi^{bj}\, \Pi^{ck} = \delta_{il} \det \Pi$$

$$\det \Pi = \frac{1}{3!} \epsilon_{ijk}\, \epsilon_{abc}\, \Pi^{ai}\, \Pi^{bj}\, \Pi^{ck}$$

$$F_{ab}^{i}\,\Pi^{aj}\,\Pi^{bk}\,\epsilon_{ijk} = \Big[\underbrace{\epsilon_{abc}\,\epsilon_{ijk}\,\Pi^{a}_{l}\,\Pi^{\ldots}}_{W_{il}}$$

$$W_{il} = \epsilon_{ijk}\,\epsilon_{abc}\,\Pi^{a}_{l}\,\Pi^{bj}\,\Pi^{ck} = \delta_{il}\det\Pi$$

$$\det\Pi = \frac{1}{3!}\epsilon_{ijk}\,\epsilon_{abc}\,\Pi^{ai}\,\Pi^{bj}\,\Pi^{ck}$$

$$\underbrace{\left[\epsilon_{abc}\,\epsilon_{ijk}\,\Pi^{c}_{l}\,\Pi^{aj}\,\Pi^{bk}\right]}_{W_{il}}\phi^{il}$$

$$\Pi^{ck} = [\;]\,\delta_{il}\,\det\Pi$$

$$\phi\,\Pi^{ck}$$

Check

ϵ's x numerical factors

$\ldots \epsilon_{abc} (\Pi_e \Pi \ldots) \epsilon \ldots$

$$\det \Pi = \frac{1}{3!} \epsilon_{ijk} \epsilon_{abc} \Pi^{ai} \Pi^{bj} \Pi^{ck}$$

$$F_{ab}^{\ i} \Pi^{aj} \Pi^{bk} \epsilon_{ijk} -$$

$$V_{il} = \epsilon_{ijk}\,\epsilon_{abc}\,\Pi^{a}_{l}\,\Pi^{bj}\,\Pi^{ck} = [\;]\,\delta_{il}$$

$$\det \Pi = \frac{1}{3!}\epsilon_{ijk}\,\epsilon_{abc}\,\Pi^{ai}\,\Pi^{bj}\,\Pi^{ck}$$

$$F^{i}_{ab}\,\Pi^{aj}\,\Pi^{bk}\,\epsilon_{ijk} - \Lambda \det \Pi = 0$$

$$W_{il} = \epsilon_{ijk}\,\epsilon_{abc}\,\Pi^{a}_{l}\,\Pi^{bj}\,\Pi^{ck} = [\;]\,\delta_{il}$$

$$\det\Pi = \frac{1}{3!}\epsilon_{ijk}\,\epsilon_{abc}\,\Pi^{ai}\,\Pi^{bj}\,\Pi^{ck}$$

$$\boxed{C = F^{i}_{ab}\,\Pi^{aj}\,\Pi^{bk}\,\epsilon_{ijk} - \Lambda\det\Pi = 0}$$

$$\tilde{\Pi}^a_i = \frac{\delta S}{\delta \bar{A}^i_4} = \frac{1}{2}\epsilon^{abc} F_{bc}{}^{j} (\bar{\phi}^{-1})$$

$$\Pi_{\phi_{ij}} = \frac{\delta S}{\delta \bar{\phi}_{ij}} = 0 \qquad E^a_i = \Pi$$

$$\delta S = \int (\ )\, \delta \bar{A} \qquad G$$

$$H = \int \Pi \bar{A} +$$

$\gamma(\bar{\phi}^{-1})_{ij}$ $\qquad \tilde{\Pi}^0_i = 0 \qquad \sim$ = density

$^a = \tilde{\Pi}^a_i - \frac{1}{2}\tilde{\Sigma}^{abc} F_{bc\,j}\, \phi^{-1\,j}_{i} = 0$

$G^i = D_a \Pi^{ai} \qquad$ "G" $= D(\Sigma F \bar{\phi}^{-1})$

$$W_{il} = \epsilon_{ijk}\,\epsilon_{abc}\,\Pi^{a}_{l}\,\Pi^{bj}\,\Pi^{ck} = [\,]\,\delta_{il}$$

$$\det \Pi = \frac{1}{3!}\epsilon_{ijk}\,\underline{\epsilon_{abc}}\,\tilde{\Pi}^{ai}\,\tilde{\Pi}^{bj}\,\tilde{\Pi}^{ck}$$

$$\tilde{\tilde{\tilde{C}}} = F_{ab}^{\ \ i}\,\tilde{\Pi}^{aj}\,\tilde{\Pi}^{bk}\,\epsilon_{ijk} - \Lambda \det \Pi = 0$$

su(2)

$$H = \int \lambda_i G^i + \underset{\sim}{N} \, \tilde{\tilde{C}}^{o}$$

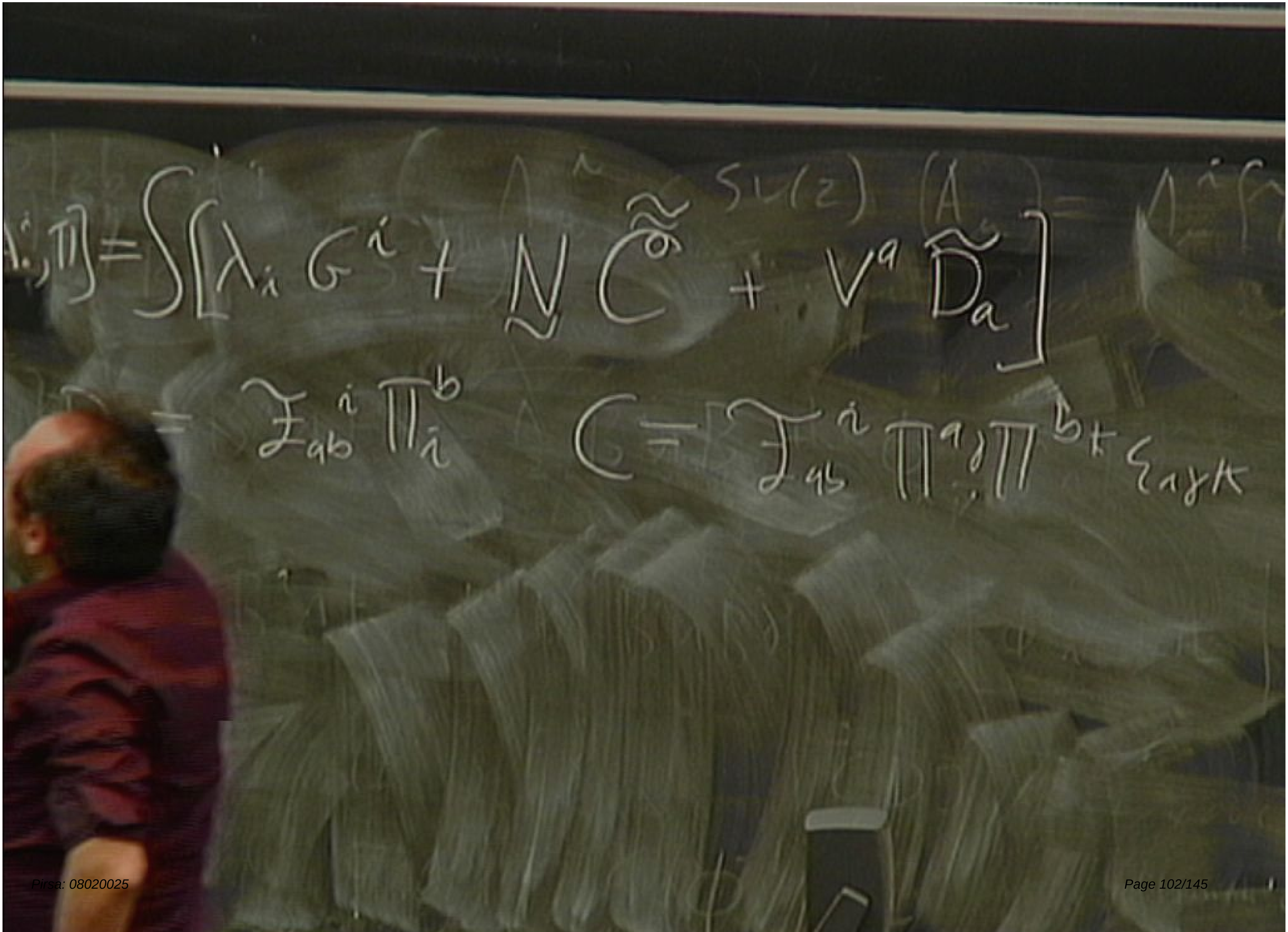
$$H = \int \left[\lambda_i G^i + \underset{\sim}{N}\, \tilde{\tilde{C}} + V^a \tilde{D}_a \right]$$

$$H[A_a^i,\Pi] = \int \left[\lambda_i G^i + \underset{\sim}{N}\,\tilde{\tilde{C}}^{o} + V^a \tilde{D}_a\right]$$

$$H[A_a^i, \Pi] = \int [\Lambda_i \; ($$

$$L(A_a, \Pi) = \int \Lambda_i G^i + N C$$

$$D_a = \mathcal{F}_{ab}{}^{i}\, \Pi^{b}_{i}$$


SU(2)
$A^i, \Pi] = \int [\lambda_i G^i + \underset{\sim}{N} \tilde{C} + V^a \tilde{D}_a]$
$= F_{ab}^{\ \ i} \Pi^b_i$
$C = F_{ab}^{\ \ i} \Pi^a_j \Pi^b_k \epsilon_{ijk}$

$$H[A^i, \Pi] = \int \left[\lambda_i G^i + \underset{\sim}{N} \tilde{\tilde{C}} + V^a \tilde{D}_a \right]$$

$$D_a = F_{ab}^{\,i} \, \Pi^b_i \qquad C = F_{ab}^{\,i} \, \Pi^a_j \Pi^b_k \, \epsilon_{ijk}$$

$D_a = \qquad \epsilon_{ab}\, \pi_i \quad ($

dof		const
A	9	7
Π	9	7
	18	

$D_a = \epsilon_{ab} \pi^i$ (

dof		const
A	9	7
π	9	7
	18	14

$18 - 14 = 4 = 2+2$

$$G(\mu) = \int_{\Sigma} G^{i} \mu_{i}$$

$\mathcal{L}_{45}\quad \| \;{}^{\circ}\; \| \qquad \varepsilon_{ijk}$

$$G(\mu) = \int_{\Sigma} G^{i} \mu_i$$

$$\{G[\mu], G(\mu')\} = G(\varepsilon_{ijk}\mu^j \mu'^k)$$

$$D(V) = \int V^a D_a$$

$$C(\underset{\sim}{N}) = \int \underset{\sim}{N}\, \widetilde{\widehat{C}}^{0}$$

$$D(V) = \int V^a D_a \qquad C(\underset{\sim}{N}) = \int \underset{\sim}{N}$$

$$\mathcal{L}_{V^a} A = V^a \partial_a A_b + A_a \partial_b V^a$$

$$D(V) = \int V^a D_a \qquad C(\underset{\sim}{N}) = \int \underset{\sim}{N}$$

$$\mathcal{L}_{V^a} A^i_b = V^a \partial_a A^i_b + A^i_a \partial_b V^a$$

$$(V^a \Pi_b \quad V \quad \partial_a A_b +$$

$$\mathcal{D}(V) = \int \left(\mathcal{L}_V A^i_a \right) \Pi^a_i$$

$$\mathcal{D}(v) = \int (\mathcal{D}_v A^i_a)\, \tilde{\Pi}^a_i$$

$\partial_b V$

$$\{A_a^i(x), \mathcal{D}(V)\} = \mathcal{L}_V A_a^i$$

$\partial_b V$

$$\{A_a^i(x), \mathcal{D}(V)\} = \mathcal{L}_V A_a^i$$

$$\{\widetilde{\Pi}^a_i(x), \mathcal{D}(V)\} = \mathcal{L}_V \widetilde{\Pi}^a_i$$

$$\partial_b V^a$$

$$\{A_a^i(x), \mathcal{D}(V)\} = \mathcal{L}_V A_a^i$$

$$\{\tilde{\Pi}_i^a(x), \mathcal{D}(V)\} = -\mathcal{L}_V \tilde{\Pi}^a_i \quad \text{check}$$

$$\mathcal{L}_{V^a} A_b = V^a \partial_a A_b^i + A_a^i \partial$$

$$D(V) = \int (\mathcal{L}_V A_a^i) \tilde{\Pi}^a_i$$

$D(V)$ generates diffeos of Σ

$$\int (\mathcal{L}_v A_a) \Pi_i \qquad \{A_a(x), \mathcal{D}(v)\}$$

$\mathcal{D}(v)$ generates diffeos of Σ $\qquad \{\hat{\Pi}^a_i(x), \mathcal{D}(v)\} =$

$$\{\mathcal{D}(v), \mathcal{D}(w)\} = \mathcal{D}([v,w])$$

Lie algebra of

Diff(Σ)

$$D(V) = \int (\mathcal{L}_V A_a^i)\, \tilde{\Pi}^a_i \qquad \{A_a^i(x), D(V)\} =$$

$D(V)$ generates diffeos of Σ $\qquad \{\tilde{\Pi}^a_i(x), D(V)\} =$

$$\boxed{\{D(V), D(W)\} = D([V,W])}$$

$$V^a F_{ab}{}^i \Pi^b_i$$

$$\int V^a F_{ab}{}^i \Pi^b_i =$$

$$\partial A + A^2$$

$$\int V^a F_{ab}{}^i \Pi_i^b \equiv D(V) = \mathcal{D}(V) + \int V^a A_a^i G_i$$

$\partial A + A^2$

$$\equiv D(V) = \mathcal{D}(V) + \int V^a A_a^i G_i$$

$$\mathcal{D}(V) = D(V) - G(V^a A_a^i)$$

$$\equiv D(V) = \mathcal{D}(V) + \int V^a A_a^i G_i$$

$$\boxed{\mathcal{D}(V) = D(V) + G(V^a A_a^i)}$$

$$\equiv D(V) = \mathcal{D}(V) \int V^a A_a^i G_i$$

$$\boxed{\mathcal{D}(V) = D(V) + G(V^a A_a^i)}$$

FTT $\quad$ $\mathcal{D}$TT

$$= V^a \partial_a A^i_b + A^i_a \partial_b V^a$$

$$(\mathcal{L}_V A^i_a)\tilde{\Pi}^a_i$$

$$\{A^i_a(x), \mathcal{D}(V)\}_0 = \mathcal{L}_V A^i_a$$

$$\{\tilde{\Pi}^a_i(x), \mathcal{D}(V)\}_0 = -\mathcal{L}_V \tilde{\Pi}^a_i \quad \text{chec}$$

enerates diffeos of Σ

$$\ldots), \mathcal{D}(W)\}_0 = \mathcal{D}([V,W])$$

Lie algebra of $\mathrm{Diff}(\Sigma)$

$$\mathcal{L}_{V} A^i_b = V^a \partial_a A^i_b + A^i_a \partial_b \ldots$$

$$\mathcal{D}(V) = \int (\mathcal{L}_V A^i_a)\tilde{\Pi}^a_i$$

$\mathcal{D}(V)$ generates diffeos of Σ

Check $\{\mathcal{D}(V), \mathcal{D}(W)\} = \mathcal{D} \ldots$

$$\partial A + A^2$$

$$\boxed{\mathcal{D}(V) = D}$$

$$\int \{C^\sigma(N), G(\lambda)\} = 0$$

$$\{C^{o}(N), G(\lambda)\} = 0$$

$$\{C(N), D(V)\}$$

$$\boxed{\mathcal{D}(v) = D(v) + \ldots}$$

FT

$$\{C(N), G(\lambda)\} = 0$$

$$\{C(N), \mathcal{D}(v)\} = \mathcal{L}_v$$

$$\boxed{\mathcal{D}(v) = D(v) +}$$

FT

$$\oint \{ C(N), G(\lambda) \} = 0$$

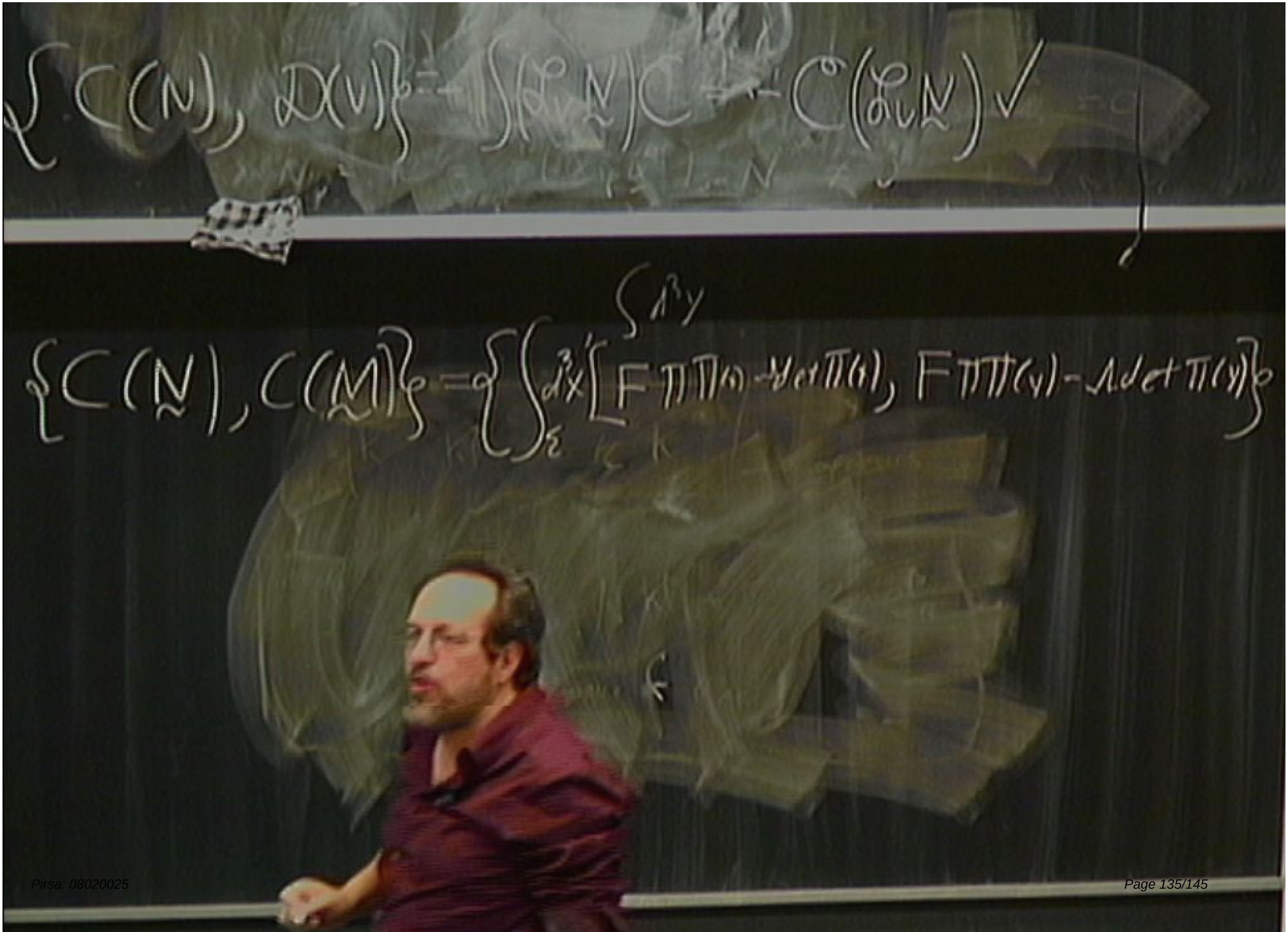
$$\oint \{ C(N), \mathcal{D}(v) \} = - \int (\mathcal{L}_v N) C =$$

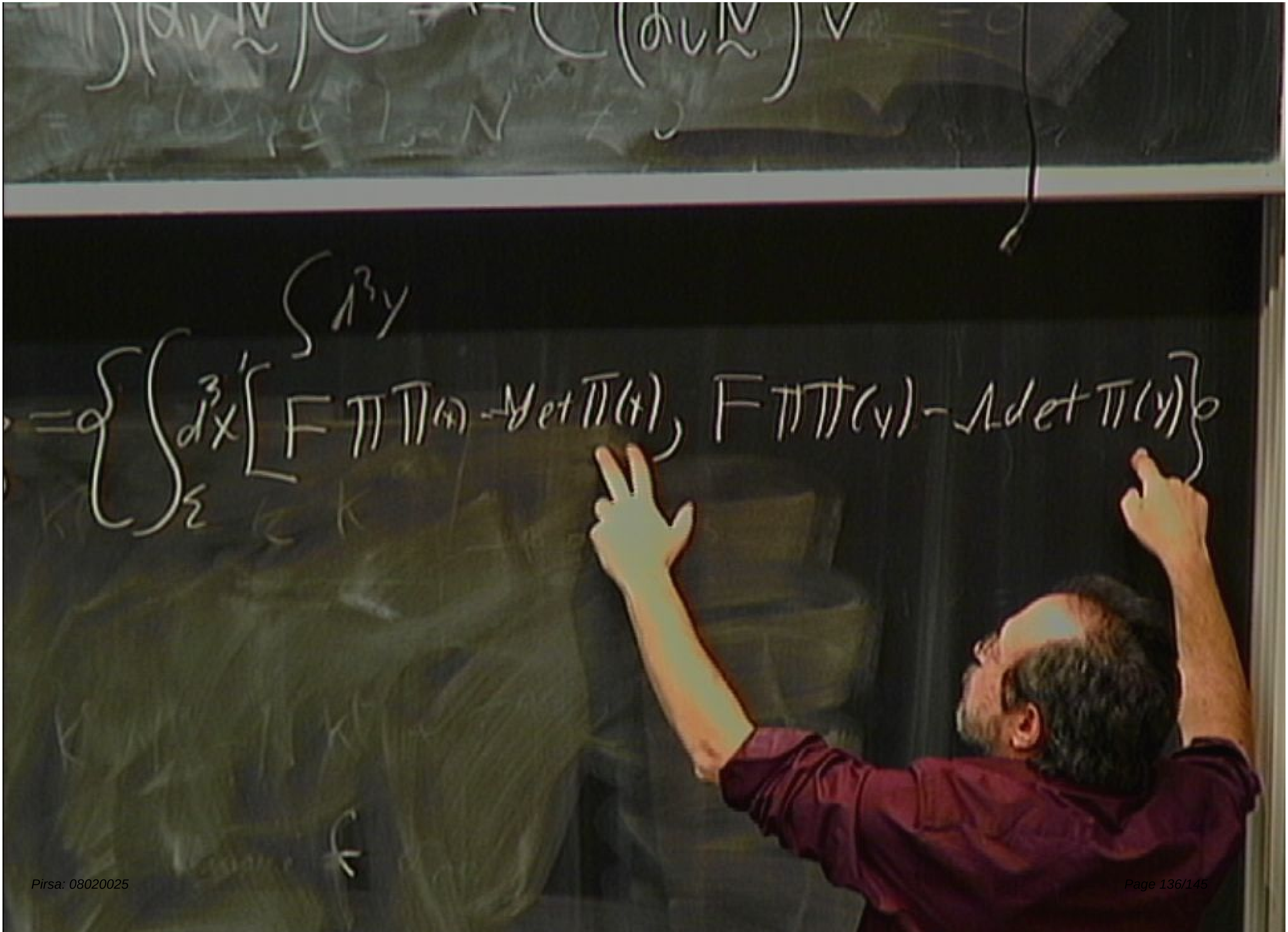
$$\boxed{\mathcal{D}(\vec{V}) = \underbrace{D(\vec{V})}_{FTT} + \underbrace{G(V^i A_a^i)}_{\mathcal{D}TT}}$$

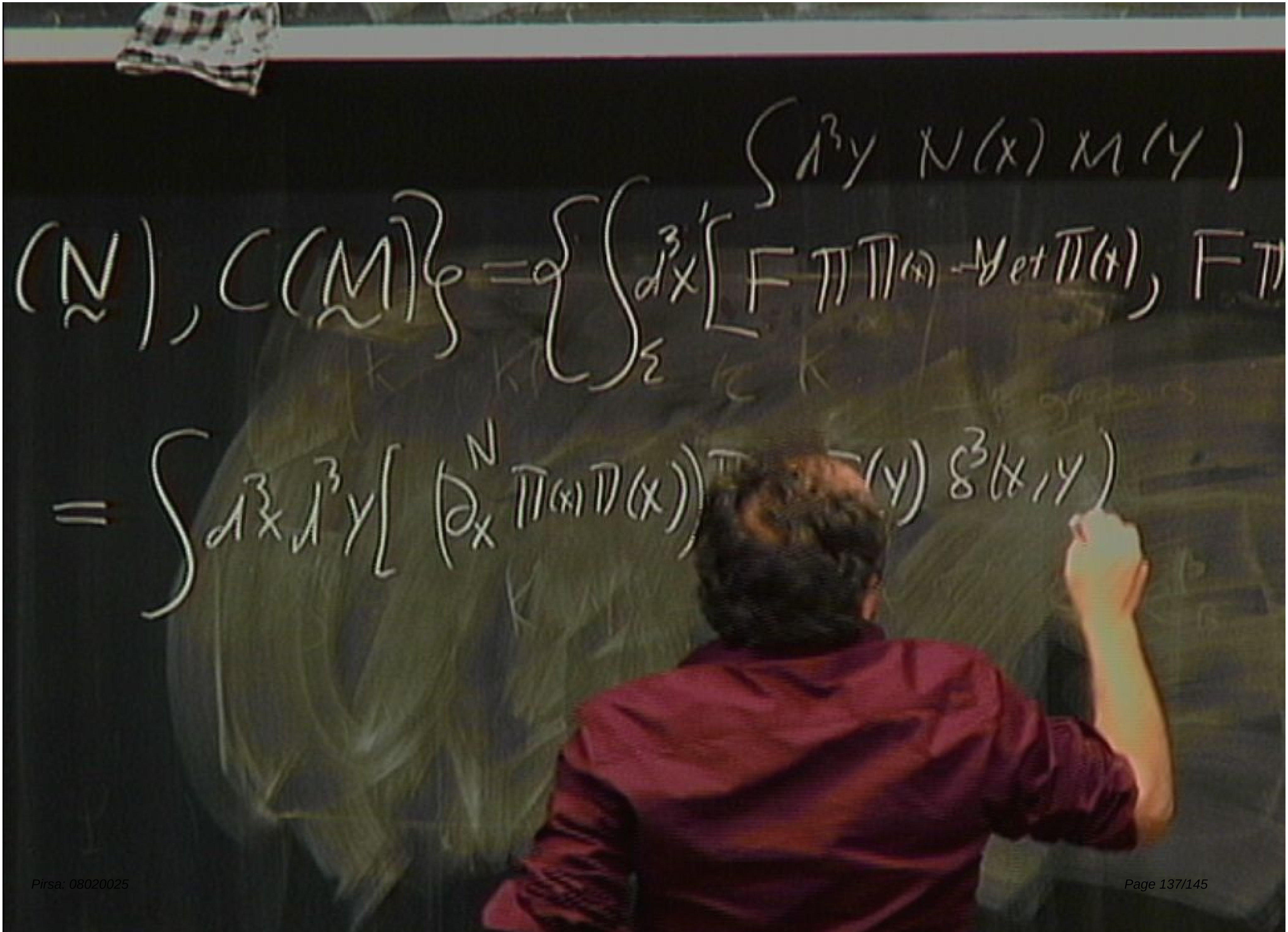
$$\{C^0(N), G(\lambda)\} = 0$$

$$\{C^0(N), \mathcal{D}(\vec{V})\} = -\int (\mathcal{L}_{\vec V} \underset{\sim}{N}) C^0 = -C^0(\mathcal{L}_{\vec V} \underset{\sim}{N})$$

$$\{C(\underset{\sim}{N}), C(\underset{\sim}{M})\}$$


{C(N), D(v)} = C(δ_v N) ✓
{C(N), C(M)} = ∫ d³x ∫ d³y [F ΠΠ(x) − Λ det Π(x), F ΠΠ(y) − Λ det Π(y)]




∫ d³y N(x) M(y)
{C(N), C(M)} = q { ∫_Σ d³x [F Π Π(x) - N et Π(x)], F Π
= ∫ d³x d³y [(∂_x^N Π(x) Π(x)) Π (y) δ³(x, y))

$$\int d^3y\, N(x)\, M(y)$$

$$\{C(N), C(M)\}_\xi = \left\{ \int_\Sigma d^3x \left[F\Pi\Pi(x) - \Lambda \det \Pi(x) \right], F\Pi\Pi(y) - \Lambda \det \Pi(y) \right\}$$

$$= \int d^3x\, d^3y \left[\left(\partial_x^N \Pi(x)\Pi(x)\right)\Pi(y) F(y)\, \delta^3(x,y)\, M(y) - (N \leftrightarrow M) \right]$$

$$= \int d^3x\, d^3y \Big[\big(\partial_x^N \pi(x) \pi(x)\big) \pi(y) F(y)\, \delta^3(x,y)\, M \ldots$$

$$= \int d^3x\, d^3y \Big[\big(M \partial_a N + N \partial_a M\big)\, \pi\, \pi\, \pi\, F\, \delta^3(x,y)$$

$$\{C(\underset{\sim}{N}), C(\underset{\sim}{M})\}_{\epsilon} = \{\int_{\Sigma} d^3x' [F\,\Pi\Pi(x) - \Lambda \det \Pi(x), F\,\Pi\Pi(y) - \Lambda \det \Pi(y)$$

$$= \int d^3x\, d^3y \left[\left(\partial_x^N \Pi(x) \Pi(x)\right) \Pi(y) F(y)\, \delta^3(x,y)\, M(y) - (N \leftrightarrow M) \right]$$

$$= \int d^3x\, d^3y \left[(M \partial_a N - N \partial_a M)\, \Pi\Pi\Pi\, F\, \delta^3(x,y) - (NM - NM)(\sim) \right)$$

$$= \int d^3x \left[M \partial_a N - N \partial_a M \right] \Pi^a_i \Pi^{jb} F^i_{bc} \Pi^c_j$$

$$\{G[n], G(n')\} = G(\epsilon_{\gamma k} n^{\gamma} n'^{k})$$

$$\{C(N), C(M)\} = \int (N \partial_q M - M \partial_q N)(\Pi^{a}_{i} \Pi^{b}_{i}) D_b$$

$$\{\hat{\Pi}^{a}_{i}(x), \mathcal{D}(V)\} = \mathcal{L}_V \hat{\Pi}^{a}_{i}$$ check

diffeo's of Σ

$$(W)\} = \mathcal{D}([V, W])$$ Lie algebra of

$$\{G[n], G(n')\} = G(\epsilon_{\gamma k}\, n^{\gamma} n'^{k})$$

2+2

$$\{C(N), C(M)\} = \int (N\partial_a M - M\partial_a N)(\tilde{\Pi}^{a}_{i}\tilde{\Pi}^{b}_{i}) D_b$$

$$\{\tilde{\Pi}^{a}_{i}(x), \mathcal{D}(V)\} = -\mathcal{L}_V \tilde{\Pi}^{a}_{i} \quad \text{check}$$

diffeo's of Σ

$$(W)\} = \mathcal{D}([V,W])$$

Lie algebra

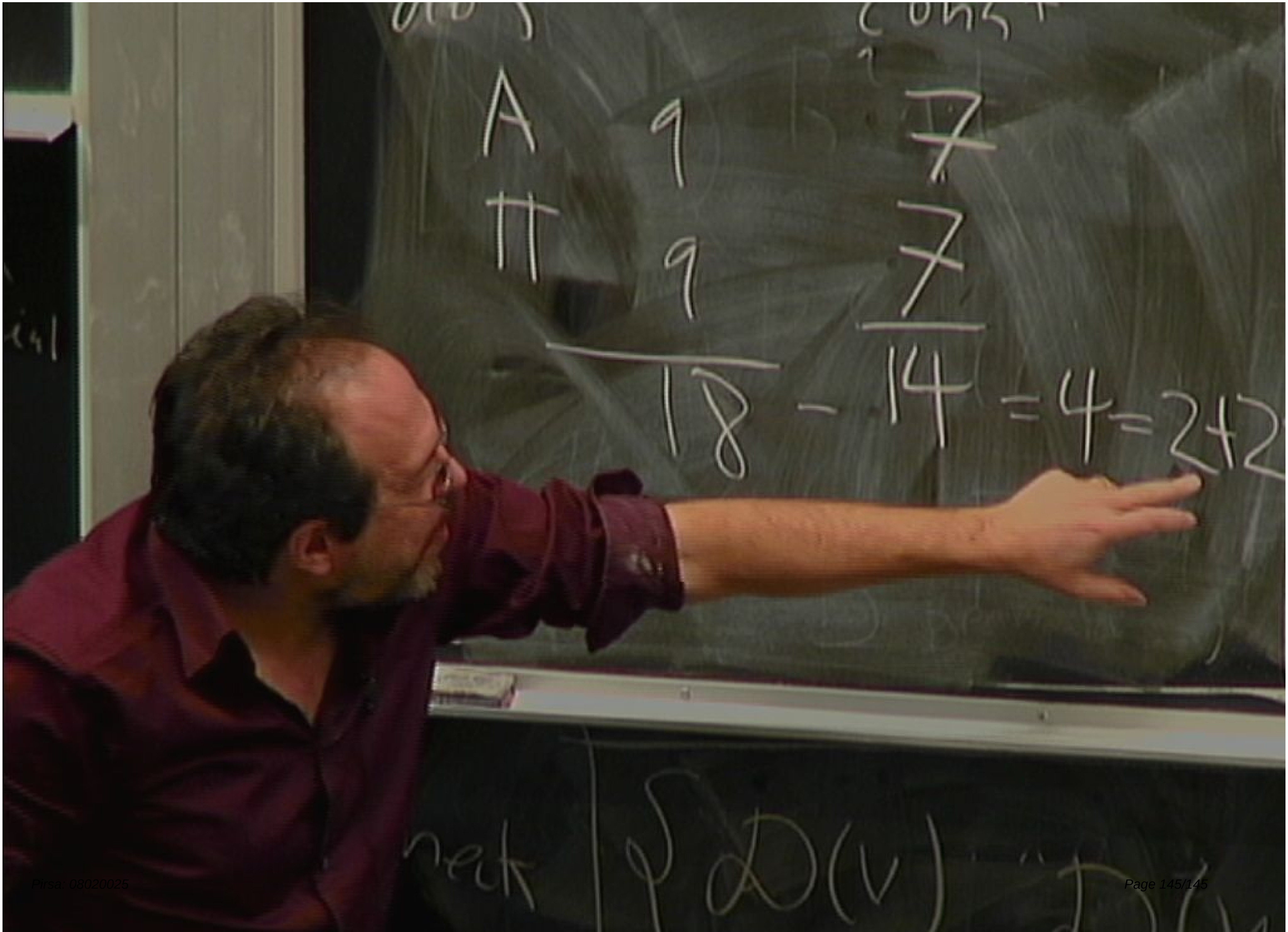
$$)) \Pi(y) F(y) \delta^3(x,y) M(y) - (N \leftrightarrow M) \Big]$$

$$N \partial_a M) \, \Pi \, \Pi \, \Pi \, F \, \delta^3(x,y) - (NM - NM)(\sim)$$

$$\Pi^a_i \Pi^{ab} F_{bc} \Pi^c_y \qquad \tilde{\Pi}^a_i \tilde{\Pi}^{bi} = (\det q) q^{ab}$$

$(x,y) - (NM - NM)(\sim)$

$$\prod_i \tilde{a}_i \prod \tilde{b}_i = (1+\varepsilon)\, q^{ab}$$


A 9 7
9 7
18 - 14 = 4 = 2+2