Title: Foundations of Quantum Mechanics #3

Date: Jan 15, 2008 06:30 PM

URL: http://pirsa.org/08010018

Abstract: Interferometry, measurement and interpretation. Beyond the quanta.

Errata. Page 3.

$$\xi(x)=\int_y \phi^*(y)\Psi(x,y)dx$$

should read

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Hidden Variables

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- Original quantum theory! De Broglie, 1924-1927.
  - Wave *or* particle?
  - Wave *and* particle!

$$\psi(x,t)=|\psi(x,t)|e^{iS(x,t)}$$

For a plane wave Ae^{ikx} so $p=\hbar k=\hbar\nabla S(x,t)$

$$m\dot{x}=\hbar\nabla S(x,t) \qquad P(x|t=t_0)=|\psi(x,t_0)|^2$$

Conservation equation $\frac{\partial P(x,t)}{\partial t}+\nabla J=0$

$$J(x,t)=\frac{\psi^*(x,t)\hbar\nabla\psi(x,t)-\psi(x,t)\hbar\nabla\psi^*(x,t)}{2im}=\frac{P(x,t)\hbar\nabla S(x,t)}{m}=P(x,t)\dot{x}$$

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Quantum Non-locality

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- The EPR argument
  - What Einstein (probably) intended
- Bell Inequalities
  - Bell's original argument
  - The derivation of the CHHS inequality
  - Locality and Non-locality
  - Loopholes: logical
  - Loopholes: empirical

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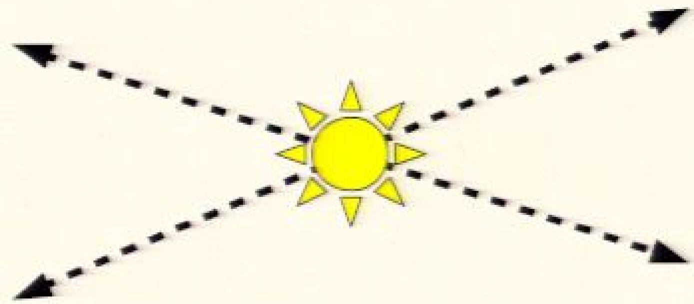

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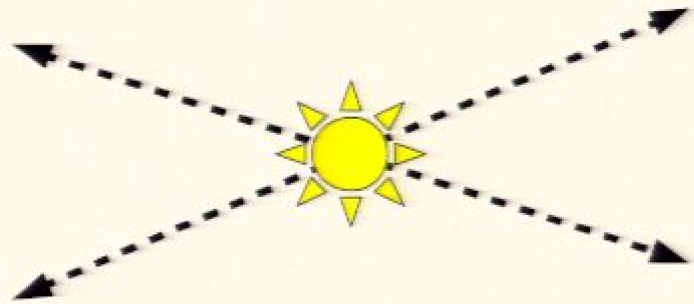

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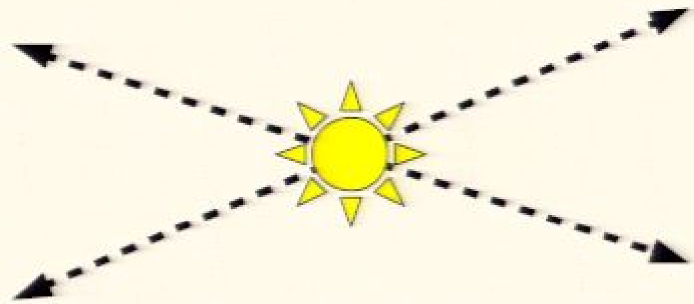

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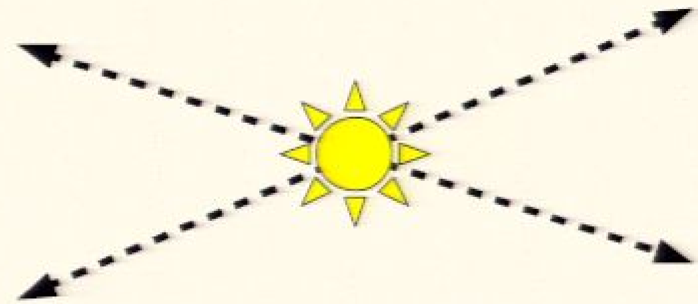

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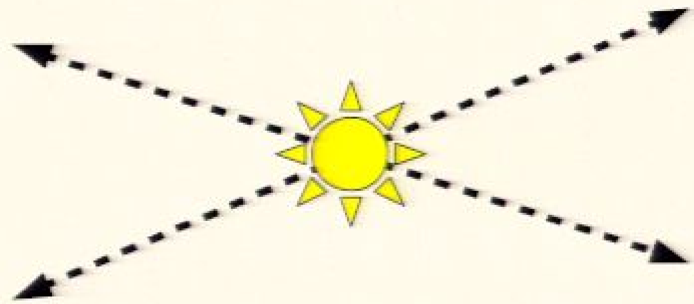

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As the second possibility is non-local, EPR opt for the first

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- 1966: Bell shows error in von Neumann proof, defending hidden variables

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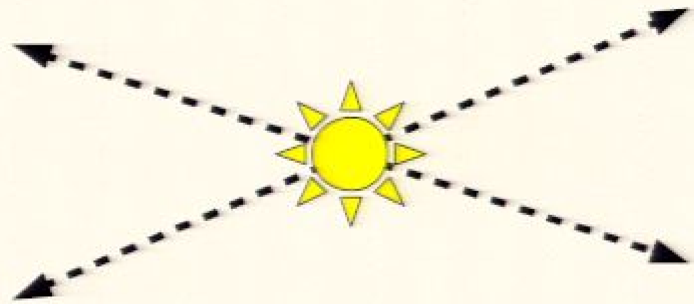

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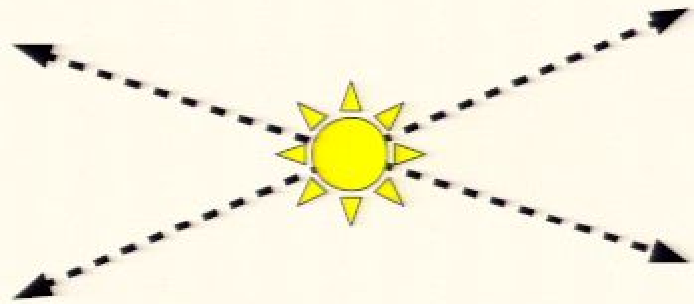

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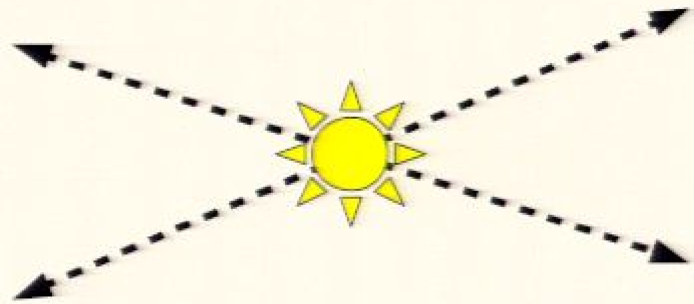

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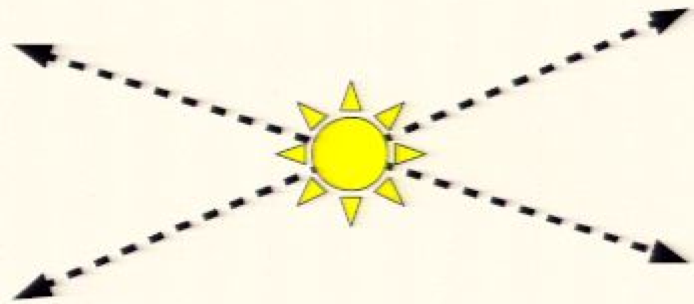

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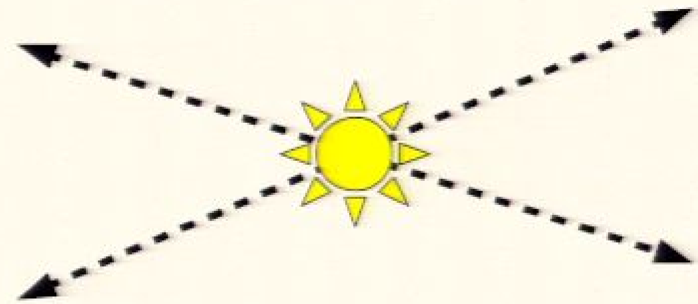

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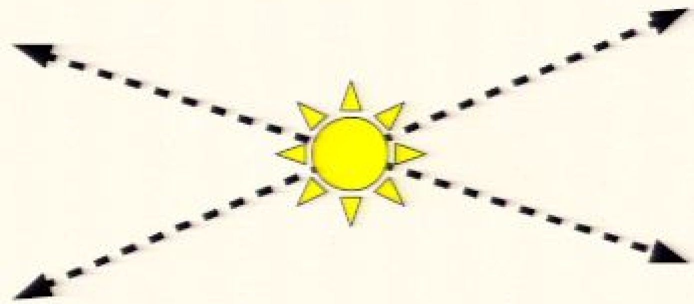

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Bell concludes from this and EPR that
quantum theory is irreducibly, ineliminably, unequivocally
non-local.

The CHHS Inequality

1969: Clauser, Holt, Horne, Shimony derive a more general inequality, also violated by the singlet state in quantum theory:

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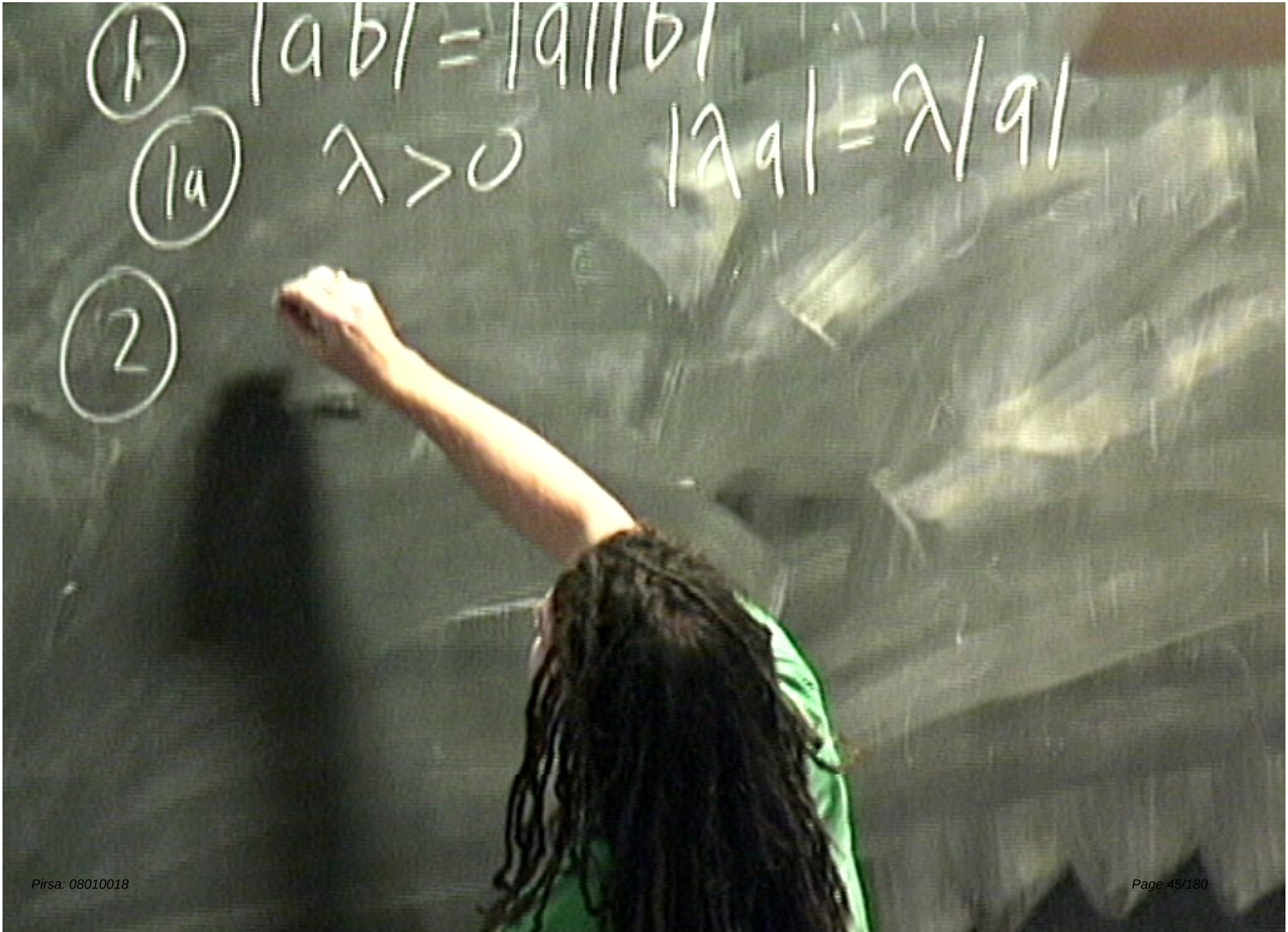
Emprical: detector efficiency, noise, time coincidence, memory, fair sampling...

"Principle of Common Cause"

Reichenbach, 1956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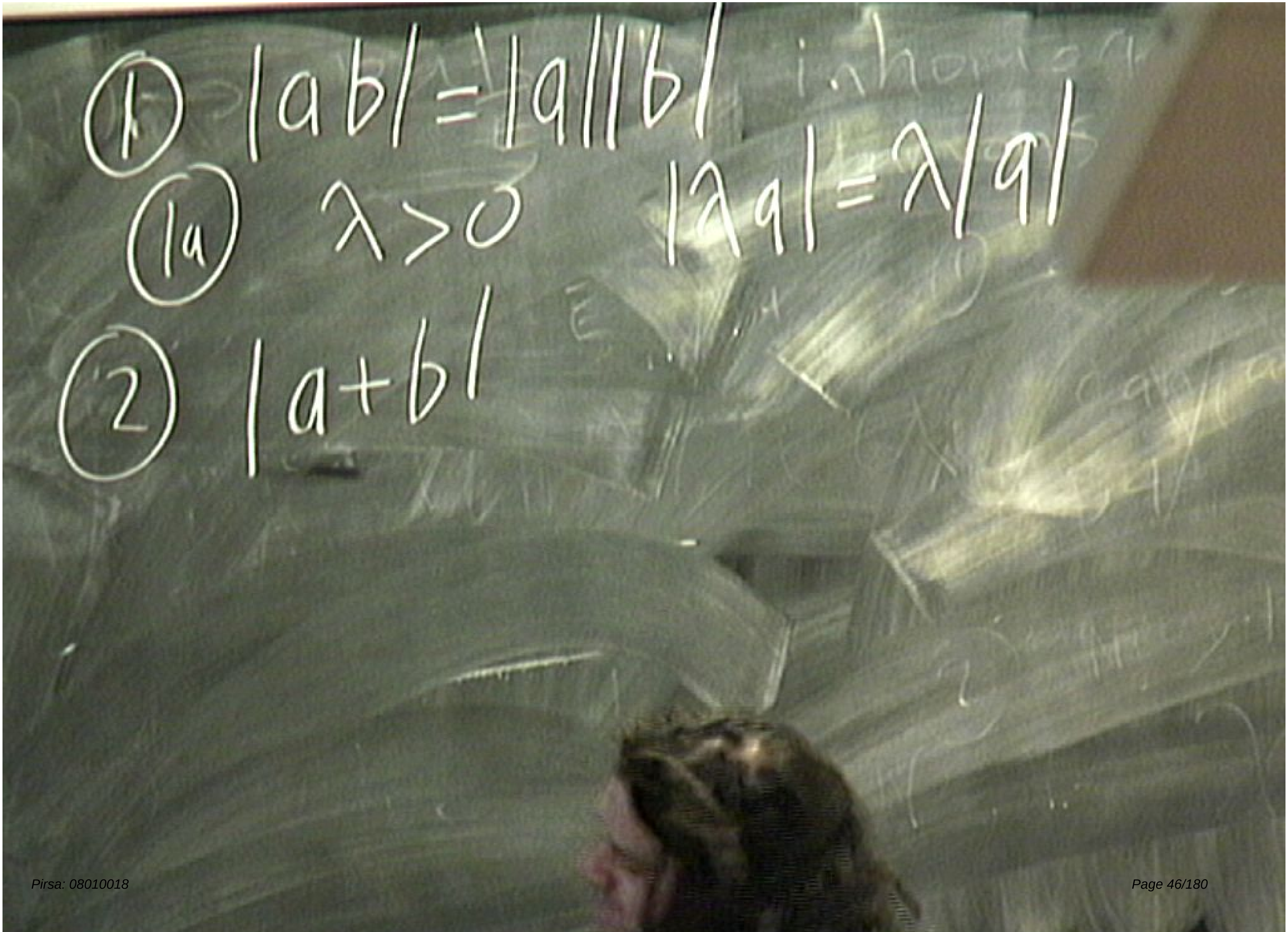
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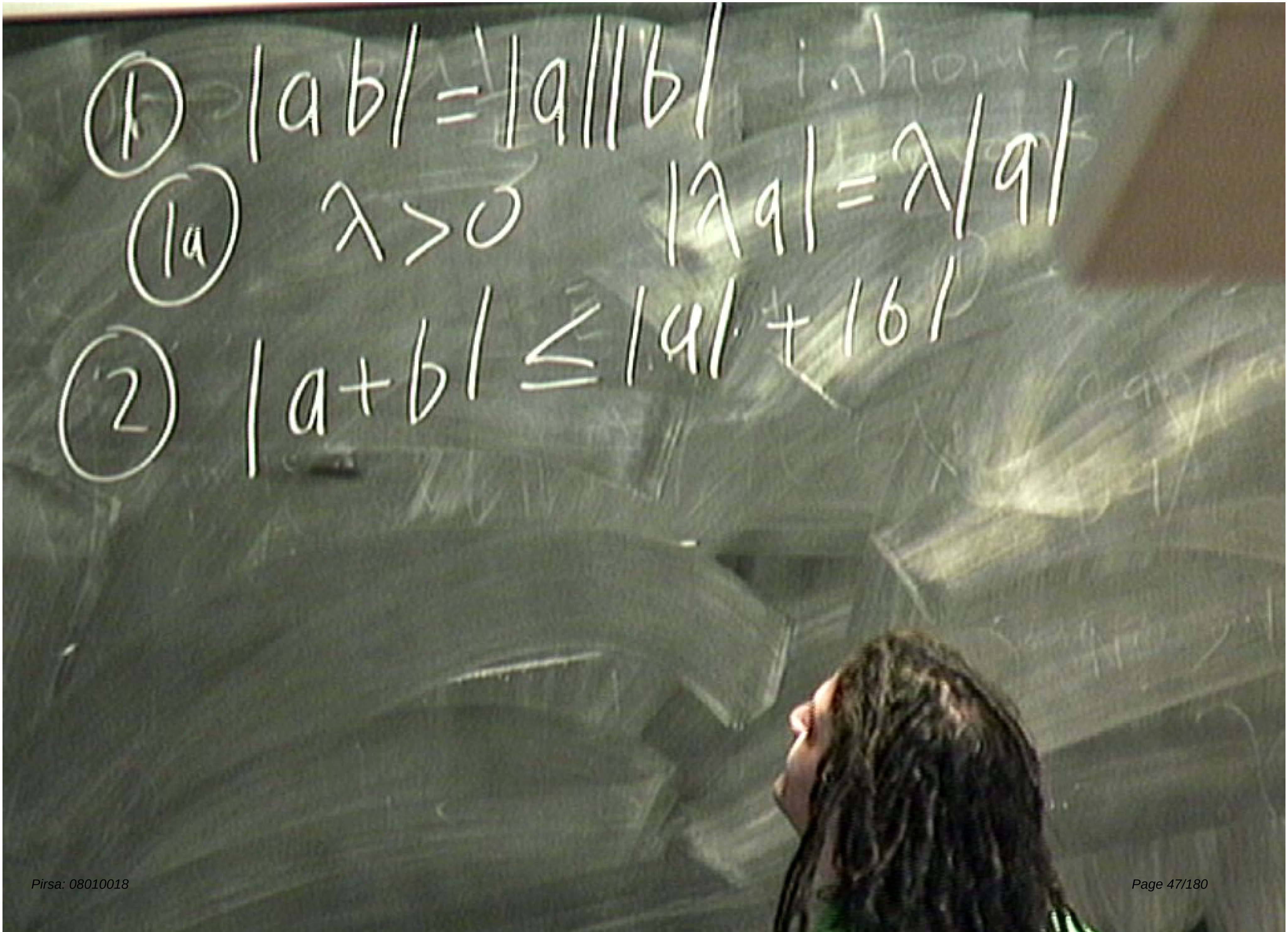

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(3) IF $0 \leq \lambda_i \leq 1$

$$\sum \lambda_i |x_i| \leq \sum |x_i|$$

"Principle of Common Cause"

Reichenbach, 1956

Suppose there is a possibility of events occurring, A,B $\qquad P(A,B)$

and suppose there is a correlation between these events: $P(A,B) \neq P(A)P(B)$

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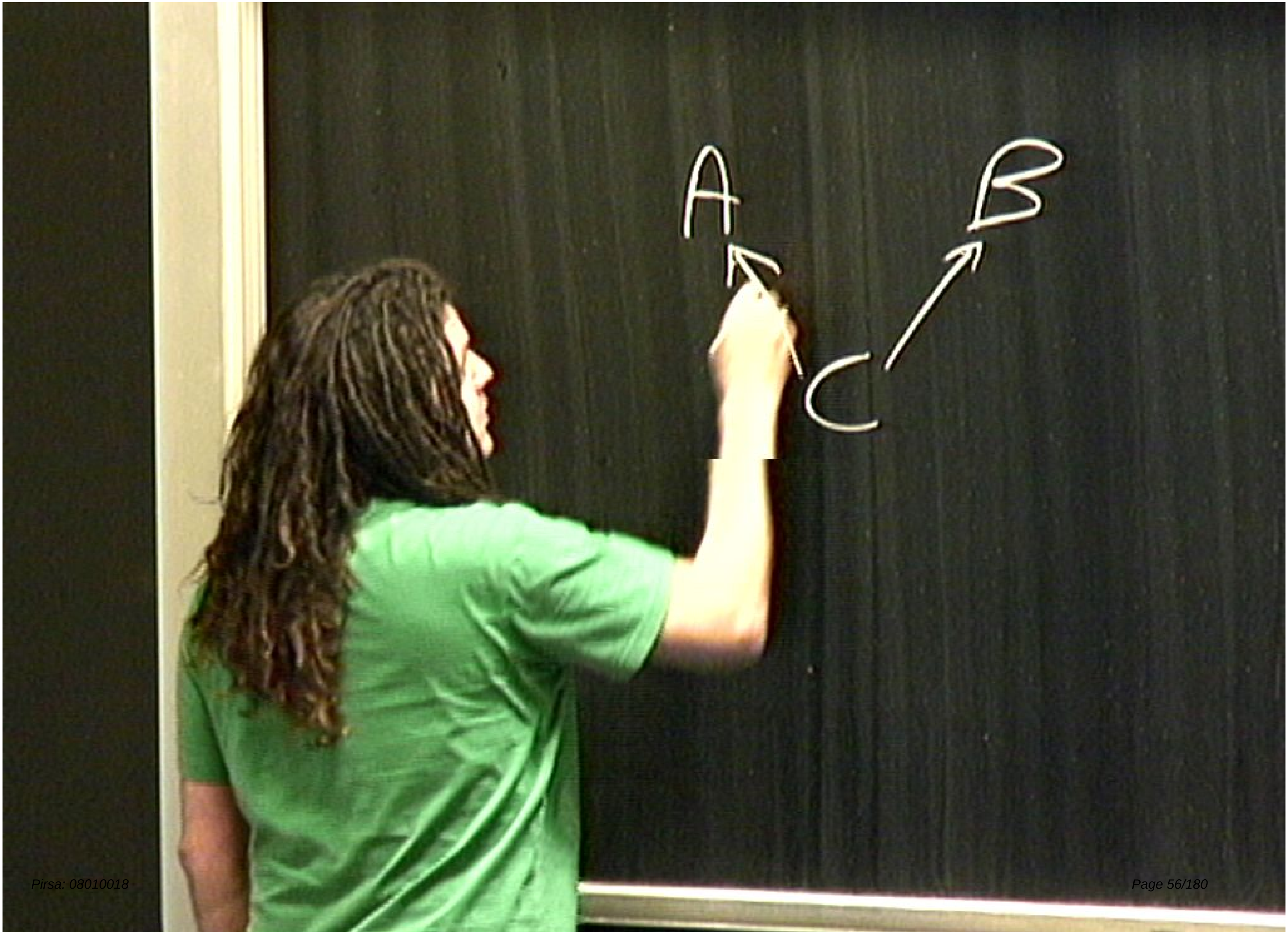
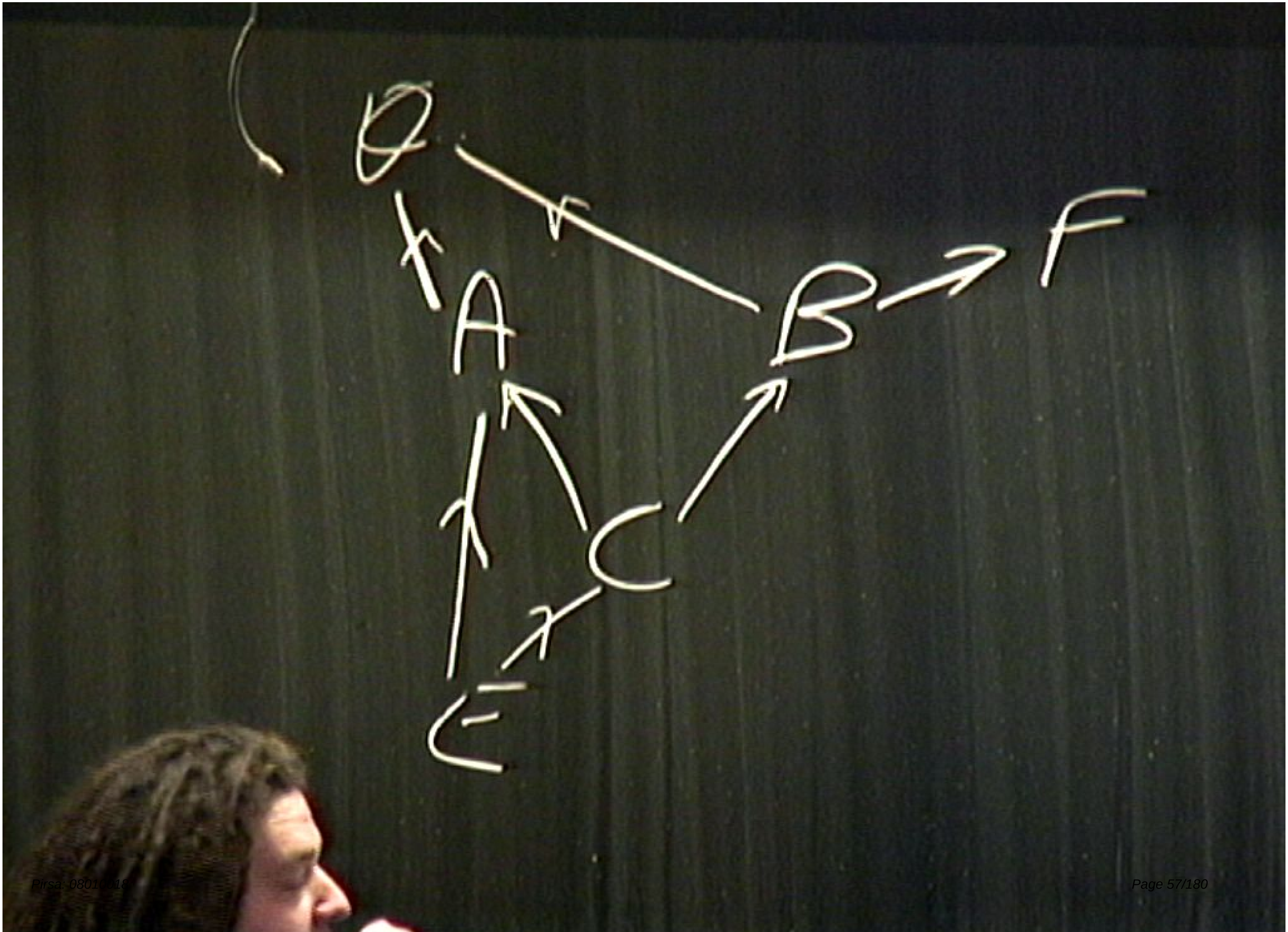
or

3) There exists another event, C, which is a common cause that has a causal influence upon the occurrence of both A and B such that:

$$P(A, B|C) = P(A|C) P(B|C)$$

Relativistic Causality


A
B
C


A
B
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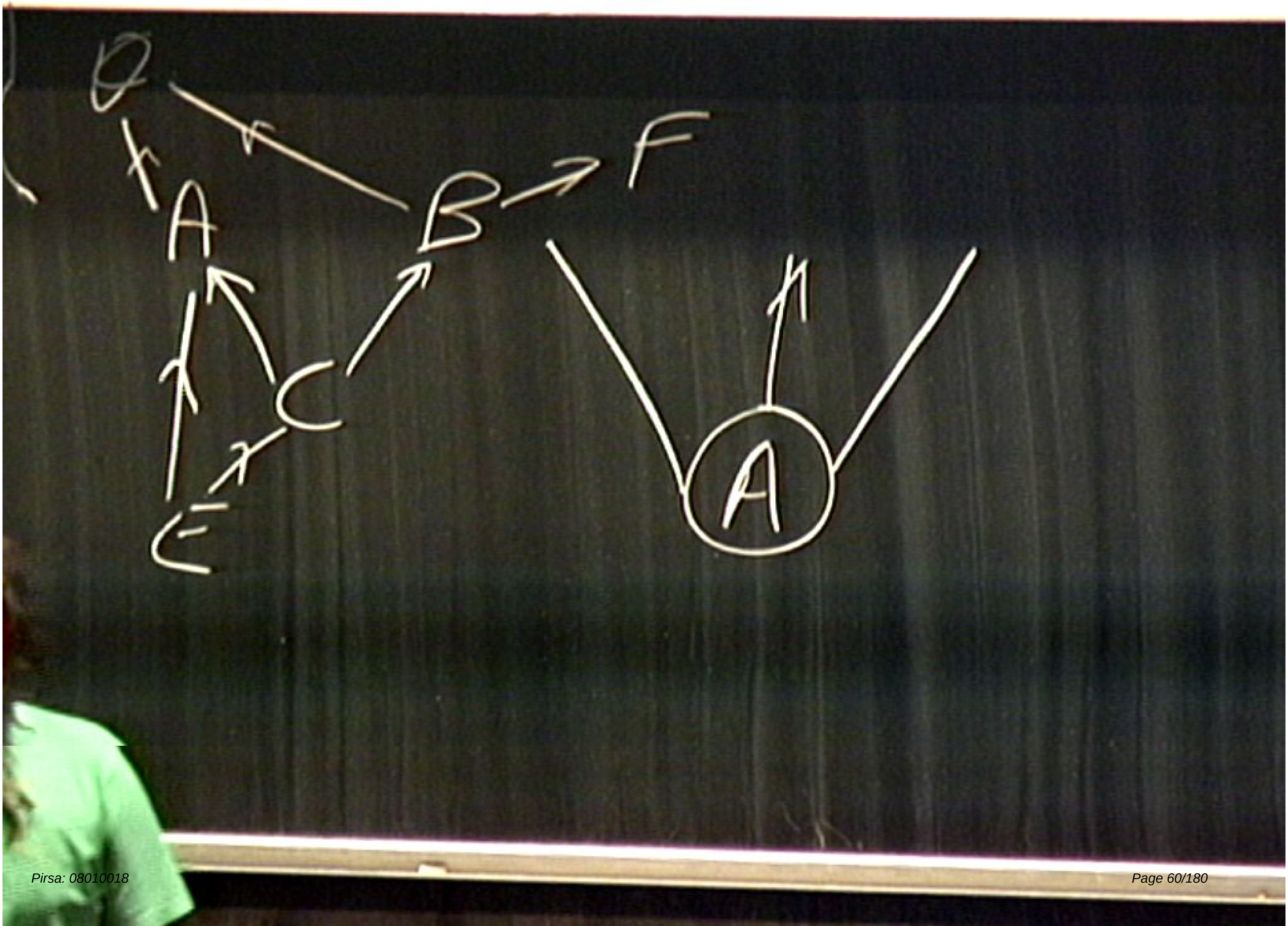
Relativistic Causality

Relativistic Causality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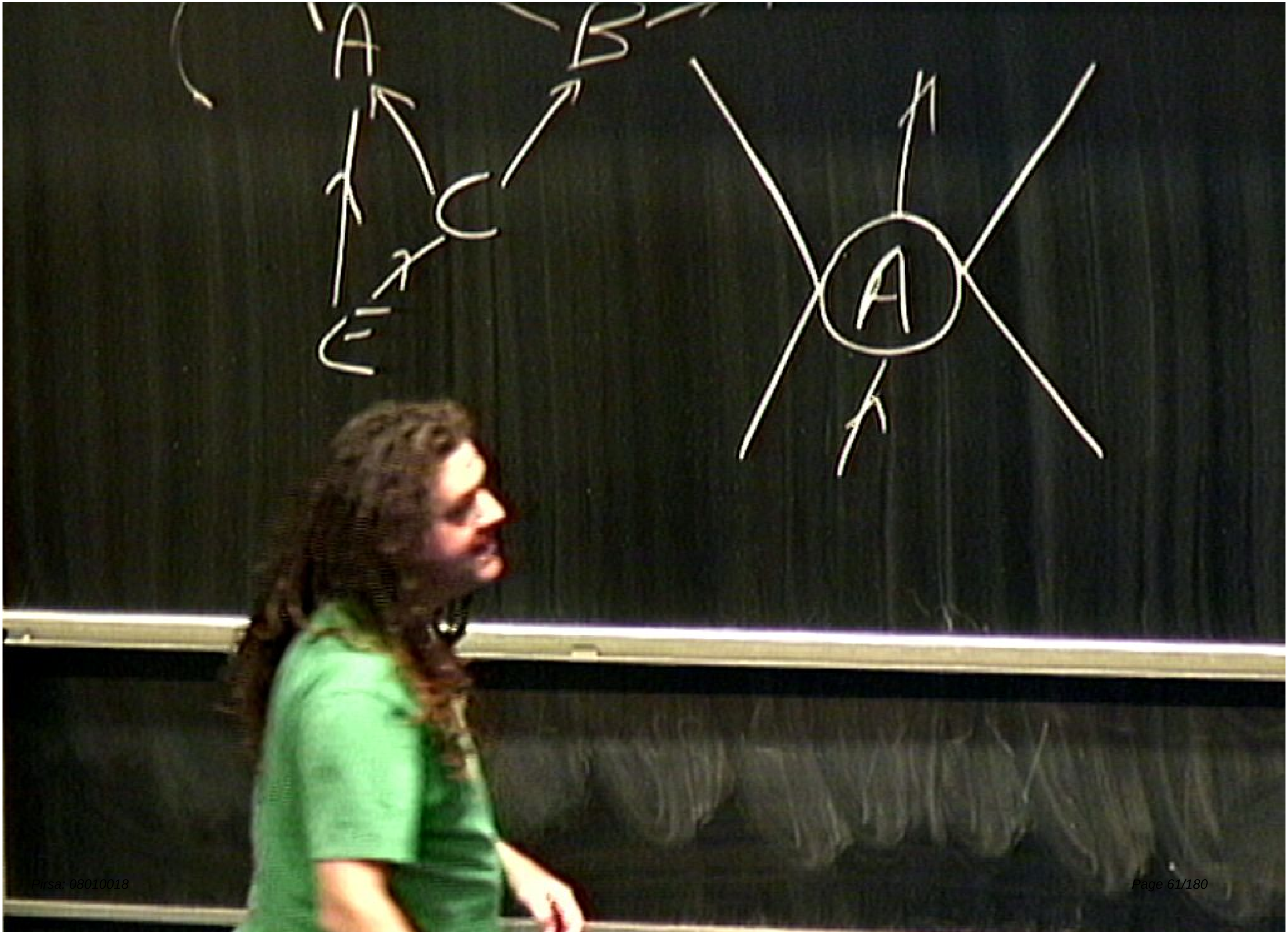
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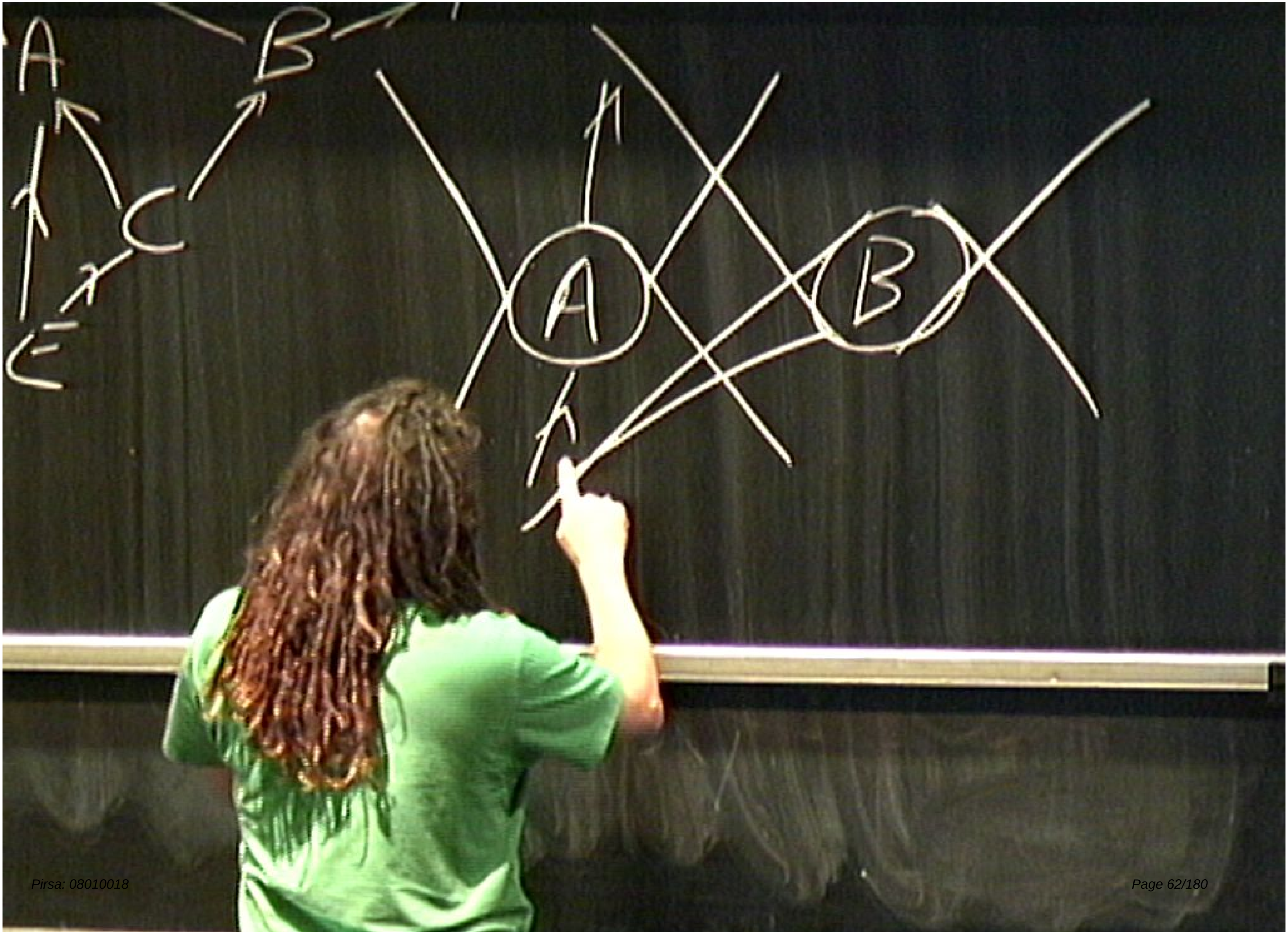
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C
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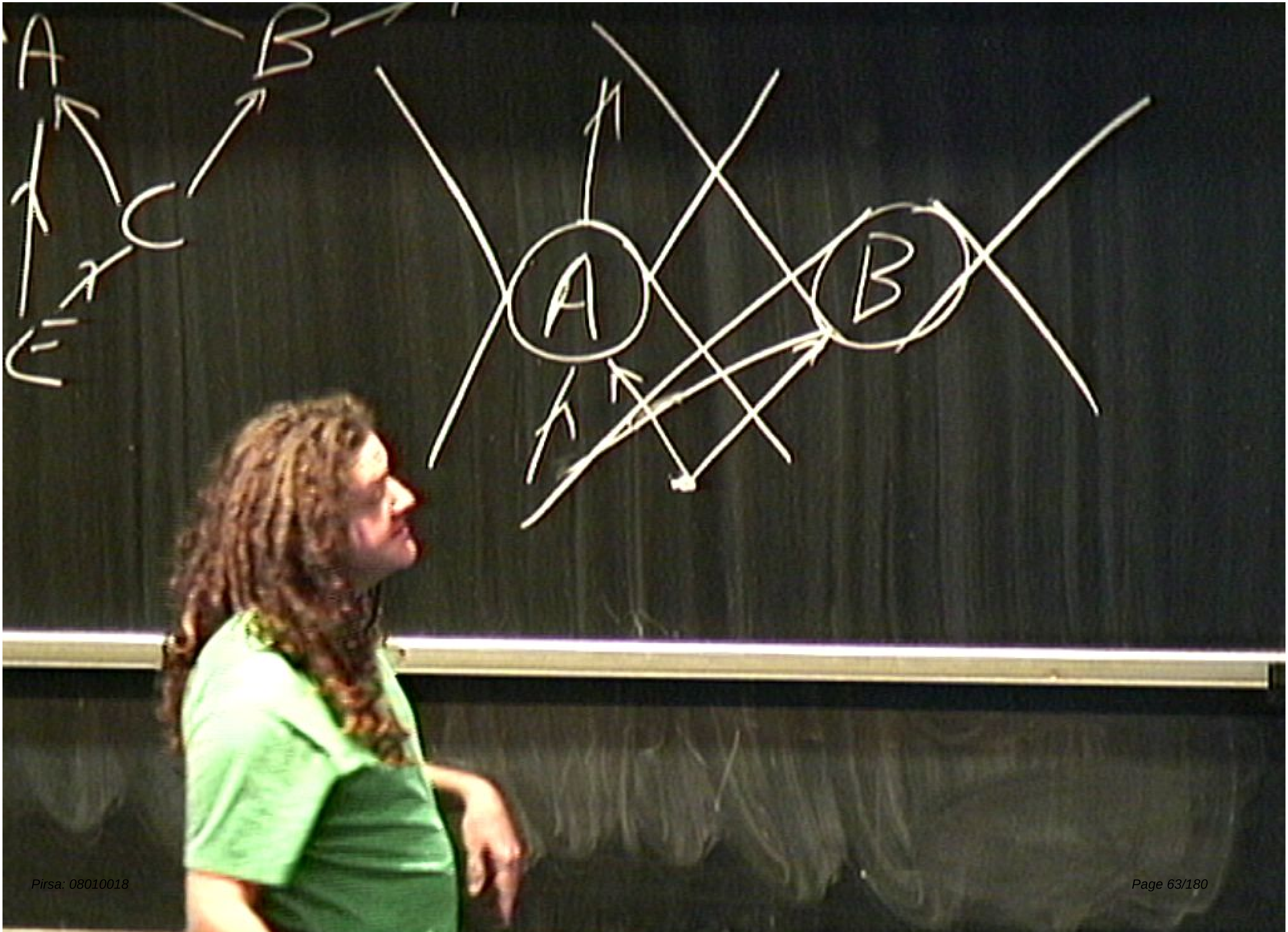
A
B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B
C
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A
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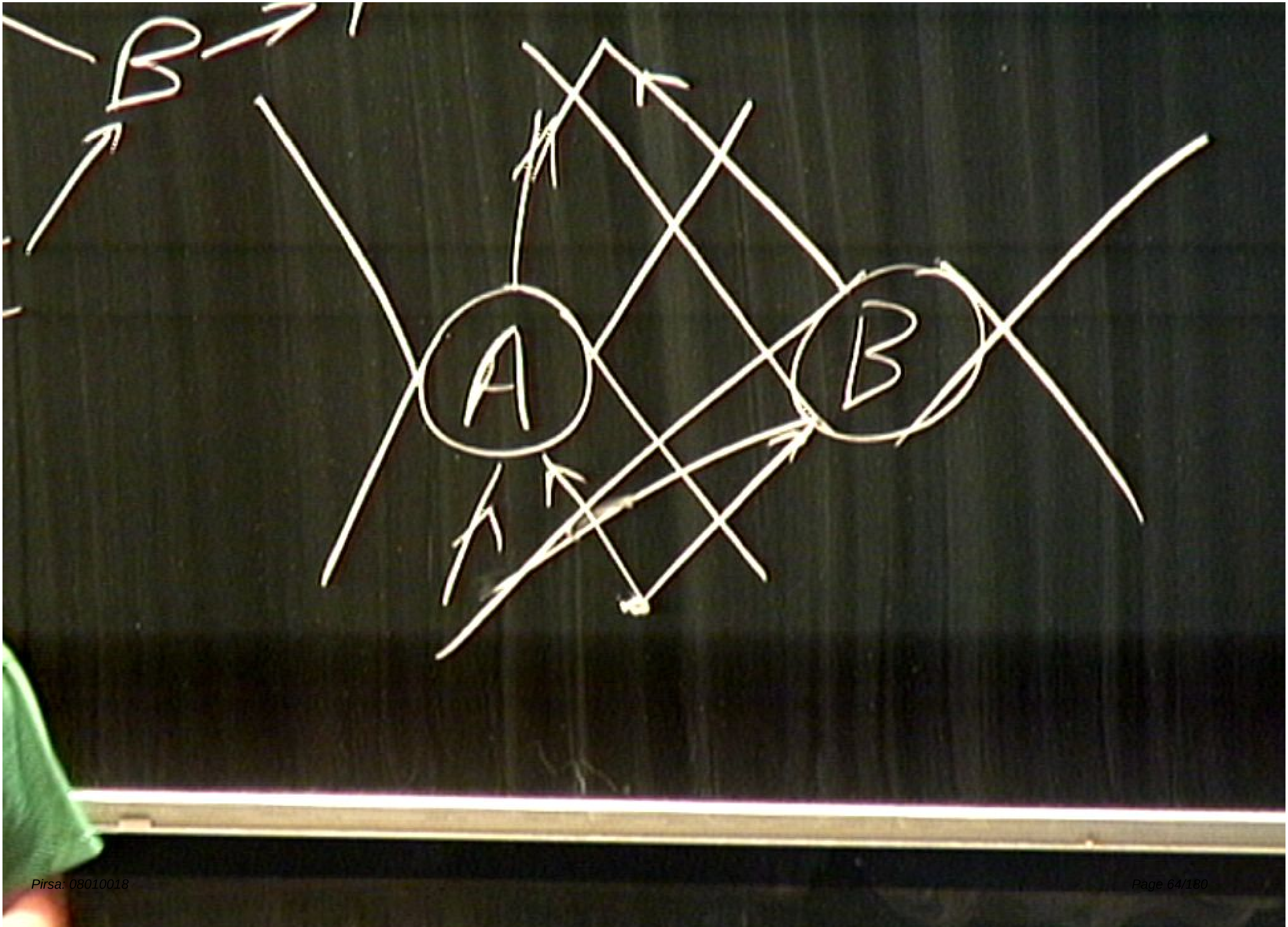
B
A
B

Relativistic Causality

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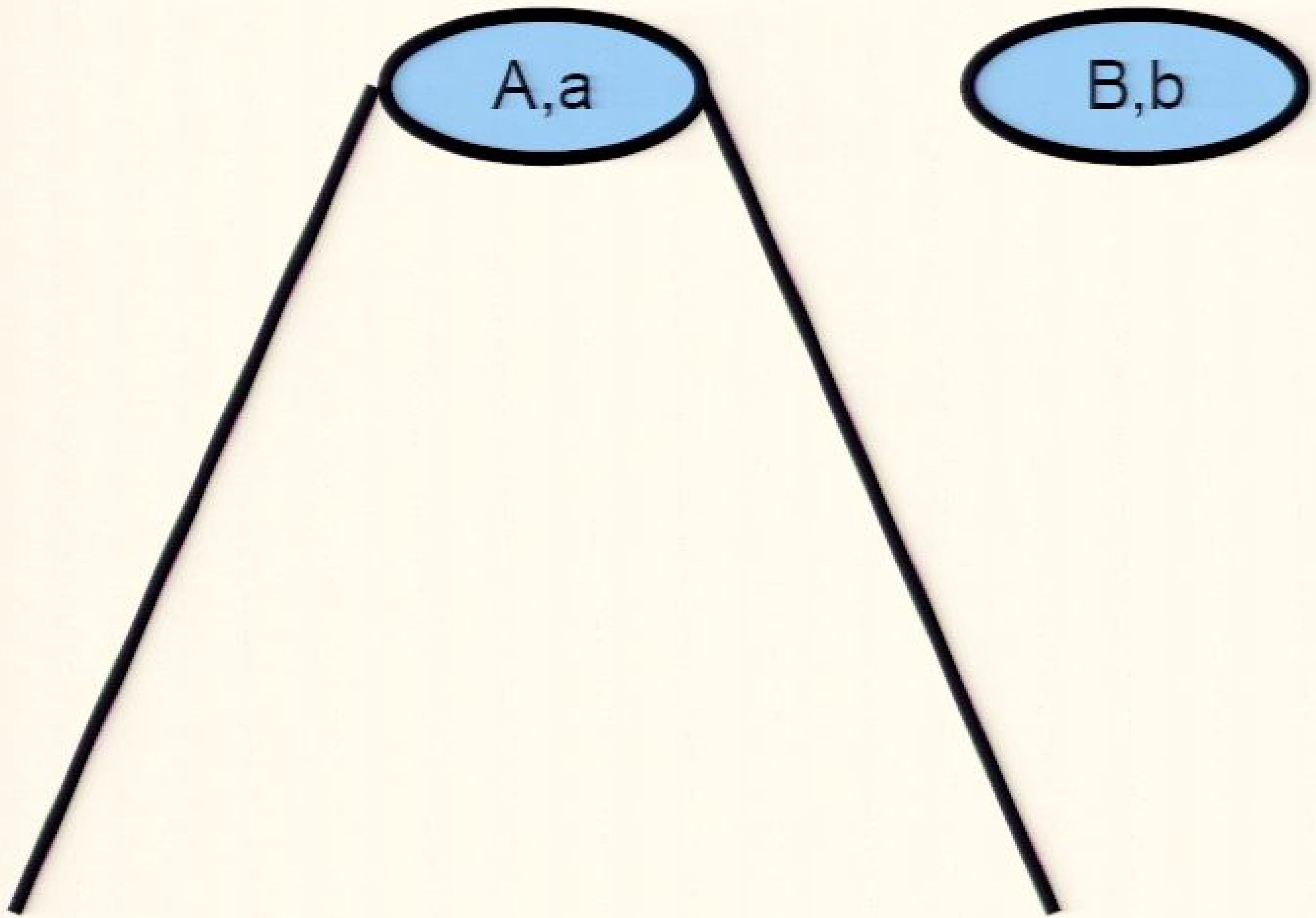
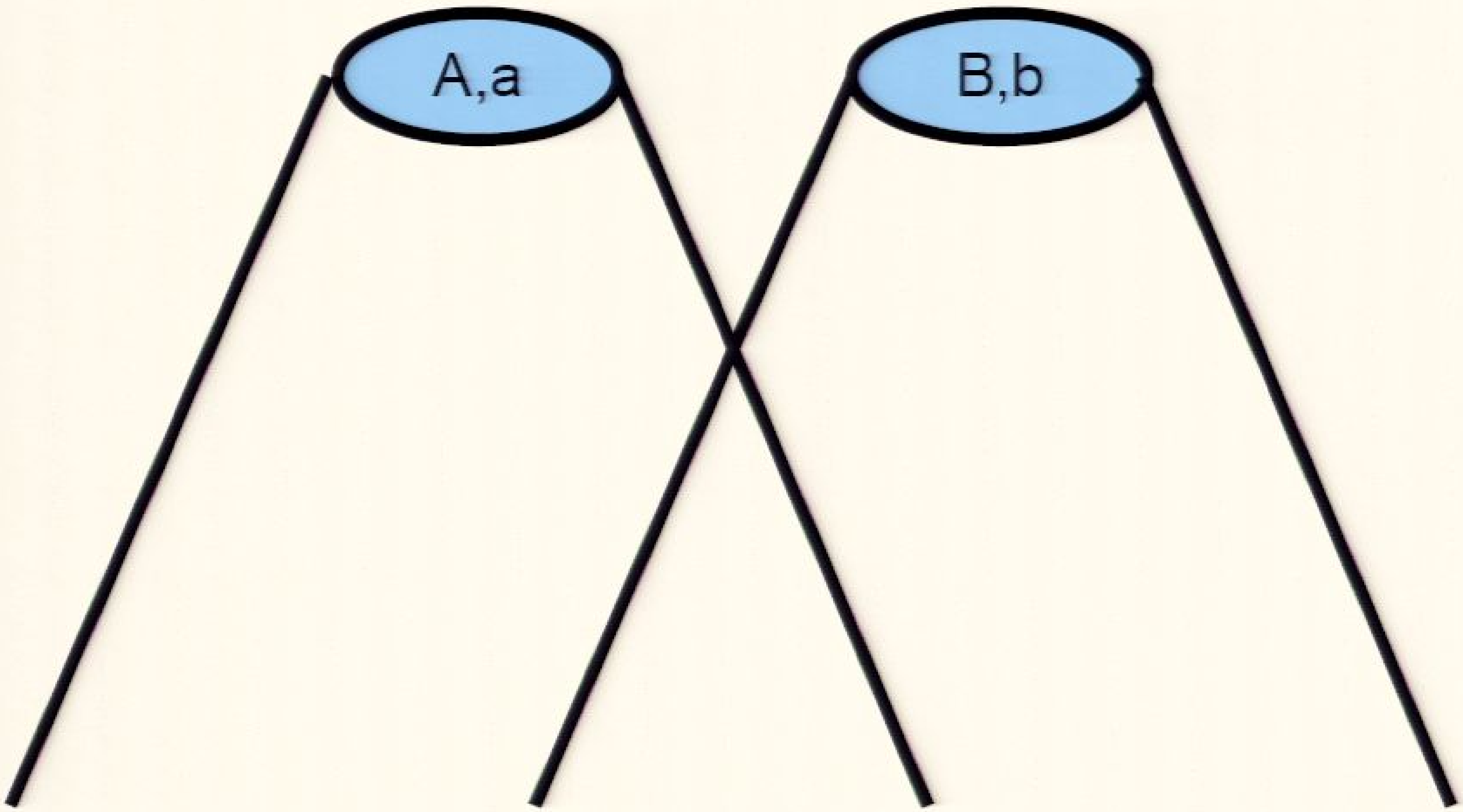
Relativistic Causality

Relativistic Causality

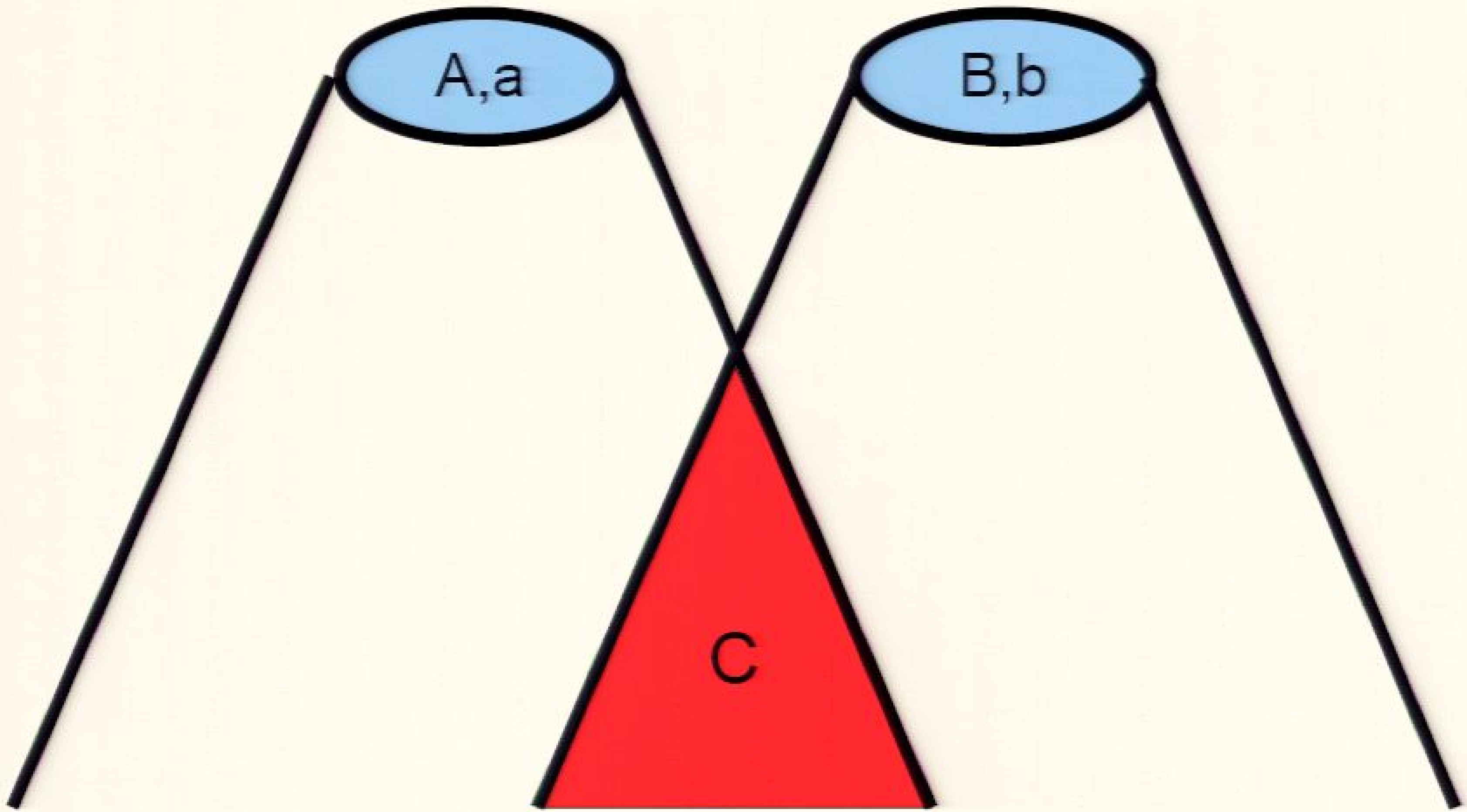
Relativistic Causality



Relativistic Causality



Relativistic Causality



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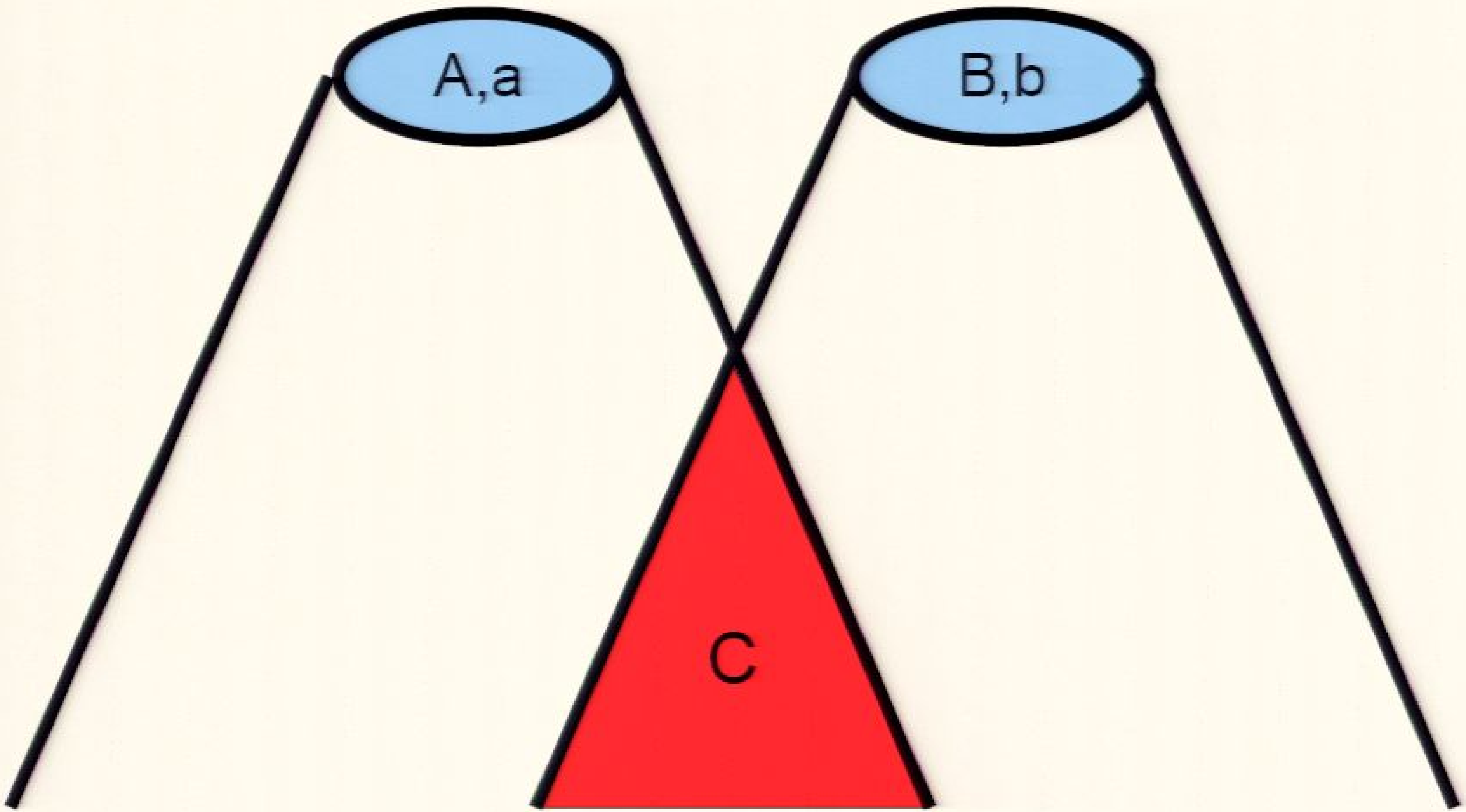

$$P(A,a,B,b|C)=P(A,a|C)P(B,b|C)$$

Free Variables

Suppose a and b are experiment settings, under the control of an experimenter

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$$P(a,b|C)=P(a|C)\,P(b|C)=P(a)\,P(b)$$ R.C.

$$P(a,b)=\sum_C P(a,b|C)\,P(C)$$

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$$P(a,b)=\sum_C P(a,b|C)P(C)=P(a)P(b)\sum_C P(C)=P(a)P(b)$$

The Bell-CHHS Inequality

$$P(A,a,B,b,C)=P(A,a,B,b|C)P(C)$$

The Bell-CHHS Inequality

$$P(A,a,B,b,C)=P(A,a,B,b|C)P(C)$$

$$P(A,a,B,b,C)=P(A,a|C)P(B,b|C)P(C)$$

The Bell-CHHS Inequality

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The Bell-CHHS Inequality

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$$P(A,a,B,b,C)=P(A|a,C)P(a|C)P(B|b,C)P(b|C)P(C)$$

$$P(A,a,B,b,C)=P(A|a,C)P(a)P(B|b,C)P(b)P(C)$$ F.V.

$$P(A,a,B,b)=\sum_C P(A,a,B,b,C)$$

$$P(A,a,B,b)=\sum_C P(A|a,C)P(a)P(B|b,C)P(b)P(C)$$

$$P(A,a,B,b)=P(A,B|a,b)P(a,b)$$

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$$P(A,a,B,b,C)=P(A|a,C)P(a|C)P(B|b,C)P(b|C)P(C)$$

$$P(A,a,B,b,C)=P(A|a,C)P(a)P(B|b,C)P(b)P(C) \quad \text{F.V.}$$

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$$P(A,a,B,b)=\sum_C P(A|a,C)P(a)P(B|b,C)P(b)P(C)$$

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$$P(A,a,B,b)=P(A,B|a,b)P(a,b)$$

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$$P(A,B|a,b)=\sum_C P(A|a,C)P(B|b,C)P(C)$$

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Count a Red light flashing as +1, Count a Green light flashing as -1

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$$E(A,B|a,b)=P(R_A,R_B|a,b)-P(G_A,R_B|a,b)-P(R_A,G_B|a,b)+P(G_A,G_B|a,b)$$

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$$E(A|a,C)=P(R_A|a,C)-P(G_A|a,C)$$

$$P(R_A|a,C)+P(G_A|a,C)=1 \qquad -1\leq E(A|a,C)\leq +1$$

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$$0\leq\left|E(A|a,C)\right|\leq 1$$

The Bell-CHHS Inequality

$$CHHS = E(A,B|a,b) + E(A,B|a',b) + E(A,B|a,b') - E(A,B|a',b')$$

The Bell-CHHS Inequality

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$$CHHS = \sum_C \left\{ E(A|a,C)\left[E(B|b,C) + E(B|b',C)\right] + E(A|a',C)\left[E(B|b,C) - E(B|b',C)\right] \right\} P(C)$$

The Bell-CHHS Inequality

$$CHHS = E(A,B|a,b) + E(A,B|a',b) + E(A,B|a,b') - E(A,B|a',b')$$

$$CHHS = \sum_C \left\{ \begin{array}{c} E(A|a,C)E(B|b,C) + E(A|a,C)E(B|b',C) \\ +E(A|a',C)E(B|b,C) - E(A|a',C)E(B|b',C) \end{array} \right\} P(C)$$

$$CHHS = \sum_C \{E(A|a,C)[E(B|b,C)+E(B|b',C)] + E(A|a',C)[E(B|b,C)-E(B|b',C)]\} P(C)$$

$$|CHHS| \le \sum_C |E(A|a,C)[E(B|b,C)+E(B|b',C)] + E(A|a',C)[E(B|b,C)-E(B|b',C)]| P(C)$$

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$$CHHS = \sum_C \left\{ \begin{array}{c} E(A|a,C)E(B|b,C) + E(A|a,C)E(B|b',C) \\ + E(A|a',C)E(B|b,C) - E(A|a',C)E(B|b',C) \end{array} \right\} P(C)$$

$$CHHS = \sum_C \{ E(A|a,C)[E(B|b,C) + E(B|b',C)] + E(A|a',C)[E(B|b,C) - E(B|b',C)] \} P(C)$$

$$|CHHS| \leq \sum_C |E(A|a,C)[E(B|b,C) + E(B|b',C)] + E(A|a',C)[E(B|b,C) - E(B|b',C)]| P(C)$$

$$|CHHS| \leq \sum_C \{ |E(A|a,C)||E(B|b,C) + E(B|b',C)| + |E(A|a',C)||E(B|b,C) - E(B|b',C)| \} P(C)$$

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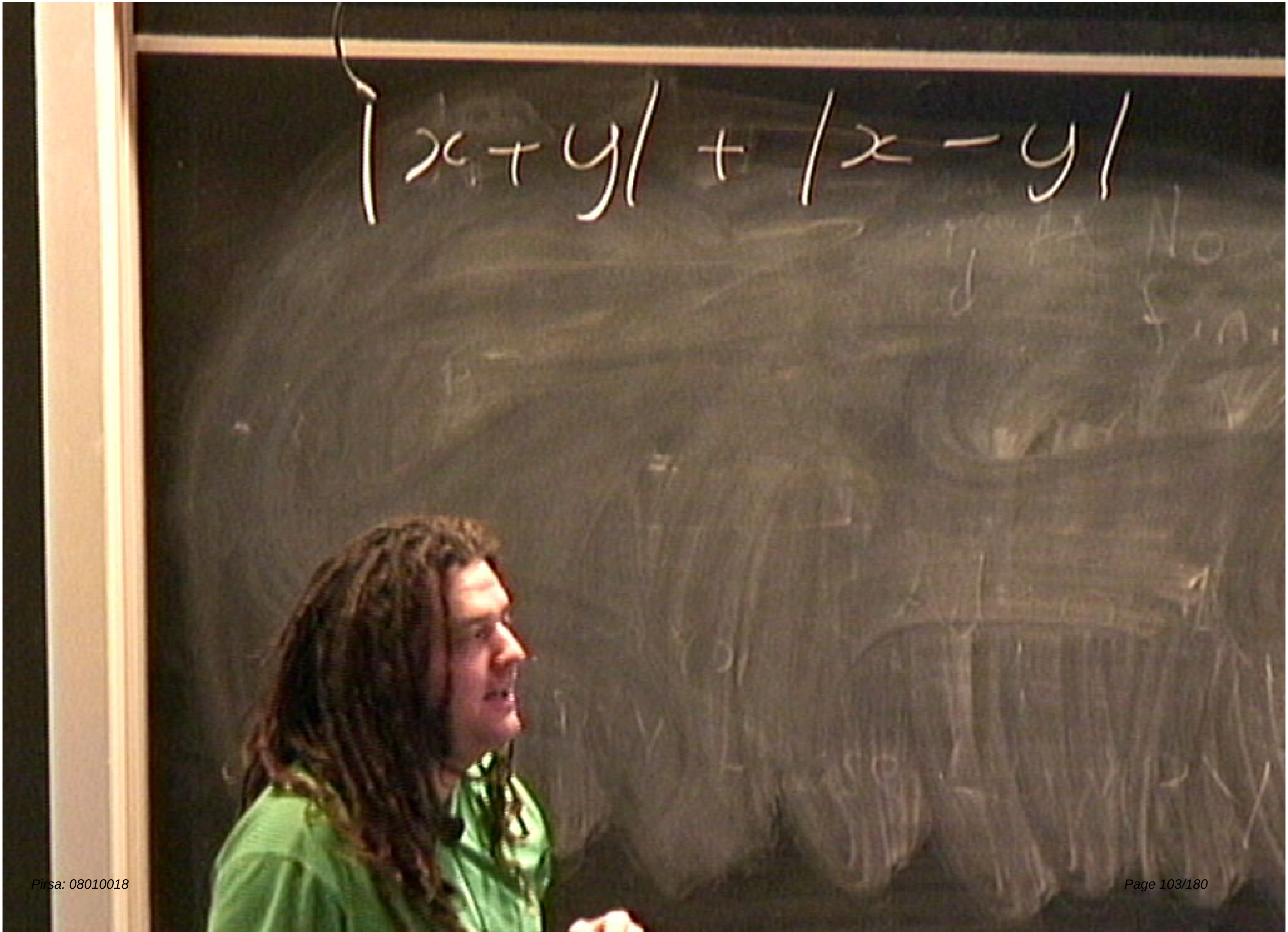
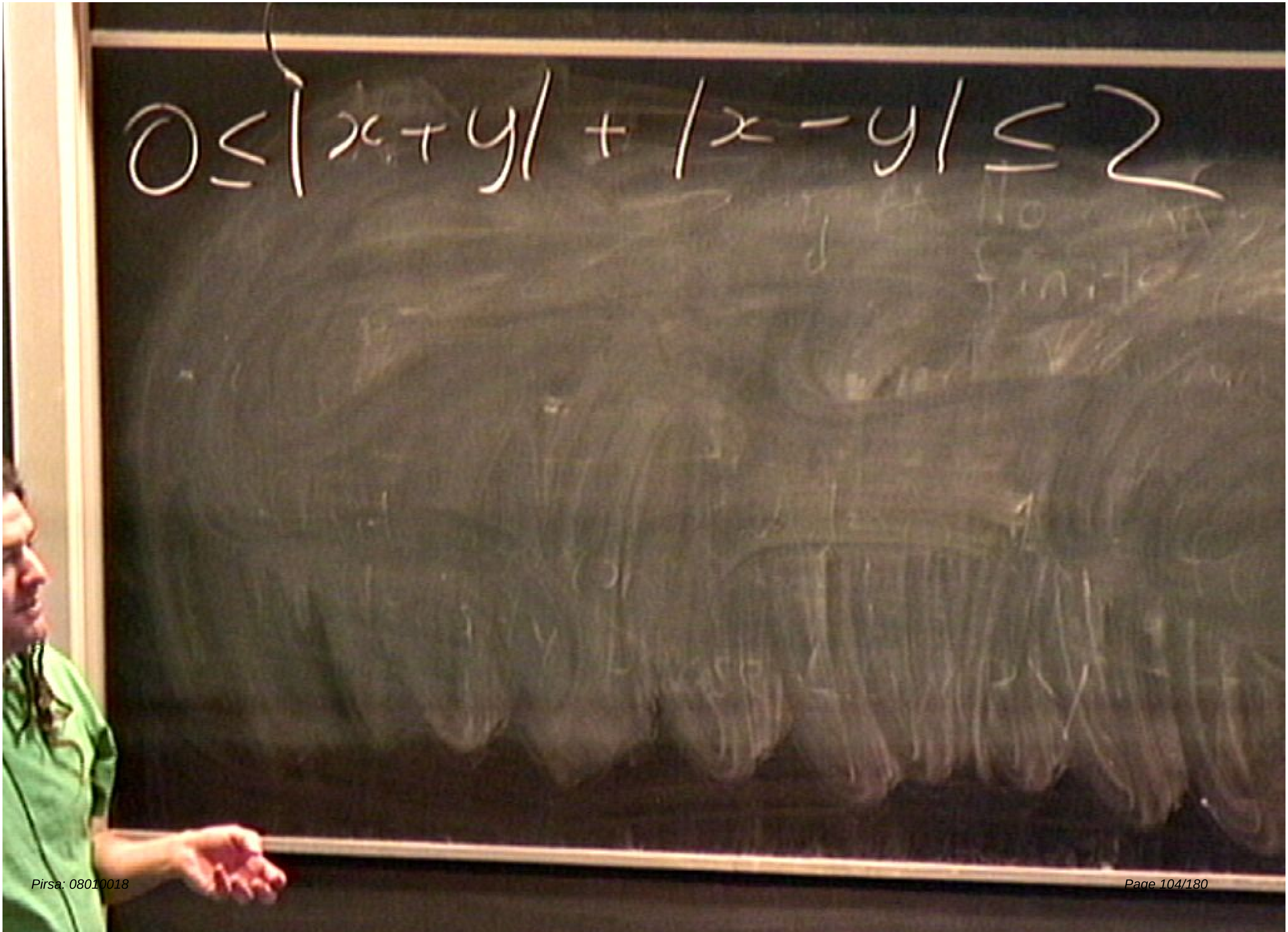
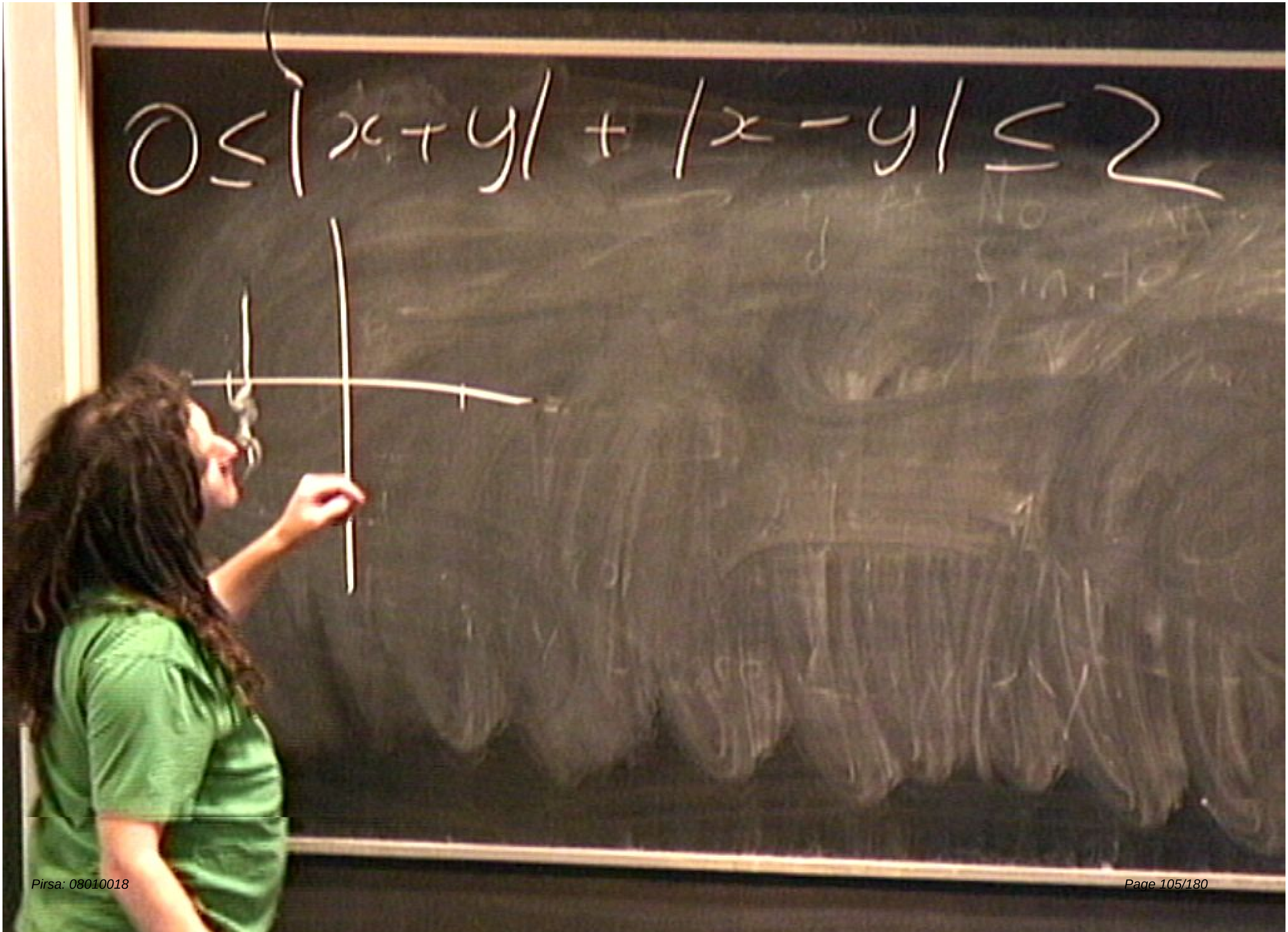
$$|CHHS| \leq \sum_C \left| E(A|a,C)\left[E(B|b,C) + E(B|b',C) \right] + E(A|a',C)\left[E(B|b,C) - E(B|b',C) \right] \right| P(C)$$

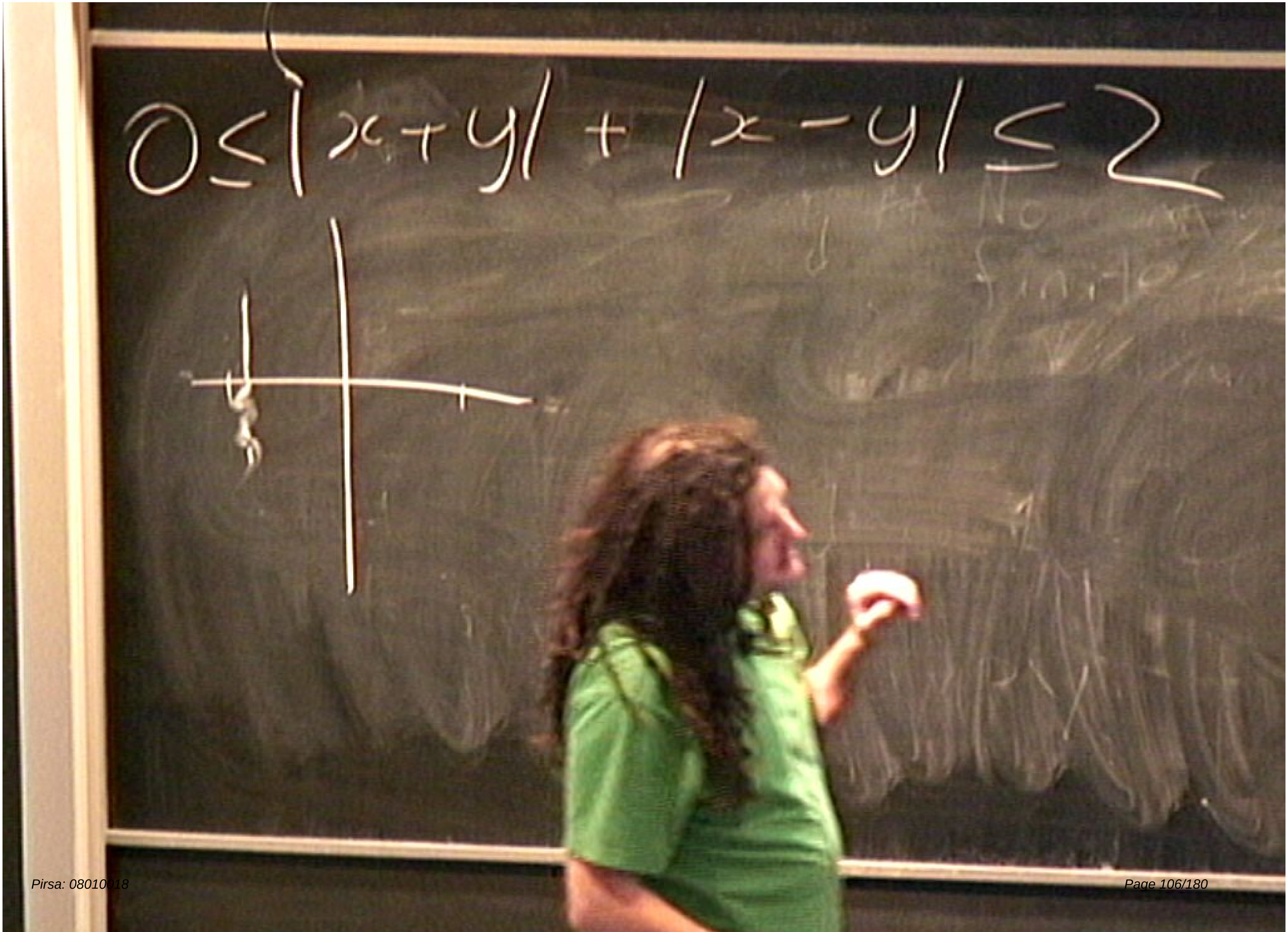
$$|CHHS| \leq \sum_C \left\{ |E(A|a,C)||E(B|b,C) + E(B|b',C)| + |E(A|a',C)||E(B|b,C) - E(B|b',C)| \right\} P(C)$$

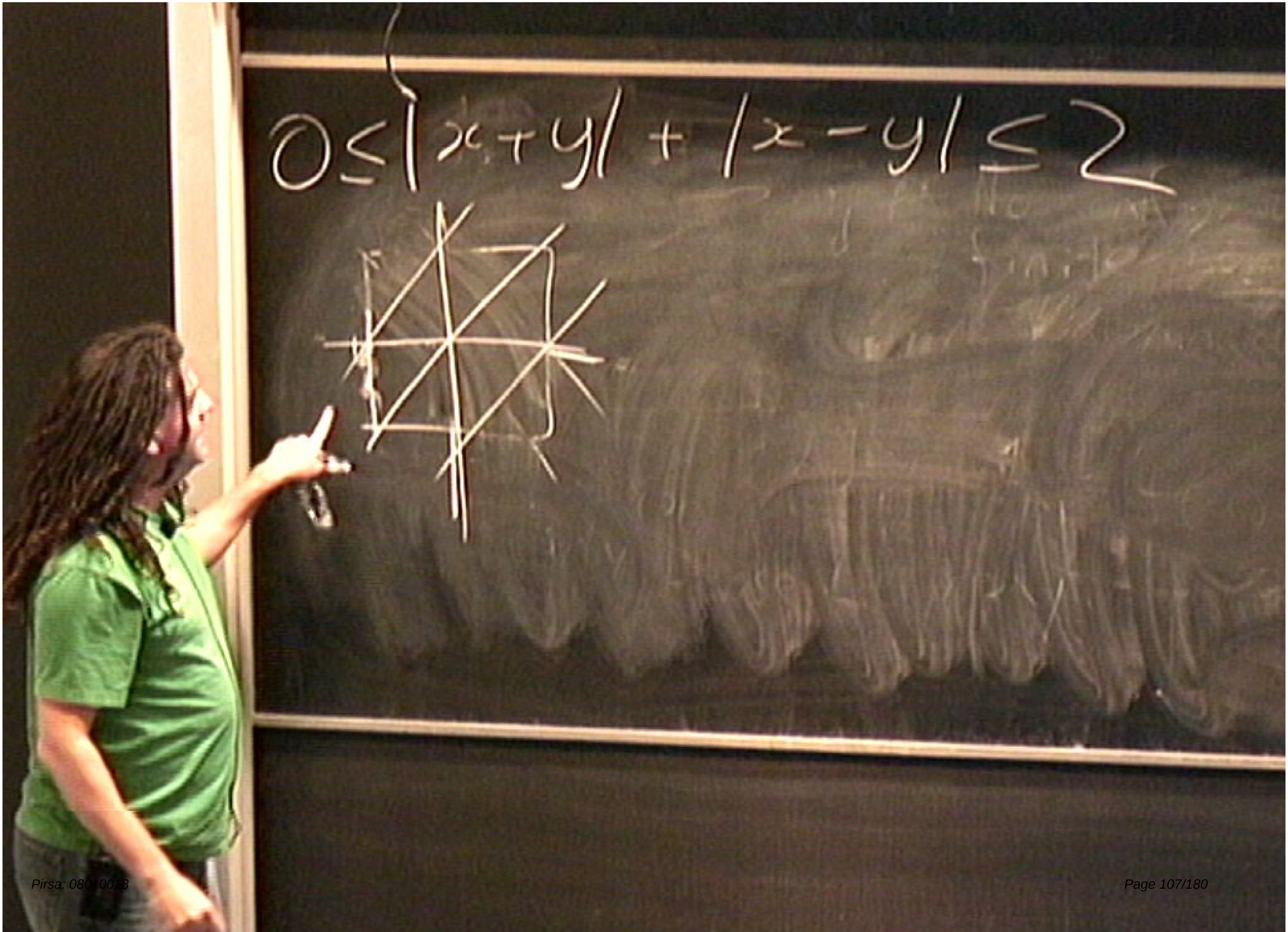
$$|CHHS| \leq \sum_C \left\{ |E(B|b,C) + E(B|b',C)| + |E(B|b,C) - E(B|b',C)| \right\} P(C)$$

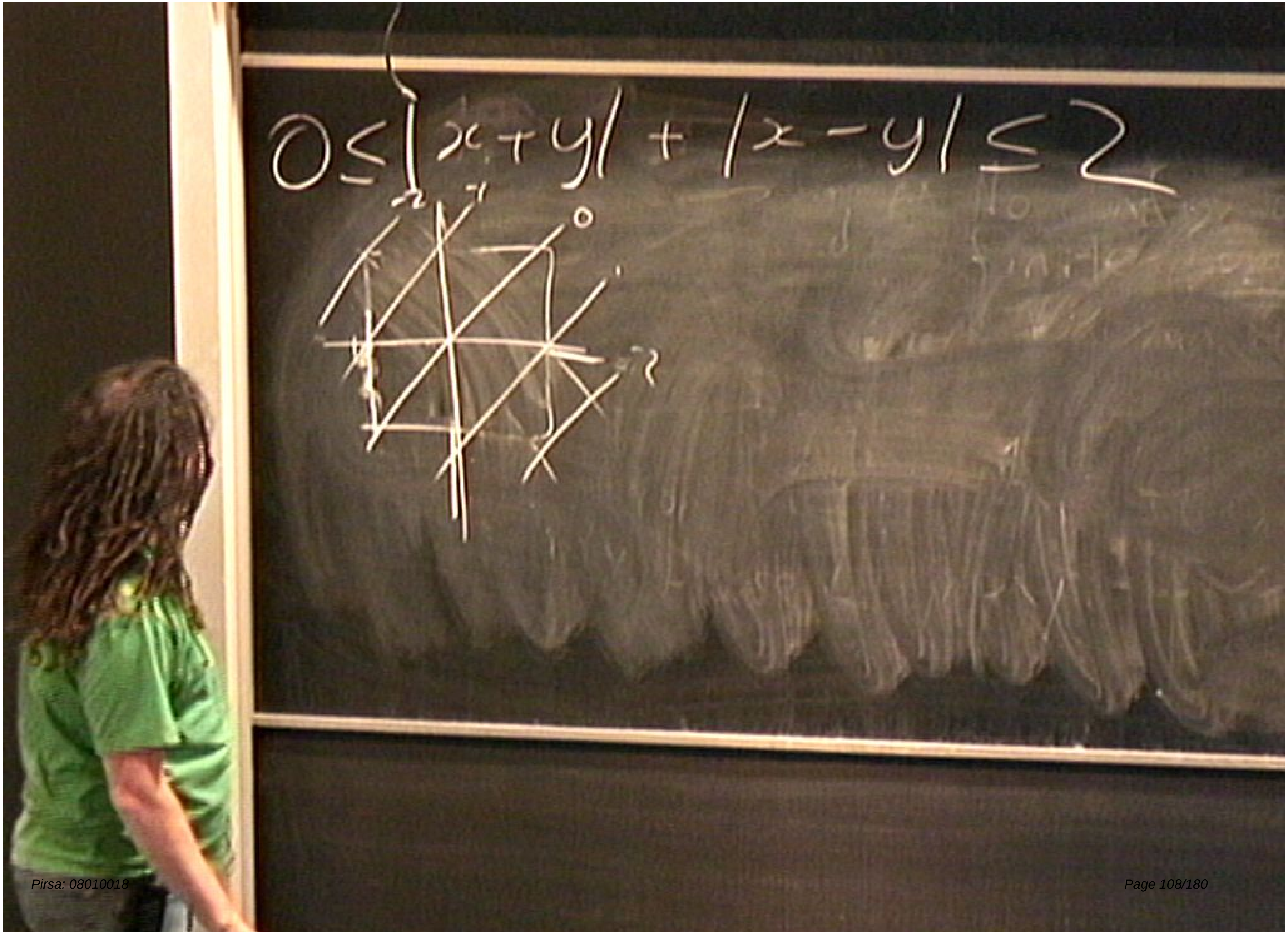

|x + y| + |x − y|

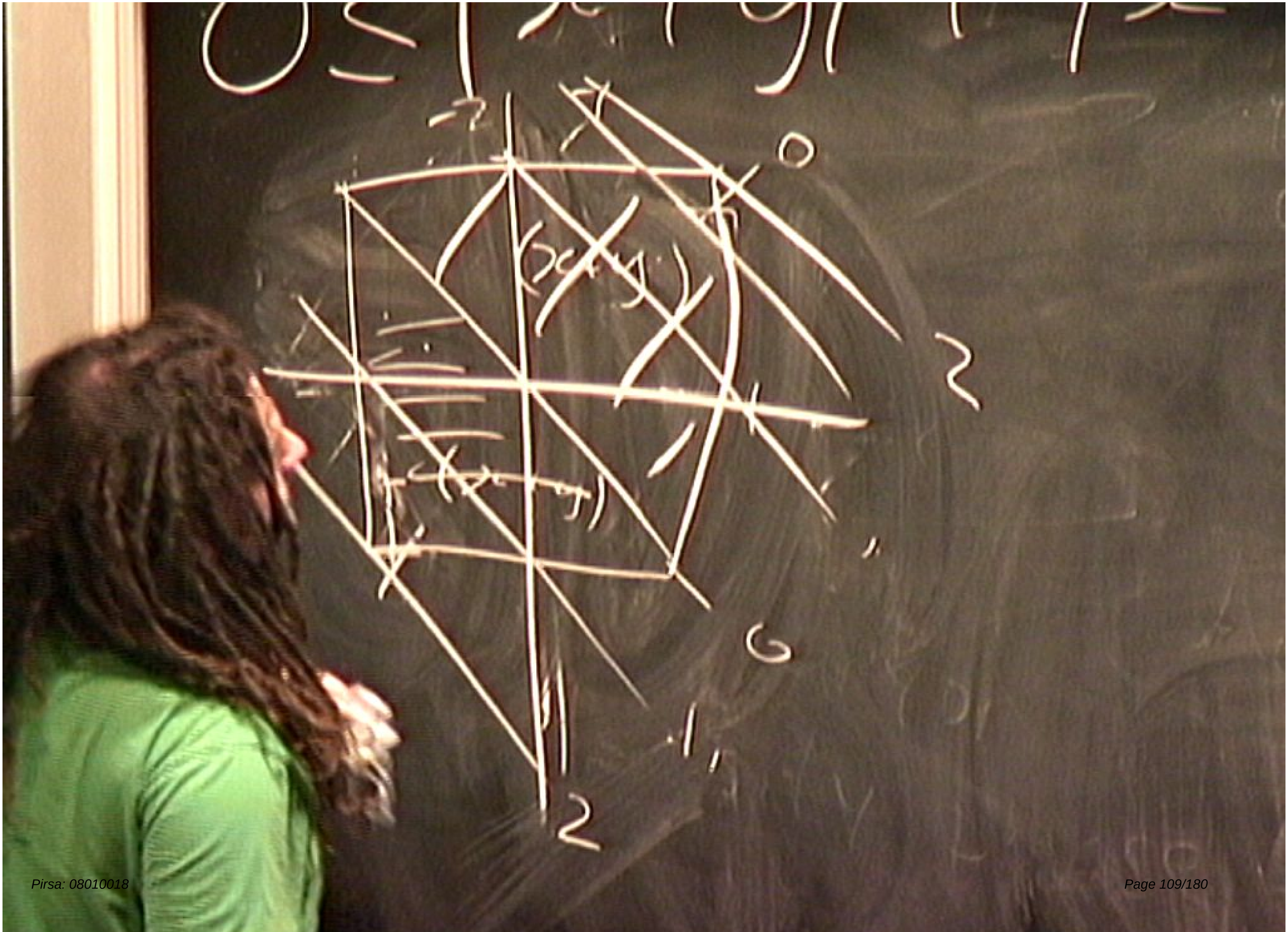

$0 \le |x+y| + |x-y| \le 2$

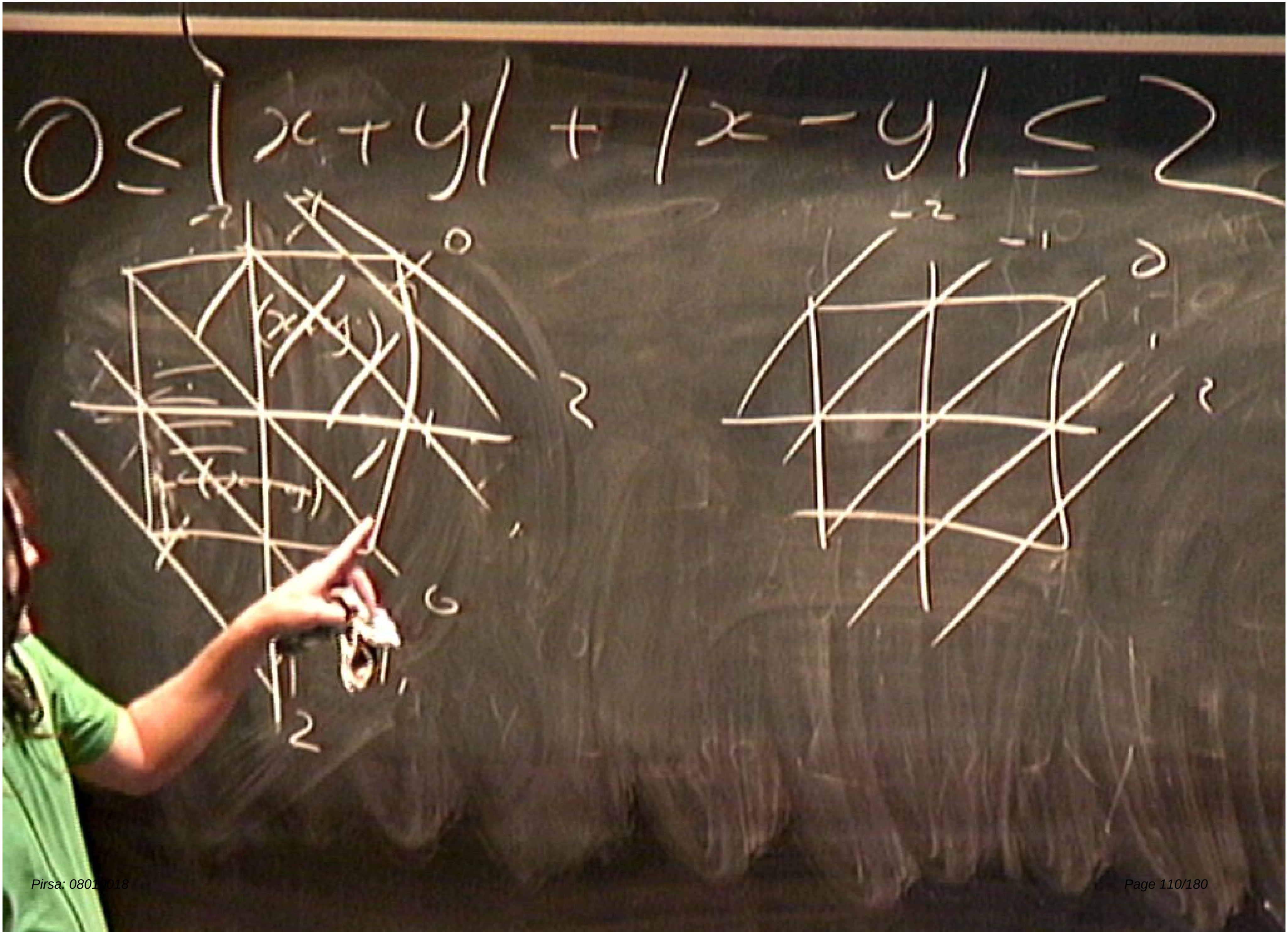

0 ≤ |x + y| + |x − y| ≤ 2


0 ≤ |x + y| + |x − y| ≤ 2


0 ≤ |x + y| + |x − y| ≤ 2


$0 \le |x+y| + |x-y| \le 2$

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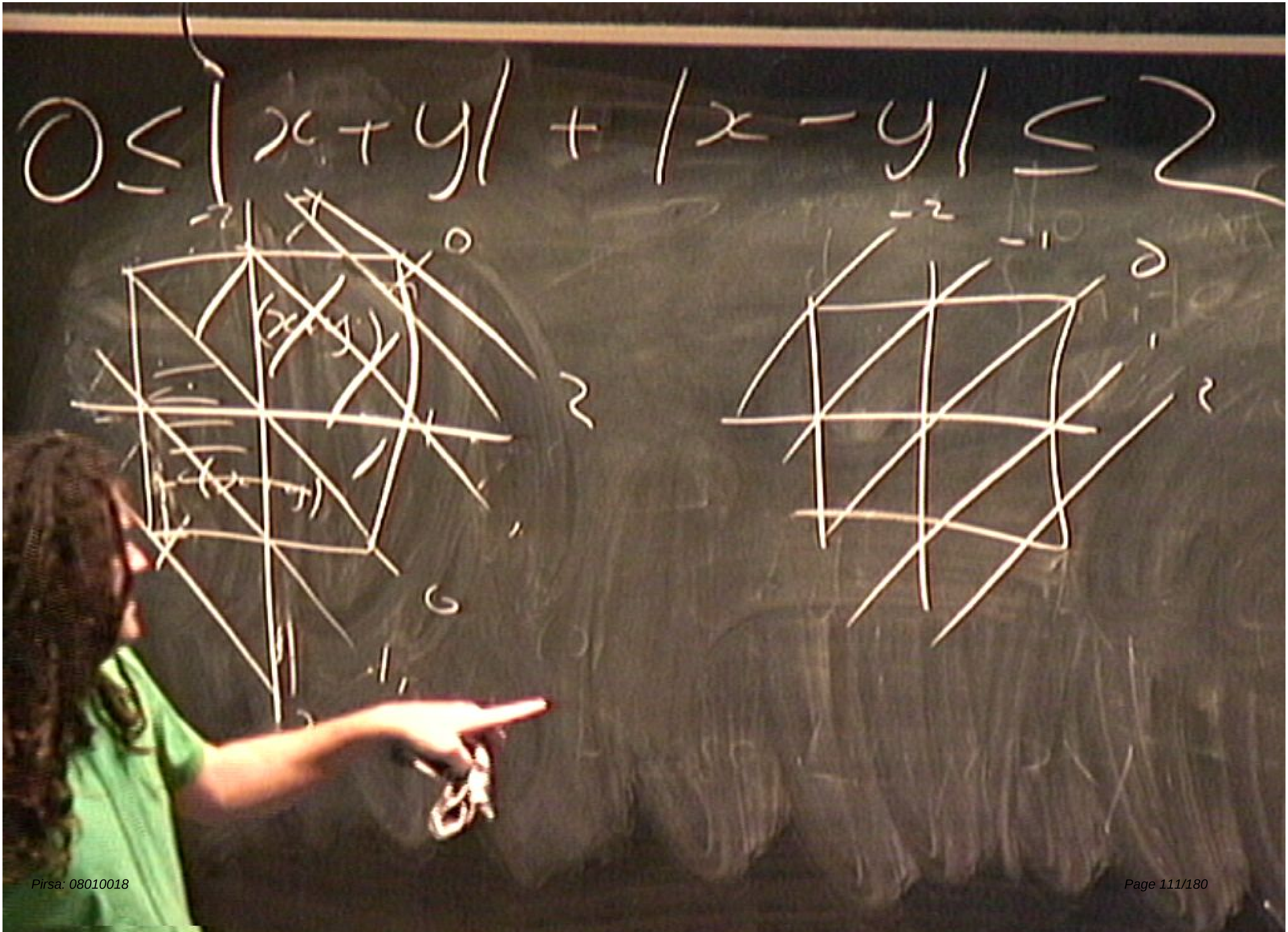
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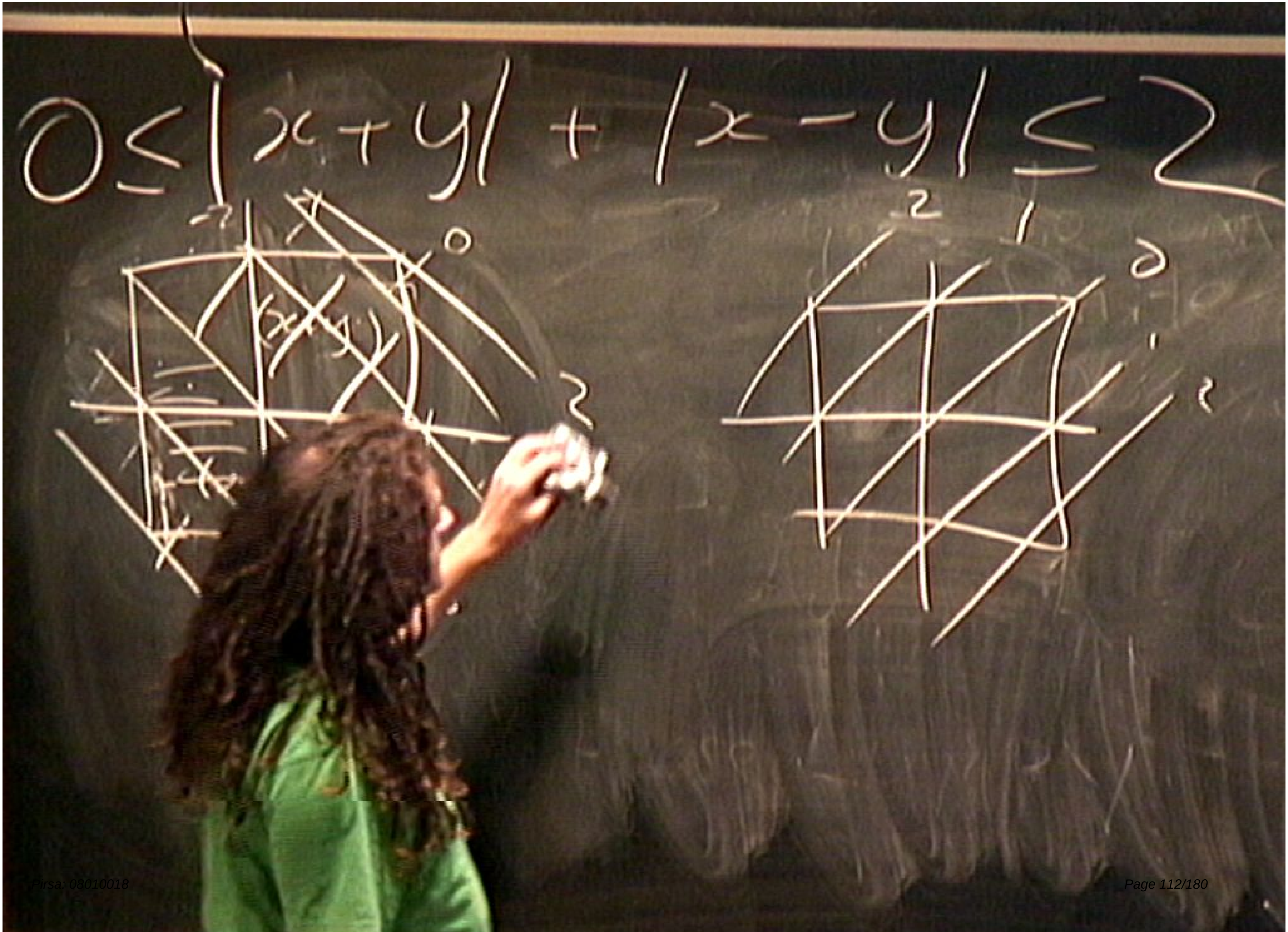
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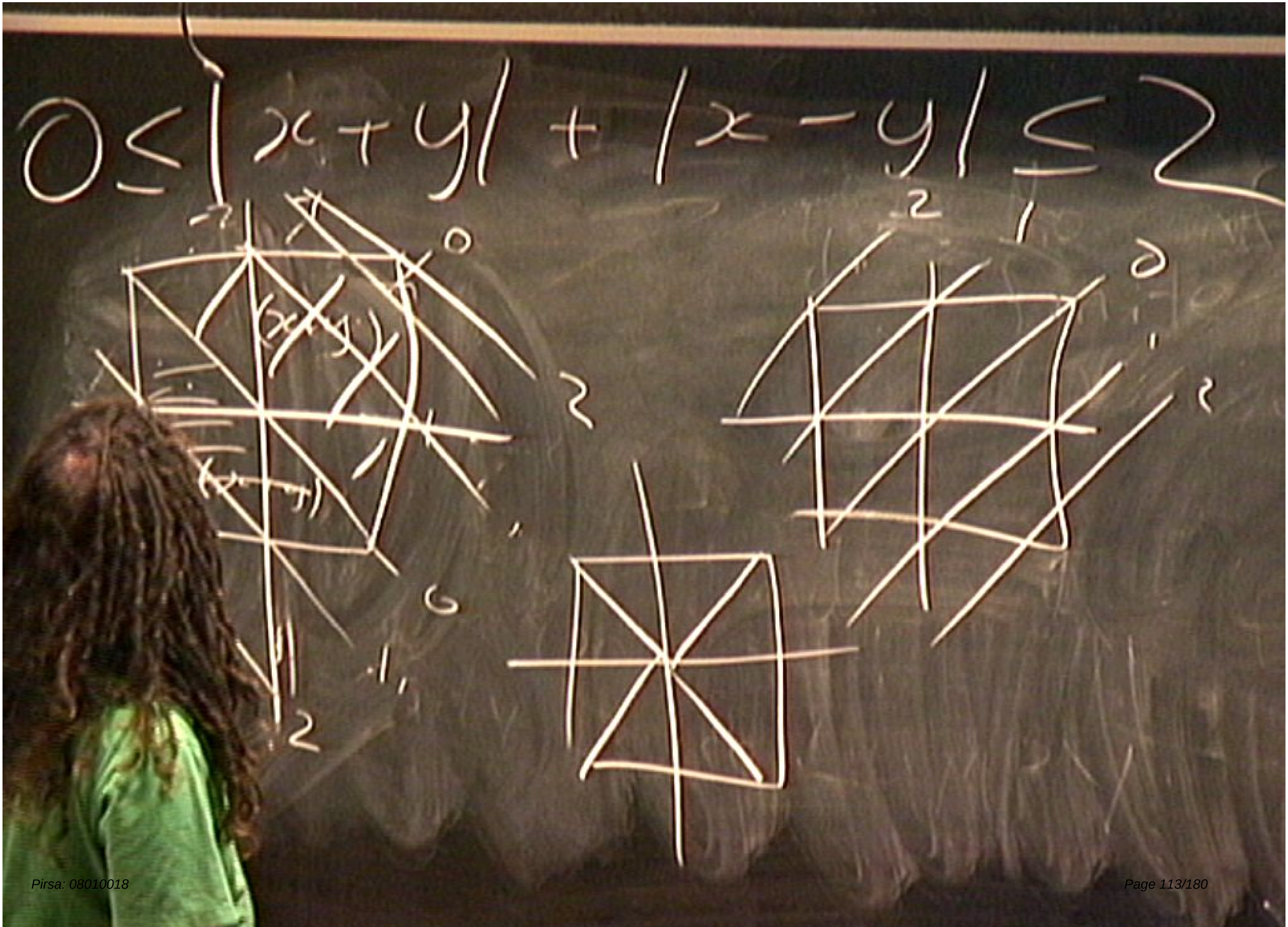
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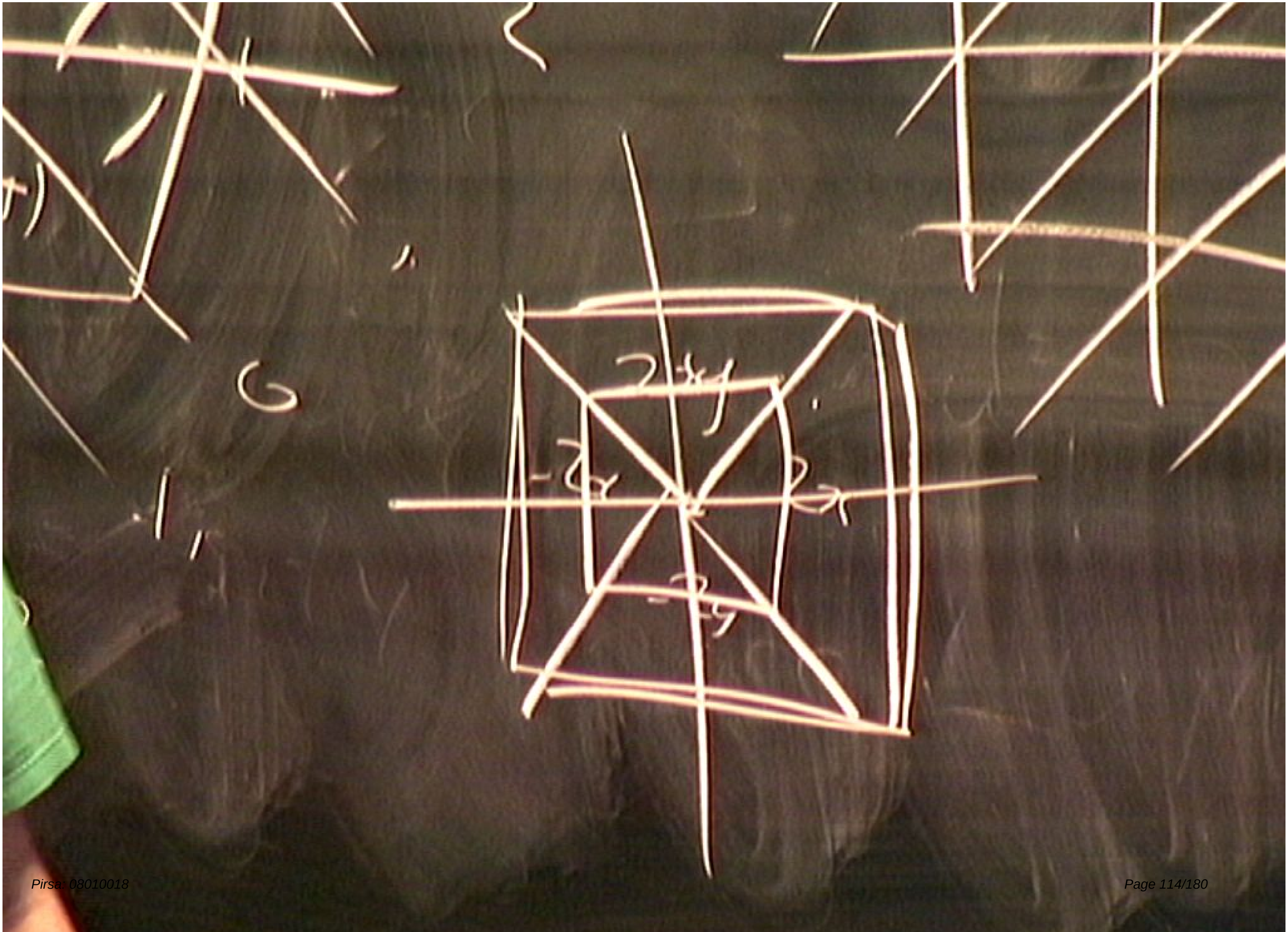
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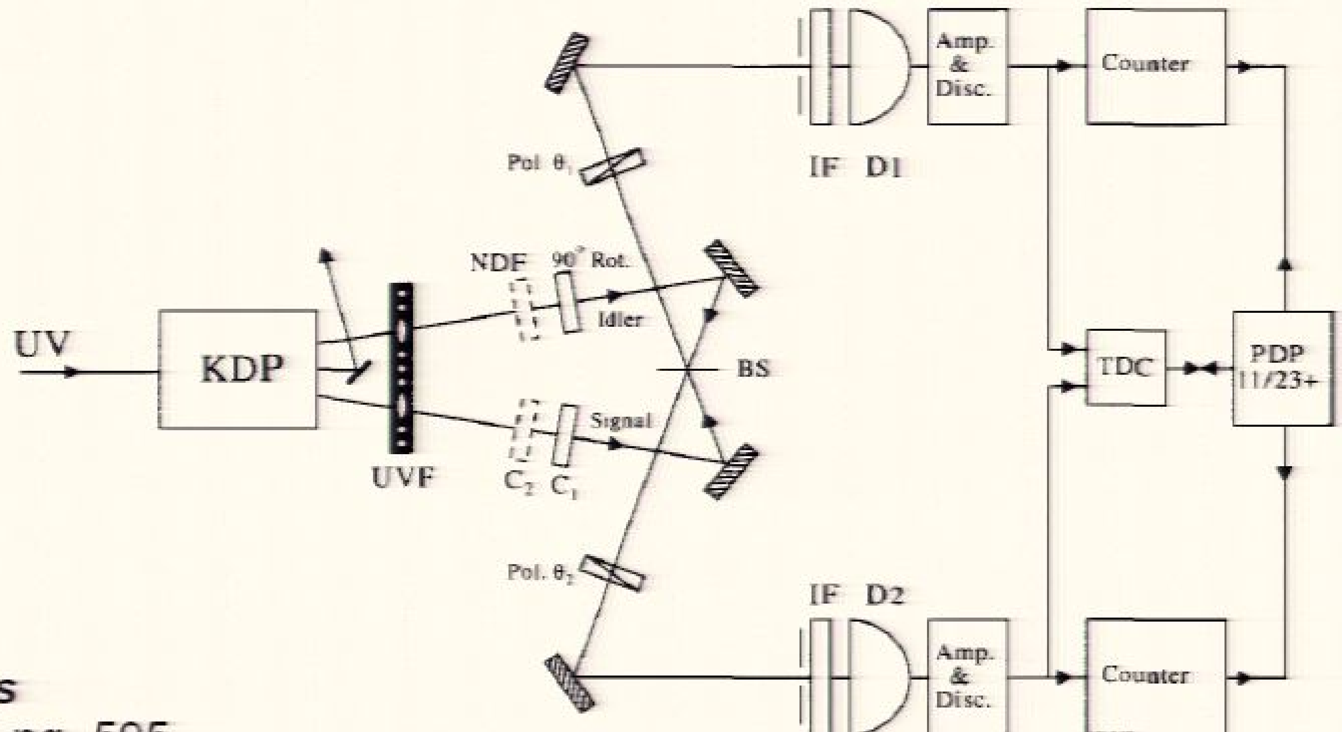
Fig. 21.6
Outline of the experiment to test Bell's inequality. (From Z. Y. Ou and L. Mandel, *Phys. Rev. Lett.* **61**, 50 (1988).)



Source:
Quantum Optics
Scully & Zubairy, pg. 595

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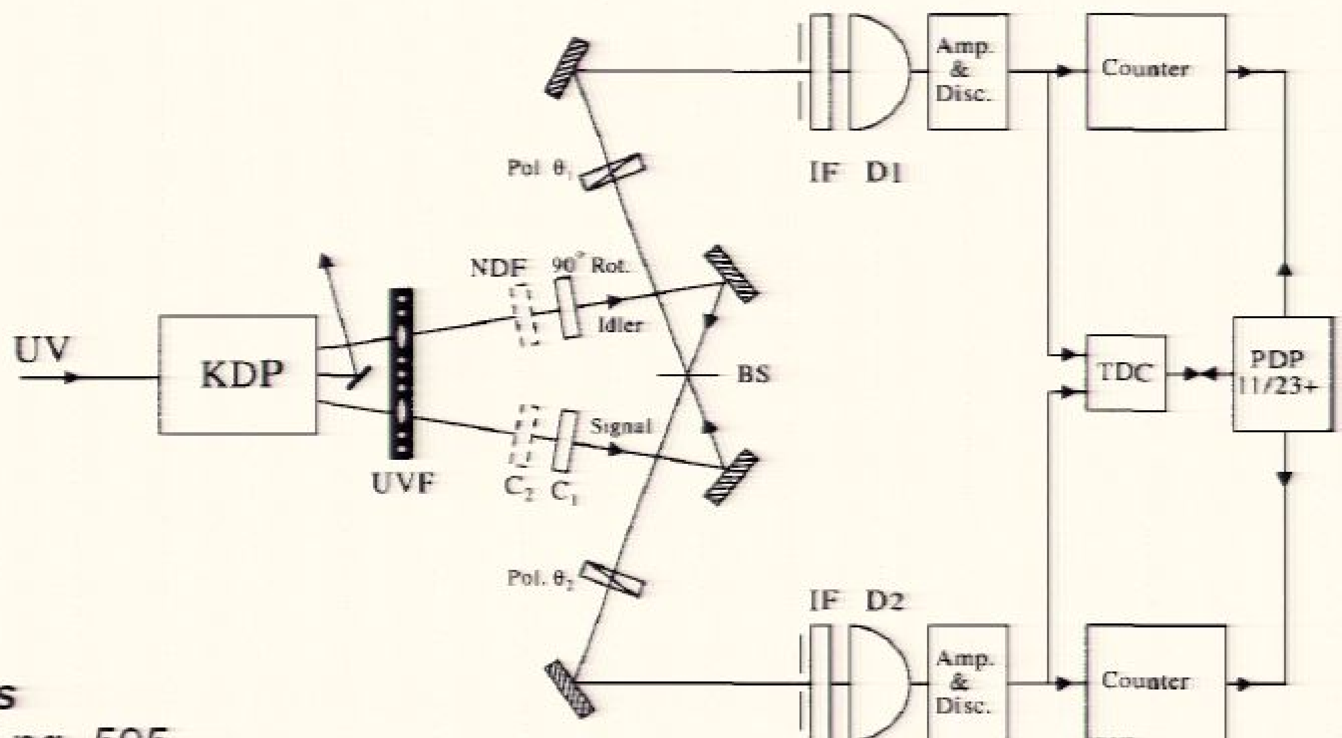

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Quantum Theory!

$$\Psi(x,y)=\frac{1}{\sqrt{2}}\left(\psi_u(x)\phi_u(y)+\psi_d(x)\phi_d(y)\right)$$

Quantum Theory!

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$$\Psi(x,y)=\frac{1}{\sqrt{2}}\left(\begin{array}{l}(\cos\alpha\cos\beta+\sin\alpha\sin\beta)\psi_R(x,\alpha)\phi_R(y,\beta)+(\cos\alpha\sin\beta-\sin\alpha\cos\beta)\psi_R(x,\alpha)\phi_G(y,\beta)\\+(\sin\alpha\cos\beta-\cos\alpha\sin\beta)\psi_G(x,\alpha)\phi_R(y,\beta)+(\cos\alpha\cos\beta+\sin\alpha\sin\beta)\psi_G(x,\alpha)\phi_G(y,\beta)\end{array}\right)$$

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$$\Psi(x,y)=\frac{1}{\sqrt{2}}\begin{pmatrix}\cos(\alpha-\beta)\psi_R(x,\alpha)\phi_R(y,\beta)-\sin(\alpha-\beta)\psi_R(x,\alpha)\phi_G(y,\beta)\\+\sin(\alpha-\beta)\psi_G(x,\alpha)\phi_R(y,\beta)+\cos(\alpha-\beta)\psi_G(x,\alpha)\phi_G(y,\beta)\end{pmatrix}$$

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$$E(A,B|\alpha,\beta)=\frac{1}{2}\left(\cos^2(\alpha-\beta)-\sin^2(\alpha-\beta)-\sin^2(\alpha-\beta)+\cos^2(\alpha-\beta)\right)=\cos(2(\alpha-\beta))$$

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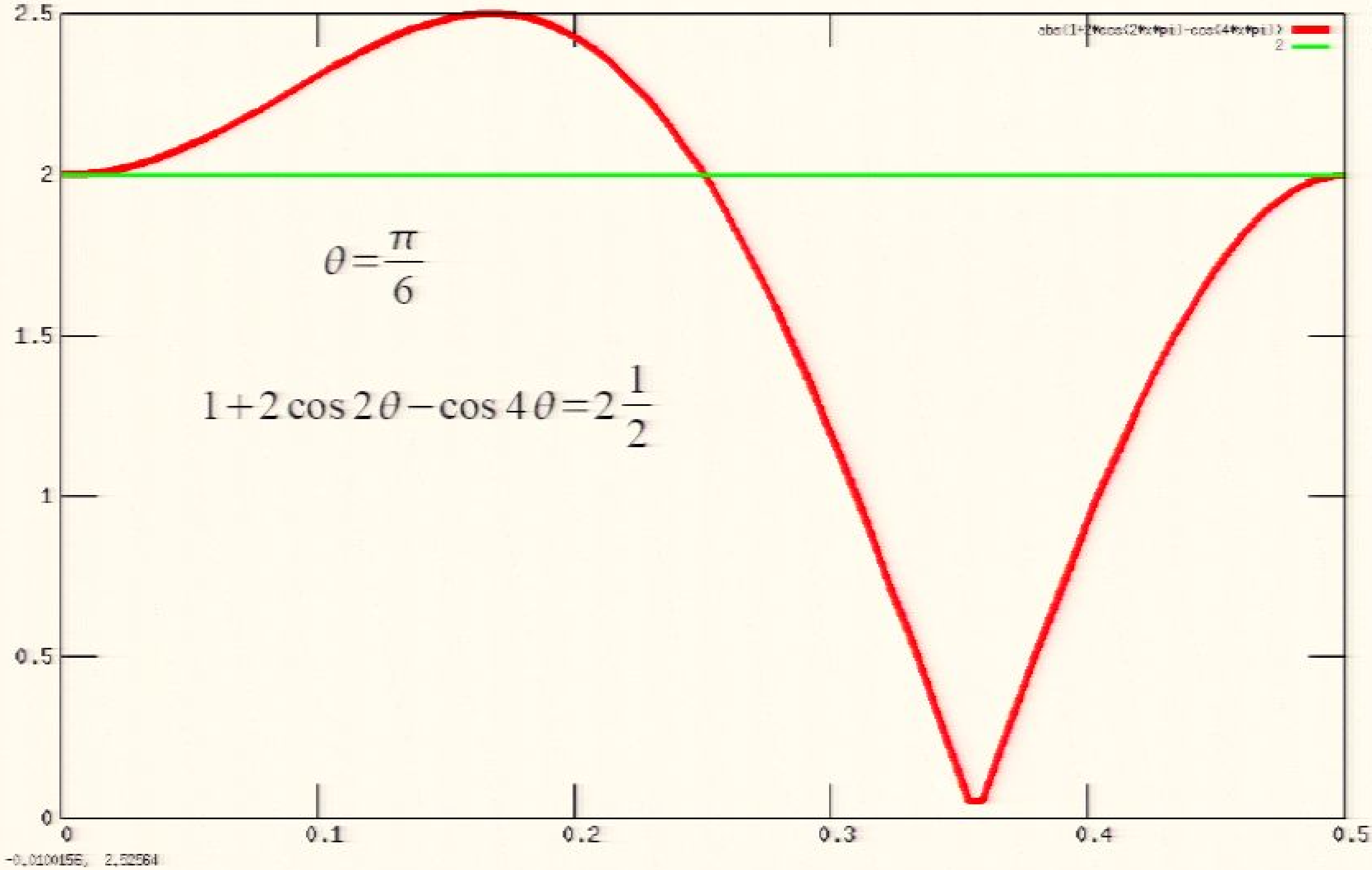
$$E(A,B|0,0)+E(A,B|\theta,0)+E(A,B|0,-\theta)-E(A,B|\theta,-\theta)=1+2\cos2\theta-\cos4\theta$$

Quantum Theory!



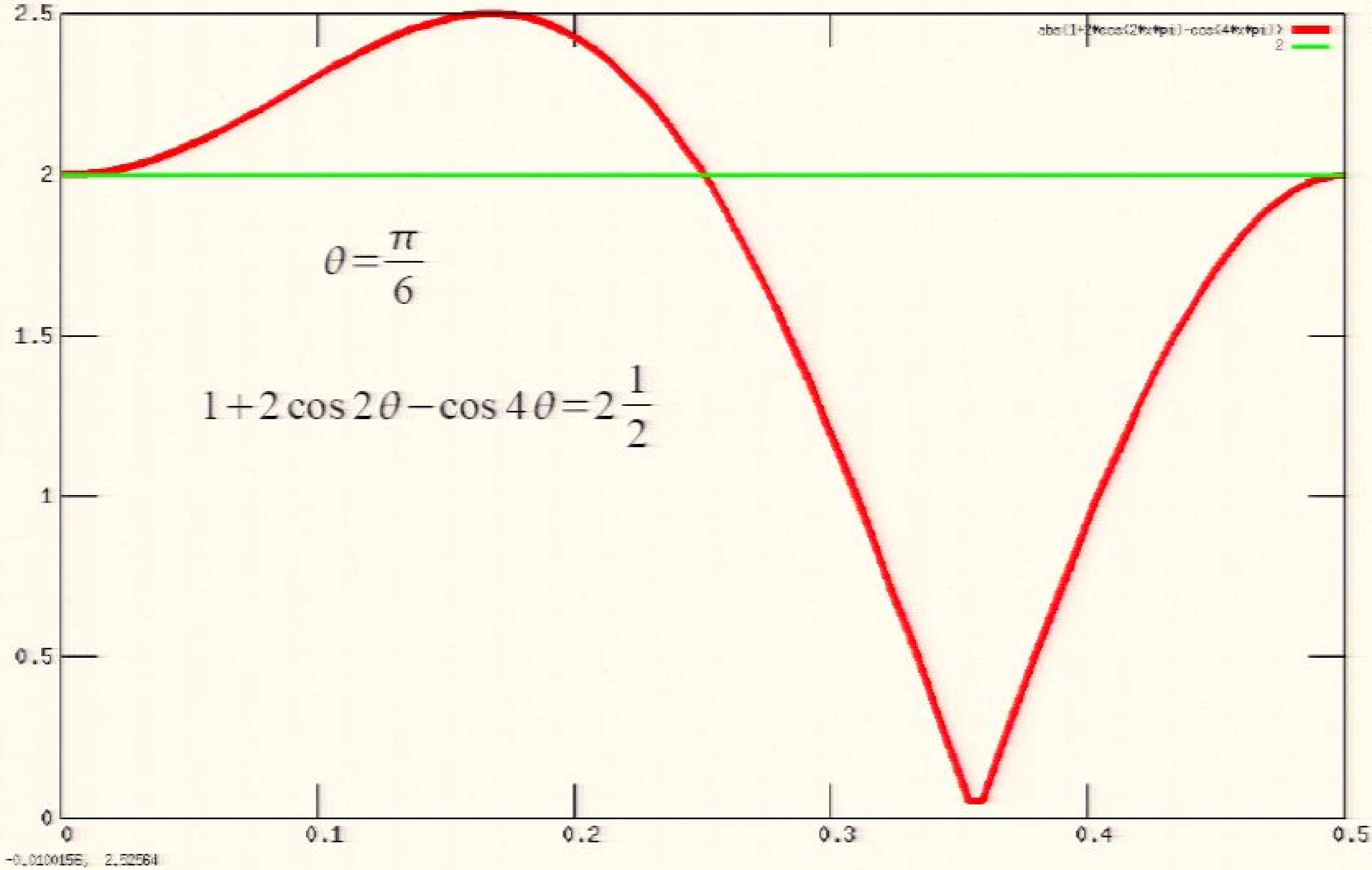
An Introduction to Quantum Foundations
Lecture 3: Quantum Non-locality

New Horizons in Fundamental Physics

No Signalling

$$\chi_\lambda(y)=\sum_n \alpha_{\lambda,n}\phi_n(y) \qquad \phi_m(y)=\sum_\mu \alpha^*_{\mu,m}\chi_\mu(y)$$

Quantum Theory!



$$\theta = \frac{\pi}{6}$$

$$1 + 2\cos 2\theta - \cos 4\theta = 2\frac{1}{2}$$

No Signalling

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$$\chi_\lambda(y)=\sum_n \alpha_{\lambda,n}\phi_n(y) \qquad \phi_m(y)=\sum_\mu \alpha^*_{\mu,m}\chi_\mu(y)$$

No Signalling

$$\chi_\lambda(y) = \sum_n \alpha_{\lambda,n}\phi_n(y) \qquad \phi_m(y) = \sum_\mu \alpha^*_{\mu,m}\chi_\mu(y)$$

$$\int_y \chi^*_\lambda(y)\chi_\mu(y)dy = \delta_{\lambda,\mu} \qquad \int_y \phi^*_n(y)\phi_m(y)dy = \delta_{n,m}$$

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$$\rho_X(x)=\sum_\lambda \int_y \chi^*_\lambda(y)\Psi(x,y)\Psi^*(x,y)\chi_\lambda(y)\,dy$$

No Signalling

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No Signalling

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No Signalling

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$$\rho_\phi(x)=\sum_\lambda \int_y \chi^*_\lambda(y)\Psi(x,y)\Psi^*(x,y)\chi_\lambda(y)\,dy=\rho_\chi(x)$$

Locality and Non-locality

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Locality and Non-locality

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- What does it mean for "the world" to be local (or non-local)?
  - All the objects or properties of "the world" have well defined locations?
  - All the objects of "the world" can be affected only by other objects in the neighbourhood of their location?
  - We cannot directly affect other objects not in our neighbourhood?

Assumptions, revisited

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- Local counterfactual definiteness

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$$P(A,a,B,b|C)=P(A,a|B,b,C)P(B,b|C)$$

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- Local counterfactual definiteness
  - Unperformed measurements have no outcomes. One cannnot say what *would have* happened with ***A***, if ***b'*** (as opposed to ***b***) *had* been measured, still less that it *would have been* the same.

Assumptions, revisited

- Local realism
  - Locality: Properties cannot be affected by remote actions
  - "Realism": Measurements have predetermined outcomes
  - Save locality by denying realism?
    - EPR requires the second to *avoid* violating first. To save locality this way one must still respond to EPR and explain perfect correlations (and non-factorisability in general) of quantum theory.
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    - *Stochastic locality conditions do not assume this.*

Loopholes

- Logical

Loopholes

- Logical
  - Action-at-a-distance

Loopholes

- Logical
  - Action-at-a-distance
  - Retro-causality

Loopholes

- Logical
  - Action-at-a-distance
  - Retro-causality
  - Superdeterminism

Loopholes

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  - Deriving the CHHS inequality

Loopholes

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- "Loopholes" to what?
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  - To non-locality?

Loopholes

- Empirical

Loopholes

- Empirical
  - Switching times

Loopholes

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- Empirical
  - Switching times (aka "locality")

Loopholes

- Empirical
  - Switching times (aka "locality")
  - Memory

Loopholes

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- Empirical
  - Switching times (aka "locality")
  - Memory
  - Detector efficiency

Loopholes

- Empirical
  - Switching times (aka "locality")
  - Memory
  - Detector efficiency, noise, time-coincidence

Loopholes

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  - Memory
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  - Fair sampling

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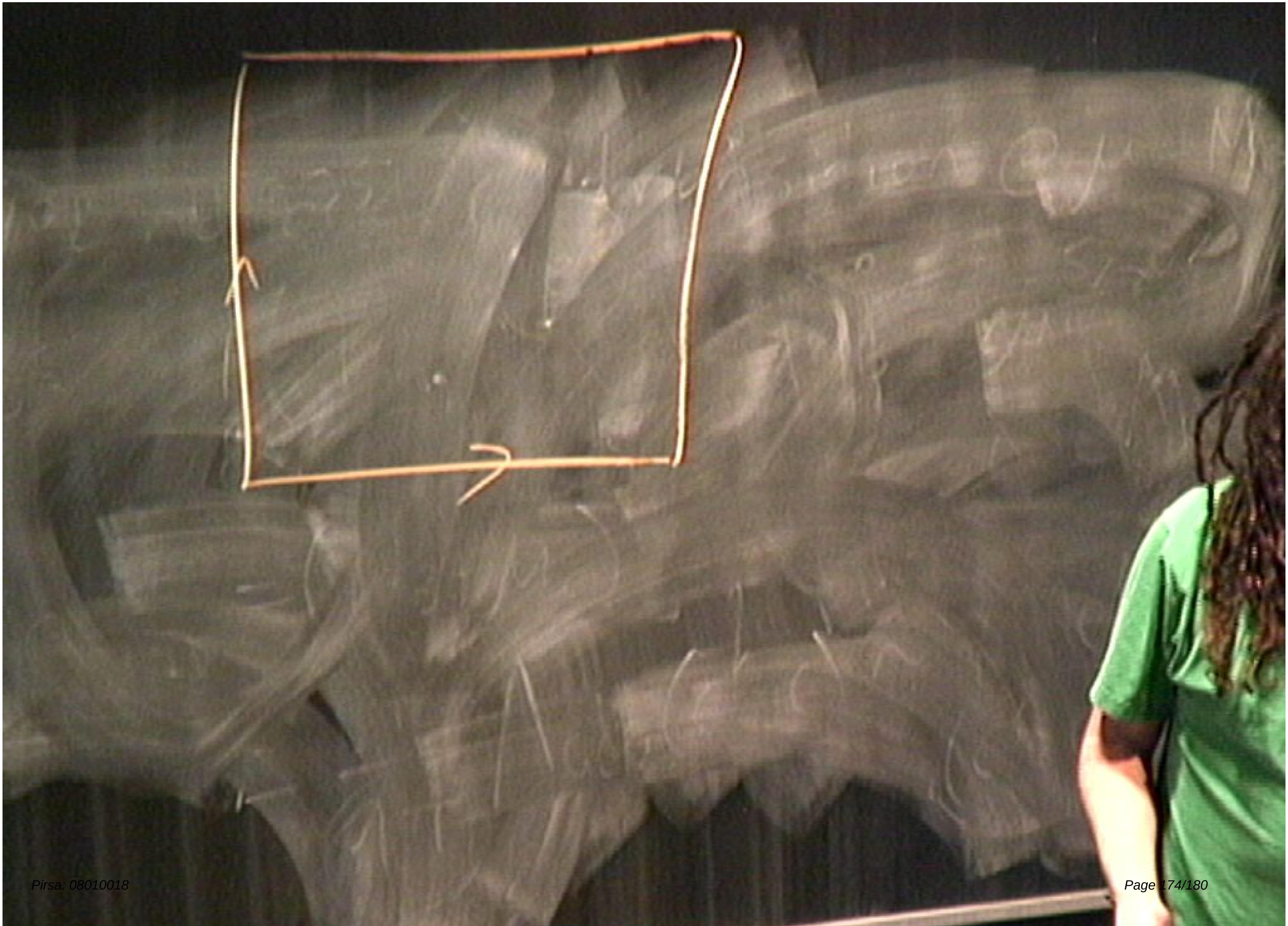
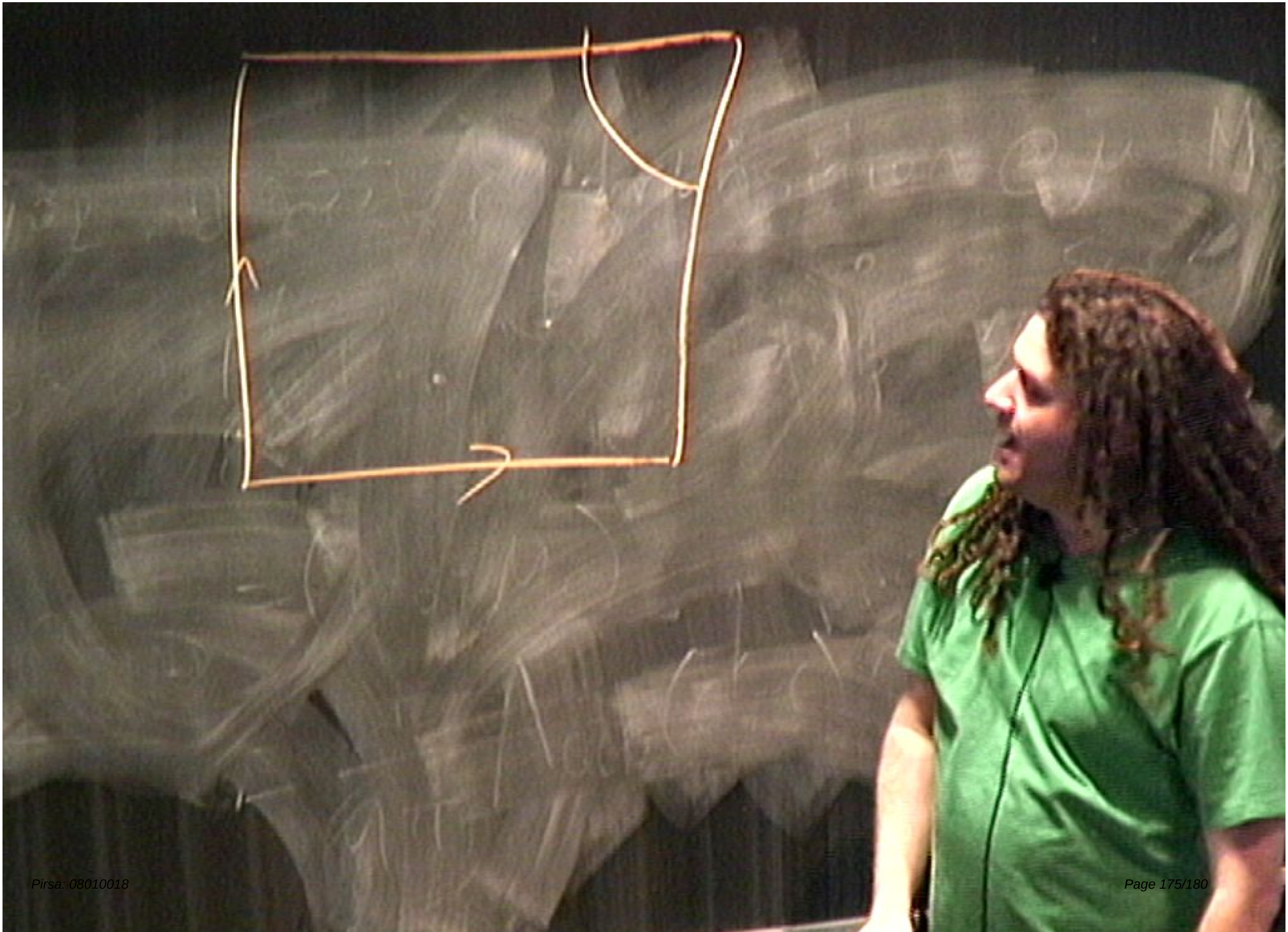
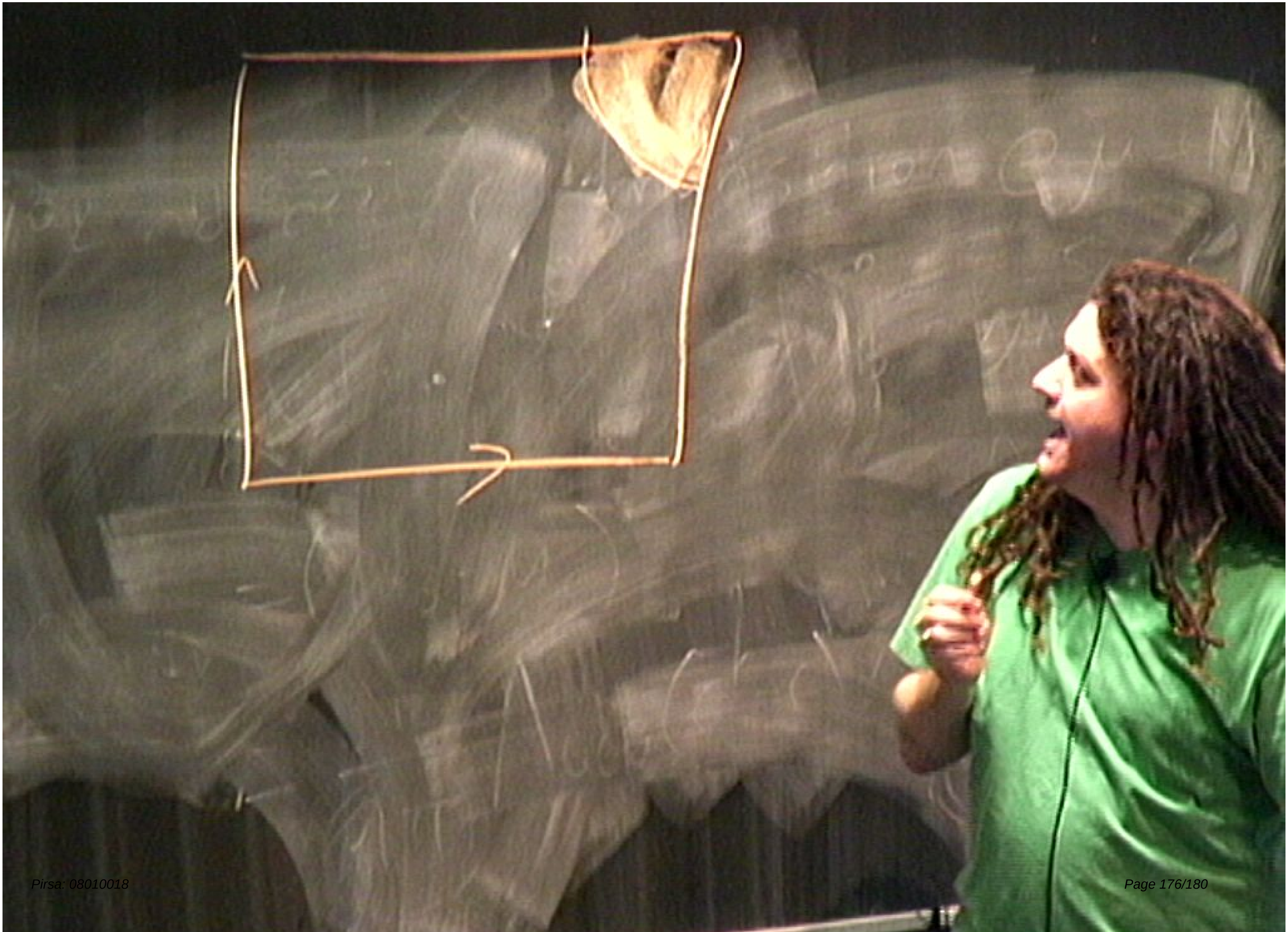
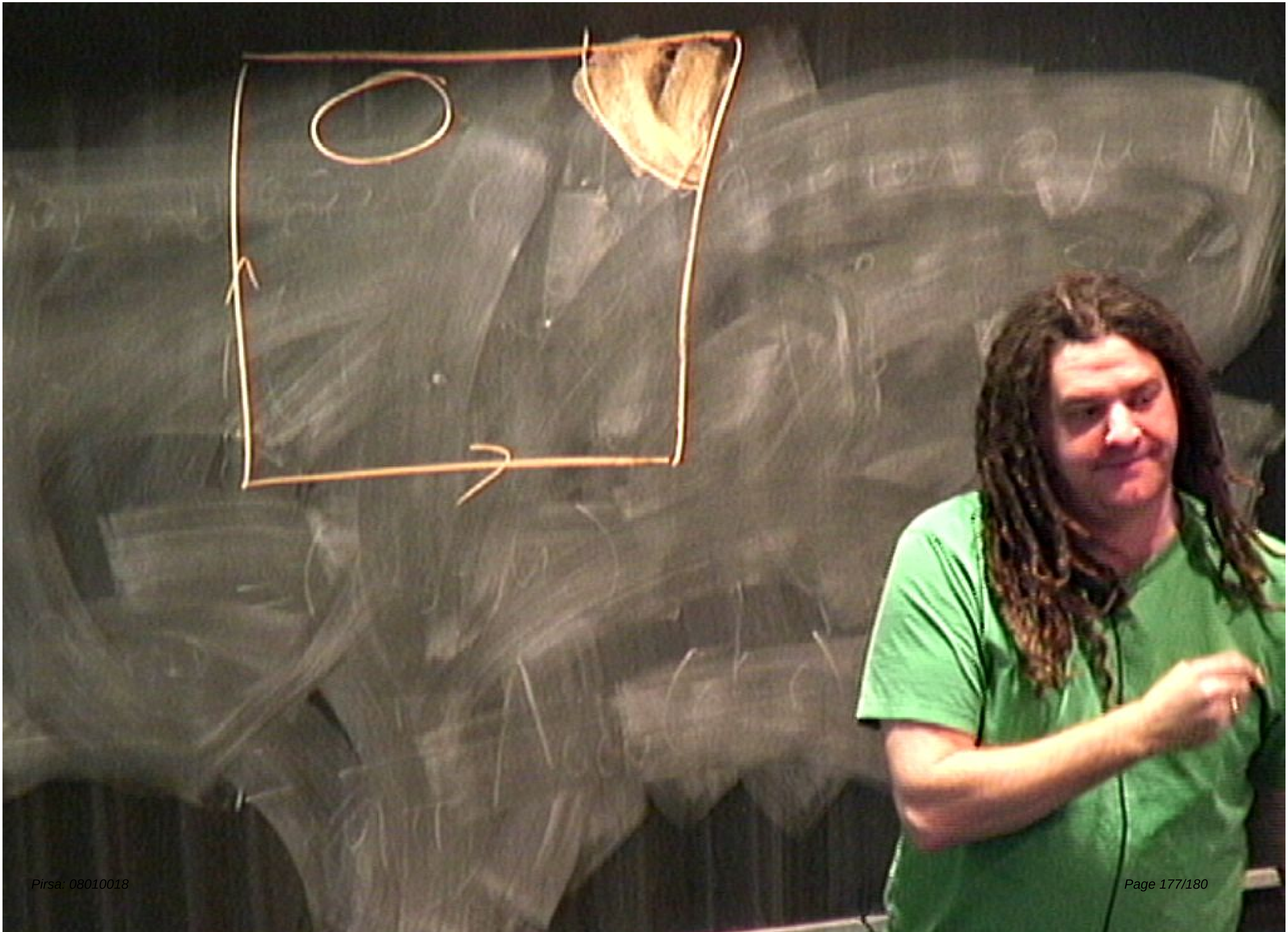
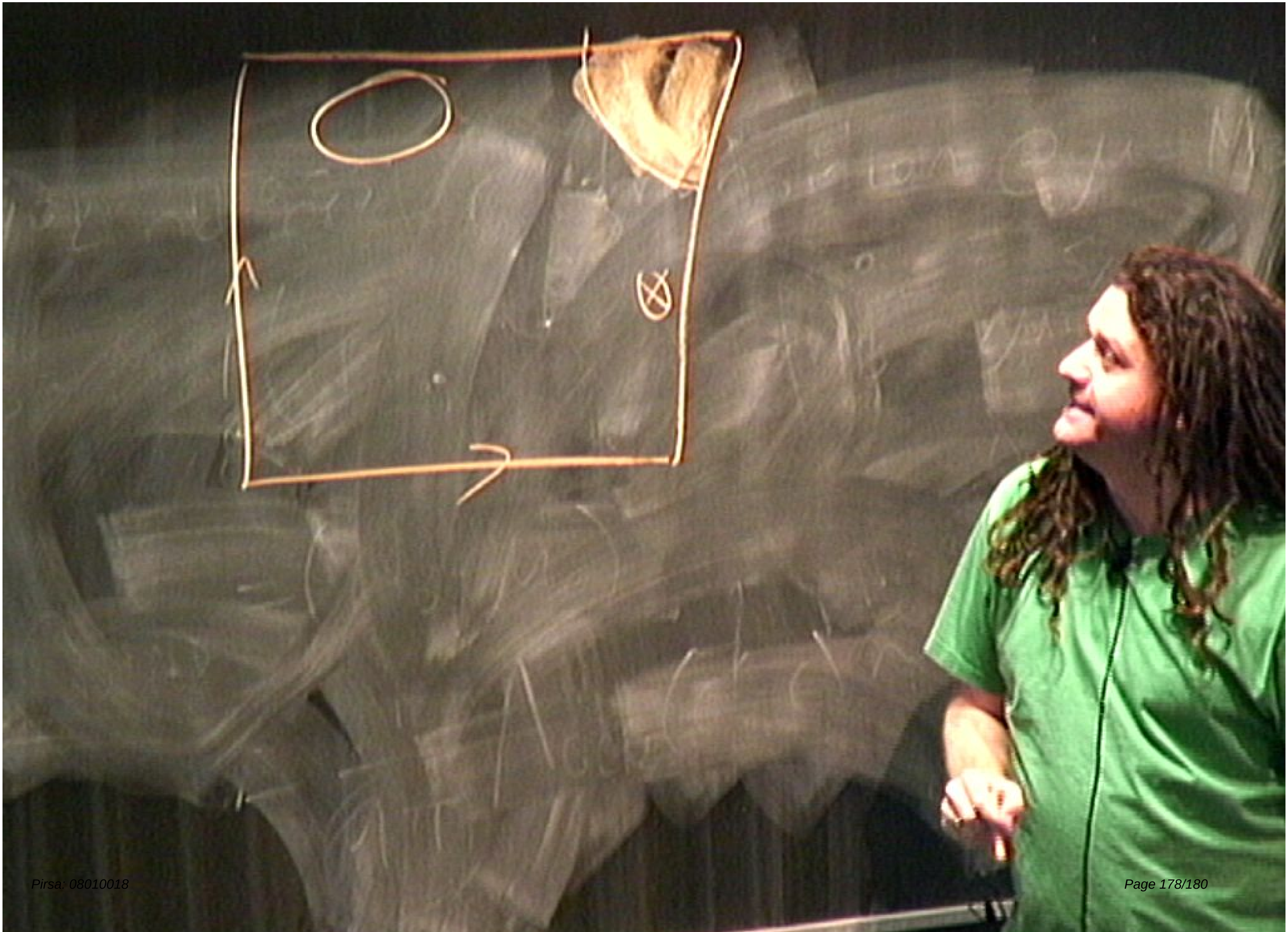
$$|E(A,B|a,b)+E(A,B|a',b)+E(A,B|a,b')-E(A,B|a',b')|\leq\frac{4}{\eta}-2$$











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