

Title: Hamiltonian Quantum Cellular Automata in 1D

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URL: <http://pirsa.org/08010008>

Abstract: We construct a simple translationally invariant, nearest-neighbor Hamiltonian on a chain of 10-dimensional qudits that makes it possible to realize universal quantum computing without any external control during the computational process, requiring only initial product state preparation. Both the quantum circuit and its input are encoded in an initial canonical basis state of the qudit chain. The computational process is then carried out by the autonomous Hamiltonian time evolution. After a time greater than a polynomial in the size of the quantum circuit has passed, the result of the computation can be obtained with high probability by measuring a few qudits in the computational basis.

This result also implies that there cannot exist efficient classical simulation methods for generic translationally invariant nearest-neighbor Hamiltonians on qudit chains, unless quantum computers can be efficiently simulated by classical computers (or, put in complexity theoretic terms, unless  $BPP=BQP$ ). This is joint work with Daniel Nagaj.

# Hamiltonian Quantum Cellular Automata in 1D

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## Daniel Nagaj



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quantum circuit model



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quantum circuit model  
single qubit gates  
two qubit gate



# Hamiltonian Quantum Cellular Automata in 1D

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quantum circuit modelled  
initialise in comp basis states  
single qubit gates  
two qubit gate  
measure in the comp basis

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Quantum circuit model  
initial in comp basis states  
single qubit gates  
two qubit gate  
measure in the comp basis  
polynomial  
performance based



COMMON  
computation is driven  
by external control  
operations



# Daniel Nagaj

quantum circuit model  
influence in comp basis states  
single qubit gates  
two qubit gate  
measure in the comp basis

topological  
measurement based

COMMON  
computation is driven  
by external control  
operations  
time-dependent Ham.

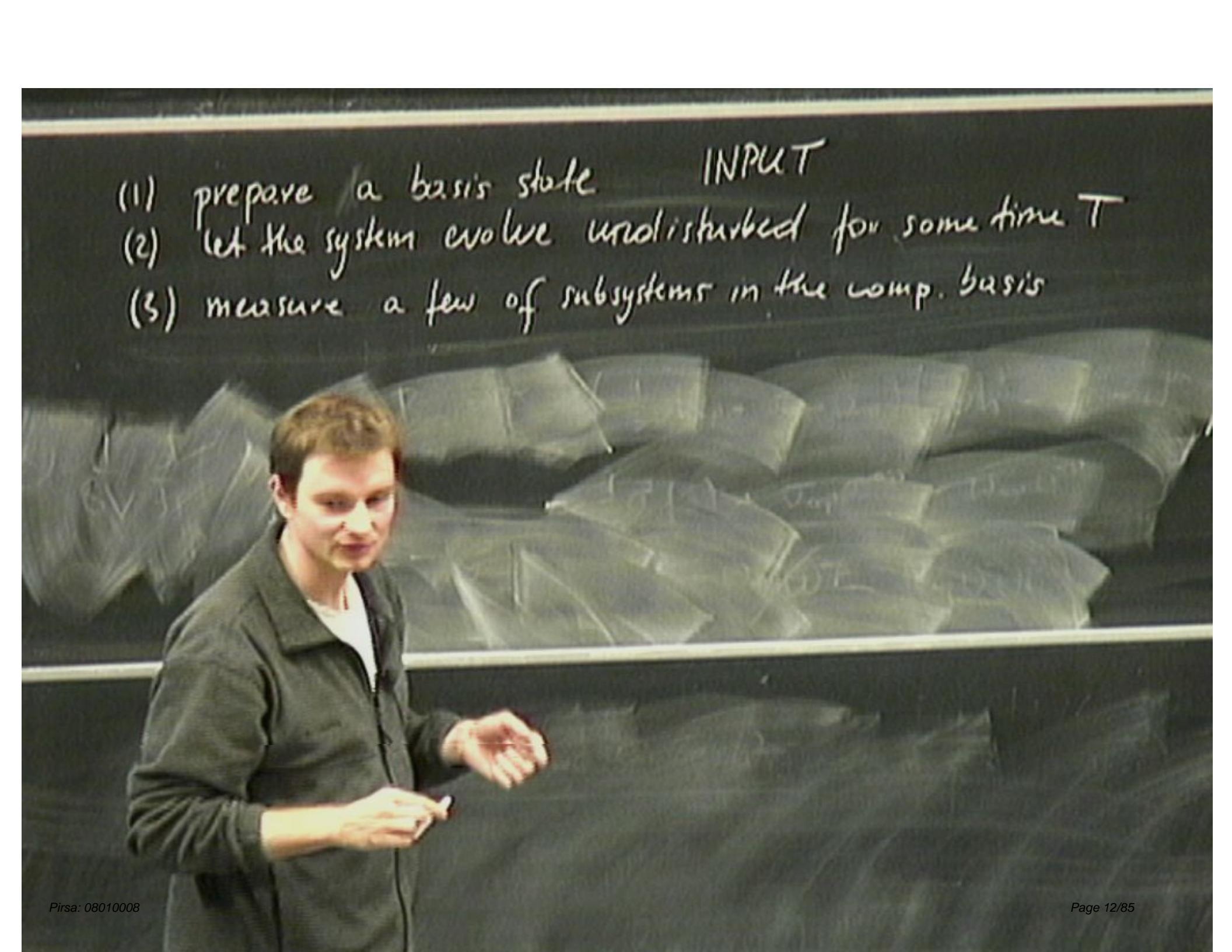
(ii) prepare a basis state



- (1) prepare a basis state
- (2) let the system evolve undisturbed for some time  $T$



- (1) prepare a basis state
- (2) let the system evolve unobserved for some time T
- (3) measure a few of subsystems in the comp. basis

- 
- (1) prepare a basis state      INPUT
- (2) let the system evolve unobserved for some time T
- (3) measure a few of subsystems in the comp. basis

- INPUT PROGRAM
- (1) prepare a basis state
  - (2) let the system evolve undisturbed for some time T
  - (3) measure a few of subsystems in the comp. basis

# Hamiltonian Quantum Cellular Automata in 1D

## Daniel Nagaj

quantum circuit model  
initialise in comp. basis states  
single qubit gates  
two qubit gate  
measure in the comp basis  
topological  
measurement based

COMMON  
computation is driven  
by external control  
operations  
time-dependent Ham.

- (1) prepare a basis state      INPUT      PROGRAM  
(2) let the system evolve undisturbed for some time T  
(3) measure a few of subsystems in the comp. basis



- (1) prepare a basis state      INPUT      PROGRAM
- (2) let the system evolve undisturbed for some time  $T$
- (3) measure a few of subsystems in the comp. basis

- INPUT PROGRAM
- (1) prepare a basis state
  - (2) let the system evolve until disturbed for some time  $T$
  - (3) measure a few of subsystems in the comp. basis

WHY

- (1) thermodynamics of comp.

- |  |       |         |
|--|-------|---------|
|  | INPUT | PROGRAM |
| (1) prepare a basis state                              |       |         |
| (2) let the system evolve unobserved for some time $T$ |       |         |
| (3) measure a few of subsystems in the comp. basis     |       |         |

WHY

- (1) thermodynamics of comp.
- (2) trigger new ideas

- |  |       |         |
|--|-------|---------|
|  | INPUT | PROGRAM |
| (1) prepare a basis state                              |       |         |
| (2) let the system evolve unobserved for some time $T$ |       |         |
| (3) measure a few of subsystems in the comp. basis     |       |         |

### WHY

- (1) thermodynamics of comp.
- (2) trigger new ideas
- (3) puts comp. bounds on methods for simulating Hamiltonian dynamics

- |  |       |         |
|--|-------|---------|
|  | INPUT | PROGRAM |
| (1) prepare a basis state                              |       |         |
| (2) let the system evolve unobserved for some time $T$ |       |         |
| (3) measure a few of subsystems in the comp. basis     |       |         |

### WHY

- (1) thermodynamics of comp.
  - (2) trigger new ideas
  - (3) puts comp. bounds on methods for simulating Hamiltonian dynamics
- classical

- INPUT PROGRAM
- (1) prepare a basis state
  - (2) let the system evolve unobserved for some time  $T$
  - (3) measure a few of subsystems in the comp. basis

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- (1) thermodynamics of comp.
- (2) trigger numbers
- (3) put comp bounds on methods for simulating Hamiltonian dynamics

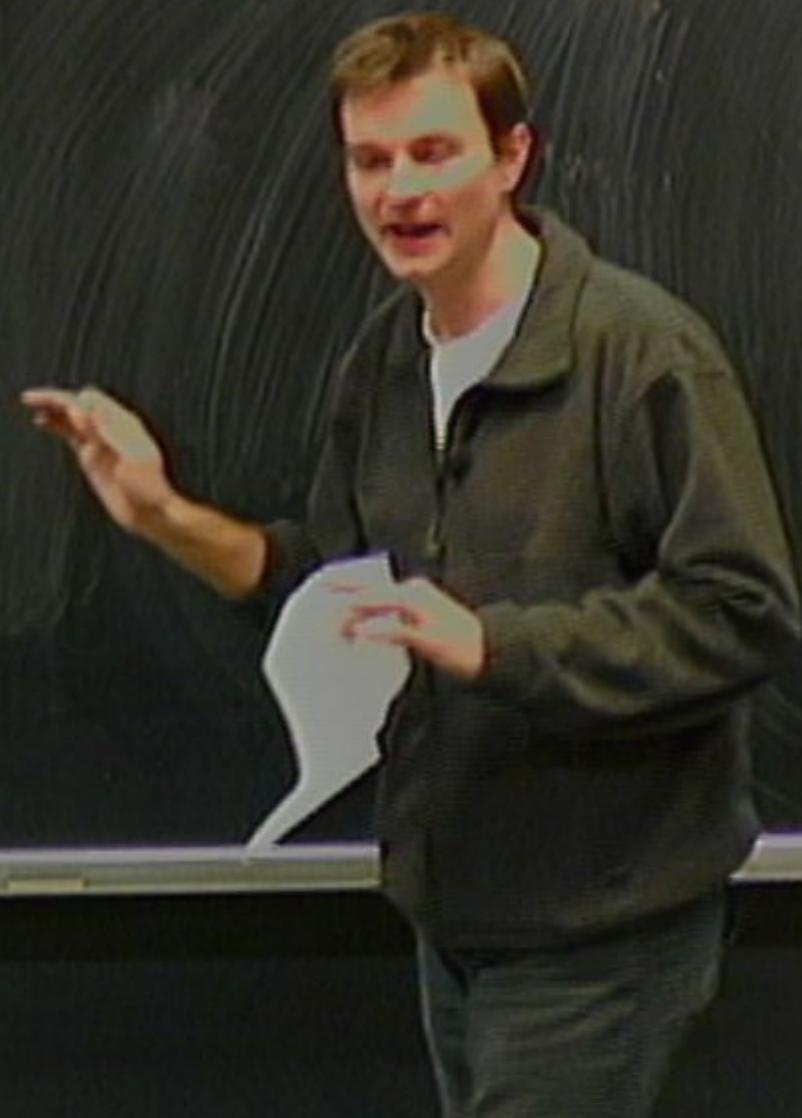
-classical

$$W = \begin{bmatrix} 1 & 1 & -\frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$$



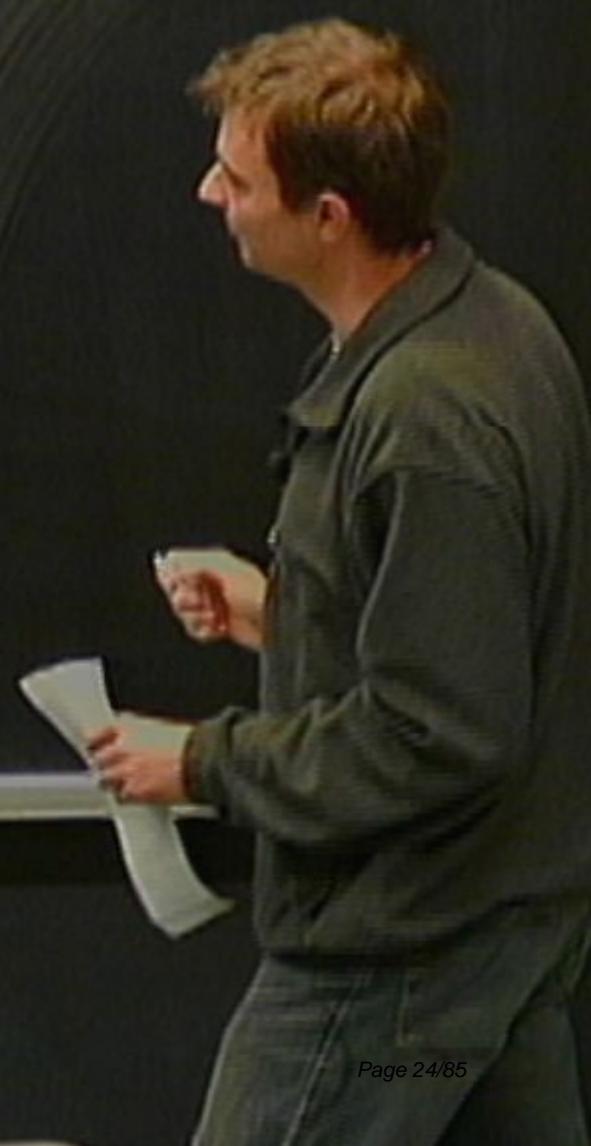
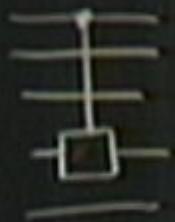
$$W = \begin{bmatrix} 1 & 1 & -\frac{1}{2} \\ 0 & \frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

universal

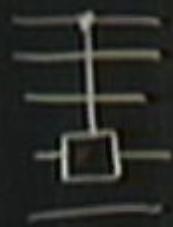


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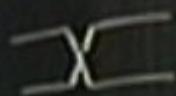
universal



$$W = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \text{ universal}$$



S swap gate

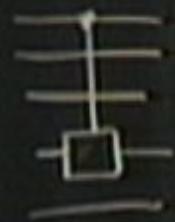




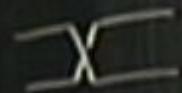
$$W = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} + \frac{i}{2} & 0 \\ 0 & 0 & \frac{1}{2} - \frac{i}{2} \end{bmatrix}$$

universal

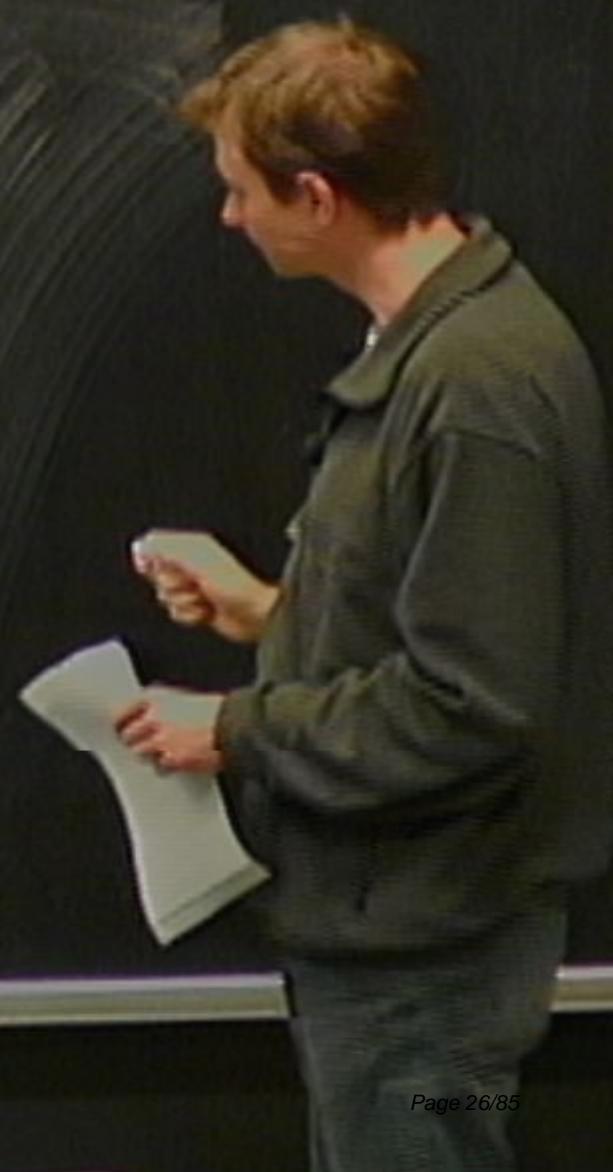
$$\begin{array}{c} w_1 \\ w_2 \\ w_3 \end{array} = \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \end{array}$$



S swap gate



I

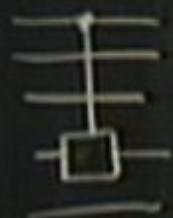


$\omega$    $\in \mathcal{C}^R(\omega_{1,1})$

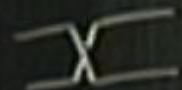
$$W = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

universal

$$\begin{array}{c|c} w_1 & \square \\ w_2 & \square \\ w_3 & \square \end{array}$$



S swap gate



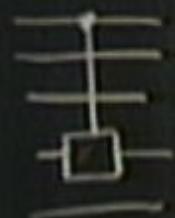
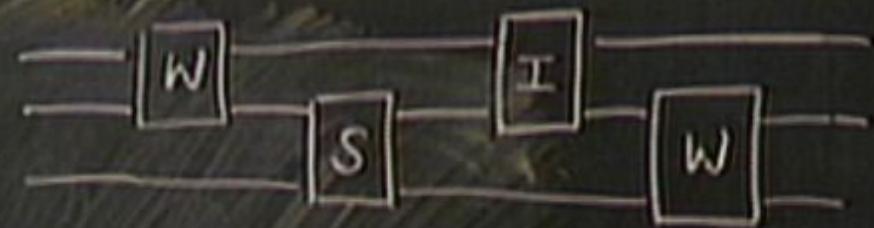
I

$$\omega \cup \text{G}^R(\omega_{1,1}) \vdash$$

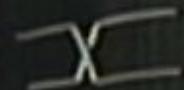
$$W = \begin{bmatrix} 1 & 1 & -\frac{1}{2} \\ 0 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$$

universal

$w_1$   
 $w_2$   
 $w_3$



S swap gate



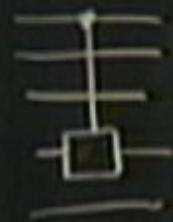
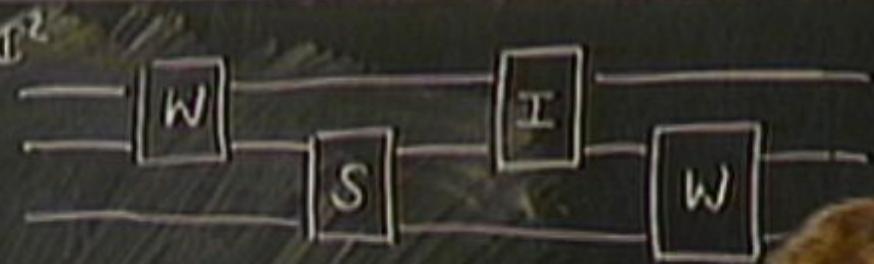
I

$$\omega \cup \mathcal{G}^R(\omega_{1,1}) \stackrel{!}{=} \dots$$

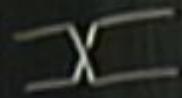
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universal

$$w_1 \\ w_2 \\ w_3$$



S swap gate



I

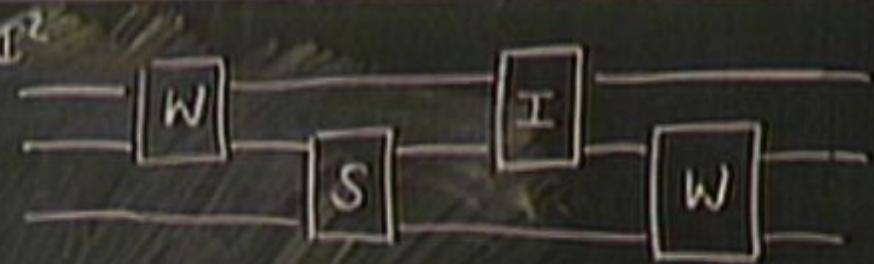


$$\omega \cup \text{arc} \in \mathcal{C}^R(\omega_{1,1})$$

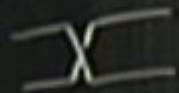
$$W = \begin{bmatrix} 1 & 1 & -\frac{1}{2} \\ 0 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$$

universal

$$w_1 \\ w_2 \\ w_3$$

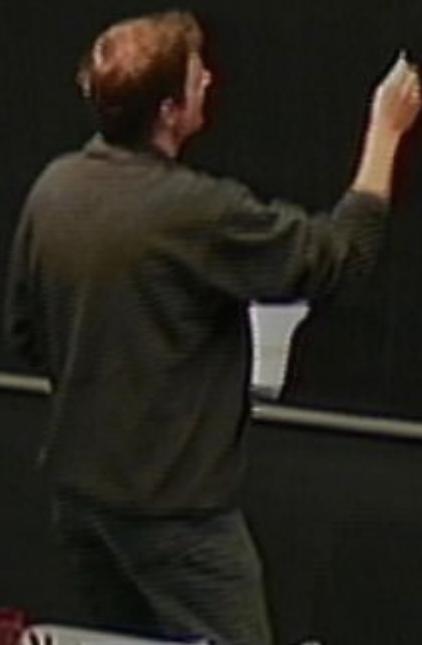


S swap gate

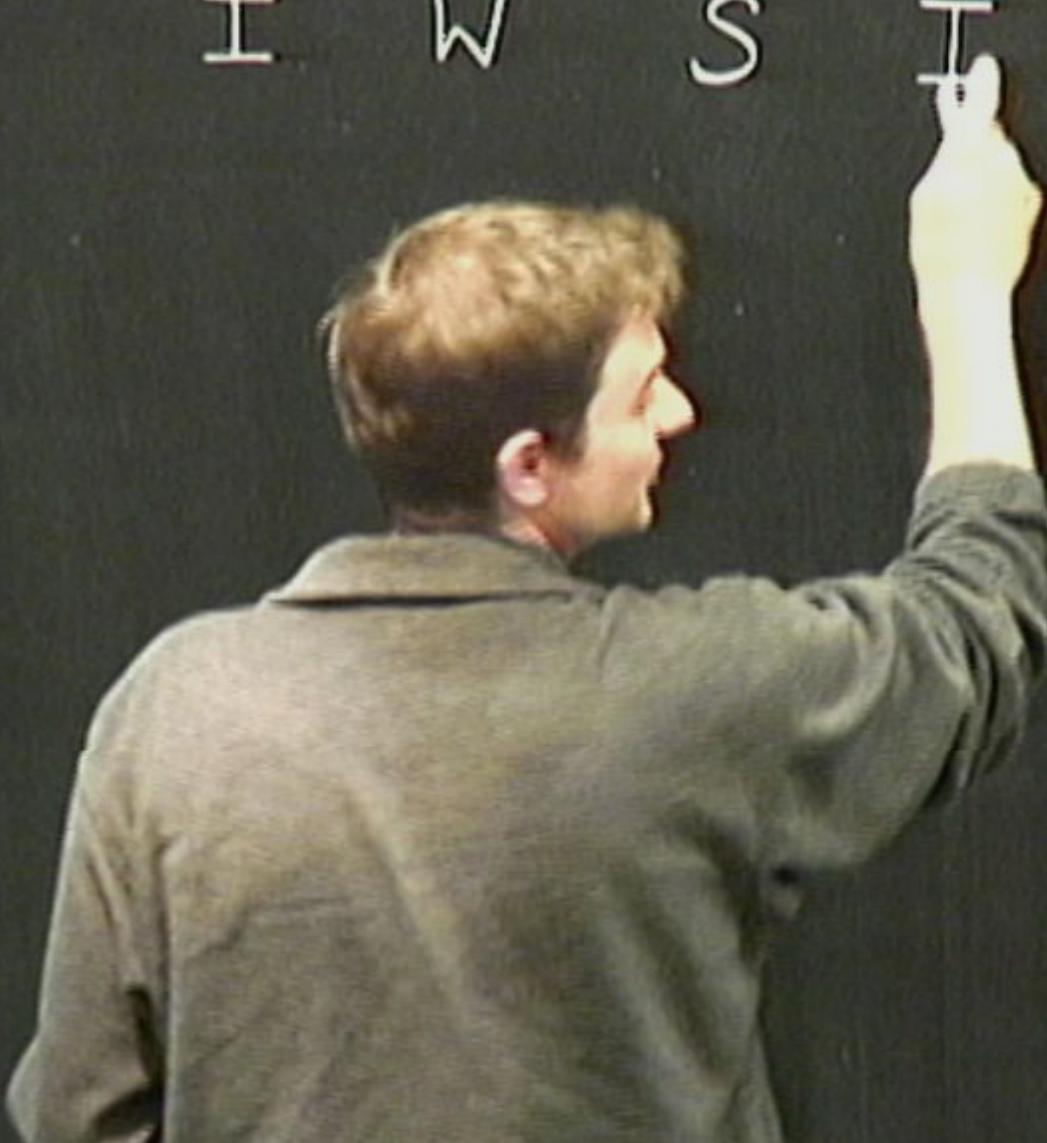


I

I W S



I W S T

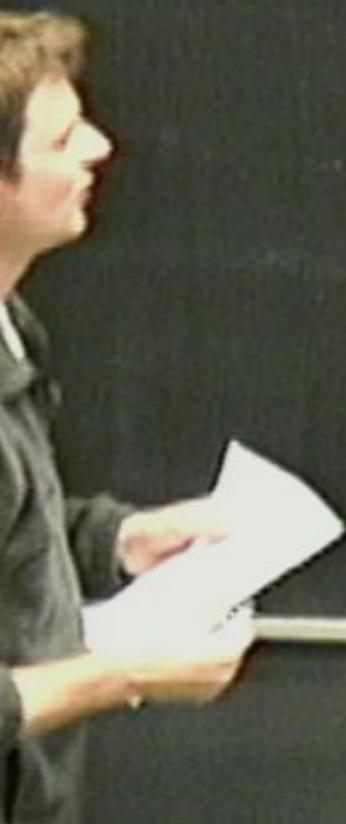


I W S I I W I I



I W S I I W I I I  
w<sub>1</sub> w<sub>2</sub> w<sub>3</sub> o o o o o o

• ► • • ► I W S I I W I I I  
○○○○○○ w<sub>1</sub> w<sub>2</sub> w<sub>3</sub> ○ ○ ○ ○ ○ ○ ○ ○



$\kappa_4$

$\bullet \bullet \blacktriangleright \bullet \bullet \blacktriangleright I W S I I W I I I$

$\circ \circ \circ \circ \circ \circ w_1 w_2 w_3 \circ \circ \circ \circ \circ \circ \circ \circ \circ$

$\mathcal{K}_d = \{$

$\overset{\bullet}{\bullet} \triangleright \overset{\bullet}{\bullet} \triangleright \overset{\bullet}{\bullet} \triangleright \overset{\bullet}{\bullet} \quad I \quad W \quad S \quad I \quad I \quad W \quad I \quad I \quad I$

$\overset{\bullet}{\bullet} \overset{\bullet}{\bullet} \overset{\bullet}{\bullet} \overset{\bullet}{\bullet} \overset{\bullet}{\bullet} \quad w_1 \quad w_2 \quad w_3 \quad o \quad o \quad o \quad o \quad o \quad o \quad o$

$$\mathcal{H}_d = \{ |0\rangle, |1\rangle \}$$

$\mathcal{H}_P$

$\begin{matrix} \cdot & \cdot & \cdot & \cdot & \cdot & I & W & S & I & I & W & I & I & I \\ 0 & 0 & 0 & 0 & 0 & w_1 & w_2 & w_3 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix}$

$$\mathcal{H}_d = \{ |0\rangle, |1\rangle \}$$

$$\mathcal{H}_P = \{ |+\rangle, |>, |x\rangle, |s\rangle, |w\rangle \}$$

•	►	•	•	►	I	W	S	I	I	W	I	I	I
0	0	0	0	0	$w_1$	$w_2$	$w_3$	0	0	0	0	0	0

$$\mathcal{H}_d = \{ |0\rangle, |1\rangle \}$$

$$\mathcal{H}_P = \{ |+\rangle, |>, |x\rangle, |s\rangle, |w\rangle \}$$

• ▶ • • ▶ I W S I I W I I I  
○ ○ ○ ○ ○ w<sub>1</sub> w<sub>2</sub> w<sub>3</sub> ○ ○ ○ ○ ○ ○ ○ ○



$$\mathcal{K}_d = \{|0\rangle, |1\rangle\} \quad \mathcal{K}_P = \{|I\rangle, |D\rangle, |T\rangle, |S\rangle, |W\rangle\}$$

$$\begin{array}{ccccccccc} \cdot & \blacktriangleright & \cdot & \blacktriangleright & I & W & S & I & I \\ 0 & 0 & 0 & 0 & w_1 & w_2 & w_3 & 0 & 0 \end{array} \quad \begin{array}{ccccccccc} W & I & I & I & W & I & I & I \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$

1:   $\rightarrow$

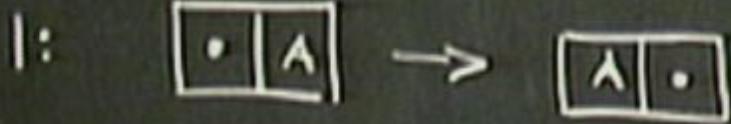
$$A \in \{I, S, W\}$$

2.

$$\mathcal{K}_d = \{ |0\rangle, |1\rangle \}$$

$$\mathcal{K}_P = \{ |I\rangle, |D\rangle, |T\rangle, |S\rangle, |W\rangle \}$$

0 0 0 0 0 0      I      W      S      T      T      W      I      I      I  
0 0 0 0 0 0       $w_1$        $w_2$        $w_3$       0      0      0      0 0 0 0

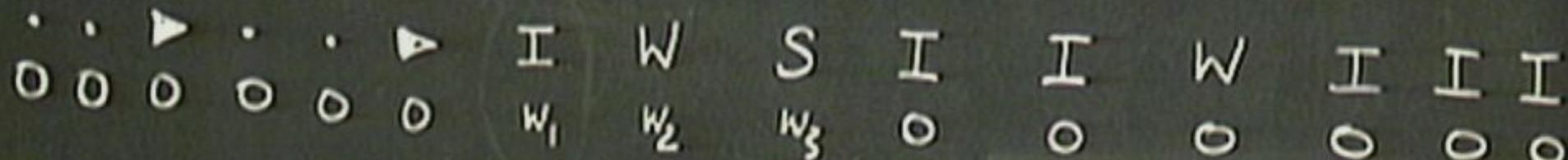


$$A \in \{I, S, W\}$$



$$\mathcal{H}_d = \{|0\rangle, |1\rangle\}$$

$$\mathcal{H}_P = \{|i\rangle, |p\rangle, |r\rangle, |s\rangle, |w\rangle\}$$



1:  $A \in \{I, S, W\}$

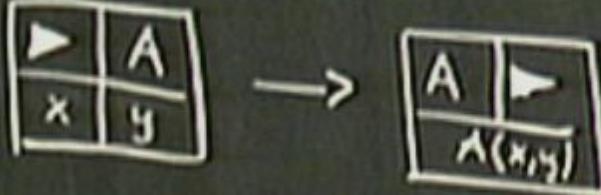
2:  $A(x,y)$

$$\mathcal{H}_d = \{|0\rangle, |1\rangle\}$$

$$\mathcal{H}_P = \{|+\rangle, |>, |x\rangle, |s\rangle, |w\rangle\}$$

$$\begin{array}{ccccccccc} & \blacktriangleright & \cdot & \cdot & \blacktriangleright & I & W & S & I \\ 0 & 0 & 0 & 0 & 0 & 0 & w_1 & w_2 & w_3 \\ & & & & & & & & \end{array}$$

1:   $A \in \{I, S, W\}$

2: 

$$\mathcal{K}_d = \{ |0\rangle, |1\rangle \}$$

$$\mathcal{K}_P = \{ |.\rangle, |\triangleright\rangle, |\tau\rangle, |s\rangle, |w\rangle \}$$

$$\begin{array}{ccccccccc} \cdot & \blacktriangleright & \cdot & \cdot & \blacktriangleright & I & W & S & I \\ 0 & 0 & 0 & 0 & 0 & 0 & w_1 & w_2 & w_3 \\ & & & & & & & & \end{array}$$

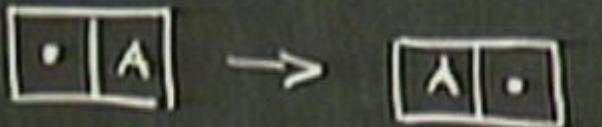


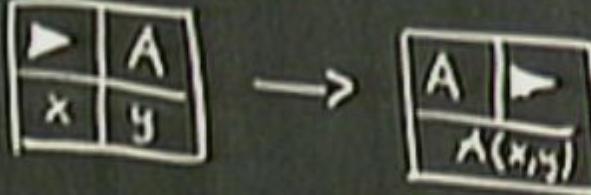
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$$\mathcal{H}_d = \{|0\rangle, |1\rangle\}$$

$$\mathcal{H}_P = \{|I\rangle, |D\rangle, |T\rangle, |S\rangle, |W\rangle\}$$

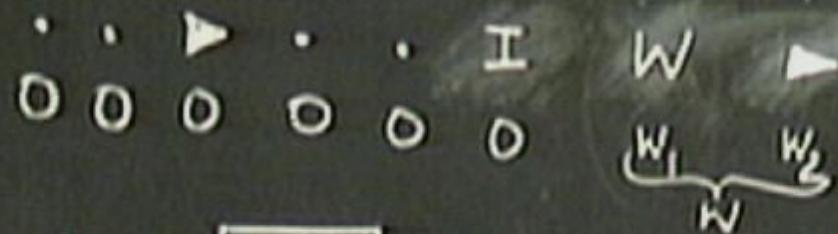
• •  $\blacktriangleright$  • • I  $\blacktriangleright$  W S I I W I I I  
0 0 0 0 0 0  $w_1$   $w_2$   $w_3$  0 0 0 0 0 0 0 0 0

1:   $A \in \{I, S, W\}$

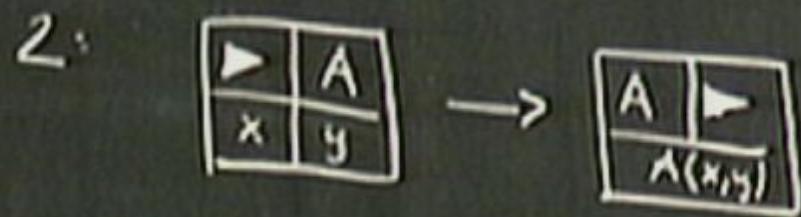
2: 

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$$\mathcal{H}_P = \{|+\rangle, |>, |1\rangle, |2\rangle, |S\rangle, |W\rangle\}$$

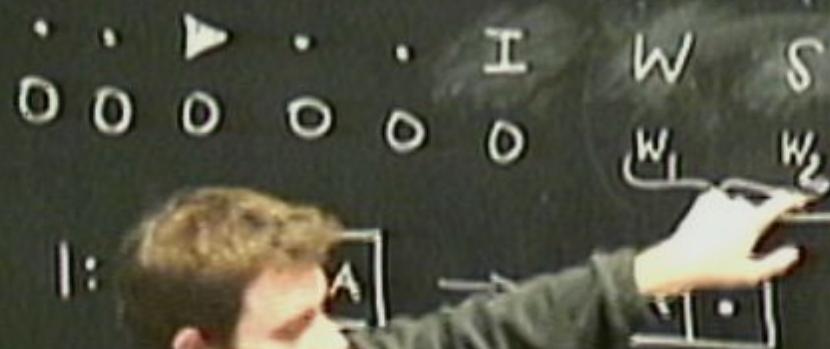


$$A \in \{I, S, W\}$$



$$\mathcal{K}_d = \{ |0\rangle, |1\rangle \}$$

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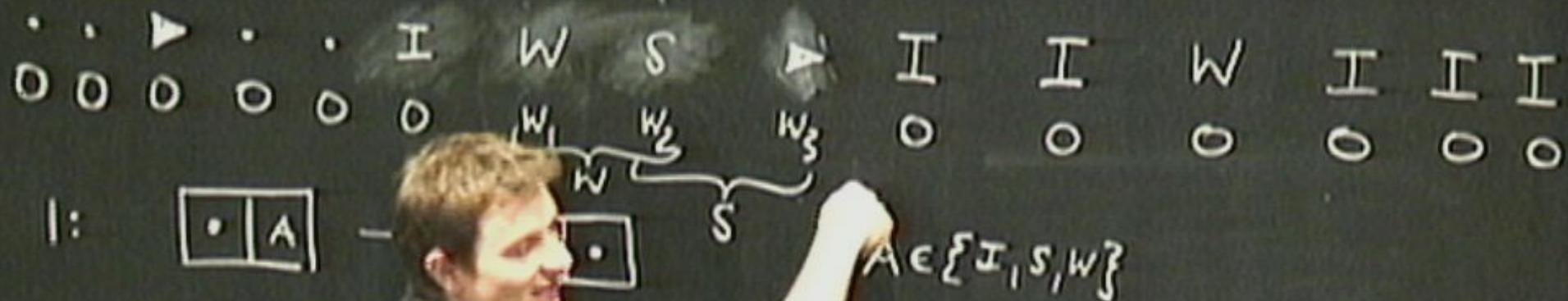


$$A \in \{I, S, W\}$$



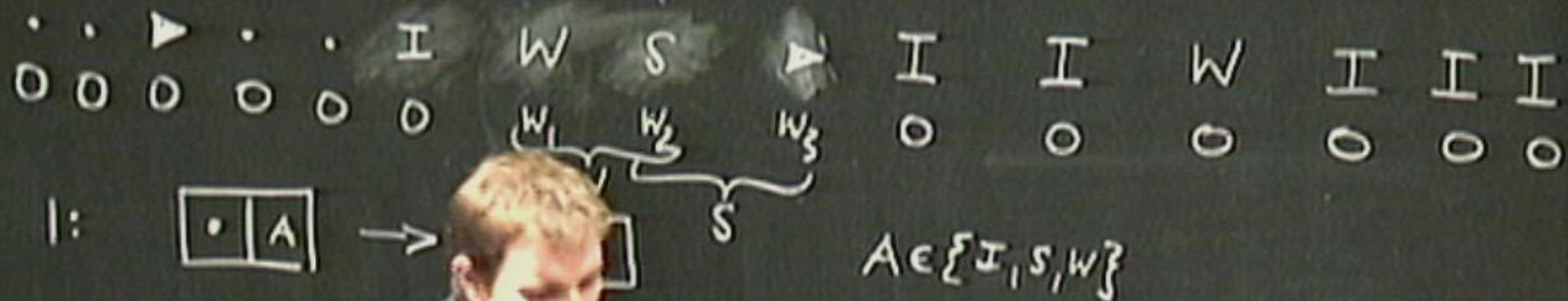
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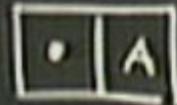


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1:

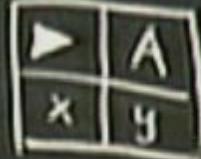


$\rightarrow$



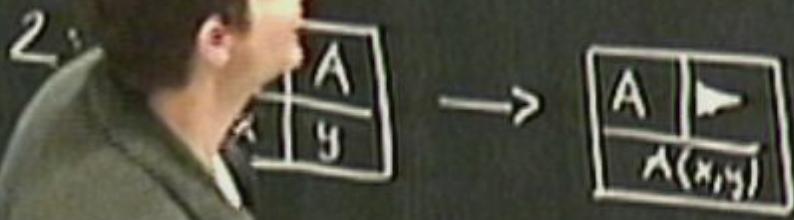
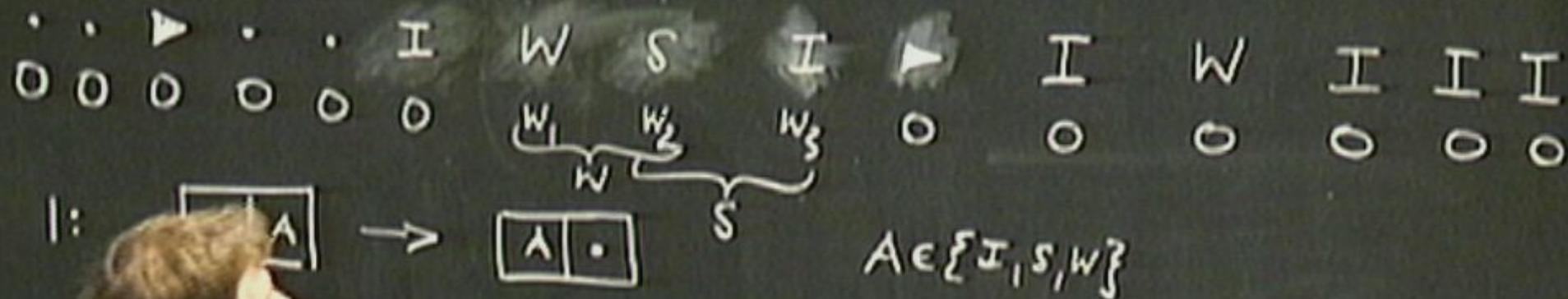
$$A \in \{I, S, W\}$$

2:



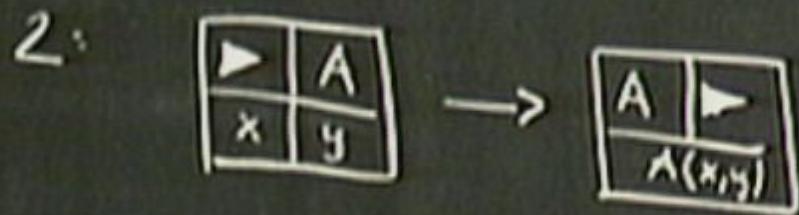
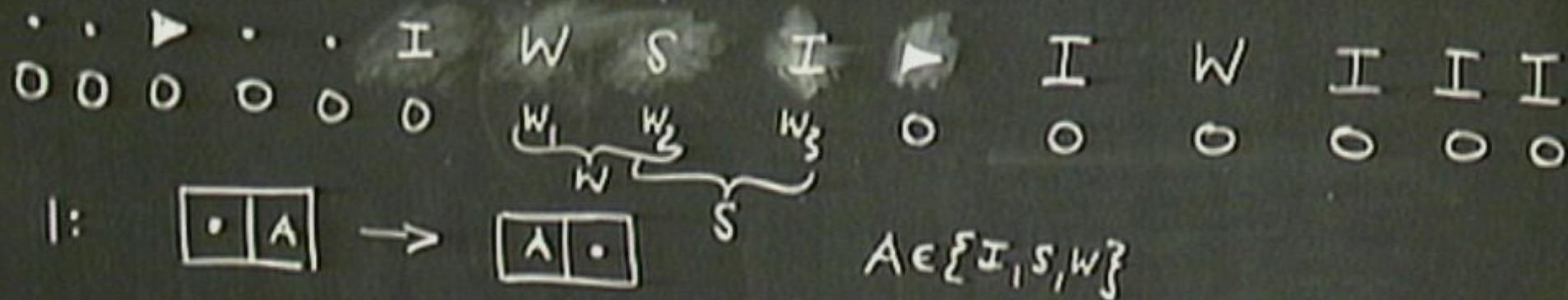
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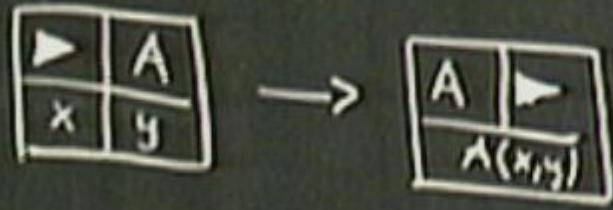
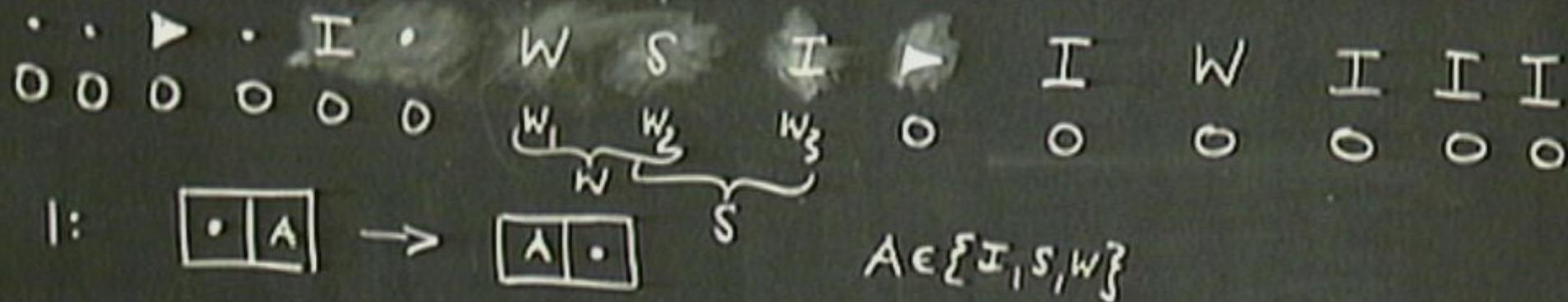
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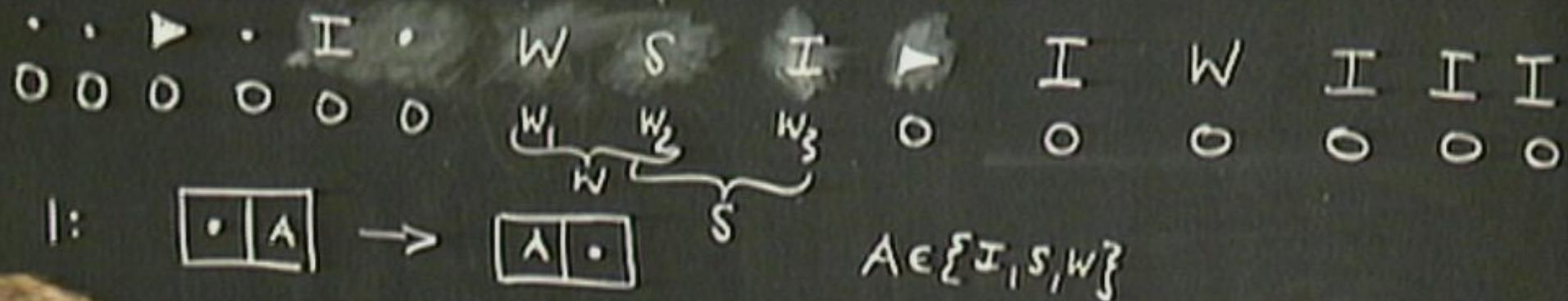
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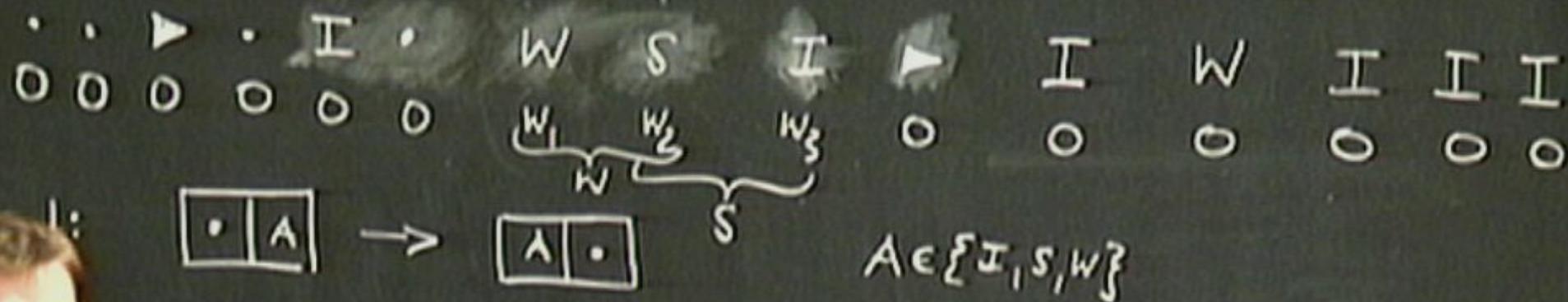
$$\mathcal{H}_d = \{ |0\rangle, |1\rangle \}$$

$$\mathcal{H}_P = \{ |\cdot\rangle, |\triangleright\rangle, |\tau\rangle, |s\rangle, |w\rangle \}$$



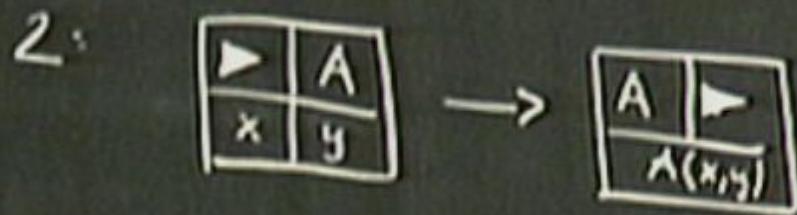
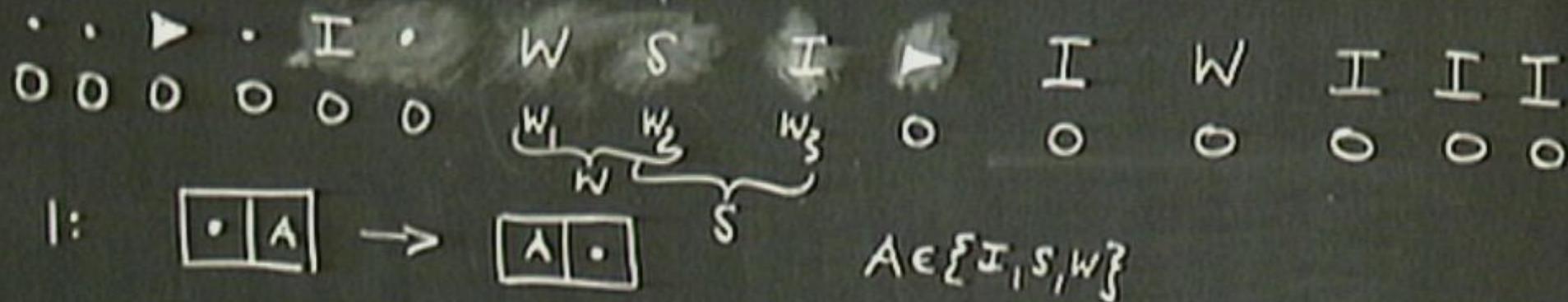
$$\mathcal{K}_d = \{|0\rangle, |1\rangle\}$$

$$\mathcal{K}_P = \{|I\rangle, |D\rangle, |T\rangle, |S\rangle, |W\rangle\}$$



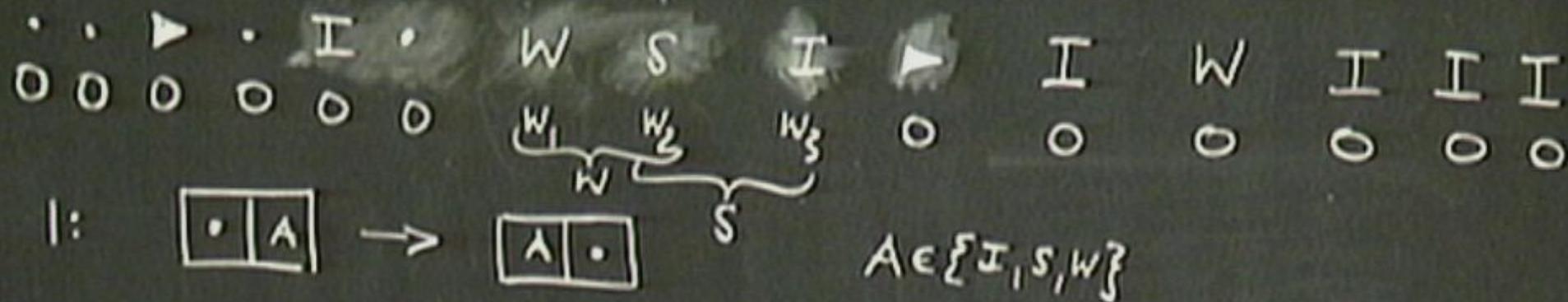
$$\mathcal{K}_d = \{ |0\rangle, |1\rangle \}$$

$$\mathcal{K}_P = \{ |\cdot\rangle, |\triangleright\rangle, |\tau\rangle, |s\rangle, |w\rangle \}$$



$$\mathcal{K}_d = \{ |0\rangle, |1\rangle \}$$

$$\mathcal{K}_P = \{ |\cdot\rangle, |\triangleright\rangle, |\tau\rangle, |s\rangle, |w\rangle \}$$

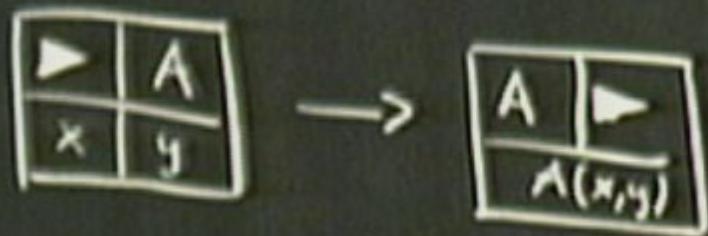
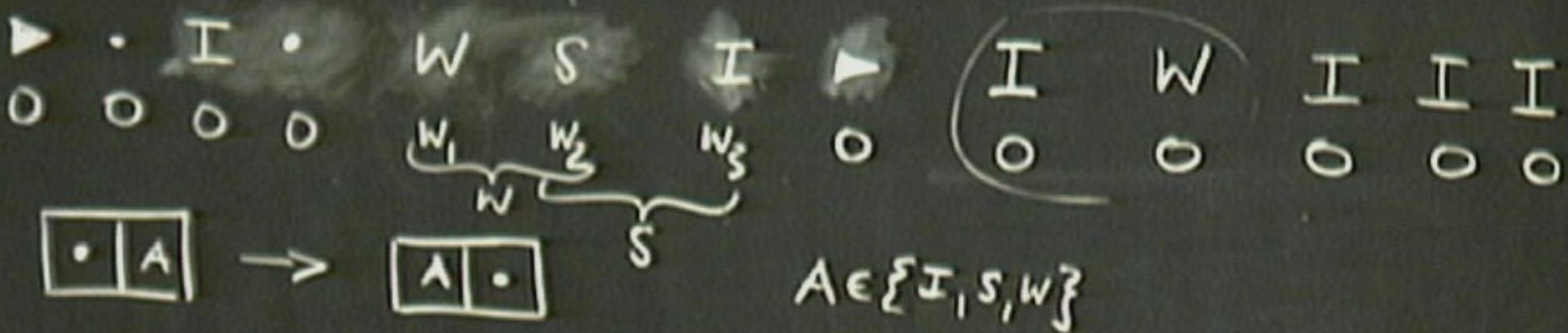


$$H = \sum_j$$

$$H = \sum_j (R_j + R_j^\dagger)$$

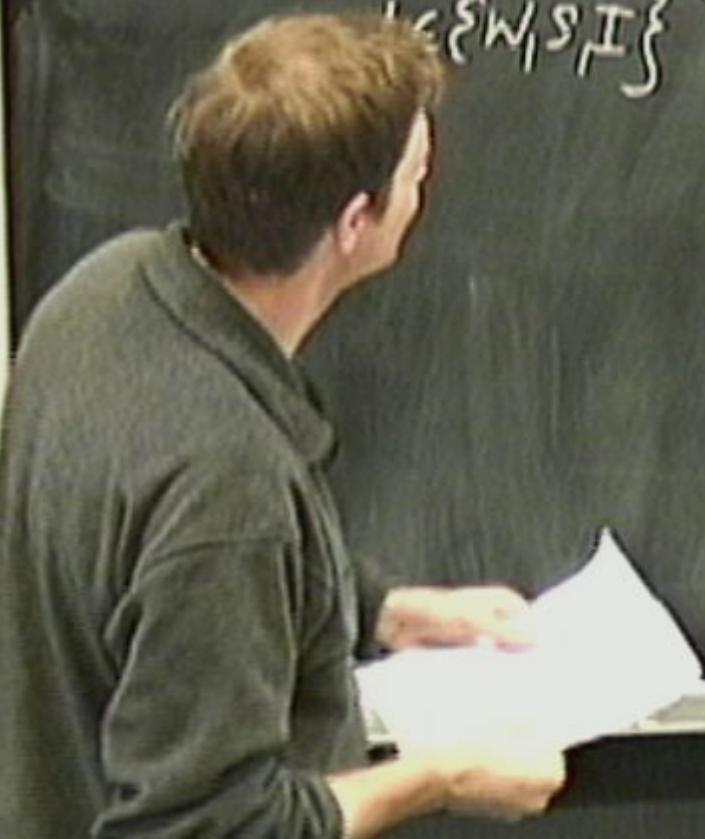


$$\mathcal{K}_d = \{ |0\rangle, |1\rangle \} \quad \mathcal{K}_P = \{ |\cdot\rangle, |\blacktriangleright\rangle, |\text{I}\rangle, |s\rangle, |w\rangle \}$$



$$H = \sum_j (R + R^+)_{j,j+1}$$

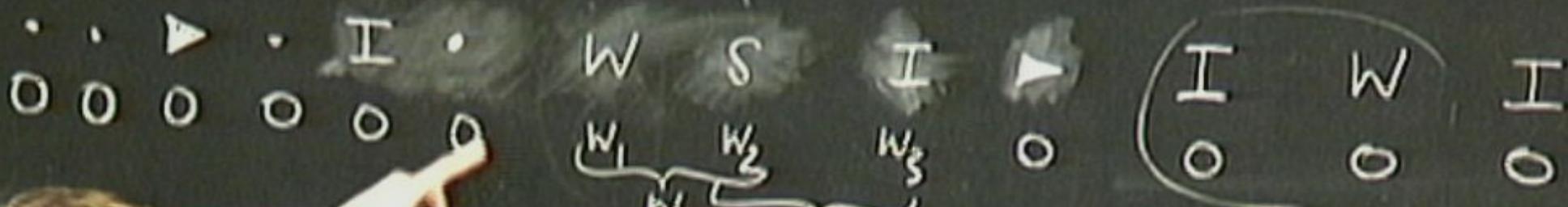
$$R = \sum_{\{e \in \mathcal{E} \mid e \neq I\}} [ |A \cdot X \cdot A| ]$$



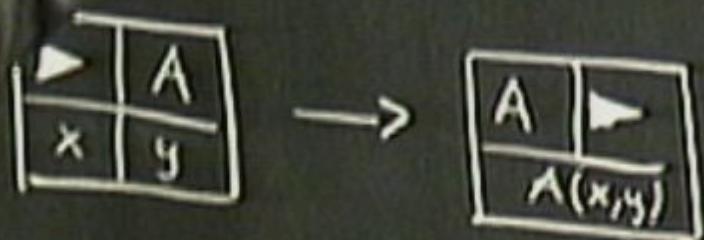
$$H = \sum_j (R + R^+)_{j,j+1}$$

$$R = \sum_{A \in \{N, S, I\}^3} [ |A \cdot X \cdot A| \otimes I ]$$

$$\mathcal{K}_d = \{|0\rangle, |1\rangle\} \quad \mathcal{K}_p = \{|+\rangle, |>\rangle, |I\rangle, |S\rangle, |W\rangle\}$$



$$A \in \{I, S, W\}$$

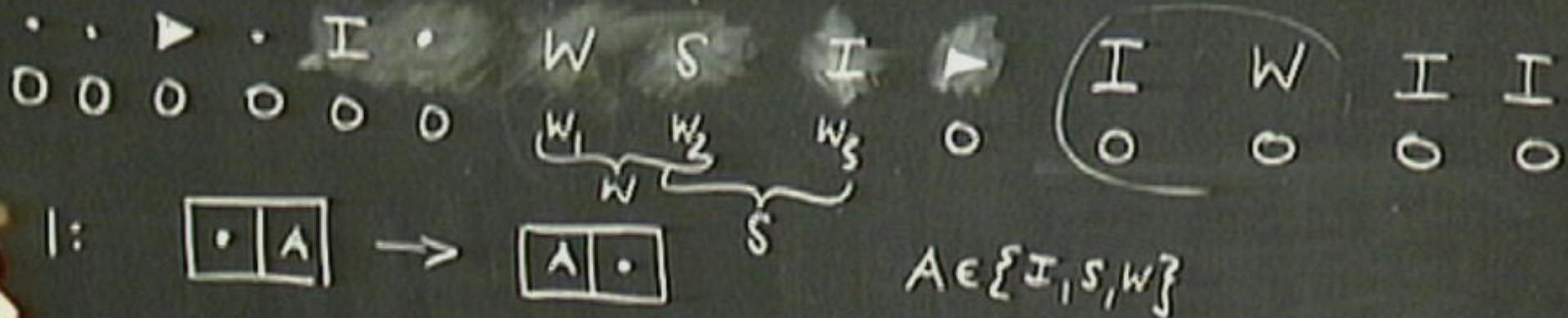


$$H = \sum_j (R + R^+)_{j,j+1}$$

$$R = \sum_{A \in \mathcal{E}_{W,S,I}} \left[ |A \cdot X \cdot A|_{p_1 p_2} \otimes I_{d_1 d_2} + |A \triangleright X \triangleright A|_{pp} \otimes A_{d_1 d_2} \right]$$

$$\mathcal{K}_d = \{ |0\rangle, |1\rangle \}$$

$$\mathcal{K}_p = \{ |+\rangle, |>\rangle, |x\rangle, |s\rangle, |w\rangle \}$$



1:



$$A \in \{I, S, W\}$$

2:

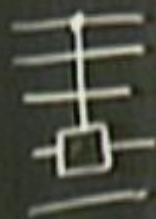
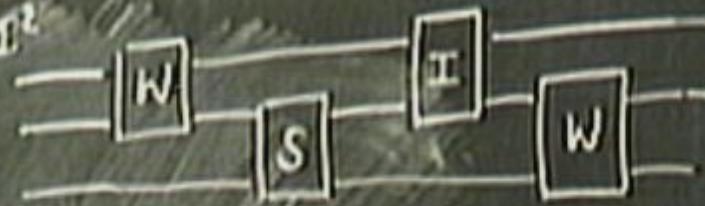


$$\omega \quad G^R(\omega_{11}) \quad \frac{1}{\omega}$$

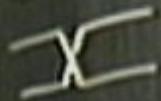
$$W = \begin{bmatrix} 1 & 0 & -\frac{1}{\alpha^2} \\ 0 & 1 & \frac{1}{\alpha^2} \\ 0 & 0 & 1 \end{bmatrix}$$

universal

$N_1$   
 $N_2$   
 $N_3$



S swap gate

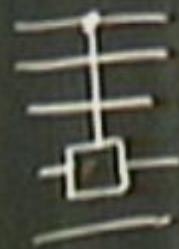
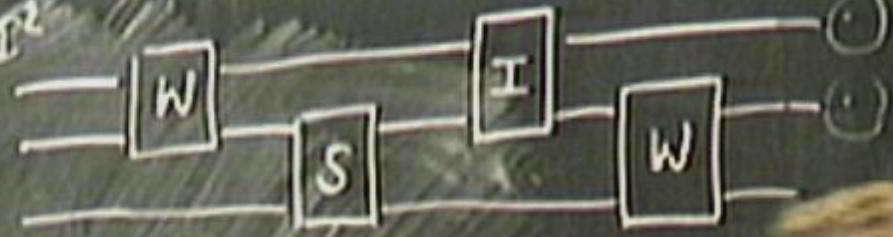


I

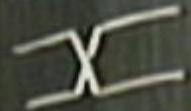
$$W = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

universal

$\oplus^2$   
 $w_1$   
 $w_2$   
 $w_3$



S swap gate



I

$$\mathcal{K}_d = \{ |0\rangle, |1\rangle \}$$

$$\mathcal{K}_p = \{ |+\rangle, |-\rangle, |I\rangle, |S\rangle, |W\rangle \}$$

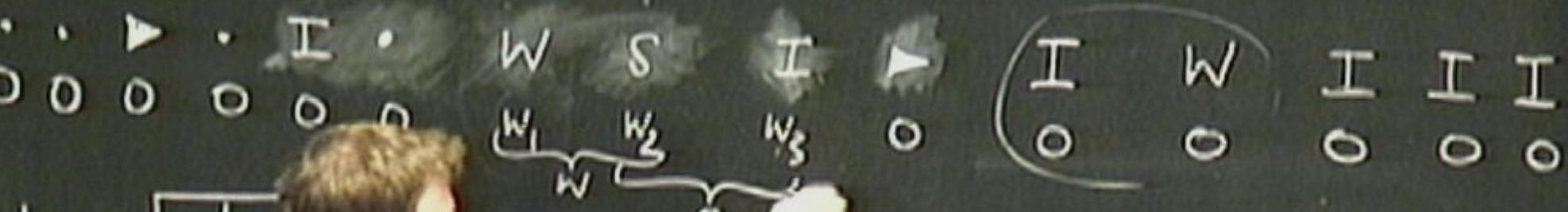


$$A \in \{I, S, W\}$$

2.

$$\mathcal{K}_d = \{ |0\rangle, |1\rangle \}$$

$$\mathcal{K}_p = \{ |.\rangle, |\triangleright\rangle, |I\rangle, |S\rangle, |W\rangle \}$$



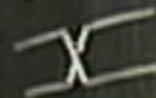
$$A \in \{I, S, W\}$$

2.

$$W = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$



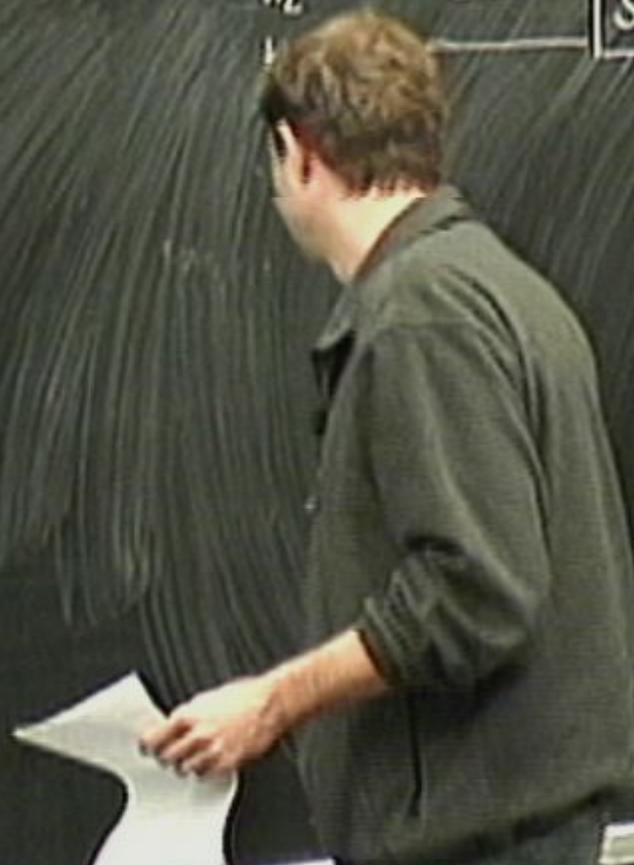
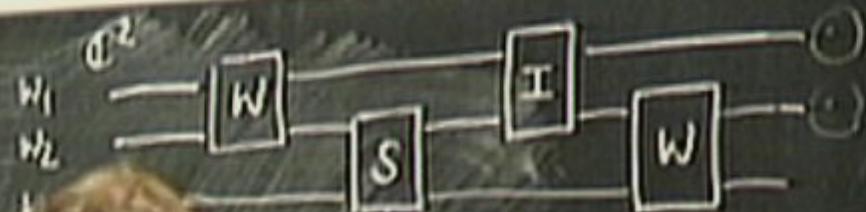
S swap rule



I

universal

$$G(\omega_1) = \dots$$



Required-Time



Required-Time

•  $\nabla \rightarrow 0$

$W, S, I \rightarrow 1$

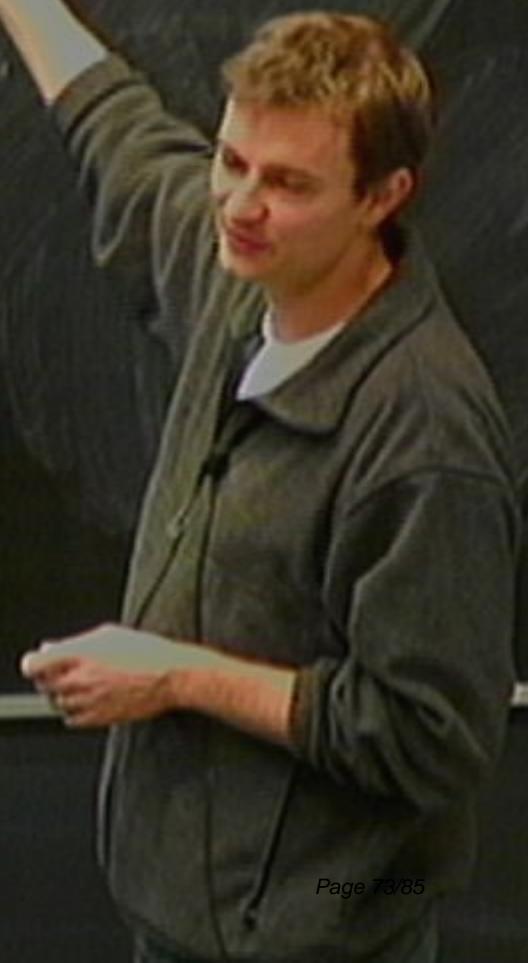


Required-Time

•  $\triangleright \rightarrow 0$

$W, S, I \rightarrow I$

$$H = \sum_j \left( (|0\rangle\langle 0| + |1\rangle\langle 1|)_{j,j+1} \right)$$



Required-Time

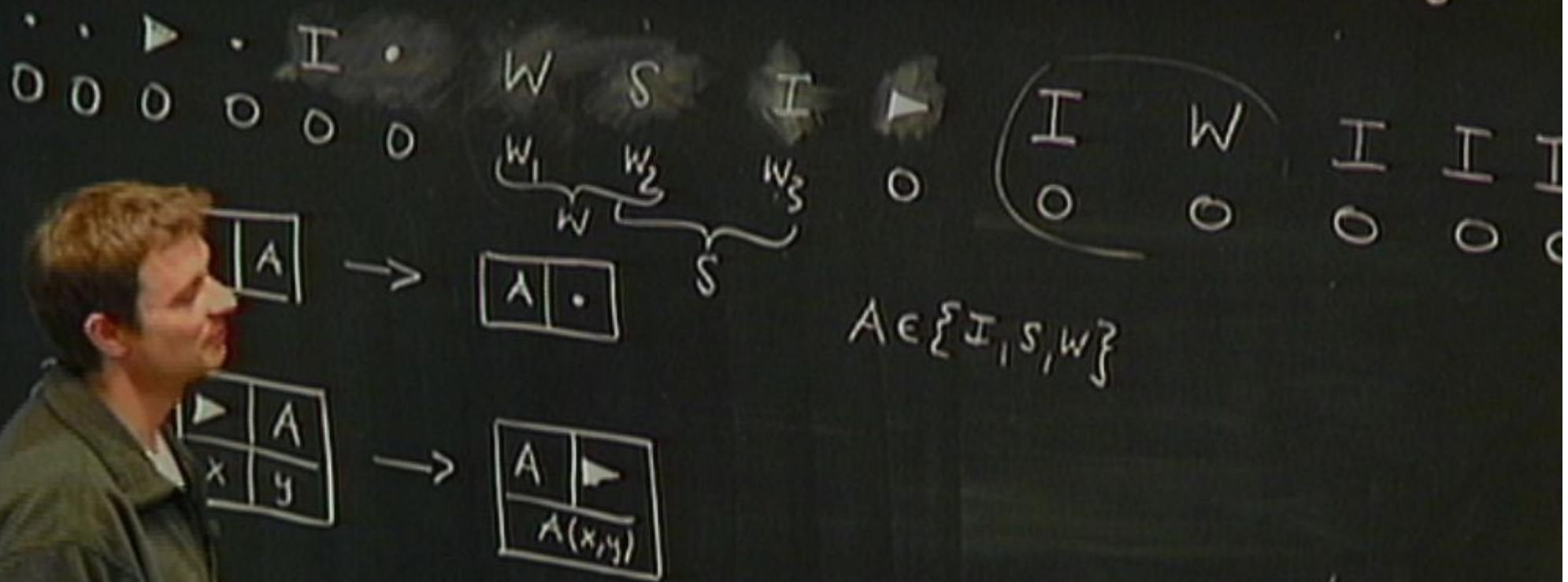
•  $\triangleright \rightarrow 0$

$W, S, I \rightarrow I$

$$H = \sum_j ((o_0 \times o_1) + (o_1 \times o_0))_{j, j+1}$$

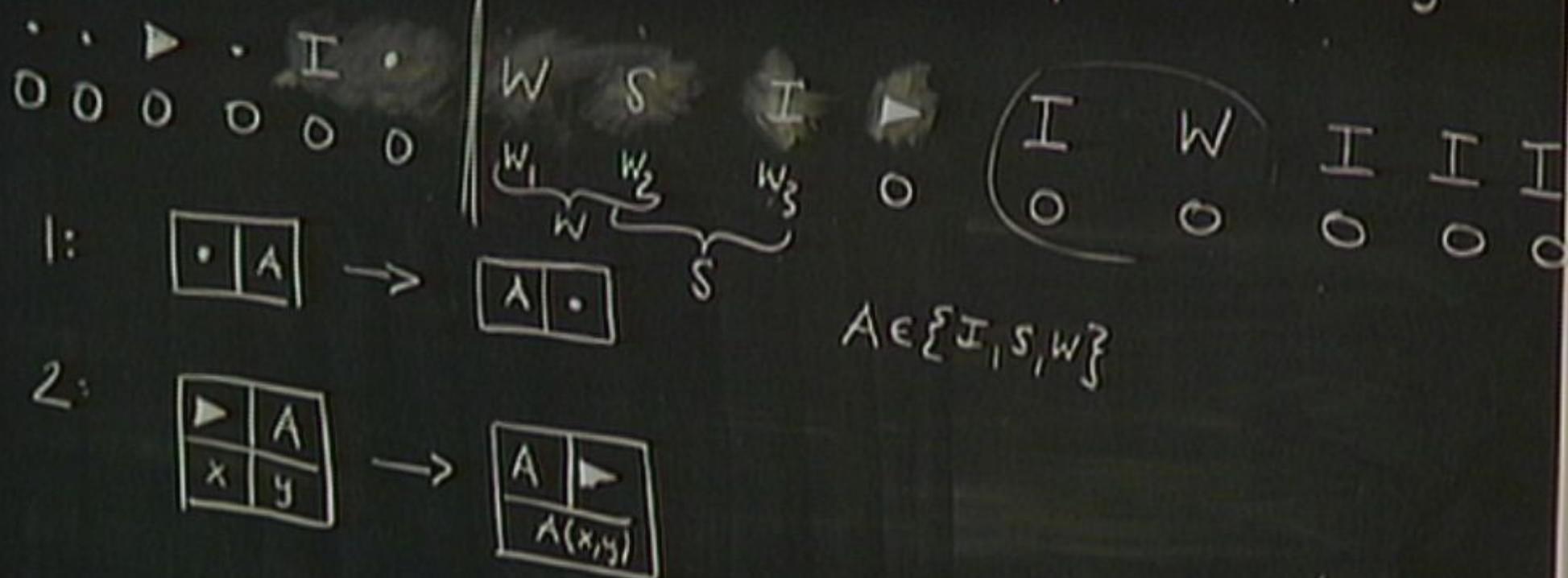
$$\mathcal{K}_d = \{|0\rangle, |1\rangle\}$$

$$\mathcal{K}_p = \{|.\rangle, |\triangleright\rangle, |I\rangle, |S\rangle, |W\rangle\}$$



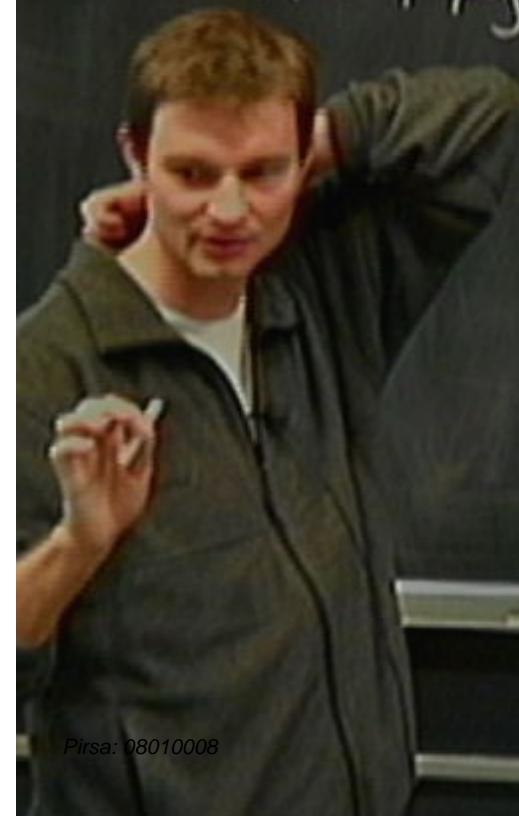
$$\mathcal{K}_d = \{ |0\rangle, |1\rangle \}$$

$$\mathcal{K}_P = \{ |\cdot\rangle, |\triangleright\rangle, |\ddot{\sqcap}\rangle, |\ddot{\sqcup}\rangle, |s\rangle, |w\rangle \}$$



$$H = \sum_j (R + R^+)_{j,j+1}$$

$$R = \sum_{A \in \{W, S, I\}^3} \left[ |A \cdot X \cdot A|_{P_1 P_2} \otimes I_{d_1 d_2} + |A \triangleright X \triangleright A|_{P_1 P_2} \otimes A_{d_1 d_2} \right]$$



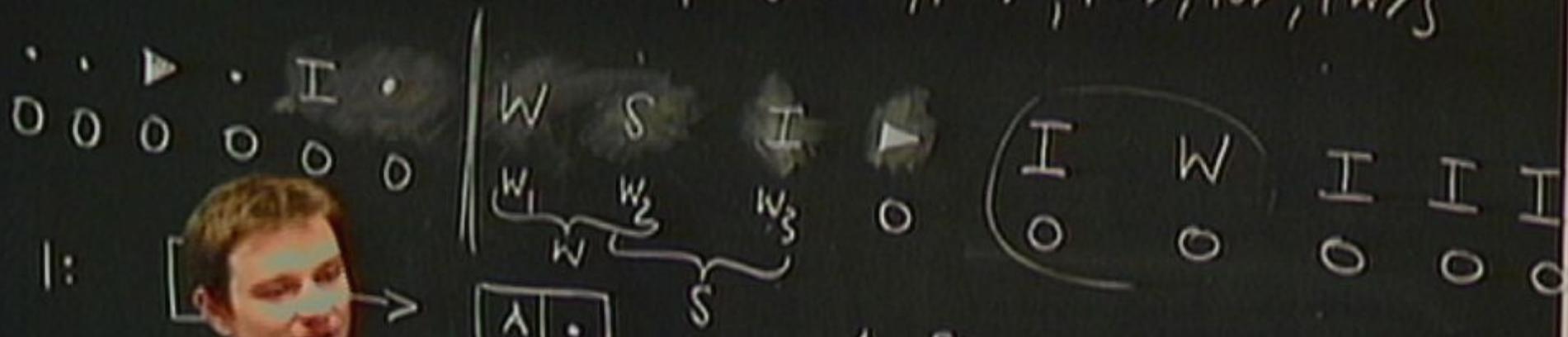
$$H = \sum_j (R + R^+)_{j,j+1}$$

$$R = \sum_{A \in \{W, S, I\}^3} \left[ |A \cdot X \cdot A|_{P_1 P_2} \otimes I_{d_1 d_2} + |A \triangleright X \triangleright A|_{P_1 P_2} \otimes A_{d_1 d_2} \right]$$

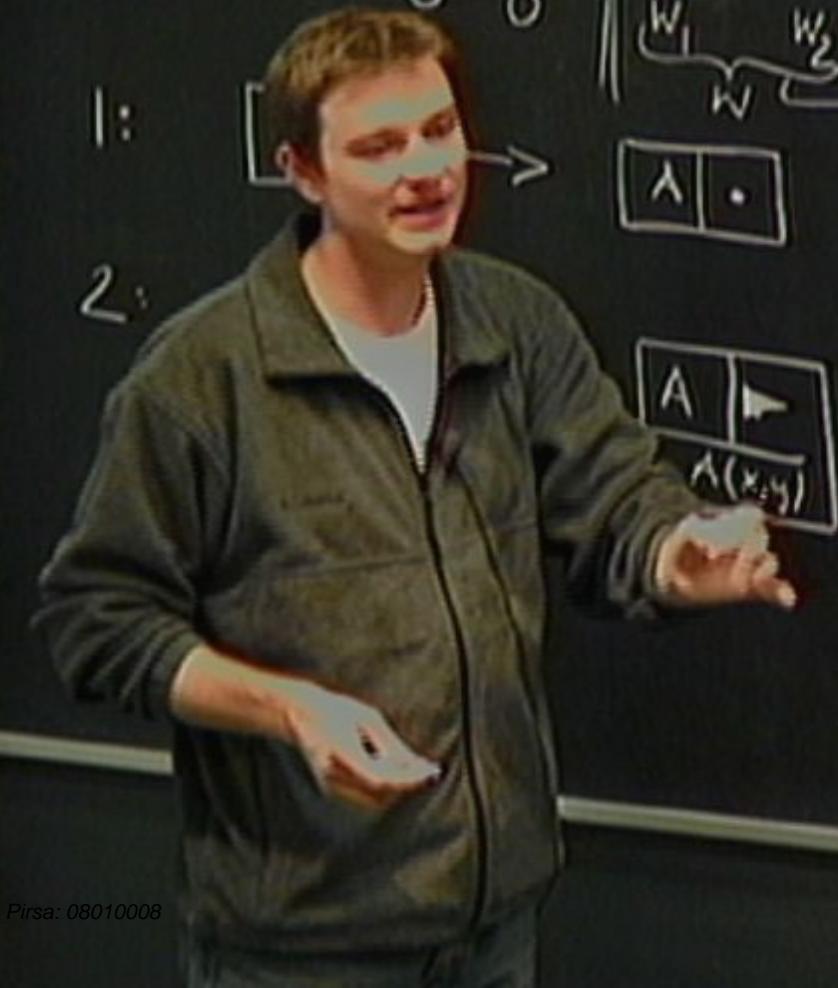
BQP

$$\mathcal{K}_d = \{ |0\rangle, |1\rangle \}$$

$$\mathcal{K}_P = \{ |.\rangle, |\blacktriangleright\rangle, |I\rangle, |S\rangle, |W\rangle \}$$



$$A \in \{I, S, W\}$$



$$H = \sum_j (R + R^+)_{j, j+1}$$

$$R = \sum_{A \in \{W, S, I\}^J} \left[ |A \cdot X \cdot A|_{P_1 P_2} \mathbb{I}_{d_1 d_2} + |A \triangleright X \triangleright A|_{P_1 P_2} \otimes A_{d_1 d_2} \right]$$

$$\begin{matrix} \text{BQP} \\ + \\ 2/3 \\ + \\ 1/3 \end{matrix}$$

$$H = \sum_j (R + R^+)_{j,j+1}$$

$$R = \sum_{A \in \{W, S, I\}^3} \left[ |A \cdot X \cdot A|_{P_1 P_2} \otimes I_{d_1 d_2} + |A \triangleright X \triangleright A|_{P_1 P_2} \otimes A_{d_1 d_2} \right]$$

BQP



$$H = \sum_j (R + R^+)_{j,j+1}$$

$$R = \sum_{A \in \{W, S, T\}} \left[ |A \cdot X \cdot A| \otimes I_{d_1 d_L} + |A \triangleright X \triangleright A| \otimes A_{d_1 d_L} \right]$$

$\beta QP$

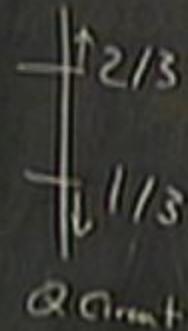
$QCA$

$QCA$

$$H = \sum_j (R + R^+)_{j,j+1}$$

$$R = \sum_{A \in \mathcal{E}_{W,S,I}} \left[ |A \cdot X \cdot A|_{P_1 P_2} \otimes I_{d_1 d_2} + |A \triangleright X \triangleright A|_{P_1 P_2} \otimes A_{d_1 d_2} \right]$$

$\beta QP$



$QCA_{\text{out}}$

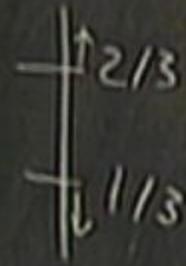


$QCA$

$$H = \sum_j (R + R^+)_{j,j+1}$$

$$R = \sum_{A \in \{W, S, I\}^{\{}} \left[ |A \cdot X \cdot A| \otimes I_{d_1 d_2} + |A \triangleright X \triangleright A| \otimes A_{d_1 d_2} \right]$$

$\beta QP$



$QCA_{int}$



$QCA$

$$\mathcal{K}_d = \{|0\rangle, |1\rangle\}$$

$$\mathcal{K}_p = \{|+\rangle, |\blacktriangleright\rangle, |\downarrow\rangle, |s\rangle, |w\rangle\}$$

