Title: Advanced General Relativity - Lecture 3A

Date: Jan 23, 2008 10:30 AM

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Abstract: Advanced General Relativity

Local flatness

$gap(p)$

Local flatness

$$g_{ab}(p) \overset{*}{=} \eta_{ab}$$

Local flatness

$$g_{\alpha\beta}(p) \overset{*}{=} \eta_{\alpha\beta}$$

$$\partial_\gamma g_{\alpha\beta}(p) \overset{*}{=} 0$$

Local flatness

$$g_{\alpha\beta}(p) \stackrel{*}{=} \eta_{\alpha\beta}$$

$$\partial_\gamma g_{\alpha\beta}(p) \stackrel{*}{=} 0 \longrightarrow \Gamma^\mu_{\alpha\beta}(p$$

Local flatness

$$g_{\alpha\beta}(p) \overset{*}{=} \eta_{\alpha\beta}$$

$$\partial_\gamma g_{\alpha\beta}(p) \overset{*}{=} 0 \longrightarrow \Gamma^\mu_{\alpha\beta}(p) \overset{*}{=} 0$$

Local flatness

$$g_{\alpha\beta}(p) \overset{*}{=} \eta_{\alpha\beta}$$

$$\partial_\gamma g_{\alpha\beta}(p) \overset{*}{=} 0 \longrightarrow \Gamma^{\mu}_{\alpha\beta}(p) \overset{*}{=} 0$$

Riemann normal coordinates

Local flatness

$$g_{\alpha\beta}(P) \overset{*}{=} \eta_{\alpha\beta}$$

$$\partial_\gamma g_{\alpha\beta}(p) \overset{*}{=} 0 \longrightarrow \Gamma^{\mu}{}_{\alpha\beta}(P) \overset{*}{=} 0$$

Riemann normal coordinates

In RNC,

$$g_{\alpha\beta} = \eta_{\alpha\beta} - \frac{1}{3} R_{\alpha\mu\beta\nu} x^\mu x^\nu$$

$$g_{\alpha\beta}(P) \overset{*}{=} \eta_{\alpha\beta}$$

$$\partial_\gamma g_{\alpha\beta}(P) \overset{*}{=} 0 \longrightarrow \Gamma^{\mu}{}_{\alpha\beta}(P) \overset{*}{=} 0$$

Riemann normal coordinates

In RNC,

$$g_{\alpha\beta} = \eta_{\alpha\beta} - \frac{1}{3} R_{\alpha\mu\beta\nu} x^\mu x^\nu$$

$$+ O(\nabla R x^3, \nabla\nabla R x^4,$$

Local flatness

$$g_{\alpha\beta}(P) \stackrel{*}{=} \eta_{\alpha\beta}$$

$$\partial_\gamma g_{\alpha\beta}(P) \stackrel{*}{=} 0 \longrightarrow \Gamma^\mu_{\alpha\beta}(P) \stackrel{*}{=} 0$$

Riemann normal coordinates

In RNC,

$$g_{\alpha\beta} = \eta_{\alpha\beta} - \frac{1}{3} R_{\alpha\mu\beta\nu} x^\mu x^\nu$$

$$+ O(\nabla R x^3, \nabla\nabla R x^4, R^2 x^4) + \cdots$$

$R_{\alpha\mu\beta\nu}$ = const tensor

Local flatness

$$g_{\alpha\beta}(P) \stackrel{*}{=} \eta_{\alpha\beta}$$

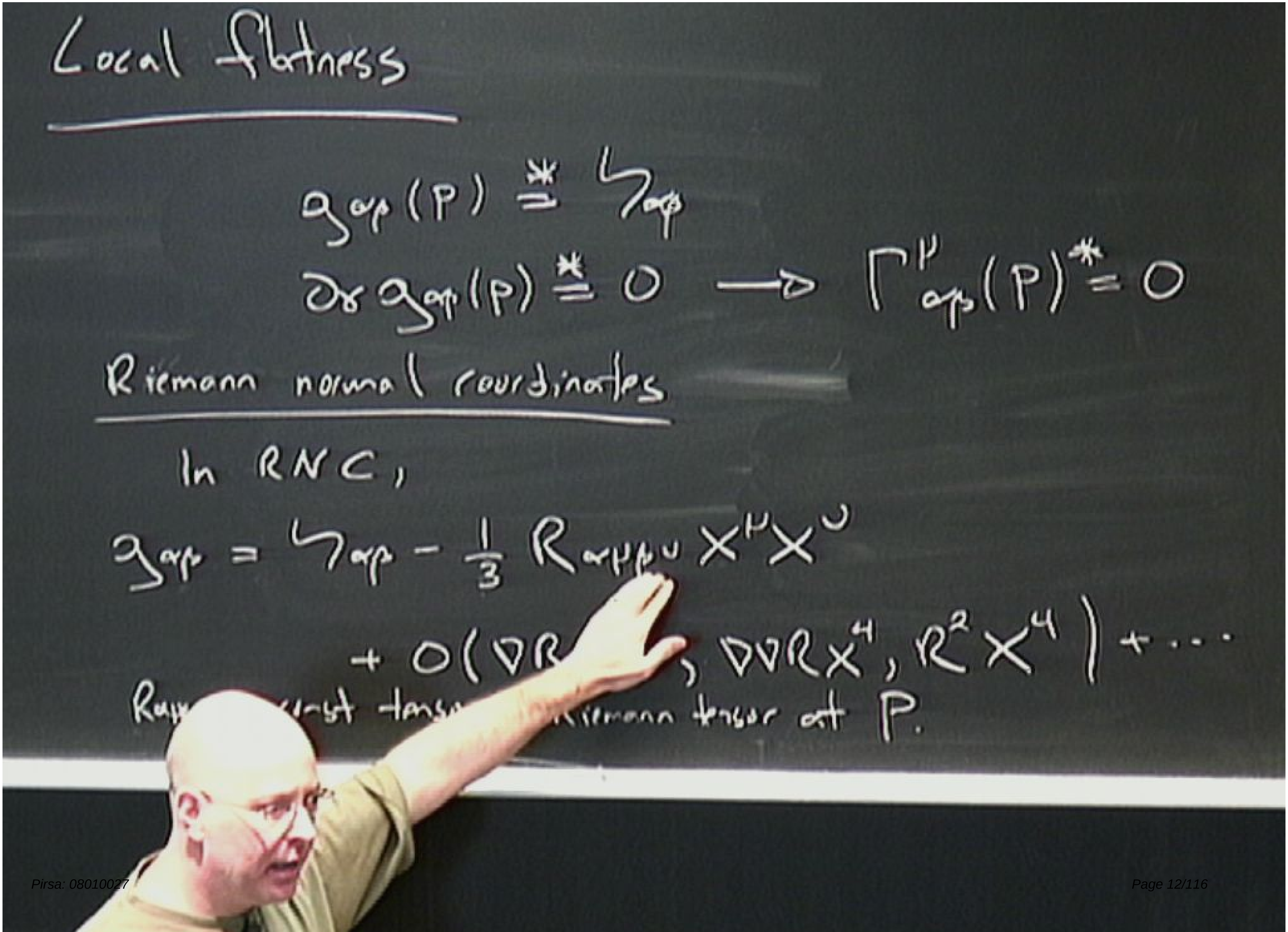
$$\partial_\gamma g_{\alpha\beta}(P) \stackrel{*}{=} 0 \longrightarrow \Gamma^\mu_{\alpha\beta}(P) \stackrel{*}{=} 0$$

Riemann normal coordinates

In RNC,

$$g_{\alpha\beta} = \eta_{\alpha\beta} - \frac{1}{3} R_{\alpha\mu\beta\nu} x^\mu x^\nu + O(\nabla R x^3, \nabla\nabla R x^4, R^2 x^4) + \cdots$$

R[illegible] const tensor = Riemann tensor at P.


Local flatness
$g_{\alpha\beta}(P) \overset{*}{=} \eta_{\alpha\beta}$
$\partial_\gamma g_{\alpha\beta}(P) \overset{*}{=} 0 \longrightarrow \Gamma^\mu_{\alpha\beta}(P) \overset{*}{=} 0$
Riemann normal coordinates
In RNC,
$g_{\alpha\beta} = \eta_{\alpha\beta} - \frac{1}{3} R_{\alpha\mu\beta\nu} x^\mu x^\nu$
$+ O(\nabla R \quad , \nabla\nabla R x^4, R^2 x^4) + \cdots$
Riemann tensor at P.

Local flatness

$$g_{\alpha\beta}(P) \overset{*}{=} \eta_{\alpha\beta}$$

$$\partial_\gamma g_{\alpha\beta}(P) \overset{*}{=} 0 \longrightarrow \Gamma^{\mu}_{\alpha\beta}(P) \overset{*}{=} 0$$

Riemann normal coordinates

In RNC,

$$g_{\alpha\beta} = \eta_{\alpha\beta} - \frac{1}{3} R_{\alpha\mu\beta\nu} x^{\mu} x^{\nu}$$

$$+ O(\nabla R x^3, \nabla\nabla R x^4, R^2 x^4) + \cdots$$

$R_{\alpha\mu\beta\nu}$ = const tensor = Riemann tensor at P.

Local flatness

$$g_{\alpha\beta}(P) \overset{*}{=} \eta_{\alpha\beta}$$

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Riemann normal coordinates

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Local flatness

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Riemann normal coordinates

In RNC, $P: x^\mu = 0$

$$g_{\alpha\beta} = \eta_{\alpha\beta} - \frac{1}{3} R_{\alpha\mu\beta\nu} x^\mu x^\nu + O\left(\nabla R x^3, \nabla\nabla R x^4, R^2 x^4\right) + \cdots$$

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Local flatness

$$g_{\alpha\beta}(P) \overset{*}{=} \eta_{\alpha\beta}$$

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Riemann normal coordinates

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$$g_{\alpha\beta} = \eta_{\alpha\beta} - \frac{1}{3} R_{\alpha\mu\beta\nu} x^\mu x^\nu$$

$$+ O(\nabla R x^3, \nabla\nabla R x^4, R^2 x^4) + \cdots$$

$R_{\alpha\mu\beta\nu}$ = const tensor = Riemann tensor at P.

$$\Gamma^{\alpha}_{\beta\gamma} = \frac{1}{2} g^{\alpha\mu}\left(g_{\mu\beta,\gamma} + g_{\mu\gamma,\beta} - g_{\beta\gamma,\mu} \right)$$

$$\Gamma^{\alpha}_{\beta\gamma} = \frac{1}{2} g^{\alpha\mu} \left(g_{\mu\beta,\gamma} + g_{\mu\gamma,\beta} - g_{\beta\gamma,\mu} \right)$$

$g_{\mu\beta,\gamma}$

$$\Gamma^{\alpha}_{\beta\gamma} = \frac{1}{2} g^{\alpha\mu} \left(g_{\mu\beta,\gamma} + g_{\mu\gamma,\beta} - g_{\beta\gamma,\mu} \right)$$

$$g_{\mu\beta,\gamma} = D_{\gamma} \left(-\frac{1}{3} R_{\mu\lambda\beta\rho} x^{\lambda} x^{\rho} \right)$$

$$\Gamma^{\alpha}_{\beta\gamma} = \frac{1}{2} g^{\alpha\mu} \left(g_{\mu\beta,\gamma} + g_{\mu\gamma,\beta} - g_{\beta\gamma,\mu} \right)$$

$$g_{\mu\beta,\gamma} = \partial_{\gamma} \left(-\frac{1}{3} R_{\mu\lambda\beta\rho} x^{\lambda} x^{\rho} \right)$$

$$= -\frac{1}{3} R_{\mu\lambda\beta\rho} \left(\delta^{\lambda}_{\gamma} x^{\rho} + x^{\lambda} \delta^{\rho}_{\gamma} \right)$$

$$=$$

$$\Gamma^{\alpha}_{\beta\gamma} = \frac{1}{2} g^{\alpha\mu} \left(g_{\mu\beta,\gamma} + g_{\mu\gamma,\beta} - g_{\beta\gamma,\mu} \right)$$

$$g_{\mu\beta,\gamma} = \partial_\gamma \left(-\frac{1}{3} R_{\mu\lambda\beta\rho} x^\lambda x^\rho \right)$$

$$= -\frac{1}{3} R_{\mu\lambda\beta\rho} \left(\delta^\lambda_\gamma x^\rho + x^\lambda \delta^\rho_\gamma \right)$$

$$= -\frac{1}{3} R_{\mu\gamma\beta\rho} x^\rho - \frac{1}{3} R_{\mu\lambda\beta\gamma} x^\lambda$$

$$\xi_{\mu\rho;\gamma} = \partial_\gamma \left(-\tfrac{1}{3} R_{\mu\lambda\rho\rho}\, x^\lambda x^\rho \right)$$

$$= -\tfrac{1}{3} R_{\mu\lambda\rho\rho} \left(\delta^\lambda_\gamma x^\rho + x^\lambda \delta^\rho_\gamma \right)$$

$$= -\tfrac{1}{3} R_{\mu\gamma\rho\rho} x^\rho - \tfrac{1}{3} R_{\mu\lambda\rho\gamma} x^\lambda$$

$$= -\tfrac{1}{3} \left(R_{\mu\gamma\rho\lambda} + R_{\mu\lambda\rho\gamma} \right) x^\lambda$$

$$\xi_{\mu\beta,\gamma} = \partial_\gamma \left(-\frac{1}{3} R_{\mu\lambda\beta\rho} x^\lambda x^\rho \right)$$

$$= -\frac{1}{3} R_{\mu\lambda\beta\rho} \left(\delta^\lambda{}_\gamma x^\rho + x^\lambda \delta^\rho{}_\gamma \right)$$

$$= -\frac{1}{3} R_{\mu\gamma\beta\rho} x^\rho - \frac{1}{3} R_{\mu\lambda\beta\gamma} x^\lambda$$

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$$\frac{1}{2}\left(-\frac{1}{3} \right)$$

$$\Gamma^{\alpha}_{\beta\gamma} = \frac{1}{2} g^{\alpha\rho} \left(g_{\rho\beta,\gamma} + g_{\rho\gamma,\beta} - g_{\beta\gamma,\rho} \right)$$

$$g_{\mu\beta,\gamma} = \partial_\gamma \left(-\frac{1}{3} R_{\mu\lambda\beta\rho} x^\lambda x^\rho \right)$$

$$= -\frac{1}{3} R_{\mu\lambda\beta\rho} \left(\delta^\lambda_\gamma x^\rho + x^\lambda \delta^\rho_\gamma \right)$$

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$$\Gamma^{\alpha}_{\beta\gamma} = \frac{1}{2}\left(-\frac{1}{3}\right) g^{\alpha\rho} \Big($$

$$\Gamma^{\alpha}_{\mu\gamma} = \frac{1}{2} g^{\alpha\beta} \left(g_{\mu\beta,\gamma} + g_{\beta\gamma,\mu} - g_{\mu\gamma,\beta} \right)$$

$$g_{\mu\beta,\gamma} = \partial_\gamma \left(-\frac{1}{3} R_{\mu\lambda\beta\rho} x^\lambda x^\rho \right)$$

$$= -\frac{1}{3} R_{\mu\lambda\beta\rho} \left(\delta^\lambda_\gamma x^\rho + x^\lambda \delta^\rho_\gamma \right)$$

$$= -\frac{1}{3} R_{\mu\gamma\beta\rho} x^\rho - \frac{1}{3} R_{\mu\lambda\beta\gamma} x^\lambda$$

$$= -\frac{1}{3} \left(R_{\mu\gamma\beta\lambda} + R_{\mu\lambda\beta\gamma} \right) x^\lambda$$

$$\Gamma^{\alpha}_{\mu\gamma} = \frac{1}{2}\left(-\frac{1}{3}\right) g^{\alpha\beta} \left(R_{\mu\gamma\beta\lambda} + R_{\mu\lambda\beta\gamma} \right.$$

$$\Gamma^{\alpha}_{\mu\gamma} = \frac{1}{2} g^{\alpha\beta}\left(g_{\mu\beta,\gamma} + g_{\beta\gamma,\mu} - g_{\mu\gamma,\beta} \right)$$

$$g_{\mu\beta,\gamma} = \partial_\gamma \left(-\frac{1}{3} R_{\mu\lambda\beta\rho} x^\lambda x^\rho \right)$$

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$$\Gamma^{\alpha}_{\mu\gamma} = \frac{1}{2}\left(-\frac{1}{3}\right) g^{\alpha\beta} \left(R_{\mu\gamma\beta\lambda} + R_{\mu\lambda\beta\gamma} \right.$$

$$\Gamma^{\alpha}_{\beta\gamma} = \frac{1}{2} g^{\alpha\rho} \left(g_{\rho\beta,\gamma} + g_{\rho\gamma,\beta} - g_{\beta\gamma,\rho} \right)$$

$$
\begin{aligned}
g_{\mu\beta,\gamma} &= \partial_\gamma \left(-\frac{1}{3} R_{\mu\lambda\beta\rho} x^\lambda x^\rho \right) \\
&= -\frac{1}{3} R_{\mu\lambda\beta\rho} \left(\delta^\lambda_\gamma x^\rho + x^\lambda \delta^\rho_\gamma \right) \\
&= -\frac{1}{3} R_{\mu\gamma\beta\rho} x^\rho - \frac{1}{3} R_{\mu\lambda\beta\gamma} x^\lambda \\
&= -\frac{1}{3} \left(R_{\mu\gamma\beta\lambda} + R_{\mu\lambda\beta\gamma} \right) x^\lambda
\end{aligned}
$$

$$\Gamma^{\alpha}_{\beta\gamma} = \frac{1}{2}\left(-\frac{1}{3}\right) g^{\alpha\mu} \left(R_{\mu\gamma\beta\lambda} + R_{\mu\lambda\beta\gamma} + R_{\mu\beta\gamma\lambda} + R_{\mu\lambda\gamma\beta} \right.$$

$$\Gamma_{\beta\gamma} = \frac{1}{2} g^{\cdot}\left(g_{\mu\beta,\gamma} + g_{\mu\gamma,\beta} - g_{\beta\gamma,\mu} \right)$$

$$g_{\mu\beta,\gamma} = \partial_\gamma \left(-\frac{1}{3} R_{\mu\lambda\beta\rho} x^\lambda x^\rho \right)$$

$$= -\frac{1}{3} R_{\mu\lambda\beta\rho} \left(\delta^\lambda_\gamma x^\rho + x^\lambda \delta^\rho_\gamma \right)$$

$$= -\frac{1}{3} R_{\mu\gamma\beta\rho} x^\rho - \frac{1}{3} R_{\mu\lambda\beta\gamma} x^\lambda$$

$$= -\frac{1}{3} \left(R_{\mu\gamma\beta\lambda} + R_{\mu\lambda\beta\gamma} \right) x^\lambda$$

$$\Gamma^\alpha_{\beta\gamma} = \frac{1}{2}\left(-\frac{1}{3}\right) g^{\alpha\mu} \left(R_{\mu\gamma\beta\lambda} + R_{\mu\lambda\beta\gamma} + R_{\mu\beta\gamma\lambda} + R_{\mu\lambda}\ldots \right.$$

$$\Gamma^{\alpha}{}_{\beta\gamma} = \frac{1}{2} g^{\alpha\rho} (g_{\rho\beta,\gamma} + g_{\rho\gamma,\beta} - g_{\beta\gamma,\rho})$$

$$g_{\mu\beta,\gamma} = \partial_\gamma \left(-\frac{1}{3} R_{\mu\lambda\beta\rho} x^\lambda x^\rho \right)$$

$$= -\frac{1}{3} R_{\mu\lambda\beta\rho} \left(\delta^\lambda_\gamma x^\rho + x^\lambda \delta^\rho_\gamma \right)$$

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$$\Gamma^{\alpha}{}_{\beta\gamma} = \frac{1}{2}\left(-\frac{1}{3}\right) g^{\alpha\mu} \left(R_{\mu\gamma\beta\lambda} + R_{\mu\lambda\beta\gamma} + R_{\mu\beta\gamma\lambda} + R_{\mu\lambda\gamma\beta} - R_{\beta\mu\gamma\lambda} \right.$$

$R_{\alpha\beta\mu\nu}$ = const tensor = Riemann tensor at P.

$$\Gamma^{\alpha}_{\beta\gamma} = \frac{1}{2} g^{\alpha\mu} \left(g_{\mu\beta,\gamma} + g_{\mu\gamma,\beta} - g_{\beta\gamma,\mu} \right)$$

$$g_{\mu\beta,\gamma} = \partial_\gamma \left(-\frac{1}{3} R_{\mu\lambda\beta\rho} x^\lambda x^\rho \right)$$

$$= -\frac{1}{3} R_{\mu\lambda\beta\rho} \left(\delta^\lambda_\gamma x^\rho + x^\lambda \delta^\rho_\gamma \right)$$

$$= -\frac{1}{3} R_{\mu\gamma\beta\rho} x^\rho - \frac{1}{3} R_{\mu\lambda\beta\gamma} x^\lambda$$

$$= -\frac{1}{3} \left(R_{\mu\gamma\beta\lambda} + R_{\mu\lambda\beta\gamma} \right) x^\lambda$$

$$\Gamma^{\alpha}_{\beta\gamma} = \frac{1}{2}\left(-\frac{1}{3}\right) g^{\alpha\mu} \left(R_{\mu\gamma\beta\lambda} + R_{\mu\lambda\beta\gamma} + R_{\mu\beta\gamma\lambda} + R_{\mu\lambda\gamma\beta} - R_{\beta\mu\gamma\lambda} - R_{\beta\lambda\gamma\mu} \right) x^\lambda$$

$R_{\alpha\mu\beta\nu}$ = curv. tensor = Riemann tensor at P.

$$= -\frac{1}{3} R_{\mu\lambda\rho\rho} \left(\delta^\lambda_\gamma x^\rho + x^\lambda \delta^\rho_\gamma \right)$$

$$= -\frac{1}{3} R_{\mu\gamma\rho\rho} x^\rho - \frac{1}{3} R_{\mu\lambda\rho\gamma} x^\lambda$$

$$= -\frac{1}{3} \left(R_{\mu\gamma\rho\lambda} + R_{\mu\lambda\rho\gamma} \right) x^\lambda$$

$$\Gamma^\alpha_{\mu\gamma} = \frac{1}{2}\left(-\frac{1}{3}\right) g^{\alpha\rho} \Big(R_{\mu\gamma\rho\lambda} + \cancel{R_{\mu\lambda\rho\gamma}} + R_{\mu\rho\gamma\lambda} + \cancel{R_{\mu\lambda\gamma\rho}} - R_{\rho\mu\gamma\lambda} - R_{\rho\lambda\gamma\mu} \Big) x^\lambda$$

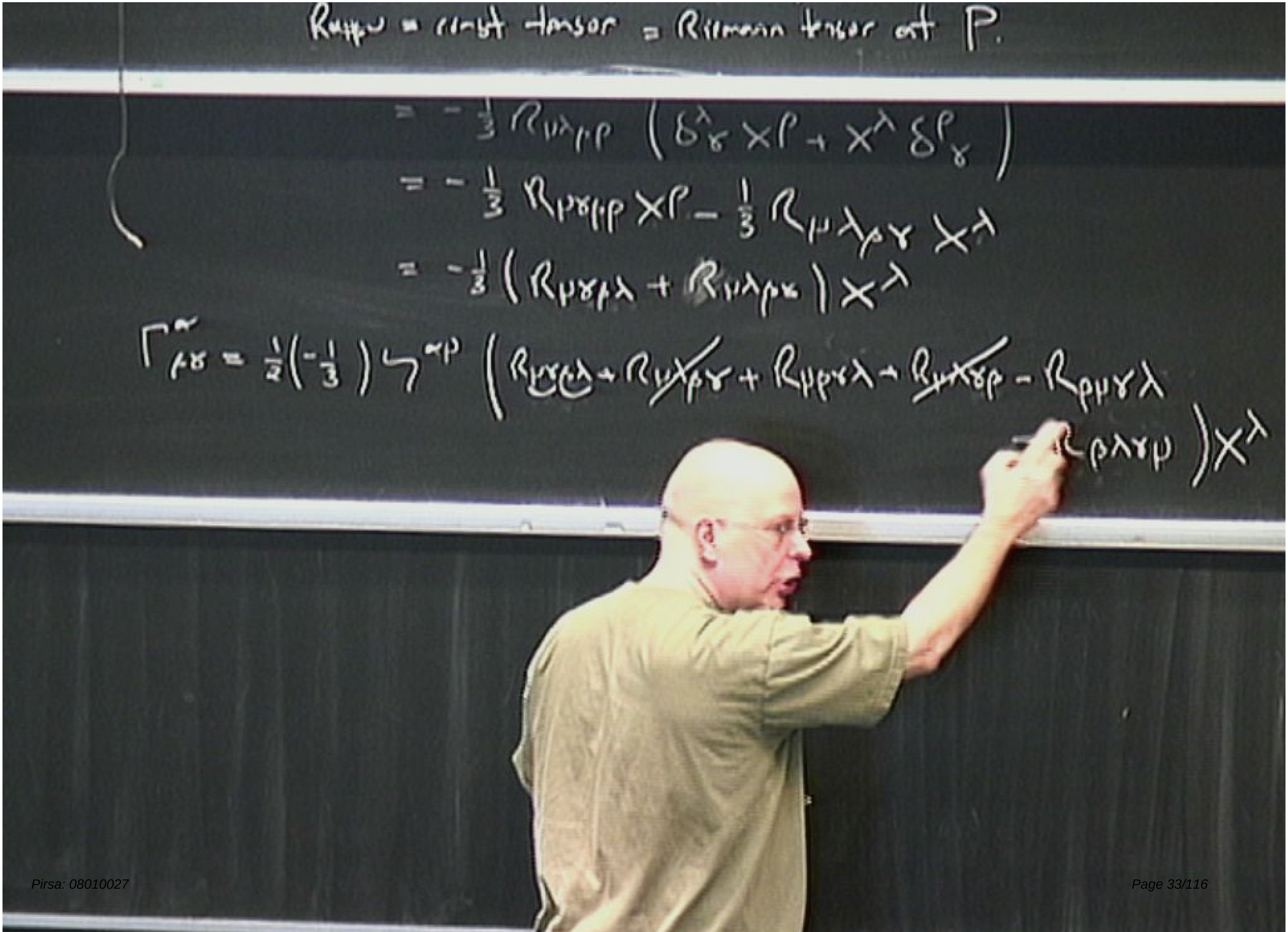
$R_{\alpha\beta\mu\nu}$ = curvt tensor = Riemann tensor at P.

$$= -\frac{1}{3} R_{\nu\lambda\mu\rho} \left(\delta^\lambda_\gamma x^\rho + x^\lambda \delta^\rho_\gamma \right)$$

$$= -\frac{1}{3} R_{\nu\gamma\mu\rho} x^\rho - \frac{1}{3} R_{\mu\lambda\rho\gamma} x^\lambda$$

$$= -\frac{1}{3} \left(R_{\mu\gamma\rho\lambda} + R_{\mu\lambda\rho\gamma} \right) x^\lambda$$

$$\Gamma^\alpha_{\mu\gamma} = \frac{1}{2}\left(-\frac{1}{3}\right) g^{\alpha\rho} \left(R_{\mu\gamma\rho\lambda} + \cancel{R_{\mu\lambda\rho\gamma}} + R_{\mu\rho\gamma\lambda} + \cancel{R_{\mu\lambda\gamma\rho}} - R_{\rho\mu\gamma\lambda} - R_{\rho\lambda\gamma\mu} \right) x^\lambda$$


$R_{\alpha\beta\mu\nu}$ = curv. tensor = Riemann tensor at P.
$= -\frac{1}{3} R_{\mu\lambda\beta\rho} \left(\delta^\lambda_\gamma x^\rho + x^\lambda \delta^\rho_\gamma \right)$
$= -\frac{1}{3} R_{\mu\gamma\beta\rho} x^\rho - \frac{1}{3} R_{\mu\lambda\beta\gamma} x^\lambda$
$= -\frac{1}{3} \left(R_{\mu\gamma\beta\lambda} + R_{\mu\lambda\beta\gamma} \right) x^\lambda$
$\Gamma^\alpha_{\beta\gamma} = \frac{1}{2}\left(-\frac{1}{3}\right) g^{\alpha\mu} \left(R_{\mu\gamma\beta\lambda} + \cancel{R_{\mu\lambda\beta\gamma}} + R_{\mu\beta\gamma\lambda} + \cancel{R_{\mu\lambda\gamma\beta}} - R_{\beta\mu\gamma\lambda} \right.$
$\left. - R_{\beta\lambda\gamma\mu} \right) x^\lambda$

$R_{\alpha\beta\gamma\delta}$ = const tensor = Riemann tensor at P.

$$= -\frac{1}{3} R_{\mu\lambda\rho\beta} \left(\delta^\lambda_\gamma X^\rho + X^\lambda \delta^\rho_\gamma \right)$$

$$= -\frac{1}{3} R_{\mu\gamma\rho\beta} X^\rho - \frac{1}{3} R_{\mu\lambda\gamma\beta} X^\lambda$$

$$= -\frac{1}{3} \left(R_{\mu\gamma\beta\lambda} + R_{\mu\lambda\beta\gamma} \right) X^\lambda$$

$$\Gamma^\alpha_{\beta\gamma} = \frac{1}{2}\left(-\frac{1}{3}\right) g^{\alpha\mu} \left(R_{\mu\gamma\beta\lambda} + \cancel{R_{\mu\lambda\beta\gamma}} + R_{\mu\beta\gamma\lambda} + \cancel{R_{\mu\lambda\gamma\beta}} - R_{\beta\mu\gamma\lambda} - R_{\beta\lambda\gamma\mu} \right) X^\lambda$$

$R_{\alpha\beta\mu\nu}$ = const tensor = Riemann tensor at P.

$$\frac{1}{3} R_{\mu\gamma\beta\rho} x^\rho - \frac{1}{3} R_{\mu\lambda\beta\gamma} x^\lambda$$

$$= -\frac{1}{3}\left(R_{\mu\gamma\beta\lambda} + R_{\mu\lambda\beta\gamma} \right) x^\lambda$$

$$\Gamma^\alpha_{\beta\gamma} = \frac{1}{2}\left(-\frac{1}{3}\right) g^{\alpha\mu} \Big(R_{\mu\gamma\beta\lambda} + \cancel{R_{\mu\lambda\beta\gamma}} + R_{\mu\beta\gamma\lambda} + \cancel{R_{\mu\lambda\gamma\beta}} - R_{\beta\mu\gamma\lambda} - R_{\beta\lambda\gamma\mu} \Big) x^\lambda$$

$$\Gamma^\alpha_{\beta\gamma} = -\frac{1}{3}$$

$R_{\alpha\beta\mu\nu}$ = const tensor = Riemann tensor at P.

$$\tfrac{1}{3} R_{\mu\gamma\lambda\rho} X^\lambda - \tfrac{1}{3} R_{\mu\lambda\rho\gamma} X^\lambda$$

$$= -\tfrac{1}{3}\left(R_{\mu\gamma\rho\lambda} + R_{\mu\lambda\rho\gamma}\right) X^\lambda$$

$$\Gamma^\alpha_{\rho\gamma} = \tfrac{1}{2}\left(-\tfrac{1}{3}\right) g^{\alpha\mu}\left(R_{\mu\gamma\rho\lambda} + \cancel{R_{\mu\lambda\rho\gamma}} + R_{\mu\rho\gamma\lambda} + \cancel{R_{\mu\lambda\gamma\rho}} - R_{\rho\mu\gamma\lambda} - R_{\rho\lambda\gamma\mu}\right) X^\lambda$$

$$\boxed{\Gamma^\alpha_{\rho\gamma} = -\tfrac{1}{3}\left(R^\alpha{}_{\gamma\rho\lambda} + R^\alpha{}_{\rho\gamma\lambda}\right) X^\lambda}$$

$R_{\alpha\beta\mu\nu}$ = const tensor = Riemann tensor at P.

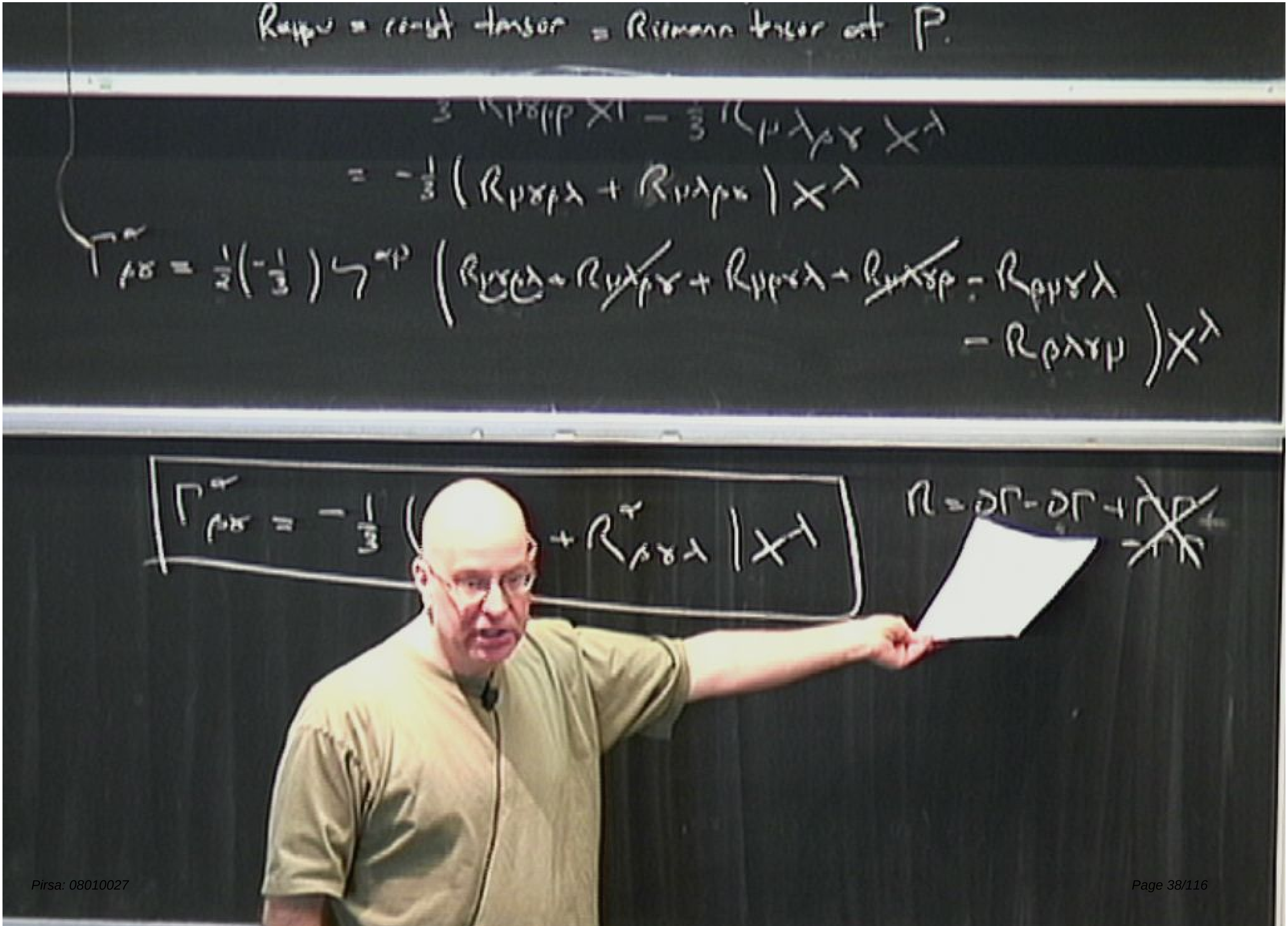
$$\frac{}{3} R_{\mu\gamma\lambda\rho} x^\lambda - \frac{1}{3} R_{\mu\lambda\rho\gamma} x^\lambda$$

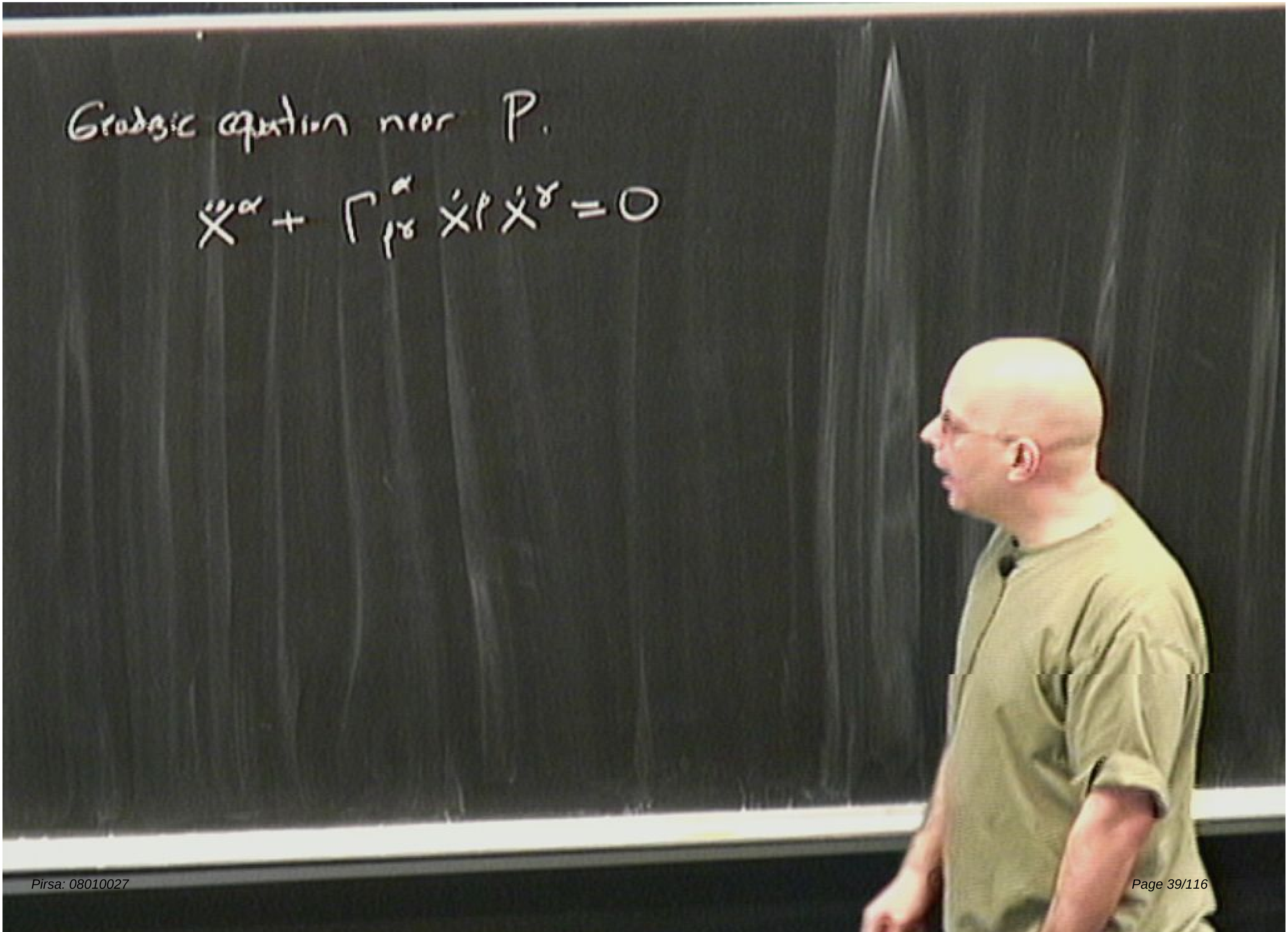
$$= -\frac{1}{3}\left(R_{\mu\gamma\rho\lambda} + R_{\mu\lambda\rho\gamma}\right) x^\lambda$$

$$\Gamma^\alpha_{\beta\gamma} = \frac{1}{2}\left(-\frac{1}{3}\right) g^{\alpha\mu}\Big(R_{\mu\gamma\beta\lambda} + \cancel{R_{\mu\lambda\beta\gamma}} + R_{\mu\beta\gamma\lambda} + \cancel{R_{\mu\lambda\gamma\beta}} - R_{\beta\mu\gamma\lambda} - R_{\beta\lambda\gamma\mu}\Big) x^\lambda$$

$$\boxed{\Gamma^\alpha_{\beta\gamma} = -\frac{1}{3}\left(R^\alpha{}_{\gamma\beta\lambda} + R^\alpha{}_{\beta\gamma\lambda}\right) x^\lambda}$$

$$R = \partial\Gamma - \partial\Gamma + \cancel{\Gamma\Gamma} - \cancel{\Gamma\Gamma}$$

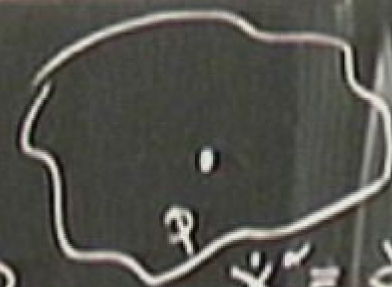

$R_{\alpha\beta\mu\nu}$ = const tensor = Riemann tensor at P.
$- \frac{1}{3} R_{\mu\lambda\rho\gamma} x^\lambda$
$= -\frac{1}{3}\left(R_{\mu\gamma\rho\lambda} + R_{\mu\lambda\rho\gamma}\right) x^\lambda$
$\Gamma^\alpha_{\rho\gamma} = \frac{1}{2}\left(-\frac{1}{3}\right) g^{\alpha\mu} \left(R_{\mu\gamma\rho\lambda} + R_{\mu\lambda\rho\gamma} + R_{\mu\rho\gamma\lambda} + R_{\mu\lambda\gamma\rho} - R_{\rho\mu\gamma\lambda} - R_{\rho\lambda\gamma\mu}\right) x^\lambda$
$\Gamma^\alpha_{\rho\gamma} = -\frac{1}{3}\left(\quad + R^\alpha{}_{\rho\gamma\lambda}\right) x^\lambda$
$R = \partial\Gamma - \partial\Gamma + \Gamma\Gamma - \Gamma\Gamma$


Geodesic equation near P.
$\ddot{x}^{\alpha} + \Gamma^{\alpha}_{\beta\gamma}\dot{x}^{\beta}\dot{x}^{\gamma} = 0$

Geodesic equation near P.

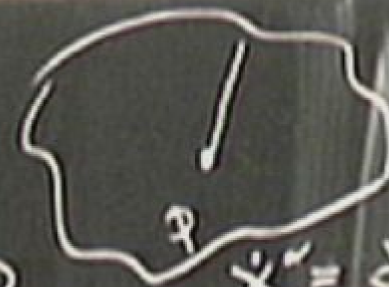
$$\ddot{x}^{\alpha} + \Gamma^{\alpha}_{\beta\gamma} \dot{x}^{\beta} \dot{x}^{\gamma} = 0 \qquad \dot{x}^{\alpha} = \frac{dx^{\alpha}}{ds}$$

Geodesic equation near P.



$$\ddot{x}^\alpha + \Gamma^\alpha_{\beta\gamma}\,\dot{x}^\beta \dot{x}^\gamma = 0 \qquad \dot{x}^\alpha = \frac{dx^\alpha}{ds} \qquad s \equiv \text{proper distance.}$$

Geodesic equation near P.

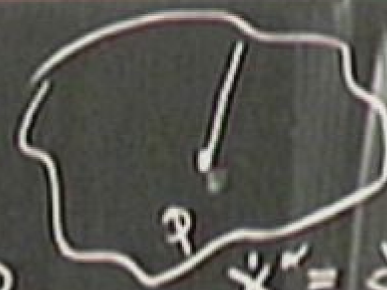


$$\ddot{x}^\alpha + \Gamma^\alpha_{\beta\gamma}\,\dot{x}^\beta \dot{x}^\gamma = 0$$

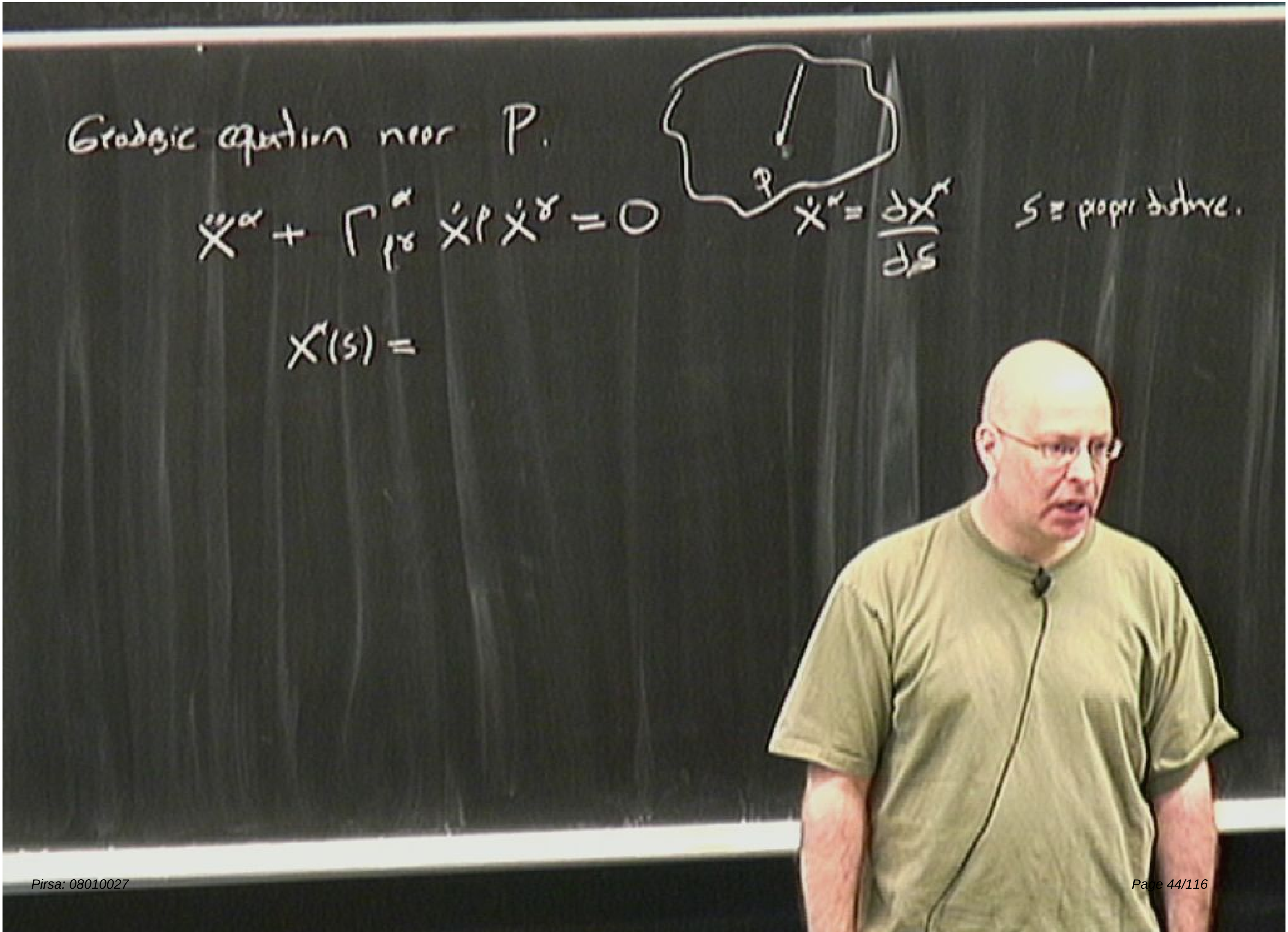
$$\dot{x}^\alpha = \frac{dx^\alpha}{ds}$$ $s \equiv$ proper distance.

Geodesic equation near P.

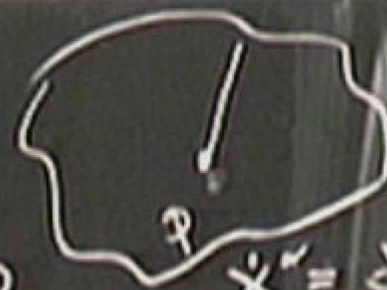
$$\ddot{x}^\alpha + \Gamma^\alpha_{\beta\gamma}\,\dot{x}^\beta\,\dot{x}^\gamma = 0$$



$\dot{x}^\alpha = \dfrac{dx^\alpha}{ds}$ $\quad s$ = proper distance.


Geodesic equation near P.
P
$\ddot{x}^\alpha + \Gamma^\alpha_{\beta\gamma}\dot{x}^\beta\dot{x}^\gamma = 0$
$\dot{x}^\alpha = \frac{dx^\alpha}{ds}$
s = proper distance.
$x^\alpha(s) =$

Geodesic equation near P.

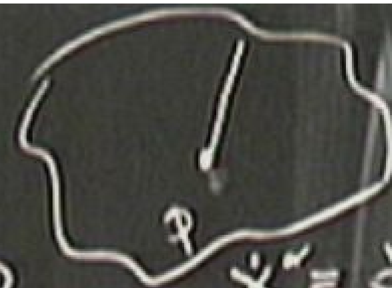


$$\ddot{x}^\alpha + \Gamma^\alpha_{\beta\gamma}\,\dot{x}^\beta \dot{x}^\gamma = 0 \qquad \dot{x}^\alpha = \frac{dx^\alpha}{ds}$$

s = proper distance.

$$x^\alpha(s) = s\,n^\alpha + s^2 A^\alpha + s^3 B^\alpha + \cdots$$

Geodesic equation near P.



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$n^\alpha, A^\alpha, B^\alpha, \dots \Rightarrow$ constant vectors (evaluated at P)

Geodesic equation near P.

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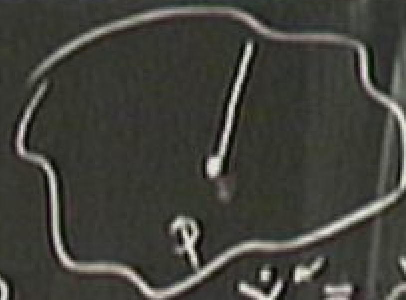
Initial condition $x^\alpha(P) = 0$, $\dot{x}^\alpha(P) = n^\alpha$

Geodesic equation near P.



$$\ddot{x}^\alpha + \Gamma^\alpha_{\beta\gamma}\,\dot{x}^\beta\,\dot{x}^\gamma = 0 \qquad \dot{x}^\alpha = \frac{dx^\alpha}{ds}$$

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$$\dot{x}^\alpha(s) = n^\alpha$$

Geodesic equation near P.

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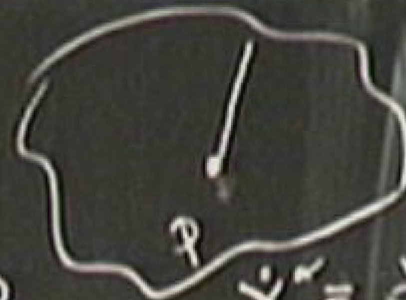
Initial condition $x^\alpha(P) = 0$, $\dot{x}^\alpha(P) = n^\alpha$

$$\dot{x}^\alpha(s) = n^\alpha + 2s\,A^\alpha + 3s^2 B^\alpha + \cdots$$

Geodesic equation near P.



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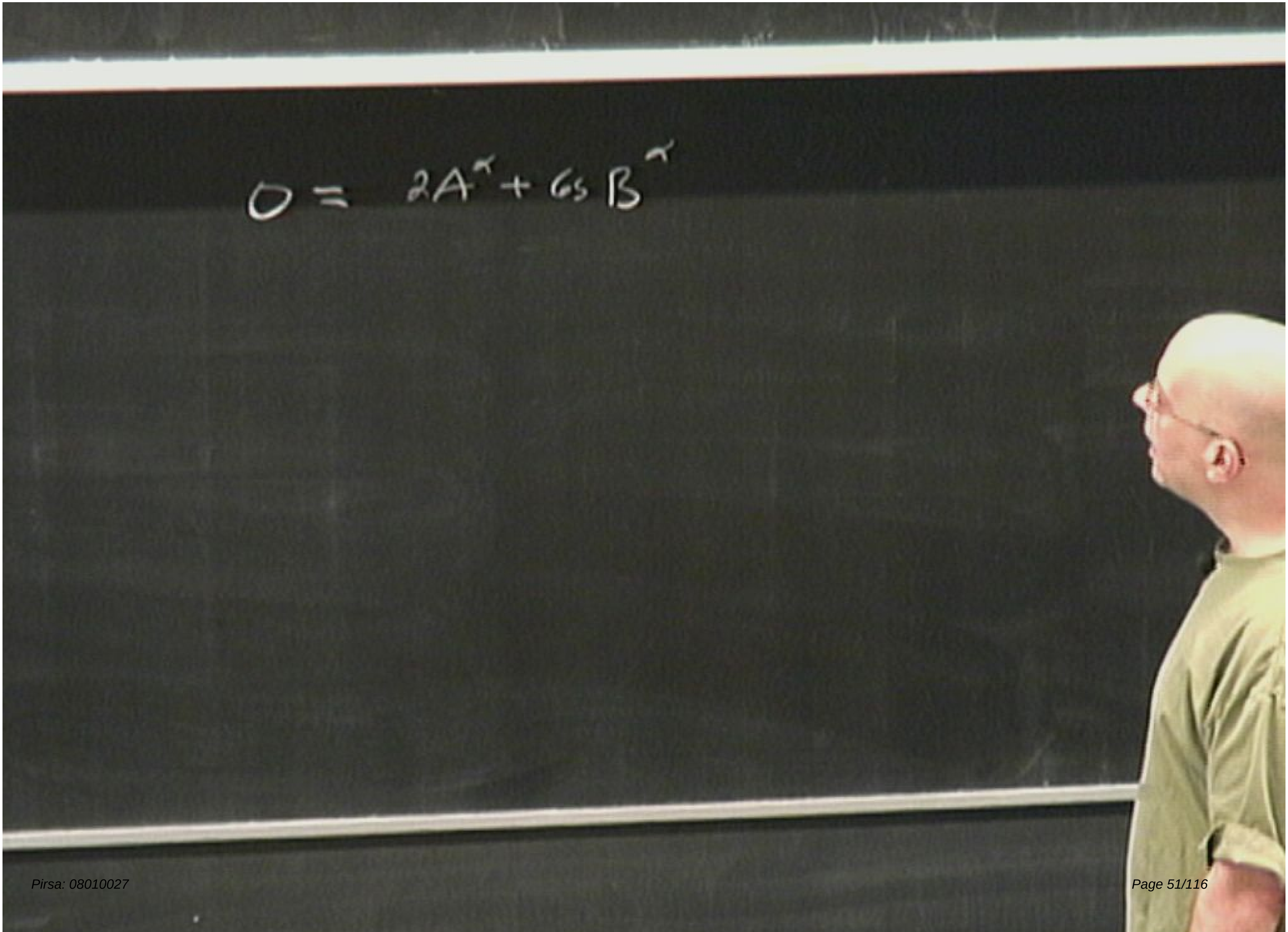
$n^\alpha, A^\alpha, B^\alpha, \ldots \Rightarrow$ constant vectors (evaluated at P)

Initial condition $x^\alpha(P) = 0$, $\dot{x}^\alpha(P) = n^\alpha$

$$\dot{x}^\alpha(s) = n^\alpha + 2s A^\alpha + 3s^2 B^\alpha + \cdots$$

$$\ddot{x}^\alpha(s) = 2A^\alpha + 6s B^\alpha + \cdots$$


$0 = 2A^{\alpha} + 6s\,B^{\alpha}$

$$0 = 2A^{\alpha} + 6s\,B^{\alpha} + \left[-\frac{1}{3}\left(R^{\alpha}{}_{\sigma\rho\lambda} + R^{\alpha}{}_{\rho\sigma\lambda} \right) s n^{\lambda} \right]$$

$$0 = 2A^{\alpha} + 6s\,B^{\alpha} + \left[-\frac{1}{3}\left(R^{\alpha}{}_{\gamma\rho\lambda} + R^{\alpha}{}_{\rho\gamma\lambda}\right) s n^{\lambda}\right]$$

$$\left[n^{\rho} + \cdots\right]\left[n^{\gamma} + \cdots\right]$$

$$0 = 2A^\alpha + Gs B^\alpha + \left[-\frac{1}{3} \left(R^\alpha{}_{\gamma\beta\lambda} + R^\alpha{}_{\beta\gamma\lambda} \right) s n^\lambda \right]$$

$$\left[n^\beta + \dots \right] \left[n^\gamma + \dots \right]$$

$$= 2A^\alpha + s \left[G B^\alpha - \frac{1}{3} \left(R^\alpha{}_{\gamma\beta\lambda} + R^\alpha{}_{\beta\gamma\lambda} \right) n^\lambda n^\beta n^\gamma + \dots \right.$$

$$0 = 2A^\alpha + 6s\,B^\alpha + \left[-\tfrac{1}{3}\left(R^\alpha{}_{\gamma\beta\lambda} + R^\alpha{}_{\beta\gamma\lambda}\right) s n^\lambda\right]$$

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$$\left[n^\beta + \cdots\right]\left\{n^\gamma + \cdots\right\}$$

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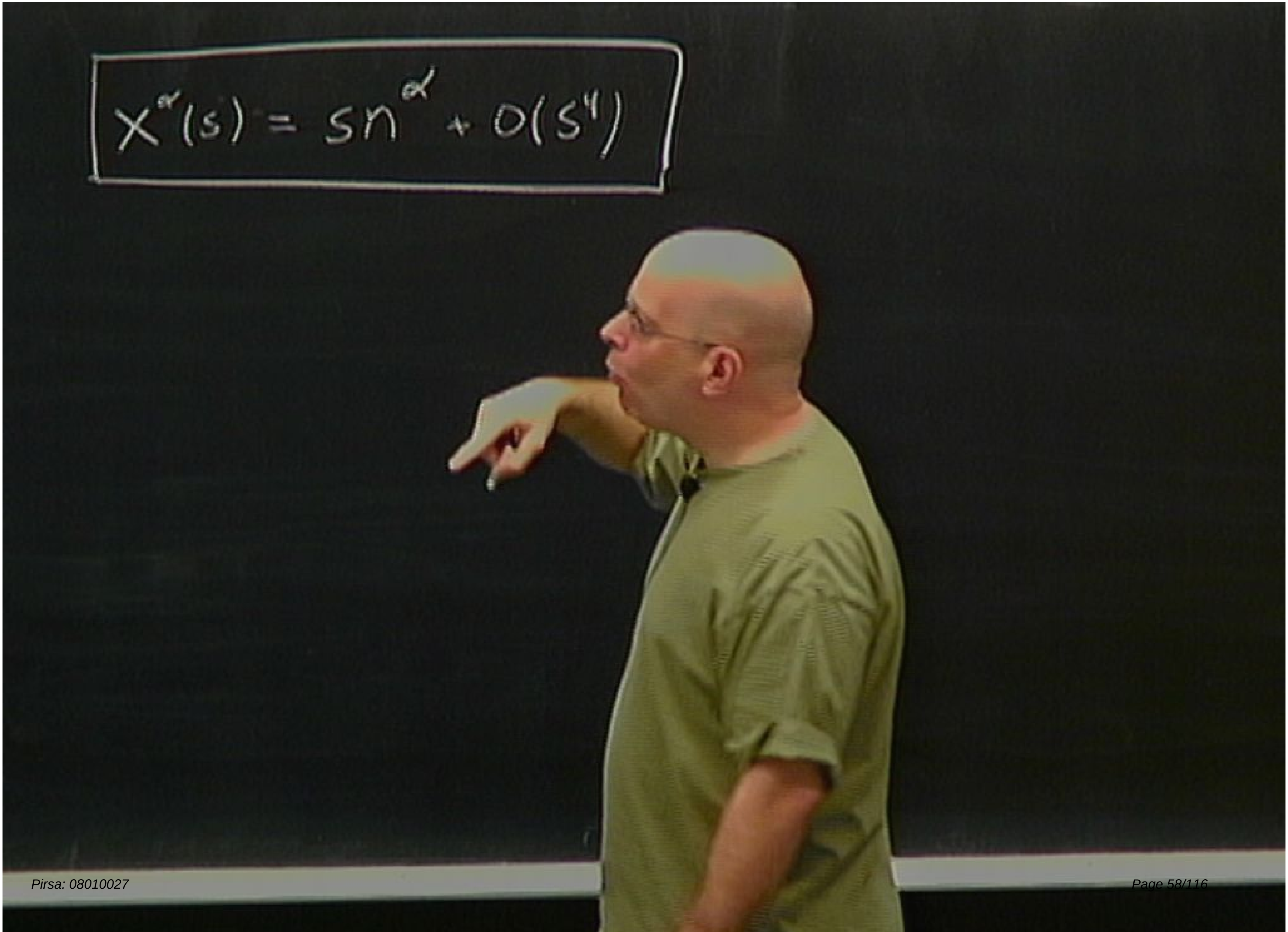
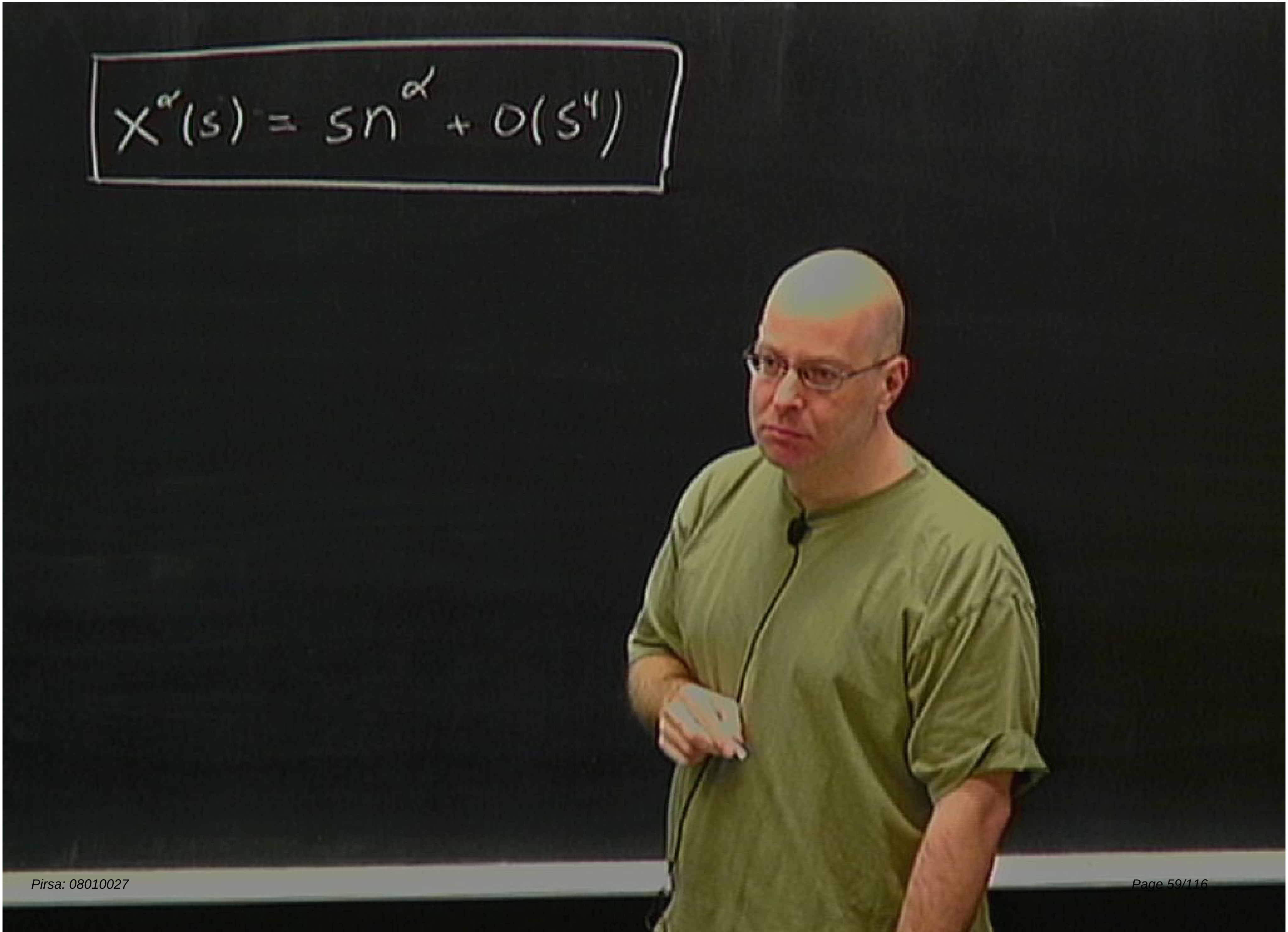
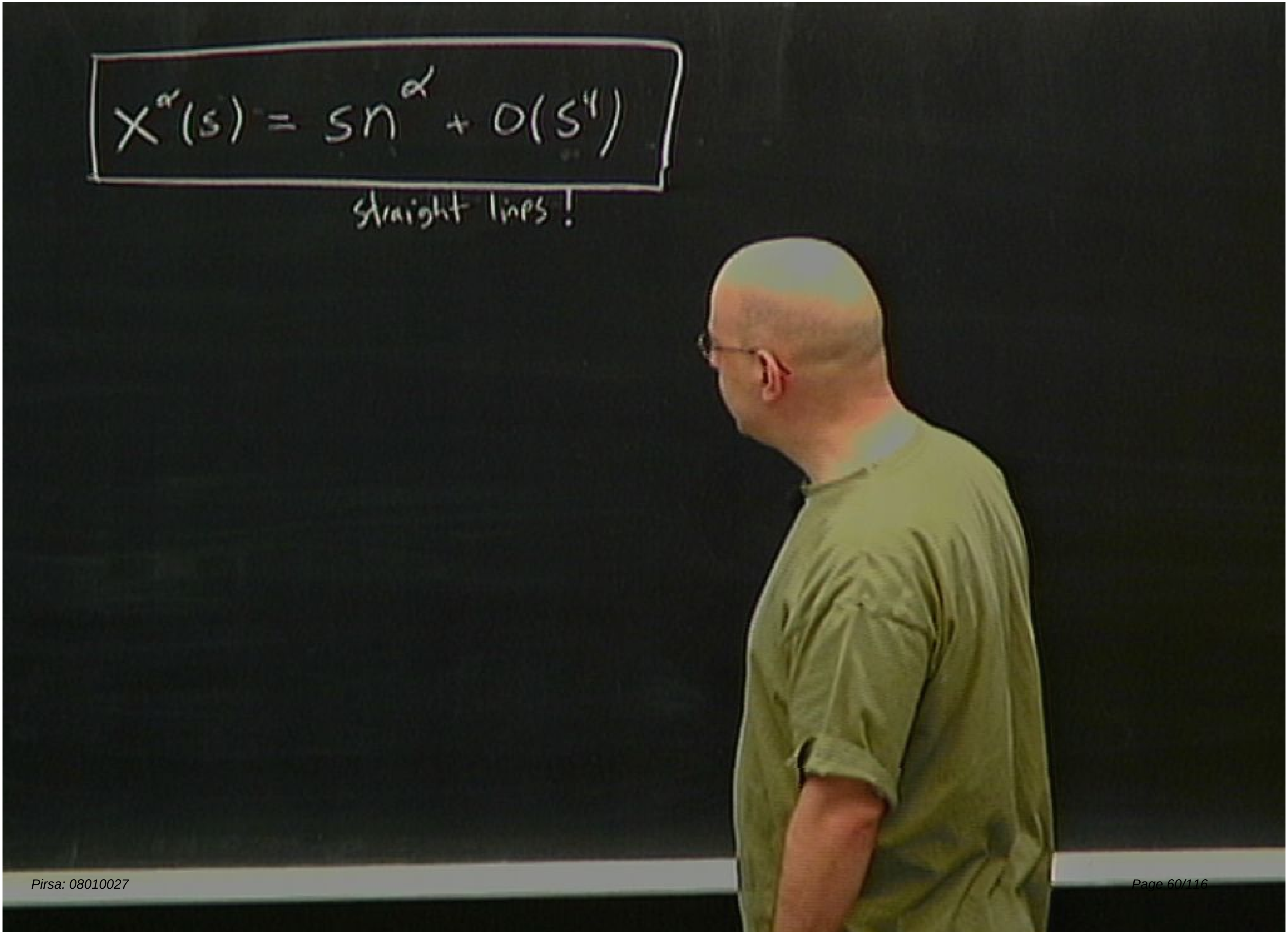
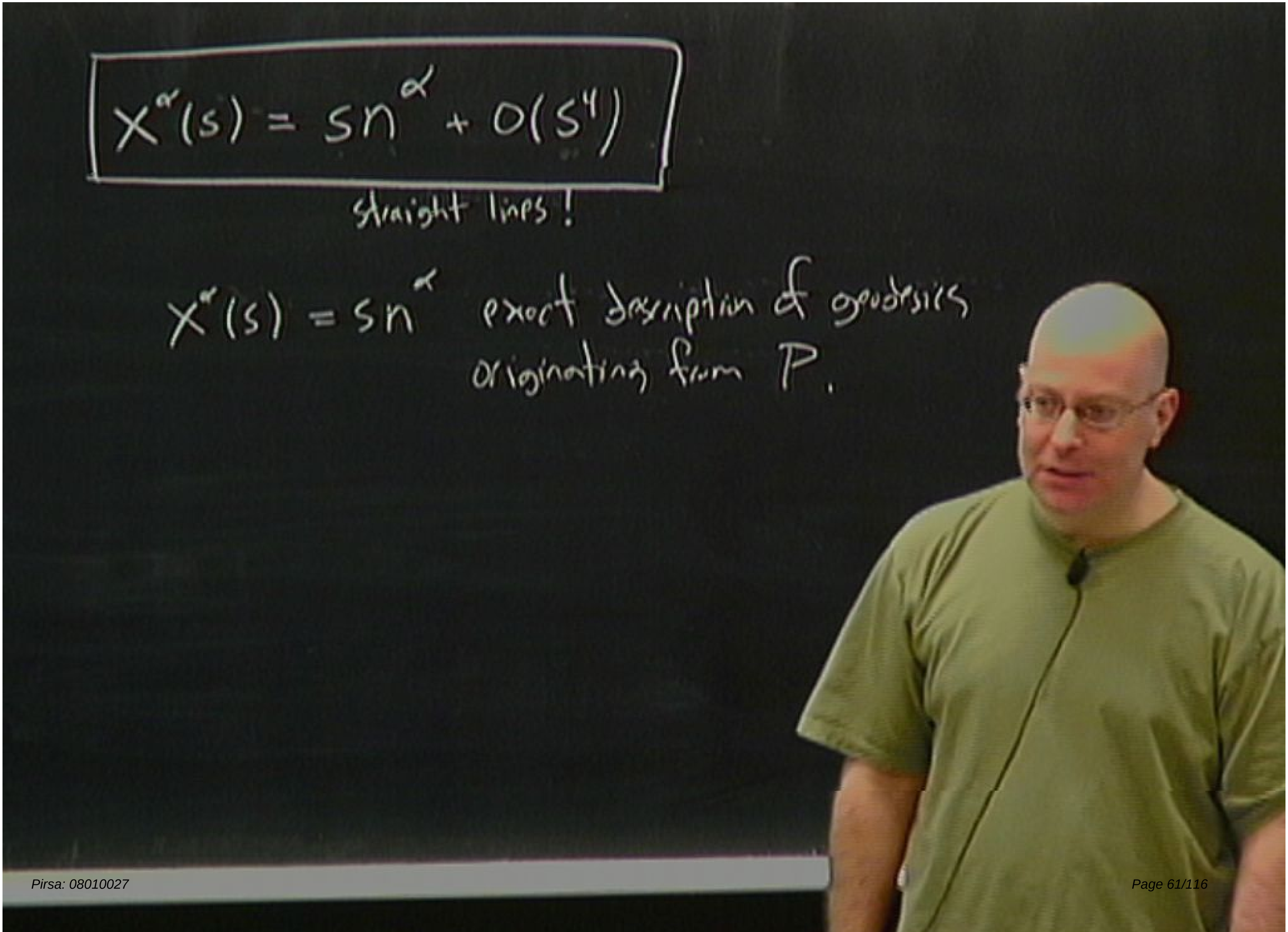
$$A^\alpha = B^\alpha = 0$$

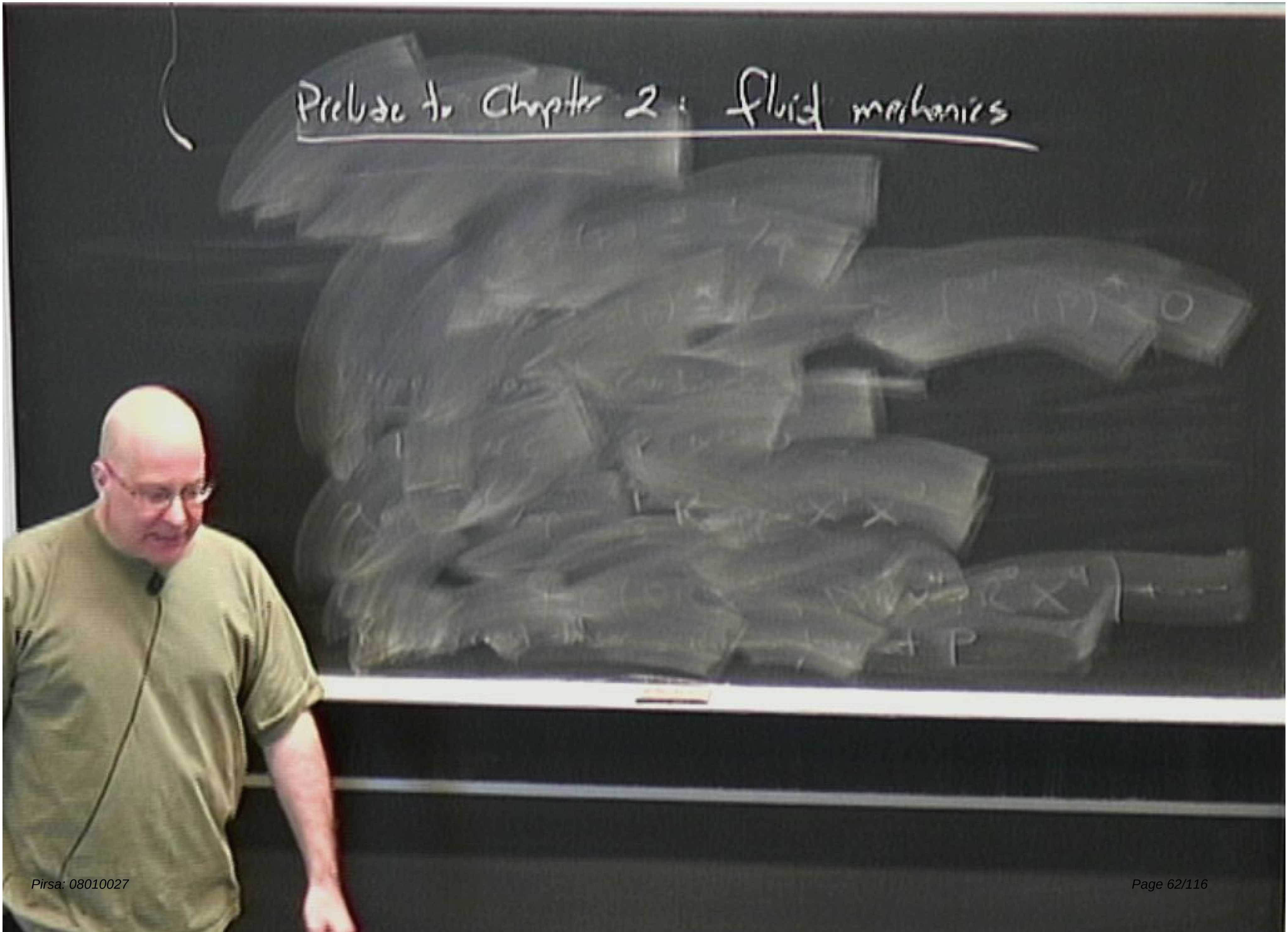

$\boxed{X^\alpha(s) = s n^\alpha + O(s^4)}$


$X^\alpha(s) = s n^\alpha + O(s^4)$



$X^\alpha(s) = s n^\alpha + O(s^4)$
straight lines!


$\boxed{x^\alpha(s) = s n^\alpha + O(s^4)}$
straight lines!
$x^\alpha(s) = s n^\alpha$ exact description of geodesics
originating from P.


Prelude to Chapter 2: fluid mechanics

Prelude to Chapter 2: fluid mechanics

Prelude to Chapter 2: fluid mechanics

fluid of density $\rho(t, \vec{x})$

and velocity $\vec{v}(t, \vec{x})$

Prelude to Chapter 2: fluid mechanics

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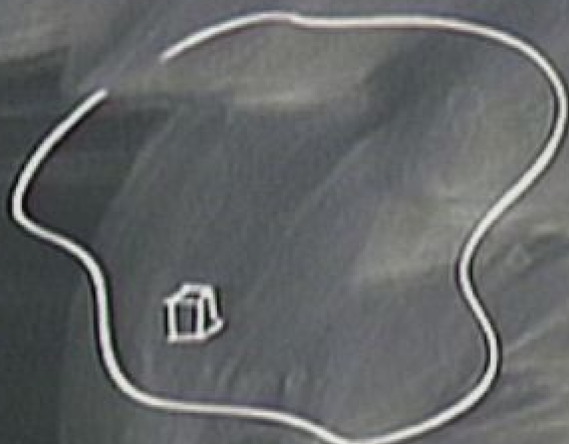
Prelude to Chapter 2: fluid mechanics



fluid of density $\rho(t,\vec{x})$
and velocity $\vec{v}(t,\vec{x})$

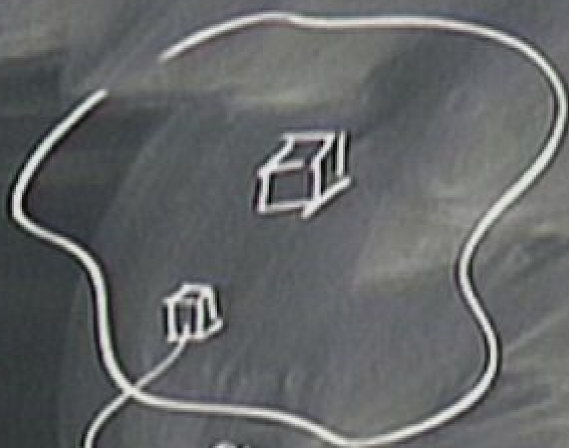
Prelude to Chapter 2: fluid mechanics

reference fluid element at $\vec{x}_0$

fluid of density $\rho(t,\vec{x})$

and velocity $\vec{v}(t,\vec{x})$

Prelude to Chapter 2: fluid mechanics



reference flow element at $\vec{x}_0$

fluid of density $\rho(t,\vec{x})$
and velocity $\vec{v}(t,\vec{x})$

Prelude to Chapter 2: fluid mechanics

reference flow element at $\vec{x}_0$

fluid of density $\rho(t, \vec{x})$

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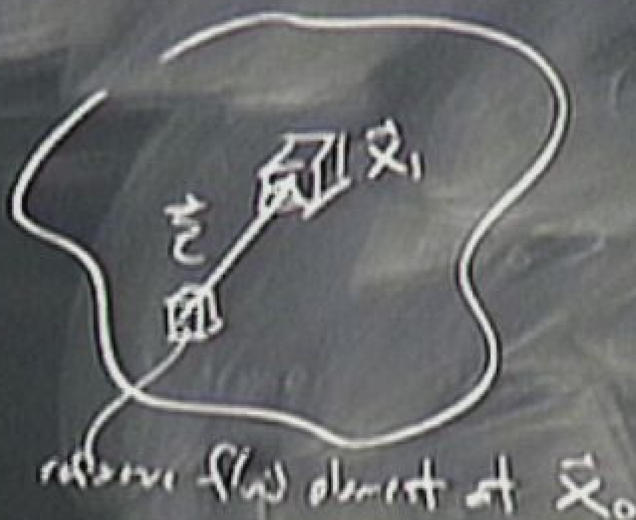
Prelude to Chapter 2: fluid mechanics



fluid of density $\rho(t, \vec{x})$
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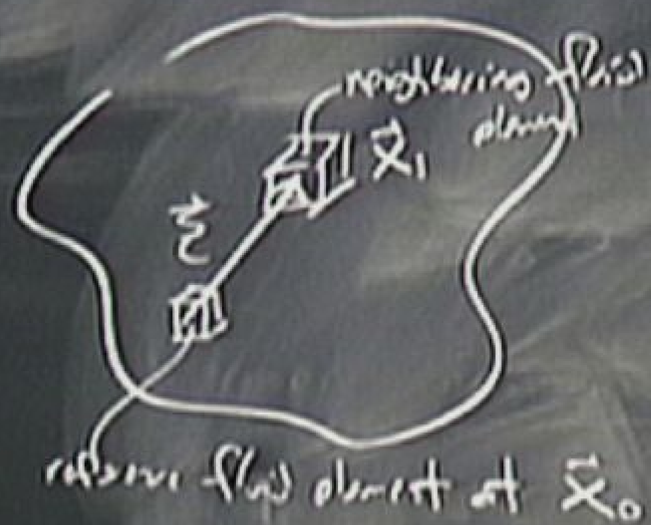
Prelude to Chapter 2: fluid mechanics

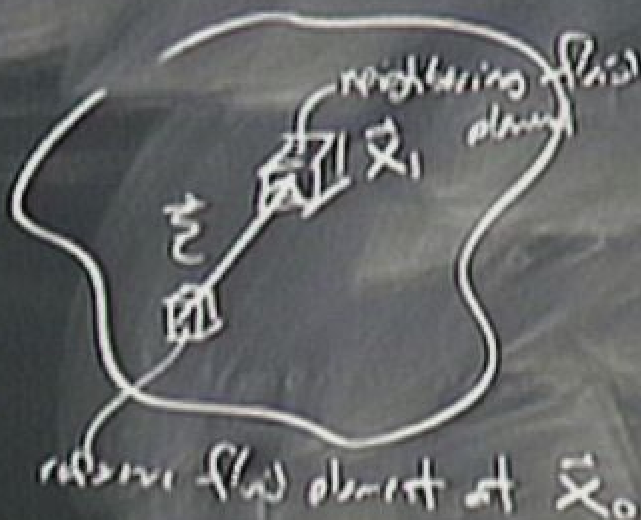


fluid of density $\rho(t,\vec{x})$

and velocity $\vec{v}(t,\vec{x})$

$$\vec{\xi}(t) = \vec{x}_1 - \vec{x}_0$$

Prelude to Chapter 2: fluid mechanics

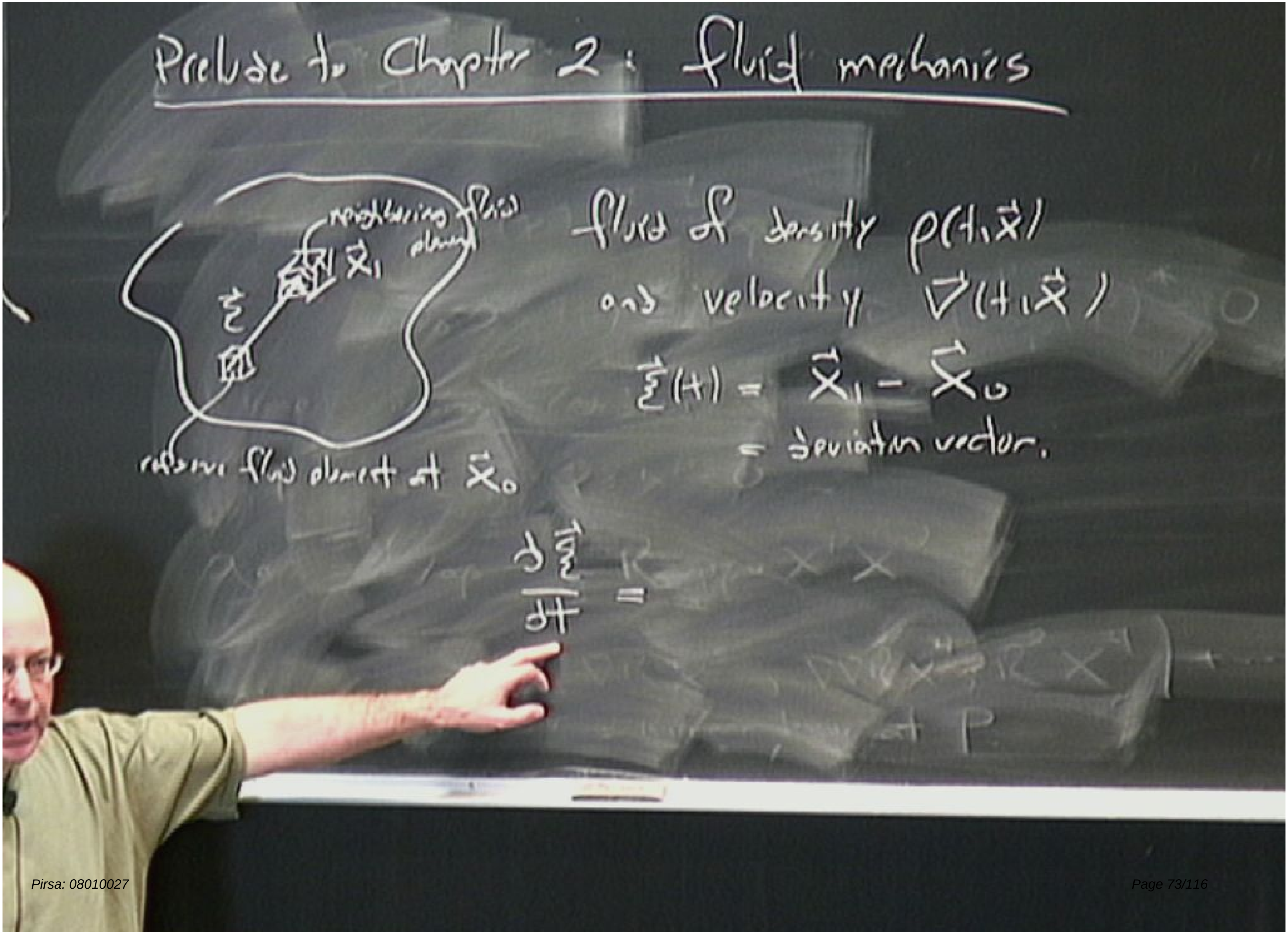


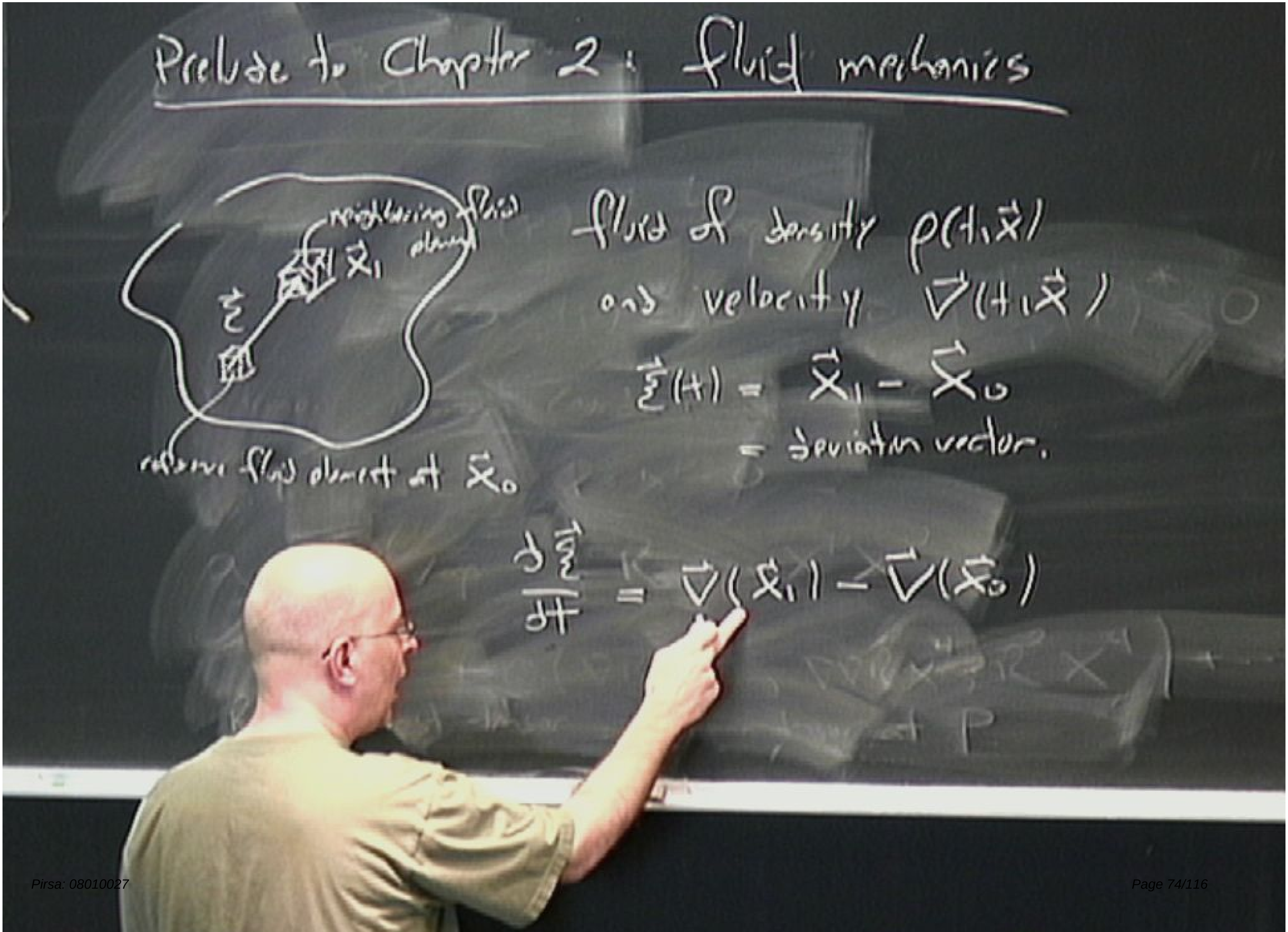
fluid of density $\rho(t, \vec{x})$

and velocity $\vec{v}(t, \vec{x})$

$$\vec{\xi}(t) = \vec{x}_1 - \vec{x}_0$$

= deviation vector.


Prelude to Chapter 2: fluid mechanics
neighboring fluid element
$\vec{x}_1$
$\vec{\xi}$
reference fluid element at $\vec{x}_0$
fluid of density $\rho(t,\vec{x})$
and velocity $\vec{v}(t,\vec{x})$
$\vec{\xi}(t) = \vec{x}_1 - \vec{x}_0$
= deviation vector.
$\frac{d\vec{\xi}}{dt} =$


Prelude to Chapter 2: fluid mechanics
neighboring fluid element
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$\vec{\xi}$
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fluid of density $\rho(t,\vec{x})$
and velocity $\vec{v}(t,\vec{x})$
$\vec{\xi}(t) = \vec{x}_1 - \vec{x}_0$
= deviation vector.
$\frac{d\vec{\xi}}{dt} = \vec{v}(\vec{x}_1) - \vec{v}(\vec{x}_0)$

$$\frac{d\xi^a}{dt} = V^a(\vec{x}_1) - V^a(\vec{x}_0)$$

$$= V^a(\vec{x}_0 + \vec{\xi}) - V^a(\vec{x}_0)$$

$$\frac{d\xi^a}{dt} = V^a(\vec{x}_1) - V^a(\vec{x}_0)$$

$$= V^a(\vec{x}_0 + \vec{\xi}) - V^a(\vec{x}_0)$$

$$= V^a{}_{;b}\, \xi^b + \cdots$$

$$\frac{d\xi^a}{dt} = B^a{}_b\, \xi^b$$

$$B^a{}_b = V^a{}_{;b}$$

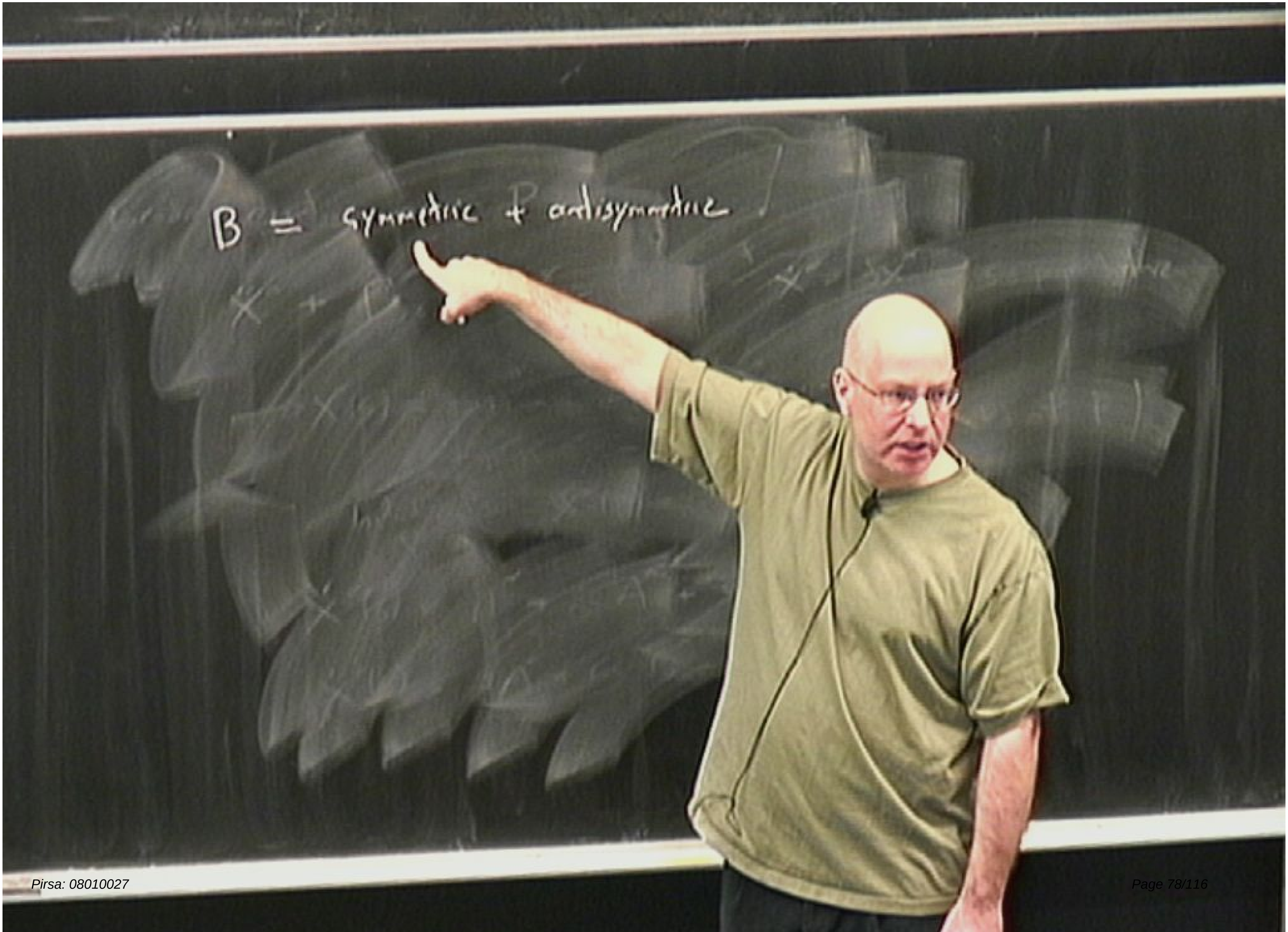
$$\frac{d\xi^a}{dt} = V^a(\vec{x}_1) - V^a(\vec{x}_0)$$

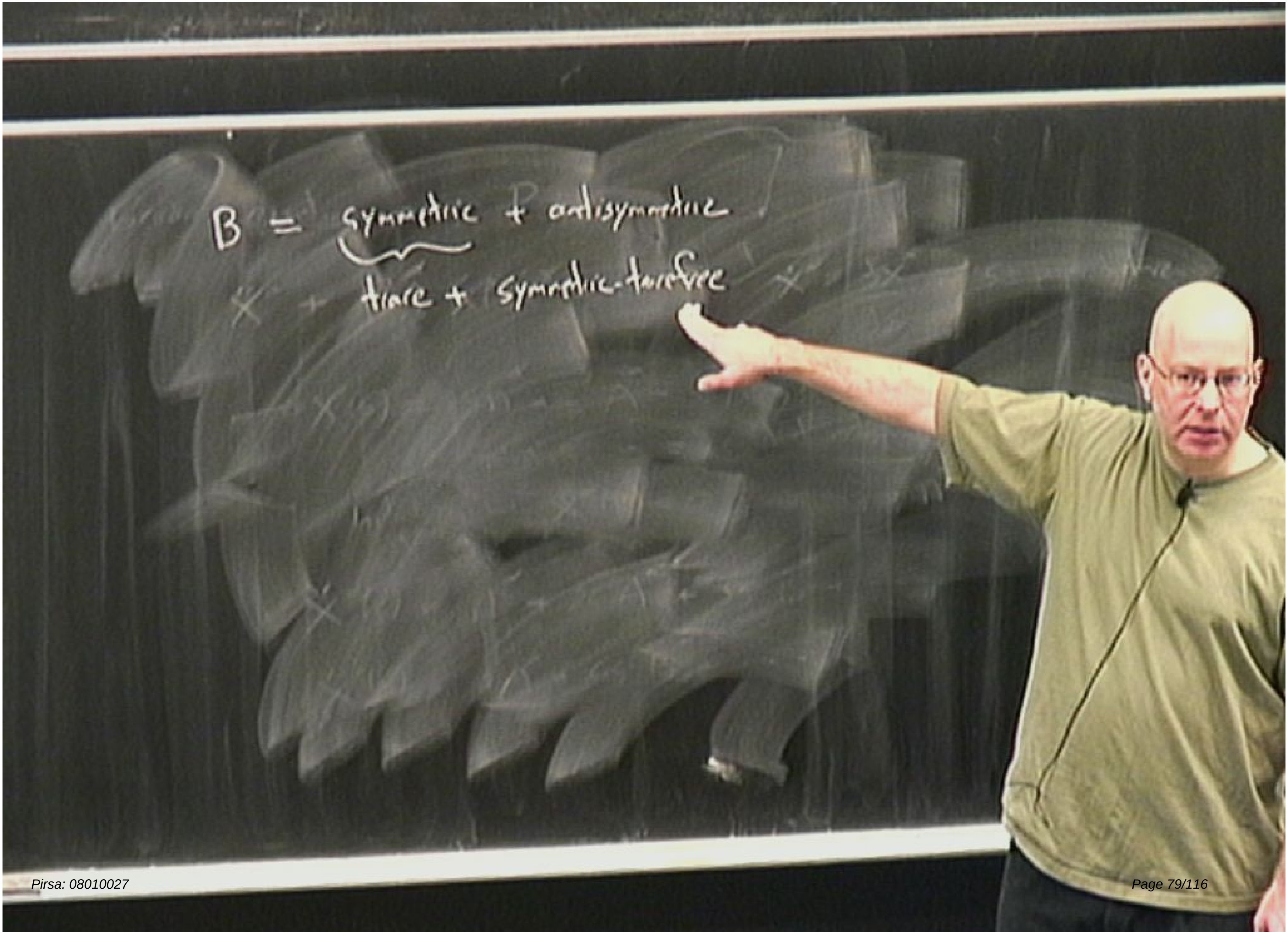
$$= V^a(\vec{x}_0 + \vec{\xi}) - V^a(\vec{x}_0)$$

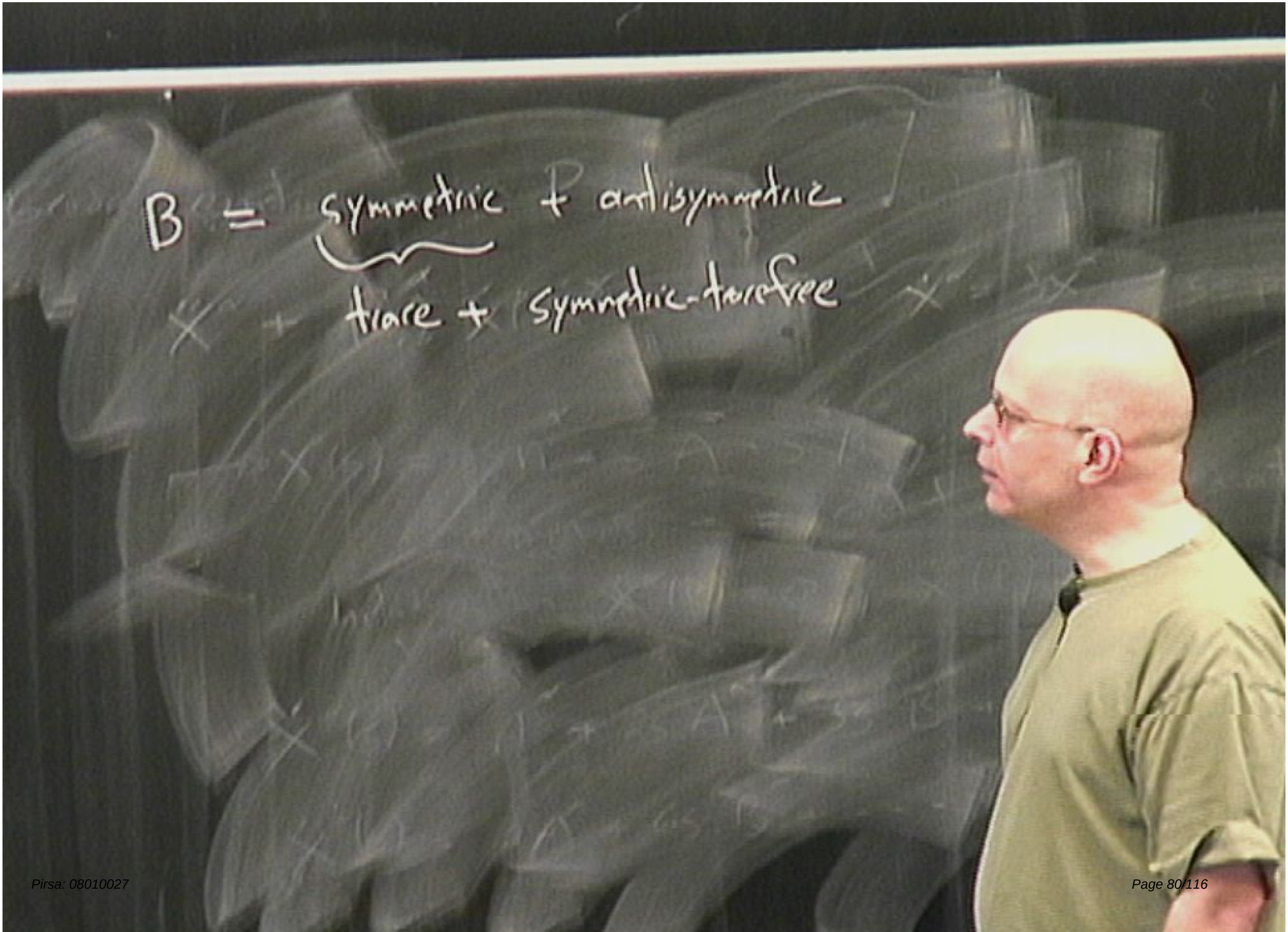
$$= V^a{}_{;b}\,\xi^b + \cdots$$

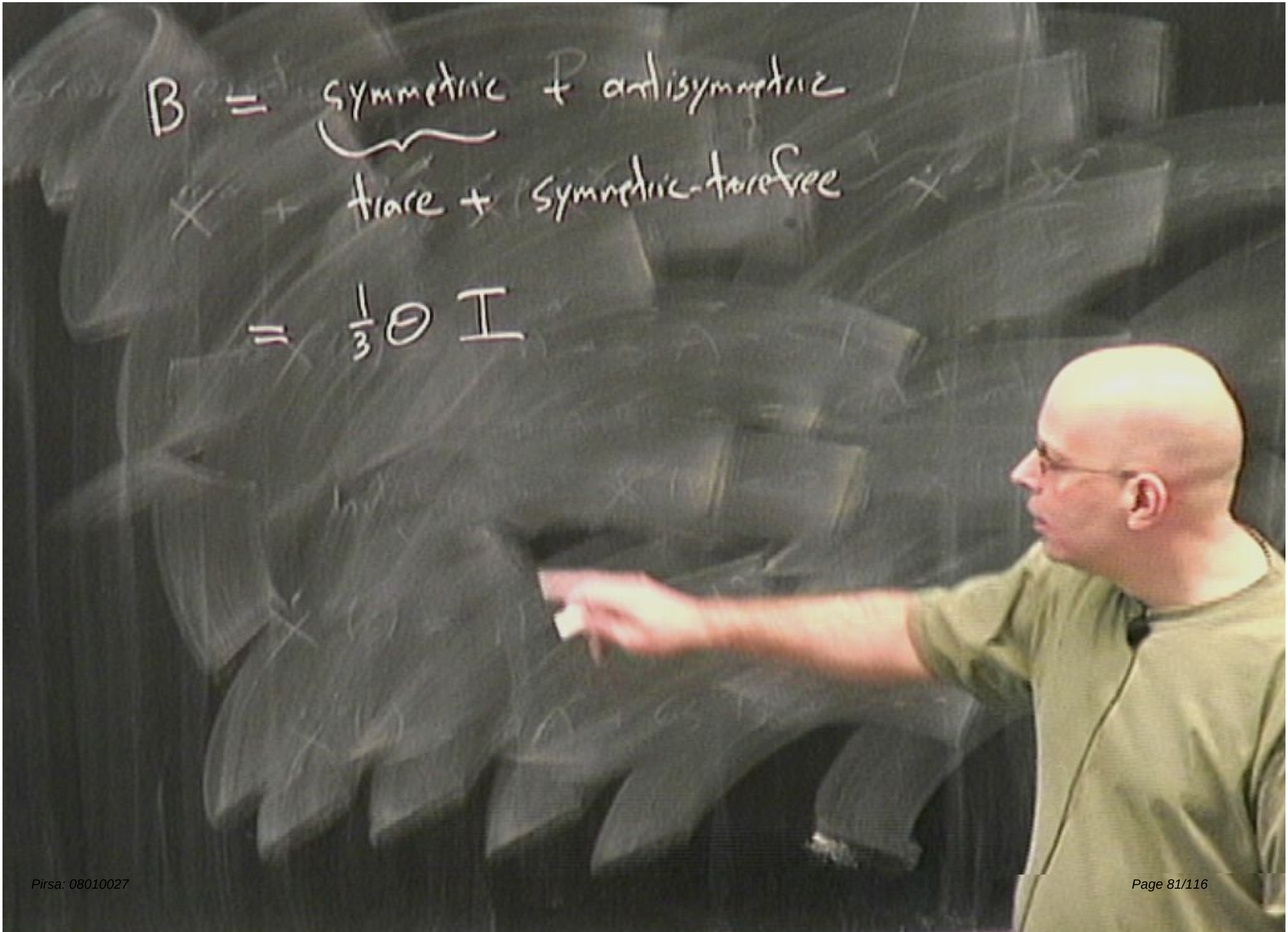
$$\boxed{\begin{aligned} \frac{d\xi^a}{dt} &= B^a{}_b\,\xi^b \\ B^a{}_b &= V^a{}_{;b} \end{aligned}}$$

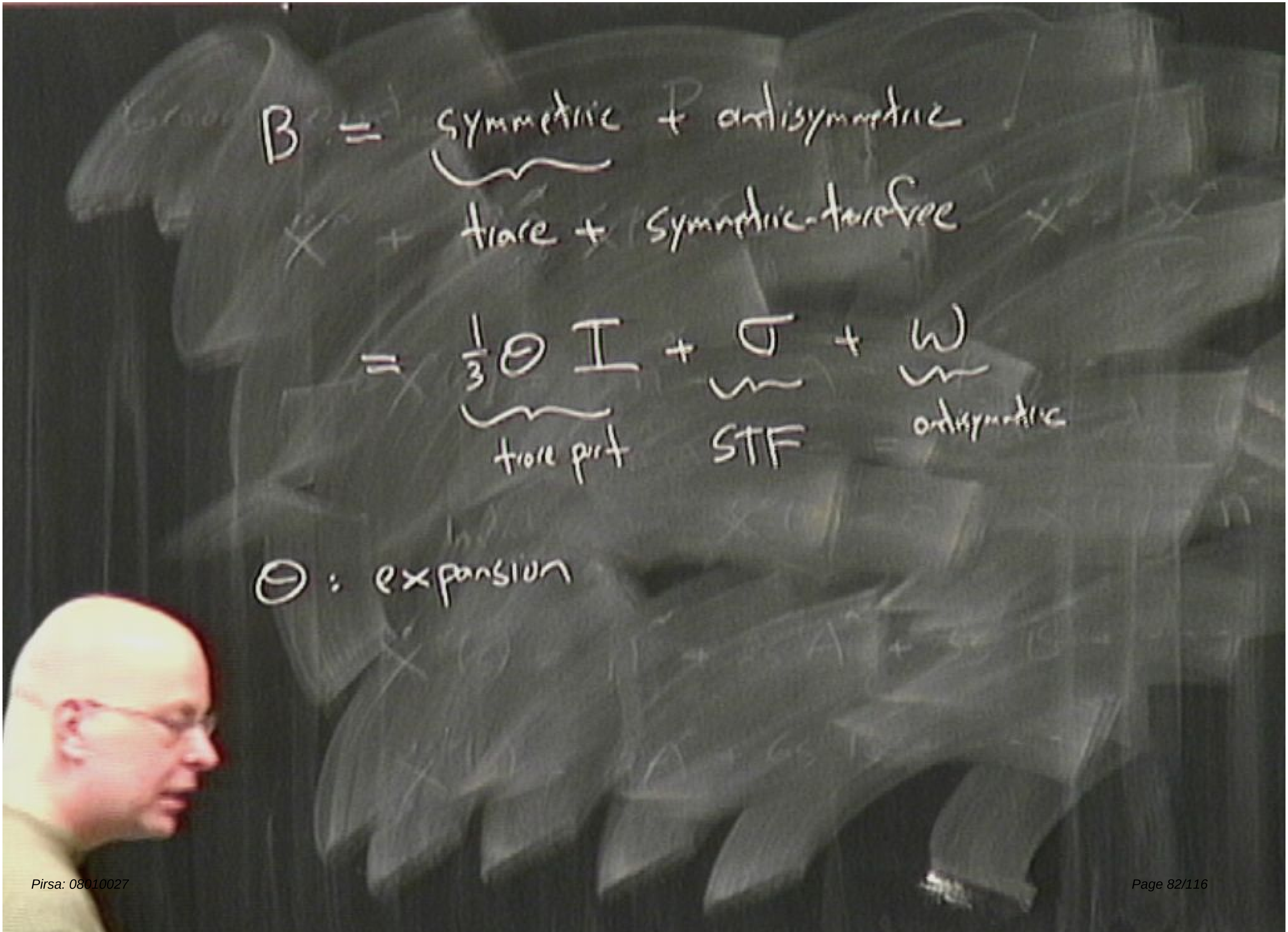
= general 3×3 matrix

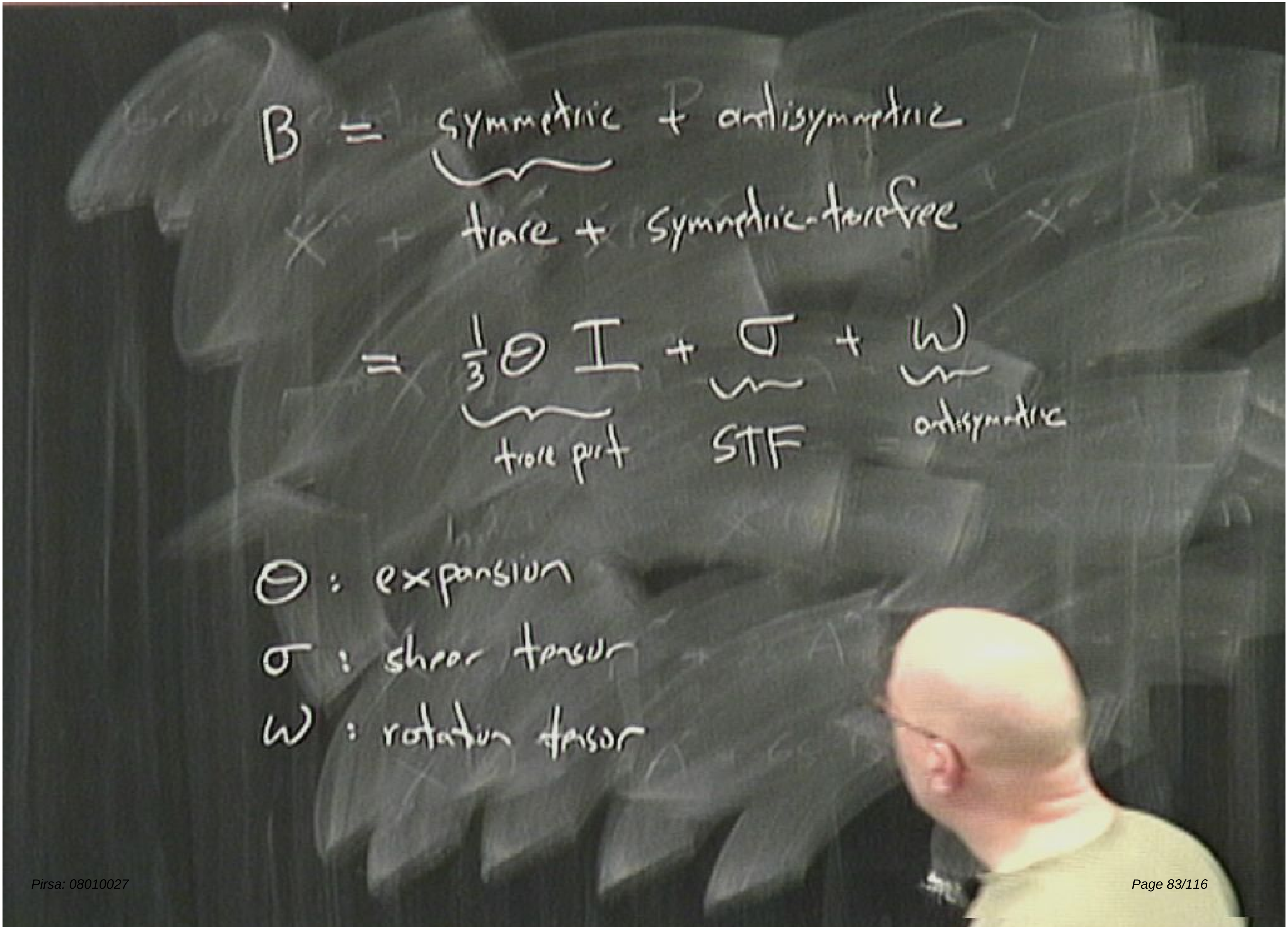

B = symmetric + antisymmetric


B = symmetric + antisymmetric
trace + symmetric-tracefree


B = symmetric + antisymmetric
trace + symmetric-tracefree


B = symmetric + antisymmetric
trace + symmetric-tracefree
= 1/3 Θ I


$B = \underbrace{\text{symmetric}}_{\text{trace + symmetric-tracefree}} + \text{antisymmetric}$
$= \underbrace{\tfrac{1}{3}\Theta\, I}_{\text{trace part}} + \underbrace{\sigma}_{\text{STF}} + \underbrace{\omega}_{\text{antisymmetric}}$
Θ : expansion


B = symmetric + antisymmetric
trace + symmetric-tracefree
= 1/3 Θ I + σ + ω
trace part
STF
antisymmetric
Θ : expansion
σ : shear tensor
ω : rotation tensor

B = $\underbrace{\text{symmetric}}_{\text{trace + symmetric-tracefree}}$ + antisymmetric

$$= \underbrace{\tfrac{1}{3}\Theta\, I}_{\text{trace part}} + \underbrace{\sigma}_{\text{STF}} + \underbrace{\omega}_{\text{antisymmetric}}$$

$$B^a{}_b = \tfrac{1}{3}\Theta\,\delta^a{}_b + \sigma^a{}_b + \omega^a{}_b$$

Θ : expansion (1)

σ : shear tensor (5)

ω : rotation tensor (3)

$$B = \text{symmetric} + \text{antisymmetric}$$

$$\underbrace{\text{symmetric}}_{\text{trace} + \text{symmetric-tracefree}}$$

$$= \underbrace{\tfrac{1}{3}\Theta\, I}_{\text{trace part}} + \underbrace{\sigma}_{\text{STF}} + \underbrace{\omega}_{\text{antisymmetric}}$$

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$$B = \underbrace{\text{symmetric}}_{\text{trace + symmetric-tracefree}} + \text{antisymmetric}$$

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θ : expansion (1)

σ : shear tensor (5)

ω : rotation tensor (3)

$$\theta = B^a{}_a$$

$$B = \underbrace{\text{symmetric}}_{\text{trace} + \text{symmetric-tracefree}} + \text{antisymmetric}$$

$$= \underbrace{\tfrac{1}{3}\Theta\, I}_{\text{trace part}} + \underbrace{\sigma}_{\text{STF}} + \underbrace{\omega}_{\text{antisymmetric}}$$

$$\boxed{B^a{}_b = \tfrac{1}{3}\Theta\,\delta^a{}_b + \sigma^a{}_b + \omega^a{}_b}$$

Θ : expansion (1)

σ : shear tensor (5)

ω : rotation tensor (3)

$$\Theta = B^a{}_a = V^a{}_{;a}$$

Expansion θ

continuity equation : $\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v}) = 0$

Expansion θ

continuity equation: $\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v}) = 0$

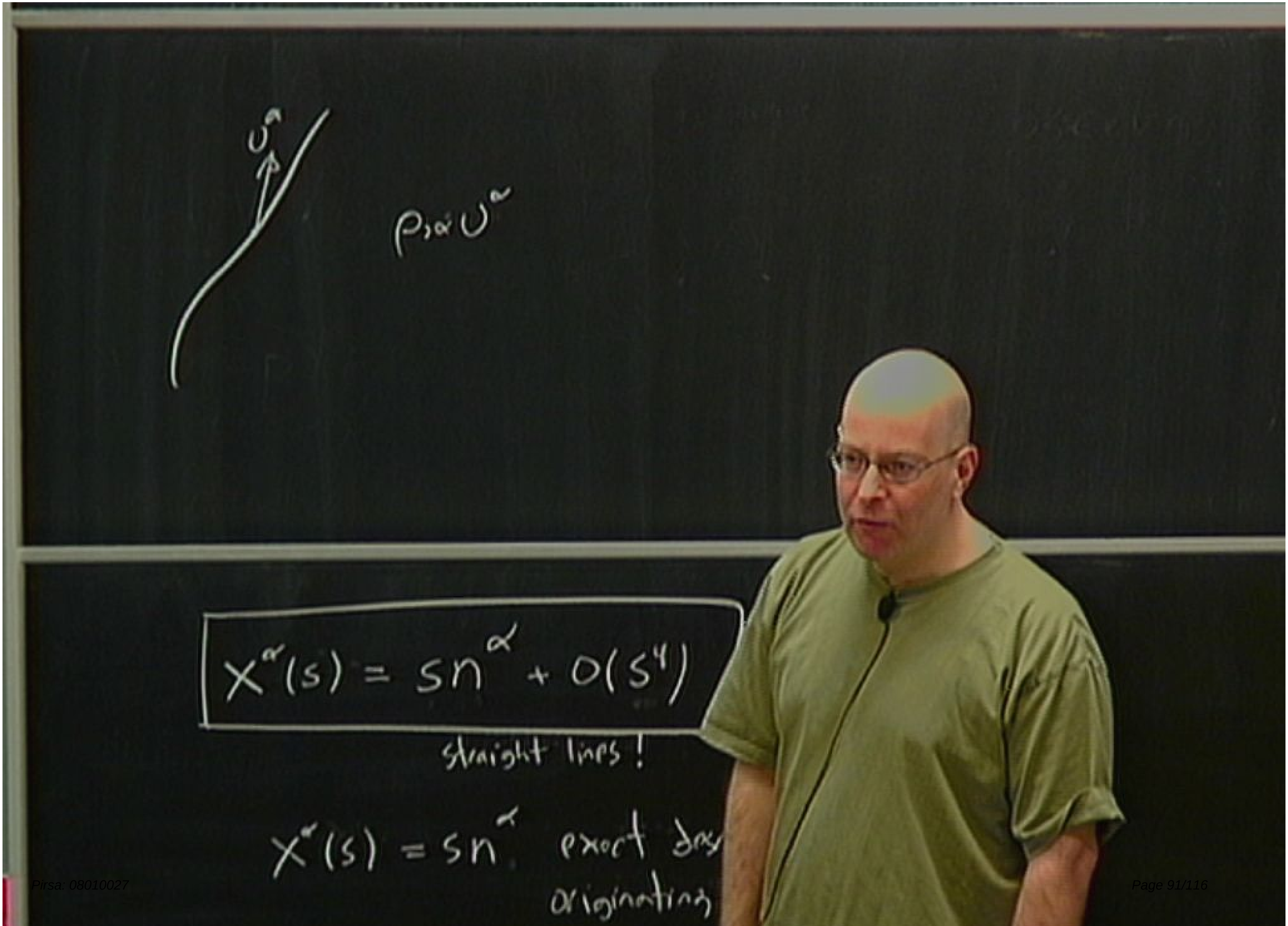
rate of change of ρ following a fluid element:

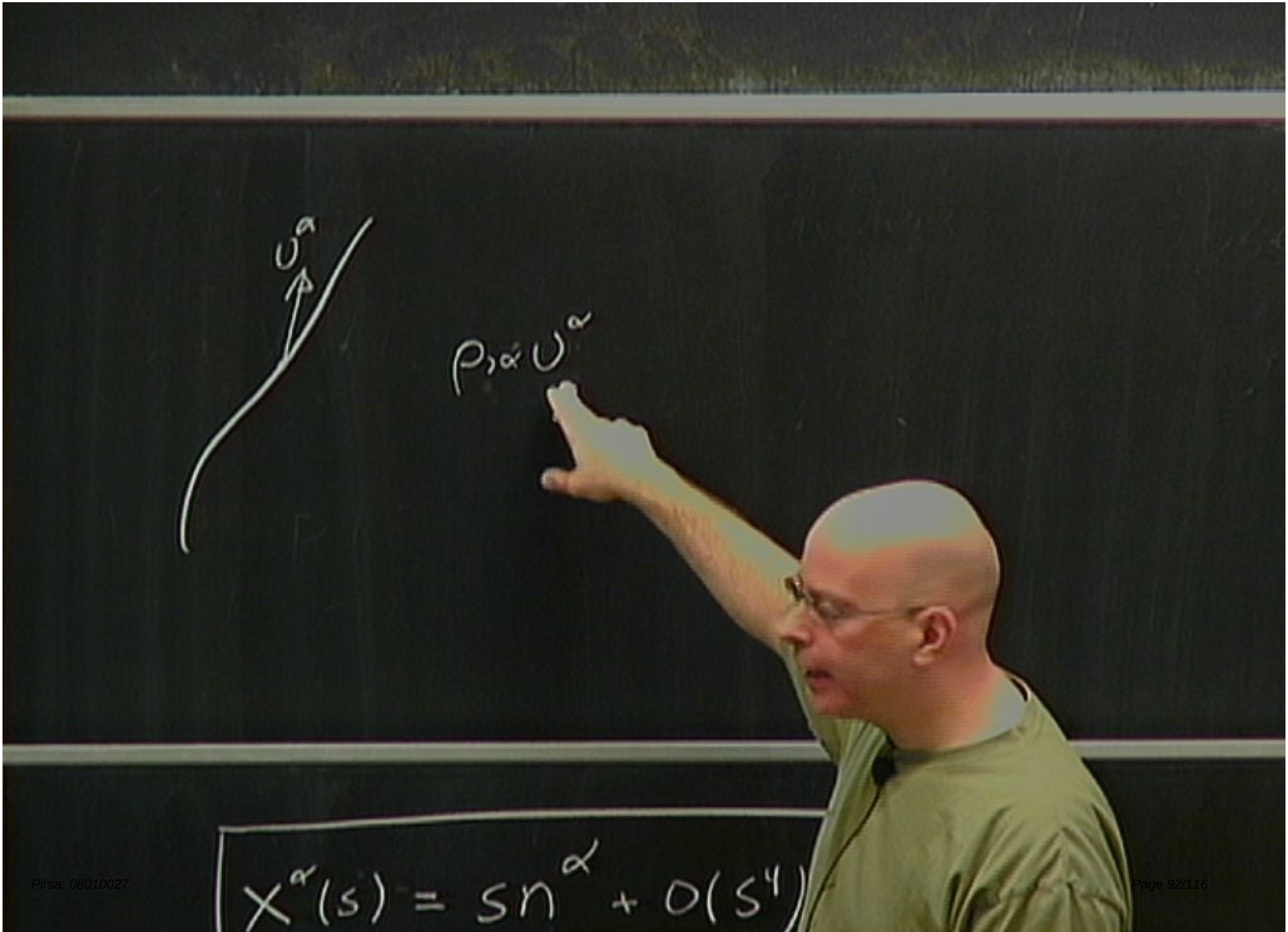
Expansion θ

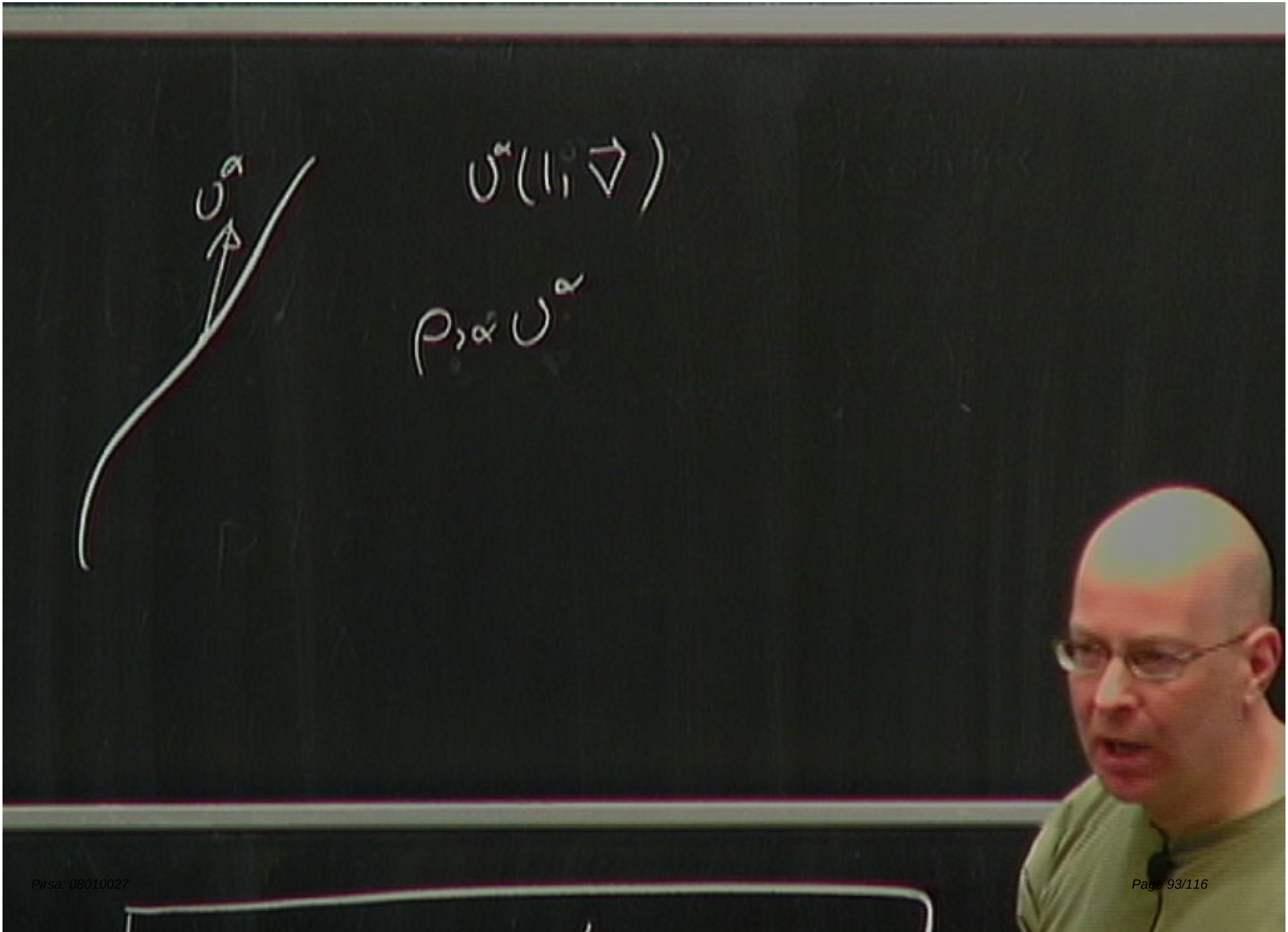
continuity equation : $\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v}) = 0$

rate of change of ρ following a fluid element :

$$\frac{d\rho}{dt} = \frac{\partial \rho}{\partial t} + \vec{v} \cdot \vec{\nabla} \rho$$

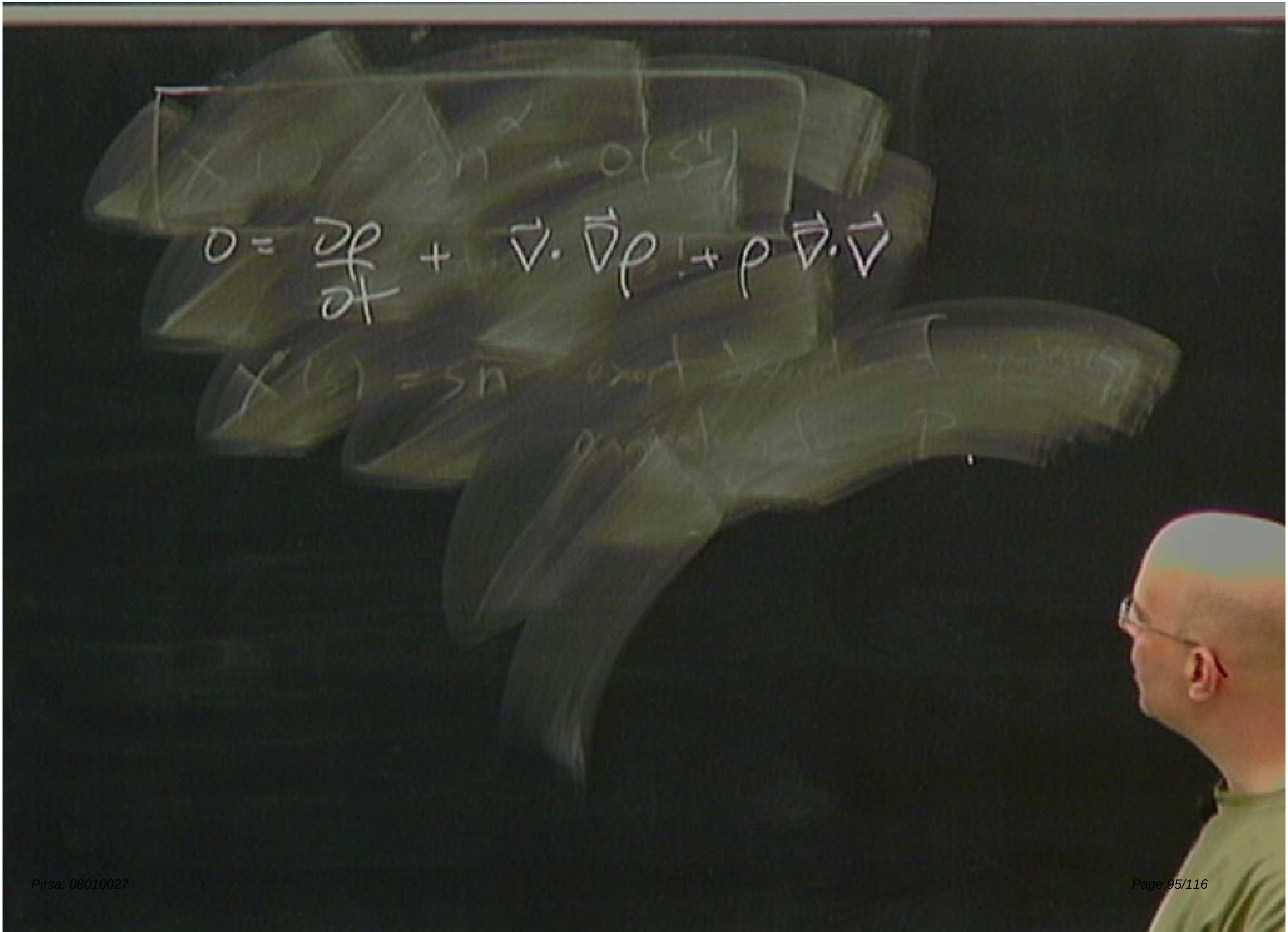

x^α(s) = sn^α + O(s^4)
straight lines!
x^α(s) = sn^α exact
originating

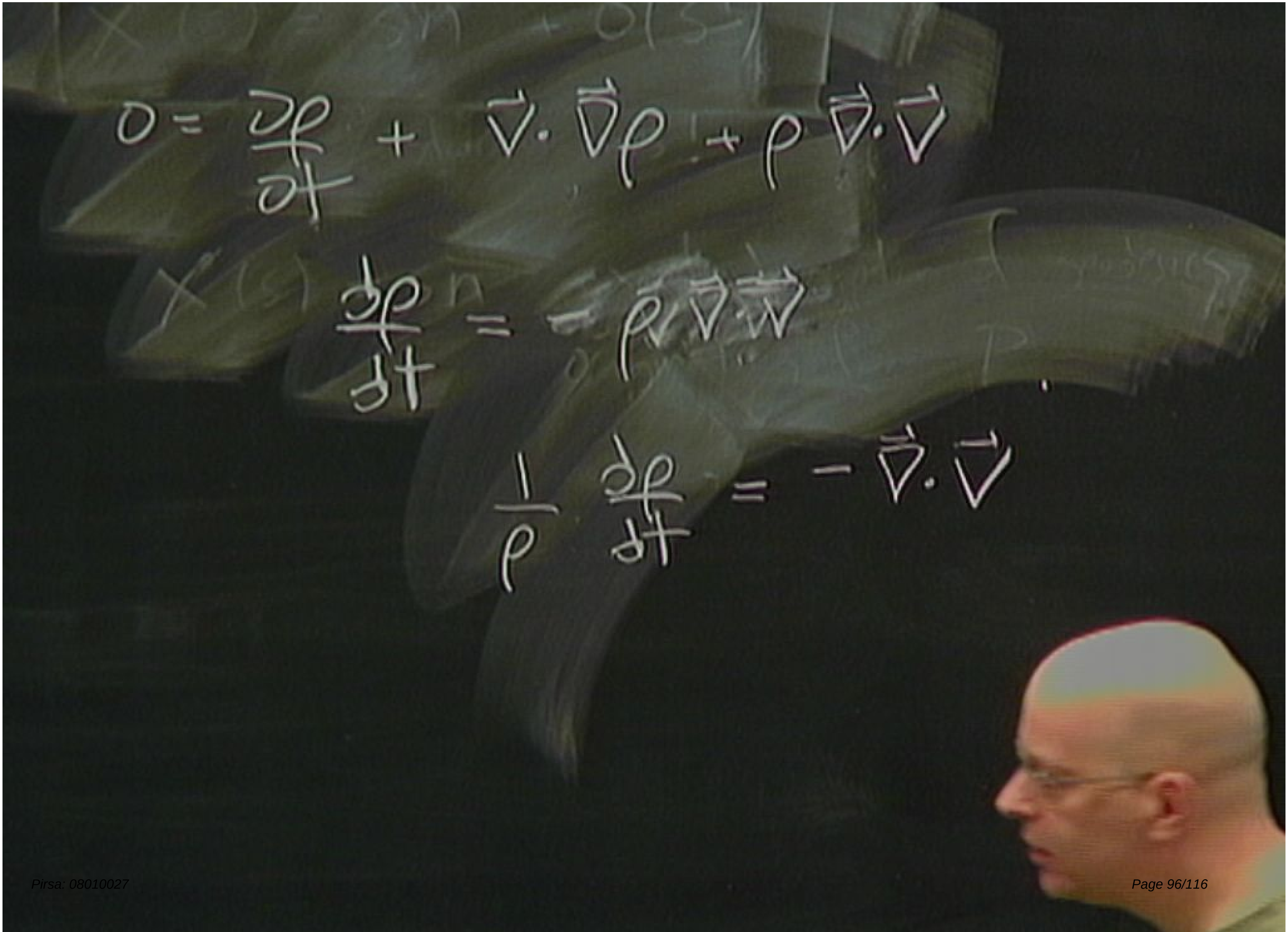

U^α
$\rho_{,\alpha} U^\alpha$
$X^\alpha(s) = s n^\alpha + O(s^4)$


u^α
$u^\alpha(1;\nabla)$
$\rho_{;\alpha} u^\alpha$

$$u^\alpha(1, \vec{v})$$

$$\rho_{,\alpha} u^\alpha = \frac{\partial \rho}{\partial t} + \vec{v} \cdot \vec{\nabla} \rho$$


$0 = \frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot\vec{\nabla}\rho + \rho\,\vec{\nabla}\cdot\vec{\nabla}$



$0 = \frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{V} \rho + \rho \vec{\nabla} \cdot \vec{V}$
$\frac{\partial \rho}{\partial t} = -\rho \vec{\nabla} \cdot \vec{V}$
$\frac{1}{\rho} \frac{\partial \rho}{\partial t} = -\vec{\nabla} \cdot \vec{V}$

$$0 = \frac{\partial \rho}{\partial t} + \vec{v}\cdot\vec{\nabla}\rho + \rho\,\vec{\nabla}\cdot\vec{v}$$

$$\frac{d\rho}{dt} = -\rho\,\vec{\nabla}\cdot\vec{v}$$

$$\frac{1}{\rho}\frac{d\rho}{dt} = -\vec{\nabla}\cdot\vec{v}$$

$$0 = \frac{\partial \rho}{\partial t} + \vec{V}\cdot\vec{\nabla}\rho + \rho\,\vec{\nabla}\cdot\vec{V}$$

$$\frac{d\rho}{dt} = -\rho\,\vec{\nabla}\cdot\vec{V}$$

$$\frac{1}{\rho}\frac{d\rho}{dt} = -\vec{\nabla}\cdot\vec{V}$$

$$\rho = \frac{\delta m}{\delta V}$$

$$\boxed{\frac{1}{\delta V}\frac{d}{dt}\delta V = \vec{\nabla}\cdot\vec{V} = \Theta}$$

antisymmetric

$$\boxed{B^a{}_b = \frac{1}{3}\Theta\delta^a{}_b + \sigma^a{}_b + \omega^a{}_b}$$

$$\Theta = B^a{}_a = V^a{}_{;a} = \vec{\nabla}\cdot\vec{V}$$

Shear

$$\sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_{22} & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & -\sigma_{11}-\sigma_{22} \end{pmatrix}$$

Shear

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$$\sigma_{12} \neq 0$$

σ : shear tensor (5)

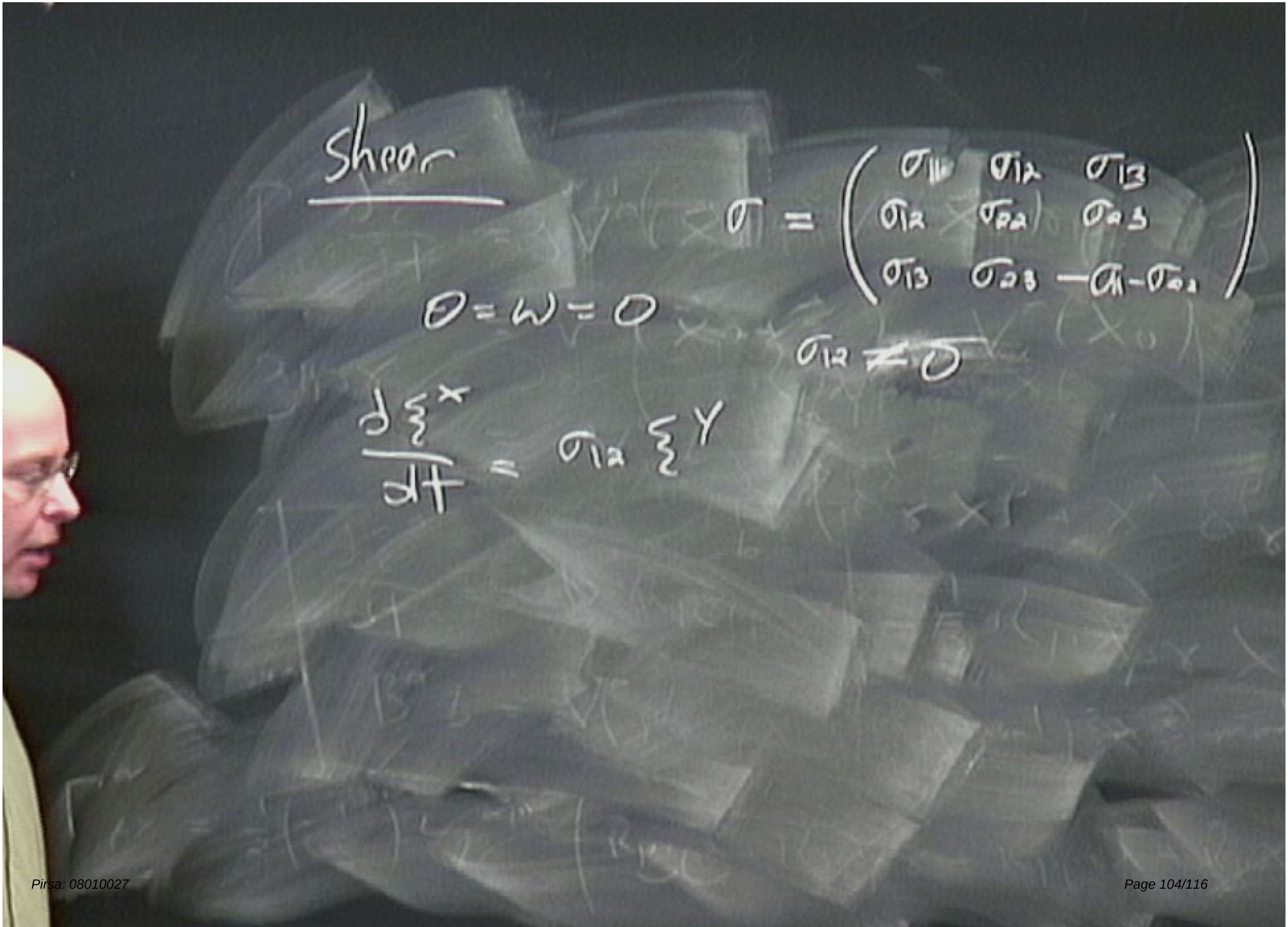
ω : rotation tensor (3)

$$\frac{d\xi^a}{dt} = B^a{}_b \, \xi^b$$

σ : shear te

ω : rotation

$$\frac{d\xi^a}{dt} = B^a{}_b\, \xi^b$$


Shear
$\sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_{22} & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & -\sigma_{11}-\sigma_{22} \end{pmatrix}$
$\theta = \omega = 0$
$\sigma_{12} \neq 0$
$\frac{d\xi^x}{dt} = \sigma_{12}\,\xi^y$

Shear

$$\sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_{22} & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & -\sigma_{11}-\sigma_{22} \end{pmatrix}$$

$$\theta = \omega = 0$$

$$\sigma_{12} \neq 0$$

$$\frac{d\xi^x}{dt} = \sigma_{12}\,\xi^y$$

$$\frac{d\xi^y}{dt} = \sigma_{12}\,\xi^x$$

Shear

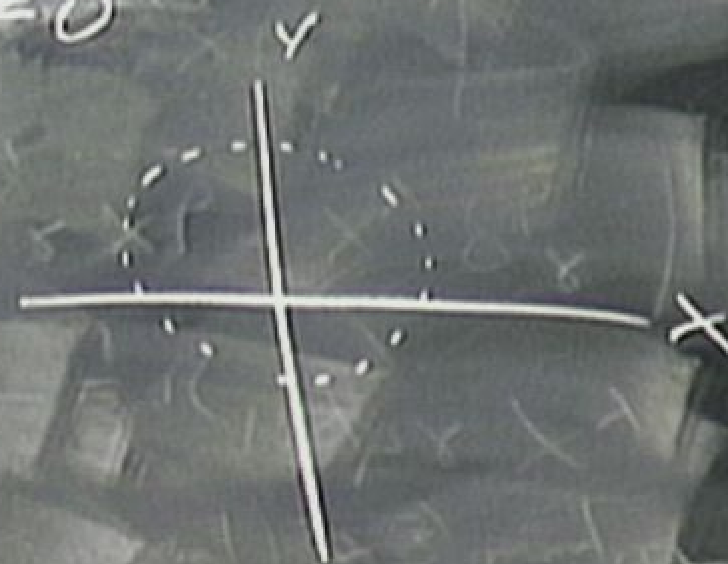
$$\sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_{22} & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & -\sigma_{11}-\sigma_{22} \end{pmatrix}$$

$$\theta = \omega = 0$$

$$\sigma_{12} \neq 0$$

$$\frac{d\xi^x}{dt} = \sigma_{12}\,\xi^Y$$

$$\frac{d\xi^Y}{dt} = \sigma_{12}\,\xi^x$$



Shear

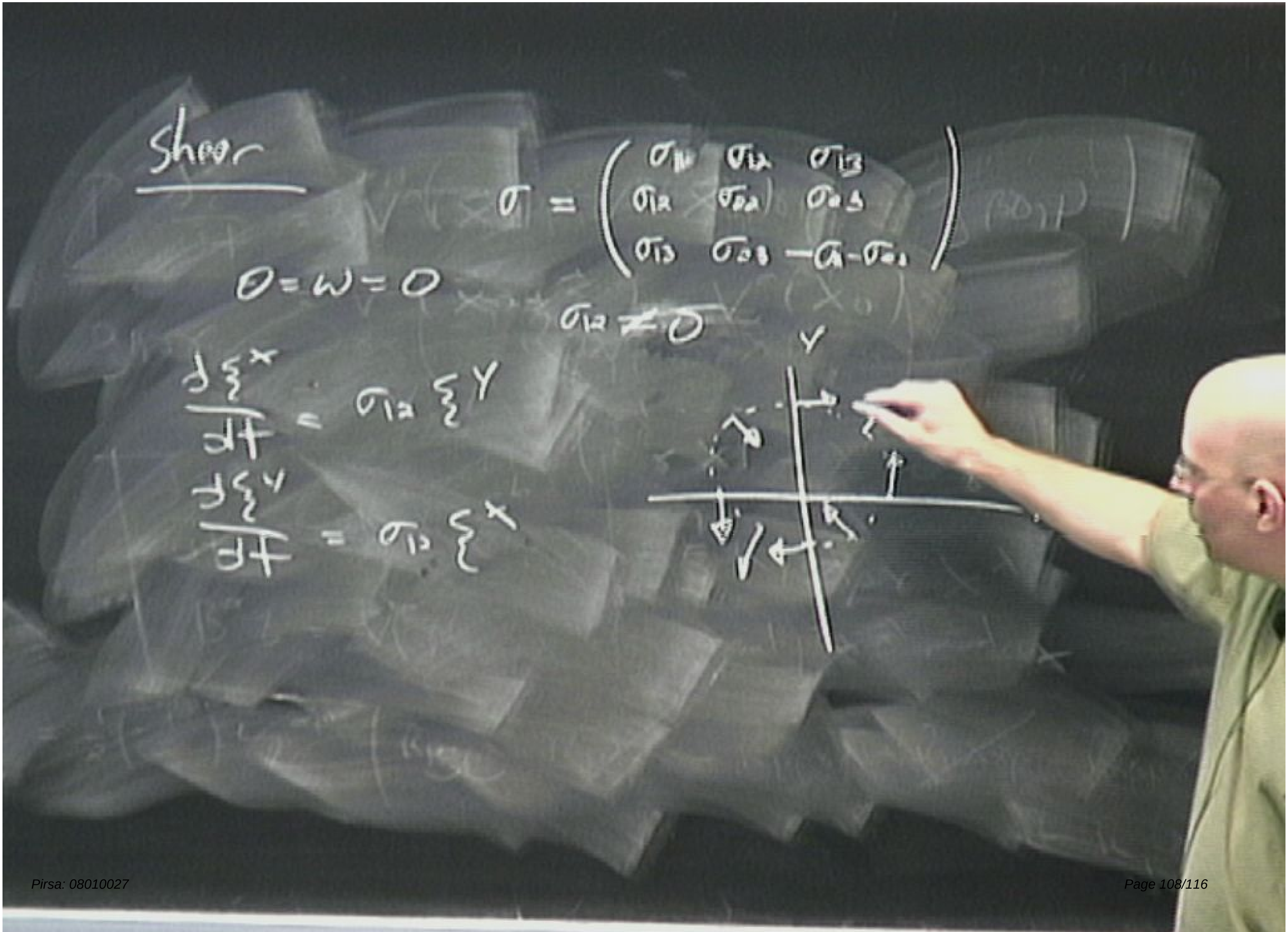
$$\sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_{22} & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & -\sigma_{11}-\sigma_{22} \end{pmatrix}$$

$$\theta = \omega = 0$$

$$\sigma_{12} \neq 0$$

$$\frac{d\xi^x}{dt} = \sigma_{12}\,\xi^Y$$

$$\frac{d\xi^Y}{dt} = \sigma_{12}\,\xi^x$$


Shear
$\sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_{22} & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & -\sigma_{11}-\sigma_{22} \end{pmatrix}$
$\theta = \omega = 0$
$\sigma_{12} \neq 0$
$\frac{d\xi^x}{dt} = \sigma_{12}\, \xi^Y$
$\frac{d\xi^Y}{dt} = \sigma_{12}\, \xi^x$
Y

Shear

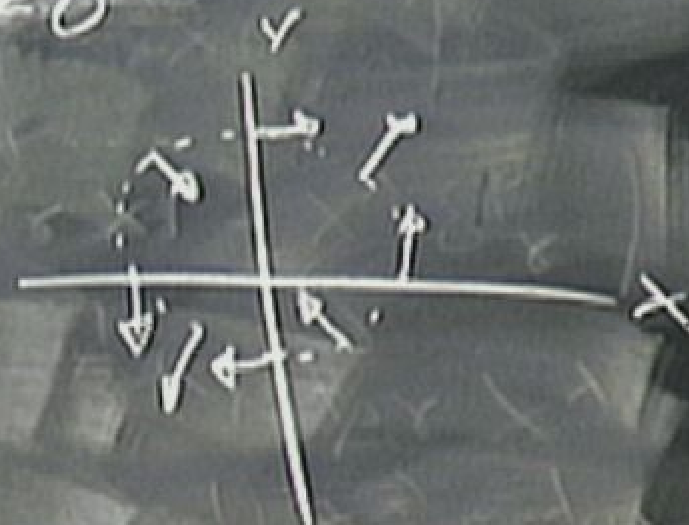
$$\sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_{22} & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & -\sigma_{11}-\sigma_{22} \end{pmatrix}$$

$$\theta = \omega = 0$$

$$\sigma_{12} \neq 0$$

$$\frac{d\xi^x}{dt} = \sigma_{12}\,\xi^y$$

$$\frac{d\xi^y}{dt} = \sigma_{12}\,\xi^x$$



Shear

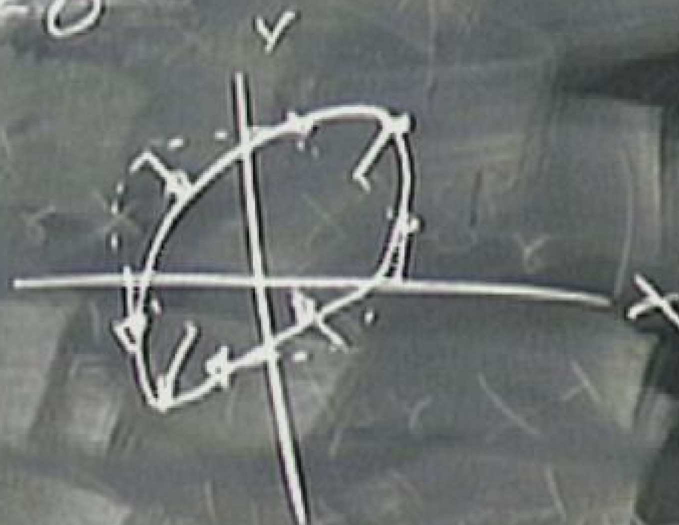
$$\sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_{22} & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & -\sigma_{11}-\sigma_{22} \end{pmatrix}$$

$$\theta = \omega = 0$$

$$\sigma_{12} \neq 0$$

$$\frac{d\xi^x}{dt} = \sigma_{12}\,\xi^Y$$

$$\frac{d\xi^Y}{dt} = \sigma_{12}\,\xi^x$$



rotation

$$\theta = \sigma = 0 \qquad W = \begin{pmatrix} 0 & w_{12} & w_{13} \\ -w_{12} & 0 & w_{23} \\ -w_{13} & -w_{23} & 0 \end{pmatrix}$$

rotation

$$\theta = \sigma = 0$$

$$W = \begin{pmatrix} 0 & W_{12} & W_{13} \\ -W_{12} & 0 & W_{23} \\ -W_{13} & -W_{23} & 0 \end{pmatrix}$$

$$W_{12} \neq 0$$

$$\frac{d\xi^x}{dt} = W_{12}\,\xi^y$$

$$\frac{d\xi^y}{dt} = -W_{12}\,\xi^x$$

rotation

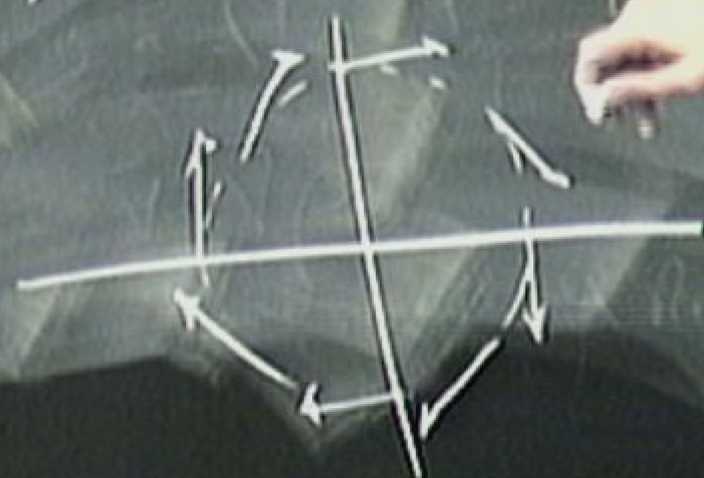
$$\theta = \sigma = 0$$

$$W = \begin{pmatrix} 0 & W_{12} & W_{13} \\ -W_{12} & 0 & W_{23} \\ -W_{13} & -W_{23} & 0 \end{pmatrix}$$

$$W_{12} \neq 0$$

$$\frac{d\xi^x}{dt} = W_{12}\,\xi^y$$

$$\frac{d\xi^y}{dt} = -W_{12}\,\xi^x$$



rotation

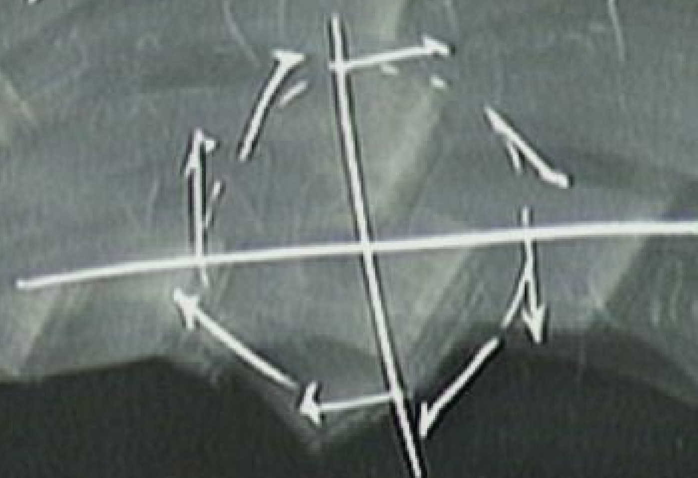
$$\theta = \sigma = 0$$

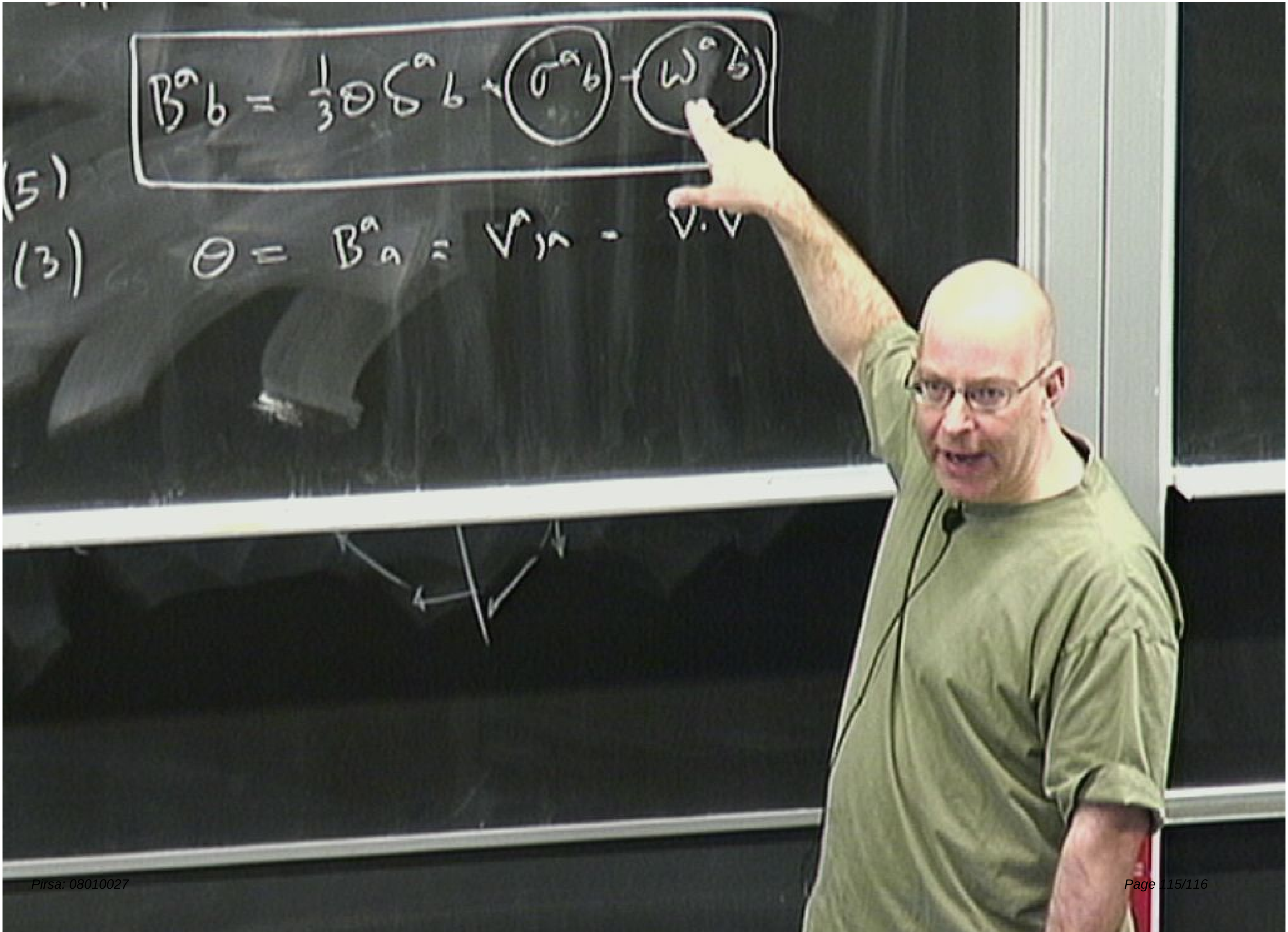
$$\omega = \begin{pmatrix} 0 & \omega_{12} & \omega_{13} \\ -\omega_{12} & 0 & \omega_{23} \\ -\omega_{13} & -\omega_{23} & 0 \end{pmatrix}$$

$$\omega_{12} \neq 0$$

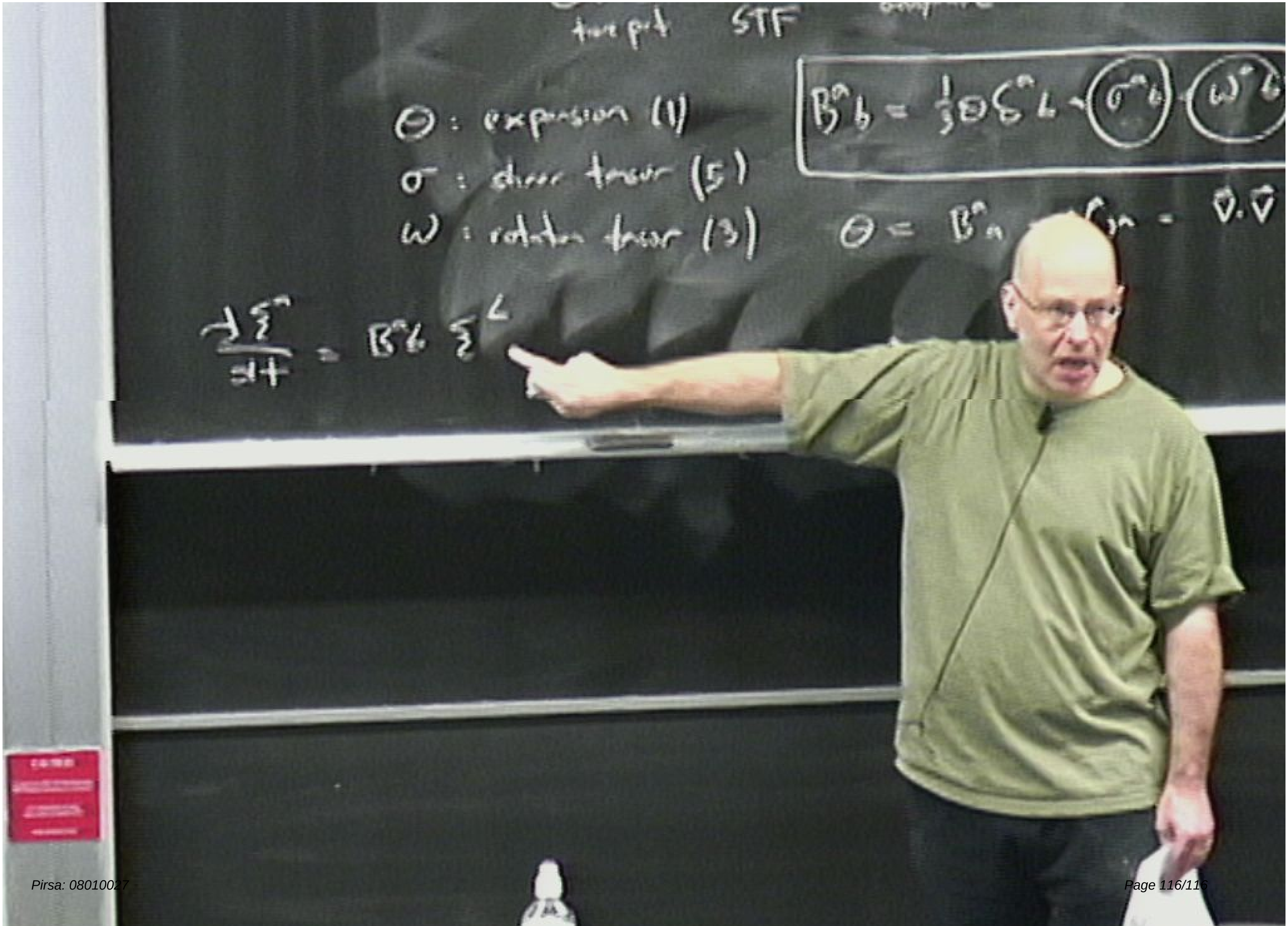
$$\frac{d\xi^x}{dt} = \omega_{12}\, \xi^y$$

$$\frac{d\xi^y}{dt} = -\omega_{12}\, \xi^x$$





(5)
$B^a{}_b = \frac{1}{3}\theta\delta^a{}_b + \sigma^a{}_b + \omega^a{}_b$
(3)
$\theta = B^a{}_a = V^a{}_{;a} = \nabla \cdot V$


two pt STF
$B^a{}_b = \frac{1}{3}\Theta\delta^a{}_b + \sigma^a{}_b + \omega^a{}_b$
Θ : expansion (1)
σ : shear tensor (5)
ω : rotation tensor (3)
$\Theta = B^a{}_a$
$\frac{d\xi^a}{dt} = B^a{}_b\,\xi^b$