Title: Advanced General Relativity - Lecture 6A

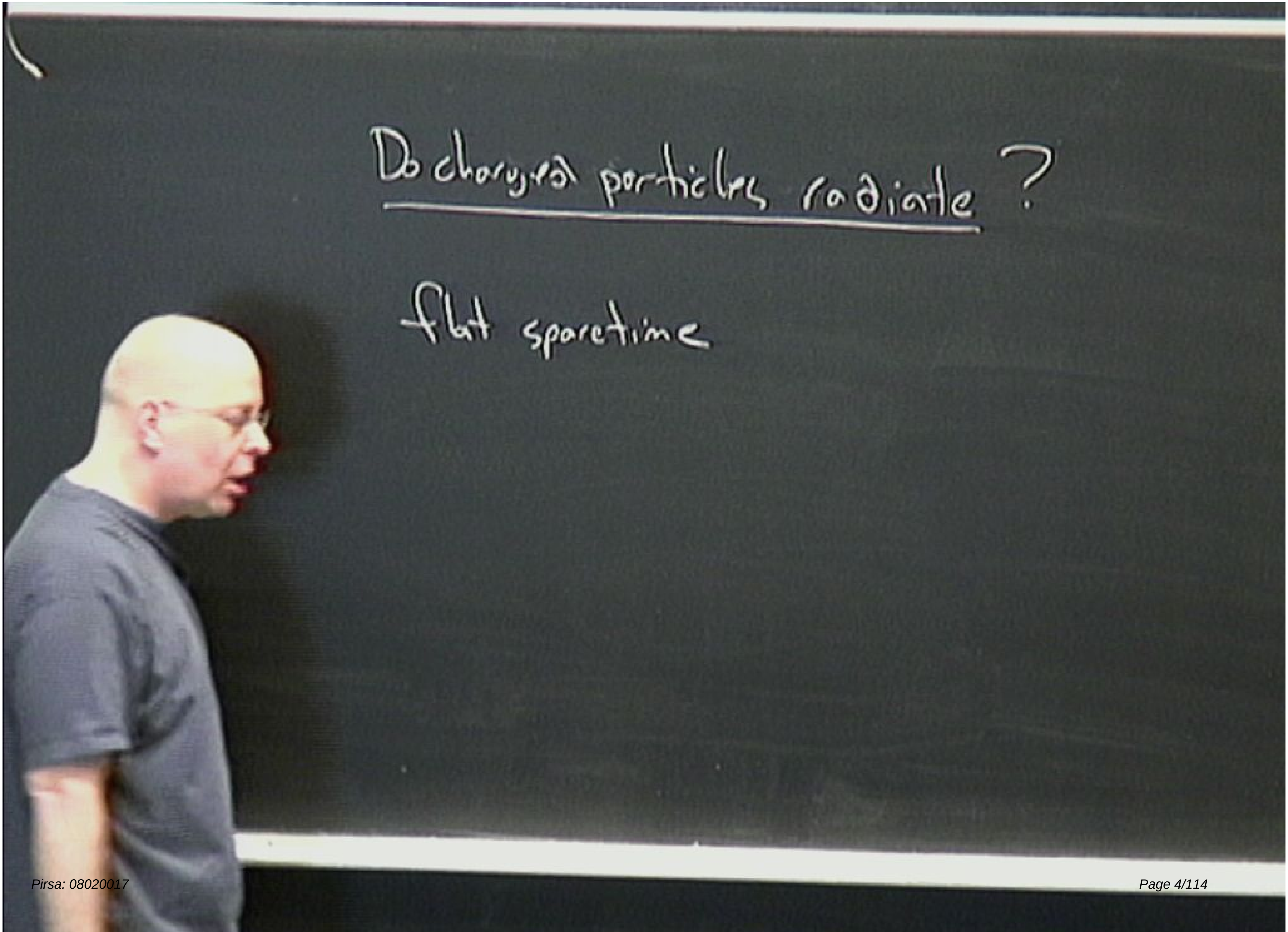
Date: Feb 13, 2008 10:30 AM

URL: http://pirsa.org/08020017

Abstract: Advanced General Relativity

Do charged particles radiate ?

Do charged particles radiate?

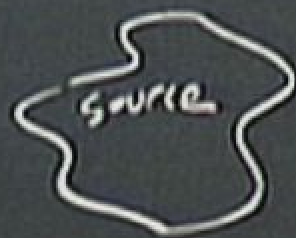

Do charged particles radiate ?
flat spacetime

Do charged particles radiate?

flat spacetime

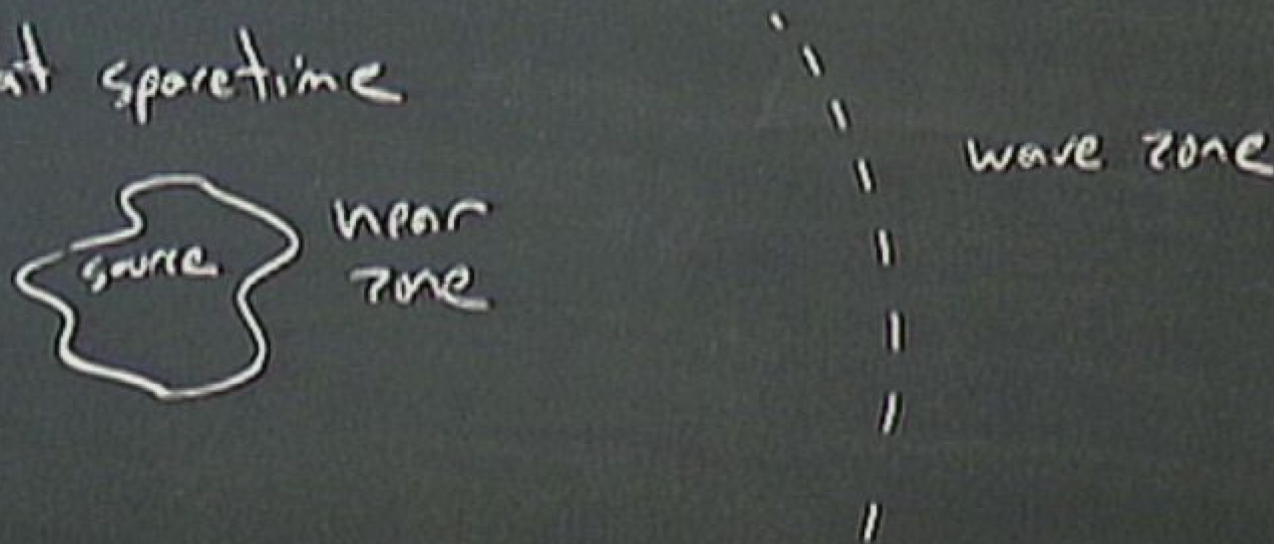
Do charged particles radiate?

flat spacetime



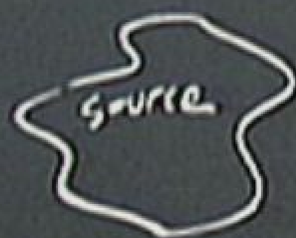
Do charged particles radiate?

flat spacetime



Do charged particles radiate?

flat spacetime



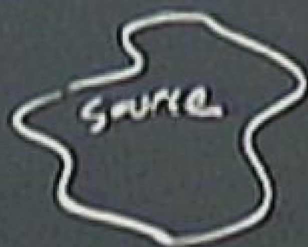
near zone

wave zone

- propagation at c
- transverse
- $\vec{E} \propto 1/r$
- $\vec{B} \propto 1/r$

Do charged particles radiate?

flat spacetime



near zone

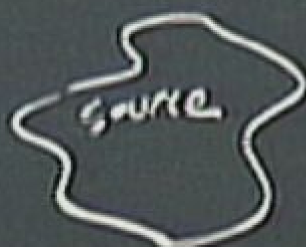
time-dependent process t_c

wave zone

- propagation at c
- transverse
- $\vec{E} \propto 1/r$
- $\vec{B} \propto 1/r$

Do charged particles radiate?

flat spacetime



near zone

time-dependent process t_c

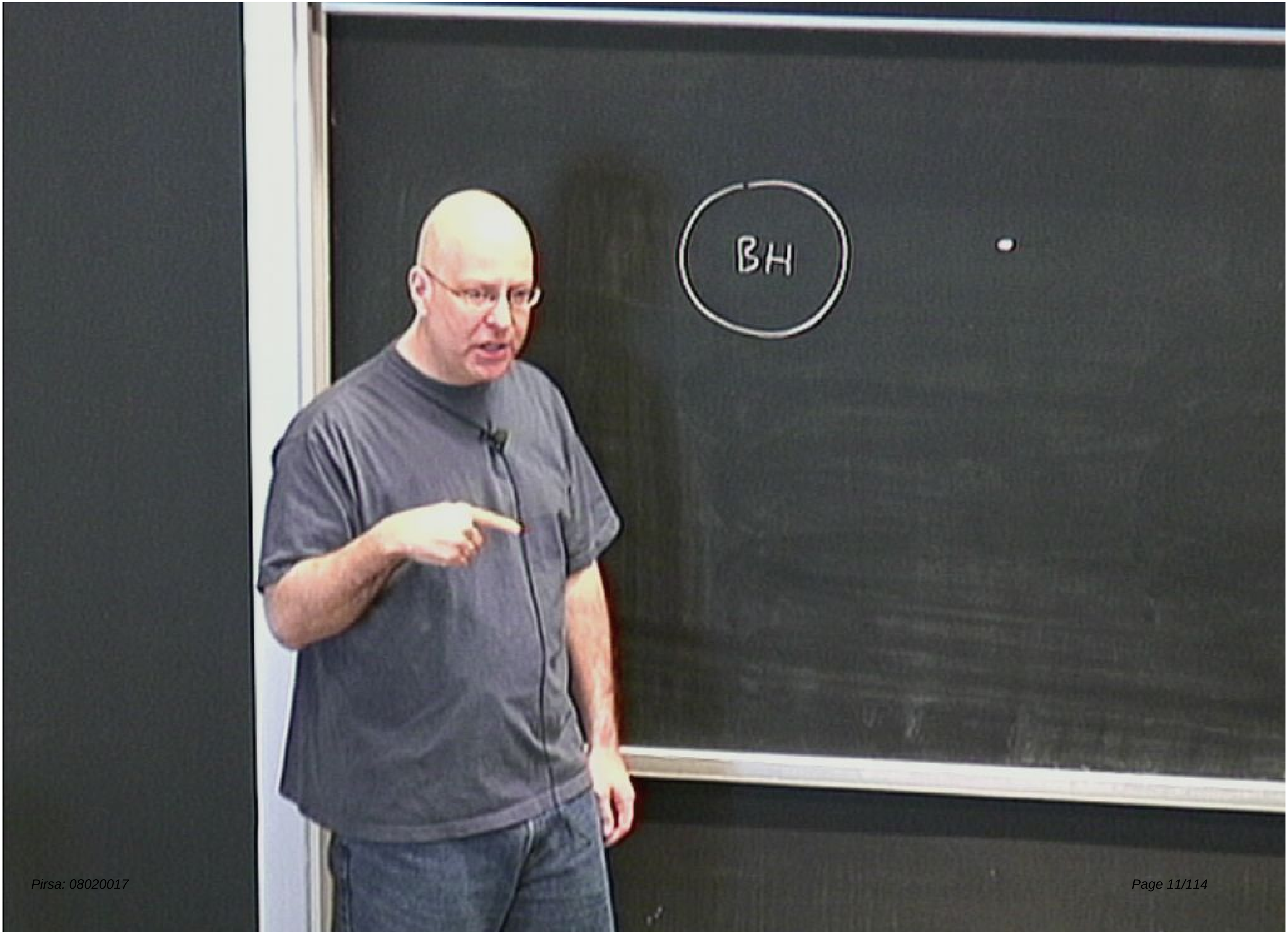
$\omega_c \sim 1/t_c$

$\lambda_c \sim c\,t_c \sim c/\omega_c$

wave zone : $r \gg \lambda_c$

wave zone

$\left\{\begin{array}{l}\text{propagation at } c \\ \text{transverse} \\ \vec{E} \propto 1/r \\ \vec{B} \propto 1/r\end{array}\right.$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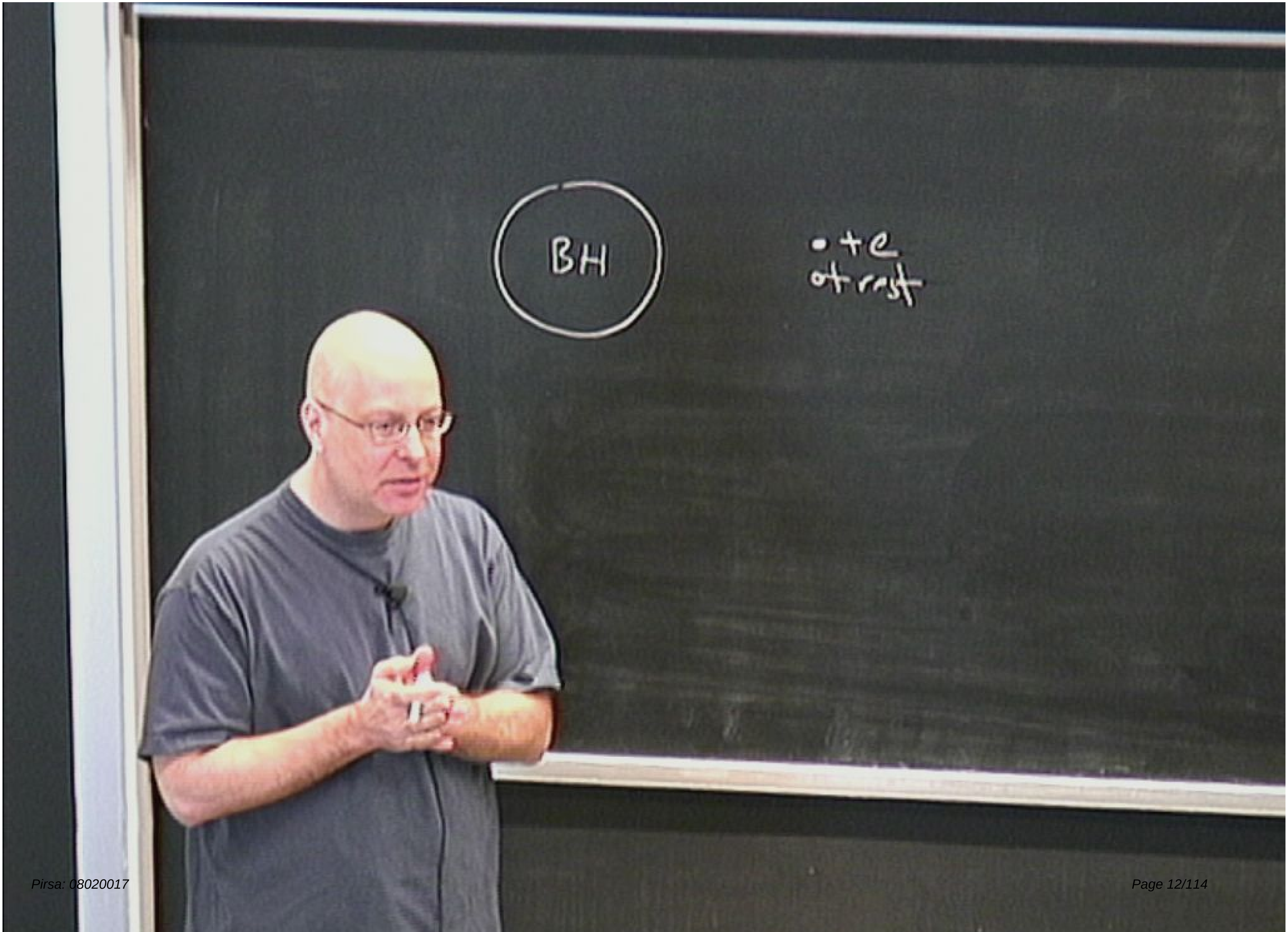
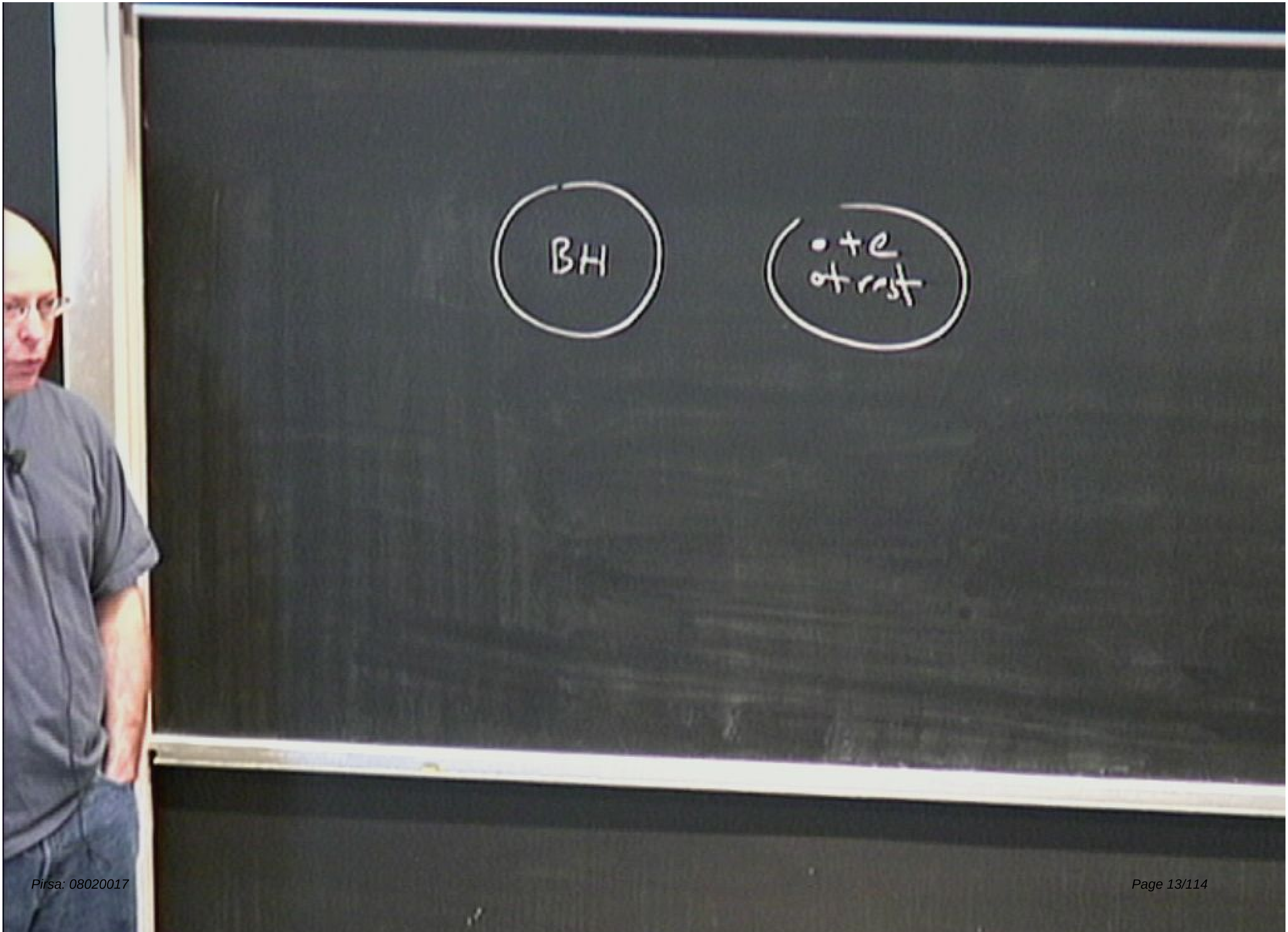
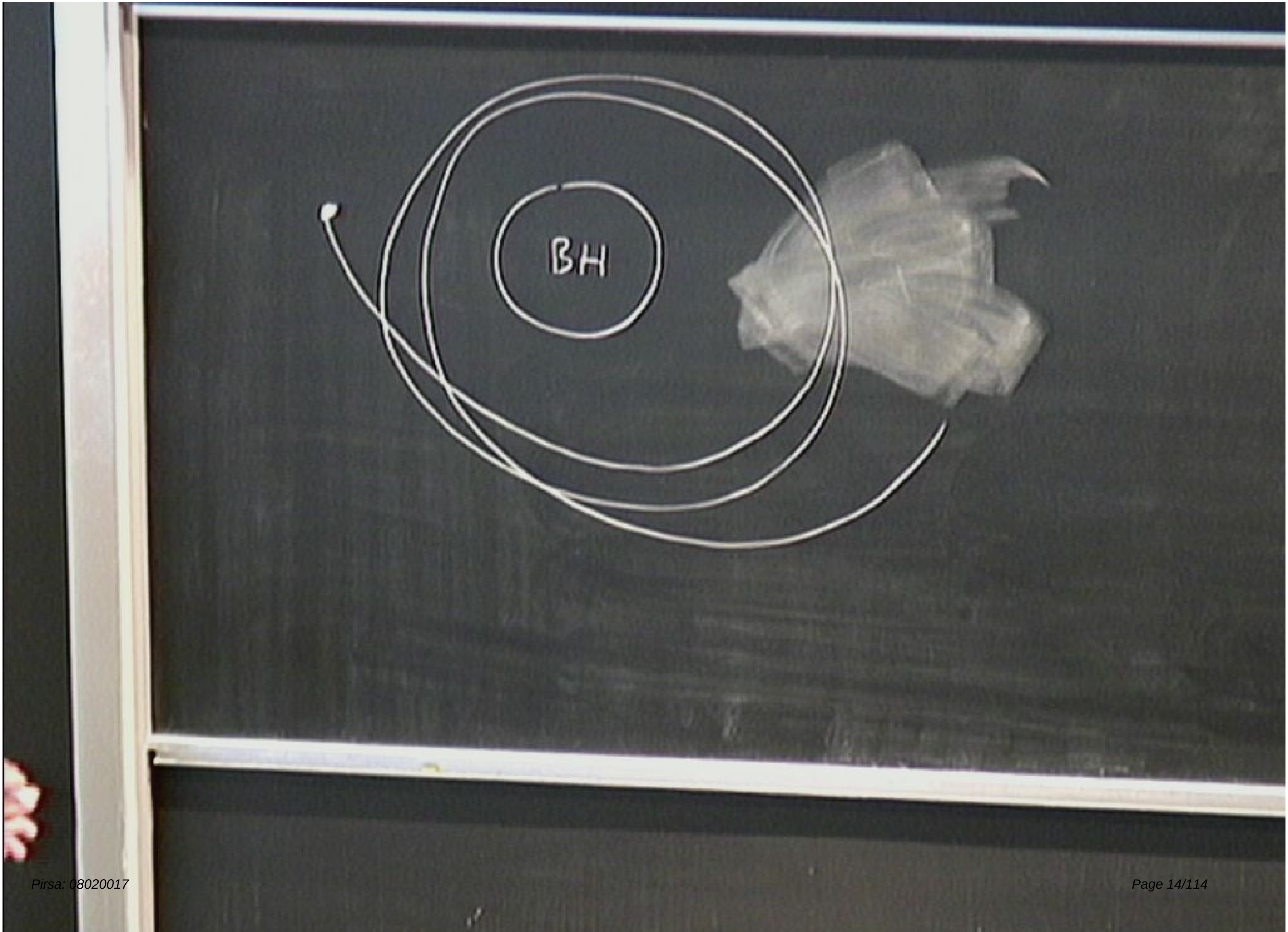

BH
• + e
at rest


BH
+e
at rest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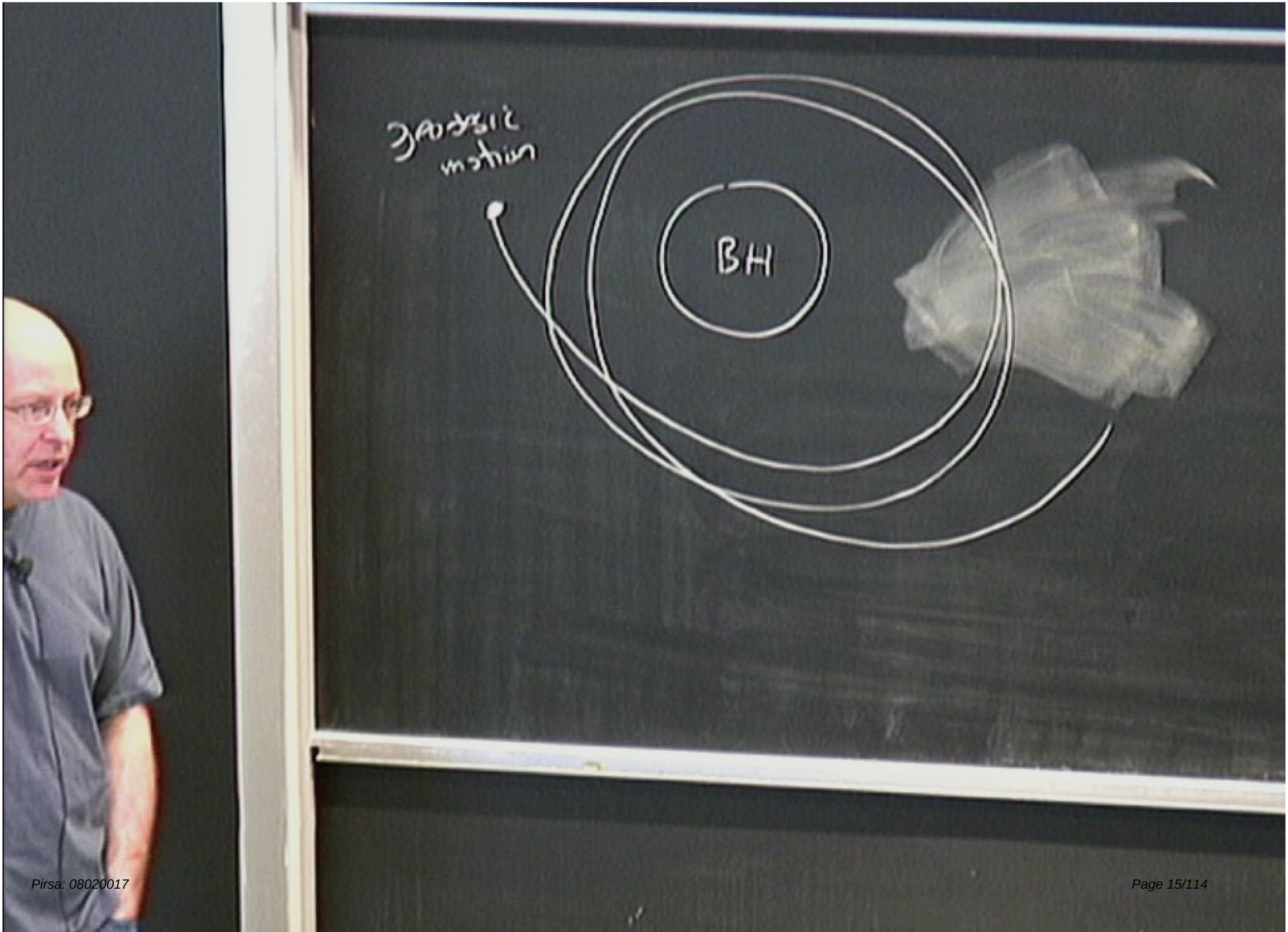
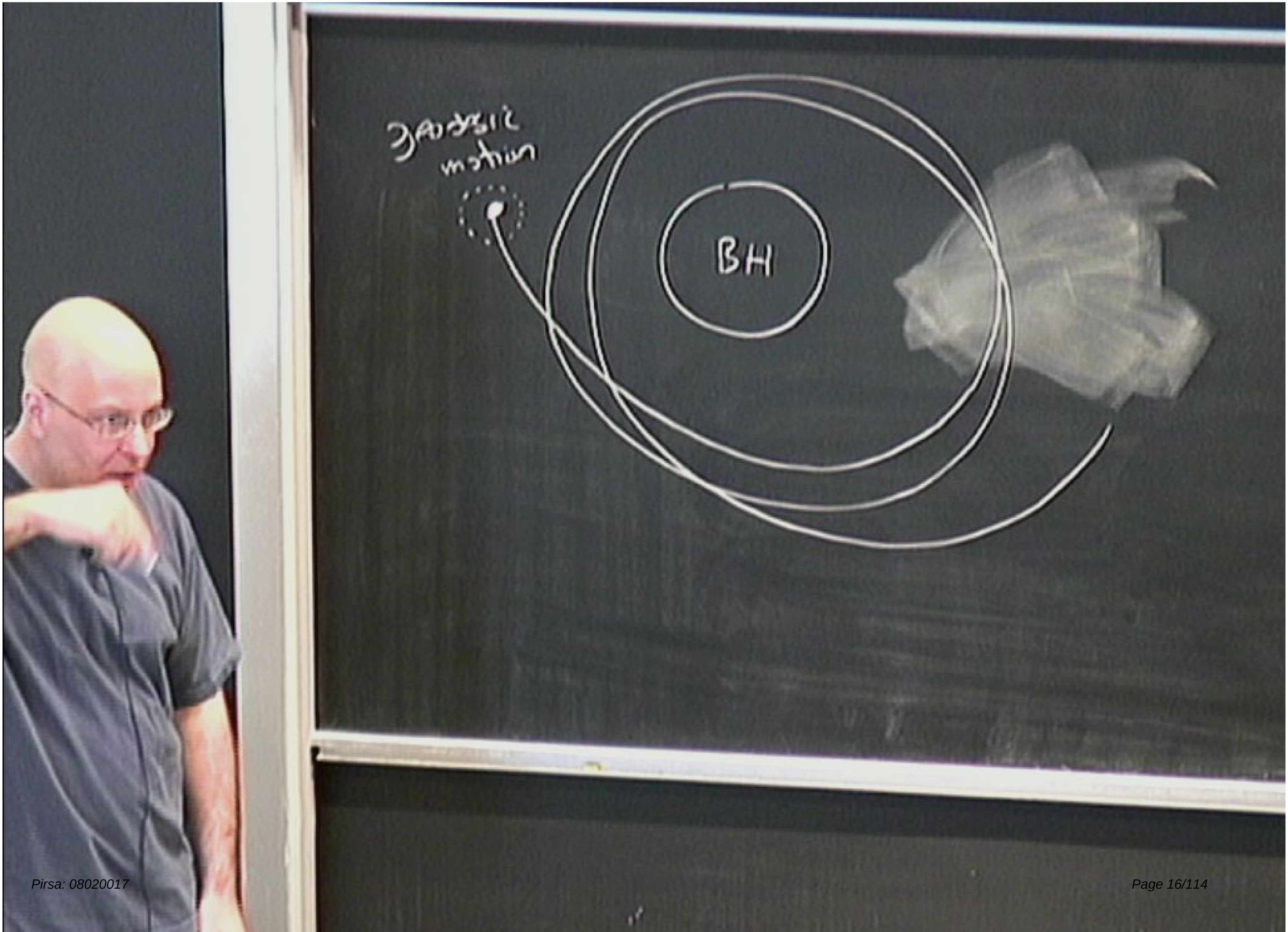
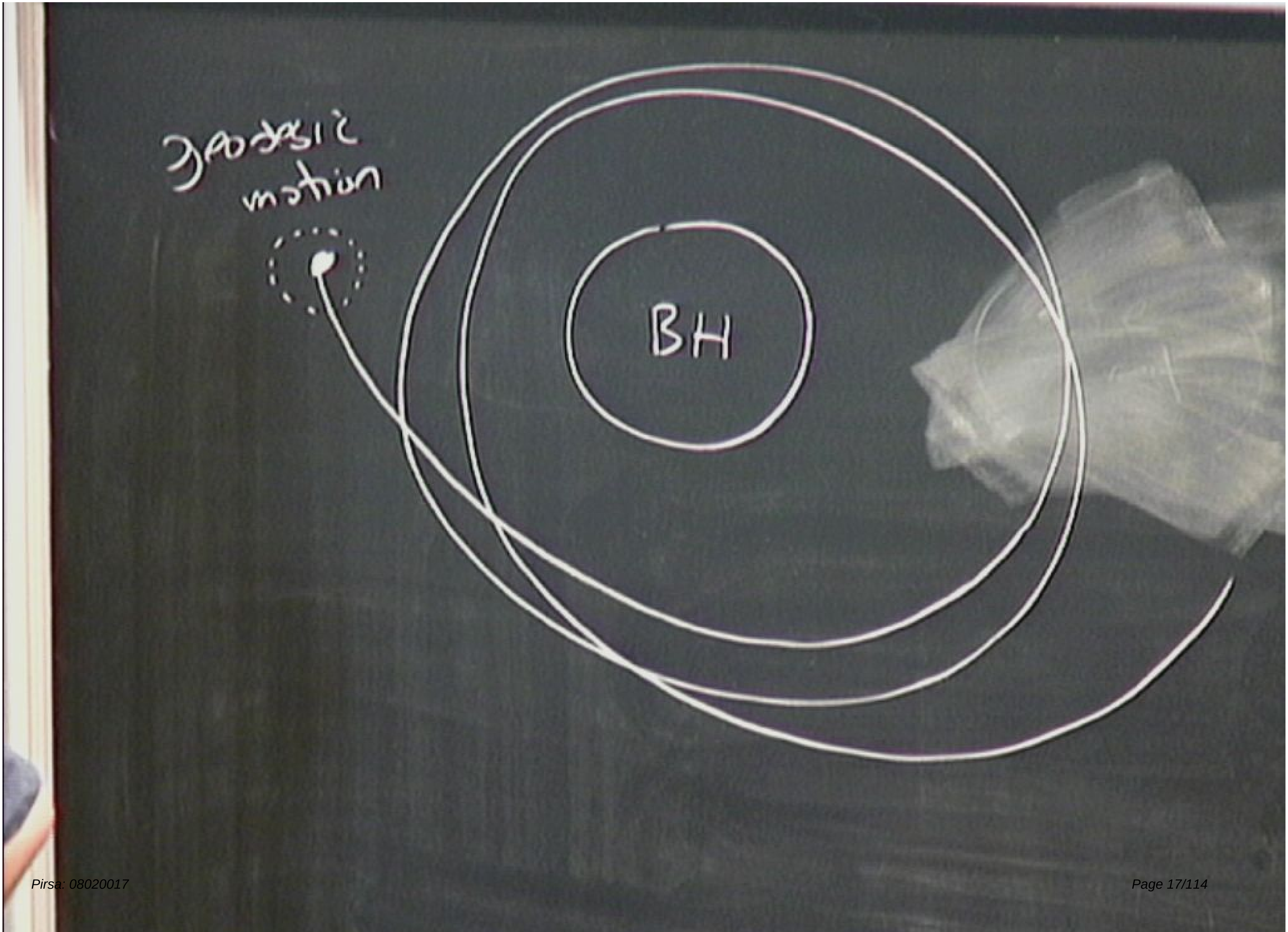
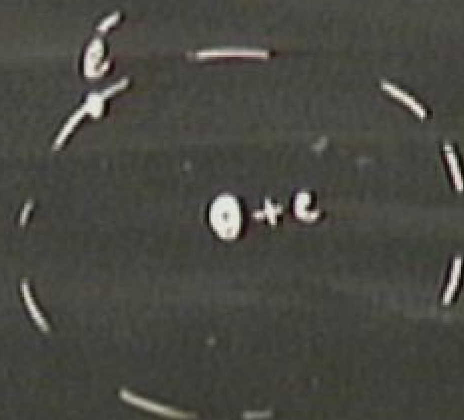

geodesic
motion
BH


geodesic
motion
BH

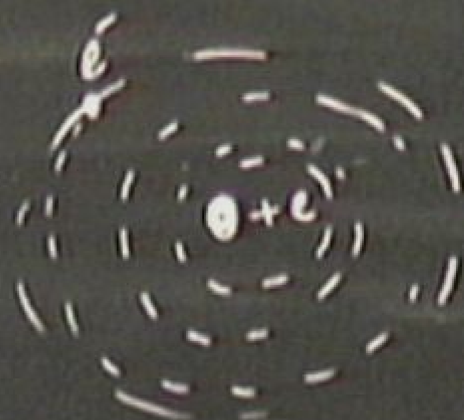

geodesic
motion
BH

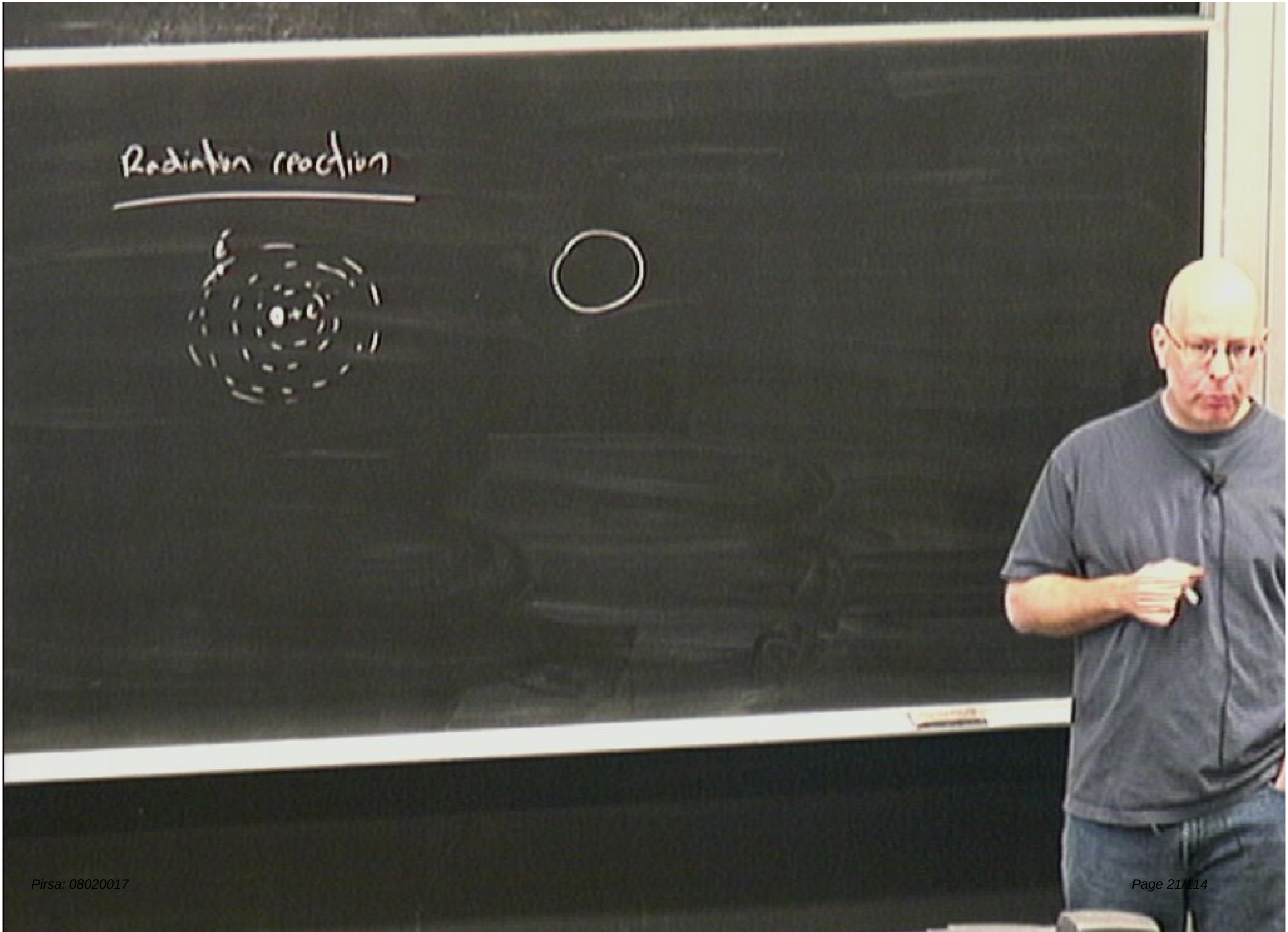
Radiation reaction

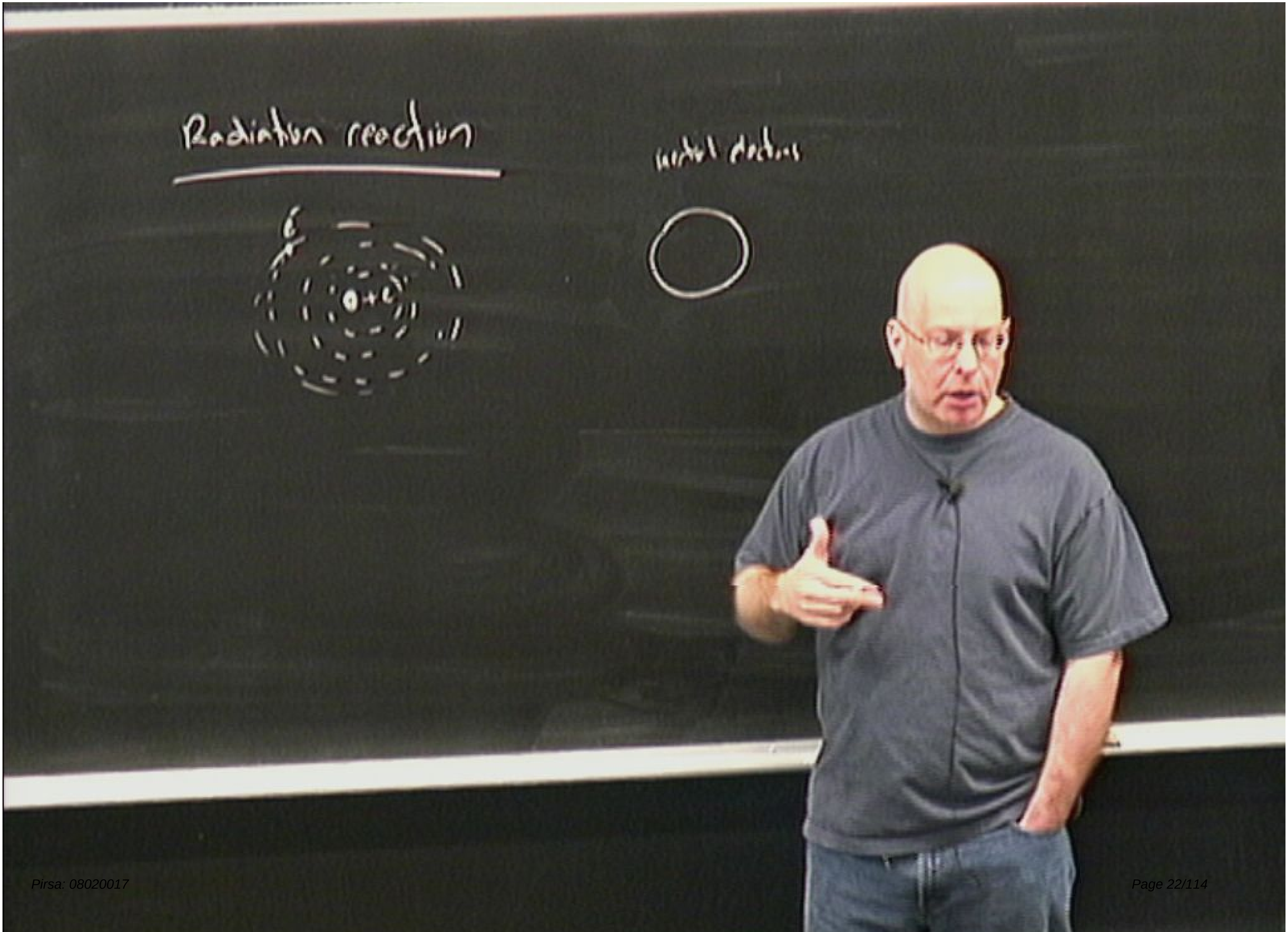
Radiation reaction



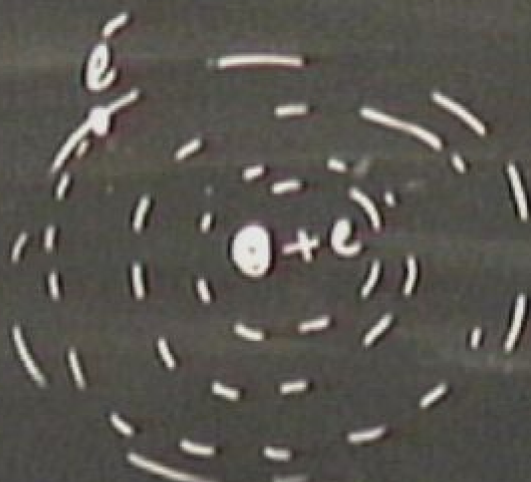
Radiation reaction




Radiation reaction

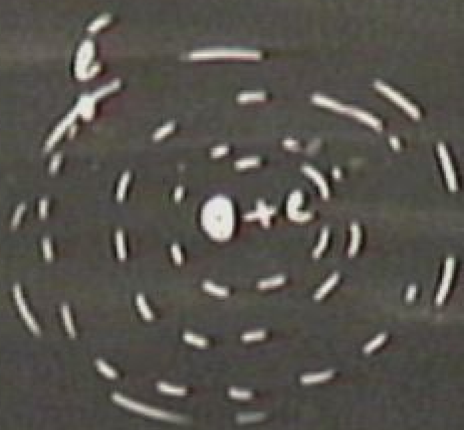

Radiation reaction

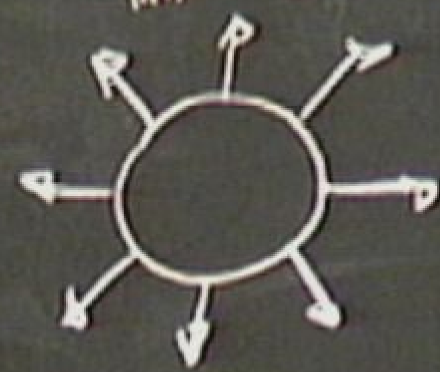
Radiation reaction



inertial electrons

Radiation reaction

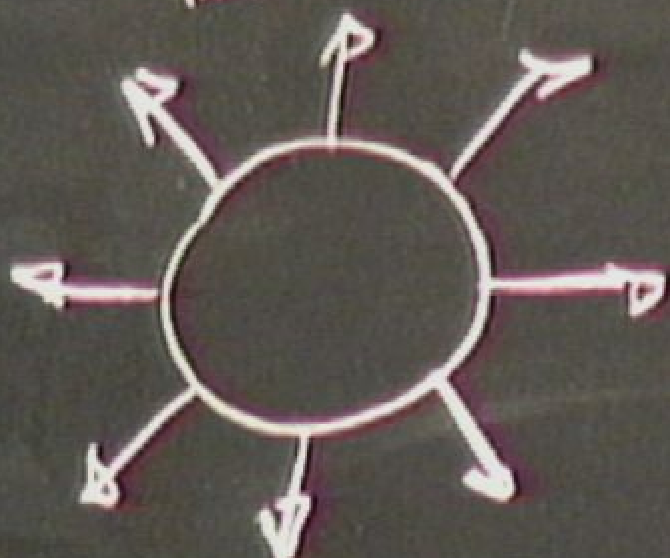




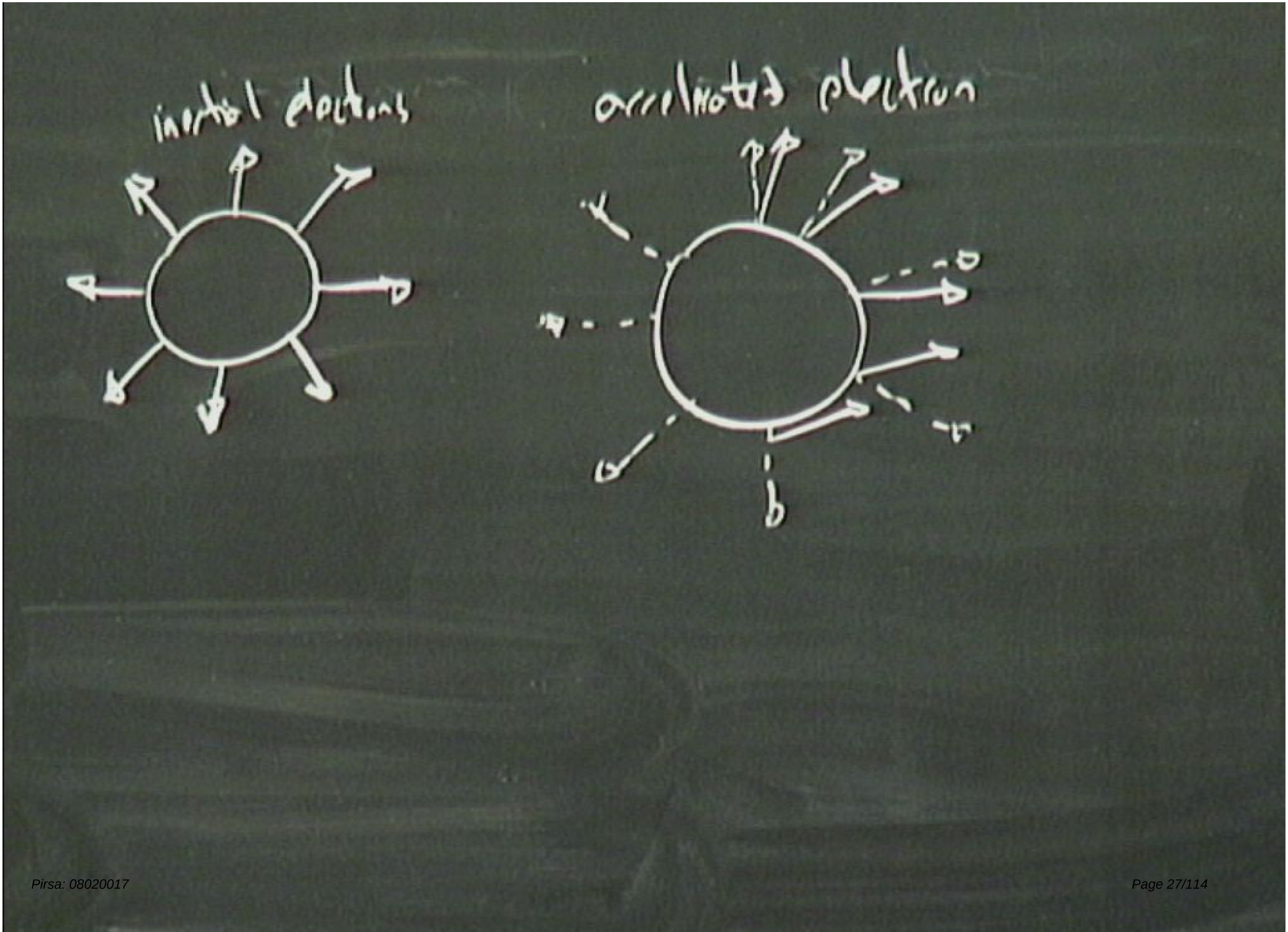
Radiation reaction

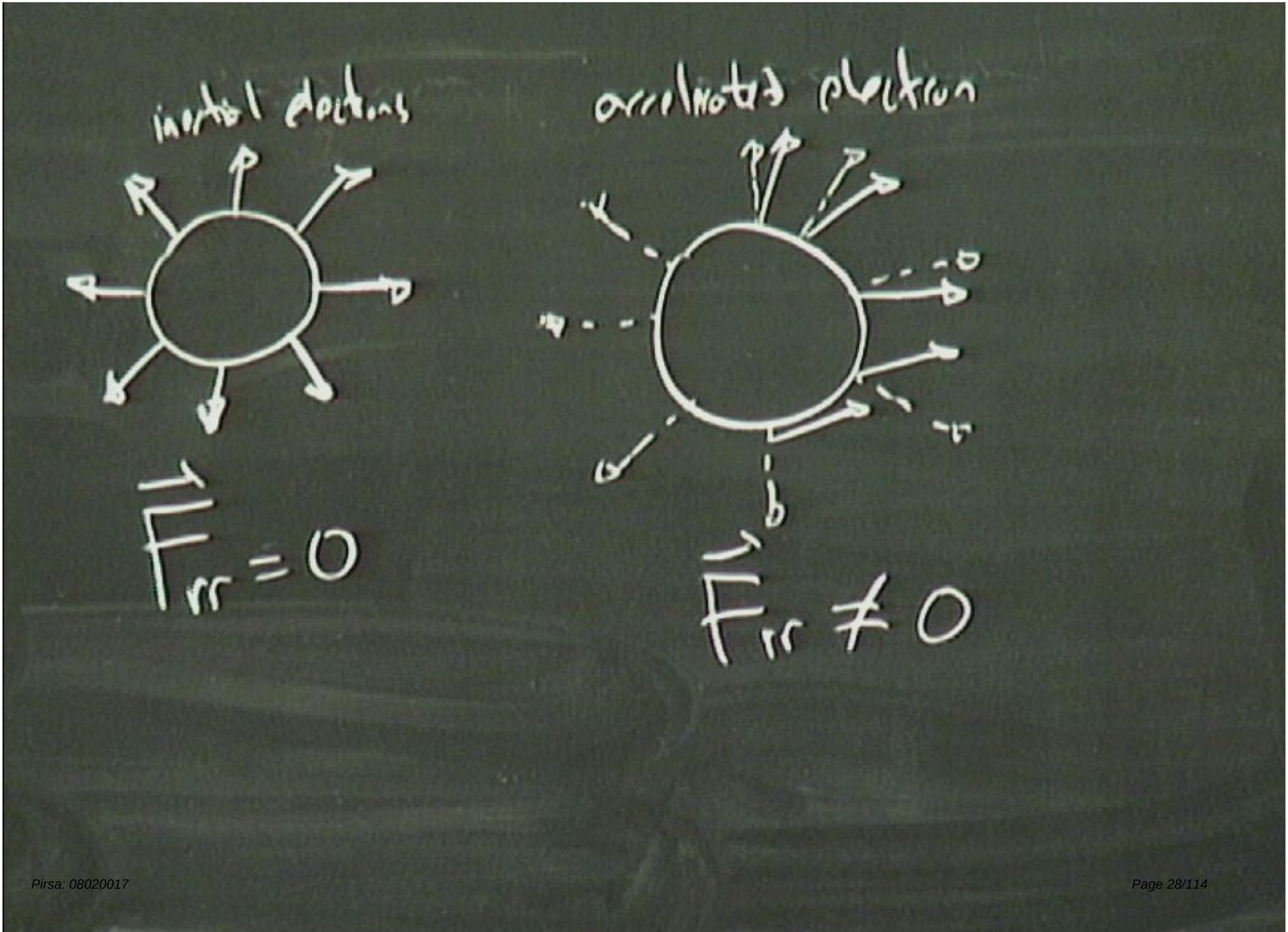
oscillating electron

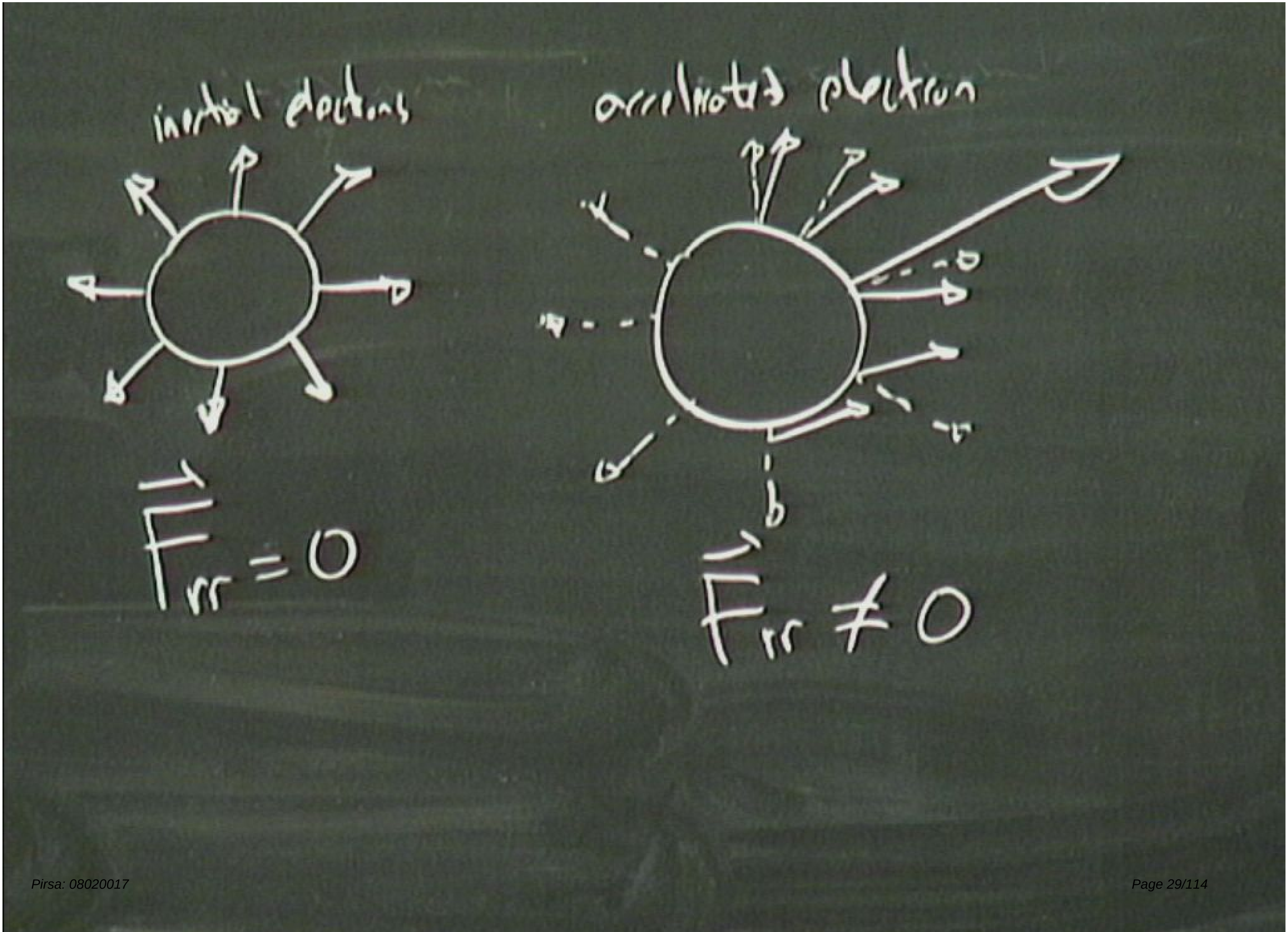
inertial electrons



accelerated electron


inertial electrons
accelerated electron
b


inertial electrons
accelerated electron
$\vec{F}_{rr} = 0$
$\vec{F}_{rr} \neq 0$


inertial electrons
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$\vec{F}_{rr} = 0$
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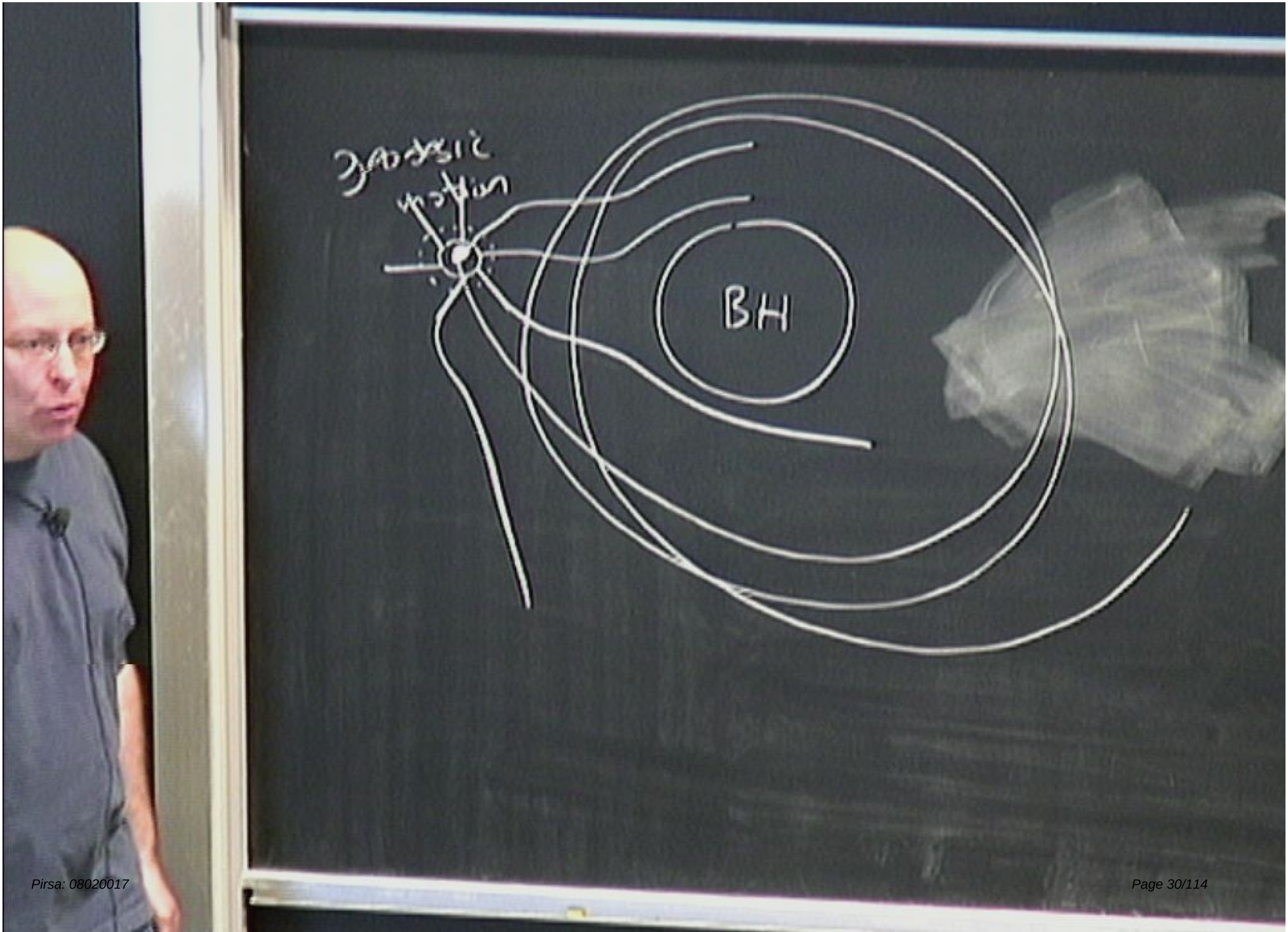
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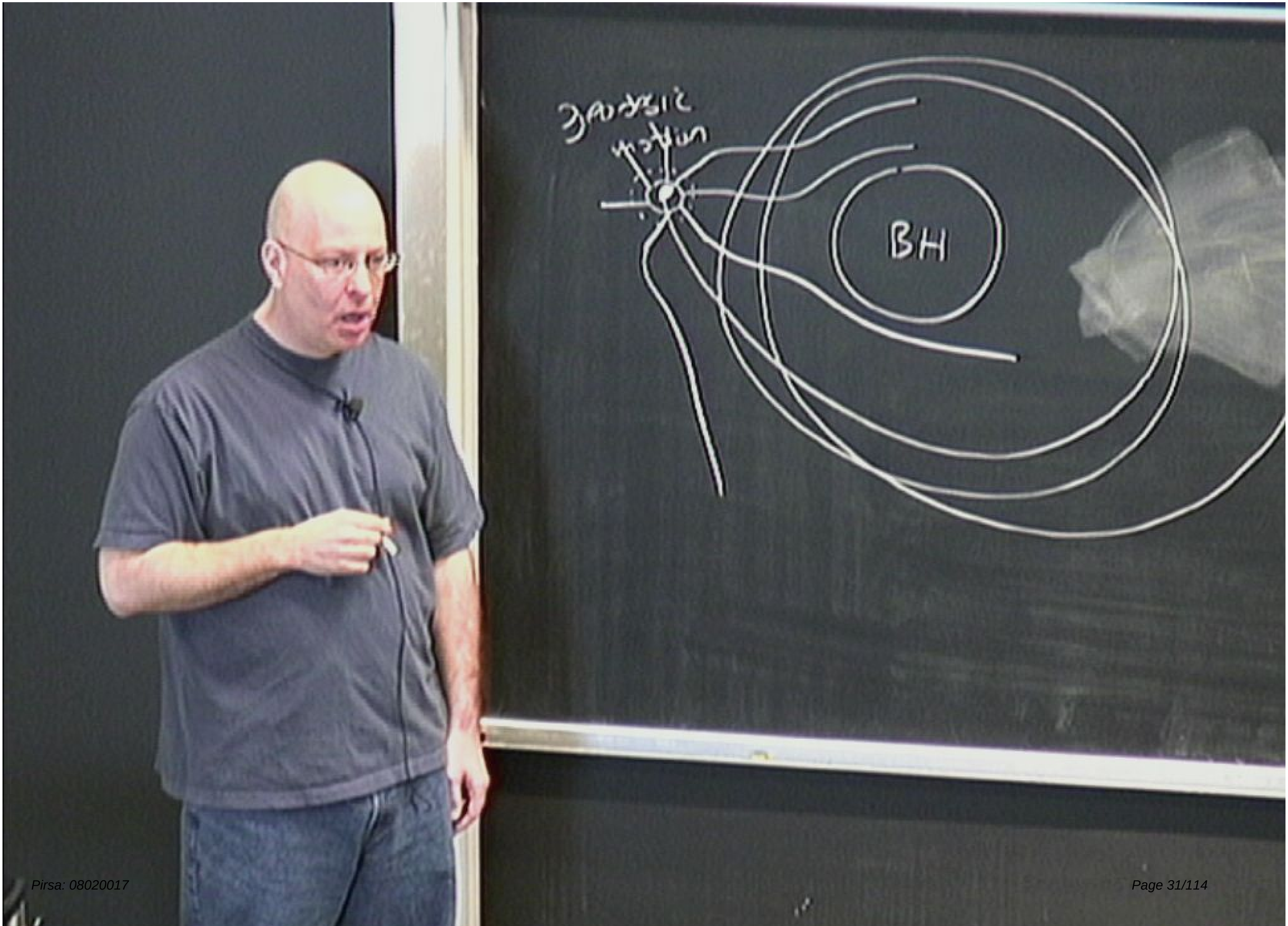
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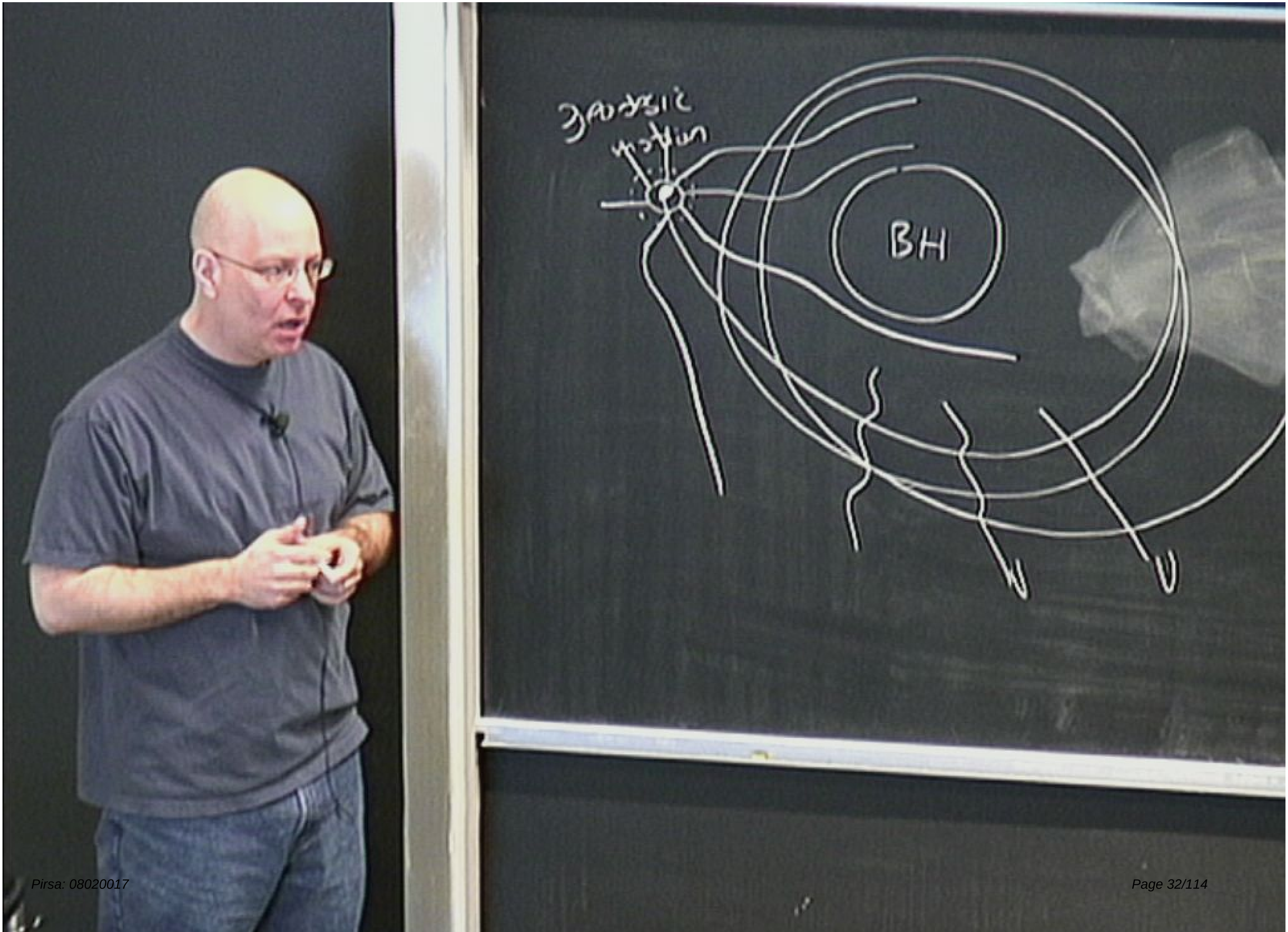
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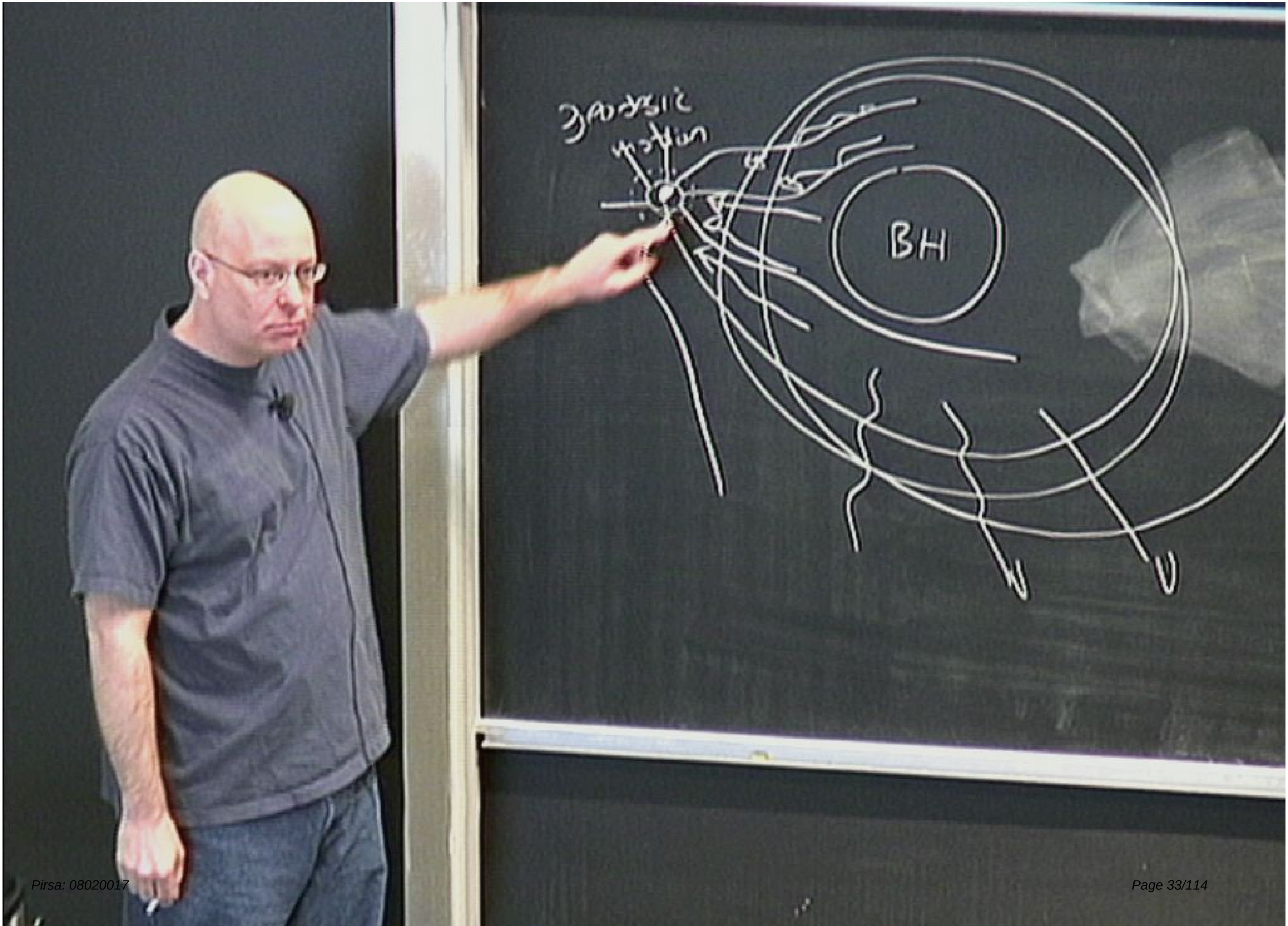
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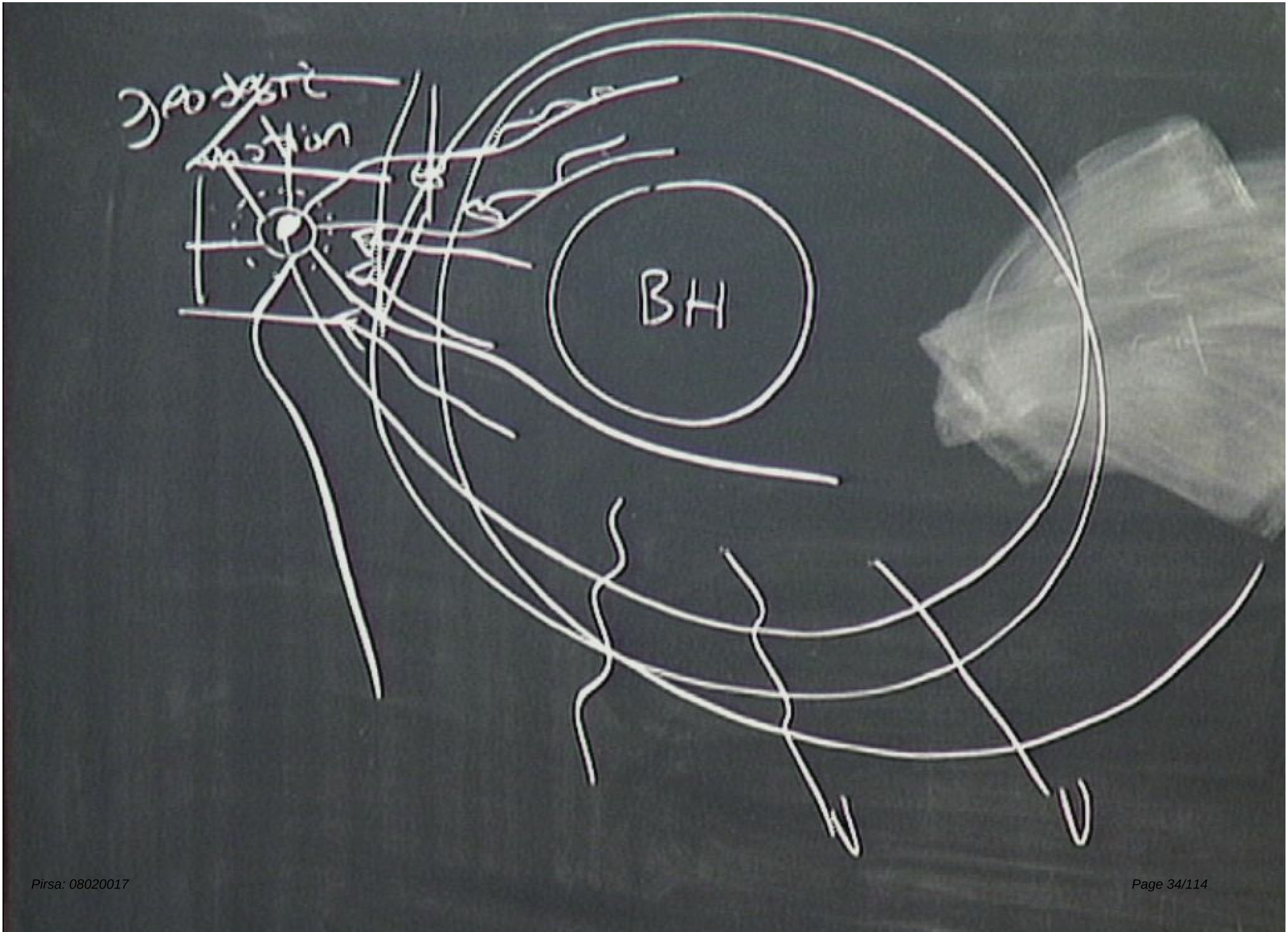
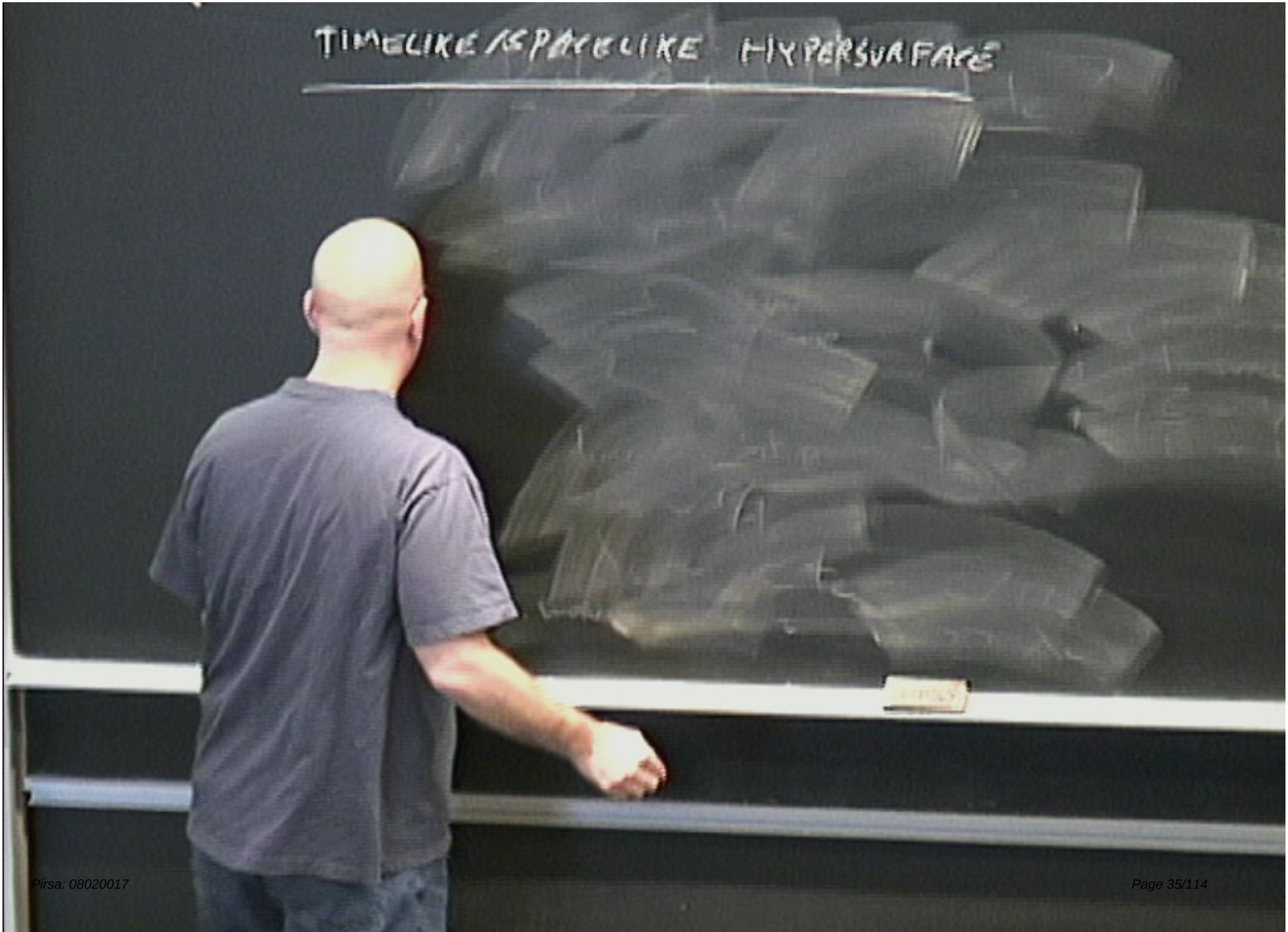
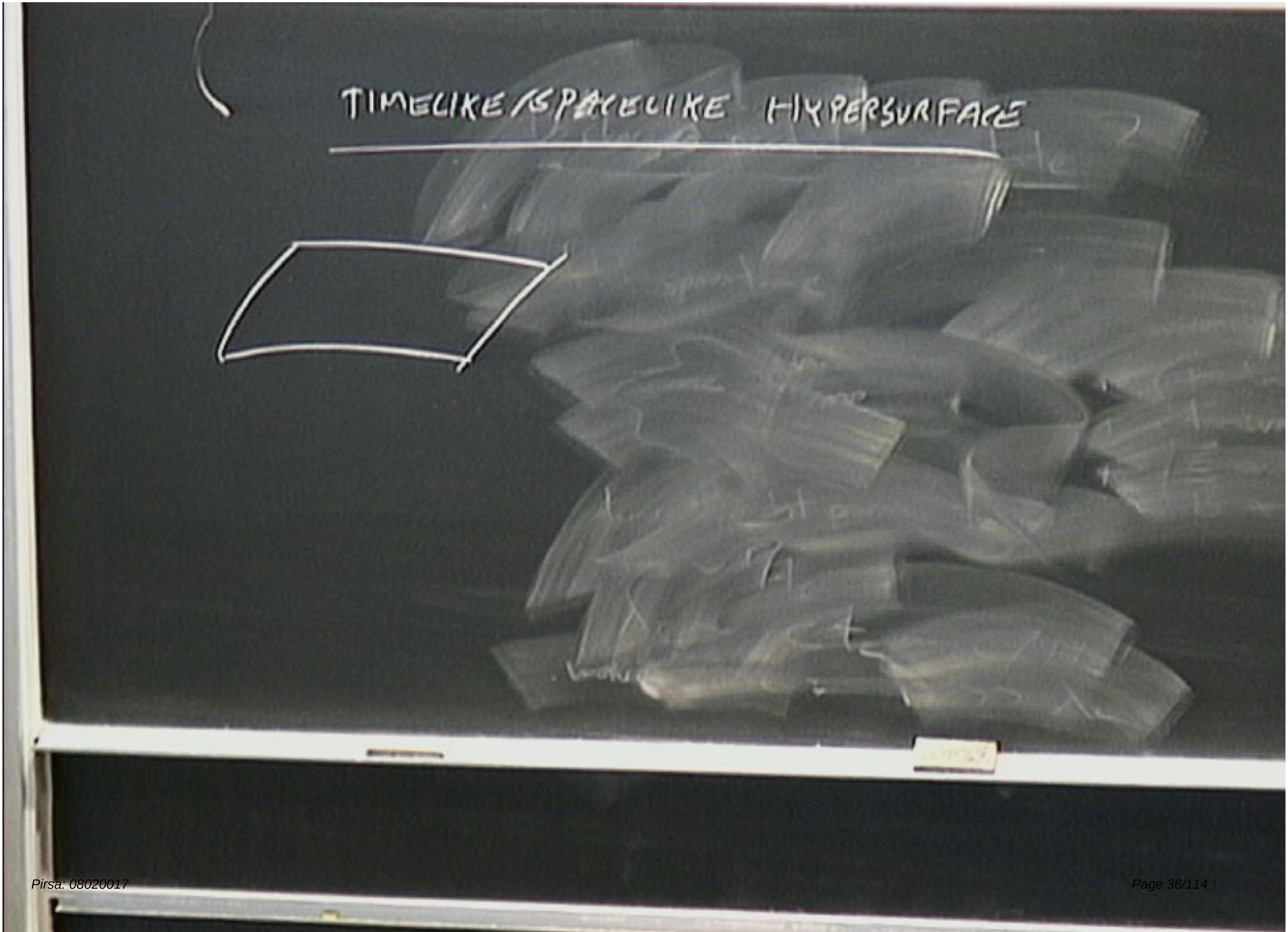
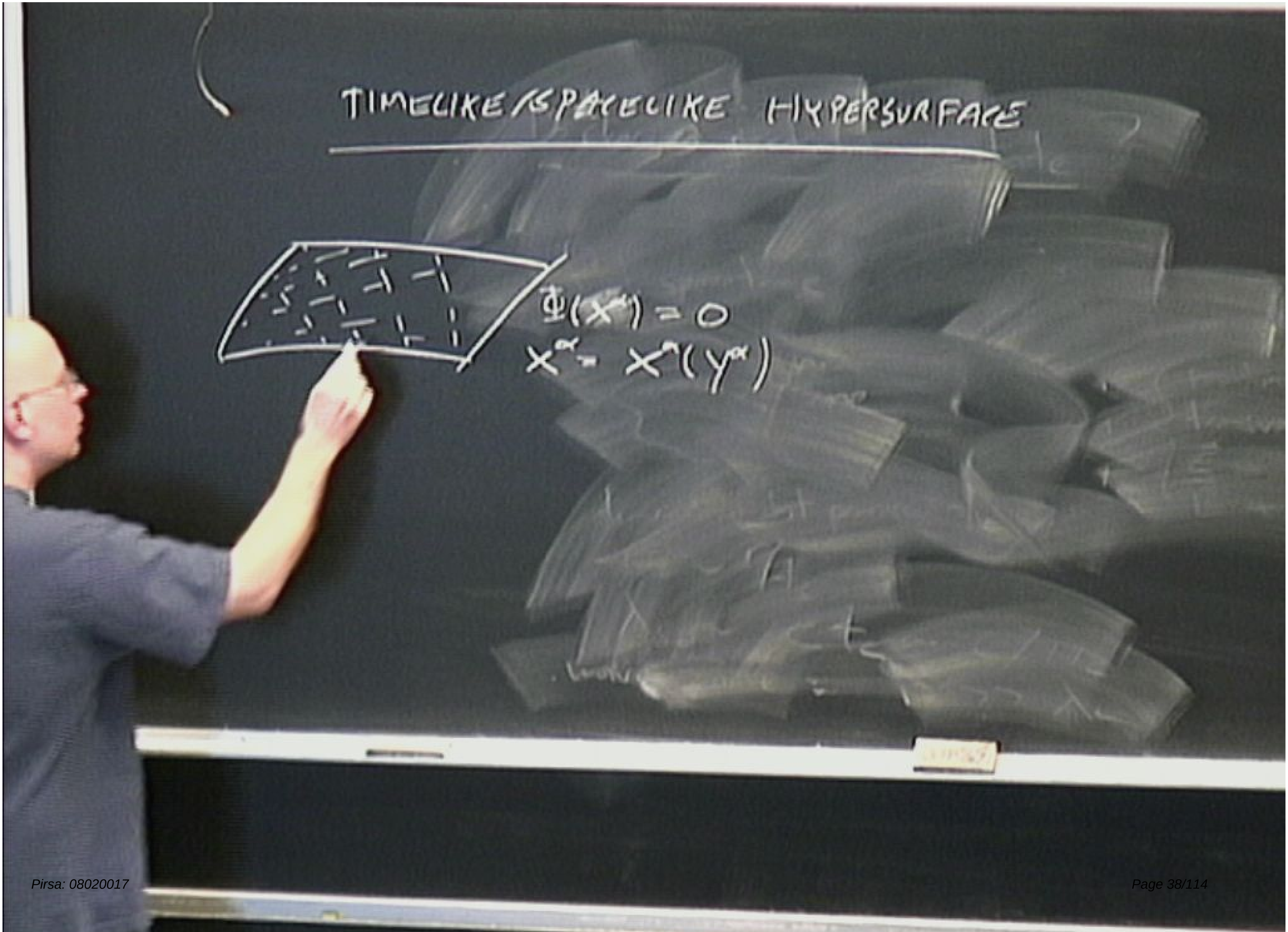

geodesic
motion
BH


TIMELIKE / SPACELIKE HYPERSURFACE


TIMELIKE/SPACELIKE HYPERSURFACE

TIMELIKE/SPACELIKE HYPERSURFACE

$$\Phi(x^\mu) = 0$$


TIMELIKE/SPACELIKE HYPERSURFACE
$\Phi(x^\alpha) = 0$
$x^\alpha = x^\alpha(y^a)$

TIMELIKE/SPACELIKE HYPERSURFACE

$$\Phi(x^\alpha) = 0$$

$$x^\alpha = x^\alpha(y^a)$$

TIMELIKE / SPACELIKE HYPERSURFACE

$\Phi(x^\alpha) = 0 \longrightarrow n_\alpha \propto \partial_\alpha \Phi$

$x^\alpha = x^\alpha(y^a)$

TIMELIKE/SPACELIKE HYPERSURFACE

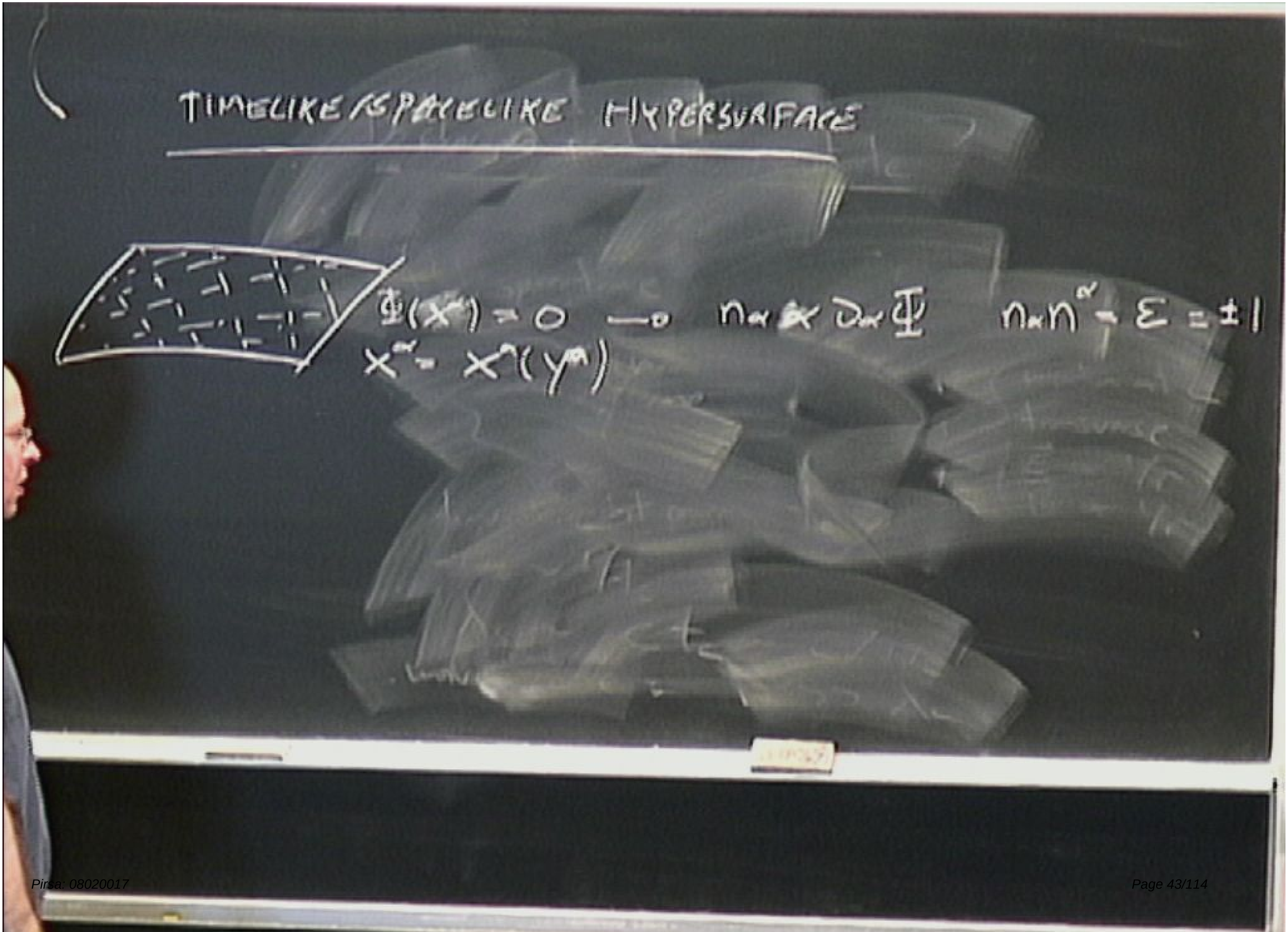
$\Phi(x^\alpha) = 0 \longrightarrow n_\alpha \propto \partial_\alpha \Phi \qquad n_\alpha n^\alpha = \varepsilon$

$x^\alpha = x^\alpha(y^a)$

TIMELIKE/SPACELIKE HYPERSURFACE

$$\Phi(x^\alpha) = 0 \longrightarrow n_\alpha \propto \partial_\alpha \Phi \qquad n_\alpha n^\alpha = \varepsilon = \pm$$

$$x^\alpha = x^\alpha(y^a)$$


TIMELIKE/SPACELIKE HYPERSURFACE
$\Phi(x^\alpha) = 0 \rightarrow n_\alpha \propto \partial_\alpha \Phi \quad n_\alpha n^\alpha = \varepsilon = \pm 1$
$x^\alpha = x^\alpha(y^a)$

TIMELIKE/SPACELIKE HYPERSURFACE

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$$x^\alpha = x^\alpha(y^a) \longrightarrow e^\alpha_a = \frac{\partial x^\alpha}{\partial y^a}$$

TIMELIKE/SPACELIKE HYPERSURFACE

$\Phi(x^\alpha) = 0 \longrightarrow n_\alpha \propto \partial_\alpha \Phi \qquad n_\alpha n^\alpha = \varepsilon = \pm 1$

$x^\alpha = x^\alpha(y^a) \longrightarrow e^\alpha_a = \dfrac{\partial x^\alpha}{\partial y^a}$

$n_\alpha e^\alpha_a = 0$

$h_{ab} = g_{\alpha\beta}\, e^\alpha_a e^\beta_b$

$ds^2 =$

TIMELIKE/SPACELIKE HYPERSURFACE

$$\Phi(x^\alpha) = 0 \longrightarrow n_\alpha \propto \partial_\alpha \Phi \qquad n_\alpha n^\alpha = \varepsilon = \pm 1$$

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TIMELIKE / SPACELIKE HYPERSURFACE

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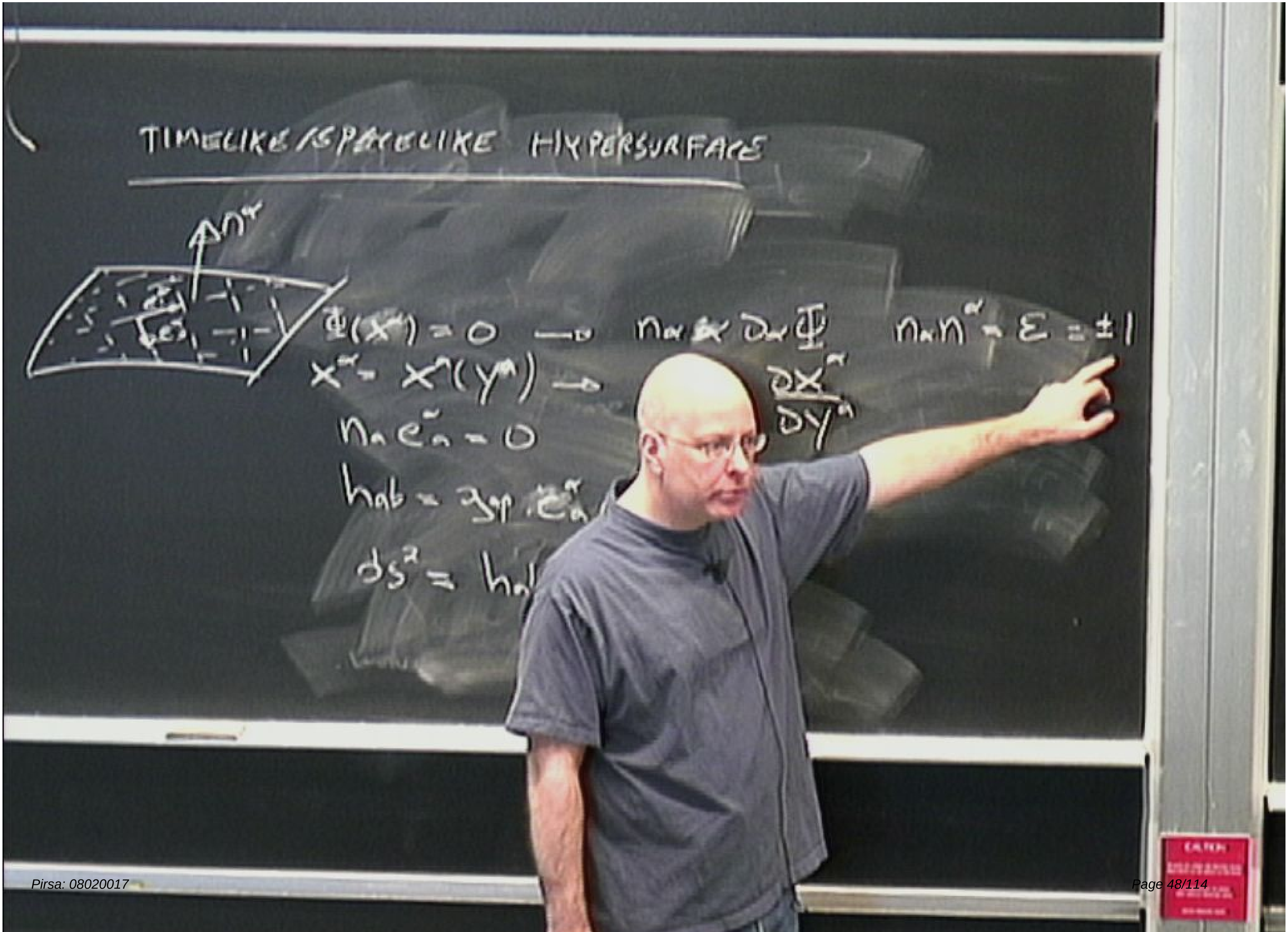
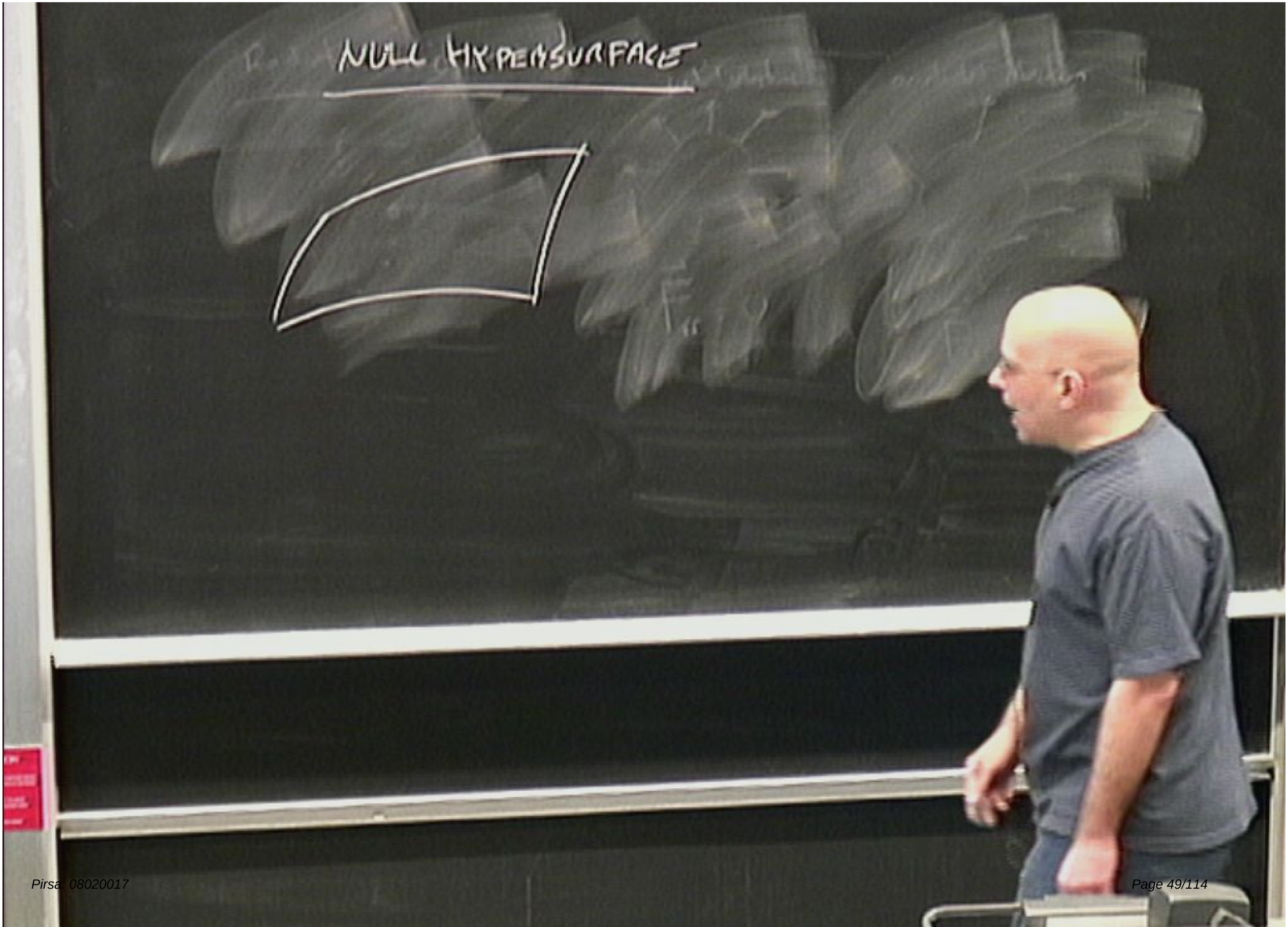
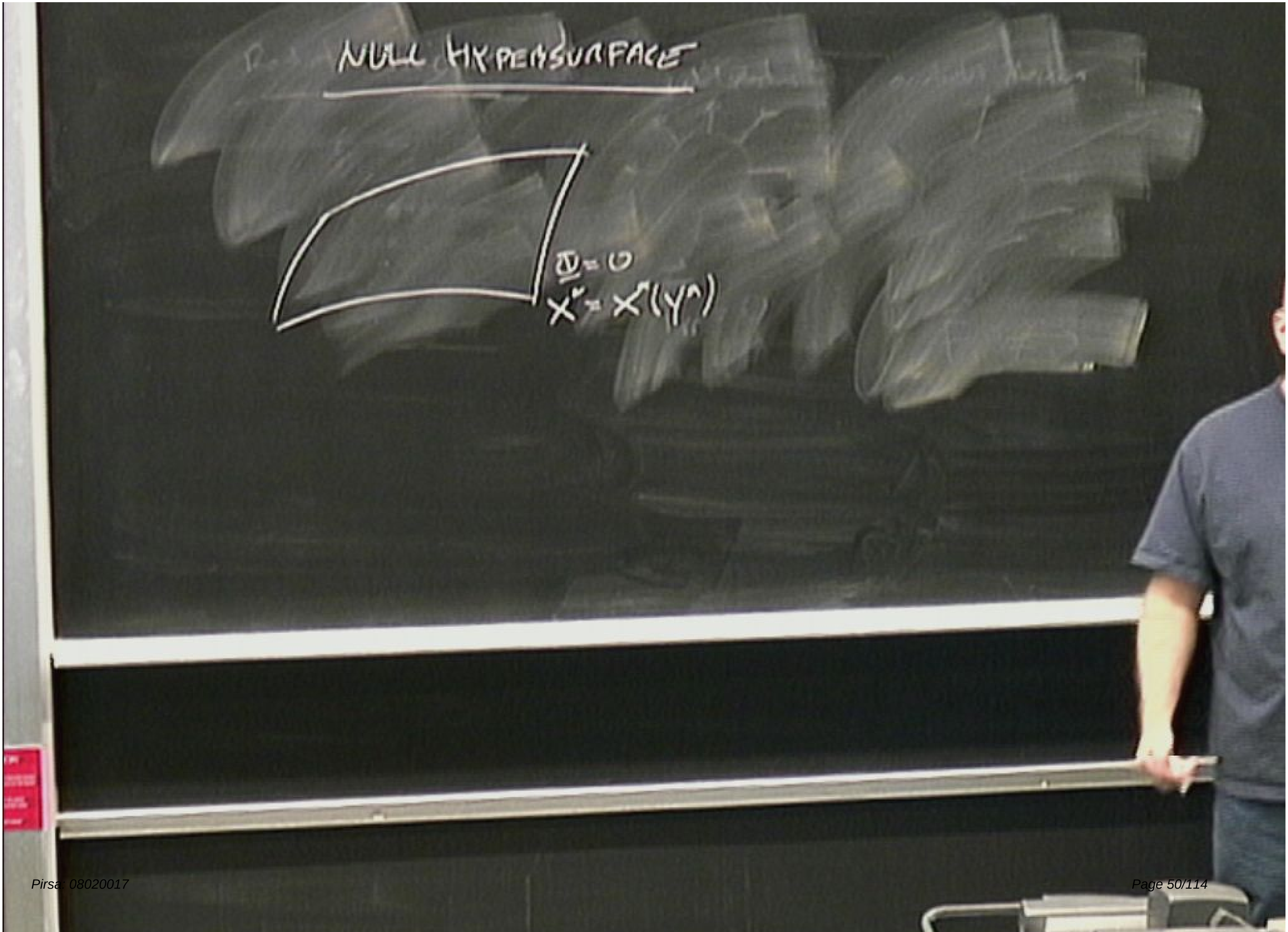
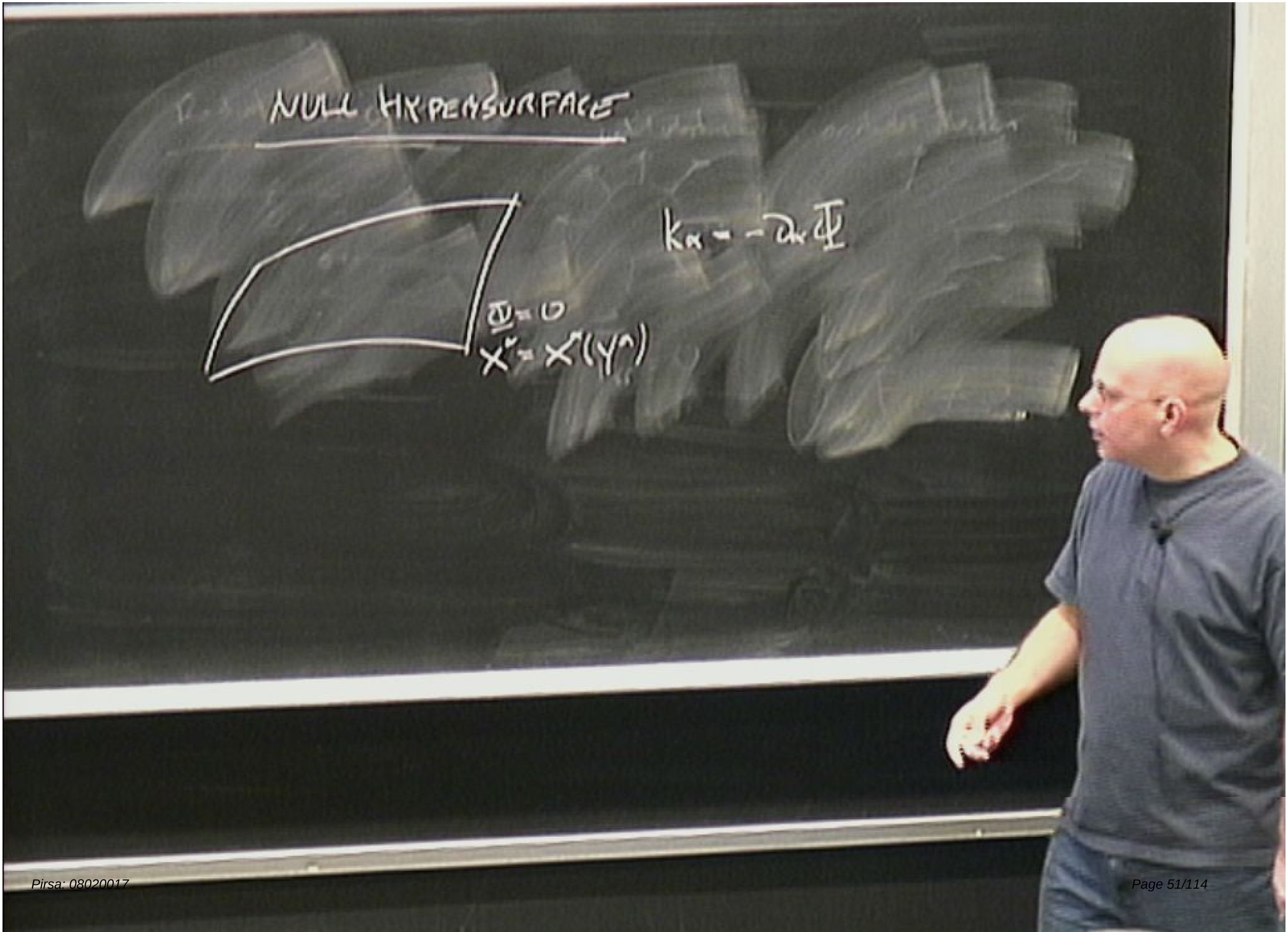

TIMELIKE/SPACELIKE HYPERSURFACE


NULL HYPERSURFACE


NULL HYPERSURFACE
Φ = 0
X^μ = X^μ(y^a)


NULL HYPERSURFACE
$K_\alpha = -\partial_\alpha \Phi$
$\Phi = 0$
$X^\mu = X^\mu(y^a)$

NULL HYPERSURFACE

$\Phi = 0$

$x^\alpha = x^\alpha(y^a)$

$k_\alpha = -\partial_\alpha \Phi \qquad k_\alpha k^\alpha = 0$

NULL HYPERSURFACE

$\Phi = 0$

$x^\alpha = x^\alpha(y^a)$

$k_\alpha = -\partial_\alpha \Phi$

$k_\alpha k^\alpha = 0$

NULL HYPERSURFACE

$\Phi = 0$

$x^\alpha = x^\alpha(y^a)$

$k_\alpha = -\partial_\alpha \Phi \qquad k_\alpha k^\alpha = 0$

NULL HYPERSURFACE

$\Phi = 0$

$x^\alpha = x^\alpha(y^A)$

$k_\alpha = -\partial_\alpha \Phi \qquad k_\alpha k^\alpha = 0$

NULL HYPERSURFACE

$$k_\alpha = -\partial_\alpha \Phi \qquad k_\alpha k^\alpha = 0$$

$\Phi = 0$

$$x^\alpha = x^\alpha(y^a) \rightarrow e^\alpha_a = \frac{\partial x^\alpha}{\partial y^a} \qquad k_\alpha e^\alpha_a = 0$$

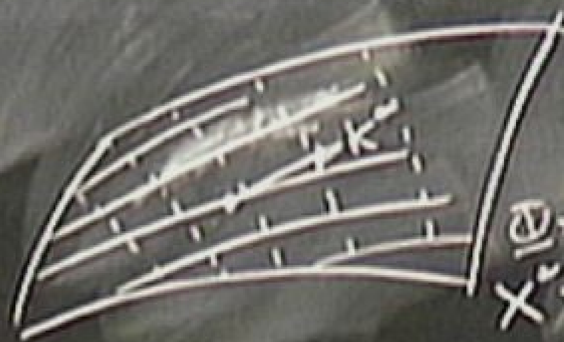
NULL HYPERSURFACE

$$k_\alpha = -\partial_\alpha \Phi \qquad k_\alpha k^\alpha = 0$$

$$\Phi = 0$$

$$x^\alpha = x^\alpha(y^a) \to e^\alpha_a = \frac{\partial x^\alpha}{\partial y^a} \qquad k_\alpha e^\alpha_a = 0$$

NULL HYPERSURFACE

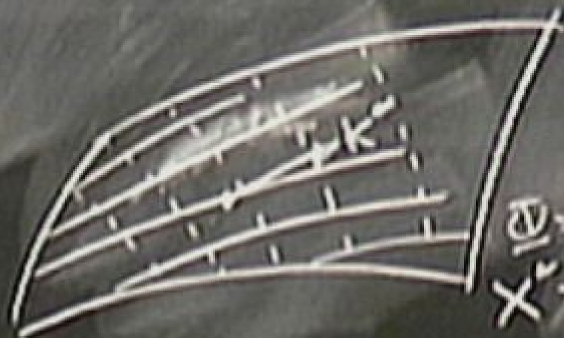


$k_\alpha = -\partial_\alpha \Phi \qquad k_\alpha k^\alpha = 0$

$\Phi = 0$

$x^\alpha = x^\alpha(y^a) \to e^\alpha_a = \dfrac{\partial x^\alpha}{\partial y^a} \qquad k_\alpha e^\alpha_a = 0$

NULL HYPERSURFACE



$$k_\alpha = -\partial_\alpha \Phi \qquad k_\alpha k^\alpha = 0$$

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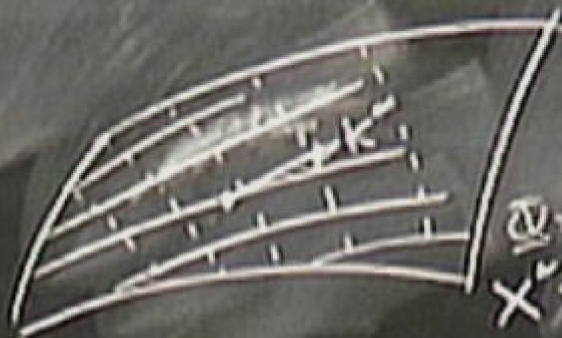
NULL HYPERSURFACE



$k_\alpha = -\partial_\alpha \Phi \qquad k_\alpha k^\alpha = 0$

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NULL HYPERSURFACE

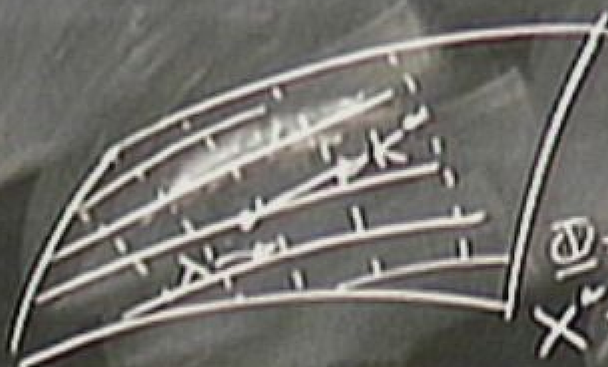
$$k_\alpha = -\partial_\alpha \Phi \qquad k_\alpha k^\alpha = 0$$

$\Phi = 0$

$$x^\alpha = x^\alpha(y^a) \longrightarrow e^\alpha_a = \frac{\partial x^\alpha}{\partial y^a} \qquad k_\alpha e^\alpha_a = 0$$

generators of null hypersurface are parameterized by λ and labelled by

NULL HYPERSURFACE



$k_\alpha = -\partial_\alpha \Phi$ $\qquad$ $k_\alpha k^\alpha = 0$

$\Phi = 0$

$x^\alpha = x^\alpha(y^a) \rightarrow e^\alpha_a = \dfrac{\partial x^\alpha}{\partial y^a}$ $\qquad$ $k_\alpha e^\alpha_a = 0$

generators of null hypersurface are parametrized by λ and labelled by y

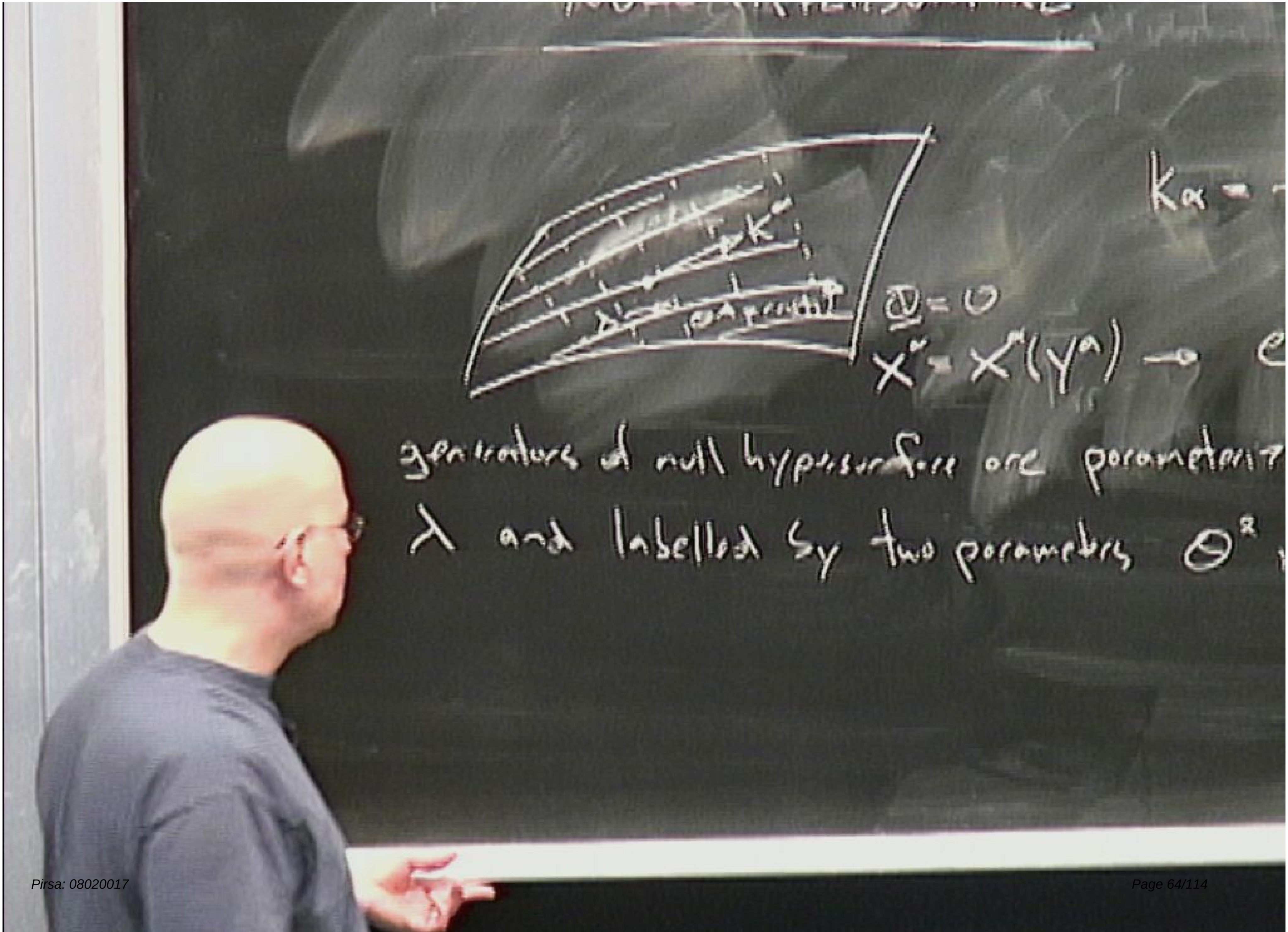
NULL HYPERSURFACE

$$k_\alpha = -\partial_\alpha \Phi \qquad k_\alpha k^\alpha = 0$$

$\Phi = 0$

$$x^\alpha = x^\alpha(y^a) \rightarrow e^\alpha_a = \frac{\partial x^\alpha}{\partial y^a} \qquad k_\alpha e^\alpha_a = 0$$

generators of null hypersurface are parameterized by λ and labelled by two parameters $\theta^2, \theta^3 \rightarrow \theta^A$


k^α
Φ = 0
x^α = x^α(y^a) → e
Kα =
generators of null hypersurface are parametri
λ and labelled by two parameters Θ^a

NULL HYPERSURFACE

$$k_\alpha = -\partial_\alpha \Phi \qquad k_\alpha k^\alpha = 0$$

$\Phi = 0$

$$x^\alpha = x^\alpha(y^a) \rightarrow e^\alpha_a = \frac{\partial x^\alpha}{\partial y^a} \qquad k_\alpha e^\alpha_a = 0$$

generators of null hypersurface are parametrized by λ and labelled by two parameters $\theta^2, \theta^3 \rightarrow \theta^A$

Description : $x^\alpha = x^\alpha(\lambda; \theta^A)$

NULL HYPERSURFACE

$\Phi = 0$

$$k_\alpha = -\partial_\alpha \Phi \qquad k_\alpha k^\alpha = 0$$

$$x^\alpha = x^\alpha(y^a) \rightarrow e^\alpha_a = \frac{\partial x^\alpha}{\partial y^a} \qquad k_\alpha e^\alpha_a = 0$$

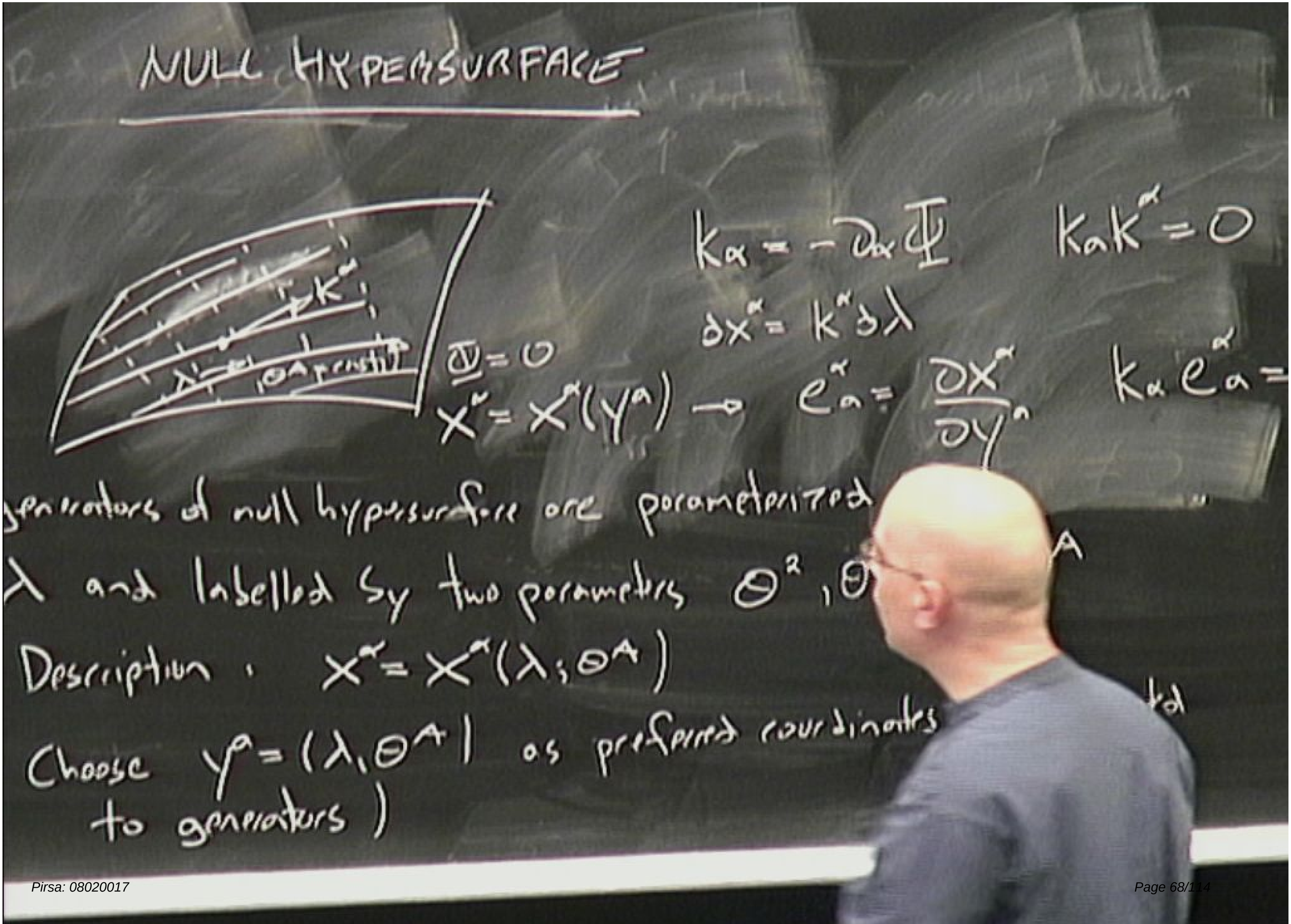
generators of null hypersurface are parametrized by λ and labelled by two parameters $\theta^2, \theta^3 \rightarrow \theta^A$

Description: $x^\alpha = x^\alpha(\lambda; \theta^A)$

Choose $y^a = (\lambda, \theta^A)$ as preferred coordinates (well adapted to generators)

to generators)

$$e^{\alpha}_{\lambda} = \left(\frac{\partial x^{\alpha}}{\partial \lambda}\right)_{0A}$$


NULL HYPERSURFACE
k^α
λ
θ^A = const
Φ = 0
k_α = −∂_α Φ
k_α k^α = 0
dx^α = k^α dλ
x^α = x^α(y^a) → e^α_a = ∂x^α/∂y^a
k_α e^α_a =
generators of null hypersurface are parameterized
λ and labelled by two parameters θ^2, θ
Description: x^α = x^α(λ; θ^A)
Choose y^a = (λ, θ^A) as preferred coordinates
to generators)

$k_\alpha = -\partial_\alpha \Phi \qquad k_\alpha k^\alpha = 0$

$dx^\alpha = k^\alpha d\lambda$ (displacement on geodesic)

$\Phi = 0$

$x^\alpha = x^\alpha(y^a) \to e^\alpha_a = \dfrac{\partial x^\alpha}{\partial y^a} \qquad k_\alpha e^\alpha_a = 0$

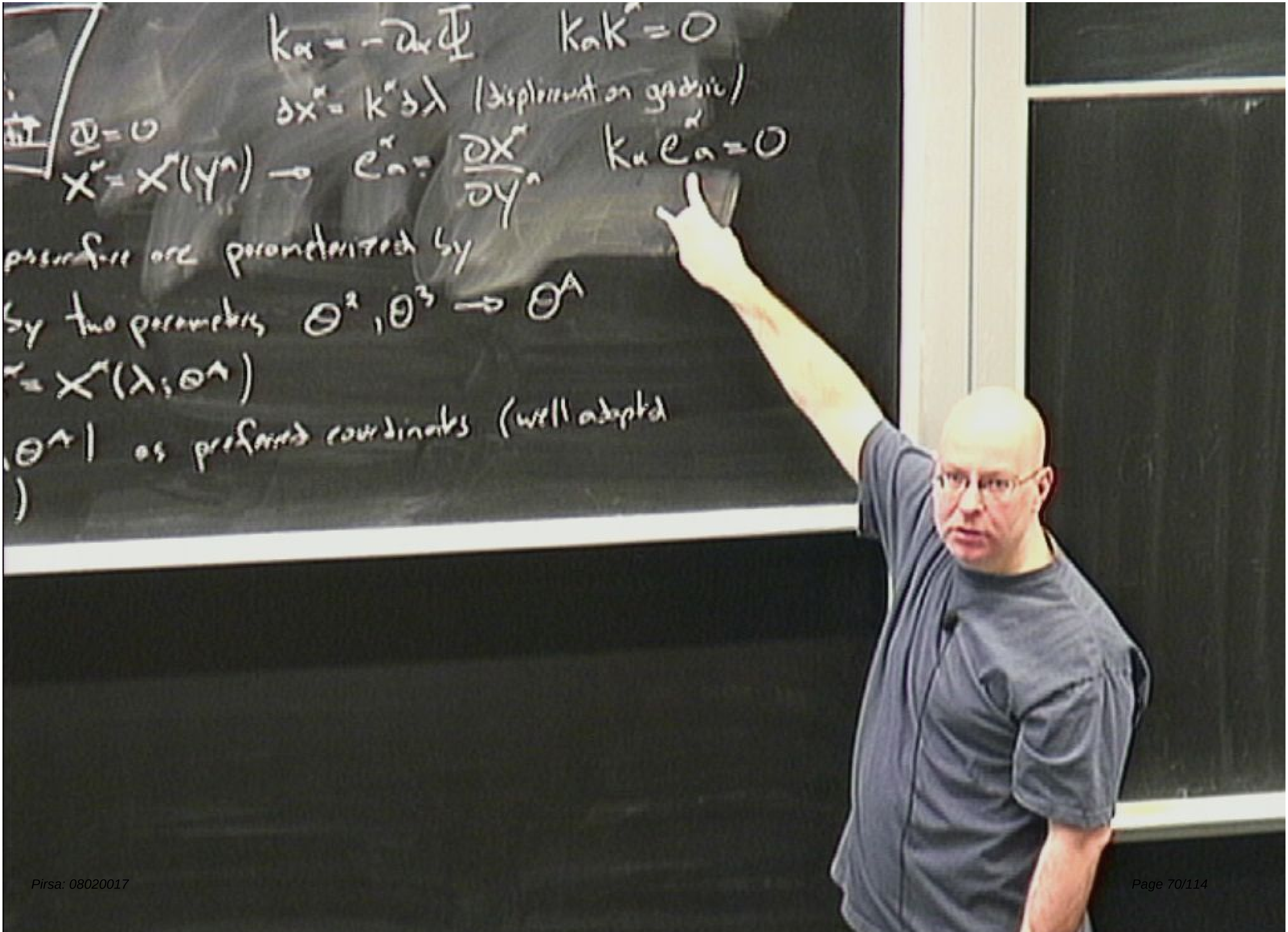
...rators of null hypersurface are parameterized by

... and labelled by two parameters $\theta^2, \theta^3 \to$

Description : $x^\alpha = x^\alpha(\lambda; \theta^A)$

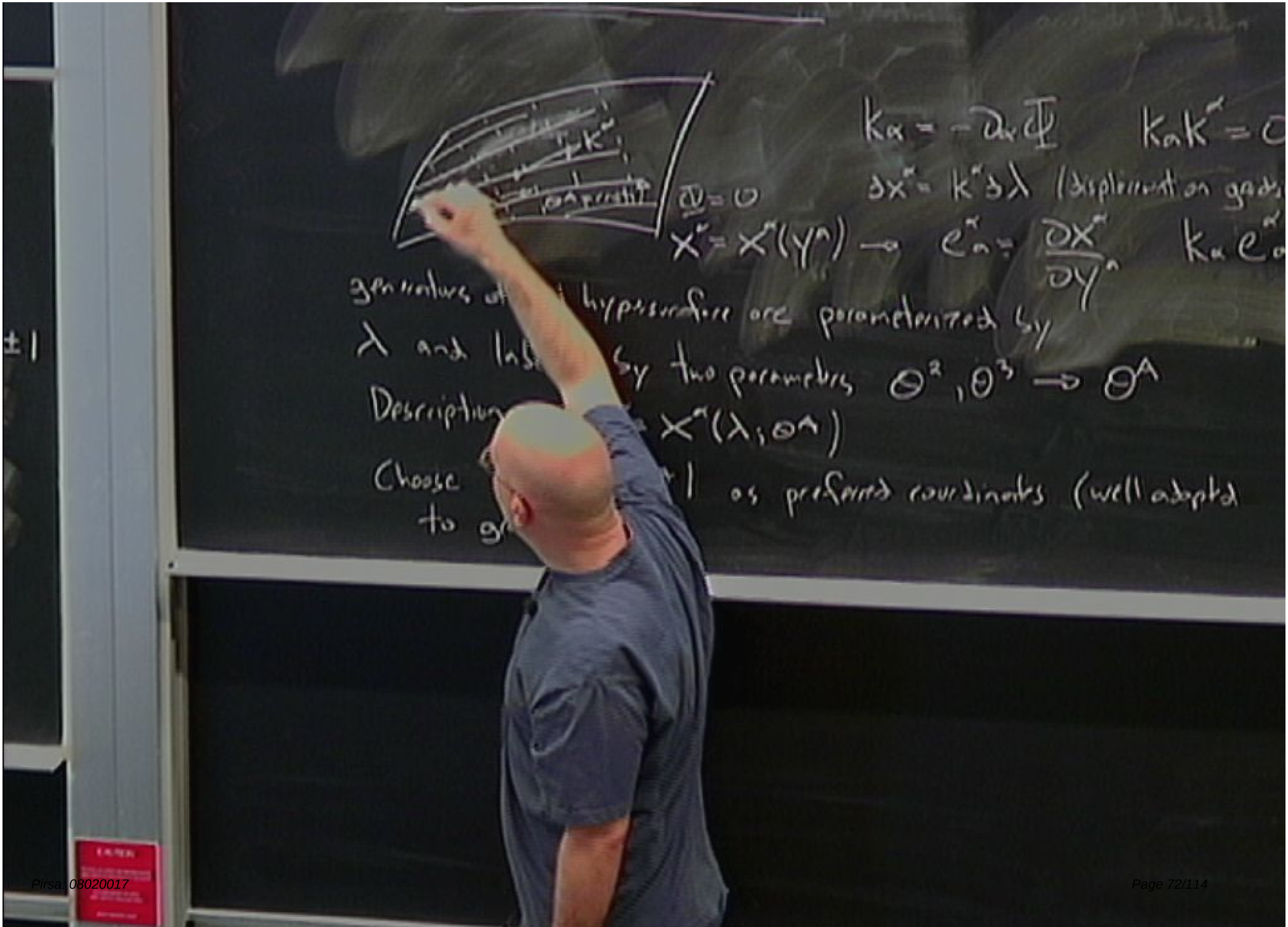
Choose $y^a = (\lambda, \theta^A)$ as preferred coordinates (wel...

to generators)

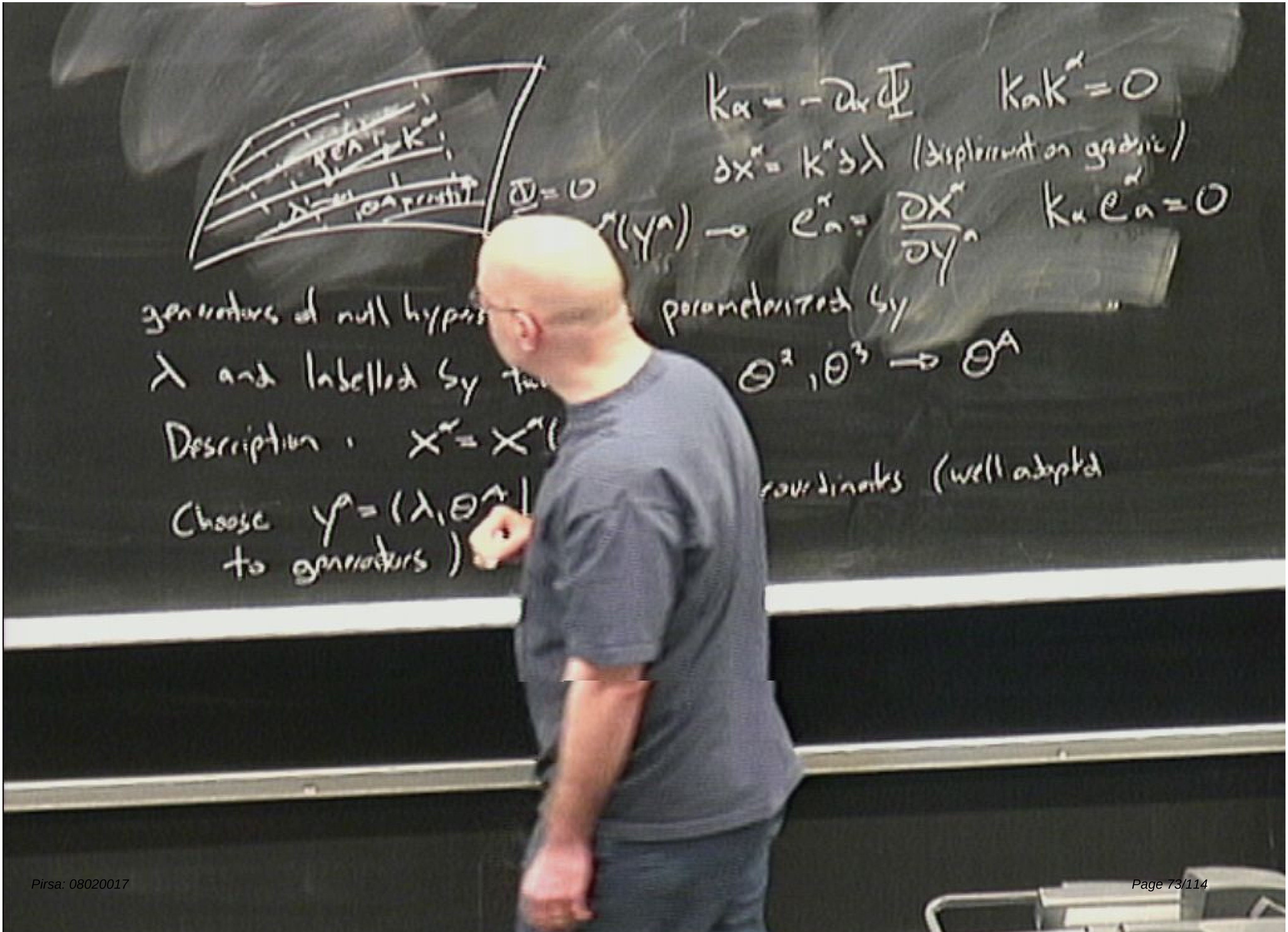

$k_\alpha = -\partial_\alpha \Phi$ $k_\alpha k^\alpha = 0$
$\delta x^\alpha = k^\alpha \delta\lambda$ (displacement on geodesic)
$\Phi = 0$
$x^\alpha = x^\alpha(y^a) \rightarrow e^\alpha_a = \frac{\partial x^\alpha}{\partial y^a}$ $k_\alpha e^\alpha_a = 0$
are parametrized by
by two parameters $\theta^2, \theta^3 \rightarrow \theta^A$
$= x^\alpha(\lambda, \theta^A)$
$\theta^A)$ as preferred coordinates (well adapted
)

$$e^{\alpha}_{\lambda} = \left(\frac{\partial x^{\alpha}}{\partial \lambda}\right)_{\theta^A} = K^{\alpha}$$

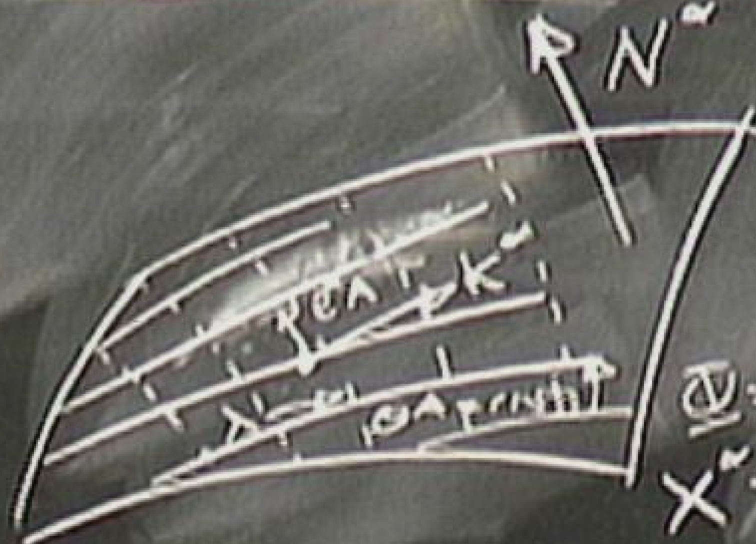
$$e^{\alpha}_{A} = \left(\frac{\partial x^{\alpha}}{\partial \theta^A}\right)_{\lambda}$$



kα = −∂α Φ
Φ = 0
dx^α = k^α dλ (displacement on gen
x^α = x^α(y^a) → e^α_a = ∂x^α/∂y^a
hypersurface are parameterized by
λ and
by two parameters θ², θ³ → θ^A
Description
x^α(λ, θ^A)
Choose
as preferred coordinates (well adapted
to


$k_\alpha = -\partial_\alpha \Phi$ $k_\alpha k^\alpha = 0$
$\delta x^\alpha = k^\alpha \delta\lambda$ (displacement on geodesic)
$\Phi = 0$
$(y^a) \to e^\alpha_a = \frac{\partial x^\alpha}{\partial y^a}$ $k_\alpha e^\alpha_a = 0$
generators of null hyp parameterized by
λ and labelled by $\theta^2, \theta^3 \to \theta^A$
Description : $x^\alpha = x^\alpha($
coordinates (well adapted
Choose $y^a = (\lambda, \theta^A)$
to generators)

NULL HYPERSURFACE



$\Phi = 0$

$k_\alpha = -\partial_\alpha \Phi$

$dx^\alpha = k^\alpha d\lambda$

$x^\alpha = x^\alpha(y^a) \to e^\alpha_a = \frac{\partial}{\partial}$

generators of null hypersurface are parameterized by λ and labelled by two parameters θ^2, θ^3 —

Description: $x^\alpha = x^\alpha(\lambda; \theta^A)$

Choose $y^a = (\lambda, \theta^A)$ as preferred coordinates

to generators)

$$e^\alpha_\lambda = \left(\frac{\partial x^\alpha}{\partial \lambda}\right)_{\theta^A} = k^\alpha$$

$$e^\alpha_A = \left(\frac{\partial x^\alpha}{\partial \theta^A}\right)_\lambda \quad ; \quad k_\alpha e^\alpha_A = 0$$

Complete basis by introducing 4th vector N^α:

$$e^{\alpha}_{\lambda} = \left(\frac{\partial x^{\alpha}}{\partial \lambda}\right)_{\theta^A} = K^{\alpha}$$

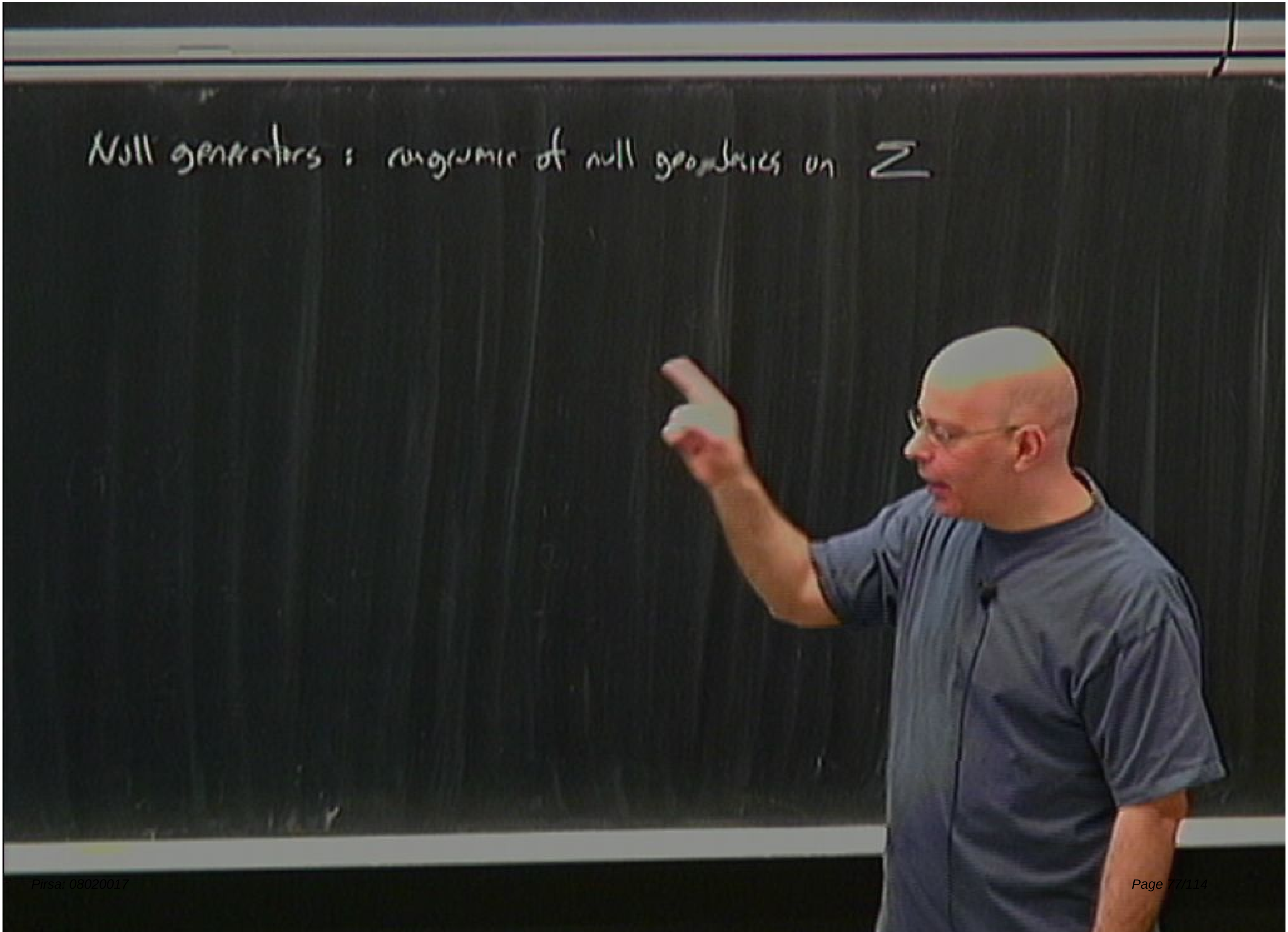
$$e^{\alpha}_{A} = \left(\frac{\partial x^{\alpha}}{\partial \theta^A}\right)_{\lambda} \quad ; \; K_{\alpha} e^{\alpha}_{A} = 0$$

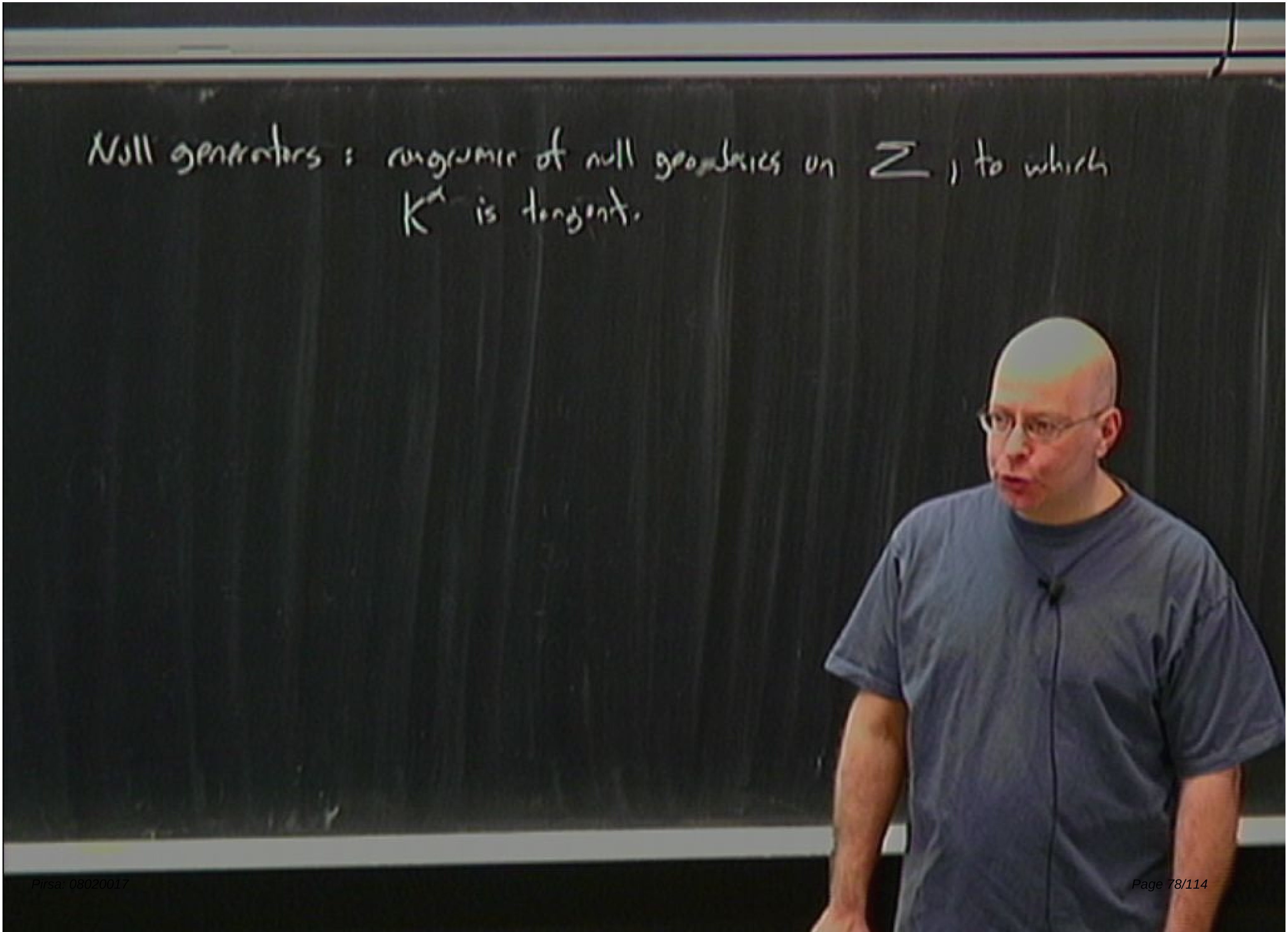
Complete basis by introducing 4th vector N^{α}:

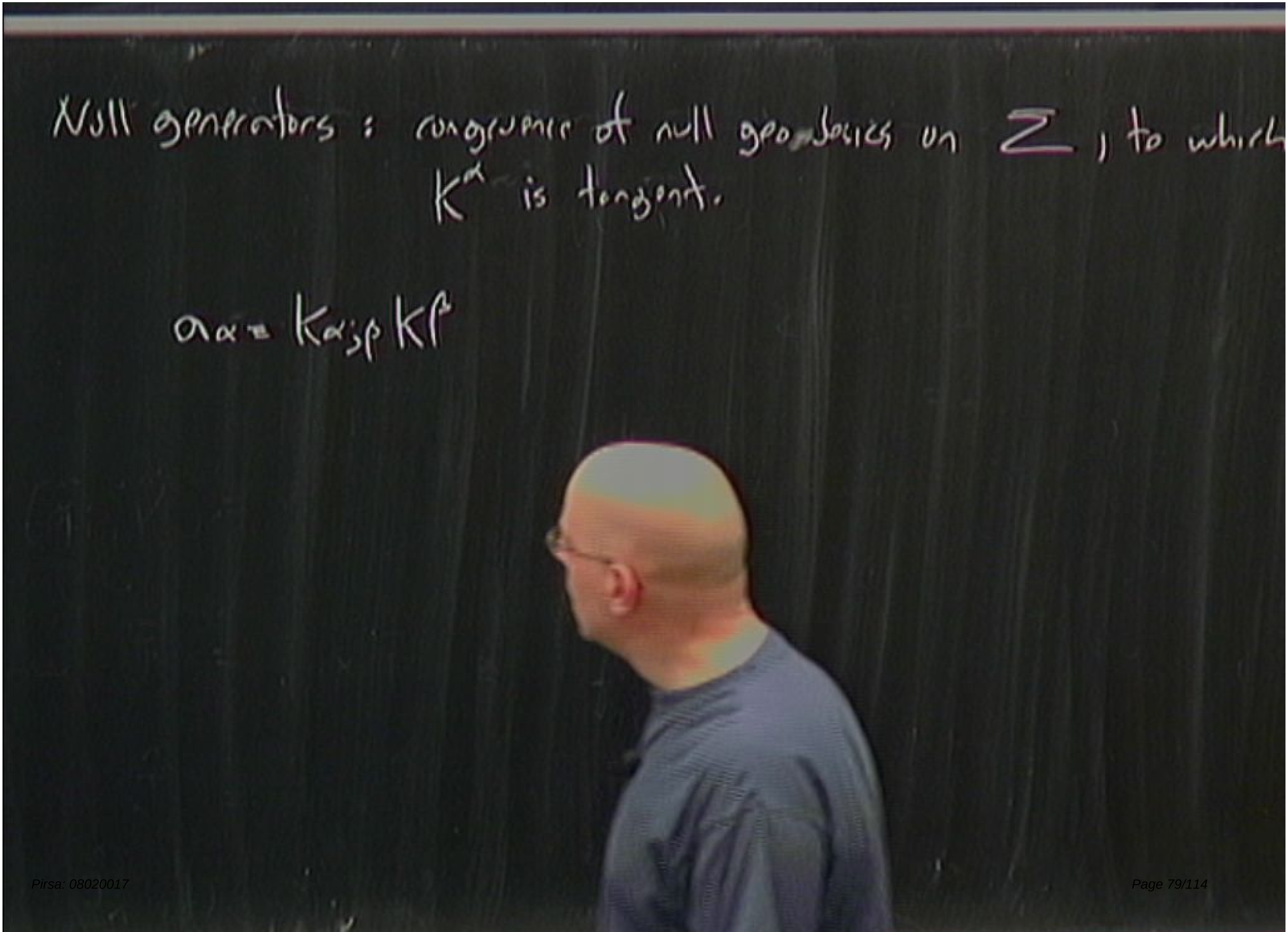
$$N_{\alpha} N^{\alpha} = 0$$

$$N_{\alpha} K^{\alpha} = -1$$

$$N_{\alpha} e^{\alpha}_{A} = 0$$


Null generators : congruence of null geodesics on $\mathcal{I}$


Null generators : congruence of null geodesics on Σ, to which
K^a is tangent.


Null generators : congruence of null geodesics on Z , to which
K^α is tangent.
$a_\alpha = K_{\alpha;\beta} K^\beta$

Null generators: congruence of null geodesics on Σ, to which K^α is tangent.

$$a_\alpha = K_{\alpha;\beta} K^\beta = \underline{\Phi}_{;\alpha\beta}\, \underline{\Phi}^{;\beta}$$

Null generators: congruence of null geodesics on Σ, to which K^α is tangent.

$$a_\alpha = K_{\alpha;\beta} K^\beta = \Phi_{;\alpha\beta} \Phi^{;\beta} = \Phi_{;\beta\alpha} \Phi^{;\beta}$$

$$= \frac{1}{2} \left(\Phi_{;\beta} \Phi^{;\beta} \right)_{;\alpha}$$

Null generators : congruence of null geodesics on Σ, to which K^α is tangent.

$$a_\alpha = K_{\alpha;\beta} K^\beta = \underline{\Phi}_{;\alpha\beta} \underline{\Phi}^{;\beta} = \underline{\Phi}_{;\beta\alpha} \underline{\Phi}^{;\beta}$$

$$= \frac{1}{2} \Big(\underbrace{\underline{\Phi}_{;\beta} \underline{\Phi}^{;\beta}}_{\text{zero on } \Sigma, \text{ not necessarily off } \Sigma.} \Big)_{;\alpha}$$

Null generators: congruence of null geodesics on Σ, to which K^α is tangent.

$$a_\alpha = K_{\alpha;\beta}K^\beta = \underline{\Phi}_{;\alpha\beta}\,\underline{\Phi}^{;\beta} = \underline{\Phi}_{;\beta\alpha}\,\underline{\Phi}^{;\beta}$$

$$= \frac{1}{2}\Big(\underbrace{\underline{\Phi}_{;\beta}\,\underline{\Phi}^{;\beta}}_{\text{zero on } \Sigma\text{, not necessarily off } \Sigma}\Big)_{;\alpha}$$

$$a_\alpha = -\kappa N_\alpha$$

Null generators: congruence of null geodesics on Σ, to which K^α is tangent.

$$a_\alpha = K_{\alpha;\beta} K^\beta = \underline{\Phi}_{;\alpha\beta}\, \underline{\Phi}^{;\beta} = \underline{\Phi}_{;\beta\alpha}\, \underline{\Phi}^{;\beta}$$

$$= \frac{1}{2} \Big(\underbrace{\underline{\Phi}_{;\beta}\, \underline{\Phi}^{;\beta}}_{\text{zero on } \Sigma \text{, not necessarily off } \Sigma} \Big)_{;\alpha}$$

$$a_\alpha = -\kappa N_\alpha + A K_\alpha$$

Null generators: congruence of null geodesics on Σ, to which K^α is tangent.

$$a_\alpha = K_{\alpha;\beta}K^\beta = \Phi_{;\alpha\beta}\,\Phi^{;\beta} = \Phi_{;\beta\alpha}\,\Phi^{;\beta}$$

$$= \frac{1}{2}\big(\underbrace{\Phi_{;\beta}\,\Phi^{;\beta}}_{\text{zero on } \Sigma,\ \text{not necessarily off } \Sigma.}\big)_{;\alpha}$$

$$a_\alpha = -\kappa N_\alpha + A K_\alpha + B^A e^\alpha_A$$

Null generators : congruence of null geodesics on Σ, to which K^α is tangent.

$$a_\alpha = K_{\alpha;\beta}K^\beta = \underline{\Phi}_{;\alpha\beta}\,\underline{\Phi}^{;\beta} = \underline{\Phi}_{;\beta\alpha}\,\underline{\Phi}^{;\beta}$$

$$= \frac{1}{2}\Big(\underbrace{\underline{\Phi}_{;\beta}\,\underline{\Phi}^{;\beta}}_{\text{zero on } \Sigma\text{, not necessarily off } \Sigma.}\Big)_{;\alpha}$$

$$a_\alpha = -\kappa N_\alpha + A K_\alpha + B^A e^\alpha_A$$

$$a_\alpha K^\alpha = \kappa$$

Null generators : congruence of null geodesics on Σ, to which K^α is tangent.

$$a_\alpha = K_{\alpha;\beta} K^\beta = \Phi_{;\alpha\beta}\, \Phi^{;\beta} = \Phi_{;\beta\alpha}\, \Phi^{;\beta}$$

$$= \frac{1}{2} \Big(\underbrace{\Phi_{;\beta}\, \Phi^{;\beta}}_{\text{zero on } \Sigma \text{, not necessarily off } \Sigma} \Big)_{;\alpha}$$

$$a_\alpha = -A N_\alpha - \kappa K_\alpha + B^A e^\alpha_A$$

Null generators: congruence of null geodesics on Σ, to which K^α is tangent.

$$a_\alpha = K_{\alpha;\beta} K^\beta = \underline{\Phi}_{;\alpha\beta} \underline{\Phi}^{;\beta} = \underline{\Phi}_{;\beta\alpha} \underline{\Phi}^{;\beta}$$

$$= \frac{1}{2} \big(\underbrace{\underline{\Phi}_{;\beta} \underline{\Phi}^{;\beta}} \big)_{;\alpha}$$

zero on Σ, not necessarily off Σ.

$$a_\alpha = -A N_\alpha - \kappa K_\alpha + B^A e^\alpha_A$$

$$a_\alpha K^\alpha = A = \frac{1}{2} \big(\underline{\Phi}_{;\beta} \underline{\Phi}^{;\beta} \big)_{;\alpha} K^\alpha$$

Null generators: congruence of null geodesics on Σ, to which K^α is tangent.

$$a_\alpha = K_{\alpha;\beta} K^\beta = \Phi_{;\alpha\beta}\,\Phi^{;\beta} = \Phi_{;\beta\alpha}\,\Phi^{;\beta}$$

$$= \frac{1}{2}\big(\underbrace{\Phi_{;\beta}\,\Phi^{;\beta}}\big)_{;\alpha}$$

zero on Σ, not necessarily off Σ.

$$a_\alpha = -\cancel{A N_\alpha} - \kappa K_\alpha + B^A e^\alpha_A$$

$$a_\alpha K^\alpha = A = \frac{1}{2}\big(\Phi_{;\beta}\,\Phi^{;\beta}\big)_{;\alpha} K^\alpha = 0$$

Null generators: congruence of null geodesics on Σ, to which K^α is tangent.

$$a_\alpha = K_{\alpha;\beta} K^\beta = \underline{\Phi}_{;\alpha\beta}\, \underline{\Phi}^{;\beta} = \underline{\Phi}_{;\beta\alpha}\, \underline{\Phi}^{;\beta}$$

$$= \frac{1}{2} \Big(\underbrace{\underline{\Phi}_{;\beta}\, \underline{\Phi}^{;\beta}}_{\text{zero on } \Sigma \text{, not necessarily off } \Sigma} \Big)_{;\alpha}$$

$$a_\alpha = - \cancel{A N_\alpha} - \kappa K_\alpha + B^A e^\alpha_A$$

$$a_\alpha K^\alpha = A = \frac{1}{2} \left(\underline{\Phi}_{;\beta}\, \underline{\Phi}^{;\beta} \right)_{;\alpha} K^\alpha = 0$$

$$a_\alpha e^\alpha_B =$$

Null generators: congruence of null geodesics on Σ, to which K^α is tangent.

$$a_\alpha = K_{\alpha;\beta}K^\beta = \Phi_{;\alpha\beta}\,\Phi^{;\beta} = \Phi_{;\beta\alpha}\,\Phi^{;\beta}$$

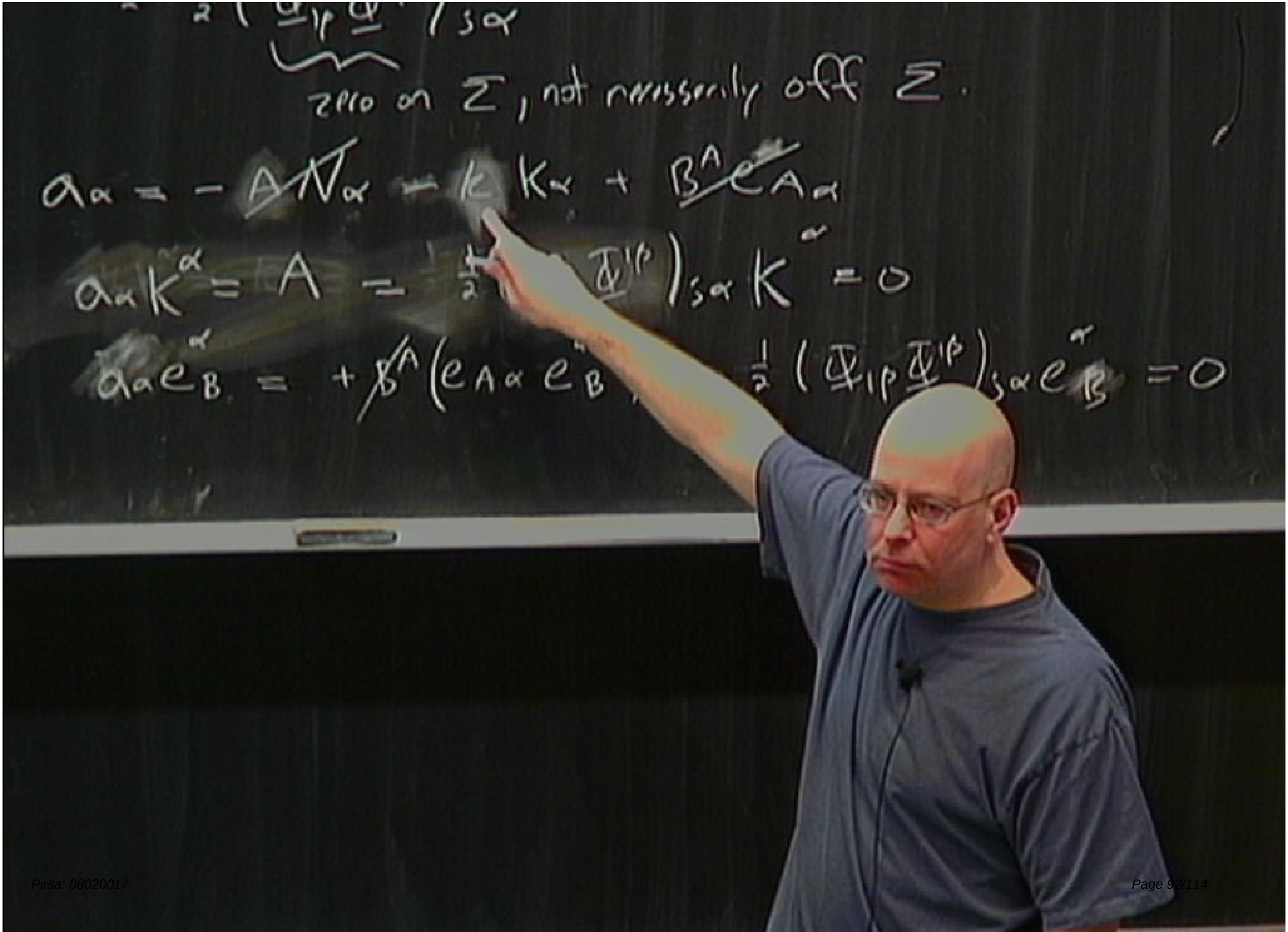
$$= \frac{1}{2}\Big(\underbrace{\Phi_{;\beta}\,\Phi^{;\beta}}\Big)_{;\alpha}$$

zero on Σ, not necessarily off Σ.

$$a_\alpha = -\cancel{A N_\alpha} + \kappa K_\alpha + B^A e_{A\alpha}$$

$$a_\alpha K^\alpha = A = \frac{1}{2}\left(\Phi_{;\beta}\,\Phi^{;\beta}\right)_{;\alpha} K^\alpha = 0$$

$$a_\alpha e^\alpha_B = +B^A\left(e_{A\alpha}\, e^\alpha_B\right)$$


zero on Σ, not necessarily off Σ.
$a_\alpha = - A N_\alpha - k K_\alpha + B^A e_{A\alpha}$
$a_\alpha K^\alpha = A$
$= 0$

Null generators: congruence of null geodesics on Σ, to which K^α is tangent.

$$a_\alpha = K_{\alpha;\beta} K^\beta = \Phi_{;\alpha\beta} \Phi^{;\beta} = \Phi_{;\beta\alpha} \Phi^{;\beta}$$

$$= \frac{1}{2} \big(\underbrace{\Phi_{;\beta} \Phi^{;\beta}} \big)_{;\alpha}$$

zero on Σ, not necessarily off Σ.

$$a_\alpha = - \cancel{A N_\alpha} - \kappa K_\alpha + \cancel{B^A e_{A\alpha}}$$

$$a_\alpha K^\alpha = A = \frac{1}{2} \left(\Phi_{;\beta} \Phi^{;\beta} \right)_{;\alpha} K^\alpha = 0$$

$$a_\alpha e^\alpha_B = + \cancel{B^A} \left(e_{A\alpha} e^\alpha_B \right) = \frac{1}{2} \left(\Phi_{;\beta} \Phi^{;\beta} \right)_{;\alpha} e^\alpha_B = 0$$

$$a_\alpha N^\alpha = \kappa =$$

Null generators : congruence of null geodesics on Σ, to which K^α is tangent.

$$a_\alpha = K_{\alpha;\beta}K^\beta = \Phi_{;\alpha\beta}\,\Phi^{;\beta} = \Phi_{;\beta\alpha}\,\Phi^{;\beta}$$

$$= \frac{1}{2}\Big(\underbrace{\Phi_{;\beta}\,\Phi^{;\beta}}_{\text{zero on } \Sigma\text{, not necessarily off } \Sigma.}\Big)_{;\alpha}$$

$$a_\alpha = -\cancel{A N_\alpha} - \kappa K_\alpha + \cancel{B^A e_{A\alpha}}$$

$$a_\alpha K^\alpha = A = \frac{1}{2}\left(\Phi_{;\beta}\Phi^{;\beta}\right)_{;\alpha}K^\alpha = 0$$

$$a_\alpha e^\alpha_B = +\cancel{B}^A\left(e_{A\alpha}\,e^\alpha_B\right) = \frac{1}{2}\left(\Phi_{;\beta}\Phi^{;\beta}\right)_{;\alpha}e^\alpha_B = 0$$

$$a_\alpha N^\alpha = \kappa = \frac{1}{2}\left(\Phi_{;\beta}\Phi^{;\beta}\right)_{;\alpha}N^\alpha$$

Null generators : congruence of null geodesics on Σ, to which K^α is tangent.

$$a_\alpha = K_{\alpha;\beta} K^\beta = \Phi_{;\alpha\beta}\, \Phi^{;\beta} = \Phi_{;\beta\alpha}\, \Phi^{;\beta}$$

$$= \frac{1}{2} \big(\underbrace{\Phi_{;\beta}\, \Phi^{;\beta}} \big)_{;\alpha}$$

zero on Σ, not necessarily off Σ.

$$a_\alpha = - \cancel{A N_\alpha} + \kappa K_\alpha + \cancel{B^A e_{A\alpha}}$$

$$a_\alpha K^\alpha = A = \frac{1}{2} \big(\Phi_{;\beta} \Phi^{;\beta} \big)_{;\alpha} K^\alpha = 0$$

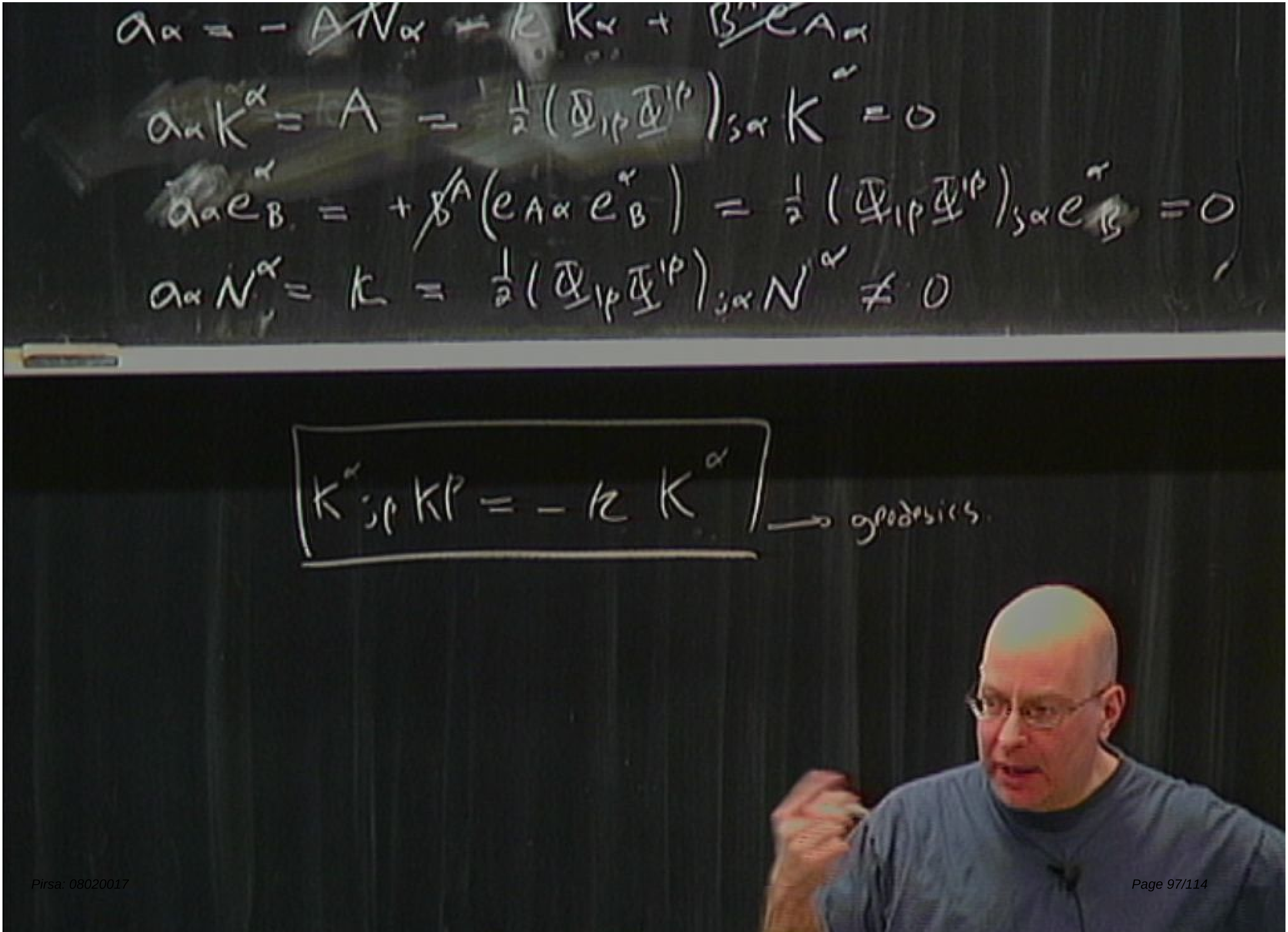
$$a_\alpha e^\alpha_B = + \cancel{B^A} \big(e_{A\alpha} e^\alpha_B \big) = \frac{1}{2} \big(\Phi_{;\beta} \Phi^{;\beta} \big)_{;\alpha} e^\alpha_B = 0$$

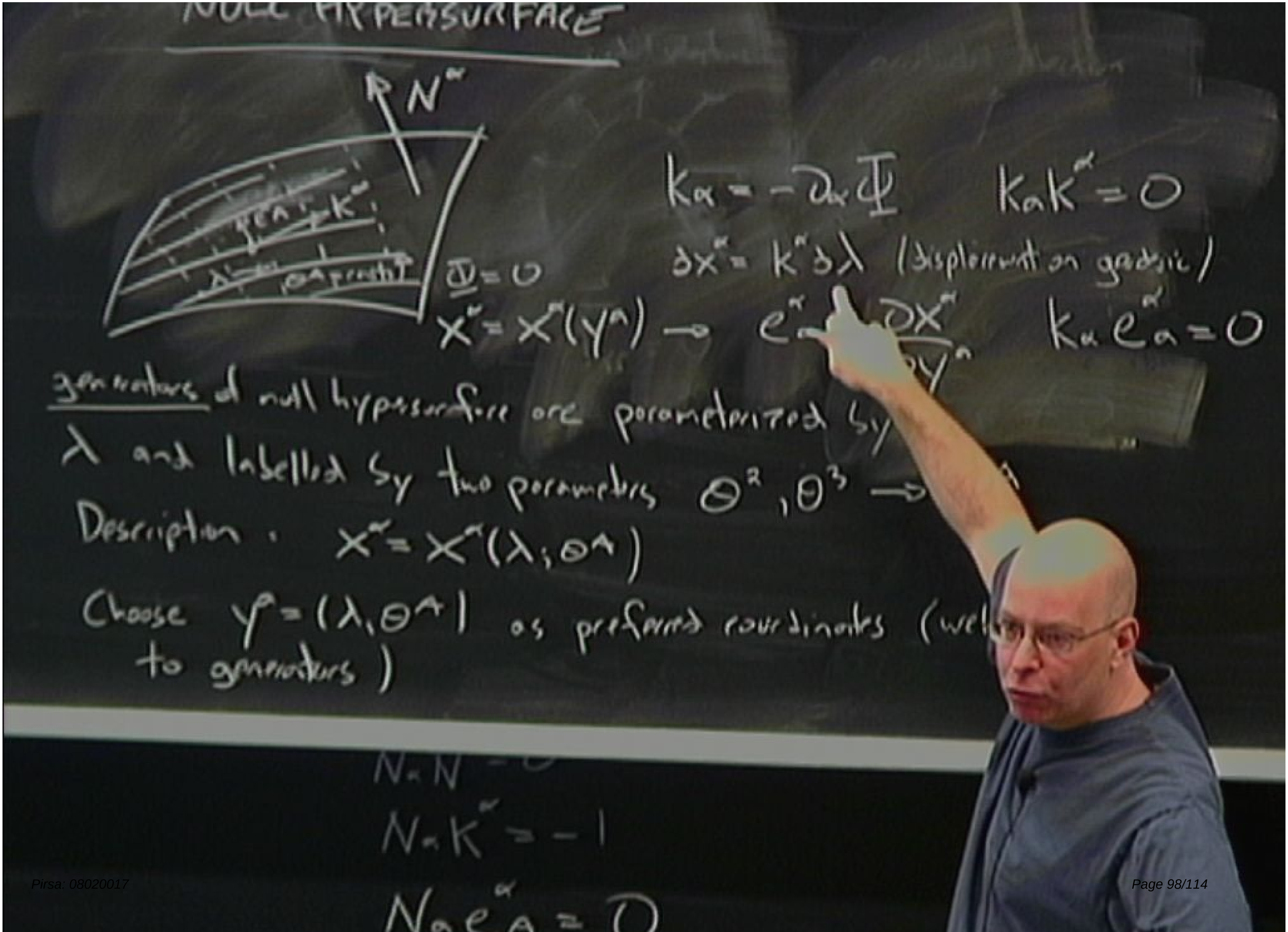
$$a_\alpha N^\alpha = \kappa = \frac{1}{2} \big(\Phi_{;\beta} \Phi^{;\beta} \big)_{;\alpha} N^\alpha \neq 0$$

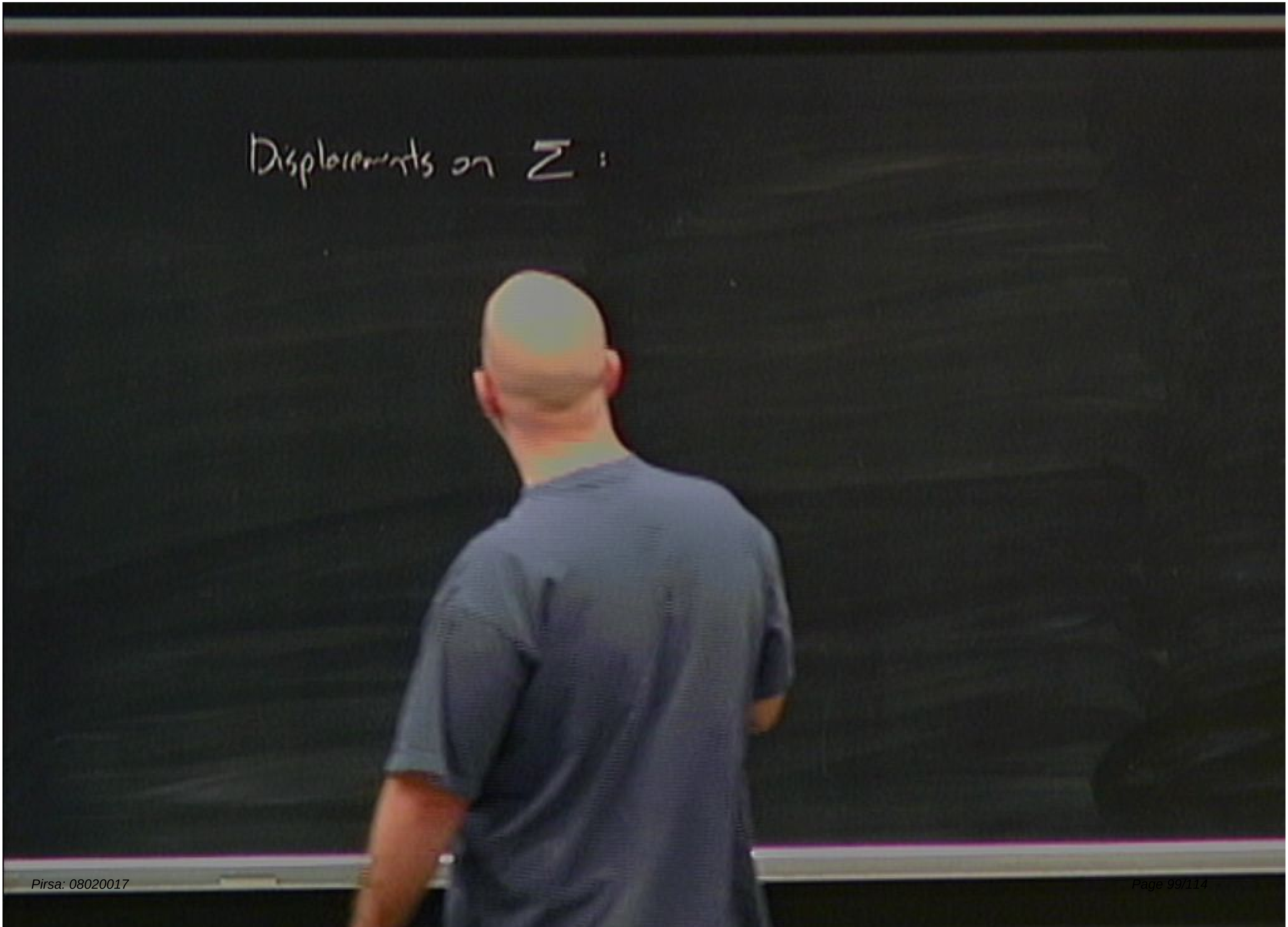
$$a_\alpha e^\alpha_B = +\cancel{\delta}^A \left(e_{A\alpha} e^\alpha_B \right) = \frac{1}{2} \left(\Phi_{|\beta} \Phi^{|\beta} \right)_{;\alpha} e^\alpha_B = 0$$

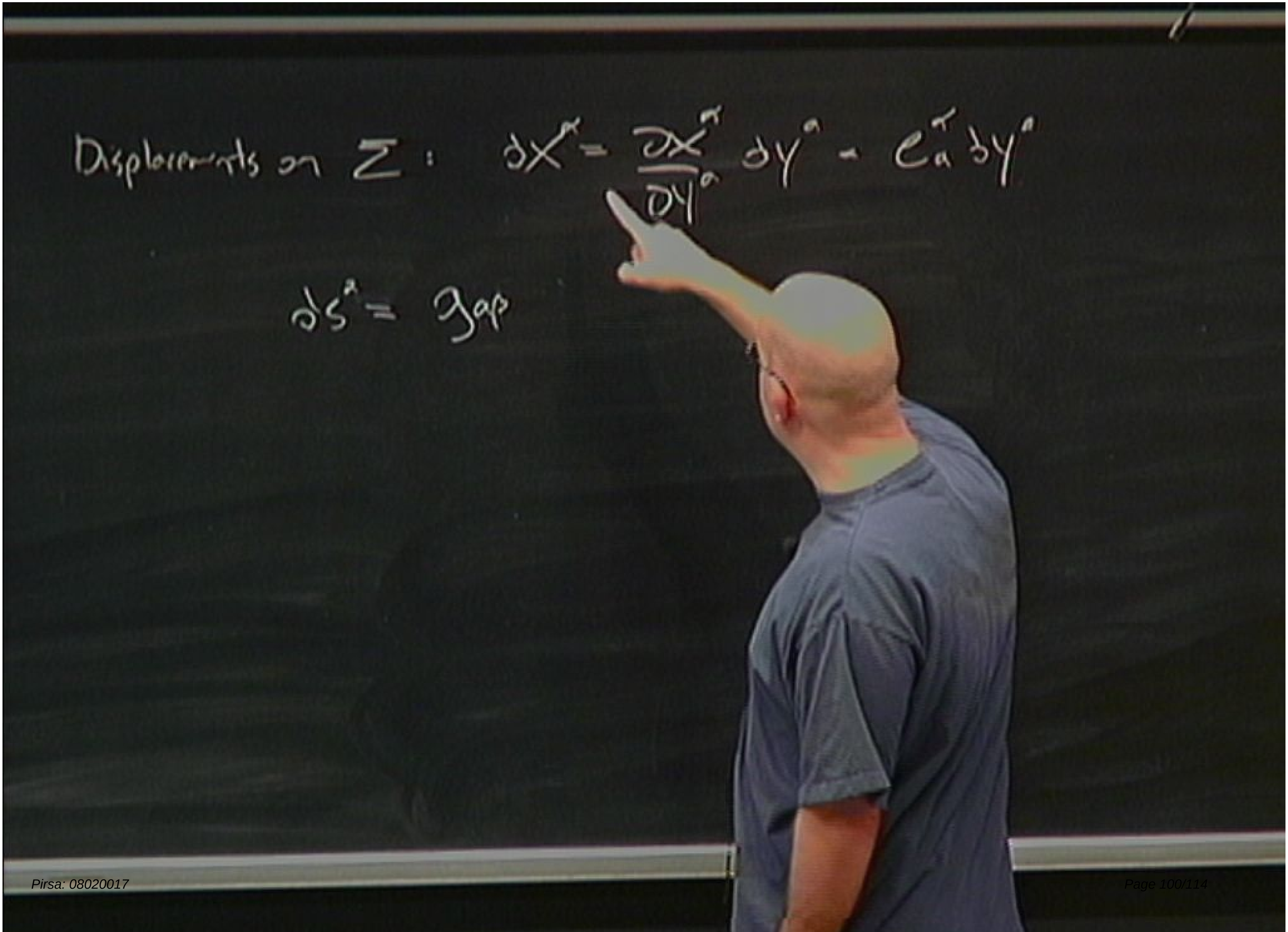
$$a_\alpha N^\alpha = k = \frac{1}{2} \left(\Phi_{|\beta} \Phi^{|\beta} \right)_{;\alpha} N^\alpha \neq 0$$

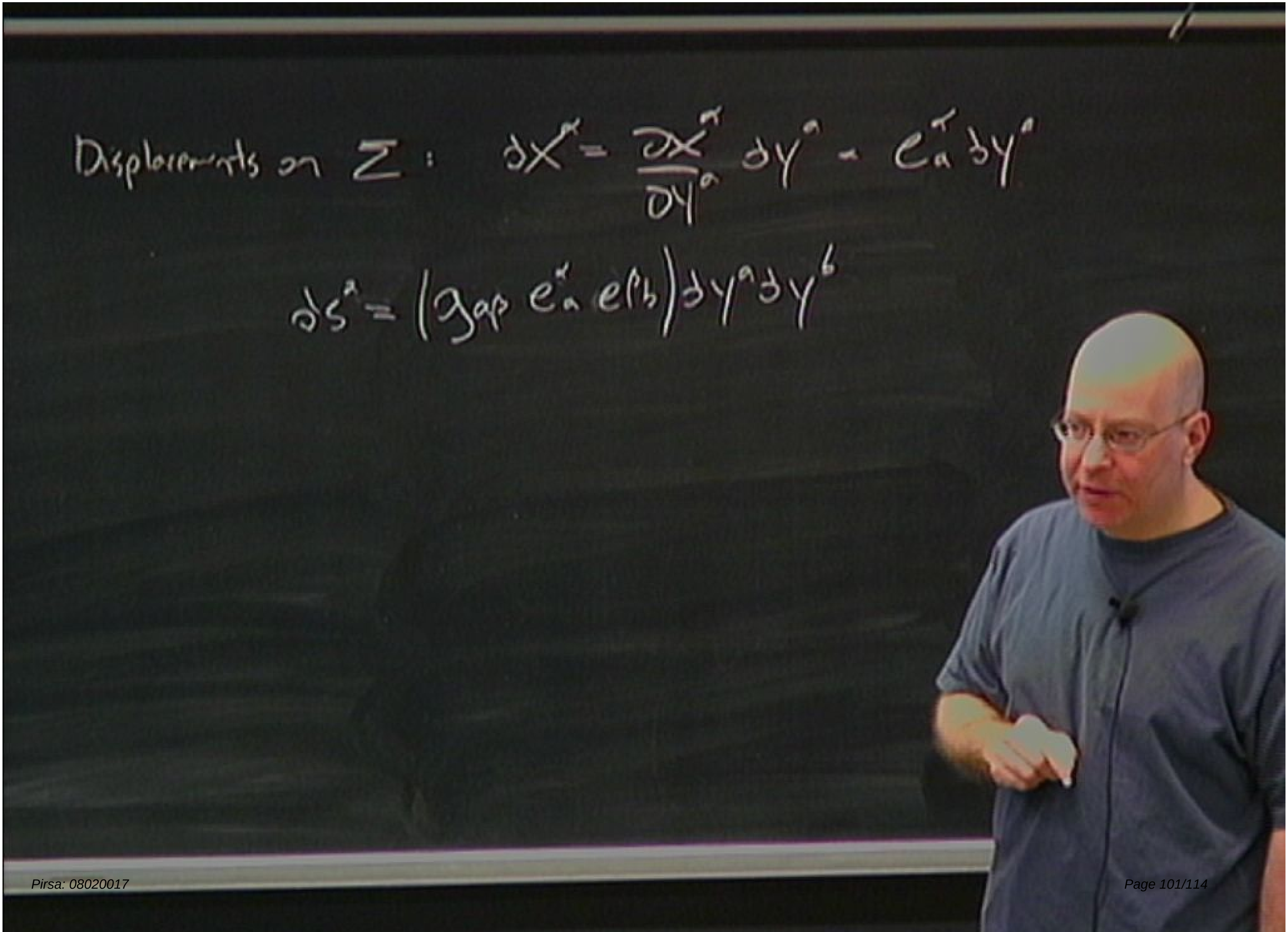
$$\boxed{K^\alpha{}_{;\beta} K^\beta = -k\, K^\alpha}$$


$a_\alpha = - A N_\alpha - k K_\alpha + B^A e_{A\alpha}$
$a_\alpha K^\alpha = A = \frac{1}{2}(\Phi_{,\rho}\Phi^{,\rho})_{;\alpha} K^\alpha = 0$
$a_\alpha e^\alpha_B = + \delta^A (e_{A\alpha} e^\alpha_B) = \frac{1}{2}(\Phi_{,\rho}\Phi^{,\rho})_{;\alpha} e^\alpha_B = 0$
$a_\alpha N^\alpha = k = \frac{1}{2}(\Phi_{,\rho}\Phi^{,\rho})_{;\alpha} N^\alpha \neq 0$
$\boxed{K^\alpha{}_{;\rho} K^\rho = - k K^\alpha} \longrightarrow$ geodesics


NULL HYPERSURFACE
N^α
k^α
λ
$\Phi = 0$
$k_\alpha = -\partial_\alpha \Phi$ $k_\alpha k^\alpha = 0$
$dx^\alpha = k^\alpha d\lambda$ (displacement on geodesic)
$x^\alpha = x^\alpha(y^a) \rightarrow e^\alpha_a = \frac{\partial x^\alpha}{\partial y^a}$ $k_\alpha e^\alpha_a = 0$
generators of null hypersurface are parameterized by
λ and labelled by two parameters $\theta^2, \theta^3 \rightarrow$
Description: $x^\alpha = x^\alpha(\lambda, \theta^A)$
Choose $y^a = (\lambda, \theta^A)$ as preferred coordinates (wel
to generators)
$N_\alpha k^\alpha = -1$
$N_\alpha e^\alpha_A = 0$

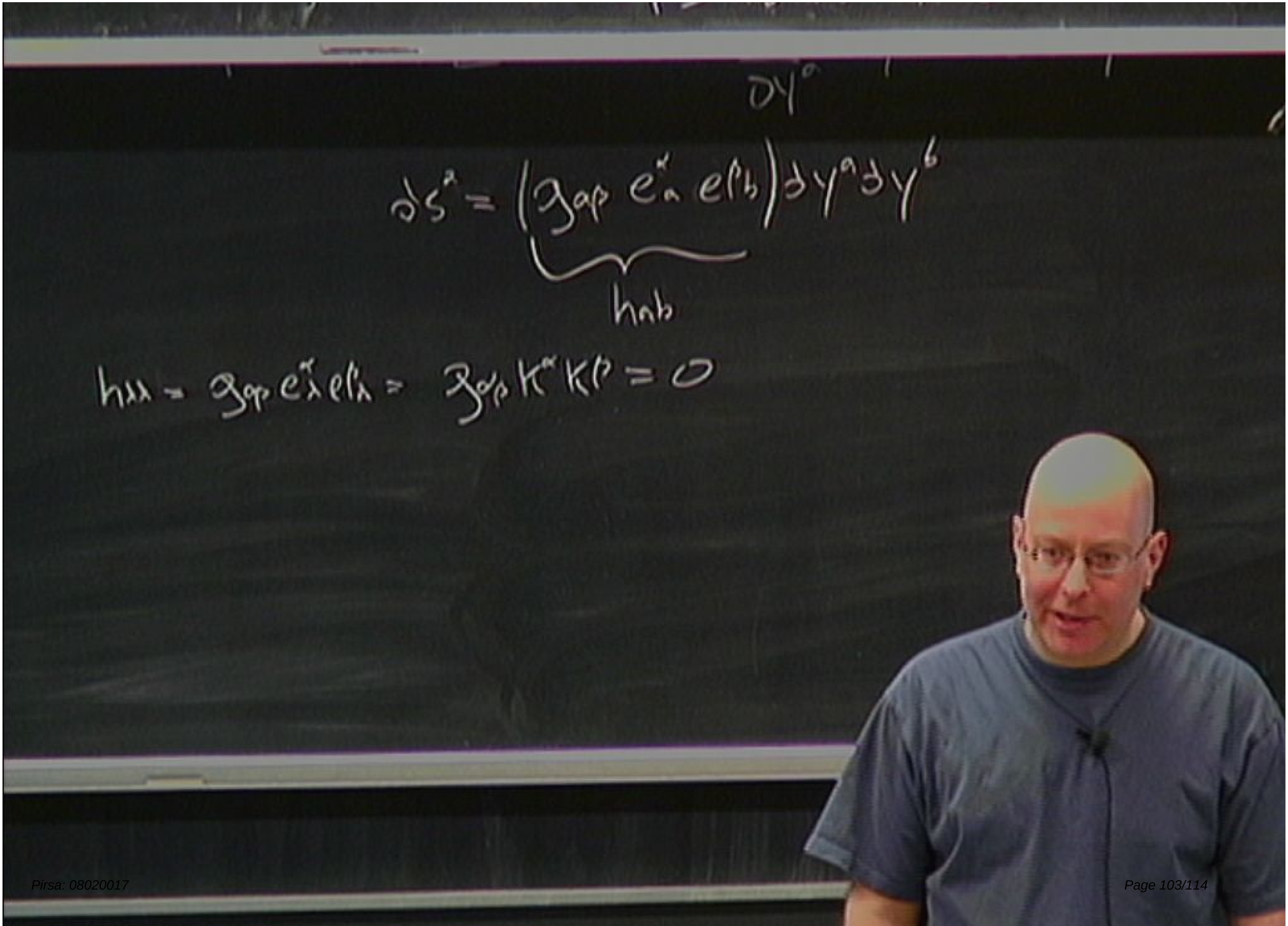

Displacements on Z :

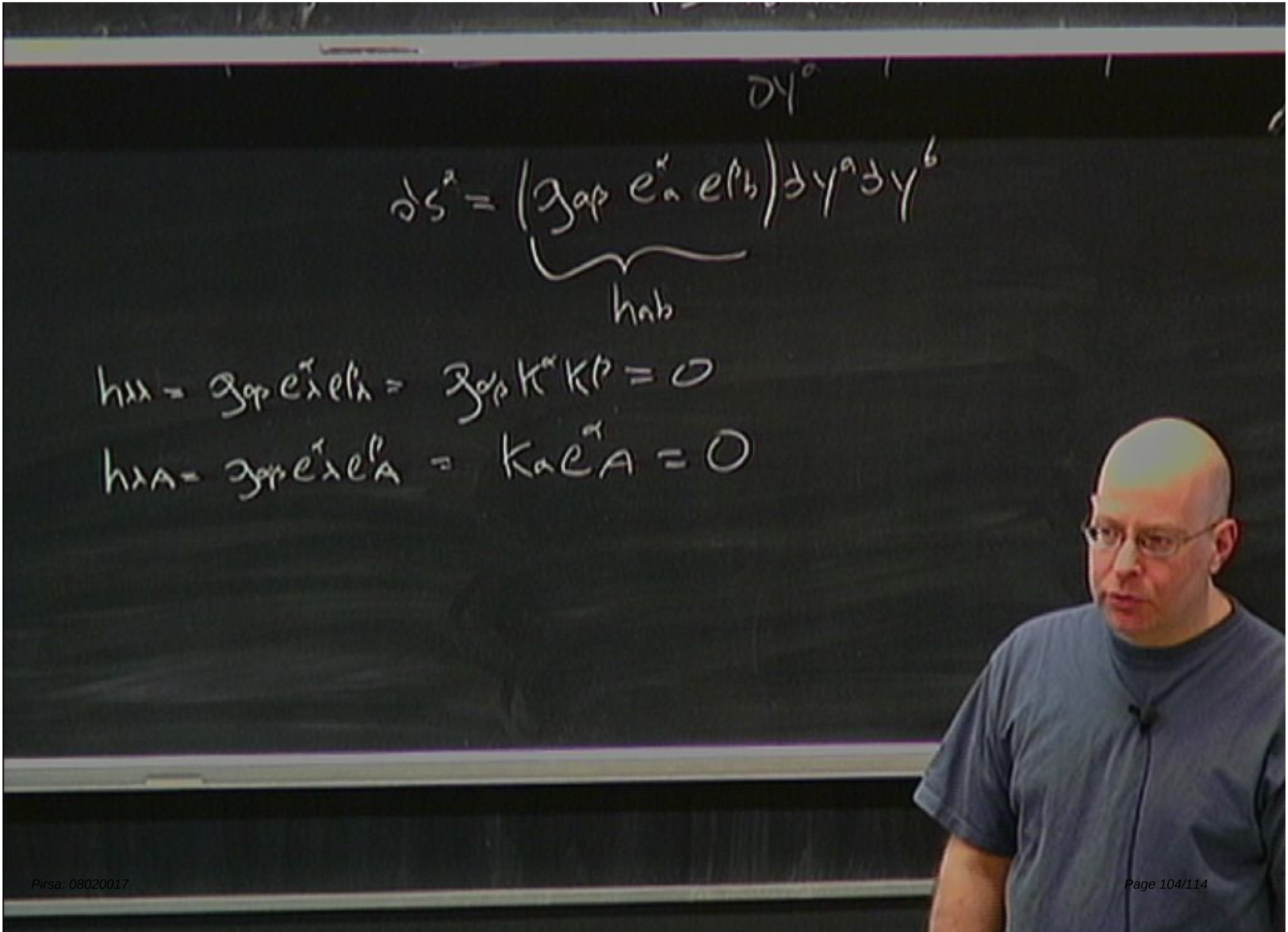

Displacements on Σ : $dx^\alpha = \frac{\partial x^\alpha}{\partial y^a} dy^a = e^\alpha_a dy^a$
$ds^2 = g_{\alpha\beta}$

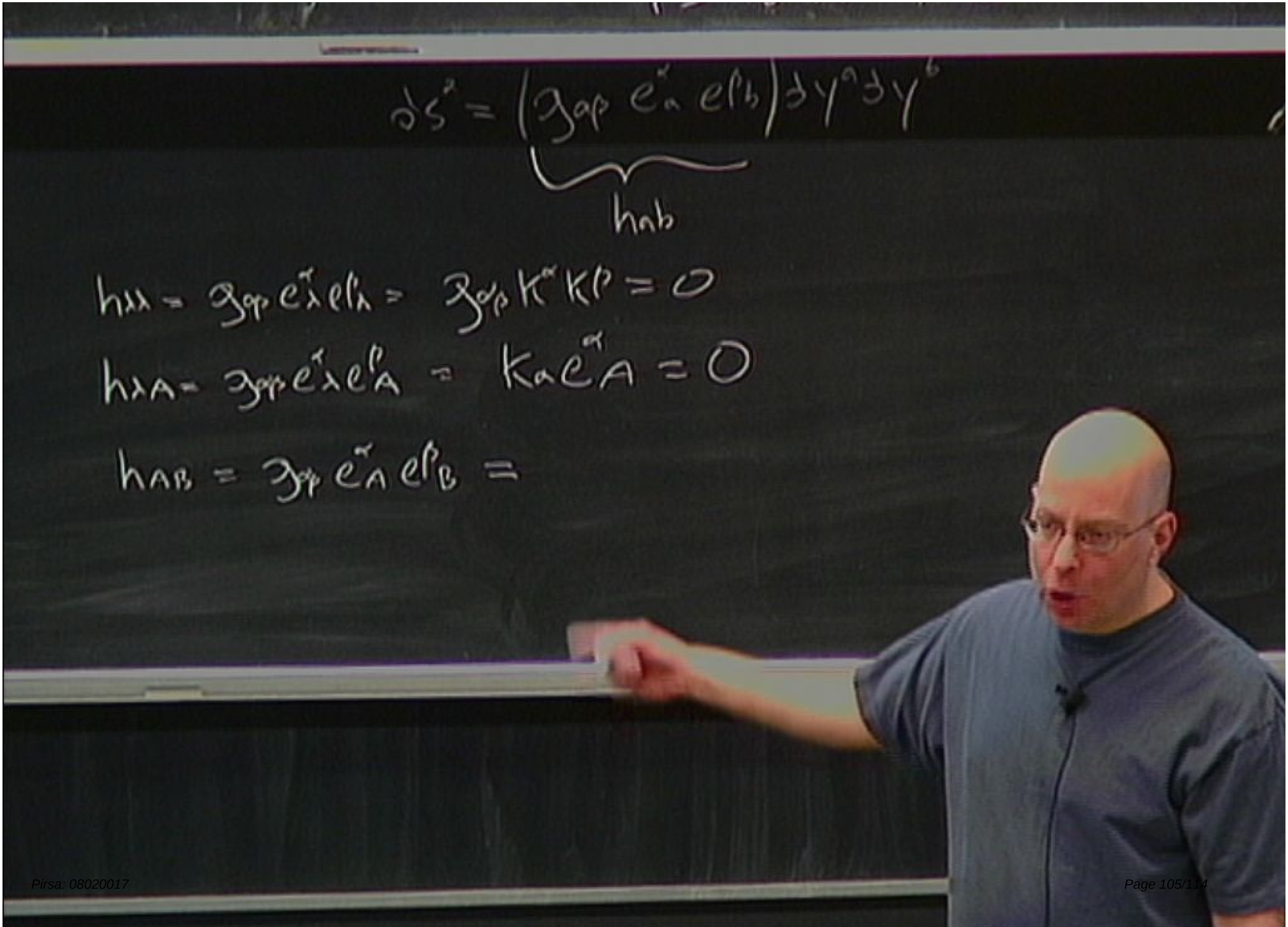

Displacements on Σ: $\delta x^\alpha = \frac{\partial x^\alpha}{\partial y^a} \delta y^a = e^\alpha_a \delta y^a$
$\delta s^2 = (g_{\alpha\beta} e^\alpha_a e^\beta_b) \delta y^a \delta y^b$

Displacements on Σ: $\quad dx^\alpha = \dfrac{\partial x^\alpha}{\partial y^a}\, dy^a = e^\alpha_a\, dy^a$

$$ds^2 = \underbrace{\left(g_{\alpha\beta}\, e^\alpha_a\, e^\beta_b\right)}_{h_{ab}} dy^a dy^b$$


dy^a
$ds^2 = \underbrace{(g_{\alpha\beta}\, e^\alpha_a\, e^\beta_b)}_{h_{ab}}\, dy^a dy^b$
$h_{\lambda\lambda} = g_{\alpha\beta}\, e^\alpha_\lambda\, e^\beta_\lambda = g_{\alpha\beta}\, k^\alpha k^\beta = 0$


dy^0
$ds^2 = \underbrace{\left(g_{\alpha\beta}\, e^\alpha_a\, e^\beta_b\right)}_{h_{ab}} dy^a dy^b$
$h_{\lambda\lambda} = g_{\alpha\beta} e^\alpha_\lambda e^\beta_\lambda = g_{\alpha\beta} k^\alpha k^\beta = 0$
$h_{\lambda A} = g_{\alpha\beta} e^\alpha_\lambda e^\beta_A = k_\alpha e^\alpha_A = 0$



$ds^2 = \underbrace{(g_{\alpha\beta} e^\alpha_a e^\beta_b)}_{h_{ab}} dy^a dy^b$
$h_{\lambda\lambda} = g_{\alpha\beta} e^\alpha_\lambda e^\beta_\lambda = g_{\alpha\beta} K^\alpha K^\beta = 0$
$h_{\lambda A} = g_{\alpha\beta} e^\alpha_\lambda e^\beta_A = K_\alpha e^\alpha_A = 0$
$h_{AB} = g_{\alpha\beta} e^\alpha_A e^\beta_B =$

h_{ab}

$$h_{\lambda\lambda} = g_{\alpha\beta}\, e^{\alpha}_{\lambda} e^{\beta}_{\lambda} = g_{\alpha\beta}\, k^{\alpha} k^{\beta} = 0$$

$$h_{\lambda A} = g_{\alpha\beta}\, e^{\alpha}_{\lambda} e^{\beta}_{A} = k_{\alpha} e^{\alpha}_{A} = 0$$

$$h_{AB} = g_{\alpha\beta}\, e^{\alpha}_{A} e^{\beta}_{B} \equiv \sigma_{AB}$$

$$\longrightarrow ds^2 = \sigma_{AB}\, d\theta^A d\theta^B$$

h_{ab}

$$h_{\lambda\lambda} = g_{\alpha\beta}\, e^{\alpha}_{\lambda} e^{\beta}_{\lambda} = g_{\alpha\beta} K^{\alpha} K^{\beta} = 0$$

$$h_{\lambda A} = g_{\alpha\beta}\, e^{\alpha}_{\lambda} e^{\beta}_{A} = K_{\alpha} e^{\alpha}_{A} = 0$$

$$h_{AB} = g_{\alpha\beta}\, e^{\alpha}_{A} e^{\beta}_{B} \equiv \sigma_{AB}$$

$$\rightarrow \boxed{ds^2 = \sigma_{AB}\, d\theta^A d\theta^B}$$

Inverse metric on Σ :

$$g^{\alpha\beta} = \varepsilon n^{\alpha} n^{\beta}$$

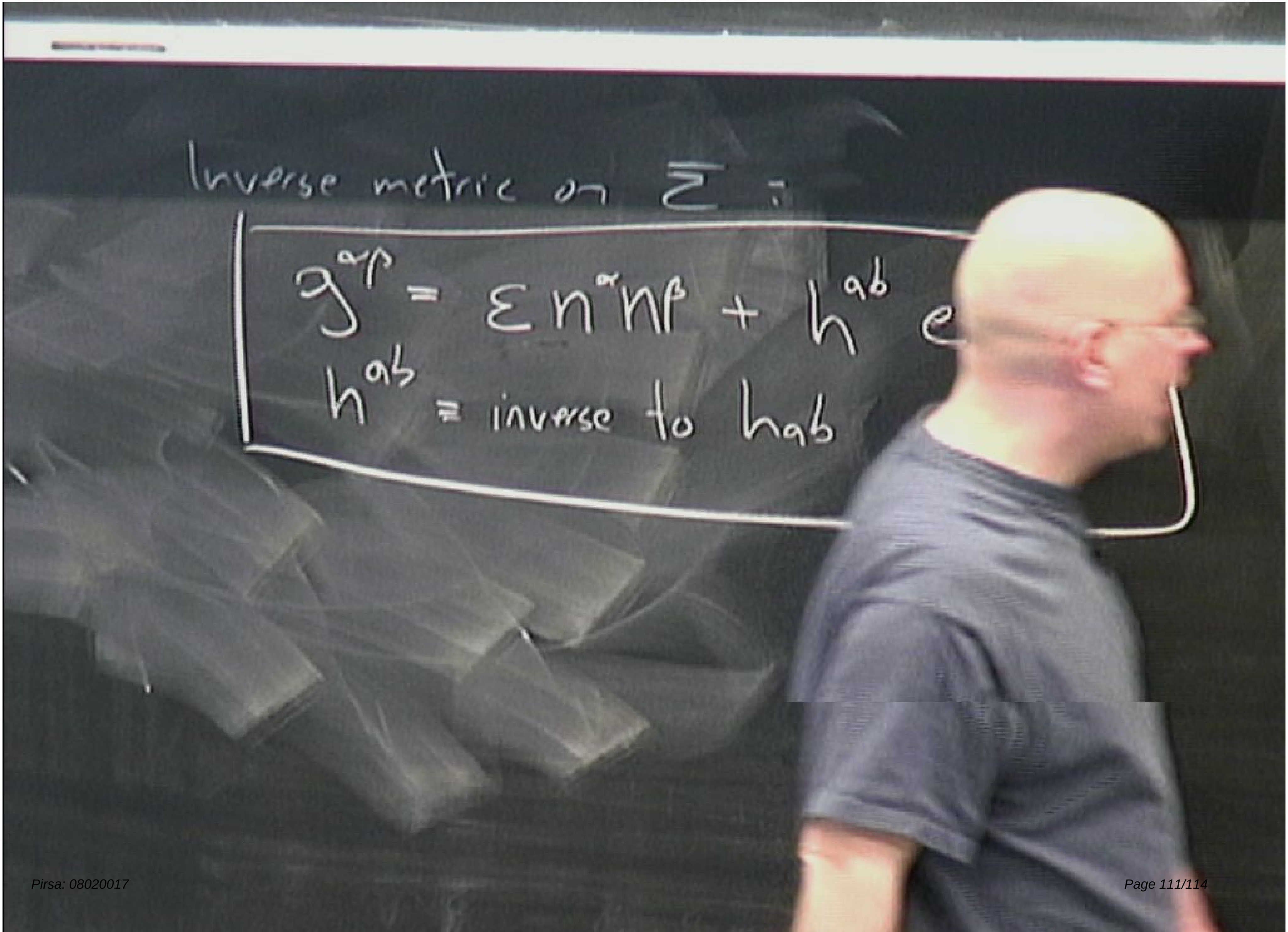
Inverse metric on Σ:

$$g^{\alpha\beta} = \varepsilon n^\alpha n^\beta + h^{ab} e^\alpha{}_a e^\beta{}_b$$

Inverse metric on Σ:

$$g^{\alpha\beta} = \varepsilon n^\alpha n^\beta + h^{ab} e^\alpha{}_a e^\beta{}_b$$

h^{ab} = inverse to h_{ab}


Inverse metric on Σ :
$g^{\alpha\beta} = \varepsilon n^\alpha n^\beta + h^{ab} e$
h^{ab} = inverse to h_{ab}

Inverse metric on Σ :

$$\boxed{g^{\alpha\beta} = \varepsilon n^{\alpha} n^{\beta} + h^{ab} e^{\alpha}{}_{a} e^{\beta}{}_{b}}$$

h^{ab} = inverse to h_{ab}

$$\boxed{K^\alpha{}_{;\beta} K^\beta = -k K^\alpha} \longrightarrow \text{geodesics.}$$

$$g^{\alpha\beta} = -K^\alpha N^\beta - N^\alpha K^\beta + \sigma^{AB} e^\alpha_A e^\beta_B$$

σ^{AB} is 2×2 inverse of σ_{AB}.

$$\boxed{K^\alpha{}_{;\beta} K^\beta = -\kappa K^\alpha} \longrightarrow \text{geodesics.}$$

$$g^{\alpha\beta} = - K^\alpha N^\beta - N^\alpha K^\beta + \sigma^{AB} e^\alpha_A e^\beta_B$$

σ^{AB} is 2×2 inverse of σ_{AB}.