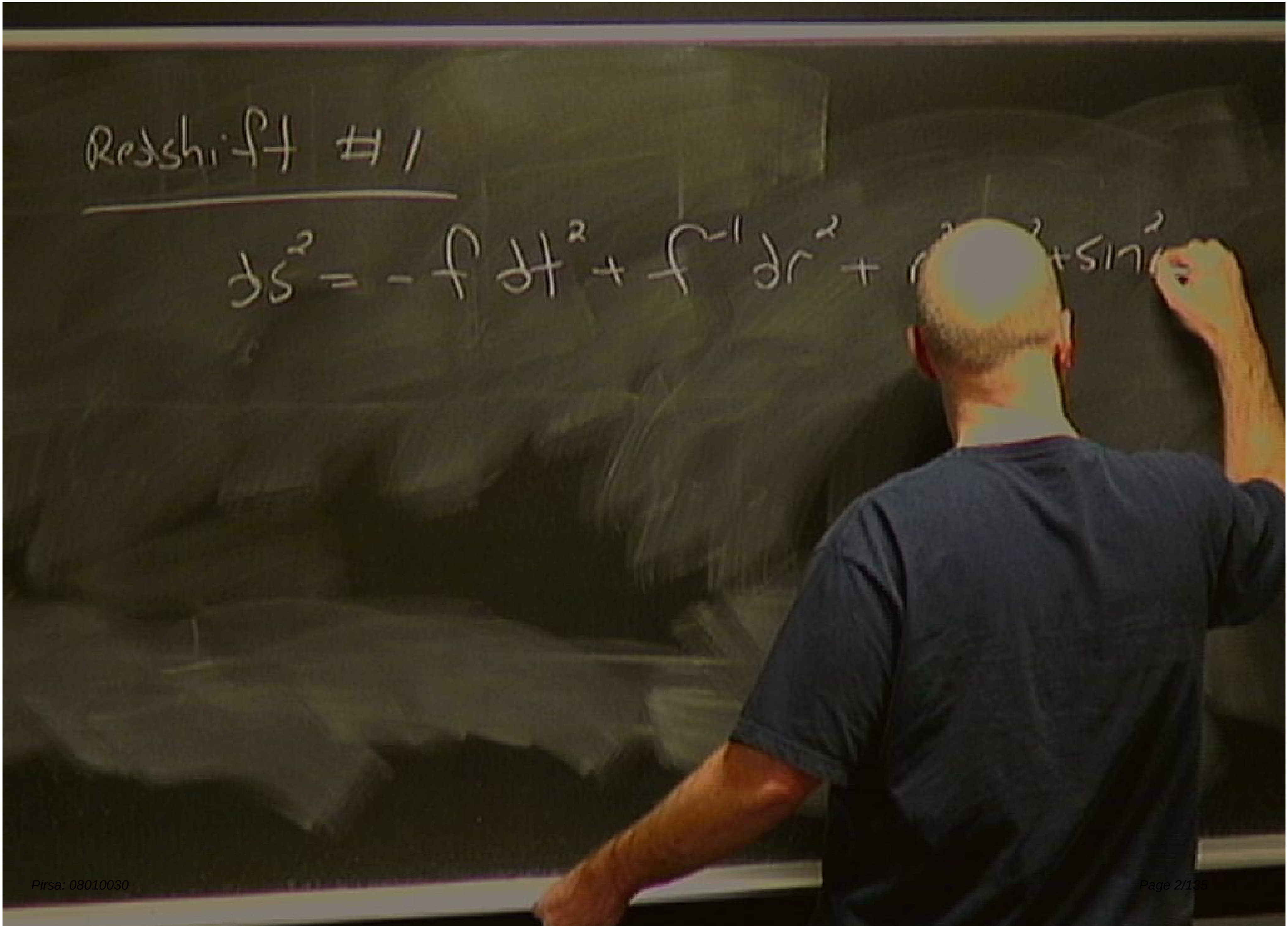
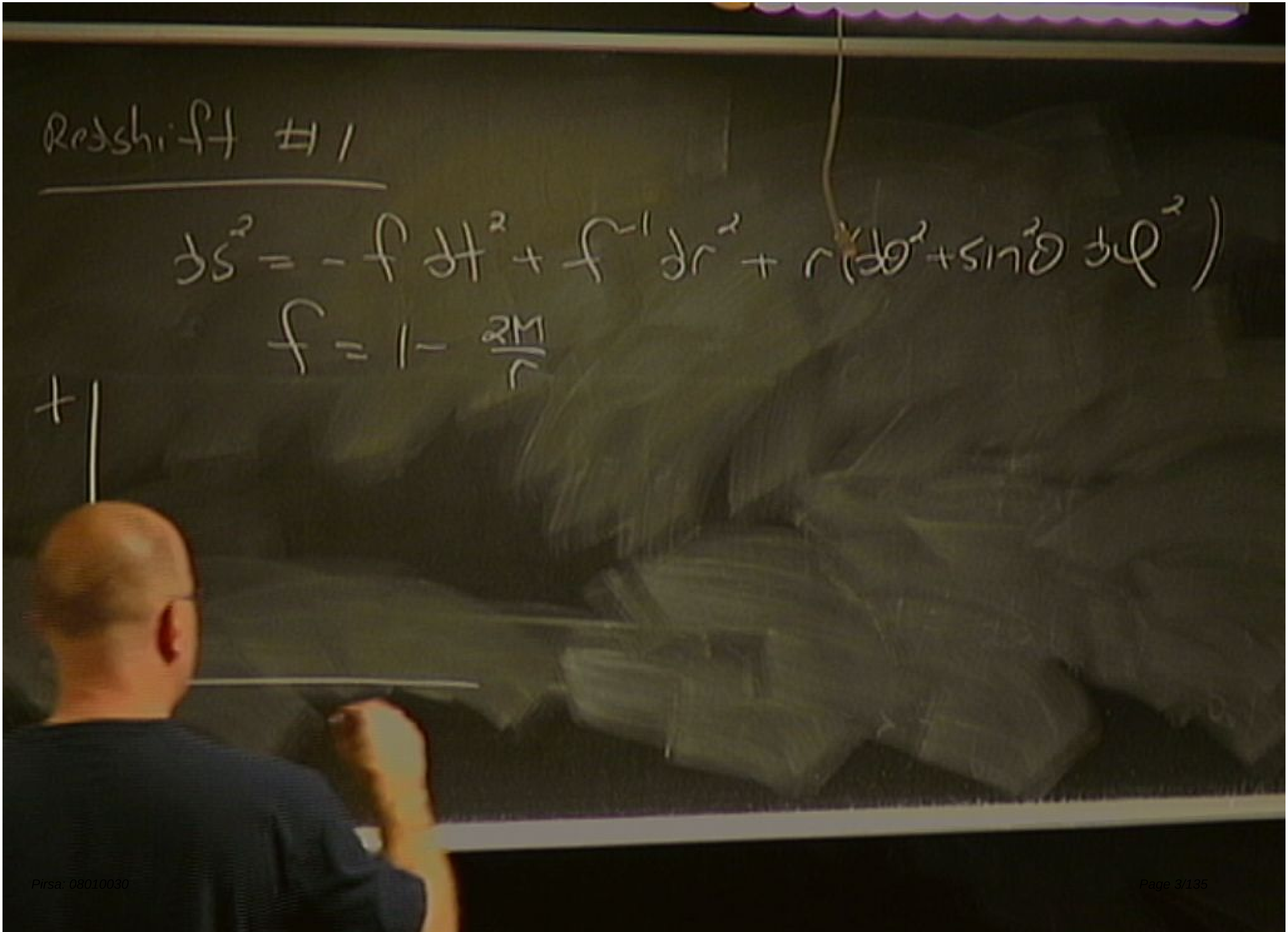
Title: Advanced General Relativity - Lecture 2B

Date: Jan 16, 2008 04:00 PM

URL: http://pirsa.org/08010030

Abstract: Advanced General Relativity

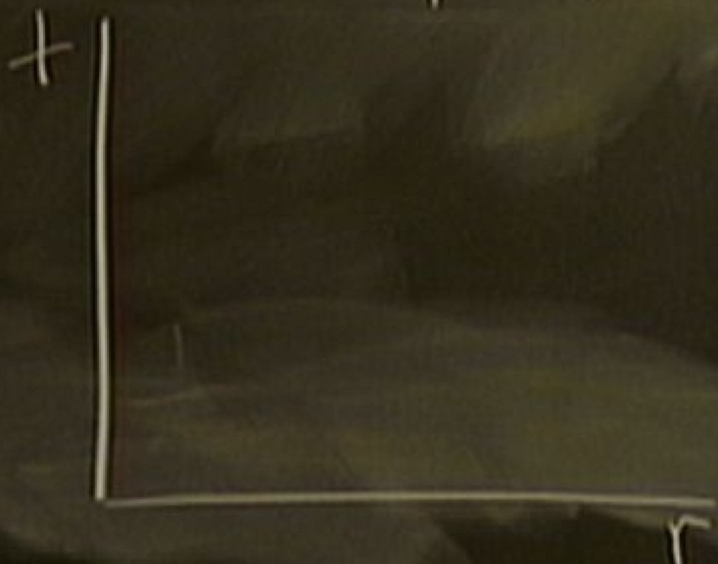

Redshift #1
ds² = - f dt² + f⁻¹ dr² +


Redshift #1
$ds^2 = -f\,dt^2 + f^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$
$f = 1 - \frac{2M}{r}$
t

Redshift #1

$$ds^2 = -f\,dt^2 + f^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

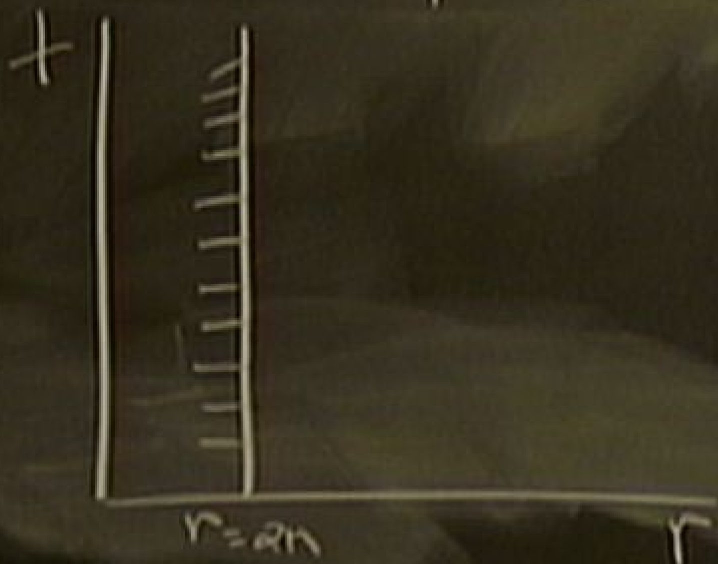
$$f = 1 - \frac{2M}{r}$$



Redshift #1

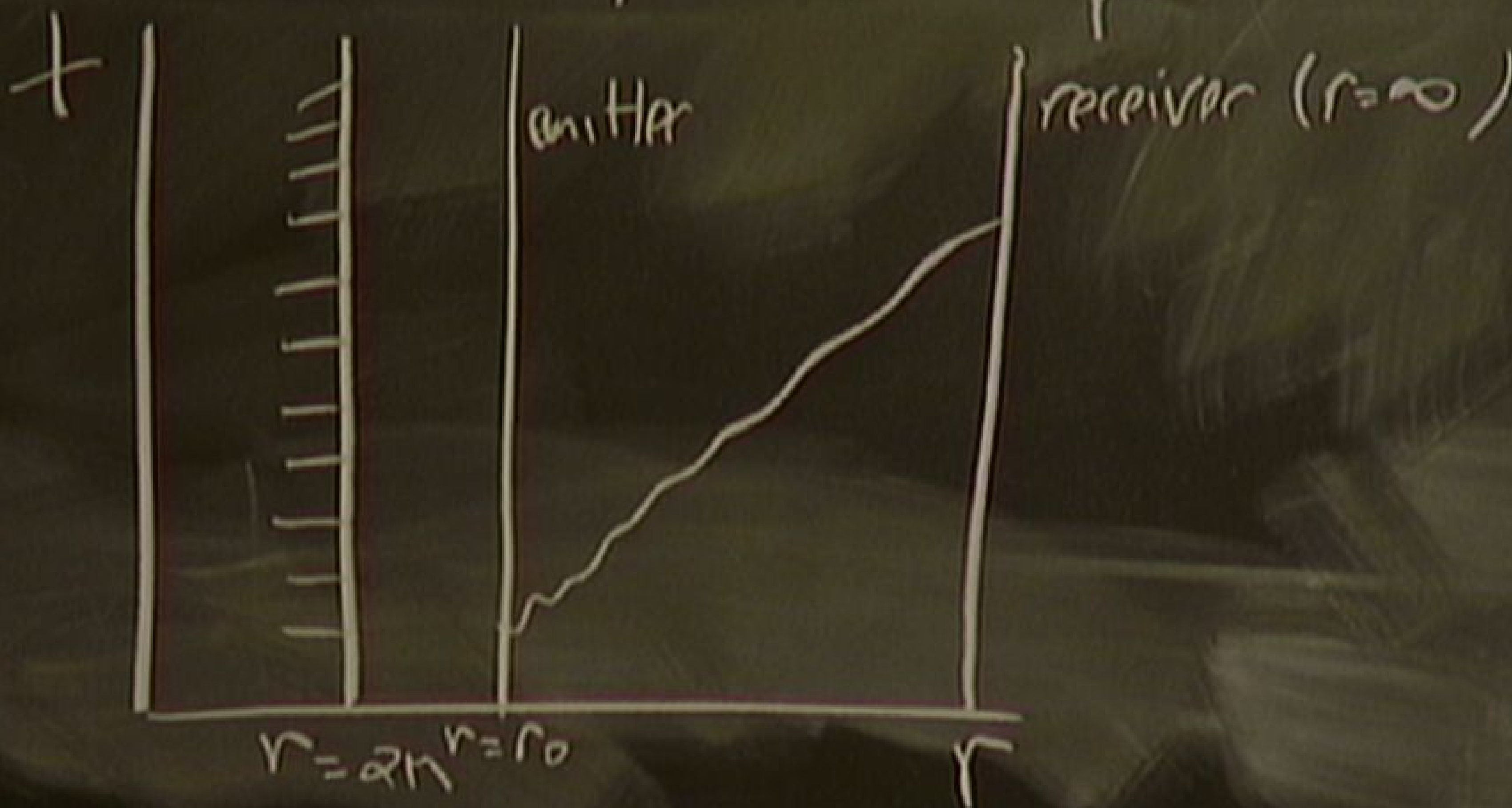
$$ds^2 = -f\,dt^2 + f^{-1}\,dr^2 + r^2(d\theta^2 + \sin^2\theta\,d\varphi^2)$$

$$f = 1 - \frac{2M}{r}$$



$$ds = -f\,dt + f\,dr +$$

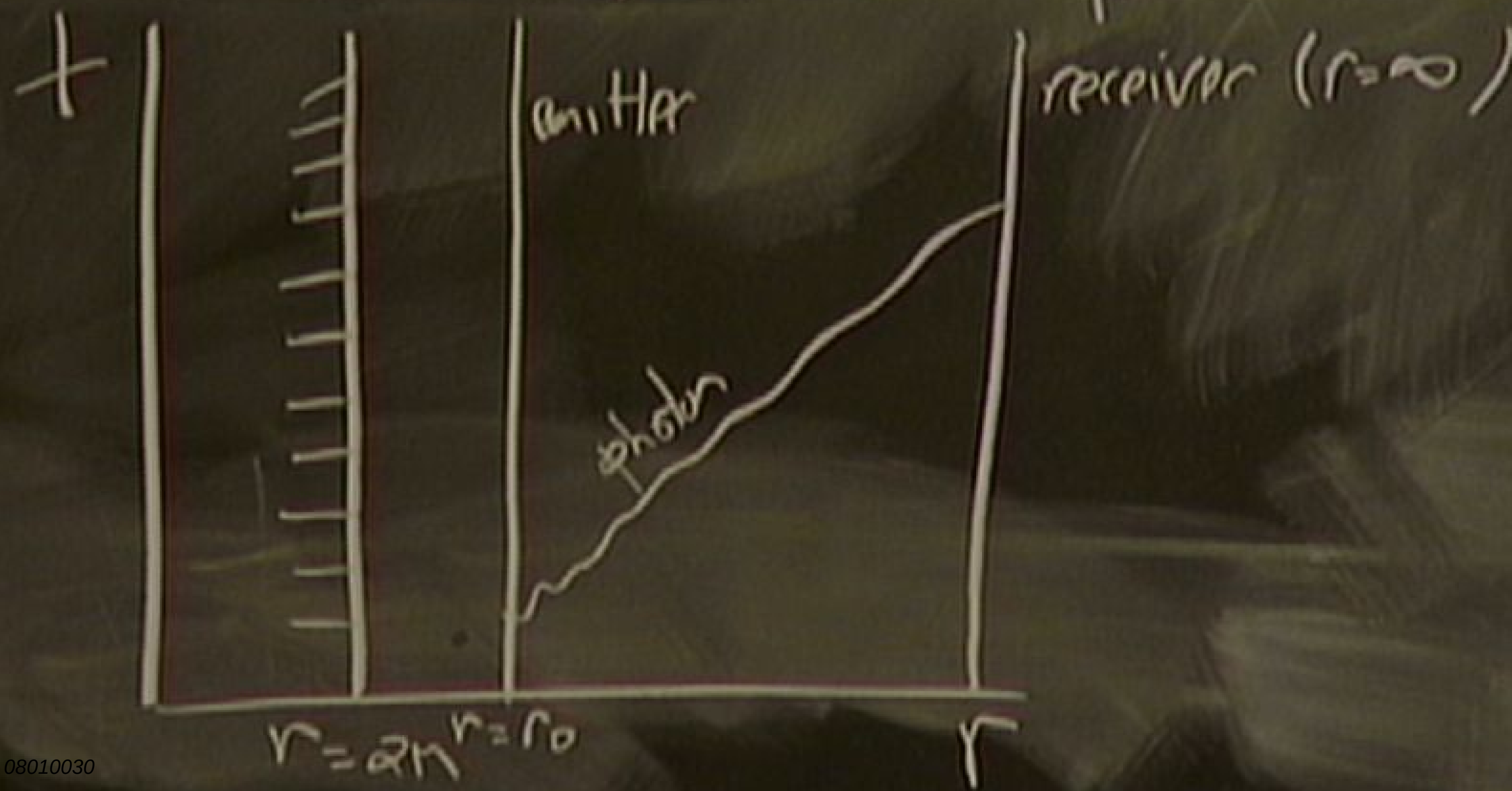
$$f = 1 - \frac{2M}{r}$$

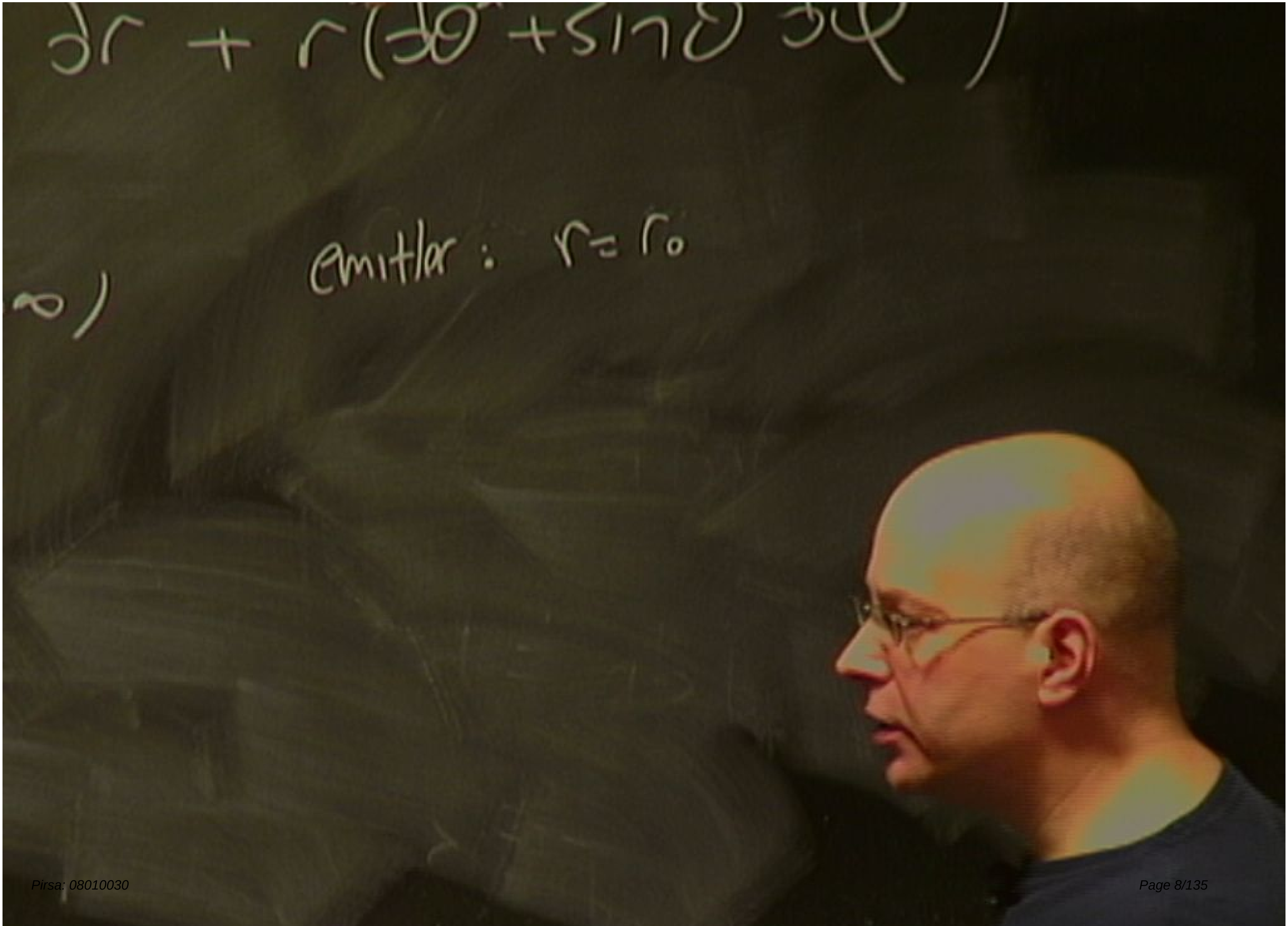


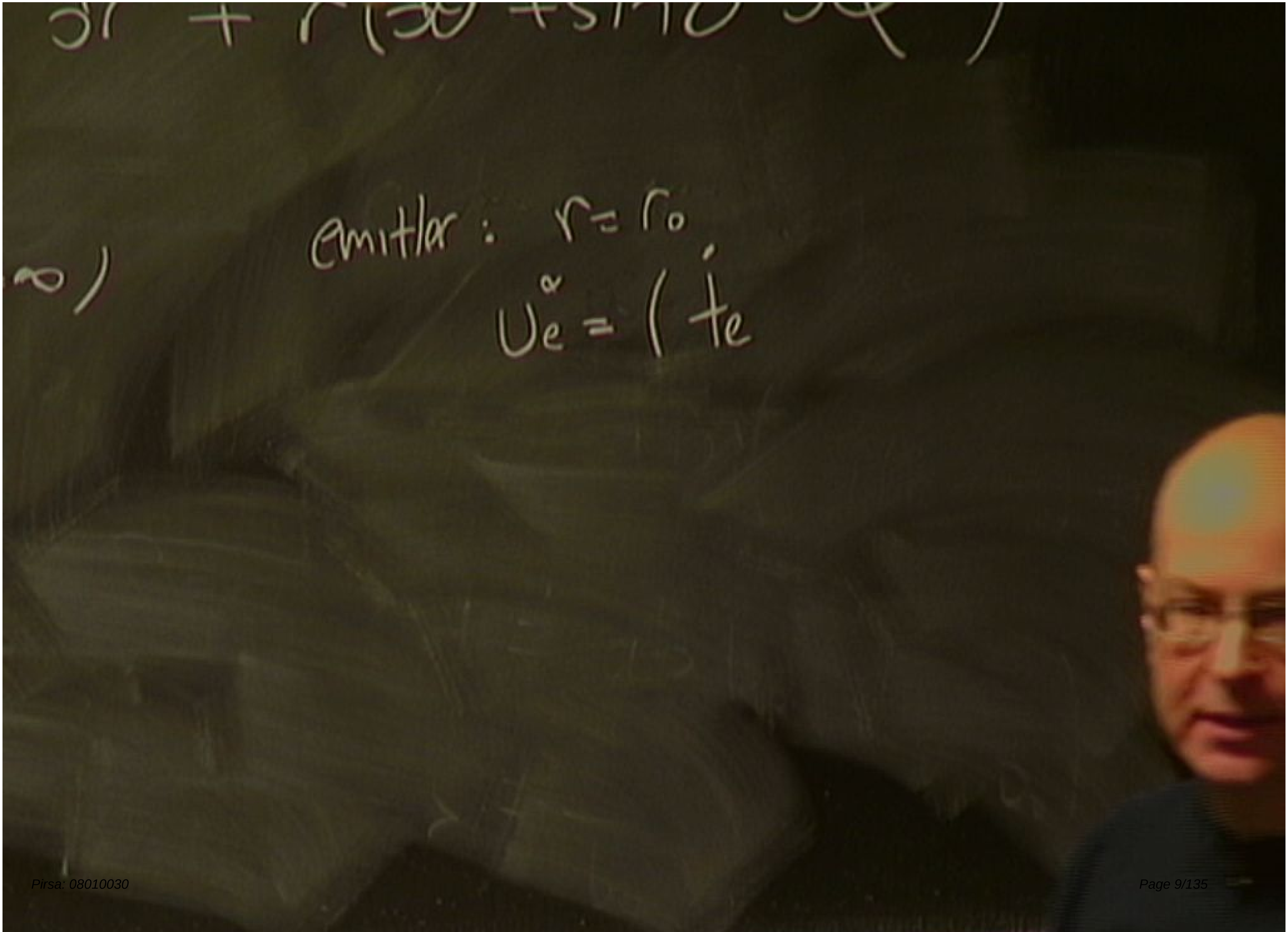
Redshift #1

$$ds^2 = -f\,dt^2 + f^{-1}\,dr^2 + r^2$$

$$f = 1 - \frac{2M}{r}$$



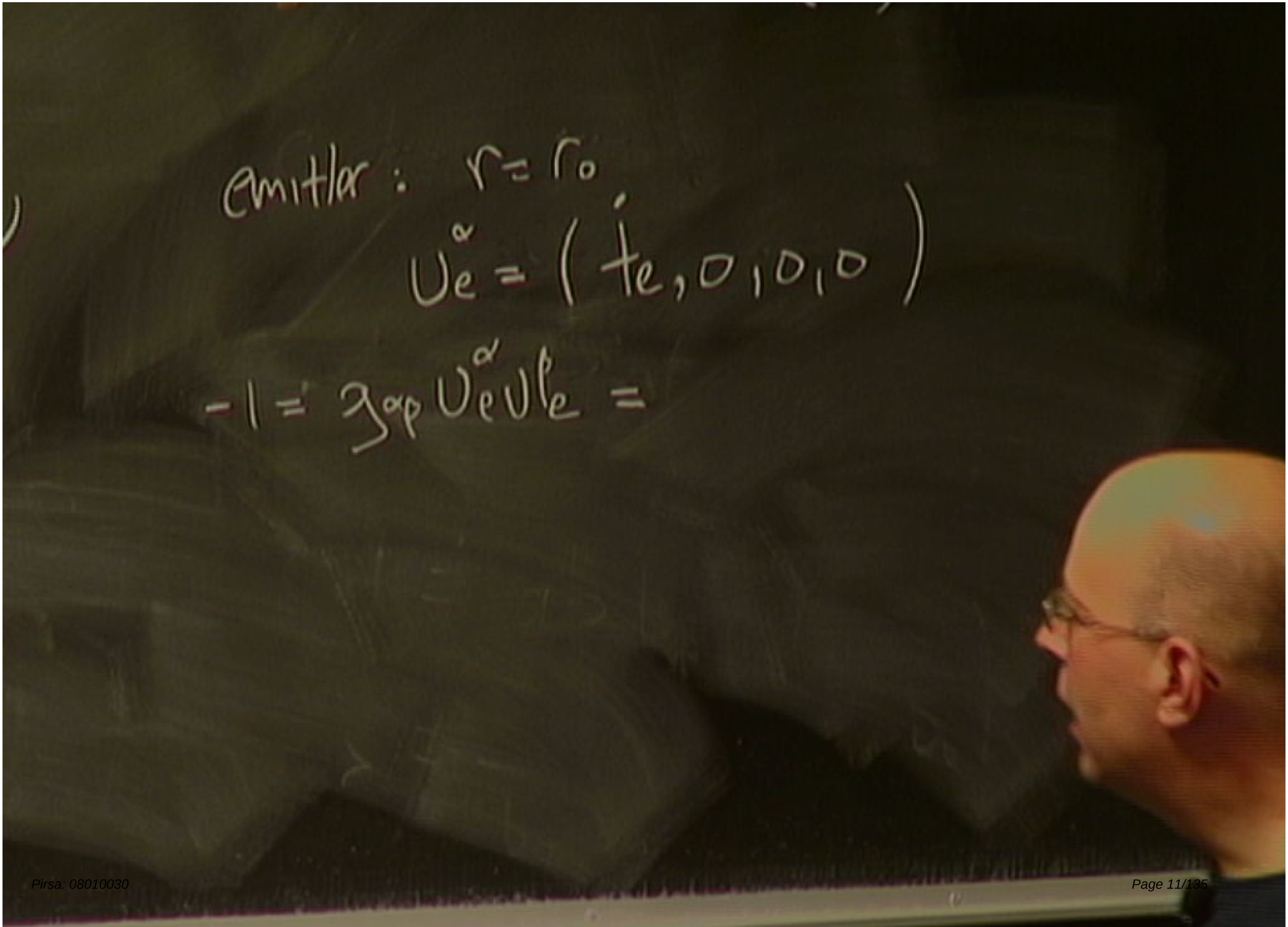

emitter : r = r0


emitter : $r = r_0$
$U_e^\alpha = (\dot{t}_e$

$\infty)$

emitter: $r = r_0$

$U_e^\alpha = (\dot{t}_e, 0, 0, 0)$


emitter: $r = r_0$
$U^\alpha_e = (\dot{t}_e, 0, 0, 0)$
$-1 = g_{\alpha\beta} U^\alpha_e U^\beta_e =$

emitter: $r = r_0$

$$U_e^\alpha = (\dot{t}_e, 0, 0, 0)$$

$$-1 = g_{\alpha\beta} U_e^\alpha U_e^\beta = g_{tt} (\dot{t}^e)^2$$

emitter: $r = r_0$

$$U_e^{\alpha} = (\dot{t}_e, 0, 0, 0)$$

$$-1 = g_{\alpha\beta} U_e^{\alpha} U_e^{\beta} = g_{tt} (\dot{t}^e)^2 = -f_e \dot{t}_e^2$$

$$\dot{t}_e = \frac{1}{\sqrt{f_e}} = \frac{1}{\sqrt{1 - \frac{2M}{r_0}}}$$

receiver : $r = \infty$

$$U_r^\alpha = (1, 0, 0, 0)$$

receiver : $r = \infty$

$$U_r^{\alpha} = (1, 0, 0, 0)$$

photon :

receiver : $r = \infty$

$$U_r^\alpha = (1, 0, 0, 0)$$

photon : $(\dot\theta = \dot\varphi = 0)$

receiver : $r = \infty$

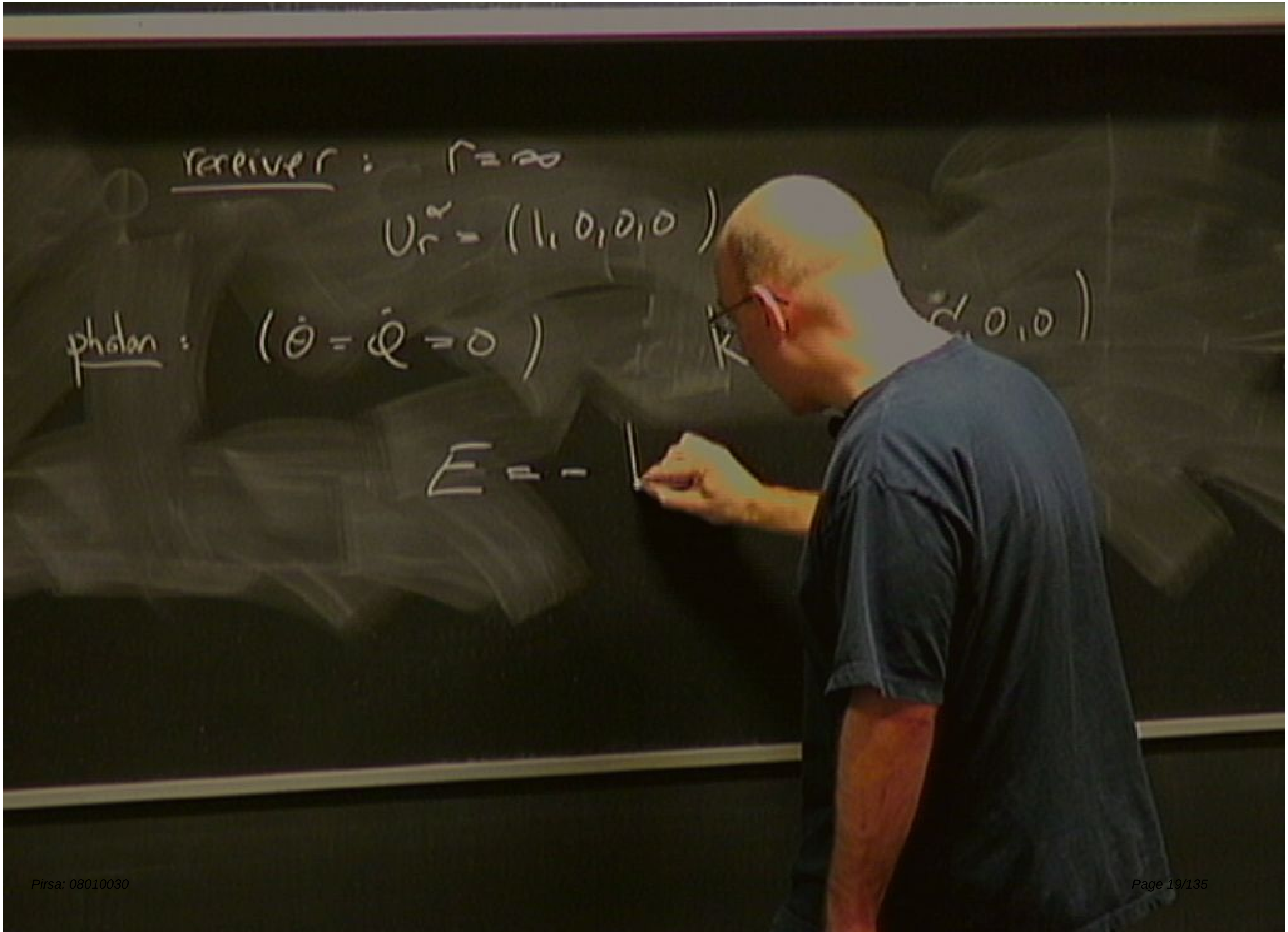
$$U_r^\alpha = (1, 0, 0, 0)$$

photon : $(\dot\theta = \dot\varphi = 0)$ $\quad K^\alpha = (\dot t, \dot r, 0, 0)$

receiver : $r = \infty$

$$U_r^\alpha = (1, 0, 0, 0)$$

photon : $(\dot\theta = \dot\varphi = 0)$ $\quad K^\alpha = (\dot t, \dot r, 0, 0)$


receiver : r = ∞
U_r^α = (1, 0, 0, 0)
photon : (θ̇ = φ̇ = 0)
E = -

receiver : $r = \infty$

$$U_r^{\alpha} = (1, 0, 0, 0)$$

photon : $(\dot{\theta} = \dot{\varphi} = 0)$ $\quad K^{\alpha} = (\dot{t}, \dot{r}, 0, 0)$

$$E = -k_{\alpha} \xi^{\alpha}_{(t)}$$

receiver : $r = \infty$

$$U_r^{\alpha} = (1, 0, 0, 0)$$

photon : $(\dot{\theta} = \dot{\varphi} = 0)$ $\quad k^{\alpha} = (\dot{t}, \dot{r}, 0, 0)$

$$E = -\, k_{\alpha}\, \xi^{\alpha}_{(t)} =$$

$: \quad r = \infty$

$$U_r^\alpha = (1, 0, 0, 0)$$

$$= \dot{Q} = 0 \,) \qquad K^\alpha = (\dot{t}, \dot{r}, 0, 0)$$

$$E = -k_\alpha \xi^\alpha_{(t)} = -k_t = -g_{tt} k^t$$

$r: \quad r = \infty$

$$U_r^\alpha = (1, 0, 0, 0)$$

$$= \dot{Q} = 0 \;) \qquad K^\alpha = (t', r', 0, 0)$$

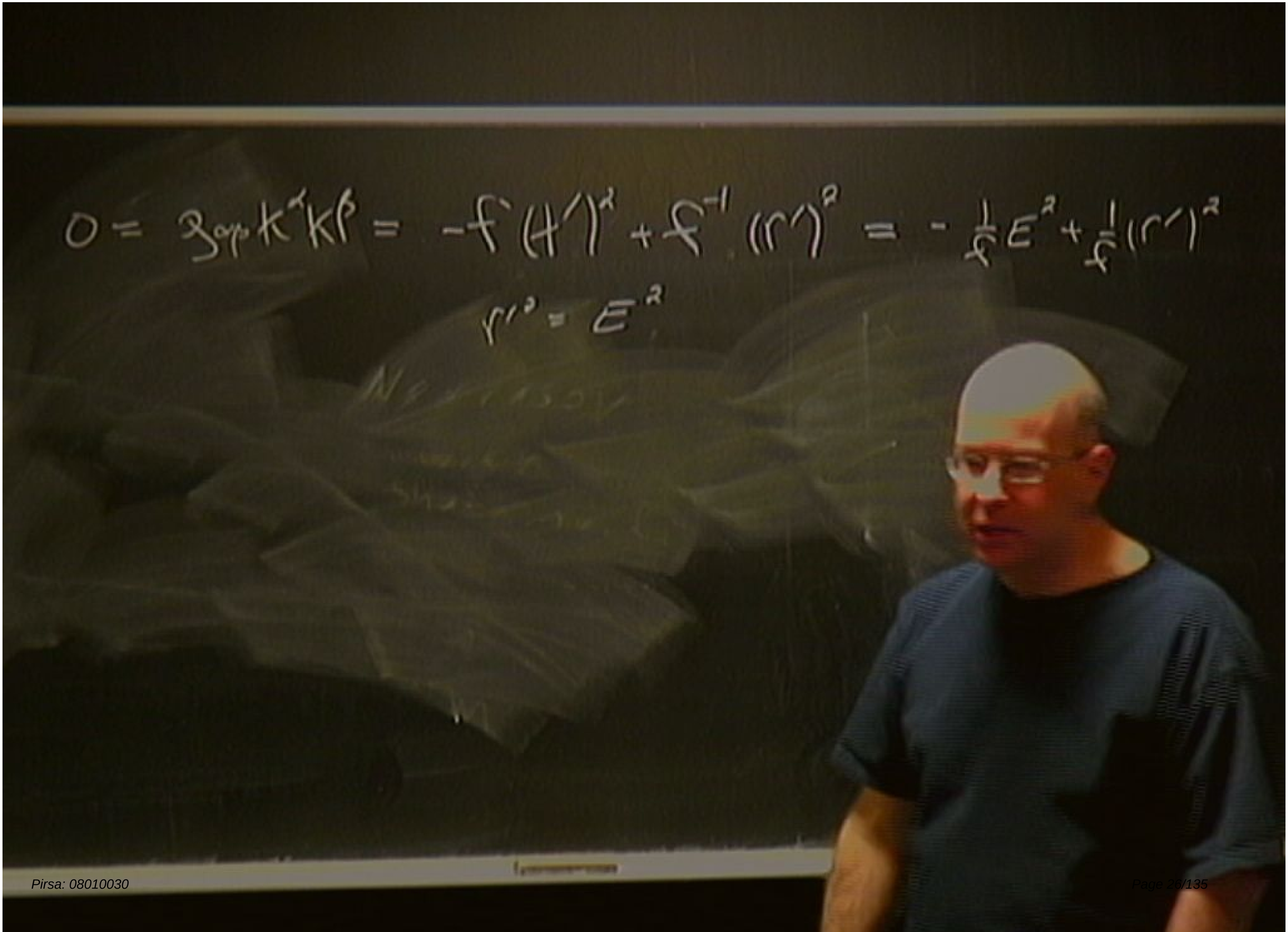
$$E = -k_\alpha \xi^\alpha_{(t)} = -K_t = -g_{tt} K^t = f t'$$

$$= 0) \qquad K^\alpha = (t', r', 0, 0)$$

$$E = -k_\alpha \xi^\alpha_{(t)} = -k_t = -g_{tt} k^t = f t'$$

$$\Rightarrow t' = \frac{E}{f}$$

$$0 = g_{\alpha\beta} k^{\alpha} k^{\beta} = -f (t')^2 + f^{-1} (r')^2$$

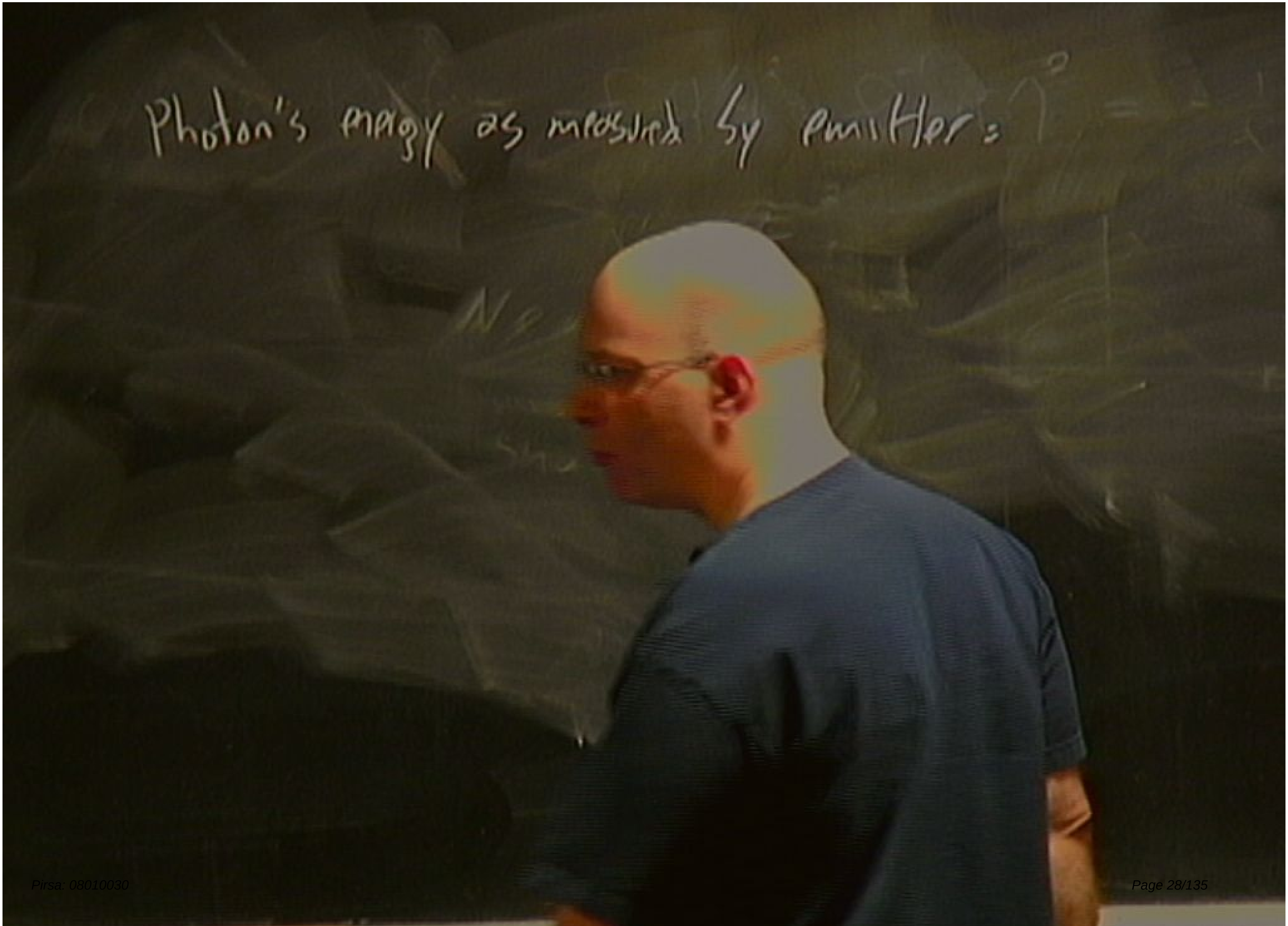

$0 = g_{\alpha\beta} k^\alpha k^\beta = -f (t')^2 + f^{-1} (r')^2 = -\frac{1}{f} E^2 + \frac{1}{f} (r')^2$
$r'^2 = E^2$

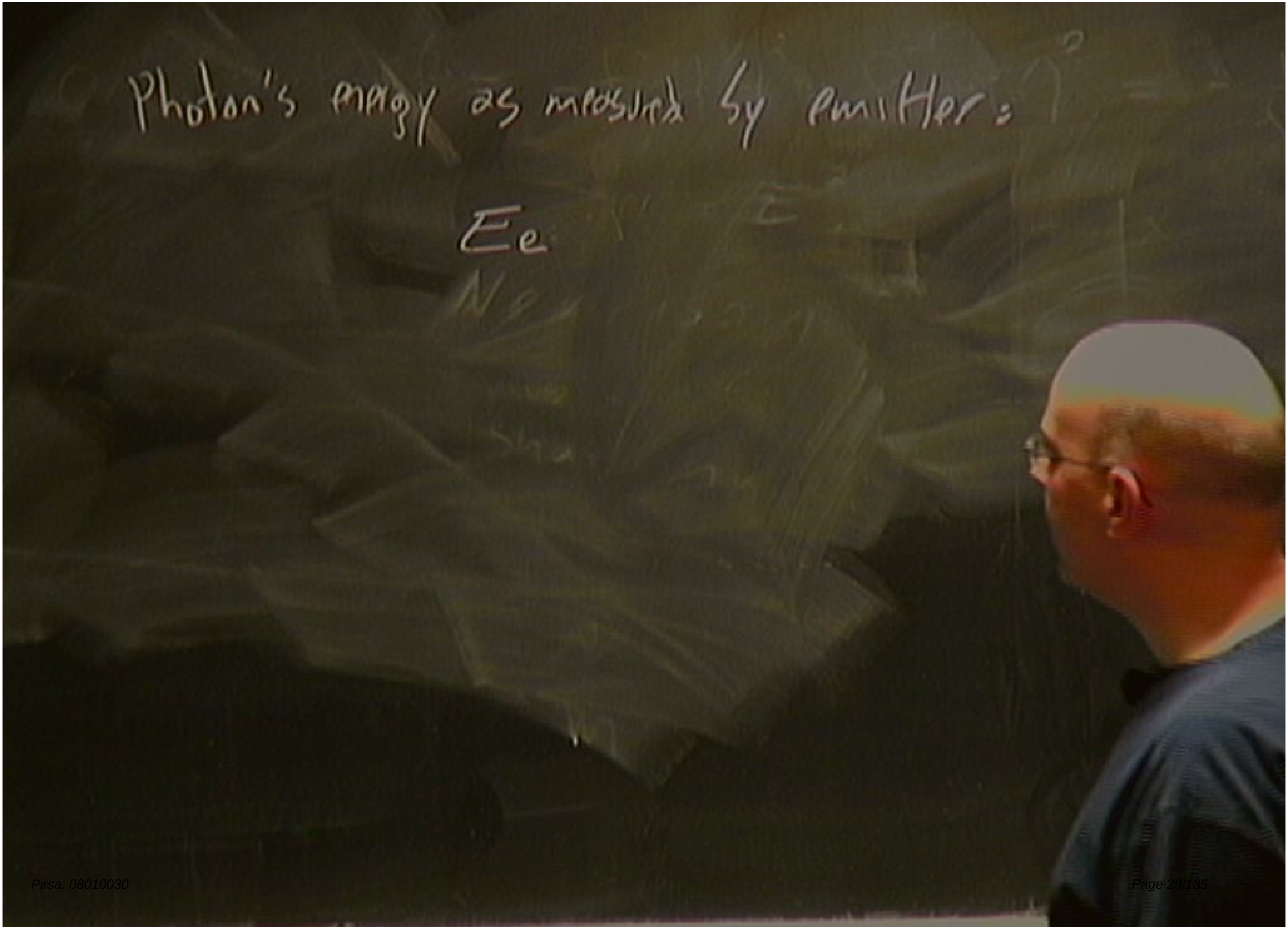
$$U_r^\alpha = (1, 0, 0, 0)$$

$$= \dot{\varphi} = 0 \,) \qquad K^\alpha = (t', r', 0, 0)$$

$$E = -k_\alpha \xi^\alpha_{(t)} = -k_t = -g_{tt} k^t = f t'$$

$$\Rightarrow t' = \frac{E}{f} \qquad r' = E$$


Photon's energy as measured by emitter:


Photon's energy as measured by emitter:
Ee

Photon's energy as measured by emitter:

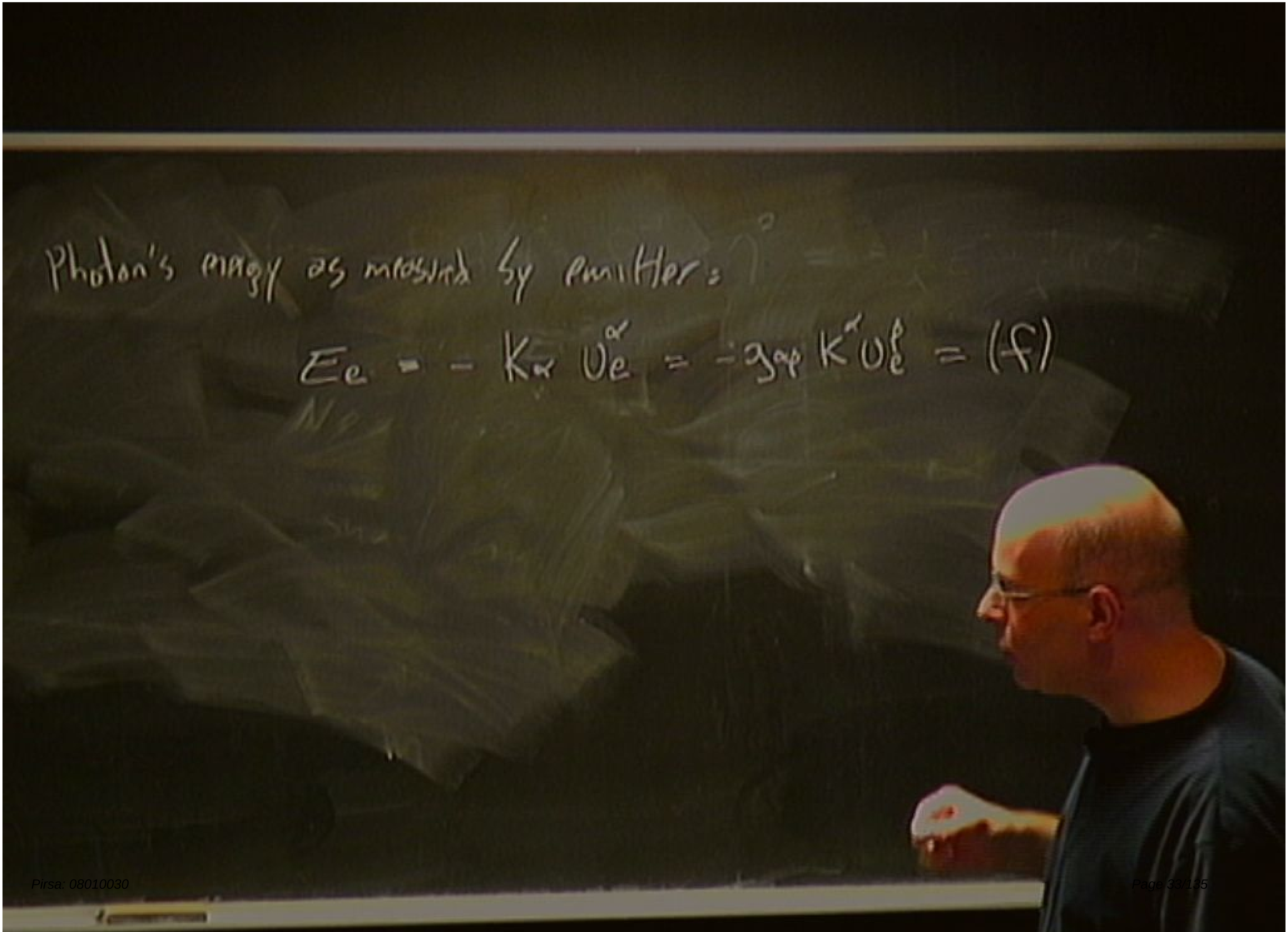
$$E_e = -K_\alpha U_e^\alpha$$

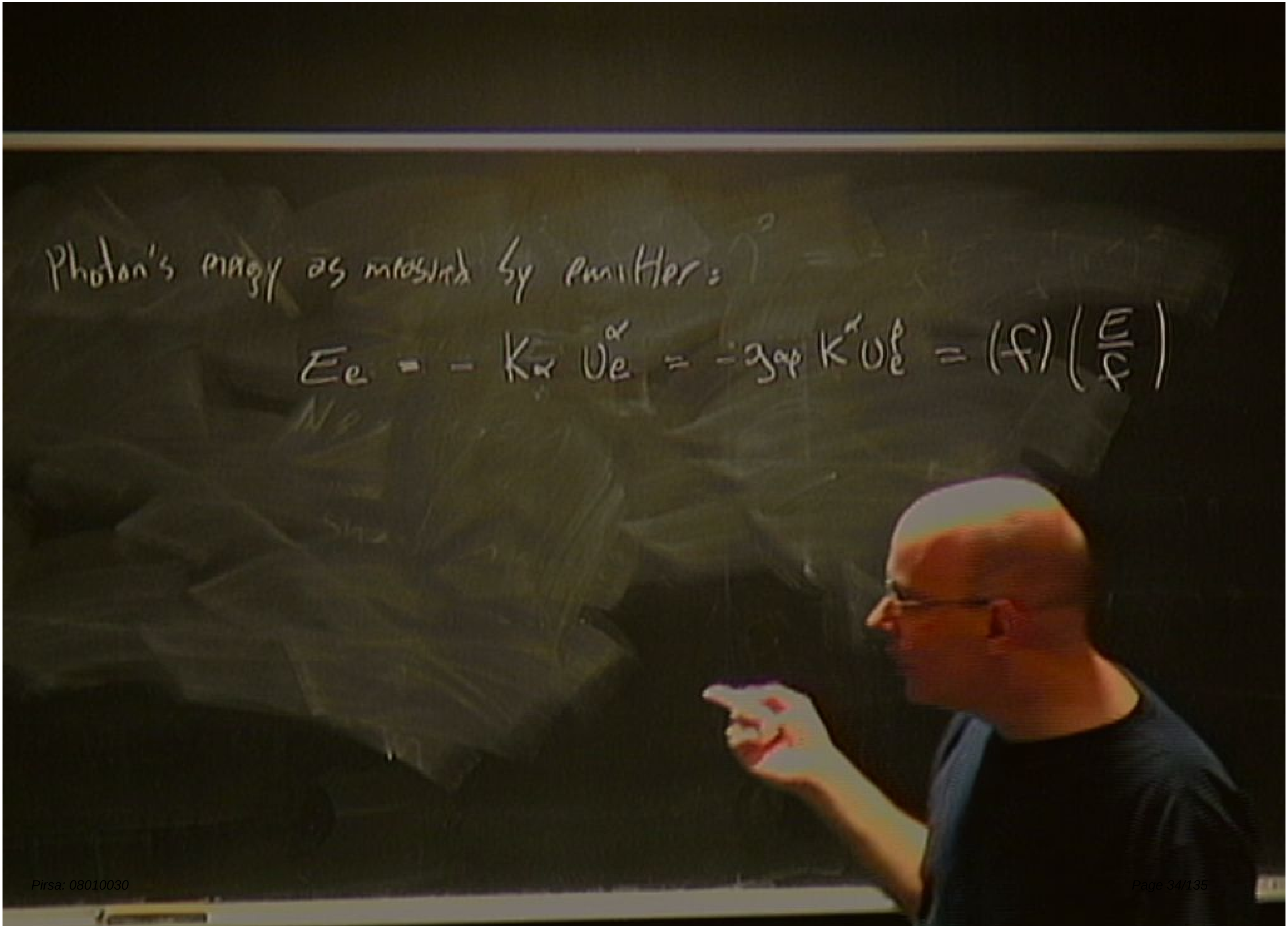
Photon's energy as measured by emitter:

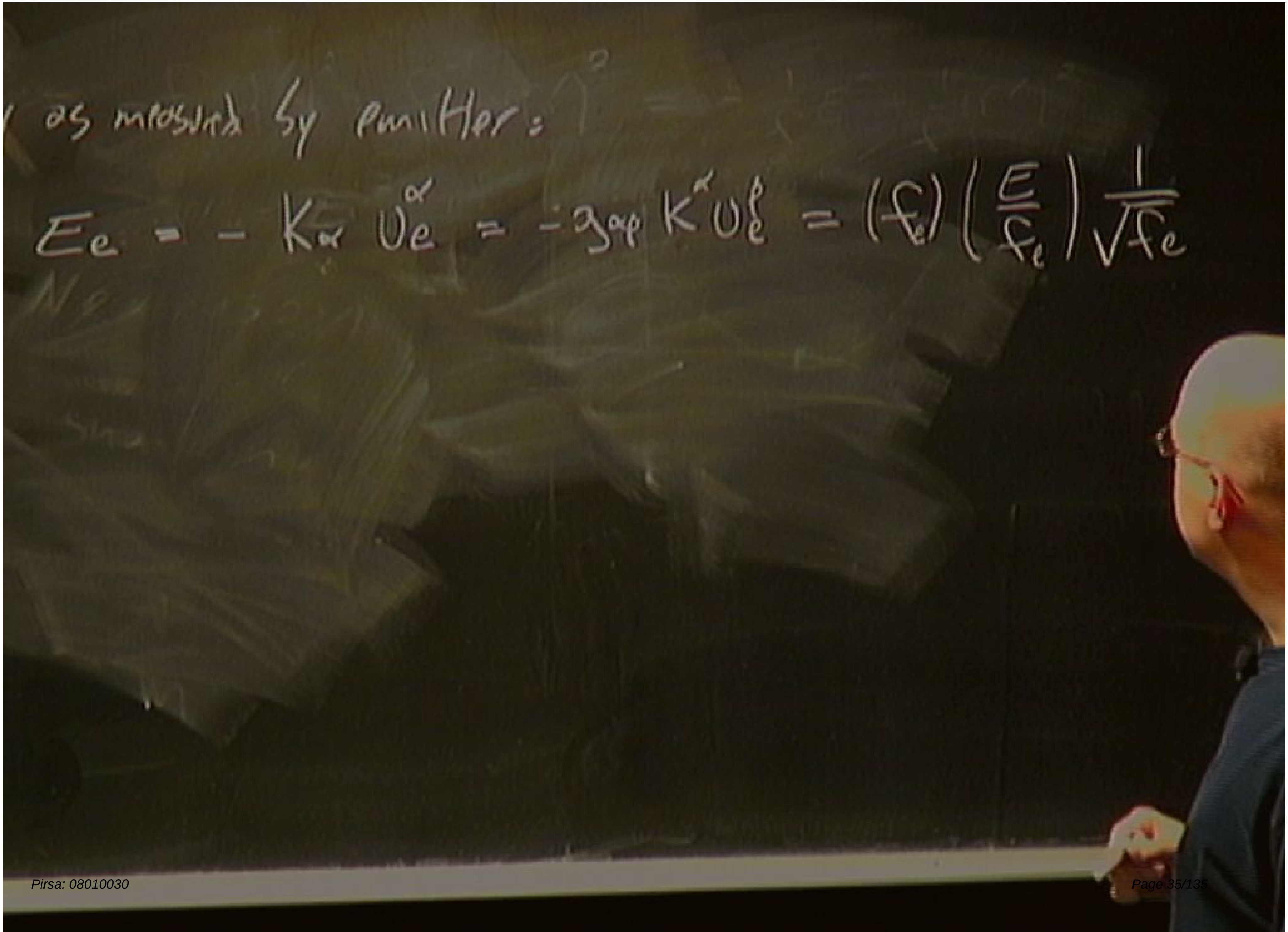
$$E_e = - K_\alpha U_e^\alpha$$

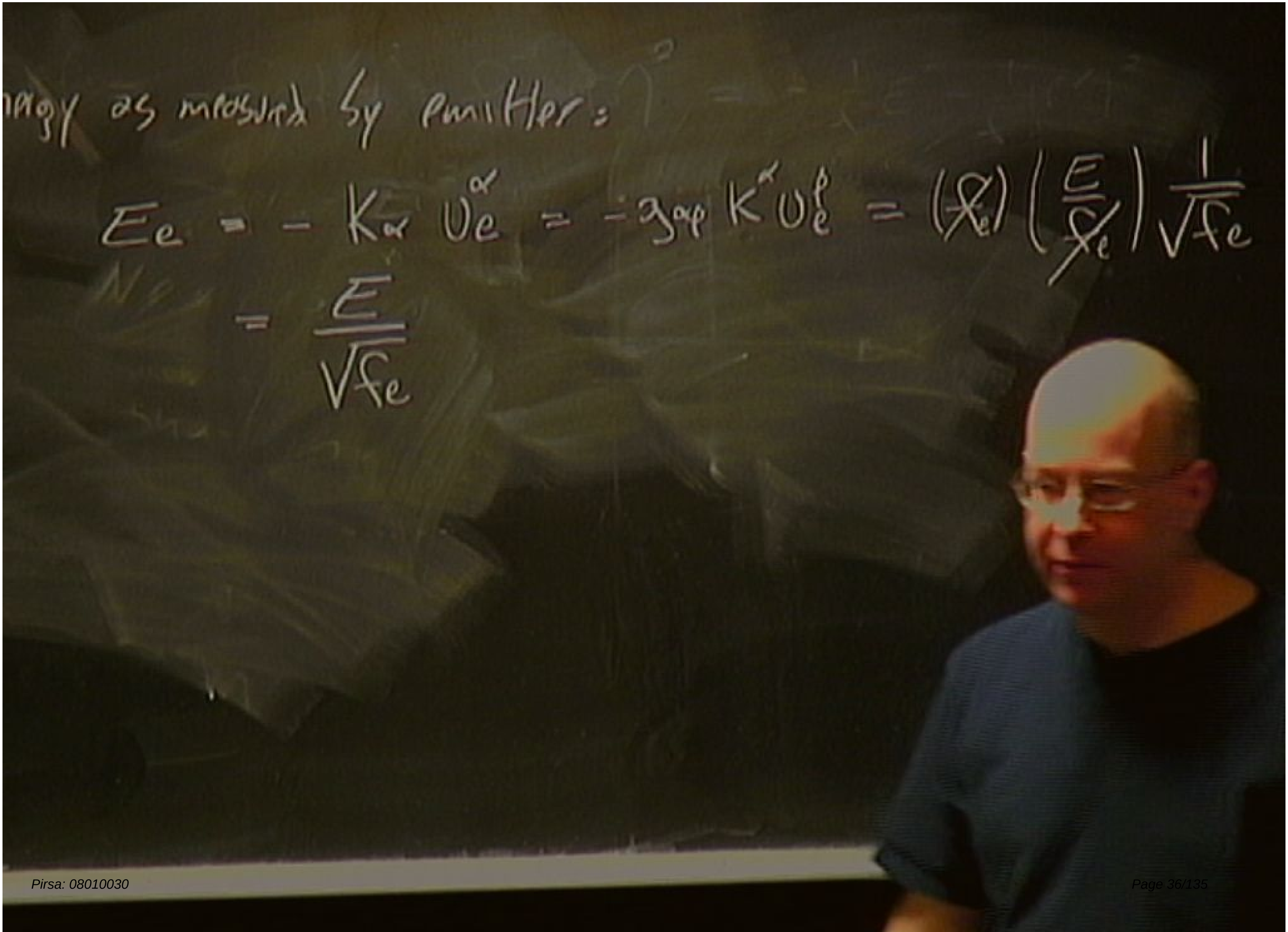
Photon's energy as measured by emitter:

$$E_e = -K_\alpha U_e^\alpha = -g_{\alpha\beta} K^\alpha U_e^\beta$$


Photon's energy as measured by emitter:
$E_e = -K_\alpha U_e^\alpha = -g_{\alpha\beta} K^\alpha U_e^\beta = (f)$


Photon's energy as measured by emitter:
$E_e = -K_\alpha U_e^\alpha = -g_{\alpha\beta} K^\alpha U_e^\beta = (-f)\left(\frac{E}{f}\right)$


as measured by emitter:
$E_e = -K_\alpha U^\alpha_e = -g_{\alpha\beta} K^\alpha U^\beta_e = (-f_e)\left(\frac{E}{f_e}\right)\frac{1}{\sqrt{f_e}}$


…rgy as measured by emitter:
$E_e = -K_\alpha U_e^\alpha = -g_{\alpha\beta} K^\alpha U_e^\beta = (\cancel{\chi_e})\left(\frac{E}{\cancel{\chi_e}}\right)\frac{1}{\sqrt{f_e}}$
$= \frac{E}{\sqrt{f_e}}$

$$E_e = -K_\alpha U_e = -g_{\alpha\beta}\ldots$$

$$= \frac{E}{\sqrt{f_e}}$$

Photon's energy as measured by receiver:

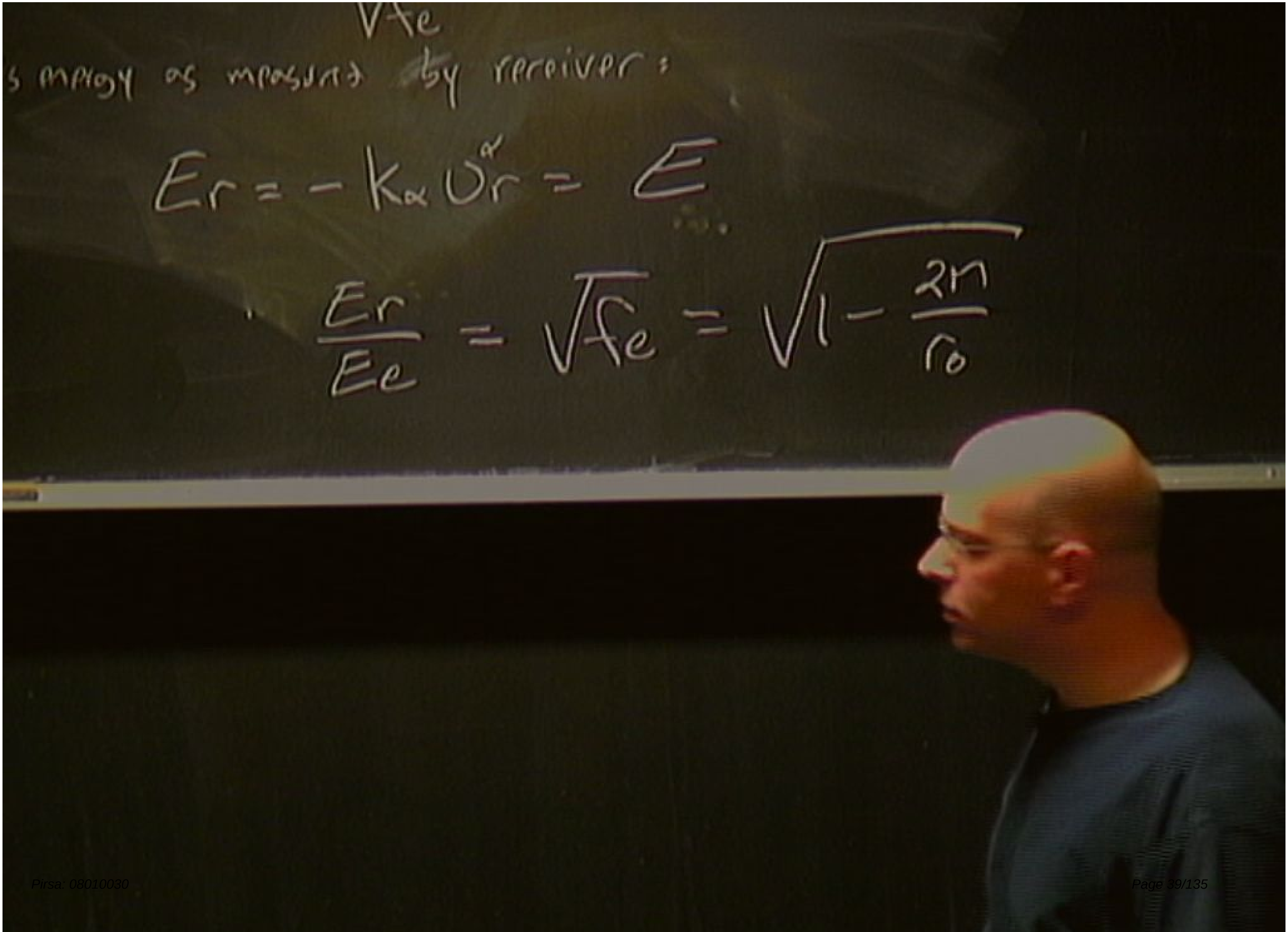
$$E_r = -K_\alpha U_r^\alpha = E$$

$$E_e = -K_\alpha U_e = -g_{\alpha\beta} K$$

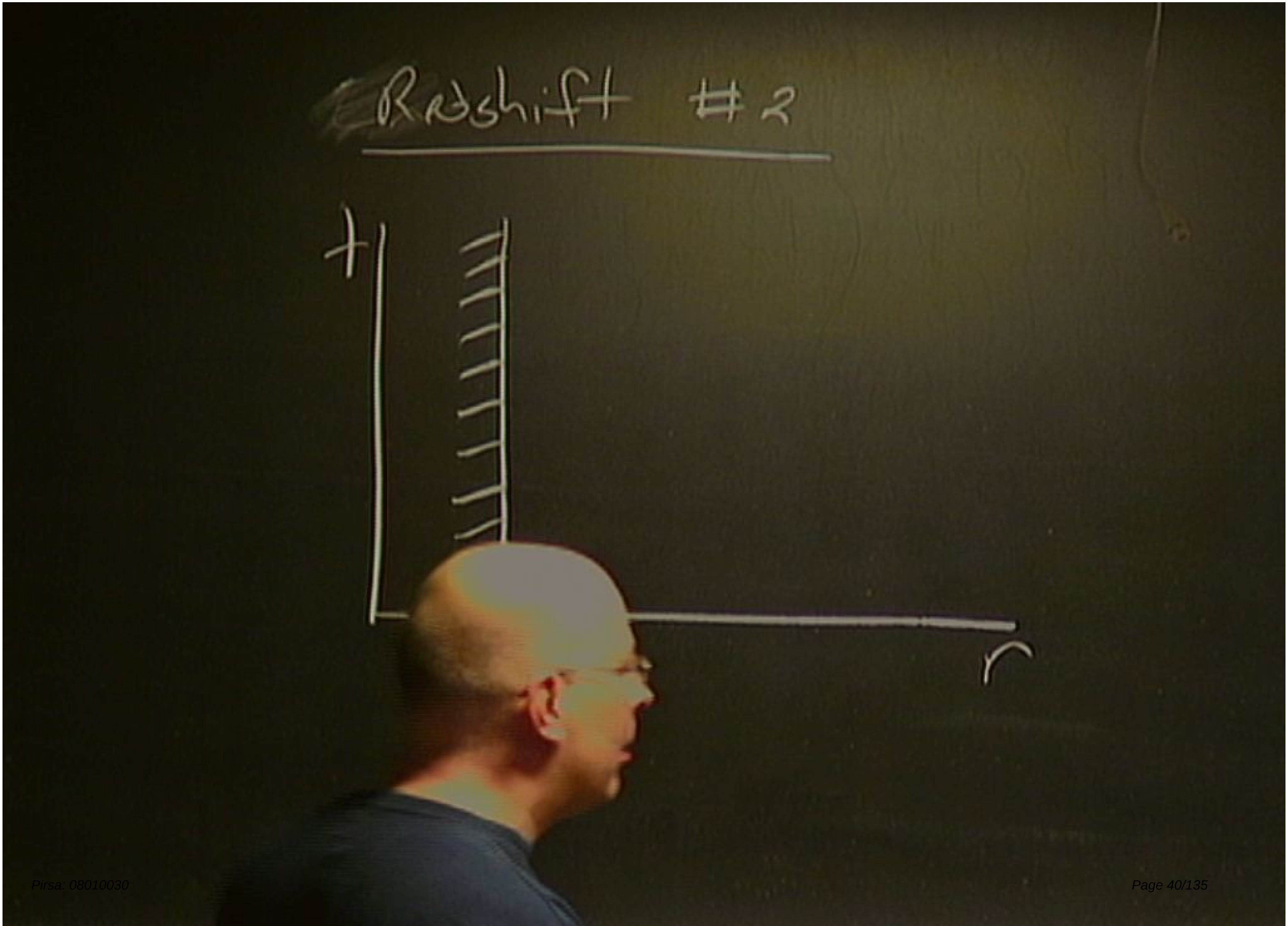
$$= \frac{E}{\sqrt{f_e}}$$

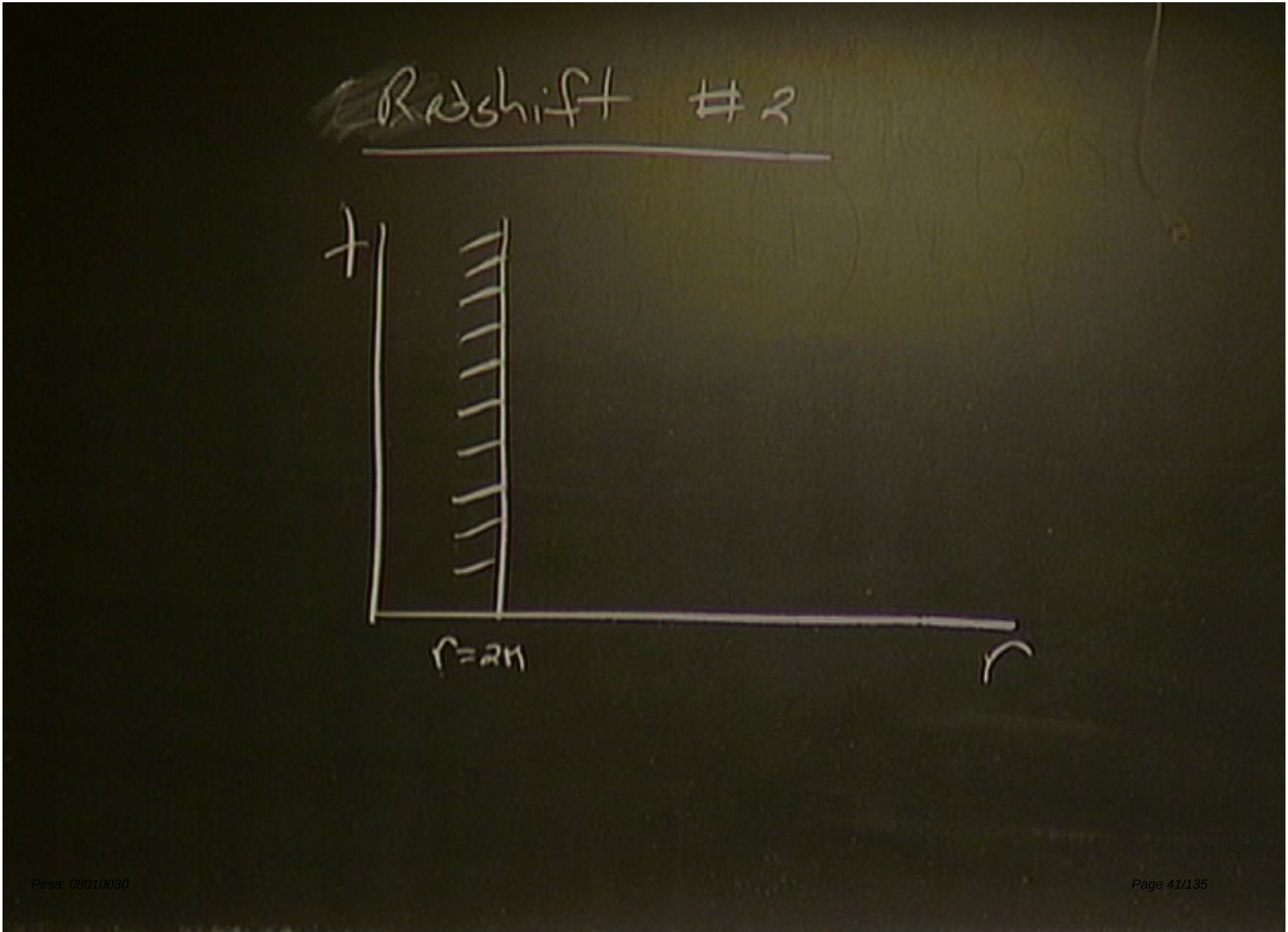
Photon's energy as measured by receiver:

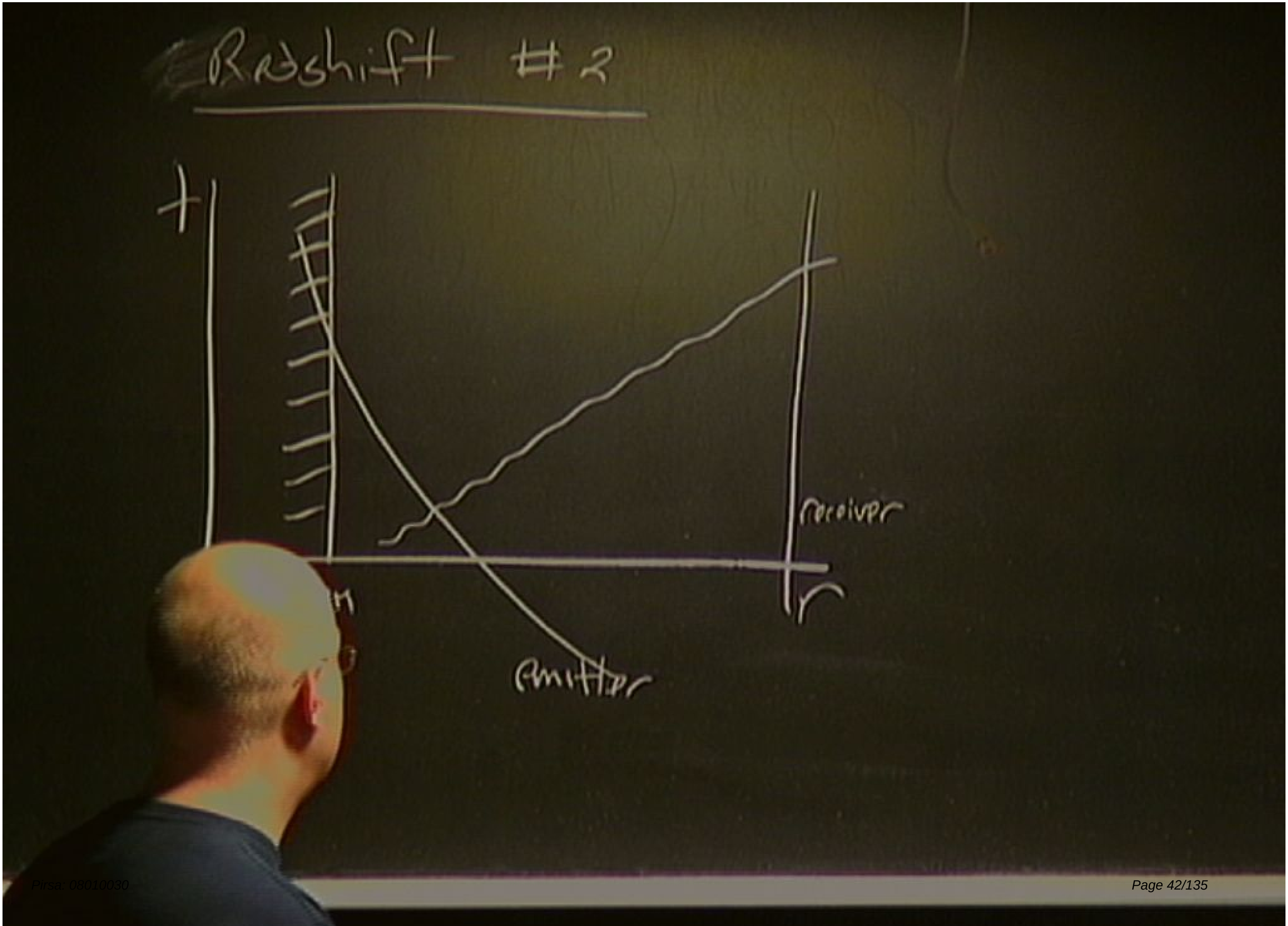
$$E_r = -K_\alpha U_r^\alpha = E$$

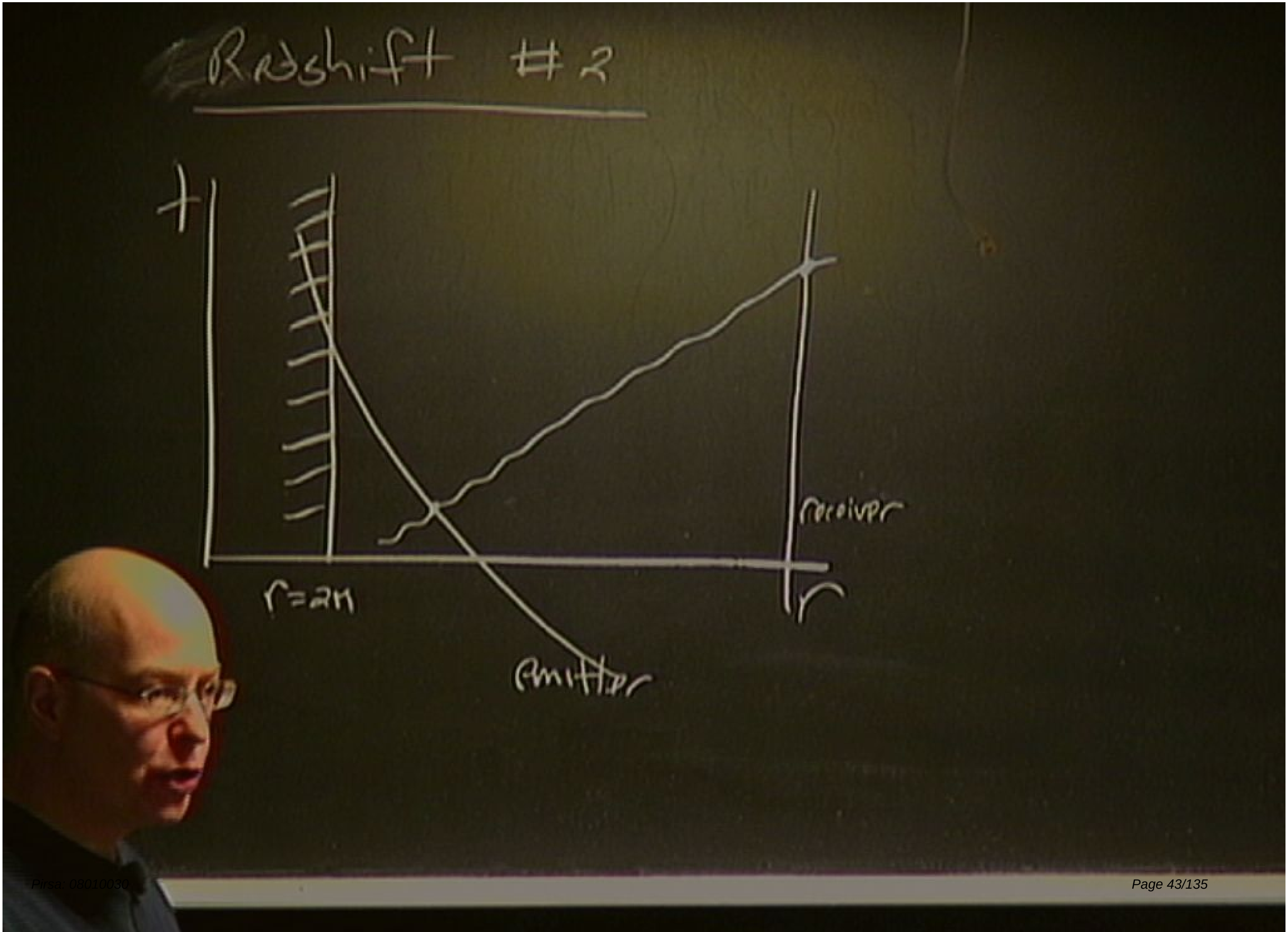


√fe
s energy as measured by receiver:
Er = - Kα Ur^α = E
Er / Ee = √fe = √(1 - 2m / r0)

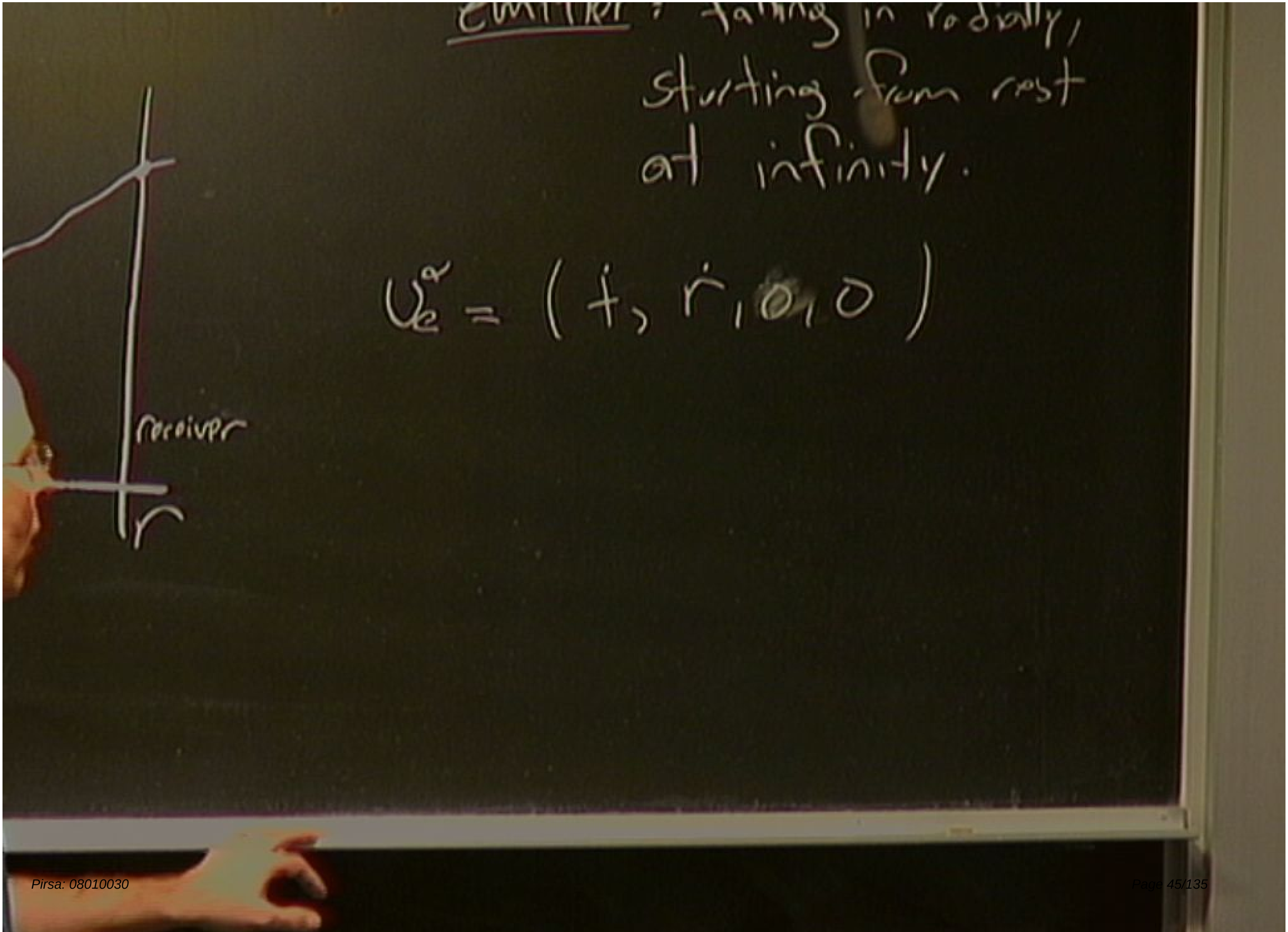

Redshift #2
t
r

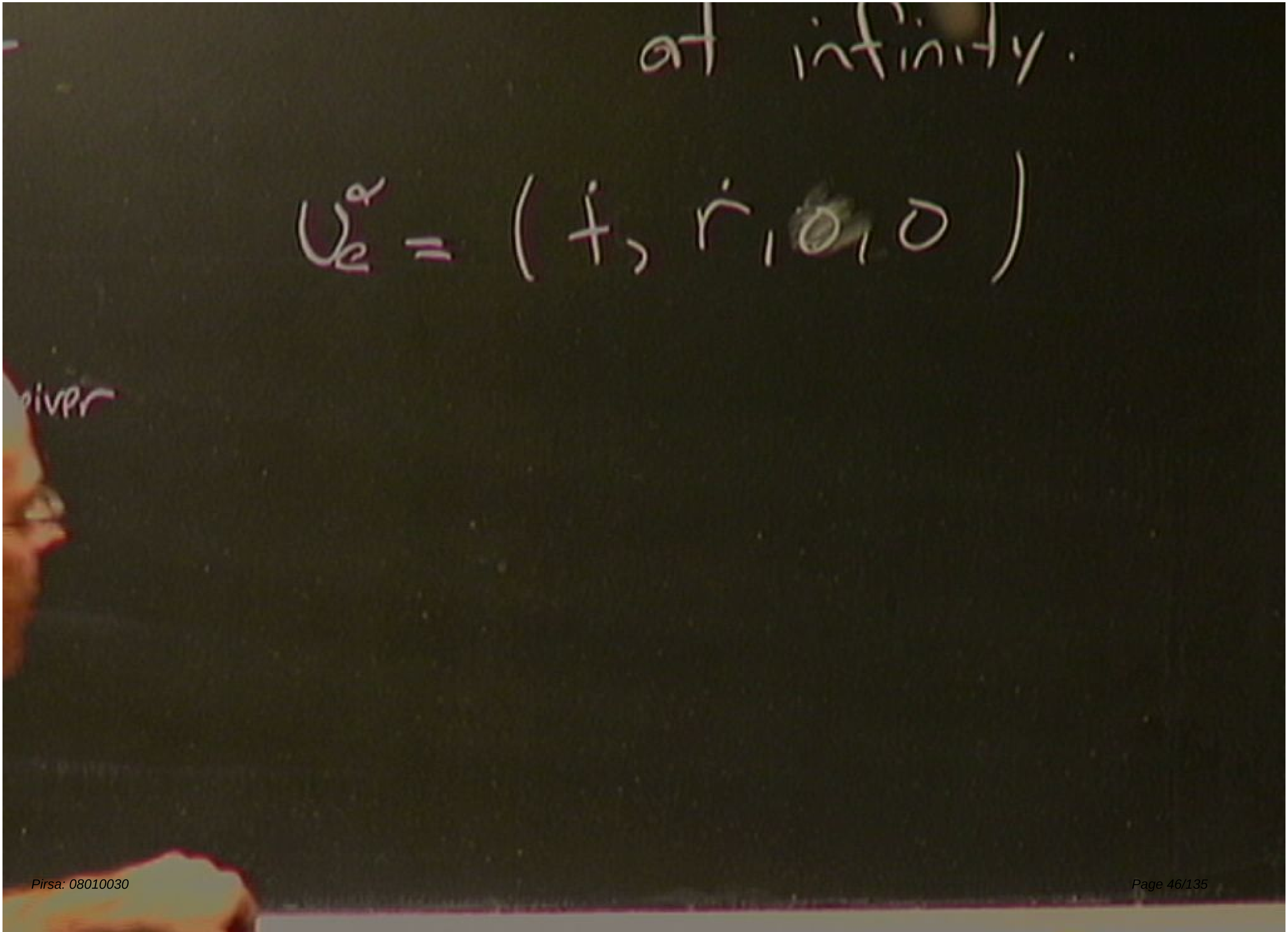

Redshift #2
t
r=2M
r

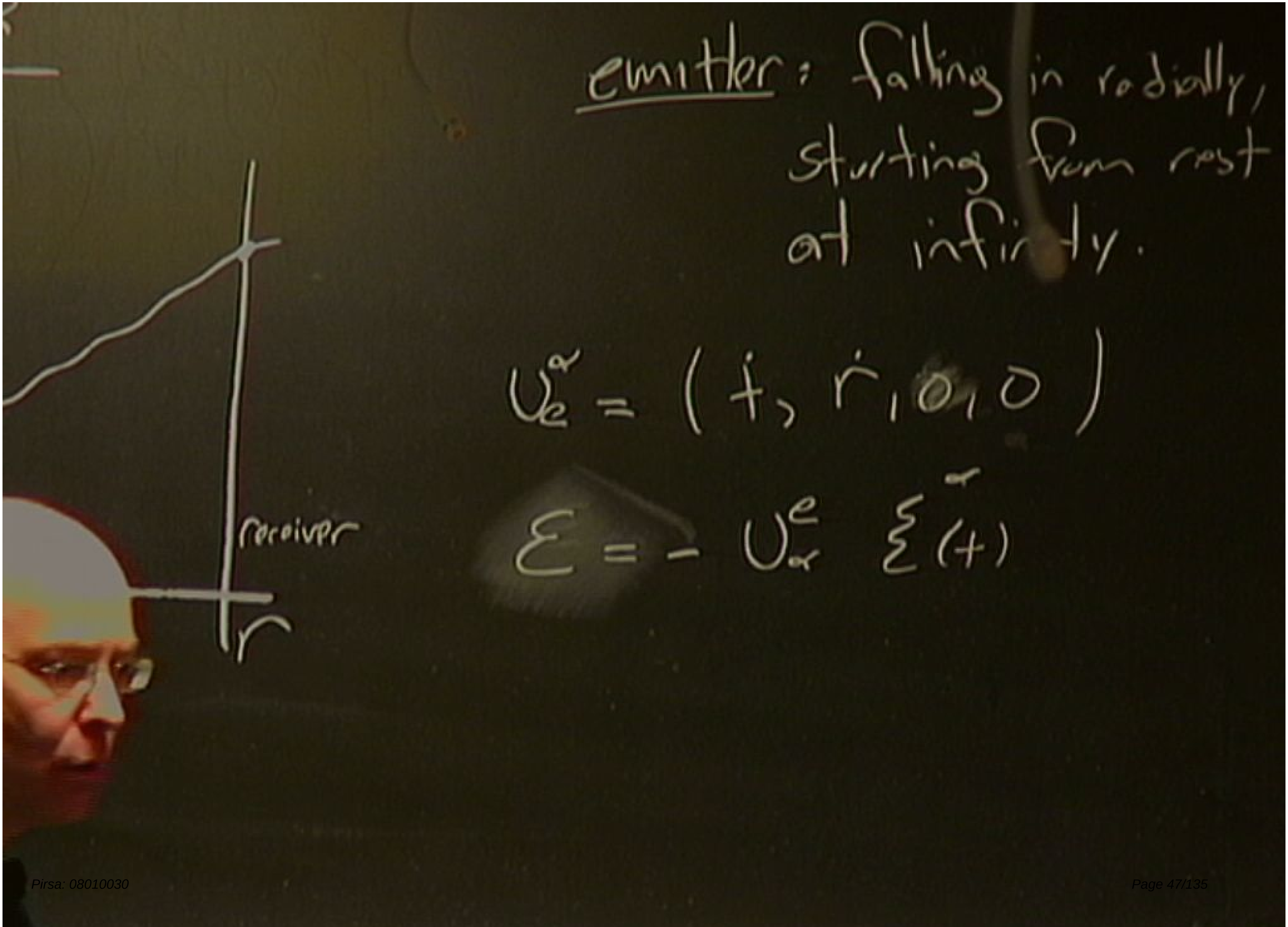

Redshift #2
t
receiver
r
emitter

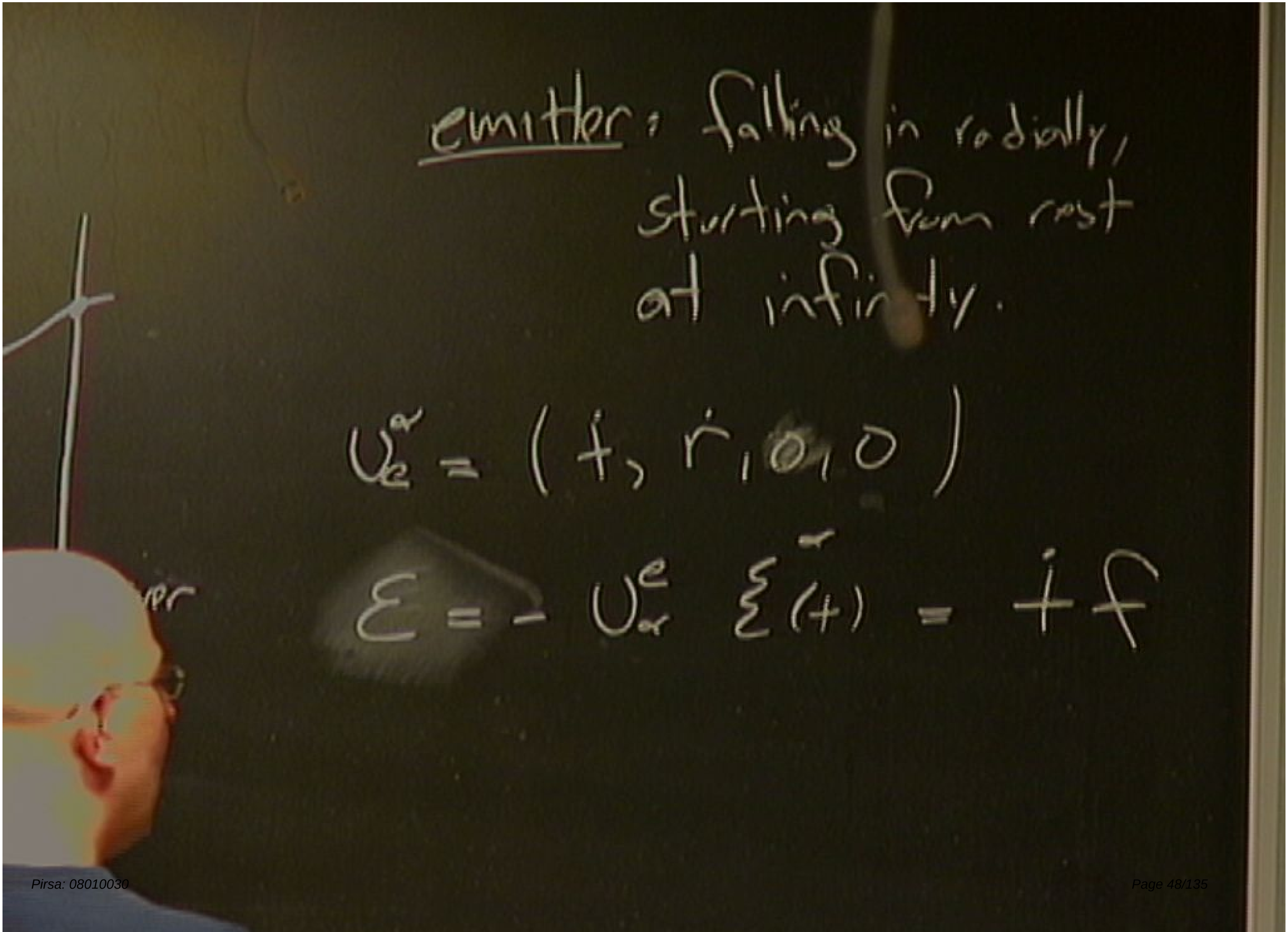

Redshift #2
t
r=2M
r
receiver
emitter

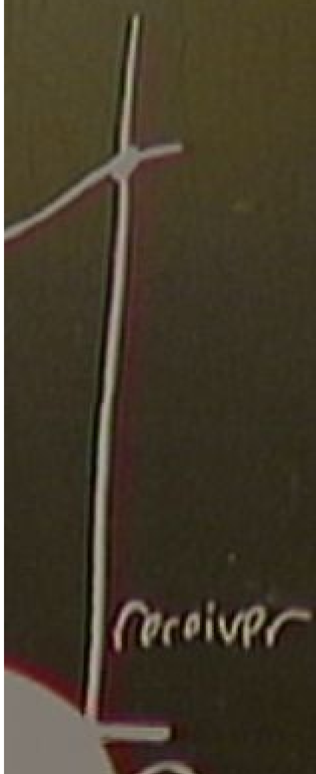
emitter: falling in radially,
starting from rest
at infinity.


Emitter: falling in radially,
starting from rest
at infinity.
$u_e^\alpha = (\dot{t}, \dot{r}, 0, 0)$
receiver
r


at infinity.
$U_e^a = (\dot{t}, \dot{r}, 0, 0)$


emitter: falling in radially, starting from rest at infinity.
$U_e^\alpha = (\dot{t}, \dot{r}, 0, 0)$
$\mathcal{E} = -U^e_\alpha \xi^\alpha_{(t)}$
receiver
r


emitter: falling in radially, starting from rest at infinity.
$U_e^\alpha = (\dot{t}, \dot{r}, 0, 0)$
$\mathcal{E} = - U^e_\alpha \, \xi^\alpha_{(t)} = \dot{t} f$

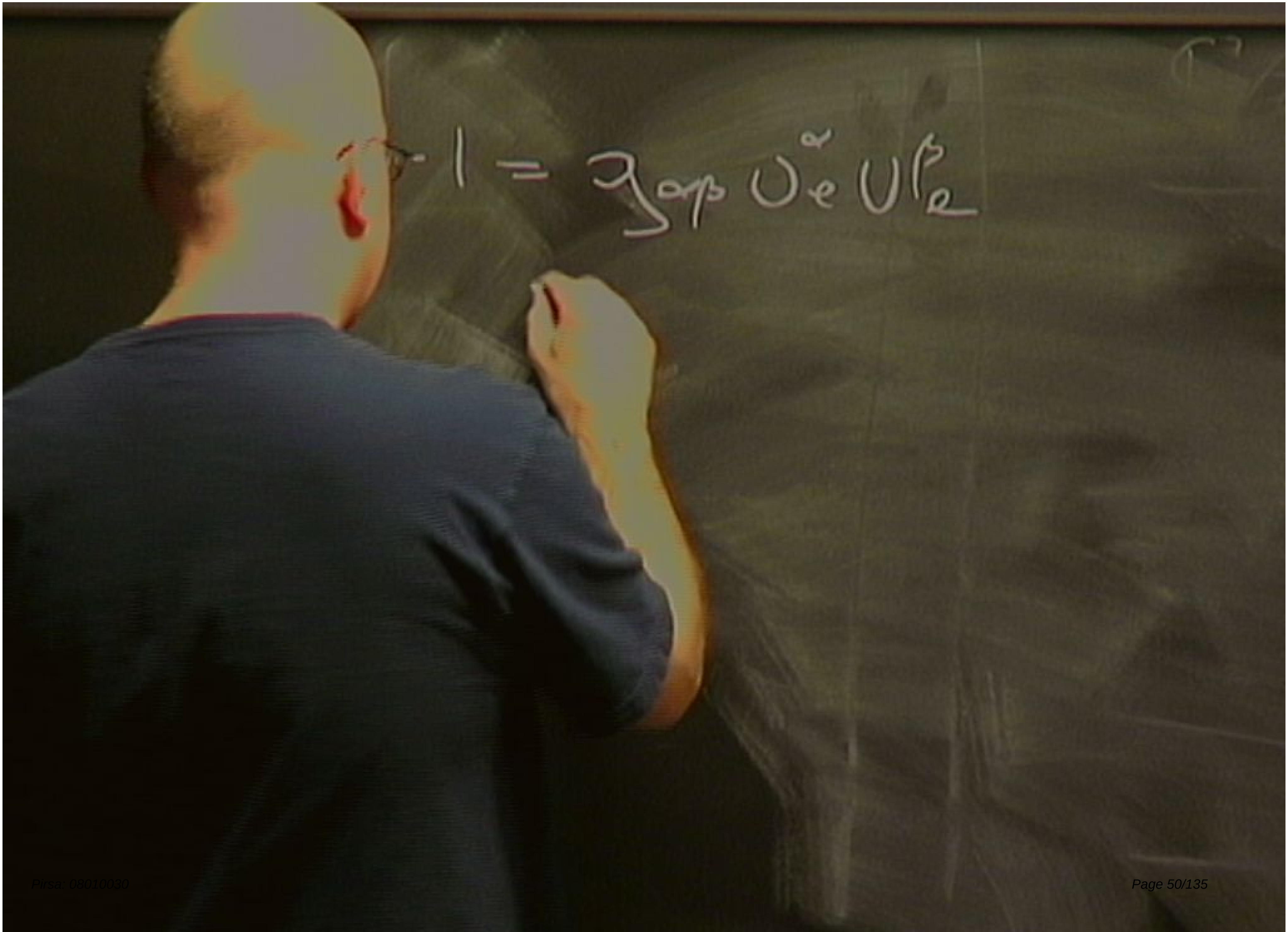


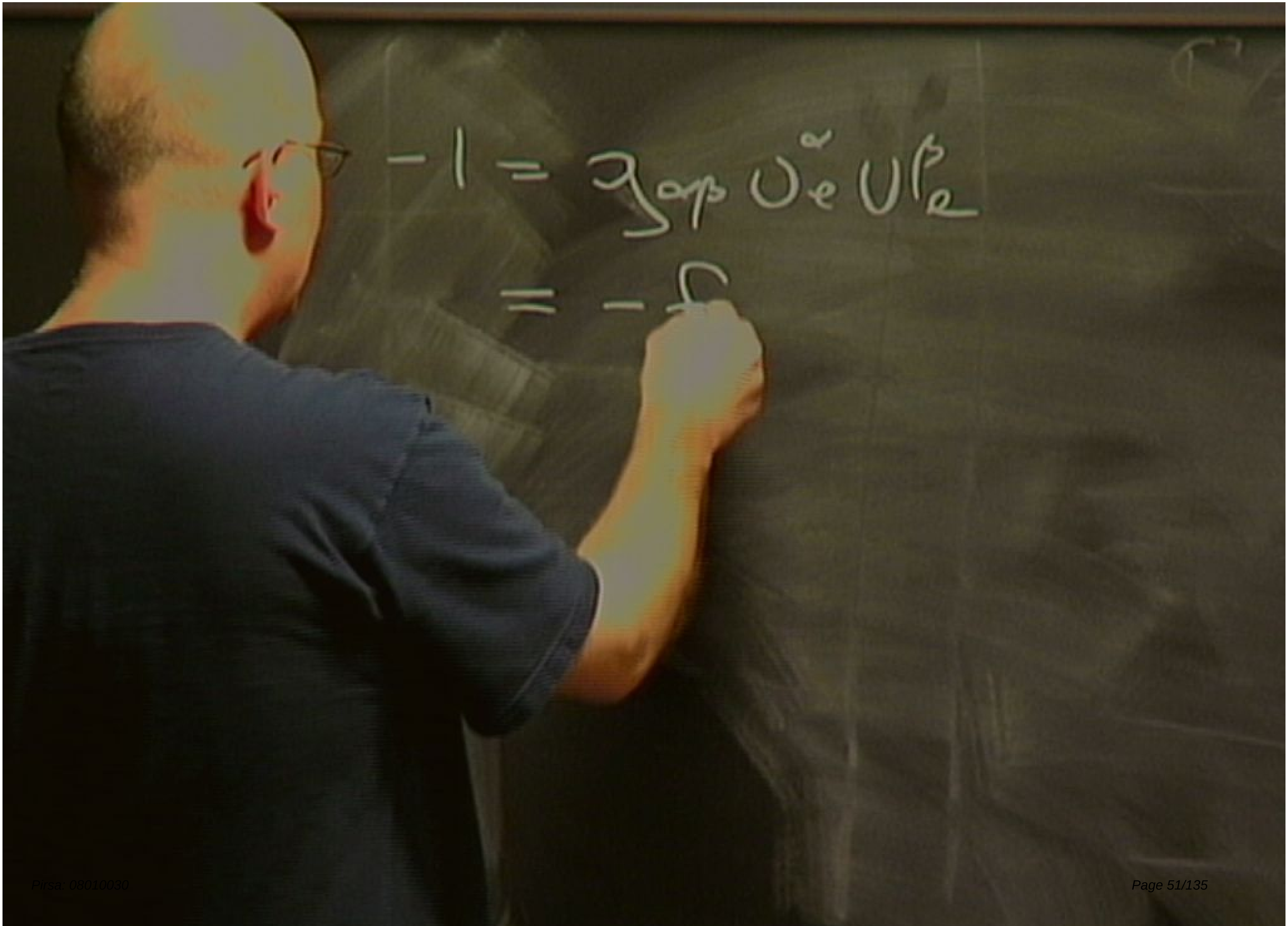
<u>emitter</u>: falling in radially, starting from rest at infinity.

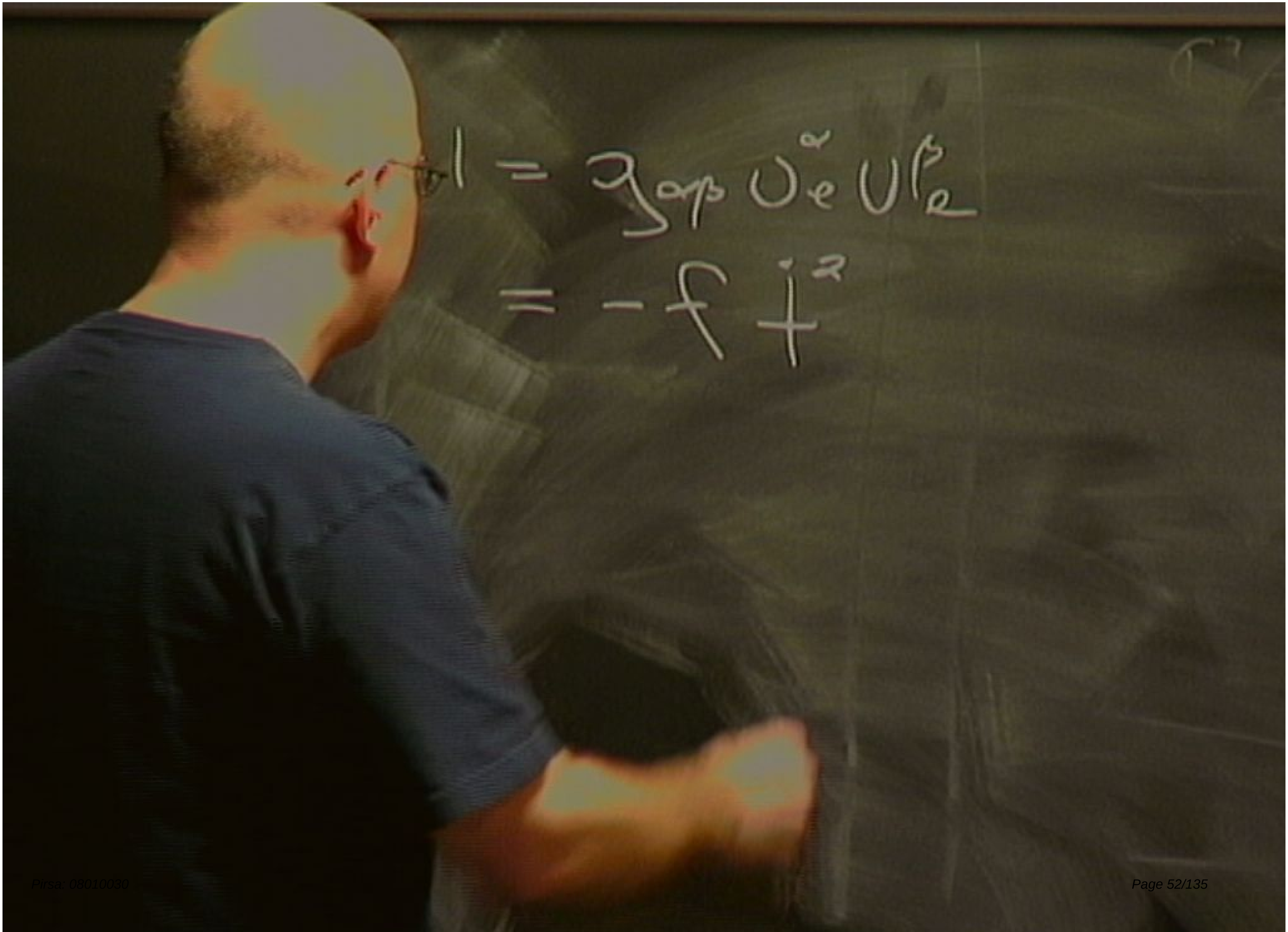
$$u_e^\alpha = (\dot t, \dot r, 0, 0)$$

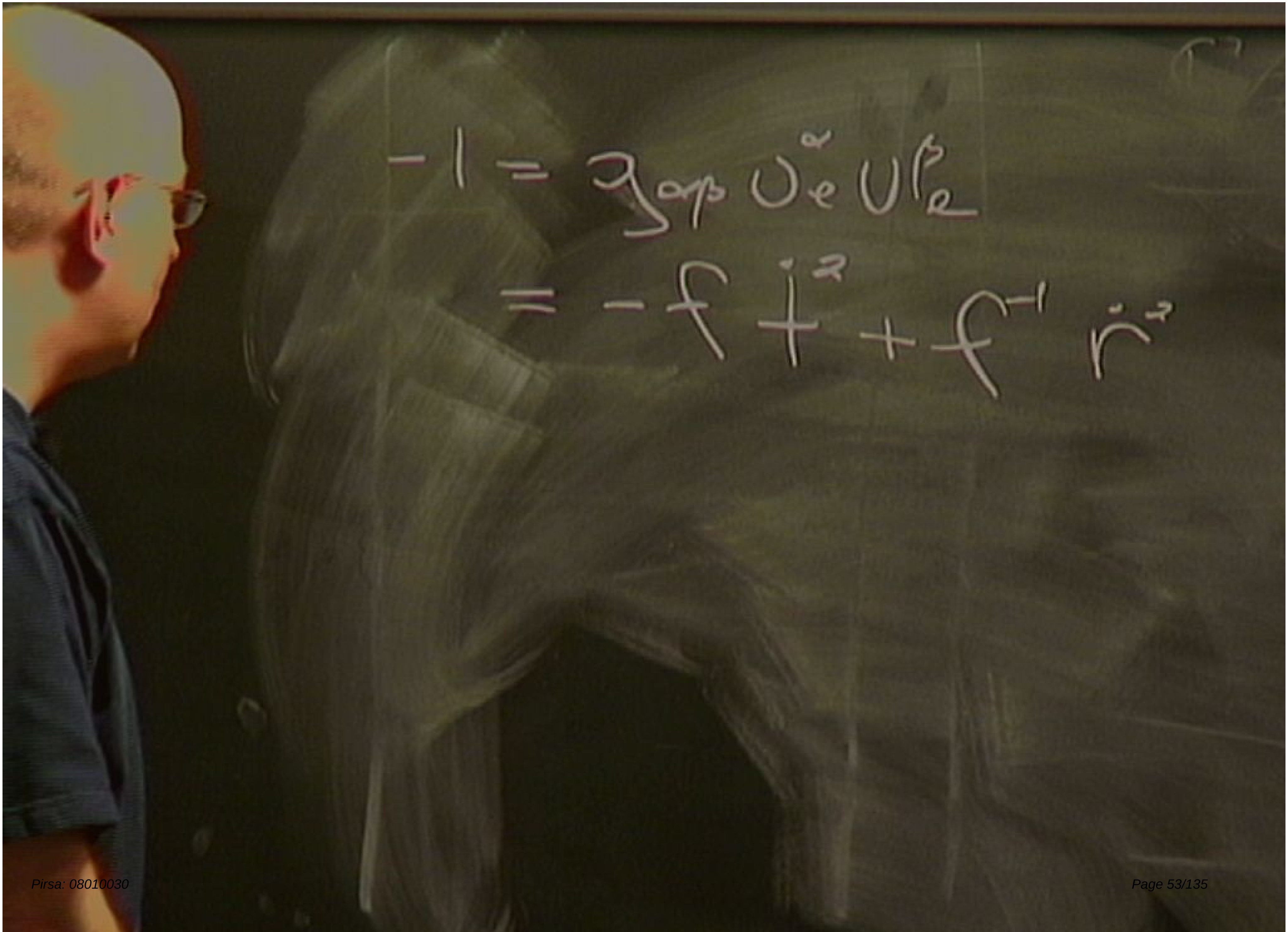
$$\mathcal{E} = - u^e_\alpha \, \xi^\alpha_{(t)} = \dot t f$$

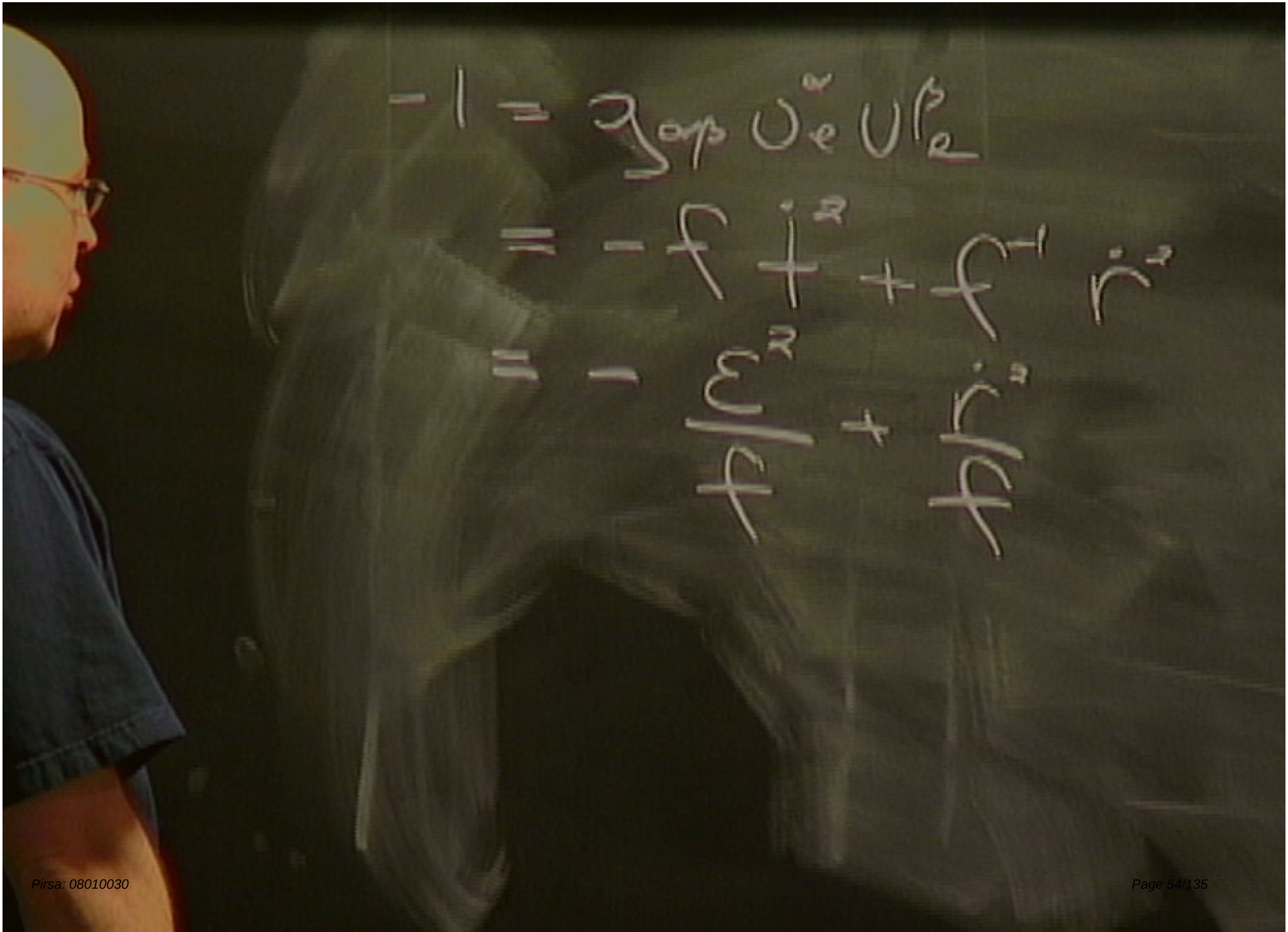
$$\dot t = \frac{\mathcal{E}}{f}$$

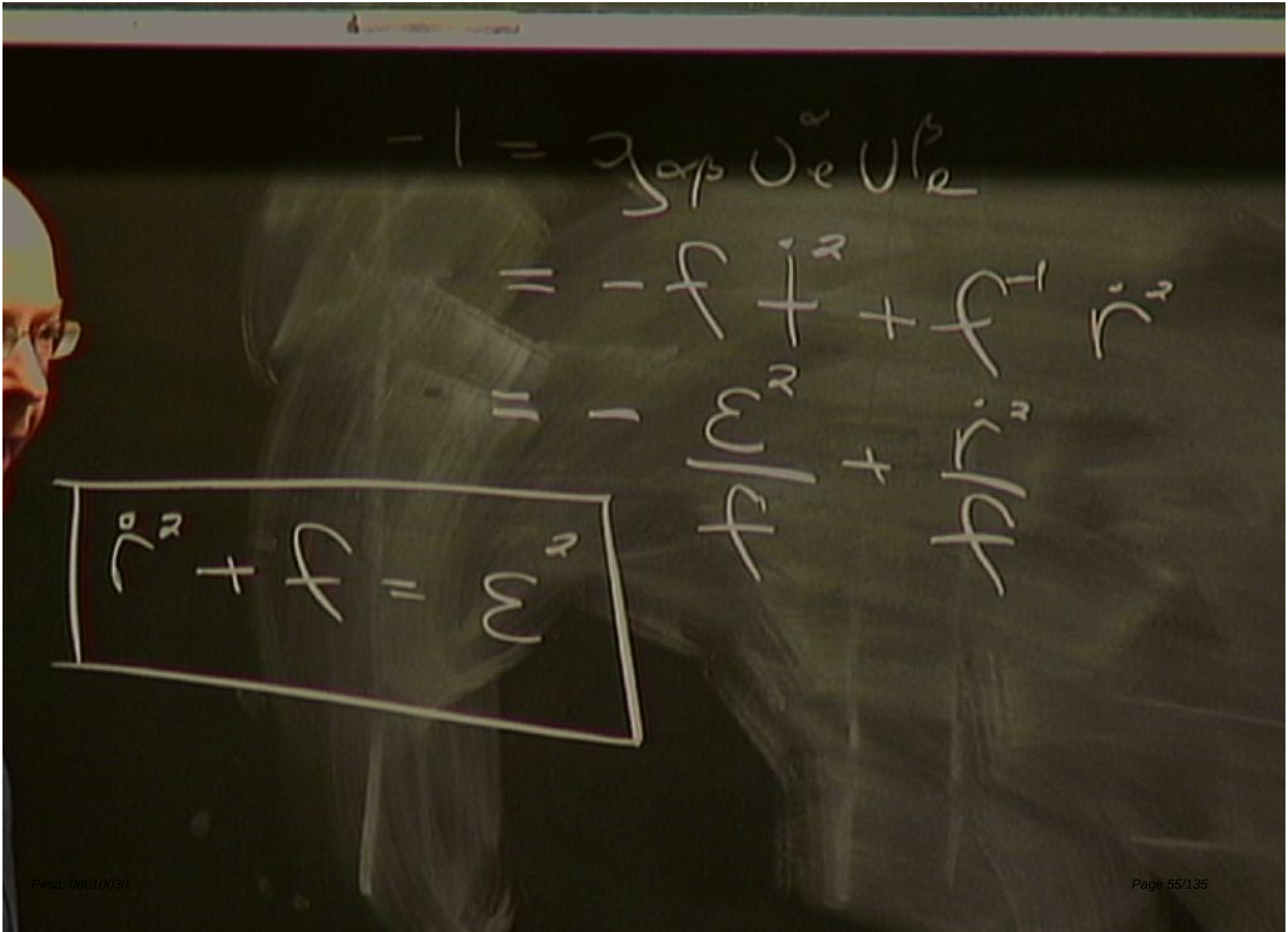


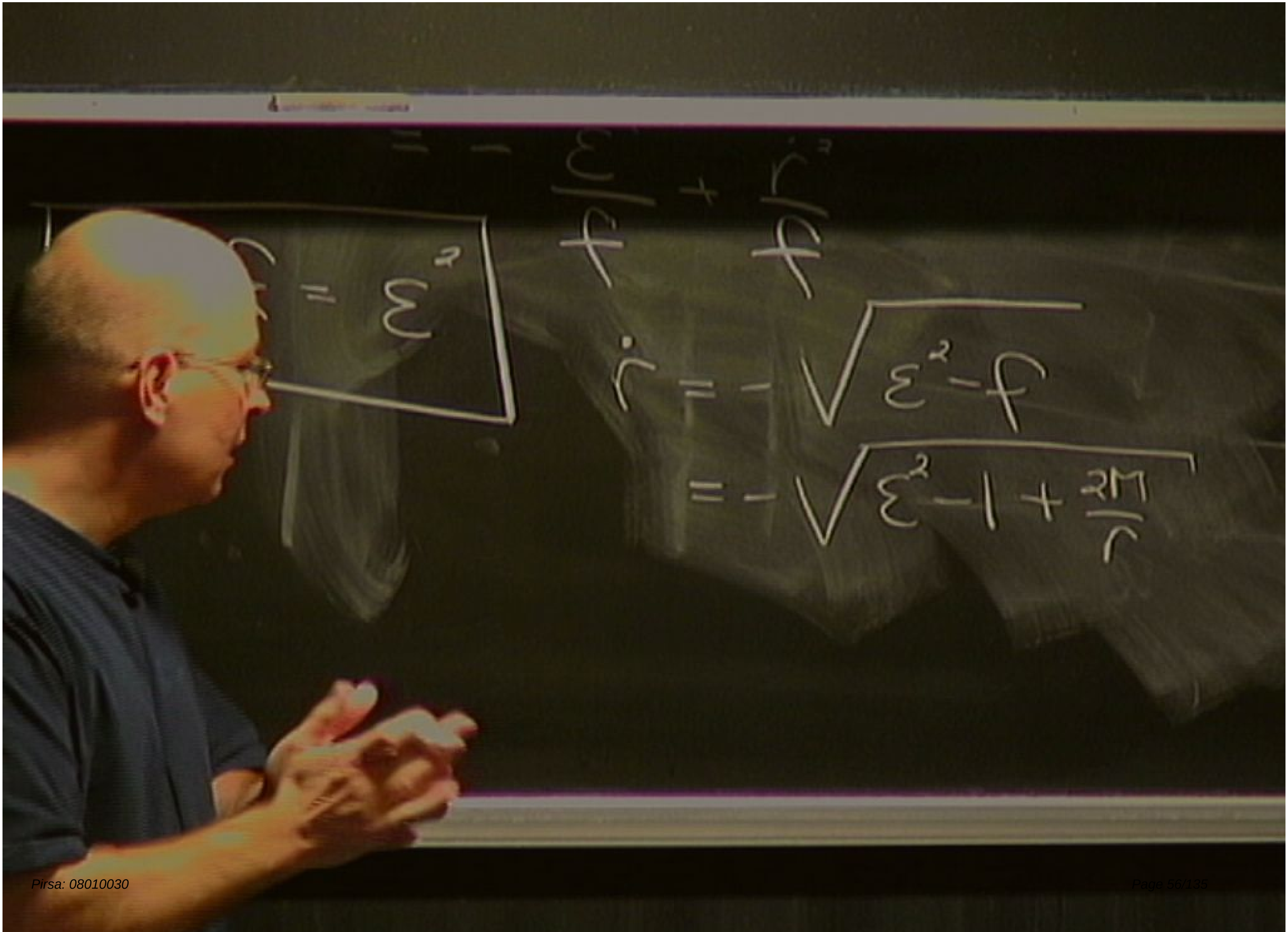

$-1 = g_{\alpha\beta} U^{\alpha}_{e} U^{\beta}_{e}$
$= -$


$l = g_{\alpha\beta} U^{\alpha}_{e} U^{\beta}_{e}$
$= -f \dot{t}^{2}$


$-1 = g_{\alpha\beta} \dot{u}^\alpha_e \dot{u}^\beta_e$
$= -f \dot{t}^2 + f^{-1} \dot{r}^2$


$-1 = g_{\alpha\beta} U^\alpha_e U^\beta_e$
$= -f\,\dot{t}^2 + f^{-1}\,\dot{r}^2$
$= -\frac{\mathcal{E}^2}{f} + \frac{\dot{r}^2}{f}$


$-1 = g_{\alpha\beta} U^\alpha_e U^\beta_e$
$= -f \dot{t}^2 + f^{-1} \dot{r}^2$
$= -\frac{\mathcal{E}^2}{f} + \frac{\dot{r}^2}{f}$
$\boxed{\dot{r}^2 + f = \mathcal{E}^2}$

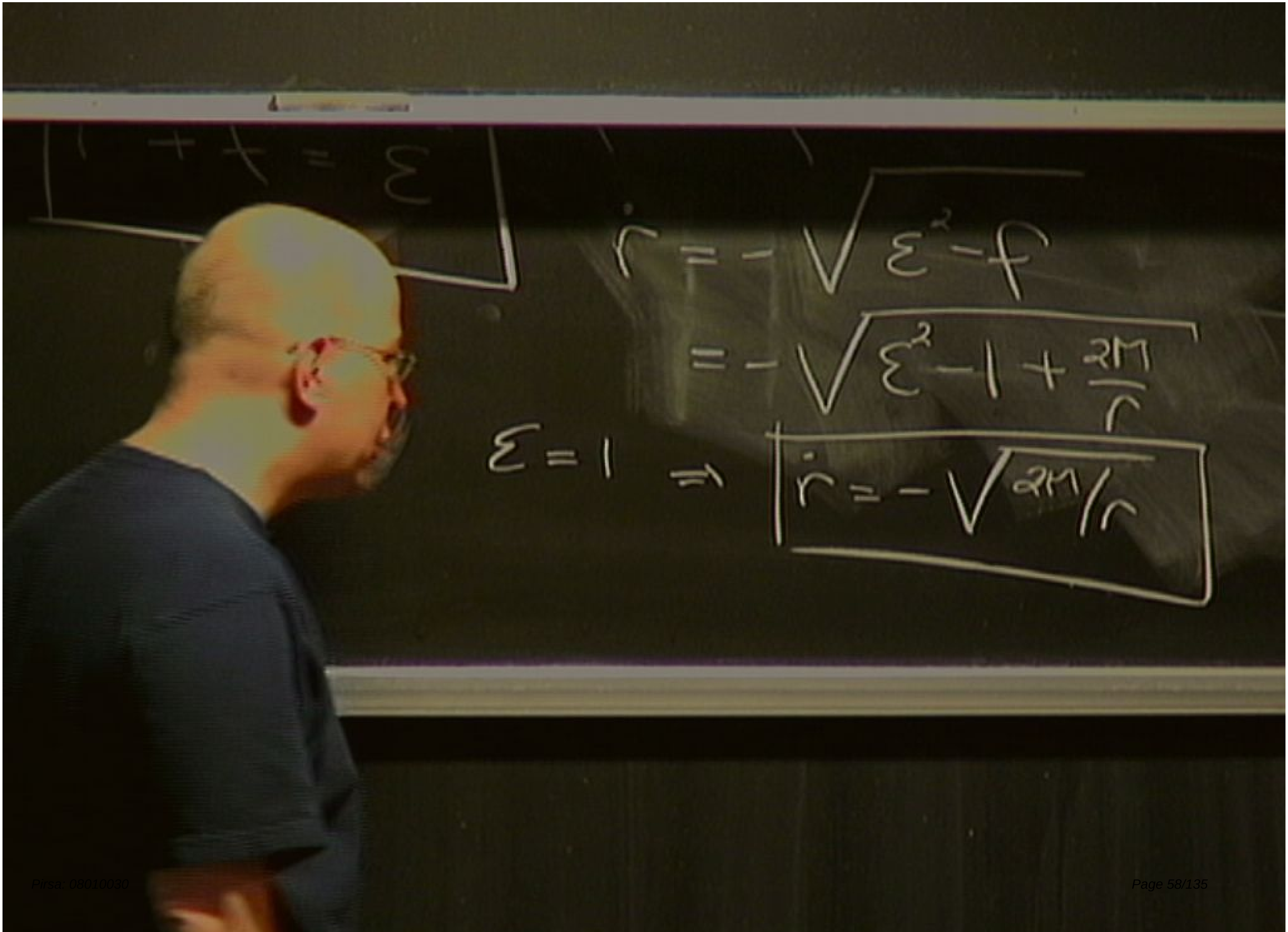

= − ε²/f + ṙ²/f
= ε²
ṙ = −√(ε² − f)
= −√(ε² − 1 + 2M/r)

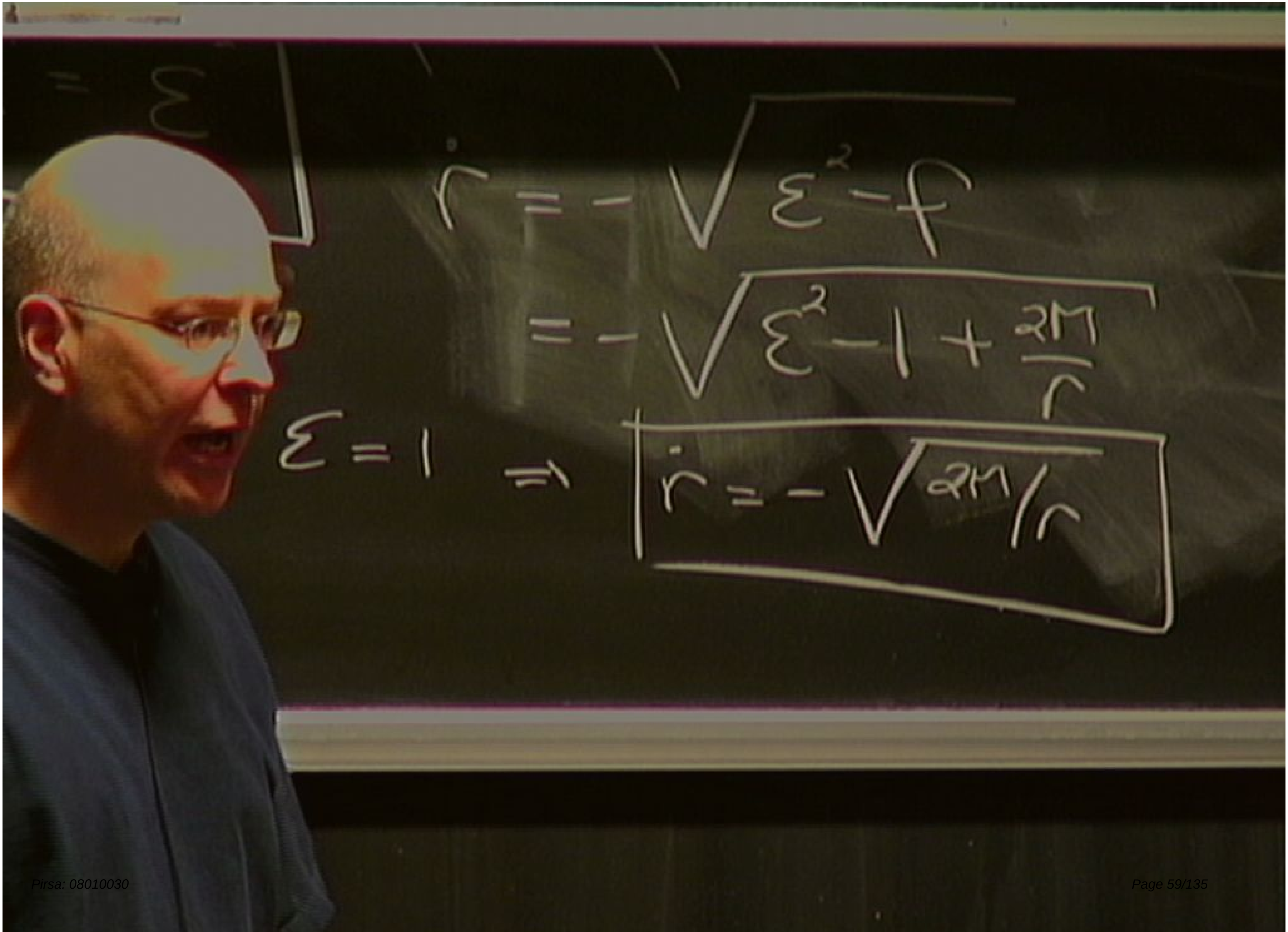
$$= -\frac{\mathcal{E}^2}{f} + \frac{\dot{r}^2}{f}$$

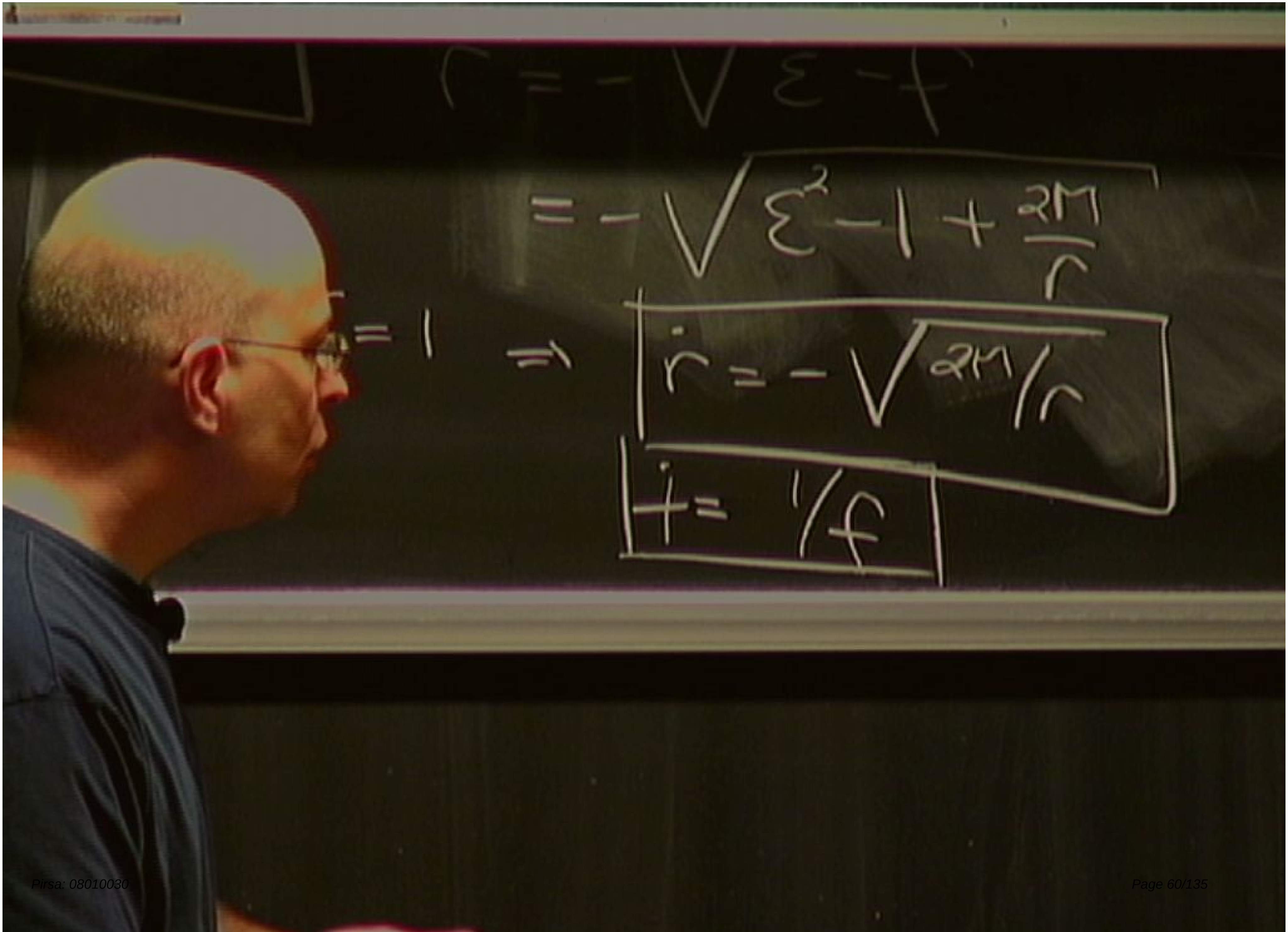
$$\boxed{\dot{r}^2 + f = \mathcal{E}^2}$$

$$\dot{r} = -\sqrt{\mathcal{E}^2 - f}$$

$$= -\sqrt{\mathcal{E}^2 - 1 + \frac{2M}{r}}$$


$\dot{r} = -\sqrt{\varepsilon^2 - f}$
$= -\sqrt{\varepsilon^2 - 1 + \frac{2M}{r}}$
$\varepsilon = 1 \Rightarrow \boxed{\dot{r} = -\sqrt{2M/r}}$


$\dot{r} = -\sqrt{\mathcal{E}^2 - f}$
$= -\sqrt{\mathcal{E}^2 - 1 + \frac{2M}{r}}$
$\mathcal{E} = 1 \Rightarrow \boxed{\dot{r} = -\sqrt{2M/r}}$


$= -\sqrt{\mathcal{E}^2 - 1 + \frac{2M}{r}}$
$= 1 \Rightarrow \boxed{\dot{r} = -\sqrt{2M/r}}$
$\boxed{\dot{t} = 1/f}$

Photon's energy as measured by emitter:

Photon's energy as measured by emitter:

$$E_e = -k_a U_e^a$$

Photon's energy as measured by emitter:

$$E_e = -k_\alpha u_e^\alpha = -g_{\alpha\beta} k^\alpha u_e^\beta$$

Photon's energy as measured by emitter:

$$E_e = -k_\alpha U_e^\alpha = -g_{\alpha\beta} k^\alpha U_e^\beta$$

$$= (\hbar)\left(\frac{E}{\hbar}\right)$$

Photon's energy as measured by emitter:

$$E_e = -k_\alpha U_e^\alpha = -g_{\alpha\beta} k^\alpha U_e^\beta$$

$$= (f)\left(\frac{E}{f}\right)\left(\frac{1}{f}\right)$$

Photon's energy as measured by emitter:

$$E_e = -k_\alpha U_e^\alpha = -g_{\alpha\beta} k^\alpha U_e^\beta$$

$$= (f)\left(\frac{E}{f}\right)\left(\frac{1}{f}\right) - \left(\frac{1}{f}\right)(E)\left(-\sqrt{\frac{2m}{r}}\right)$$

Photon's energy as measured by emitter:

$$E_e = -k_\alpha U_e^\alpha = -g_{\alpha\beta} k^\alpha U_e^\beta$$

$$= (\cancel{f})\left(\frac{E}{f}\right)\left(\frac{1}{\cancel{f}}\right) - \left(\frac{1}{f}\right)(E)\left(-\sqrt{\frac{2m}{r}}\right)$$

$$= \frac{E}{f}\left(1-\sqrt{\frac{2m}{r}}\right)$$

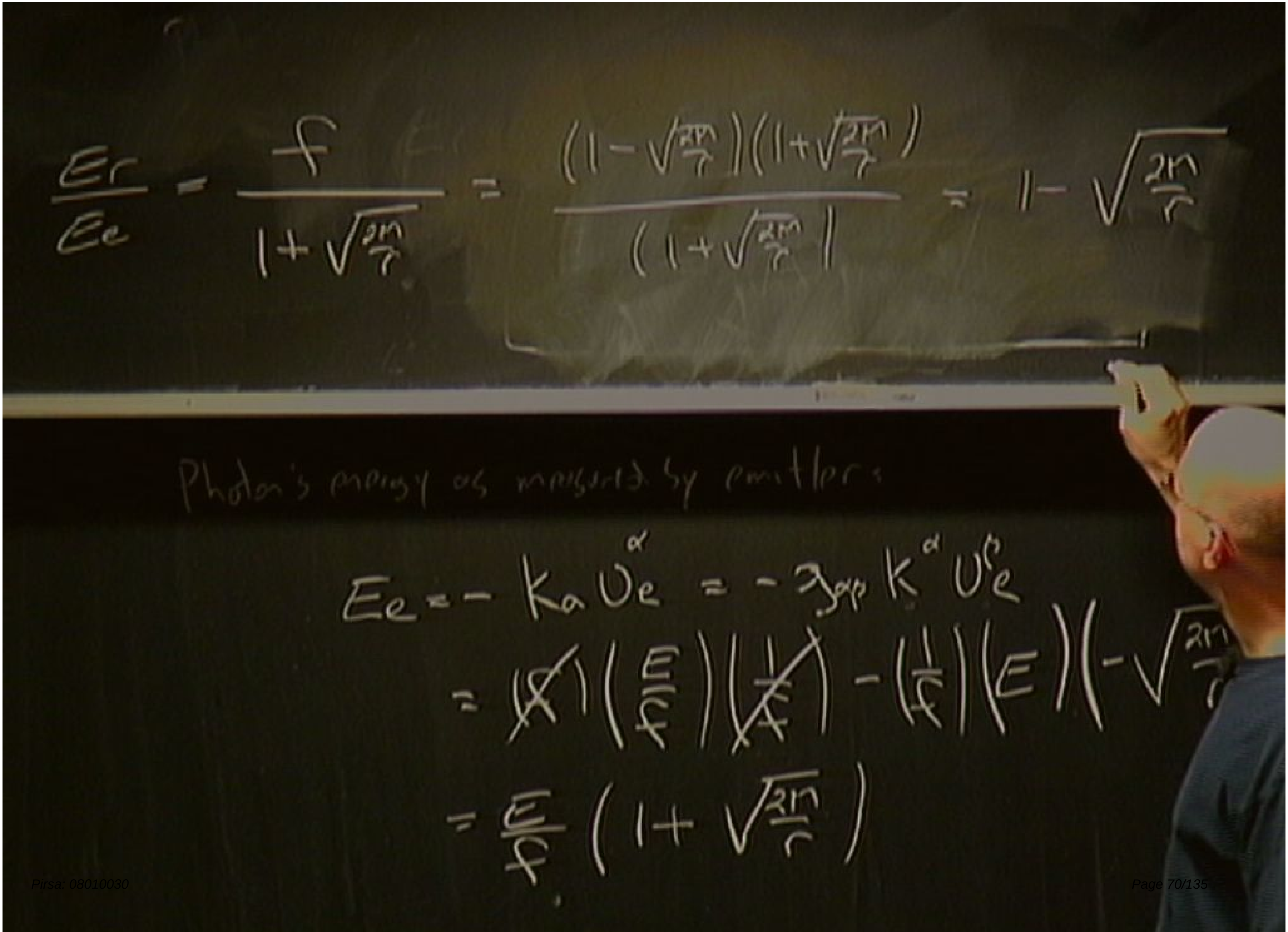
$$\frac{E_r}{E_e} = \frac{f}{1-\sqrt{\frac{2m}{r}}}$$

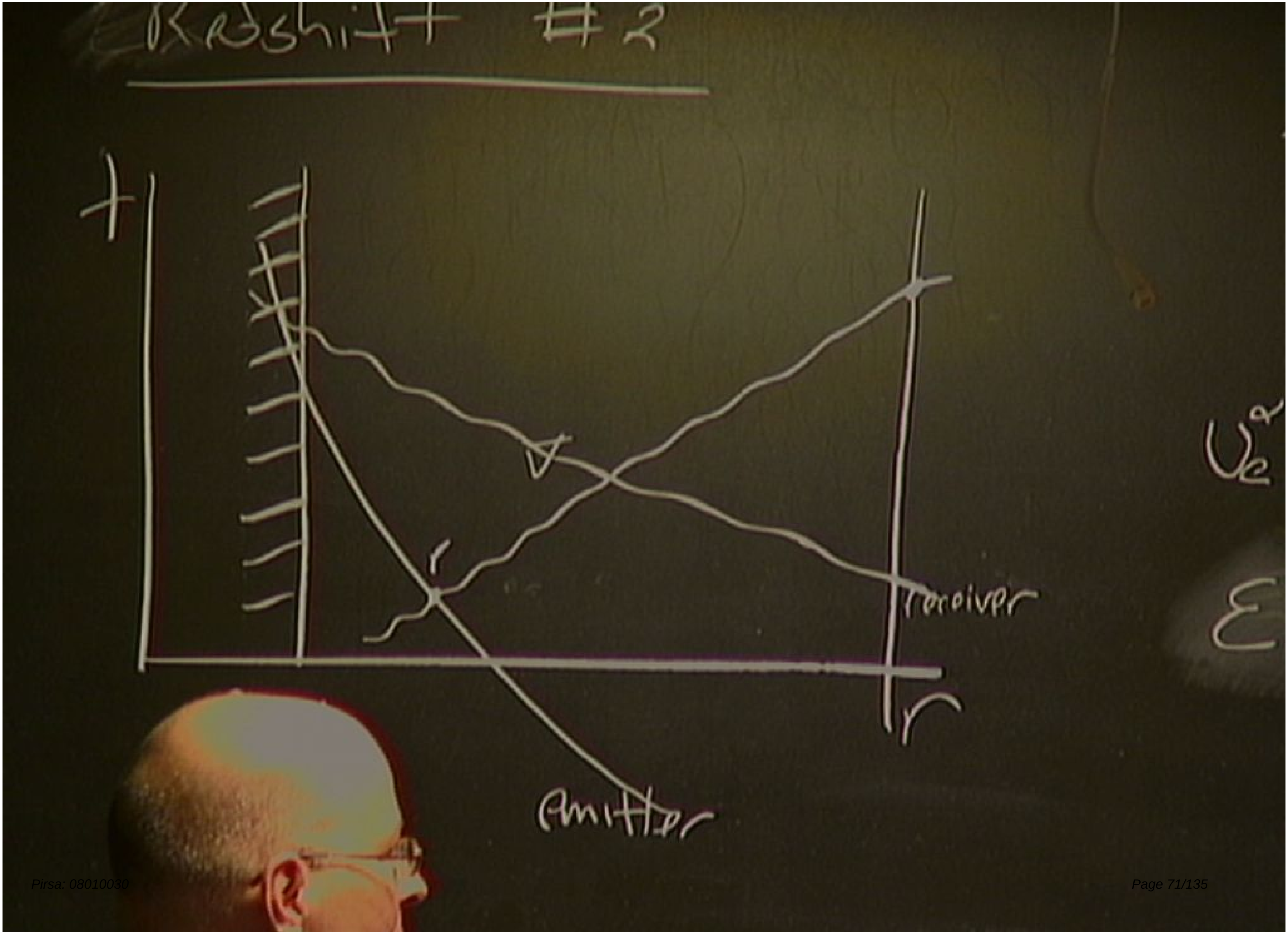
$$E_e = -K a U_e = -g$$

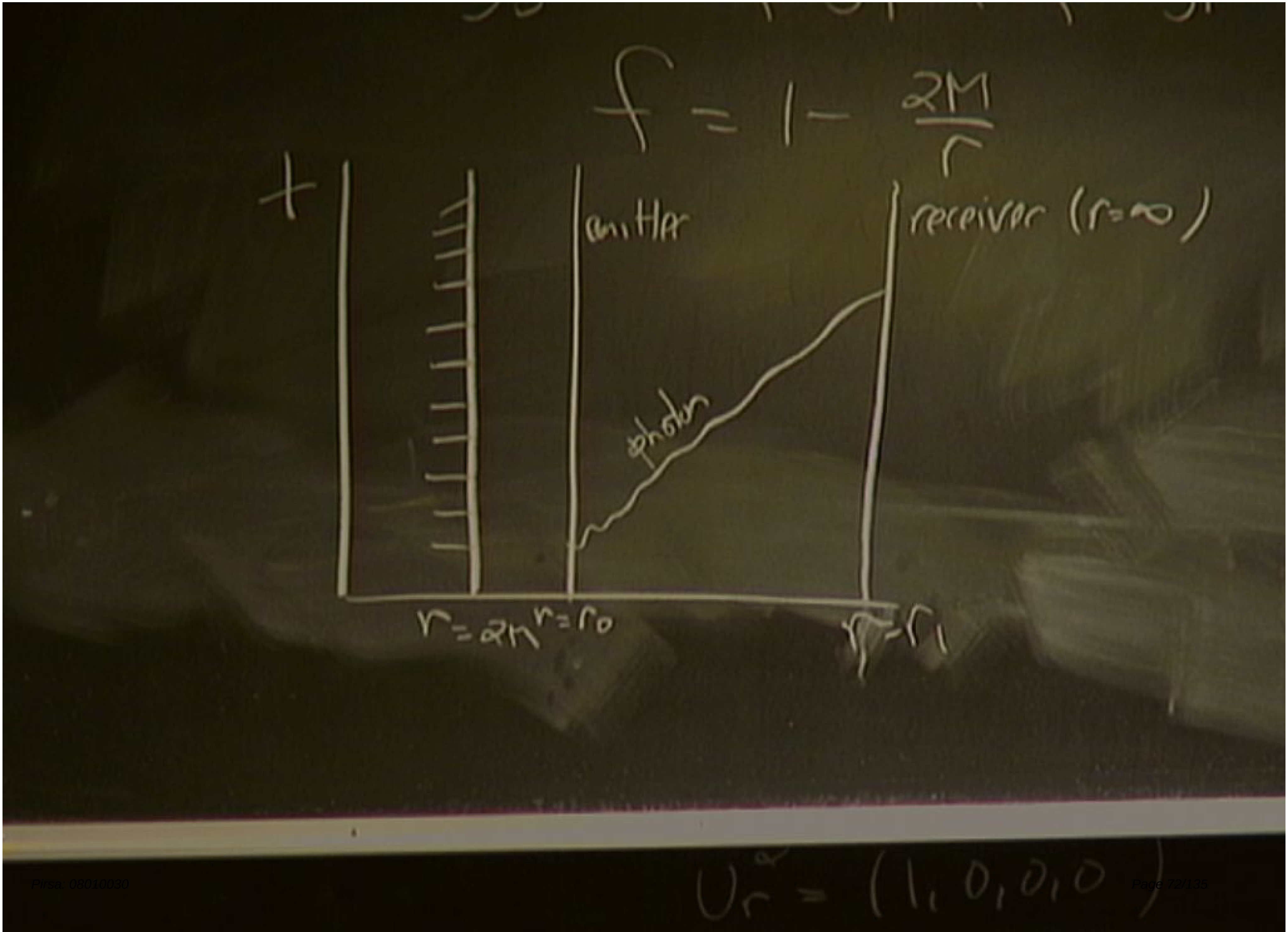
$$= \cancel{(K)}\left(\frac{E}{f}\right)\left(\frac{1}{\cancel{K}}\right)$$

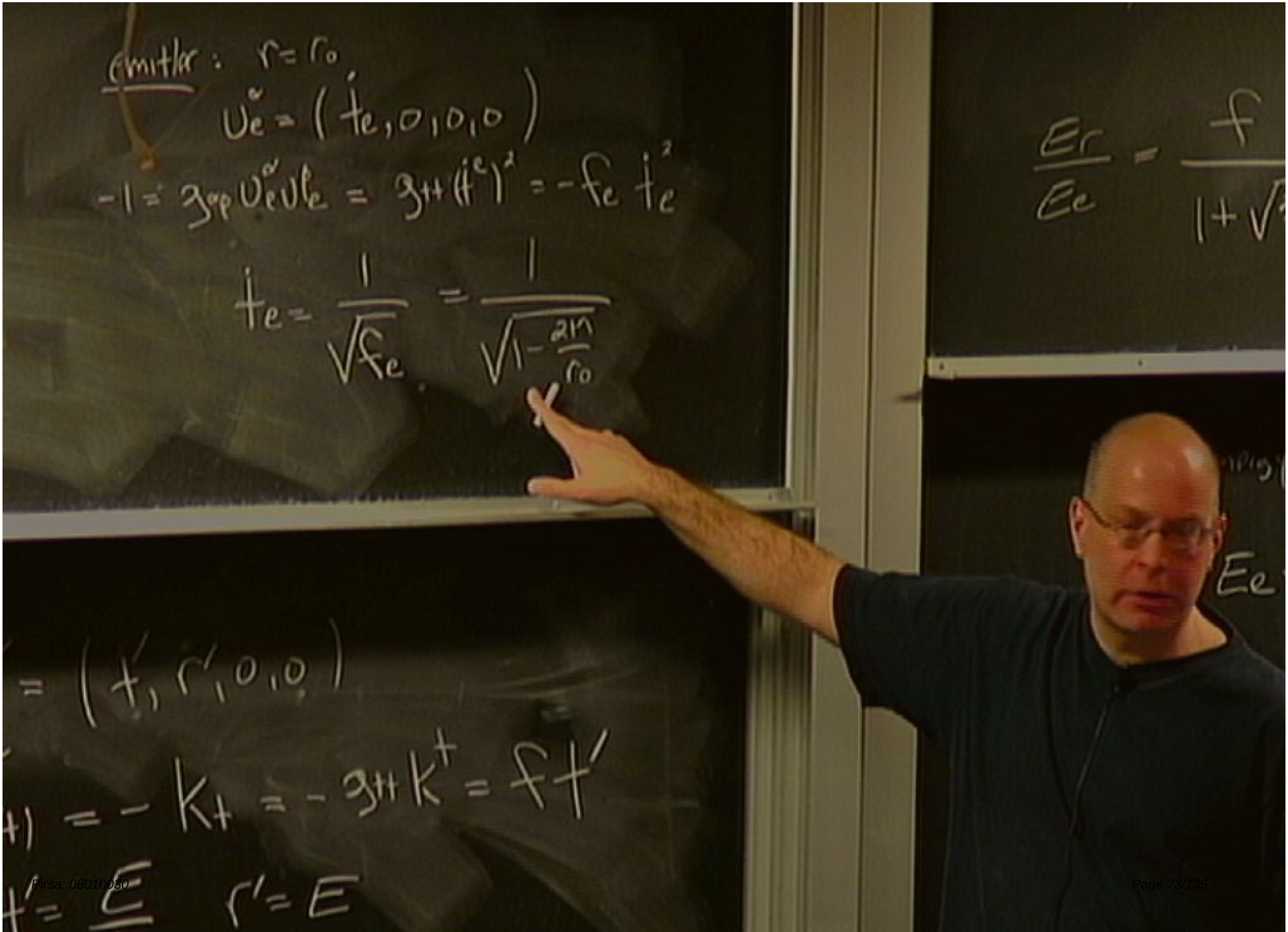
$$\frac{E_r}{E_e} = \frac{f}{1 + \sqrt{\frac{2m}{r}}}$$

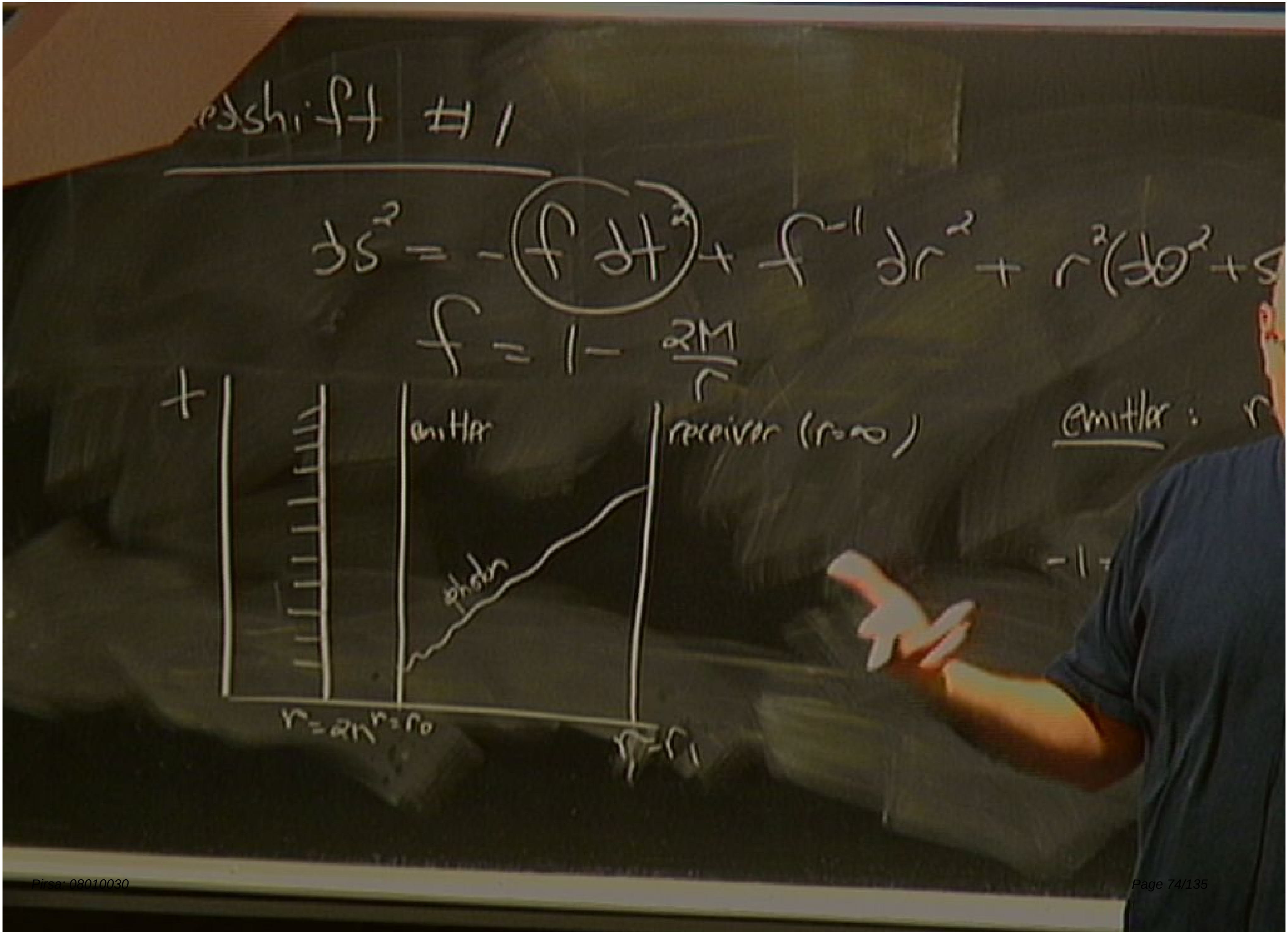
$$E_e = - K a V_e^{\alpha}$$

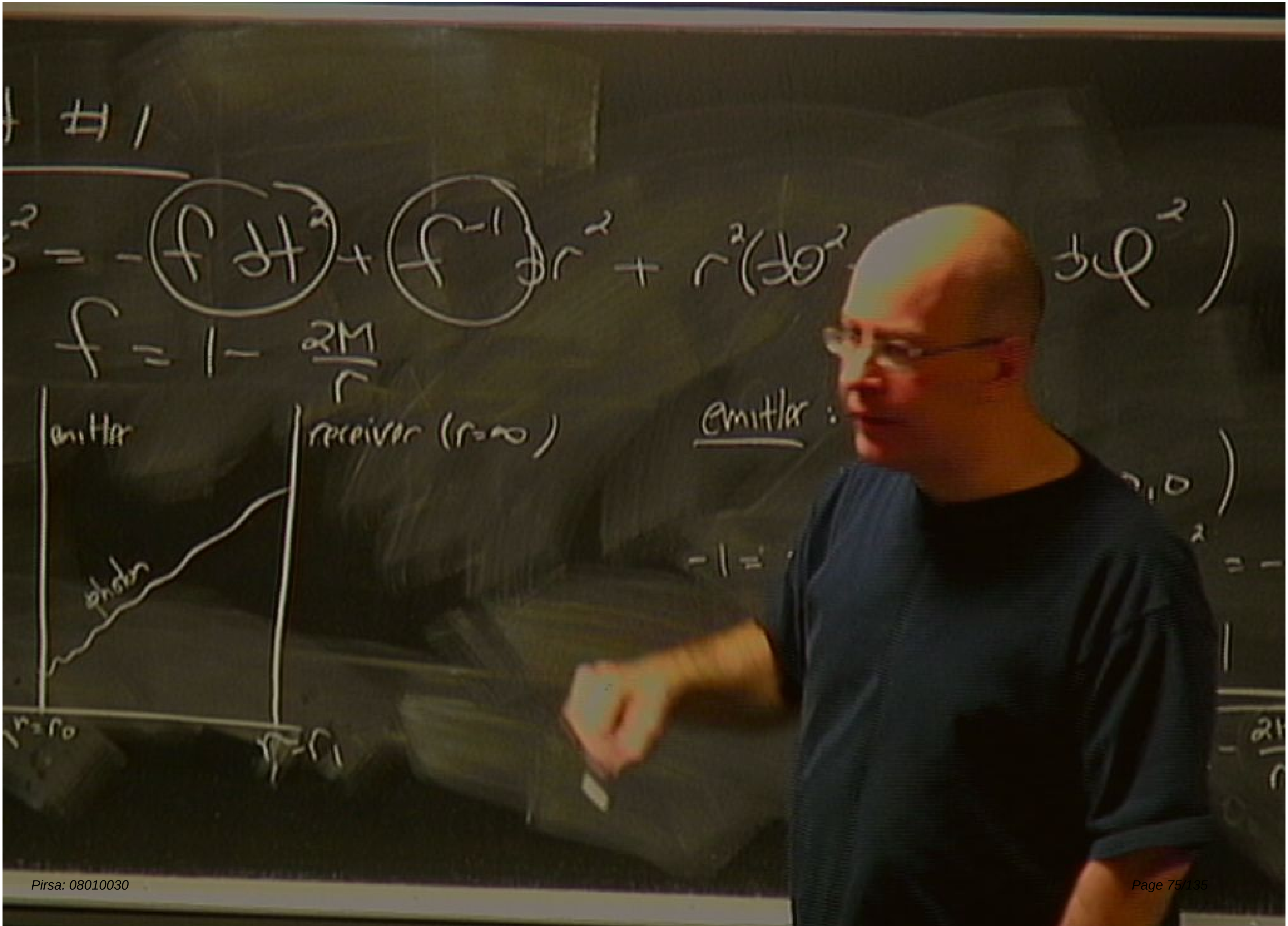

$\frac{E_r}{E_e} = \frac{f}{1+\sqrt{\frac{2m}{r}}} = \frac{\left(1-\sqrt{\frac{2m}{r}}\right)\left(1+\sqrt{\frac{2m}{r}}\right)}{\left(1+\sqrt{\frac{2m}{r}}\right)} = 1-\sqrt{\frac{2m}{r}}$
Photon's energy as measured by emitters
$E_e = -K_\alpha U_e^\alpha = -g_{\alpha\beta} K^\alpha U_e^\beta$
$= \frac{E}{f}\left(1+\sqrt{\frac{2m}{r}}\right)$


Redshift #2
t
receiver
r
emitter


f = 1 - 2M/r
t
emitter
receiver (r=∞)
photon
r=2M
r=r0
r=r1
Ur = (1,0,0,0)


Emitter : r = r0
Ue = (te, 0, 0, 0)
-1 = gαβ Ue Ue = gtt (te)² = -fe te²
te = 1/√fe = 1/√(1 - 2M/r0)
= (t, r, 0, 0)
= - Kt = - gtt k^t = f t'
t' = E   r' = E
Er/Ee = f
Ee


edshift #1
$ds^2 = -f\,dt^2 + f^{-1}\,dr^2 + r^2(d\theta^2 + s$
$f = 1 - \frac{2M}{r}$
t
emitter
receiver (r=∞)
photon
r=2M
r=r0
r=r1
emitter : r
-1 +


#1
f = 1 − 2M/r
emitter
receiver (r=∞)
photon
emitter :

$$ds^2 = g_{\alpha\beta}\, dx^\alpha\, dx^\beta$$

$$ds^2 = g_{\alpha\beta}\, dx^\alpha\, dx^\beta$$

$$ds^2 = g_{\alpha\beta}\, dx^\alpha\, dx^\beta$$

$$\sqrt{-g}$$

$$ds^2 = g_{\alpha\beta}\, dx^\alpha\, dx^\beta$$

$$\sqrt{-g} \qquad g = \det[g_{\alpha\beta}]$$

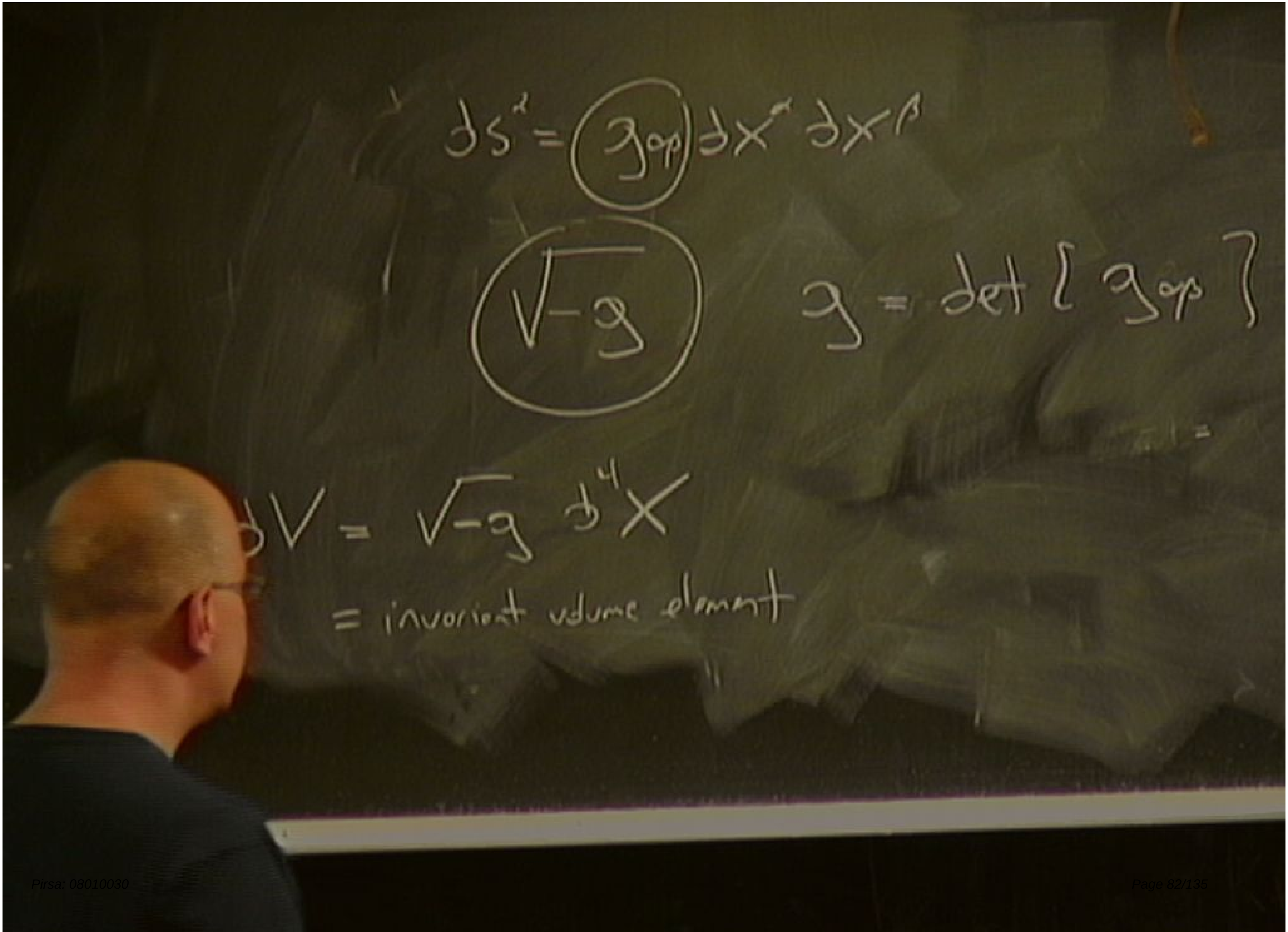
$$ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta$$

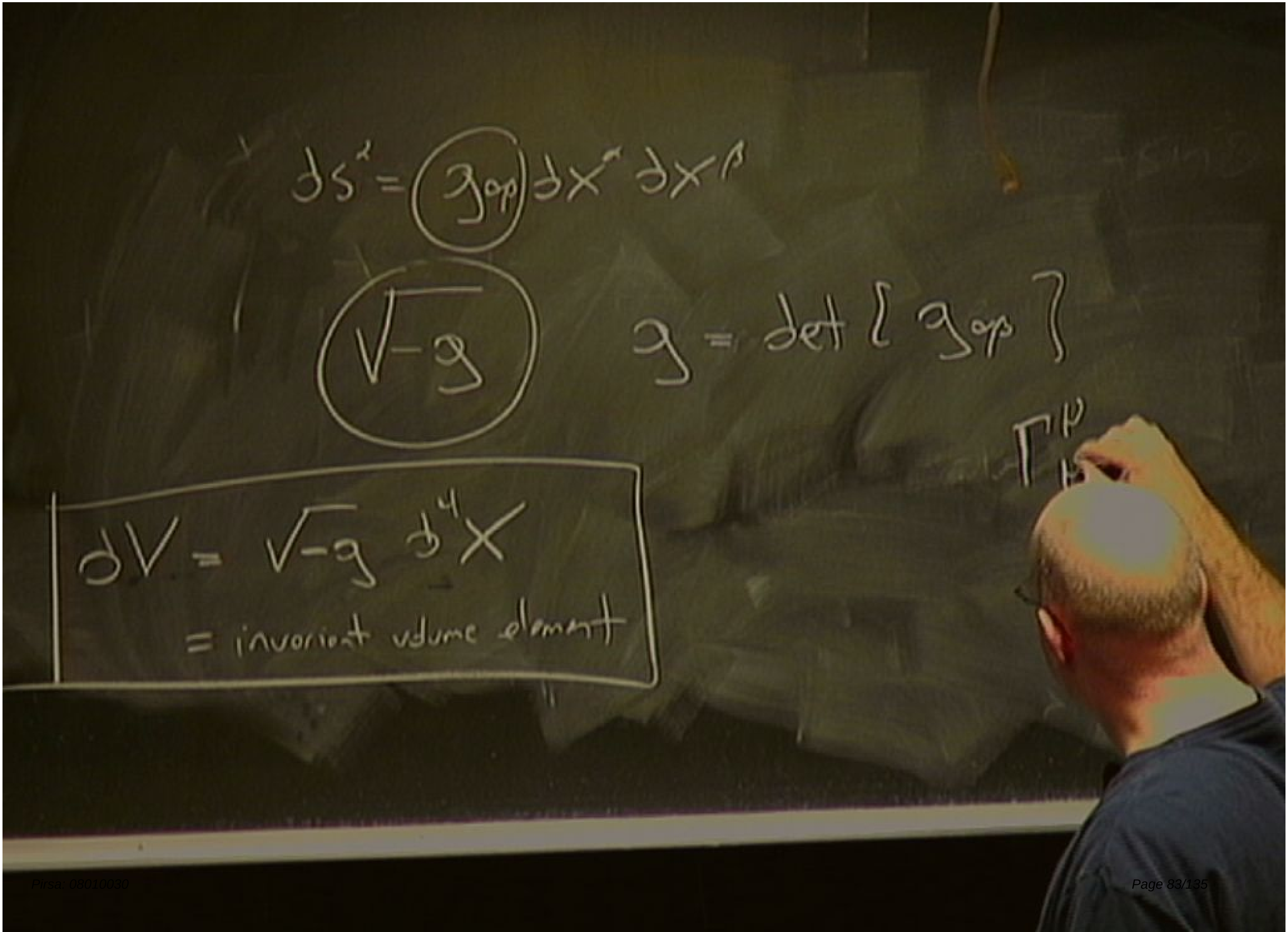
$$\sqrt{-g} \qquad g = \det[g_{\alpha\beta}]$$

$$ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta$$

$$\sqrt{-g} \qquad g = \det[g_{\alpha\beta}]$$

$$dV = \sqrt{-g}\, d^4x$$


$ds^2 = g_{\alpha\beta}\, dx^\alpha\, dx^\beta$
$\sqrt{-g}$
$g = \det[g_{\alpha\beta}]$
$dV = \sqrt{-g}\, d^4x$
= invariant volume element


$ds^2 = g_{\alpha\beta} \, dx^\alpha \, dx^\beta$
$\sqrt{-g}$
$g = \det [g_{\alpha\beta}]$
Γ^{μ}
$\boxed{dV = \sqrt{-g} \, d^4 x}$
$=$ invariant volume element

$$[\, g_{\alpha\beta} \,]$$

$$\Gamma^{\mu}_{\mu\alpha} = \frac{1}{\sqrt{-g}}\,\partial_\alpha\left(\sqrt{-g}\right)$$

$$\Gamma^{\mu}_{\mu\alpha} = \frac{1}{\sqrt{-g}}\,\partial_\alpha\left(\sqrt{-g}\right)$$

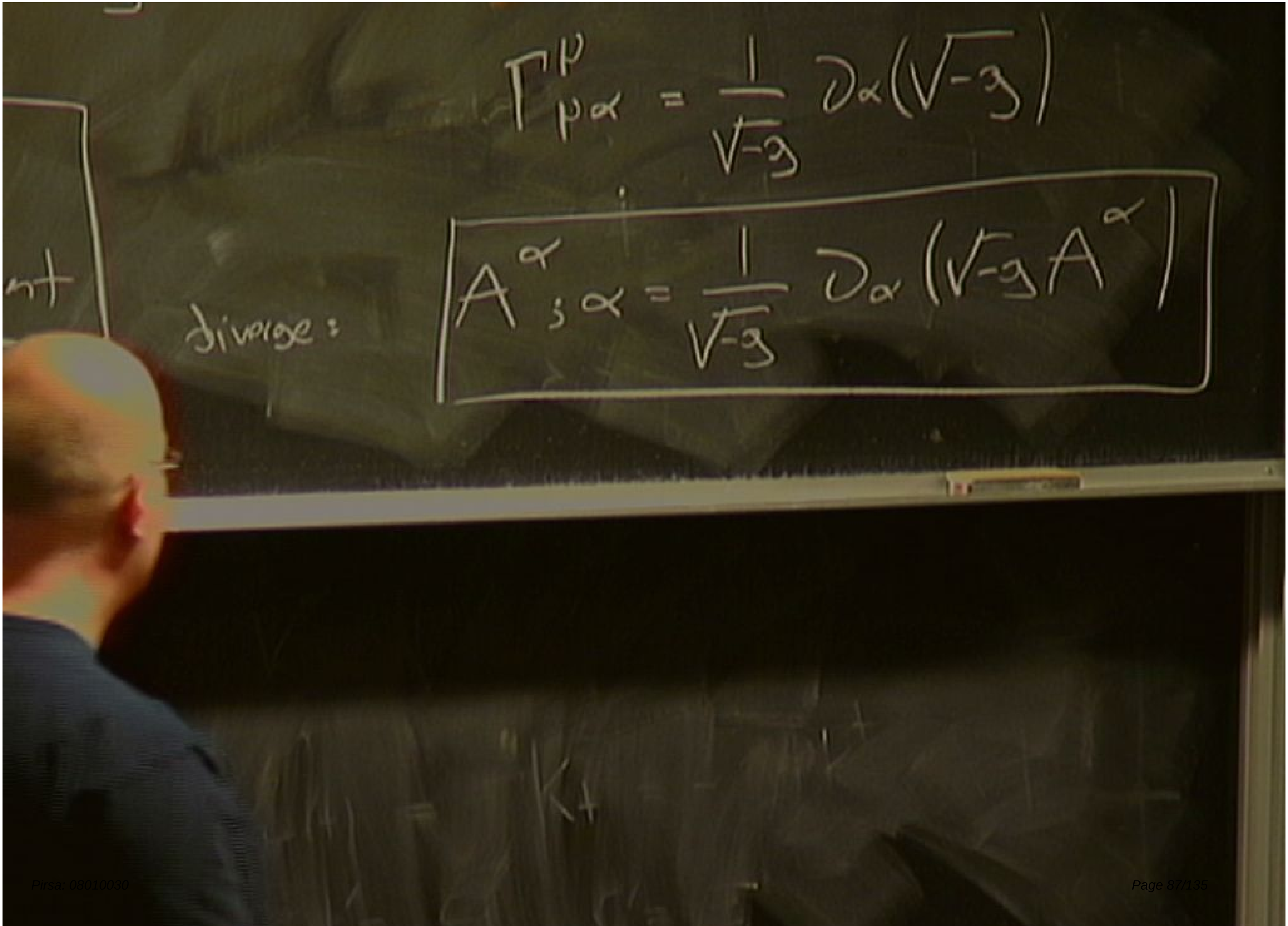
$$A^{\alpha}{}_{;\alpha}$$

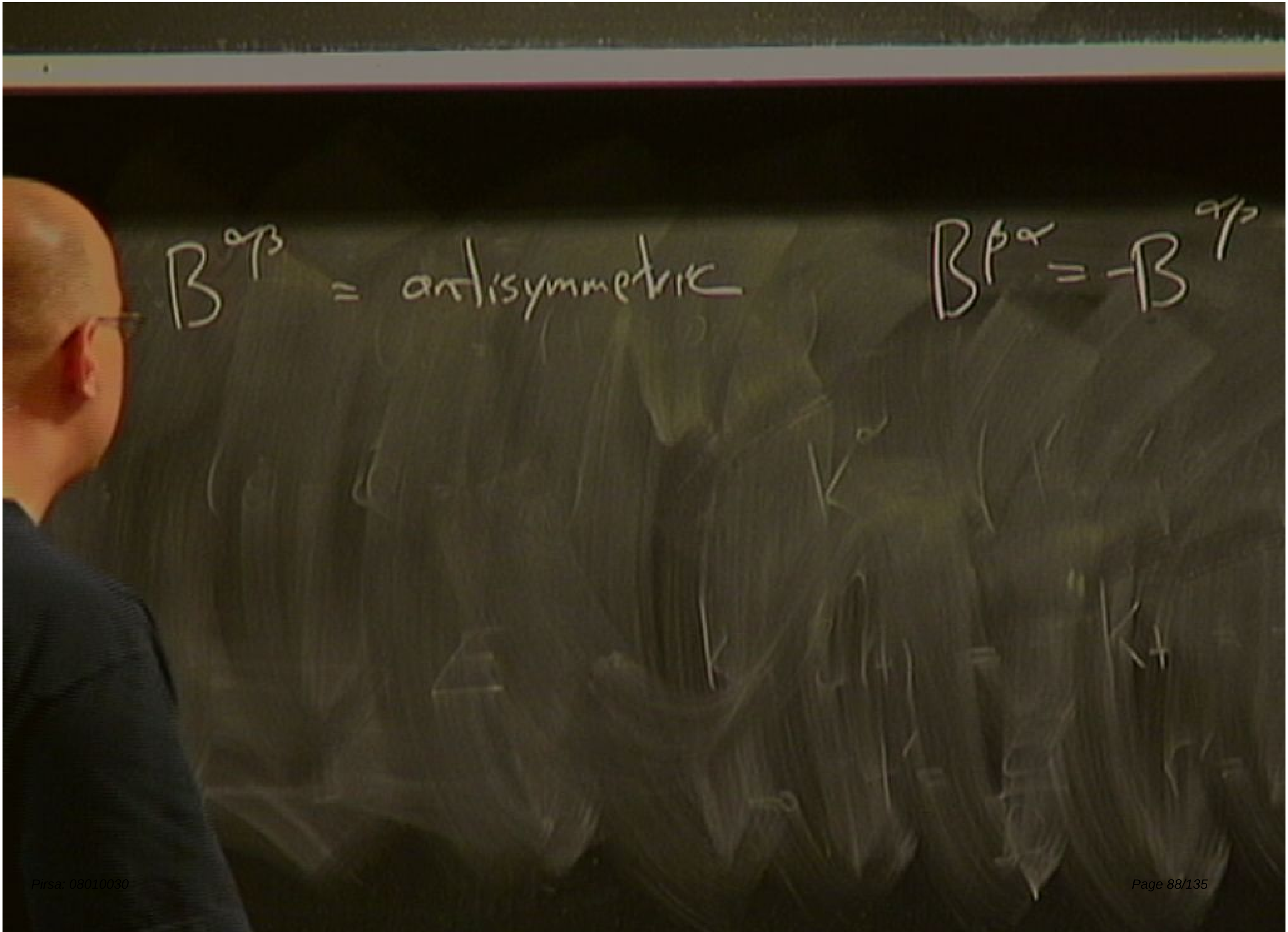
$$g = \det[g_{\alpha\beta}]$$

$$\Gamma^{\mu}_{\mu\alpha} = \frac{1}{\sqrt{-g}}\partial_\alpha\left(\sqrt{-g}\right)$$

diverge:

$$A^{\alpha}_{;\alpha} = \frac{1}{\sqrt{-g}}\partial_\alpha\left(\sqrt{-g}A^{\alpha}\right)$$


$\Gamma^{\mu}_{\mu\alpha} = \frac{1}{\sqrt{-g}} \partial_\alpha\left(\sqrt{-g}\right)$
diverge:
$\boxed{A^{\alpha}{}_{;\alpha} = \frac{1}{\sqrt{-g}} \partial_\alpha\left(\sqrt{-g}\,A^{\alpha}\right)}$


$B^{\alpha\beta}$ = antisymmetric
$B^{\beta\alpha} = -B^{\alpha\beta}$

$B^{\alpha\beta}$ = antisymmetric $\qquad B^{\beta\alpha} = -B^{\alpha\beta}$

$B^{\alpha\beta}{}_{;\beta}$

$B^{\alpha\beta}$ = antisymmetric $\quad B^{\beta\alpha} = -B^{\alpha\beta}$

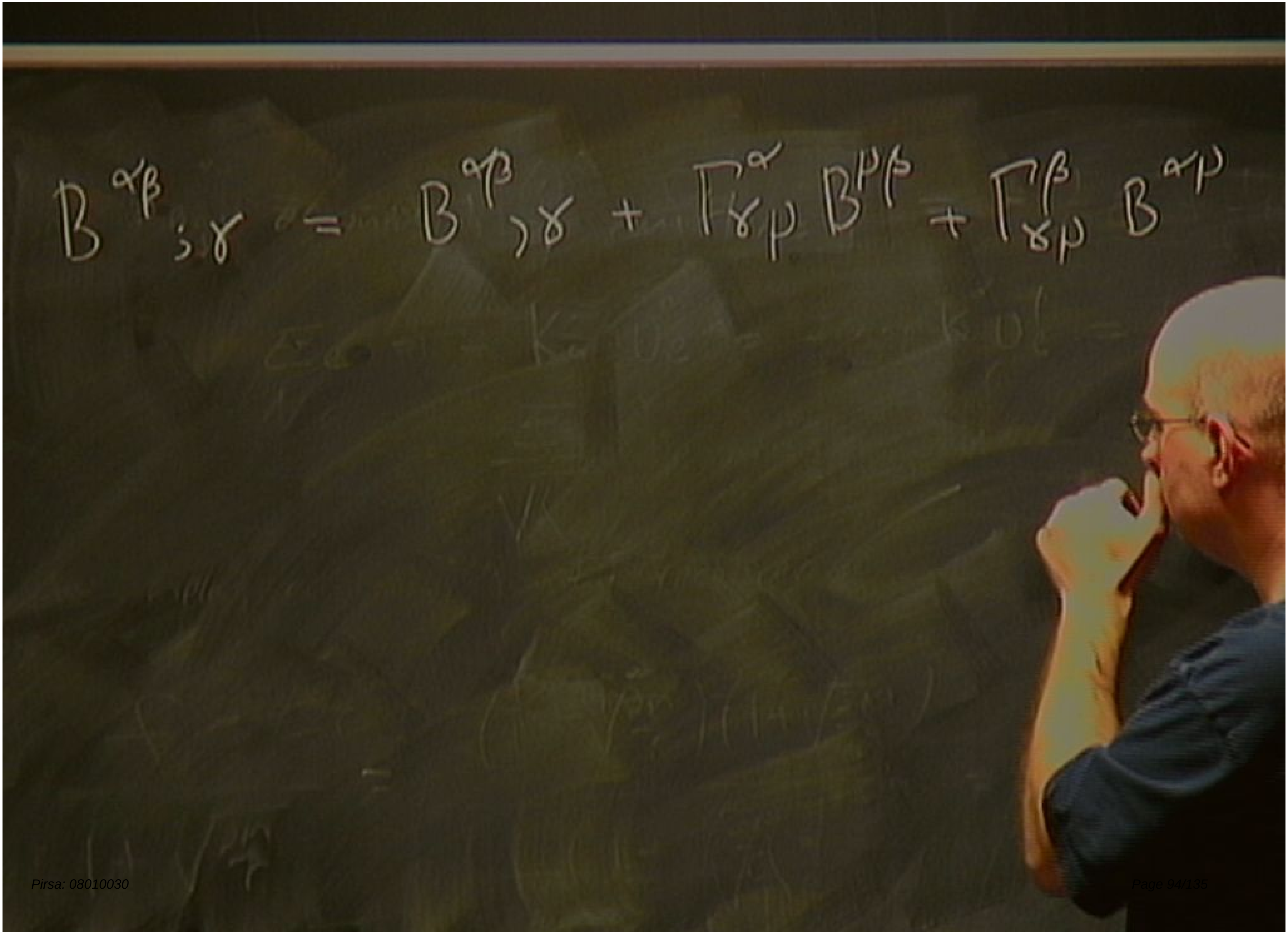
$$B^{\alpha\beta}{}_{;\beta} = \frac{1}{\sqrt{-g}}\,\partial_\beta\left(\sqrt{-g}\,B^{\alpha\beta}\right)$$

$B^{\alpha\beta}$ = antisymmetric $\quad B^{\beta\alpha} = -B^{\alpha\beta}$

$$\boxed{B^{\alpha\beta}{}_{;\beta} = \frac{1}{\sqrt{-g}}\,\partial_\beta\left(\sqrt{-g}\,B^{\alpha\beta}\right)}$$

$$B^{\alpha\beta}{}_{;\gamma} = B^{\alpha\beta}{}_{,\gamma}$$

$$B^{\alpha\beta}{}_{;\gamma} = B^{\alpha\beta}{}_{,\gamma} + \Gamma^{\alpha}{}_{\gamma\mu} B^{\mu\beta}$$


$B^{\alpha\beta}{}_{;\gamma} = B^{\alpha\beta}{}_{,\gamma} + \Gamma^{\alpha}_{\gamma\mu} B^{\mu\beta} + \Gamma^{\beta}_{\gamma\mu} B^{\alpha\mu}$

$$B^{\alpha\beta}{}_{;\gamma} = B^{\alpha\beta}{}_{,\gamma} + \Gamma^{\alpha}_{\gamma\mu} B^{\mu\beta} + \Gamma^{\beta}_{\gamma\mu} B^{\alpha\mu}$$

$$B^{\alpha\beta}{}_{;\beta} = B^{\alpha\beta}{}_{,\beta} + \Gamma^{\alpha}_{\beta\mu} B^{\mu\beta} + \Gamma^{\beta}_{\beta\mu} B^{\alpha\mu}$$

$$B^{\alpha\beta}{}_{;\gamma} = B^{\alpha\beta}{}_{,\gamma} + \Gamma^{\alpha}_{\gamma\mu} B^{\mu\beta} + \Gamma^{\beta}_{\gamma\mu} B^{\alpha\mu}$$

$$B^{\alpha\beta}{}_{;\beta} = B^{\alpha\beta}{}_{,\beta} + \Gamma^{\alpha}_{\beta\mu} B^{\mu\beta} + \underbrace{\Gamma^{\beta}_{\beta\mu}}_{\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g})} B^{\alpha\mu}$$

$$+ \Gamma^{\alpha}_{\gamma\mu} B^{\rho\beta} + \Gamma^{\beta}_{\gamma\mu} B^{\alpha\mu}$$

$$\Gamma^{\alpha}_{\rho\mu} B^{\mu\rho} + \underbrace{\Gamma^{\beta}_{\beta\mu}}_{\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}\right)} B^{\alpha\mu}$$

$$B^{\alpha\beta}{}_{;\gamma} = B^{\alpha\beta}{}_{,\gamma} + \Gamma^{\alpha}_{\gamma\mu} B^{\mu\beta} + \Gamma^{\beta}_{\gamma\mu} B^{\alpha\mu}$$

$$B^{\alpha\beta}{}_{;\beta} = B^{\alpha\beta}{}_{,\beta} + \Gamma^{\alpha}_{\beta\mu} B^{\mu\beta} + \underbrace{\Gamma^{\beta}_{\beta\mu}}_{\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g})} B^{\alpha\mu}$$

$$B^{\alpha\beta}{}_{;\gamma} = B^{\alpha\beta}{}_{,\gamma} + \Gamma^{\alpha}_{\gamma\mu} B^{\mu\beta} + \Gamma^{\beta}_{\gamma\mu} B^{\alpha\mu}$$

$$B^{\alpha\beta}{}_{;\beta} = B^{\alpha\beta}{}_{,\beta} + \underset{0}{\Gamma^{\alpha}_{\beta\mu} B^{\mu\beta}} + \underbrace{\Gamma^{\beta}_{\beta\mu}}_{\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g})} B^{\alpha\mu}$$

$[\alpha\ \beta\ \gamma\ \delta \}$

$$[\alpha\beta\gamma\delta] = \text{permutation symbol}$$

$$= \begin{cases} +1 \\ \end{cases}$$

$$[\alpha\beta\gamma\delta] = \text{permutation symbol}$$

$$= \begin{cases} +1 & \text{even perm of } 0123 \\ -1 & \text{odd} \quad '' \quad '' \quad '' \\ 0 & \end{cases}$$

$$[\alpha\beta\gamma\delta] = \text{permutation symbol}$$

$$= \begin{cases} +1 & \text{even perm of } 0\,1\,2\,3 \\ -1 & \text{odd} \quad " \quad " \quad " \\ 0 & \text{if any indices agree} \end{cases}$$

$[\alpha\beta\gamma\delta]$ = permutation symbol

$$= \begin{cases} +1 & \text{even perm of } 0123 \\ -1 & \text{odd} \quad " \quad " \quad " \\ 0 & \text{if any indices agree} \end{cases}$$

$[0123] = 1$

$[1023] = -1$

$[0213] = -1$

$[2013] = +1$

$[\alpha\beta\gamma\delta]$ = permutation symbol

$$= \begin{cases} +1 & \text{even perm of } 0123 \\ -1 & \text{odd} \\ 0 & \text{if any indices agree} \end{cases}$$

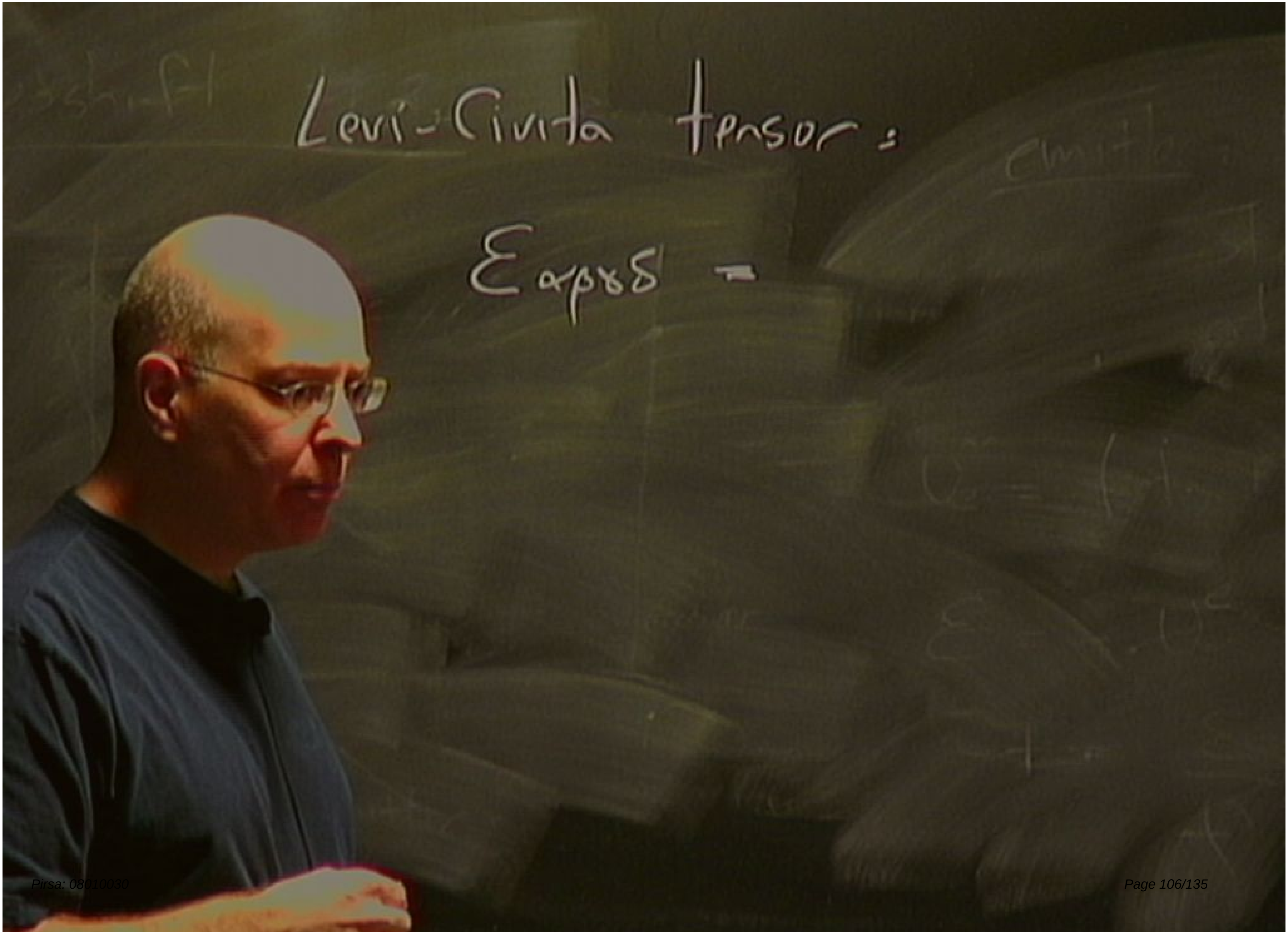
$[0123] = 1$

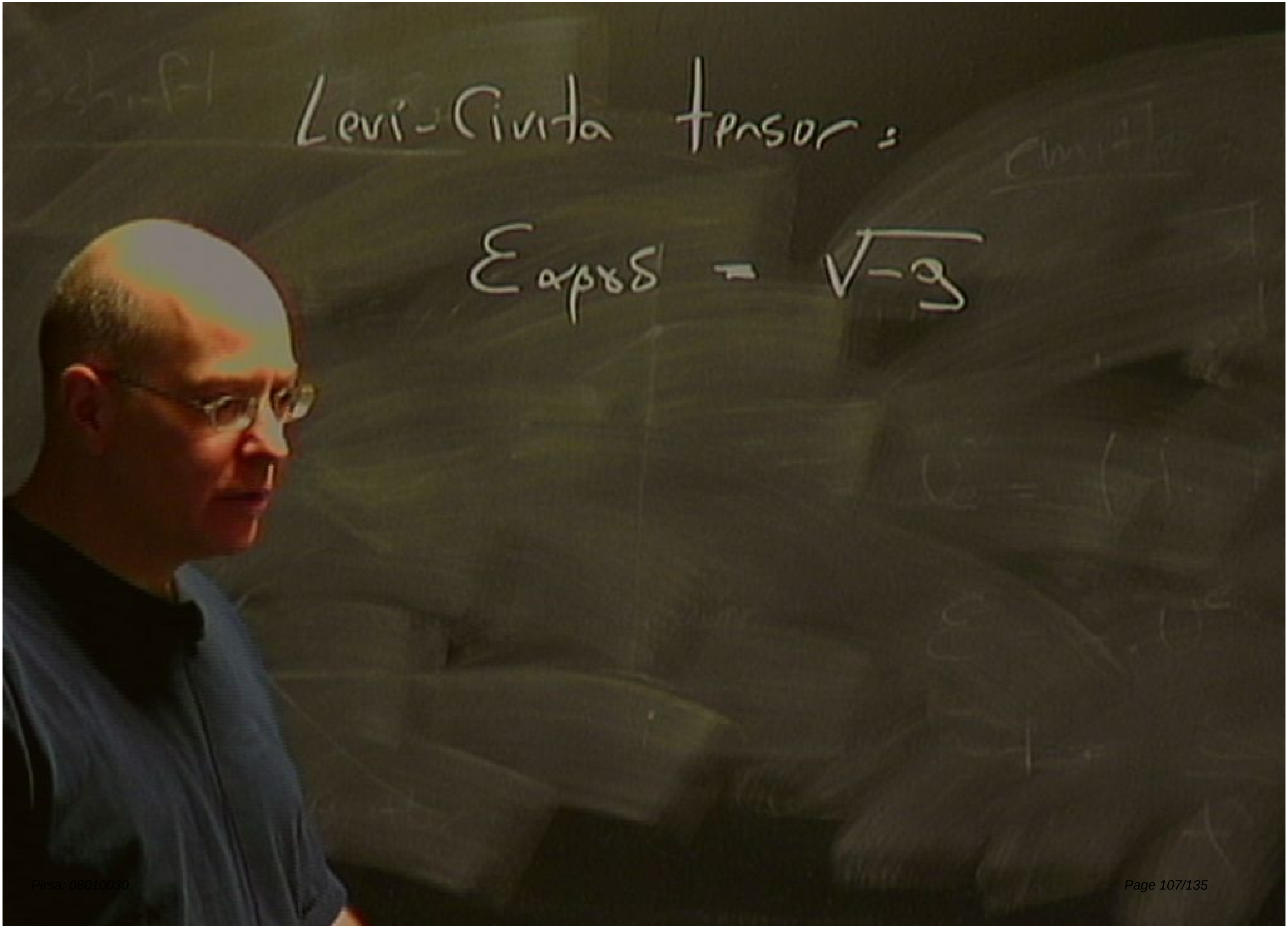
$[1023] = -1$

$[0213] = -1$

$[2013] = +1$

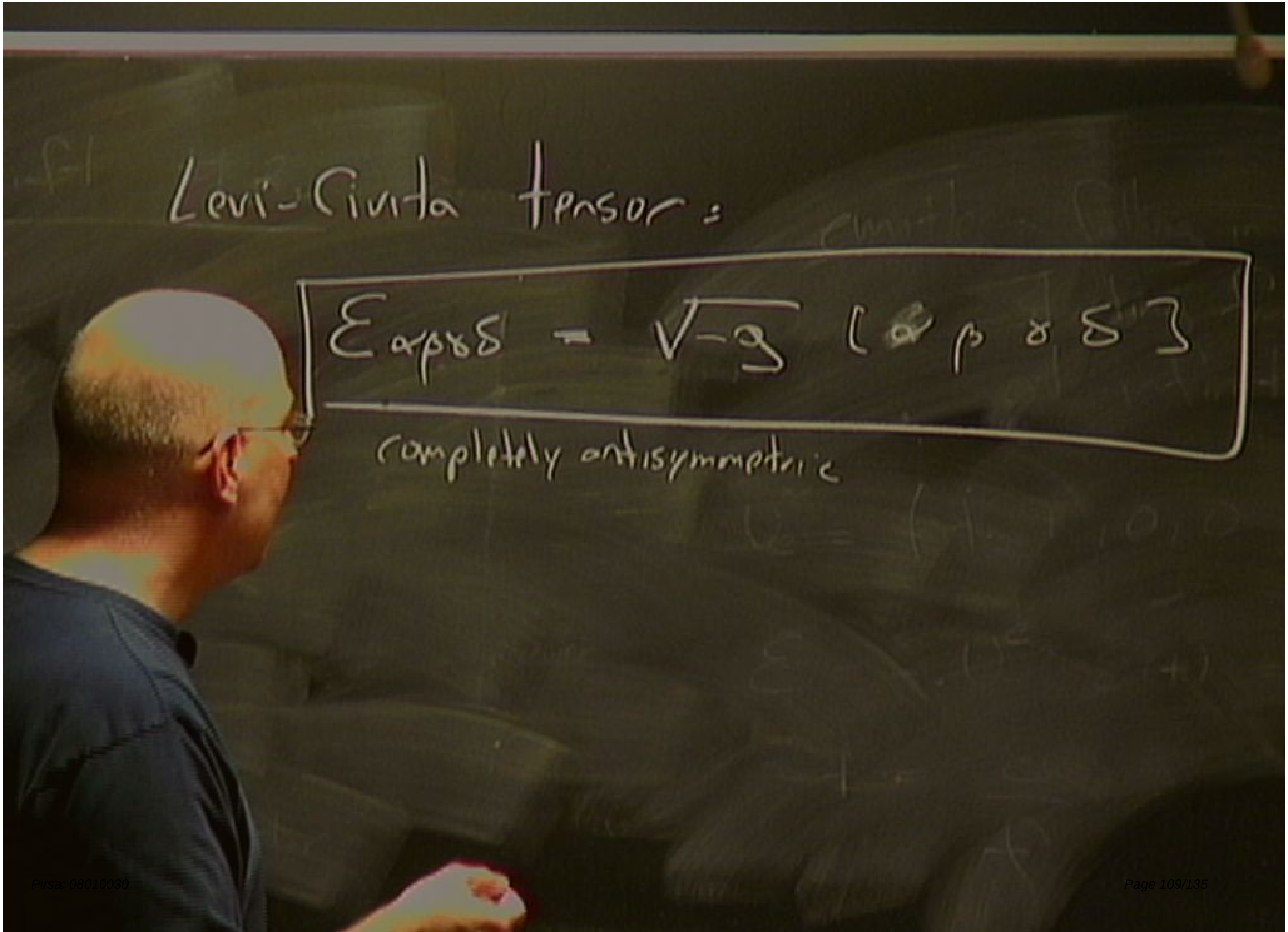
$[1123] = 0$


Levi-Civita tensor:
$\varepsilon_{\alpha\beta\gamma\delta} =$


Levi-Civita tensor:
$\epsilon_{\alpha\beta\gamma\delta} = \sqrt{-g}$

Levi-Civita tensor:

$$\boxed{\epsilon_{\alpha\beta\gamma\delta} = \sqrt{-g}\,[\alpha\,\beta\,\gamma\,\delta]}$$


Levi-Civita tensor:
$\epsilon_{\alpha\beta\gamma\delta} = \sqrt{-g}\,[\alpha\,\beta\,\gamma\,\delta]$
completely antisymmetric

Curvature

$$A^{\mu}{}_{;\alpha\beta} - A^{\mu}{}_{;\beta\alpha}$$

Curvature

$$A^{\mu}{}_{;\alpha\beta} - A^{\mu}{}_{;\beta\alpha} = -R^{\mu}{}_{\nu\alpha\beta} A^{\nu}$$

Curvature

$$A^{\mu}{}_{;\alpha\beta} - A^{\mu}{}_{;\beta\alpha} = - R^{\mu}{}_{\nu\alpha\beta} A^{\nu}$$

$$A^{\mu}{}_{;\alpha\beta} - A^{\mu}{}_{;\beta\alpha} = - R^{\mu}{}_{\nu\alpha\beta} A^{\nu}$$

$$R = \partial\Gamma - \partial\Gamma + \Gamma\Gamma - \Gamma\Gamma$$

$$P_{\mu;\alpha\beta} - P_{\mu;\beta\alpha} =$$

$$A^{\mu}{}_{;\alpha\beta} - A^{\mu}{}_{;\beta\alpha} = -R^{\mu}{}_{\nu\alpha\beta}A^{\nu}$$

$$R = \partial\Gamma - \partial\Gamma + \Gamma\Gamma - \Gamma\Gamma$$

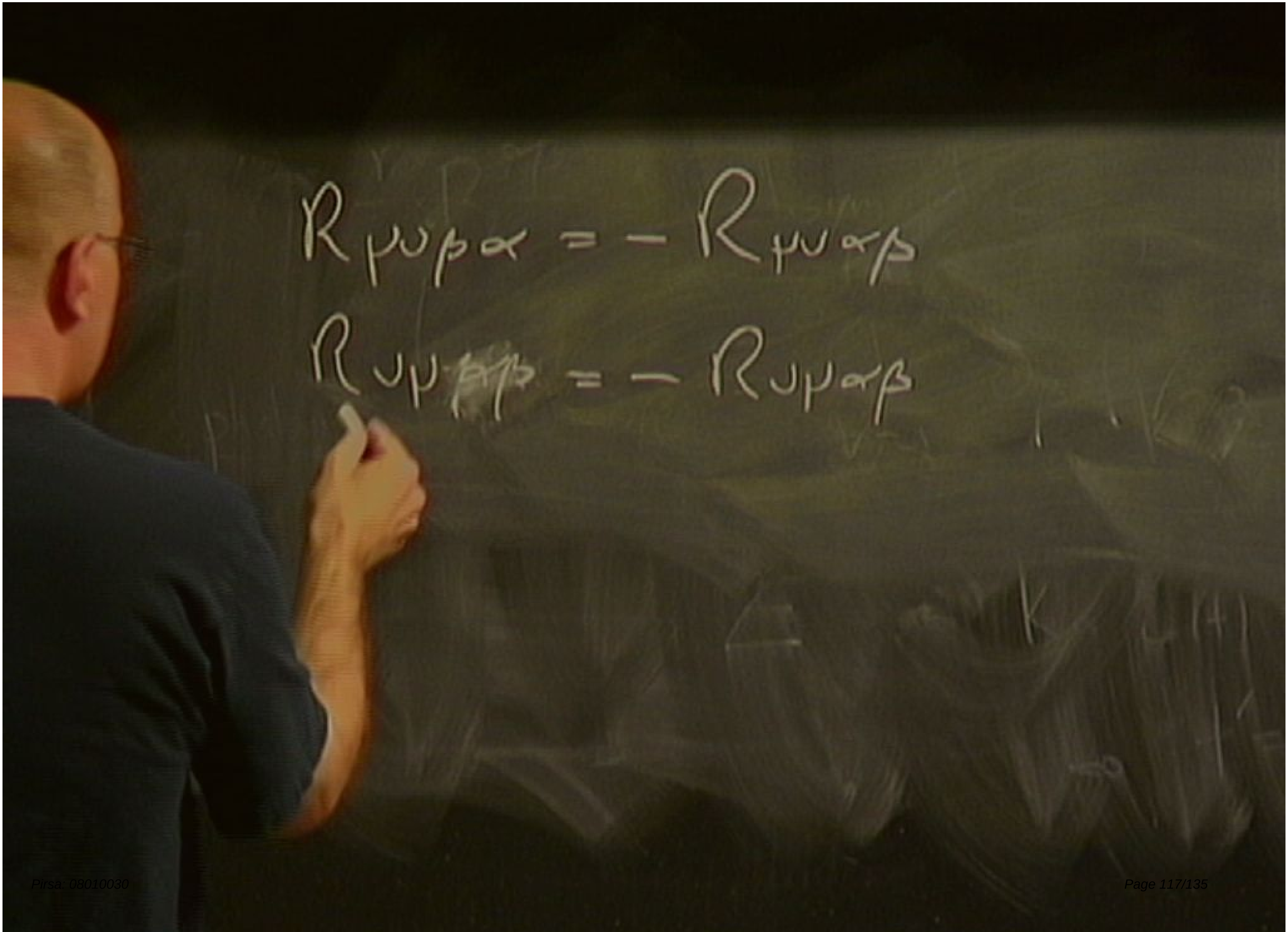
$$P_{\mu;\alpha\beta} - P_{\mu;\beta\alpha} = R$$

$$A^{\mu}{}_{;\alpha\beta} - A^{\mu}{}_{;\beta\alpha} = - R^{\mu}{}_{\nu\alpha\beta} A^{\nu}$$

$$R = \partial\Gamma - \partial\Gamma + \Gamma\Gamma - \Gamma\Gamma$$

$$p_{\mu;\alpha\beta} - p_{\mu;\beta\alpha} = R^{\nu}{}_{\mu\alpha\beta}\, p_{\nu}$$

$$R_{\mu\nu\beta\alpha} = -R_{\mu\nu\alpha\beta}$$


$R_{\mu\nu\beta\alpha} = - R_{\mu\nu\alpha\beta}$
$R_{\nu\mu} = - R_{\nu\mu\alpha\beta}$

$$R_{\mu\nu\beta\alpha} = -R_{\mu\nu\alpha\beta}$$

$$R_{\nu\mu\alpha\beta} = -R_{\nu\mu\alpha\beta}$$

$$R_{\alpha\beta\mu\nu} = R_{\mu\nu\alpha\beta}$$

$$R_{\mu\nu\beta\alpha} = -R_{\mu\nu\alpha\beta}$$

$$R_{\nu\mu\alpha\beta} = -R_{\nu\mu\alpha\beta}$$

$$R_{\alpha\beta\mu\nu} = R_{\mu\nu\alpha\beta}$$

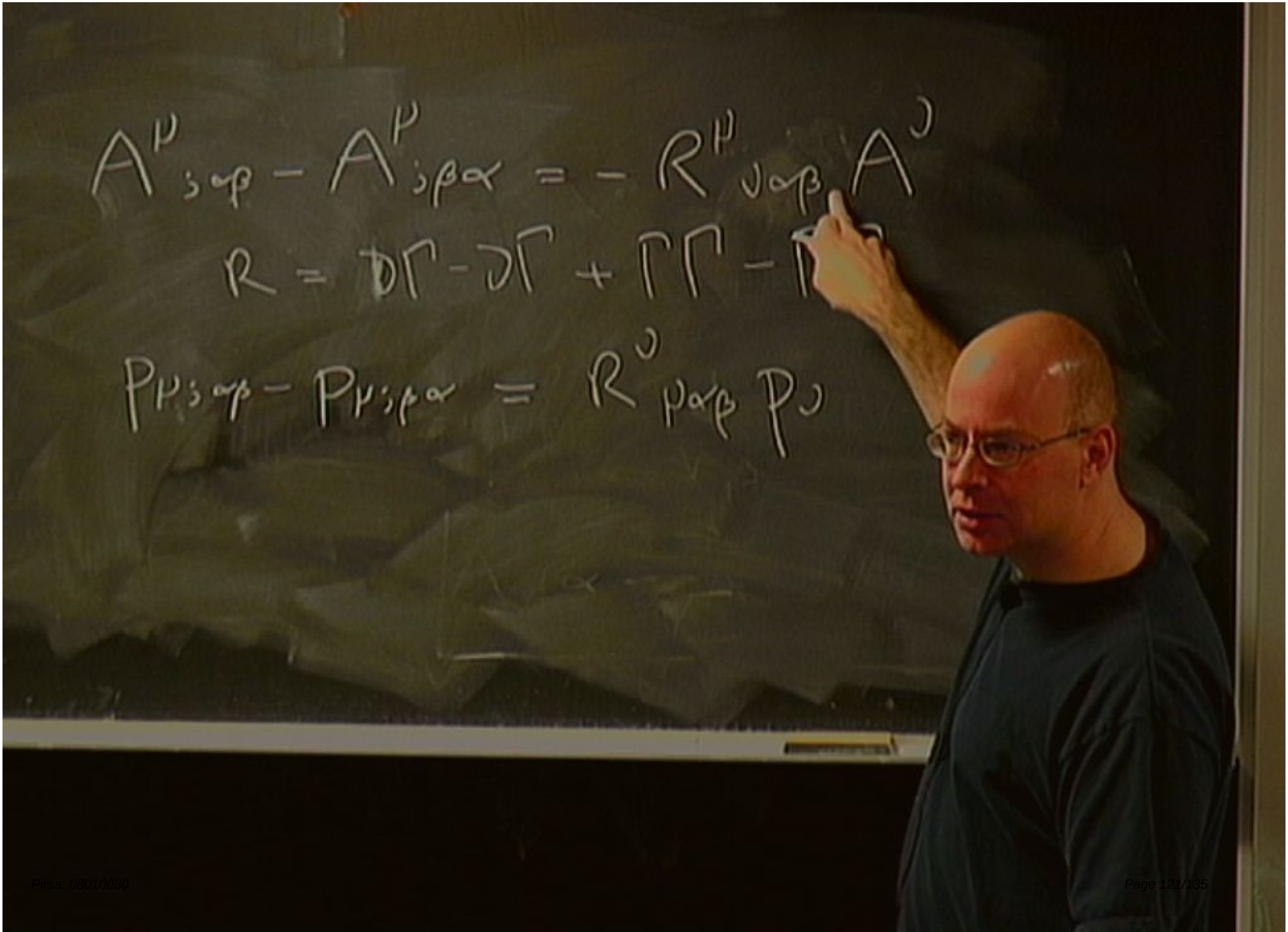
$$R_{\mu\alpha\beta\gamma} + R_{\mu\gamma\alpha\beta} + R_{\mu\beta\gamma\alpha}$$

$$R_{\mu\nu\beta\alpha} = -R_{\mu\nu\alpha\beta}$$

$$R_{\nu\mu\alpha\beta} = -R_{\nu\mu\alpha\beta}$$

$$R_{\alpha\beta\mu\nu} = R_{\mu\nu\alpha\beta}$$

$$R_{\mu\alpha\beta\gamma} + R_{\mu\gamma\alpha\beta} + R_{\mu\beta\gamma\alpha} = 0$$

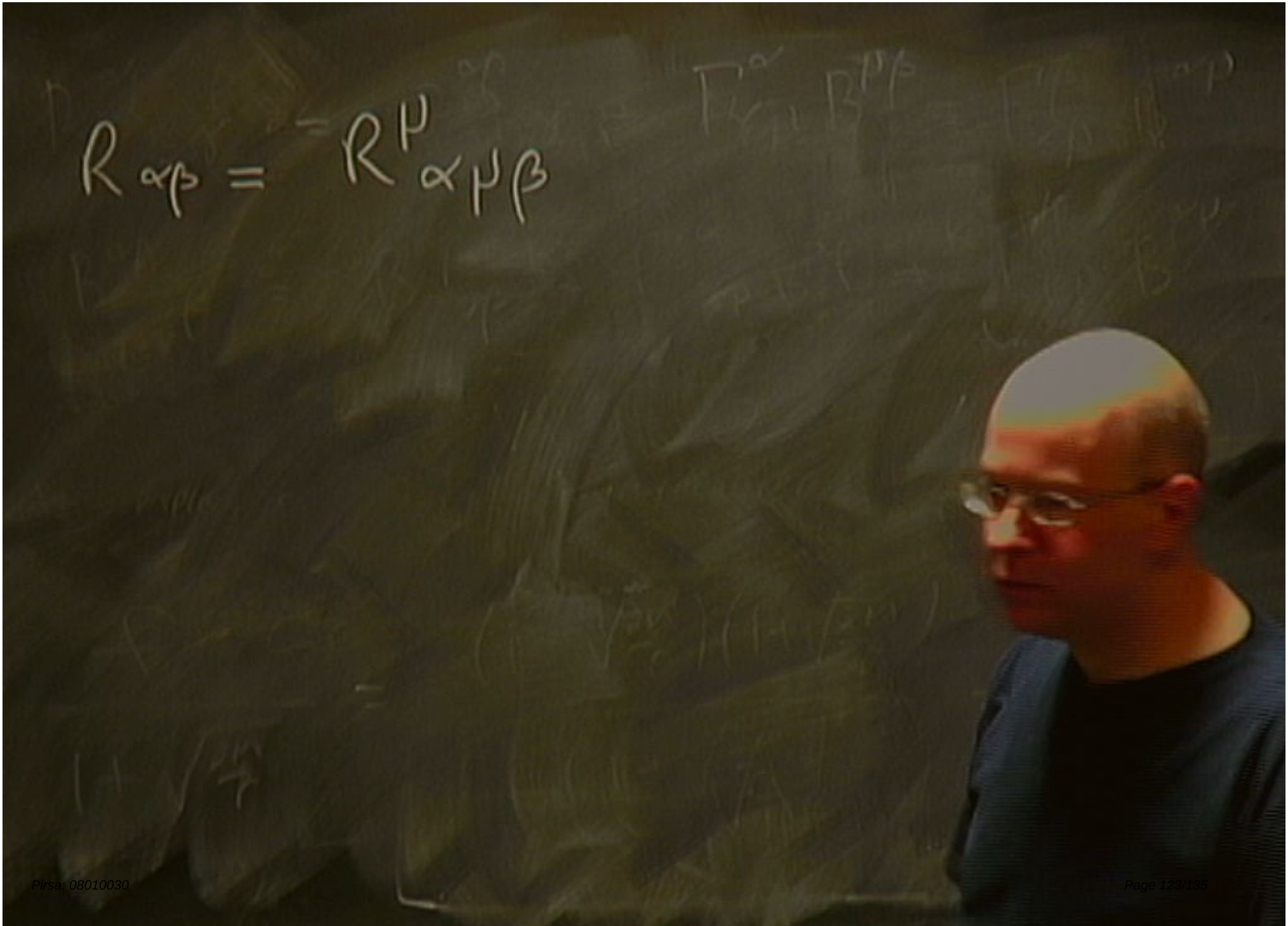

$A^{\mu}{}_{;\alpha\beta} - A^{\mu}{}_{;\beta\alpha} = - R^{\mu}{}_{\nu\alpha\beta} A^{\nu}$
$R = \partial\Gamma - \partial\Gamma + \Gamma\Gamma - \Gamma$
$p_{\mu;\alpha\beta} - p_{\mu;\beta\alpha} = R^{\nu}{}_{\mu\alpha\beta}\, p_{\nu}$

$$R_{\mu\nu\beta\alpha} = -R_{\mu\nu\alpha\beta}$$

$$R_{\nu\mu\beta\alpha} = -R_{\nu\mu\alpha\beta}$$

$$R_{\alpha\beta\mu\nu} = R_{\mu\nu\alpha\beta}$$

$$R_{\mu\alpha\beta\gamma} + R_{\mu\gamma\alpha\beta} + R_{\mu\beta\gamma\alpha} = 0$$


$R_{\alpha\beta} = R^{\mu}{}_{\alpha\mu\beta}$

$$R_{\alpha\beta} = R^{\mu}{}_{\alpha\mu\beta}$$

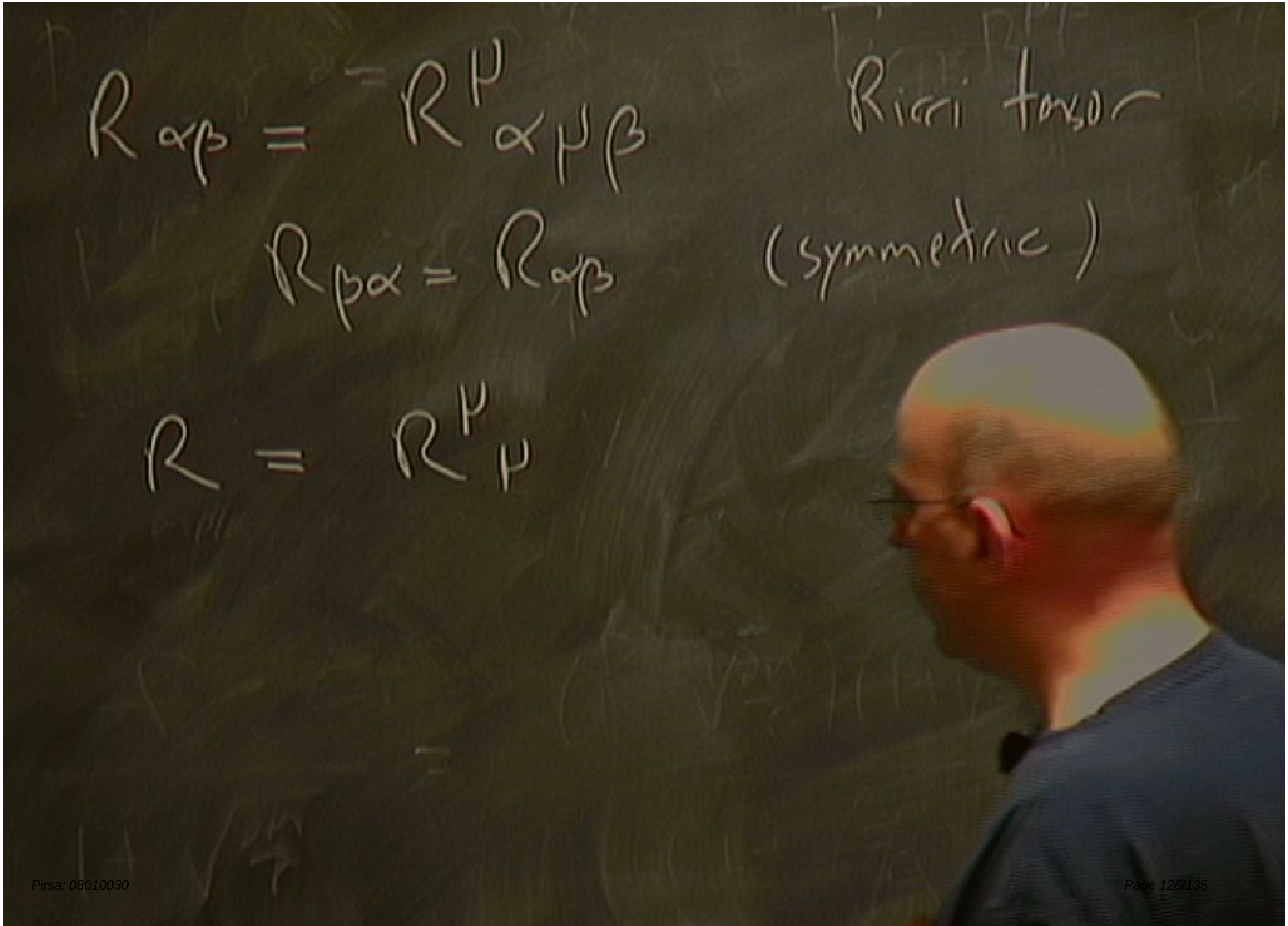
Ricci tensor

$$R_{\alpha\beta} = R^{\mu}{}_{\alpha\mu\beta}$$

Ricci tensor

$$R_{\beta\alpha} = R_{\alpha\beta}$$

(symmetric)



$R_{\alpha\beta} = R^{\mu}{}_{\alpha\mu\beta}$
Ricci tensor
$R_{\beta\alpha} = R_{\alpha\beta}$
(symmetric)
$R = R^{\mu}{}_{\mu}$

$$R_{\alpha\beta} = R^{\mu}{}_{\alpha\mu\beta}$$ Ricci tensor

$$R_{\beta\alpha} = R_{\alpha\beta}$$ (symmetric)

$$R = R^{\mu}{}_{\mu} = g^{\alpha\beta} R_{\alpha\beta}$$ (Ricci scalar)

$$R_{\alpha\beta} = R^{\mu}{}_{\alpha\mu\beta}$$ Ricci tensor

$$R_{\beta\alpha} = R_{\alpha\beta}$$ (symmetric)

$$R = R^{\mu}{}_{\mu} = g^{\alpha\beta} R_{\alpha\beta}$$ (Ricci scalar)

$$G_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R$$

$$G_{\beta\alpha} = G_{\alpha\beta}$$

$$R_{\alpha\beta} = R^{\mu}{}_{\alpha\mu\beta}$$

$$R_{\beta\alpha} = R_{\alpha\beta} \quad \text{(symmetric)}$$

$$R = R^{\mu}{}_{\mu} = g^{\alpha\beta} R_{\alpha\beta} \quad \text{(Ricci scalar)}$$

$$G_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R \quad \text{(Einstein)}$$

$$G_{\beta\alpha} = G_{\alpha\beta}$$

$$G_{\alpha\beta} = 8\pi T_{\alpha\beta}$$

$R_{\beta\alpha} = R_{\alpha\beta}$ (symmetric)

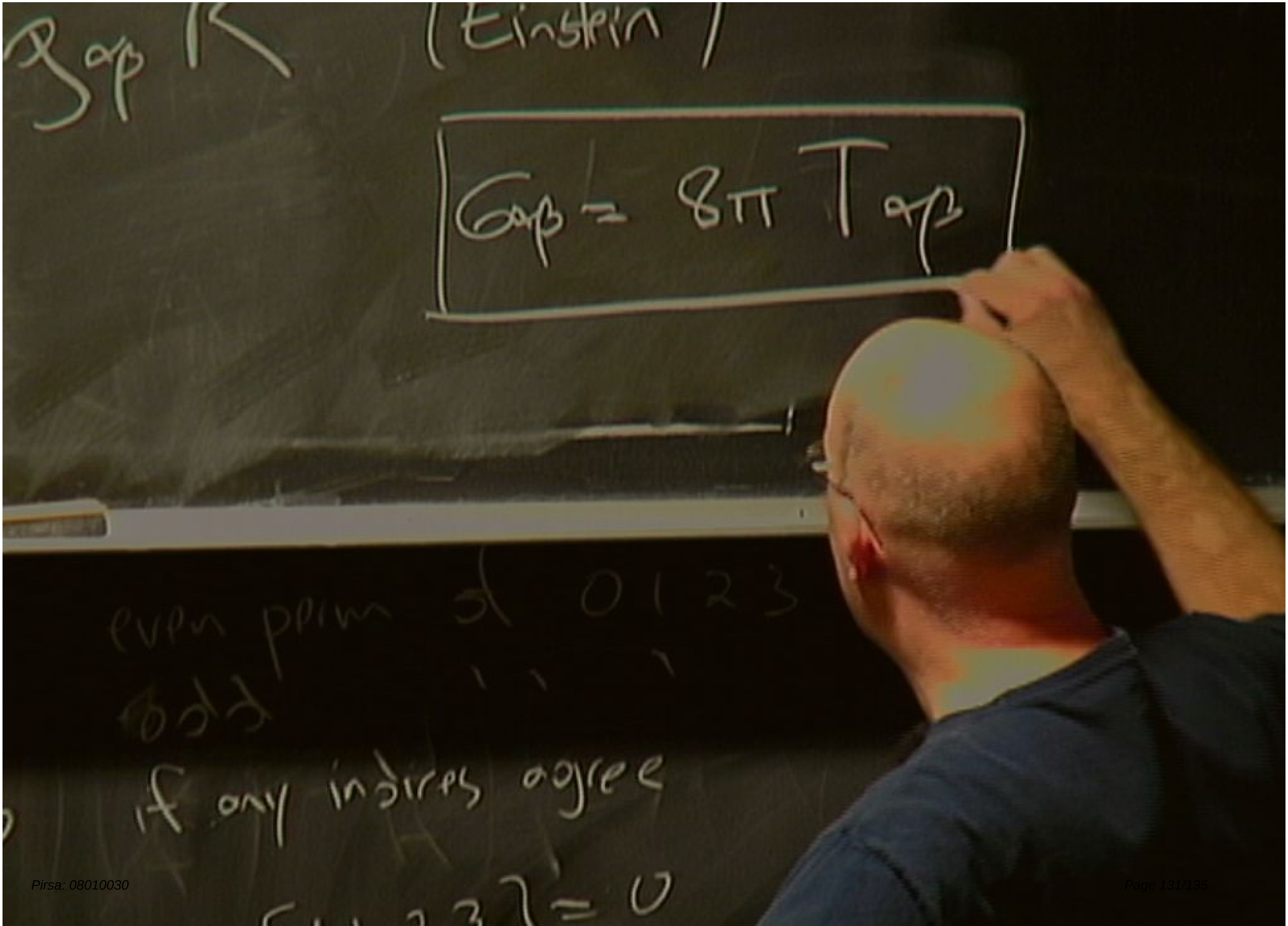
$$R = R^{\mu}{}_{\mu} = g^{\alpha\beta} R_{\alpha\beta} \quad \text{(Ricci scalar)}$$

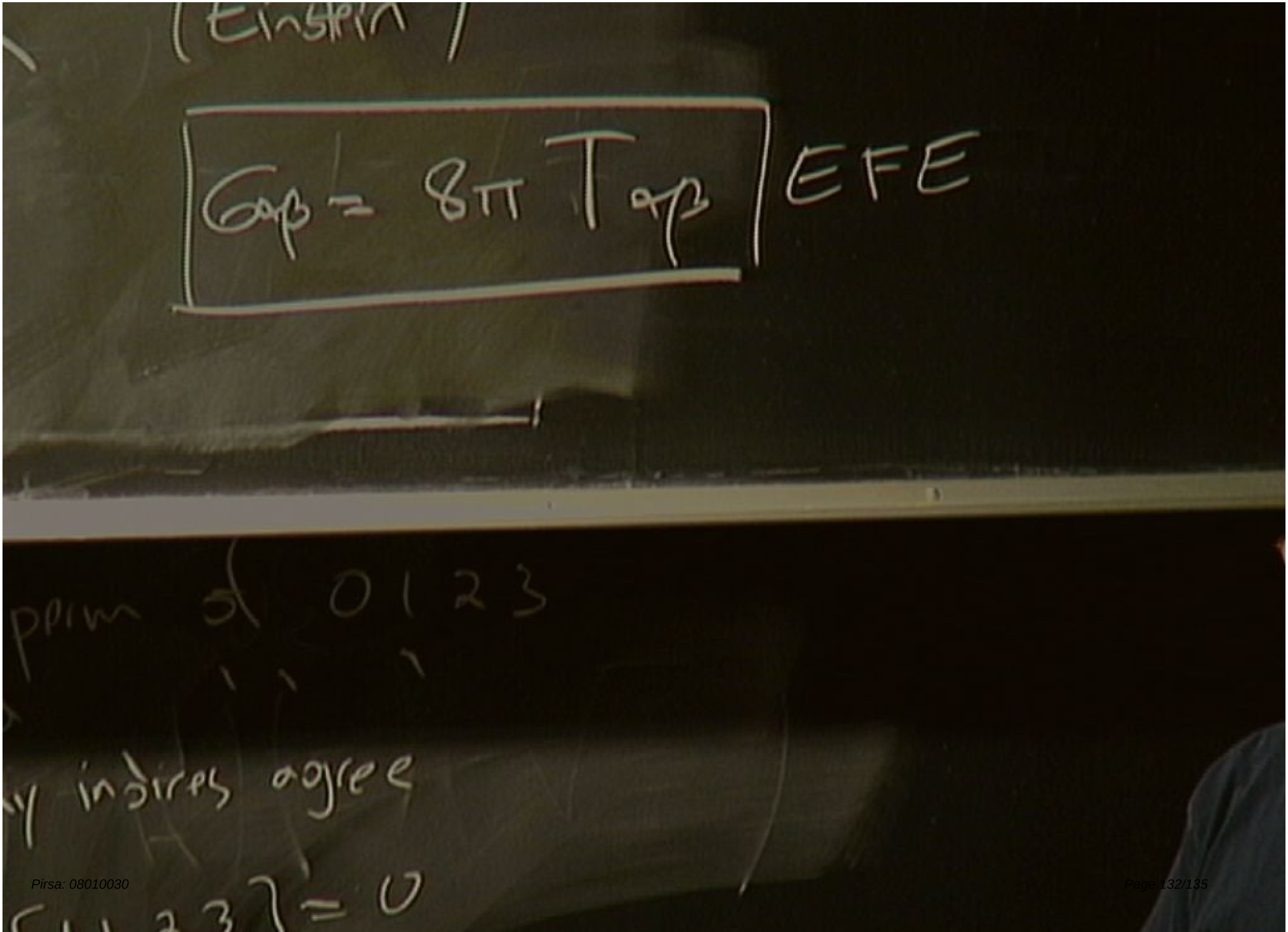
$$G_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R \quad \text{(Einstein)}$$

$$G_{\beta\alpha} = G_{\alpha\beta} \qquad G_{\alpha\beta} = 8\pi T_{\alpha\beta}$$

$$= \begin{cases} +1 & \text{even perm of } 0123 \\ -1 & \text{odd " "} \\ 0 & \text{if any indices agree} \end{cases}$$

$[0123] = 1 \qquad [1123] = 0$


(Einstein)
$G_{\alpha\beta} = 8\pi T_{\alpha\beta}$
even perm
odd
if any indices agree


(Einstein)
Gαβ = 8π Tαβ
EFE
0 1 2 3
indices agree
= 0

$= G_{\alpha\beta}$

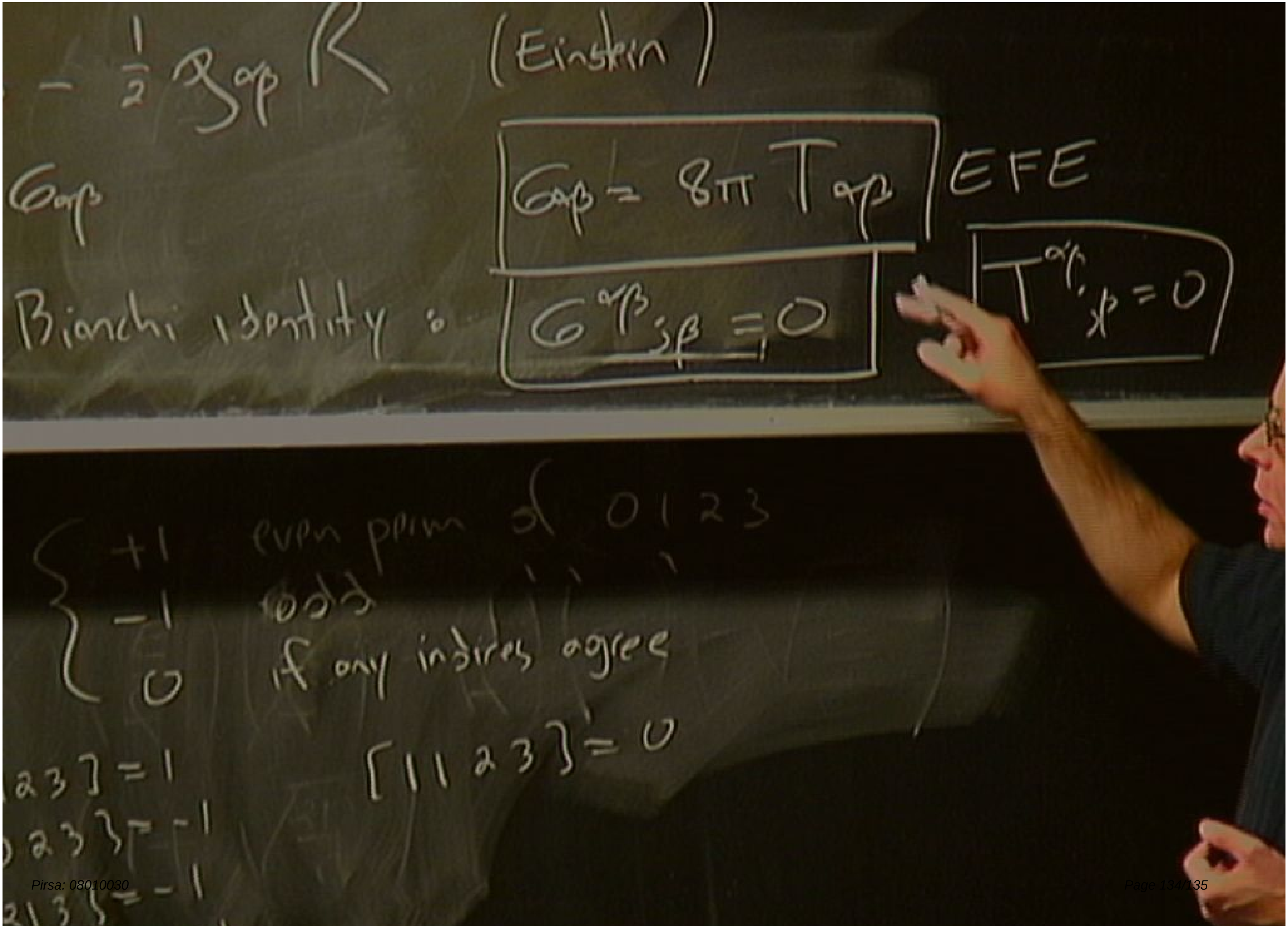
$$\boxed{G_{\alpha\beta} = 8\pi T_{\alpha\beta}}$$

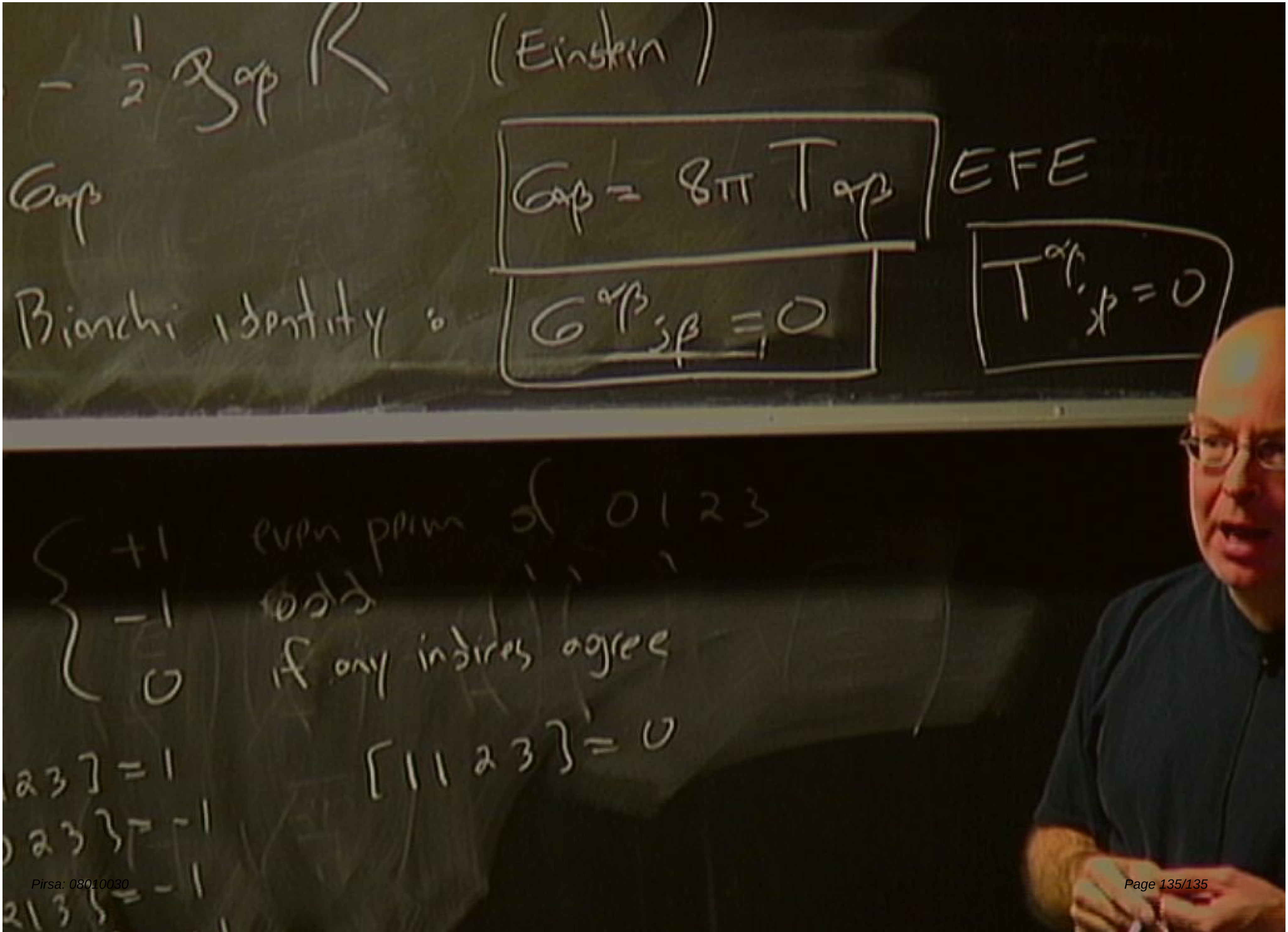
Bianchi identity : $\boxed{G^{\alpha\beta}{}_{;\beta} = 0}$

$$= \begin{cases} +1 & \text{even perm of } 0123 \\ -1 & \text{odd} \\ 0 & \text{if any indices agree} \end{cases}$$

$[0123] = 1$ $\quad [1123] = 0$

$[1023] = -1$


(Einstein)
EFE
Bianchi identity :
even perm of 0123
odd
if any indices agree


- 1/2 g_αβ R (Einstein)
G_αβ
G_αβ = 8π T_αβ EFE
Bianchi identity : G^αβ_;β = 0
T^αβ_;β = 0
+1 even perm of 0123
-1 odd
0 if any indices agree
[1123] = 0
123] = 1
023] = -1
213] = -1