Title: Special Topics in Physics - Lecture 2A

Date: Jan 16, 2008 07:00 PM

URL: http://pirsa.org/08010034

Abstract: The Problem of Time in Quantum Gravity and Cosmology

Class mech $\xrightarrow{\text{Hamiltonian}}$ QM

$$\dot{A} = \{A, H\}$$

$$i\dot{\Psi} = H\Psi$$

$$\text{Class mech} \xrightarrow{\text{Hamiltonian}} \text{QM}$$

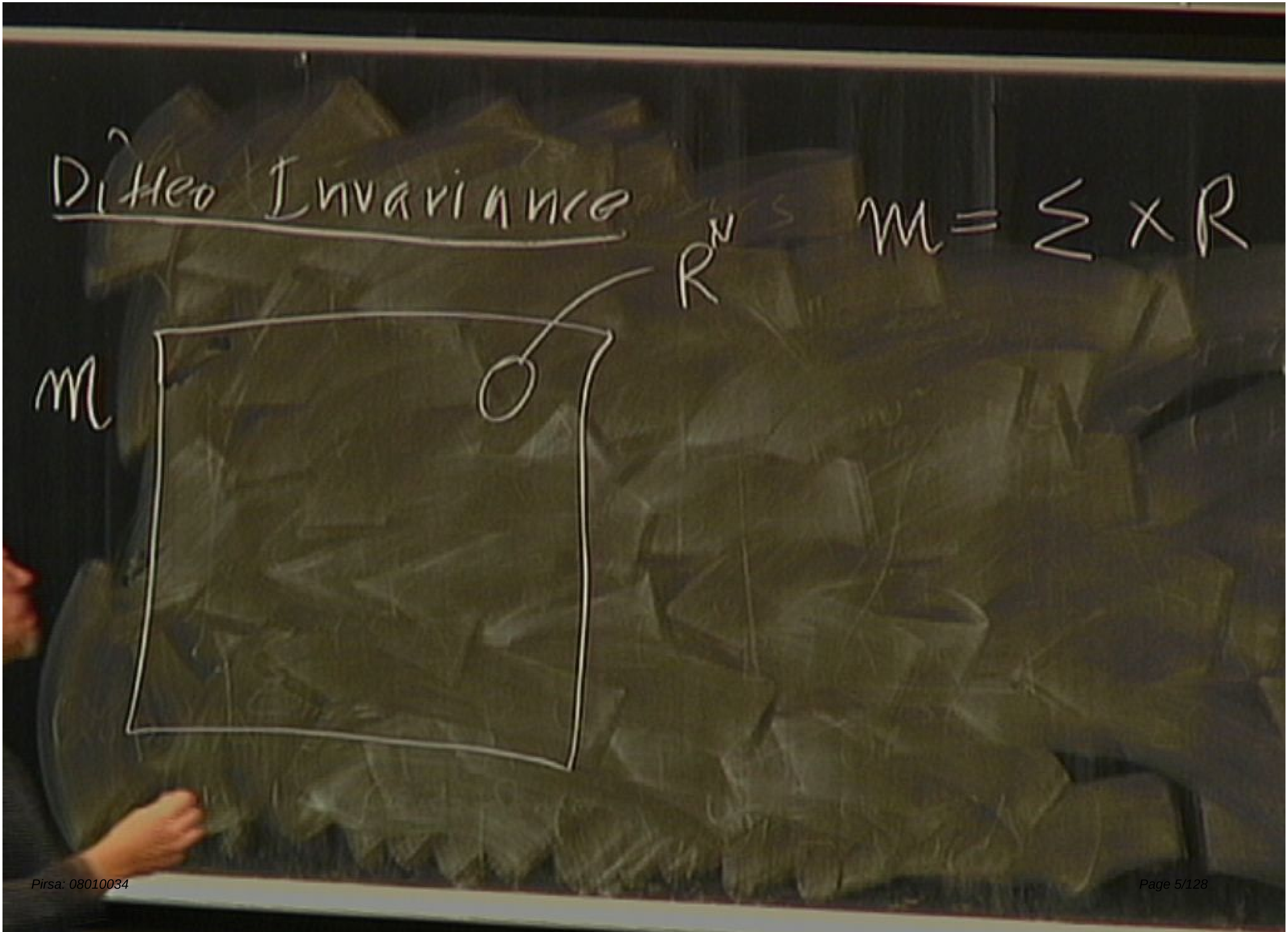
$$\dot{A} = \{A, H\} \qquad \int d\phi \, e^{iS} \qquad i\dot{\Psi} = H\Psi$$

Class mech $\xrightarrow{\text{Hamiltonian}}$ QM

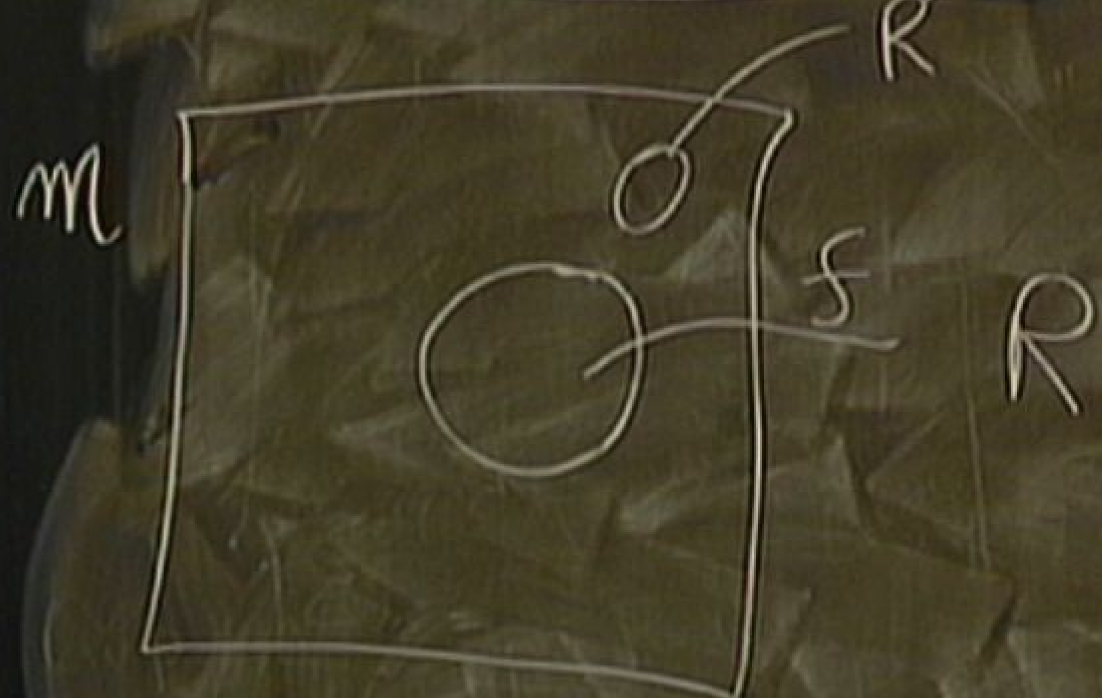
$\dot{A} = \{A, H\}$ $\quad \int d\phi\, e^{iS}$ $\quad i\dot{\Psi} = H\Psi$

Dirac — constrained Hamiltonian system

~ diffeo inv. ~ gauge invariance


Diffeo Invariance
M = Σ × R
R^N
M

Diffeo Invariance



$M = \Sigma \times R$

Fix topology

Fix differential structu

R^N

$$M^{d+1} = \Sigma^d \times R$$

Fix topology

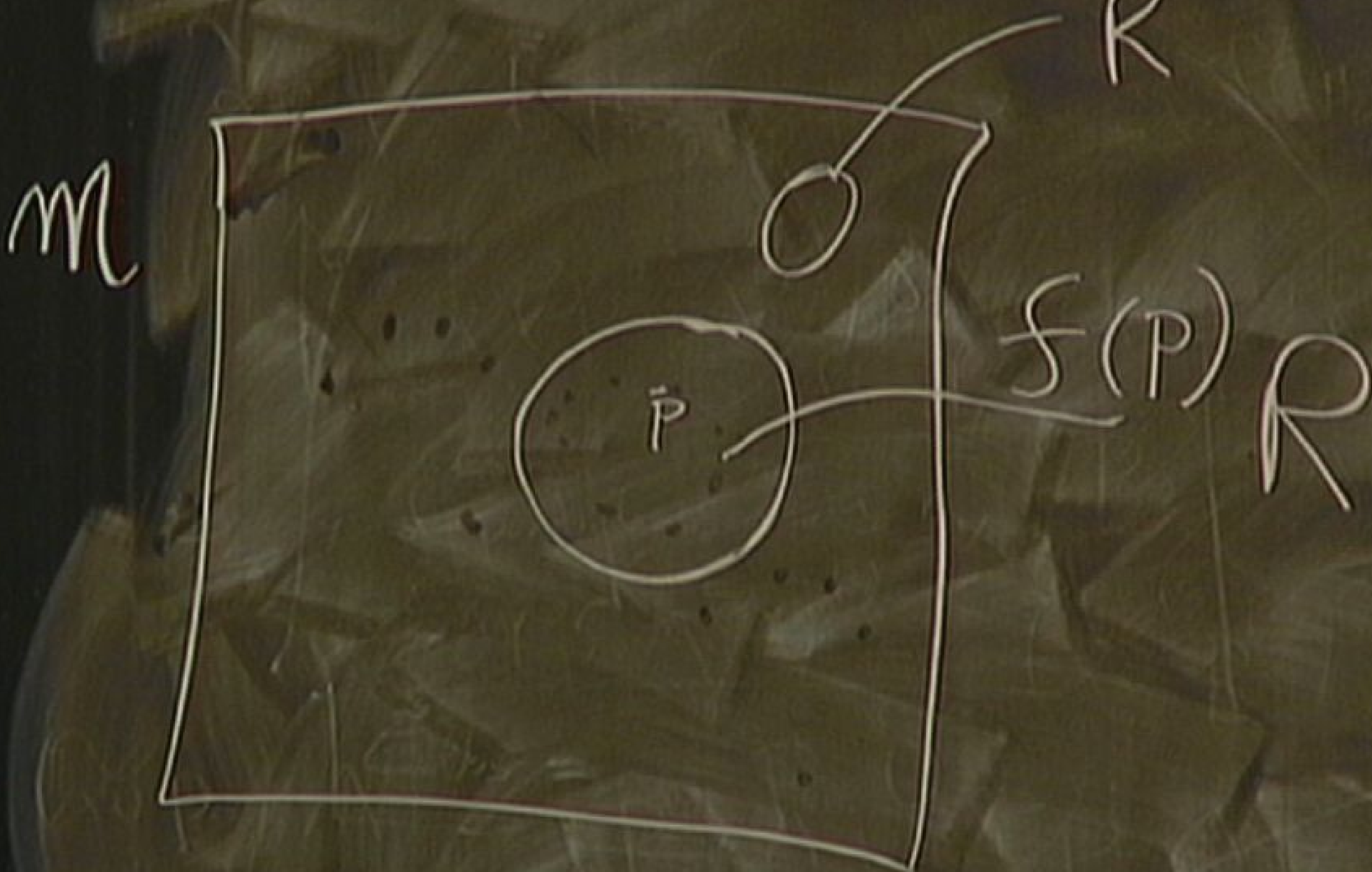
Fix differential structure

(P)

R

∂f

Diffeo Invariance

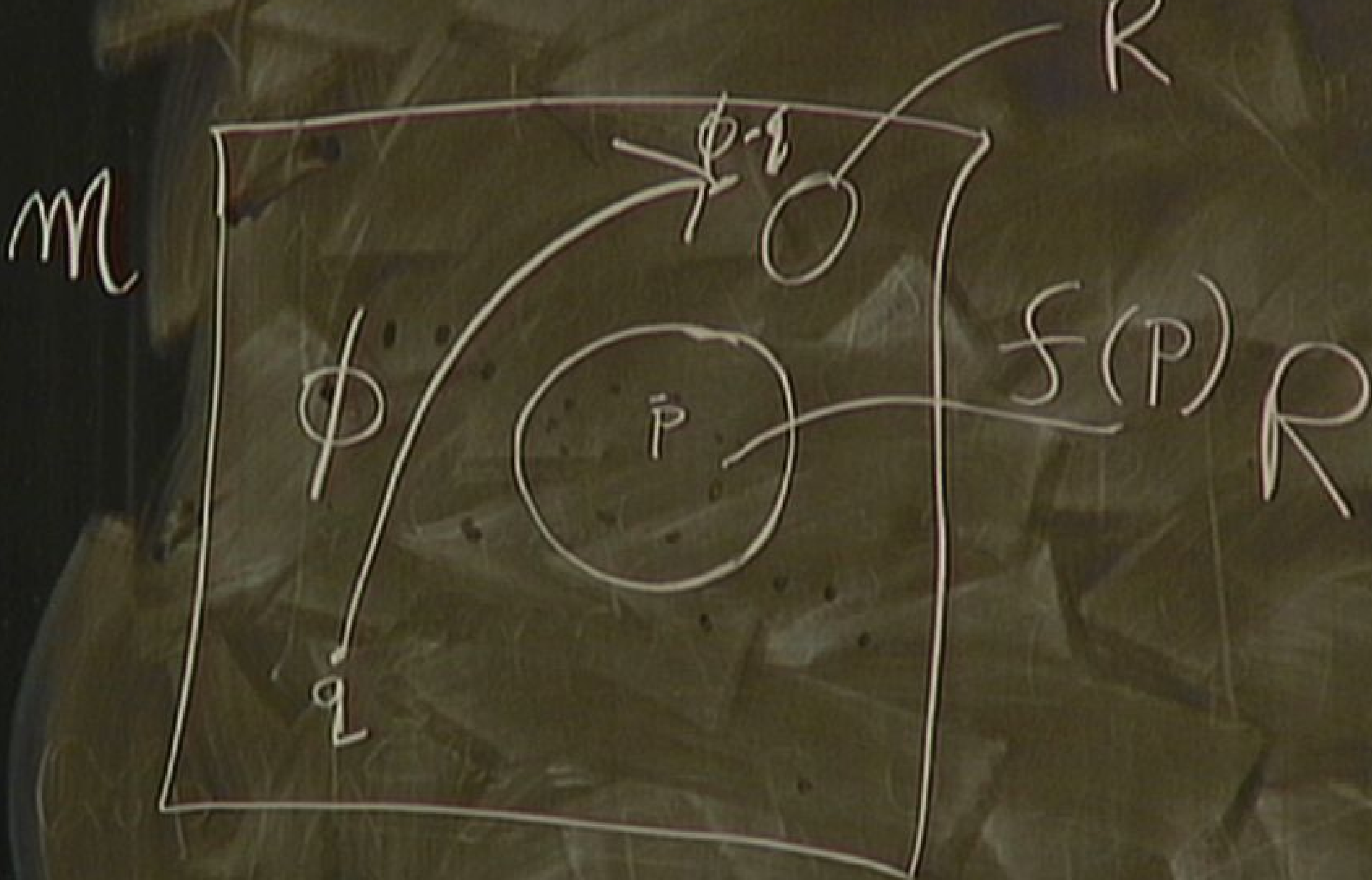


$\mathcal{M}^{d+}$

Fix

Fix

Diffeo Invariance



M^{d+}

Fix

Fix

Fix differential structure

$f(p)$ $\mathbb{R}$ ∂f C^∞

Def: Diffeo on $\mathcal{M}$

$\phi: \mathcal{M} \to \mathcal{M}$

ϕ^{-1} exists

$C^\infty \to C^\infty$

$$\phi f(q) = f(\phi^{-1} q)$$

$$\phi f(q) = f(\phi^{-1} q)$$

$$G_{ab} = R_{ab} - \frac{1}{2} g_{ab} R = T_{ab}$$

(g_{ab}, M)

metrics on M

connections M

fields, ψ on M

$\psi) = f(\phi \circ \psi)$

metrics on

(connections

fields, ψ or

$R_{ab} - \frac{1}{2} g_{ab} R = T_{ab}$

$\{g_{ab}, M\}$ equivalence class under all the diffeo's

$\mathrm{Diff}(M)$ $\quad \phi^{*} f(q) = f(\phi^{-1} \circ q$

$$G_{ab} = R_{ab} - \frac{1}{2} g_{ab} R = T$$

(g_{ab}, M) $\quad \{g_{ab}, M\}$ equiv

the

Diff(M)

diffeos on M

form a group

$$\phi^{*} f(q) = f(\phi^{-1} \circ q)$$

$$G_{ab} = R_{ab} - \frac{1}{2} g_{ab} R = T$$

$$(g_{ab}, M)$$

$$\{g_{ab}, M\}$$ equiv

the

$\mathrm{Diff}(M)$ ϕ $f(q) = f(\phi^{-1} \circ q)$

diffeos on M
form a group

$$G_{ab} = R_{ab} - \frac{1}{2} g_{ab} R = T_{ab}$$

(g_{ab}, M) $\{g_{ab}, M\}$ equivale[nce] the di[ffeos]

What are the real laws?

$\mathrm{Diff}(M)$ $\phi f(q) = f(\phi^{-1}\ldots$

diffeos on M

form a group

$$G_{ab} = R_{ab} - \frac{1}{2} g_{ab} R = \ldots$$

(g_{ab}, M) $\{g_{ab}, M\}$ equ... th...

What are the real laws?

What are initial data

$\{g_{ab}, m\}$ equivalence class under all the diffeo's

laws?

data?

ables?

EdM vector potential A_a

$\epsilon \, \partial_{ab} R = T_{ab}$

$\{ g_{ab}, M \}$ equivalence class under all the diffeo's

real laws?
... data?
observables?

E&M vector potential $A_a \to A'_a$

$$F_{ab} = \partial_a A_b - \partial_b A_a$$

$$A'_a = A_a + \delta A_a$$

$$\delta A_a = \partial_a f$$

$\tfrac{1}{2} g_{ab} R = T_{ab}$

$\{g_{ab}, M\}$ equivalence class under all the diffeo's

real laws?

al data?

observables?

E&M vector potential $A_a \rightarrow A'_a$

$F_{ab} = \partial_a A_b - \partial_b A_a$ $A'_a = A_a + \delta A_a$

$\delta A_a = \partial_a f$ $\delta F_{ab} = 0$

$\ldots \mathrm{is}\ \varepsilon \partial_{ab} R = T_{ab}$

$\{g_{ab}, m\}$ equivalence class under all the diffeo's

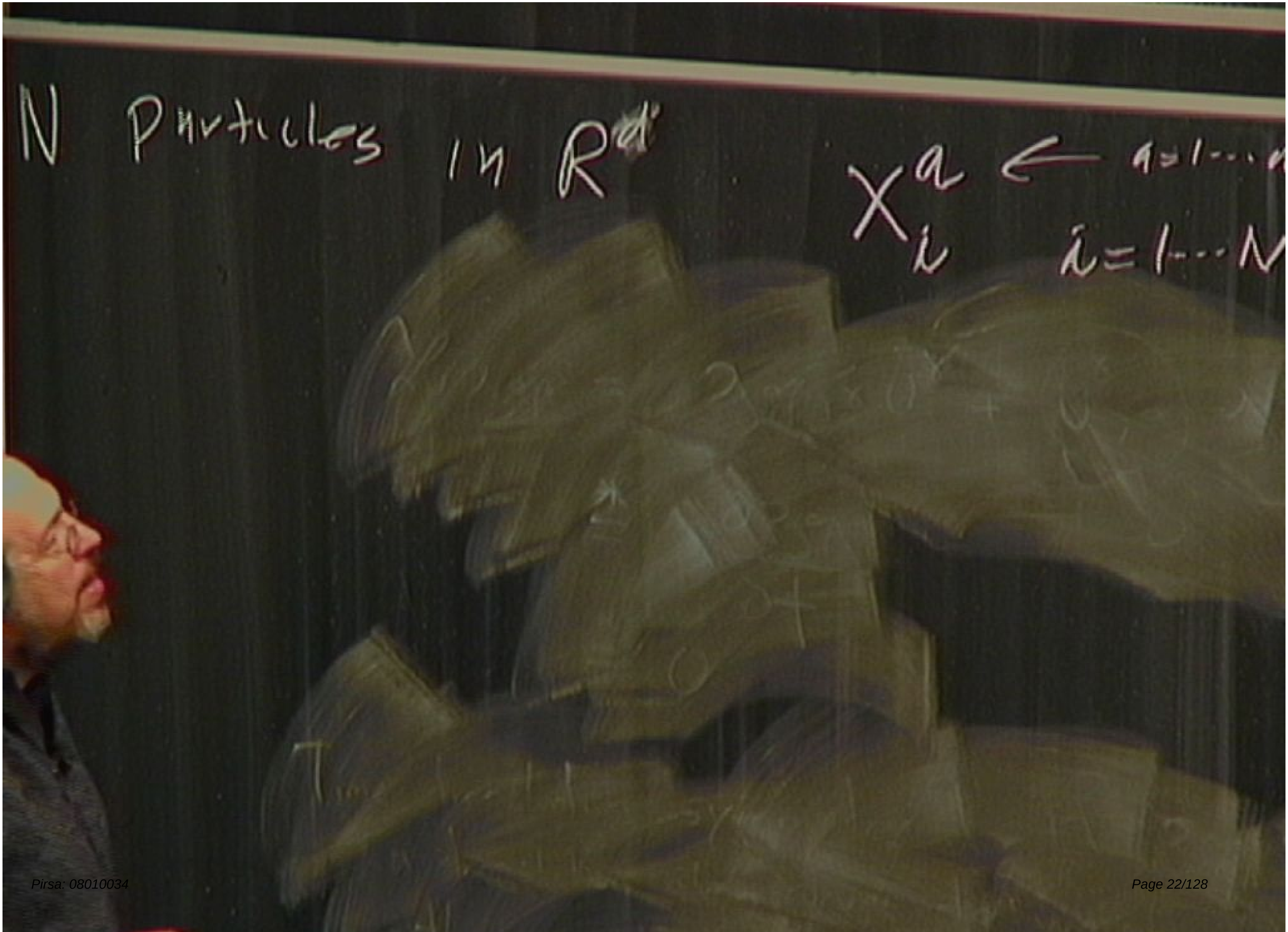
...ical laws? / ...al data? / ...observables?

EdM vector potential $A_a \to A'_a$

$$\boxed{F_{ab} = \partial_a A_b - \partial_b A_a}$$

$A'_a = A_a + \delta A_a$

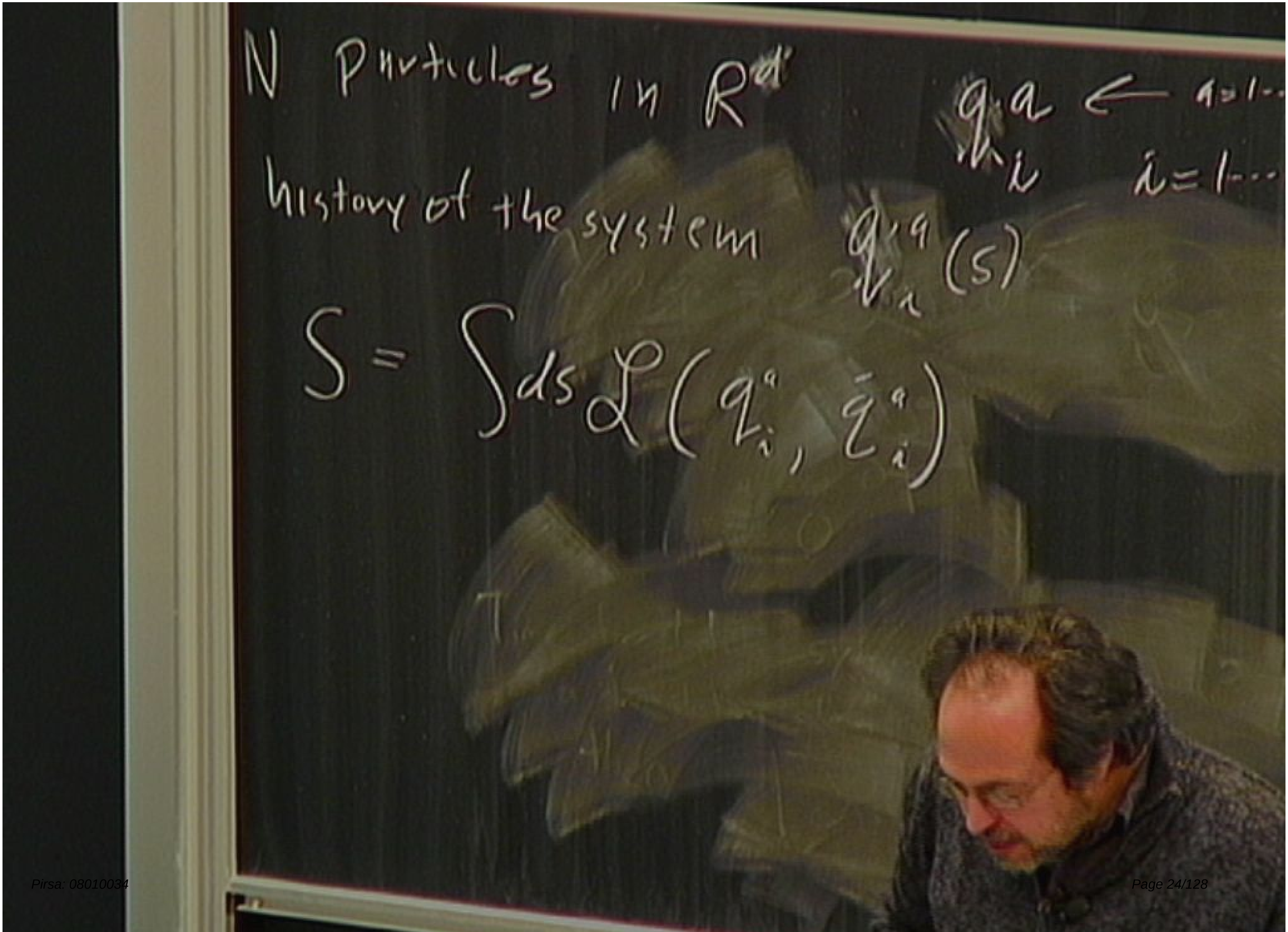
$\delta A_a = \partial_a f \qquad \delta F_{ab} = 0$


N Particles in R
X
a = 1 ...
i = 1 ... N

N particles in R^d $\quad X^a_i \leftarrow a=1\cdots d$ $\quad$ time parameter

$i=1\cdots N$

history of the system $\quad X^a_i(s)$

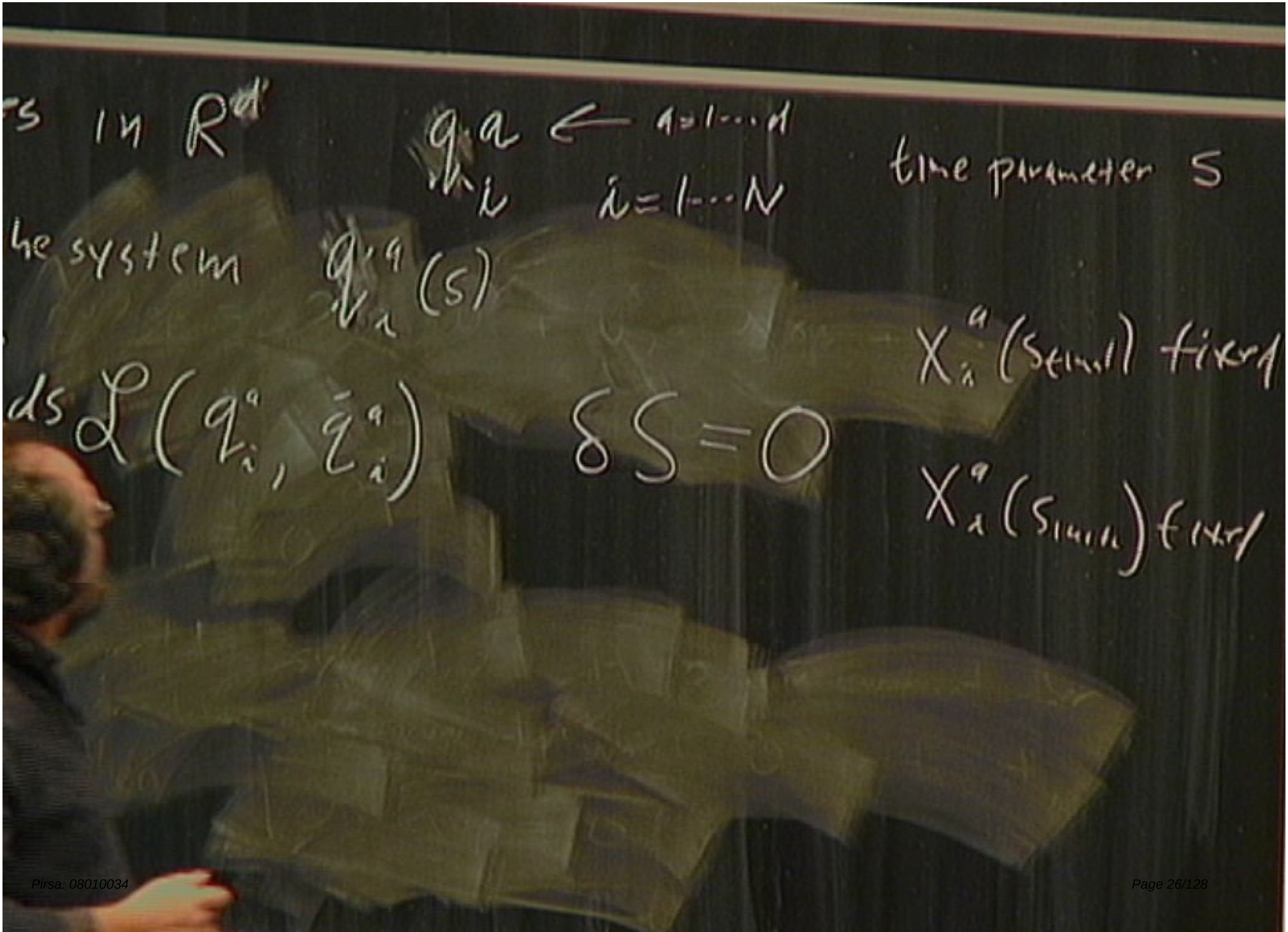

N Particles in R^d
$q_i^a \leftarrow a=1\ldots$
$i=1\ldots$
history of the system $q_i^a(s)$
$S = \int ds\, \mathcal{L}(q_i^a, \dot{q}_i^a)$

N particles in R^d $\quad q^a_i \leftarrow a = 1 \dots d$

$i = 1 \dots N$

history of the system $\quad q^a_i(s)$

$$S = \int ds\, \mathcal{L}(q^a_i, \dot{q}^a_i) \qquad \delta S = 0$$


s in R^d
$q_i^a \leftarrow a=1\ldots d$
$i=1\ldots N$
time parameter s
he system $q_i^a(s)$
$X_i^a(s_{\text{final}})$ fixed
$ds\,\mathcal{L}(q_i^a, \dot{q}_i^a)$
$\delta S = 0$
$X_i^a(s_{\text{initial}})$ fixed

s in R^d $q^a \leftarrow a=1\ldots d$

q^a_i $i=1\ldots N$

time parameter s

he system $q^a_i(s)$

$q^a_i(s_{\text{final}})$ fixed

$ds\,\mathcal{L}(q^a_i, \dot{q}^a_i)$ $\delta S=0$

$q^a_i(s_{\text{init}})$ fixed

$$S = \int ds\, \mathcal{L}(q^a_i, \bar{q}^a_i)$$

$$\Rightarrow$$

E-L $$\frac{d}{ds}\frac{\delta S}{\delta \bar{q}^a_i} - \frac{\delta S}{\delta q^a_i} = 0$$

$\dot{q}_i^a(s)$

$q_i^a(s_{final})$ fixed

$\mathcal{L}(q_i^a, \dot{q}_i^a)$ $\quad \delta S = 0$

$q_i^a(s_{initial})$ fixed

$$\frac{\delta S}{\delta \dot{q}_i^a} - \frac{\delta S}{\delta q_i^a} = 0$$

$$P_a^i(s) = \frac{\delta S}{\delta \dot{q}_i^a}$$

$q_i^a(s)$

$$S = \int ds\, \mathcal{L}(q_i^a, \dot{q}_i^a) \qquad \delta S = 0$$

$\Rightarrow$

E-L $$\frac{d}{ds}\frac{\delta S}{\delta \dot{q}_i^a} - \frac{\delta S}{\delta q_i^a} = 0$$

$$\boxed{P_a^i(s) = \frac{\delta S}{\delta \dot{q}_i^a}}$$

$$\dot{p}_a^i = \frac{\delta S}{\delta q_i^a}$$

$\bar{q}^a_i)$ $\delta S = 0$

$q^a_i(s_{\text{initial}})$ fixed

$=0$

$P^i_a(s) = \dfrac{\delta S}{\delta \bar{q}^a_i}$ $\cdots$ $\bar{q}$

N_i $\quad i = 1 \dots N$ $\quad$ time parameter s

the system $q_i^a(s)$

$$\int ds\, \mathcal{L}(q_i^a, \dot{q}_i^a) \qquad \delta S = 0$$

$q_i^a(s_{\text{initial}})$ fix

$q_i^a(s_{\text{final}})$ fix

$$\frac{d}{ds}\frac{\delta S}{\delta \dot{q}_i^a} - \frac{\delta S}{\delta q_i^a} = 0$$

$$\dot{p}^i_a = \frac{\delta S}{\delta q_i^a}$$

$$p_a^i(s) = \frac{\delta S}{\delta \dot{q}_i^a} \quad \therefore \dot{q}$$

$$H(q,p) = p\dot{q} - \mathcal{L}(q,\dot{q})$$

$$\frac{\delta \ldots}{\delta q^q_i} \quad \frac{\delta \ldots}{\delta q^q_i} = 0$$

$$\dot{p}_i = \frac{\delta S}{\delta q^q_i}$$

$$H(q,p) = p\dot{q} - \mathcal{L}(q,\dot{q})$$

$$\delta \oint ds \left[p\dot{q} - H(q,p) \right]$$

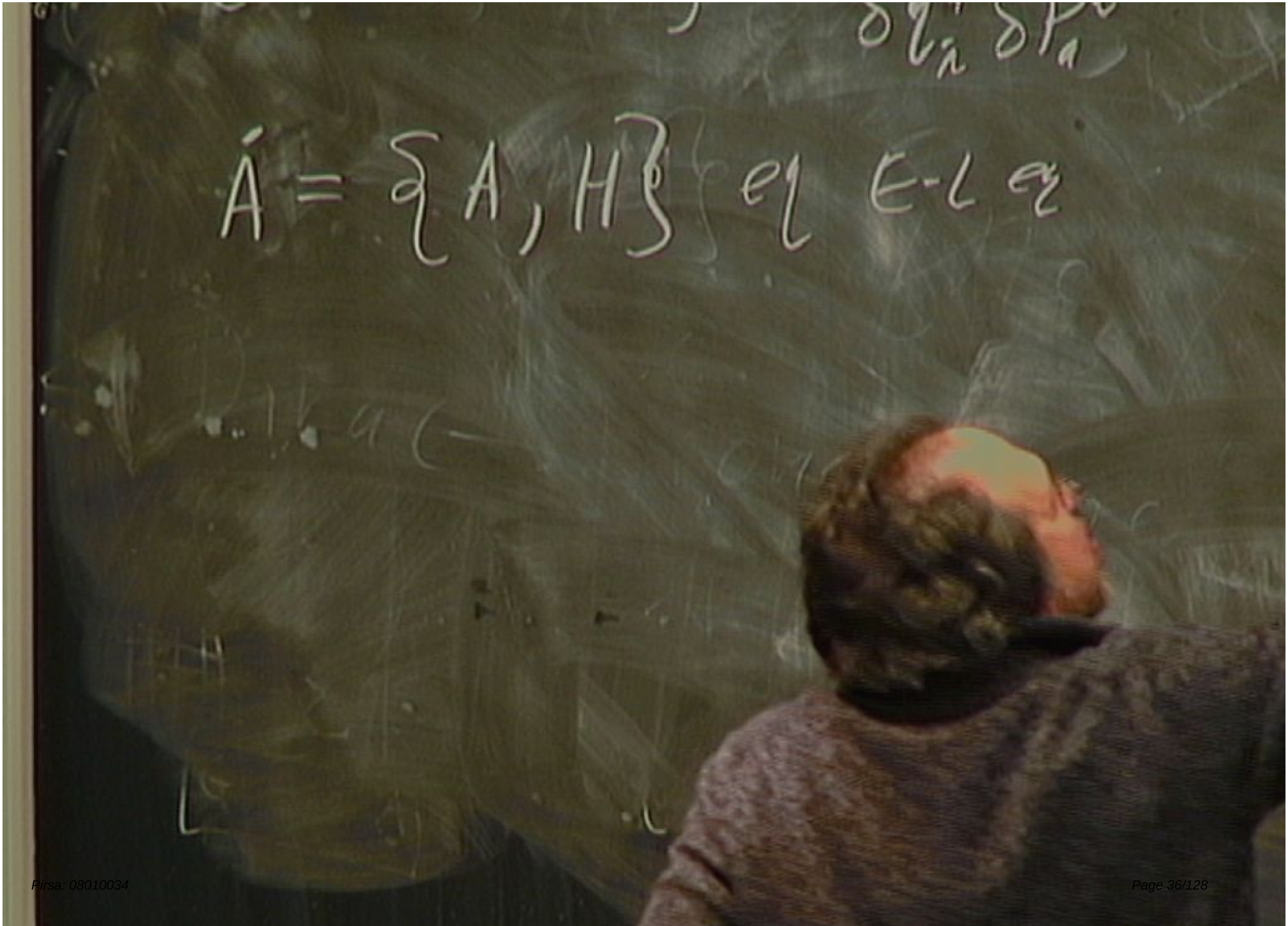
$$\{A(q,p), B(q,p)\} = \frac{\delta A}{\delta q^a_i}\frac{\delta B}{\delta p^i_a} - A \leftrightarrow B$$

$A \leftrightarrow B$

P_{q^a} $\bar{q}^{q^a}$

$\dot{q}^a \dot{q}^a$

system


Á = {A, H} eq E-L eq

$$\frac{\partial}{\partial q_i} \quad \frac{\partial}{\partial p_a}$$

$$\dot{A} = \{A, H\} \quad \text{eq} \quad \text{E-L eq}$$

$$\dot{q}^a_i = \frac{\delta H}{\delta p^i_a} \qquad \dot{p}^i_a = -\frac{\delta H}{\delta q^a_i}$$

m equivalence class under all the diffeo's

EdM vector potential $A_a \rightarrow A'_a$

$$\boxed{F_{ab} = \partial_a A_b - \partial_b A_a} \qquad A'_a = A_a + \delta A_a$$

$$\delta A_a = \partial_a f(t) \qquad \delta F_{ab} = 0$$

$p^i_a.$ $\quad \dot{p}_a^i = -\frac{\delta H}{\delta q^a_i}$

$$\mathcal{L} = \frac{1}{2}\dot{q}^a \dot{q}^b g_{ab} - V(q)$$

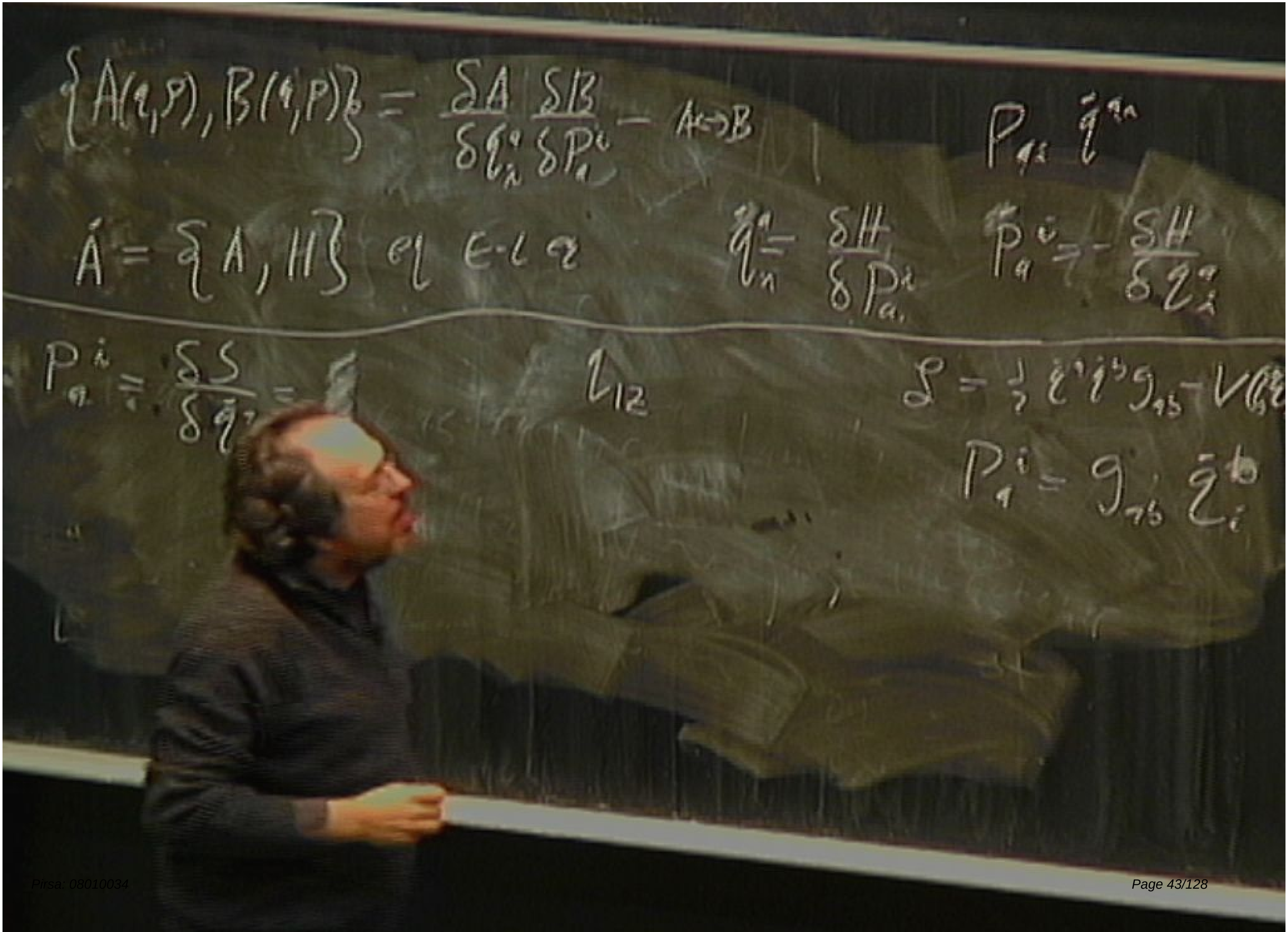
$$\dot{A} = \{A, H\} \text{ eq}$$

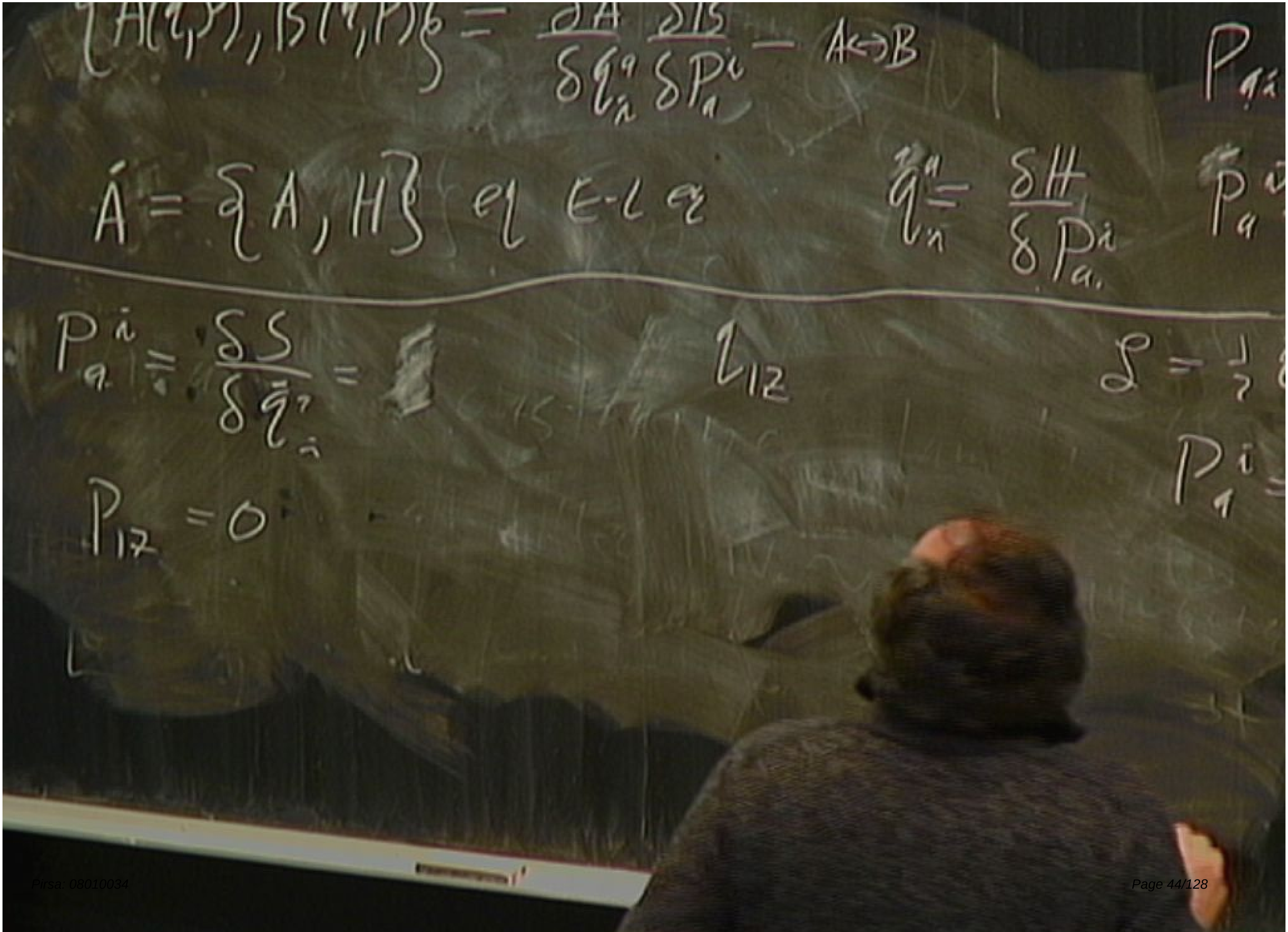
$$P_a^i = \frac{\delta S}{\delta \dot{q}^a_i} =$$

$$\frac{\delta H}{\delta P^i_a}, \qquad \dot{P}^i_a = -\frac{\delta H}{\delta q^a_i}$$

$$\mathcal{L} = \frac{1}{2}\dot{q}^a_i \dot{q}^b_i g_{ab} - V(q)$$

$$P^i_a = g_{ab}\,\dot{q}^b_i$$




A↔B
Á = {A, H} eq E-L eq
P = δS/δq = 0
P = 0
L

$$\{A(q,p), B(q,p)\} = \frac{\delta A}{\delta q^a_i}\frac{\delta B}{\delta p^i_a} - A \leftrightarrow B$$

$$\dot{A} = \{A, H\} \quad \text{eq E-L eq} \qquad \dot{q}^a_i = \frac{\delta}{\delta}$$

$$p^i_a = \frac{\delta S}{\delta \dot{q}^a_i} =$$

$$L_{12}$$

$$P_{17} = 0$$

$$P^{i}_{q_a} = \frac{\delta S}{\delta \bar{q}^{a}_{i}} =$$

$$P_{17} = 0$$

$$P_{17} = f(q, p)$$

L_{12}

$\mathcal{L} =$

$\Rightarrow \dot{q} = (\ )\, p$

P

$f(q,p)$

$P \bar{q}) \Rightarrow MPP$

$$p^{i}_{q} = \frac{\delta S}{\delta \dot{q}_{i}} = \text{linear} \Rightarrow \dot{q} = (\;)$$

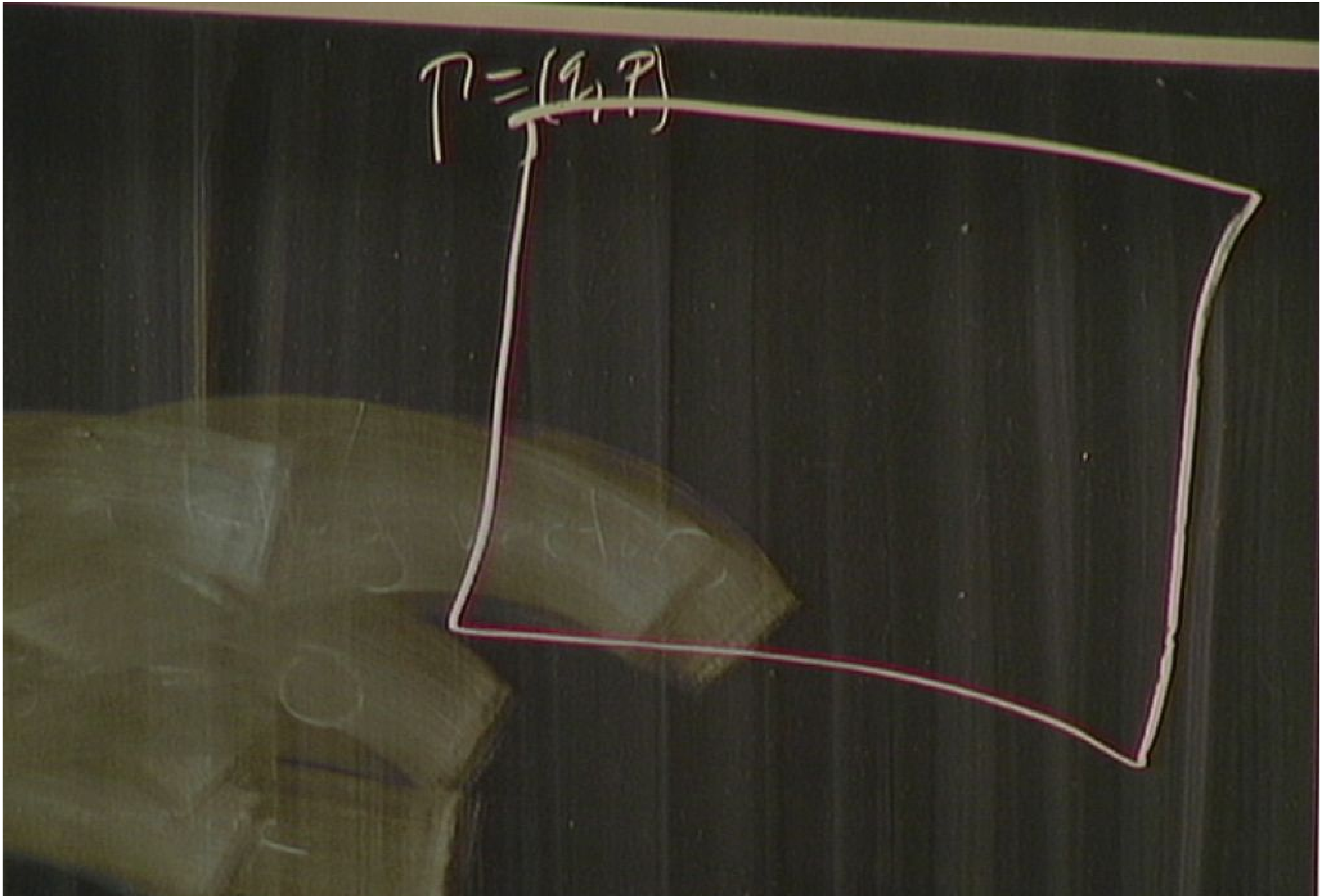
$$P_{17} = 0 \qquad P_{17} = f(q, p)$$

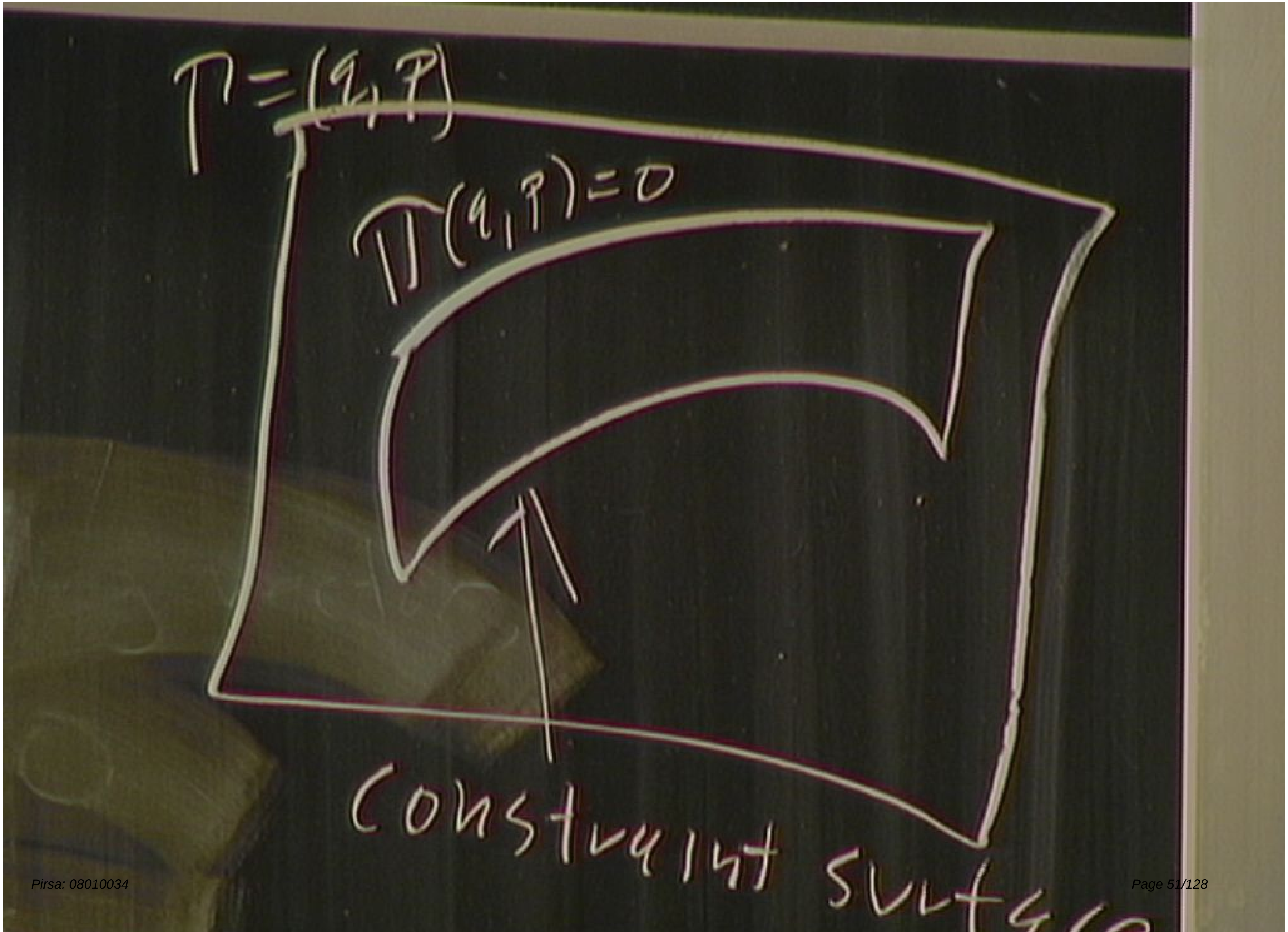
L

$$P^{i}_{q} = \frac{\delta S}{\delta \dot{q}^{i}} = \xrightarrow{\text{linear}} \dot{q} = (\ )\,P \qquad q_{1z}$$

$$P_{17} = 0 \qquad P_{17} = f(q,p)$$

Primary constraints $\Pi(q,p) = 0$


Γ = (q, p)

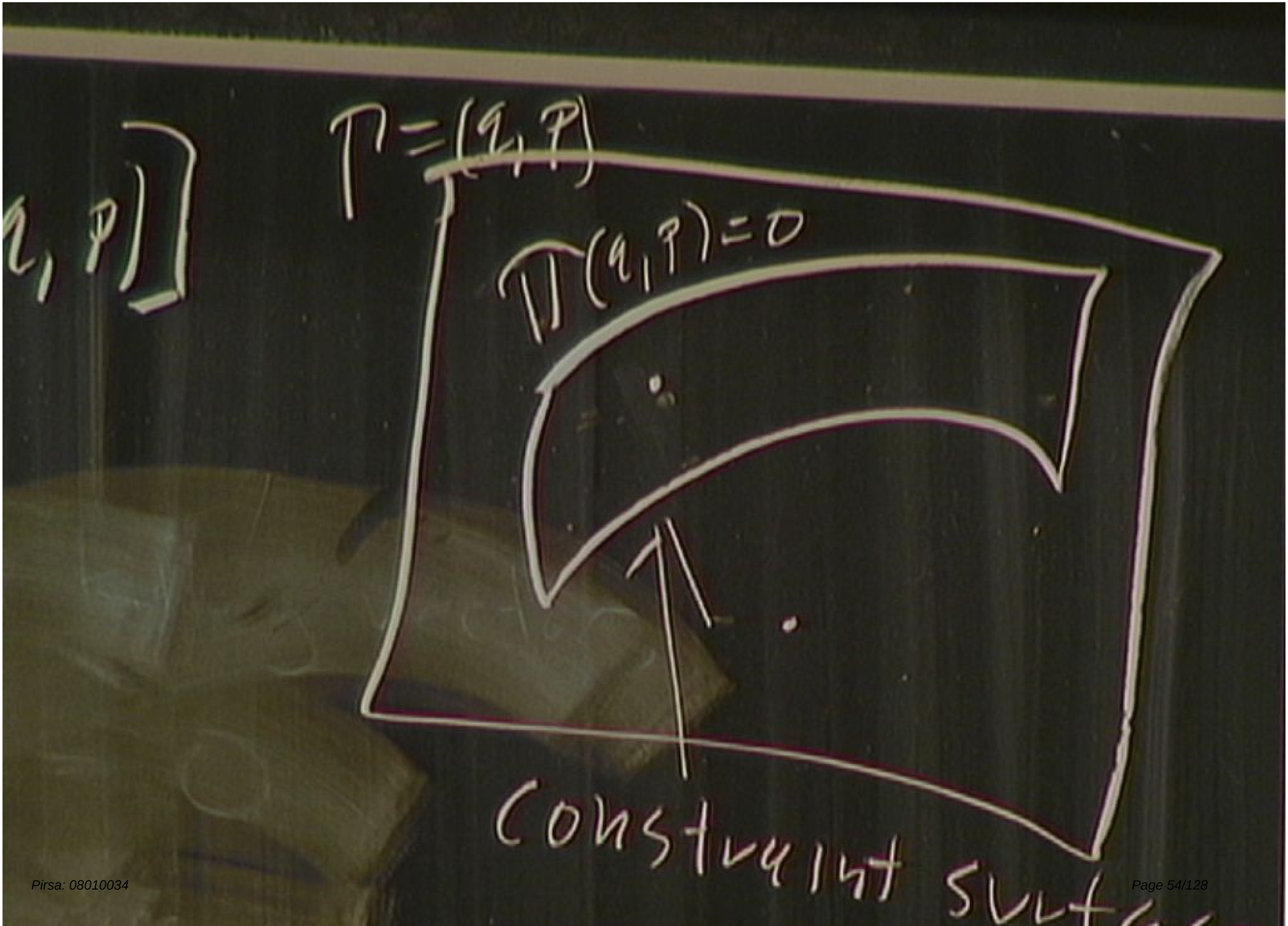

Γ=(q,p)
Γ(q,p)=0
constraint surface

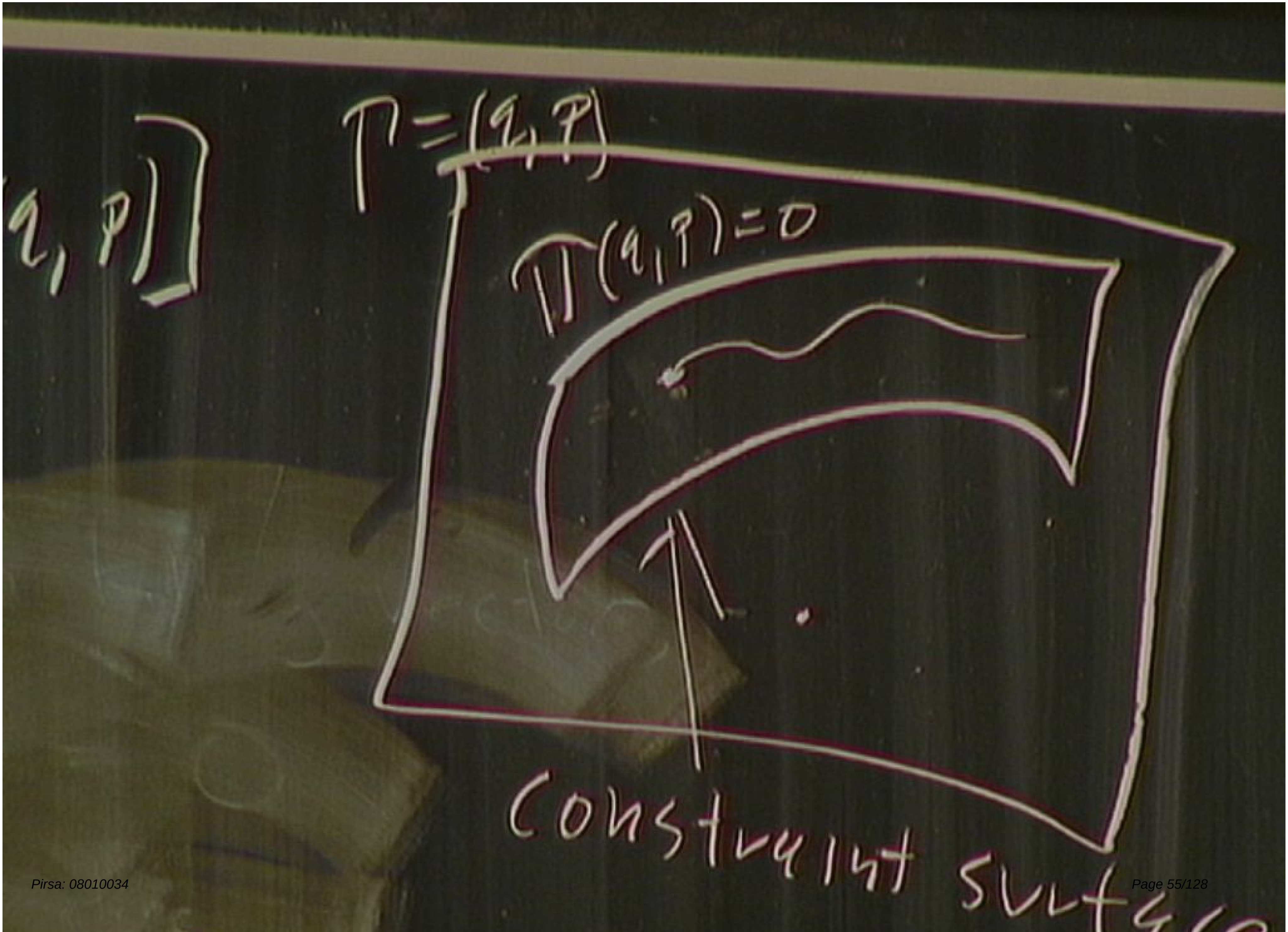
$$0 = \delta \overbrace{\int \left[p\dot{q} - H(p,q) + \lambda \Pi(q,p) \right]}^{S'}$$

$$\frac{\delta S'}{\delta \lambda} = \Pi(p,q) = 0$$

$$\frac{\delta S'}{\delta \lambda} = \Pi(p, q) = 0$$

$$H' = H^{0} + \lambda \Pi$$


Γ=(q,p)
Φ(q,p)=0
constraint


q, p]
Γ = (q, p)
Π(q, p) = 0
Constraint

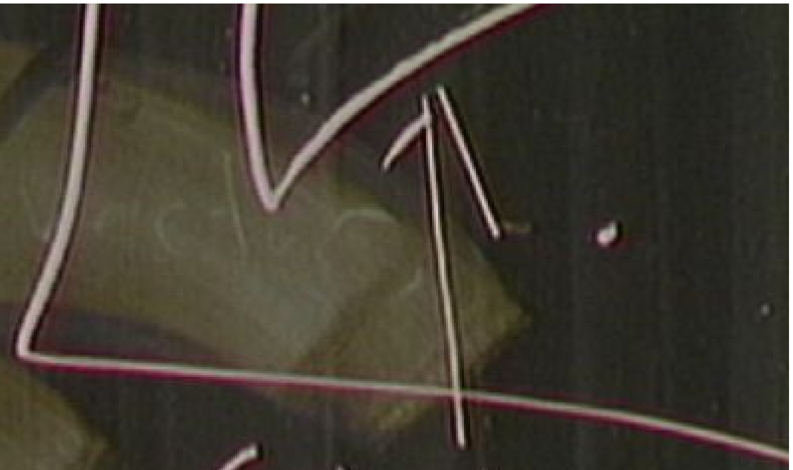
$$\frac{\delta S'}{\delta \lambda} = \Pi(p, q) = 0$$

$$H' = H^0 + \lambda \Pi$$

$$\dot{q} = \{q, H'\} = \{q, H^0\} + \lambda \{q, \Pi\}$$

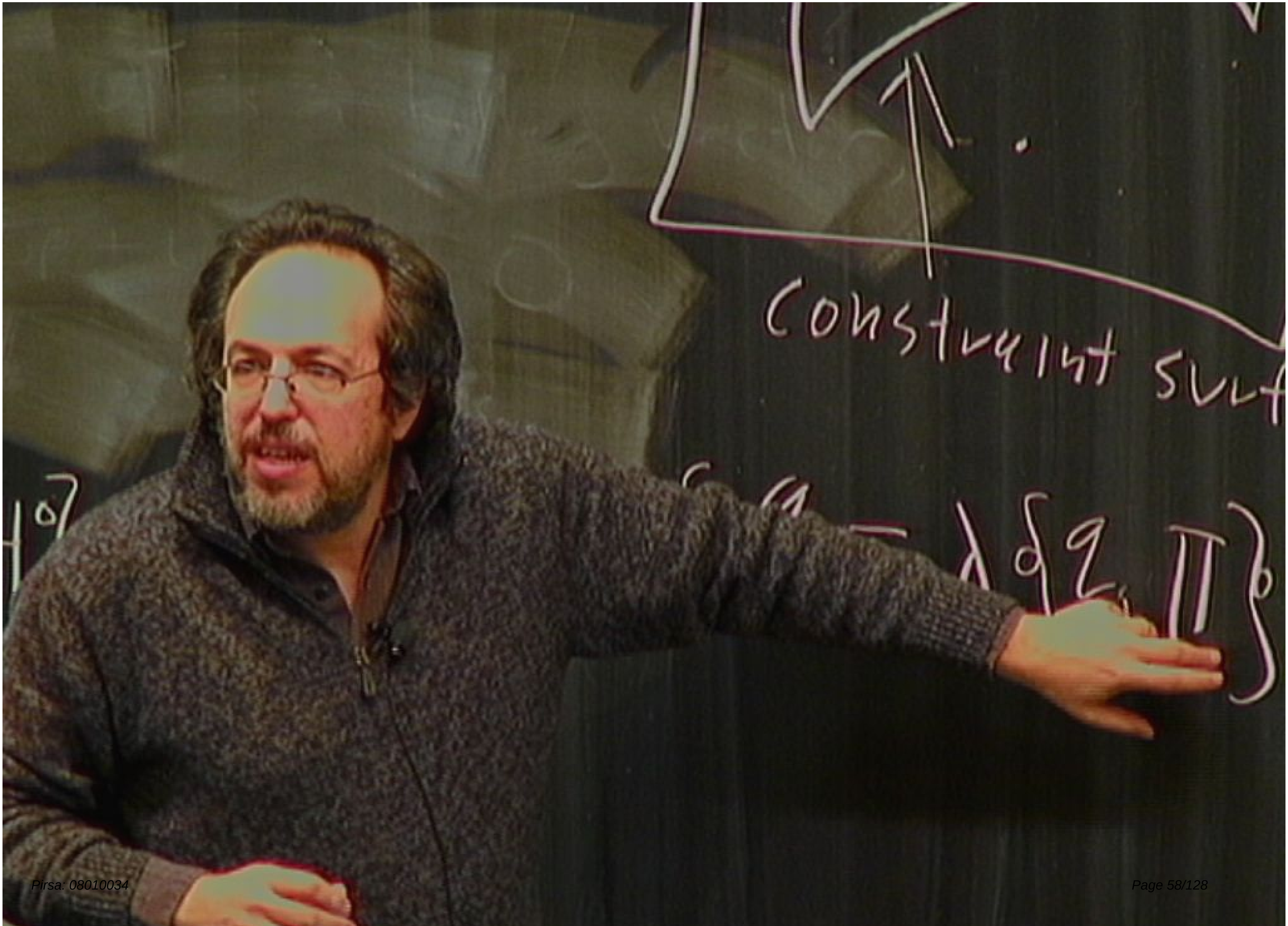
$= 0$

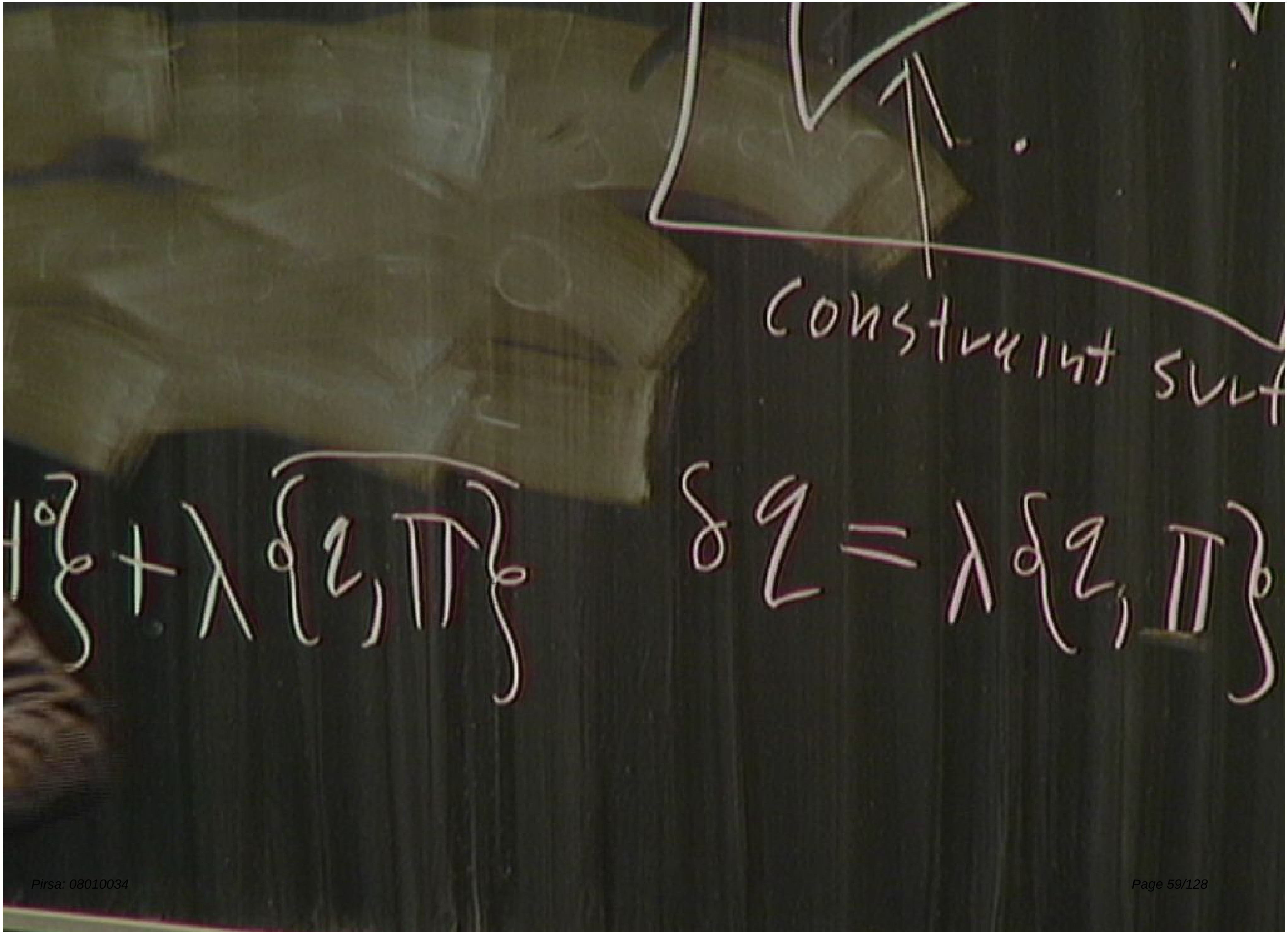
Π



$\{q, H_0\} + \overbrace{\lambda \{q, \Pi\}}$

$\delta q = \lambda \{q,$


constraint surf


constraint surf
$H^0\} + \lambda\{q, \Pi^0\}$
$\delta q = \lambda\{q, \Pi^0\}$

$$H' = H^0 + \lambda \Pi$$

$$\dot{q} = \{q, H'\} = \{q, H^0\} + \lambda \{q, \Pi\}$$

$$\dot{\Pi} = 0 = \{\Pi, H'\}$$

$$H' = H^0 + \lambda \Pi$$

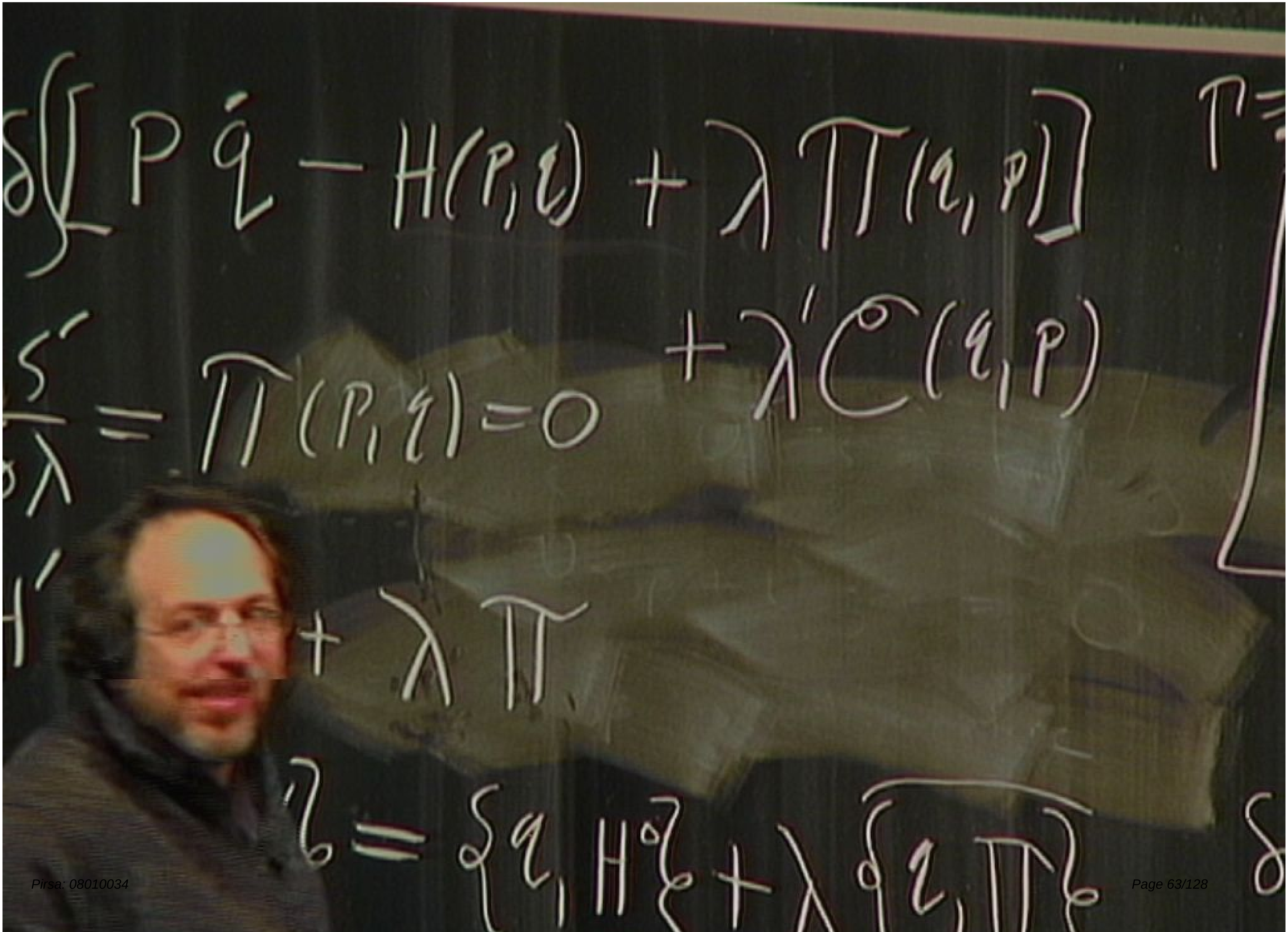
$$\dot{q} = \{q, H'\} = \{q, H^0\} + \lambda \{q, \Pi$$

$$\dot{\Pi} = 0 = \{\Pi, H'\} \overset{?}{=} 0$$

$$H' = H^0 + \lambda \Pi$$

$$\dot{q} = \{q, H'\} = \{q, H^0\} + \lambda \{q, \Pi$$

$$\dot{\Pi} = 0 = \{\Pi, H'\} \overset{?}{=} 0$$

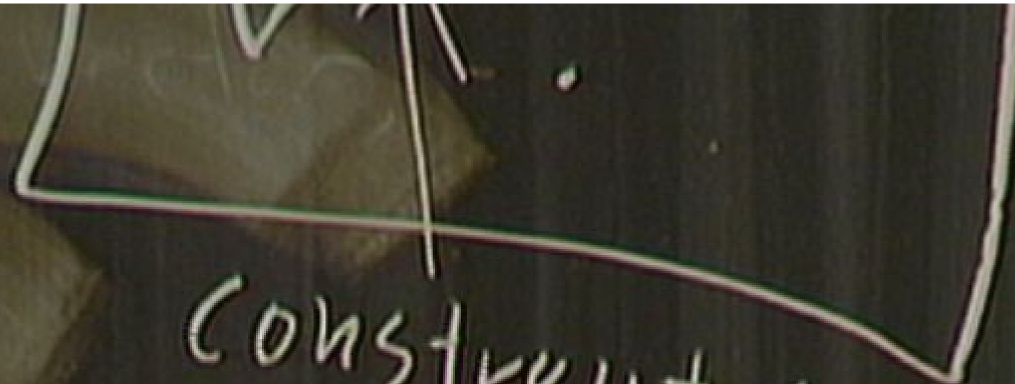


$\delta \int [p\dot{q} - H(p,q) + \lambda \Pi(q,p)]$
$\frac{\delta S}{\delta \lambda} = \Pi(p,q) = 0$
$+ \lambda' C(q,p)$
$H' = H + \lambda \Pi$
$\dot{q} = \{q, H\} + \lambda \{q, \Pi\}$

$$H' = H^0 + \lambda \Pi$$

$$\dot{q} = \{q, H'\} = \{q, H^0\} + \lambda \{q, \Pi\}$$

$$\dot{\Pi} = 0 = \{\Pi, H'\} \overset{?}{=} \phi(q, p) = 0$$



constraint surface

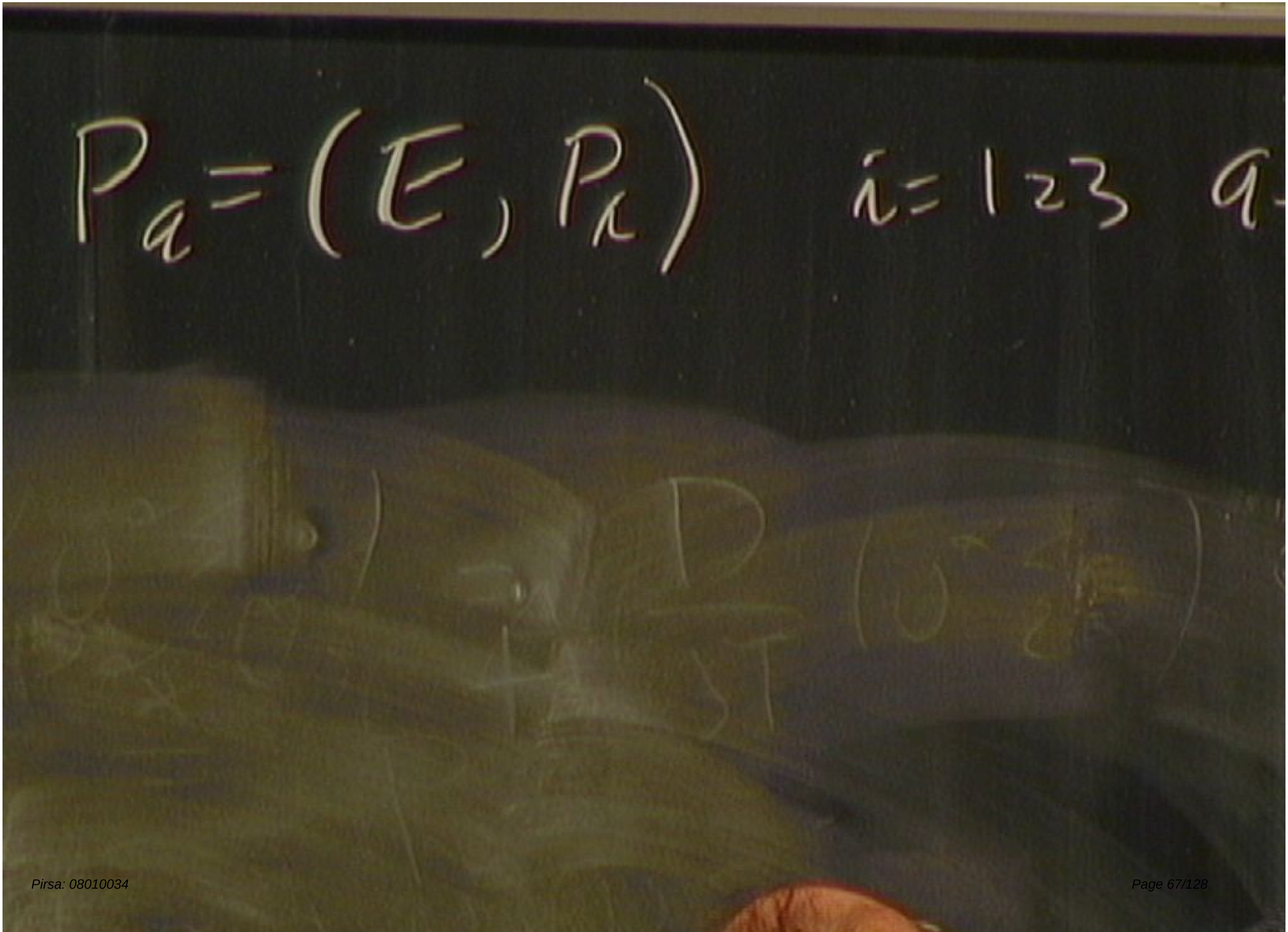
$$\ldots\} + \lambda\{q, \Pi\}$$

$$\delta q = \lambda \{q, \Pi\}$$

$$= C(q, p) = 0$$

secondary constraints

Free relativistic particle

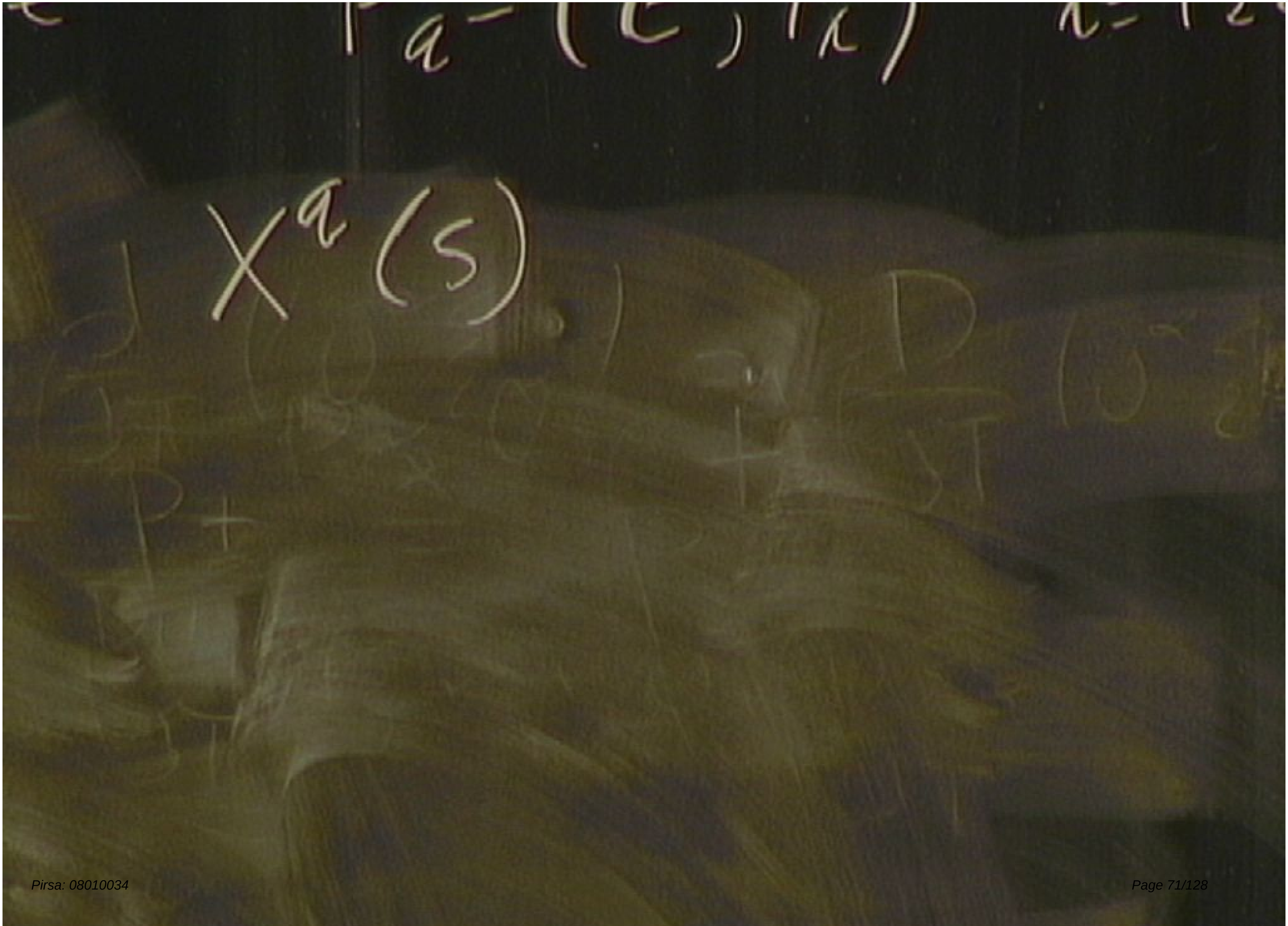

$P_a = (E, P_i)$
$i = 1\,2\,3$

$(E, P_i) \quad i = 1 2 3 \quad a = 0, i$

relativistic particle

$$E^2 = p^2 + m^2$$

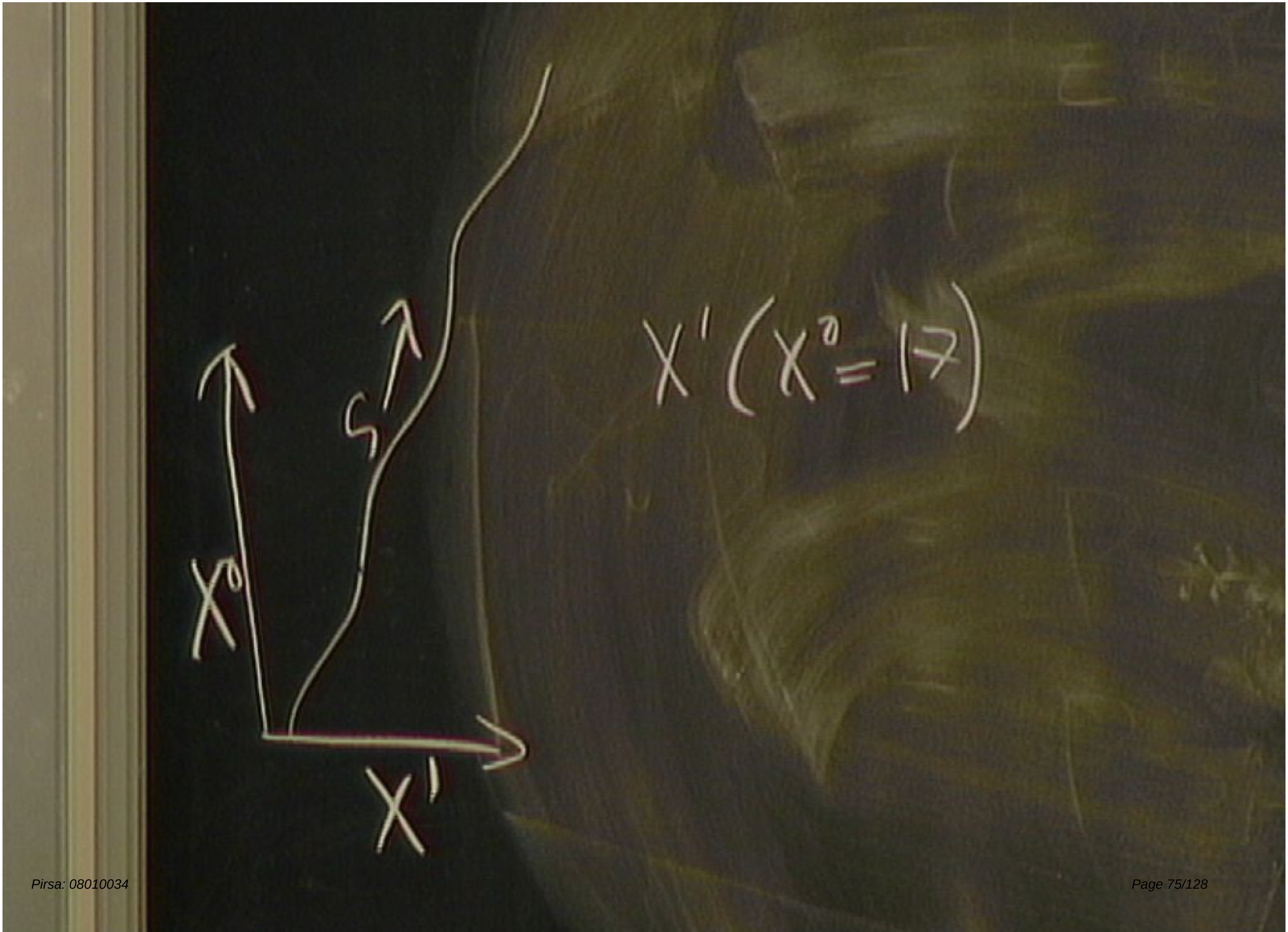
$i = 1\,2\,3 \qquad a = 0, i \qquad C = I$


$X^a(s)$


S


X^0
s
X^1

$$S \to S' = f(S)$$


X^0
s
X^1
$X^1(X^0 = 17)$

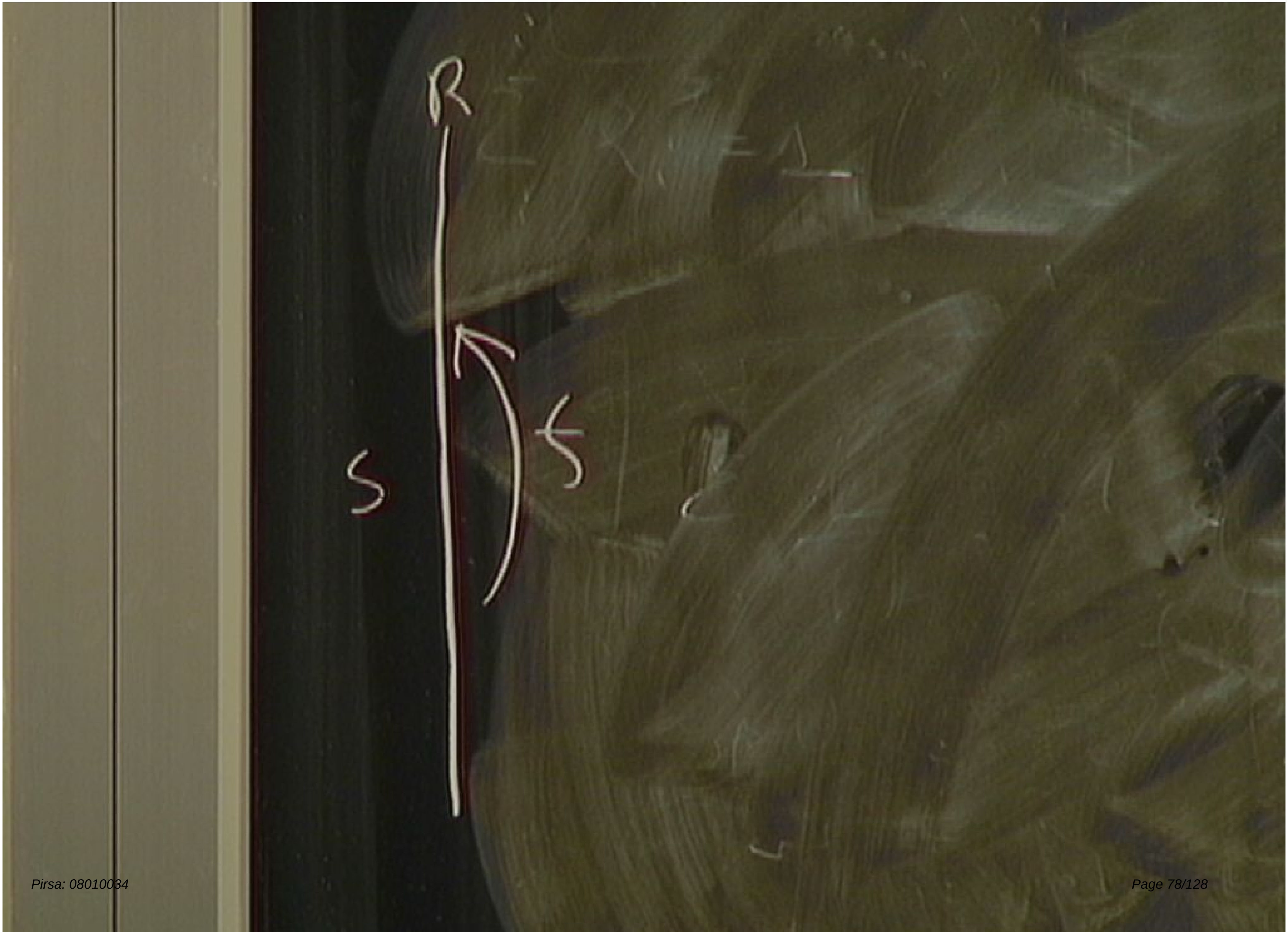
$$s \to s' = f(s)$$ monotonic

reparameterization

$s \to s' = f(s)$ monotonic

reparameterization

repar. invariant


R
S

repara

rep

$$S = \int ds \left[p_a \dot{x}^a \right.$$

repara

rep

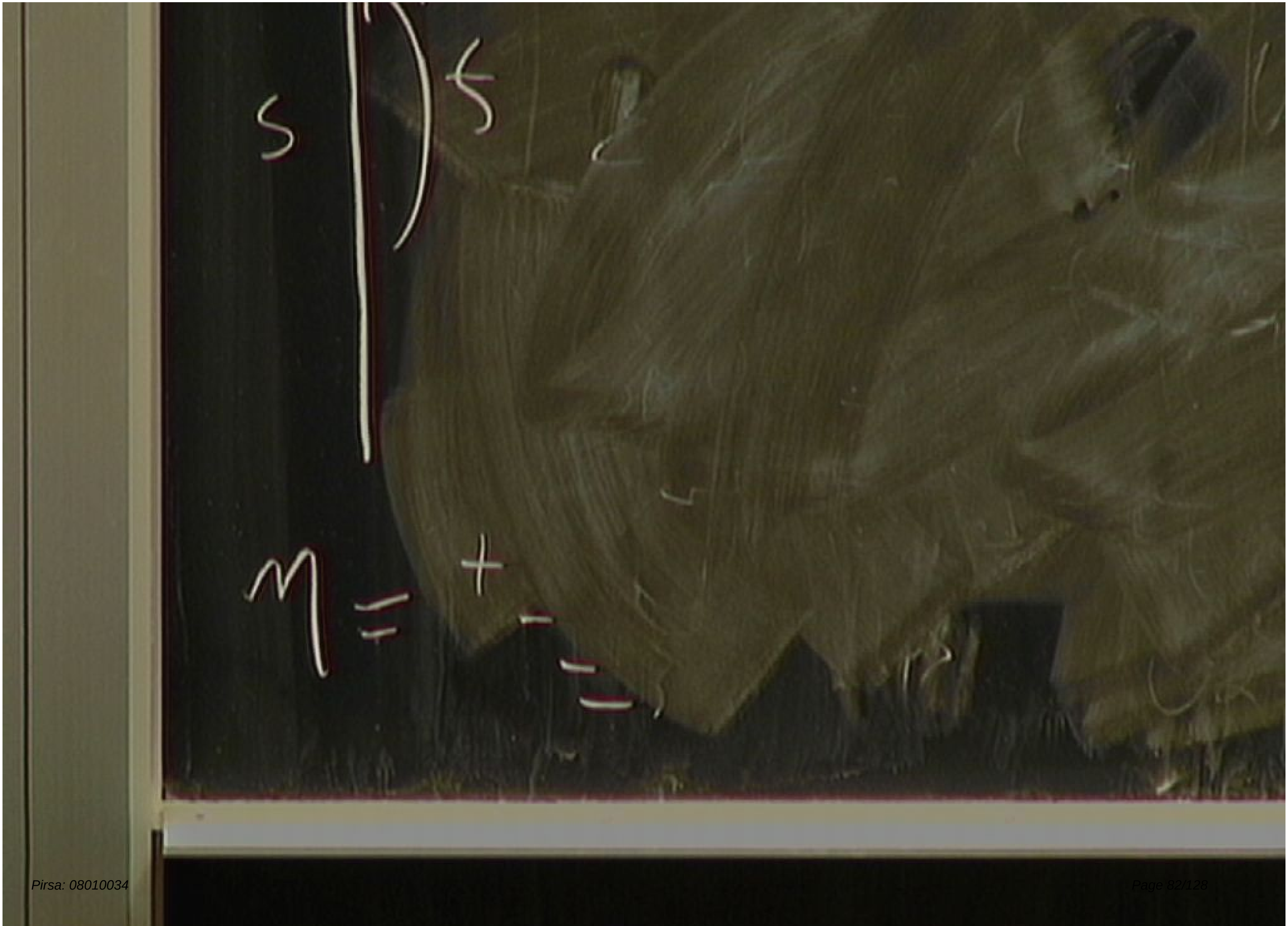
$$S = \int ds \left[p_a \dot{x}^a - \lambda \mathcal{H} \right.$$

ree relativistic particle $\quad P_a$

$$E^2 = p^2 + m^2 \qquad X^a(s$$

$$\mathcal{H} = E^2 - p^2 - m^2 =$$

$s \uparrow \qquad x^1(x^0 = 17) \qquad S = \int ds \left[P_a \right.$



relativistic particle $P_a = (E$

$E^2 = P^2 + m^2$ $X^a(s)$

$\mathcal{X} = E^2 - P^2 - m^2 = \eta^{ab} P_a P_b - m^2$

$X^1(X^0 = 17)$ $S = \int ds \, [P_a \dot{X}^a -$

...ativistic particle $\qquad P_a = (E, P\ldots$

$\ldots = P^2 + m^2 \qquad X^a(s)$

$\ldots = E^2 - P^2 - m^2 = \eta^{ab} P_a P_b - m^2$

$\ldots x'(x^0 = 17) \qquad S = \int ds \left[P_a \dot{x}^a - \lambda \ldots \right.$

$$s \rightarrow s' = f(s) \qquad S = \int ds\,[$$

$$ds \rightarrow ds' = f'\,ds$$

$$s \to s' = f(s) \qquad S = \int ds \,[$$

$$ds \to ds' = f' ds$$

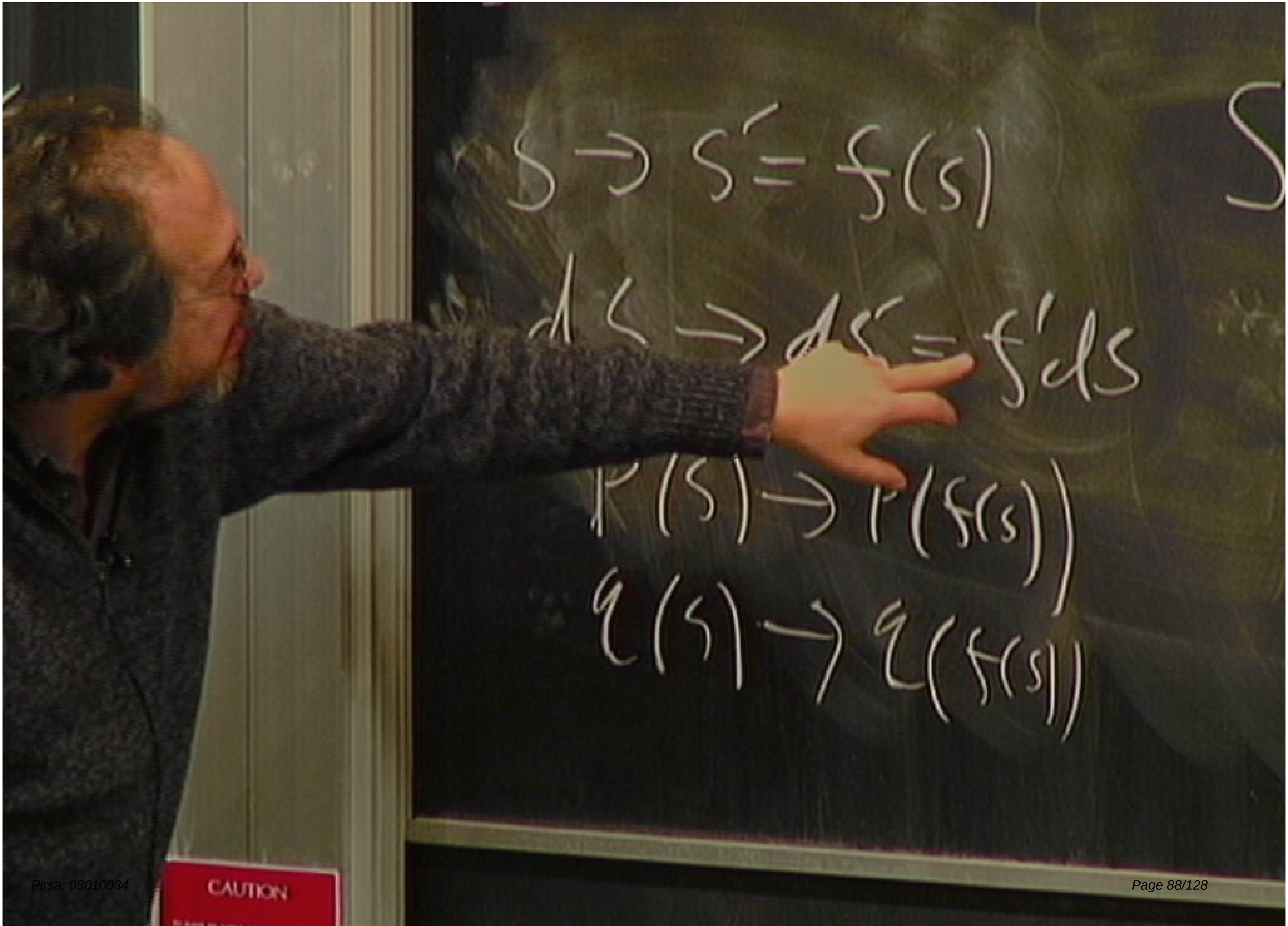
$$p(s) \to p(f(s))$$

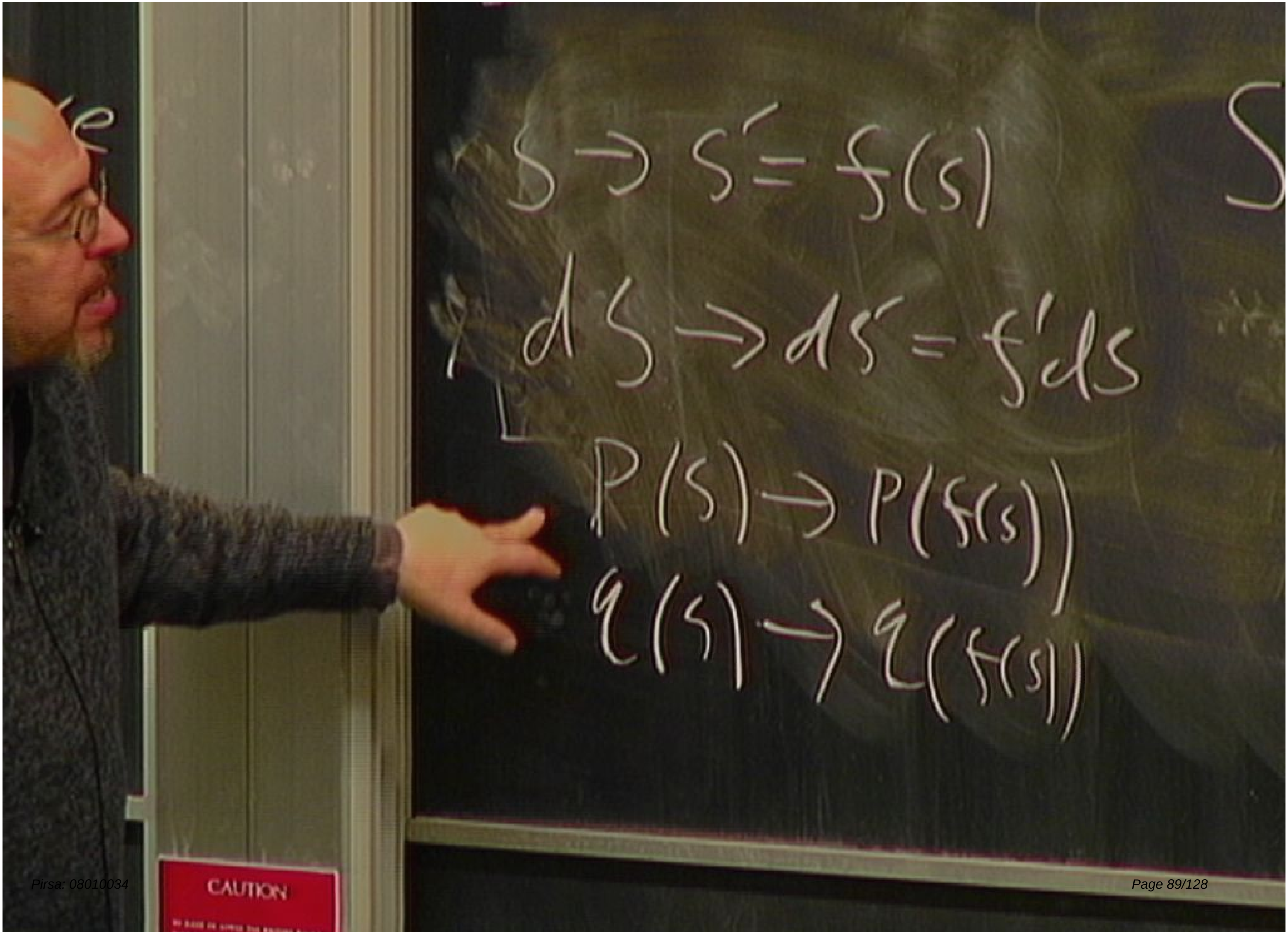
$$q(s) \to q(f(s))$$

$$S = \int ds \left[\underbrace{P_a \dot{x}'^a} - \lambda \mathcal{H} \right]$$

$$\int ds \frac{dx^a}{ds} P_a$$

$$\Rightarrow \int ds' \cancel{f'}^{-1} \frac{dx}{ds'} \cancel{f'}$$


s → s' = f(s)
ds → ds' = f' ds
p(s) → p(f(s))
q(s) → q(f(s))
CAUTION


s → s' = f(s)
ds → ds' = f' ds
P(s) → P(f(s))
q(s) → q(f(s))
CAUTION

$\lambda \phi x]$

$$\lambda(s) \rightarrow \lambda'(s') = \lambda(s) f'$$

$$\frac{dx}{ds'} = \lambda \frac{dx}{ds}$$

$$\lambda(s) \rightarrow \lambda(s') = \lambda(s) f'$$

$$\frac{dx}{ds'} = \lambda \frac{dx}{ds}$$

$\lambda(s)$ is a density on R

m^2

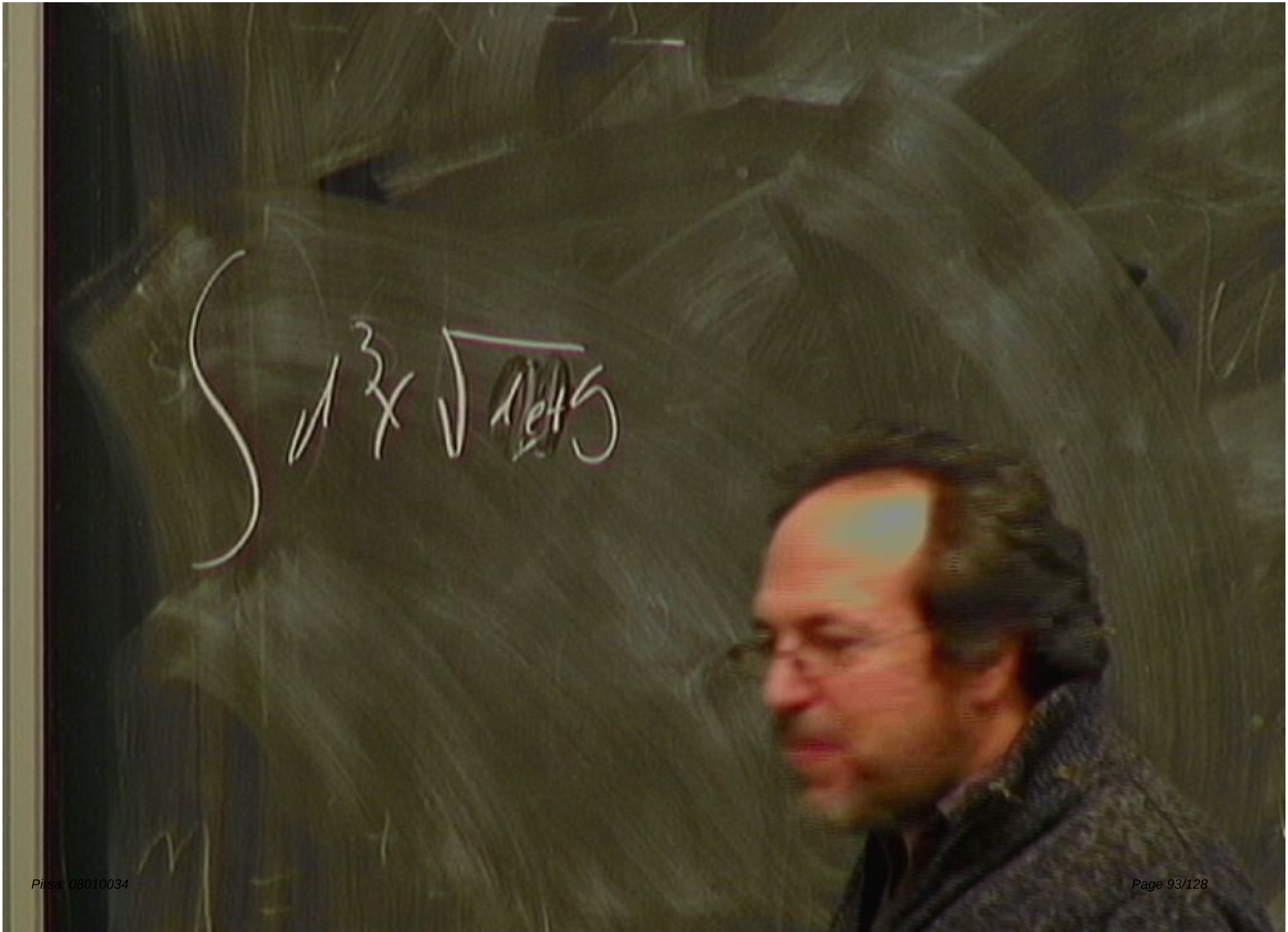
reparameterization

repar. invariant

$$-\lambda \cancel{m}] \rightarrow \int ds' [\; \ldots \;]$$

$$\lambda(s) \rightarrow \lambda'(s') = \lambda(s) f'$$

$$\frac{dx}{ds'} = \lambda \frac{dx}{\ldots}$$

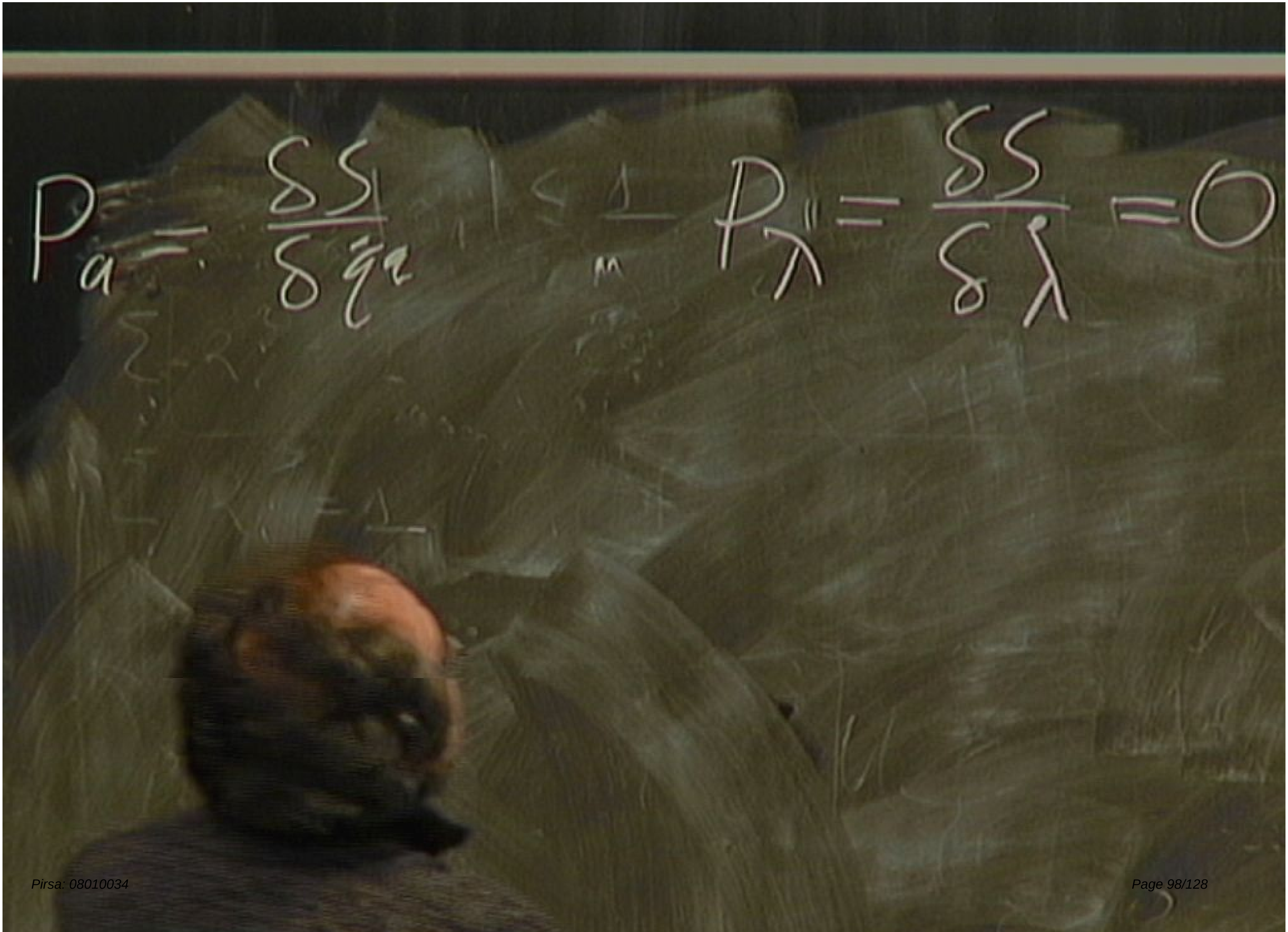


$$\int d^3x \sqrt{g}$$

$$P_a = \frac{\delta S}{\delta \dot{q}_a}$$

$$P_a = \frac{\delta S}{\delta \tilde{q}^a} \qquad P_\lambda = \frac{\delta S}{\delta \dot{\lambda}}$$

$$P_a = \frac{\delta S}{\delta \dot{q}^a} \qquad P_\lambda = \frac{\delta S}{\delta \dot{\lambda}}$$


$P_a = \frac{\delta S}{\delta \tilde{q}^a}$
$P_\lambda = \frac{\delta S}{\delta \lambda} = 0$

$$P_a = \frac{\delta S}{\delta \xi^a} \qquad P_\lambda = \frac{\delta S}{\delta \dot{\lambda}} = 0$$

$$\Pi \equiv P_x = 0$$

$$\frac{\delta \ldots}{\delta \lambda} = 0 \qquad H \equiv P_\lambda = 0$$

$$H' = H^0 + \lambda' P_\lambda$$

$$H^{0} = p\dot{q} - \mathcal{L} = \lambda \mathcal{H}$$

$$H' = \lambda \mathcal{H} + \lambda' p_{T}$$

$$H \equiv p + \lambda\lambda$$

$$\dot{\Pi} = 0 = \{\Pi_\lambda, H'\}$$

$$\{q^a, p_b\} = \delta^a_b$$

$$\{\lambda, p_\lambda\} = 1$$

$$\dot{\pi} = 0 = \{p_\lambda, H'\} = -\mathcal{H}$$

$$= 0 \qquad \Pi \equiv P_\lambda = 0$$

$$\mathcal{K} = \eta^{ab} p_a p_b - m$$

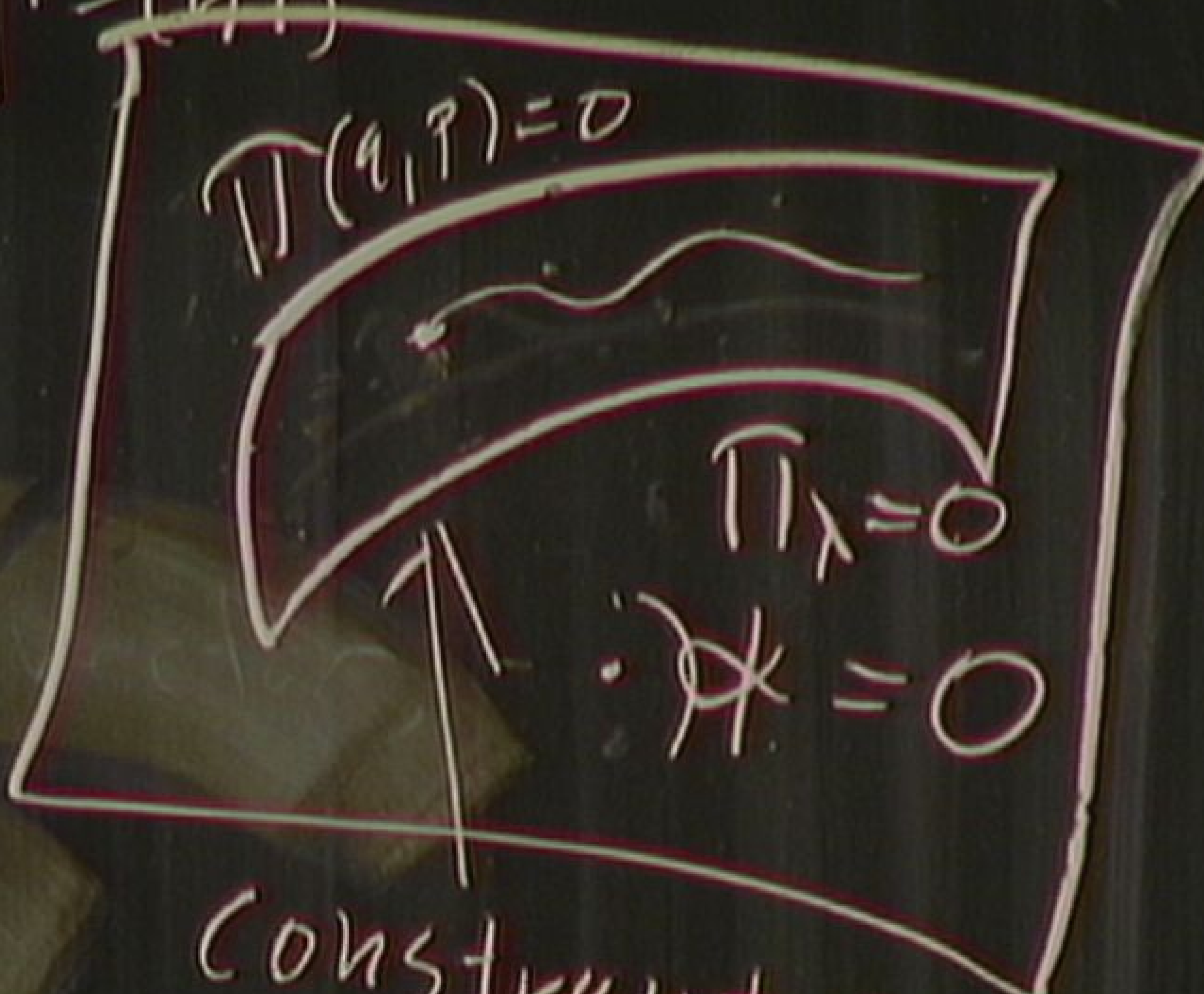
$$H' = H^0 + \lambda' P_\lambda$$

$$\dot{\Pi} = 0 = \{P_\lambda, H'\} = -\mathcal{K}$$

$[(q, p)]$

$\Gamma = (q, p)$

$C(q, p)$



Constraint surface

$$\boxed{H' = \lambda \mathcal{H} + \lambda' P_\lambda}$$

$$\{q^a, p_b\} = \delta^a_b$$

$$\{\lambda, P_\lambda\} = 1$$

$$\dot{\pi} \stackrel{?}{=} 0 = \{\lambda, H'\} = -\mathcal{H}$$

$$\dot{\mathcal{H}} \stackrel{?}{=} 0 = \{\mathcal{H}, H'\}$$

$$\dot{\Pi} \overset{?}{=} 0 = \{P_\lambda, H'\} = -\mathcal{H}$$

$$\dot{\mathcal{H}} \overset{?}{=} 0 = \{\mathcal{H}, H'\} = 0$$

$$H^{0} = p\dot{q} - \mathcal{L} = \lambda \mathcal{H}$$

$$\boxed{H' = \lambda \mathcal{H} + \lambda' P_{\lambda}}$$

$$\{q^{a}, p_{b}\} = \delta^{a}_{b}$$

$$\{\lambda, P_{\lambda}\} = 1$$

$$\ldots = \delta^a_b$$

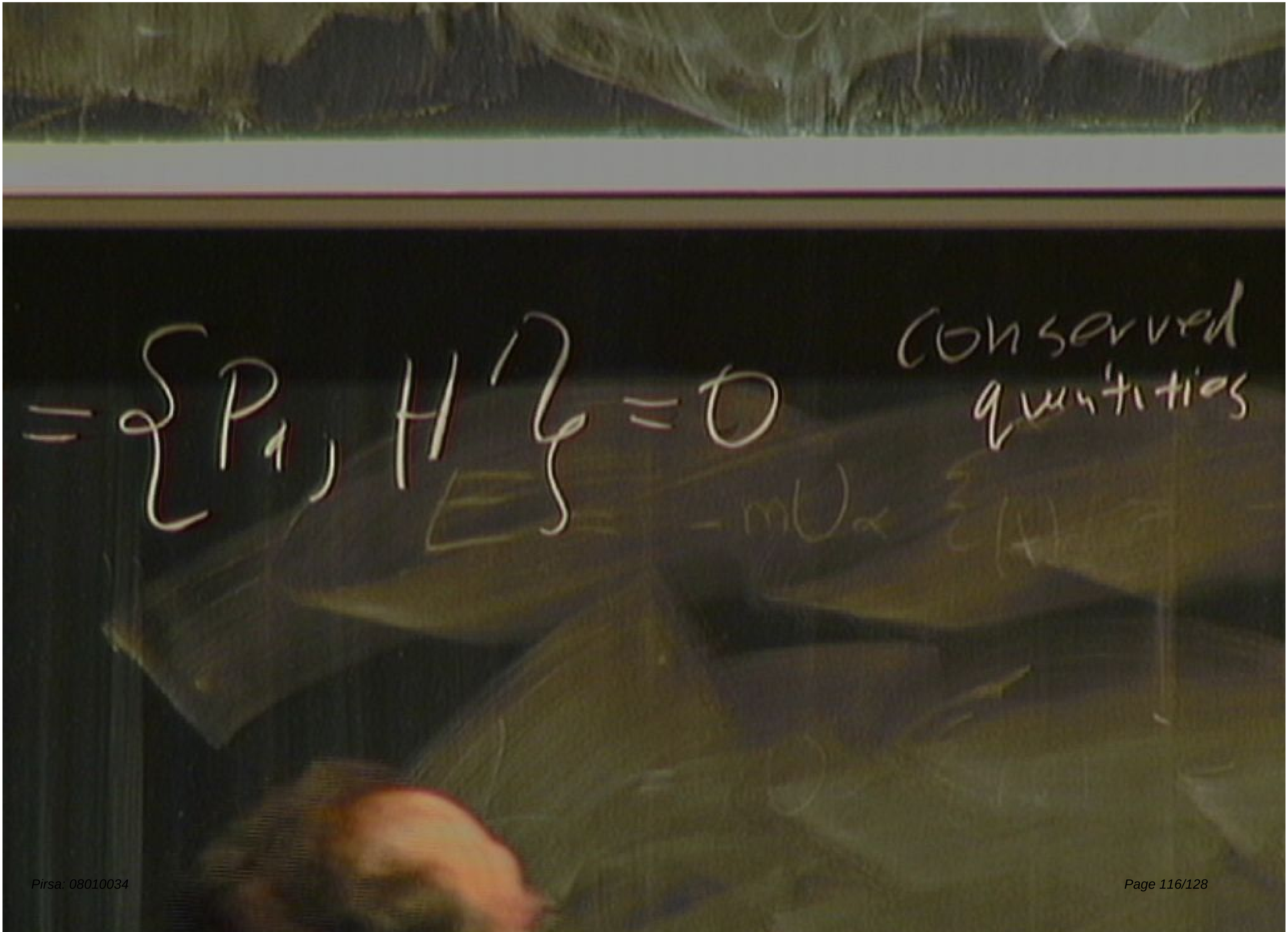
$$\ldots = 1 \Rightarrow H' = 0$$

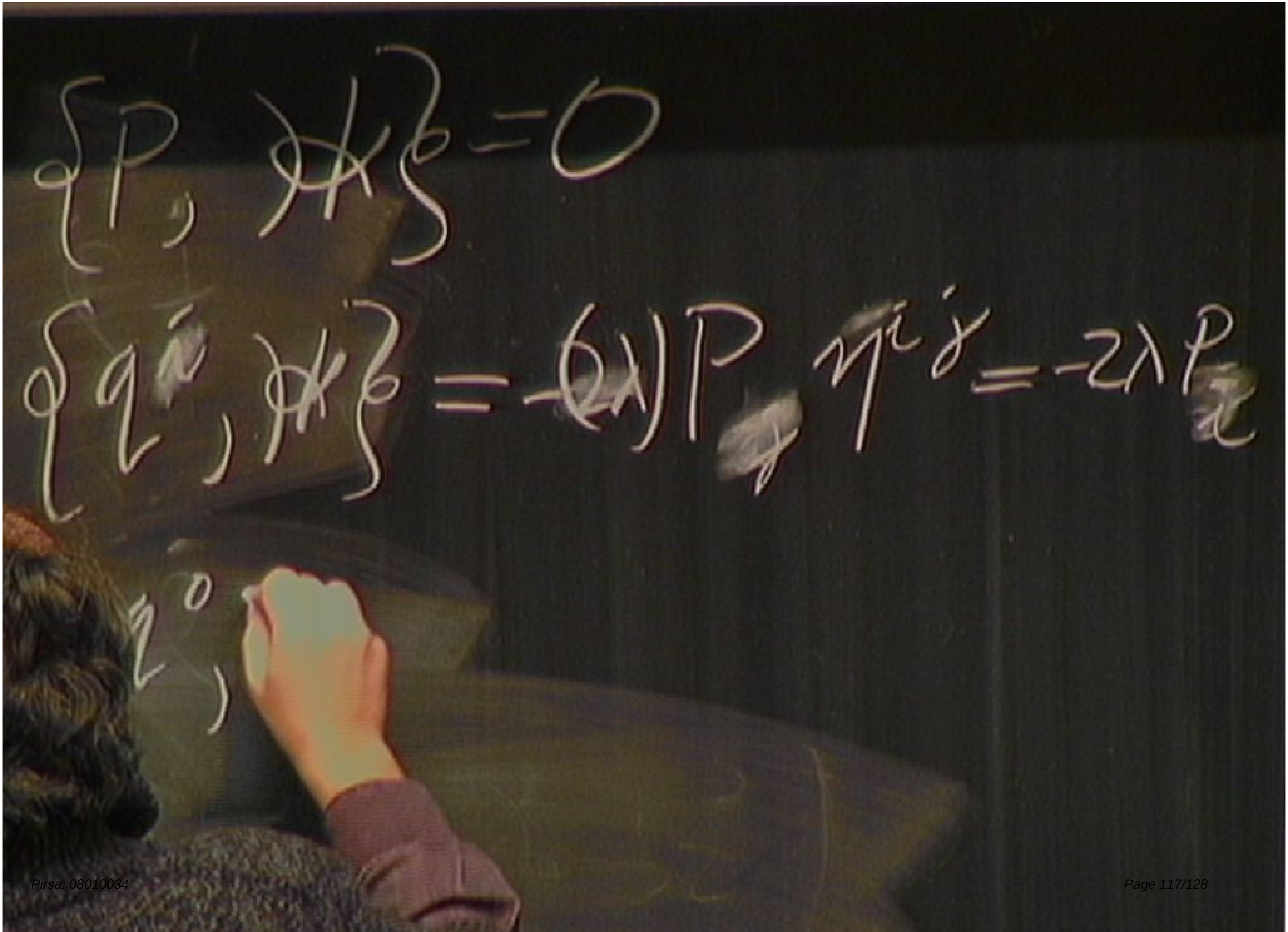
$$\dot{H}' \doteq 0 = \{H' \ldots$$

$$\dot{p}_a = \{ p_a , H \}$$

$$\{P, \mathcal{H}\} = 0$$

$$\dot{P}_a = \{P_a, H\} = 0$$


$= \{P_1, H'\} = 0$
conserved quantities


{P, H} = 0
{q^i, H} = -(2λ)P_j η^{ij} = -2λP_i
q^0,

$$\{p, \mathcal{H}\} = 0$$

$$\{q^i, \mathcal{H}\} = -(2\lambda) p_j \eta^{ij} = -2\lambda p$$

$$\{q^0, \mathcal{H}\} = -2\lambda p_0 = 2\lambda E$$

$$\bar{q}^{i} = -2\lambda P_{i}$$

$$\bar{q}^{0} = 2\lambda E$$

$$p_a = \{p_a, H\} = 0$$

$$\boxed{\begin{aligned} \dot{q}^i &= -2\lambda p_i \\ \dot{q}^0 &= 2\lambda E \end{aligned}}$$

$$\{q^i, \mathcal{H}\}_b = -2\lambda p^i \ldots = -2\lambda \ldots$$

$$\{q^0, \mathcal{H}\}_b = -2\lambda P_b = -2\lambda E$$

$$\lambda < 0$$

$$q^0 = 2\lambda E$$

$$\boxed{\begin{aligned} q^i(s) &= (-2\lambda) s\, P_i + a^i \\ q^0(s) &= (2\lambda s) E + a^0 \end{aligned}}$$

$$\{P, \mathcal{H}\} = 0$$

$$\{q^i, \mathcal{H}\} = -(\;)P_j \eta^{ij} = -2$$

$$\{q^0, \mathcal{H}\} = -2 \quad P_0 = 2 \quad E$$

$$< 0$$

$$\boxed{\begin{aligned} q^i(s) &= (-2\lambda)s\,P_i + a^i \\ q^0(s) &= (2\lambda s)E + a^0 \end{aligned}}$$

$$2\lambda s = \frac{q^0 - a^0}{E}$$

$$\boxed{q^i[q^0] = \frac{p_i}{E}\left(q^0 - q^0\right) + a^i}$$

$$\boxed{s \to s' = f(s)}$$ monoton

reparameterization

m^2 repar. invariant

$$\lambda \,\cancel{x}\, \Big] \to \int ds' \Big[\cdots$$

$$\lambda(s) \to \lambda(s') = \lambda(s) f'$$

conserved quantities

$$\{P, \mathcal{H}\}_b = 0$$

$$\{q^i, \mathcal{H}\}_b = -2 P_j \eta^{ij} = -2 P_i$$

$$\{q^0, \mathcal{H}\}_b = -2 P_0 = -2 E$$

$$< 0$$

$$\boxed{q^i[q^0] = \frac{P_i}{E}(q^0 - a^0) + a^i}$$

$$\boxed{q^i[q^0] = \frac{p^i}{E}(q^0 - a^0) + a^i}$$

$$\{q^i[q^0], \phi^k\} = 0$$

CHECK