

Title: Advanced General Relativity - Lecture 7A

Date: Feb 27, 2008 10:30 AM

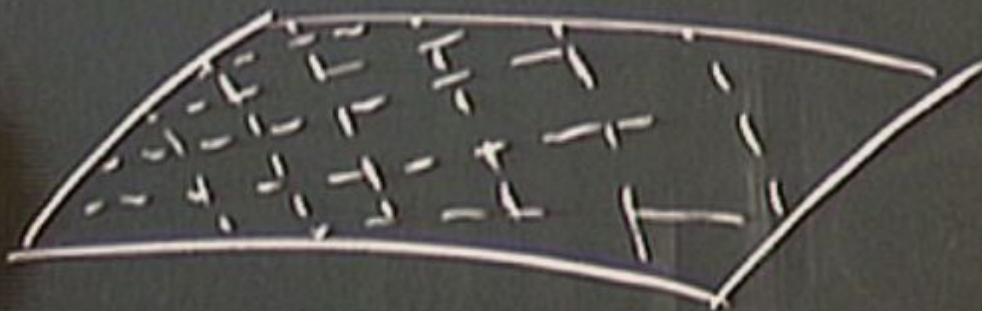
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Abstract: Advanced General Relativity

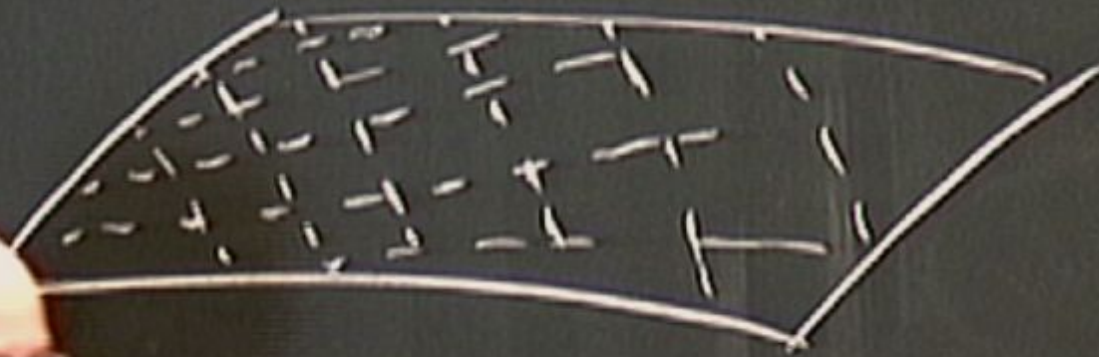
INTRINSIC VS EXTRINSIC CURVATURE



INTRINSIC VS EXTRINSIC CURVATURE



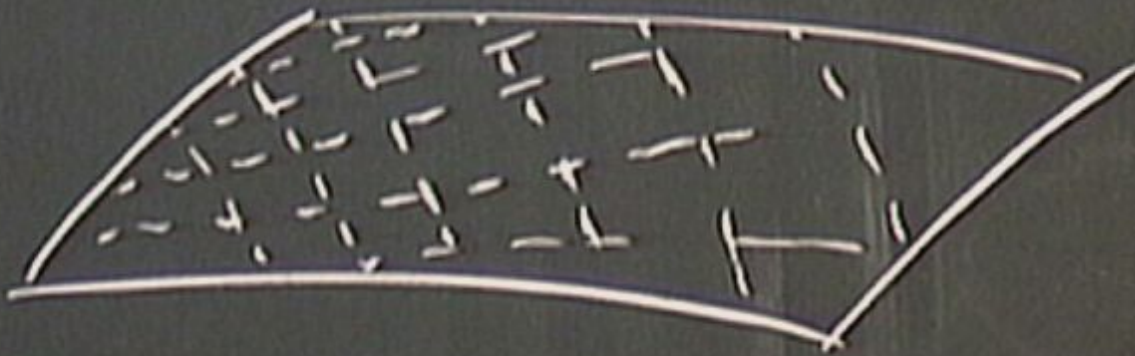
INTRINSIC VS EXTRINSIC CURVATURE



$$\underline{\partial}(X^{\alpha}) = 0$$

$$X^{\alpha}(Y^{\beta}) = 0$$

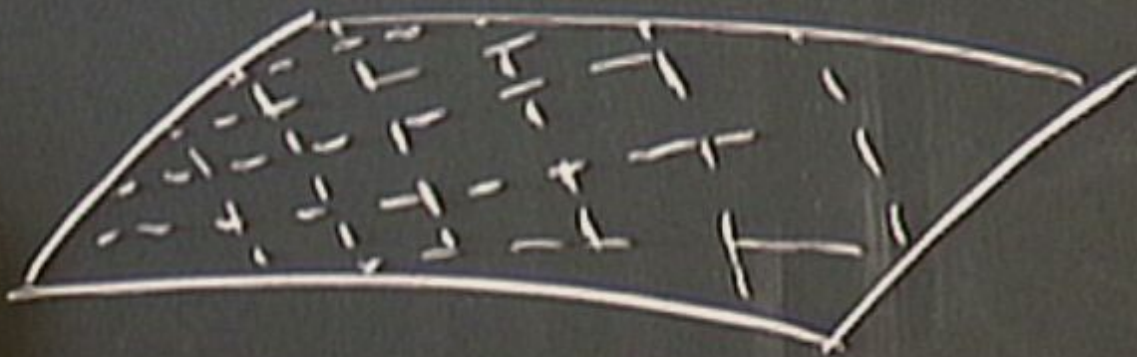
INTRINSIC VS EXTRINSIC CURVATURE



$$\underline{\partial}(x^\alpha) = 0$$

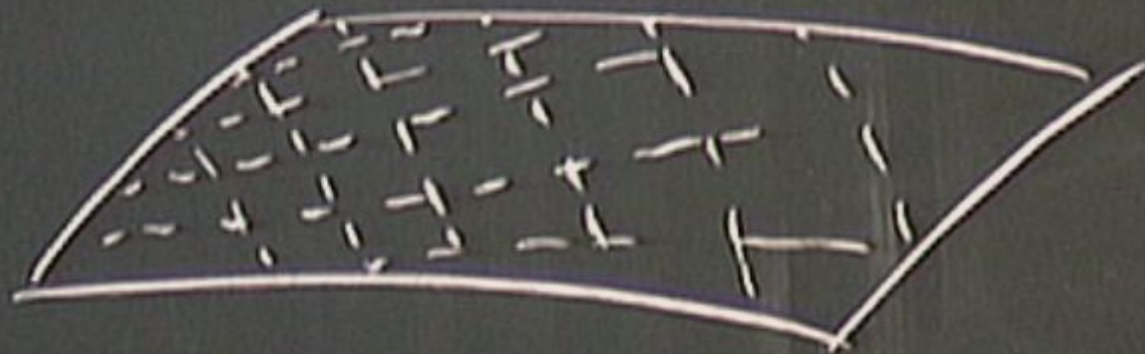
$$x^\alpha(y^\mu) = 0$$

INTRINSIC VS EXTRINSIC CURVATURE



$$\begin{aligned} \partial(x^\alpha) &= 0 \\ x^\alpha(y^\beta) &= 0 \\ e_\alpha &= \frac{\partial x^\alpha}{\partial y^\beta} \\ n_\alpha &\propto \partial_\alpha \Phi \end{aligned}$$

INTRINSIC VS EXTRINSIC CURVATURE



$$h_{ab} = g_{\alpha\beta} e^{\alpha}_a e^{\beta}_b$$

$$\underline{\partial}(x^{\alpha}) = 0$$

$$x^{\alpha}(y^{\beta}) = 0$$

$$e^{\alpha}_a = \frac{\partial x^{\alpha}}{\partial y^a}$$

$$\eta_{\alpha\beta} \propto \partial_x \partial_x \Phi$$

1 - Gaussian normal coordinates

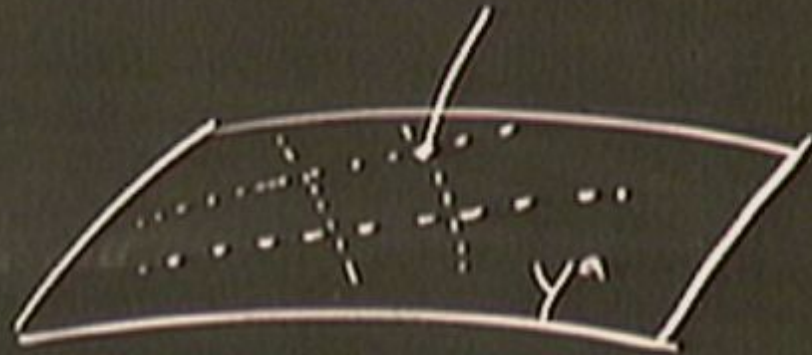
1 - Gaussian normal coordinates

1- Gaussian normal coordinates



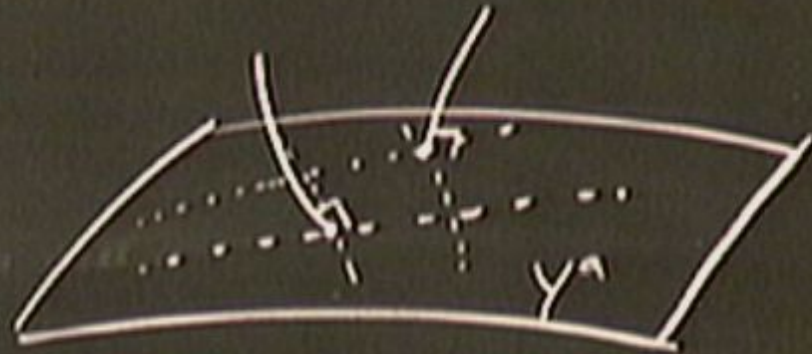
$$x^a = y^a$$

1- Gaussian normal coordinates



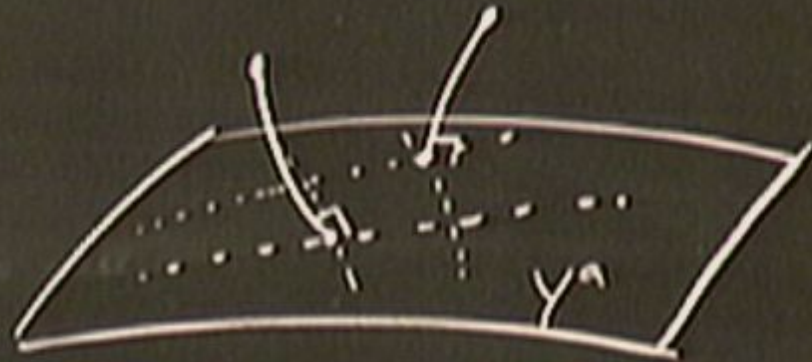
$$x^a = y^a$$

1- Gaussian normal coordinates



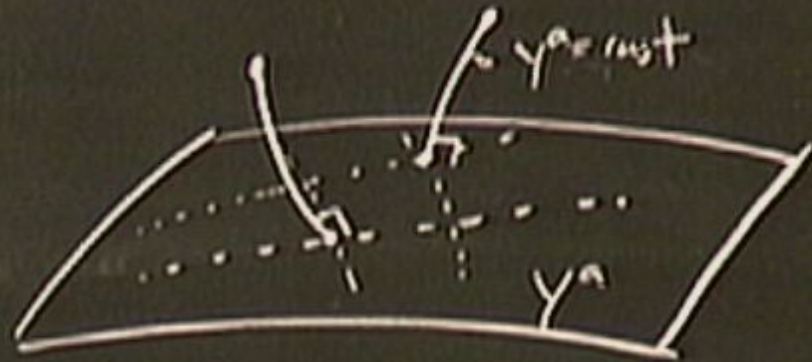
$$x^a = y^a$$

1- Gaussian normal coordinates



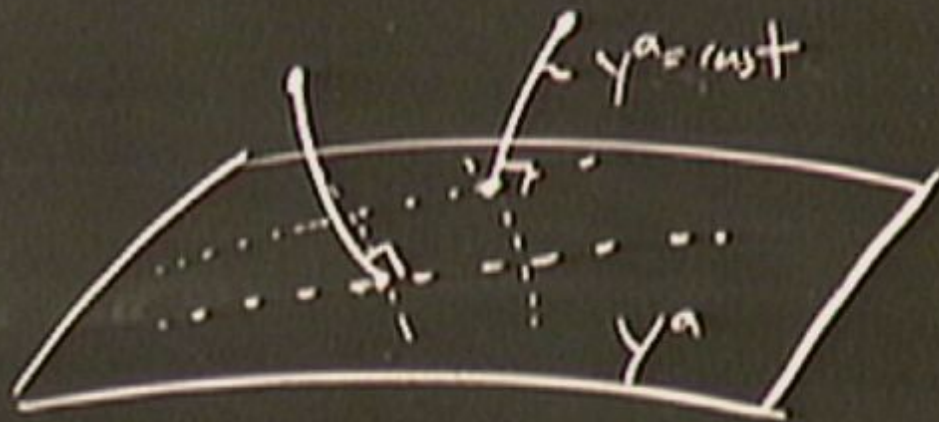
$$x^2 = y^2$$

1- Gaussian normal coordinates



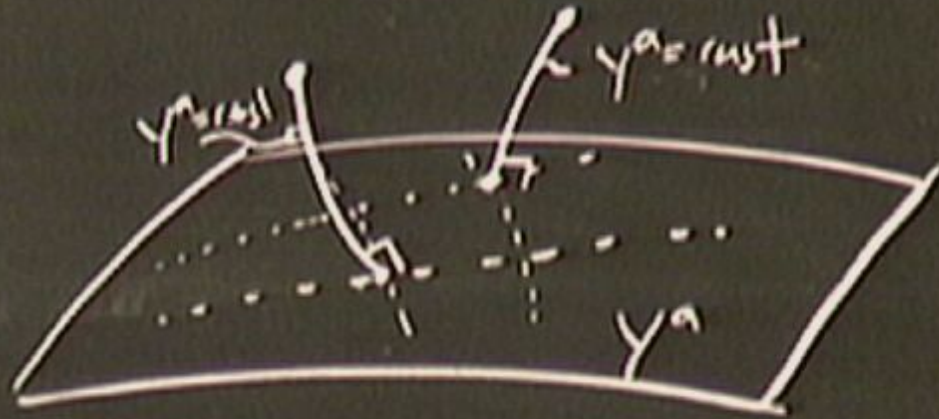
$$x^a = y^a$$

1- Gaussian normal coordinates



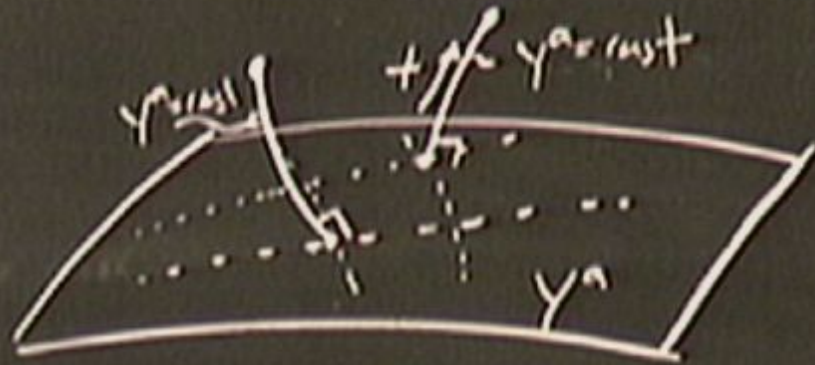
$$x^a = y^a$$

1- Gaussian normal coordinates



$$x^a = y^a$$

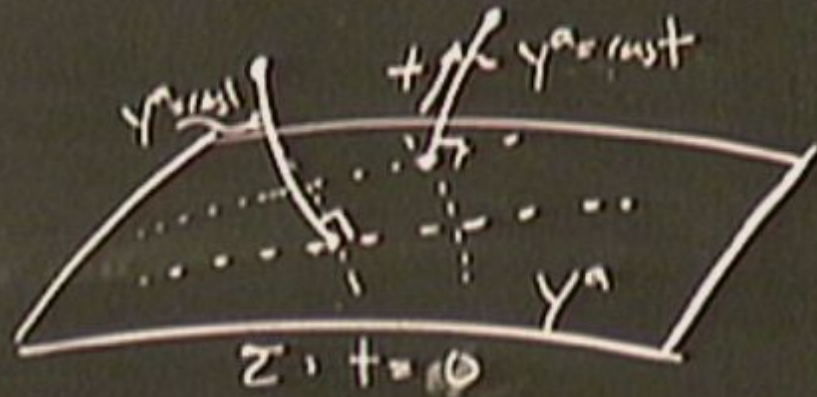
1 - Gaussian normal coordinates



$$x^a = y^a$$

t = proper time on curves $y^a = \text{const}$

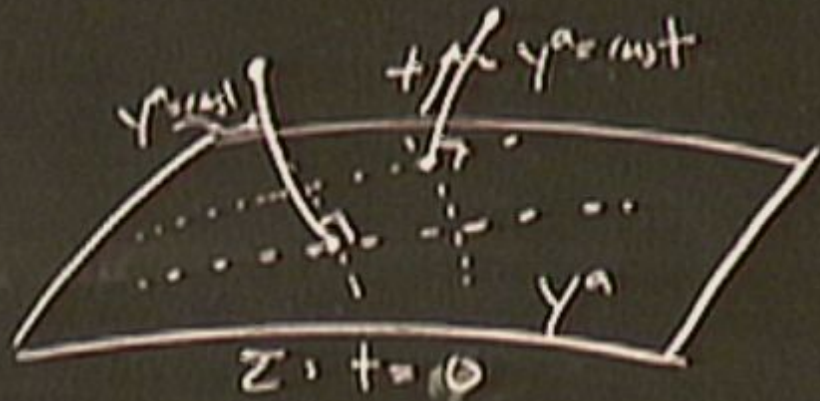
1 - Gaussian normal coordinates



$$x^a = y^a$$

$t = \text{proper time on curves } y^a = \text{const}$

1- Gaussian normal coordinates

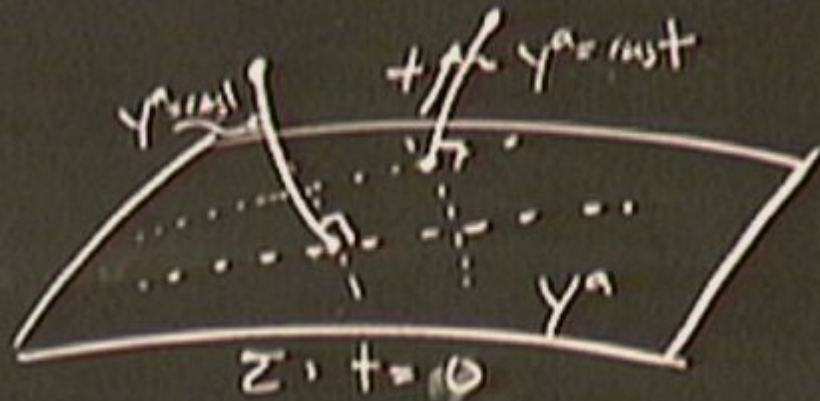


$$x^\alpha = y^a$$

$t \Rightarrow$ proper time on curves $y^a = \text{const}$

$$\left. \begin{array}{l} x^\alpha = y^a \\ t \Rightarrow \text{proper time on curves } y^a = \text{const} \end{array} \right\} x^\alpha = (t, y^a)$$

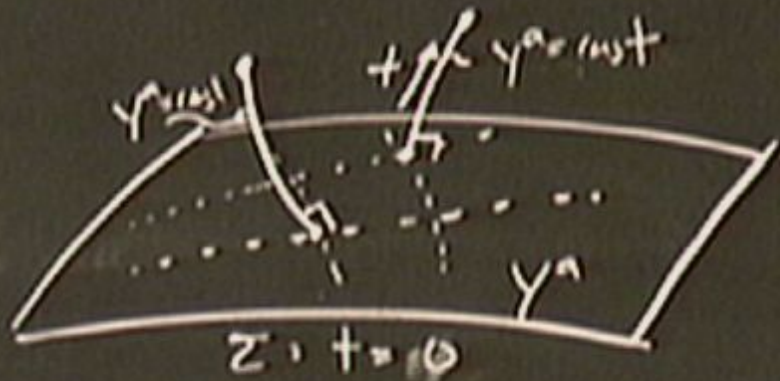
1- Gaussian normal coordinates



$$\left. \begin{array}{l} x^a = y^a \\ t \Rightarrow \text{proper time on curves } y^a = \text{const} \end{array} \right\} x^a = (t, y^a)$$

$$ds^2 = -dt^2$$

1- Gaussian normal coordinates



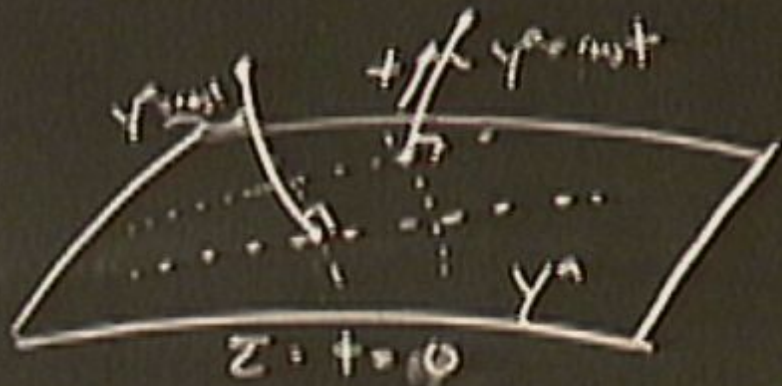
$$x^a = y^a$$

$t =$ proper time on curves $y^a = \text{const}$

$$\left. \begin{array}{l} x^a = y^a \\ t = \text{proper time on curves } y^a = \text{const} \end{array} \right\} x^\alpha = (t, y^a)$$

$$ds^2 = -dt^2 + g_{ab}(t) dy^a dy^b$$

1 - Gaussian normal coordinates



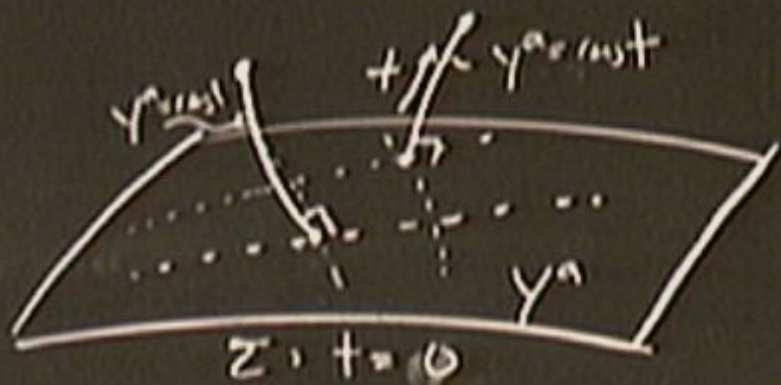
$$x^a = Y^a$$

$t \Rightarrow$ proper time on curves $y^a = \text{const}$

$$\left. \begin{array}{l} x^a = Y^a \\ t \Rightarrow \text{proper time on curves } y^a = \text{const} \end{array} \right\} x^a = (t, Y^a)$$

$$ds^2 = -dt^2 + g_{ab}(t, Y^a) dY^a dY^b$$

1- Gaussian normal coordinates

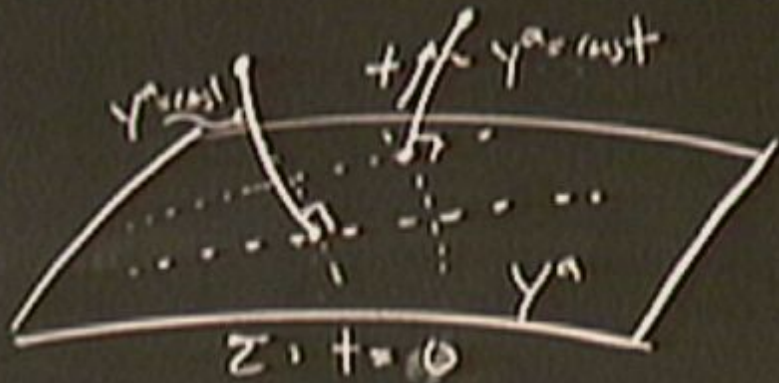


$$x^a = y^a \quad \left. \begin{array}{l} t \Rightarrow \text{proper time on curves } y^a = c^a + v^a t \end{array} \right\} x^a = (t, y^a)$$

$$ds^2 = -dt^2 + g_{ab}(t, y^a) dy^a dy^b$$

$$g_{ab}(0, y^a) = h_{ab}$$

1 - Gaussian normal coordinates



$$\left. \begin{array}{l} x^a = y^a \\ t \Rightarrow \text{proper time on curves } y^a = \text{const} \end{array} \right\} x^a = (t, y^a)$$

$$\boxed{ds^2 = -dt^2 + g_{ab}(t, y^a) dy^a dy^b}$$
$$g_{ab}(0, y^a) = h_{ab}$$

2 - Connections

2-connections

$$4 \int_{\mathcal{P}^8} =$$

2-connections

$H\Gamma_{\mathcal{P}\mathcal{S}}^\alpha = \text{spacetime connection (compatible with } \mathcal{Z}_{\mathcal{P}\mathcal{S}})$

2-connections

$H\Gamma^{\alpha}$
 $\rho\sigma =$ spacetime connection (compatible with $\mathcal{F}_{\rho\sigma}$)

$$\hookrightarrow \nabla_{\mu}$$

2-connections

$4\Gamma_{\rho\sigma}^{\alpha}$ = spacetime connection (compatible with \mathbb{Z}_{eff})

$$\hookrightarrow \nabla_{\mu} = \mathbb{Z}_{\mu}$$

2-connections

$4 \Gamma_{\rho\sigma}^{\alpha}$ = spacetime connection (compatible with $\mathcal{L}_{\text{grav}}$)

$$\hookrightarrow \nabla_{\mu} = \partial_{\mu}$$

$$\Gamma_{bc}^a$$

2-connections

${}^4\Gamma_{\rho\sigma}^{\alpha}$ = spacetime connection (compatible with $g_{\rho\sigma}$)

$$\hookrightarrow \nabla_{\mu} = \nabla_{\mu}$$

Γ_{bc}^a = hypersurface connection (compatible with h_{ab})

$$\hookrightarrow D_n$$

2-connections

${}^4\Gamma_{\rho\sigma}^{\alpha}$ = spacetime connection (compatible with $\mathcal{Z}_{\rho\sigma}$)

$$\hookrightarrow \nabla_{\mu} = \mathcal{D}_{\mu}$$

Γ_{bc}^a = hypersurface connection (compatible with h_{ab})

$$\hookrightarrow D_n = \mathcal{D}_n$$

$$\text{with } \gamma^a = \text{const} \left. \vphantom{\text{with}} \right\} x^\alpha = (t, \gamma^a)$$

$$\boxed{ds^2 = -dt^2 + g_{ab}(t, \gamma^a) d\gamma^a d\gamma^b}$$

$$g_{ab}(0, \gamma^a) = h_{ab}$$

$$g_{\alpha\beta} = \begin{pmatrix} -1 & 0 \\ 0 & g_{ab} \end{pmatrix}$$

$$g^{\alpha\beta} = \begin{pmatrix} -1 & 0 \\ 0 & g^{ab} \end{pmatrix}$$

spacetime connection (compatible with $g_{\alpha\beta}$)

$$\nabla_\mu \equiv \partial_\mu$$

hypersurface connection (compatible with h_{ab})

curves $\gamma^a = \text{const}$ } $x^\alpha = (t, \gamma^a)$

$$ds^2 = -dt^2 + g_{ab}(t, \gamma^a) d\gamma^a d\gamma^b$$

$$g_{ab}(0, \gamma^a) = h_{ab}$$

$$g_{\alpha\beta} = \begin{pmatrix} -1 & 0 \\ 0 & g_{ab} \end{pmatrix}$$

$$g^{\alpha\beta} = \begin{pmatrix} -1 & 0 \\ 0 & g^{ab} \end{pmatrix}$$

spacetime connection (compatible with $g_{\alpha\beta}$)

$$\nabla_\mu = \partial_\mu$$

= hypersurface connection (compatible with h_{ab})

$$\Gamma^+_{p\gamma} = \frac{1}{2} \beta^+$$



$${}^4\Gamma_{\rho\sigma}^{\mu\nu} = \frac{1}{2} \mathfrak{Z}^{\mu\nu} (\mathfrak{Z}_{\rho\mu}\delta + \mathfrak{Z}_{\sigma\nu}\rho - \mathfrak{Z}_{\rho\sigma}\mu)$$

$$\Gamma_{\rho\gamma}^{++} = \frac{1}{2} \mathcal{Z}^{++} \left(\cancel{\mathcal{Z}_{\rho,1,0}^{++}} + \cancel{\mathcal{Z}_{\rho,0,1}^{++}} - \mathcal{Z}_{\rho 0,1}^{++} \right)$$

$${}^4\Gamma_{\rho\gamma}^{\alpha} = \frac{1}{2} g^{\alpha\beta} \left(\cancel{\partial_{\beta} g_{\rho\gamma}} + \cancel{\partial_{\rho} g_{\beta\gamma}} - \partial_{\rho} g_{\beta\gamma} \right)$$

$${}^4\Gamma_{ab}^{\alpha} = \frac{1}{2} \partial^{\alpha} g_{ab}$$

$${}^4\Gamma_{\rho\gamma}^{\tau} = \frac{1}{2} g^{\tau\delta} (g_{\delta\rho, \gamma} + g_{\delta\gamma, \rho} - g_{\rho\gamma, \delta})$$

$${}^4\Gamma_{ab}^{\tau} = \frac{1}{2} \partial_{\tau} g_{ab}$$

$${}^4\Gamma_{+b}^a = \frac{1}{2} g^{am}$$



$${}^4\Gamma_{\rho\gamma}^{\tau} = \frac{1}{2} g^{\tau\delta} (g_{\delta\rho,\gamma} + g_{\delta\gamma,\rho} - g_{\rho\gamma,\delta})$$

$${}^4\Gamma_{ab}^{\tau} = \frac{1}{2} \partial_{\tau} g_{ab}$$

$${}^4\Gamma_{+b}^a = \frac{1}{2} g^{am} \partial_{\tau} g_{mb}$$

$${}^4\Gamma_{\rho\gamma}^{\dagger} = \frac{1}{2} g^{\dagger\mu} (g_{\mu\rho, \gamma} + g_{\mu\gamma, \rho} - g_{\rho\gamma, \mu})$$

$${}^4\Gamma_{ab}^{\dagger} = \frac{1}{2} \partial_t g_{ab}$$

$${}^4\Gamma_{+b}^a = \frac{1}{2} g^{am} \partial_t g_{mb}$$

$${}^4\Gamma_{bc}^a = \frac{1}{2} g^{am} (g_{mb,c} + g_{mc,b} - g_{bc,m})$$



$${}^4\Gamma_{\rho\gamma}^{\dagger} = \frac{1}{2} g^{\dagger\dagger} \left(\cancel{\partial_{\gamma\rho, \delta}} + \cancel{\partial_{\delta\rho, \gamma}} - \partial_{\rho\delta, \gamma} \right)$$

$${}^4\Gamma_{ab}^{\dagger} = \frac{1}{2} \partial_{\dagger} g_{ab}$$

$${}^4\Gamma_{\dagger b}^a = \frac{1}{2} g^{am} \partial_{\dagger} g_{mb}$$

$${}^4\Gamma_{bc}^a = \frac{1}{2} g^{am} \left(\partial_{mb, c} + \partial_{mc, b} - \partial_{bc, m} \right)$$

on Σ :

$${}^4\Gamma_{ab}^{\dagger} = \frac{1}{2} K_{ab}$$

$${}^4\Gamma_{\rho\gamma}^{\dagger} = \frac{1}{2} g^{\dagger\dagger} \left(\cancel{\partial_{\dagger\rho}\gamma} + \cancel{\partial_{\dagger\sigma}\rho} - \partial_{\rho\dagger}\gamma \right)$$

$${}^4\Gamma_{ab}^{\dagger} = \frac{1}{2} \partial_{\dagger} g_{ab}$$

$${}^4\Gamma_{+b}^a = \frac{1}{2} g^{am} \partial_{\dagger} g_{mb}$$

$${}^4\Gamma_{bc}^a = \frac{1}{2} g^{am} \left(\partial_{mb}c + \partial_{mc}b - \partial_{bc}m \right)$$

on Σ :

$${}^4\Gamma_{ab}^{\dagger} = \frac{1}{2} K_{ab}$$

$${}^4\Gamma_{+b}^a = \frac{1}{2} K^a_b$$

$${}^4\Gamma_{bc}^a = \Gamma_{bc}^a$$

$${}^4\Gamma_{\rho\gamma}^{\dagger} = \frac{1}{2} g^{\dagger\dagger} (g_{\dagger\rho, \gamma} + g_{\dagger\gamma, \rho} - g_{\rho\dagger, \dagger})$$

$${}^4\Gamma_{ab}^{\dagger} = \frac{1}{2} \partial_t g_{ab}$$

$${}^4\Gamma_{+b}^a = \frac{1}{2} g^{am} \partial_t g_{mb}$$

$${}^4\Gamma_{bc}^a = \frac{1}{2} g^{am} (g_{mb,c} + g_{mc,b} - g_{bc,m})$$

on Σ :

$${}^4\Gamma_{ab}^{\dagger} = K_{ab}$$

$$K_{ab} = \frac{1}{2} \partial_t g_{ab}(t=0)$$

$${}^4\Gamma_{+b}^a = K^a_b$$

$${}^4\Gamma_{bc}^a = \Gamma_{bc}^a$$

$${}^4\Gamma_{\rho\gamma}^{\dagger} = \frac{1}{2} g^{\dagger\dagger} (g_{\dagger\rho, \gamma} + g_{\dagger\gamma, \rho} - g_{\rho\sigma, \dagger})$$

$${}^4\Gamma_{ab}^{\dagger} = \frac{1}{2} \partial_t g_{ab}$$

$${}^4\Gamma_{+b}^a = \frac{1}{2} g^{am} \partial_t g_{mb}$$

$${}^4\Gamma_{bc}^a = \frac{1}{2} g^{am} (g_{mb,c} + g_{mc,b} - g_{bc,m})$$

on Σ :

$${}^4\Gamma_{ab}^{\dagger} = K_{ab}$$

$$K_{ab} = \frac{1}{2} \partial_t g_{ab}(t=0)$$

$${}^4\Gamma_{+b}^a = K^a_b$$

$$K^a_b = h^{ac} K_{cb}$$

$${}^4\Gamma_{bc}^a = \Gamma_{bc}^a$$

$${}^4\Gamma_{\rho\gamma}^{\dagger} = \frac{1}{2} g^{\dagger\dagger} (g_{\dagger\rho, \gamma} + g_{\dagger\gamma, \rho} - g_{\rho\dagger, \gamma})$$

$${}^4\Gamma_{ab}^{\dagger} = \frac{1}{2} \partial_t g_{ab}$$

$${}^4\Gamma_{+b}^a = \frac{1}{2} g^{am} \partial_t g_{mb}$$

$${}^4\Gamma_{bc}^a = \frac{1}{2} g^{am} (g_{mb,c} + g_{mc,b} - g_{bc,m})$$

on Σ :

$${}^4\Gamma_{ab}^{\dagger} = K_{ab}$$

$$K_{ab} = \frac{1}{2} \partial_t g_{ab}(t=0)$$

$${}^4\Gamma_{+b}^a = K^a_b$$

$$K^a_b = h^{ac} K_{cb}$$

$${}^4\Gamma_{bc}^a = \Gamma_{bc}^a$$

$${}^4\Gamma_{\rho\gamma}^{\dagger} = \frac{1}{2} g^{\dagger\dagger} (g_{\dagger\rho, \gamma} + g_{\dagger\gamma, \rho} - g_{\rho\dagger, \gamma})$$

$${}^4\Gamma_{ab}^{\dagger} = \frac{1}{2} \partial_{\dagger} g_{ab}$$

$${}^4\Gamma_{\dagger b}^a = \frac{1}{2} g^{am} \partial_{\dagger} g_{mb}$$

$${}^4\Gamma_{bc}^a = \frac{1}{2} g^{am} (g_{mb, c} + g_{mc, b} - g_{bc, m})$$

on Σ :

$${}^4\Gamma_{ab}^{\dagger} = K_{ab}$$

$${}^4\Gamma_{\dagger b}^a = K^a_b$$

$${}^4\Gamma_{bc}^a = \Gamma_{bc}^a$$

$$K_{ab} = \frac{1}{2} \partial_{\dagger} g_{ab}(\dagger)$$

$$K^a_b = h^{ac} K_{cb}$$

→ extrinsic curvature

$${}^4\Gamma^+_{\rho\gamma} = \frac{1}{2} g^{+\mu} (g_{\mu\rho, \gamma} + g_{\mu\gamma, \rho} - g_{\rho\gamma, \mu})$$

$${}^4\Gamma^+_{ab} = \frac{1}{2} \partial_t g_{ab}$$

$${}^4\Gamma^a_{+b} = \frac{1}{2} g^{am} \partial_t g_{mb}$$

$${}^4\Gamma^a_{bc} = \frac{1}{2} g^{am} (g_{mb,c} + g_{mc,b} - g_{bc,m})$$

on Σ :

$${}^4\Gamma^+_{ab} = K_{ab}$$

$$K_{ab} = \frac{1}{2} \partial_t g_{ab}(t=0)$$

$${}^4\Gamma^a_{+b} = K^a_b$$

$$K^a_b = h^{ac} K_{cb}$$

$${}^4\Gamma^a_{bc} = \Gamma^a_{bc}$$

→ extrinsic "curvature"

$${}^4\Gamma^t_{\mu\nu} = \frac{1}{2} g^{tt} \left(\cancel{\partial_{\mu\nu} t} + \cancel{\partial_{\nu\mu} t} - \partial_{\mu\nu} t \right)$$

$${}^4\Gamma^t_{ab} = \frac{1}{2} \partial_t g_{ab}$$

$${}^4\Gamma^a_{tb} = \frac{1}{2} g^{am} \partial_t g_{mb}$$

$${}^4\Gamma^a_{bc} = \frac{1}{2} g^{am} \left(\partial_{mb} c + \partial_{mc} b - \partial_{bc} m \right)$$

on Σ :

$${}^4\Gamma^t_{ab} = K_{ab}$$

$$K_{ab} = \frac{1}{2} \partial_t g_{ab} (t=0)$$

$${}^4\Gamma^a_{tb} = K^a_b$$

$$K^a_b = K^a_c b^c$$

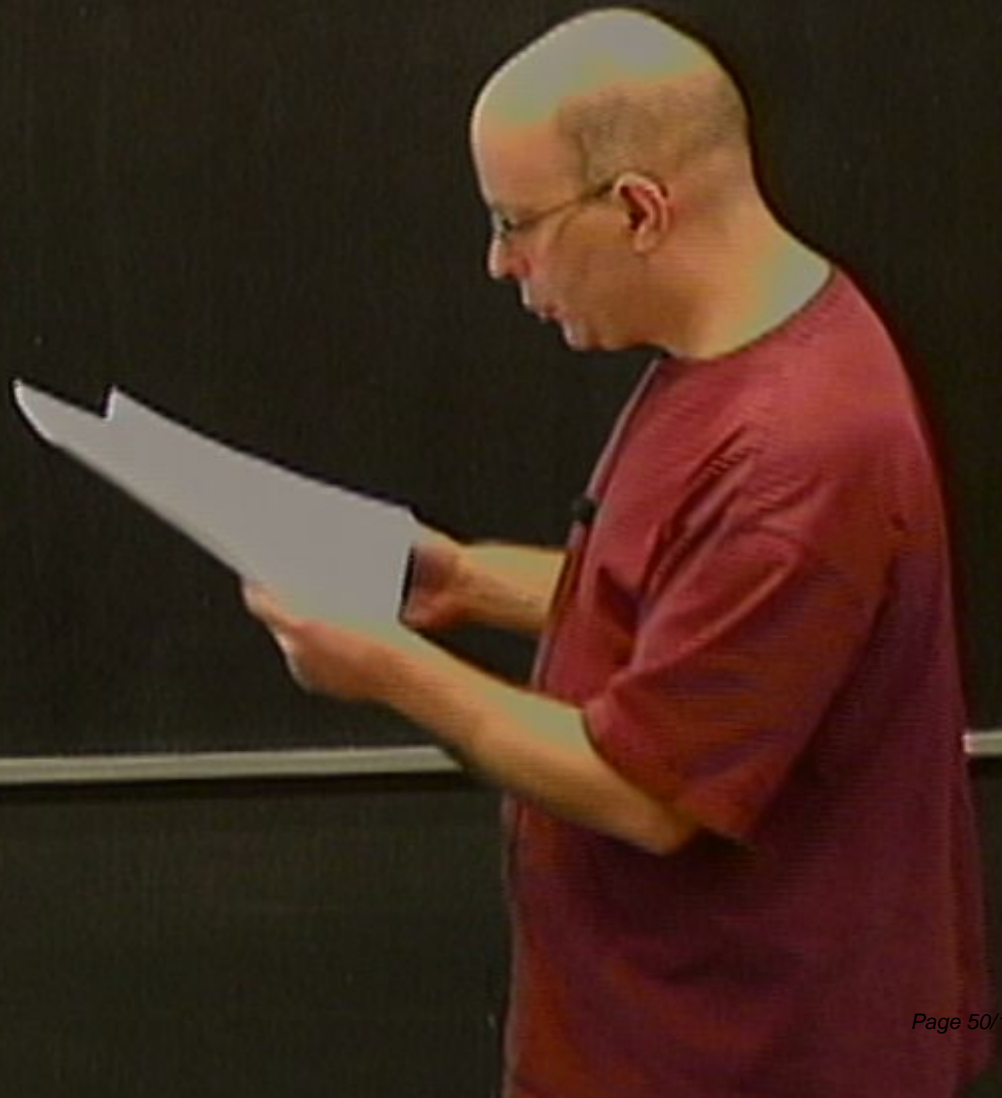
$${}^4\Gamma^a_{bc} = \Gamma^a_{bc}$$

→ extrinsic curvature

3-Counters

On Z :

$4R + atb$



3- Curvatures

On Z :

$$4R_{tatb} = -\frac{1}{2} \partial_t^2 g_{ab}(t, x)$$

3- Curvatures

On Σ :

$$4R_{tatb} = -\frac{1}{2} \partial_t^2 g_{ab}(t=0) + K_{am} K^m_b$$



3- Curvatures

On Σ :

$$4R_{tatb} = -\frac{1}{2} \partial_t^2 g_{ab}(t=0) + K_{am} K^m{}_b$$

$$4R_{tabc} =$$



3-Curvature

On Σ :

$$4R_{tatb} = -\frac{1}{2} \partial_t^2 g_{ab}(t=0) + K_{am} K^m{}_b$$

$$4R_{tabc} = D_c K_{ab}$$

3- Curvatures

On Σ :

$$4R_{tatb} = -\frac{1}{2} \partial_t^2 g_{ab}(t=0) + K_{am} K^m{}_b$$

$$4R_{tabc} = D_c K_{ab} - D_b K_{ac}$$

3- Curvatures

On Σ :

$$4R_{tatb} = -\frac{1}{2} \partial_t^2 g_{ab}(t=0) + K_{am} K^m{}_b$$

$$4R_{tabc} = D_c K_{ab} - D_b K_{ac}$$

3- Curvatures

On Σ :

$$4R_{tatb} = -\frac{1}{2} \partial_t^2 g_{ab}(t=0) + K_a m^m b$$

$$4R_{tabc} = D_c K_{ab} - D_b K_{ac}$$

$$4R_{abca} = R_{abcd}$$

3 - Curvatures

On Σ :

$$4R_{tatb} = -\frac{1}{2} \partial_t^2 g_{ab}(t, x) + K_{am} K^m{}_b$$

$$4R_{tabc} = D_c K_{ab} - D_b K_{ac}$$

$$4R_{abcd} = R_{abcd} + K_{ac} K_{bd} - K_{ad} K_{bc}$$

↓
spatial
components
of spacetime
curvature

↳ intrinsic
curvature obtained
from h_{ab}

3 - Curvatures

On Σ :

$$4 R_{t a t b} = -\frac{1}{2} \partial_t^2 g_{ab}(t=0) + K_{am} K^m_b$$

$$4 R_{t a b c} = D_c K_{ab} - D_b K_{ac}$$

$$4 R_{abcd} = R_{abcd} + K_{ac} K_{bd} - K_{ad} K_{bc}$$

↓
spatial
components
of spacetime
curvature

↳ intrinsic
curvature obtained
from h_{ab}

$\wedge \Sigma:$

$${}^4\Gamma_{ab}^t = K_{ab} \quad K_{ab} = \frac{1}{2} \partial_t g_{ab}(t=0)$$

$${}^4\Gamma_{tb}^a = K^a_b \quad h^{ac} K_{cb}$$

$${}^4\Gamma_{bc}^a = \Gamma_{bc}^a$$

→ extrinsic curvature

$${}^4R_{tabc} = D_c K_{ab} - D_b K_{ac}$$

$${}^4R_{abcd} = R_{abcd} + K_{ac} K_{bd} - K_{ad} K_{bc}$$

↓
spatial
components
of spacetime
curvature

↳ intrinsic
curvature obtained
from h_{ab}

$$4\Gamma^a{}_{bc} = \Gamma^a{}_{bc}$$

Curvatures

On Σ :

$$4R_{tatb} = -\frac{1}{2} \partial_t^2 g_{ab}(t=0) + K_{am} K^m{}_b$$

$$4R_{tabc} = D_c K_{ab} - D_b K_{ac}$$

$$4R_{abcs} = R_{abcs} + K_{ac} K_{bs} - K_{ad} K_{bc}$$

↓
spatial
components
of spacetime
curvature

↳ intrinsic
curvature obtained
from h_{ab}

$$4G_H = \frac{1}{2} (R)$$

[The rest of the page is heavily obscured by a large, dense scribble of white marks, likely representing a complex mathematical derivation or a diagram. Some faint characters like 'x' and '(x, y)' are visible within the scribble.]

$$4G_{tt} = \frac{1}{2} (R - K^{ab} K_{ab} + K^a_a)$$

$$4G_{tt} = \frac{1}{2} (R - K^{ab} K_{ab} + K^2)$$

$$4G_{ta} = D_b K^b_a - D_a K$$

$$4G_{tt} = \frac{1}{2} (R - K^{ab} K_{ab} + K^2)$$

$$4G_{ta} = D_b K^b_a - D_a K$$

$$4G_{aa} =$$

$$4G_{tt} = \frac{1}{2} (R - K^{ab} K_{ab} + K^2)$$

$$4G_{ta} = D_b K^b_a - D_a K$$

$$4G_{ab} = G_{ab}$$

$$4G_{tt} = \frac{1}{2} (R - K^{ab} K_{ab} + K^2)$$

$$4G_{ta} = D_b K^b_a - D_a K$$

$$4G_{nn} = G_{ab} + \frac{1}{2} \partial_t^2 g_{ab} - \frac{1}{2} \text{tr} g (\partial_t^2 g_{ab})$$

$$4G_{tt} = \frac{1}{2} (R - K^{ab} K_{ab} + K^2)$$

$$4G_{ta} = D_b K^b_a - D_a K$$

$$4G_{aa} = G_{ab} + \frac{1}{2} D_t^2 g_{ab} - \frac{1}{2} h_{ab} (h^{cd} D_t^2 g_{cd}) \\ - 2 K_{ac} K^c_b + \frac{3}{2} h_{ab} (K^{cd} K_{cd}) + K K_{ab} - \frac{1}{2} h_{ab} K^2$$

$$4G_{tt} = \frac{1}{2} (R - K^{ab} K_{ab} + K^2)$$

$$4G_{ta} = D_b K^b_a - D_a K$$

$$4G_{ab} = G_{ab} + \frac{1}{6} \partial_t^2 g_{ab} - \frac{1}{2} h_{ab} (h^{cd} \partial_t^2 g_{cd}) \\ - 2 K_{ac} K^c_b + \frac{3}{2} h_{ab} (K^{cd} K_{cd}) + K K_{ab} - \frac{1}{2} h_{ab} K^2$$

$$4G_{tt} = \frac{1}{2} (R - K^{ab} K_{ab} + K^2)$$

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R_{ab} = intrinsic Ricci tensor (from h_{ab})

$$\rightarrow D_a = \nabla_a$$

$$4G_{tt} = \frac{1}{2} (R - K^{ab} K_{ab} + K^2)$$

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R_{ab} = intrinsic Ricci tensor (from h_{ab})

$$R = h^{ab} R_{ab}$$

$$G_{ab} = R_{ab} - \frac{1}{2} R h_{ab}$$

$$\hookrightarrow D_a = \nabla_a$$

$$4G_{tt} = \frac{1}{2} (R - K^{ab} K_{ab} + K^2)$$

$$4G_{ta} = D_b K^b_a - D_a K$$

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R_{ab} = intrinsic Ricci tensor (from h_{ab})

$$R = h^{ab} R_{ab}$$

$$G_{ab} = R_{ab} - \frac{1}{2} R h_{ab}$$

$$K = h^{ab} K_{ab}$$



$$h_{ab} = g_{\mu\nu} e^{\mu}_{\alpha} e^{\nu}_{\beta}$$

$$e^{\alpha}_{\mu} = \frac{\partial x^{\alpha}}{\partial y^{\mu}}$$

$$n_{\alpha} \propto \partial_{\alpha} \Phi$$

$$n_{\alpha} = -\partial_{\alpha} t$$



$$h_{ab} = g_{pp} e^{\alpha} e^{\beta}$$

$$n_{\alpha} \propto \partial_{\alpha} \Phi$$

$$n_{\alpha} = -\partial_{\alpha} t$$

$$e^{\alpha} =$$

$$h_{ab} = g_{\mu\nu} e^{\mu}{}_{a} e^{\nu}{}_{b}$$

$$n_a \propto \partial_x \phi$$

$$n_a \propto \partial_x \phi$$

$$e^{\mu}{}_{a} \propto \delta^{\mu}_a$$

4-Extrinsic curvature

$$k_{ab} = {}^4 \Gamma_{ab}^{\dagger} = \frac{1}{2} \partial_t \gamma_{ab}$$

4-Extrinsic curvature

$$K_{ab} = {}^4\Gamma_{ab}^{\perp} = \frac{1}{2} \partial_t g_{ab}$$

Consider :

Musp

4 - Extrinsic curvature

$$K_{ab} = {}^4 \Gamma_{ab}^{\dagger} = \frac{1}{2} \partial_t g_{ab}$$

Consider :

$$\eta_{\nu\mu} e^{\nu}_a e^{\mu}_b$$

4 - Extrinsic curvature

$$K_{ab} = {}^4 \Gamma_{ab}^\dagger = \frac{1}{2} \partial_t g_{ab}$$

Consider :

$$n_{\mu\nu\rho} e^{\mu}_a e^{\nu}_b$$

$$U^\mu_{;\nu} U^\nu$$

4-Extrinsic curvature

$$K_{ab} = {}^4\Gamma_{ab}^{\dagger} = \frac{1}{2} \partial_t g_{ab}$$

Consider :

$$n_{\nu;\rho} e^{\alpha} e^{\beta}_{\nu} \stackrel{*}{=} (n_{\alpha\rho} + {}^4\Gamma_{\alpha\rho}^{\gamma} n_{\gamma}) e^{\alpha} e^{\beta}_{\nu}$$

4-Extrinsic curvature

$$k_{ab} = {}^4\Gamma_{ab}^{\dagger} = \frac{1}{2} \partial_t g_{ab}$$

Consider :

$$\begin{array}{l} n_{\mu\nu\rho} e^{\alpha} e^{\beta} \parallel^* (n_{\alpha\rho} + {}^4\Gamma_{\alpha\rho}^{\gamma} n_{\gamma}) e^{\alpha} e^{\beta} \\ \parallel^* \end{array}$$

4 - Extrinsic curvature

$$k_{ab} = {}^4 \Gamma_{ab}^{\dagger} = \frac{1}{2} \partial_t g_{ab}$$

Consider :

$$\begin{array}{c} n_{\alpha\beta} e^{\alpha} e^{\beta} \\ \parallel * \\ \parallel * \end{array} (n_{\alpha\beta} + {}^4 \Gamma_{\alpha\beta}^{\gamma} n_{\gamma}) e^{\alpha} e^{\beta}$$

4 - Extrinsic curvature

$$K_{ab} = {}^4\Gamma_{ab}^{\dagger} = \frac{1}{2} \partial_t g_{ab}$$

Consider :

$$n_{\mu;\nu} e^{\alpha} e^{\beta}_{\nu} \parallel^* (n_{\alpha;\rho} \Gamma^{\rho}_{\gamma} n_{\gamma})$$

\parallel^*

4 - Extrinsic curvature

$$K_{ab} = {}^4\Gamma_{ab}^{\dagger} = \frac{1}{2} \partial_t g_{ab}$$

Consider :

$$\begin{aligned} n_{\mu;\nu} e^{\alpha} e^{\beta}_{\alpha} & \parallel^{*} (n_{\alpha;\beta} + {}^4\Gamma_{\alpha\beta}^{\gamma} n_{\gamma}) e^{\alpha} e^{\beta} \\ & \parallel^{*} \end{aligned}$$

4 - Extrinsic curvature

$$K_{ab} = {}^4\Gamma_{ab}^\dagger = \frac{1}{2} \partial_t g_{ab}$$

Consider :

$$\begin{aligned} n_{\mu\nu\rho} e^{\alpha} e^{\beta} &\stackrel{1}{=} (n_{\alpha\rho} {}^4\Gamma_{\alpha\beta}^{\gamma} n_{\gamma}) e^{\alpha} e^{\beta} \\ &\stackrel{2}{=} {}^4\Gamma_{ab}^{\dagger} \end{aligned}$$

4 - Extrinsic curvature

$$K_{ab} = {}^4\Gamma_{ab}^{\dagger} = \frac{1}{2} \partial_t g_{ab}$$

Consider :

$$\begin{aligned} n_{\mu;\nu} e^{\alpha} e^{\beta}_{\nu} &\stackrel{1}{=} (n_{\alpha;\rho} \Gamma_{\rho\sigma}^{\dagger} n_{\sigma}) e^{\alpha} e^{\beta}_{\nu} \\ &\stackrel{2}{=} {}^4\Gamma_{ab}^{\dagger} \\ &= K_{ab} \end{aligned}$$

4 - Extrinsic curvature

$$K_{ab} = \Gamma_{ab}^{\perp} = \frac{1}{2} \partial_t g_{ab}$$

Consider

$$n_{\mu;\nu\rho} e^{\alpha} e^{\beta} \parallel^* (n_{\alpha\rho} \Gamma_{\alpha\rho}^{\gamma} n_{\gamma}) e^{\alpha} e^{\beta}$$

$$\parallel^* \Gamma_{ab}^{\perp}$$
$$\parallel^* K_{ab}$$

4-Extrinsic curvature

$$K_{ab} = {}^4\Gamma_{ab}^{\dagger} = \frac{1}{2} \partial_t g_{ab}$$

Consider :

$$n_{\alpha;\beta} e^{\alpha} e^{\beta} \stackrel{||*}{=} (n_{\alpha;\beta} + \Gamma_{\alpha\beta}^{\gamma} n_{\gamma}) e^{\alpha} e^{\beta}$$

$$K_{ab} \equiv n_{\alpha;\beta} e^{\alpha} e^{\beta}$$

$$\stackrel{||*}{=} \Gamma_{ab}^{\dagger} \stackrel{||*}{=} K_{ab}$$

4 - Extrinsic curvature

$$K_{ab} = {}^4\Gamma_{ab}^\dagger = \frac{1}{2} \partial_t g_{ab}$$

Consider :

$$n_{\alpha;\beta} e^{\alpha} e^{\beta} \stackrel{||*}{=} (n_{\alpha;\beta} + \Gamma_{\alpha\beta}^{\gamma} n_{\gamma}) e^{\alpha} e^{\beta}$$

$$\boxed{K_{ab} \equiv n_{\alpha;\beta} e^{\alpha} e^{\beta} = K_{ba}}$$

$$\stackrel{||*}{=} {}^4\Gamma_{ab}^\dagger$$
$$\stackrel{||*}{=} K_{ab}$$

$$\sum_n Z_{\text{op}} = n_{\alpha j p} + n_{p j \alpha}$$

$$\mathcal{L}_n \mathcal{Z}_{\alpha\beta} = n_{\alpha\beta} + n_{\beta\alpha}$$

$$(\mathcal{L}_n \mathcal{Z}_{\alpha\beta}) e^{\alpha} e^{\beta} =$$

$$\mathcal{L}_n \mathcal{Z}_{\text{op}} = n_{\alpha\beta} + n_{\beta\alpha}$$

$$(\mathcal{L}_n \mathcal{Z}_{\text{op}}) e^{\alpha} e^{\beta} = 2n_{(\alpha\beta)} e^{\alpha} e^{\beta}$$

$$\mathcal{L}_n \mathcal{Z}_{op} = n_{\alpha\beta} + n_{\beta\alpha}$$

$$(\mathcal{L}_n \mathcal{Z}_{op}) e^{\alpha} e^{\beta} = 2n_{(\alpha\beta)} e^{\alpha} e^{\beta} = 2K_{\alpha\beta}$$

$$\mathcal{L}_n \mathcal{G}_{\alpha\beta} = n_{\alpha\beta} + n_{\beta\alpha}$$

$$(\mathcal{L}_n \mathcal{G}_{\alpha\beta}) e^{\alpha} e^{\beta} = 2 n_{(\alpha\beta)} e^{\alpha} e^{\beta} = 2 K_{\alpha\beta}$$

$$K_{\alpha\beta} = \frac{1}{2} (\mathcal{L}_n \mathcal{G}_{\alpha\beta}) e^{\alpha} e^{\beta}$$

$$4G_{tt} = \frac{1}{2} (R - K^{ab} K_{ab} + K^2) = 64\pi^2 \rho$$

$$4G_{ta} = D_b K^b_a - D_a K$$

$$4G_{ab} = G_{ab} + \frac{1}{2} D_t^2 g_{ab} - \frac{1}{2} h_{ab} (h^{cd} D_t^2 g_{cd})$$

$$- 2 K_{ac} K^c_b + \frac{3}{2} h_{ab} (K^{cd} K_{cd}) + K K_{ab} -$$

R_{ab} = intrinsic Ricci tensor (from h_{ab})

$$R = h^{ab} R_{ab}$$

$$G_{ab} = R_{ab} - \frac{1}{2} R h_{ab}$$

$$K = h^{ab} K_{ab}$$

$$4G_{tt} = \frac{1}{2} (R - K^{ab} K_{ab} + K^2) = \text{"Gunn"}^t$$

$$4G_{ta} = D_b K^b_a - D_a K = \text{"Gunn"}^a$$

$$4G_{ab} = G_{ab} + \frac{1}{2} D_t^2 g_{ab} - \frac{1}{2} h_{ab} (h^{cd} D_t^2 g_{cd}) \\ - 2 K_{ac} K^c_b + \frac{3}{2} h_{ab} (K^{cd} K_{cd}) + K K_{ab}$$

R_{ab} = intrinsic Ricci tensor (from h_{ab})

$$R = h^{ab} R_{ab}$$

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$$K = h^{ab} K_{ab}$$

$$4G_{tt} = \frac{1}{2} (R - K^{ab} K_{ab} + K^a{}_a) = {}^4G_{tt} n^t n^t$$

$$4G_{ta} = D_b K^b{}_a - D_a K = {}^4G_{ta} e^a$$

$$4G_{ab} = G_{ab} + \frac{1}{2} \partial_t^2 g_{ab} - \frac{1}{2} h_{ab} (h^{cd} \partial_t^2 g_{cd}) \\ - 2 K^c{}_a K^c{}_b + \frac{3}{2} h_{ab} (K^{cd} K_{cd}) + K K_{ab} - \frac{1}{2} h_{ab} K^c{}_c$$

R_{ab} = intrinsic Ricci tensor (from h_{ab})

$$R = h^{ab} R_{ab}$$

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$$\mathcal{L}_n \mathcal{Z}_{\alpha\beta} = n_{\alpha\beta} + n_{\beta\alpha}$$

$$(\mathcal{L}_n \mathcal{Z}_{\alpha\beta}) e_a^\alpha e_b^\beta = 2 n_{(\alpha\beta)} e_a^\alpha e_b^\beta = 2 K_{ab}$$

$$K_{ab} = \frac{1}{2} (\mathcal{L}_n \mathcal{Z}_{\alpha\beta}) e_a^\alpha e_b^\beta$$

On Σ :

$$4R_{tatb} = -\frac{1}{2} \partial_t^2 g_{ab}(t=0) + K_{am} K^m{}_b$$

$$4R_{tabc} = D_c K_{ab} - D_b K_{ac}$$

$$4R_{abcd} = R_{abcd} + K_{ac} K_{bd} - K_{ad} K_{bc}$$

↓
spatial
components
of spacetime
curvature

↳ intrinsic
curvature obtained
from h_{ab}

On Σ :

$$4R_{tatb} = -\frac{1}{2} \partial_t^2 g_{ab}(t=0) + K_{am} K^m{}_b$$

$$4R_{tabc} = D_c K_{ab} - D_b K_{ac}$$

$$4R_{ab\dot{c}} = R_{abcd} + K_{ac} K_{bd} - K_{ad} K_{bc}$$

↓
spatial
components
of spacetime
curvature

↳ intrinsic
curvature obtained
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5- Tangent vector fields

$$A^\alpha = A^a e_a^\alpha$$

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$$A^\alpha = A^\alpha e_\alpha \equiv (0, A^1, A^2, A^3)$$

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$$A^\alpha; \beta e_b^\beta$$

components
of spacetime
curvature

from

5- Tangent vector fields

$$A^\alpha = A^a e_a^\alpha \equiv (0, A^1, A^2, A^3)$$

$$A^\alpha_{;\beta} e_b^\beta \equiv (A^\alpha_{;\beta} + \Gamma_{\beta\delta}^\alpha A^\delta)$$

components
of spacetime
curvature

from h

4 - Extrinsic curvature

$$K_{ab} = {}^4 \Gamma_{ab}^{\dagger} = \frac{1}{2} \partial_t g_{ab}$$

Consider :

$$n_{\alpha;\beta} e^{\alpha} e^{\beta} \stackrel{||*}{=} (n_{\alpha;\beta} + \Gamma_{\alpha\beta}^{\gamma} n_{\gamma}) e^{\alpha}$$

$$\boxed{K_{ab} \equiv n_{\alpha;\beta} e^{\alpha} e^{\beta} = K_{ba}}$$

$$\stackrel{||*}{=} {}^4 \Gamma_{ab}^{\dagger} = K_{ab}$$

5- Tangent vector fields

$$A^\alpha = A^a e_a^\alpha \cong (0, A^1, A^2, A^3)$$

$$A^\alpha_{;\beta} e^\beta \cong (A^\alpha_{;\beta} + \Gamma_{\mu\beta}^\alpha A^\mu) e^\beta$$
$$\cong (A^\alpha_{;\beta}) e^\beta$$

components
of spacetime
curvature

sum has

5- Tangent vector fields

$$A^\alpha = A^a e_a^\alpha \equiv (0, A^1, A^2, A^3)$$

$$A^\alpha_{;\beta} e^\beta \equiv (A^\alpha_{;\beta} + \Gamma_{\mu\beta}^\alpha A^\mu) e^\beta$$
$$\equiv (A^\alpha_{;\beta} +$$

components
of spacetime
curvature

5- Tangent vector (Liebs)

$$A^\alpha = A^a e^a, \quad \equiv^* (0, A^1, A^2, A^3)$$

$$A^\alpha{}_{;\beta} e^\beta \equiv^* (A^\alpha{}_{;\beta} + {}^4\Gamma_{\beta\delta}^\alpha A^\delta) e^\beta$$
$$\equiv^* (A^\alpha{}_{;b} + {}^4\Gamma_{bc}^\alpha A^c)$$

5- Tangent vector (Fields)

$$A^\alpha = A^a e^\alpha, \quad \equiv (0, A^1, A^2, A^3)$$

$$A^\alpha_{; \beta} e^\beta \equiv (A^\alpha_{; \beta} + \Gamma_{\beta\delta}^\alpha A^\delta) e^\beta$$
$$\equiv (A^\alpha_{; b} + \Gamma_{bc}^\alpha A^c)$$

$$\alpha = t: \quad A^+_{; \beta} e^\beta \equiv$$

5- Tangent vector (Fields)

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$$A^\alpha_{;\beta} e^\beta \equiv (A^\alpha_{;\beta} + \Gamma_{\beta\delta}^\alpha A^\delta) e^\beta$$

$$\equiv (A^\alpha_{;\beta} + \Gamma_{bc}^\alpha A^c)$$

$$\alpha = t: \quad A^+_{;\beta} e^\beta \equiv \Gamma_{bc}^+ A^c = K_{bc} A^c$$

↓
 spatial
 intrinsic
 curvature of space

5- Tangent vector (fields)

$$A^\alpha = A^a e^\alpha \equiv (0, A^1, A^2, A^3)$$

$$A^\alpha{}_{;\beta} e^\beta \equiv (A^\alpha{}_{;\beta} + {}^4\Gamma_{\beta\delta}^\alpha A^\delta) e^\beta$$
$$\equiv (A^\alpha{}_{;\beta} + {}^4\Gamma_{bc}^\alpha A^c)$$

$$\alpha = + : A^+{}_{;\beta} e^\beta \equiv {}^4\Gamma_{bc}^+ A^c = K_{bc} A^c$$

$$\alpha = a : A^a{}_{;\beta} e^\beta \equiv A^a{}_{;\beta} + \Gamma_{bc}^a A^c = D_b A^a$$

5- tangent vector (x^i, \dot{x}^i)

$$A^\alpha = A^a e^\alpha \equiv (0, A^1, A^2, A^3)$$

$$A^\alpha_{; \beta} e^\beta \equiv (A^\alpha_{; \beta} + \Gamma_{\beta\delta}^\alpha A^\delta) e^\beta$$

$$\equiv (A^\alpha_{; b} + \Gamma_{bc}^\alpha A^c)$$

$$\alpha = + : A^+_{; \beta} e^\beta \equiv \Gamma_{bc}^+ A^c = K_{bc} A^c$$

$$\alpha = a : A^a_{; \beta} e^\beta \equiv A^a_{; b} + \Gamma_{bc}^a A^c = D_b A^a$$

$$A^a_{; \beta} e^\beta =$$

$$\Gamma_{bcd} = R_{bcd} + K_{ac} K_{bd} - K_{ad} K_{bc}$$

5- Tangent vector (Srlbs)

$$A^\alpha = A^a e_a^\alpha \equiv (0, A^1, A^2, A^3)$$

$$A^\alpha_{; \beta} e^\beta_b \equiv (A^\alpha_{; \beta} + {}^4 \Gamma_{\beta\delta}^\alpha A^\delta) e^\beta_b \\ \equiv (A^\alpha_{; \beta} + {}^4 \Gamma_{bc}^\alpha A^c)$$

$$\alpha = + : A^+_{; \beta} e^\beta_b \equiv {}^4 \Gamma_{bc}^+ A^c - K_{bc} A^c$$

$$\alpha = a : A^a_{; \beta} e^\beta_b \equiv A^a_{; b} + \Gamma_{bc}^a A^c = D_b A^a$$

$$A^a_{; \beta} e^\beta_b = D_b A^a e_a^\alpha + A^a K_{ab} n^\alpha$$

3 - Curvatures

On Σ :

$$4 R_{tatb} = -\frac{1}{2} \partial_t^2 g_{ab}(t=0) + K_{am} K^m{}_b$$

$$4 R_{tabc} = D_c K_{ab} - D_b K_{ac}$$

$$4 R_{abcd} = R_{abcd} + K_{ac} K_{bd} - K_{bc} K_{ad}$$

↓
spatial
components
of spacetime
curvature

↳ intrinsic
curvature obtained
from h_{ab}

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3 - Curvatures

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↓
spatial
components
of spacetime
curvature

↳ intrinsic
curvature obtained
from h_{ab}

6- Gauss-Codazzi equations

$$2 \nabla_i \nabla_j e_{\alpha}^i e_{\alpha}^j = 2 K_{\alpha\beta}$$

6- Gauss-Codazzi equations

$$4R_{\alpha\beta\gamma\delta}e^{\alpha}e^{\beta}e^{\gamma}e^{\delta}$$

6 - Gauss-Codazzi equations

$$4R_{abcd}e^{\pi}_ae^{\rho}_be^{\delta}_ce^{\delta}_d = R_{abcd} + K_{ac}K_{bd} - K_{ad}K_{bc}$$

$$4R_{abcd}n^{\pi}e^{\pi}_ae^{\rho}_be^{\delta}_c$$

6 - Gauss-Codazzi equations

$$4 R_{\alpha\beta\gamma\delta} e^\alpha e^\beta e^\gamma e^\delta = R_{abcd} + K_{ac} K_{bd} - K_{ab} K_{cd}$$

$$4 R_{\mu\nu\rho\sigma} n^\mu e^\alpha e^\nu e^\rho e^\sigma = D_c K_{ab} - D_b K_{ac}$$

6 - Gauss-Codazzi equations

$$4 R_{abcd} e^a e^b e^c e^d = R_{abcd} + K_{ac} K_{bd} - K_{ab} K_{cd}$$

$$4 R_{\mu\nu\rho\sigma} n^\mu e^a e^b e^c = D_c K_{ab} - D_b K_{ac}$$

$$4 G_{\mu\nu} n^\mu n^\nu =$$

6 - Gauss-Codazzi equations

$$4 R_{\alpha\beta\gamma\delta} e^\alpha e^\beta e^\gamma e^\delta = R_{ab\cdots} + K_{ac} K_{bd} - K_{ab} K_{cd}$$

$$4 R_{\mu\nu\rho\sigma} n^\mu e^\alpha e^\beta e^\gamma e^\delta = D_c K_{ab} - D_b K_{ac}$$

$$4 G_{\alpha\beta} n^\alpha n^\beta = \frac{1}{2} (R - K^{\alpha\beta} K_{\alpha\beta} + K^2)$$

$$4 G_{\alpha\beta} n^\alpha e^\beta =$$

6- Gauss-Codazzi equations

$$4R_{\alpha\beta\gamma\delta} e^\alpha e^\beta e^\gamma e^\delta = R_{abcd} + K_{ac} K_{bd} - K_{ab} K_{cd}$$

$$4R_{\mu\nu\rho\sigma} n^\mu e^\alpha e^\nu e^\rho e^\sigma = D_c K_{ab} - D_b K_{ac}$$

$$4G_{\alpha\beta} n^\alpha n^\beta = \frac{1}{2} (R - K^{ab} K_{ab} + K^2)$$

$$4G_{\alpha\beta} n^\alpha e^\beta k = D_b K_a^b - D_a K$$