Title: Quantifying the quantumness of correlations: beyond entanglement and back

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Abstract: We define a measure of the quantumness of correlations, based on the operative task of local broadcasting of a bipartite state. Such a task is feasible for a state if and only if it corresponds to a joint classical probability distribution, or, in other terms, it is strictly classically correlated. A gap, defined in terms of quantum mutual information, can quantify the degree of failure in fulfilling such a task, therefore providing a measure of how non-classical a given state is. We are led to consider the asymptotic average mutual information of a state, defined as the minimal per-copy mutual information between parties, when they share an infinite amount of broadcast copies of the state. We analyze the properties of such quantity, and find that it satifies many of the properties required for an entanglement measure. We show that it lies between the quantum- and the classical-conditioned versions of squashed entanglement. The non-vanishing of asymptotic average mutual information for entangled states may be interpreted as a signature of monogamy of entanglement.

Waterloo, 12th January 2007

Quantifying quantumness of correlations

...beyond entanglement and back...

Caterina-Eloisa Mora

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Joint work with Marco Piani and Pawel Horodecki

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Motivation

- It is important to analyze features that distinguish the quantum world from the classical one (from both a fundamental and a practical point of view)

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- Is there more than entanglement in the quantumness of correlations? In what sense?

Outline

- What makes correlations quantum?
  - Correlations and mutual information
  - More than ``just" separable states: (QQ), CC, and CQ

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  - $I^{(\infty)}$: a candidate entanglement measure

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- Outlook and conclusions

Correlations

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A measure of correlations: quantum mutual information

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It satisfies the two axioms:

- ☑ $I(\rho_{AB})=0 \Leftrightarrow \rho_{AB} = \rho_A \otimes \rho_B$
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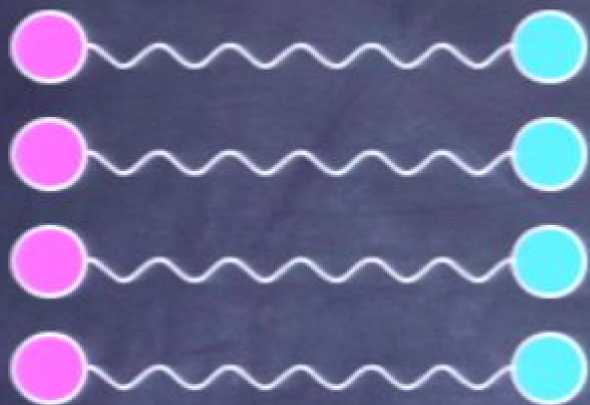

$\rho_{AB}^{\otimes n}$

Decorrelating by random unitaries [Groisman et al.]

$$\frac{1}{N}\sum_{i=1}^{N}(U_i^A \otimes I_B)\rho_{AB}^{\otimes n}(U_i^A \otimes I_B)^\dagger \approx \omega_A \otimes \omega_B$$

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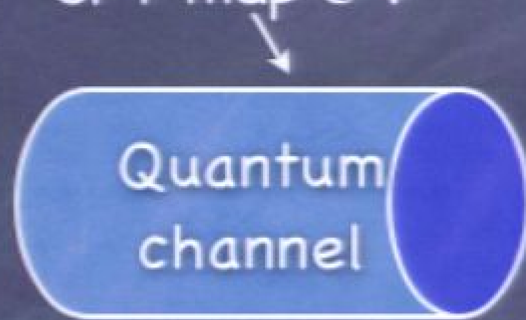
Asymptotically $\lim_n \frac{\log N}{n} = I(\rho_{AB})$ bits of randomness per copy are necessary and sufficient

Remarks on quantum MI (III)

Entanglement assisted capacity of quantum channels

Sender (A) | CPT map $\mathcal{N}$ | Receiver (B)



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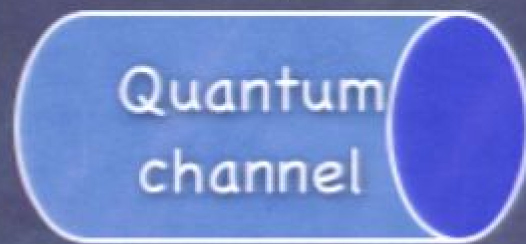

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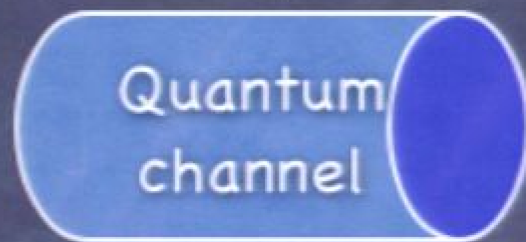
↑

Classical-to-quantum encoding



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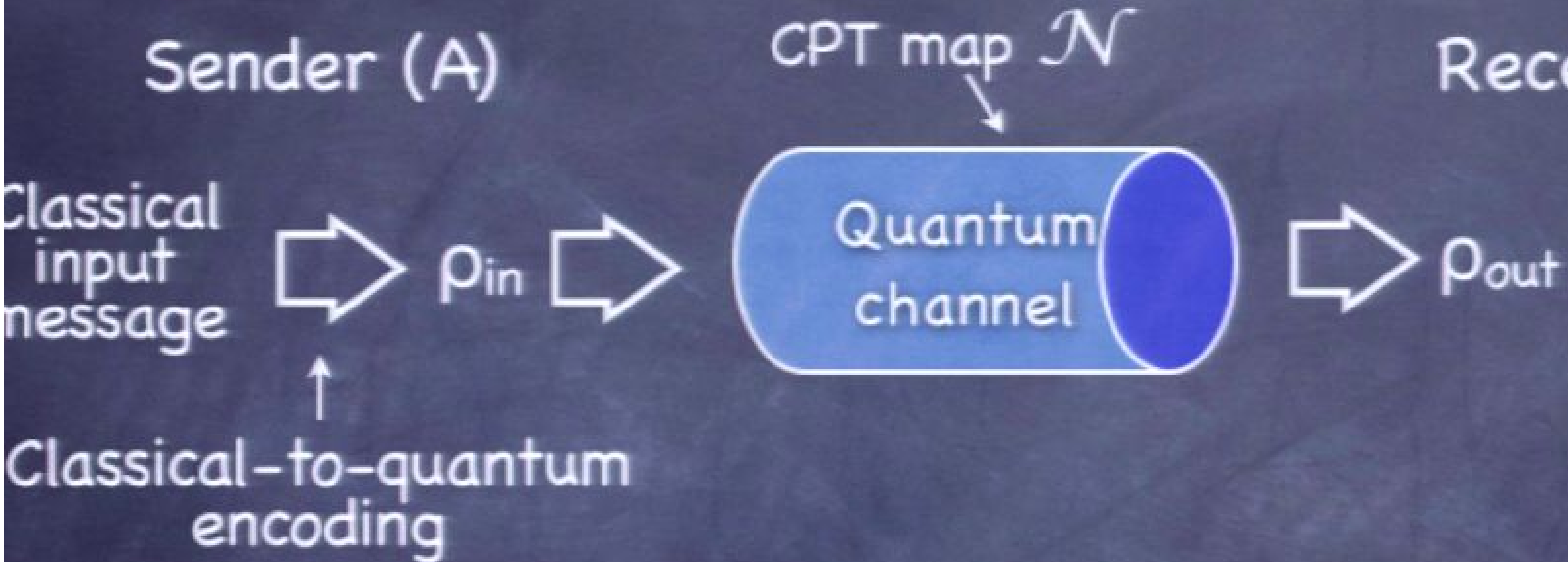

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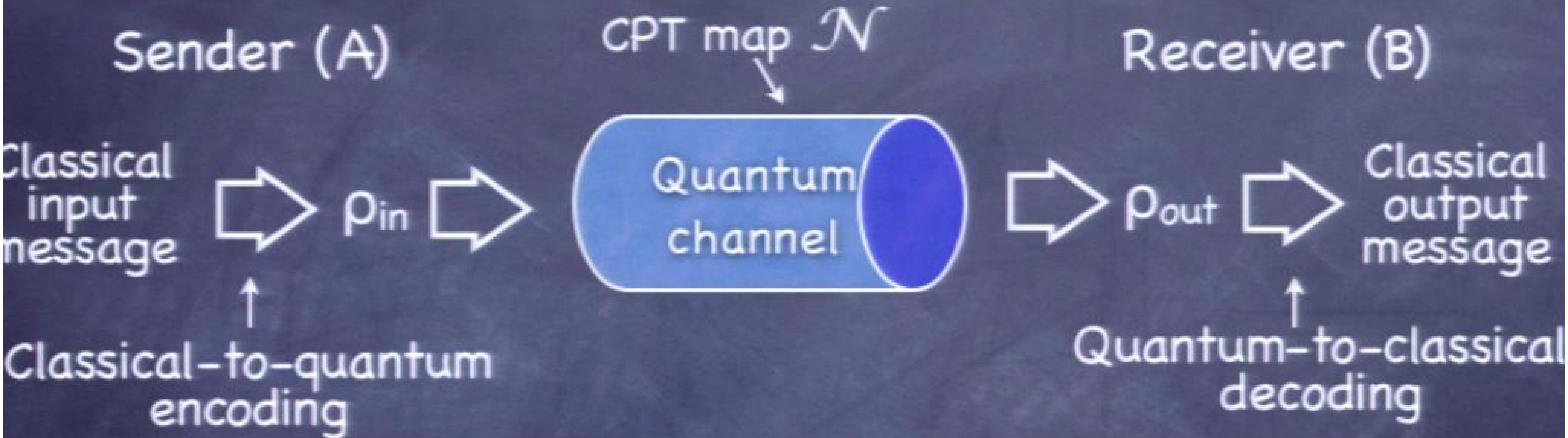

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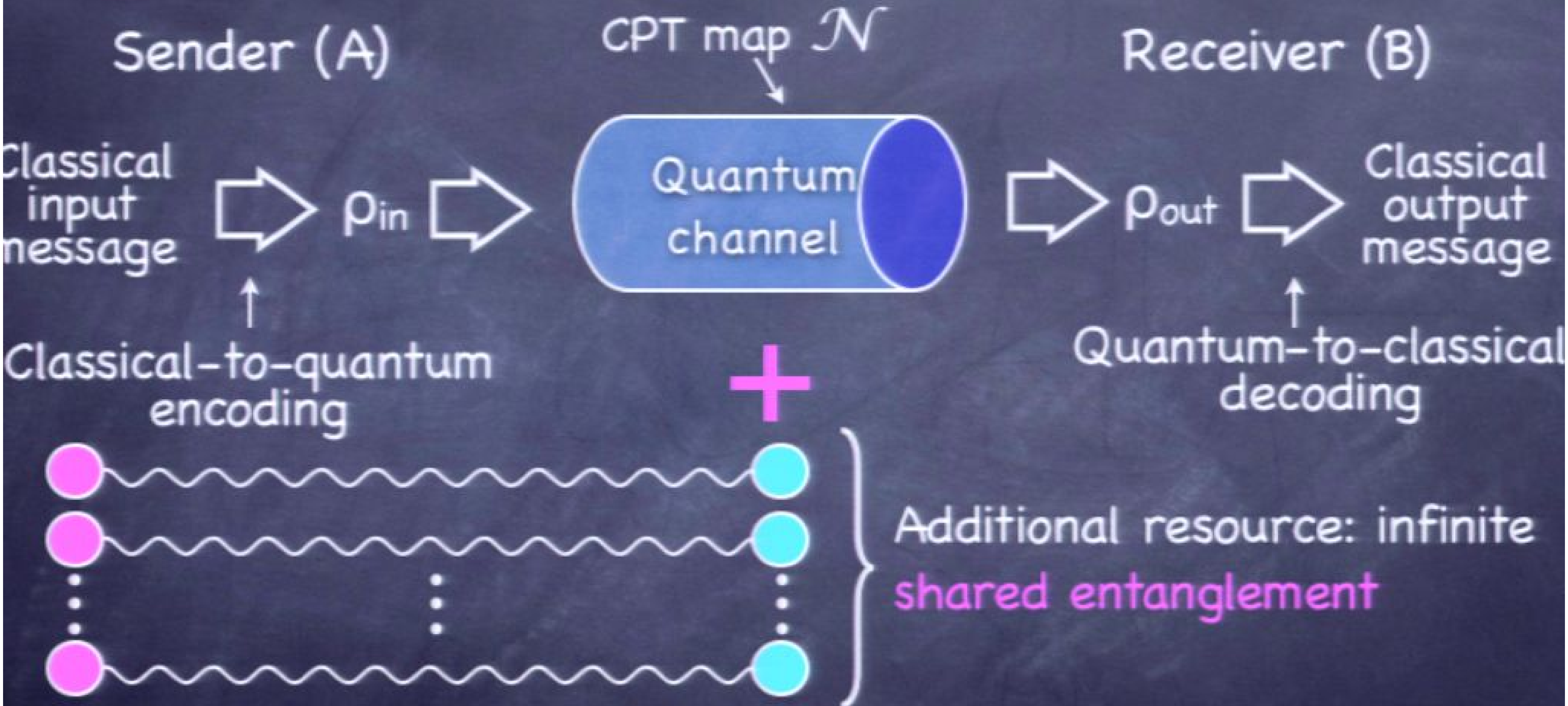

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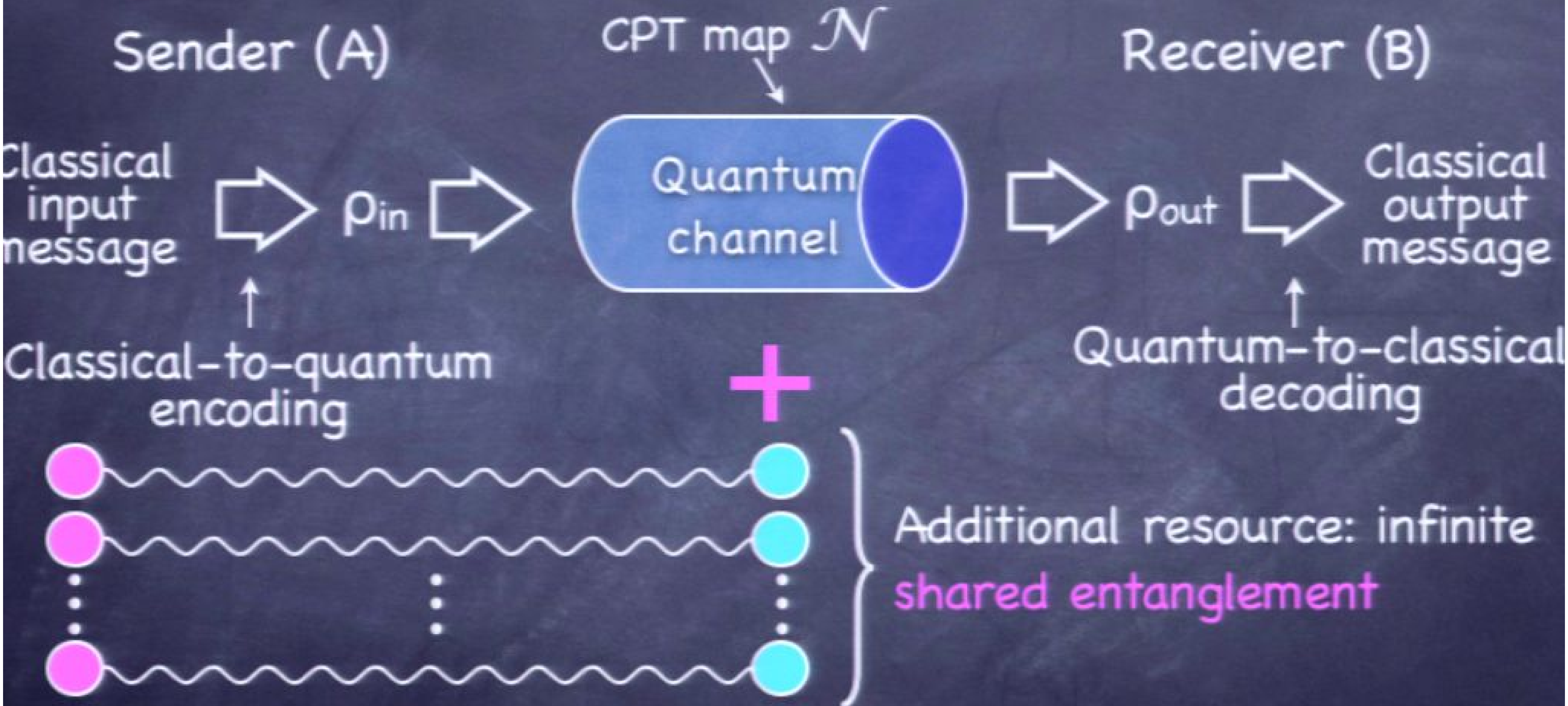

ntanglement-assisted capacity of a quantum channel

$$C_E(\mathcal{N}) = \max_{\mathrm{Tr}_B \psi_{AB}} I((\mathcal{N}_A \otimes \mathrm{id}_B)[\psi_{AB}])$$ [Bennett et al.]

Separable states

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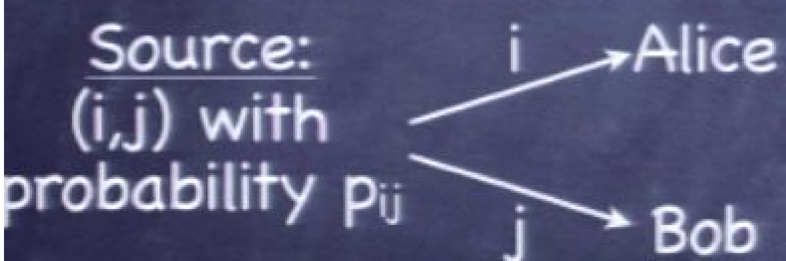

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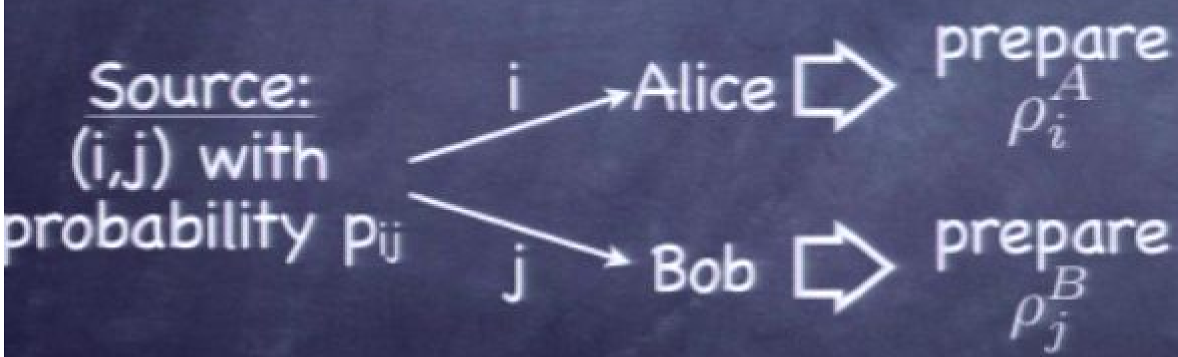

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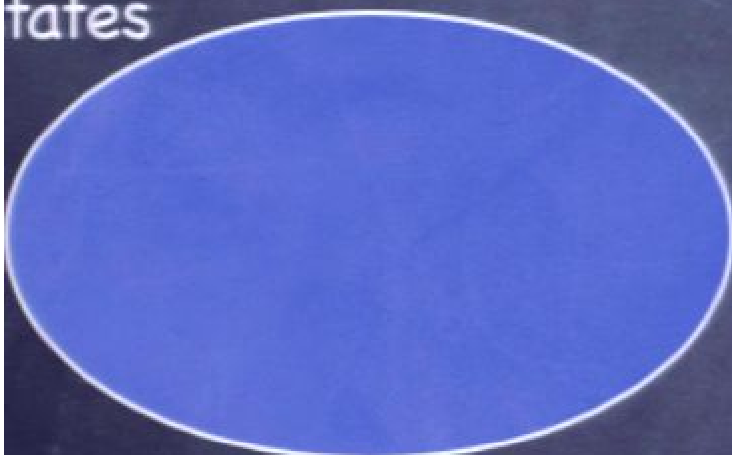
Quantum state of the classical register

Hierarchy of correlations

Set of quantum
tates



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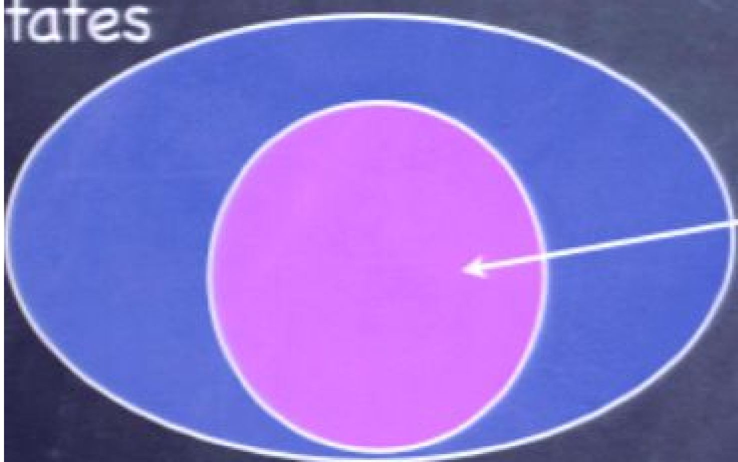

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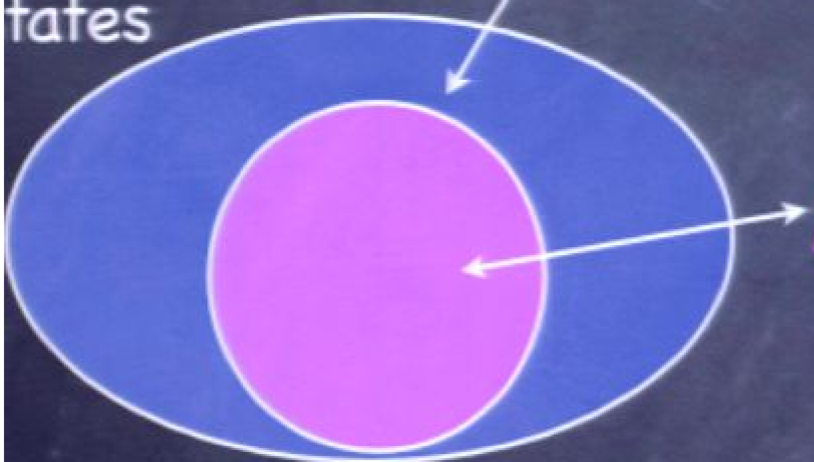

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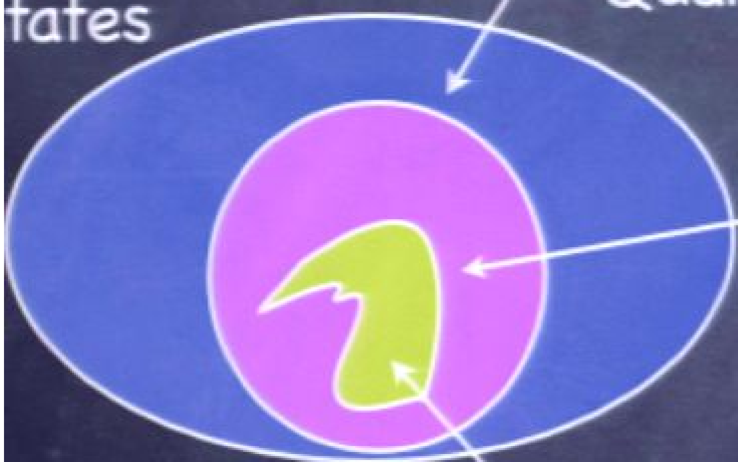

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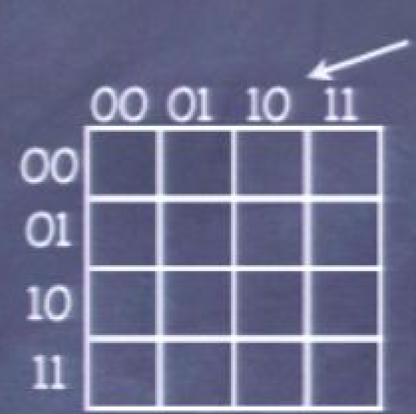

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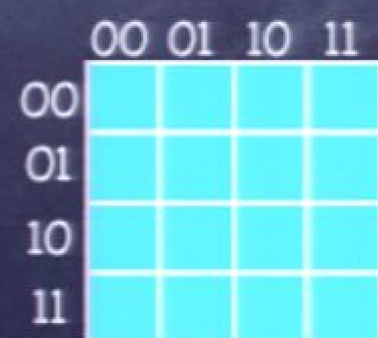

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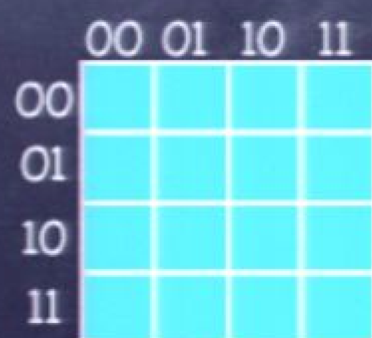

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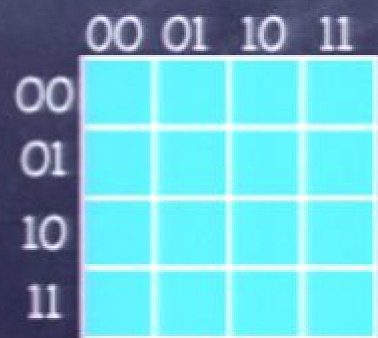

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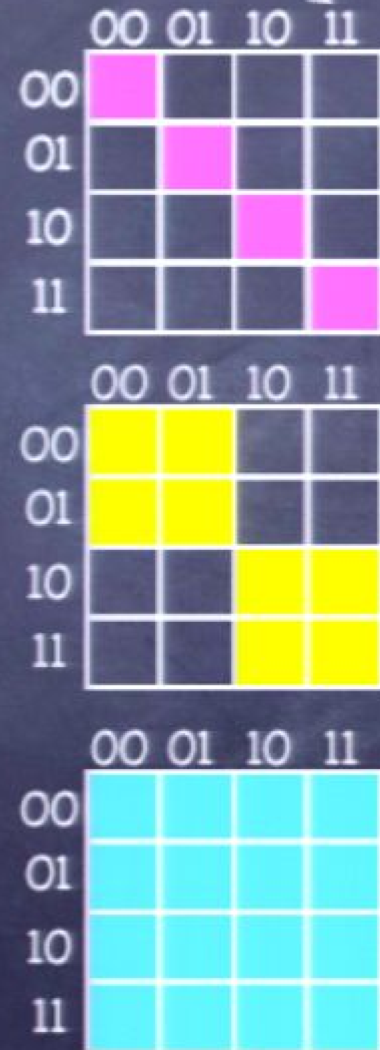

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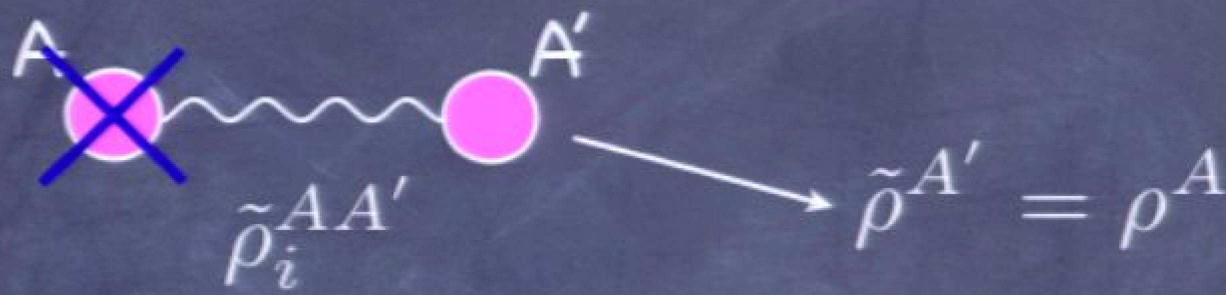

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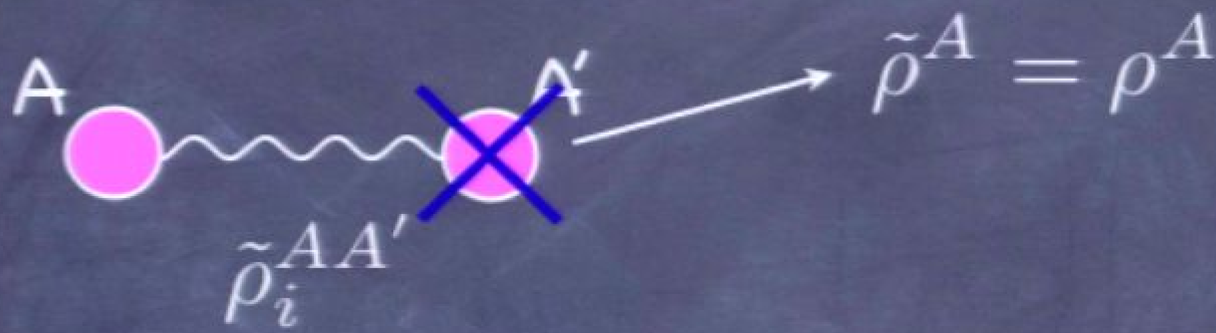

Standard broadcasting

Single system A, many states $\rho_1^A, \rho_2^A, \cdots$

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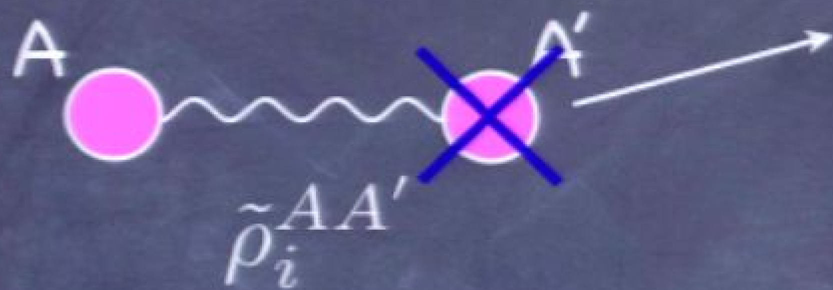

$$\tilde{\rho}^A = \rho^A$$

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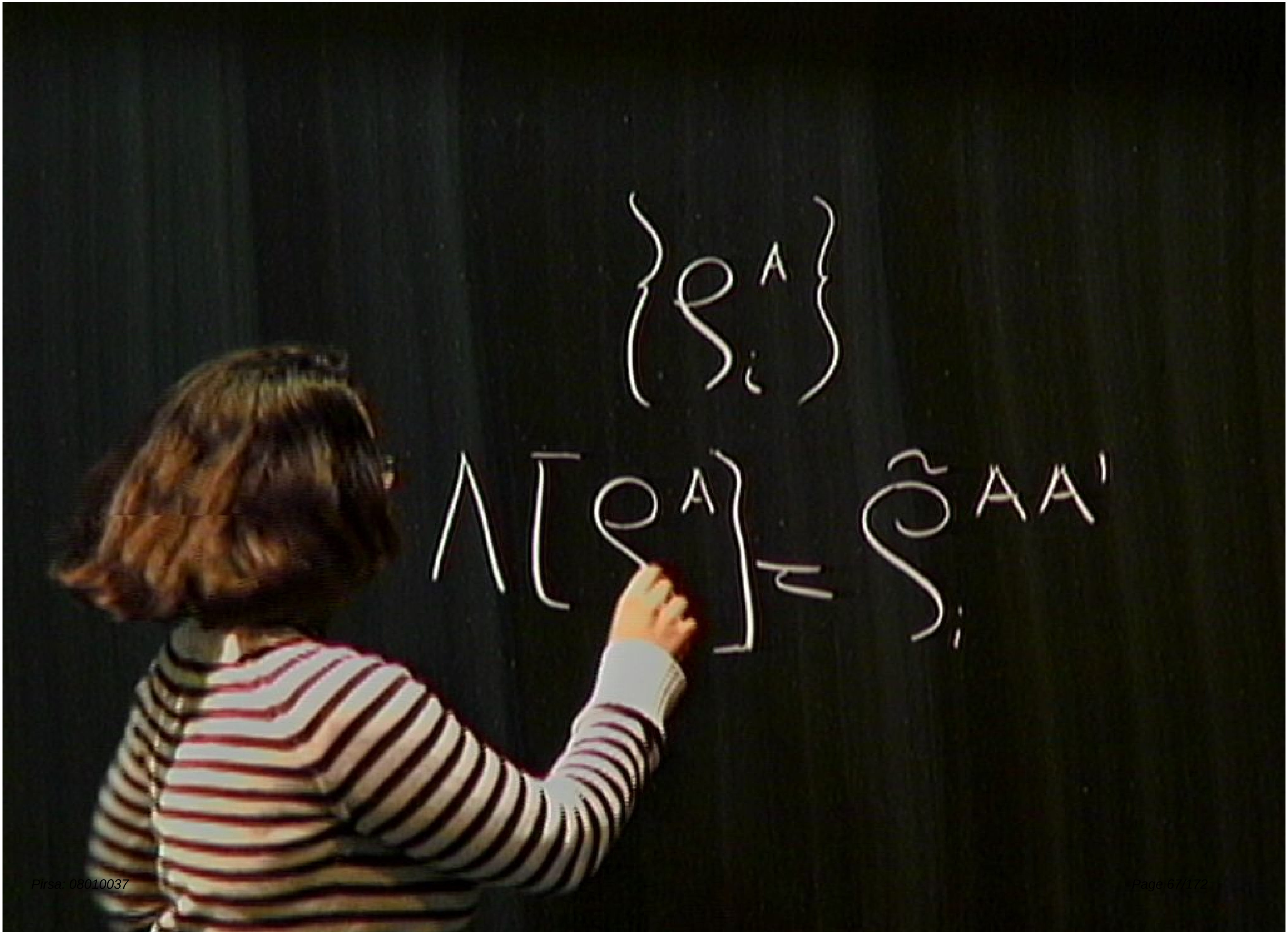
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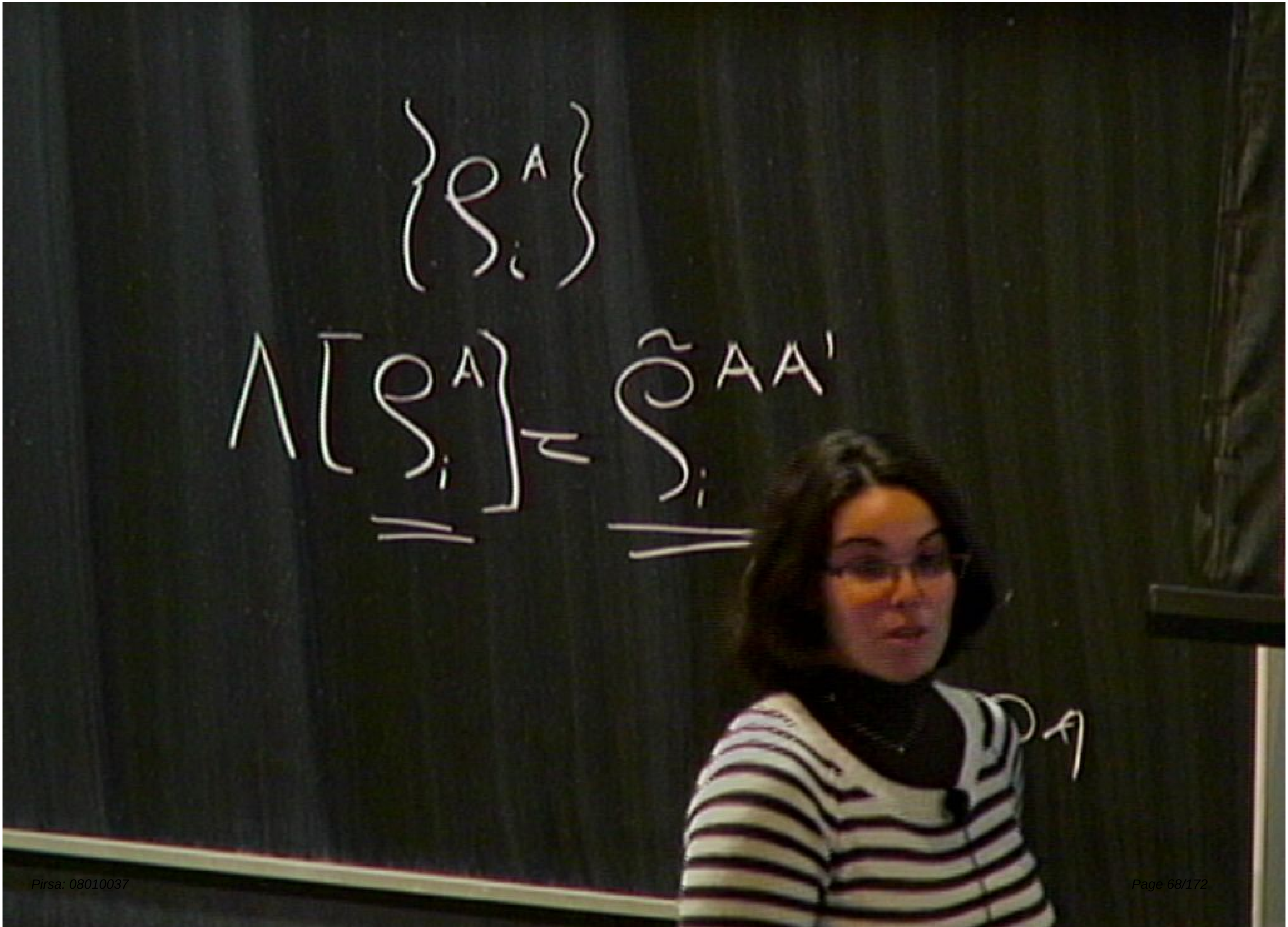
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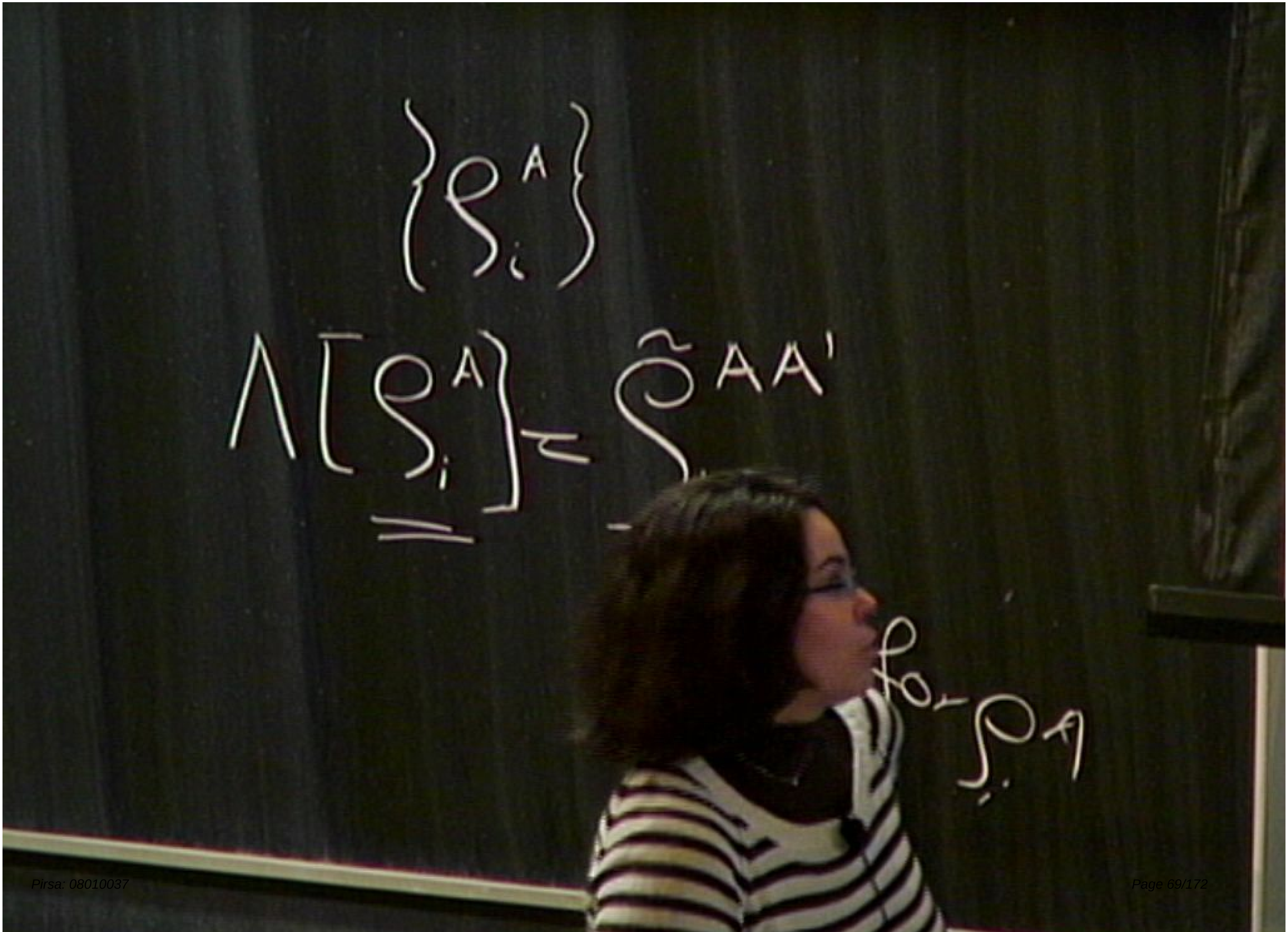
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No broadcasting theorem: [Barnum et al.] broadcasting of $\{\rho_i^A\}$ is possible iff $[\rho_i, \rho_j] = 0$ (classicality of set)

Local broadcasting

Two systems A and B, single state ρ^{AB}

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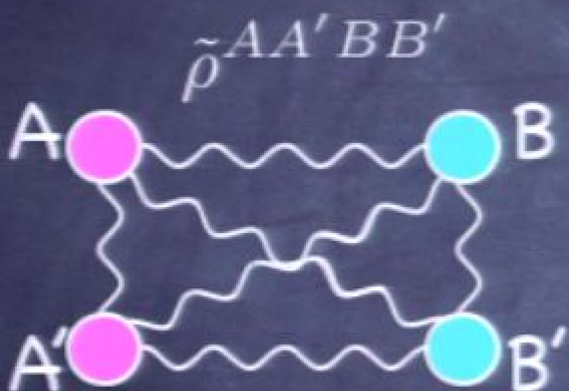

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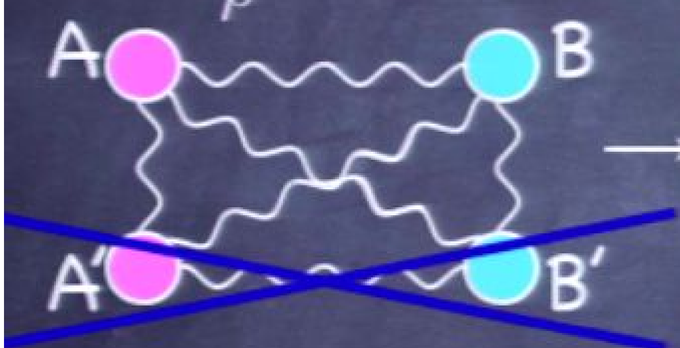

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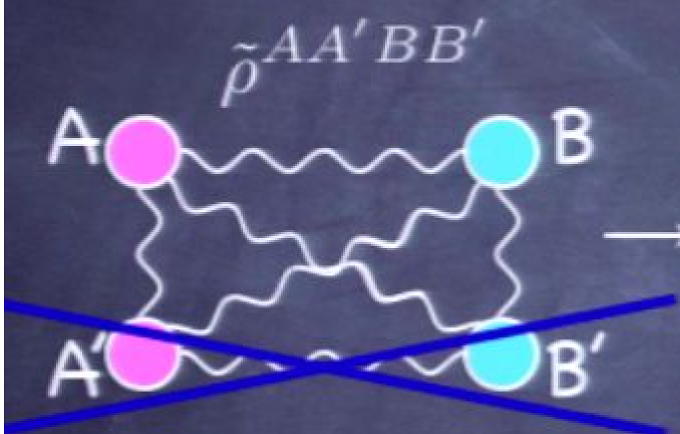

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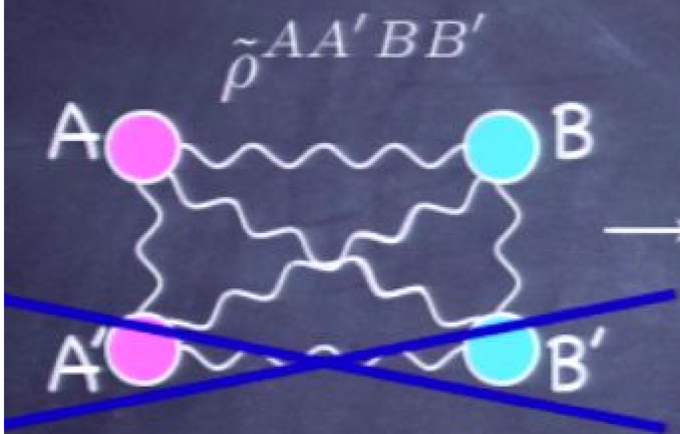

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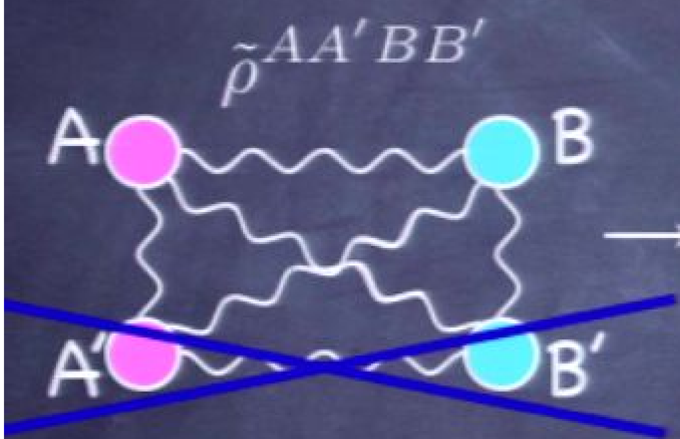

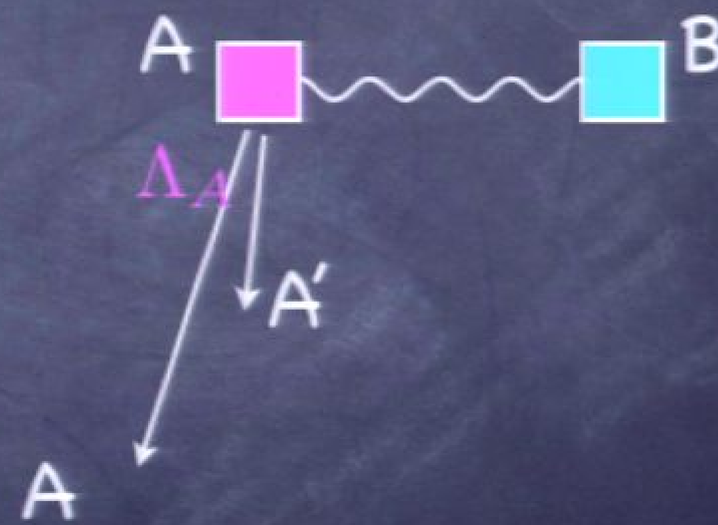

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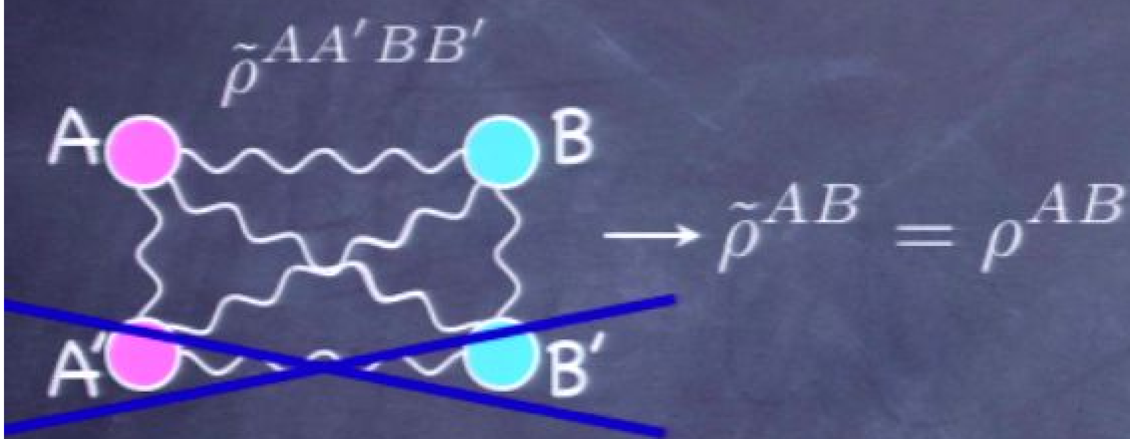

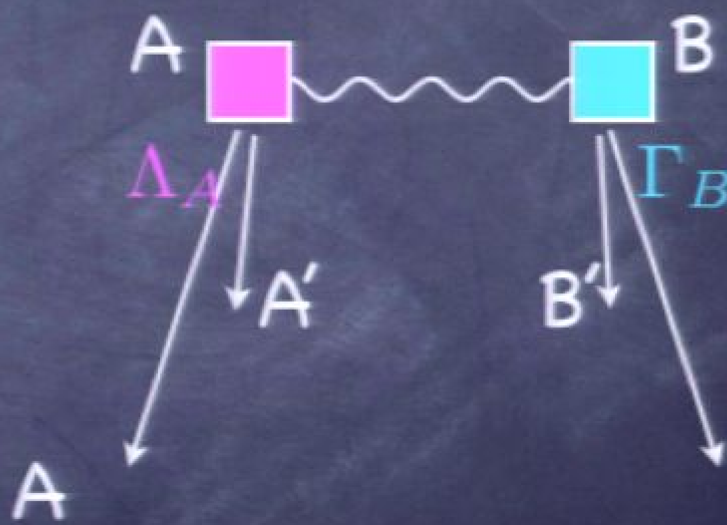

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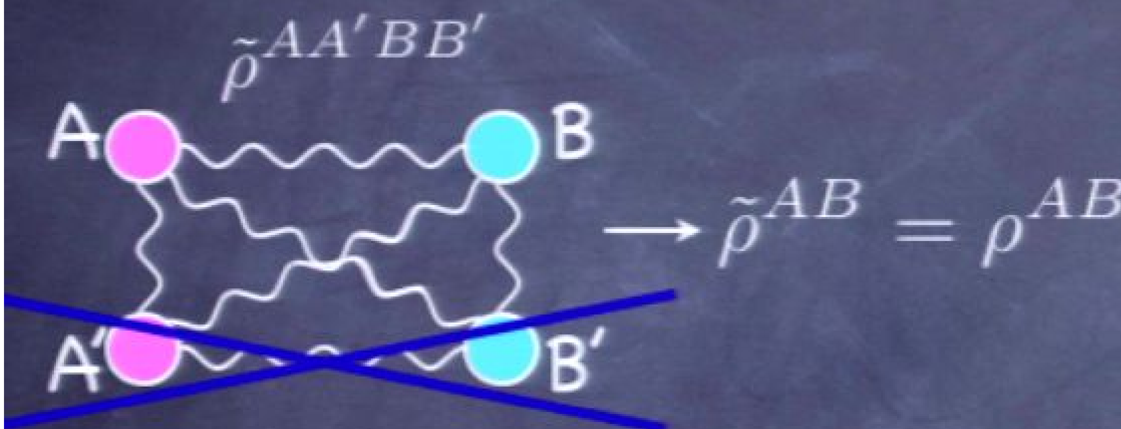

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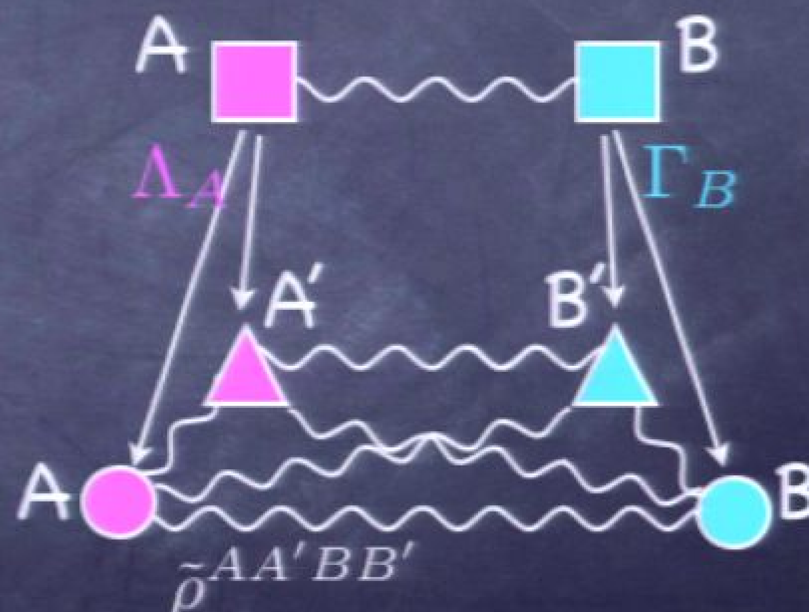

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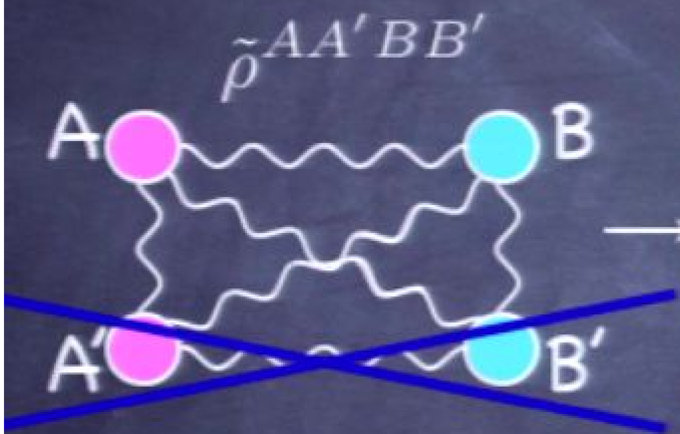

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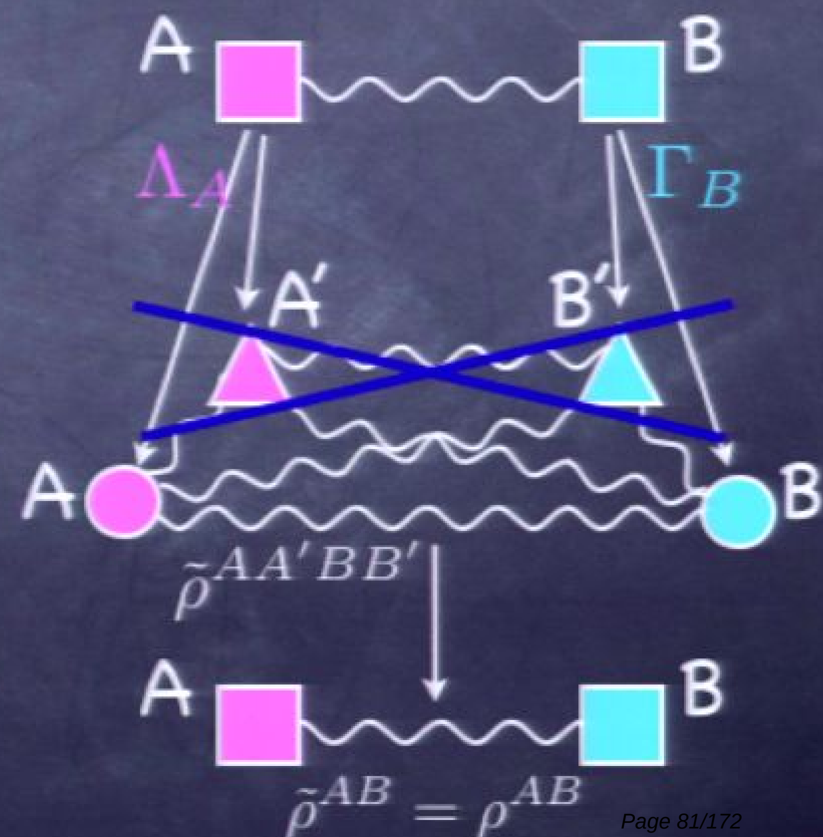

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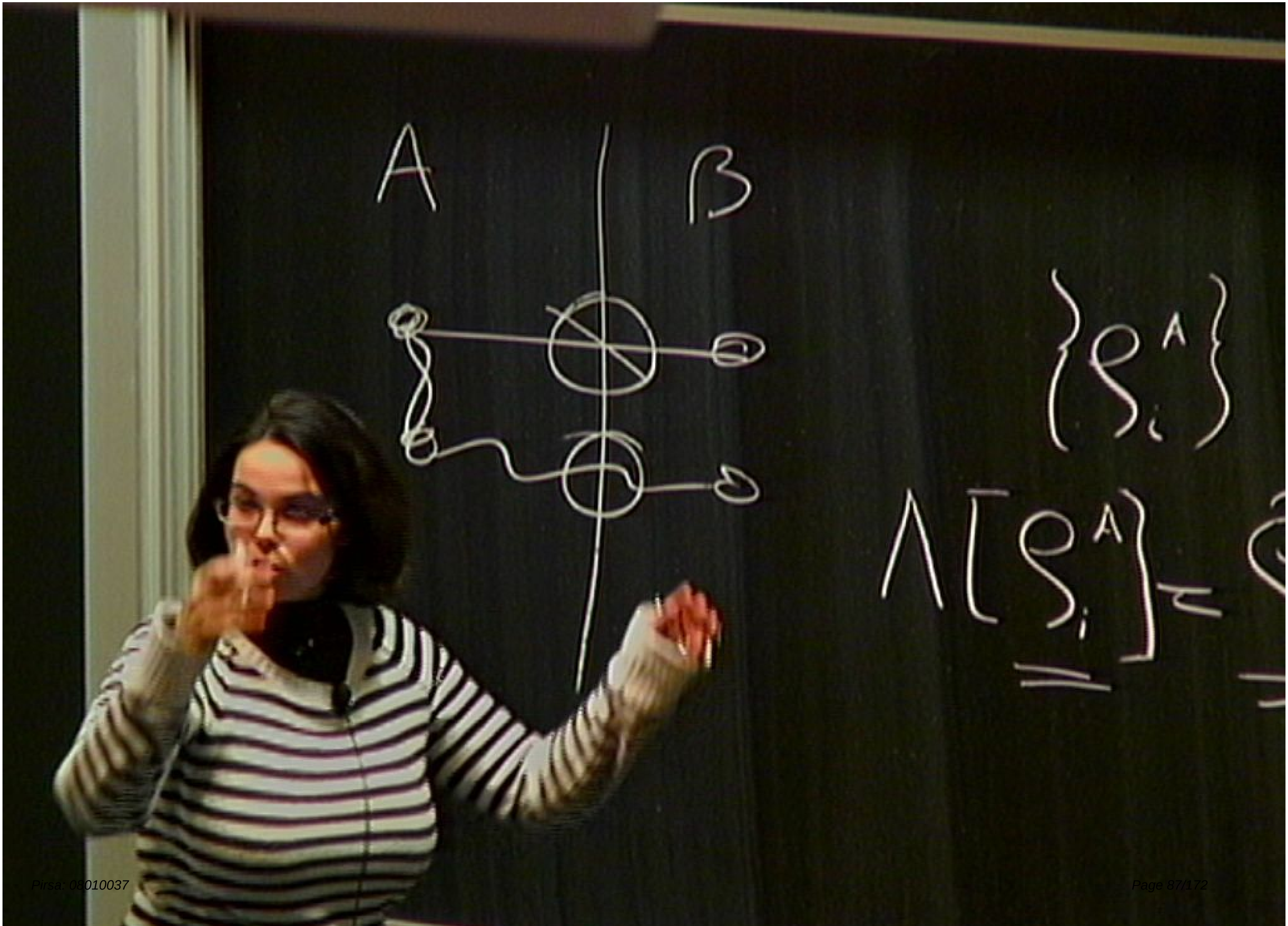
A
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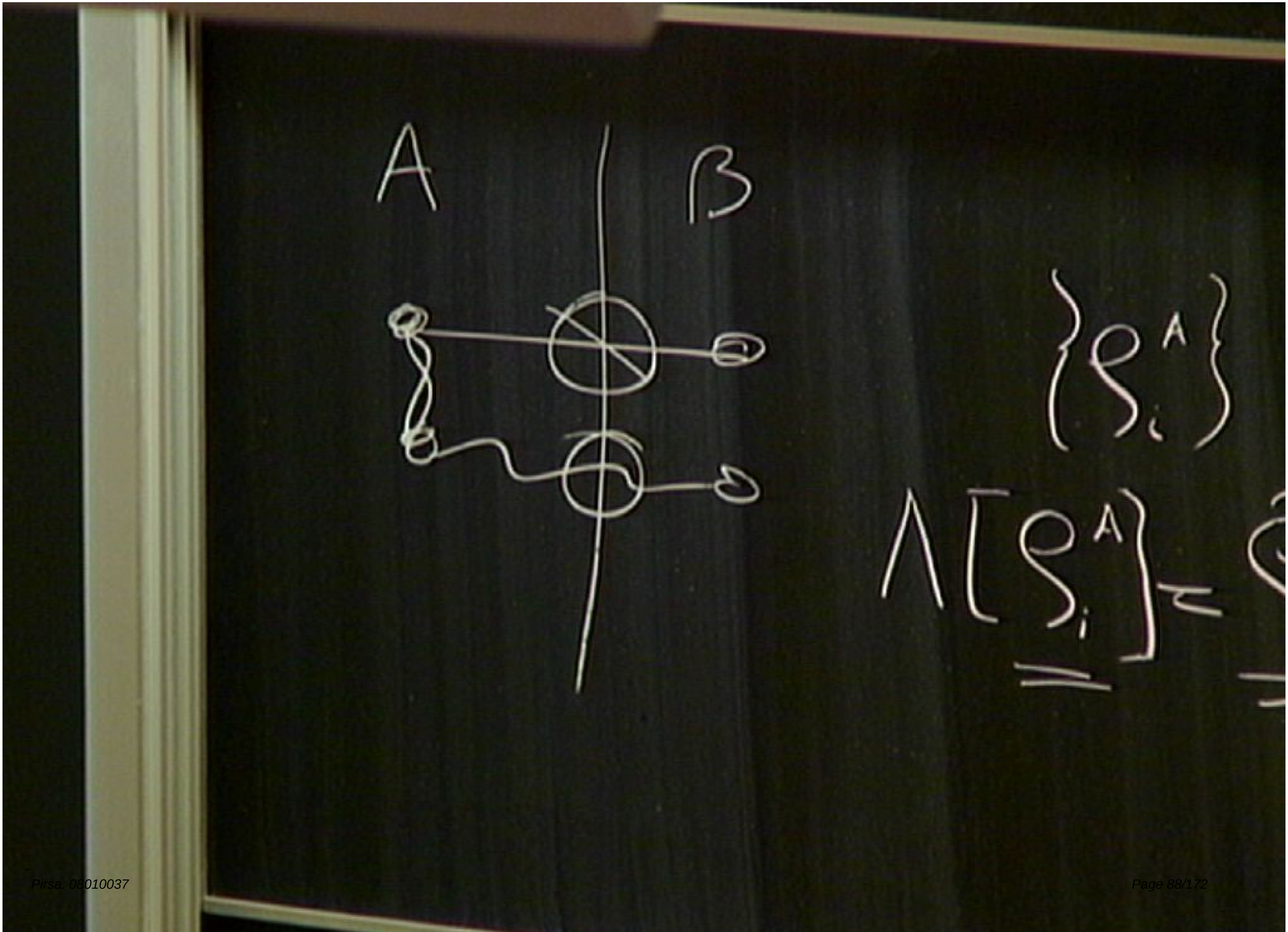
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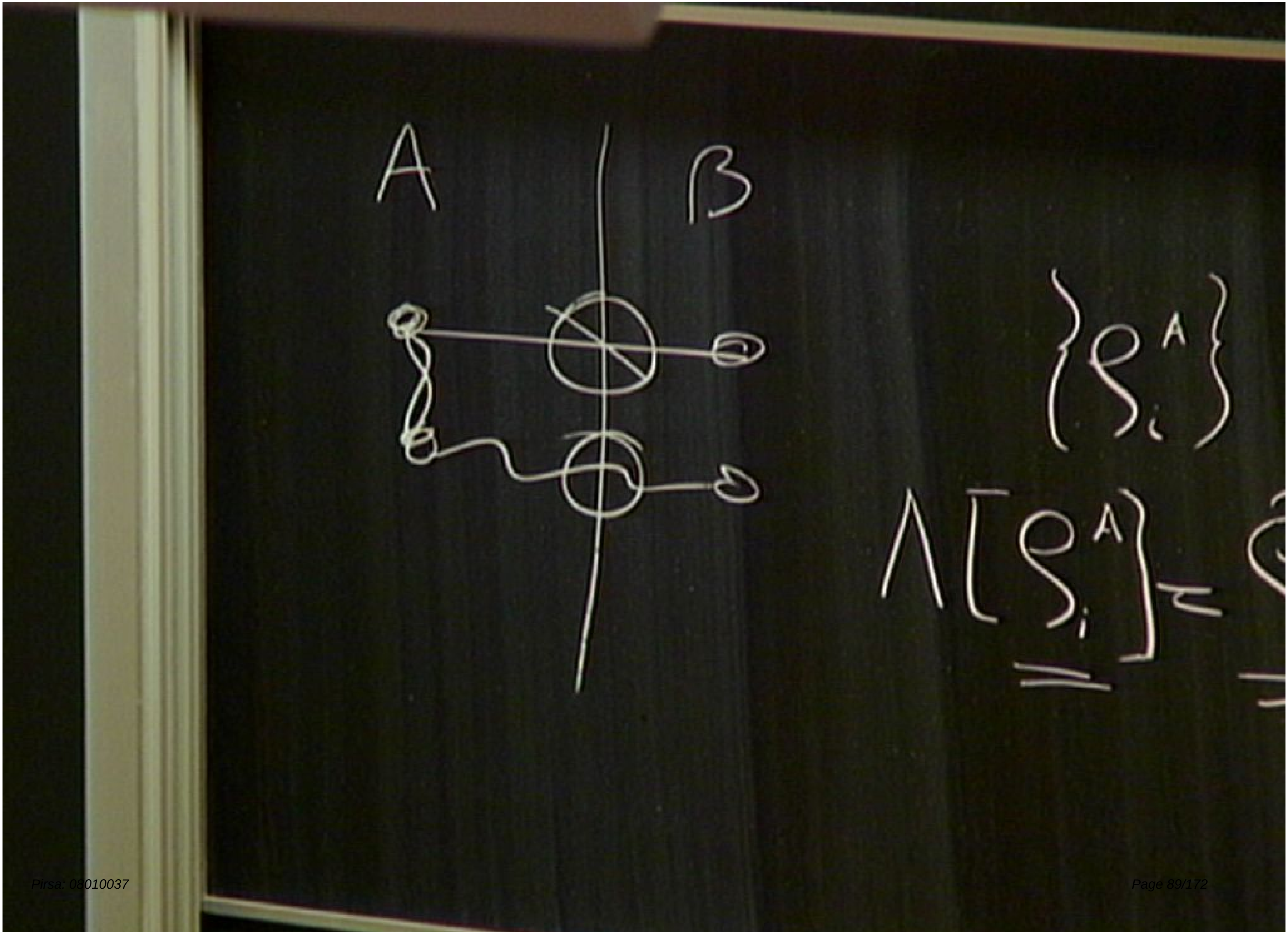
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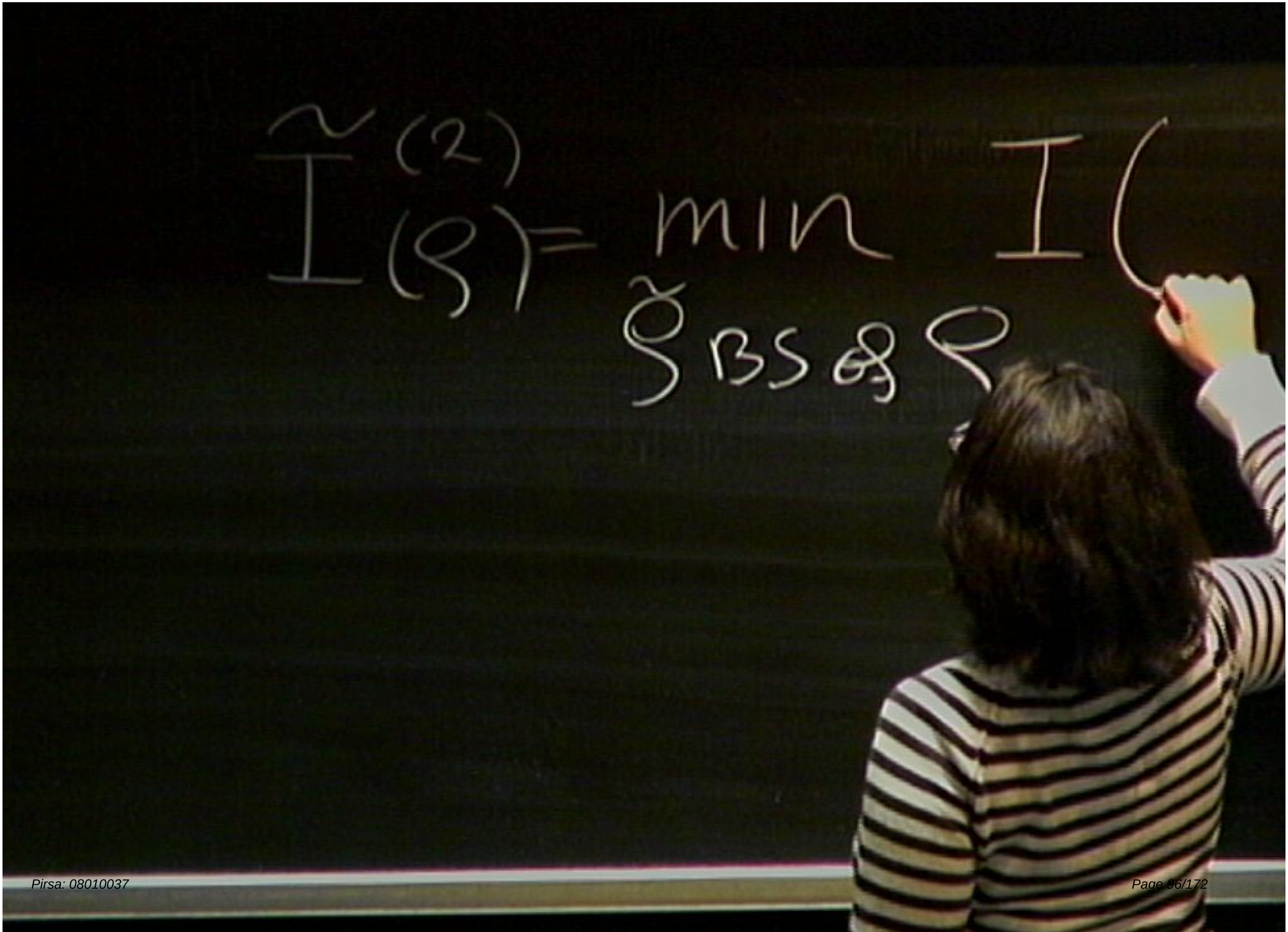
[redacted]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$$\sum_{ij} p_{ij} |i\rangle\langle i| \otimes |j\rangle\langle j| \rightarrow \sum_{ij} p_{ij} (\Lambda_i \otimes \Gamma_j)[|i\rangle\langle i| \otimes |j\rangle\langle j|] \rightarrow \sum_{ij} p_{ij} \rho_i \otimes \rho_j$$

$$\tilde{I}^{(2)}(\rho) = \min_{\tilde{\rho}_{BS} \otimes \rho} I(\tilde{\rho})$$

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$\rho^{\otimes 2}$

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Examples (I)

- CC states $\rho_{cc} = \sum_{ij} p_{ij} |i\rangle_A\langle i| \otimes |j\rangle_B\langle j|$

Possible broadcast state: clone local orthonormal basis

$$\tilde{I}^{(2)}(\rho_{CC}) \leq I\Big(\sum_{ij} p_{ij} |ii\rangle_{AA}\langle ii| \otimes |jj\rangle_{BB}\langle jj|\Big) = I(\rho_{CC})$$

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$\Rightarrow$ $\tilde{\Delta}^{(2)}(\rho_{\rm pure}) = I(\rho_{\rm pure})$ (maximal)

$E_F(\psi) = S(\mathrm{Tr}_B \psi)$

Examples (II)

- Uni-locally distinguishable mixtures

$$\rho^{AB} = \sum_i p_i |\psi_i\rangle\langle\psi_i| \text{ s.t. } \exists \{P_k^A\} : P_k^A |\psi_i\rangle = \delta_{ik} |\psi_k\rangle$$

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$$\tilde{\rho}_{\text{opt}} = \sum_{ij} q_{ij} |\psi_i\rangle_{AB}\langle\psi_i| \otimes |\psi_j\rangle_{A'B'}\langle\psi_j|$$

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$$\Rightarrow \quad \tilde{I}^{(2)}(\rho) = 2E_F(\rho) + \min_{\tilde{\rho}_{\text{opt}}} S(\tilde{\rho}_{B_1 B_2})$$

Examples (IIa)

A special case is given by states (on $\mathbb{C}^4 \otimes \mathbb{C}^2$)

$$\rho = \frac{1}{2}|\psi_0\rangle\langle\psi_0| + \frac{1}{2}|\psi_1\rangle\langle\psi_1|$$

Examples (II)

- Uni-locally distinguishable mixtures

$$\rho^{AB} = \sum_i p_i |\psi_i\rangle\langle\psi_i| \text{ s.t. } \exists \{P_k^A\} : P_k^A |\psi_i\rangle = \delta_{ik} |\psi_k\rangle$$

$$\Rightarrow \quad I(\rho) = E_F(\rho) + S(\rho_B)$$

decomposition is optimal!!!

Where $E_F(\rho) = \min_{\{q_k,\phi_k\}} \sum_k q_k E_F(\phi_k) = \sum_i p_i E_F(\psi_i)$

We know the form of the optimal broadcast state!

$$\tilde{\rho}_{\rm opt} = \sum_{ij} q_{ij} |\psi_i\rangle_{AB}\langle\psi_i| \otimes |\psi_j\rangle_{A'B'}\langle\psi_j|$$

$$\Rightarrow \quad \tilde{I}^{(2)}(\rho) = 2E_F(\rho) + \min_{\tilde{\rho}_{\rm opt}} S(\tilde{\rho}_{B_1 B_2})$$

Examples (IIa)

A special case is given by states (on $\mathbb{C}^4 \otimes \mathbb{C}^2$)

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Examples (IIb: plots)

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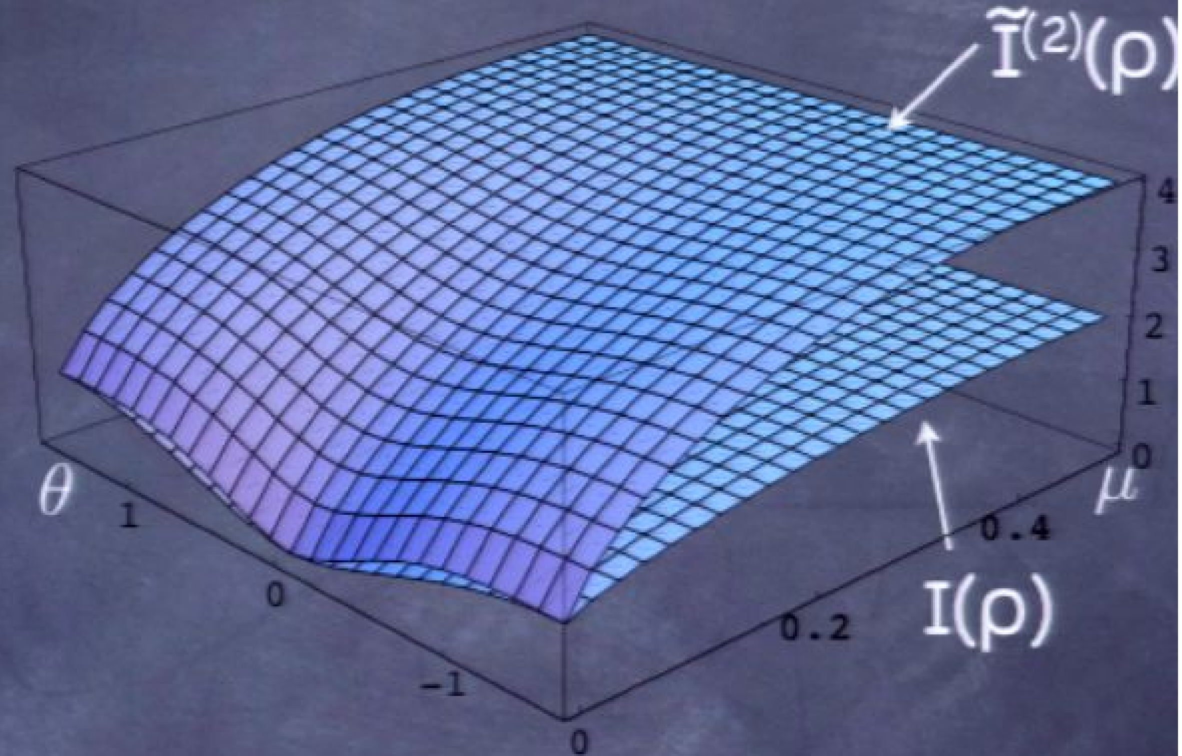

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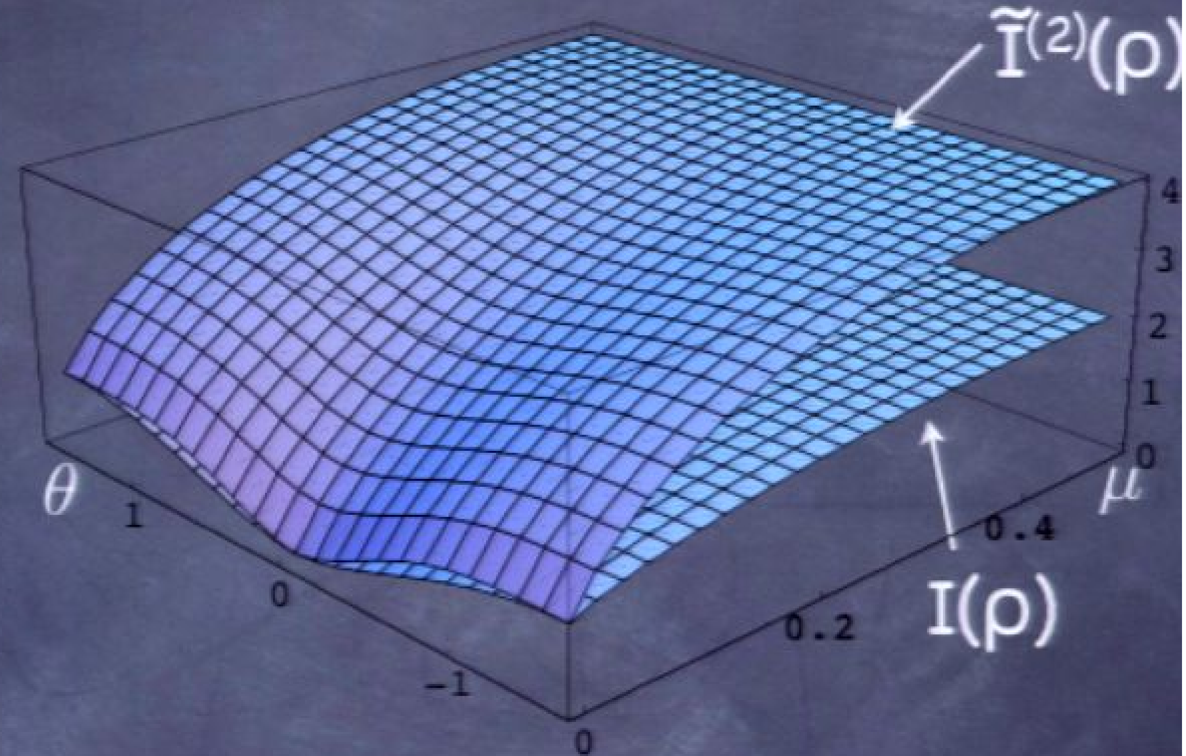

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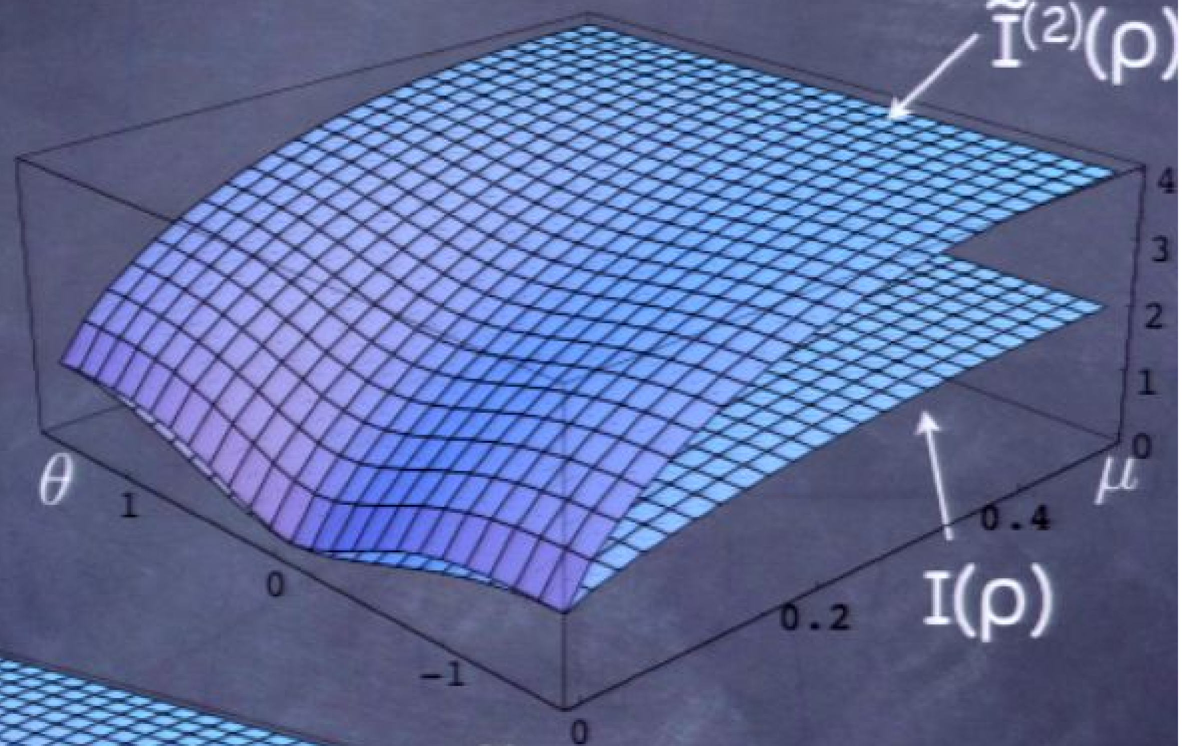
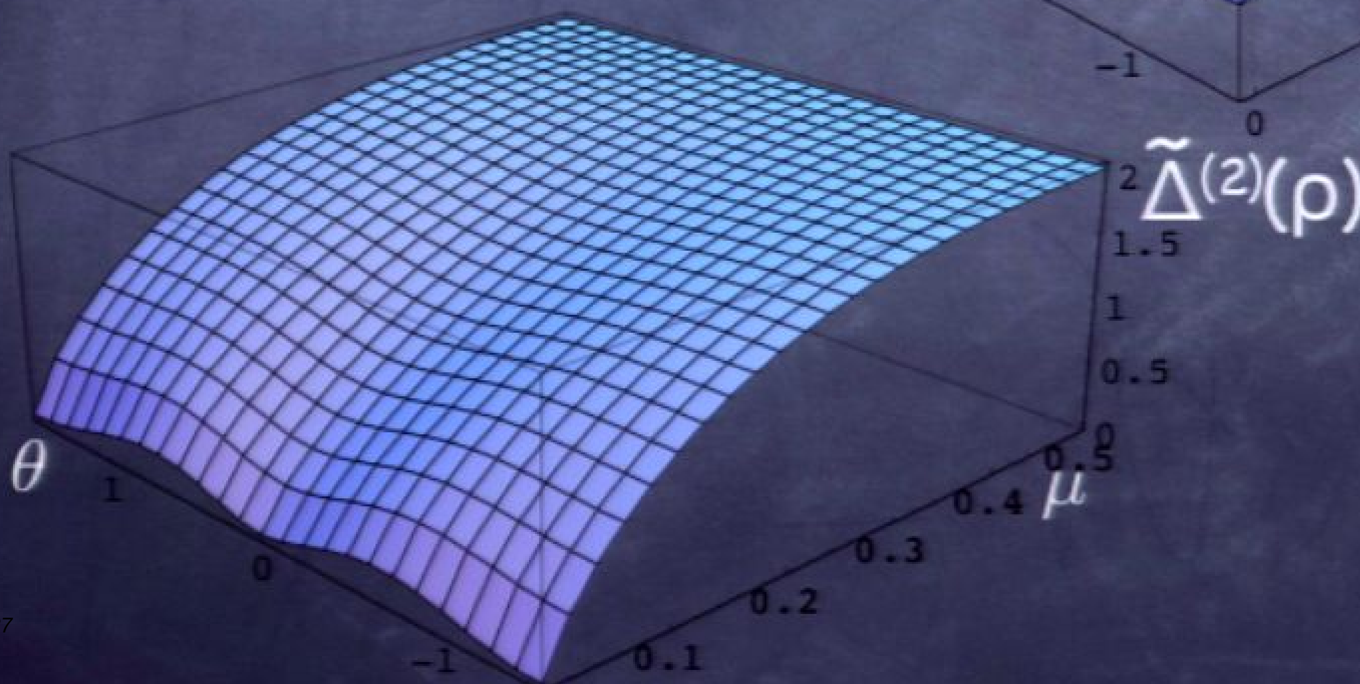
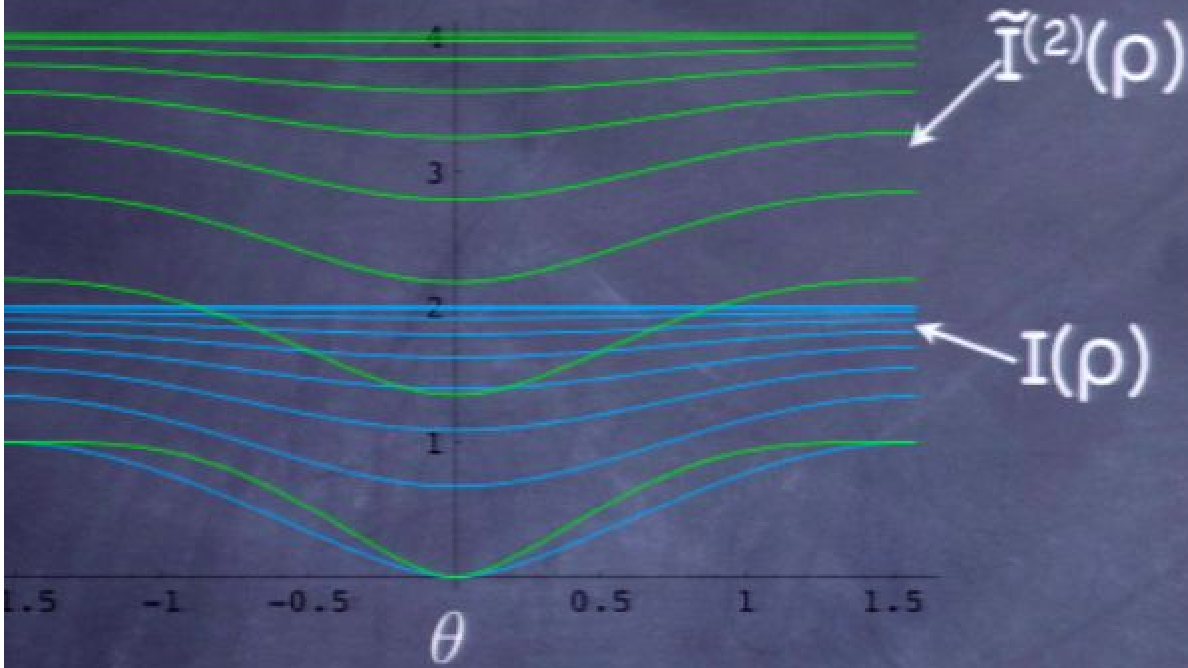




Examples (IIc: sections)



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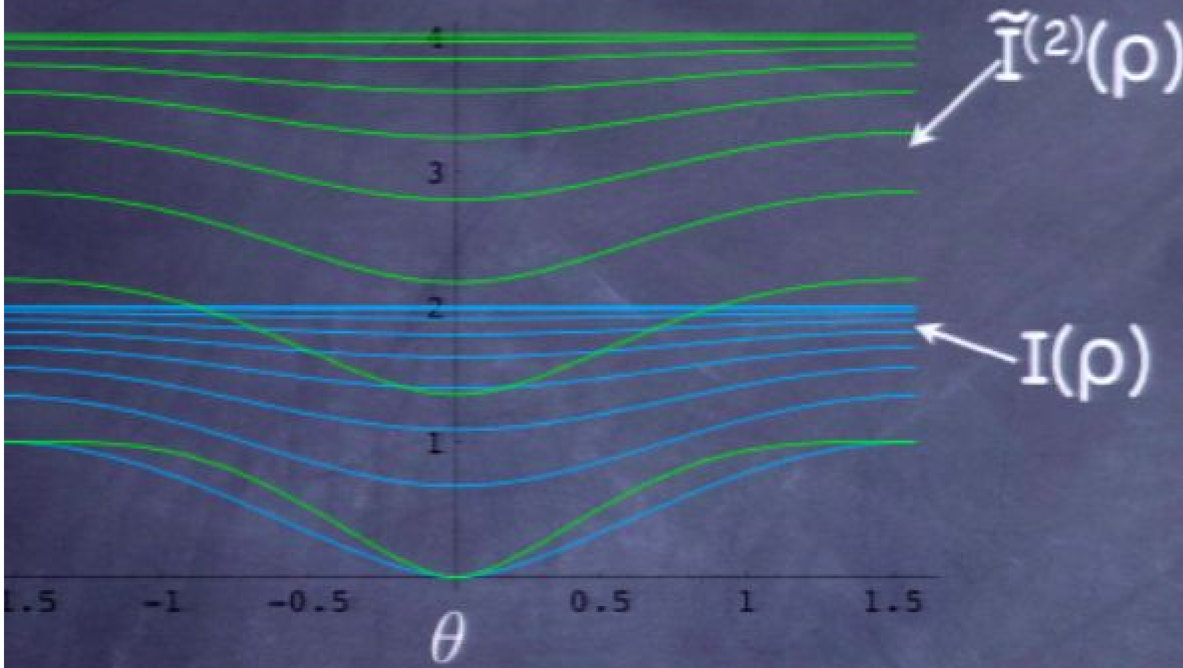

As μ grows (state becomes more entangled) the amount of correlations become independent of θ

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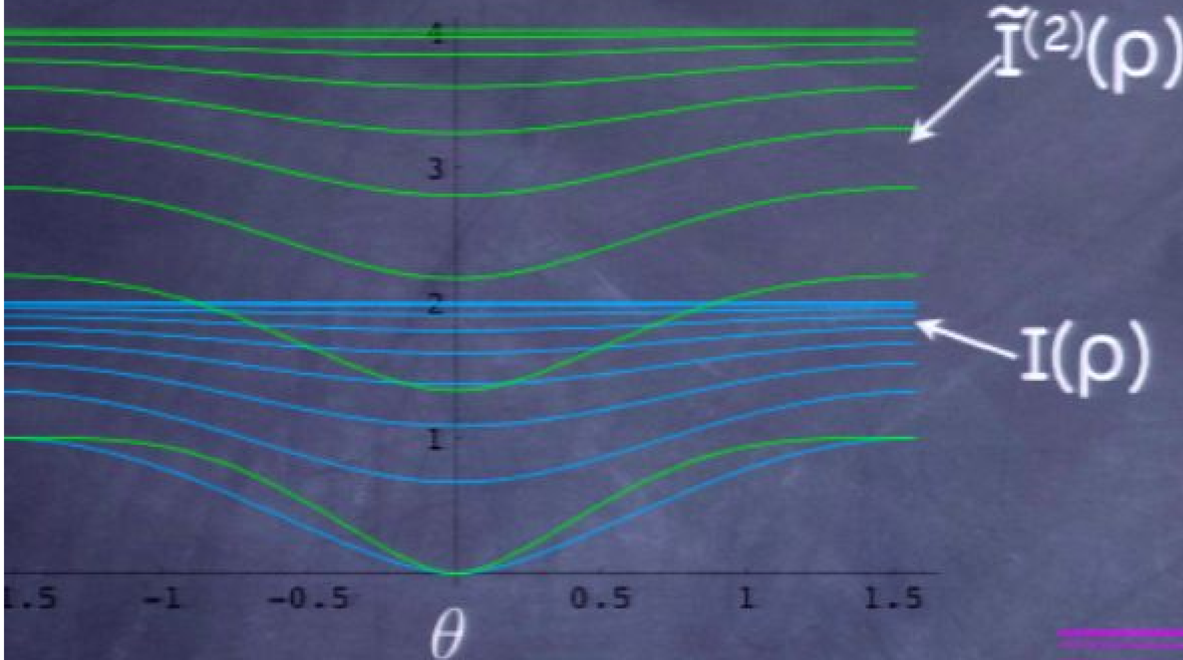

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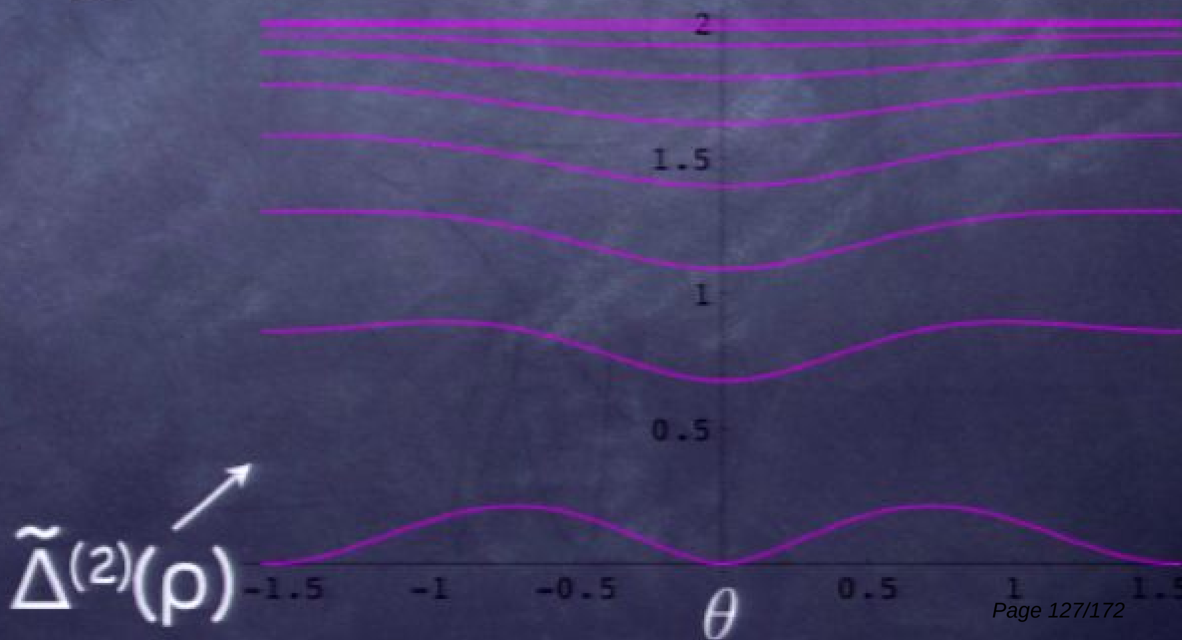

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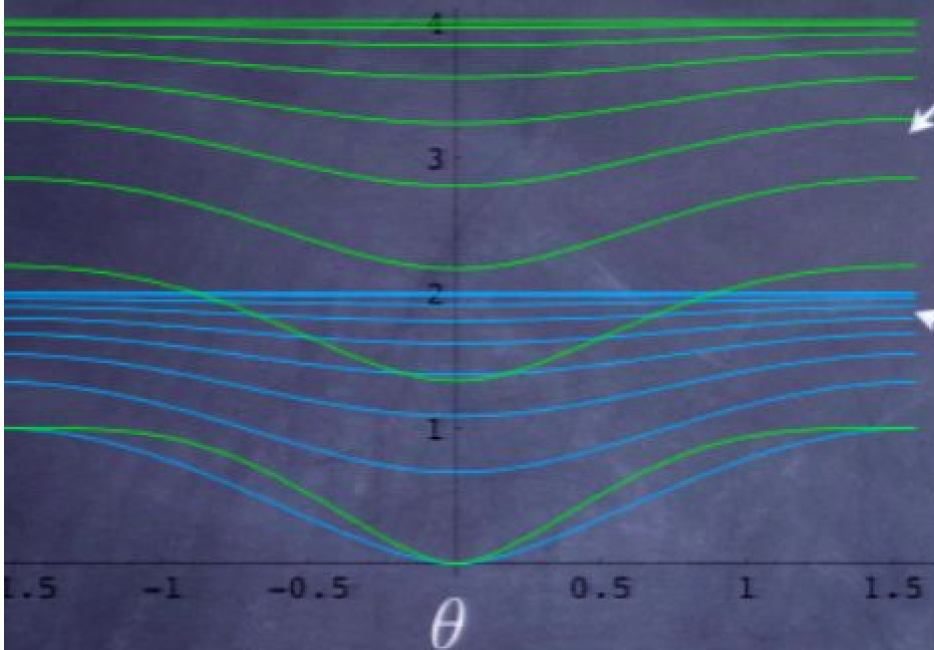

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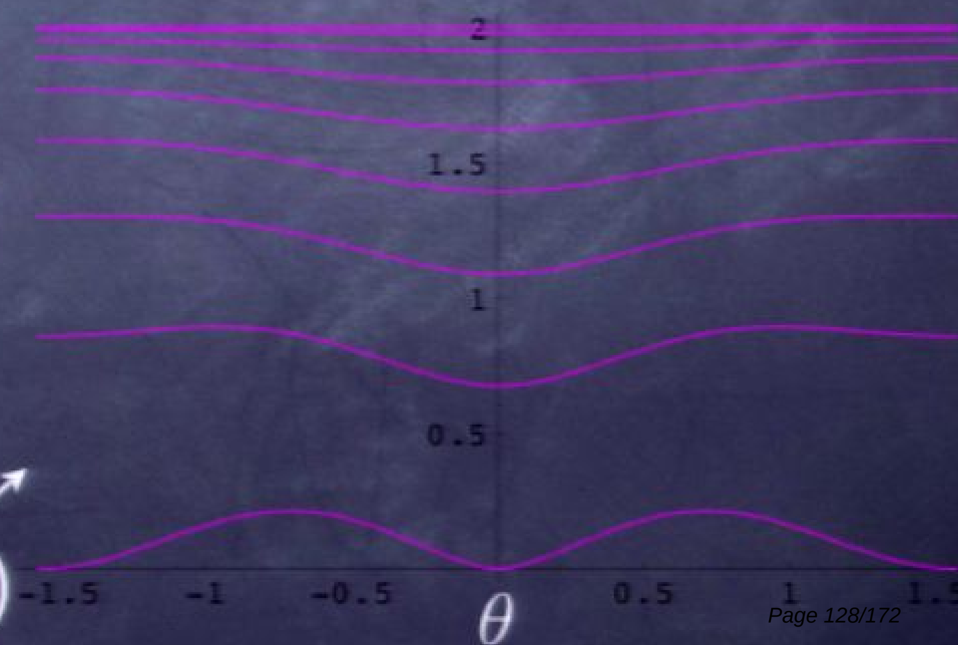
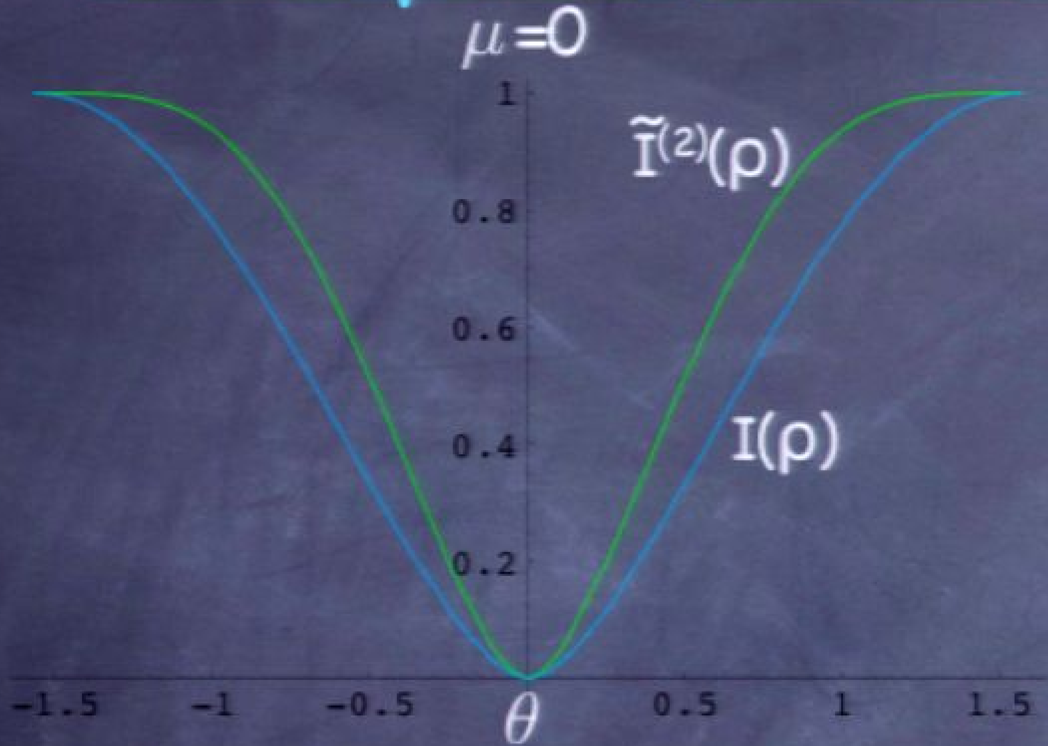
As μ grows the state behaves more like a pure state $\tilde{I}^{(2)}(\rho) \to 2I(\rho)$



Examples (IId: separable cut)



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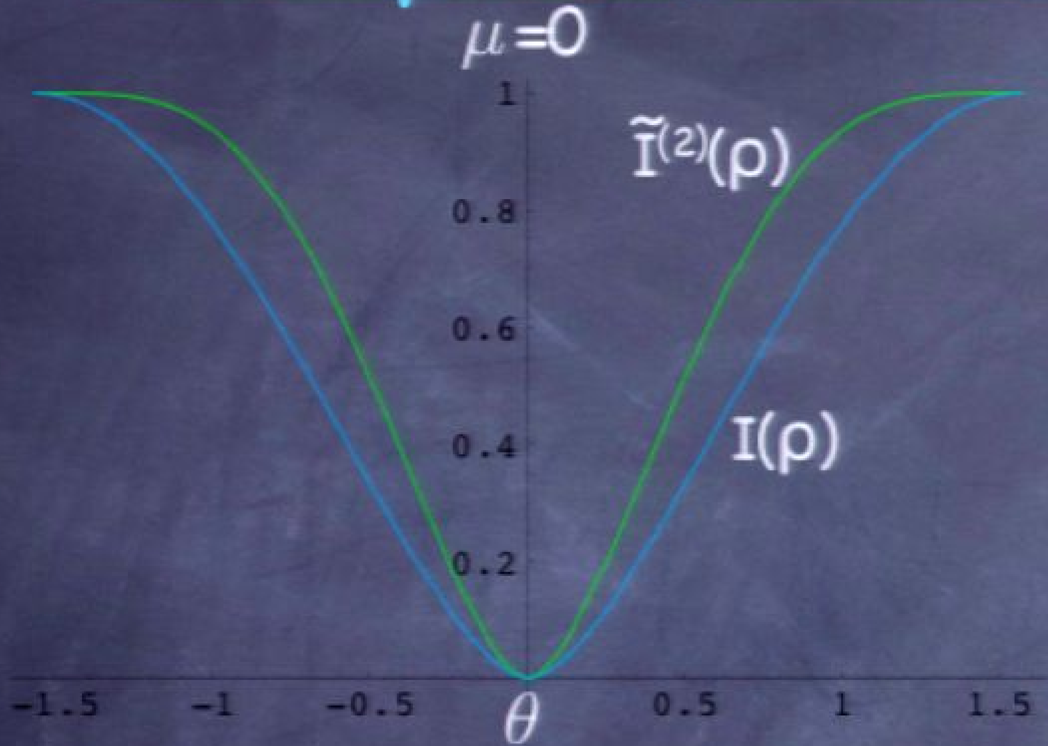

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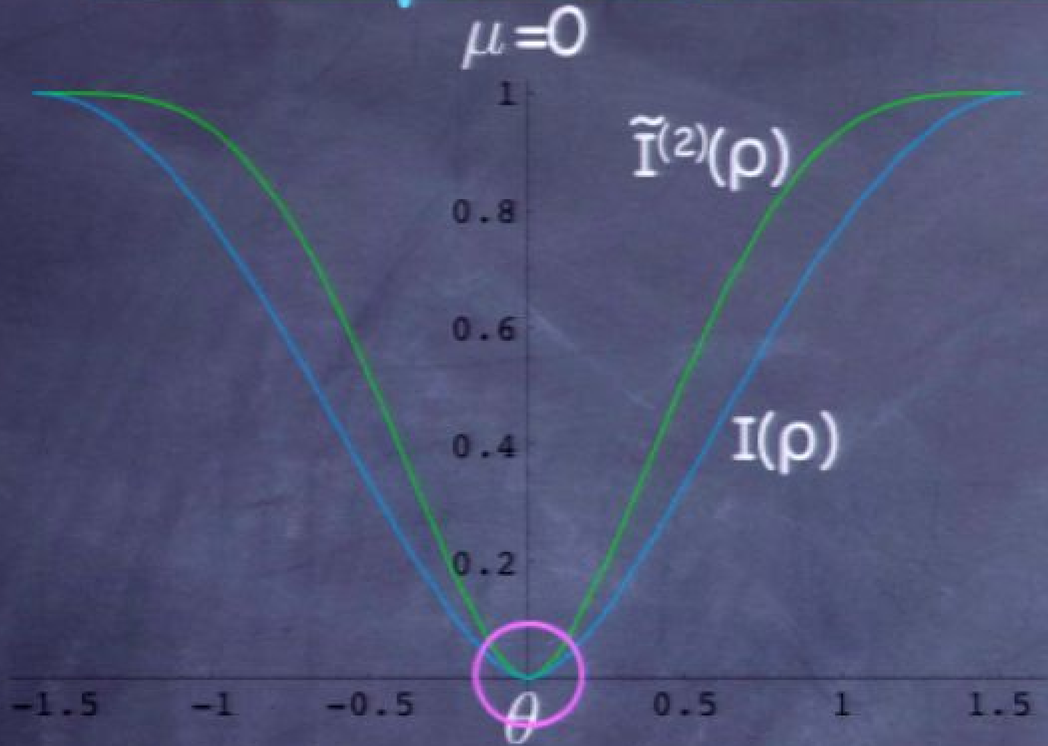

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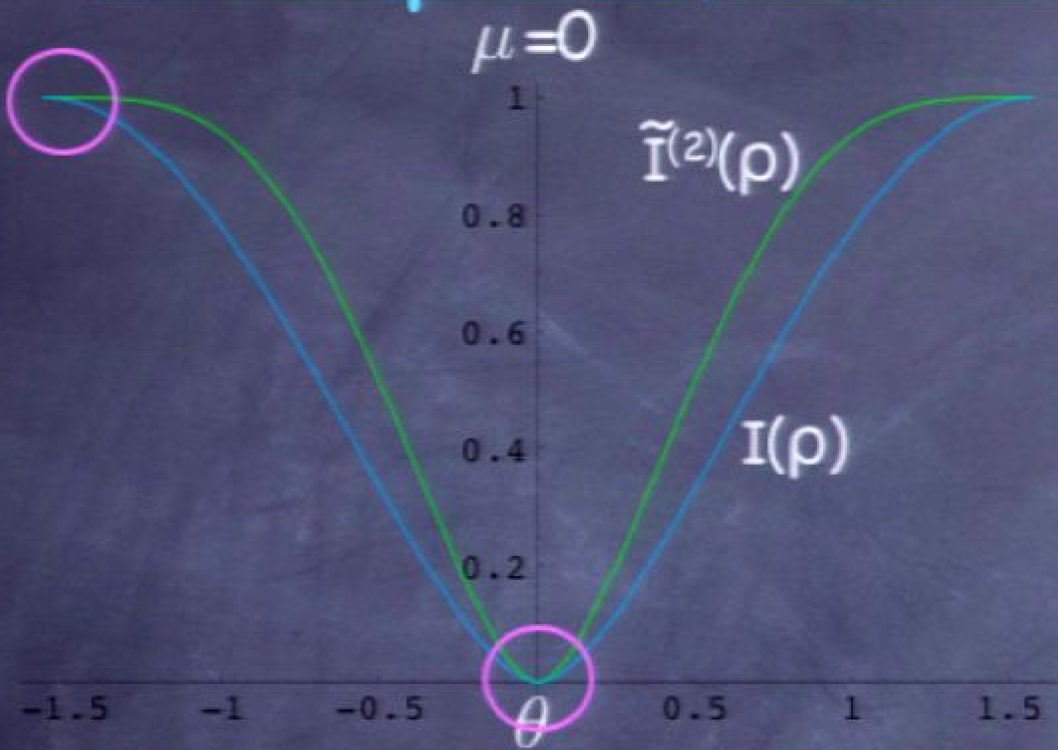

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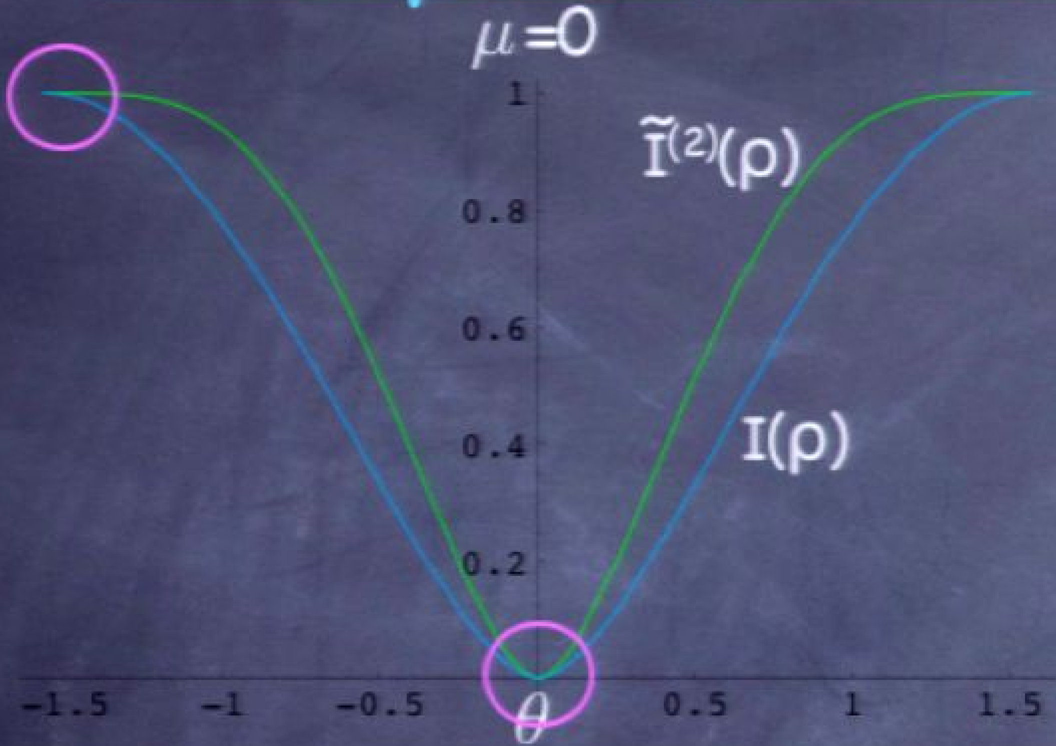

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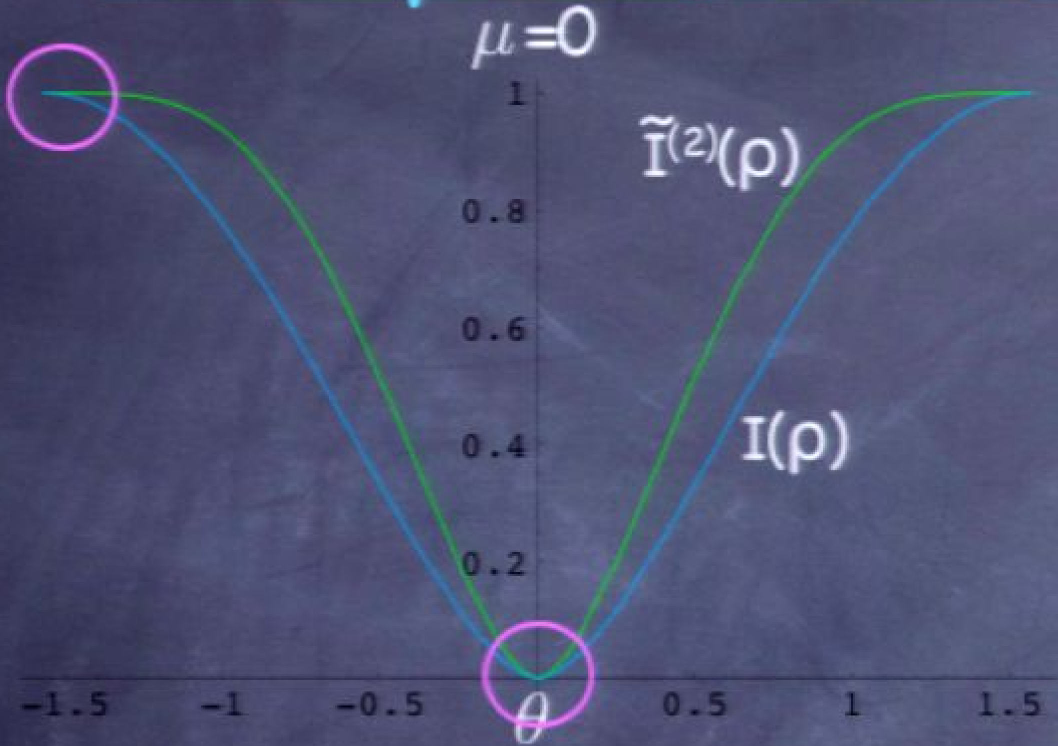

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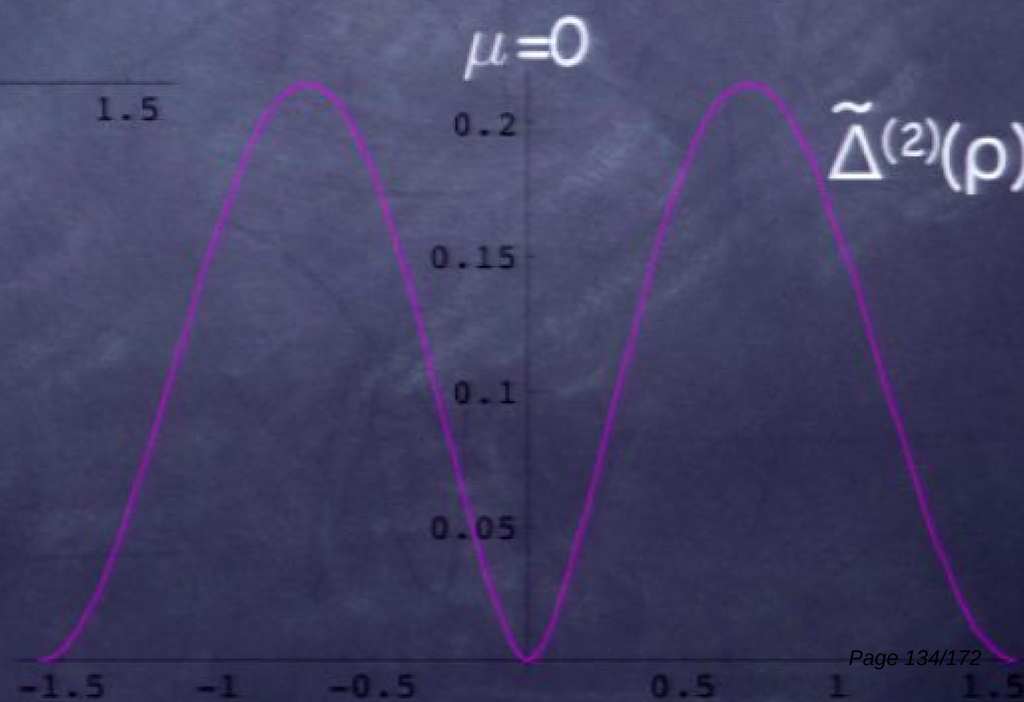

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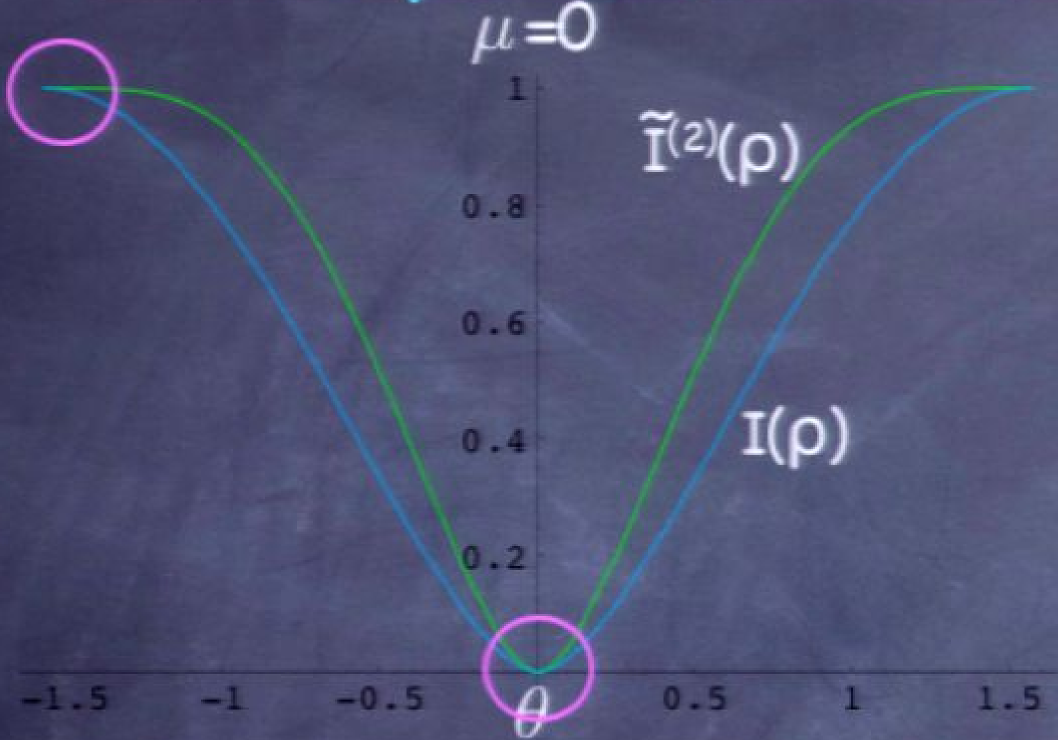

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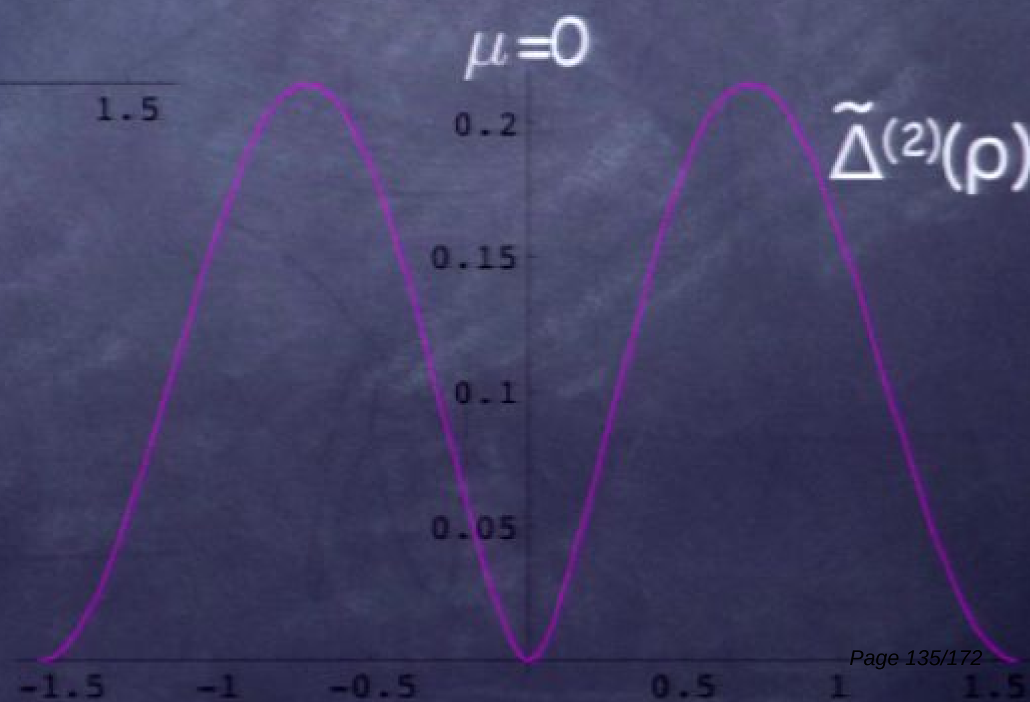
Along this cut the state is separable, yet it is still non-classical

$\tilde{\Delta}^{(2)}(\rho) > 0$



Why just two copies?

An n-copy broadcast state is

$$\tilde{\rho}^{(n)}_{A_1 B_1 \ldots A_n B_n} \text{ such that } \mathrm{Tr}_{\setminus\{A_i B_i\}}(\tilde{\rho}^{(n)}) = \rho$$

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$$\tilde{I}^{(n)}(\rho)=\min_{\substack{\tilde{\rho}^{(n)}\text{ BS}\\ \text{of } \rho}} I(\tilde{\rho}^{(n)}) \quad \text{and} \quad \tilde{\Delta}^{(n)}(\rho)=\tilde{I}^{(n)}(\rho)-I(\rho)$$

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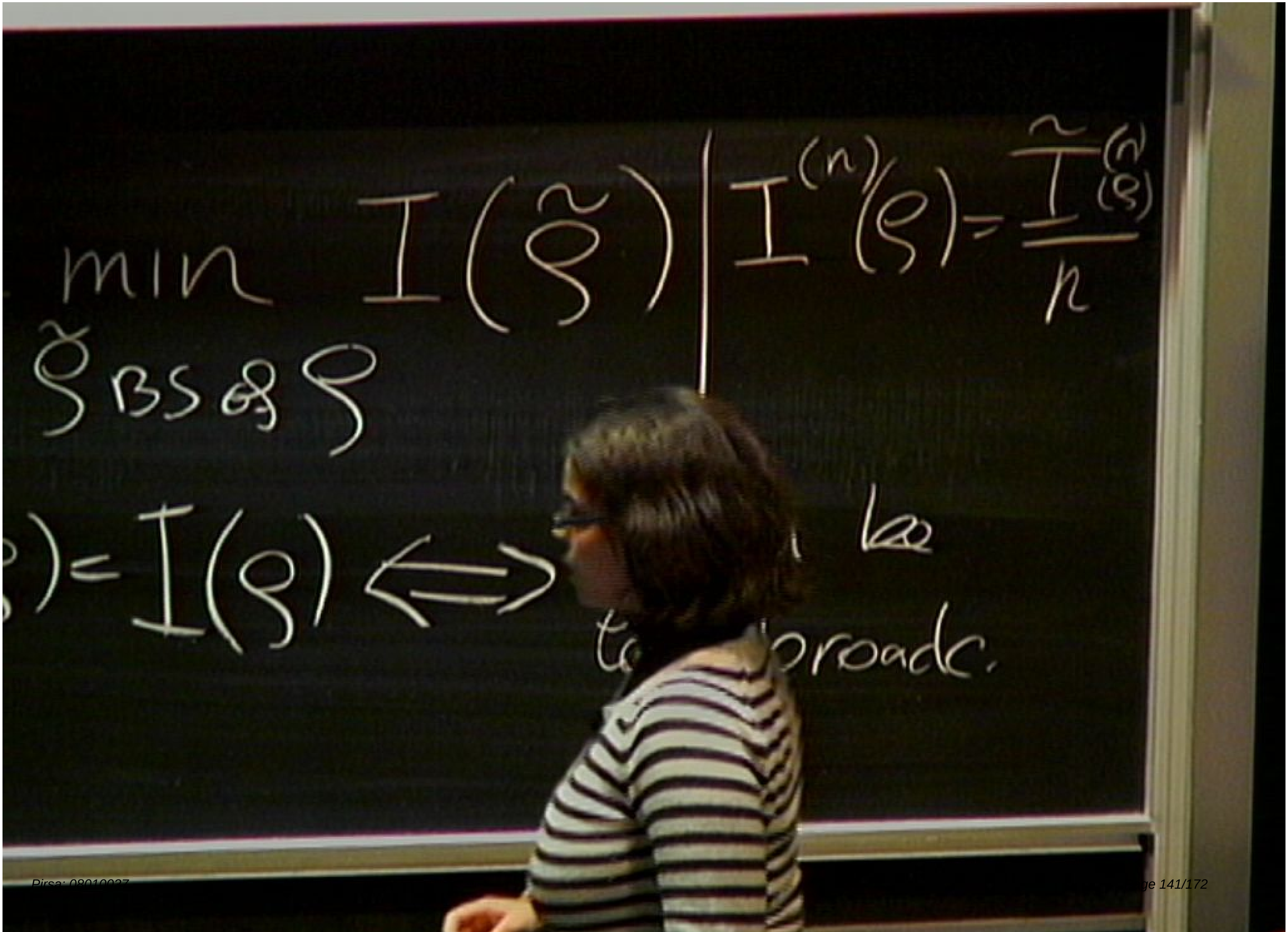
⇨ We consider the regularized versions:

$$I^{(n)}(\rho)=\frac{1}{n}\tilde{I}^{(n)}(\rho) \quad \text{and} \quad \Delta^{(n)}(\rho)=I^{(n)}(\rho)-\frac{1}{n}I(\rho)$$


min

Asymptotic average MI

We are led to consider the asymptotic case:

$$\Delta^{(\infty)}(\rho) = \lim_{n\to\infty}\left(I^{(n)}(\rho) - \frac{1}{n} I(\rho)\right) = \lim_n I^{(n)}(\rho) = I^{(\infty)}(\rho)$$

Entanglement measures

Any entanglement measure E should satisfy the following properties:

- ☐ Vanishing on separable states: $E(\rho_{sep})=0$
- ☐ "Weak" monotonicity under LOCC: $E(\Lambda_{LOCC}[\rho]) \leq E(\rho)$
- ☐ Convexity: $E(\Sigma_k p_k \rho_k) \leq \Sigma_k p_k E(\rho_k)$

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We have seen that $I^{(\infty)}$ satisfies the first one...what about the others?

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Note: MI is neither convex nor concave

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$I^{(\infty)}$ lies between two known entanglement measures:

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⚠ Unknown if $E_{sq}^{Q}=E_{sq}^{C}$! ⇨ Problem solved if $I^{(\infty)}$ is NOT a measure!

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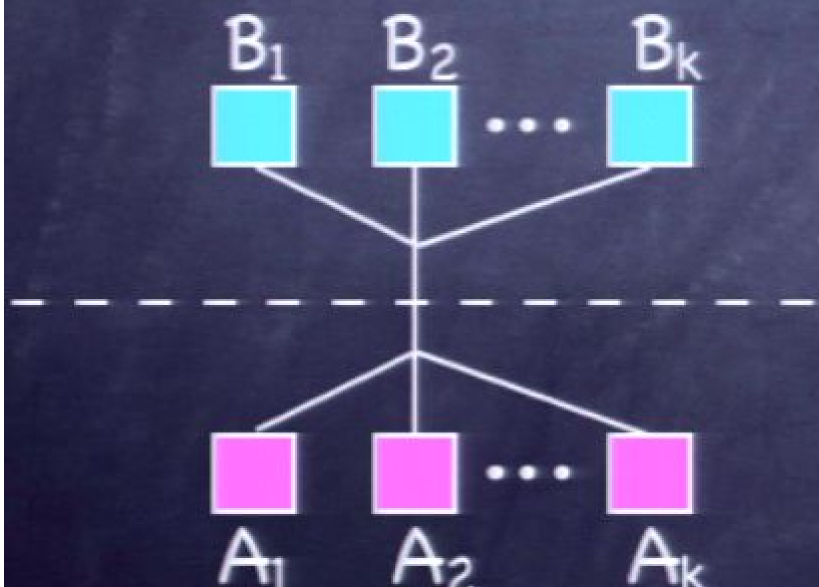

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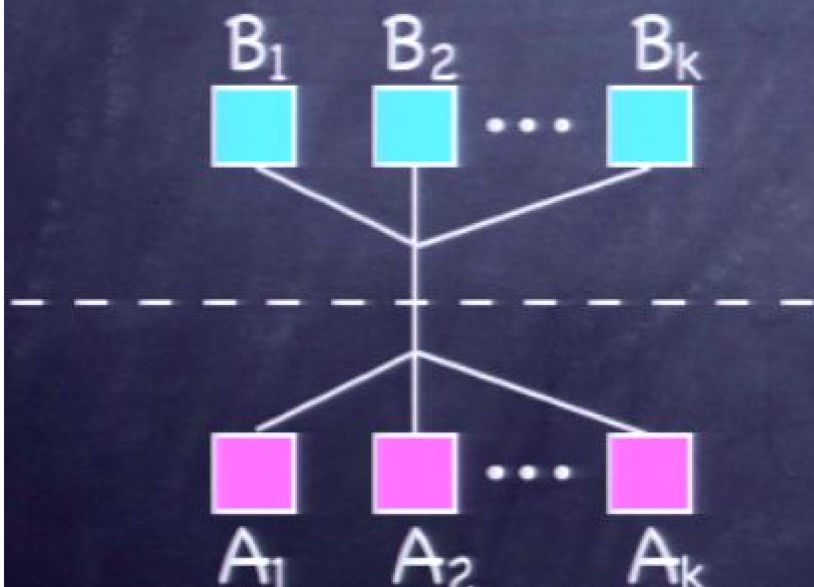

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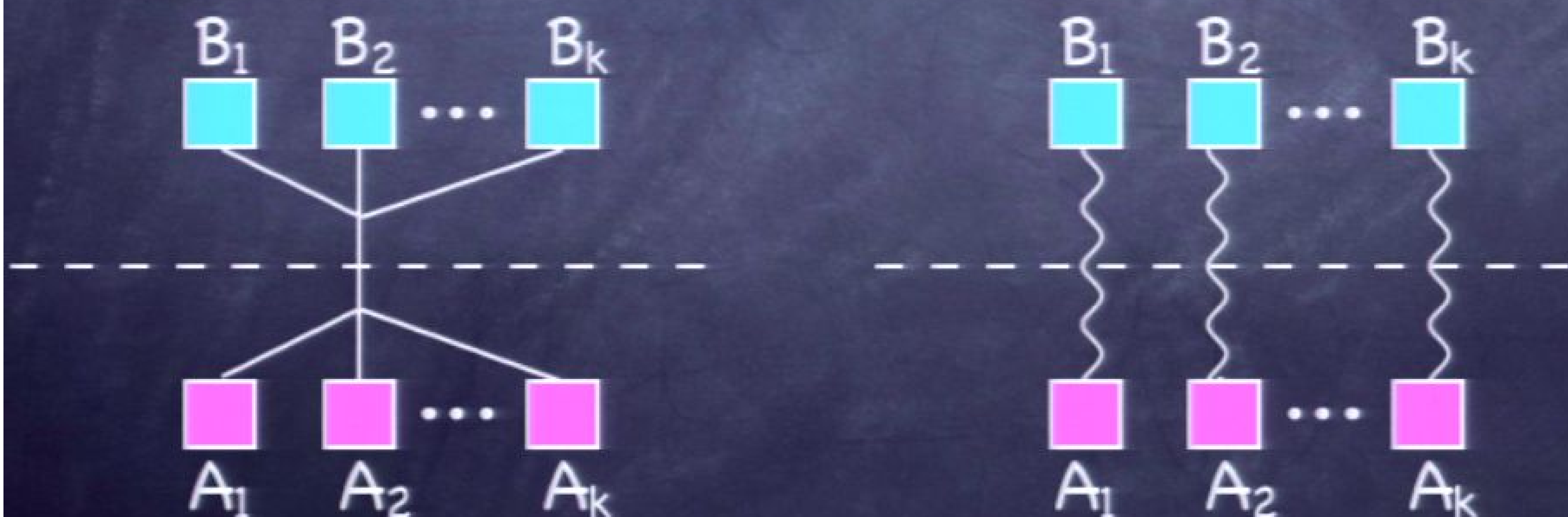

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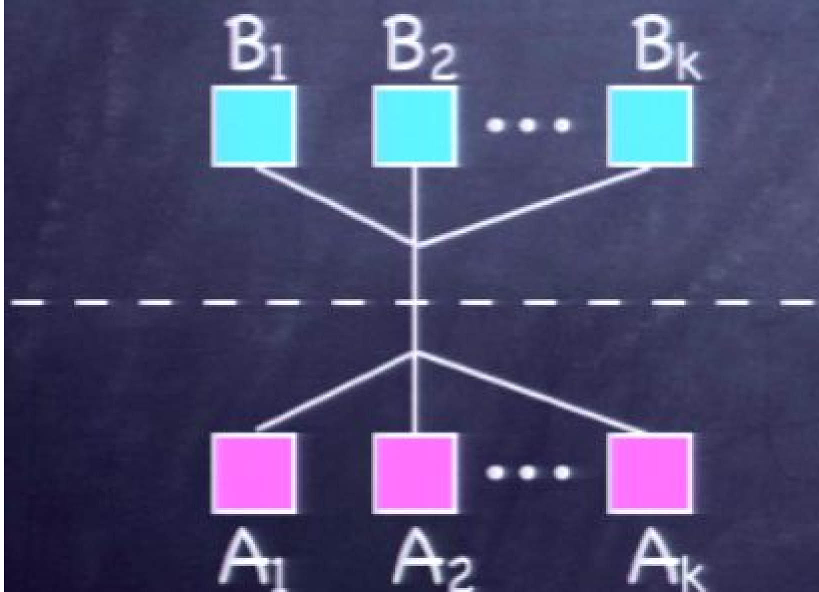
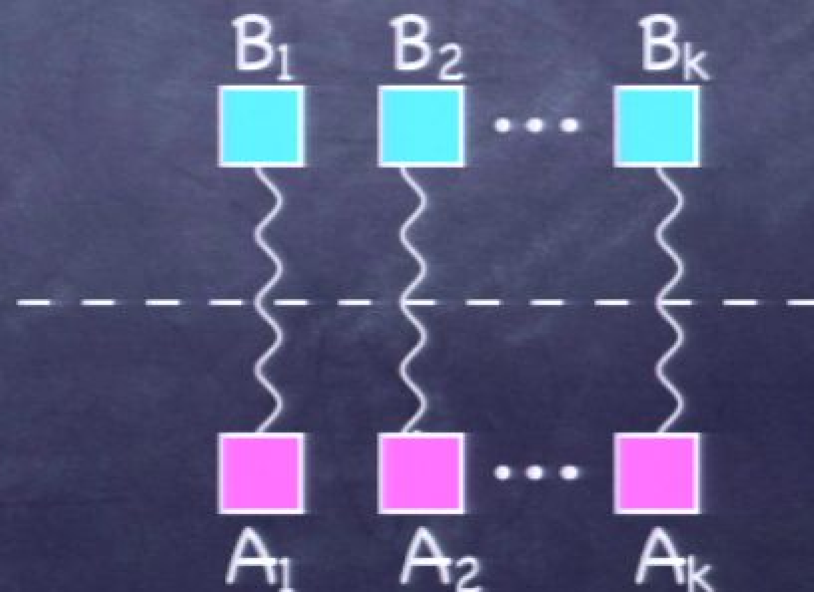
⇒ it seems that there must be a finite amount of correlations per each copy





Multipartite case

Define the multipartite mutual information for an n-partite state $\rho_{A_1,\ldots,A_n}$ as:

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- We have shown how it is possible to compute such a measure for some non-trivial classes of states.
- We have discussed the role of entanglement
  - deriving a candidate entanglement measure based on the previous quantity
  - showing that there is a copy-copy monogamy for (all?) entangled states

Open questions

- Quantumness can increase under LO (CC → non-CC), but there is loss of MI. Is there always a trade-off?
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- Are there applications for this measure of quantumness? (e.g. other measures have been used to study the correlations in some computational models)
- What are the (other) properties of $I^{(\infty)}$?
  - Is it an entanglement measure (weak/strong monotonicity under LOCC)?
  - Is it always non-zero for entangled states?