

Title: Deformations of $N=2$ superconformal field theories

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Abstract: TBA

Deformations of $N=2$ SCFTs

P.C. Argyres w/ N. Seiberg, J. Wittig

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and to appear w/ P. Esposito, J. Vázquez-Poritz, J. Wittig

Motivation

What: Explore the space of 4-d $N=2$ SCFTs.

Why: Because we can. Also

- Start of RG flows to many $N \leq 2$ field theories.
- Alternative to strings as route to non-Lagrangian FTs.

How: Exact field theory techniques:

- $N=2$ susy selection rules.
- Low energy effective action on moduli space of vacua (SW theory).
- Representation theory of $N=2$ superconformal algebra.

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- I** Review of $N=2$ susy
- II** Marginal deformations (gauge couplings)
- III** Central charges
- IV** Relevant deformations (masses, mostly)
- V** Connection to string constructions
- VI** Further directions

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I Review of N=2 susy

All known interacting N=2 conformal theories have neutral scalar chiral multiplets.

N=2 susy implies no potential is allowed for their vevs, u_i , $i = 1, \dots, r$. So they parameterize a moduli space of vacua with unbroken $U(1)^r$ gauge symmetry: Coulomb branch. "Rank" of SCFT is r .



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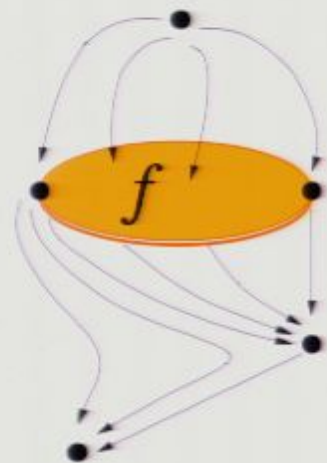
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Review of $N=2$ susy (cont)

Can flow between SCFTs by turning on relevant operators (masses or AF gauge couplings):



Each point in theory space includes accompanying Coulomb branch.

Exactly marginal operators \Rightarrow complex manifolds of fixed points.

The marginal coupling f is a coordinate on this manifold.

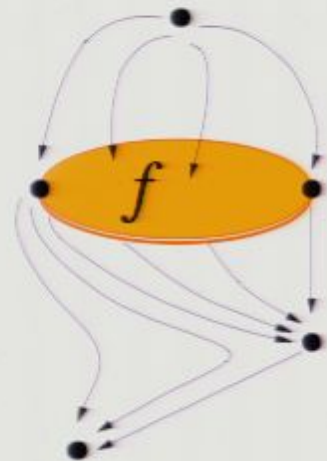
S-duality = complex geometry of space of marginal couplings.

If no exactly marginal operators \Rightarrow "isolated SCFT".

Isolated rank 1 SCFTs \simeq Kodaira classification (\sim A-D-E)...

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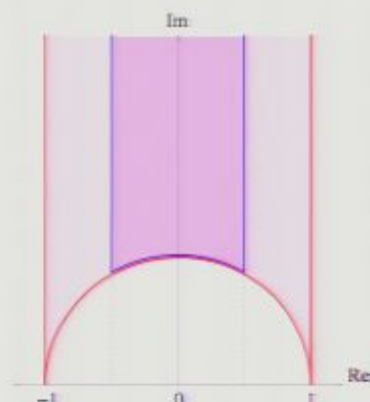
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II Marginal deformations

Some old puzzles:

- What happens as $g \rightarrow \infty$ in $N=2$ Lagrangian theories?
E.g., $su(3)$ w/ $6 \cdot 3$ has S-duality group $\Gamma^0(2) \subset sl(2, \mathbb{Z})$ whose fundamental domain contains $g = \infty$:

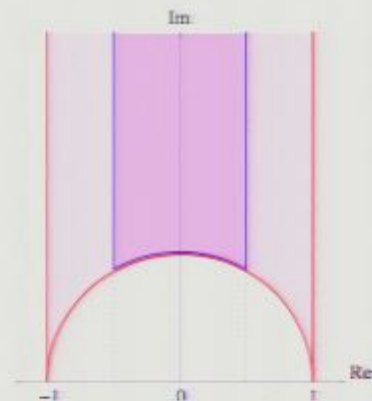


- Can the E_n isolated rank 1 SCFTs be realized in field theory?
E.g., by RG flow from appropriate Lagrangian SCFTs?
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A surprise:

In addition to finding the known E_n isolated rank 1 SCFTs as decoupled factors in various $g \rightarrow \infty$ limits of Lagrangian SCFTs, we also find **new isolated rank 1 SCFTs**.

They share the same conformal singularity as the E_n theories, but they have **different global symmetries** (and different central charges).

More about these later...

- Compare $f \rightarrow 1$ degeneration to $f \rightarrow 0$ (weak cpling) degeneration:
 - As $f \rightarrow 0$ in $\text{su}(3)$ SW curve, genus 2 curve pinches to 2 genus 0 curves **everywhere on moduli space**. Pinching cycle \leftrightarrow charged W^\pm bosons becoming massless, corresponding to expected $\text{su}(3) \rightarrow \text{u}(1) \times \text{u}(1)$ Higgsing. (Figure on left.)



- But as $f \rightarrow 1$, only one cycle vanishes everywhere on moduli space. (Figure on right.) Not enough W -bosons for a weakly-coupled Higgs mechanism, so some **new "phase"** is indicated for this theory at $f = 1$.

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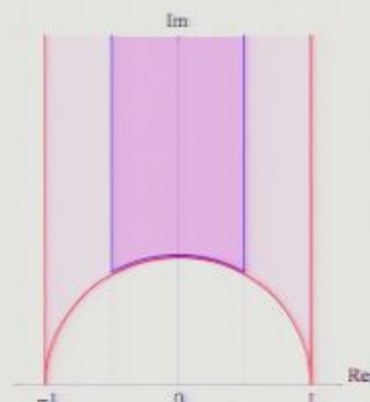


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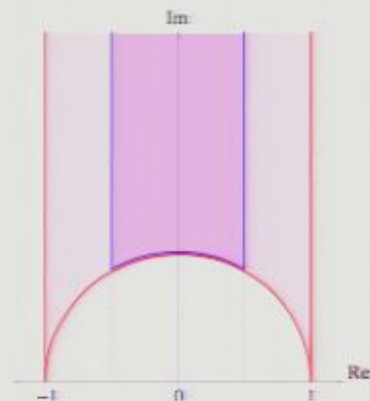


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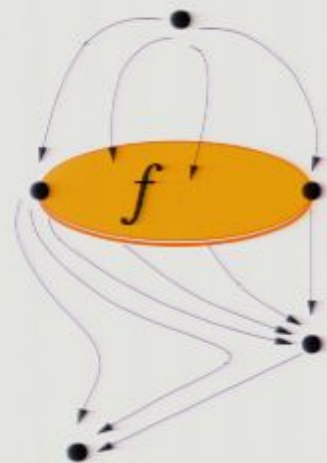
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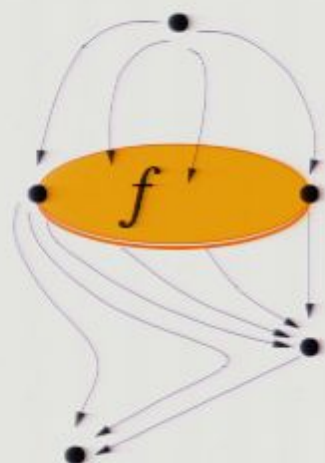
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Instead we adduce evidence from:

- from exact IR effective actions (SW curves),
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These fix \tilde{g} , the SCFT, and their coupling, given the scale-invariant theory g .

- Can be made much more precise.

E.g., if g w/ $r = su(3)$ w/ $6 \cdot 3$ (*i.e.*, with 6 fund. hyper-plets):

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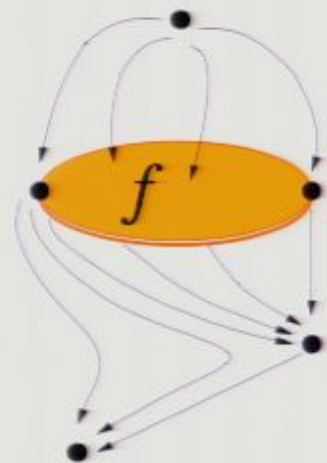
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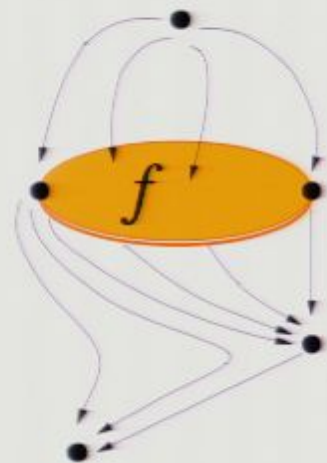
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	g	w/ r	$= \tilde{g}$	w/ \tilde{r}	\oplus	SCFT $[d : h]$
1.	$sp(3)$	$14 \oplus 11 \cdot 6$	$= sp(2)$			$[6 : E_8]$
2.	$su(6)$	$20 \oplus 15 \oplus \overline{15} \oplus 5 \cdot 6 \oplus 5 \cdot \overline{6}$	$= su(5)$	$5 \oplus \overline{5} \oplus 10 \oplus \overline{10}$		$[6 : E_8]$
3.	$so(12)$	$3 \cdot 32 \oplus 32' \oplus 4 \cdot 12$	$= so(11)$	$3 \cdot 32$		$[6 : E_8]$
4.	G_2	$8 \cdot 7$	$= su(2)$	2		$[6 : sp(5)]$
5.	$so(7)$	$4 \cdot 8 \oplus 6 \cdot 7$	$= sp(2)$	$5 \cdot 4$		$[6 : sp(5)]$
6.	$su(6)$	$21 \oplus \overline{21} \oplus 20 \oplus 6 \oplus \overline{6}$	$= su(5)$	$10 \oplus \overline{10}$		$[6 : sp(5)]$
7.	$sp(2)$	$12 \cdot 4$	$= su(2)$			$[4 : E_7]$
8.	$su(4)$	$2 \cdot 6 \oplus 6 \cdot 4 \oplus 6 \cdot \overline{4}$	$= su(3)$	$2 \cdot 3 \oplus 2 \cdot \overline{3}$		$[4 : E_7]$
9.	$so(7)$	$6 \cdot 8 \oplus 4 \cdot 7$	$= G_2$	$4 \cdot 7$		$[4 : E_7]$
10.	$so(8)$	$6 \cdot 8 \oplus 4 \cdot 8' \oplus 2 \cdot 8''$	$= so(7)$	$6 \cdot 8$		$[4 : E_7]$
11.	$so(8)$	$6 \cdot 8 \oplus 6 \cdot 8'$	$= G_2$			$[4 : E_7] \oplus [4 : E_7]$
12.	$sp(2)$	$6 \cdot 5$	$= su(2)$			$[4 : sp(3) \oplus su(2)]$
13.	$sp(2)$	$4 \cdot 4 \oplus 4 \cdot 5$	$= su(2)$	$3 \cdot 2$		$[4 : sp(3) \oplus su(2)]$
14.	$su(4)$	$10 \oplus \overline{10} \oplus 2 \cdot 4 \oplus 2 \cdot \overline{4}$	$= su(3)$	$3 \oplus \overline{3}$		$[4 : sp(3) \oplus su(2)]$
15.	$su(3)$	$6 \cdot 3 \oplus 6 \cdot \overline{3}$	$= su(2)$	$2 \cdot 2$		$[3 : E_6]$
16.	$su(4)$	$4 \cdot 6 \oplus 4 \cdot 4 \oplus 4 \cdot \overline{4}$	$= sp(2)$	$6 \cdot 4$		$[3 : E_6]$
17.	$su(3)$	$3 \oplus \overline{3} \oplus 6 \oplus \overline{6}$	$= su(2)$	$n \cdot 2$		$[3 : h]$

	g	w/ r	$= \tilde{g}$	w/ \tilde{r}	\oplus	SCFT $[d : h]$
18.	$su(2) \oplus su(3)$	$2 \cdot (2, 1) \oplus (2, 3 \oplus \overline{3}) \oplus 4 \cdot (1, 3 \oplus \overline{3})$	$= su(2) \oplus su(2)$	$2 \cdot (2, 1) \oplus 2 \cdot (1, 2)$		$[3 : E_6]$
19.	$su(2) \oplus sp(2)$	$2 \cdot (2, 4) \oplus 8 \cdot (1, 4)$	$= su(2) \oplus su(2)$			$[4 : E_7]$
20.	$su(2) \oplus sp(2)$	$3 \cdot (2, 1) \oplus (2, 5) \oplus 4 \cdot (1, 5)$	$= su(2) \oplus su(2)$	$3 \cdot (2, 1)$		$[4 : sp(3) \oplus su(2)]$
21.	$su(2) \oplus G_2$	$(2, 1) \oplus (2, 7) \oplus 6 \cdot (1, 7)$	$= su(2) \oplus su(2)$	$(2, 1) \oplus (1, 2)$		$[6 : sp(5)]$
22.	$su(3) \oplus su(3)$	$2 \cdot (3, 3) \oplus 2 \cdot (3, \overline{3})$	$= su(2) \oplus su(3)$	$2 \cdot (2, 1)$		$[3 : E_6]$
23.	$su(3) \oplus su(3)$	$(3 \oplus \overline{3}, 3 \oplus \overline{3})$	$= su(2) \oplus su(3)$	$2 \cdot (2, 1)$		$[3 : E_6]$
24.	$su(3) \oplus su(3)$	$3 \cdot (3 \oplus \overline{3}, 1) \oplus (3, \overline{3}) \oplus (\overline{3}, 3) \oplus 3 \cdot (1, 3 \oplus \overline{3})$	$= su(2) \oplus su(3)$	$2 \cdot (2, 1) \oplus 3 \cdot (1, 3 \oplus \overline{3})$		$[3 : E_6]$
25.	$su(3) \oplus sp(2)$	$(3 \oplus \overline{3}, 1) \oplus (3 \oplus \overline{3}, 5)$	$= su(2) \oplus sp(2)$	$2 \cdot (2, 1)$		$[3 : E_6]$
			$= su(3) \oplus su(2)$	$(3 \oplus \overline{3}, 1)$		$[4 : sp(3) \oplus su(2)]$
26.	$su(3) \oplus sp(2)$	$2 \cdot (3 \oplus \overline{3}, 1) \oplus (3 \oplus \overline{3}, 4) \oplus 6 \cdot (1, 4)$	$= su(2) \oplus sp(2)$	$2 \cdot (2, 1) \oplus 6 \cdot (1, 4)$		$[3 : E_6]$
			$= su(3) \oplus su(2)$	$2 \cdot (3 \oplus \overline{3}, 1)$		$[4 : E_7]$
27.	$sp(2) \oplus sp(2)$	$2 \cdot (5, 1) \oplus (5, 4) \oplus 7 \cdot (1, 4)$	$= su(2) \oplus sp(2)$	$7 \cdot (1, 4)$		$[4 : sp(3) \oplus su(2)]$
			$= sp(2) \oplus su(2)$	$2 \cdot (5, 1)$		$[4 : E_7]$
28.	$sp(2) \oplus sp(2)$	$4 \cdot (4, 1) \oplus 2 \cdot (4, 4) \oplus 4 \cdot (1, 4)$	$= su(2) \oplus sp(2)$	$4 \cdot (1, 4)$		$[4 : E_7]$
29.	$sp(2) \oplus G_2$	$5 \cdot (4, 1) \oplus (4, 7) \oplus 4 \cdot (1, 7)$	$= su(2) \oplus G_2$	$4 \cdot (1, 7)$		$[4 : E_7]$
			$= sp(2) \oplus su(2)$	$5 \cdot (4, 1) \oplus (1, 2)$		$[6 : sp(5)]$

	\mathfrak{g}	w/ r	$= \tilde{\mathfrak{g}}$	w/ \tilde{r}	\oplus	SCFT $[d : \mathfrak{h}]$
1.	$\mathfrak{sp}(3)$	$14 \oplus 11 \cdot 6$	$= \mathfrak{sp}(2)$			$[6 : E_8]$
2.	$\mathfrak{su}(6)$	$20 \oplus 15 \oplus \overline{15} \oplus 5 \cdot 6 \oplus 5 \cdot \overline{6}$	$= \mathfrak{su}(5)$	$5 \oplus \overline{5} \oplus 10 \oplus \overline{10}$		$[6 : E_8]$
3.	$\mathfrak{so}(12)$	$3 \cdot 32 \oplus 32' \oplus 4 \cdot 12$	$= \mathfrak{so}(11)$	$3 \cdot 32$		$[6 : E_8]$
4.	G_2	$8 \cdot 7$	$= \mathfrak{su}(2)$	2		$[6 : \mathfrak{sp}(5)]$
5.	$\mathfrak{so}(7)$	$4 \cdot 8 \oplus 6 \cdot 7$	$= \mathfrak{sp}(2)$	$5 \cdot 4$		$[6 : \mathfrak{sp}(5)]$
6.	$\mathfrak{su}(6)$	$21 \oplus \overline{21} \oplus 20 \oplus 6 \oplus \overline{6}$	$= \mathfrak{su}(5)$	$10 \oplus \overline{10}$		$[6 : \mathfrak{sp}(5)]$
7.	$\mathfrak{sp}(2)$	$12 \cdot 4$	$= \mathfrak{su}(2)$			$[4 : E_7]$
8.	$\mathfrak{su}(4)$	$2 \cdot 6 \oplus 6 \cdot 4 \oplus 6 \cdot \overline{4}$	$= \mathfrak{su}(3)$	$2 \cdot 3 \oplus 2 \cdot \overline{3}$		$[4 : E_7]$
9.	$\mathfrak{so}(7)$	$6 \cdot 8 \oplus 4 \cdot 7$	$= G_2$	$4 \cdot 7$		$[4 : E_7]$
10.	$\mathfrak{so}(8)$	$6 \cdot 8 \oplus 4 \cdot 8' \oplus 2 \cdot 8''$	$= \mathfrak{so}(7)$	$6 \cdot 8$		$[4 : E_7]$
11.	$\mathfrak{so}(8)$	$6 \cdot 8 \oplus 6 \cdot 8'$	$= G_2$			$[4 : E_7] \oplus [4 : E_7]$
12.	$\mathfrak{sp}(2)$	$6 \cdot 5$	$= \mathfrak{su}(2)$			$[4 : \mathfrak{sp}(3) \oplus \mathfrak{su}(2)]$
13.	$\mathfrak{sp}(2)$	$4 \cdot 4 \oplus 4 \cdot 5$	$= \mathfrak{su}(2)$	$3 \cdot 2$		$[4 : \mathfrak{sp}(3) \oplus \mathfrak{su}(2)]$
14.	$\mathfrak{su}(4)$	$10 \oplus \overline{10} \oplus 2 \cdot 4 \oplus 2 \cdot \overline{4}$	$= \mathfrak{su}(3)$	$3 \oplus \overline{3}$		$[4 : \mathfrak{sp}(3) \oplus \mathfrak{su}(2)]$
15.	$\mathfrak{su}(3)$	$6 \cdot 3 \oplus 6 \cdot \overline{3}$	$= \mathfrak{su}(2)$	$2 \cdot 2$		$[3 : E_6]$
16.	$\mathfrak{su}(4)$	$4 \cdot 6 \oplus 4 \cdot 4 \oplus 4 \cdot \overline{4}$	$= \mathfrak{sp}(2)$	$6 \cdot 4$		$[3 : E_6]$
17.	$\mathfrak{su}(3)$	$3 \oplus \overline{3} \oplus 6 \oplus \overline{6}$	$= \mathfrak{su}(2)$	$n \cdot 2$		$[3 : \mathfrak{h}]$

	\mathfrak{g}	w/ r	$= \tilde{\mathfrak{g}}$	w/ \tilde{r}	\oplus	SCFT $[d : \mathfrak{h}]$
18.	$\mathfrak{su}(2) \oplus \mathfrak{su}(3)$	$2 \cdot (2, 1) \oplus (2, 3 \oplus \overline{3}) \oplus 4 \cdot (1, 3 \oplus \overline{3})$	$= \mathfrak{su}(2) \oplus \mathfrak{su}(2)$	$2 \cdot (2, 1) \oplus 2 \cdot (1, 2)$		$[3 : E_6]$
19.	$\mathfrak{su}(2) \oplus \mathfrak{sp}(2)$	$2 \cdot (2, 4) \oplus 8 \cdot (1, 4)$	$= \mathfrak{su}(2) \oplus \mathfrak{su}(2)$			$[4 : E_7]$
20.	$\mathfrak{su}(2) \oplus \mathfrak{sp}(2)$	$3 \cdot (2, 1) \oplus (2, 5) \oplus 4 \cdot (1, 5)$	$= \mathfrak{su}(2) \oplus \mathfrak{su}(2)$	$3 \cdot (2, 1)$		$[4 : \mathfrak{sp}(3) \oplus \mathfrak{su}(2)]$
21.	$\mathfrak{su}(2) \oplus G_2$	$(2, 1) \oplus (2, 7) \oplus 6 \cdot (1, 7)$	$= \mathfrak{su}(2) \oplus \mathfrak{su}(2)$	$(2, 1) \oplus (1, 2)$		$[6 : \mathfrak{sp}(5)]$
22.	$\mathfrak{su}(3) \oplus \mathfrak{su}(3)$	$2 \cdot (3, 3) \oplus 2 \cdot (3, \overline{3})$	$= \mathfrak{su}(2) \oplus \mathfrak{su}(3)$	$2 \cdot (2, 1)$		$[3 : E_6]$
23.	$\mathfrak{su}(3) \oplus \mathfrak{su}(3)$	$(3 \oplus \overline{3}, 3 \oplus \overline{3})$	$= \mathfrak{su}(2) \oplus \mathfrak{su}(3)$	$2 \cdot (2, 1)$		$[3 : E_6]$
24.	$\mathfrak{su}(3) \oplus \mathfrak{su}(3)$	$3 \cdot (3 \oplus \overline{3}, 1) \oplus (3, \overline{3}) \oplus (\overline{3}, 3) \oplus 3 \cdot (1, 3 \oplus \overline{3})$	$= \mathfrak{su}(2) \oplus \mathfrak{su}(3)$	$2 \cdot (2, 1) \oplus 3 \cdot (1, 3 \oplus \overline{3})$		$[3 : E_6]$
25.	$\mathfrak{su}(3) \oplus \mathfrak{sp}(2)$	$(3 \oplus \overline{3}, 1) \oplus (3 \oplus \overline{3}, 5)$	$= \mathfrak{su}(2) \oplus \mathfrak{sp}(2)$	$2 \cdot (2, 1)$		$[3 : E_6]$
			$= \mathfrak{su}(3) \oplus \mathfrak{su}(2)$	$(3 \oplus \overline{3}, 1)$		$[4 : \mathfrak{sp}(3) \oplus \mathfrak{su}(2)]$
26.	$\mathfrak{su}(3) \oplus \mathfrak{sp}(2)$	$2 \cdot (3 \oplus \overline{3}, 1) \oplus (3 \oplus \overline{3}, 4) \oplus 6 \cdot (1, 4)$	$= \mathfrak{su}(2) \oplus \mathfrak{sp}(2)$	$2 \cdot (2, 1) \oplus 6 \cdot (1, 4)$		$[3 : E_6]$
			$= \mathfrak{su}(3) \oplus \mathfrak{su}(2)$	$2 \cdot (3 \oplus \overline{3}, 1)$		$[4 : E_7]$
27.	$\mathfrak{sp}(2) \oplus \mathfrak{sp}(2)$	$2 \cdot (5, 1) \oplus (5, 4) \oplus 7 \cdot (1, 4)$	$= \mathfrak{su}(2) \oplus \mathfrak{sp}(2)$	$7 \cdot (1, 4)$		$[4 : \mathfrak{sp}(3) \oplus \mathfrak{su}(2)]$
			$= \mathfrak{sp}(2) \oplus \mathfrak{su}(2)$	$2 \cdot (5, 1)$		$[4 : E_7]$
28.	$\mathfrak{sp}(2) \oplus \mathfrak{sp}(2)$	$4 \cdot (4, 1) \oplus 2 \cdot (4, 4) \oplus 4 \cdot (1, 4)$	$= \mathfrak{su}(2) \oplus \mathfrak{sp}(2)$	$4 \cdot (1, 4)$		$[4 : E_7]$
29.	$\mathfrak{sp}(2) \oplus G_2$	$5 \cdot (4, 1) \oplus (4, 7) \oplus 4 \cdot (1, 7)$	$= \mathfrak{su}(2) \oplus G_2$	$4 \cdot (1, 7)$		$[4 : E_7]$
			$= \mathfrak{sp}(2) \oplus \mathfrak{su}(2)$	$5 \cdot (4, 1) \oplus (1, 2)$		$[6 : \mathfrak{sp}(5)]$

	\mathfrak{g}	w/ r	$= \tilde{\mathfrak{g}}$	w/ \tilde{r}	\oplus	SCFT $[d : h]$
1.	$\mathfrak{sp}(3)$	$14 \oplus 11 \cdot 6$	$= \mathfrak{sp}(2)$			$[6 : E_8]$
2.	$\mathfrak{su}(6)$	$20 \oplus 15 \oplus \overline{15} \oplus 5 \cdot 6 \oplus 5 \cdot \overline{6}$	$= \mathfrak{su}(5)$	$5 \oplus \overline{5} \oplus 10 \oplus \overline{10}$		$[6 : E_8]$
3.	$\mathfrak{so}(12)$	$3 \cdot 32 \oplus 32' \oplus 4 \cdot 12$	$= \mathfrak{so}(11)$	$3 \cdot 32$		$[6 : E_8]$
4.	G_2	$8 \cdot 7$	$= \mathfrak{su}(2)$	2		$[6 : \mathfrak{sp}(5)]$
5.	$\mathfrak{so}(7)$	$4 \cdot 8 \oplus 6 \cdot 7$	$= \mathfrak{sp}(2)$	$5 \cdot 4$		$[6 : \mathfrak{sp}(5)]$
6.	$\mathfrak{su}(6)$	$21 \oplus \overline{21} \oplus 20 \oplus 6 \oplus \overline{6}$	$= \mathfrak{su}(5)$	$10 \oplus \overline{10}$		$[6 : \mathfrak{sp}(5)]$
7.	$\mathfrak{sp}(2)$	$12 \cdot 4$	$= \mathfrak{su}(2)$			$[4 : E_7]$
8.	$\mathfrak{su}(4)$	$2 \cdot 6 \oplus 6 \cdot 4 \oplus 6 \cdot \overline{4}$	$= \mathfrak{su}(3)$	$2 \cdot 3 \oplus 2 \cdot \overline{3}$		$[4 : E_7]$
9.	$\mathfrak{so}(7)$	$6 \cdot 8 \oplus 4 \cdot 7$	$= G_2$	$4 \cdot 7$		$[4 : E_7]$
10.	$\mathfrak{so}(8)$	$6 \cdot 8 \oplus 4 \cdot 8' \oplus 2 \cdot 8''$	$= \mathfrak{so}(7)$	$6 \cdot 8$		$[4 : E_7]$
11.	$\mathfrak{so}(8)$	$6 \cdot 8 \oplus 6 \cdot 8'$	$= G_2$			$[4 : E_7] \oplus [4 : E_7]$
12.	$\mathfrak{sp}(2)$	$6 \cdot 5$	$= \mathfrak{su}(2)$			$[4 : \mathfrak{sp}(3) \oplus \mathfrak{su}(2)]$
13.	$\mathfrak{sp}(2)$	$4 \cdot 4 \oplus 4 \cdot 5$	$= \mathfrak{su}(2)$	$3 \cdot 2$		$[4 : \mathfrak{sp}(3) \oplus \mathfrak{su}(2)]$
14.	$\mathfrak{su}(4)$	$10 \oplus \overline{10} \oplus 2 \cdot 4 \oplus 2 \cdot \overline{4}$	$= \mathfrak{su}(3)$	$3 \oplus \overline{3}$		$[4 : \mathfrak{sp}(3) \oplus \mathfrak{su}(2)]$
15.	$\mathfrak{su}(3)$	$6 \cdot 3 \oplus 6 \cdot \overline{3}$	$= \mathfrak{su}(2)$	$2 \cdot 2$		$[3 : E_6]$
16.	$\mathfrak{su}(4)$	$4 \cdot 6 \oplus 4 \cdot 4 \oplus 4 \cdot \overline{4}$	$= \mathfrak{sp}(2)$	$6 \cdot 4$		$[3 : E_6]$
17.	$\mathfrak{su}(3)$	$3 \oplus \overline{3} \oplus 6 \oplus \overline{6}$	$= \mathfrak{su}(2)$	$n \cdot 2$		$[3 : h]$

	\mathfrak{g}	w/ r	$= \tilde{\mathfrak{g}}$	w/ \tilde{r}	\oplus	SCFT $[d : h]$
18.	$\mathfrak{su}(2) \oplus \mathfrak{su}(3)$	$2 \cdot (2, 1) \oplus (2, 3 \oplus \overline{3}) \oplus 4 \cdot (1, 3 \oplus \overline{3})$	$= \mathfrak{su}(2) \oplus \mathfrak{su}(2)$	$2 \cdot (2, 1) \oplus 2 \cdot (1, 2)$		$[3 : E_6]$
19.	$\mathfrak{su}(2) \oplus \mathfrak{sp}(2)$	$2 \cdot (2, 4) \oplus 8 \cdot (1, 4)$	$= \mathfrak{su}(2) \oplus \mathfrak{su}(2)$			$[4 : E_7]$
20.	$\mathfrak{su}(2) \oplus \mathfrak{sp}(2)$	$3 \cdot (2, 1) \oplus (2, 5) \oplus 4 \cdot (1, 5)$	$= \mathfrak{su}(2) \oplus \mathfrak{su}(2)$	$3 \cdot (2, 1)$		$[4 : \mathfrak{sp}(3) \oplus \mathfrak{su}(2)]$
21.	$\mathfrak{su}(2) \oplus G_2$	$(2, 1) \oplus (2, 7) \oplus 6 \cdot (1, 7)$	$= \mathfrak{su}(2) \oplus \mathfrak{su}(2)$	$(2, 1) \oplus (1, 2)$		$[6 : \mathfrak{sp}(5)]$
22.	$\mathfrak{su}(3) \oplus \mathfrak{su}(3)$	$2 \cdot (3, 3) \oplus 2 \cdot (3, \overline{3})$	$= \mathfrak{su}(2) \oplus \mathfrak{su}(3)$	$2 \cdot (2, 1)$		$[3 : E_6]$
23.	$\mathfrak{su}(3) \oplus \mathfrak{su}(3)$	$(3 \oplus \overline{3}, 3 \oplus \overline{3})$	$= \mathfrak{su}(2) \oplus \mathfrak{su}(3)$	$2 \cdot (2, 1)$		$[3 : E_6]$
24.	$\mathfrak{su}(3) \oplus \mathfrak{su}(3)$	$3 \cdot (3 \oplus \overline{3}, 1) \oplus (3, \overline{3}) \oplus (\overline{3}, 3) \oplus 3 \cdot (1, 3 \oplus \overline{3})$	$= \mathfrak{su}(2) \oplus \mathfrak{su}(3)$	$2 \cdot (2, 1) \oplus 3 \cdot (1, 3 \oplus \overline{3})$		$[3 : E_6]$
25.	$\mathfrak{su}(3) \oplus \mathfrak{sp}(2)$	$(3 \oplus \overline{3}, 1) \oplus (3 \oplus \overline{3}, 5)$	$= \mathfrak{su}(2) \oplus \mathfrak{sp}(2)$	$2 \cdot (2, 1)$		$[3 : E_6]$
			$= \mathfrak{su}(3) \oplus \mathfrak{su}(2)$	$(3 \oplus \overline{3}, 1)$		$[4 : \mathfrak{sp}(3) \oplus \mathfrak{su}(2)]$
26.	$\mathfrak{su}(3) \oplus \mathfrak{sp}(2)$	$2 \cdot (3 \oplus \overline{3}, 1) \oplus (3 \oplus \overline{3}, 4) \oplus 6 \cdot (1, 4)$	$= \mathfrak{su}(2) \oplus \mathfrak{sp}(2)$	$2 \cdot (2, 1) \oplus 6 \cdot (1, 4)$		$[3 : E_6]$
			$= \mathfrak{su}(3) \oplus \mathfrak{su}(2)$	$2 \cdot (3 \oplus \overline{3}, 1)$		$[4 : E_7]$
27.	$\mathfrak{sp}(2) \oplus \mathfrak{sp}(2)$	$2 \cdot (5, 1) \oplus (5, 4) \oplus 7 \cdot (1, 4)$	$= \mathfrak{su}(2) \oplus \mathfrak{sp}(2)$	$7 \cdot (1, 4)$		$[4 : \mathfrak{sp}(3) \oplus \mathfrak{su}(2)]$
			$= \mathfrak{sp}(2) \oplus \mathfrak{su}(2)$	$2 \cdot (5, 1)$		$[4 : E_7]$
28.	$\mathfrak{sp}(2) \oplus \mathfrak{sp}(2)$	$4 \cdot (4, 1) \oplus 2 \cdot (4, 4) \oplus 4 \cdot (1, 4)$	$= \mathfrak{su}(2) \oplus \mathfrak{sp}(2)$	$4 \cdot (1, 4)$		$[4 : E_7]$
29.	$\mathfrak{sp}(2) \oplus G_2$	$5 \cdot (4, 1) \oplus (4, 7) \oplus 4 \cdot (1, 7)$	$= \mathfrak{su}(2) \oplus G_2$	$4 \cdot (1, 7)$		$[4 : E_7]$
			$= \mathfrak{sp}(2) \oplus \mathfrak{su}(2)$	$5 \cdot (4, 1) \oplus (1, 2)$		$[6 : \mathfrak{sp}(5)]$

A surprise:

In addition to finding the known E_n isolated rank 1 SCFTs as decoupled factors in various $g \rightarrow \infty$ limits of Lagrangian SCFTs, we also find **new isolated rank 1 SCFTs**.

They share the same conformal singularity as the E_n theories, but they have **different global symmetries** (and different central charges).

More about these later...

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3.	$so(12)$	$3 \cdot 32 \oplus 32' \oplus 4 \cdot 12$	$= so(11)$	$3 \cdot 32$		$[6 : E_8]$
4.	G_2	$8 \cdot 7$	$= su(2)$	2		$[6 : sp(5)]$
5.	$so(7)$	$4 \cdot 8 \oplus 6 \cdot 7$	$= sp(2)$	$5 \cdot 4$		$[6 : sp(5)]$
6.	$su(6)$	$21 \oplus \overline{21} \oplus 20 \oplus 6 \oplus \overline{6}$	$= su(5)$	$10 \oplus \overline{10}$		$[6 : sp(5)]$
7.	$sp(2)$	$12 \cdot 4$	$= su(2)$			$[4 : E_7]$
8.	$su(4)$	$2 \cdot 6 \oplus 6 \cdot 4 \oplus 6 \cdot \overline{4}$	$= su(3)$	$2 \cdot 3 \oplus 2 \cdot \overline{3}$		$[4 : E_7]$
9.	$so(7)$	$6 \cdot 8 \oplus 4 \cdot 7$	$= G_2$	$4 \cdot 7$		$[4 : E_7]$
10.	$so(8)$	$6 \cdot 8 \oplus 4 \cdot 8' \oplus 2 \cdot 8''$	$= so(7)$	$6 \cdot 8$		$[4 : E_7]$
11.	$so(8)$	$6 \cdot 8 \oplus 6 \cdot 8'$	$= G_2$			$[4 : E_7] \oplus [4 : E_7]$
12.	$sp(2)$	$6 \cdot 5$	$= su(2)$			$[4 : sp(3) \oplus su(2)]$
13.	$sp(2)$	$4 \cdot 4 \oplus 4 \cdot 5$	$= su(2)$	$3 \cdot 2$		$[4 : sp(3) \oplus su(2)]$
14.	$su(4)$	$10 \oplus \overline{10} \oplus 2 \cdot 4 \oplus 2 \cdot \overline{4}$	$= su(3)$	$3 \oplus \overline{3}$		$[4 : sp(3) \oplus su(2)]$
15.	$su(3)$	$6 \cdot 3 \oplus 6 \cdot \overline{3}$	$= su(2)$	$2 \cdot 2$		$[3 : E_6]$
16.	$su(4)$	$4 \cdot 6 \oplus 4 \cdot 4 \oplus 4 \cdot \overline{4}$	$= sp(2)$	$6 \cdot 4$		$[3 : E_6]$
17.	$su(3)$	$3 \oplus \overline{3} \oplus 6 \oplus \overline{6}$	$= su(2)$	$n \cdot 2$		$[3 : h]$

	g	w/ r	$= \tilde{g}$	w/ \tilde{r}	\oplus	SCFT $[d : h]$
18.	$su(2) \oplus su(3)$	$2 \cdot (2, 1) \oplus (2, 3 \oplus \overline{3}) \oplus 4 \cdot (1, 3 \oplus \overline{3})$	$= su(2) \oplus su(2)$	$2 \cdot (2, 1) \oplus 2 \cdot (1, 2)$		$[3 : E_6]$
19.	$su(2) \oplus sp(2)$	$2 \cdot (2, 4) \oplus 8 \cdot (1, 4)$	$= su(2) \oplus su(2)$			$[4 : E_7]$
20.	$su(2) \oplus sp(2)$	$3 \cdot (2, 1) \oplus (2, 5) \oplus 4 \cdot (1, 5)$	$= su(2) \oplus su(2)$	$3 \cdot (2, 1)$		$[4 : sp(3) \oplus su(2)]$
21.	$su(2) \oplus G_2$	$(2, 1) \oplus (2, 7) \oplus 6 \cdot (1, 7)$	$= su(2) \oplus su(2)$	$(2, 1) \oplus (1, 2)$		$[6 : sp(5)]$
22.	$su(3) \oplus su(3)$	$2 \cdot (3, 3) \oplus 2 \cdot (3, \overline{3})$	$= su(2) \oplus su(3)$	$2 \cdot (2, 1)$		$[3 : E_6]$
23.	$su(3) \oplus su(3)$	$(3 \oplus \overline{3}, 3 \oplus \overline{3})$	$= su(2) \oplus su(3)$	$2 \cdot (2, 1)$		$[3 : E_6]$
24.	$su(3) \oplus su(3)$	$3 \cdot (3 \oplus \overline{3}, 1) \oplus (3, \overline{3}) \oplus (\overline{3}, 3) \oplus 3 \cdot (1, 3 \oplus \overline{3})$	$= su(2) \oplus su(3)$	$2 \cdot (2, 1) \oplus 3 \cdot (1, 3 \oplus \overline{3})$		$[3 : E_6]$
25.	$su(3) \oplus sp(2)$	$(3 \oplus \overline{3}, 1) \oplus (3 \oplus \overline{3}, 5)$	$= su(2) \oplus sp(2)$	$2 \cdot (2, 1)$		$[3 : E_6]$
			$= su(3) \oplus su(2)$	$(3 \oplus \overline{3}, 1)$		$[4 : sp(3) \oplus su(2)]$
26.	$su(3) \oplus sp(2)$	$2 \cdot (3 \oplus \overline{3}, 1) \oplus (3 \oplus \overline{3}, 4) \oplus 6 \cdot (1, 4)$	$= su(2) \oplus sp(2)$	$2 \cdot (2, 1) \oplus 6 \cdot (1, 4)$		$[3 : E_6]$
			$= su(3) \oplus su(2)$	$2 \cdot (3 \oplus \overline{3}, 1)$		$[4 : E_7]$
27.	$sp(2) \oplus sp(2)$	$2 \cdot (5, 1) \oplus (5, 4) \oplus 7 \cdot (1, 4)$	$= su(2) \oplus sp(2)$	$7 \cdot (1, 4)$		$[4 : sp(3) \oplus su(2)]$
			$= sp(2) \oplus su(2)$	$2 \cdot (5, 1)$		$[4 : E_7]$
28.	$sp(2) \oplus sp(2)$	$4 \cdot (4, 1) \oplus 2 \cdot (4, 4) \oplus 4 \cdot (1, 4)$	$= su(2) \oplus sp(2)$	$4 \cdot (1, 4)$		$[4 : E_7]$
29.	$sp(2) \oplus G_2$	$5 \cdot (4, 1) \oplus (4, 7) \oplus 4 \cdot (1, 7)$	$= su(2) \oplus G_2$	$4 \cdot (1, 7)$		$[4 : E_7]$
			$= sp(2) \oplus su(2)$	$5 \cdot (4, 1) \oplus (1, 2)$		$[6 : sp(5)]$

	\mathfrak{g}	w/ r	$= \tilde{\mathfrak{g}}$	w/ \tilde{r}	\oplus	SCFT $[d : \mathfrak{h}]$
1.	$\mathfrak{sp}(3)$	$14 \oplus 11 \cdot 6$	$= \mathfrak{sp}(2)$			$[6 : E_8]$
2.	$\mathfrak{su}(6)$	$20 \oplus 15 \oplus \overline{15} \oplus 5 \cdot 6 \oplus 5 \cdot \overline{6}$	$= \mathfrak{su}(5)$	$5 \oplus \overline{5} \oplus 10 \oplus \overline{10}$		$[6 : E_8]$
3.	$\mathfrak{so}(12)$	$3 \cdot 32 \oplus 32' \oplus 4 \cdot 12$	$= \mathfrak{so}(11)$	$3 \cdot 32$		$[6 : E_8]$
4.	G_2	$8 \cdot 7$	$= \mathfrak{su}(2)$	2		$[6 : \mathfrak{sp}(5)]$
5.	$\mathfrak{so}(7)$	$4 \cdot 8 \oplus 6 \cdot 7$	$= \mathfrak{sp}(2)$	$5 \cdot 4$		$[6 : \mathfrak{sp}(5)]$
6.	$\mathfrak{su}(6)$	$21 \oplus \overline{21} \oplus 20 \oplus 6 \oplus \overline{6}$	$= \mathfrak{su}(5)$	$10 \oplus \overline{10}$		$[6 : \mathfrak{sp}(5)]$
7.	$\mathfrak{sp}(2)$	$12 \cdot 4$	$= \mathfrak{su}(2)$			$[4 : E_7]$
8.	$\mathfrak{su}(4)$	$2 \cdot 6 \oplus 6 \cdot 4 \oplus 6 \cdot \overline{4}$	$= \mathfrak{su}(3)$	$2 \cdot 3 \oplus 2 \cdot \overline{3}$		$[4 : E_7]$
9.	$\mathfrak{so}(7)$	$6 \cdot 8 \oplus 4 \cdot 7$	$= G_2$	$4 \cdot 7$		$[4 : E_7]$
10.	$\mathfrak{so}(8)$	$6 \cdot 8 \oplus 4 \cdot 8' \oplus 2 \cdot 8''$	$= \mathfrak{so}(7)$	$6 \cdot 8$		$[4 : E_7]$
11.	$\mathfrak{so}(8)$	$6 \cdot 8 \oplus 6 \cdot 8'$	$= G_2$			$[4 : E_7] \oplus [4 : E_7]$
12.	$\mathfrak{sp}(2)$	$6 \cdot 5$	$= \mathfrak{su}(2)$			$[4 : \mathfrak{sp}(3) \oplus \mathfrak{su}(2)]$
13.	$\mathfrak{sp}(2)$	$4 \cdot 4 \oplus 4 \cdot 5$	$= \mathfrak{su}(2)$	$3 \cdot 2$		$[4 : \mathfrak{sp}(3) \oplus \mathfrak{su}(2)]$
14.	$\mathfrak{su}(4)$	$10 \oplus \overline{10} \oplus 2 \cdot 4 \oplus 2 \cdot \overline{4}$	$= \mathfrak{su}(3)$	$3 \oplus \overline{3}$		$[4 : \mathfrak{sp}(3) \oplus \mathfrak{su}(2)]$
15.	$\mathfrak{su}(3)$	$6 \cdot 3 \oplus 6 \cdot \overline{3}$	$= \mathfrak{su}(2)$	$2 \cdot 2$		$[3 : E_6]$
16.	$\mathfrak{su}(4)$	$4 \cdot 6 \oplus 4 \cdot 4 \oplus 4 \cdot \overline{4}$	$= \mathfrak{sp}(2)$	$6 \cdot 4$		$[3 : E_6]$
17.	$\mathfrak{su}(3)$	$3 \oplus \overline{3} \oplus 6 \oplus \overline{6}$	$= \mathfrak{su}(2)$	$n \cdot 2$		$[3 : \mathfrak{h}]$

	\mathfrak{g}	w/ r	$= \tilde{\mathfrak{g}}$	w/ \tilde{r}	\oplus	SCFT $[d : \mathfrak{h}]$
18.	$\mathfrak{su}(2) \oplus \mathfrak{su}(3)$	$2 \cdot (2, 1) \oplus (2, 3 \oplus \overline{3}) \oplus 4 \cdot (1, 3 \oplus \overline{3})$	$= \mathfrak{su}(2) \oplus \mathfrak{su}(2)$	$2 \cdot (2, 1) \oplus 2 \cdot (1, 2)$		$[3 : E_6]$
19.	$\mathfrak{su}(2) \oplus \mathfrak{sp}(2)$	$2 \cdot (2, 4) \oplus 8 \cdot (1, 4)$	$= \mathfrak{su}(2) \oplus \mathfrak{su}(2)$			$[4 : E_7]$
20.	$\mathfrak{su}(2) \oplus \mathfrak{sp}(2)$	$3 \cdot (2, 1) \oplus (2, 5) \oplus 4 \cdot (1, 5)$	$= \mathfrak{su}(2) \oplus \mathfrak{su}(2)$	$3 \cdot (2, 1)$		$[4 : \mathfrak{sp}(3) \oplus \mathfrak{su}(2)]$
21.	$\mathfrak{su}(2) \oplus G_2$	$(2, 1) \oplus (2, 7) \oplus 6 \cdot (1, 7)$	$= \mathfrak{su}(2) \oplus \mathfrak{su}(2)$	$(2, 1) \oplus (1, 2)$		$[6 : \mathfrak{sp}(5)]$
22.	$\mathfrak{su}(3) \oplus \mathfrak{su}(3)$	$2 \cdot (3, 3) \oplus 2 \cdot (3, \overline{3})$	$= \mathfrak{su}(2) \oplus \mathfrak{su}(3)$	$2 \cdot (2, 1)$		$[3 : E_6]$
23.	$\mathfrak{su}(3) \oplus \mathfrak{su}(3)$	$(3 \oplus \overline{3}, 3 \oplus \overline{3})$	$= \mathfrak{su}(2) \oplus \mathfrak{su}(3)$	$2 \cdot (2, 1)$		$[3 : E_6]$
24.	$\mathfrak{su}(3) \oplus \mathfrak{su}(3)$	$3 \cdot (3 \oplus \overline{3}, 1) \oplus (3, \overline{3}) \oplus (\overline{3}, 3) \oplus 3 \cdot (1, 3 \oplus \overline{3})$	$= \mathfrak{su}(2) \oplus \mathfrak{su}(3)$	$2 \cdot (2, 1) \oplus 3 \cdot (1, 3 \oplus \overline{3})$		$[3 : E_6]$
25.	$\mathfrak{su}(3) \oplus \mathfrak{sp}(2)$	$(3 \oplus \overline{3}, 1) \oplus (3 \oplus \overline{3}, 5)$	$= \mathfrak{su}(2) \oplus \mathfrak{sp}(2)$	$2 \cdot (2, 1)$		$[3 : E_6]$
			$= \mathfrak{su}(3) \oplus \mathfrak{su}(2)$	$(3 \oplus \overline{3}, 1)$		$[4 : \mathfrak{sp}(3) \oplus \mathfrak{su}(2)]$
26.	$\mathfrak{su}(3) \oplus \mathfrak{sp}(2)$	$2 \cdot (3 \oplus \overline{3}, 1) \oplus (3 \oplus \overline{3}, 4) \oplus 6 \cdot (1, 4)$	$= \mathfrak{su}(2) \oplus \mathfrak{sp}(2)$	$2 \cdot (2, 1) \oplus 6 \cdot (1, 4)$		$[3 : E_6]$
			$= \mathfrak{su}(3) \oplus \mathfrak{su}(2)$	$2 \cdot (3 \oplus \overline{3}, 1)$		$[4 : E_7]$
27.	$\mathfrak{sp}(2) \oplus \mathfrak{sp}(2)$	$2 \cdot (5, 1) \oplus (5, 4) \oplus 7 \cdot (1, 4)$	$= \mathfrak{su}(2) \oplus \mathfrak{sp}(2)$	$7 \cdot (1, 4)$		$[4 : \mathfrak{sp}(3) \oplus \mathfrak{su}(2)]$
			$= \mathfrak{sp}(2) \oplus \mathfrak{su}(2)$	$2 \cdot (5, 1)$		$[4 : E_7]$
28.	$\mathfrak{sp}(2) \oplus \mathfrak{sp}(2)$	$4 \cdot (4, 1) \oplus 2 \cdot (4, 4) \oplus 4 \cdot (1, 4)$	$= \mathfrak{su}(2) \oplus \mathfrak{sp}(2)$	$4 \cdot (1, 4)$		$[4 : E_7]$
29.	$\mathfrak{sp}(2) \oplus G_2$	$5 \cdot (4, 1) \oplus (4, 7) \oplus 4 \cdot (1, 7)$	$= \mathfrak{su}(2) \oplus G_2$	$4 \cdot (1, 7)$		$[4 : E_7]$
			$= \mathfrak{sp}(2) \oplus \mathfrak{su}(2)$	$5 \cdot (4, 1) \oplus (1, 2)$		$[6 : \mathfrak{sp}(5)]$

A surprise:

In addition to finding the known E_n isolated rank 1 SCFTs as decoupled factors in various $g \rightarrow \infty$ limits of Lagrangian SCFTs, we also find **new isolated rank 1 SCFTs**.

They share the same conformal singularity as the E_n theories, but they have **different global symmetries** (and different central charges).

More about these later...

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More about these later...

IIa Example: $su(3)$ w/ 6.3

- Ingredients:

Coulomb branch moduli: $\{u, v\}$

Coupling constant: $f \sim e^{2\pi i \tau}$. ($f \rightarrow 0$ is $g \rightarrow 0$, $f \rightarrow 1$ is $g \rightarrow \infty$.)

SW curve: $y^2 = (x^3 - ux - v)^2 - f^2 x^6$.

Holomorphic 1-forms: $\omega_u = xdx/y$, $\omega_v = dx/y$.

- Limits:

Set $u=0$ and $f=1$: degenerates to genus 1.

Change variables: $(\tilde{y}^2 = \tilde{x}^3 - \tilde{v}^4, \omega_{\tilde{v}} = d\tilde{x}/\tilde{y})$

This is the E_6 SCFT. (Minahan, Nemeschansky)

Set $v=0$ and $f \sim 1$: degenerates to

$$y^2 = \underbrace{x^2}_{\text{pinch cycle}} \cdot \underbrace{[(x^2 - u)^2 - f^2 x^4]}_{\equiv \tilde{y}^2}, \quad \underbrace{\omega_u = xdx/y}_{\updownarrow \omega_u = dx/\tilde{y}}.$$

weak $su(2)$ SCFT

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weak $\mathfrak{su}(2)$ SCFT

- Compare $f \rightarrow 1$ degeneration to $f \rightarrow 0$ (weak cpling) degeneration:
 - As $f \rightarrow 0$ in $su(3)$ SW curve, genus 2 curve pinches to 2 genus 0 curves **everywhere on moduli space**. Pinching cycle \leftrightarrow charged W^\pm bosons becoming massless, corresponding to expected $su(3) \rightarrow u(1) \times u(1)$ Higgsing. (Figure on left.)



- But as $f \rightarrow 1$, only one cycle vanishes everywhere on moduli space. (Figure on right.) Not enough W -bosons for a weakly-coupled Higgs mechanism, so some **new "phase"** is indicated for this theory at $f = 1$.

- Global symmetries \Leftrightarrow mass deformations:

E_6 masses: $M_n \sim m^n$ are explicit basis of E_6 Casimirs.

$$\tilde{y}^2 = \tilde{x}^3 - (\tilde{v}^2 M_2 + \tilde{v} M_5 + M_8) \tilde{x} - (\tilde{v}^4 + \tilde{v}^2 M_6 + \tilde{v} M_9 + M_{12}).$$

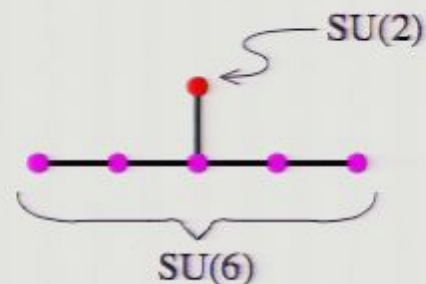
Turn on u at $f = 1$: same c.o.v. gives

$$\tilde{y}^2 = \tilde{x}^3 - (\tilde{v}^2 u + \frac{1}{48} u^4) \tilde{x} - (\tilde{v}^4 + \tilde{v}^2 \frac{1}{12} u^3 + \frac{1}{864} u^6).$$

Implies: u is mass deformation of E_6 SCFT with

$$\{M_2=u, M_5=0, M_6=\frac{1}{12}u^3, M_8=\frac{1}{48}u^4, M_9=0, M_{12}=\frac{1}{864}u^6\}.$$

Since $u \neq 0$ higgses $su(2) \rightarrow u(1)$, some **group theory** implies that the $su(2)$ gauges the \bullet subgroup of E_6 :



The unbroken $su(6)$ is part of the **global symmetry** of the $su(3)$ w/ $6 \cdot 3$ theory. But the **full global symmetry** of the theory is

$$u(1)_B \times su(6) \times u(2)_R.$$

- The R symmetry is realized at infinite coupling as a certain combination of the R symmetries of the rank 1 $su(2)$ and E_6 SCFTs.
- The $su(6)$ flavor symmetry was identified above as the “un-gauged” part of the E_6 group.
- Which leaves the $u(1)_B$ “baryon number” unaccounted for.

The solution can only be that there is a single **SU(2)-doublet hypermultiplet** at infinite coupling:

$$su(3)_g \text{ w/ } 6 \cdot 3 \sim su(2)_{1/g} \text{ w/ } 1 \cdot 2 \oplus (E_6\text{-CFT}).$$

- Further checks: anomaly matching, beta functions ...

III Central charges

- **Flavor current algebra central charge:**

By weakly gauging the global symmetries of the g theory both at weak and infinite gauge coupling, and comparing its beta function to the one computed in the dual description allows us to compute the contribution of the SCFT “matter” to the \tilde{g} gauge coupling beta function.

This is governed by the central charge, k , of the global flavor current algebra of the SCFT:

$$“J^a(x)J^b(0) \sim \frac{k \delta^{ab}}{x^6} + \dots + \frac{f_c^{ab} J^c(0)}{x^3} + \dots”.$$

k is thus a **new exactly computed observable of isolated N=2 SCFTs**.

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2.	$\mathfrak{su}(6)$	$20 \oplus 15 \oplus \overline{15} \oplus 5 \cdot 6 \oplus 5 \cdot \overline{6}$	$= \mathfrak{su}(5)$	$5 \oplus \overline{5} \oplus 10 \oplus \overline{10}$		$[6 : E_8]$
3.	$\mathfrak{so}(12)$	$3 \cdot 32 \oplus 32' \oplus 4 \cdot 12$	$= \mathfrak{so}(11)$	$3 \cdot 32$		$[6 : E_8]$
4.	G_2	$8 \cdot 7$	$= \mathfrak{su}(2)$	2		$[6 : \mathfrak{sp}(5)]$
5.	$\mathfrak{so}(7)$	$4 \cdot 8 \oplus 6 \cdot 7$	$= \mathfrak{sp}(2)$	$5 \cdot 4$		$[6 : \mathfrak{sp}(5)]$
6.	$\mathfrak{su}(6)$	$21 \oplus \overline{21} \oplus 20 \oplus 6 \oplus \overline{6}$	$= \mathfrak{su}(5)$	$10 \oplus \overline{10}$		$[6 : \mathfrak{sp}(5)]$
7.	$\mathfrak{sp}(2)$	$12 \cdot 4$	$= \mathfrak{su}(2)$			$[4 : E_7]$
8.	$\mathfrak{su}(4)$	$2 \cdot 6 \oplus 6 \cdot 4 \oplus 6 \cdot \overline{4}$	$= \mathfrak{su}(3)$	$2 \cdot 3 \oplus 2 \cdot \overline{3}$		$[4 : E_7]$
9.	$\mathfrak{so}(7)$	$6 \cdot 8 \oplus 4 \cdot 7$	$= G_2$	$4 \cdot 7$		$[4 : E_7]$
10.	$\mathfrak{so}(8)$	$6 \cdot 8 \oplus 4 \cdot 8' \oplus 2 \cdot 8''$	$= \mathfrak{so}(7)$	$6 \cdot 8$		$[4 : E_7]$
11.	$\mathfrak{so}(8)$	$6 \cdot 8 \oplus 6 \cdot 8'$	$= G_2$			$[4 : E_7] \oplus [4 : E_7]$
12.	$\mathfrak{sp}(2)$	$6 \cdot 5$	$= \mathfrak{su}(2)$			$[4 : \mathfrak{sp}(3) \oplus \mathfrak{su}(2)]$
13.	$\mathfrak{sp}(2)$	$4 \cdot 4 \oplus 4 \cdot 5$	$= \mathfrak{su}(2)$	$3 \cdot 2$		$[4 : \mathfrak{sp}(3) \oplus \mathfrak{su}(2)]$
14.	$\mathfrak{su}(4)$	$10 \oplus \overline{10} \oplus 2 \cdot 4 \oplus 2 \cdot \overline{4}$	$= \mathfrak{su}(3)$	$3 \oplus \overline{3}$		$[4 : \mathfrak{sp}(3) \oplus \mathfrak{su}(2)]$
15.	$\mathfrak{su}(3)$	$6 \cdot 3 \oplus 6 \cdot \overline{3}$	$= \mathfrak{su}(2)$	$2 \cdot 2$		$[3 : E_6]$
16.	$\mathfrak{su}(4)$	$4 \cdot 6 \oplus 4 \cdot 4 \oplus 4 \cdot \overline{4}$	$= \mathfrak{sp}(2)$	$6 \cdot 4$		$[3 : E_6]$
17.	$\mathfrak{su}(3)$	$3 \oplus \overline{3} \oplus 6 \oplus \overline{6}$	$= \mathfrak{su}(2)$	$n \cdot 2$		$[3 : \mathfrak{h}]$

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18.	$\mathfrak{su}(2) \oplus \mathfrak{su}(3)$	$2 \cdot (2, 1) \oplus (2, 3 \oplus \overline{3}) \oplus 4 \cdot (1, 3 \oplus \overline{3})$	$= \mathfrak{su}(2) \oplus \mathfrak{su}(2)$	$2 \cdot (2, 1) \oplus 2 \cdot (1, 2)$		$[3 : E_6]$
19.	$\mathfrak{su}(2) \oplus \mathfrak{sp}(2)$	$2 \cdot (2, 4) \oplus 8 \cdot (1, 4)$	$= \mathfrak{su}(2) \oplus \mathfrak{su}(2)$			$[4 : E_7]$
20.	$\mathfrak{su}(2) \oplus \mathfrak{sp}(2)$	$3 \cdot (2, 1) \oplus (2, 5) \oplus 4 \cdot (1, 5)$	$= \mathfrak{su}(2) \oplus \mathfrak{su}(2)$	$3 \cdot (2, 1)$		$[4 : \mathfrak{sp}(3) \oplus \mathfrak{su}(2)]$
21.	$\mathfrak{su}(2) \oplus G_2$	$(2, 1) \oplus (2, 7) \oplus 6 \cdot (1, 7)$	$= \mathfrak{su}(2) \oplus \mathfrak{su}(2)$	$(2, 1) \oplus (1, 2)$		$[6 : \mathfrak{sp}(5)]$
22.	$\mathfrak{su}(3) \oplus \mathfrak{su}(3)$	$2 \cdot (3, 3) \oplus 2 \cdot (3, \overline{3})$	$= \mathfrak{su}(2) \oplus \mathfrak{su}(3)$	$2 \cdot (2, 1)$		$[3 : E_6]$
23.	$\mathfrak{su}(3) \oplus \mathfrak{su}(3)$	$(3 \oplus \overline{3}, 3 \oplus \overline{3})$	$= \mathfrak{su}(2) \oplus \mathfrak{su}(3)$	$2 \cdot (2, 1)$		$[3 : E_6]$
24.	$\mathfrak{su}(3) \oplus \mathfrak{su}(3)$	$3 \cdot (3 \oplus \overline{3}, 1) \oplus (3, \overline{3}) \oplus (\overline{3}, 3) \oplus 3 \cdot (1, 3 \oplus \overline{3})$	$= \mathfrak{su}(2) \oplus \mathfrak{su}(3)$	$2 \cdot (2, 1) \oplus 3 \cdot (1, 3 \oplus \overline{3})$		$[3 : E_6]$
25.	$\mathfrak{su}(3) \oplus \mathfrak{sp}(2)$	$(3 \oplus \overline{3}, 1) \oplus (3 \oplus \overline{3}, 5)$	$= \mathfrak{su}(2) \oplus \mathfrak{sp}(2)$	$2 \cdot (2, 1)$		$[3 : E_6]$
			$= \mathfrak{su}(3) \oplus \mathfrak{su}(2)$	$(3 \oplus \overline{3}, 1)$		$[4 : \mathfrak{sp}(3) \oplus \mathfrak{su}(2)]$
26.	$\mathfrak{su}(3) \oplus \mathfrak{sp}(2)$	$2 \cdot (3 \oplus \overline{3}, 1) \oplus (3 \oplus \overline{3}, 4) \oplus 6 \cdot (1, 4)$	$= \mathfrak{su}(2) \oplus \mathfrak{sp}(2)$	$2 \cdot (2, 1) \oplus 6 \cdot (1, 4)$		$[3 : E_6]$
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27.	$\mathfrak{sp}(2) \oplus \mathfrak{sp}(2)$	$2 \cdot (5, 1) \oplus (5, 4) \oplus 7 \cdot (1, 4)$	$= \mathfrak{su}(2) \oplus \mathfrak{sp}(2)$	$7 \cdot (1, 4)$		$[4 : \mathfrak{sp}(3) \oplus \mathfrak{su}(2)]$
			$= \mathfrak{sp}(2) \oplus \mathfrak{su}(2)$	$2 \cdot (5, 1)$		$[4 : E_7]$
28.	$\mathfrak{sp}(2) \oplus \mathfrak{sp}(2)$	$4 \cdot (4, 1) \oplus 2 \cdot (4, 4) \oplus 4 \cdot (1, 4)$	$= \mathfrak{su}(2) \oplus \mathfrak{sp}(2)$	$4 \cdot (1, 4)$		$[4 : E_7]$
29.	$\mathfrak{sp}(2) \oplus G_2$	$5 \cdot (4, 1) \oplus (4, 7) \oplus 4 \cdot (1, 7)$	$= \mathfrak{su}(2) \oplus G_2$	$4 \cdot (1, 7)$		$[4 : E_7]$
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17.	$su(3)$	$3 \oplus \overline{3} \oplus 6 \oplus \overline{6}$	$= su(2)$	$n \cdot 2$		$[3 : h]$

	g	w/ r	$= \tilde{g}$	w/ \tilde{r}	\oplus	SCFT $[d : h]$
18.	$su(2) \oplus su(3)$	$2 \cdot (2, 1) \oplus (2, 3 \oplus \overline{3}) \oplus 4 \cdot (1, 3 \oplus \overline{3})$	$= su(2) \oplus su(2)$	$2 \cdot (2, 1) \oplus 2 \cdot (1, 2)$		$[3 : E_6]$
19.	$su(2) \oplus sp(2)$	$2 \cdot (2, 4) \oplus 8 \cdot (1, 4)$	$= su(2) \oplus su(2)$			$[4 : E_7]$
20.	$su(2) \oplus sp(2)$	$3 \cdot (2, 1) \oplus (2, 5) \oplus 4 \cdot (1, 5)$	$= su(2) \oplus su(2)$	$3 \cdot (2, 1)$		$[4 : sp(3) \oplus su(2)]$
21.	$su(2) \oplus G_2$	$(2, 1) \oplus (2, 7) \oplus 6 \cdot (1, 7)$	$= su(2) \oplus su(2)$	$(2, 1) \oplus (1, 2)$		$[6 : sp(5)]$
22.	$su(3) \oplus su(3)$	$2 \cdot (3, 3) \oplus 2 \cdot (3, \overline{3})$	$= su(2) \oplus su(3)$	$2 \cdot (2, 1)$		$[3 : E_6]$
23.	$su(3) \oplus su(3)$	$(3 \oplus \overline{3}, 3 \oplus \overline{3})$	$= su(2) \oplus su(3)$	$2 \cdot (2, 1)$		$[3 : E_6]$
24.	$su(3) \oplus su(3)$	$3 \cdot (3 \oplus \overline{3}, 1) \oplus (3, \overline{3}) \oplus (\overline{3}, 3) \oplus 3 \cdot (1, 3 \oplus \overline{3})$	$= su(2) \oplus su(3)$	$2 \cdot (2, 1) \oplus 3 \cdot (1, 3 \oplus \overline{3})$		$[3 : E_6]$
25.	$su(3) \oplus sp(2)$	$(3 \oplus \overline{3}, 1) \oplus (3 \oplus \overline{3}, 5)$	$= su(2) \oplus sp(2)$	$2 \cdot (2, 1)$		$[3 : E_6]$
			$= su(3) \oplus su(2)$	$(3 \oplus \overline{3}, 1)$		$[4 : sp(3) \oplus su(2)]$
26.	$su(3) \oplus sp(2)$	$2 \cdot (3 \oplus \overline{3}, 1) \oplus (3 \oplus \overline{3}, 4) \oplus 6 \cdot (1, 4)$	$= su(2) \oplus sp(2)$	$2 \cdot (2, 1) \oplus 6 \cdot (1, 4)$		$[3 : E_6]$
			$= su(3) \oplus su(2)$	$2 \cdot (3 \oplus \overline{3}, 1)$		$[4 : E_7]$
27.	$sp(2) \oplus sp(2)$	$2 \cdot (5, 1) \oplus (5, 4) \oplus 7 \cdot (1, 4)$	$= su(2) \oplus sp(2)$	$7 \cdot (1, 4)$		$[4 : sp(3) \oplus su(2)]$
			$= sp(2) \oplus su(2)$	$2 \cdot (5, 1)$		$[4 : E_7]$
28.	$sp(2) \oplus sp(2)$	$4 \cdot (4, 1) \oplus 2 \cdot (4, 4) \oplus 4 \cdot (1, 4)$	$= su(2) \oplus sp(2)$	$4 \cdot (1, 4)$		$[4 : E_7]$
29.	$sp(2) \oplus G_2$	$5 \cdot (4, 1) \oplus (4, 7) \oplus 4 \cdot (1, 7)$	$= su(2) \oplus G_2$	$4 \cdot (1, 7)$		$[4 : E_7]$
			$= sp(2) \oplus su(2)$	$5 \cdot (4, 1) \oplus (1, 2)$		$[6 : sp(5)]$

The unbroken $su(6)$ is part of the **global symmetry** of the $su(3)$ w/ $6 \cdot 3$ theory. But the **full global symmetry** of the theory is

$$u(1)_B \times su(6) \times u(2)_R.$$

- The R symmetry is realized at infinite coupling as a certain combination of the R symmetries of the rank 1 $su(2)$ and E_6 SCFTs.
- The $su(6)$ flavor symmetry was identified above as the “un-gauged” part of the E_6 group.
- Which leaves the $u(1)_B$ “baryon number” unaccounted for.

The solution can only be that there is a single **SU(2)-doublet hypermultiplet** at infinite coupling:

$$su(3)_g \text{ w/ } 6 \cdot 3 \sim su(2)_{1/g} \text{ w/ } 1 \cdot 2 \oplus (E_6\text{-CFT}).$$

- Further checks: anomaly matching, beta functions ...

III Central charges

- **Flavor current algebra central charge:**

By weakly gauging the global symmetries of the g theory both at weak and infinite gauge coupling, and comparing its beta function to the one computed in the dual description allows us to compute the contribution of the SCFT “matter” to the \tilde{g} gauge coupling beta function.

This is governed by the central charge, k , of the global flavor current algebra of the SCFT:

$$“J^a(x)J^b(0) \sim \frac{k \delta^{ab}}{x^6} + \dots + \frac{f_c^{ab} J^c(0)}{x^3} + \dots”.$$

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- Conformal algebra central charges:

$$T(x)T(0) \sim \frac{c \delta^{ab}}{x^8} + \dots + \frac{(1+a)T(0)}{x^4} + \dots$$

a and c are computed in many cases by 't Hooft anomaly matching arguments for R-symmetries.

a is believed to decrease along RG flows.

- New relations:

Find (w/ J. Wittig) for all known N=2 SCFTs

$$2a - c = \frac{1}{4} \sum_{i=1}^r (2d_i - 1)$$

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This is analogous to the Sugawara construction in 2-d CFT.

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IV Relevant deformations

Rank 1 SCFTs traditionally classified (via SW theory) by the **Kodaira classification** of singularities of families of elliptic curves fibered over \mathbb{P}^1 :

singularity	curve	flavor symmetry f
E_8	$y^2 = x^3 + x(M_2u^3 + M_8u^2 + M_{14}u + M_{20}) + (2u^5 + M_{12}u^3 + M_{18}u^2 + M_{24}u + M_{30})$	e_8
E_7	$y^2 = x^3 + x(u^3 + M_8u + M_{12}) + (M_2u^4 + M_6u^3 + M_{10}u^2 + M_{14}u + M_{18})$	e_7
E_6	$y^2 = x^3 + x(M_2u^2 + M_5u + M_8) + (u^4 + M_6u^2 + M_9u + M_{12})$	e_6
D_4	$y^2 = x^3 + x(3M_0u^2 + M_2u + M_4) + (2u^3 + \tilde{M}_4u + M_6)$	$so(8)$
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Even at rank 1 many new theories are predicted, but techniques for their construction need to be developed.
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With more complete knowledge of $N=2$ FTs, can explore if all can be realized as low energy limits of string theory.
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