Title: Deformations of N=2 superconformal field theories

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Abstract: TBA

Deformations of N=2 SCFTs

P.C. Argyres w/ N. Seiberg, J. Wittig

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and to appear w/ P. Esposito, J. Vázquez-Poritz, J. Wittig

Motivation

What: Explore the space of 4-d N=2 SCFTs.

Why: Because we can. Also

- Start of RG flows to many N$\leq$2 field theories.
- Alternative to strings as route to non-Lagrangian FTs.

How: Exact field theory techniques:

- N=2 susy selection rules.
- Low energy effective action on moduli space of vacua (SW theory).
- Representation theory of N=2 superconformal algebra.

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Outline

I Review of N=2 susy

II Marginal deformations (gauge couplings)

III Central charges

IV Relevant deformations (masses, mostly)

V Connection to string constructions

VI Further directions

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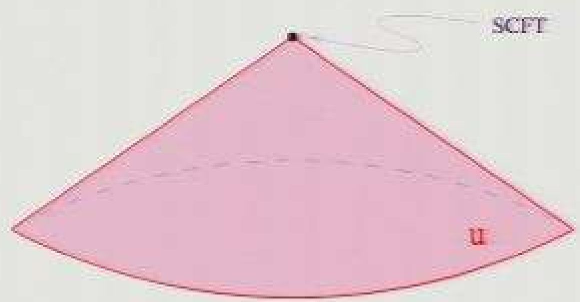
VI — Further directions

I Review of N=2 susy

All known interacting N=2 conformal theories have neutral scalar chiral multiplets.

N=2 susy implies no potential is allowed for their vevs, u_i, $i = 1, \ldots, r$. So they parameterize a moduli space of vacua with unbroken $U(1)^r$ gauge symmetry: Coulomb branch. "Rank" of SCFT is r.



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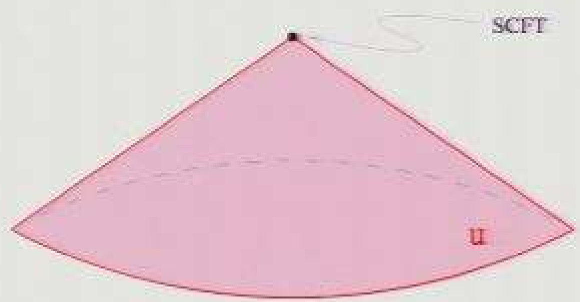

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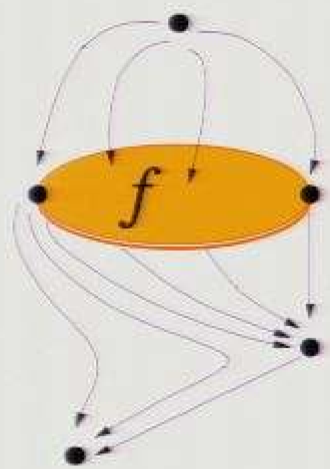

Each point in theory space includes accompanying Coulomb branch.

Exactly marginal operators $\Rightarrow$ complex manifolds of fixed points.
The marginal coupling f is a coordinate on this manifold.
S-duality = complex geometry of space of marginal couplings.

If no exactly marginal operators $\Rightarrow$ "isolated SCFT".
Isolated rank 1 SCFTs $\simeq$ Kodaira classification ($\sim$ A-D-E)...

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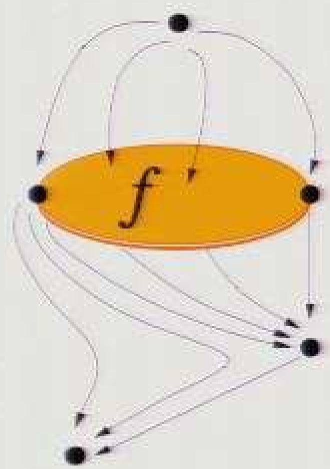

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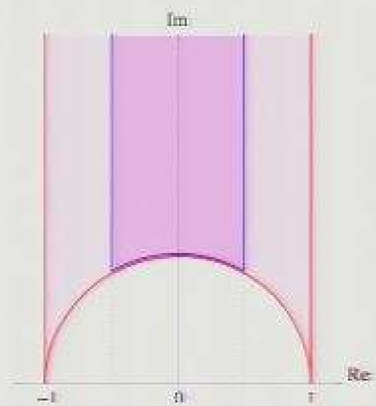
Some old puzzles:

- What happens as $g \to \infty$ in N=2 Lagrangian theories?
  E.g., su(3) w/ $6 \cdot 3$ has S-duality group $\Gamma^0(2) \subset sl(2,\mathbb{Z})$ whose fundamental domain contains $g = \infty$:



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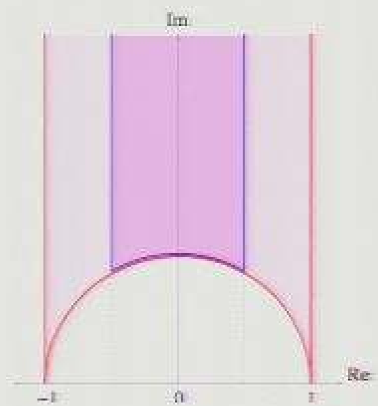

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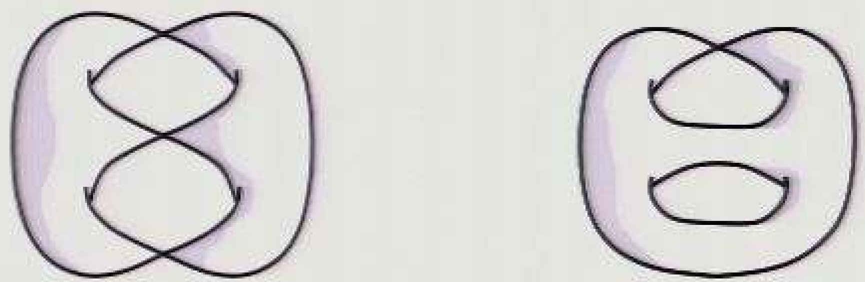
A surprise:

In addition to finding the known E_n isolated rank 1 SCFTs as decoupled factors in various $g \to \infty$ limits of Lagrangian SCFTs, we also find new isolated rank 1 SCFTs.

They share the same conformal singularity as the E_n theories, but they have different global symmetries (and different central charges).

More about these later...

- Compare $f \to 1$ degeneration to $f \to 0$ (weak cpling) degeneration:
  - As $f \to 0$ in su(3) SW curve, genus 2 curve pinches to 2 genus 0 curves everywhere on moduli space. Pinching cycle $\leftrightarrow$ charged $W^{\pm}$ bosons becoming massless, corresponding to expected su(3) $\to$ u(1) $\times$ u(1) Higgsing. (Figure on left.)



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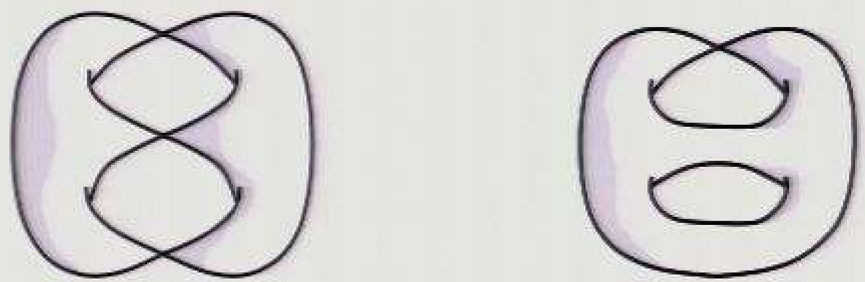

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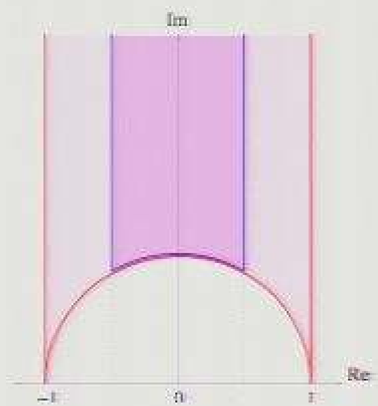

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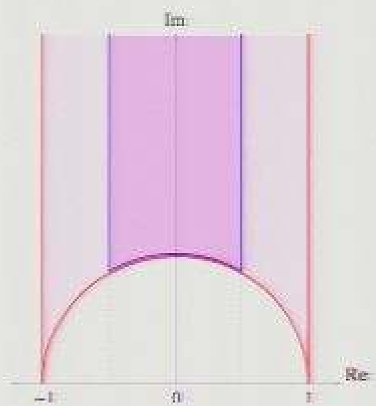

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- Generalizes S-duality from N=4 to N=2 scale-invariant field theories.
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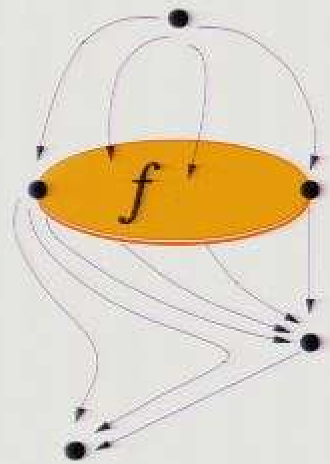

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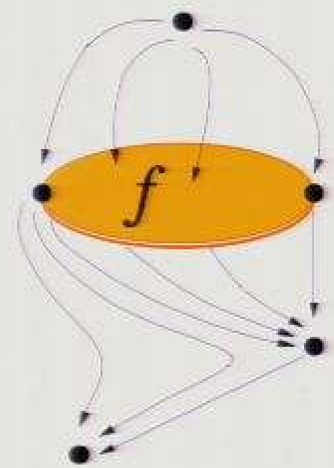

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  E.g., if $\mathfrak{g}$ w/ $\mathbf{r} = \mathrm{su}(3)$ w/ $6 \cdot \mathbf{3}$ (*i.e.*, with 6 fund. hyper-plets):

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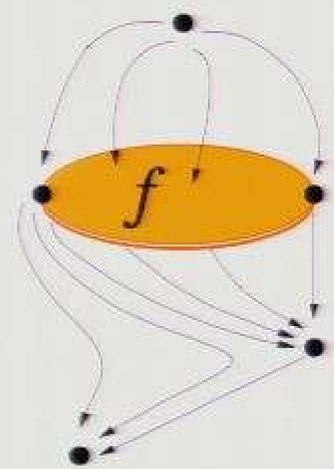

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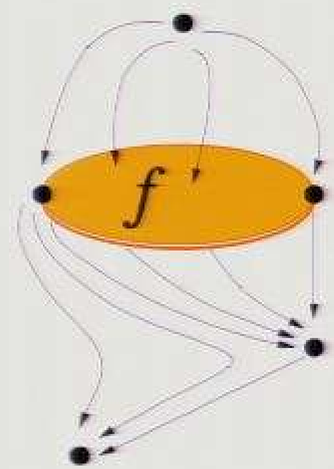

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  Many more examples $\longrightarrow$ $\longrightarrow$ $\longrightarrow$ $\longrightarrow$ $\longrightarrow$

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  E.g., the $\widetilde{\mathfrak{g}} = \mathrm{su}(2)$ W-bosons are magnetically charged under $\mathfrak{g} = \mathrm{su}(3)$ in the above example.

	$\mathfrak{g}$	w/ $\mathbf{r}$	= $\tilde{\mathfrak{g}}$	w/ $\tilde{\mathbf{r}}$	$\oplus$ SCFT $[d:\mathfrak{h}]$
1.	sp(3)	$\mathbf{14}\oplus 11\cdot\mathbf{6}$	= sp(2)		$[6:E_8]$
2.	su(6)	$\mathbf{20}\oplus\mathbf{15}\oplus\overline{\mathbf{15}}\oplus 5\cdot\mathbf{6}\oplus 5\cdot\bar{\mathbf{6}}$	= su(5)	$\mathbf{5}\oplus\bar{\mathbf{5}}\oplus\mathbf{10}\oplus\overline{\mathbf{10}}$	$[6:E_8]$
3.	so(12)	$3\cdot\mathbf{32}\oplus\mathbf{32}'\oplus 4\cdot\mathbf{12}$	= so(11)	$3\cdot\mathbf{32}$	$[6:E_8]$
4.	G_2	$8\cdot\mathbf{7}$	= su(2)	$\mathbf{2}$	$[6:\mathrm{sp}(5)]$
5.	so(7)	$4\cdot\mathbf{8}\oplus 6\cdot\mathbf{7}$	= sp(2)	$5\cdot\mathbf{4}$	$[6:\mathrm{sp}(5)]$
6.	su(6)	$\mathbf{21}\oplus\overline{\mathbf{21}}\oplus\mathbf{20}\oplus\mathbf{6}\oplus\bar{\mathbf{6}}$	= su(5)	$\mathbf{10}\oplus\overline{\mathbf{10}}$	$[6:\mathrm{sp}(5)]$
7.	sp(2)	$12\cdot\mathbf{4}$	= su(2)		$[4:E_7]$
8.	su(4)	$2\cdot\mathbf{6}\oplus 6\cdot\mathbf{4}\oplus 6\cdot\bar{\mathbf{4}}$	= su(3)	$2\cdot\mathbf{3}\oplus 2\cdot\bar{\mathbf{3}}$	$[4:E_7]$
9.	so(7)	$6\cdot\mathbf{8}\oplus 4\cdot\mathbf{7}$	= G_2	$4\cdot\mathbf{7}$	$[4:E_7]$
10.	so(8)	$6\cdot\mathbf{8}\oplus 4\cdot\mathbf{8}'\oplus 2\cdot\mathbf{8}''$	= so(7)	$6\cdot\mathbf{8}$	$[4:E_7]$
11.	so(8)	$6\cdot\mathbf{8}\oplus 6\cdot\mathbf{8}'$	= G_2		$[4:E_7]\oplus[4:E_7]$
12.	sp(2)	$6\cdot\mathbf{5}$	= su(2)		$[4:\mathrm{sp}(3)\oplus\mathrm{su}(2)]$
13.	sp(2)	$4\cdot\mathbf{4}\oplus 4\cdot\mathbf{5}$	= su(2)	$3\cdot\mathbf{2}$	$[4:\mathrm{sp}(3)\oplus\mathrm{su}(2)]$
14.	su(4)	$\mathbf{10}\oplus\overline{\mathbf{10}}\oplus 2\cdot\mathbf{4}\oplus 2\cdot\bar{\mathbf{4}}$	= su(3)	$\mathbf{3}\oplus\bar{\mathbf{3}}$	$[4:\mathrm{sp}(3)\oplus\mathrm{su}(2)]$
15.	su(3)	$6\cdot\mathbf{3}\oplus 6\cdot\bar{\mathbf{3}}$	= su(2)	$2\cdot\mathbf{2}$	$[3:E_6]$
16.	su(4)	$4\cdot\mathbf{6}\oplus 4\cdot\mathbf{4}\oplus 4\cdot\bar{\mathbf{4}}$	= sp(2)	$6\cdot\mathbf{4}$	$[3:E_6]$
17.	su(3)	$\mathbf{3}\oplus\bar{\mathbf{3}}\oplus\mathbf{6}\oplus\bar{\mathbf{6}}$	= su(2)	$n\cdot\mathbf{2}$	$[3:\mathfrak{h}]$

	$\mathfrak{g}$	w/ $\mathbf{r}$	= $\tilde{\mathfrak{g}}$	w/ $\tilde{\mathbf{r}}$	$\oplus$ SCFT $[d:\mathfrak{h}]$
18.	su(2) ⊕ su(3)	$2\cdot(\mathbf{2},\mathbf{1})\oplus(\mathbf{2},\mathbf{3}\oplus\bar{\mathbf{3}})\oplus 4\cdot(\mathbf{1},\mathbf{3}\oplus\bar{\mathbf{3}})$	= su(2) ⊕ su(2)	$2\cdot(\mathbf{2},\mathbf{1})\oplus 2\cdot(\mathbf{1},\mathbf{2})$	$[3:E_6]$
19.	su(2) ⊕ sp(2)	$2\cdot(\mathbf{2},\mathbf{4})\oplus 8\cdot(\mathbf{1},\mathbf{4})$	= su(2) ⊕ su(2)		$[4:E_7]$
20.	su(2) ⊕ sp(2)	$3\cdot(\mathbf{2},\mathbf{1})\oplus(\mathbf{2},\mathbf{5})\oplus 4\cdot(\mathbf{1},\mathbf{5})$	= su(2) ⊕ su(2)	$3\cdot(\mathbf{2},\mathbf{1})$	$[4:\mathrm{sp}(3)\oplus\mathrm{su}(2)]$
21.	su(2) ⊕ G_2	$(\mathbf{2},\mathbf{1})\oplus(\mathbf{2},\mathbf{7})\oplus 6\cdot(\mathbf{1},\mathbf{7})$	= su(2) ⊕ su(2)	$(\mathbf{2},\mathbf{1})\oplus(\mathbf{1},\mathbf{2})$	$[6:\mathrm{sp}(5)]$
22.	su(3) ⊕ su(3)	$2\cdot(\mathbf{3},\mathbf{3})\oplus 2\cdot(\mathbf{3},\mathbf{3})$	= su(2) ⊕ su(3)	$2\cdot(\mathbf{2},\mathbf{1})$	$[3:E_6]$
23.	su(3) ⊕ su(3)	$(\mathbf{3}\oplus\bar{\mathbf{3}},\mathbf{3}\oplus\bar{\mathbf{3}})$	= su(2) ⊕ su(3)	$2\cdot(\mathbf{2},\mathbf{1})$	$[3:E_6]$
24.	su(3) ⊕ su(3)	$3\cdot(\mathbf{3}\oplus\bar{\mathbf{3}},\mathbf{1})\oplus(\mathbf{3},\bar{\mathbf{3}})\oplus(\bar{\mathbf{3}},\mathbf{3})$ $\oplus 3\cdot(\mathbf{1},\mathbf{3}\oplus\bar{\mathbf{3}})$	= su(2) ⊕ su(3)	$2\cdot(\mathbf{2},\mathbf{1})$ $\oplus 3\cdot(\mathbf{1},\mathbf{3}\oplus\bar{\mathbf{3}})$	$[3:E_6]$
25.	su(3) ⊕ sp(2)	$(\mathbf{3}\oplus\bar{\mathbf{3}},\mathbf{1})\oplus(\mathbf{3}\oplus\bar{\mathbf{3}},\mathbf{5})$	= su(2) ⊕ sp(2)	$2\cdot(\mathbf{2},\mathbf{1})$	$[3:E_6]$
			= su(3) ⊕ su(2)	$(\mathbf{3}\oplus\bar{\mathbf{3}},\mathbf{1})$	$[4:\mathrm{sp}(3)\oplus\mathrm{su}(2)]$
26.	su(3) ⊕ sp(2)	$2\cdot(\mathbf{3}\oplus\bar{\mathbf{3}},\mathbf{1})\oplus(\mathbf{3}\oplus\bar{\mathbf{3}},\mathbf{4})\oplus 6\cdot(\mathbf{1},\mathbf{4})$	= su(2) ⊕ sp(2)	$2\cdot(\mathbf{2},\mathbf{1})\oplus 6\cdot(\mathbf{1},\mathbf{4})$	$[3:E_6]$
			= su(3) ⊕ su(2)	$2\cdot(\mathbf{3}\oplus\bar{\mathbf{3}},\mathbf{1})$	$[4:E_7]$
27.	sp(2) ⊕ sp(2)	$2\cdot(\mathbf{5},\mathbf{1})\oplus(\mathbf{5},\mathbf{4})\oplus 7\cdot(\mathbf{1},\mathbf{4})$	= su(2) ⊕ sp(2)	$7\cdot(\mathbf{1},\mathbf{4})$	$[4:\mathrm{sp}(3)\oplus\mathrm{su}(2)]$
			= sp(2) ⊕ su(2)	$2\cdot(\mathbf{5},\mathbf{1})$	$[4:E_7]$
28.	sp(2) ⊕ sp(2)	$4\cdot(\mathbf{4},\mathbf{1})\oplus 2\cdot(\mathbf{4},\mathbf{4})\oplus 4\cdot(\mathbf{1},\mathbf{4})$	= su(2) ⊕ sp(2)	$4\cdot(\mathbf{1},\mathbf{4})$	$[4:E_7]$
29.	sp(2) ⊕ G_2	$5\cdot(\mathbf{4},\mathbf{1})\oplus(\mathbf{4},\mathbf{7})\oplus 4\cdot(\mathbf{1},\mathbf{7})$	= su(2) ⊕ G_2	$4\cdot(\mathbf{1},\mathbf{7})$	$[4:E_7]$
			= sp(2) ⊕ su(2)	$5\cdot(\mathbf{4},\mathbf{1})\oplus(\mathbf{1},\mathbf{2})$	$[6:\mathrm{sp}(5)]$

	$\mathfrak{g}$	w/ $\mathfrak{r}$	$=$	$\tilde{\mathfrak{g}}$	w/ $\tilde{\mathfrak{r}}$	$\oplus$ SCFT $[d:\mathfrak{h}]$
1.	sp(3)	$\mathbf{14} \oplus 11 \cdot \mathbf{6}$	$=$	sp(2)		$[6:E_8]$
2.	su(6)	$\mathbf{20} \oplus \mathbf{15} \oplus \overline{\mathbf{15}} \oplus 5 \cdot \mathbf{6} \oplus 5 \cdot \bar{\mathbf{6}}$	$=$	su(5)	$\mathbf{5} \oplus \bar{\mathbf{5}} \oplus \mathbf{10} \oplus \overline{\mathbf{10}}$	$[6:E_8]$
3.	so(12)	$3 \cdot \mathbf{32} \oplus \mathbf{32}' \oplus 4 \cdot \mathbf{12}$	$=$	so(11)	$3 \cdot \mathbf{32}$	$[6:E_8]$
4.	G_2	$8 \cdot \mathbf{7}$	$=$	su(2)	$\mathbf{2}$	$[6:\mathrm{sp}(5)]$
5.	so(7)	$4 \cdot \mathbf{8} \oplus 6 \cdot \mathbf{7}$	$=$	sp(2)	$5 \cdot \mathbf{4}$	$[6:\mathrm{sp}(5)]$
6.	su(6)	$\mathbf{21} \oplus \overline{\mathbf{21}} \oplus \mathbf{20} \oplus \mathbf{6} \oplus \bar{\mathbf{6}}$	$=$	su(5)	$\mathbf{10} \oplus \overline{\mathbf{10}}$	$[6:\mathrm{sp}(5)]$
7.	sp(2)	$12 \cdot \mathbf{4}$	$=$	su(2)		$[4:E_7]$
8.	su(4)	$2 \cdot \mathbf{6} \oplus 6 \cdot \mathbf{4} \oplus 6 \cdot \bar{\mathbf{4}}$	$=$	su(3)	$2 \cdot \mathbf{3} \oplus 2 \cdot \bar{\mathbf{3}}$	$[4:E_7]$
9.	so(7)	$6 \cdot \mathbf{8} \oplus 4 \cdot \mathbf{7}$	$=$	G_2	$4 \cdot \mathbf{7}$	$[4:E_7]$
10.	so(8)	$6 \cdot \mathbf{8} \oplus 4 \cdot \mathbf{8}' \oplus 2 \cdot \mathbf{8}''$	$=$	so(7)	$6 \cdot \mathbf{8}$	$[4:E_7]$
11.	so(8)	$6 \cdot \mathbf{8} \oplus 6 \cdot \mathbf{8}'$	$=$	G_2		$[4:E_7] \oplus [4:E_7]$
12.	sp(2)	$6 \cdot \mathbf{5}$	$=$	su(2)		$[4:\mathrm{sp}(3) \oplus \mathrm{su}(2)]$
13.	sp(2)	$4 \cdot \mathbf{4} \oplus 4 \cdot \mathbf{5}$	$=$	su(2)	$3 \cdot \mathbf{2}$	$[4:\mathrm{sp}(3) \oplus \mathrm{su}(2)]$
14.	su(4)	$\mathbf{10} \oplus \overline{\mathbf{10}} \oplus 2 \cdot \mathbf{4} \oplus 2 \cdot \bar{\mathbf{4}}$	$=$	su(3)	$\mathbf{3} \oplus \bar{\mathbf{3}}$	$[4:\mathrm{sp}(3) \oplus \mathrm{su}(2)]$
15.	su(3)	$6 \cdot \mathbf{3} \oplus 6 \cdot \bar{\mathbf{3}}$	$=$	su(2)	$2 \cdot \mathbf{2}$	$[3:E_6]$
16.	su(4)	$4 \cdot \mathbf{6} \oplus 4 \cdot \mathbf{4} \oplus 4 \cdot \bar{\mathbf{4}}$	$=$	sp(2)	$6 \cdot \mathbf{4}$	$[3:E_6]$
17.	su(3)	$\mathbf{3} \oplus \bar{\mathbf{3}} \oplus \mathbf{6} \oplus \bar{\mathbf{6}}$	$=$	su(2)	$n \cdot \mathbf{2}$	$[3:\mathfrak{h}]$

	$\mathfrak{g}$	w/ $\mathfrak{r}$	$=$	$\tilde{\mathfrak{g}}$	w/ $\tilde{\mathfrak{r}}$	$\oplus$ SCFT$[d:\mathfrak{h}]$
18.	su(2) $\oplus$ su(3)	$2\cdot(\mathbf{2},\mathbf{1}) \oplus (\mathbf{2},\mathbf{3}\oplus\bar{\mathbf{3}}) \oplus 4\cdot(\mathbf{1},\mathbf{3}\oplus\bar{\mathbf{3}})$	$=$	su(2) $\oplus$ su(2)	$2\cdot(\mathbf{2},\mathbf{1}) \oplus 2\cdot(\mathbf{1},\mathbf{2})$	$[3:E_6]$
19.	su(2) $\oplus$ sp(2)	$2\cdot(\mathbf{2},\mathbf{4}) \oplus 8\cdot(\mathbf{1},\mathbf{4})$	$=$	su(2) $\oplus$ su(2)		$[4:E_7]$
20.	su(2) $\oplus$ sp(2)	$3\cdot(\mathbf{2},\mathbf{1}) \oplus (\mathbf{2},\mathbf{5}) \oplus 4\cdot(\mathbf{1},\mathbf{5})$	$=$	su(2) $\oplus$ su(2)	$3\cdot(\mathbf{2},\mathbf{1})$	$[4:\mathrm{sp}(3)\oplus\mathrm{su}(2)]$
21.	su(2) $\oplus$ G_2	$(\mathbf{2},\mathbf{1}) \oplus (\mathbf{2},\mathbf{7}) \oplus 6\cdot(\mathbf{1},\mathbf{7})$	$=$	su(2) $\oplus$ su(2)	$(\mathbf{2},\mathbf{1}) \oplus (\mathbf{1},\mathbf{2})$	$[6:\mathrm{sp}(5)]$
22.	su(3) $\oplus$ su(3)	$2\cdot(\mathbf{3},\mathbf{3}) \oplus 2\cdot(\mathbf{3},\mathbf{3})$	$=$	su(2) $\oplus$ su(3)	$2\cdot(\mathbf{2},\mathbf{1})$	$[3:E_6]$
23.	su(3) $\oplus$ su(3)	$(\mathbf{3}\oplus\bar{\mathbf{3}},\mathbf{3}\oplus\bar{\mathbf{3}})$	$=$	su(2) $\oplus$ su(3)	$2\cdot(\mathbf{2},\mathbf{1})$	$[3:E_6]$
24.	su(3) $\oplus$ su(3)	$3\cdot(\mathbf{3}\oplus\bar{\mathbf{3}},\mathbf{1}) \oplus (\mathbf{3},\bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}},\mathbf{3})$ $\oplus 3\cdot(\mathbf{1},\mathbf{3}\oplus\bar{\mathbf{3}})$	$=$	su(2) $\oplus$ su(3)	$2\cdot(\mathbf{2},\mathbf{1})$ $\oplus 3\cdot(\mathbf{1},\mathbf{3}\oplus\bar{\mathbf{3}})$	$[3:E_6]$
25.	su(3) $\oplus$ sp(2)	$(\mathbf{3}\oplus\bar{\mathbf{3}},\mathbf{1}) \oplus (\mathbf{3}\oplus\bar{\mathbf{3}},\mathbf{5})$	$=$	su(2) $\oplus$ sp(2)	$2\cdot(\mathbf{2},\mathbf{1})$	$[3:E_6]$
			$=$	su(3) $\oplus$ su(2)	$(\mathbf{3}\oplus\bar{\mathbf{3}},\mathbf{1})$	$[4:\mathrm{sp}(3)\oplus\mathrm{su}(2)]$
26.	su(3) $\oplus$ sp(2)	$2\cdot(\mathbf{3}\oplus\bar{\mathbf{3}},\mathbf{1}) \oplus (\mathbf{3}\oplus\bar{\mathbf{3}},\mathbf{4}) \oplus 6\cdot(\mathbf{1},\mathbf{4})$	$=$	su(2) $\oplus$ sp(2)	$2\cdot(\mathbf{2},\mathbf{1}) \oplus 6\cdot(\mathbf{1},\mathbf{4})$	$[3:E_6]$
			$=$	su(3) $\oplus$ su(2)	$2\cdot(\mathbf{3}\oplus\bar{\mathbf{3}},\mathbf{1})$	$[4:E_7]$
27.	sp(2) $\oplus$ sp(2)	$2\cdot(\mathbf{5},\mathbf{1}) \oplus (\mathbf{5},\mathbf{4}) \oplus 7\cdot(\mathbf{1},\mathbf{4})$	$=$	su(2) $\oplus$ sp(2)	$7\cdot(\mathbf{1},\mathbf{4})$	$[4:\mathrm{sp}(3)\oplus\mathrm{su}(2)]$
			$=$	sp(2) $\oplus$ su(2)	$2\cdot(\mathbf{5},\mathbf{1})$	$[4:E_7]$
28.	sp(2) $\oplus$ sp(2)	$4\cdot(\mathbf{4},\mathbf{1}) \oplus 2\cdot(\mathbf{4},\mathbf{4}) \oplus 4\cdot(\mathbf{1},\mathbf{4})$	$=$	su(2) $\oplus$ sp(2)	$4\cdot(\mathbf{1},\mathbf{4})$	$[4:E_7]$
29.	sp(2) $\oplus$ G_2	$5\cdot(\mathbf{4},\mathbf{1}) \oplus (\mathbf{4},\mathbf{7}) \oplus 4\cdot(\mathbf{1},\mathbf{7})$	$=$	su(2) $\oplus$ G_2	$4\cdot(\mathbf{1},\mathbf{7})$	$[4:E_7]$
			$=$	sp(2) $\oplus$ su(2)	$5\cdot(\mathbf{4},\mathbf{1}) \oplus (\mathbf{1},\mathbf{2})$	$[6:\mathrm{sp}(5)]$

	$\mathfrak{g}$ w/ $\mathbf{r}$		=	$\widetilde{\mathfrak{g}}$ w/ $\widetilde{\mathbf{r}}$		⊕ SCFT $[d:\mathfrak{h}]$
1.	sp(3)	$\mathbf{14} \oplus 11 \cdot \mathbf{6}$	=	sp(2)		$[6 : E_8]$
2.	su(6)	$\mathbf{20} \oplus \mathbf{15} \oplus \overline{\mathbf{15}} \oplus 5 \cdot \mathbf{6} \oplus 5 \cdot \overline{\mathbf{6}}$	=	su(5)	$\mathbf{5} \oplus \overline{\mathbf{5}} \oplus \mathbf{10} \oplus \overline{\mathbf{10}}$	$[6 : E_8]$
3.	so(12)	$3 \cdot \mathbf{32} \oplus \mathbf{32}' \oplus 4 \cdot \mathbf{12}$	=	so(11)	$3 \cdot \mathbf{32}$	$[6 : E_8]$
4.	G_2	$8 \cdot \mathbf{7}$	=	su(2)	$\mathbf{2}$	[6 : sp(5)]
5.	so(7)	$4 \cdot \mathbf{8} \oplus 6 \cdot \mathbf{7}$	=	sp(2)	$5 \cdot \mathbf{4}$	[6 : sp(5)]
6.	su(6)	$\mathbf{21} \oplus \overline{\mathbf{21}} \oplus \mathbf{20} \oplus \mathbf{6} \oplus \overline{\mathbf{6}}$	=	su(5)	$\mathbf{10} \oplus \overline{\mathbf{10}}$	[6 : sp(5)]
7.	sp(2)	$12 \cdot \mathbf{4}$	=	su(2)		$[4 : E_7]$
8.	su(4)	$2 \cdot \mathbf{6} \oplus 6 \cdot \mathbf{4} \oplus 6 \cdot \overline{\mathbf{4}}$	=	su(3)	$2 \cdot \mathbf{3} \oplus 2 \cdot \overline{\mathbf{3}}$	$[4 : E_7]$
9.	so(7)	$6 \cdot \mathbf{8} \oplus 4 \cdot \mathbf{7}$	=	G_2	$4 \cdot \mathbf{7}$	$[4 : E_7]$
10.	so(8)	$6 \cdot \mathbf{8} \oplus 4 \cdot \mathbf{8}' \oplus 2 \cdot \mathbf{8}''$	=	so(7)	$6 \cdot \mathbf{8}$	$[4 : E_7]$
11.	so(8)	$6 \cdot \mathbf{8} \oplus 6 \cdot \mathbf{8}'$	=	G_2		$[4 : E_7] \oplus [4 : E_7]$
12.	sp(2)	$6 \cdot \mathbf{5}$	=	su(2)		[4 : sp(3) ⊕ su(2)]
13.	sp(2)	$4 \cdot \mathbf{4} \oplus 4 \cdot \mathbf{5}$	=	su(2)	$3 \cdot \mathbf{2}$	[4 : sp(3) ⊕ su(2)]
14.	su(4)	$\mathbf{10} \oplus \overline{\mathbf{10}} \oplus 2 \cdot \mathbf{4} \oplus 2 \cdot \overline{\mathbf{4}}$	=	su(3)	$\mathbf{3} \oplus \overline{\mathbf{3}}$	[4 : sp(3) ⊕ su(2)]
15.	su(3)	$6 \cdot \mathbf{3} \oplus 6 \cdot \overline{\mathbf{3}}$	=	su(2)	$2 \cdot \mathbf{2}$	$[3 : E_6]$
16.	su(4)	$4 \cdot \mathbf{6} \oplus 4 \cdot \mathbf{4} \oplus 4 \cdot \overline{\mathbf{4}}$	=	sp(2)	$6 \cdot \mathbf{4}$	$[3 : E_6]$
17.	su(3)	$\mathbf{3} \oplus \overline{\mathbf{3}} \oplus \mathbf{6} \oplus \overline{\mathbf{6}}$	=	su(2)	$n \cdot \mathbf{2}$	$[3 : \mathfrak{h}]$

	$\mathfrak{g}$	w/ $\mathbf{r}$	=	$\widetilde{\mathfrak{g}}$	w/ $\widetilde{\mathbf{r}}$	⊕ SCFT$[d:\mathfrak{h}]$
18.	su(2) ⊕ su(3)	$2 \cdot (\mathbf{2},\mathbf{1}) \oplus (\mathbf{2},\mathbf{3} \oplus \overline{\mathbf{3}}) \oplus 4 \cdot (\mathbf{1},\mathbf{3} \oplus \overline{\mathbf{3}})$	=	su(2) ⊕ su(2)	$2 \cdot (\mathbf{2},\mathbf{1}) \oplus 2 \cdot (\mathbf{1},\mathbf{2})$	$[3 : E_6]$
19.	su(2) ⊕ sp(2)	$2 \cdot (\mathbf{2},\mathbf{4}) \oplus 8 \cdot (\mathbf{1},\mathbf{4})$	=	su(2) ⊕ su(2)		$[4 : E_7]$
20.	su(2) ⊕ sp(2)	$3 \cdot (\mathbf{2},\mathbf{1}) \oplus (\mathbf{2},\mathbf{5}) \oplus 4 \cdot (\mathbf{1},\mathbf{5})$	=	su(2) ⊕ su(2)	$3 \cdot (\mathbf{2},\mathbf{1})$	[4 : sp(3)⊕su(2)]
21.	su(2) ⊕ G_2	$(\mathbf{2},\mathbf{1}) \oplus (\mathbf{2},\mathbf{7}) \oplus 6 \cdot (\mathbf{1},\mathbf{7})$	=	su(2) ⊕ su(2)	$(\mathbf{2},\mathbf{1}) \oplus (\mathbf{1},\mathbf{2})$	[6 : sp(5)]
22.	su(3) ⊕ su(3)	$2 \cdot (\mathbf{3},\mathbf{3}) \oplus 2 \cdot (\mathbf{3},\mathbf{3})$	=	su(2) ⊕ su(3)	$2 \cdot (\mathbf{2},\mathbf{1})$	$[3 : E_6]$
23.	su(3) ⊕ su(3)	$(\mathbf{3} \oplus \overline{\mathbf{3}},\mathbf{3} \oplus \mathbf{3})$	=	su(2) ⊕ su(3)	$2 \cdot (\mathbf{2},\mathbf{1})$	$[3 : E_6]$
24.	su(3) ⊕ su(3)	$3 \cdot (\mathbf{3} \oplus \overline{\mathbf{3}},\mathbf{1}) \oplus (\mathbf{3},\overline{\mathbf{3}}) \oplus (\overline{\mathbf{3}},\mathbf{3})$ $\oplus 3 \cdot (\mathbf{1},\mathbf{3} \oplus \overline{\mathbf{3}})$	=	su(2) ⊕ su(3)	$2 \cdot (\mathbf{2},\mathbf{1})$ $\oplus 3 \cdot (\mathbf{1},\mathbf{3} \oplus \overline{\mathbf{3}})$	$[3 : E_6]$
25.	su(3) ⊕ sp(2)	$(\mathbf{3} \oplus \overline{\mathbf{3}},\mathbf{1}) \oplus (\mathbf{3} \oplus \overline{\mathbf{3}},\mathbf{5})$	=	su(2) ⊕ sp(2)	$2 \cdot (\mathbf{2},\mathbf{1})$	$[3 : E_6]$
			=	su(3) ⊕ su(2)	$(\mathbf{3} \oplus \overline{\mathbf{3}},\mathbf{1})$	[4 : sp(3)⊕su(2)]
26.	su(3) ⊕ sp(2)	$2 \cdot (\mathbf{3} \oplus \overline{\mathbf{3}},\mathbf{1}) \oplus (\mathbf{3} \oplus \overline{\mathbf{3}},\mathbf{4}) \oplus 6 \cdot (\mathbf{1},\mathbf{4})$	=	su(2) ⊕ sp(2)	$2 \cdot (\mathbf{2},\mathbf{1}) \oplus 6 \cdot (\mathbf{1},\mathbf{4})$	$[3 : E_6]$
			=	su(3) ⊕ su(2)	$2 \cdot (\mathbf{3} \oplus \overline{\mathbf{3}},\mathbf{1})$	$[4 : E_7]$
27.	sp(2) ⊕ sp(2)	$2 \cdot (\mathbf{5},\mathbf{1}) \oplus (\mathbf{5},\mathbf{4}) \oplus 7 \cdot (\mathbf{1},\mathbf{4})$	=	su(2) ⊕ sp(2)	$7 \cdot (\mathbf{1},\mathbf{4})$	[4 : sp(3)⊕su(2)]
			=	sp(2) ⊕ su(2)	$2 \cdot (\mathbf{5},\mathbf{1})$	$[4 : E_7]$
28.	sp(2) ⊕ sp(2)	$4 \cdot (\mathbf{4},\mathbf{1}) \oplus 2 \cdot (\mathbf{4},\mathbf{4}) \oplus 4 \cdot (\mathbf{1},\mathbf{4})$	=	su(2) ⊕ sp(2)	$4 \cdot (\mathbf{1},\mathbf{4})$	$[4 : E_7]$
29.	sp(2) ⊕ G_2	$5 \cdot (\mathbf{4},\mathbf{1}) \oplus (\mathbf{4},\mathbf{7}) \oplus 4 \cdot (\mathbf{1},\mathbf{7})$	=	su(2) ⊕ G_2	$4 \cdot (\mathbf{1},\mathbf{7})$	$[4 : E_7]$
			=	sp(2) ⊕ su(2)	$5 \cdot (\mathbf{4},\mathbf{1}) \oplus (\mathbf{1},\mathbf{2})$	[6 : sp(5)]

A surprise:

In addition to finding the known E_n isolated rank 1 SCFTs as decoupled factors in various $g \to \infty$ limits of Lagrangian SCFTs, we also find new isolated rank 1 SCFTs.

They share the same conformal singularity as the E_n theories, but they have different global symmetries (and different central charges).

More about these later...

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	$\mathfrak{g}$	w/ $\mathbf{r}$	=	$\widetilde{\mathfrak{g}}$	w/ $\widetilde{\mathbf{r}}$	$\oplus$ SCFT $[d:\mathfrak{h}]$
1.	sp(3)	$\mathbf{14}\oplus 11\cdot\mathbf{6}$	=	sp(2)		$[6:E_8]$
2.	su(6)	$\mathbf{20}\oplus\mathbf{15}\oplus\overline{\mathbf{15}}\oplus 5\cdot\mathbf{6}\oplus 5\cdot\overline{\mathbf{6}}$	=	su(5)	$\mathbf{5}\oplus\overline{\mathbf{5}}\oplus\mathbf{10}\oplus\overline{\mathbf{10}}$	$[6:E_8]$
3.	so(12)	$3\cdot\mathbf{32}\oplus\mathbf{32}'\oplus 4\cdot\mathbf{12}$	=	so(11)	$3\cdot\mathbf{32}$	$[6:E_8]$
4.	G_2	$8\cdot\mathbf{7}$	=	su(2)	$\mathbf{2}$	$[6:\mathrm{sp}(5)]$
5.	so(7)	$4\cdot\mathbf{8}\oplus 6\cdot\mathbf{7}$	=	sp(2)	$5\cdot\mathbf{4}$	$[6:\mathrm{sp}(5)]$
6.	su(6)	$\mathbf{21}\oplus\overline{\mathbf{21}}\oplus\mathbf{20}\oplus\mathbf{6}\oplus\overline{\mathbf{6}}$	=	su(5)	$\mathbf{10}\oplus\overline{\mathbf{10}}$	$[6:\mathrm{sp}(5)]$
7.	sp(2)	$12\cdot\mathbf{4}$	=	su(2)		$[4:E_7]$
8.	su(4)	$2\cdot\mathbf{6}\oplus 6\cdot\mathbf{4}\oplus 6\cdot\overline{\mathbf{4}}$	=	su(3)	$2\cdot\mathbf{3}\oplus 2\cdot\overline{\mathbf{3}}$	$[4:E_7]$
9.	so(7)	$6\cdot\mathbf{8}\oplus 4\cdot\mathbf{7}$	=	G_2	$4\cdot\mathbf{7}$	$[4:E_7]$
10.	so(8)	$6\cdot\mathbf{8}\oplus 4\cdot\mathbf{8}'\oplus 2\cdot\mathbf{8}''$	=	so(7)	$6\cdot\mathbf{8}$	$[4:E_7]$
11.	so(8)	$6\cdot\mathbf{8}\oplus 6\cdot\mathbf{8}'$	=	G_2		$[4:E_7]\oplus[4:E_7]$
12.	sp(2)	$6\cdot\mathbf{5}$	=	su(2)		$[4:\mathrm{sp}(3)\oplus\mathrm{su}(2)]$
13.	sp(2)	$4\cdot\mathbf{4}\oplus 4\cdot\mathbf{5}$	=	su(2)	$3\cdot\mathbf{2}$	$[4:\mathrm{sp}(3)\oplus\mathrm{su}(2)]$
14.	su(4)	$\mathbf{10}\oplus\overline{\mathbf{10}}\oplus 2\cdot\mathbf{4}\oplus 2\cdot\overline{\mathbf{4}}$	=	su(3)	$\mathbf{3}\oplus\overline{\mathbf{3}}$	$[4:\mathrm{sp}(3)\oplus\mathrm{su}(2)]$
15.	su(3)	$6\cdot\mathbf{3}\oplus 6\cdot\overline{\mathbf{3}}$	=	su(2)	$2\cdot\mathbf{2}$	$[3:E_6]$
16.	su(4)	$4\cdot\mathbf{6}\oplus 4\cdot\mathbf{4}\oplus 4\cdot\overline{\mathbf{4}}$	=	sp(2)	$6\cdot\mathbf{4}$	$[3:E_6]$
17.	su(3)	$\mathbf{3}\oplus\overline{\mathbf{3}}\oplus\mathbf{6}\oplus\overline{\mathbf{6}}$	=	su(2)	$n\cdot\mathbf{2}$	$[3:\mathfrak{h}]$

	$\mathfrak{g}$	w/ $\mathbf{r}$	=	$\widetilde{\mathfrak{g}}$	w/ $\widetilde{\mathbf{r}}$	$\oplus$ SCFT$[d:\mathfrak{h}]$
18.	su(2) $\oplus$ su(3)	$2\cdot(\mathbf{2},\mathbf{1})\oplus(\mathbf{2},\mathbf{3}\oplus\overline{\mathbf{3}})\oplus 4\cdot(\mathbf{1},\mathbf{3}\oplus\overline{\mathbf{3}})$	=	su(2) $\oplus$ su(2)	$2\cdot(\mathbf{2},\mathbf{1})\oplus 2\cdot(\mathbf{1},\mathbf{2})$	$[3:E_6]$
19.	su(2) $\oplus$ sp(2)	$2\cdot(\mathbf{2},\mathbf{4})\oplus 8\cdot(\mathbf{1},\mathbf{4})$	=	su(2) $\oplus$ su(2)		$[4:E_7]$
20.	su(2) $\oplus$ sp(2)	$3\cdot(\mathbf{2},\mathbf{1})\oplus(\mathbf{2},\mathbf{5})\oplus 4\cdot(\mathbf{1},\mathbf{5})$	=	su(2) $\oplus$ su(2)	$3\cdot(\mathbf{2},\mathbf{1})$	$[4:\mathrm{sp}(3)\oplus\mathrm{su}(2)]$
21.	su(2) $\oplus$ G_2	$(\mathbf{2},\mathbf{1})\oplus(\mathbf{2},\mathbf{7})\oplus 6\cdot(\mathbf{1},\mathbf{7})$	=	su(2) $\oplus$ su(2)	$(\mathbf{2},\mathbf{1})\oplus(\mathbf{1},\mathbf{2})$	$[6:\mathrm{sp}(5)]$
22.	su(3) $\oplus$ su(3)	$2\cdot(\mathbf{3},\mathbf{3})\oplus 2\cdot(\mathbf{3},\mathbf{3})$	=	su(2) $\oplus$ su(3)	$2\cdot(\mathbf{2},\mathbf{1})$	$[3:E_6]$
23.	su(3) $\oplus$ su(3)	$(\mathbf{3}\oplus\overline{\mathbf{3}},\mathbf{3}\oplus\overline{\mathbf{3}})$	=	su(2) $\oplus$ su(3)	$2\cdot(\mathbf{2},\mathbf{1})$	$[3:E_6]$
24.	su(3) $\oplus$ su(3)	$3\cdot(\mathbf{3}\oplus\overline{\mathbf{3}},\mathbf{1})\oplus(\mathbf{3},\overline{\mathbf{3}})\oplus(\overline{\mathbf{3}},\mathbf{3})$ $\oplus 3\cdot(\mathbf{1},\mathbf{3}\oplus\overline{\mathbf{3}})$	=	su(2) $\oplus$ su(3)	$2\cdot(\mathbf{2},\mathbf{1})$ $\oplus 3\cdot(\mathbf{1},\mathbf{3}\oplus\overline{\mathbf{3}})$	$[3:E_6]$
25.	su(3) $\oplus$ sp(2)	$(\mathbf{3}\oplus\overline{\mathbf{3}},\mathbf{1})\oplus(\mathbf{3}\oplus\overline{\mathbf{3}},\mathbf{5})$	=	su(2) $\oplus$ sp(2)	$2\cdot(\mathbf{2},\mathbf{1})$	$[3:E_6]$
			=	su(3) $\oplus$ su(2)	$(\mathbf{3}\oplus\overline{\mathbf{3}},\mathbf{1})$	$[4:\mathrm{sp}(3)\oplus\mathrm{su}(2)]$
26.	su(3) $\oplus$ sp(2)	$2\cdot(\mathbf{3}\oplus\overline{\mathbf{3}},\mathbf{1})\oplus(\mathbf{3}\oplus\overline{\mathbf{3}},\mathbf{4})\oplus 6\cdot(\mathbf{1},\mathbf{4})$	=	su(2) $\oplus$ sp(2)	$2\cdot(\mathbf{2},\mathbf{1})\oplus 6\cdot(\mathbf{1},\mathbf{4})$	$[3:E_6]$
			=	su(3) $\oplus$ su(2)	$2\cdot(\mathbf{3}\oplus\overline{\mathbf{3}},\mathbf{1})$	$[4:E_7]$
27.	sp(2) $\oplus$ sp(2)	$2\cdot(\mathbf{5},\mathbf{1})\oplus(\mathbf{5},\mathbf{4})\oplus 7\cdot(\mathbf{1},\mathbf{4})$	=	su(2) $\oplus$ sp(2)	$7\cdot(\mathbf{1},\mathbf{4})$	$[4:\mathrm{sp}(3)\oplus\mathrm{su}(2)]$
			=	sp(2) $\oplus$ su(2)	$2\cdot(\mathbf{5},\mathbf{1})$	$[4:E_7]$
28.	sp(2) $\oplus$ sp(2)	$4\cdot(\mathbf{4},\mathbf{1})\oplus 2\cdot(\mathbf{4},\mathbf{4})\oplus 4\cdot(\mathbf{1},\mathbf{4})$	=	su(2) $\oplus$ sp(2)	$4\cdot(\mathbf{1},\mathbf{4})$	$[4:E_7]$
29.	sp(2) $\oplus$ G_2	$5\cdot(\mathbf{4},\mathbf{1})\oplus(\mathbf{4},\mathbf{7})\oplus 4\cdot(\mathbf{1},\mathbf{7})$	=	su(2) $\oplus$ G_2	$4\cdot(\mathbf{1},\mathbf{7})$	$[4:E_7]$
			=	sp(2) $\oplus$ su(2)	$5\cdot(\mathbf{4},\mathbf{1})\oplus(\mathbf{1},\mathbf{2})$	$[6:\mathrm{sp}(5)]$

	$\mathfrak{g}$	w/ $\mathbf{r}$	=	$\tilde{\mathfrak{g}}$	w/ $\tilde{\mathbf{r}}$	$\oplus$ SCFT $[d:\mathfrak{h}]$
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2.	$\mathsf{su}(6)$	$\mathbf{20} \oplus \mathbf{15} \oplus \overline{\mathbf{15}} \oplus 5 \cdot \mathbf{6} \oplus 5 \cdot \overline{\mathbf{6}}$	=	$\mathsf{su}(5)$	$\mathbf{5} \oplus \overline{\mathbf{5}} \oplus \mathbf{10} \oplus \overline{\mathbf{10}}$	$[6:E_8]$
3.	$\mathsf{so}(12)$	$3 \cdot \mathbf{32} \oplus \mathbf{32}' \oplus 4 \cdot \mathbf{12}$	=	$\mathsf{so}(11)$	$3 \cdot \mathbf{32}$	$[6:E_8]$
4.	G_2	$8 \cdot \mathbf{7}$	=	$\mathsf{su}(2)$	$\mathbf{2}$	$[6:\mathsf{sp}(5)]$
5.	$\mathsf{so}(7)$	$4 \cdot \mathbf{8} \oplus 6 \cdot \mathbf{7}$	=	$\mathsf{sp}(2)$	$5 \cdot \mathbf{4}$	$[6:\mathsf{sp}(5)]$
6.	$\mathsf{su}(6)$	$\mathbf{21} \oplus \overline{\mathbf{21}} \oplus \mathbf{20} \oplus \mathbf{6} \oplus \overline{\mathbf{6}}$	=	$\mathsf{su}(5)$	$\mathbf{10} \oplus \overline{\mathbf{10}}$	$[6:\mathsf{sp}(5)]$
7.	$\mathsf{sp}(2)$	$12 \cdot \mathbf{4}$	=	$\mathsf{su}(2)$		$[4:E_7]$
8.	$\mathsf{su}(4)$	$2 \cdot \mathbf{6} \oplus 6 \cdot \mathbf{4} \oplus 6 \cdot \overline{\mathbf{4}}$	=	$\mathsf{su}(3)$	$2 \cdot \mathbf{3} \oplus 2 \cdot \overline{\mathbf{3}}$	$[4:E_7]$
9.	$\mathsf{so}(7)$	$6 \cdot \mathbf{8} \oplus 4 \cdot \mathbf{7}$	=	G_2	$4 \cdot \mathbf{7}$	$[4:E_7]$
10.	$\mathsf{so}(8)$	$6 \cdot \mathbf{8} \oplus 4 \cdot \mathbf{8}' \oplus 2 \cdot \mathbf{8}''$	=	$\mathsf{so}(7)$	$6 \cdot \mathbf{8}$	$[4:E_7]$
11.	$\mathsf{so}(8)$	$6 \cdot \mathbf{8} \oplus 6 \cdot \mathbf{8}'$	=	G_2		$[4:E_7] \oplus [4:E_7]$
12.	$\mathsf{sp}(2)$	$6 \cdot \mathbf{5}$	=	$\mathsf{su}(2)$		$[4:\mathsf{sp}(3) \oplus \mathsf{su}(2)]$
13.	$\mathsf{sp}(2)$	$4 \cdot \mathbf{4} \oplus 4 \cdot \mathbf{5}$	=	$\mathsf{su}(2)$	$3 \cdot \mathbf{2}$	$[4:\mathsf{sp}(3) \oplus \mathsf{su}(2)]$
14.	$\mathsf{su}(4)$	$\mathbf{10} \oplus \overline{\mathbf{10}} \oplus 2 \cdot \mathbf{4} \oplus 2 \cdot \overline{\mathbf{4}}$	=	$\mathsf{su}(3)$	$\mathbf{3} \oplus \overline{\mathbf{3}}$	$[4:\mathsf{sp}(3) \oplus \mathsf{su}(2)]$
15.	$\mathsf{su}(3)$	$6 \cdot \mathbf{3} \oplus 6 \cdot \overline{\mathbf{3}}$	=	$\mathsf{su}(2)$	$2 \cdot \mathbf{2}$	$[3:E_6]$
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17.	$\mathsf{su}(3)$	$\mathbf{3} \oplus \overline{\mathbf{3}} \oplus \mathbf{6} \oplus \overline{\mathbf{6}}$	=	$\mathsf{su}(2)$	$n \cdot \mathbf{2}$	$[3:\mathfrak{h}]$

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20.	$\mathsf{su}(2) \oplus \mathsf{sp}(2)$	$3 \cdot (\mathbf{2},\mathbf{1}) \oplus (\mathbf{2},\mathbf{5}) \oplus 4 \cdot (\mathbf{1},\mathbf{5})$	=	$\mathsf{su}(2) \oplus \mathsf{su}(2)$	$3 \cdot (\mathbf{2},\mathbf{1})$	$[4:\mathsf{sp}(3) \oplus \mathsf{su}(2)]$
21.	$\mathsf{su}(2) \oplus G_2$	$(\mathbf{2},\mathbf{1}) \oplus (\mathbf{2},\mathbf{7}) \oplus 6 \cdot (\mathbf{1},\mathbf{7})$	=	$\mathsf{su}(2) \oplus \mathsf{su}(2)$	$(\mathbf{2},\mathbf{1}) \oplus (\mathbf{1},\mathbf{2})$	$[6:\mathsf{sp}(5)]$
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25.	$\mathsf{su}(3) \oplus \mathsf{sp}(2)$	$(\mathbf{3} \oplus \overline{\mathbf{3}}, \mathbf{1}) \oplus (\mathbf{3} \oplus \overline{\mathbf{3}}, \mathbf{5})$	=	$\mathsf{su}(2) \oplus \mathsf{sp}(2)$	$2 \cdot (\mathbf{2},\mathbf{1})$	$[3:E_6]$
			=	$\mathsf{su}(3) \oplus \mathsf{su}(2)$	$(\mathbf{3} \oplus \overline{\mathbf{3}}, \mathbf{1})$	$[4:\mathsf{sp}(3) \oplus \mathsf{su}(2)]$
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			=	$\mathsf{sp}(2) \oplus \mathsf{su}(2)$	$2 \cdot (\mathbf{5},\mathbf{1})$	$[4:E_7]$
28.	$\mathsf{sp}(2) \oplus \mathsf{sp}(2)$	$4 \cdot (\mathbf{4},\mathbf{1}) \oplus 2 \cdot (\mathbf{4},\mathbf{4}) \oplus 4 \cdot (\mathbf{1},\mathbf{4})$	=	$\mathsf{su}(2) \oplus \mathsf{sp}(2)$	$4 \cdot (\mathbf{1},\mathbf{4})$	$[4:E_7]$
29.	$\mathsf{sp}(2) \oplus G_2$	$5 \cdot (\mathbf{4},\mathbf{1}) \oplus (\mathbf{4},\mathbf{7}) \oplus 4 \cdot (\mathbf{1},\mathbf{7})$	=	$\mathsf{su}(2) \oplus G_2$	$4 \cdot (\mathbf{1},\mathbf{7})$	$[4:E_7]$
			=	$\mathsf{sp}(2) \oplus \mathsf{su}(2)$	$5 \cdot (\mathbf{4},\mathbf{1}) \oplus (\mathbf{1},\mathbf{2})$	$[6:\mathsf{sp}(5)]$

A surprise:

In addition to finding the known E_n isolated rank 1 SCFTs as decoupled factors in various $g \to \infty$ limits of Lagrangian SCFTs, we also find new isolated rank 1 SCFTs.

They share the same conformal singularity as the E_n theories, but they have different global symmetries (and different central charges).

More about these later...

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IIa Example: su(3) w/ 6·3

- Ingredients:

Coulomb branch moduli: $\{u, v\}$
Coupling constant: $f \sim e^{2\pi i\tau}$. ($f \to 0$ is $g \to 0$, $f \to 1$ is $g \to \infty$.)
SW curve: $y^2 = (x^3 - ux - v)^2 - f^2 x^6$.
Holomorphic 1-forms: $\omega_u = x dx/y$, $\omega_v = dx/y$.

- Limits:

Set u=0 and f=1: degenerates to genus 1.
Change variables: $\left(\tilde{y}^2 = \tilde{x}^3 - \tilde{v}^4,\ \omega_{\tilde{v}} = d\tilde{x}/\tilde{y}\right)$
This is the E_6 SCFT. (Minahan, Nemeschansky)

Set v=0 and $f\sim$1: degenerates to

$$y^2 = \underbrace{x^2}_{\text{pinch cycle}} \cdot \underbrace{\left[(x^2-u)^2 - f^2x^4\right]}_{\equiv \widetilde{y}^2}, \qquad \underbrace{\omega_u = x dx/y}.$$

weak su(2) SCFT

$$\Updownarrow$$

$$\overbrace{\omega_u = dx/\widetilde{y}}.$$

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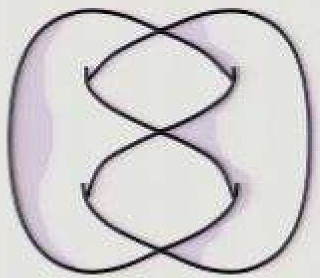
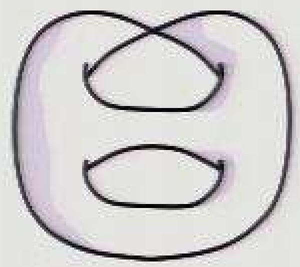
Set v=0 **and** $f \sim 1$**:** degenerates to

$$y^2 = \underbrace{x^2}_{\text{pinch cycle}} \cdot \underbrace{\left[(x^2 - u)^2 - f^2 x^4\right]}_{\equiv \tilde{y}^2}, \qquad \underbrace{\omega_u = x dx/y}_{} . \quad \Updownarrow \quad \overbrace{\omega_u = dx/\tilde{y}}^{} .$$

weak su(2) SCFT

- Compare $f \to 1$ degeneration to $f \to 0$ (weak cpling) degeneration:
  - As $f \to 0$ in su(3) SW curve, genus 2 curve pinches to 2 genus 0 curves everywhere on moduli space. Pinching cycle $\leftrightarrow$ charged $W^{\pm}$ bosons becoming massless, corresponding to expected su(3) $\to$ u(1) $\times$ u(1) Higgsing. (Figure on left.)





  - But as $f \to 1$, only one cycle vanishes everywhere on moduli space. (Figure on right.) Not enough W-bosons for a weakly-coupled Higgs mechanism, so some new "phase" is indicated for this theory at $f = 1$.

- Global symmetries ⇔ mass deformations:

E$_6$ **masses:** $M_n \sim m^n$ are explicit basis of E$_6$ Casimirs.

$$\tilde{y}^2 = \tilde{x}^3 - (\tilde{v}^2 M_2 + \tilde{v} M_5 + M_8)\tilde{x} - (\tilde{v}^4 + \tilde{v}^2 M_6 + \tilde{v} M_9 + M_{12}).$$

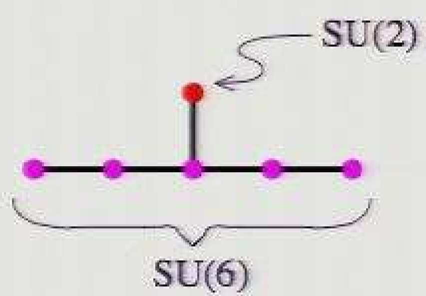
Turn on u at $f = 1$: same c.o.v. gives

$$\tilde{y}^2 = \tilde{x}^3 - (\tilde{v}^2 u + \tfrac{1}{48}u^4)\tilde{x} - (\tilde{v}^4 + \tilde{v}^2 \tfrac{1}{12}u^3 + \tfrac{1}{864}u^6).$$

Implies: u is mass deformation of E$_6$ SCFT with

$$\{M_2 = u, M_5 = 0, M_6 = \tfrac{1}{12}u^3, M_8 = \tfrac{1}{48}u^4, M_9 = 0, M_{12} = \tfrac{1}{864}u^6\}.$$

Since $u \neq 0$ higgses su(2) → u(1), some **group theory** implies that the su(2) gauges the • subgroup of E$_6$:



The unbroken su(6) is part of the **global symmetry** of the su(3) w/ 6·3 theory. But the **full global symmetry** of the theory is

$$u(1)_B \times su(6) \times u(2)_R.$$

— The R symmetry is realized at infinite coupling as a certain combination of the R symmetries of the rank 1 su(2) and E_6 SCFTs.

— The su(6) flavor symmetry was identified above as the "un-gauged" part of the E_6 group.

— Which leaves the $u(1)_B$ "baryon number" unaccounted for.

The solution can only be that there is a single **SU(2)-doublet hypermultiplet** at infinite coupling:

$$su(3)_g \text{ w/ } 6\cdot\mathbf{3} \sim su(2)_{1/g} \text{ w/ } 1\cdot\mathbf{2} \oplus (E_6\text{-CFT}).$$

- Further checks: anomaly matching, beta functions ...

III Central charges

- **Flavor current algebra central charge**:

  By weakly gauging the global symmetries of the $\mathfrak{g}$ theory both at weak and infinite gauge coupling, and comparing its beta function to the one computed in the dual description allows us to compute the contribution of the SCFT "matter" to the $\tilde{\mathfrak{g}}$ gauge coupling beta function.

  This is governed by the central charge, k, of the global flavor current algebra of the SCFT:

$$\text{“}J^a(x)J^b(0) \sim \frac{k\,\delta^{ab}}{x^6} + \cdots + \frac{f^{ab}_c J^c(0)}{x^3} + \cdots\text{”}.$$

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6.	su(6)	$\mathbf{21}\oplus\overline{\mathbf{21}}\oplus\mathbf{20}\oplus\mathbf{6}\oplus\bar{\mathbf{6}}$	=	su(5)	$\mathbf{10}\oplus\overline{\mathbf{10}}$	[6 : sp(5)]
7.	sp(2)	$12\cdot\mathbf{4}$	=	su(2)		$[4:E_7]$
8.	su(4)	$2\cdot\mathbf{6}\oplus 6\cdot\mathbf{4}\oplus 6\cdot\bar{\mathbf{4}}$	=	su(3)	$2\cdot\mathbf{3}\oplus 2\cdot\bar{\mathbf{3}}$	$[4:E_7]$
9.	so(7)	$6\cdot\mathbf{8}\oplus 4\cdot\mathbf{7}$	=	G_2	$4\cdot\mathbf{7}$	$[4:E_7]$
10.	so(8)	$6\cdot\mathbf{8}\oplus 4\cdot\mathbf{8}'\oplus 2\cdot\mathbf{8}''$	=	so(7)	$6\cdot\mathbf{8}$	$[4:E_7]$
11.	so(8)	$6\cdot\mathbf{8}\oplus 6\cdot\mathbf{8}'$	=	G_2		$[4:E_7]\oplus[4:E_7]$
12.	sp(2)	$6\cdot\mathbf{5}$	=	su(2)		[4 : sp(3) ⊕ su(2)]
13.	sp(2)	$4\cdot\mathbf{4}\oplus 4\cdot\mathbf{5}$	=	su(2)	$3\cdot\mathbf{2}$	[4 : sp(3) ⊕ su(2)]
14.	su(4)	$\mathbf{10}\oplus\overline{\mathbf{10}}\oplus 2\cdot\mathbf{4}\oplus 2\cdot\bar{\mathbf{4}}$	=	su(3)	$\mathbf{3}\oplus\bar{\mathbf{3}}$	[4 : sp(3) ⊕ su(2)]
15.	su(3)	$6\cdot\mathbf{3}\oplus 6\cdot\bar{\mathbf{3}}$	=	su(2)	$2\cdot\mathbf{2}$	$[3:E_6]$
16.	su(4)	$4\cdot\mathbf{6}\oplus 4\cdot\mathbf{4}\oplus 4\cdot\bar{\mathbf{4}}$	=	sp(2)	$6\cdot\mathbf{4}$	$[3:E_6]$
17.	su(3)	$\mathbf{3}\oplus\bar{\mathbf{3}}\oplus\mathbf{6}\oplus\bar{\mathbf{6}}$	=	su(2)	$n\cdot\mathbf{2}$	$[3:\mathfrak{h}]$

	$\mathfrak{g}$	w/ $\mathbf{r}$	=	$\tilde{\mathfrak{g}}$	w/ $\tilde{\mathbf{r}}$	$\oplus$ SCFT $[d:\mathfrak{h}]$
18.	su(2) ⊕ su(3)	$2\cdot(\mathbf{2},\mathbf{1})\oplus(\mathbf{2},\mathbf{3}\oplus\bar{\mathbf{3}})\oplus 4\cdot(\mathbf{1},\mathbf{3}\oplus\bar{\mathbf{3}})$	=	su(2) ⊕ su(2)	$2\cdot(\mathbf{2},\mathbf{1})\oplus 2\cdot(\mathbf{1},\mathbf{2})$	$[3:E_6]$
19.	su(2) ⊕ sp(2)	$2\cdot(\mathbf{2},\mathbf{4})\oplus 8\cdot(\mathbf{1},\mathbf{4})$	=	su(2) ⊕ su(2)		$[4:E_7]$
20.	su(2) ⊕ sp(2)	$3\cdot(\mathbf{2},\mathbf{1})\oplus(\mathbf{2},\mathbf{5})\oplus 4\cdot(\mathbf{1},\mathbf{5})$	=	su(2) ⊕ su(2)	$3\cdot(\mathbf{2},\mathbf{1})$	[4 : sp(3)⊕su(2)]
21.	su(2) ⊕ G_2	$(\mathbf{2},\mathbf{1})\oplus(\mathbf{2},\mathbf{7})\oplus 6\cdot(\mathbf{1},\mathbf{7})$	=	su(2) ⊕ su(2)	$(\mathbf{2},\mathbf{1})\oplus(\mathbf{1},\mathbf{2})$	[6 : sp(5)]
22.	su(3) ⊕ su(3)	$2\cdot(\mathbf{3},\mathbf{3})\oplus 2\cdot(\bar{\mathbf{3}},\bar{\mathbf{3}})$	=	su(2) ⊕ su(3)	$2\cdot(\mathbf{2},\mathbf{1})$	$[3:E_6]$
23.	su(3) ⊕ su(3)	$(\mathbf{3}\oplus\bar{\mathbf{3}},\mathbf{3}\oplus\bar{\mathbf{3}})$	=	su(2) ⊕ su(3)	$2\cdot(\mathbf{2},\mathbf{1})$	$[3:E_6]$
24.	su(3) ⊕ su(3)	$3\cdot(\mathbf{3}\oplus\bar{\mathbf{3}},\mathbf{1})\oplus(\mathbf{3},\bar{\mathbf{3}})\oplus(\bar{\mathbf{3}},\mathbf{3})$	=	su(2) ⊕ su(3)	$2\cdot(\mathbf{2},\mathbf{1})$	$[3:E_6]$
		$\oplus\, 3\cdot(\mathbf{1},\mathbf{3}\oplus\bar{\mathbf{3}})$			$\oplus\, 3\cdot(\mathbf{1},\mathbf{3}\oplus\bar{\mathbf{3}})$	
25.	su(3) ⊕ sp(2)	$(\mathbf{3}\oplus\bar{\mathbf{3}},\mathbf{1})\oplus(\mathbf{3}\oplus\bar{\mathbf{3}},\mathbf{5})$	=	su(2) ⊕ sp(2)	$2\cdot(\mathbf{2},\mathbf{1})$	$[3:E_6]$
			=	su(3) ⊕ su(2)	$(\mathbf{3}\oplus\bar{\mathbf{3}},\mathbf{1})$	[4 : sp(3)⊕su(2)]
26.	su(3) ⊕ sp(2)	$2\cdot(\mathbf{3}\oplus\bar{\mathbf{3}},\mathbf{1})\oplus(\mathbf{3}\oplus\bar{\mathbf{3}},\mathbf{4})\oplus 6\cdot(\mathbf{1},\mathbf{4})$	=	su(2) ⊕ sp(2)	$2\cdot(\mathbf{2},\mathbf{1})\oplus 6\cdot(\mathbf{1},\mathbf{4})$	$[3:E_6]$
			=	su(3) ⊕ su(2)	$2\cdot(\mathbf{3}\oplus\bar{\mathbf{3}},\mathbf{1})$	$[4:E_7]$
27.	sp(2) ⊕ sp(2)	$2\cdot(\mathbf{5},\mathbf{1})\oplus(\mathbf{5},\mathbf{4})\oplus 7\cdot(\mathbf{1},\mathbf{4})$	=	su(2) ⊕ sp(2)	$7\cdot(\mathbf{1},\mathbf{4})$	[4 : sp(3)⊕su(2)]
			=	sp(2) ⊕ su(2)	$2\cdot(\mathbf{5},\mathbf{1})$	$[4:E_7]$
28.	sp(2) ⊕ sp(2)	$4\cdot(\mathbf{4},\mathbf{1})\oplus 2\cdot(\mathbf{4},\mathbf{4})\oplus 4\cdot(\mathbf{1},\mathbf{4})$	=	su(2) ⊕ sp(2)	$4\cdot(\mathbf{1},\mathbf{4})$	$[4:E_7]$
29.	sp(2) ⊕ G_2	$5\cdot(\mathbf{4},\mathbf{1})\oplus(\mathbf{4},\mathbf{7})\oplus 4\cdot(\mathbf{1},\mathbf{7})$	=	su(2) ⊕ G_2	$4\cdot(\mathbf{1},\mathbf{7})$	$[4:E_7]$
			=	sp(2) ⊕ su(2)	$5\cdot(\mathbf{4},\mathbf{1})\oplus(\mathbf{1},\mathbf{2})$	[6 : sp(5)]

	$\mathfrak{g}$	w/ $\mathbf{r}$	=	$\tilde{\mathfrak{g}}$	w/ $\tilde{\mathbf{r}}$	$\oplus$ SCFT $[d:\mathfrak{h}]$
1.	sp(3)	$\mathbf{14} \oplus 11 \cdot \mathbf{6}$	=	sp(2)		$[6:E_8]$
2.	su(6)	$\mathbf{20} \oplus \mathbf{15} \oplus \overline{\mathbf{15}} \oplus 5 \cdot \mathbf{6} \oplus 5 \cdot \bar{\mathbf{6}}$	=	su(5)	$\mathbf{5} \oplus \bar{\mathbf{5}} \oplus \mathbf{10} \oplus \overline{\mathbf{10}}$	$[6:E_8]$
3.	so(12)	$3 \cdot \mathbf{32} \oplus \mathbf{32}' \oplus 4 \cdot \mathbf{12}$	=	so(11)	$3 \cdot \mathbf{32}$	$[6:E_8]$
4.	G_2	$8 \cdot \mathbf{7}$	=	su(2)	$\mathbf{2}$	$[6:\mathrm{sp}(5)]$
5.	so(7)	$4 \cdot \mathbf{8} \oplus 6 \cdot \mathbf{7}$	=	sp(2)	$5 \cdot \mathbf{4}$	$[6:\mathrm{sp}(5)]$
6.	su(6)	$\mathbf{21} \oplus \overline{\mathbf{21}} \oplus \mathbf{20} \oplus \mathbf{6} \oplus \bar{\mathbf{6}}$	=	su(5)	$\mathbf{10} \oplus \overline{\mathbf{10}}$	$[6:\mathrm{sp}(5)]$
7.	sp(2)	$12 \cdot \mathbf{4}$	=	su(2)		$[4:E_7]$
8.	su(4)	$2 \cdot \mathbf{6} \oplus 6 \cdot \mathbf{4} \oplus 6 \cdot \bar{\mathbf{4}}$	=	su(3)	$2 \cdot \mathbf{3} \oplus 2 \cdot \bar{\mathbf{3}}$	$[4:E_7]$
9.	so(7)	$6 \cdot \mathbf{8} \oplus 4 \cdot \mathbf{7}$	=	G_2	$4 \cdot \mathbf{7}$	$[4:E_7]$
10.	so(8)	$6 \cdot \mathbf{8} \oplus 4 \cdot \mathbf{8}' \oplus 2 \cdot \mathbf{8}''$	=	so(7)	$6 \cdot \mathbf{8}$	$[4:E_7]$
11.	so(8)	$6 \cdot \mathbf{8} \oplus 6 \cdot \mathbf{8}'$	=	G_2		$[4:E_7] \oplus [4:E_7]$
12.	sp(2)	$6 \cdot \mathbf{5}$	=	su(2)		$[4:\mathrm{sp}(3) \oplus \mathrm{su}(2)]$
13.	sp(2)	$4 \cdot \mathbf{4} \oplus 4 \cdot \mathbf{5}$	=	su(2)	$3 \cdot \mathbf{2}$	$[4:\mathrm{sp}(3) \oplus \mathrm{su}(2)]$
14.	su(4)	$\mathbf{10} \oplus \overline{\mathbf{10}} \oplus 2 \cdot \mathbf{4} \oplus 2 \cdot \bar{\mathbf{4}}$	=	su(3)	$\mathbf{3} \oplus \bar{\mathbf{3}}$	$[4:\mathrm{sp}(3) \oplus \mathrm{su}(2)]$
15.	su(3)	$6 \cdot \mathbf{3} \oplus 6 \cdot \bar{\mathbf{3}}$	=	su(2)	$2 \cdot \mathbf{2}$	$[3:E_6]$
16.	su(4)	$4 \cdot \mathbf{6} \oplus 4 \cdot \mathbf{4} \oplus 4 \cdot \bar{\mathbf{4}}$	=	sp(2)	$6 \cdot \mathbf{4}$	$[3:E_6]$
17.	su(3)	$\mathbf{3} \oplus \bar{\mathbf{3}} \oplus \mathbf{6} \oplus \bar{\mathbf{6}}$	=	su(2)	$n \cdot \mathbf{2}$	$[3:\mathfrak{h}]$

	$\mathfrak{g}$	w/ $\mathbf{r}$	=	$\tilde{\mathfrak{g}}$	w/ $\tilde{\mathbf{r}}$	$\oplus$ SCFT$[d:\mathfrak{h}]$
18.	su(2) $\oplus$ su(3)	$2 \cdot (\mathbf{2},\mathbf{1}) \oplus (\mathbf{2},\mathbf{3}\oplus\bar{\mathbf{3}}) \oplus 4 \cdot (\mathbf{1},\mathbf{3}\oplus\bar{\mathbf{3}})$	=	su(2) $\oplus$ su(2)	$2 \cdot (\mathbf{2},\mathbf{1}) \oplus 2 \cdot (\mathbf{1},\mathbf{2})$	$[3:E_6]$
19.	su(2) $\oplus$ sp(2)	$2 \cdot (\mathbf{2},\mathbf{4}) \oplus 8 \cdot (\mathbf{1},\mathbf{4})$	=	su(2) $\oplus$ su(2)		$[4:E_7]$
20.	su(2) $\oplus$ sp(2)	$3 \cdot (\mathbf{2},\mathbf{1}) \oplus (\mathbf{2},\mathbf{5}) \oplus 4 \cdot (\mathbf{1},\mathbf{5})$	=	su(2) $\oplus$ su(2)	$3 \cdot (\mathbf{2},\mathbf{1})$	$[4:\mathrm{sp}(3)\oplus\mathrm{su}(2)]$
21.	su(2) $\oplus$ G_2	$(\mathbf{2},\mathbf{1}) \oplus (\mathbf{2},\mathbf{7}) \oplus 6 \cdot (\mathbf{1},\mathbf{7})$	=	su(2) $\oplus$ su(2)	$(\mathbf{2},\mathbf{1}) \oplus (\mathbf{1},\mathbf{2})$	$[6:\mathrm{sp}(5)]$
22.	su(3) $\oplus$ su(3)	$2 \cdot (\mathbf{3},\mathbf{3}) \oplus 2 \cdot (\mathbf{3},\mathbf{3})$	=	su(2) $\oplus$ su(3)	$2 \cdot (\mathbf{2},\mathbf{1})$	$[3:E_6]$
23.	su(3) $\oplus$ su(3)	$(\mathbf{3}\oplus\bar{\mathbf{3}},\mathbf{3}\oplus\bar{\mathbf{3}})$	=	su(2) $\oplus$ su(3)	$2 \cdot (\mathbf{2},\mathbf{1})$	$[3:E_6]$
24.	su(3) $\oplus$ su(3)	$3 \cdot (\mathbf{3}\oplus\bar{\mathbf{3}},\mathbf{1}) \oplus (\mathbf{3},\bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}},\mathbf{3})$ $\oplus 3 \cdot (\mathbf{1},\mathbf{3}\oplus\bar{\mathbf{3}})$	=	su(2) $\oplus$ su(3)	$2 \cdot (\mathbf{2},\mathbf{1})$ $\oplus 3 \cdot (\mathbf{1},\mathbf{3}\oplus\bar{\mathbf{3}})$	$[3:E_6]$
25.	su(3) $\oplus$ sp(2)	$(\mathbf{3}\oplus\bar{\mathbf{3}},\mathbf{1}) \oplus (\mathbf{3}\oplus\bar{\mathbf{3}},\mathbf{5})$	=	su(2) $\oplus$ sp(2)	$2 \cdot (\mathbf{2},\mathbf{1})$	$[3:E_6]$
			=	su(3) $\oplus$ su(2)	$(\mathbf{3}\oplus\bar{\mathbf{3}},\mathbf{1})$	$[4:\mathrm{sp}(3)\oplus\mathrm{su}(2)]$
26.	su(3) $\oplus$ sp(2)	$2 \cdot (\mathbf{3}\oplus\bar{\mathbf{3}},\mathbf{1}) \oplus (\mathbf{3}\oplus\bar{\mathbf{3}},\mathbf{4}) \oplus 6 \cdot (\mathbf{1},\mathbf{4})$	=	su(2) $\oplus$ sp(2)	$2 \cdot (\mathbf{2},\mathbf{1}) \oplus 6 \cdot (\mathbf{1},\mathbf{4})$	$[3:E_6]$
			=	su(3) $\oplus$ su(2)	$2 \cdot (\mathbf{3}\oplus\bar{\mathbf{3}},\mathbf{1})$	$[4:E_7]$
27.	sp(2) $\oplus$ sp(2)	$2 \cdot (\mathbf{5},\mathbf{1}) \oplus (\mathbf{5},\mathbf{4}) \oplus 7 \cdot (\mathbf{1},\mathbf{4})$	=	su(2) $\oplus$ sp(2)	$7 \cdot (\mathbf{1},\mathbf{4})$	$[4:\mathrm{sp}(3)\oplus\mathrm{su}(2)]$
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28.	sp(2) $\oplus$ sp(2)	$4 \cdot (\mathbf{4},\mathbf{1}) \oplus 2 \cdot (\mathbf{4},\mathbf{4}) \oplus 4 \cdot (\mathbf{1},\mathbf{4})$	=	su(2) $\oplus$ sp(2)	$4 \cdot (\mathbf{1},\mathbf{4})$	$[4:E_7]$
29.	sp(2) $\oplus$ G_2	$5 \cdot (\mathbf{4},\mathbf{1}) \oplus (\mathbf{4},\mathbf{7}) \oplus 4 \cdot (\mathbf{1},\mathbf{7})$	=	su(2) $\oplus$ G_2	$4 \cdot (\mathbf{1},\mathbf{7})$	$[4:E_7]$
			=	sp(2) $\oplus$ su(2)	$5 \cdot (\mathbf{4},\mathbf{1}) \oplus (\mathbf{1},\mathbf{2})$	$[6:\mathrm{sp}(5)]$

The unbroken su(6) is part of the **global symmetry** of the su(3) w/ $6\cdot\mathbf{3}$ theory. But the **full global symmetry** of the theory is

$$u(1)_B \times su(6) \times u(2)_R.$$

— The R symmetry is realized at infinite coupling as a certain combination of the R symmetries of the rank 1 su(2) and E_6 SCFTs.

— The su(6) flavor symmetry was identified above as the "ungauged" part of the E_6 group.

— Which leaves the $u(1)_B$ "baryon number" unaccounted for.

The solution can only be that there is a single **SU(2)-doublet hypermultiplet** at infinite coupling:

$$\boxed{su(3)_g \text{ w/ } 6\cdot\mathbf{3} \sim su(2)_{1/g} \text{ w/ } 1\cdot\mathbf{2} \oplus (E_6\text{-CFT}).}$$

- Further checks: anomaly matching, beta functions ...

III Central charges

- Flavor current algebra central charge:

  By weakly gauging the global symmetries of the $\mathfrak{g}$ theory both at weak and infinite gauge coupling, and comparing its beta function to the one computed in the dual description allows us to compute the contribution of the SCFT "matter" to the $\widetilde{\mathfrak{g}}$ gauge coupling beta function.

  This is governed by the central charge, k, of the global flavor current algebra of the SCFT:

$$\text{“} J^a(x) J^b(0) \sim \frac{k\,\delta^{ab}}{x^6} + \cdots + \frac{f^{ab}_c J^c(0)}{x^3} + \cdots \text{”}.$$

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- Conformal algebra central charges:

$$\text{``}T(x)T(0) \sim \frac{c\,\delta^{ab}}{x^8} + \cdots + \frac{(1+a)\,T(0)}{x^4} + \cdots\text{''}.$$

a and c are computed in many cases by 't Hooft anomaly matching arguments for R-symmetries.
a is believed to decrease along RG flows.

- New relations:
Find (w/ J. Wittig) for all known N=2 SCFTs

$$2a - c = \frac{1}{4}\sum_{i=1}^{r}(2d_i - 1)$$

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N=2 conformal bootstrap?

We have a rich- and detailed-enough set of N=2 SCFTs to try to find special subsets with particularly simple OPEs (e.g., closing on just T, J^a, and Φ_i in their most singular terms).

This is analogous to the Sugawara construction in 2-d CFT.

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IV Relevant deformations

Rank 1 SCFTs traditionally classified (via SW theory) by the Kodaira classification of singularities of families of elliptic curves fibered over $\mathbb{P}^1$:

singularity	curve	flavor symmetry $\mathfrak{f}$
E_8	$y^2 = x^3 + x(M_2u^3 + M_8u^2 + M_{14}u + M_{20}) + (2u^5 + M_{12}u^3 + M_{18}u^2 + M_{24}u + M_{30})$	$\mathfrak{e}_8$
E_7	$y^2 = x^3 + x(u^3 + M_8u + M_{12}) + (M_2u^4 + M_6u^3 + M_{10}u^2 + M_{14}u + M_{18})$	$\mathfrak{e}_7$
E_6	$y^2 = x^3 + x(M_2u^2 + M_5u + M_8) + (u^4 + M_6u^2 + M_9u + M_{12})$	$\mathfrak{e}_6$
D_4	$y^2 = x^3 + x(3M_0u^2 + M_2u + M_4) + (2u^3 + \widetilde{M}_4u + M_6)$	$\mathfrak{so}(8)$
H_3	$y^2 = x^3 + x(M_{1/2}u + M_2) + (u^2 + M_3)$	$\mathfrak{u}(3)$
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H_1	$y^2 = x^3 + x(M_{4/5}) + (u)$	$\mathfrak{u}(1)$
$D_{n>4}$	$y^2 = x^3 + 3ux^2 + \Lambda^{-(n-4)}\widetilde{M}_nx + 4\Lambda^{-2(n-4)}(u^{n-1} + M_2u^{n-2} + \cdots + M_{2n-2})$	$\mathfrak{so}(2n)$
$A_{n\geq 0}$	$y^2 = (x-1)(x^2 + \Lambda^{-(n+1)}[u^{n+1} + M_1u^n + M_2u^{n-1} + \cdots + M_{n+1}])$	$\mathfrak{u}(n+1)$

Their maximal mass deformations (shown) correspond to known SCFTs.

However, our new duality also predicts that the Coulomb branch singularities of the rank 1 SCFTs have other, inequivalent, mass deformations.

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For Lagrangian FTs this is familiar: inequivalent mass deformations correspond to different matter content (hypermultiplets) which contribute the same amount to the gauge coupling beta function(s).

For example:

- su(2) w/ $4 \cdot 2$ is conformal, described by the D_4 singularity, but has $\mathfrak{so}(8)$ flavor symmetry.

- su(2) w/ $1 \cdot 3$ is conformal, described by the D_4 singularity, but has $\mathfrak{sp}(1)$ flavor symmetry.

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E_7	$y^2 = x^3 + x(u^3 + M_8u + M_{12}) + (M_2u^4 + M_6u^3 + M_{10}u^2 + M_{14}u + M_{18})$	$\mathfrak{e}_7$
E_6	$y^2 = x^3 + x(M_2u^2 + M_5u + M_8) + (u^4 + M_6u^2 + M_9u + M_{12})$	$\mathfrak{e}_6$
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$A_{n\geq 0}$	$y^2 = (x-1)(x^2 + \Lambda^{-(n+1)}[u^{n+1} + M_1u^n + M_2u^{n-1} + \cdots + M_{n+1}])$	$\mathfrak{u}(n+1)$

Their maximal mass deformations (shown) correspond to known SCFTs.

However, our new duality also predicts that the Coulomb branch singularities of the rank 1 SCFTs have other, inequivalent, mass deformations.

IV Relevant deformations

Rank 1 SCFTs traditionally classified (via SW theory) by the Kodaira classification of singularities of families of elliptic curves fibered over $\mathbb{P}^1$:

singularity	curve	flavor symmetry $\mathfrak{f}$
E_8	$y^2 = x^3 + x(M_2u^3 + M_8u^2 + M_{14}u + M_{20}) + (2u^5 + M_{12}u^3 + M_{18}u^2 + M_{24}u + M_{30})$	$\mathfrak{e}_8$
E_7	$y^2 = x^3 + x(u^3 + M_8u + M_{12}) + (M_2u^4 + M_6u^3 + M_{10}u^2 + M_{14}u + M_{18})$	$\mathfrak{e}_7$
E_6	$y^2 = x^3 + x(M_2u^2 + M_5u + M_8) + (u^4 + M_6u^2 + M_9u + M_{12})$	$\mathfrak{e}_6$
D_4	$y^2 = x^3 + x(3M_0u^2 + M_2u + M_4) + (2u^3 + \widetilde{M}_4u + M_6)$	$\mathfrak{so}(8)$
H_3	$y^2 = x^3 + x(M_{1/2}u + M_2) + (u^2 + M_3)$	$\mathfrak{u}(3)$
H_2	$y^2 = x^3 + x(u) + (M_{2/3}u + M_2)$	$\mathfrak{u}(2)$
H_1	$y^2 = x^3 + x(M_{4/5}) + (u)$	$\mathfrak{u}(1)$
$D_{n>4}$	$y^2 = x^3 + 3ux^2 + \Lambda^{-(n-4)}\widetilde{M}_nx + 4\Lambda^{-2(n-4)}(u^{n-1} + M_2u^{n-2} + \cdots + M_{2n-2})$	$\mathfrak{so}(2n)$
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However, our new duality also predicts that the Coulomb branch singularities of the rank 1 SCFTs have other, inequivalent, mass deformations.

For Lagrangian FTs this is familiar: inequivalent mass deformations correspond to different matter content (hypermultiplets) which contribute the same amount to the gauge coupling beta function(s).

For example:

- su(2) w/ $4 \cdot 2$ is conformal, described by the D_4 singularity, but has $\mathfrak{so}(8)$ flavor symmetry.

- su(2) w/ $1 \cdot 3$ is conformal, described by the D_4 singularity, but has $\mathfrak{sp}(1)$ flavor symmetry.

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V String constructions

- F-theory realizations.
  –Aharony and Tachikawa (arXiv:0711.4532) compute central charges of the E_n SCFTs using (large N) F-theory realization and AdS/CFT.
  –F-theory realization of sub-maximal mass deformations is unclear.

- M-theory realizations.
  –With J. Vázquez-Poritz, M-theory lift of a IIA construction (D4's suspended between NS5's in background of $O6^-$'s) gives non-singular description of infinite-coupling points.
  –Isolated SCFTs correspond to a novel strongly coupled NS5-$O6^-$ bound state in IIA string theory.)
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VI Summary/Further directions

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  Even at rank 1 many new theories are predicted, but techniques for their construction need to be developed.

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