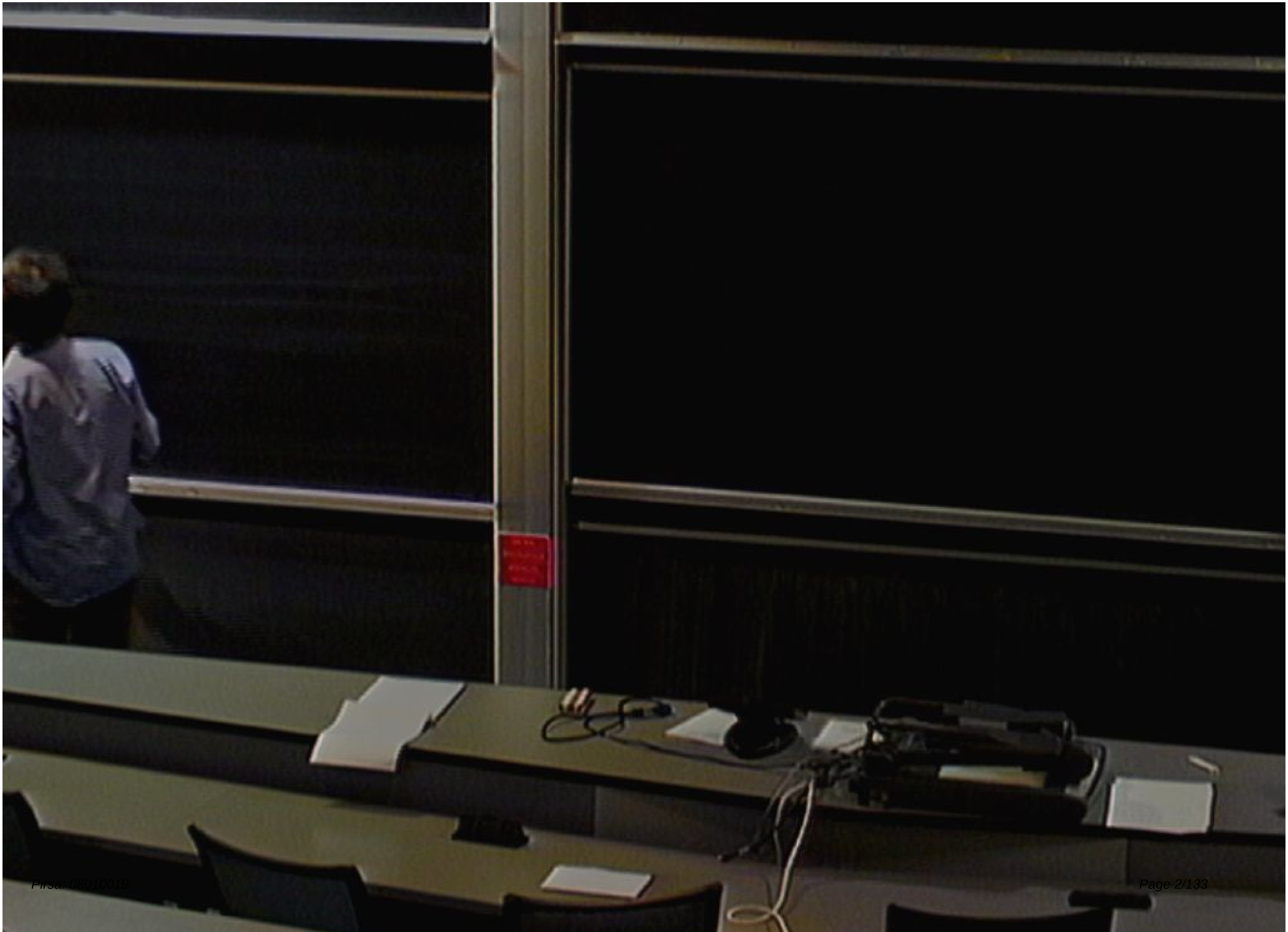
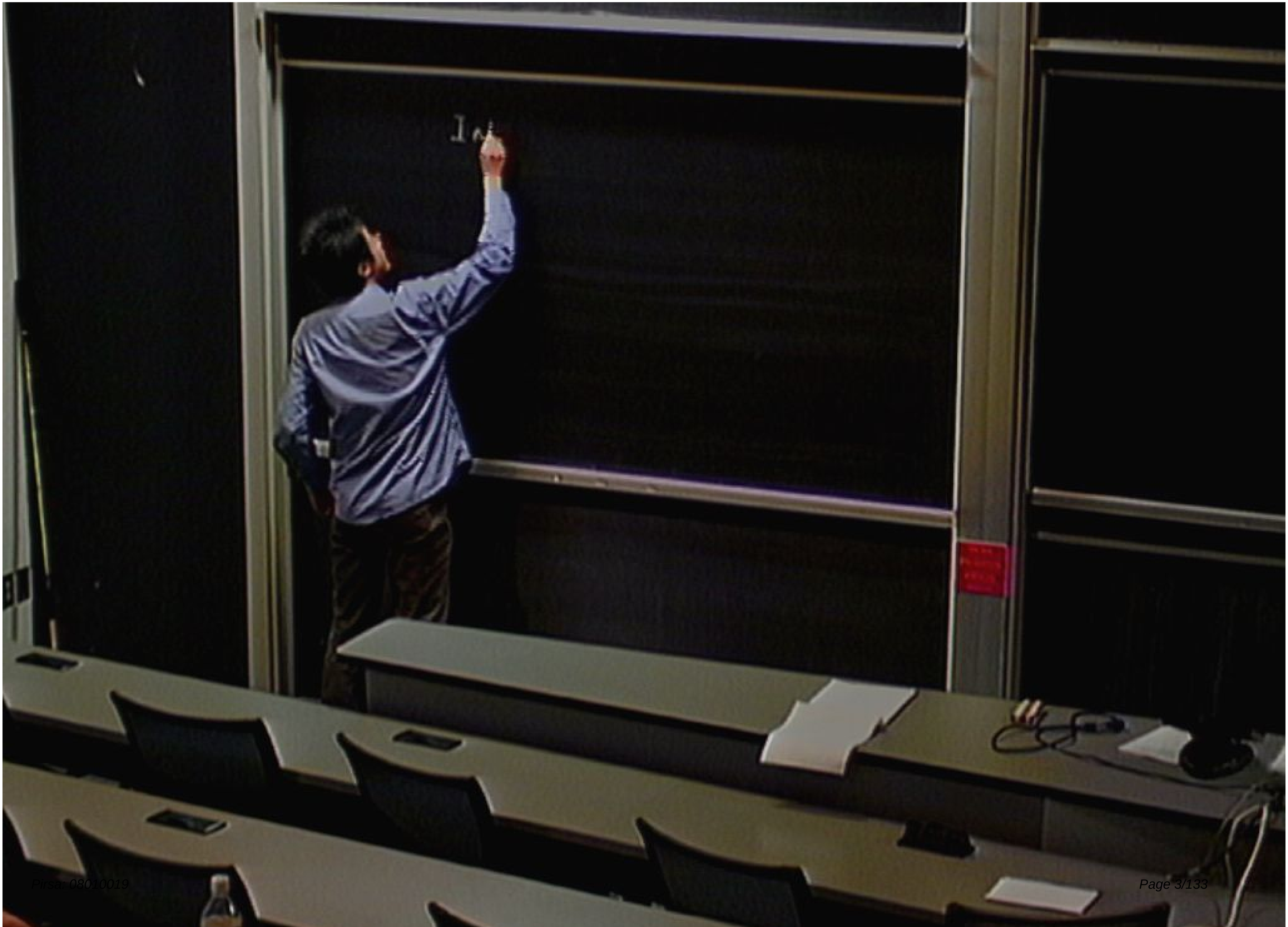
Title: String Theory #1

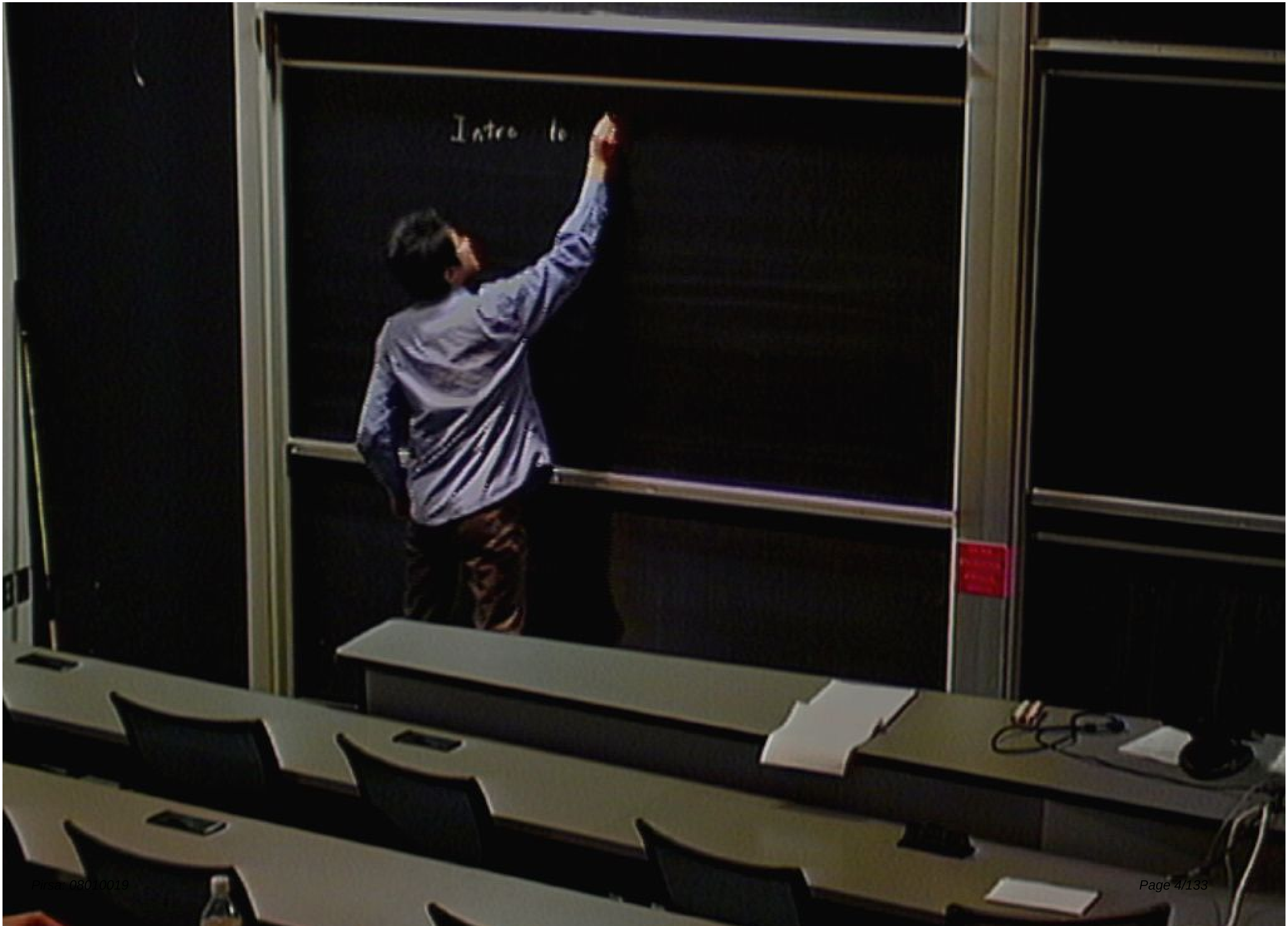
Date: Jan 22, 2008 06:30 PM

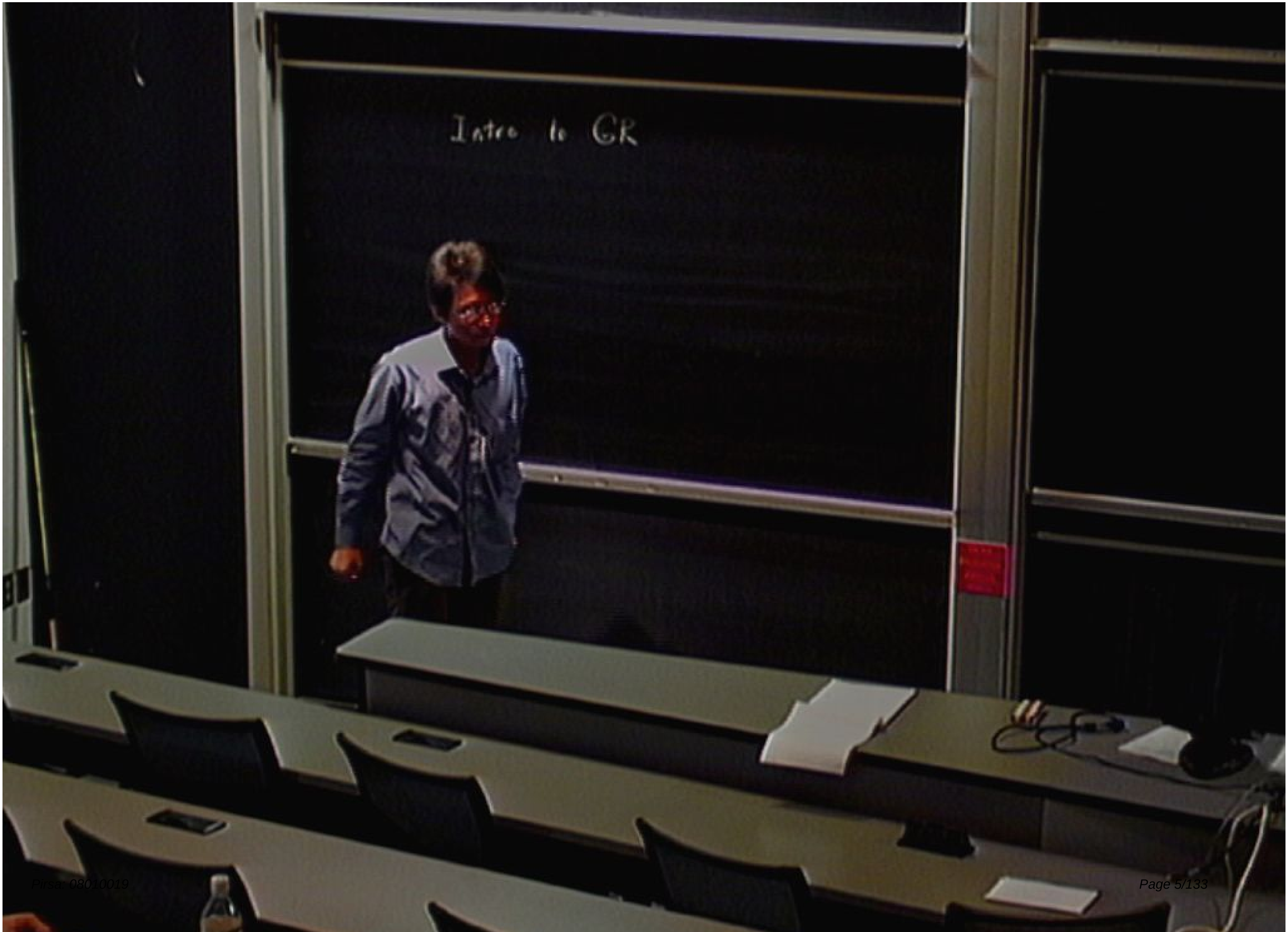
URL: http://pirsa.org/08010019

Abstract: Strings vs. particles. Branes and Holography in quantum gravity.






Intro to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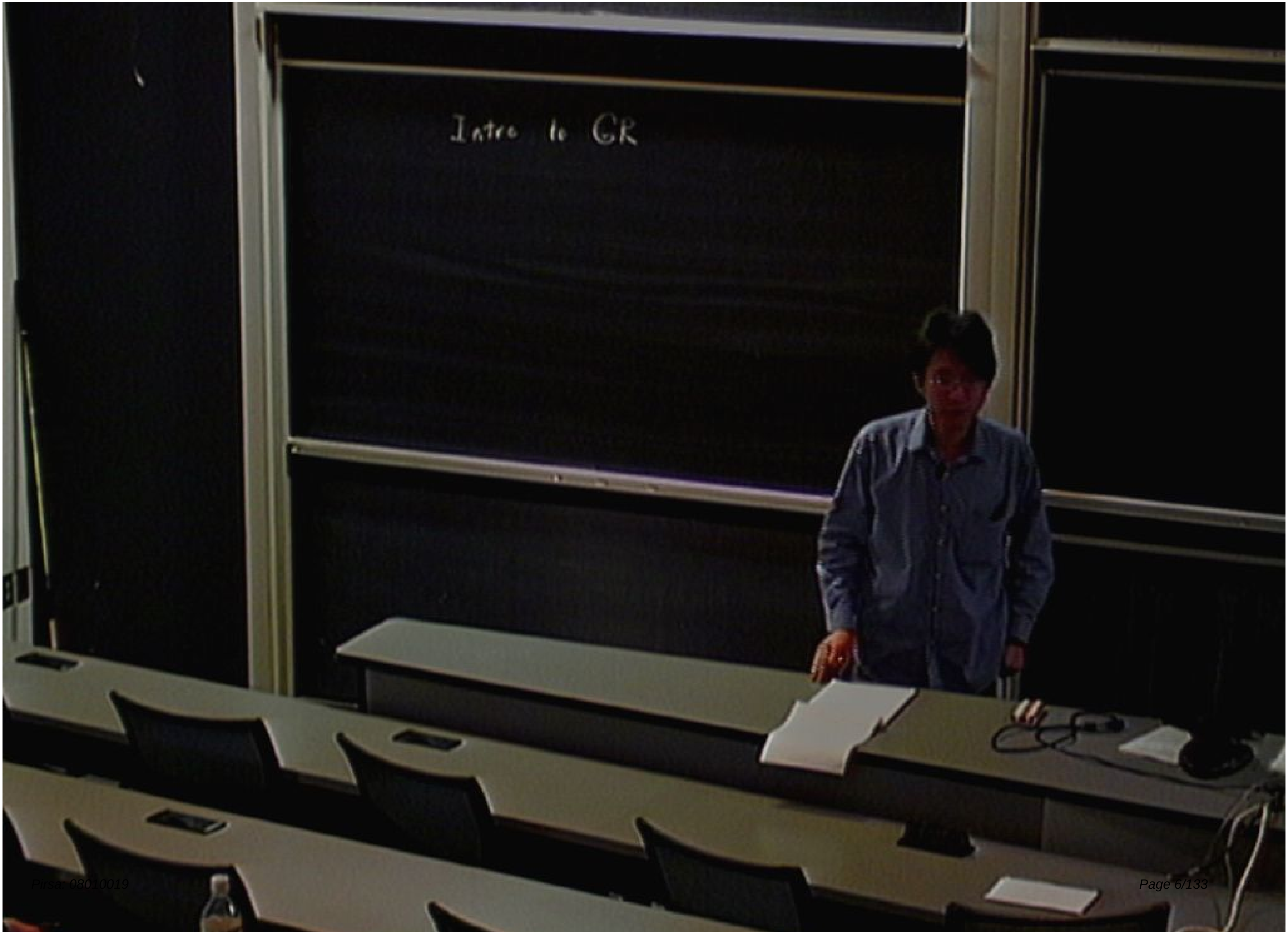
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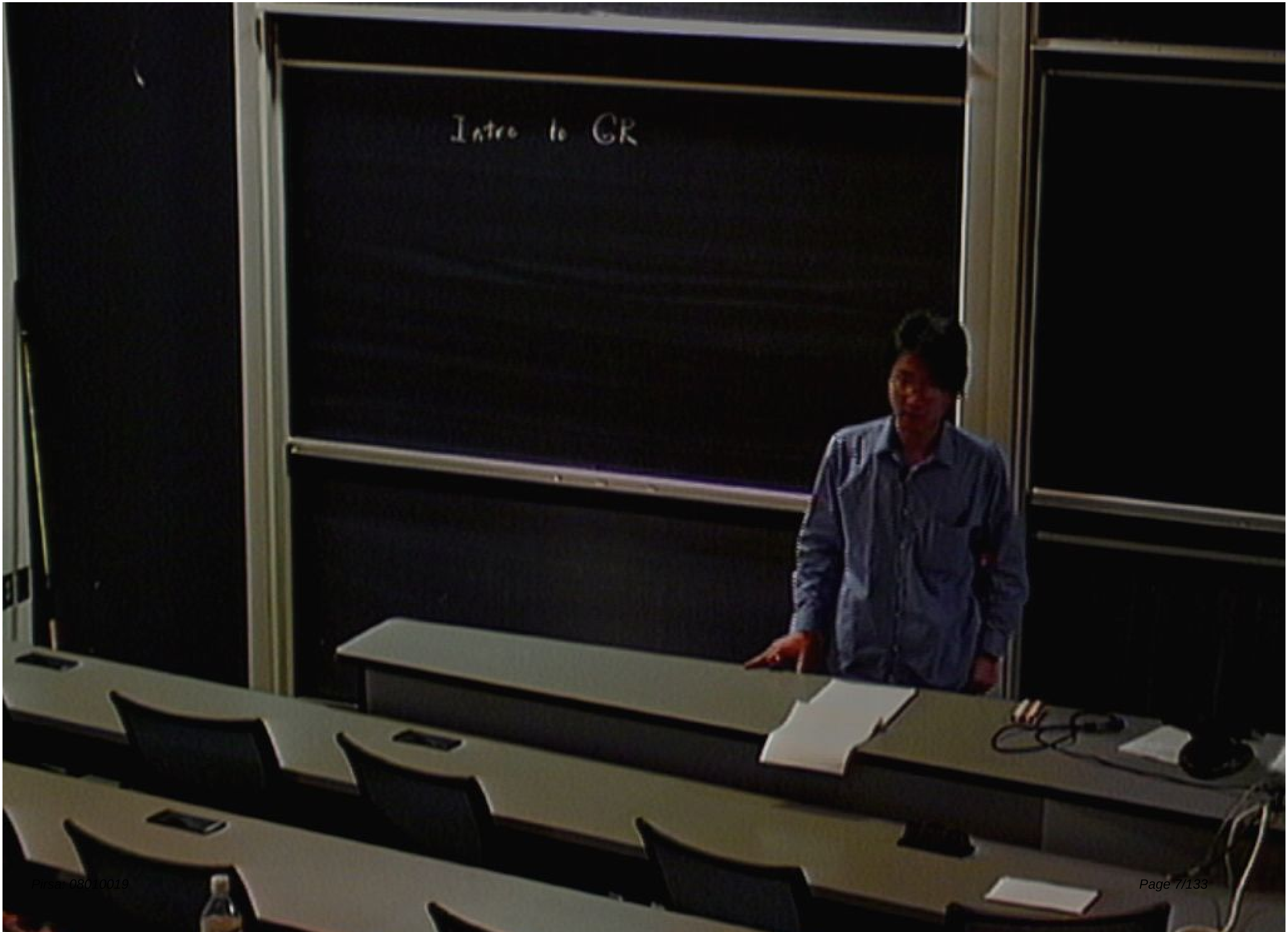
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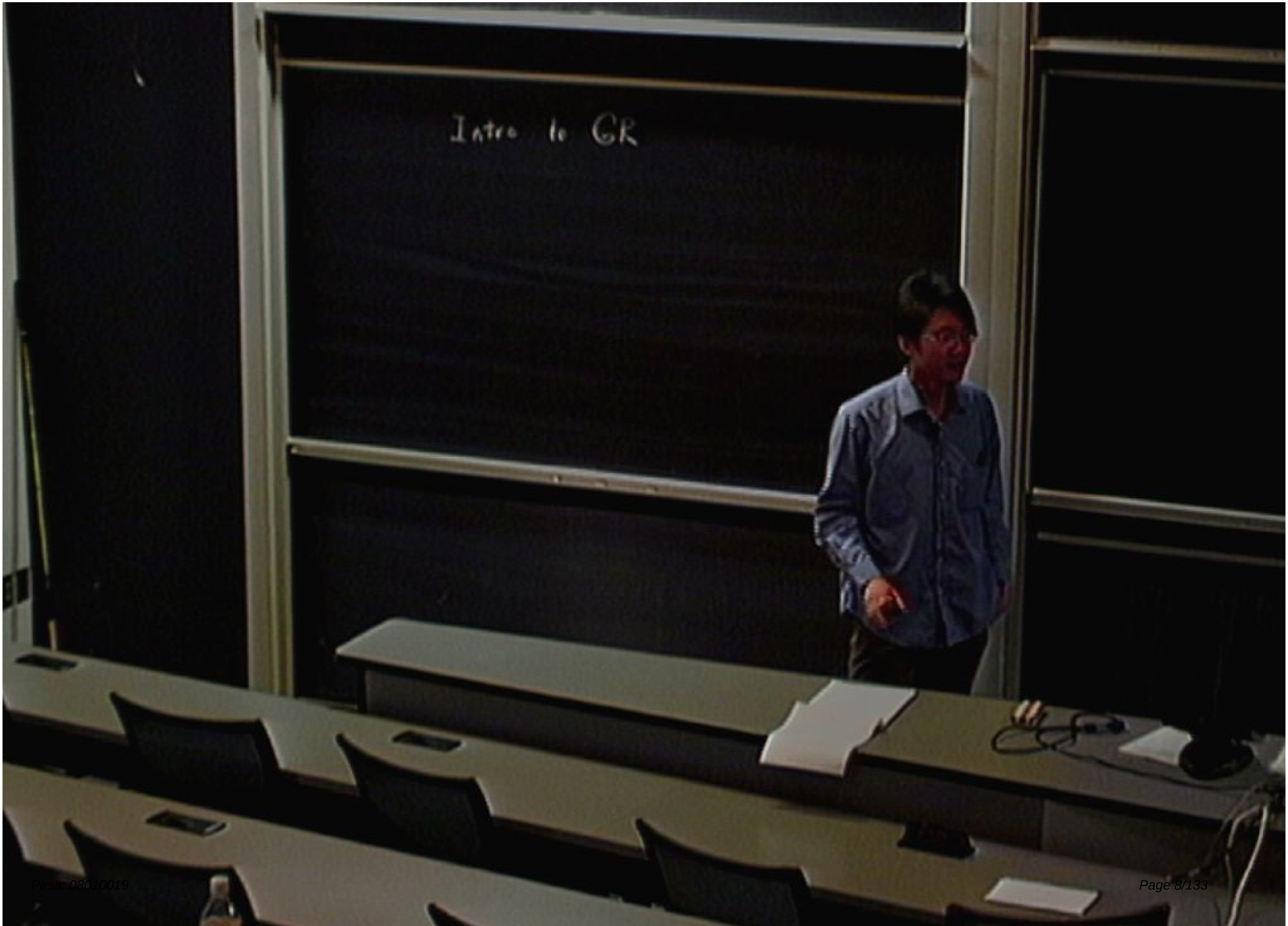
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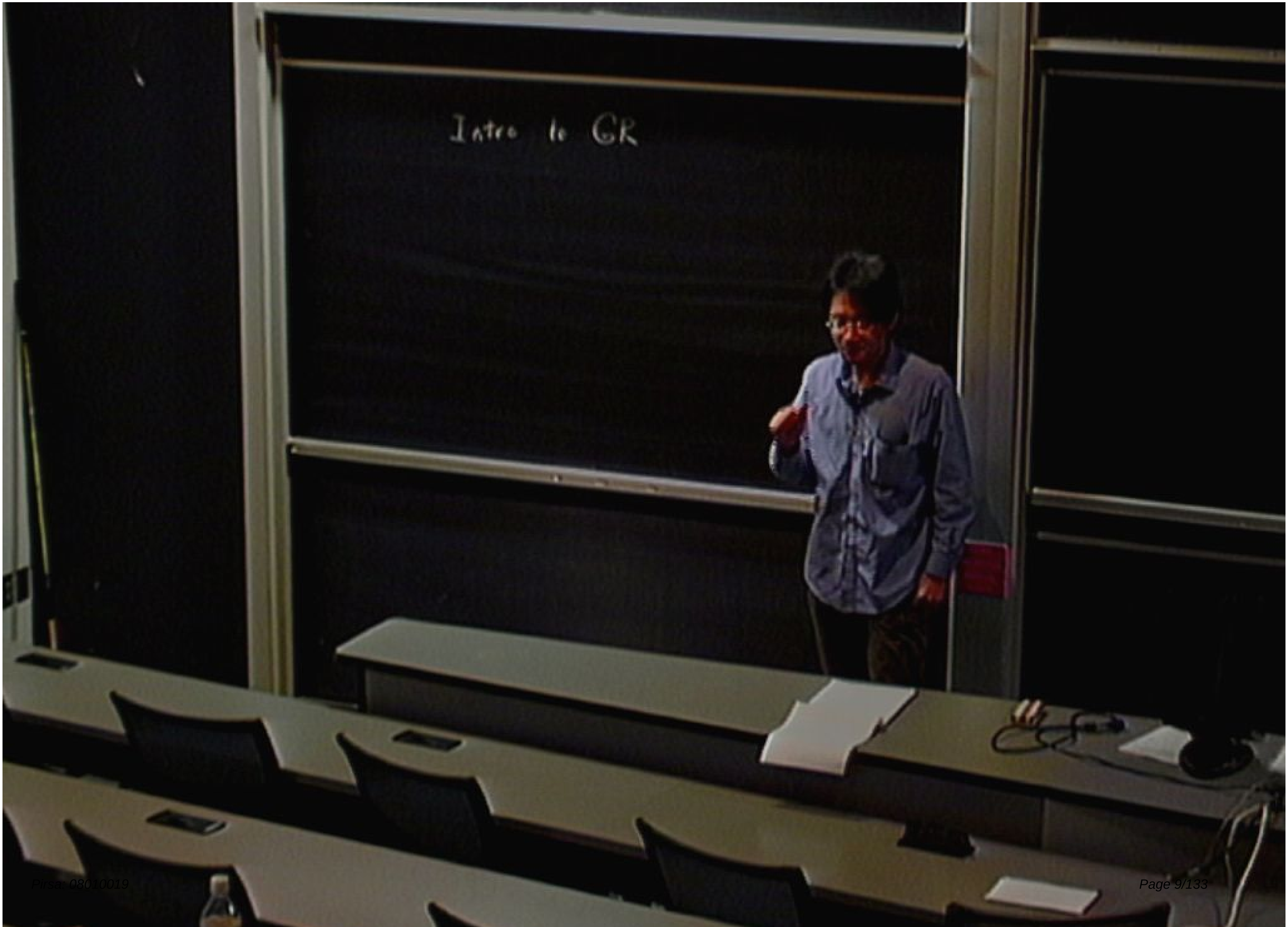
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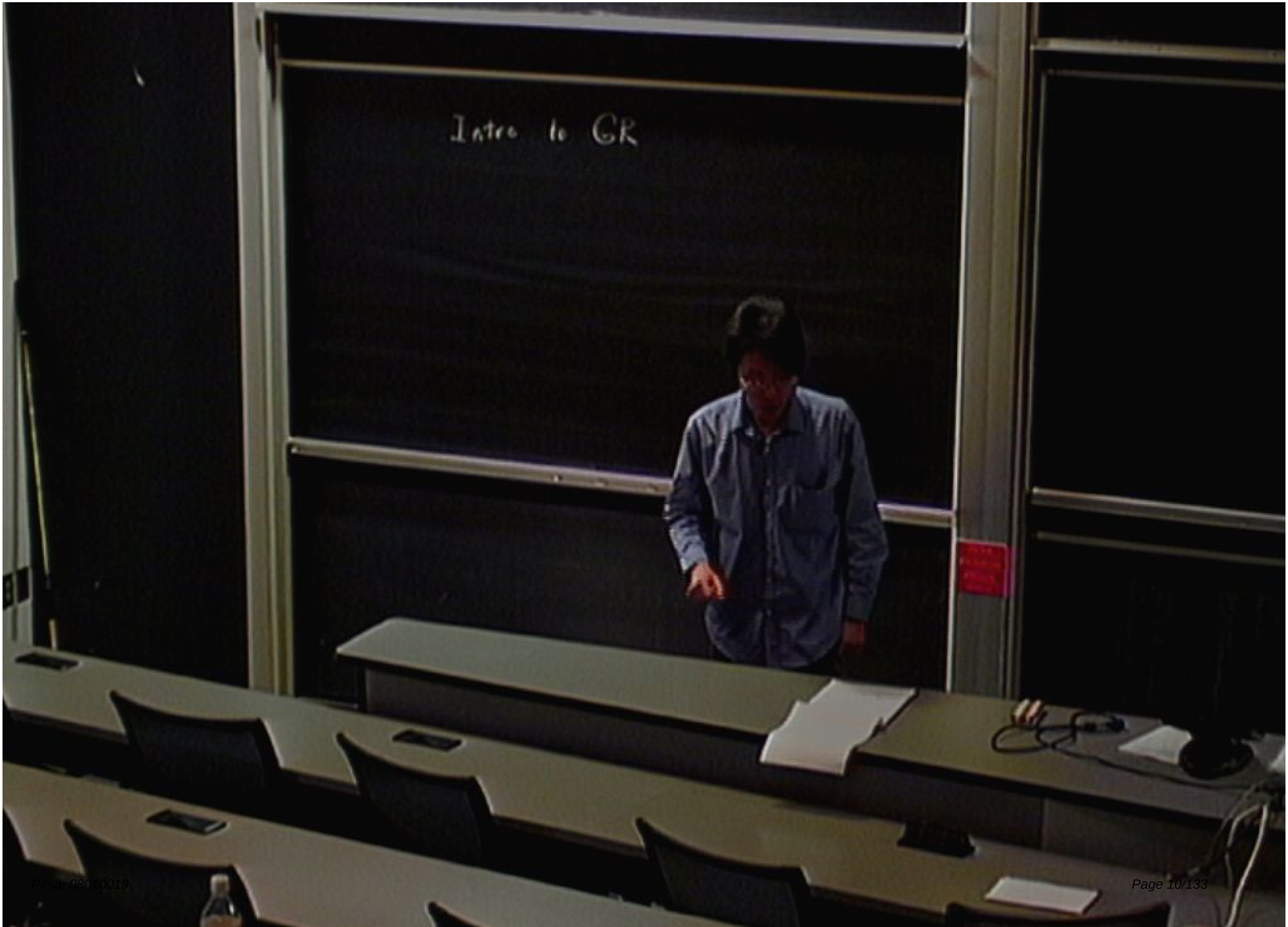
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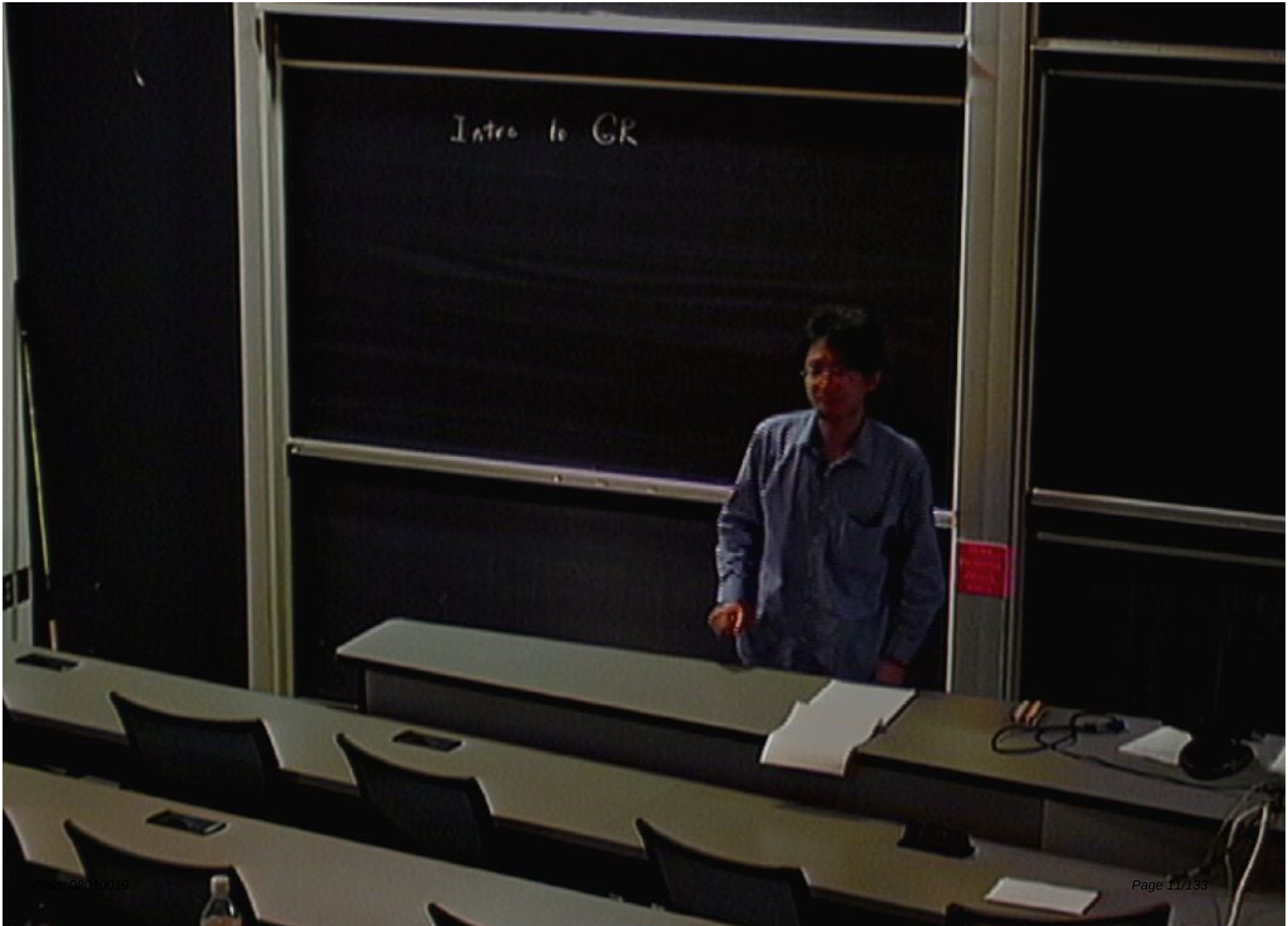
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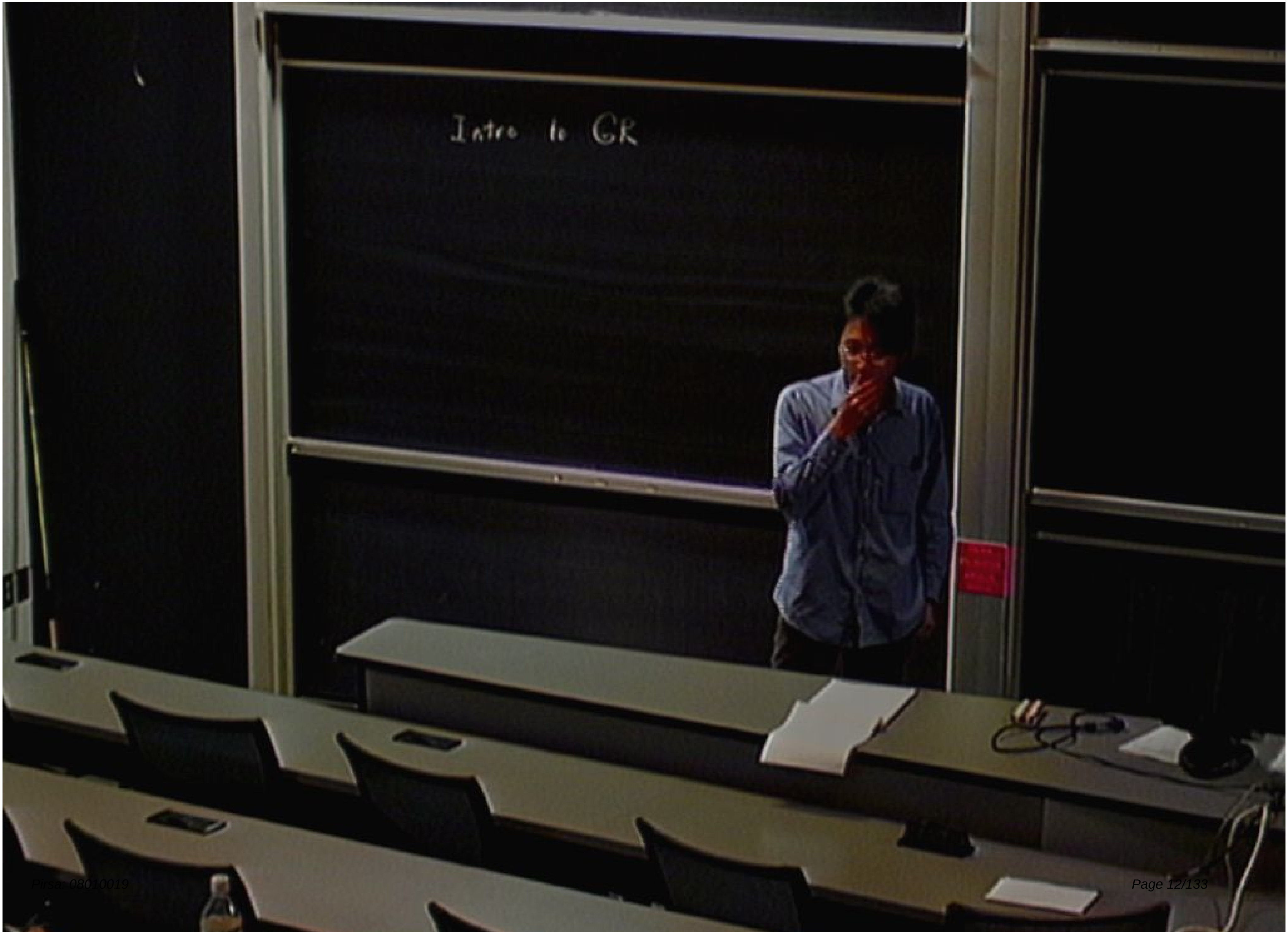
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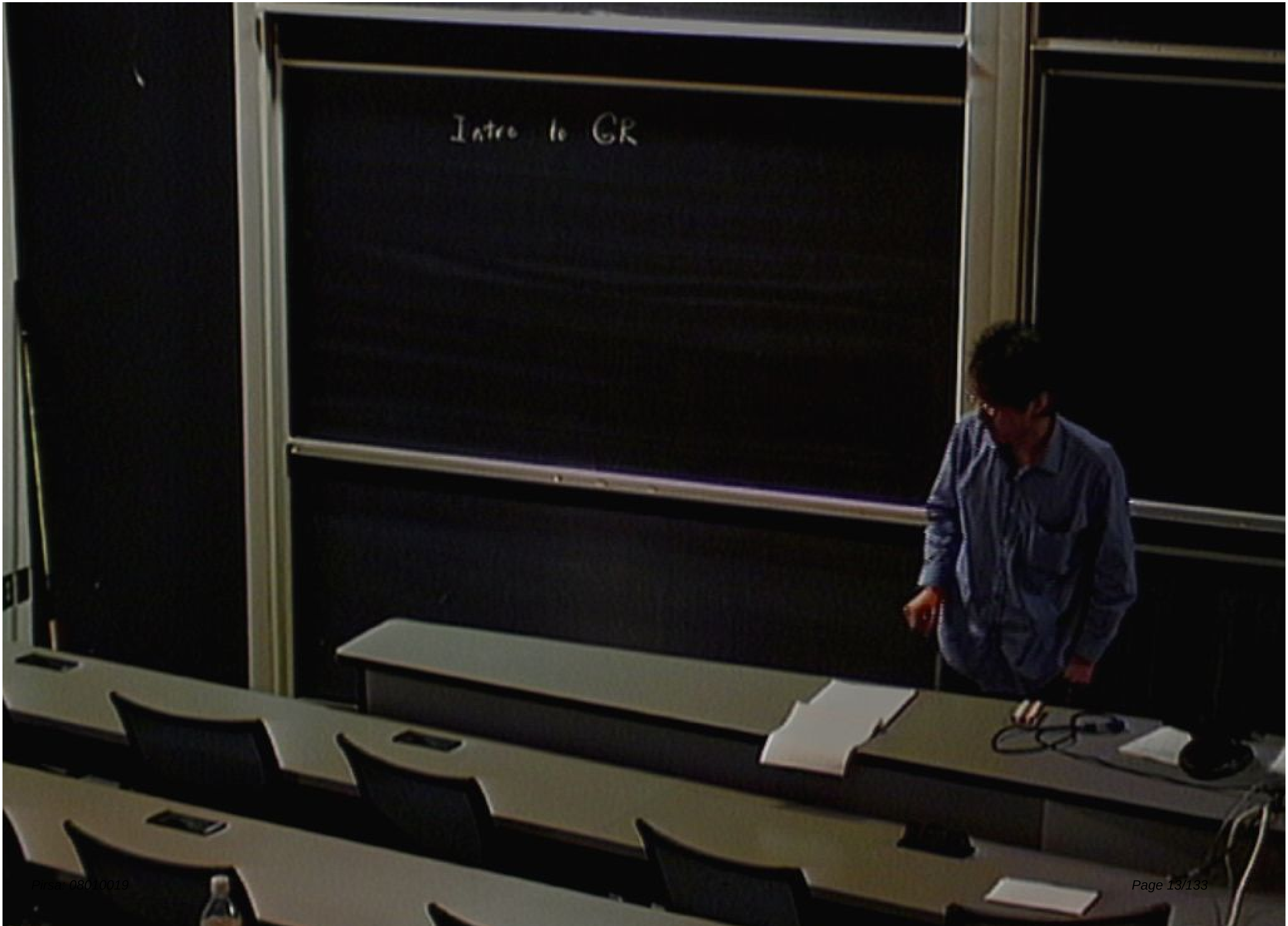
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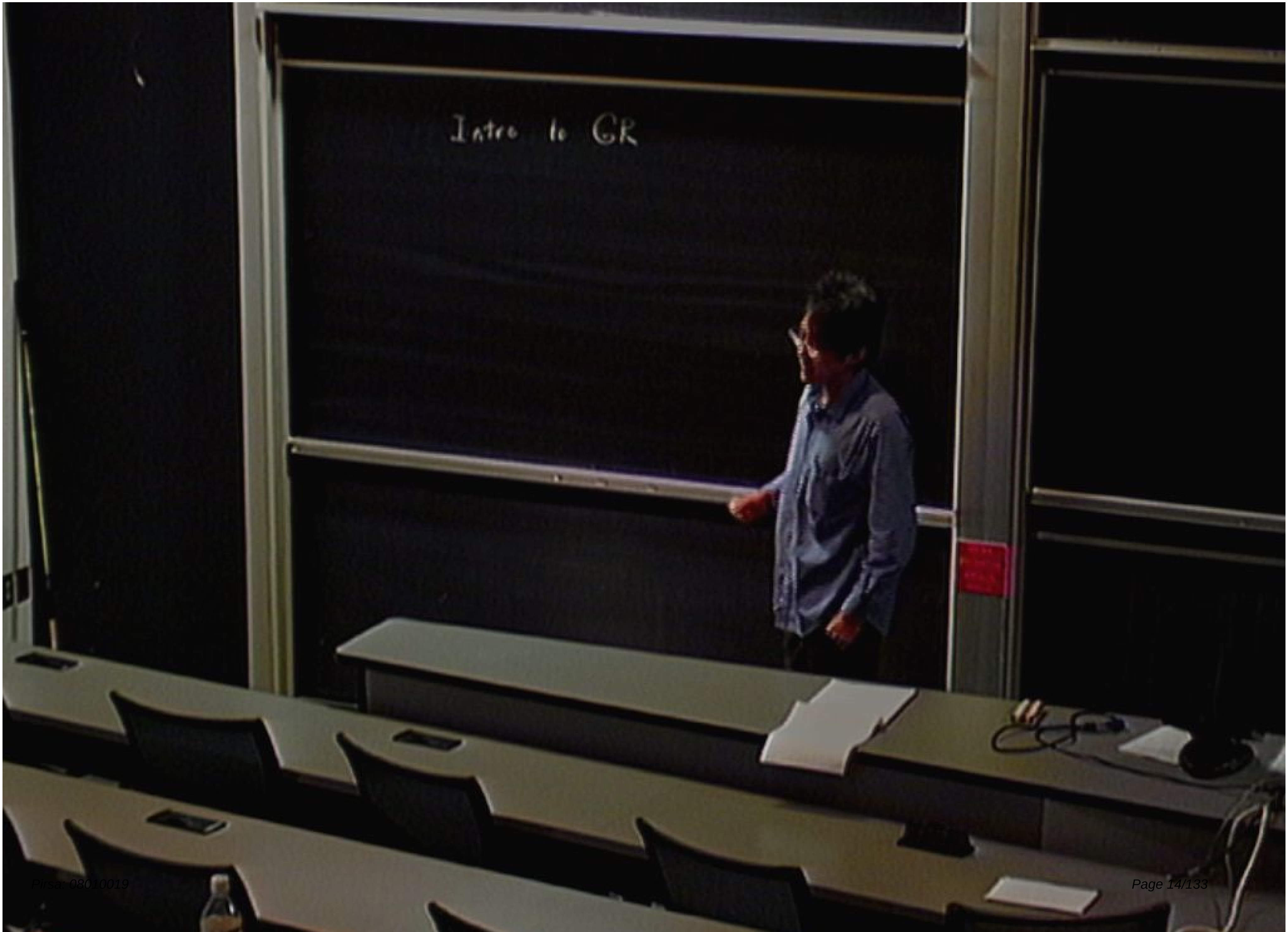
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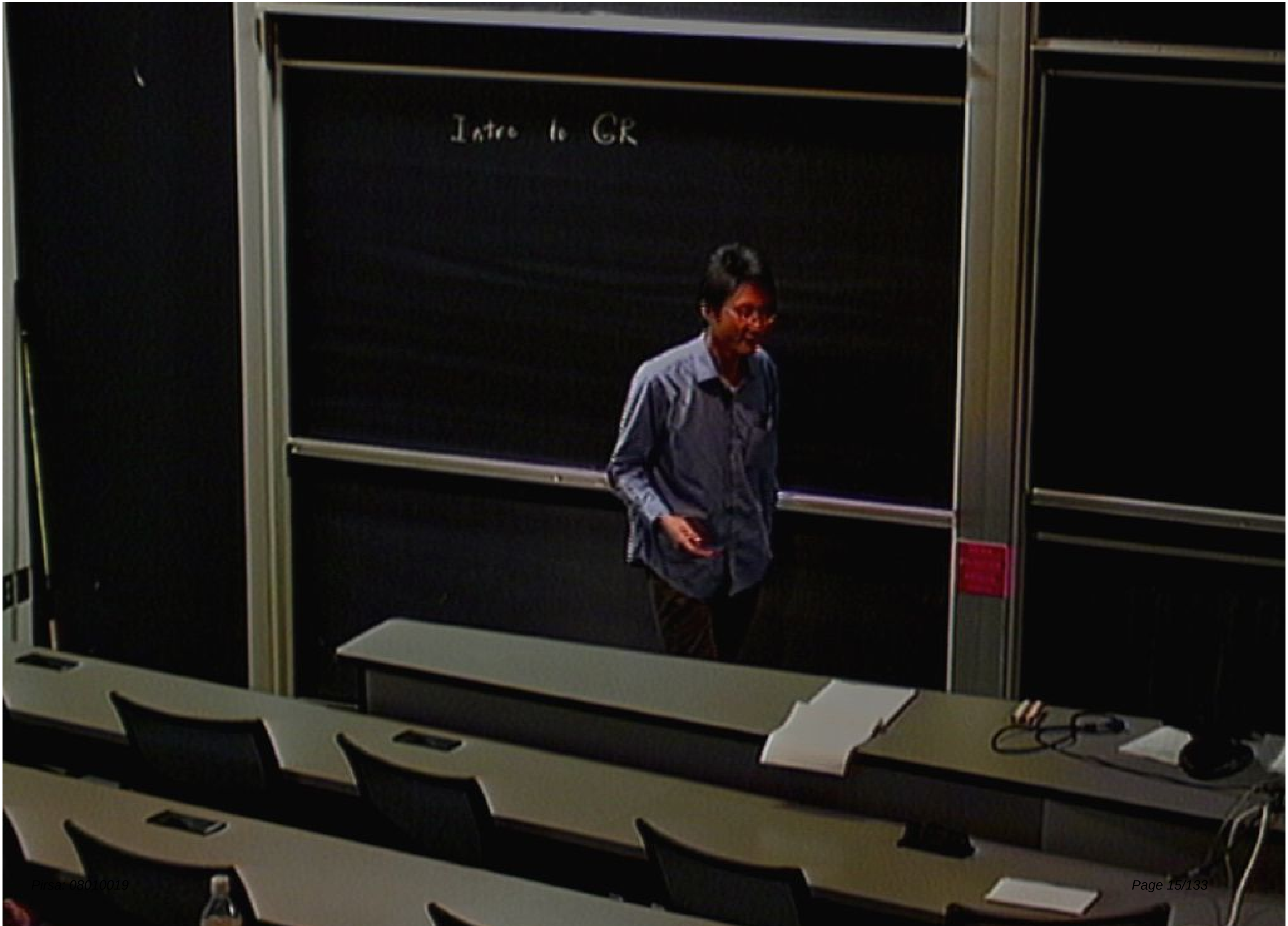
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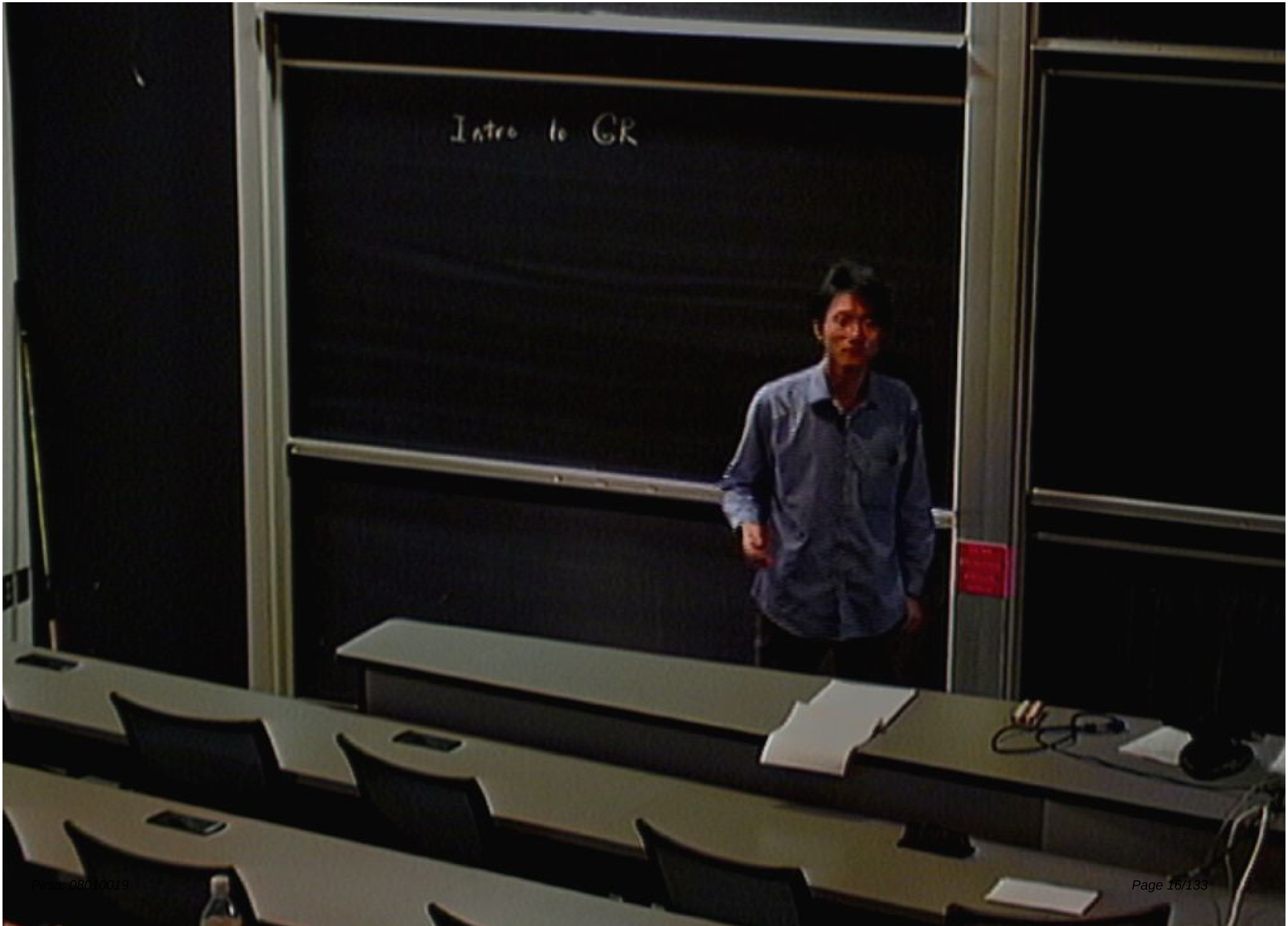
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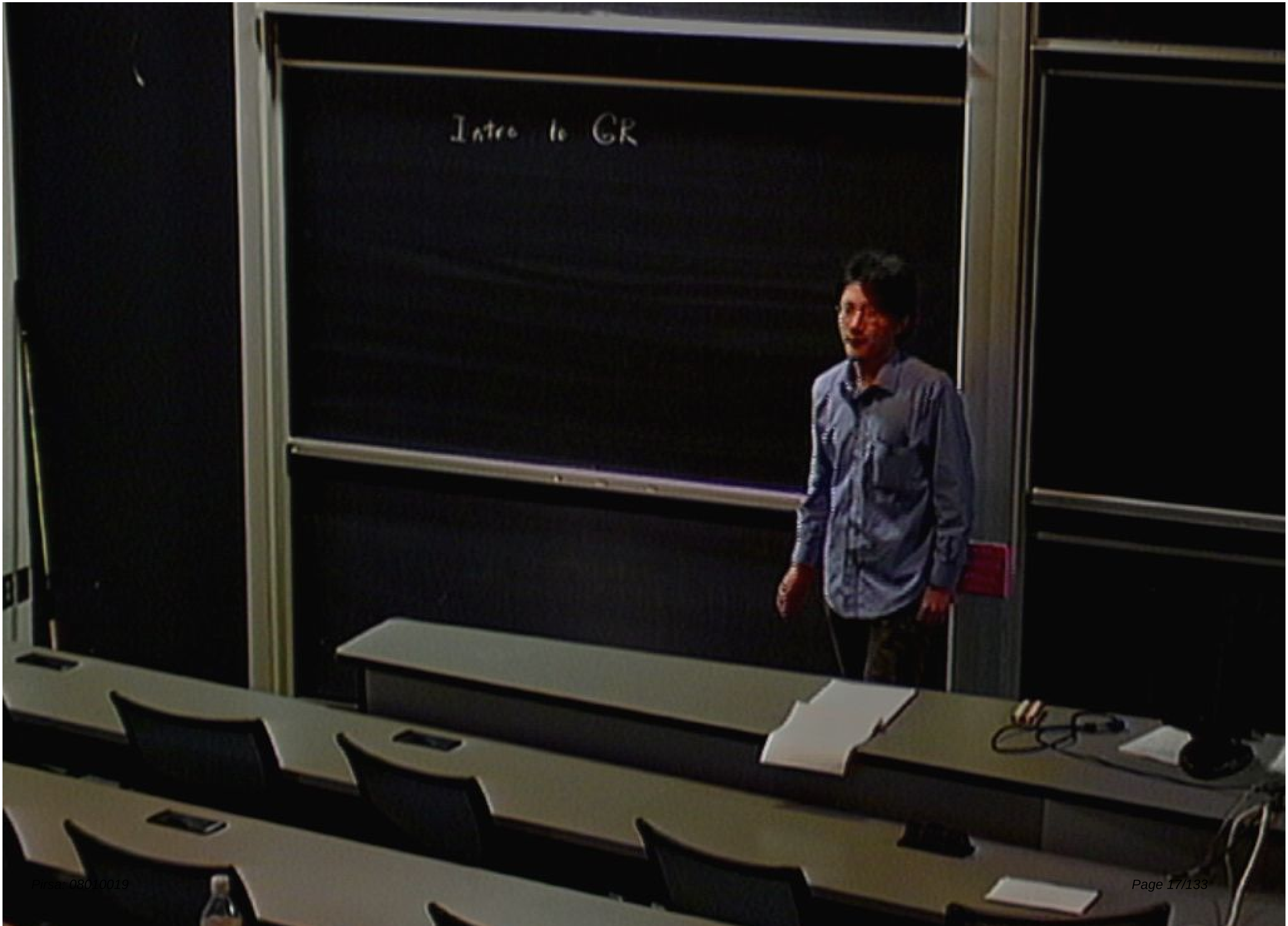
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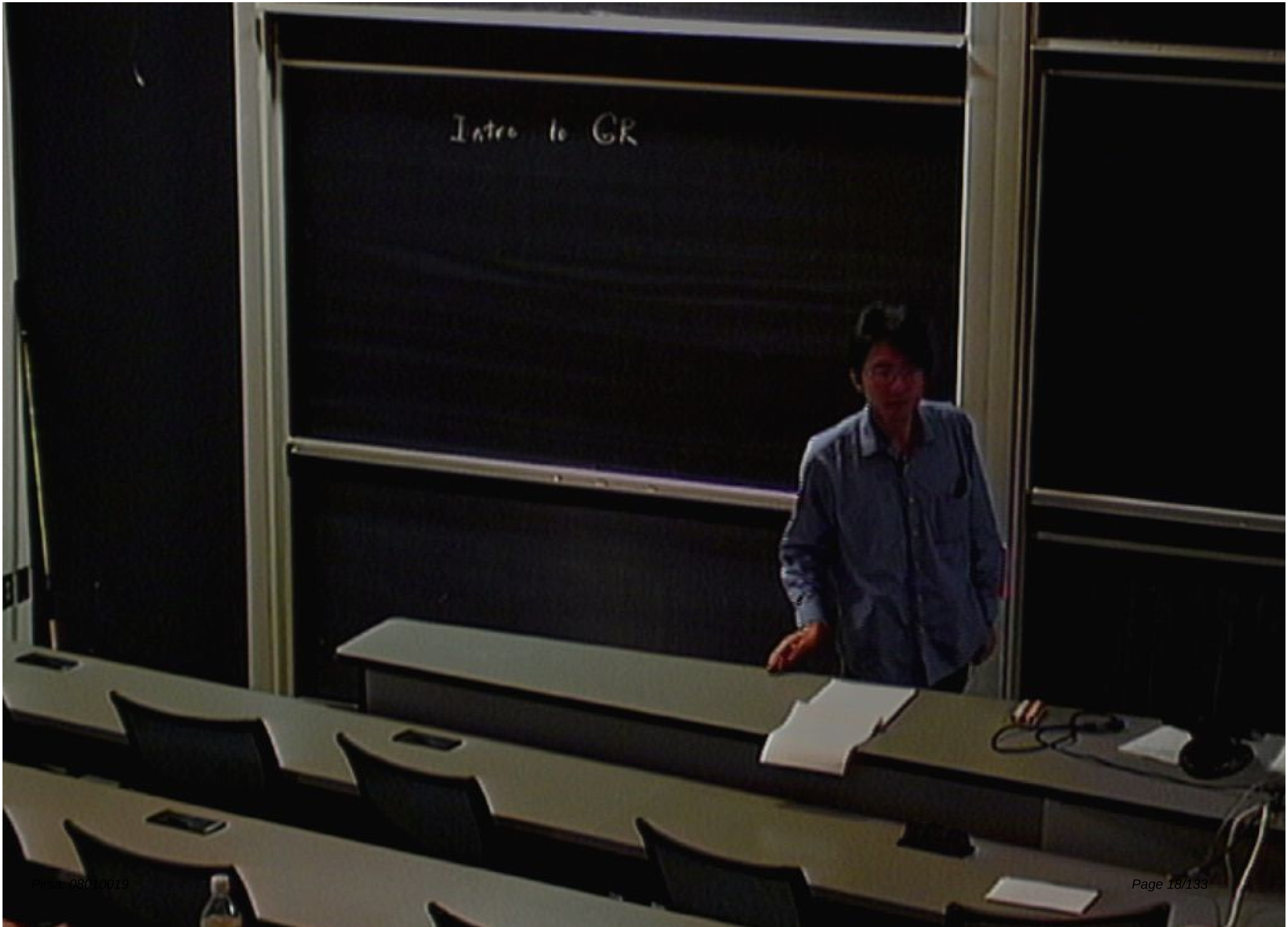
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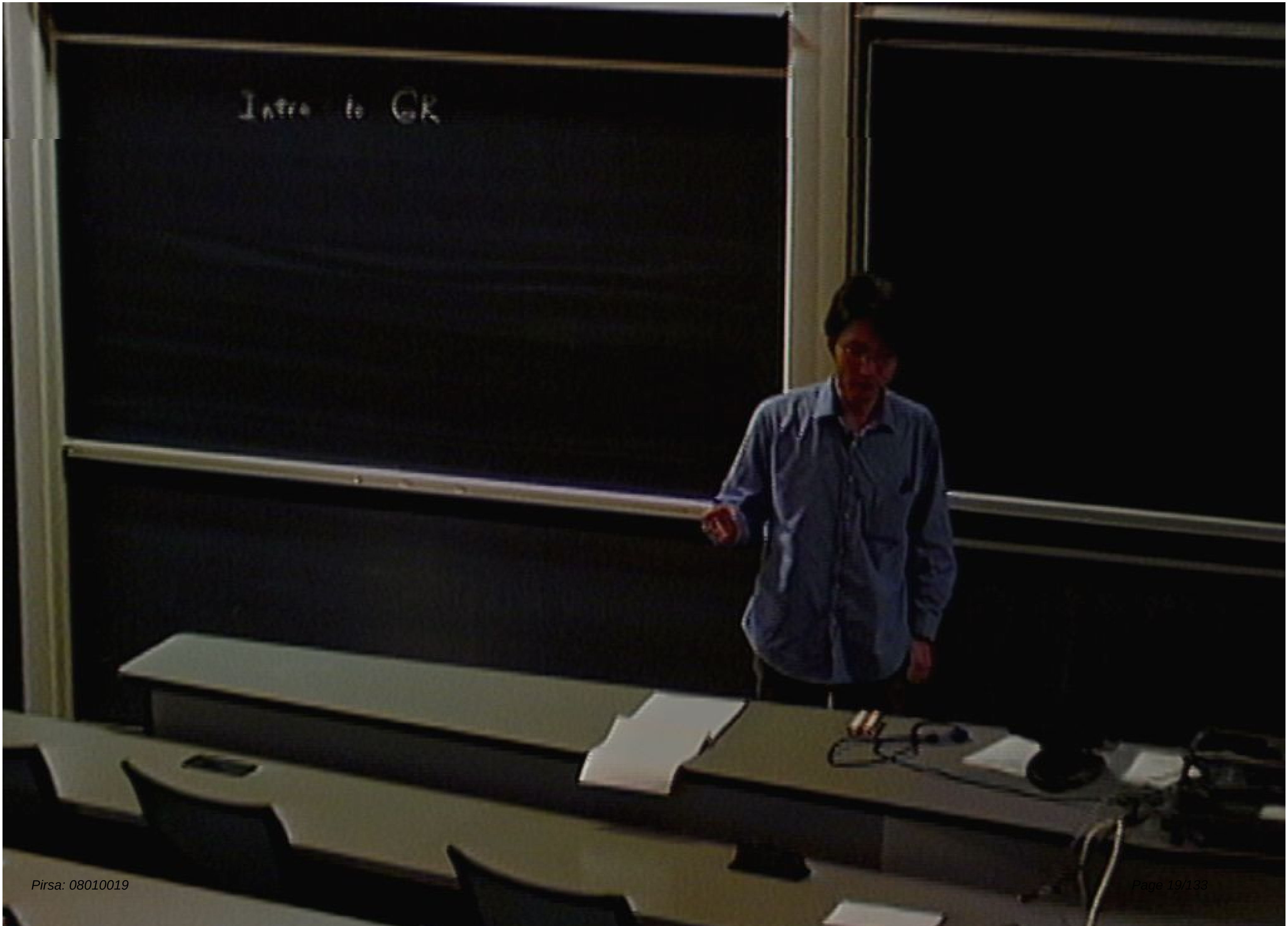
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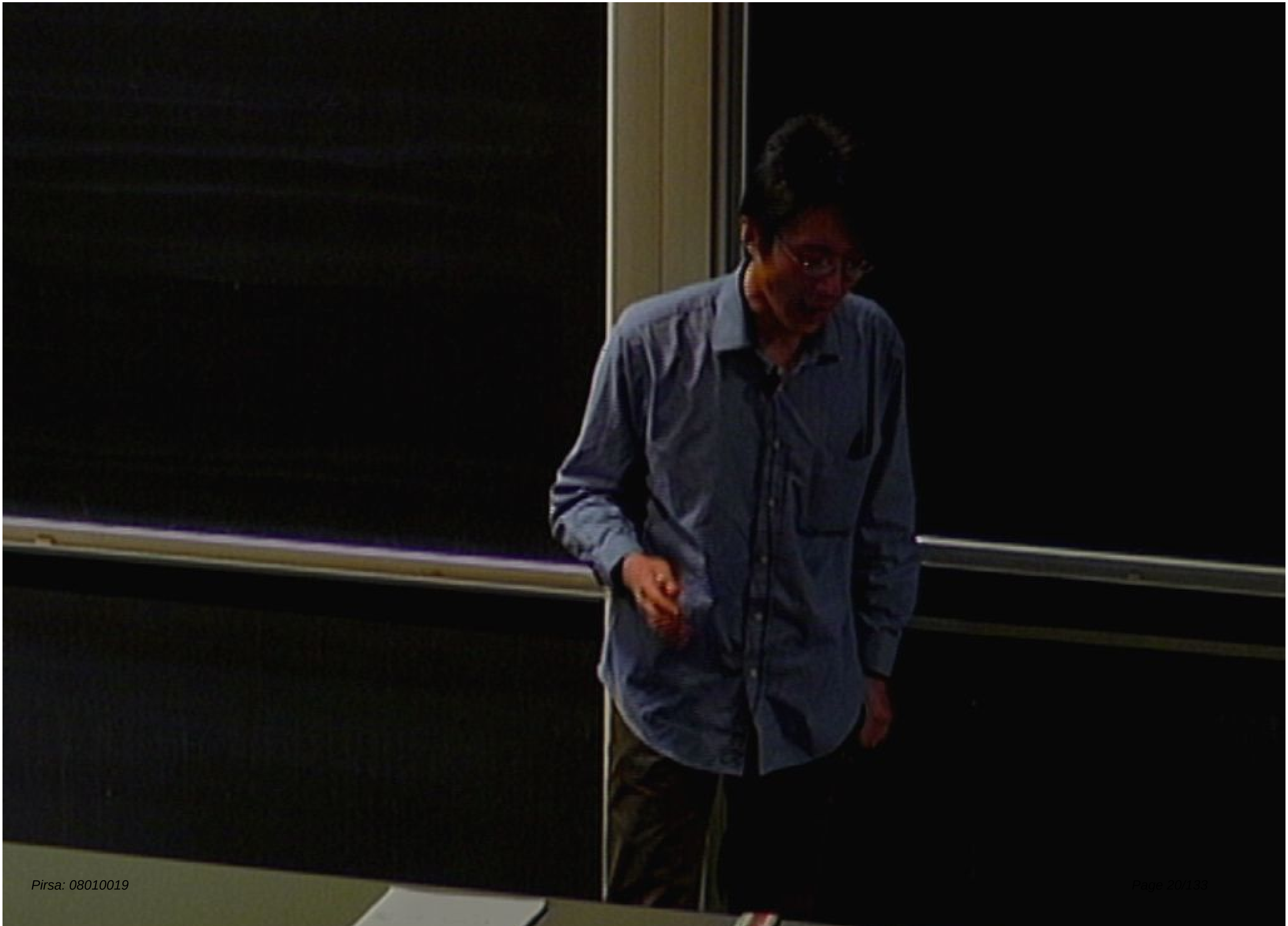

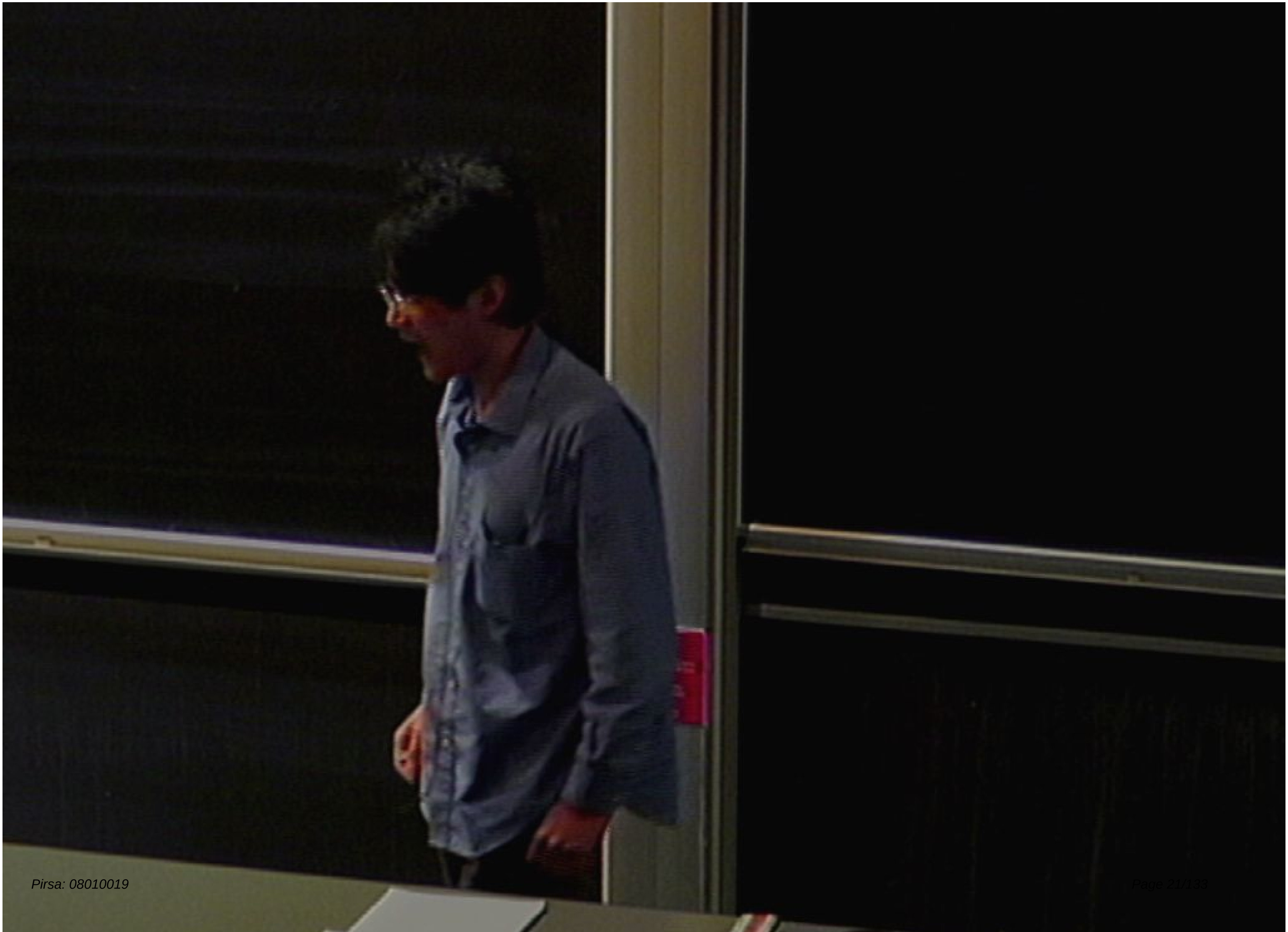

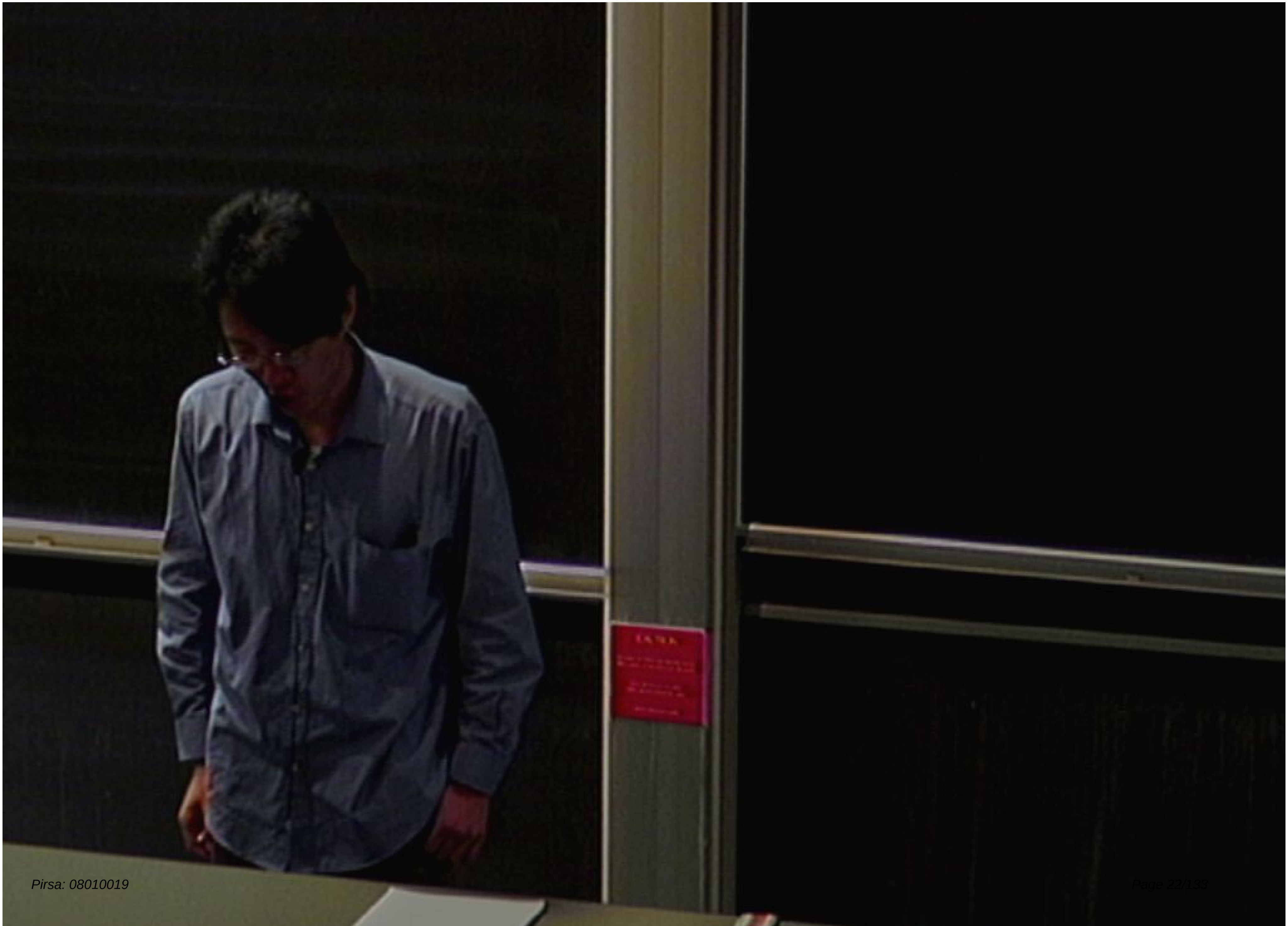

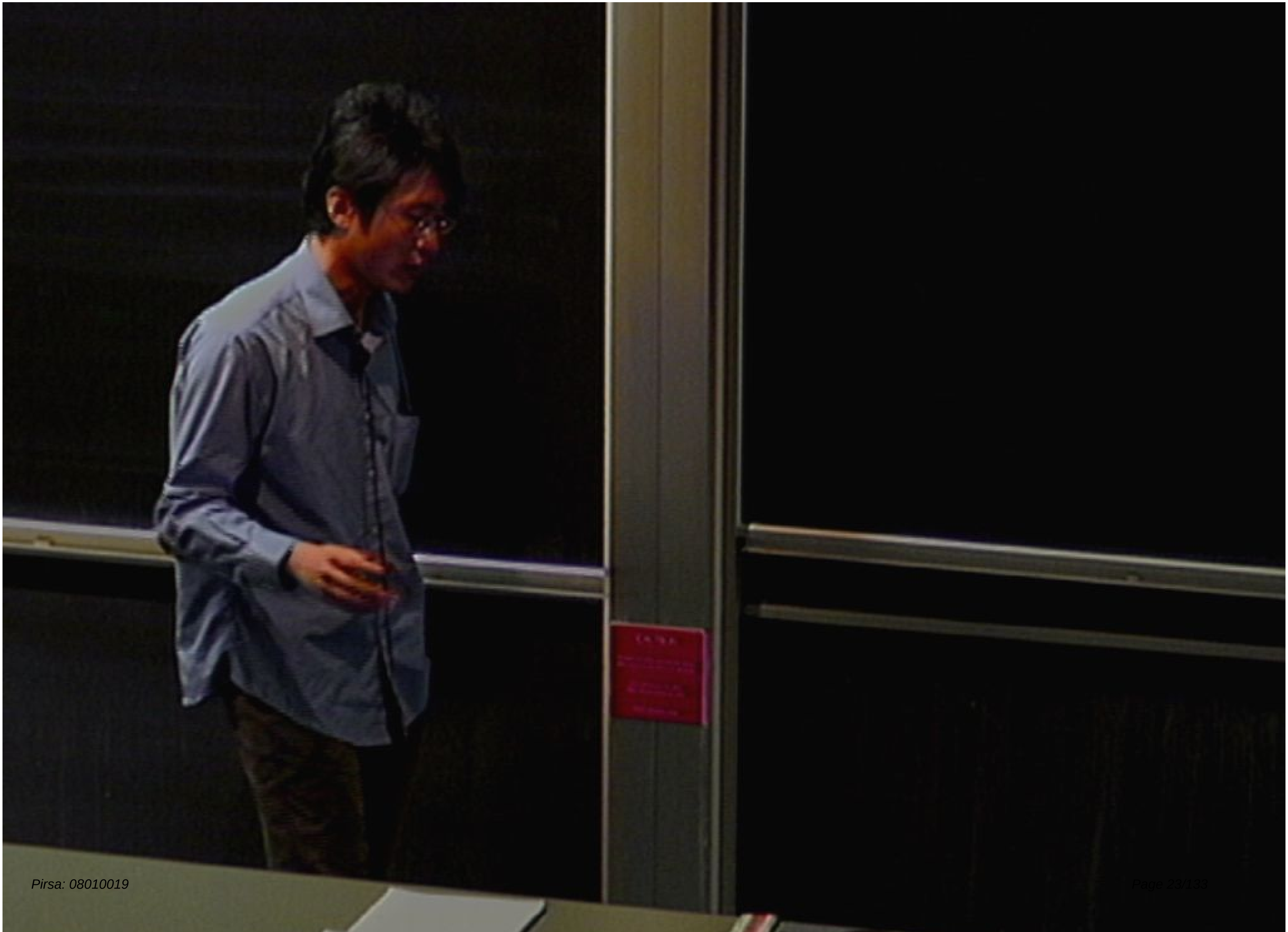

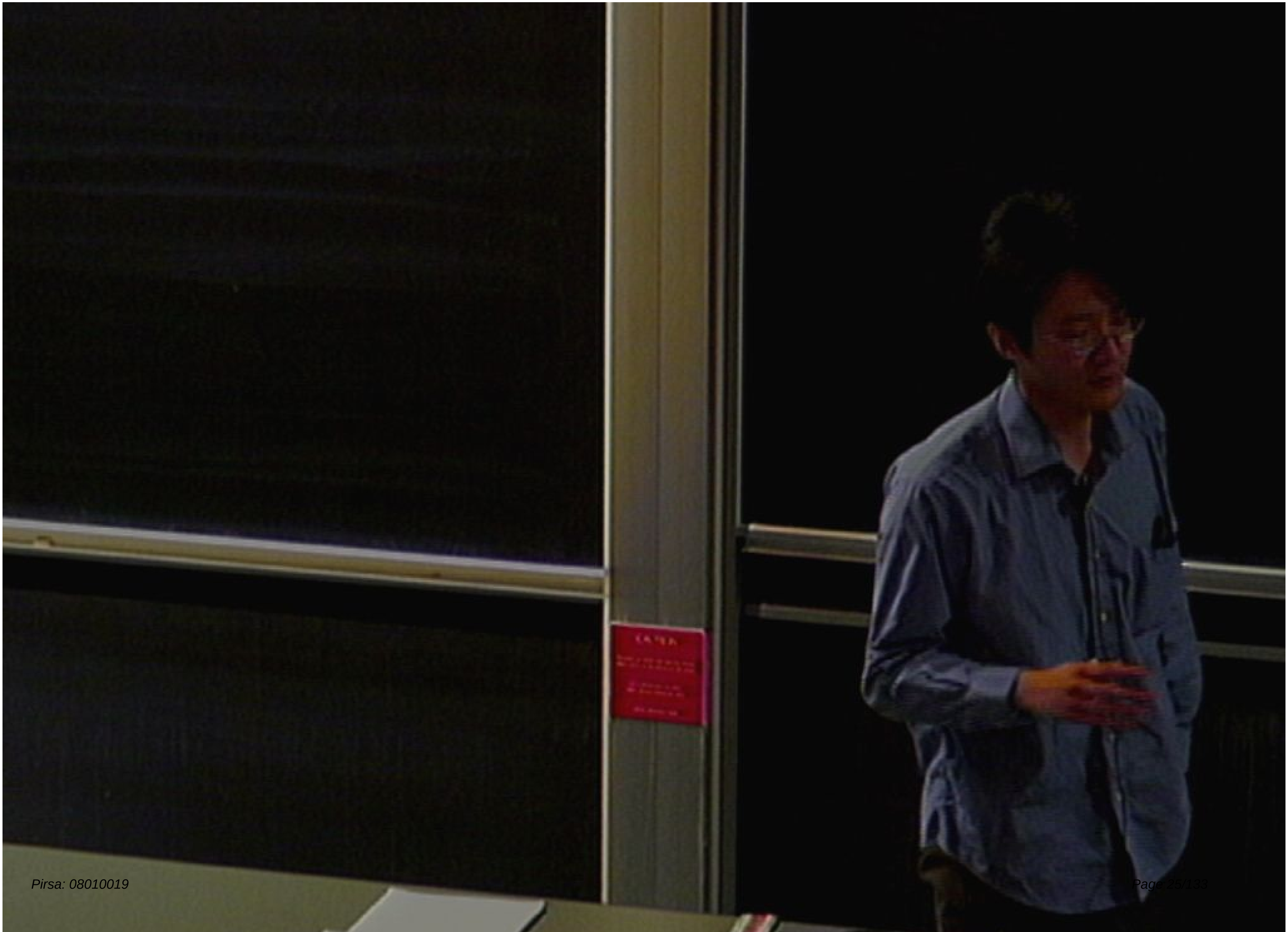

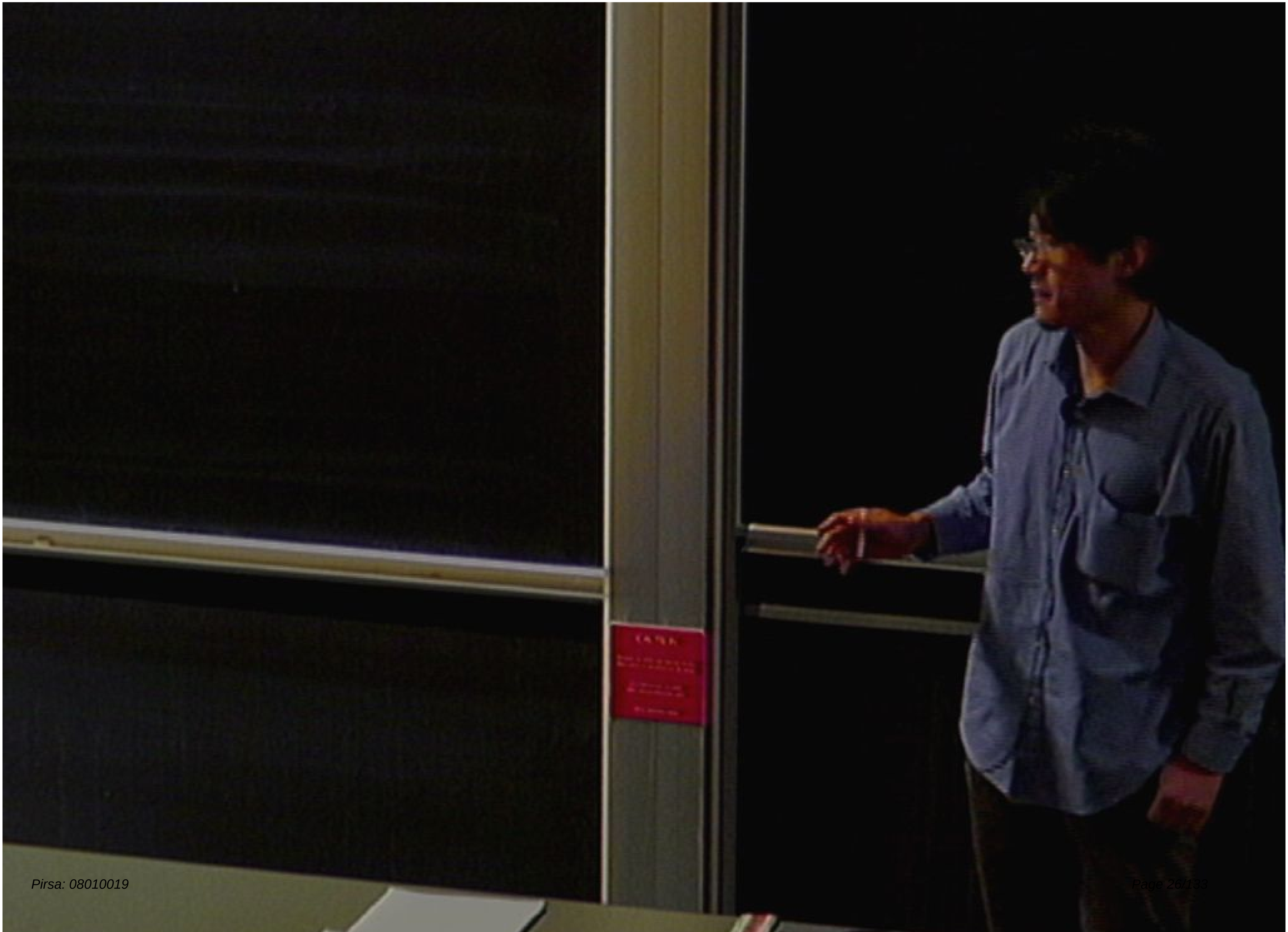

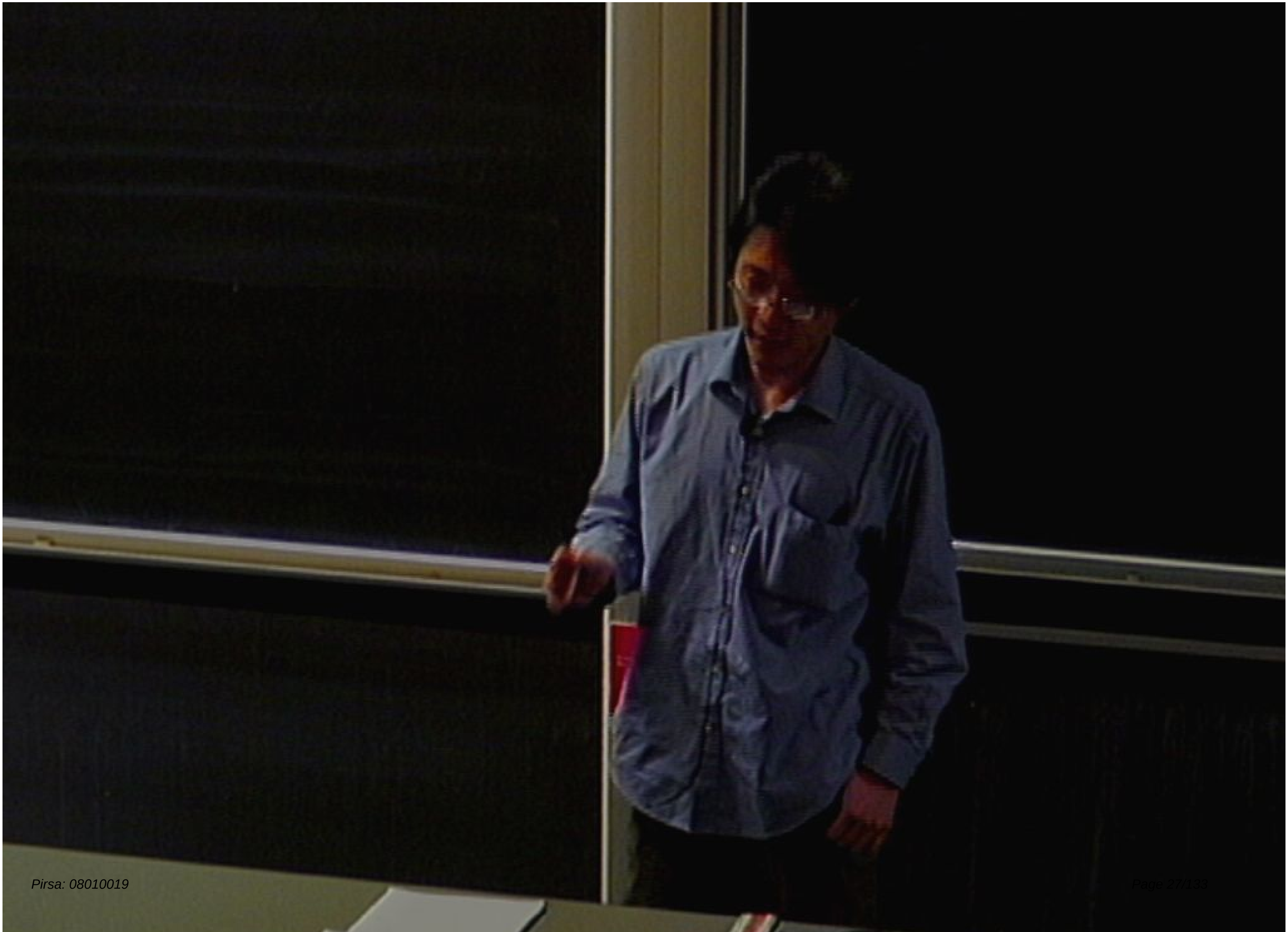

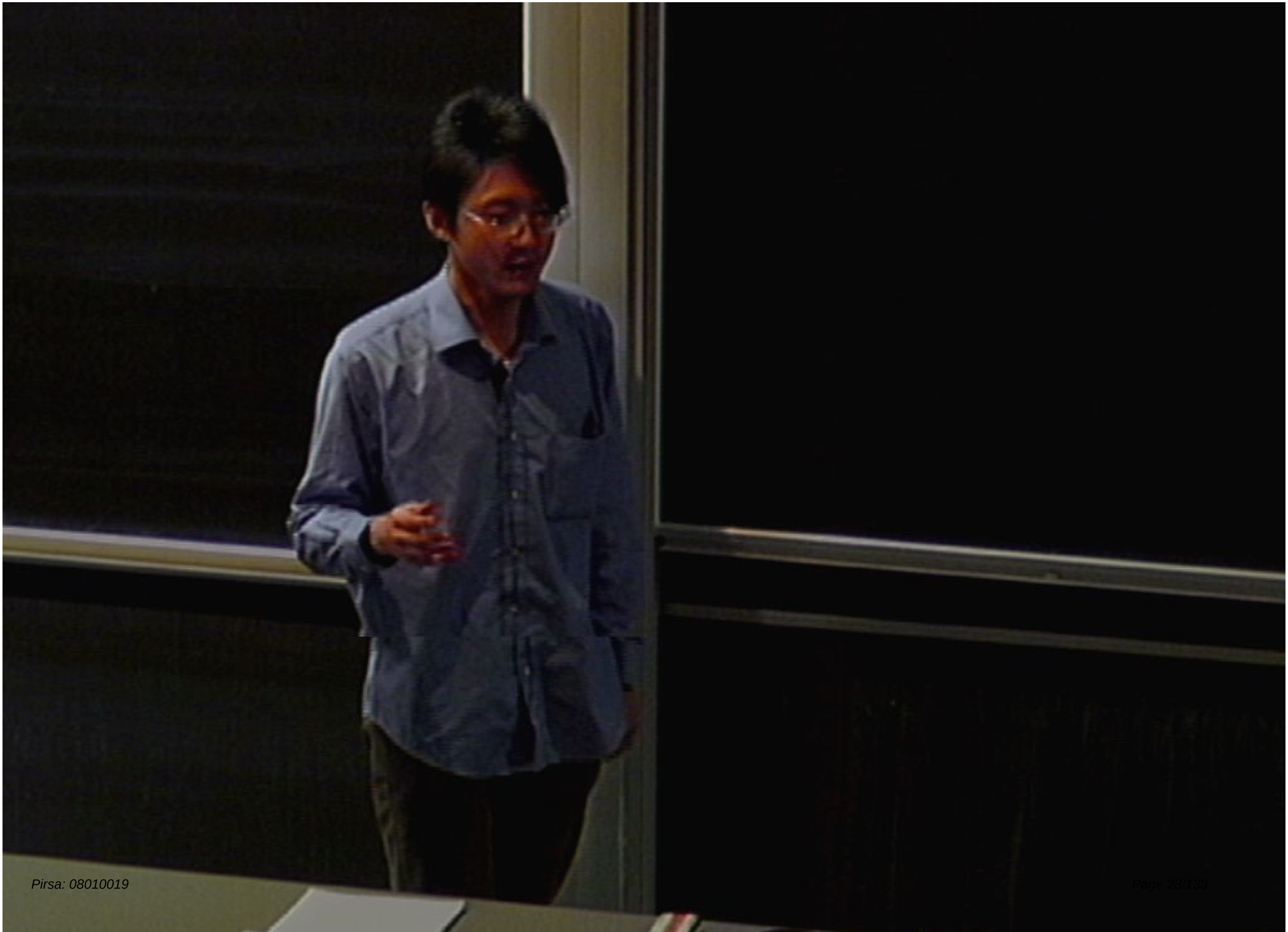

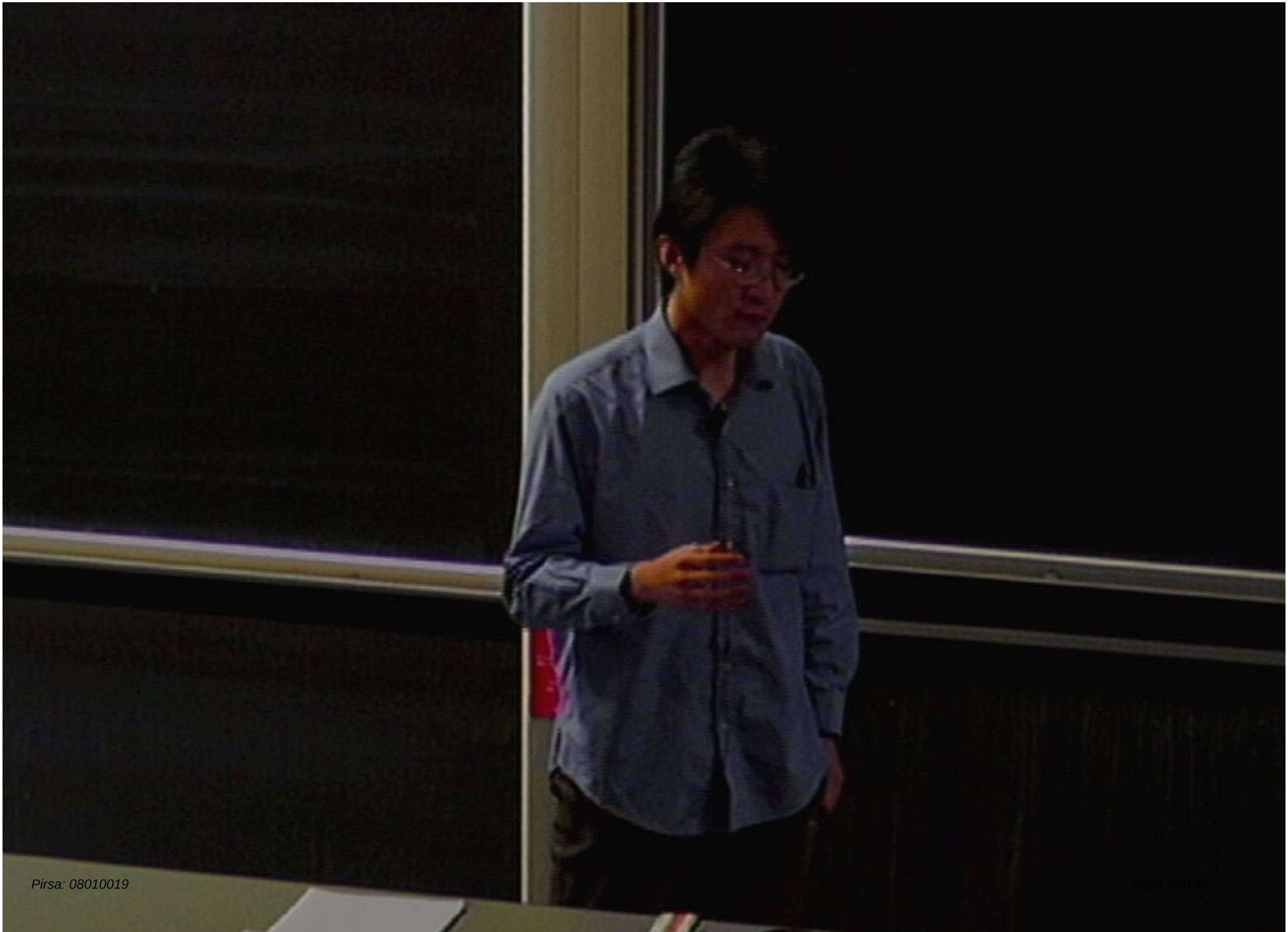

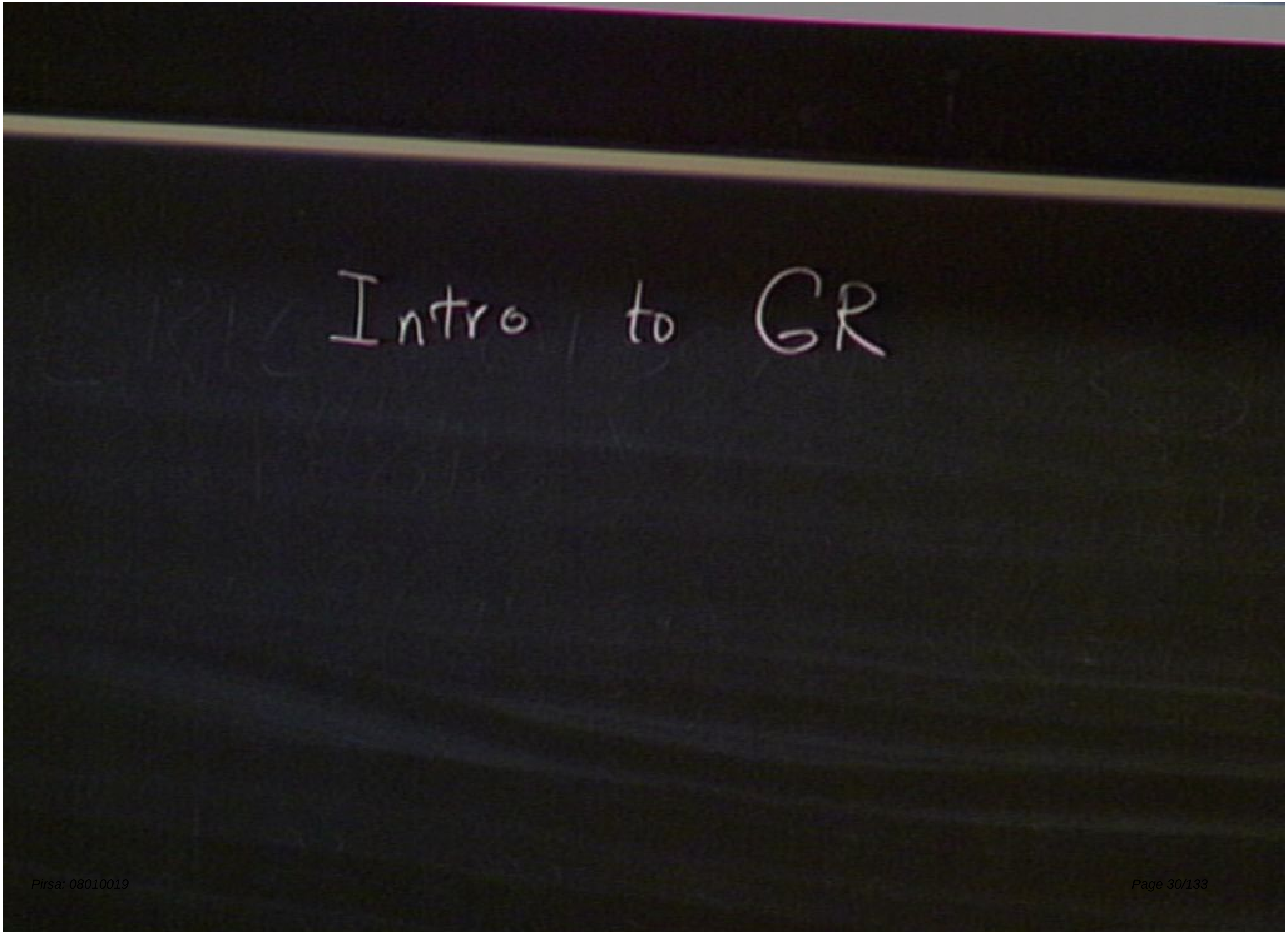
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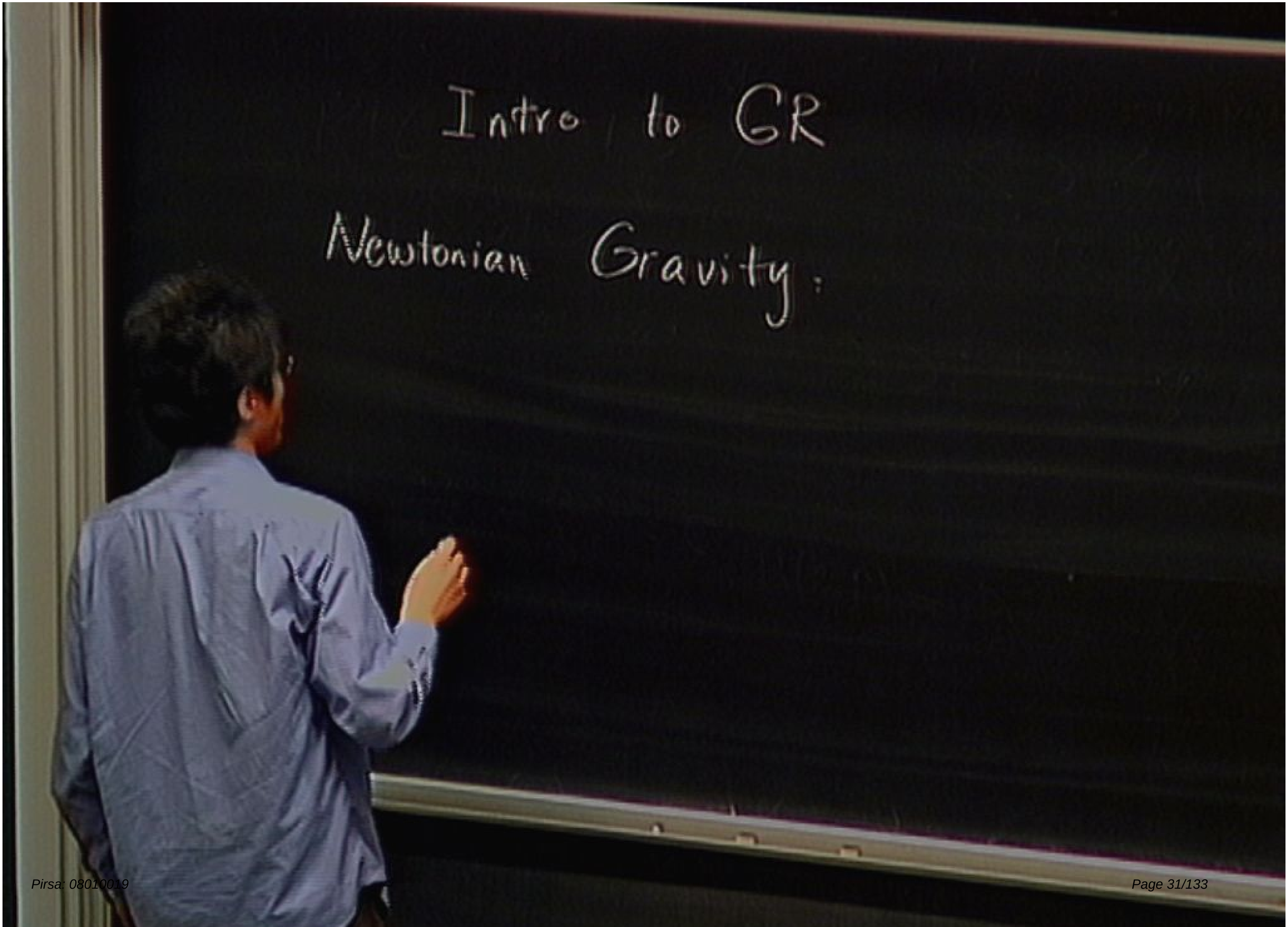
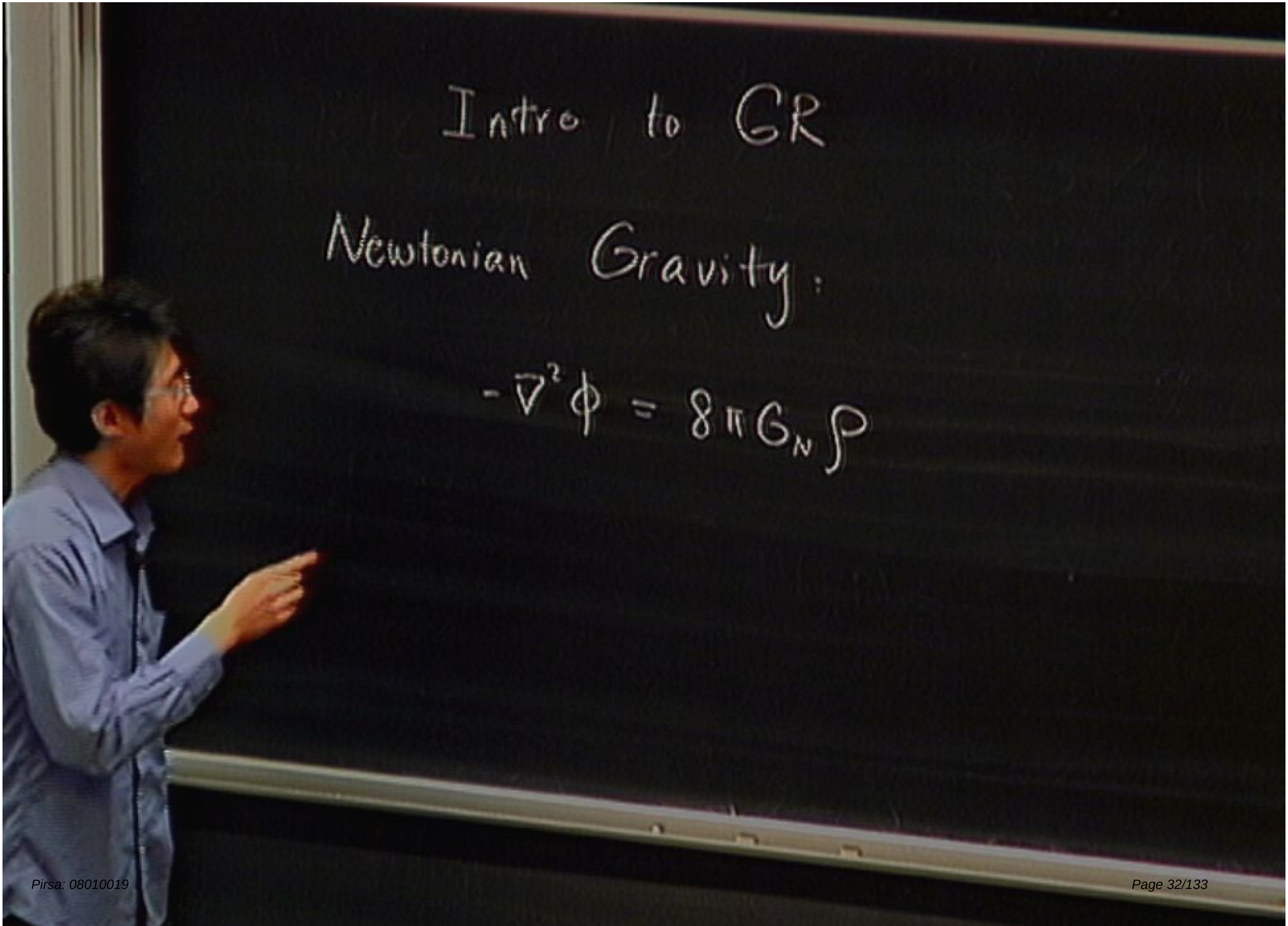

Intro to GR
Newtonian Gravity:


Intro to GR
Newtonian Gravity:
$-\nabla^2\phi = 8\pi G_N \rho$

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$$\Box$$

$$\frac{\partial^2}{\partial t^2} - \nabla^2$$

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$$\Box \phi = 8\pi G_N ?$$

$$\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2$$

Newtonian Gravity:

$$-\nabla^2 \phi = 8\pi G_N \rho$$

$$\downarrow$$

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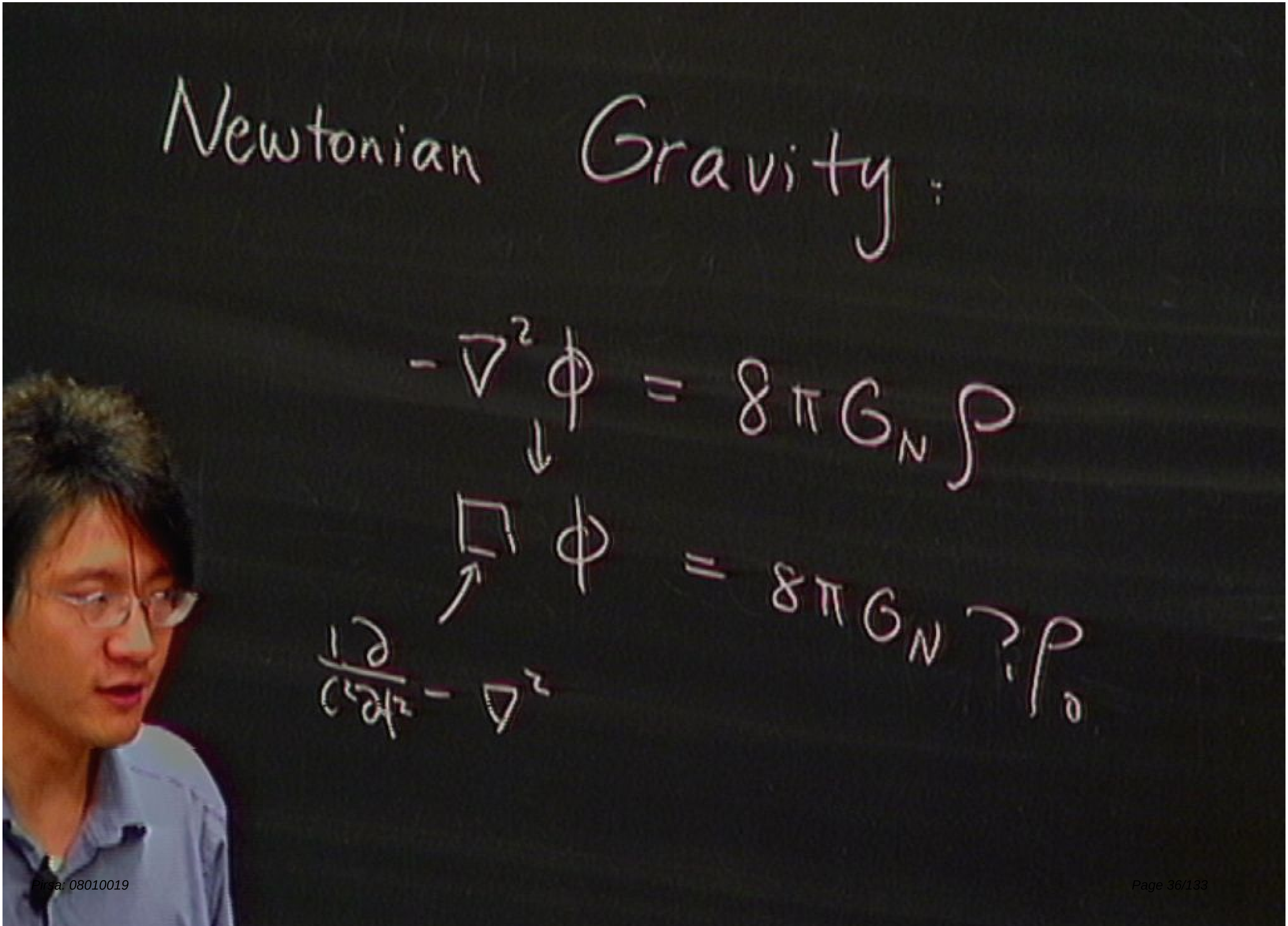
$$\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2$$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$$g_{\mu\nu} \qquad\qquad T_{\mu\nu}$$

Newtonian Gravity:

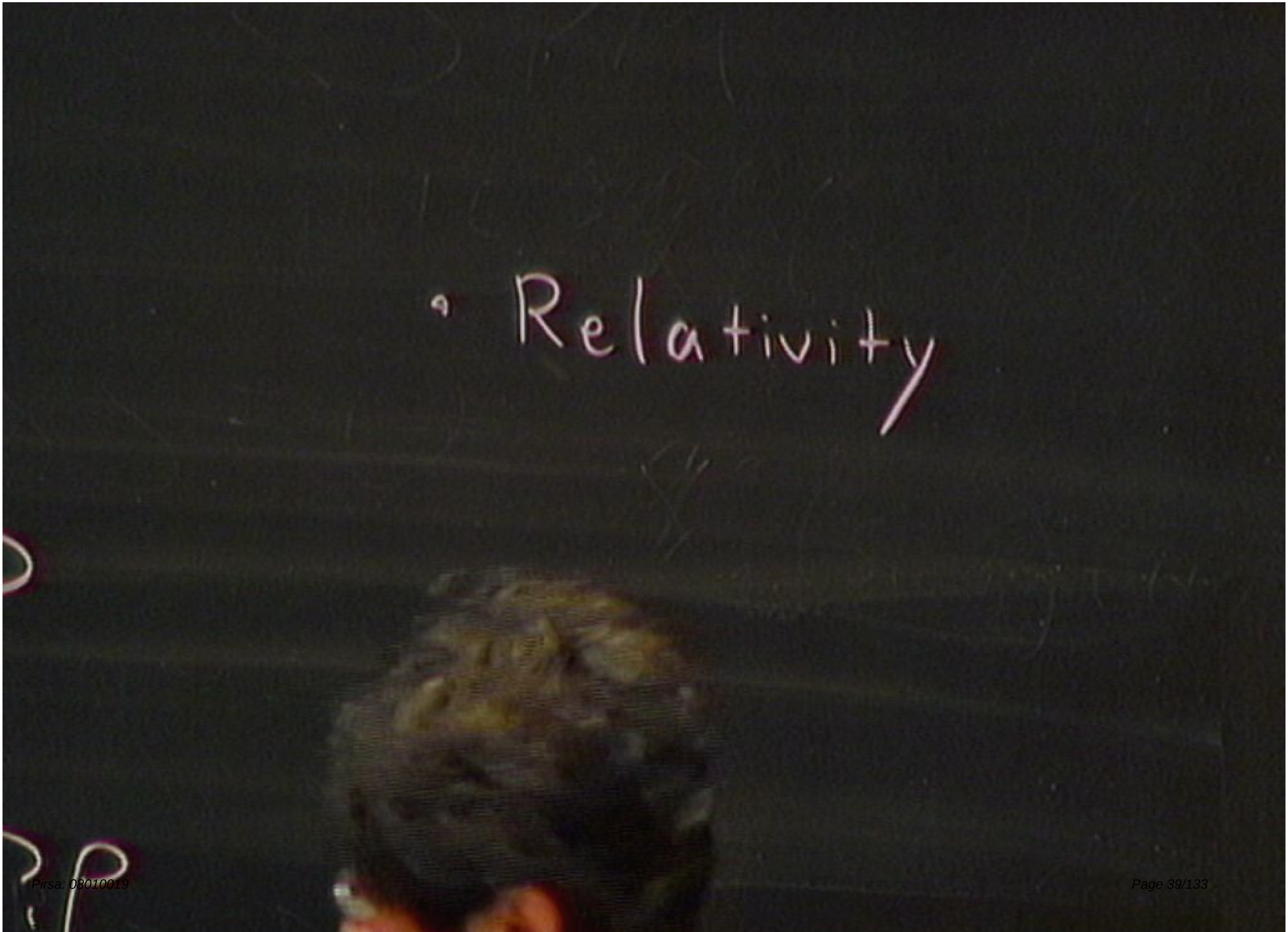
$$-\nabla^2 \phi = 8\pi G_N \rho$$

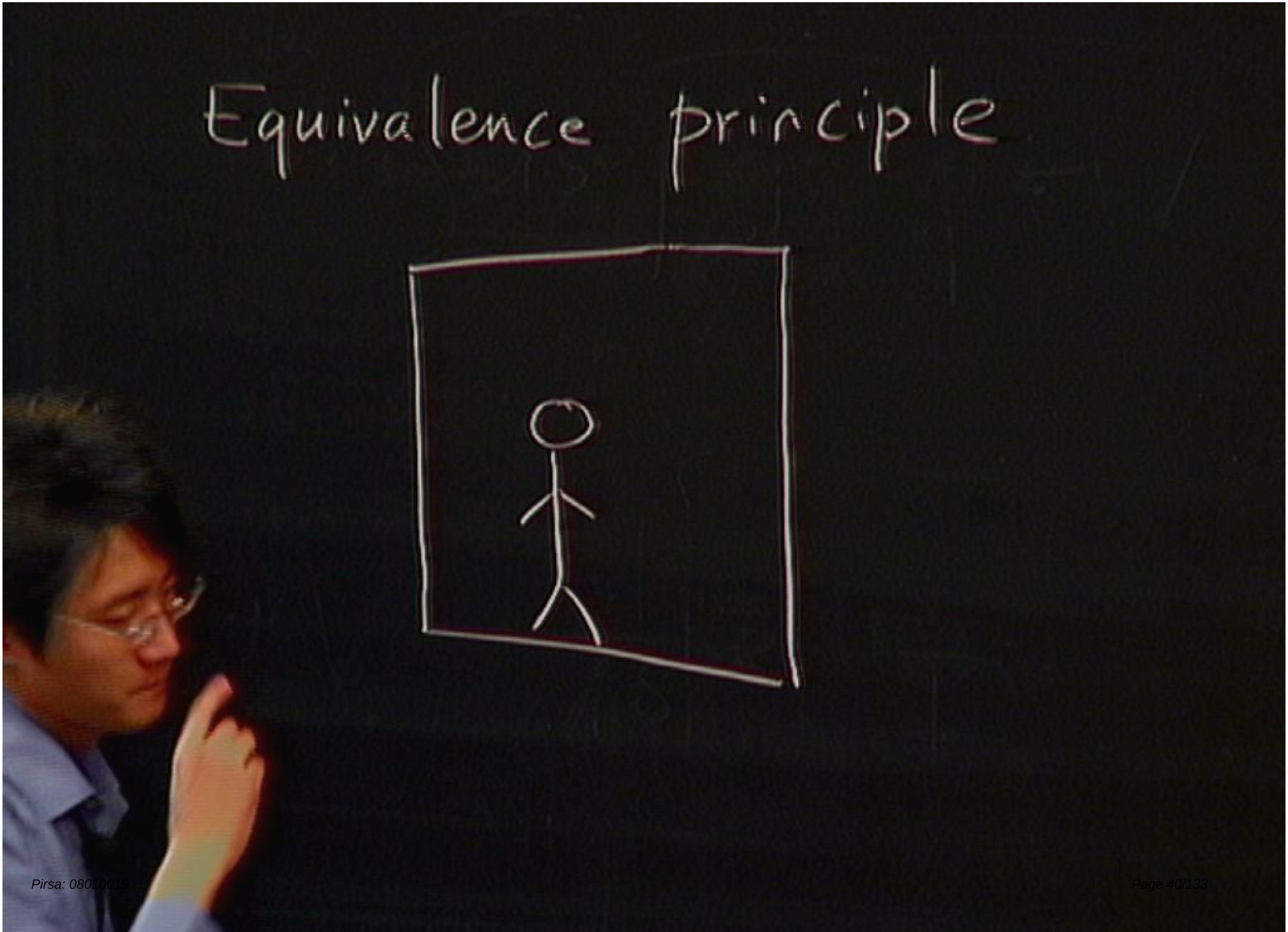
$$\downarrow$$

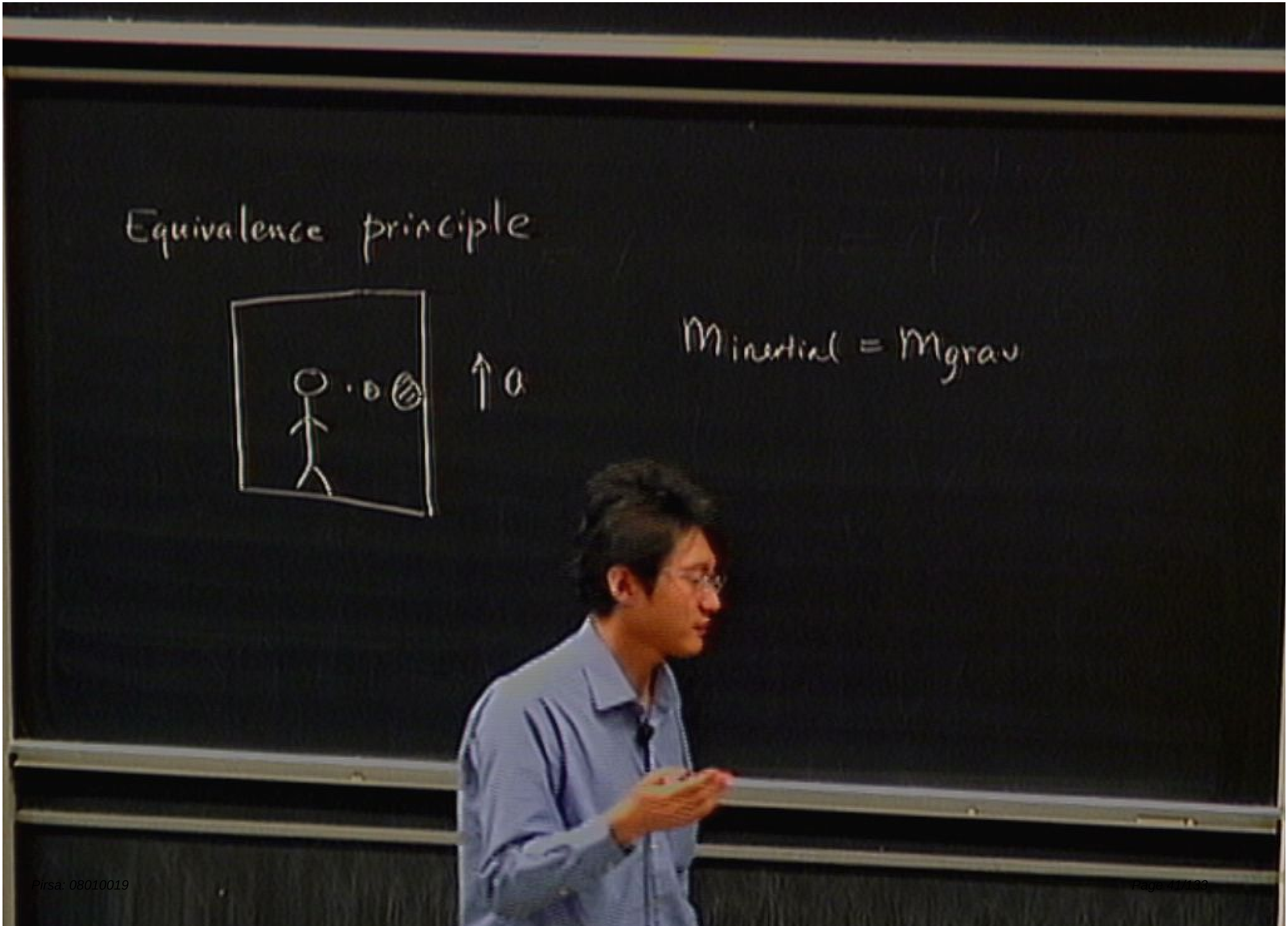
$$\Box \phi = 8\pi G_N \; ?\; \rho_0$$

$$\frac{1\,\partial}{c^2\partial t^2} - \nabla^2 \qquad \phi \to g_{\mu\nu}$$

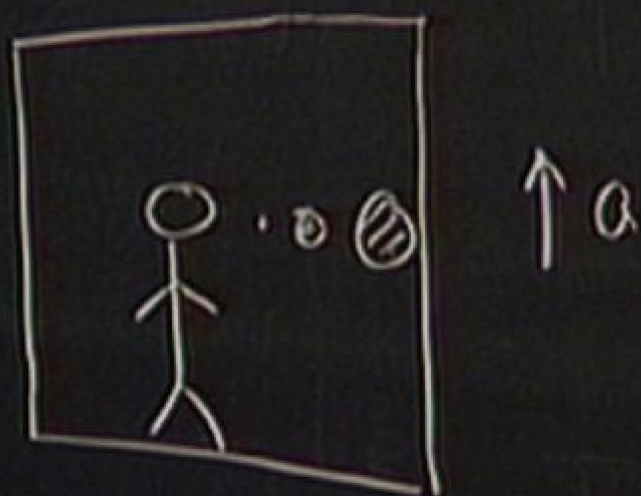
$$\rho \to T_{\mu\nu}$$


• Relativity


Equivalence principle


Equivalence principle
a
m inertial = m grav

Equivalence principle



$$m_{inertial} = m_{grav}$$

$$m_{\text{inertial}} = m_{\text{grav}}$$

$\uparrow a$

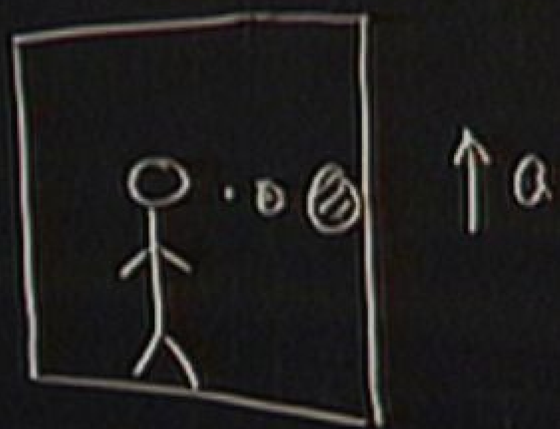
$$F = ma$$

$\downarrow$

$$m_{\text{grav.}} \cdot a = m_{\text{int}} \cdot a'$$

$$a' = a$$

Equivalence principle



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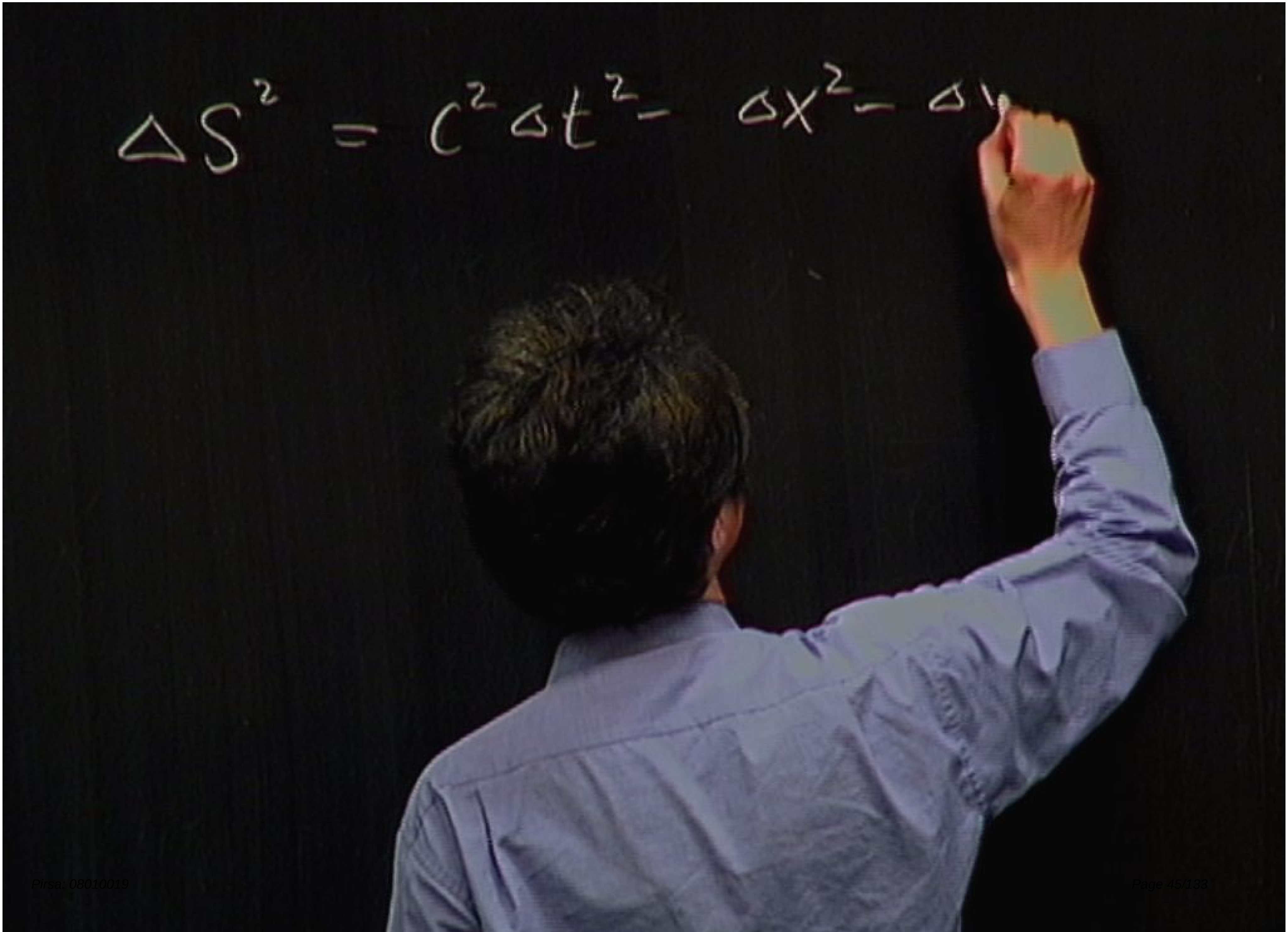
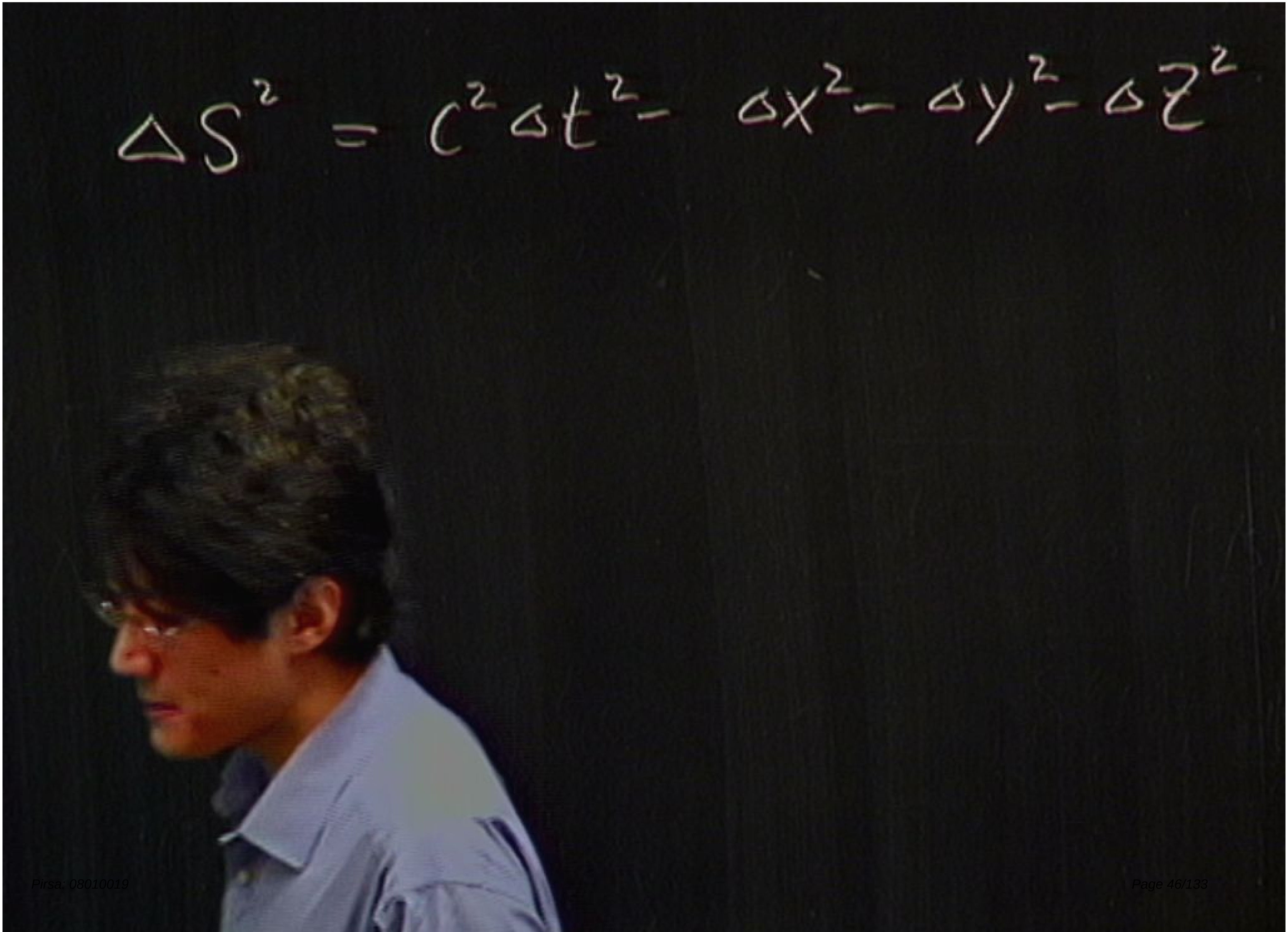

$\Delta S^2 = c^2 \Delta t^2 - \Delta x^2 -$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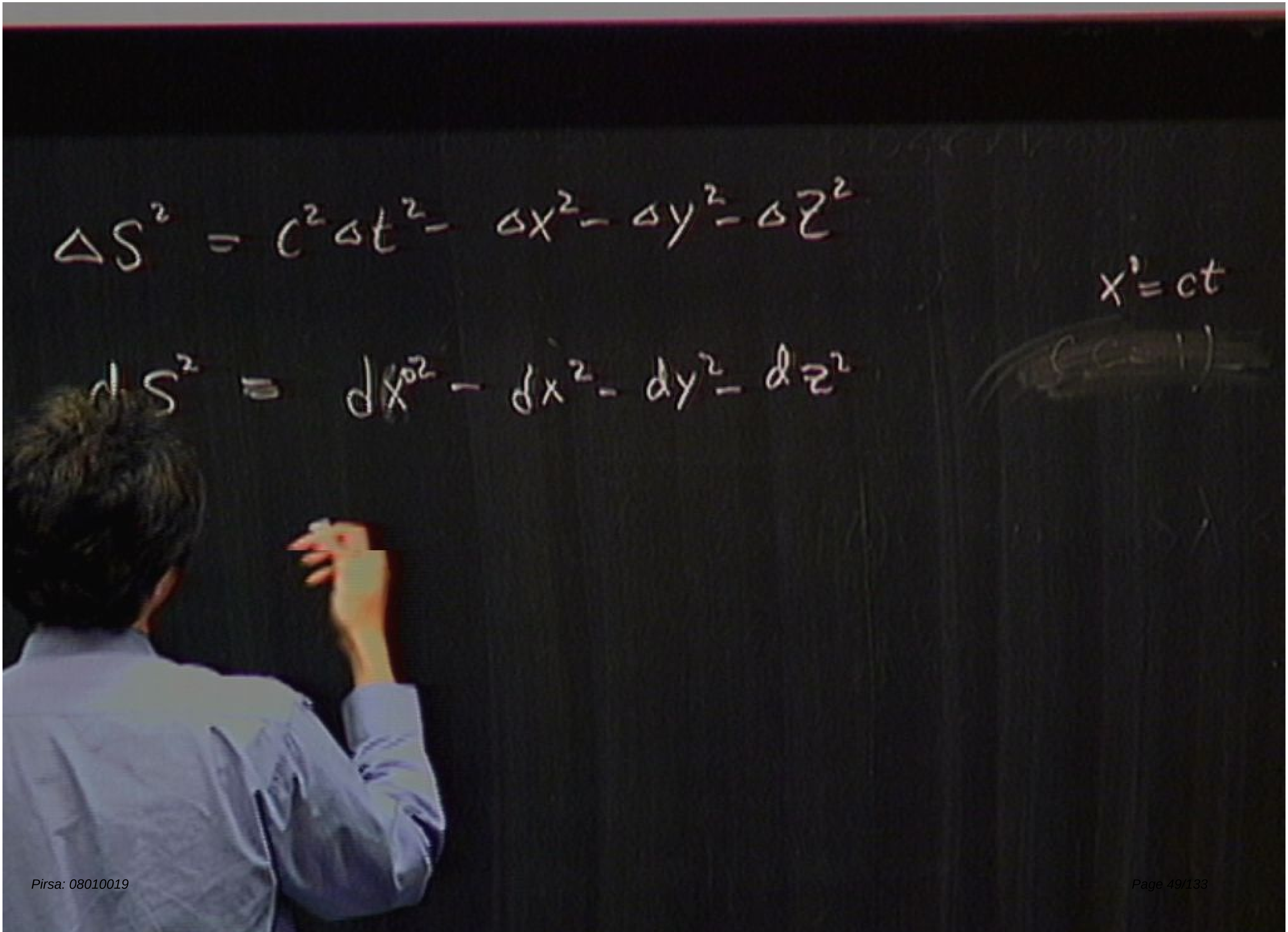
$\Delta S^2 = c^2 \Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2$
$x^0 = ct$
$dS^2 = dx^{0\,2} - dx^2 - dy^2 - dz^2$

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$$= \eta_{\mu\nu}\, dx^\mu dx^\nu$$

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$$= \eta_{\mu\nu}\, dx^\mu\, dx^\nu$$

$$\eta_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

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$$(c = 1)$$

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$$x^0 = ct$$

$$x^\mu \longrightarrow \bar{x}^\mu$$

$$\eta_{\mu\nu} \longrightarrow \frac{\partial x^\rho}{\partial \bar{x}^\mu} \frac{\partial x^\sigma}{\partial \bar{x}^\nu} \eta_{\rho\sigma}$$

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$$\Delta S^2 = c^2 \Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2$$

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$$= \eta_{\mu\nu}\, dx^\mu dx^\nu$$

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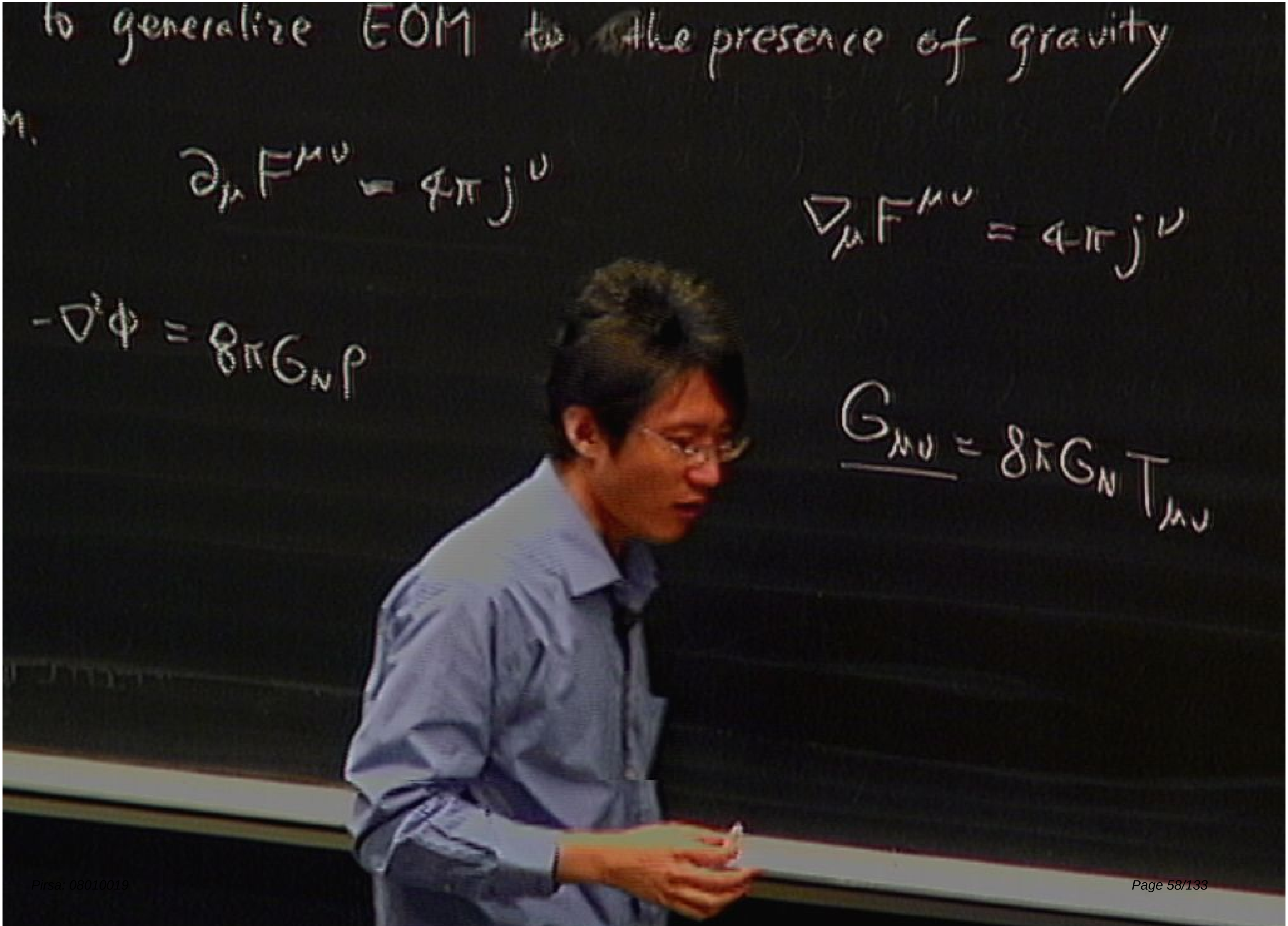
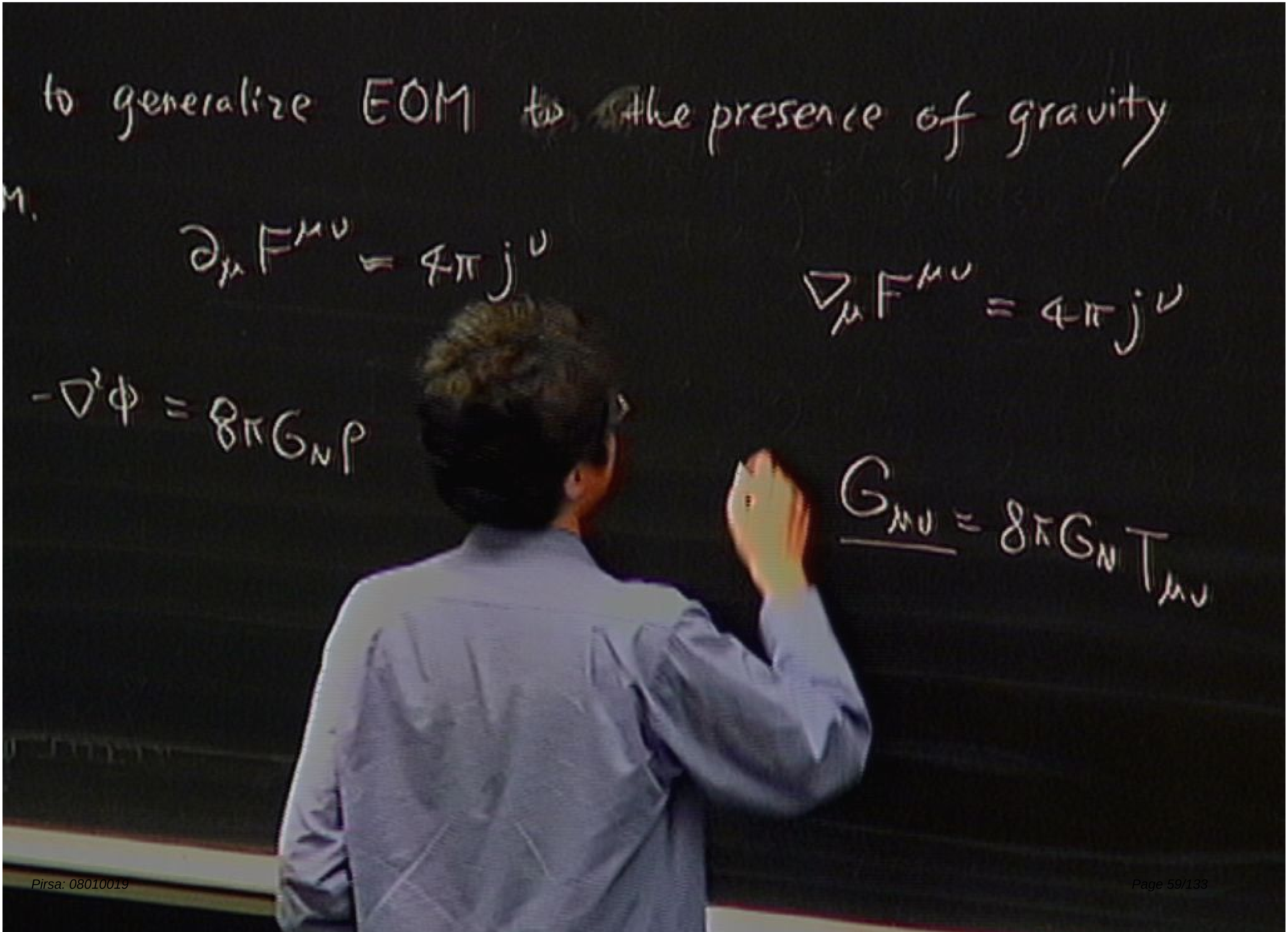
· How to generalize EOM to the presence of gravity

How to generalize EOM to the presence of gravity

E.M.

$$\partial_\mu F^{\mu\nu} = 4\pi j^\nu$$


to generalize EOM to the presence of gravity
$\partial_\mu F^{\mu\nu} = 4\pi j^\nu$
$\nabla_\mu F^{\mu\nu} = 4\pi j^\nu$
$-\nabla^2\phi = 8\pi G_N \rho$
$G_{\mu\nu} = 8\pi G_N T_{\mu\nu}$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M.
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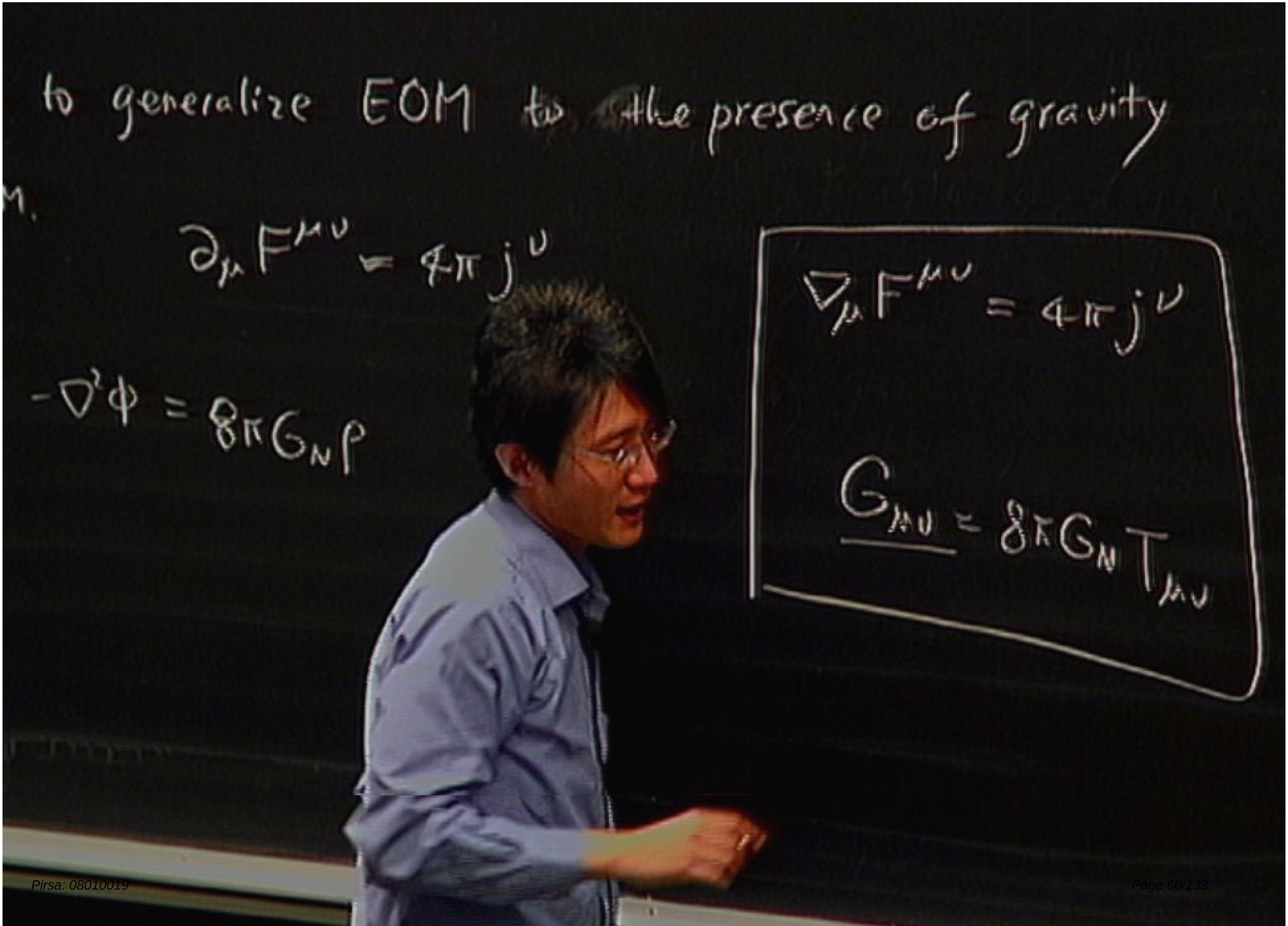
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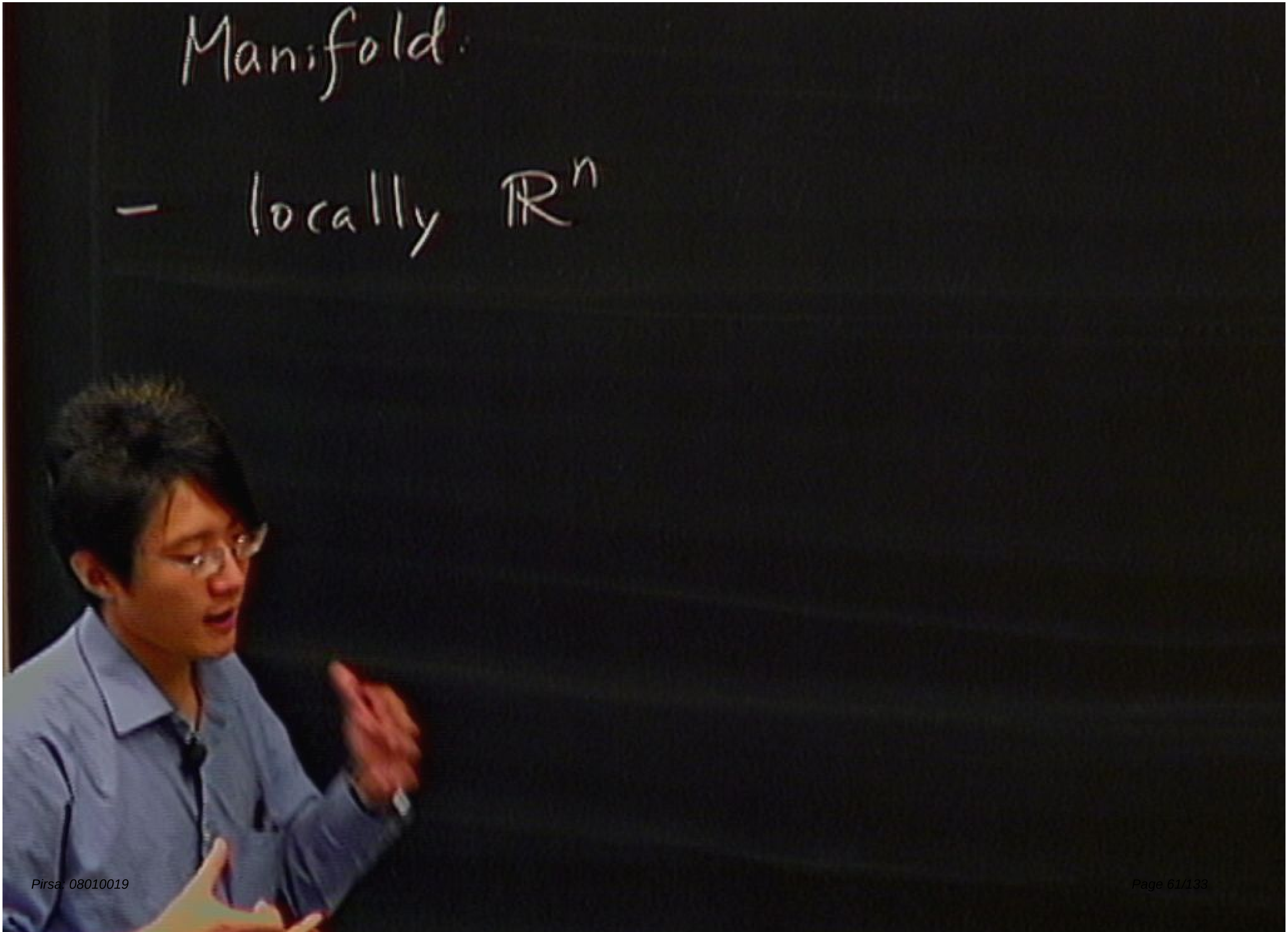
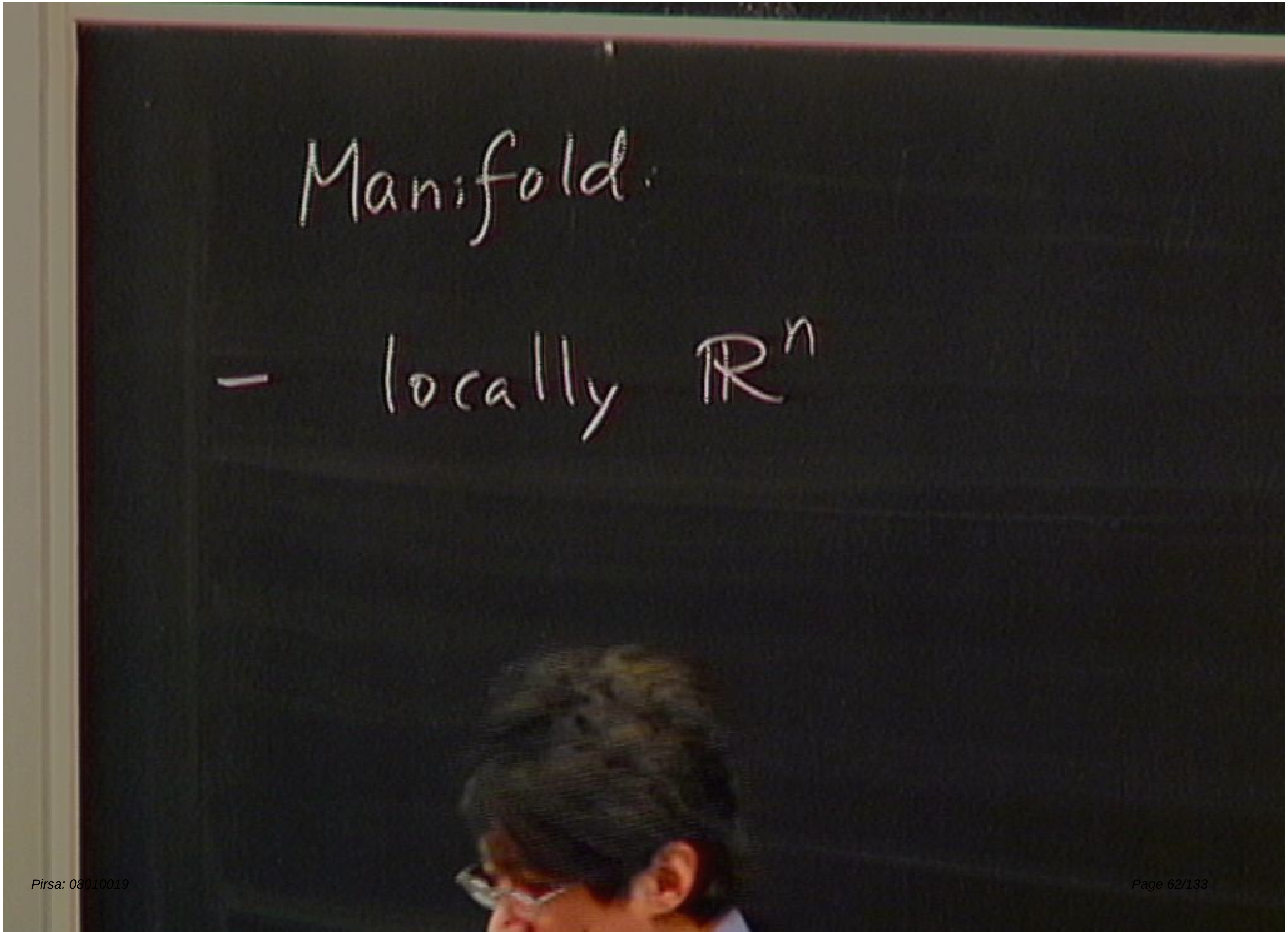

Manifold:
- locally $\mathbb{R}^n$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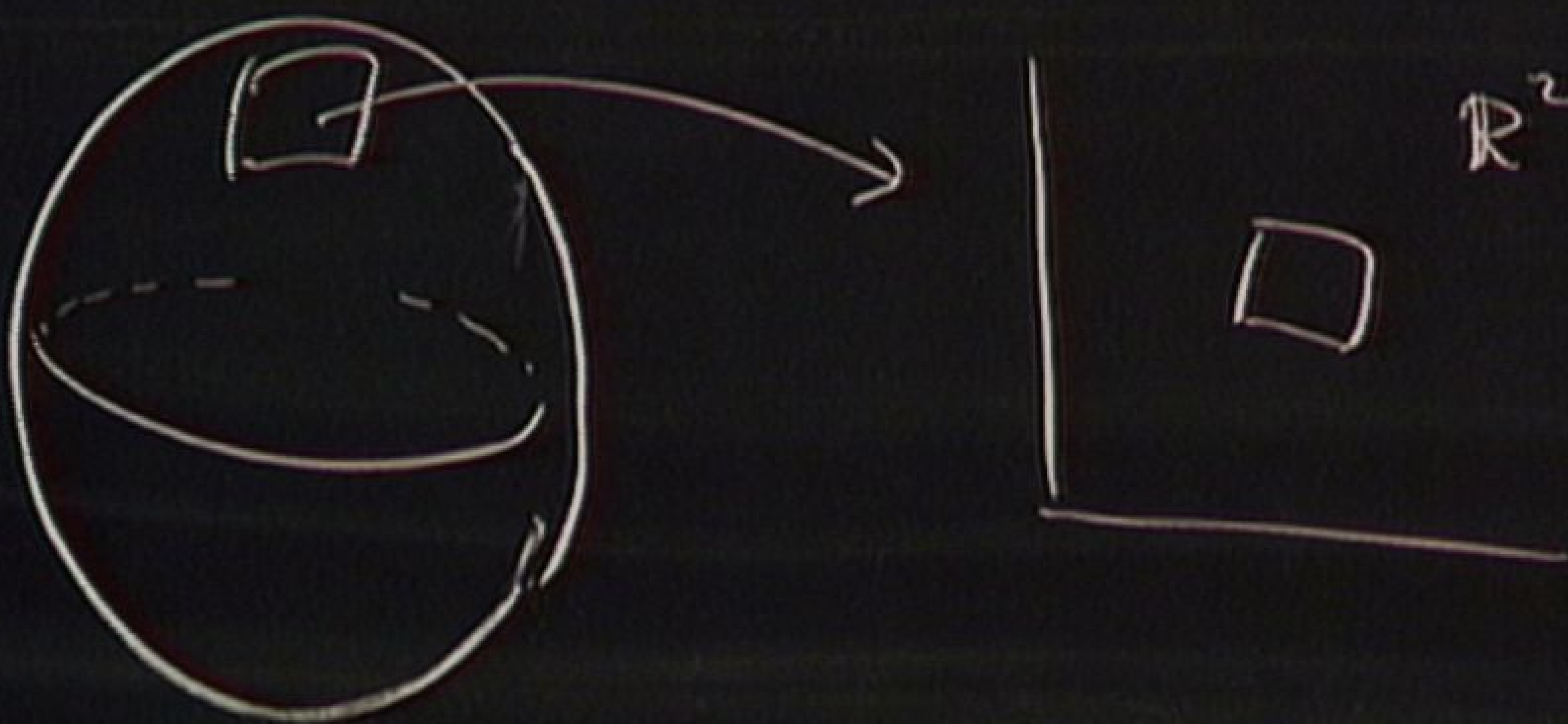
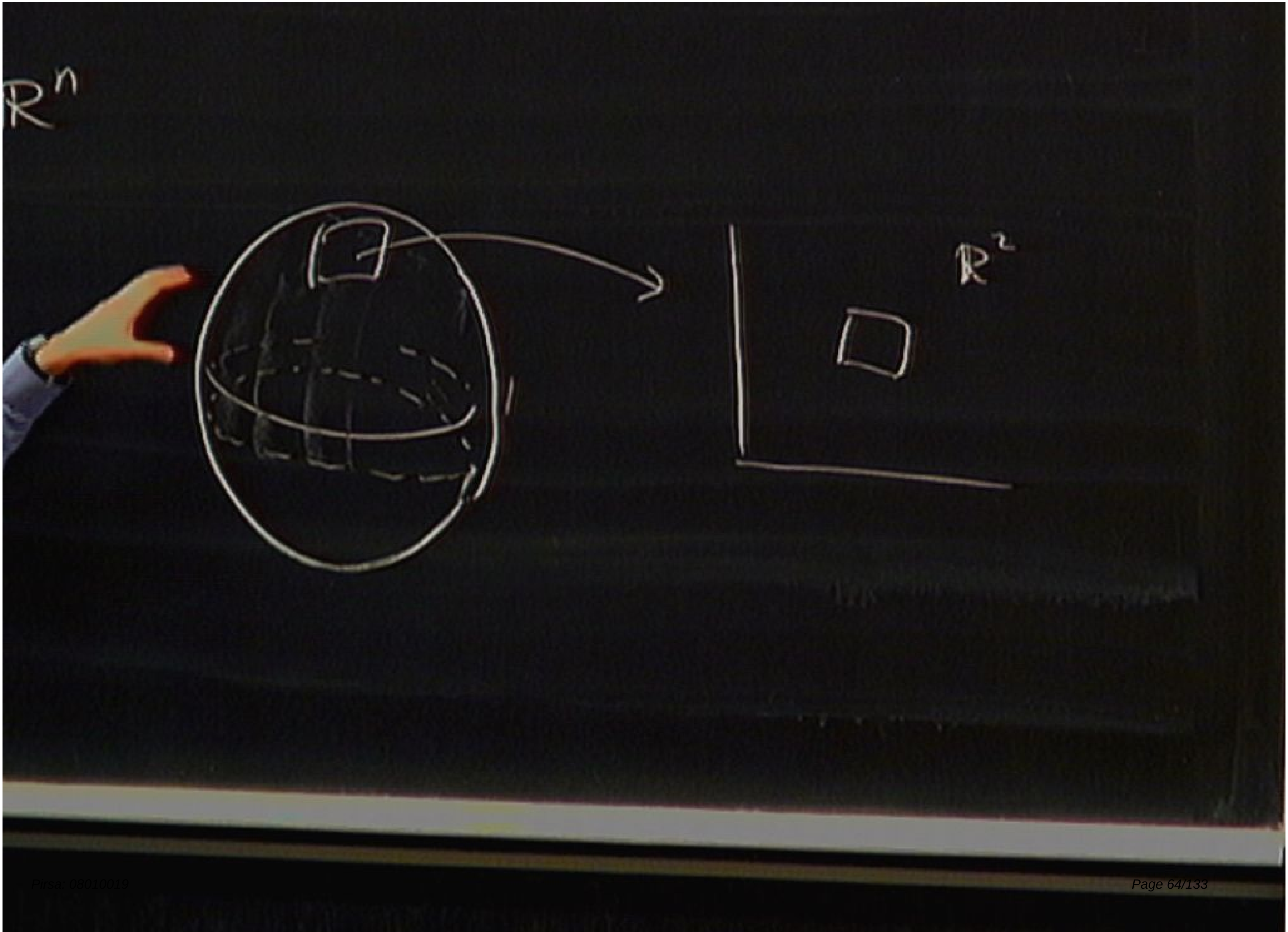
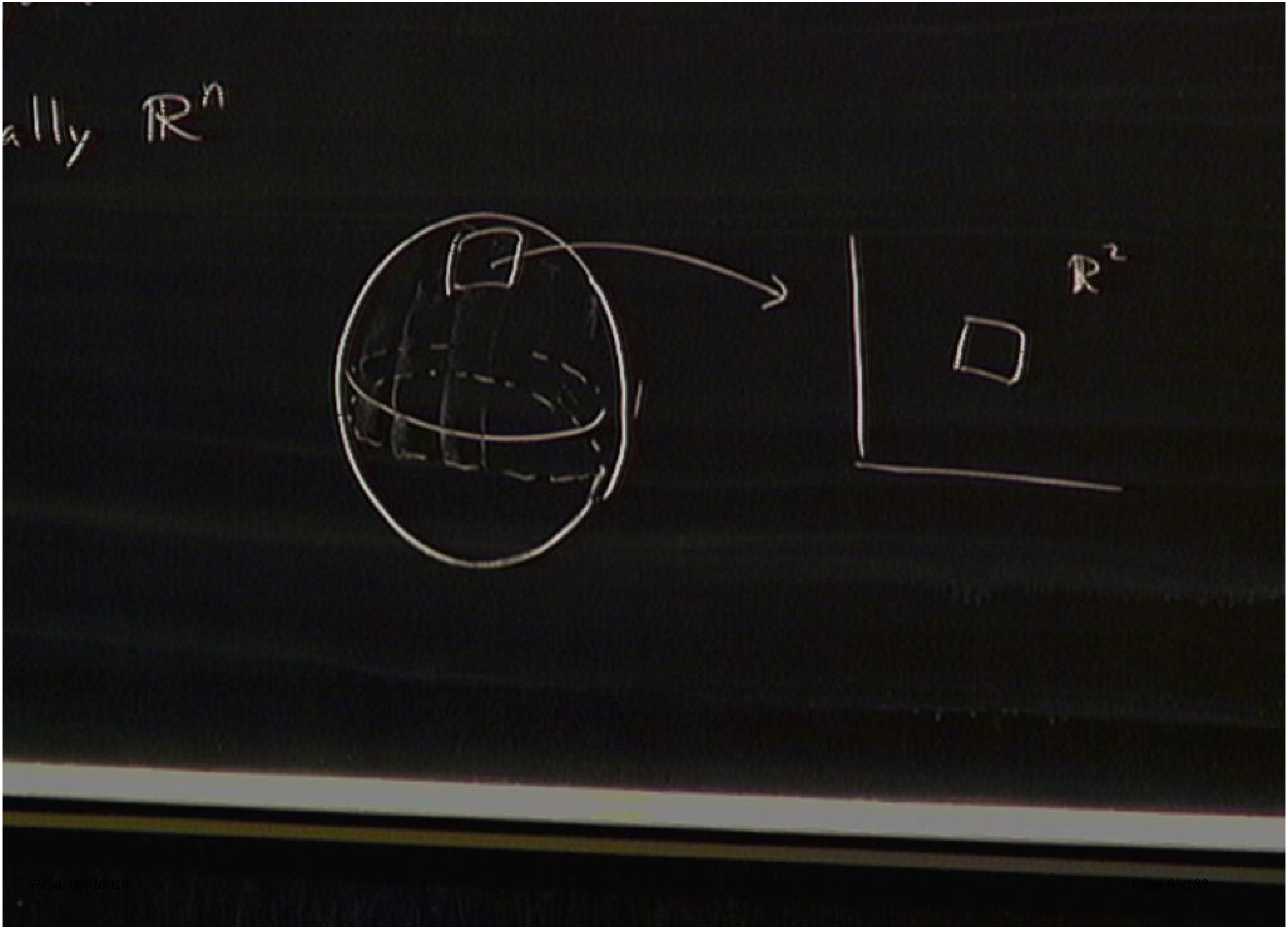
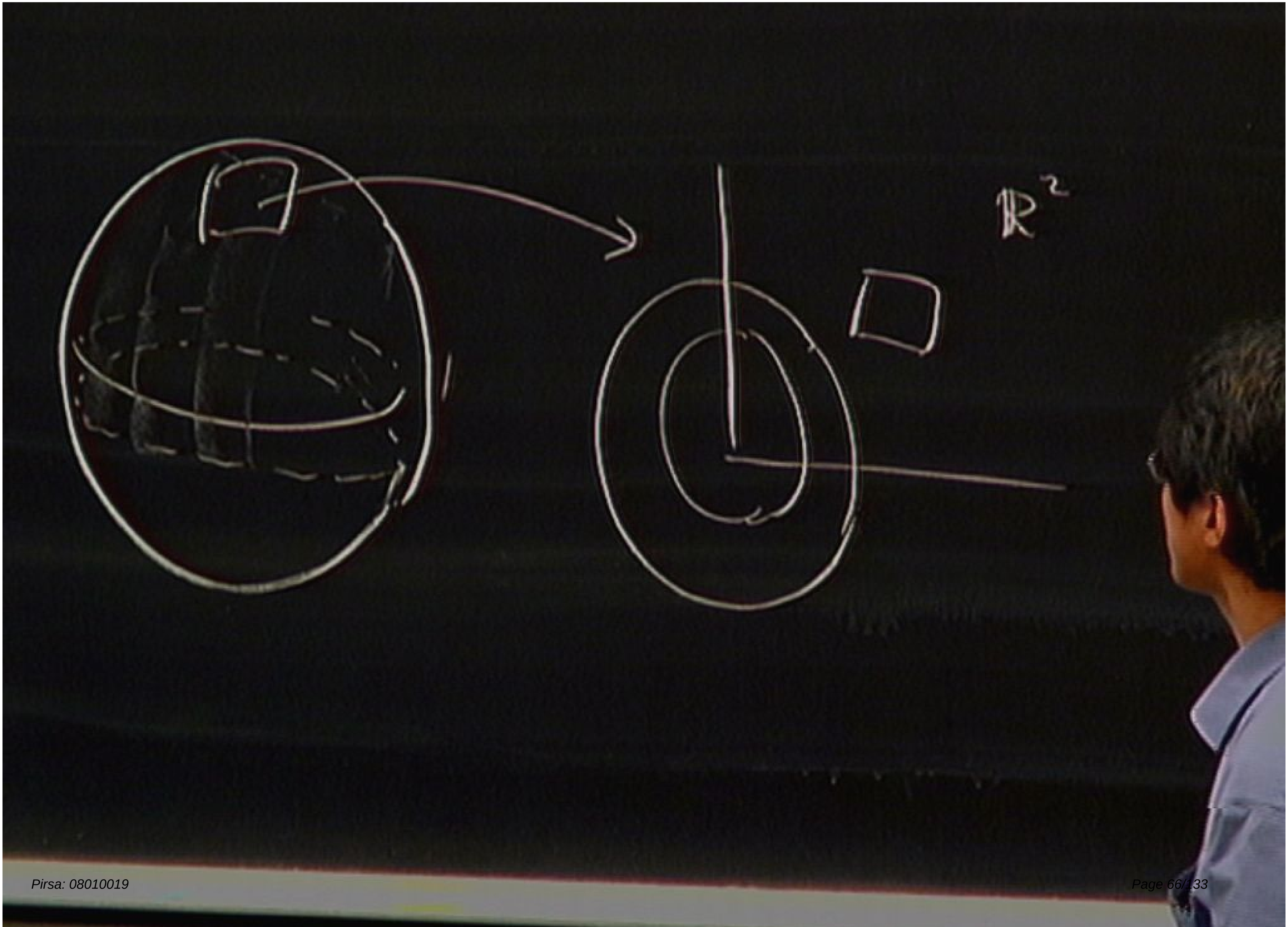
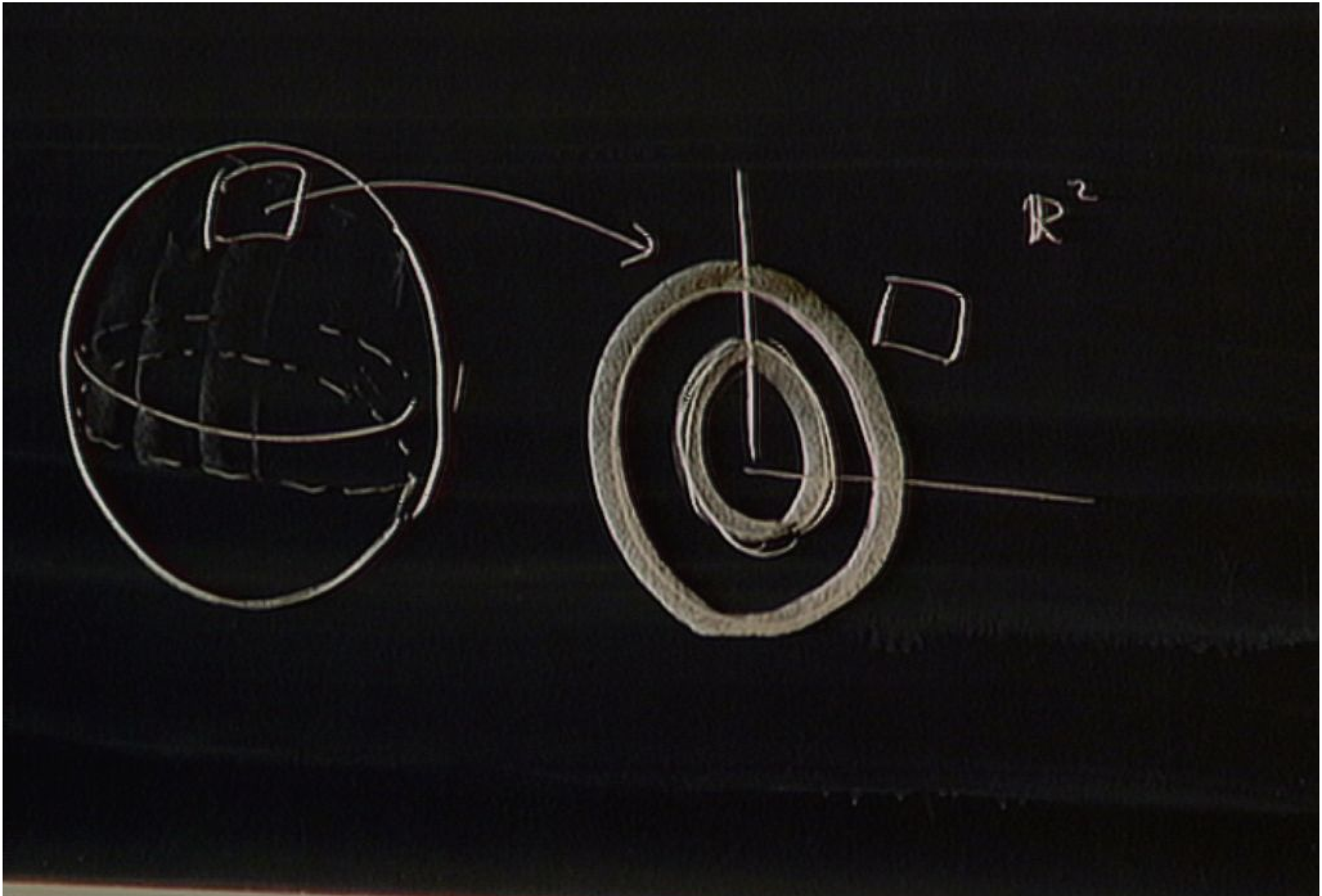

$\mathbb{R}^2$


R^n
$\mathbb{R}^2$


ally $\mathbb{R}^n$
$\mathbb{R}^2$


$\mathbb{R}^2$


$\mathbb{R}^2$

Manifold:

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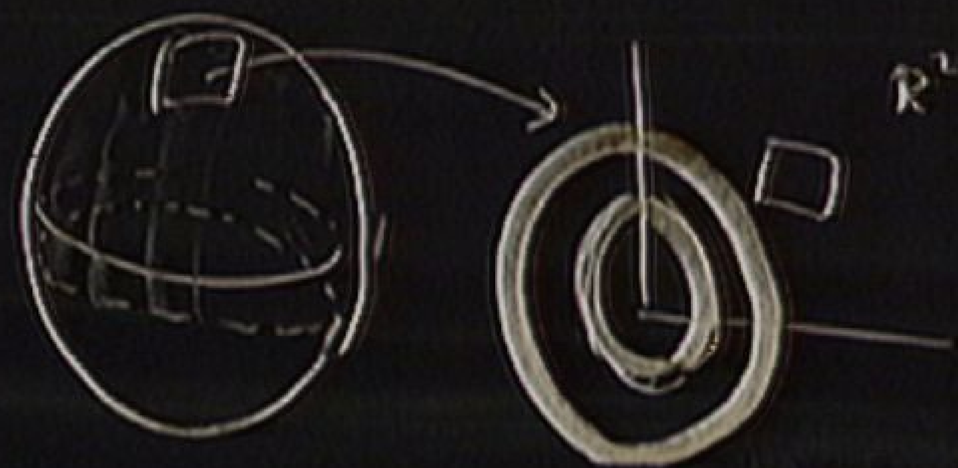
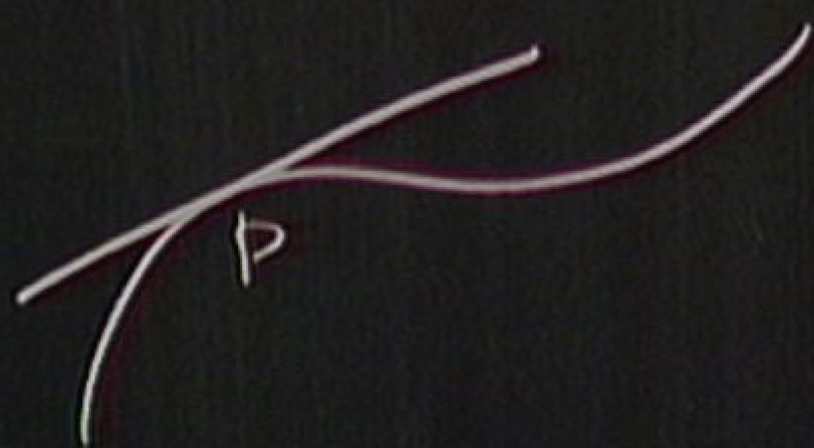
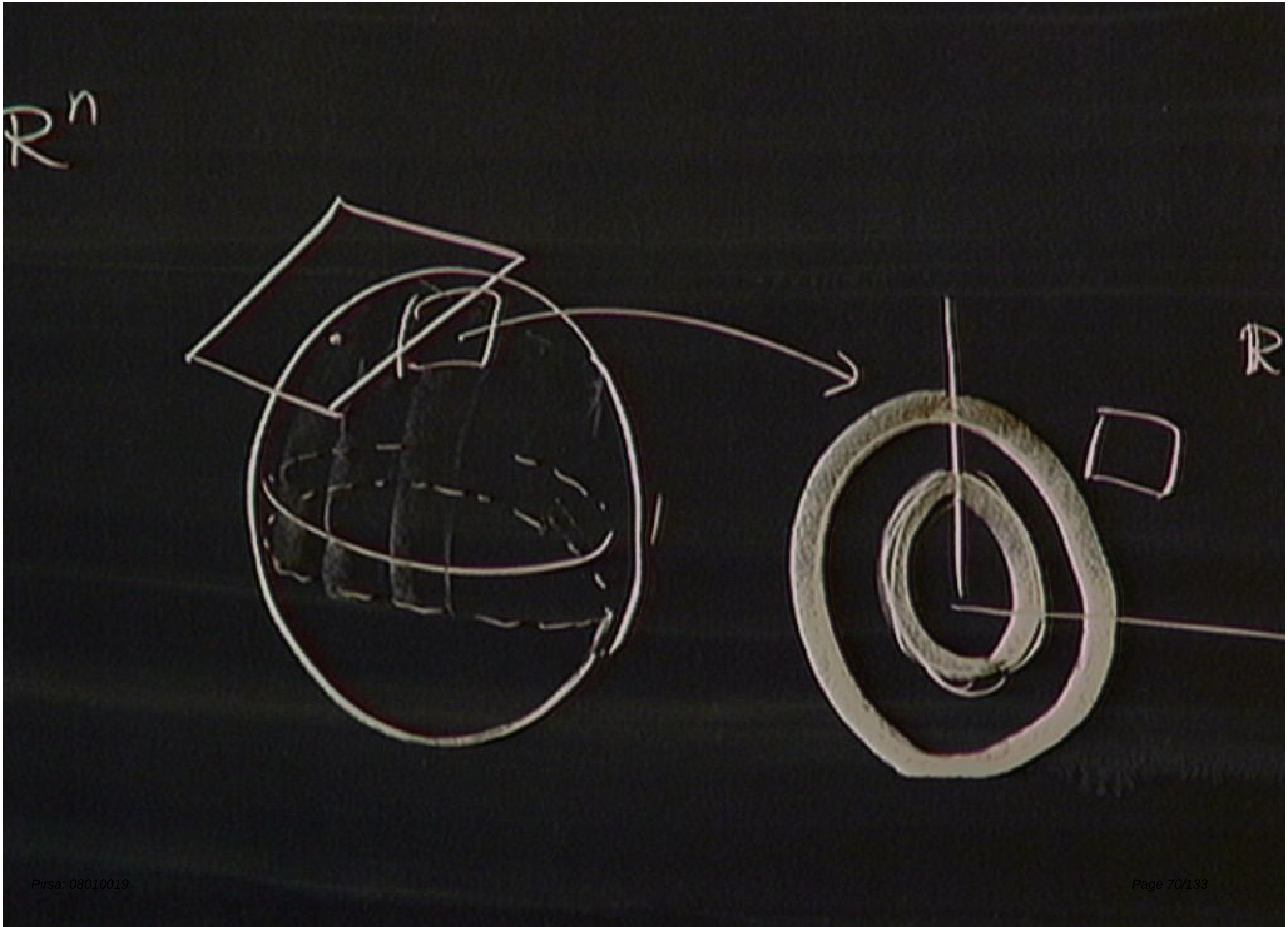
-

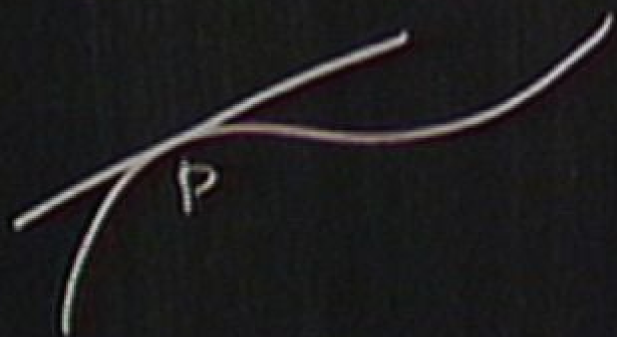


- tangent space at a pt

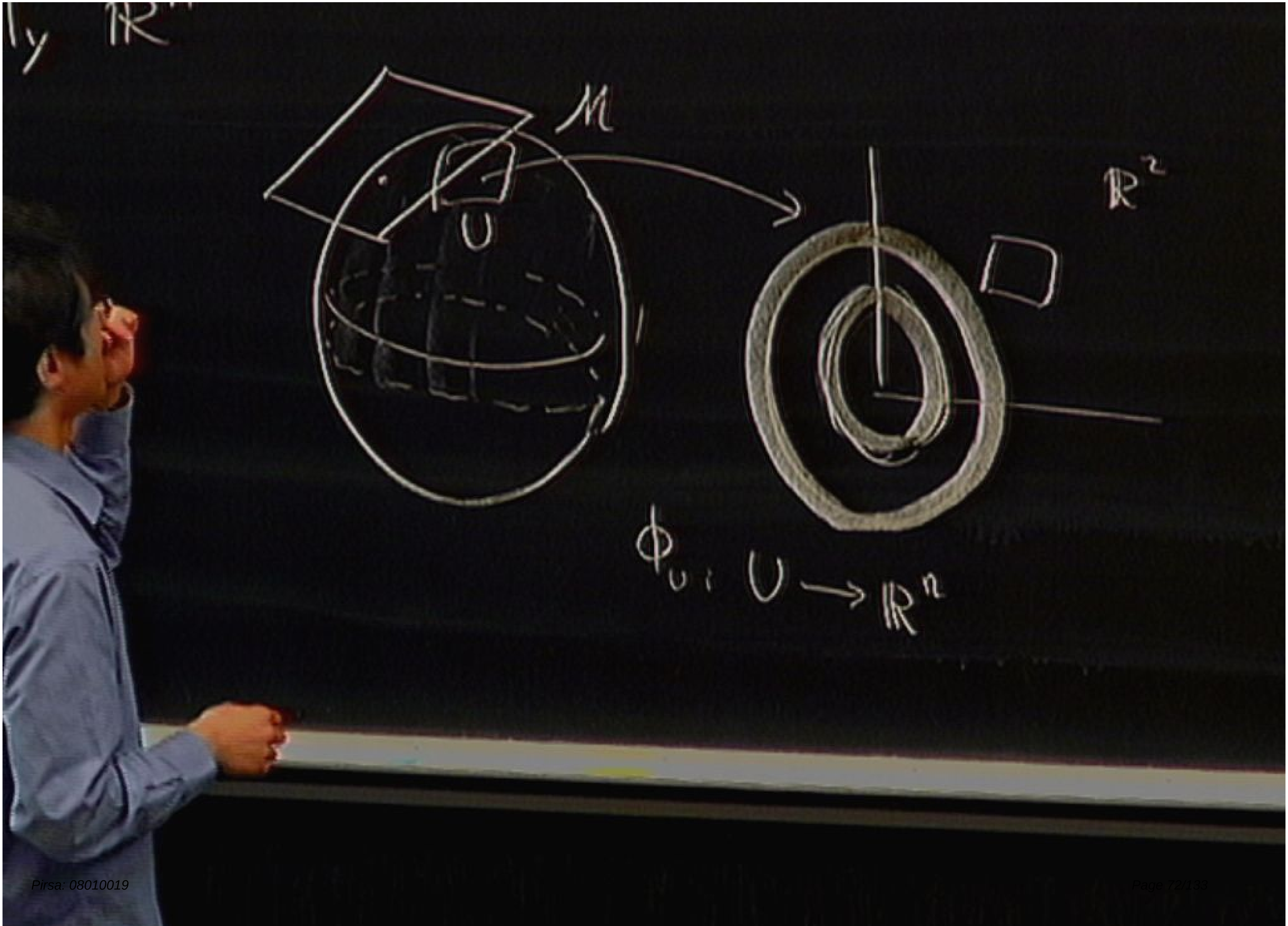



$\mathbb{R}^n$
$\mathbb{R}$

- tangent space at a pt

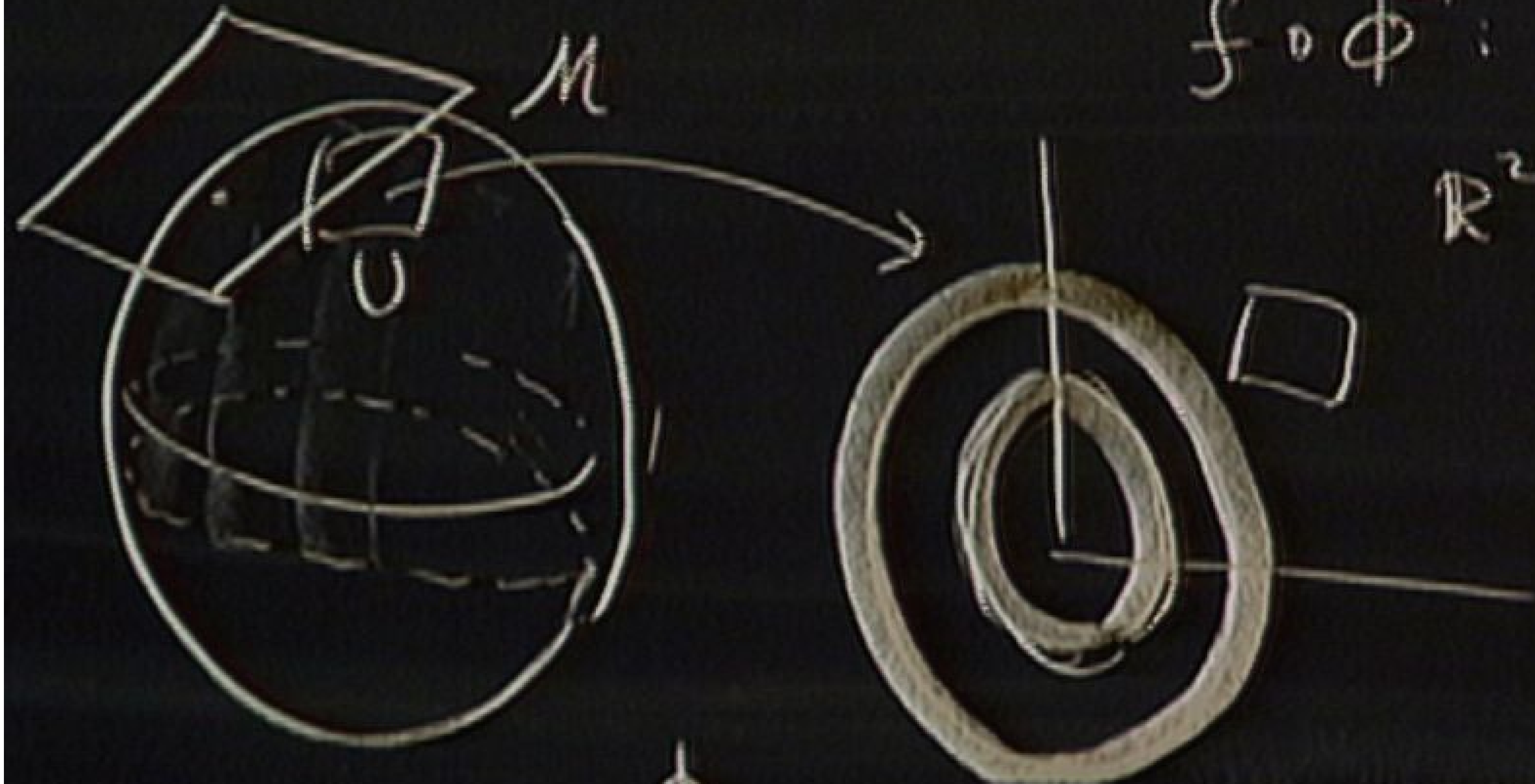


· Vector space dim n

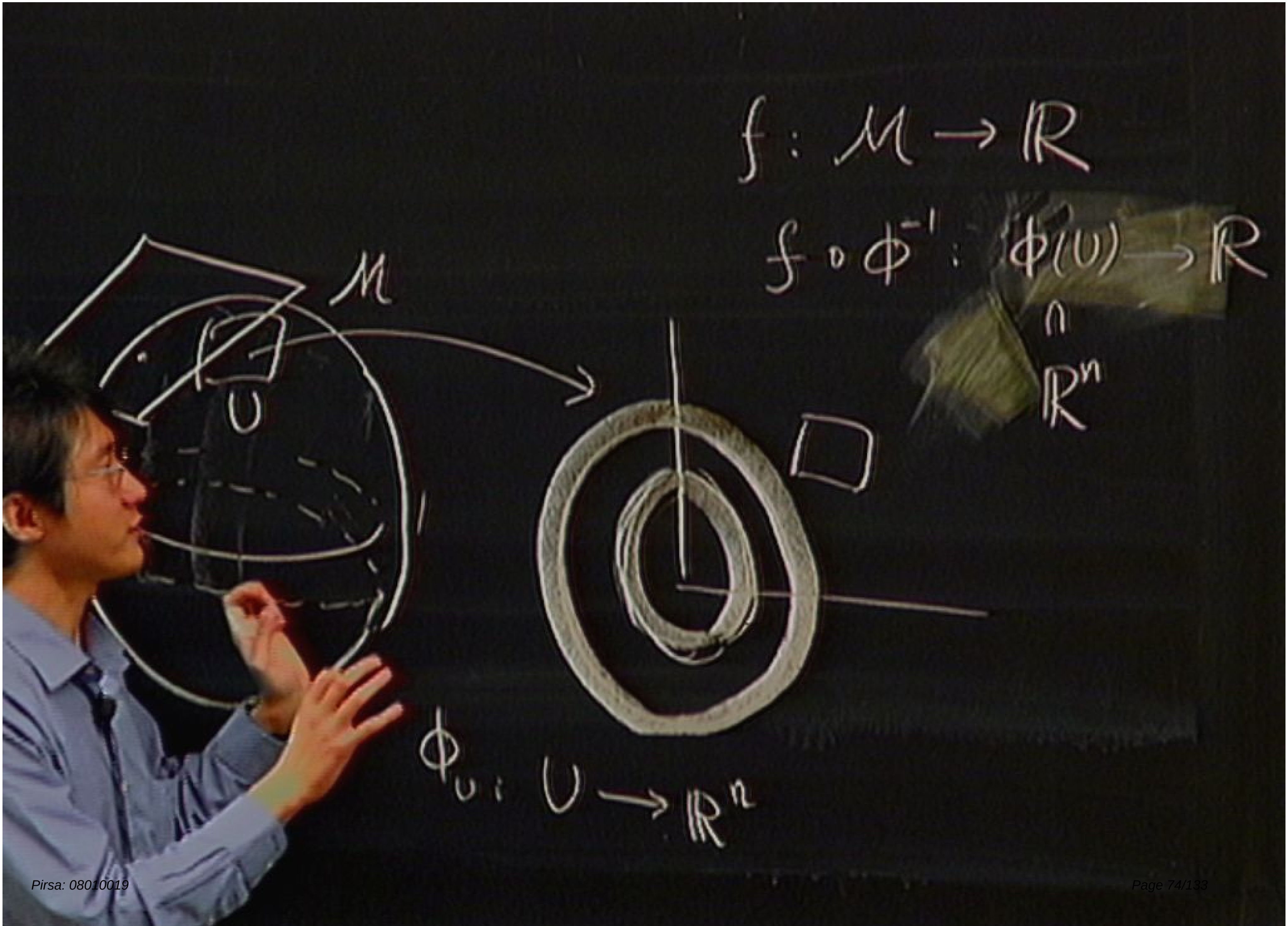

M
U
$\mathbb{R}^2$
$\phi_U : U \to \mathbb{R}^n$

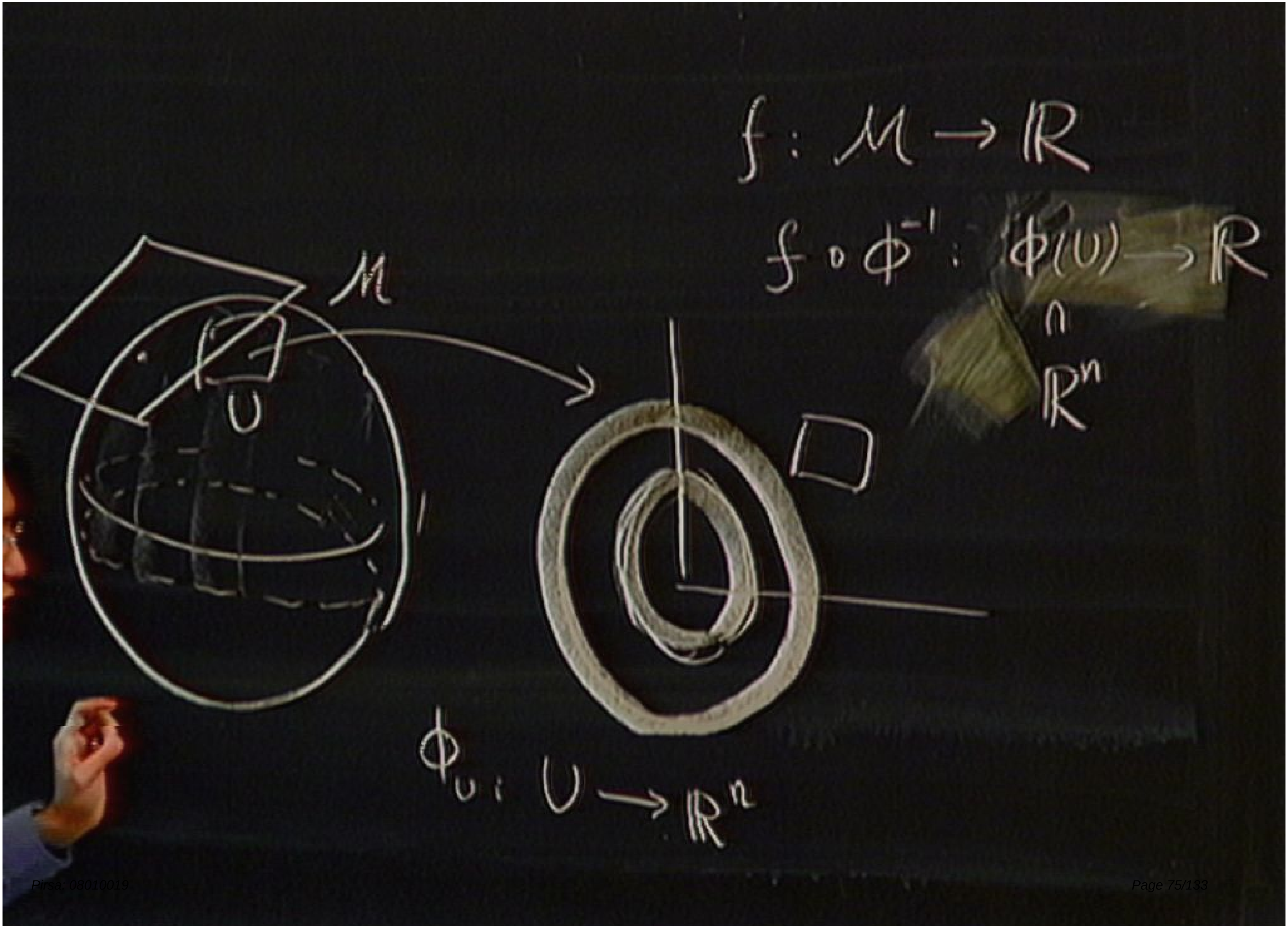
$$f : \mathcal{M} \to \mathbb{R}$$

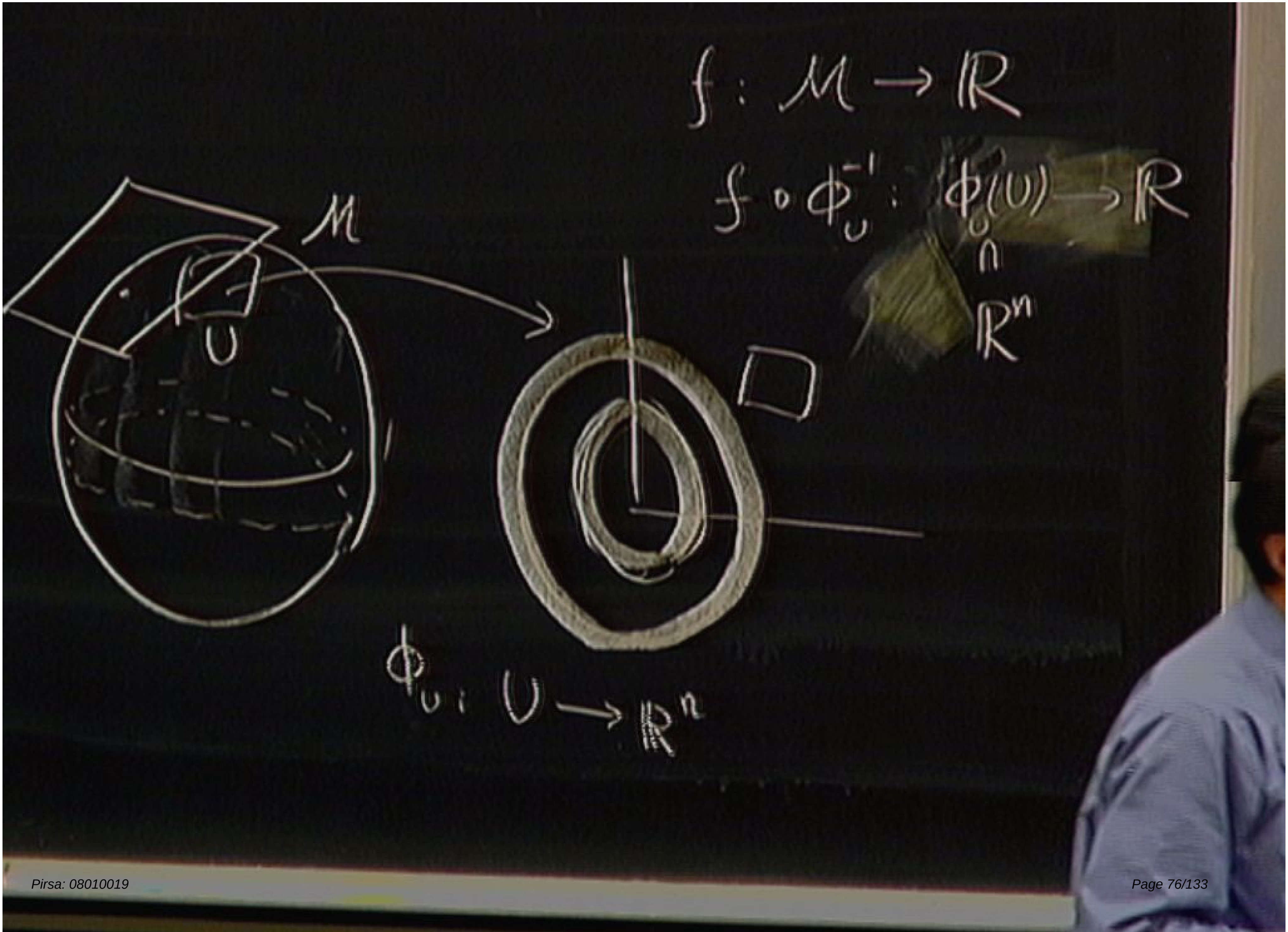
$$f \circ \phi^{-1} : \mathbb{R}^n \to R$$



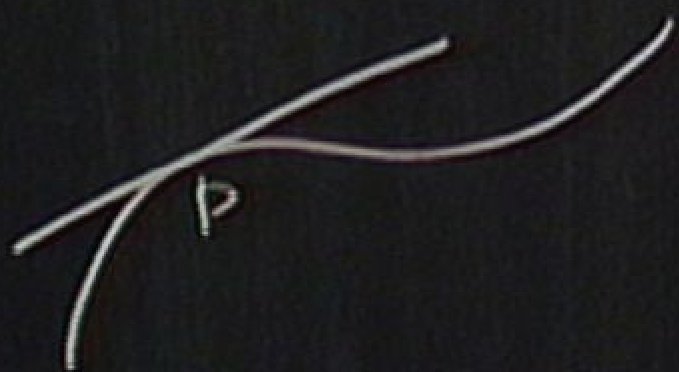
$$\phi_U : U \to \mathbb{R}^n$$


$f : \mathcal{M} \to \mathbb{R}$
$f \circ \phi^{-1} : \phi(U) \to \mathbb{R}$
$\cap$
$\mathbb{R}^n$
$\mathcal{M}$
U
$\phi_U : U \to \mathbb{R}^n$


$f : \mathcal{M} \to \mathbb{R}$
$f \circ \phi^{-1} : \phi(U) \to \mathbb{R}$
$\cap$
$\mathbb{R}^n$
$\mathcal{M}$
U
$\phi_U : U \to \mathbb{R}^n$


$f : \mathcal{M} \to \mathbb{R}$
$f \circ \phi_U^{-1} : \phi_U(U) \to \mathbb{R}$
$\cap$
$\mathbb{R}^n$
$\mathcal{M}$
U
$\phi_U : U \to \mathbb{R}^n$

- tangent space at a p^t



- Vector space dim n

- $\{x^i, \; i=1,\dots n\}$

- Vector space dim n

- $\{x^i, \; i=1, \cdots n\}$

$$\sum_{i=1}^{n} a^i \frac{\partial}{\partial x^i}\Big|_p f(x)$$

· Vector space dim n

· $\{x^i,\ i=1,\cdots n\}$

$$\sum_{i=1}^{n} a^i \frac{\partial}{\partial x^i}\bigg|_p f(x)$$

$$(a+b)V = aV + bV$$

e a^+ a p^+

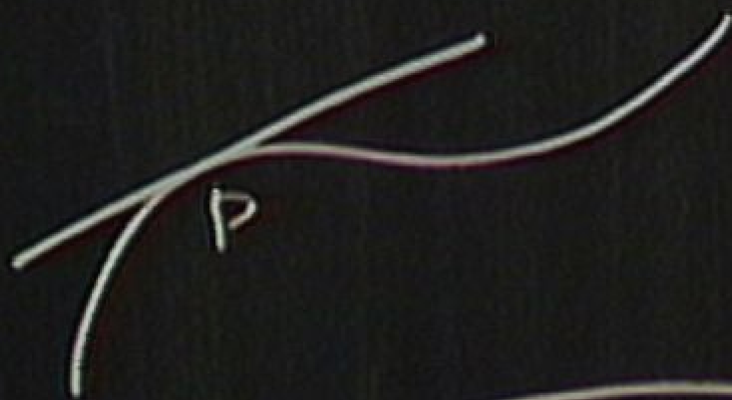
- Vector space dim n

- $\{x^i,\ i=1,\dots n\}$

$$\sum_{i=1}^{n} a^i \frac{\partial}{\partial x^i}\Big|_p f(x)$$

$$(a+b)V = aV + bV$$

- tangent space at a p



- Vector space dim

- $\{x^i,\ i = 1, \cdots n\}$

$$\sum_{i=1}^{n} a^i \frac{\partial}{\partial x^i}\Big|_p f(x)$$

Cotangent space.

ce a^+ a p^+ T_p

· Vector space dim

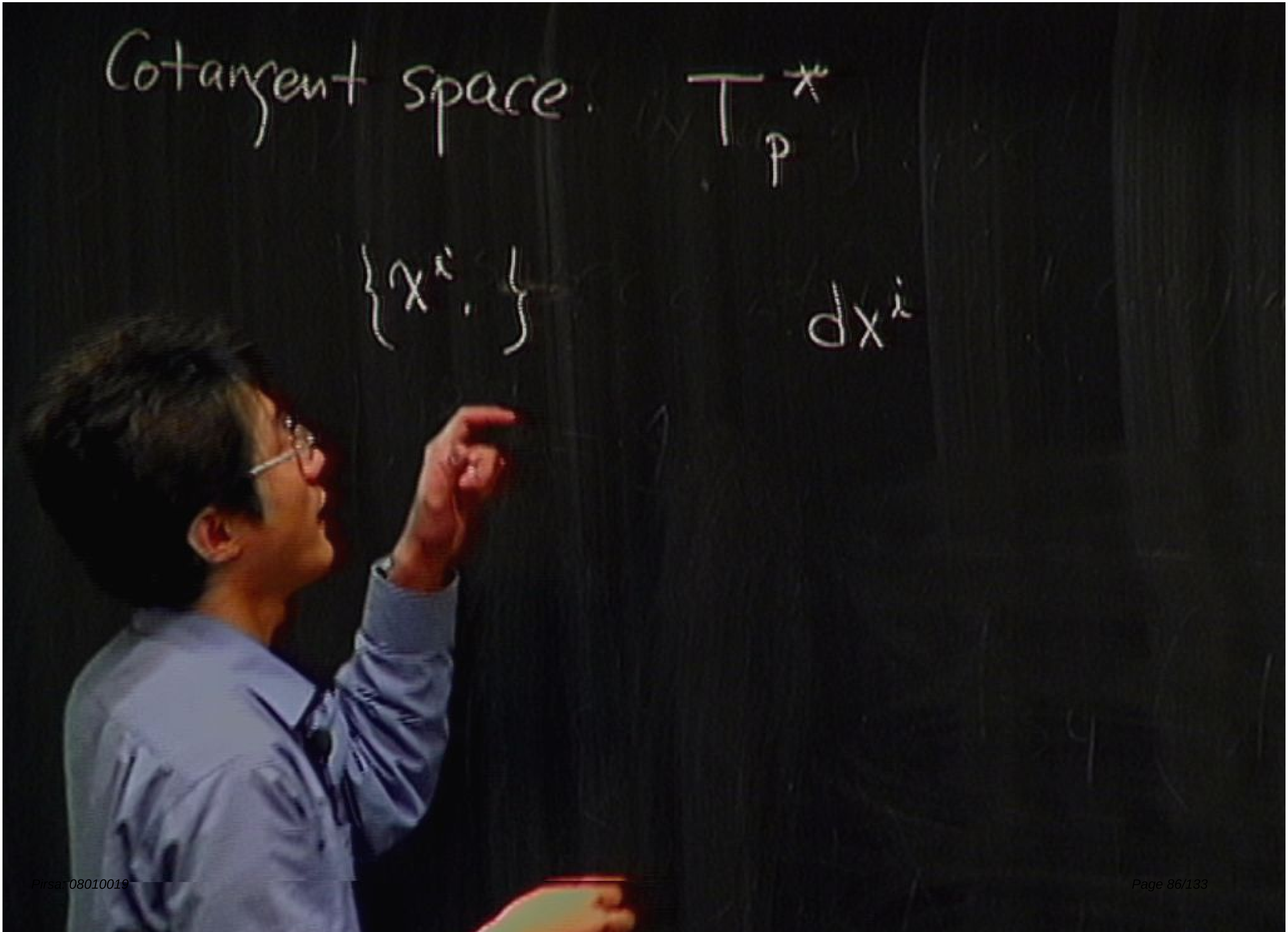
· $\{x^i,\ i = 1, \cdots n\}$

$\sum^n \alpha^i \partial$ $f(x)$

$f(x)$ $\quad$ $(a+b)V = aV$

p

$$\frac{\partial}{\partial x^i}, \quad i = 1, \dots, n$$


Cotangent space. T_p^*
$\{x^i\}$
dx^i

space. T_p^*

$\{x^i\}$ $\langle dx^i, \frac{\partial}{\partial x^j} \rangle = \delta^i_j$

Cotangent space. T_p^*

$\{x^i\}$

$\langle dx^i, \frac{\partial}{\partial x^j} \rangle = \delta^i_j$

Tensor field

A ,

Cotangent space. T_p^*

$\{x^i\}$ $\quad$ $\langle dx^i, \frac{\partial}{\partial x^j} \rangle = \delta^i{}_j$

Tensor field

A_μ, $\quad F_{\mu\nu}$

Cotangent space: T_p^*

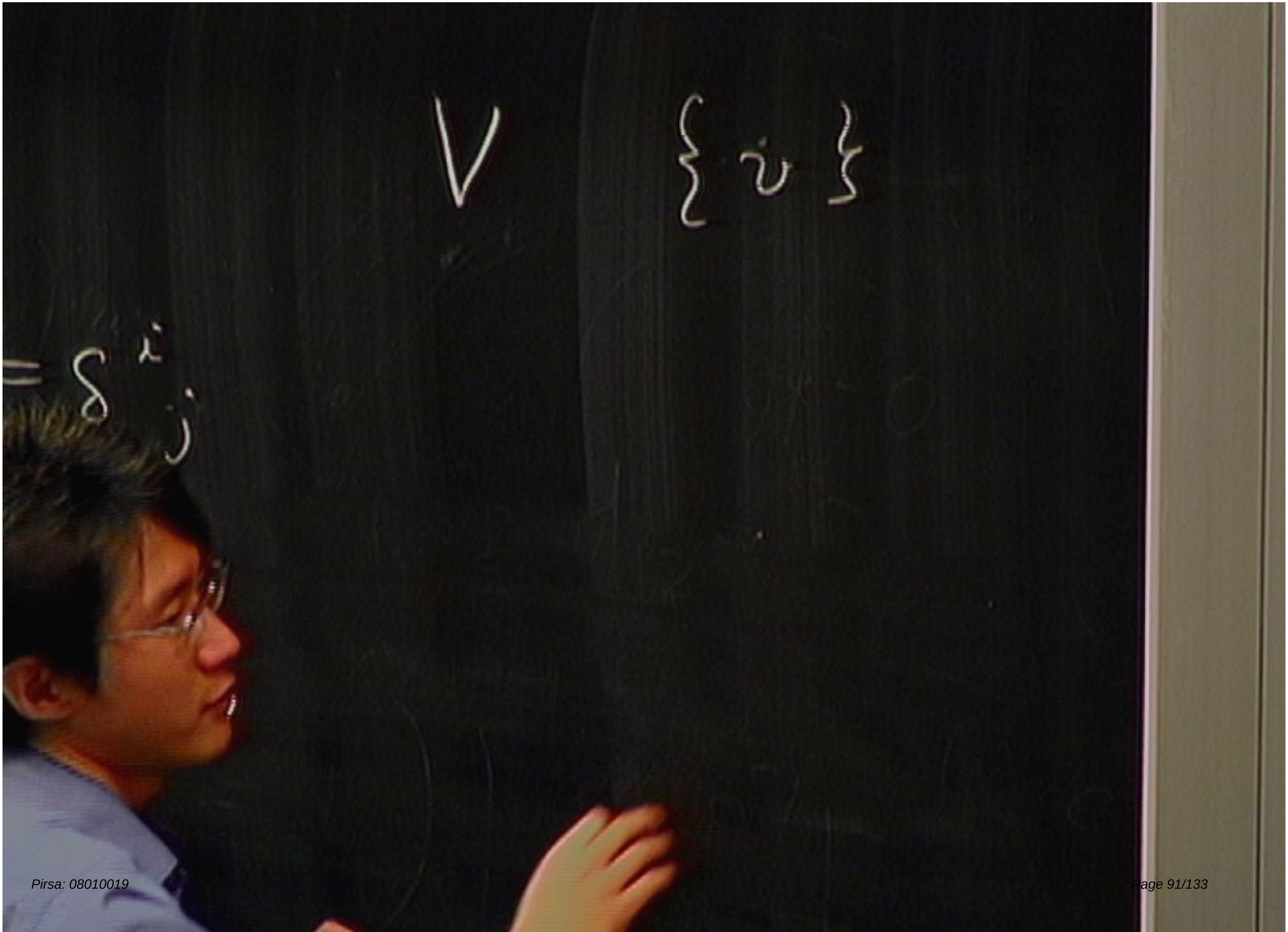
$\{x^i\}$ $\quad \langle dx^i, \frac{\partial}{\partial x^j} \rangle = \delta^i_j$

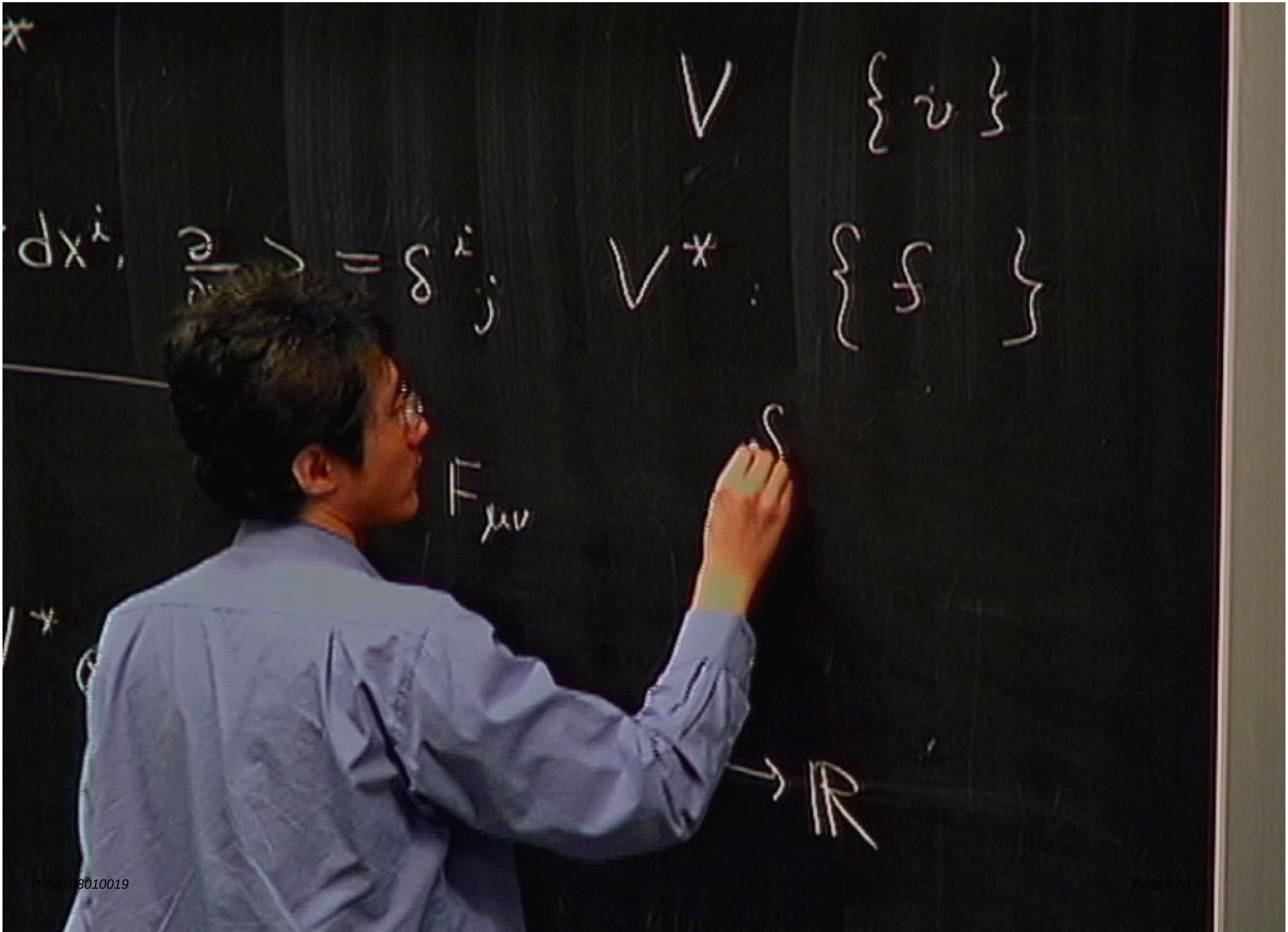
Tensor field:

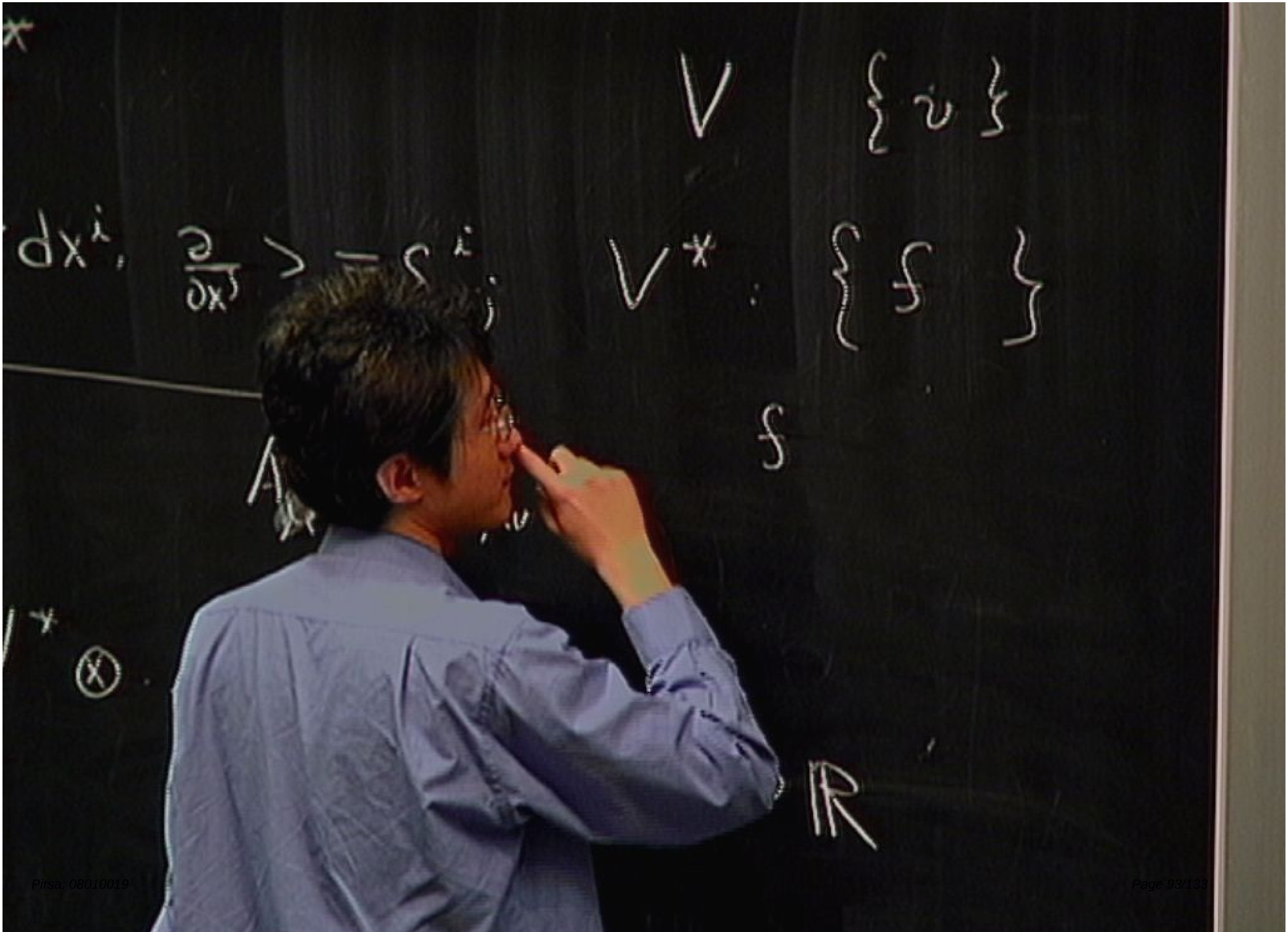
A_μ, $F_{\mu\nu}$

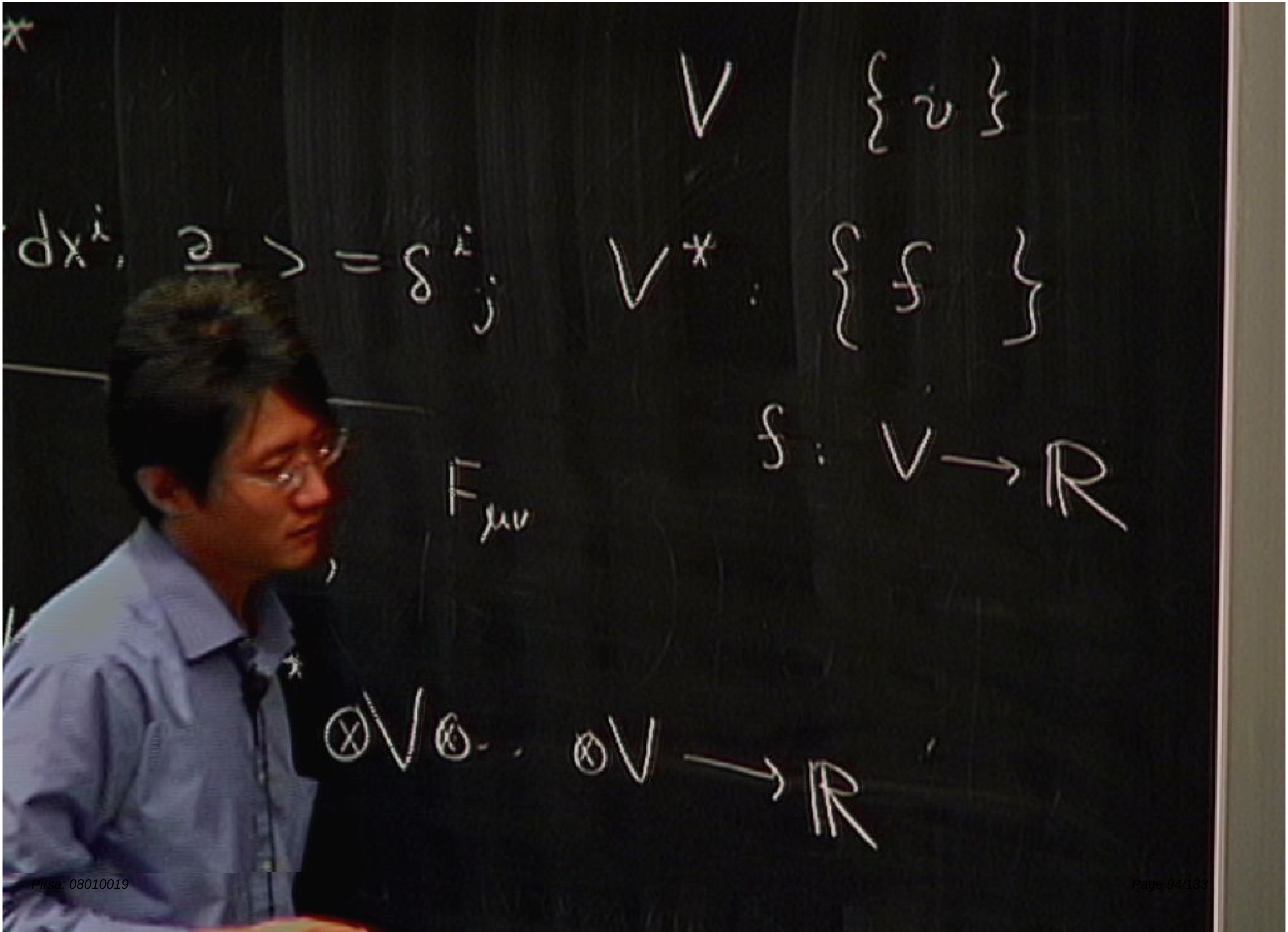
tensor on V $\quad X : V^* \otimes \cdots \otimes V^* \otimes V \otimes \cdots \otimes V \to \mathbb{R}$

linear


V
$= \delta^i_j$


V {v}
dx^i, ∂/∂ > = δ^i_j
V* : {f}
F_μν
→ R


V {v}
V* : {f}
f
R


dx^i, ∂ > = δ^i_j
V {v}
V* : {f}
f : V → R
F_μν
⊗V⊗ .. ⊗V → R

$*$

$V \quad \{v\}$

$\langle dx^i, \frac{\partial}{\partial x^j} \rangle = \delta^i{}_j \qquad V^* : \{f\}$

$f : V \to \mathbb{R}$

$A \qquad F_{\mu\nu}$

$f(av) = af(v)$

$f(v_1 + v_2) = f(v_1) + f(v_2)$

$V^* \otimes \cdots \otimes V^* \otimes V \otimes \cdots \otimes V \to \mathbb{R}$

$V \quad \{v\}$

$\frac{\partial}{\partial x^j} \rangle = \delta^i_j ; \quad V^* : \{f\}$

$f : V \to \mathbb{R}$

$A \quad F_{\mu\nu}$

$f(av) = af(v)$

$f(v_1 + v_2) = f(v_1) + f(v_2)$

$\otimes V^* \otimes V \otimes \cdots \otimes V \to \mathbb{R}$

$V.$ $\{v_i,\ i=1,\cdots,n\}$

$$v=\sum_{i=1}^{n}a^i v_i$$

$V.$ $\{v_i,\ i=1,\cdots,n\}$

$$v=\sum_{i=1}^{n} a^i v_i$$

$f_i \in V^*$ $f_i(v_j)=\delta_{ij}$

$V.$ $\{v_i.\ i=1,\dots,n\}$

$$v=\sum_{i=1}^{n} a^i v_i$$

$f_i \in V^*$

$$f_i(v_j)=\delta_{ij}$$

$$f_i\left(v=\sum_j a^j v_j\right)=a^i$$

$V.$ $\{v_i \,.\, i = 1, \cdots, n\}$

$$v = \sum_{i=1}^{n} a^i v_i$$

$f_i \in V^*$ $\qquad f_i(v_j) = \delta_{ij}$

$$f_i\left(v = \sum_j a^j v_j\right) = a^i$$

n $\quad f_i$'s

V $\{v_i \; i = 1, \cdots, n\}$

$v = \sum_{i=1}^{n} a^i v_i$

$f_i \in V^*$ $f_i(v_j) = \delta_{ij}$

$f_i(v = \sum_j a^j v_j) = a^i$

n f_i's $f(v = \sum_j a^j v_j)$

$$f_i(v_j) = \delta_{ij}$$

$$f_i\left(v = \sum_j a^j v_j\right) = a^i$$

$$f\left(v = \sum_j a^j v_j\right) = \sum_j a^j f(v_j)$$

$$= \sum_{j=1}^{n} f(v_j)\, f_j(v)$$

T_p

$\{x^i,\}$ $\quad \langle dx^i, \frac{\partial}{\partial x^j} \rangle = \delta^i{}_j$

Tensor field :

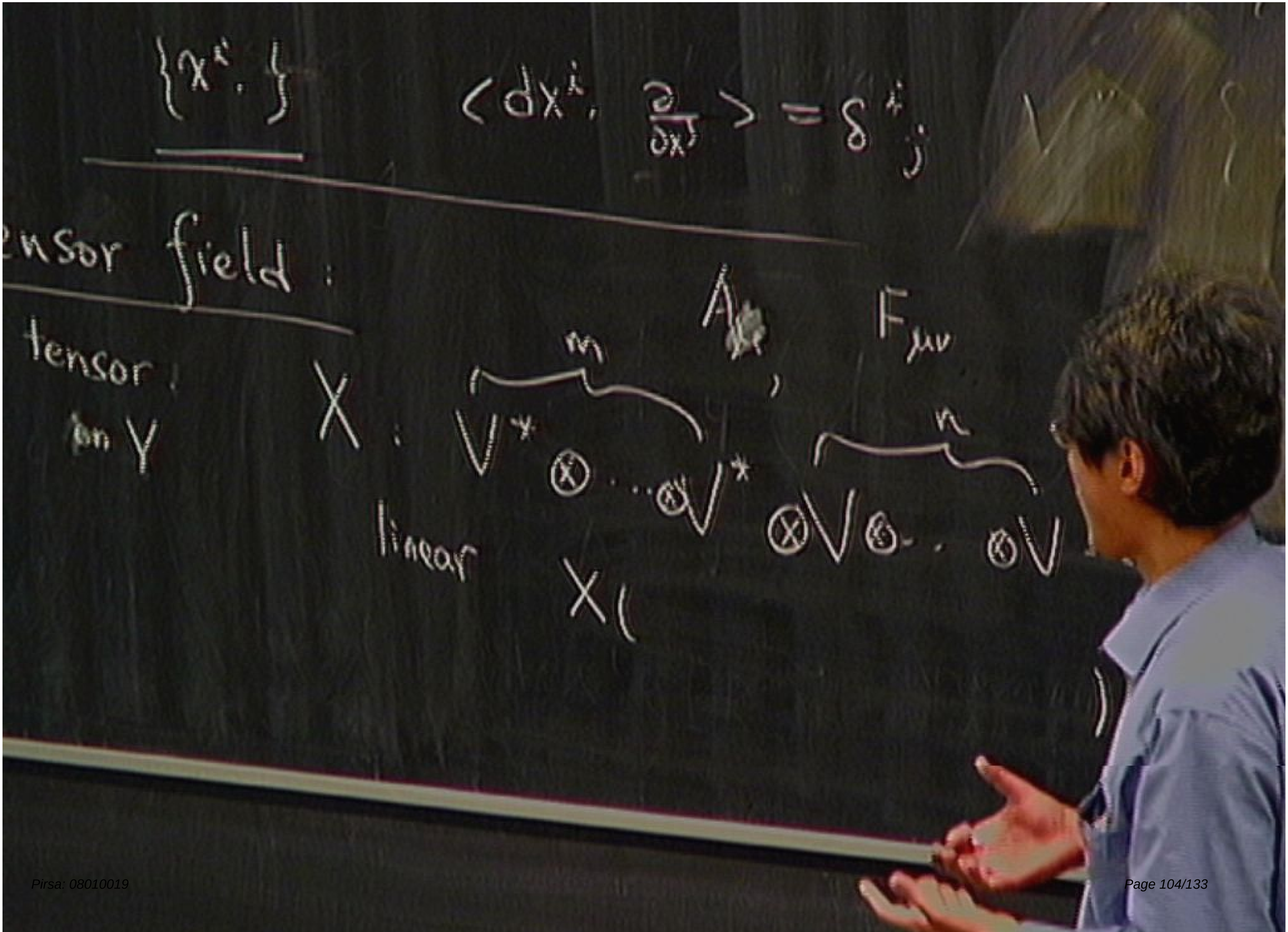
A_μ , $F_{\mu\nu}$

tensor :

in Y

$X : \overbrace{V^* \otimes \cdots \otimes V^*}^{m} \otimes \overbrace{V \otimes \cdots \otimes V}^{n} \to \mathbb{R}$

linear


$\{x^i\}$
$\langle dx^i, \frac{\partial}{\partial x^j} \rangle = \delta^i{}_j$
ensor field :
A_μ , $F_{\mu\nu}$
tensor :
on V
$X : \overbrace{V^* \otimes \cdots \otimes V^*}^{m} \otimes \overbrace{V \otimes \cdots \otimes V}^{n}$
linear
X(

$\{x^i\}$

$\langle dx^i, \frac{\partial}{\partial x^j} \rangle = \delta^i{}_j$

ensor field:

$A_\mu, \quad F_{\mu\nu}$

tensor $X : \overbrace{V^* \otimes \cdots \otimes V^*}^{m} \otimes \overbrace{V \otimes \cdots \otimes V}^{n} \longrightarrow \mathbb{R}$

linear

$X(\qquad)$

$$\langle dx^i, \frac{\partial}{\partial x^j} \rangle = \delta^i{}_j$$

$$\partial_i = \frac{\partial}{\partial x^i}$$

$$A_\mu \qquad F_{\mu\nu}$$

$$: \overbrace{V^* \otimes \cdots \otimes V^*}^{m}, \overbrace{\otimes V \otimes \cdots \otimes V}^{n} \longrightarrow \mathbb{R}$$

inear

$$X(dx^{i_1}, \ldots, dx^{i_m}, \partial_{j_1}, \ldots, \partial_{j_n})$$

$\partial_i = \frac{\partial}{\partial x^i}$

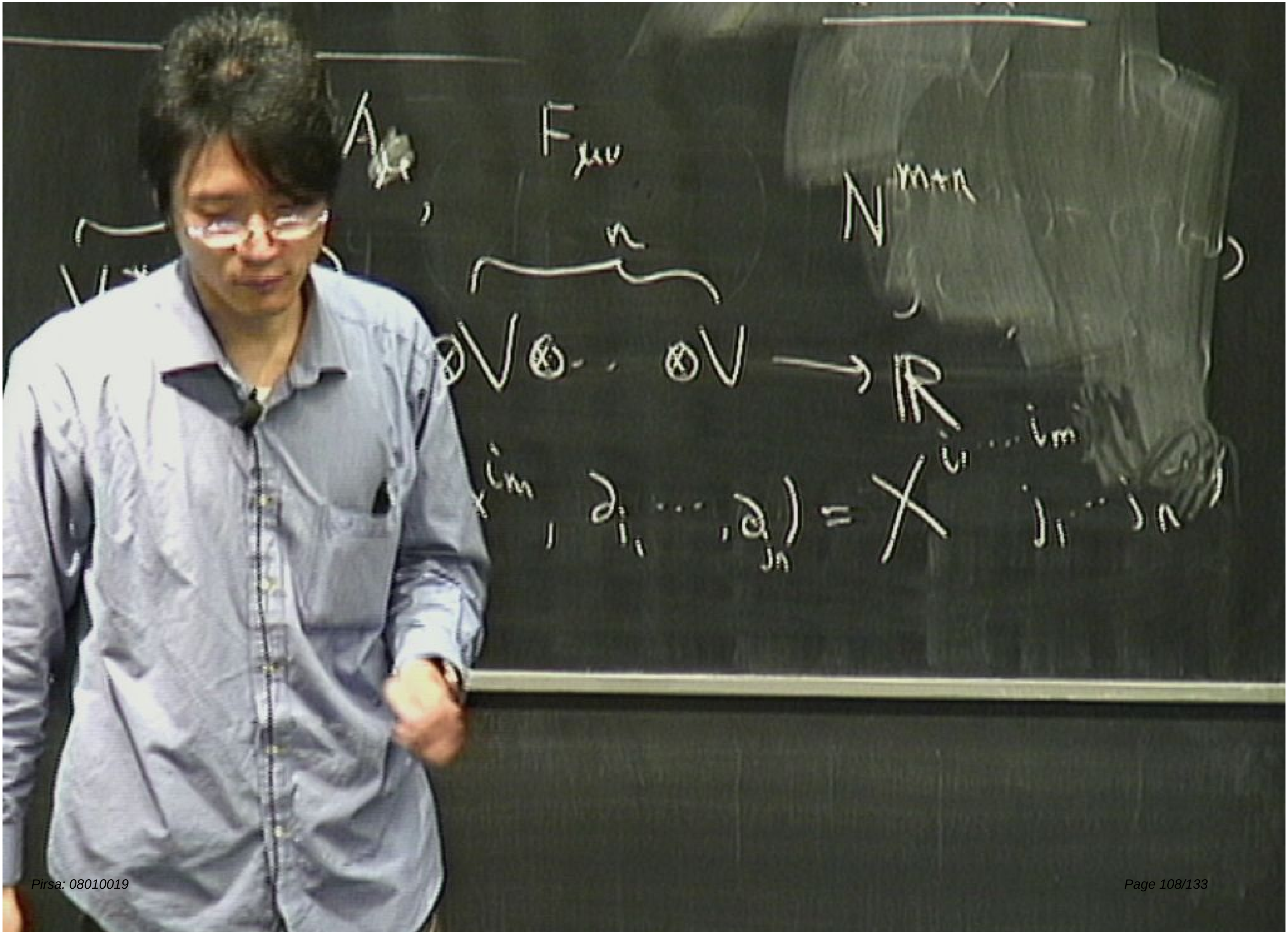
d :

A_μ , $F_{\mu\nu}$

$$X : \overbrace{V^* \otimes \cdots \otimes V^*}^{m} \otimes \overbrace{V \otimes \cdots \otimes V}^{n} \longrightarrow \mathbb{R}$$

linear

$$X(dx^{i_1}, \ldots, dx^{i_m}, \partial_{j_1}, \ldots, \partial_{j_n}) = X^{i_1 \cdots i_m}{}_{j_1 \cdots}$$

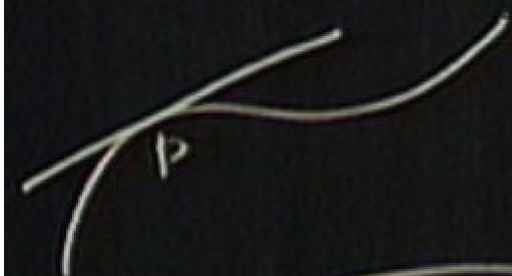


A_μ ,
F_μν
N^{m+n}
⊗V⊗···⊗V → R
, ∂_{i_1} ··· ,∂_{j_n}) = X^{i_1···i_m}_{j_1···j_n}

$\otimes V \longrightarrow \mathbb{R}$

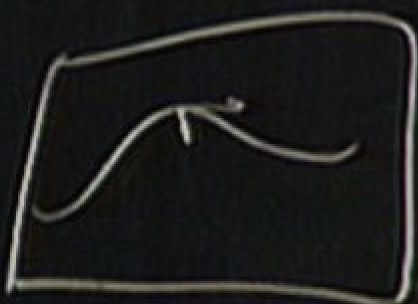
$(\partial_{i_1}, \dots, \partial_{j_n}) = \boxed{X^{i_1 \dots i_m}_{\;\;\;\;\;\;\;\;\;\;\; j_1 \dots j_n}}$

tangent space at a pt T_p



- Vector space dim n

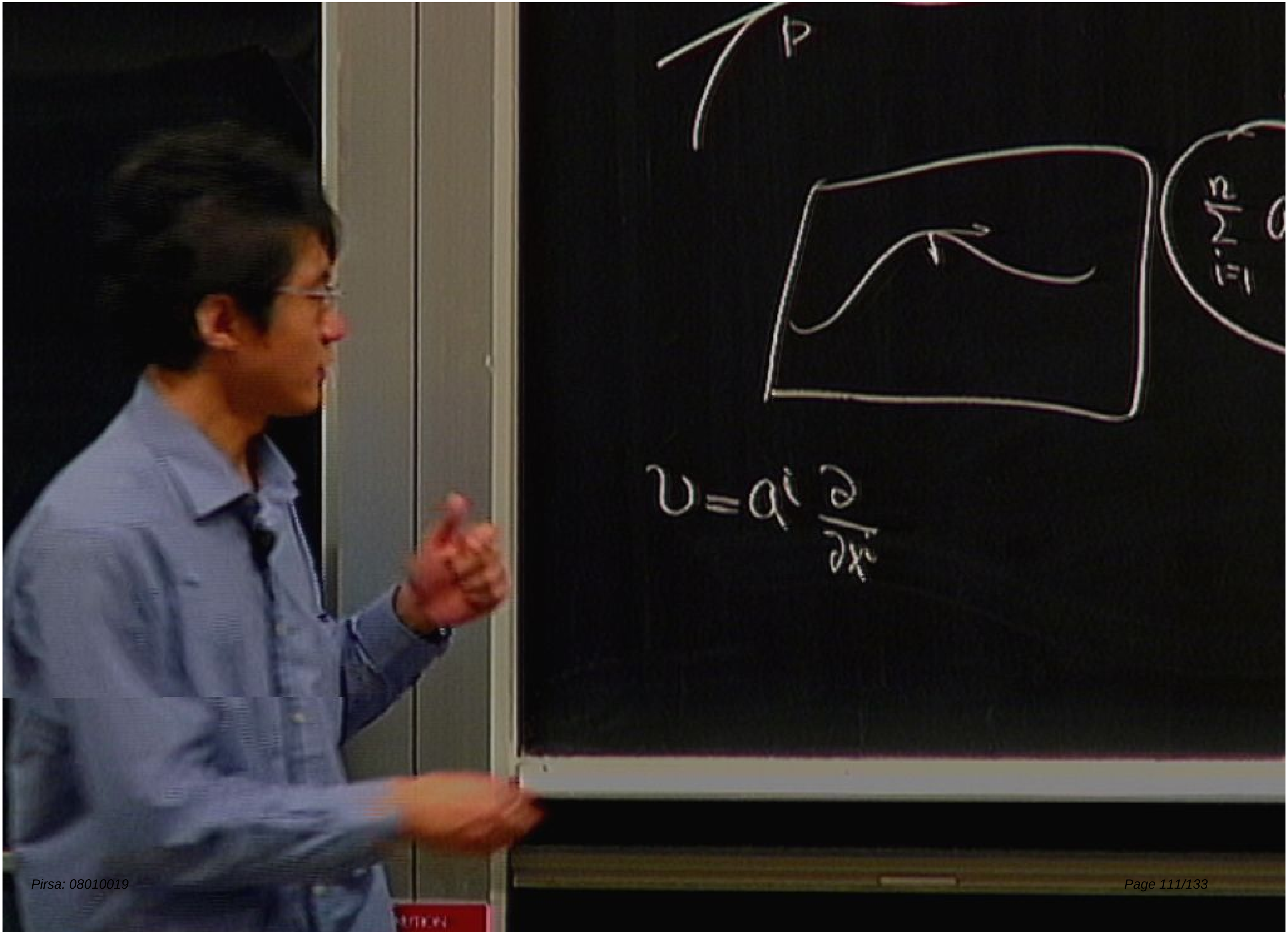
- $\{x^i,\ i=1,\cdots n\}$

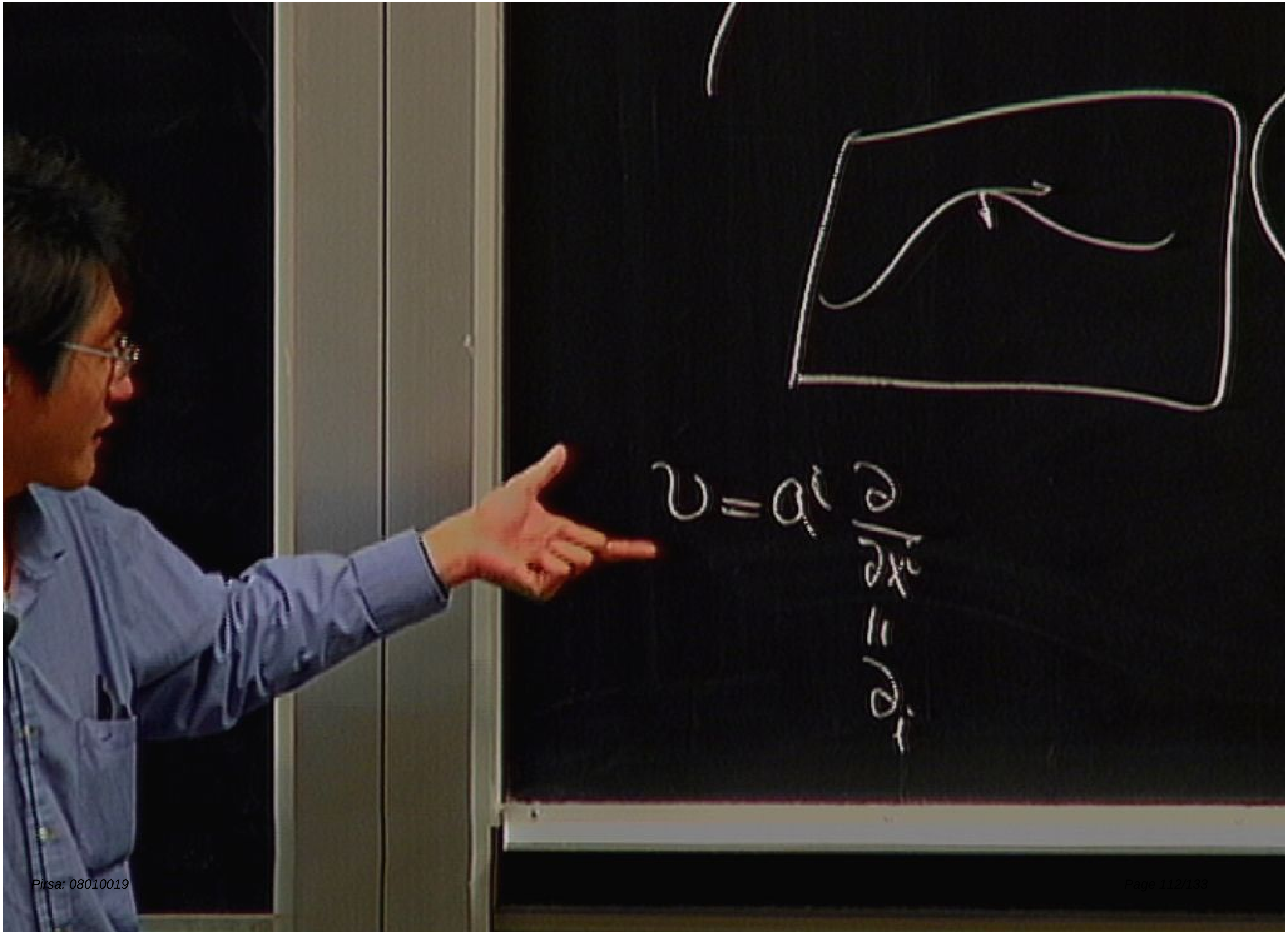


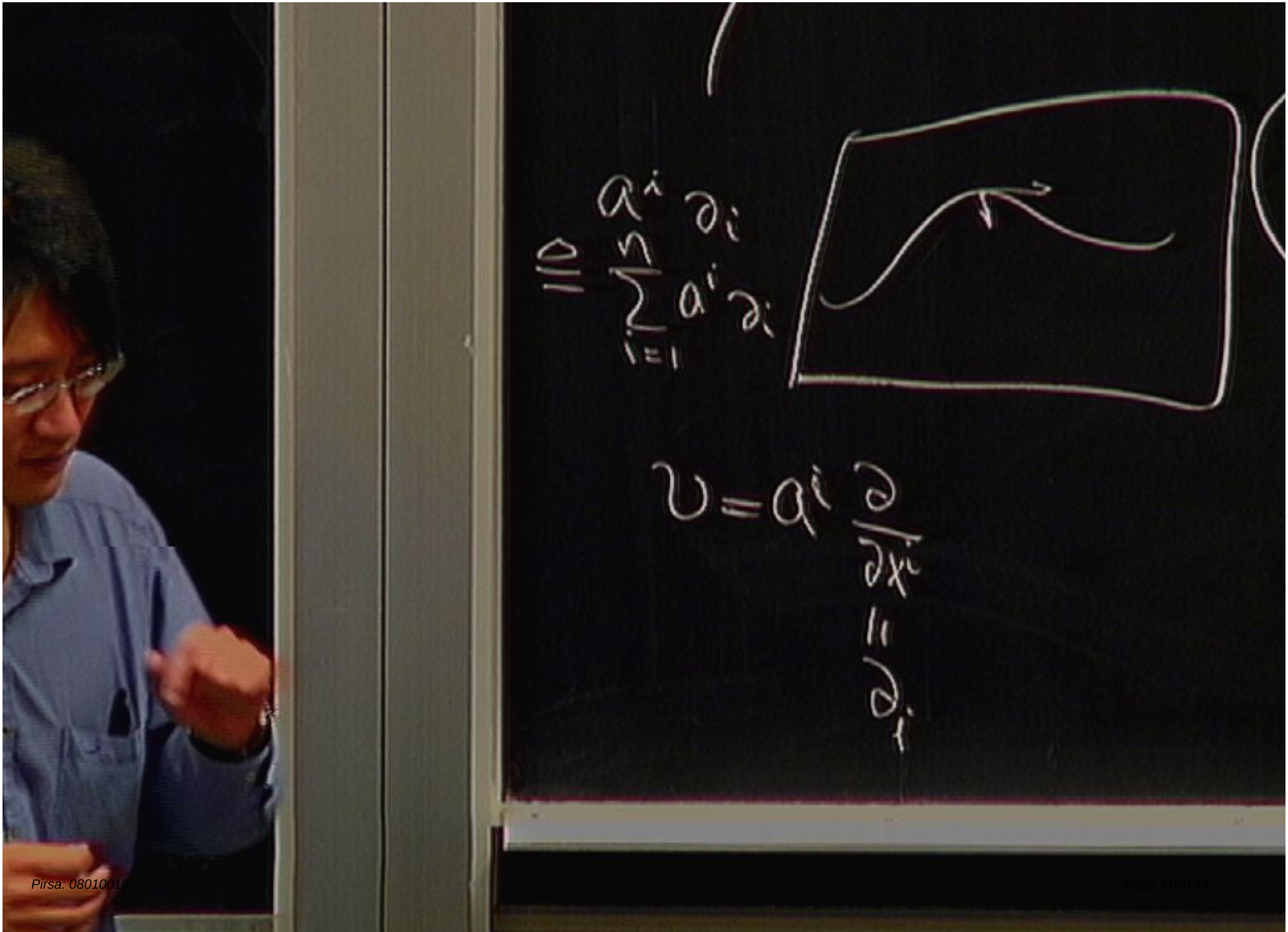
$$\sum_{i=1}^{n} a^i \frac{\partial}{\partial x^i}\Big|_p f(x)$$

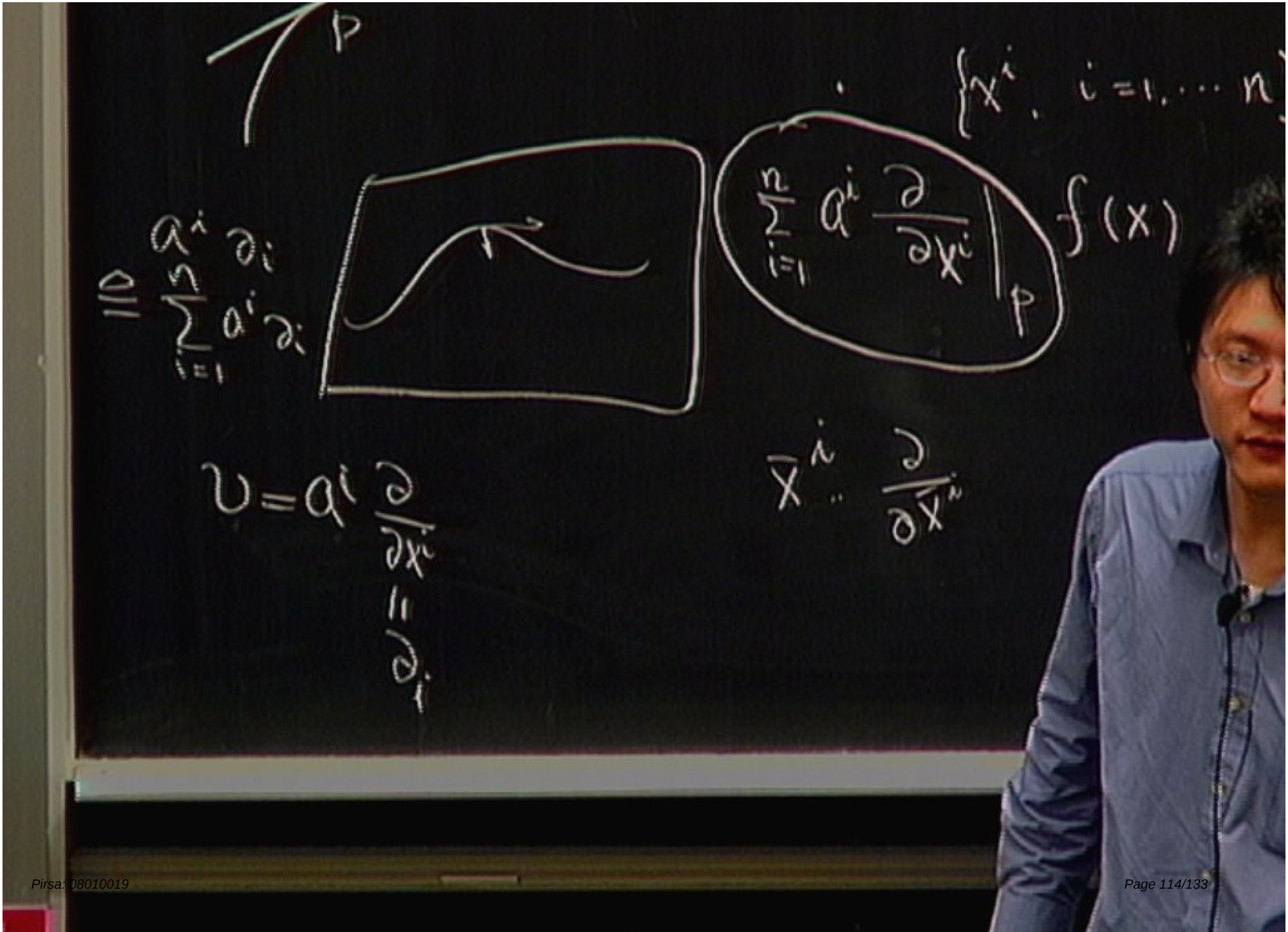
$$(a+b)V = aV + bV$$

$$\frac{\partial}{\partial x^i},\quad i=1,\cdots,n$$


p
$v = a^i \frac{\partial}{\partial x^i}$


$v = a^i \frac{\partial}{\partial x^i}$
$=$
∂_i


$a^i \partial_i$
$= \sum_{i=1}^{n} a^i \partial_i$
$v = a^i \frac{\partial}{\partial x^i}$
$\frac{\partial}{\partial x^i} = \partial_i$


p
{x^i : i = 1, ... n}
Σ_{i=1}^{n} a^i ∂/∂x^i |_p f(x)
a^i ∂_i
≡ Σ_{i=1}^{n} a^i ∂_i
v = a^i ∂/∂x^i = ∂_i
x̄^i ∂/∂x̄^i

p

$\{x^i,\ i = 1, \cdots n\}$

$a^i \partial_i \equiv \sum_{i=1}^{n} a^i \partial_i$

$\sum_{i=1}^{n} a^i \frac{\partial}{\partial x^i}\Big|_p f(x)$

$v = a^i \frac{\partial}{\partial x^i} = b^i \bar{\partial}_i$

$\frac{\partial}{\partial x^i} = \partial_i$

$\bar{x}^i : \frac{\partial}{\partial \bar{x}^i}$

$$a^i \partial_i f = b^i \bar{\partial}_i f \qquad \forall f$$

$$\|$$

$$a^i \frac{\partial f}{\partial x^i}$$

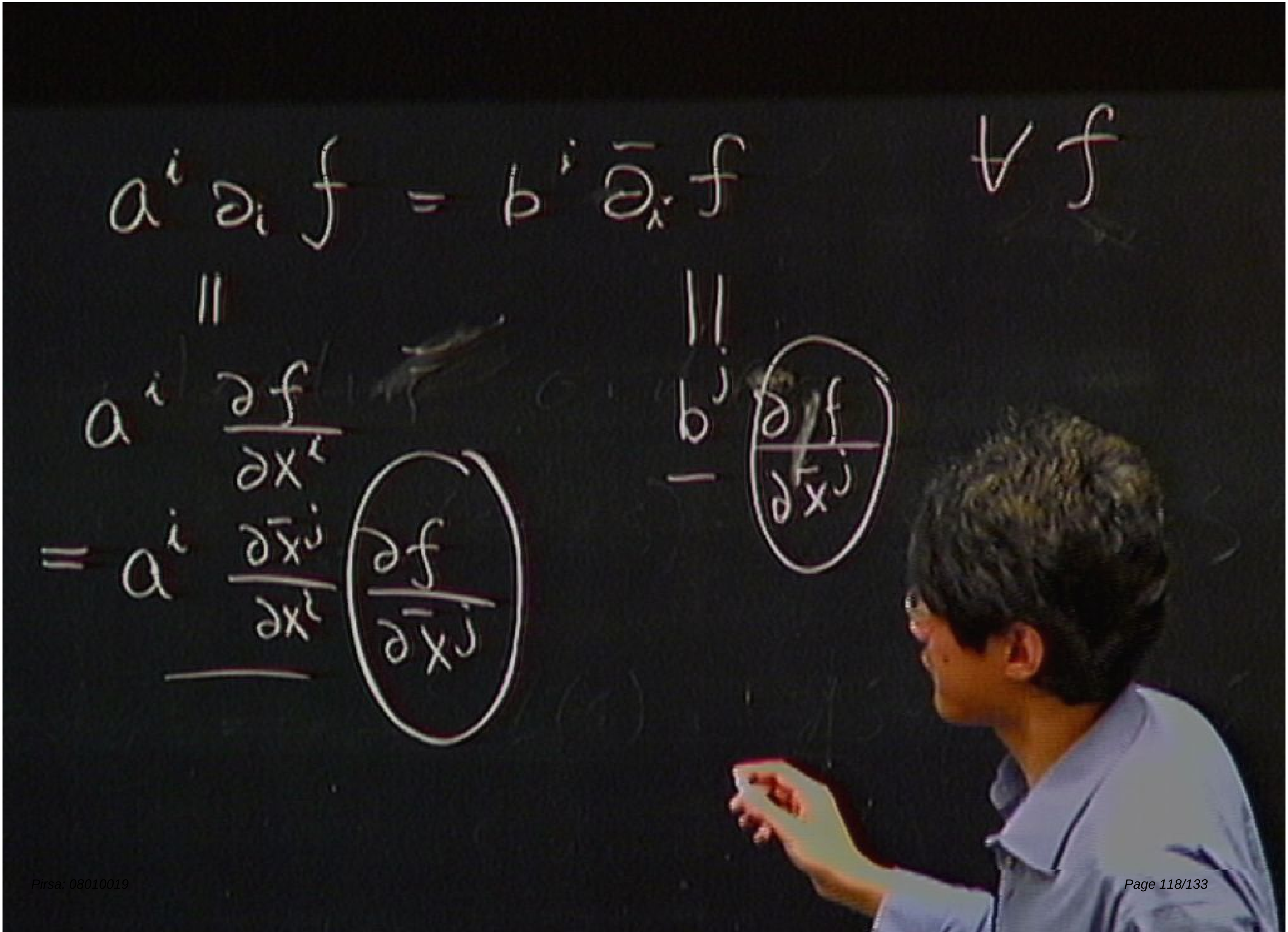
$$= a^i \frac{\partial \bar{x}^j}{\partial x^i} \frac{\partial f}{\partial \bar{x}^j}$$

$$a^i \partial_i f = b^i \bar{\partial}_i f \qquad \forall f$$

$$\| \qquad\qquad\qquad \|$$

$$a^i \frac{\partial f}{\partial x^i} \qquad\qquad b^j \frac{\partial f}{\partial \bar{x}^j}$$

$$= a^i \frac{\partial \bar{x}^j}{\partial x^i} \frac{\partial f}{\partial \bar{x}^j}$$


$a^i \partial_i f = b^i \bar{\partial}_i f \quad \forall f$
$a^i \frac{\partial f}{\partial x^i}$
$b^j \frac{\partial f}{\partial \bar{x}^j}$
$= a^i \frac{\partial \bar{x}^j}{\partial x^i} \frac{\partial f}{\partial \bar{x}^j}$

$$a^i \partial_i f = b^i \bar{\partial}_i f \qquad \forall f$$

$$a^i \frac{\partial f}{\partial x^i} = b^j \frac{\partial f}{\partial \bar{x}^j}$$

$$= a^i \frac{\partial \bar{x}^j}{\partial x^i} \frac{\partial f}{\partial \bar{x}^j}$$

$$b^j = a^i \frac{\partial \bar{x}^j}{\partial x^i}$$

$$a^i \partial_i f = b^i \bar{\partial}_i f \qquad \forall f$$

$$a^i \frac{\partial f}{\partial x^i} \qquad\qquad b^j \frac{\partial f}{\partial \bar{x}^j}$$

$$= a^i \frac{\partial \bar{x}^j}{\partial x^i} \frac{\partial f}{\partial \bar{x}^j}$$

$$\boxed{b^j = a^i \frac{\partial \bar{x}^j}{\partial x^i}}$$

$$a^i \partial_i f = b^i \bar{\partial}_i f \quad \forall f$$

$$a^i \frac{\partial f}{\partial x^i} \qquad\qquad b^j \frac{\partial f}{\partial \bar{x}^j}$$

$$= a^i \frac{\partial \bar{x}^j}{\partial x^i} \frac{\partial f}{\partial \bar{x}^j}$$

$$\boxed{b^j\Big|_p = a^i\Big|_p \frac{\partial \bar{x}^j}{\partial x^i}\Big|_p}$$

$$a^i \partial_i f = b^i \bar{\partial}_i f \qquad \forall f$$

$$a^i \frac{\partial f}{\partial x^i} \qquad\qquad b^j \frac{\partial f}{\partial \bar{x}^j}$$

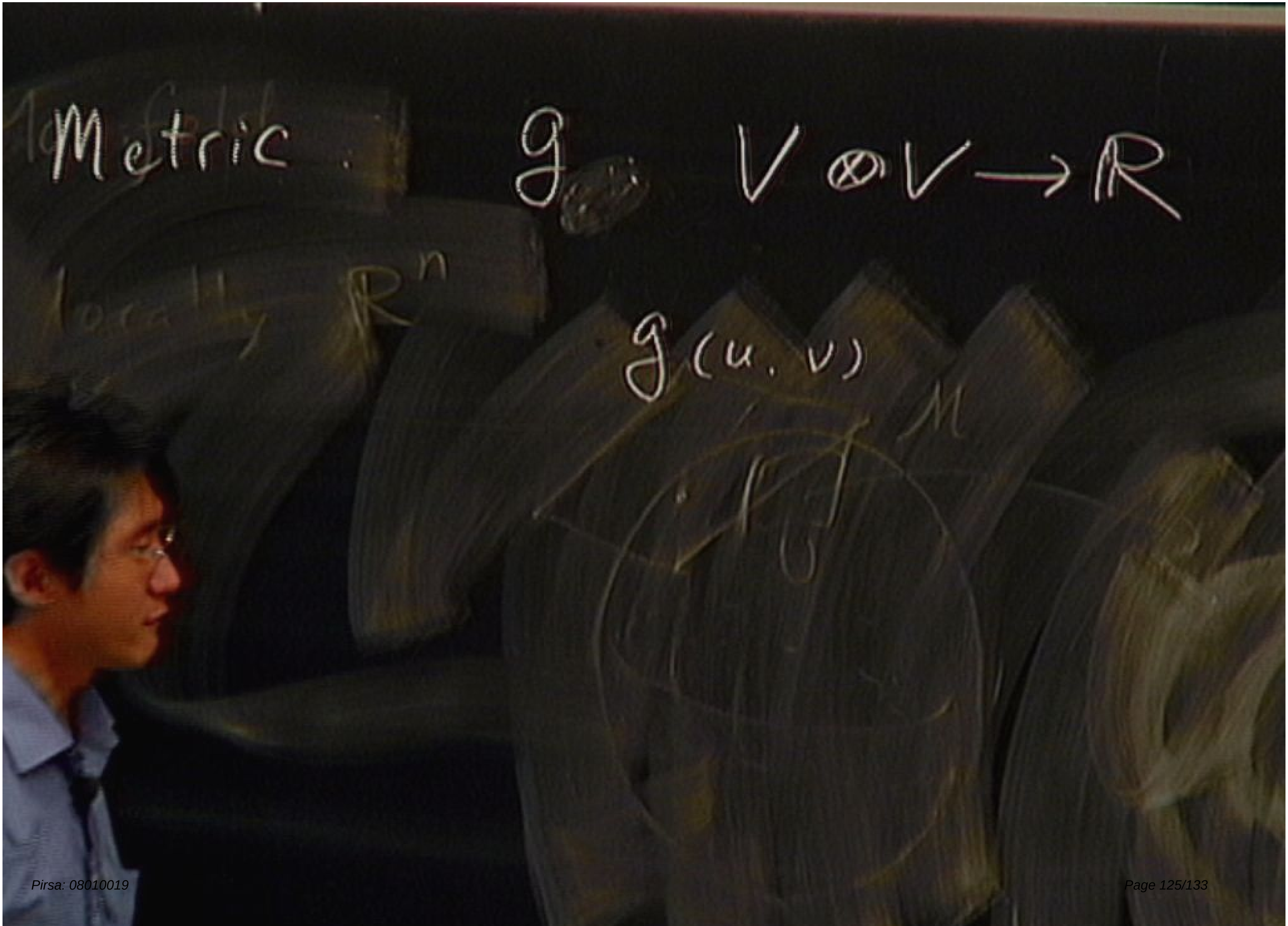
$$= a^i \frac{\partial \bar{x}^j}{\partial x^i} \frac{\partial f}{\partial \bar{x}^j}$$

$$\boxed{b^j\big|_p = a^i\big|_p \frac{\partial \bar{x}^j}{\partial x^i}\bigg|_p}$$

$$\bar{X}^{ij}{}_{k} = X^{lm}{}_{n} \frac{\partial \bar{x}^i}{\partial x^l} \frac{\partial \bar{x}^j}{\partial x^m} \frac{\partial x^n}{\partial \bar{x}^k}$$

$$\boxed{b^j\big|_p = a}$$

$$\bar{X}^{ij}{}_{k} = X^{lm}{}_{n} \frac{\partial \bar{x}^{i}}{\partial x^{l}} \frac{\partial \bar{x}^{j}}{\partial x^{m}} \frac{\partial x^{n}}{\partial \bar{x}^{k}}$$

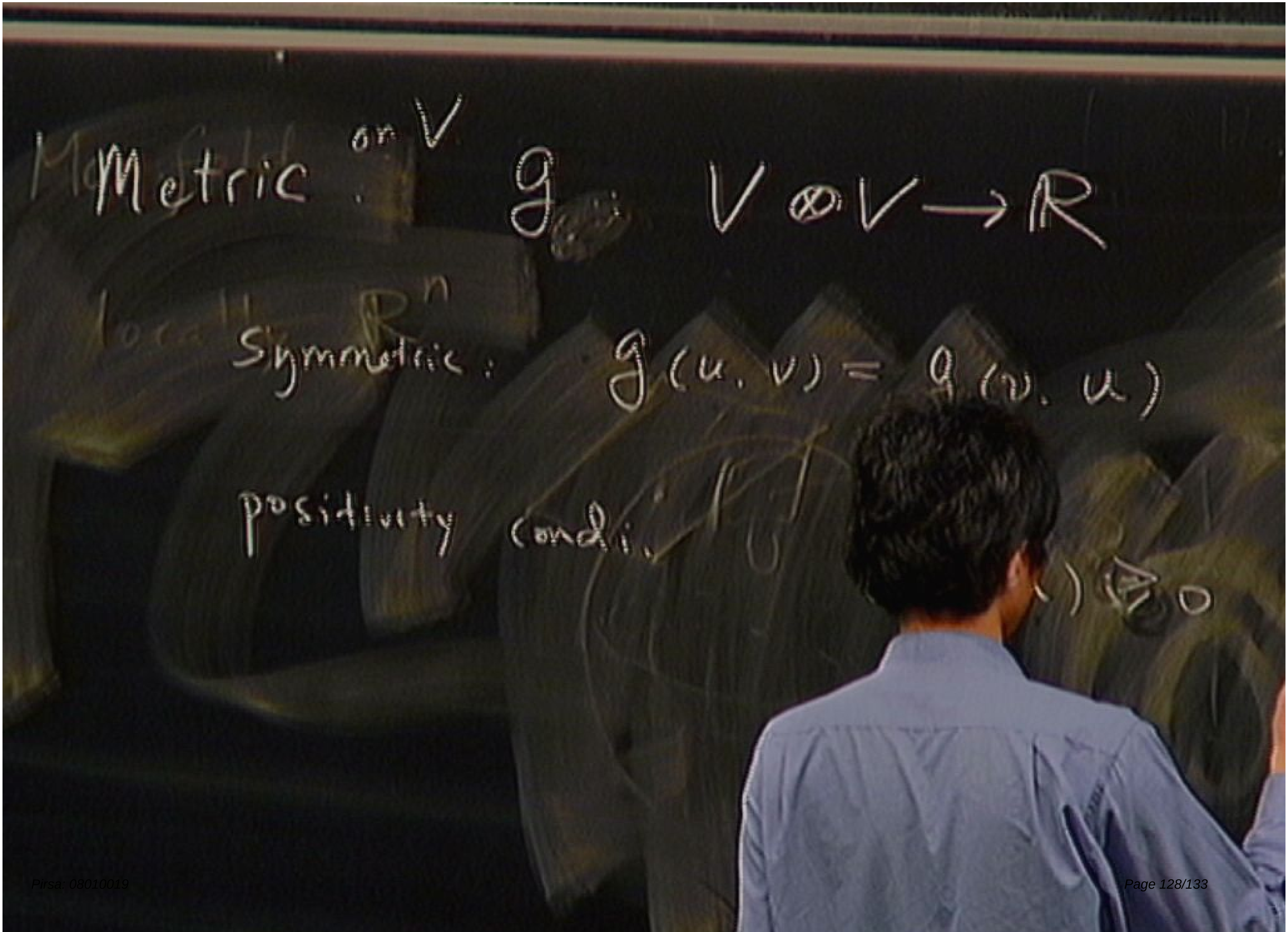

Metric :
g : V ⊗ V → R
g(u, v)

Metric: $g : V \otimes V \to \mathbb{R}$

Symmetric: $g(u, v) = g(v, u)$

Metric on V : $g : V \otimes V \to \mathbb{R}$

Symmetric : $g(u, v) = g(v, u)$

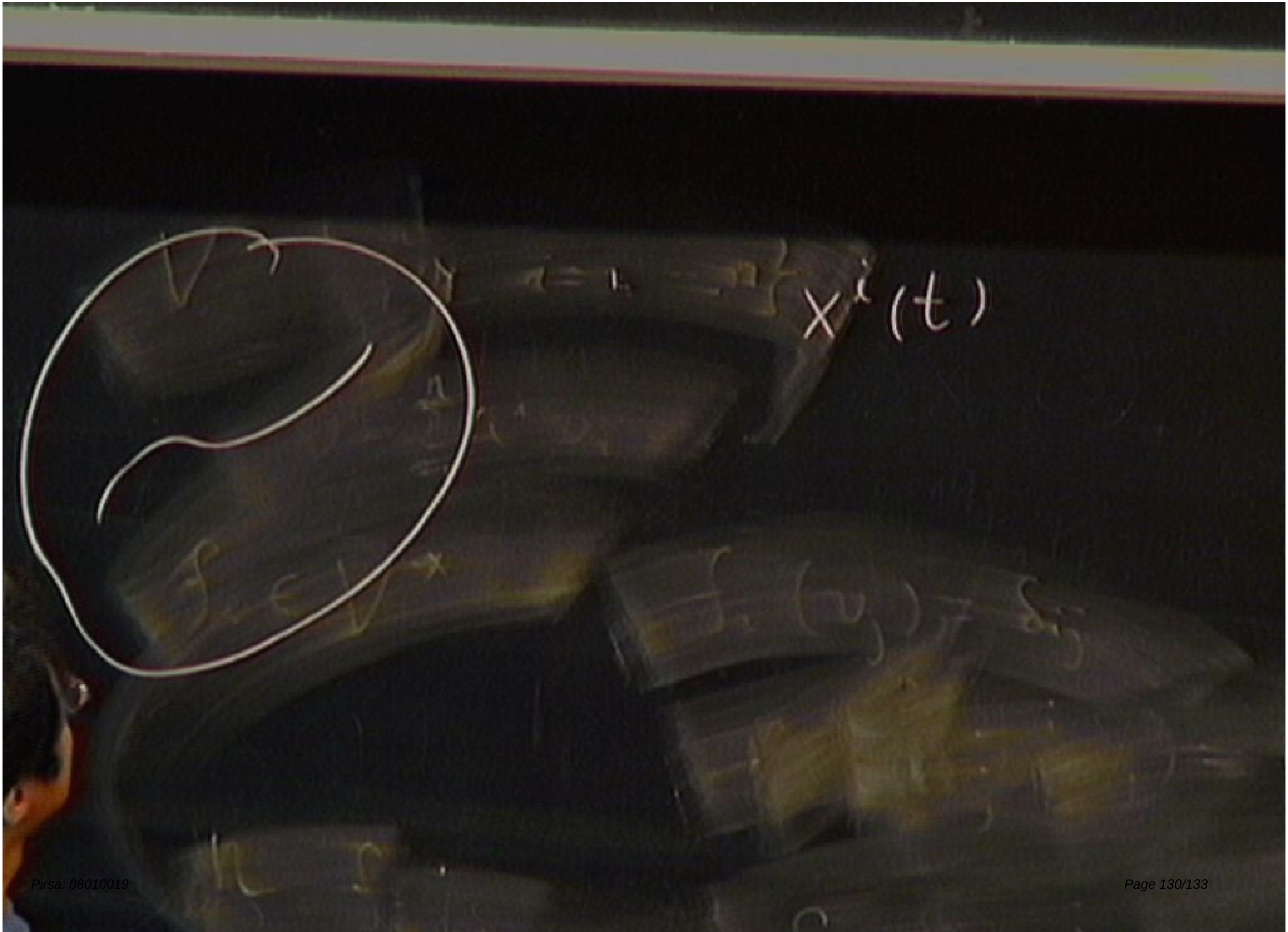

Metric on V : g : V ⊗ V → ℝ
Symmetric : g(u, v) = g(v, u)
positivity condi.

Metric on V g : $V \otimes V \to \mathbb{R}$

Symmetric : $g(u,v) = g(v,u)$

positivity condi. $g(u,u) \geq 0$

$= 0$ iff $u = 0$

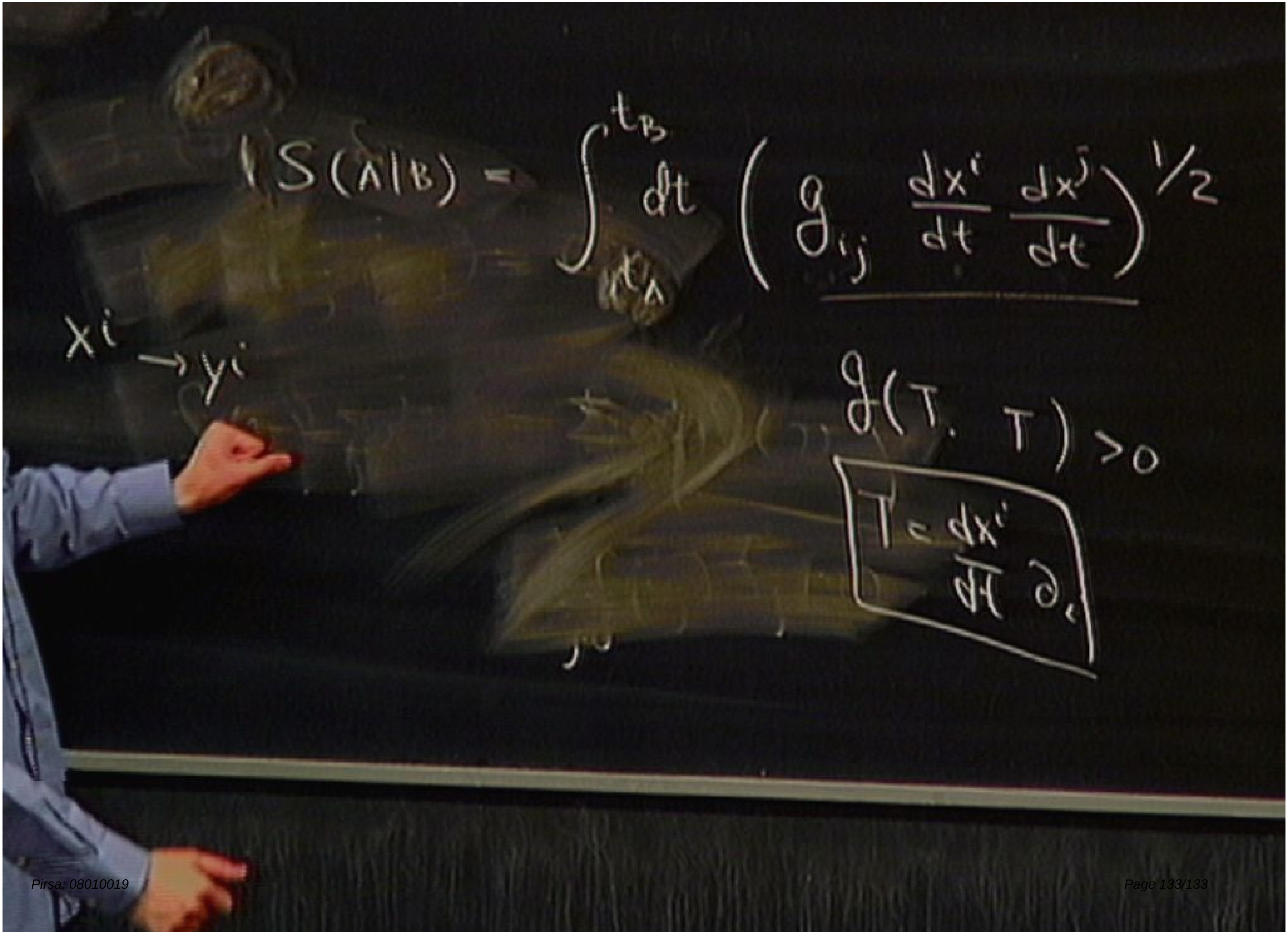

$x^i(t)$

$$S(A|B) = \int_{t_A}^{t_B} dt \left(g_{ij} \frac{dx^i}{dt} \frac{dx^j}{dt} \right)^{1/2}$$

$$S(A|B) = \int_{t_A}^{t_B} dt \left(g_{ij} \frac{dx^i}{dt} \frac{dx^j}{dt} \right)^{1/2}$$

$$g(T, T) > 0$$

$$\boxed{T = \frac{dx^i}{dt} \partial_i}$$


$S(A|B) = \int_{t_A}^{t_B} dt \left(g_{ij} \frac{dx^i}{dt} \frac{dx^j}{dt} \right)^{1/2}$
$x^i \to y^i$
$g(T, T) > 0$
$\boxed{T = \frac{dx^i}{dt} \partial_i}$