Title: Supersymmetric codimension-two branes in six-dimensional gauged supergravity

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Abstract: We consider a consistent construction of the supersymmetric action for a codimension-two brane in six-dimensional Salam-Sezgin supergravity. When the brane carries a tension, we supersymmetrize the brane tension action by introducing a localized Fayet-Iliopoulos term on the brane and modifying the bulk SUSY transformations.

As a result, we find that among the axisymmetric vacua of the system, the unwarped background with football-shaped extra dimensions respects N=1 supersymmetry. We extend the analysis to include the brane multiplets with the couplings to the bulk fields.

Supersymmetric codimension-two branes in six-dimensional gauged supergravity

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Ref: Antonios Papazoglou, HML, arXiv: 0710.4319 [hep-th];
Antonios Papazoglou, HML, to appear.

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Outline

1. Introduction
2. SUSY brane action with a brane tension
3. SUSY brane solutions
4. Matter multiplets on the brane

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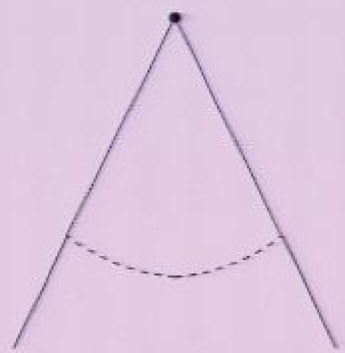
Introduction

- Can we use flux compactifications to solve the cosmological constant problem?
- What is the implication of flux compactifications for particle physics model building?
- For the simplest example, we consider the flux compactifications with a codimension-two brane in six dimensions.

Codimension-two branes



- The tension(T) of codimension-two brane deforms the geometry of extra dimensions by a deficit angle without curving the 4D spacetime: locally, $ds^2 \simeq dr^2 + \beta^2 r^2 d\theta^2$, with a deficit angle $2\pi(1-\beta) = T$.
- When the Euler number χ of compact dimensions is kept, the bulk curvature should respond to the change of a brane tension: $\chi = \frac{1}{4\pi}\int d^2y R_2 + \frac{1}{2\pi}\sum_i T_i$.

Flux compactifications

[Cremmer,Scherk(1976); Carroll,Guica(2003); Navarro(2003)]

- In 6D Einstein-Maxwell theory, the bulk action is

$$S = \int d^6x\sqrt{-G}\Big(\frac{1}{2}R - \Lambda_b - \frac{1}{4}F_{MN}F^{MN}\Big). \quad (1)$$

- The factorizable solution is

$$ds^2 = g_{\mu\nu}(x)dx^\mu dx^\nu + R_0^2(d\theta^2 + \beta^2\sin^2\theta d\phi^2), \quad (2)$$

$$F_{\theta\phi} = qR_0^2\beta\sin\theta, \quad (3)$$

where $g_{\mu\nu}(x)$ is the maximally symmetric solution with $\Lambda_{\rm eff} = \frac{1}{6}(\Lambda_b - \frac{1}{2}q^2)$, and $R_0^{-2} = \frac{1}{2}(\Lambda_b + \frac{3}{2}q^2)$.

- Tuning $q^2 = 2\Lambda_b$ with $\Lambda_b > 0$ is required for the 4D flatness.
- Two equal brane tensions $T_1 = T_2 = 2\pi(1-\beta)$ are located at the poles.[cf. This is the condition to keep $\chi = 2$.]

- The gauge potential in the patch surrounding the north(south) pole is

$$A_\phi = -qR_0^2\beta(\cos\theta \mp 1). \tag{4}$$

- Under the gauge transformation in the overlapping region, $\Lambda(\phi) = 2qR_0^2\beta\phi$, a bulk field with charge g transforms by a phase $e^{ig\Lambda}$. Then, the single-valuedness for the charge bulk field requires the flux to be quantized as $q = \frac{n}{2g\beta R_0^2}$ with integer n.
- The bulk tuning condition $q^2 = 2\Lambda_b$ becomes

$$\left(1 - \frac{T_1}{2\pi}\right)^2 = \frac{\Lambda_b n^2}{2g^2}. \tag{5}$$

[I.Navarro(2003); Nilles et al(2003); HML(2003); Garriga,Porrati(2004)]

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Nishino-Sezgin gauged supergravity

[Nishino,Sezgin(1984)]

- 6D chiral supergravity: "gravity" $(e^A_M, \psi_M, B^+_{MN})$+ "tensor" (ϕ, χ, B^-_{MN})
- Gauging $U(1)_R$: Add "vector" (A_M, λ) + $n_H = 245$ neutral "hypers" $(\Phi^\alpha, \Psi^a; \alpha = 1, \cdots, 4n_H, a = 1, \cdots, 2n_H)$.
- The bosonic action of the 6D $U(1)_R$ gauged supergravity is

$$S = \int d^6x\sqrt{-G}\Big(R - \frac{1}{4}(\partial_M\phi)^2 - 2G_{ab}(\Phi)D_M\Phi^a D^M\Phi^b - e^{-\frac{1}{2}\phi}v(\Phi) - \frac{1}{4}e^{\frac{1}{2}\phi}F_{MN}F^{MN} - \frac{1}{12}e^{\phi}G_{MNP}G^{MNP}\Big)$$

where G_{ab} is the metric on $Sp(1,245)/Sp(245)\times Sp(1)$ and

$$F_{MN} = 2\partial_{[M}A_{N]}, \quad G_{MNP} = 3\partial_{[M}B_{NP]} + 3F_{[MN}A_{P]}. \qquad (6)$$

Salam-Sezgin compactification

[Salam,Sezgin(1984)]

- For $\Phi^a = 0$, the bulk potential is positive definite as $v(0) = 8g^2$ with g being the $U(1)_R$ gauge coupling. Thus, the 6D Minkowski space such as $M_4 \times T^2$ is not a solution.
- For $\Phi^a = B_{MN} = 0$ and constant $\phi = \phi_0$, the Minkowski solution is a unique solution with 4D maximal symmetry:

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu + R_0^2(d\theta^2 + \sin^2\theta d\psi^2), \quad (7)$$

$$F_{\theta\psi} = e^{-\frac{1}{2}\phi_0} q R_0^2 \sin\theta, \ \text{others} = 0 \quad (8)$$

where $R_0^{-2} = 8g^2 e^{-\frac{1}{2}\phi_0}$ and $q^2 = 16g^2$.
- Dilaton equation guarantees the bulk tuning $q^2 = 16g^2$, which is consistent with the quantized flux with $n = \pm 1$.
- 4D $\mathcal{N} = 1$ SUSY is preserved.

- Deform the Salam-Sezgin solution with a deficit angle $2\pi(1-\beta)$:

$$ds^2 = R_0^2(d\theta^2 + \sin^2\theta d\psi^2) \Rightarrow R_0^2(d\theta^2 + \beta^2\sin^2\theta d\psi^2). \quad (9)$$

[Burgess et al(2003)]

  - The bulk tuning $q^2 = 16g^2$ is maintained.
  - Two equal brane tensions $T_1 = T_2 = 2\pi(1-\beta)$ are located at the poles.
  - Flux quantization condition $\beta = |n|$ requires the quantized tensions.

- Other factorizable solutions with fluxes:
  - $\Phi^a = 0, B_{MN} \neq 0 \Rightarrow$ supersymmetric $AdS_3 \times$ squashed S^3.

    [Güven et al(2003)]
  - $B_{MN} = 0, \Phi^a \neq 0 \Rightarrow$ singular supersymmetric unwarped solutions with 4D flatness. [Paramaswaran et al(2005)]

General warped solutions

[Gibbons et al(2003); Aghababaie et al(2003); Lee,Lüdeling(2005)]

- Assuming the axial symmetry of extra dimensions, the general warped solution has been found to be

$$ds^2 = W^2(r)\eta_{\mu\nu}dx^\mu dx^\nu + R^2(r)(dr^2 + \lambda^2\Theta^2(r)d\theta^2), \quad (10)$$

$$F_{mn} = qe^{-\frac{1}{2}\phi_0}W^{-6}\epsilon_{mn}, \quad \phi = \phi_0 + 4\ln W, \quad (11)$$

with $R = \frac{W}{f_0}, \Theta = \frac{r}{W^4}, W^4 = \frac{f_1}{f_0}$ and $f_0 = 1 + \frac{r^2}{r_0^2}, f_1 = 1 + \frac{r^2}{r_1^2}$.

Here λ, q, ϕ_0 are constants, $r_0^2 = \frac{1}{2g^2}e^{\frac{1}{2}\phi_0}$ and $r_1^2 = \frac{8}{q^2}e^{\frac{1}{2}\phi_0}$.

- Two different brane tensions are located at the conical singularities, $r = 0$ and $r = \infty$:

$$\frac{T_0}{4\pi} = 1 - \lambda, \quad \frac{T_\infty}{4\pi} = 1 - \lambda\frac{r_1^2}{r_0^2}. \quad (12)$$

- The gauge potential: in the patch surrounding $r = 0$,

$$A_\theta = -\frac{4\lambda}{q}\Big(\frac{1}{f_1} - 1\Big); \tag{13}$$

in the patch surrounding $r = \infty$,

$$A_\theta = -\frac{4\lambda}{q}\frac{1}{f_1}. \tag{14}$$

- The flux quantization condition leads to the relation between brane tensions,

$$\Big(1 - \frac{T_0}{4\pi}\Big)\Big(1 - \frac{T_\infty}{4\pi}\Big) = n^2. \tag{15}$$

- The warped solution is marginally stable under perturbations.

[HML,Papazoglou(2006); Burgess et al(2006)]

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Brane world supergravity

- The tension action for a brane located at $y = 0$ is given by

$$\mathcal{L}_{\rm brane} = -e_4 \delta^2(y) T_0 \tag{16}$$

with T_0 being the brane tension.

- It is not invariant under the bulk SUSY transformation as

$$\delta\mathcal{L}_{\rm brane} = -e_4 \delta^2(y) \frac{1}{4} T_0 (\bar{\psi}_\mu \gamma^\mu \varepsilon + {\rm h.c.}). \tag{17}$$

SUSY is broken explicitly at the energy scale of the brane tension which is of order the 6D fundamental scale. This is the case with a decoupling of brane SUSY partners.

- In the case with explicit SUSY breaking, the SUSY breaking scale may be suppressed by the large volume of extra dimensions like $M^2_{SUSY} \sim \frac{T_0}{M_6^4 V} \sim \frac{1}{V}$. [See Burgess hep-th/0510123 for a review.]

- However, SUSY can be broken spontaneously at the brane. This requires a construction of the SUSY brane action. Brane SUSY gives rise to a fixed coupling of the brane multiplets to the bulk fields. In the case where the SUSY breaking scale is lower than the compactification scale, one can compute the soft mass parameters in the 4D effective theory.

  [Falkowski,HML,Lüdeling(2005)]

- We first consider the supersymmetrization of the brane tension action and discuss on including the matter multiplets on the brane.

Outline

Bulk SUSY

- The bulk Langrangian of Salam-Sezgin supergravity is

$$e_6^{-1}\mathcal{L}_{\rm bulk} = R - \frac{1}{4}(\partial_M\phi)^2 - \frac{e^{\phi}}{12}G_{MNP}G^{MNP} - \frac{e^{\frac{1}{2}\phi}}{4}F_{MN}F^{MN} - 8g^2e^{-\frac{1}{2}\phi}$$
$$+\bar\psi_M\Gamma^{MNP}\mathcal{D}_N\psi_P + \bar\chi\Gamma^M\mathcal{D}_M\chi + \bar\lambda\Gamma^M\mathcal{D}_M\lambda$$
$$+\frac{1}{4}(\partial_M\phi)(\bar\psi_N\Gamma^M\Gamma^N\chi + \bar\chi\Gamma^N\Gamma^M\psi_N)$$
$$+\frac{1}{24}e^{\frac{1}{2}\phi}G_{MNP}(\bar\psi^R\Gamma_{[R}\Gamma^{MNP}\Gamma_{S]}\psi^S + \bar\psi_R\Gamma^{MNP}\Gamma^R\chi$$
$$-\bar\chi\Gamma^R\Gamma^{MNP}\psi_R - \bar\chi\Gamma^{MNP}\chi + \bar\lambda\Gamma^{MNP}\lambda)$$
$$-\frac{1}{4\sqrt{2}}e^{\frac{1}{4}\phi}F_{MN}(\bar\psi_Q\Gamma^{MN}\Gamma^Q\lambda + \bar\lambda\Gamma^Q\Gamma^{MN}\psi_Q + \bar\chi\Gamma^{MN}\lambda - \bar\lambda\Gamma^{MN}\chi)$$
$$+i\sqrt{2}ge^{-\frac{1}{4}\phi}(\bar\psi_M\Gamma^M\lambda + \bar\lambda\Gamma^M\psi_M - \bar\chi\lambda + \bar\lambda\chi). \quad (18)$$

- The field strength tensors are

$$F_{MN} = \partial_M A_N - \partial_N A_N, \quad G_{MNP} = 3\partial_{[M} B_{NP]} + \frac{3}{2} F_{[MN} A_{P]}, \tag{19}$$

with the Bianchi identities

$$\partial_{[Q} F_{MN]} = 0, \quad \partial_{[Q} G_{MNP]} = \frac{3}{4} F_{[MN} F_{QP]}. \tag{20}$$

- For $\delta A_M = \partial_M \Lambda$ under the $U(1)_R$, the Kalb-Ramond field B_{MN} transforms as $\delta_{\rm gauge} B_{MN} = -\frac{1}{2}\Lambda F_{MN}$.
- All the spinors have the same charge normalized to $+1$ under $U(1)_R$, so the covariant derivative of the gravitino, for instance, is given by

$$\mathcal{D}_M \psi_N = (\partial_M + \frac{1}{4}\omega_{MAB}\Gamma^{AB} - igA_M)\psi_N. \tag{21}$$

- The bulk SUSY transformations (up to the trilinear fermion terms) are

$$\begin{aligned}
\delta e^A_M &= -\frac{1}{4}\bar{\varepsilon}\Gamma^A\psi_M + \text{h.c.}, \quad \delta\phi = \frac{1}{2}\bar{\varepsilon}\chi + \text{h.c.}, \\
\delta B_{MN} &= A_{[M}\delta A_{N]} + \frac{e^{-\frac{1}{2}\phi}}{4}(\bar{\varepsilon}\Gamma_M\psi_N - \bar{\varepsilon}\Gamma_N\psi_M + \bar{\varepsilon}\Gamma_{MN}\chi + \text{h.c.}), \\
\delta\chi &= -\frac{1}{4}(\partial_M\phi)\Gamma^M\varepsilon + \frac{1}{24}e^{\frac{1}{2}\phi}G_{MNP}\Gamma^{MNP}\varepsilon, \\
\delta\psi_M &= \mathcal{D}_M\varepsilon + \frac{1}{48}e^{\frac{1}{2}\phi}G_{PQR}\Gamma^{PQR}\Gamma_M\varepsilon, \\
\delta A_M &= \frac{1}{2\sqrt{2}}e^{-\frac{1}{4}\phi}(\bar{\varepsilon}\Gamma_M\lambda + \text{h.c.}), \\
\delta\lambda &= \frac{1}{4\sqrt{2}}e^{\frac{1}{4}\phi}F_{MN}\Gamma^{MN}\varepsilon - i\sqrt{2}g\, e^{-\frac{1}{4}\phi}\varepsilon.
\end{aligned}$$

- The bulk action is invariant up to the Bianchi identities as follows,

$$\delta\mathcal{L}_{\rm bulk} = e_6\Big[-\frac{1}{24}e^{\frac{1}{2}\phi}\Big(\partial^S G_{MNP}-\frac{3}{4}F_{MN}F^S{}_P\Big)\times\Big(\bar\psi^R\Gamma_{[R}\Gamma^{MNP}\Gamma_{S]}\varepsilon-\bar\chi\Gamma_S\Gamma^{MNP}\varepsilon+{\rm h.c.}\Big)+\frac{1}{4\sqrt{2}}e^{\frac{1}{4}\phi}\Big(\partial_Q F_{MN}\bar\lambda\Gamma^Q\Gamma^{MN}\varepsilon+{\rm h.c.}\Big)\Big]. \quad (22)$$

- We can modify the Bianchi identities to cancel the SUSY variation of the brane action.

Supersymmetrizing the brane tension action

[HML,Papazoglou(2007)]

- Replace the field strength tensors with the hatted ones in the bulk action and the SUSY transformations (but keep A_M as it is):

$$\hat{G}_{\mu mn} = G_{\mu mn} - \xi_0 A_\mu \epsilon_{mn} \frac{\delta^2(y)}{e_2}, \tag{23}$$

$$\hat{F}_{mn} = F_{mn} - \xi_0 \epsilon_{mn} \frac{\delta^2(y)}{e_2}, \tag{24}$$

where $\xi_0 = \frac{T_0}{4g}$ is the localized Fayet-Iliopoulos(FI) term and other components are unmodified. Then the modified Bianchi identities are

$$\partial_{[\mu} \hat{G}_{\nu mn]} = \frac{1}{2} \hat{F}_{[\mu\nu} \hat{F}_{mn]}, \quad \partial_{[\mu} \hat{F}_{mn]} = 0. \tag{25}$$

- Impose the Z_2 orbifold symmetry around the brane to project out the half SUSY. The Z_2 symmetry is a discrete subgroup of the axial symmetry.
- Then, the Z_2 parities for the bulk fields ($\lambda = (\tilde{\lambda}, 0)^T$ with $\tilde{\lambda} = (\tilde{\lambda}_L, \tilde{\lambda}_R)^T$, etc, α, β: 4D Lorentz, a, b: 2D Lorentz) are

$$\text{even} \;:\; \tilde{\psi}_{\alpha L},\ \tilde{\psi}_{aR},\ \tilde{\lambda}_L,\ \tilde{\chi}_R,\ \tilde{\varepsilon}_L,\ A_\alpha,\ B_{\alpha\beta},\ B_{ab},\ \phi, \tag{26}$$

$$\text{odd} \;:\; \tilde{\psi}_{\alpha R},\ \tilde{\psi}_{aL},\ \tilde{\lambda}_R,\ \tilde{\chi}_L,\ \tilde{\varepsilon}_R,\ A_a,\ B_{\alpha a}. \tag{27}$$

- For changing ξ_0 to $-\xi_0$, the 4D chiralities of all Z_2 eigenstates are reversed.
- We also need to modify the gauge transformation of the KR field as

$$\delta_{\text{gauge}} B_{mn} = \Lambda\Big(-\frac{1}{2} F_{mn} + \xi_0 \frac{\delta^2(y)}{e_2} \Big). \tag{28}$$

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Modified equations and warped solutions

- Setting the KR field and A_μ to zero, the Einstein equation is

$$R_{MN} = 2g^2 e^{-\frac{1}{2}\phi} g_{MN} + \frac{1}{2} e^{\frac{1}{2}\phi}(\hat{F}_{MP}\hat{F}_N{}^P - \frac{1}{8} g_{MN}\hat{F}^2_{PQ}) + \frac{1}{4}\partial_M\phi\partial_N\phi + T^i_{MN} \,, \quad (29)$$

where $T^i_{MN} = -\frac{1}{2}\frac{\sqrt{g_4}}{\sqrt{g_6}} T_i (g^{(4)}_{\mu\nu}\delta^\mu_M\delta^\nu_N - g_{MN})\delta^2(y-y_i)$ is the brane tension contribution (with $g^{(4)}_{\mu\nu}$ the 4D induced metric). Furthermore, the dilaton and the gauge field equations read

$$\Box^{(6)}\phi = \frac{1}{4} e^{\frac{1}{2}\phi}\hat{F}^2_{PQ} - 8g^2 e^{-\frac{1}{2}\phi} \,, \quad (30)$$

$$\partial_M(\sqrt{-g}\, e^{\frac{1}{2}\phi}\hat{F}^{MN}) = 0 \,. \quad (31)$$

- The warped solutions is maintained, except the solution for F_{mn} is replaced with the hatted one:

$$\hat{F}_{r\theta} = F_{r\theta} - \frac{\xi_0}{2\pi}\delta(r) = \lambda e^{-\frac{1}{2}\phi_0} q \frac{\Theta R^2}{W^6}. \qquad (32)$$

- The localized FI term affects the VEV of F_{mn} such that the gauge potential becomes nonzero at $r = 0$ and $r = \infty$:

$$A_\theta = -\frac{4\lambda}{q}\left(\frac{1}{f_1} - 1\right) + \frac{\xi_0}{2\pi}; \;\; A_\theta = -\frac{4\lambda}{q}\frac{1}{f_1} + \frac{\xi_\infty}{2\pi}. \qquad (33)$$

- The quantization condition becomes $\frac{4\lambda g}{q} = n + \frac{g}{2\pi}(\xi_\infty - \xi_0)$ with $n \in \mathbf{Z}$. $\Rightarrow \left(1 - \frac{T_0}{4\pi}\right)\left(1 - \frac{T_\infty}{4\pi}\right) = \left[n + \frac{g}{2\pi}(\xi_\infty - \xi_0)\right]^2$
- The bulk terms involving $\hat{F}_{mn}$ remain free of the singular term.

SUSY brane solutions

- The warped solutions break the bulk SUSY completely: e.g.

$$\delta\chi = -\frac{W'}{W}[\cos\theta\,\sigma^1\otimes\gamma^5+\sin\theta\,\sigma^2\otimes\mathbf{1}]\varepsilon\neq 0. \tag{34}$$

- The football solution has a constant warp factor. In the patch surrounding $r=0$, the nontrivial fermionic SUSY transformations are

$$\delta\lambda = i2\sqrt{2}g(P_R\varepsilon), \tag{35}$$

$$\begin{aligned}\delta\psi_\theta &= \left[\partial_\theta+\frac{i}{2}\left\{1+n\left(1-\frac{2}{f_0}\right)\right\}\gamma^5+in\left(\frac{1}{f_0}-1\right)-i\frac{g\xi_0}{2\pi}\right]\varepsilon \\ &= \partial_\theta(P_L\varepsilon).\end{aligned} \tag{36}$$

Then, 4D $\mathcal{N}=1$ SUSY is preserved due to a constant $\tilde{\varepsilon}_L$ satisfying both equations.

Gravitino spectrum

- The solution of the massless left-handed 4D gravitino with $\tilde{\psi}_{\mu L}(x,r,\theta)=\tilde{\psi}^{(m)}_{\mu L}(x)\varphi^{(m)}_L(r)e^{im\theta}$ is

$$\varphi^{(m)}_L=\frac{N_m}{W\sqrt{\lambda R\Theta}}\left(\frac{r}{r_0}\right)^{\frac{s}{2}}\left(1+\frac{r^2}{r_0^2}\right)^{\frac{1-t}{2}}; \qquad (37)$$

$$s = \frac{1}{\lambda}(1+2m)-\frac{g\xi_0}{\pi\lambda},$$

$$t = \frac{1}{\lambda}(m+\frac{1}{2}-n-\frac{g\xi_\infty}{2\pi})\left(1-\frac{r_0^2}{r_1^2}\right)+\frac{1}{\lambda}\left[n+\frac{g}{2\pi}(\xi_\infty-\xi_0)\right]+1,$$

where N_m is the normalization constant.

- The solution of the massless right-handed 4D gravitino is given by the one for the left-handed gravitino with (m,n,ξ_0,ξ_∞) being replaced by $(-m,-n,-\xi_0,-\xi_\infty)$.

- The normalizability condition

$$\int d\theta \int dr\ \lambda W R^2 \Theta\ |\varphi^{(m)}_{L,R}|^2 < \infty, \tag{38}$$

gives the condition for the left-handed zero mode as

$$s > -1, \quad s - 2t < -1, \tag{39}$$

or

$$-\frac{1}{2}(1+\lambda) + \frac{g\xi_0}{2\pi} < m < n - \frac{1}{2}\left(1 - \lambda\frac{r_1^2}{r_0^2}\right) + \frac{g\xi_\infty}{2\pi}\ . \tag{40}$$

- There exist generically multiple zero modes.
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- There is the $U(1)_Q$ isometry associated with the extra dimensions:

$$\hat{Q} = -i\partial_\theta + \frac{1}{2}\sigma^3 \otimes \gamma^5. \qquad (41)$$

- Take only one combination Q_1 of $U(1)_R$ and $U(1)_Q$ to survive such that $Q_1 = R - 2Q = -2m$ for the left-handed zero mode with m winding number.
- The left-handed zero modes with nonzero even and opposite m or Q_1 charges can be paired up to make a 4D Dirac spinor $\Psi^{(m)}_\mu = (\tilde{\psi}^{(m)}_{\mu L}, -i\sigma^2 \tilde{\psi}^{(-m)*}_{\mu L})^T$, so that they get decoupled by their large Dirac masses. Each pair makes an $\mathcal{N} = 1$ massive spin-$\frac{3}{2}$ multiplet, together with a pair of zero modes with the same $|m|$ of an $U(1)_R$ charged spin-$\frac{1}{2}$ fermion.
- Consequently, there can be only one chiral massless mode of the gravitino with $m = 0$ in the low energy spectrum.

Outline

The SUSY brane action

[Lee,Papazoglou(to appear)]

- Introduce an $U(1)_R$ charged brane chiral multiplet (ψ, Q).
- The SUSY brane action for that is obtained as

$$\begin{aligned}\mathcal{L}_{\rm brane} &= e_4\Big[e^{\frac{1}{2}\phi}\Big(-(D^\mu Q)^\dagger D_\mu Q+\frac{1}{2}\bar{\psi}\gamma^\mu D_\mu\psi+{\rm h.c.}\Big)\\ &\quad -\sqrt{2}irge^{\frac{1}{4}\phi}\bar{\psi}\lambda Q+{\rm h.c.}+4rg^2|Q|^2-T\\ &\quad +e^{\frac{1}{2}\phi}\Big(\frac{1}{2}\bar{\psi}_\mu\gamma^\nu\gamma^\mu\psi(D_\nu Q)^\dagger+\frac{1}{2}\bar{\psi}\gamma^\mu\chi D_\mu Q+{\rm h.c.}\Big)\Big]\end{aligned}$$

with SUSY transformations $\delta Q=\frac{1}{2}\bar{\varepsilon}\psi$, $\delta\psi=-\frac{1}{2}\gamma^\mu\varepsilon D_\mu Q$,

$$D_\mu Q = (\partial_\mu - irgA_\mu)Q, \qquad (42)$$

$$D_\mu\psi = (\partial_\mu - i(r+1)gA_\mu - \frac{1}{4}\omega_{\mu\alpha\beta}\gamma^{\alpha\beta})\psi. \qquad (43)$$

- The bulk action and the SUSY transformation are modified by replacing G_{MNP} and F_{MN} with the hatted ones (keeping A_M as it is):

$$\hat{G}_{\mu mn} = G_{\mu mn} + (j_\mu - \xi_0 A_\mu)\epsilon_{mn}\frac{\delta^2(y)}{e_2}, \quad (44)$$

$$\hat{G}_{\tau\rho\sigma} = G_{\tau\rho\sigma} + \frac{\delta^2(y)}{e_2} j_{\tau\rho\sigma}, \quad (45)$$

$$\hat{F}_{mn} = F_{mn} + (rg|Q|^2 - \xi_0)\epsilon_{mn}\frac{\delta^2(y)}{e_2} \quad (46)$$

with

$$j_\mu = \frac{1}{2} i\Big[Q^\dagger D_\mu Q - (D_\mu Q)^\dagger Q + \frac{1}{2}\bar{\psi}\gamma_\mu\psi\Big], \quad (47)$$

$$j_{\tau\rho\sigma} = -\frac{1}{4}\bar{\psi}\gamma_{\tau\rho\sigma}\psi. \quad (48)$$

- The modified Bianchi identities are given by

$$\partial_{[\mu}\hat{G}_{\nu mn]} = \frac{3}{4}\hat{F}_{[\mu\nu}\hat{F}_{mn]} + \frac{i}{2}(D_{[\mu}Q)^\dagger(D_{\nu]}Q)\epsilon_{mn}\frac{\delta^2(y)}{e_2} \quad (49)$$

$$\partial_{[\mu}\hat{F}_{mn]} = \frac{1}{3}rg\partial_\mu|Q|^2\epsilon_{mn}\frac{\delta^2(y)}{e_2}. \quad (50)$$

- The SUSY and gauge transformations of the KR field have an additional term as

$$\delta B_{mn} = \frac{1}{4}i\bar{\psi}\varepsilon Q\epsilon_{mn}\frac{\delta^2(y)}{e_2} + \text{h.c.}, \quad (51)$$

$$\delta_{\text{gauge}}B_{mn} = \Lambda\xi_0\epsilon_{mn}\frac{\delta^2(y)}{e_2}. \quad (52)$$

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Scale invariance of the brane action

- The bulk equations of motion have the scaling invariance under the following transformations:

$$g_{MN} \to \Omega g_{MN}, \quad \phi \to \phi + 2\ln\Omega, \quad B_{MN} \to B_{MN}, \quad A_M \to A_M.$$

- From the bosonic part of the SUSY brane action

$$\mathcal{L} = e_4 \delta^2(y)\Big(- e^{\frac{1}{2}\phi}(D^\mu Q)^\dagger (D_\mu Q) + 4rg^2|Q|^2 - T\Big), \quad (53)$$

  we find that the scaling invariance is unbroken by the SUSY brane action for $Q \to Q$.

- In order to stabilize the bulk modulus, a bulk non-perturbative correction is necessary. [Aghababaie et al(2002); Falkowski,HML,Lüdeling(2005)]

Conclusion

- We constructed the SUSY brane action for a codimension-two brane with nonzero tension in 6D gauged supergravity.
- The modification of the bulk action and the SUSY transformation is necessary, particularly, the field strength tensors are modified by the localized terms proportional to the brane tension.
- The football solution preserves 4D $\mathcal{N}=1$ SUSY due to the cancellation between the spin and gauge connections.
- The nonzero FI term affects the spectrum of any $U(1)_R$ charged bulk field. We showed that there can be only one massless mode of gravitino for the football solution.

- We derived the SUSY brane action for an $U(1)_R$ charged brane chiral multiplet. The brane chiral multiplet has a nontrivial coupling to the dilaton such that the scaling invariance remains.
- The brane vector multiplet can be also introduced along the same line in the ungauged supergravity. [Falkowski,HML,Lüdeling (2005)]
- The analysis on the 4D effective supergravity for the modulus stabilization and the SUSY breaking is in progress.

  [HML,Papazoglou(to appear)]

Example: Heterotic M-theory

[Horava,Witten(1996)]

- In heterotic M-theory, the 11D supergravity multiplet is composed of e_I^m, ψ_I and C_{IJK}, and an E_8 super Yang-Mills is localized on each boundary of $M_{10} \times S^1/Z_2$.
- In the SUSY brane action for the E_8 Yang-Mills, a Noether term must be added as

$$\mathcal{L}_{YM} = -\frac{1}{g^2}\int d^{10}x\sqrt{g}\,\mathrm{tr}\Big(\frac{1}{4}F_{AB}F^{AB} + \frac{1}{2}\bar{\lambda}\Gamma^A D_A\lambda + \frac{1}{4}\bar{\psi}_A\Gamma^{BC}\Gamma^A F_{BC}\lambda\Big). \quad (54)$$

- A similar modification has been observed for the SUSY brane action in 5D and 6D ungauged supergravities on orbifolds.

- The modified field strength and Bianchi identity are

$$\hat{G}_{11ABC} = 4\partial_{[11}C_{ABC]} + \frac{\kappa^2}{\sqrt{2}g^2}\delta(x^{11})\omega_{BCD}, \quad (55)$$

$$d\hat{G}_{11ABCD} = -3\sqrt{2}\frac{\kappa^2}{g^2}\delta(x^{11})F^a_{[AB}F^a_{CD]}. \quad (56)$$

- Modifications of gauge and SUSY transformations are necessary:

$$\delta_{\rm gauge}C_{11AB} = \frac{\kappa^2}{6\sqrt{2}g^2}\delta(x^{11})\mathrm{tr}(\Lambda F_{AB}), \quad (57)$$

$$\delta C_{11AB} = -\frac{\kappa^2}{12\sqrt{2}g^2}\delta(x^{11})\mathrm{tr}(A_A\bar{\varepsilon}\Gamma_B\lambda - A_B\bar{\varepsilon}\Gamma_A\lambda). \quad (58)$$

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The singular terms in the linearized action

- The singular term in $\hat{G}_{\mu mn}$ remains uncancelled as

$$\hat{G}_{\mu mn} = \partial_\mu B_{mn} + 2\partial_{[m}B_{n]\mu} + \frac{1}{2}F_{mn}A_\mu + \frac{1}{2}F_{\mu[m}A_{n]} - \sum_i \xi_i A_\mu \epsilon_{mn}\frac{\delta^2(y-y_i)}{e_2}$$

- From the equation for the KR field,

$$\partial_M(\sqrt{-g}e^\phi \hat{G}^{MNP}) = 0, \qquad (59)$$

for the football solution, we get the solution

$$\hat{G}_{\mu mn} = C_\mu \epsilon_{mn}, \quad \partial_m C_\mu = 0. \qquad (60)$$

- Redefining the KR field as $B^m{}_\mu \equiv -\epsilon^{mn}\partial_n W_\mu$ and $B_{mn} \equiv b(x)\epsilon_{mn}$, for $\partial_m A_\mu = 0$, we get the equation

$$\Box^{(2)} W_\mu = \partial_\mu b + \frac{q}{2} e^{-\frac{1}{2}\phi_0} A_\mu - C_\mu - \frac{1}{2}\sum_i \xi_i A_\mu \frac{\delta^2(y-y_i)}{e_2}. \quad (61)$$

- Since the 2D volume integral of the left-hand side vanishes for extra dimensions with no boundary, we determine C_μ as

$$C_\mu = \partial_\mu b + \frac{\lambda\pi q r_0^2}{2} e^{-\frac{1}{2}\phi_0} A_\mu - \frac{1}{2}(\xi_0 - \xi_\infty)A_\mu. \quad (62)$$

The last term vanishes for $T_0 = T_\infty$.

- The singular terms in $\hat{G}^2_{\mu mn}$ disappear after integrating out the 4D vector modes of the KR field.
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- Redefining the KR field as $B^m{}_\mu \equiv -\epsilon^{mn}\partial_n W_\mu$ and $B_{mn} \equiv b(x)\epsilon_{mn}$, for $\partial_m A_\mu = 0$, we get the equation

$$\Box^{(2)} W_\mu = \partial_\mu b + \frac{q}{2} e^{-\frac{1}{2}\phi_0} A_\mu - C_\mu - \frac{1}{2}\sum_i \xi_i A_\mu \frac{\delta^2(y-y_i)}{e_2}. \quad (61)$$

- Since the 2D volume integral of the left-hand side vanishes for extra dimensions with no boundary, we determine C_μ as

$$C_\mu = \partial_\mu b + \frac{\lambda\pi q r_0^2}{2} e^{-\frac{1}{2}\phi_0} A_\mu - \frac{1}{2}(\xi_0 - \xi_\infty) A_\mu. \quad (62)$$

The last term vanishes for $T_0 = T_\infty$.

- The singular terms in $\hat{G}^2_{\mu mn}$ disappear after integrating out the 4D vector modes of the KR field.
- The axion $b = \frac{1}{2} B_{mn}\epsilon^{mn}$ is the Goldstone boson eaten up by A_μ by a Green-Schwarz mechanism.

Example: Heterotic M-theory

[Horava,Witten(1996)]

- In heterotic M-theory, the 11D supergravity multiplet is composed of e_I^m, ψ_I and C_{IJK}, and an E_8 super Yang-Mills is localized on each boundary of $M_{10} \times S^1/Z_2$.
- In the SUSY brane action for the E_8 Yang-Mills, a Noether term must be added as

$$\mathcal{L}_{YM} = -\frac{1}{g^2}\int d^{10}x\sqrt{g}\,\mathrm{tr}\Big(\frac{1}{4}F_{AB}F^{AB} + \frac{1}{2}\bar{\lambda}\Gamma^A D_A\lambda\Big) + \frac{1}{4}\bar{\psi}_A\Gamma^{BC}\Gamma^A F_{BC}\lambda\Big). \quad (54)$$

- A similar modification has been observed for the SUSY brane action in 5D and 6D ungauged supergravities on orbifolds.

- The bulk action and the SUSY transformation are modified by replacing G_{MNP} and F_{MN} with the hatted ones (keeping A_M as it is):

$$\hat{G}_{\mu mn} = G_{\mu mn} + (j_\mu - \xi_0 A_\mu)\epsilon_{mn}\frac{\delta^2(y)}{e_2}, \tag{44}$$

$$\hat{G}_{\tau\rho\sigma} = G_{\tau\rho\sigma} + \frac{\delta^2(y)}{e_2} j_{\tau\rho\sigma}, \tag{45}$$

$$\hat{F}_{mn} = F_{mn} + (rg|Q|^2 - \xi_0)\epsilon_{mn}\frac{\delta^2(y)}{e_2} \tag{46}$$

with

$$j_\mu = \frac{1}{2} i\Big[Q^\dagger D_\mu Q - (D_\mu Q)^\dagger Q + \frac{1}{2}\bar{\psi}\gamma_\mu\psi\Big], \tag{47}$$

$$j_{\tau\rho\sigma} = -\frac{1}{4}\bar{\psi}\gamma_{\tau\rho\sigma}\psi. \tag{48}$$

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- The modified field strength and Bianchi identity are

$$\hat{G}_{11ABC} = 4\partial_{[11}C_{ABC]} + \frac{\kappa^2}{\sqrt{2}g^2}\delta(x^{11})\omega_{BCD}, \quad (55)$$

$$d\hat{G}_{11ABCD} = -3\sqrt{2}\frac{\kappa^2}{g^2}\delta(x^{11})F^a_{[AB}F^a_{CD]}. \quad (56)$$

- Modifications of gauge and SUSY transformations are necessary:

$$\delta_{\rm gauge}C_{11AB} = \frac{\kappa^2}{6\sqrt{2}g^2}\delta(x^{11}){\rm tr}(\Lambda F_{AB}), \quad (57)$$

$$\delta C_{11AB} = -\frac{\kappa^2}{12\sqrt{2}g^2}\delta(x^{11}){\rm tr}(A_A\bar{\varepsilon}\Gamma_B\lambda - A_B\bar{\varepsilon}\Gamma_A\lambda). \quad (58)$$

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