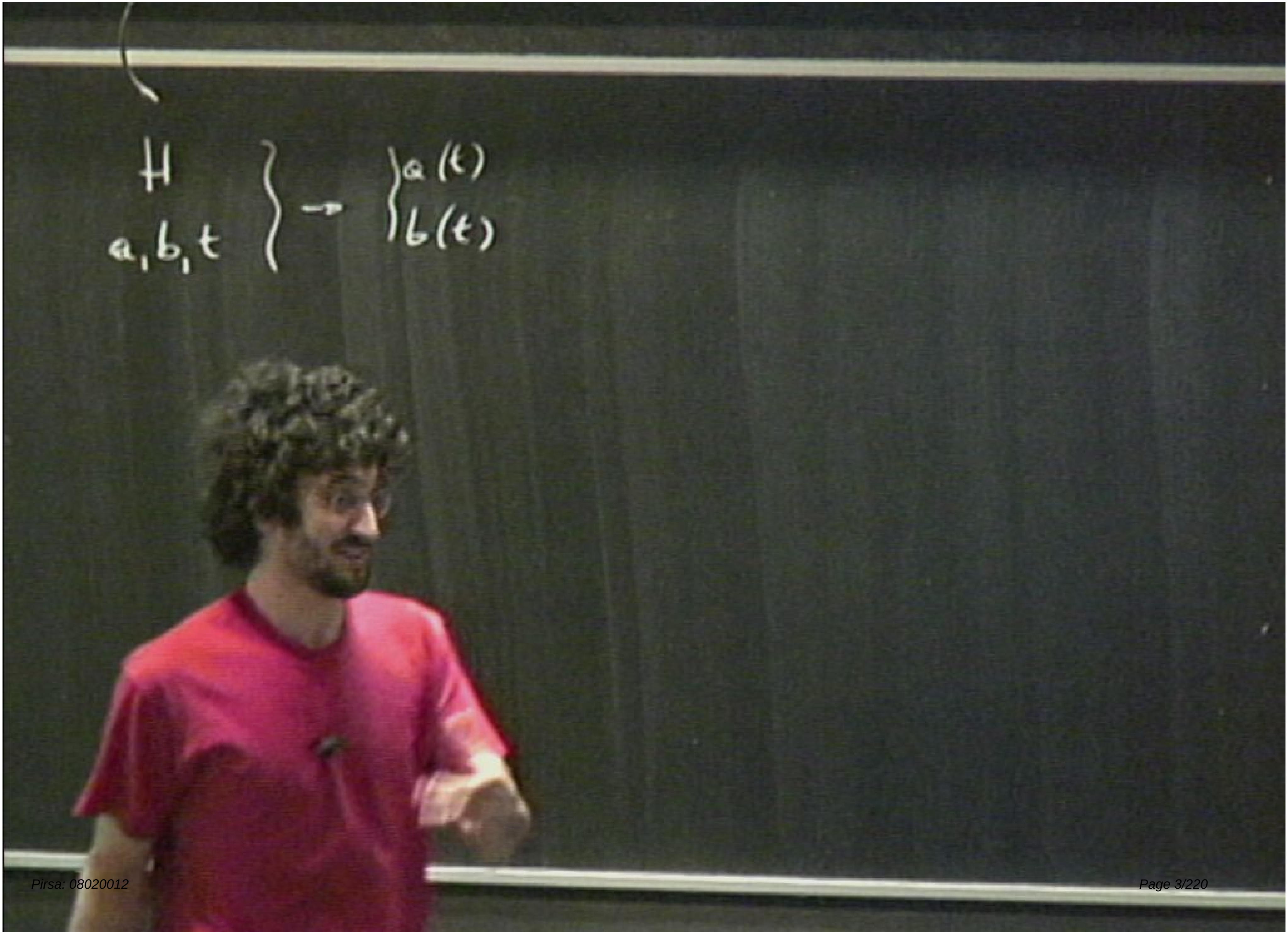
Title: Loop Quantum gravity #2

Date: Feb 07, 2008 06:30 PM

URL: http://pirsa.org/08020012

Abstract: Constrained systems, meaning of diffeomorphism invariance, loops and spin networks.


H
a, b, t
a(t)
b(t)

$$\left.\begin{matrix} H \\ a, b, t \end{matrix}\right\} \to \begin{cases} a(t) \\ b(t) \end{cases}$$

$$\left.\begin{matrix} H(a,b)=0 \\ a, b \end{matrix}\right\} \longrightarrow a(b)$$

$$\left.\begin{array}{c} H \\ a, b, t \end{array}\right\} \to \left\{\begin{array}{l} a(t) \\ b(t) \end{array}\right.$$

$$\left.\begin{array}{c} H(a,b)=0 \\ a, b \end{array}\right\} \to a(b)$$

$$\left.\begin{array}{c} H \\ a, b, t \end{array}\right\} \longrightarrow \left\{\begin{array}{l} a(t) \\ b(t) \end{array}\right.$$

$$\left.\begin{array}{c} \mathcal{H}(a,b) = 0 \\ a, b \end{array}\right\} \longrightarrow a(b)$$

$$\psi[a,b] \in \mathcal{H}$$

$$\psi[a,b] \in \mathcal{H} = L_2[\mathbb{R}^2, d^2x]$$

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$$i\partial_t \psi = \hat{H}\psi$$

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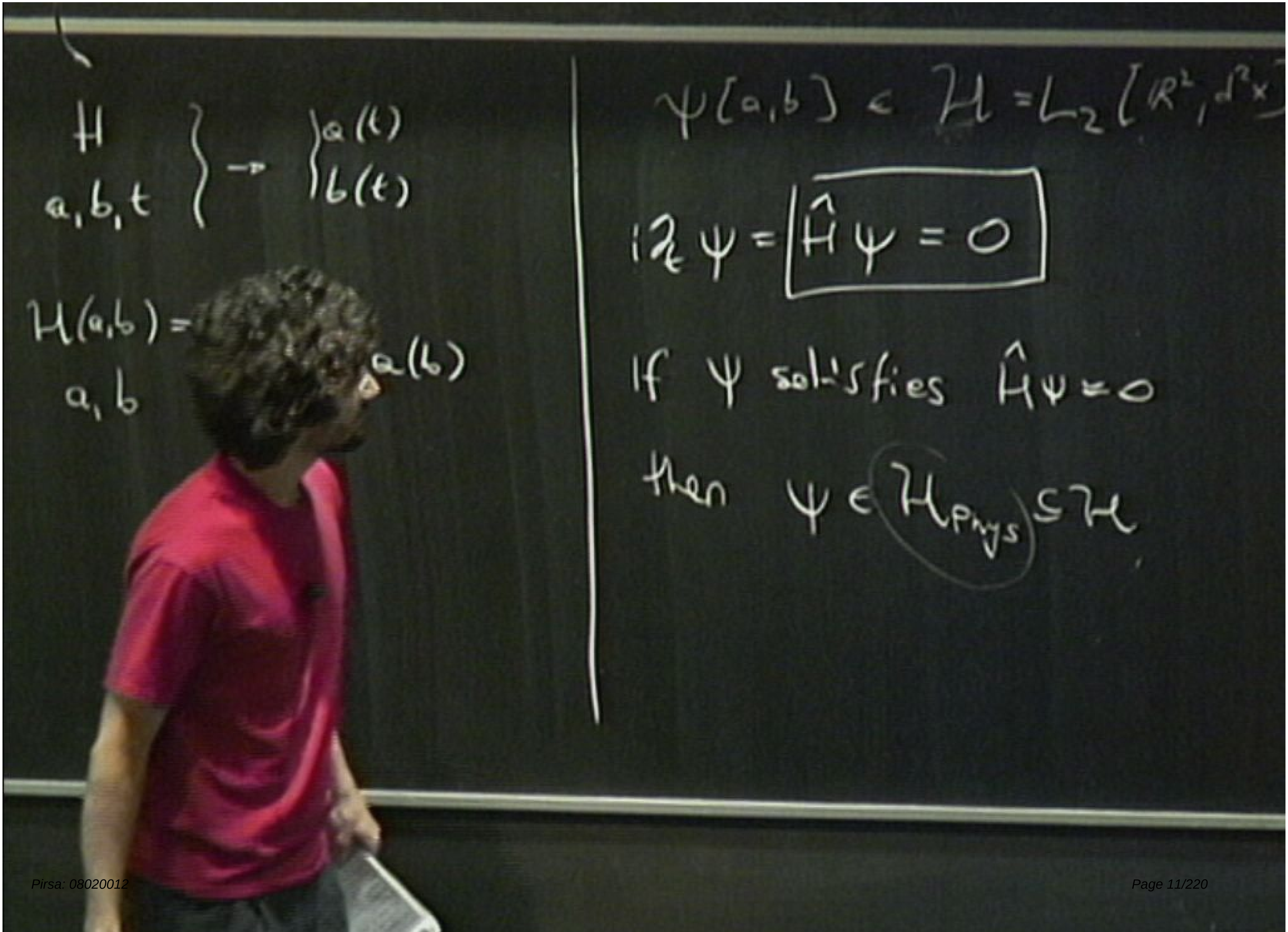
$$i\partial_t \psi = \boxed{\hat{H}\psi = 0}$$

$$\psi[a,b] \in \mathcal{H} = L_2[\mathbb{R}^2, d^2x]$$

$$i\partial_t \psi = \boxed{\hat{H}\psi = 0}$$

If ψ satisfies $\hat{H}\psi = 0$

then $\psi \in \mathcal{H}_{phys} \subseteq \mathcal{H}$


H
a, b, t → a(t), b(t)
$\mathcal{H}(a,b) =$
a(b)
a, b
$\psi[a,b] \in \mathcal{H} = L_2[\mathbb{R}^2, d^2x]$
$i\partial_t \psi = \hat{H}\psi = 0$
If ψ satisfies $\hat{H}\psi = 0$
then $\psi \in \mathcal{H}_{phys} \subseteq \mathcal{H}$

$\} a(t)$

$\} b(t)$

$\to a(b)$

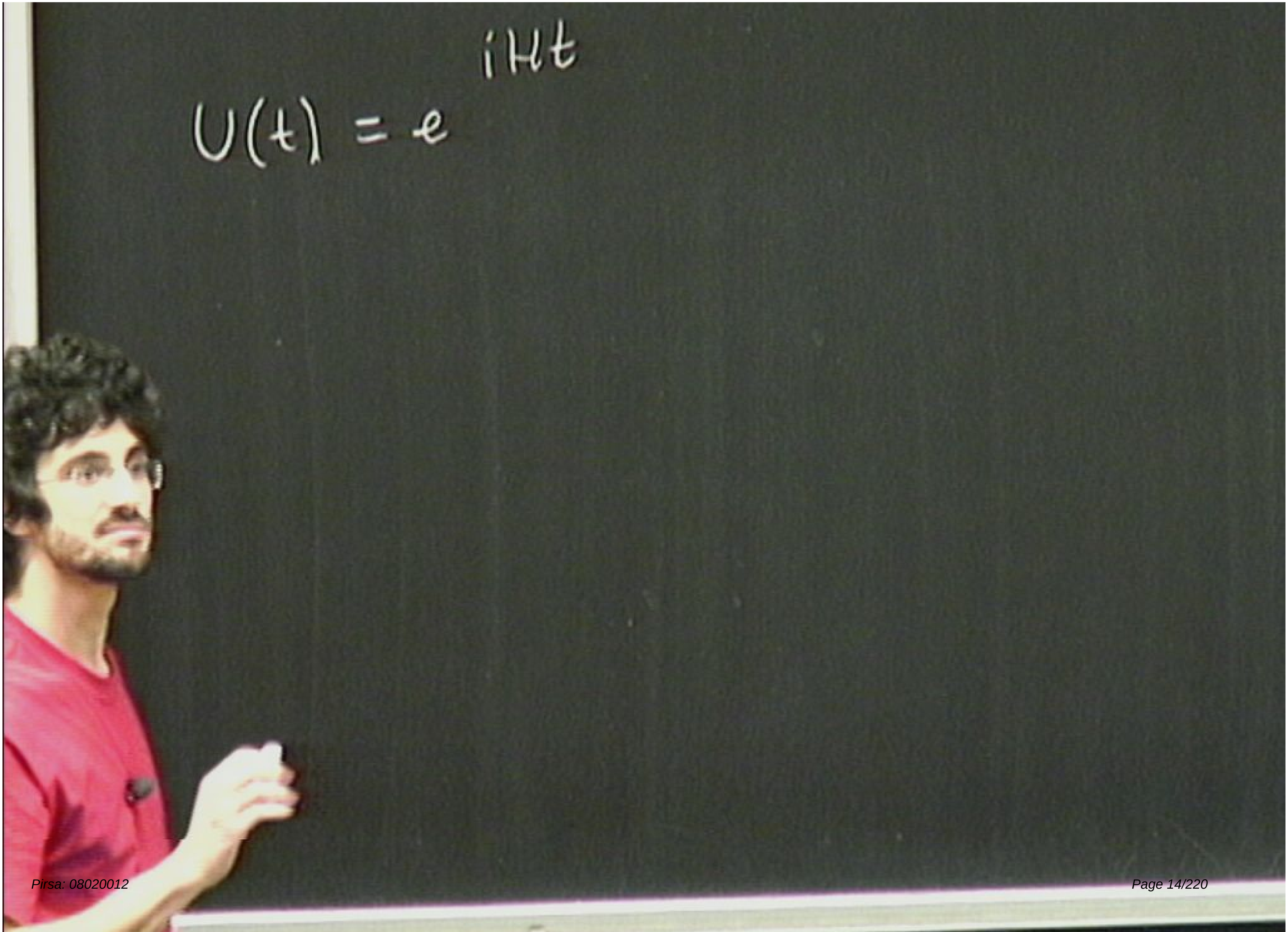
$\psi[a,b] \in \mathcal{H} = L_2[\mathbb{R}^2, d^2x]$

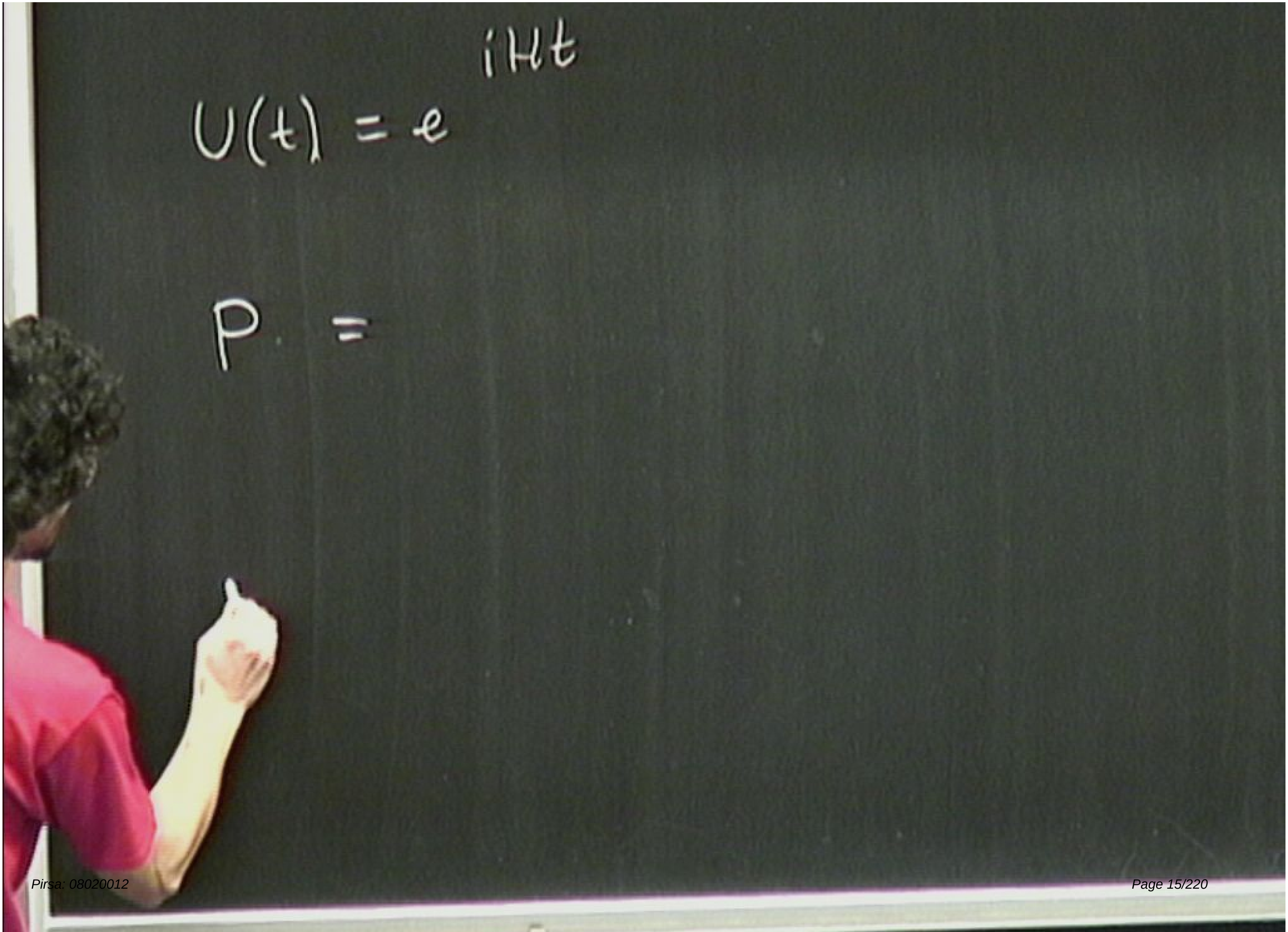
$i\partial_t \psi = \boxed{\hat{H}\psi = 0}$

If ψ satisfies $\hat{H}\psi = 0$

then $\psi \in \mathcal{H}_{phys} \subseteq \mathcal{H}$

$$U(t) = e^{iHt}$$


$U(t) = e^{iHt}$


$U(t) = e^{iHt}$
$P =$

$$U(t) = e^{iHt}$$

$$P =$$

$$\delta(x - x_0) = \int$$

$$U(t) = e^{iHt}$$

$$P =$$

$$\delta(x - x_0) = \int dp \, e^{ip(x - x_0)}$$

$$U(t) = e^{iHt}$$

$$P = \text{"}\delta(H)\text{"} = \int_{-\infty}^{+\infty} d\lambda \, e^{iH\lambda}$$

$$\delta(x - x_0) = \int dp \, e^{ip(x - x_0)}$$

$$\left.\begin{matrix} H \\ a, b, t \end{matrix}\right\} \rightarrow \left\{\begin{matrix} a(t) \\ b(t) \end{matrix}\right.$$

$$\left.\begin{matrix} H(a,b) = 0 \\ a, b \end{matrix}\right\} \rightarrow a(b)$$

$$\psi[a,b] \in \mathcal{H} = L_2[\mathbb{R}^2, d^2x]$$

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If ψ satisfies $\hat{H}\psi = 0$

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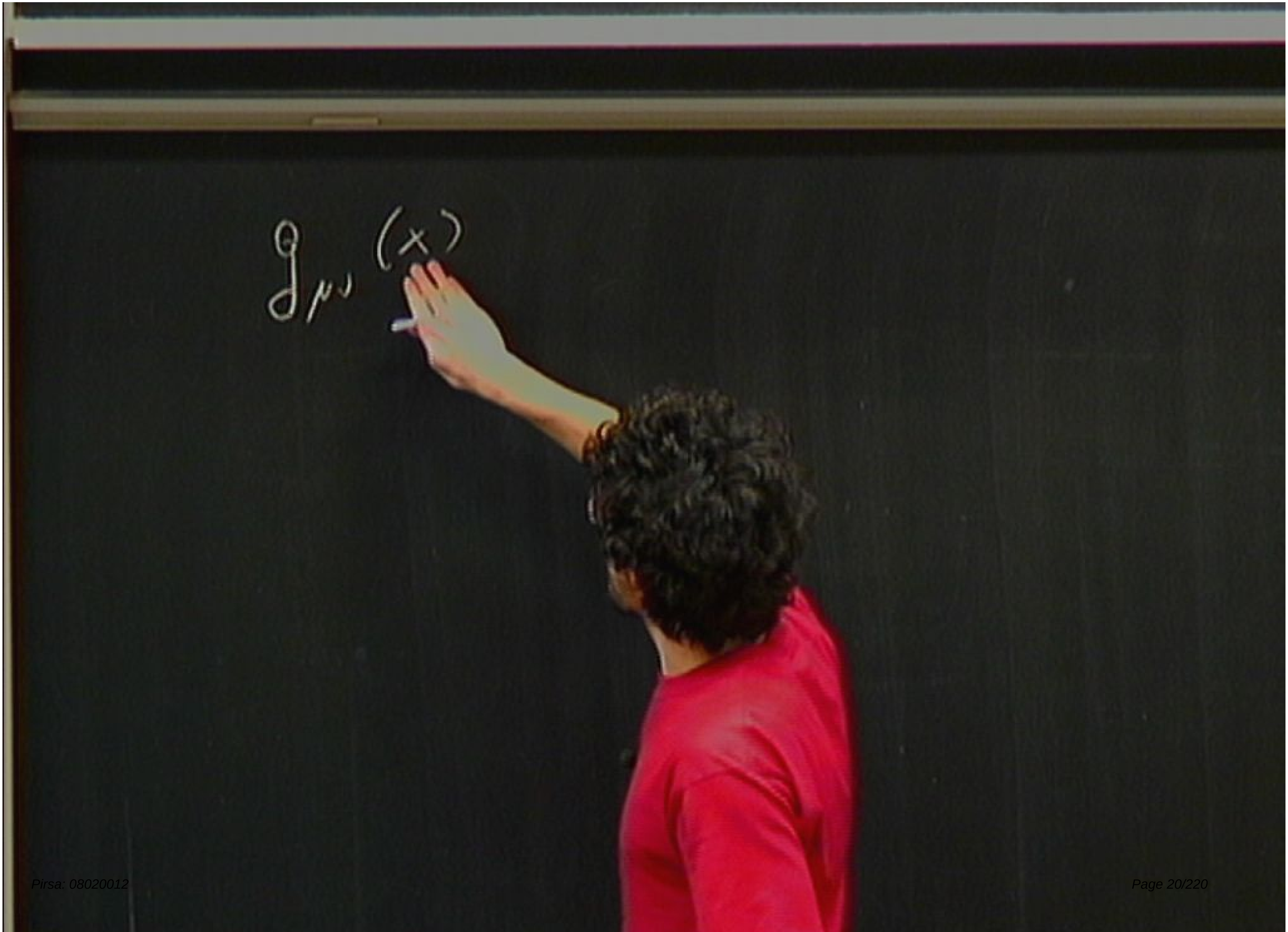
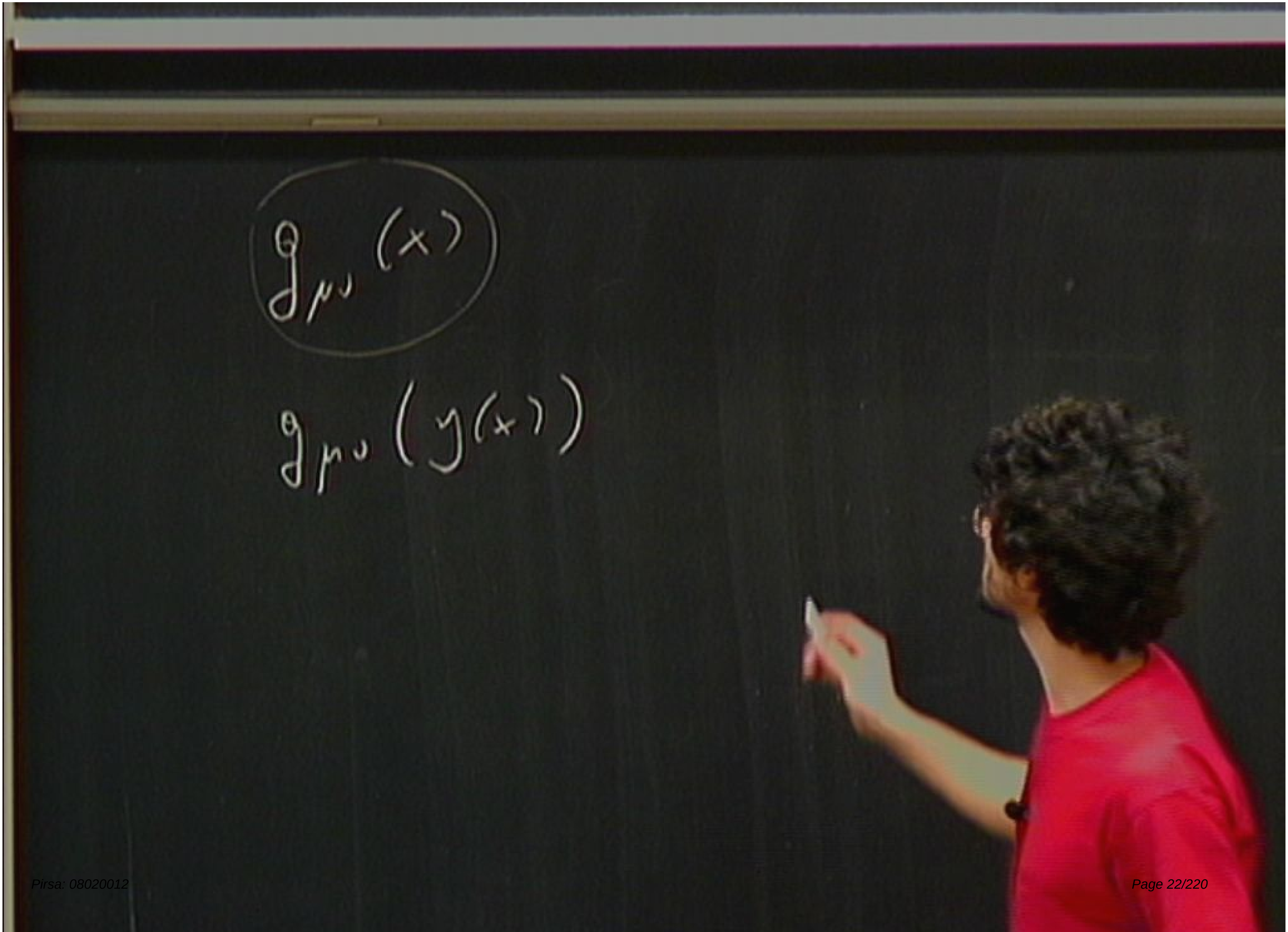

$g_{\mu\nu}(x)$

$g_{\mu\nu}(x)$

$g_{\mu\nu}(y(x))$


$g_{\mu\nu}(x)$
$g_{\mu\nu}(y(x))$

a, b, t $\Big($ $b(t)$

$$\left.\begin{matrix} H(a,b)=0 \\ a, b \end{matrix}\right\} \rightarrow a(b)$$

$$g_{\mu\nu}(x)$$

$$g_{\mu\nu}(y(x))$$

$$\mathcal{H}(g_{ab}, \pi^{ab}) = 0$$

$$H \atop a, b, t \Bigg\} \to \Bigg\{ a(t) \atop b(t)$$

$$\mathcal{H}(a,b) = 0 \atop a, b \Bigg\} \to a(b)$$

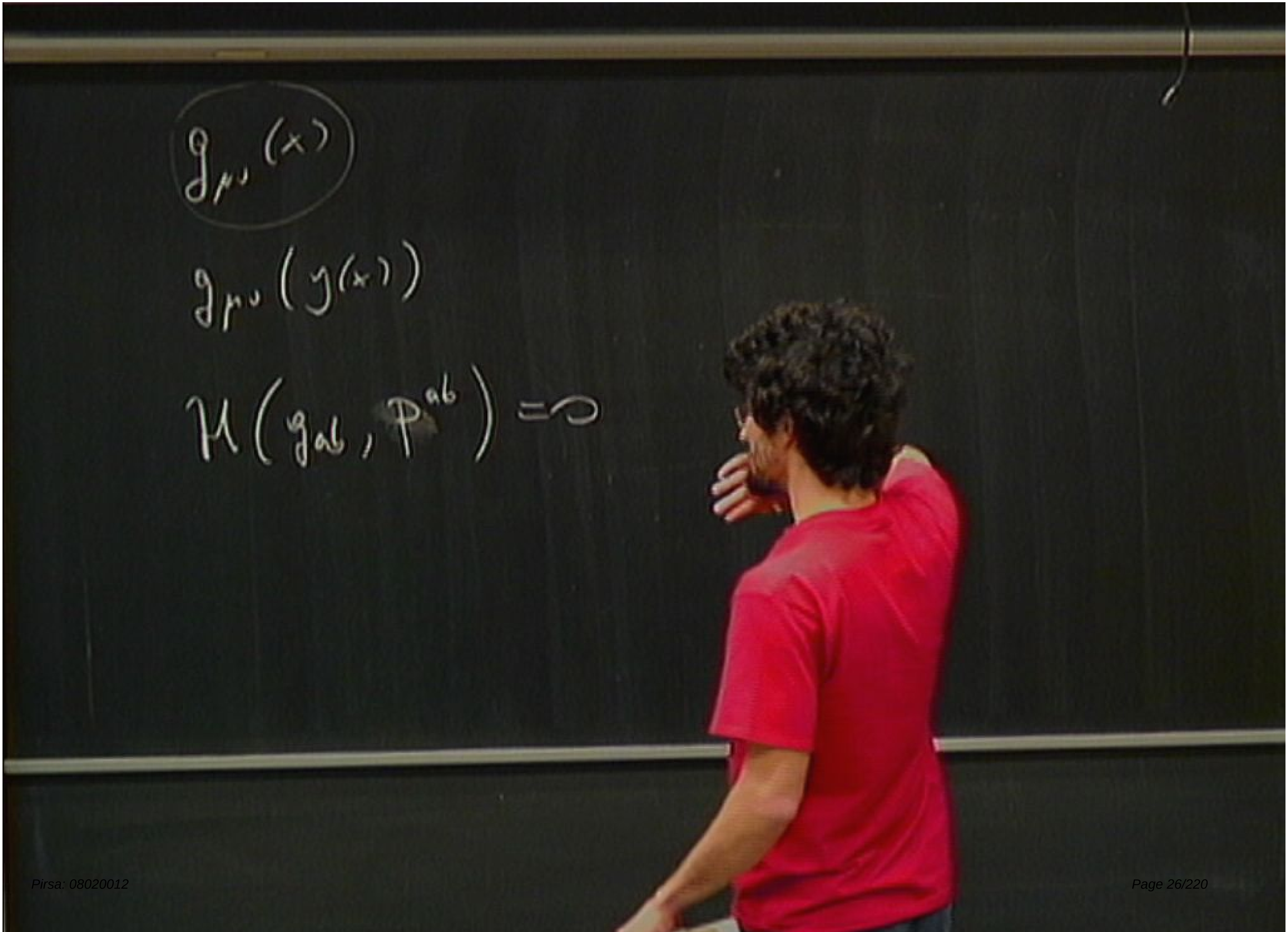
$$\mathcal{H}(a, b, p_a, p_b)$$

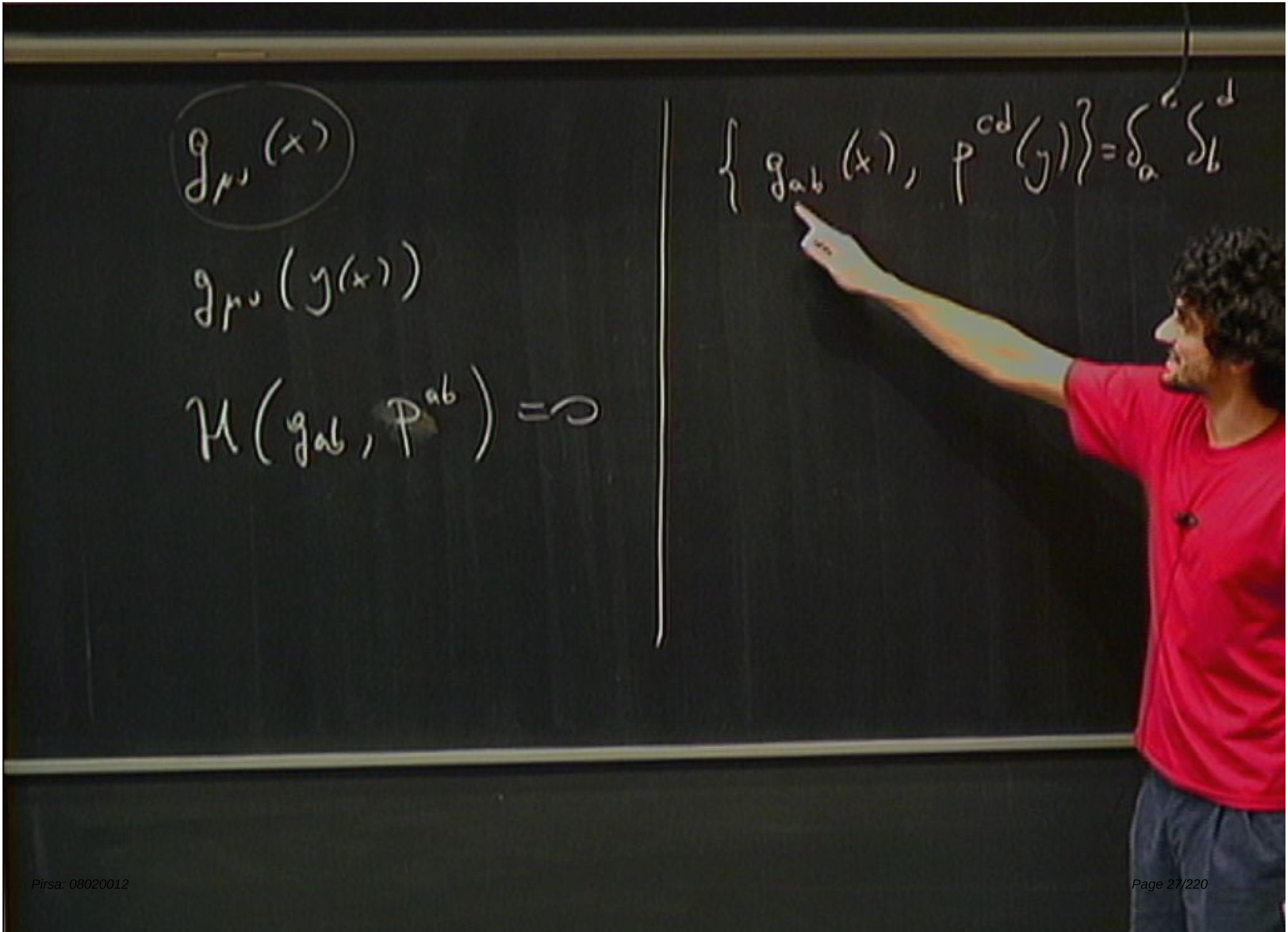
$\psi[a, b] \in$

$$i\partial_t \psi = \boxed{\hat{H}\psi =}$$

If ψ satisfies

then $\psi \in \mathcal{H}$


$g_{\mu\nu}(x)$
$g_{\mu\nu}(y(x))$
$\mathcal{H}(g_{ab}, P^{ab}) = 0$


$g_{\mu\nu}(x)$
$g_{\mu\nu}(y(x))$
$\mathcal{H}(g_{ab}, p^{ab}) = 0$
$\{ g_{ab}(x), p^{cd}(y) \} = \delta_a^c \delta_b^d$

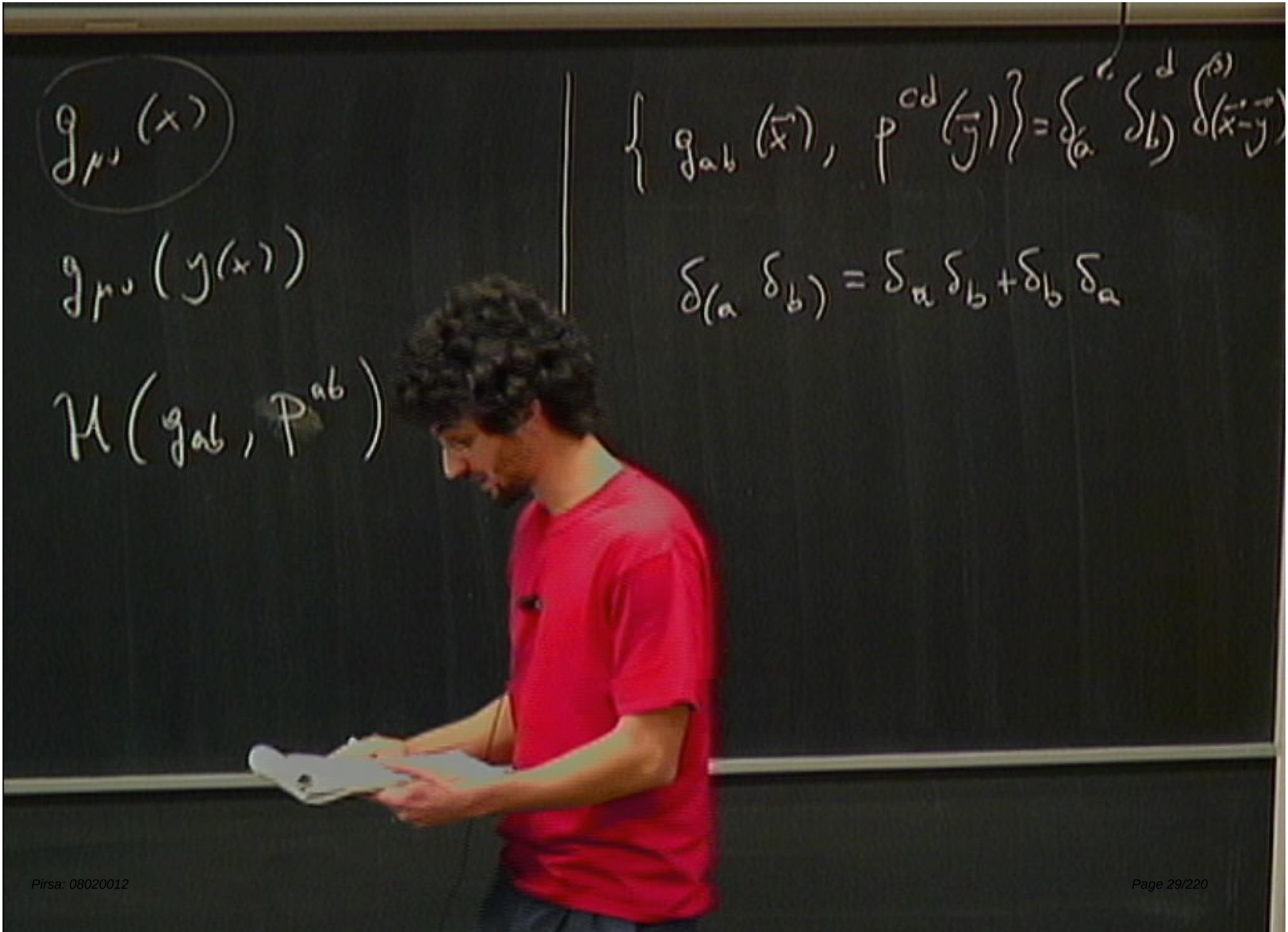
$$g_{\mu\nu}(x)$$

$$g_{\mu\nu}(y(x))$$

$$\mathcal{H}(g_{ab}, p^{ab}) = 0$$

$$\{ g_{ab}(x), p^{cd}(y) \} = \delta_{(a}^{c} \delta_{b)}^{d}$$

$$\delta_{(a} \delta_{b)} = \delta_a \delta_b + \delta_b \delta_a$$


$g_{\mu\nu}(x)$
$g_{\mu\nu}(y(x))$
$\mathcal{H}(g_{ab}, p^{ab})$
$\{g_{ab}(\vec{x}), p^{cd}(\vec{y})\} = \delta_{(a}^{c}\delta_{b)}^{d}\delta^{(3)}(\vec{x}-\vec{y})$
$\delta_{(a}\delta_{b)} = \delta_a\delta_b + \delta_b\delta_a$

$$g_{\mu\nu}(x)$$

$$g_{\mu\nu}(y(x))$$

$$\mathcal{H}(g_{ab}, p^{ab}) = 0$$

$$\{ g_{ab}(\vec{x}), p^{cd}(\vec{y}) \} = \delta_{(a}^{\ c} \delta_{b)}^{\ d} \delta^{(3)}(\vec{x}-\vec{y})$$

$$\delta_{(a} \delta_{b)} = \delta_a \delta_b + \delta_b \delta_a$$

$g_{\mu\nu}(x)$

$g_{\mu\nu}(y(x))$

$\mathcal{H}(g_{ab}, p^{ab}) = 0$

$S = \int dt\,[p\dot{q} - H]$

$\{g_{ab}(\vec{x}), p^{cd}(\vec{y})\} = \delta_{(a}^{c}\,\delta_{b)}^{d}\,\delta^{(3)}(\vec{x}-\vec{y})$

$\delta_{(a}\,\delta_{b)} = \delta_a \delta_b + \delta_b \delta_a$

$g_{\mu\nu}(x)$

$g_{\mu\nu}(y(x))$

$H(g_{ab}, p^{ab}) = 0$

$S = \int dt \, [p\dot{q} - H]$

$S[g_{ab}, p^{ab}, N, N_a]$

$\{ g_{ab}(\vec{x}), p^{cd}(\vec{y}) \} = \delta_{(a}^{c} \delta_{b)}^{d} \delta^{(3)}(\vec{x}-\vec{y})$

$\delta_{(a} \delta_{b)} = \delta_a \delta_b + \delta_b \delta_a$

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$g_{\mu\nu}(y(x))$

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$\{ g_{ab}(\vec{x}), p^{cd}(\vec{y}) \} = \delta_{(a}^{c} \delta_{b)}^{d} \delta^{(3)}(\vec{x}-\vec{y})$

$\delta_{(a} \delta_{b)} = \delta_a \delta_b + \delta_b \delta_a$

$$g_{\mu\nu} = \begin{pmatrix} g_{00} & g_{0a} \\ & \begin{pmatrix} \\ g_{ab} \\ \\ \end{pmatrix} \end{pmatrix}$$

$$\begin{cases} g_{0a} = N_a \\ g_{00} = -N^2 \end{cases}$$

$$g_{\mu\nu}(x)$$

$$g_{\mu\nu}(y(x))$$

$$\mathcal{H}(g_{ab}, p^{ab}) = 0$$

$$S = \int dt\,[p\dot{q} - H]$$

$$S[g_{ab}, p^{ab}, N, N_a] = \int dt \int d^3x\,[\dot{g}_{ab}\, p^{ab} - N\mathcal{H}(g,p) - N_a \mathcal{H}^a(g,p)]$$

$$\{ g_{ab}(\vec{x}), p^{cd}(\vec{y}) \} = \delta_{(a}^{\ c}\, \delta_{b)}^{\ d}$$

$$\delta_{(a}\, \delta_{b)} = \delta_a \delta_b + \delta_b \delta_a$$

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$g_{\mu\nu}(y(x))$

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$\{g_{ab}(\vec{x}), p^{cd}(\vec{y})\} = \delta_{(a}^{\ c}\,\delta_{b)}^{\ d}$

$\delta_{(a}\,\delta_{b)} = \delta_a\,\delta_b + \delta_b\,\delta_a$

$$\begin{cases} \mathcal{H}(g, p) = 0 \\ \mathcal{H}^a(g, p) = 0 \end{cases} \quad a = 1, 2$$

$S[g_{ab}, p^{ab}, N, N_a] = \int dt \int d^3x\,[\dot{g}_{ab}\,p^{ab} - N\mathcal{H}(g, p) - N_a\mathcal{H}^a(g, p)]$

$$g_{\mu\nu}(x)$$

$$g_{\mu\nu}(y(x))$$

$$\mathcal{H}(g_{ab}, p^{ab}) = 0$$

$$S = \int dt\,[p\dot{q} - H]$$

$$\{ g_{ab}(\vec{x}), p^{cd}(\vec{y}) \} = \delta_{(a}^{c}\,\delta_{b)}^{d}\,\delta^{(3)}(\vec{x}-\vec{y})$$

$$\delta_{(a}\delta_{b)} = \delta_a\delta_b + \delta_b\delta_a$$

$$\begin{cases} \mathcal{H}(g,p) = 0 \\ \mathcal{H}^a(g,p) = 0 \end{cases} \qquad a = 1,2,3$$

$$S[g_{ab}, p^{ab}, N, N_a] = \int dt \int d^3x\,[\dot{g}_{ab}p^{ab} - N\mathcal{H}(g,p) - N_a\mathcal{H}^a(g,p)]$$

$$\{q, p\} = \delta$$

$$\mathcal{H}(g, p) = 0$$

$$\mathcal{H}^a(g, p)$$

$$\{q, p\} = \delta$$

$$\begin{cases} \mathcal{H}(q,p) = 0 \\ \mathcal{H}^a(q,p) = 0 \end{cases}$$

$$[\hat{q}, \hat{p}] = \hbar\delta$$

$\Big\} \to \begin{cases} a(t) \\ b(t) \end{cases}$

$\Big\} \to a(b)$

$(p_a, p_b) = 0$

$\psi[a,b] \in \mathcal{H} = L_2[\mathbb{R}^2, d^2x]$

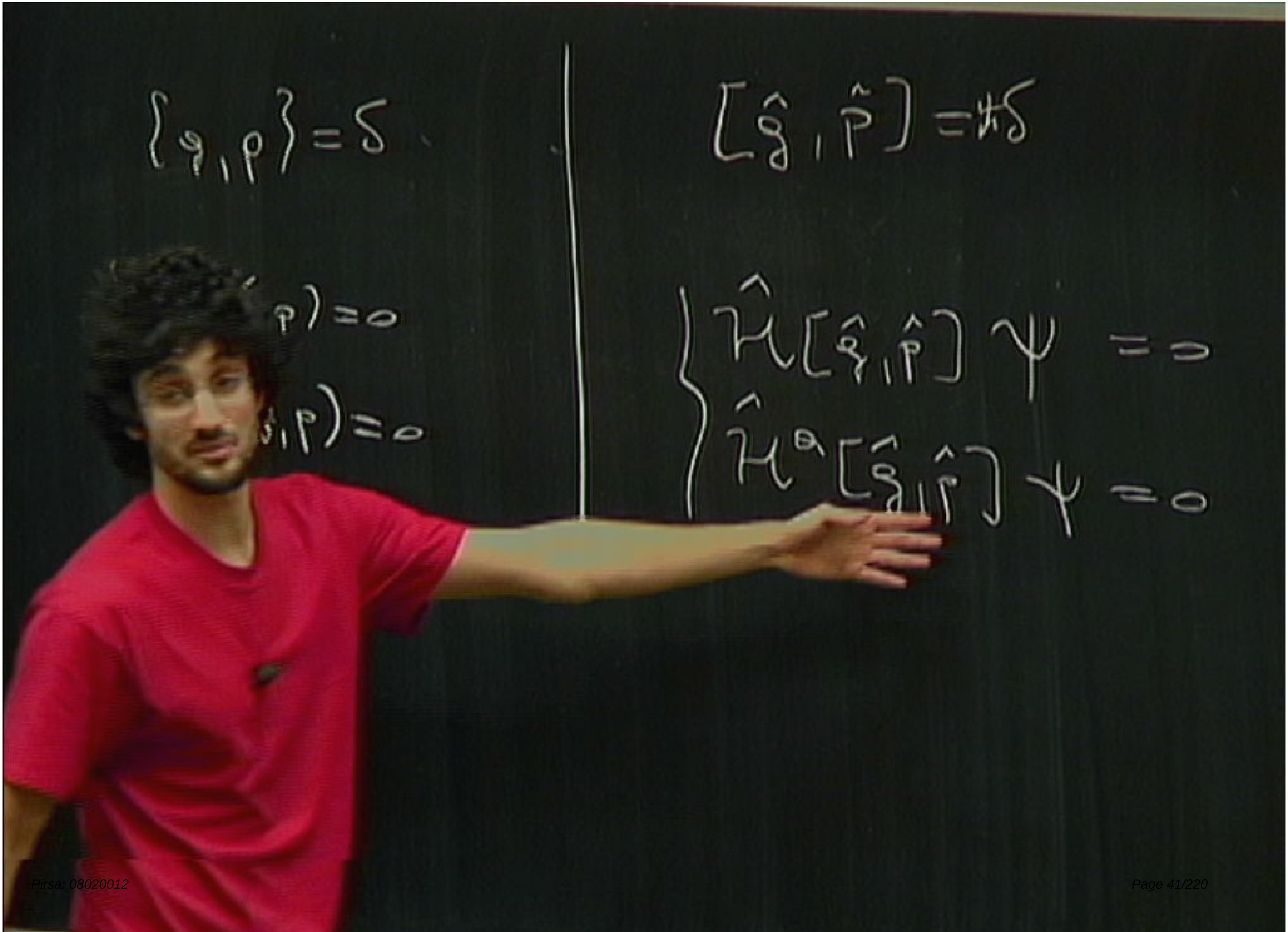
$i\partial_t \psi = \boxed{\hat{H}\psi = 0}$

If ψ satisfies $\hat{H}\psi = 0$

then $\psi \in \mathcal{H}_{phys} \subseteq \mathcal{H}$

$\{x^i, p^j\} = \delta^i_j$

$[\hat{x}^i, \hat{p}^j] = i\hbar\,\delta^i_j$


{q, p} = δ
[q̂, p̂] = iħδ
Ĥ[q̂, p̂] ψ = 0
Ĥ^a[q̂, p̂] ψ = 0

$$\{q, p\} = \delta$$

$$\begin{cases} \mathcal{H}(q, p) = 0 \\ \mathcal{H}^a(q, p) = 0 \end{cases}$$

$$[\hat{q}, \hat{p}] = i\hbar\delta$$

$$\begin{cases} \hat{\mathcal{H}}[\hat{q}, \hat{p}]\,\psi = 0 \\ \hat{\mathcal{H}}^a[\hat{q}, \hat{p}]\,\psi = 0 \end{cases}$$

$$\{q, p\} = \delta$$

$$\begin{cases} \mathcal{H}(q, p) = 0 \\ \mathcal{H}^a(q, p) = 0 \end{cases}$$

$$[\hat{q}, \hat{p}] = i\hbar\delta$$

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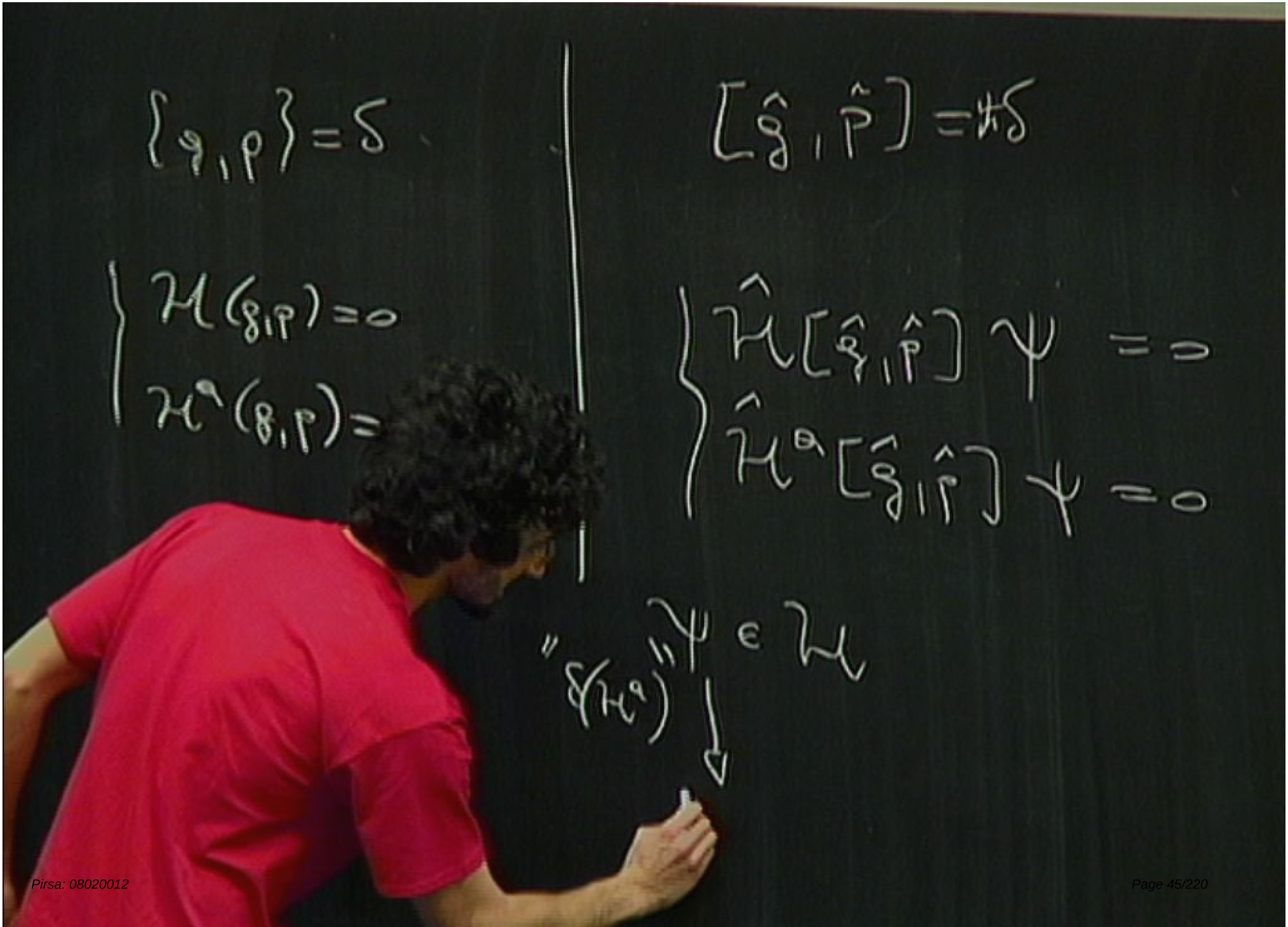
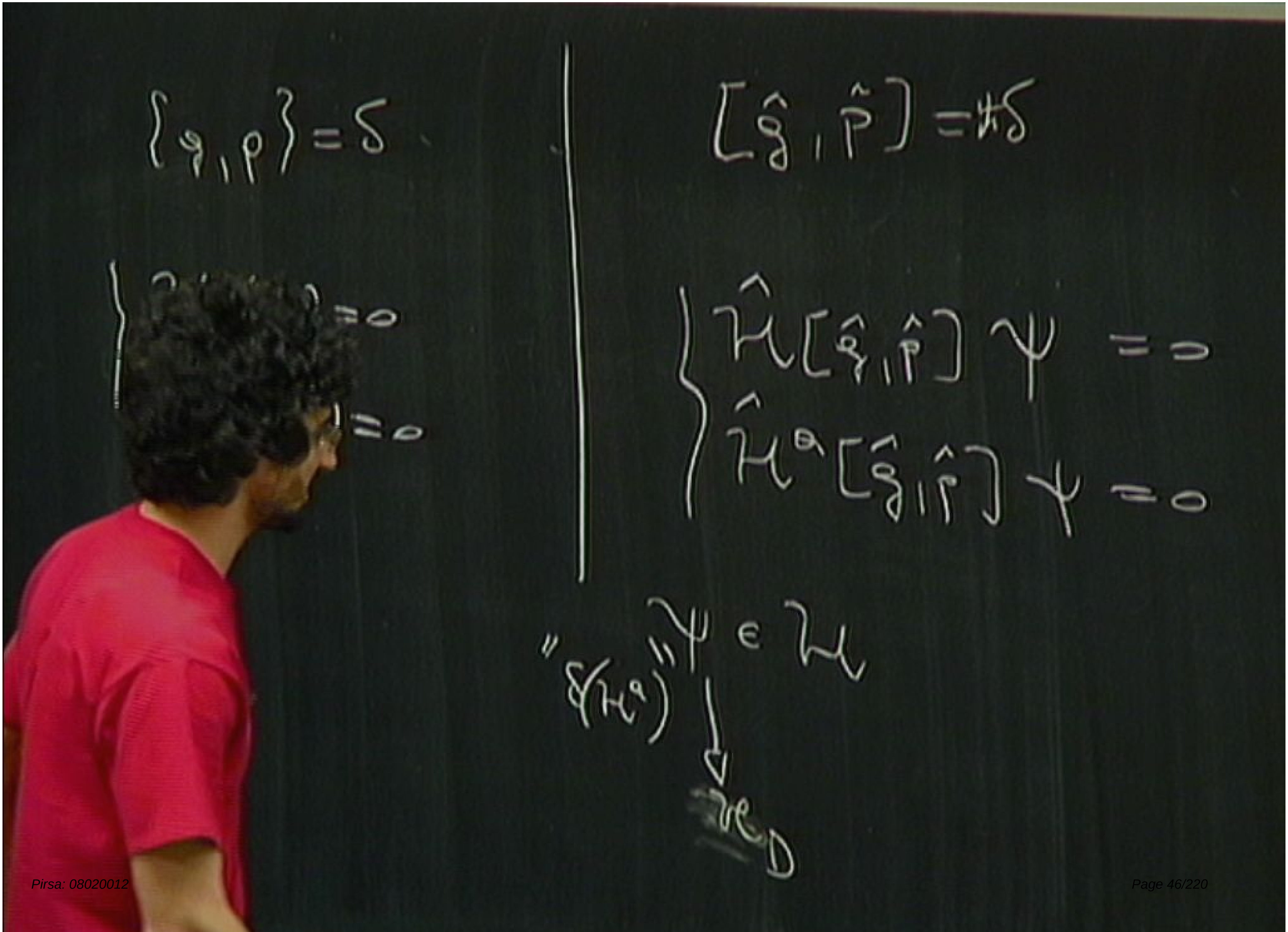
$$\begin{cases} \hat{\mathcal{H}}[\hat{q}, \hat{p}]\,\psi = 0 \\ \hat{\mathcal{H}}^a[\hat{q}, \hat{p}]\,\psi = 0 \end{cases}$$


$\{q, p\} = \delta$
$[\hat{q}, \hat{p}] = i\hbar\delta$
$\mathcal{H}(q, p) = 0$
$\mathcal{H}^a(q, p) =$
$\hat{\mathcal{H}}[\hat{q}, \hat{p}]\,\psi = 0$
$\hat{\mathcal{H}}^a[\hat{q}, \hat{p}]\,\psi = 0$
"$\delta(\mathcal{H}^a)$" $\psi \in \mathcal{H}$


$\{q, p\} = \delta$
$[\hat{q}, \hat{p}] = i\hbar\delta$
$\hat{\mathcal{H}}[\hat{q}, \hat{p}]\,\psi = 0$
$\hat{\mathcal{H}}^a[\hat{q}, \hat{p}]\,\psi = 0$
"$\delta(\mathcal{H}^a)$" $\psi \in \mathcal{H}$
$\mathcal{H}_D$

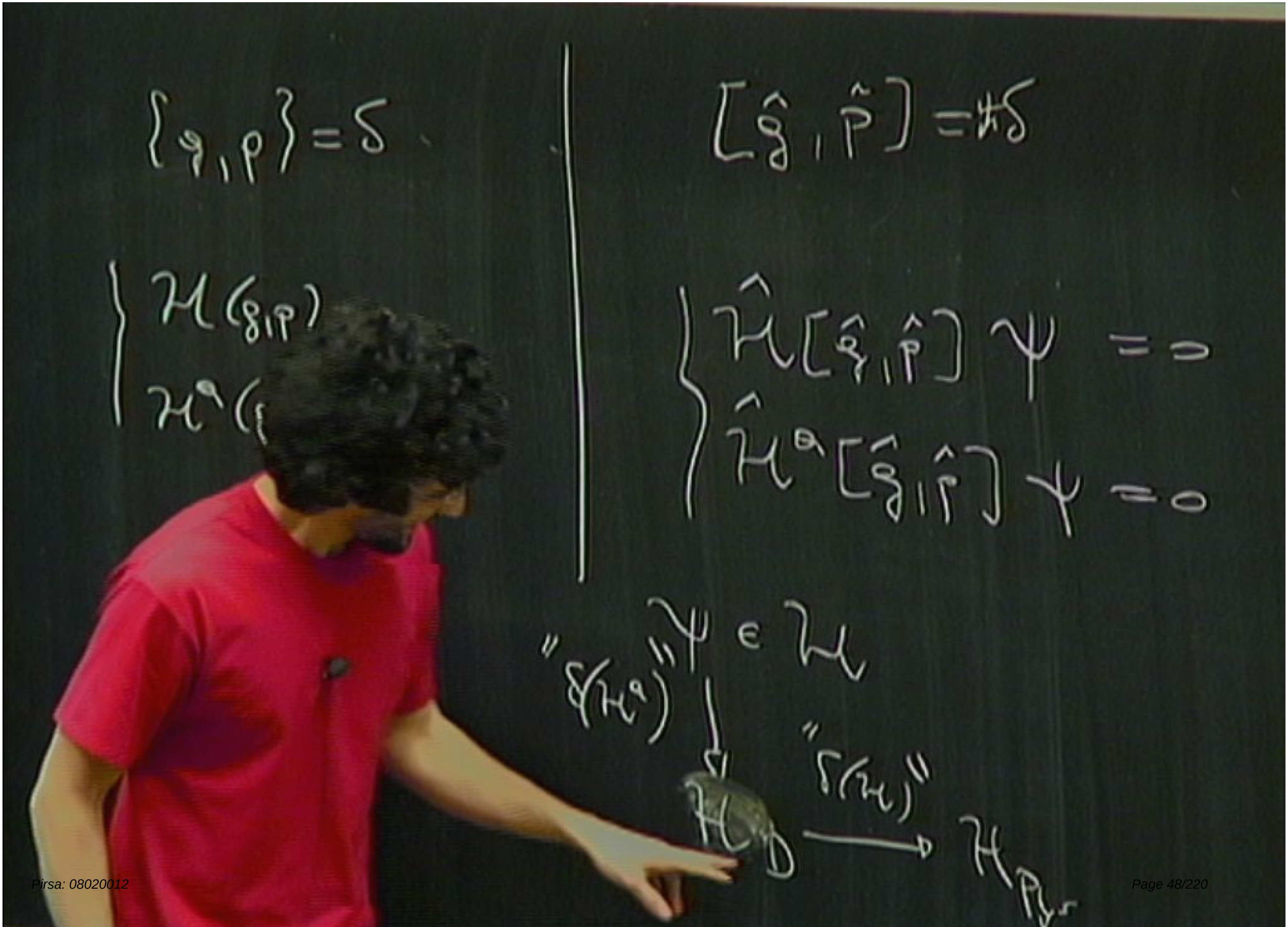
$$\{q, p\} = \delta \qquad\qquad [\hat{q}, \hat{p}] = i\hbar\delta$$

$$\begin{cases} \mathcal{H}(q,p) = 0 \\ \mathcal{H}^a(q,p) = 0 \end{cases} \qquad\qquad \begin{cases} \hat{\mathcal{H}}[\hat{q}, \hat{p}]\,\Psi = 0 \\ \hat{\mathcal{H}}^a[\hat{q}, \hat{p}]\,\Psi = 0 \end{cases}$$

$$\Psi \in \mathcal{H}$$

$$\Big\downarrow \text{“}\delta(\mathcal{H}^a)\text{”}$$

$$\mathcal{H}_D \xrightarrow{\text{“}\delta(\mathcal{H})\text{”}} \mathcal{H}_{Phys}$$


{q, p} = δ
[q̂, p̂] = iħδ
H(q, p)
Hᵃ(
Ĥ[q̂, p̂] ψ = 0
Ĥᵃ[q̂, p̂] ψ = 0
ψ ∈ H
"δ(Hᵃ)"
"δ(H)"
H_Diff → H_Phys

$$\{q, p\} = \delta \qquad\qquad [\hat{q}, \hat{p}] = i\hbar\delta$$

$$\begin{cases} \mathcal{H}(q, p) = 0 \\ \mathcal{H}^a(q, p) = 0 \end{cases} \qquad \begin{cases} \hat{\mathcal{H}}[\hat{q}, \hat{p}]\, \psi = 0 \\ \hat{\mathcal{H}}^a[\hat{q}, \hat{p}]\, \psi = 0 \end{cases}$$

$$\psi \in \mathcal{H} = L_2(?)$$

$$\big\downarrow \text{“}\delta(\mathcal{H}^a)\text{”}$$

$$\mathcal{H}_D \xrightarrow{\text{“}\delta(\mathcal{H})\text{”}} \mathcal{H}_{Phys}$$

$t \Big\} \to \Big\{ \begin{matrix} a(t) \\ b(t) \end{matrix}$

$\left.\begin{matrix} \ldots) = 0 \\ \ldots \end{matrix}\right\} \longrightarrow a(b)$

$\ldots b, p_a, p_b) = 0$

$\psi[a,b] \in \mathcal{H} = L_2[\mathbb{R}^2, d^2x]$

$i\partial_t \psi = \boxed{\hat{H}\psi = 0}$

If ψ satisfies $\hat{H}\psi = 0$

then $\psi \in \mathcal{H}_{phys} \subseteq \mathcal{H}$

$\{x^i, p^j\} = \delta^i_j$

$[\hat{x}^i, \hat{p}^j] = i\hbar\delta^i_j$

$$\{q,p\}=\delta$$

$$\begin{cases} \mathcal{H}(g,p)=0 \\ \mathcal{H}^a(g,p)=0 \end{cases}$$

$$[\hat{g},\hat{p}]=i\hbar\delta$$

$$\begin{cases} \hat{\mathcal{H}}[\hat{g},\hat{p}]\,\Psi = 0 \\ \hat{\mathcal{H}}^a[\hat{g},\hat{p}]\,\Psi = 0 \end{cases}$$

$$\Psi \in \mathcal{H} = L_2(\,?\,)$$

$$\mathcal{H} \xrightarrow{\text{"}\delta(\mathcal{H}^a)\text{"}} \mathcal{H}_{\text{Diff}} \xrightarrow{\text{"}\delta(\mathcal{H})\text{"}} \mathcal{H}_{\text{Phys}}$$

$$\{q, p\} = \delta$$

$$\begin{cases} \mathcal{H}(q,p) = 0 \\ \mathcal{H}^a(q,p) = 0 \end{cases}$$

$$[\hat{q}, \hat{p}] = i\hbar\delta$$

$$\begin{cases} \hat{\mathcal{H}}[\hat{q},\hat{p}]\,\Psi = 0 \\ \hat{\mathcal{H}}^a[\hat{q},\hat{p}]\,\Psi = 0 \end{cases}$$

$$\Psi \in \mathcal{H} = L_2(\,?\,)$$

$$\mathcal{H} \xrightarrow{\text{"}\delta(\mathcal{H}^a)\text{"}} \mathcal{H}_{D} \xrightarrow{\text{"}\delta(\mathcal{H})\text{"}} \mathcal{H}_{Phys}$$

$\{q, p\} = \delta$

$$\begin{cases} \mathcal{H}(g, p) = 0 \\ \mathcal{H}^a(g, p) = 0 \end{cases}$$

$[\hat{g}, \hat{p}] = i\hbar\delta$

$$\begin{cases} \hat{\mathcal{H}}[\hat{g}, \hat{p}]\,\psi = 0 \\ \hat{\mathcal{H}}^a[\hat{g}, \hat{p}]\,\psi = 0 \end{cases}$$

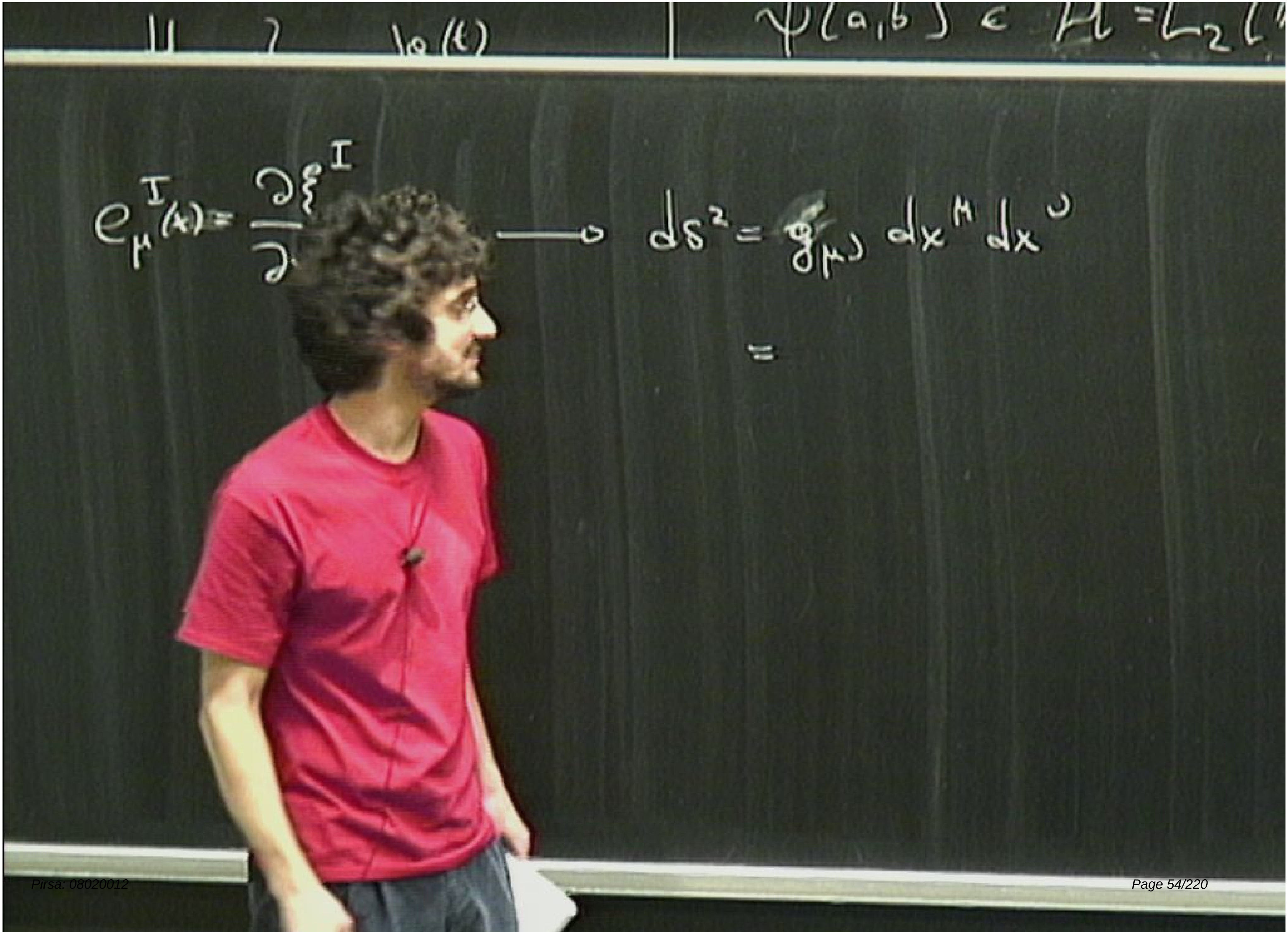
$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu$

$\psi \in \mathcal{H} = L_2(?)$

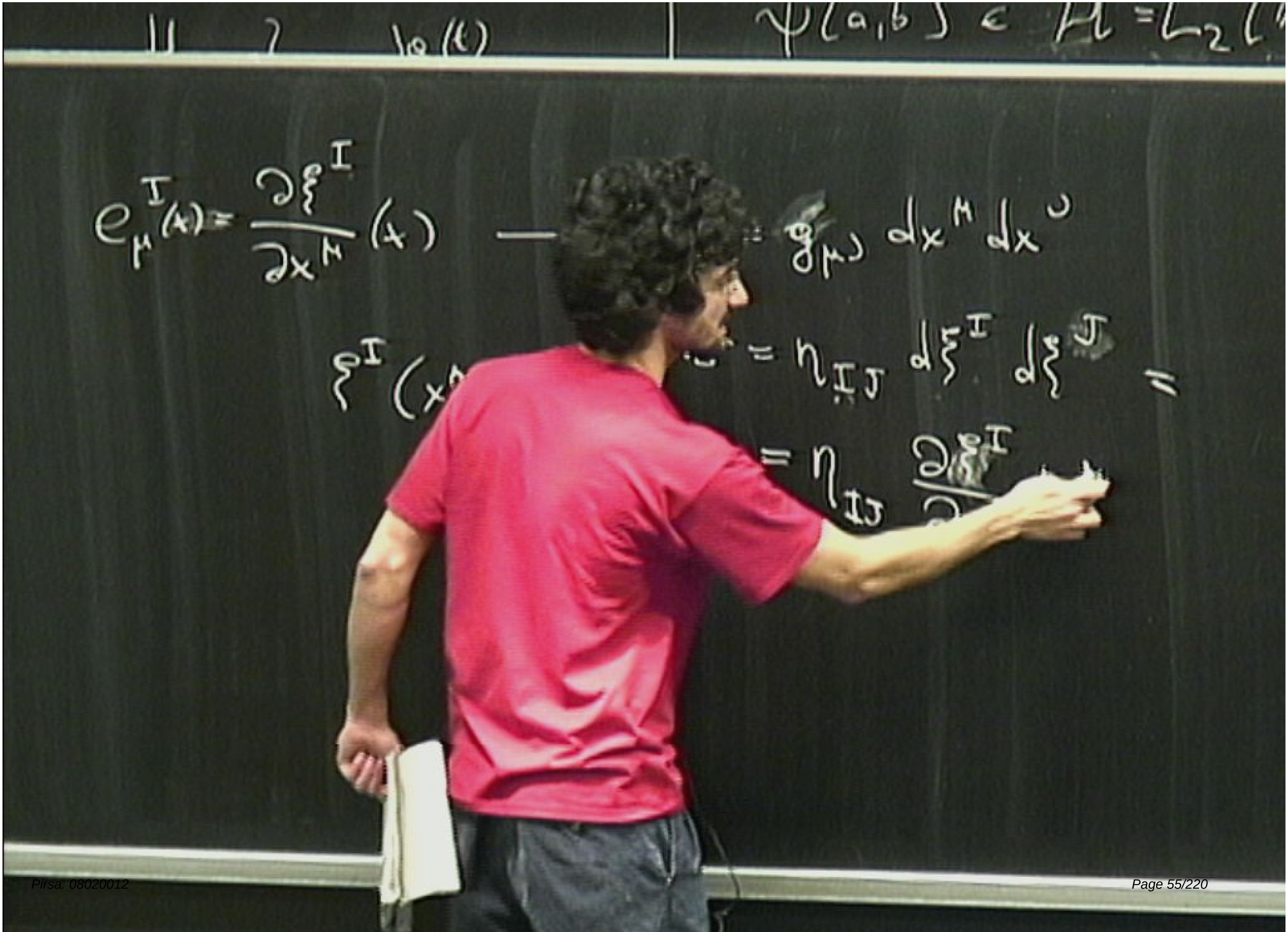
"$\delta(\mathcal{H}^a)$"

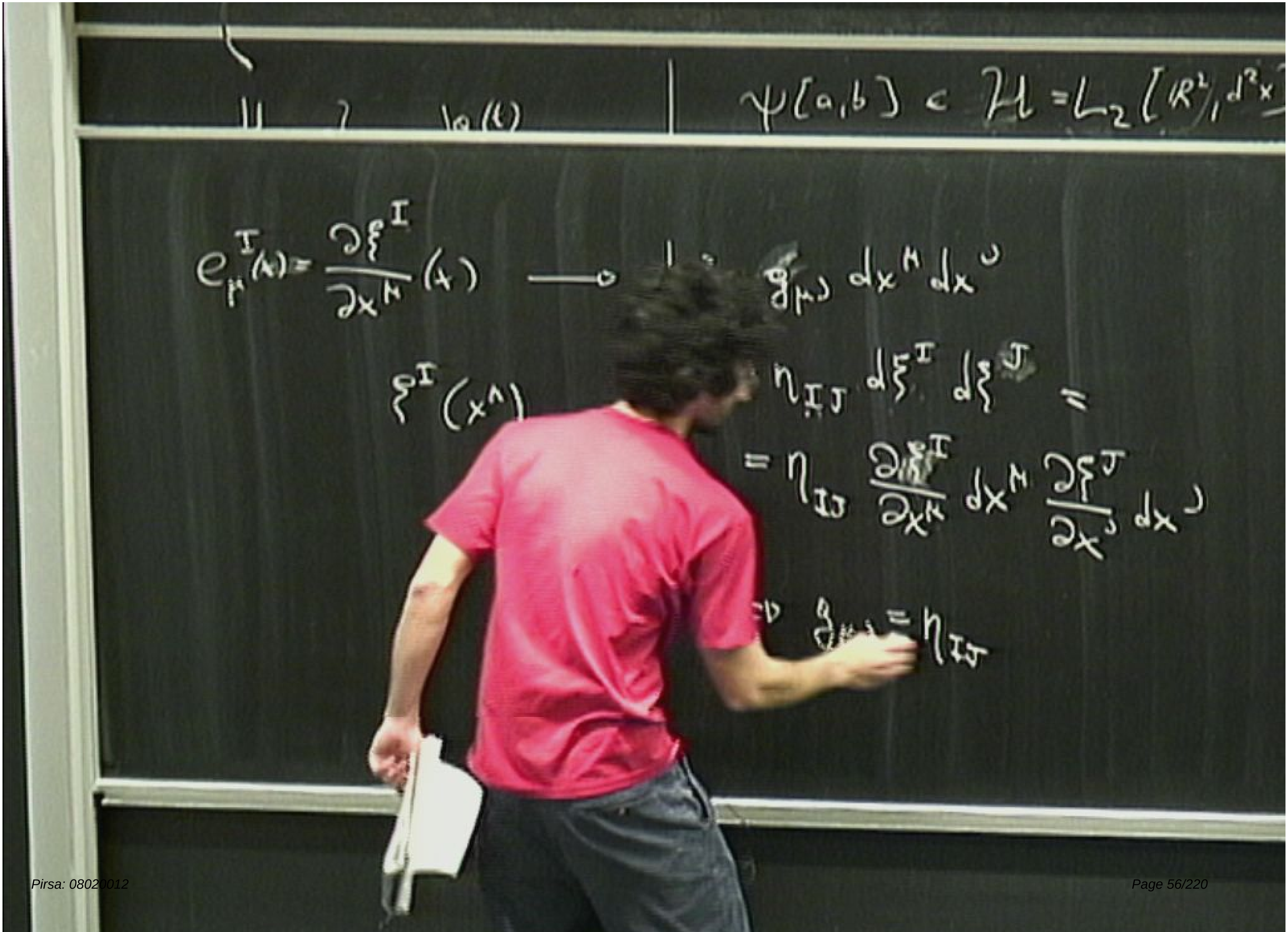
$\downarrow$

$\mathcal{H}_{Diff} \xrightarrow{\text{"}\delta(\mathcal{H})\text{"}} \mathcal{H}_{Phys}$



$\psi(a,b) \in \mathcal{H} = L_2($
$e_\mu^I(x) = \frac{\partial \xi^I}{\partial}$
$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu$
$=$


$\psi[a,b] \in \mathcal{H} = L_2($
$e_\mu^I(x) = \frac{\partial \xi^I}{\partial x^\mu}(x)$
$g_{\mu\nu}\, dx^\mu dx^\nu$
$\xi^I(x$
$= \eta_{IJ}\, d\xi^I d\xi^J =$
$= \eta_{IJ} \frac{\partial \xi^I}{\partial}$



$\psi[a,b] \in \mathcal{H} = L_2(\mathbb{R}^2, d^2x)$
$e_\mu^I(x) = \frac{\partial \xi^I}{\partial x^\mu}(x) \longrightarrow$
$g_{\mu\nu}\, dx^\mu dx^\nu$
$\xi^I(x^\mu)$
$\eta_{IJ}\, d\xi^I d\xi^J =$
$= \eta_{IJ} \frac{\partial \xi^I}{\partial x^\mu} dx^\mu \frac{\partial \xi^J}{\partial x^\nu} dx^\nu$
$\Rightarrow g_{\mu\nu} = \eta_{IJ}$

$\psi[a,b] \in \mathcal{H} = L_2[\mathbb{R}^2, d^2x]$

$$e_\mu^I(x) = \frac{\partial \xi^I}{\partial x^\mu}(x) \longrightarrow ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu$$

tet

$$\xi^I(x^\mu)$$

$$ds^2 = \eta_{IJ}\, d\xi^I\, d\xi^J =$$

$$= \eta_{IJ} \frac{\partial \xi^I}{\partial x^\mu} dx^\mu \frac{\partial \xi^J}{\partial x^\nu} dx^\nu$$

$$d\xi^\mu = \nabla \xi \cdot \vec{dx}$$

$$\Rightarrow g_{\mu\nu} = \eta_{IJ}\, e_\mu^I e_\nu^J$$

$$\psi[a,b] \in \mathcal{H} = L_2(\mathbb{R}^2, d^2x)$$

$$e_\mu^I(x) = \frac{\partial \xi^I}{\partial x^\mu}(x) \longrightarrow ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu$$

tetrad

$$\xi^I(x^\mu)$$

$$ds^2 = \eta_{IJ}\, d\xi^I\, d\xi^J =$$

$$= \eta_{IJ} \frac{\partial \xi^I}{\partial x^\mu} dx^\mu \frac{\partial \xi^J}{\partial x^\nu} dx^\nu$$

$$d\xi = \nabla \xi \cdot \vec{dx}$$

$$\Rightarrow g_{\mu\nu} = \eta_{IJ}\, e^I_\mu e^J_\nu$$

$\psi[a,b] \in \mathcal{H} = L_2[\mathbb{R}^2, d^2x]$

$I = 0,1,2,3 \; ; \; \mu = 0,1,2,3$

$$e_\mu^I(x) = \frac{\partial \xi^I}{\partial x^\mu}(x) \longrightarrow ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu$$

grad $\quad \xi^I(x^\mu)$

$$ds^2 = \eta_{IJ}\, d\xi^I\, d\xi^J =$$

$$= \eta_{IJ} \frac{\partial \xi^I}{\partial x^\mu} dx^\mu \frac{\partial \xi^J}{\partial x^\nu} dx^\nu$$

$$d\xi(t) = \nabla \xi \cdot \vec{dx}$$

$$\Rightarrow g_{\mu\nu} = \eta_{IJ}\, e_\mu^I\, e_\nu^J$$

$\psi[a,b] \in \mathcal{H} = L_2(\mathbb{R}^2, d^2x)$

$g_{\mu\nu} = \eta$

$I = 0,1,2,3 \; ; \; \mu = 0,1,2,3$

$$e_\mu^I(x) = \frac{\partial \xi^I}{\partial x^\mu}(x) \longrightarrow ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu$$

<u>tetrad</u>

$\xi^I(x^\mu)$

$$ds^2 = \eta_{IJ}\, d\xi^I\, d\xi^J =$$

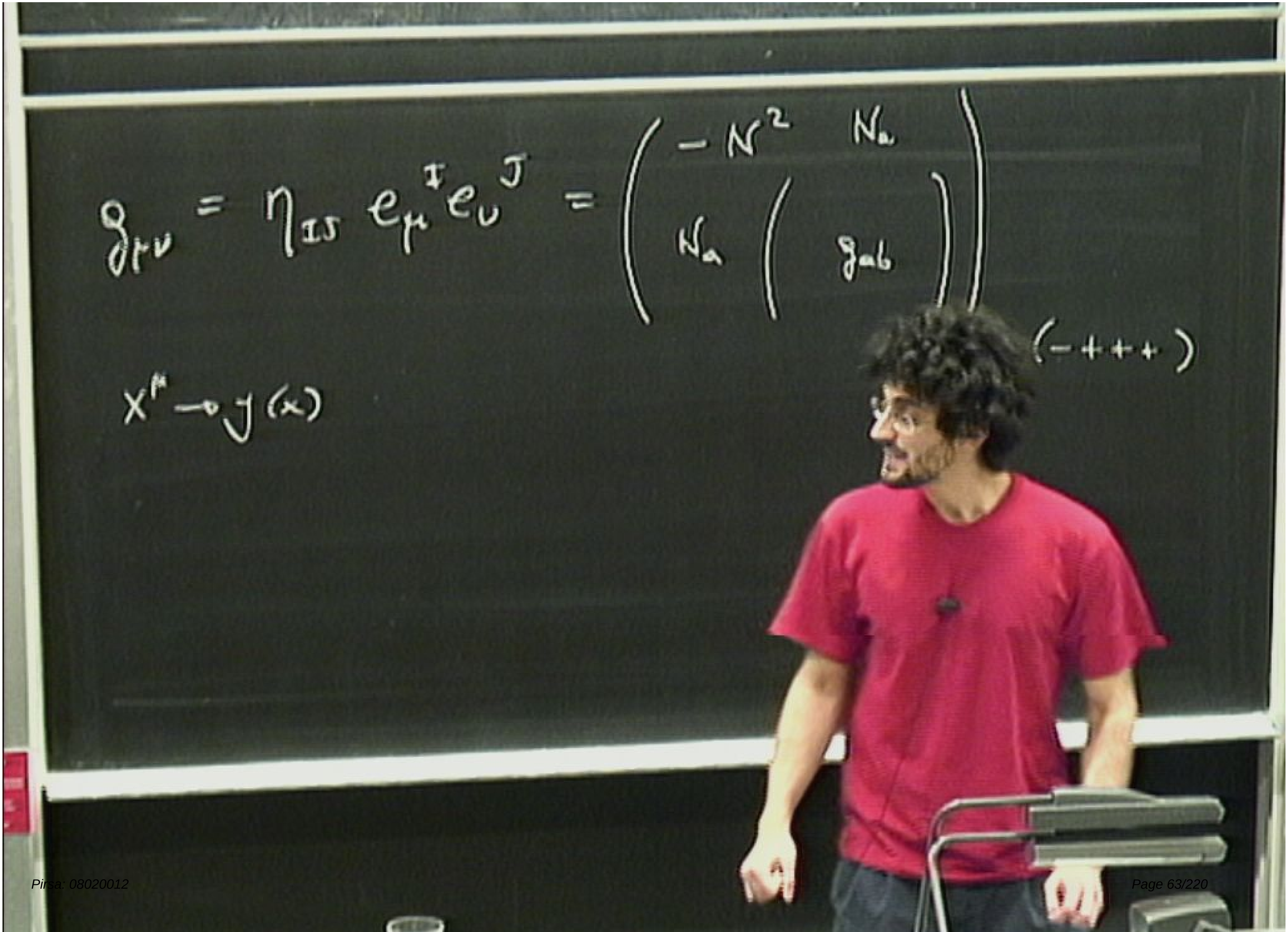
$$= \eta_{IJ} \frac{\partial \xi^I}{\partial x^\mu} dx^\mu \frac{\partial \xi^J}{\partial x^\nu} dx^\nu$$

$d\xi = \nabla \xi \cdot \vec{dx}$

$$\Rightarrow \boxed{g_{\mu\nu} = \eta_{IJ}\, e_\mu^I\, e_\nu^J}$$

$$g_{\mu\nu} = \eta_{IJ}\, e_\mu{}^I e_\nu{}^J = \begin{pmatrix} -N^2 & N_a \\ N_a & \begin{pmatrix} & g_{ab} & \end{pmatrix} \end{pmatrix}$$

$(-+++)$


$g_{\mu\nu} = \eta_{IJ} e_\mu{}^I e_\nu{}^J = \begin{pmatrix} -N^2 & N_a \\ N_a & \begin{pmatrix} g_{ab} \end{pmatrix} \end{pmatrix}$
$(-+++)$
$x^\mu \to y(x)$

$$g_{\mu\nu} = \eta_{IJ}\, e_\mu{}^I e_\nu{}^J = \begin{pmatrix} -N^2 & N_a \\ N_a & \begin{pmatrix} g_{ab} \end{pmatrix} \end{pmatrix}$$

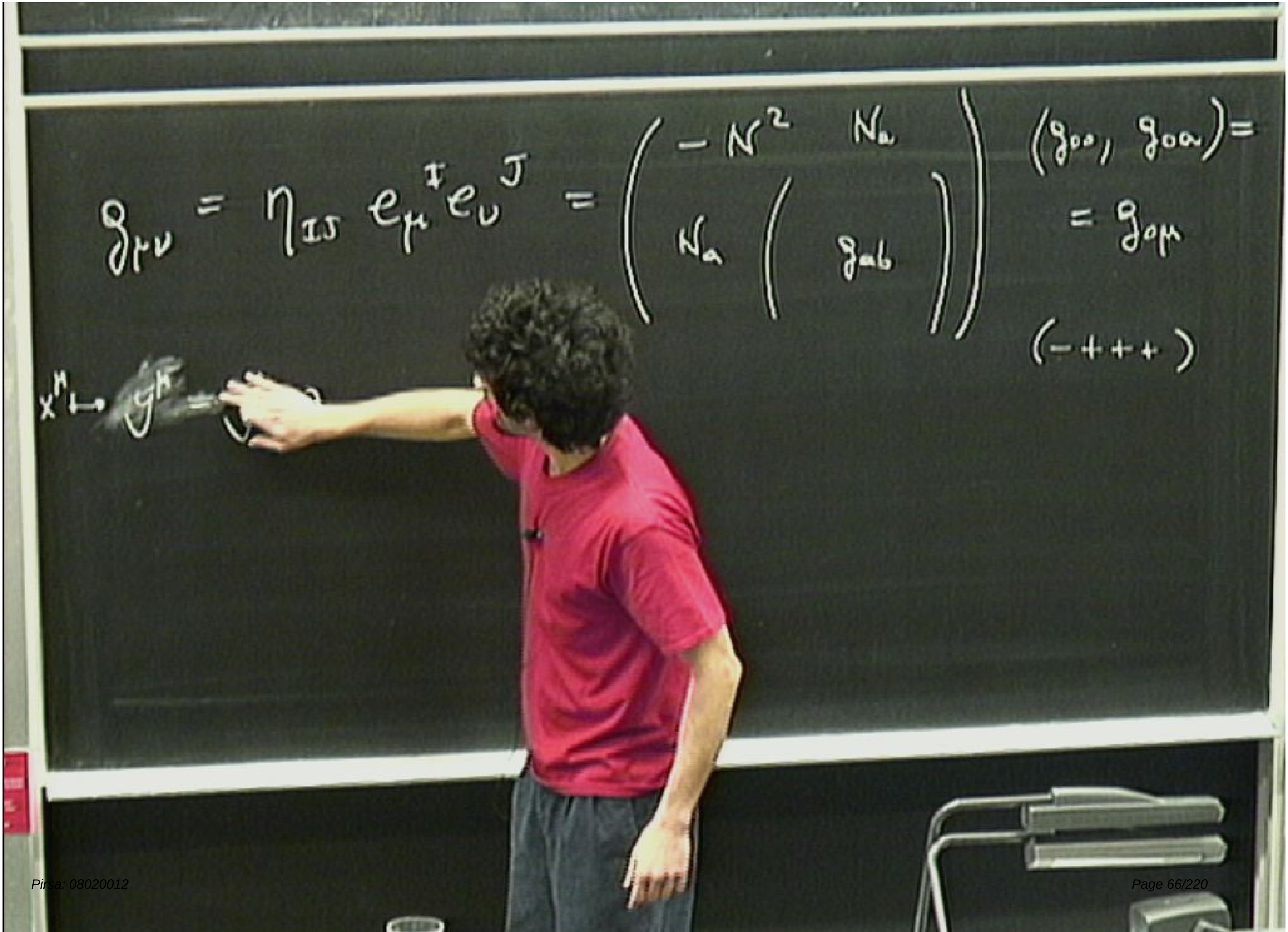
$(-+++)$

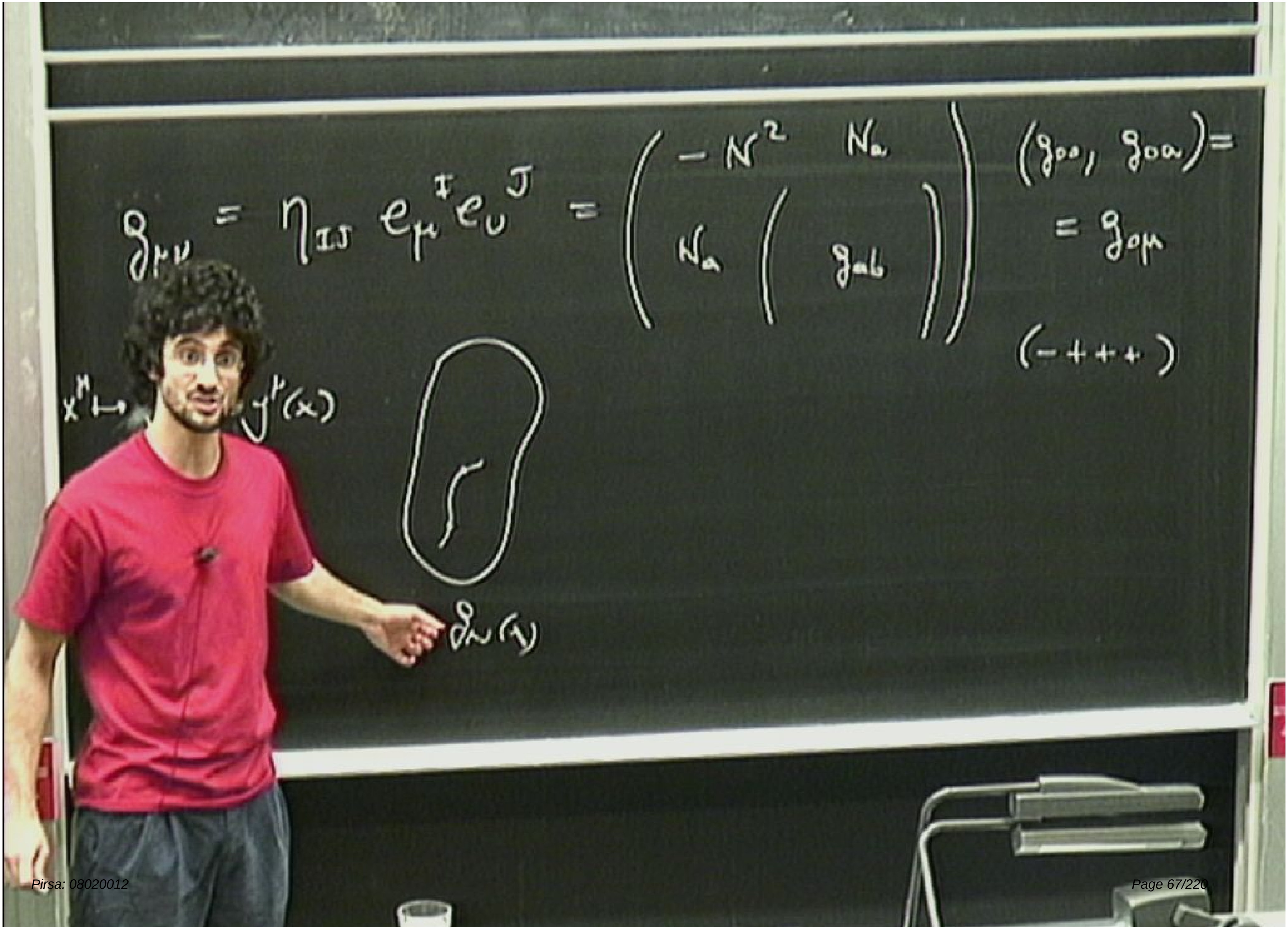
$$x^\mu \mapsto y^\mu = y^\mu(x)$$

$$g_{\mu\nu} = \eta_{IJ}\, e_\mu{}^I e_\nu{}^J = \begin{pmatrix} -N^2 & N_a \\ N_a & \begin{pmatrix} g_{ab} \end{pmatrix} \end{pmatrix}$$

$(-+++)$

$x^\mu \mapsto y^\mu = y^\mu(x)$


$g_{\mu\nu} = \eta_{IJ}\, e_\mu{}^I e_\nu{}^J = \begin{pmatrix} -N^2 & N_a \\ N_a & \begin{pmatrix} g_{ab} \end{pmatrix} \end{pmatrix}$
$(g_{00}, g_{0a}) =$
$= g_{0\mu}$
$(-+++)$
$x^\mu \mapsto y^\mu$


$g_{\mu\nu} = \eta_{IJ}\, e_\mu{}^I e_\nu{}^J = \begin{pmatrix} -N^2 & N_a \\ N_a & (g_{ab}) \end{pmatrix}$
$(g_{00}, g_{0a}) =$
$= g_{0\mu}$
$(- + + +)$
$x^\mu \mapsto y^\mu(x)$
$g_{\mu\nu}(y)$

$$g_{\mu\nu} = \eta_{IJ}\, e_\mu{}^I e_\nu{}^J = \begin{pmatrix} -N^2 & N_a \\ N_a & \left(g_{ab} \right) \end{pmatrix}$$

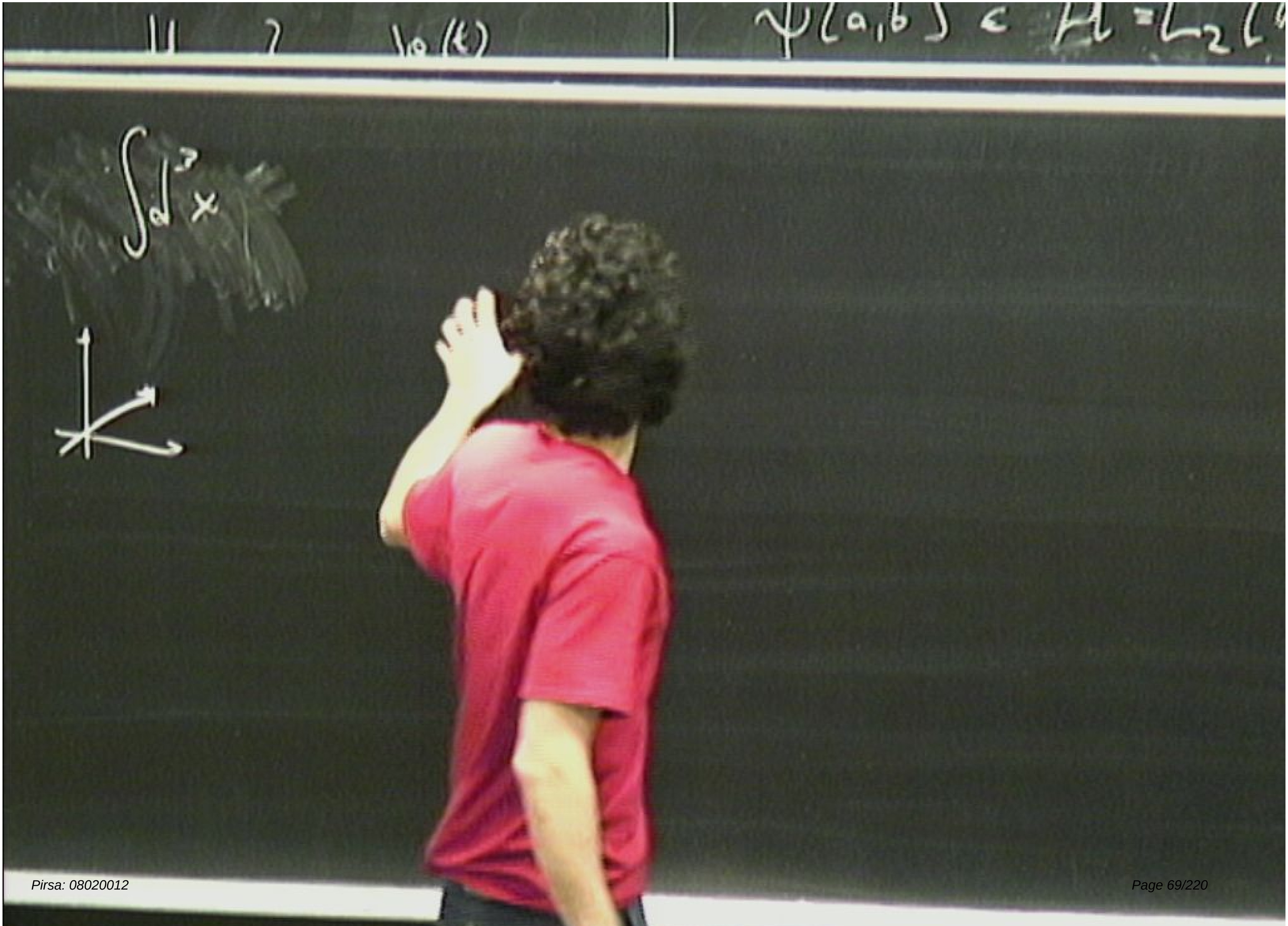
$(g_{00}, g_{0a}) =$

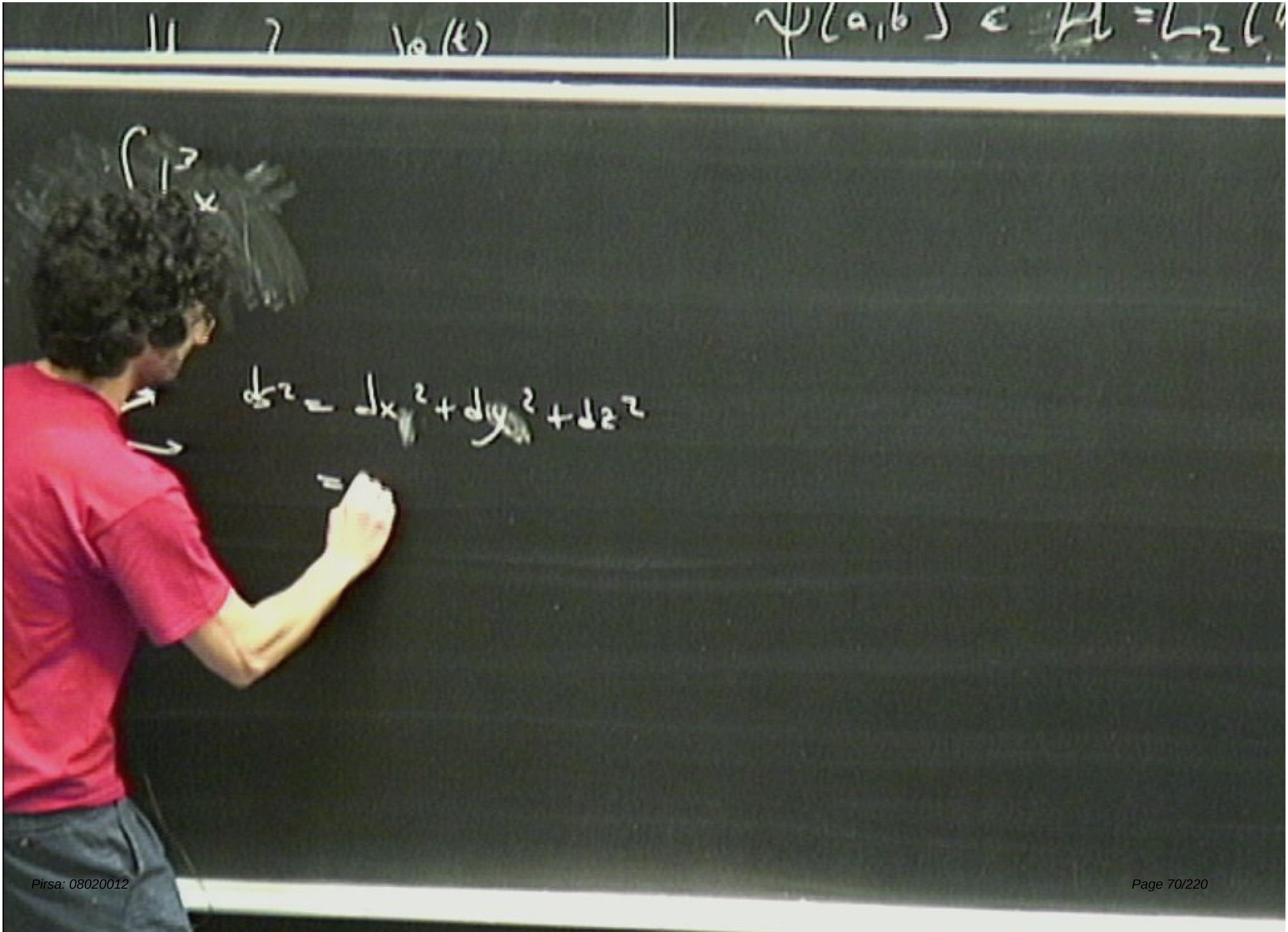
$= g_{0\mu}$

$(-+++)$

$x^\mu \mapsto \cancel{y^\mu} = y^\mu(x)$

$g_{\mu\nu}(y)$


$\psi(a,b) \in \mathcal{H} = L_2($
$\int d^3x$


ψ(a,b) ∈ H = L_2(
ds^2 = dx^2 + dy^2 + dz^2
=

$\psi(a,b) \in \mathcal{H} = L_2($

$\int d^3x$

$$ds^2 = dx^2 + dy^2 + dz^2$$

$$= g_{IJ}\, dx^I\, dx^J$$

$$g_{IJ} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\psi(a,b) \in \mathcal{H} = L_2($

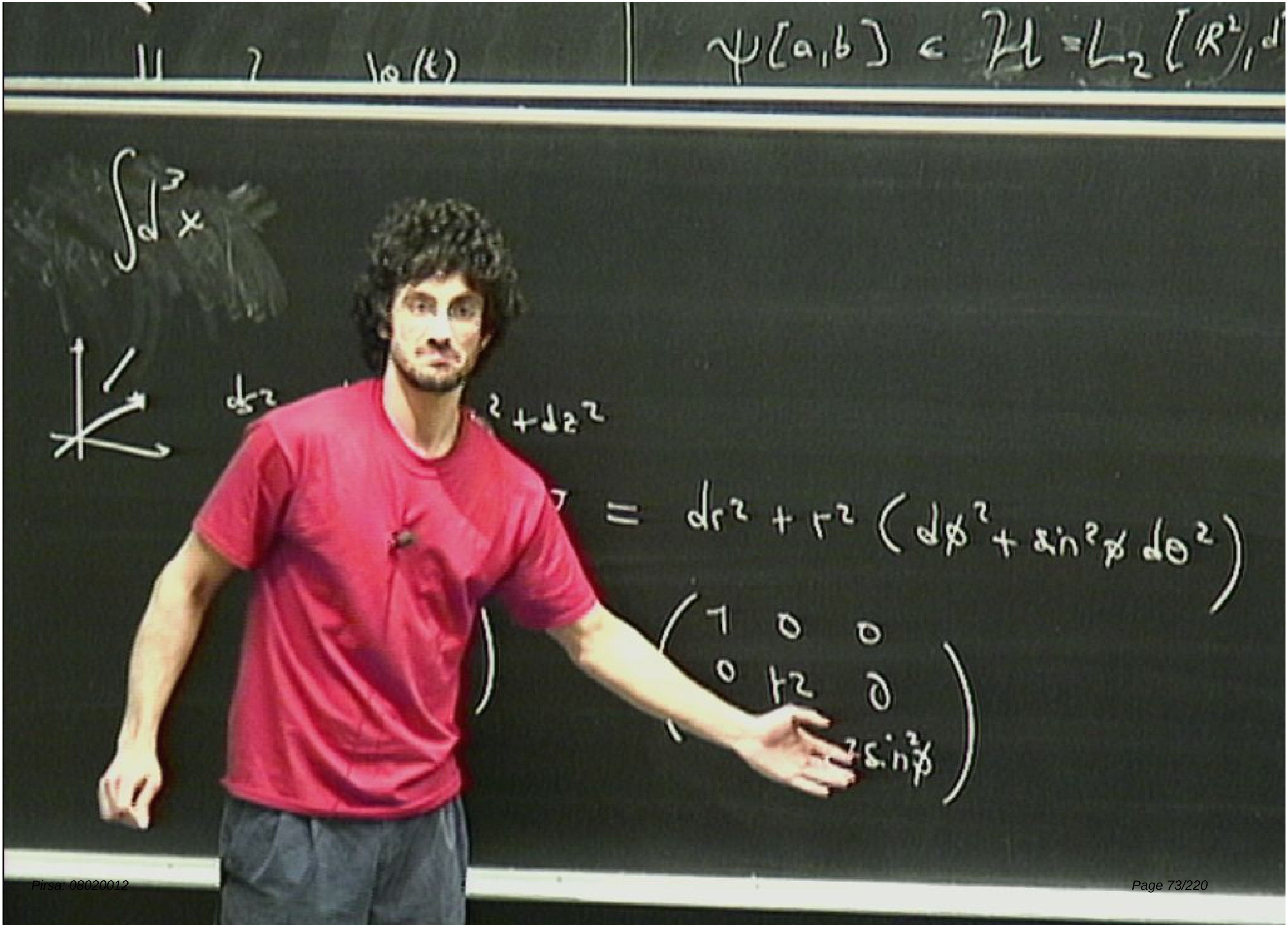
$$\int d^3x$$

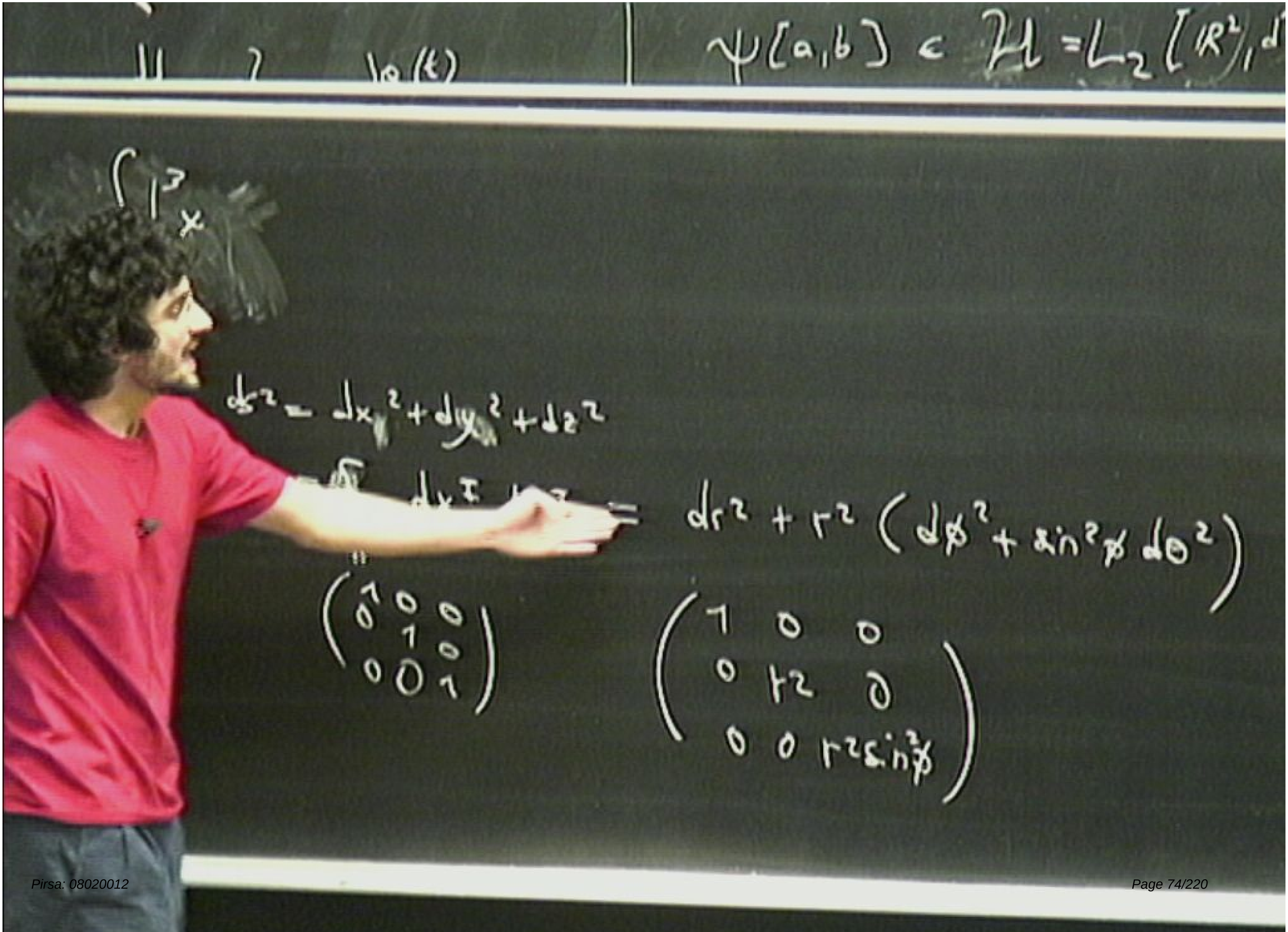
$$ds^2 = dx^2 + dy^2 + dz^2$$

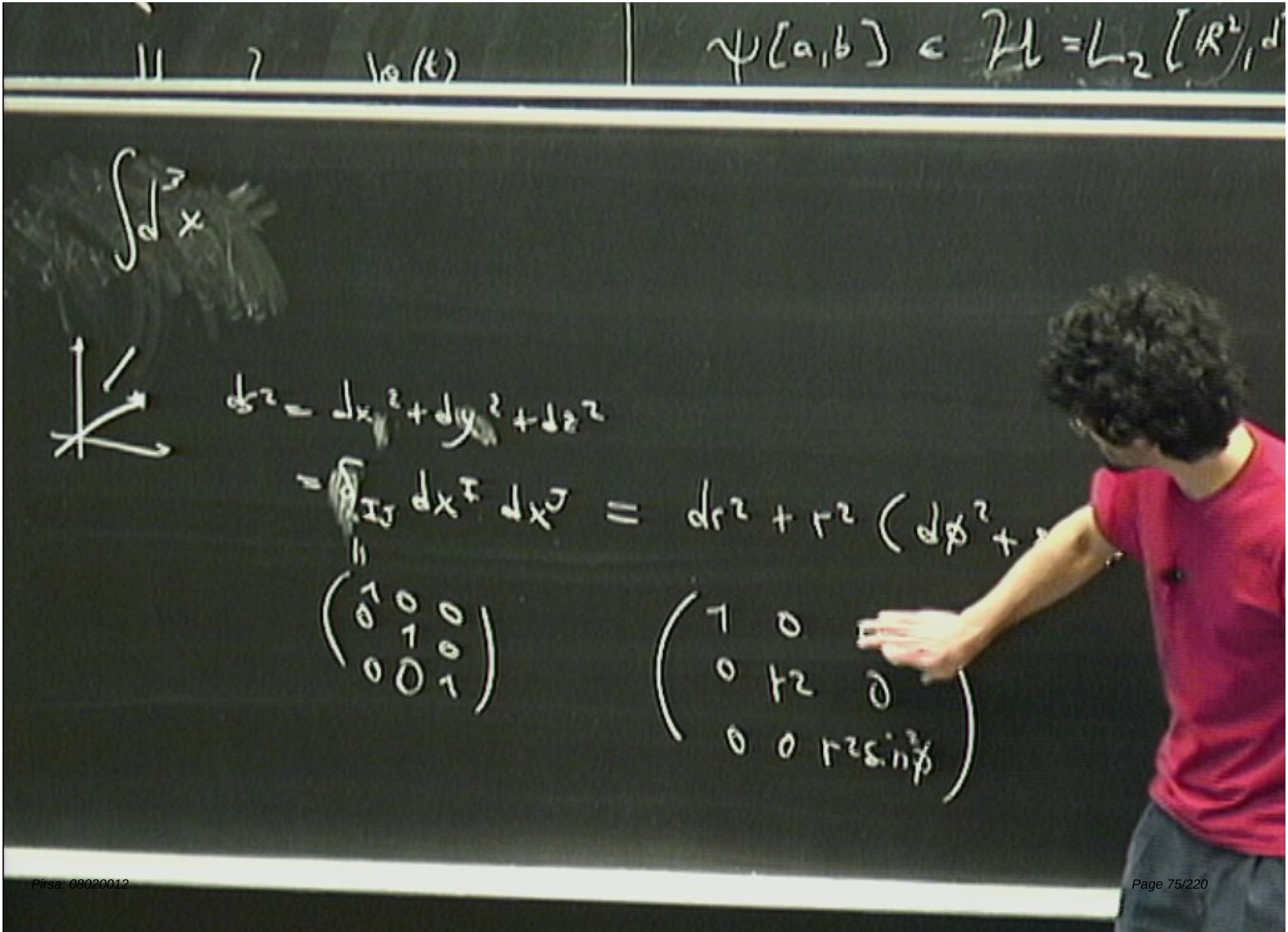
$$= g_{IJ}\, dx^I\, dx^J$$

$$\| $$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$


ψ[a,b] ∈ H = L₂[(R²), d
∫d³x
+ dz²
= dr² + r² (dφ² + sin²φ dθ²)


ψ[a,b] ∈ ℋ = L₂[ℝ²,
ds² = dx² + dy² + dz²
= dr² + r² (dφ² + sin²φ dθ²)
(1 0 0 / 0 1 0 / 0 0 1)
(1 0 0 / 0 r² 0 / 0 0 r² sin²φ)



ψ[a,b] ∈ H = L₂[R², d
∫d³x
ds² = dx² + dy² + dz²
= g_IJ dx^I dx^J = dr² + r²(dφ² +
1 0 0
0 1 0
0 0 1
1 0
0 r² 0
0 0 r²sin²φ

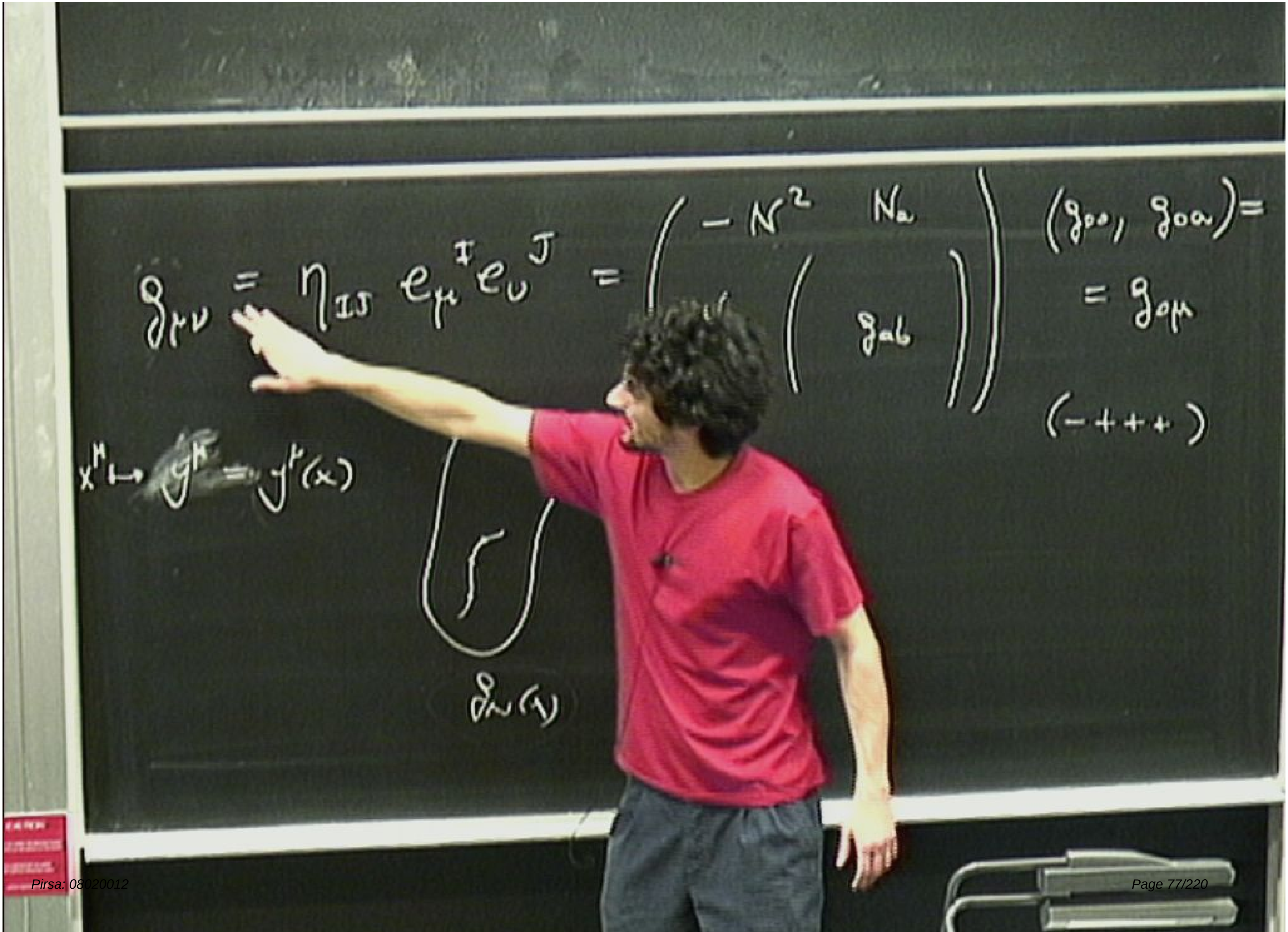
$\psi[a,b] \in \mathcal{H} = L_2(\mathbb{R}^2, d$

$\int d^3x$

$$ds^2 = dx^2 + dy^2 + dz^2$$

$$= g_{IJ}\, dx^I dx^J = dr^2 + r^2 (d\phi^2 + \sin^2\phi\, d\theta^2)$$

$$g_{IJ} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \begin{pmatrix} 1 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & r^2\sin^2\phi \end{pmatrix}$$



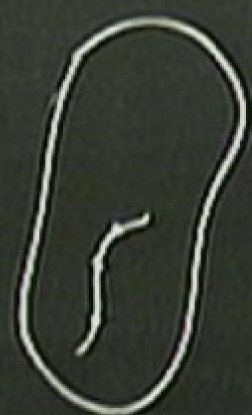
$g_{\mu\nu} = \eta_{IJ}\, e_\mu{}^I e_\nu{}^J = \begin{pmatrix} -N^2 & N_a \\ & g_{ab} \end{pmatrix}$
$(g_{00}, g_{0a}) = g_{0\mu}$
$(-+++)$
$x^\mu \mapsto y^\mu = y^\mu(x)$
$g_{\mu\nu}(y)$

$$g_{\mu\nu} = \eta_{IJ}\, e_\mu{}^I e_\nu{}^J = \begin{pmatrix} -N^2 & N_a \\ N_a & \begin{pmatrix} & \\ & g_{ab} \end{pmatrix} \end{pmatrix}$$

$$(g_{00}, g_{0a}) = \\ = g_{0\mu}$$

$$(-+++)$$

$$x^\mu \mapsto y^\mu = y^\mu(x)$$



$g_{\mu\nu}(y)$

$$g_{\mu\nu} = \eta_{IJ}\, e_\mu{}^I e_\nu{}^J = \begin{pmatrix} -N^2 & N_a \\ N_a & \begin{pmatrix} & g_{ab} & \end{pmatrix} \end{pmatrix}$$

$(g_{00}, g_{0a}) =$

$= g_{0\mu}$

$(-+++)$

$x^\mu \mapsto y^\mu = y^\mu(x)$

$$= \begin{pmatrix} -1 & 0 \; 0 \; 0 \\ 0 & \\ 0 & \begin{pmatrix} & g_{ab} & \end{pmatrix} \\ 0 & \end{pmatrix}$$

$g_{\mu\nu}(y)$

$$g_{\mu\nu} = \eta_{IJ}\, e_\mu{}^I e_\nu{}^J = \begin{pmatrix} -N^2 & N_a \\ N_a & \begin{pmatrix} g_{ab} \end{pmatrix} \end{pmatrix}$$

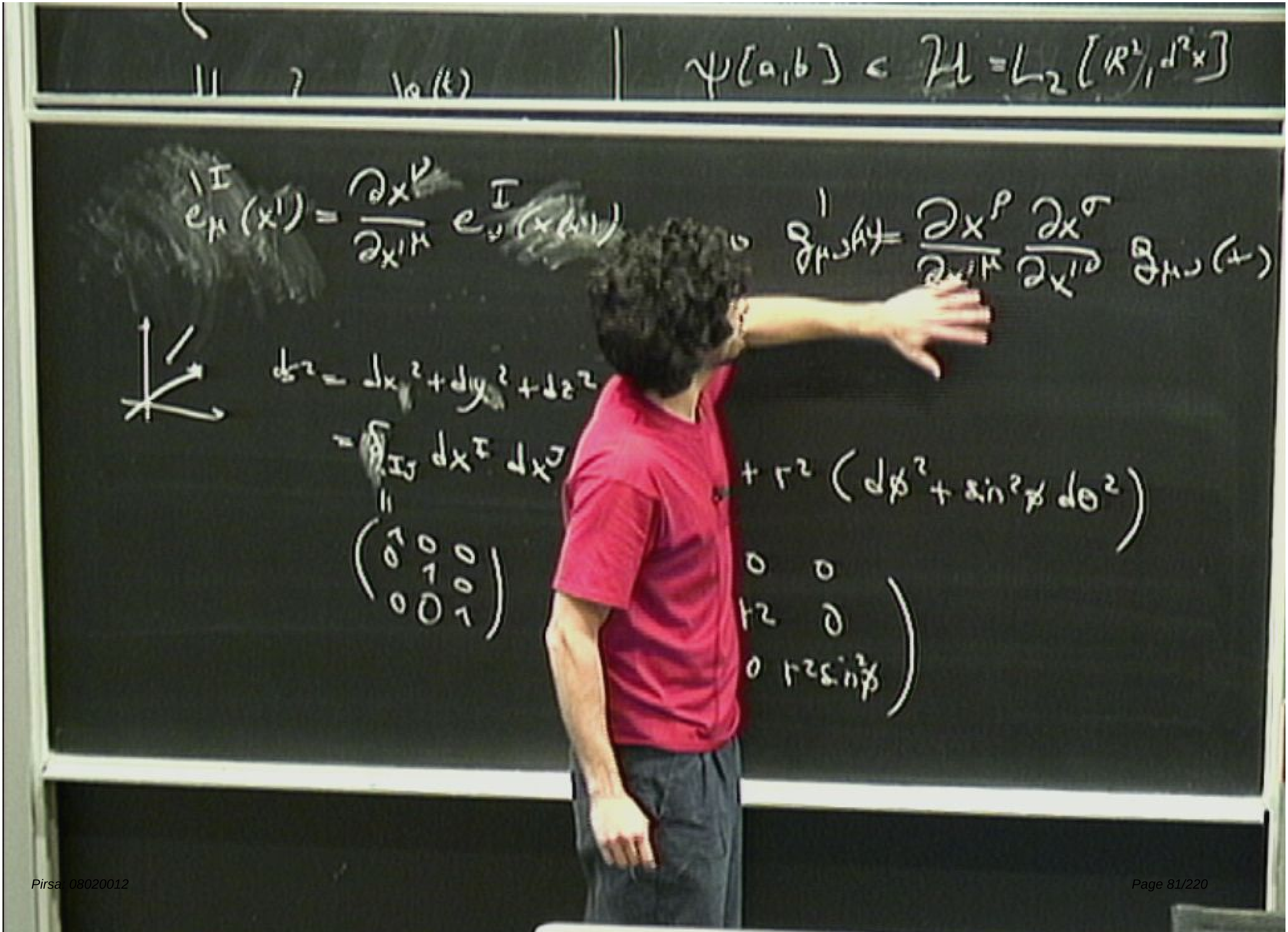
$(g_{00}, g_{0a}) =$
$= g_{0\mu}$

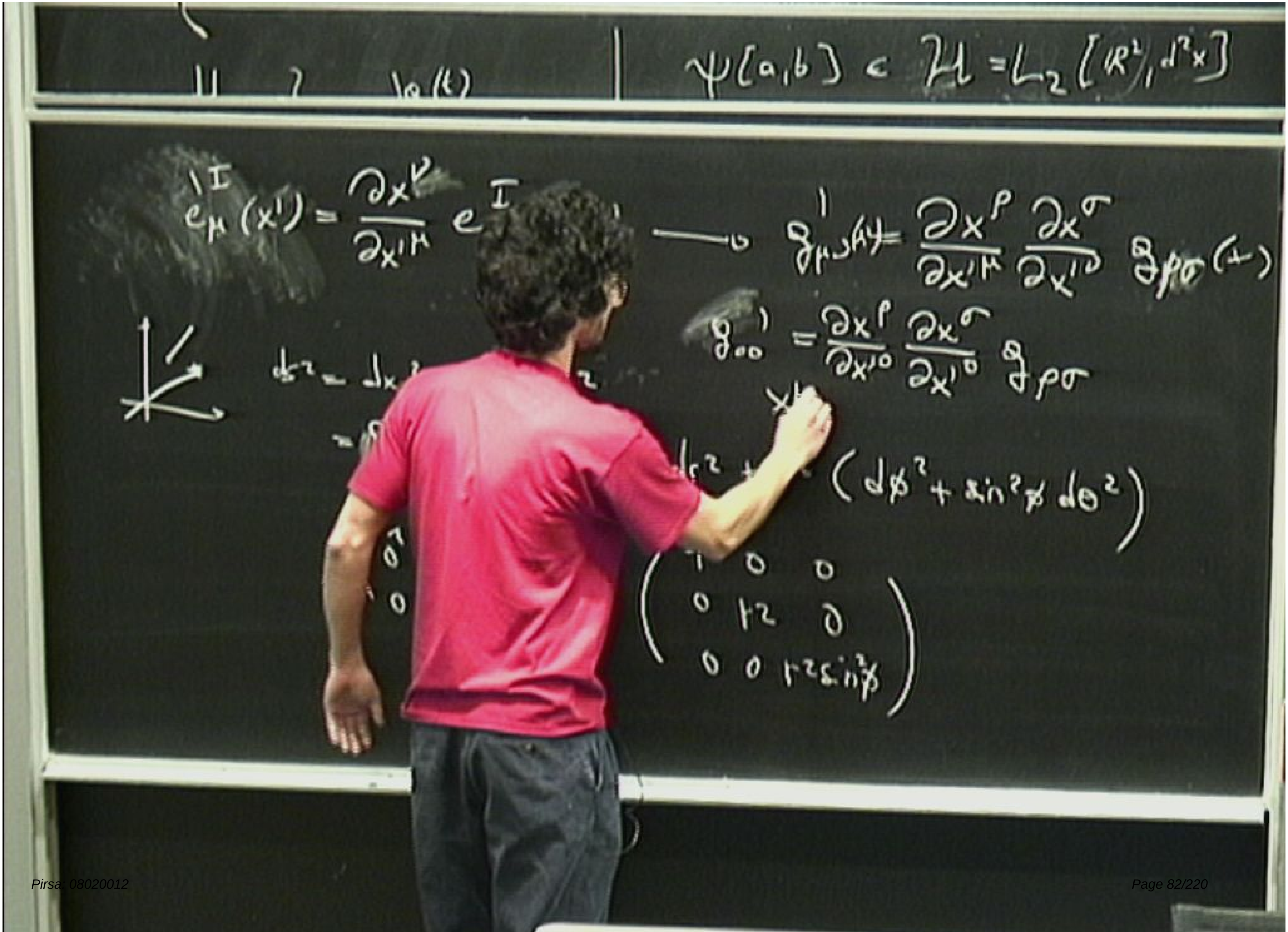
$(-+++)$

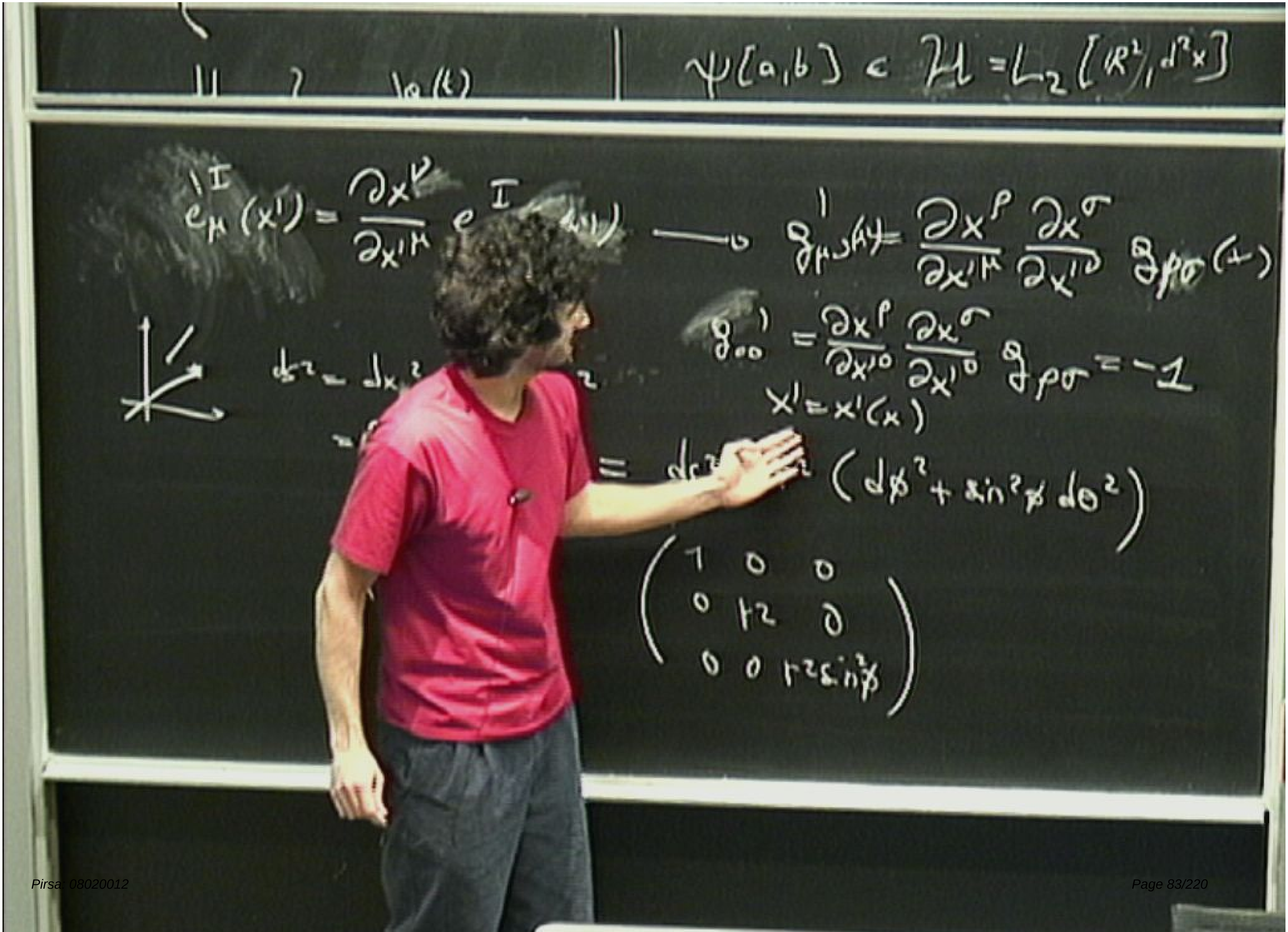
$x^\mu \mapsto y^\mu = y^\mu(x)$

$g_{\mu\nu}(y)$

$$= \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & & & \\ 0 & & g_{ab} & \\ 0 & & & \end{pmatrix}$$


ψ[a,b] ∈ H = L2(R², d²x)
e'^I_μ(x') = ∂x^ρ/∂x'^μ e^I_ν(x(x'))
g'_μν(x') = ∂x^ρ/∂x'^μ ∂x^σ/∂x'^ν g_μν(x)
ds² = dx² + dy² + dz²
= δ_IJ dx^I dx^J
+ r² (dφ² + sin²φ dθ²)


ψ[a,b] ∈ H = L_2[R^2, d^2x]
e'_μ^I(x') = ∂x^ρ/∂x'^μ e^I
g'_μν(x') = ∂x^ρ/∂x'^μ ∂x^σ/∂x'^ν g_ρσ(x)
g'_00 = ∂x^ρ/∂x'^0 ∂x^σ/∂x'^0 g_ρσ
(dφ^2 + sin^2φ dθ^2)
0 r^2 0
0 0 r^2 sin^2φ


$\psi[a,b] \in \mathcal{H} = L_2[\mathbb{R}^2, d^2x]$
$e'^{I}_{\mu}(x') = \frac{\partial x^{\rho}}{\partial x'^{\mu}} e^{I}$
$g'_{\mu\nu}(x') = \frac{\partial x^{\rho}}{\partial x'^{\mu}} \frac{\partial x^{\sigma}}{\partial x'^{\nu}} g_{\rho\sigma}(x)$
$g'_{00} = \frac{\partial x^{\rho}}{\partial x'^{0}} \frac{\partial x^{\sigma}}{\partial x'^{0}} g_{\rho\sigma} = -1$
$x' = x'(x)$
$(d\phi^2 + \sin^2\phi \, d\theta^2)$
$\begin{pmatrix} 1 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & r^2\sin^2\phi \end{pmatrix}$

$\| \quad ? \quad |a(t)\rangle$

$\psi[a,b] \in \mathcal{H} = L_2[\mathbb{R}^2, d^2x]$

$$e'^{I}_{\mu}(x') = \frac{\partial x^\nu}{\partial x'^\mu} e^{I}_{\nu}(x(x')) \longrightarrow g'_{\mu\nu}(x') = \frac{\partial x^\rho}{\partial x'^\mu}\frac{\partial x^\sigma}{\partial x'^\nu} g_{\rho\sigma}(x)$$

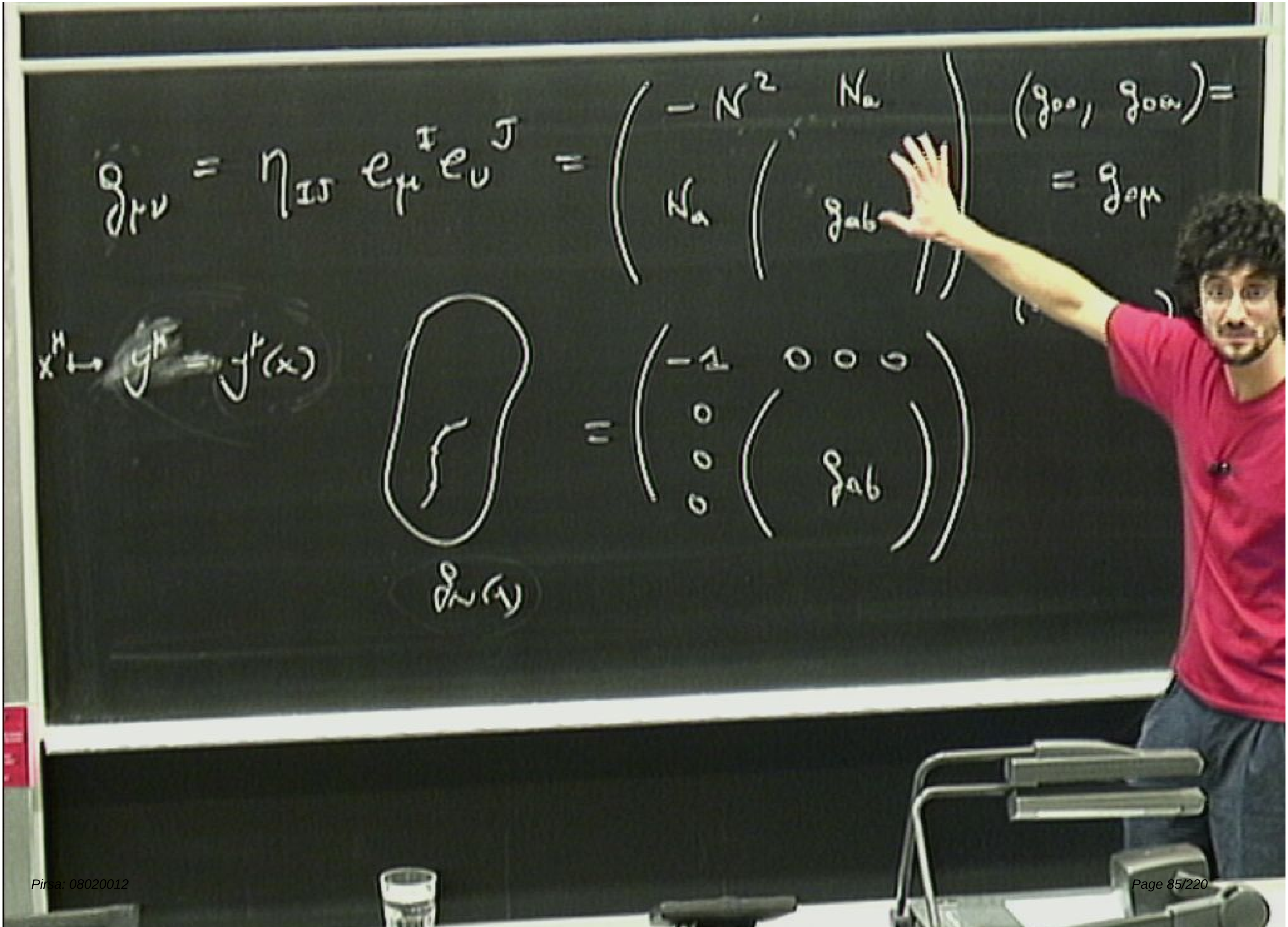
$$g'_{00} = \frac{\partial x^\rho}{\partial x'^0}\frac{\partial x^\sigma}{\partial x'^0} g_{\rho\sigma} = -1$$

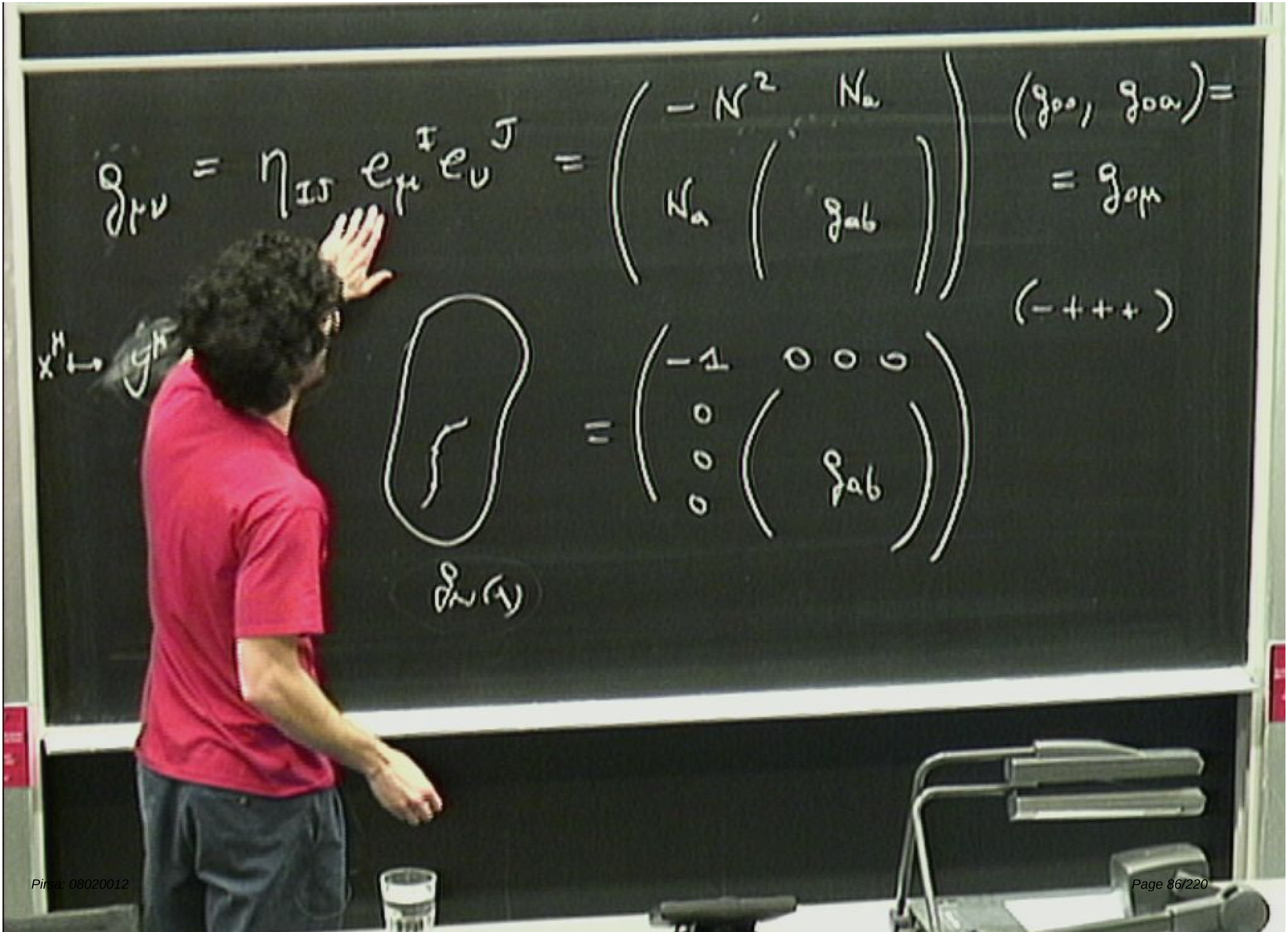
$$x' = x'(x)$$

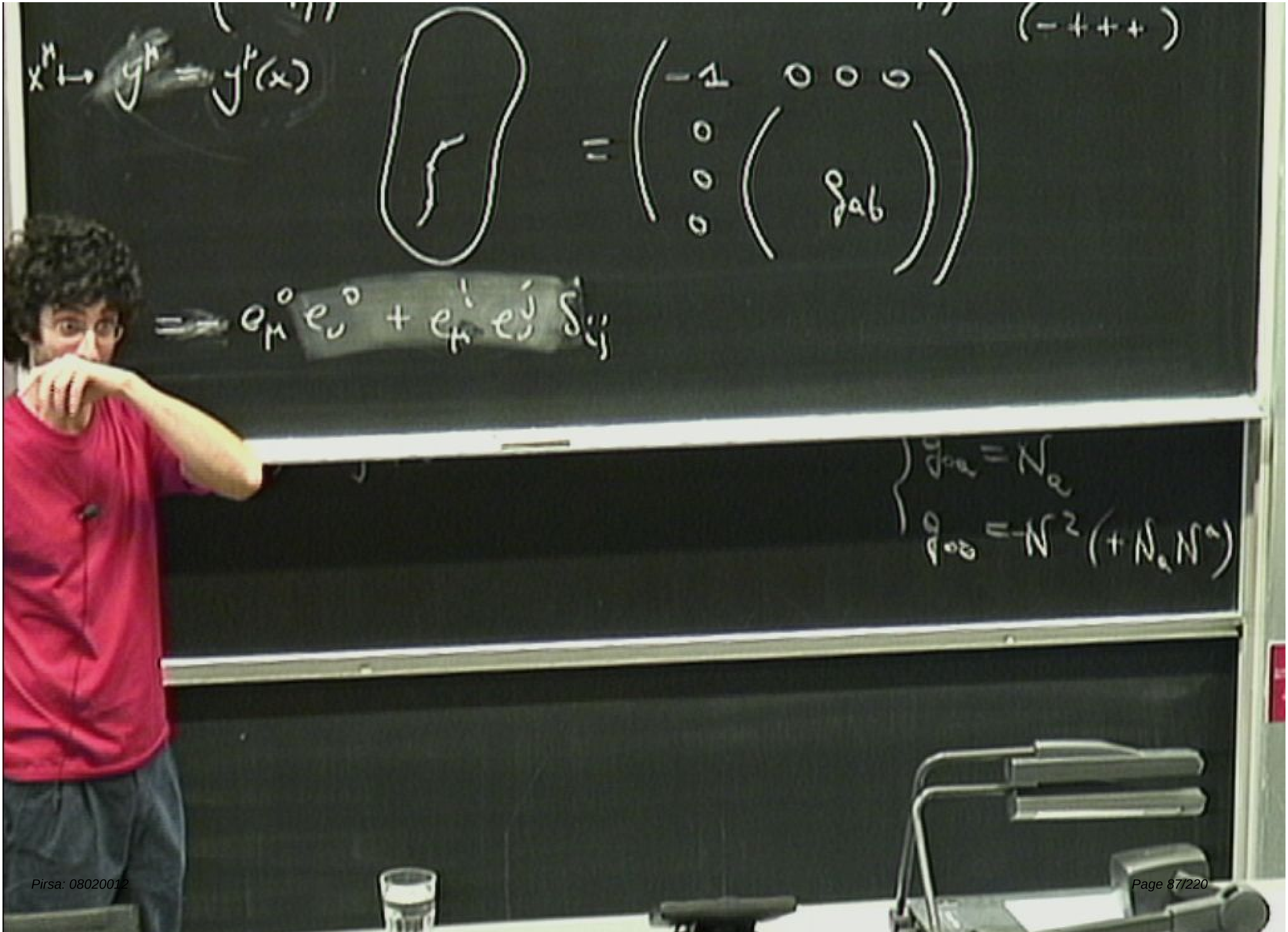
$$ds^2 = dx^2 + dy^2 + dz^2$$

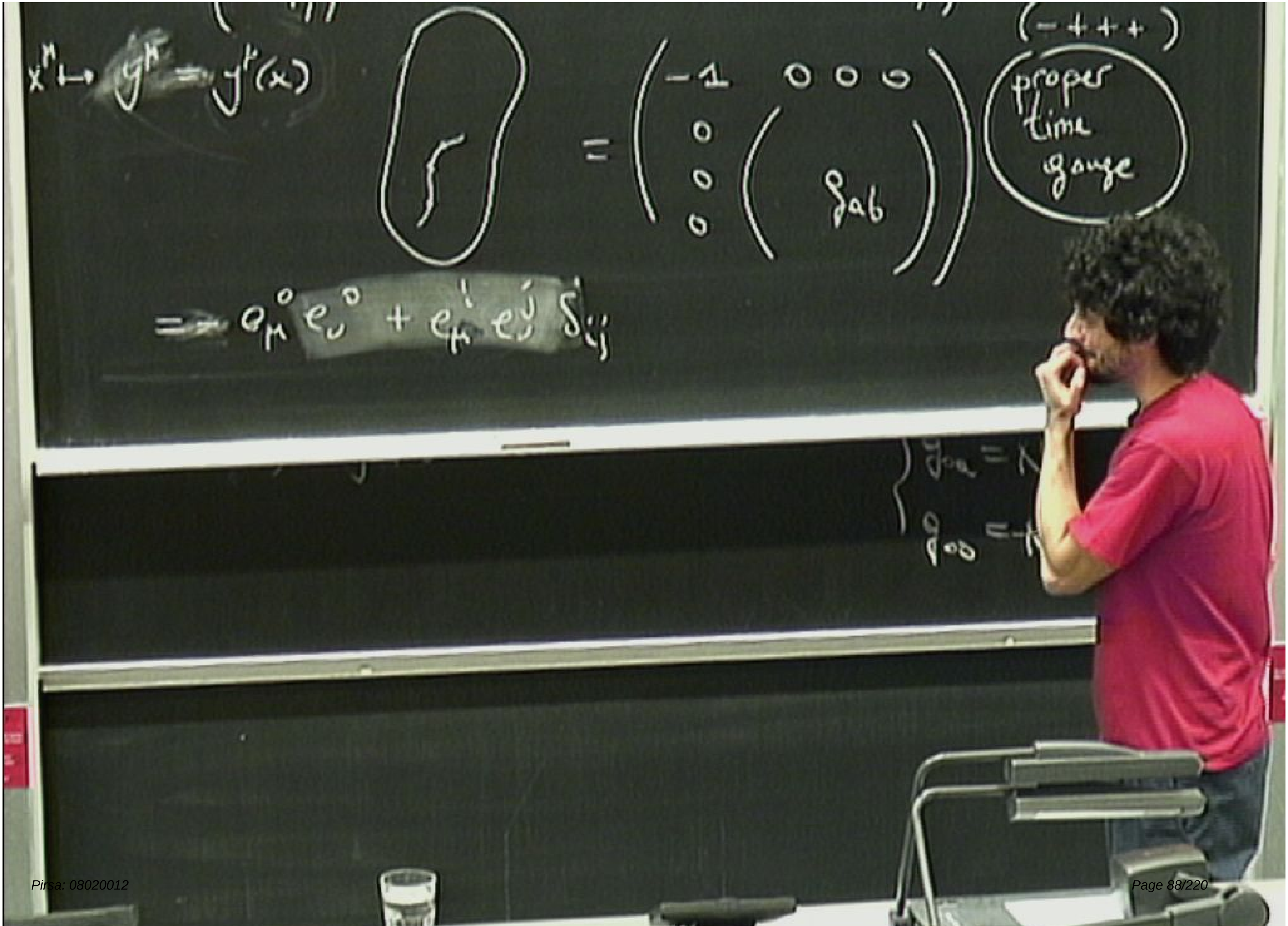
$$= \delta_{IJ}\, dx^I dx^J = dr^2 + r^2\left(d\phi^2 + \sin^2\phi\, d\theta^2\right)$$

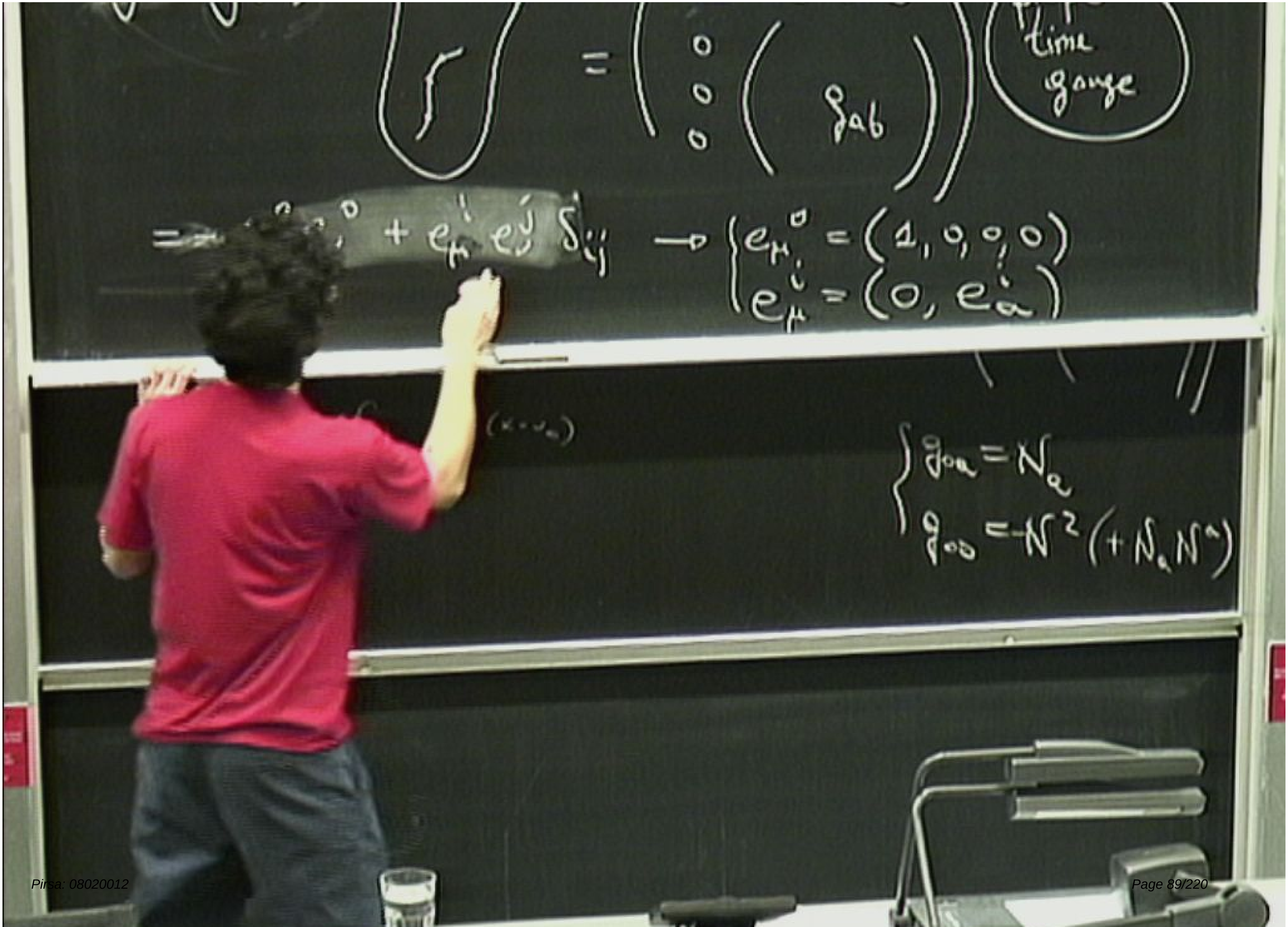
$$\delta_{IJ} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \begin{pmatrix} 1 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & r^2\sin^2\phi \end{pmatrix}$$

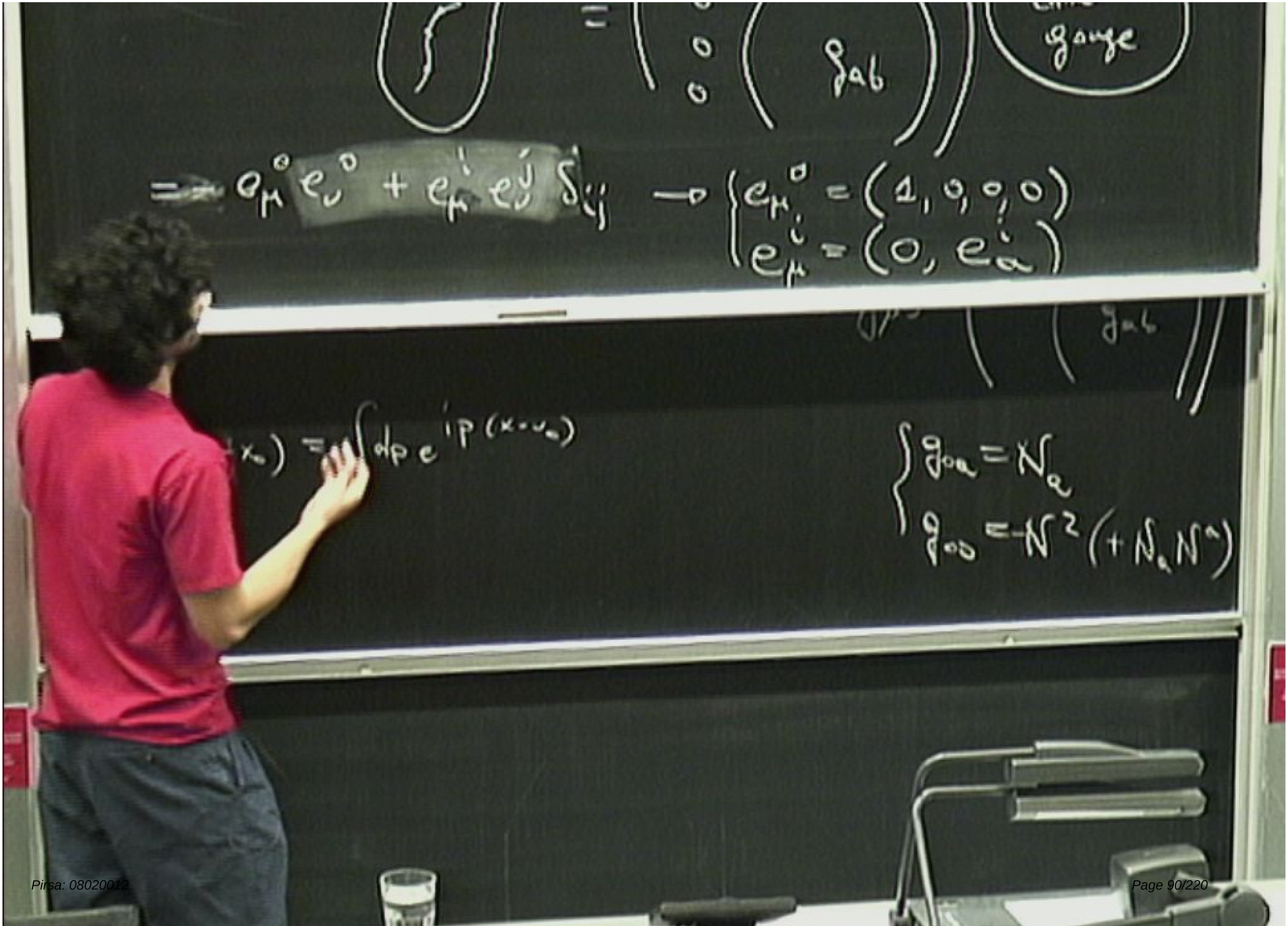

$g_{\mu\nu} = \eta_{IJ}\, e_\mu{}^I e_\nu{}^J = \begin{pmatrix} -N^2 & N_a \\ N_a & \begin{pmatrix} g_{ab} \end{pmatrix} \end{pmatrix}$
$(g_{00}, g_{0a}) =$
$= g_{0\mu}$
$x^\mu \mapsto y^\mu = y^\mu(x)$
$= \begin{pmatrix} -1 & 0\ 0\ 0 \\ 0 & \\ 0 & \begin{pmatrix} g_{ab} \end{pmatrix} \\ 0 & \end{pmatrix}$
$g_{\mu\nu}(y)$

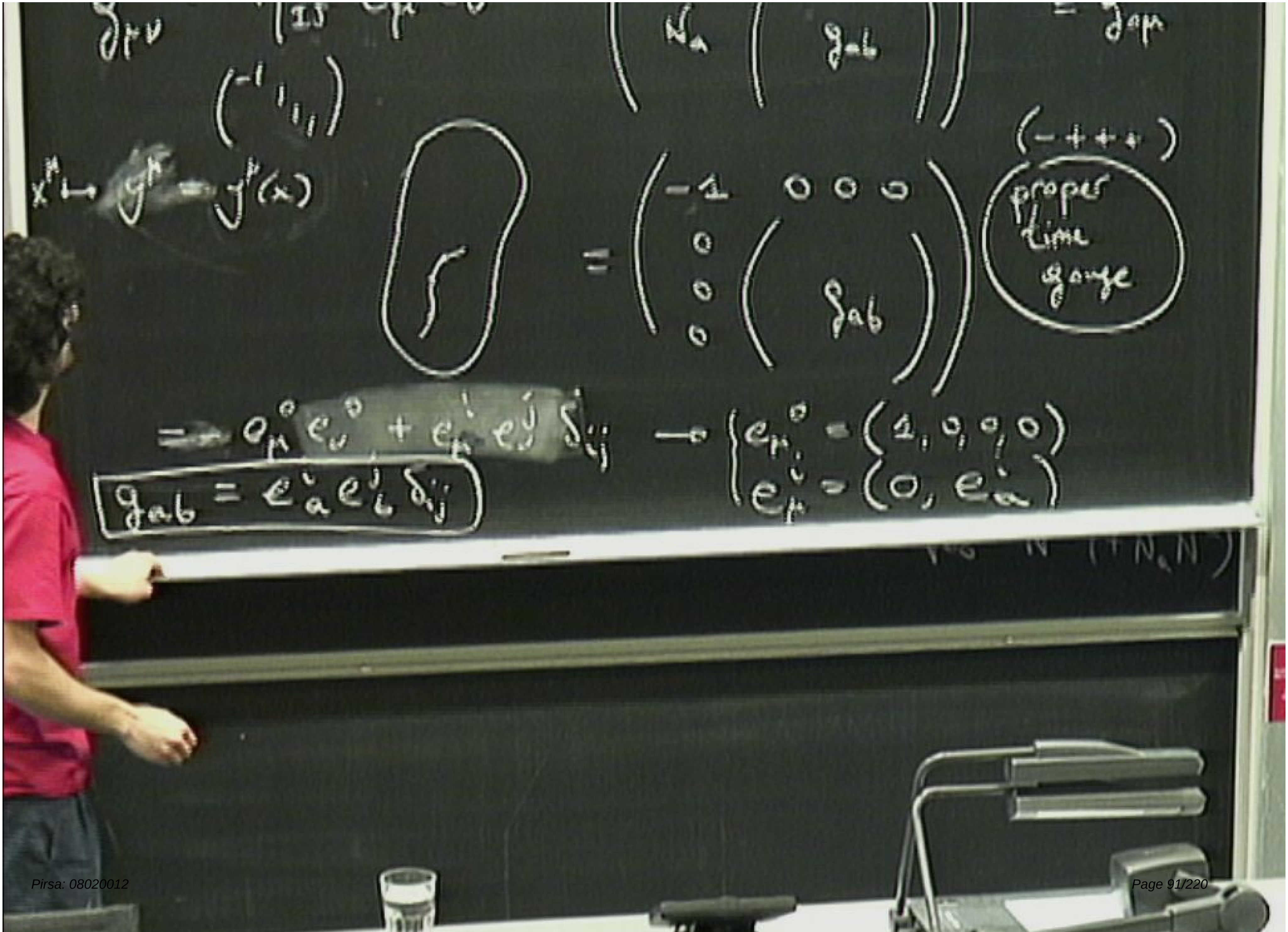

$g_{\mu\nu} = \eta_{IJ}\, e_\mu{}^I e_\nu{}^J = \begin{pmatrix} -N^2 & N_a \\ N_a & \begin{pmatrix} g_{ab} \end{pmatrix} \end{pmatrix}$
$(g_{00}, g_{0a}) =$
$= g_{0\mu}$
$(-+++)$
$= \begin{pmatrix} -1 & 0\ 0\ 0 \\ 0 & \\ 0 & \begin{pmatrix} g_{ab} \end{pmatrix} \\ 0 & \end{pmatrix}$
$g_{\mu\nu}(\gamma)$


(− + + +)
x^μ ↦ y^μ = y^μ(x)
= (−1 0 0 0 / 0 0 0 g_ab)
= − e_μ^0 e_ν^0 + e_μ^i e_ν^j δ_ij
g_0a = N_a
g_00 = −N^2 (+ N_a N^a)


(- + + +)
proper time gauge
g_ab


time
gauge
g_{ab}
$e_\mu^{\ 0} = (1, 0, 0, 0)$
$e_\mu^{\ i} = (0, e_a^{\ i})$
$g_{0a} = N_a$
$g_{00} = -N^2 (+ N_a N^a)$


gauge
g_{ab}
$= e_\mu^0 e_\nu^0 + e_\mu^i e_\nu^j \delta_{ij} \rightarrow \begin{cases} e_\mu^0 = (1,0,0,0) \\ e_\mu^i = (0, e_a^i) \end{cases}$
g_{ab}
$x_0) = \int dp\, e^{ip(x-x_0)}$
$\begin{cases} g_{0a} = N_a \\ g_{00} = -N^2 (+ N_a N^a) \end{cases}$



(- + + +)
proper time gauge
$g_{ab} = e^i_a e^j_b \delta_{ij}$

$$\begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

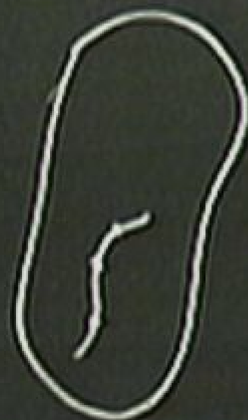
$x^\mu \mapsto y^\mu = y^\mu(x)$

$I = 0, \dots, 3$

$i = 1, 2, 3$

μ

a $1, 2, 3$



$$= \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & & & \\ 0 & & g_{ab} & \\ 0 & & & \end{pmatrix}$$

$(- + + +)$

proper time gauge

$$= e_\mu^0 e_\nu^0 + e_\mu^i e_\nu^j \delta_{ij} \rightarrow \begin{cases} e_\mu^0 = (1, 0, 0, 0) \\ e_\mu^i = (0, e_a^i) \end{cases}$$

$$\boxed{g_{ab} = e_a^i e_b^j \delta_{ij}}$$

$g_{00} = -N^2 (+ N_a N^a)$

$$\begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

$$x^\mu \mapsto y^\mu = y^\mu(x)$$

$I = 0, \dots, 3$

$i = 1, 2, 3$

$\mu = 0, \dots 3$

$a = 1, 2, 3$

$$= \begin{pmatrix} -1 & 0 \; 0 \; 0 \\ \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} & \begin{pmatrix} & & \\ & g_{ab} & \\ & & \end{pmatrix} \end{pmatrix}$$

$(- + + +)$

proper time gauge

$$= e_\mu^0 e_\nu^0 + e_\mu^i e_\nu^j \delta_{ij} \longrightarrow \begin{cases} e_\mu^0 = (1, 0, 0, 0) \\ e_\mu^i = (0, e_a^i) \end{cases}$$

$$\boxed{g_{ab} = e_a^i e_b^j \delta_{ij}}$$

$$g_{00} = -N^2 (+ N_a N^a)$$

$$\begin{pmatrix}-1&&&\\&1&&\\&&1&\\&&&1\end{pmatrix}$$

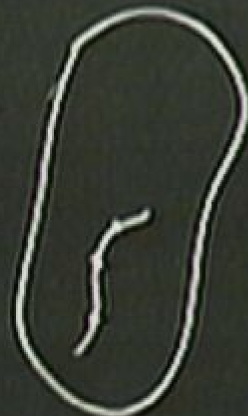
$x^\mu \mapsto y^\mu = y^\mu(x)$

$I = 0, \dots 3$

$i = 1, 2, 3$

$\mu = 0, \dots 3$

$a = 1, 2, 3$



$$= \begin{pmatrix} -1 & 0 \; 0 \; 0 \\ \begin{matrix}0\\0\\0\end{matrix} & \begin{pmatrix} \\ g_{ab} \\ \\ \end{pmatrix} \end{pmatrix}$$

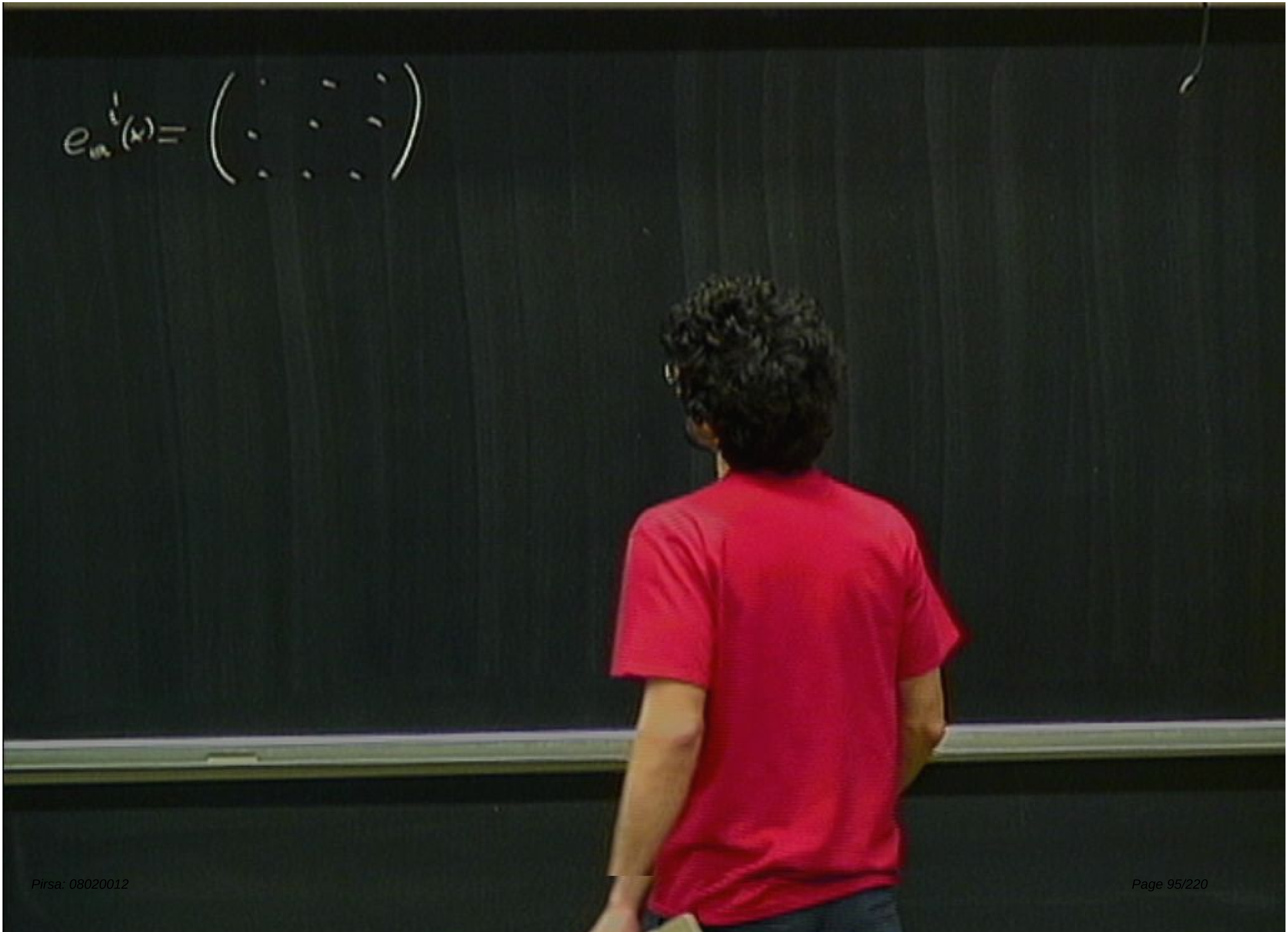
$(- + + +)$

proper time gauge

$$= e_\mu{}^0 e_\nu{}^0 + e_\mu{}^i e_\nu{}^j \delta_{ij} \rightarrow \begin{cases} e_\mu{}^0 = (1, 0, 0, 0) \\ e_\mu{}^i = (0, e_a{}^i) \end{cases}$$

$$\boxed{g_{ab} = e^i_a e^j_b \delta_{ij}}$$

$g_{00} = -N^2 (+ N_a N^a)$


$e_m{}^i(x) = \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$

$$e_a{}^i(x) = \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

$$e(x) = \det e_a{}^i(x)$$

$$e_a{}^i(x) = \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

$$e(x) = \det e_a{}^i(x) =$$

$$\varepsilon^{ijk}$$

$$e_a{}^i(x) = \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

$$e(x) = \det e_a{}^i(x) =$$

$$\varepsilon^{ijk}$$

$$\vec{a} \cdot (\vec{b} \wedge \vec{c})$$

$$e_a{}^i(x) = \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

$$e(x) = \det e_a{}^i(x) =$$

$$\varepsilon^{ijk}$$

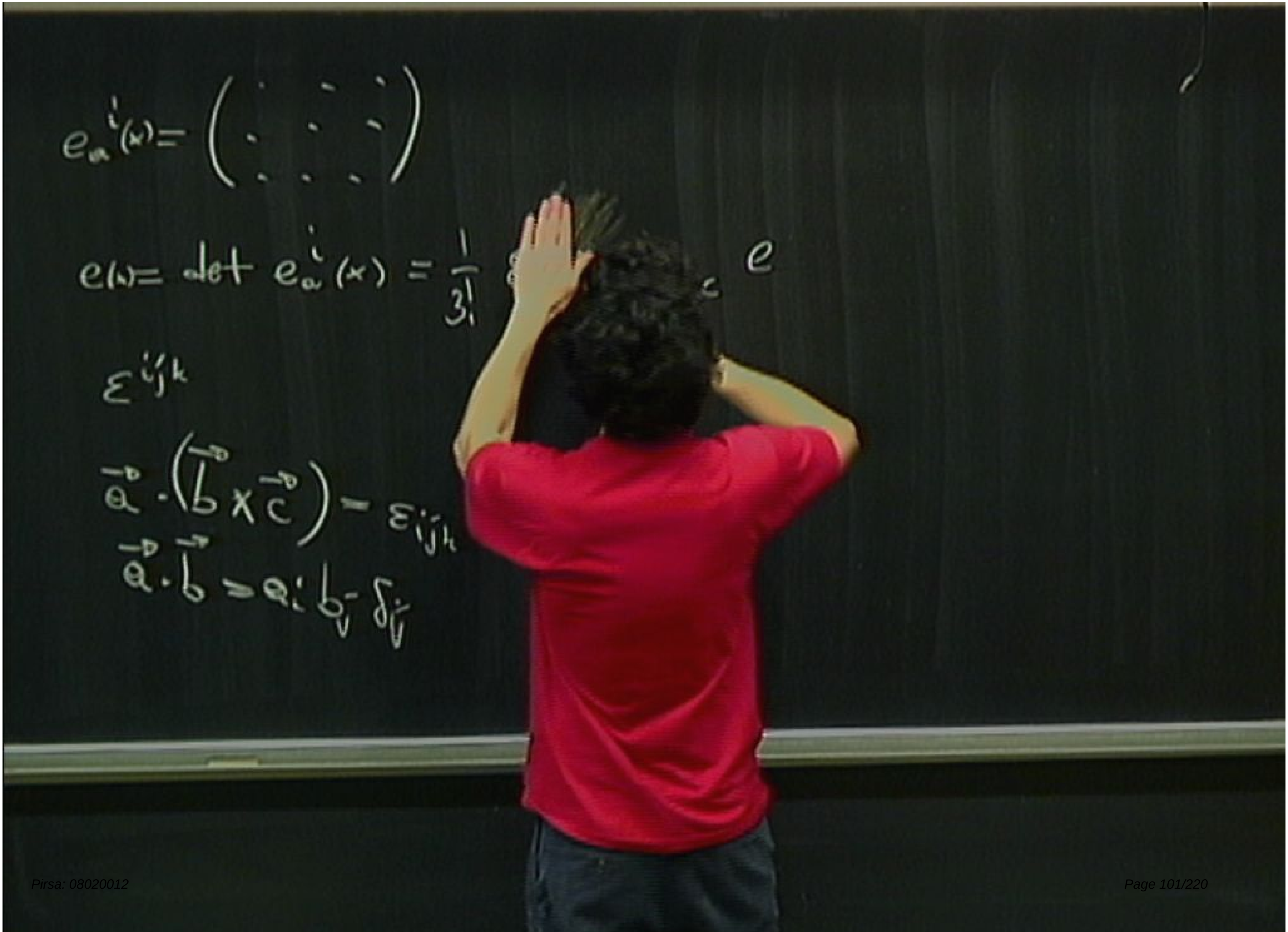
$$\vec{a} \cdot (\vec{b} \times \vec{c})$$

$$e_a{}^i(x) = \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

$$e(x) = \det e_a{}^i(x) =$$

$$\varepsilon^{ijk}$$

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = \varepsilon_{ijk}\, a^i b^j c^k$$


$e_a{}^i(x) = \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$
$e(x) = \det e_a{}^i(x) = \frac{1}{3!}$
e
ε^{ijk}
$\vec{a} \cdot (\vec{b} \times \vec{c}) = \varepsilon_{ijk}$
$\vec{a} \cdot \vec{b} = a_i b_j \delta_{ij}$

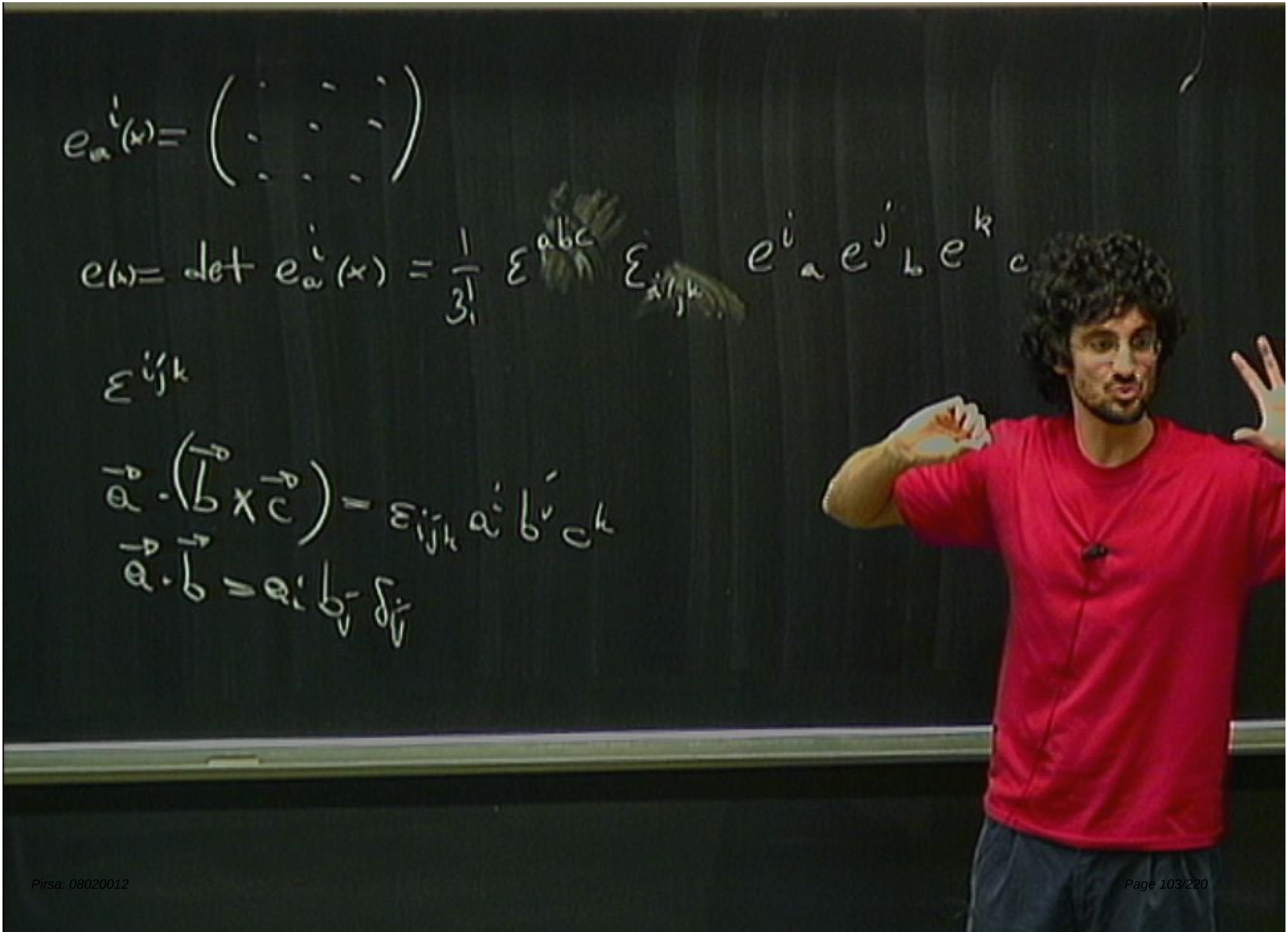
$$e_a{}^i(x) = \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

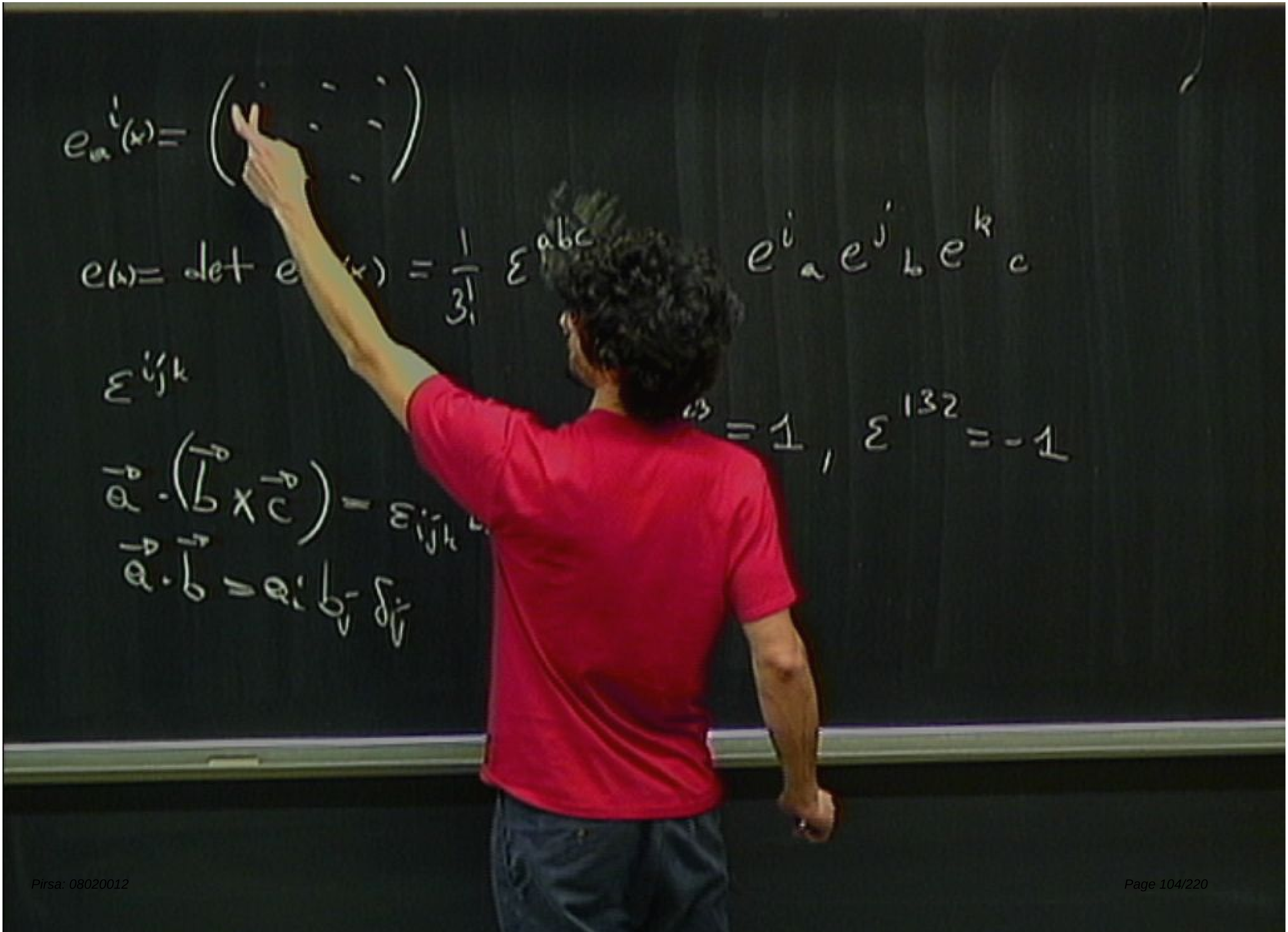
$$e(x) = \det e_a{}^i(x) = \frac{1}{3!}\,\varepsilon^{abc}\,\varepsilon_{ijk}$$

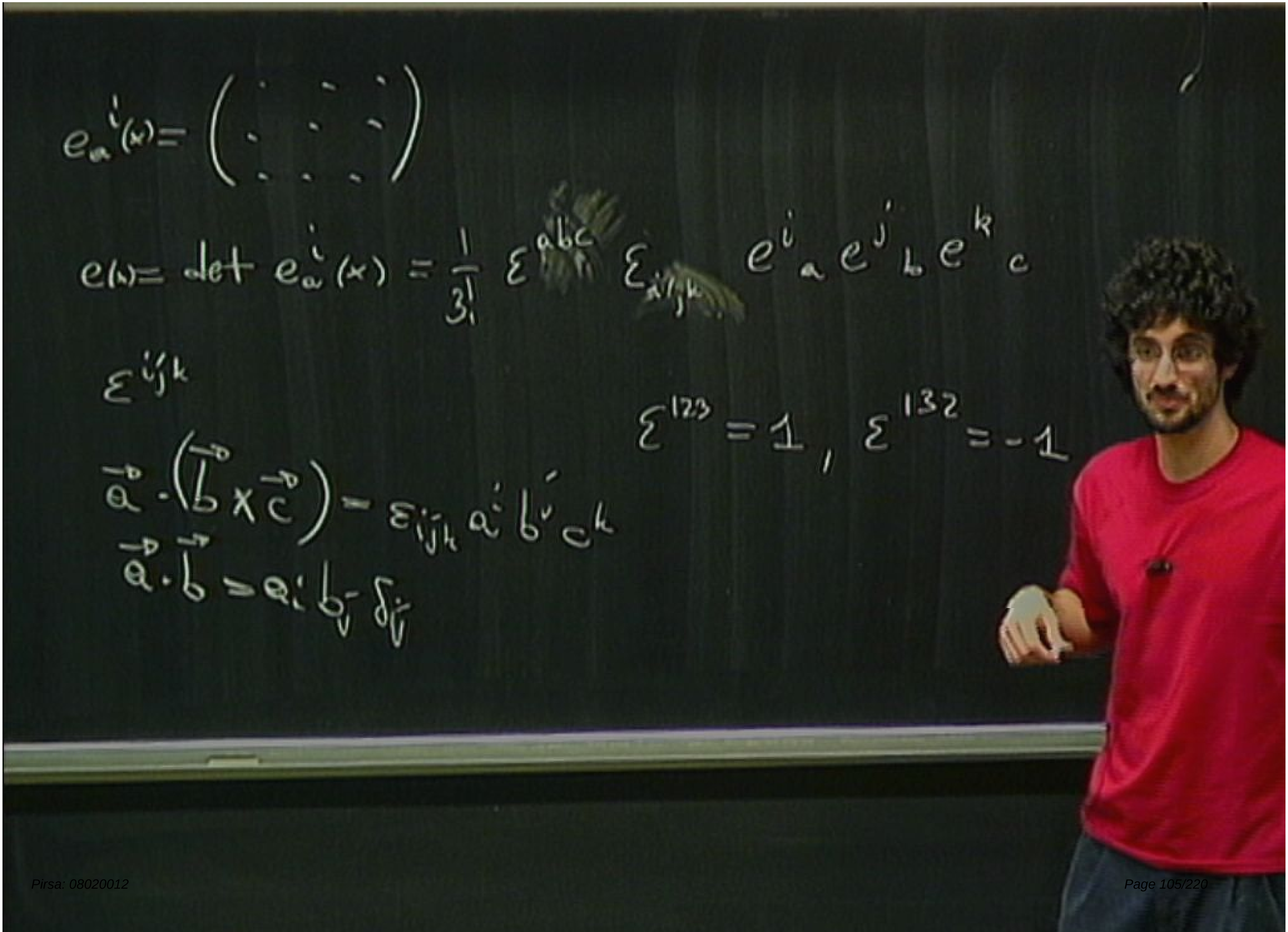
$$\varepsilon^{ijk}$$

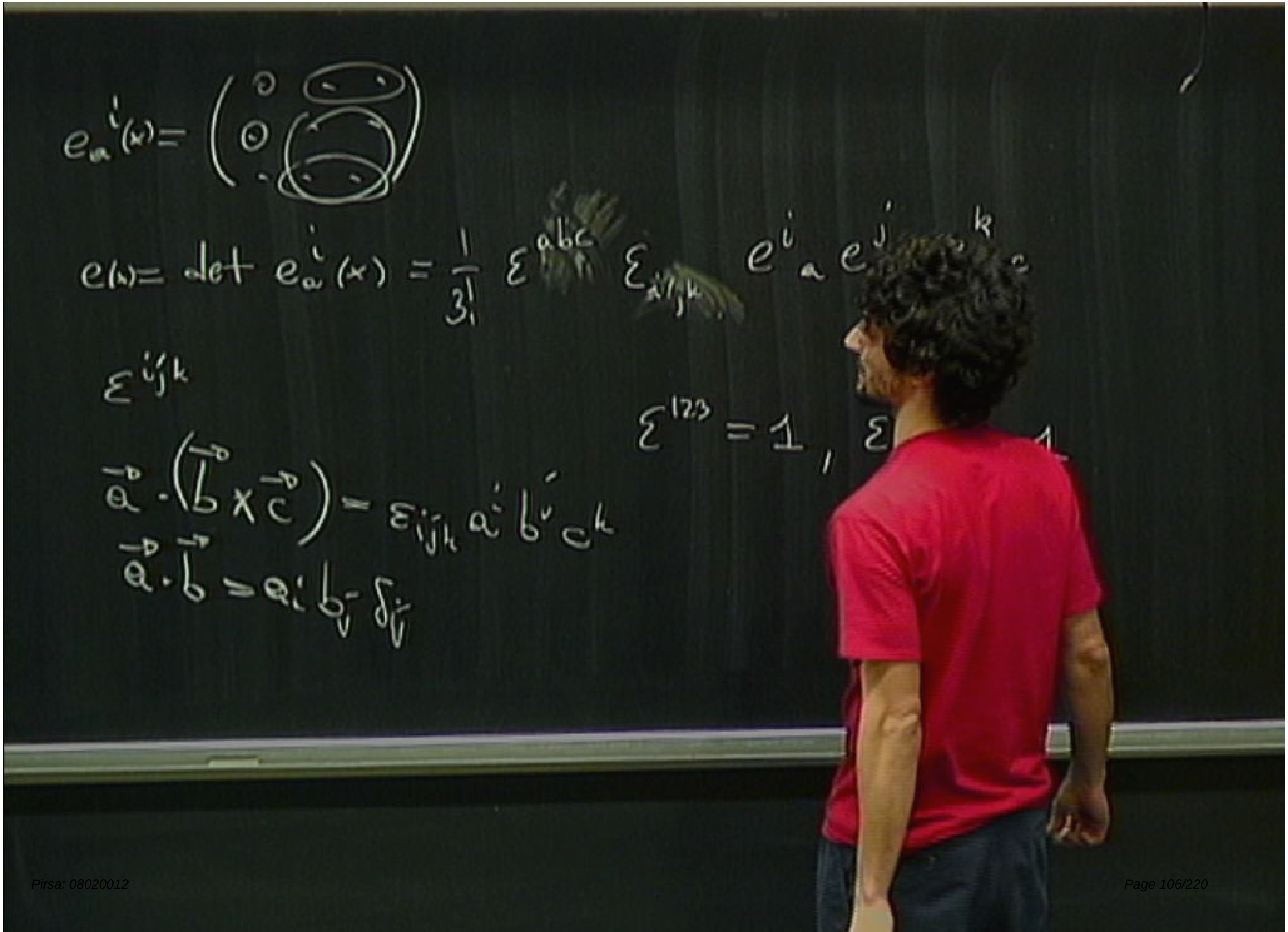
$$\vec{a}\cdot(\vec{b}\times\vec{c}) = \varepsilon_{ijk}\, a^i b^j c^k$$

$$\vec{a}\cdot\vec{b} = a^i b_j \delta_{ij}$$


$e_a^{\ i}(x) = \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$
$e(x) = \det e_a^{\ i}(x) = \frac{1}{3!} \varepsilon^{abc} \varepsilon_{ijk} e^i_{\ a} e^j_{\ b} e^k_{\ c}$
ε^{ijk}
$\vec{a} \cdot (\vec{b} \times \vec{c}) = \varepsilon_{ijk} a^i b^j c^k$
$\vec{a} \cdot \vec{b} = a^i b^j \delta_{ij}$


e_a^i(x) =
e(x) = det e(x) = 1/3! ε^abc e^i_a e^j_b e^k_c
ε^ijk
= 1 , ε^132 = -1
a · (b × c) = ε_ijk
a · b = a_i b_j δ_ij


$e_a^{\ i}(x) = \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$
$e(x) = \det e_a^{\ i}(x) = \frac{1}{3!} \varepsilon^{abc} \varepsilon_{ijk} e^i_{\ a} e^j_{\ b} e^k_{\ c}$
ε^{ijk}
$\varepsilon^{123} = 1, \ \varepsilon^{132} = -1$
$\vec{a} \cdot (\vec{b} \times \vec{c}) = \varepsilon_{ijk} a^i b^j c^k$
$\vec{a} \cdot \vec{b} = a_i b_j \delta_{ij}$


$e_a{}^i(x) =$
$e(x) = \det e_a{}^i(x) = \frac{1}{3!}\varepsilon^{abc}\varepsilon_{ijk}\, e^i{}_a e^j$
ε^{ijk}
$\varepsilon^{123} = 1$, ε
$\vec{a}\cdot(\vec{b}\times\vec{c}) = \varepsilon_{ijk}\, a^i b^j c^k$
$\vec{a}\cdot\vec{b} = a^i b^j \delta_{ij}$

$$e_a{}^i(x) = \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

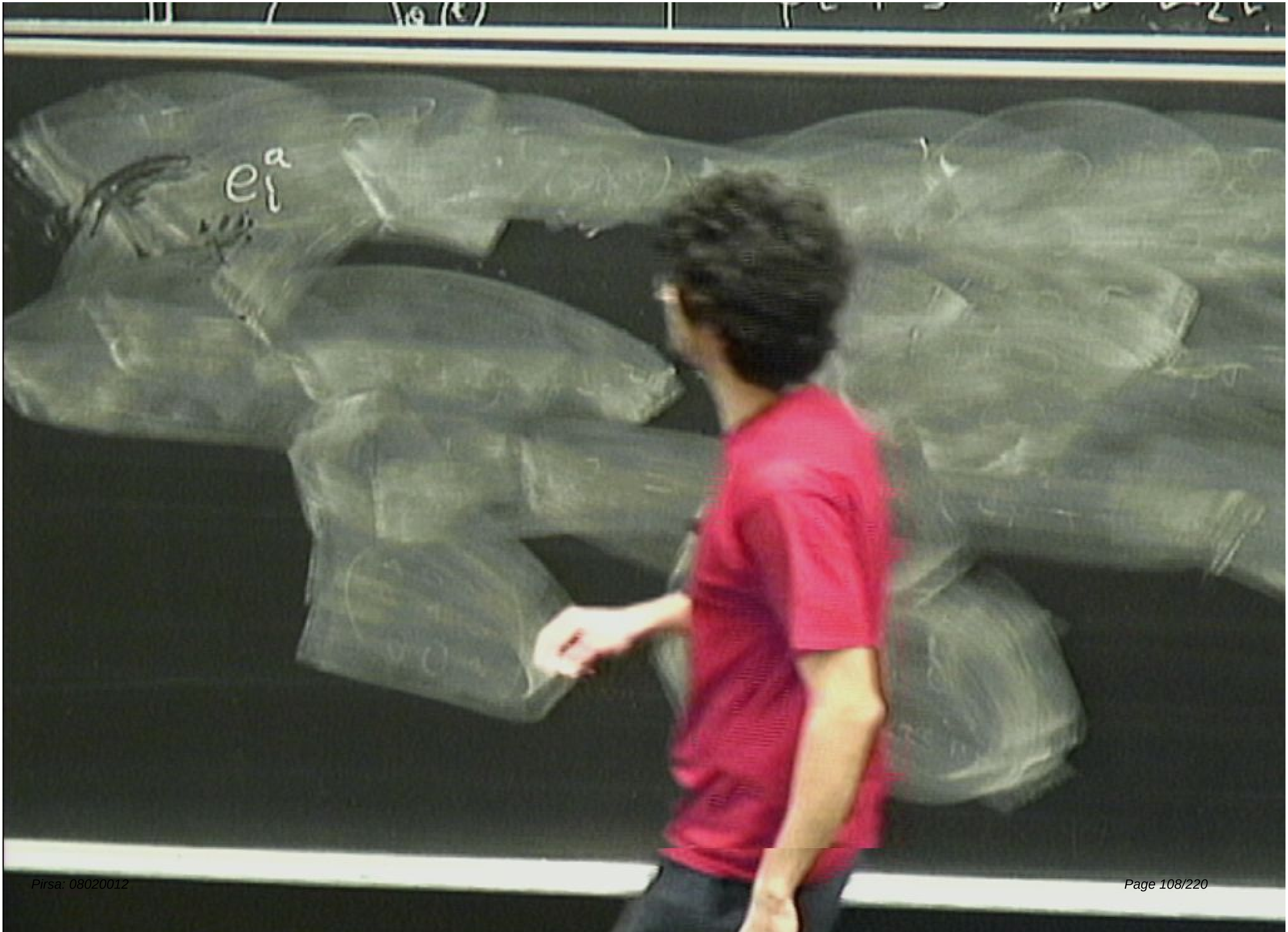
$$e(x) = \det e_a{}^i(x) = \frac{1}{3!}\,\varepsilon^{abc}\,\varepsilon_{ijk}\; e^i{}_a\, e^j{}_b\, e^k{}_c$$

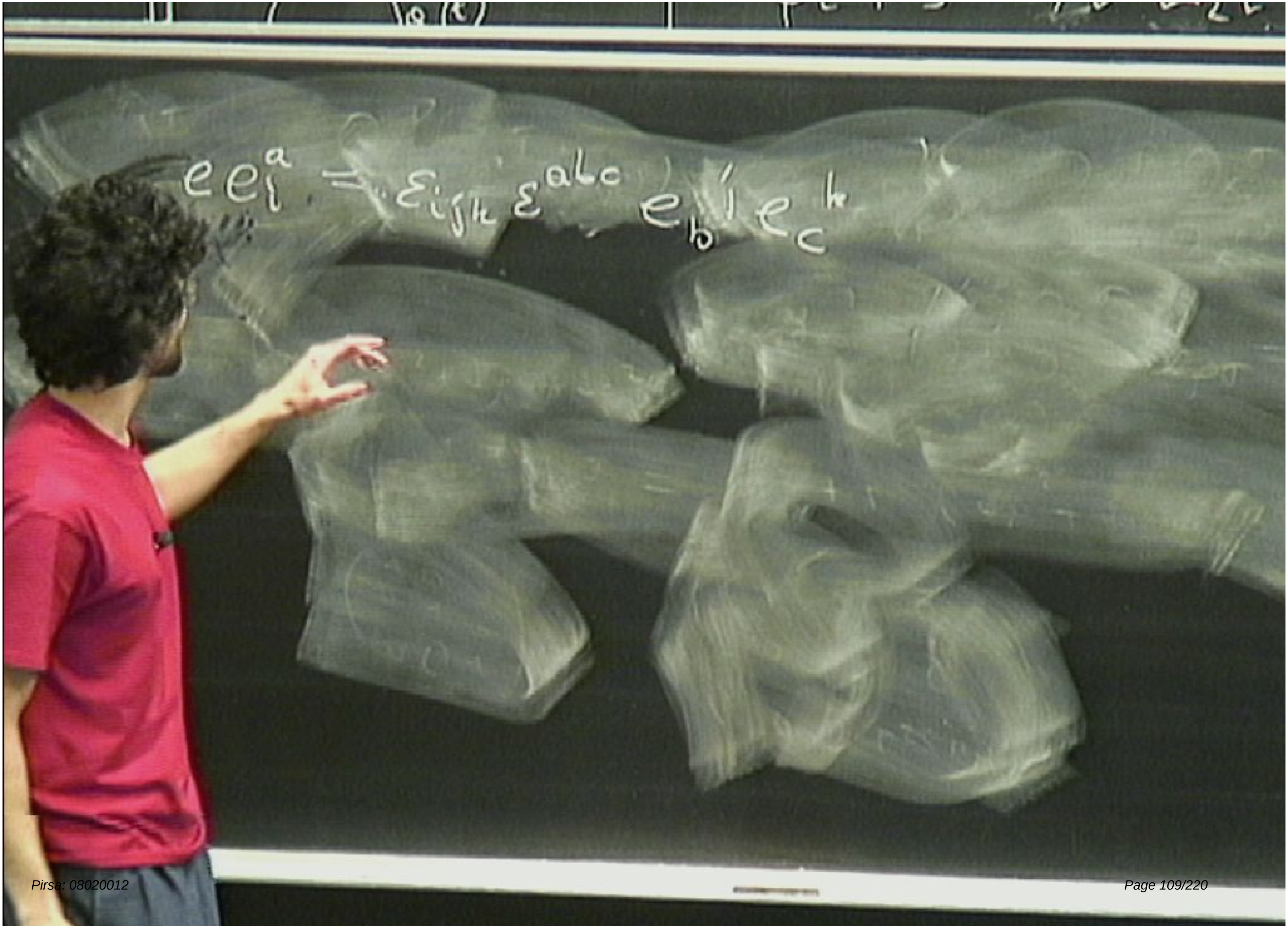
$$\varepsilon^{ijk}$$

$$\varepsilon^{123} = 1\,, \quad \varepsilon^{132} = -1$$

$$\vec{a}\cdot(\vec{b}\times\vec{c}) = \varepsilon_{ijk}\, a^i\, b^j\, c^k$$

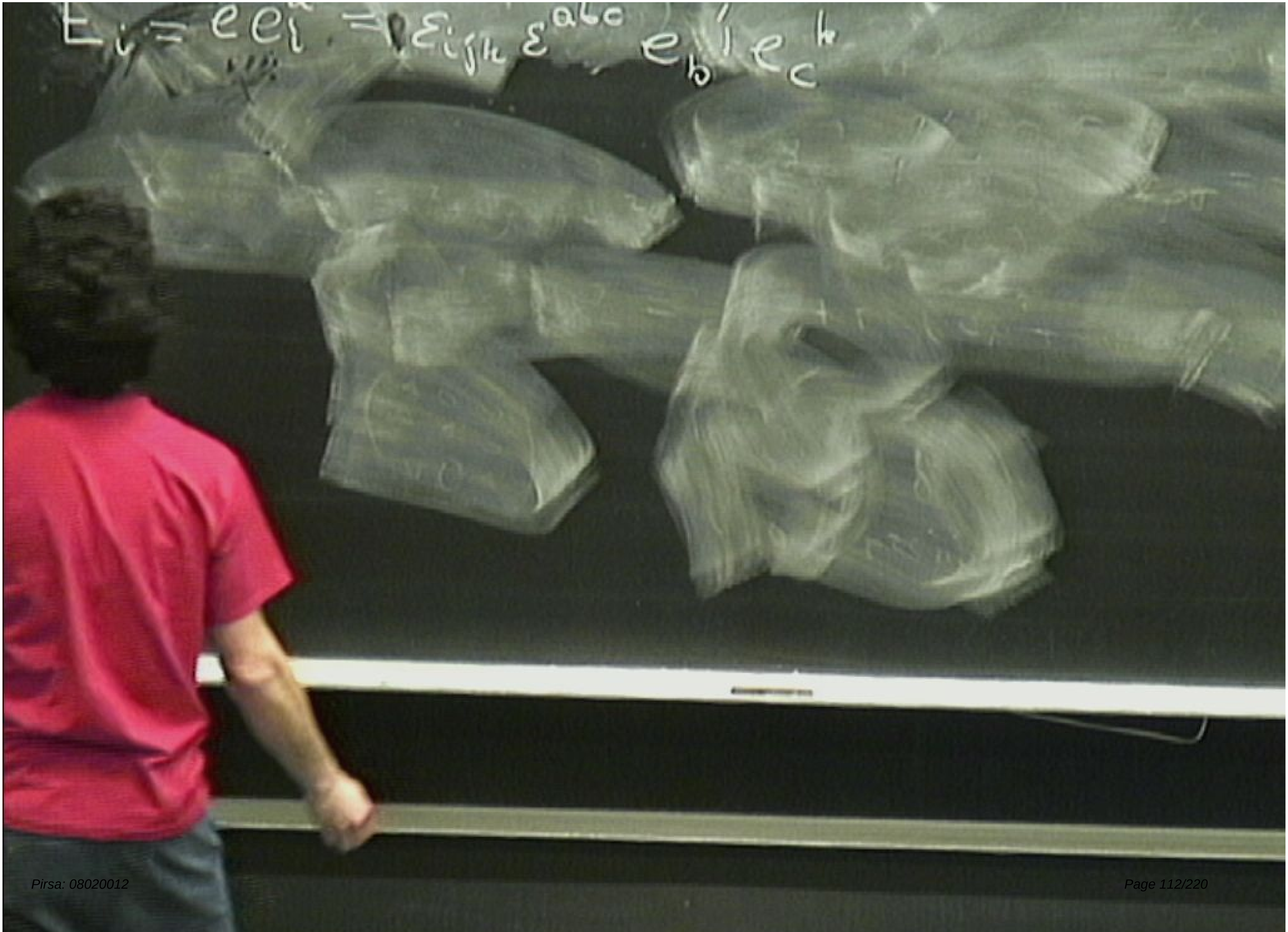
$$\vec{a}\cdot\vec{b} = a_i\, b_j\, \delta_{ij}$$

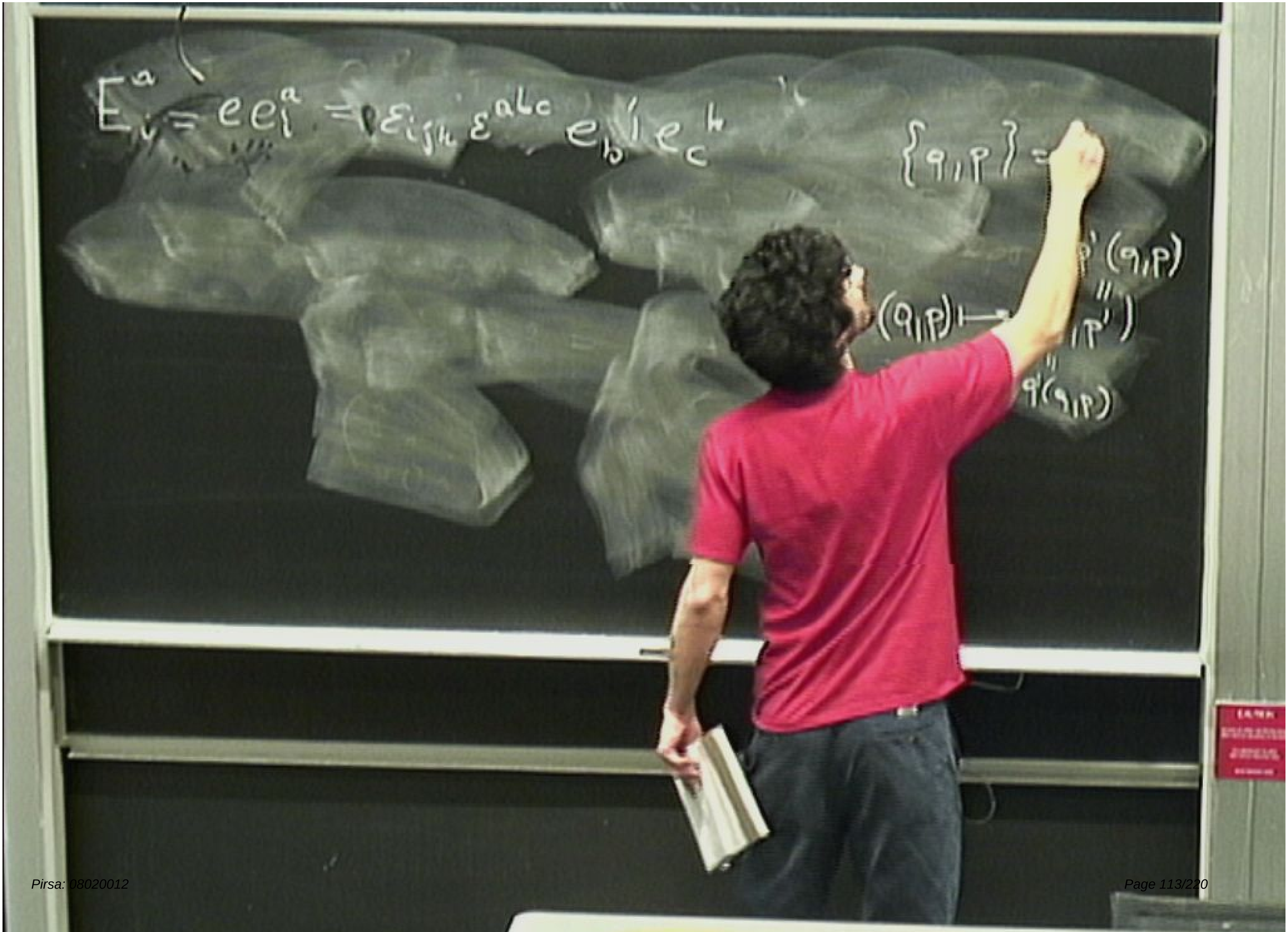

e_i^a

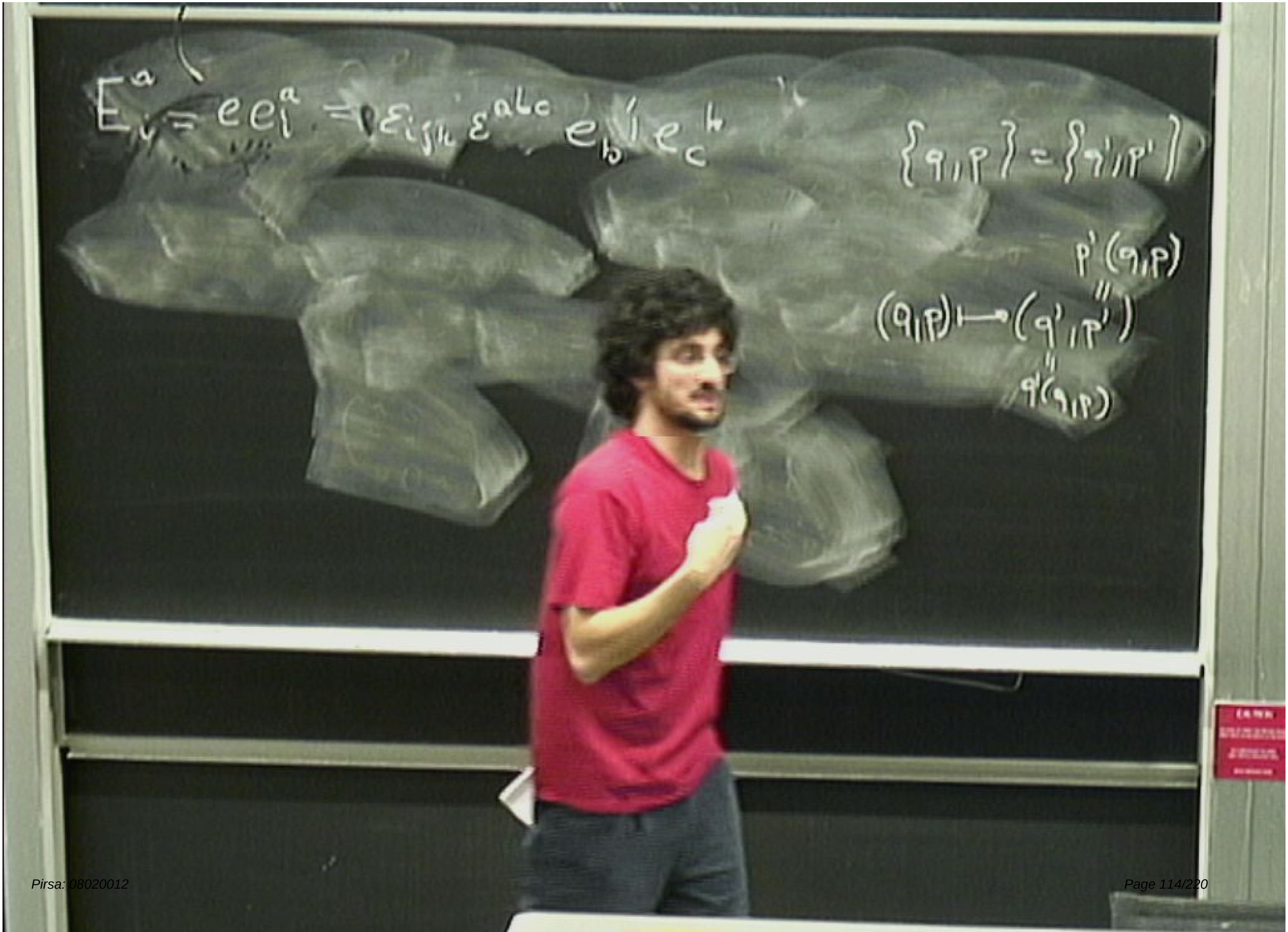

$e e_i^a = \varepsilon_{ijk} \varepsilon^{abc} e_b^j e_c^k$

$$e\, e_i^a = \varepsilon_{ijk}\, \varepsilon^{abc}\, e_b^j\, e_c^k$$

$$E^a_i = e\, e^a_i = \epsilon_{ijk}\, \epsilon^{abc}\, e^j_b\, e^k_c$$


$E_i^a = e e_i^a = \varepsilon_{ijk} \varepsilon^{abc} e_b^j e_c^k$


$E^a_i = e e_i^a = \varepsilon_{ijk} \varepsilon^{abc} e_b^j e_c^k$
$\{q,p\} =$
$p'(q,p)$
$(q,p) \longmapsto (q', p')$
$q'(q,p)$


$E^a_i = e e^a_i = \varepsilon_{ijk} \varepsilon^{abc} e_b^j e_c^k$
$\{q, p\} = \{q', p'\}$
$p'(q,p)$
$(q,p) \mapsto (q', p')$
$q'(q,p)$

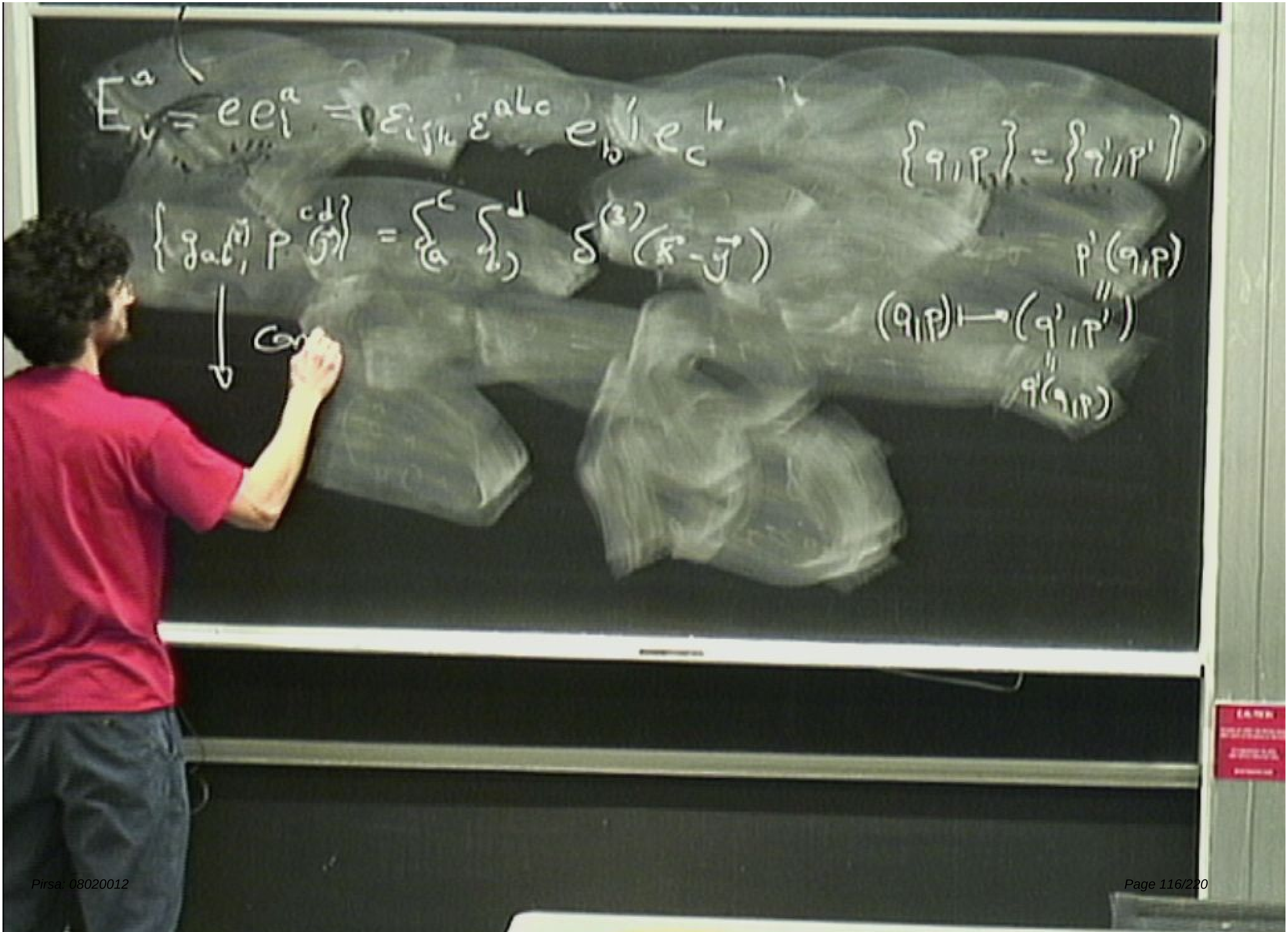
$$E^a_i = e e^a_i = \epsilon_{ijk} \epsilon^{abc} e_b^j e_c^k$$

$$\{q, p\} = \{q', p'\}$$

$$p'(q,p)$$

$$(q,p) \longmapsto (q', p')$$

$$q'(q,p)$$



$E^a_i = e e^a_i = \varepsilon_{ijk} \varepsilon^{abc} e^j_b e^k_c$
$\{q, p\} = \{q', p'\}$
$\{g_{ab}(\vec{x}), p^{cd}(\vec{y})\} = \delta^c_{(a} \delta^d_{b)} \delta^{(3)}(\vec{x} - \vec{y})$
$p'(q, p)$
$(q, p) \mapsto (q', p')$
$q'(q, p)$
Con

$$E^a_i = e e^a_i = \varepsilon_{ijk}\, \varepsilon^{abc}\, e^j_b\, e^k_c$$

$$\{q, p\} = \{q', p'\}$$

$$\{g_{ab}(\vec{x}), p^{cd}(\vec{y})\} = \delta^c_{(a}\, \delta^d_{b)}\, \delta^{(3)}(\vec{x} - \vec{y})$$

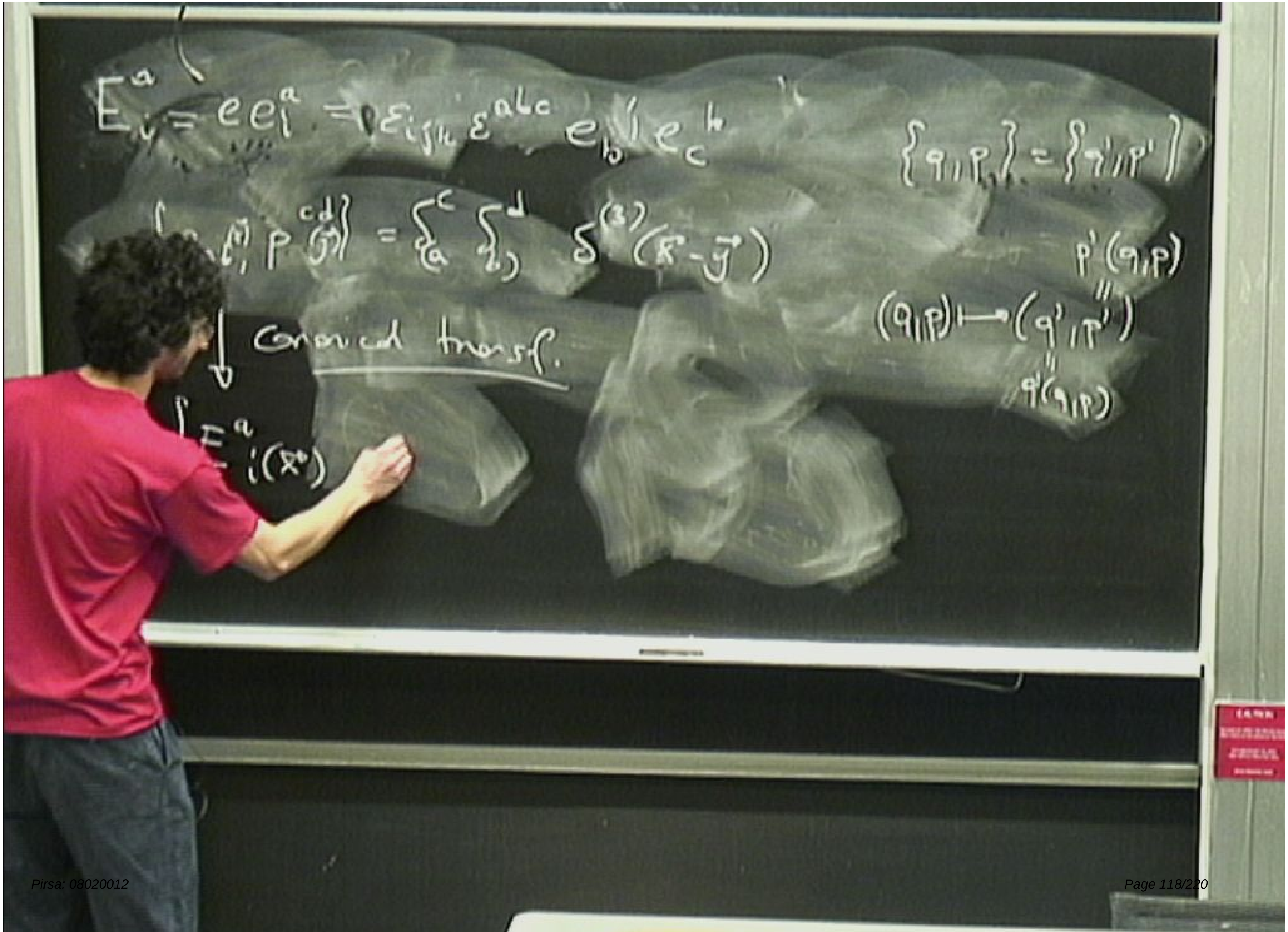
$$p'(q, p)$$

$$(q, p) \longmapsto (q', p')$$

$$q'(q, p)$$

$\downarrow$ Canonical transf.

$\{$


$E^a_i = e e^a_i = \varepsilon_{ijk} \varepsilon^{abc} e_b^j e_c^k$
$\{q, p\} = \{q', p'\}$
$\delta^c_a \delta^d_b \, \delta^{(3)}(\vec{x} - \vec{y})$
$p'(q,p)$
$(q,p) \longmapsto (q', p')$
Canonical transf.
$q'(q,p)$

$$E^a_i = e e^a_i = \varepsilon_{ijk}\,\varepsilon^{abc}\, e_b^j\, e_c^k$$

$$\{q, p\} = \{q', p'\}$$

$$\{g_{ab}(\vec{x}), p^{cd}(\vec{y})\} = \delta^c_{(a}\,\delta^d_{b)}\,\delta^{(3)}(\vec{x}-\vec{y})$$

$$p'(q,p)$$

$$(q,p) \longmapsto (q', p')$$

$$q'(q,p)$$

Canonical transf.

$$\{E^a_i(\vec{x}), A$$

$$E^a_i = e\, e^a_i = \varepsilon_{ijk}\, \varepsilon^{abc}\, e_b^j\, e_c^k$$

$$\{g_{ab}(\vec{x}), p^{cd}(\vec{y})\} = \delta^c_{(a}\, \delta^d_{b)}\, \delta^{(3)}(\vec{x}-\vec{y})$$

$\Big\downarrow$ canonical transf.

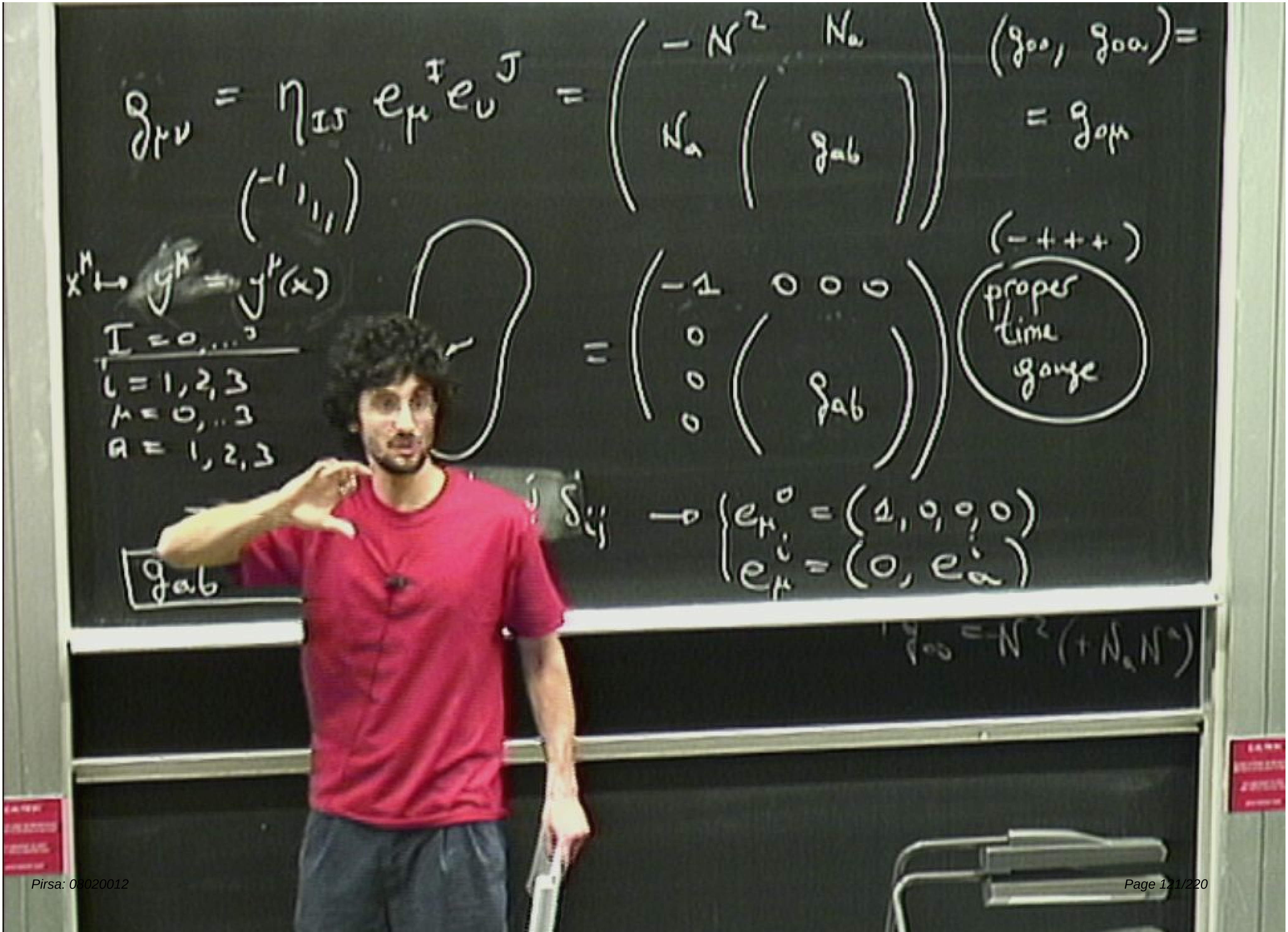
$$\{E^a_i(\vec{x}), A_b$$

$$\{q,p\} = \{q',p'\}$$

$$p'(q,p)$$

$$(q,p) \longmapsto (q',p')$$

$$q' = q'(q,p)$$


$g_{\mu\nu} = \eta_{IJ}\, e_\mu{}^I e_\nu{}^J = \begin{pmatrix} -N^2 & N_a \\ N_a & g_{ab} \end{pmatrix}$
$(g_{00}, g_{0a}) = g_{0\mu}$
$x^\mu \mapsto y^\mu = y^\mu(x)$
$I = 0, \dots 3$
$i = 1, 2, 3$
$\mu = 0, \dots 3$
$a = 1, 2, 3$
$(-+++)$
$= \begin{pmatrix} -1 & 0\,0\,0 \\ 0 & \\ 0 & g_{ab} \\ 0 & \end{pmatrix}$
proper time gauge
$\delta_{ij} \to \begin{cases} e_\mu{}^0 = (1,0,0,0) \\ e_\mu{}^i = (0, e_a^i) \end{cases}$
g_{ab}
$g_{00} = -N^2 (+ N_a N^a)$

$$g_{\mu\nu} = \eta_{IJ}\, e_\mu{}^I e_\nu{}^J = \begin{pmatrix} -N^2 & N_a \\ N_a & \begin{pmatrix} & & \\ & g_{ab} & \\ & & \end{pmatrix} \end{pmatrix}$$

$(g_{00}, g_{0a}) = $
$= g_{0\mu}$

$$\begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

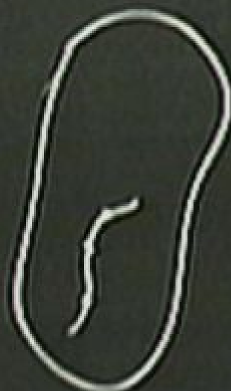
$x^\mu \mapsto y^\mu = y^\mu(x)$

$I = 0, \dots, 3$

$i = 1, 2, 3$

$\mu = 0, \dots, 3$

$a = 1, 2, 3$



$(-+++)$

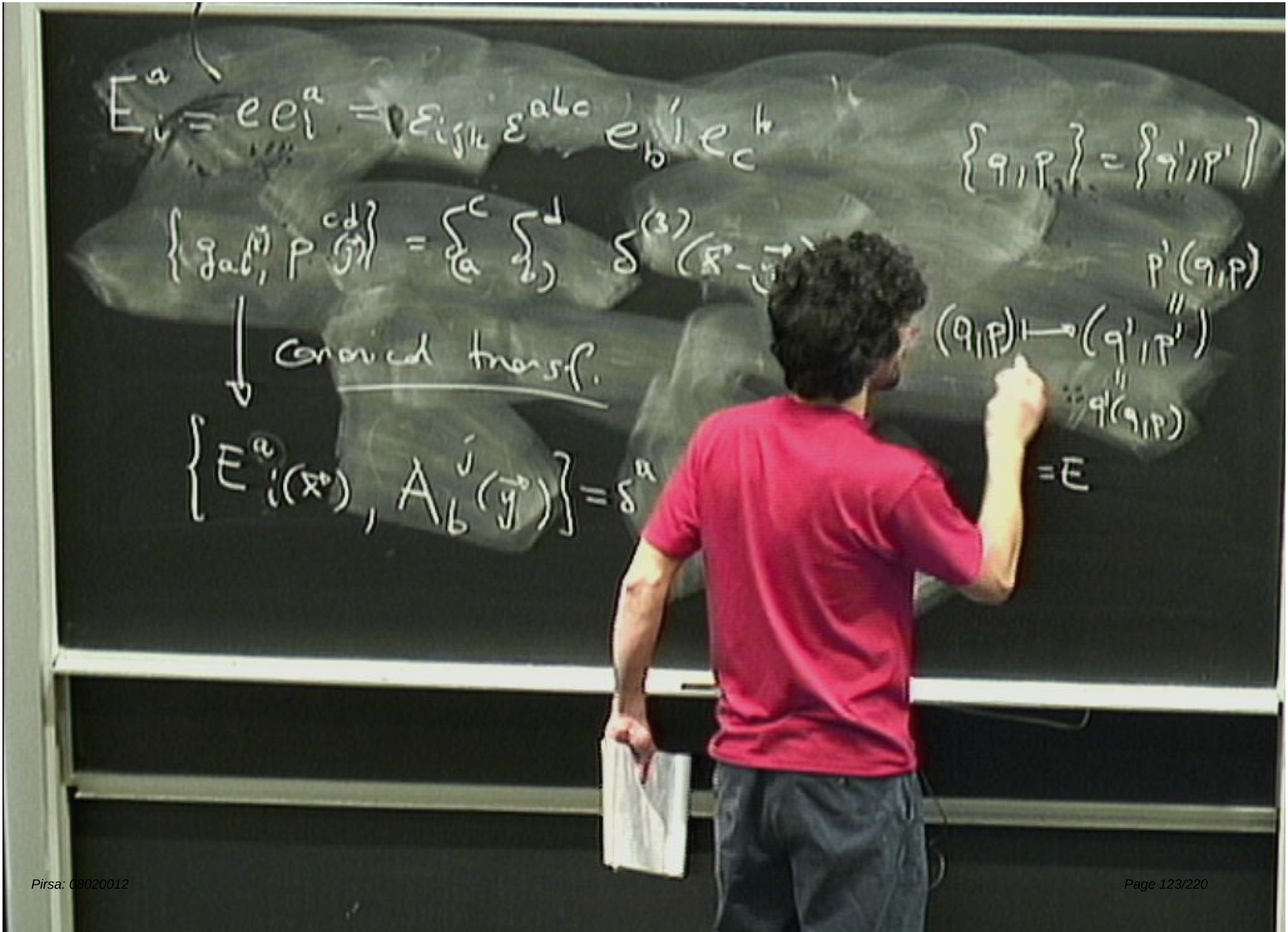
$$= \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & & & \\ 0 & & g_{ab} & \\ 0 & & & \end{pmatrix}$$

proper time gauge

$$= e_\mu{}^0 e_\nu{}^0 + e_\mu{}^i e_\nu{}^j \delta_{ij} \to \begin{cases} e_\mu{}^0 = (1, 0, 0, 0) \\ e_\mu{}^i = (0, e_a{}^i) \end{cases}$$

$$\boxed{g_{ab} = e_a{}^i e_b{}^j \delta_{ij}}$$

$g_{00} = -N^2 (+ N_a N^a)$


$E^a_i = e e^a_i = \epsilon_{ijk} \epsilon^{abc} e^j_b e^k_c$
$\{q,p\} = \{q',p'\}$
$\{g_{ab}(\vec{x}), p^{cd}(\vec{y})\} = \delta^c_{(a} \delta^d_{b)} \delta^{(3)}(\vec{x}-\vec{y})$
$p'(q,p)$
canonical transf.
$(q,p) \mapsto (q',p')$
$q'(q,p)$
$\{E^a_i(\vec{x}), A^j_b(\vec{y})\} = \delta^a$
$= E$

$$E^a_i = e e^a_i = \varepsilon_{ijk}\varepsilon^{abc} e_b^j e_c^k$$

$$\{q, p\} = \{q', p'\}$$

$$\{g_{ab}(\vec{x}), p^{cd}(\vec{y})\} = \delta^c_{(a}\delta^d_{b)}\,\delta^{(3)}(\vec{x}-\vec{y})$$

$$p'(q,p)$$

$$(q,p) \longmapsto (q', p')$$

canonical transf.

$$q'(q,p)$$

$$\{E^a_i(\vec{x}), A^j_b(\vec{y})\} = \delta^a_b\,\delta^j_i\,\delta^{(3)}(\vec{x}-\vec{y})$$

$$\begin{cases} E = E(g_{ab}, p^{cd}) \\ A = A(g_{ab}, p^{cd}) \end{cases}$$

$$E^a_i = e e^a_i = \epsilon_{ijk}\, \varepsilon^{abc}\, e_b{}^j e_c{}^k$$

$$\{g_{ab}(\vec{x}), P^{cd}(\vec{y})\} = \delta^c_{(a}\, \delta^d_{b)}\, \delta^{(3)}(\vec{x}-\vec{y})$$

$\Big\downarrow$ canonical transf.

$$\{E^a_i(\vec{x}), A^j_b(\vec{y})\} = \delta^a_b\, \delta^j_i\, \delta^{(3)}(\vec{x}-\vec{y})$$

$$\{E, E\} = 0\ ;\ \{A, A\} = 0$$

$$\{q, q\} = 0$$

$$\{q, p\} = \{q', p'\}$$

$$\{p, p\} = 0 \qquad p'(q,p)$$

$$(q,p) \longmapsto (q', p')$$

$$q'(q,p)$$

$$\begin{cases} E = E(g_{ab}, P^{cd}) \\ A = A(g_{ab}, P^{cd}) \end{cases}$$

$$E^a_i = e e^a_i = e\,\varepsilon_{ijk}\,\varepsilon^{abc}\,e_b^j\,e_c^k$$

$$\{g_{ab}(\vec{x}), p^{cd}(\vec{y})\} = \delta^c_{(a}\delta^d_{b)}\,\delta^{(3)}(\vec{x}-\vec{y})$$

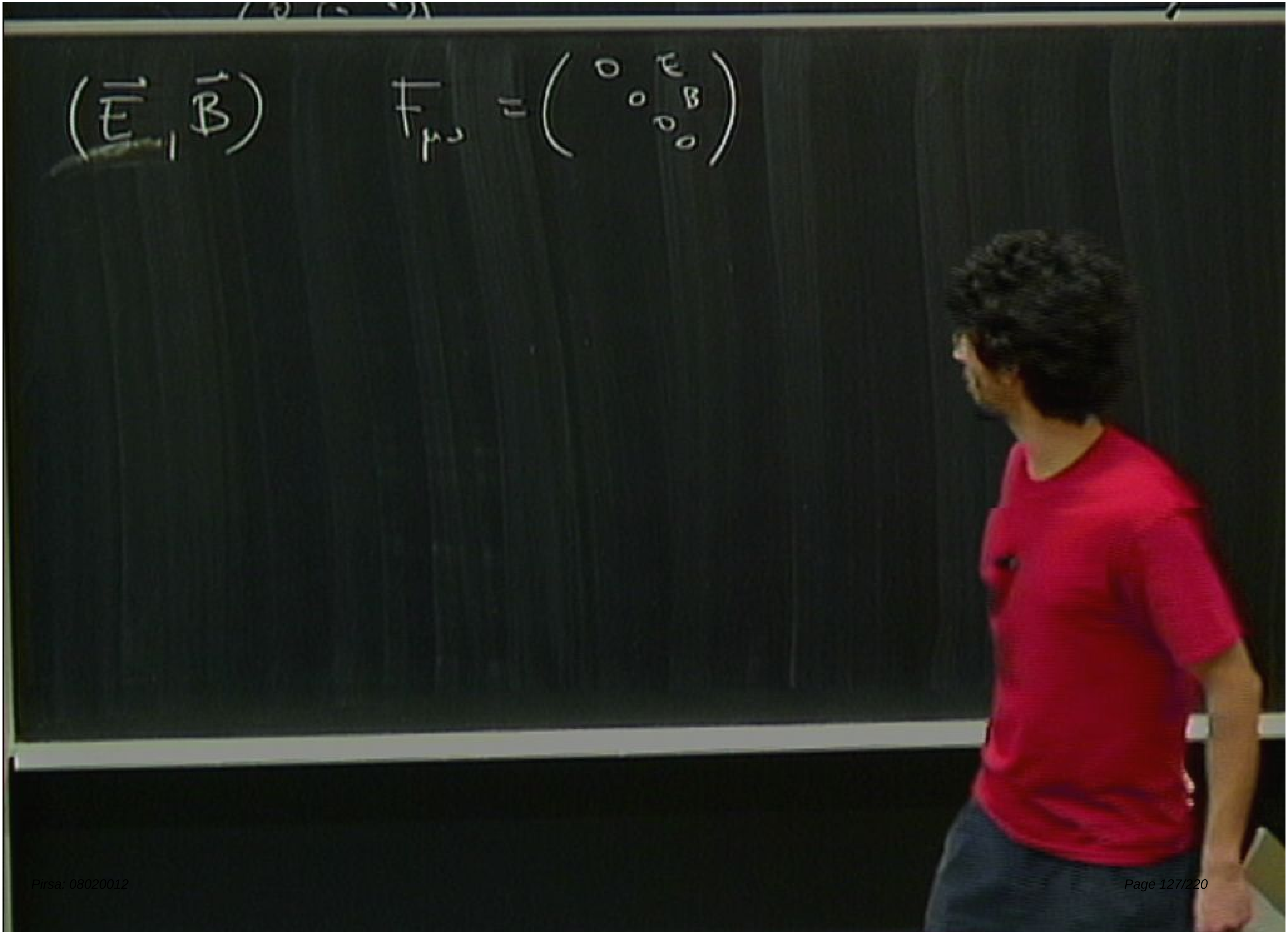
$\downarrow$ canonical transf.

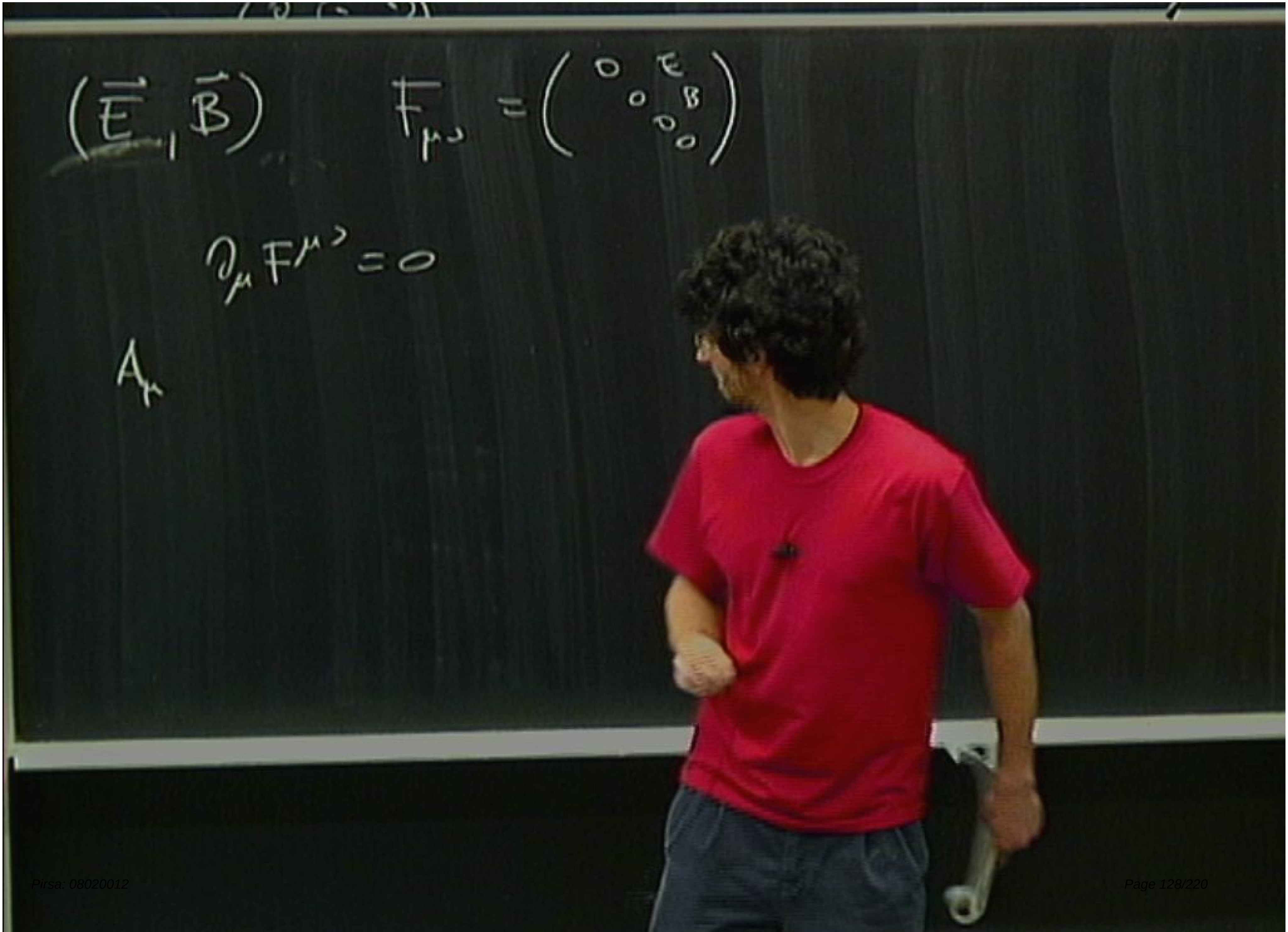
$$\{E^a_i(\vec{x}), A^j_b(\vec{y})\} = \delta^a_b\,\delta^j_i\,\delta^{(3)}(\vec{x}-\vec{y})$$

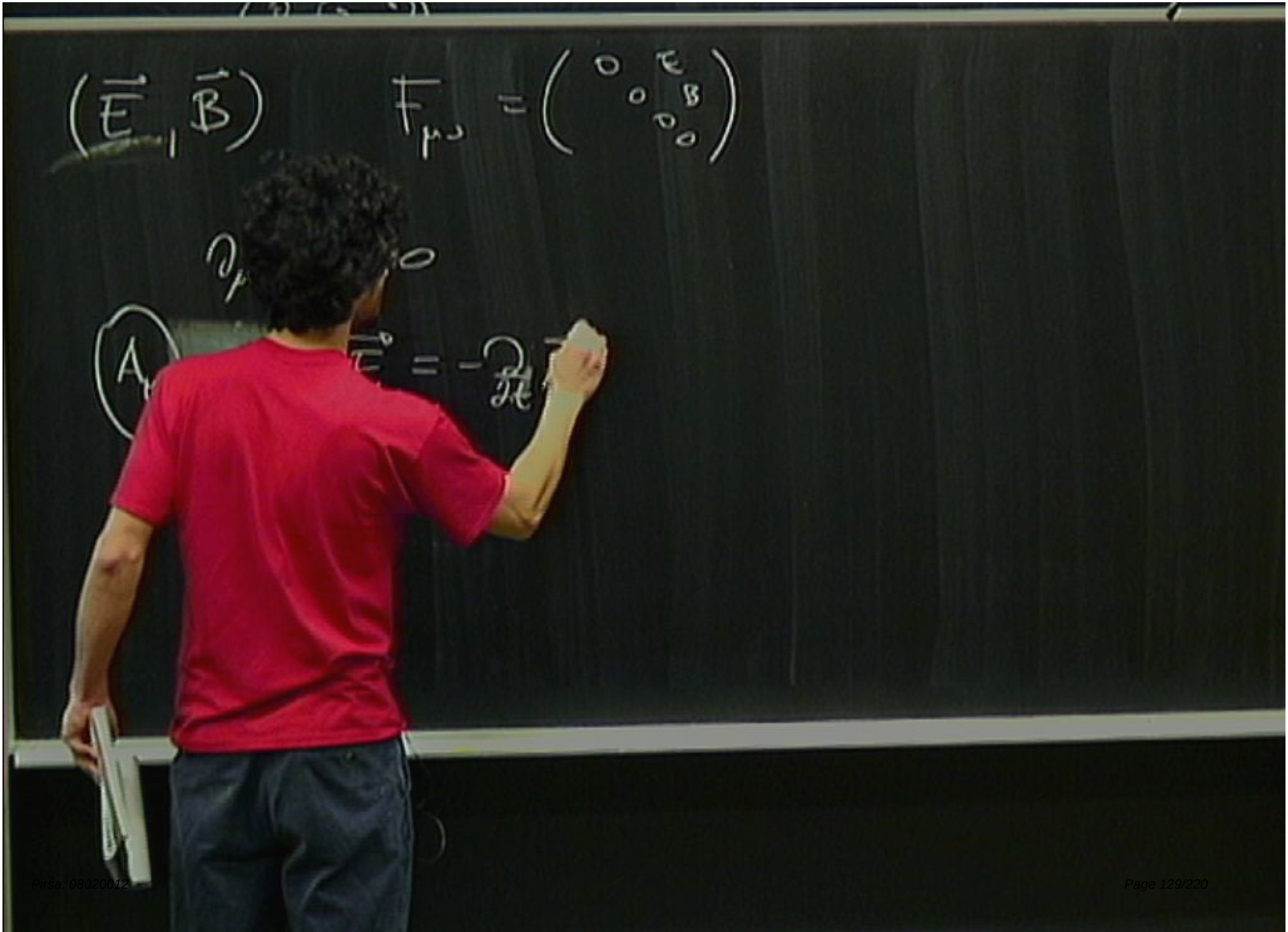
$$\{E, E\} = 0\ ;\ \{A, A\} = 0$$

$$\{q, q\} = 0$$
$$\{q, p\} = \{q', p'\}$$
$$\{p, p\} = 0$$
$$p'(q,p)$$
$$(q,p) \longmapsto (q', p')$$
$$q'(q,p)$$

$$\begin{cases} E = E(g_{ab}, p^{cd}) \\ A = A(g_{ab}, p^{cd}) \end{cases}$$


$(\vec{E}, \vec{B})$
$F_{\mu\nu} = \begin{pmatrix} 0 & E & & \\ & 0 & B & \\ & & 0 & \\ & & & 0 \end{pmatrix}$


$(\vec{E}, \vec{B})$
$F_{\mu\nu} = \begin{pmatrix} 0 & & E \\ & 0 & B \\ & & 0 \\ & & & 0 \end{pmatrix}$
$\partial_\mu F^{\mu\nu} = 0$
A_μ


(E, B)
Fμν =
A
E = -∂/∂t

$(\vec{E}, \vec{B})$ $\qquad F_{\mu\nu} = \begin{pmatrix} 0 & E & & \\ & 0 & B & \\ & & 0 & \\ & & & 0 \end{pmatrix}$

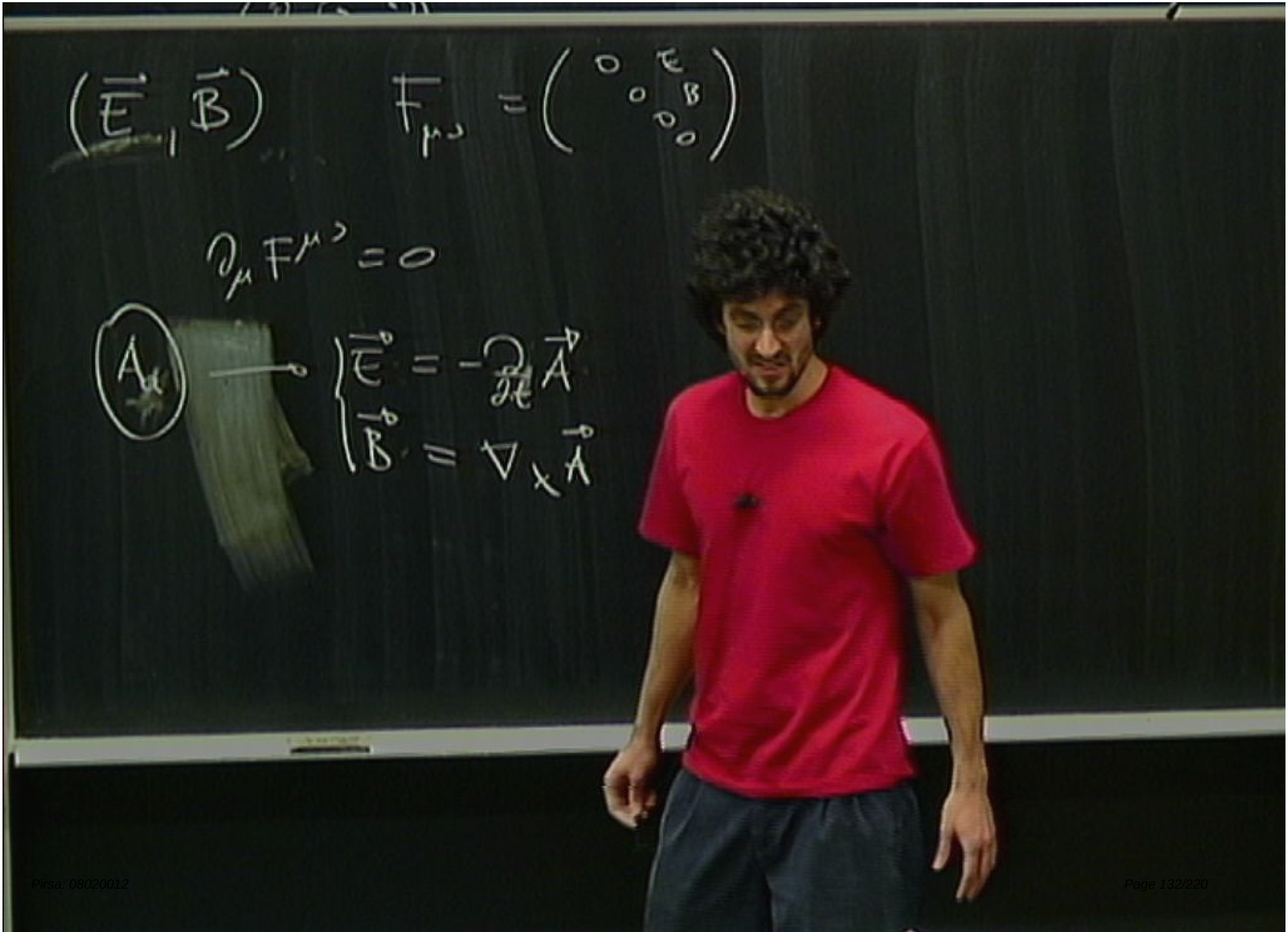
$$\partial_\mu F^{\mu\nu} = 0$$

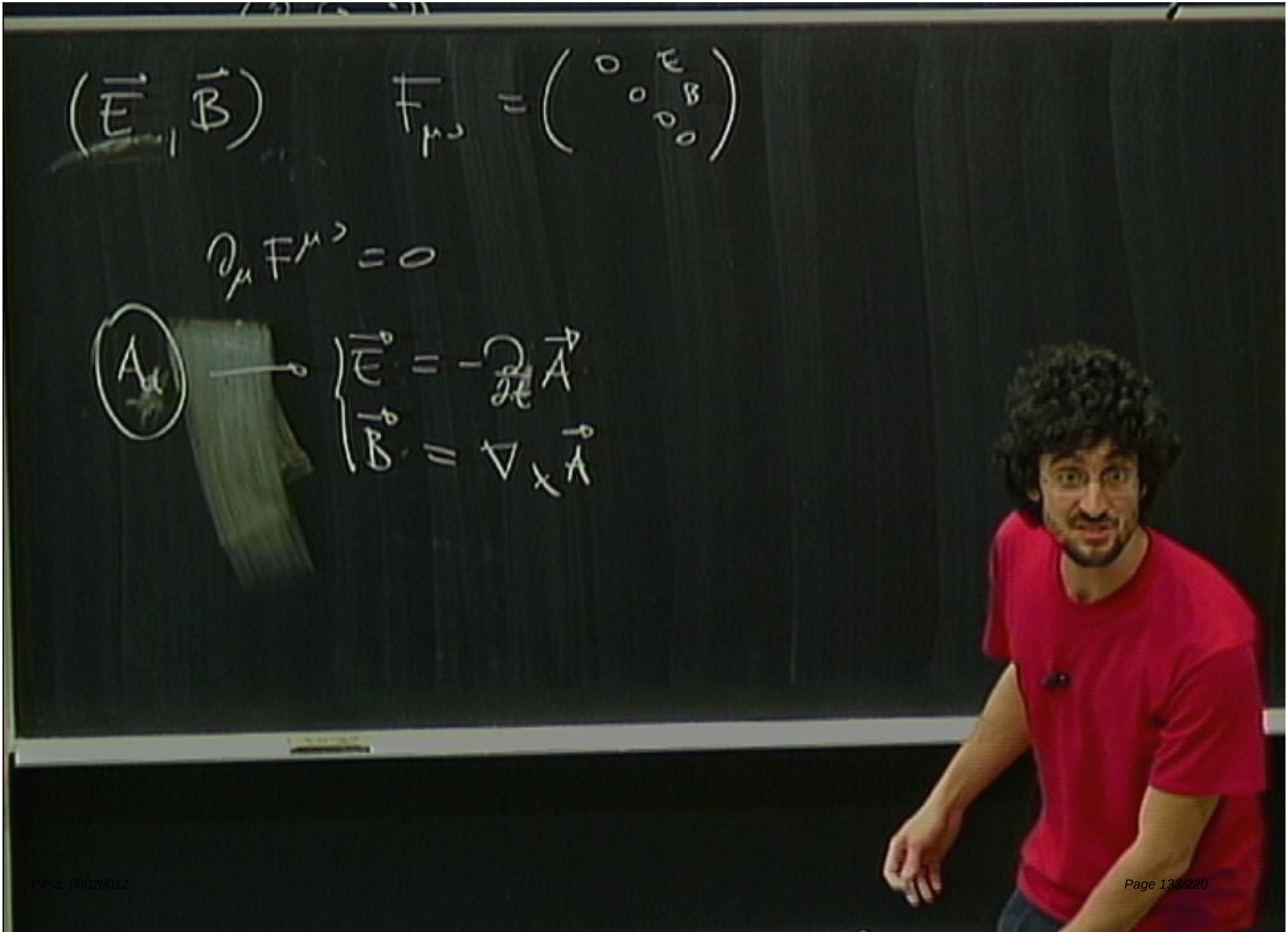
$$A_\mu \longrightarrow \vec{E} = -\partial_t \vec{A}$$

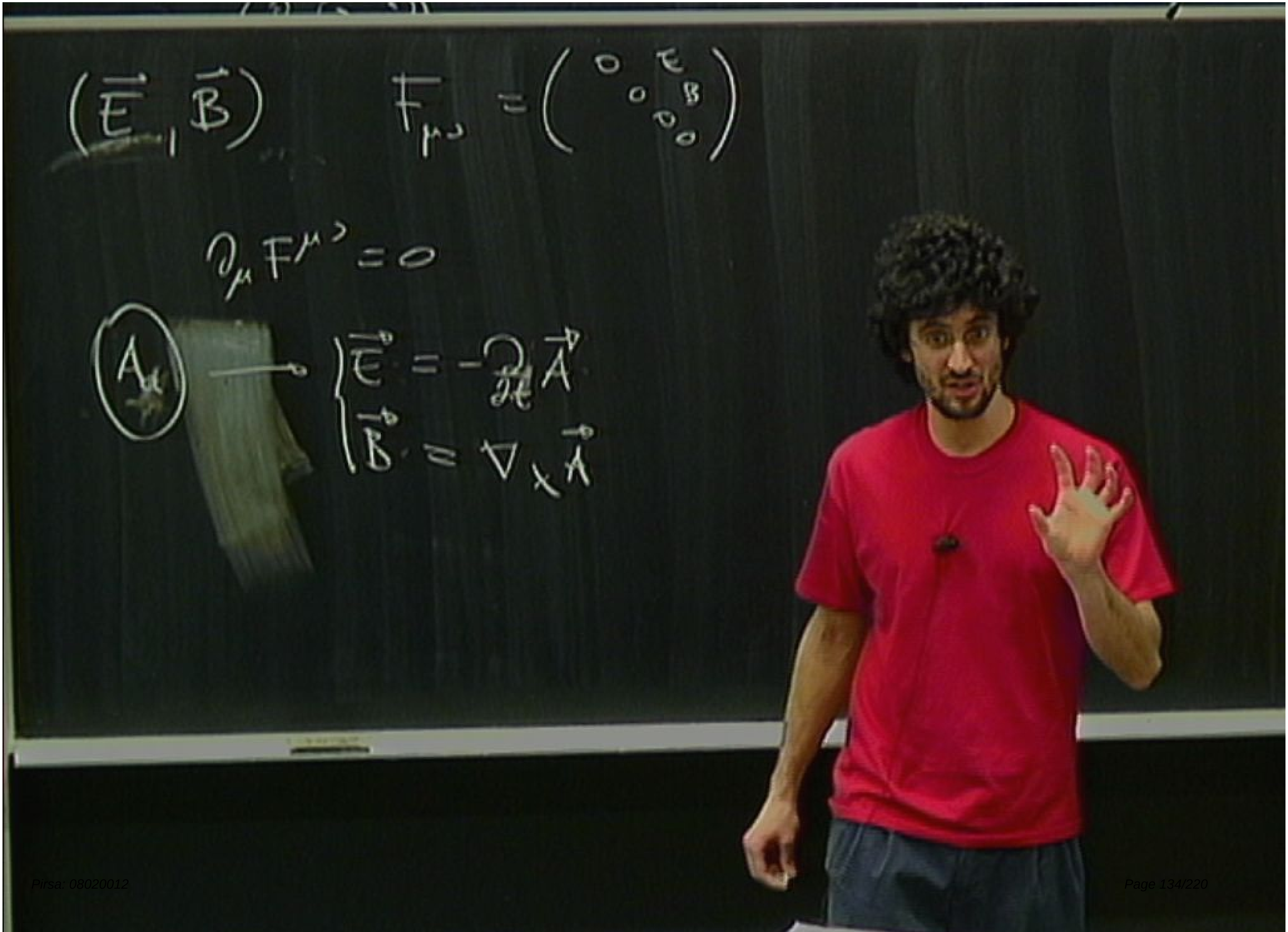
$$(\vec{E}, \vec{B}) \qquad F_{\mu\nu} = \begin{pmatrix} 0 & E & & \\ & 0 & B & \\ & & 0 & \\ & & & 0 \end{pmatrix}$$

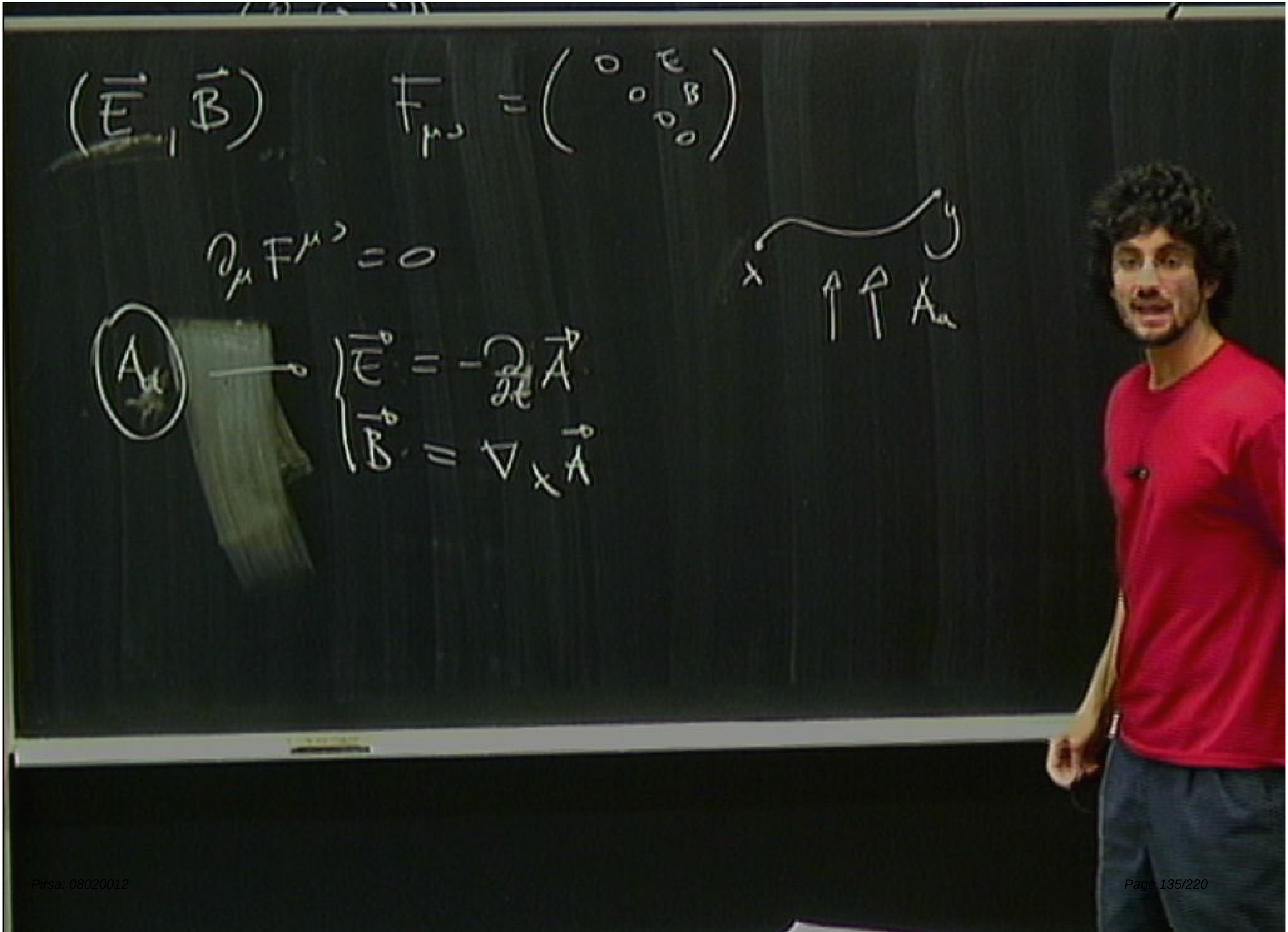
$$\partial_\mu F^{\mu\nu} = 0$$

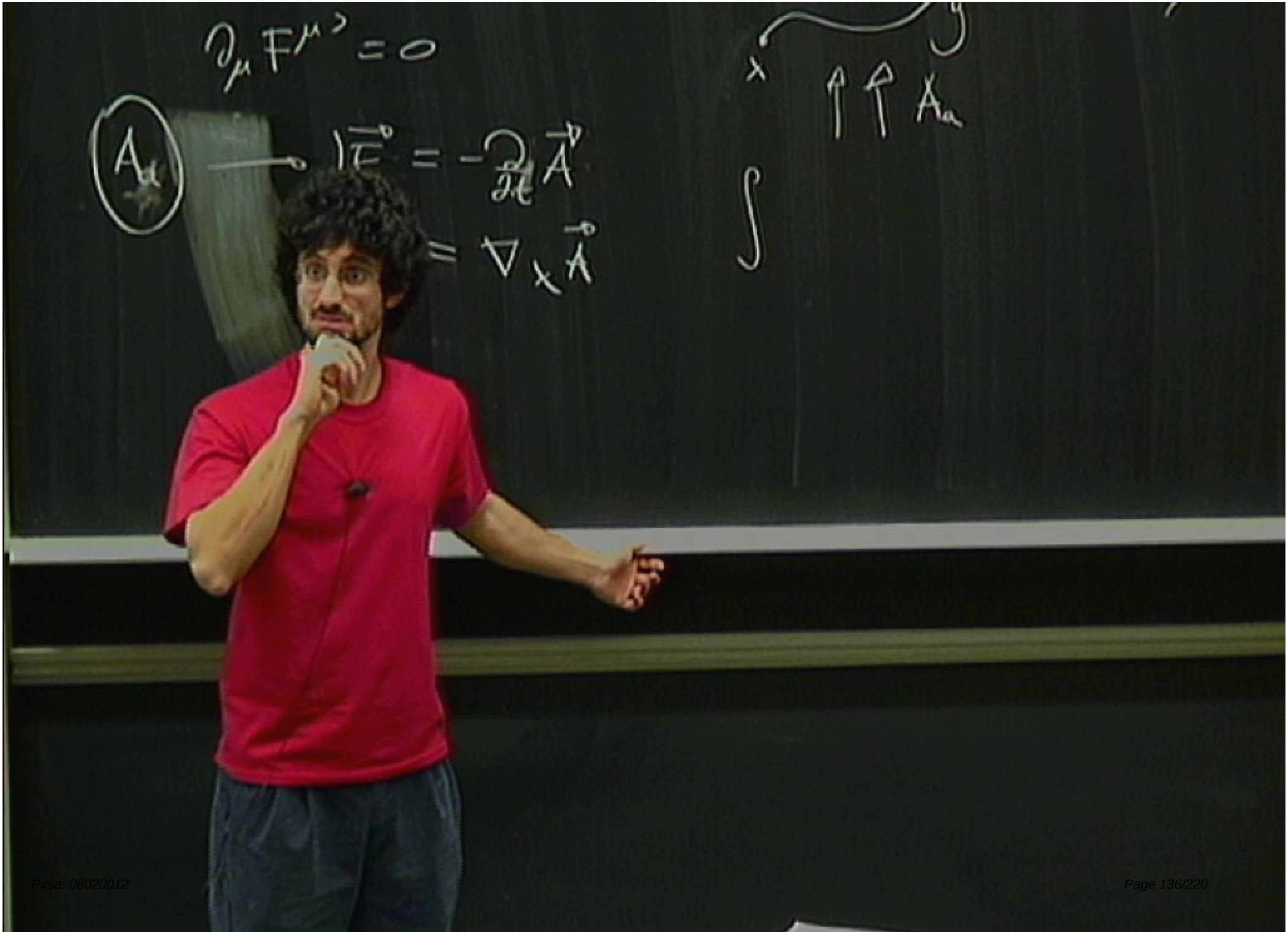
$$A_\mu \longrightarrow \begin{cases} \vec{E} = -\dfrac{\partial \vec{A}}{\partial t} \\ \vec{B} = \nabla \times \vec{A} \end{cases}$$

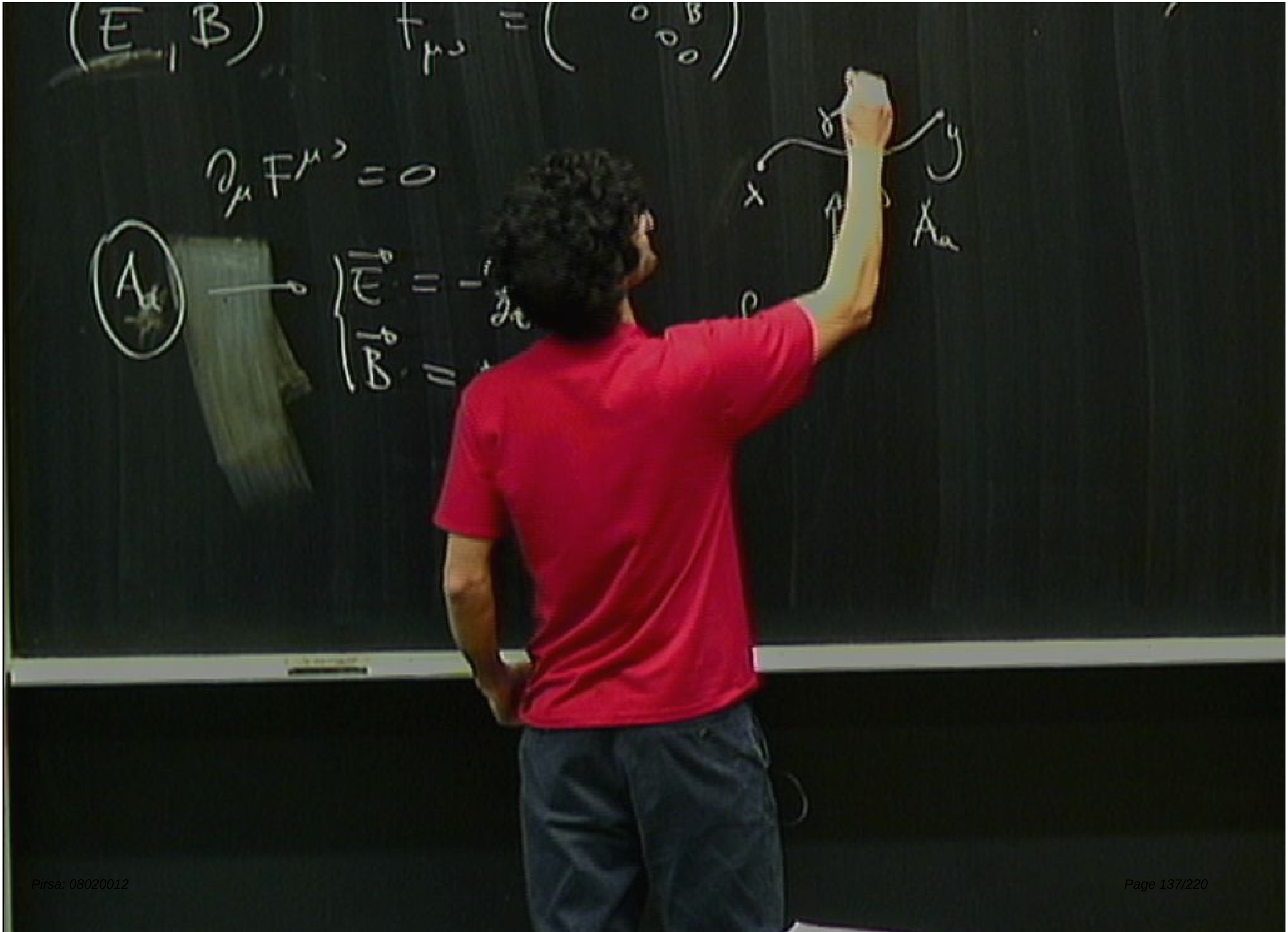

(E, B)
F_μν = (0 E 0 B 0 0)
∂_μ F^μν = 0
A_μ
E = -∂A/∂t
B = ∇ × A


$(\vec{E}, \vec{B})$
$F_{\mu\nu} = \begin{pmatrix} 0 & E \\ & 0 & B \\ & & 0 \\ & & & 0 \end{pmatrix}$
$\partial_\mu F^{\mu\nu} = 0$
$A_\mu \to \begin{cases} \vec{E} = -\frac{\partial \vec{A}}{\partial t} \\ \vec{B} = \nabla \times \vec{A} \end{cases}$


$(\vec{E}, \vec{B})$
$F_{\mu\nu} = \begin{pmatrix} 0 & E & & \\ & 0 & B & \\ & & 0 & \\ & & & 0 \end{pmatrix}$
$\partial_\mu F^{\mu\nu} = 0$
$A_\mu \longrightarrow \begin{cases} \vec{E} = -\frac{\partial \vec{A}}{\partial t} \\ \vec{B} = \nabla \times \vec{A} \end{cases}$


$(\vec{E}, \vec{B})$
$F_{\mu\nu} = \begin{pmatrix} 0 & E & & \\ & 0 & B & \\ & & 0 & \\ & & & 0 \end{pmatrix}$
$\partial_\mu F^{\mu\nu} = 0$
A_μ
$\vec{E} = -\frac{\partial}{\partial t}\vec{A}$
$\vec{B} = \nabla \times \vec{A}$
x
y
A_μ


$\partial_\mu F^{\mu\nu} = 0$
A_α
$\vec{E} = -\frac{\partial \vec{A}}{\partial t}$
$= \nabla \times \vec{A}$
x
y
A_α
$\int$


(E, B)
∂μ Fμν = 0
Aα
x
y
γ
Aα

$(\vec{E}, \vec{B})$ $\qquad F_{\mu\nu} = \begin{pmatrix} 0 & E & & \\ & 0 & B & \\ & & 0 & \\ & & & 0 \end{pmatrix}$

$s \in [0,1]$: $\gamma^a(s)$

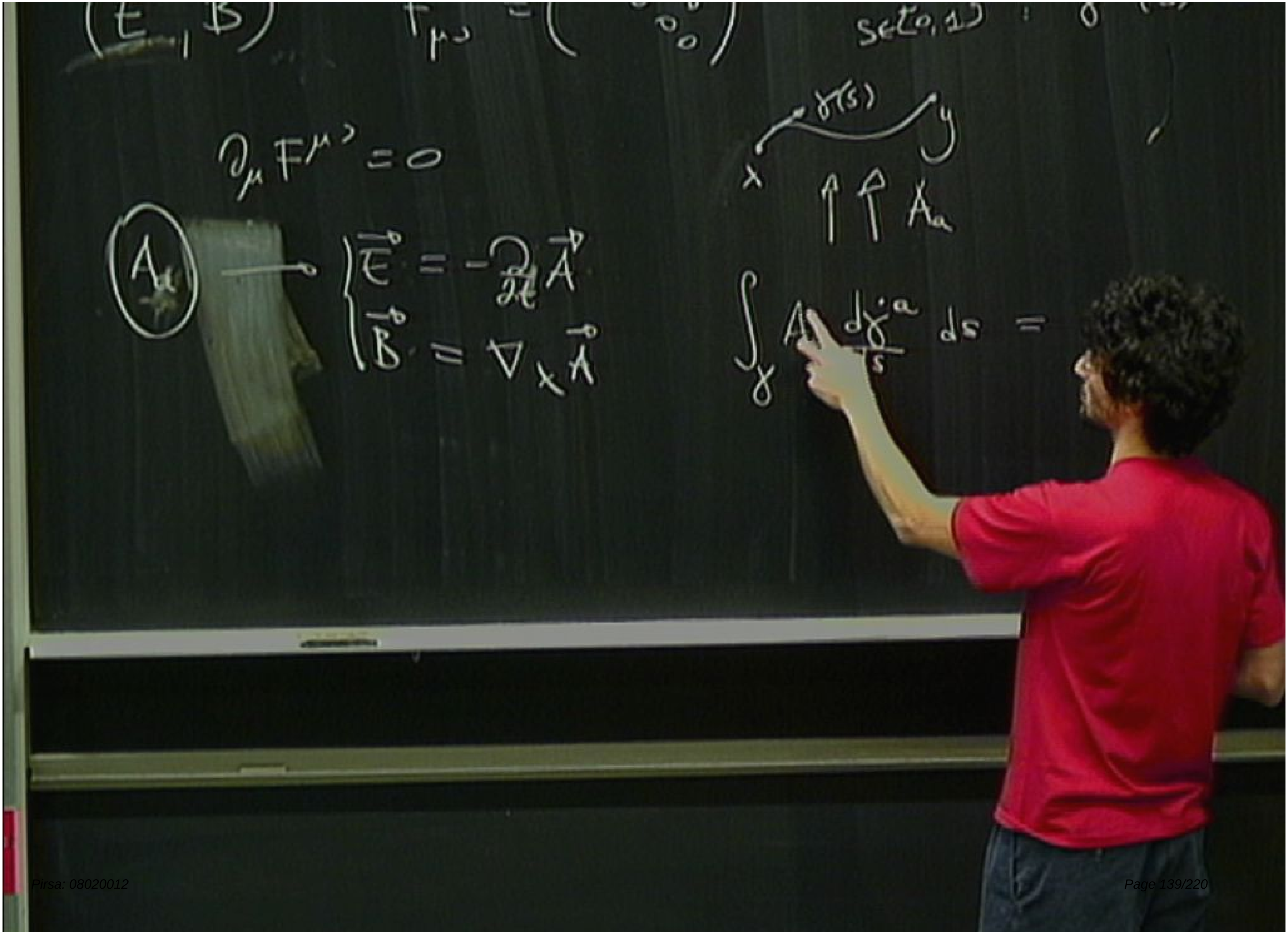
$\partial_\mu F^{\mu\nu} = 0$

$x \xrightarrow{\gamma(s)} y$

$\uparrow \; \uparrow \; A_a$

$$A_a \longrightarrow \begin{cases} \vec{E} = -\frac{\partial}{\partial t} \vec{A} \\ \vec{B} = \nabla \times \vec{A} \end{cases}$$

$\int$


$\partial_\mu F^{\mu\nu} = 0$
A_a
$\vec{E} = -\frac{\partial}{\partial t}\vec{A}$
$\vec{B} = \nabla \times \vec{A}$
$s \in [0,1]$
$\gamma(s)$
x
y
A_a
$\int_\gamma A_a \frac{d\gamma^a}{ds} ds =$

$(\vec{E}, \vec{B})$ $\quad F_{\mu\nu} = \left(\dots \right)$ $\quad s \in [0,1] : \gamma(s)$

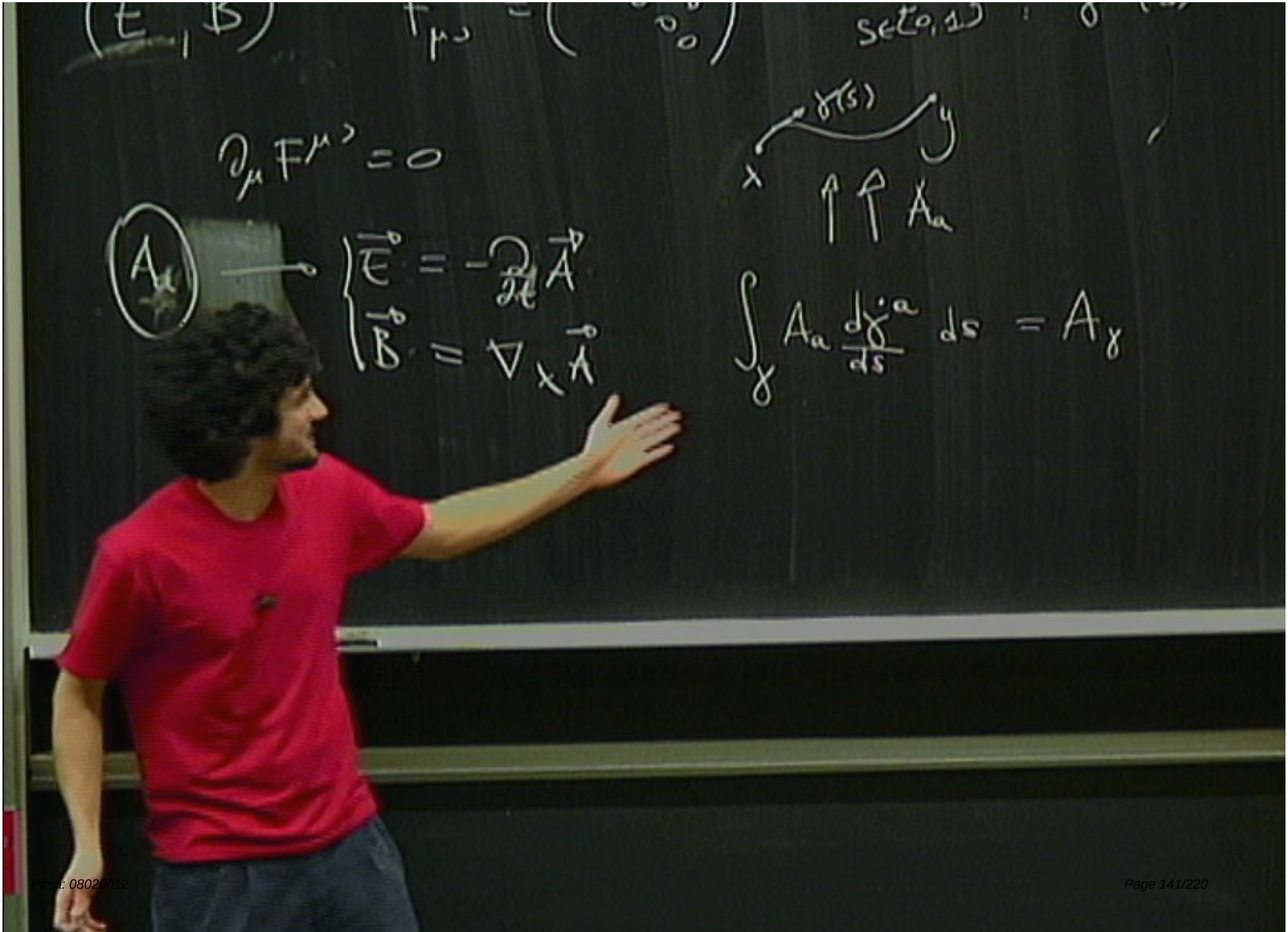
$$\partial_\mu F^{\mu\nu} = 0$$

$$A_a \longrightarrow \begin{cases} \vec{E} = -\frac{\partial}{\partial t}\vec{A} \\ \vec{B} = \nabla \times \vec{A} \end{cases}$$

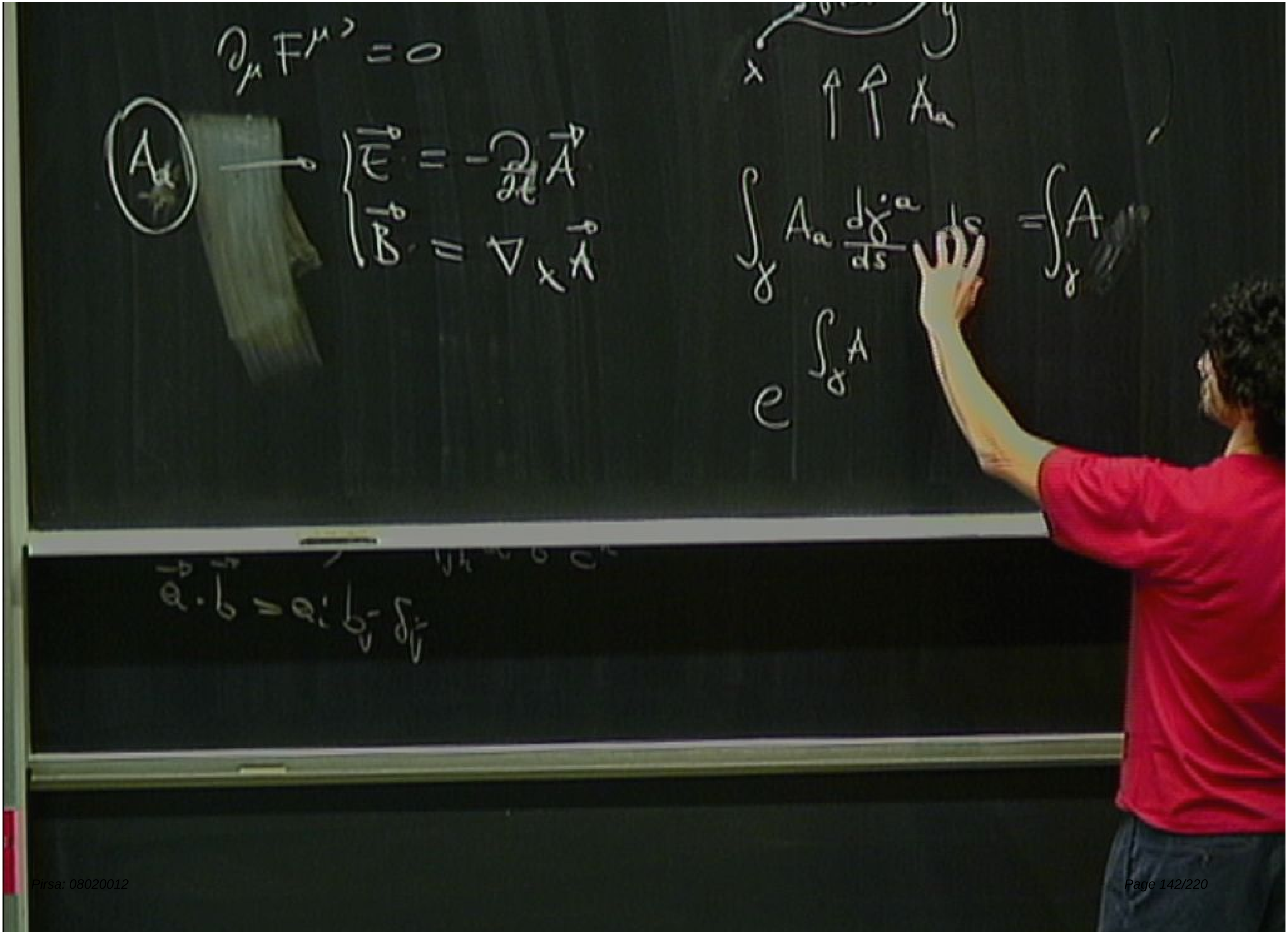
x $\quad \gamma(s)$ $\quad y$

$\uparrow \uparrow A_a$

$$\int_\gamma A_a \frac{d\gamma^a}{ds} ds$$



$(\vec{E}, \vec{B})$
$F_{\mu\nu} =$
$s \in [0,1]$
$\gamma(s)$
x
y
$\partial_\mu F^{\mu\nu} = 0$
A_a
A_μ
$\vec{E} = -\frac{\partial}{\partial t}\vec{A}$
$\vec{B} = \nabla \times \vec{A}$
$\int_\gamma A_a \frac{d\gamma^a}{ds} ds = A_\gamma$


$\partial_\mu F^{\mu\nu} = 0$
A_a
$\vec{E} = -\partial_t \vec{A}$
$\vec{B} = \nabla \times \vec{A}$
A_a
$\int_\gamma A_a \frac{d\gamma^a}{ds} ds = \int_\gamma A$
$e^{\int_\gamma A}$
$\vec{a} \cdot \vec{b} = a_i b_j \delta_{ij}$

A_a

$$\begin{cases} \vec{E} = -\partial_t \vec{A} \\ \vec{B} = \nabla \times \vec{A} \end{cases}$$

A_a

$$\int_\gamma A_a \frac{d\gamma^a}{ds}\, ds = \int_\gamma A$$

$$e^{\int_\gamma A}$$

$\vec{a} \cdot (\vec{b} \times \vec{c}) = \varepsilon_{ijk}\, a^i b^j c^k$

$\vec{a} \cdot \vec{b} = a^i b^j \delta_{ij}$

A_a

$$A_a \longrightarrow \begin{cases} \vec{E} = -\partial_t \vec{A} \\ \vec{B} = \nabla \times \vec{A} \end{cases}$$

$$\phi(x)$$

$$\int_\gamma A_a \frac{d\gamma^a}{ds}\, ds = \int_\gamma A$$

$$e^{\int_\gamma A}$$

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = \varepsilon_{ijk}\, a^i b^j c^k$$

$$\vec{a} \cdot \vec{b} = a^i b^j \delta_{ij}$$

A_a

$$\begin{cases} \vec{E} = -\partial_t \vec{A} \\ \vec{B} = \nabla \times \vec{A} \end{cases}$$

$\phi(x)$

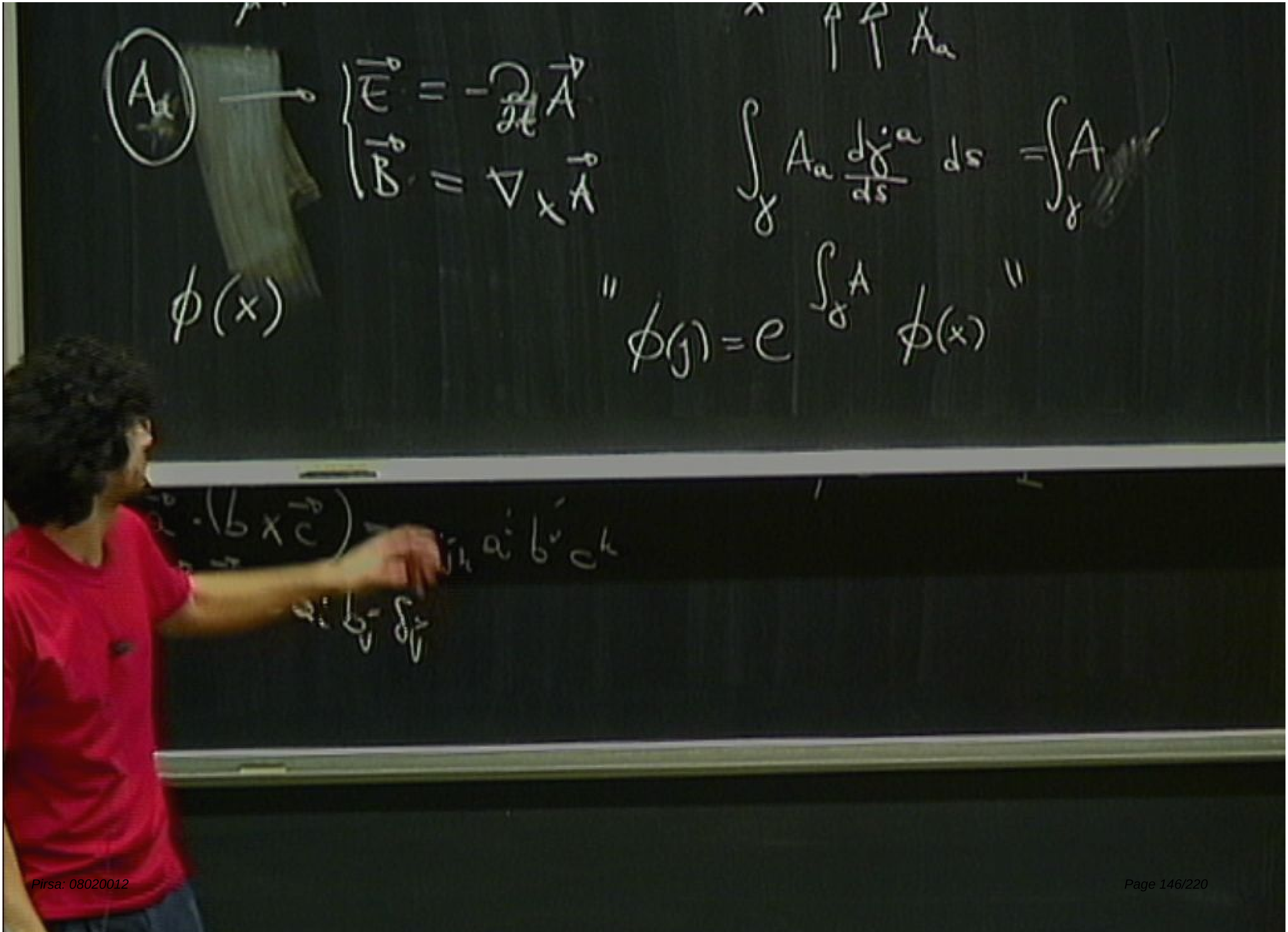
A_a

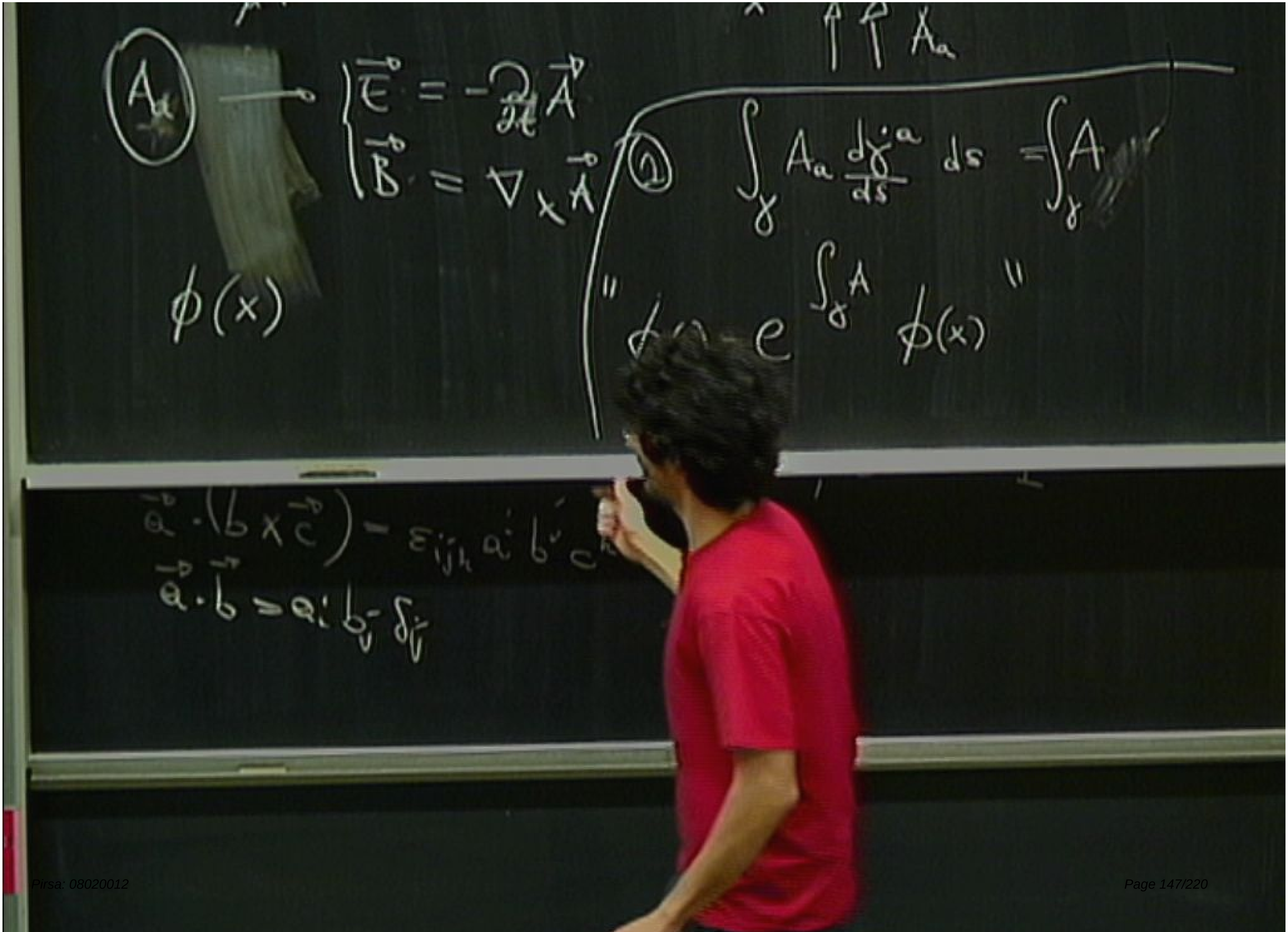
$$\int_\gamma A_a \frac{d\gamma^a}{ds}\, ds = \int_\gamma A$$

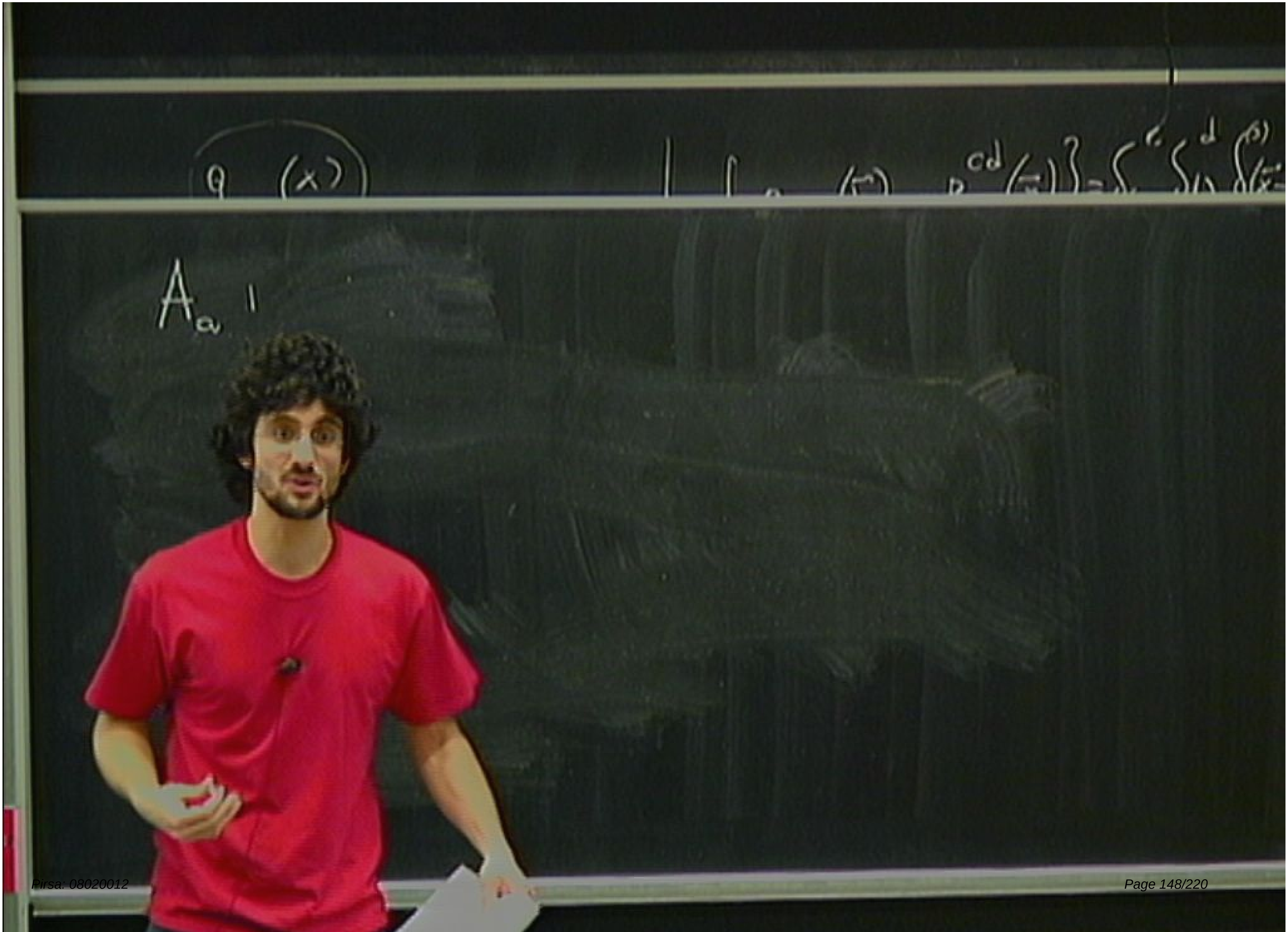
$$e^{\int_\gamma A}$$

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = \varepsilon_{ijk}\, a^i b^j c^k$$

$$\vec{a} \cdot \vec{b} = a^i b_j \delta_i^j$$

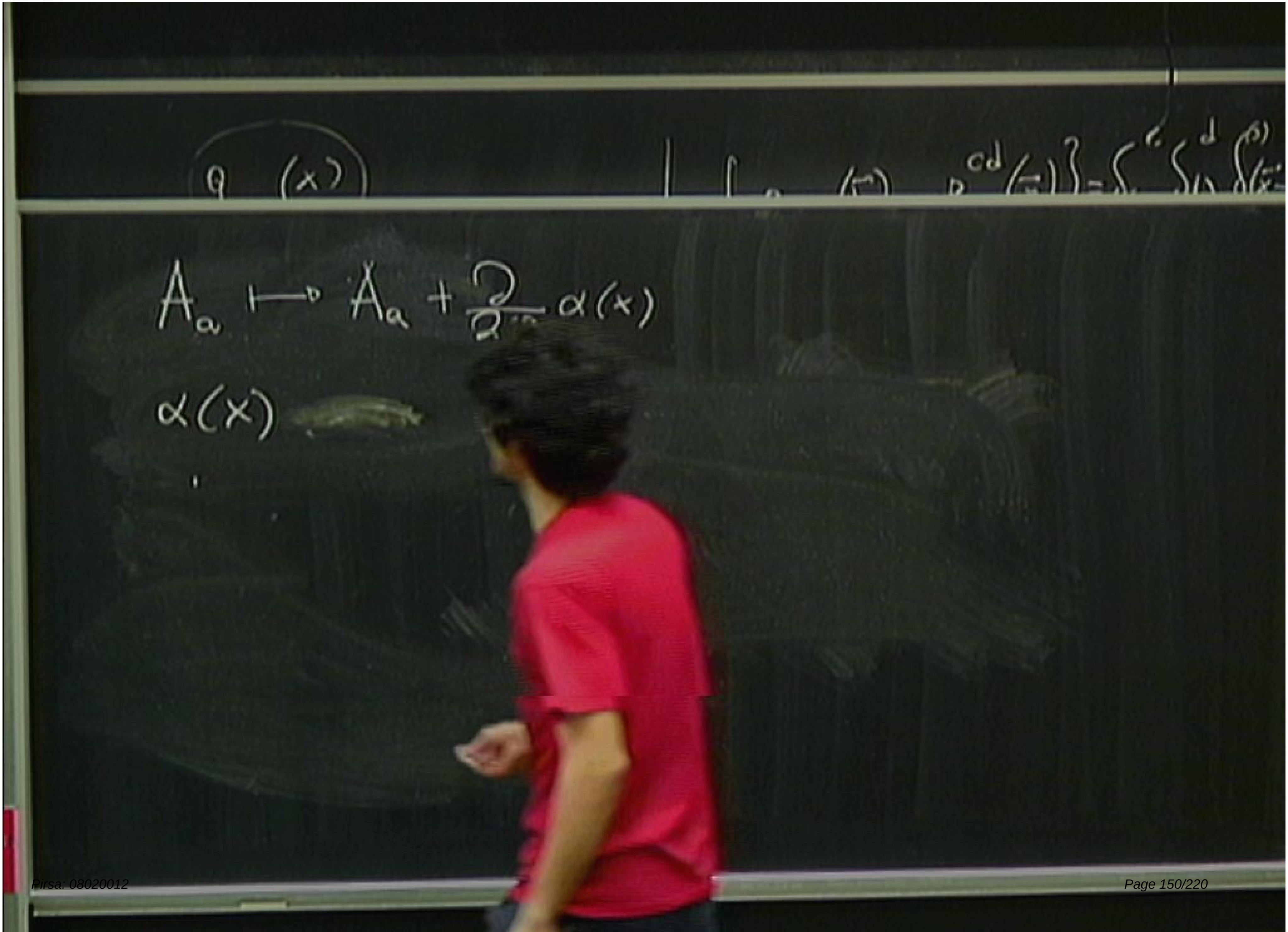

A_a
E = -∂_t A
B = ∇ × A
∫_γ A_a dγ^a/ds ds = ∫_γ A
φ(x)
"φ(γ) = e^{∫_γ A} φ(x)"
a·(b × c) = ε_ijk a^i b^j c^k


A_a
$\vec{E} = -\frac{\partial}{\partial t}\vec{A}$
$\vec{B} = \nabla \times \vec{A}$
$\int_\gamma A_a \frac{d\gamma^a}{ds} ds = \int_\gamma A$
$\phi(x)$
" $\phi(y)\, e^{\int_\gamma A}\, \phi(x)$ "
$\vec{a}\cdot(\vec{b}\times\vec{c}) = \varepsilon_{ijk} a^i b^j c^k$
$\vec{a}\cdot\vec{b} = a^i b^j \delta_{ij}$

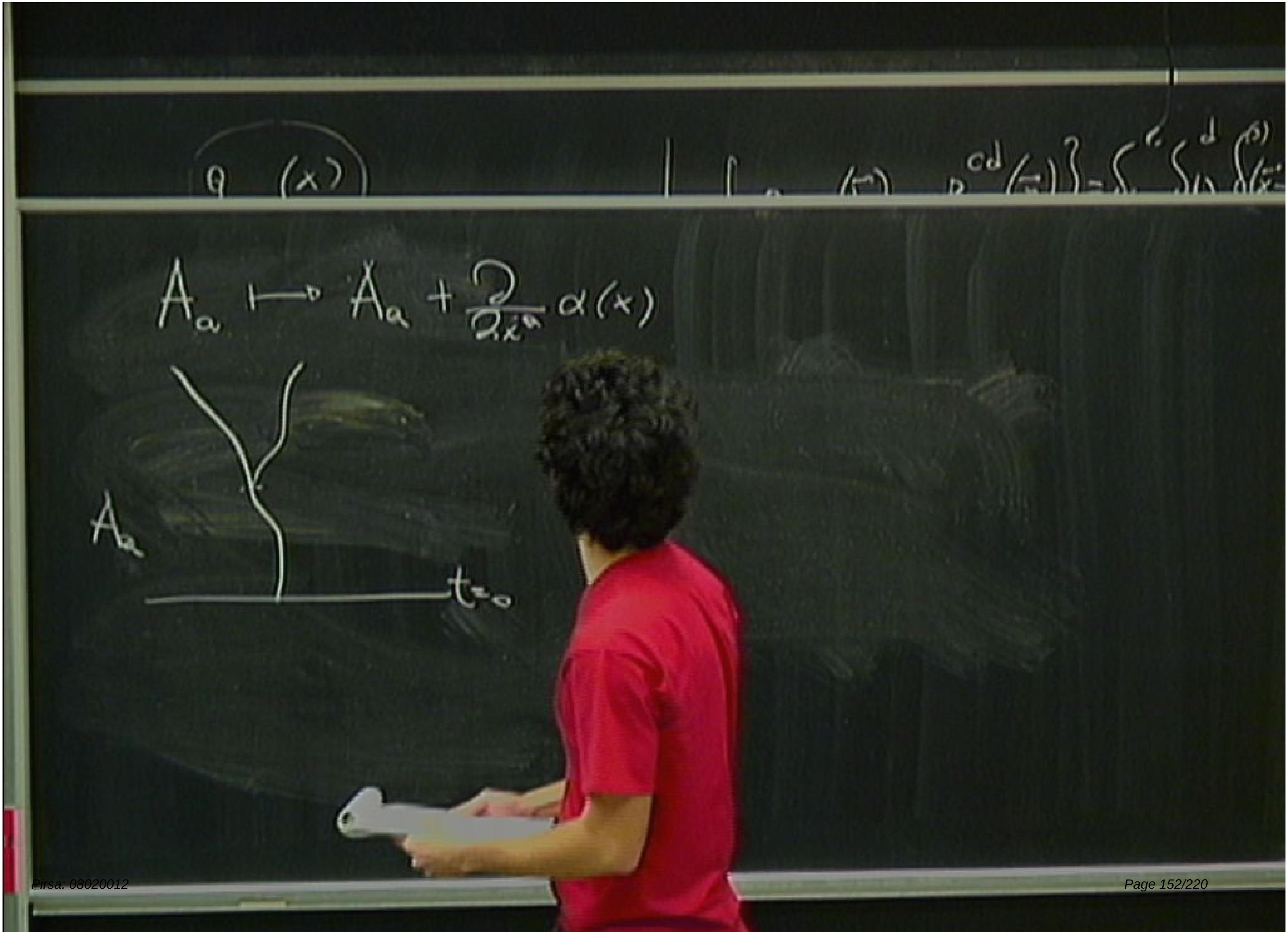


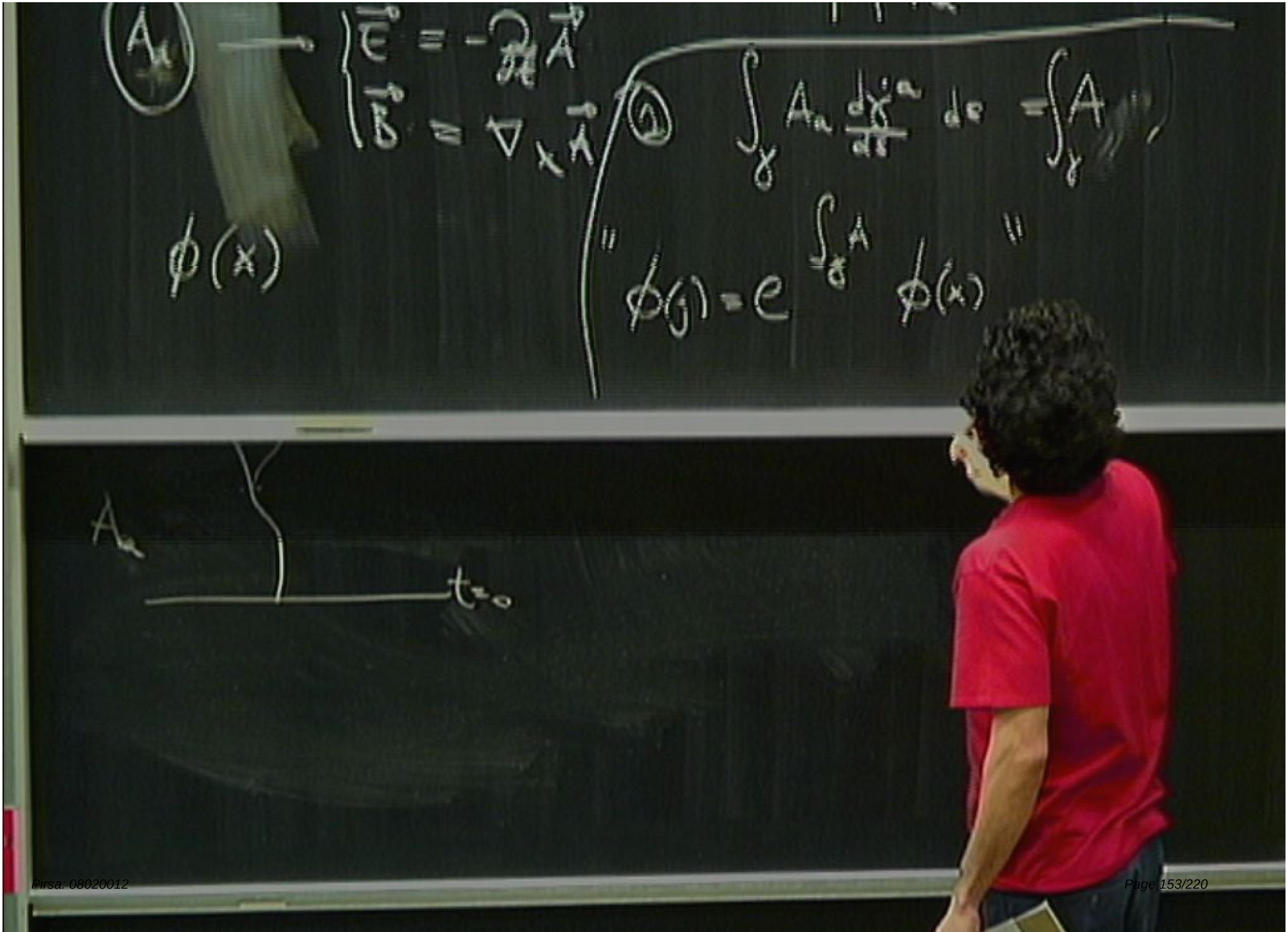
$$A_a \mapsto A_a + \partial$$

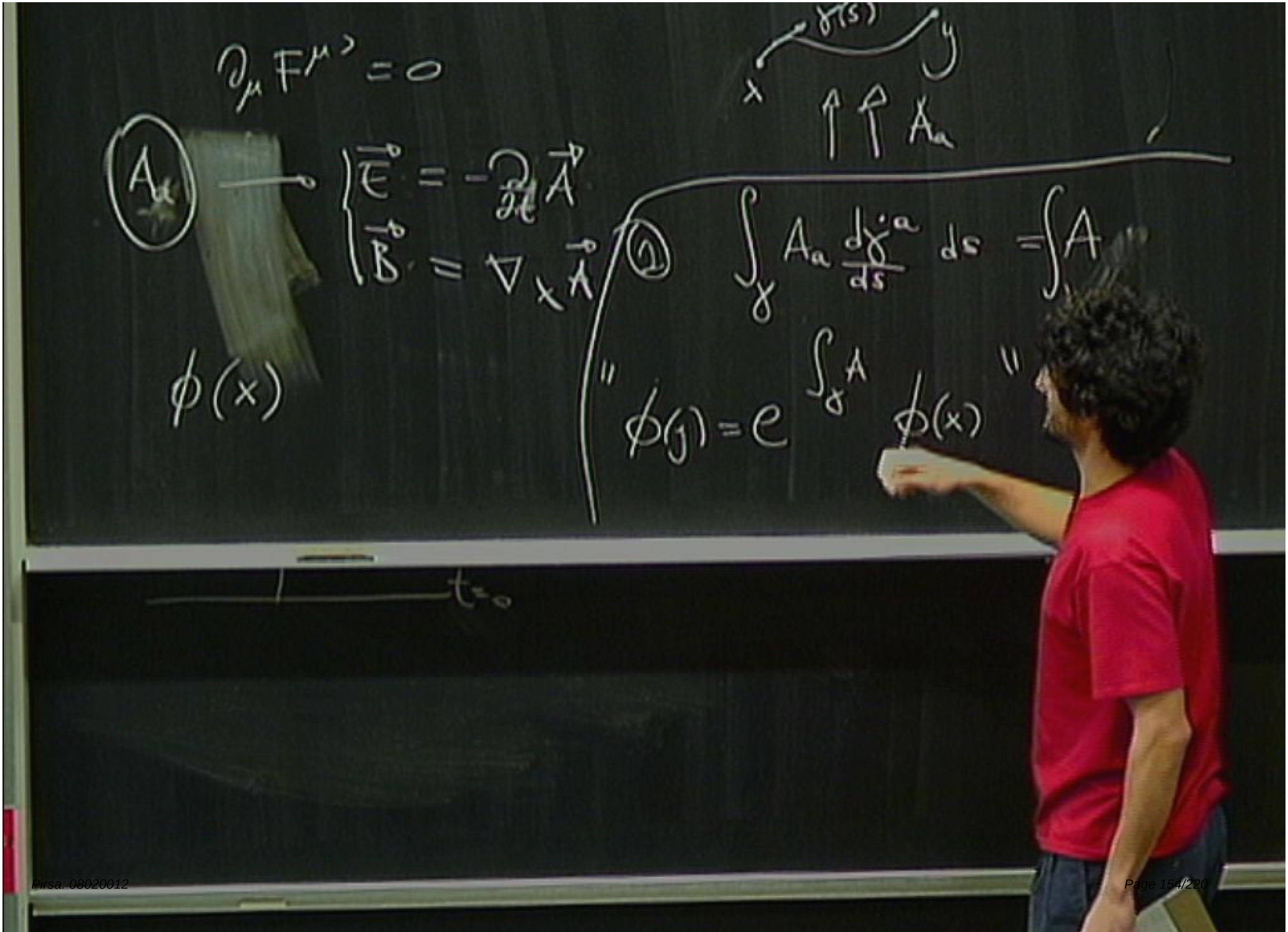
$$\alpha(x)$$

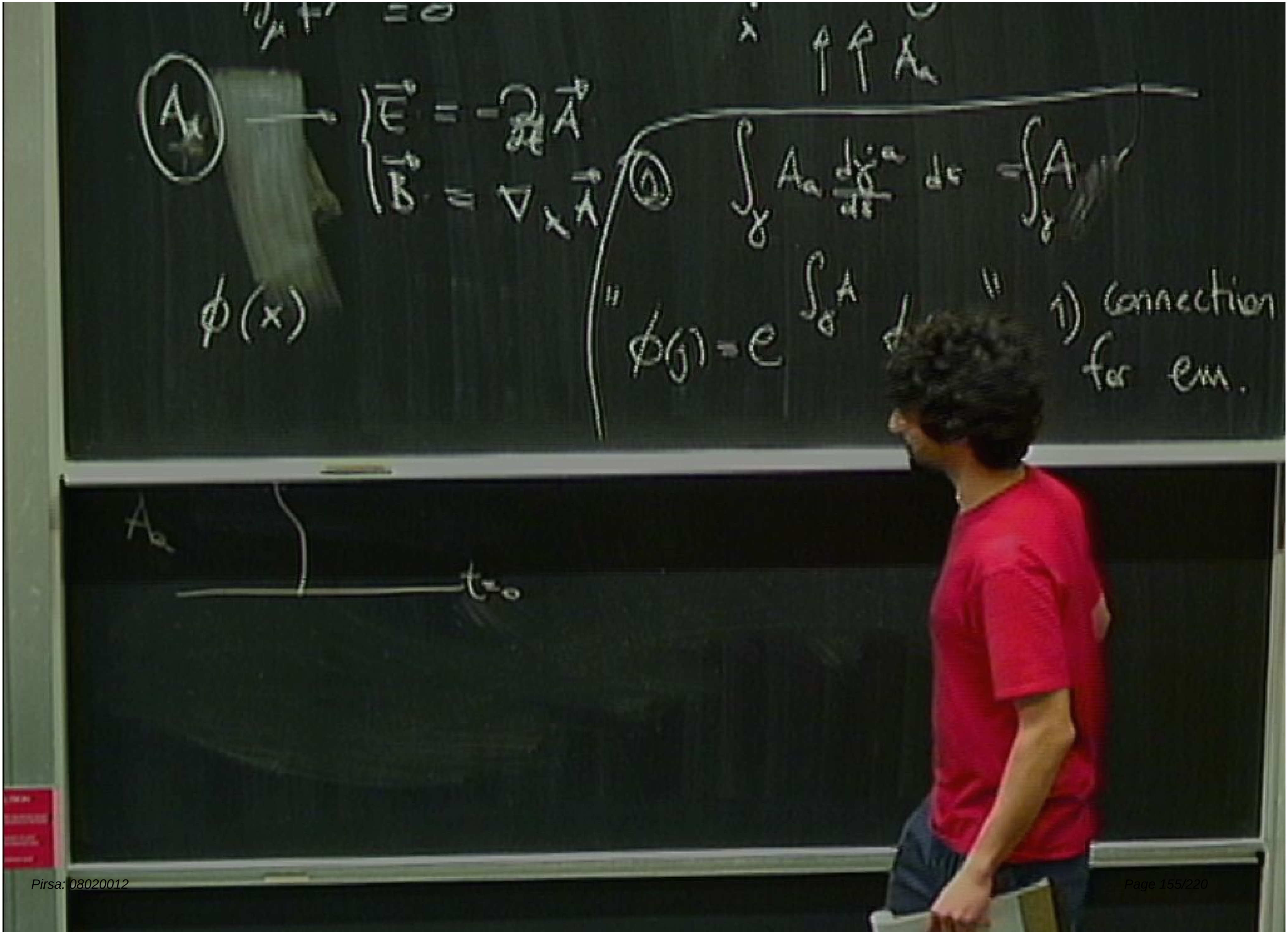

$A_a \longmapsto A_a + \partial \alpha(x)$
$\alpha(x)$

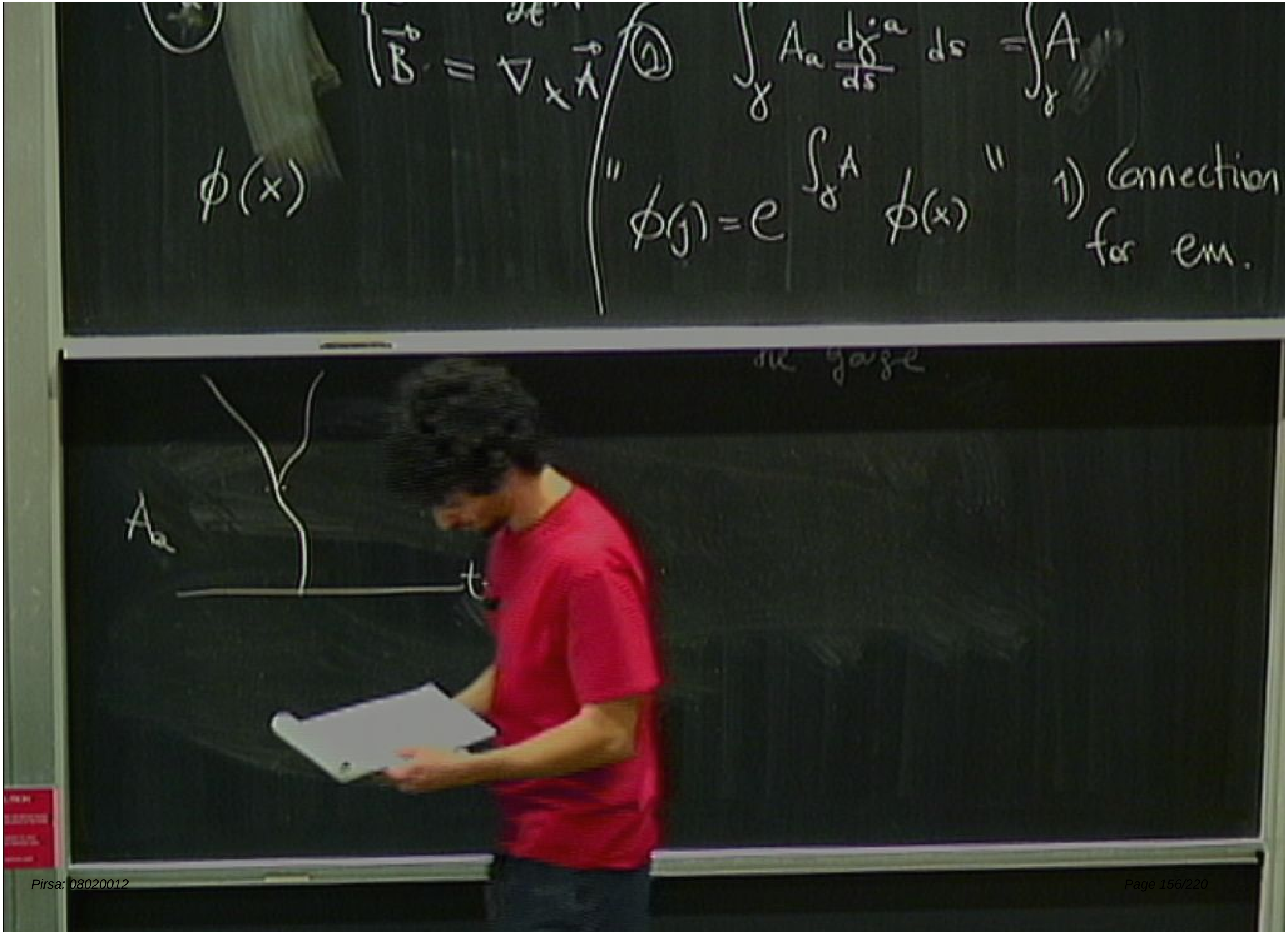
$$A_a \longmapsto A_a + \frac{\partial}{\partial x^a}\alpha(x)$$


$A_a \mapsto A_a + \frac{\partial}{\partial x^a} \alpha(x)$
A_a
$t=0$


A_a
$\vec{E} = -\partial_t \vec{A}$
$\vec{B} = \nabla \times \vec{A}$
$\int_\gamma A_a \frac{d\gamma^a}{ds} ds = \int_\gamma A$
$\phi(x)$
" $\phi(\gamma) = e^{\int_\gamma A} \phi(x)$ "
A_a
$t=0$


∂μ Fμν = 0
Aμ
E = −∂t A
B = ∇ × A
φ(x)
γ(s)
x
y
Aa
∫γ Aa dγa/ds ds = ∫A
" φ(y) = e^∫γ A φ(x) "
t=0


A_a
E = -∂_t A
B = ∇ × A
∫_γ A_a dγ^a/dt dt = ∫_γ A
φ(x)
"φ(γ) = e^{∫_γ A} φ"
1) connection for em.
A_a
t=0


B = ∇ × A
∫γ Aa dγ^a/ds ds = ∫γ A
φ(x)
"φ(γ) = e^∫γA φ(x)"
1) Connection for em.
Aa
t

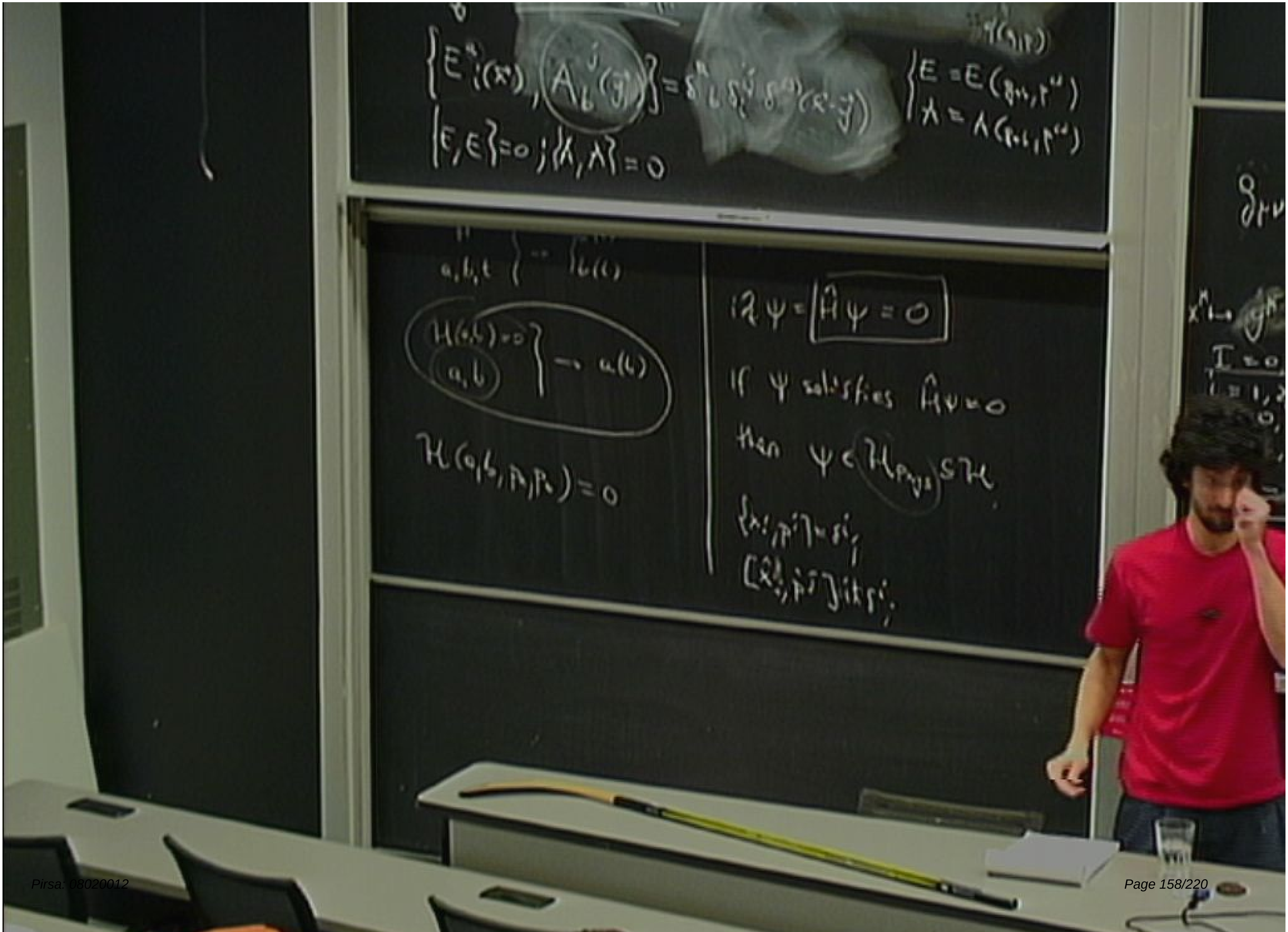
$\vec{B} = \nabla \times \vec{A}$ (2) $\int_\gamma A_a \frac{d\gamma^a}{ds} ds = \int_\gamma A$

$\phi(x)$

" $\phi(\gamma) = e^{\int_\gamma A} \phi(x)$ "

1) Connection for em.

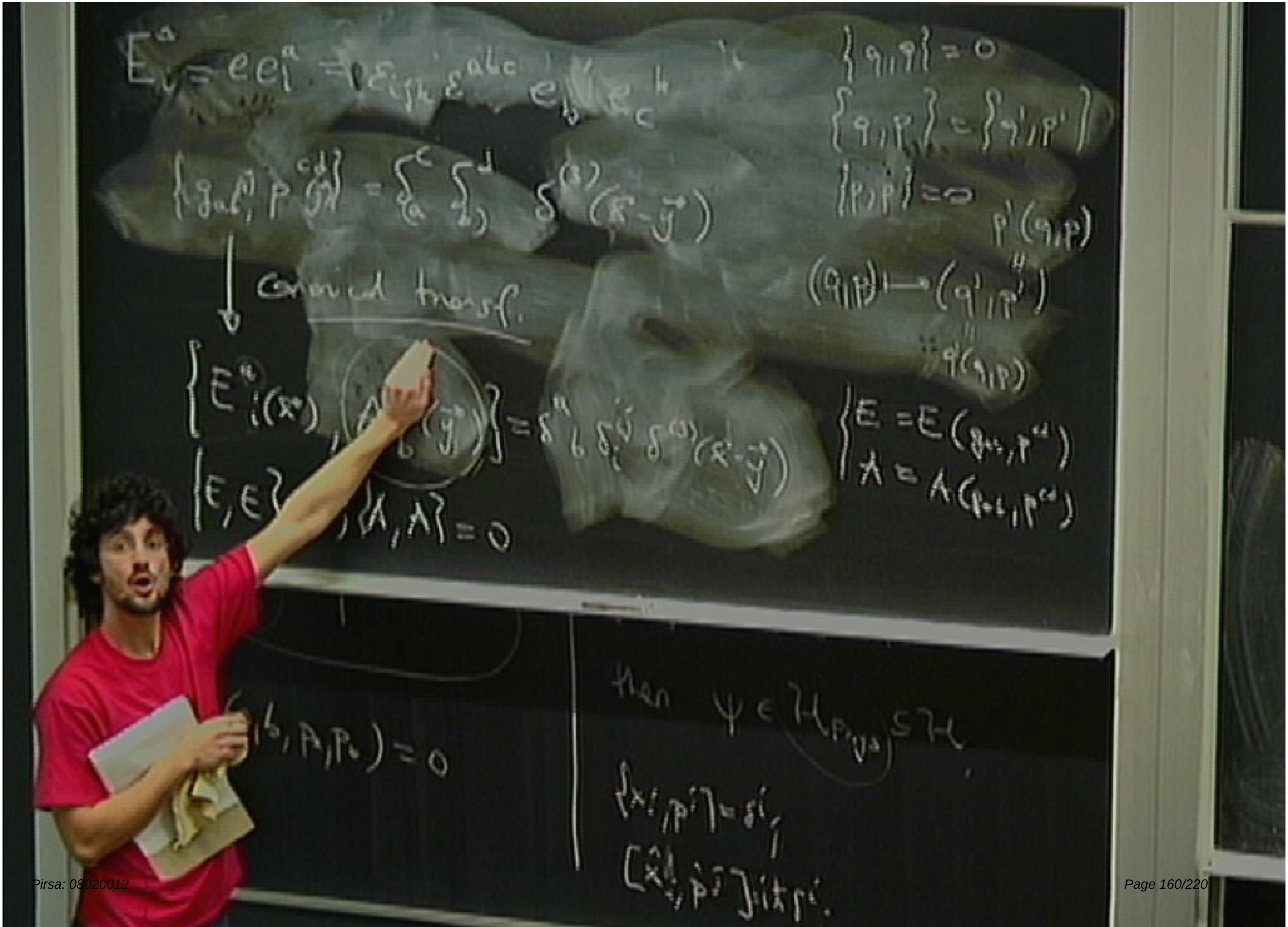
the gauge

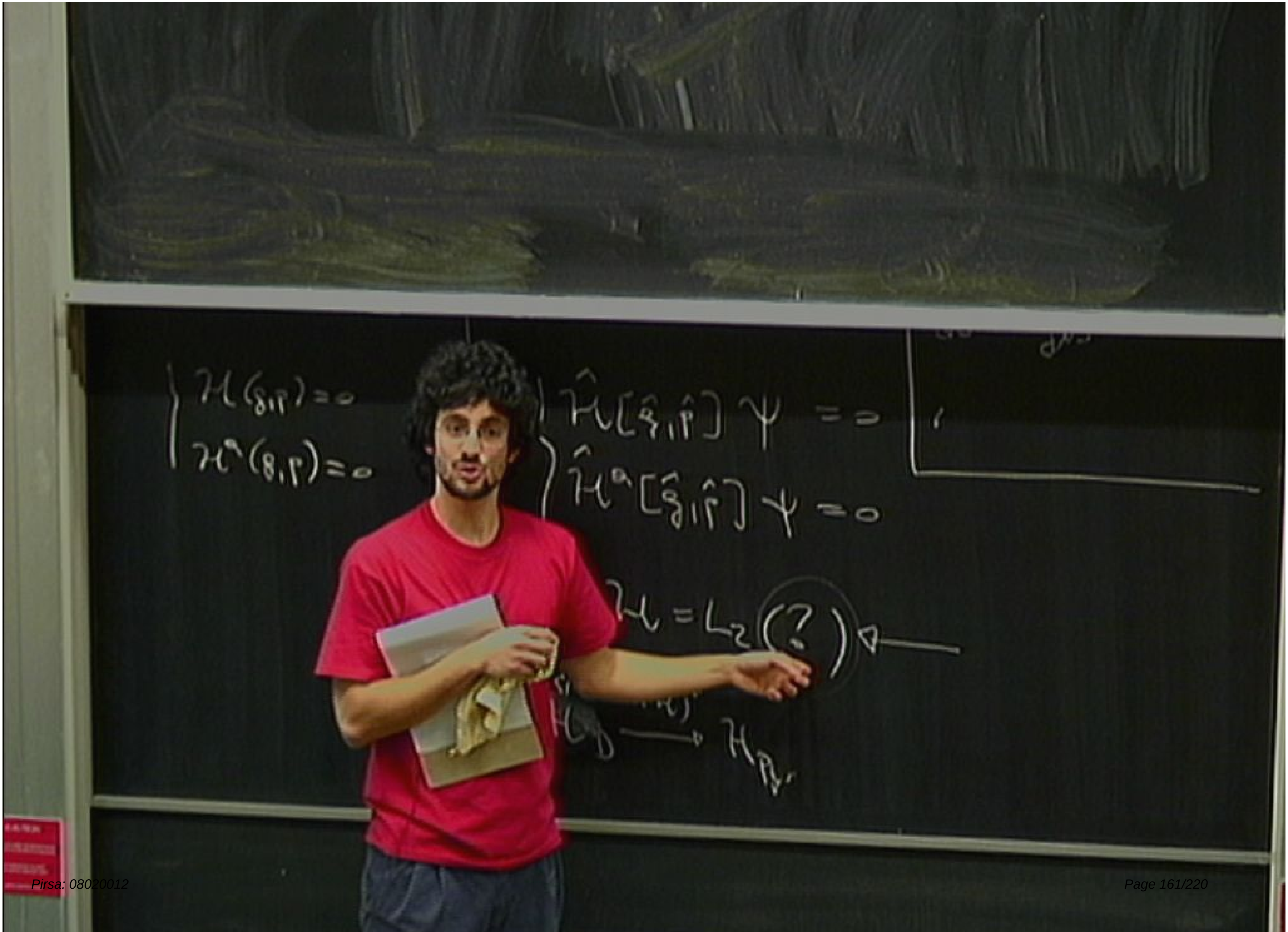

{E,E}=0 ; {A,A}=0
H(a,b)=0
a,b
H(q,b,p_a,p_b)=0
i∂_t ψ = Ĥψ = 0
If ψ satisfies Ĥψ=0
then ψ ∈ H_phys ⊆ H

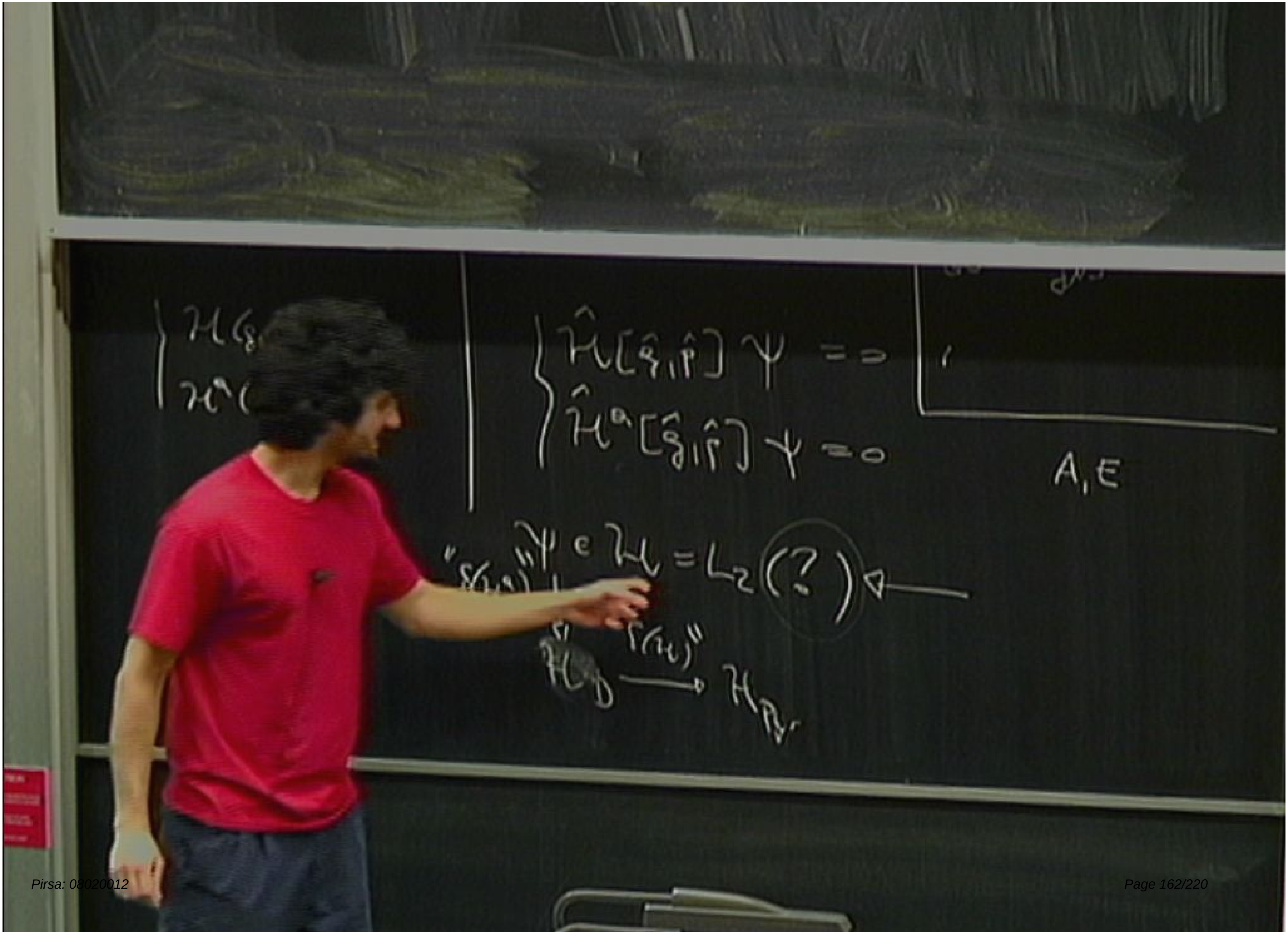
$$g_{\mu\nu} = \eta_{IJ}\, e_\mu{}^I e_\nu{}^J = \begin{pmatrix} -N^2 & N_a \\ N_a & \begin{pmatrix} g_{ab} \end{pmatrix} \end{pmatrix} \quad (g_{00}, g_{0a}) = = g_{0\mu}$$

$$\begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

$$g_{00} = -N^2 (+ N_a N^a)$$


{q,q} = 0
{q,p} = {q',p'}
{p,p} = 0
Canonical transf.
{A,A} = 0
then


$\mathcal{H}(q,p)=0$
$\mathcal{H}^a(q,p)=0$
$\hat{\mathcal{H}}[\hat{q},\hat{p}]\,\psi = 0$
$\hat{\mathcal{H}}^a[\hat{q},\hat{p}]\,\psi = 0$
$\mathcal{H} = L_2\,(?)$
$\mathcal{H}_{Phys}$



Ĥ[q̂, p̂] ψ = 0
Ĥ^a[q̂, p̂] ψ = 0
A, E
ψ ∈ H = L2(?)

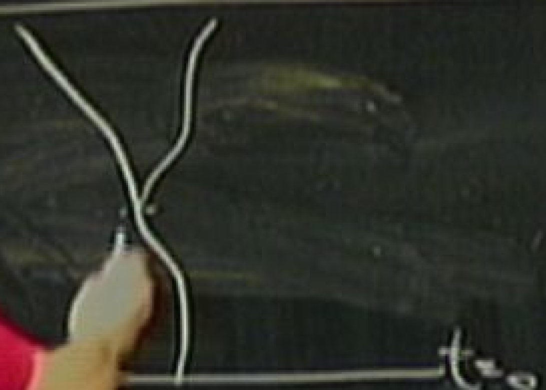
$\phi(x)$

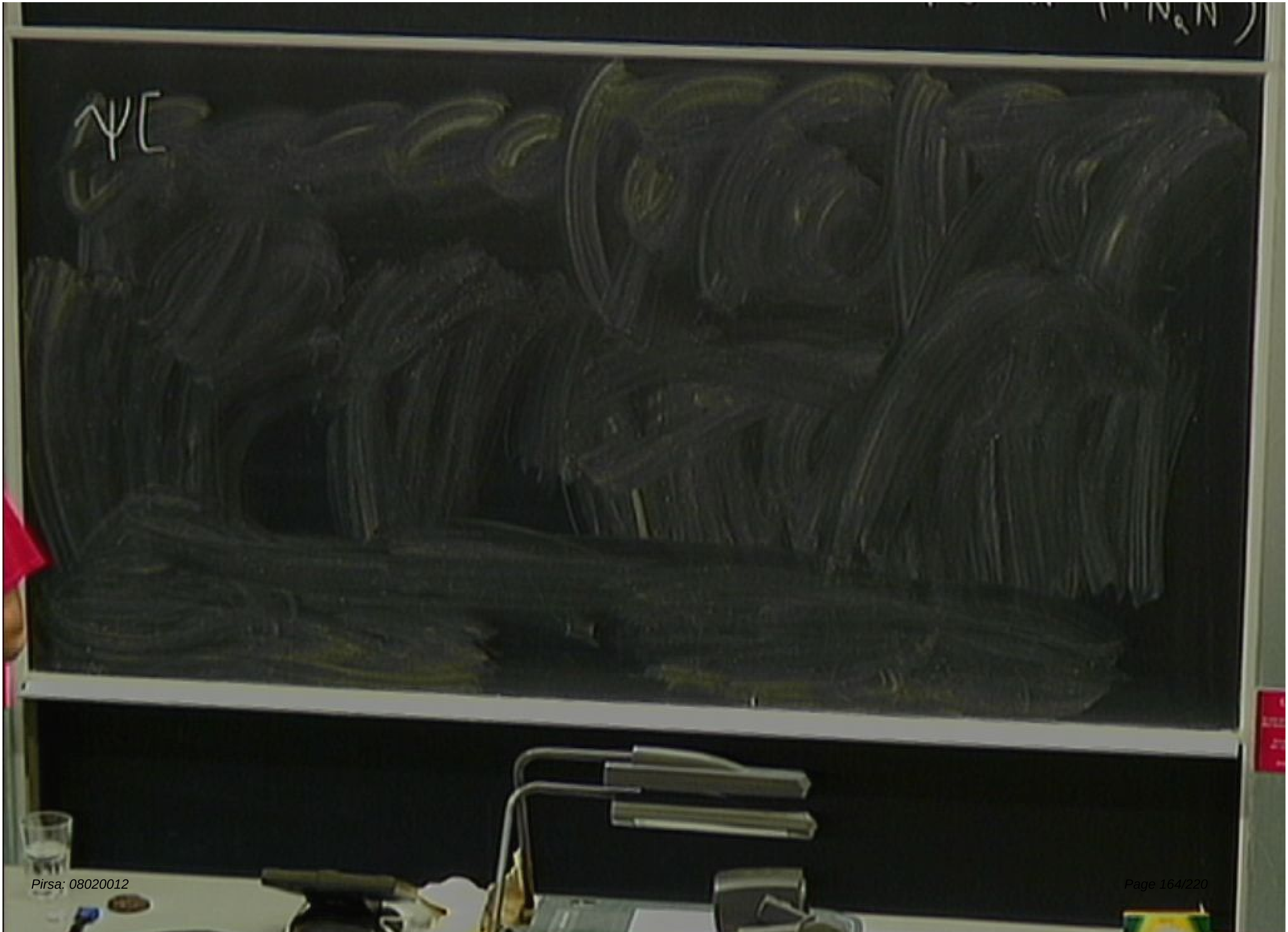
" $\phi(\gamma) = e^{\int_\gamma A} \phi(x)$ "

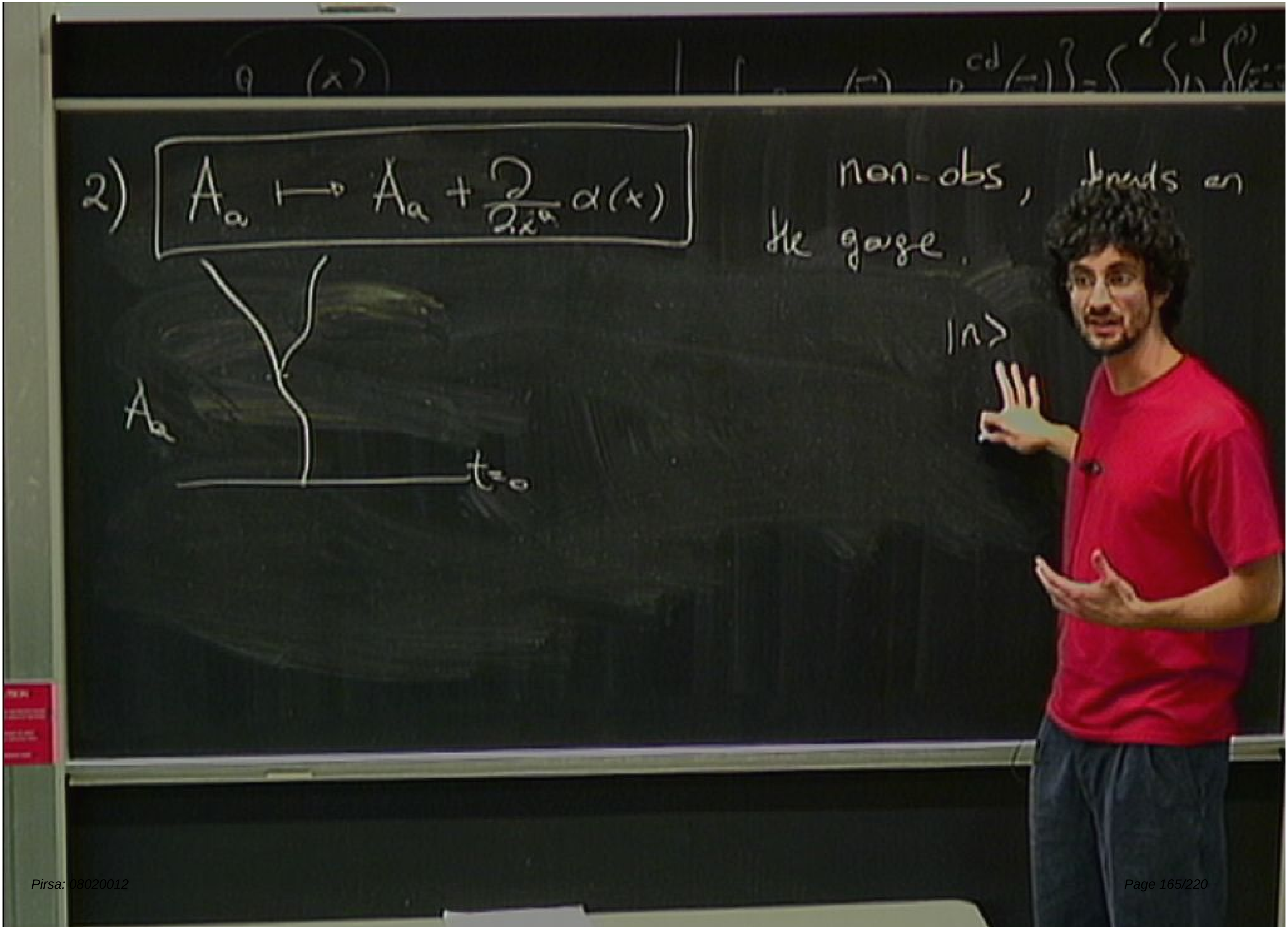
1) connection for em.

2) $\boxed{A_a \longmapsto A_a + \frac{\partial}{\partial x^a}\alpha(x)}$

non-obs, depends on the gauge.

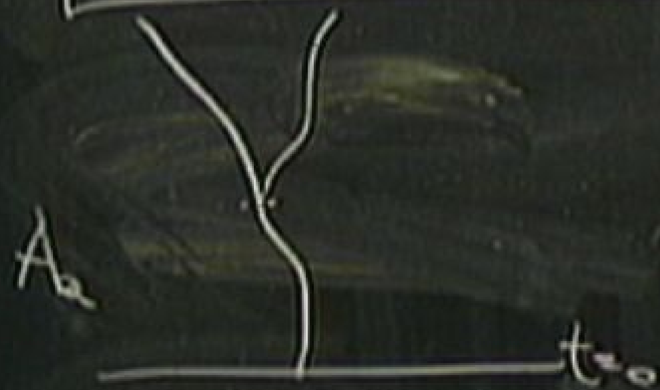





2) $A_a \mapsto A_a + \frac{\partial}{\partial x^a}\alpha(x)$
non-obs, depends on
the gauge.
$|n\rangle$
A_a
$t=0$

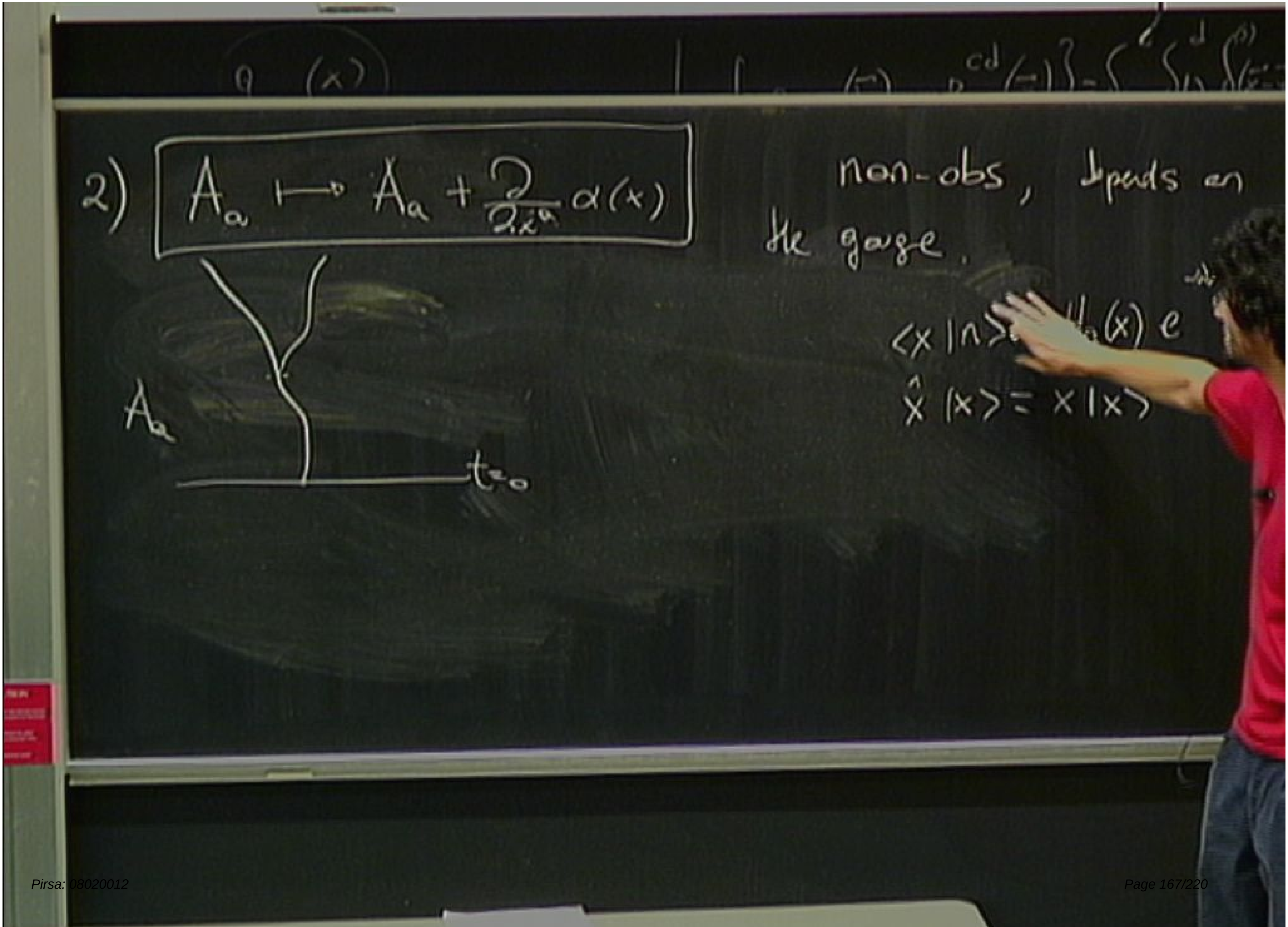
2) $\boxed{A_a \longmapsto A_a + \frac{\partial}{\partial x^a}\alpha(x)}$

non-obs, depends on the gauge.



$\langle x | n \rangle = H_n(x)\, e$

$\hat{x}\,|x\rangle = x\,|x\rangle$

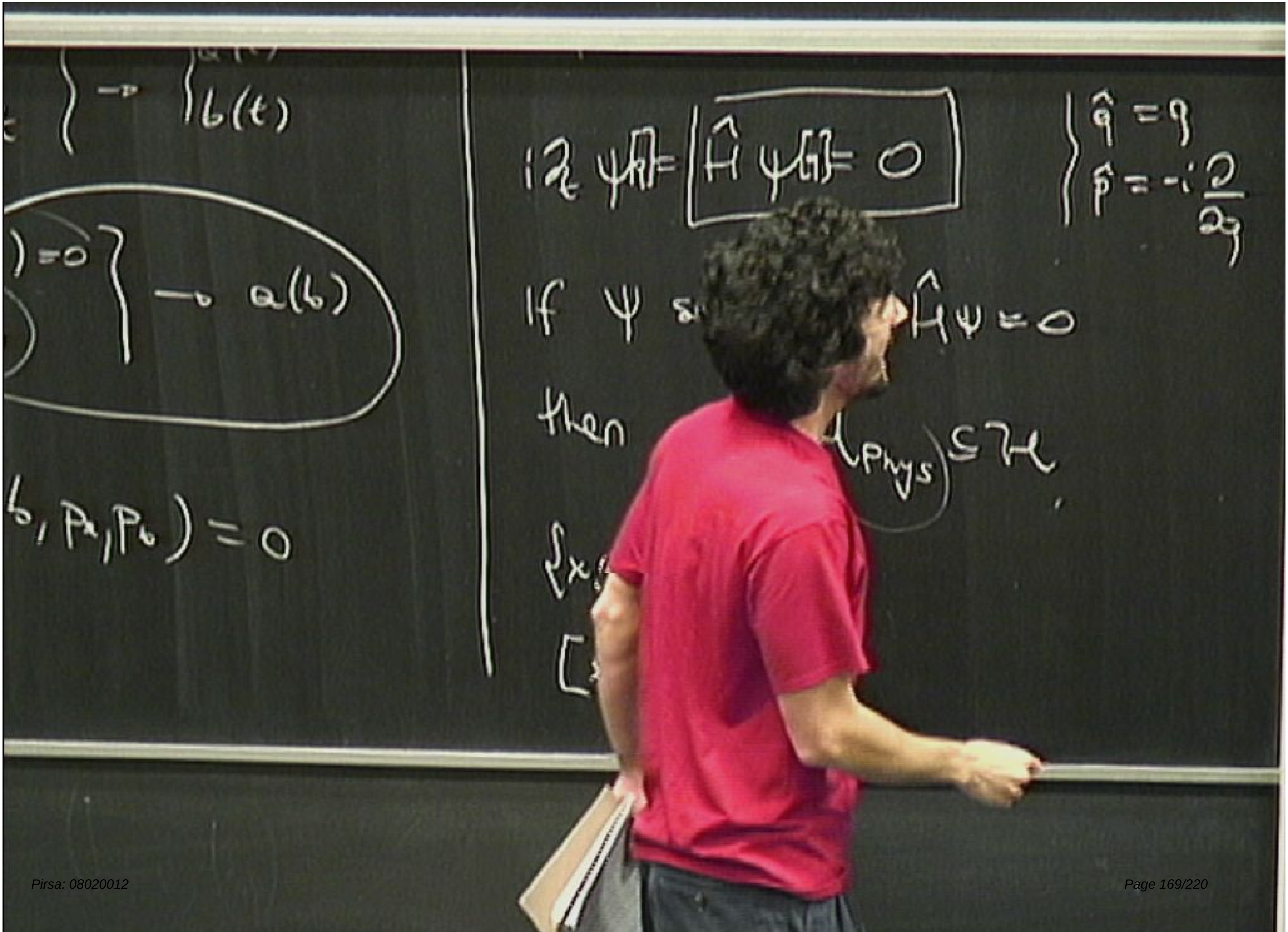

2) $A_a \mapsto A_a + \frac{\partial}{\partial x^a}\alpha(x)$
non-obs, depends on the gauge.
$\langle x|n\rangle$
$\psi_n(x)\, e$
$\hat{x}|x\rangle = x|x\rangle$
A_a
$t=0$

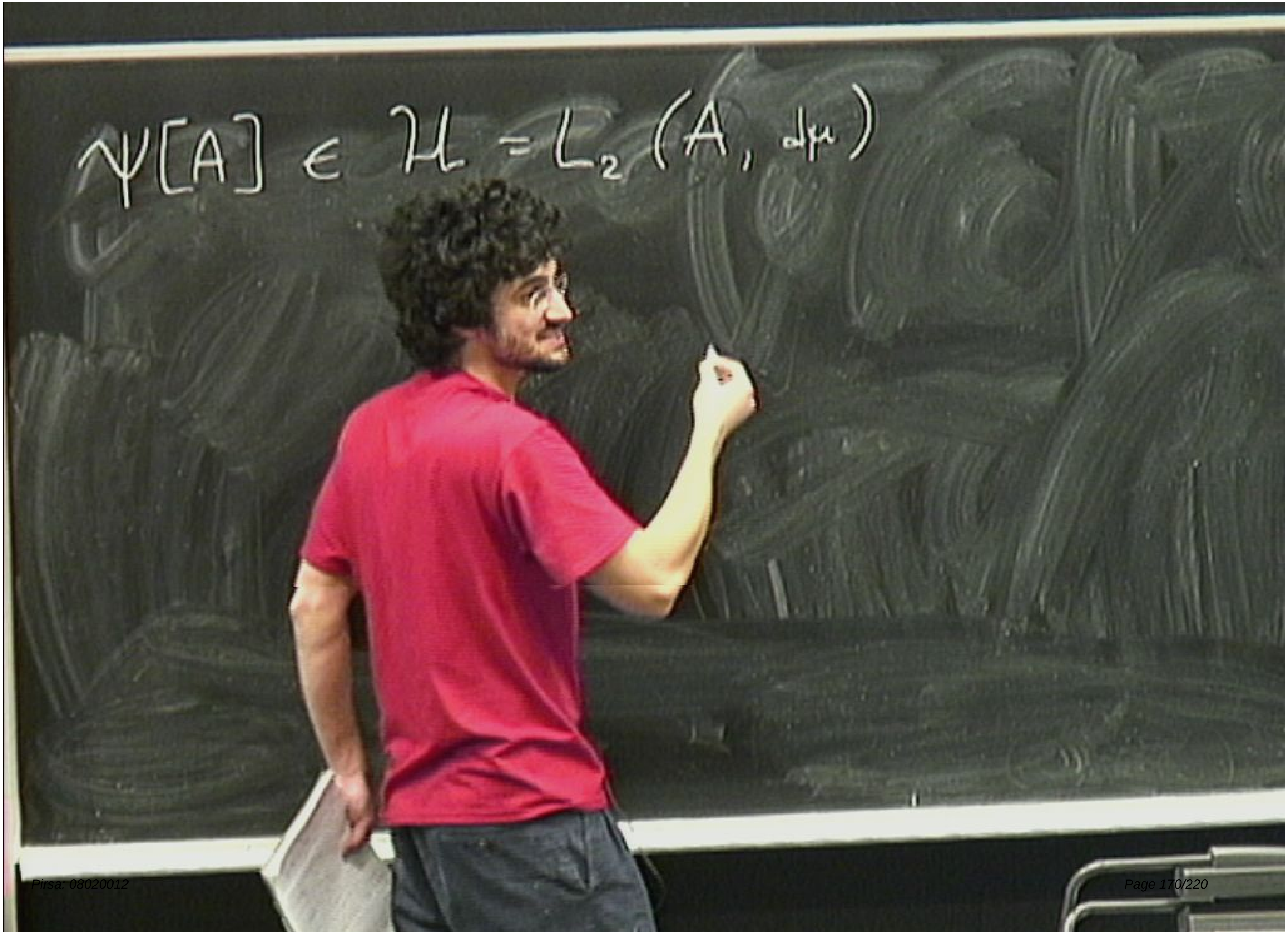
2) $\boxed{A_a \mapsto A_a + \frac{\partial}{\partial x^a}\alpha(x)}$

non-obs, depends on the gauge

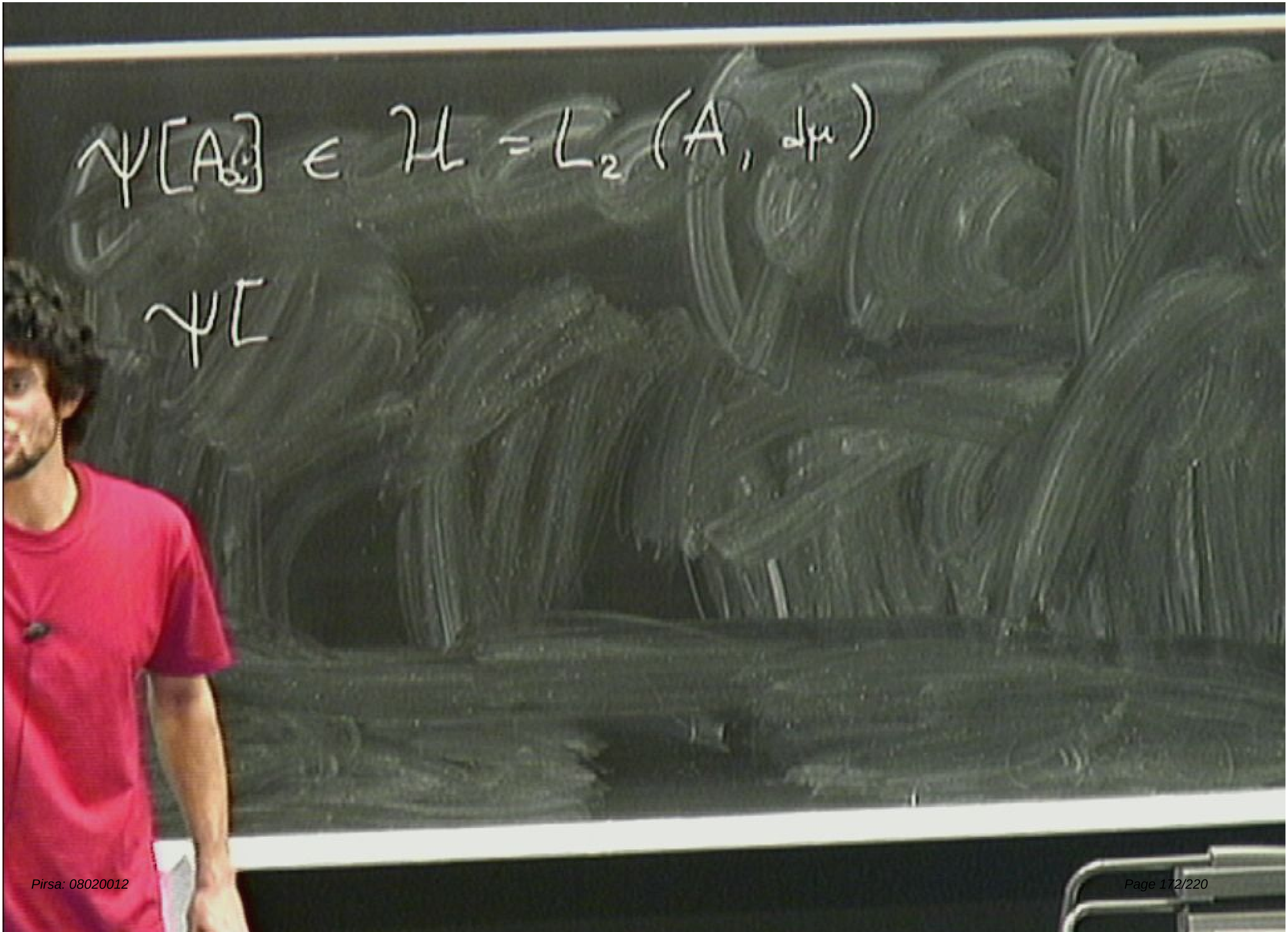
$$\langle x|n\rangle = H_n(x)\, e^{-\alpha x^2}$$

$$\hat{x}\,|x\rangle = x\,|x\rangle$$


b(t)
)=0
→ a(b)
b, Pa, Pb) = 0
i∂t ψ[q] = Ĥ ψ[q] = 0
q̂ = q
p̂ = -i ∂/∂q
If Ψ
Ĥψ = 0
then
phys ⊆ H

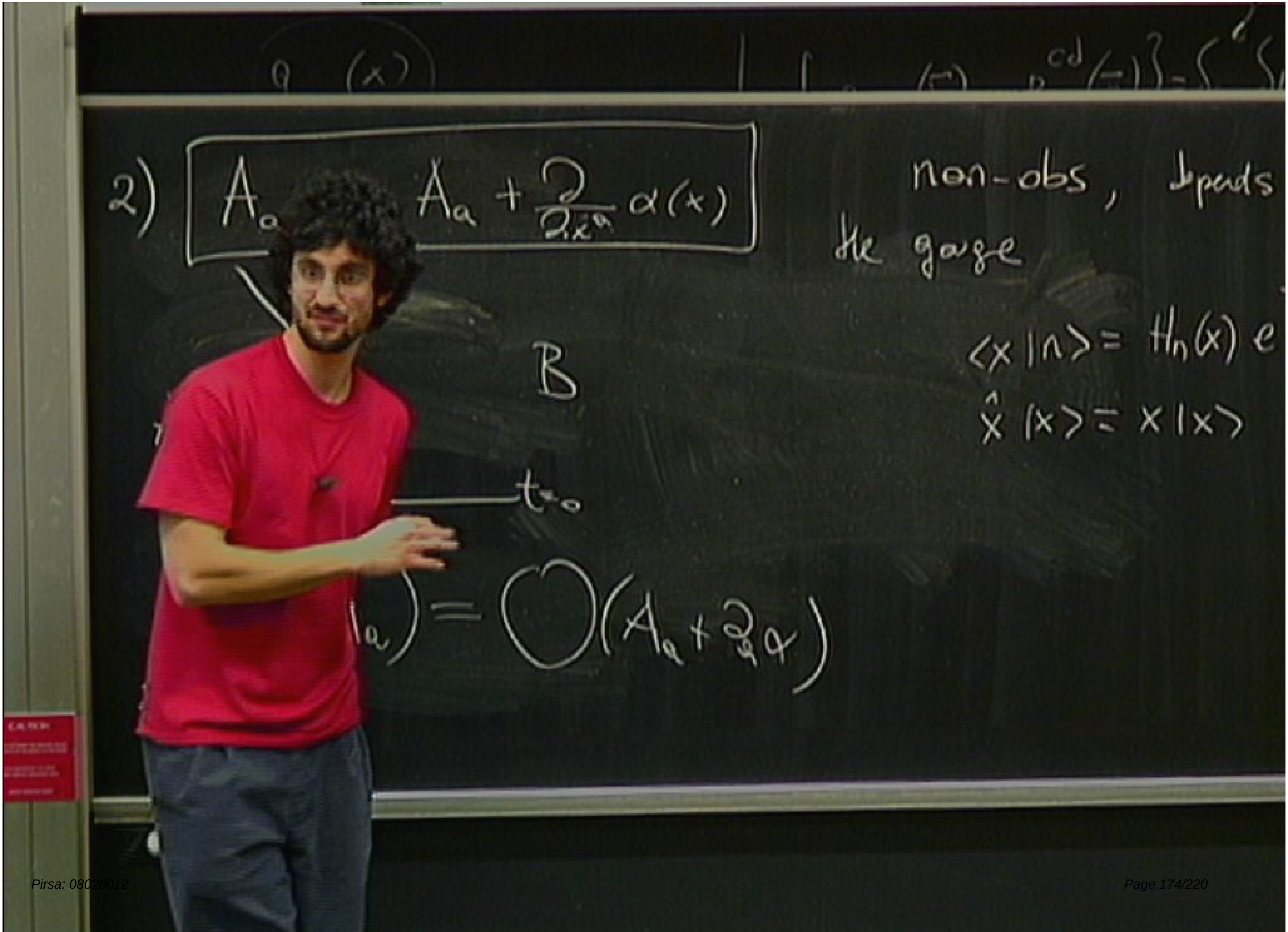

$\Psi[A] \in \mathcal{H} = L_2(\mathcal{A}, d\mu)$

$$\Psi[A] \in \mathcal{H} = L_2(A, d\mu)$$


$\Psi[A_{ab}] \in \mathcal{H} = L_2(A, d\mu)$
$\Psi[$

$\psi[A_a] \in \mathcal{H} = L_2(A, d\mu)$

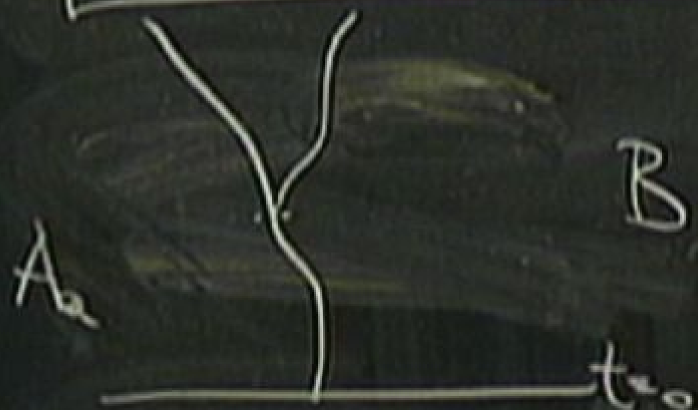
$\psi[A_a]$


2) A_a → A_a + ∂/∂x^a α(x)
non-obs, depends
the gauge
B
t=0
= ∮(A_a + ∂_a α)
⟨x|n⟩ = H_n(x) e
x̂ |x⟩ = x |x⟩

2) $\boxed{A_a \longmapsto A_a + \frac{\partial}{\partial x^a}\alpha(x)}$

non-obs, depends

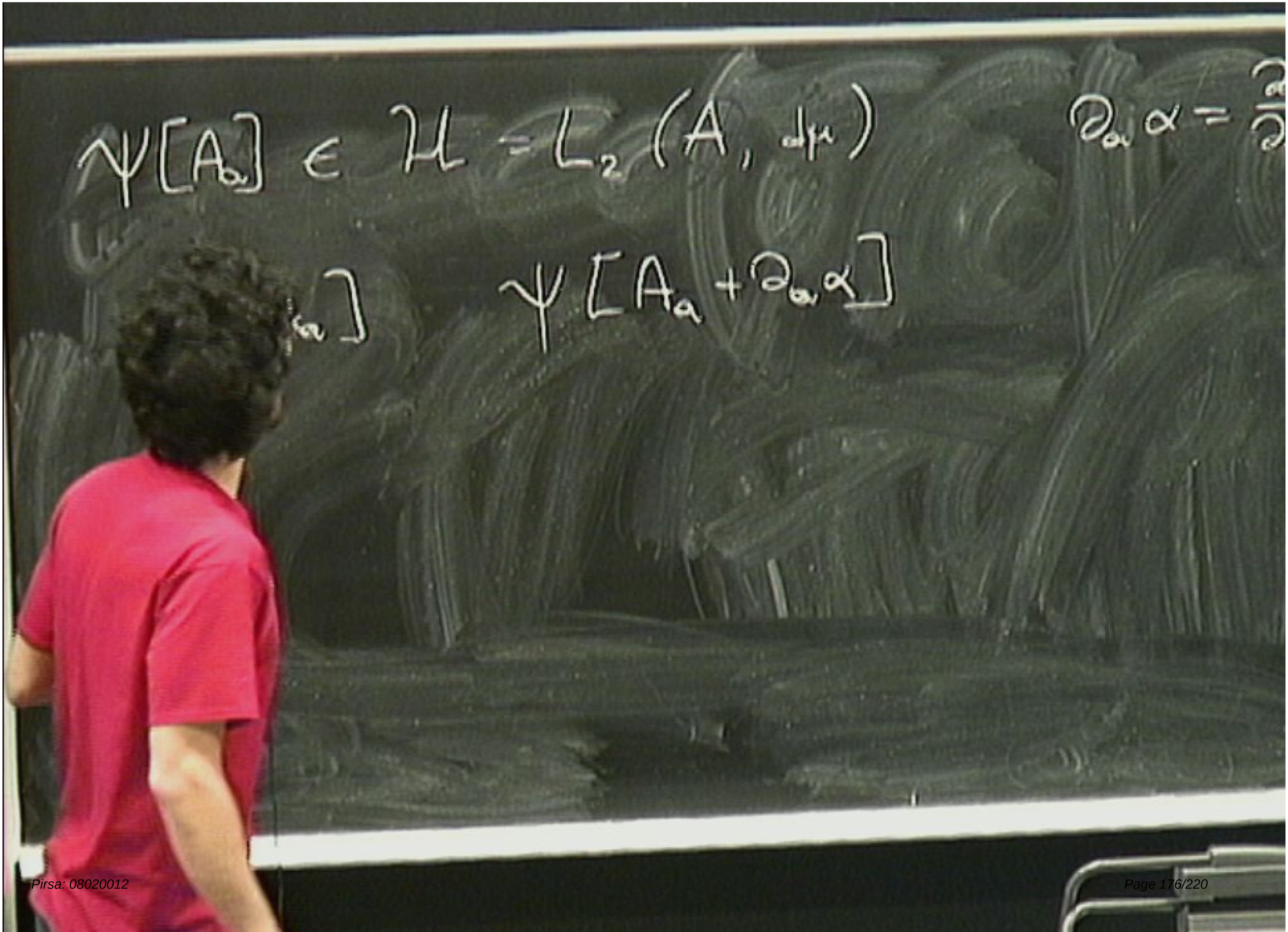
the gauge



$$\mathcal{O}(A_a) = \mathcal{O}(A_a + \partial_a \alpha)$$

$\langle x | n \rangle = H_n(x)\, e$

$\hat{x}\,|x\rangle = x\,|x\rangle$


ψ[A_a] ∈ H = L_2 (A, dμ)
∂_a α =
a]
ψ[A_a + ∂_a α]

$$g_{00} = -N^2 (+N_a N$$

$$\Psi[A_a] \in \mathcal{H} = L_2(A, d\mu) \qquad \partial_a \alpha = \frac{\partial \alpha}{\partial x^a}$$

$$\Psi[A_a] = \Psi[A_a + \partial_a \alpha]$$

$$q_{00} = -N^2 (1 + N_a N$$

$$\Psi[A_a] \in \mathcal{H} = L_2(A, d\mu) \qquad \partial_a \alpha = \frac{\partial \alpha}{\partial x^a}$$

$$\Psi_P[A_a] = \Psi_P[A_a + \partial_a \alpha]$$

$$\Psi_P \in \mathcal{H}_{Phys}$$

$$a, b, t \; \Big\} \rightarrow \Big\{ \; b(t)$$

$$\left.\begin{matrix} \mathcal{H}(a,b) = 0 \\ a, b \end{matrix}\right\} \rightarrow a(b)$$

$$\mathcal{H}(a, b, p_a, p_b) = 0$$

$$i\partial_t \psi[q] = \boxed{\hat{H}\psi[q] = 0}$$

$$\hat{q} = q, \quad \hat{p} = -i\frac{\partial}{\partial q}$$

If ψ satisfies $\hat{H}\psi = 0$

then $\psi \in \mathcal{H}_{phys} \subseteq \mathcal{H}$

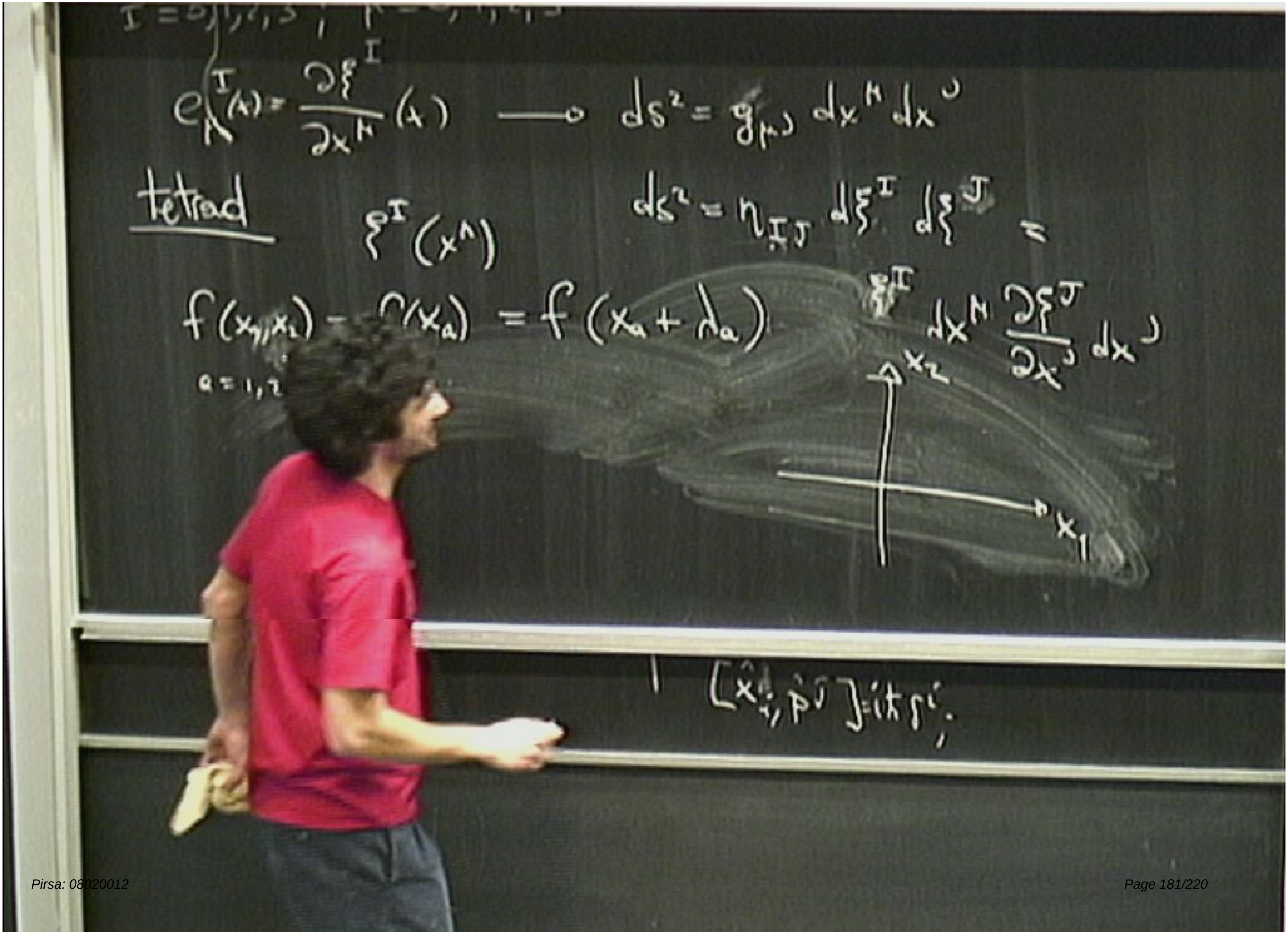
$$\{x^i, p^j\} = \delta^i_j$$

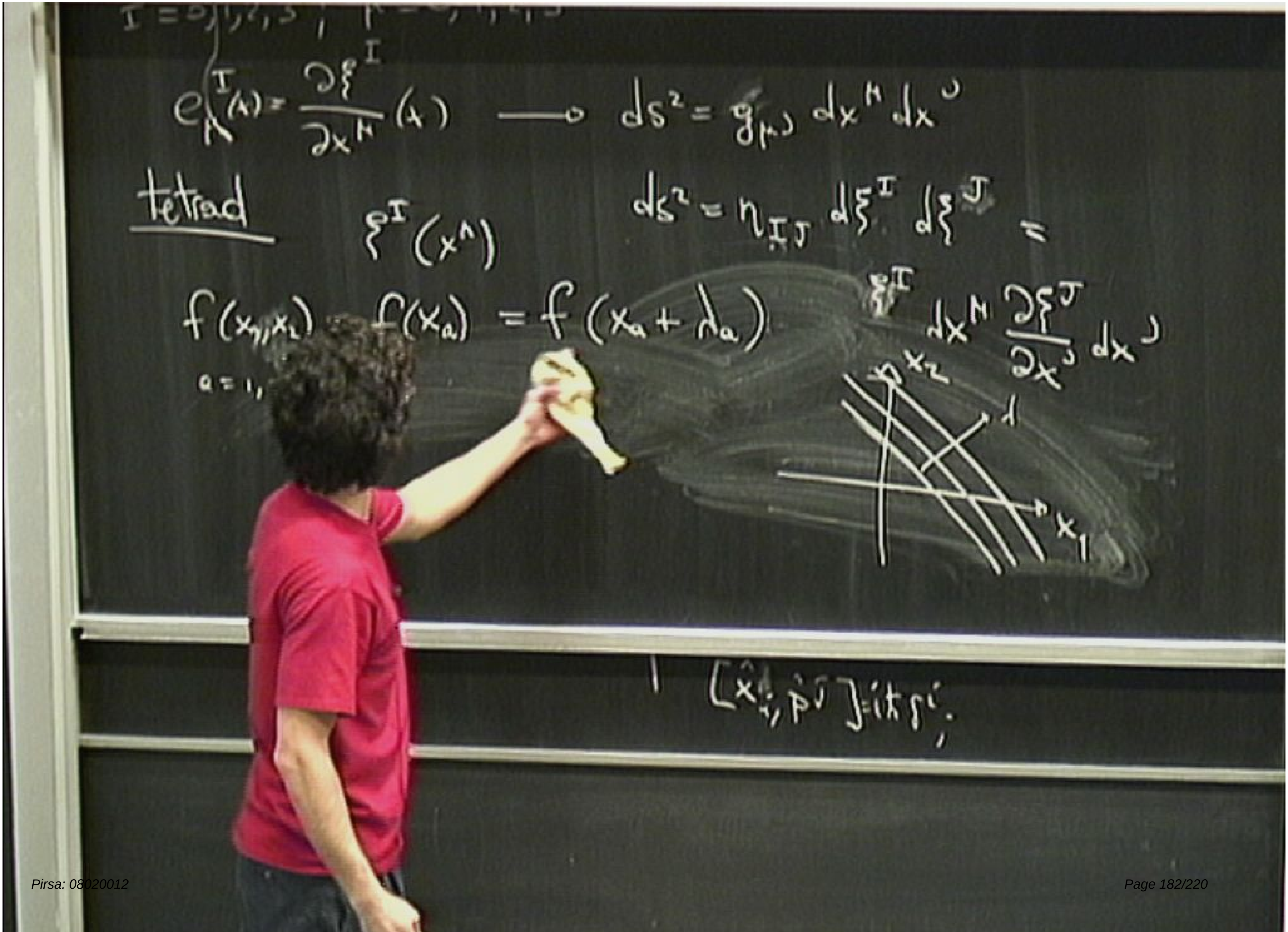
$$[\hat{x}^i, \hat{p}^j] = i\hbar \delta^i_j$$

$$\Psi[A_a] \in \mathcal{H} = L_2(\mathcal{A}, d\mu) \qquad \partial_a \alpha = \frac{\partial \alpha}{\partial x^a}$$

$$\Psi_p[A_a] = \Psi_p[A_a + \partial_a \alpha]$$

$$\Psi_p \in \mathcal{H}_{phys}$$


$e_\mu^I(x) = \frac{\partial \xi^I}{\partial x^\mu}(x) \longrightarrow ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu$
tetrad
$\xi^I(x^\mu)$
$ds^2 = \eta_{IJ}\, d\xi^I d\xi^J =$
$\frac{\partial \xi^I}{\partial x^\mu} dx^\mu \frac{\partial \xi^J}{\partial x^\nu} dx^\nu$
$f(x_1, x_2)$
$a = 1, 2$
$(x_a) = f(x_a + \lambda_a)$
x_2
x_1
$[\hat{x}^i, \hat{p}^j] = i\hbar\, \delta^i_j$


$e_\mu^I(x) = \frac{\partial \xi^I}{\partial x^\mu}(x) \longrightarrow ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu$
tetrad
$\xi^I(x^\mu)$
$ds^2 = \eta_{IJ}\, d\xi^I\, d\xi^J =$
$\frac{\partial \xi^I}{\partial x^\mu} dx^\mu \frac{\partial \xi^J}{\partial x^\nu} dx^\nu$
$f(x_1, x_2)$
$f(x_a) = f(x_a + \lambda_a)$
$a = 1,$
x_2
λ
x_1
$[\hat{x}^i, \hat{p}^j] = i\hbar \delta^{ij}$

$I = 0,1,2,3 \; ; \; \mu = 0,1,2,3$

$$e^I_\mu(x) = \frac{\partial \xi^I}{\partial x^\mu}(x) \longrightarrow ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu$$

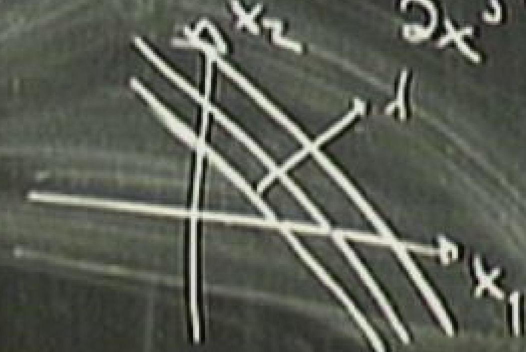
tetrad

$$\xi^I(x^\mu)$$

$$ds^2 = \eta_{IJ}\, d\xi^I d\xi^J = \frac{\partial \xi^I}{\partial x^\mu} dx^\mu \frac{\partial \xi^J}{\partial x^\nu} dx^\nu$$

$$f(x_1, x_2) = f(x_a) = f(x_a + \lambda_a)$$

$a = 1,2$



$$\lambda^a \partial_a f(x) = 0$$

$$f(x) = f(x+a)$$

$$[\hat{x}^i, \hat{p}^j] = i\hbar\delta^{ij}$$

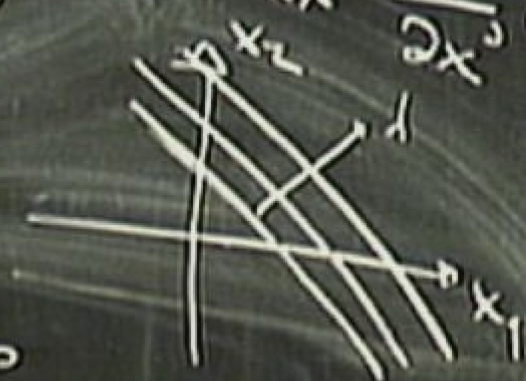
$$e^I_\mu(x) = \frac{\partial \xi^I}{\partial x^\mu}(x) \longrightarrow ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu$$

<u>tetrad</u>

$$\xi^I(x^\mu) \qquad ds^2 = \eta_{IJ}\, d\xi^I d\xi^J = \xi^I dx^\mu \frac{\partial \xi^J}{\partial x^\nu} dx^\nu$$

$$f(x_1, x_2) = f(x_a) = f(x_a + \lambda_a)$$

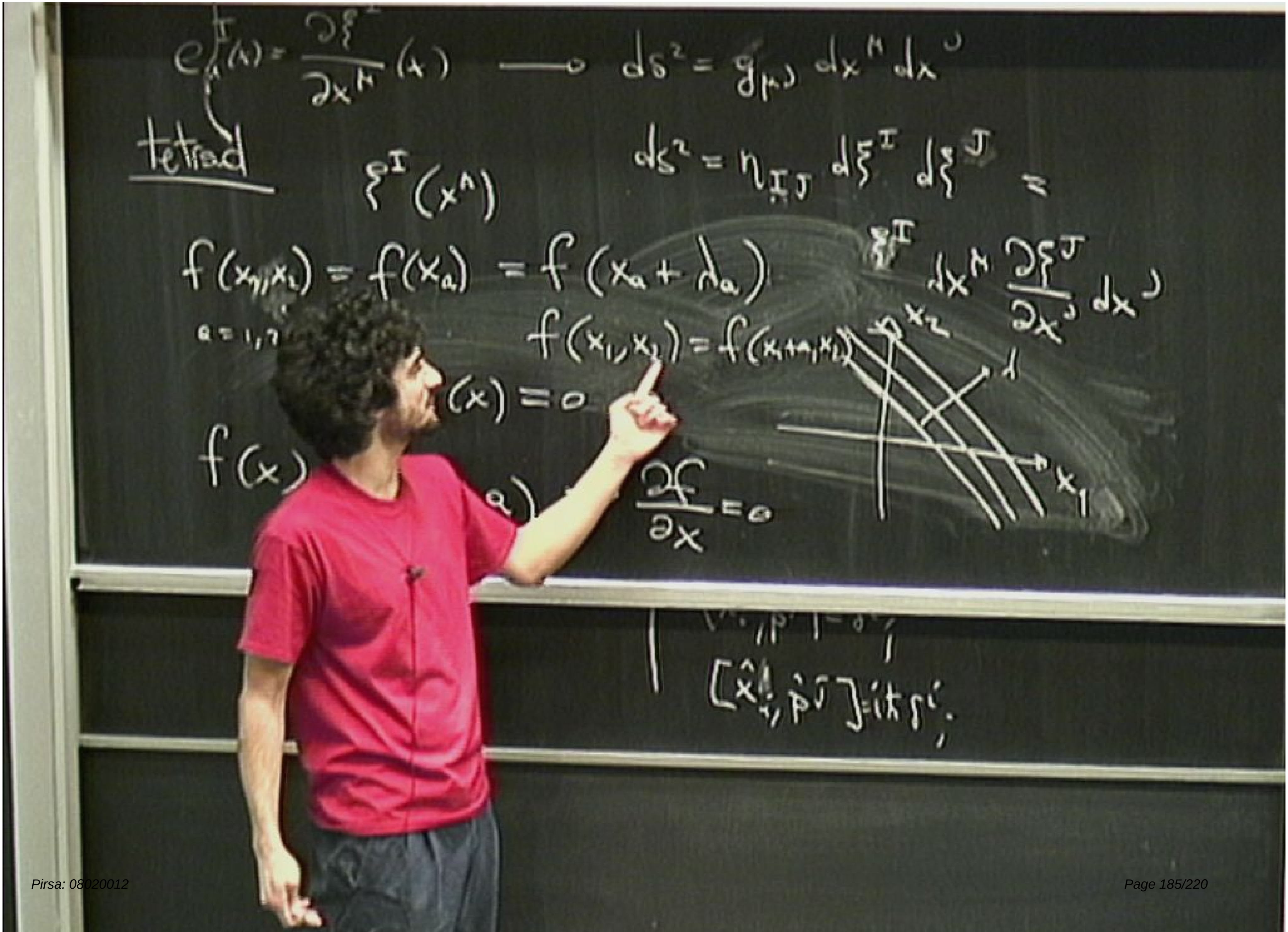
$a = 1, 2$



$$\lambda^a \partial_a f(x) = 0$$

$$f(x) = f(x+a) \to \frac{\partial f}{\partial x} = 0$$

$$[\hat{x}^i, \hat{p}^j] = i\hbar \delta^i_j$$



$e^I_\mu(x) = \frac{\partial \xi^I}{\partial x^\mu}(x) \longrightarrow ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu$
tetrad
$\xi^I(x^\mu)$
$ds^2 = \eta_{IJ}\, d\xi^I\, d\xi^J =$
$f(x_1, x_2) = f(x_a) = f(x_a + \lambda_a)$
$f(x_1, x_2) = f(x_1 + a, x_2)$
$dx^\mu \frac{\partial \xi^J}{\partial x^\nu} dx^\nu$
$\frac{\partial f}{\partial x} = 0$
x_2
x_1
λ

$e^I_\mu(x) = \frac{\partial \xi^I}{\partial x^\mu}(x) \longrightarrow ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu$

tetrad

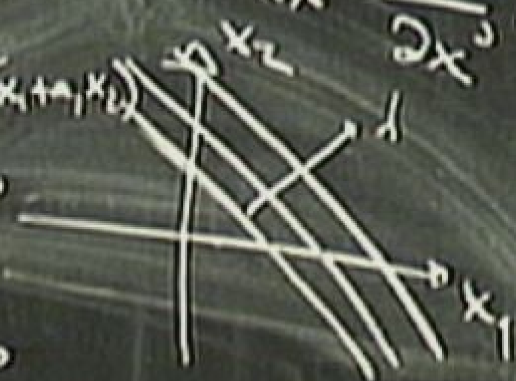
$\xi^I(x^\mu)$

$ds^2 = \eta_{IJ}\, d\xi^I d\xi^J = \xi^I dx^\mu \frac{\partial \xi^J}{\partial x^\nu} dx^\nu$

$f(x_1,x_2) = f(x_a) = f(x_a + \lambda_a)$

$a = 1,2$

$f(x_1,x_2) = f(x_1+a, x_2)$



$\lambda^a \partial_a f(x) = 0 \quad \frac{\partial f}{\partial x_1} = 0$

$f(x) = f(x+a) \to \frac{\partial f}{\partial x} = 0$

$[\hat{x}^i, \hat{p}^j] = i\hbar\delta^{ij}$

$$e^I_\mu(x) = \frac{\partial \xi^I}{\partial x^\mu}(x) \longrightarrow ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu$$

tetrad

$$\xi^I(x^\mu)$$

$$ds^2 = \eta_{IJ}\, d\xi^I d\xi^J = \xi^I dx^\mu \frac{\partial \xi^J}{\partial x^\nu} dx^\nu$$

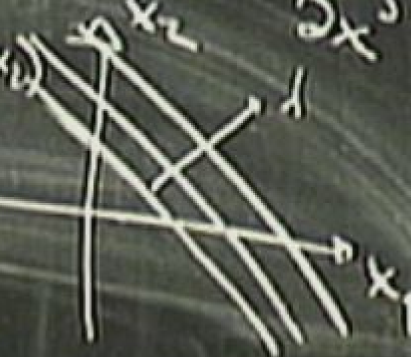
$$\boxed{f(x_1,x_2) = f(x_a) = f(x_a+\lambda_a)}$$

$a = 1,2$

$$\boxed{f(x_1,x_2) = f(x_1+a, x_2)}$$

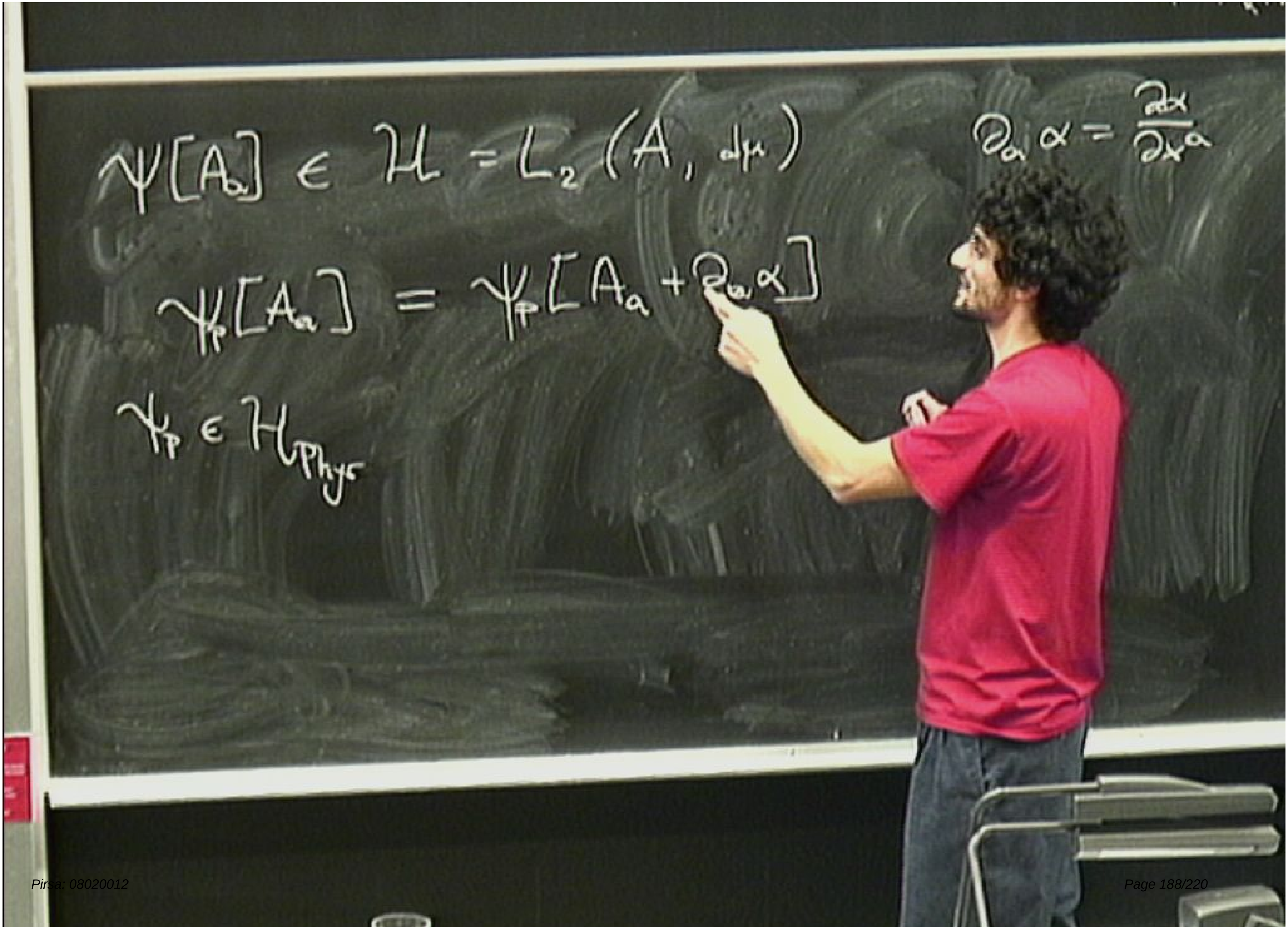
$$\boxed{\lambda^a \partial_a f(x) = 0}$$

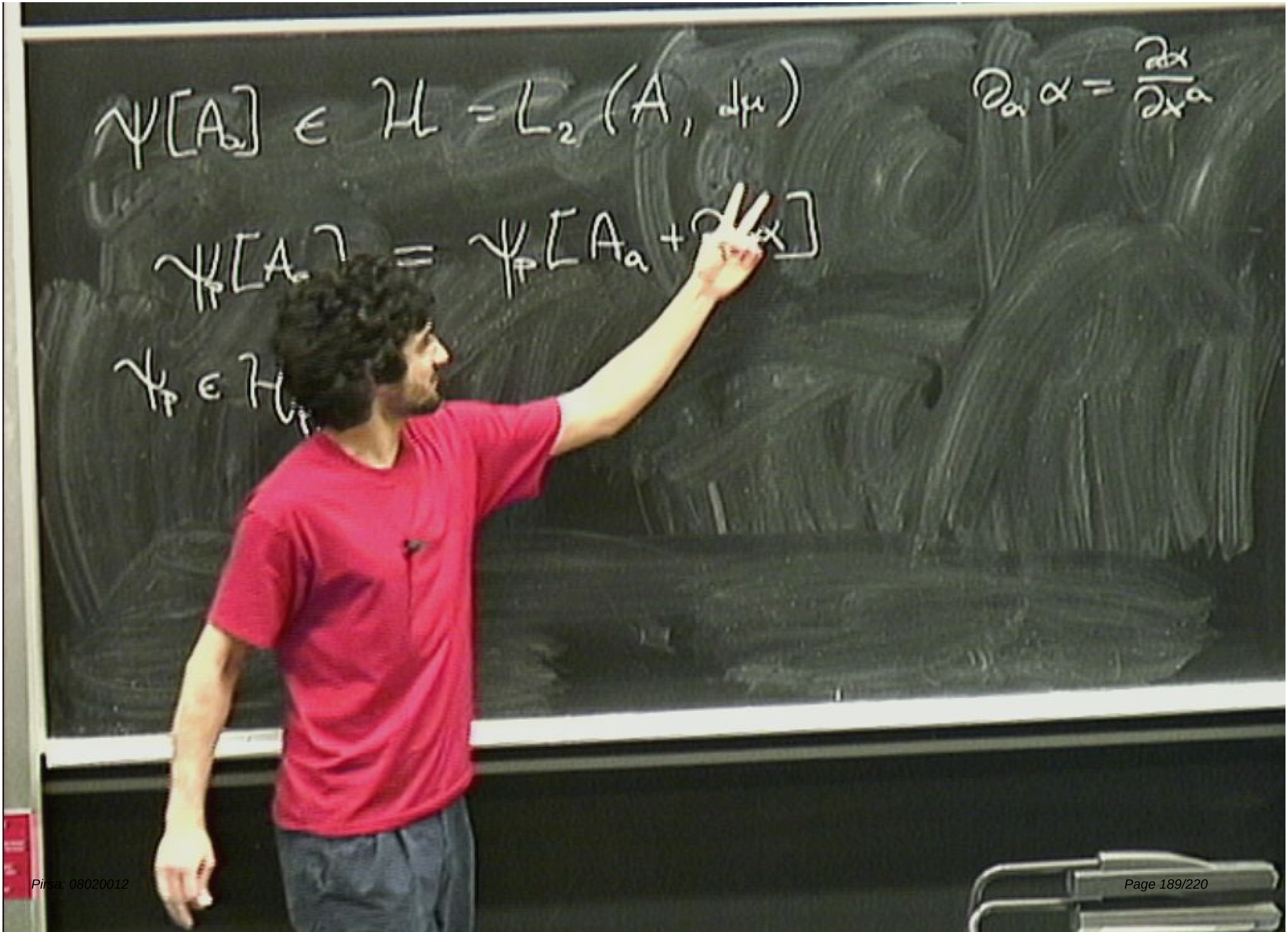
$$\frac{\partial f}{\partial x_1} = 0$$



$$f(x) = f(x+a) \to \frac{\partial f}{\partial x} = 0$$

$$[\hat{x}^i, \hat{p}^j] = i\hbar\delta^{ij}$$


$\Psi[A_a] \in \mathcal{H} = L_2(A, d\mu)$
$\partial_a \alpha = \frac{\partial \alpha}{\partial x^a}$
$\Psi_p[A_a] = \Psi_p[A_a + \partial_a \alpha]$
$\Psi_p \in \mathcal{H}_{Phys}$


$\Psi[A_a] \in \mathcal{H} = L_2(A, d\mu)$
$\partial_a \alpha = \frac{\partial \alpha}{\partial x^a}$
$\Psi_P[A$
$= \Psi_P[A_a +$
$\alpha]$
$\Psi_P \in \mathcal{H}$

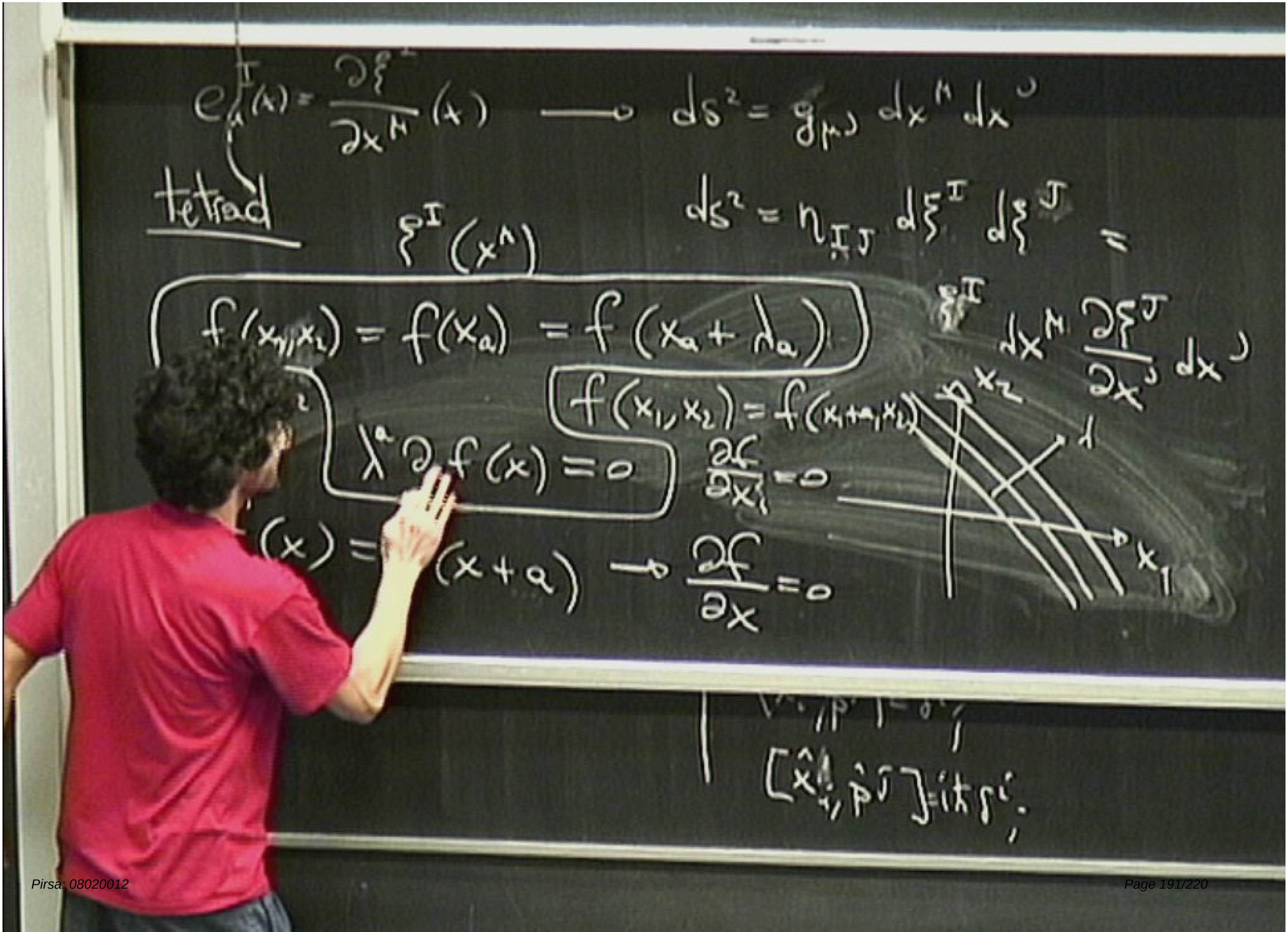
$$\Psi[A_a] \in \mathcal{H} = L_2(A, d\mu)$$

$$\partial_a \alpha = \frac{\partial \alpha}{\partial x^a}$$

$$\Psi_p[A_a] = \Psi_p[A_a + \partial_a \alpha]$$

$$\Psi_p \in \mathcal{H}_{Phys}$$

$$\partial_a \alpha(x)$$


tetrad
ds² = g_μν dx^μ dx^ν
ds² = η_IJ dξ^I dξ^J =
ξ^I (x^μ)
f(x_1, x_2) = f(x_a) = f(x_a + λ_a)
f(x_1, x_2) = f(x_1 + a, x_2)
λ^a ∂_a f(x) = 0
∂f/∂x_1 = 0
f(x) = f(x + a) → ∂f/∂x = 0
x_1
x_2
λ

$$e^I_\mu(x) = \frac{\partial \xi^I}{\partial x^\mu}(x) \longrightarrow ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu$$

tetrad

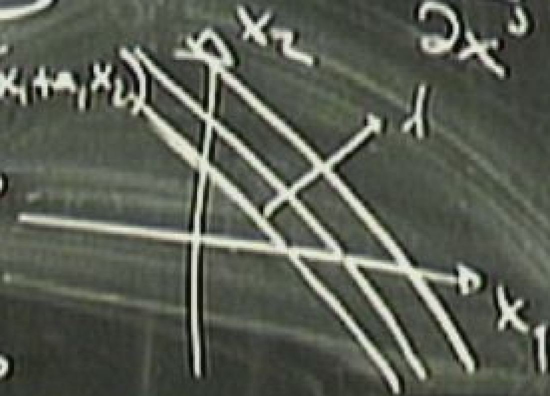
$$\xi^I(x^\mu) \qquad ds^2 = \eta_{IJ}\, d\xi^I d\xi^J = \frac{\partial \xi^I}{\partial x^\mu} dx^\mu \frac{\partial \xi^J}{\partial x^\nu} dx^\nu$$

$$\boxed{f(x_1,x_2) = f(x_a) = f(x_a + \lambda_a)}$$

$a = 1,2$

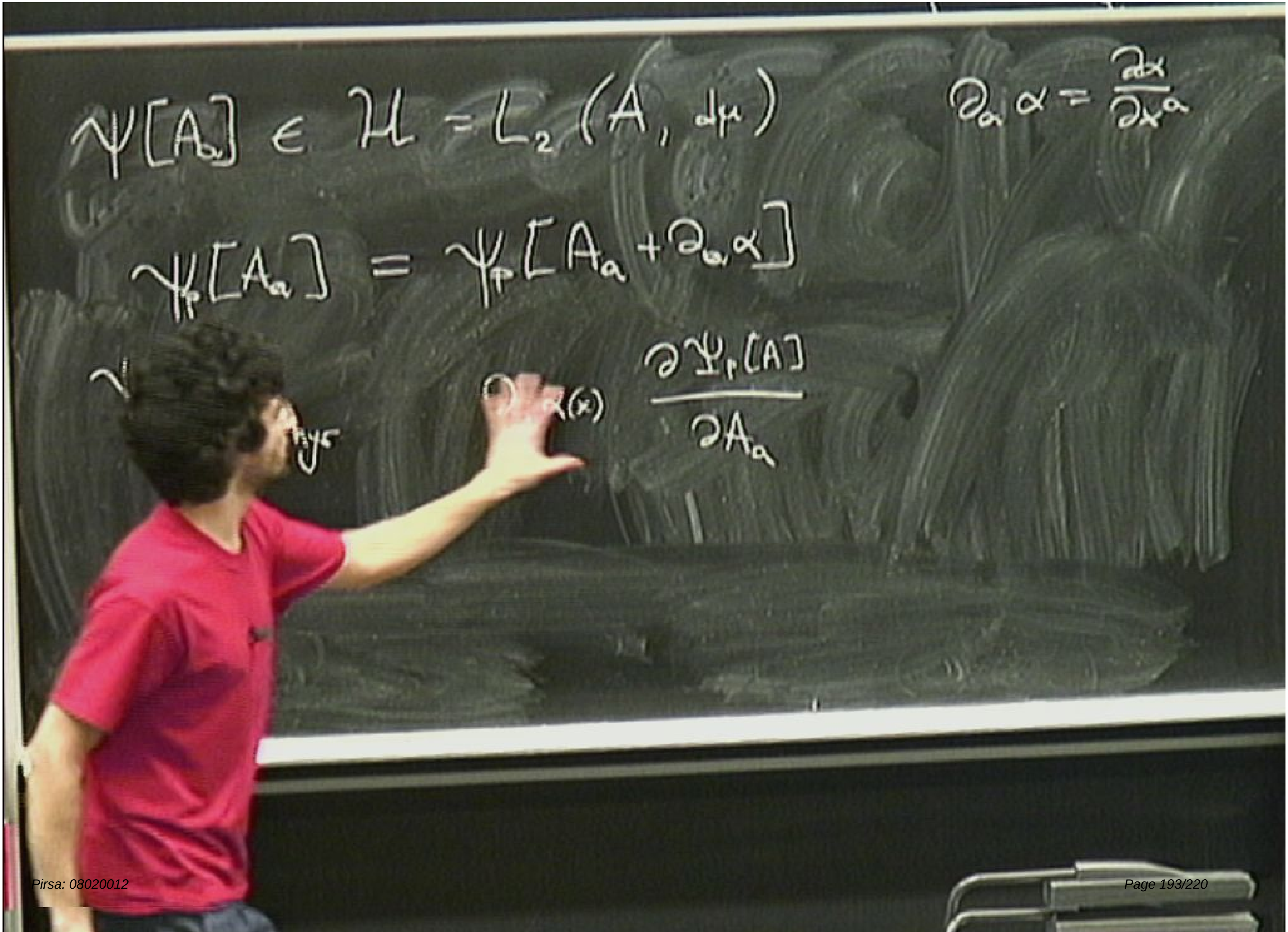
$$f(x_1,x_2) = f(x_1 + a, x_2)$$

$$\boxed{\lambda^a \frac{\partial}{\partial x^a} f(x) = 0} \qquad \frac{\partial f}{\partial x_1} = 0$$

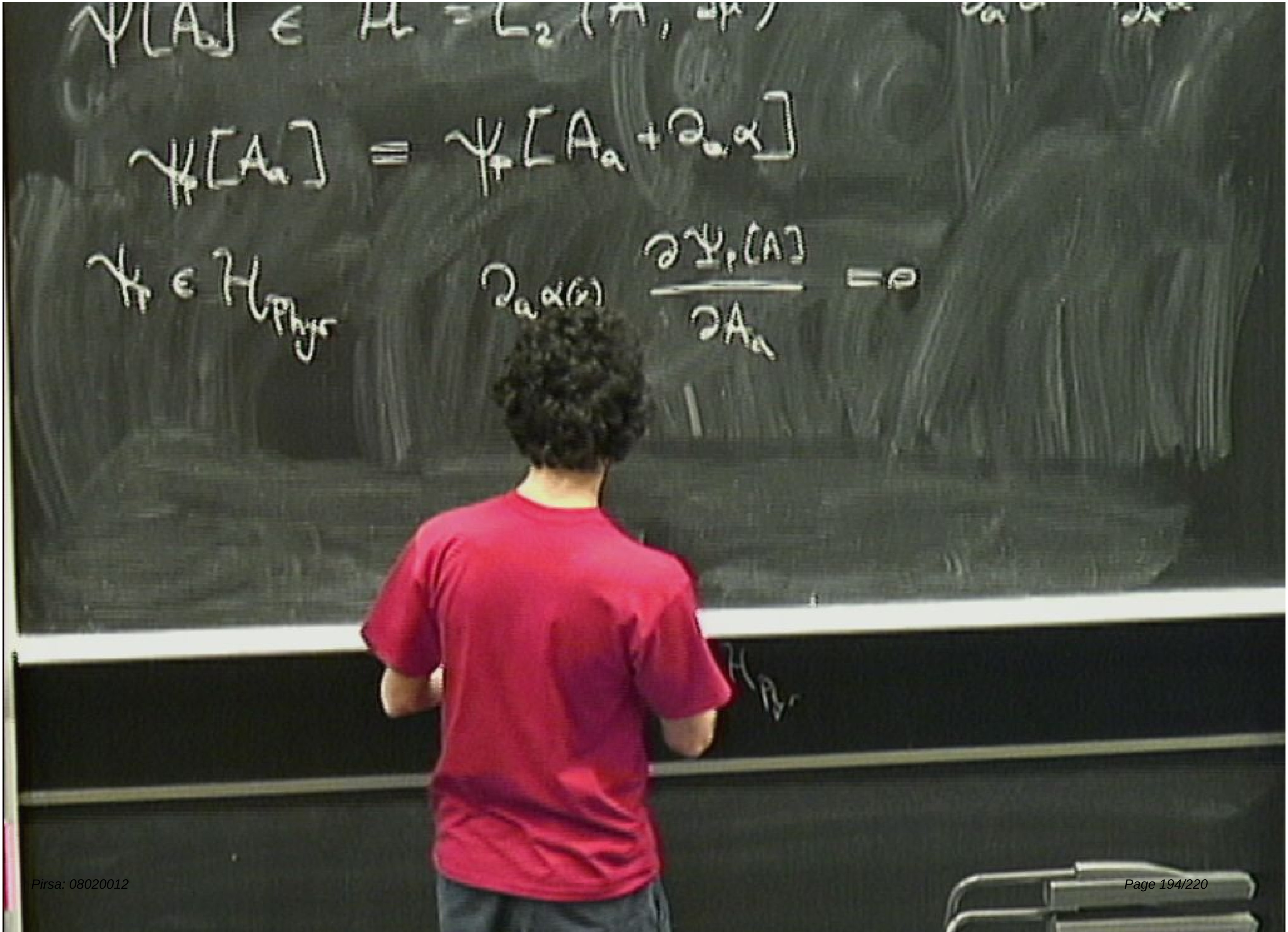


$$f(x) = f(x+a) \to \frac{\partial f}{\partial x} = 0$$

$$[\hat{x}^i, \hat{p}^j] = i\hbar\delta^{ij}$$



$\Psi[A_a] \in \mathcal{H} = L_2(A, d\mu)$
$\partial_a \alpha = \frac{\partial \alpha}{\partial x^a}$
$\Psi_P[A_a] = \Psi_P[A_a + \partial_a \alpha]$
$\alpha(x) \frac{\partial \Psi_P[A]}{\partial A_a}$


$\Psi[A_a] \in \mathcal{H} = L_2(\mathcal{A}, d\mu)$
$\Psi_P[A_a] = \Psi_P[A_a + \partial_a \alpha]$
$\Psi_P \in \mathcal{H}_{Phys}$
$\partial_a \alpha(x) \frac{\partial \Psi_P(A)}{\partial A_a} = 0$

$$\Psi_p[A_a] = \Psi_p[A_a + \partial_a \alpha]$$

$$\Psi_p \in \mathcal{H}_{phys} \qquad \partial_a \alpha(x) \frac{\partial \Psi_p[A]}{\partial A_a} = 0$$

$$\mathcal{H}_g \xrightarrow{\text{"}\Gamma(\mathcal{H})\text{"}} \mathcal{H}_{phys}$$

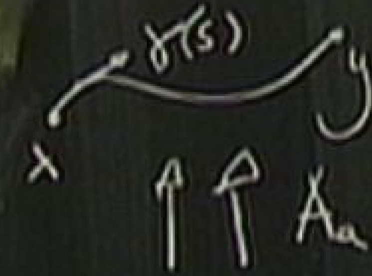
$$\Psi[A_a] \in \mathcal{H} = L_2(\mathcal{A}, d\mu) \qquad \partial_a \alpha = \frac{\partial \alpha}{\partial x^a}$$

$$\psi_p[A_a] = \psi_p[A_a + \partial_a \alpha]$$

$$\psi_p \in \mathcal{H}_{phys} \qquad \partial_a \alpha(x) \frac{\partial \Psi_p[A]}{\partial A_a} = 0$$

$f(x) \to \frac{df}{dx}$

$s \in [0,1] \; : \; \gamma^a(s)$



① $\int_\gamma A_a \frac{d\gamma^a}{ds} \, ds = \int_\gamma A$

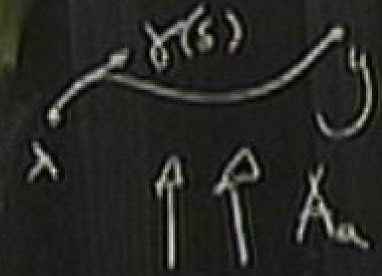
" $\phi(y) = e^{\int_\gamma A} \phi(x)$ "

1) connecti
for em

$f(x) \to \frac{df}{dx}$

$f(x_i) \to \frac{\partial f}{\partial x_i}$

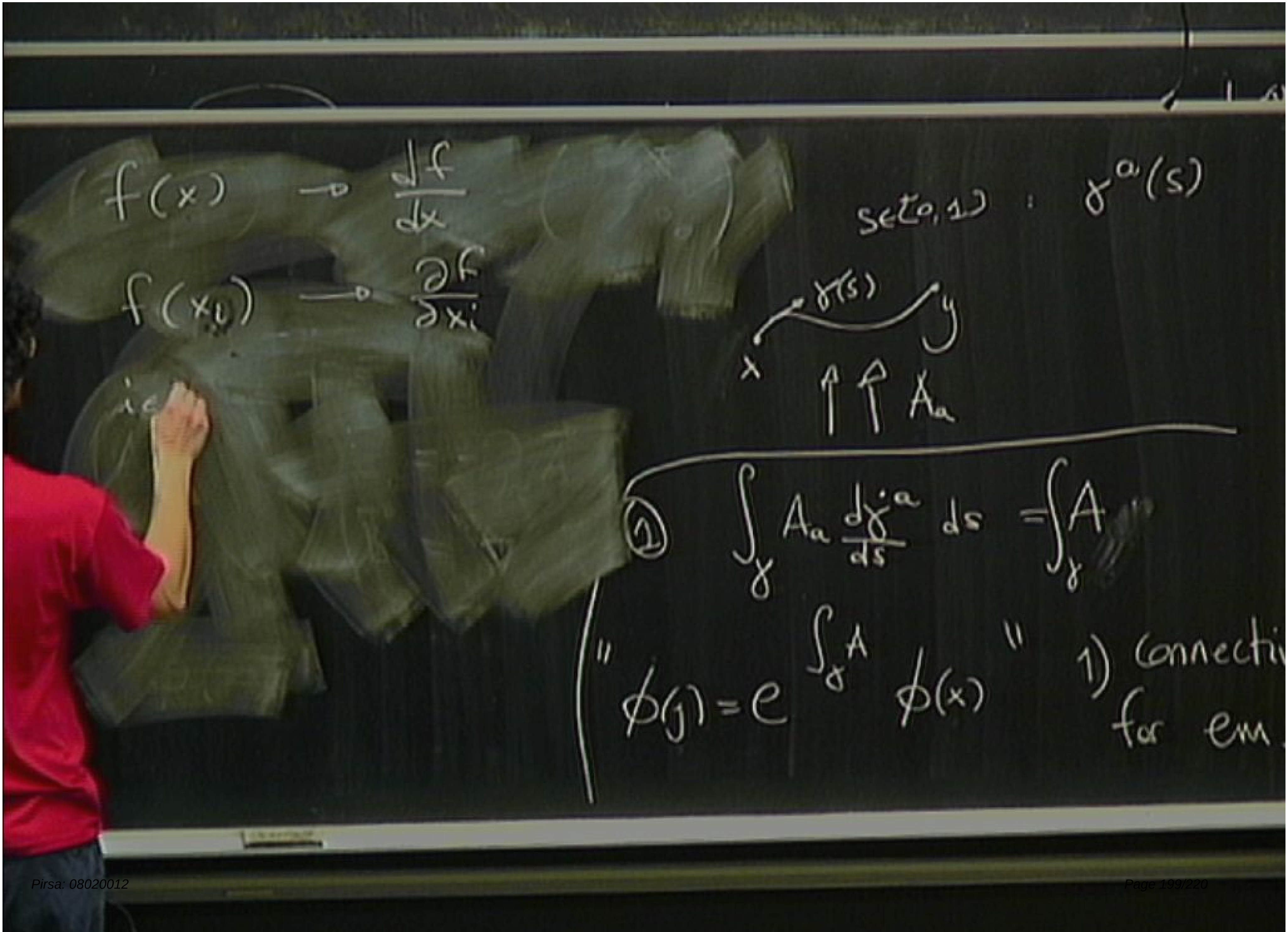
$s \in [0,1] : \gamma^a(s)$



(1) $\int_\gamma A_a \frac{d\gamma^a}{ds} ds = \int_\gamma A$

" $\phi(y) = e^{\int_\gamma A} \phi(x)$ "

1) connecti
for em.


f(x) → df/dx
f(x_i) → ∂f/∂x_i
s∈[0,1] : γ^a(s)
γ(s)
x
y
A_a
① ∫_γ A_a dγ^a/ds ds = ∫_γ A
" φ(y) = e^{∫_γ A} φ(x) "
1) connecti
for em

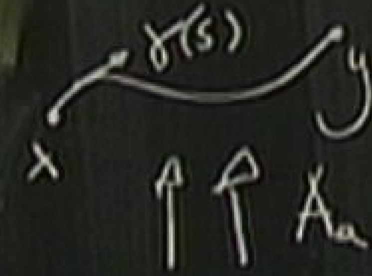
$f(a) \to \frac{df}{da}$

$f(a_i) \to \frac{\partial f}{\partial a_i}$

$i \in N$

$f($

$s \in [0,1] : \gamma^a(s)$



① $\int_\gamma A_a \frac{d\gamma^a}{ds} ds = \int_\gamma A$

"$\phi(y) = e^{\int_\gamma A} \phi(x)$"

1) Connecti
for em

$f(a) \to \frac{df}{da}$

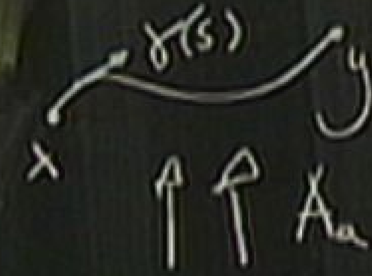
$f(a_i) \to \frac{\partial f}{\partial a_i}$

$i \in \mathbb{N}$

$f($

$i \in \mathbb{R}$

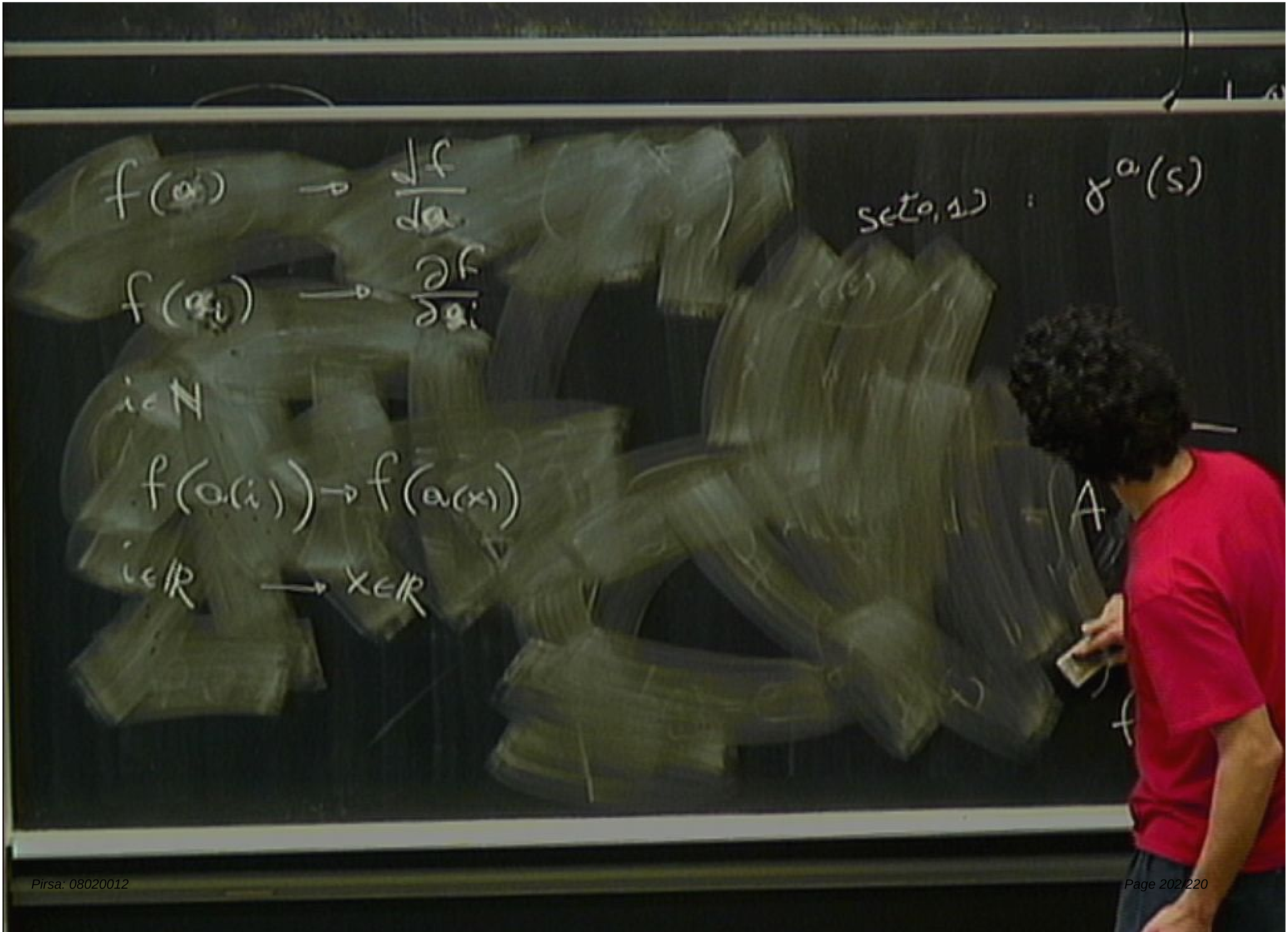
$s \in [0,1] : \gamma^a(s)$

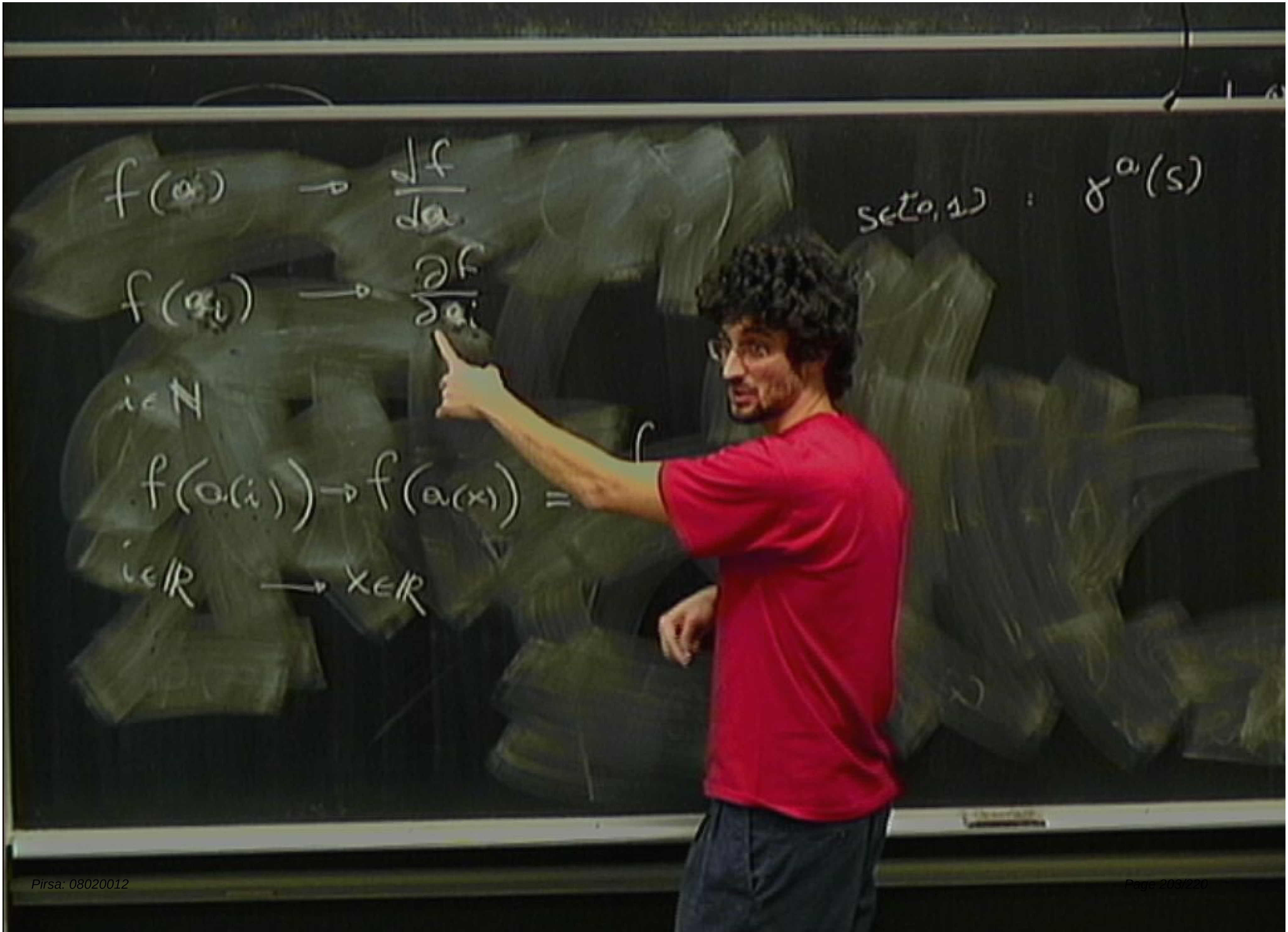


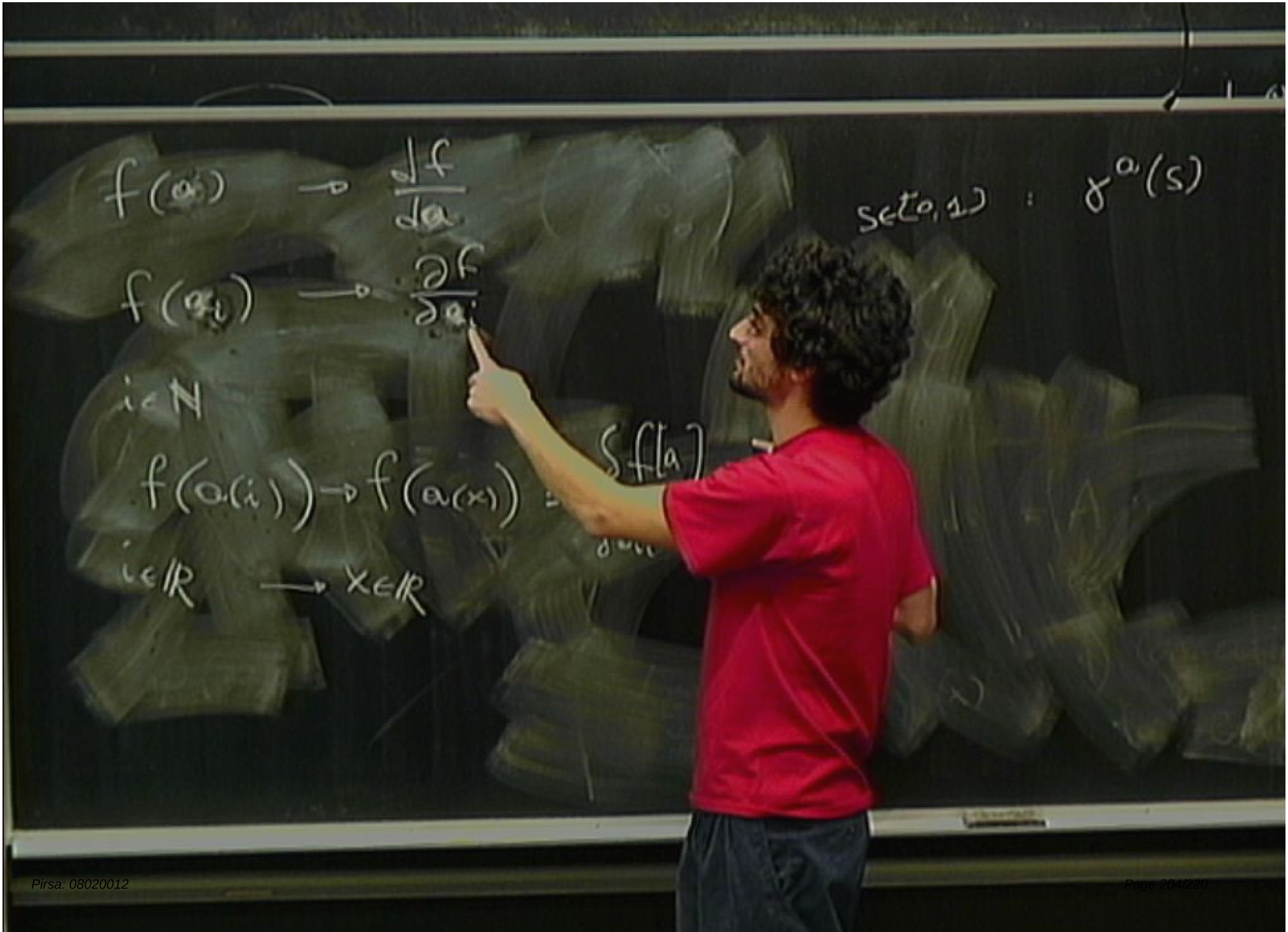
① $\int_\gamma A_a \frac{d\gamma^a}{ds} ds = \int_\gamma A$

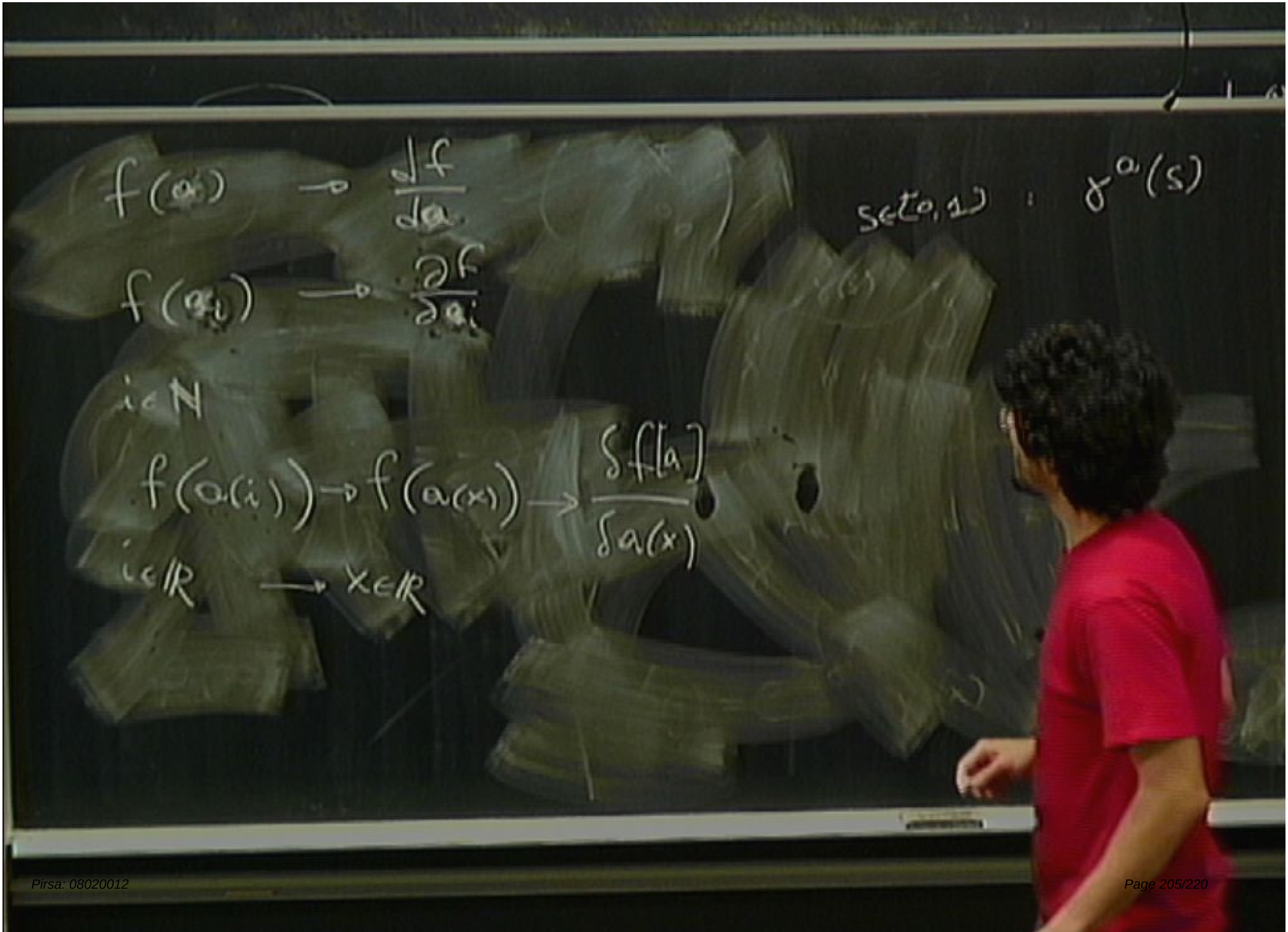
" $\phi(y) = e^{\int_\gamma A} \phi(x)$ "

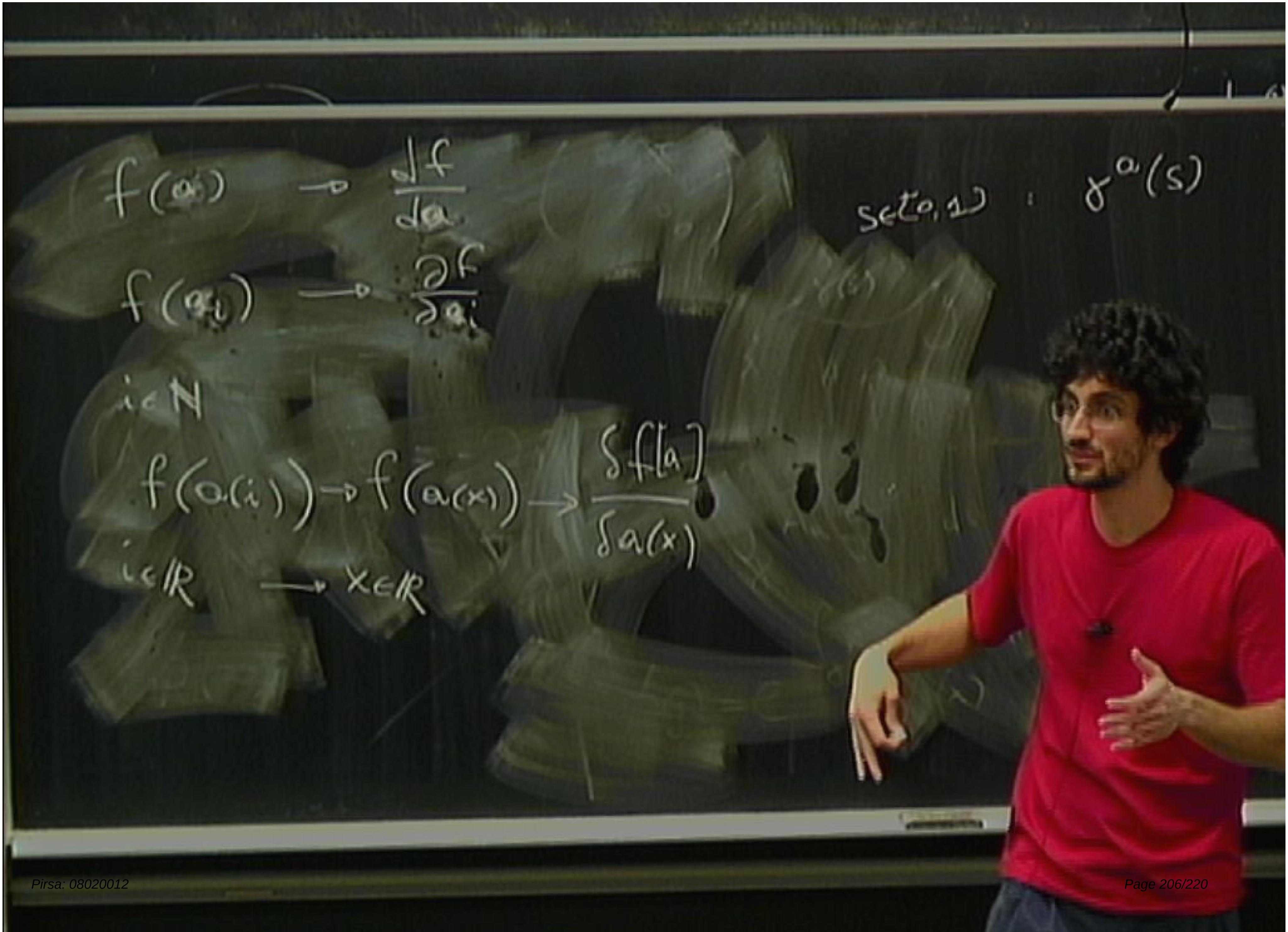
1) connecti
for em.

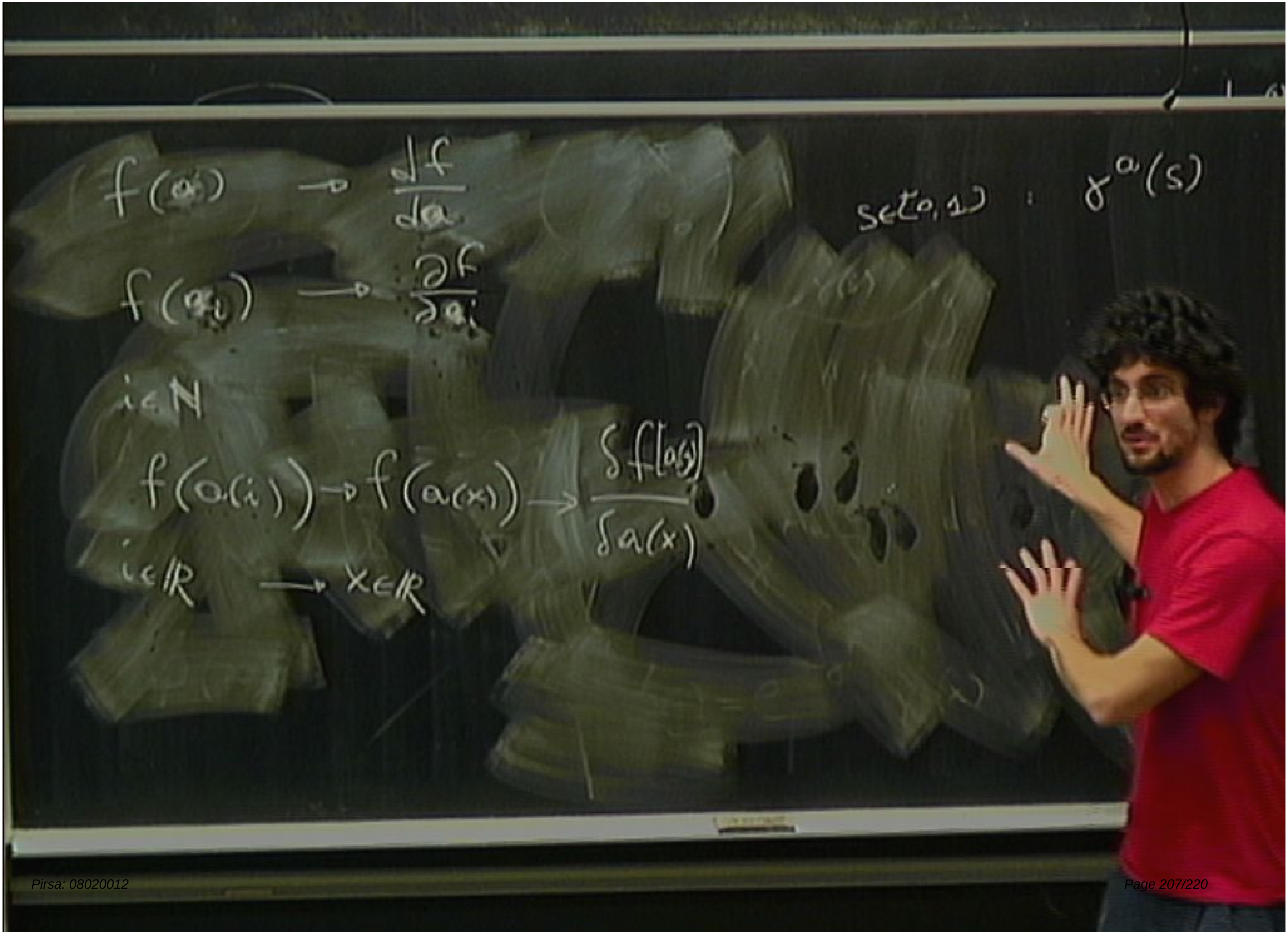

f(a) → df/da
f(a_i) → ∂f/∂a_i
i∈N
f(a(i)) → f(a(x))
i∈R → x∈R
s∈[0,1] : γ^a(s)
A


f(a) → df/da
f(a_i) → ∂f/∂a_i
i ∈ ℕ
f(a(i)) → f(a(x)) =
i ∈ ℝ → x ∈ ℝ
s ∈ [0,1] : γ^a(s)

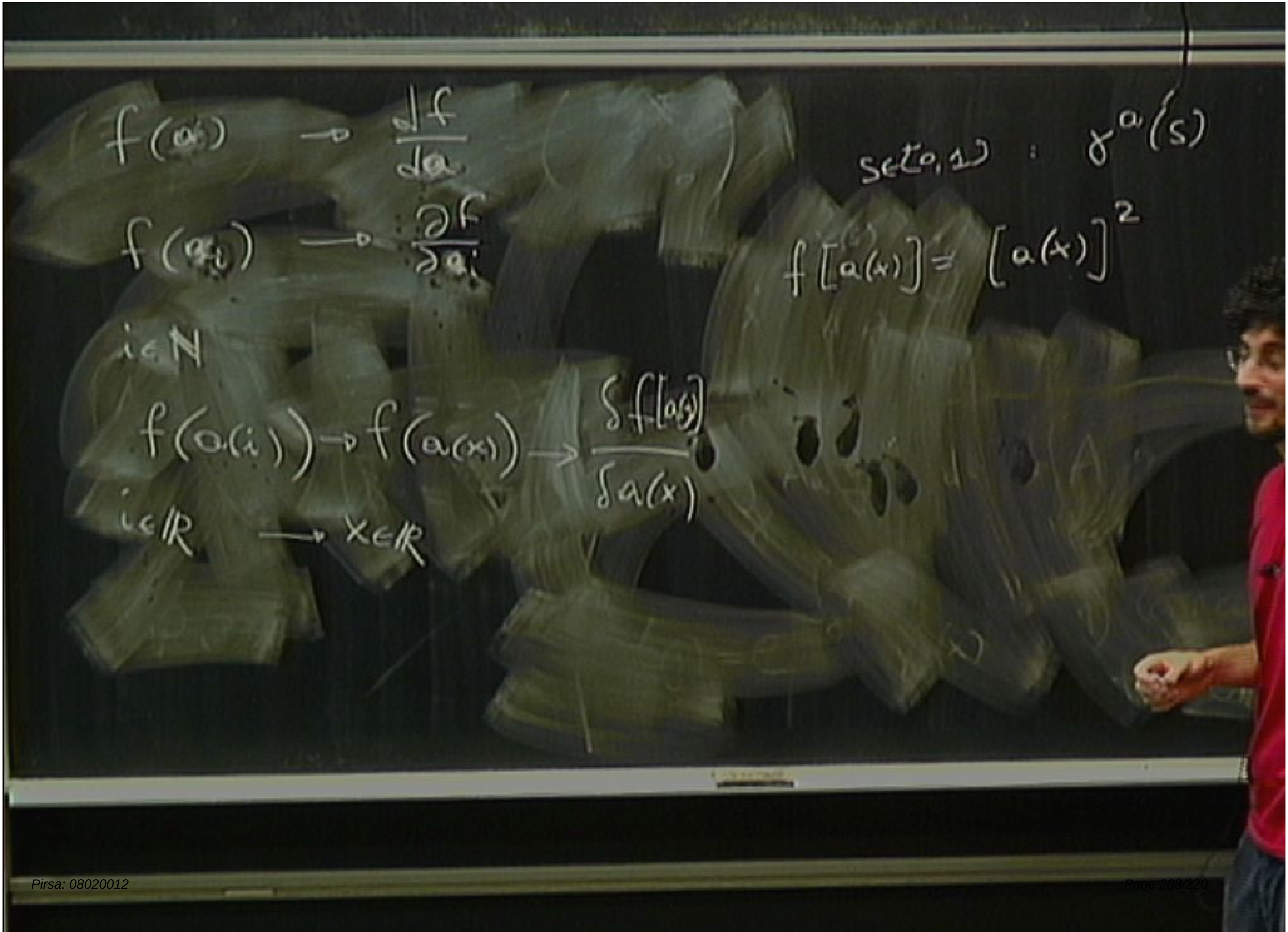

f(a) → df/da
f(a_i) → ∂f/∂a_i
i∈N
f(a(i)) → f(a(x))
i∈R → x∈R
s∈[0,1] : γ^a(s)
∫f[a]

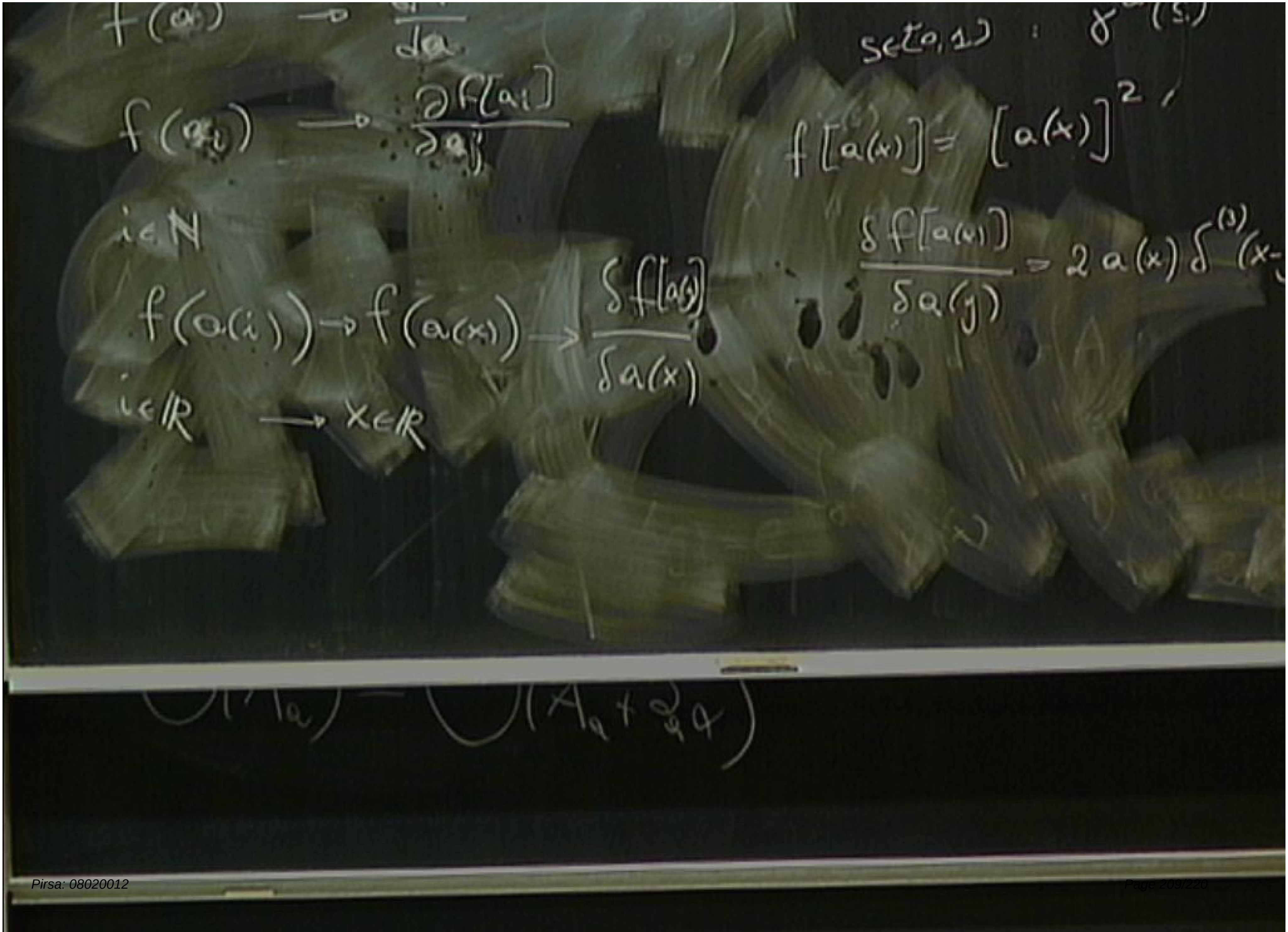

f(a) → df/da
f(a_i) → ∂f/∂a_i
i ∈ N
f(a(i)) → f(a(x)) → δf[a]/δa(x)
i ∈ R → x ∈ R
s ∈ [0,1] : γ^a(s)


f(a) → df/da
f(a_i) → ∂f/∂a_i
i∈N
f(a(i)) → f(a(x)) → δf[a]/δa(x)
i∈R → x∈R
set[0,1] : γ^a(s)



$f(a) \rightarrow \frac{df}{da}$
$f(a_i) \rightarrow \frac{\partial f}{\partial a_i}$
$i \in \mathbb{N}$
$f(a(i)) \rightarrow f(a(x)) \rightarrow \frac{\delta f[a(s)]}{\delta a(x)}$
$i \in \mathbb{R} \rightarrow x \in \mathbb{R}$
$s \in [0,1] \; : \; \gamma^a(s)$


$f(a) \to \frac{df}{da}$
$f(a_i) \to \frac{\partial f}{\partial a_i}$
$i \in \mathbb{N}$
$f(a(i)) \to f(a(x)) \to \frac{\delta f[a(x)]}{\delta a(x)}$
$i \in \mathbb{R} \to x \in \mathbb{R}$
$s \in [0,1] : \gamma^a(s)$
$f[a(x)] = [a(x)]^2$


set(a,1)
f[a(x)] = [a(x)]^2
i∈N
i∈R → x∈R

$$f(a_i) \to \frac{df}{da}$$

$$f(a_i) \to \frac{\partial f(a_i)}{\partial a_i}$$

$i \in \mathbb{N}$

$$f(a(i)) \to f(a(x)) \to \frac{\delta f[a(y)]}{\delta a(x)}$$

$i \in \mathbb{R} \longrightarrow x \in \mathbb{R}$

$s \in [0,1]$: $\gamma^a(s)$

$$f[a(x)] = [a(x)]^2,$$

$$\frac{\delta f[a(x)]}{\delta a(y)} = 2\, a(x)\, \delta^{(1)}(x-y)$$

$$\mathcal{O}(A_a) = \mathcal{O}(A_a + \partial_a \alpha)$$

$$f(a_i) \rightarrow \frac{df}{da}$$

$$f(a_i) \rightarrow \frac{\partial f(a_i)}{\partial a_j}$$

$i \in \mathbb{N}$

$$f(a(i)) \rightarrow f(a(x)) \rightarrow \frac{\delta f[a(y)]}{\delta a(x)}$$

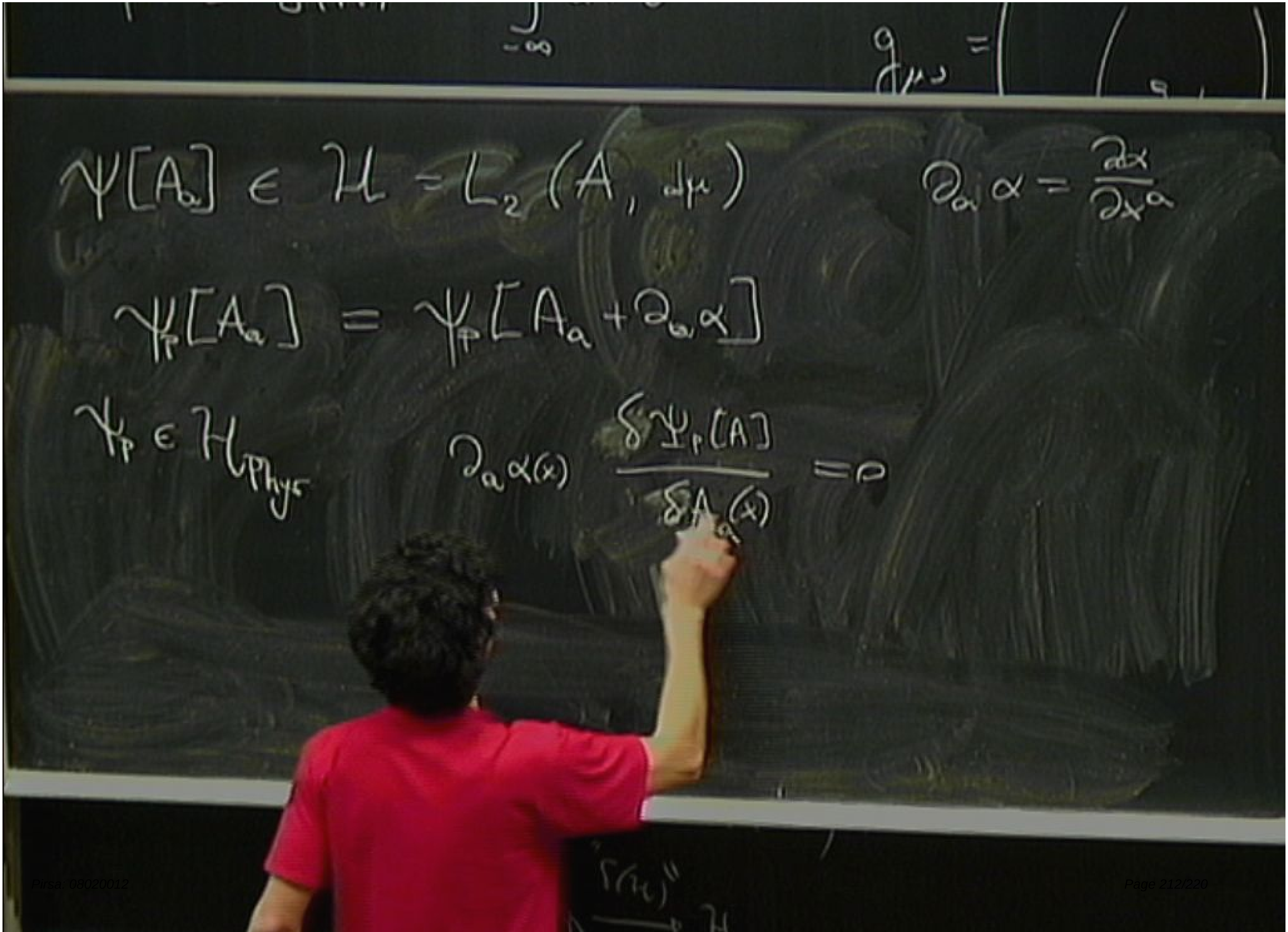
$i \in \mathbb{R} \longrightarrow x \in \mathbb{R}$

$s \in [0,1] \; : \; \gamma^a(s)$

$$f[a(x)] = [a(x)]^2 ,$$

$$\frac{\delta f[a(x)]}{\delta a(y)} = 2\, a(x)\, \delta^{(1)}(x-y)$$

$$O(A_a) = O(A_a + \partial_a \phi)$$


$\Psi[A_a] \in \mathcal{H} = L_2(\mathcal{A}, d\mu)$
$\partial_a \alpha = \frac{\partial \alpha}{\partial x^a}$
$\Psi_p[A_a] = \Psi_p[A_a + \partial_a \alpha]$
$\Psi_p \in \mathcal{H}_{phys}$
$\partial_a \alpha(x) \frac{\delta \Psi_p[A]}{\delta A_a(x)} = 0$

$$\Psi[A_a] \in \mathcal{H} = L_2(\mathcal{A}, d\mu) \qquad \partial_a \alpha = \frac{\partial \alpha}{\partial x^a}$$

$$\Psi_p[A_a] = \Psi_p[A_a + \partial_a \alpha]$$

$$\Psi_p \in \mathcal{H}_{phys} \qquad \partial_a \alpha(x) \frac{\delta \Psi_p[A]}{\delta A_a(x)} = 0$$

$$g_{\mu\nu} = \left(\; g_{ab} \; \right)$$

$$\Psi[A_a] \in \mathcal{H} = L_2(\mathcal{A}, d\mu) \qquad \partial_a \alpha = \frac{\partial \alpha}{\partial x^a}$$

$$\Psi_P[A_a] = \Psi_P[A_a + \partial_a \alpha]$$

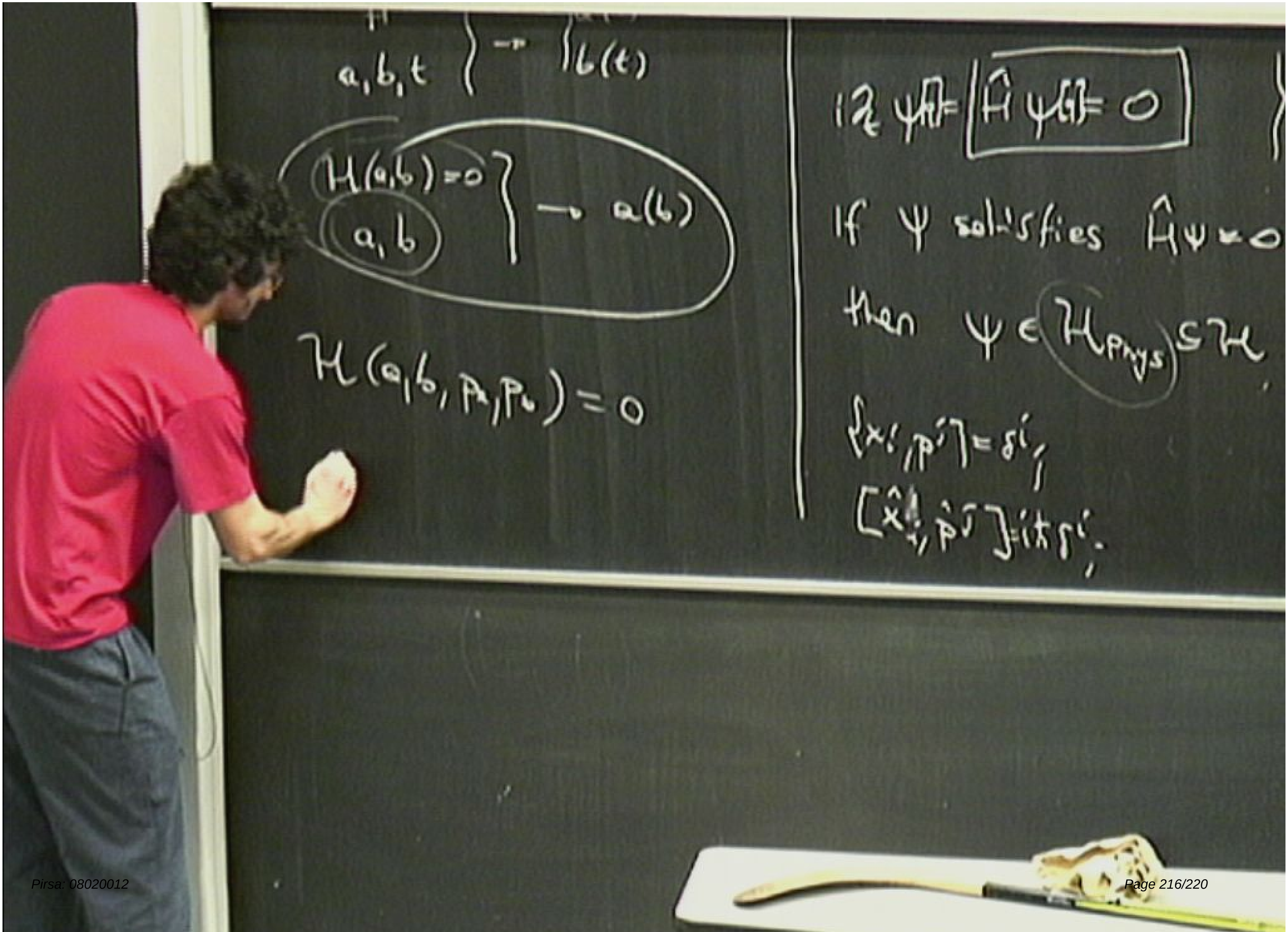
$$\Psi_P \in \mathcal{H}_{Phys} \qquad \partial_a \alpha(x) \frac{\delta \Psi_P[A]}{\delta A_a(x)} = 0$$

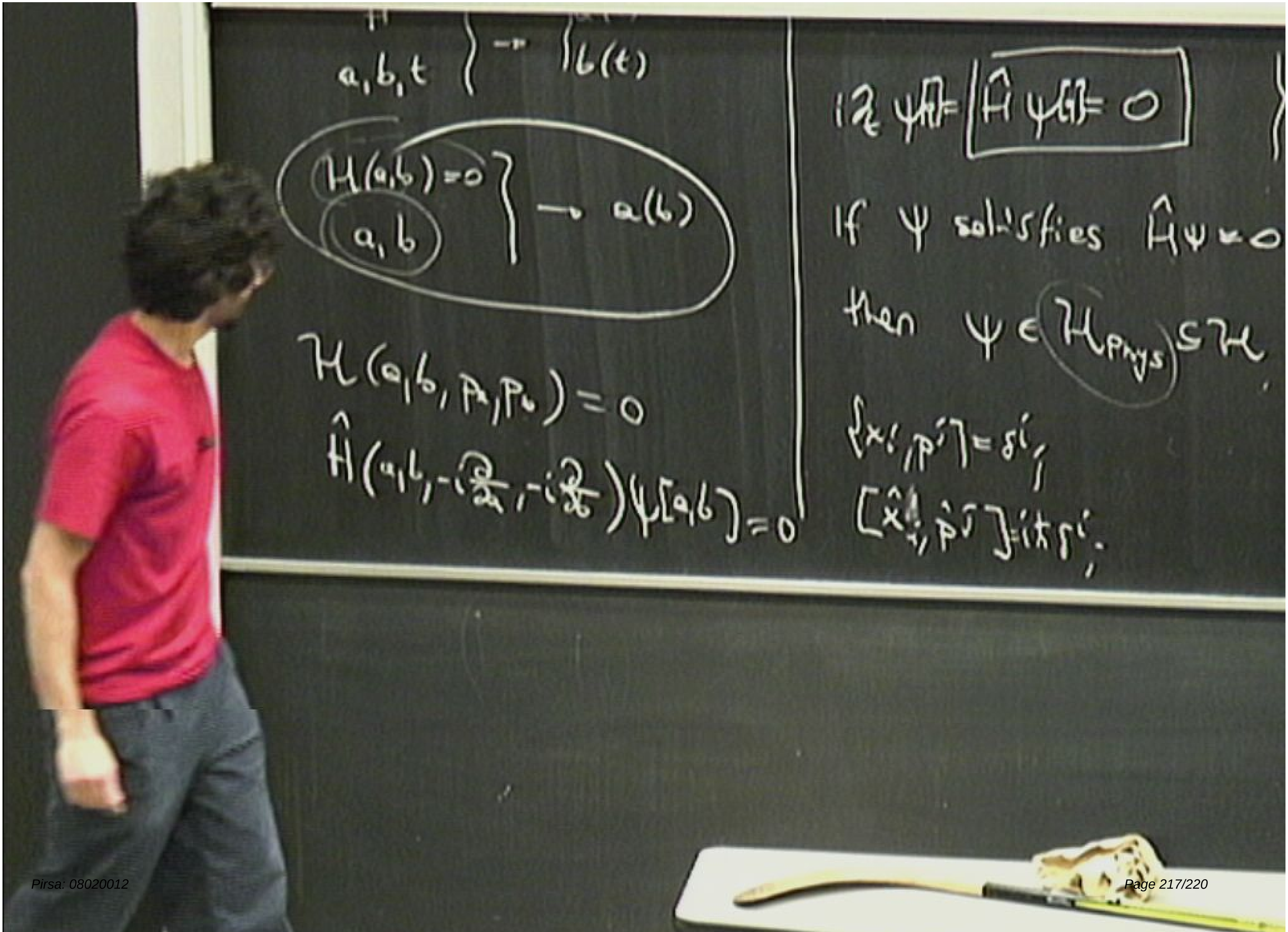
$$\Psi[A_a] \in \mathcal{H} = L_2(\mathcal{A}, d\mu) \qquad \partial_a \alpha = \frac{\partial \alpha}{\partial x^a}$$

$$\Psi_p[A_a] = \Psi_p[A_a + \partial_a \alpha]$$

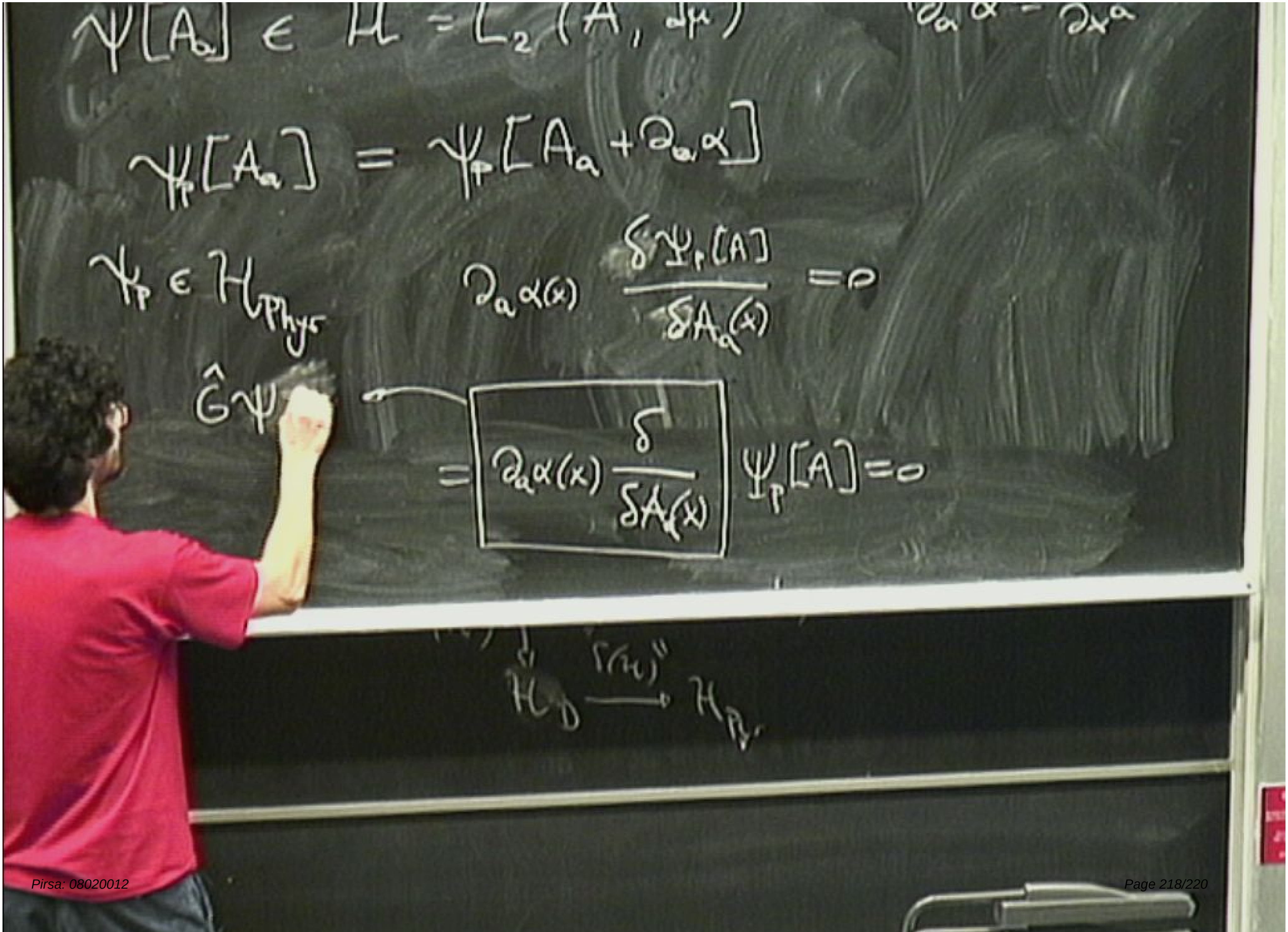
$$\Psi_p \in \mathcal{H}_{Phys} \qquad \partial_a \alpha(x) \frac{\delta \Psi_p[A]}{\delta A_a(x)} = 0$$

$$= \partial_a \alpha(x) \frac{\delta}{\delta A_a(x)} \Psi_p[A] = 0$$


a, b, t
b(t)
H(a,b) = 0
a, b
→ a(b)
H(a, b, p_a, p_b) = 0
If Ψ satisfies ĤΨ = 0
then Ψ ∈ H_phys ⊆ H



a, b, t → b(t)
H(a,b)=0
a, b
→ a(b)
H(a,b,p_a,p_b) = 0
Ĥ(a,b,-i∂/∂a,-i∂/∂b)ψ[a,b] = 0
i∂_t ψ(t) = Ĥψ(t) = 0
If ψ satisfies Ĥψ=0
then ψ ∈ H_phys ⊆ H
{x^i, p^j} = δ^i_j
[x̂^i, p̂^j] = iħδ^i_j


$\Psi[A_a] \in \mathcal{H} = L_2(A, d\mu)$
$\partial_a \alpha - \partial_x a$
$\Psi_p[A_a] = \Psi_p[A_a + \partial_a \alpha]$
$\Psi_p \in \mathcal{H}_{phys}$
$\partial_a \alpha(x) \frac{\delta \Psi_p[A]}{\delta A_a(x)} = 0$
$\hat{G}\Psi$
$= \boxed{\partial_a \alpha(x) \frac{\delta}{\delta A_a(x)}} \Psi_p[A] = 0$
$\mathcal{H}_g \xrightarrow{} \mathcal{H}_{Ph}$

$$\Psi[A_a] \in \mathcal{H} = L_2(A, d\mu) \qquad \partial_a \alpha = \partial_{x^a}$$

$$\Psi_p[A_a] = \Psi_p[A_a + \partial_a \alpha]$$

$$\Psi_p \in \mathcal{H}_{phys} \qquad \partial_a \alpha(x) \frac{\delta \Psi_p[A]}{\delta A_a(x)} = 0$$

$$\hat{G}\psi = 0 \qquad = \boxed{\partial_a \alpha(x) \frac{\delta}{\delta A_a(x)}} \Psi_p[A] = 0$$

$$\mathcal{H}_g \xrightarrow{\text{"}\Gamma(\mathcal{H})\text{"}} \mathcal{H}_{Ph}$$

$$\delta(x-x_0) = \int dp\, e \ldots \qquad \int g_{0a} = N_0$$

$$\Psi[A_a] \in \mathcal{H} = L_2(A, d\mu) \qquad \partial_a \alpha = \frac{\partial \alpha}{\partial x^a}$$

$$\Psi_p[A_a] = \Psi_p[A_a + \partial_a \alpha]$$

$$\Psi_p \in \mathcal{H}_{phys} \qquad \partial_a \alpha(x) \frac{\delta \Psi_p[A]}{\delta A_a(x)} = 0$$

$$\hat{G}\psi = 0$$

$$= \boxed{\partial_a \alpha(x) \frac{\delta}{\delta A_a(x)}} \Psi_p[A] = 0$$