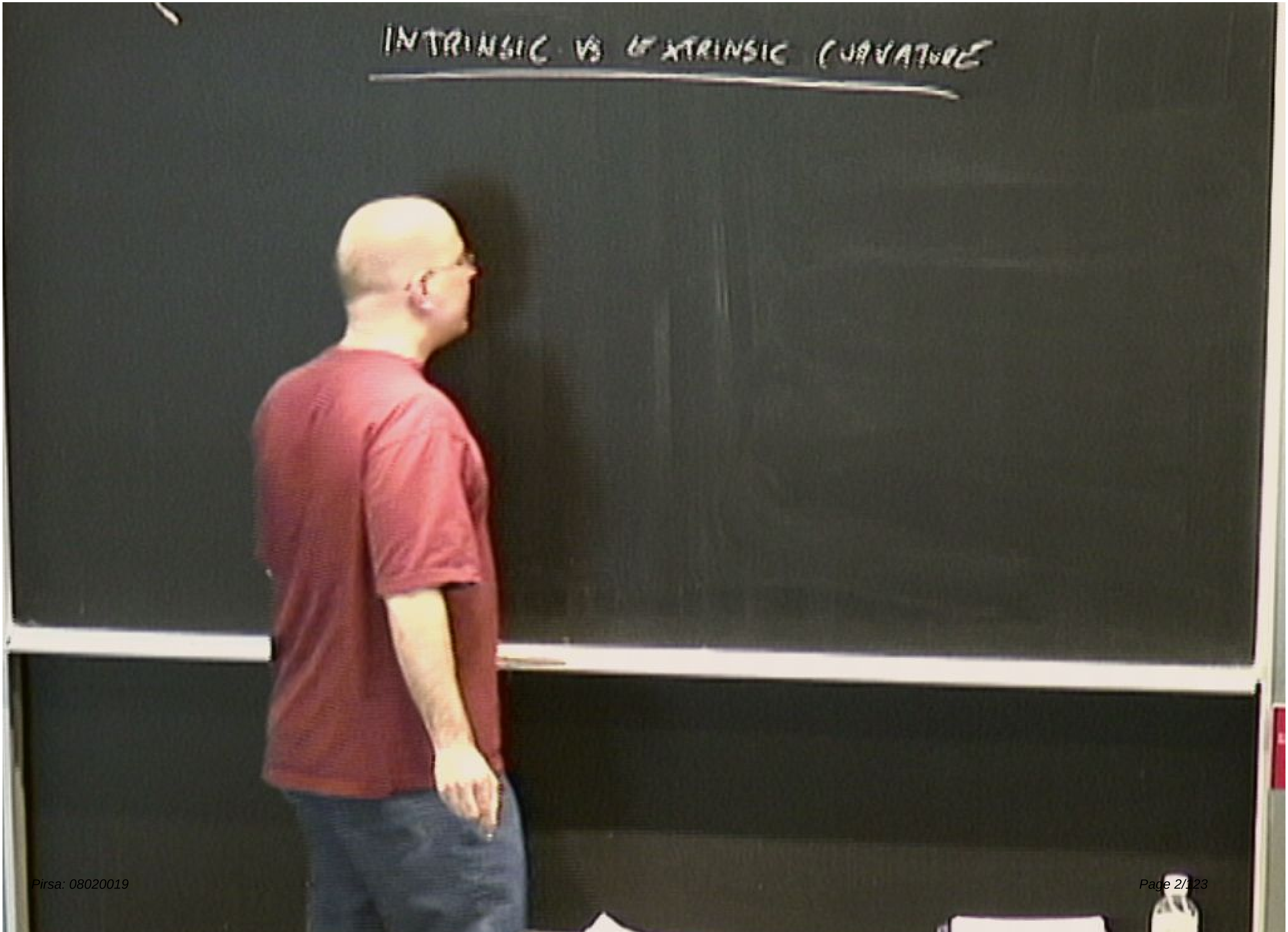
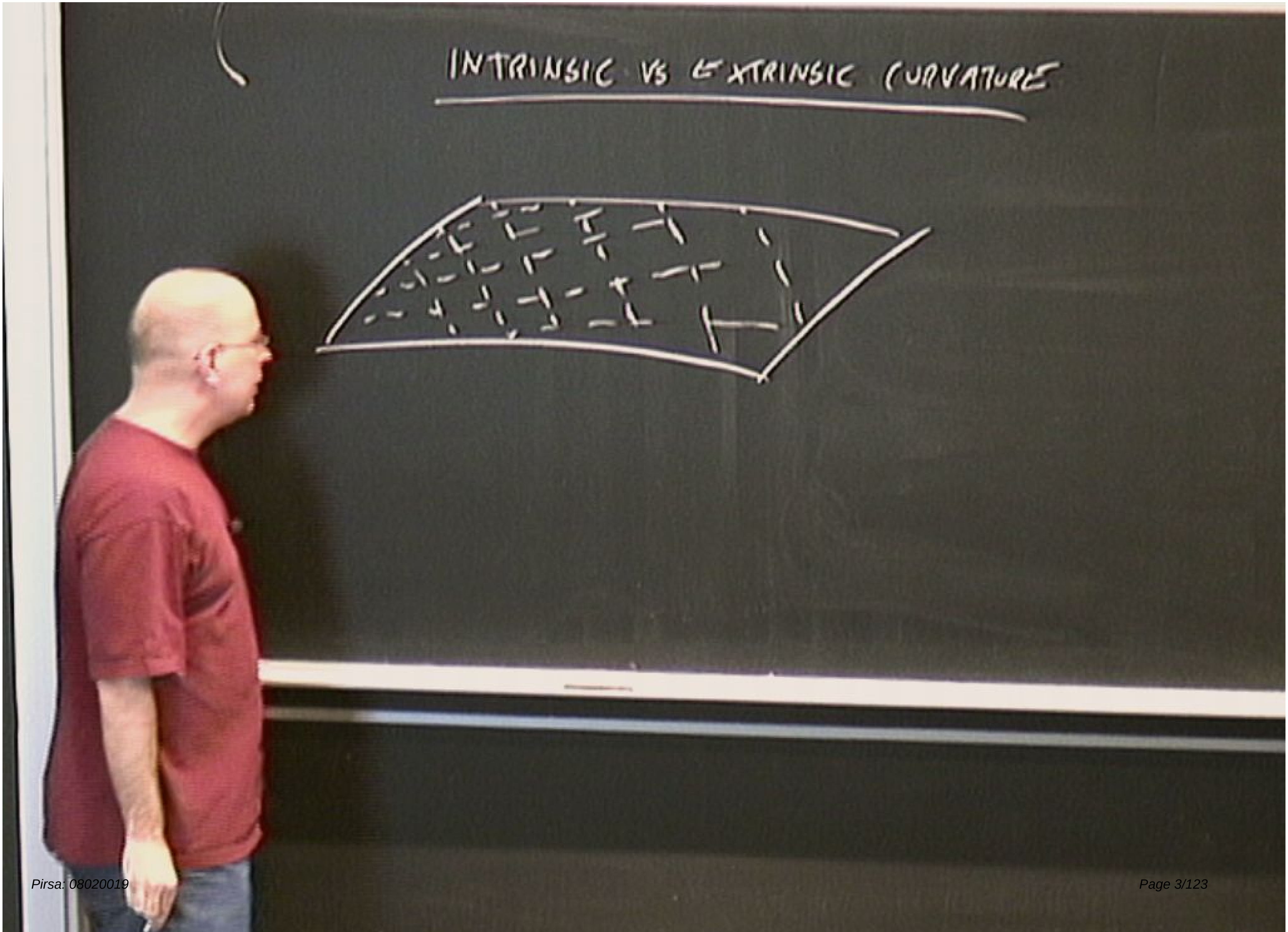
Title: Advanced General Relativity - Lecture 7A

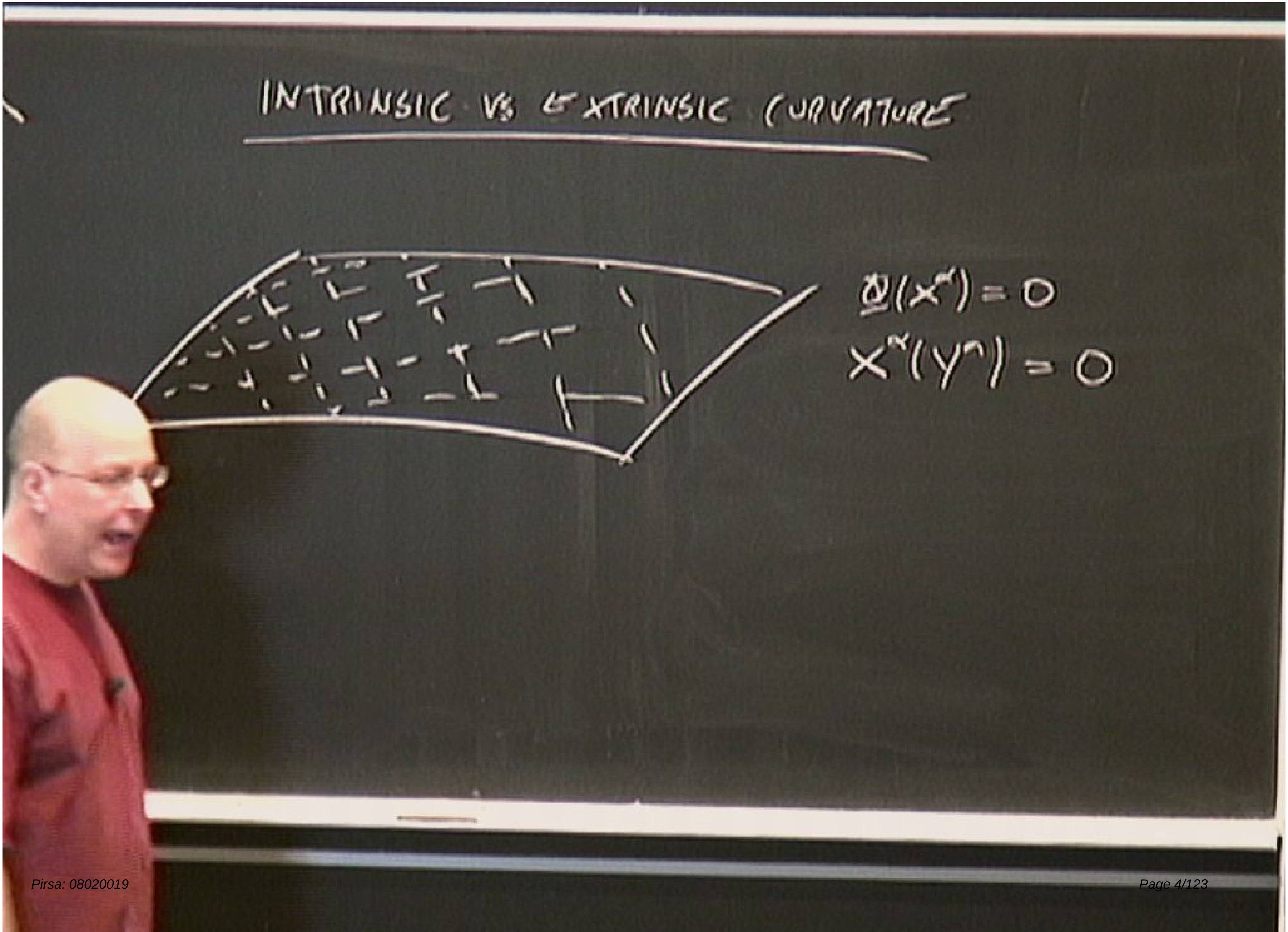
Date: Feb 27, 2008 10:30 AM

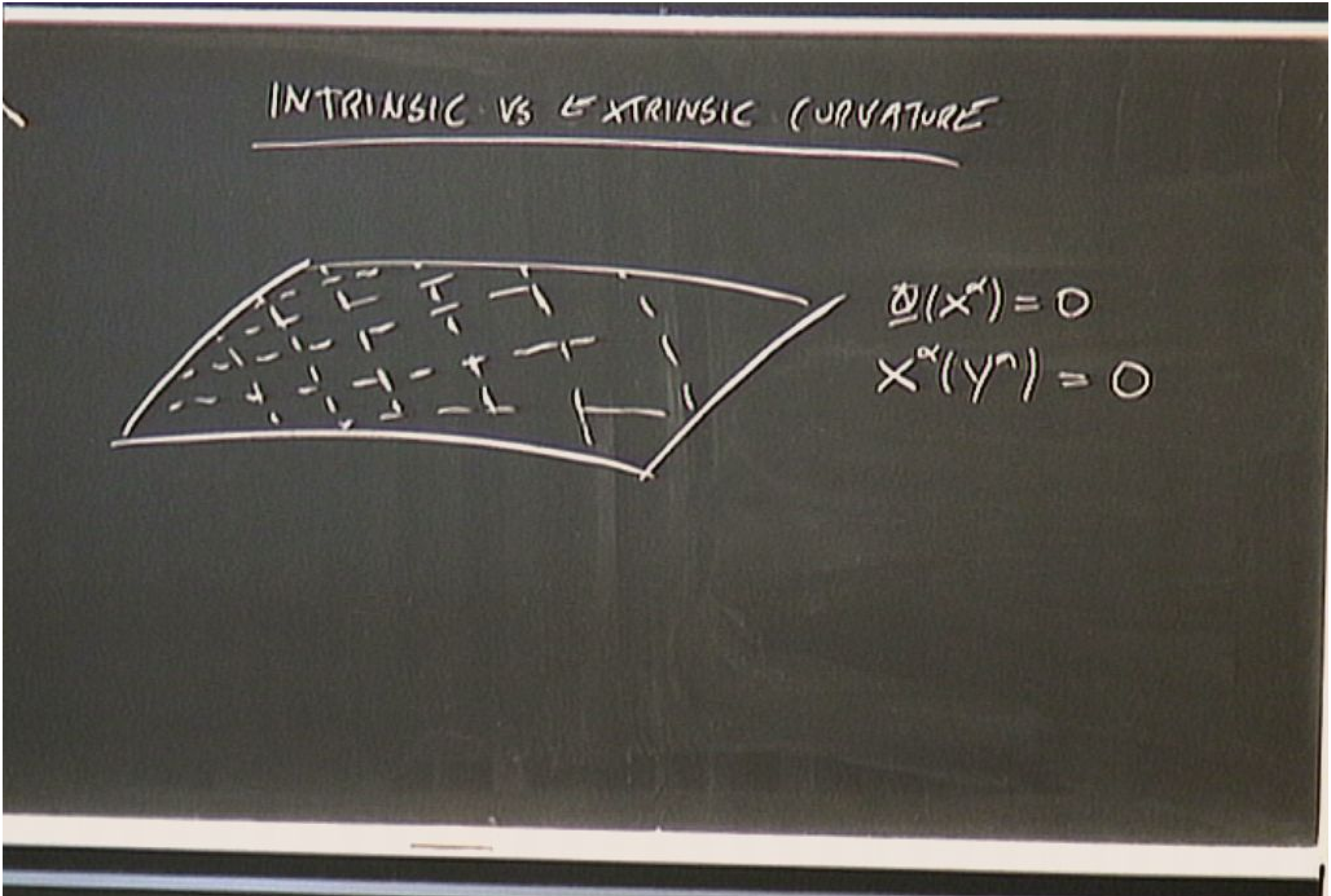
URL: http://pirsa.org/08020019

Abstract: Advanced General Relativity

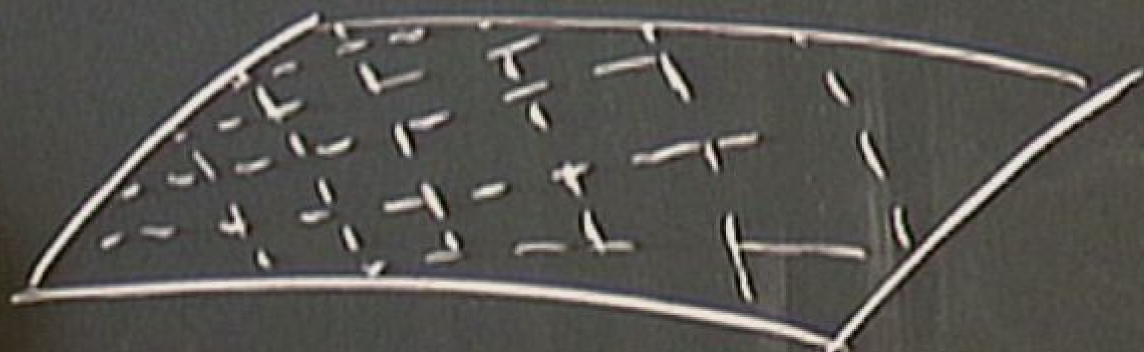

INTRINSIC VS EXTRINSIC CURVATURE


INTRINSIC VS EXTRINSIC CURVATURE


INTRINSIC VS EXTRINSIC CURVATURE
$\Phi(x^\alpha) = 0$
$X^\alpha(Y^a) = 0$


INTRINSIC VS EXTRINSIC CURVATURE
$\underline{\Phi}(X^\alpha) = 0$
$X^\alpha(Y^n) = 0$

INTRINSIC VS EXTRINSIC CURVATURE



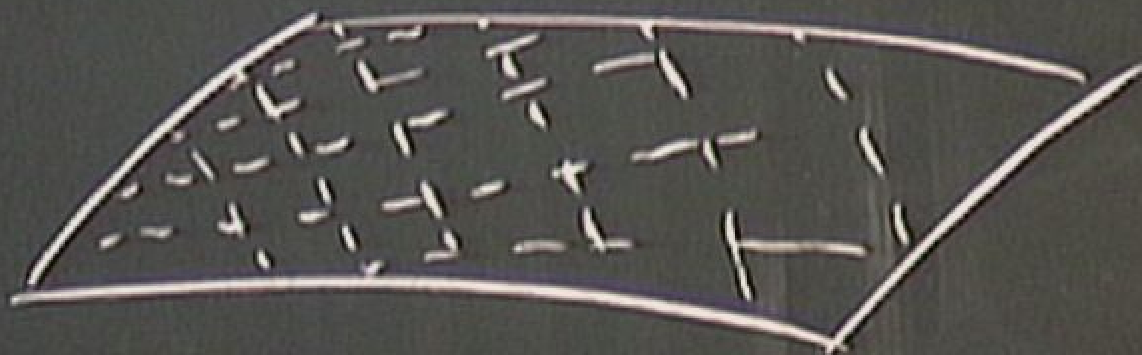
$$\underline{\Phi}(x^\alpha) = 0$$

$$x^\alpha(y^a) = 0$$

$$e^\alpha_a = \frac{\partial x^\alpha}{\partial y^a}$$

$$n_\alpha \propto \partial_\alpha \underline{\Phi}$$

INTRINSIC VS EXTRINSIC CURVATURE



$$\Phi(x^\alpha) = 0$$

$$x^\alpha(y^a) = 0$$

$$e^\alpha_a = \frac{\partial x^\alpha}{\partial y^a}$$

$$n_\alpha \propto \partial_\alpha \Phi$$

$$h_{ab} = g_{\alpha\beta} e^\alpha_a e^\beta_b$$

1- Gaussian normal coordinates

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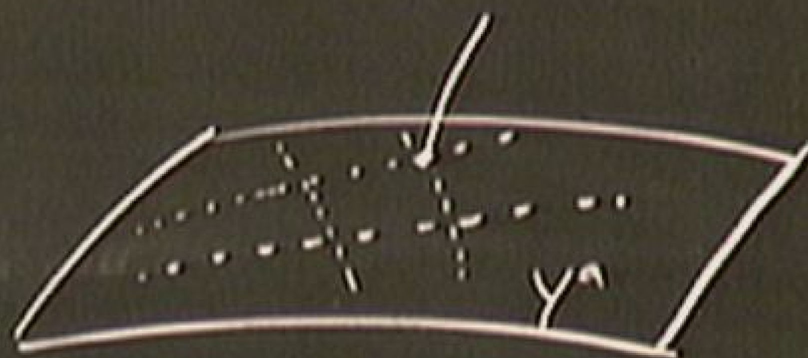
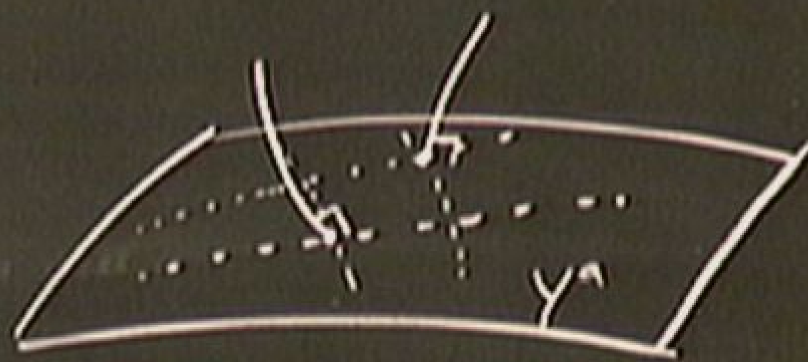
$$X^a = Y^a$$

1- Gaussian normal coordinates



$$x^a = y^a$$

1- Gaussian normal coordinates



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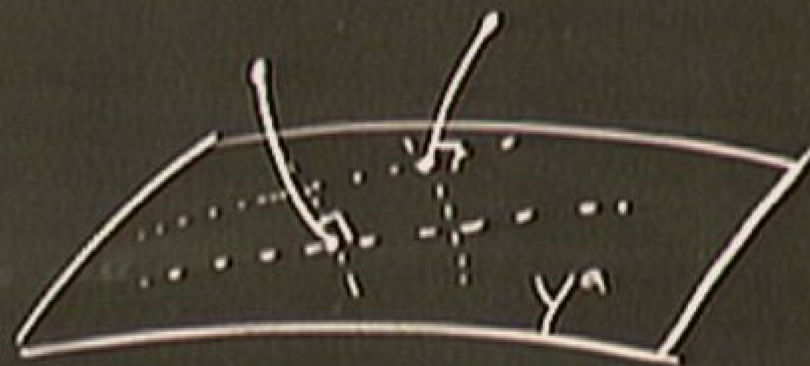
1- Gaussian normal coordinates



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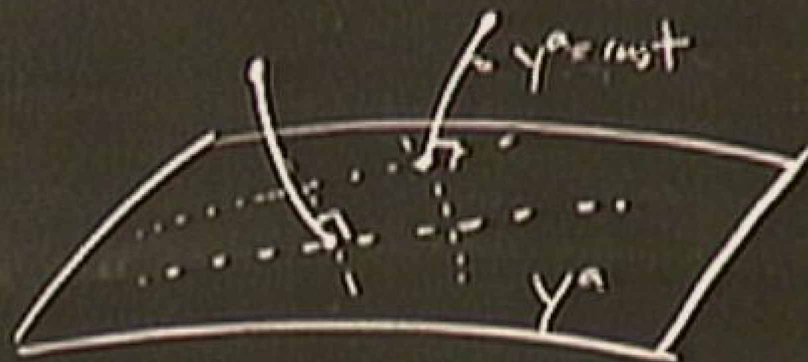

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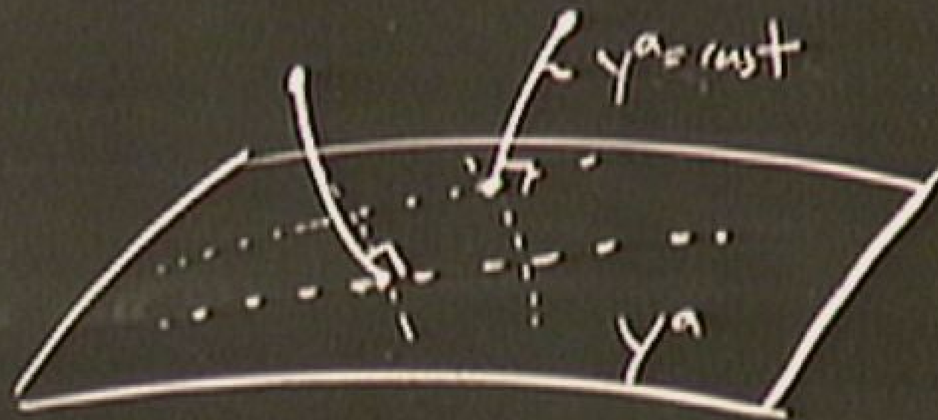

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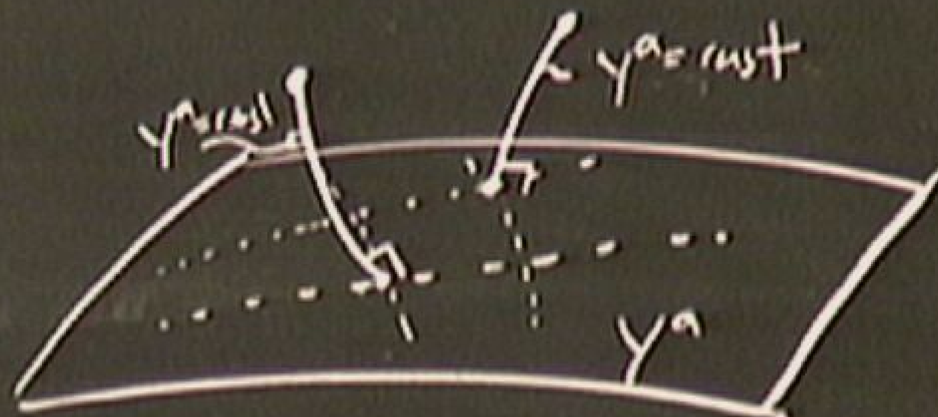

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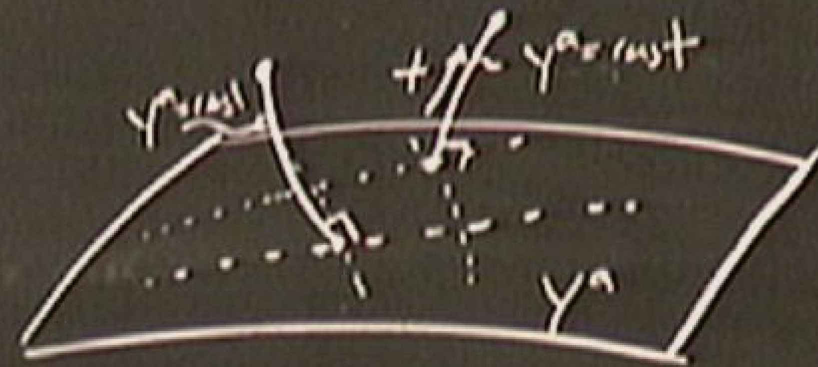
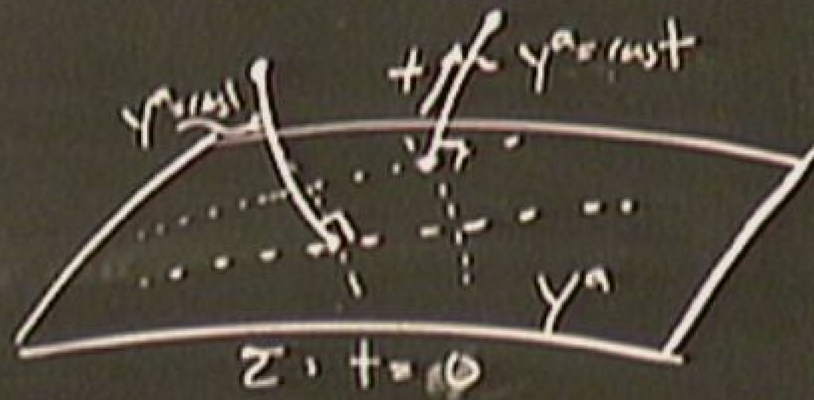


$$X^a = Y^a$$

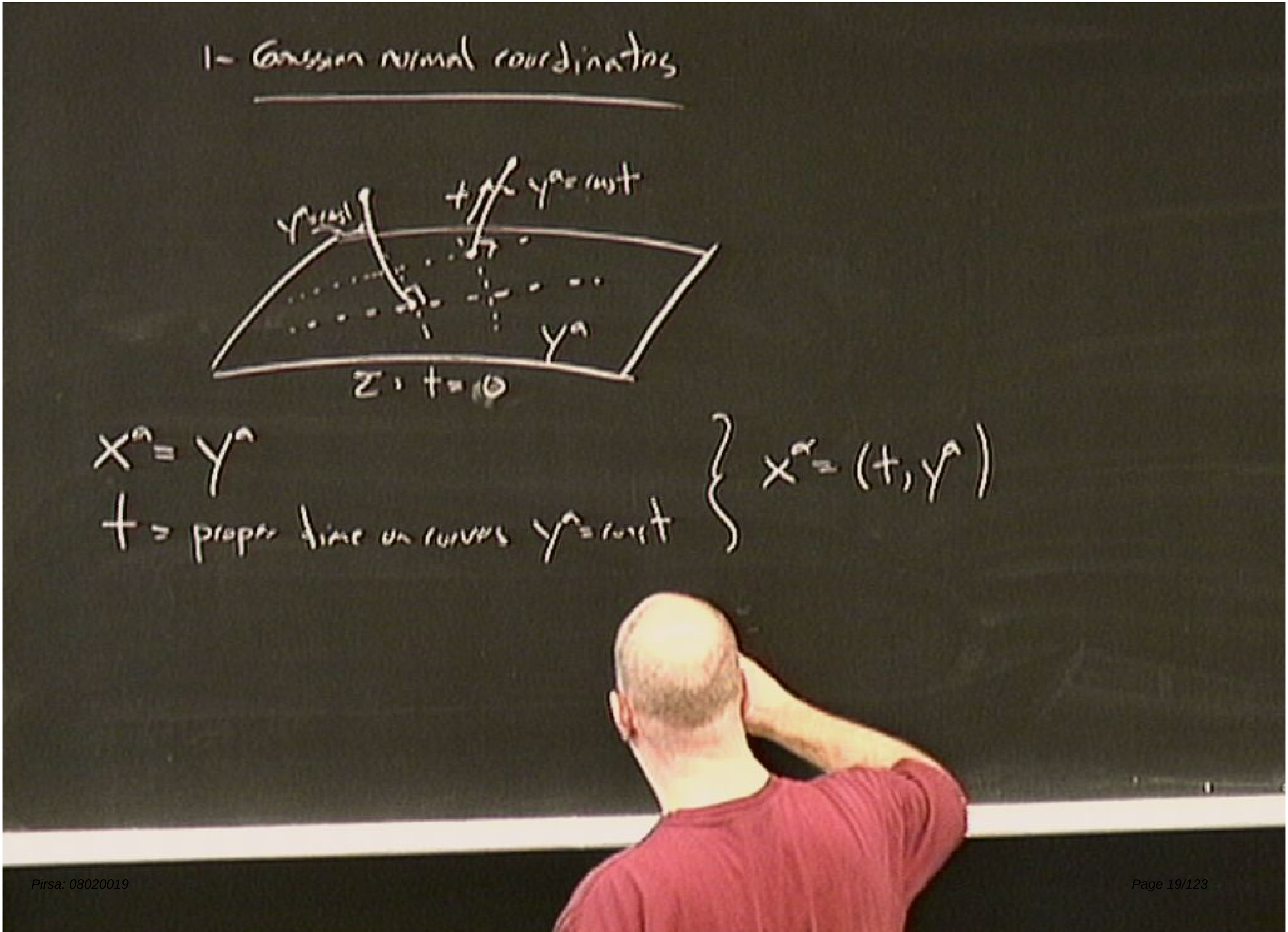
t = proper time on curves $Y^a = \text{const}$

1- Gaussian normal coordinates

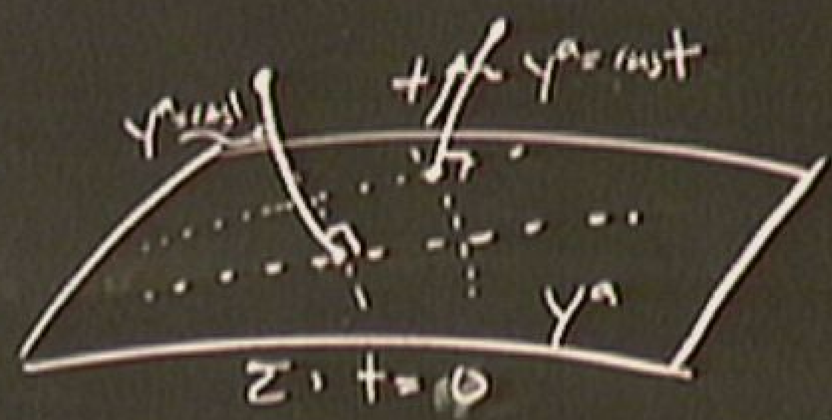


$X^a = Y^a$

t = proper time on curves y^a = const


1- Gaussian normal coordinates
t, y^a = const
y^a = const
y^a
Σ : t = 0
X^a = Y^a
t = proper time on curves y^a = const
x^α = (t, y^a)

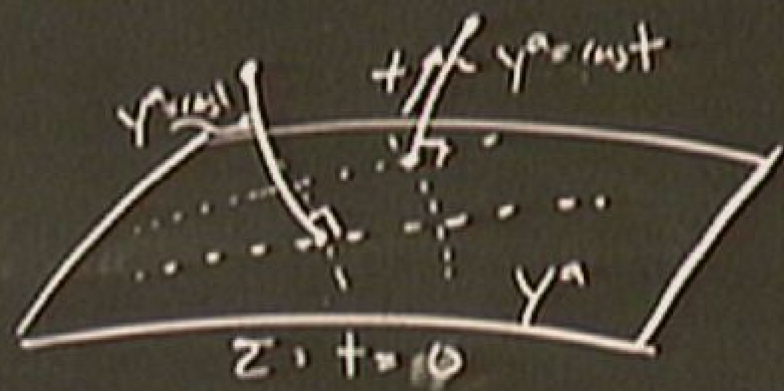
1- Gaussian normal coordinates



$$\left.\begin{array}{l} x^a = y^a \\ t = \text{proper time on curves } y^a = \text{const} \end{array}\right\} \; x^\alpha = (t, y^a)$$

$$ds^2 = -dt^2$$

1- Gaussian normal coordinates



$$x^a = y^a$$

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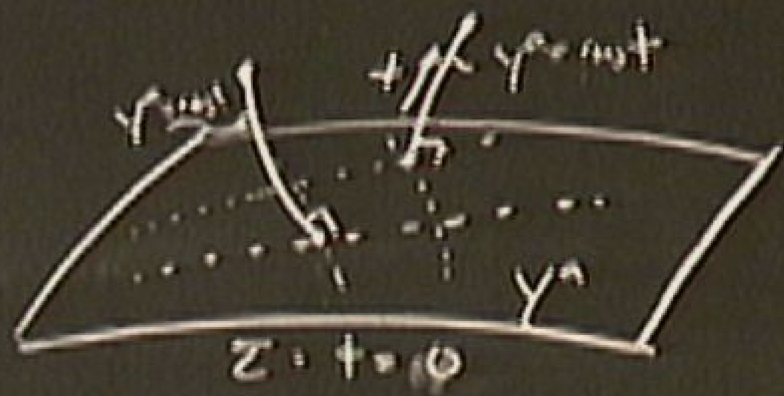
$$ds^2 = -dt^2 + g_{ab}(t)\, dy^a dy^b$$

1- Gaussian normal coordinates



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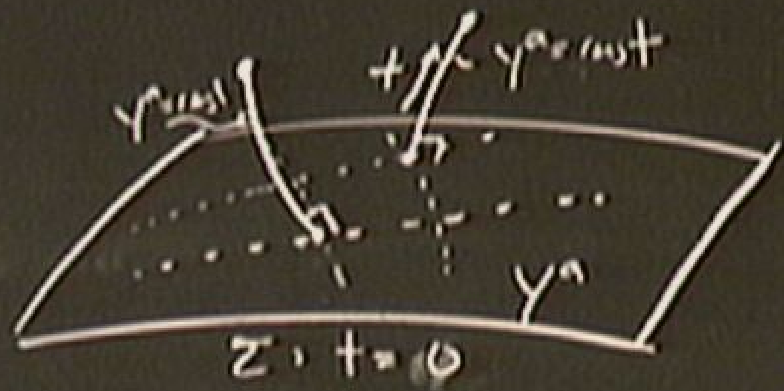
$$ds^2 = -dt^2 + g_{ab}(t, y)\, dy^a dy^b$$

1- Gaussian normal coordinates



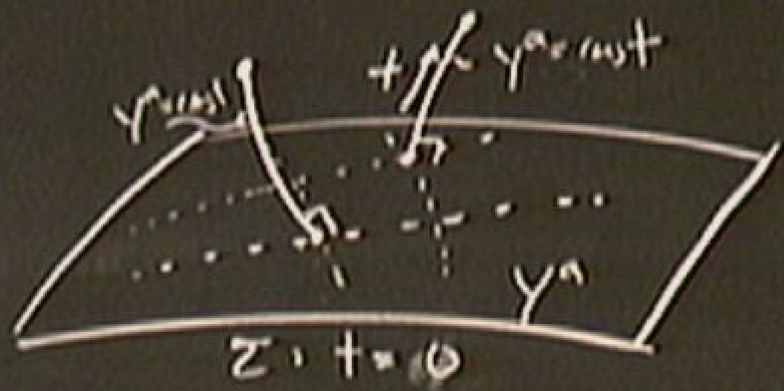
$$x^a = y^a$$

t = proper time on curves y^a=const $\Big\}$ $x^\alpha = (t, y^a)$

$$ds^2 = -dt^2 + g_{ab}(t, y^a)\, dy^a dy^b$$

$$g_{ab}(0, y^a) = h_{ab}$$

1 - Gaussian normal coordinates



$$\left.\begin{array}{l} x^a = y^a \\ t = \text{proper time on curves } y^a = \text{const} \end{array}\right\} \; x^\alpha = (t, y^a)$$

$$\boxed{\begin{array}{c} ds^2 = -dt^2 + g_{ab}(t, y)\, dy^a dy^b \\ g_{ab}(0, y^a) = h_{ab} \end{array}}$$

2- Connections

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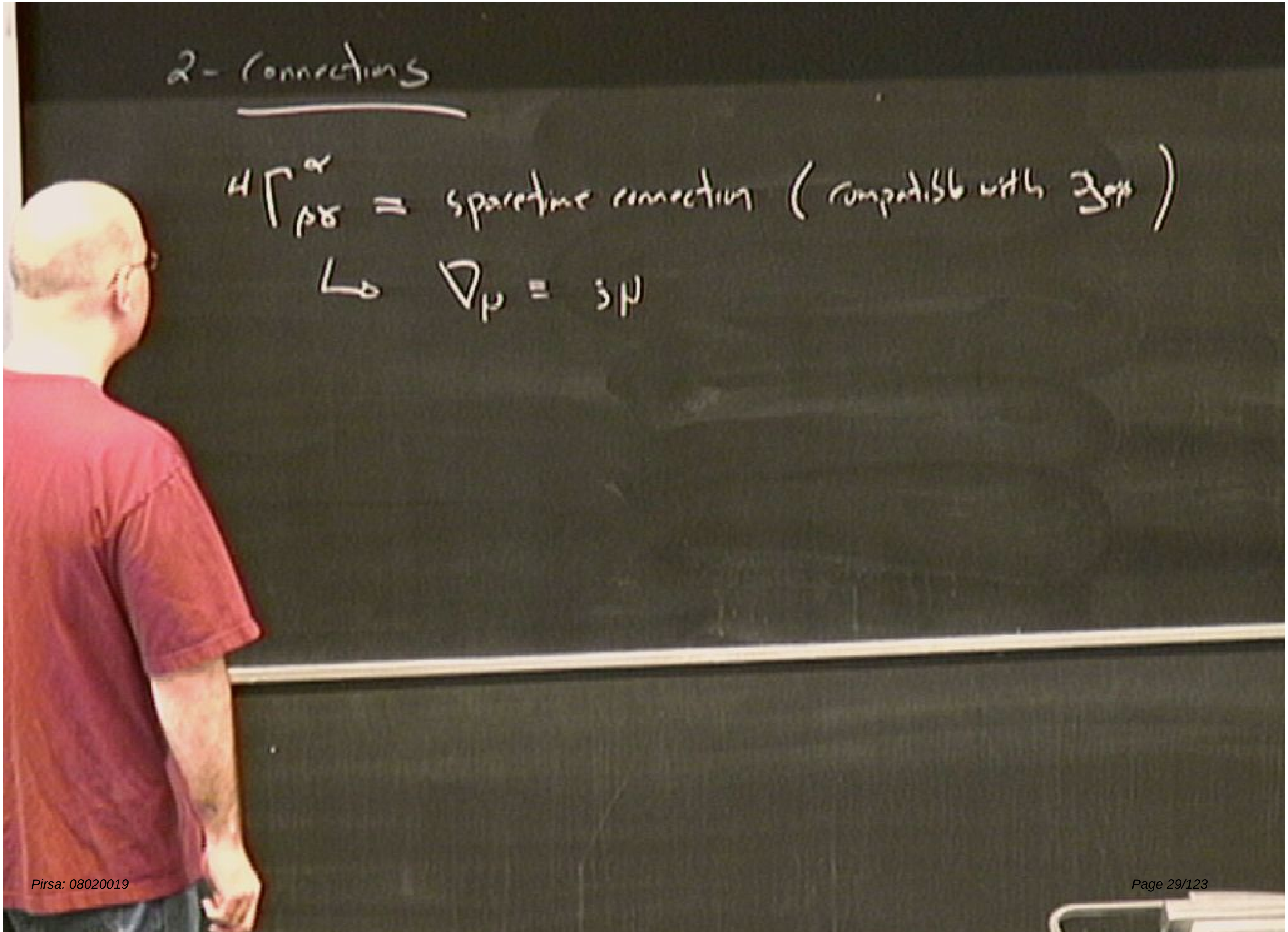
$\hookrightarrow \nabla_\mu$


2 - Connections
$^4\Gamma^\alpha_{\beta\gamma}$ = spacetime connection (compatible with $g_{\alpha\beta}$)
↳ $\nabla_\mu = \partial_\mu$

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$$ \hookrightarrow \nabla_{\mu} = \partial_{\mu} $$

$$ \Gamma^{a}_{bc} $$

2- Connections

$^4\Gamma^{\alpha}_{\beta\gamma}$ = spacetime connection (compatible with $g_{\alpha\beta}$)

↳ $\nabla_\mu = \partial_\mu$

Γ^{a}_{bc} = hypersurface connection (compatible with h_{ab})

↳ D_a

2 - Connections

$${}^{4}\Gamma^{\alpha}_{\beta\gamma} = \text{spacetime connection (compatible with } g_{\alpha\beta})$$

$$\hookrightarrow \nabla_\mu \equiv ;\mu$$

$$\Gamma^{a}_{bc} = \text{hypersurface connection (compatible with } h_{ab})$$

$$\hookrightarrow D_a \equiv |a$$

curves $y^a = \text{const}$ $\Big\}$ $x^\alpha = (t, y^a)$

$$\boxed{\begin{aligned} ds^2 &= -dt^2 + g_{ab}(t, y^a)\, dy^a dy^b \\ g_{ab}(0, y^a) &= h_{ab} \end{aligned}}$$

$g_{\alpha\beta} = \begin{pmatrix} -1 & 0 \\ 0 & g_{ab} \end{pmatrix}$

$g^{\alpha\beta} = ($

spacetime connection (compatible with $g_{\alpha\beta}$)

$\nabla_\mu = \partial_\mu$

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$$g_{\alpha\beta} = \begin{pmatrix} -1 & 0 \\ 0 & g_{ab} \end{pmatrix}$$

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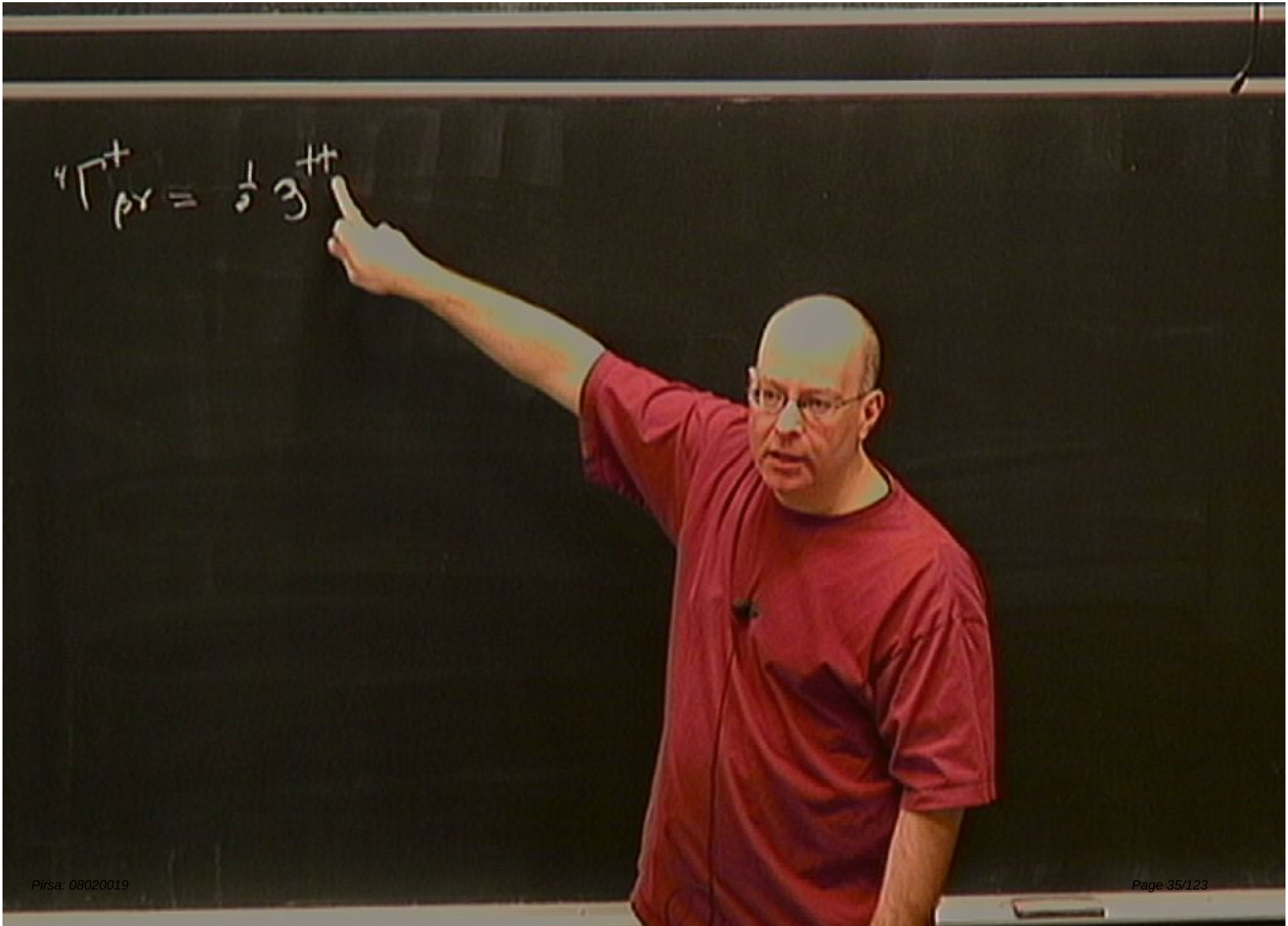
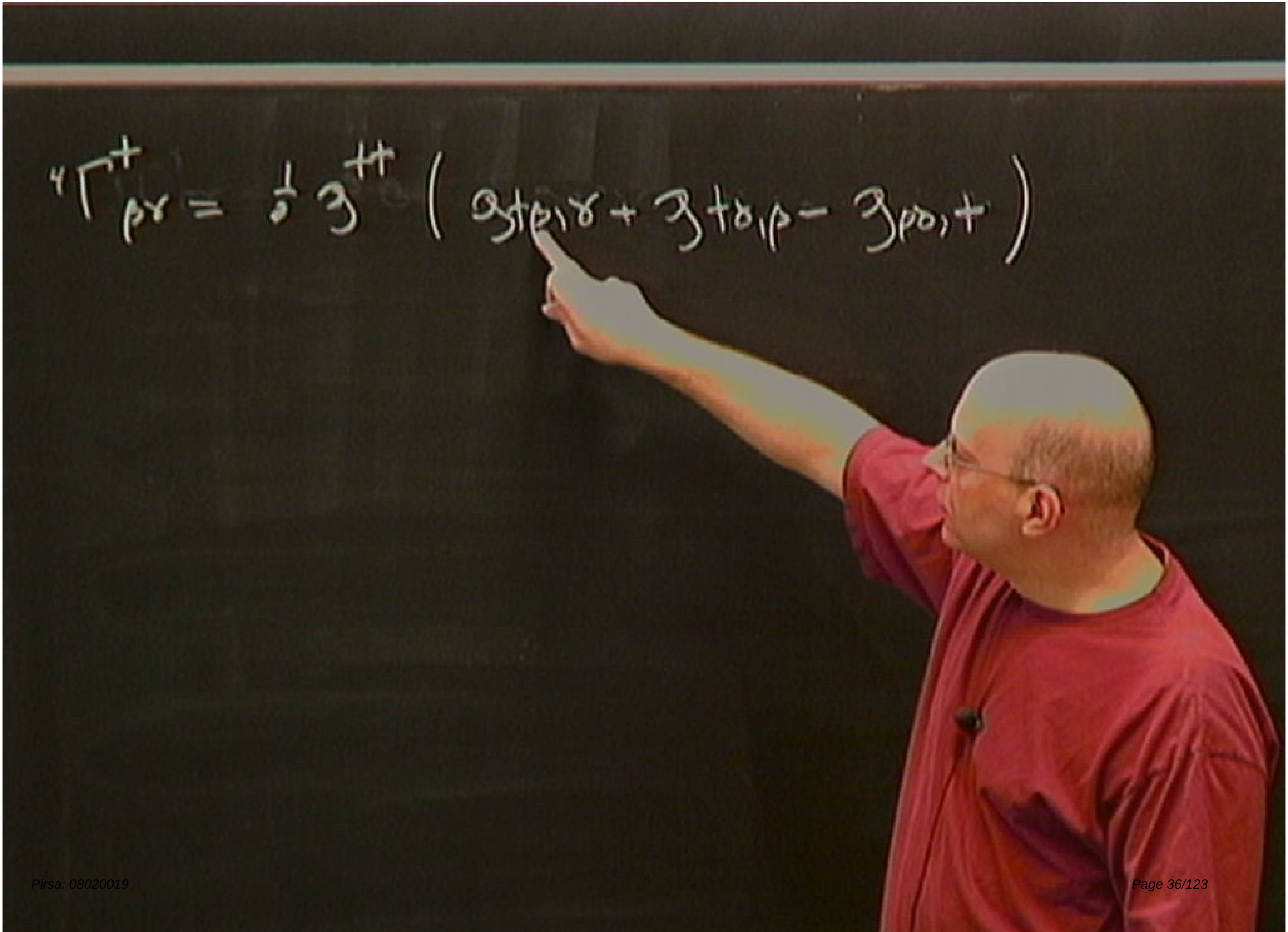
spacetime connection (compatible with $g_{\alpha\beta}$)

$\nabla_\mu = \partial_\mu$

= hypersurface connection (compatible with h_{ab})


${}^{4}\Gamma^{+}_{\rho\gamma} = \frac{1}{2} g^{++}$


${}^4\Gamma^{t}_{\rho\gamma} = \frac{1}{2} g^{tt} \left(g_{t\rho,\gamma} + g_{t\gamma,\rho} - g_{\rho\gamma,t} \right)$

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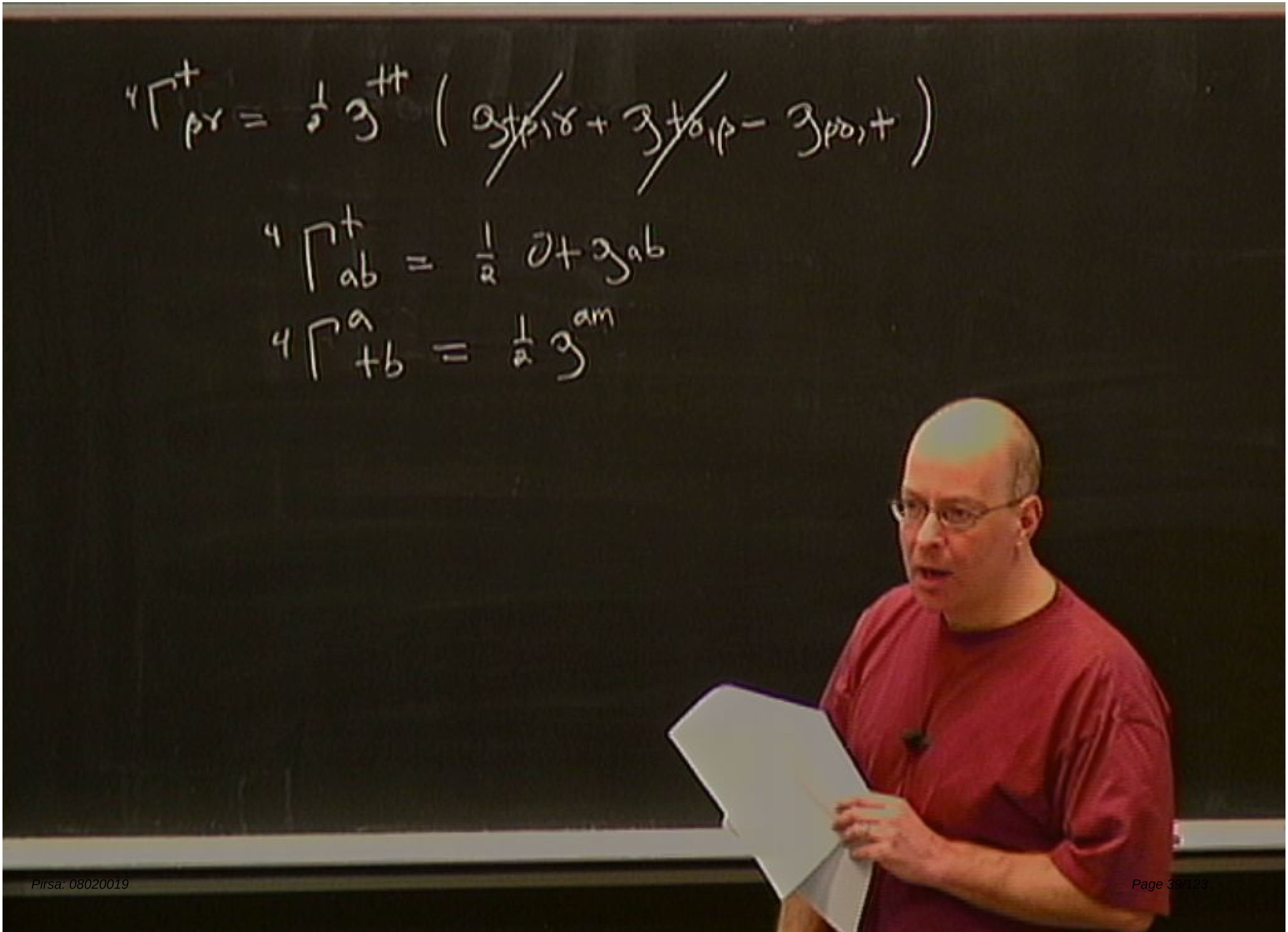
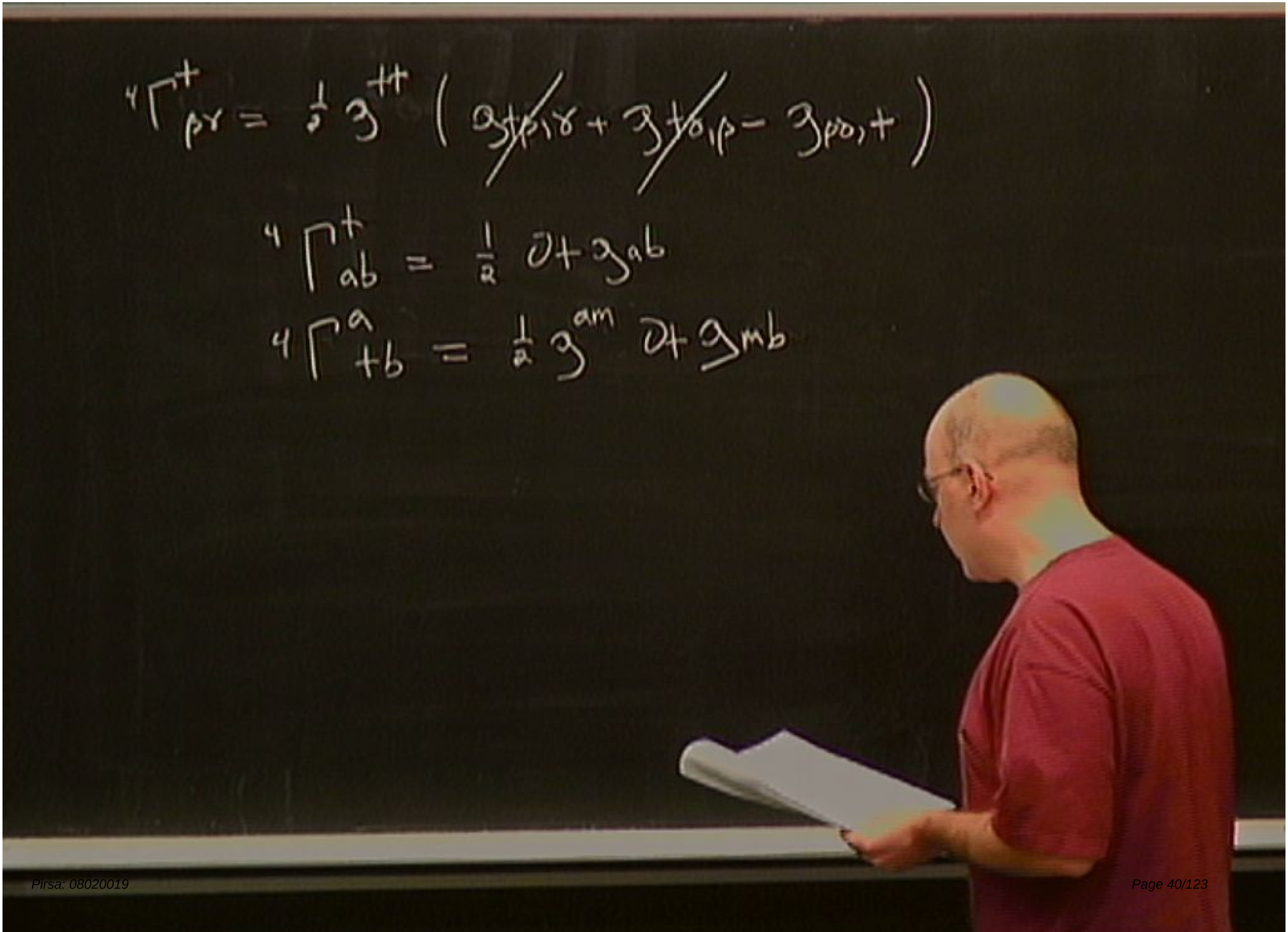
$${}^4\Gamma^{t}_{ab} = \frac{1}{2} \partial_t g_{ab}$$


$^4\Gamma^t_{\rho\gamma} = \frac{1}{2} g^{tt} \left(\cancel{g_{t\rho,\gamma}} + \cancel{g_{t\gamma,\rho}} - g_{\rho\gamma,t} \right)$
$^4\Gamma^t_{ab} = \frac{1}{2} \partial_t g_{ab}$
$^4\Gamma^a_{tb} = \frac{1}{2} g^{am}$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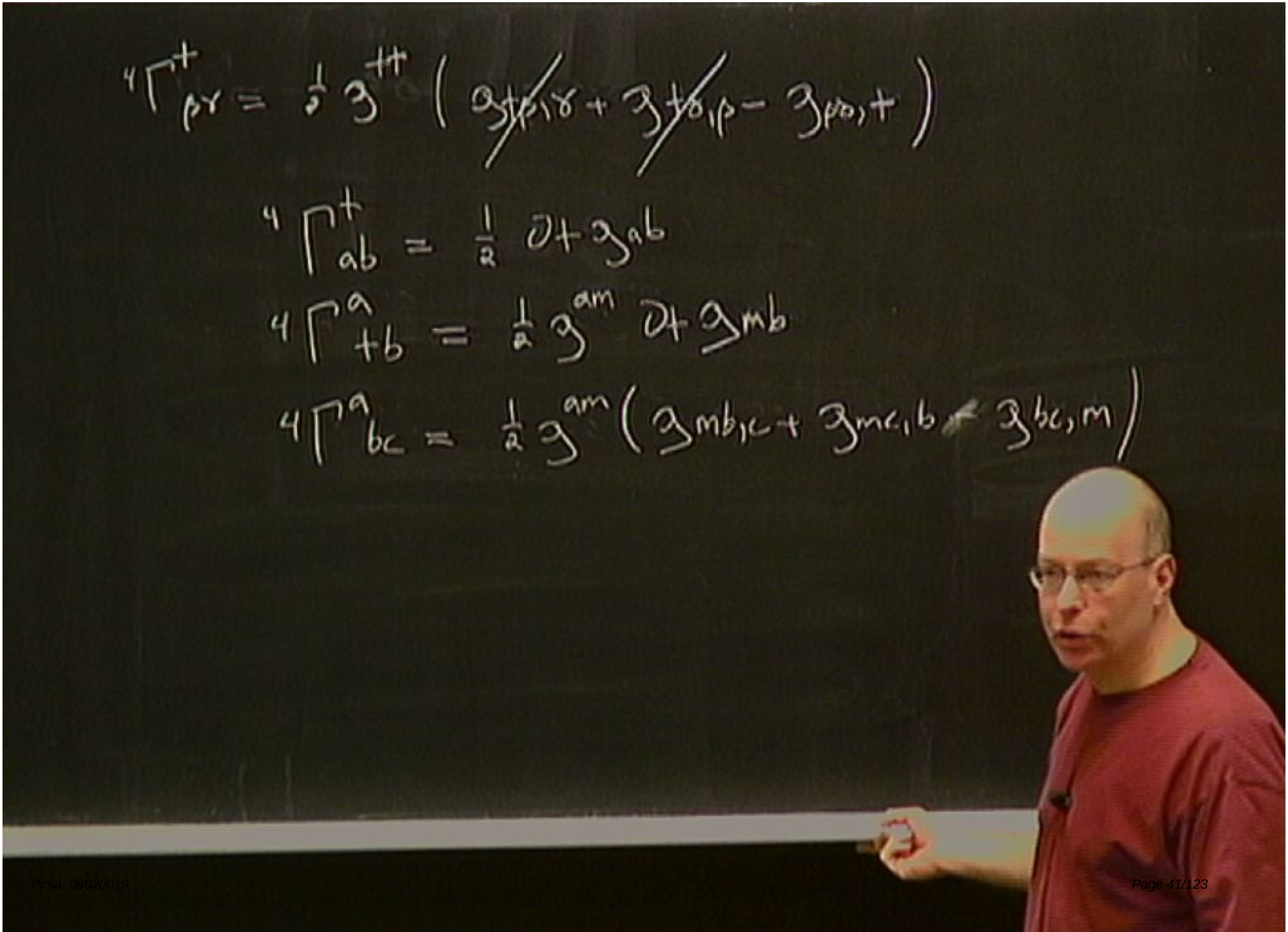
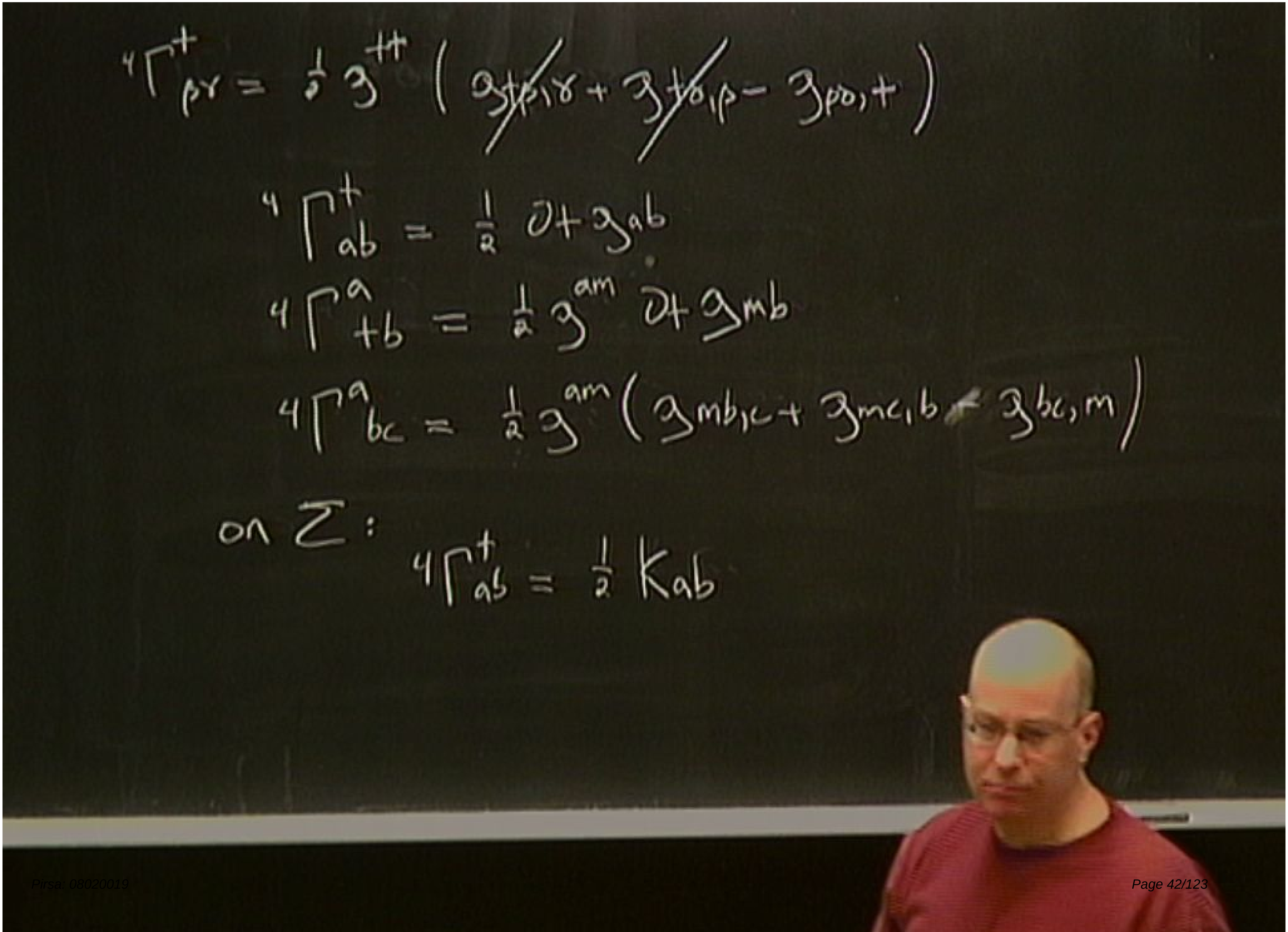

${}^4\Gamma^t_{pr} = \frac{1}{2} g^{tt} \left(\cancel{g_{tp,r}} + \cancel{g_{tr,p}} - g_{pr,t} \right)$
${}^4\Gamma^t_{ab} = \frac{1}{2} \partial_t g_{ab}$
${}^4\Gamma^a_{tb} = \frac{1}{2} g^{am} \partial_t g_{mb}$
${}^4\Gamma^a_{bc} = \frac{1}{2} g^{am} \left(g_{mb,c} + g_{mc,b} - g_{bc,m} \right)$


$^4\Gamma^t_{\rho\gamma} = \frac{1}{2} g^{tt} \left(\cancel{g_{t\rho,\gamma}} + \cancel{g_{t\sigma,\rho}} - g_{\rho\sigma,t} \right)$
$^4\Gamma^t_{ab} = \frac{1}{2} \partial_t g_{ab}$
$^4\Gamma^a_{tb} = \frac{1}{2} g^{am} \partial_t g_{mb}$
$^4\Gamma^a_{bc} = \frac{1}{2} g^{am} \left(g_{mb,c} + g_{mc,b} - g_{bc,m} \right)$
on Σ:
$^4\Gamma^t_{ab} = \frac{1}{2} K_{ab}$

$$ {}^4\Gamma^t{}_{\rho\gamma} = \frac{1}{2} g^{tt}\left(\cancel{g_{t\rho,\gamma}} + \cancel{g_{t\gamma,\rho}} - g_{\rho\gamma,t} \right) $$

$$ {}^4\Gamma^t{}_{ab} = \frac{1}{2}\, \partial_t g_{ab} $$

$$ {}^4\Gamma^a{}_{tb} = \frac{1}{2} g^{am}\, \partial_t g_{mb} $$

$$ {}^4\Gamma^a{}_{bc} = \frac{1}{2} g^{am}\left(g_{mb,c} + g_{mc,b} - g_{bc,m} \right) $$

on Σ:

$$ {}^4\Gamma^t{}_{ab} = \frac{1}{2} K_{ab} $$

$$ {}^4\Gamma^a{}_{tb} = \frac{1}{2} K^a{}_b $$

$$ {}^4\Gamma^a{}_{bc} = \Gamma^a{}_{bc} $$

$$^4\Gamma^t_{\rho\gamma} = \frac{1}{2} g^{tt}\left(\cancel{g_{t\rho,\gamma}} + \cancel{g_{t\gamma,\rho}} - g_{\rho\gamma,t}\right)$$

$$^4\Gamma^t_{ab} = \frac{1}{2}\partial_t g_{ab}$$

$$^4\Gamma^a_{tb} = \frac{1}{2} g^{am}\partial_t g_{mb}$$

$$^4\Gamma^a_{bc} = \frac{1}{2} g^{am}\left(g_{mb,c} + g_{mc,b} - g_{bc,m}\right)$$

on Σ:

$$^4\Gamma^t_{ab} = K_{ab} \qquad K_{ab} = \frac{1}{2}\partial_t g_{ab}(t=0)$$

$$^4\Gamma^a_{tb} = K^a{}_b$$

$$^4\Gamma^a_{bc} = \Gamma^a_{bc}$$

$${}^4\Gamma^t{}_{\rho\gamma} = \frac{1}{2} g^{tt} \left(\cancel{g_{t\rho,\gamma}} + \cancel{g_{t\gamma,\rho}} - g_{\rho\gamma,t} \right)$$

$${}^4\Gamma^t{}_{ab} = \frac{1}{2} \partial_t g_{ab}$$

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on Σ:

$${}^4\Gamma^t{}_{ab} = K_{ab} \qquad K_{ab} = \frac{1}{2} \partial_t g_{ab} (t=0)$$

$${}^4\Gamma^a{}_{tb} = K^a{}_b \qquad K^a{}_b = h^{ac} K_{cb}$$

$${}^4\Gamma^a{}_{bc} = \Gamma^a{}_{bc}$$

$$ {}^4\Gamma^t_{\rho\sigma} = \frac{1}{2} g^{tt}\left(\cancel{g_{t\rho,\sigma}} + \cancel{g_{t\sigma,\rho}} - g_{\rho\sigma,t}\right) $$

$$ {}^4\Gamma^t_{ab} = \frac{1}{2}\partial_t g_{ab} $$

$$ {}^4\Gamma^a_{tb} = \frac{1}{2} g^{am}\,\partial_t g_{mb} $$

$$ {}^4\Gamma^a_{bc} = \frac{1}{2} g^{am}\left(g_{mb,c} + g_{mc,b} - g_{bc,m}\right) $$

on Σ:

$$ \boxed{\begin{array}{ll} {}^4\Gamma^t_{ab} = K_{ab} & K_{ab} = \frac{1}{2}\partial_t g_{ab}\,(t=0) \\ {}^4\Gamma^a_{tb} = K^a{}_b & K^a{}_b = h^{ac}K_{cb} \\ {}^4\Gamma^a_{bc} = \Gamma^a_{bc} & \end{array}} $$

$${}^4\Gamma^t_{\rho\gamma} = \frac{1}{2} g^{tt}\left(\cancel{g_{t\rho,\gamma}} + \cancel{g_{t\gamma,\rho}} - g_{\rho\gamma,t}\right)$$

$${}^4\Gamma^t_{ab} = \frac{1}{2}\partial_t g_{ab}$$

$${}^4\Gamma^a_{tb} = \frac{1}{2} g^{am}\partial_t g_{mb}$$

$${}^4\Gamma^a_{bc} = \frac{1}{2} g^{am}\left(g_{mb,c} + g_{mc,b} - g_{bc,m}\right)$$

→ extrinsic [illegible]ture

on Σ:

$${}^4\Gamma^t_{ab} = K_{ab} \qquad K_{ab} = \frac{1}{2}\partial_t g_{ab}(t\ [illegible])$$

$${}^4\Gamma^a_{tb} = K^a{}_b \qquad K^a{}_b = h^{ac}K_{cb}$$

$${}^4\Gamma^a_{bc} = \Gamma^a_{bc}$$

$$ {}^4\Gamma^t_{\rho\gamma} = \frac{1}{2} g^{tt}\left(\cancel{g_{t\rho,\gamma}} + \cancel{g_{t\gamma,\rho}} - g_{\rho\gamma,t}\right) $$

$$ {}^4\Gamma^t_{ab} = \frac{1}{2}\partial_t g_{ab} $$

$$ {}^4\Gamma^a_{tb} = \frac{1}{2} g^{am}\partial_t g_{mb} $$

$$ {}^4\Gamma^a_{bc} = \frac{1}{2} g^{am}\left(g_{mb,c} + g_{mc,b} - g_{bc,m}\right) $$

extrinsic "curvature"

on Σ:

$$ \boxed{\begin{array}{ll} {}^4\Gamma^t_{ab} = K_{ab} & K_{ab} = \frac{1}{2}\partial_t g_{ab}(t=0) \\ {}^4\Gamma^a_{tb} = K^a{}_b & K^a{}_b = h^{ac}K_{cb} \\ {}^4\Gamma^a_{bc} = \Gamma^a_{bc} & \end{array}} $$

$${}^4\Gamma^t{}_{\rho\gamma} = \frac{1}{2} g^{tt} \left(\cancel{g_{t\rho,\gamma}} + \cancel{g_{t\gamma,\rho}} - g_{\rho\gamma,t} \right)$$

$${}^4\Gamma^t{}_{ab} = \frac{1}{2} \partial_t g_{ab}$$

$${}^4\Gamma^a{}_{tb} = \frac{1}{2} g^{am} \partial_t g_{mb}$$

$${}^4\Gamma^a{}_{bc} = \frac{1}{2} g^{am} \left(g_{mb,c} + g_{mc,b} - g_{bc,m} \right)$$

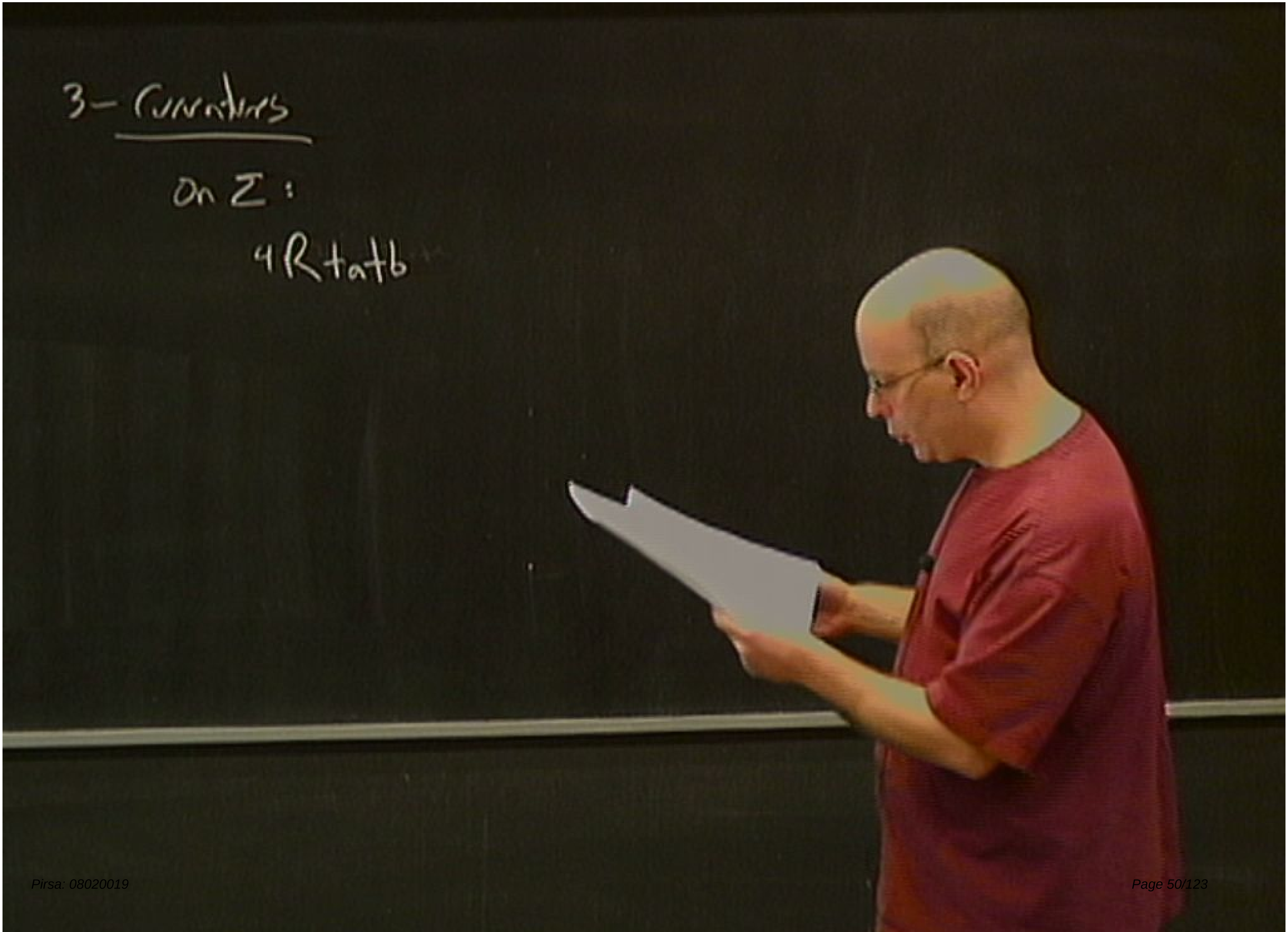
extrinsic "curvature"

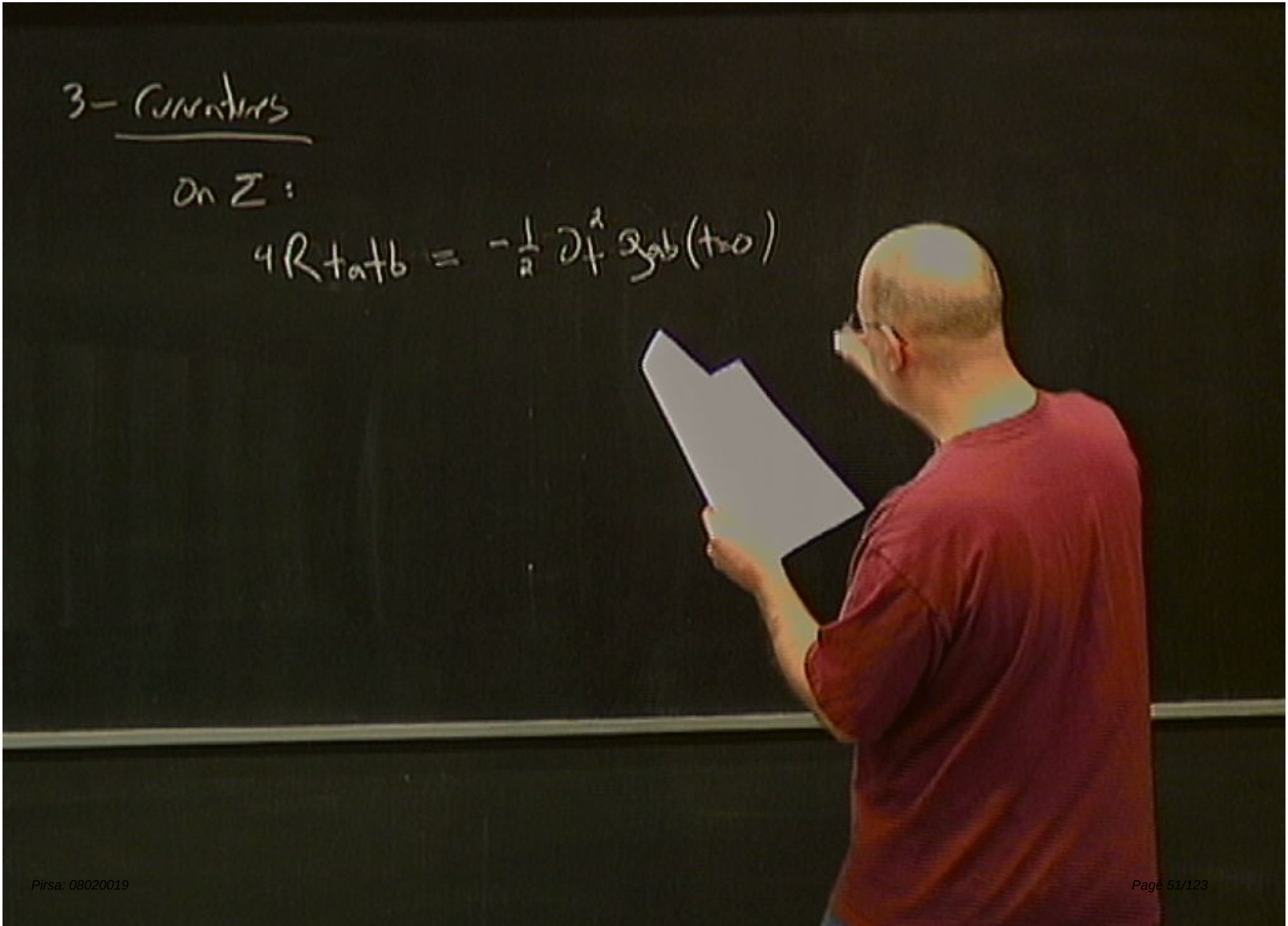
on Σ:

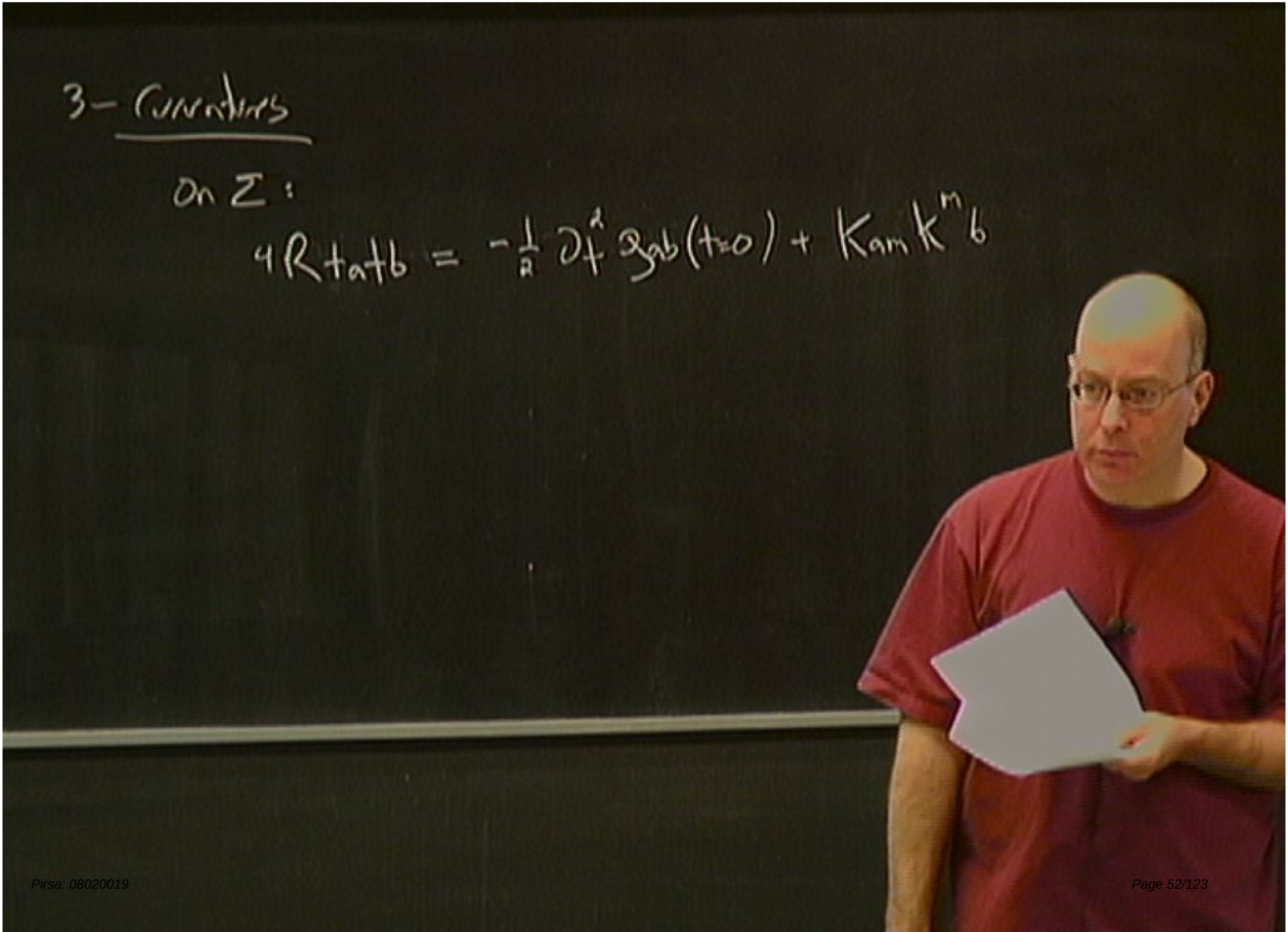
$${}^4\Gamma^t{}_{ab} = K_{ab} \qquad K_{ab} = \frac{1}{2} \partial_t g_{ab} (t=0)$$

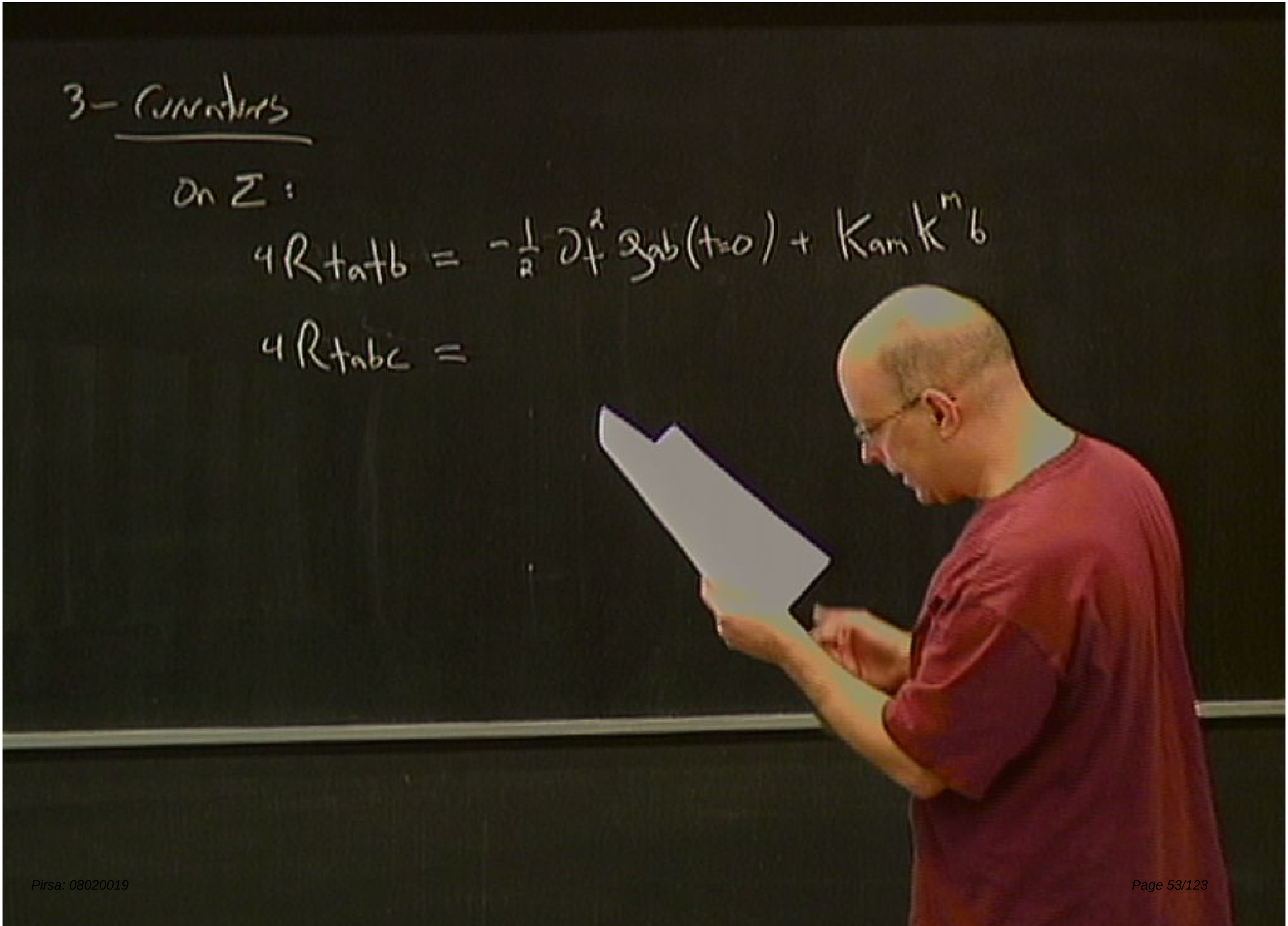
$${}^4\Gamma^a{}_{tb} = K^a{}_b \qquad K^a{}_b = [illegible] K_{cb}$$

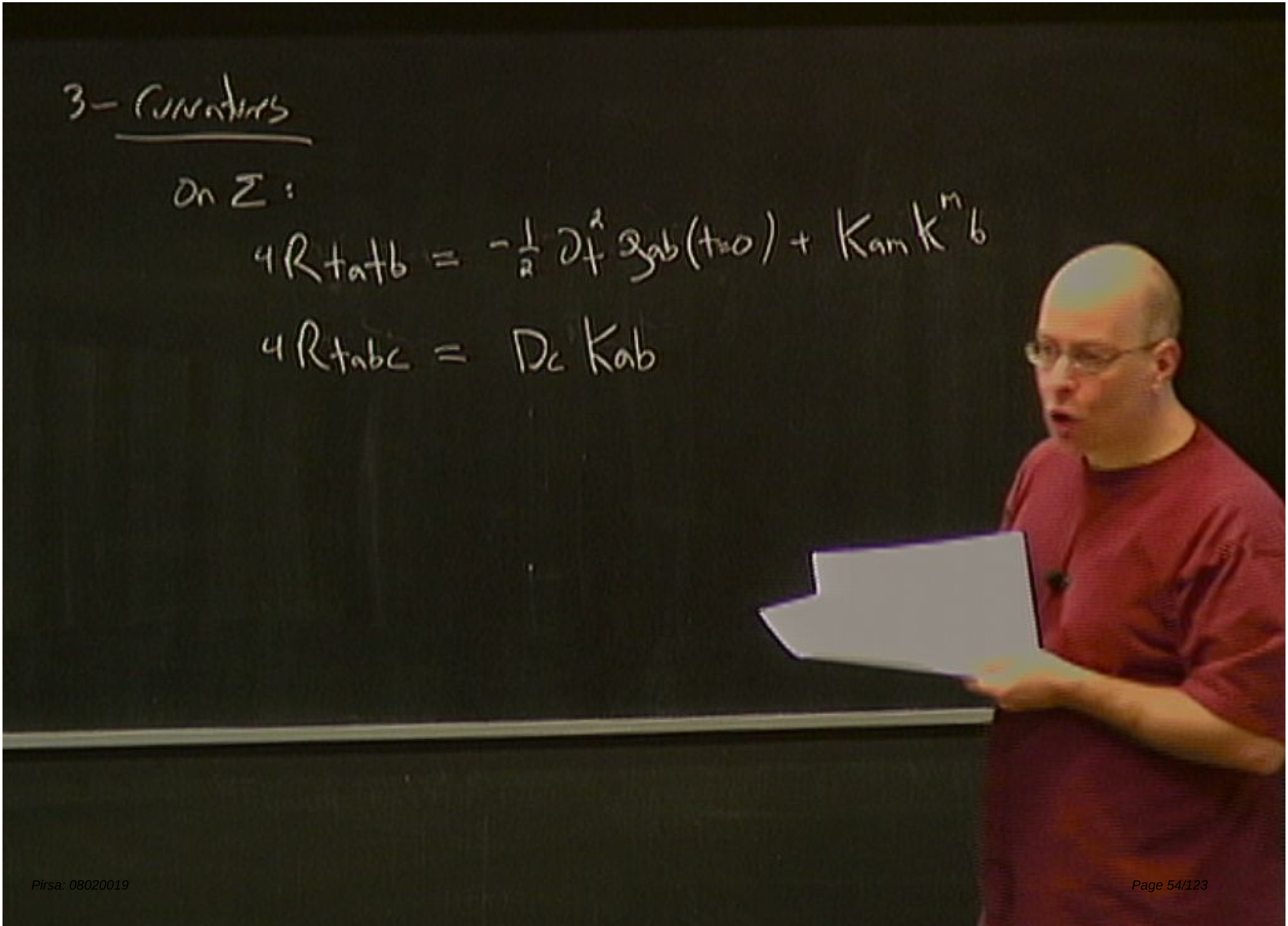
$${}^4\Gamma^a{}_{bc} = \Gamma^a{}_{bc}$$

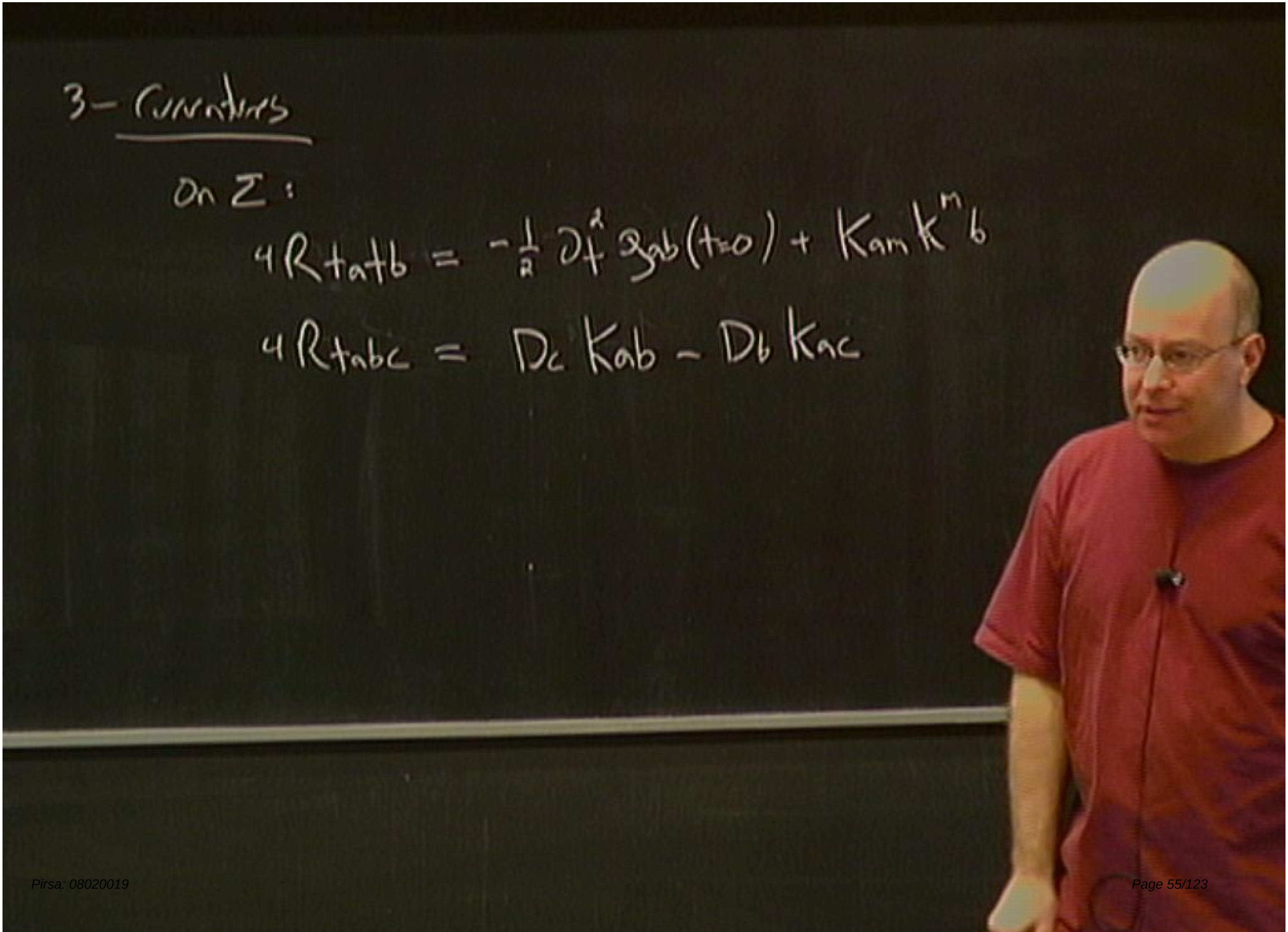

3- Curvatures
On Z:
4R+a+b


3 - Curvatures
On Σ:
$^4R_{tatb} = -\frac{1}{2}\partial_t^2 g_{ab}(t=0)$


3– Curvatures
On Σ :
$^{4}R_{tatb} = -\frac{1}{2}\partial_t^2 g_{ab}(t=0) + K_{am}K^{m}{}_{b}$


3- Curvatures
On Σ :
${}^4R_{tatb} = -\frac{1}{2}\partial_t^2 g_{ab}(t=0) + K_{am}K^m{}_b$
${}^4R_{tabc} =$


3 – Curvatures
On Σ:
$^4R_{tatb} = -\frac{1}{2}\partial_t^2 g_{ab}(t=0) + K_{am}K^m{}_b$
$^4R_{tabc} = D_c K_{ab}$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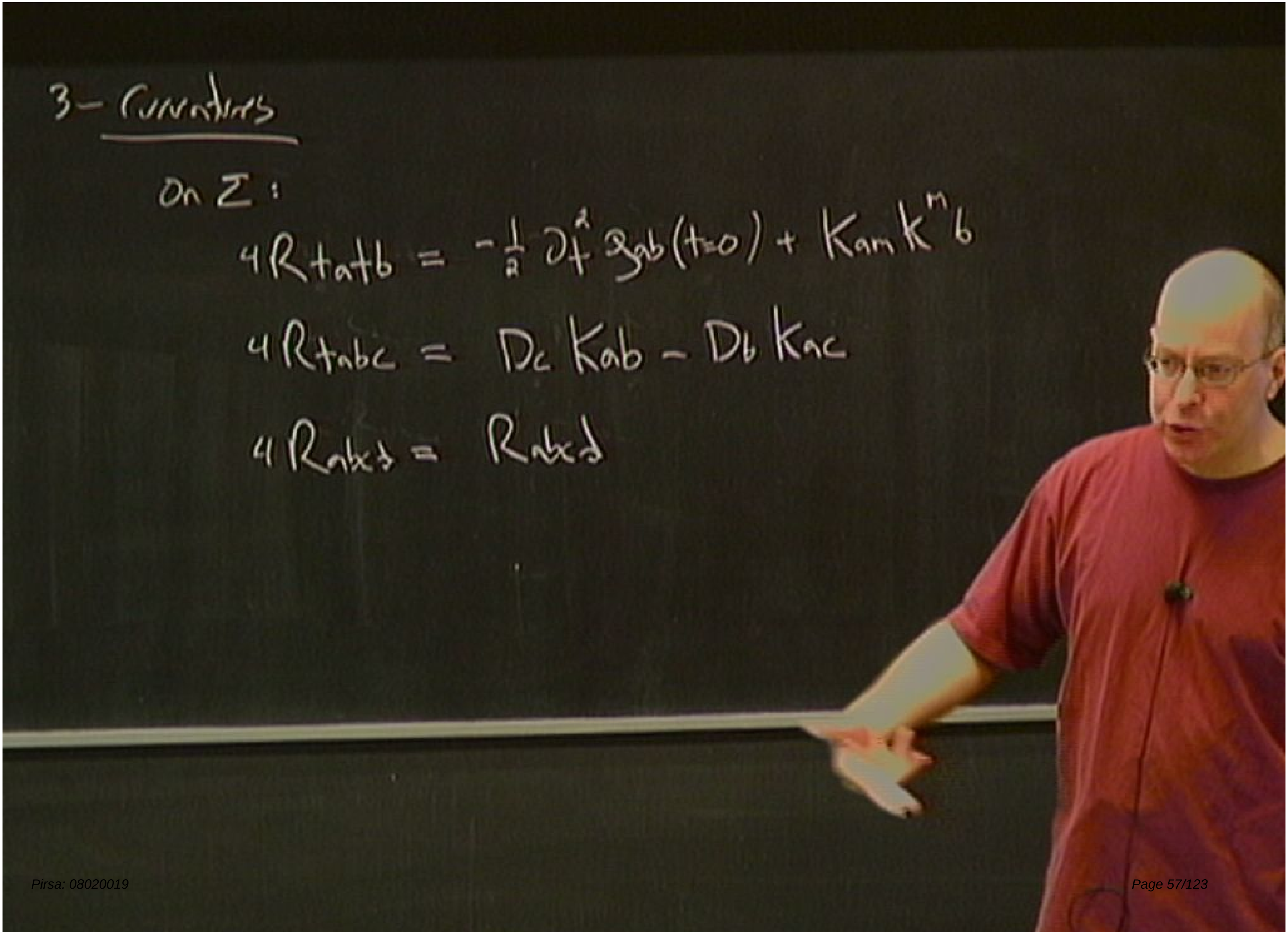
3 – Curvatures
On Σ :
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${}^4R_{abcd} = R_{abcd}$

3– Curvatures

On Σ:

$$\boxed{\begin{aligned} {}^4R_{tatb} &= -\tfrac{1}{2}\,\partial_t^2 g_{ab}(t=0) + K_{am}K^m{}_b \\ {}^4R_{tabc} &= D_c K_{ab} - D_b K_{ac} \\ {}^4R_{abcd} &= R_{abcd} + K_{ac}K_{bd} - K_{ad}K_{bc} \end{aligned}}$$

spatial components of spacetime curvature

intrinsic curvature obtained from h_{ab}

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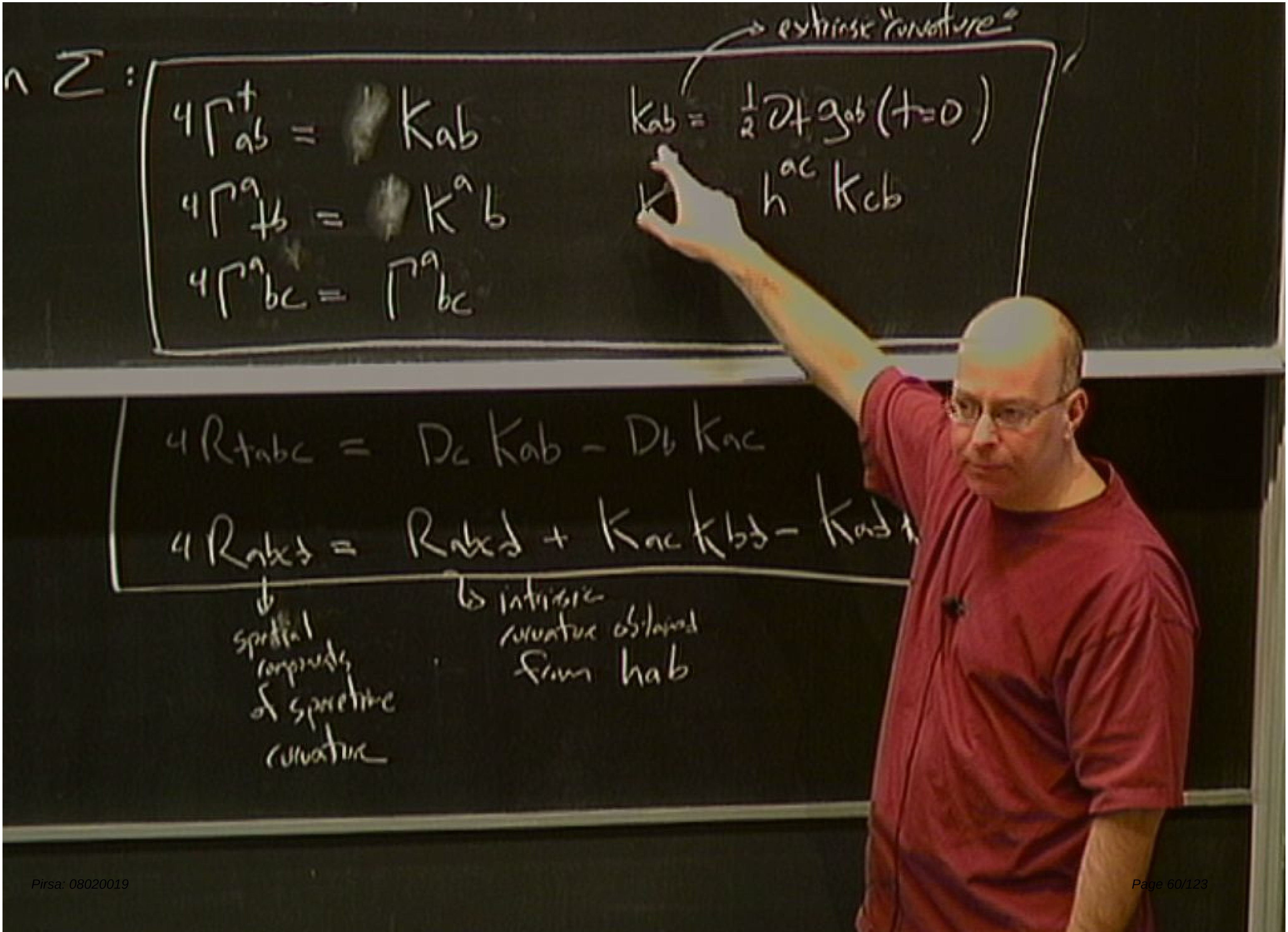
extrinsic "curvature"
Σ :
$^4\Gamma^t{}_{ab} = K_{ab}$
$^4\Gamma^a{}_{tb} = K^a{}_b$
$^4\Gamma^a{}_{bc} = \Gamma^a{}_{bc}$
$K_{ab} = \frac{1}{2}\partial_t g_{ab}(t=0)$
$h^{ac} K_{cb}$
$^4R_{tabc} = D_c K_{ab} - D_b K_{ac}$
$^4R_{abcd} = R_{abcd} + K_{ac}K_{bd} - K_{ad}$
spatial components of spacetime curvature
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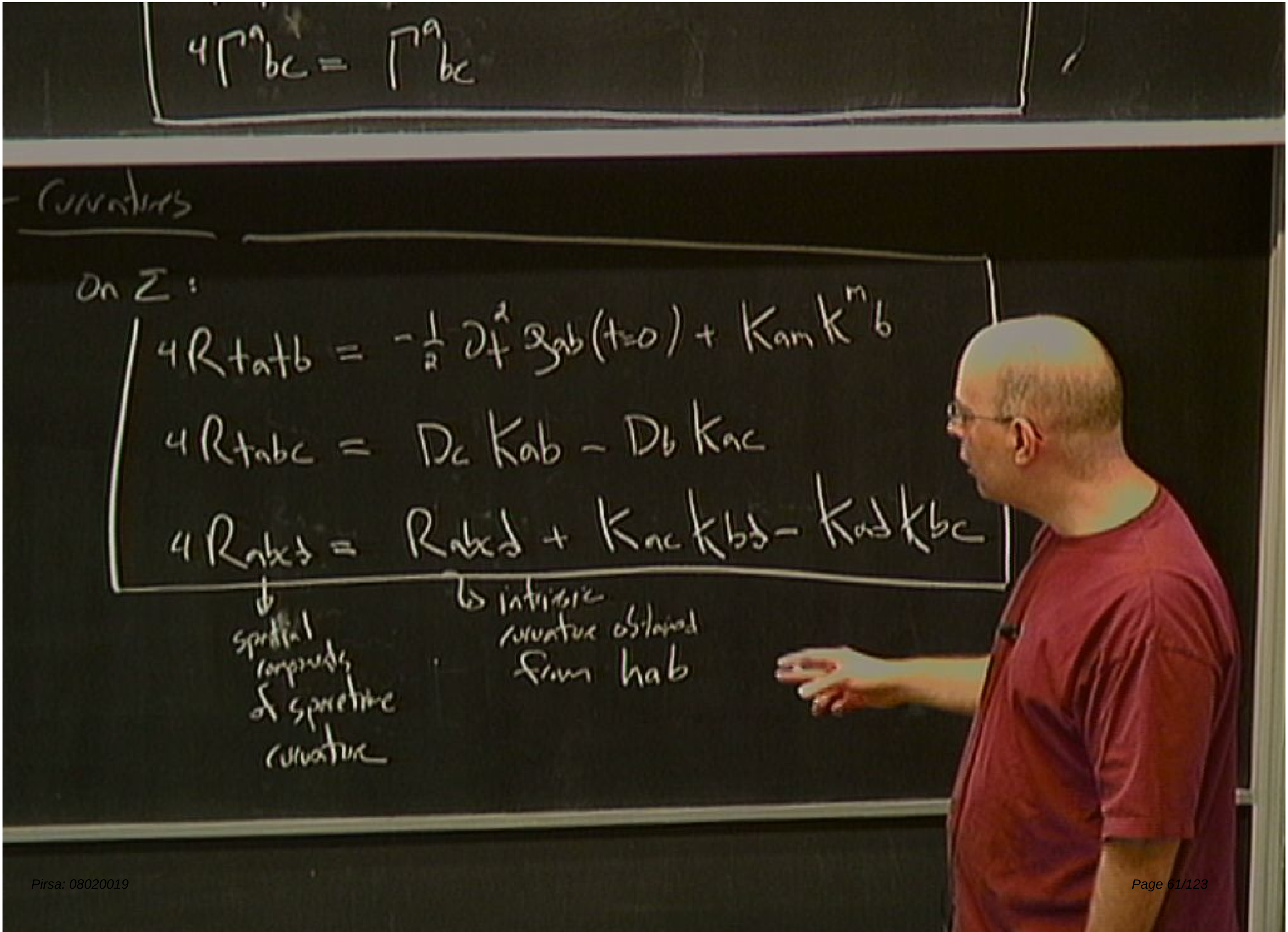
${}^4\Gamma^a{}_{bc} = \Gamma^a{}_{bc}$
Curvatures
On Σ:
${}^4R_{tatb} = -\frac{1}{2}\partial_t^2 g_{ab}(t=0) + K_{am}K^m{}_b$
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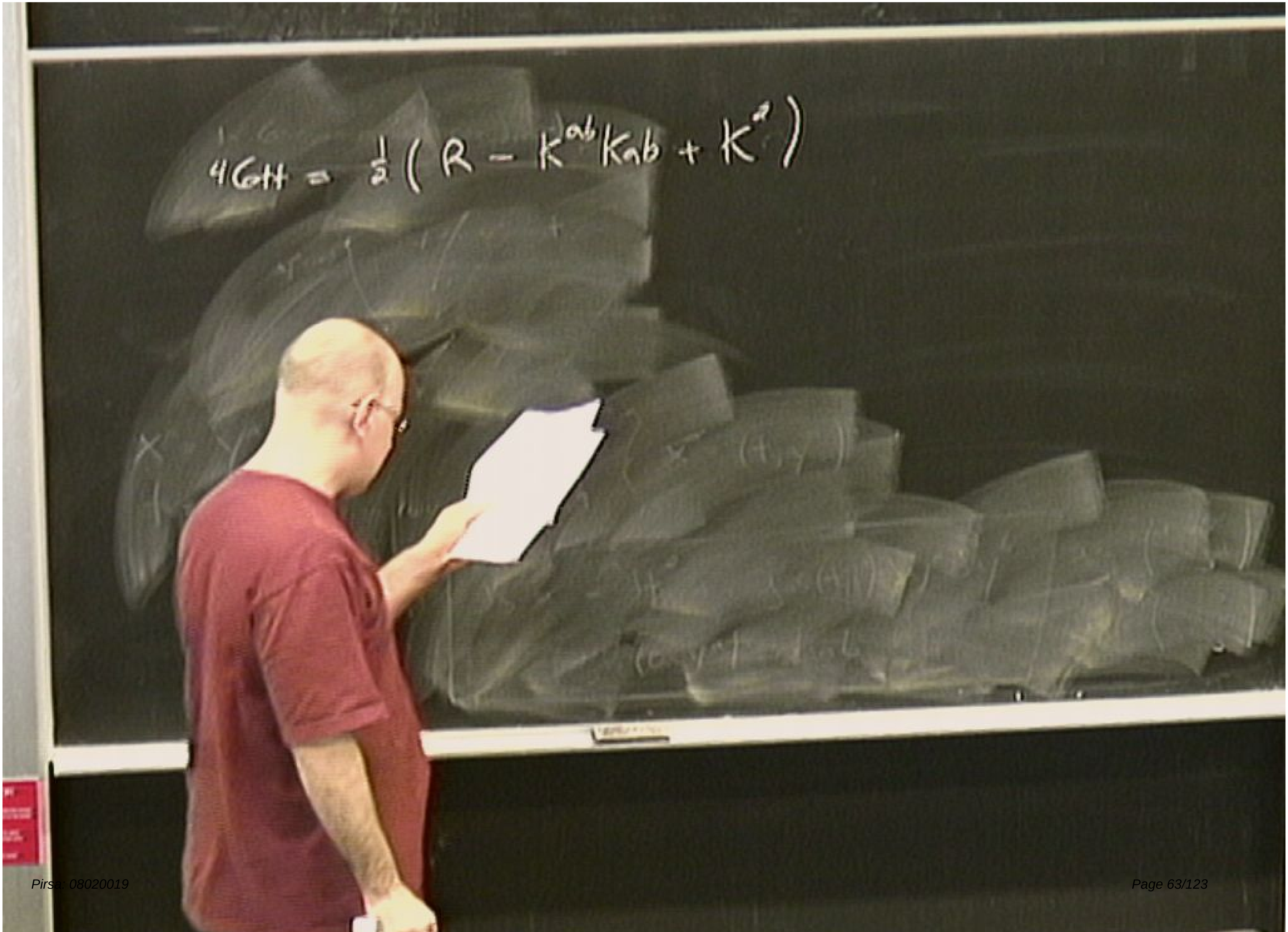
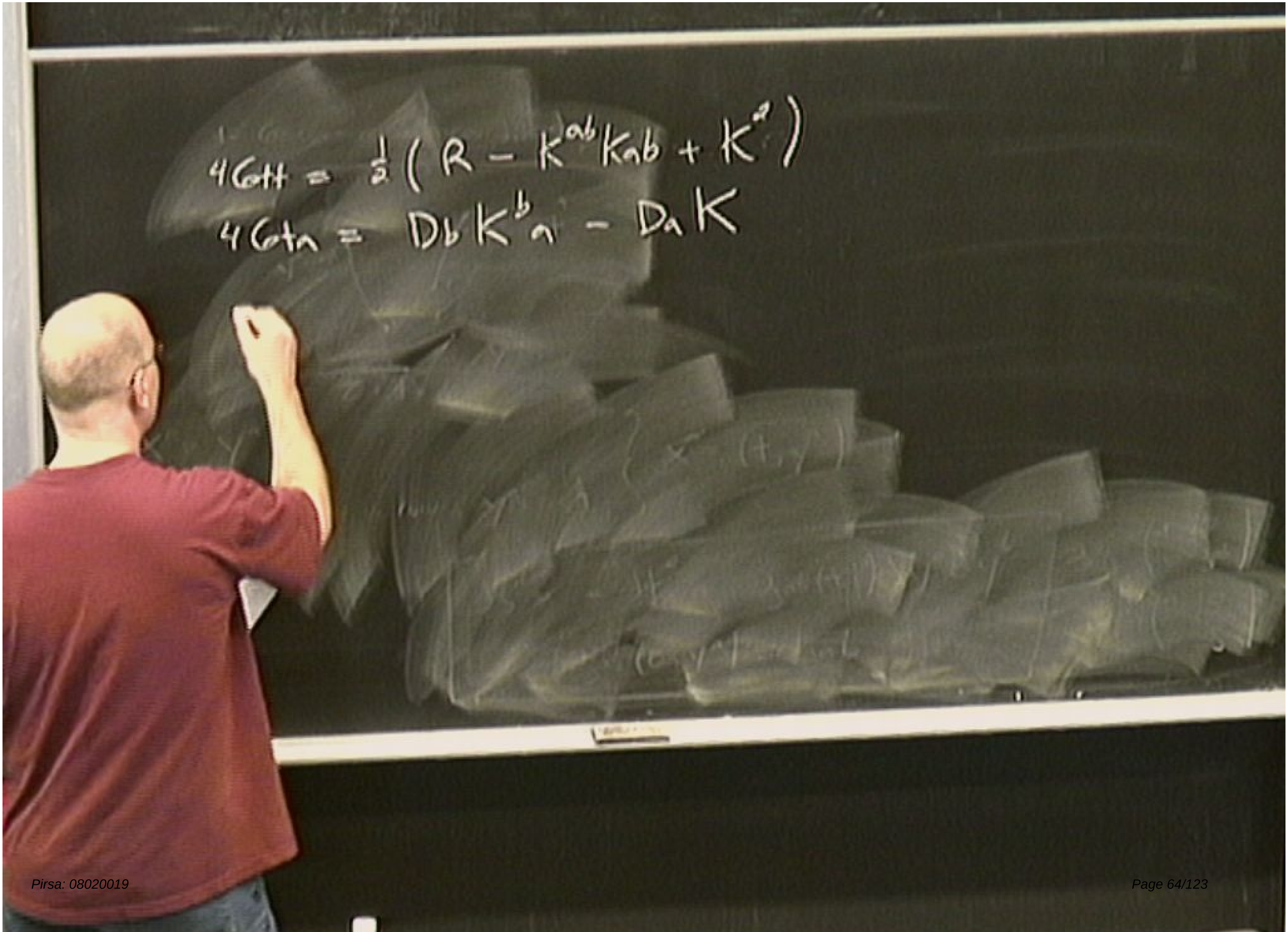
$$4GH = \frac{1}{2}\left(R\right.$$


$4GH = \frac{1}{2}\left(R - K^{ab}K_{ab} + K^2\right)$


$4GH = \frac{1}{2}\left(R - K^{ab}K_{ab} + K^2\right)$
$4Gt_a = D_b K^b{}_a - D_a K$

$$4G_{tt} = \frac{1}{2}\left(R - K^{ab}K_{ab} + K^2 \right)$$

$$4G_{ta} = D_b K^b{}_a - D_a K$$

$$4G_{nn} =$$

$$4G_{tt} = \frac{1}{2}\left(R - K^{ab}K_{ab} + K^2\right)$$

$$4G_{ta} = D_b K^b{}_a - D_a K$$

$$4G_{ab} = G_{ab}$$

$$4G_{tt} = \frac{1}{2}\left(R - K^{ab}K_{ab} + K^2\right)$$

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$$4G_{ab} = G_{ab} + \frac{1}{2}\partial_t^2 g_{ab} - \frac{1}{2} h_{ab}\left(h^{cd}\partial_t^2 g_{cd}\right)$$

$$4G_{tt} = \frac{1}{2}\left(R - K^{ab}K_{ab} + K^2 \right)$$

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R_{ab} = intrinsic Ricci tensor (from h_{ab})

$\to D_a \equiv |_a$

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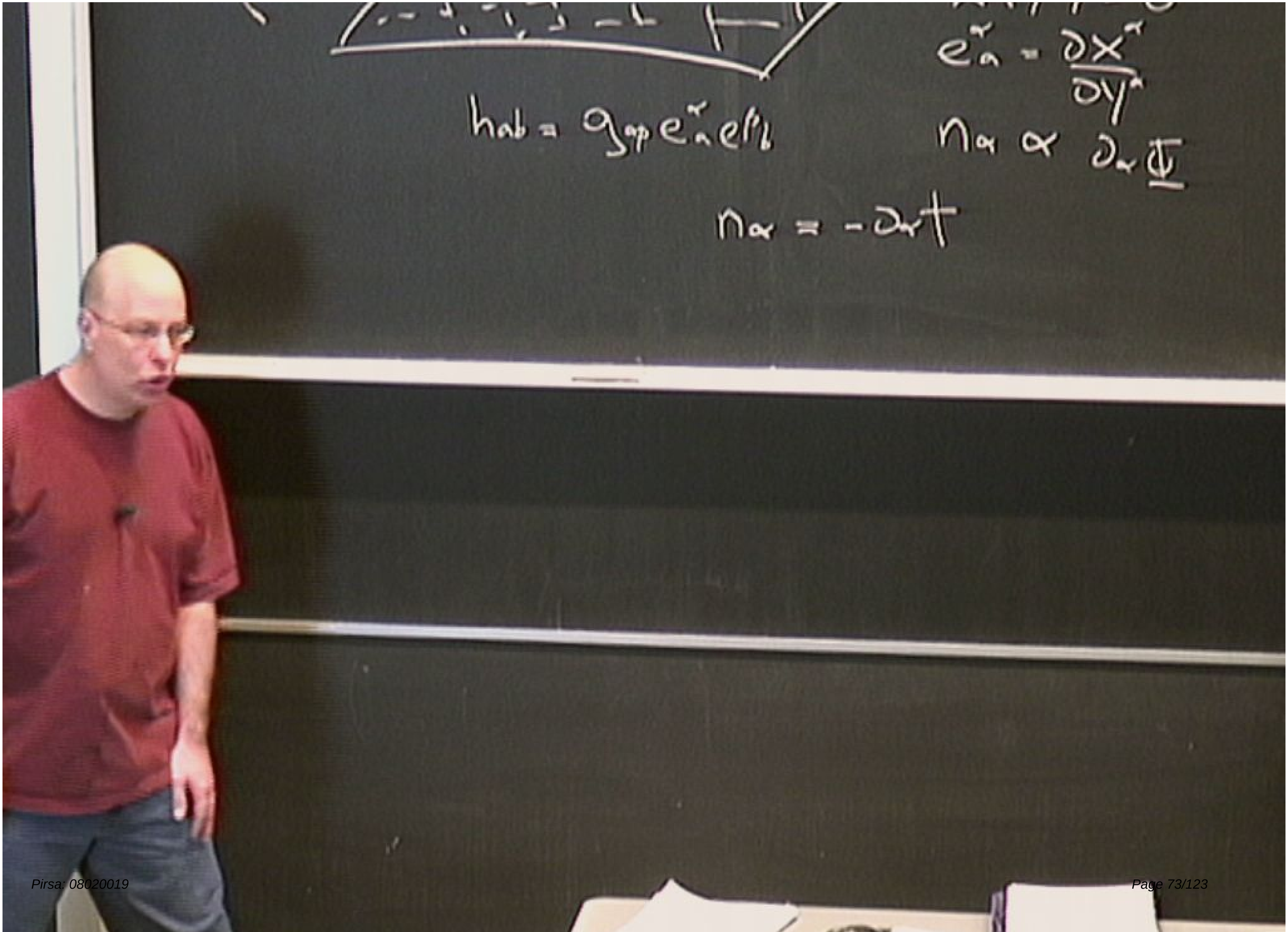
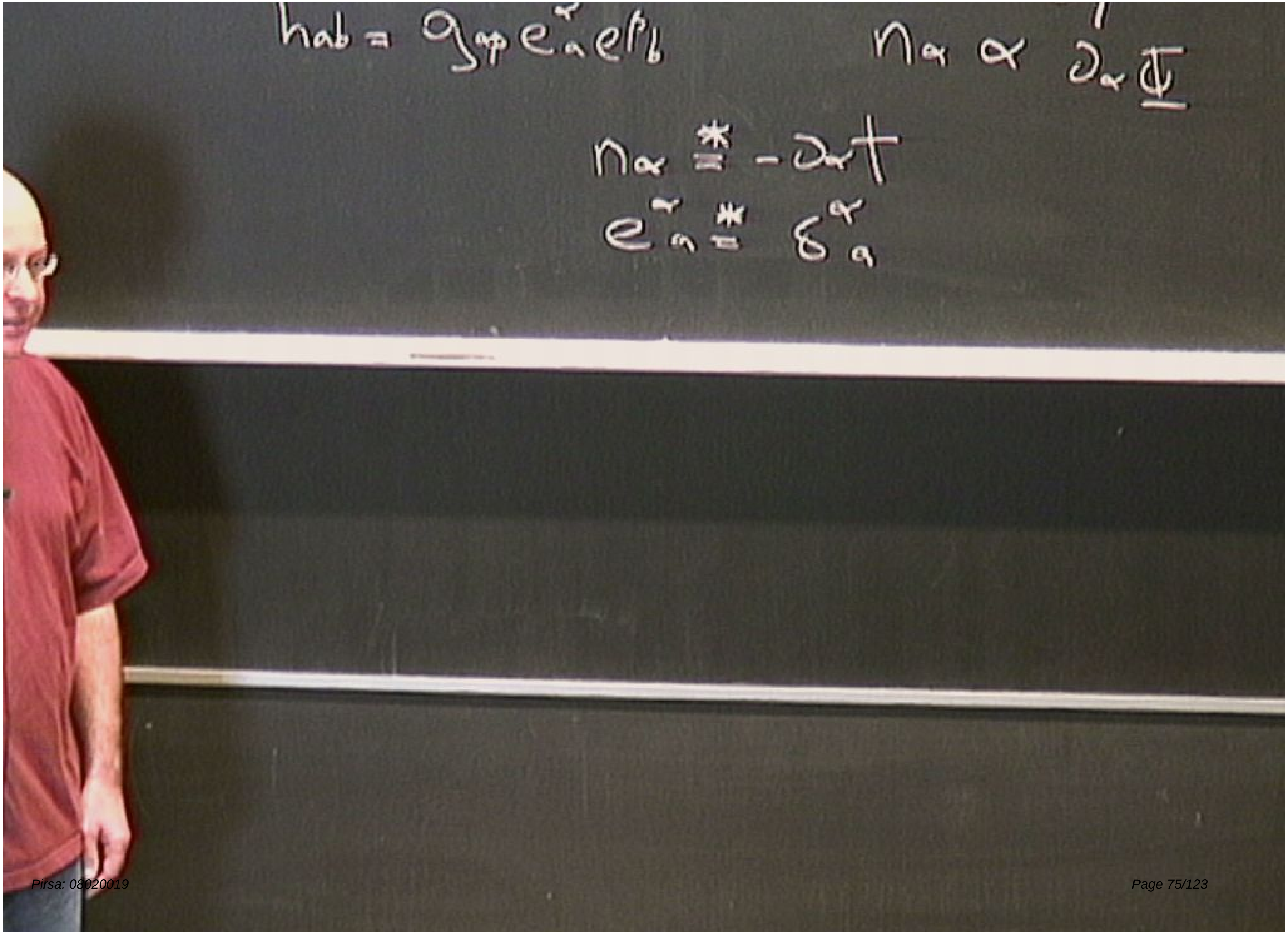
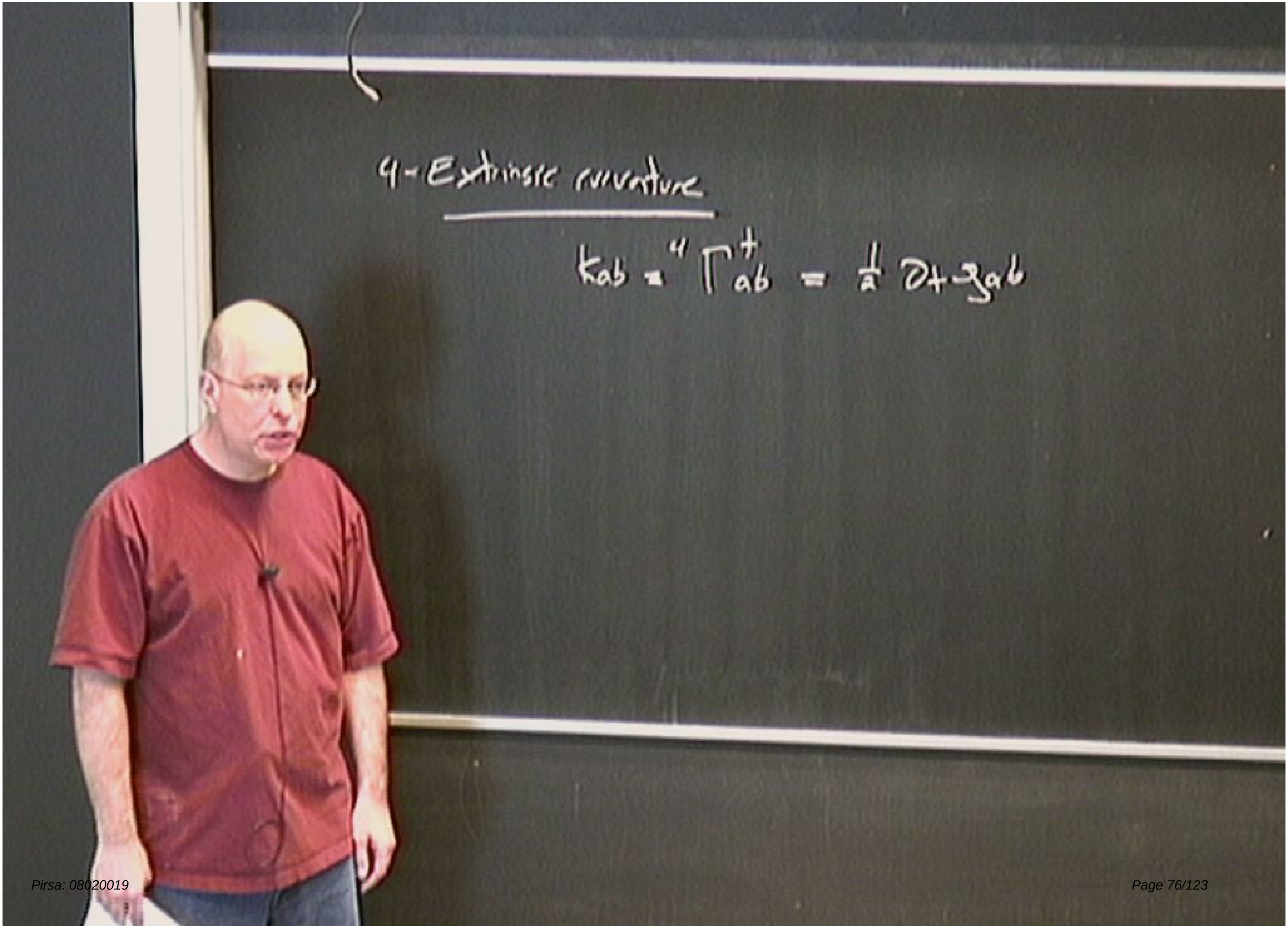

$e^\alpha_a = \frac{\partial x^\alpha}{\partial y^a}$
$h_{ab} = g_{\alpha\beta} e^\alpha_a e^\beta_b$
$n_\alpha \propto \partial_\alpha \Phi$
$n_\alpha = -\partial_\alpha t$

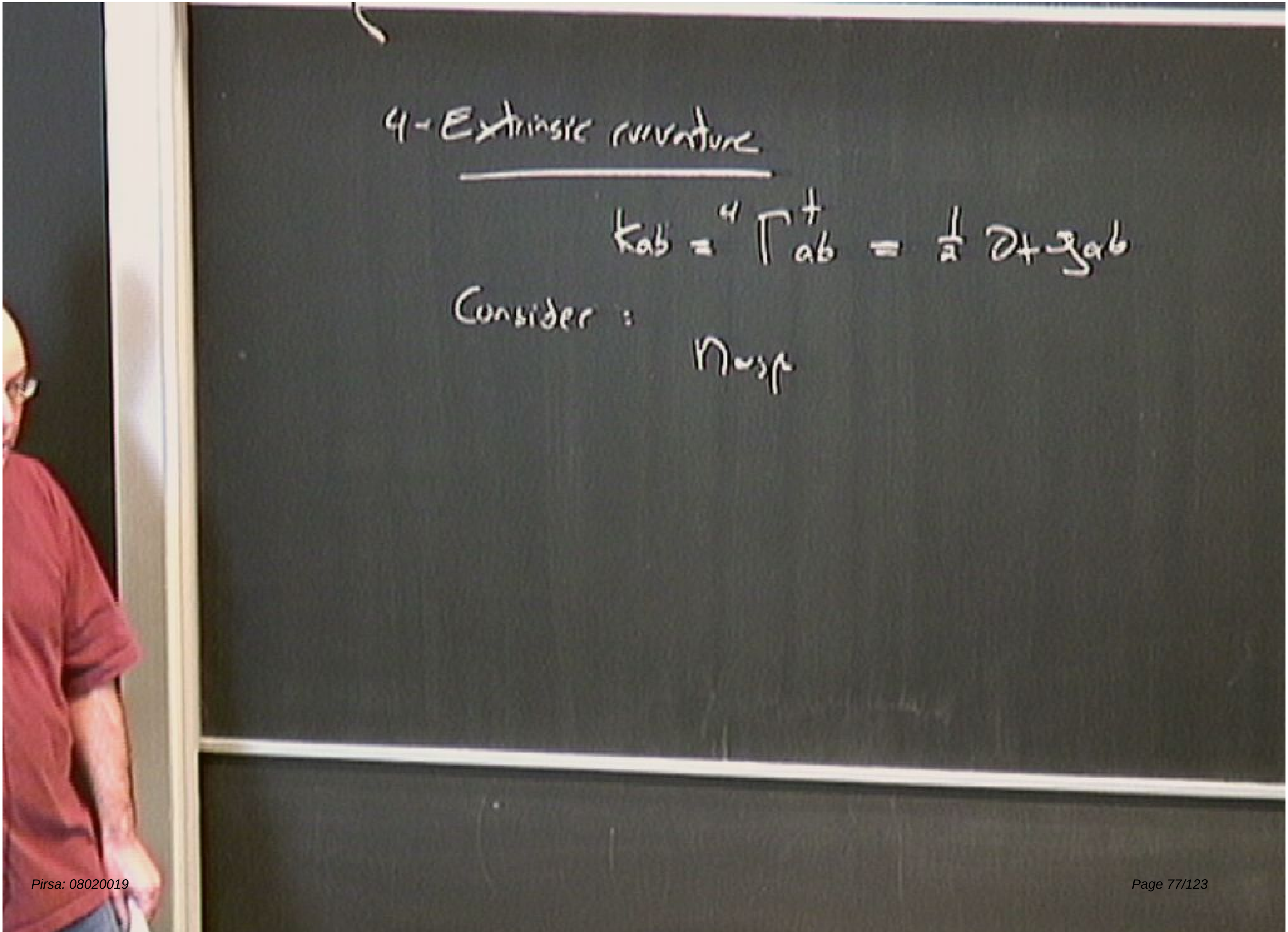
$h_{ab} = g_{\alpha\beta} e^{\alpha}_{a} e^{\beta}_{b}$ $\qquad$ $n_\alpha \propto \partial_\alpha \Phi$

$n_\alpha \overset{*}{=} -\partial_\alpha t$

$e^{\alpha}_{a} =$


$h_{ab} = g_{\alpha\beta} e^\alpha_a e^\beta_b$
$n_\alpha \propto \partial_\alpha \Phi$
$n_\alpha \overset{*}{=} -\partial_\alpha t$
$e^\alpha_a \overset{*}{=} \delta^\alpha_a$


4-Extrinsic curvature
$K_{ab} = {}^{4}\Gamma^{t}_{ab} = \frac{1}{2}\partial_t g_{ab}$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Consider :
$\eta_{\alpha\beta}$

4 - Extrinsic curvature

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Consider :

$$\eta_{\alpha\beta} e^\alpha_a e^\beta_b$$

$u^\alpha{}_{;\beta} u^\beta$

4 - Extrinsic curvature

$$K_{ab} = {}^4\Gamma^t_{ab} = \frac{1}{2}\partial_t g_{ab}$$

Consider :

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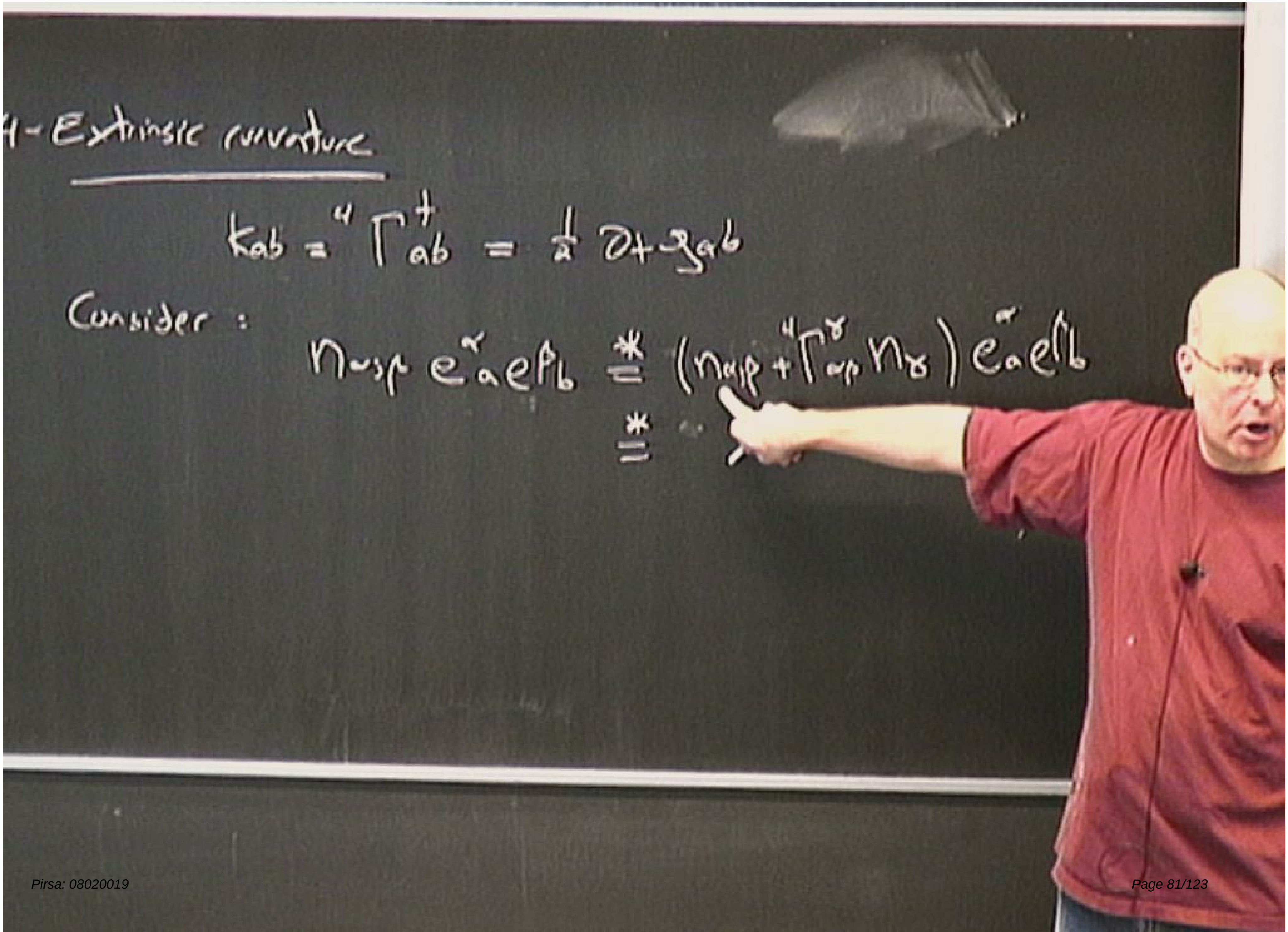
4-Extrinsic curvature

$$K_{ab} = {}^4\Gamma^t_{ab} = \frac{1}{2}\partial_t g_{ab}$$

Consider :

$$n_{\alpha;\beta}\, e^\alpha_a e^\beta_b \stackrel{*}{=} \left(n_{\alpha,\beta} + {}^4\Gamma^\gamma_{\alpha\beta} n_\gamma\right) e^\alpha_a e^\beta_b$$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Consider :
$n_{\alpha;\beta}\, e^\alpha_a e^\beta_b \overset{*}{=} (n_{\alpha,\beta} + {}^4\Gamma^\gamma_{\alpha\beta} n_\gamma)\, e^\alpha_a e^\beta_b$
$\overset{*}{=}$

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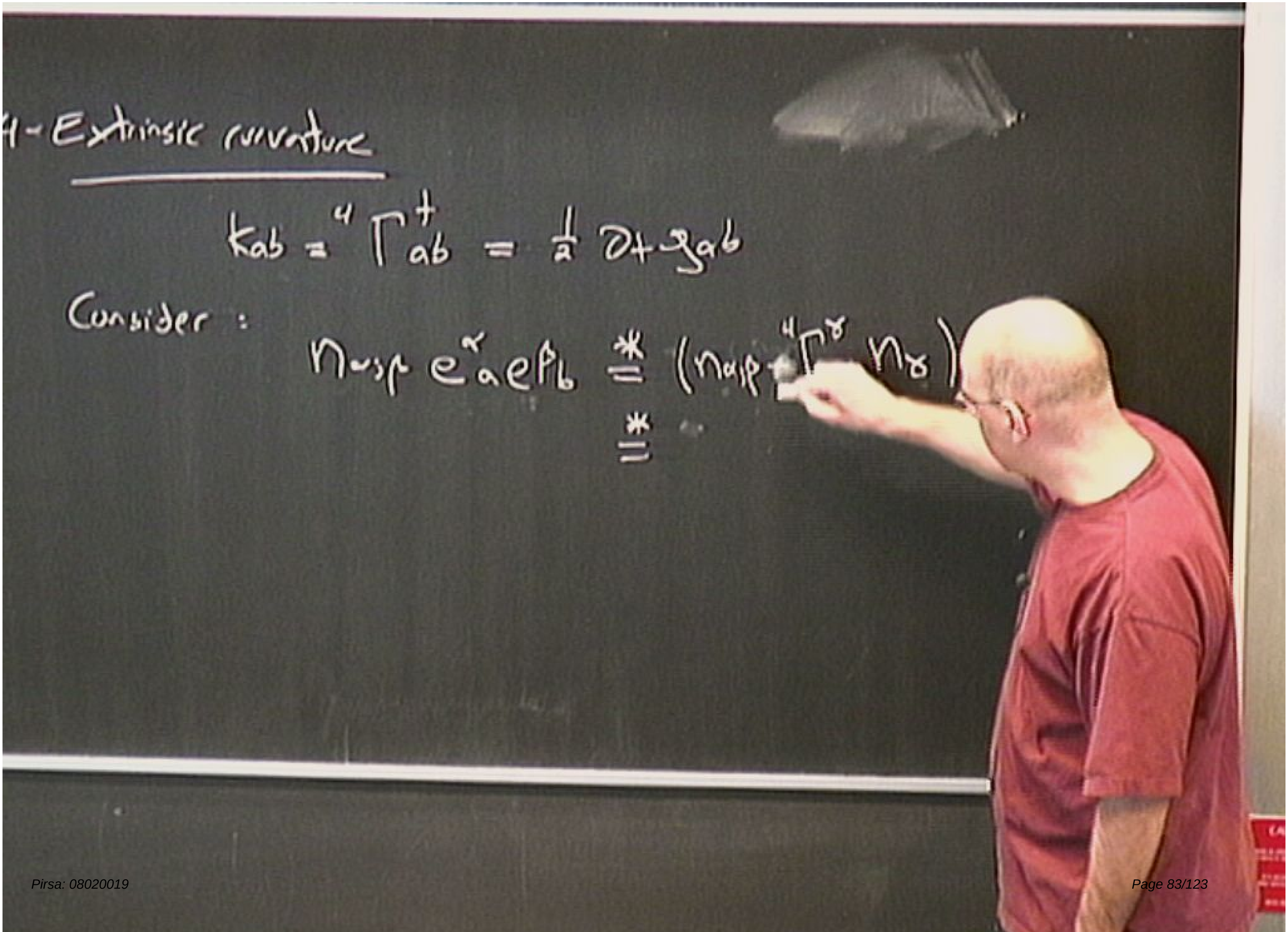
$$\stackrel{*}{=}$$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$K_{ab} = {}^4\Gamma^t_{ab} = \frac{1}{2}\partial_t g_{ab}$
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$\overset{*}{=}$

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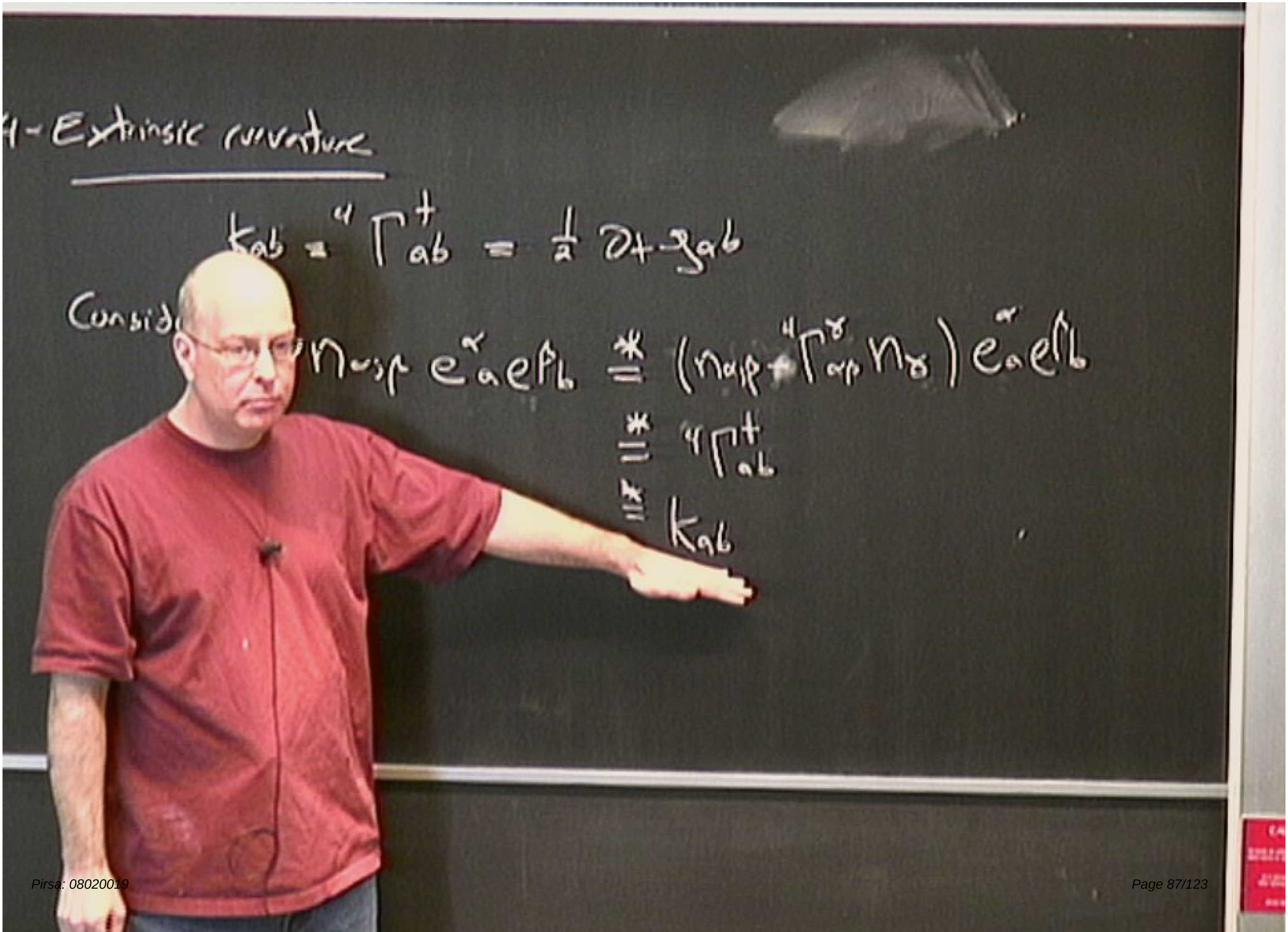
$$\overset{*}{=} {}^4\Gamma^{t}_{ab}$$

$$= K_{ab}$$


Extrinsic curvature
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Consid
$n_{\alpha;\beta}\, e^\alpha_a e^\beta_b \overset{*}{=} (n_{\alpha,\beta} - {}^4\Gamma^\gamma_{\alpha\beta} n_\gamma)\, e^\alpha_a e^\beta_b$
$\overset{*}{=} {}^4\Gamma^t_{ab}$
$\overset{*}{=} K_{ab}$

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$$K_{ab} \equiv n_{\alpha;\beta}\, e^\alpha_a e^\beta_b \qquad \overset{*}{=} {}^4\Gamma^t_{ab}$$

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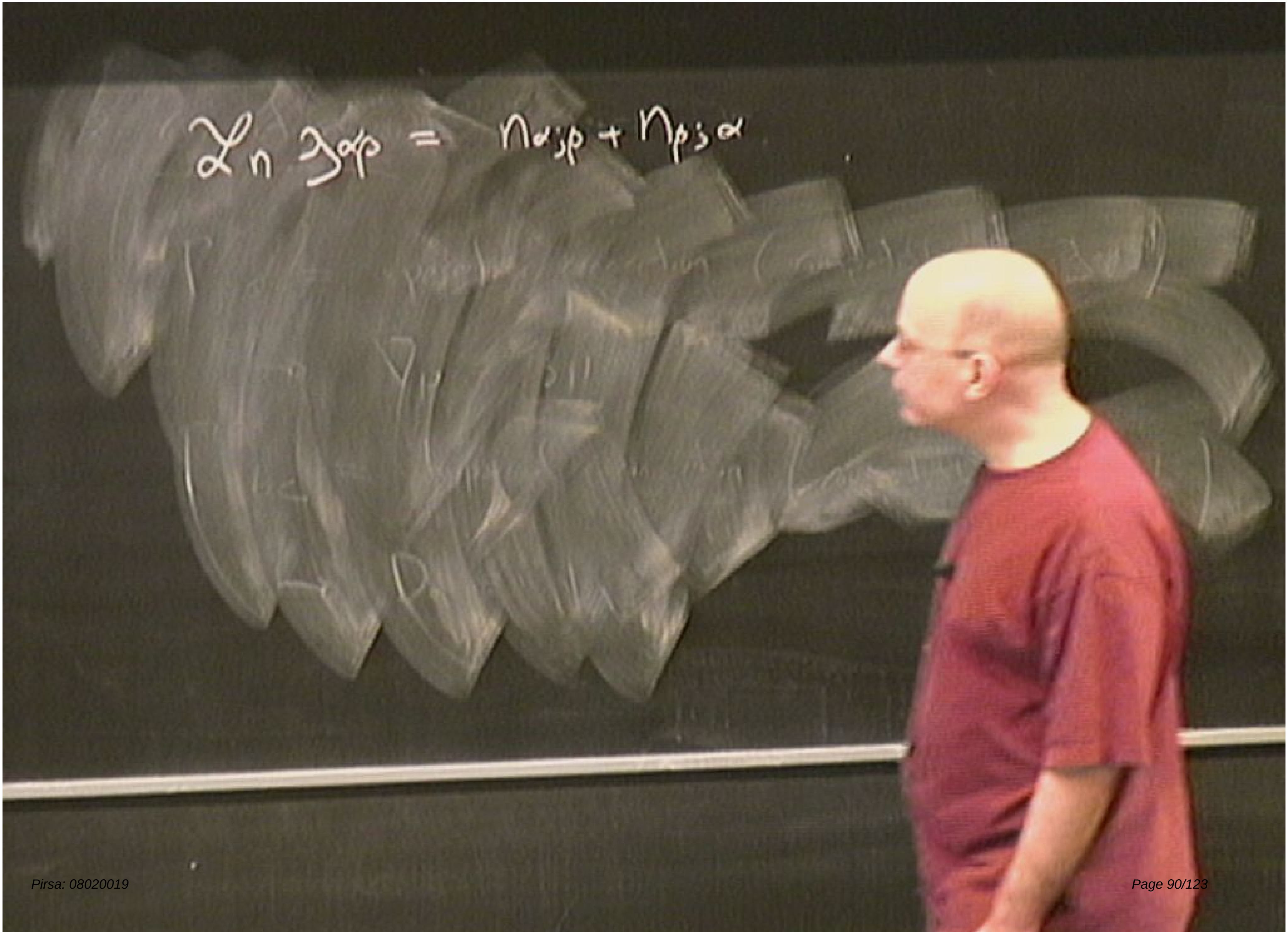
$$\stackrel{*}{=} {}^4\Gamma^t_{ab}$$

$$\stackrel{*}{=} K_{ab}$$

$$\boxed{\begin{aligned} K_{ab} &\equiv n_{\alpha;\beta}\, e^\alpha_a e^\beta_b \\ &= K_{ba} \end{aligned}}$$


$\mathcal{L}_n g_{\alpha\beta} = n_{\alpha;\beta} + n_{\beta;\alpha}$

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$$(\mathcal{L}_n g_{\alpha\beta})\, e^{\alpha}_{a} e^{\beta}_{b} = 2\, n_{(\alpha;\beta)}\, e^{\alpha}_{a} e^{\beta}_{b}$$

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$$\boxed{K_{ab} = \frac{1}{2}(\mathcal{L}_n g_{\alpha\beta})\, e^\alpha_a e^\beta_b}$$

$$4G_{tt} = \frac{1}{2}\left(R - K^{ab}K_{ab} + K^2\right) = G_{\mu\nu}n^\mu n^\nu$$

$$4G_{ta} = D_b K^b{}_a - D_a K$$

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R_{ab} = intrinsic Ricci tensor (from h_{ab})

$R = h^{ab}R_{ab}$

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$$\mathcal{L}_n g_{\alpha\beta} = n_{\alpha;\beta} + n_{\beta;\alpha}$$

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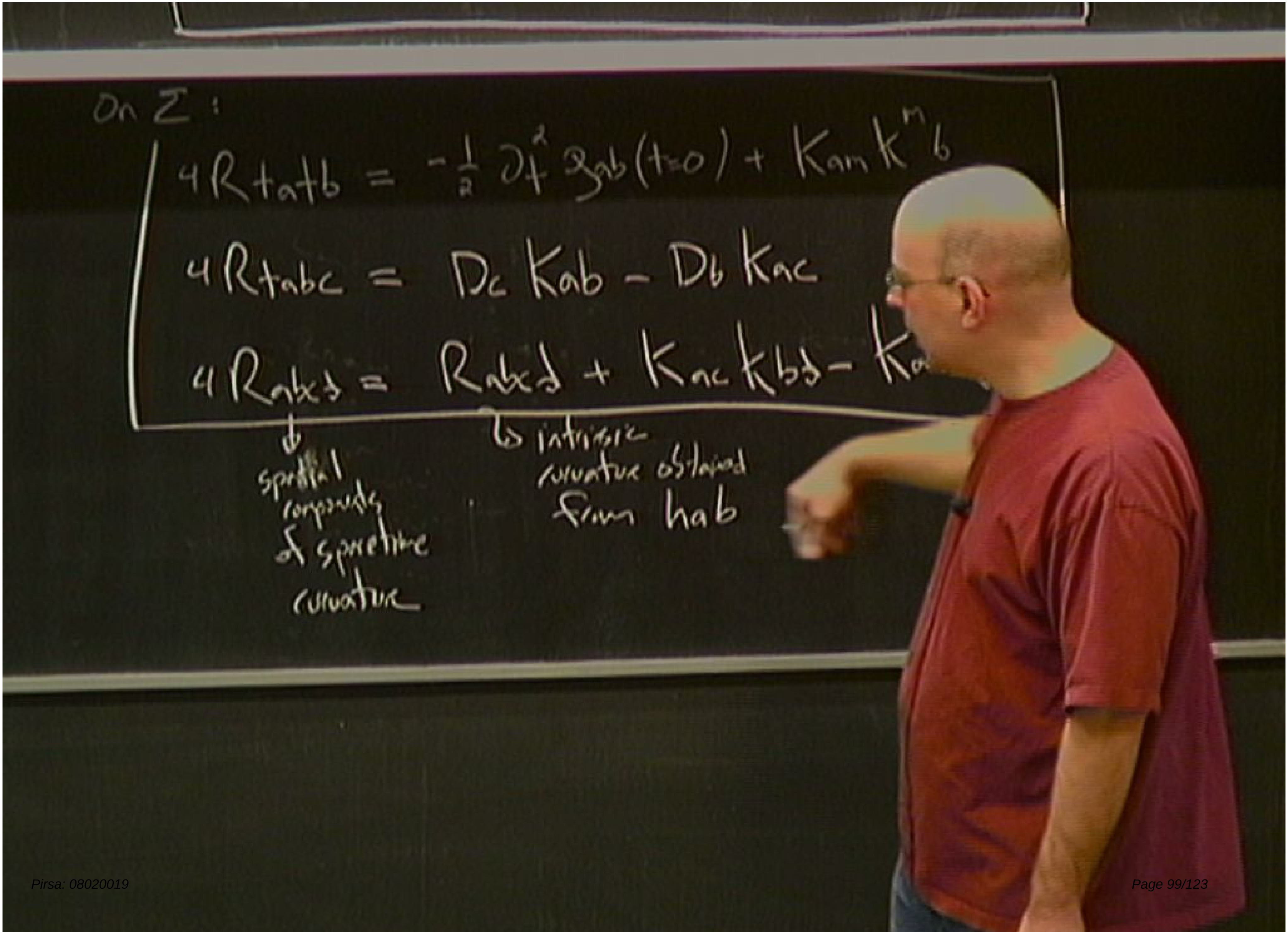
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spatial components of spacetime curvature
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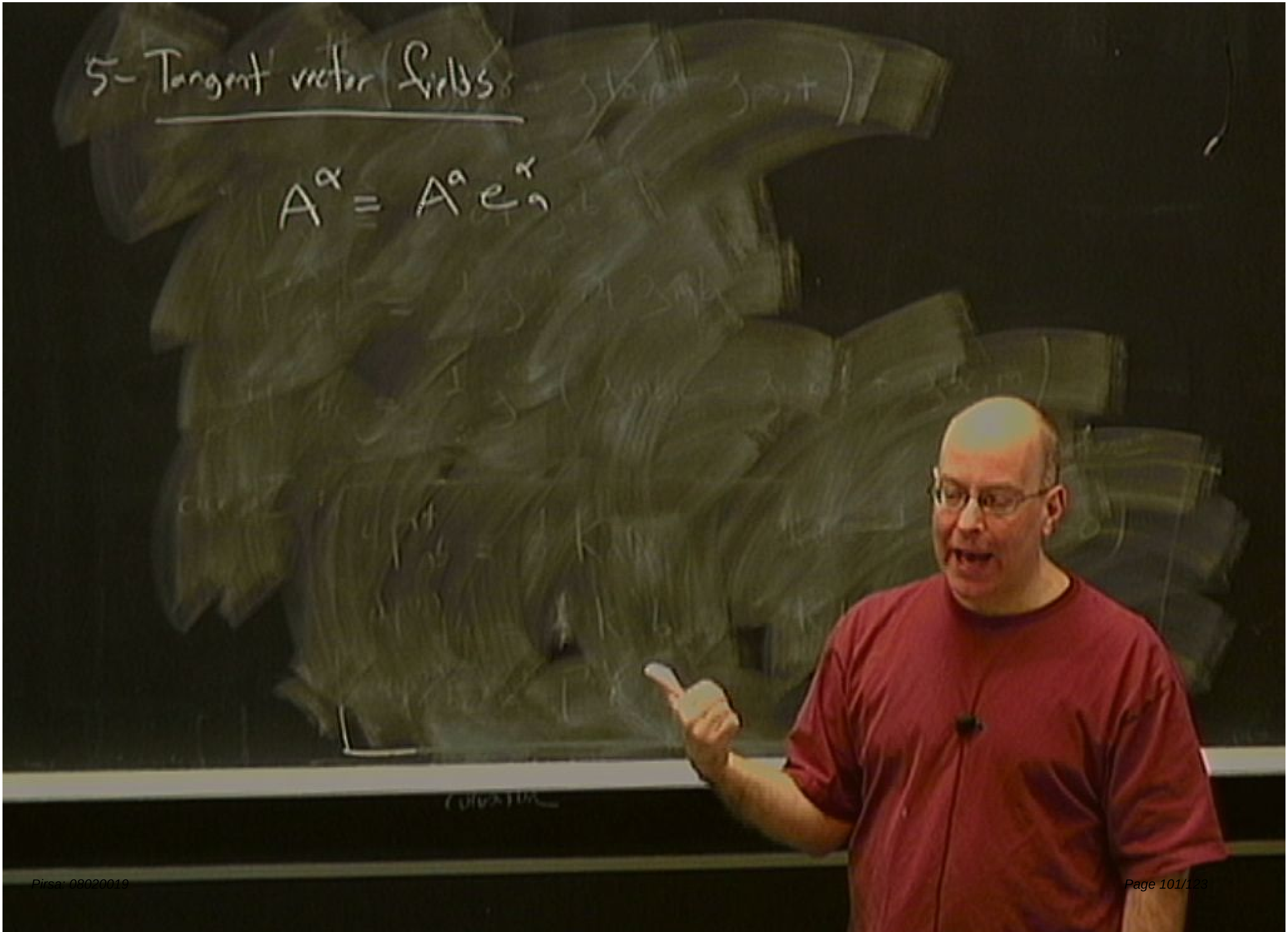
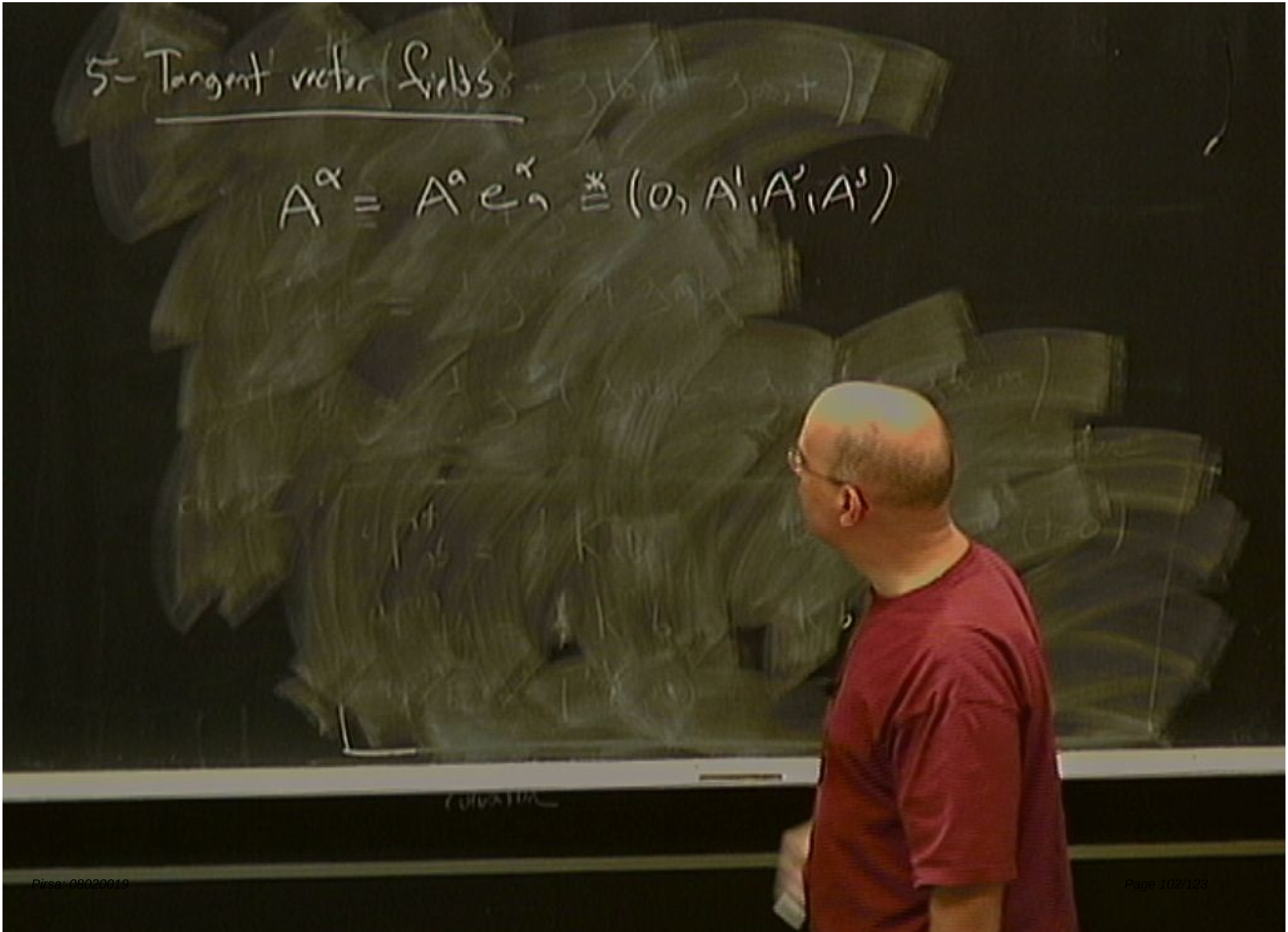

5- Tangent vector fields
$A^\alpha = A^a e_a^\alpha$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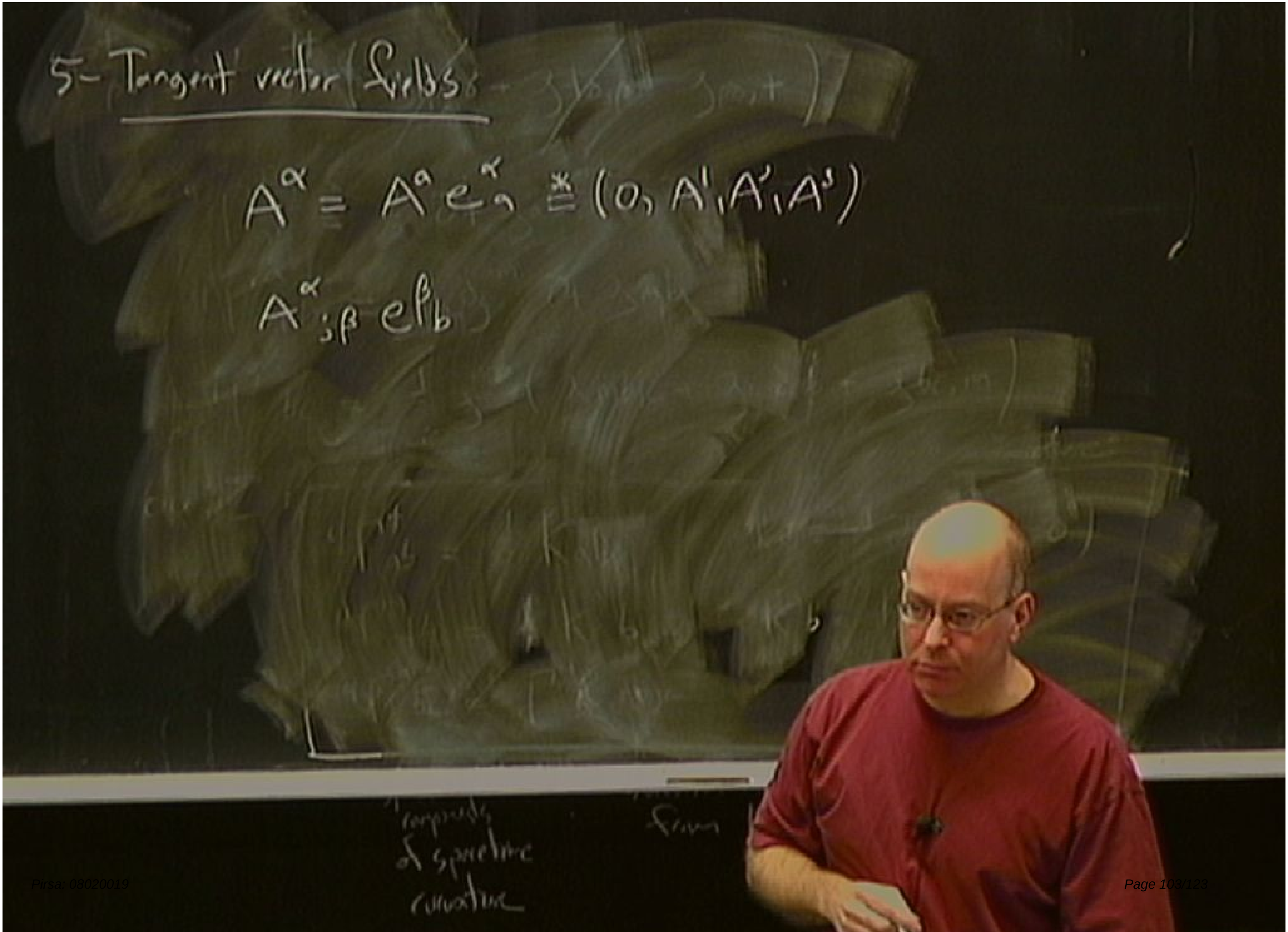
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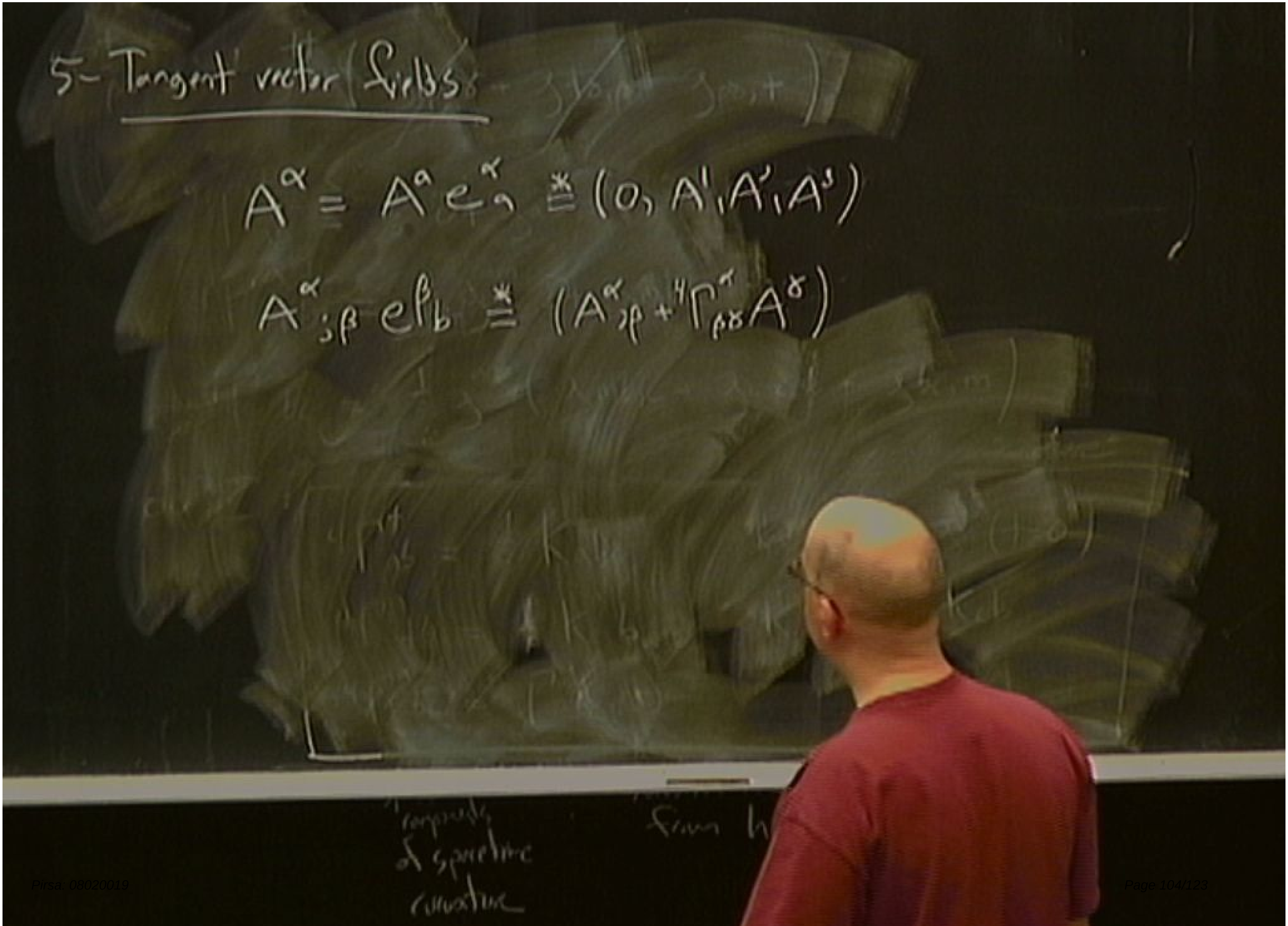
$A^\alpha{}_{;\beta}\, e^\beta_b$
of spacetime



5- Tangent vector fields:
$A^\alpha = A^a e^\alpha_a \overset{*}{=} (0, A^1, A^2, A^3)$
$A^\alpha_{;\beta} e^\beta_b \overset{*}{=} (A^\alpha_{,\beta} + {}^4\Gamma^\alpha_{\beta\gamma} A^\gamma)$

4-Extrinsic curvature

$$K_{ab} = {}^4\Gamma^{t}_{ab} = \frac{1}{2}\partial_t g_{ab}$$

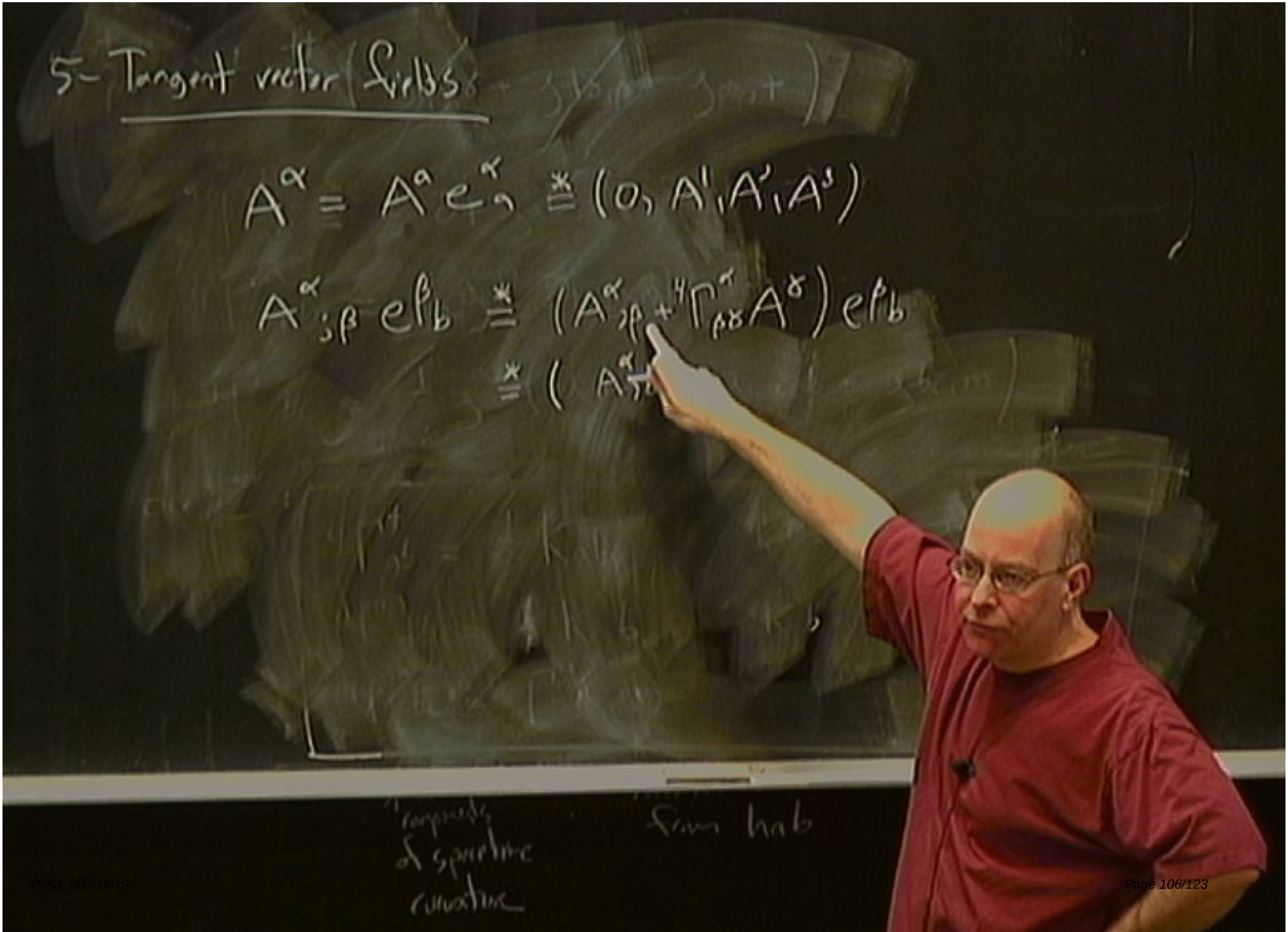
Consider :

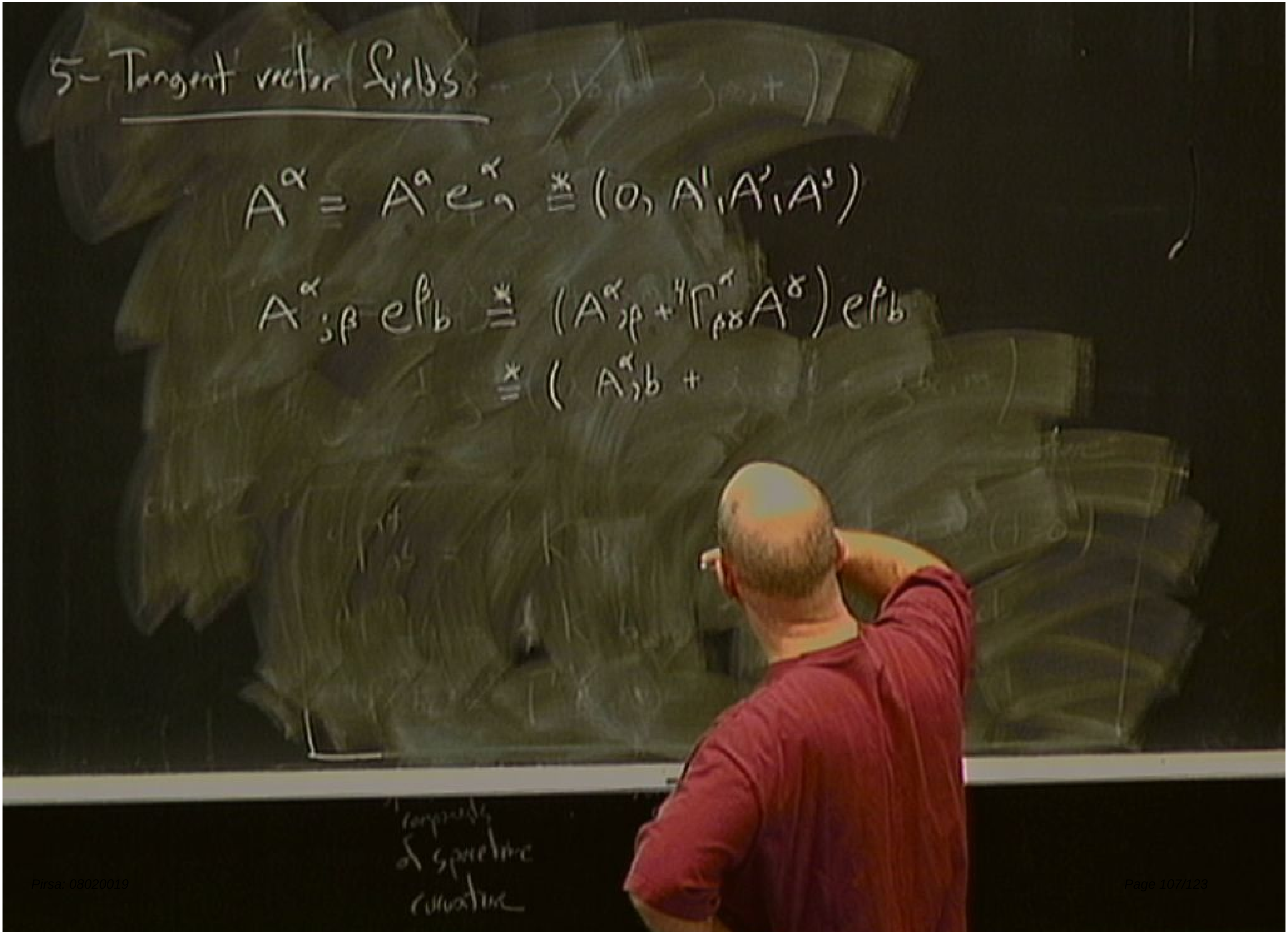
$$n_{\alpha;\beta}\, e^{\alpha}_{a} e^{\beta}_{b} \overset{*}{=} \left(n_{\alpha,\beta} - {}^4\Gamma^{\gamma}_{\alpha\beta} n_{\gamma}\right) e^{\alpha}_{a} \ldots$$

$$\overset{*}{=} {}^4\Gamma^{t}_{ab}$$

$$\overset{*}{=} K_{ab}$$

$$\boxed{\begin{aligned} K_{ab} &\equiv n_{\alpha;\beta}\, e^{\alpha}_{a} e^{\beta}_{b} \\ &= K_{ba} \end{aligned}}$$


5- Tangent vector fields
A^α = A^a e_a^α ≝ (0, A^1, A^2, A^3)
A^α_{;β} e^β_b ≝ (A^α_{,β} + ^4Γ^α_{βγ} A^γ) e^β_b
≝ (A^α_{;b}
from hab
of spacetime
curvature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$$A^\alpha{}_{;\beta}\, e^\beta{}_b =$$

$$\boxed{{}^4R_{abcd} = R_{abcd} + K_{ac}K_{bd} - K_{ad}K_{b}}$$

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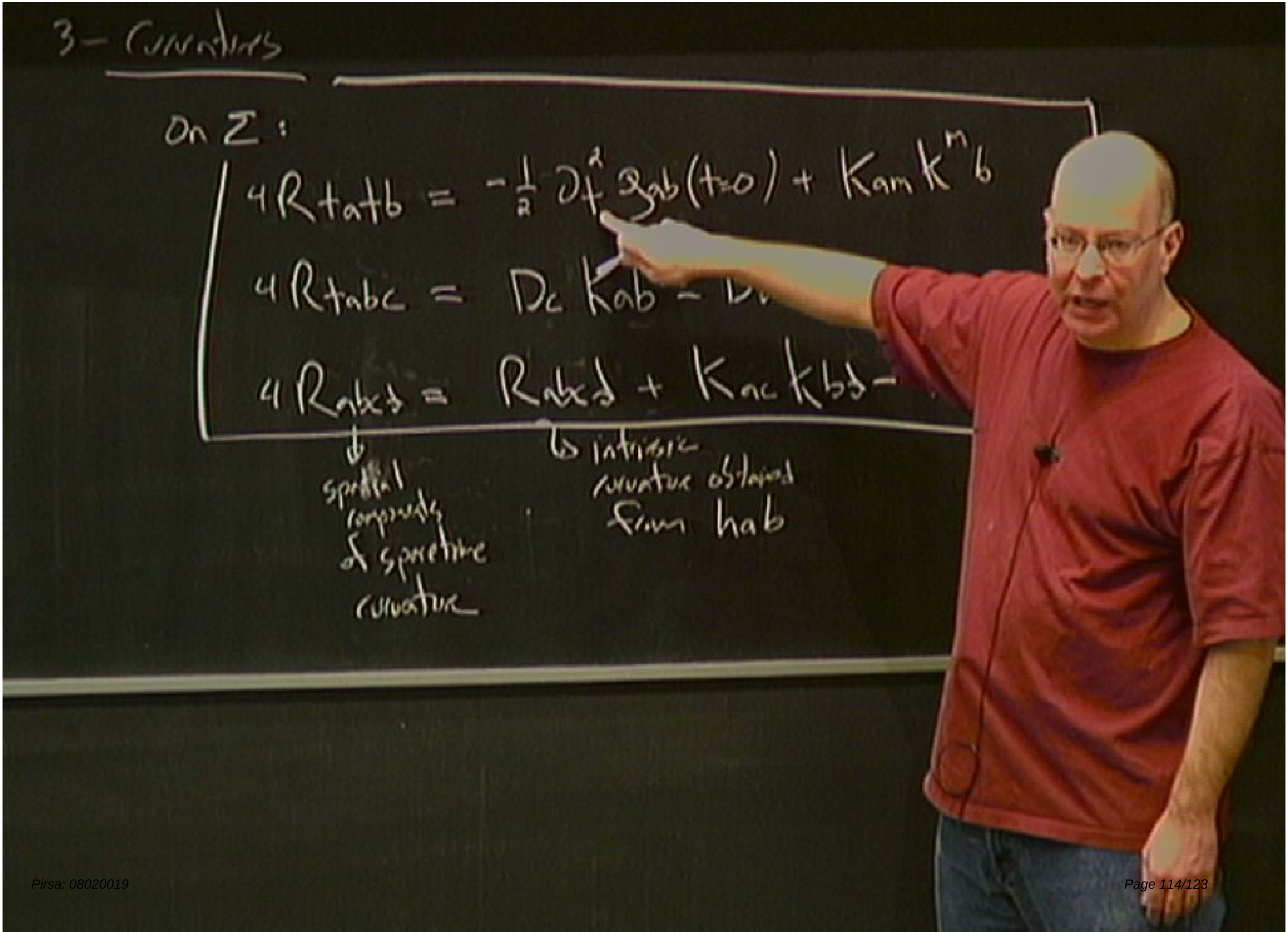
$$\boxed{A^\alpha{}_{;\beta} e^\beta_b = D_b A^a e^\alpha_a + A^a K_{ab} n^\alpha}$$


3 – Curvatures
On Σ :
${}^4R_{tatb} = -\frac{1}{2}\partial_t^2 g_{ab}(t=0) + K_{am}K^m{}_b$
${}^4R_{tabc} = D_c K_{ab} -$
${}^4R_{abcd} = R_{abcd} + K_{ac}K_{bd} -$
spatial components of spacetime curvature
intrinsic curvature obtained from h_{ab}

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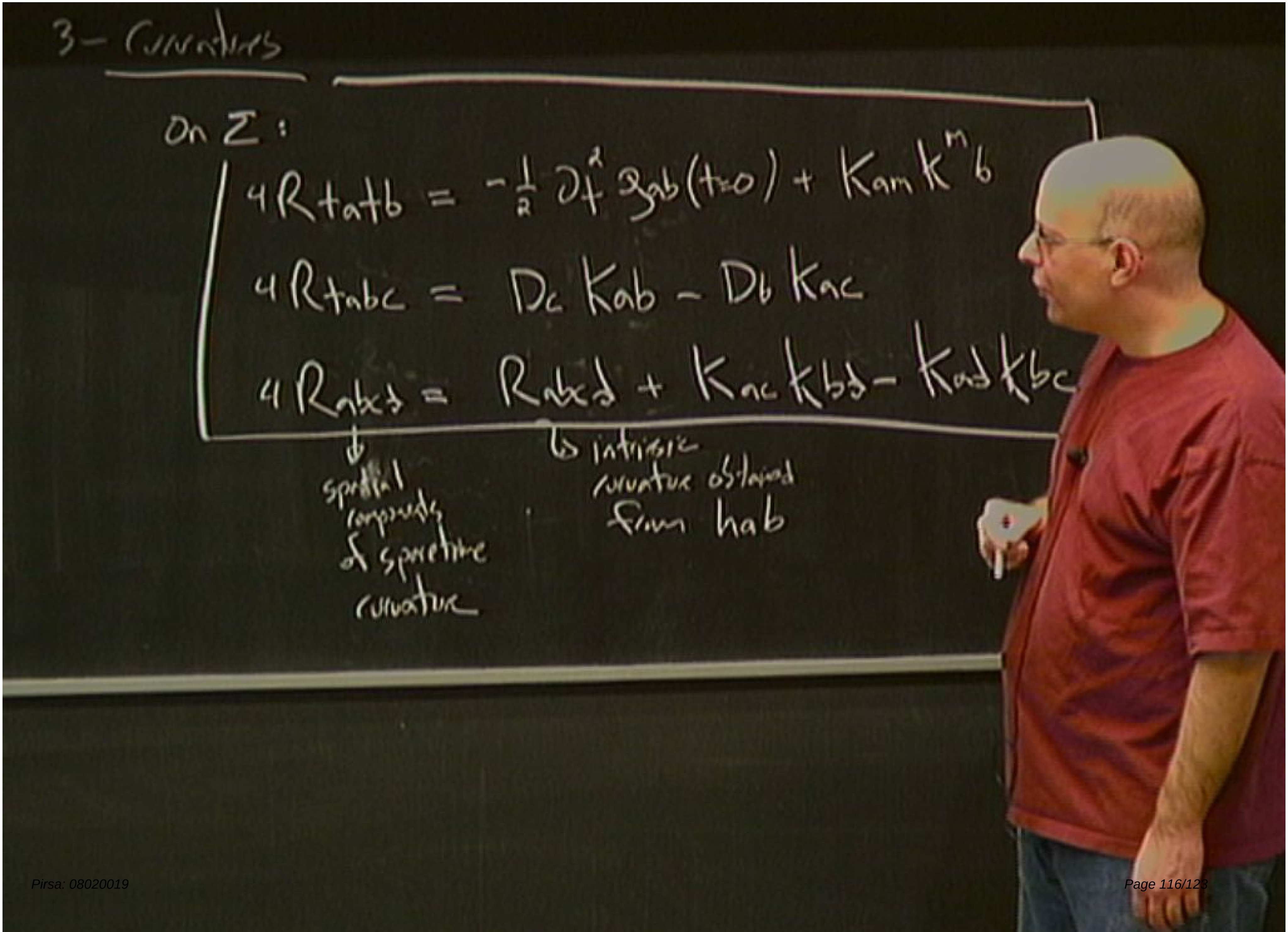
$$\boxed{\begin{aligned} {}^4R_{tatb} &= -\tfrac{1}{2}\,\partial_t^2 g_{ab}(t=0) + K_{am}K^m{}_b \\ {}^4R_{tabc} &= D_c K_{ab} - D_b K_{ac} \\ {}^4R_{abcd} &= R_{abcd} + K_{ac}K_{bd} - K_{ad}K \end{aligned}}$$

${}^4R_{abcd}$: spatial components of spacetime curvature

R_{abcd}: intrinsic curvature obtained from h_{ab}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6 - Gauss - Codazzi equations

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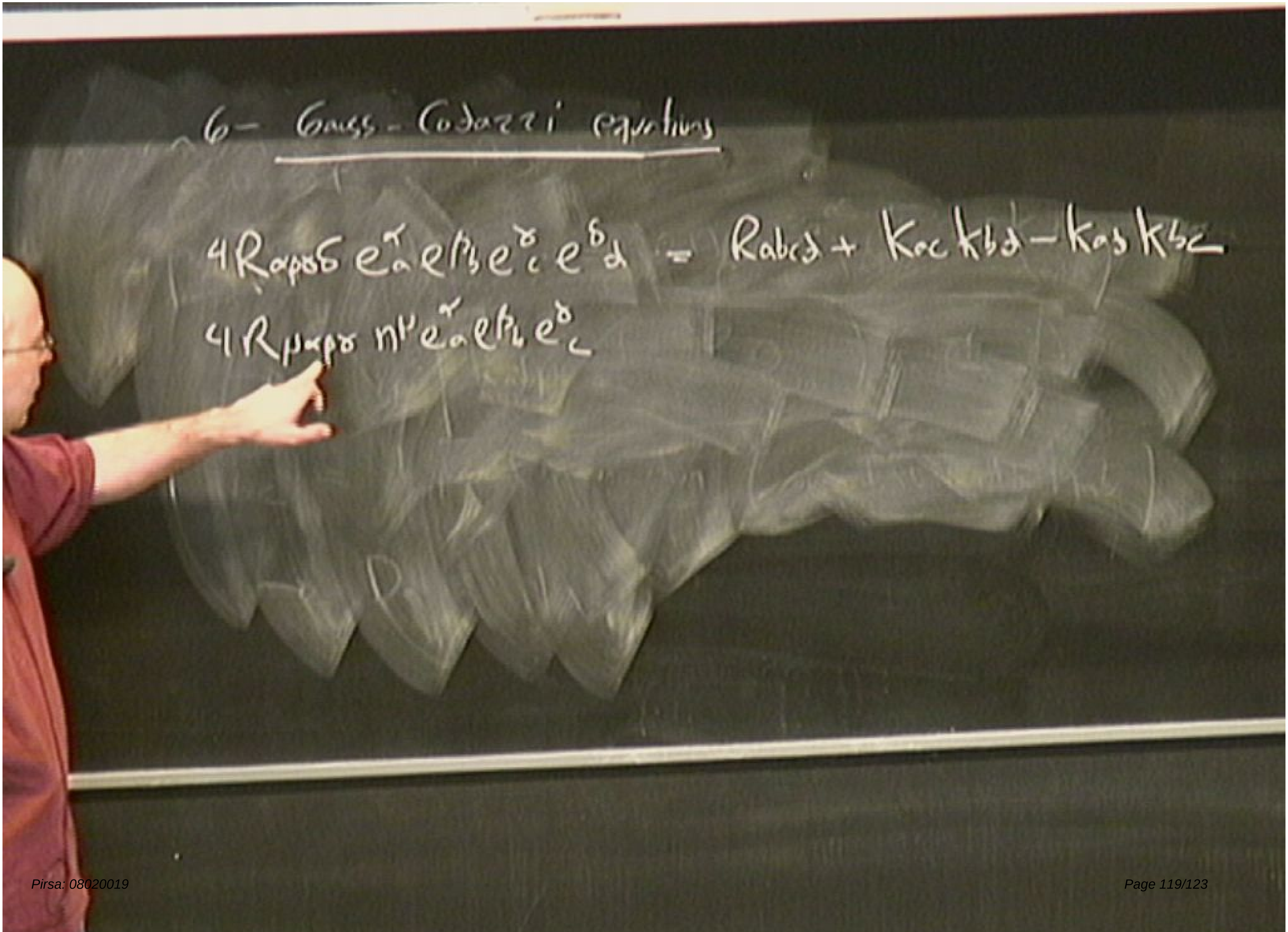
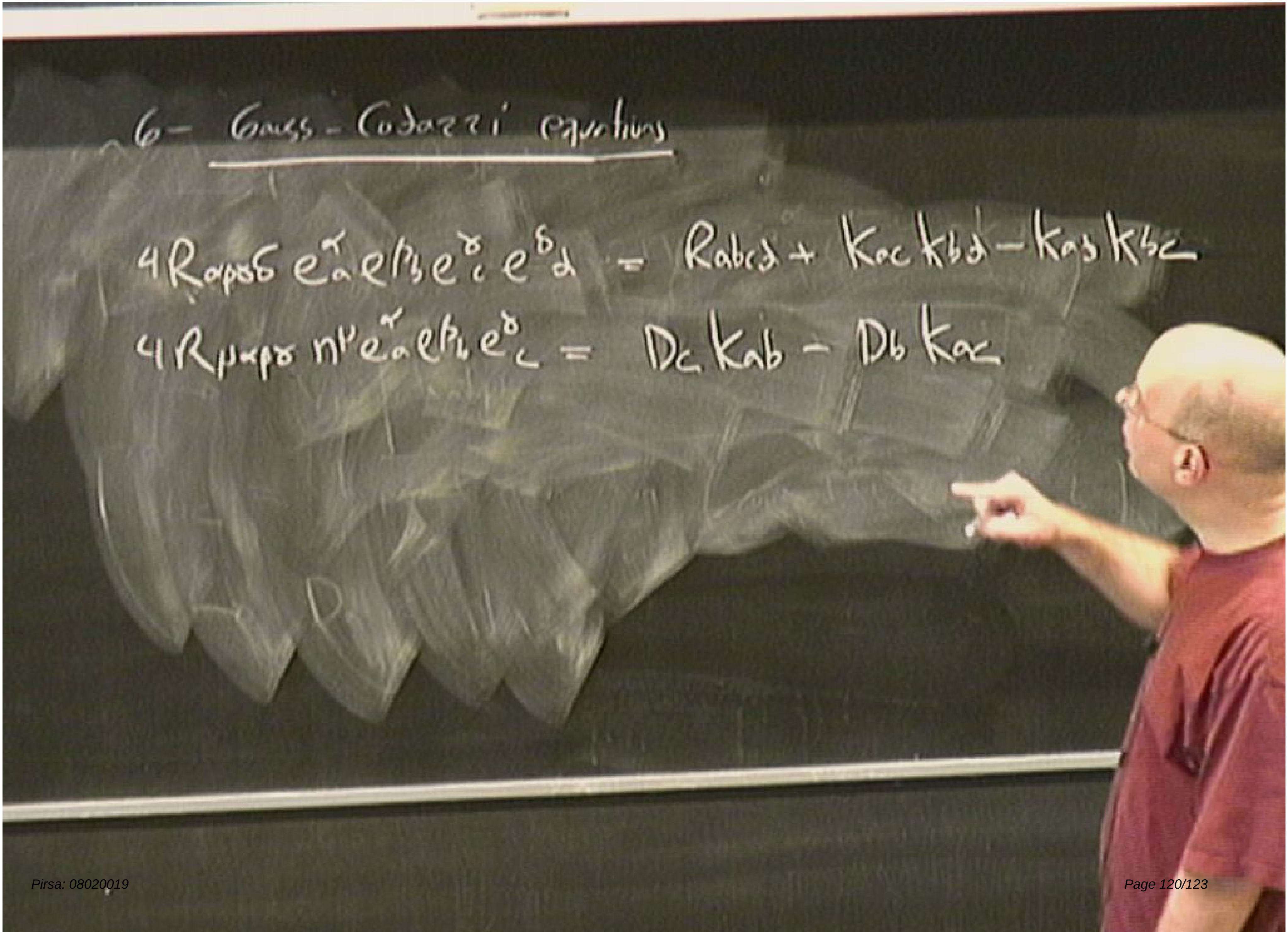
$$^4R_{\alpha\beta\gamma\delta}\, e^{\alpha}_{a}\, e^{\beta}_{b}\, e^{\gamma}_{c}\, e^{\delta}_{d}$$


6 - Gauss - Codazzi equations
${}^4R_{\alpha\beta\gamma\delta}\, e^\alpha_a e^\beta_b e^\gamma_c e^\delta_d = R_{abcd} + K_{ac}K_{bd} - K_{ad}K_{bc}$
${}^4R_{\mu\alpha\beta\gamma}\, n^\mu e^\alpha_a e^\beta_b e^\gamma_c$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$$^4G_{\alpha\beta}\, n^\alpha n^\beta =$$

6 - Gauss - Codazzi equations

$${}^4R_{\alpha\beta\gamma\delta}\, e^{\alpha}{}_{a}\, e^{\beta}{}_{b}\, e^{\gamma}{}_{c}\, e^{\delta}{}_{d} = R_{abcd} + K_{ac}K_{bd} - K_{ad}K_{bc}$$

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$${}^4G_{\alpha\beta}\, n^{\alpha} n^{\beta} = \frac{1}{2}\left(R - K^{ab}K_{ab} + K^2 \right)$$

$${}^4G_{\alpha\beta}\, n^{\alpha} e^{\beta}{}_{b} =$$

6 - Gauss-Codazzi equations

$$\boxed{\begin{aligned}
{}^4R_{\alpha\beta\gamma\delta}\, e^\alpha_a e^\beta_b e^\gamma_c e^\delta_d &= R_{abcd} + K_{ac}K_{bd} - K_{ad}K_{bc} \\
{}^4R_{\mu\alpha\beta\gamma}\, n^\mu e^\alpha_a e^\beta_b e^\gamma_c &= D_c K_{ab} - D_b K_{ac} \\
{}^4G_{\alpha\beta}\, n^\alpha n^\beta &= \frac{1}{2}\left(R - K^{ab}K_{ab} + K^2 \right) \\
{}^4G_{\alpha\beta}\, n^\alpha e^\beta_a &= D_b K^b{}_a - D_a K
\end{aligned}}$$