Title: Entanglement Renormalization, Quantum Criticality and Topological Order

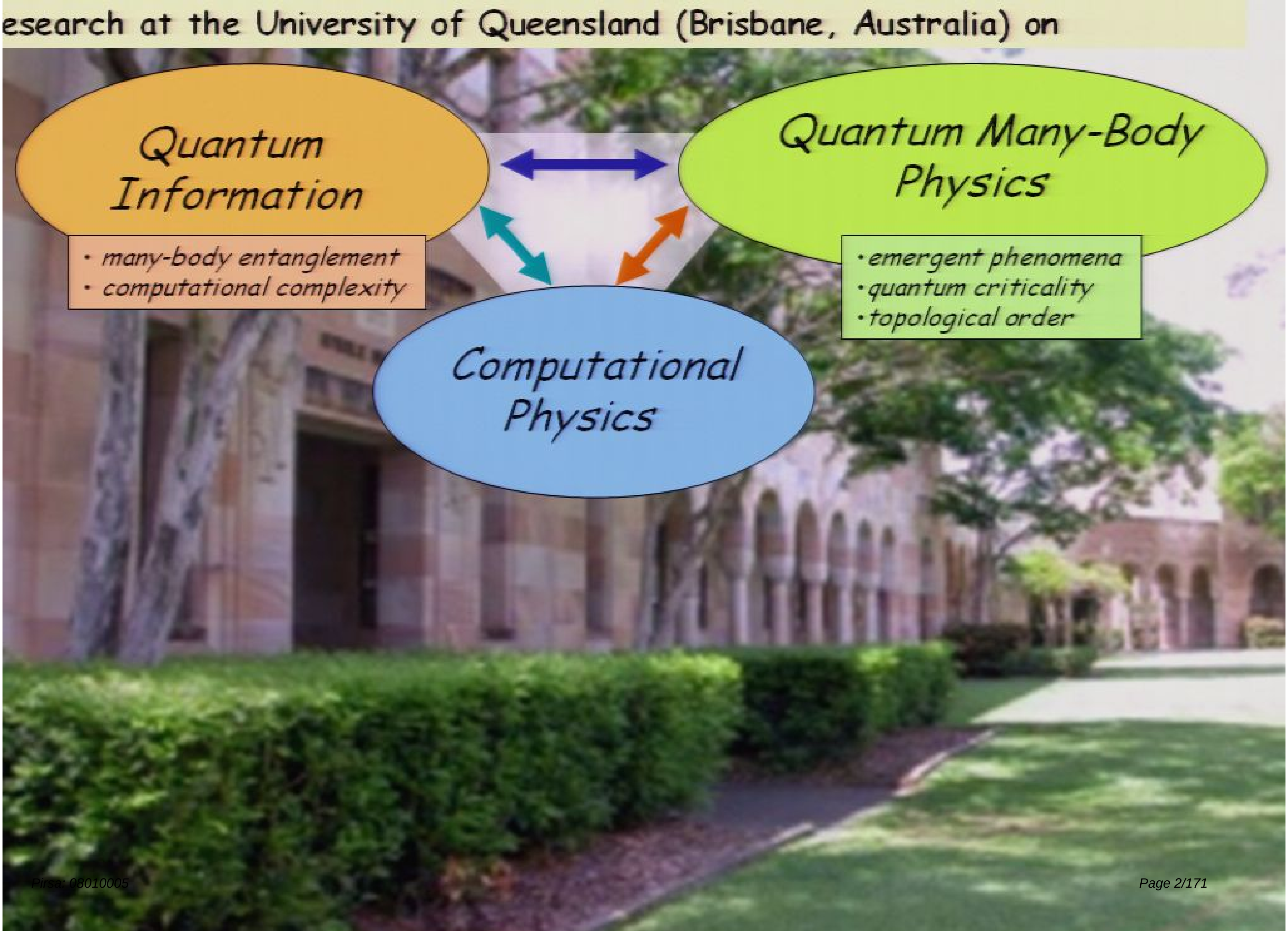
Date: Jan 23, 2008 04:00 PM

URL: http://pirsa.org/08010005

Abstract: The renormalization group (RG) is one of the conceptual pillars of statistical mechanics and quantum field theory, and a key theoretical element in the modern formulation of critical phenomena and phase transitions. RG transformations are also the basis of numerical approaches to the study of low energy properties and emergent phenomena in quantum many-body systems. In this colloquium I will introduce the notion of \\\"entanglement renormalization\\\" and use it to define a coarse-graining transformation for quantum systems on a lattice [G.Vidal, Phys. Rev. Lett.

99, 220405 (2007)]. The resulting real-space RG approach is able to numerically address 1D and 2D lattice systems with thousands of quantum spins using only very modest computational resources. From the theoretical point of view, entanglement renormalization sheds new light into the structure of correlations in the ground state of extended quantum systems.

I will discuss how it leads to a novel, efficient representation for the ground state of a system at a quantum critical point or with topological order.


esearch at the University of Queensland (Brisbane, Australia) on
Quantum Information
• many-body entanglement
• computational complexity
Quantum Many-Body Physics
• emergent phenomena
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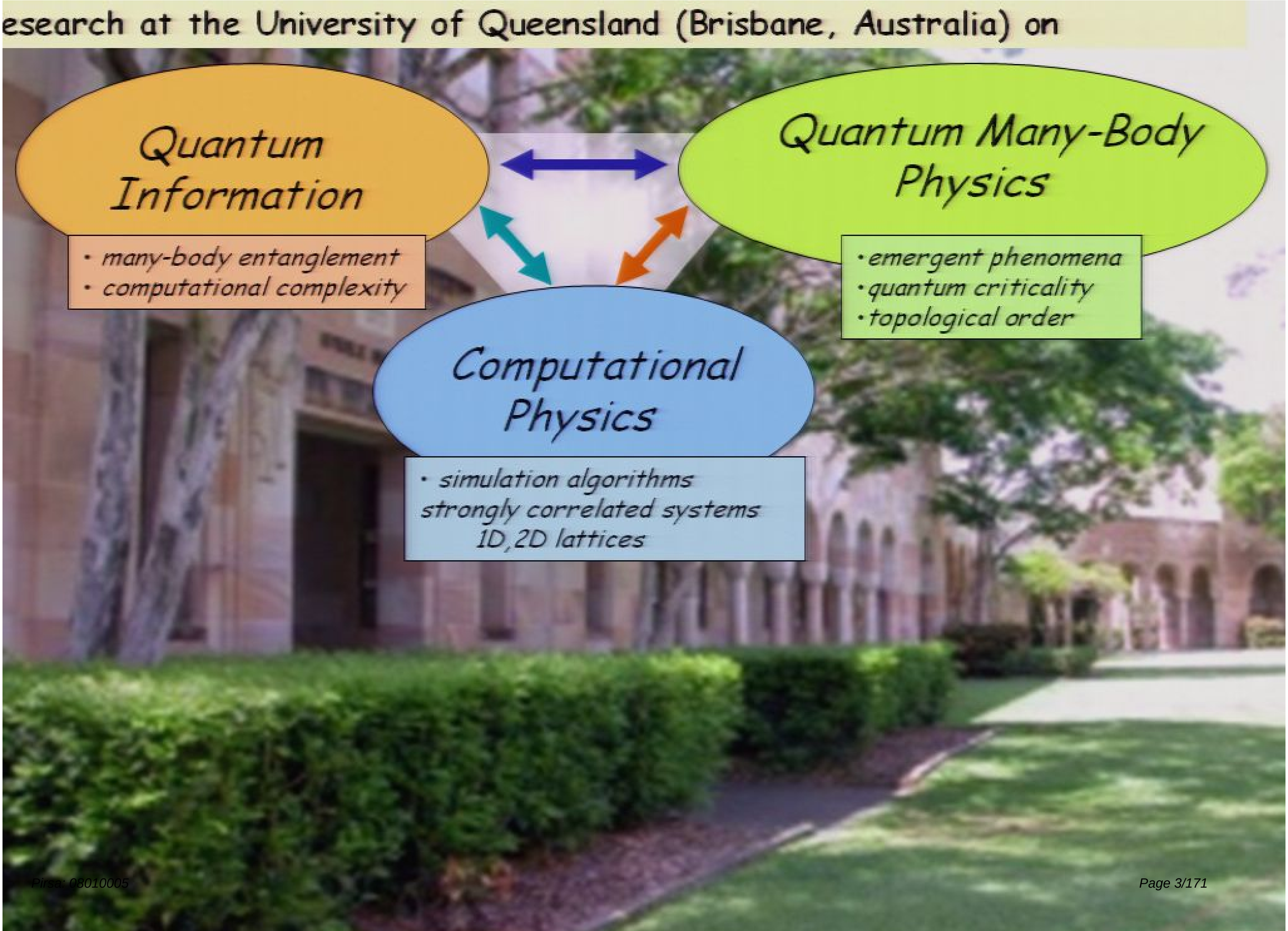
• topological order
Computational Physics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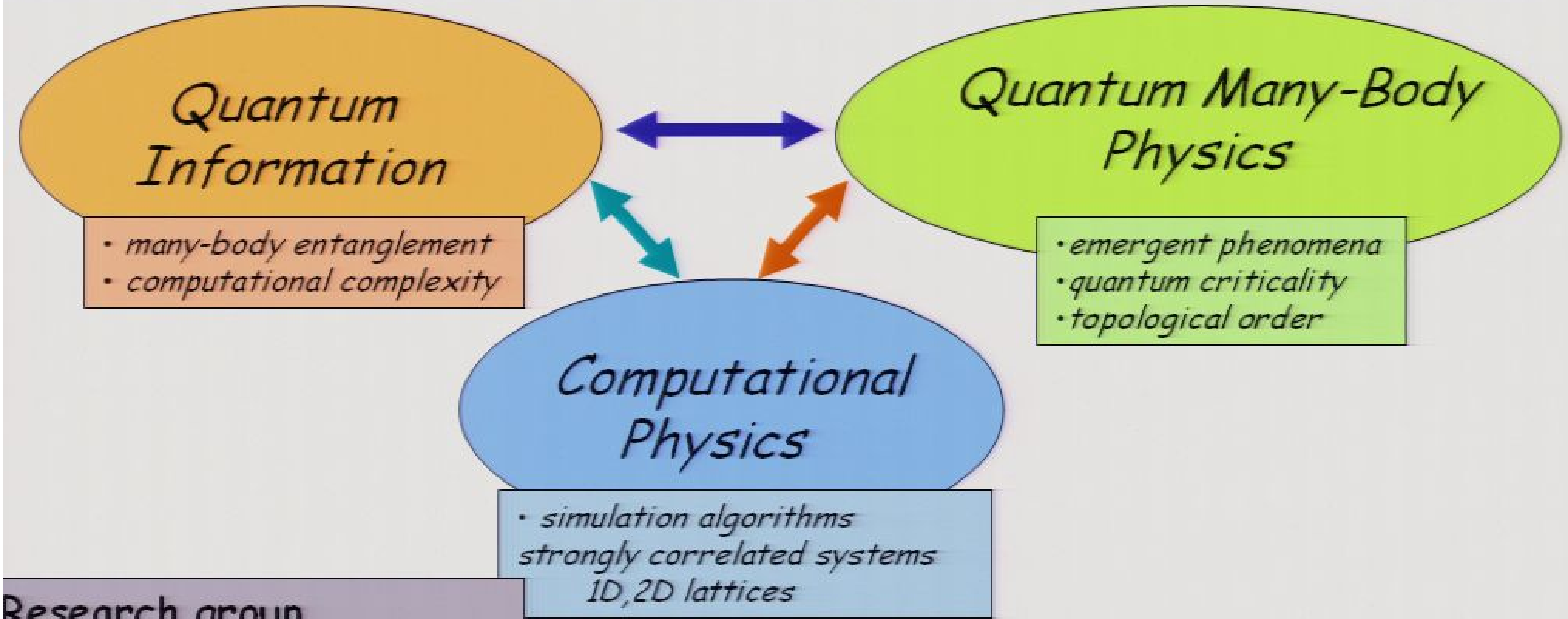

Research group

Richard Davis (postdoc)
Ian McCulloch (postdoc)
Roman Orus (postdoc)

Glen Evenbly (PhD)
Jacob Jordan (PhD)
Sukhi Singh (PhD)

Guifre Vidal

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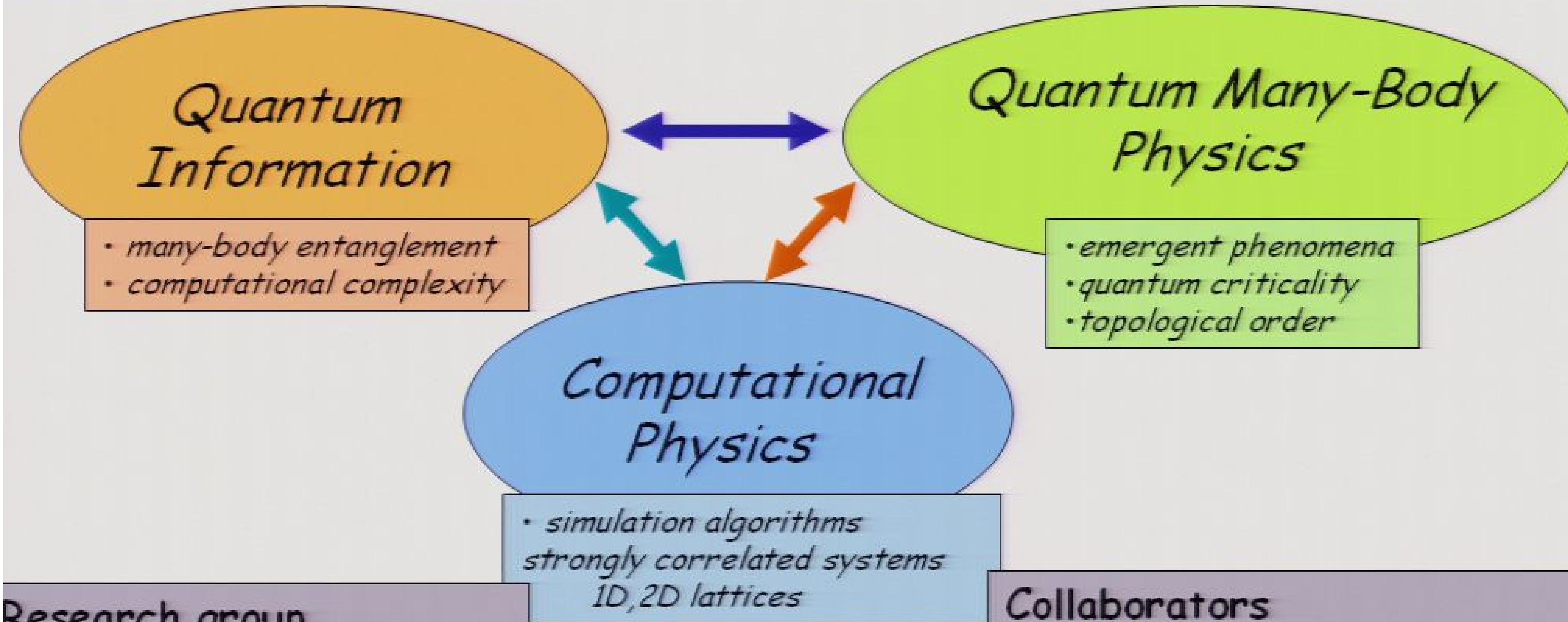

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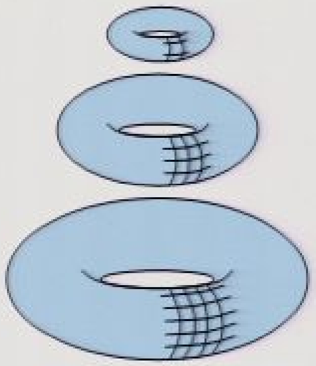
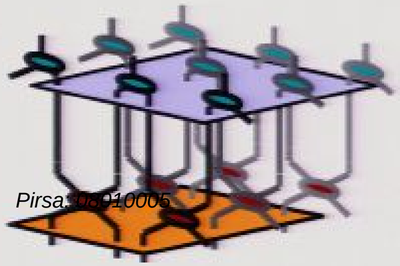
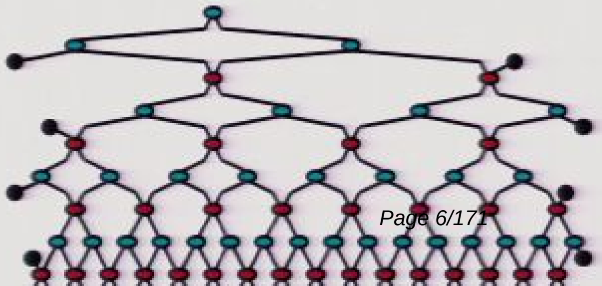
Collaborators

Andrew Doherty (UQ)
Frank Verstraete (U. Vienna)
Ignacio Cirac (MPQ Garching)
Miguel Aguado (MPQ Garching)
Mateo Rizzi (MPQ Garching)
Luca Tagliacozzo (U. Barcelona)
Simone Montangero (U. Pisa)
Greg Crosswhite (U. Washington)
Anders Sandvik (U. Boston)

Outline







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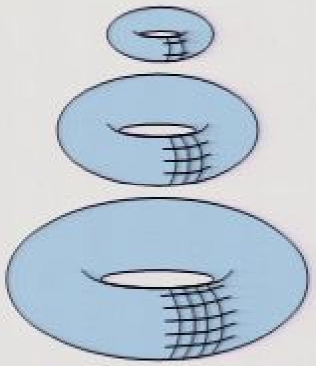
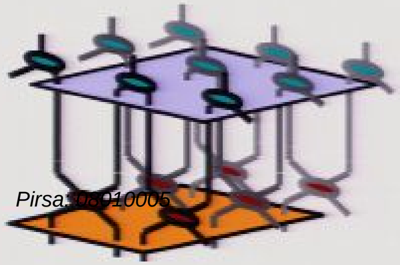
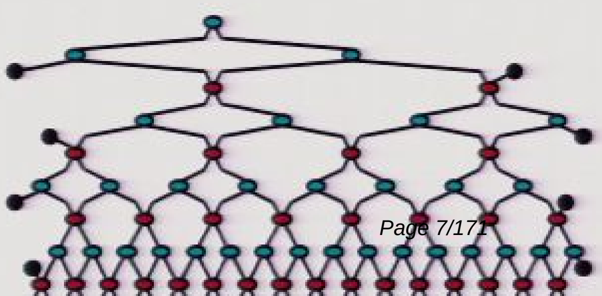


- Renormalization Group transformations





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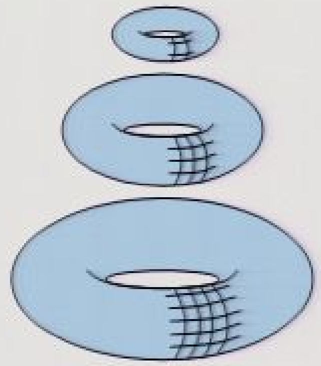
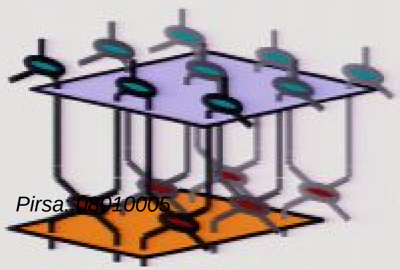
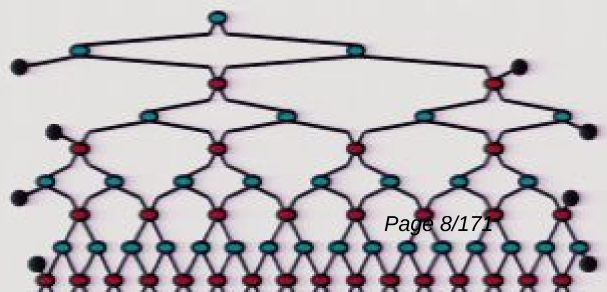


- Renormalization Group transformations

- Ground State Entanglement





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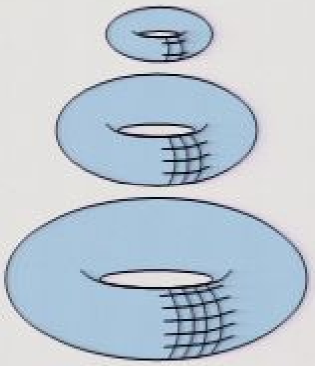
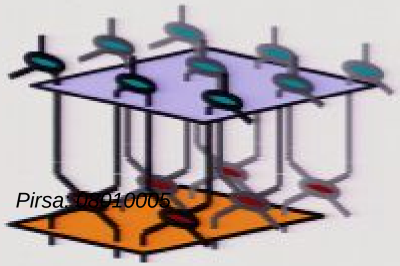
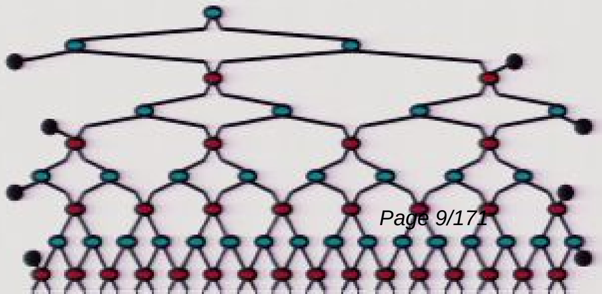


- Renormalization Group transformations

- Ground State Entanglement

- Entanglement Renormalization





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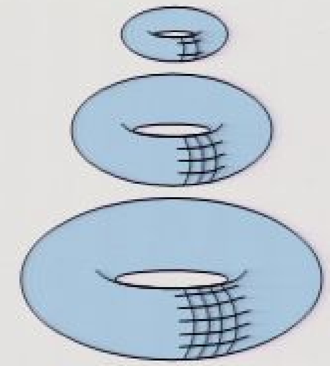
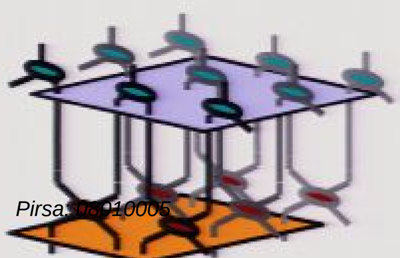
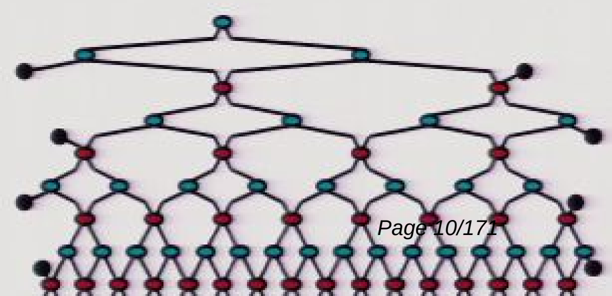


- Renormalization Group transformations
- Ground State Entanglement
- Entanglement Renormalization
- Quantum Criticality





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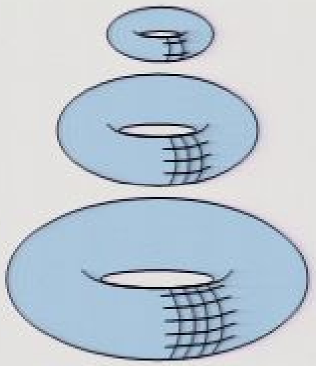
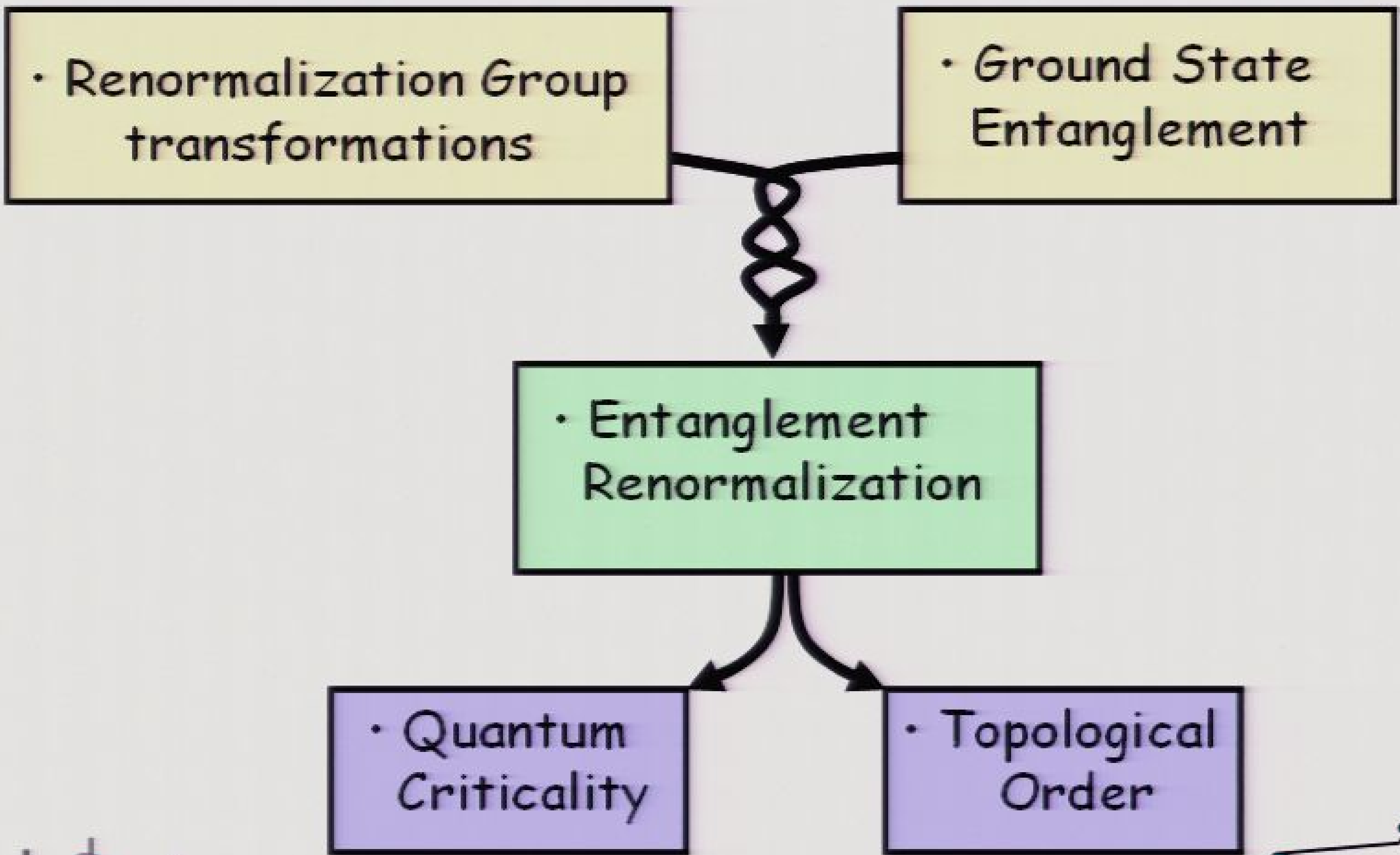
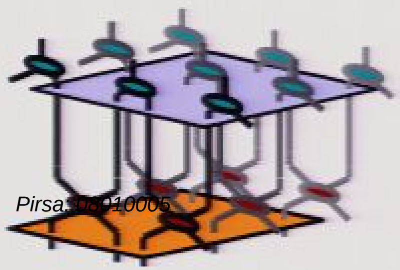
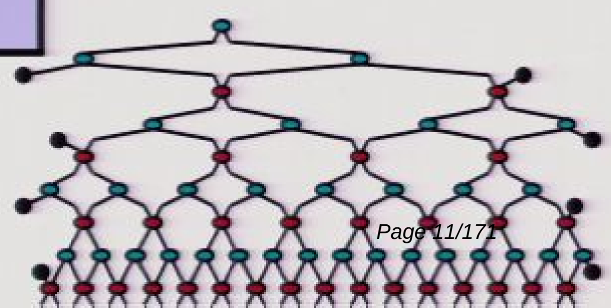
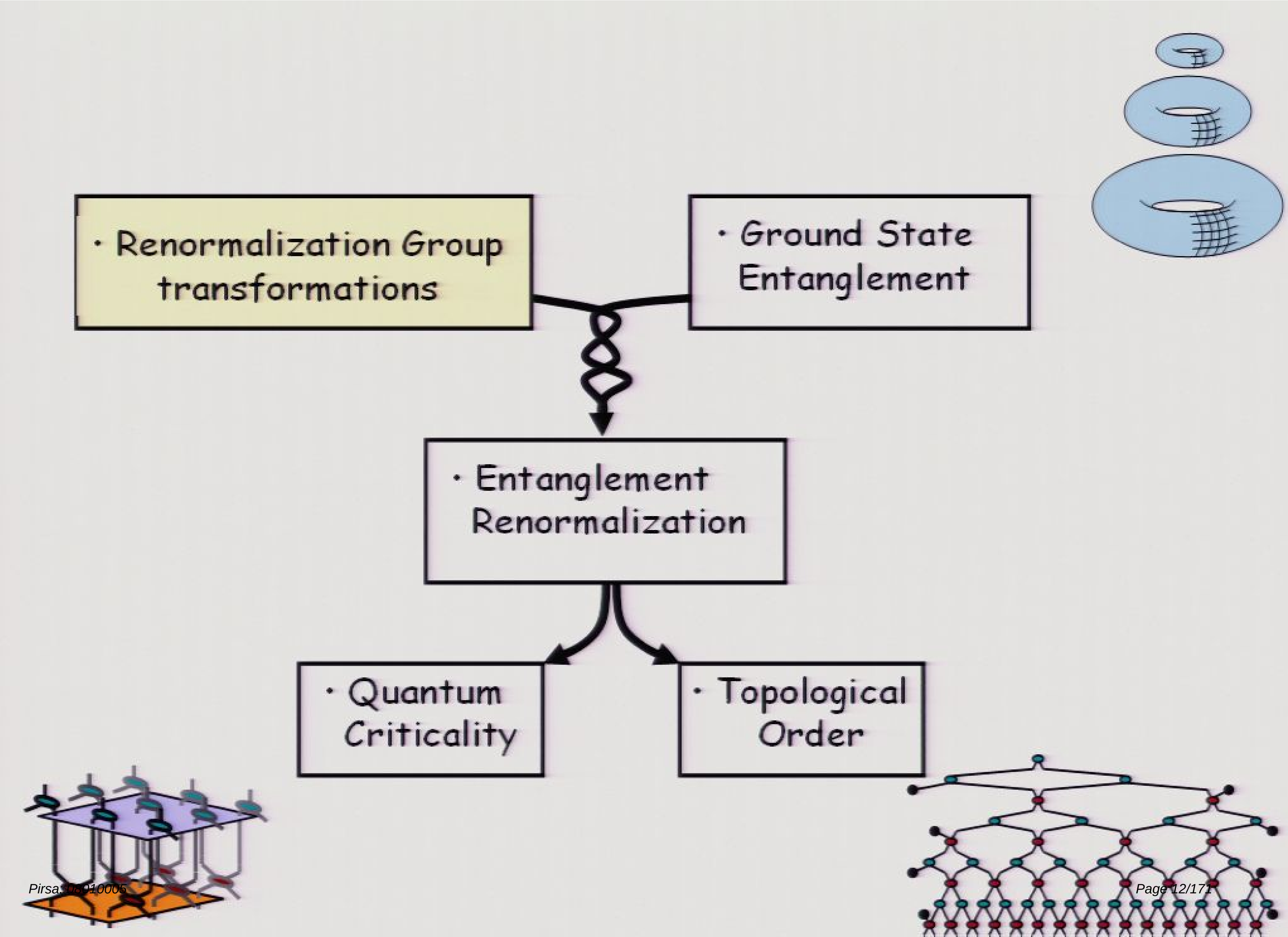




• Renormalization Group transformations
• Ground State Entanglement
• Entanglement Renormalization
• Quantum Criticality
• Topological Order







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Kadanoff (66), Wilson (74), and many others

- Extended system: spin lattice, field theory...

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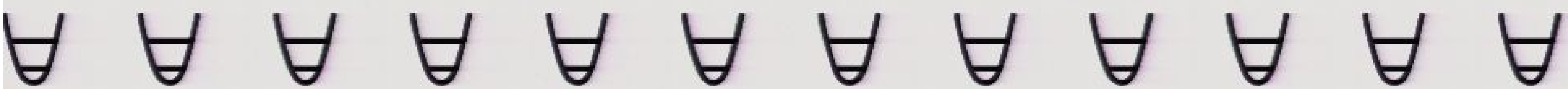
- spin $\frac{1}{2}$
- hopping fermion
- hopping boson
- atomic energy levels



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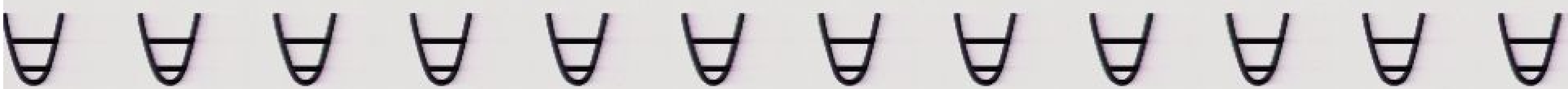
- Extended system: spin lattice, field theory...
- Change of **Physics** with **observation scale**



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he Renormalization Group

- RG transformation = coarse-graining + rescaling

$$\mathcal{L}_0 \quad \cdots \quad \forall \quad \forall \quad \forall \quad \forall \quad \forall \quad \forall \quad \cdots \quad H_0$$

lattice Hamiltonian

he Renormalization Group

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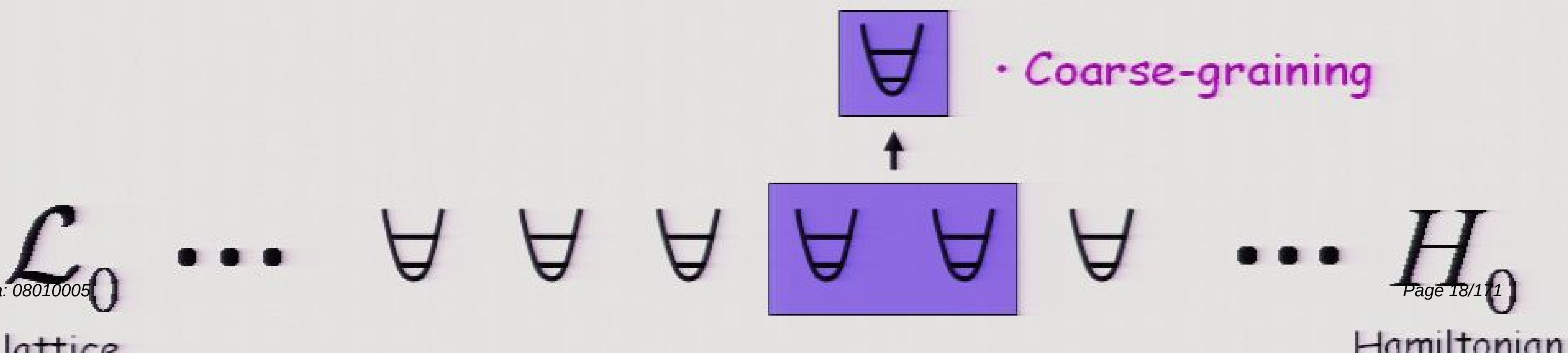

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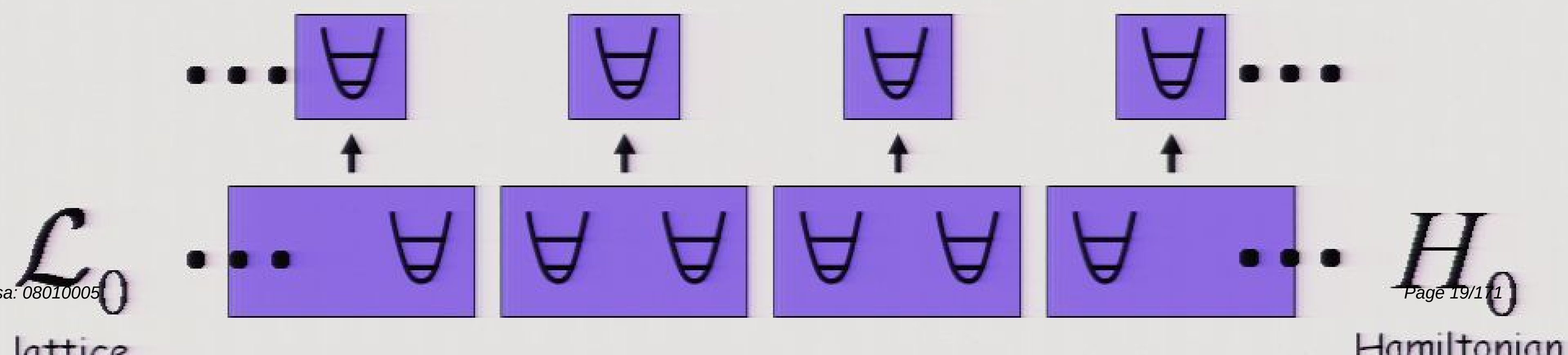

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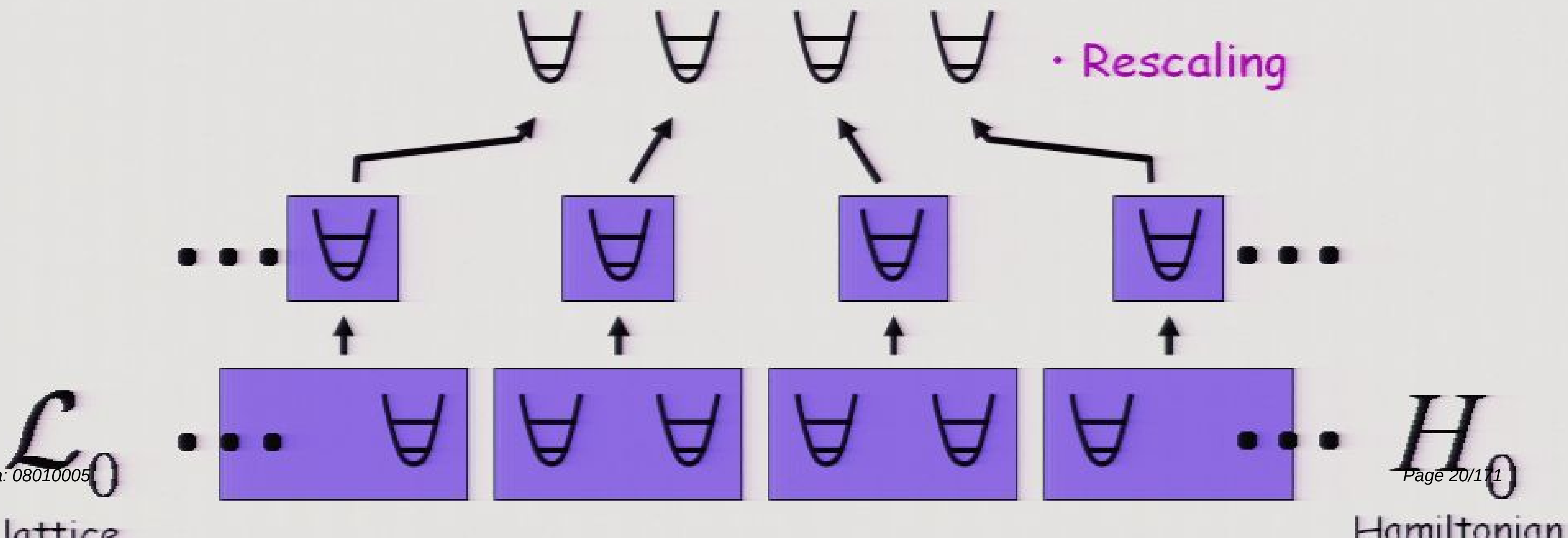

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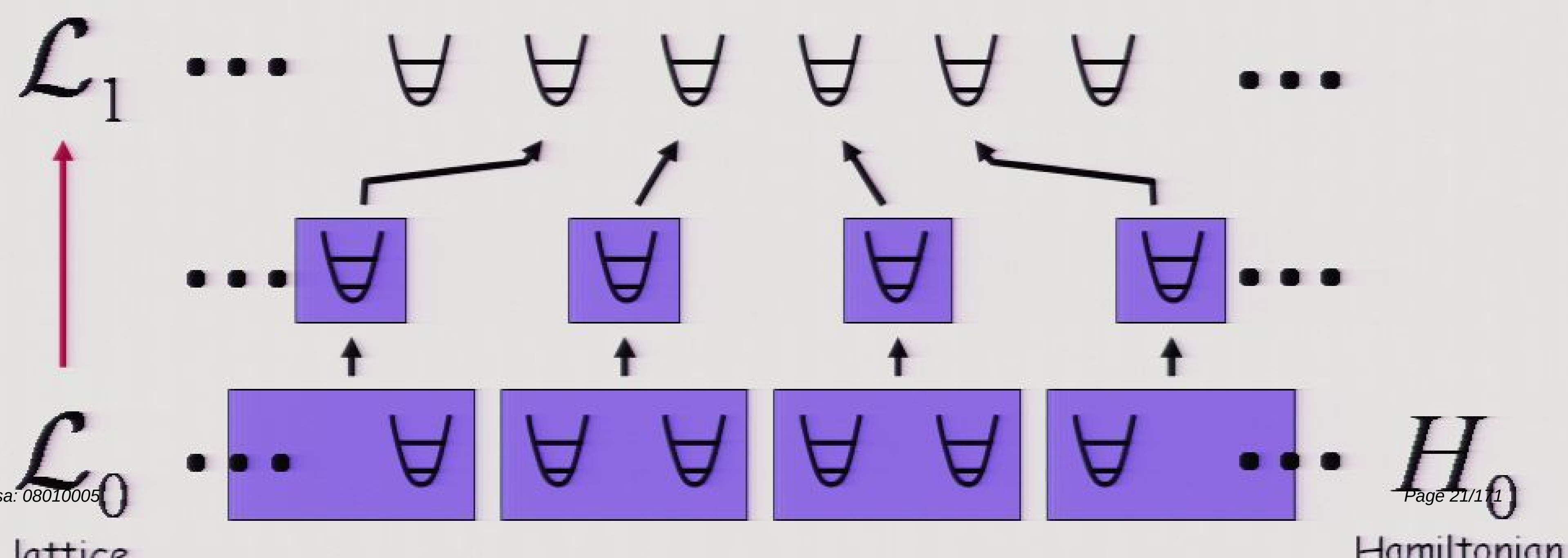

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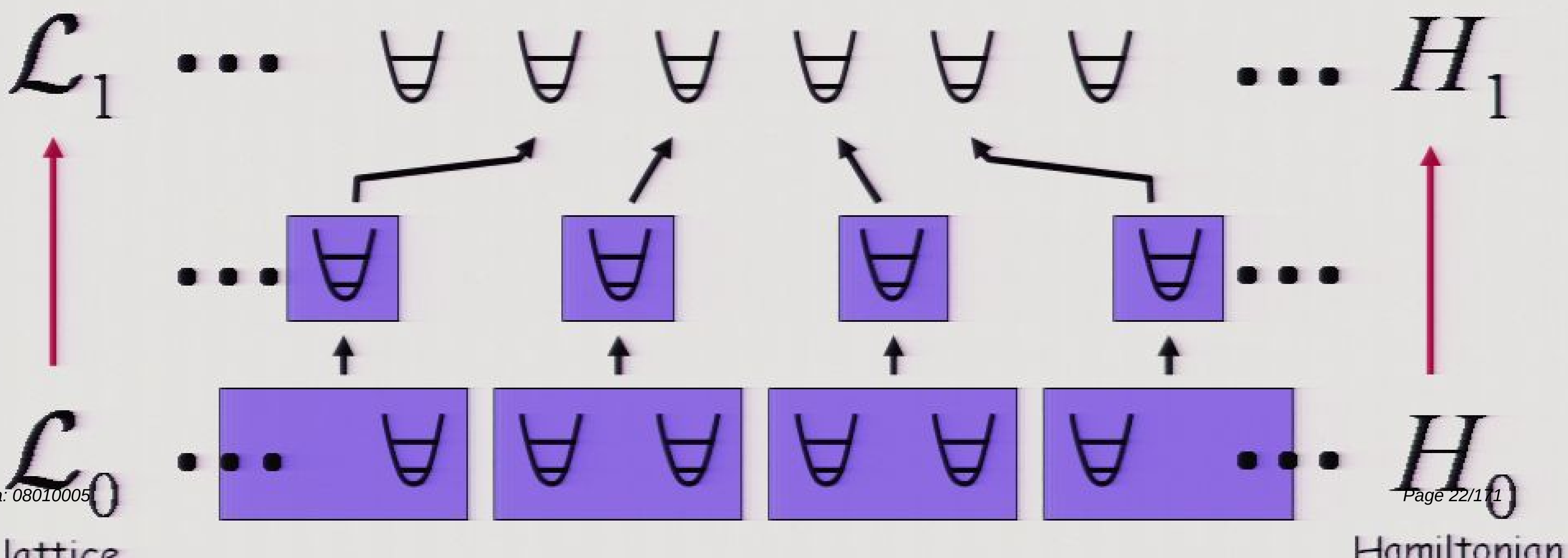

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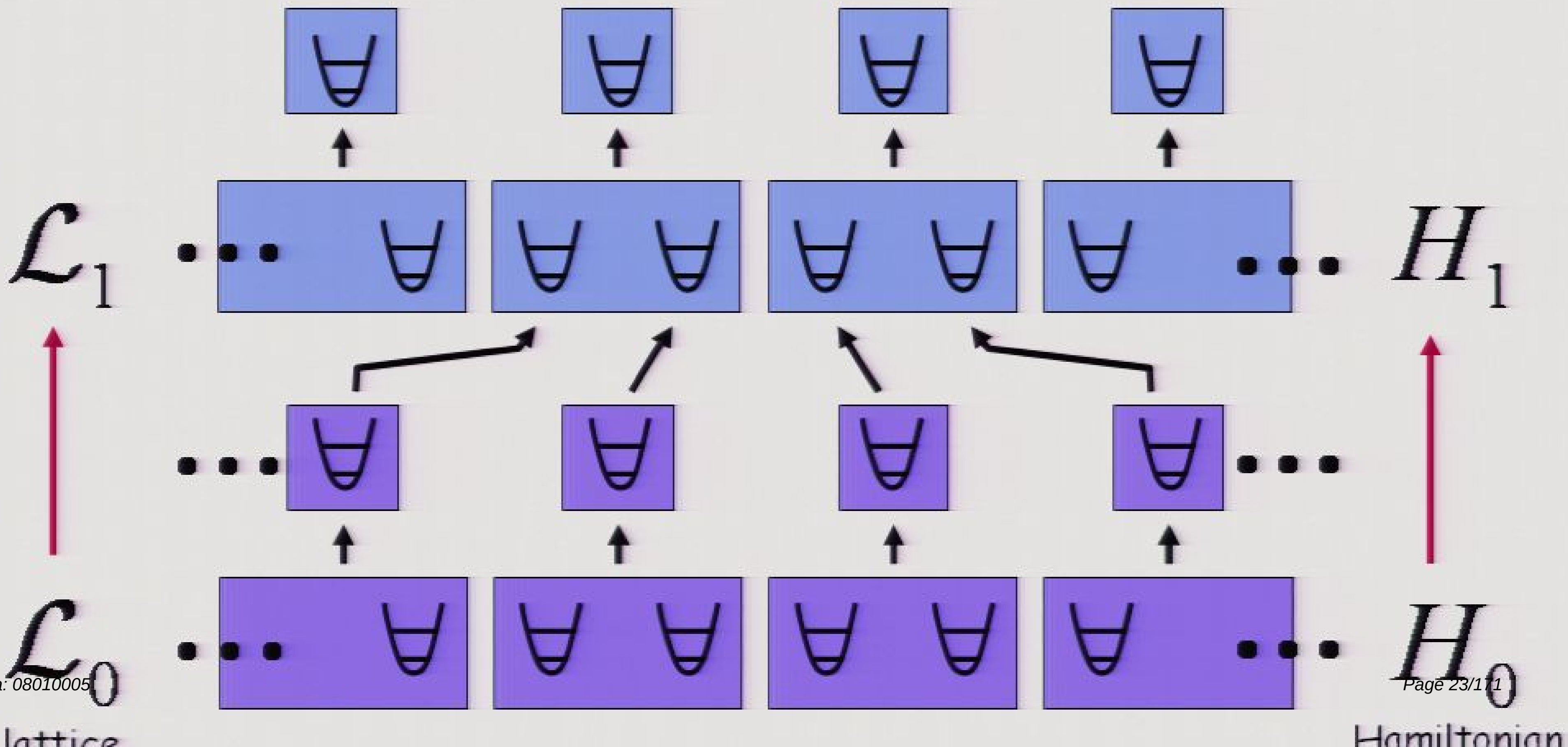

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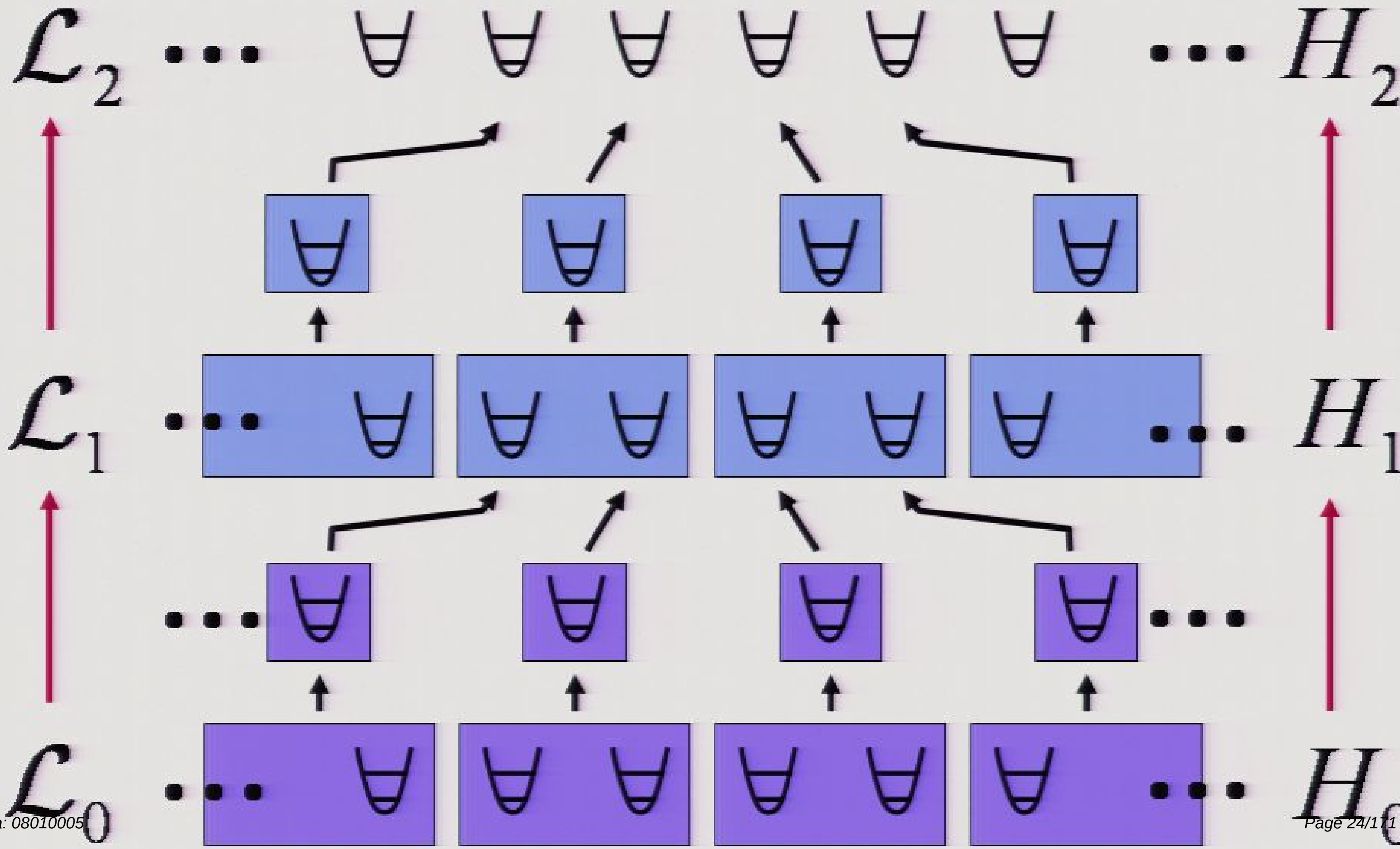

lattice
Hamiltonian

he Renormalization Group

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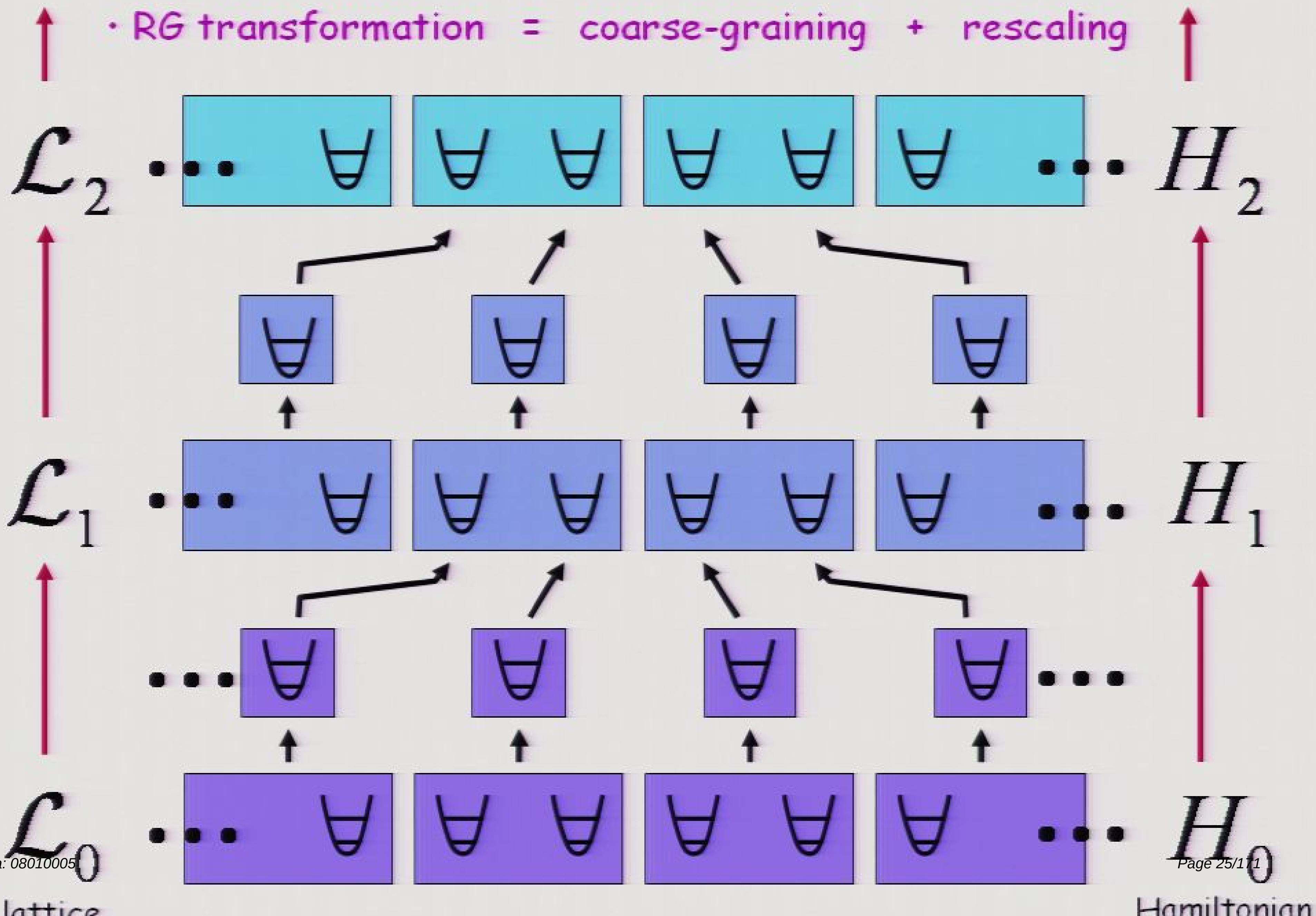

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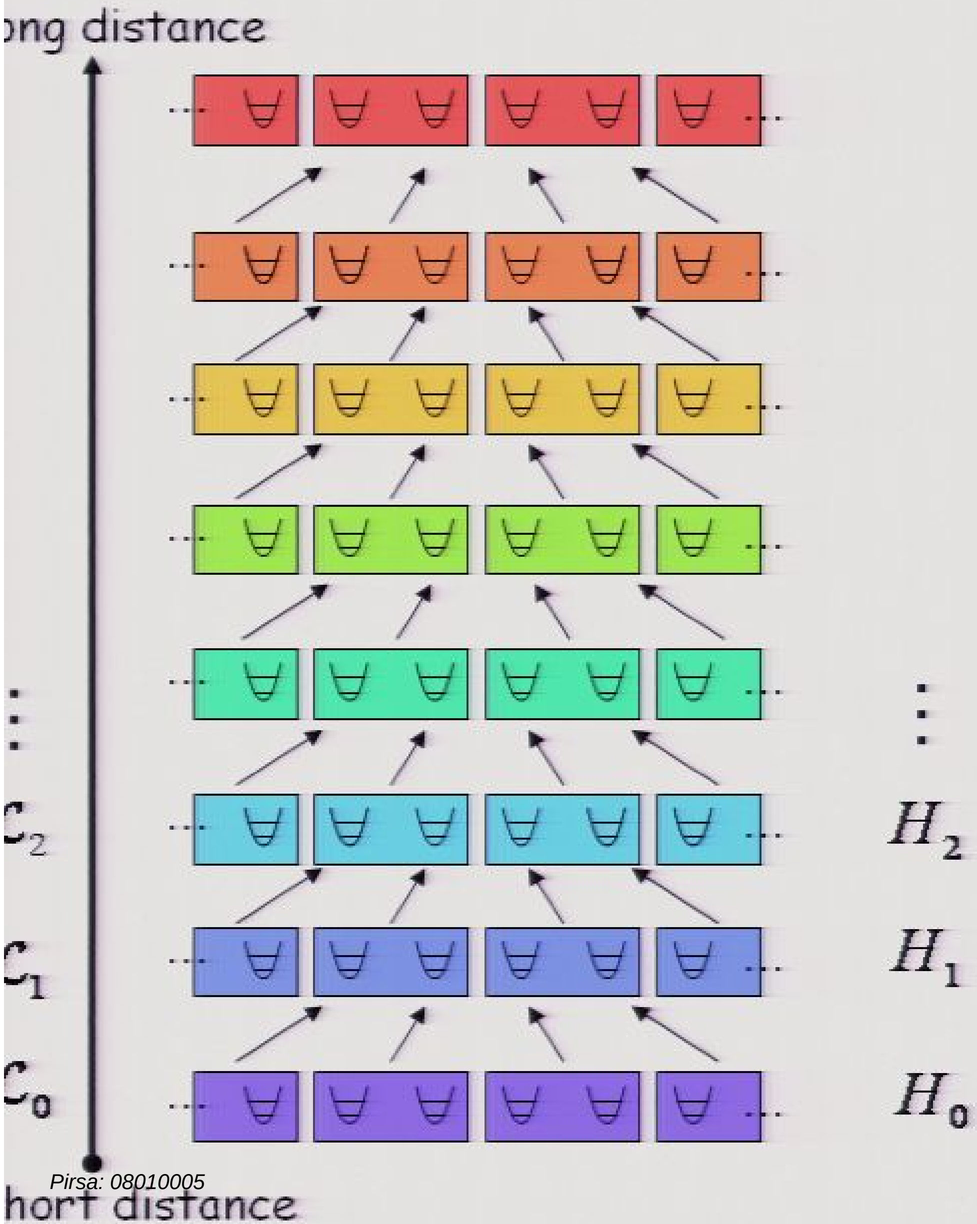

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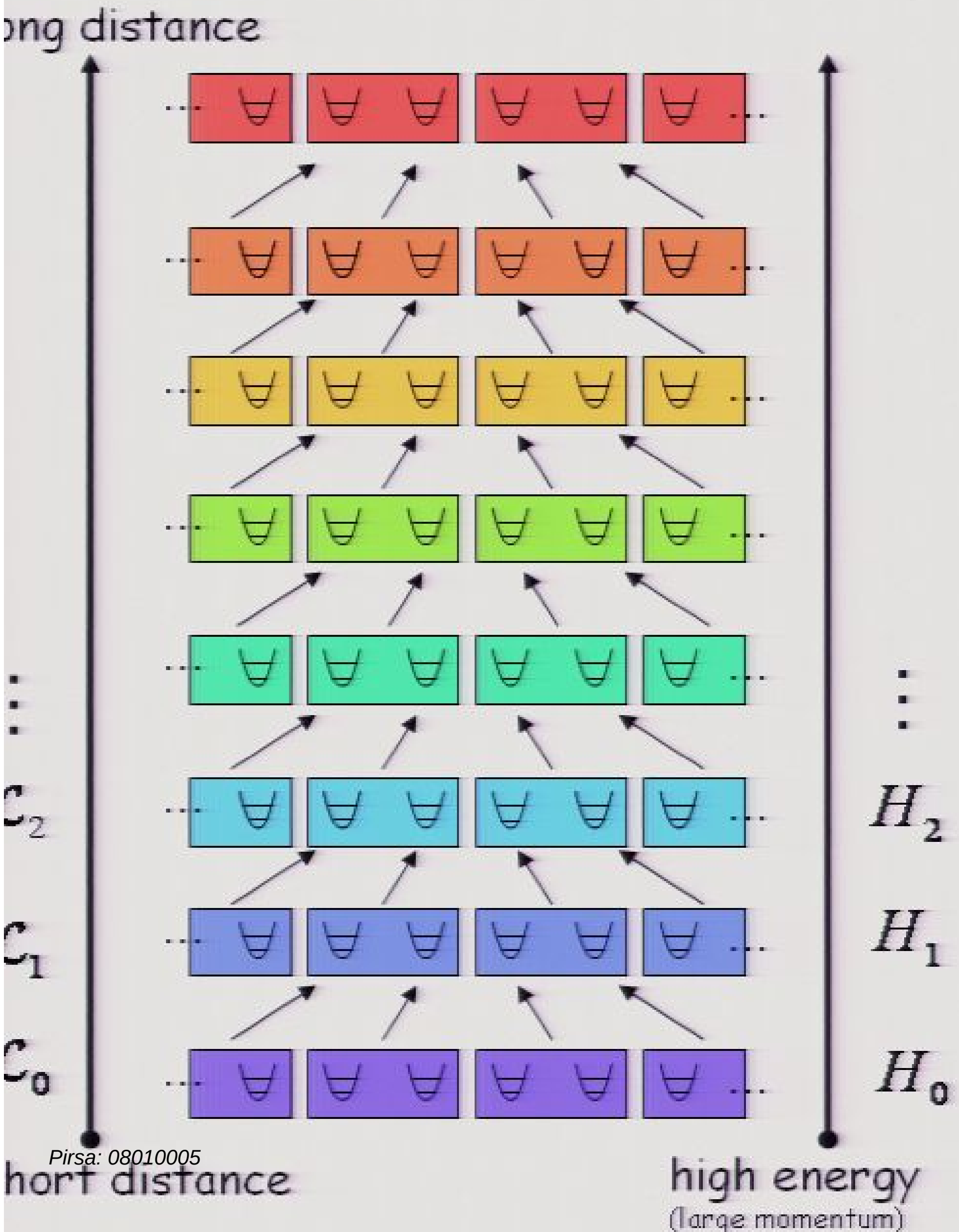

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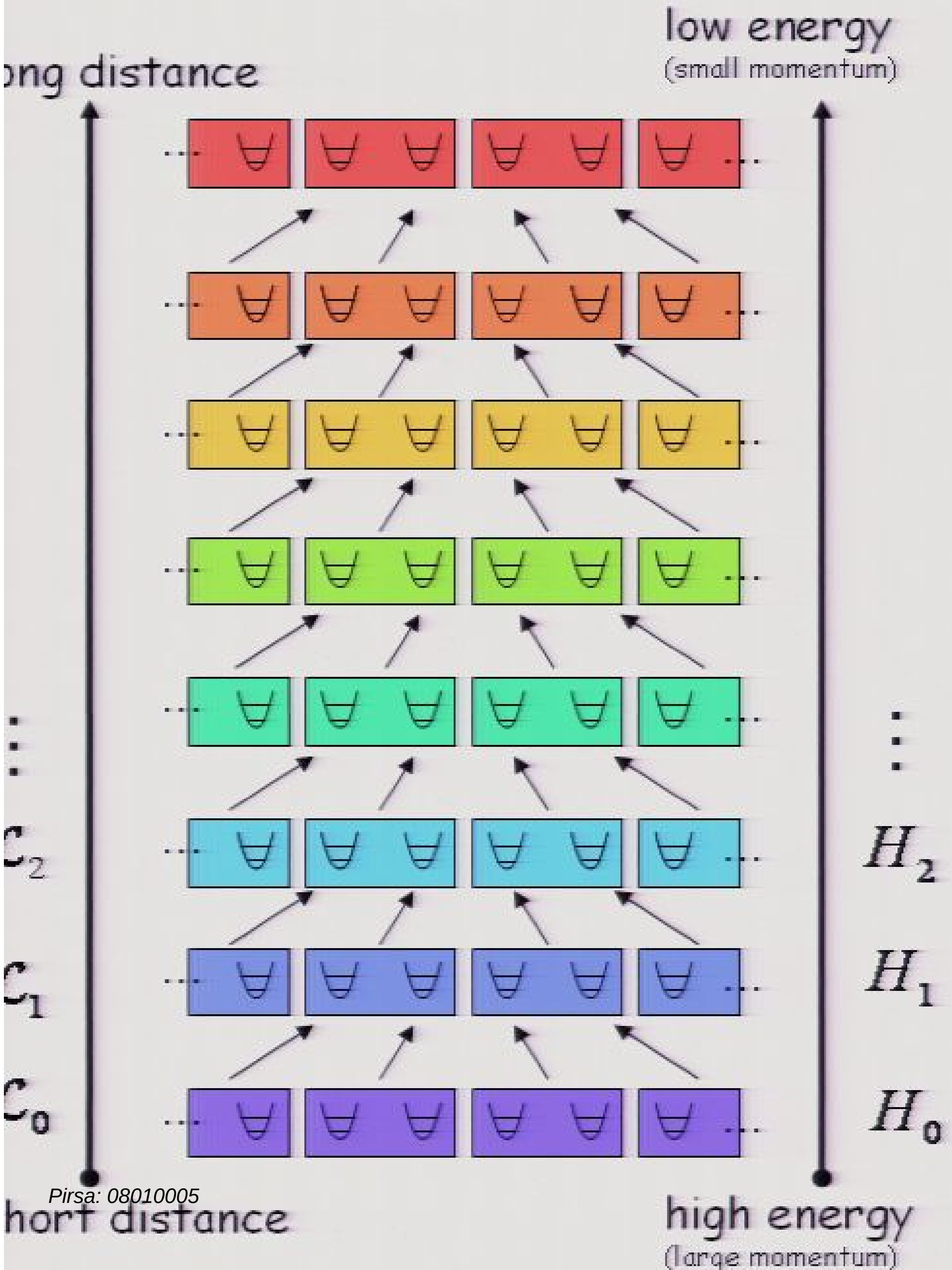

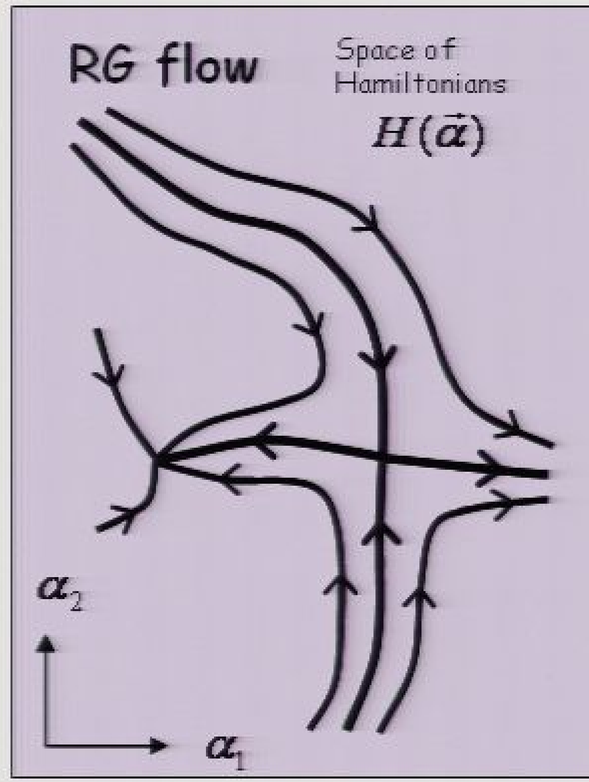

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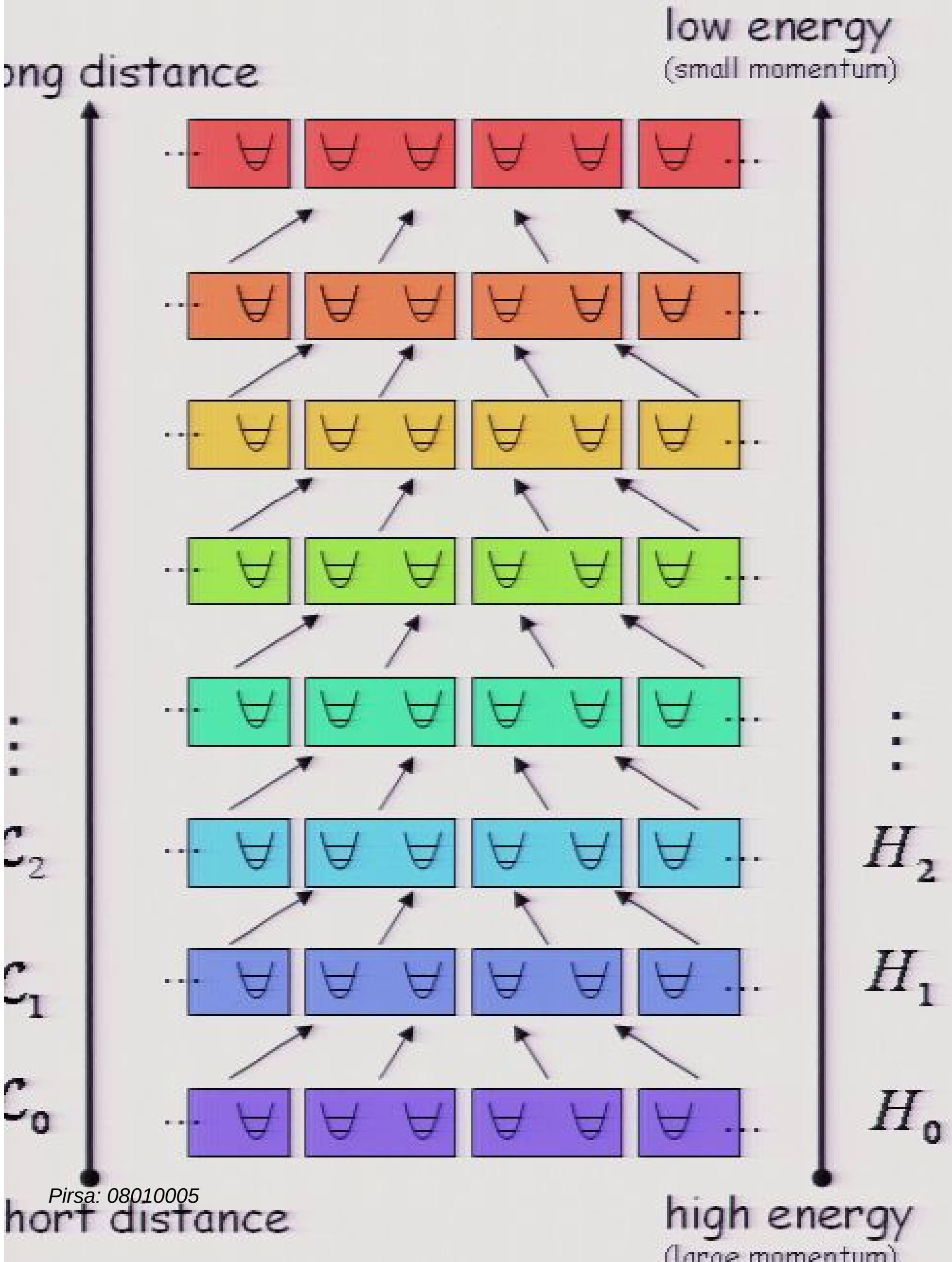
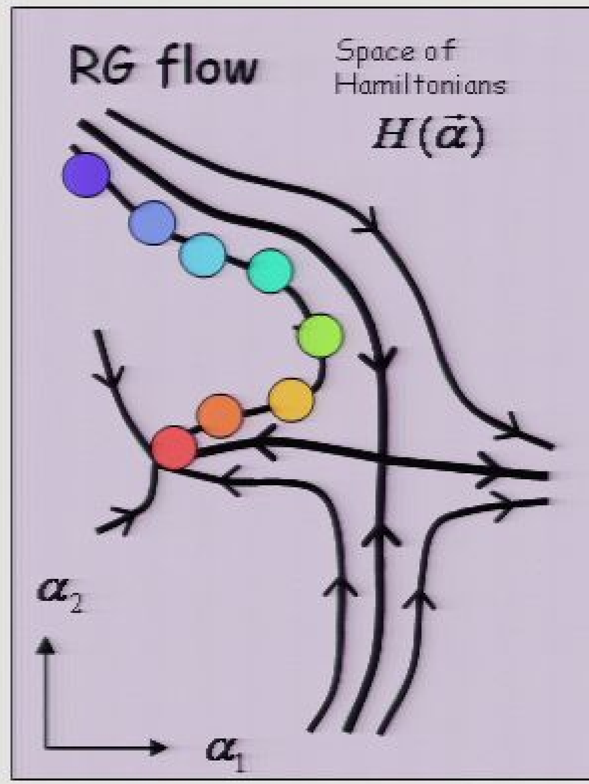
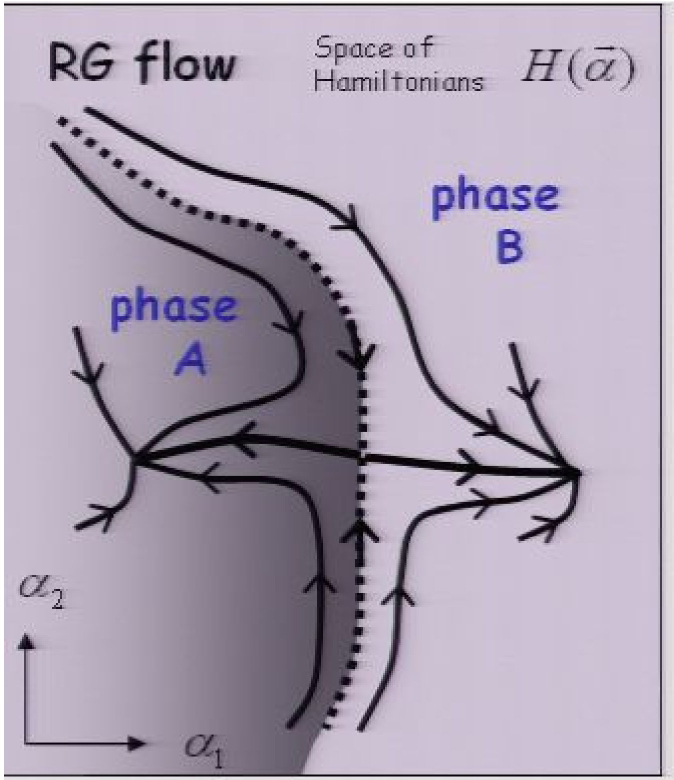
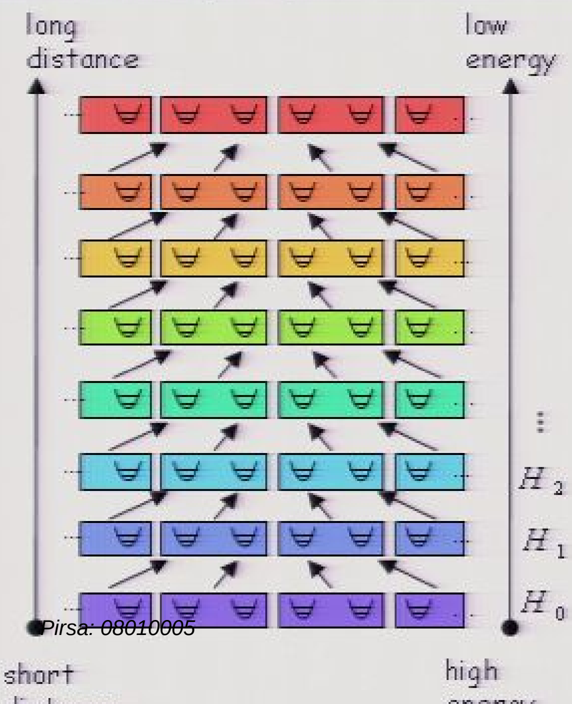
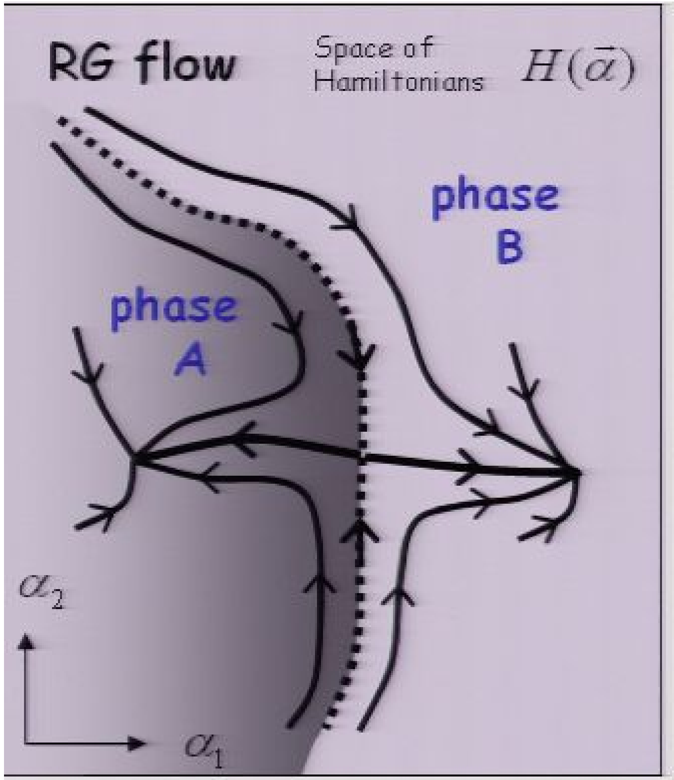
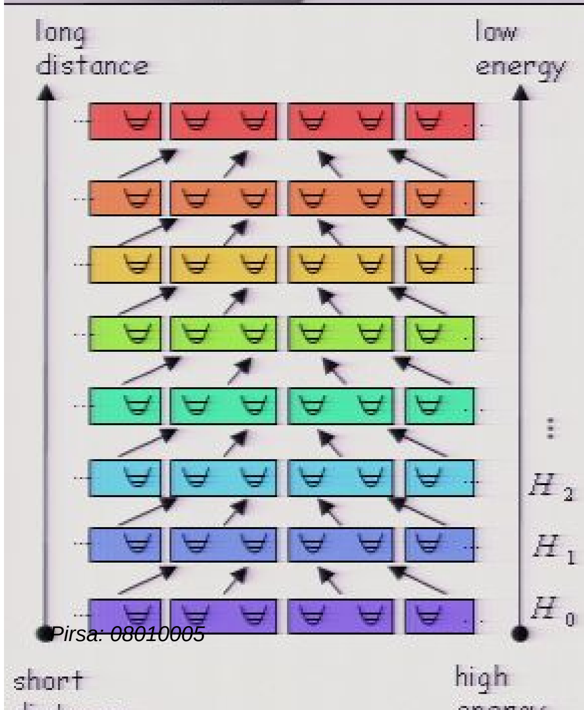




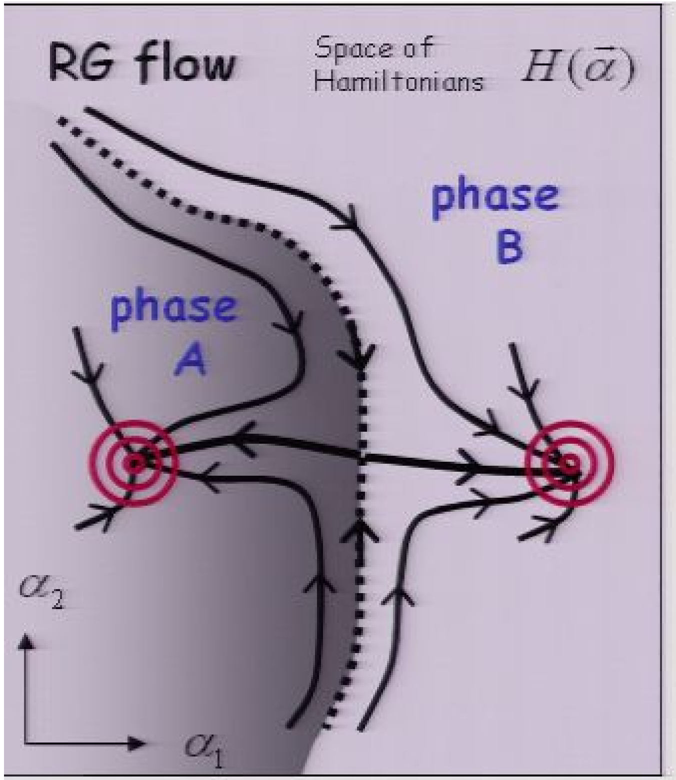

RG flow
Space of Hamiltonians
$H(\vec{\alpha})$
phase B
phase A
α_2
α_1

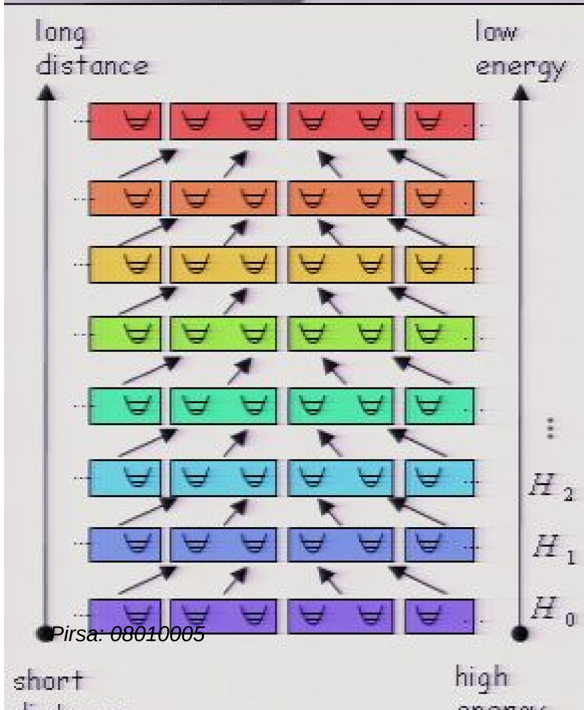

long distance
low energy
H_2
H_1
H_0
short
high



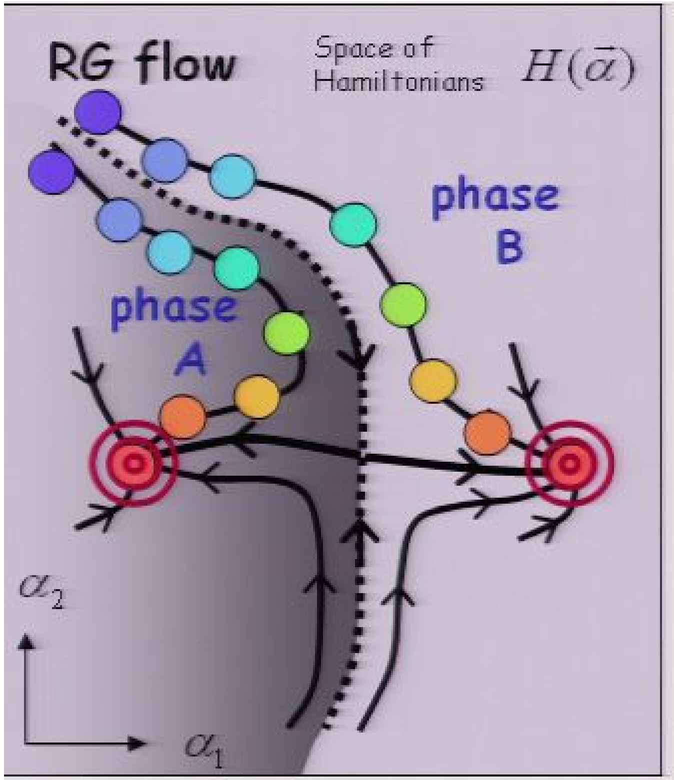


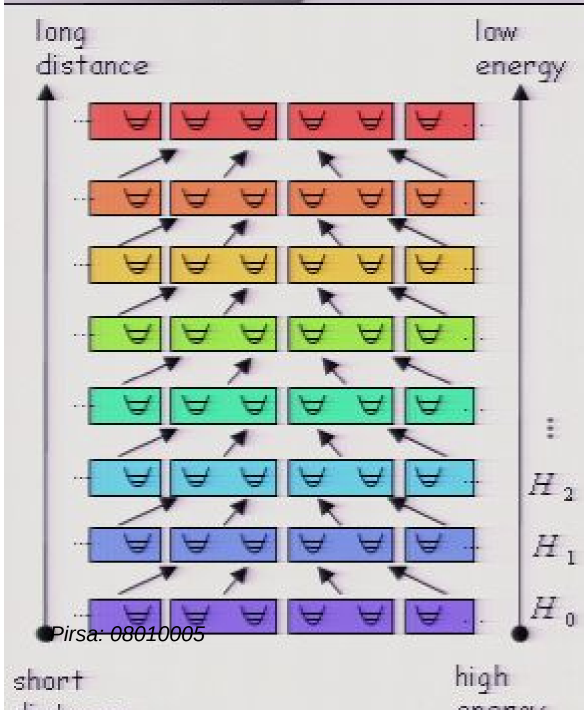
• fixed points of the RG flow





- fixed points of the RG flow
  - well-defined phase (e.g. ordered, disordered phase)
    "trivial" fixed point





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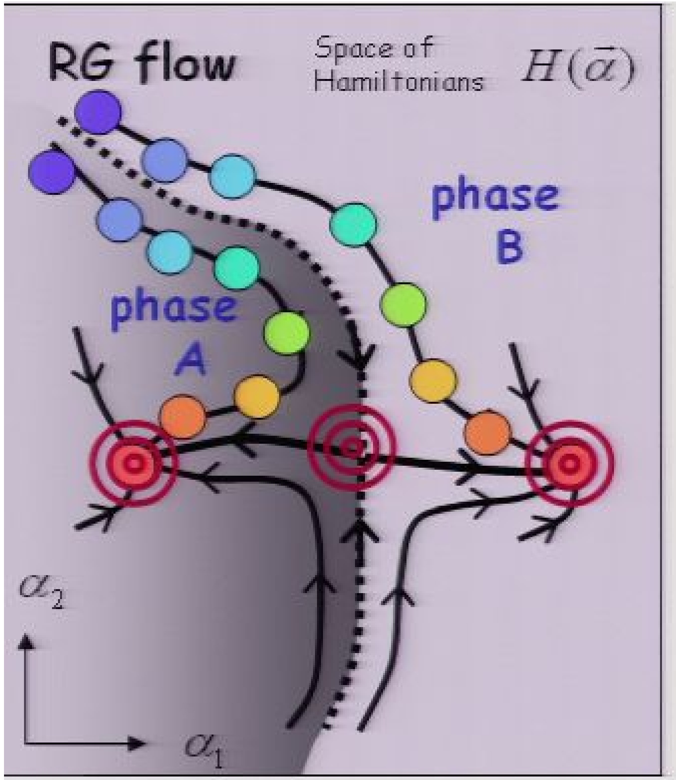
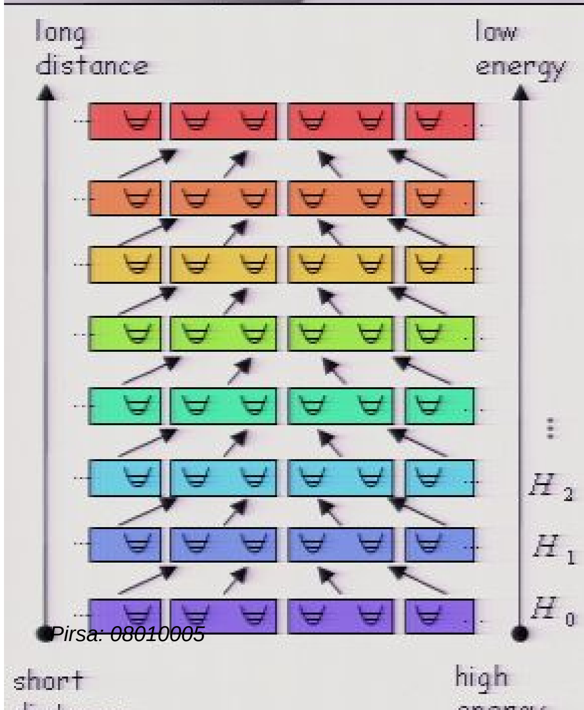
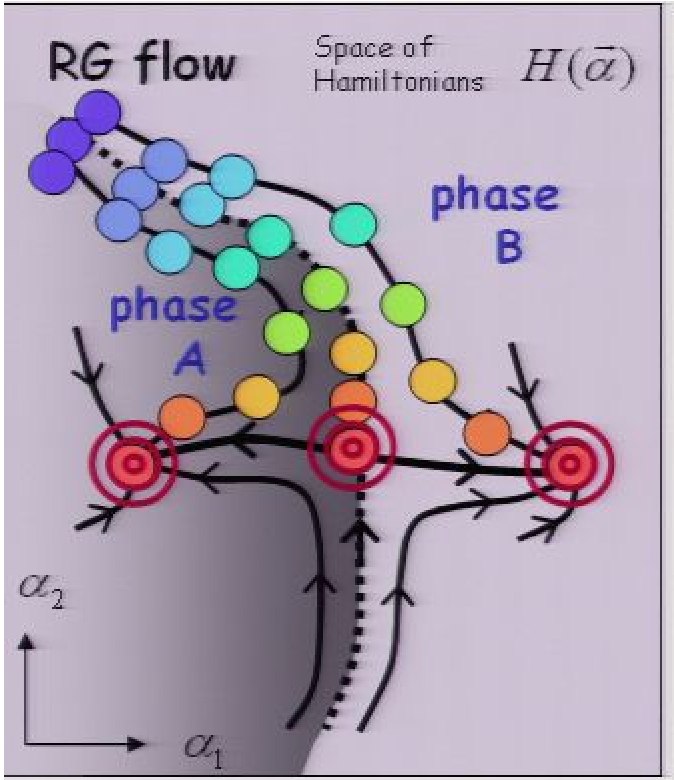
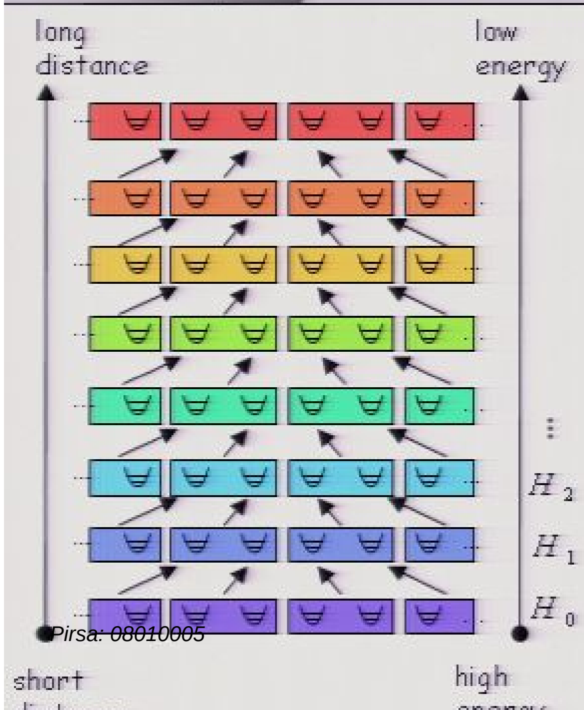


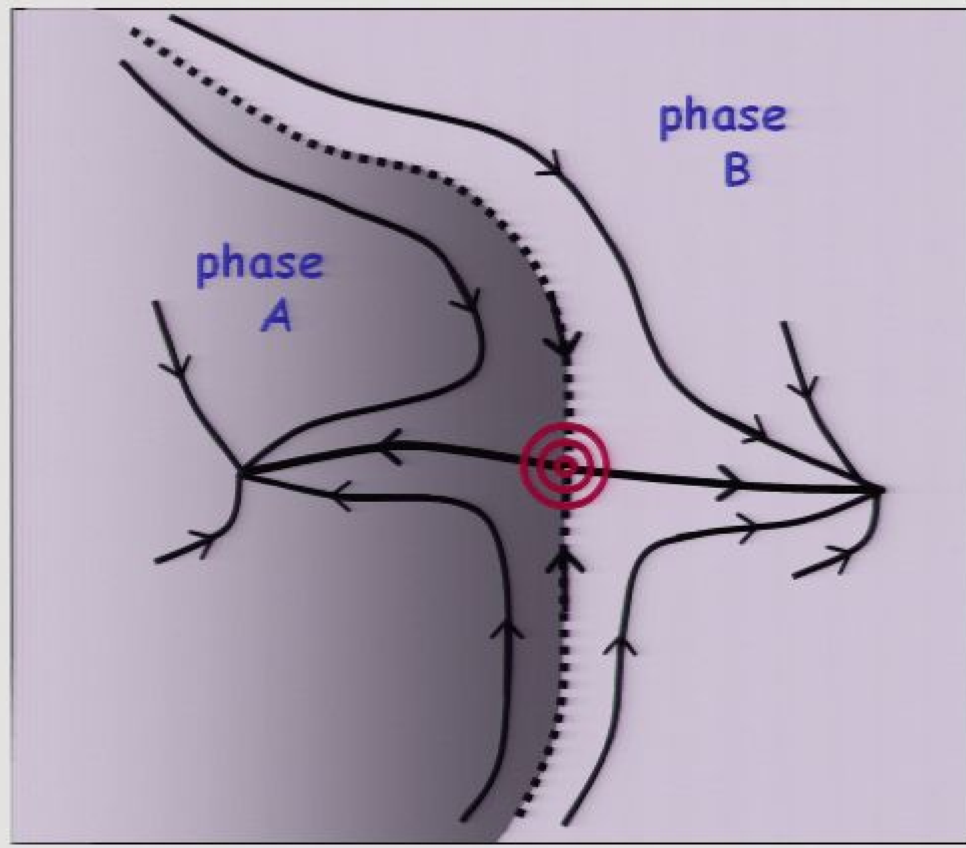


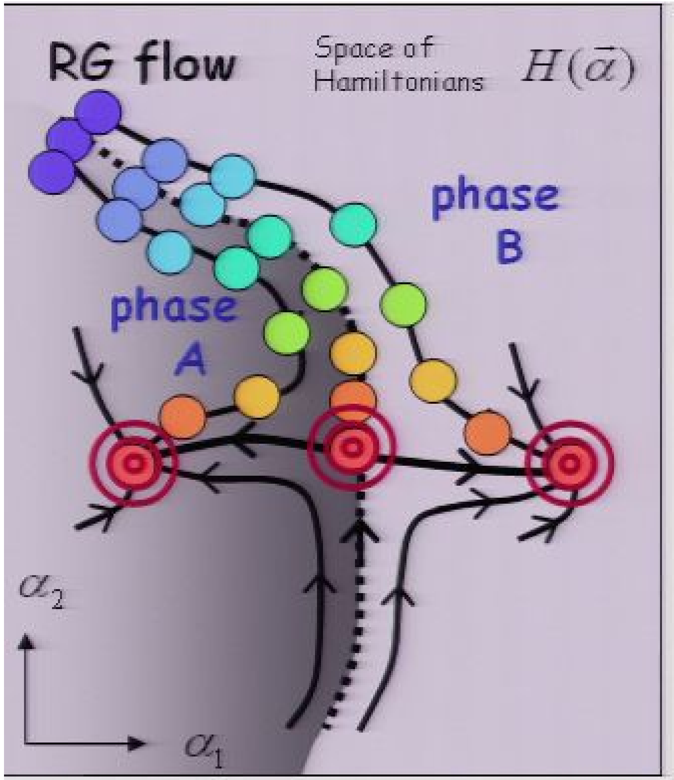
- fixed points of the RG flow
  - well-defined phase (e.g. ordered, disordered phase)
    "trivial" fixed point
  - critical point (phase transition)
    "non-trivial" fixed point

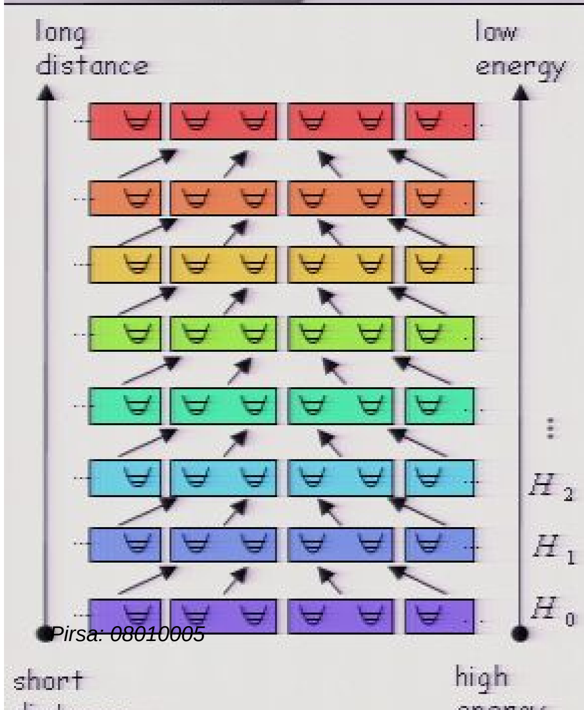




- fixed points of the RG flow
  - well-defined phase (e.g. ordered, disordered phase) "trivial" fixed point
  - critical point (phase transition) "non-trivial" fixed point
- scale invariance at a fixed point:







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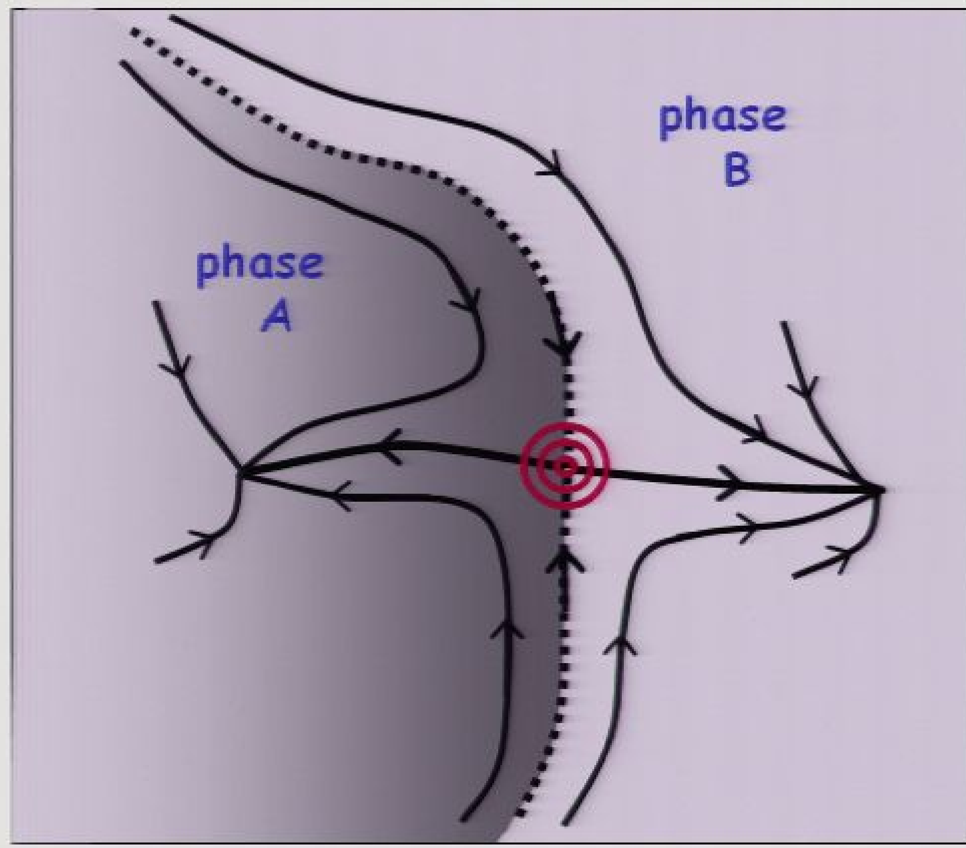


$$H_0 = H_1 = H_2 = \ldots$$

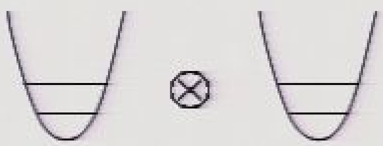
Real Space RG transformation

Coarse-graining:

Real Space RG transformation

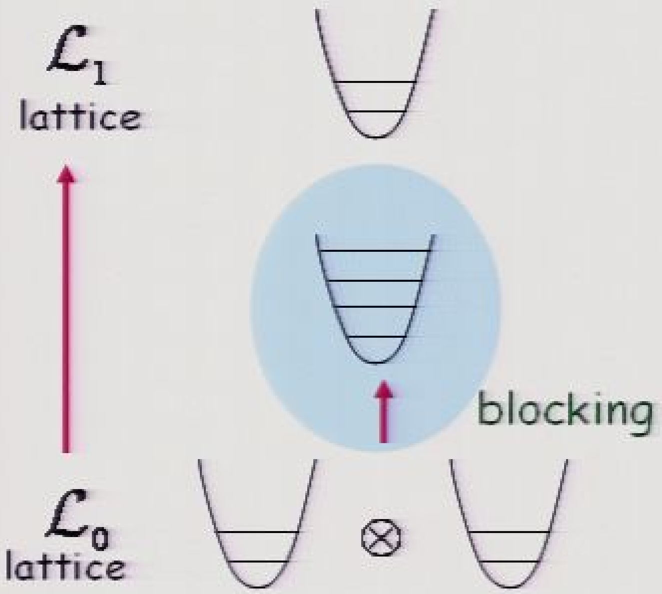
Coarse-graining:

$\mathcal{L}_0$
lattice



Real Space RG transformation

Coarse-graining:



Real Space RG transformation

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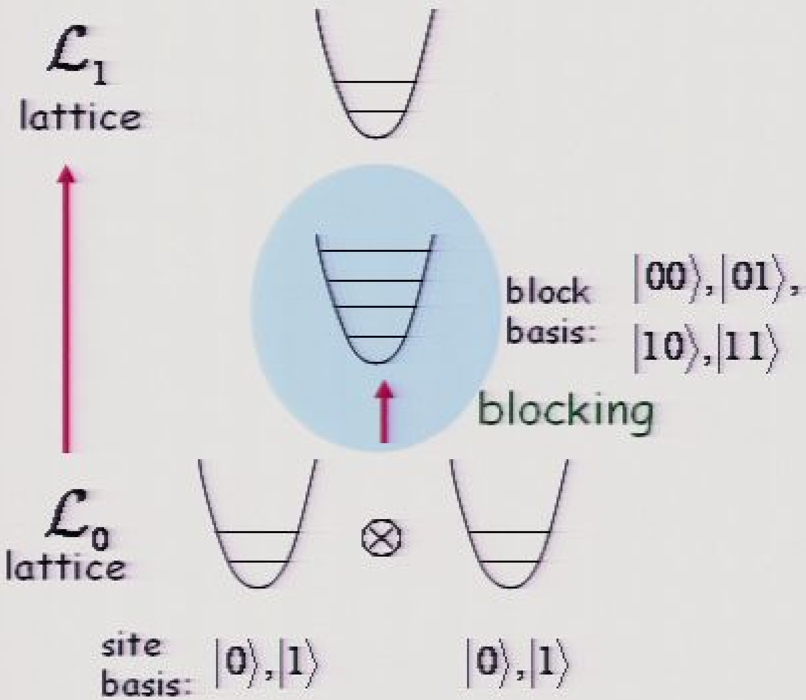

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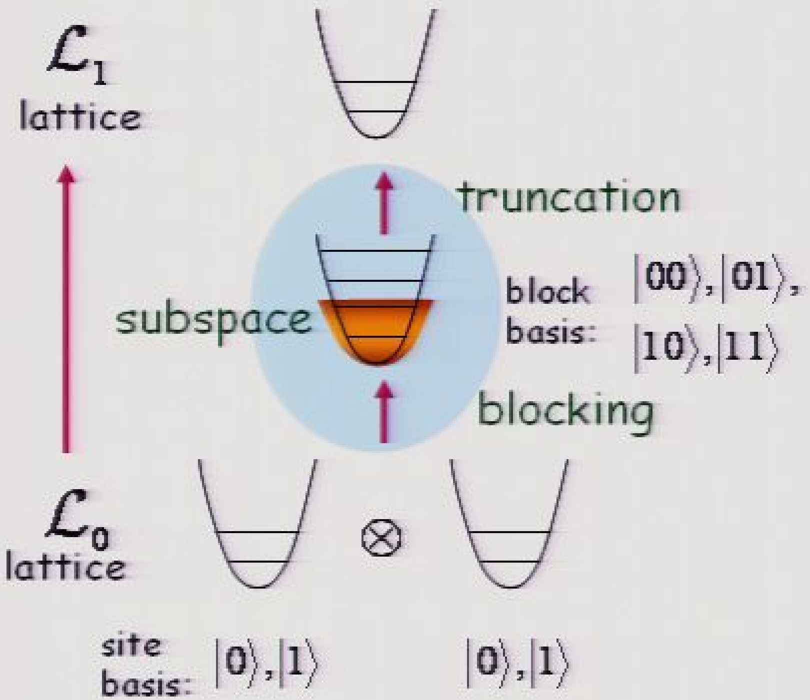

Real Space RG transformation

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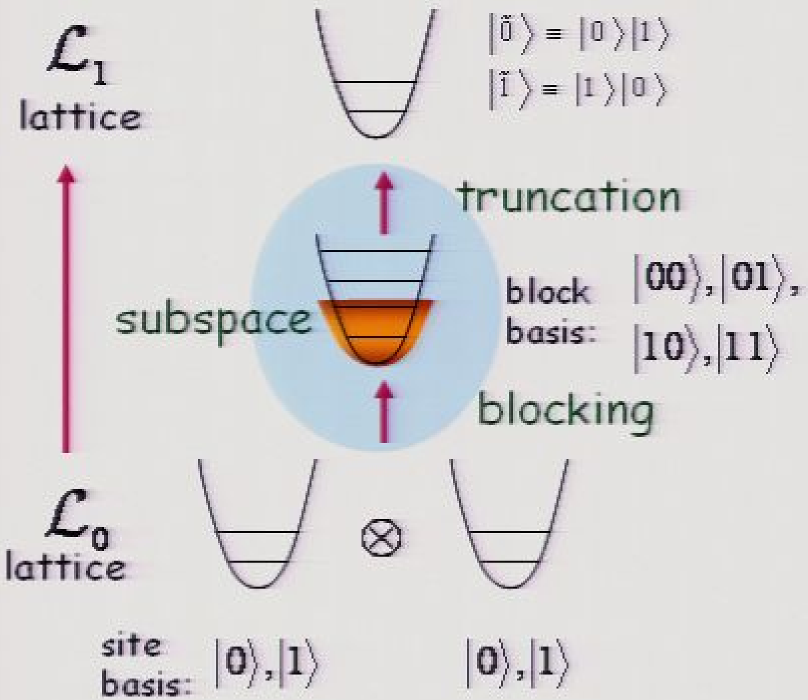
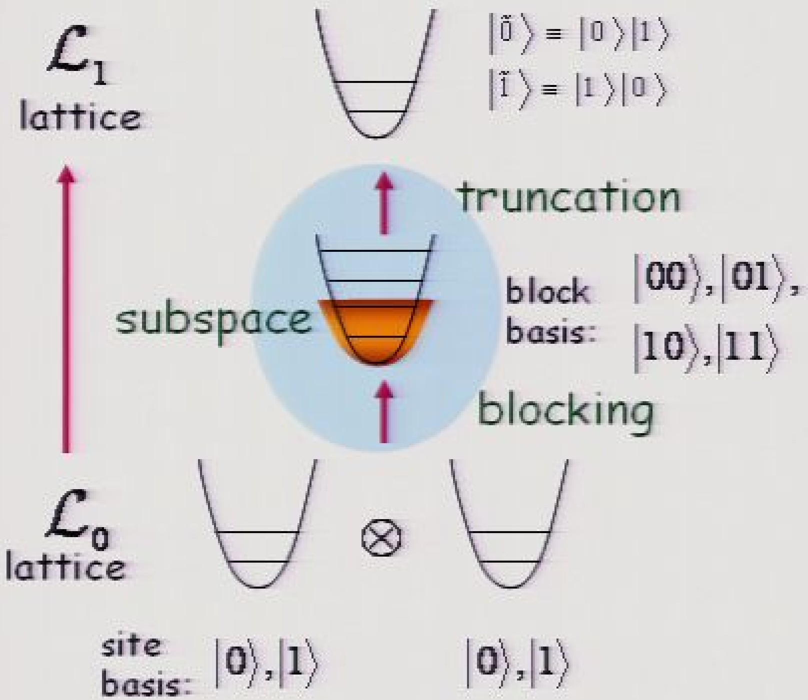


- crucial: choose correct subspace 

Real Space RG transformation

Coarse-graining:



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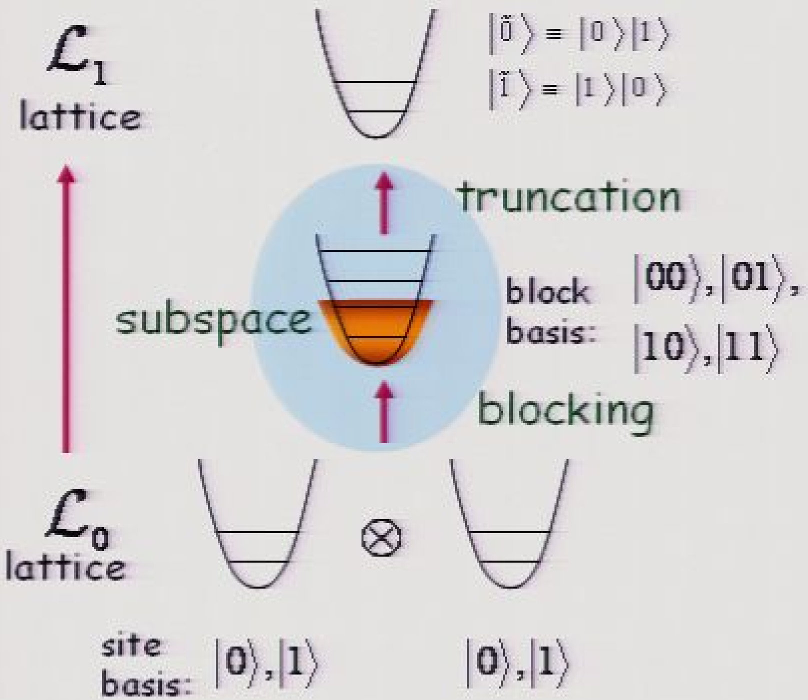

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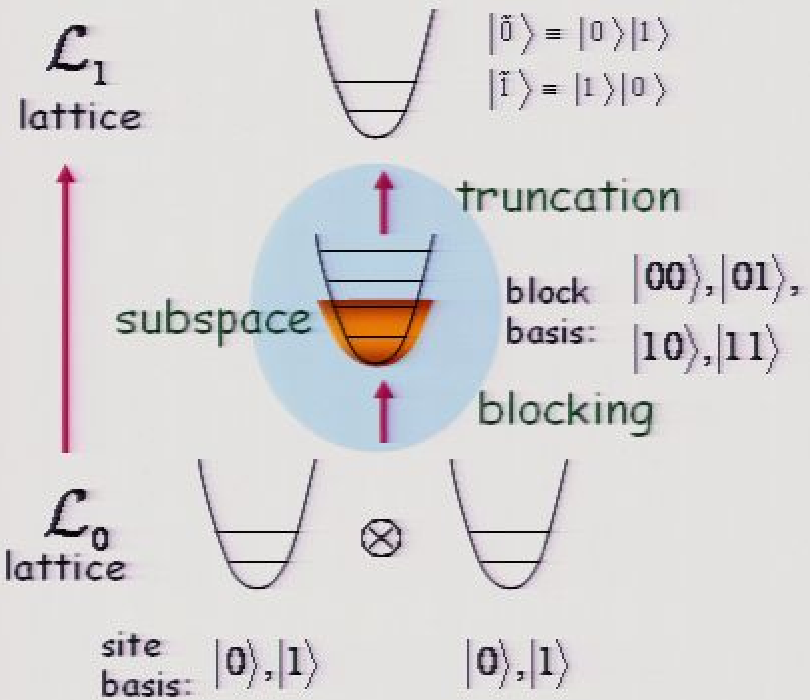

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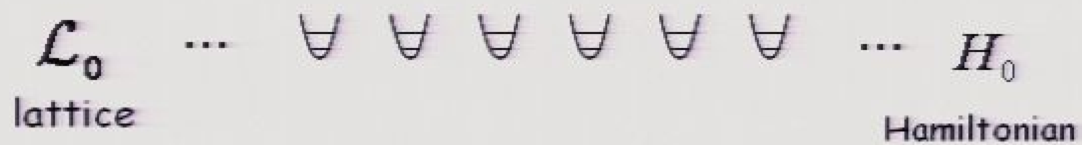

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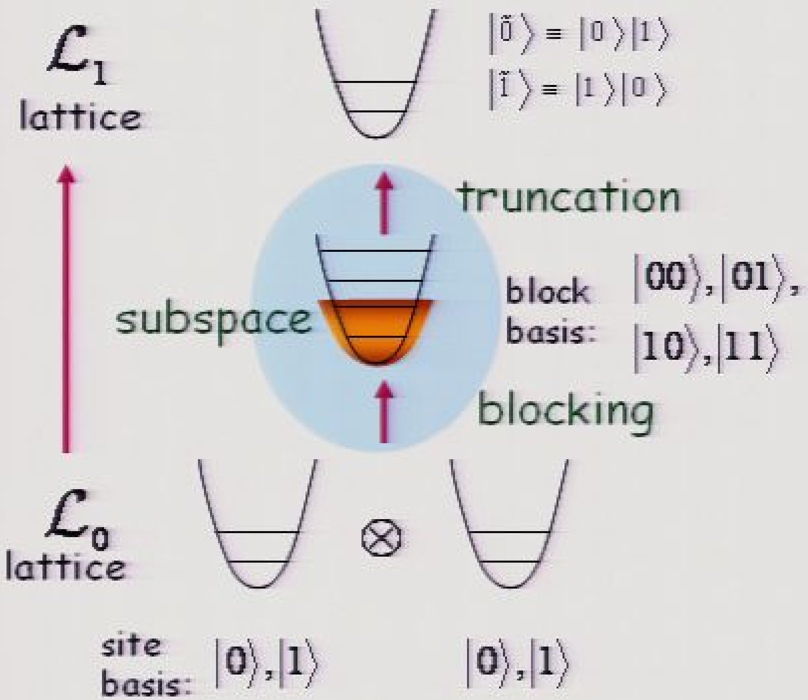

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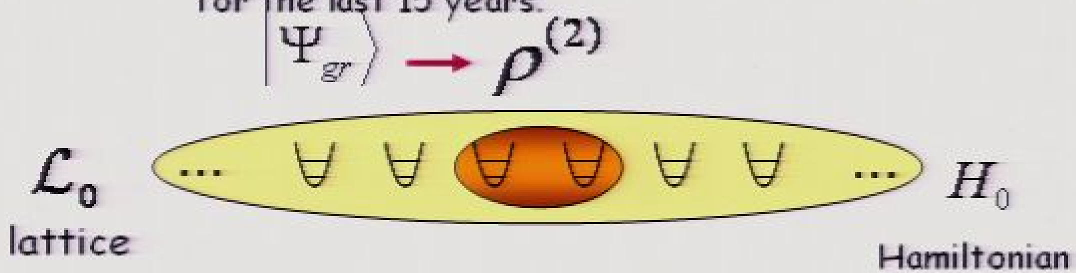

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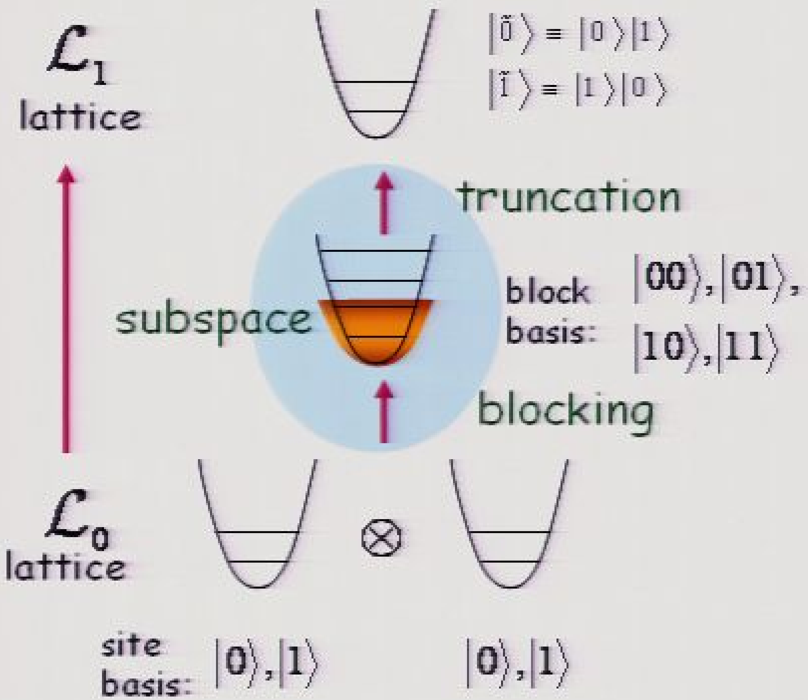

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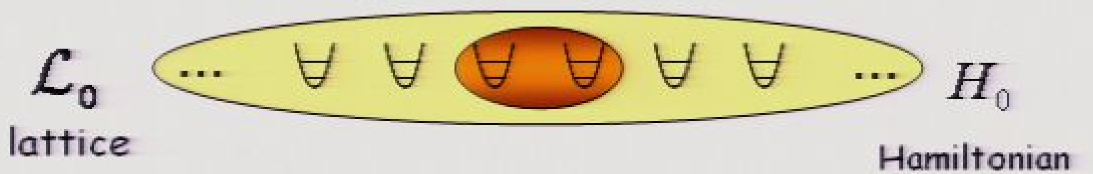
$$|\Psi_{gr}\rangle \rightarrow \rho^{(2)}$$



$$\rho^{(2)} = U \begin{bmatrix} p_1 & & & \\ & p_2 & & \\ & & p_3 \sim 0 & \\ & & & p_4 \sim 0 \end{bmatrix} U^\dagger$$

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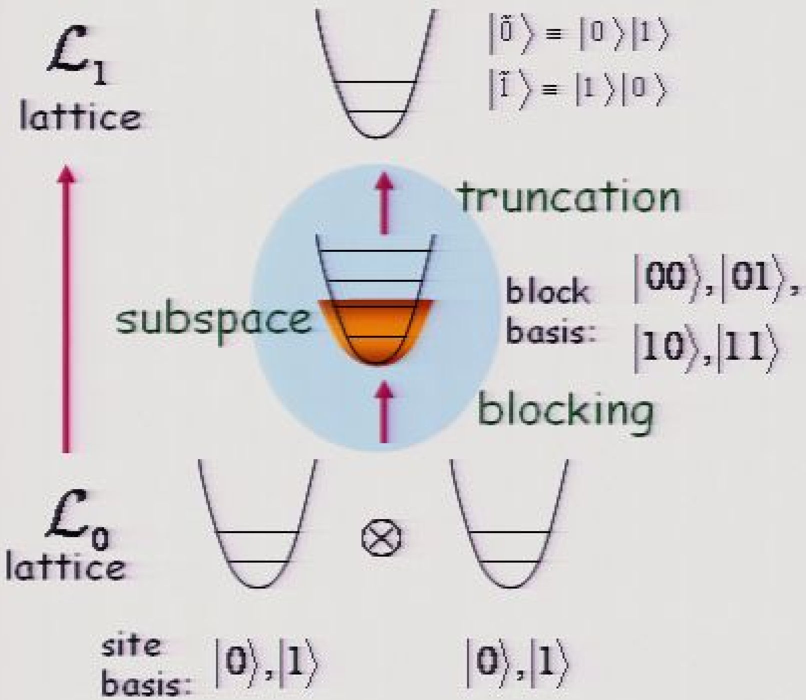

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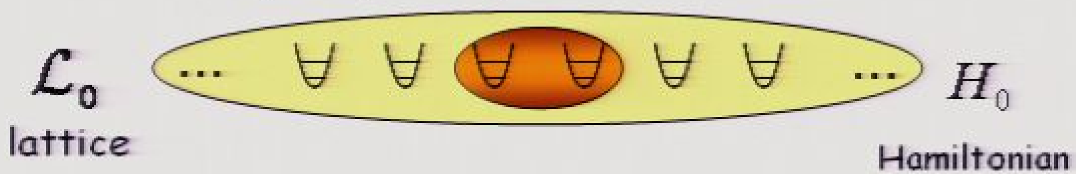

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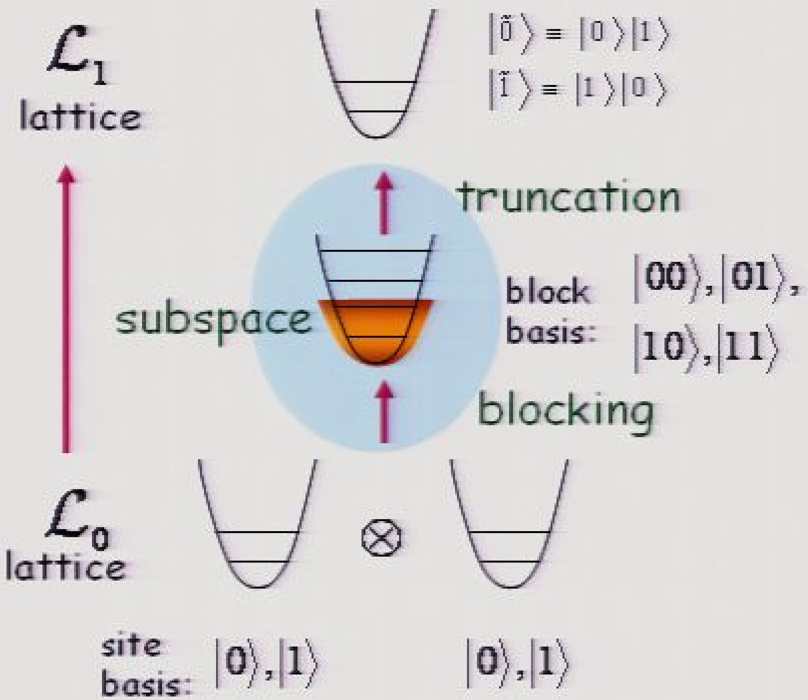

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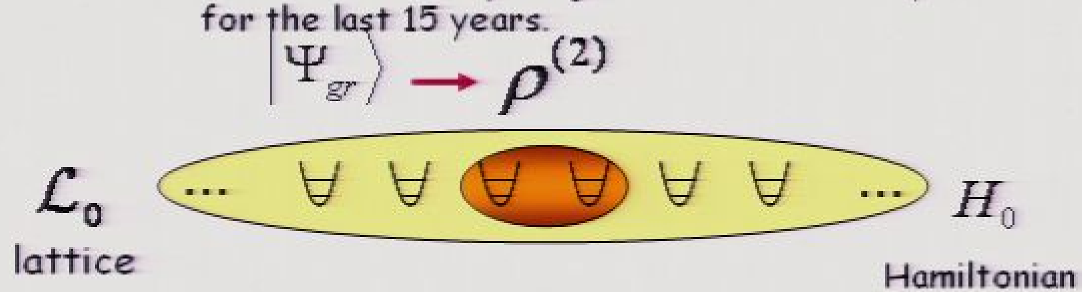

With this choice, if $|\Psi_{gr,0}\rangle, O_0 \downarrow |\Psi_{gr,1}\rangle, O_1$ then $\langle \Psi_{gr,0} | O_0 | \Psi_{gr,0} \rangle = \langle \Psi_{gr,1} | O_1 | \Psi_{gr,1} \rangle$

effective theory

Real Space RG transformation

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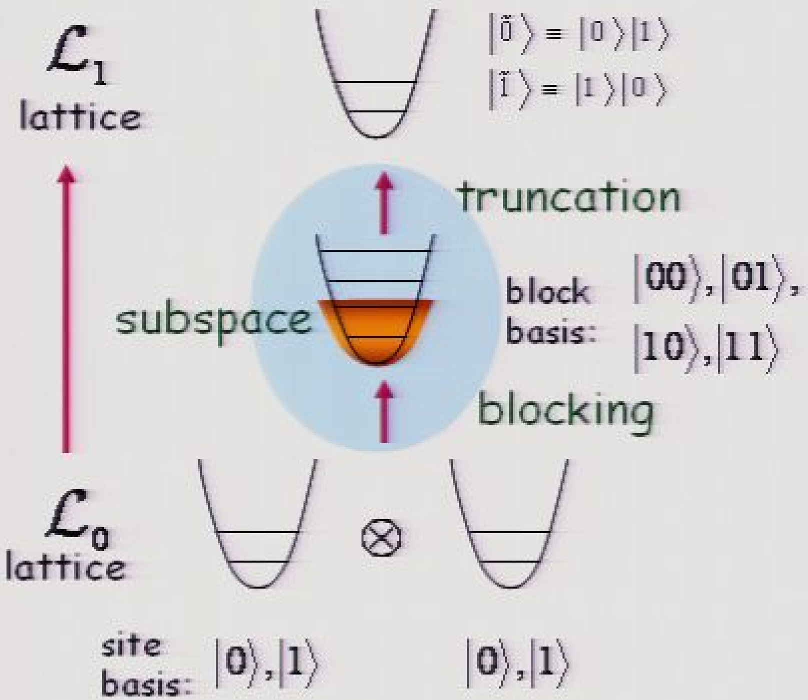

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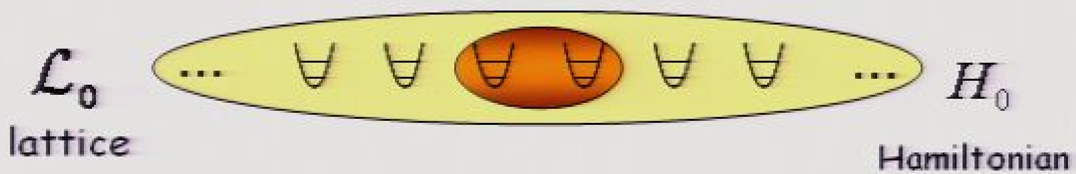

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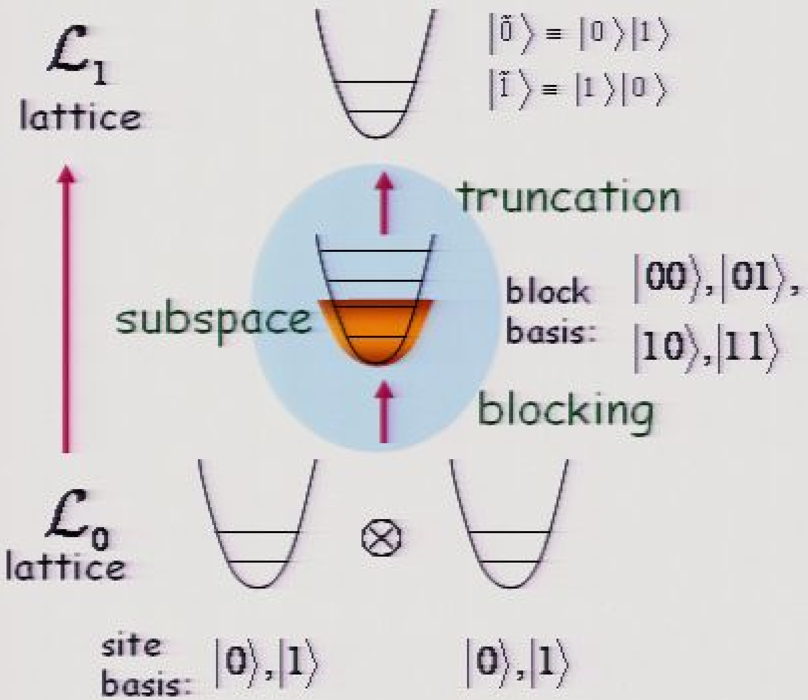

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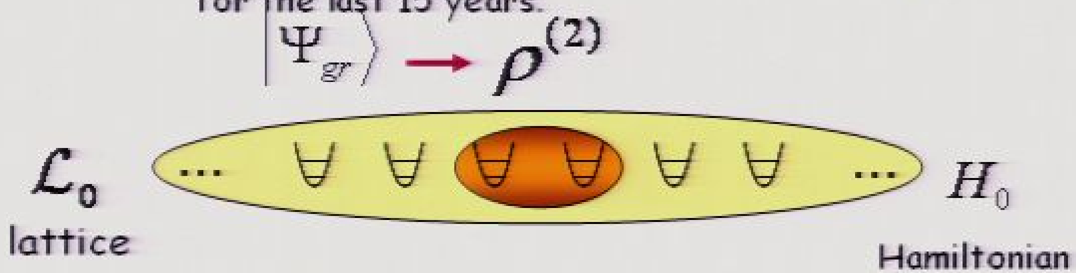

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• the required dimension of the effective site depends on a ***property of the ground state***'s reduced density matrices

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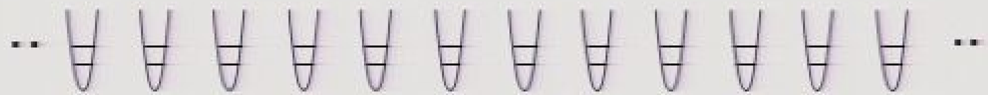

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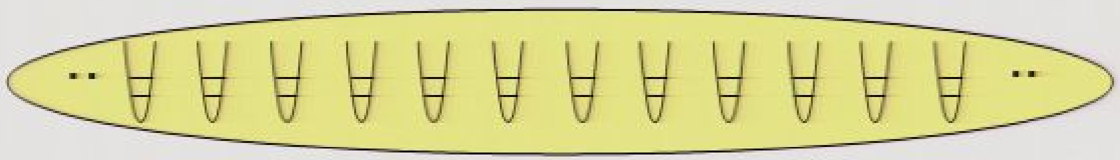

$$\left|\Psi_{gr}\right\rangle$$

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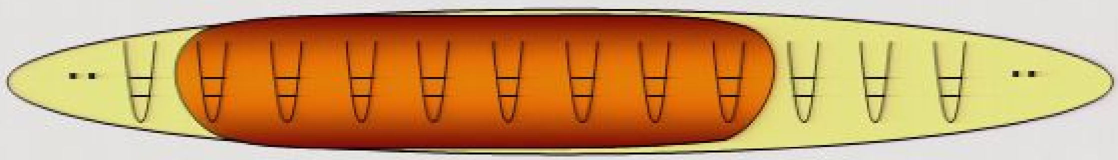

$$|\Psi_{gr}\rangle \rightarrow \rho$$

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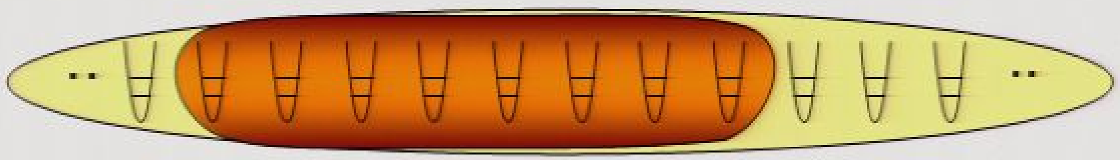
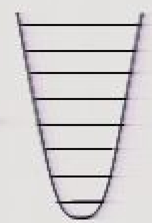
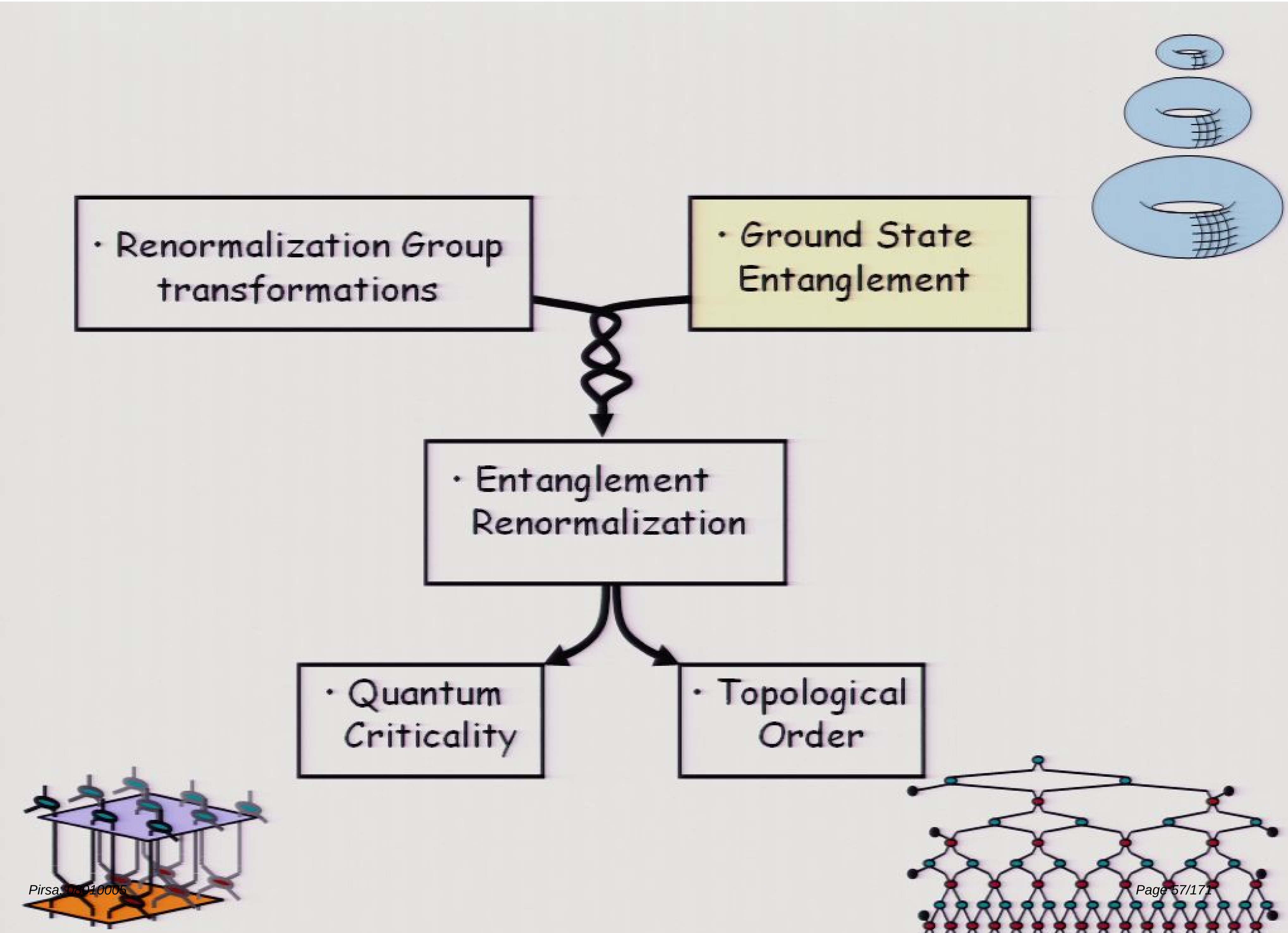


$$|\Psi_{gr}\rangle \to \rho \to \chi \to$$



χ- dimensional Hilbert space for the effective lattice site


• Renormalization Group transformations
• Ground State Entanglement
• Entanglement Renormalization
• Quantum Criticality
• Topological Order

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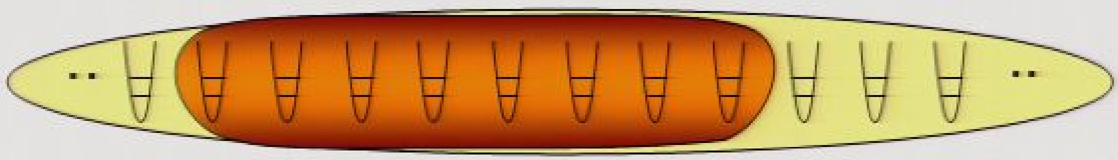

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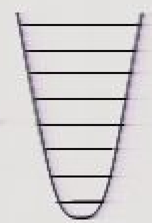

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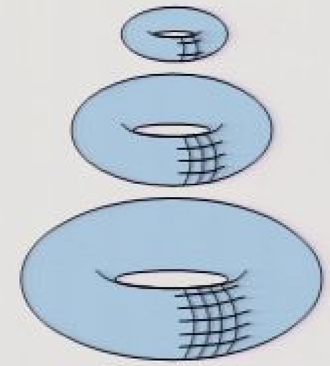

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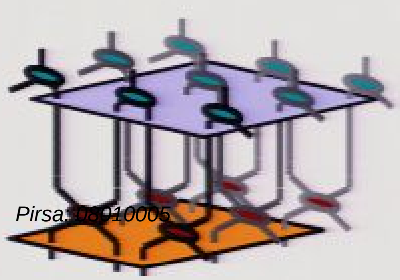

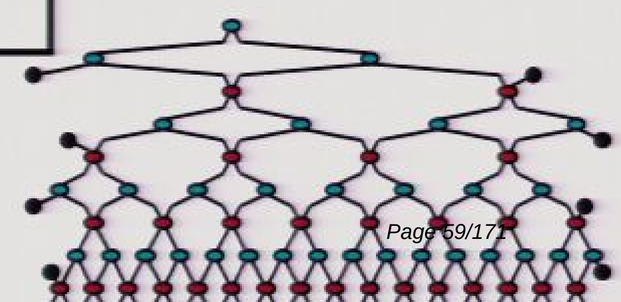

Ground state entanglement

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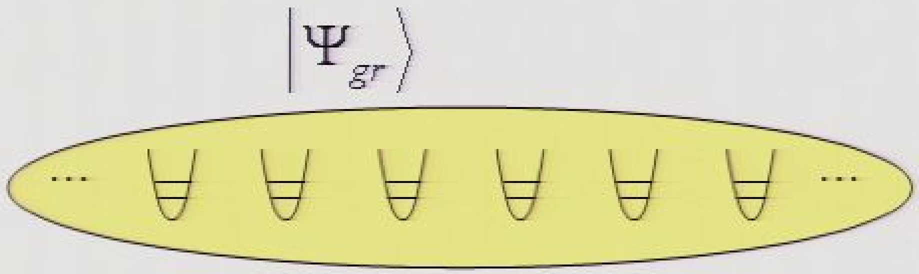

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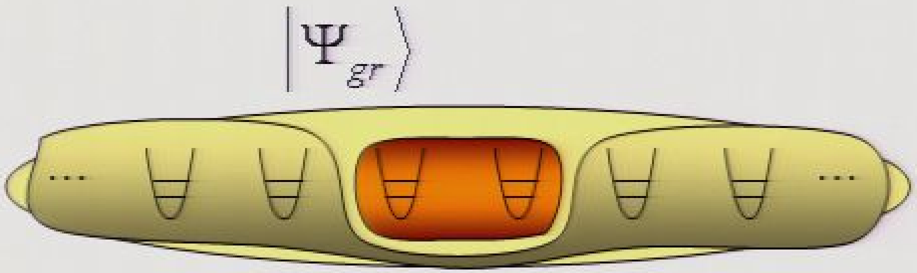
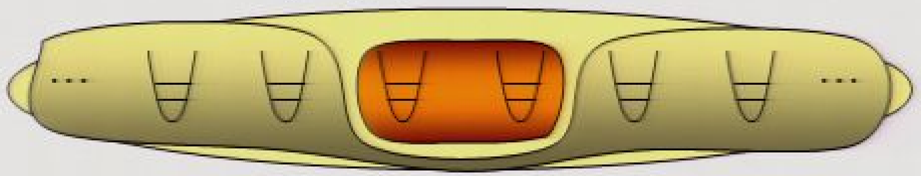


Ground state entanglement

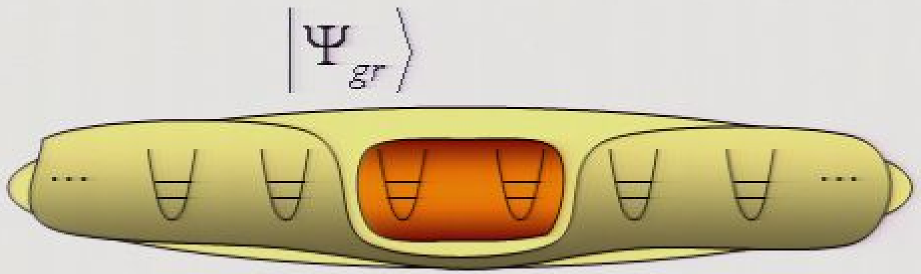
$$\left|\Psi_{gr}\right\rangle$$



$$\left|\Psi_{gr}\right\rangle = \sum_{\alpha=1}^{\chi} \sqrt{p_\alpha} \left|\varphi_\alpha^{(2)}\right\rangle \left|\psi_\alpha^{(rest)}\right\rangle$$

Schmidt decomposition

Ground state entanglement



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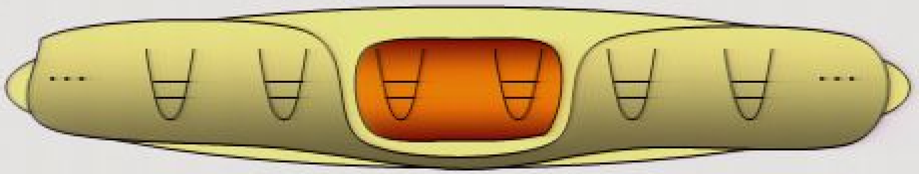

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Schmidt decomposition

reduced density matrix

Ground state entanglement

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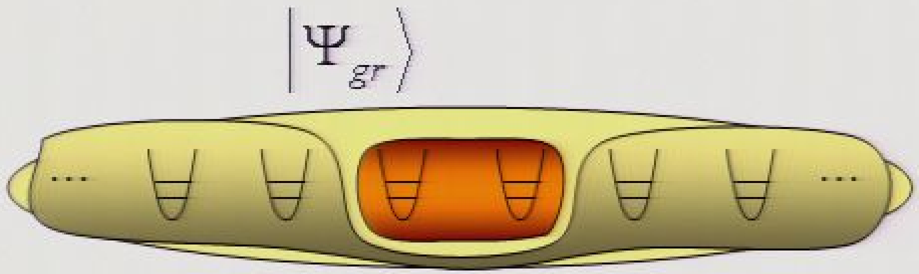
Schmidt decomposition — reduced density matrix

$$S = -\sum_\alpha p_\alpha \log p_\alpha$$

Entanglement Entropy

- quantifies the entanglement between the block (2) and the rest of the chain

Ground state entanglement



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is the effective size (vector space dimension) of the coarse-grained sites

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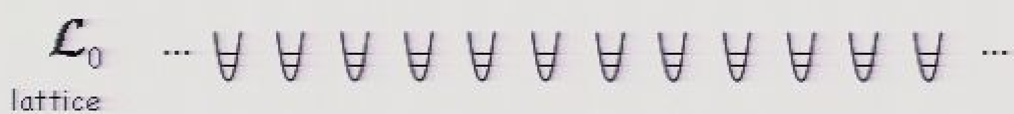
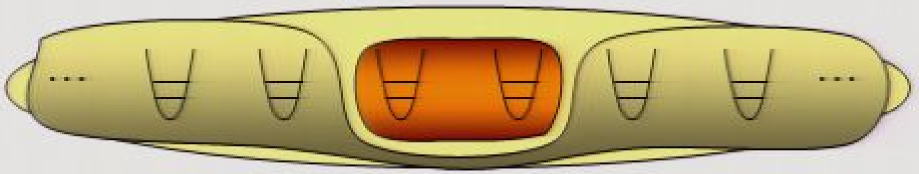


Ground state entanglement

$|\Psi_{gr}\rangle$



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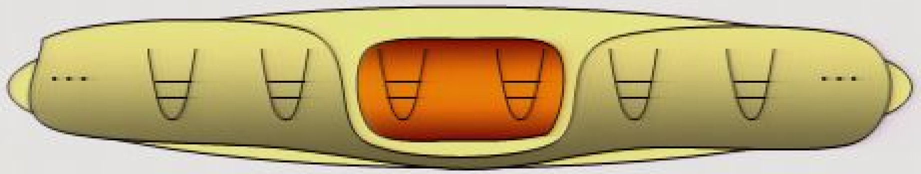

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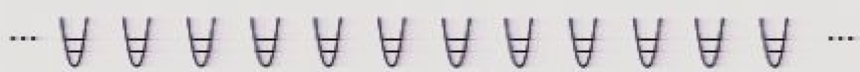
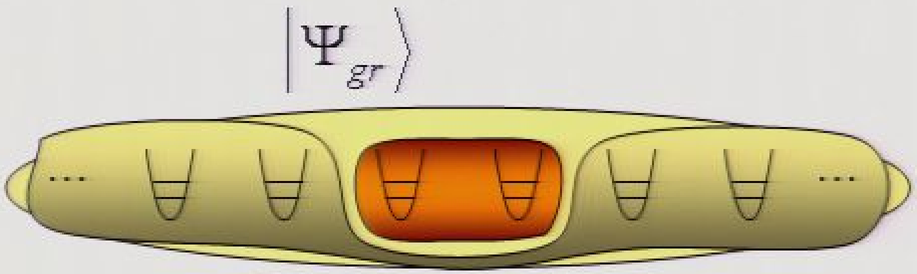
$\mathcal{L}_0$

lattice



Ground state entanglement



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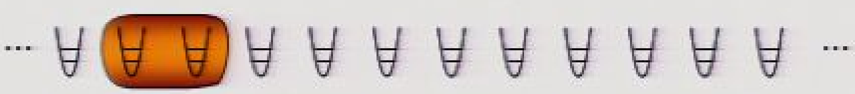

$\rho^{(2^1)}$

Ground state entanglement

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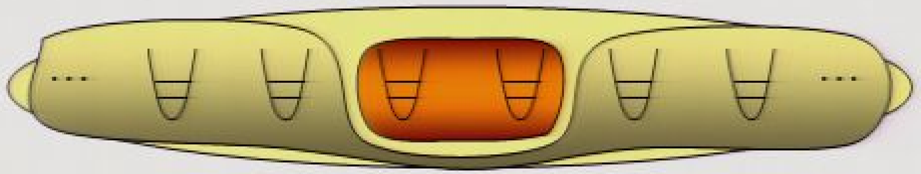

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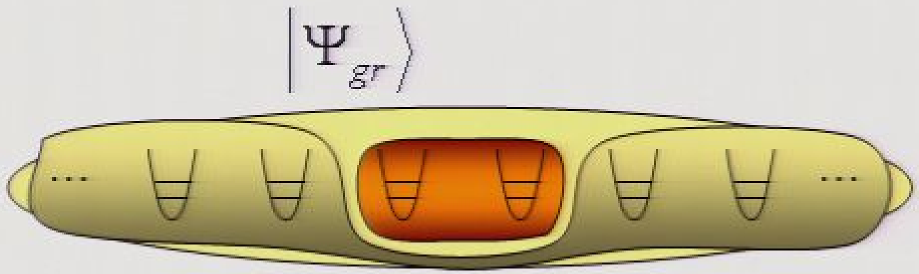
lattice

$\rho^{(2^1)} \rightarrow S_1 \rightarrow$ $\mathcal{L}_1$

$\rho^{(2^2)} \rightarrow S_2 \rightarrow$ $\mathcal{L}_2$

Ground state entanglement



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caling of entanglement in ground states

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1D lattice
(critical)

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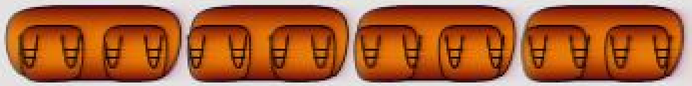
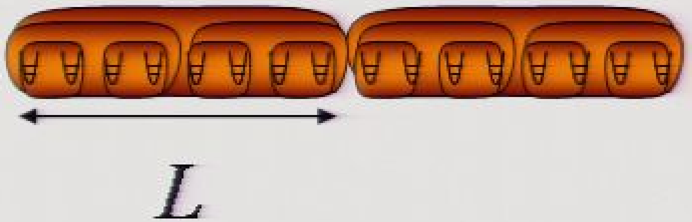


caling of entanglement in ground states

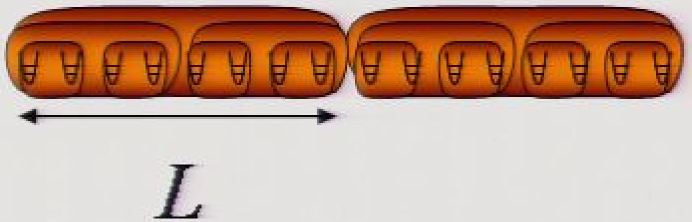
1D lattice
(critical)



caling of entanglement in ground states

$$S_L \sim \log L$$

1D lattice
(critical)

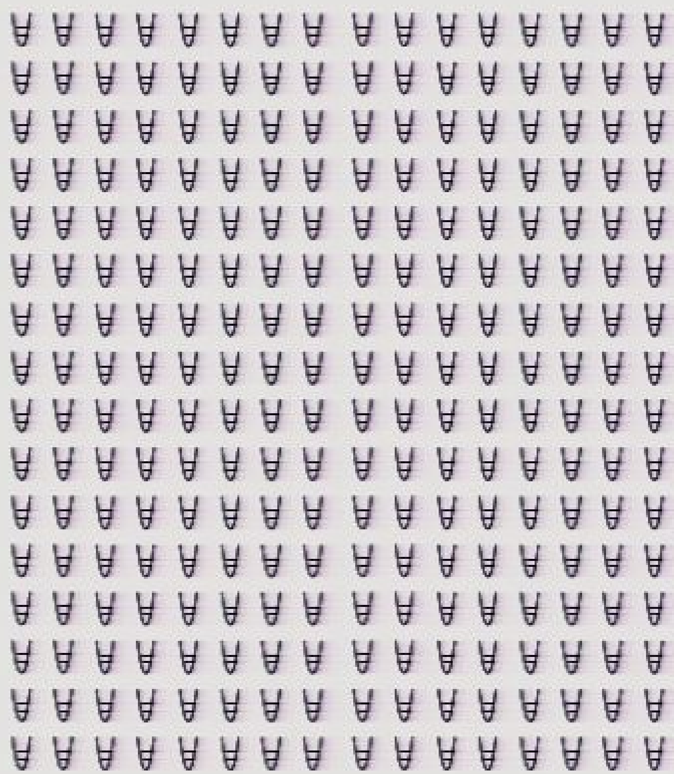


caling of entanglement in ground states

$$S_L \sim \log L \;\Rightarrow\; \chi_L \sim L^q$$

1D lattice
(critical)

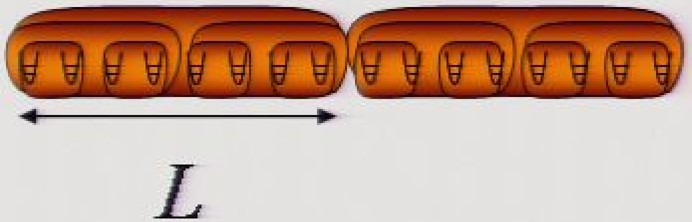
2D lattice
(critical or
non-critical)



caling of entanglement in ground states

$$S_L \sim \log L \;\Rightarrow\; \chi_L \sim L^q$$

1D lattice
(critical)



2D lattice
(critical or
non-critical)

caling of entanglement in ground states

$$S_L \sim \log L \;\Rightarrow\; \chi_L \sim L^q$$

1D lattice
(critical)

2D lattice
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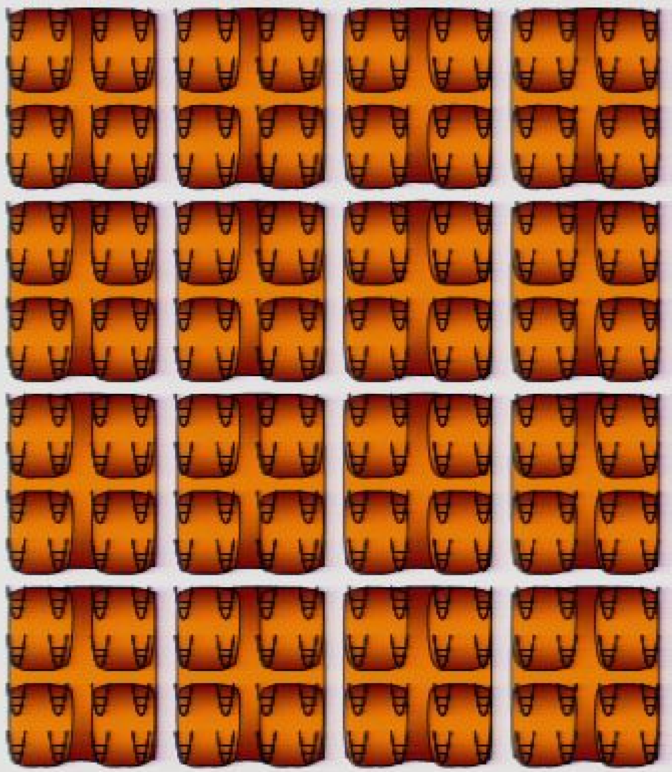

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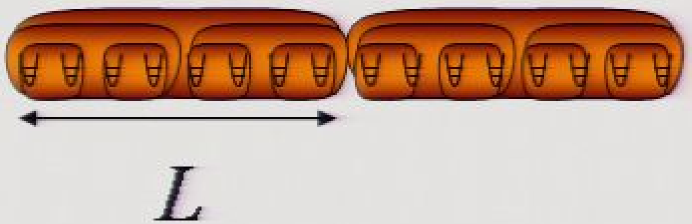
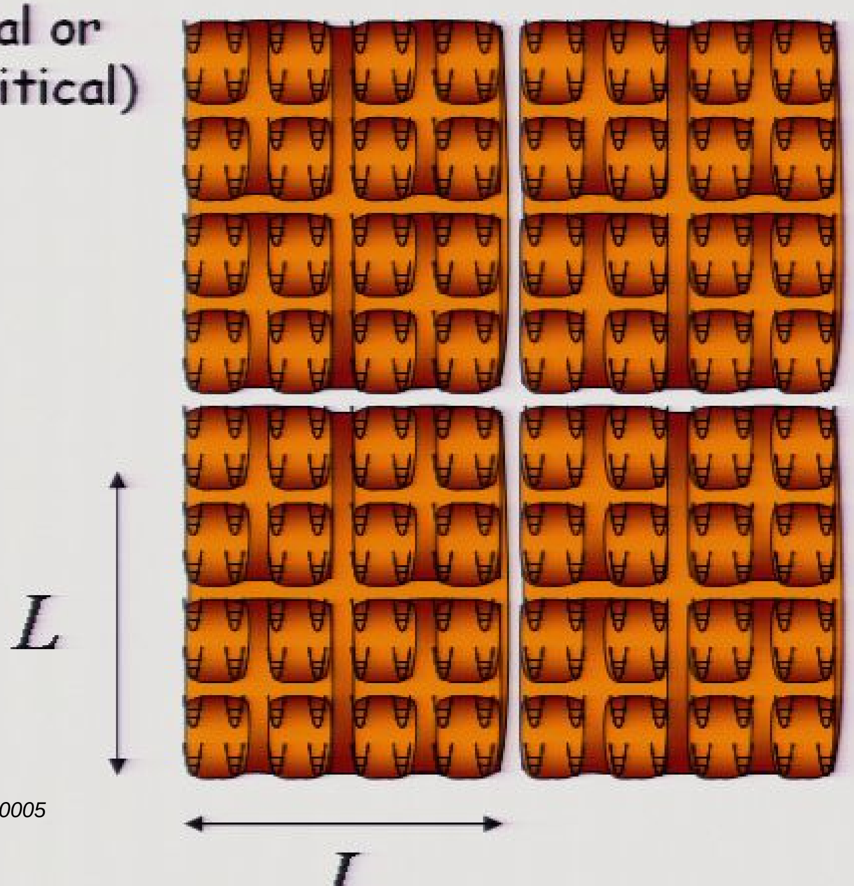


2D lattice
(critical or
non-critical)



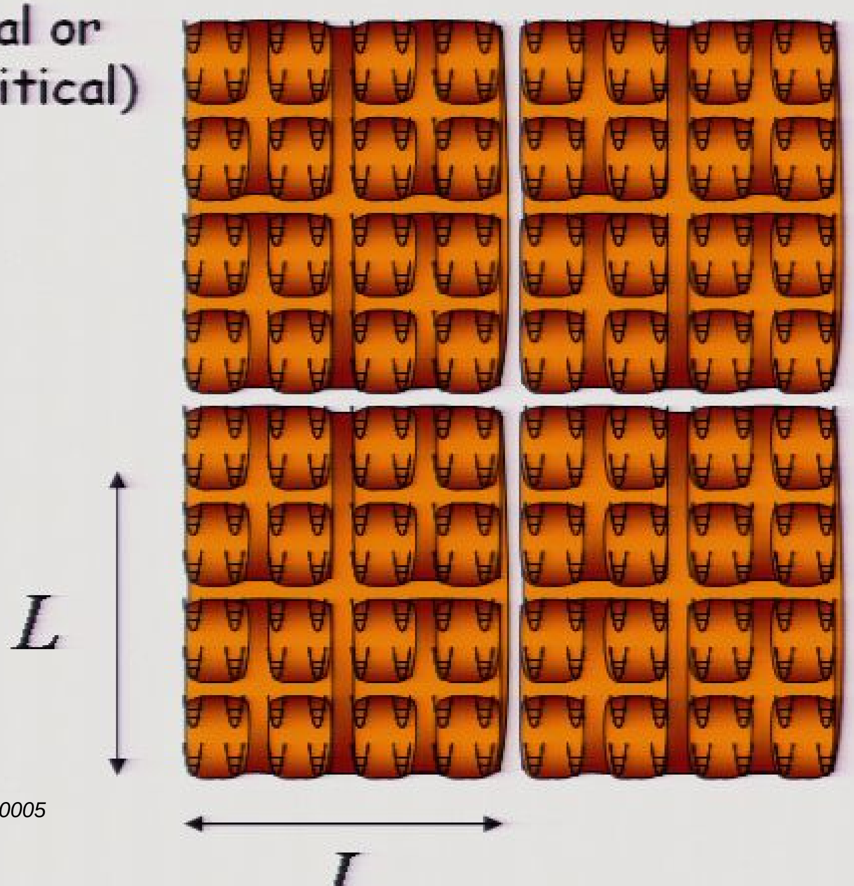
caling of entanglement in ground states

$$S_L \sim \log L \;\Rightarrow\; \chi_L \sim L^q$$

1D lattice
(critical)

$$S_{L^2} \sim L \;\Rightarrow\; \chi_{L^2} \sim e^L$$

2D lattice
(critical or
non-critical)

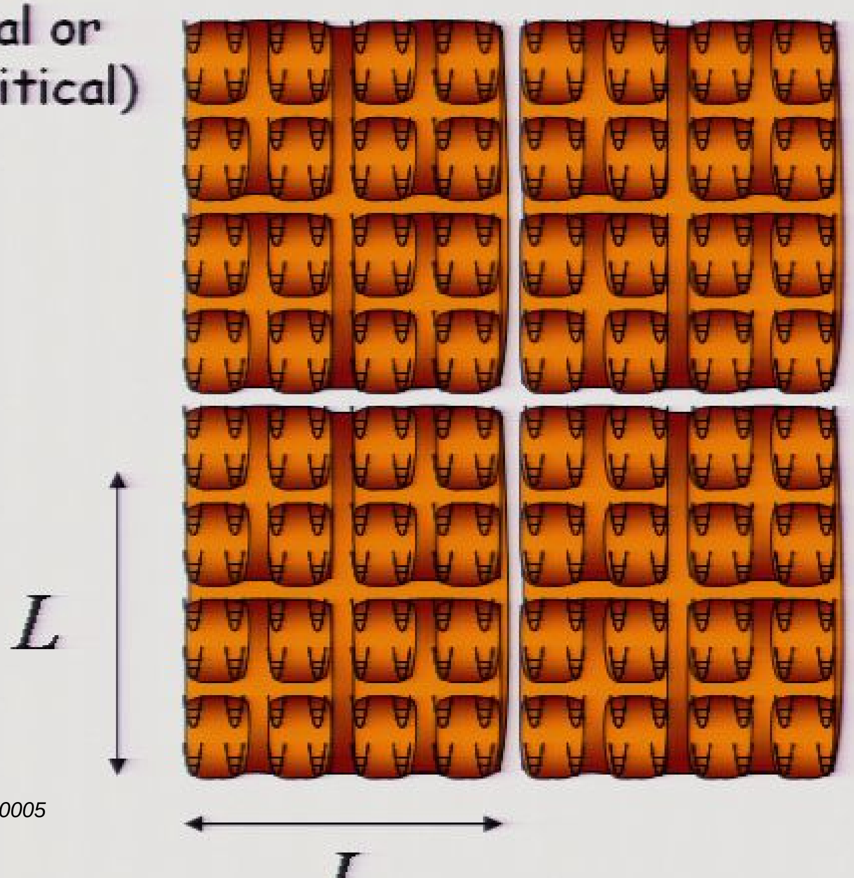


caling of entanglement in ground states

1D lattice (critical)

$$S_L \sim \log L \;\Rightarrow\; \chi_L \sim L^q$$

2D lattice (critical or non-critical)



$$S_{L^2} \sim L \;\Rightarrow\; \chi_{L^2} \sim e^L$$

in D dimensions

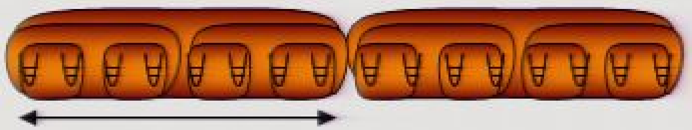
$$S_{L^D} \sim L^{D-1}$$

boundary law

caling of entanglement in ground states

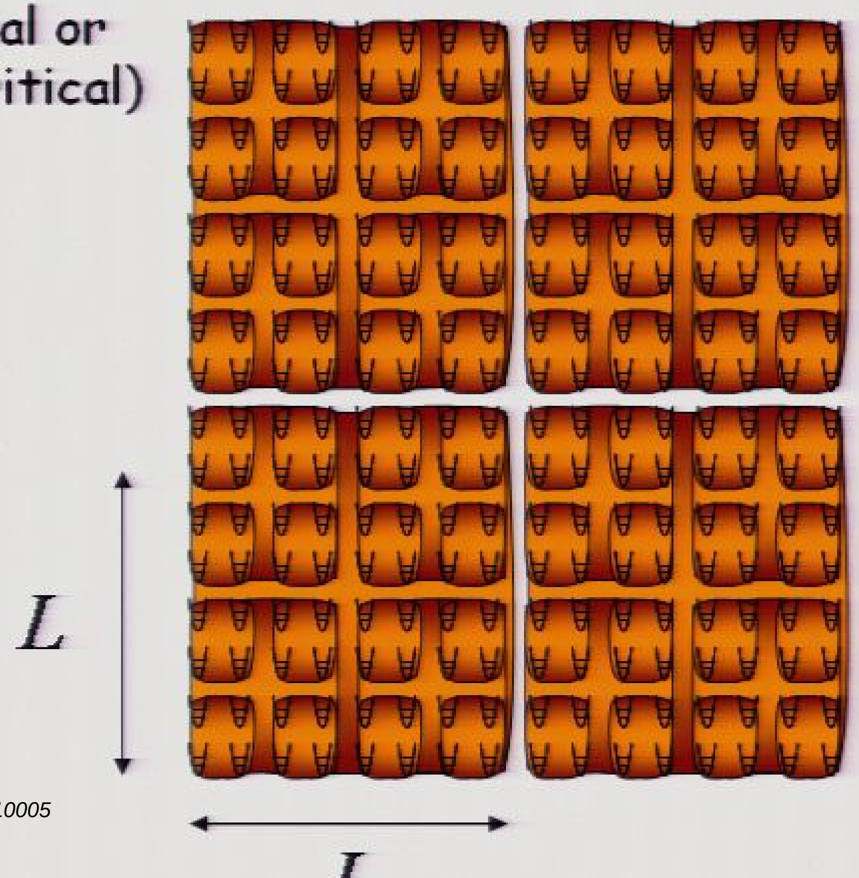
$$S_L \sim \log L \;\rightarrow\; \chi_L \sim L^q$$

1D lattice
(critical)



2D lattice
(critical or
non-critical)

$$S_{L^2} \sim L \;\rightarrow\; \chi_{L^2} \sim e^L$$



in D dimensions

$$S_{L^D} \sim L^{D-1}$$

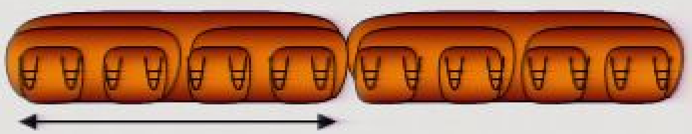
boundary law

$$\rightarrow\; \chi_{L^D} \sim e^{L^{D-1}}$$

caling of entanglement in ground states

$$S_L \sim \log L \;\rightarrow\; \chi_L \sim L^q$$

1D lattice
(critical)



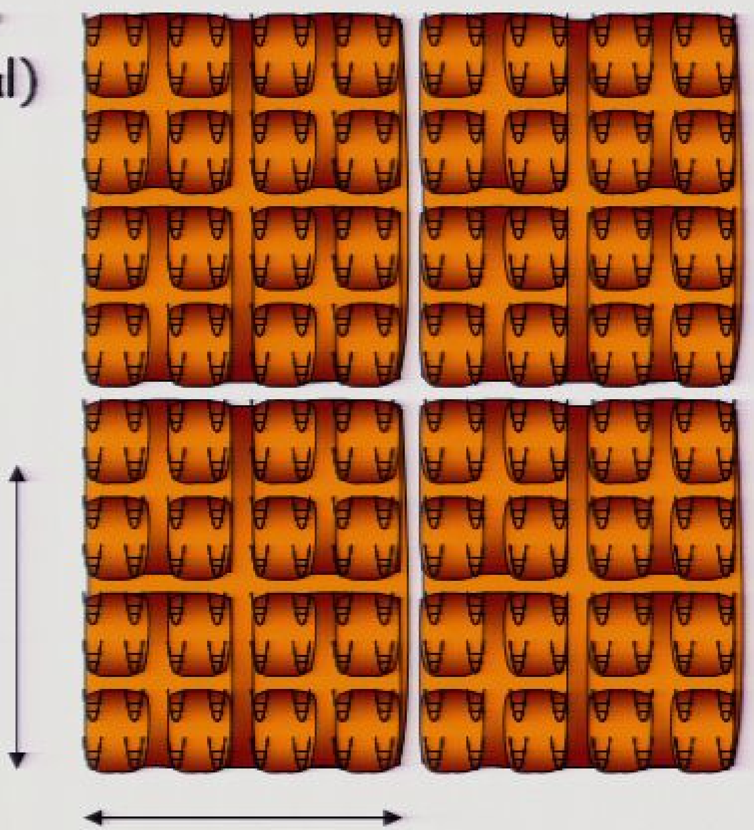
L

$$\chi_\tau \sim e^\tau$$

after τ coarse-graining steps

2D lattice
(critical or
non-critical)

$$S_{L^2} \sim L \;\rightarrow\; \chi_{L^2} \sim e^L$$



L

L

in D dimensions

$$S_{L^D} \sim L^{D-1}$$

boundary law

$$\rightarrow\; \chi_{L^D} \sim e^{L^{D-1}}$$

caling of entanglement in ground states

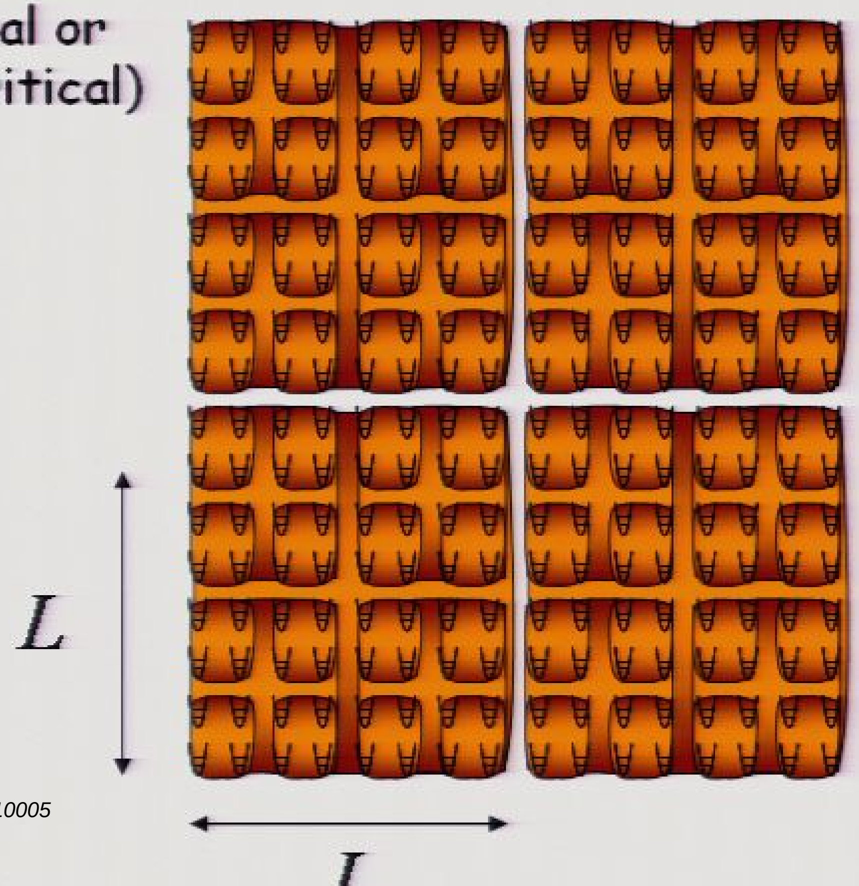
$$S_L \sim \log L \;\Rightarrow\; \chi_L \sim L^q$$

1D lattice (critical)

$$\chi_\tau \sim e^\tau$$

after τ coarse-graining steps

2D lattice (critical or non-critical)



$$S_{L^2} \sim L \;\Rightarrow\; \chi_{L^2} \sim e^L$$

$$\chi_\tau \sim e^{e^\tau}$$

after τ coarse-graining steps

in D dimensions

$$S_{L^D} \sim L^{D-1}$$

boundary law

$$\Rightarrow\; \chi_{L^D} \sim e^{L^{D-1}}$$

- As we apply the RG transformation...

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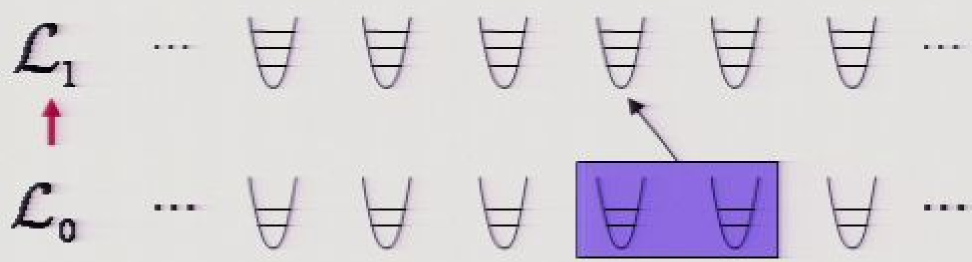

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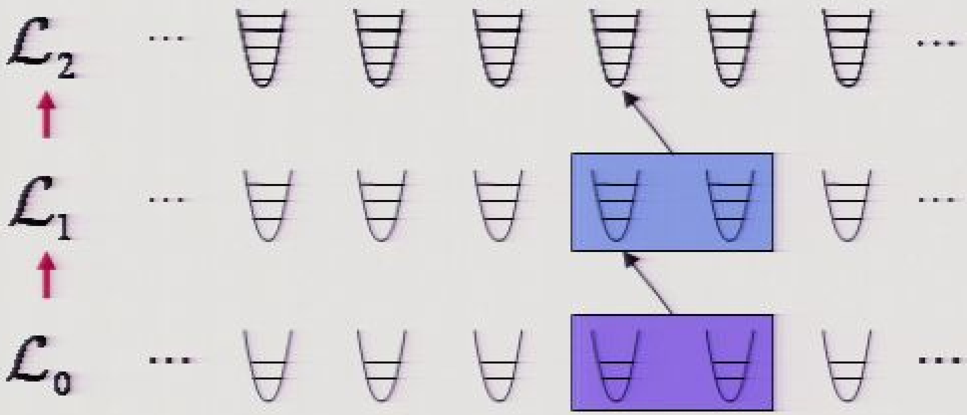

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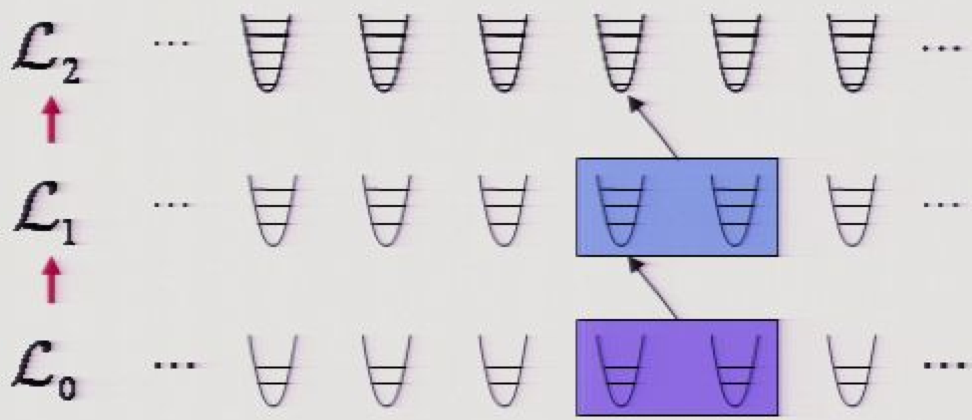
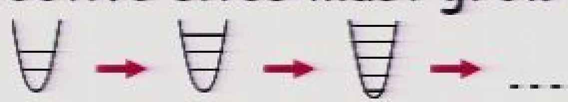


... the effective sites must grow (a lot!)



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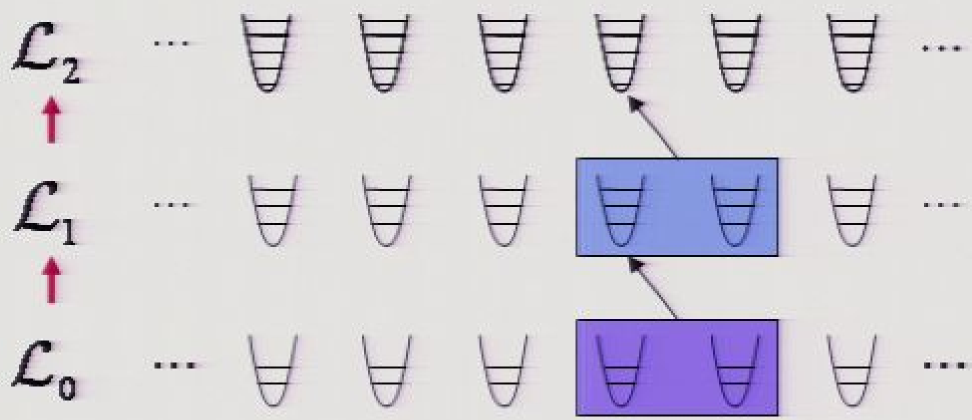

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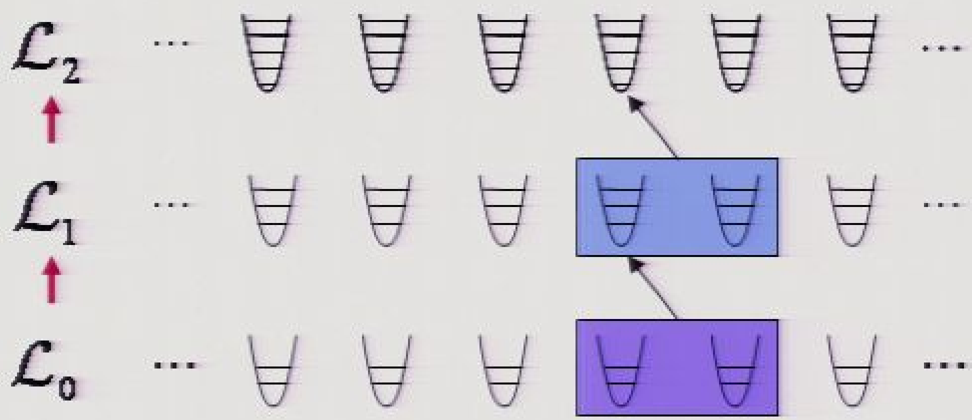
So what?

- As we apply the RG transformation…



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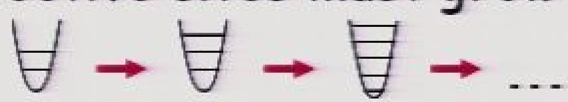
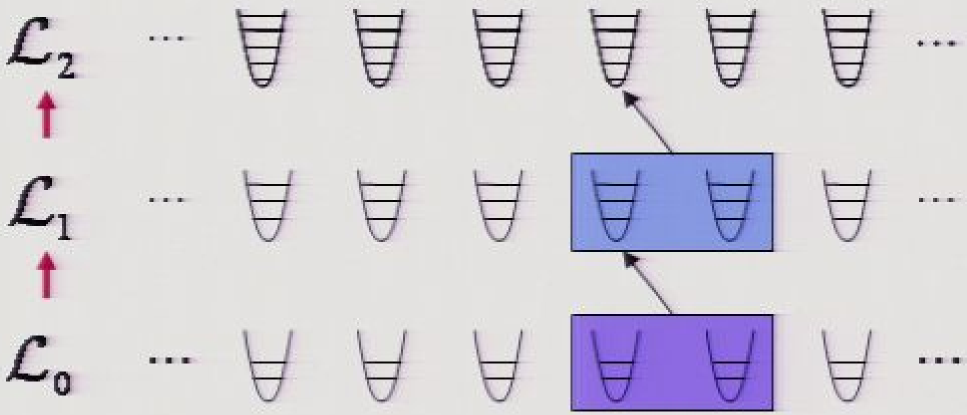


So what?

✓ • Correct results $\langle \Psi_{gr,0} | O_0 | \Psi_{gr,0} \rangle = \langle \Psi_{gr,1} | O_1 | \Psi_{gr,1} \rangle$

- As we apply the RG transformation…



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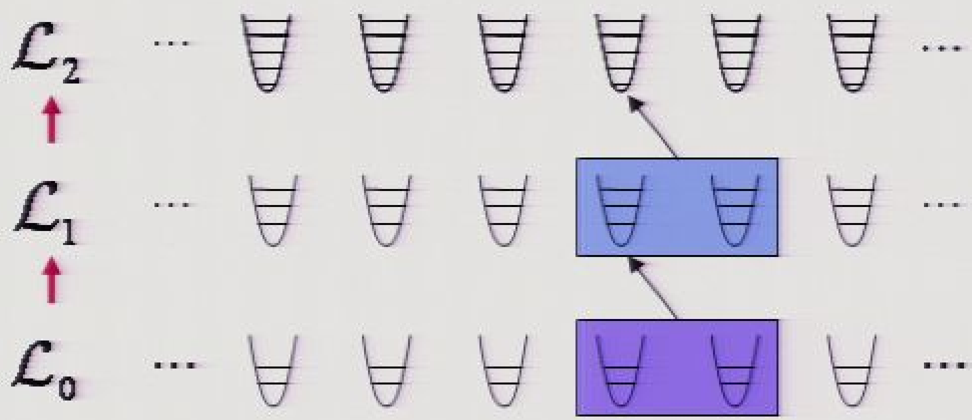
So what?

✓ • Correct results $\left\langle \Psi_{gr,0} \middle| O_0 \middle| \Psi_{gr,0} \right\rangle = \left\langle \Psi_{gr,1} \middle| O_1 \middle| \Psi_{gr,1} \right\rangle$

✗ • Conceptually unsatisfactory

- RG flow? (incomparable systems)
- Fixed points? (phases, critical)
- Scale invariance? (at critical point)

- As we apply the RG transformation...



... the effective sites must grow (a lot!)

So what?

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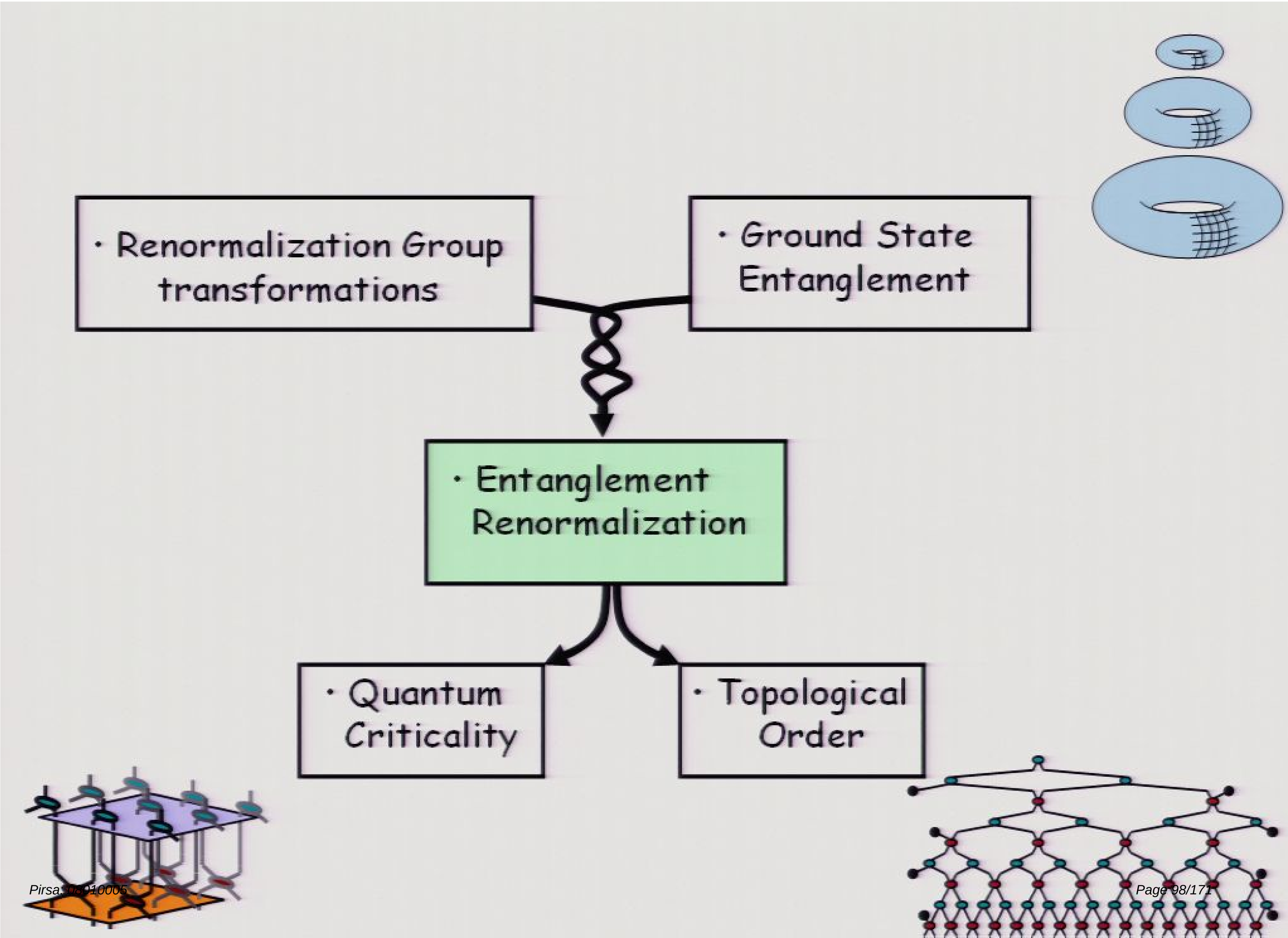
- ✗ Conceptually unsatisfactory
  - RG flow? (incomparable systems)
  - Fixed points? (phases, critical)
  - Scale invariance? (at critical point)
- ✗ Severe computational limitation
  - Only 1D non-critical
  - 1D critical: only a few iterations
  - 2D: only marginally small systems



• Renormalization Group transformations
• Ground State Entanglement
• Entanglement Renormalization
• Quantum Criticality
• Topological Order

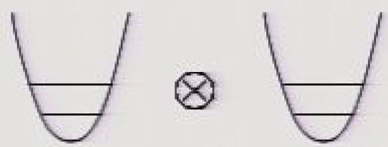
oarse-graining with disentanglers

- diagrammatic representation:

oarse-graining with disentanglers

- diagrammatic representation:

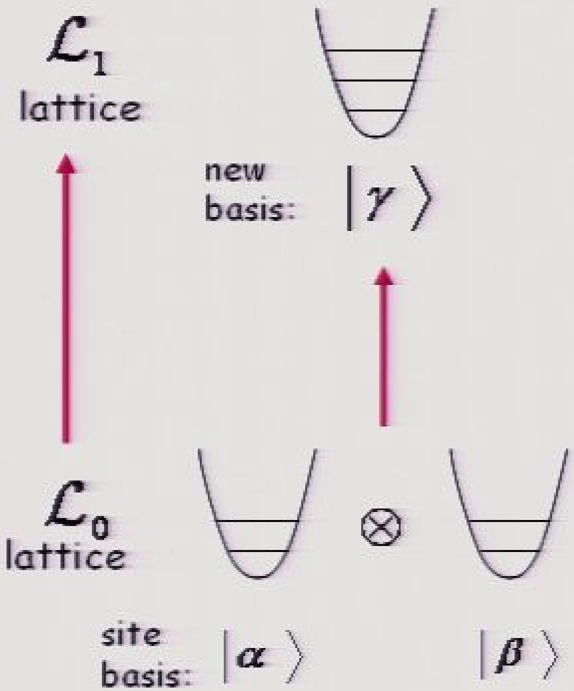
$\mathcal{L}_0$
lattice



site
basis: $|\alpha\rangle$ $|\beta\rangle$

oarse-graining with disentanglers

- diagrammatic representation:



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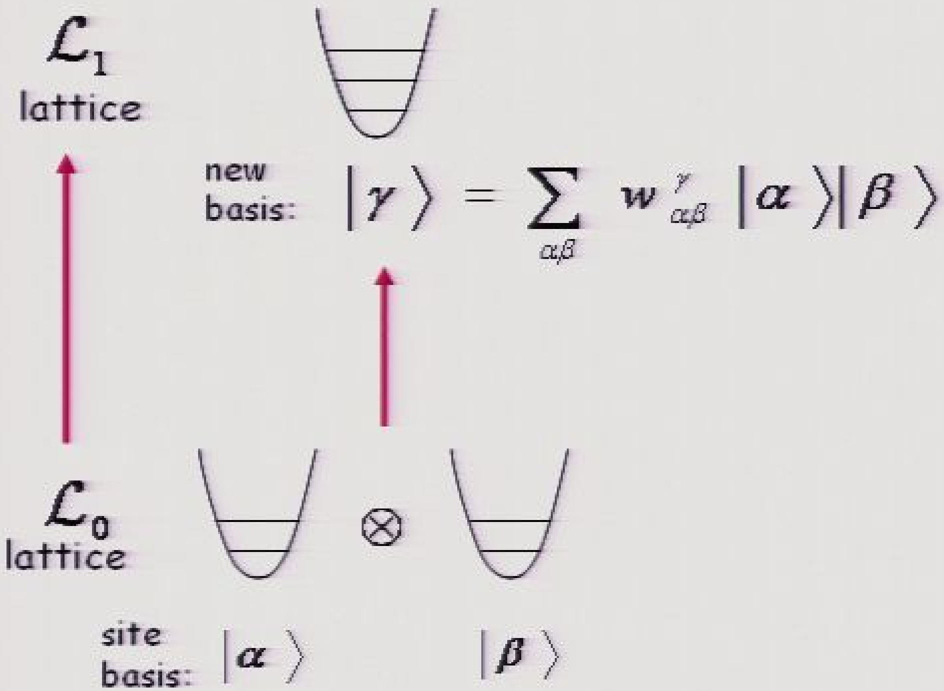

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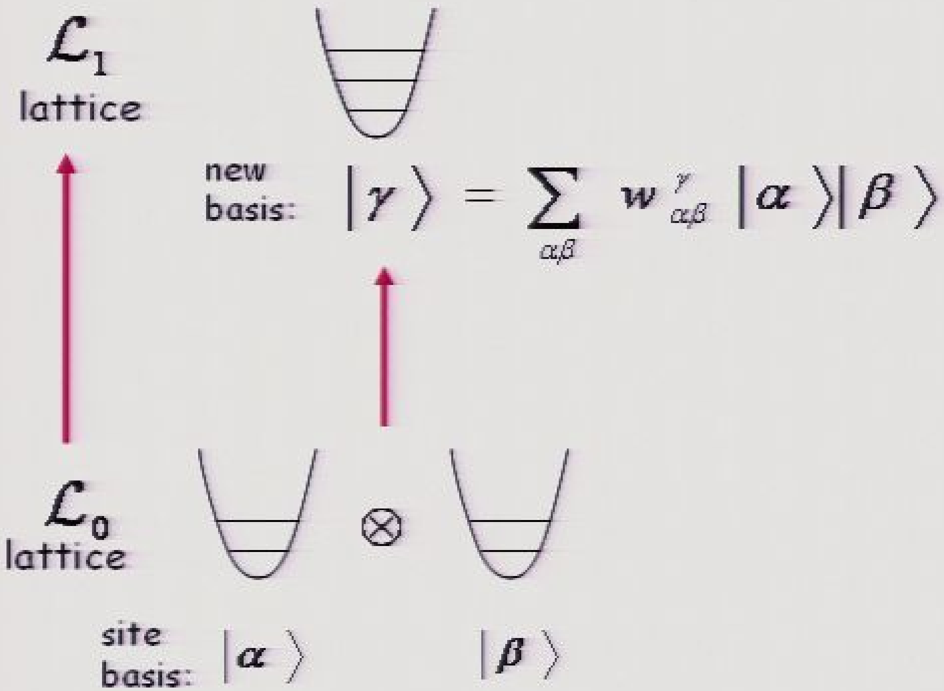
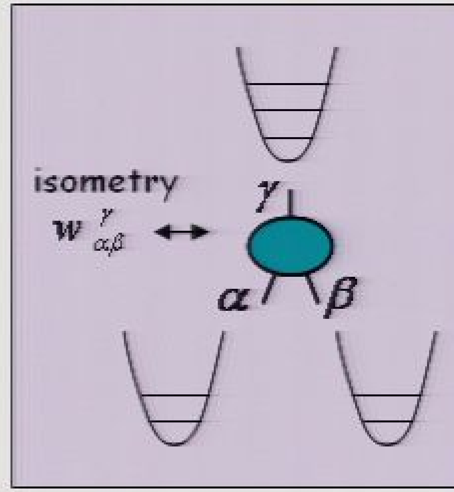


$$|\gamma\rangle = \sum_{\alpha\beta} w_{\alpha\beta}^{\gamma} |\alpha\rangle |\beta\rangle$$



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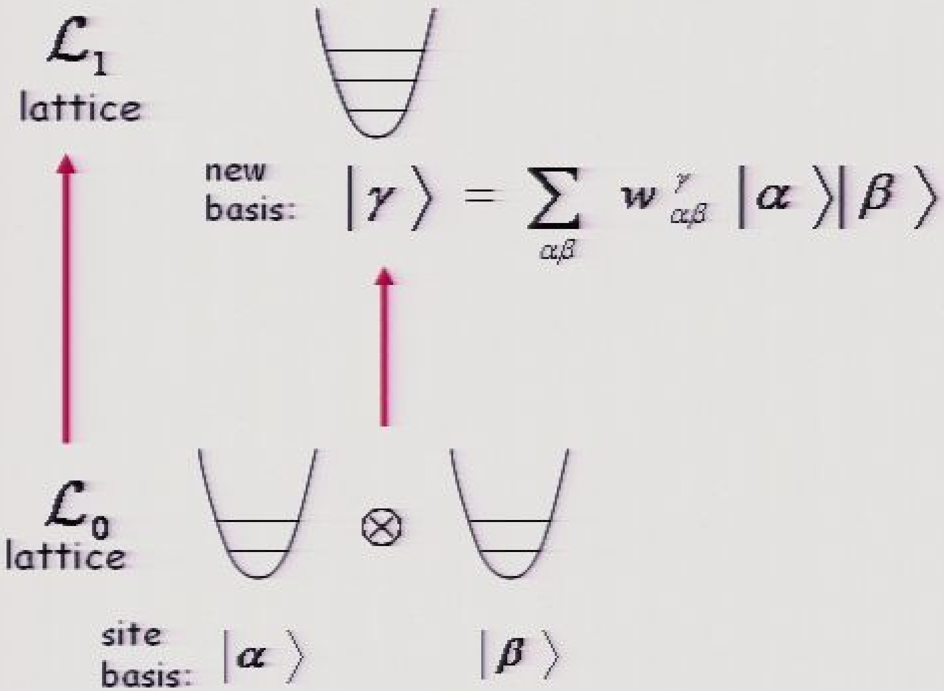
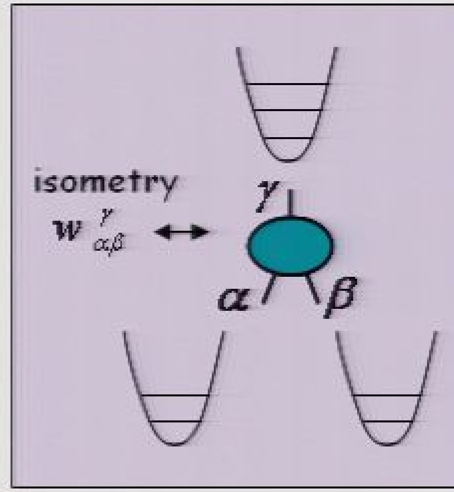
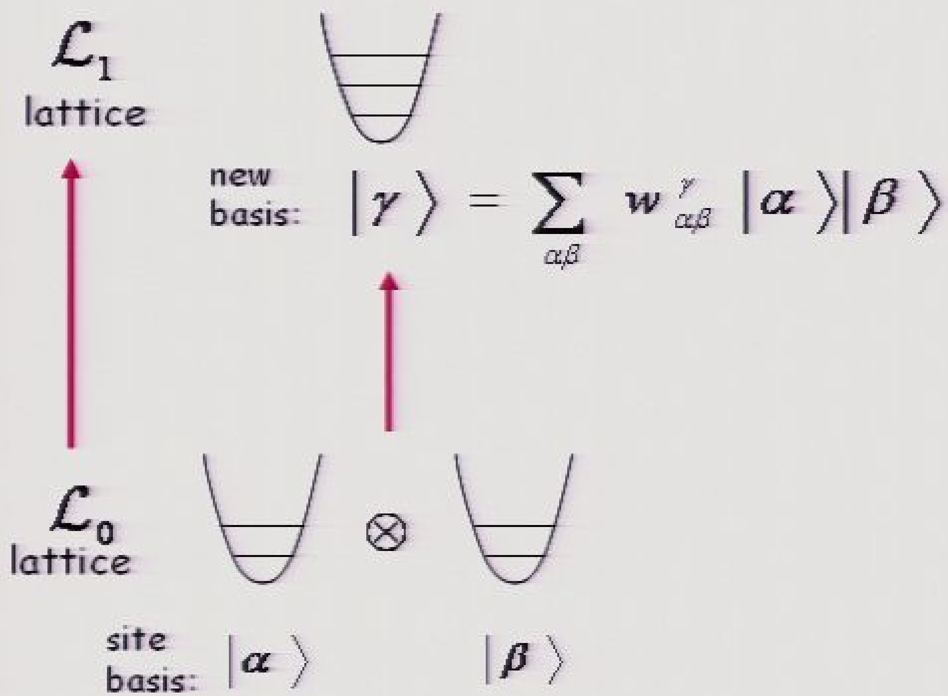
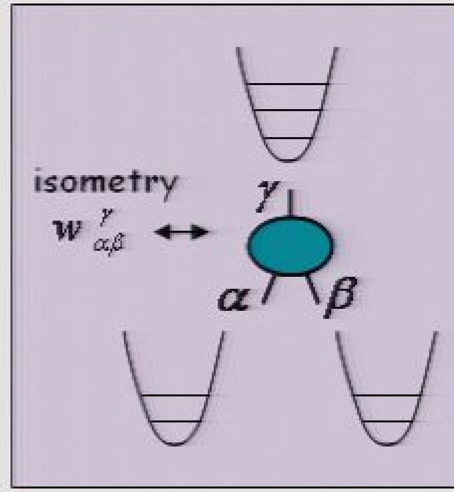


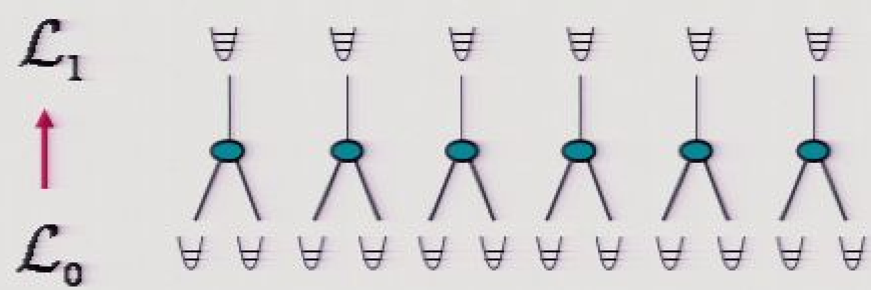


oarse-graining with disentanglers

- **diagrammatic representation:**







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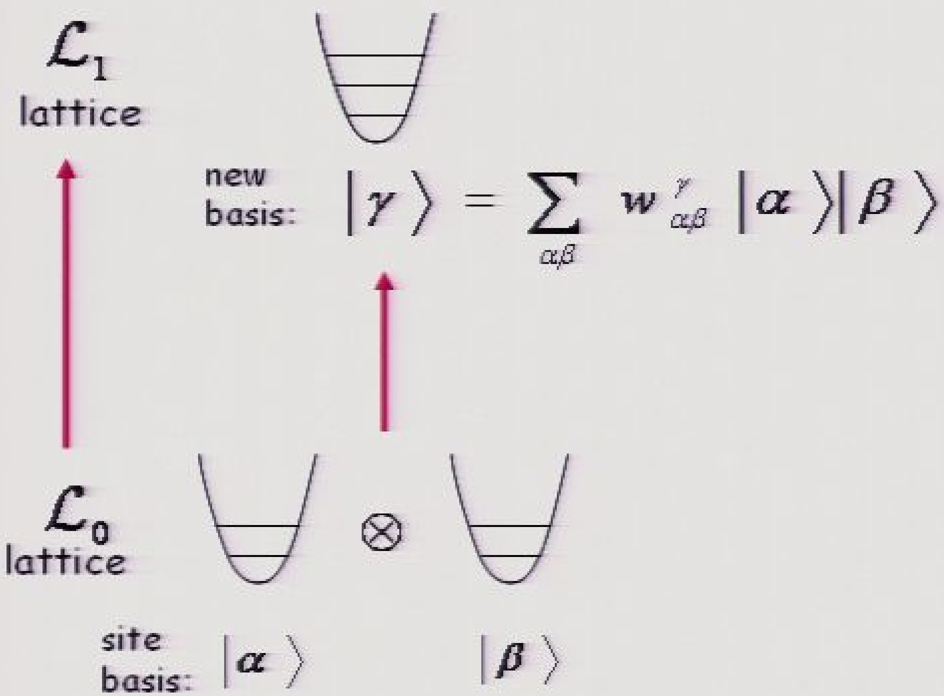
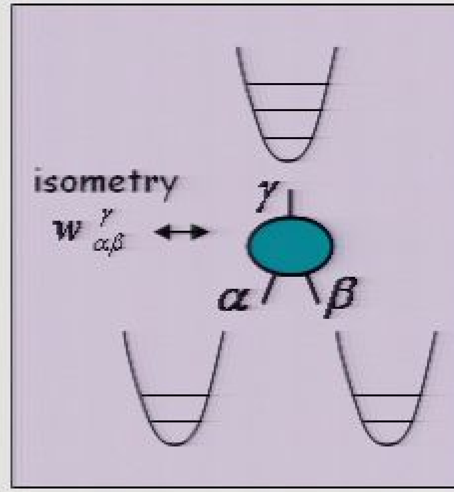
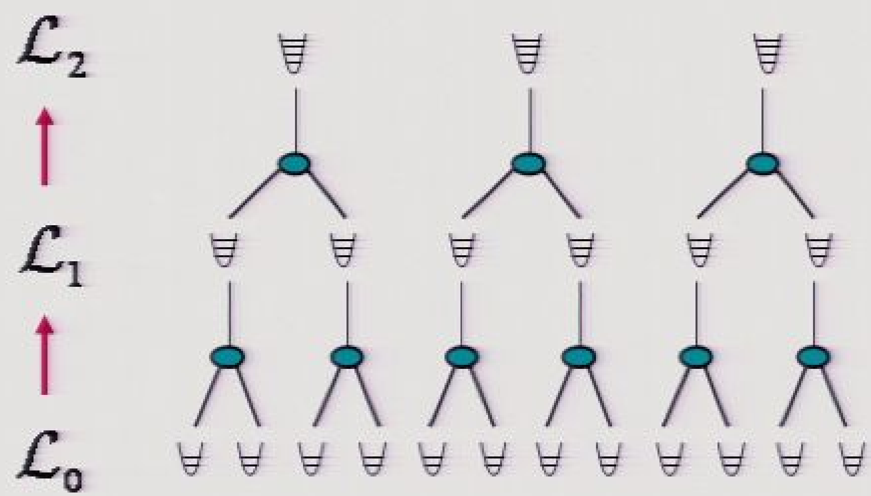
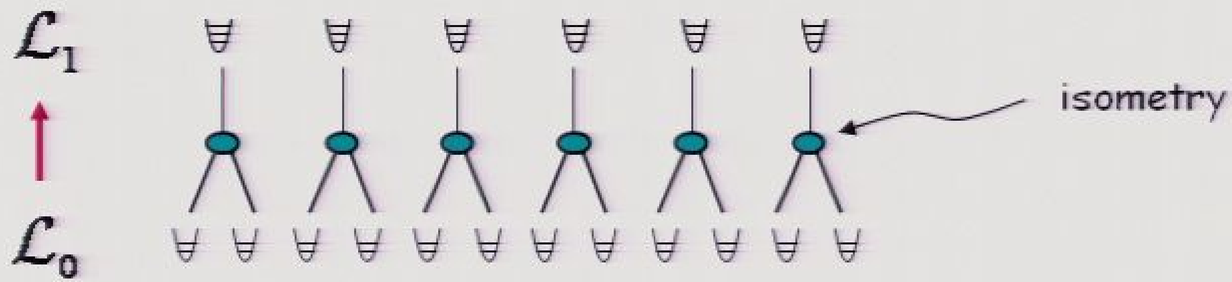




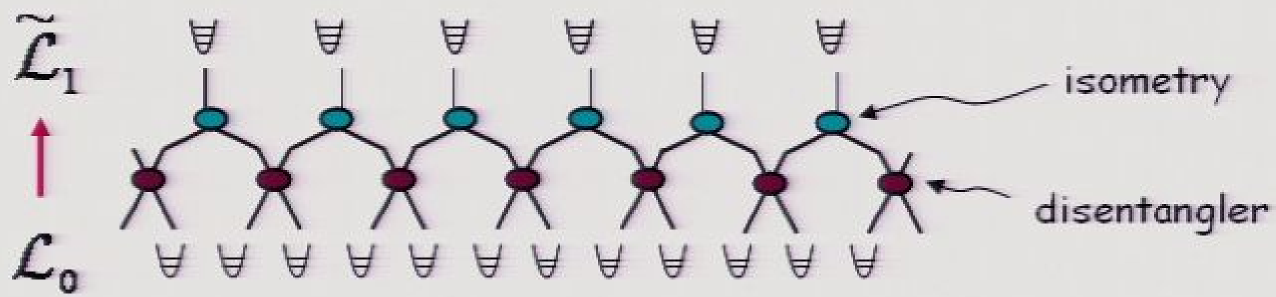


Disentanglers:

Standard coarse-graining



Coarse-graining with disentanglers



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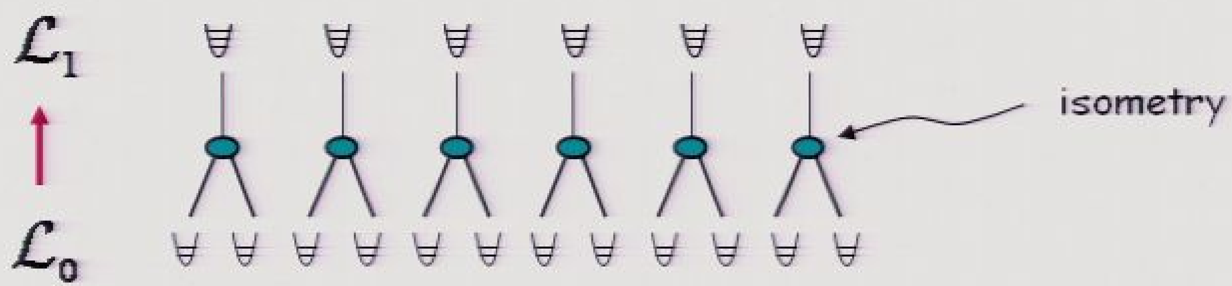
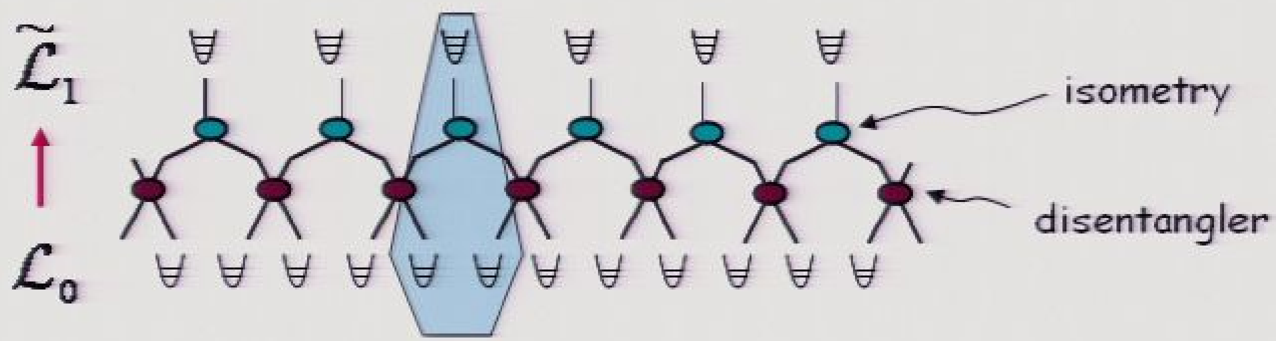


Coarse-graining with disentanglers



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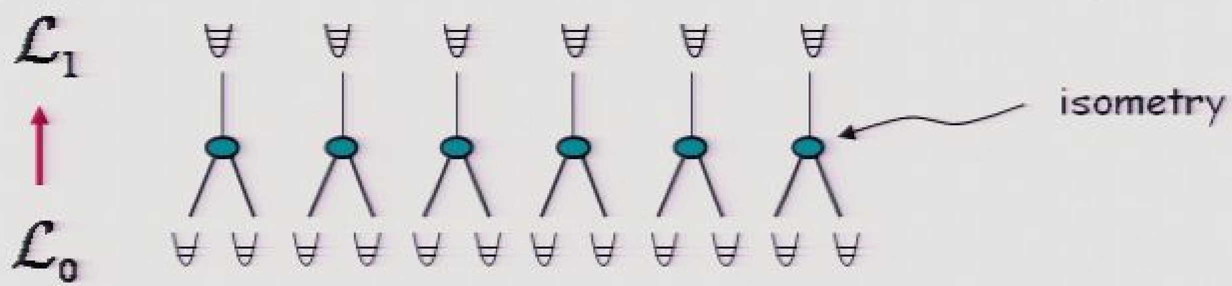

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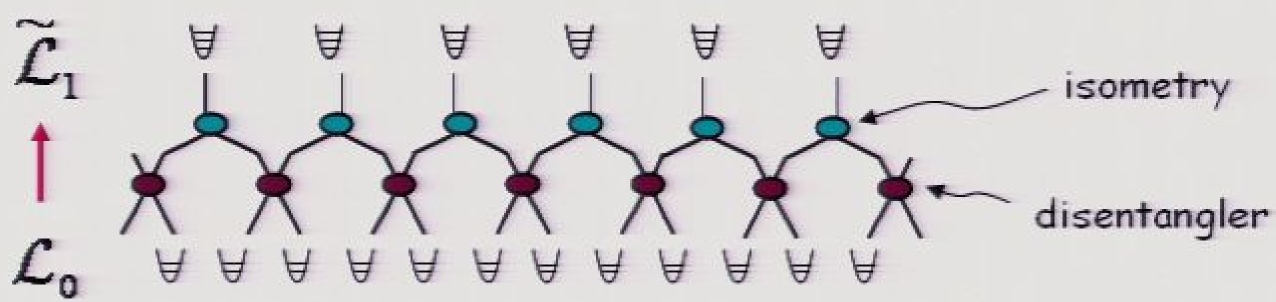
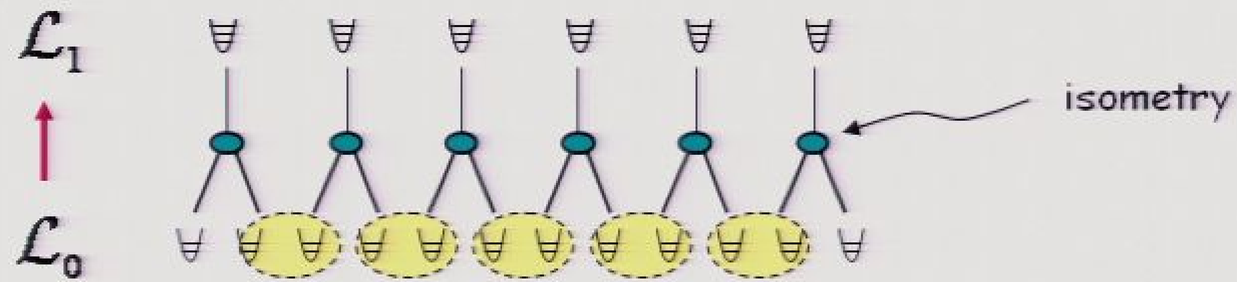


Disentanglers:

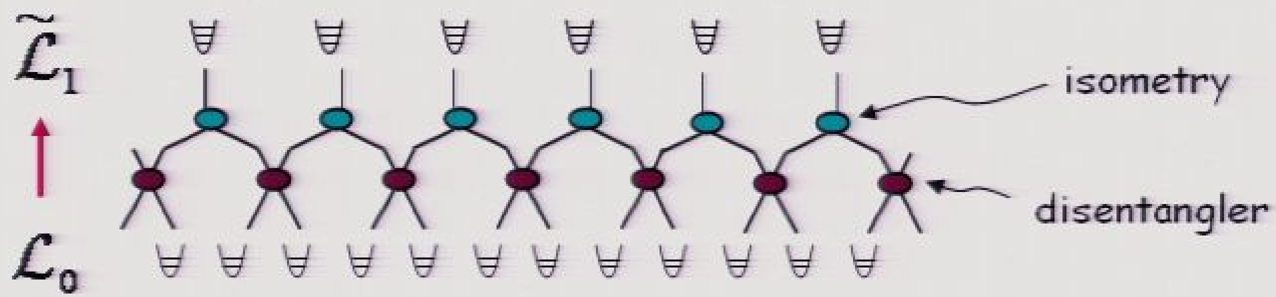
Example

Standard coarse-graining



$$\bigcirc \equiv \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

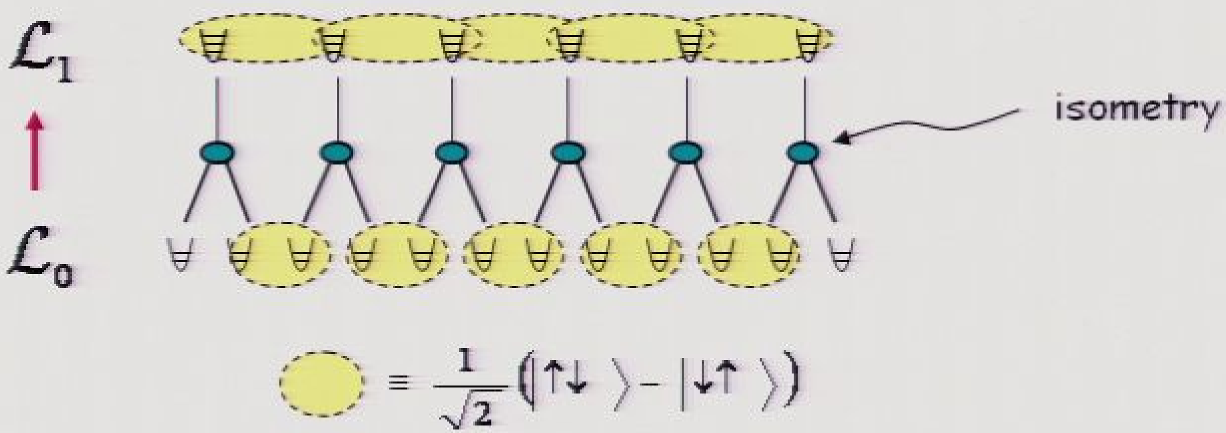
Coarse-graining with disentanglers



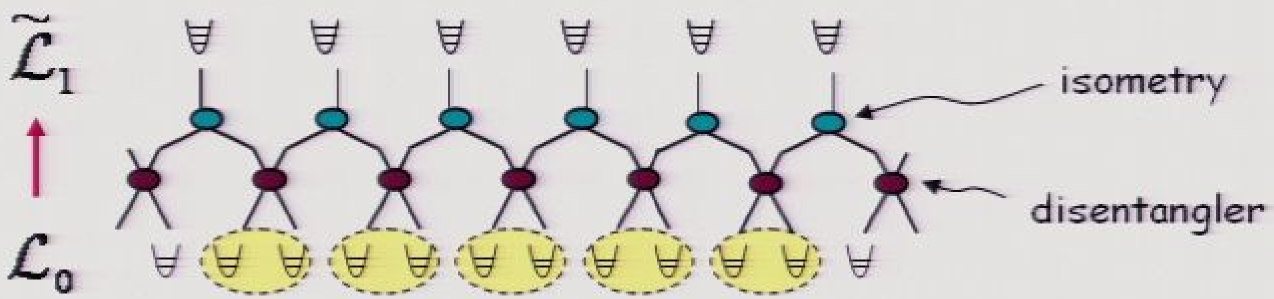
Disentanglers:

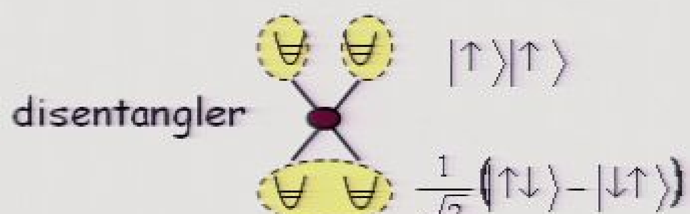
Example

Standard coarse-graining



Coarse-graining with disentanglers

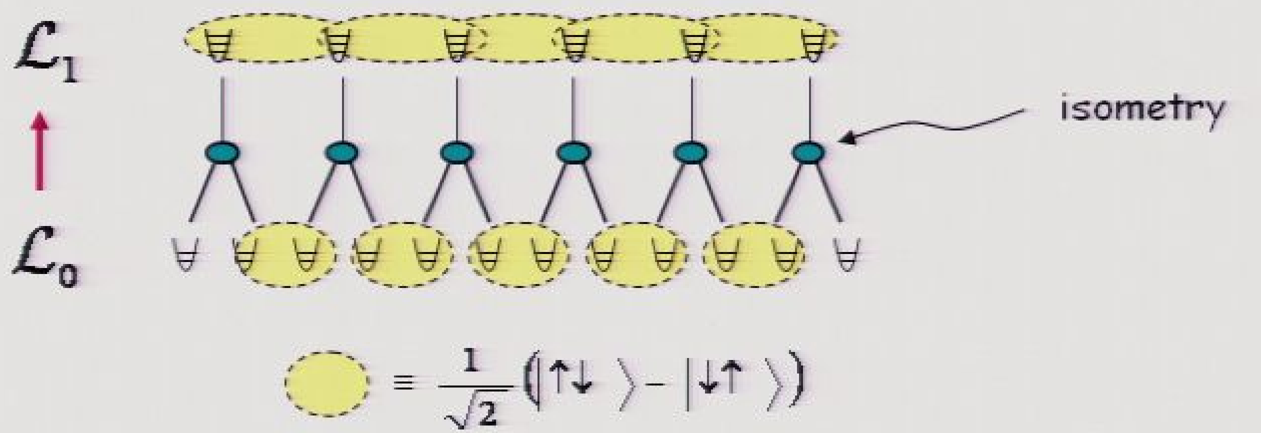




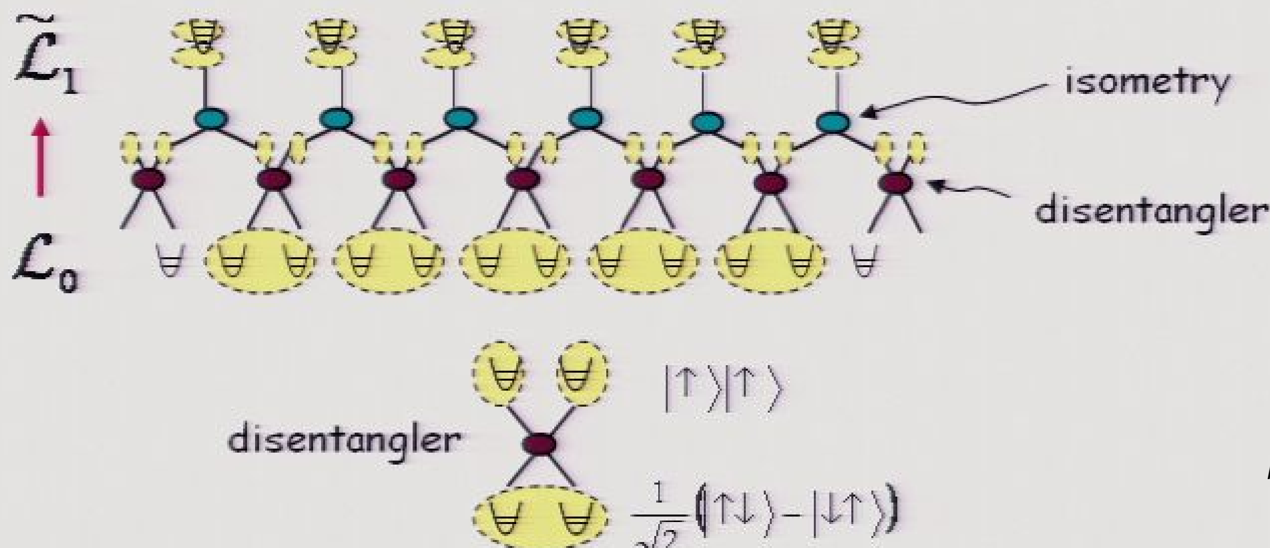
Disentanglers:

Example

Standard coarse-graining



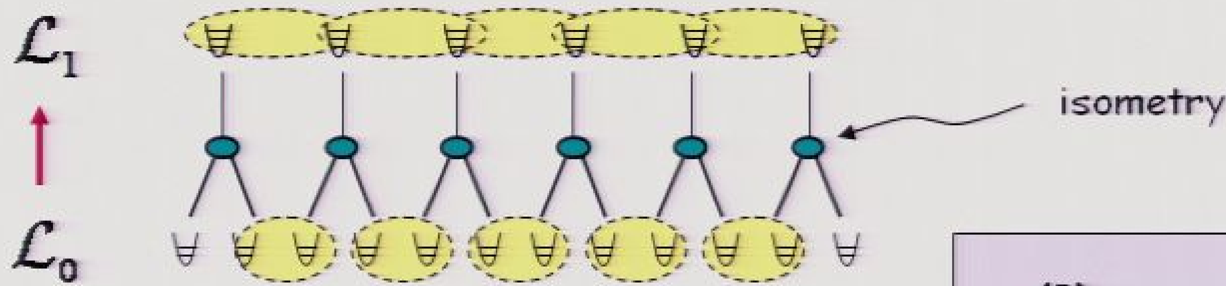
Coarse-graining with disentanglers



Disentanglers:

Example

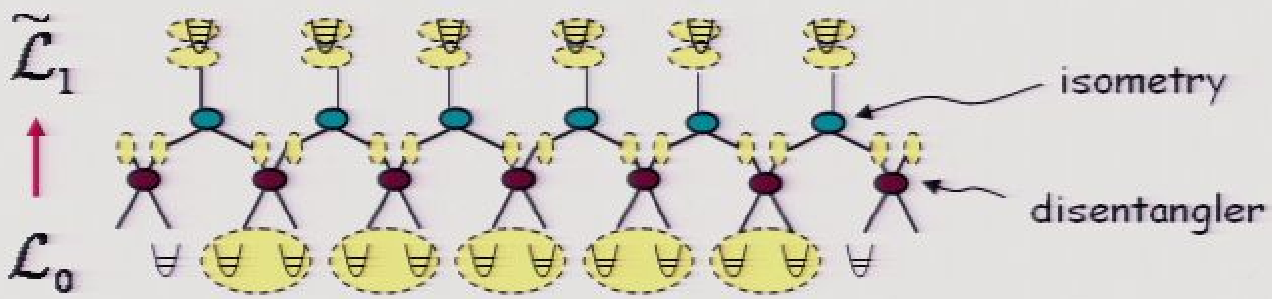
Standard coarse-graining

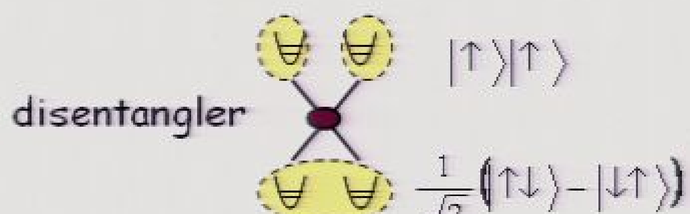


$$\equiv \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

$$\rho^{(2)} \rightarrow \quad \chi = 4$$

Coarse-graining with disentanglers

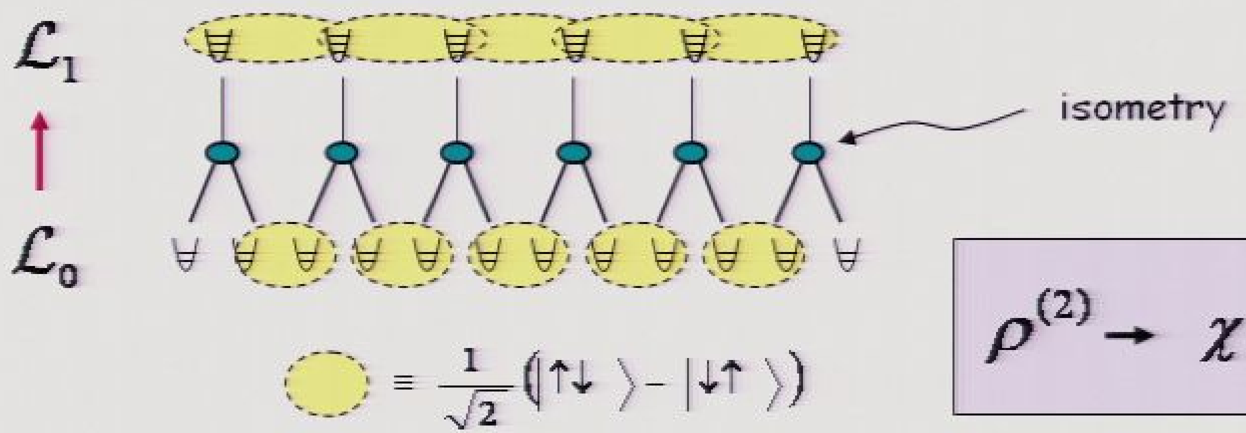




Disentanglers:

Example

Standard coarse-graining



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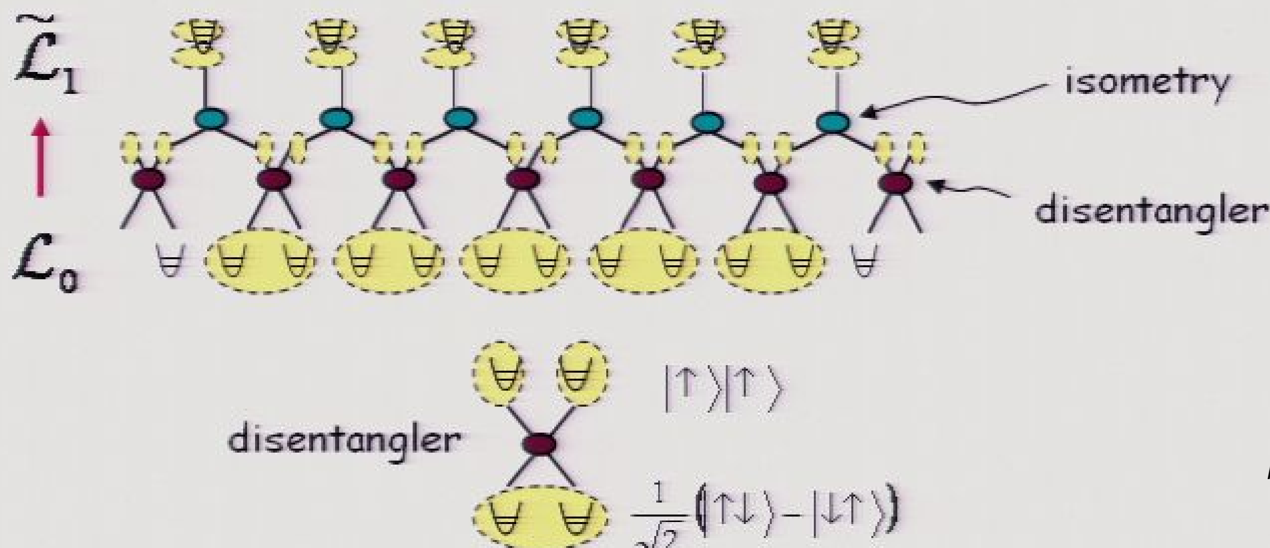
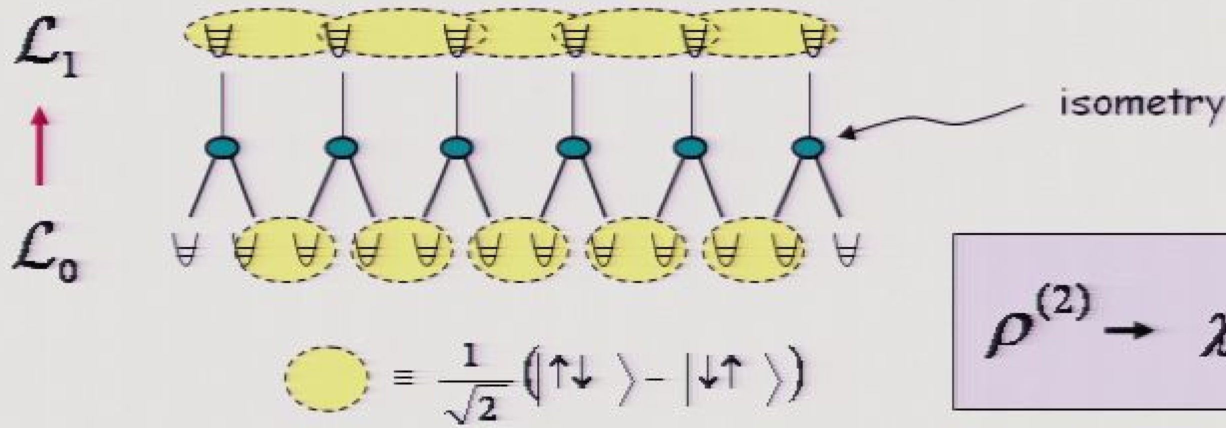
Coarse-graining with disentanglers



Disentanglers:

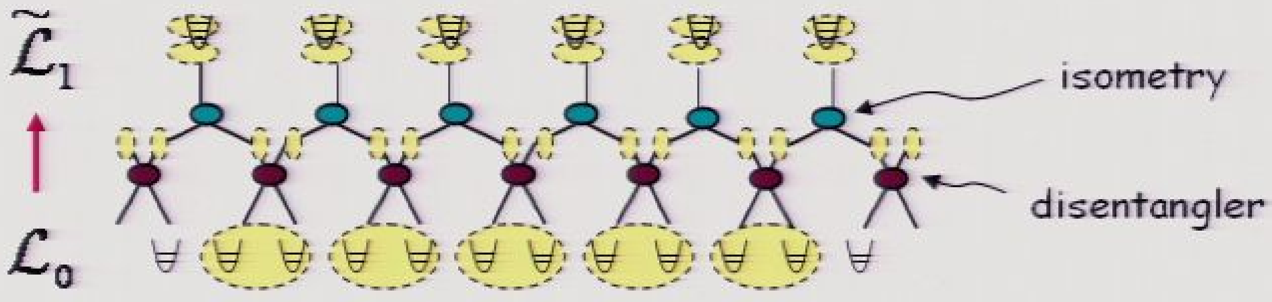
Example

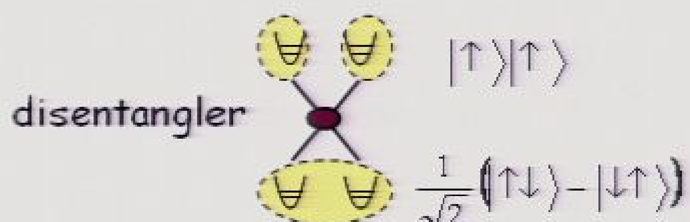
Standard coarse-graining



$$\rho^{(2)} \rightarrow \quad \chi = 4$$

Coarse-graining with disentanglers



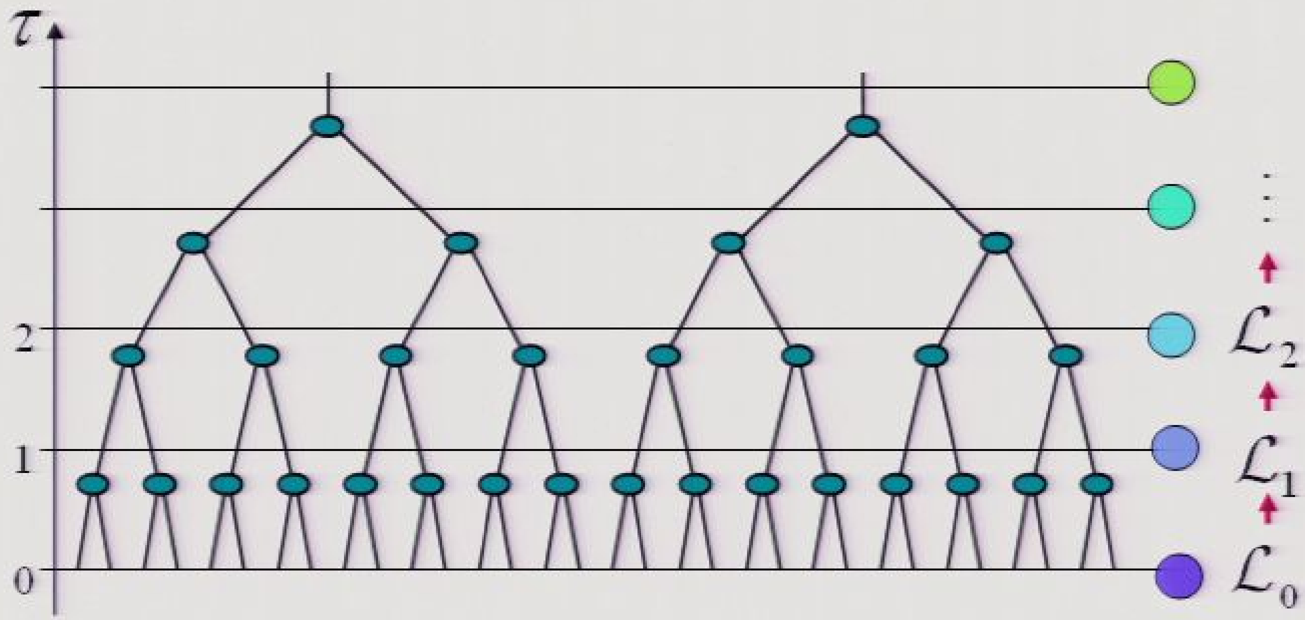


$$\tilde{\rho}^{(2)} \rightarrow \quad \tilde{\chi} = 1$$

ntanglement renormalization

G. Vidal, Phys. Rev. Lett. 99, 220405 (2007)
G. Vidal, quant-phxxxxxx

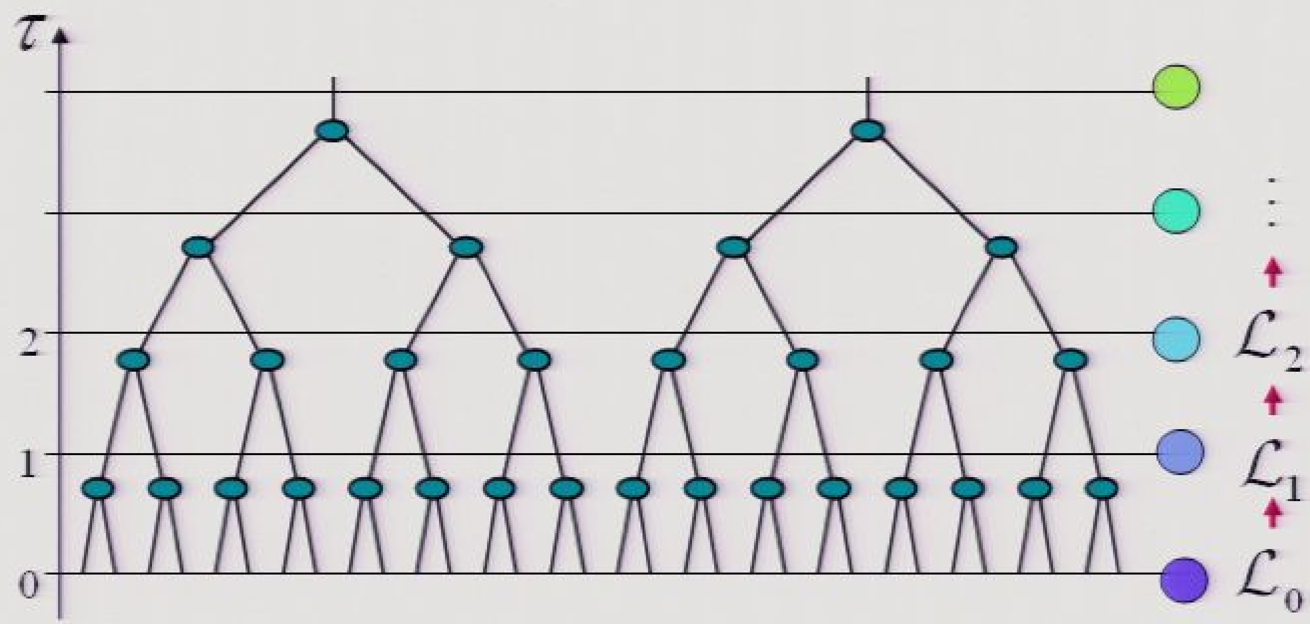
Standard
real-space
RG approach



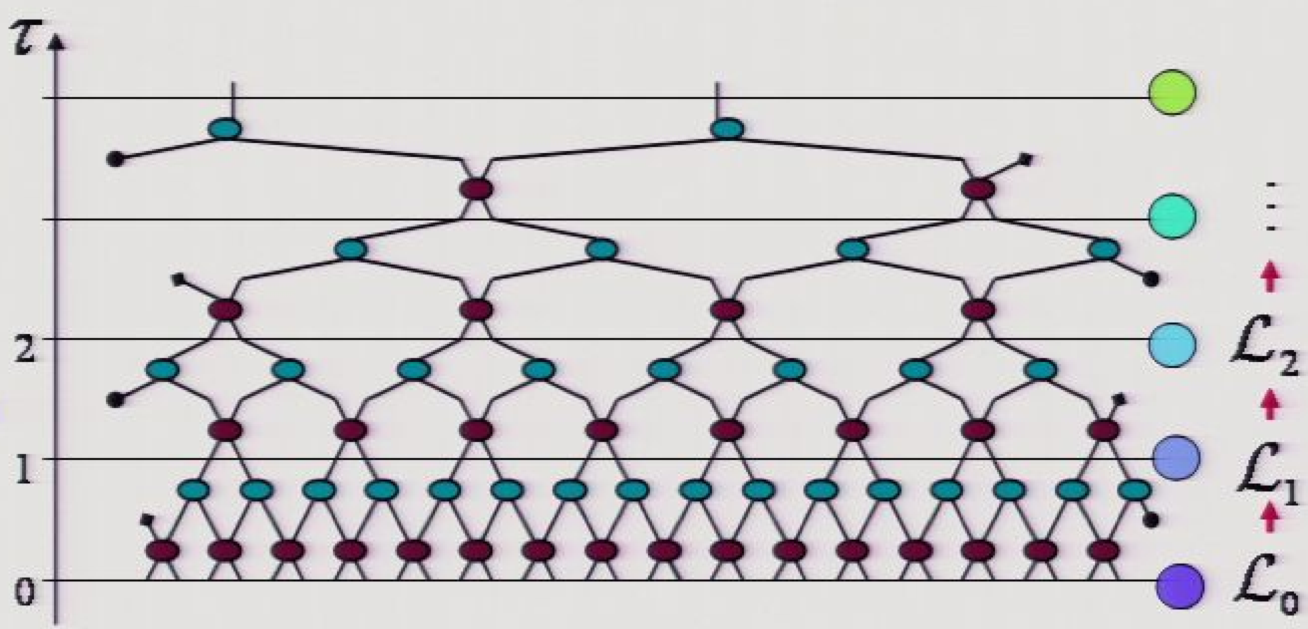
Entanglement renormalization

G. Vidal, Phys. Rev. Lett. 99, 220405 (2007)
G. Vidal, quant-phxxxxxx

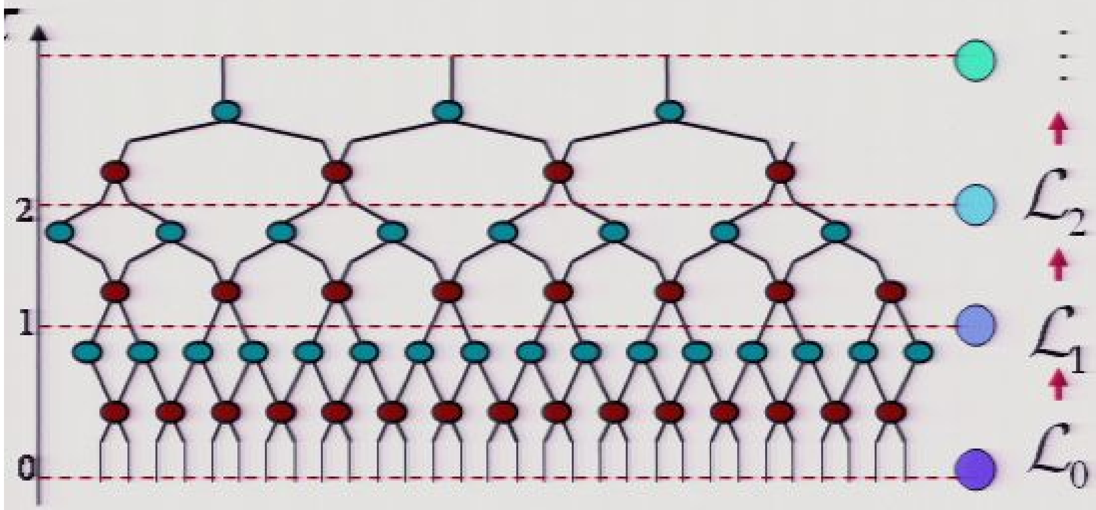
Standard real-space RG approach



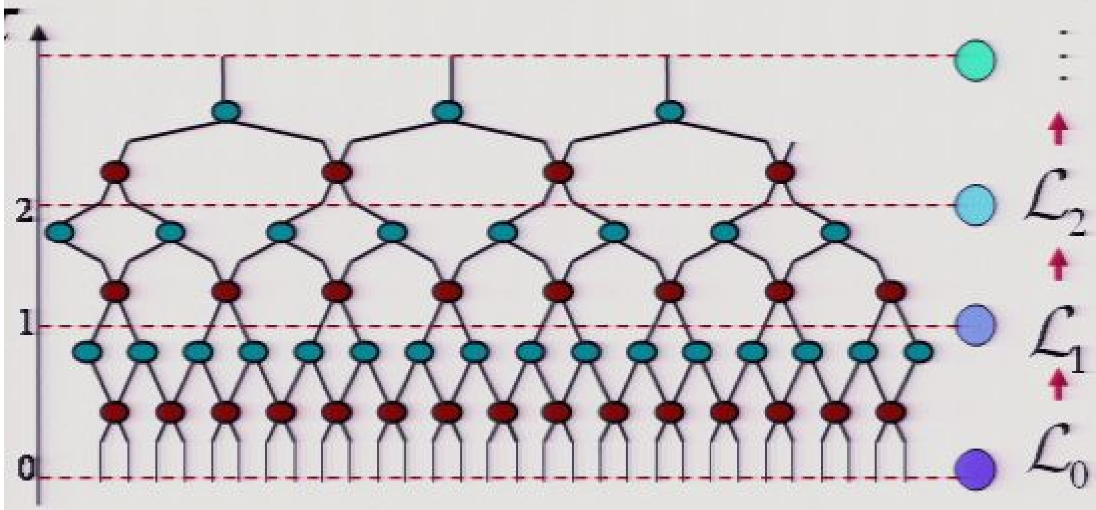
Real-space RG approach with **entanglement renormalization**



ntanglement renormalization

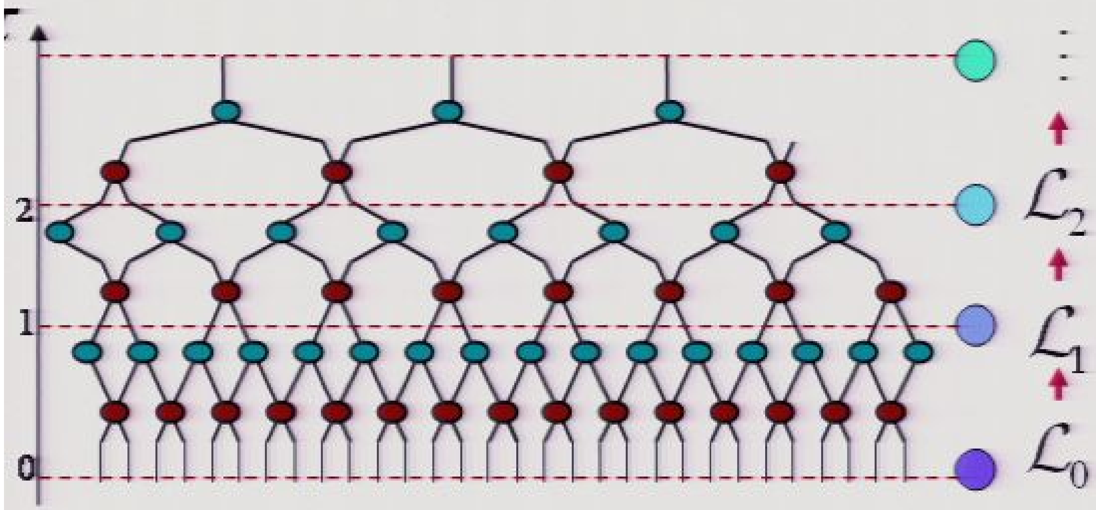


ntanglement renormalization



Properties of this RG transformation:

ntanglement renormalization

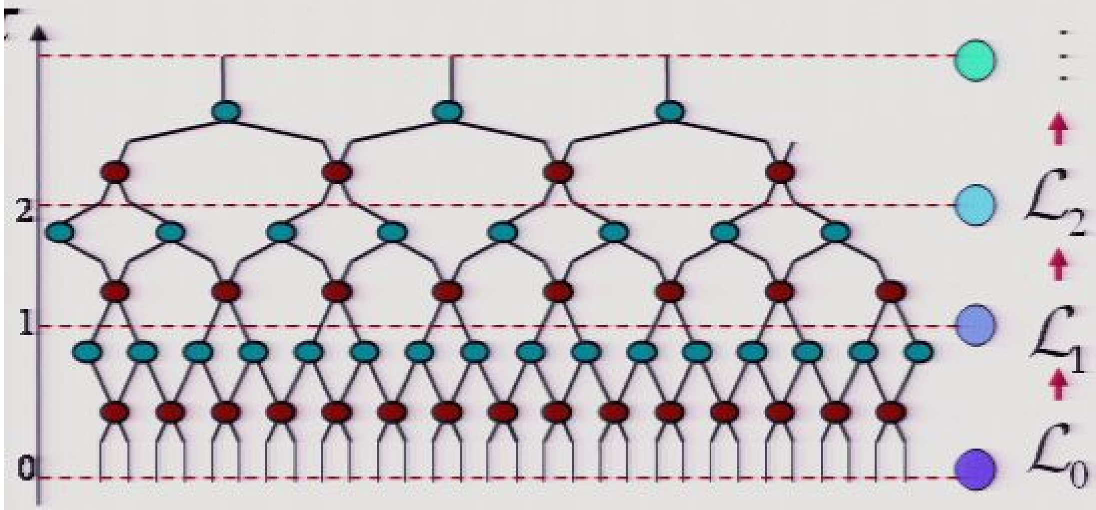


Properties of this RG transformation:

- Constant $\chi_\tau = \chi$ (in any D):

  numerical approach to arbitrary system sizes

ntanglement renormalization



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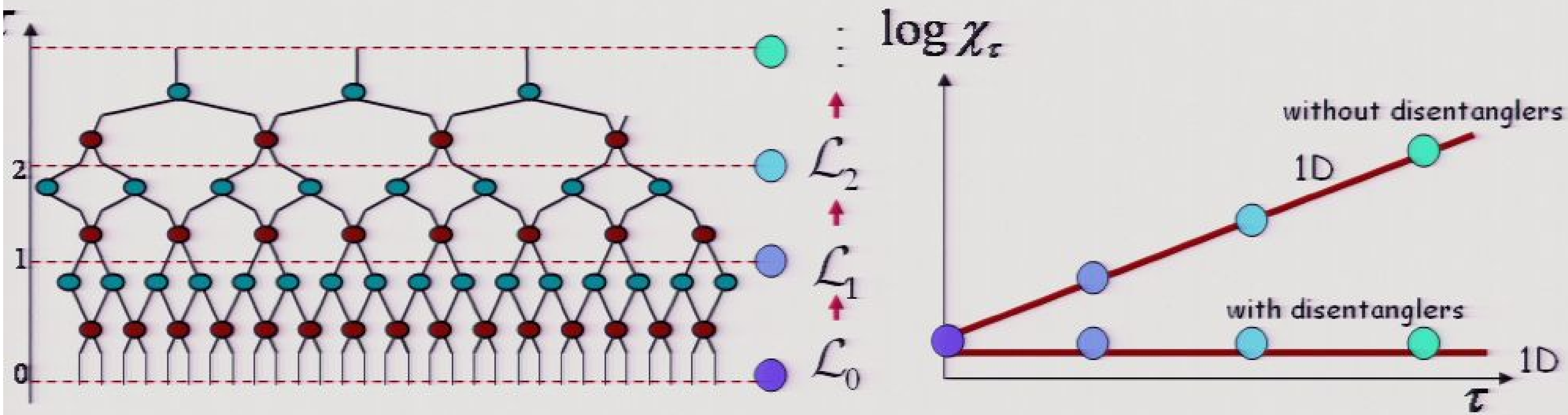

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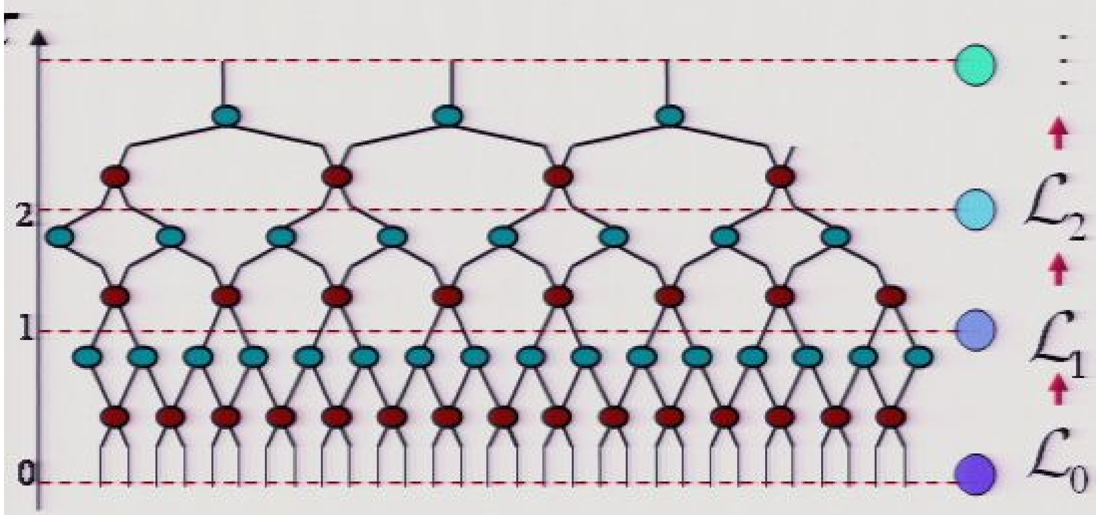

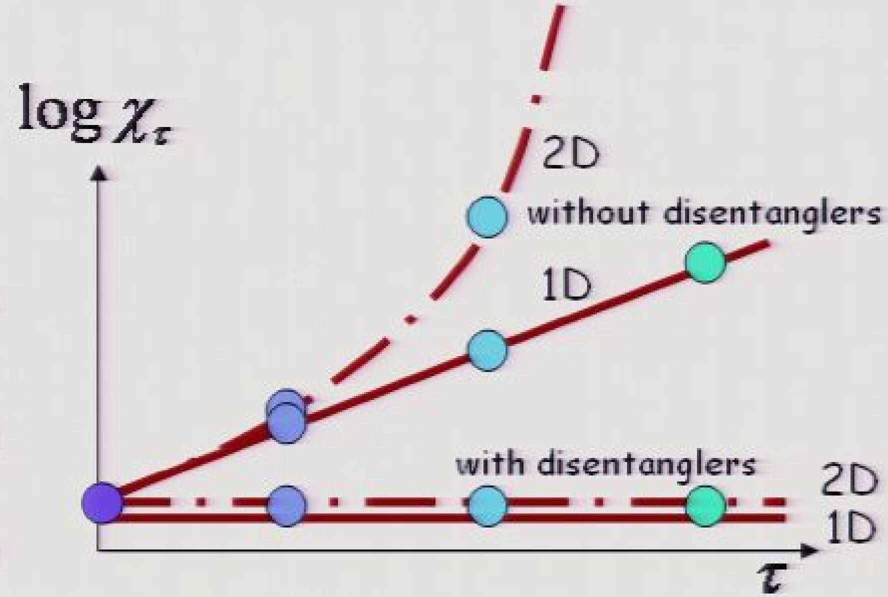

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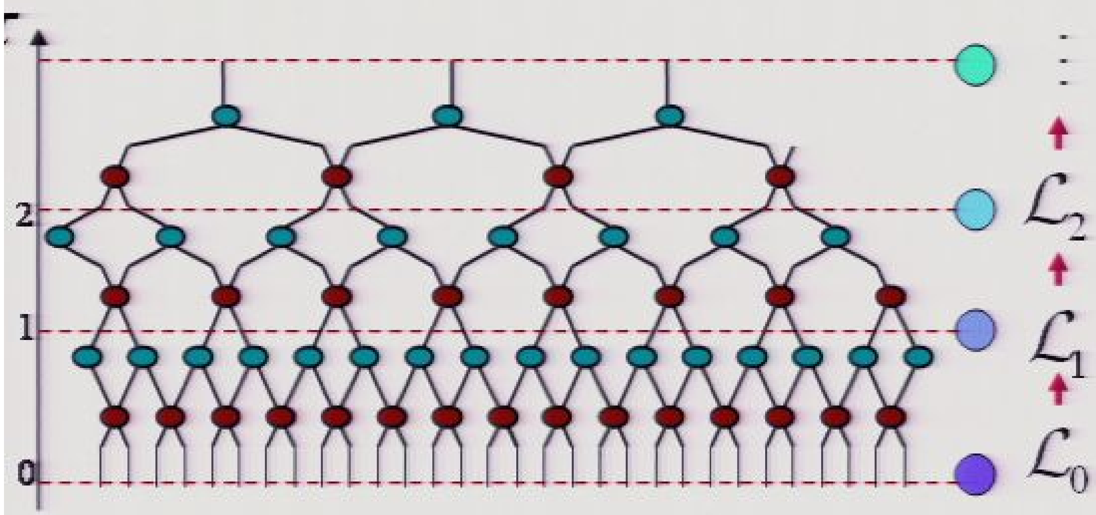
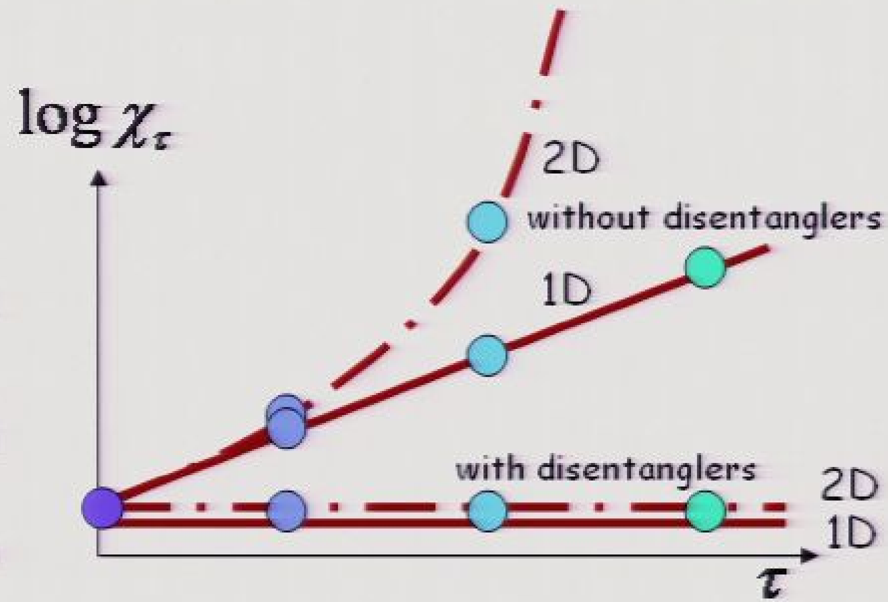




Properties of this RG transformation:

- Constant $\chi_\tau = \chi$ (in any D):

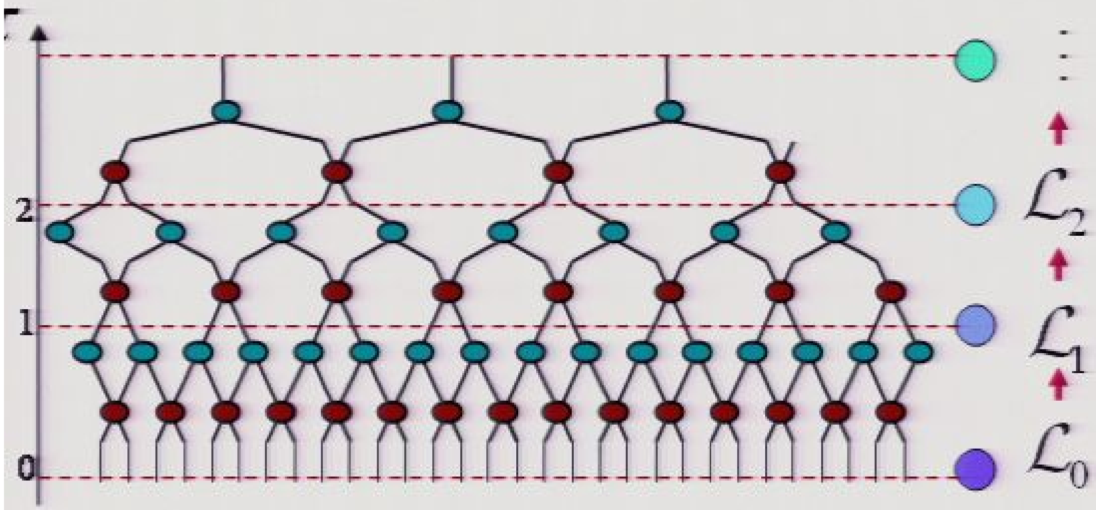
  numerical approach to arbitrary system sizes

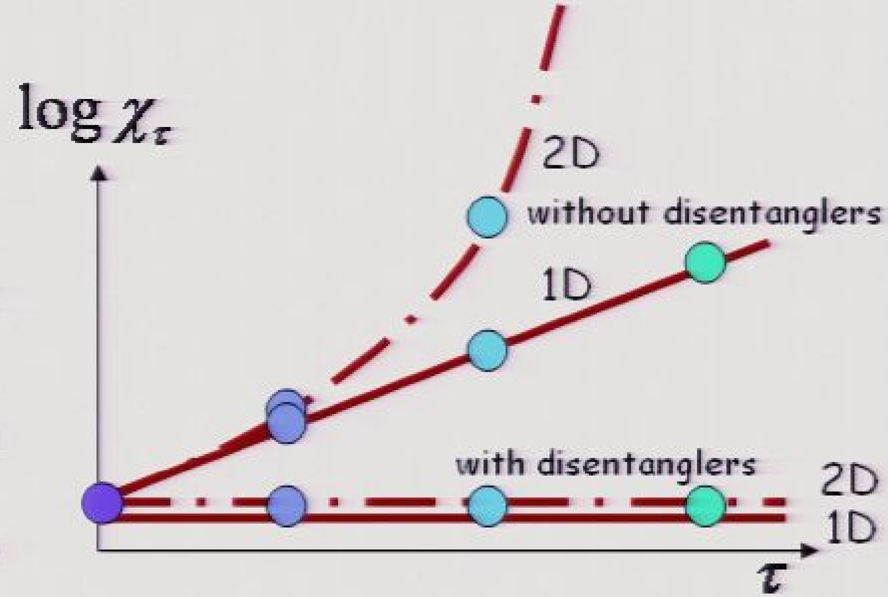
- Well-defined RG flow:

  $\mathcal{L}_\tau$ and $\mathcal{L}_{\tau+1}$ can be compared

  H_τ " $H_{\tau+1}$ "

ntanglement renormalization





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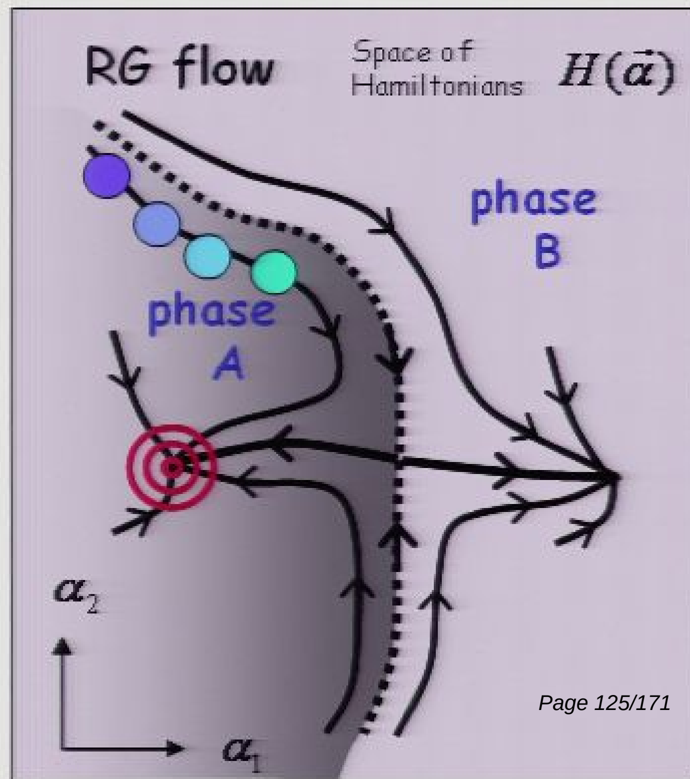
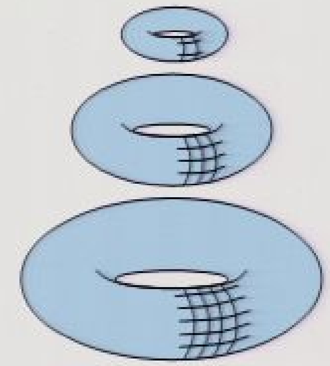
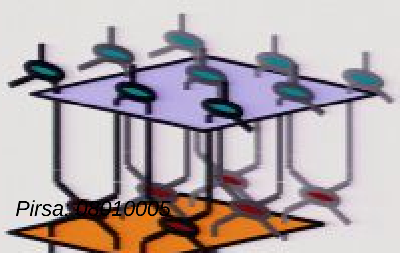
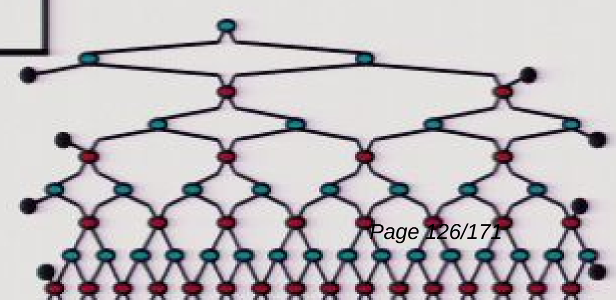


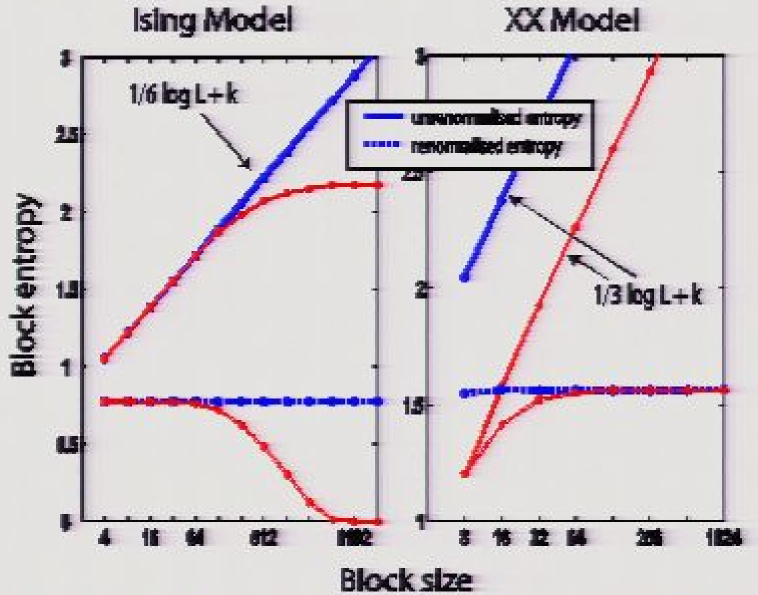


- Renormalization Group transformations
- Ground State Entanglement
- Entanglement Renormalization
- Quantum Criticality
- Topological Order

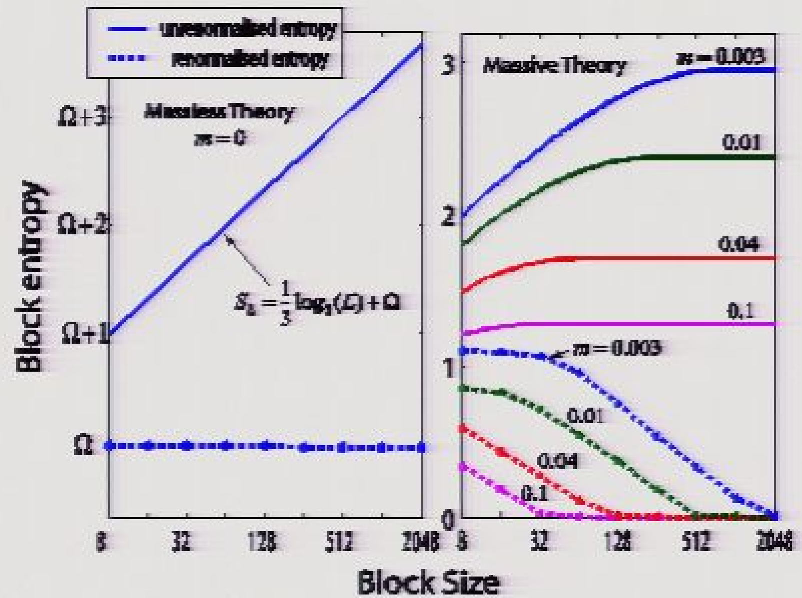




1D lattices

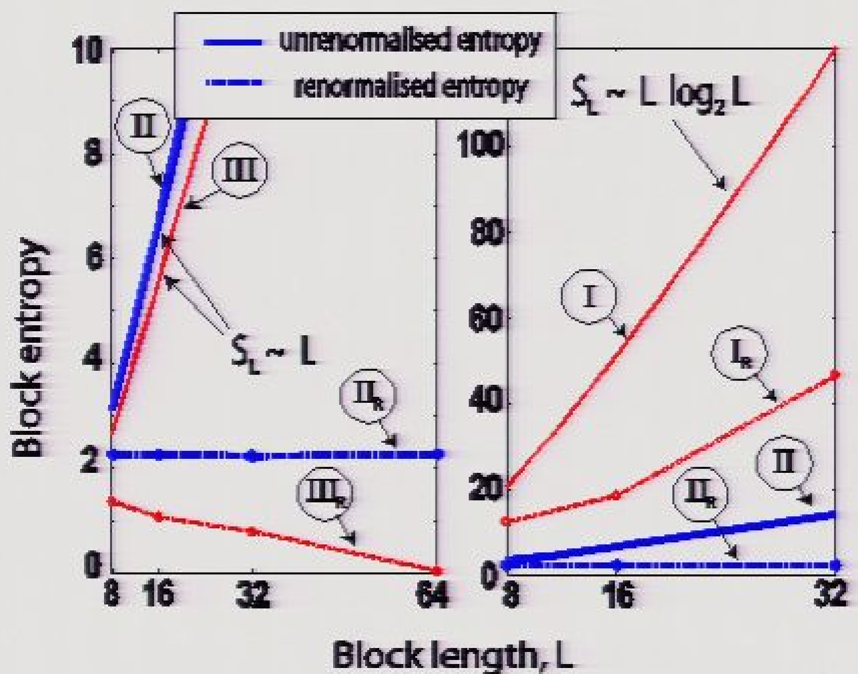


(from G. Evenbly and G. Vidal, arXiv:0710.0692)



(from G. Evenbly and G. Vidal, arXiv:0801.2449v1)

2D lattices



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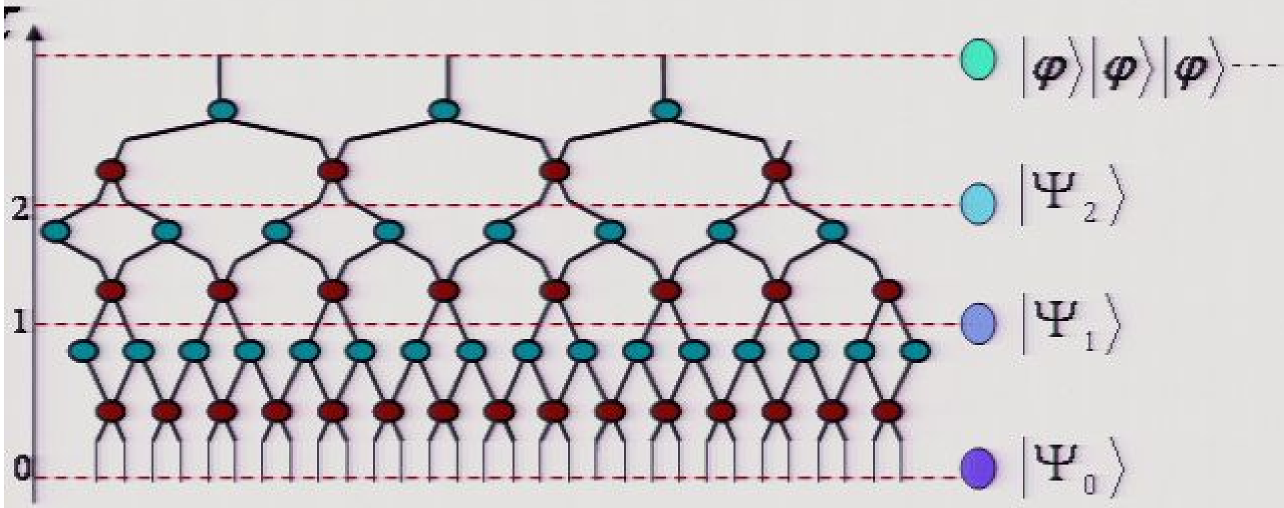
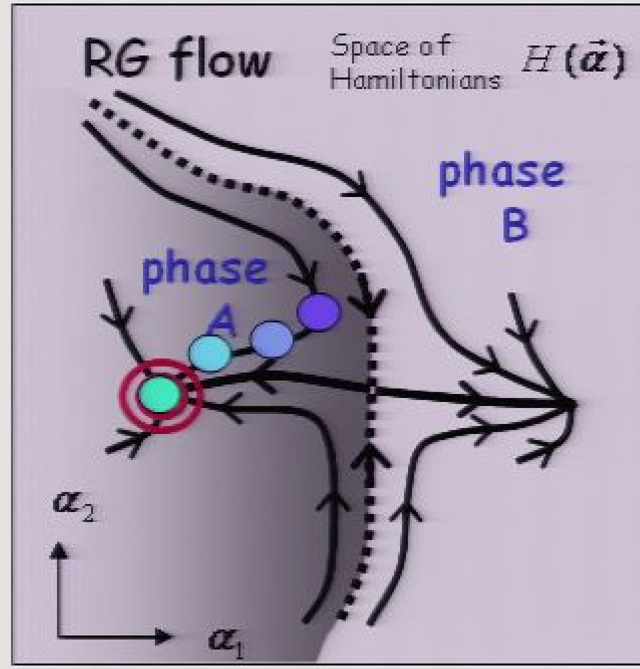
Quantum criticality

G. Vidal, Phys. Rev. Lett. 99, 220405 (2007)

Gapped system





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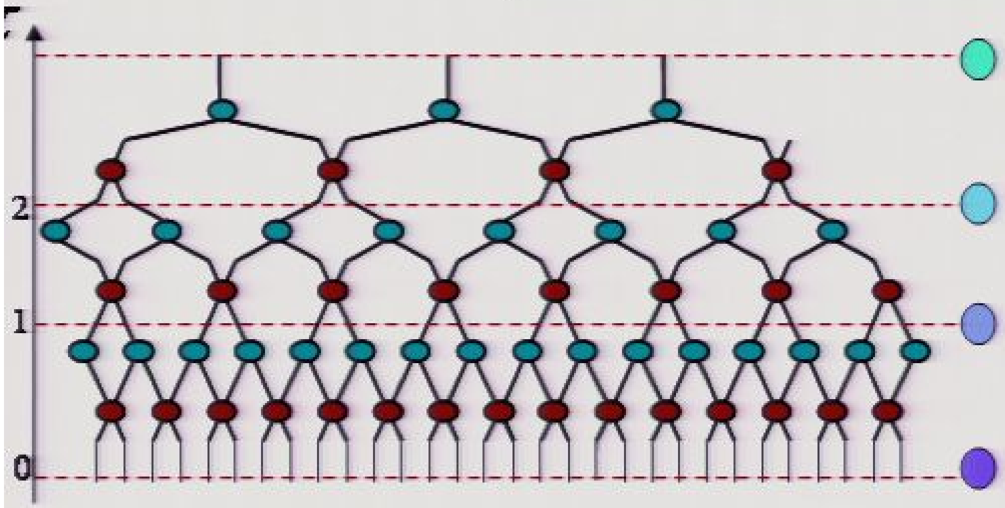
Gapped system

- trivial fixed point:

product state $|\varphi\rangle|\varphi\rangle|\varphi\rangle\cdots$

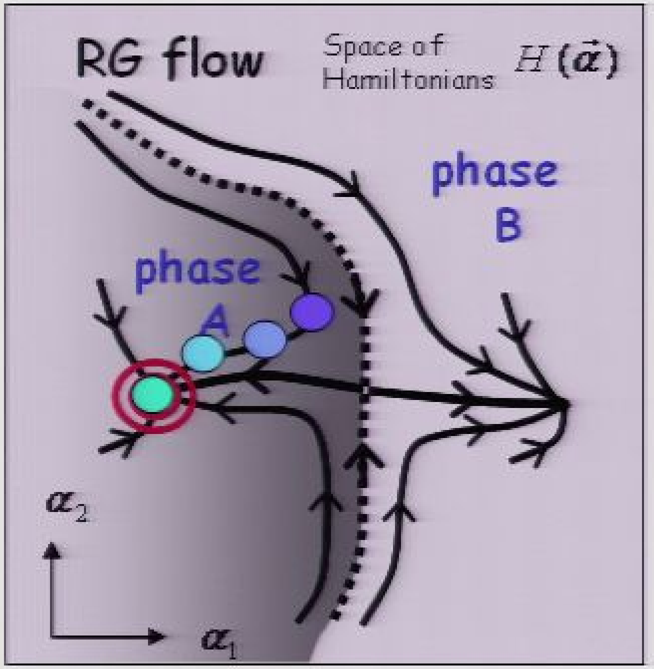


$|\varphi\rangle|\varphi\rangle|\varphi\rangle\cdots$

$|\Psi_2\rangle$

$|\Psi_1\rangle$

$|\Psi_0\rangle$



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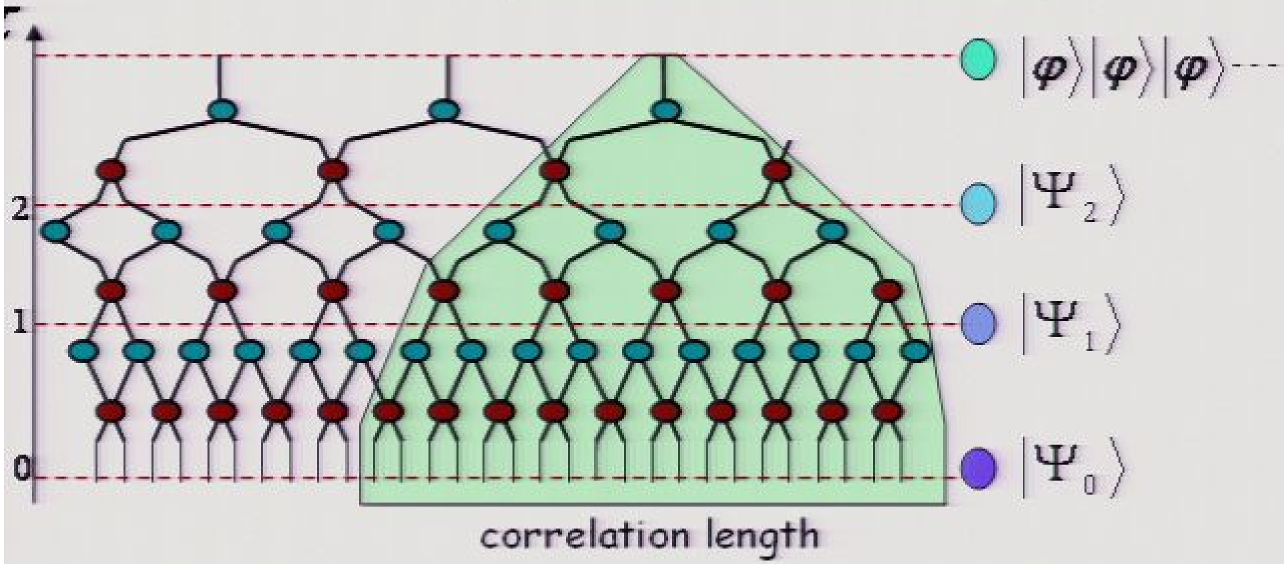

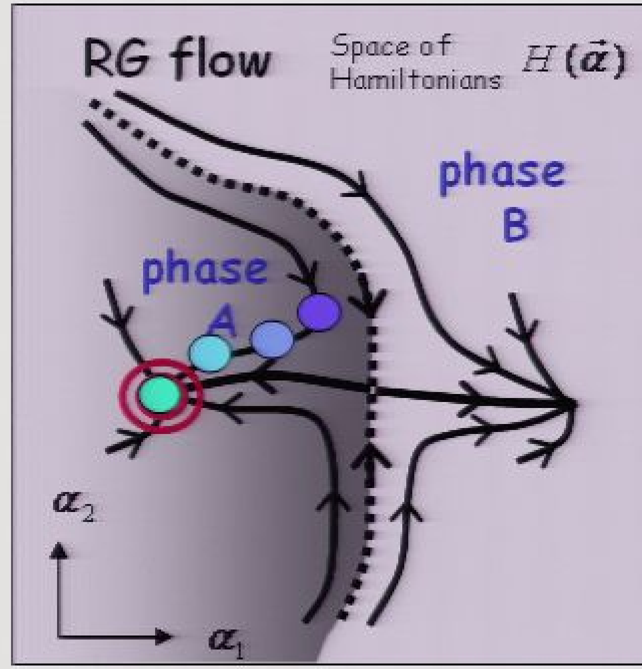

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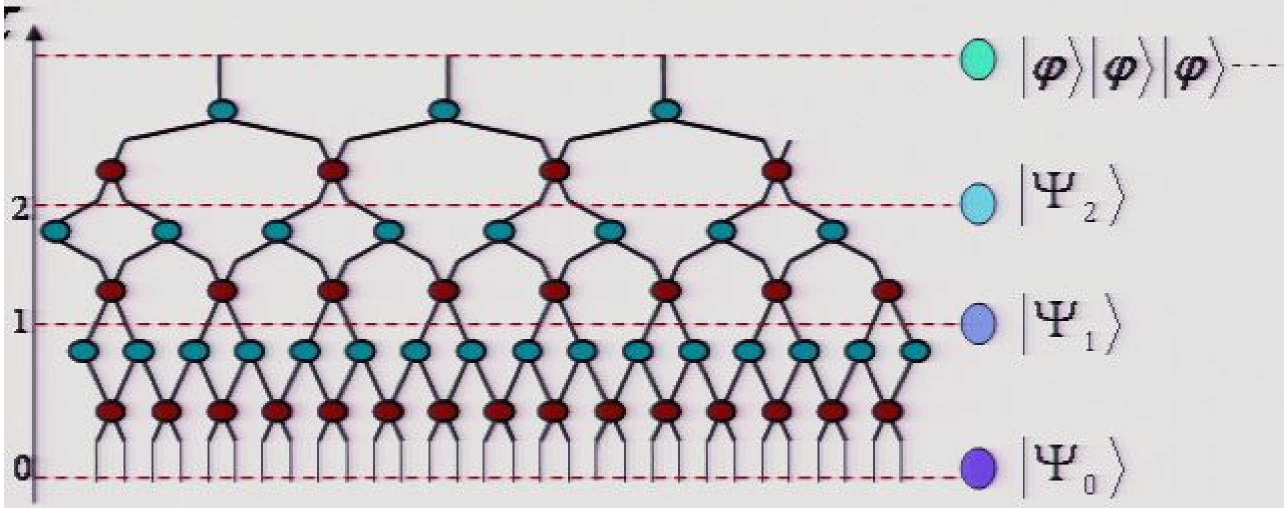
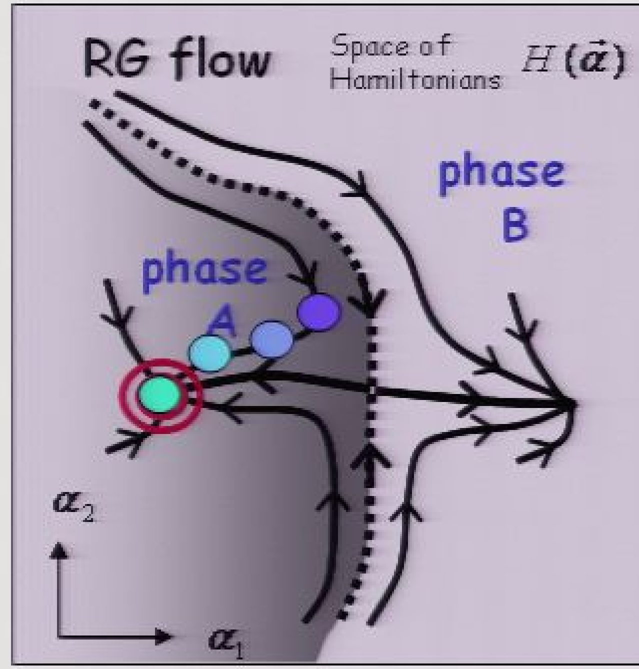
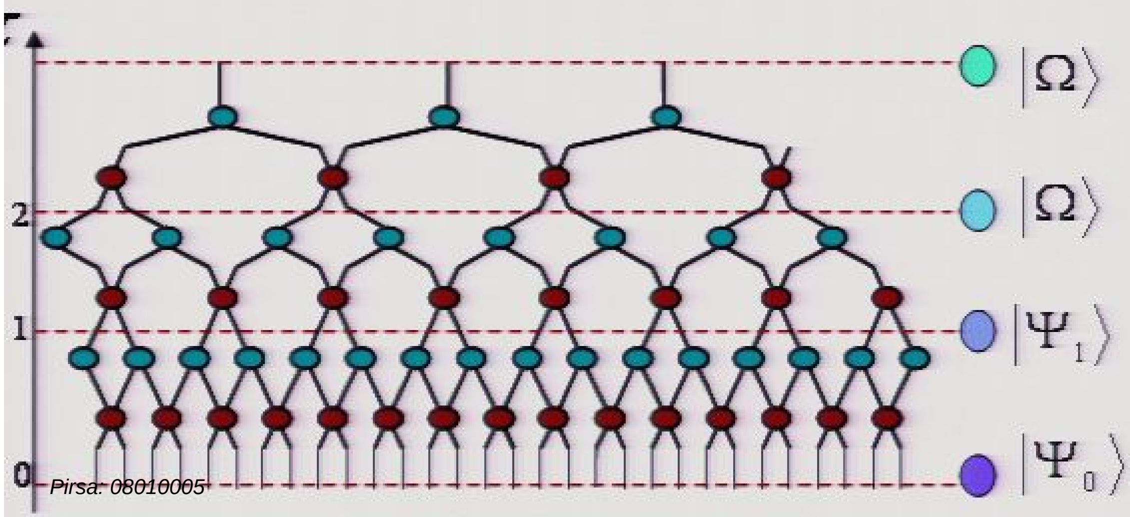




Critical system



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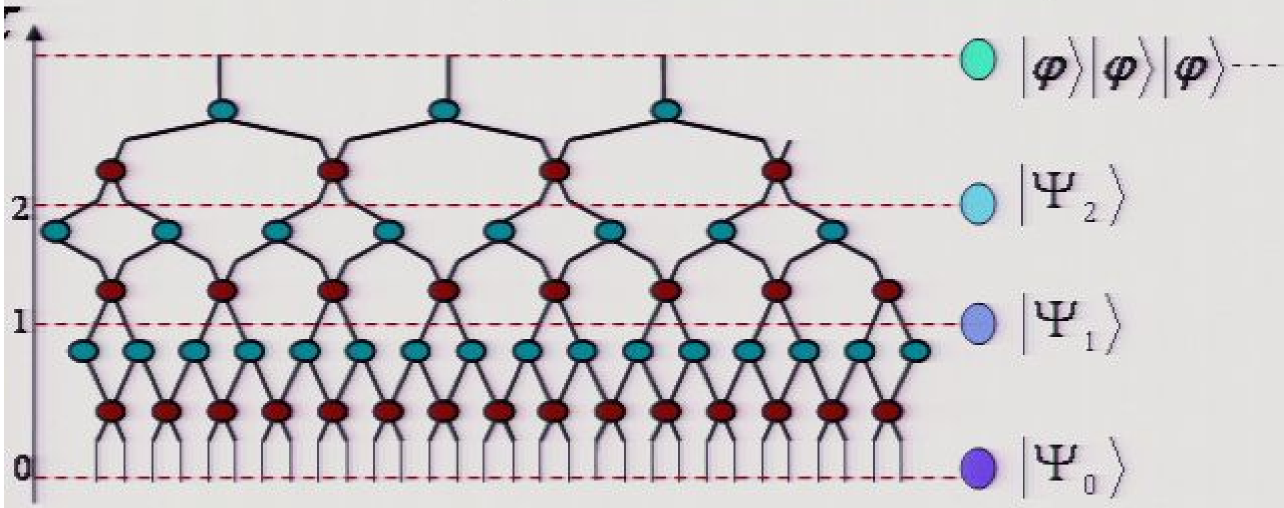

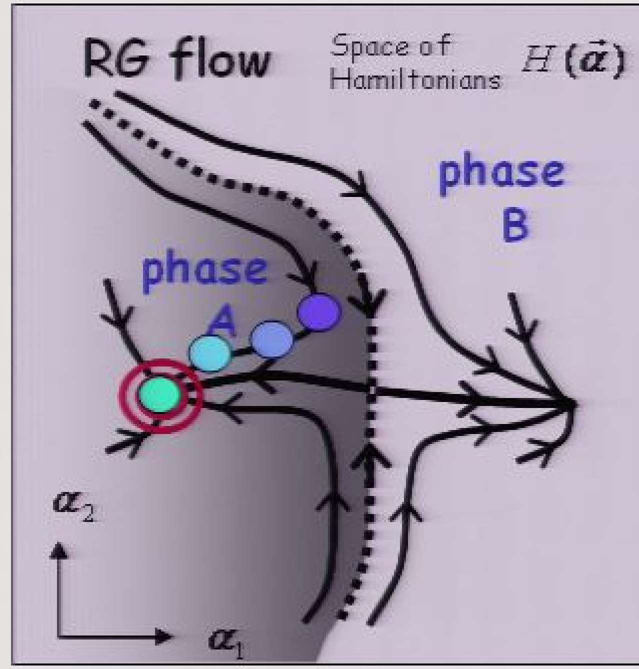

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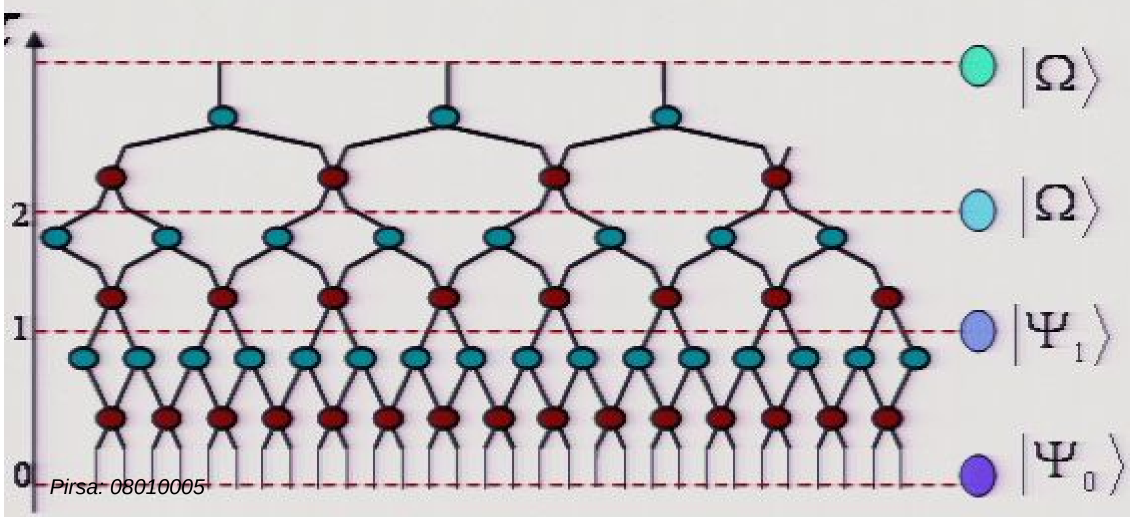

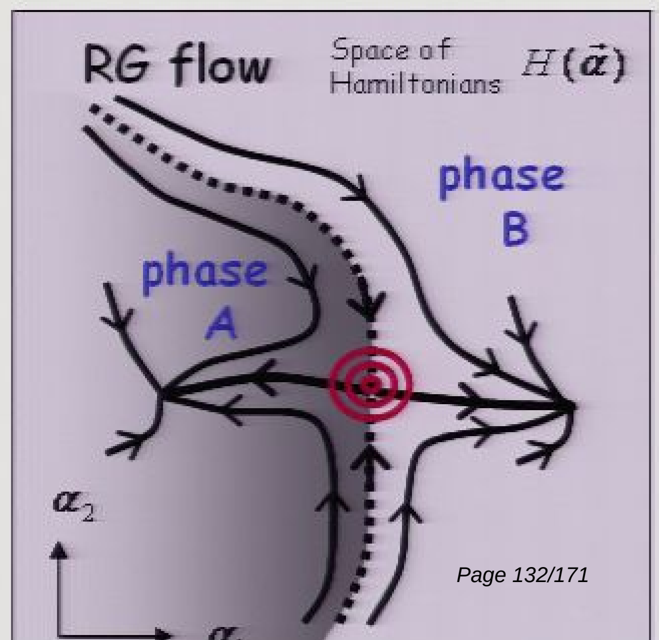

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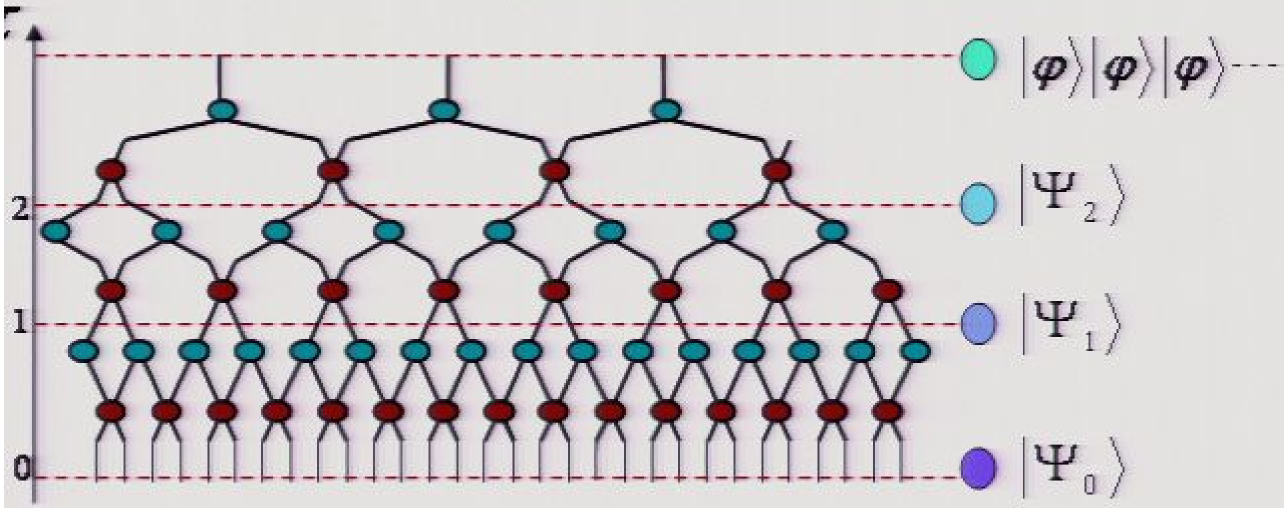
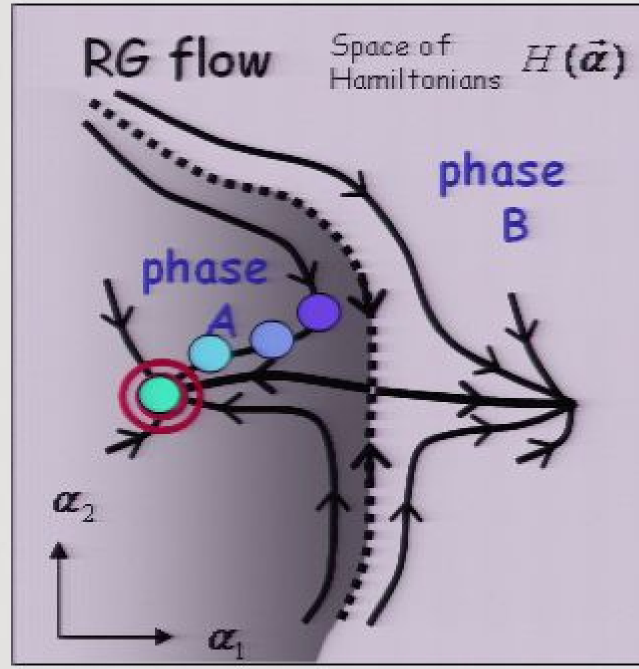
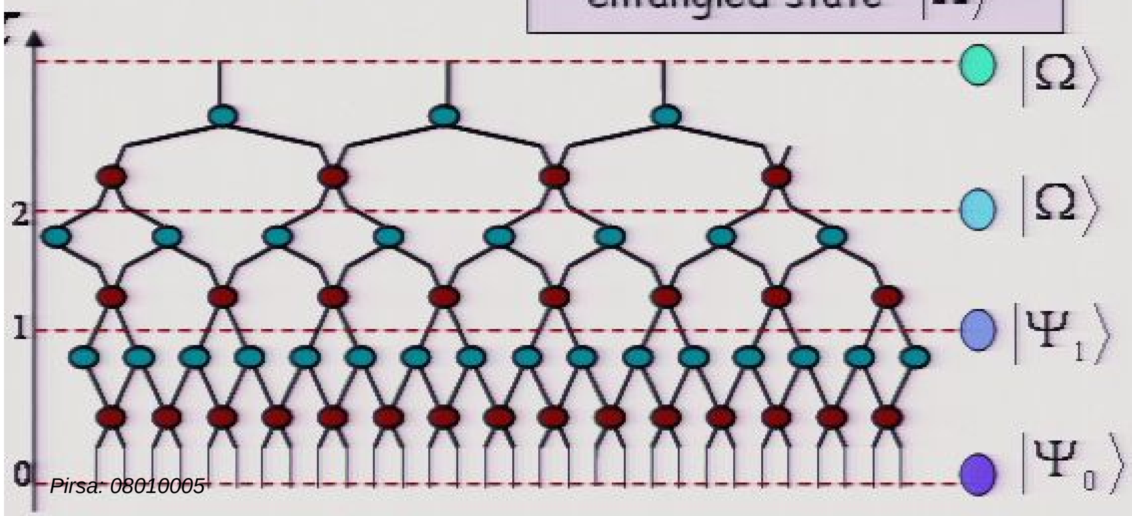


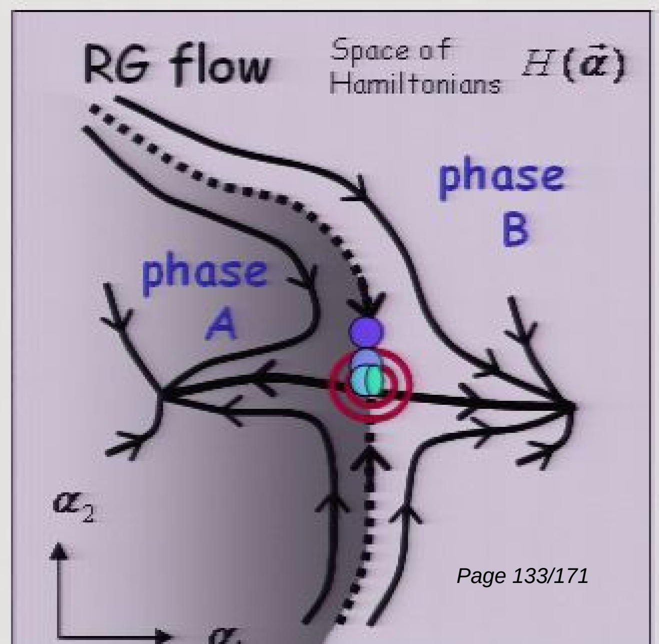


Critical system

- non-trivial fixed point:

entangled state $|\Omega\rangle$





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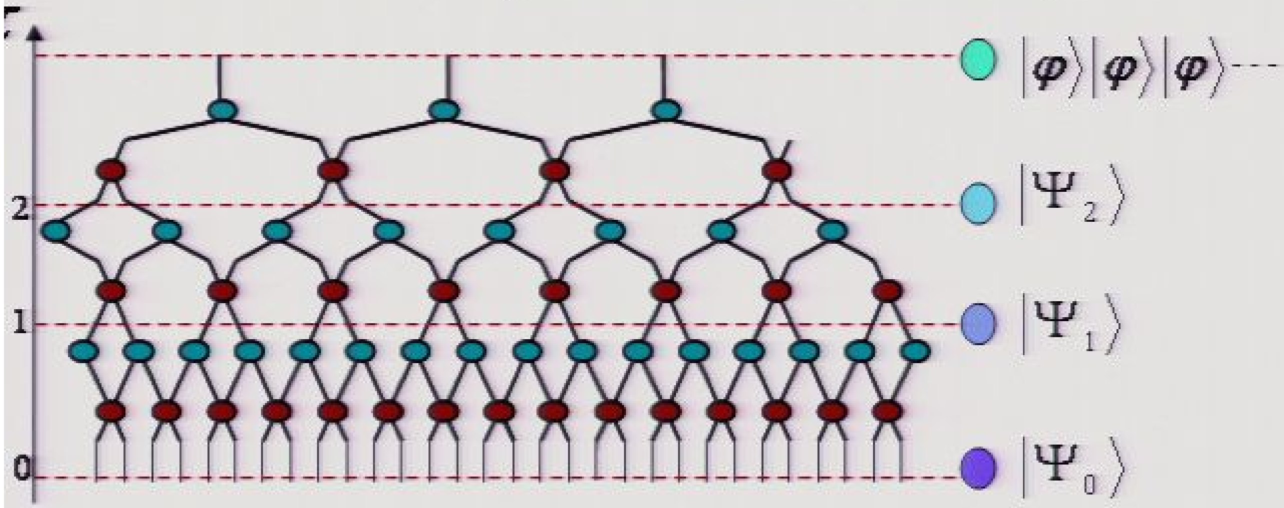

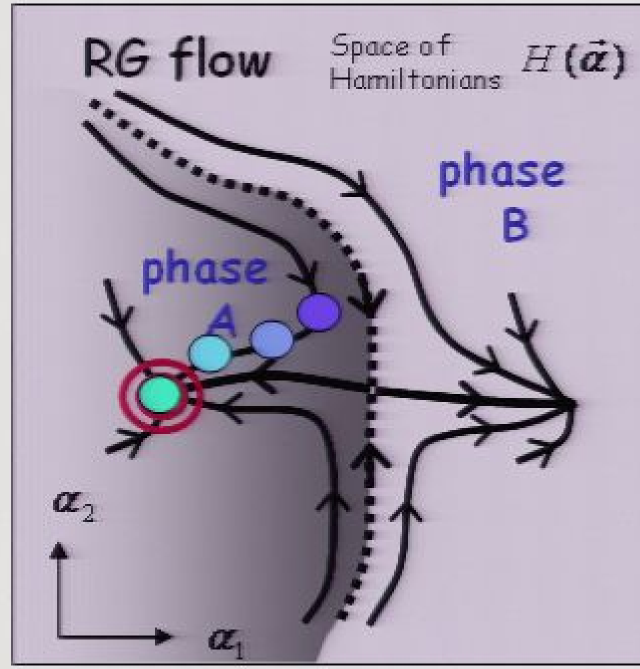

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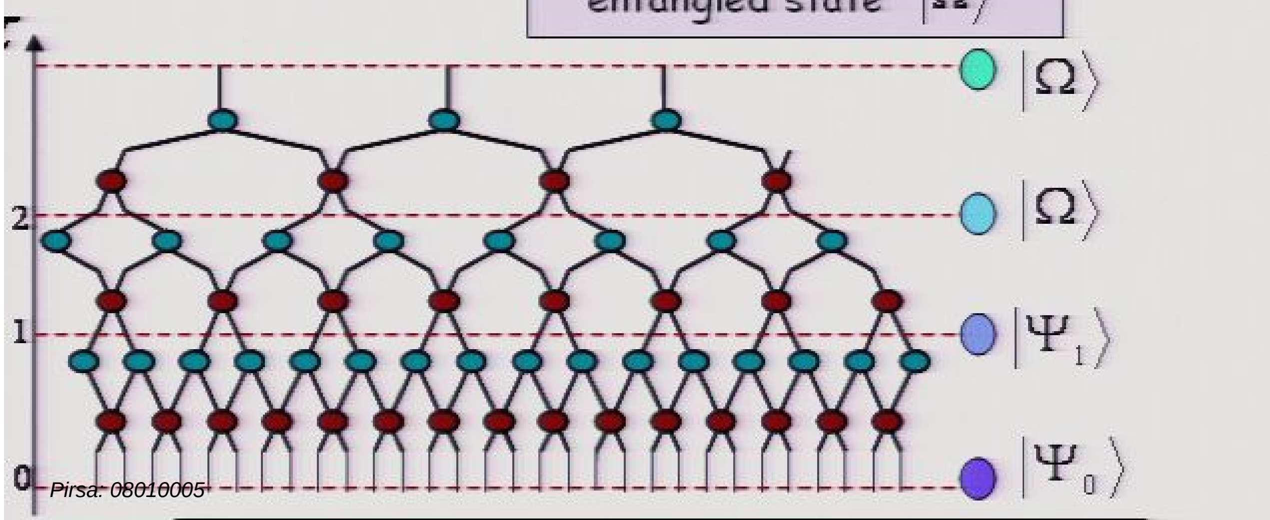
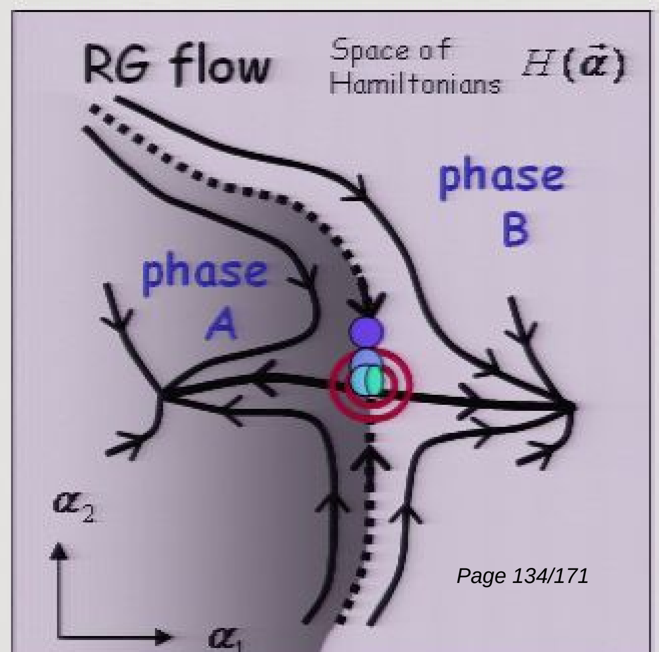
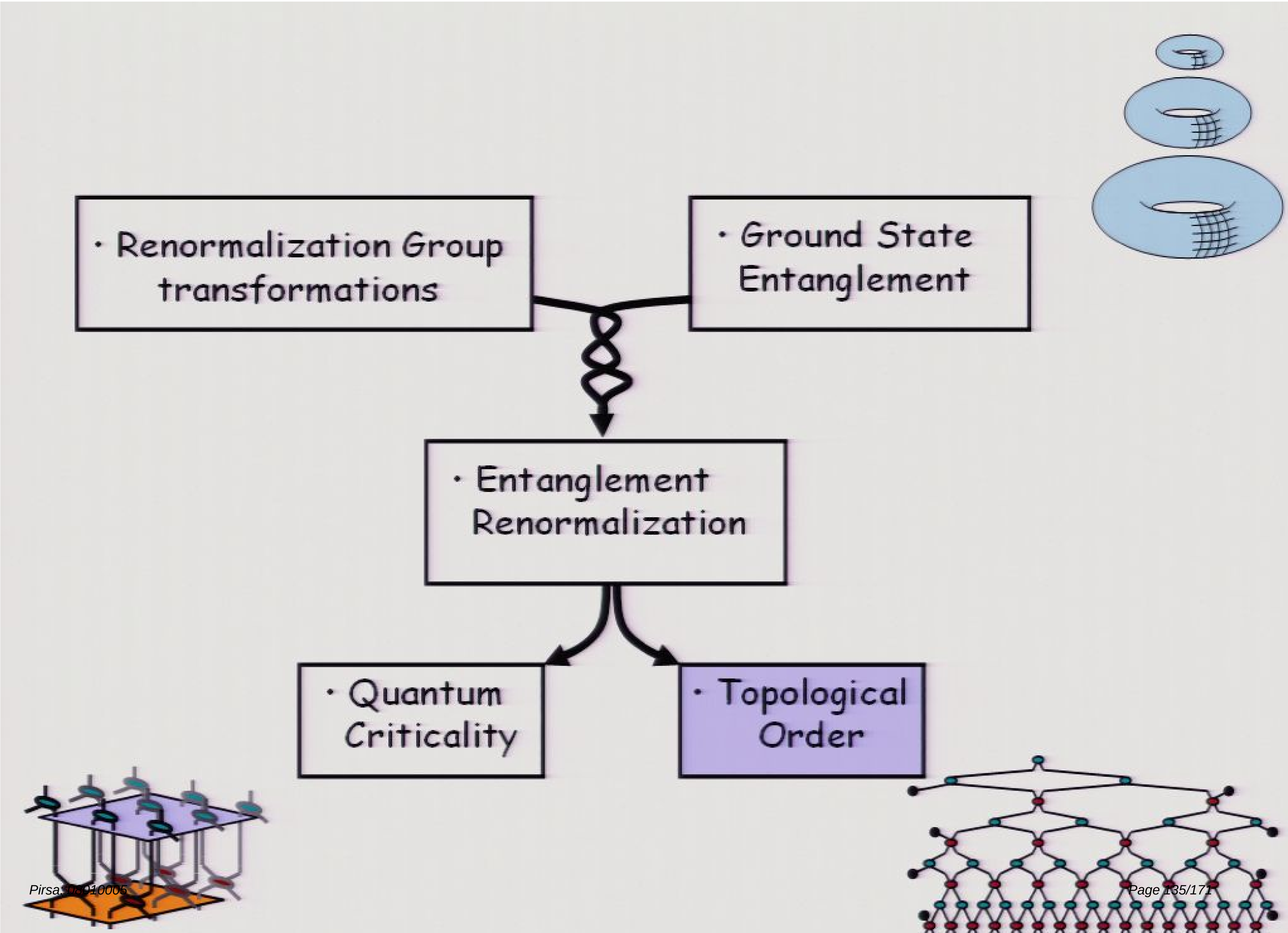




·MERA becomes a fractal structure

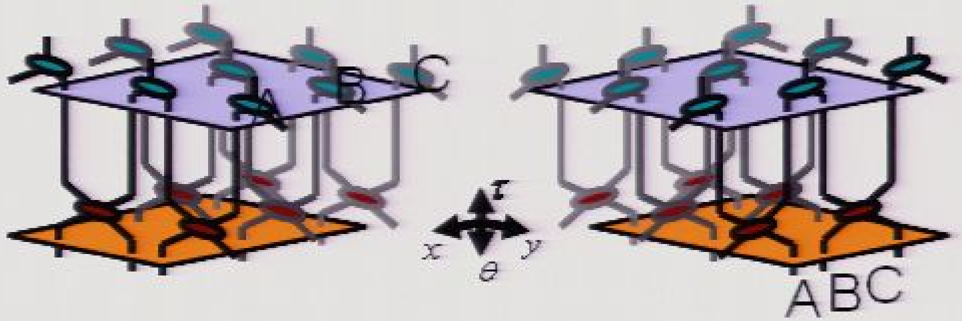


• Renormalization Group transformations
• Ground State Entanglement
• Entanglement Renormalization
• Quantum Criticality
• Topological Order

VI) Topological order

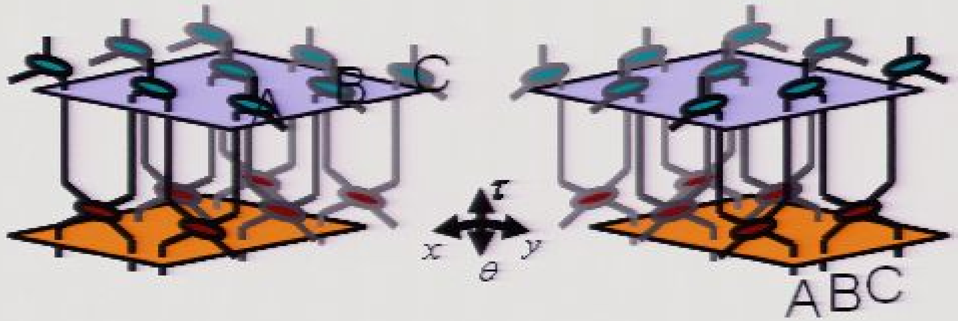
VI) Topological order

- 2D systems

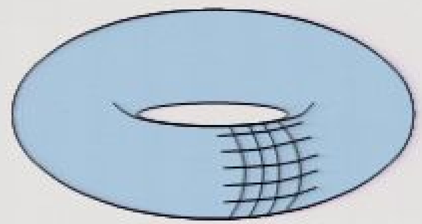


VI) Topological order

- 2D systems

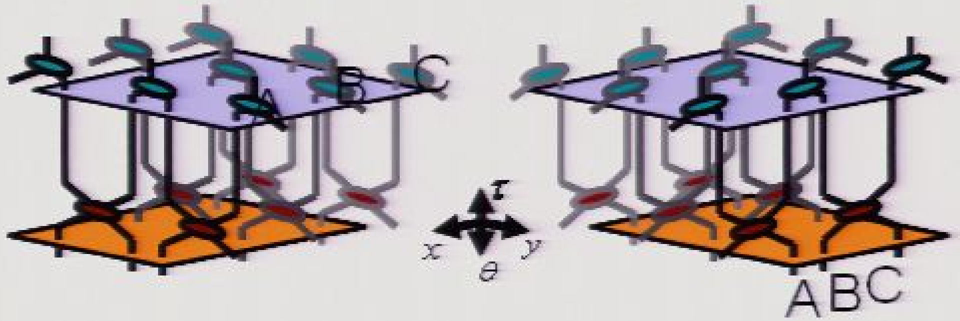


- 2D lattice systems on a torus



VI) Topological order

- 2D systems



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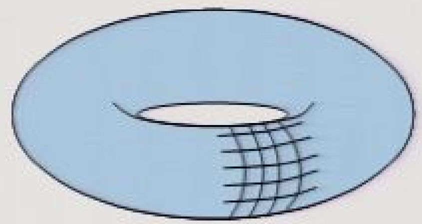


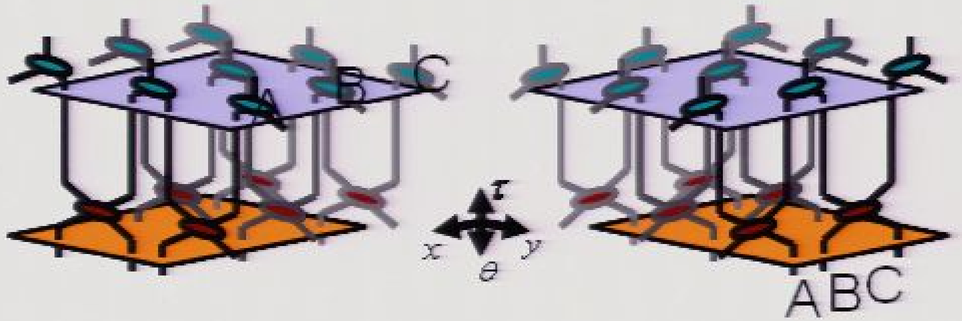
- Topological order

degenerate ground states

$$\left|\Psi_{gr}^{(1)}\right\rangle \quad \left|\Psi_{gr}^{(2)}\right\rangle$$

VI) Topological order

- 2D systems



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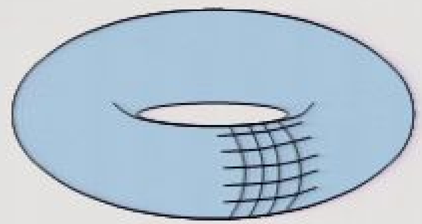
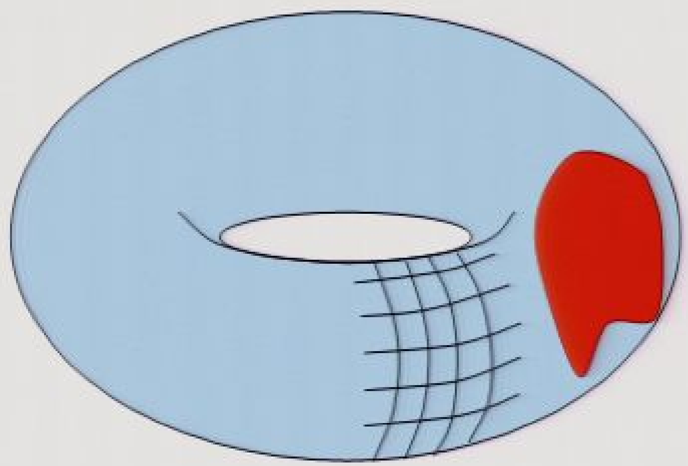


- Topological order

degenerate ground states

$|\Psi_{gr}^{(1)}\rangle$ $|\Psi_{gr}^{(2)}\rangle$



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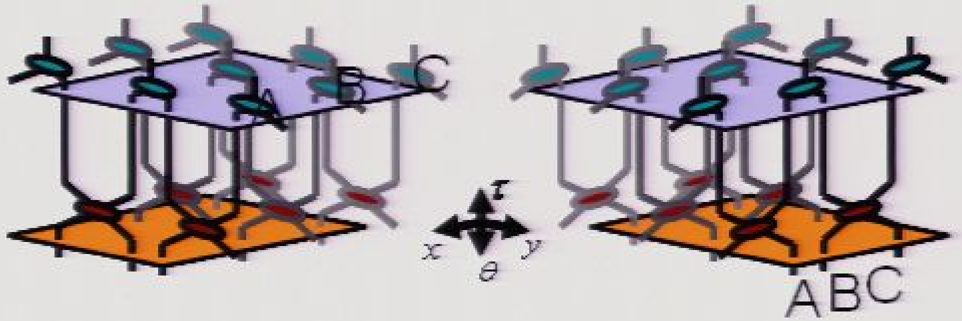
- 2D systems



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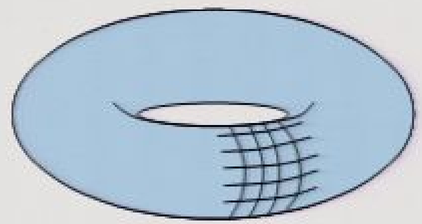

- Topological order

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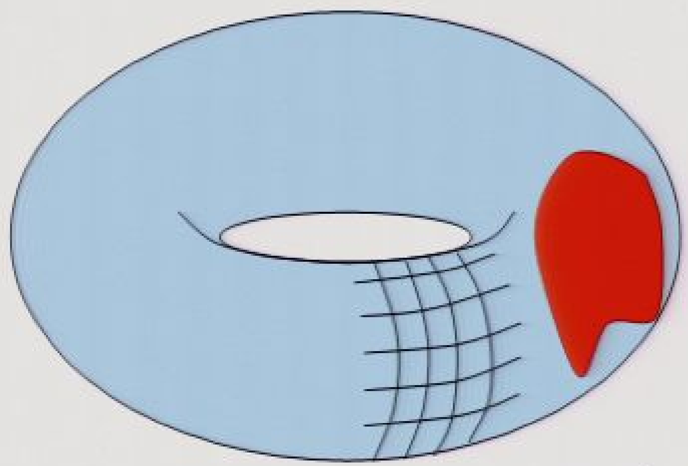
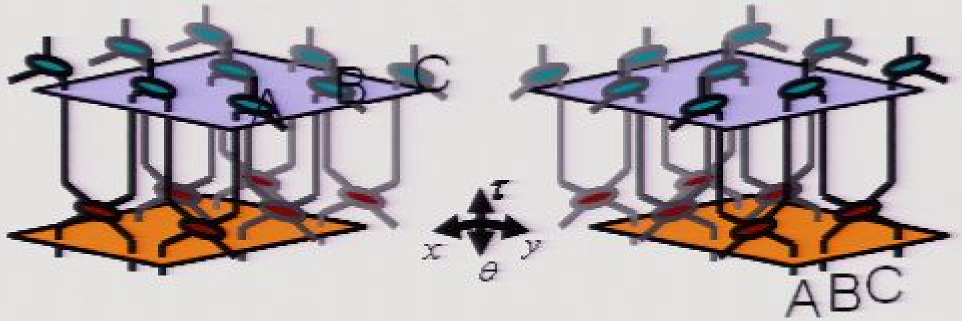


region
contractible
to a point

VI) Topological order

- 2D systems



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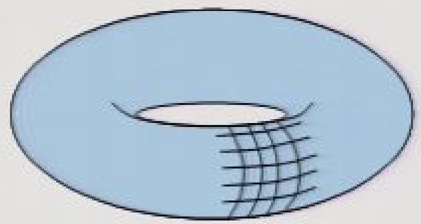
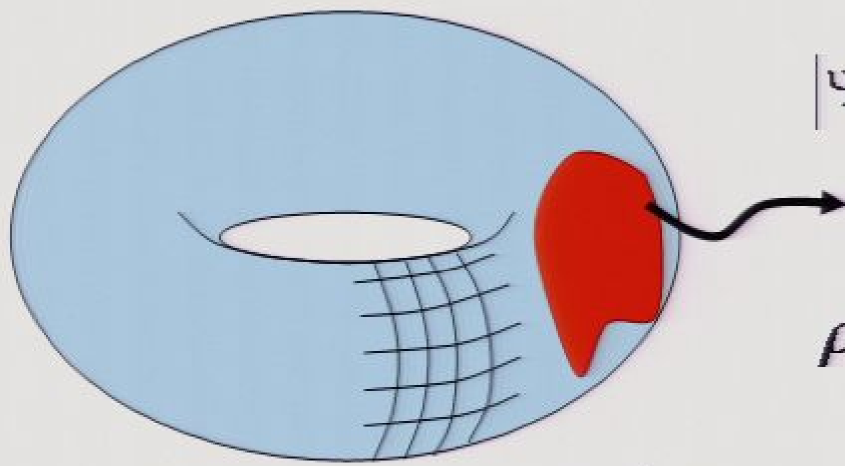


- Topological order

degenerate ground states

$$\left|\Psi_{gr}^{(1)}\right\rangle \quad \left|\Psi_{gr}^{(2)}\right\rangle$$



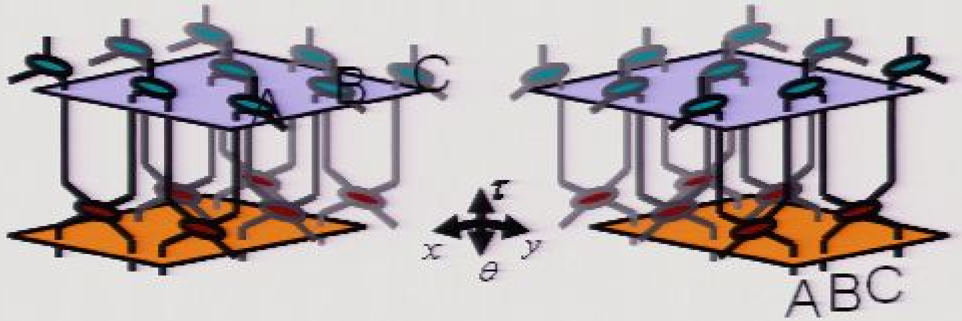
$$\left|\Psi_{gr}^{(1)}\right\rangle \quad \left|\Psi_{gr}^{(2)}\right\rangle$$

$$\boldsymbol{\rho}_{gr}^{(1)} = \boldsymbol{\rho}_{gr}^{(2)}$$

region contractible to a point

VI) Topological order

- 2D systems



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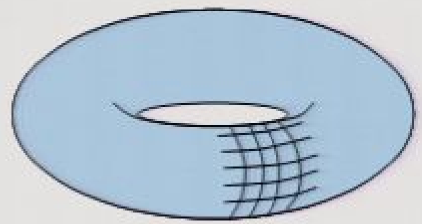
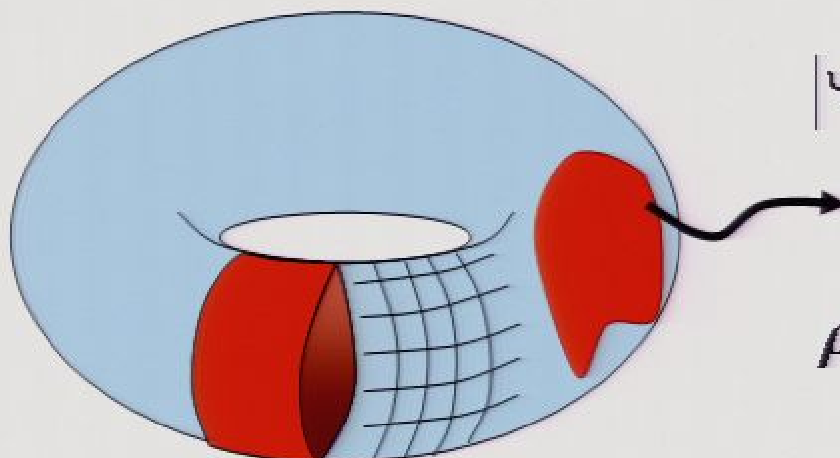


- Topological order

degenerate ground states

$\left|\Psi_{gr}^{(1)}\right\rangle \quad \left|\Psi_{gr}^{(2)}\right\rangle$



$\left|\Psi_{gr}^{(1)}\right\rangle \quad \left|\Psi_{gr}^{(2)}\right\rangle$

$\downarrow \quad \downarrow$

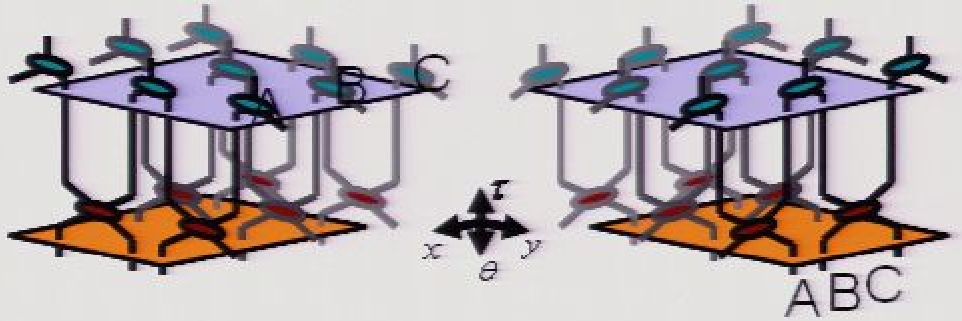
$$\rho_{gr}^{(1)} = \rho_{gr}^{(2)}$$

region not contractible to a point

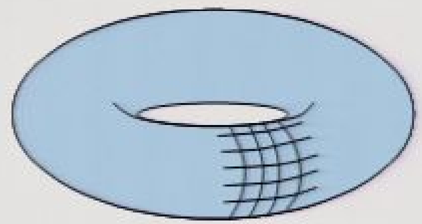
region contractible to a point

VI) Topological order

- 2D systems



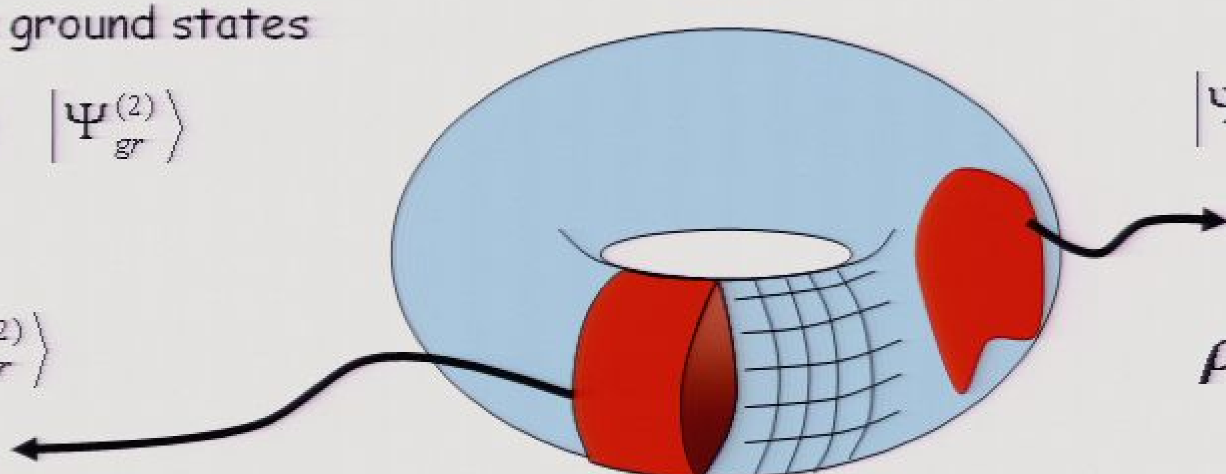
- 2D lattice systems on a torus



- Topological order

degenerate ground states

$\left|\Psi_{gr}^{(1)}\right\rangle \quad \left|\Psi_{gr}^{(2)}\right\rangle$



$\left|\Psi_{gr}^{(1)}\right\rangle \quad \left|\Psi_{gr}^{(2)}\right\rangle$

$\downarrow \quad \downarrow$

$\boldsymbol{\rho}_{gr}^{(1)} = \boldsymbol{\rho}_{gr}^{(2)}$

$\left|\Psi_{gr}^{(1)}\right\rangle \quad \left|\Psi_{gr}^{(2)}\right\rangle$

$\downarrow \quad \downarrow$

$\boldsymbol{\rho}_{gr}^{(1)} \neq \boldsymbol{\rho}_{gr}^{(2)}$

region not contractible to a point

region contractible to a point

- Topological order

degenerate ground states

$$\left|\Psi_{gr}^{(1)}\right\rangle$$

$$\left|\Psi_{gr}^{(2)}\right\rangle$$

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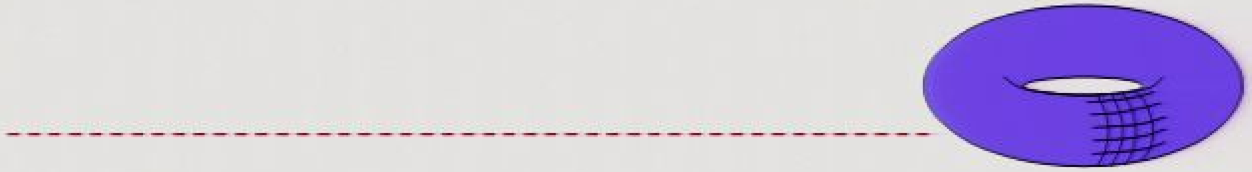

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- Topological order

degenerate ground states

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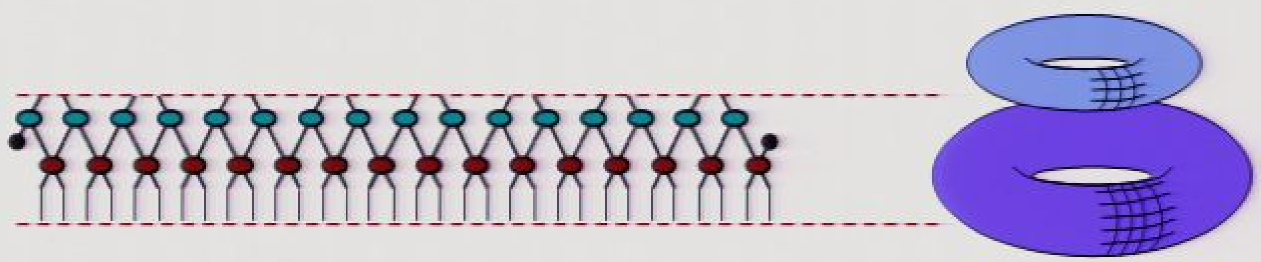

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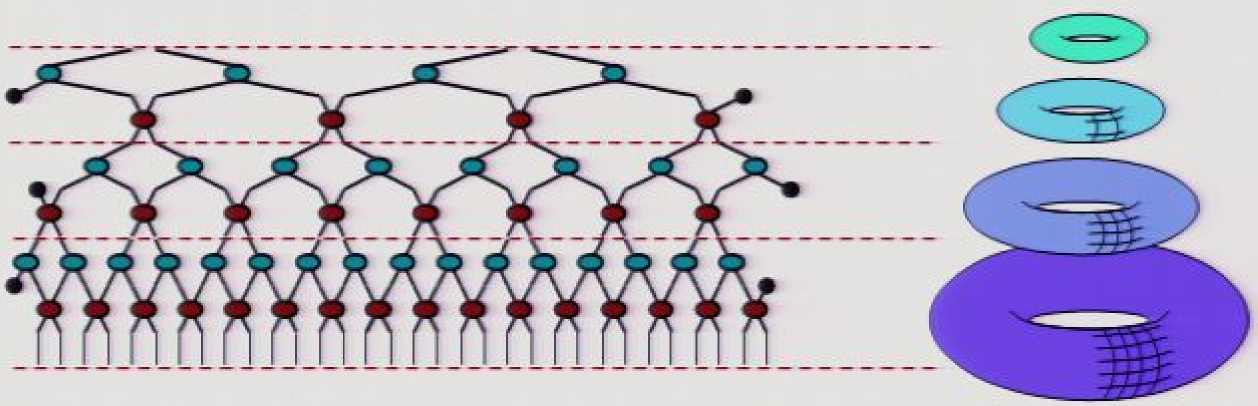

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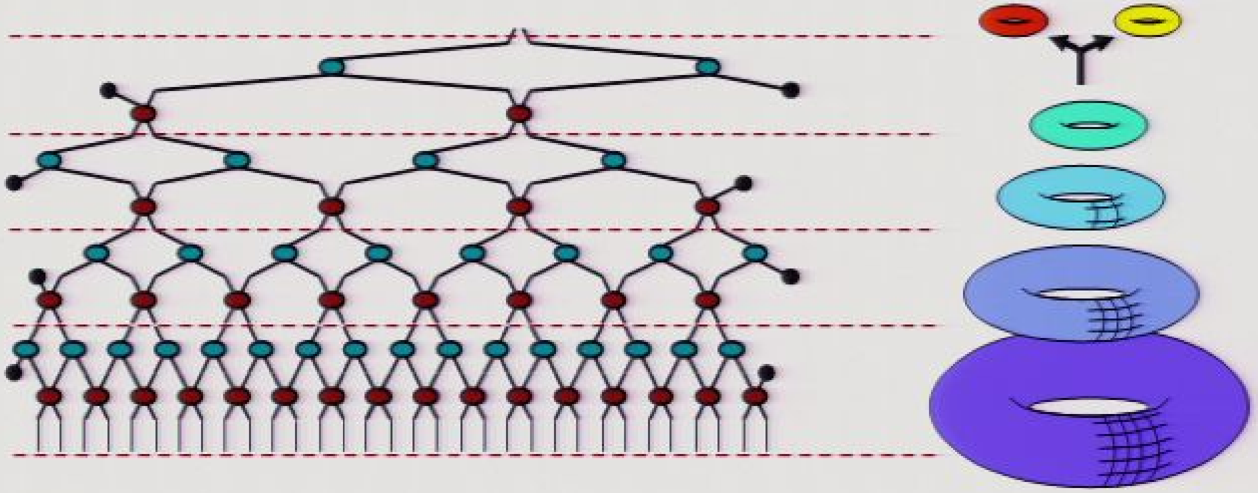

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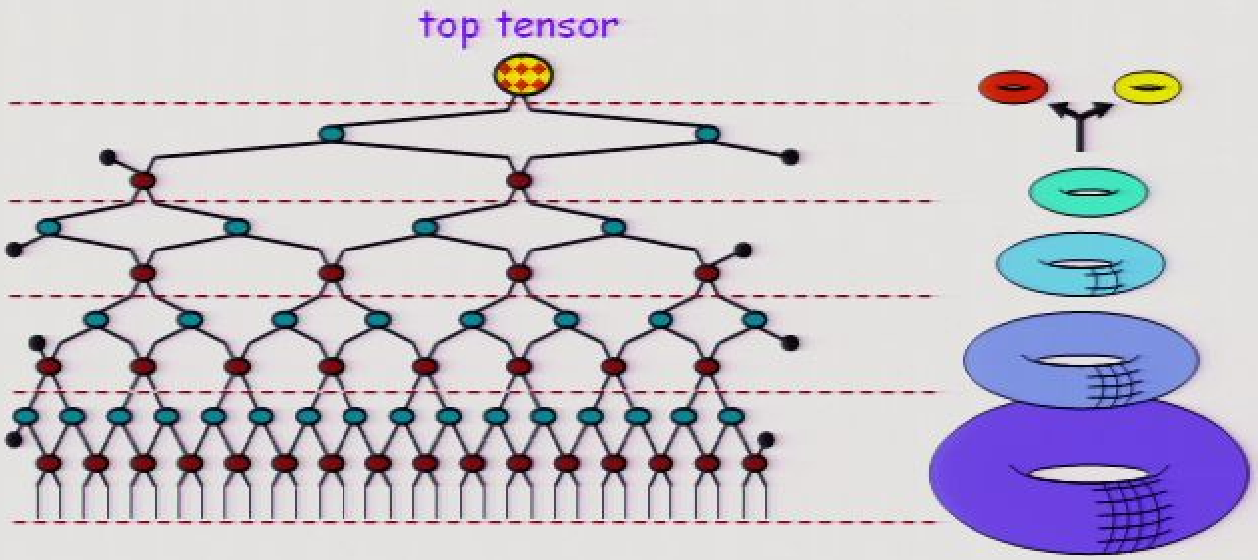
Topological information is stored in the top tensor of the MERA

M. Aguado and G. Vidal, arXiv:0712.0348, PRL



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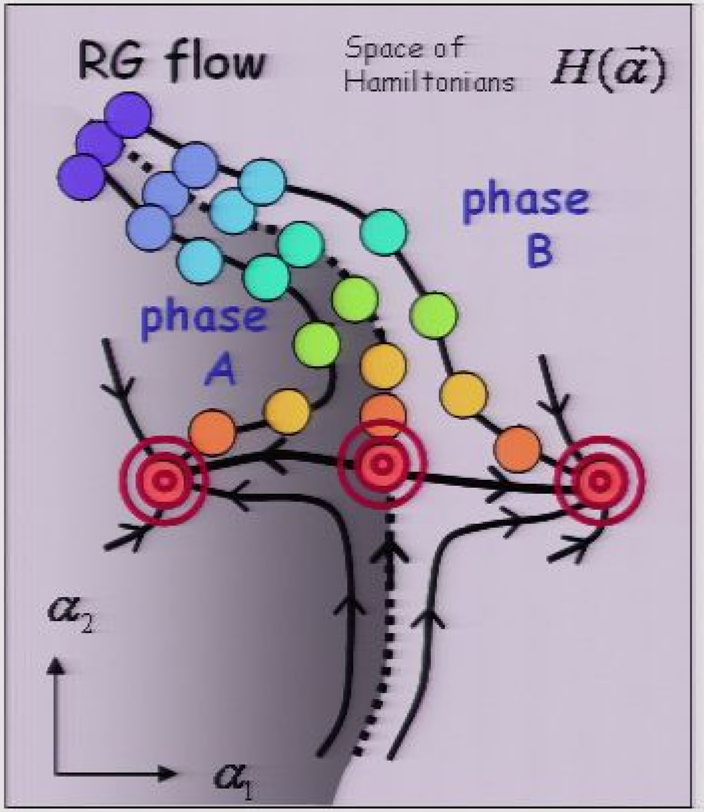
- Algorithms:
  - Flow Equations (C. Dawson, J. Eisert, T. Osborne)
  - Time Evolution (M. Rizzi, S. Montangero, G.V.) -> 1D
  - Variational Approach (G. Evenbly, G.V) -> 1D, 2D
  - Interacting Fermions (F. Verstraete, G.V.)

Summary and conclusions

•The RG studies how the properties of an extended system change with the scale of observation.



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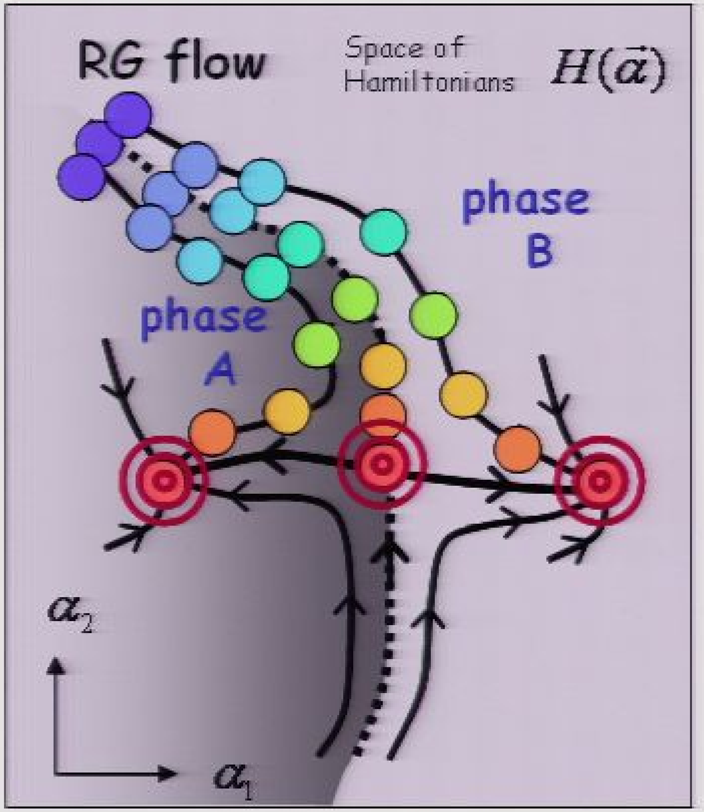
✗ RG flow? ✗ Scale invariance? ✗ Small systems



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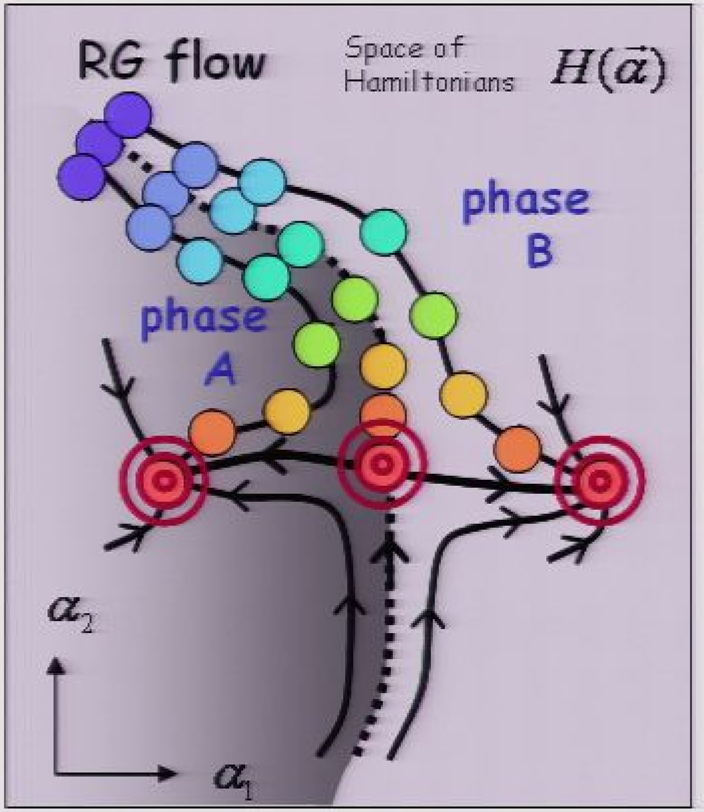

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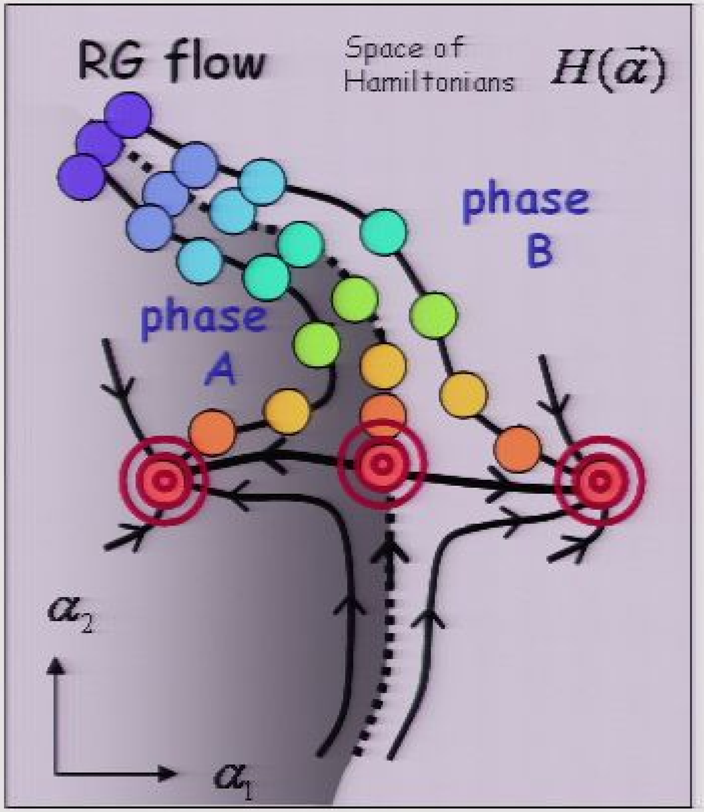

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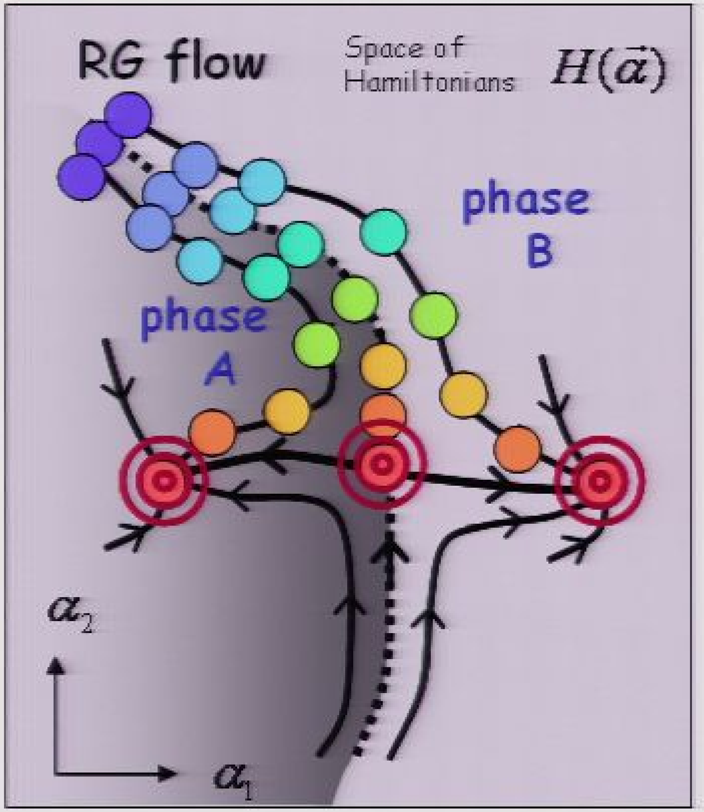

• The MERA offers a good description of systems that are **scale invariant** (critical points) or have **topological order**

- **Topological order**

degenerate ground states

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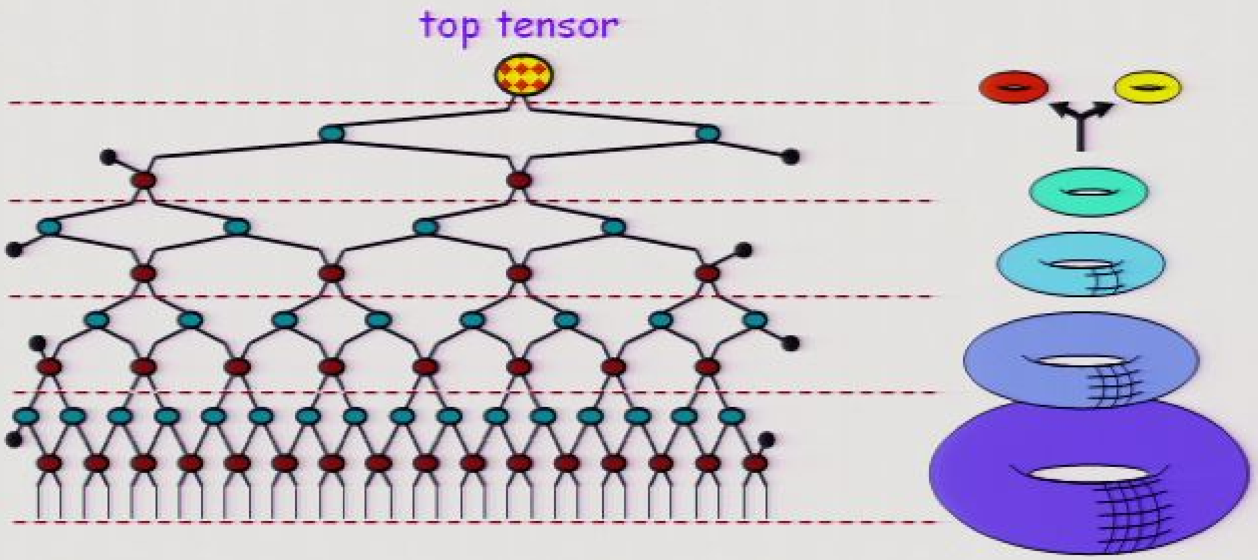

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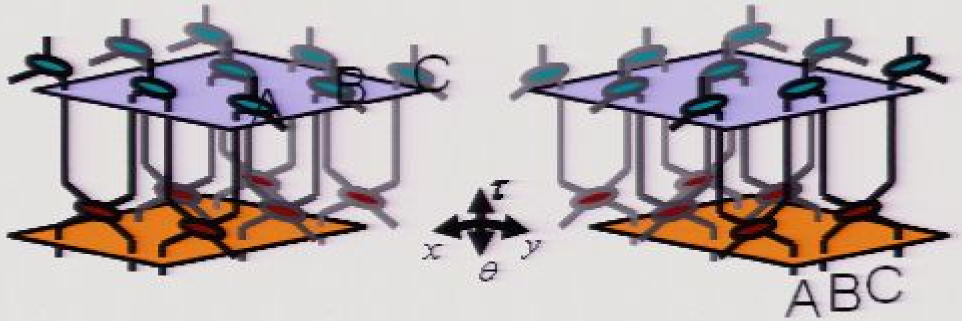
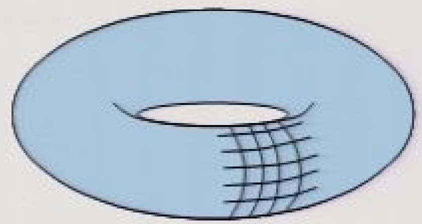
VI) Topological order

- 2D systems



- 2D lattice systems on a torus



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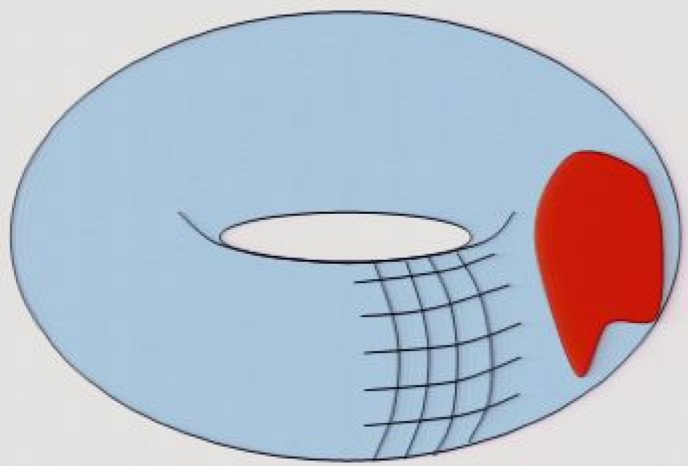
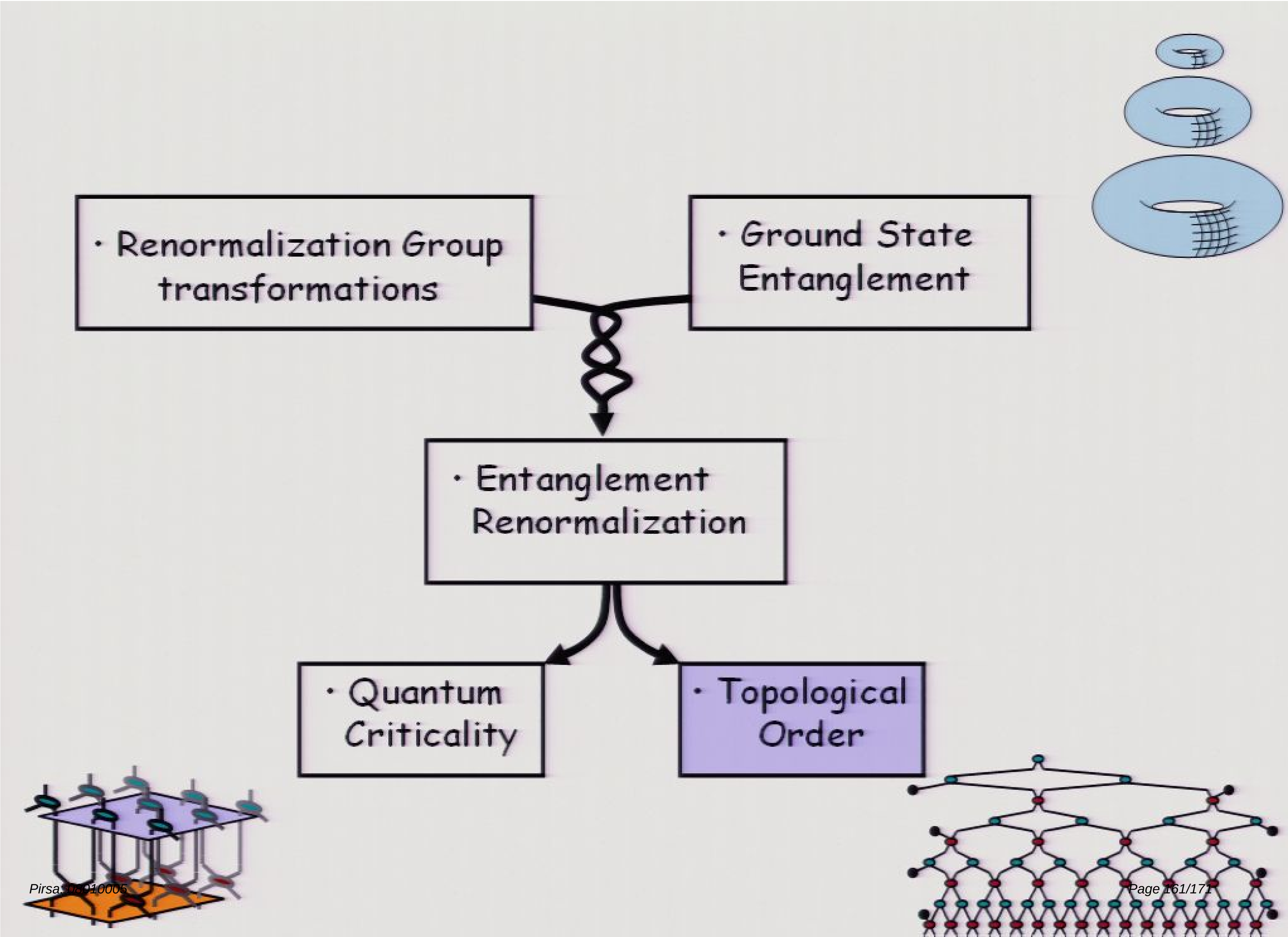




• Renormalization Group transformations
• Ground State Entanglement
• Entanglement Renormalization
• Quantum Criticality
• Topological Order

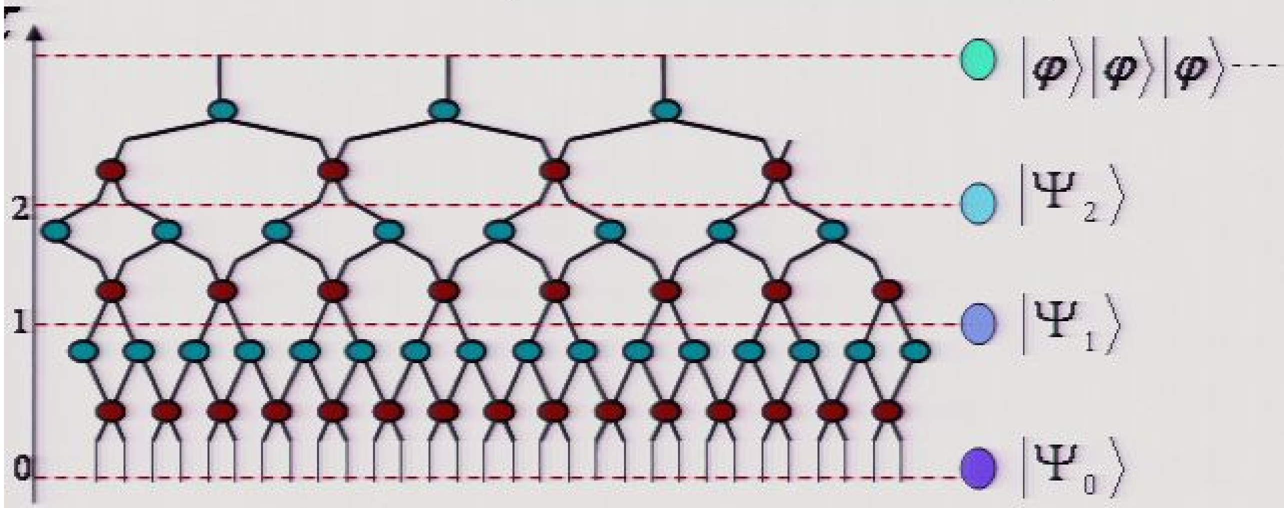
Quantum criticality

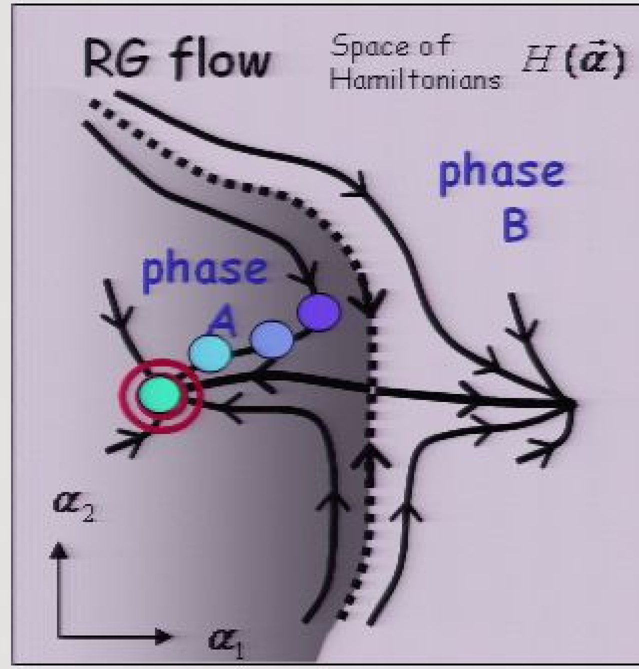
G. Vidal, Phys. Rev. Lett. 99, 220405 (2007)

Gapped system

- trivial fixed point:

product state $|\varphi\rangle|\varphi\rangle|\varphi\rangle\cdots$

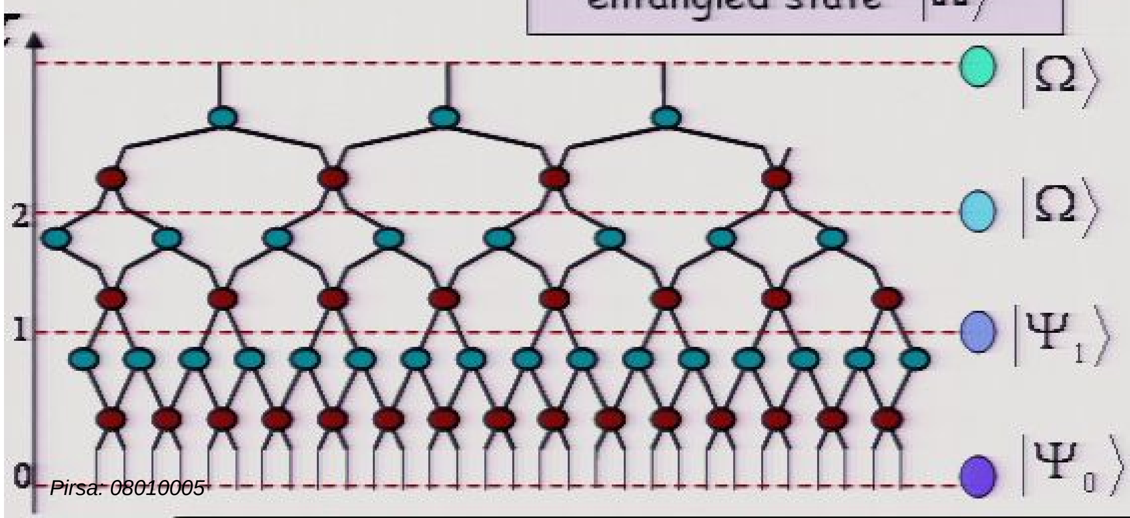


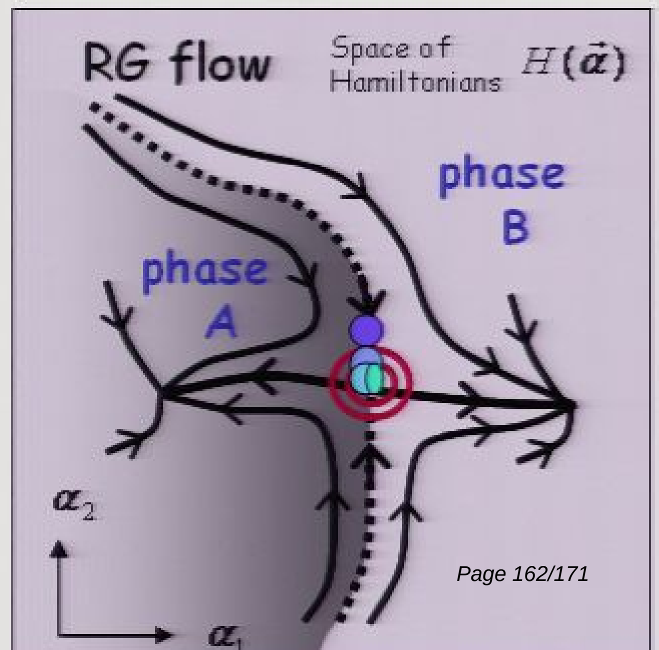


Critical system

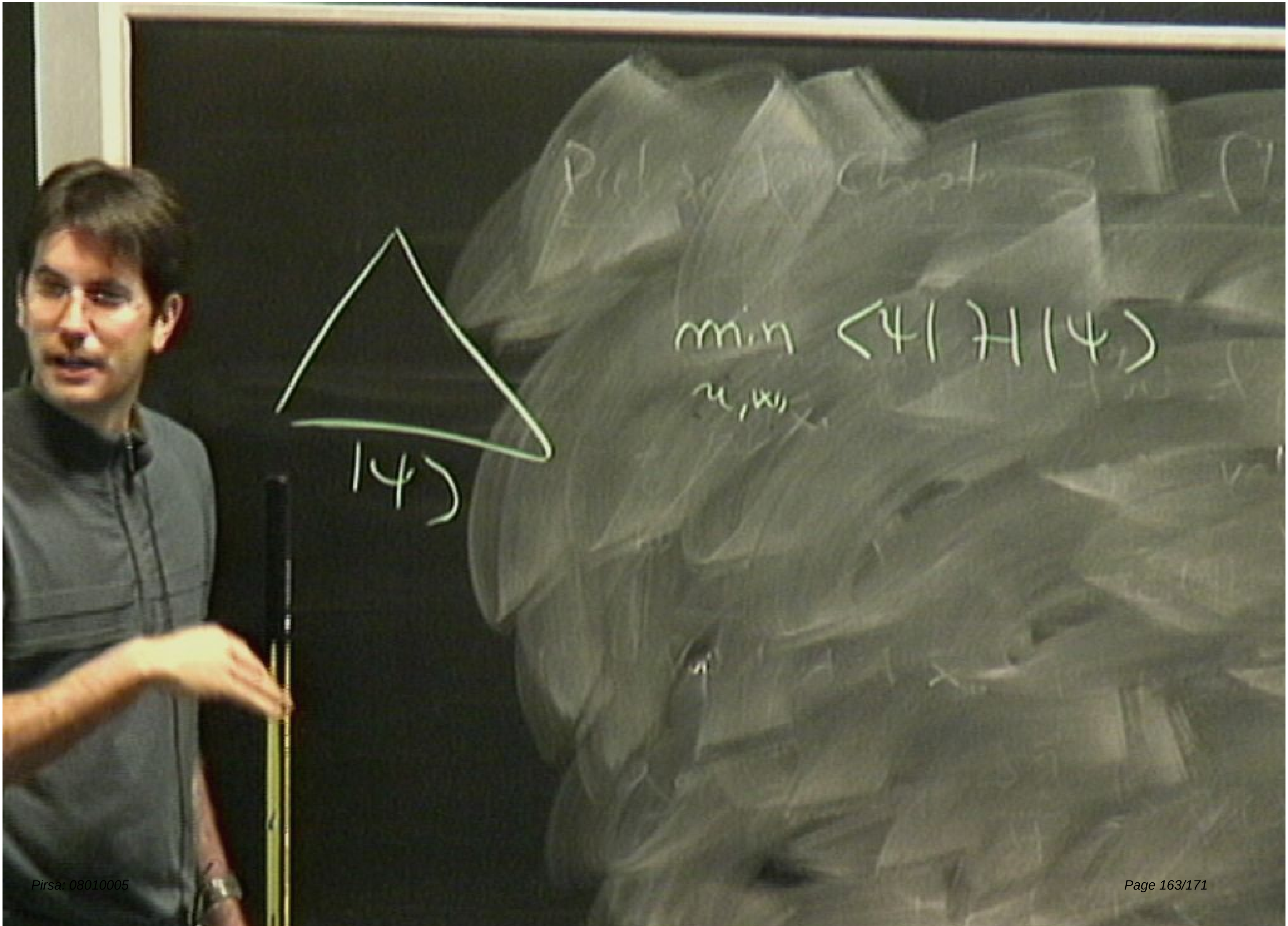
- non-trivial fixed point:

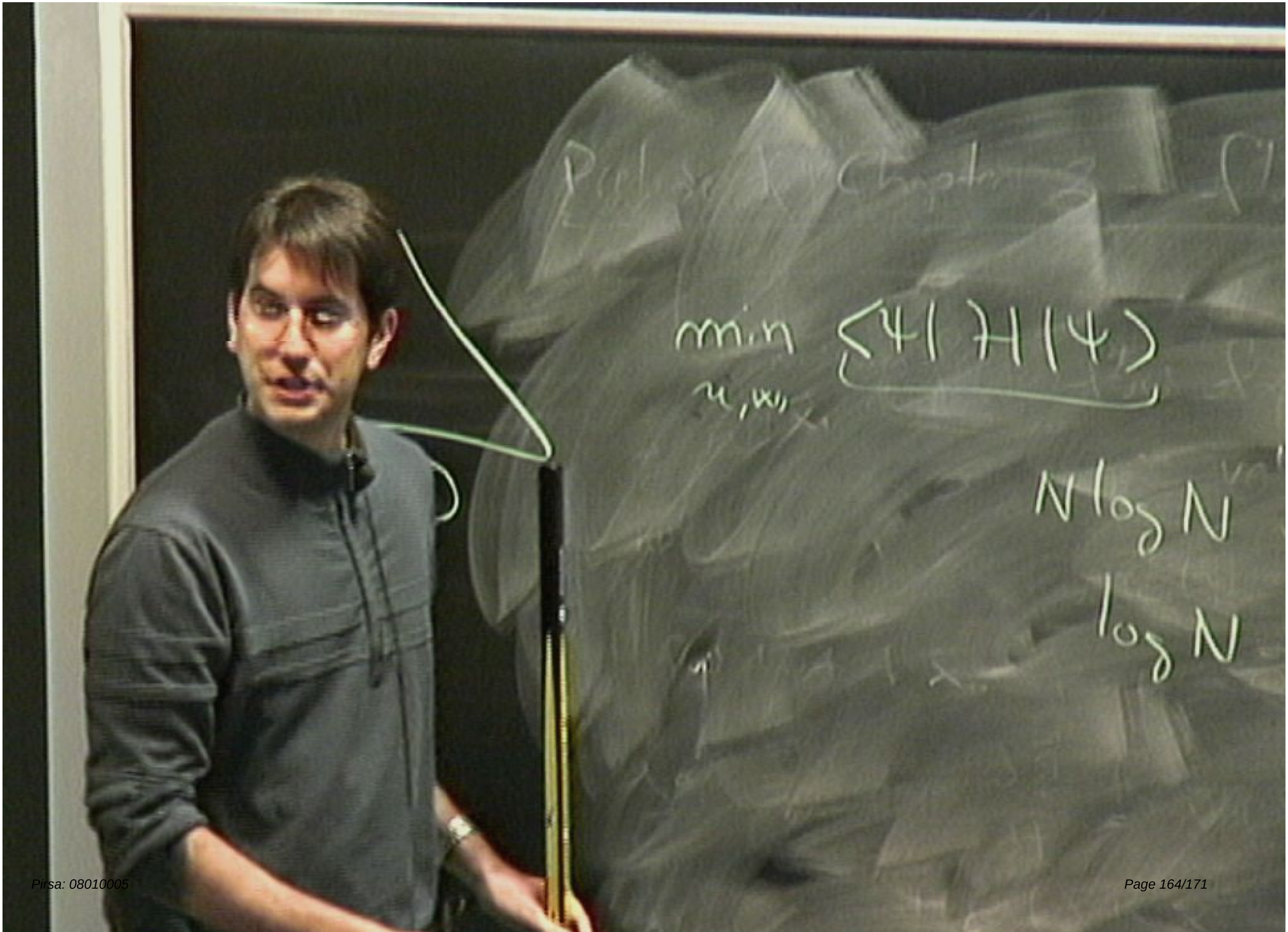
entangled state $|\Omega\rangle$

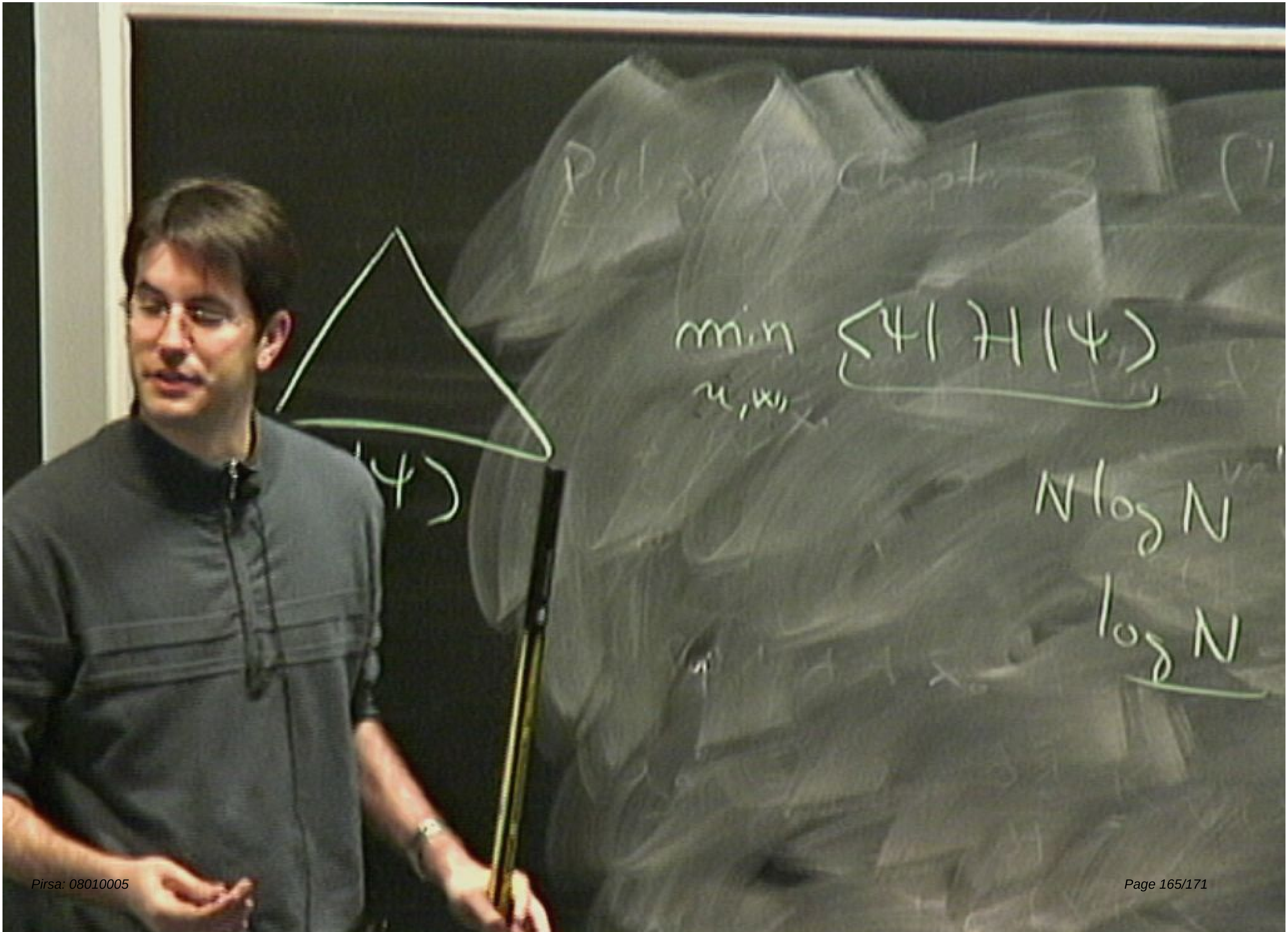


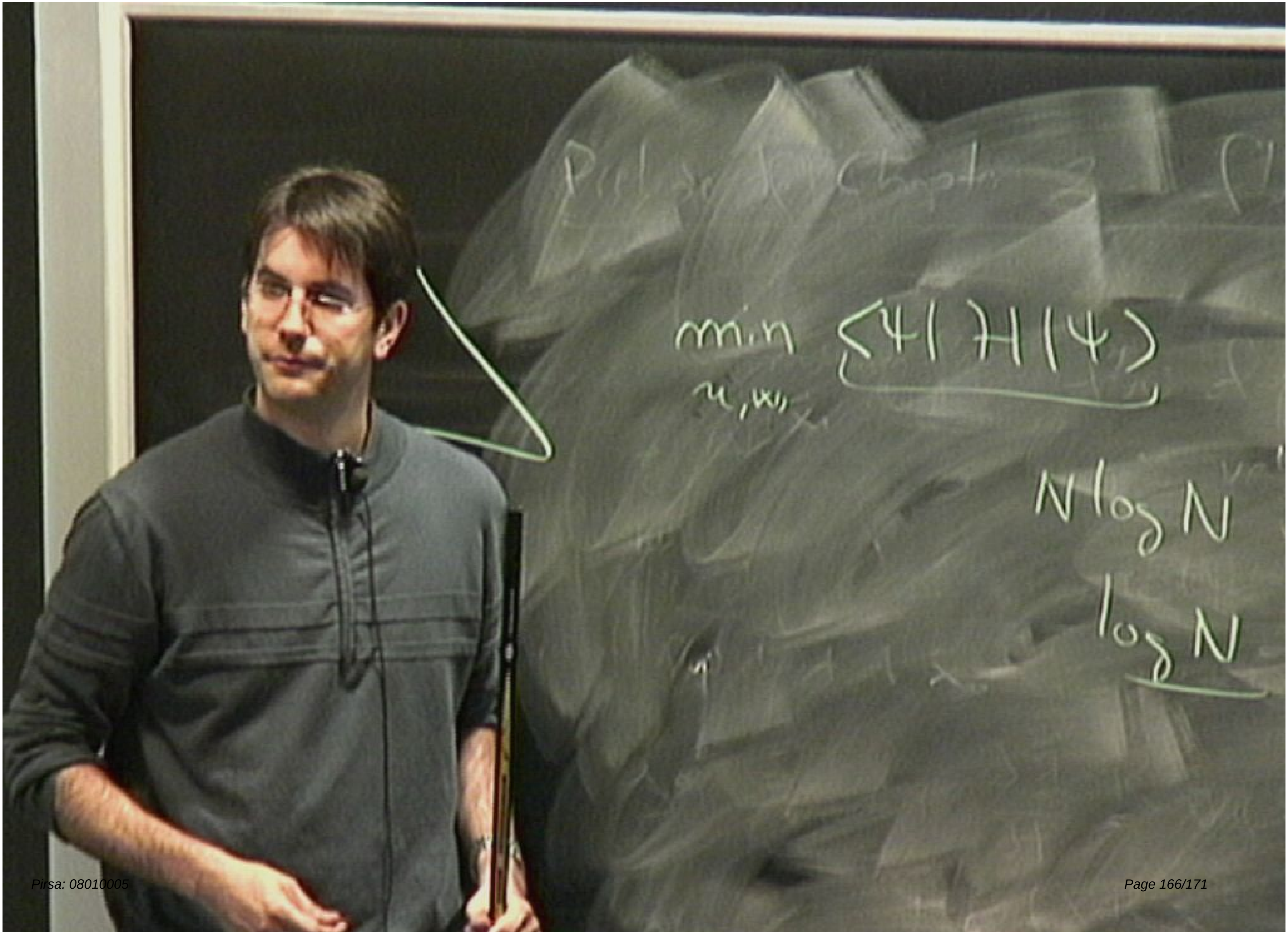


·MERA becomes a fractal structure


|ψ⟩
min ⟨ψ| H |ψ⟩


min ⟨Ψ| H |Ψ⟩
N log N
log N


min ⟨ψ| H |ψ⟩
N log N
log N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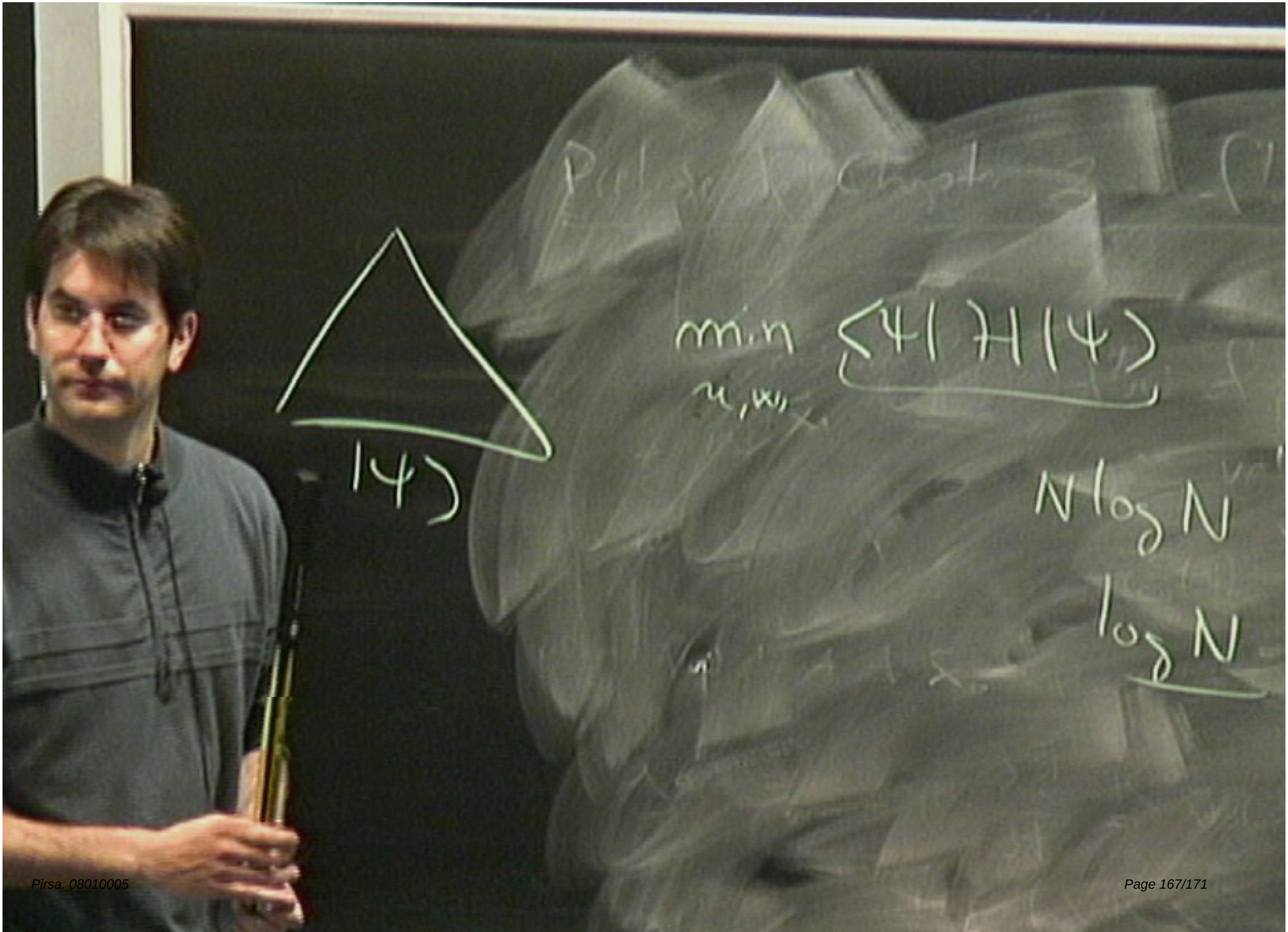
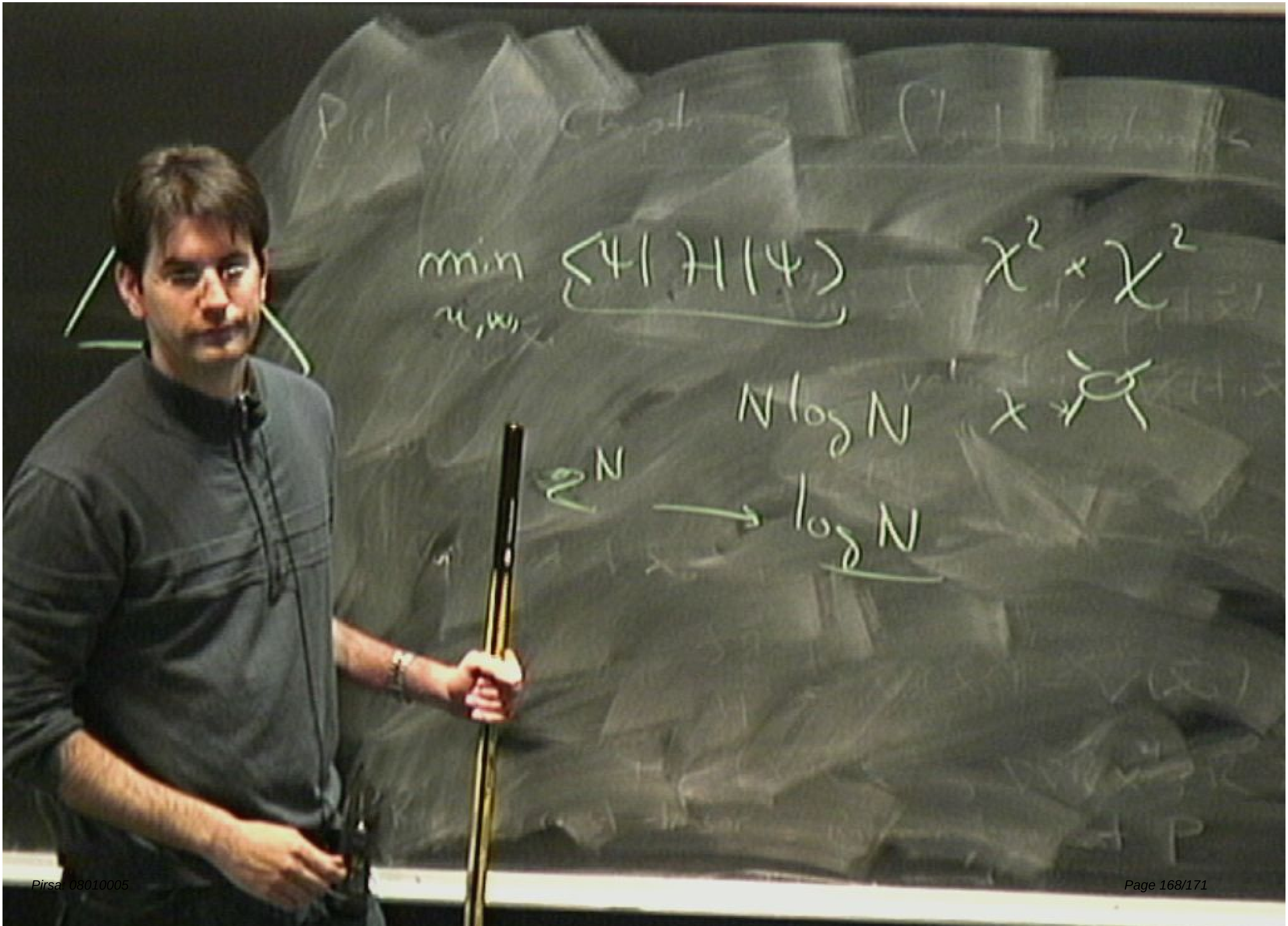
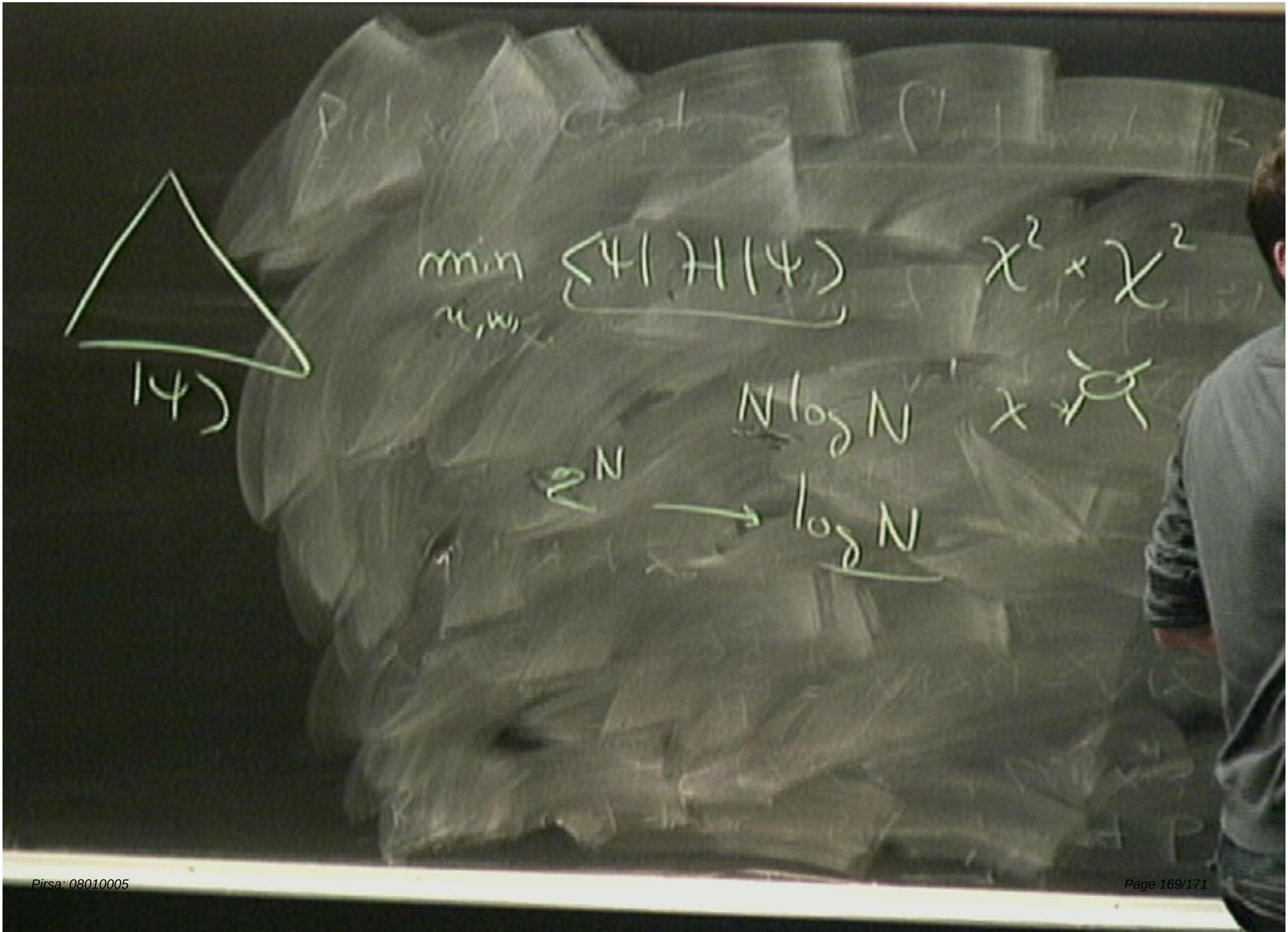
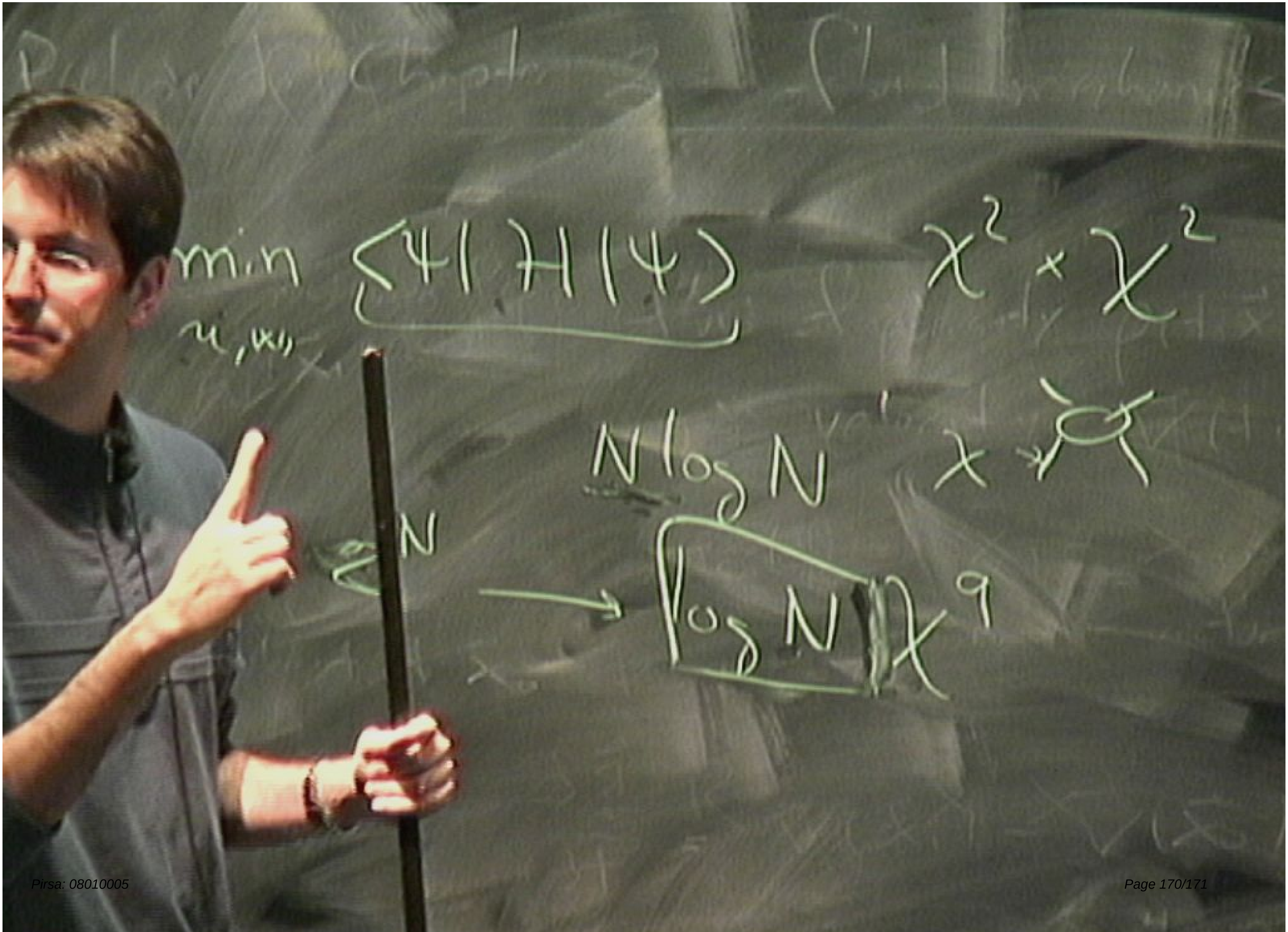
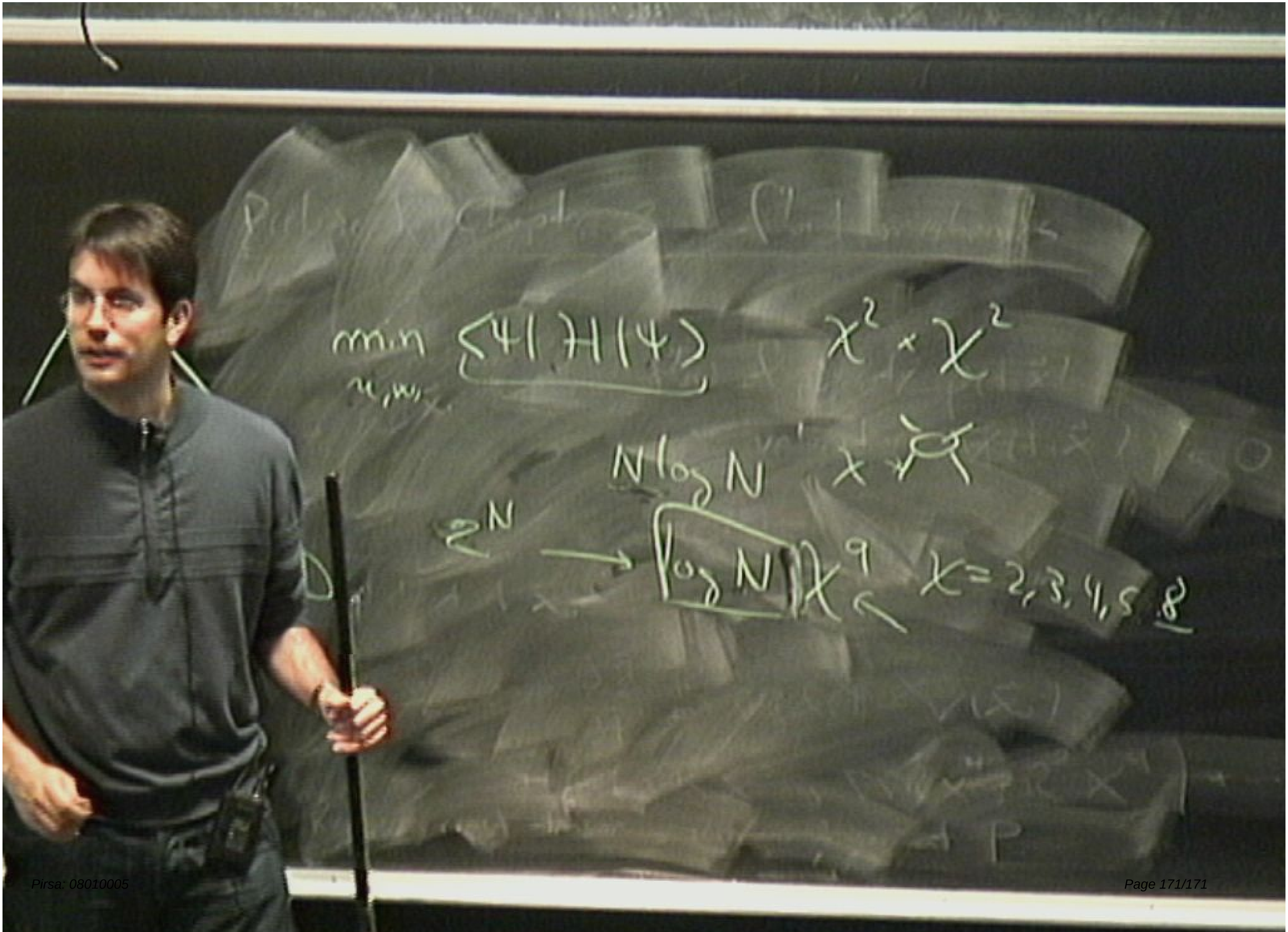

min ⟨ψ| H |ψ⟩
|ψ⟩
N log N
log N


min ⟨Ψ| H |Ψ⟩
χ² × χ²
N log N
2^N
log N


|ψ⟩
min ⟨ψ|H|ψ⟩
χ² × χ²
N log N
2^N → log N


min ⟨ψ| H |ψ⟩
N log N
log N


min ⟨ψ| H |ψ⟩
N log N
2^N
log N
x = 2, 3, 4, 5, 8