Title: Loop Quantum gravity #1

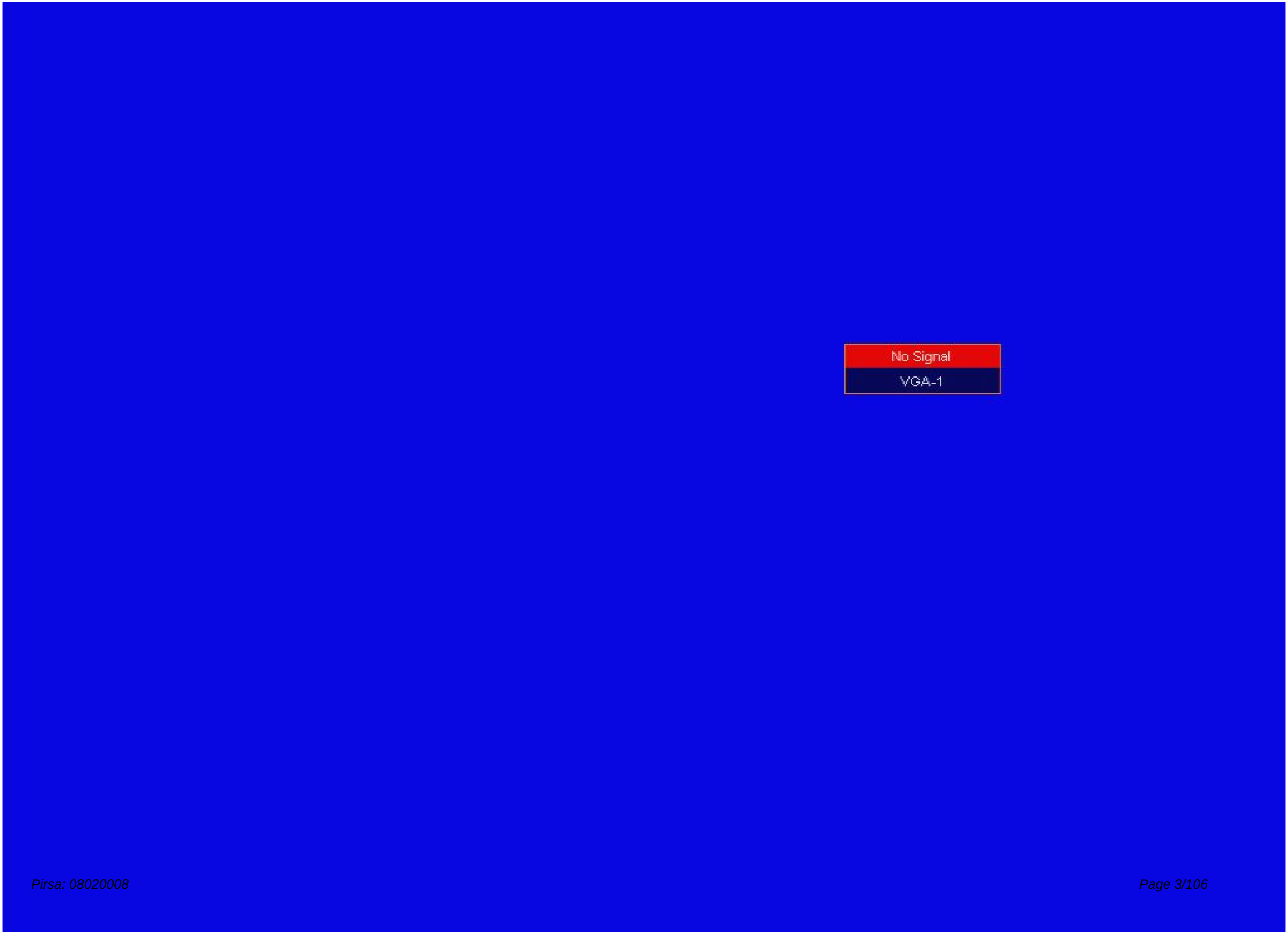
Date: Feb 05, 2008 06:30 PM

URL: http://pirsa.org/08020008

Abstract: Constrained systems, meaning of diffeomorphism invariance, loops and spin networks.

No Signal

VGA-1

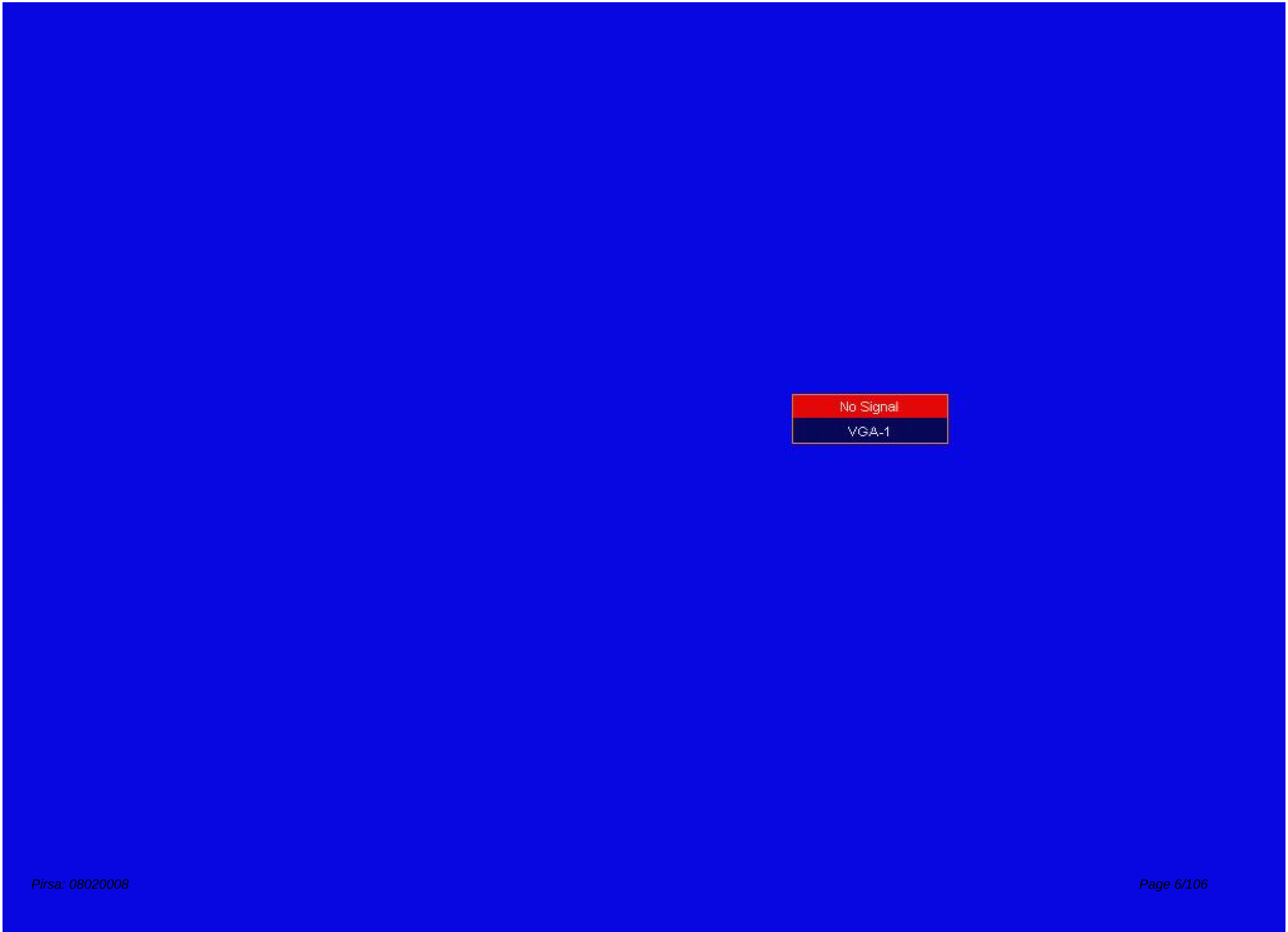

No Signal
VGA-1

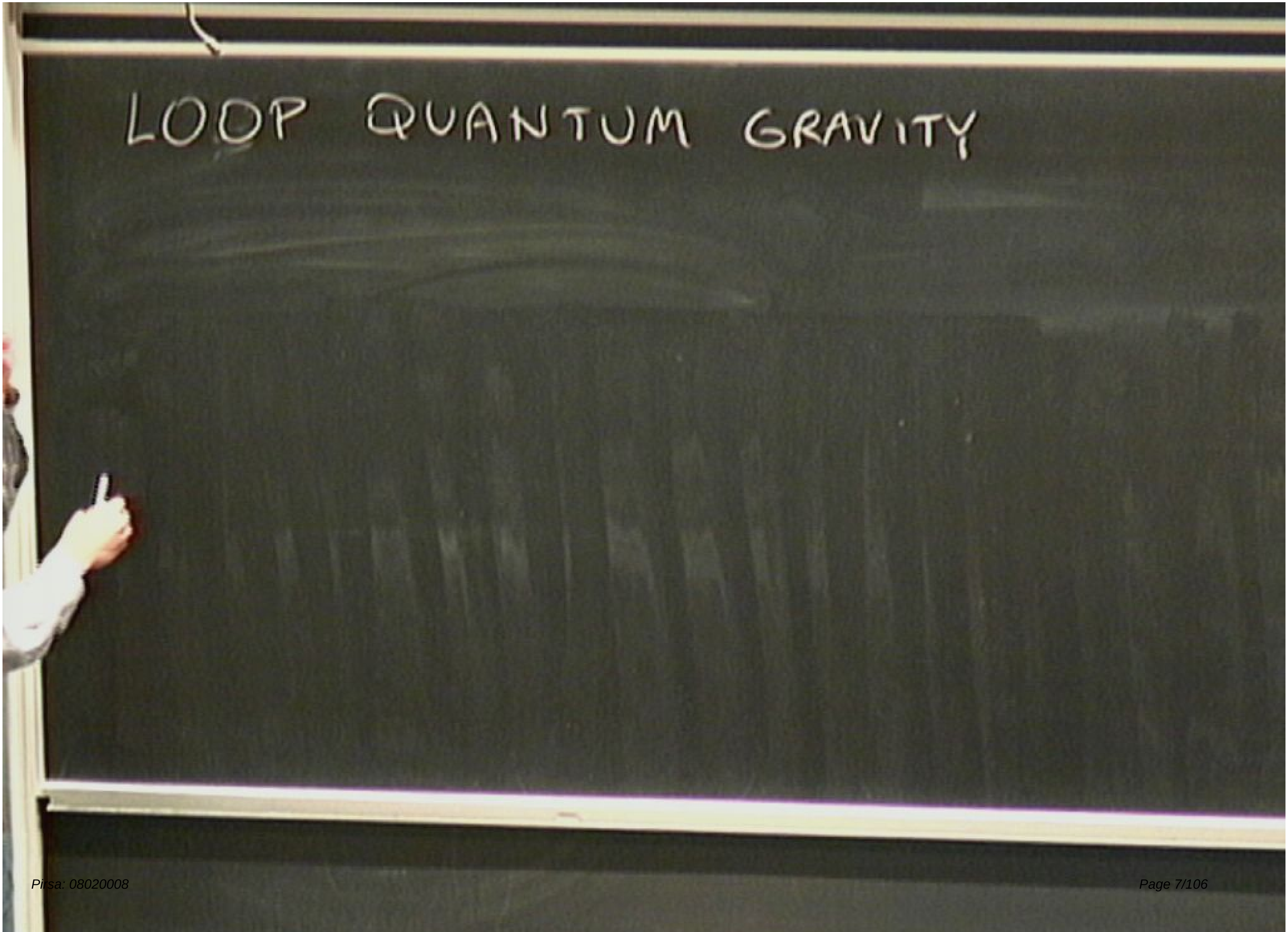
No Signal

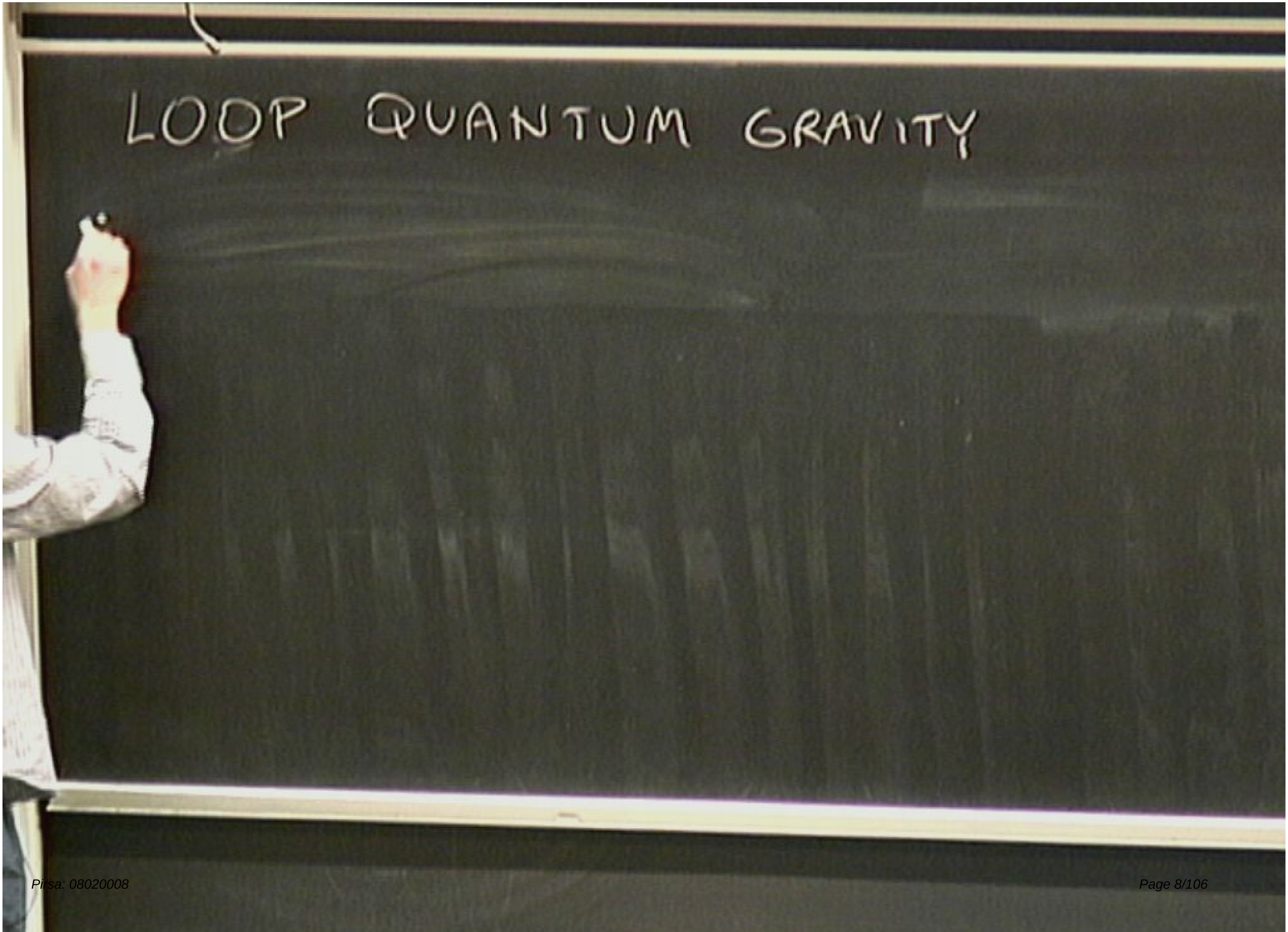
VGA-1

No Signal

VGA-1


No Signal
VGA-1


LOOP QUANTUM GRAVITY


LOOP QUANTUM GRAVITY

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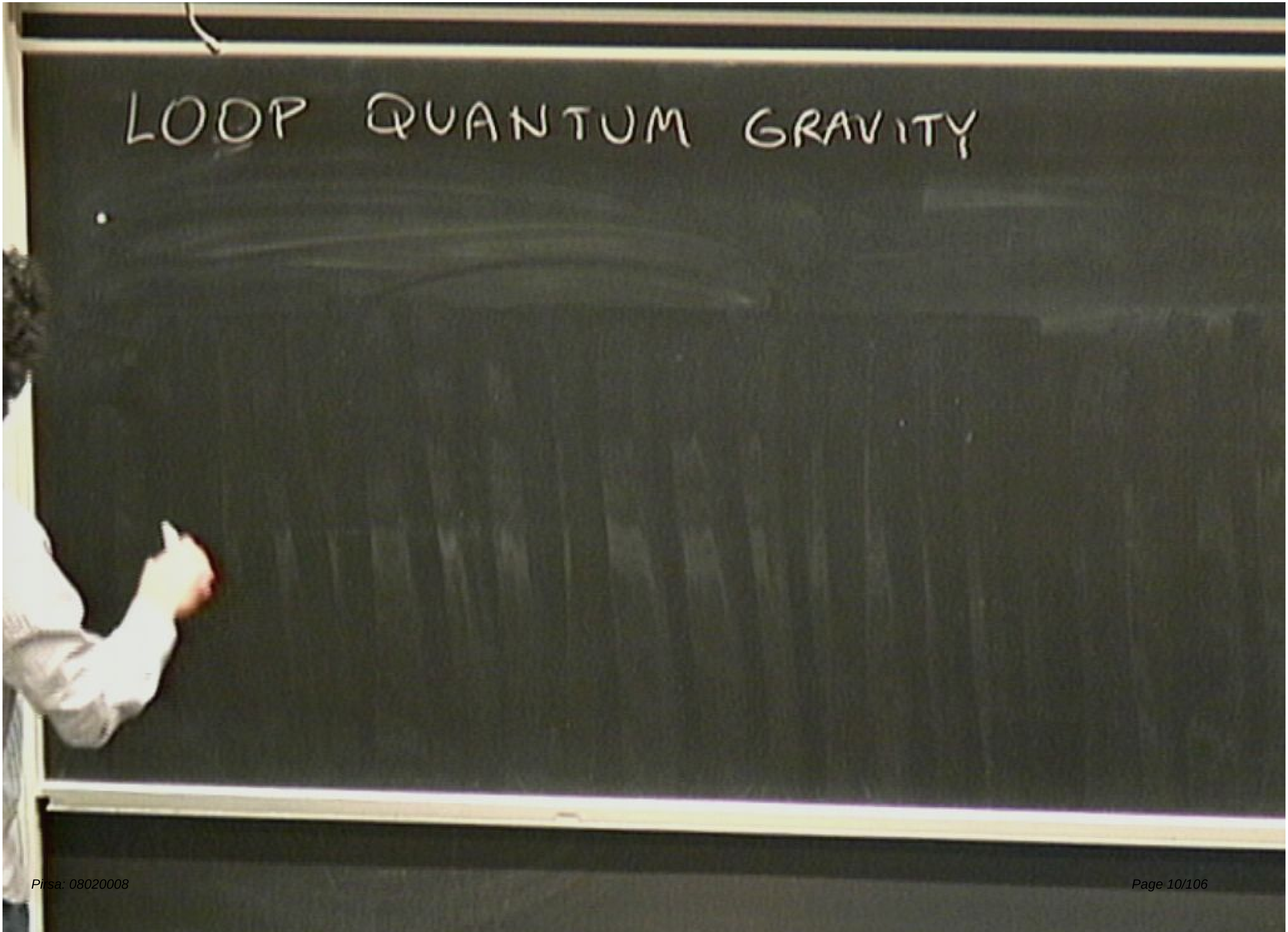
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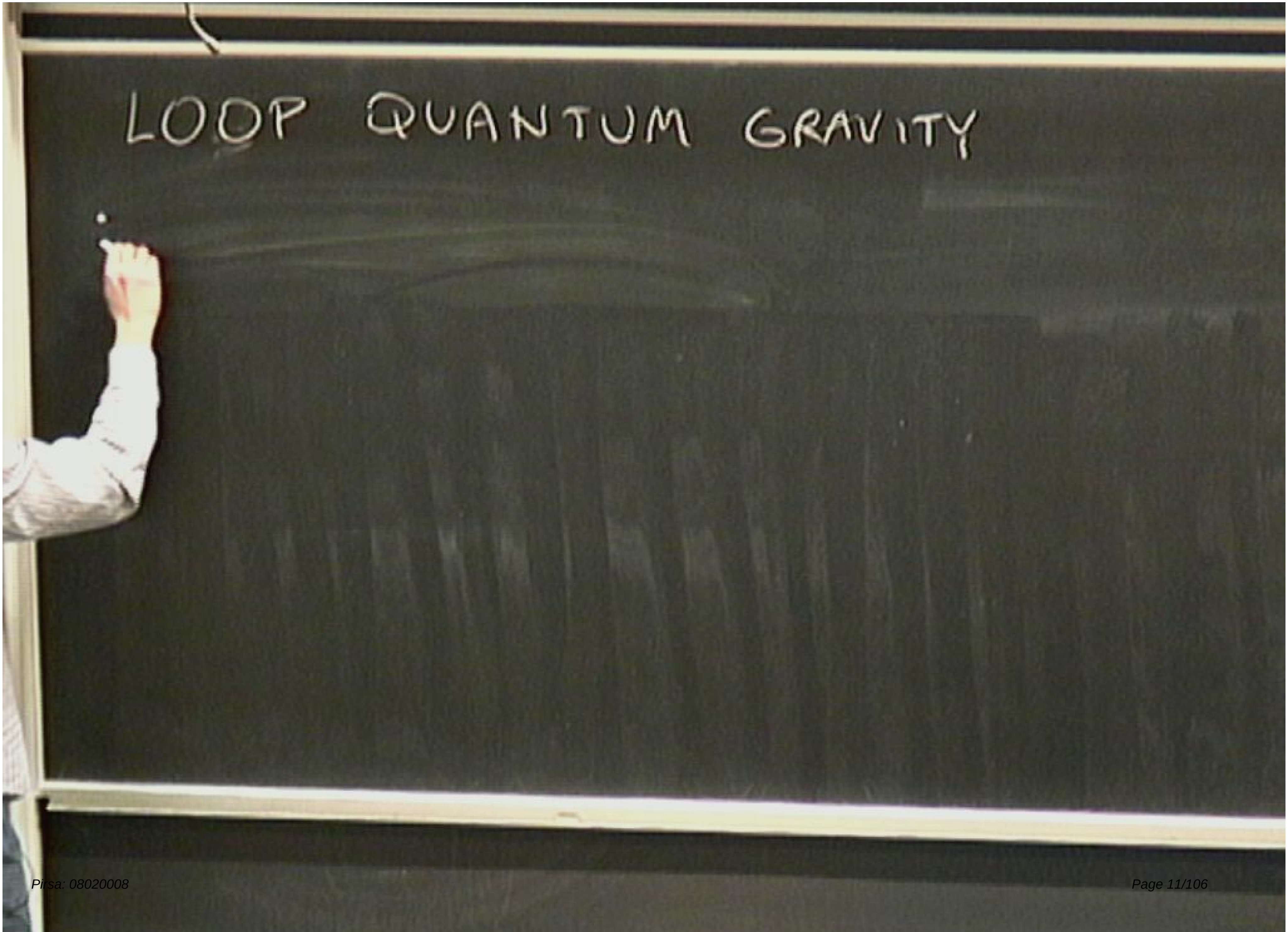
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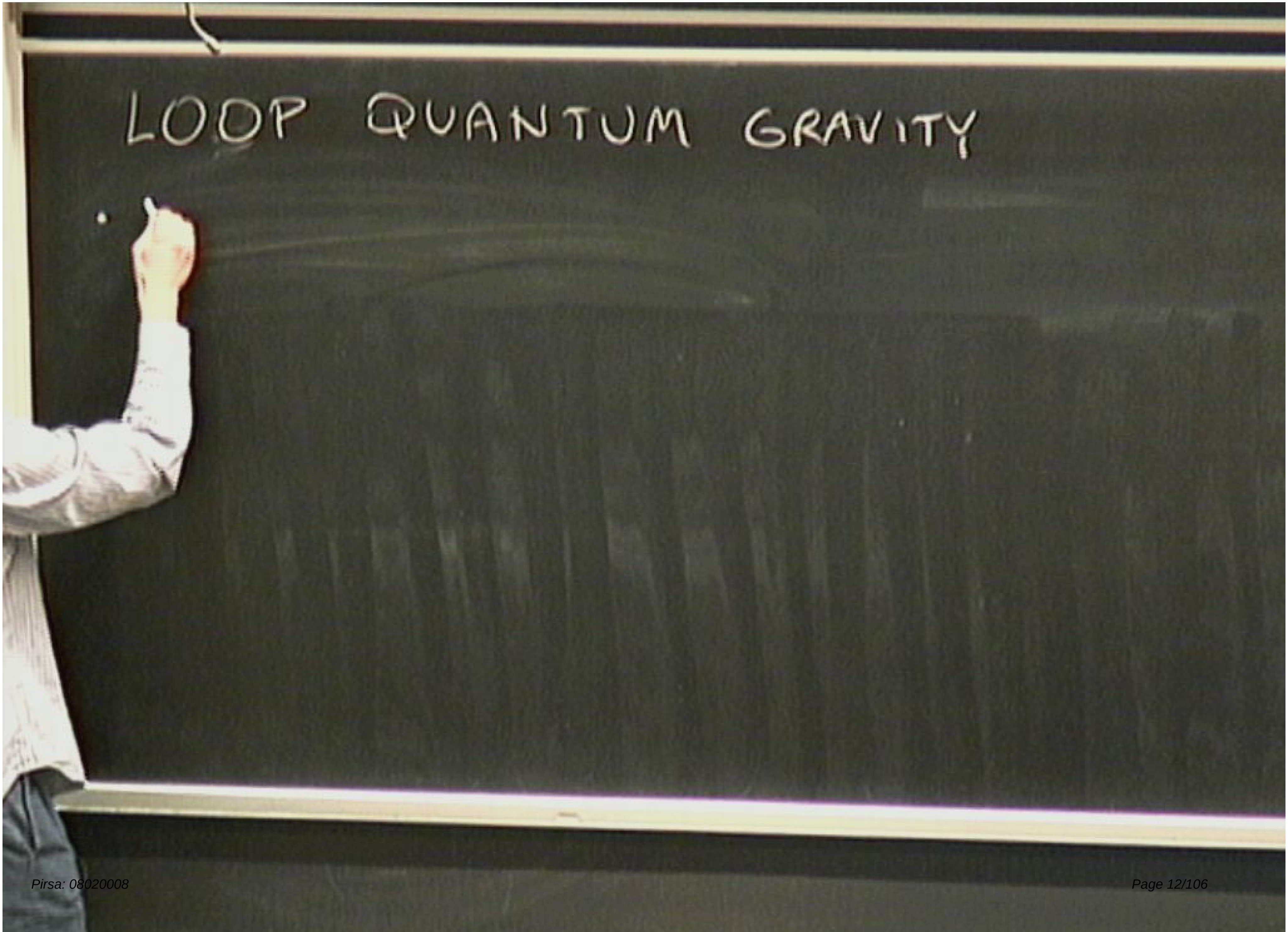
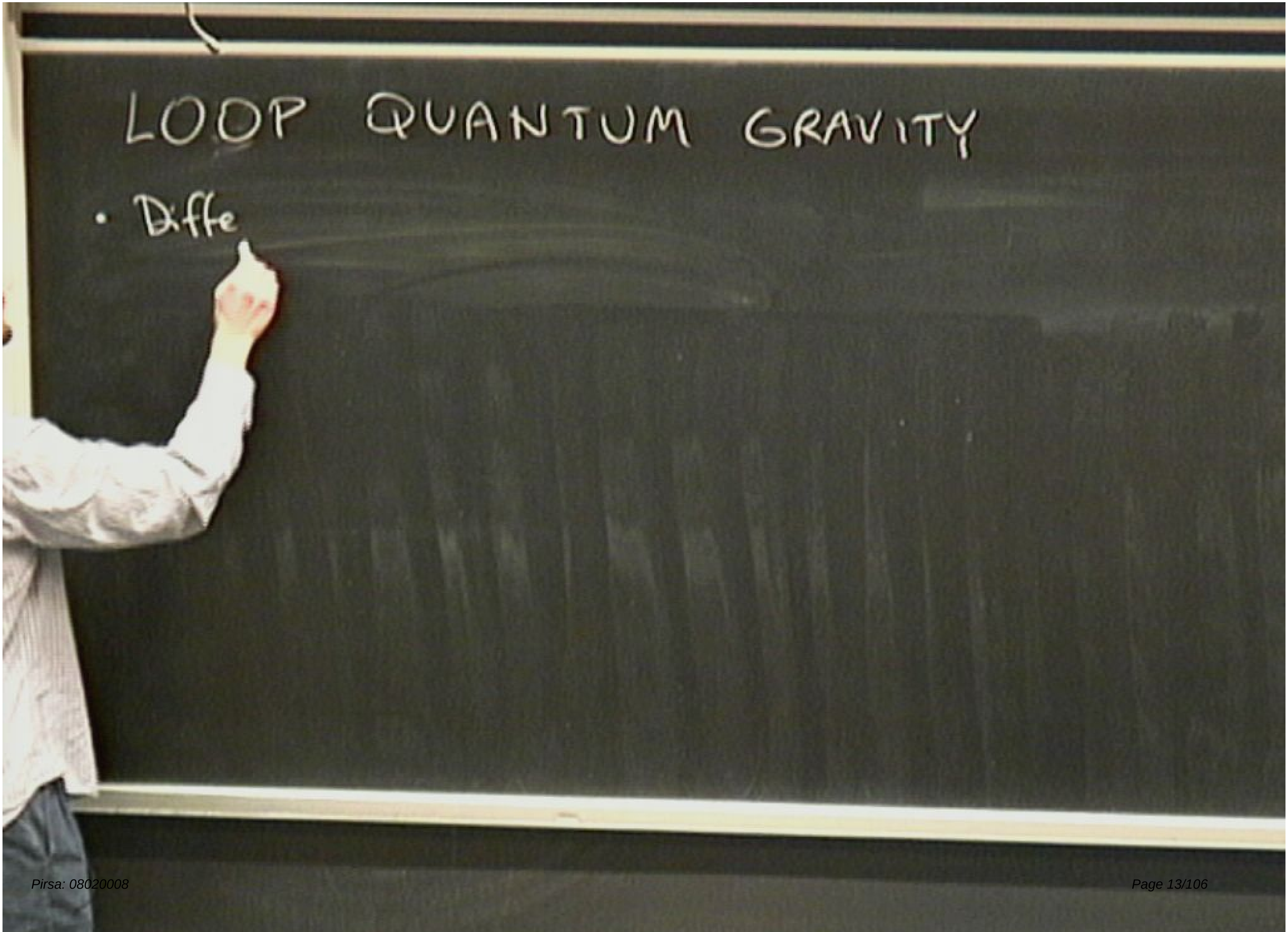
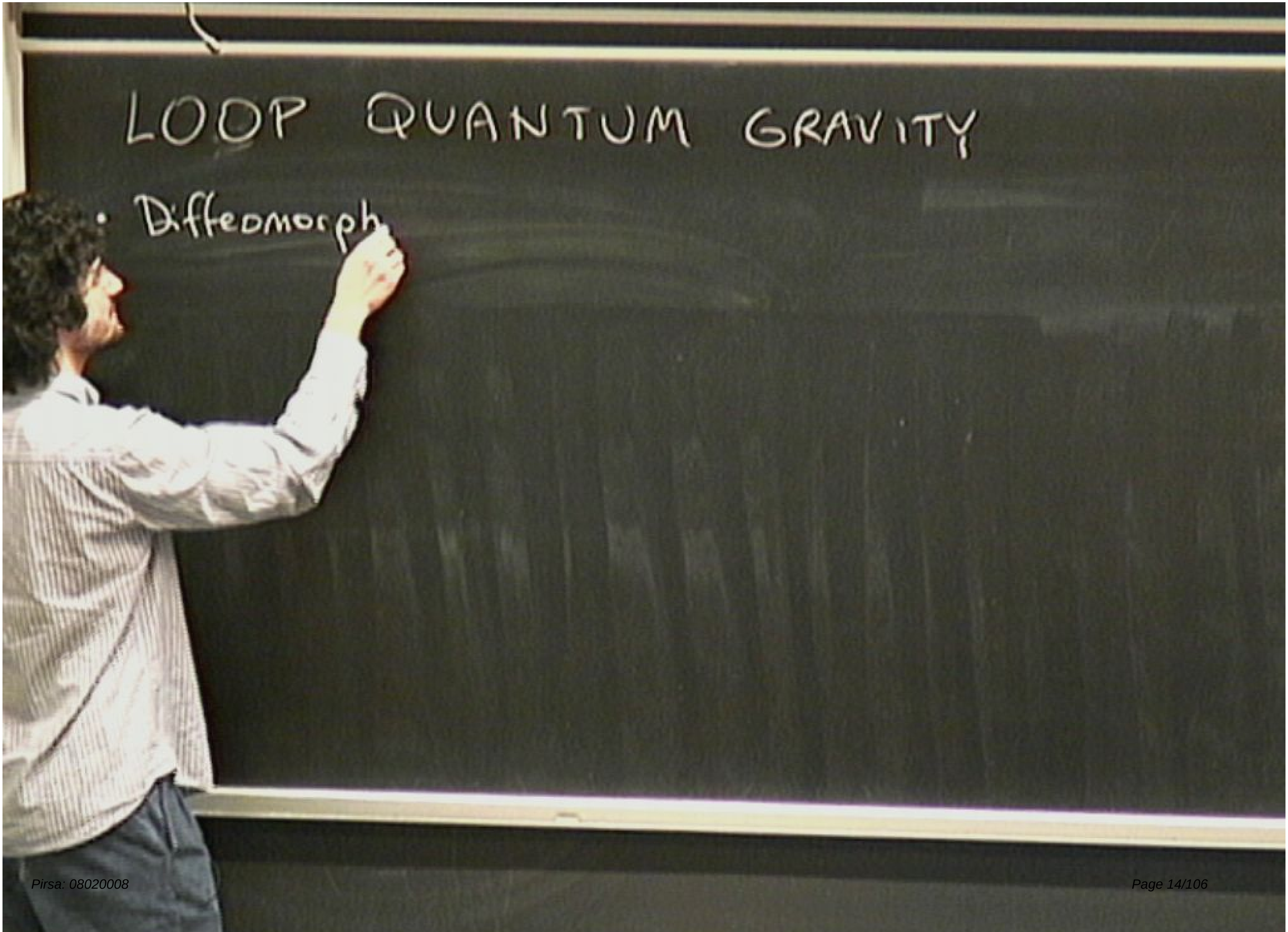
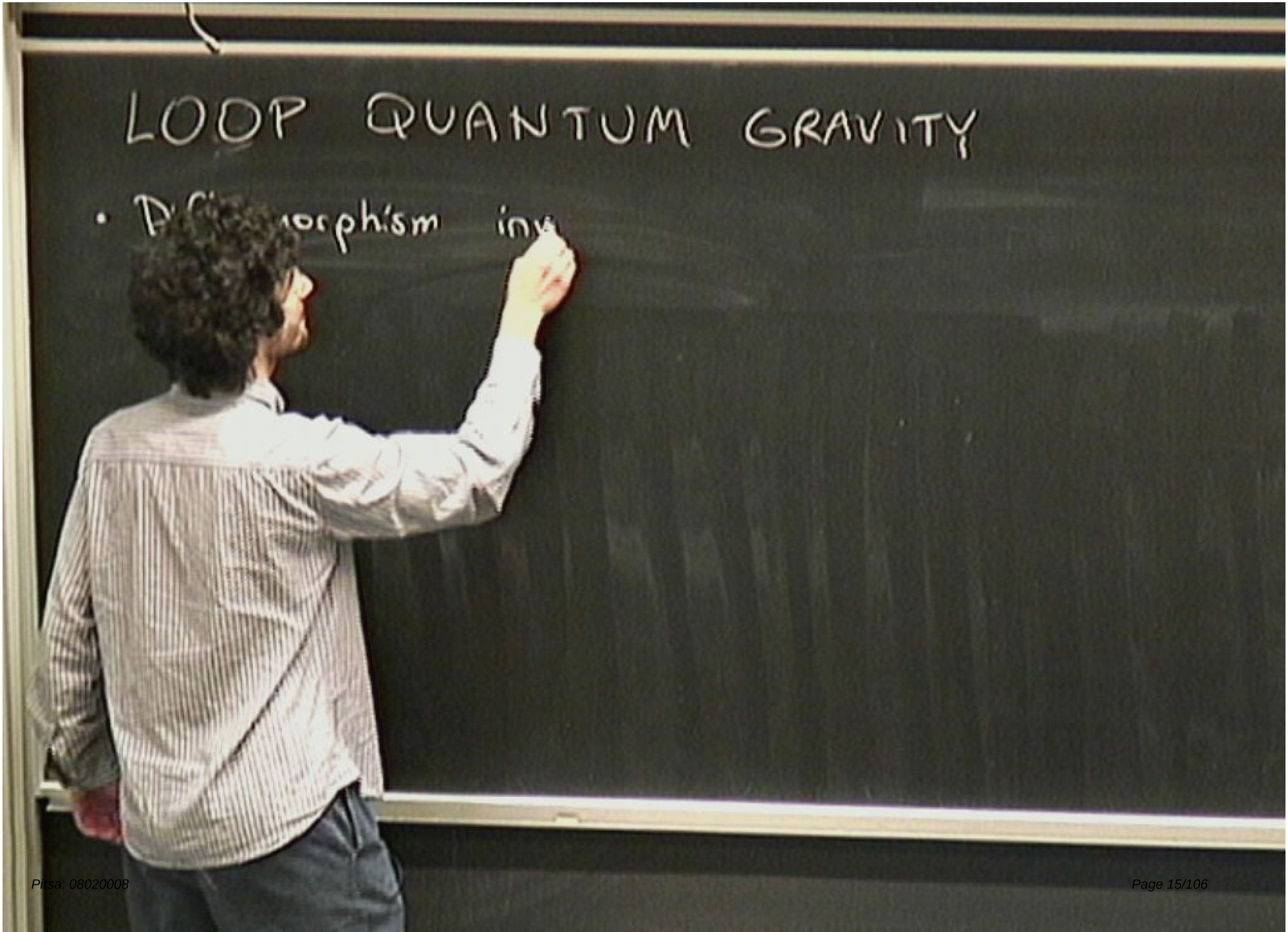
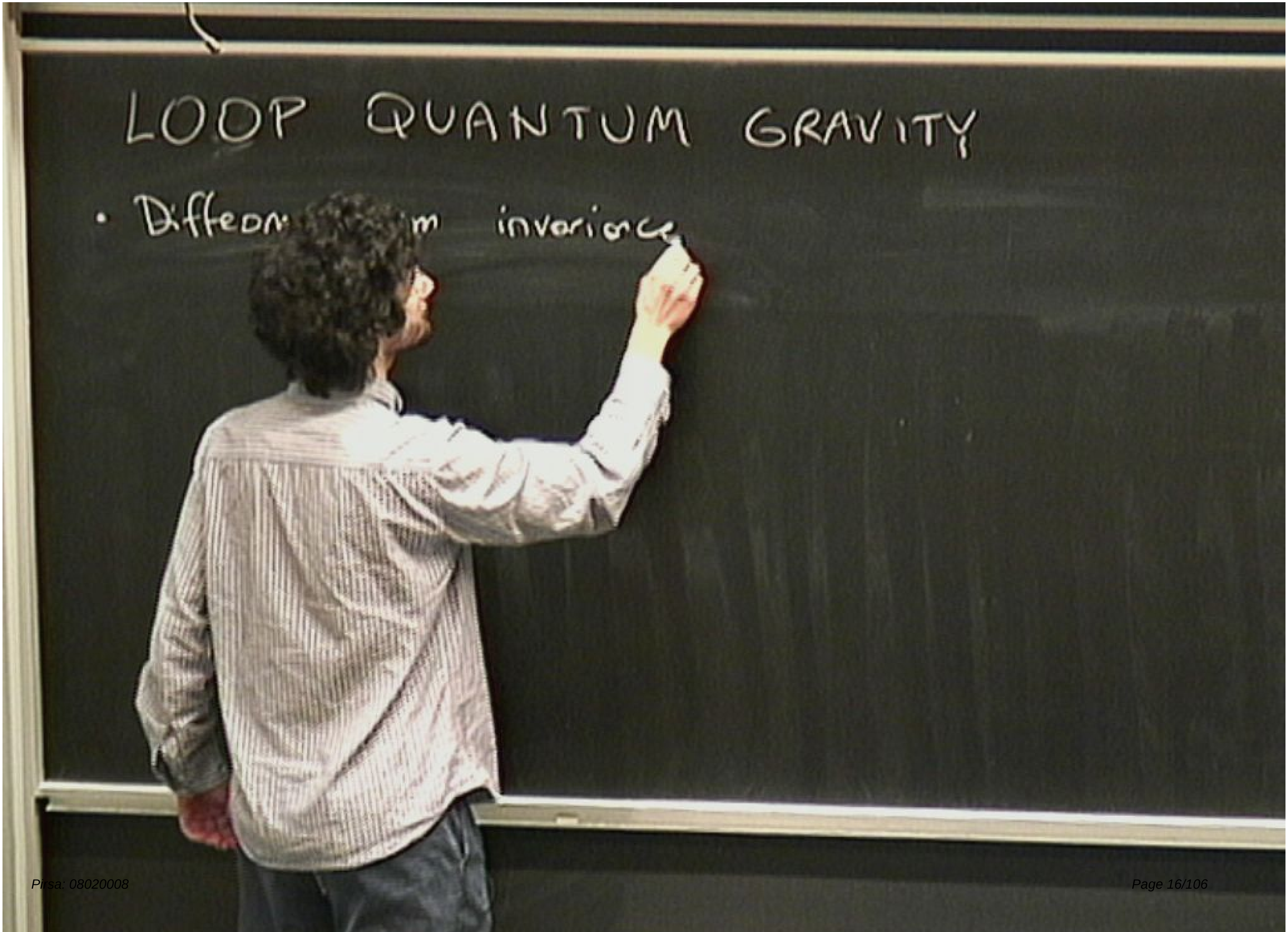
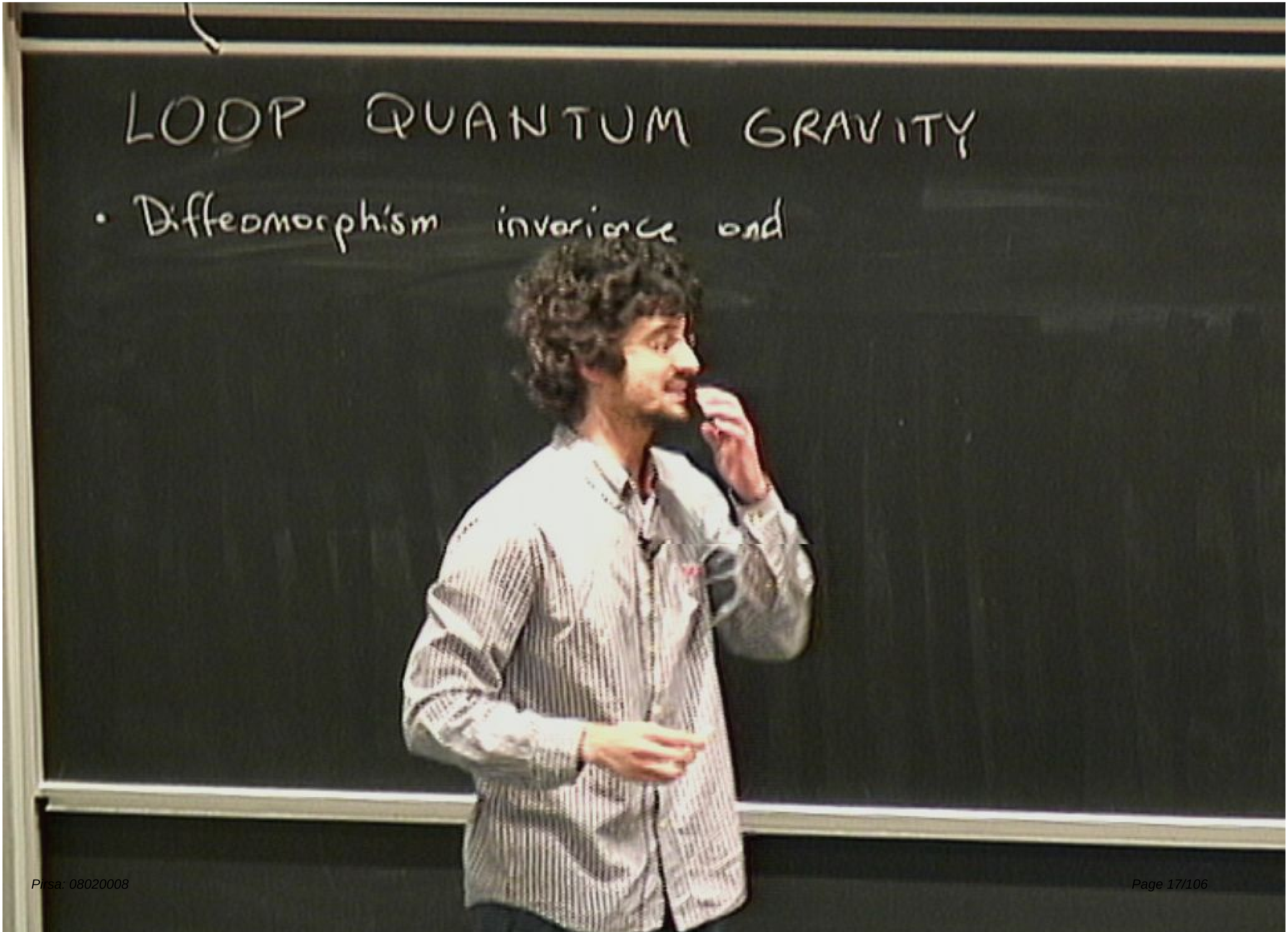

LOOP QUANTUM GRAVITY

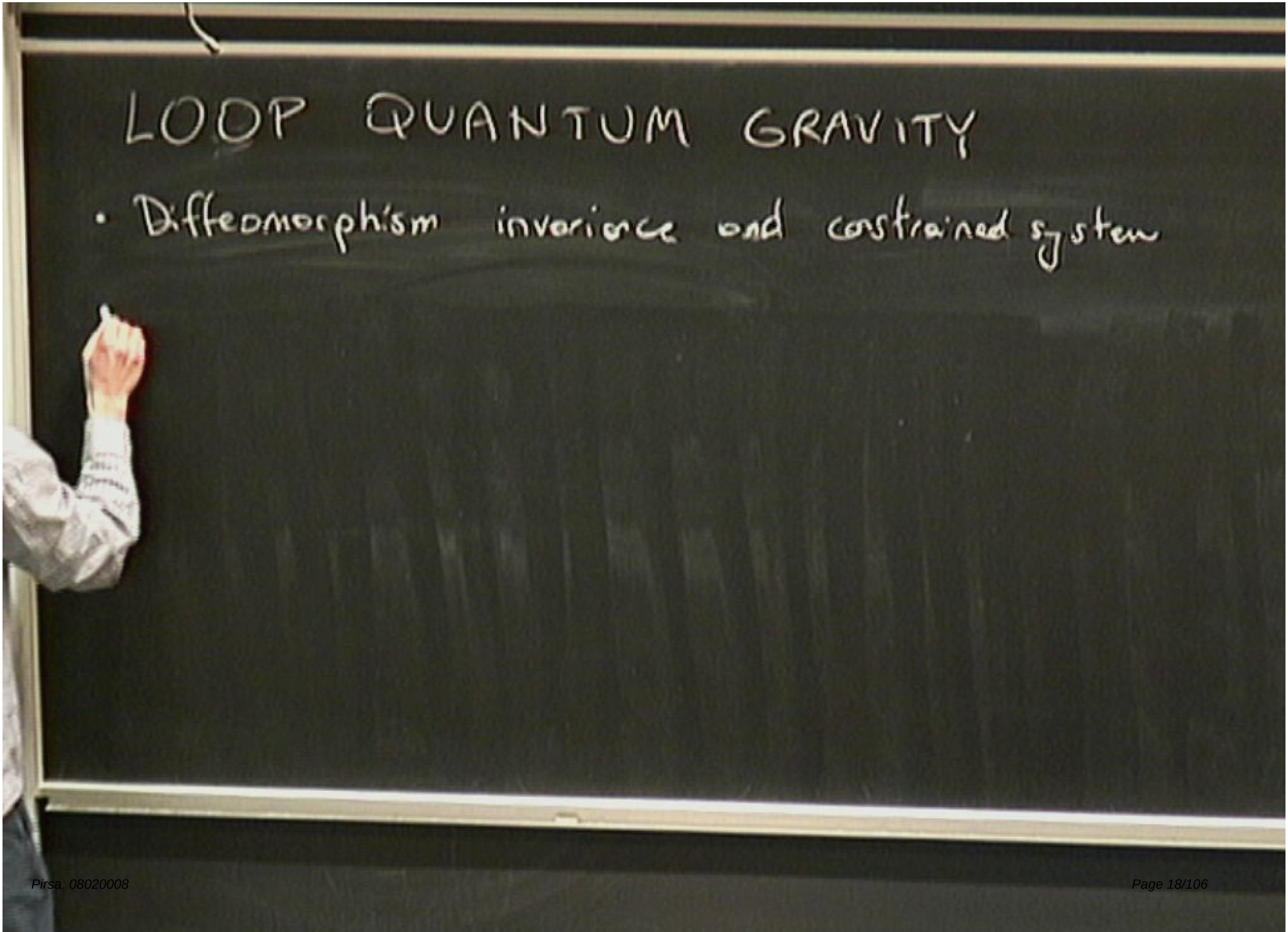

LOOP QUANTUM GRAVITY
• Diffe

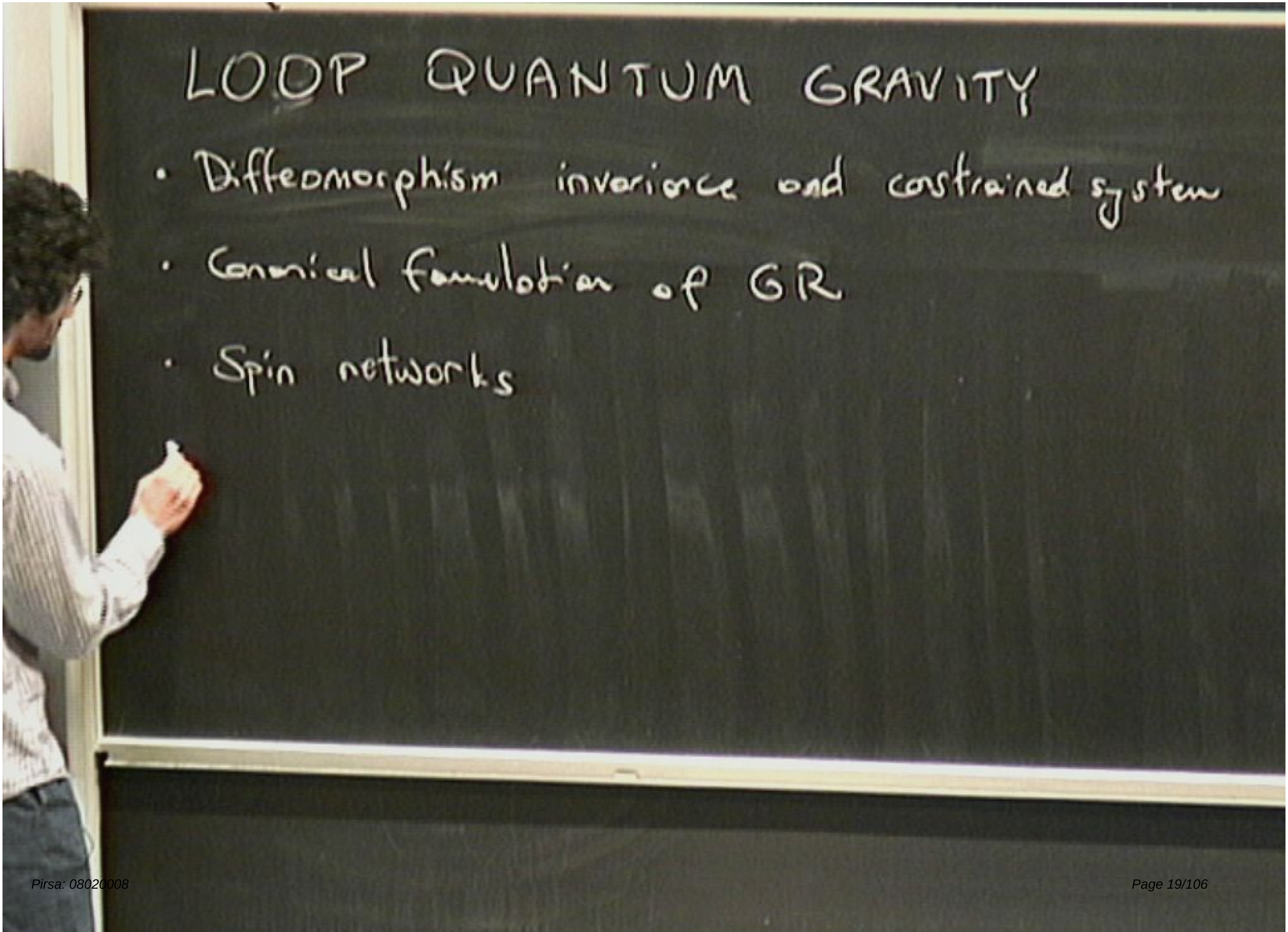

LOOP QUANTUM GRAVITY
• Diffeomorph

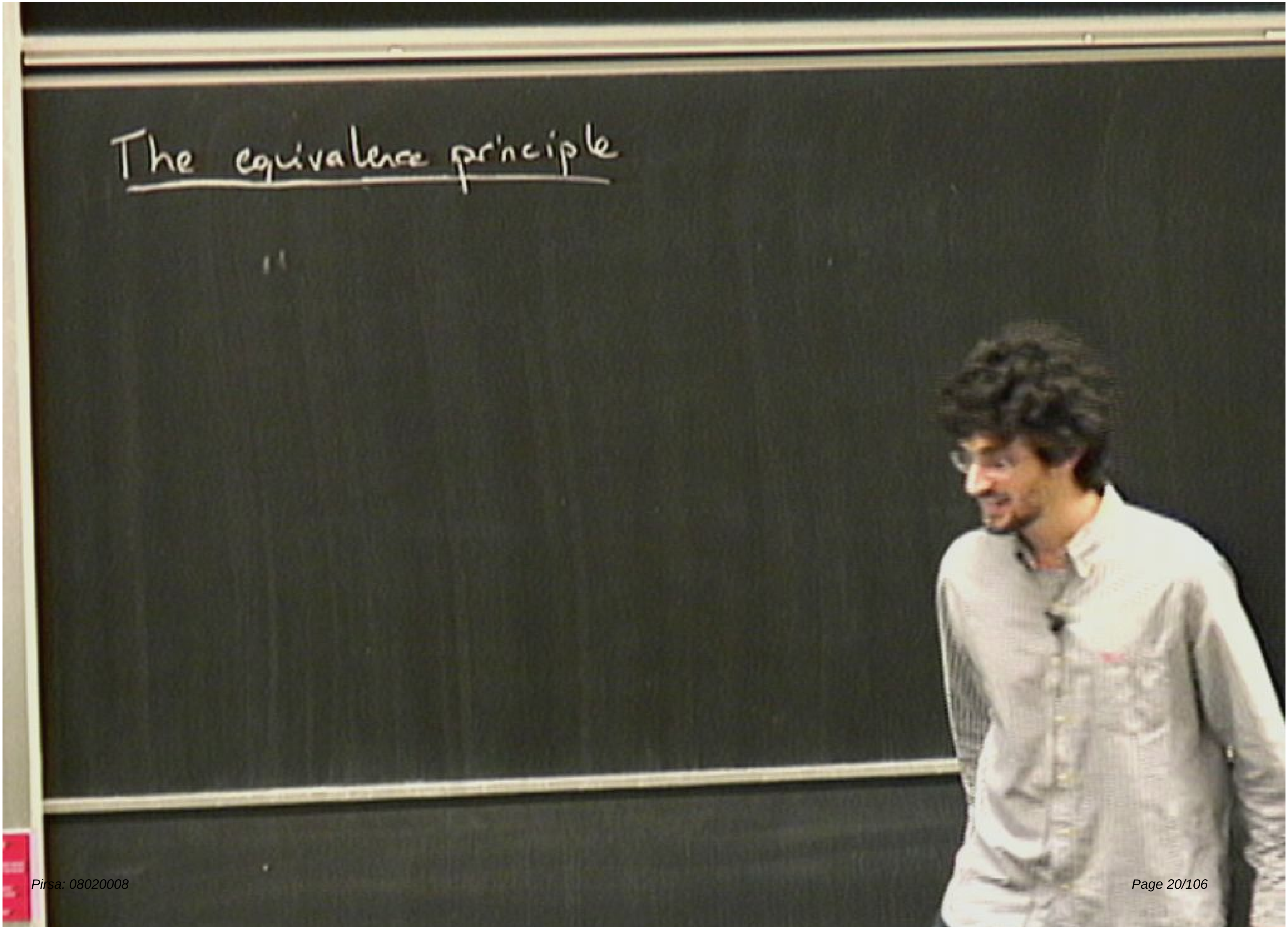

LOOP QUANTUM GRAVITY

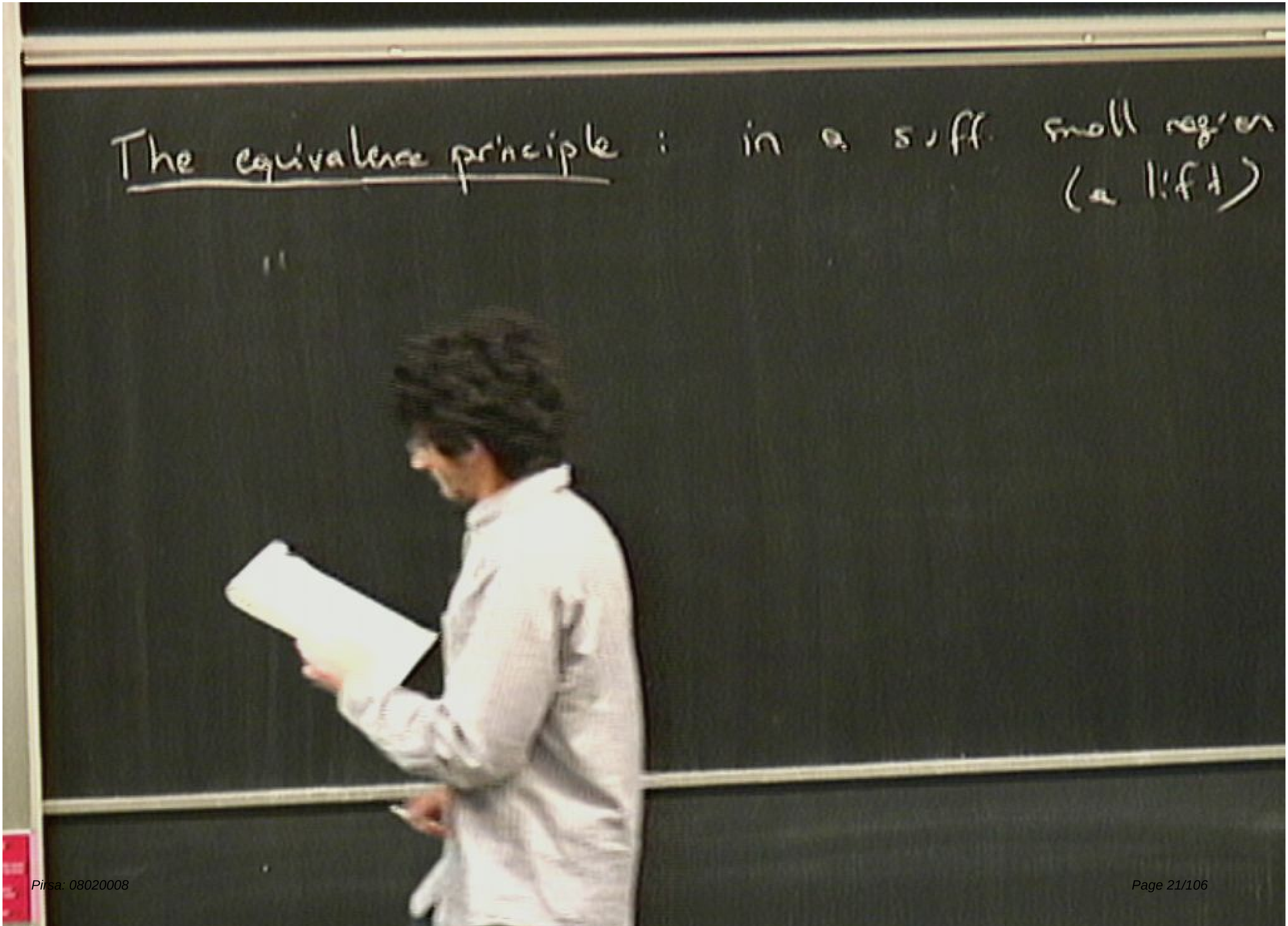

LOOP QUANTUM GRAVITY
• Diffeom
m invariance

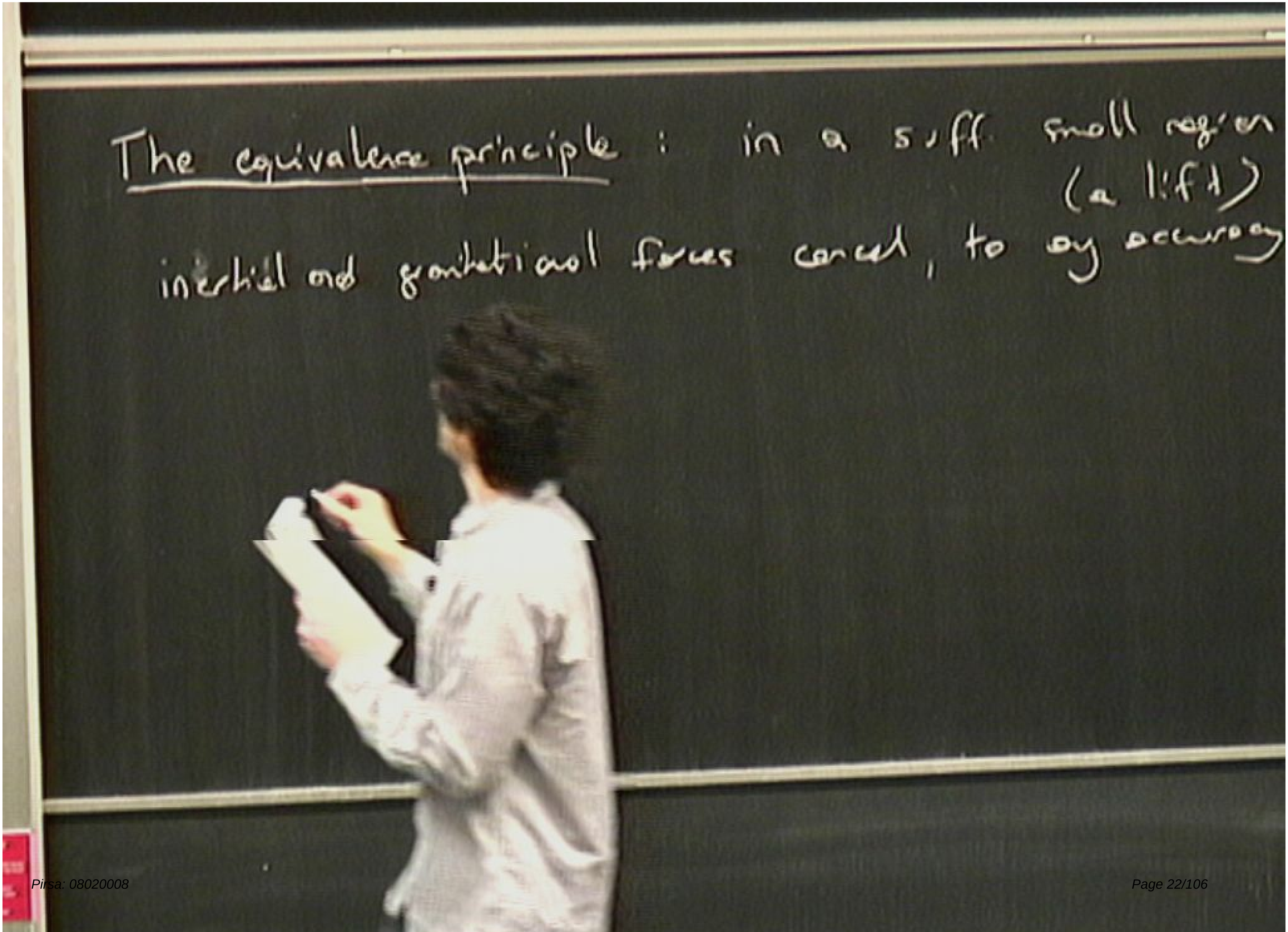

LOOP QUANTUM GRAVITY
• Diffeomorphism invariance and


LOOP QUANTUM GRAVITY
• Diffeomorphism invariance and constrained system


LOOP QUANTUM GRAVITY
• Diffeomorphism invariance and constrained system
• Canonical formulation of GR
• Spin networks


The equivalence principle


The equivalence principle : in a suff. small region
(a lift)


The equivalence principle : in a suff. small region
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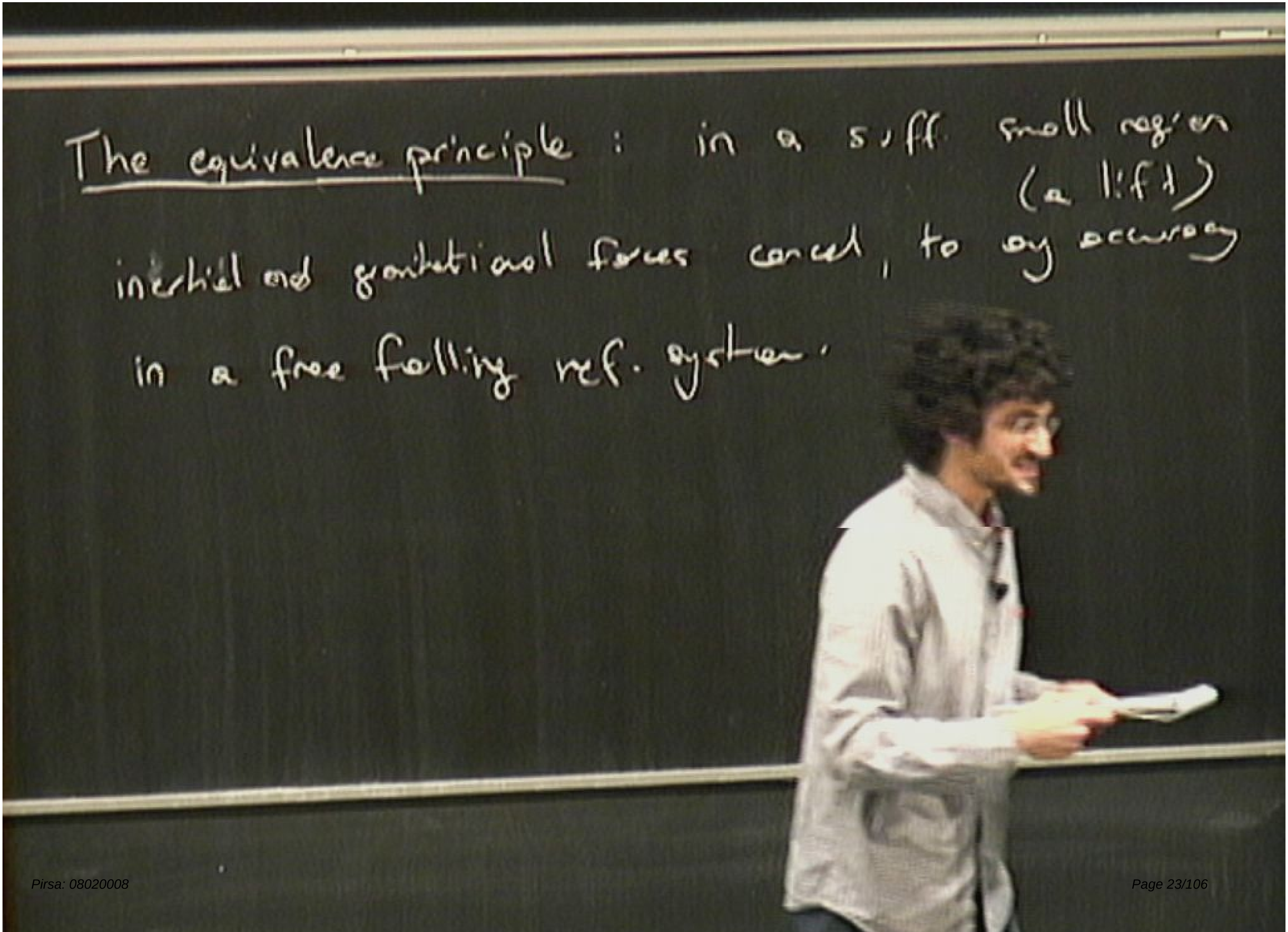
The equivalence principle : in a suff. small region
(a lift)
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in a free falling ref. system.

The equivalence principle : in a suff. small region (a lift) inertial and gravitational forces cancel, to any accuracy in a free falling ref. system.

L $\quad F = -G \frac{Mm}{r^2}$

<u>The equivalence principle</u> : in a suff. small region (a lift)

inertial and gravitational forces cancel, to any accuracy

in a free falling ref. system.

$$F = -G \frac{M m_g}{r^2} \qquad m_i = m_g$$

$$F = m_i a$$

The equivalence principle : in a suff. small regi
(a lift)

inertial and gravitational forces cancel, to any accur

in a free falling ref. system.

$$\hookrightarrow \quad F = -G\frac{M m_g}{r^2} \qquad m_i = m_g$$

$$F = m_i a \qquad ds^2 = \sum_{\mu,\nu=0}^{3} \eta_{\mu\nu}\, d\xi^\mu d\xi^\nu$$

x^μ

$$\xi^I_{x_0} = \xi^I_{x_0}(x^\mu) \simeq \underbrace{\xi^I_{x_0}(x_0^\mu)}_{0} + \sum_{\mu}^{3} \underline{\frac{\partial \xi^I}{\partial x^\mu}\Big|_{x_0}} (x^\mu - x_0^\mu)$$

$$e_\mu{}^I(x_0) = \frac{\partial \xi^I}{\partial x^\mu}\Big|_{x_0}$$

The equivalence principle : in a suff. small region (a lift)

inertial and gravitational forces cancel, to any accuracy

in a free falling ref. system.

$$g_{\mu\nu}$$

$$F = -G\frac{M m_g}{r^2} \qquad m_i = m_g$$

$$F = m_i a$$

$$ds^2 = \sum_{I,J=0}^{3} \eta_{IJ}\, d\xi^I d\xi^J$$

$$= \sum_{I,J} \eta_{IJ} \frac{\partial \xi^I}{\partial x^\mu}\frac{\partial \xi^J}{\partial x^\nu} dx^\mu dx^\nu$$

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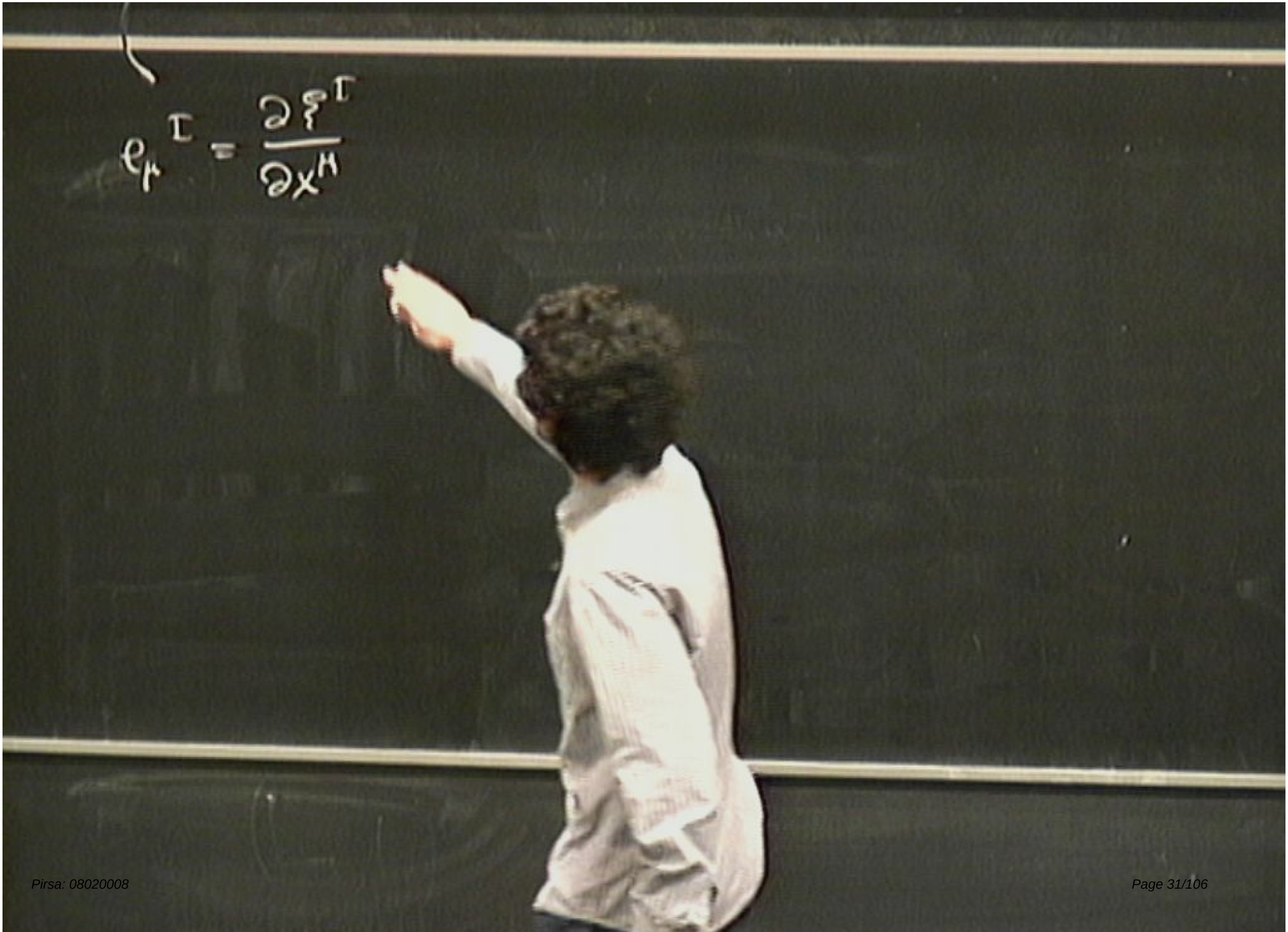
$$ds^2 = g_{\mu\nu}\, dx^\mu\, dx^\nu$$

$$g_{\mu\nu} = \sum_{I,J} \eta_{IJ}\, e_\mu^{\ I}\, e_\nu^{\ J}$$

spacetime metric.


$e_\mu{}^I = \frac{\partial \xi^I}{\partial x^\mu}$

$$e_\mu{}^{I} = \frac{\partial \xi^{I}}{\partial x^{\mu}}$$

$$\xi^{I} \longrightarrow \xi'^{I} = \Lambda^{I}{}_{J}\, \xi^{J}$$

$$x^{\mu}$$

$$e_\mu{}^{I} = \frac{\partial \xi^I}{\partial x^\mu}$$

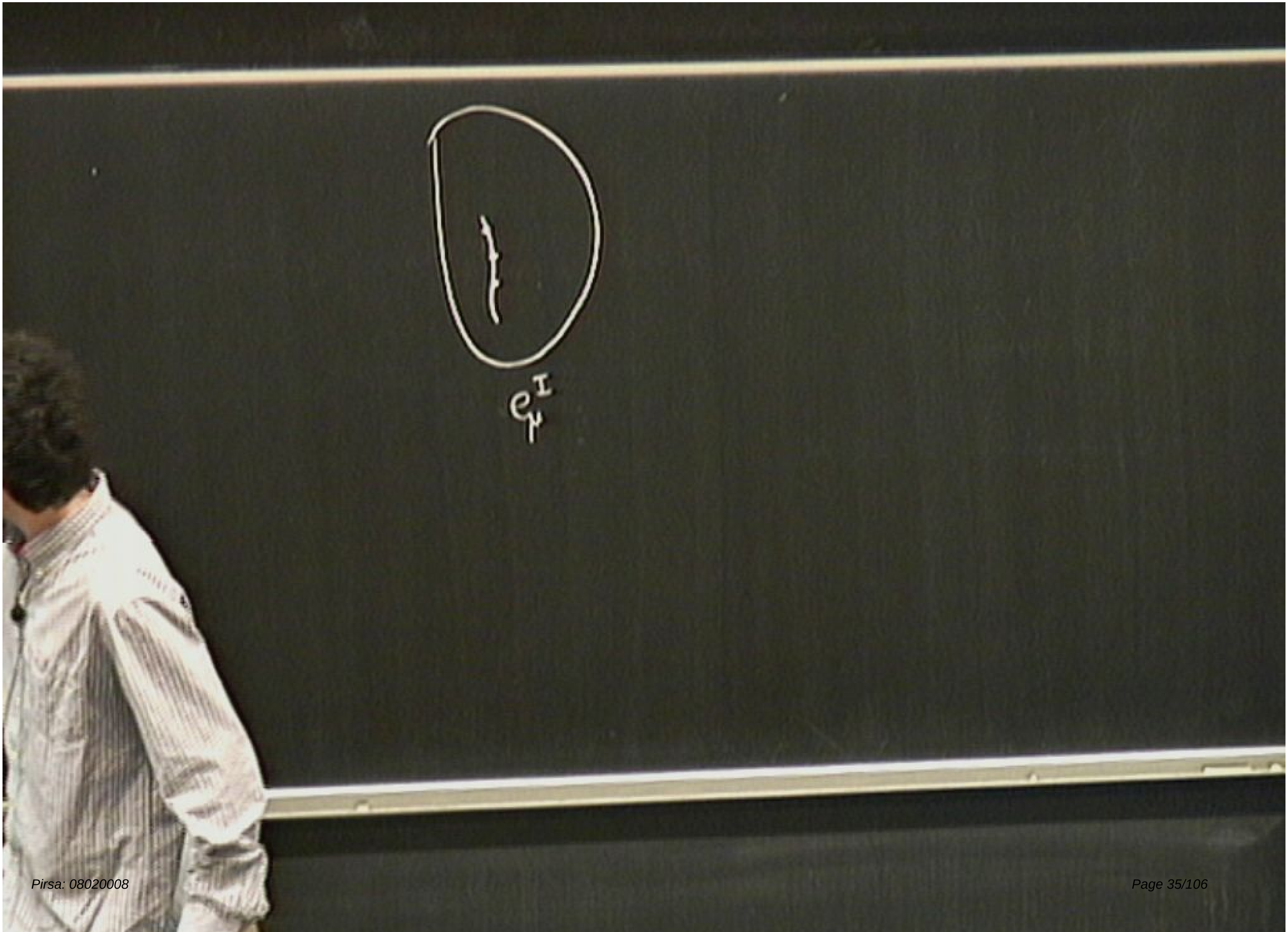
$$\xi^I \longrightarrow \xi'^I = \Lambda^I{}_J \, \xi^J$$

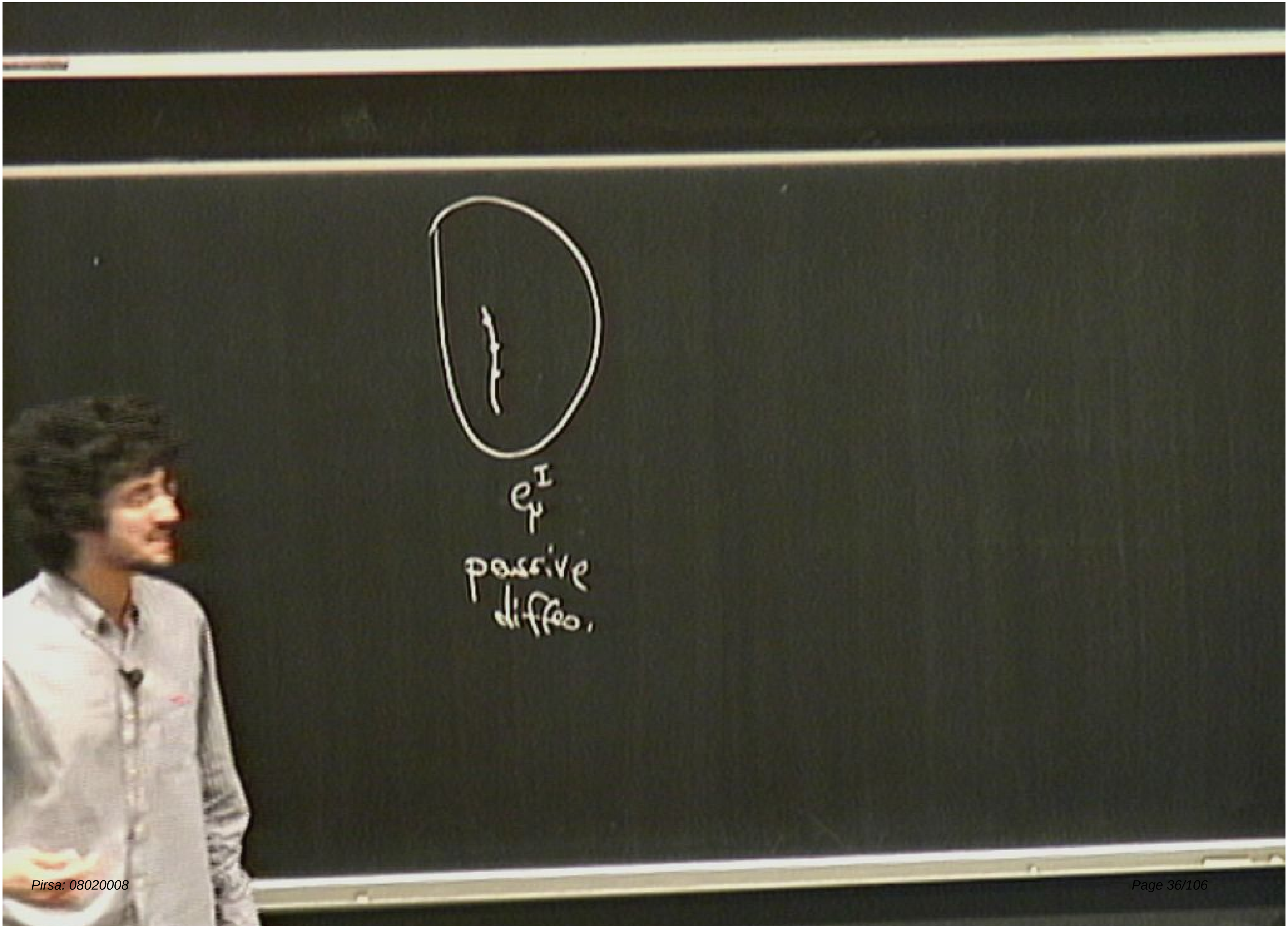
$$x^\mu \longrightarrow y^\mu(x)$$

$$e_\mu{}^{I} = \frac{\partial \xi^I}{\partial x^\mu}$$

$$\xi^I \longrightarrow \xi'^I = \Lambda^I{}_J\, \xi^J \qquad \Rightarrow \qquad e_\mu{}^I \longmapsto e'_\mu{}^I = \Lambda^I{}_J\, e_\mu{}^J$$

$$x^\mu \longrightarrow y^\mu(x) \qquad \Rightarrow \qquad e_\mu{}^I \longmapsto e'_\mu{}^I = \frac{\partial y^\nu}{\partial x^\mu}\, e_\nu{}^I$$

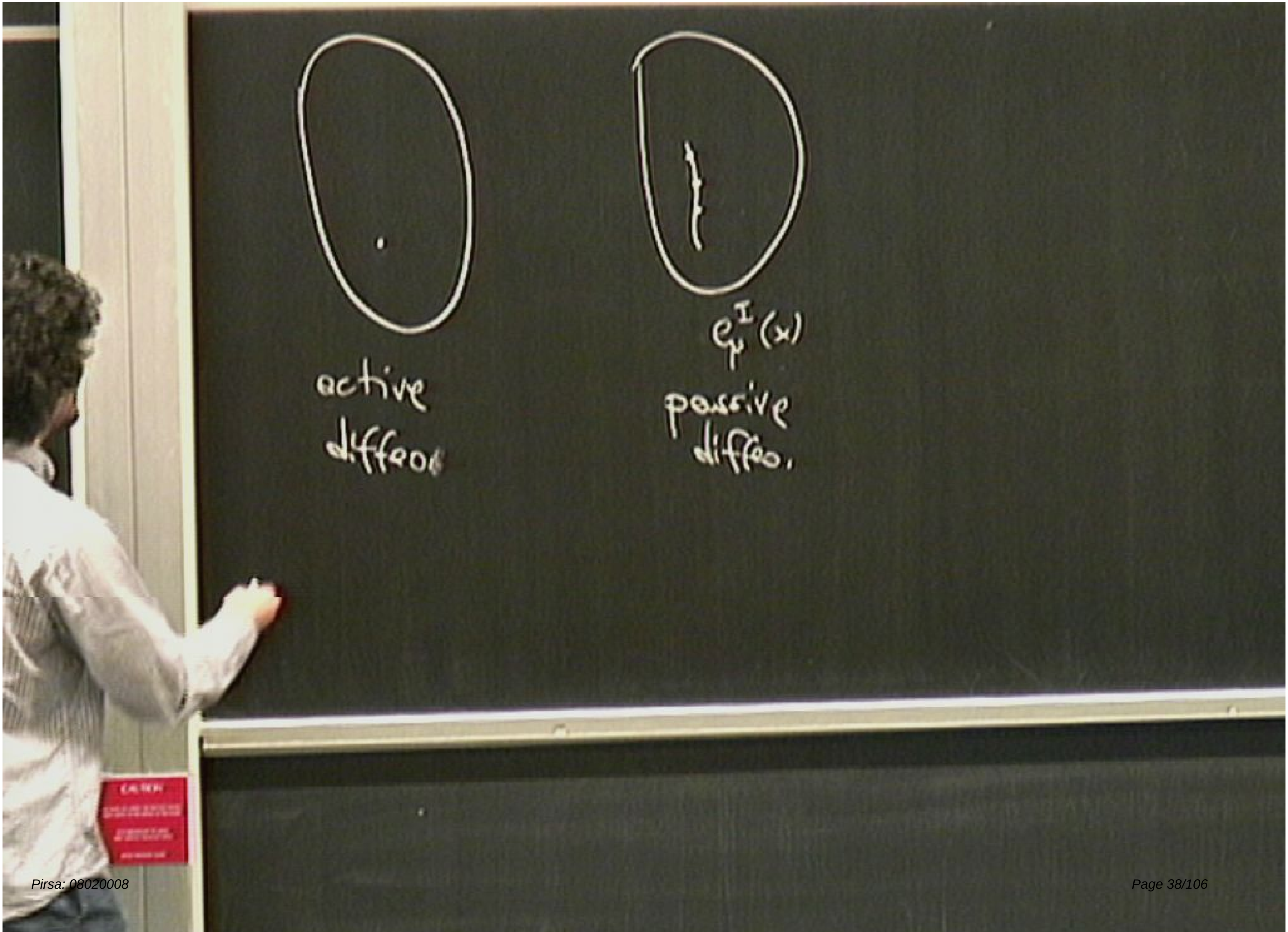

e_μ^I

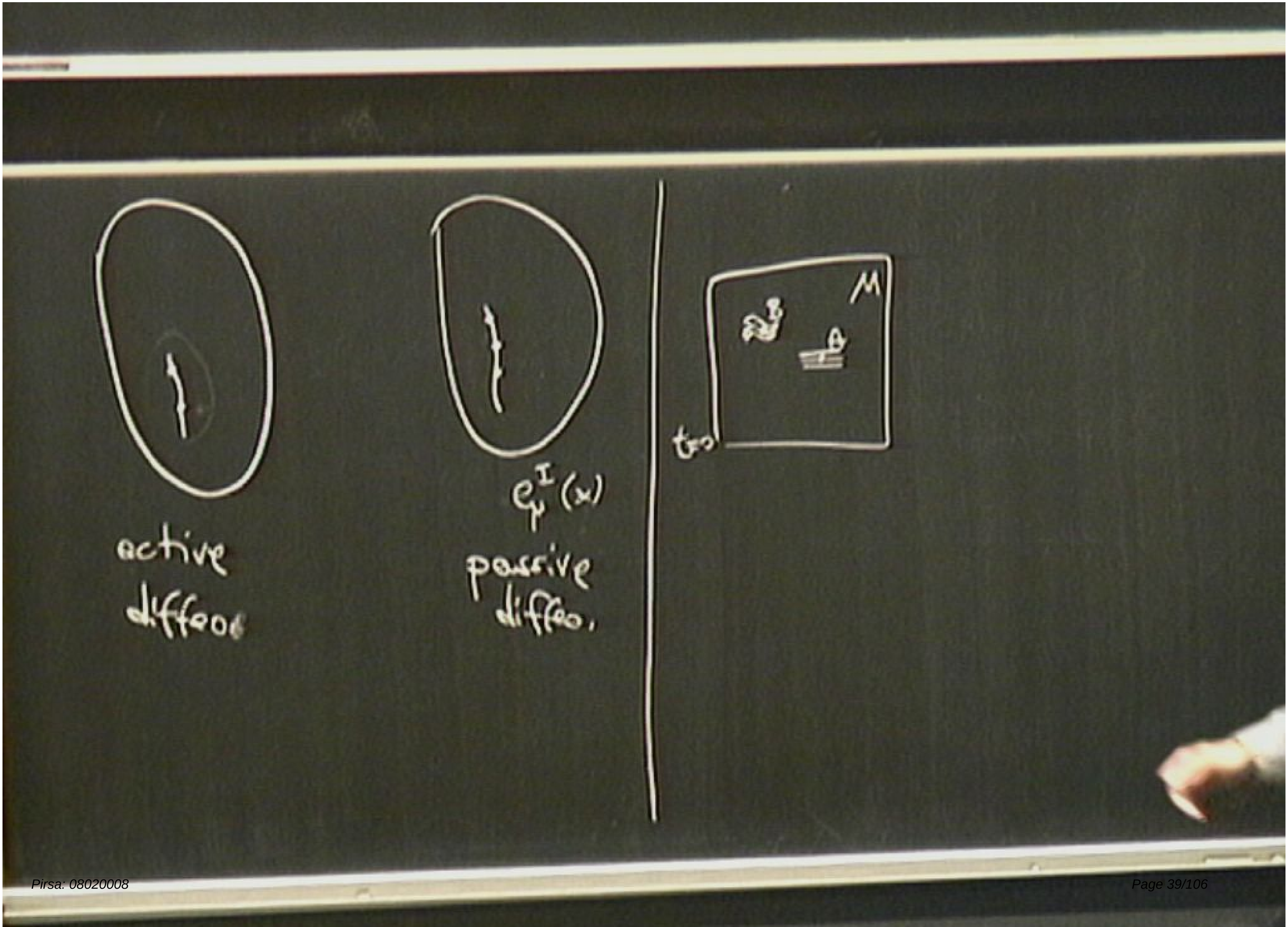

e_μ^I
passive
diffeo.

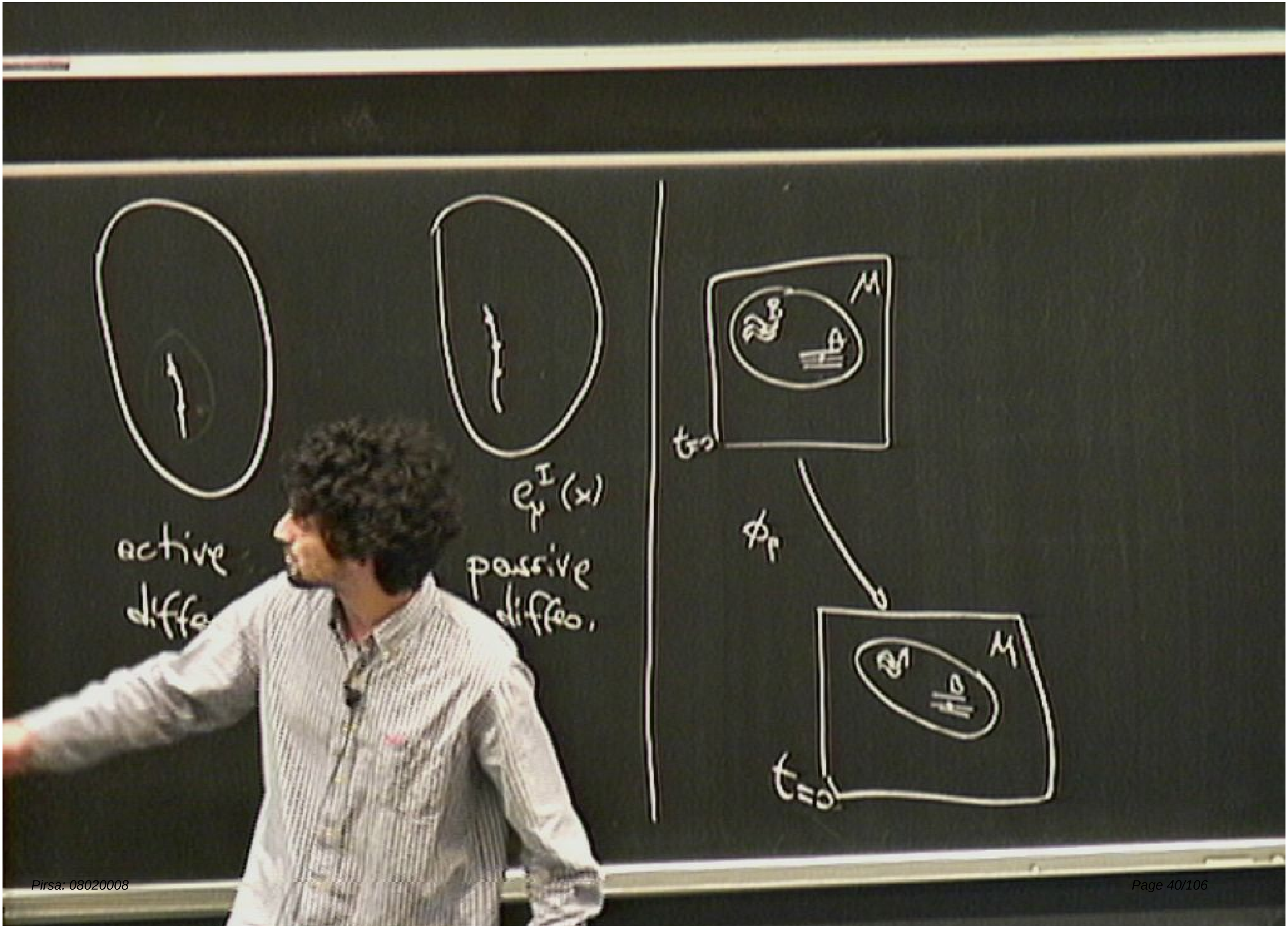
$$e_\mu{}^I = \frac{\partial \xi^I}{\partial x^\mu}$$

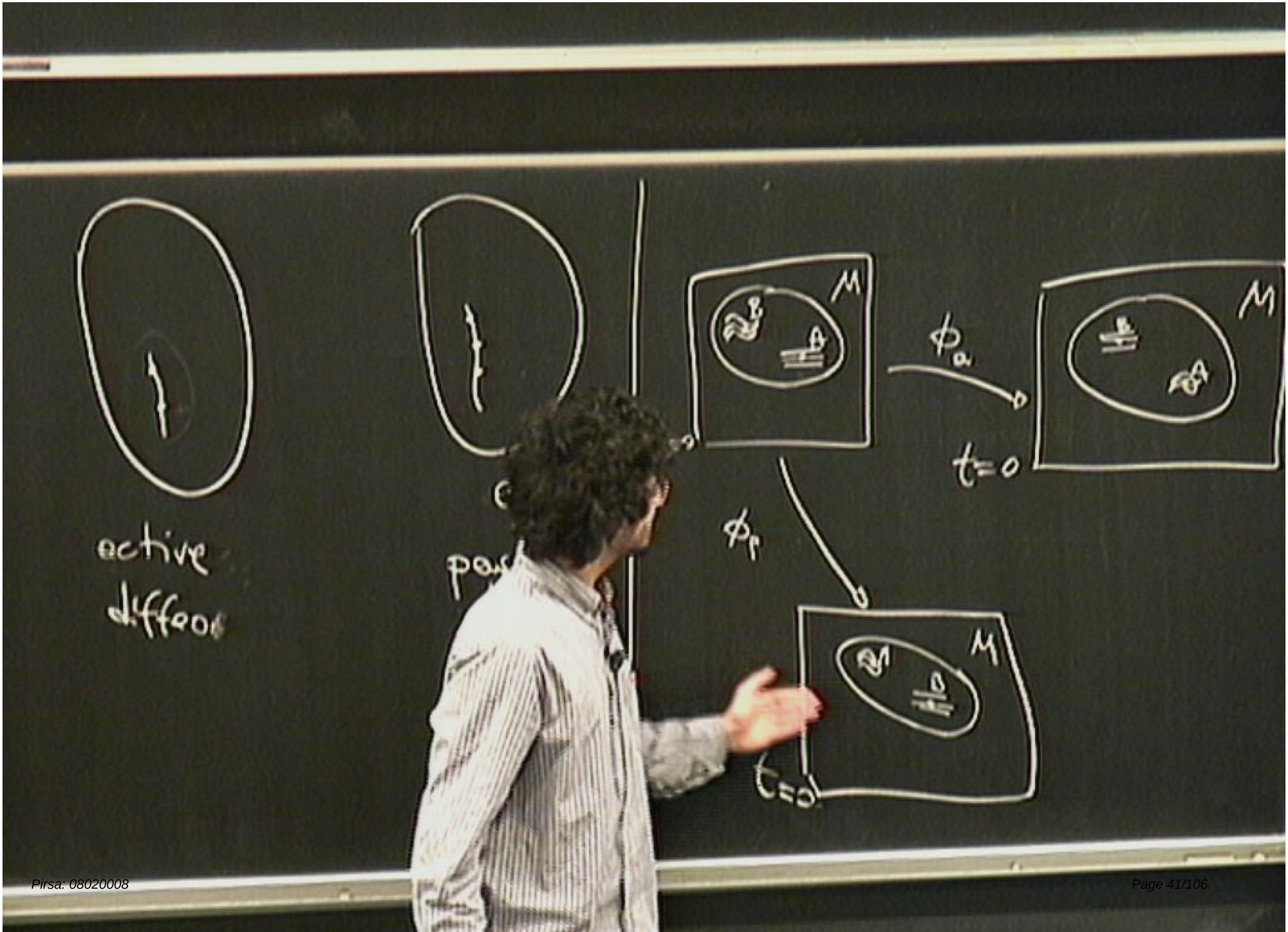
$$\xi^I \longrightarrow \xi'^I = \Lambda^I{}_J\, \xi^J \quad \Rightarrow \quad e_\mu{}^I \longmapsto e'_\mu{}^I = \Lambda^I{}_J\, e_\mu{}^J$$

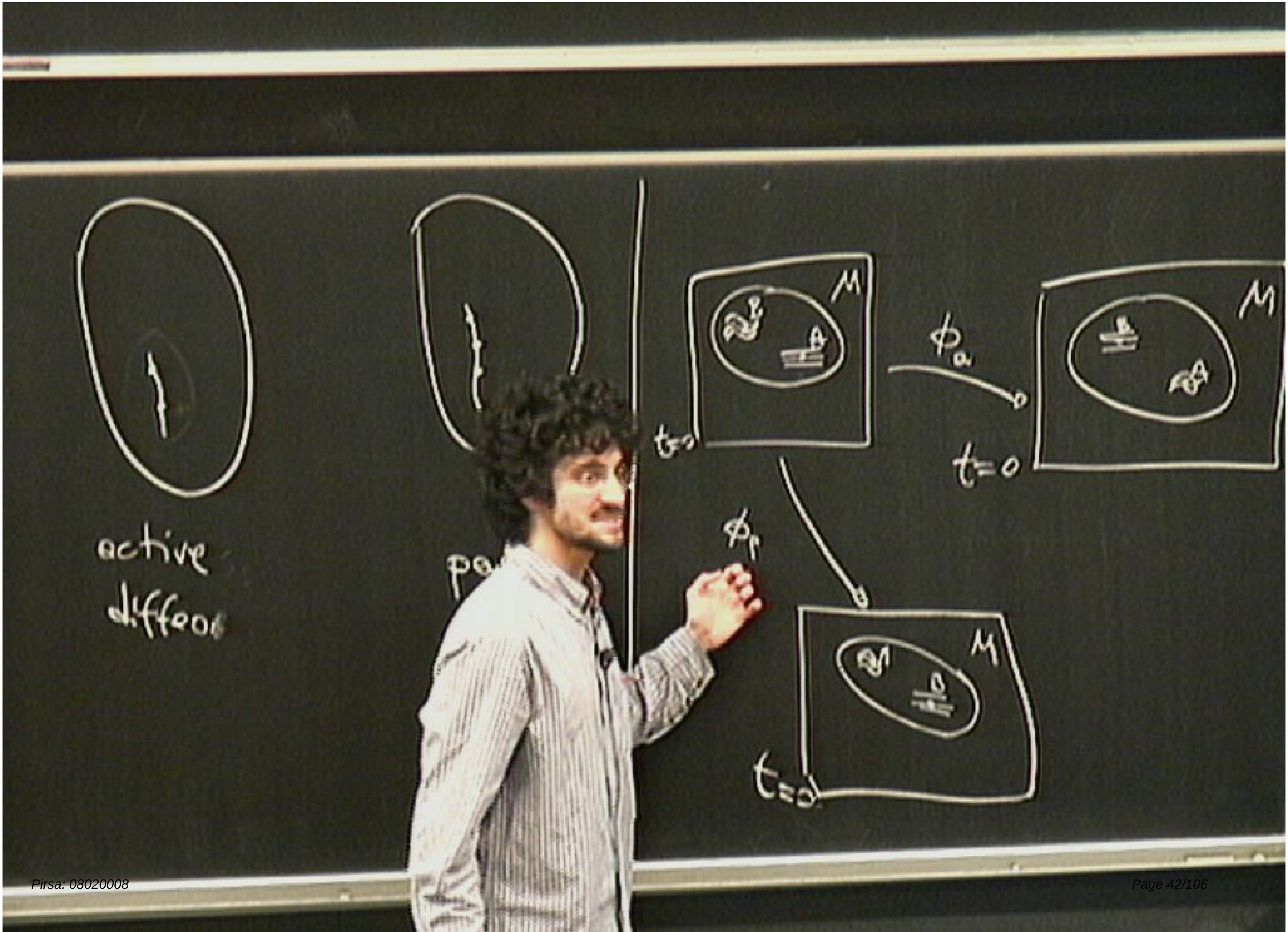
$$x^\mu \xrightarrow{\phi} y^\mu(x) \quad \Rightarrow \quad \boxed{e_\mu{}^I \longmapsto e'_\mu{}^I = \frac{\partial y^\nu}{\partial x^\mu}\, e_\nu{}^I}$$


$e_\mu^I(x)$
active diffeos
passive diffeo.

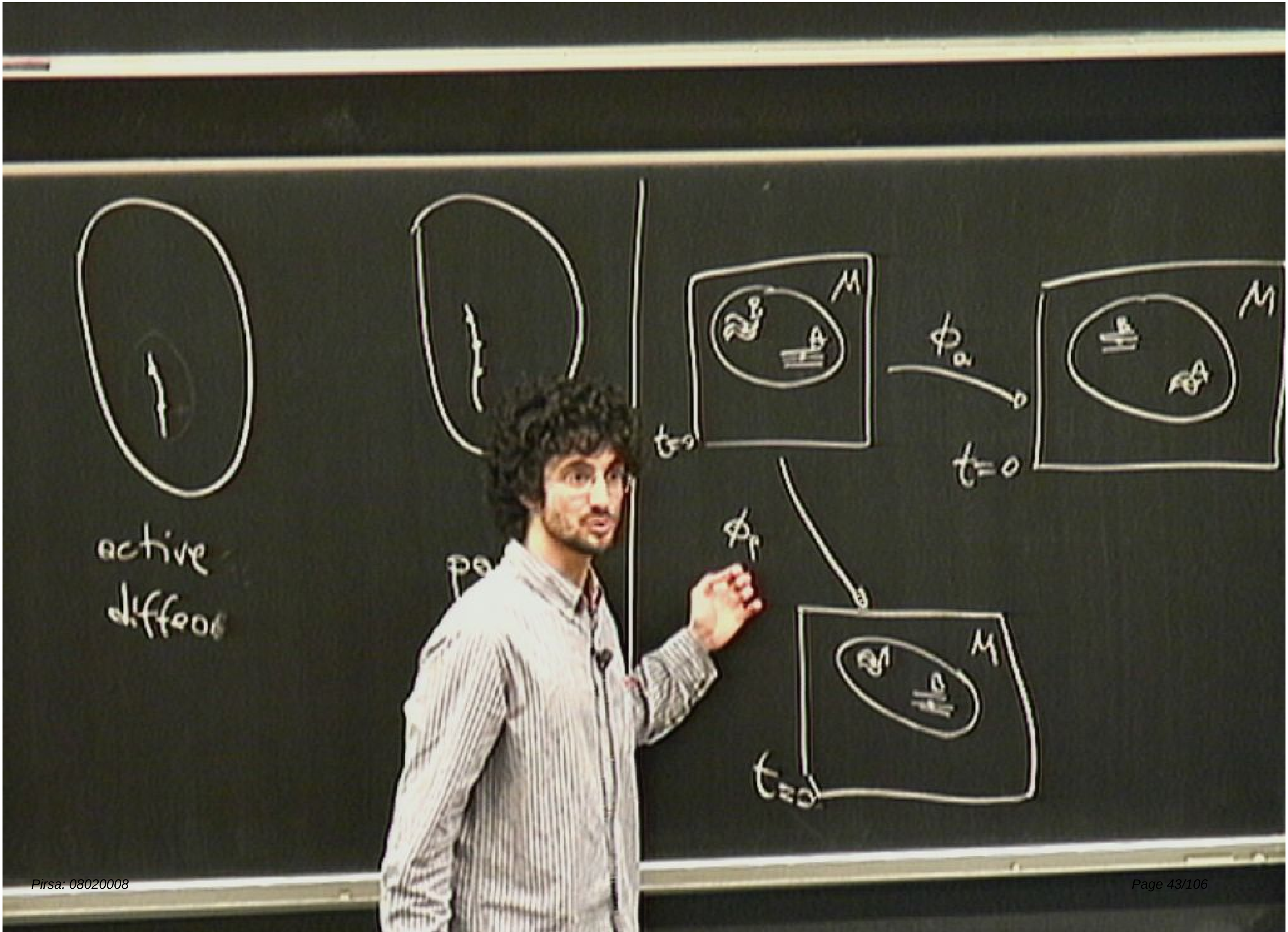

active
diffeos
$e^I_\mu(x)$
passive
diffeo.
M
t=0


active
diffe
$e_\mu^I(x)$
passive
diffeo.
M
t=0
ϕ_t
M
t=0

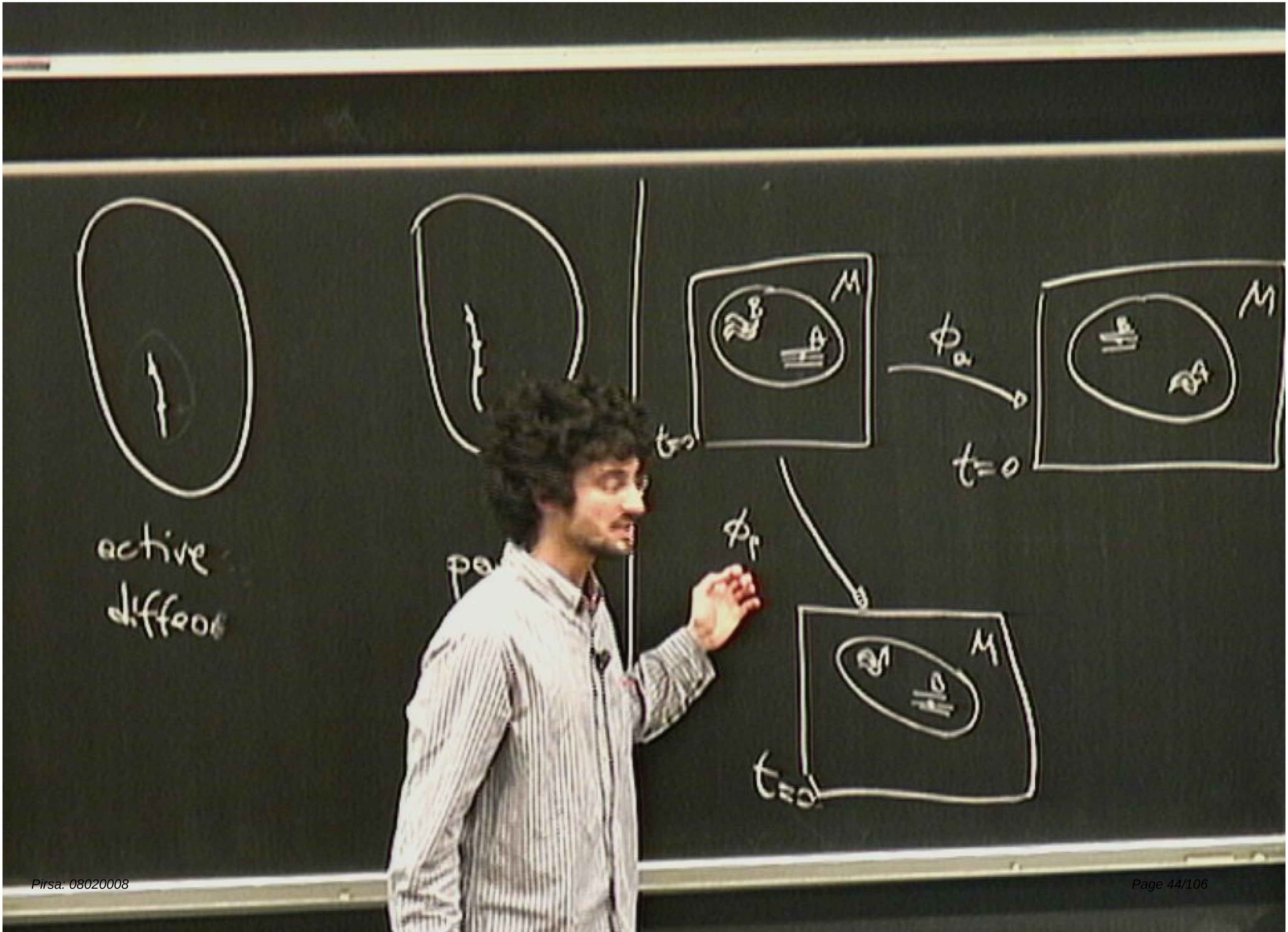

M
t=0
M
M
t=0
active
diffeos



active
diffeos
M
ϕ_a
$t=0$
M
$t=0$
ϕ_b
M
$t=0$

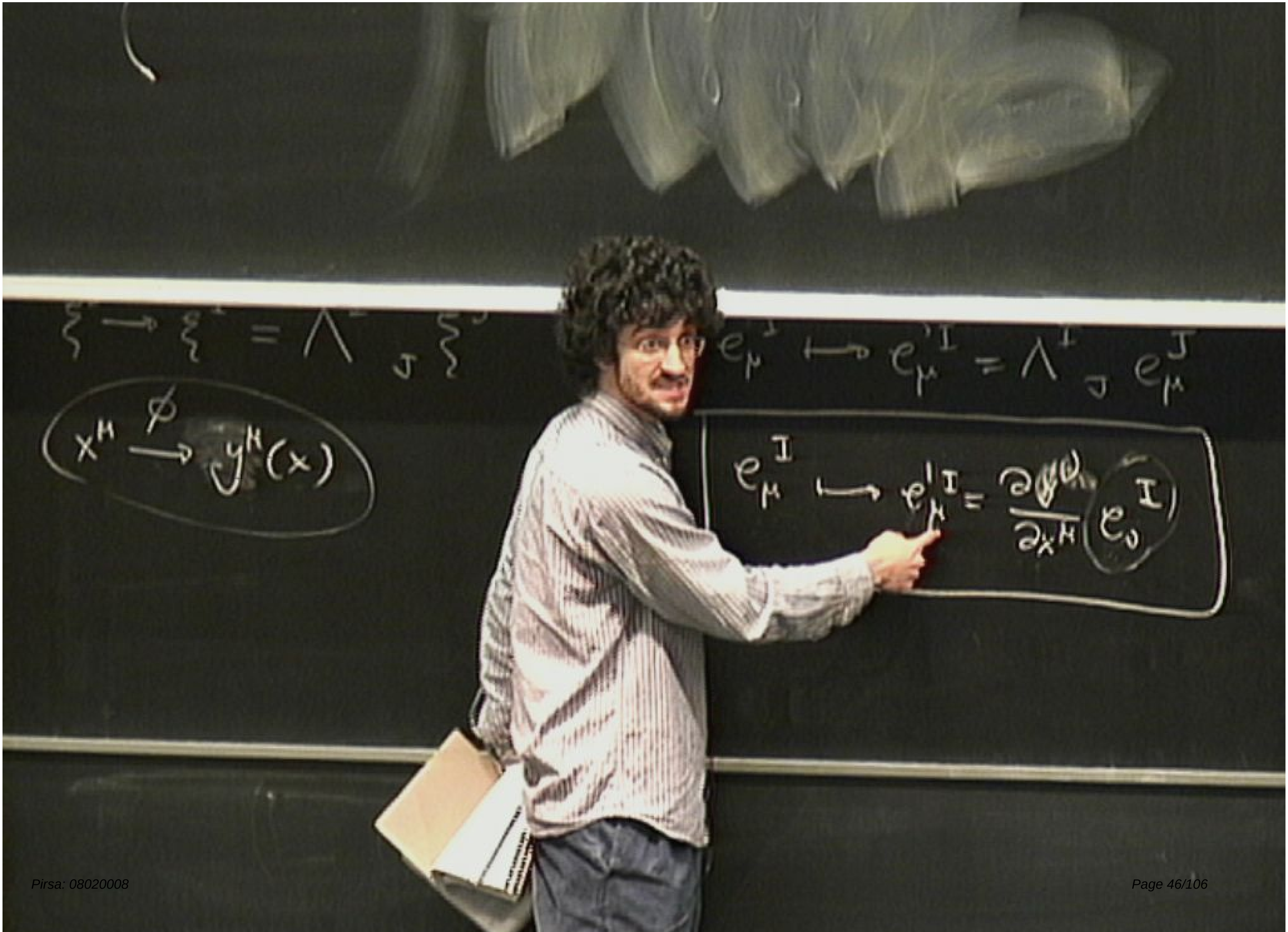


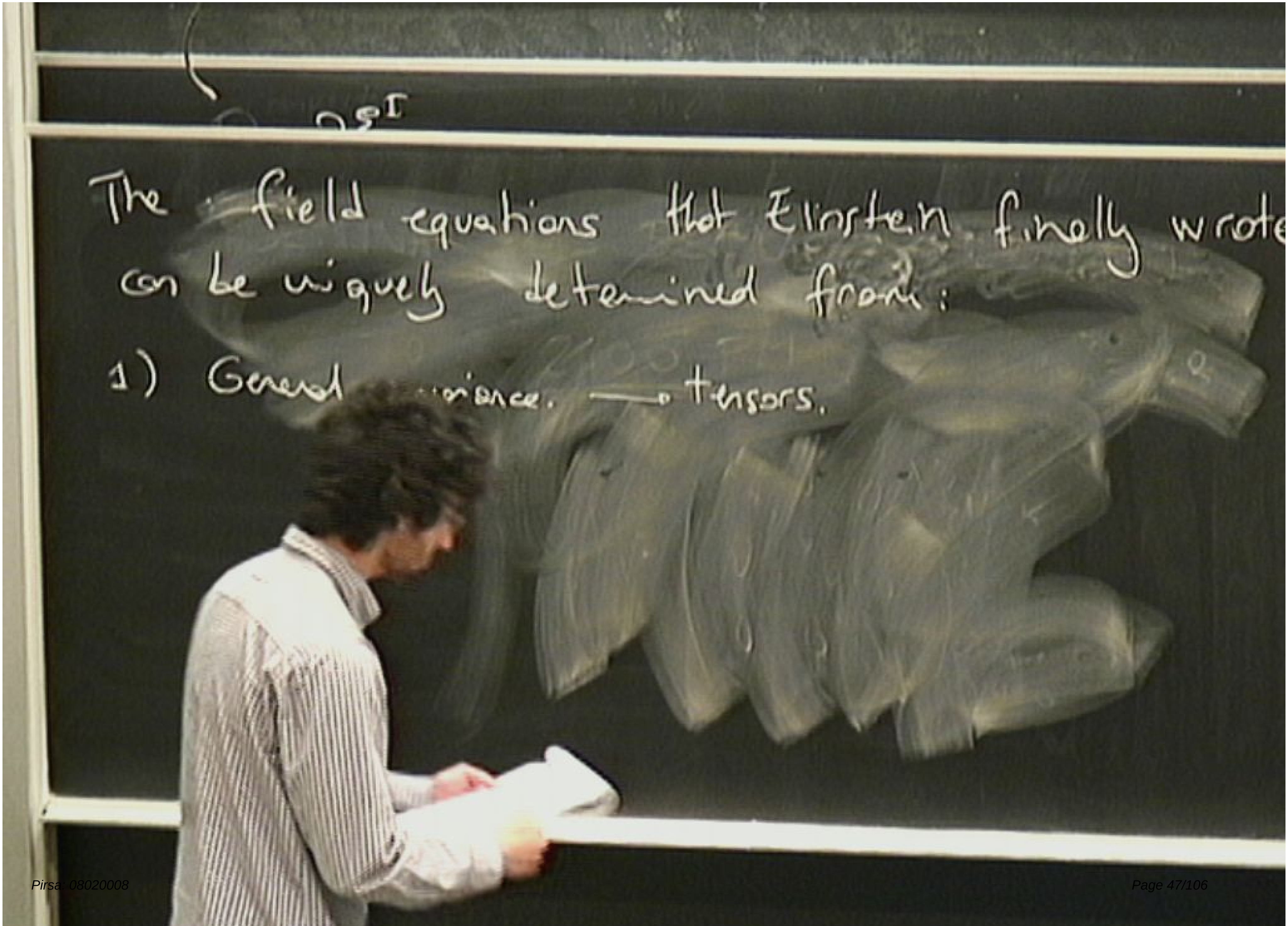
M
ϕ_α
M
t=0
t=0
ϕ_β
M
t=0
active
diffeos

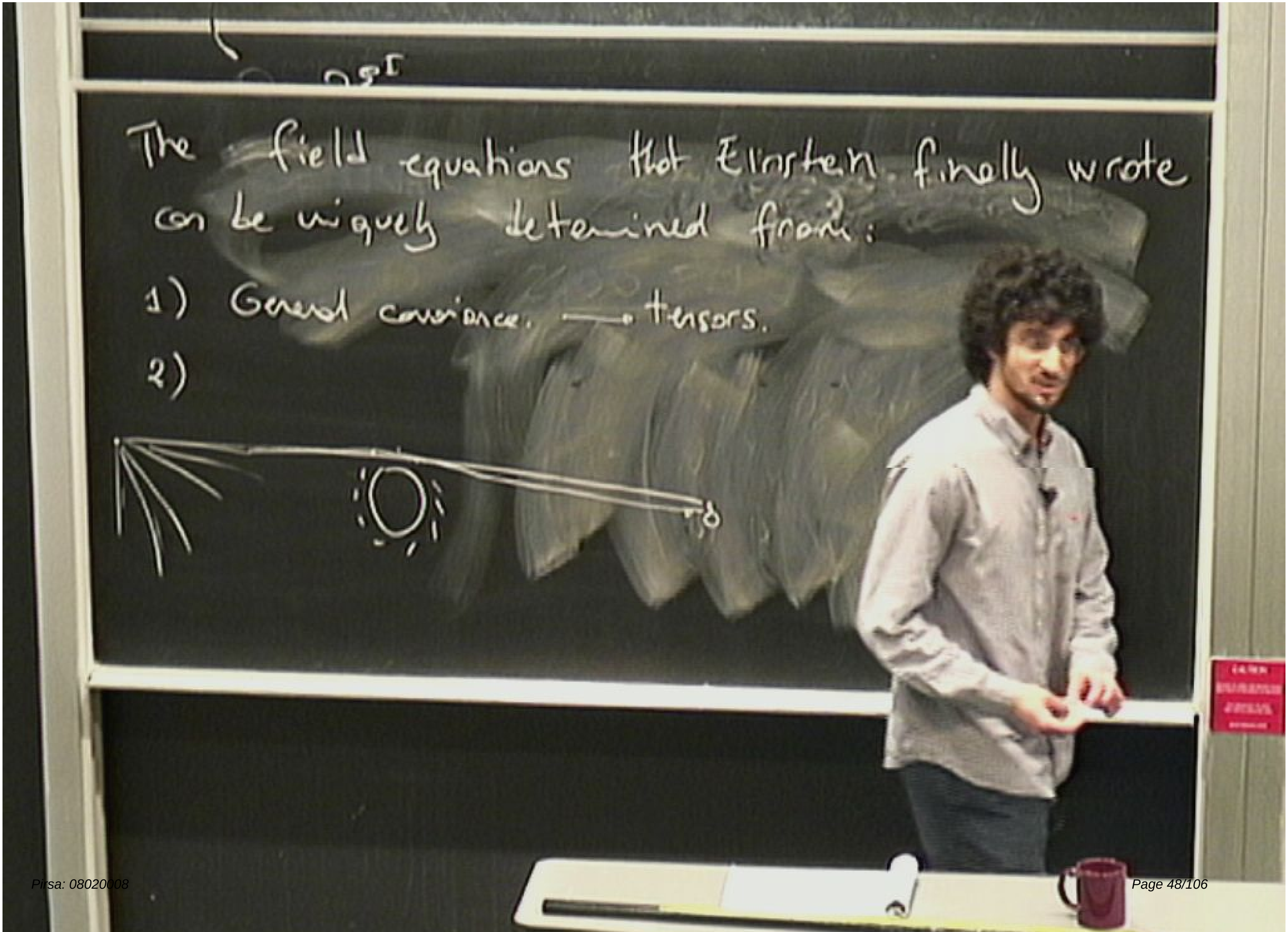


active
diffeos
M
M
M
t=0
t=0
t=0

The field equations that Einstein finally wrote can be uniquely determined from:


$x^\mu \xrightarrow{\phi} y^\mu(x)$
$e_\mu{}^I \longmapsto e'_\mu{}^I = \Lambda^I{}_J \, e_\mu{}^J$
$e_\mu{}^I \longmapsto e'_\mu{}^I = \frac{\partial y^\nu}{\partial x^\mu} e_\nu{}^I$


The field equations that Einstein finally wrote
can be uniquely determined from:
1) General
riance. → tensors.


The field equations that Einstein finally wrote
can be uniquely determined from:
1) General covariance. → tensors.
2)

The field equations that Einstein finally wrote can be uniquely determined from:

1) General covariance. $\longrightarrow$ tensors.

2) The source is energy. Energy tensor.

3) Newton's law in $c \to \infty$

4) The eqs should be at most second order, differential eqs.

$H(p,q)$

(quant

$H(p,q)$

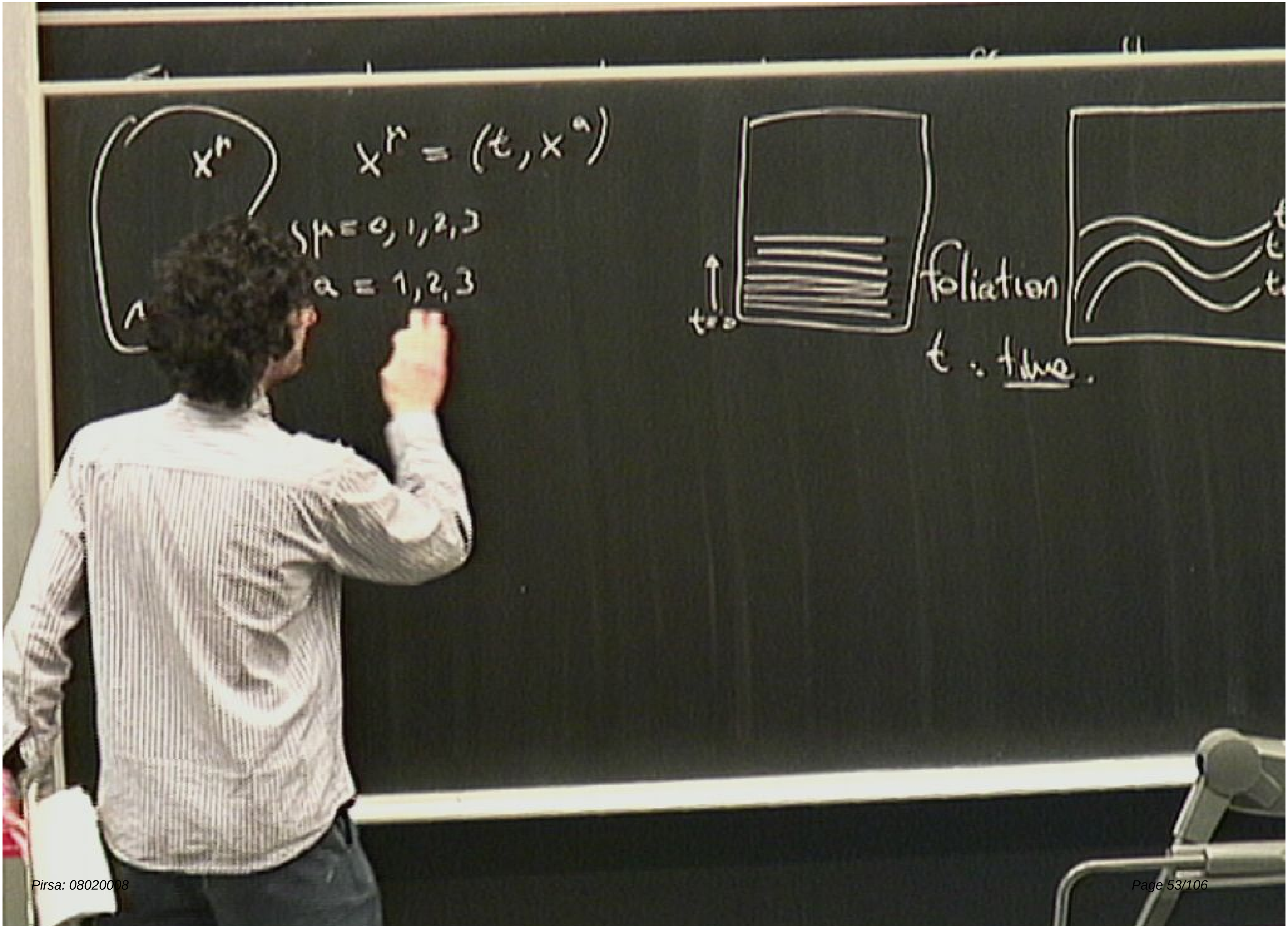
(quant.

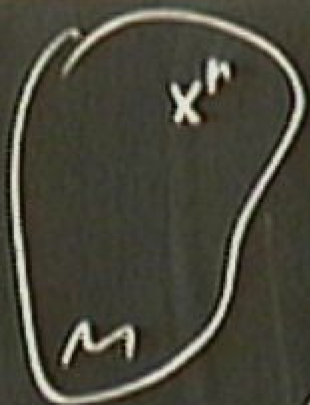
$\hat{H}(\hat{p},\hat{q})$

$H(p,q)$ $\quad \{q,p\} = 1 \quad p = \frac{\partial L}{\partial \dot{q}}$.

(quant. $\quad [\hat{q},\hat{p}] = i\hbar \quad \dot{q} = \frac{dq}{dt}$.

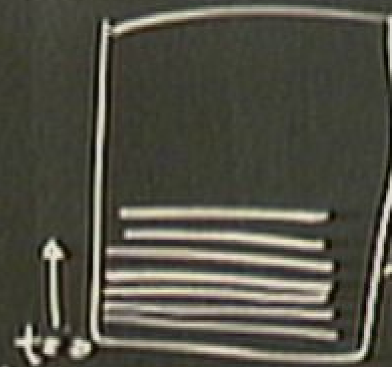
$\hat{H}(\hat{p},\hat{q}) \longrightarrow i\partial_t \psi = \hat{H}\psi$ $\quad$ S.e.


x^μ
$x^\mu = (t, x^a)$
$\mu = 0, 1, 2, 3$
$a = 1, 2, 3$
$t=0$
foliation
t : time.



$$x^\mu = (t, x^a)$$

$$\begin{cases} \mu = 0,1,2,3 \\ a = 1,2,3 \end{cases}$$



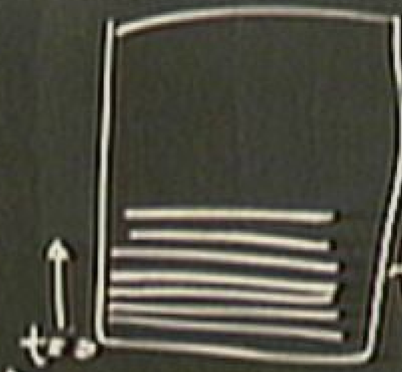
foliation

t : <u>time</u>.

$$g_{\mu\nu} = \begin{pmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{01} & \\ g_{02} & & \begin{pmatrix} g_{11} & & \\ & g_{ab} & \\ & & \end{pmatrix} \\ g_{03} & \end{pmatrix} \longrightarrow g_{ab}$$

$$x^\mu = (t, x^a)$$

$$\begin{cases} \mu = 0,1,2,3 \\ a = 1,2,3 \end{cases}$$



foliation

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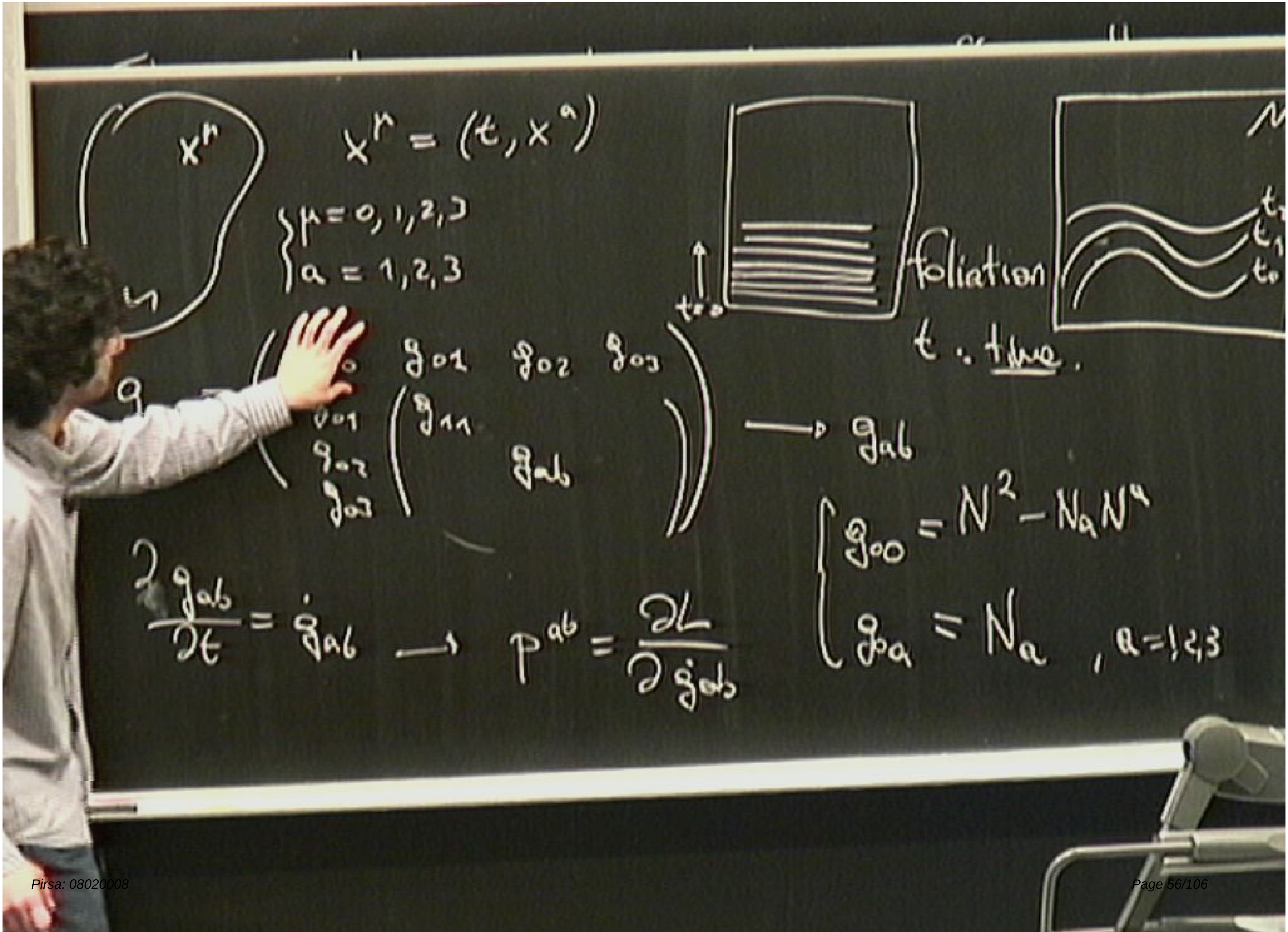
$$\frac{\partial g_{ab}}{\partial t} = \dot{g}_{ab} \longrightarrow p^{ab} = \frac{\partial L}{\partial \dot{g}_{ab}}$$



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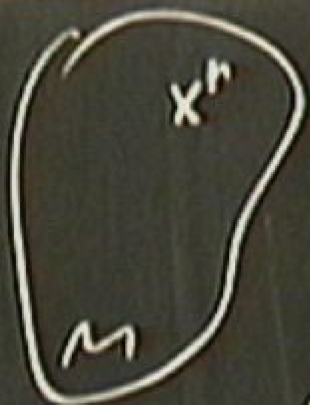
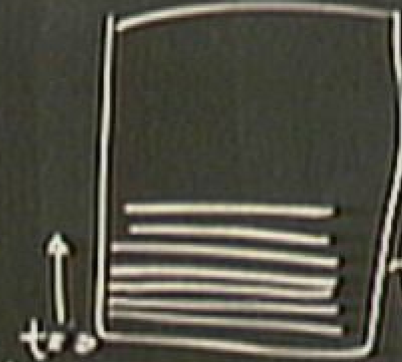
foliation
t_0 t_1
t : time.
g_{01} g_{02} g_{03}
g_{01} g_{11}
g_{02} g_{ab}
g_{03}
$\longrightarrow g_{ab}$
$g_{00} = N^2 - N_a N^a$
$g_{0a} = N_a$, $a = 1,2,3$
$\frac{\partial g_{ab}}{\partial t} = \dot{g}_{ab} \longrightarrow p^{ab} = \frac{\partial L}{\partial \dot{g}_{ab}}$



$$x^\mu = (t, x^a)$$

$$\begin{cases} \mu = 0,1,2,3 \\ a = 1,2,3 \end{cases}$$



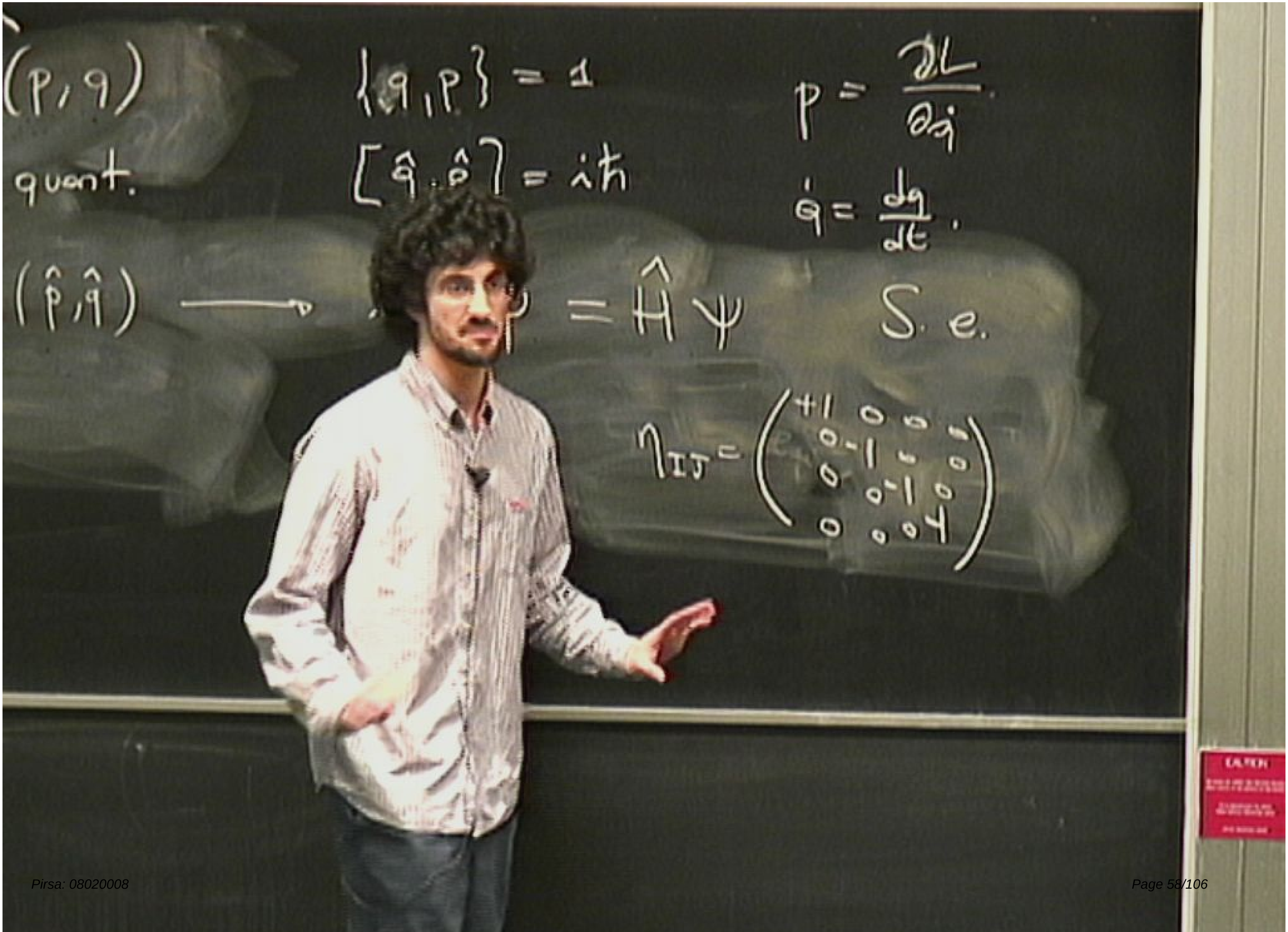
foliation

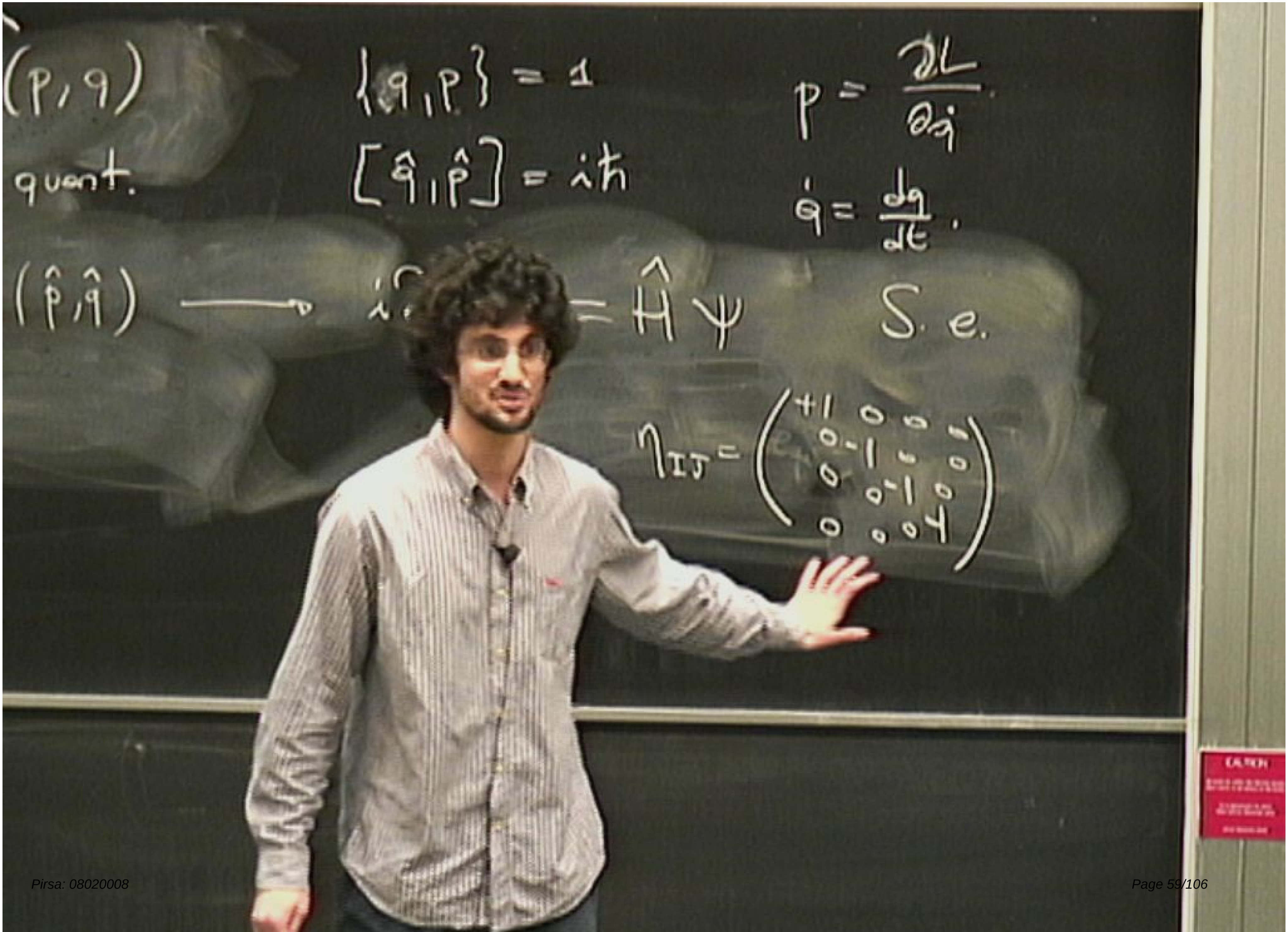
t : time

$$g_{\mu\nu} = \begin{pmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{01} & g_{11} & & \\ g_{02} & & g_{ab} & \\ g_{03} & & & \end{pmatrix} \longrightarrow g_{ab}$$

$$\begin{cases} g_{00} = N^2 - N_a N^a \\ g_{0a} = N_a \quad , a = 1,2,3 \end{cases}$$

$$\frac{\partial g_{ab}}{\partial t} = \dot{g}_{ab} \longrightarrow p^{ab} = \frac{\partial L}{\partial \dot{g}_{ab}}$$


(p, q)
{q, p} = 1
p = ∂L/∂q̇
quant.
[q̂, p̂] = iħ
q̇ = dq/dt
(p̂, q̂) →
= Ĥψ
S. e.
η_IJ =


(p, q)
{q, p} = 1
p = ∂L/∂q̇
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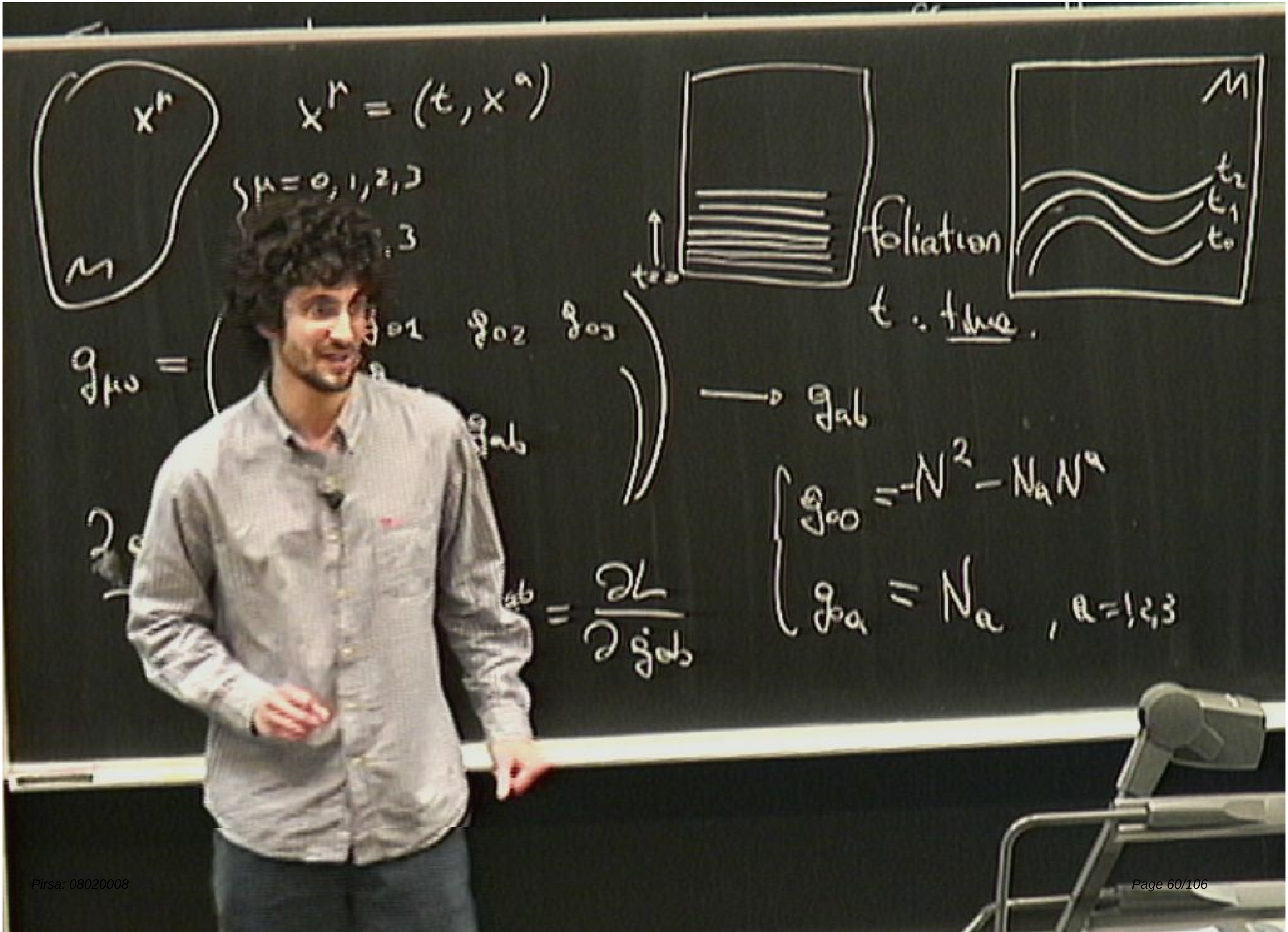
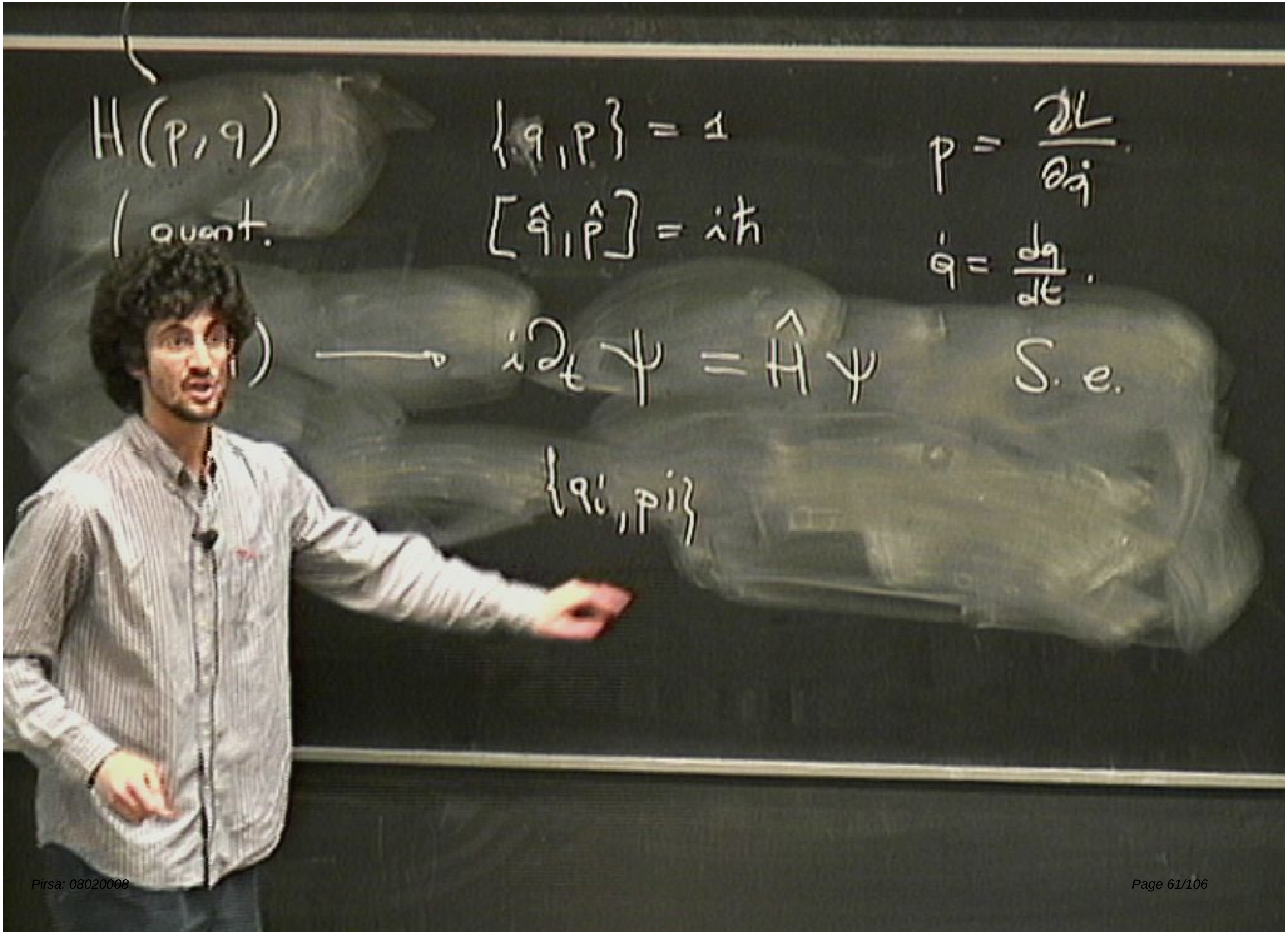
[q̂, p̂] = iħ
q̇ = dq/dt
(p̂, q̂) ⟶
= Ĥψ
S.e.
η_IJ =
+1 0 0 0
0 -1 0 0
0 0 -1 0
0 0 0 -1


x^μ
M
x^μ = (t, x^a)
μ = 0, 1, 2, 3
t=0
foliation
t : time
M
t_2
t_1
t_0
g_μν =
g_01 g_02 g_03
g_ab
→ g_ab
g_00 = -N^2 - N_a N^a
g_0a = N_a , a = 1, 2, 3
∂L / ∂ġ_ab


$H(p,q)$
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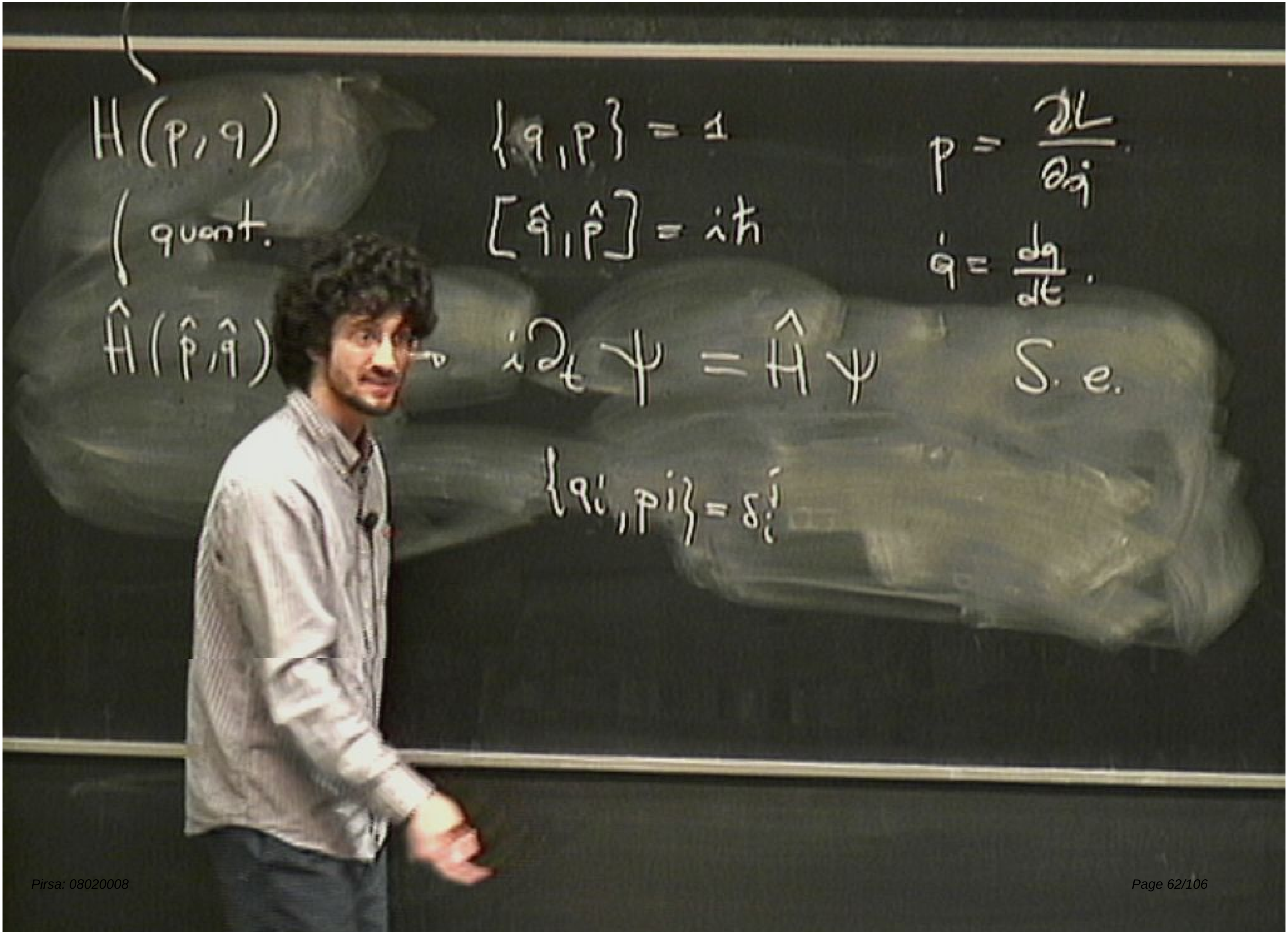
(quant.
$[\hat{q},\hat{p}] = i\hbar$
$\dot{q} = \frac{dq}{dt}$
$i\partial_t \psi = \hat{H}\psi$
S.e.
$\{q^i, p^i\}$


$H(p,q)$
quant.
$\hat{A}(\hat{p},\hat{q})$
$\{q,p\} = 1$
$[\hat{q},\hat{p}] = i\hbar$
$i\partial_t \psi = \hat{A}\psi$
$\{q^i,p^j\} = \delta_i^j$
$p = \frac{\partial L}{\partial \dot{q}}$
$\dot{q} = \frac{dq}{dt}$
S.e.

$H(p,q)$

$\Big($ quant.

$\hat{H}(\hat{p},\hat{q}) \longrightarrow i\partial_t \psi = \hat{H}\psi$ S.e.

$\{q, p\} = 1$

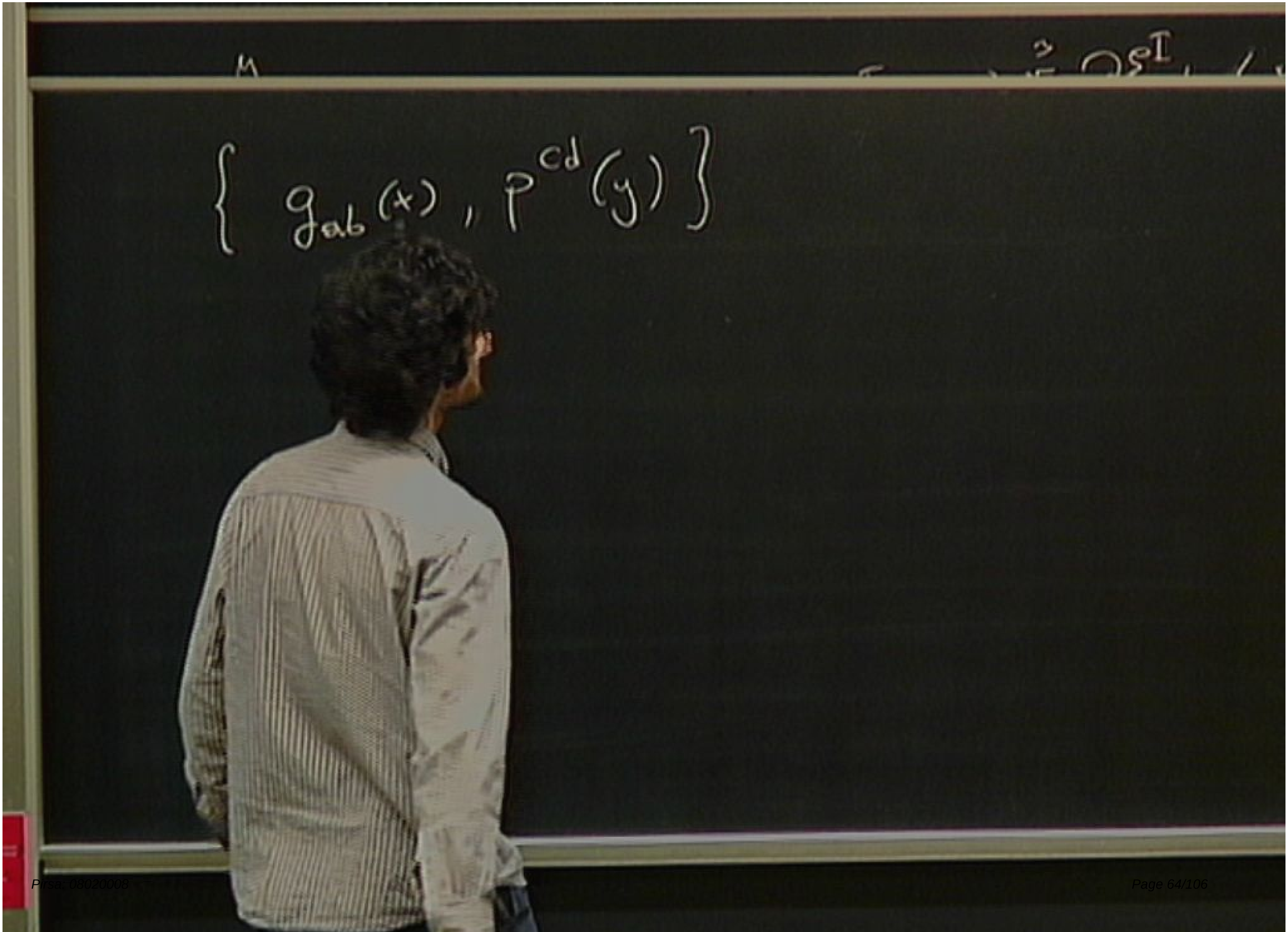
$[\hat{q}, \hat{p}] = i\hbar$

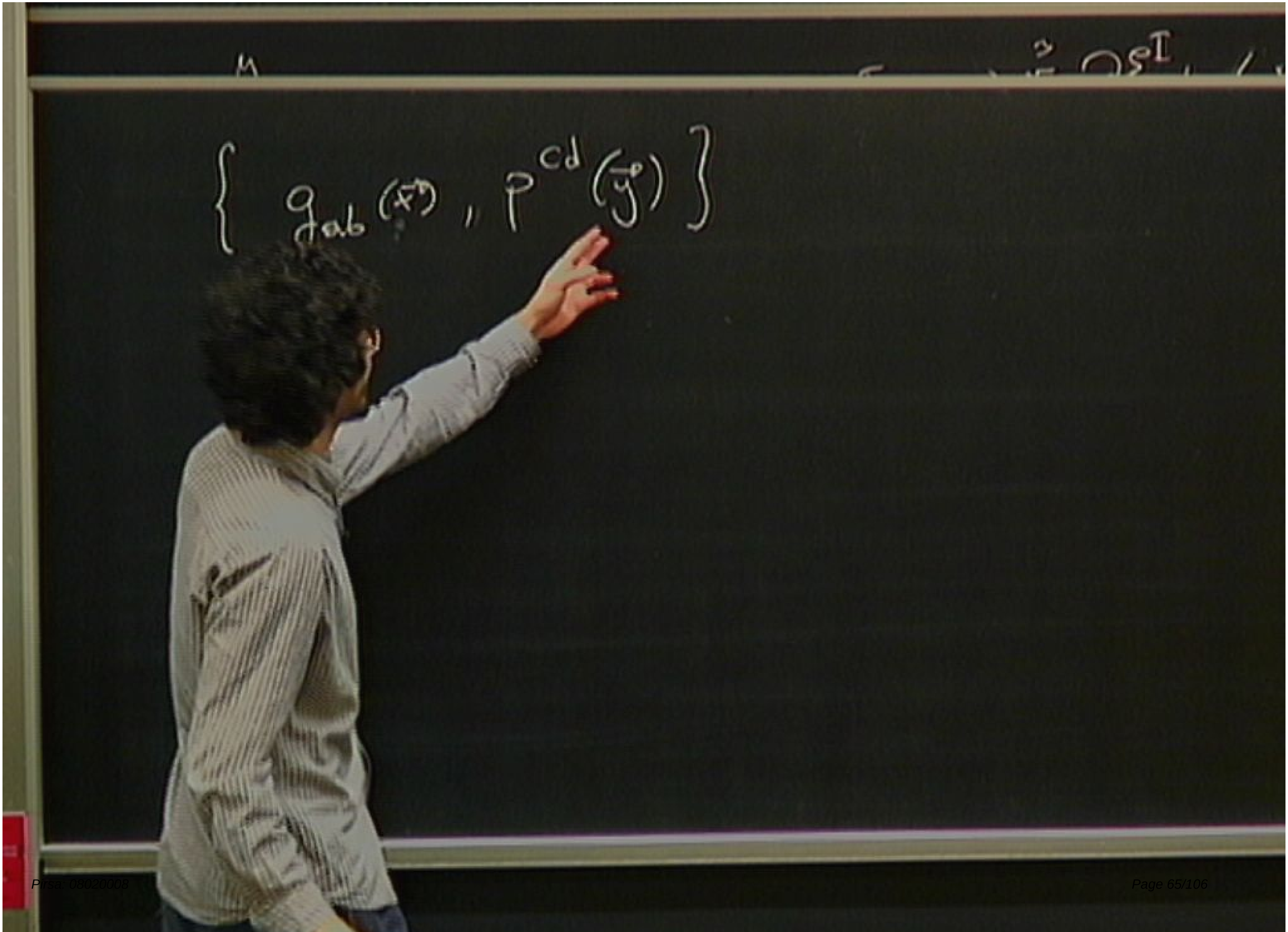
$p = \dfrac{\partial L}{\partial \dot{q}}$

$\dot{q} = \dfrac{dq}{dt}$

$\{q^i, p^j\} = \delta_i^j$

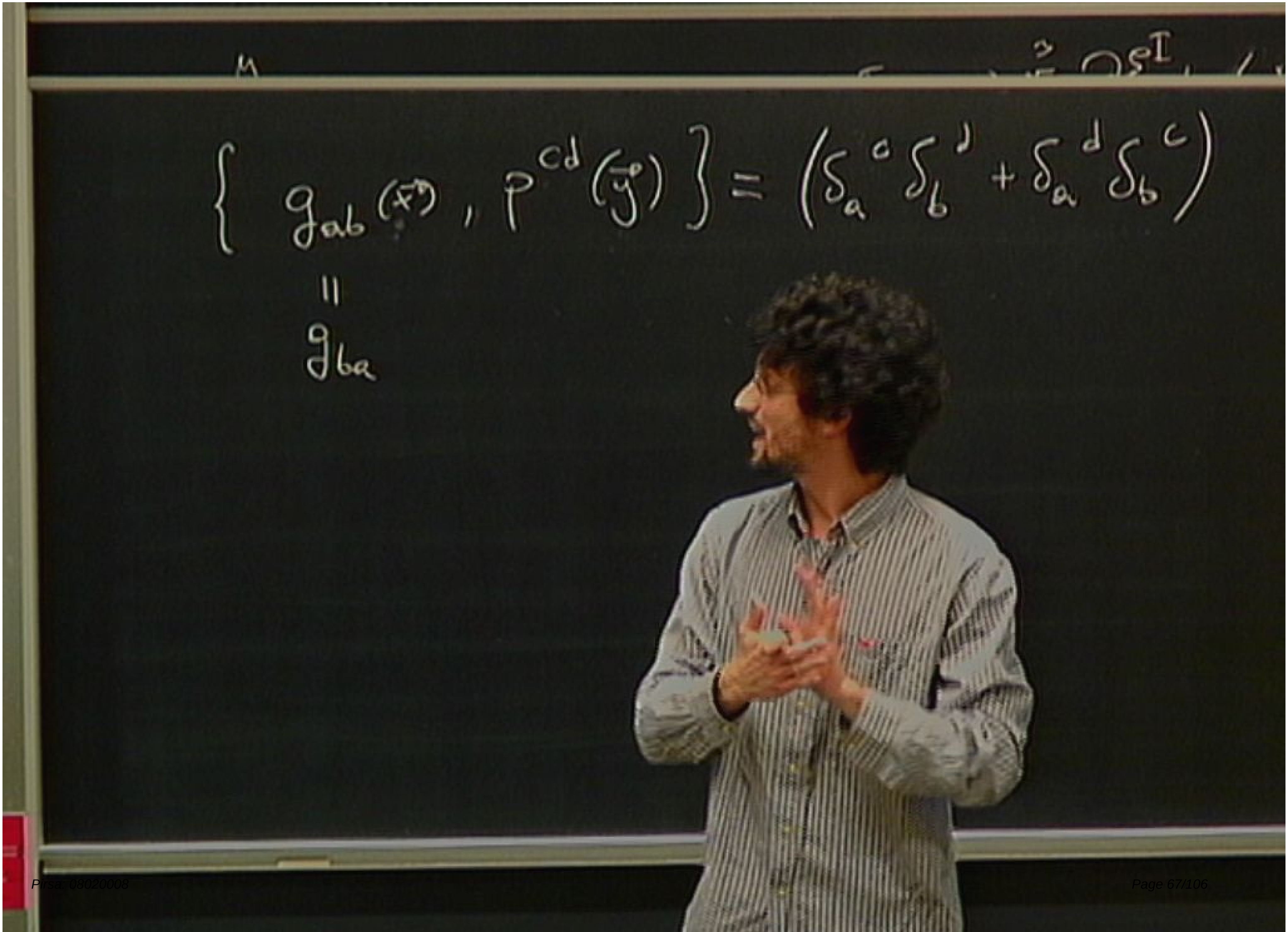
$[\hat{q}^i, \hat{p}^j] = i\hbar\, \delta_i^j$

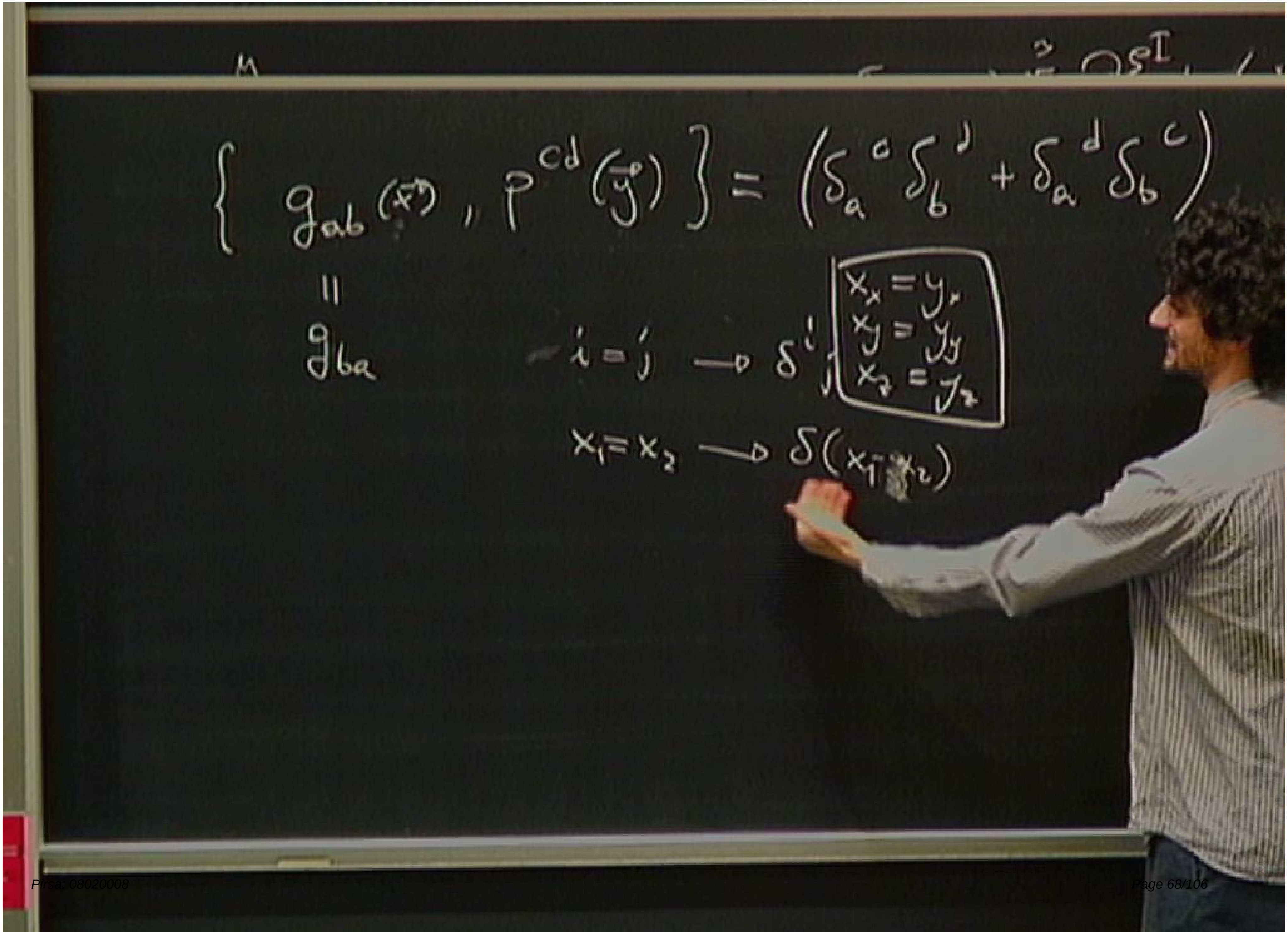

$\{ g_{ab}(x), p^{cd}(y) \}$


$\{ g_{ab}(\vec{x}), p^{cd}(\vec{y}) \}$

$$\left\{ g_{ab}(\vec{x}),\ p^{cd}(\vec{y}) \right\} = \delta_a{}^c \delta_b{}^d$$

$$\begin{matrix} g_{ab} \\ \| \\ g_{ba} \end{matrix}$$

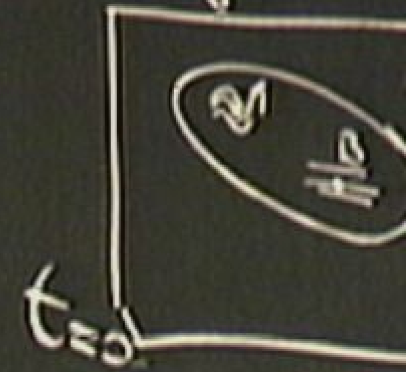

$\{ g_{ab}(\vec{x}), p^{cd}(\vec{y}) \} = (\delta_a^{\ c}\delta_b^{\ d} + \delta_a^{\ d}\delta_b^{\ c})$
$\parallel$
g_{ba}


$\{ g_{ab}(\vec{x}), p^{cd}(\vec{y}) \} = (\delta_a{}^c \delta_b{}^d + \delta_a{}^d \delta_b{}^c)$
$\|$
g_{ba}
$i = j \longrightarrow \delta^i{}_j$
$x_x = y_x$
$x_y = y_y$
$x_z = y_z$
$x_1 = x_2 \longrightarrow \delta(x_1 - x_2)$

diffeos

diffeo.

$$S[p,q] = \int dt\,[p\dot{q} - H(p,q)]$$



in a free falling ref. system:

$$F = -G\frac{M m_g}{r^2}$$

$m_i = m_g$

$$F = m_i a$$

$$ds^2 = \sum_{I,J=0}^{3} \eta_{IJ}\, d\xi^I d\xi^J$$

$$= \sum_{I,J} \eta_{IJ} \frac{\partial \xi^I}{\partial x^\mu}$$

g_{ab}

$$\begin{cases} g_{00} = -N^2 + N_a N^a \\ g_{0a} = N_a \quad , \ a = 1,2,3 \end{cases}$$

$g_{\mu\nu}$

$$\{ g_{ab}(\vec{x}), p^{cd}(\vec{y}) \} = (\delta_a{}^c \delta_b{}^d + \delta_a{}^d \delta_b{}^c)\, \delta^{(3)}(\vec{x}-\vec{y})$$

$\int d^3x$

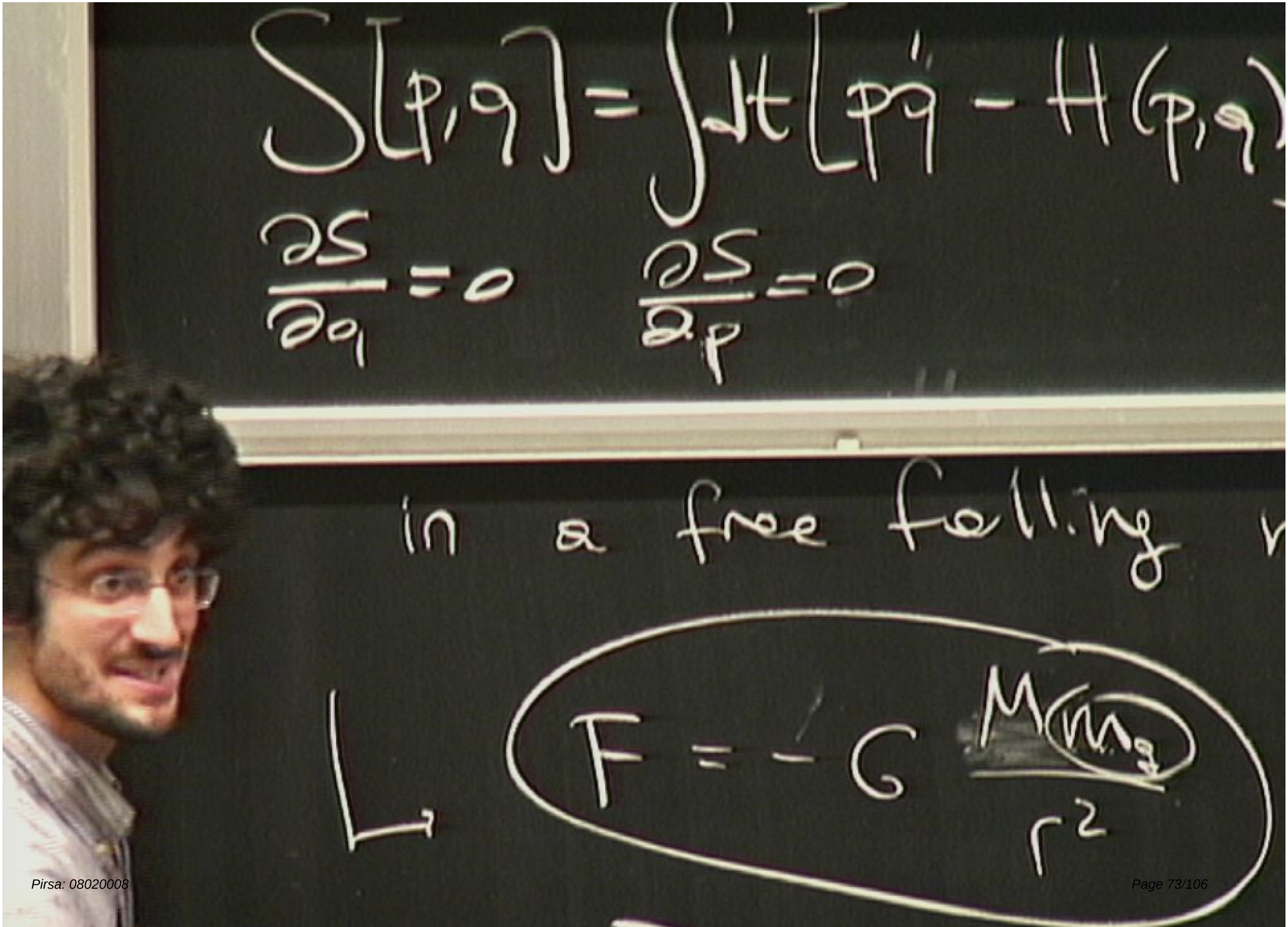
$$S[g_{ab}, p^{ab}, N, N_a] = \int dt \left[\dot{g}_{ab}\, p^{ab} - N\,\mathcal{H}(g,p) - N_a\,\mathcal{H}^a(g,p) \right]$$

$$\{ g_{ab}(\vec{x}),\ p^{cd}(\vec{y}) \} = \left(\delta_a{}^c \delta_b{}^d + \delta_a{}^d \delta_b{}^c\right) \delta^{(3)}(\vec{x}-\vec{y})$$

$$\int d^3x$$

$$S[g_{ab}, p^{ab}, N, N_a] = \int dt \left[\dot{g}_{ab} p^{ab} - N \mathcal{H}(q,p) - N_a \mathcal{H}^a(q,p) \right]$$

$$\text{"}H\text{"} = N \mathcal{H}(q,p) + N_a \mathcal{H}^a(q,p)$$


$S[p,q] = \int dt\,[p\dot{q} - H(p,q)$
$\frac{\partial S}{\partial q} = 0$ $\frac{\partial S}{\partial p} = 0$
in a free falling
L
$F = -G\frac{Mm_g}{r^2}$

$$\left\{ g_{ab}(\vec{x}),\, p^{cd}(\vec{y}) \right\} = \left(\delta_a{}^c \delta_b{}^d + \delta_a{}^d \delta_b{}^c \right) \delta^{(3)}(\vec{x}-\vec{y})$$

$$\int d^3x$$

$$S'\left[g_{ab}(x), p^{ab}(x), N(x), N_a(x) \right] = \int dt \left[\dot{g}_{ab}(x)\, p^{ab}(x) - N \mathcal{H}(q,p) - N_a \mathcal{H}^a(q,p) \right]$$

$$\text{“}H\text{''} = N \mathcal{H}(q,p) + N_a \mathcal{H}^a(q,p)$$

S

$$\{ g_{ab}(\vec{x}), p^{cd}(\vec{y}) \} = \left(\delta_a{}^c \delta_b{}^d + \delta_a{}^d \delta_b{}^c\right) \delta^{(3)}(\vec{x}-\vec{y})$$

$$\int d^3x$$

$$S'[g_{ab}(x), p^{ab}(x), N(x), N_a(x)] = \int dt \left[\dot{g}_{ab}(x) p^{ab}(x) - N \mathcal{H}(q,p) - N_a \mathcal{H}^a(q,p) \right]$$

$$\text{“}H\text{”} = N \mathcal{H}(q,p) + N_a \mathcal{H}^a(q,p)$$

$$\frac{\delta S}{\delta N}$$

$$\{ g_{ab}(\vec{x}),\ p^{cd}(\vec{y}) \} = (\delta_a{}^c \delta_b{}^d + \delta_a{}^d \delta_b{}^c)\, \delta^{(3)}(\vec{x}-\vec{y})$$

$$\int d^3x$$

$$S'[g_{ab}(x), p^{ab}(x), N(x), N_a(x)] = \int dt \left[\dot{g}_{ab}\, p^{ab} - N \mathcal{H}(g,p) - N_a \mathcal{H}^a(g,p) \right]$$

$$\text{'}H\text{'} = N \mathcal{H}(g,p) + N_a \mathcal{H}^a(g,p)$$

$$\frac{\delta S}{\delta N} = \mathcal{H}(g,p) = 0 \quad ; \quad \frac{\delta S}{\delta N_a} = \mathcal{H}^a(g,p) = 0$$

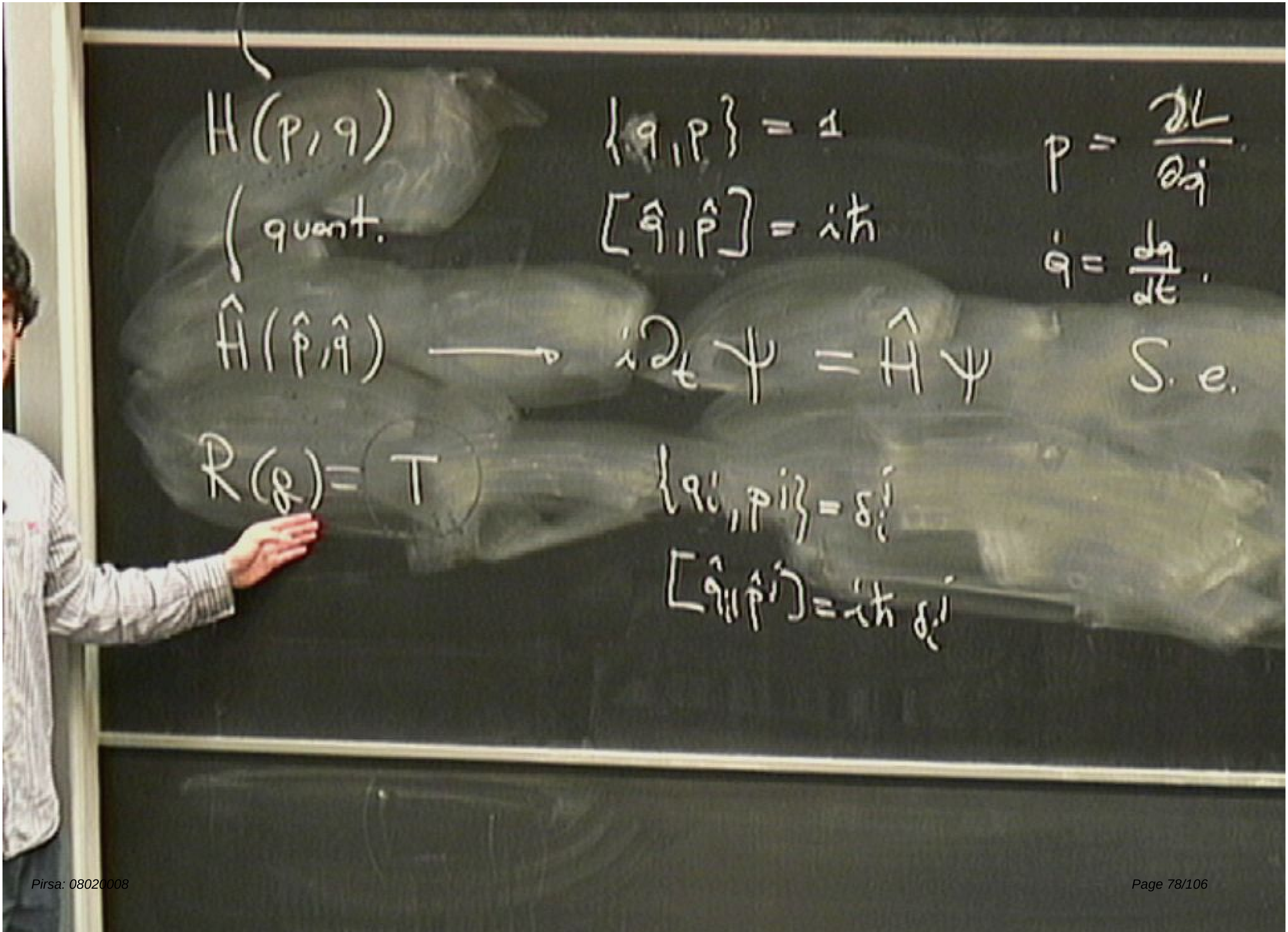
$$\{ g_{ab}(\vec{x}), p^{cd}(\vec{y}) \} = (\delta_a{}^c \delta_b{}^d + \delta_a{}^d \delta_b{}^c)\, \delta^{(3)}(\vec{x}-\vec{y})$$

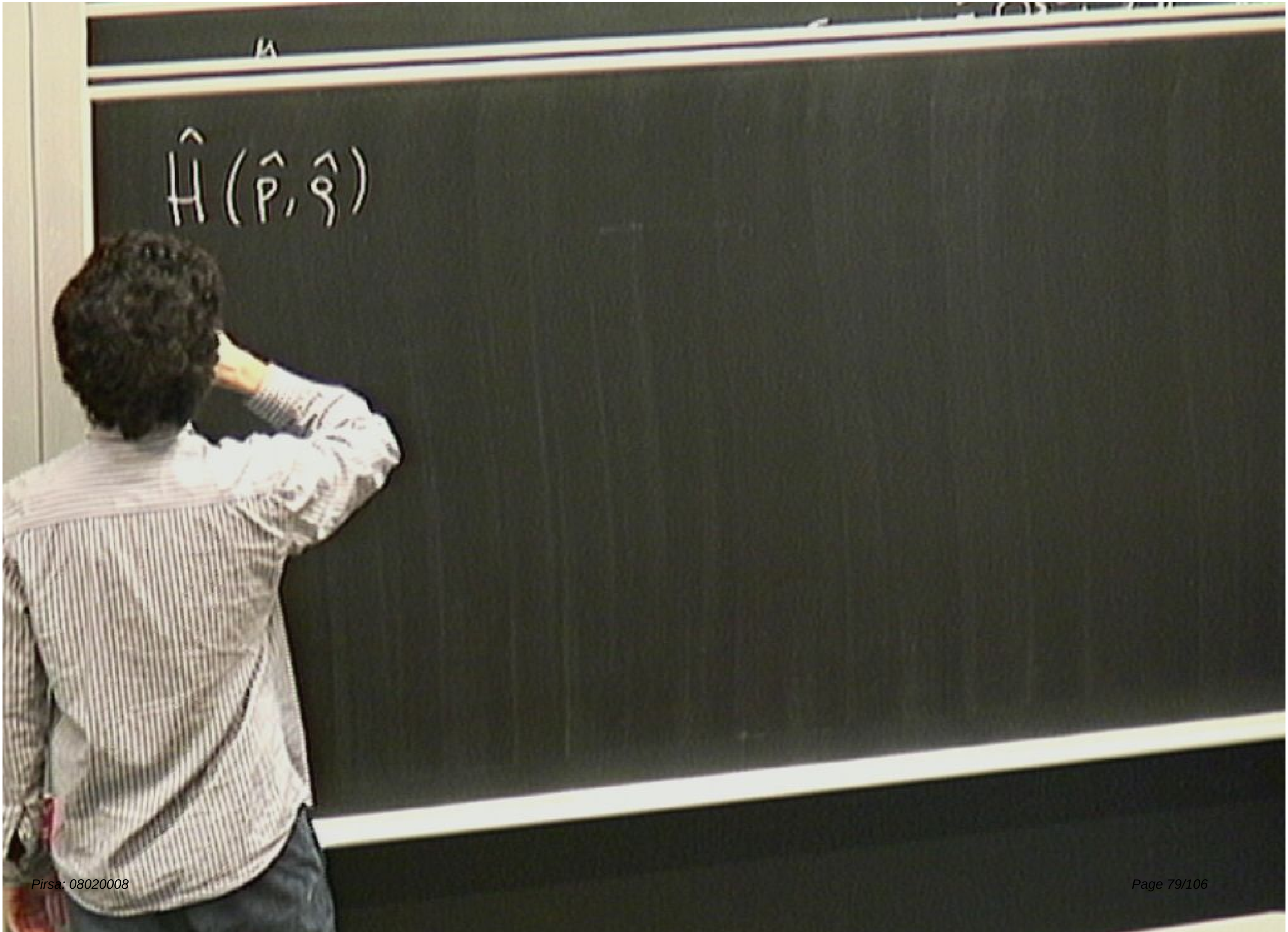
$$\int d^3x$$

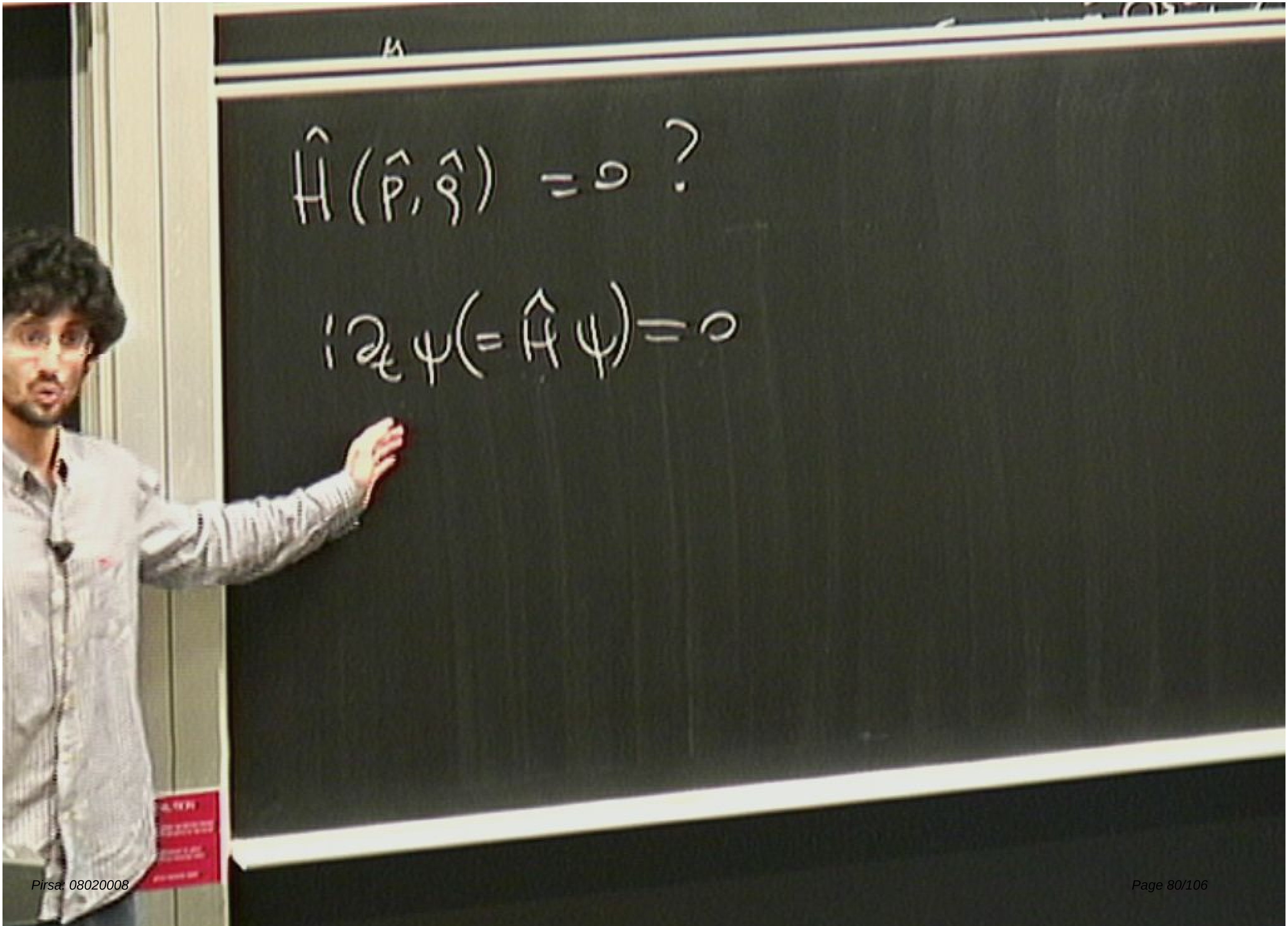
$$S'[g_{ab}(x), p^{ab}(x), N(x), N_a(x)] = \int dt \left[\dot{g}_{ab}(x) p^{ab}(x) - N \mathcal{H}(g,p) - N_a \mathcal{H}^a(g,p) \right]$$

$$\text{`}H\text{'} = N \mathcal{H}(g,p) + N_a \mathcal{H}^a(g,p) = 0.$$

$$\frac{\delta S}{\delta N} = \underline{\mathcal{H}(g,p)} = 0 \quad ; \quad \frac{\delta S}{\delta N_a} = \underline{\mathcal{H}^a(g,p)} = 0.$$


$H(p,q)$
quant.
$\hat{H}(\hat{p},\hat{q}) \longrightarrow i\partial_t \psi = \hat{H}\psi$ S. e.
$R(g) = T$
$\{q,p\} = 1$
$[\hat{q},\hat{p}] = i\hbar$
$p = \frac{\partial L}{\partial \dot{q}}$
$\dot{q} = \frac{dq}{dt}$
$\{q^i, p_j\} = \delta^i_j$
$[\hat{q}^i, \hat{p}_j] = i\hbar\, \delta^i_j$


$\hat{H}(\hat{p}, \hat{q})$


$\hat{H}(\hat{p},\hat{q}) = 0$?
$i\partial_t \psi (= \hat{H}\psi) = 0$

$$\hat{H}(\hat{p}, \hat{q}) = 0 \ ?$$

$$i\partial_t \psi (= \hat{H}\psi) = 0$$

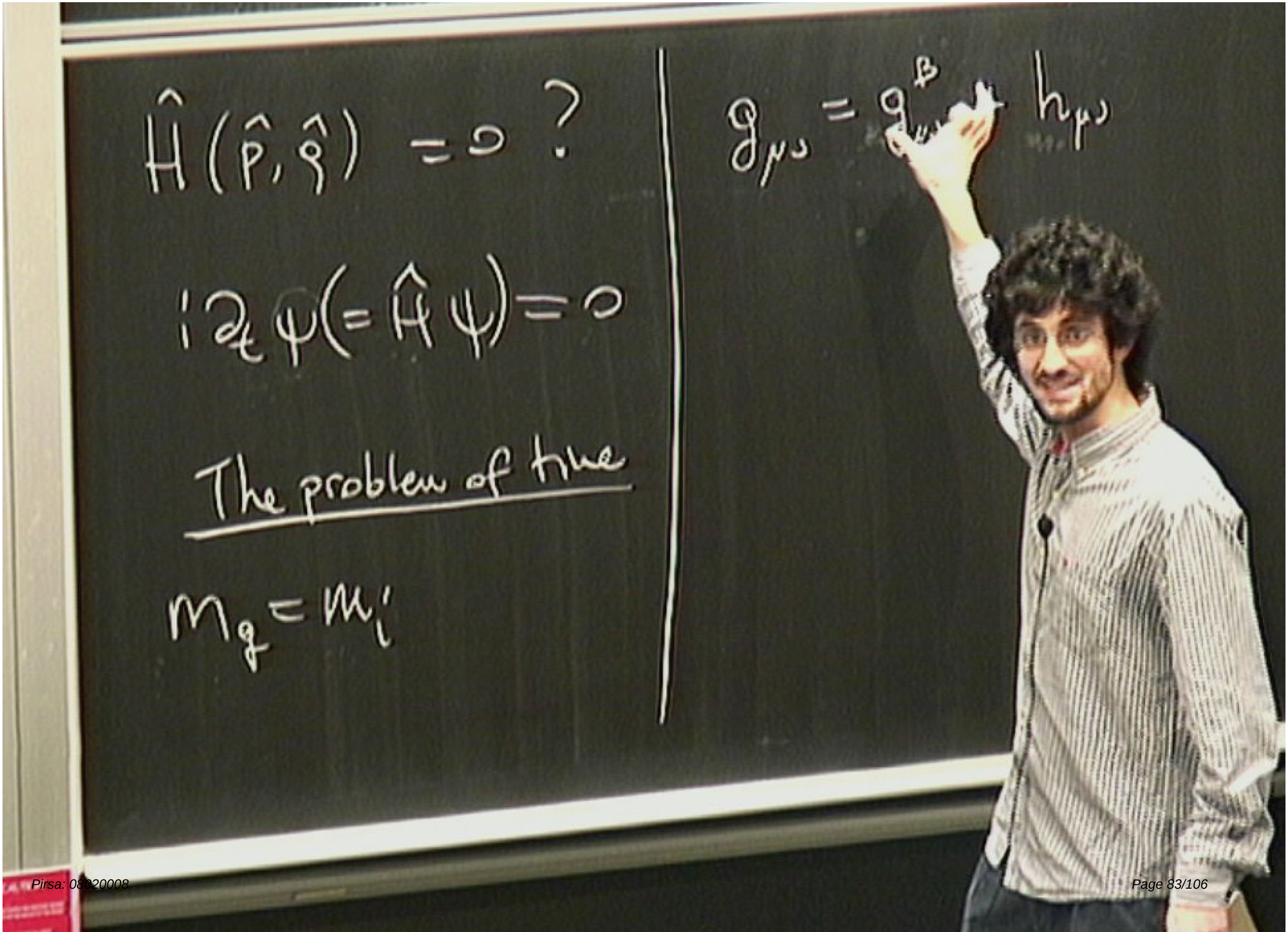
The problem of time

$$\hat{H}(\hat{p}, \hat{q}) = 0 \; ?$$

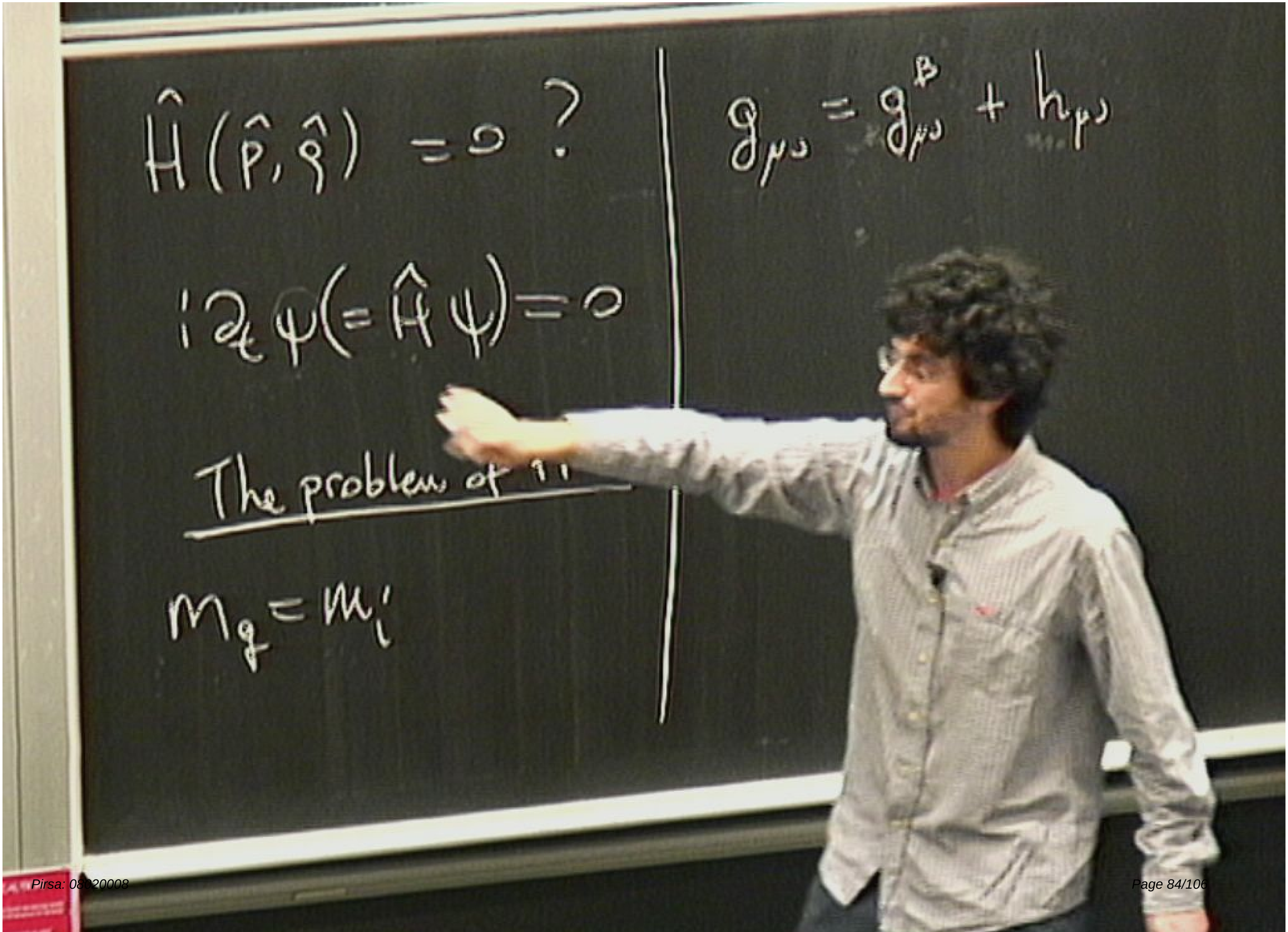
$$i \partial_t \psi \left(= \hat{H} \psi \right) = 0$$

The problem of time

$$m_g = m_i$$



$\hat{H}(\hat{p}, \hat{q}) = 0$?
$i\partial_t \psi (= \hat{H}\psi) = 0$
The problem of time
$m_g = m_i$
$g_{\mu\nu} = g^B_{\mu\nu} + h_{\mu\nu}$


$\hat{H}(\hat{p}, \hat{q}) = 0$?
$i\partial_t \psi (= \hat{H} \psi) = 0$
The problem of
$m_g = m_i$
$g_{\mu\nu} = g^B_{\mu\nu} + h_{\mu\nu}$

$$\hat{H}(\hat{p},\hat{q}) = 0 \;?$$

$$i\partial_t \psi \left(= \hat{H}\psi\right) = 0$$

<u>The problem of time</u>

$$m_g = m_i$$

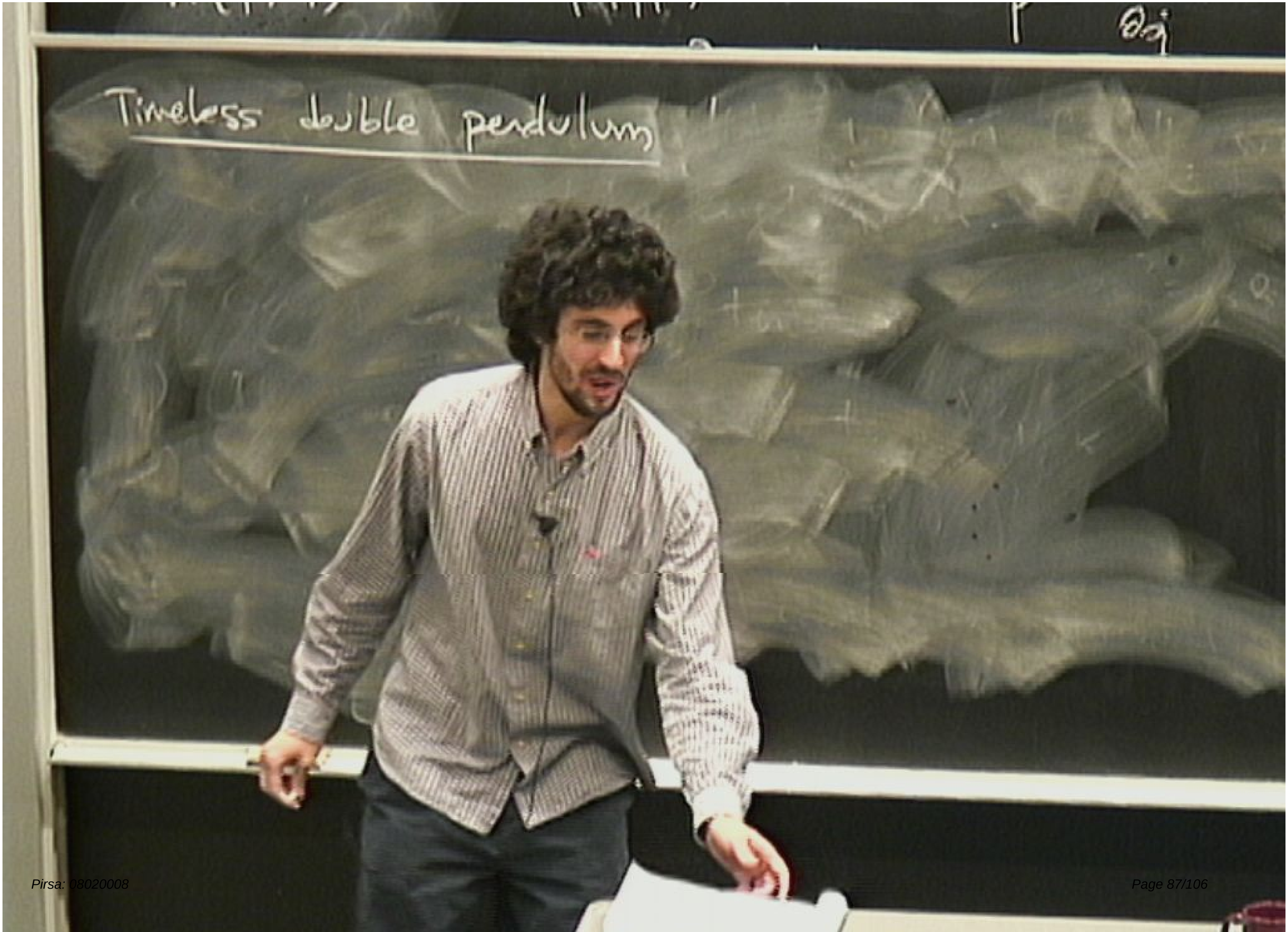
$$g_{\mu\nu} = g^{B}_{\mu\nu} + h_{\mu\nu}$$

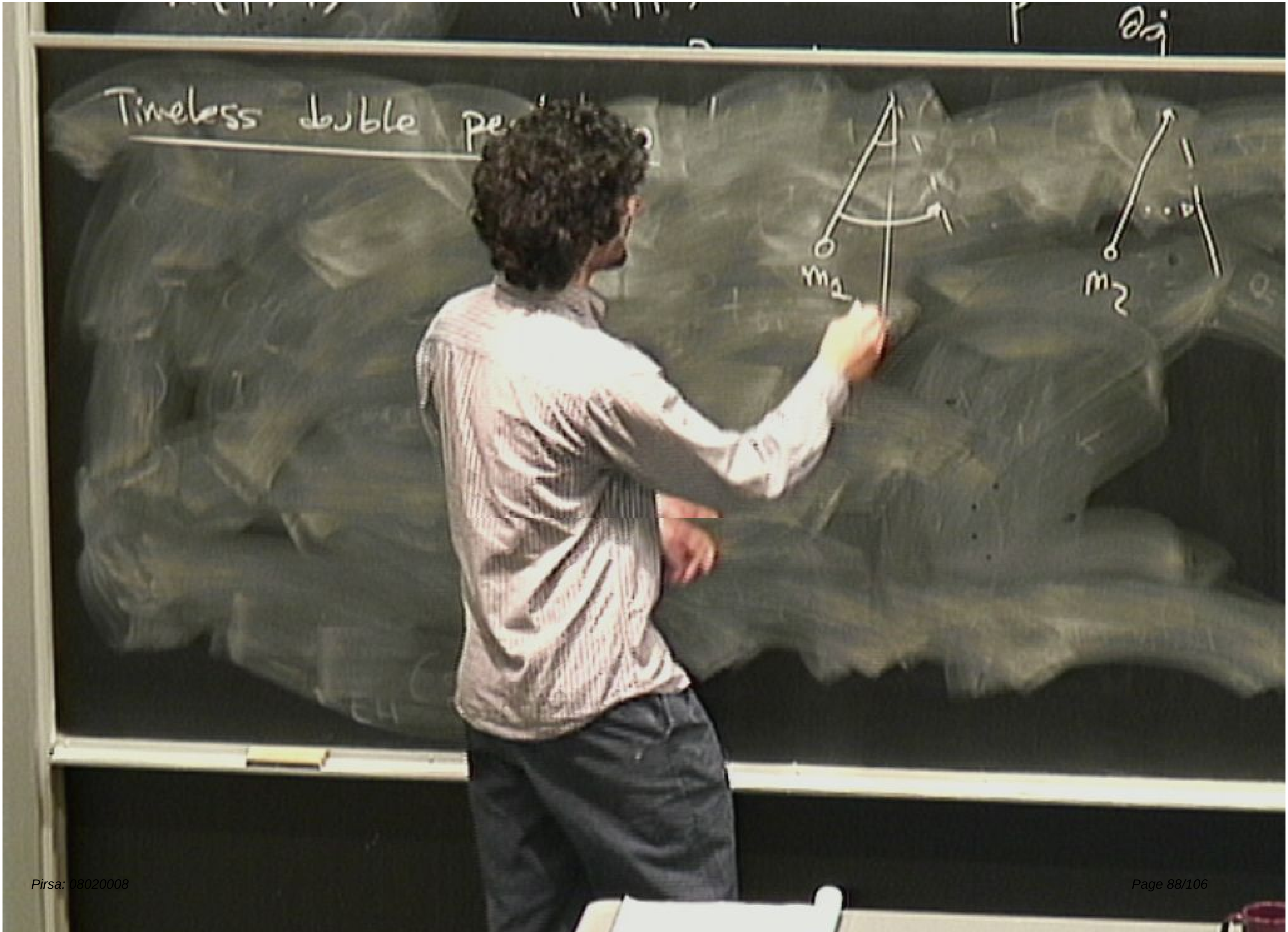
$M_q = m_i$

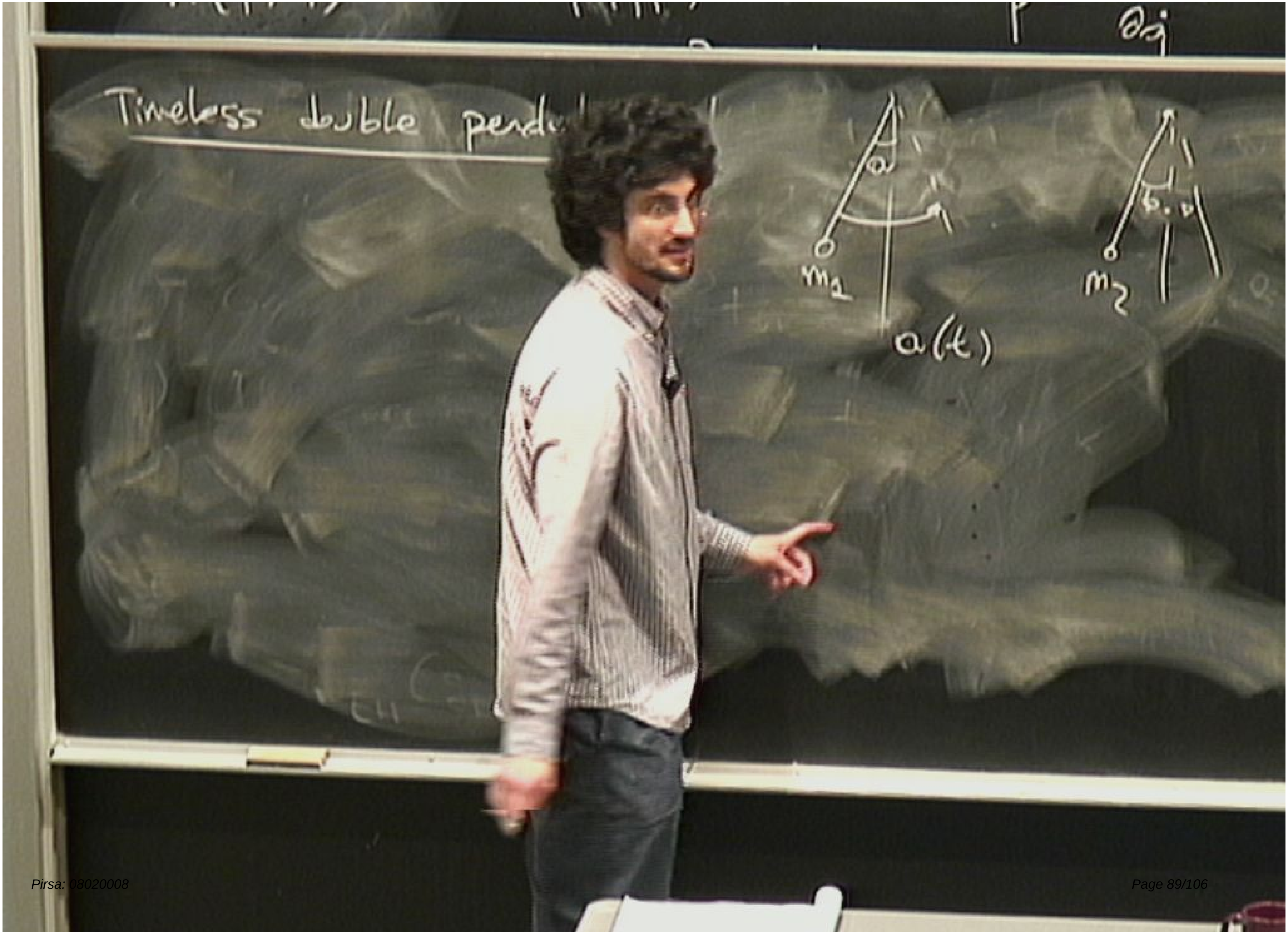
$S[q_{ab}(x), p^{ab}(x), N(x), N_a(x)] = \int dt \left[\dot{q}_{ab}(x) p^{ab}(x) - N\mathcal{H}(q,p) - N_a \mathcal{H}^a(q, \ldots \right.$

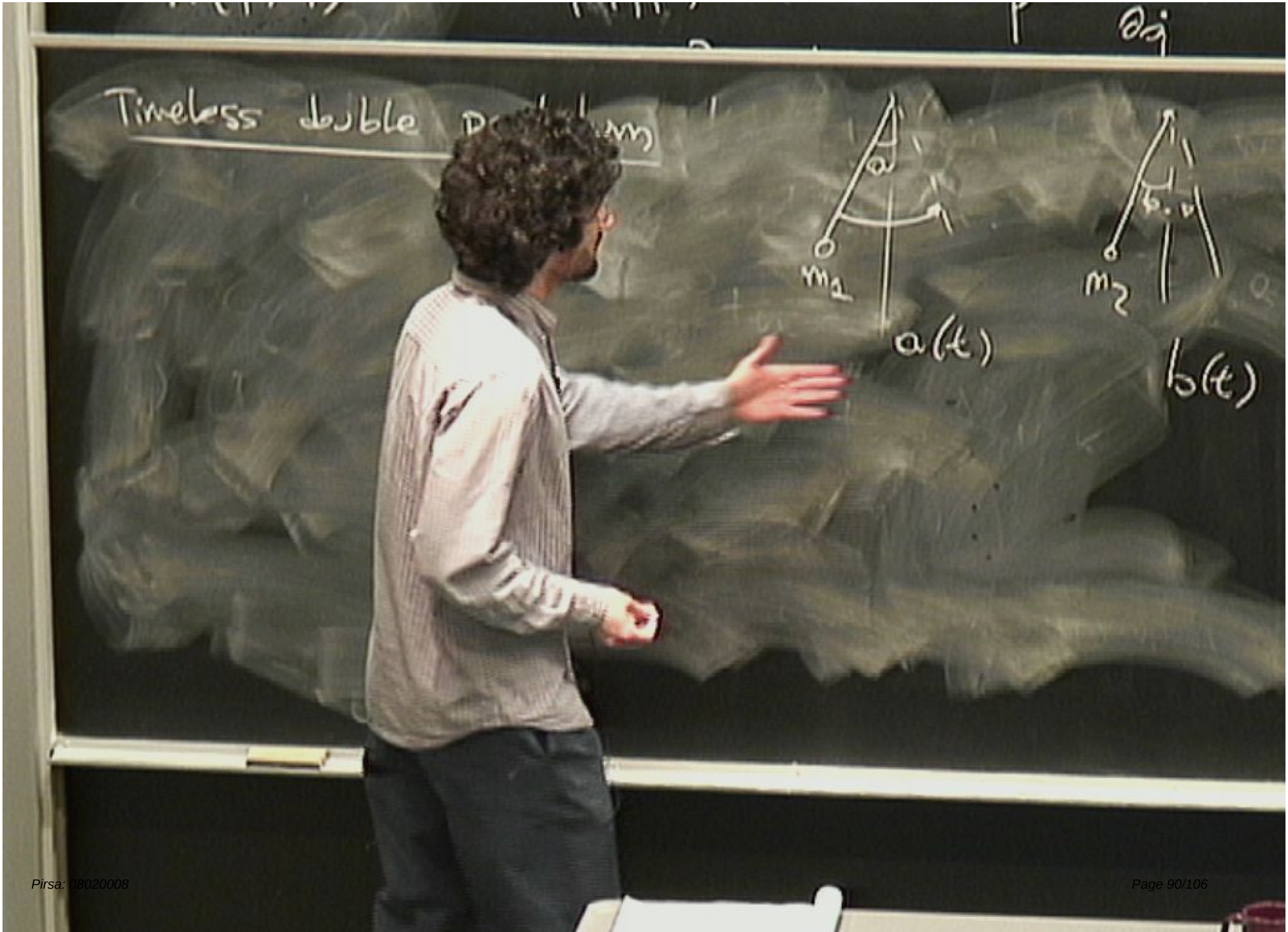
$$\text{“}H\text{”} = N\mathcal{H}(q,p) + N_a\mathcal{H}^a(q,p) = 0$$

$$\frac{\delta S}{\delta N} = \mathcal{H}(q,p) = 0 \quad ; \quad \frac{\delta S}{\delta N_a} = \mathcal{H}^a(q,p) = 0$$


Timeless double pendulum

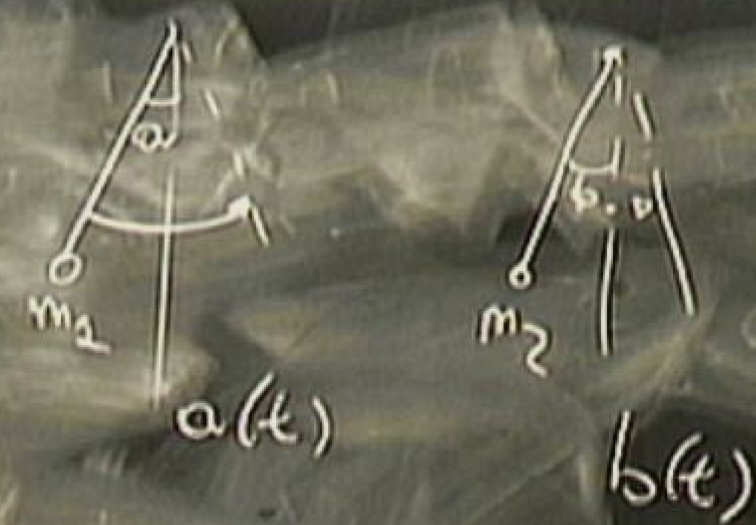

Timeless
m_1
m_2

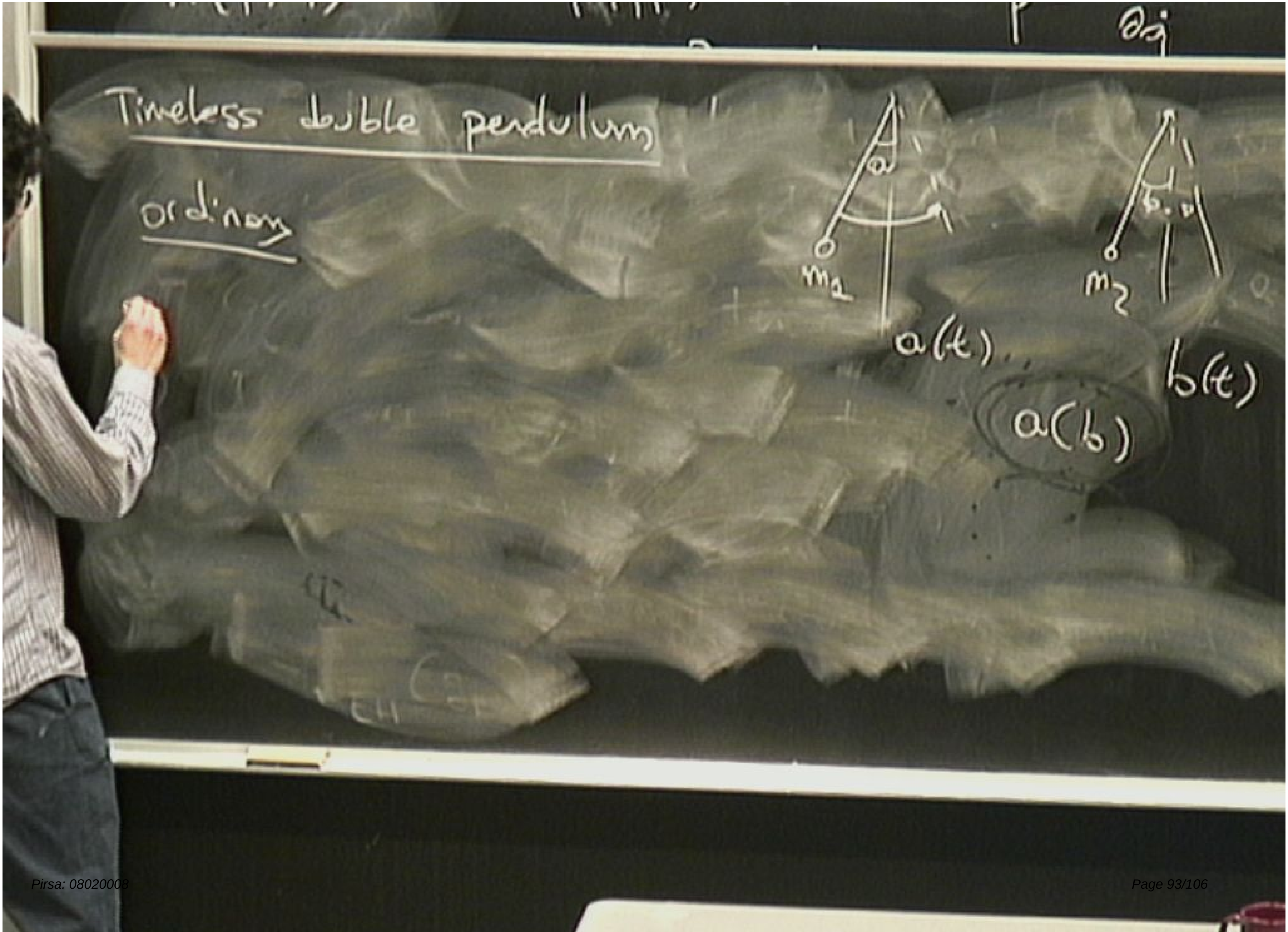

Timeless double pend
m_1
m_2
$a(t)$

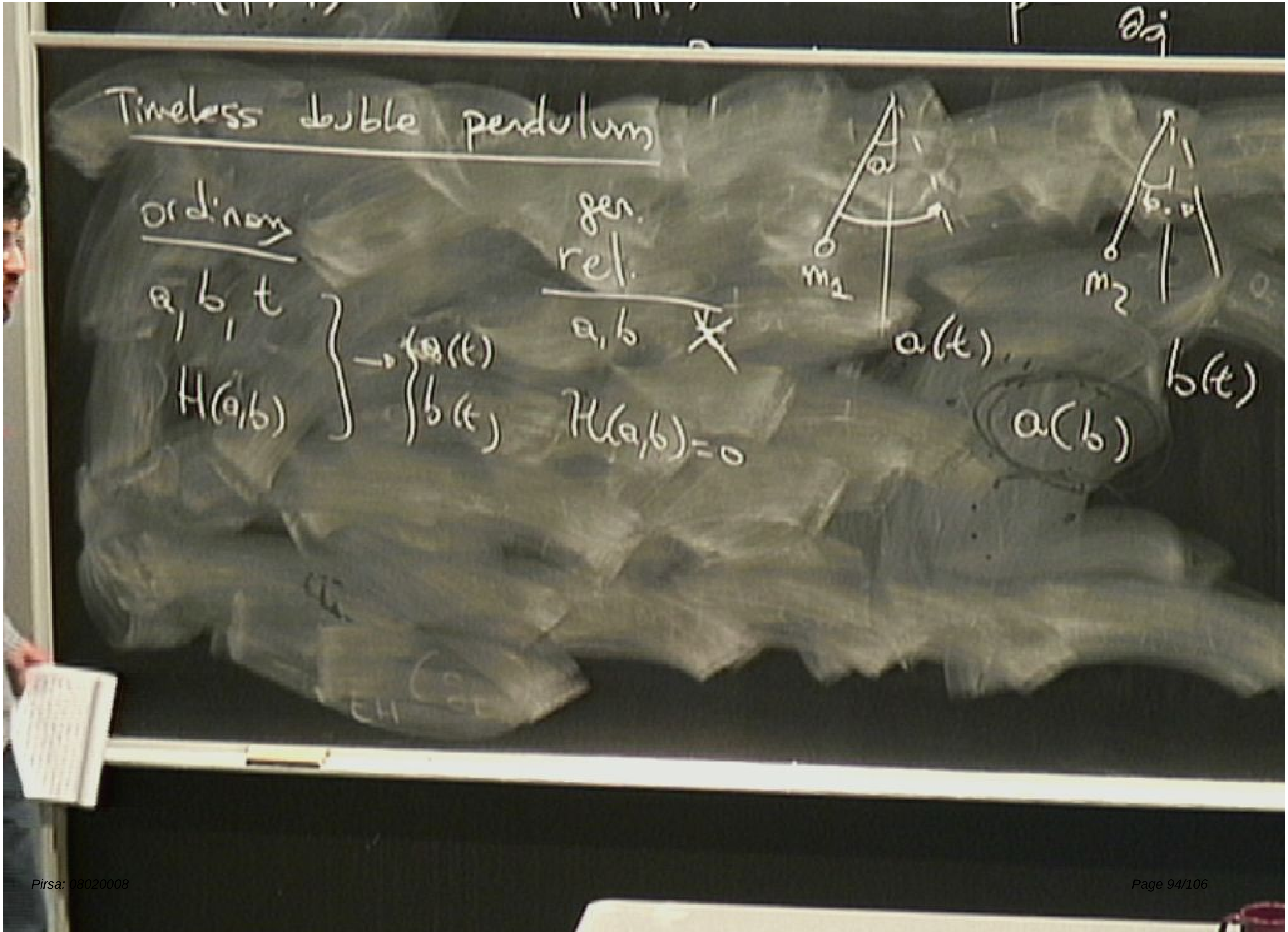

Timeless
m1
m2
a(t)
b(t)

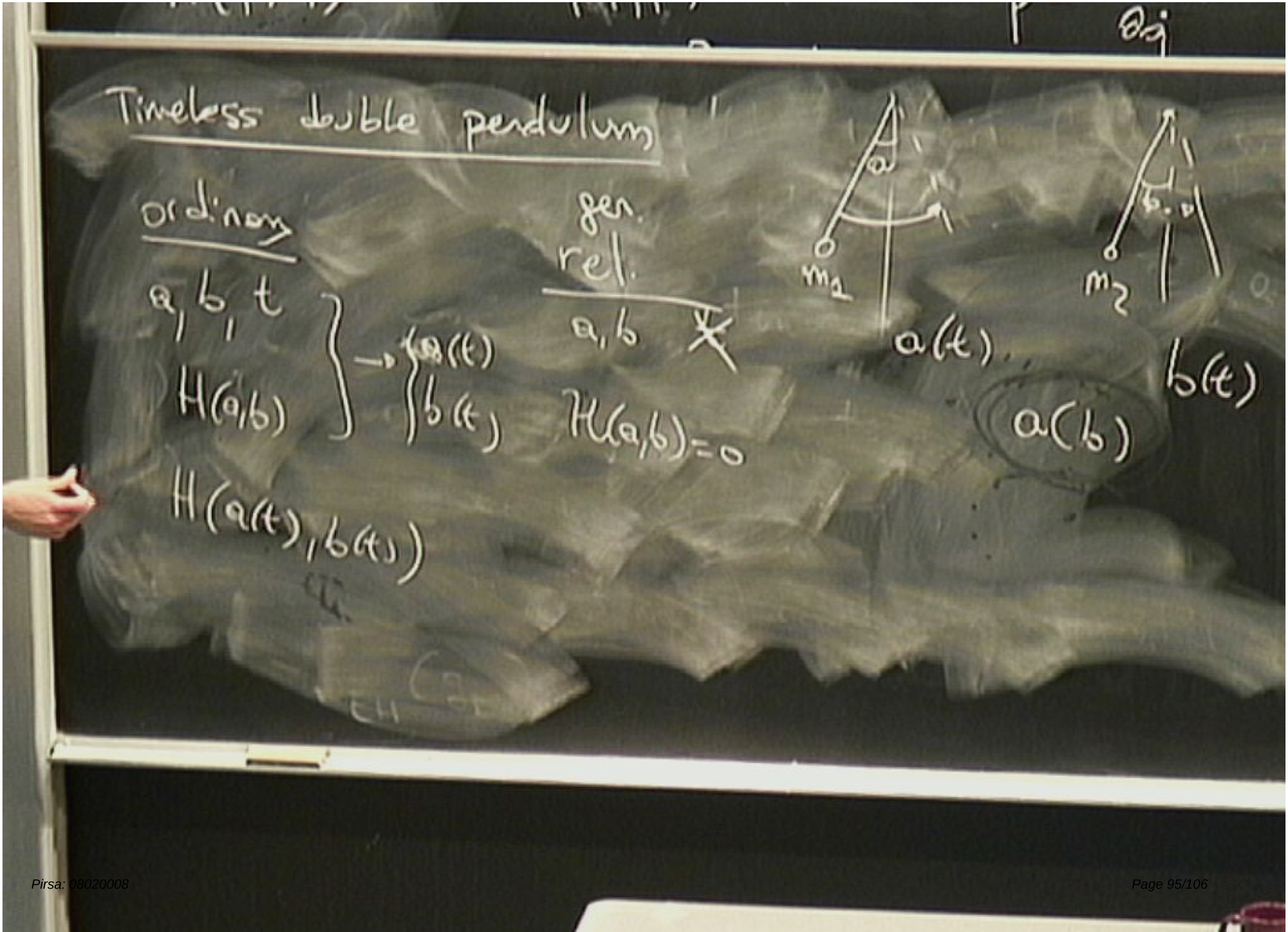
Timeless double pendulum
m_1
$a(t)$
m_2
$b(t)$

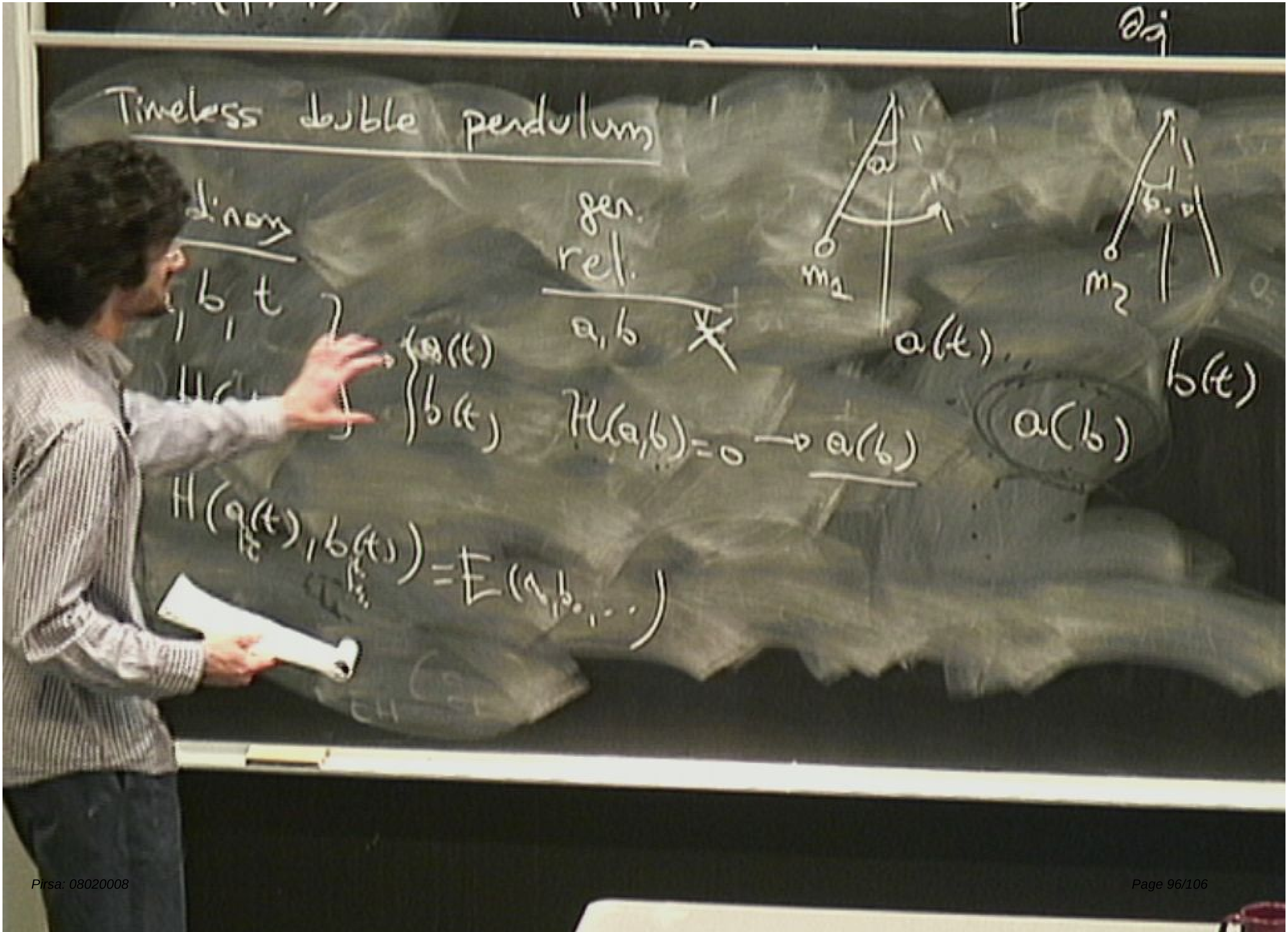
Timeless double pendulum




Timeless double pendulum
ordinary
a
m1
a(t)
b
m2
b(t)
a(b)


Timeless double pendulum
ordinary
a, b, t
H(a,b)
a(t)
b(t)
gen. rel.
a, b
t
H(a,b)=0
m1
m2
a(t)
b(t)
a(b)


Timeless double pendulum
ordinary
a, b, t
H(a,b)
a(t)
b(t)
H(a(t), b(t))
gen. rel.
a, b
H(a,b) = 0
a
b
m1
m2
a(t)
b(t)
a(b)


Timeless double pendulum
gen. rel.
a, b
a(t)
b(t)
m2
m2
a(t)
b(t)
H(a,b)=0 → a(b)
a(b)
H(a(t), b(t)) = E(a, b, ...)

Timeless double pendulum

ordinary

a, b, t

$H(a,b)$ $\Big\} \rightarrow \Big\{ \begin{matrix} a(t) \\ b(t) \end{matrix}$

gen. rel.

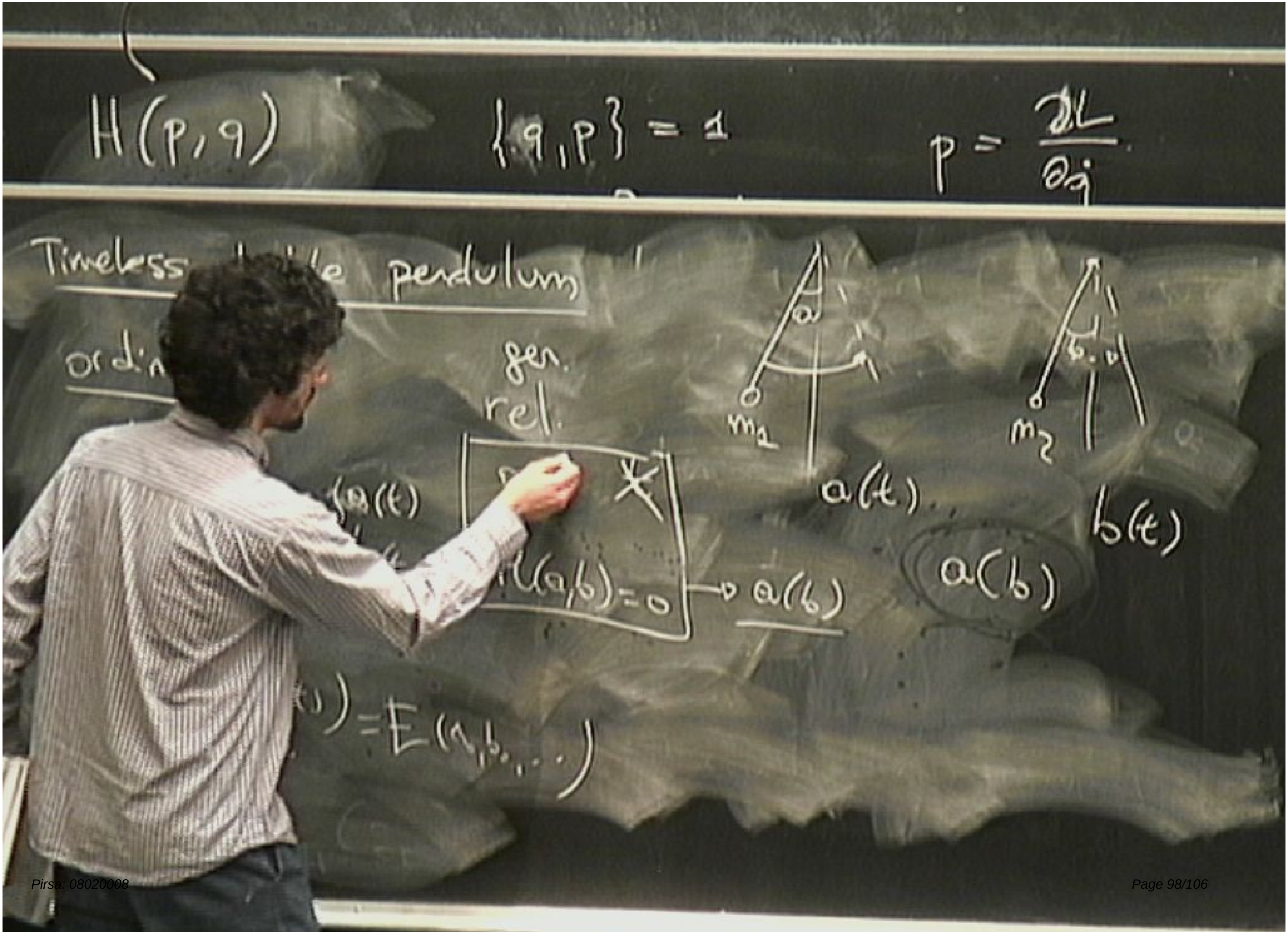
$a, b, \cancel{t}$

$\mathcal{H}(a,b) = 0 \rightarrow \underline{a(b)}$

$a(t)$, $b(t)$

$a(b)$

$H(a(t), b(t)) = E(a_0, b_0, \ldots)$



H(p,q)
{q,p} = 1
p = ∂L/∂q̇
Timeless
pendulum
gen.
rel.
m1
m2
a(t)
b(t)
a(b)
(a,b) = 0
a(b)
= E(a,b,...)

$H(p,q) \qquad \{q,p\} = 1 \qquad p = \frac{\partial L}{\partial \dot q}$

Timeless double pendulum

ordinary

$a,\ b,\ t$

$H(a,b)$

$\left.\begin{matrix} a,\ b,\ t \\ H(a,b) \end{matrix}\right\} \rightarrow \begin{cases} a(t) \\ b(t) \end{cases}$

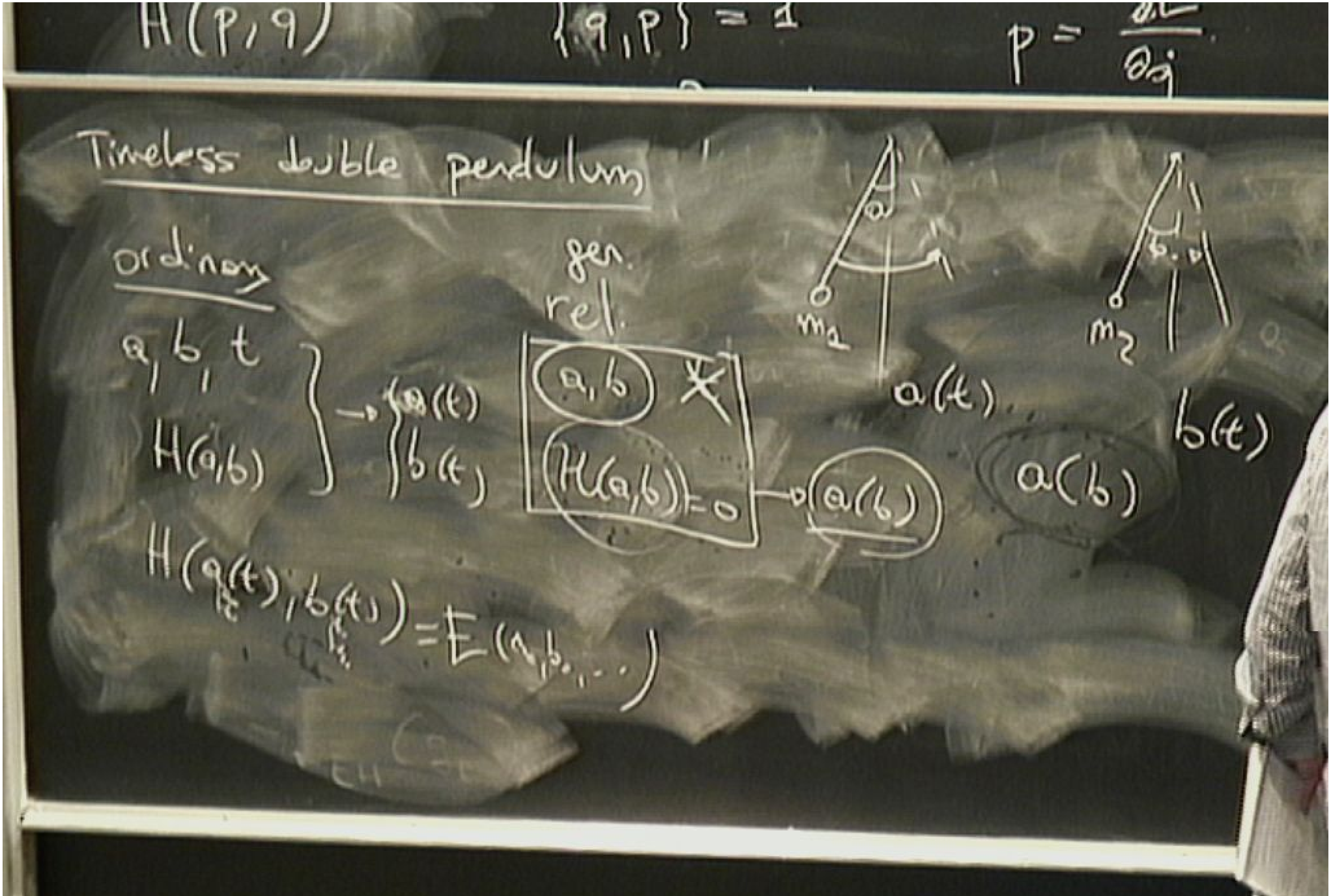
gen. rel.

$\boxed{\begin{matrix} a, b & \cancel{t} \\ H(a,b) = 0 \end{matrix}} \rightarrow a(b)$

$a(t) \qquad b(t)$

$a(b)$

$H(a(t), b(t)) = E(a, b, \ldots)$


H(p,q)
{q,p} = 1
Timeless double pendulum
ordinary
a, b, t
H(a,b)
a(t)
b(t)
H(q(t), b(t)) = E(a,b,...)
gen. rel.
a, b
H(a,b)=0
a(b)
m2
a(t)
b(t)
a(b)

$$\psi(a,b) \in \mathcal{H} = L_2(\mathbb{R}^2, d^2x)$$

$$i\partial_t \psi - \hat{H}\psi = 0$$

$$\hat{H}(\hat{a}, \hat{p}_a, \hat{b}, \hat{p}_b)$$

$$\psi(a,b) \in \mathcal{H} = L_2(\mathbb{R}^2, d^2x)$$

$$i\partial_t \psi = \hat{H}\psi = 0$$

$$\hat{H}(\hat{a}, \hat{p}_a, \hat{b}, \hat{p}_b)$$

$$\{a, p_a\} = 1 \to [\hat{a}, \hat{p}_a] = i\hbar \to \begin{cases} \hat{a} = a \\ \hat{p}_a = -i\frac{\partial}{\partial a} \end{cases}$$

$$\psi(a,b) \in \mathcal{H} = L_2(\mathbb{R}^2, d^2x)$$

$$i\partial_t \psi = \boxed{\hat{H}\,\psi(a,b) = 0}$$

$$\hat{H}(\hat{a}, \hat{p}_a, \hat{b}, \hat{p}_b)$$

$$\{a, p_a\} = 1 \to [\hat{a}, \hat{p}_a] = i\hbar \to \begin{cases} \hat{a} = a \\ \hat{p}_a = -i\frac{\partial}{\partial a} \end{cases}$$

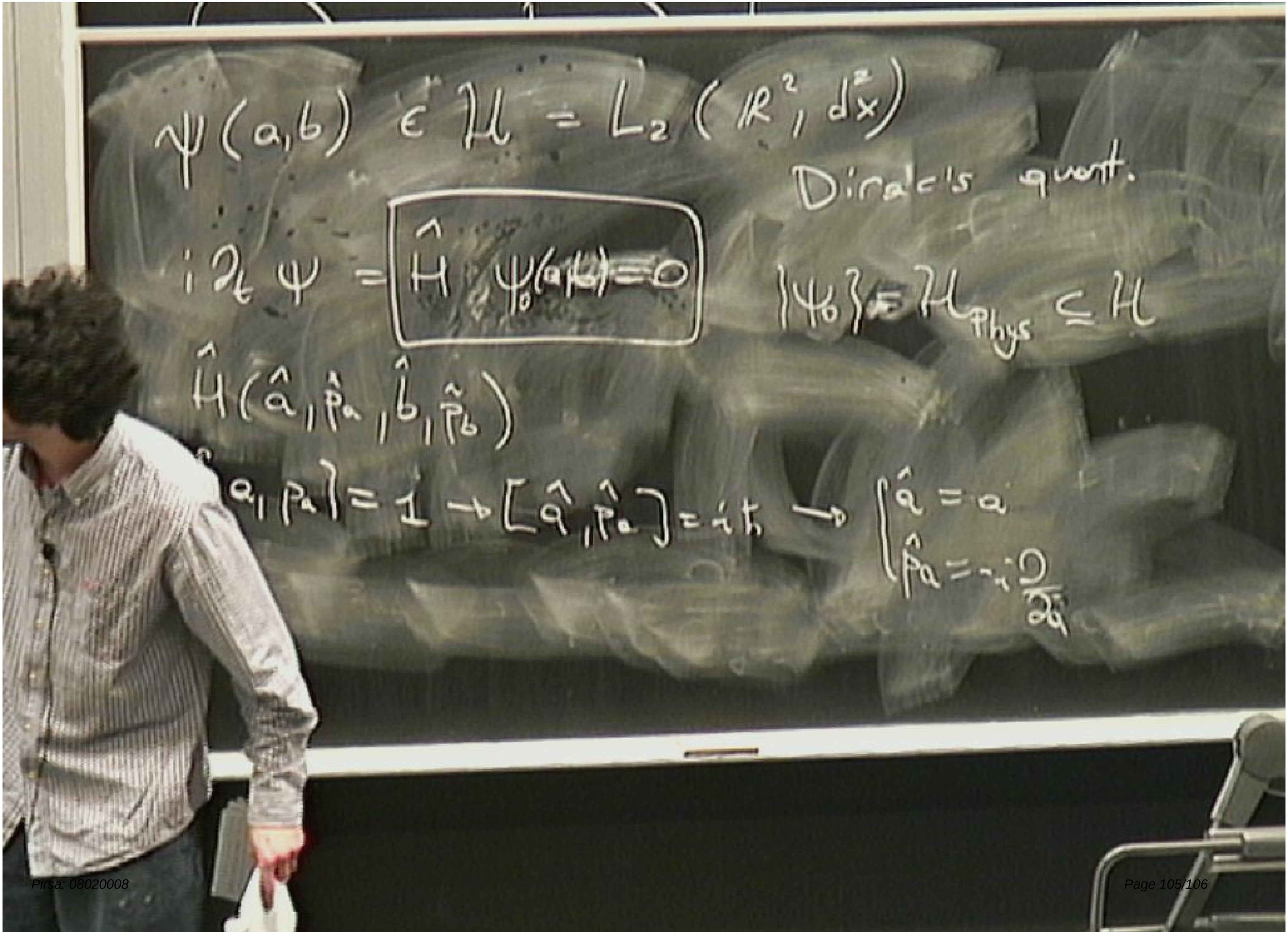
$$\psi(a,b) \in \mathcal{H} = L_2(\mathbb{R}^2, d^2x)$$

Dirac's quant.

$$i \; \psi = \boxed{\hat{H}\,\psi(a,b) = 0}$$

$$\hat{H}(\hat{a}, \hat{p}_a, \hat{b}, \hat{p}_b)$$

$$\{a, p_a\} = 1 \to [\hat{a}, \hat{p}_a] = i\hbar \to \begin{cases} \hat{a} = a \\ \hat{p}_a = -i\hbar \dfrac{\partial}{\partial a} \end{cases}$$


$\psi(a,b) \in \mathcal{H} = L_2(\mathbb{R}^2, d^2x)$
Dirac's quant.
$i\partial_t \psi = \boxed{\hat{H}\,\psi_0(a,b) = 0}$
$\{\psi_0\} = \mathcal{H}_{phys} \subseteq \mathcal{H}$
$\hat{H}(\hat{a}, \hat{p}_a, \hat{b}, \hat{p}_b)$
$\{a, p_a\} = 1 \to [\hat{a}, \hat{p}_a] = i\hbar \to \begin{cases} \hat{a} = a \\ \hat{p}_a = -i\frac{\partial}{\partial a} \end{cases}$

$$\psi(a,b) \in \mathcal{H} = L_2(\mathbb{R}^2, d^2x)$$

Dirac's quant.

$$i\partial_t \psi = \boxed{\hat{H}\,\psi_0(a,b) = 0}$$

$$\{\psi_0\} = \mathcal{H}_{phys} \subseteq \mathcal{H}$$

$$\hat{H}(\hat{a}, \hat{p}_a, \hat{b}, \hat{p}_b)$$

$$\{a, p_a\} = 1 \rightarrow [\hat{a}, \hat{p}_a] = i\hbar \rightarrow \begin{cases} \hat{a} = a \\ \hat{p}_a = -i\hbar \dfrac{\partial}{\partial a} \end{cases}$$