Title: Relativistic viscous hydrodynamics and holography

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Abstract: TBA

# Relativistic viscous hydrodynamics and holography

Andrei Starinets
University of Southampton

R.Baier, P.Romatschke, D.T.Son, A.O.S.,M.A.Stephanov, 0712.2451 [hep-th]

Perimeter Institute
January 29, 2008

# Experimental and theoretical motivation

- Heavy ion collision program at RHIC, LHC (2000-2008-2020 ??)

- Studies of hot and dense nuclear matter

- Abundance of experimental results, poor theoretical understanding:

  - the collision apparently creates a fireball of "quark-gluon fluid"

  - need to understand both thermodynamics and kinetics

-in particular, need theoretical predictions for parameters entering equations of relativistic hydrodynamics – viscosity etc – computed from the underlying microscopic theory (thermal QCD)

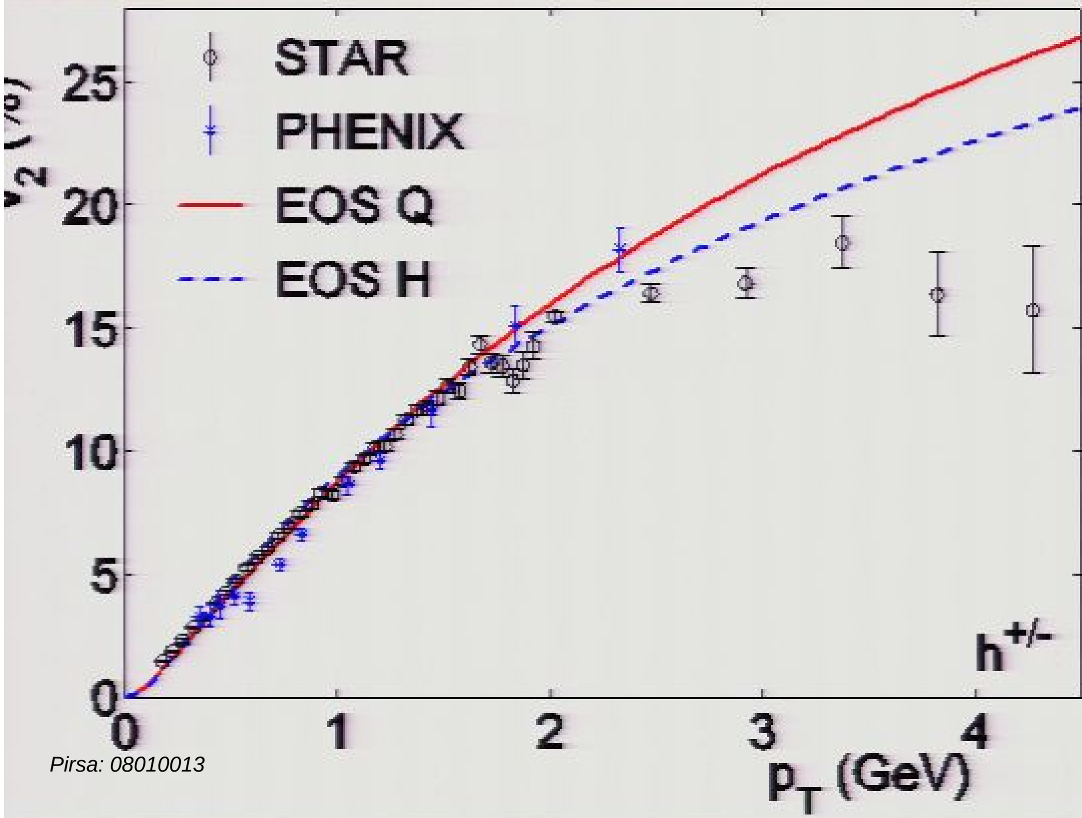
-this is difficult since the fireball is a strongly interacting nuclear fluid, not a dilute gas

# QCD kinetics

Viscosity is ONE of the parameters used in the hydro models escribing the azimuthal anisotropy of particle distribution at RHIC

$$\frac{d^2N^i}{dp_T d\phi} = N_0^i \left[1 + 2v_2^i(p_T)\cos 2\phi + \cdots\right]$$

$v_2^i(p_T)$ -elliptic flow for particle species "i"




Elliptic flow reproduced for

$$0 < \eta/s \leq 0.3$$

e.g. Baier, Romatschke, nucl-th/0610108

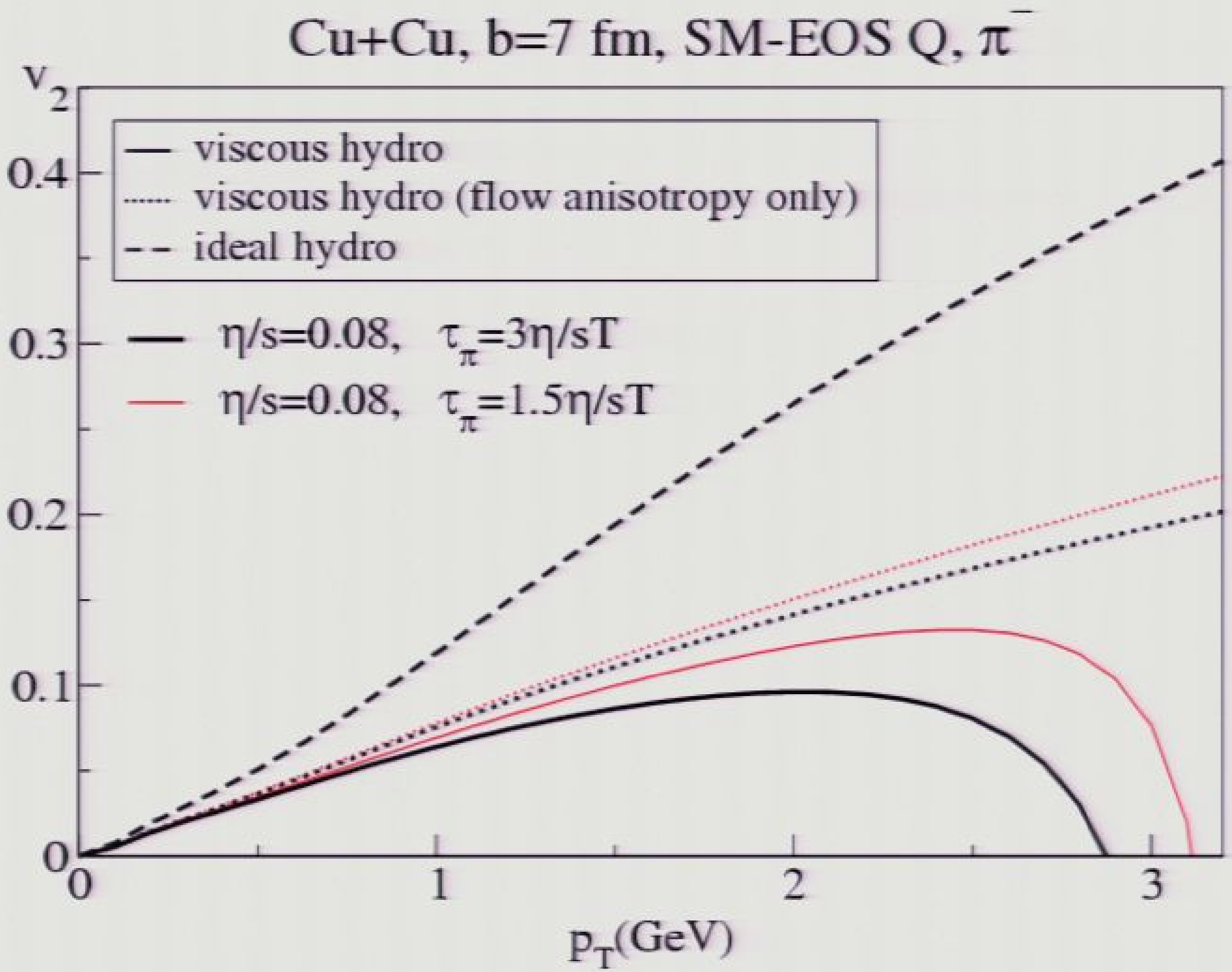
Perturbative QCD:

$$\eta/s\,(T_{\mathrm{RHIC}}) \approx 1.6 \sim 1.8$$

Chernai, Kapusta, McLerran, nucl-th/0604032

SYM: $\eta/s \approx 0.09 \sim 0.28$

# Elliptic flow from relativistic hydro simulations
## (Israel-Stewart formalism)






H.Song and U.Heinz, 0712.3715 [nucl-th]

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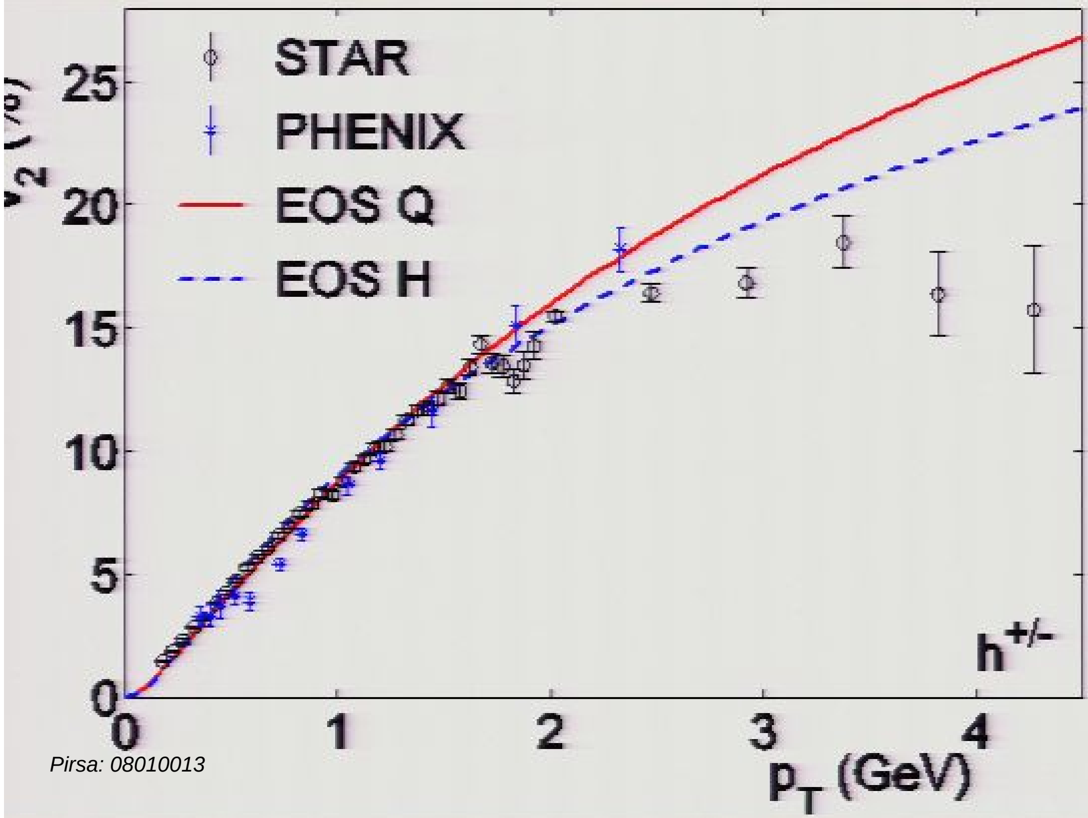


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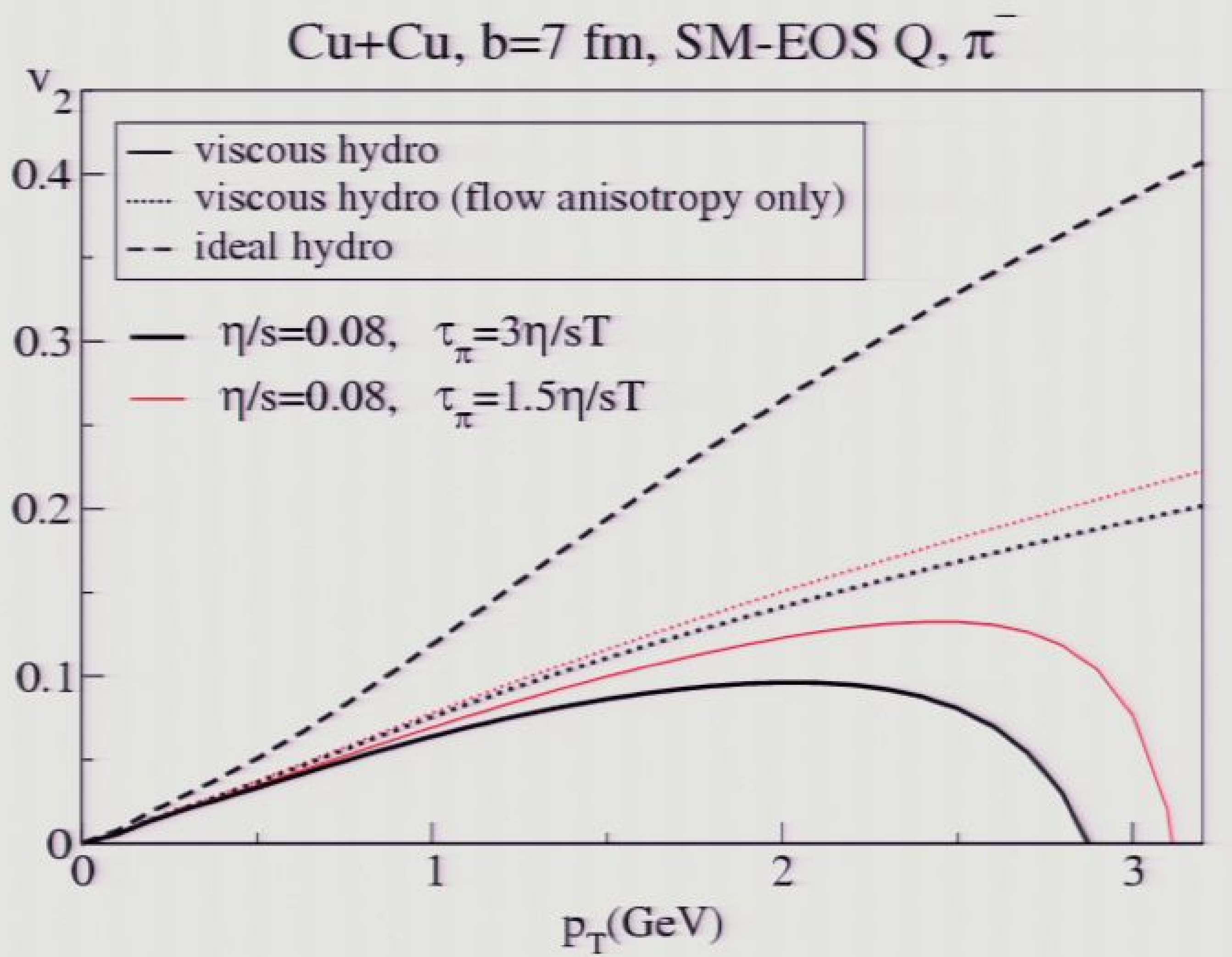


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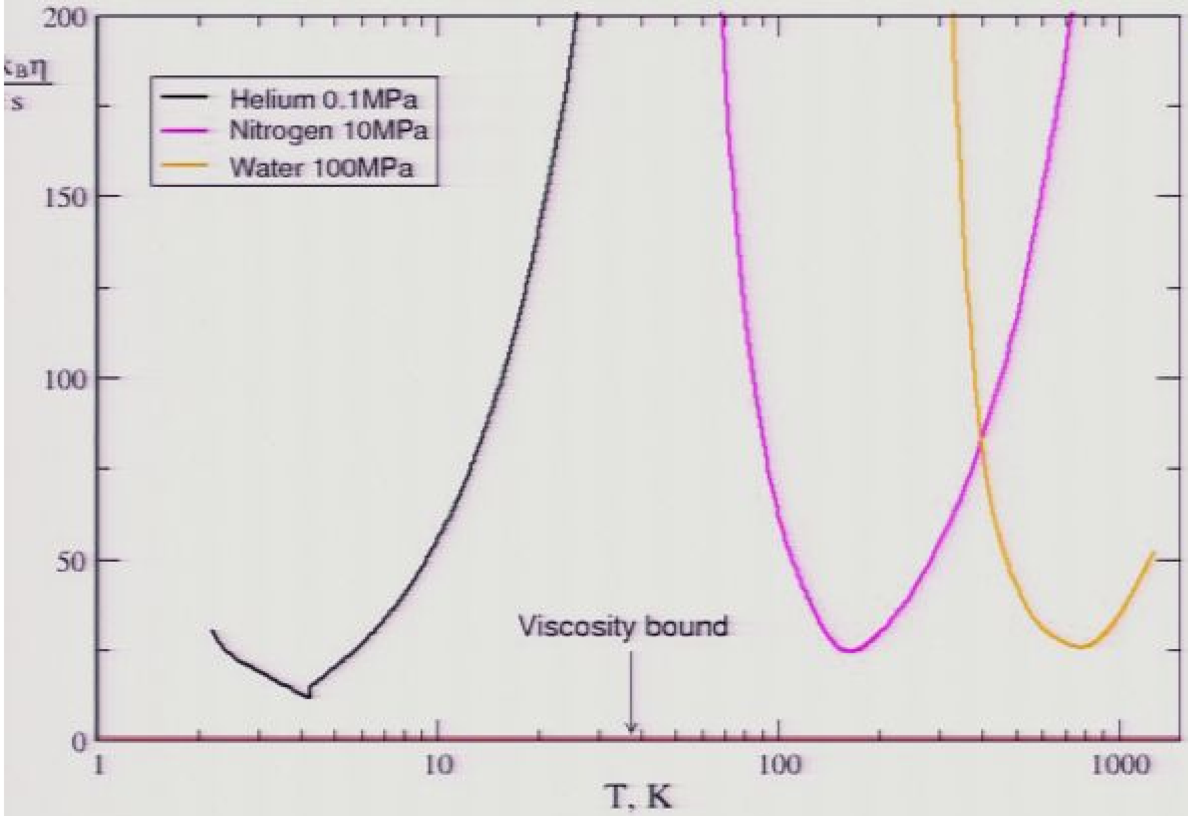
# A viscosity bound conjecture

$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi k_B} \approx 6.08 \cdot 10^{-13} \, K \cdot s$$




Minimum of $\frac{\eta}{s}$ in units of $\frac{\hbar}{4\pi k_B}$

Xe 84

Kr 57

$CO_2$ 32

$H_2O$ 25

$C_2H_5OH$ 22

Ne 17

He 8.8





P.Kovtun, D.Son, A.S., hep-th/0309213, hep-th/0405231

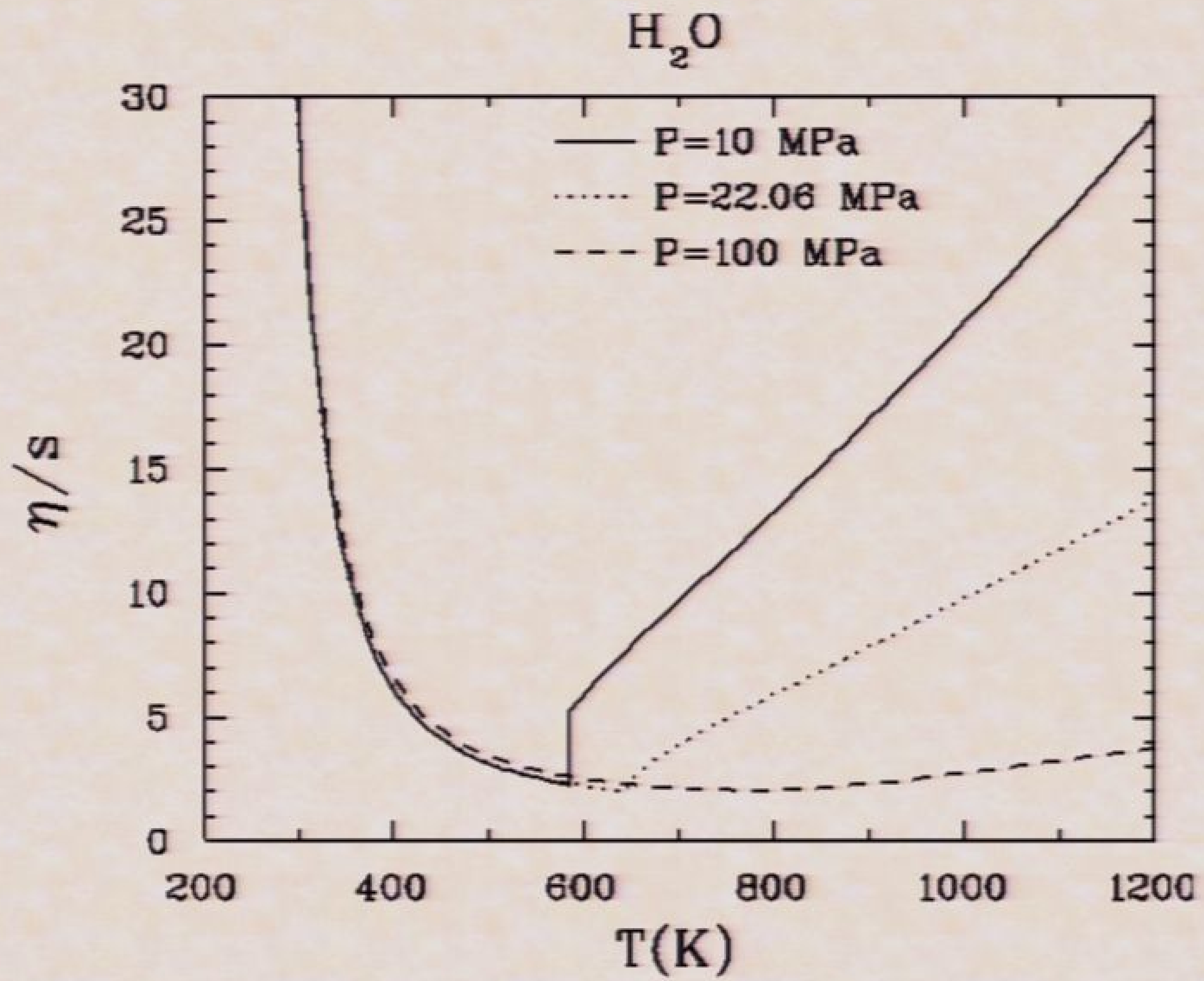
## Can we test

$$\eta/s \geq 1/4\pi$$

## experimentally?

A characteristic feature of systems saturating the bound:
strong interactions

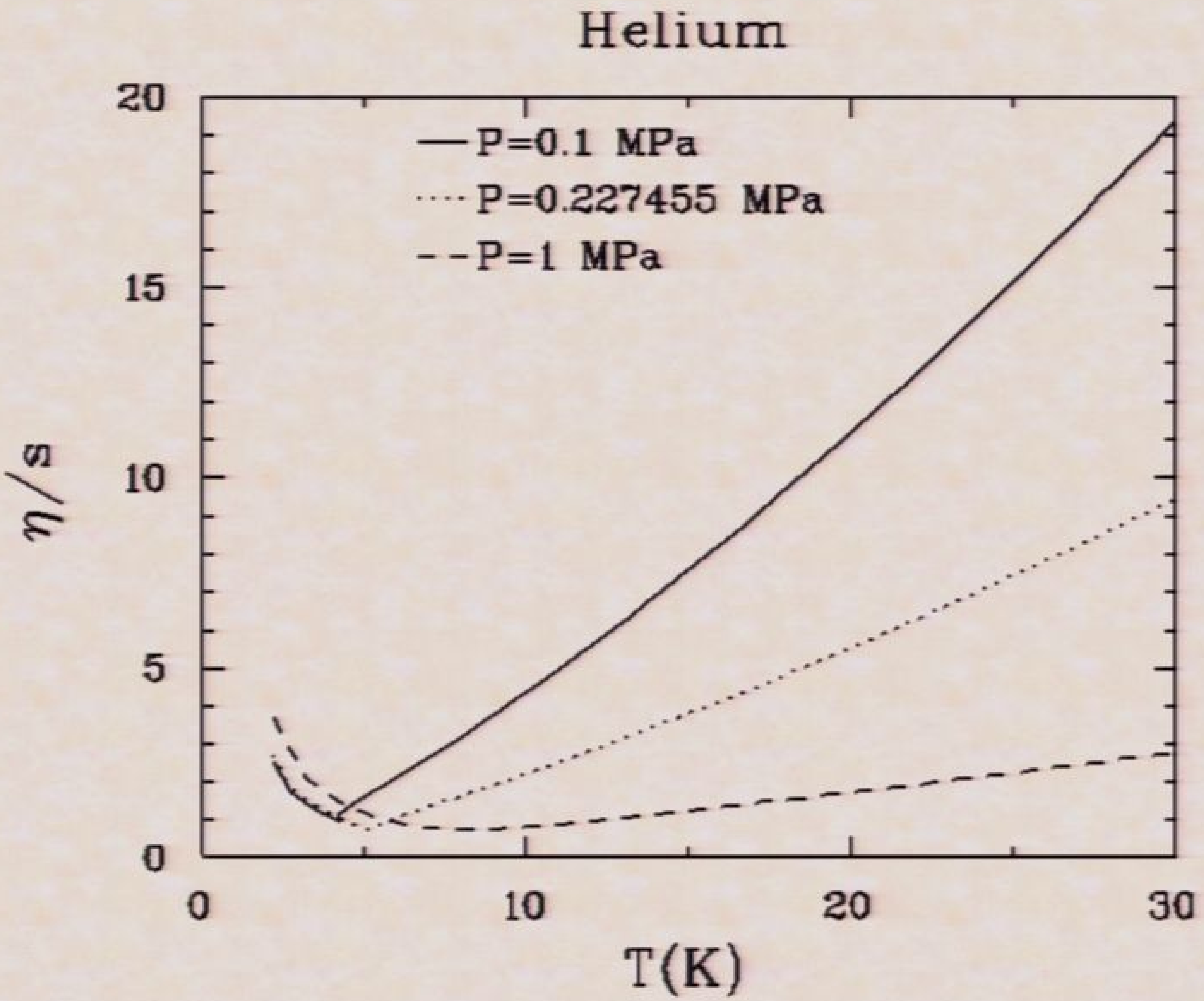
- Heavy ion collisions - experiments at RHIC
- (Indirect) lattice QCD simulations
- Trapped atoms – strongly interacting Fermi systems




$(\eta/s)_{\text{min}} \sim 25$ in units of $\frac{\hbar}{4\pi k_B}$



Chernai, Kapusta, McLerran, nucl-th/0604032

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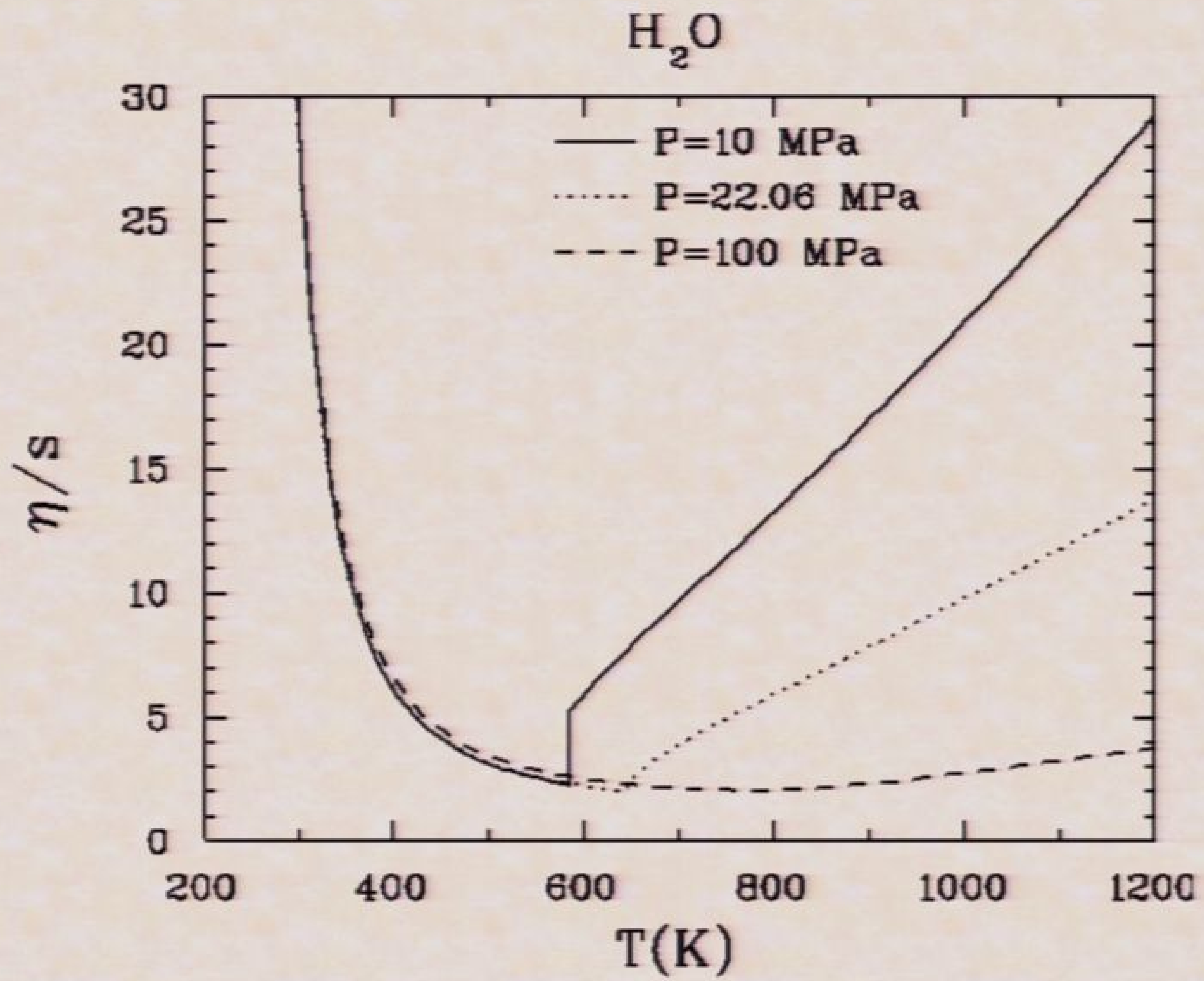
Chernai, Kapusta, McLerran, nucl-th/0604032

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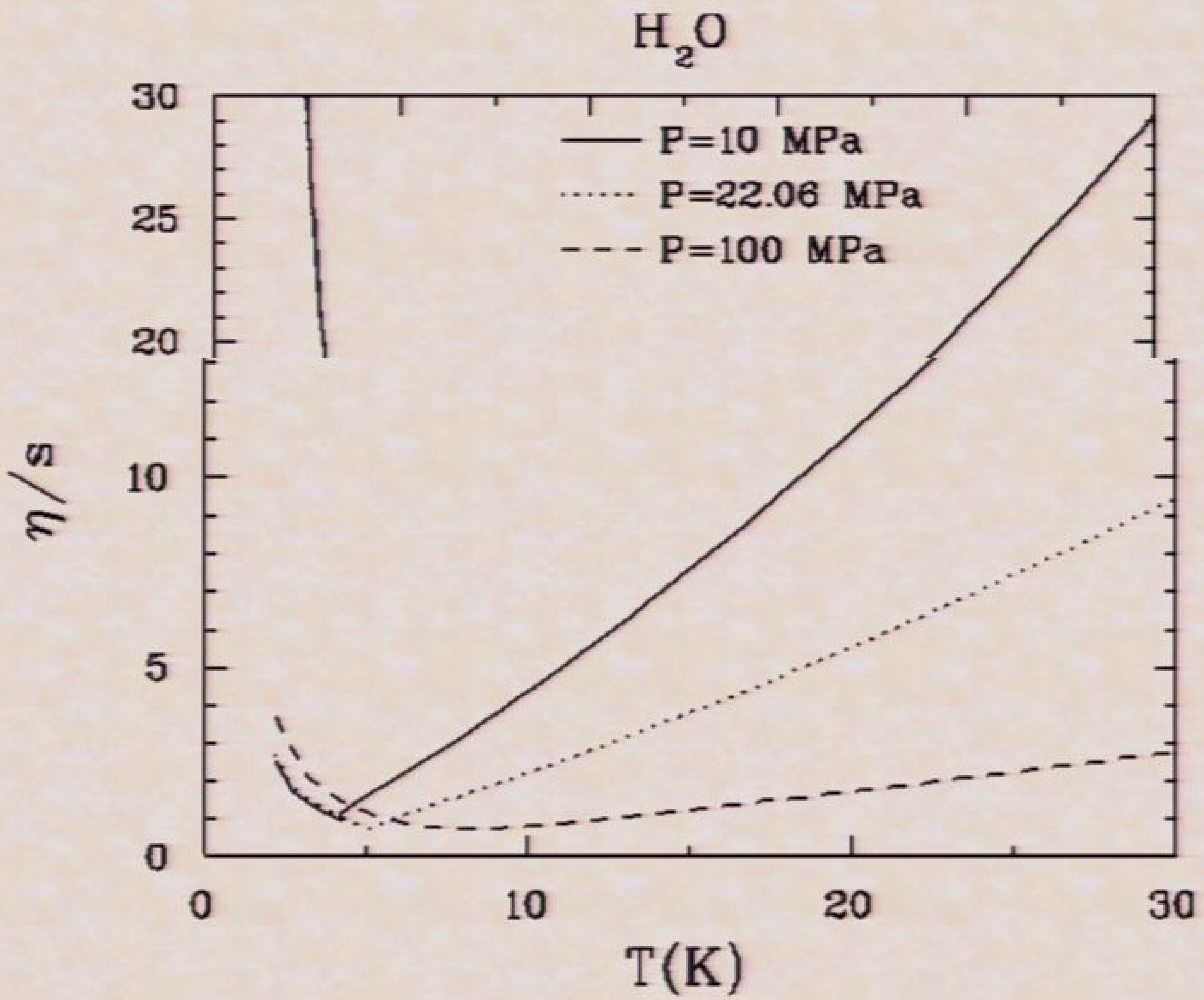


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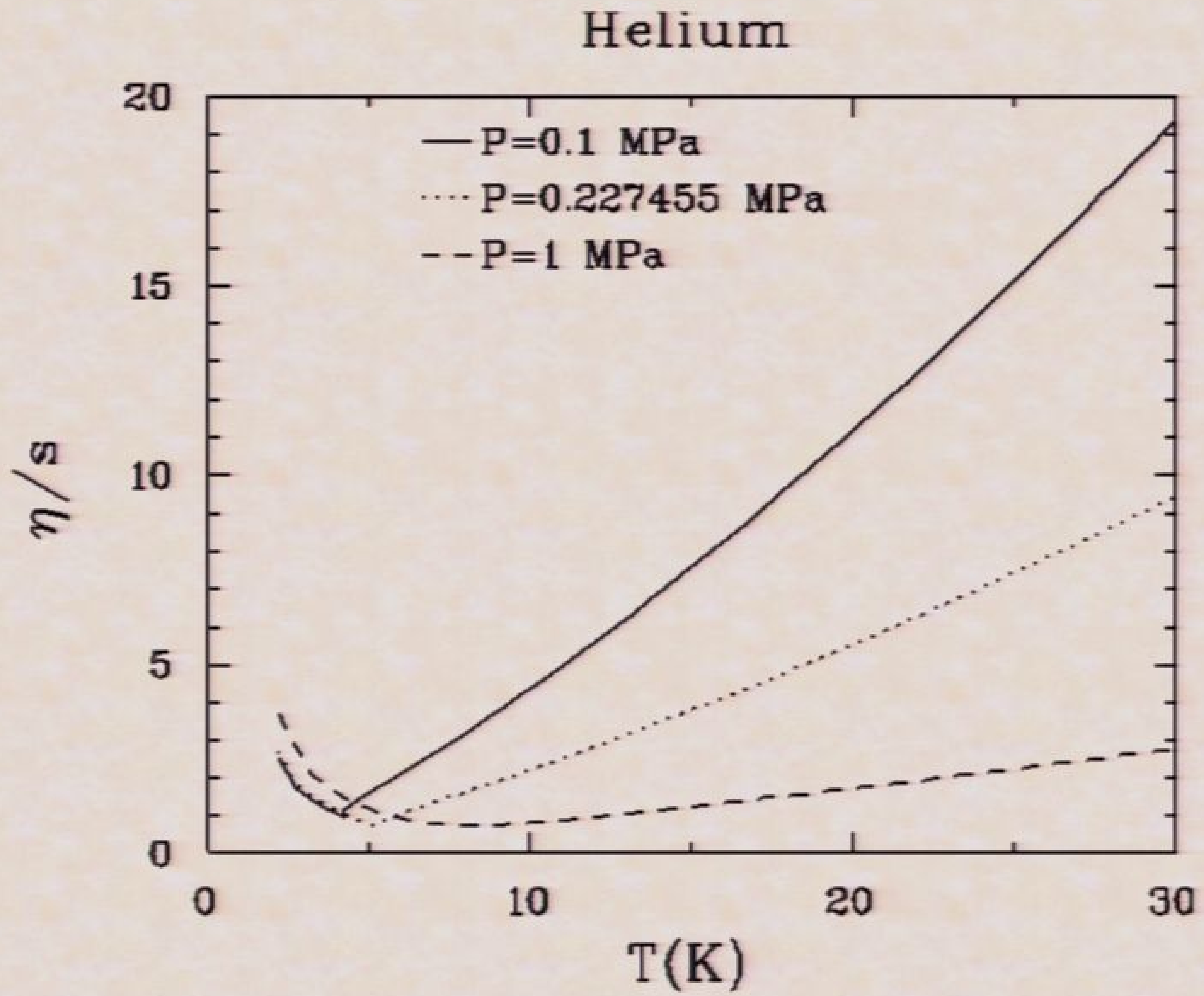


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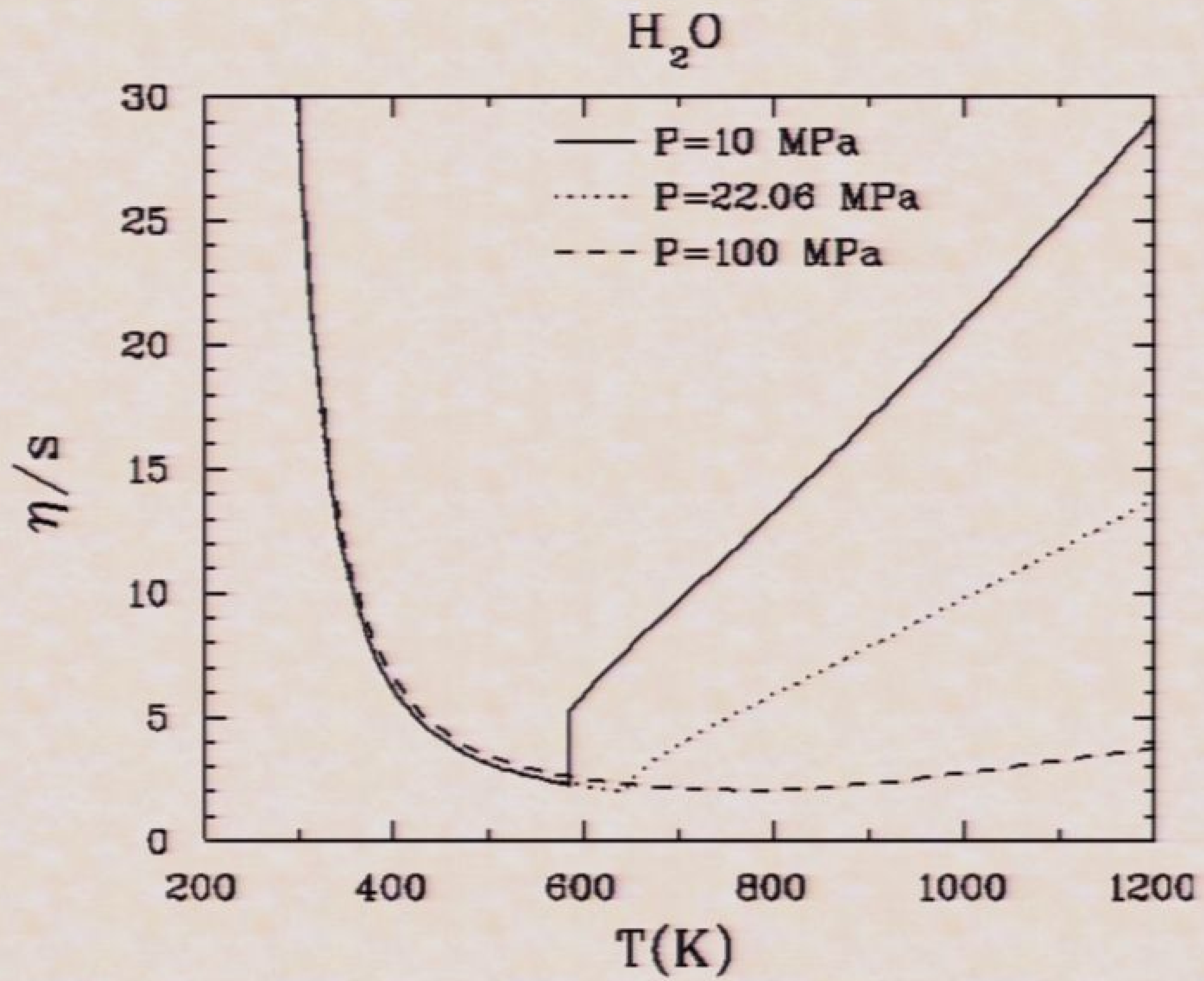


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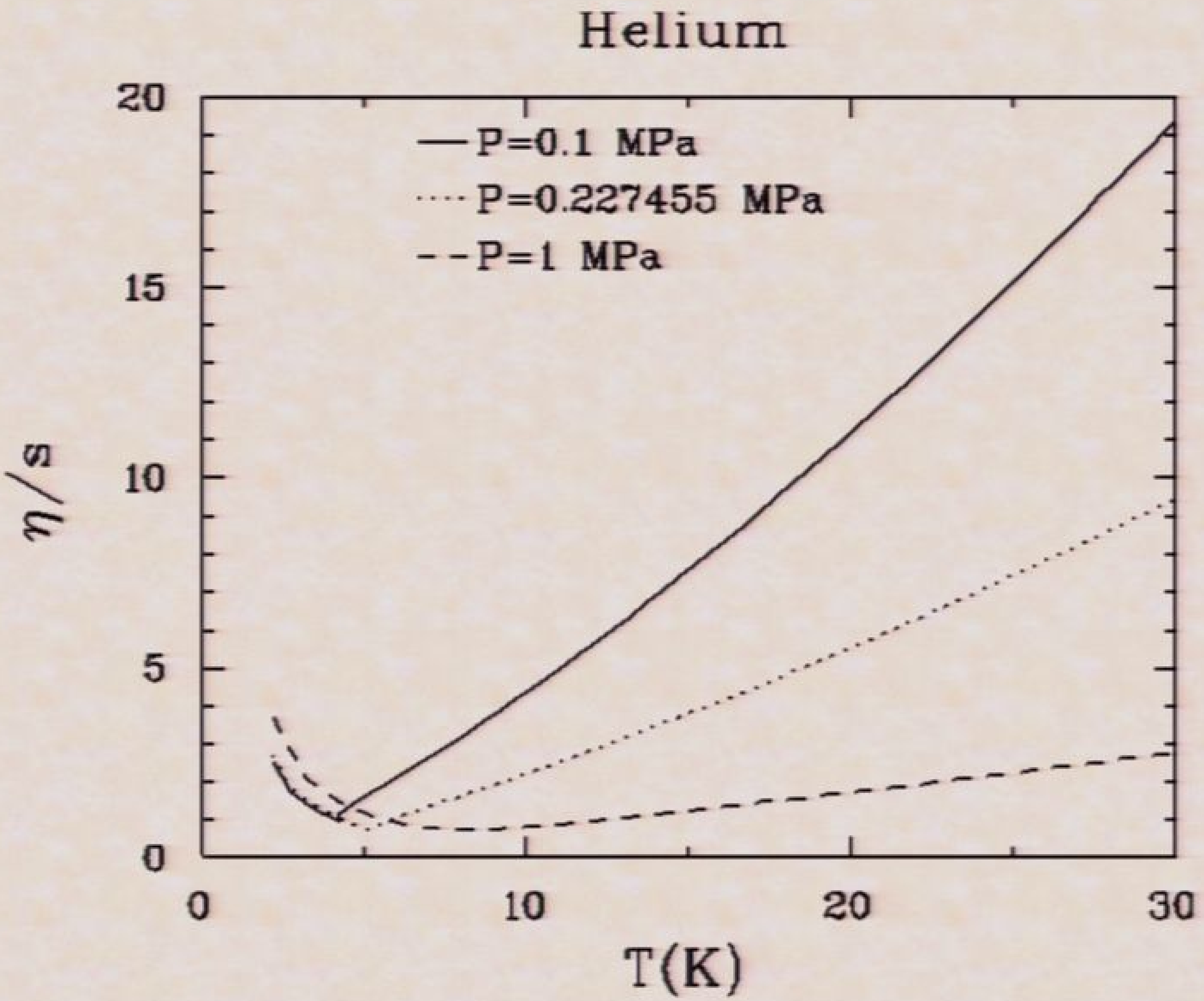


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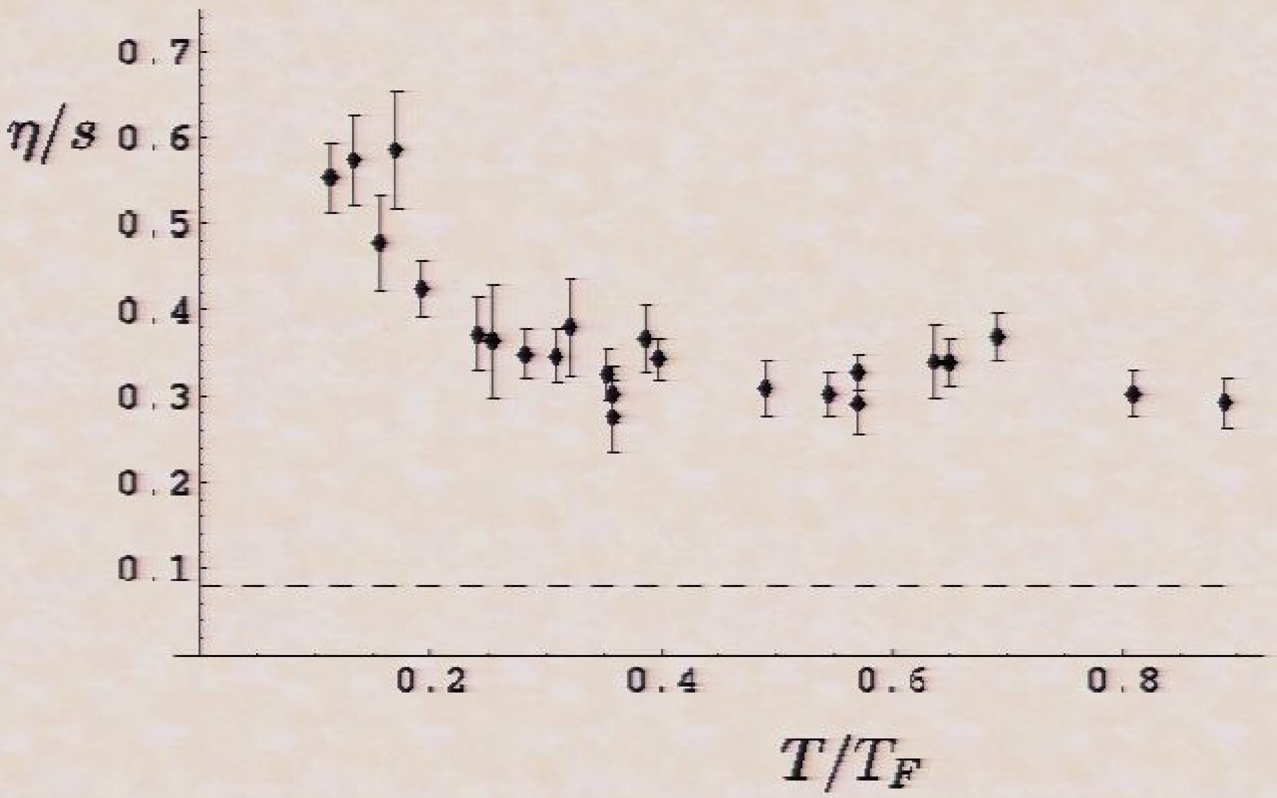
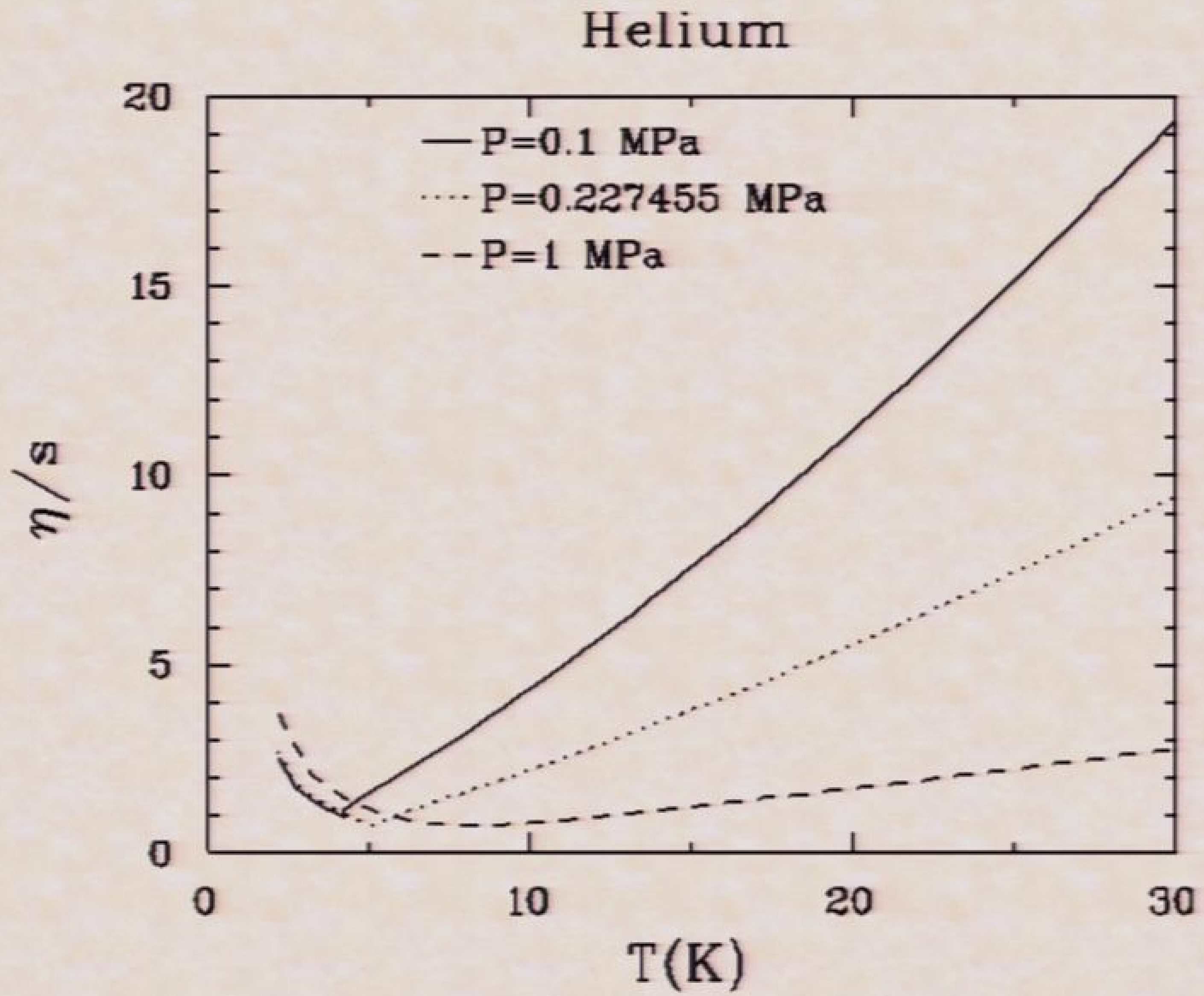
# Viscosity-entropy ratio of a trapped Fermi gas




$\eta/s \sim 4.2$ in units of $\frac{\hbar}{4\pi k_B}$  T.Schafer, cond-mat/0701251

(based on experimental results by Duke U. group, J.E.Thomas et al., 2005-06)

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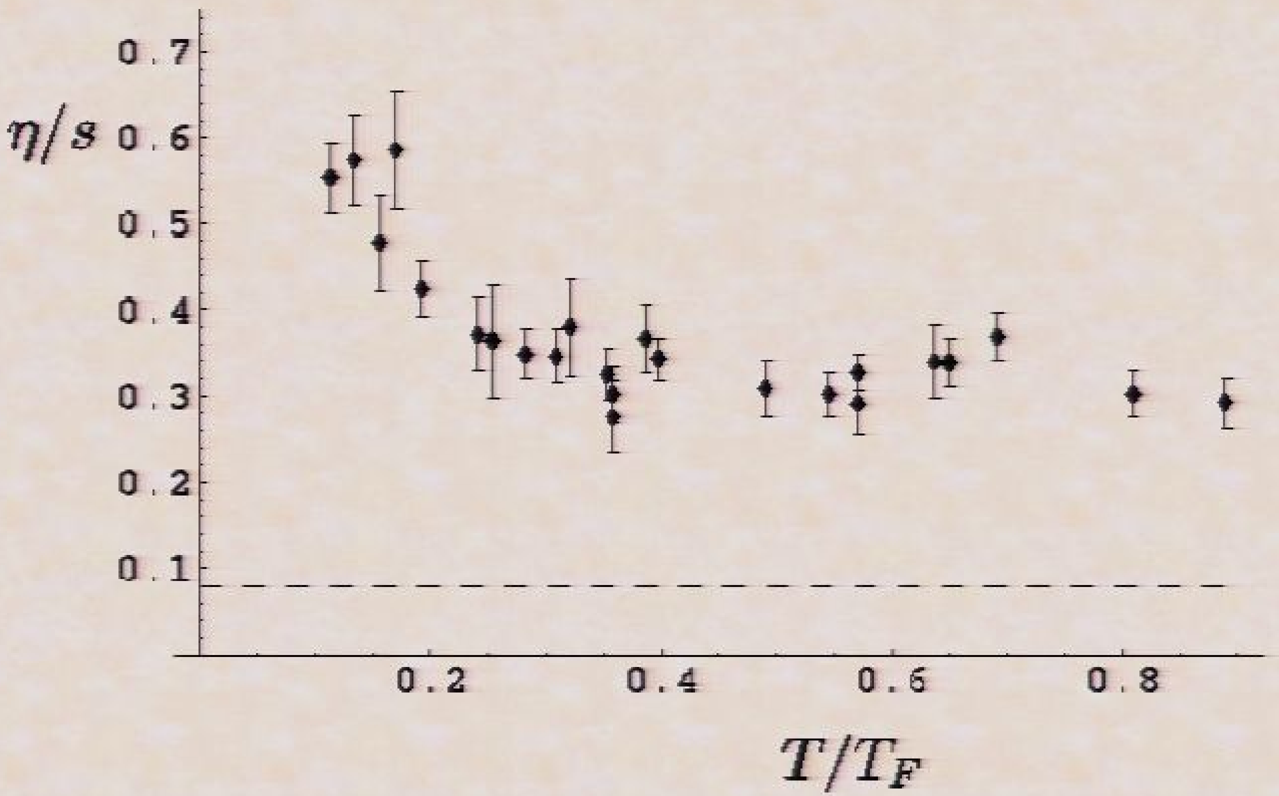


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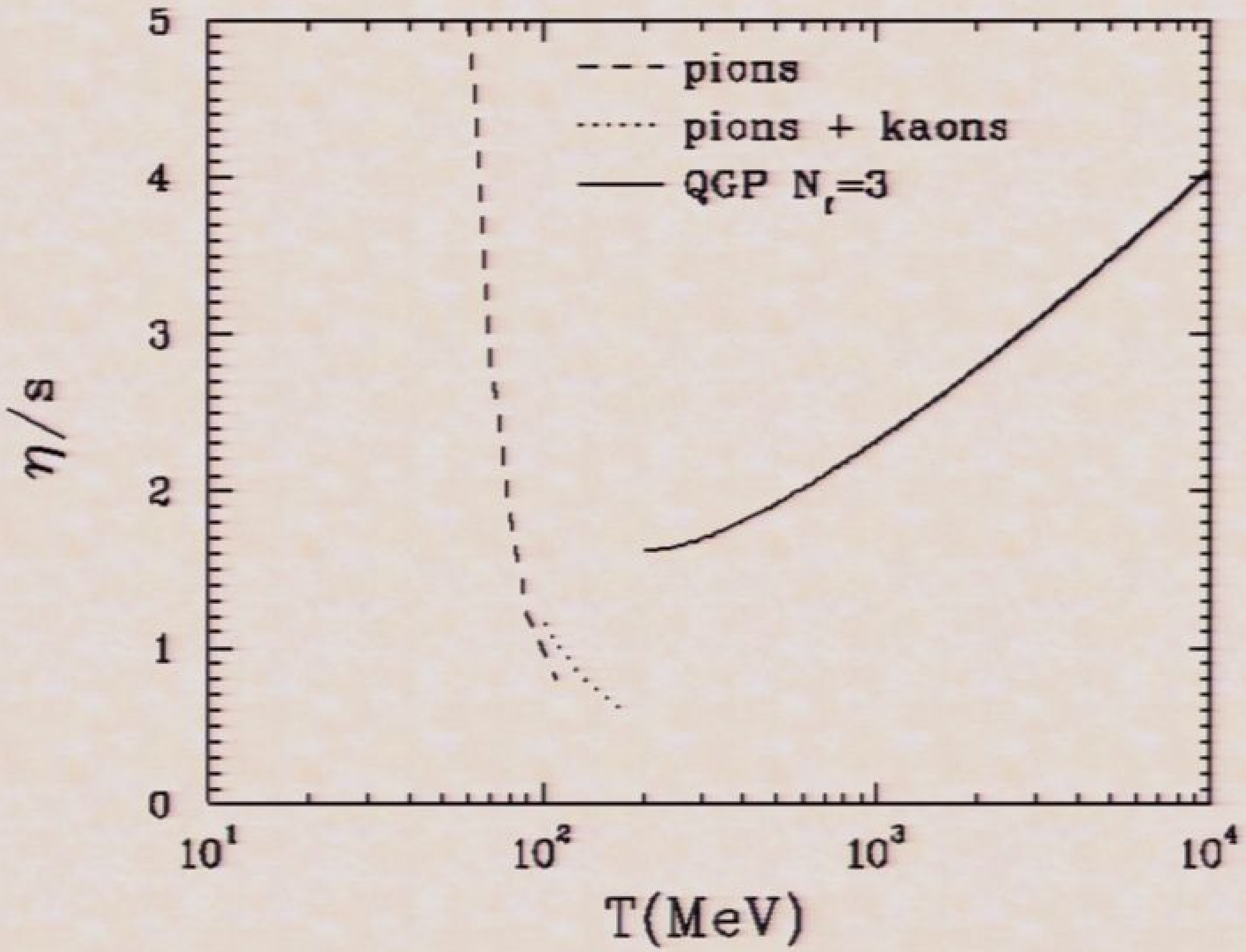
# QCD




Chernai, Kapusta, McLerran, nucl-th/0604032

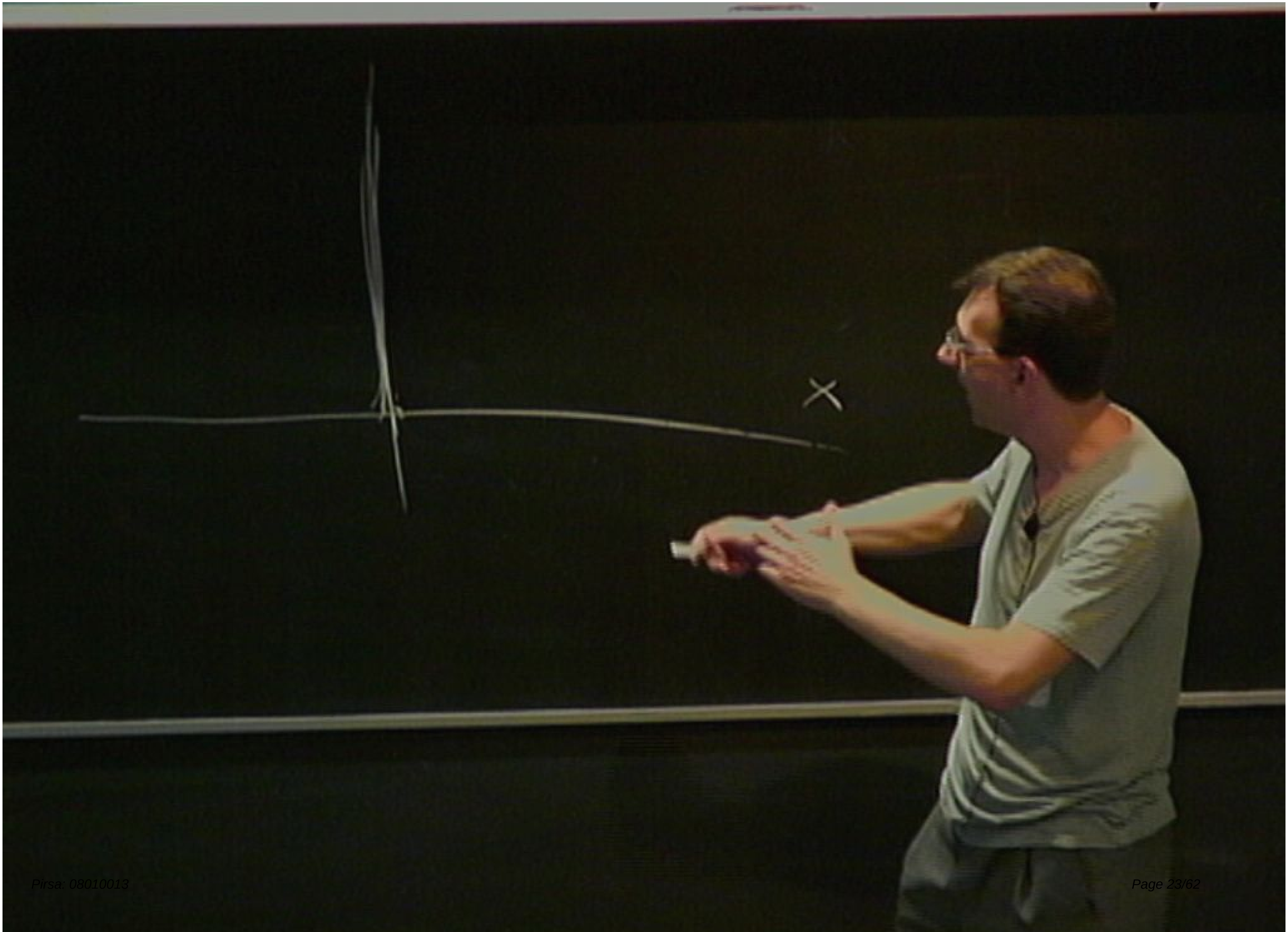
Hydrodynamics is an effective theory, valid for sufficiently small momenta

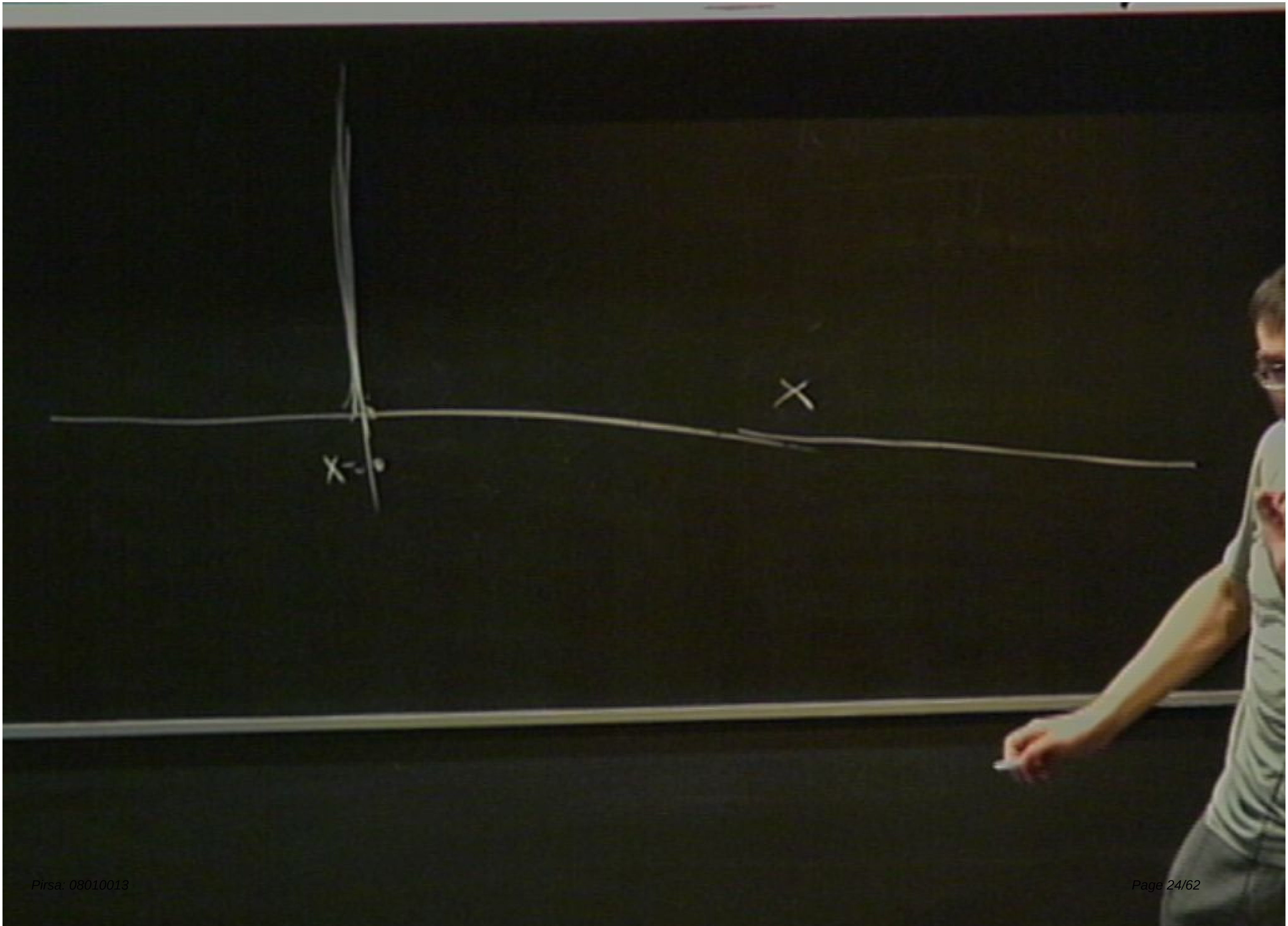
$$k\, l_{mfp} \ll 1$$

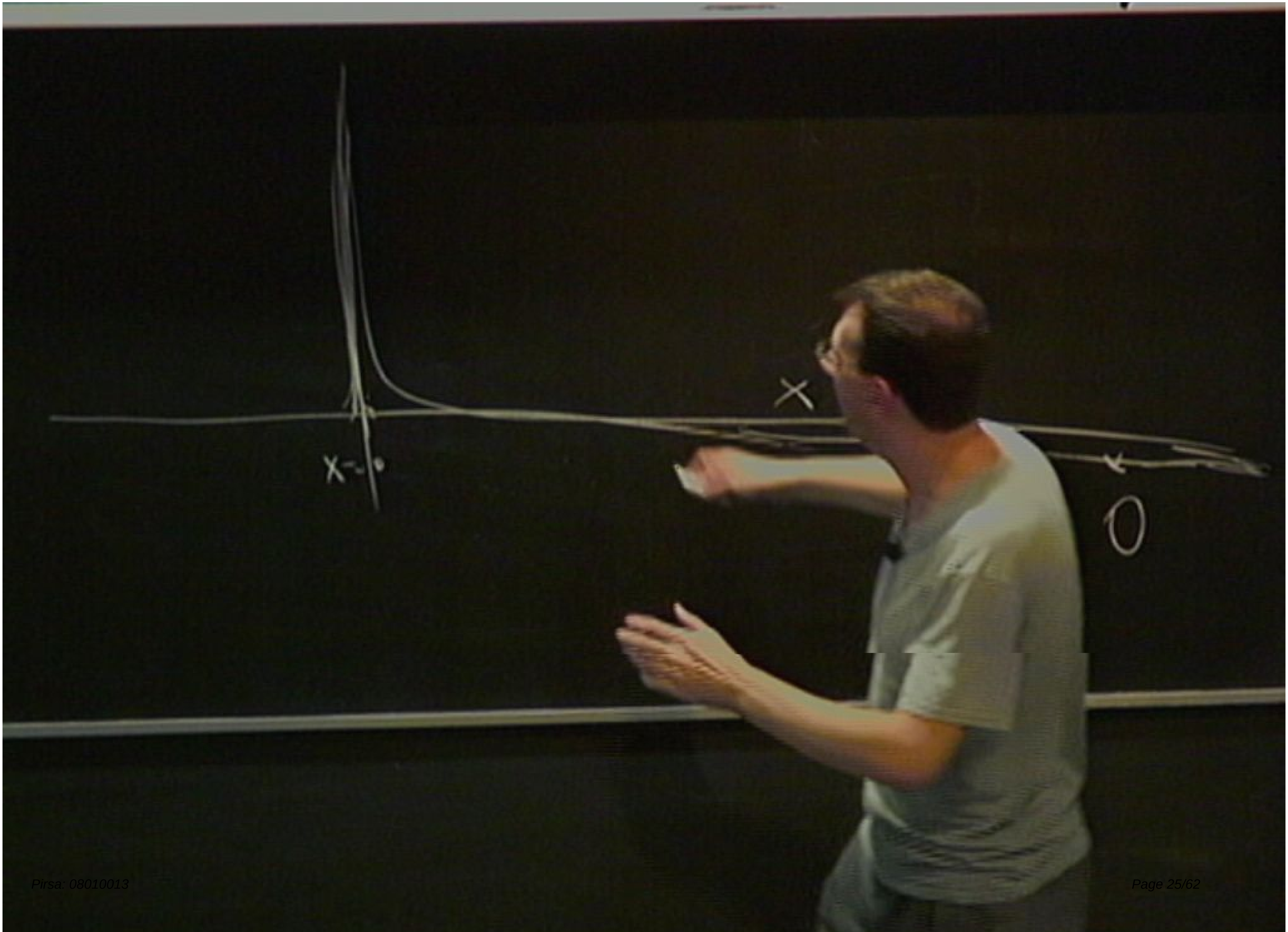
rst-order hydro eqs are parabolic. They imply instant propagation of signals.

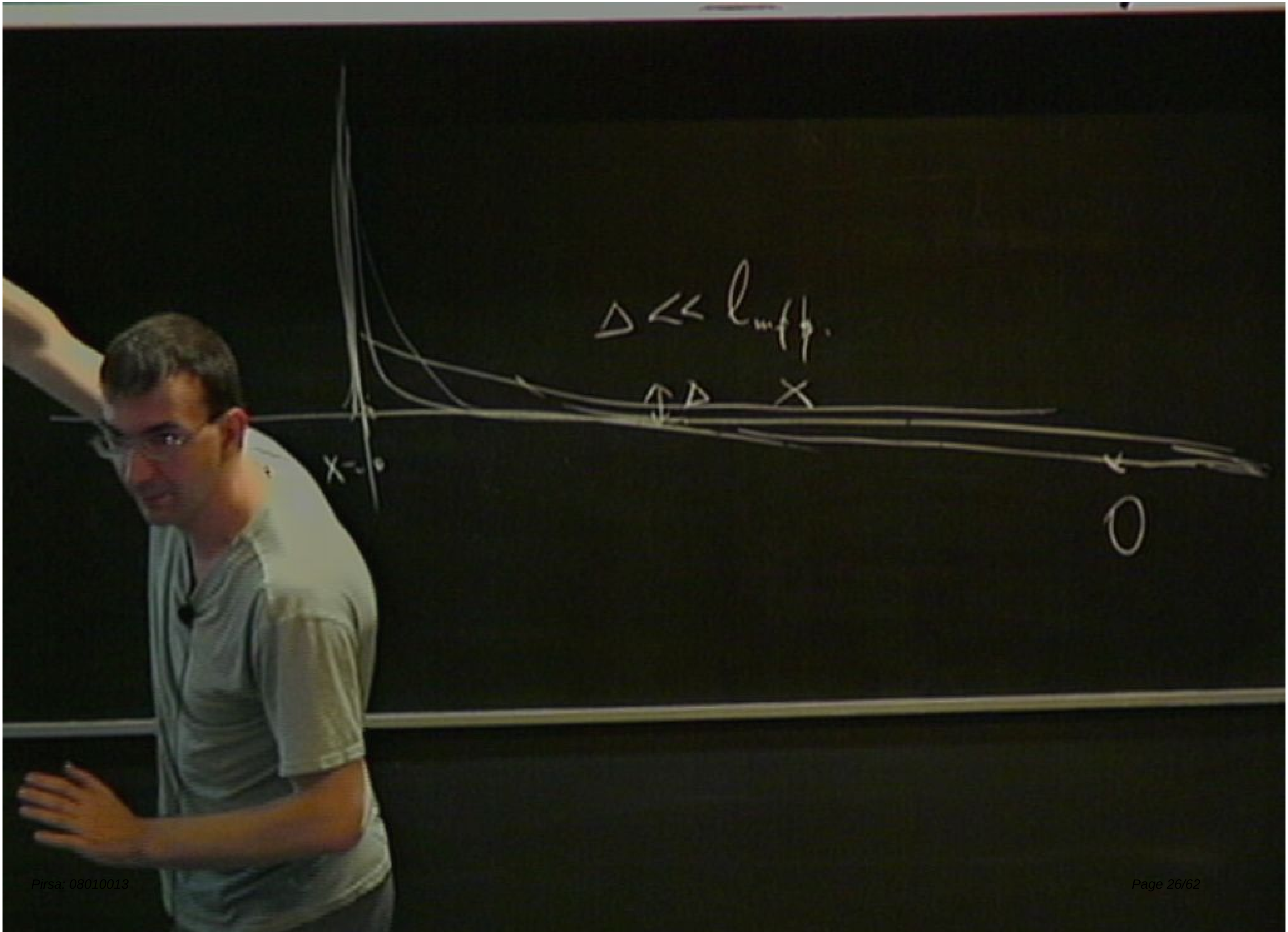
This is incompatible with special relativity.

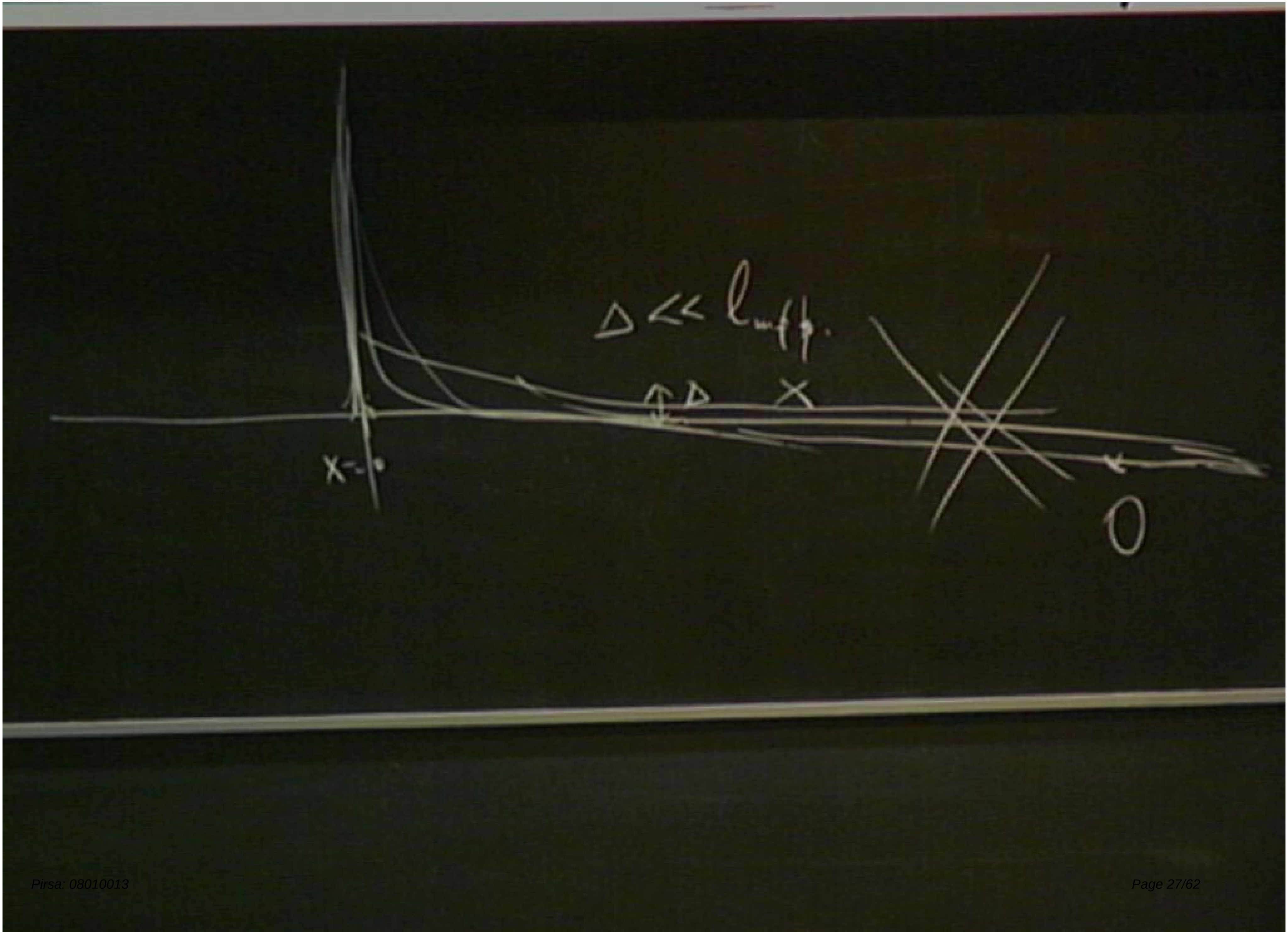
owever, the disaster strikes only in the UV, where hydro description is invalid








Δ << l_mfp.
Δ
X
X=0
O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Still, acausality is unpleasant, especially for numerical simulations, where it leads to instabilities

would be convenient to have a UV completion (at strong and weak coupling)

Israel-Stewart theory is such a completion built with e.g. Boltzmann eq

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Hydrodynamics: fundamental d.o.f. = densities of conserved charges

Need to add constitutive relations!

## Example: charge diffusion

Conservation law $$\partial_t j^0 + \partial_i j^i = 0$$

Constitutive relation

[Fick's law (1855)] $$j_i = -D\,\partial_i j^0 + O[(\nabla j^0)^2, \nabla^2 j^0]$$

Diffusion equation $$\partial_t j^0 = D \nabla^2 j^0$$

Dispersion relation $$\omega = -i\,D\,q^2 + \cdots$$



Expansion parameters: $\omega \ll T, \quad q \ll T$

# Derivative expansion in hydrodynamics: first order

Hydrodynamic d.o.f. = densities of conserved charges

$\partial_\mu T^{\mu\nu} = 0$ (4 equations)

$T^{00}\,,\ T^{0i}$ (4 d.o.f.) or $\varepsilon\,,\ u^\mu$

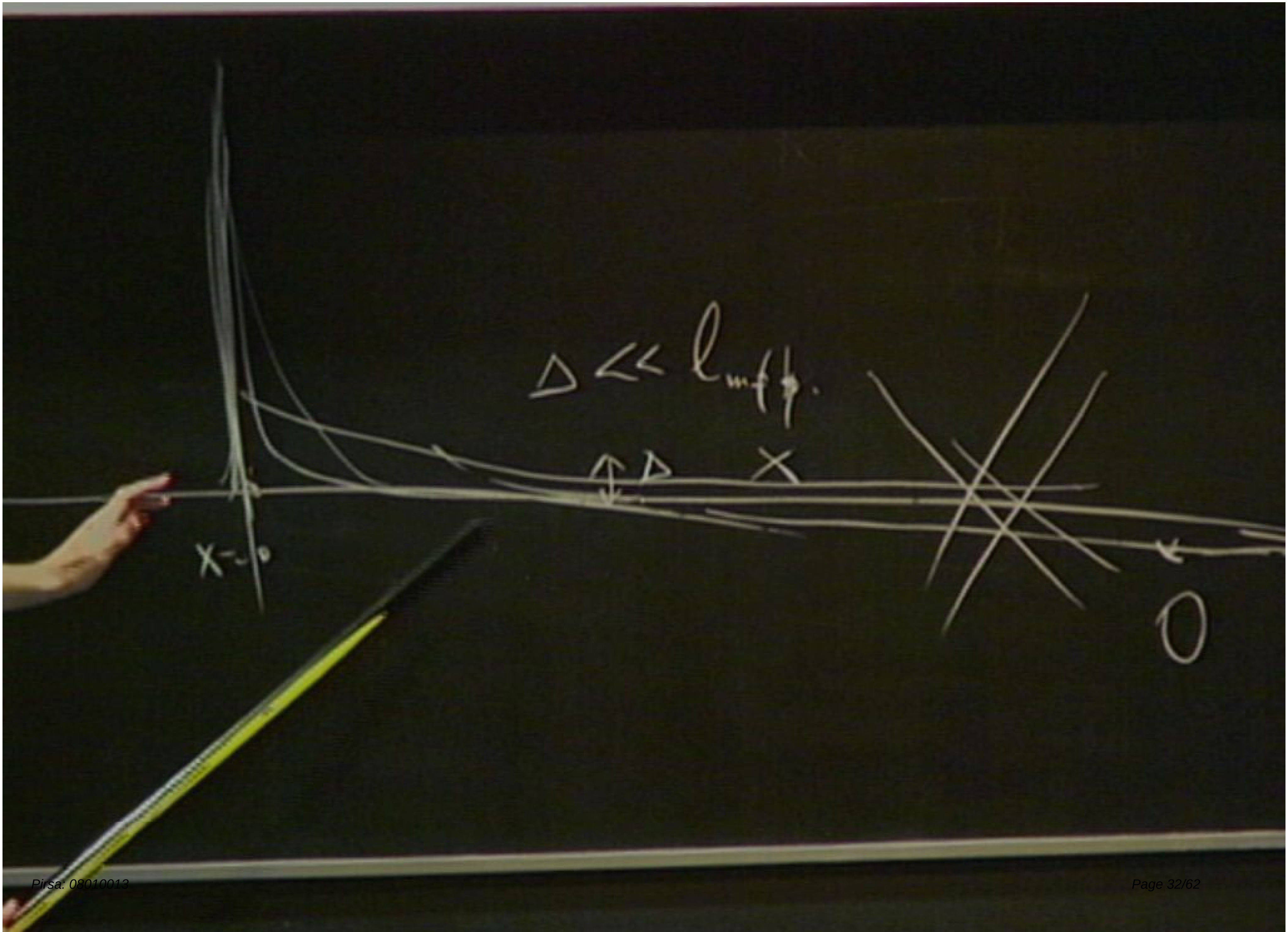
$$T^{\mu\nu} = \varepsilon\, u^\mu\, u^\nu + P(\varepsilon)\, \Delta^{\mu\nu} + \Pi^{\mu\nu}$$

$$\Pi^{\mu\nu} = -\eta(\varepsilon)\, \sigma^{\mu\nu} - \zeta(\varepsilon)\, \Delta^{\mu\nu}\, (\nabla \cdot u) + \cdots$$

$$\Delta^{\mu\nu} = g^{\mu\nu} + u^\mu\, u^\nu \qquad \sigma^{\mu\nu} = 2\nabla^{<\mu}u^{\nu>}$$

$$A^{<\mu\nu>} = \frac{1}{2}\Delta^{\mu\alpha}\Delta^{\nu\beta}\left(A_{\alpha\beta} + A_{\beta\alpha}\right) - \frac{1}{d-1}\Delta^{\mu\nu}\Delta^{\alpha\beta}A_{\alpha\beta}$$

$$u^\mu u_\mu = -1$$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# First-order conformal hydrodynamics

Weyl transformations: $g_{\mu\nu} \to e^{-2\omega} g_{\mu\nu}$

$$T^{\mu\nu} \to e^{(d+2)\omega} T^{\mu\nu}$$

$$T^{\mu}_{\mu} = 0$$

In first-order hydro this implies: $\varepsilon = (d-1)P\,,\ \zeta = 0$

$$u^{\mu} \to e^{\omega} u^{\mu} \qquad T \to e^{\omega} T \qquad \sigma^{\mu\nu} \to e^{3\omega} \sigma^{\mu\nu}$$

Thus, in first-order hydro:

$$T^{\mu\nu}_{\text{conformal}} = \varepsilon\, u^{\mu} u^{\nu} + \frac{\varepsilon}{d-1} \Delta^{\mu\nu} - \eta\, \sigma^{\mu\nu}$$

# Second-order conformal hydrodynamics

$$T^{\mu\nu}_{\text{conformal}} = \varepsilon\, u^\mu\, u^\nu + \frac{\varepsilon}{d-1}\,\Delta^{\mu\nu} + \Pi^{\mu\nu}$$

$$\Pi^{\mu\nu} = -\eta\sigma^{\mu\nu} - \tau_\Pi \left[ D\Pi^{<\mu\nu>} + \frac{d}{d-1}\Pi^{\mu\nu}\left(\nabla\cdot u\right)\right]$$

$$+\kappa\left[R^{<\mu\nu>} - (d-2)u_\alpha R^{\alpha<\mu\nu>\beta}u_\beta\right]$$

$$+\frac{\lambda_1}{\eta^2}\Pi^{<\mu}_{\lambda}\Pi^{\nu>\lambda} - \frac{\lambda_2}{\eta}\Pi^{<\mu}_{\lambda}\Omega^{\nu>\lambda} + \lambda_3\,\Omega^{<\mu}_{\lambda}\Omega^{\nu>\lambda}$$

$$D \equiv u^\mu \nabla_\mu \qquad\qquad \Omega^{\mu\nu} = \frac{1}{2}\Delta^{\mu\alpha}\Delta^{\nu\beta}\left(\nabla_\alpha u_\beta - \nabla_\beta u_\alpha\right)$$

# Second-order Israel-Stewart conformal hydrodynamics

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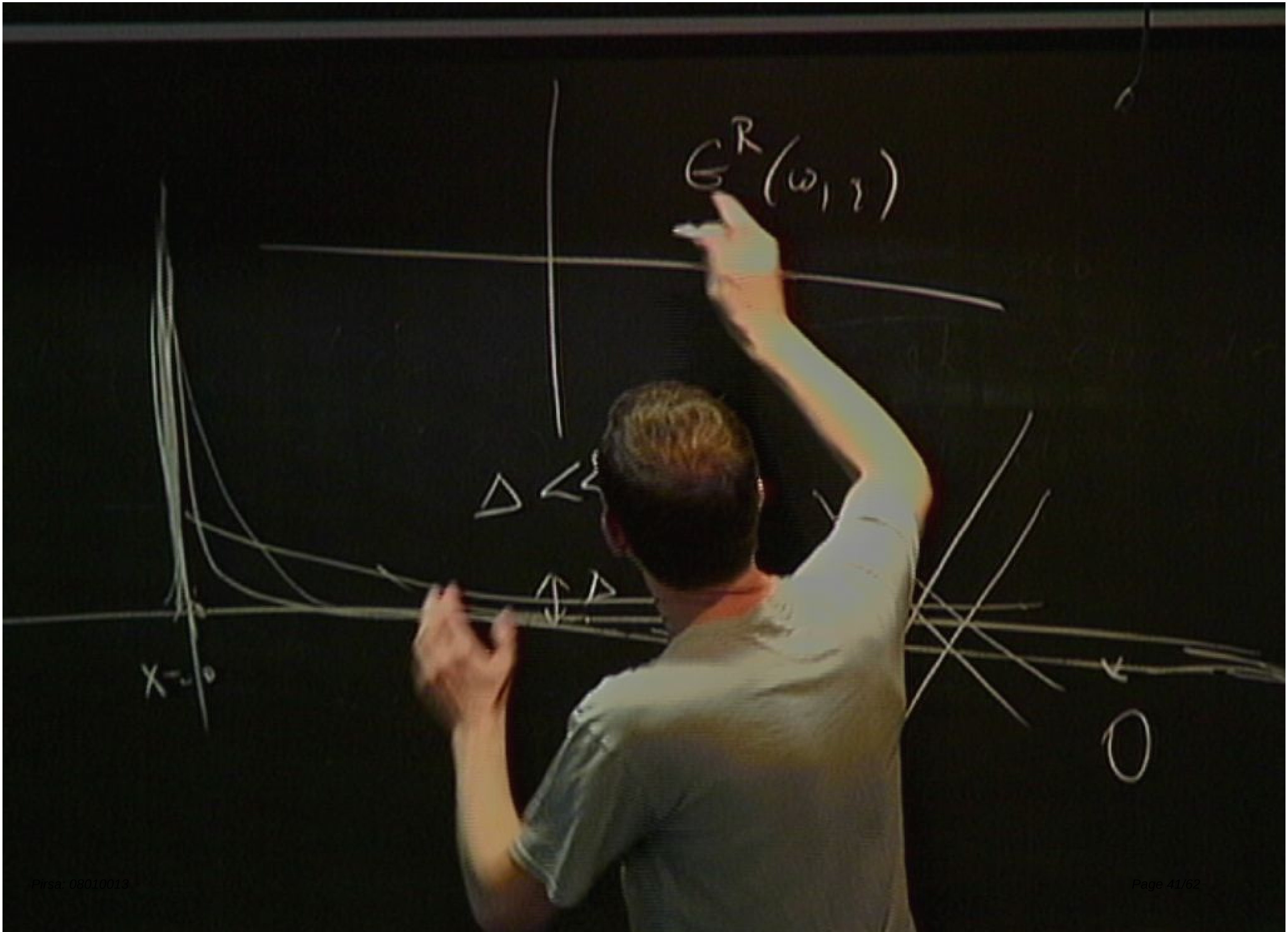
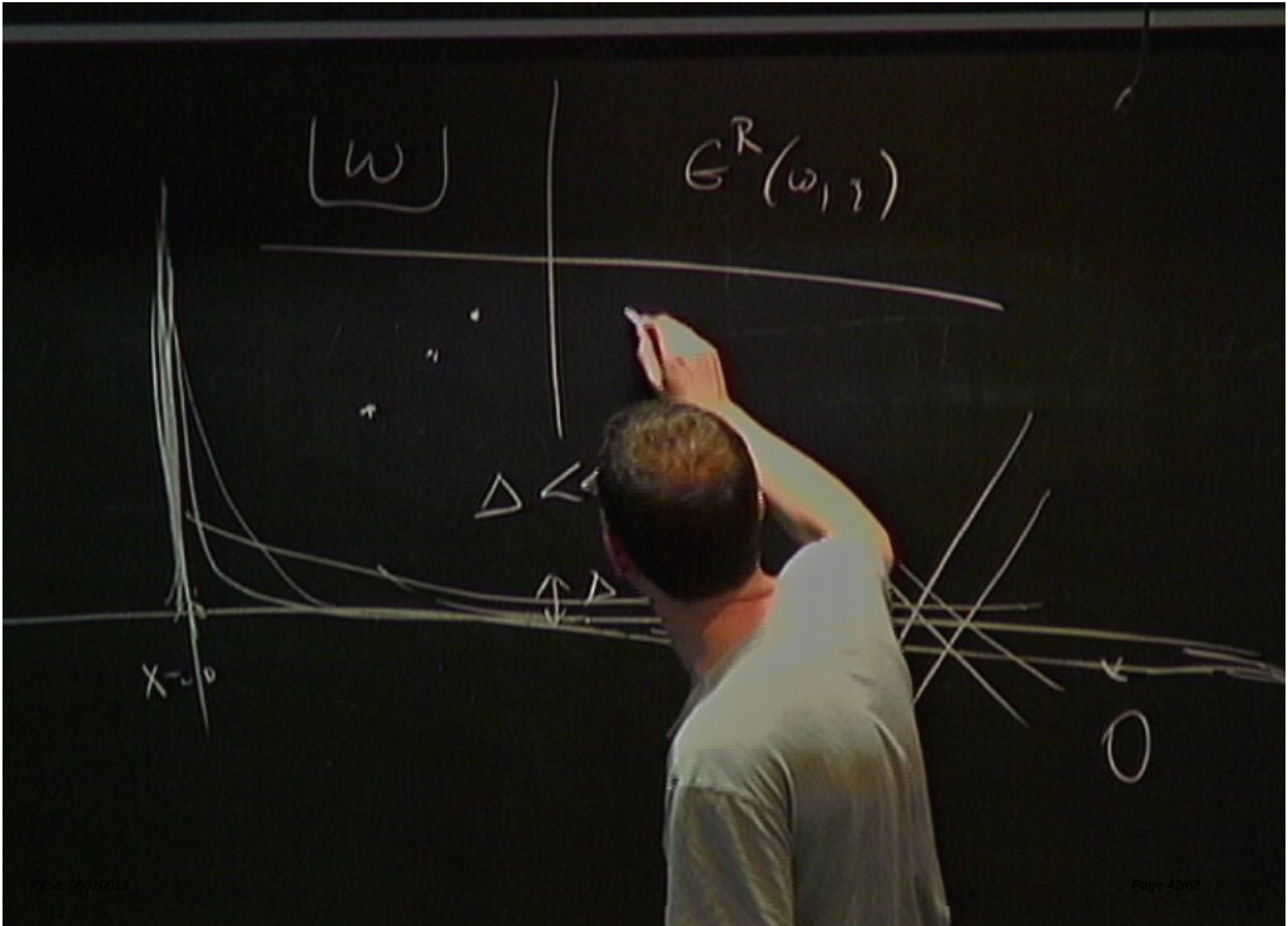
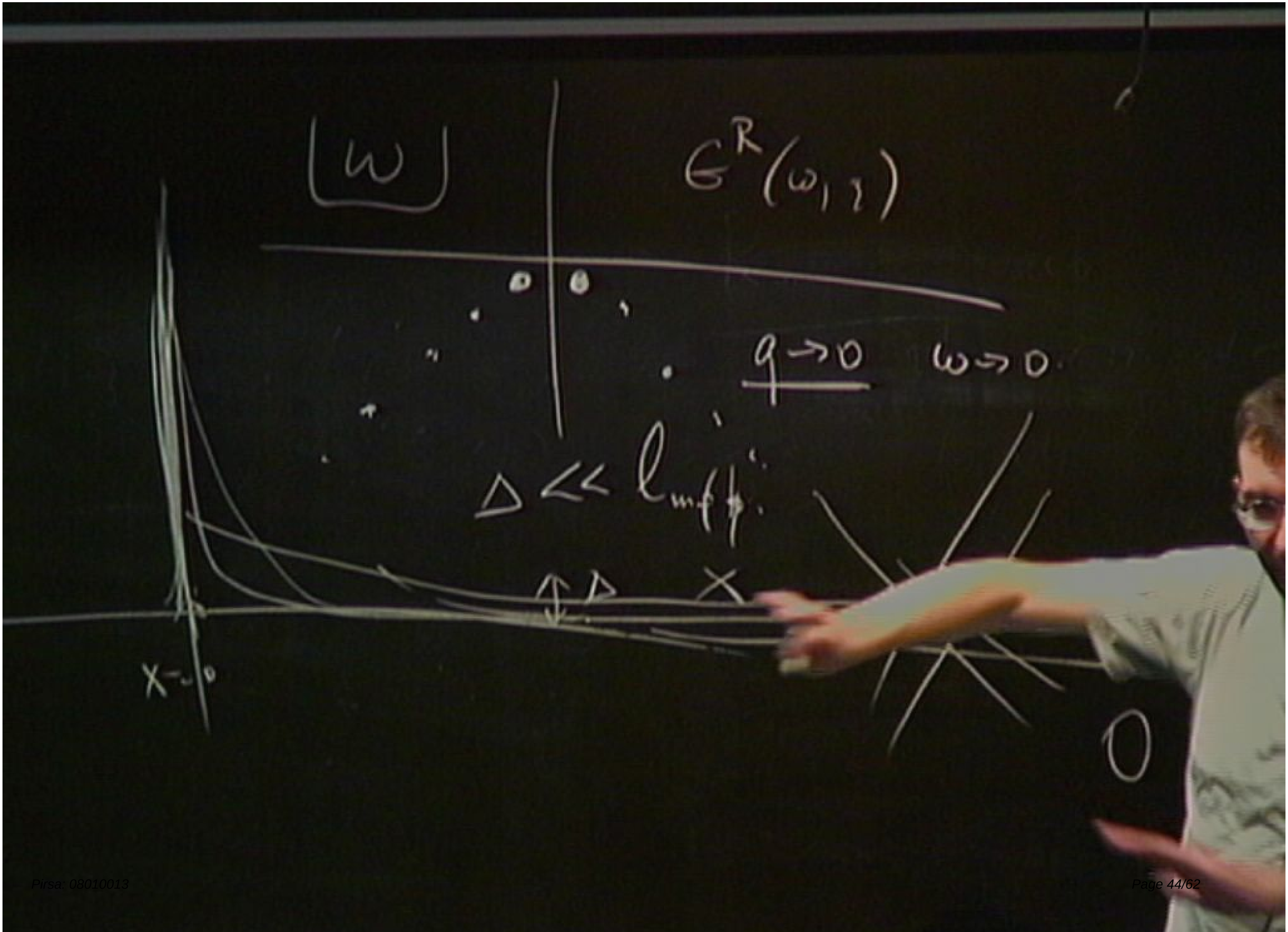
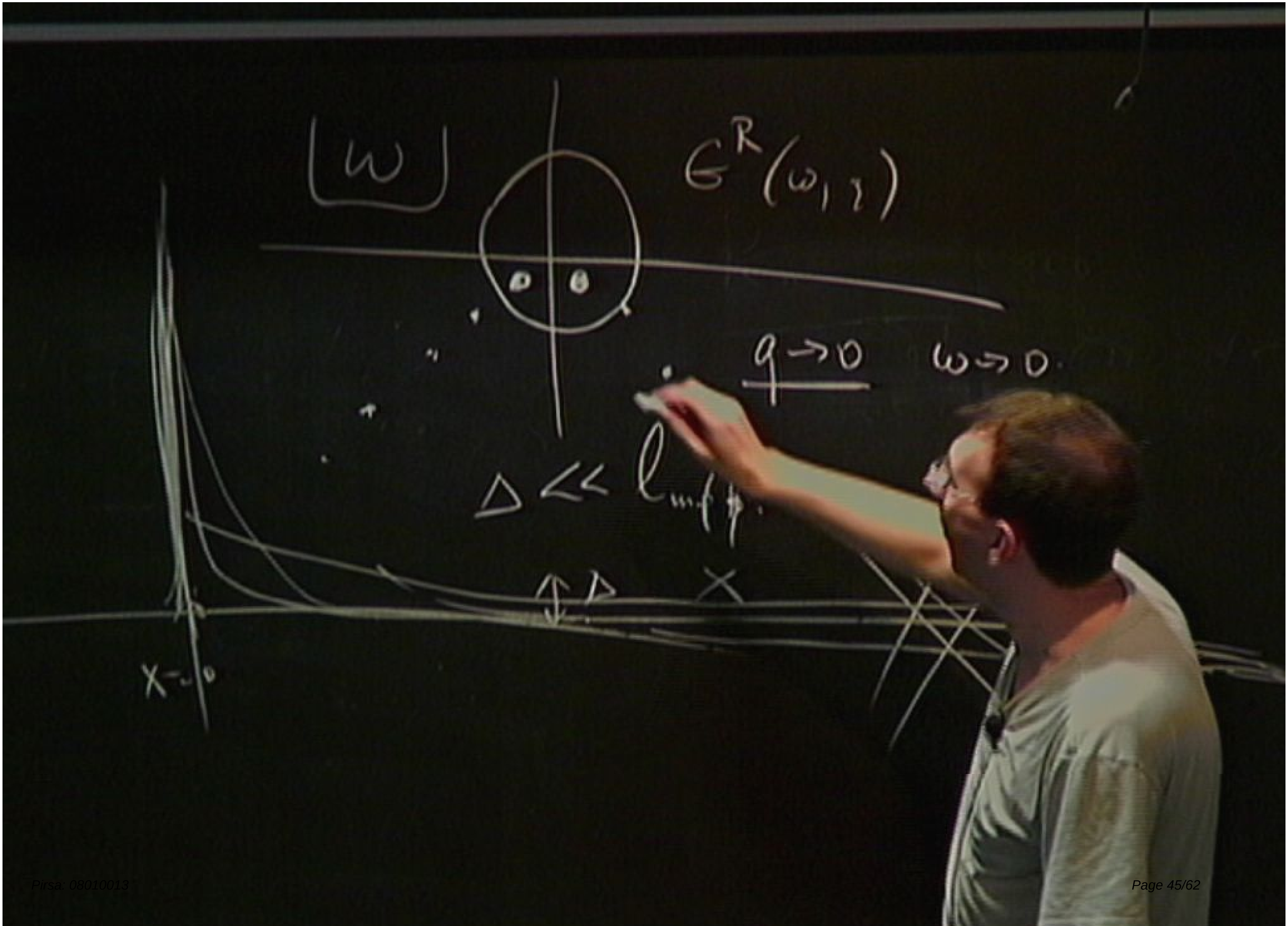
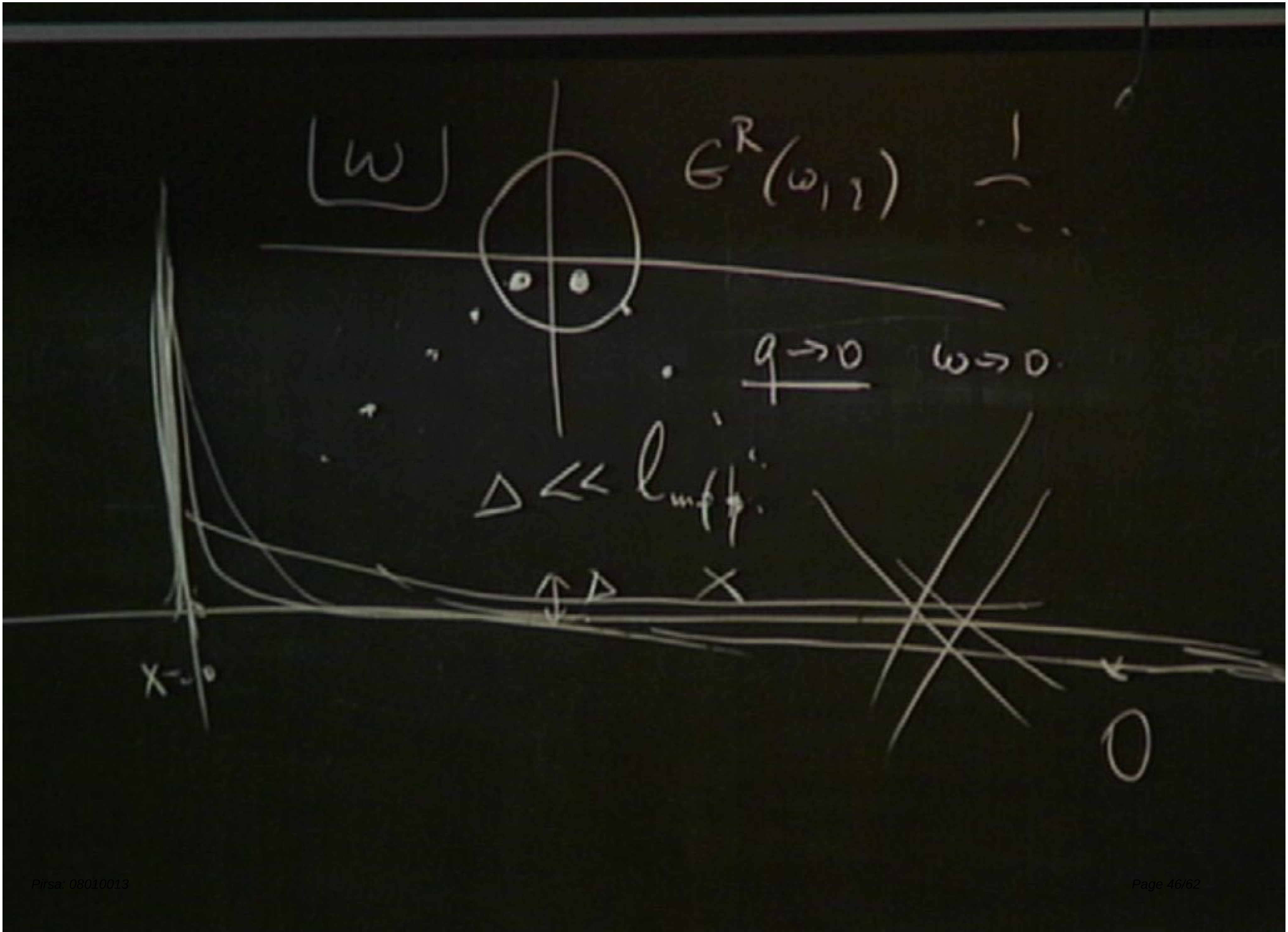

G^R(ω, q)


ω
$G^R(\omega, q)$

ω
$G^R(\omega, q)$
$\Delta \ll l_{m.f.p.}$


ω
$G^R(\omega, q)$
$q \to 0$
$\omega \to 0$
$\Delta << l_{mfp}$


ω
G^R(ω, q)
q→0 ω→0
Δ << l_mfp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G^R(ω, q)
q→0
ω→0
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# Predictions of the second-order conformal hydrodynamics

Sound dispersion: $$\omega_{1,2} = \pm c_s\, q - i\Gamma\, q^2 \pm \frac{\Gamma}{c_s}\left(c_s^2 \tau_\Pi - \frac{\Gamma}{2}\right) q^3 + O(q^4)$$

$$\Gamma = \frac{d-2}{d-1}\frac{\eta}{\varepsilon + P}$$

Kubo: $$G_R^{xy,xy}(\omega, q) = P - i\,\eta\,\omega + \eta\,\tau_\Pi\,\omega^2 - \frac{\kappa}{2}\left[(d-3)\omega^2 + q^2\right]$$

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Israel-Stewart

$$\Pi^{\mu\nu} = -\eta\sigma^{\mu\nu} - \tau_\Pi\left[D\Pi^{<\mu\nu>} + \frac{d}{d-1}\Pi^{\mu\nu}\,(\nabla\cdot u)\right]$$

$$+\kappa\left[R^{<\mu\nu>} - (d-2)u_\alpha R^{\alpha<\mu\nu>\beta}u_\beta\right]$$

$$+\frac{\lambda_1}{\eta^2}\Pi_\lambda^{<\mu}\Pi^{\nu>\lambda} - \frac{\lambda_2}{\eta}\Pi_\lambda^{<\mu}\Omega^{\nu>\lambda} + \lambda_3\,\Omega_\lambda^{<\mu}\Omega^{\nu>\lambda}$$

$$D \equiv u^\mu\nabla_\mu \qquad \Omega^{\mu\nu} = \frac{1}{2}\Delta^{\mu\alpha}\Delta^{\nu\beta}\left(\nabla_\alpha u_\beta - \nabla_\beta u_\alpha\right)$$

## Predictions of the second-order conformal hydrodynamics

Sound dispersion: $\omega_{1,2} = \pm c_s\, q - i\Gamma\, q^2 \pm \frac{\Gamma}{c_s}\left(c_s^2\tau_\Pi - \frac{\Gamma}{2}\right) q^3 + O(q^4)$

$$\Gamma = \frac{d-2}{d-1}\frac{\eta}{\varepsilon + P}$$

Kubo: $G_R^{xy,xy}(\omega, q) = P - i\,\eta\,\omega + \eta\,\tau_\Pi\,\omega^2 - \frac{\kappa}{2}\left[(d-3)\omega^2 + q^2\right]$

# Computing transport coefficients from dual gravity

Assuming validity of AdS/CFT,
all transport coefficients are completely determined
by the lowest frequencies
in quasinormal spectra of the dual gravitational background

(D.Son, A.S., hep-th/0205051, P.Kovtun, A.S., hep-th/0506184)

This determines kinetics in the regime of a thermal theory
where the dual gravity description is applicable

# Computing transport coefficients from dual gravity – various methods

1. Green-Kubo formulas (+ retarded correlator from gravity)

2. Poles of the retarded correlators

3. Lowest quasinormal frequency of the dual background

4. The membrane paradigm

## Example: stress-energy tensor correlator in $4d\ \mathcal{N}=4$ SYM in the limit $N_c \to \infty, \quad g_{YM}^2 N_c \to \infty$

Zero temperature, Euclid: $$G_E(k) = \frac{N_c^2 k_E^4}{32\pi^2} \ln k_E^2$$

Finite temperature, Mink:

$$< T_{tt}(-\omega,-q), T_{tt}(\omega,q) >^{\text{ret}} = \frac{3N_c^2\pi^2T^4q^2}{2(\omega^2 - q^2/3 + i\omega q^2/3\pi T)} + \cdots$$

(in the limit $\omega/T \ll 1,\ q/T \ll 1$ )

The pole
(or the lowest quasinormal freq.) $$\omega = \pm\frac{1}{\sqrt{3}}q - \frac{i}{6\pi T}q^2 + \frac{3-2\ln 2}{24\pi^2\sqrt{3}T^2}q^3 + \cdot$$

Compare with hydro: $$\omega = \pm v_s q - \frac{i}{2sT}\left(\zeta + \frac{4}{3}\eta\right) q^2 + \cdots$$

In CFT: $v_s = \frac{1}{\sqrt{3}}, \quad \zeta = 0$

$\Rightarrow \quad \eta = \pi N_c^2 T^3/8$

Also, $s = \pi^2 N_c^2 T^3/2$ (Gubser, Klebanov, Peet, 1996)

## Example 2 (continued): stress-energy tensor correlator in

$4d\ \mathcal{N}=4$ SYM in the limit $N_c \to \infty, \quad g_{YM}^2 N_c \to \infty$

Zero temperature, Euclid: $G_E(k) = \frac{N_c^2 k_E^4}{32\pi^2} \ln k_E^2$

Finite temperature, Mink:

$$< T_{tx}(-\omega,-q), T_{tx}(\omega,q) >^{\text{ret}} \sim \frac{N_c^2 T^4 \omega^2}{\omega - iq^2/4\pi T} + \cdots$$

(in the limit $\omega/T \ll 1, \ q/T \ll 1$ )

The pole
(or the lowest quasinormal freq.) $\omega = -\frac{i}{4\pi T} q^2 - \frac{i(1-\ln 2)}{32\pi^3 T^3} q^4 + \cdots$

Compare with hydro: $\omega = -\frac{i\,\eta}{sT} q^2 + \cdots$

$$s = \pi^2 N_c^2 T^3/2 \qquad \Rightarrow \quad \eta = \pi N_c^2 T^3/8$$

# New transport coefficients in $\mathcal{N} = 4$ SYM

Sound dispersion: $\omega = \pm \frac{1}{\sqrt{3}} q - \frac{i}{6\pi T} q^2 + \frac{3 - 2\ln 2}{24\pi^2 \sqrt{3} T^2} q^3 + \cdots$

Kubo:

$$G_R^{xy,xy}(\omega, q) = -\frac{\pi^2 N_c^2 T^4}{4} \left[ iw - w^2 + k^2 + w^2 \ln 2 - \frac{1}{2} \right] + O(w^3, wk^2)$$

$$w = \omega / 2\pi T \,, \quad k = q / 2\pi T$$

$$P = \frac{\pi^2}{8} N_c^2 T^4 \,, \quad \eta = \frac{\pi}{8} N_c^2 T^3 \,, \quad \tau_\Pi = \frac{2 - \ln 2}{2\pi T} \,, \quad \kappa = \frac{\eta}{\pi T}$$

# Example 2 (continued): stress-energy tensor correlator in $4d\ \mathcal{N}=4$ SYM in the limit $N_c \to \infty, \quad g^2_{YM} N_c \to \infty$

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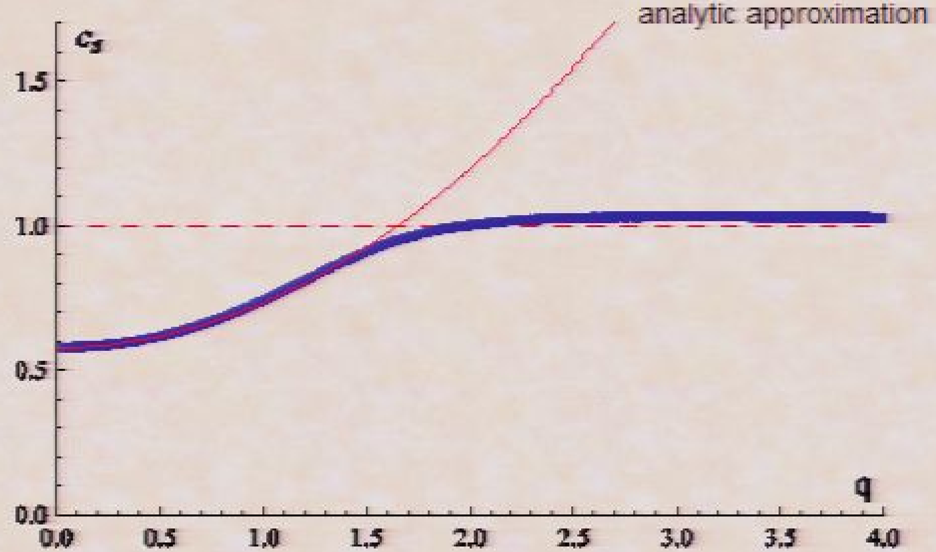
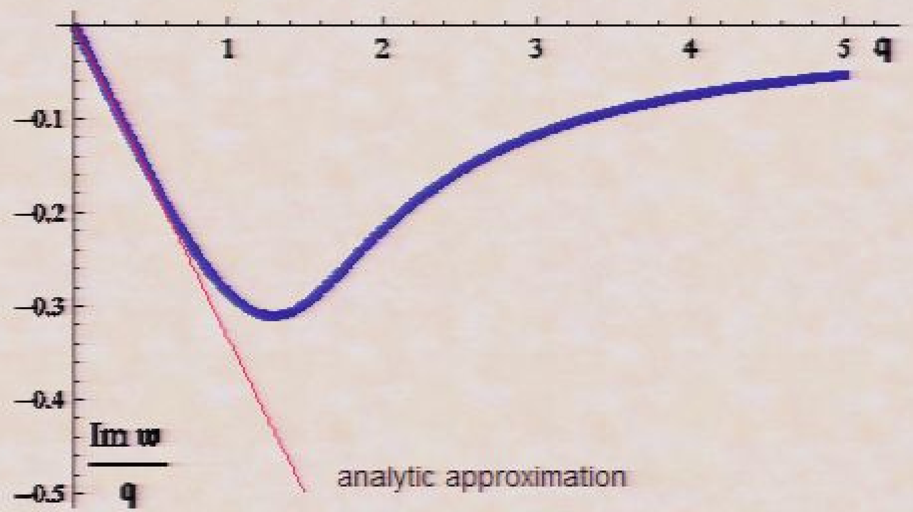
# Sound dispersion in $4d$ $\mathcal{N} = 4$ SYM

$\mathrm{Re}\,(\omega)/q$




$q$



# New transport coefficients in $\mathcal{N} = 4$ SYM

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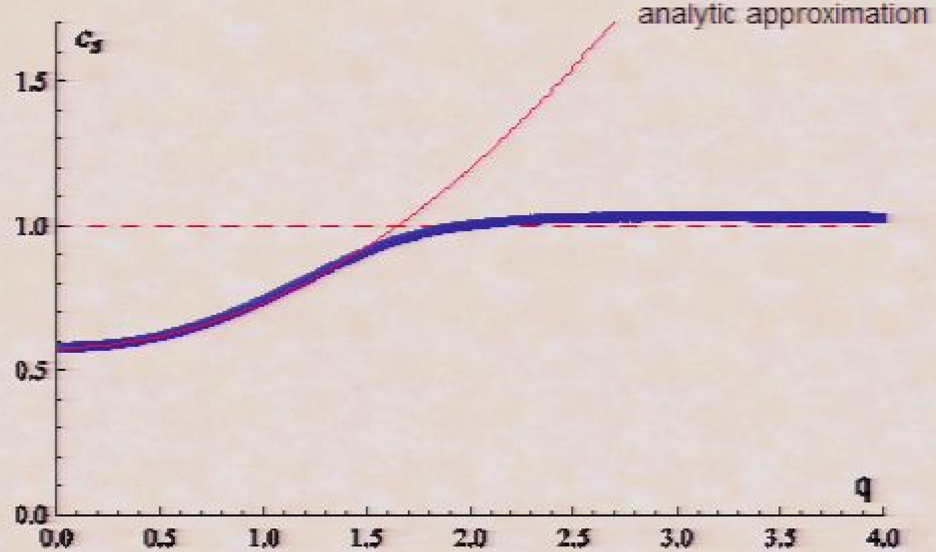
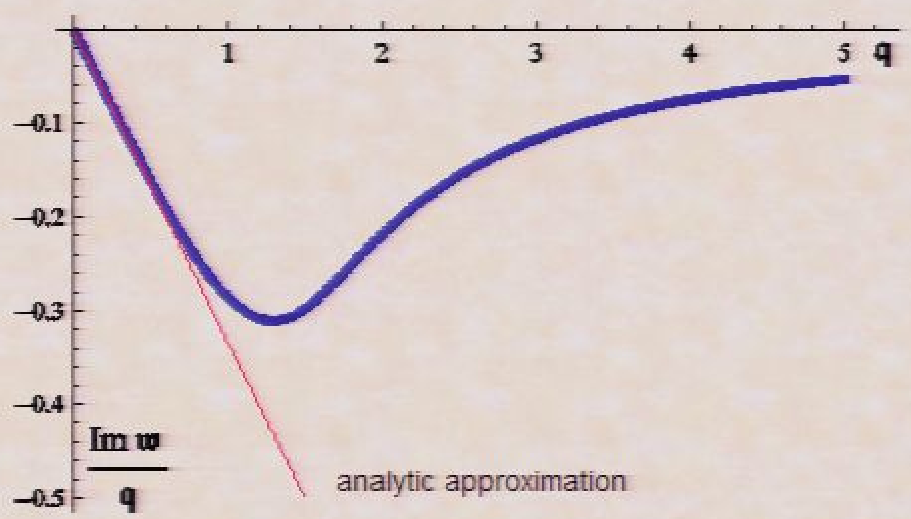



$q$



# New transport coefficients in $\mathcal{N} = 4$ SYM

Sound dispersion: $$\omega = \pm \frac{1}{\sqrt{3}} q - \frac{i}{6\pi T} q^2 + \frac{3 - 2\ln 2}{24\pi^2 \sqrt{3} T^2} q^3 + \cdots$$

Kubo:

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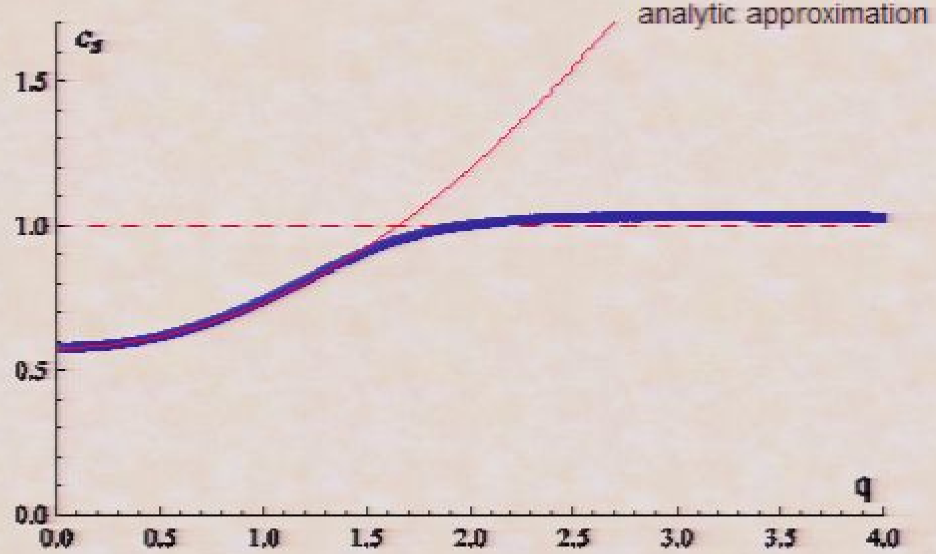
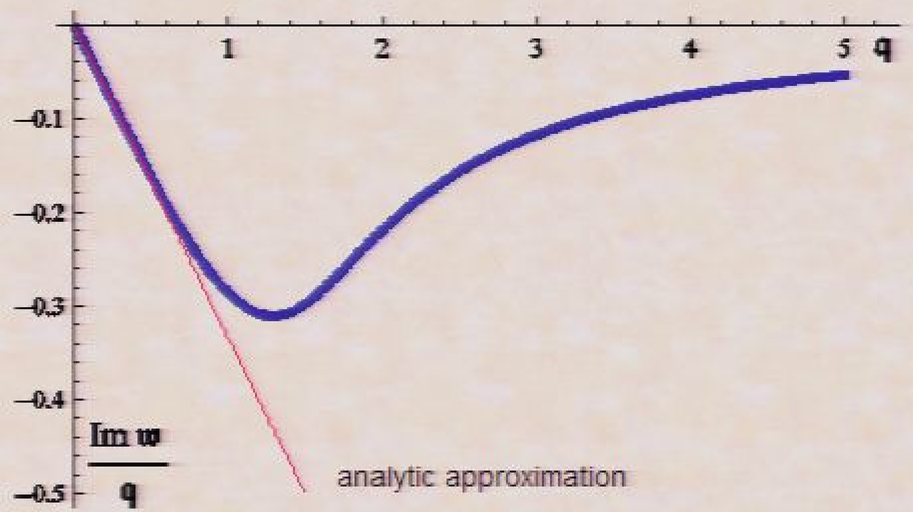
# Sound dispersion in $4d$ $\mathcal{N} = 4$ SYM

$\mathrm{Re}\,(\omega)/q$




$q$



# Conclusions

Second-order conformal hydrodynamics can be systematically constructed

It turns out to be more general than the Israel-Stewart theory

Using AdS/CFT, all new transport coefficients for N=4 SYM can be computed

$$\eta = \frac{\pi}{8} N_c^2 T^3, \;\; \tau_\Pi = \frac{2 - \ln 2}{2\pi T}, \;\; \kappa = \frac{N_c^2 T^2}{8}$$

$$\lambda_1 = \frac{\eta}{2\pi T}, \;\; \lambda_2 = -\frac{\eta \ln 2}{\pi T}, \;\; \lambda_3 = 0$$

Here we also used results from S.Bhattacharyya, V.Hubeny, S.Minwalla, M.Rangamani, 0712.2456 [hep-th]

Open question: does this affect RHIC numerics?