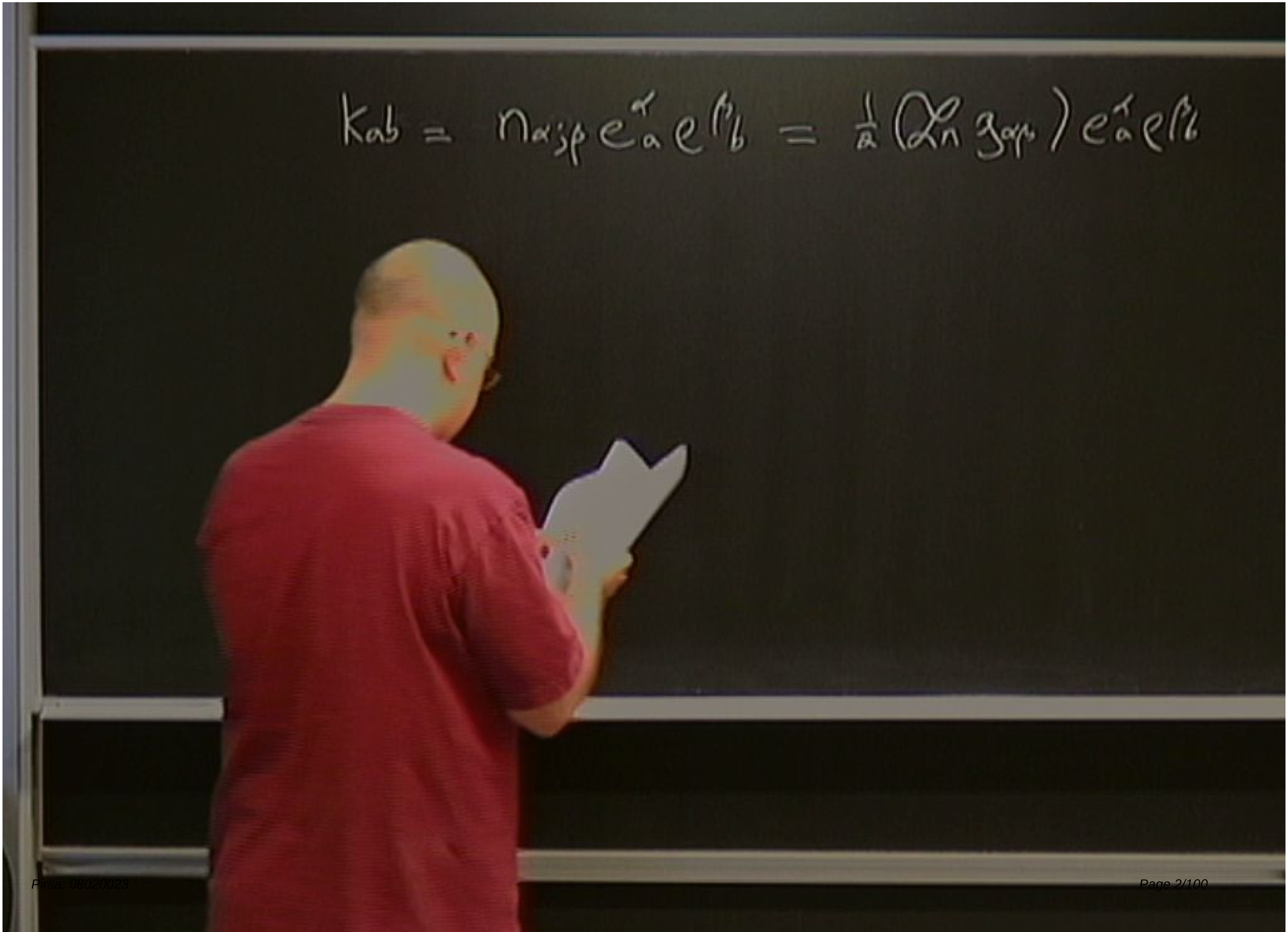
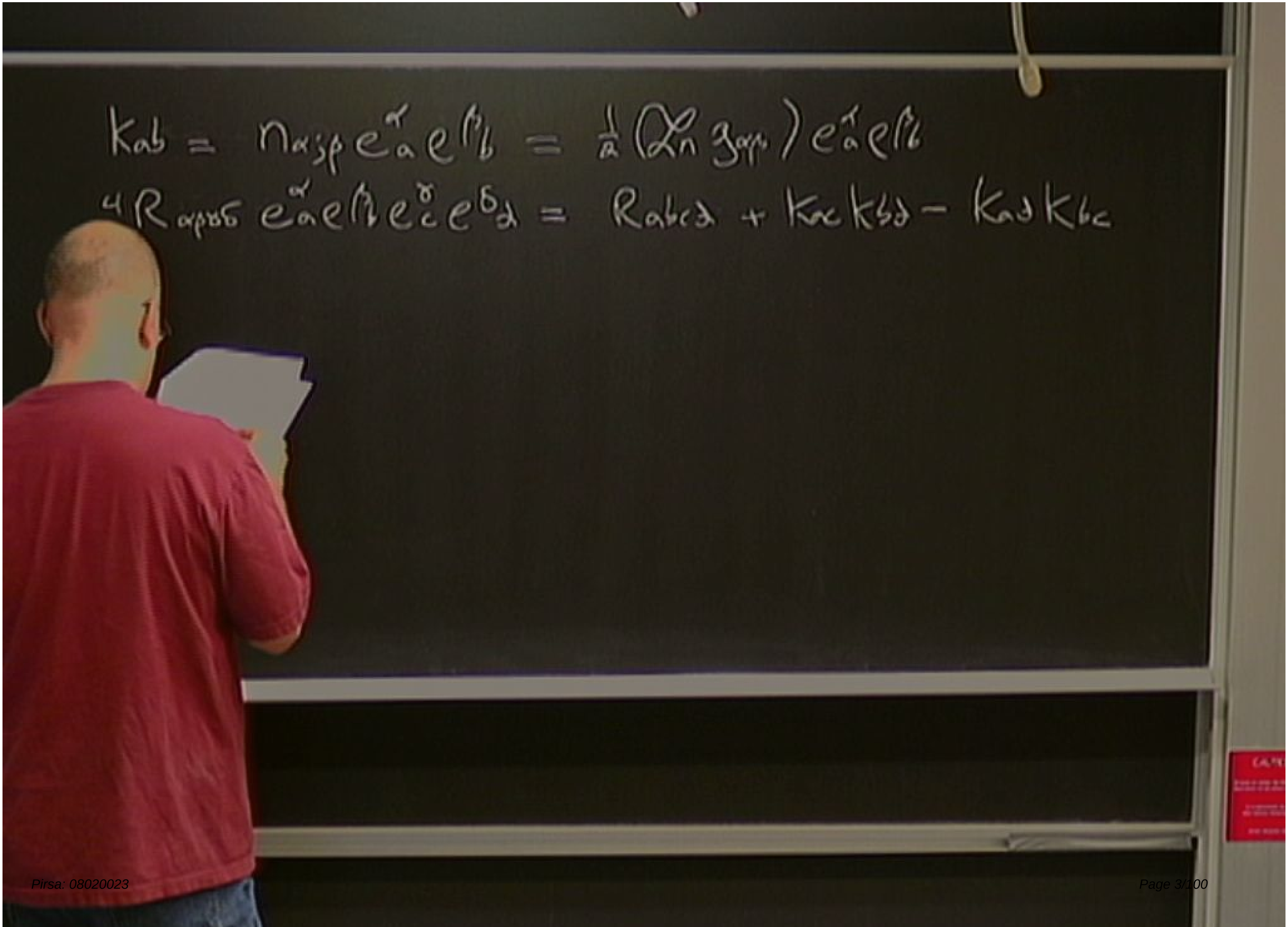
Title: Advanced General Relativity - Lecture 7B

Date: Feb 27, 2008 04:00 PM

URL: http://pirsa.org/08020023

Abstract: Advanced General Relativity


$K_{ab} = n_{\alpha;\beta} e^\alpha_a e^\beta_b = \frac{1}{2}(\mathcal{L}_n g_{\alpha\beta}) e^\alpha_a e^\beta_b$

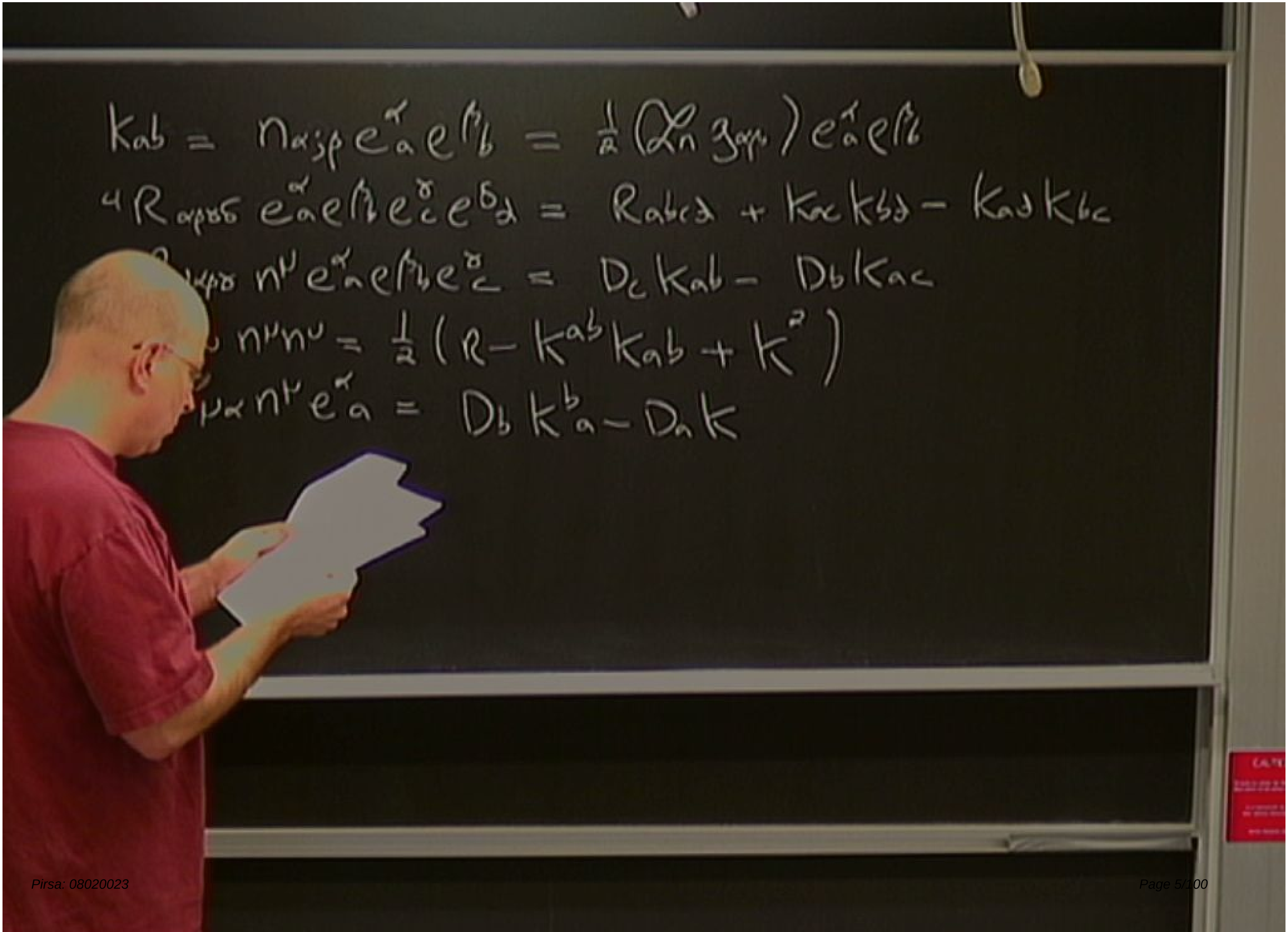

$K_{ab} = n_{\alpha;\beta} e^{\alpha}{}_{a} e^{\beta}{}_{b} = \frac{1}{2}(\mathcal{L}_n g_{\alpha\beta}) e^{\alpha}{}_{a} e^{\beta}{}_{b}$
${}^4R_{\alpha\beta\gamma\delta} e^{\alpha}{}_{a} e^{\beta}{}_{b} e^{\gamma}{}_{c} e^{\delta}{}_{d} = R_{abcd} + K_{ac}K_{bd} - K_{ad}K_{bc}$

$$K_{ab} = n_{\alpha;\beta}\, e^\alpha_a e^\beta_b = \frac{1}{2}\left(\mathcal{L}_n g_{\alpha\beta}\right) e^\alpha_a e^\beta_b$$

$$^4R_{\alpha\beta\gamma\delta}\, e^\alpha_a e^\beta_b e^\gamma_c e^\delta_d = R_{abcd} + K_{ac}K_{bd} - K_{ad}K_{bc}$$

$$R_{\mu\alpha\beta\gamma}\, n^\mu e^\alpha_a e^\beta_b e^\gamma_c = D_c K_{ab} - D_b K_{ac}$$

$$G_{\mu\nu}\, n^\mu n^\nu = \frac{1}{2}\left(R - K^{ab}K_{ab} + K^2\right)$$


$K_{ab} = n_{\alpha;\beta} e^\alpha_a e^\beta_b = \frac{1}{2} (\mathcal{L}_n g_{\alpha\beta}) e^\alpha_a e^\beta_b$
${}^4R_{\alpha\beta\gamma\delta} e^\alpha_a e^\beta_b e^\gamma_c e^\delta_d = R_{abcd} + K_{ac} K_{bd} - K_{ad} K_{bc}$
$n^\mu e^\alpha_a e^\beta_b e^\gamma_c = D_c K_{ab} - D_b K_{ac}$
$n^\mu n^\nu = \frac{1}{2} (R - K^{ab} K_{ab} + K^2)$
$n^\mu e^\alpha_a = D_b K^b_a - D_a K$

$$K_{ab} = n_{\alpha;\beta} e^{\alpha}_{a} e^{\beta}_{b} = \frac{1}{2}(\mathcal{L}_n g_{\alpha\beta}) e^{\alpha}_{a} e^{\beta}_{b}$$

$$^4R_{\alpha\beta\gamma\delta} e^{\alpha}_{a} e^{\beta}_{b} e^{\gamma}_{c} e^{\delta}_{d} = R_{abcd} + K_{ac}K_{bd} - K_{ad}K_{bc}$$

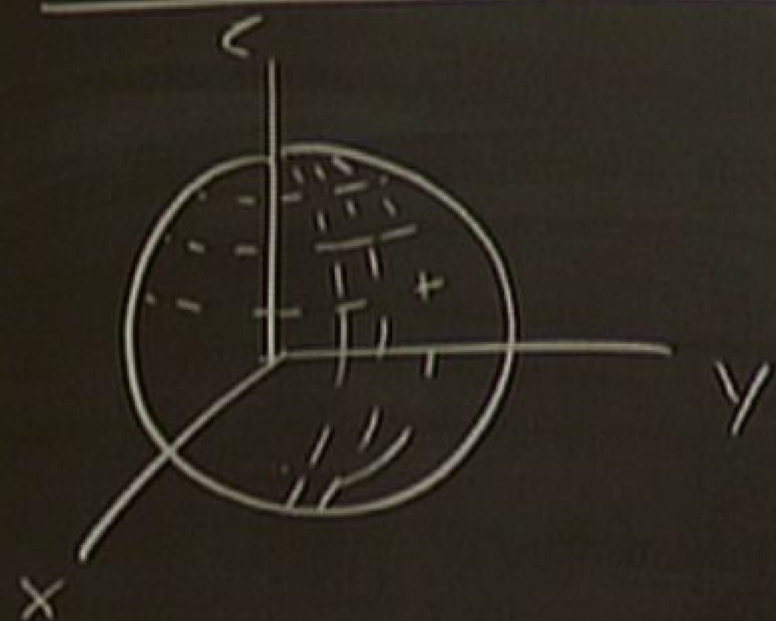
$$^4R_{\mu\alpha\beta\gamma} n^{\mu} e^{\alpha}_{a} e^{\beta}_{b} e^{\gamma}_{c} = D_c K_{ab} - D_b K_{ac}$$

$$^4G_{\mu\nu} n^{\mu} n^{\nu} = \frac{1}{2}(R - K^{ab}K_{ab} + K^2)$$

$$^4G_{\mu\alpha} n^{\mu} e^{\alpha}_{a} = D_b K^{b}_{a} - D_a K$$

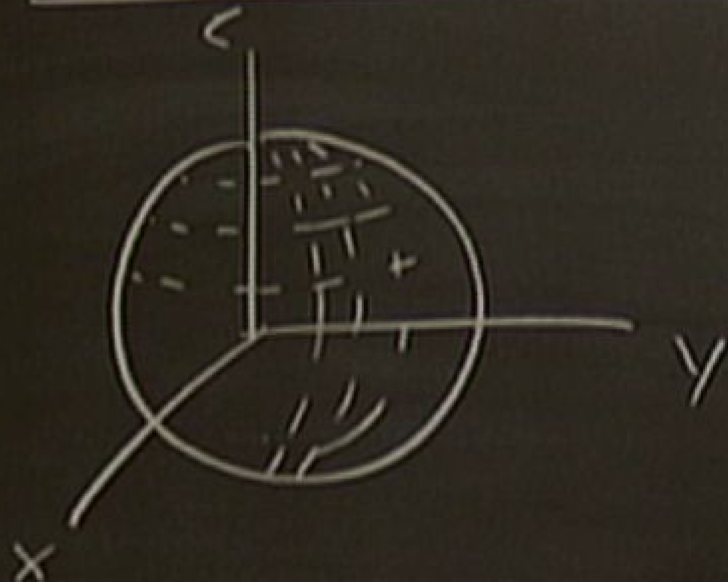
Extrinsic curvature examples:

1- 2D sphere in 3D flat space



Extrinsic curvature examples:

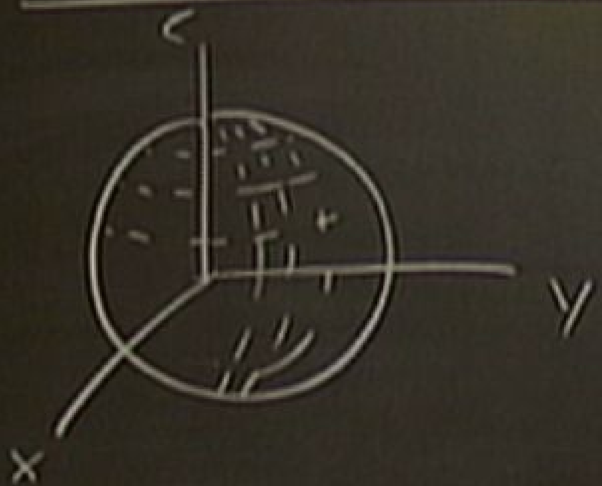
1- 2D sphere in 3D flat space



$$\Phi = x^2 + y^2 + z^2 - R^2 = 0$$

Extrinsic curvature examples:

1- 2D sphere in 3D flat space



$$\Phi = x^2+y^2+z^2-R^2 = 0$$

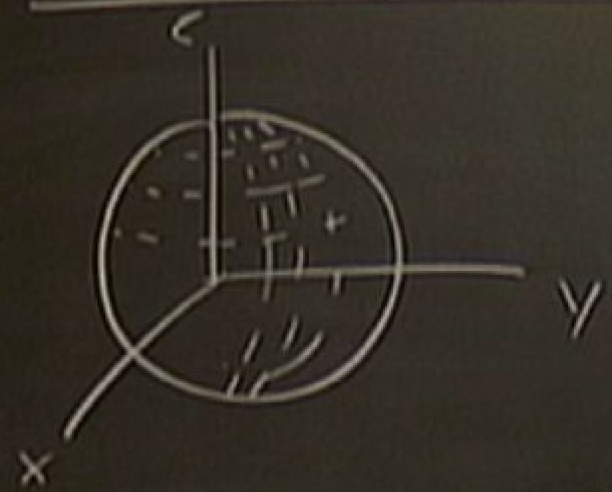
$$x = R\sin\theta\cos\varphi$$

$$y = R\sin\theta\sin\varphi$$

$$z$$

Extrinsic curvature examples:

1- 2D sphere in 3D flat space

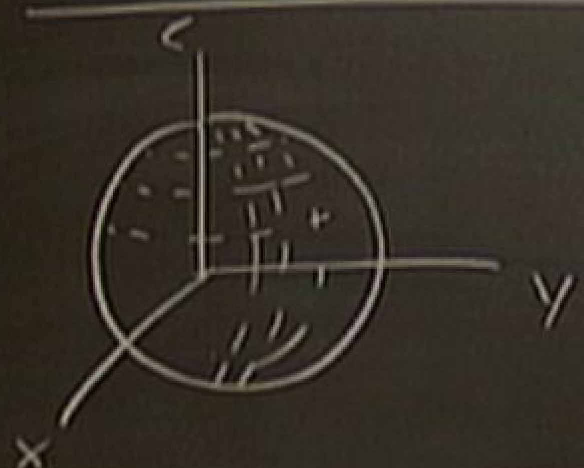


$$\Phi = x^2 + y^2 + z^2 - R^2 = 0$$

$$x = R \sin\theta \cos\varphi$$
$$y = R \sin\theta \sin\varphi$$
$$z = R \cos\theta$$

Extrinsic curvature examples:

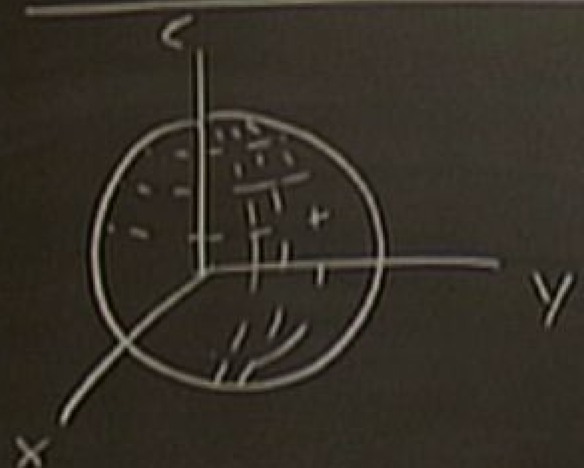
1- 2D sphere in 3D flat space



$$\left.\begin{aligned} \Phi &= x^2+y^2+z^2-R^2 = 0 \\ x &= R\sin\theta\cos\varphi \\ y &= R\sin\theta\sin\varphi \\ z &= R\cos\theta \end{aligned}\right\} \quad n_\alpha \propto (x, y, z)$$

Extrinsic curvature examples:

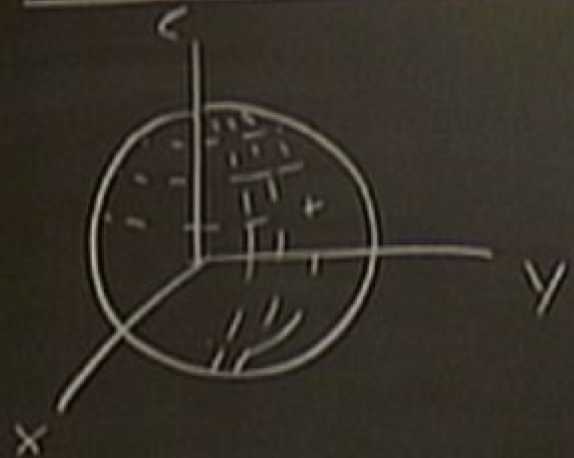
1- 2D sphere in 3D flat space



$$\left.\begin{aligned} \Phi &= x^2+y^2+z^2-R^2 = 0 \\ x &= R\sin\theta\cos\varphi \\ y &= R\sin\theta\sin\varphi \\ z &= R\cos\theta \end{aligned}\right\} \quad \begin{aligned} n_\alpha &\propto (x, y, z) \\ &= \left(\frac{x}{\sqrt{x^2+y^2+z^2}}, \frac{y}{r}, \frac{z}{r}\right) \end{aligned}$$

Extrinsic curvature examples:

1- 2D sphere in 3D flat space



$$\Phi = x^2+y^2+z^2-R^2 = 0$$

$$x = R\sin\theta\cos\varphi$$
$$y = R\sin\theta\sin\varphi$$
$$z = R\cos\theta$$

$$n_\alpha \propto (x, y, z) = \left(\frac{x}{\sqrt{x^2+y^2+z^2}}, \frac{y}{r}, \frac{z}{r}\right)$$

$$e_\theta^\alpha =$$

$(\ldots\alpha)$ space

$$\Phi = x^2+y^2+z^2-R^2 = 0$$

$$x = R\sin\theta\cos\varphi$$

$$y = R\sin\theta\sin\varphi$$

$$z = R\cos\theta$$

$$n_\alpha \propto (x, y, z) = \left(\frac{x}{\sqrt{x^2+y^2+z^2}}, \frac{y}{\sqrt{\ }}, \frac{z}{\sqrt{\ }}\right)$$

$$e^\alpha_\theta = (R\cos\theta\cos\varphi, R\cos\theta\sin\varphi, -R\sin\theta)$$

$$e^\alpha_\varphi = (-R\sin\theta\sin\varphi, R\sin\theta\cos\varphi, 0)$$

$$n^\alpha \propto (x, y, z)$$

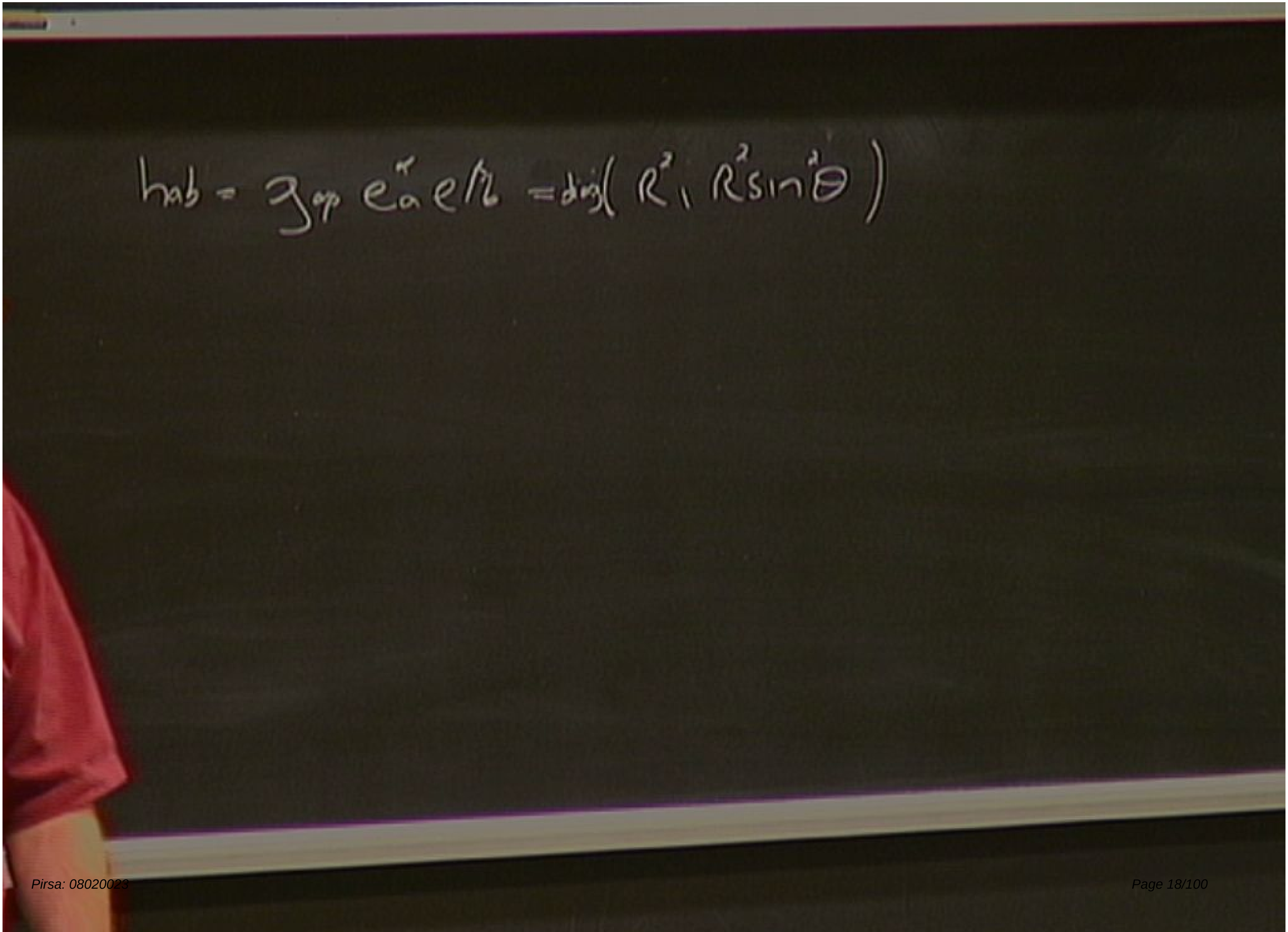
$$= \left(\frac{x}{\sqrt{x^2+y^2+z^2}}, \frac{y}{\sqrt{\ }}, \frac{z}{\sqrt{\ }} \right)$$

$$e_\theta^\alpha = (R\cos\theta\cos\phi, R\cos\theta\sin\phi, -R\sin\theta)$$

$$e_\phi^\alpha = (-R\sin\theta\sin\phi, R\sin\theta\cos\phi, 0)$$

$$h_{ab} = g_{\mu\nu} e^{\mu}_{a} e^{\nu}_{b}$$

$$h_{ab} = g_{\alpha\beta}\, e^{\alpha}_{a}\, e^{\beta}_{b}$$


$h_{ab} = g_{\alpha\beta}\, e^{\alpha}_{a}\, e^{\beta}_{b} = \mathrm{diag}(R^2 , R^2 \sin^2\theta)$

$$h_{ab} = g_{\alpha\beta}\, e^{\alpha}_{a}\, e^{\beta}_{b} = \mathrm{diag}(R^2,\ R^2 \sin^2\theta)$$

$$K_{ab} = n_{(\alpha;\beta)}\, e^{\alpha}_{a}\, e^{\beta}_{b}$$

$$h_{ab} = g_{\alpha\beta}\, e^{\alpha}{}_{a}\, e^{\beta}{}_{b} = \mathrm{diag}(R^2, R^2 \sin^2\theta)$$

$$K_{ab} = n_{(\alpha;\beta)}\, e^{\alpha}{}_{a}\, e^{\beta}{}_{b} = \mathrm{diag}(R, R\sin^2\theta)$$

$$h_{ab} = g_{\alpha\beta}\, e^{\alpha}_{a} e^{\beta}_{b} = \mathrm{diag}\left(R^2,\ R^2 \sin^2\theta \right)$$

$$K_{ab} = n_{(\alpha;\beta)}\, e^{\alpha}_{a} e^{\beta}_{b} = \mathrm{diag}\left(R,\ R \sin^2\theta \right)$$

$$= \frac{1}{R} h_{ab}$$

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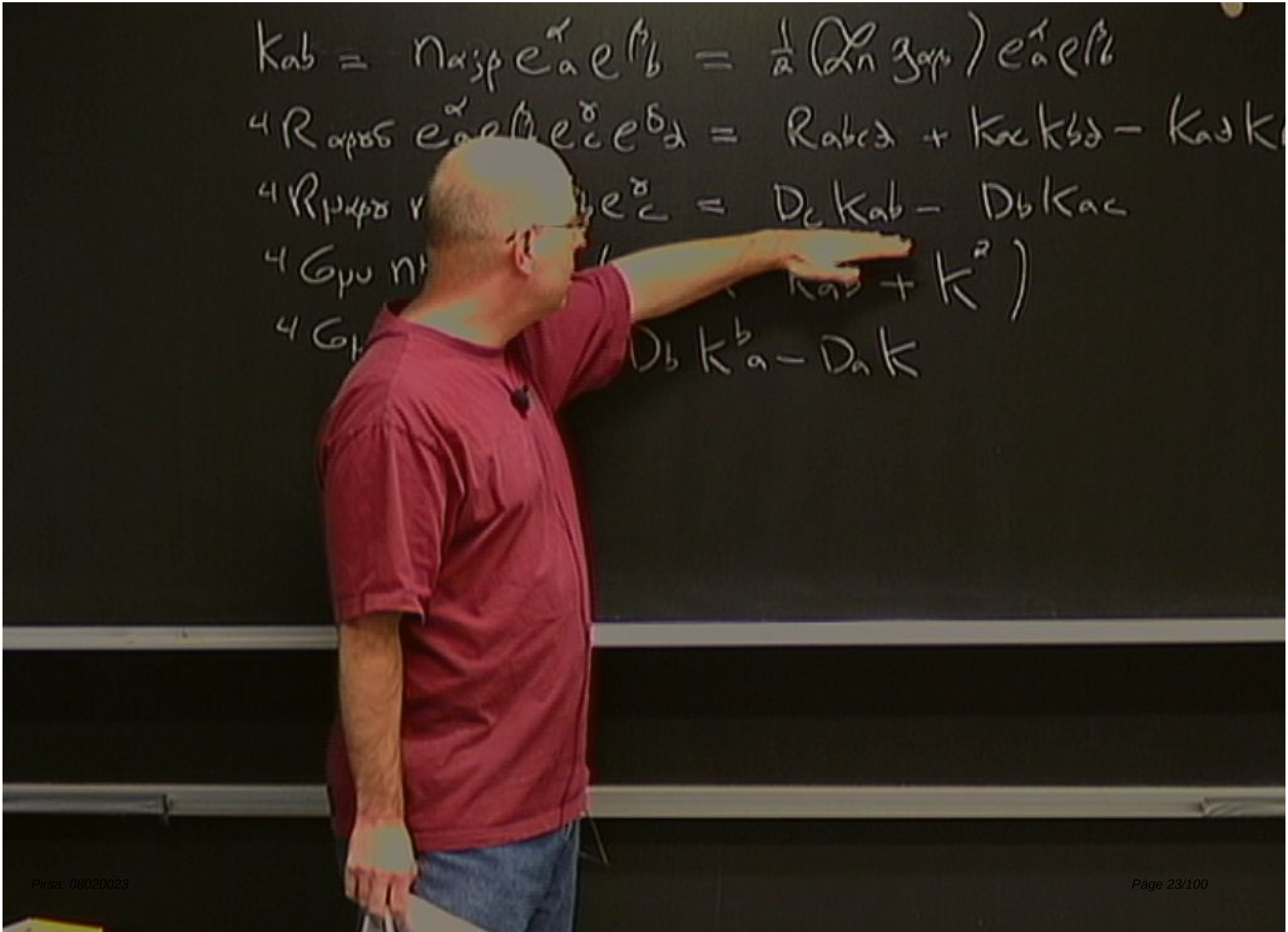
$$K_{ab} = n_{(\alpha;\beta)}\, e^{\alpha}_{a} e^{\beta}_{b} = \mathrm{diag}\left(R, R\sin^2\theta\right)$$

$$= \frac{1}{R} h_{ab}$$

$$D_c K_{ab} = 0$$


$K_{ab} = n_{\alpha;\beta} e^\alpha_a e^\beta_b = \frac{1}{2} (\mathcal{L}_n g_{\alpha\beta}) e^\alpha_a e^\beta_b$
$^4R_{\alpha\beta\gamma\delta} e^\alpha_a e^\beta_b e^\gamma_c e^\delta_d = R_{abcd} + K_{ac}K_{bd} - K_{ad}K$
$= D_c K_{ab} - D_b K_{ac}$
$+ K^2)$
$D_b K^b_a - D_a K$

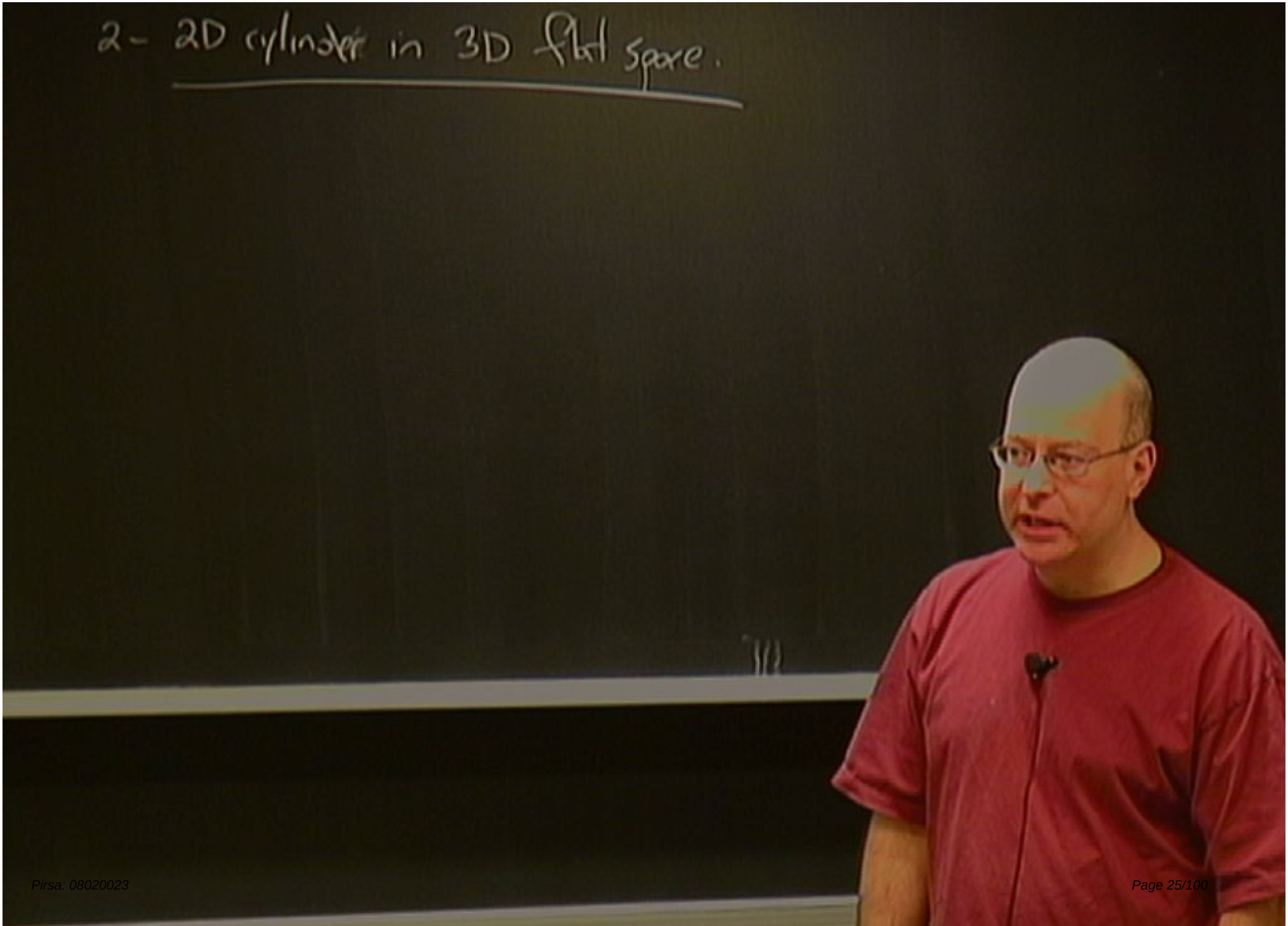
$$h_{ab} = g_{\alpha\beta}\, e^{\alpha}{}_{a}\, e^{\beta}{}_{b} = \mathrm{diag}(R^2, R^2 \sin^2\theta)$$

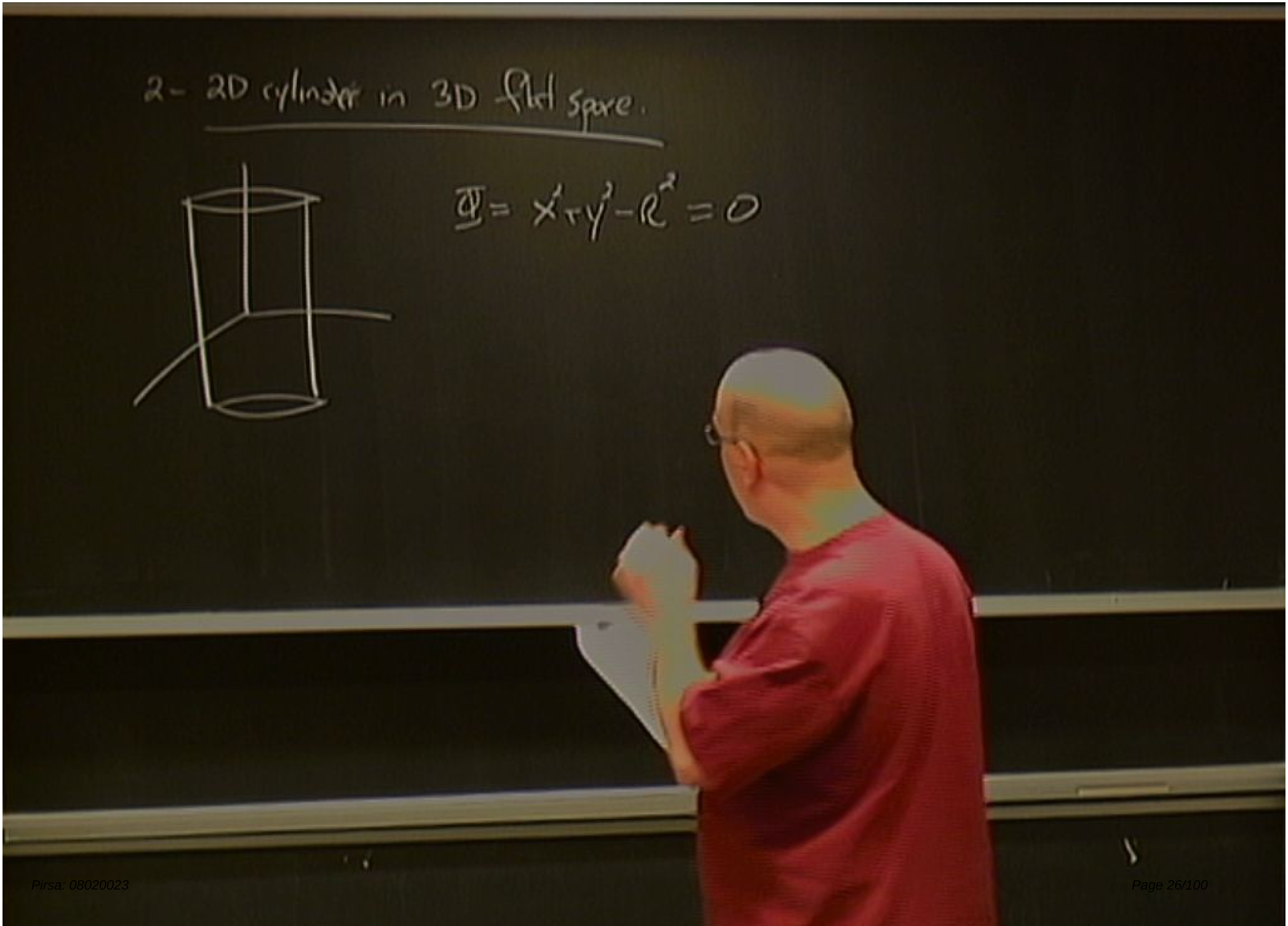
$$K_{ab} = n_{(\alpha;\beta)}\, e^{\alpha}{}_{a}\, e^{\beta}{}_{b} = \mathrm{diag}(R, R\sin^2\theta)$$

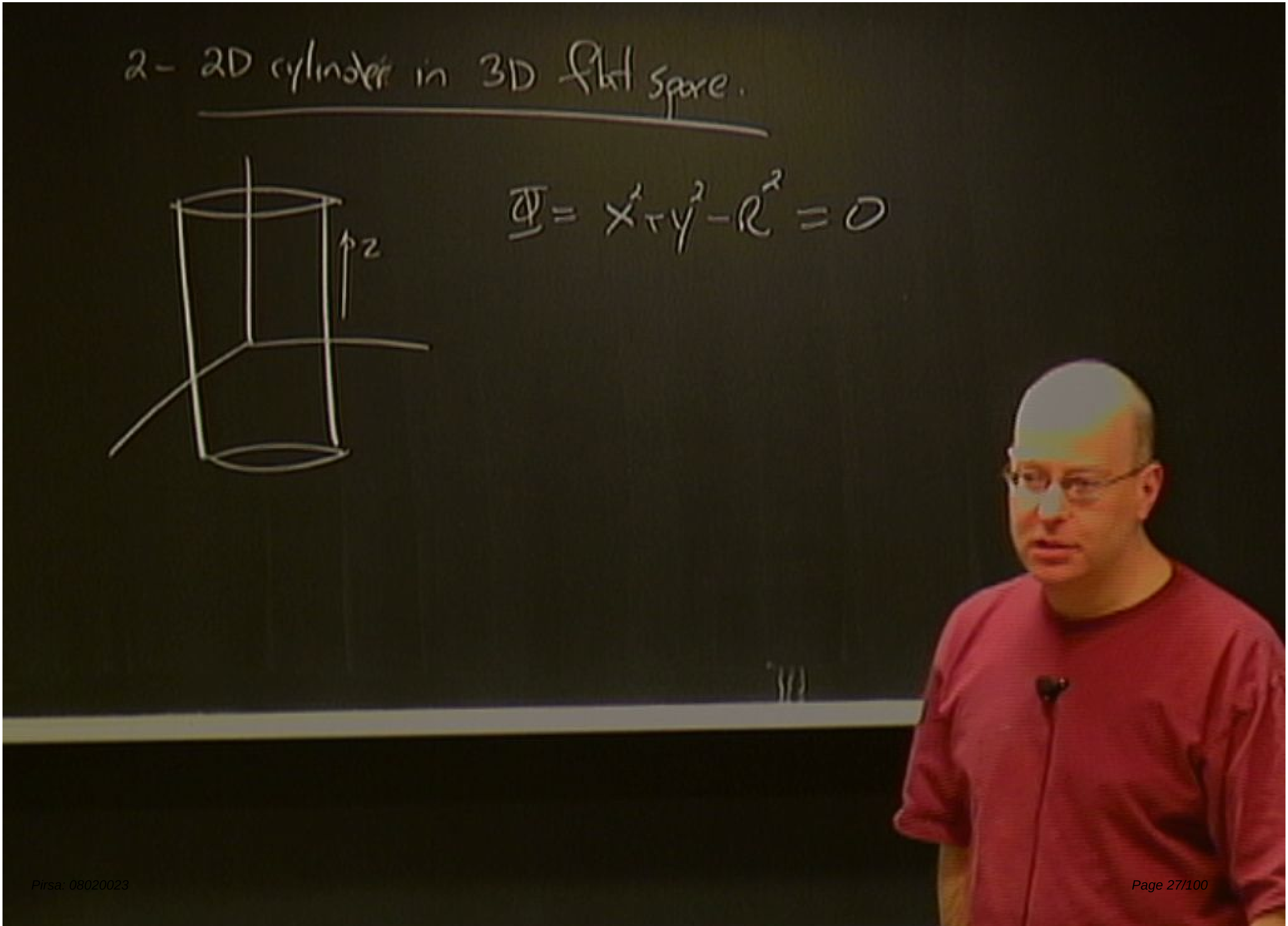
$$= \frac{1}{R} h_{ab}$$

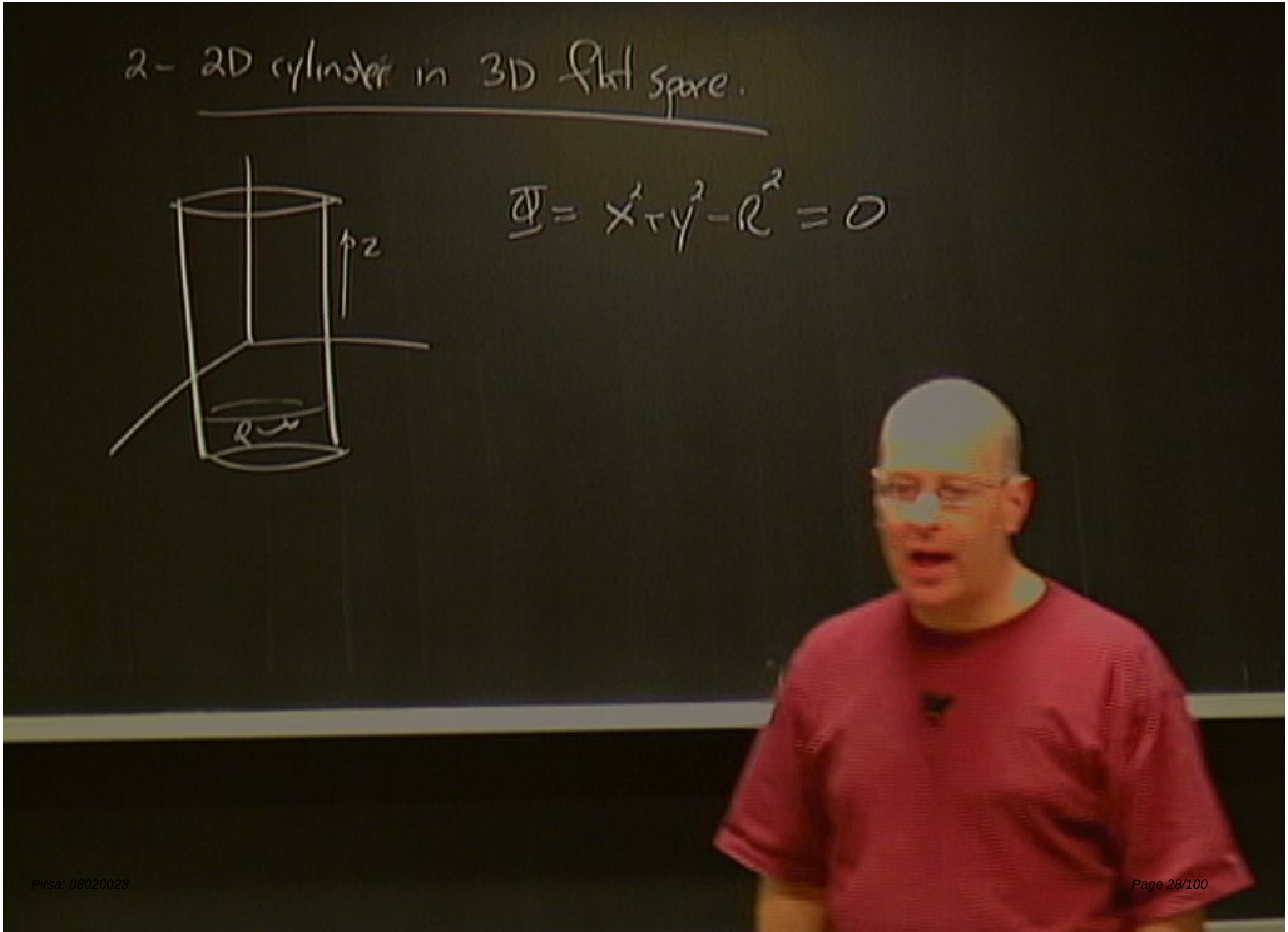
$$D_c K_{ab} = 0$$

$$R_{abcd} = \frac{1}{R^2}\left(h_{ac}h_{bd} - h_{ad}h_{bc} \right)$$

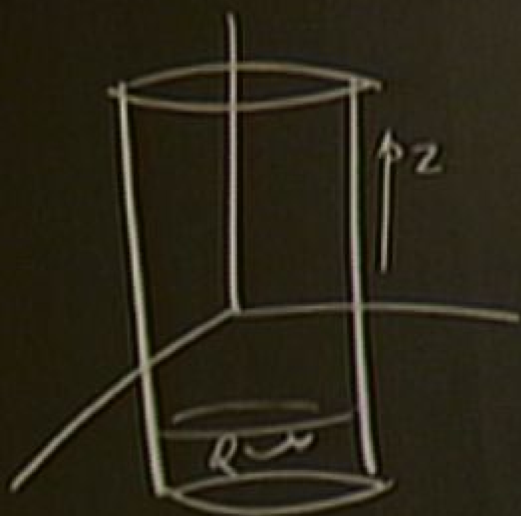

2- 2D cylinder in 3D flat space.


2- 2D cylinder in 3D flat space.
$\Phi = x^2 + y^2 - R^2 = 0$


2- 2D cylinder in 3D flat space.
z
$\underline{\Phi} = x^2 + y^2 - R^2 = 0$


2 - 2D cylinder in 3D flat space.
z
R
$\Phi = x^2 + y^2 - R^2 = 0$

2 - 2D cylinder in 3D flat space.

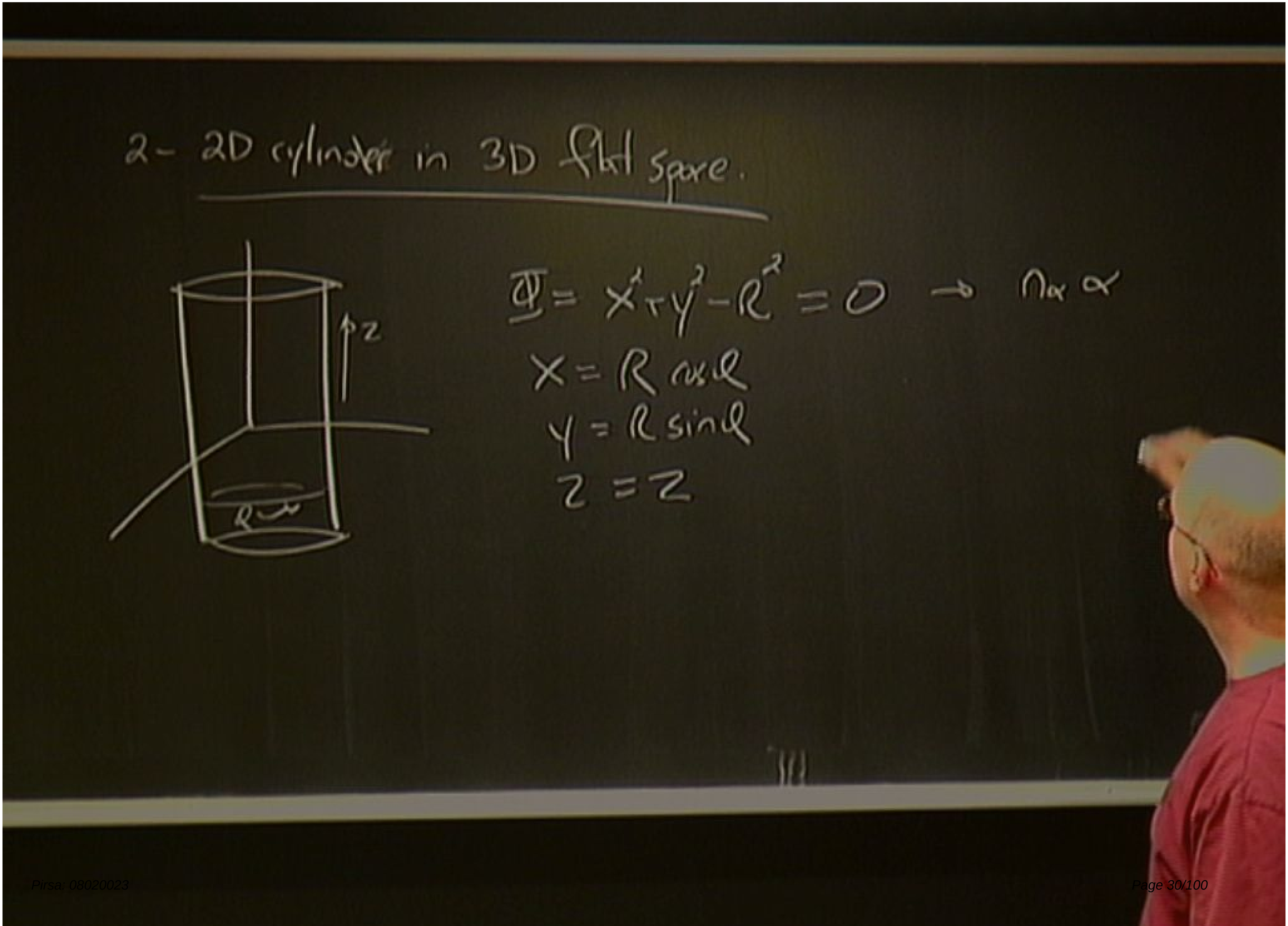


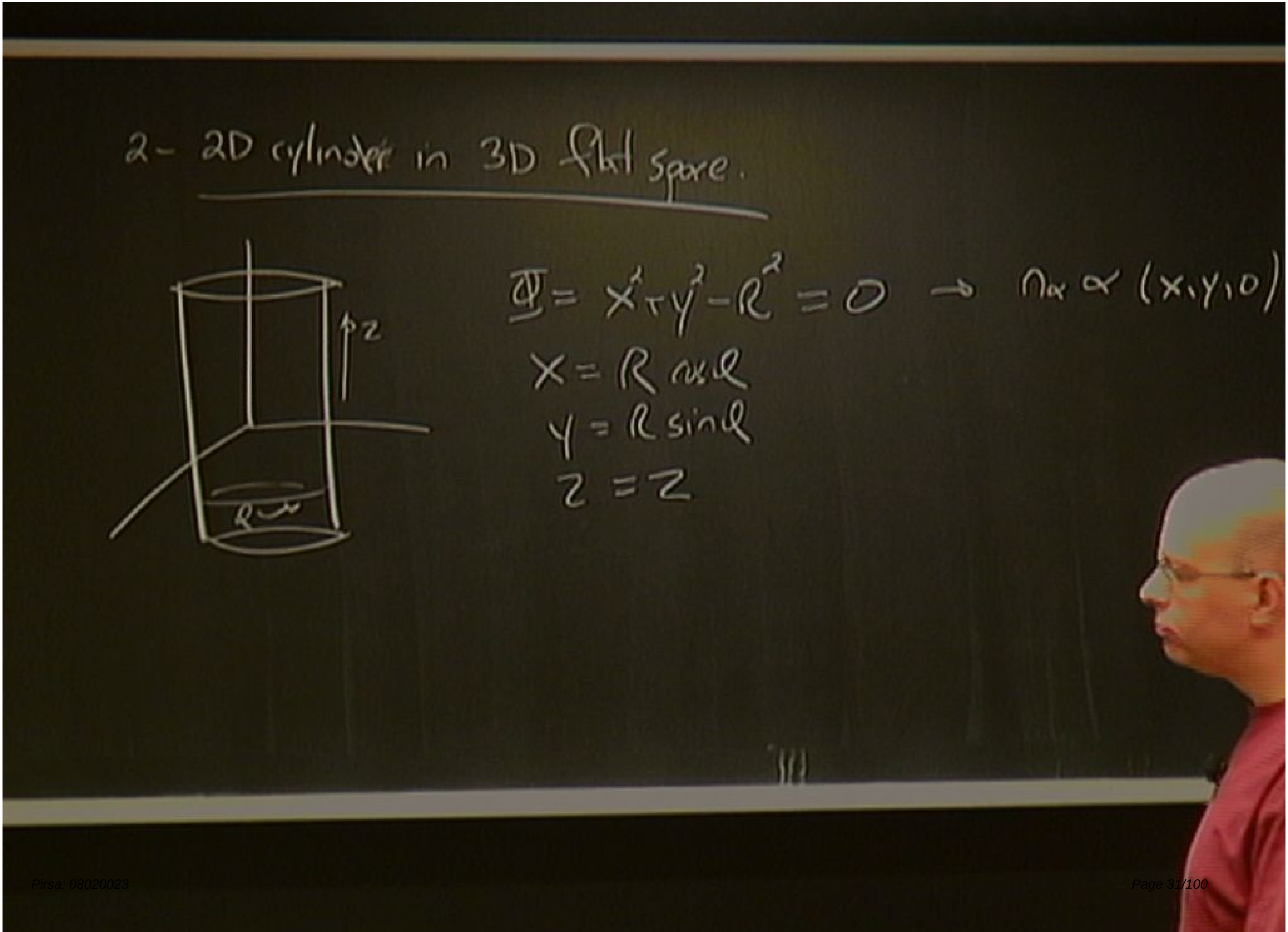
$$\Phi = x^2 + y^2 - R^2 = 0$$

$$x = R\cos\theta$$

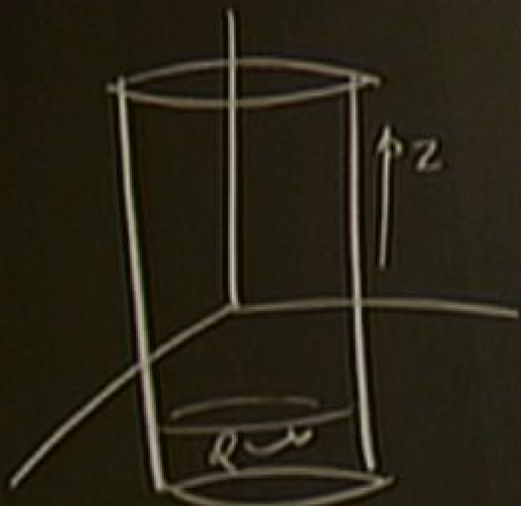
$$y = R\sin\theta$$

$$z = z$$


2- 2D cylinder in 3D flat space.
z
R
Φ = x² + y² − R² = 0 → n_α α
x = R cos θ
y = R sin θ
z = z

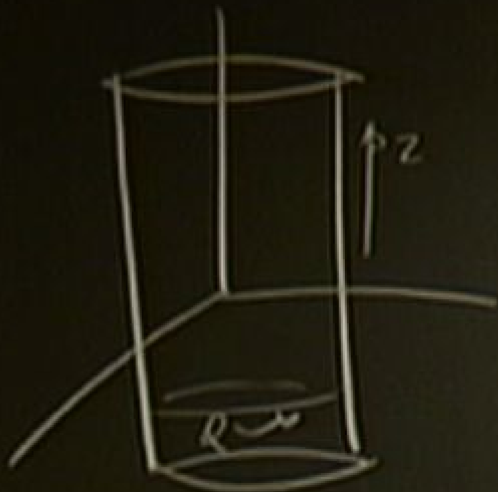

2- 2D cylinder in 3D flat space.
z
R
$\Phi = x^2 + y^2 - R^2 = 0 \rightarrow n_\alpha \propto (x, y, 0)$
$x = R\cos\varphi$
$y = R\sin\varphi$
$z = z$

2 - 2D cylinder in 3D flat space.



$$\Phi = x^2 + y^2 - R^2 = 0 \rightarrow n_\alpha \propto (x, y, 0)$$

$$\left.\begin{aligned} x &= R\cos\theta \\ y &= R\sin\theta \\ z &= z \end{aligned}\right\} \quad \begin{aligned} e^\alpha_z &= (0,0,1) \\ e^\alpha_\theta &= (-R\sin\theta, R\cos\theta, \end{aligned}$$

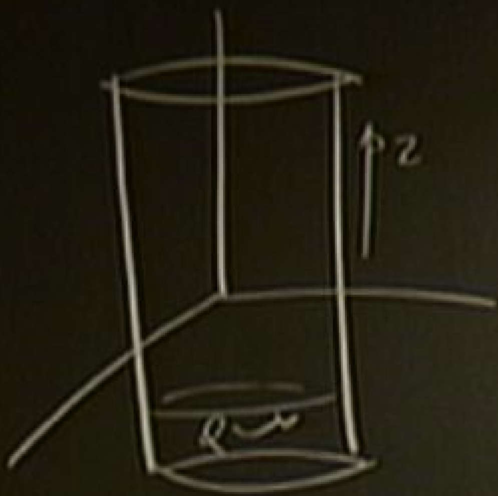


$$\Phi = x^2 + y^2 - R^2 = 0 \rightarrow n_\alpha \propto (x, y, 0)$$

$$\left.\begin{aligned} x &= R\cos\theta \\ y &= R\sin\theta \\ z &= z \end{aligned}\right\} \quad \begin{aligned} e^\alpha_z &= (0, 0, 1) \\ e^\alpha_\theta &= (-R\sin\theta, R\cos\theta, 0) \end{aligned}$$

$$y^a = (z, \theta)$$

$$h_{ab}\, dy^a dy^b =$$

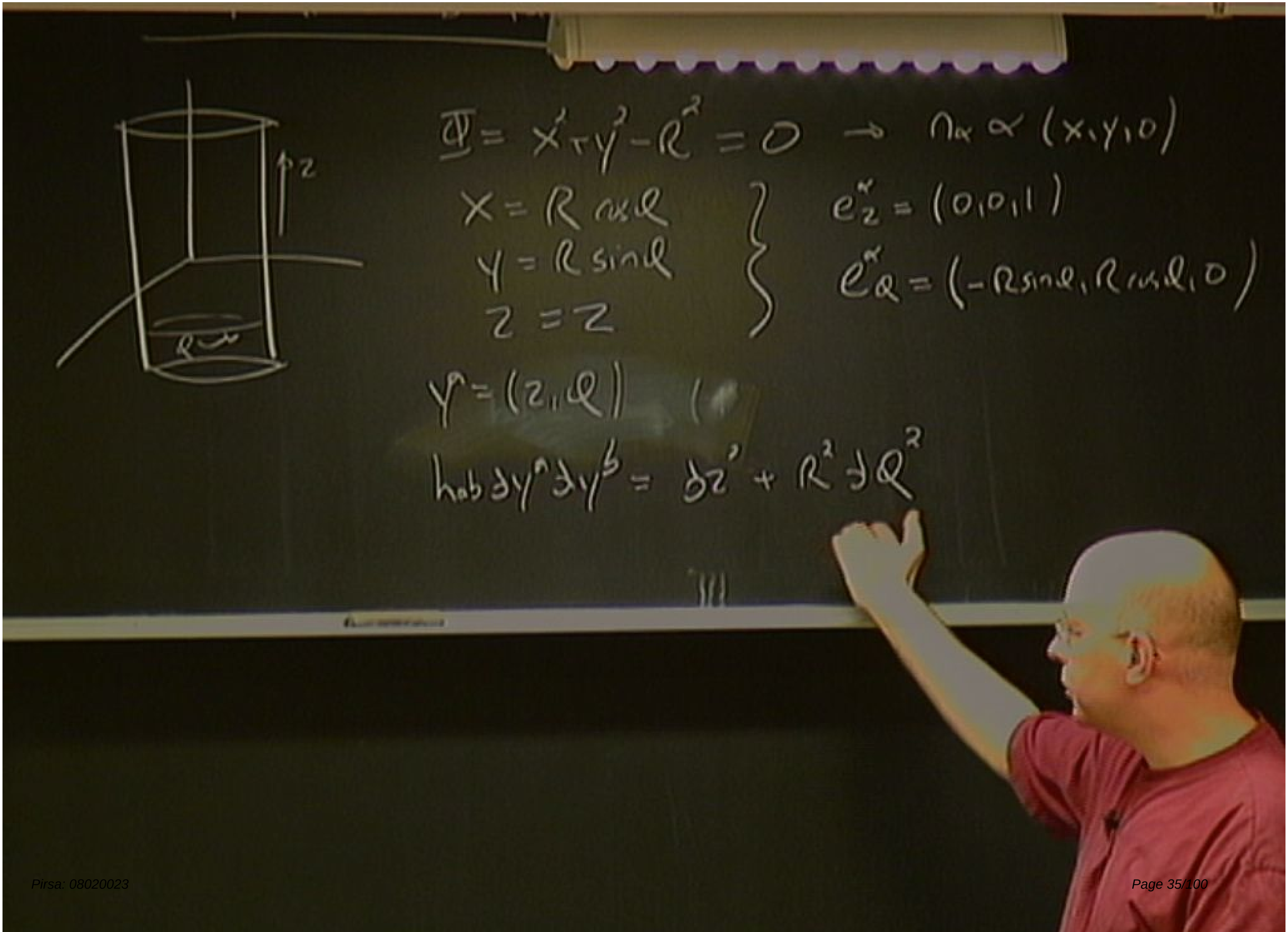


$$\Phi = x^2 + y^2 - R^2 = 0 \rightarrow n_\alpha \propto (x, y, 0)$$

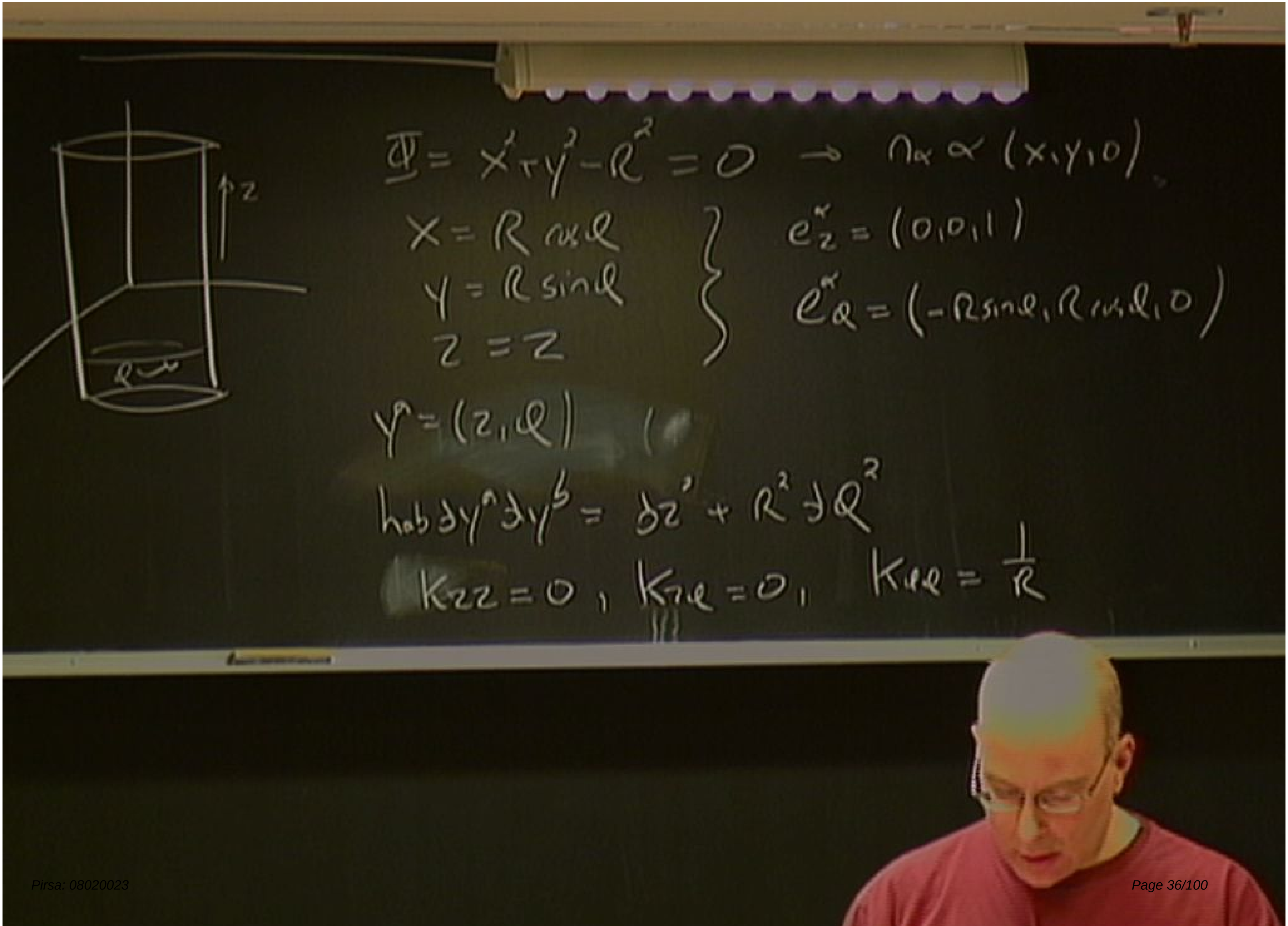
$$\left.\begin{aligned} x &= R\cos\theta \\ y &= R\sin\theta \\ z &= z \end{aligned}\right\} \quad \begin{aligned} e^\alpha_z &= (0,0,1) \\ e^\alpha_\theta &= (-R\sin\theta, R\cos\theta, 0) \end{aligned}$$

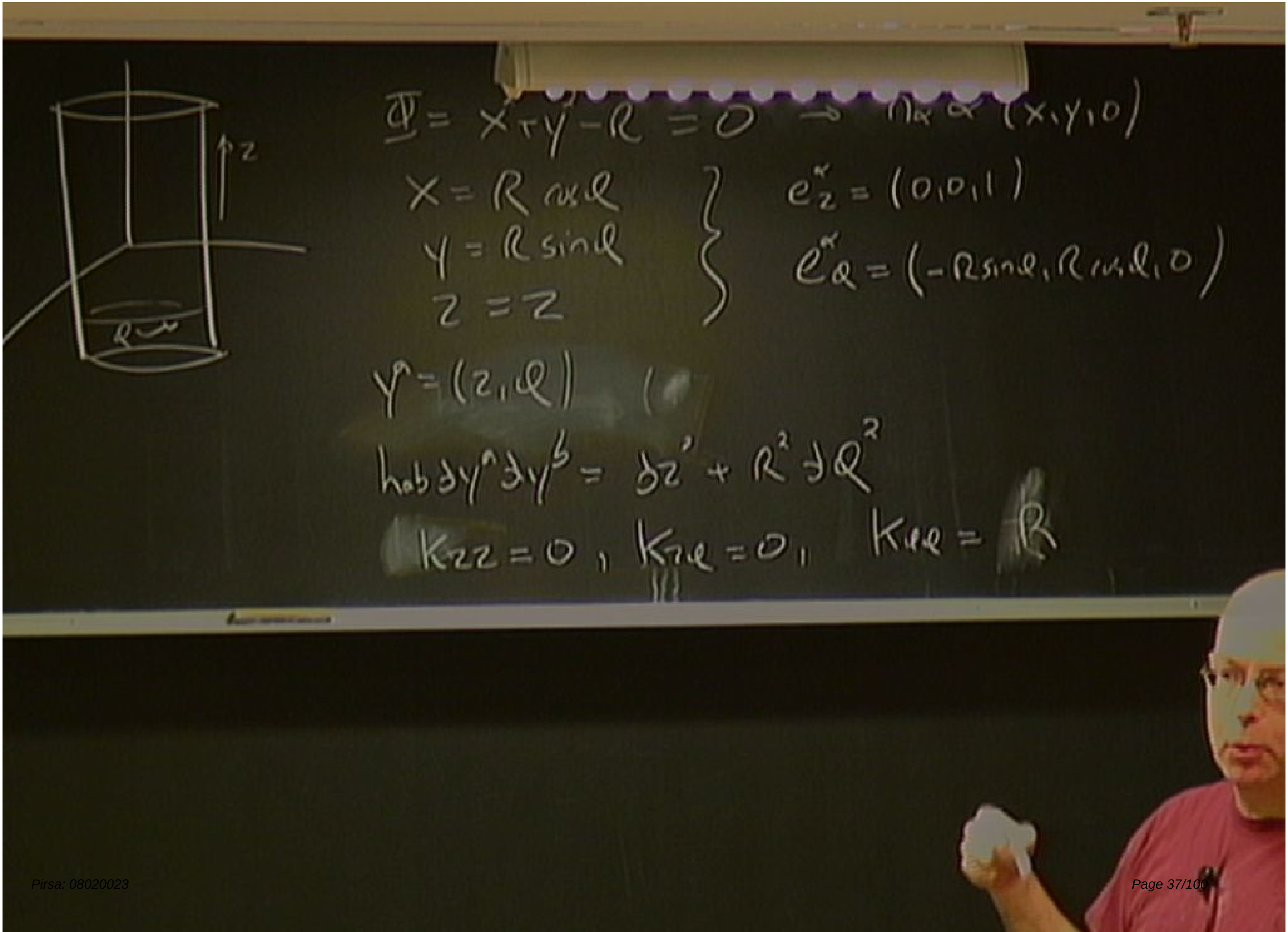
$$y^a = (z, \theta)$$

$$h_{ab}\, dy^a dy^b = dz^2 + R^2 d\theta^2$$

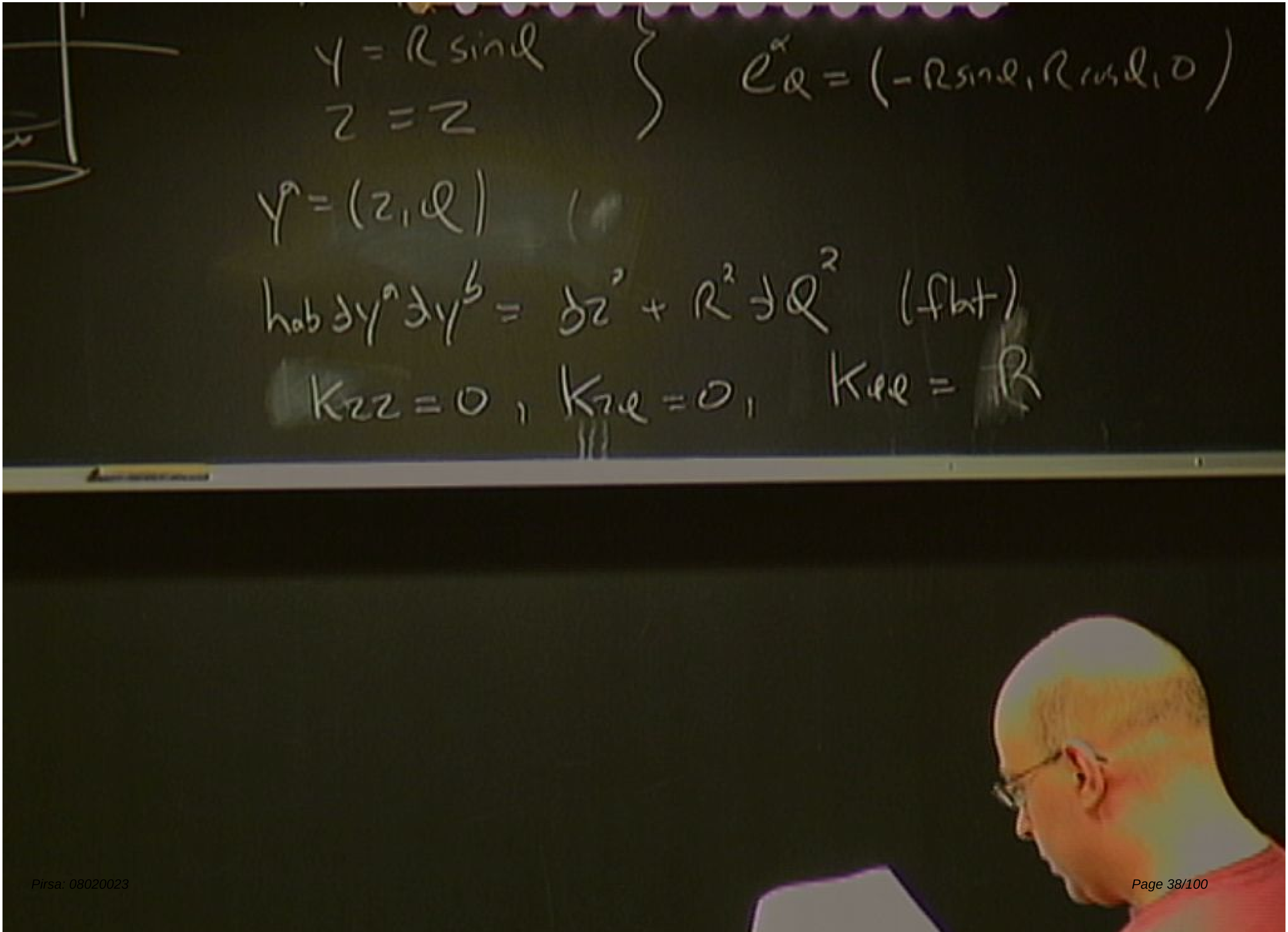


z
R
$\Phi = x^2 + y^2 - R^2 = 0 \rightarrow n_\alpha \propto (x, y, 0)$
$x = R\cos\varphi$
$y = R\sin\varphi$
$z = z$
$e^\alpha_z = (0, 0, 1)$
$e^\alpha_\varphi = (-R\sin\varphi, R\cos\varphi, 0)$
$y^a = (z, \varphi)$
$h_{ab}\, dy^a dy^b = dz^2 + R^2 d\varphi^2$

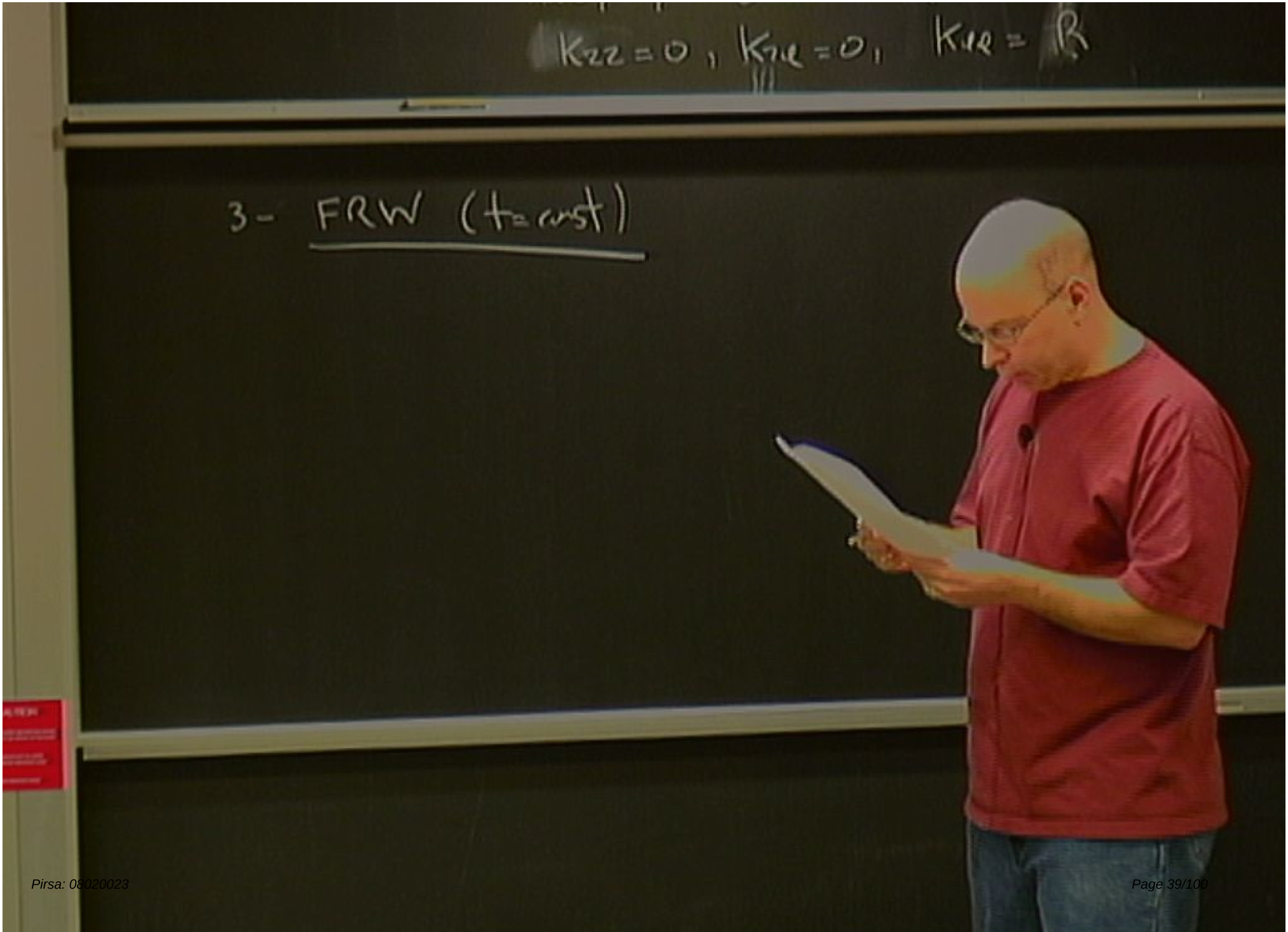

z
$\Phi = x^2 + y^2 - R^2 = 0 \rightarrow n_\alpha \propto (x, y, 0)$
$x = R\cos\varphi$
$y = R\sin\varphi$
$z = z$
$e^\alpha_z = (0,0,1)$
$e^\alpha_\varphi = (-R\sin\varphi, R\cos\varphi, 0)$
$y^a = (z, \varphi)$
$h_{ab}\,dy^a dy^b = dz^2 + R^2 d\varphi^2$
$K_{zz} = 0, \quad K_{z\varphi} = 0, \quad K_{\varphi\varphi} = \frac{1}{R}$



z
R
$\Phi = x^2 + y^2 - R^2 = 0 \rightarrow n_\alpha \propto (x, y, 0)$
$x = R\cos\varphi$
$y = R\sin\varphi$
$z = z$
$e^\alpha_z = (0, 0, 1)$
$e^\alpha_\varphi = (-R\sin\varphi, R\cos\varphi, 0)$
$y^a = (z, \varphi)$
$h_{ab}\, dy^a dy^b = dz^2 + R^2 d\varphi^2$
$K_{zz} = 0, \quad K_{z\varphi} = 0, \quad K_{\varphi\varphi} = R$



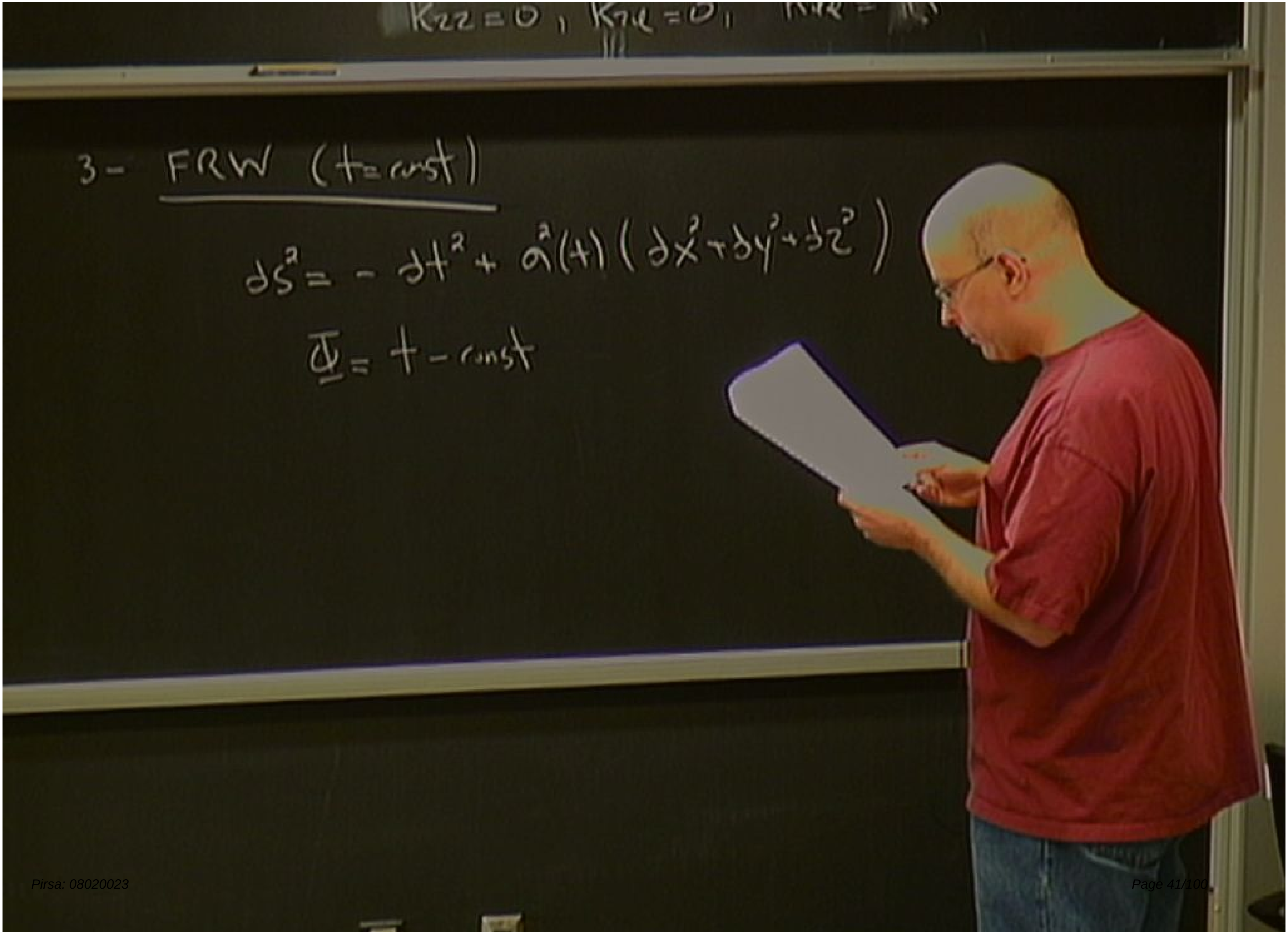
$y = R\sin\varphi$
$z = z$
$e^{\alpha}_{\varphi} = (-R\sin\varphi, R\cos\varphi, 0)$
$y^a = (z, \varphi)$
$h_{ab}\,dy^a dy^b = dz^2 + R^2 d\varphi^2$ (flat)
$K_{zz} = 0$, $K_{z\varphi} = 0$, $K_{\varphi\varphi} = R$


Kzz = 0 , Kzφ = 0, Kφφ = R
3- FRW (t=const)

$K_{zz} = 0\,,\quad K_{z\varphi} = 0\,,\quad K_{\varphi\varphi} = R$

3- FRW ($t = \text{const}$)

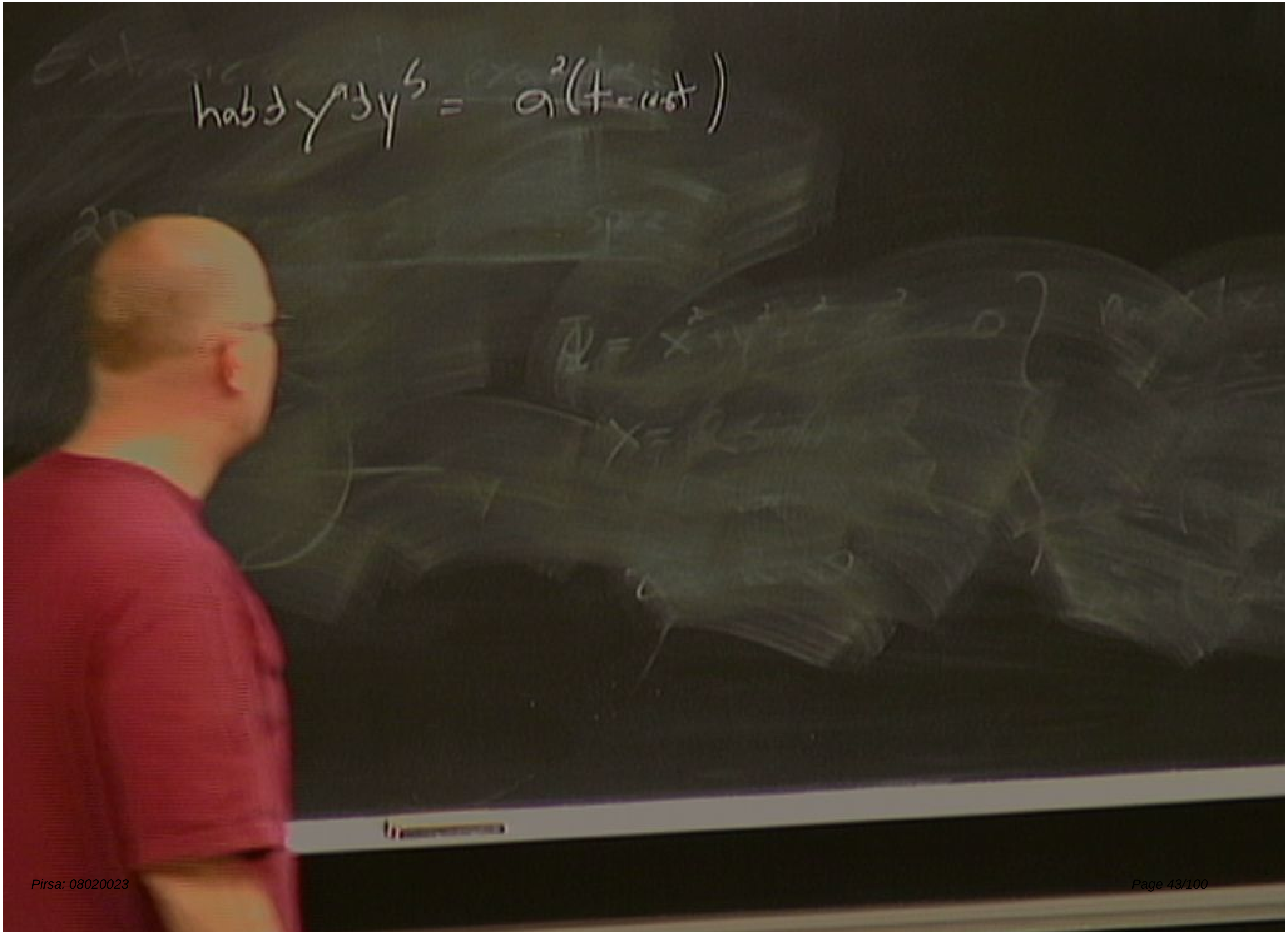
$$ds^2 = -dt^2 + a^2(t)\left(dx^2 + dy^2 + dz^2\right)$$

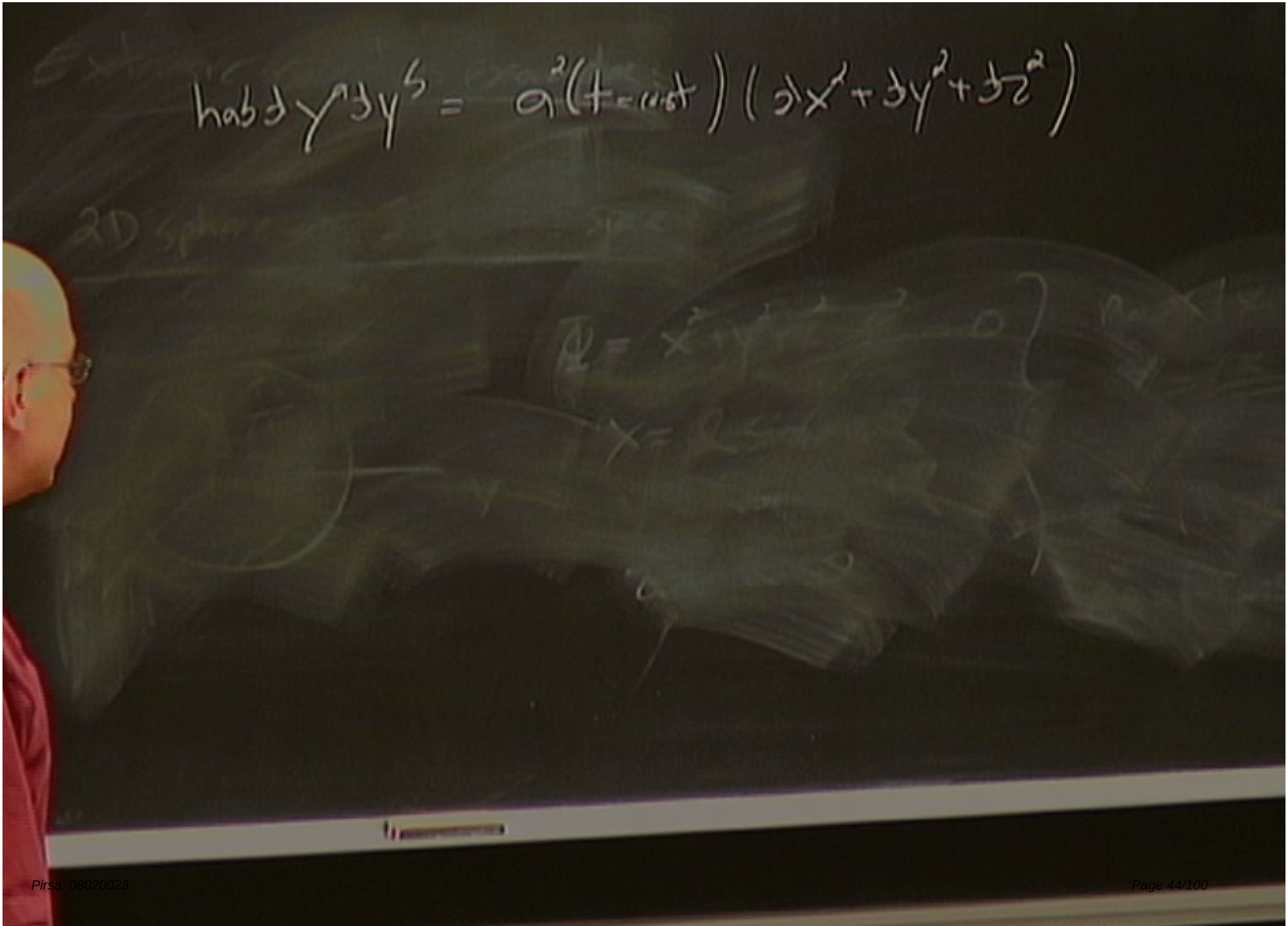

3- FRW (t=const)
ds² = - dt² + a²(t) (dx² + dy² + dz²)
Φ = t - const

3- FRW ($t = \text{const}$)

$$ds^2 = -dt^2 + a^2(t)\left(dx^2 + dy^2 + dz^2\right)$$

$$\left.\begin{aligned} \Phi &= t - \text{const} \\ x &= X \\ y &= Y \\ z &= Z \end{aligned}\right\} \quad \begin{aligned} n_a &= (-1, 0, 0, 0) \\ e^\alpha_x &= (0, 1, 0, 0) \\ e^\alpha_y &= (0, 0, 1, 0) \\ e^\alpha_z &= (0, 0, 0, 1) \end{aligned}$$


$h_{ab}\,\partial y^a \partial y^b = a^2(t=const)$


h_ab dy^a dy^b = a^2(t=const)(dx^2+dy^2+dz^2)

$$h_{ab}\, dy^a dy^b = a^2(t=\text{const})\,(dx^2+dy^2+dz^2)$$

$$K_{ab} = n_{\alpha;\beta}\, e^\alpha_a e^\beta_b$$

$\longrightarrow$

$$h_{ab}\, dy^a dy^b = a^2(t=\text{const})\,(dx^2+dy^2+dz^2)$$

$$K_{ab} = n_{\alpha;\beta}\, e^\alpha_a e^\beta_b$$

$$\rightarrow \quad K_{xx} = K_{yy} = K_{zz} = a\dot{a}\,\Big|_{t=\text{const}}$$

$$h_{ab}\, dy^a dy^b = a^2(t=\text{const})\,(dx^2+dy^2+dz^2)$$

$$K_{ab} = \eta_{\alpha\beta}\, e^{\alpha}_{a}\, e^{\beta}_{b}$$

$$\rightarrow \quad K_{xx} = K_{yy} = K_{zz} = a\dot{a}\,\Big|_{t=\text{const}}$$

$$K_{ab} = \frac{\dot{a}}{a}\, h_{ab}$$

$$h_{ab}\, dy^a dy^b = a^2(t=\text{const})\,(dx^2 + dy^2 + dz^2)$$

$$K_{ab} = n_{\alpha;\beta}\, e^{\alpha}_{a} e^{\beta}_{b}$$

$$\rightarrow \quad K_{xx} = K_{yy} = K_{zz} = a\dot{a}\,\Big|_{t=\text{const}}$$

$$K_{ab} = \frac{\dot{a}}{a} h_{ab}$$

$$D_c K_{ab} = 0$$

$$h_{ab}\, dy^a dy^b = a^2(t=\text{const})\,(dx^2+dy^2+dz^2)$$

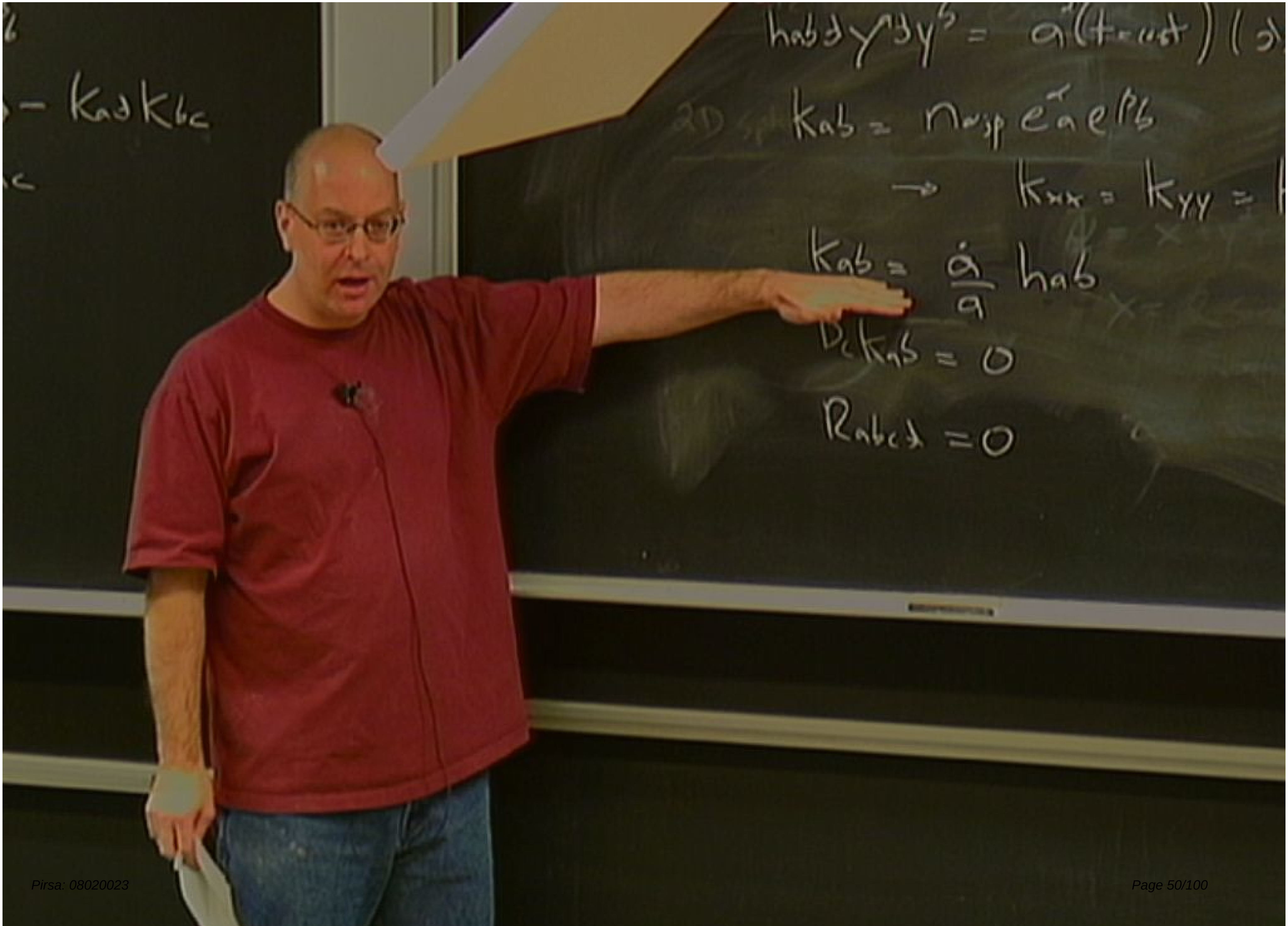
$$K_{ab} = n_{\alpha;\beta}\, e^{\alpha}_{a}\, e^{\beta}_{b}$$

$$\rightarrow \quad K_{xx} = K_{yy} = K_{zz} = a\dot{a}\,\Big|_{t=\text{const}}$$

$$K_{ab} = \frac{\dot{a}}{a}\, h_{ab}$$

$$D_c K_{ab} = 0$$

$$R_{abcd} = 0$$


Kab = ȧ/a hab
DcKab = 0
Rabcd = 0

4 - Schwarzschild ($t =$ const)

$$ds^2 = -f\,dt^2 + f^{-1}\,dr^2 + r^2\,d\Omega^2$$

4 - Schwarzschild (t = const)

$$ds^2 = -f\,dt^2 + f^{-1}\,dr^2 + r^2\,d\Omega^2 \qquad f = 1 - \frac{2m}{r}$$

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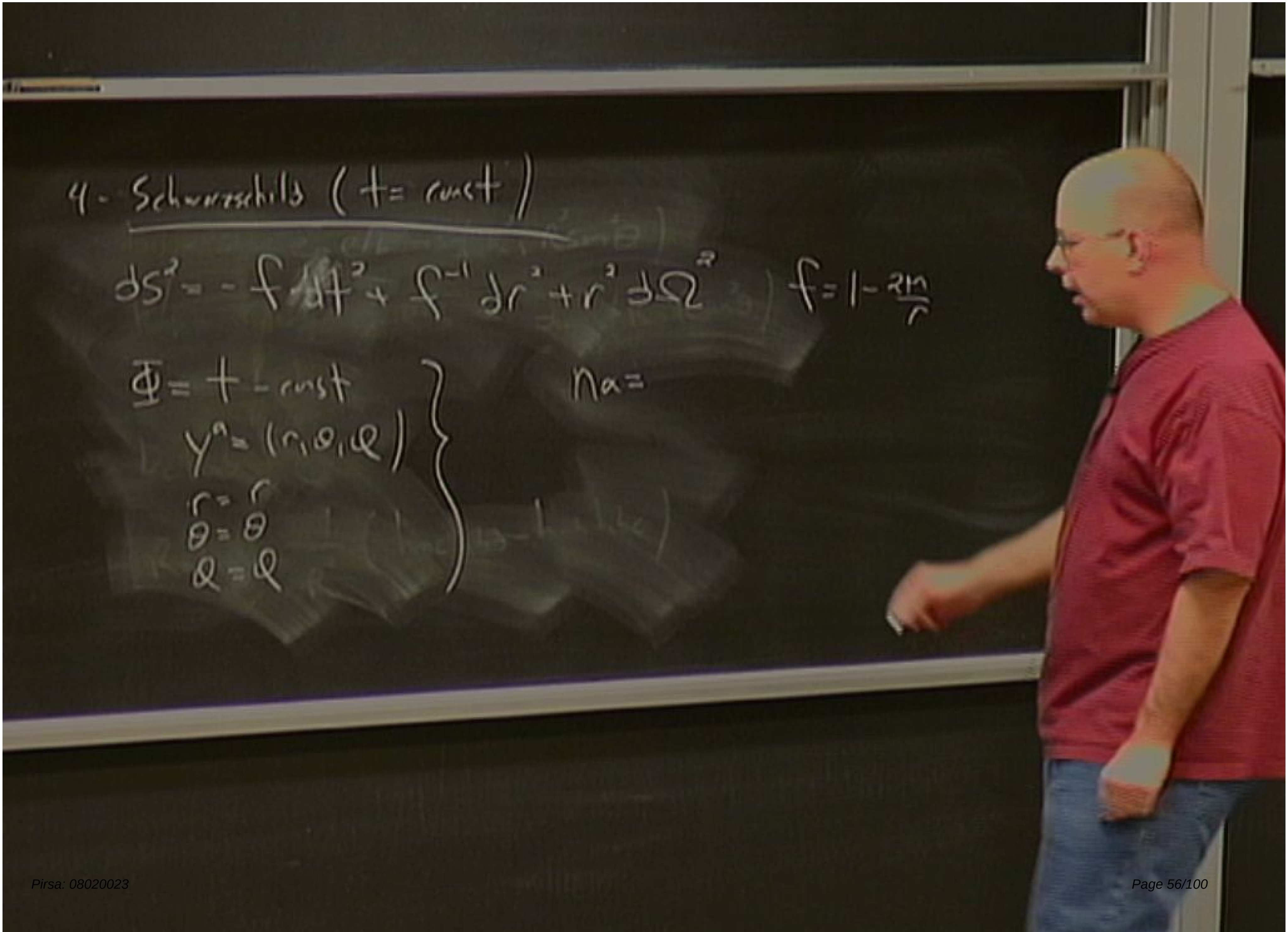
$$\Phi = t - \text{const}$$

$$y^a = (r, \theta, \varphi)$$

4 - Schwarzschild ($t = \text{const}$)

$$ds^2 = -f\,dt^2 + f^{-1}dr^2 + r^2 d\Omega^2 \qquad f = 1 - \frac{2m}{r}$$

$$\left.\begin{aligned} &\Phi = t - \text{const} \\ &y^a = (r, \theta, \varphi) \\ &r = r \\ &\theta = \theta \\ &\varphi = \varphi \end{aligned}\right\}$$


4 - Schwarzschild (t = const)
ds² = - f dt² + f⁻¹ dr² + r² dΩ²   f = 1 - 2m/r
Φ = t - const
yᵃ = (r, θ, φ)
r = r
θ = θ
φ = φ
nα =

4 - Schwarzschild ($t = \text{const}$)

$$ds^2 = -f\,dt^2 + f^{-1}dr^2 + r^2 d\Omega^2 \qquad f = 1 - \frac{2m}{r}$$

$$\left.\begin{aligned} &\Phi = t - \text{const} \\ &y^a = (r, \theta, \varphi) \\ &r = r \\ &\theta = \theta \\ &\varphi = \varphi \end{aligned}\right\} \qquad n_\alpha = \left(\frac{1}{\sqrt{f}}\right.$$

4 - Schwarzschild ($t = \text{const}$)

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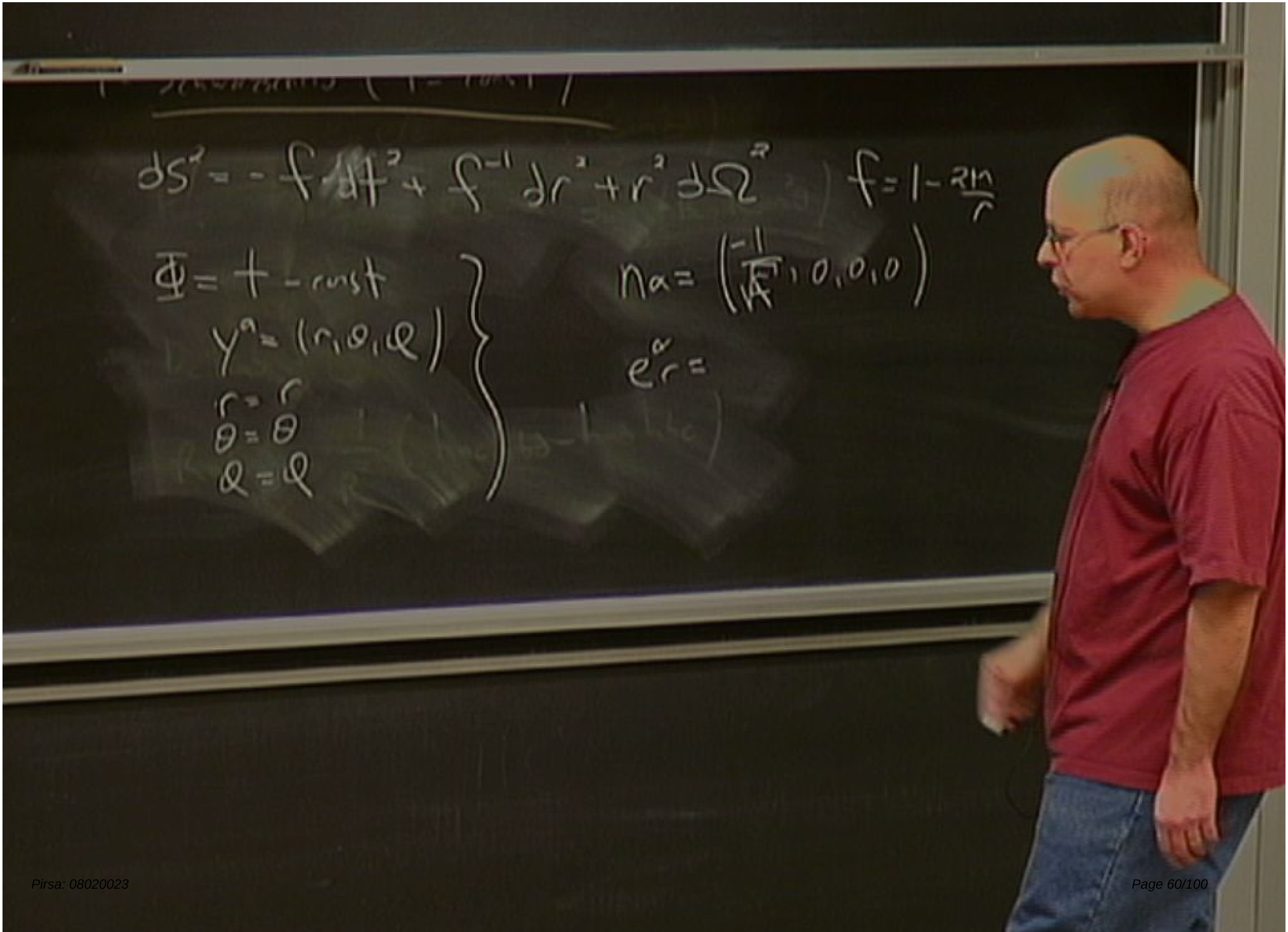
4 - Schwarzschild ($t = \text{const}$)

$$ds^2 = -f\,dt^2 + f^{-1}\,dr^2 + r^2\,d\Omega^2 \qquad f = 1 - \frac{2m}{r}$$

$$\left.\begin{aligned} &\Phi = t - \text{const} \\ &y^a = (r, \theta, \varphi) \\ &r = r \\ &\theta = \theta \\ &\varphi = \varphi \end{aligned}\right\} \qquad n_\alpha = \left(\frac{-1}{\sqrt{f}}, 0, 0, 0\right)$$


ds² = -f dt² + f⁻¹ dr² + r² dΩ²
f = 1 - 2m/r
Φ = t - const
yᵃ = (r, θ, φ)
r = r
θ = θ
φ = φ
nα = (-1/√f, 0, 0, 0)
eᵅr =

4. Schwarzschild ($t = \text{const}$)

$$ds^2 = -f\,dt^2 + f^{-1}dr^2 + r^2 d\Omega^2 \qquad f = 1 - \frac{2m}{r}$$

$$\left.\begin{aligned} &\Phi = t - \text{const} \\ &y^a = (r, \theta, \varphi) \\ &r = r \\ &\theta = \theta \\ &\varphi = \varphi \end{aligned}\right\} \qquad \begin{aligned} &n_\alpha = (-\sqrt{f}, 0, 0, 0) \\ &e^\alpha_r = (0, 1, 0, 0) \\ &e^\alpha_\theta = (0, 0, 1, 0) \\ &e^\alpha_\varphi = (0, 0, 0, 1) \end{aligned}$$

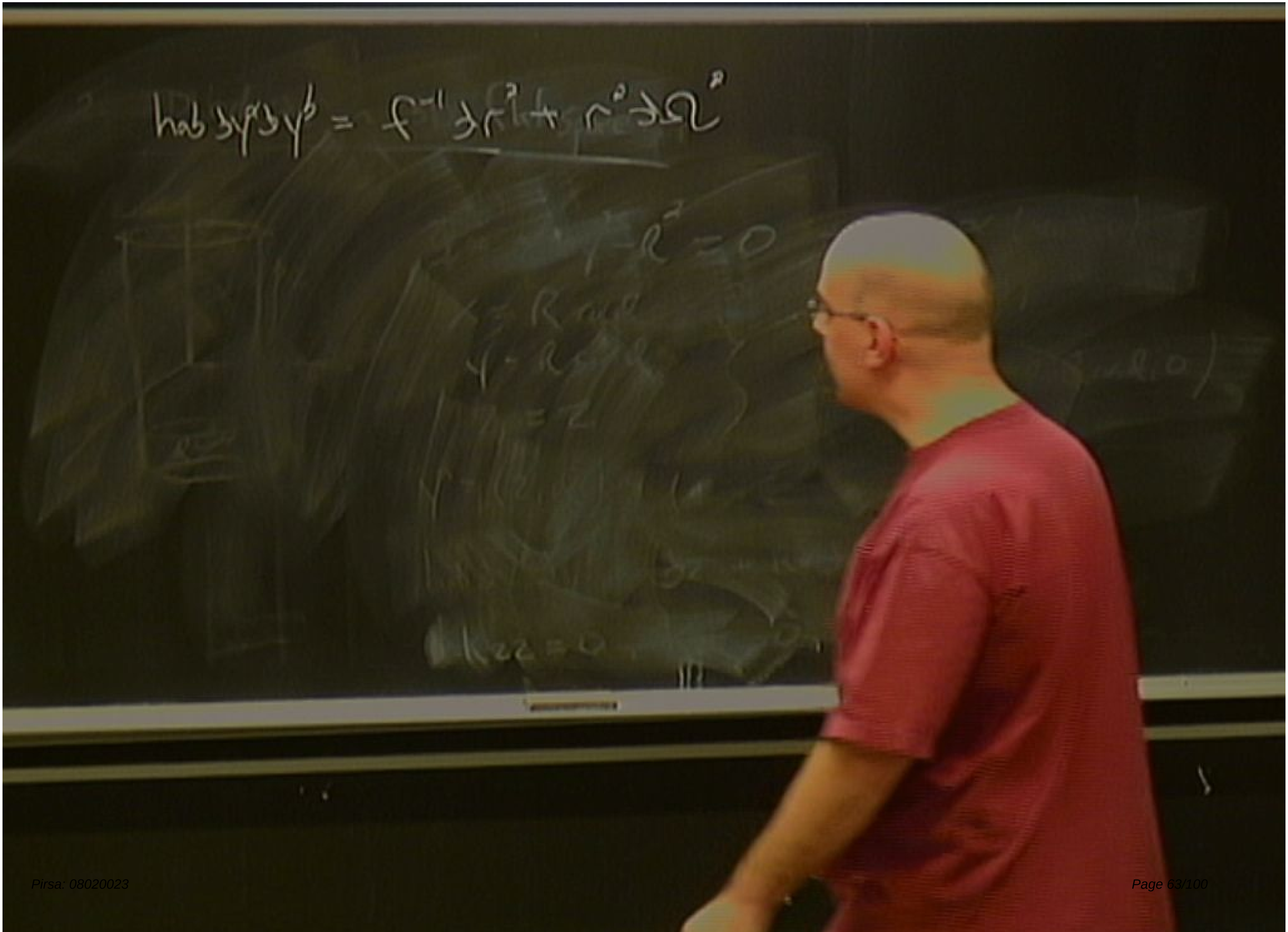
$$^{-1}dr^2 + r^2 d\Omega^2 \qquad f = 1 - \frac{2m}{r}$$

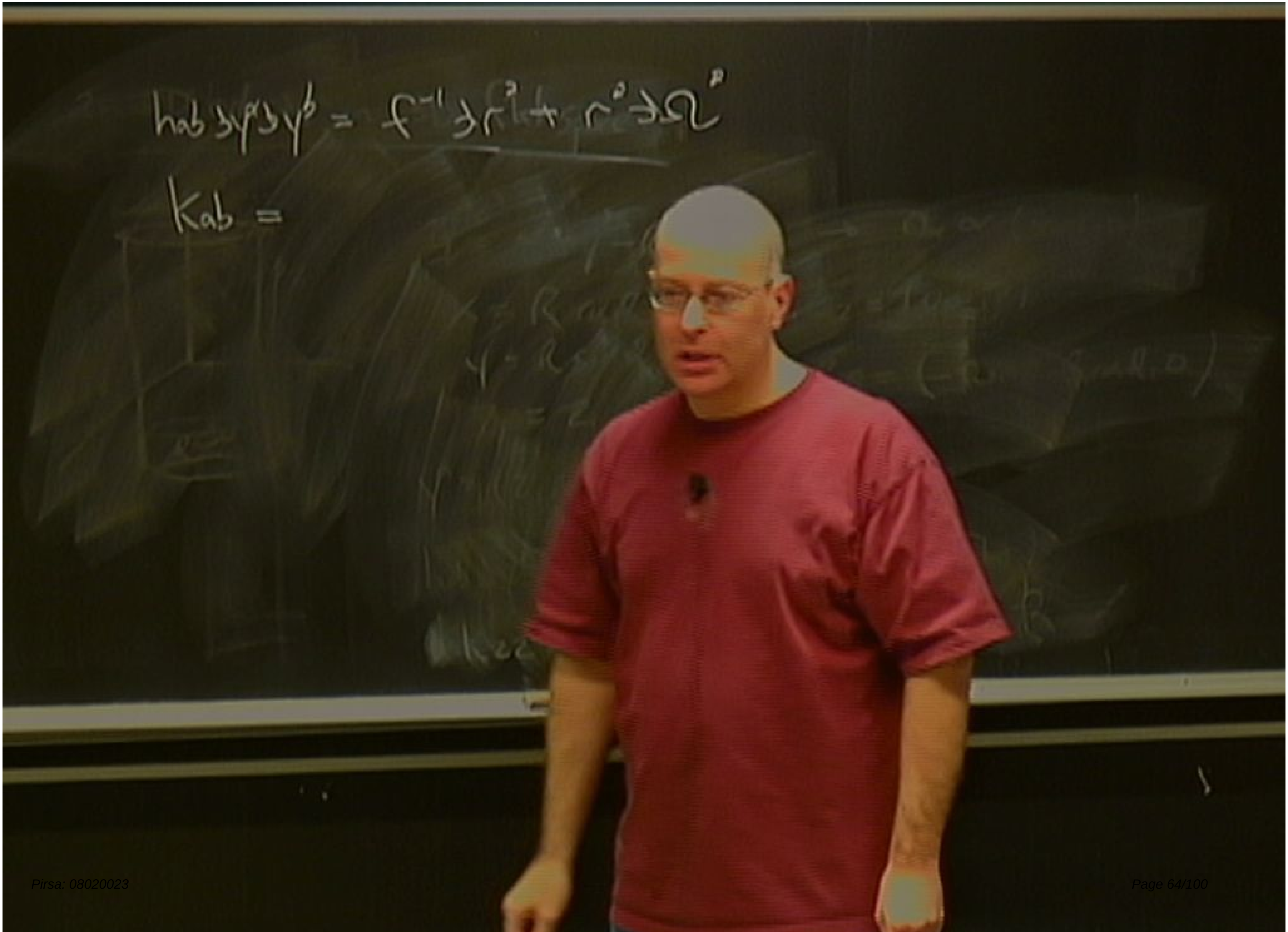
$$n_\alpha = \left(-\sqrt{f}, 0, 0, 0\right)$$

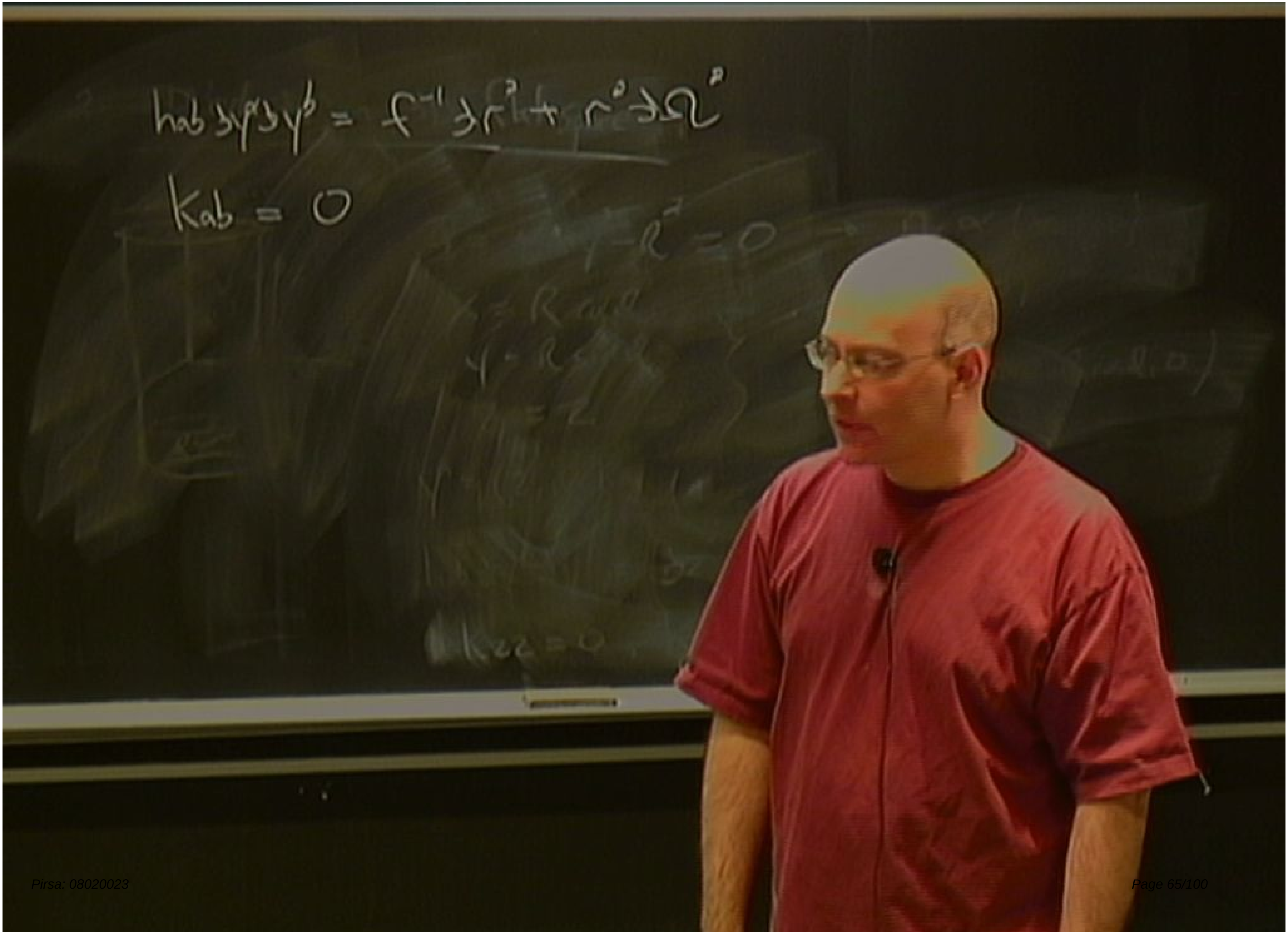
$$e^\alpha_r = (0, 1, 0, 0)$$

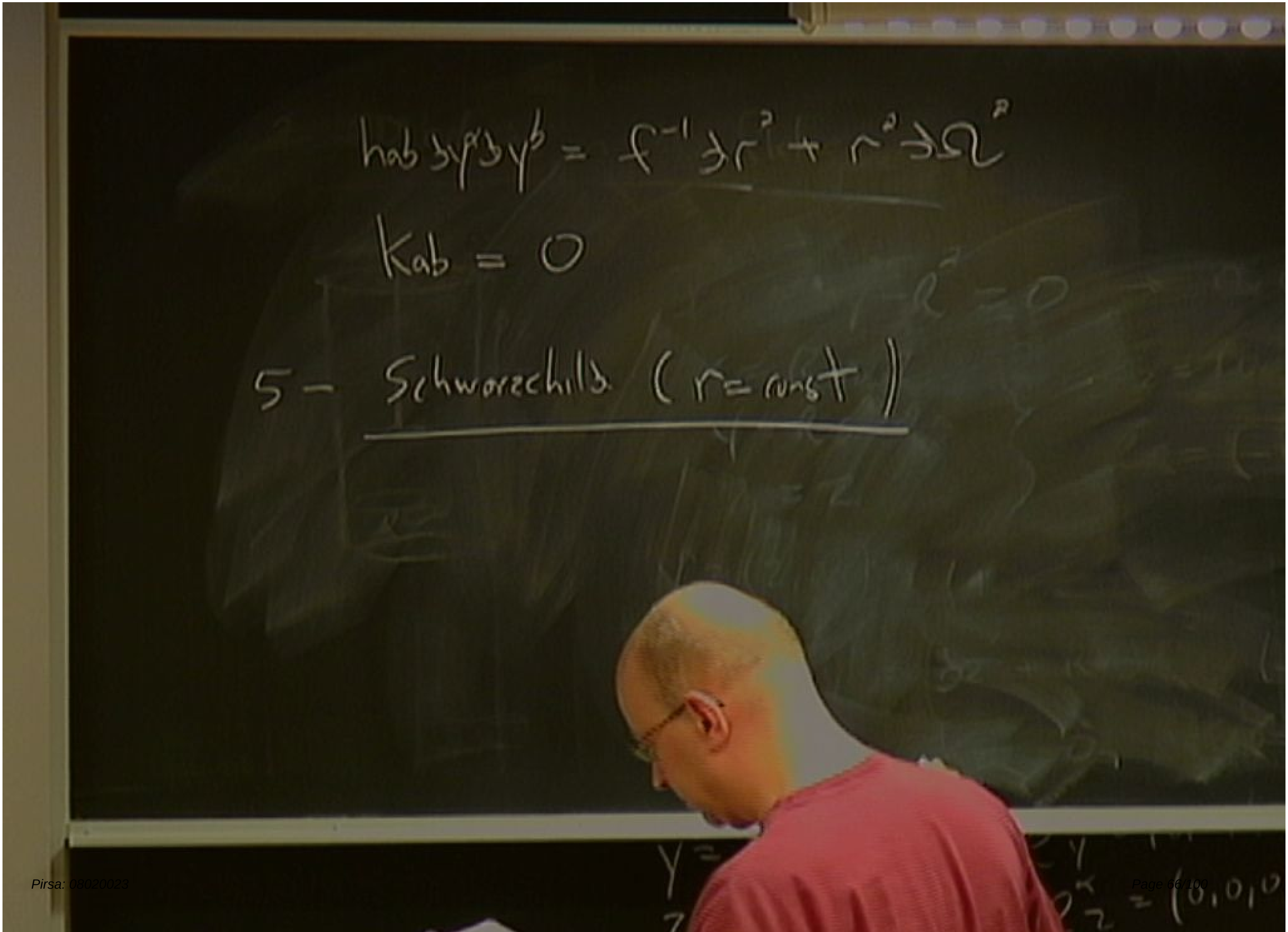
$$e^\alpha_\theta = (0, 0, 1, 0)$$

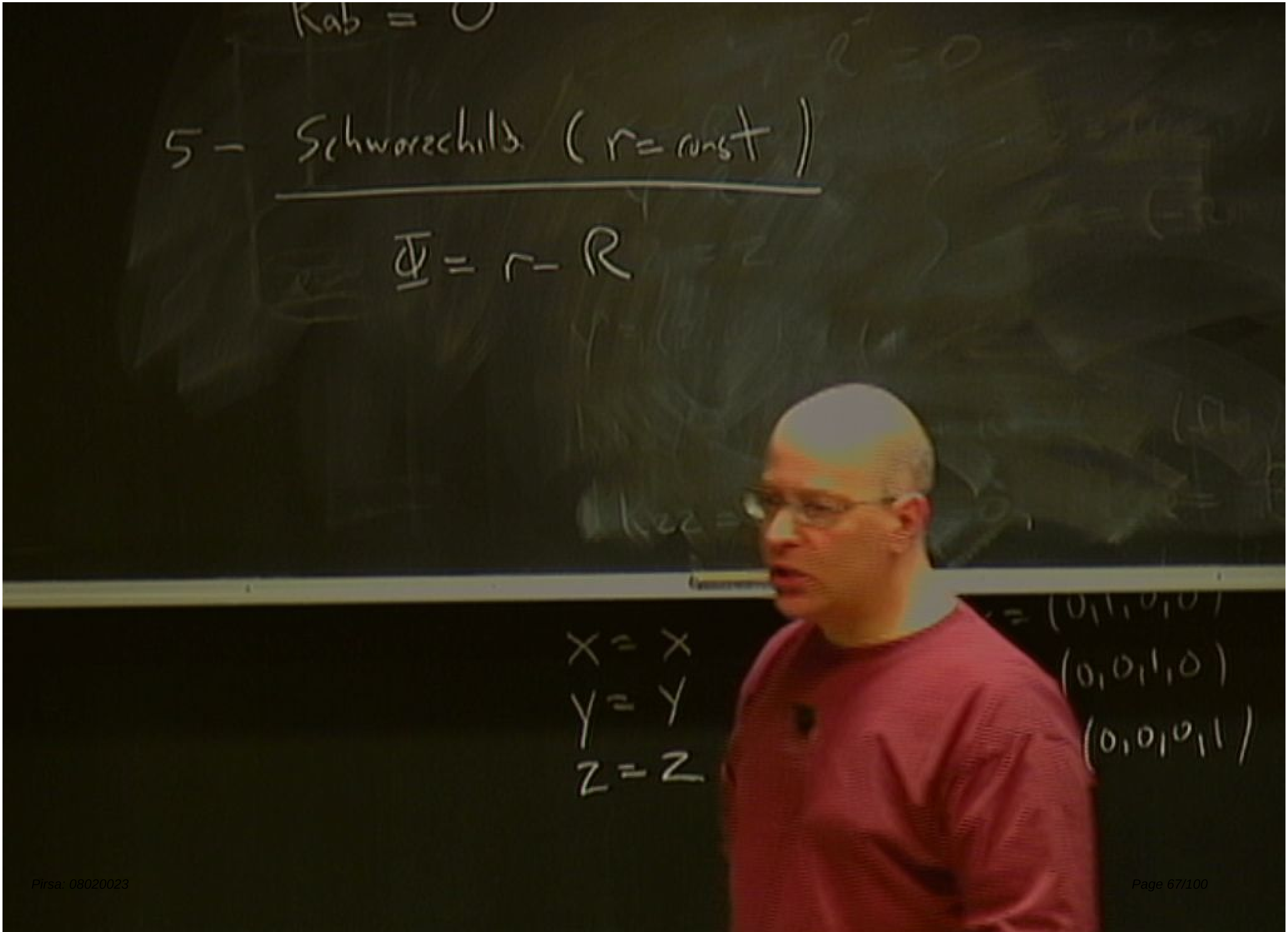
$$e^\alpha_\varphi = (0, 0, 0, 1)$$


$h_{ab}\, \partial y^a \partial y^b = f^{-1} \partial r^2 + r^2 \partial \Omega^2$

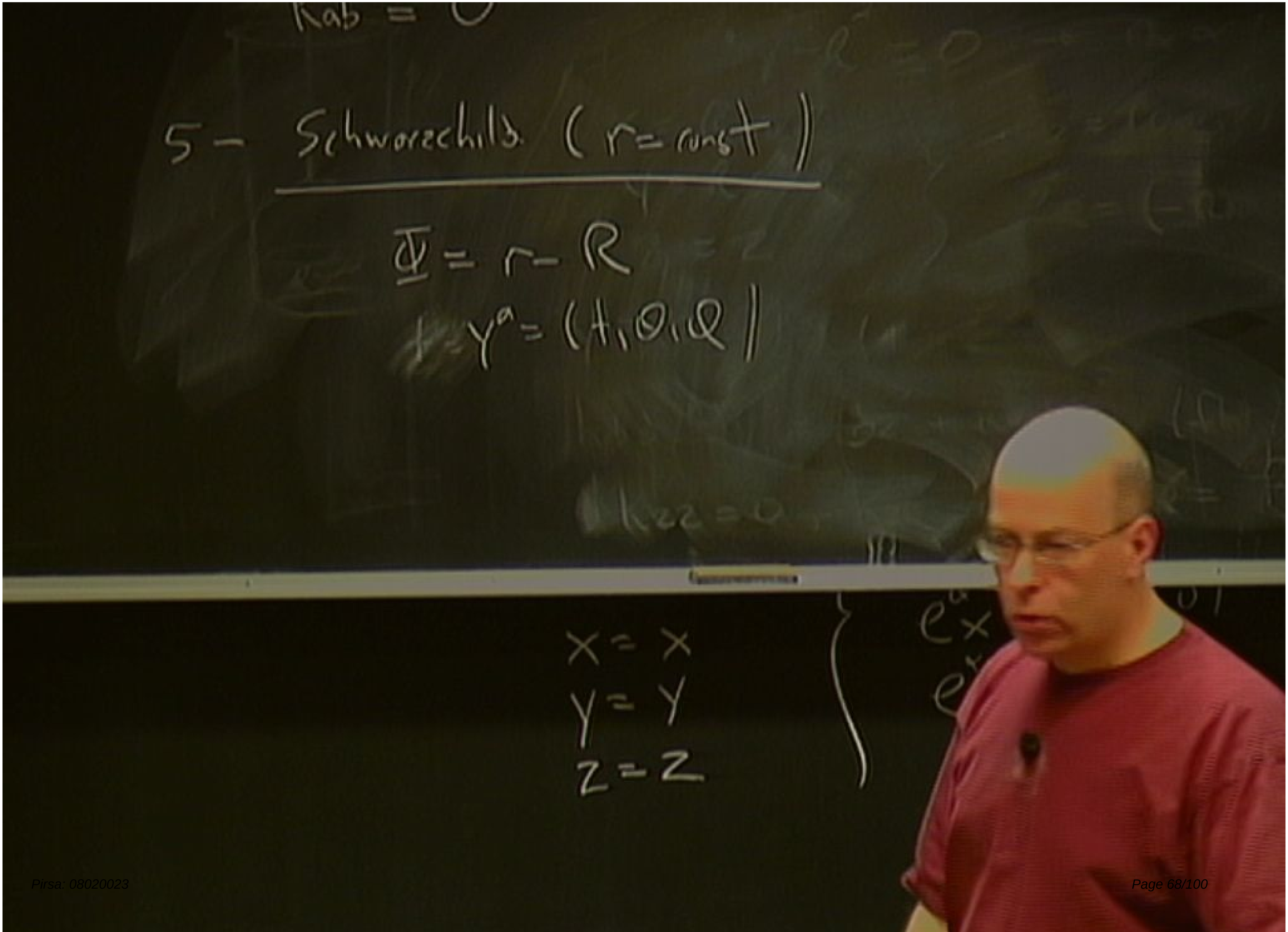

$h_{ab} \partial y^a \partial y^b = f^{-1} \partial r^2 + r^2 \partial \Omega^2$
$K_{ab} =$

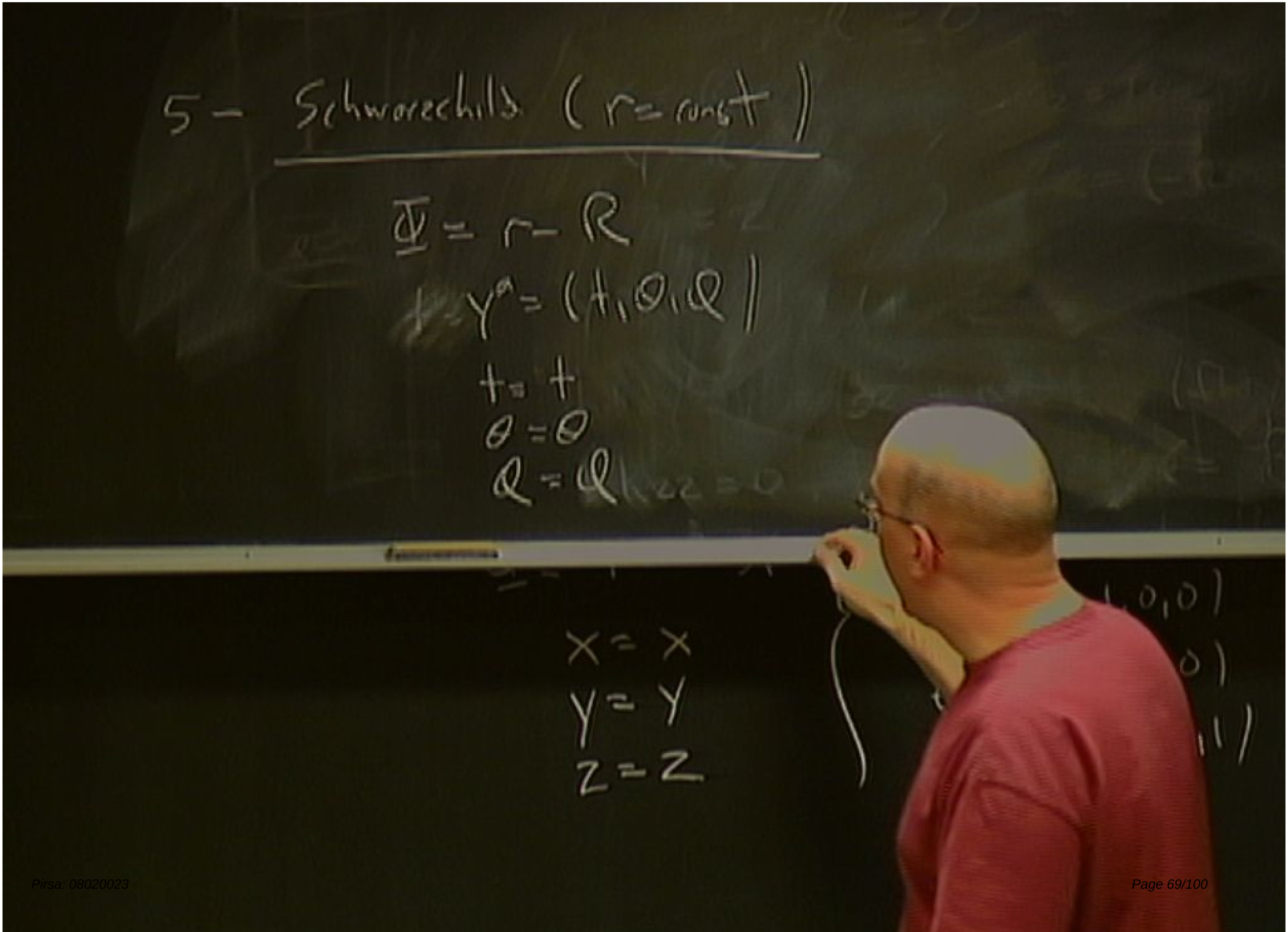

$h_{ab}\,\partial y^a \partial y^b = f^{-1}\,\partial r^2 + r^2\,\partial\Omega^2$
$K_{ab} = 0$

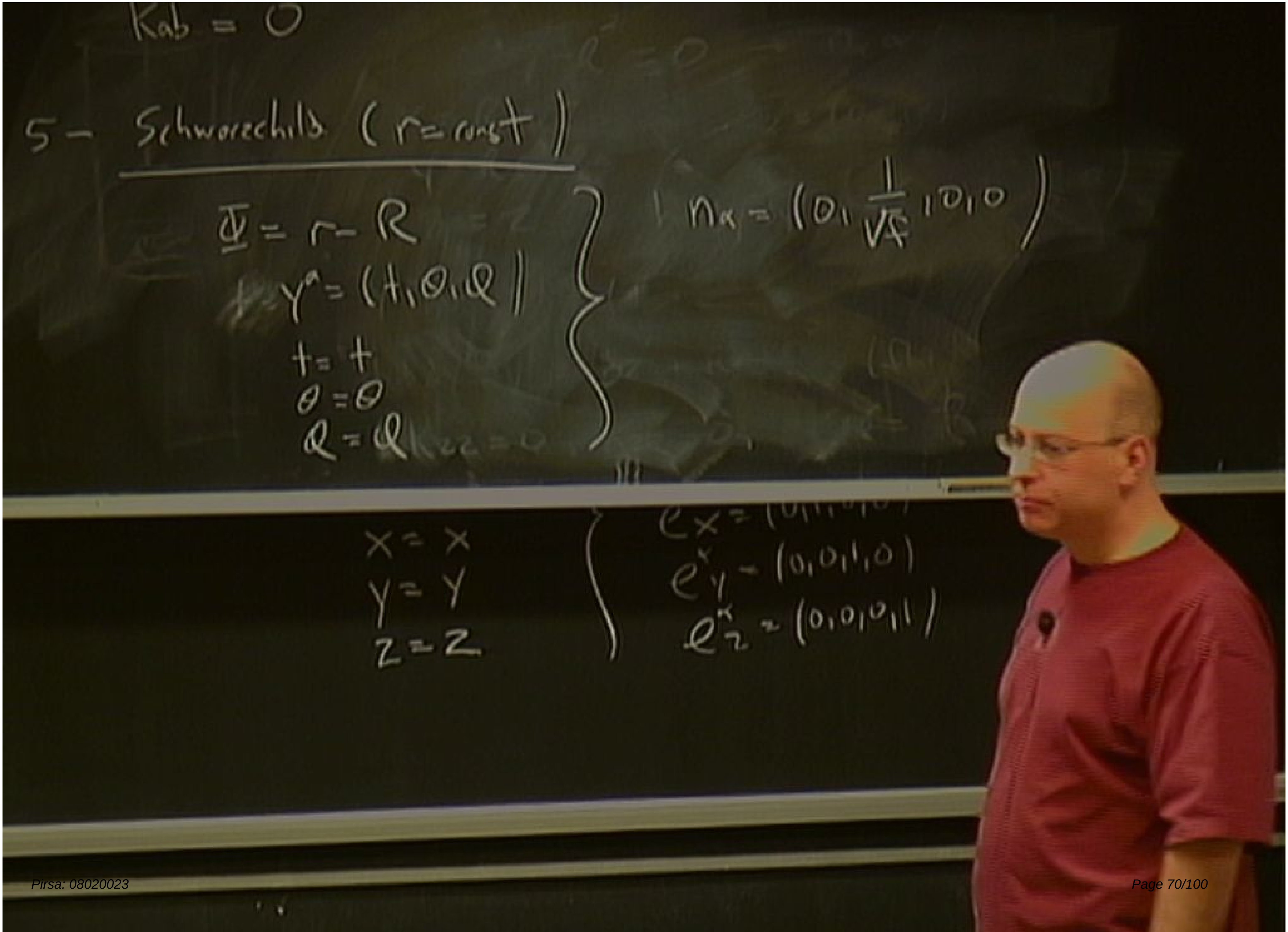

$h_{ab}\, \partial y^a \partial y^b = f^{-1} \partial r^2 + r^2 \partial \Omega^2$
$K_{ab} = 0$
5 - Schwarzchild (r = const)



Rab = 0
5 – Schwarzchild (r = const)
Φ = r – R
X = x
Y = y
Z = z
(0,1,0,0)
(0,0,1,0)
(0,0,0,1)


Rab = 0
5 − Schwarzchild (r = const)
Φ = r − R
y^a = (t, θ, φ)
x = X
y = Y
z = Z


5 – Schwarzschild (r = const)
Φ = r − R
Y^a = (t, θ, φ)
t = t
θ = θ
φ = φ
X = X
Y = Y
Z = Z


Kab = 0
5 - Schwarzschild (r = const)
Φ = r - R
y^a = (t, θ, φ)
t = t
θ = θ
φ = φ
n_α = (0, 1/√f, 0, 0)
x = X
y = Y
z = Z
e^α_y = (0, 0, 1, 0)
e^α_z = (0, 0, 0, 1)

Schwarzschild ($r = r_0$

$$\Phi = r - R$$

$$y^a = (t, \theta, \varphi)$$

$$t = t$$
$$\theta = \theta$$
$$\varphi = \varphi$$

$$n_\alpha = \left(0, \frac{1}{\sqrt{f}}, 0, 0\right)$$

$$e^\alpha_t = (1, 0, 0, 0)$$
$$e^\alpha_\theta = (0, 0, 1, 0)$$
$$e^\alpha_\varphi = (0, 0, 0, 1)$$

$$\Phi = r - R$$

$$y^a = (t, \theta, \varphi)$$

$$t = t$$

$$\theta = \theta$$

$$\varphi = \varphi$$

$$n_\alpha = \left(0, \frac{1}{\sqrt{f}}, 0, 0\right)$$

$$e^\alpha_t = (1, 0, 0, 0)$$

$$e^\alpha_\theta = (0, 0, 1, 0)$$

$$e^\alpha_\varphi = (0, 0, 0, 1)$$

$$h_{ab}\, dy^a dy^b = -f(R)\, dt^2 + R^2 \left(d\theta^2 + \sin^2\theta\, d\varphi^2\right.$$

$$t = t \qquad e^{\alpha}_{t} = (1,0,0,0)$$
$$\theta = \theta \qquad e^{\alpha}_{\theta} = (0,0,1,0)$$
$$\varphi = \varphi \qquad e^{\alpha}_{\varphi} = (0,0,0,1)$$

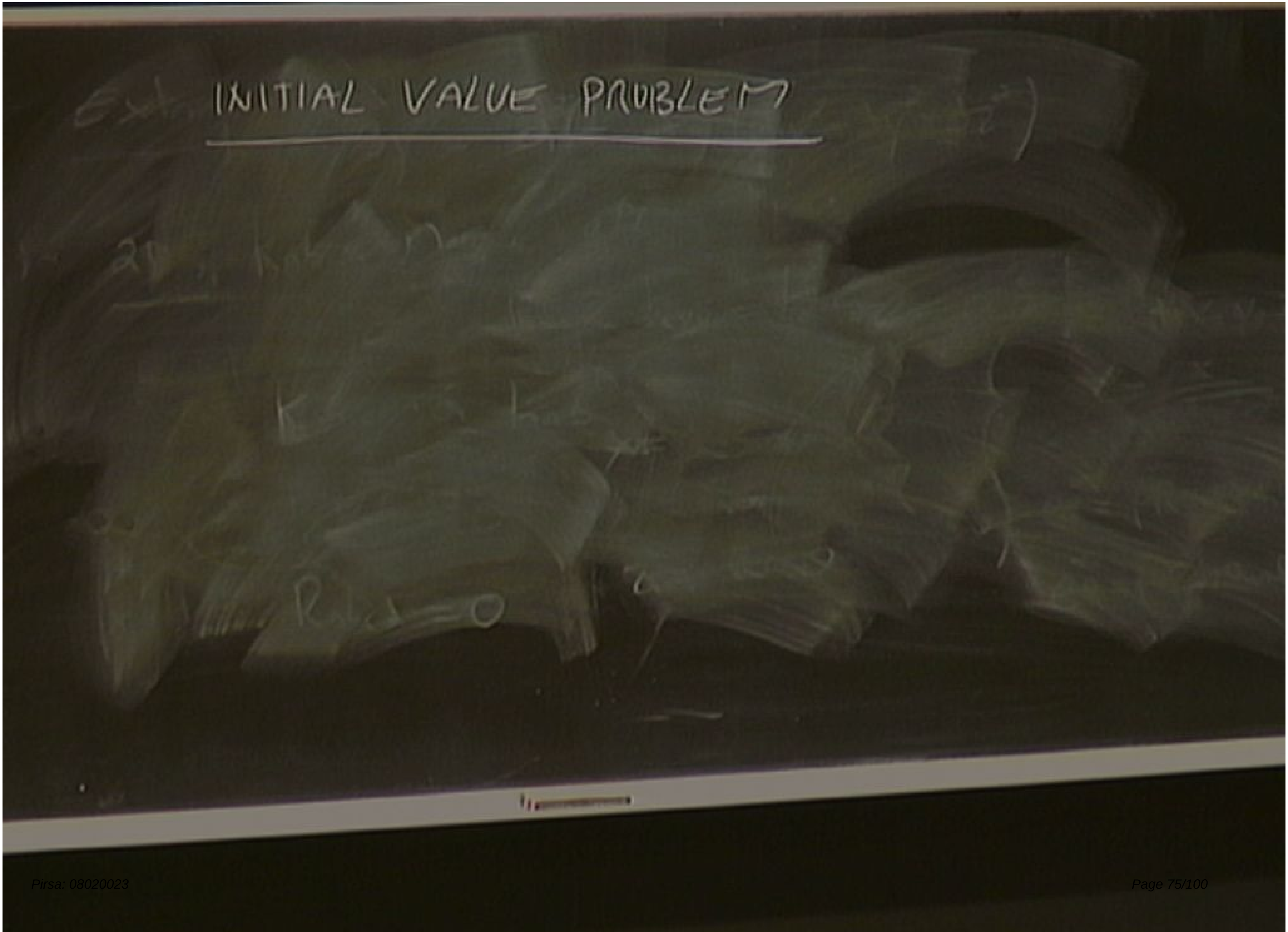
$$h_{ab}\, dy^a dy^b = -f(R)\, dt^2 + R^2 (d\theta^2 + \sin^2\theta\, d\varphi^2)$$

$$h_{ab}\, dy^a dy^b = -f(R)\, dt^2 + R^2 \left(d\theta^2 + \sin^2\theta\, d\varphi^2 \right)$$

$$K_{tt} = -\frac{1}{2}\sqrt{f(R)}\, f'(R)$$

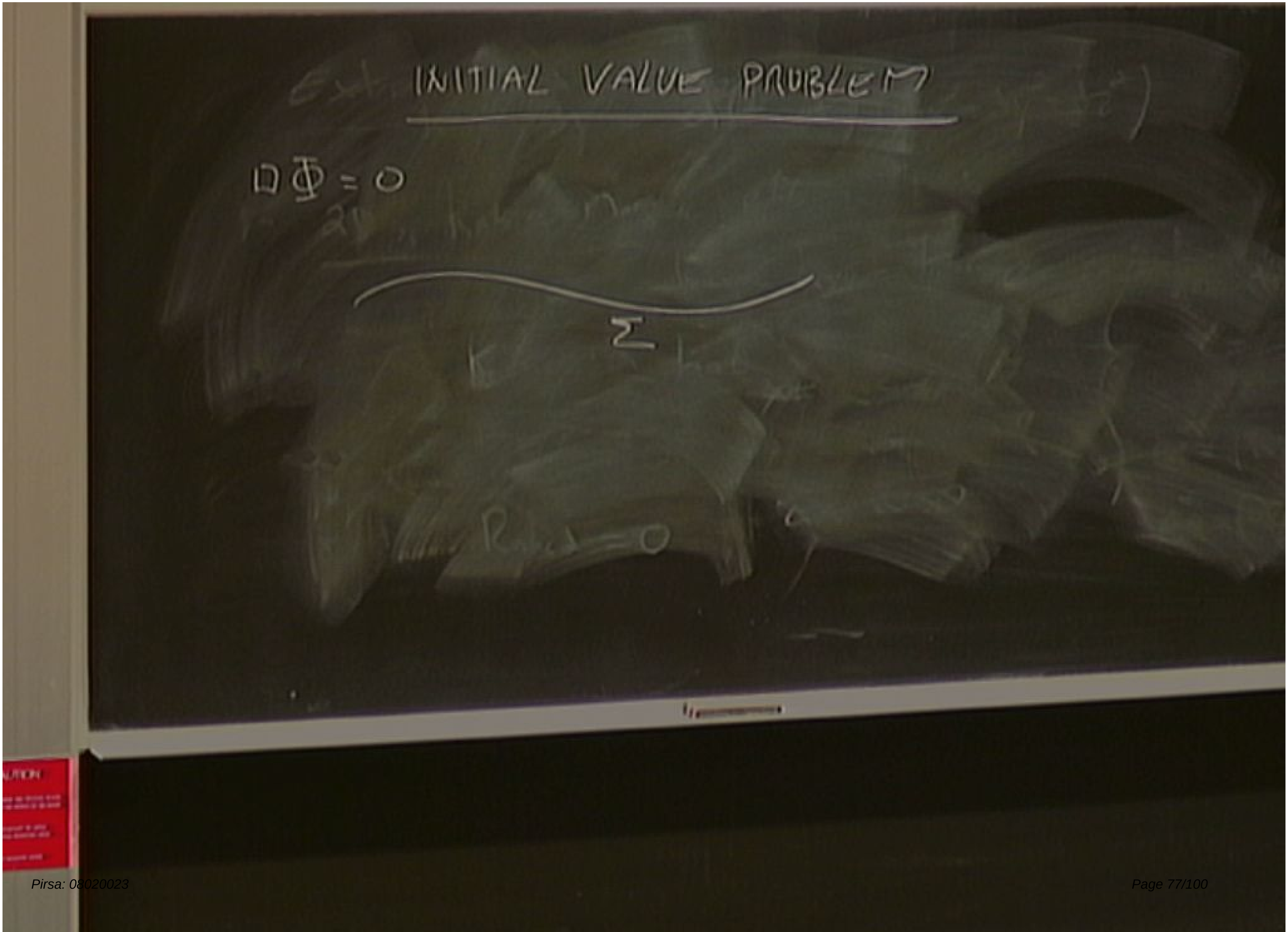
$$K_{\theta\theta} = R\sqrt{f(R)}$$

$$K_{\varphi\varphi} = R\sqrt{f(R)}\, \sin^2\theta$$


INITIAL VALUE PROBLEM

INITIAL VALUE PROBLEM

$\Box \Phi = 0$


INITIAL VALUE PROBLEM
□Φ = 0
Σ

INITIAL VALUE PROBLEM

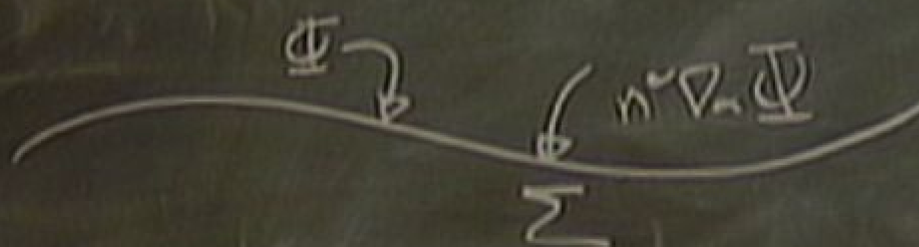
$\Box \Phi = 0$

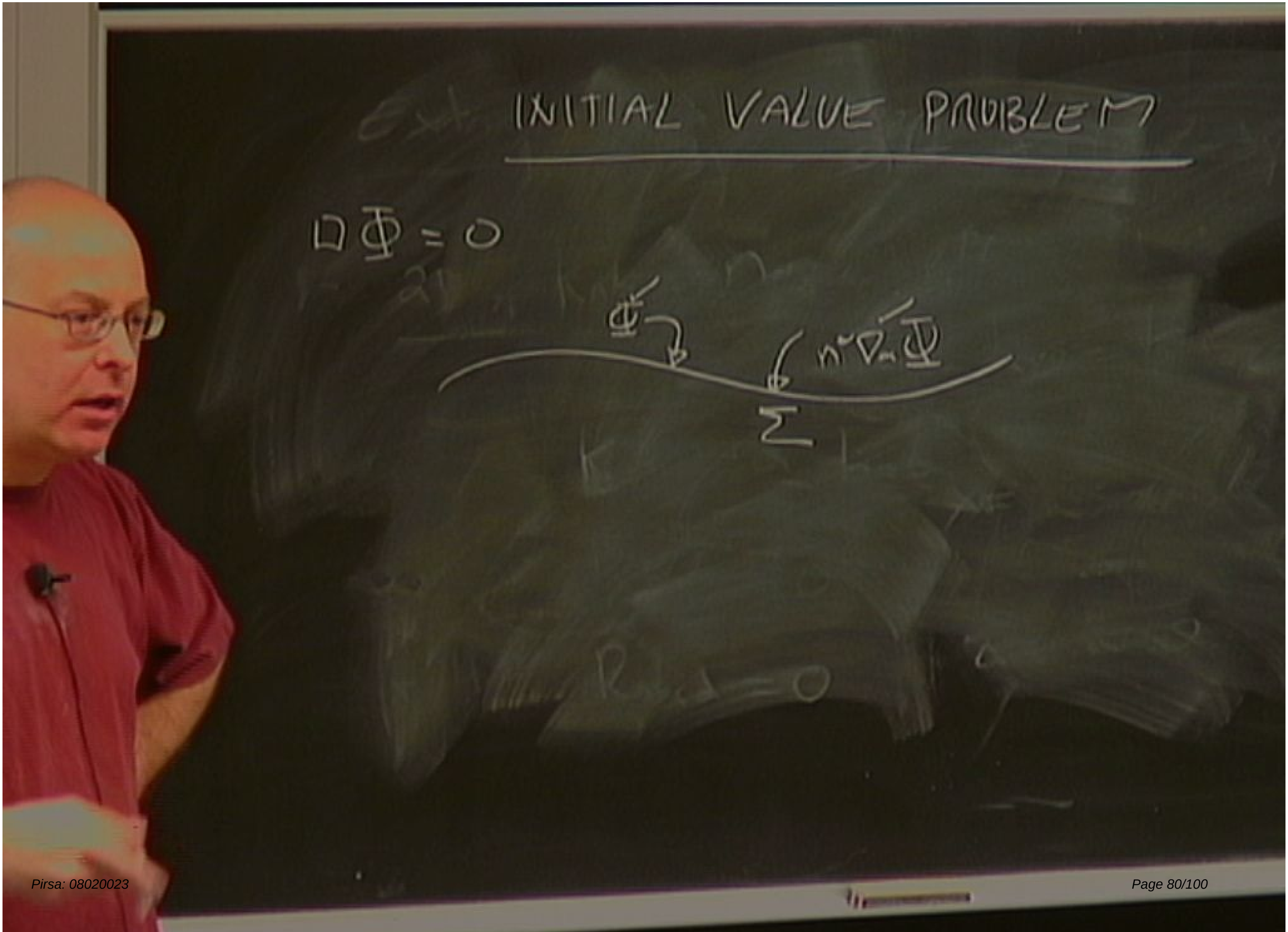
Φ

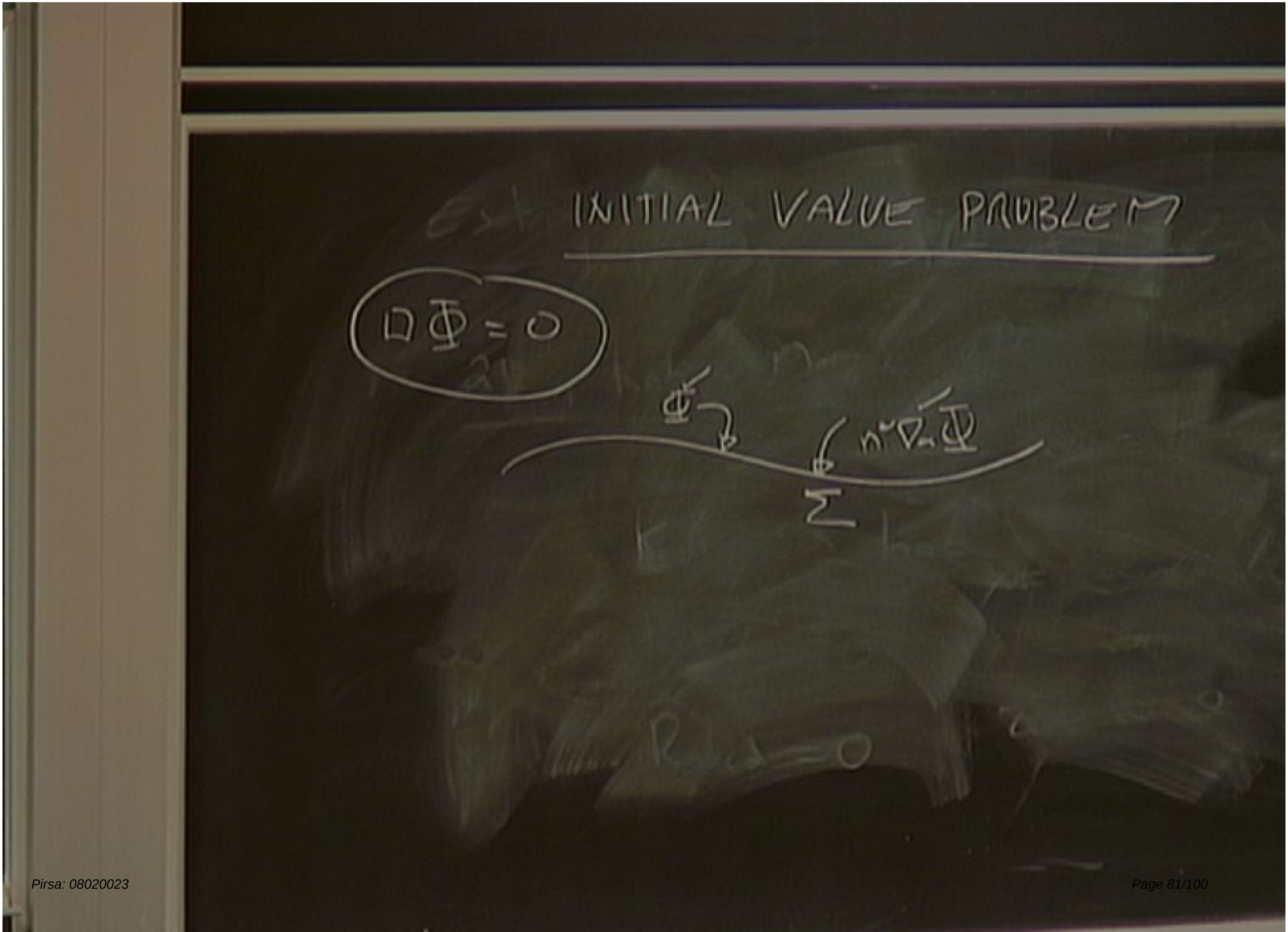
Σ

INITIAL VALUE PROBLEM

$$\Box \Phi = 0$$

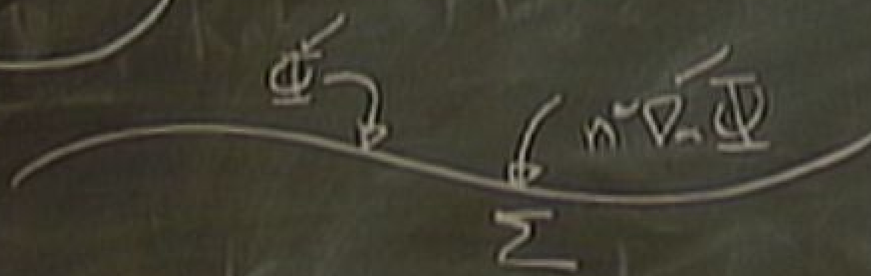


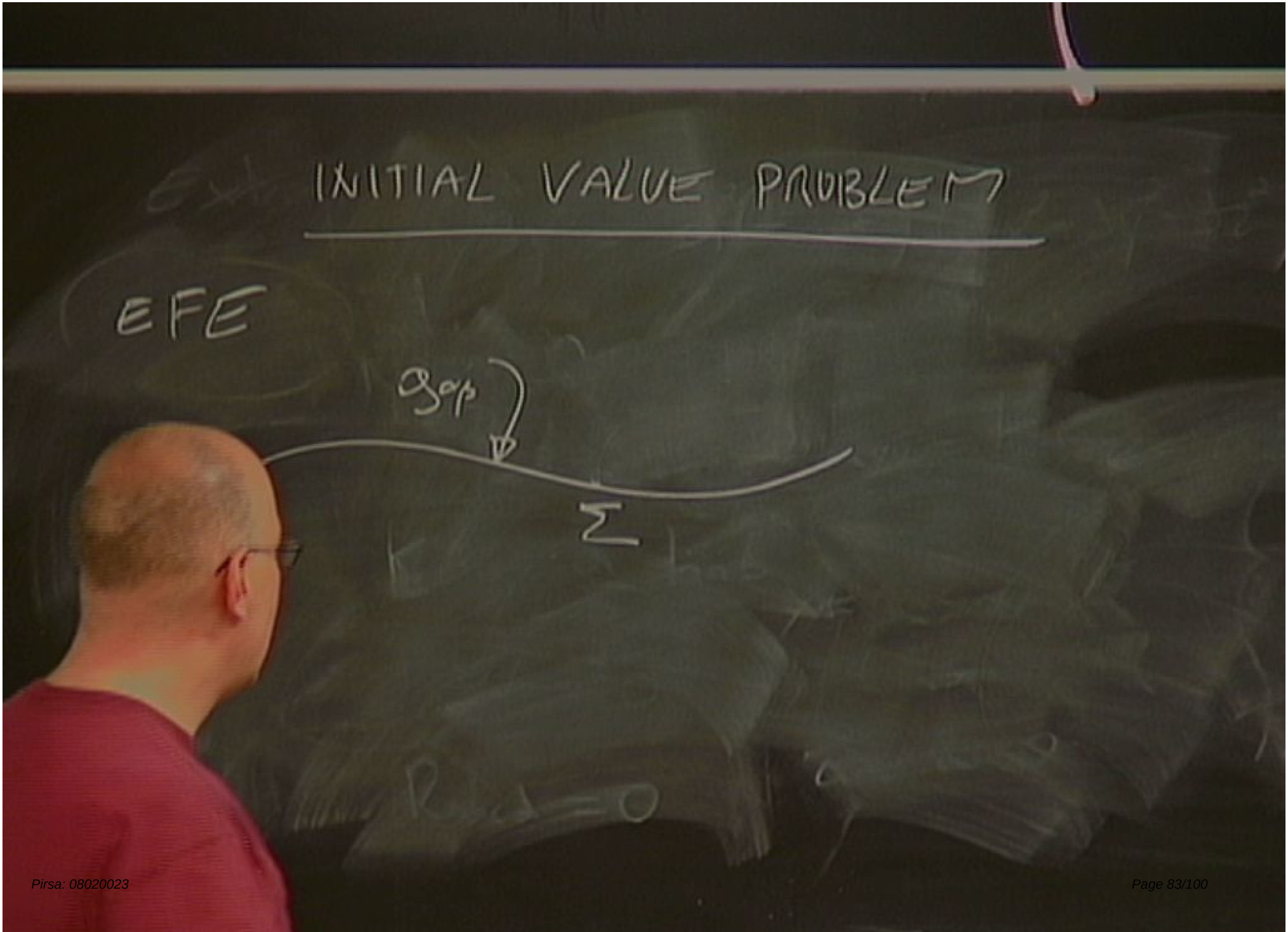

INITIAL VALUE PROBLEM
$\Box\Phi = 0$
Φ
$n^\mu\nabla_\mu\Phi$
Σ

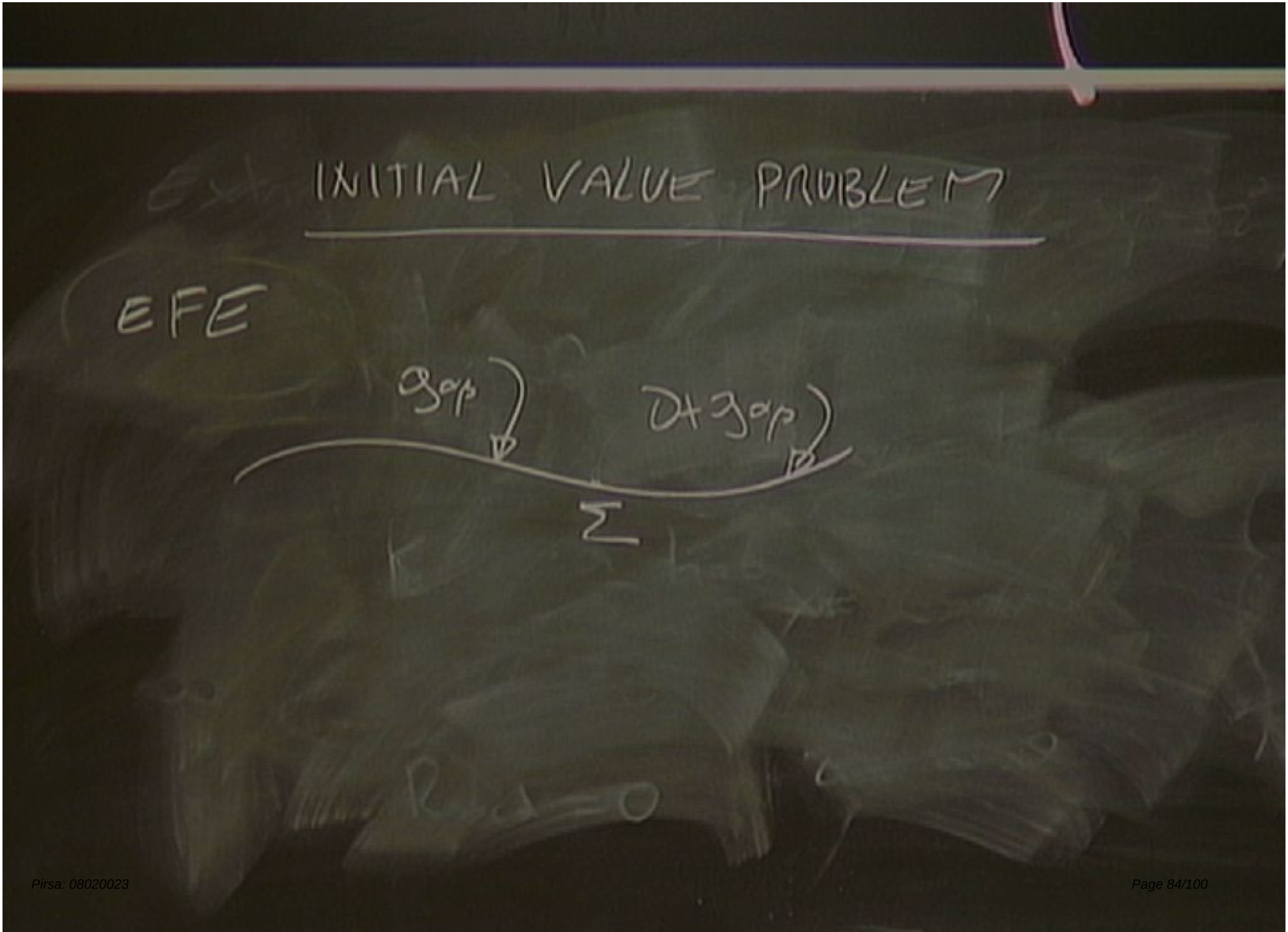

INITIAL VALUE PROBLEM
$\Box \Phi = 0$
Φ
$n^a \nabla_a \Phi$
Σ

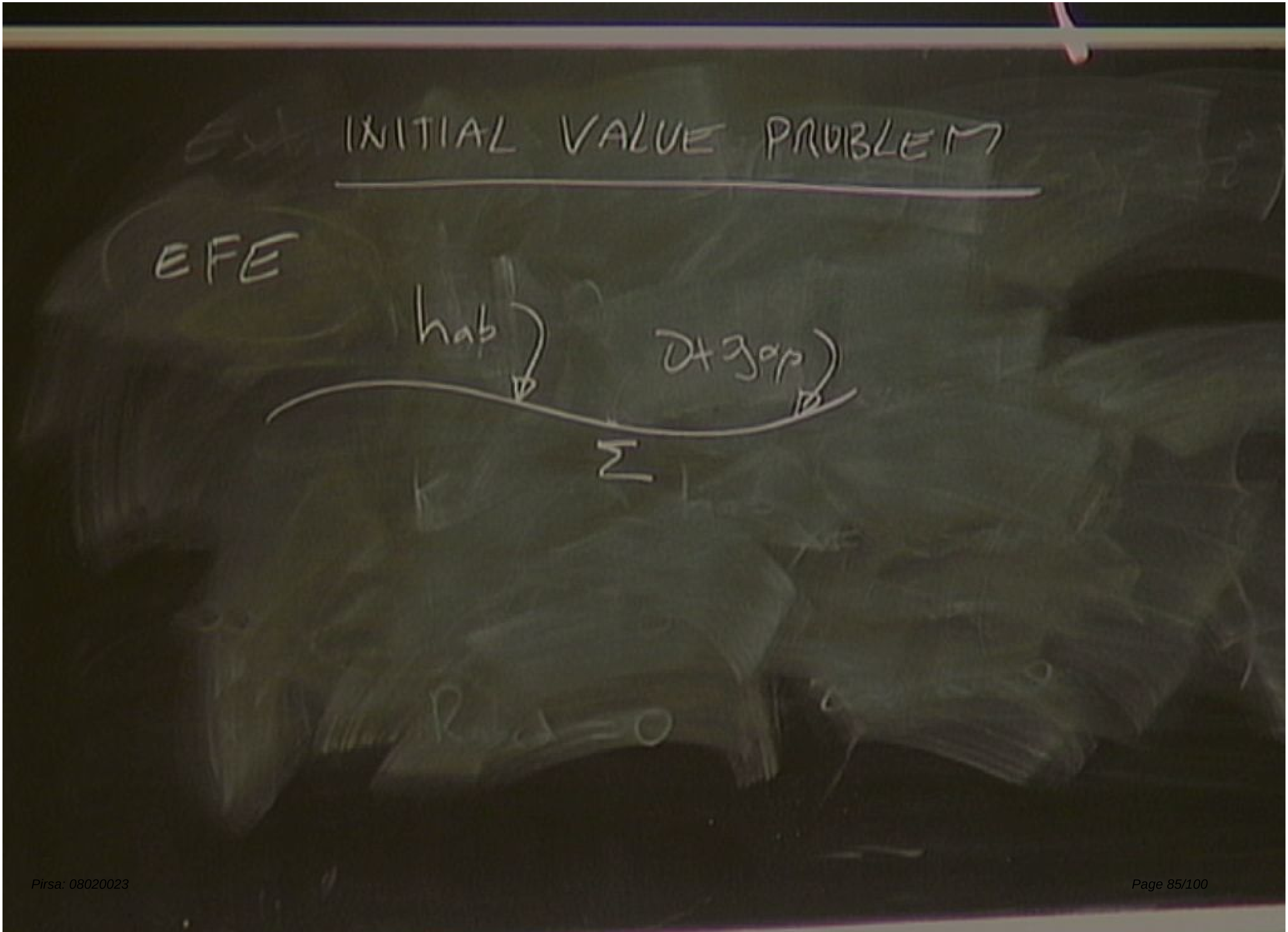
INITIAL VALUE PROBLEM

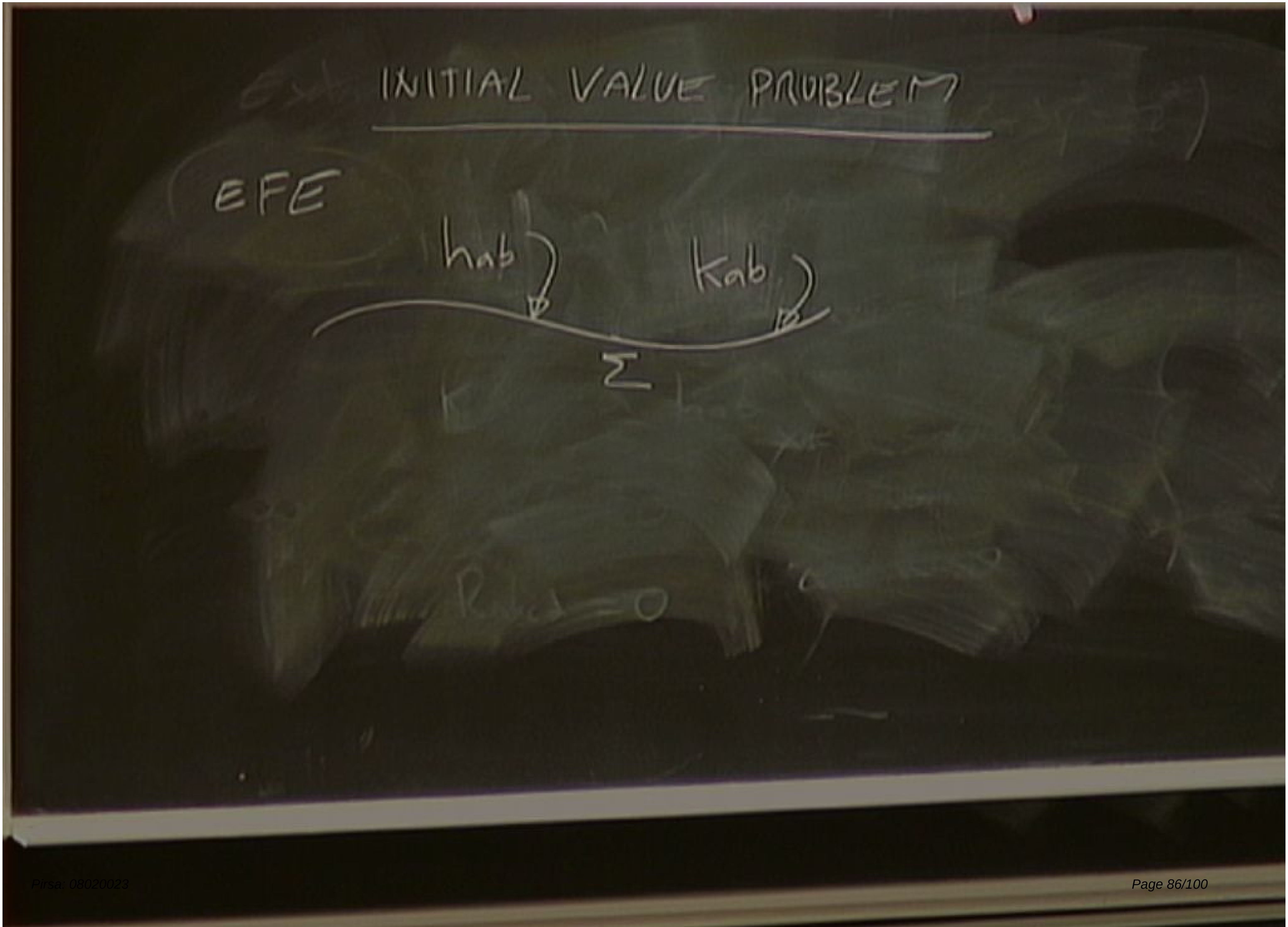
$$\Box \Phi = 0$$




INITIAL VALUE PROBLEM
EFE
$g_{\alpha\beta}$
Σ

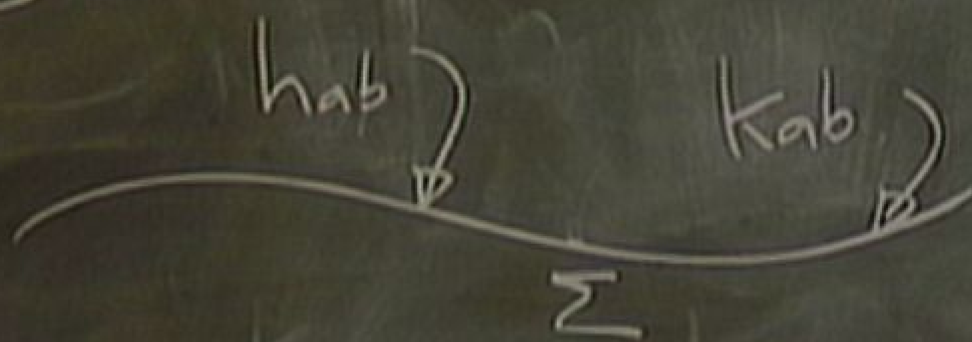

INITIAL VALUE PROBLEM
EFE
$g_{\alpha\beta}$
$\partial_t g_{\alpha\beta}$
Σ


INITIAL VALUE PROBLEM
EFE
hab
∂t gab
Σ

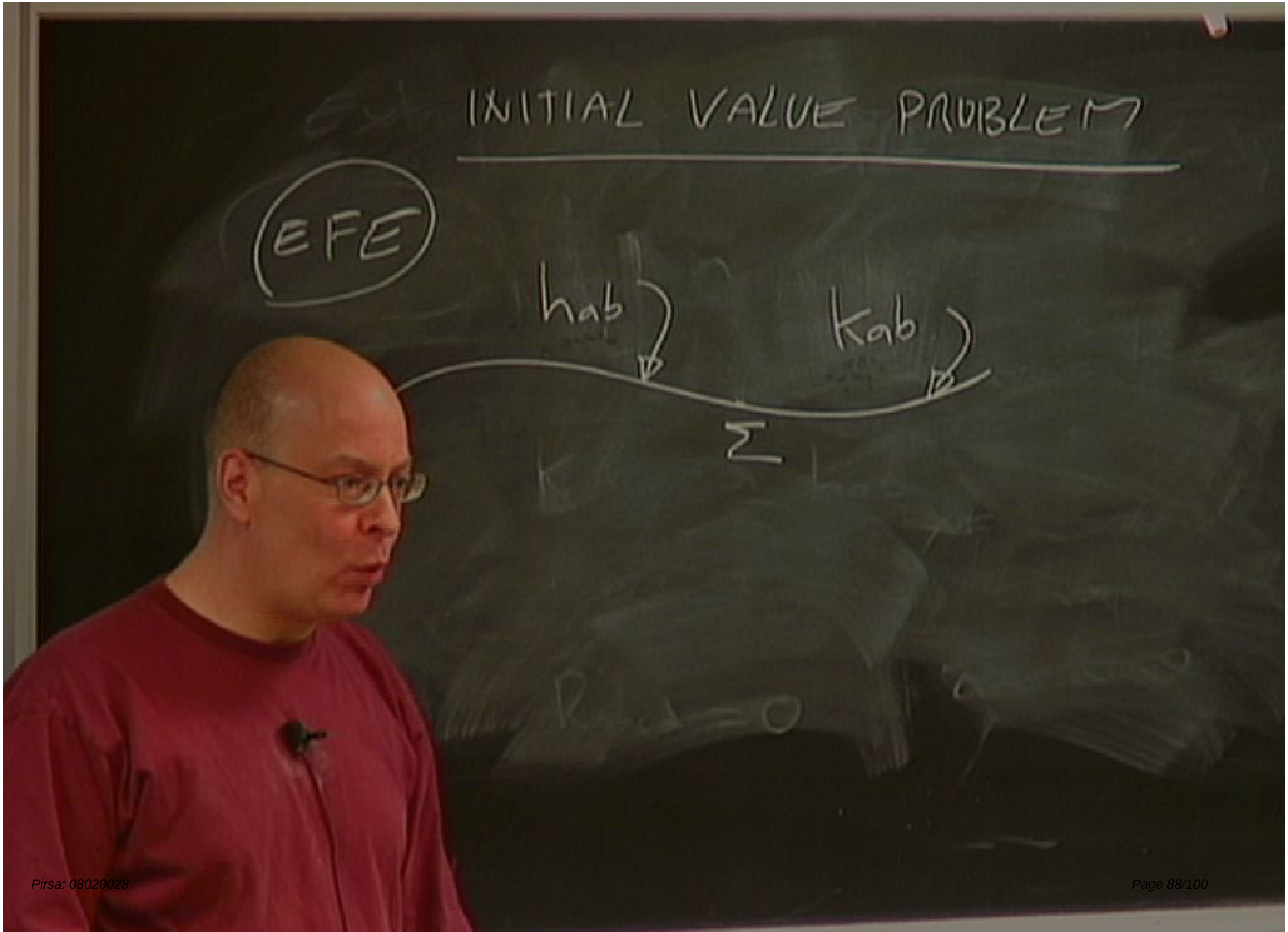

INITIAL VALUE PROBLEM
EFE
h_{ab}
K_{ab}
Σ

INITIAL VALUE PROBLEM

EFE



$R_{ab} = 0$


INITIAL VALUE PROBLEM
EFE
hab
kab
Σ

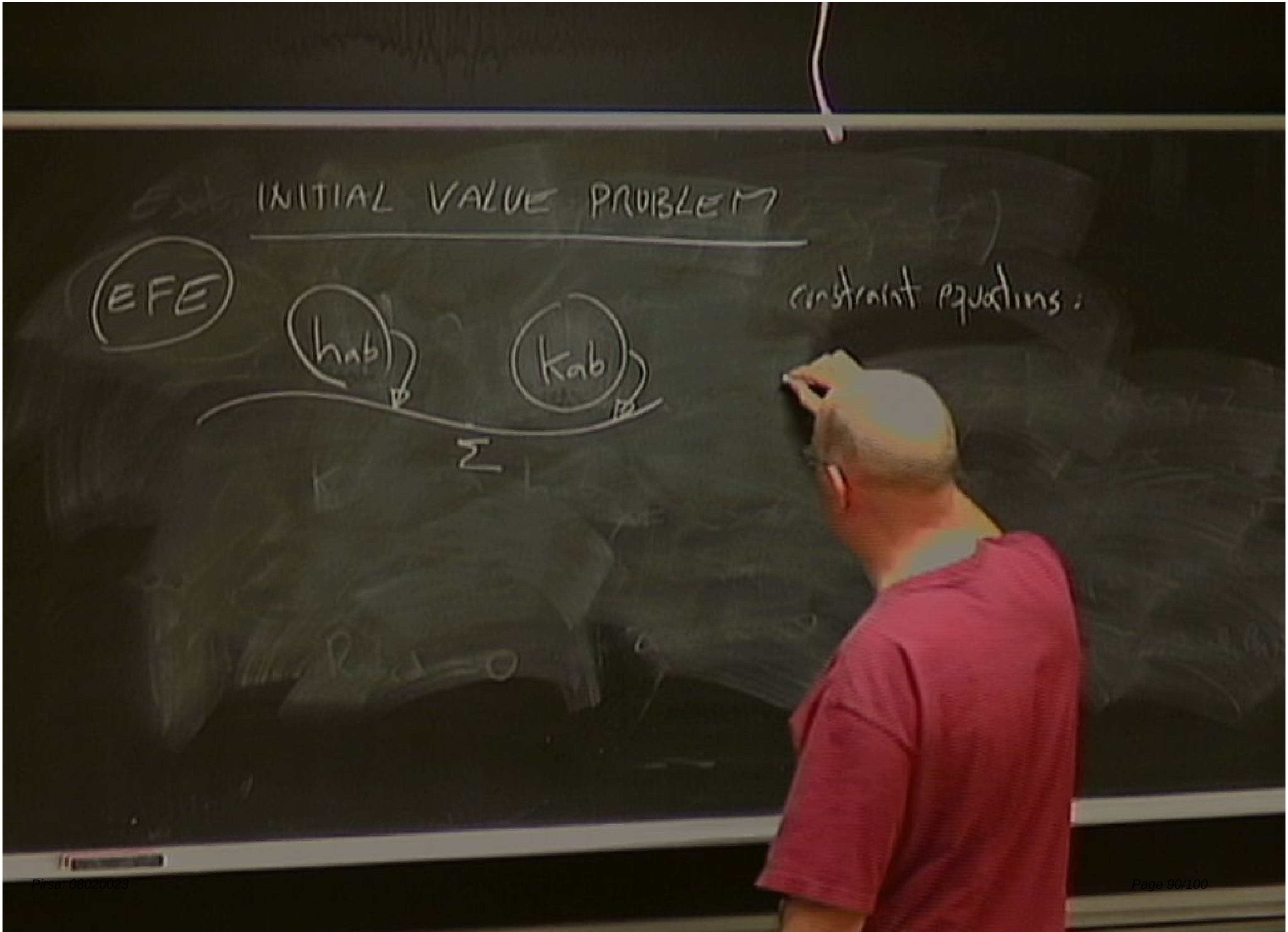
$$K_{ab} = n_{\alpha;\beta}\, e^{\alpha}_{a} e^{\beta}_{b} = \frac{1}{2}(\mathcal{L}_n g_{\alpha\beta})\, e^{\alpha}_{a} e^{\beta}_{b}$$

$${}^4R_{\alpha\beta\gamma\delta}\, e^{\alpha}_{a} e^{\beta}_{b} e^{\gamma}_{c} e^{\delta}_{d} = R_{abcd} + K_{ac}K_{bd} - K_{ad}K$$

$${}^4R_{\mu\alpha\beta\gamma}\, n^{\mu} e^{\alpha}_{a} e^{\beta}_{b} e^{\gamma}_{c} = D_c K_{ab} - D_b K_{ac}$$

$${}^4G_{\mu\nu}\, n^{\mu} n^{\nu} = \frac{1}{2}\left(R - K^{ab}K_{ab} + K^2\right)$$

$${}^4G_{\mu\alpha}\, n^{\mu} e^{\alpha}_{a} = D_b K^{b}_{a} - D_a K$$


INITIAL VALUE PROBLEM
EFE
hab
Kab
Σ
constraint equations:

INITIAL VALUE PROBLEM

EFE

h_{ab} $\quad$ K_{ab}

Σ

constraint equations:

$$R - K^{ab}K_{ab} + K^2$$

INITIAL VALUE PROBLEM

EFE

h_{ab} K_{ab}

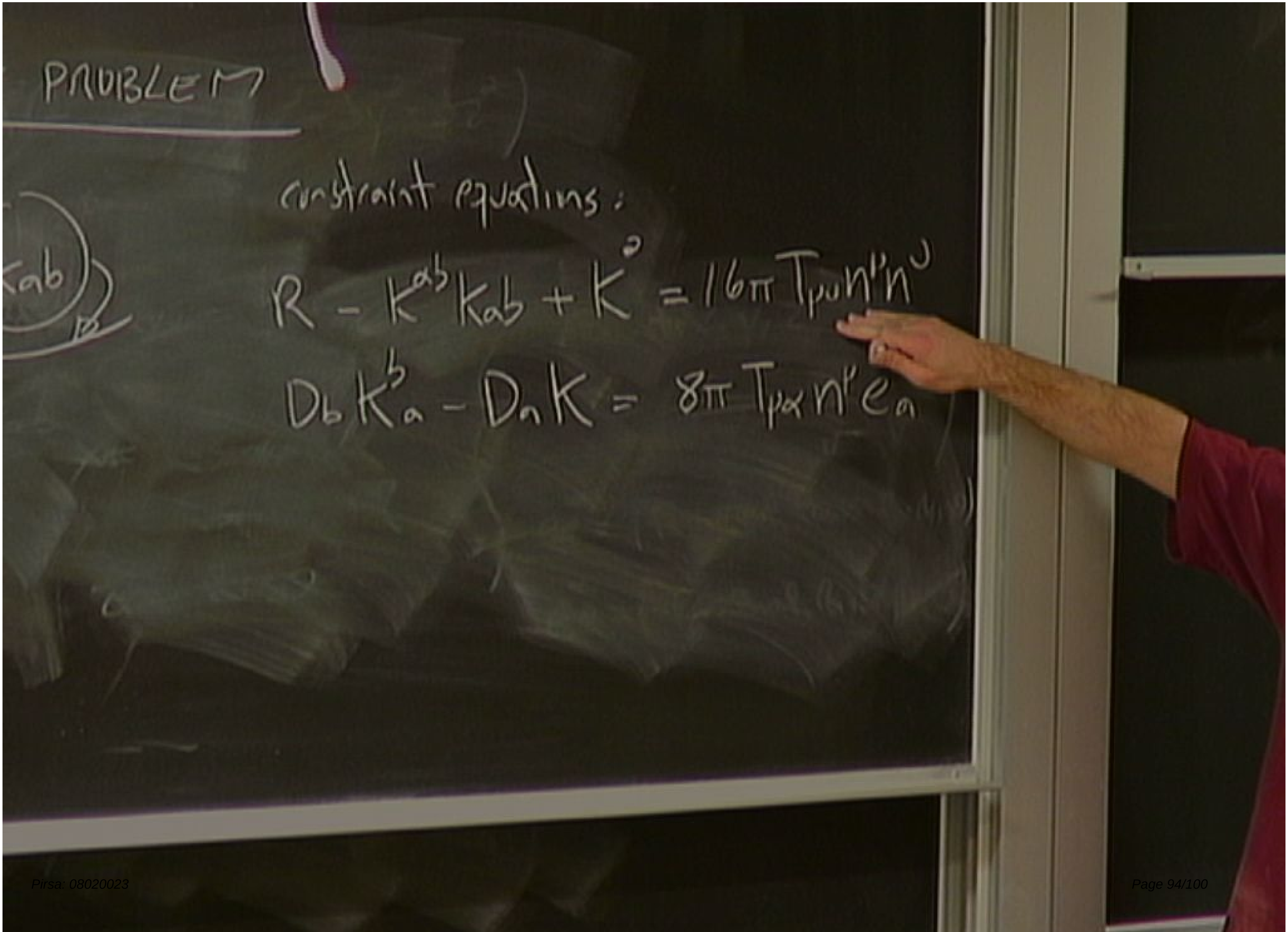
Σ

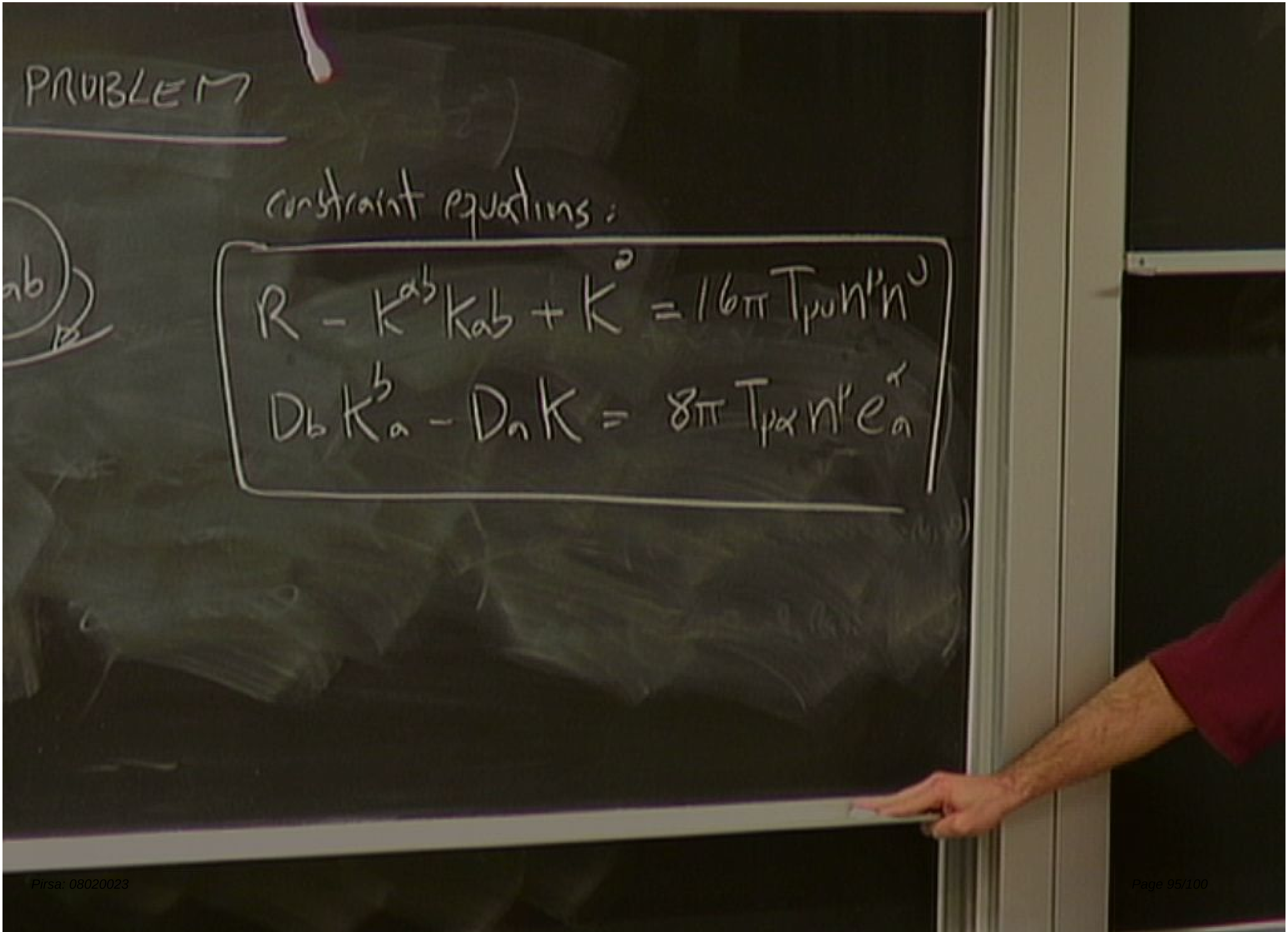
constraint equations:

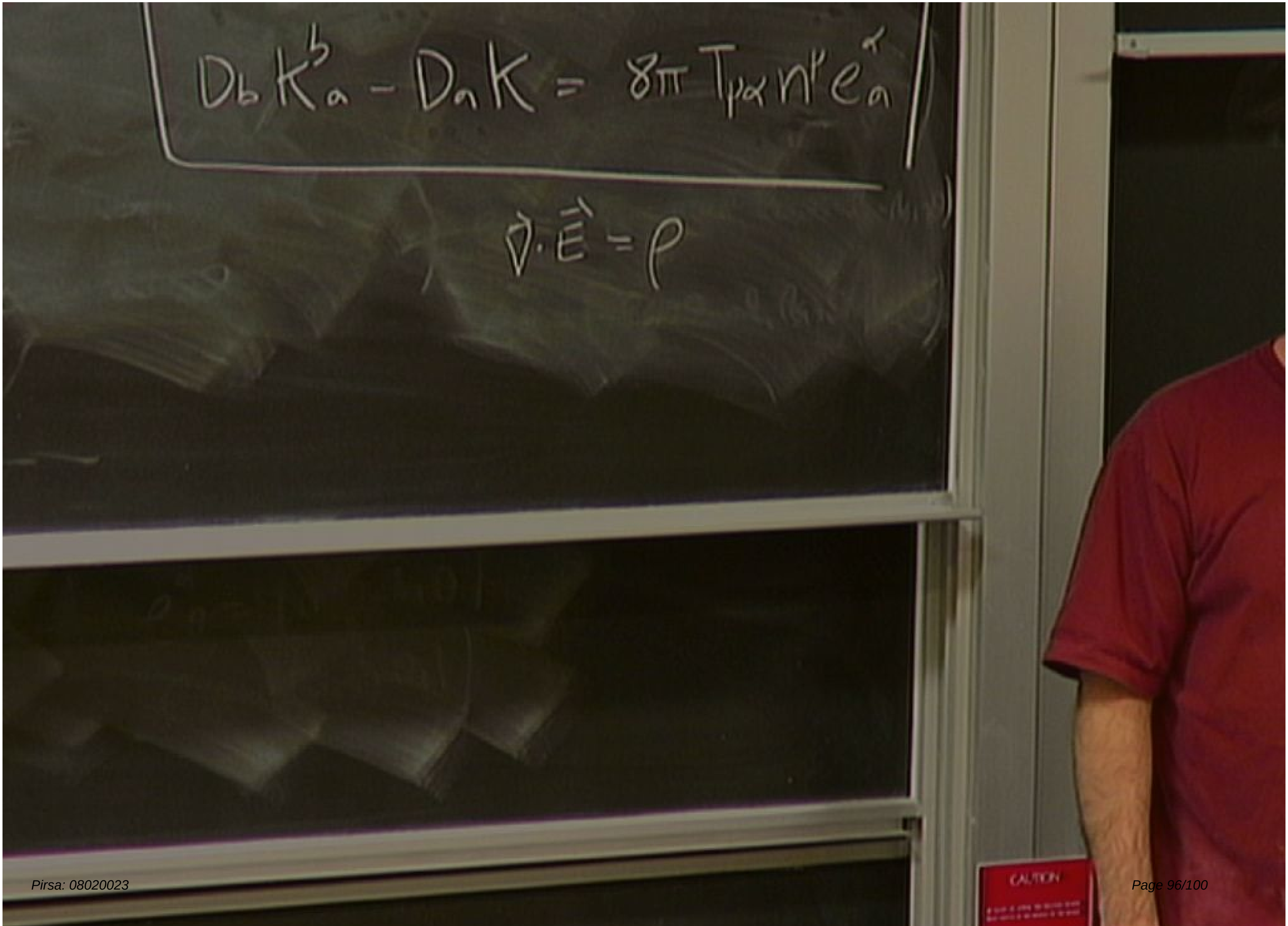
$$R - K^{ab}K_{ab} + K^2 = 16\pi T_{\mu\nu} n^\mu n^\nu$$

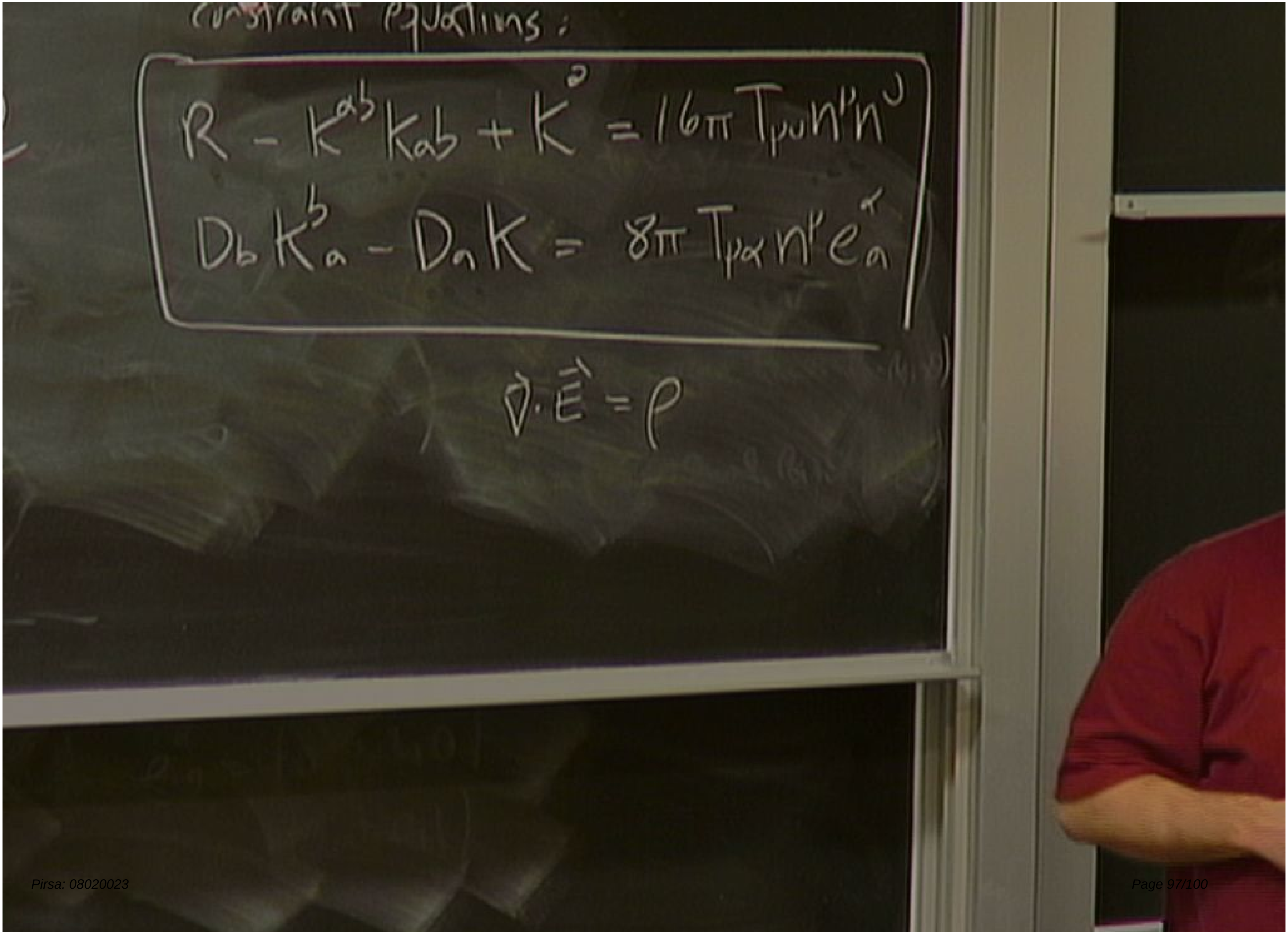
constraint equations:

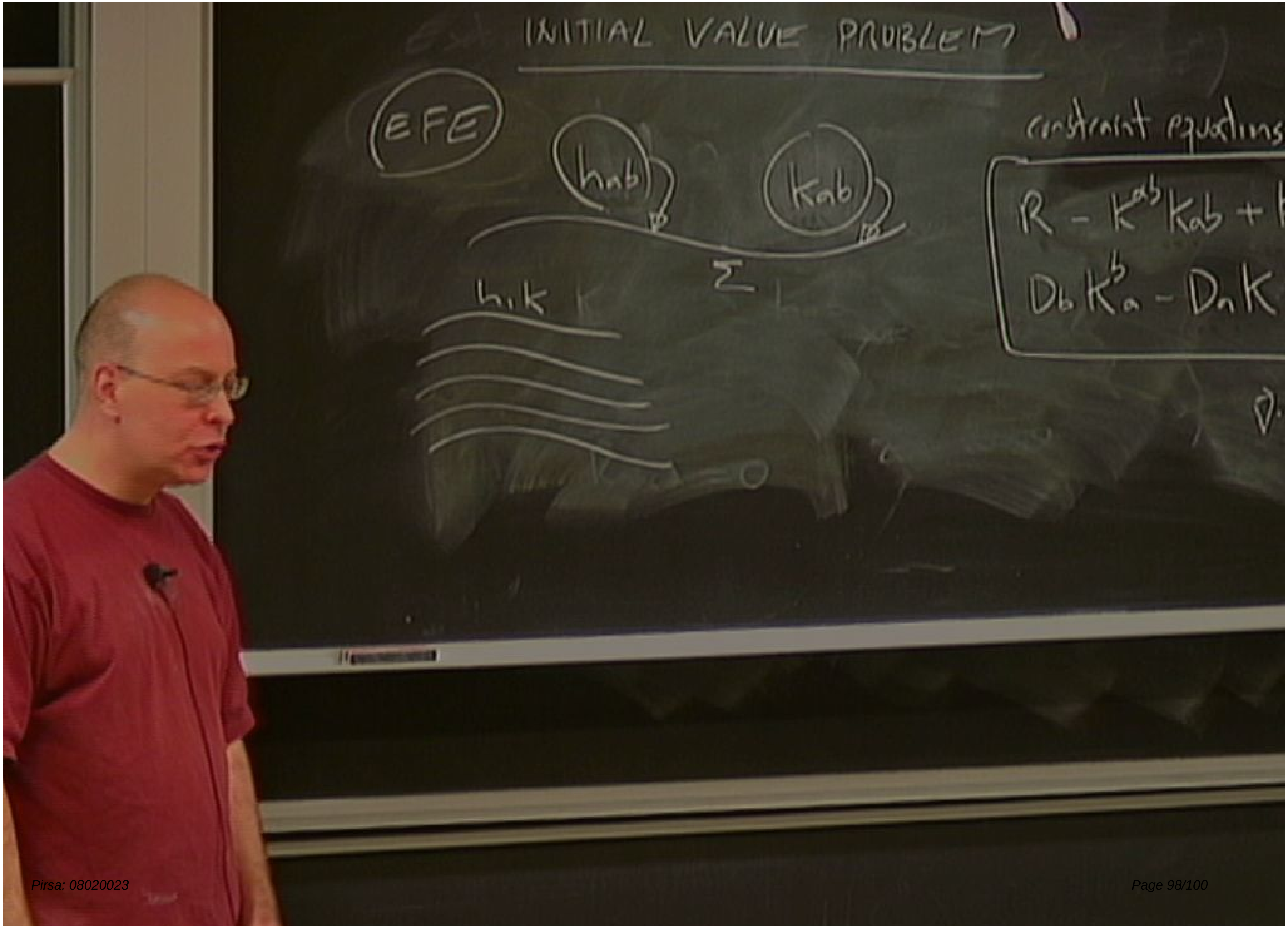
$$R - K^{ab}K_{ab} + K^2 = 16\pi T_{\mu\nu} n^\mu n^\nu$$

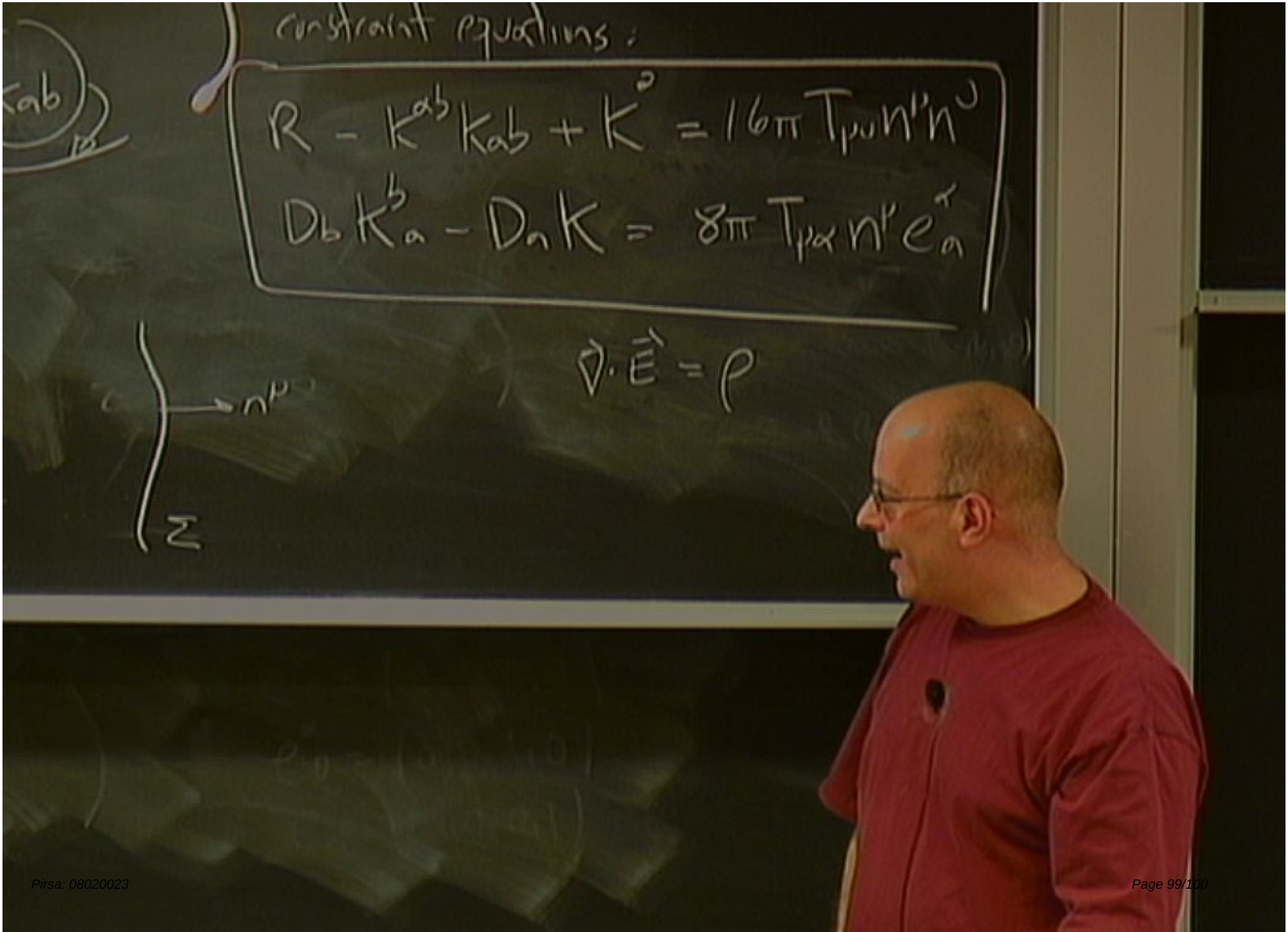

PROBLEM
constraint equations:
$R - K^{ab}K_{ab} + K^2 = 16\pi T_{\mu\nu} n^\mu n^\nu$
$D_b K^b{}_a - D_a K = 8\pi T_{\mu\alpha} n^\mu e_a$
K_{ab}

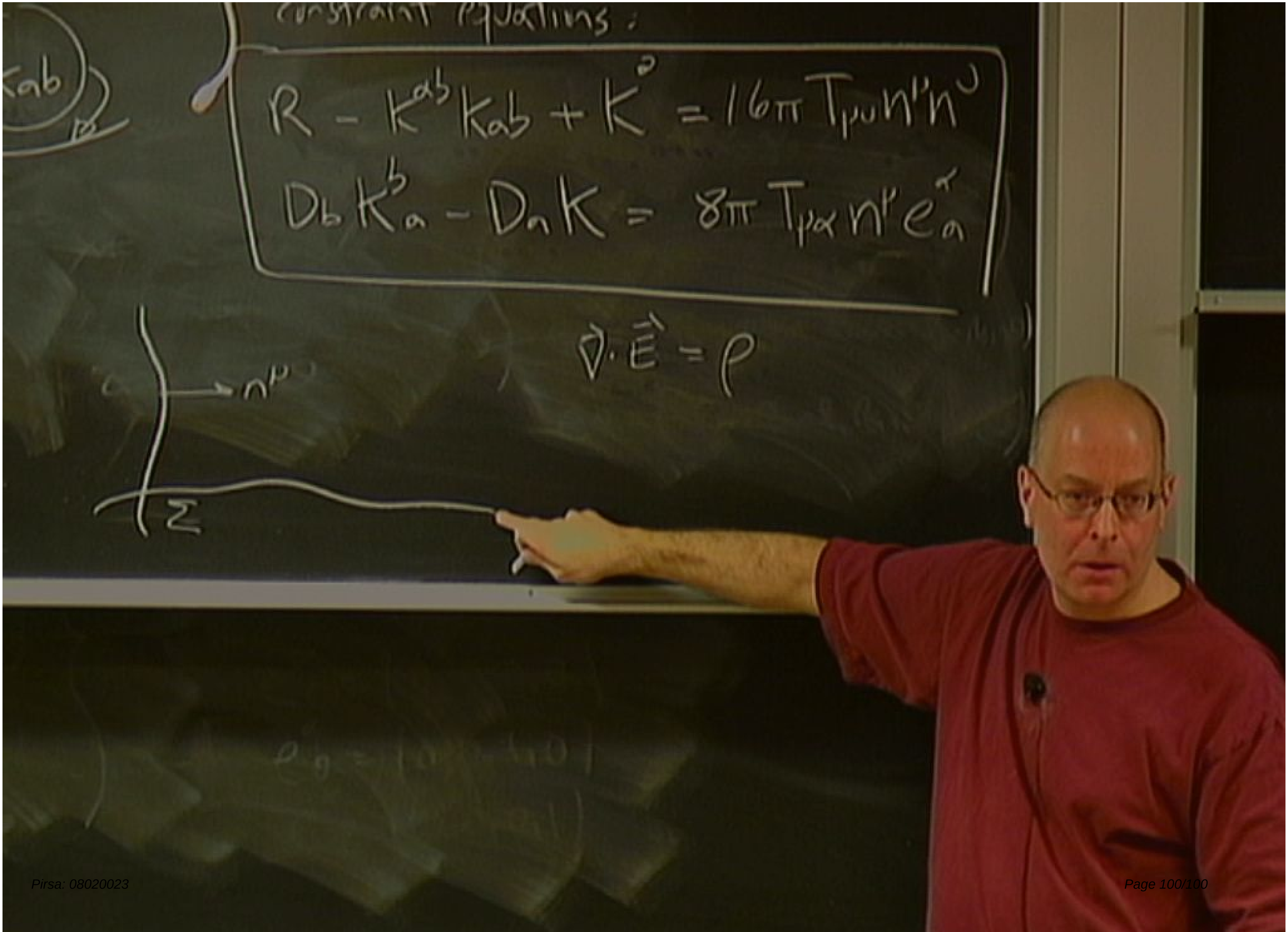

PROBLEM
constraint equations:
$R - K^{ab}K_{ab} + K^2 = 16\pi T_{\mu\nu} n^\mu n^\nu$
$D_b K^b_a - D_a K = 8\pi T_{\mu\alpha} n^\mu e^\alpha_a$


$D_b K^b{}_a - D_a K = 8\pi T_{\mu\alpha} n^\mu e^\alpha_a$
$\vec{\nabla} \cdot \vec{E} = \rho$
CAUTION


Constraint equations:
$R - K^{ab}K_{ab} + K^2 = 16\pi T_{\mu\nu} n^\mu n^\nu$
$D_b K^b{}_a - D_a K = 8\pi T_{\mu\alpha} n^\mu e^\alpha_a$
$\vec{\nabla}\cdot\vec{E} = \rho$


INITIAL VALUE PROBLEM
EFE
hab
Kab
Σ
h, k
constraint equations
R - K^ab Kab +
Db K^b_a - Da K


constraint equations:
$R - K^{ab}K_{ab} + K^2 = 16\pi T_{\mu\nu} n^\mu n^\nu$
$D_b K^b_a - D_a K = 8\pi T_{\mu\alpha} n^\mu e^\alpha_a$
$\nabla \cdot \vec{E} = \rho$
n^μ
Σ
K_{ab}


Constraint Equations:
$R - K^{ab}K_{ab} + K^2 = 16\pi T_{\mu\nu}n^\mu n^\nu$
$D_b K^b_a - D_a K = 8\pi T_{\mu\alpha} n^\mu e^\alpha_a$
$\nabla \cdot \vec{E} = \rho$
n^μ
Σ
K_{ab}