Title: The physics of the antisymmetric tensor field

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Abstract: I shall review the potential relevance of antisymmetric tensor fields in physics, perhaps the most intriguing being a massive antisymmetric tensor as dark matter. Next, based on the most general quadratic action for the antisymmetric tensor field, I shall discuss what are possible extensions of Einstein\\\'s theory which include antisymmetric tensor field and thus torsion in a dynamical fashion.

# THE PHYSICS OF ANTISYMMETRIC TENSOR FIELDS

Tomislav Prokopec
ITP & Spinoza Institute, Utrecht University

Based on publications:
**T. Prokopec and Wessel Valkenburg, Phys. Lett. B636, 1-4 (2006) [astro-ph/0503289]; astro-ph/0606315;**
**T. Prokopec and Tomas Janssen,** gr-qc/0604094, Class. Quant. Grav. 23 1-15 (2006)
Christiaan Mantz and T. Prokopec, in preparation (2008) & M.Sc. thesis



Perimeter Institute, Waterloo, Jan 15, 200

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# NONSYMMETRIC GRAVITATIONAL THEORY

- add an antisymmetric component to the metric tensor

$$\bar{g}_{\mu\nu} = g_{\mu\nu} + B_{\mu\nu}$$

$$\longrightarrow \quad g_{\mu\nu} = \bar{g}_{(\mu\nu)}, \qquad B_{\mu\nu} = \bar{g}_{[\mu\nu]} = \frac{1}{2}\left(\bar{g}_{\mu\nu} - \bar{g}_{\nu\mu}\right)$$

In 1925 Einstein proposed it as a unified theory of gravity and electromagnetism

- It does not work since
  - (a) Geodesic equation does not reproduce the Lorentz force
  - (b) Equations of motion do not impose divergenceless magnetic field
- In 1979 Moffat proposed it as a generalised theory of gravitation:
  Nonsymmetric Theory of Gravitation (NGT)
- change Newton´s Law on large scales ⇒ away with DM?



J. Moffat

°2

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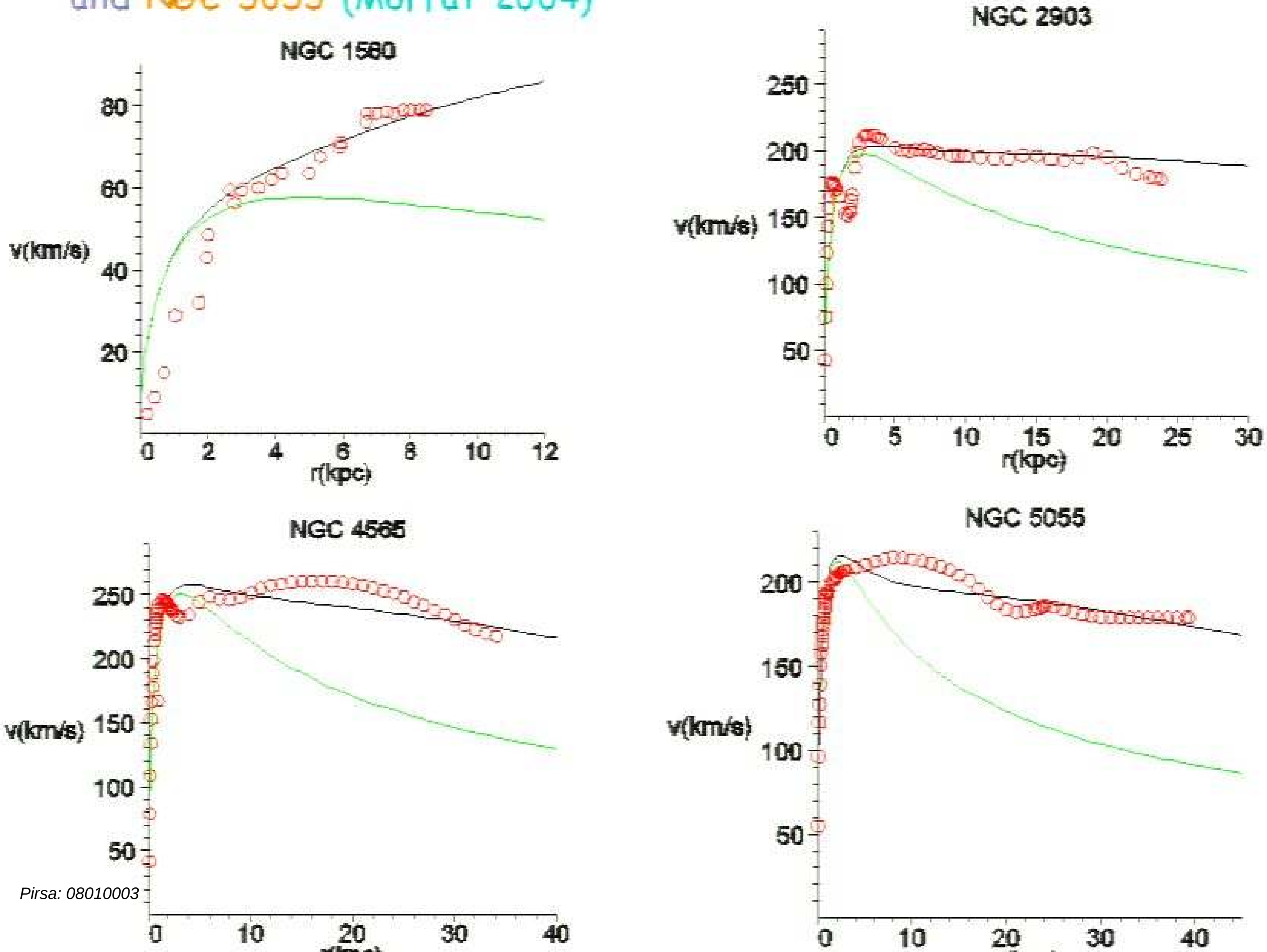
# GALAXY ROTATION CURVES

- Fits to the rotation curves for the galaxies: NGC1560, NGC 2903, NGC 4565 and NGC 5055 (Moffat 2004)




$r_0 \approx 14\,\mathrm{kpc},$

$M_0 \approx 10^{12} M$

# INSTABILITIES OF NGT



- Problems with Nonsymmetric Theory of Gravitation

  (a) When quantised, in its simplest disguise, NGT contains ghosts

  (b) There are instabilities (when B couples to Riemann/Ricci Tensor )

  Damour, Deser & McCarthy (1993) ; Moffat & Clayton

- Most general problem-free quadratic action in B

$$\Rightarrow \boldsymbol{S} = \boldsymbol{S}_{EH} + \boldsymbol{S}_{NGT} + \boldsymbol{S}_{mat}$$

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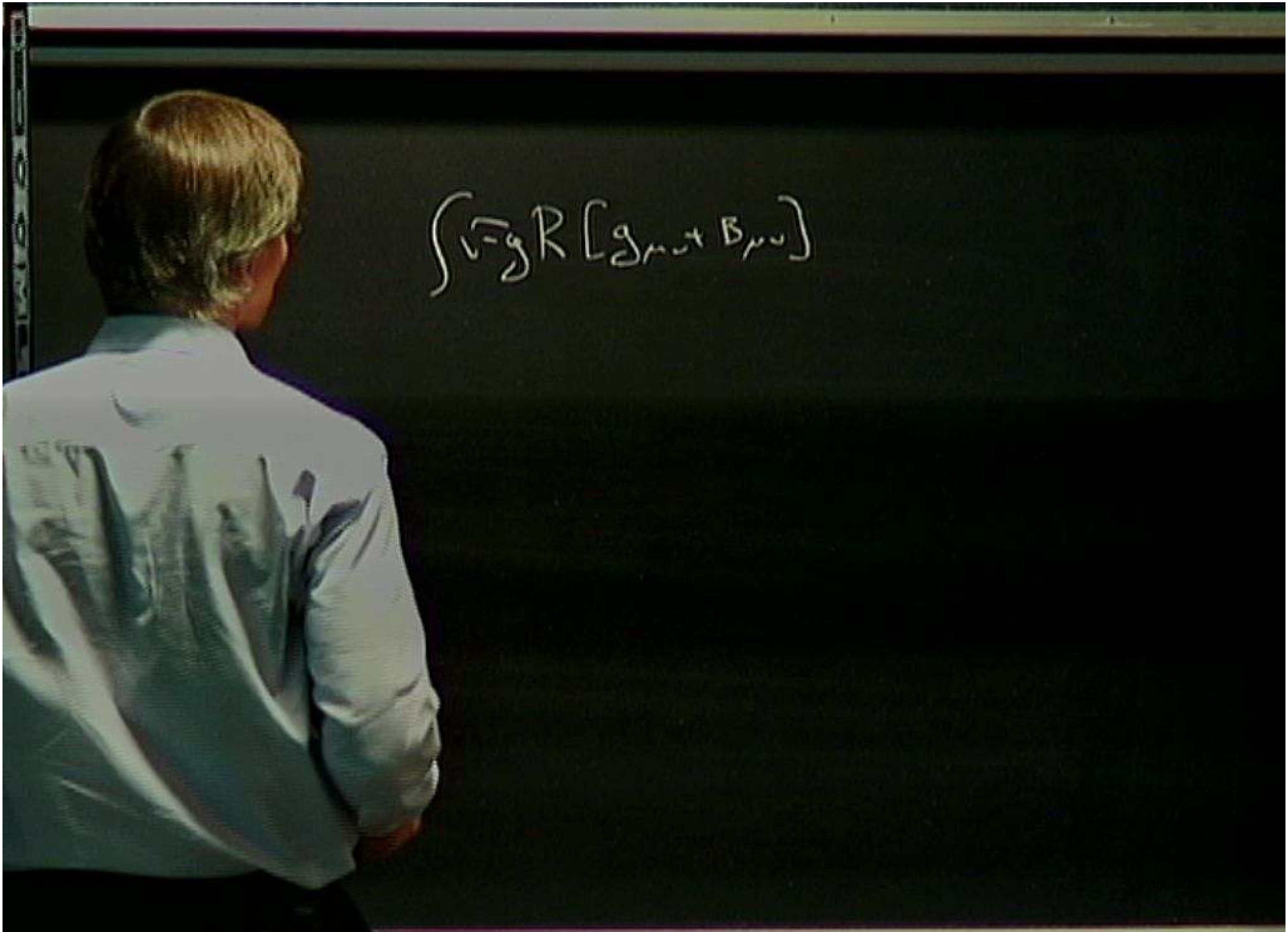
$$S_{EH} = -\frac{1}{16\pi G_N}\int d^4x\sqrt{-g}\,(\mathcal{R} - 2\Lambda)$$

$$\boldsymbol{S}_{NGT} = \int \boldsymbol{d}^4\boldsymbol{x}\sqrt{-\boldsymbol{g}}\left(-\frac{1}{12}\boldsymbol{H}_{\mu\nu\rho}\boldsymbol{H}^{\mu\nu\rho} - \left[\frac{1}{4}\boldsymbol{m}_B^2 + \xi\mathcal{R}\right]\boldsymbol{B}_{\mu\nu}\boldsymbol{B}^{\mu\nu}\right), \qquad \boldsymbol{H}_{\mu\nu\rho} = \partial_\mu \boldsymbol{B}_{\nu\rho} + \partial_\nu \boldsymbol{B}_{\rho\mu} + \partial_\rho \boldsymbol{B}_{\mu\nu}$$

$$\boldsymbol{m}_B = 2\Lambda(1 - 2\rho + 8\sigma), \quad \bar{\boldsymbol{g}}_{\mu\nu} = \boldsymbol{g}_{\mu\nu} + \boldsymbol{B}_{\mu\nu} + \rho \boldsymbol{B}_\mu{}^\alpha \boldsymbol{B}_{\alpha\nu} + \sigma \boldsymbol{B}^2 \boldsymbol{g}_{\mu\nu}$$

- If other terms present $\left(\mathcal{R}_{\mu\nu\rho\sigma}B^{\mu\nu}B^{\rho\sigma},\ \ \mathcal{R}_{\mu\nu}B^{\mu}{}_{\alpha}B^{\alpha\nu}\right)$ ghosts and/or instabilities may develop in FLRW and/or Schwarzschild space-times

- The above NGT action cannot be obtained from a geometric theory! However when one generalises Einstein theory to complex spaces which possess a new symmetry (holomorphy), this program may be attainable [in progress with Christiaan Mantz]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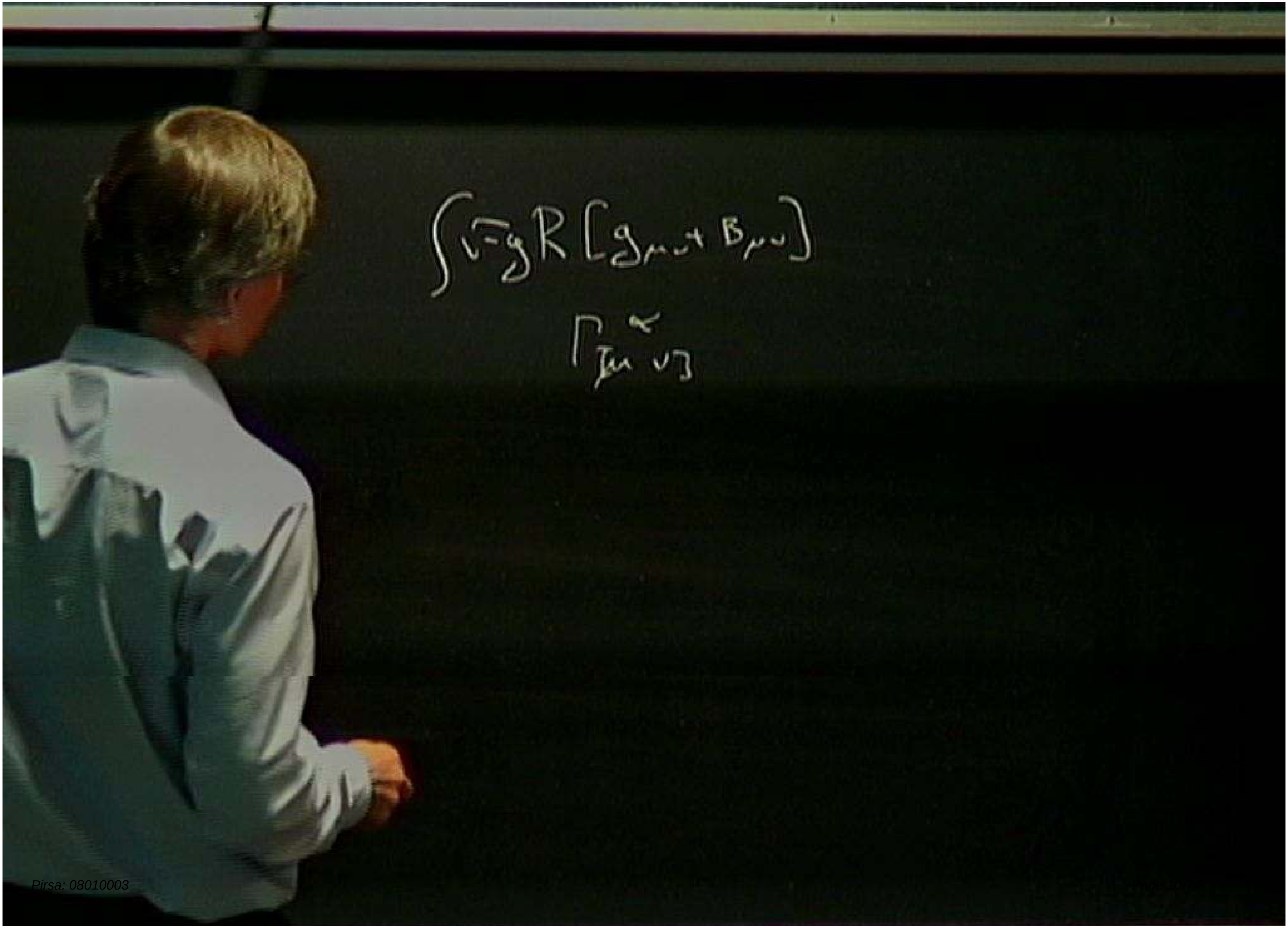
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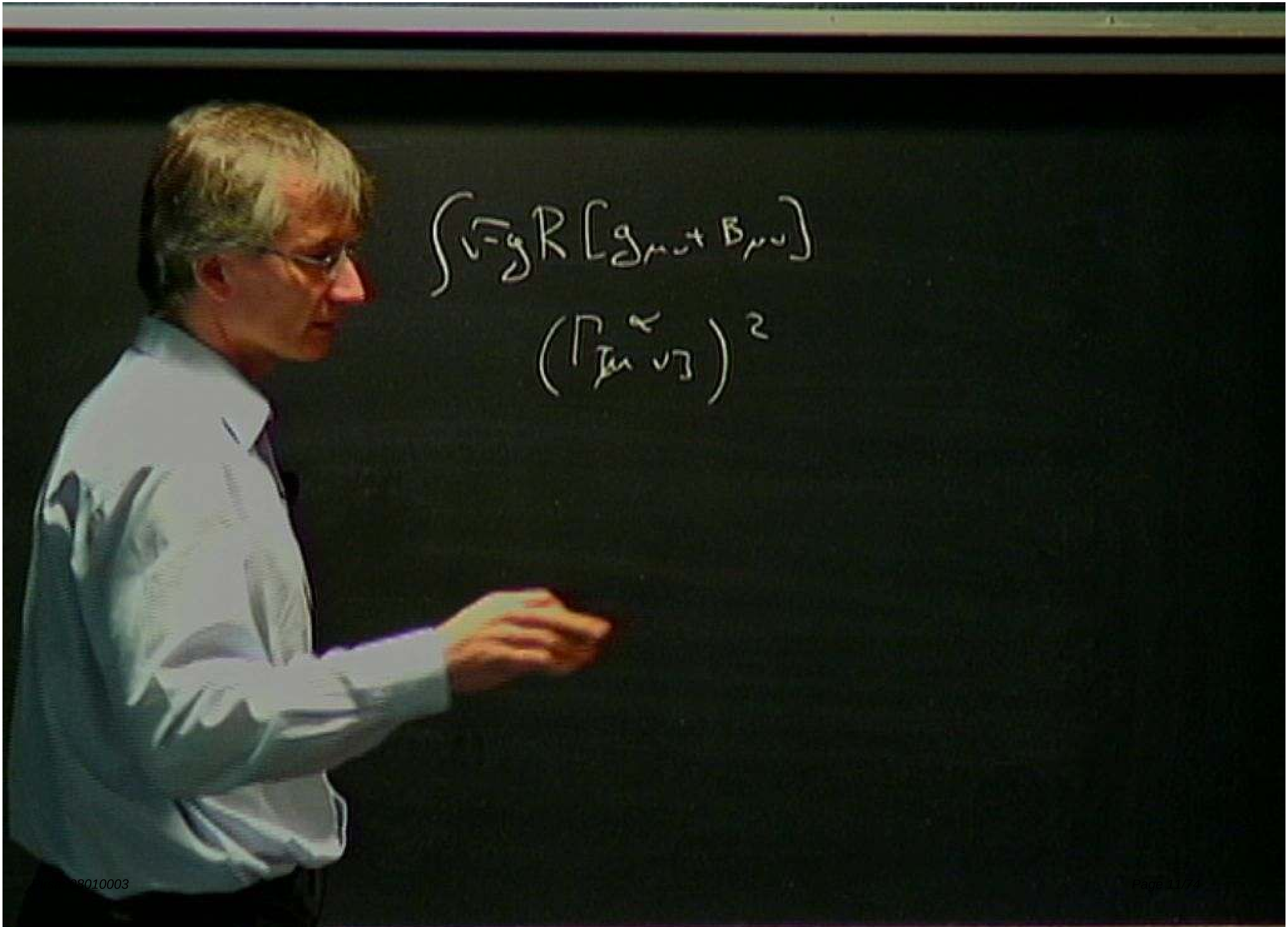
Γ^α_{[μ ν]}


$\int \sqrt{-g} R[g_{\mu\nu} + B_{\mu\nu}]$
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# KALB-RAMOND AXION

- In the massless gauge invariant limit, one can define a pseudoscalar field (Kalb-Ramond axion) $\sigma$ as

$$S_{NGT} = \int d^4x \sqrt{-g}\left(-\frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho}\right) \quad \Rightarrow \quad H^{\mu\nu\alpha} = \varepsilon^{\mu\nu\alpha\beta} \nabla_\beta \sigma$$

- The action reduces to that of a massless minimally coupled scalar

$$S[\sigma] = \int d^4x \frac{1}{2} g^{\mu\nu} (\partial_\mu \sigma)(\partial_\nu \sigma)$$

NB: the equivalence holds only on-shell, i.e. when the eqns of motion hold

NB2: **when the B field is massive then the dual of B is a massive vector field; if B couples to curvature tensor or to sources, no local duality transformation exists**

NB3: COMMON MISCONCEPTION: **since the B field couples conformally during (de Sitter) inflation, it lives in conformal vacuum during inflation, and no (observable) scale invariant spectrum is generated, contrary to what is claimed in literature based on the Kalb-Ramond axion studies**

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# CONFORMAL SPACETIMES

- The (symmetric part of) the metric tensor in conformal space time is

$$g_{\mu\nu} = a^2 \eta_{\mu\nu}, \quad \eta_{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$$

$\Longrightarrow$ a = scale factor

- The NGT action is then

$$S_{NGT} \to \int d^4x \left( \frac{1}{12a^2} \eta^{\mu\alpha}\eta^{\nu\beta}\eta^{\rho\gamma} H_{\mu\nu\rho} H_{\alpha\beta\gamma} - \frac{1}{4} m_s^2 \eta^{\mu\alpha}\eta^{\nu\beta} B_{\mu\nu} B_{\alpha\beta} \right)$$

$\Rightarrow$ Note that in the limit when $a \to \infty$ (late time inflation), the **kinetic term** drops out, and the field **fluctuations** can grow without a limit

- This is to be contrasted with the free scalar field action (kinetic term), for which fluctuations get frozen in:

$$S_{scalar} \to \int d^4x \frac{1}{2} a^2 \eta^{\mu\alpha} (\partial_\mu \phi)(\partial_\alpha \phi)$$

# PHYSICAL MODES

Consider the electric-magnetic decomposition of the Kalb-Ramond B-field

$$B_{\mu\nu} = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ -E_1 & 0 & -B_3 & B_2 \\ -E_2 & B_3 & 0 & -B_1 \\ -E_3 & -B_2 & B_1 & 0 \end{pmatrix}$$

$E_i$ = spin 1, parity -

$B_i$ = spin 1, parity +

- equations of motion

$$(\partial^2 + a^2 m_B^2)\vec{E} = 0$$

$$(\partial^2 + a^2 m_B^2)\vec{B} - \frac{2a'}{a}(\partial_\eta \vec{B} + \vec{\partial} x \vec{E}) = 0$$

- Lorentz "gauge" (consistency) condition $\eta^{\mu\nu}\partial_\mu B_{\nu\rho} = 0$ implies

$$\partial_\eta \vec{E} - \vec{\partial} x \vec{B} = 0, \qquad \vec{\partial} \cdot \vec{E} = 0$$

NB1: $\vec{\partial} \cdot \vec{B} = 0$ is missing $\rightarrow \vec{B}^{T\text{L}}$ may be dynamical

NB2: $\vec{B}^T$ equation is not independent (given by the transverse electric field)

NB3: $\vec{E}^L = 0$

NB4: From $\partial_\eta \vec{E}^T - \vec{\partial} x \vec{B}^T = 0 \rightarrow \vec{B}^T$ is a function of $\vec{E}^T$

- Physical DOFs (massive case): pseudovector $\vec{B}$ (spin = 1, parity = +)



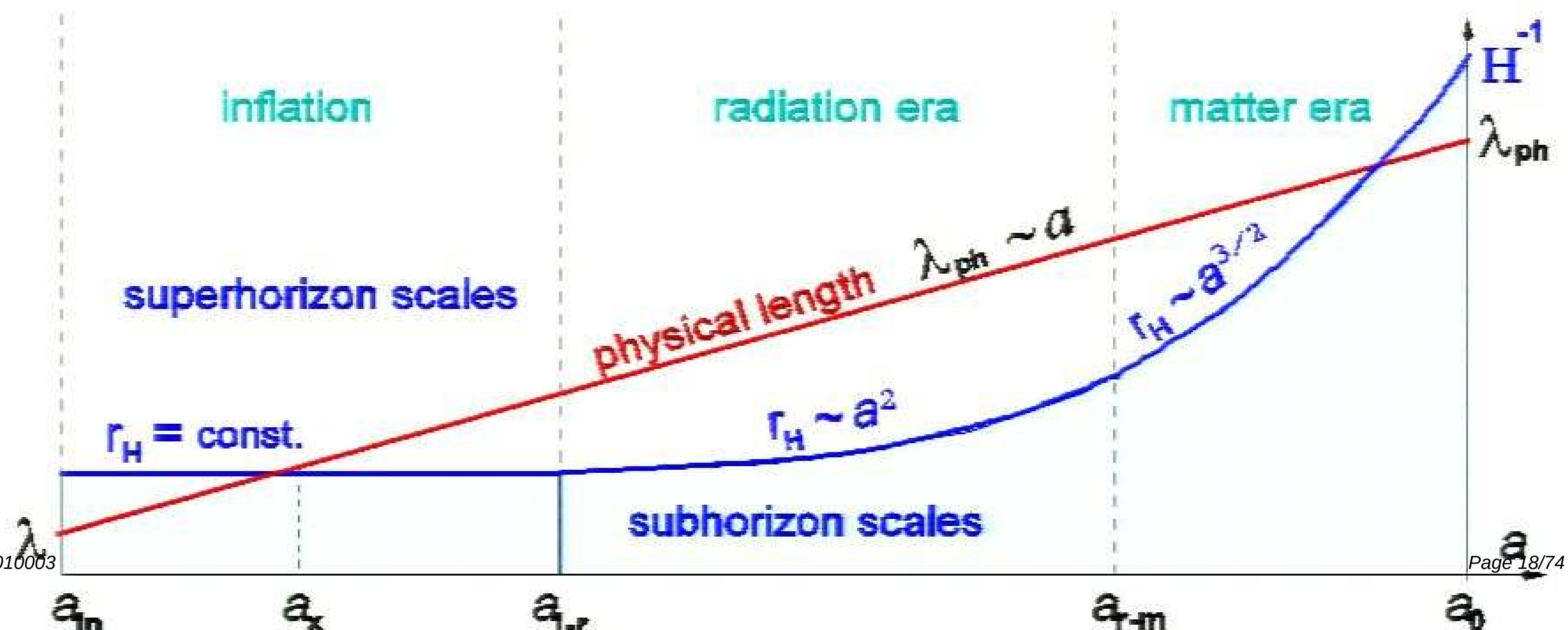
- Physical DOF (massless case): longitudinal magnetic field $\vec{B}^L$

# EVOLUTION OF FLUCTUATIONS IN COSMOLOGICAL SPACETIMES



- antisymmetric metric (tensor) particles are produced by enhancing vacuum fluctuations produced in inflation during radiation and matter era

→ Evolution of scales in the primordial Universe



# CANONICAL QUANTISATION

Impose canonical commutation relation on B-field and its canonical momentum

$$\hat{\vec{B}}^{L}(\vec{x},\eta) = a(\eta)\int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}.\vec{x}} \vec{\varepsilon}^{L}(\vec{k}) \left[ B_{\vec{k}}(\eta)\hat{b}_{\vec{k}} + B^{*}_{-\vec{k}}(\eta)\hat{b}^{\dagger}_{-\vec{k}} \right]$$

$$\left[ \hat{b}_{\vec{k}}, \hat{b}^{\dagger}_{\vec{k}'} \right] = (2\pi)^3 \delta^3(\vec{k} - \vec{k}')$$

$$\vec{k} \times \vec{\varepsilon}^{L}(\vec{k}) = 0$$

- Momentum space equation of motion for the modes

$$\left( \partial_\eta^2 + k^2 + \frac{a''}{a} - 2\left(\frac{a'}{a}\right)^2 + a^2 m_B^2 \right) B_k^L(\eta) = 0$$

- **NB**: Contrary to a scalar field (which decays with the scale factor), the Kalb-Ramond B-field grows with the scale factor **a**

# VACUUM FLUCTUATIONS IN DE SITTER INFLATION

In De Sitter inflation the scale factor is, $a = -\frac{1}{H_I \eta}$ $(\eta < -1/H_I)$ such that

$$\frac{a''}{a} - 2\left(\frac{a'}{a}\right)^2 = 0$$

- When $m_B \sim 0$ the mode equation of motion reduces to conformal vacuum

$$\left(\partial_\eta^2 + k^2 + \frac{m_B^2}{H^2 \eta^2}\right) B_k^L(\eta) = 0$$

- Conformal rescaling of the longitudinal B-mode

$$\vec{B}^L(\vec{x}, \eta) \to \vec{B}_c^L(\vec{x}, \eta) = \frac{\vec{B}^L(\vec{x}, \eta)}{a}$$

- Mode functions approach those of conformal vacuum ($m_B \ll H$)

$$B_k^L(\eta) = -\sqrt{-\frac{\pi \eta}{4}} H^{(2)}_{\frac{1}{2}\sqrt{1-4m_B^2/H^2}}(k\eta) = \frac{1}{\sqrt{2k}} e^{-ik\eta} + O\left(\frac{m_B^2}{H^2}\right)$$

NB: In conformal vacuum there is very little particle production during inflatio

# SPECTRUM OF ENERGY DENSITY

°12

- The energy density is

$$T_0^{0NGT} = \frac{1}{2a^6}\left[\left(\partial_\eta \vec{B} + \vec{\partial} \times \vec{E}\right)^2 + (\vec{\partial}\cdot\vec{B})^2 + a^2 m_B^2 \left(\vec{E}^2 + \vec{B}^2\right)\right] \equiv \rho_{NGT}$$

- When $m_B^2 \to 0$ the only contribution to the spectrum comes from long. B-fiel

$$\left\langle 0 \,|\, T_0^{0NGT} \,|\, 0 \right\rangle = \int \frac{dk}{k} P_{NGT}(\vec{k}, \eta)$$

$$P_{NGT}(\vec{k},\eta) = \frac{k^3}{2\pi^2} T_0^{0NGT}(\vec{k},\eta)$$

$$P_{NGT}(\vec{k},\eta) = \frac{H_I^4}{4\pi^2 a^4}\left\{\left|\partial_\eta B_k^L(\eta) + \frac{a'}{a} B_k^L(\eta)\right|^2 + \left(k^2 + a^2 m_B^2\right) | B_k^L(\eta) |^2\right\}$$

NB1: **this spectrum is relevant for coupling to (Einstein's) gravity (small scale cosmological perts.)**

# RADIATION ERA



- In radiation era, $a = H_I \eta \quad (\eta > 1/H_I)$

- Solutions are Whittaker functions (confluent Hypergeometric functions), which in the massless limit reduce to

$$B_{\bar{k}}(\eta) = \frac{1}{\sqrt{2k}}\left[\alpha_{\bar{k}}\left(1 - \frac{i}{k\eta}\right)e^{-ik\eta} + \beta_{\bar{k}}\left(1 + \frac{i}{k\eta}\right)e^{+ik\eta}\right]$$

with the "Wronskian" condition

$$|\alpha_{\bar{k}}|^2 - |\beta_{\bar{k}}|^2 = 1$$

- The matching coefficients are approximately

$$\alpha_{\bar{k}} = -\frac{1}{2}\frac{H_I^2}{k^2}\left(1 - 2i\frac{k}{H_I} - 2\frac{k^2}{H_I^2}\right)e^{2ik/H_I}, \qquad \beta_{\bar{k}} = -\frac{1}{2}\frac{H_I^2}{k^2}$$




NB: In matter era no exact solutions to the mode equations are known

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°12'

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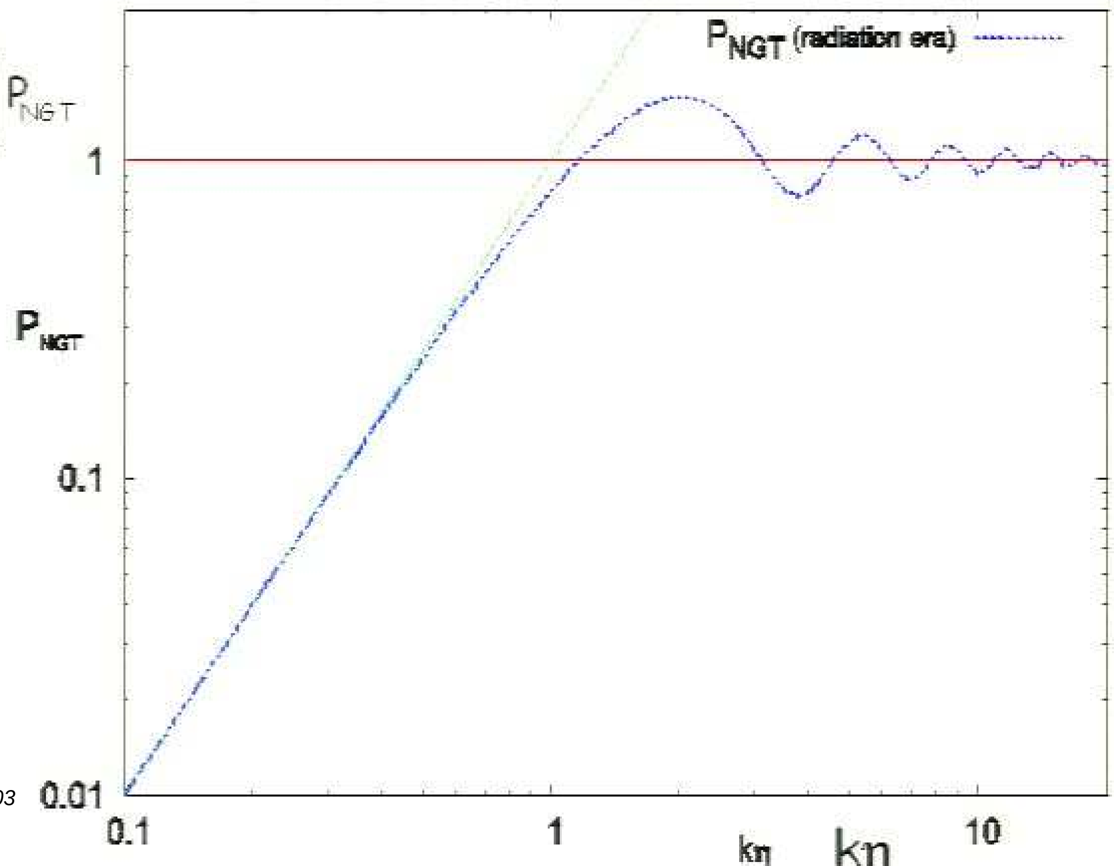
# SPECTRUM IN RADIATION ERA

°13

- **The spectrum in radiation era for a massless Kalb-Ramond field**

$$P_{NGT}(\vec{k},\eta) = \frac{H_I^4}{8\pi^2 a^4}\left[\left(1+\frac{1}{2}\frac{1}{(k\eta)^2}\right) - \frac{\sin(2k\eta)}{k\eta} - \frac{1}{2}\frac{\cos(2k\eta)}{(k\eta)^2}\right]$$

$\left(\frac{8\pi^2 a^4}{H_I^4}\right) P_{NGT}$



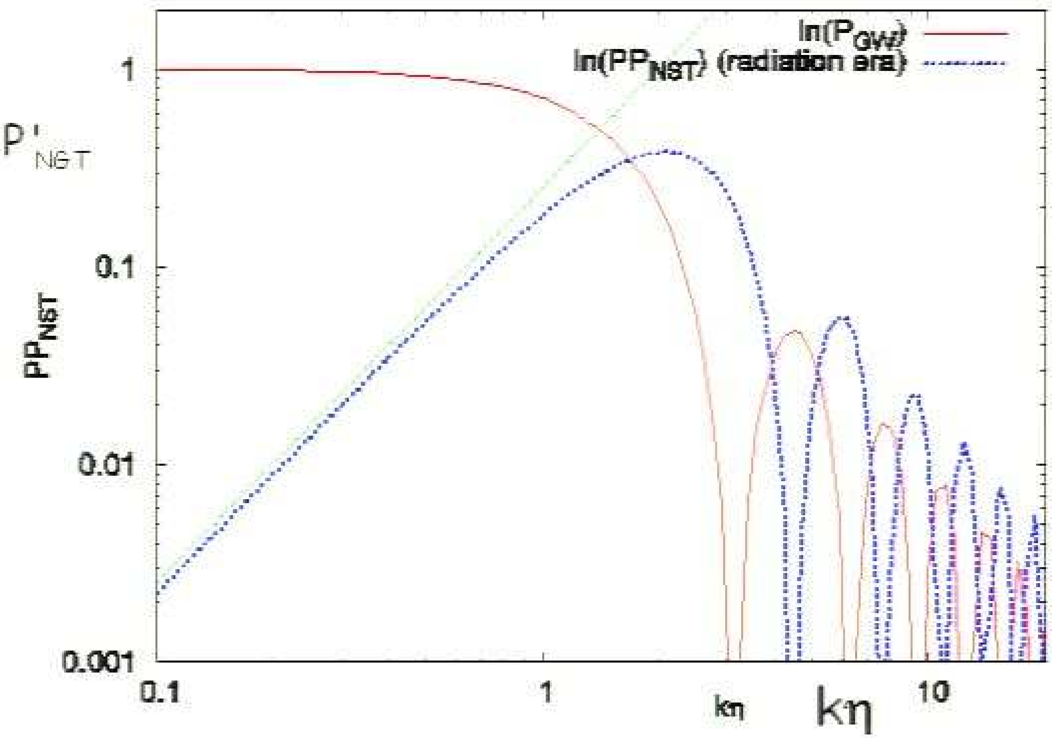

- on superhorizon scales ($k\eta<1$) the energy density spectrum scales as $P \sim k^2$.
- on subhorizon scales ($k\eta>1$), $P \sim$ const.+ small oscillations

# COMPARISON WITH GRAVITATIONAL WAVES

- **massless NGT field spectrum:** $\left\langle 0 \,|\, [\hat{B}^{L}(\vec{x},\eta)]^2 \,|\, 0 \right\rangle = \int \frac{dk}{k} P'_{NGT}(\vec{k},\eta)$

$$P'_{NGT}(\vec{k},\eta) = \frac{a^2 H_I^4}{8\pi^2 k^2}\left[\left(1+\frac{1}{(k\eta)^2}\right) - \frac{2}{k\eta}\sin(2k\eta) + \left(1-\frac{1}{(k\eta)^2}\right)\cos(2k\eta)\right]$$

$\left(\frac{8\pi^2 a^4}{H_I^4}\right) P'_{NGT}$

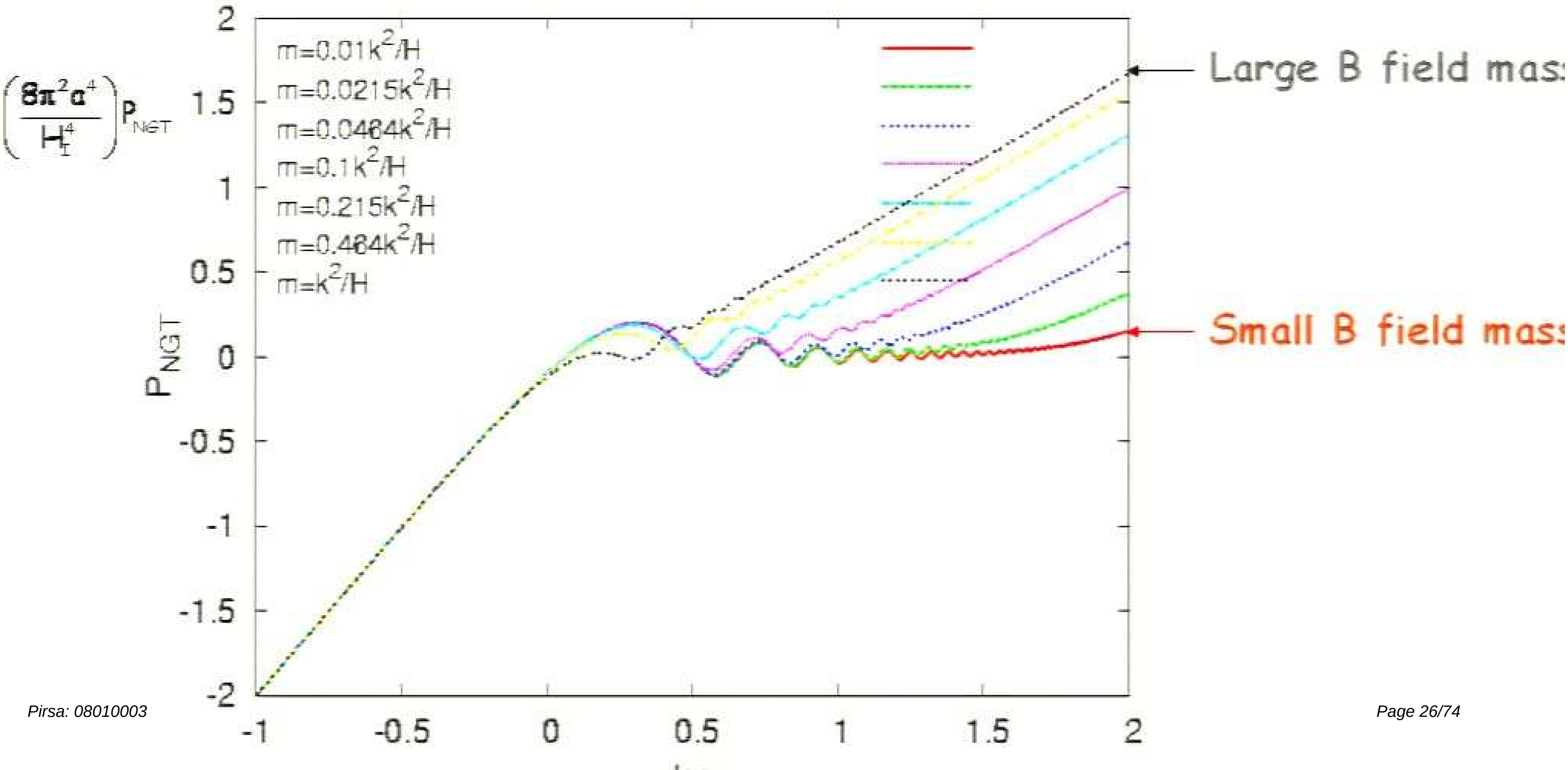



$$P'_{GW}(\vec{k},\eta) \propto \frac{\sin^2(k\eta)}{k^2}$$

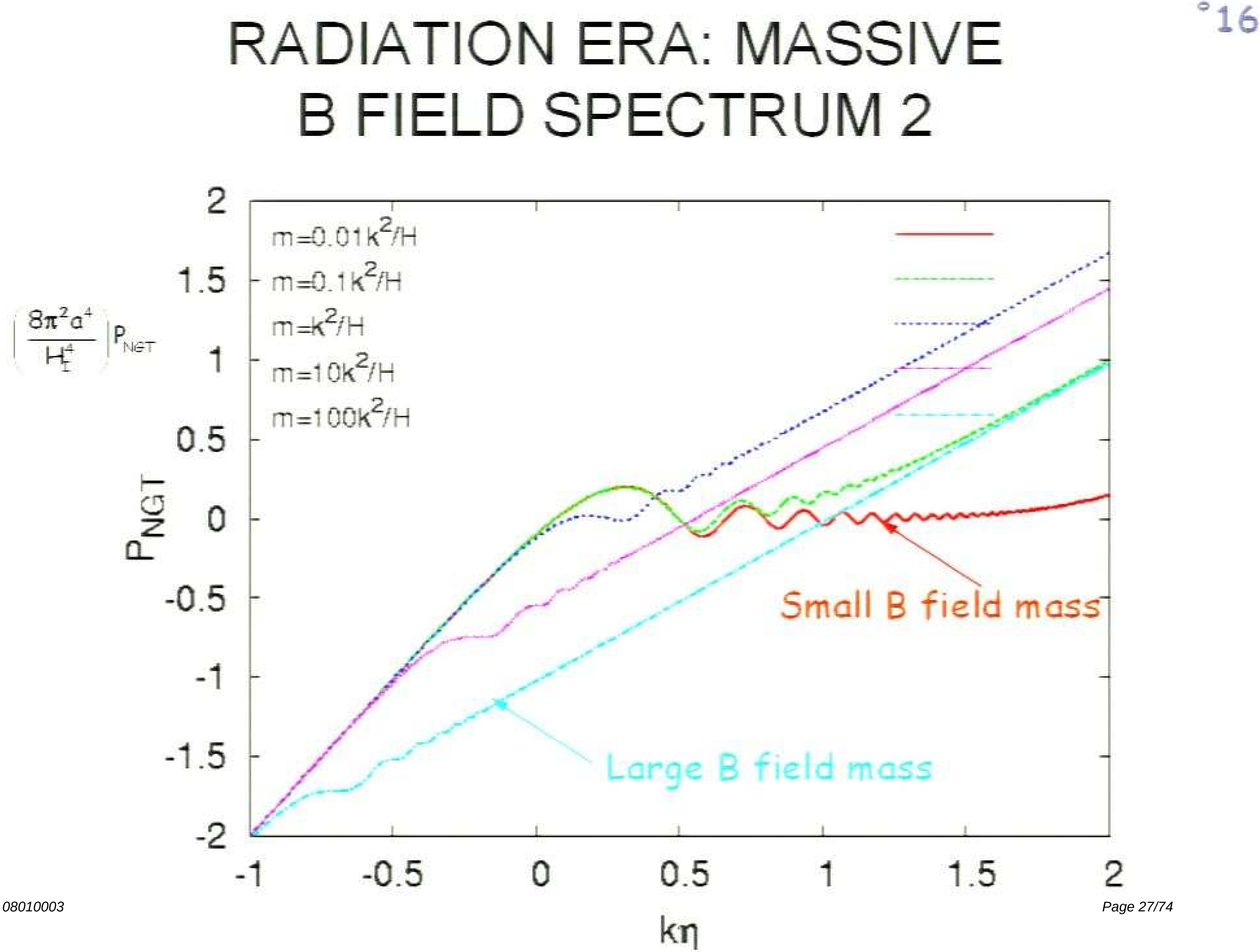
- NB: NGT field is important at horizon crossing, gravitational waves dominate on superhorizon scales

# RADIATION ERA: MASSIVE B FIELD SPECTRUM

- $B_k^L \propto$ Hankel function (de Sitter inflation)
- $B_k^L \propto$ Whittaker function (radiation era)

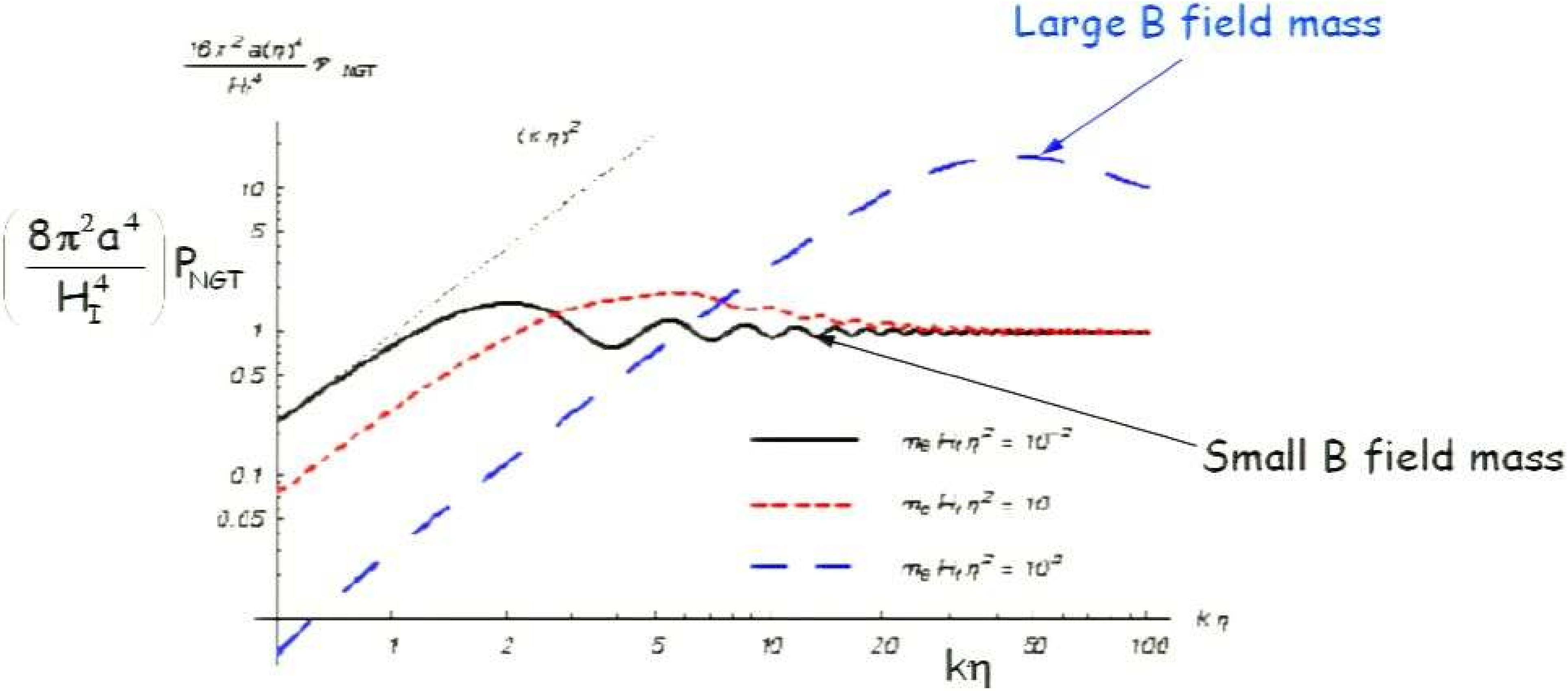


# RADIATION ERA: MASSIVE B FIELD SPECTRUM 2



# SNAPSHOT OF SPECTRUM IN RADIATION ERA

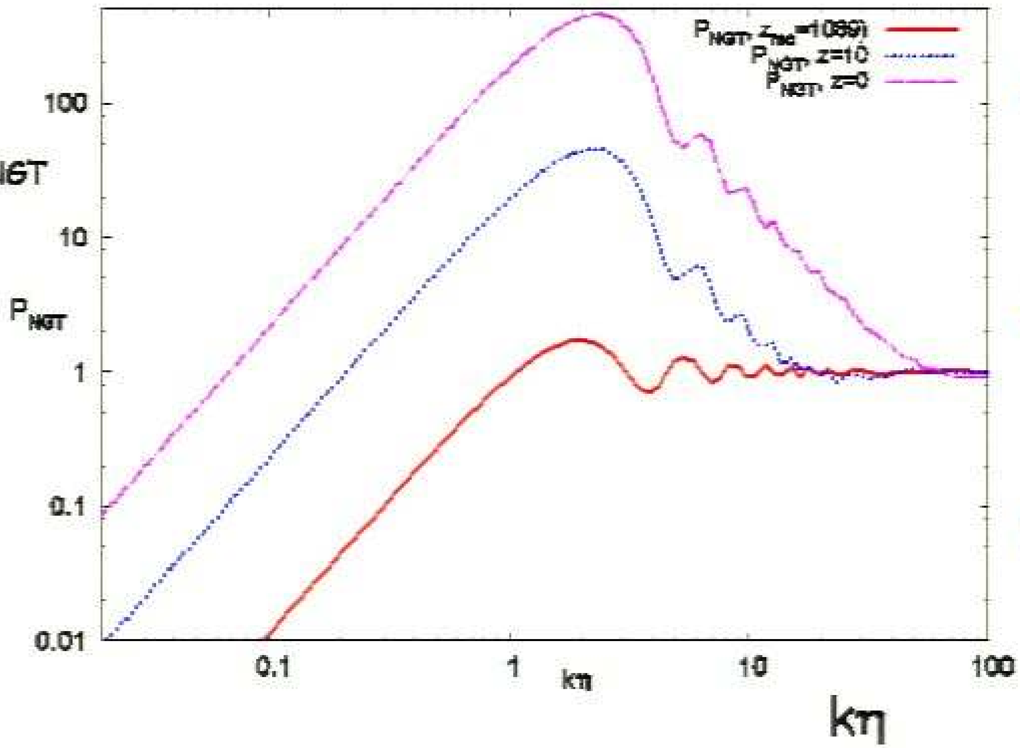




# SPECTRUM IN MATTER ERA

The spectrum in matter era is shown in figure (log-log plot)

$$\left(\frac{8\pi^2 a^4}{H_I^4}\right) P_{NGT}$$




- on superhorizon scales ($k\eta<1$) the energy density spectrum scales as $P\sim k^2$.
- on subhorizon scales ($1<k\eta<a/a_{eq}$), $P\sim 1/k^2$
- on subhorizon scales ($k\eta>a/a_{eq}$), $P\sim$const.

NB1: The bump in the power spectrum in matter era is caused by the modes which are superhorizon at equality, and which after equality begin scaling as nonrelativistic matter

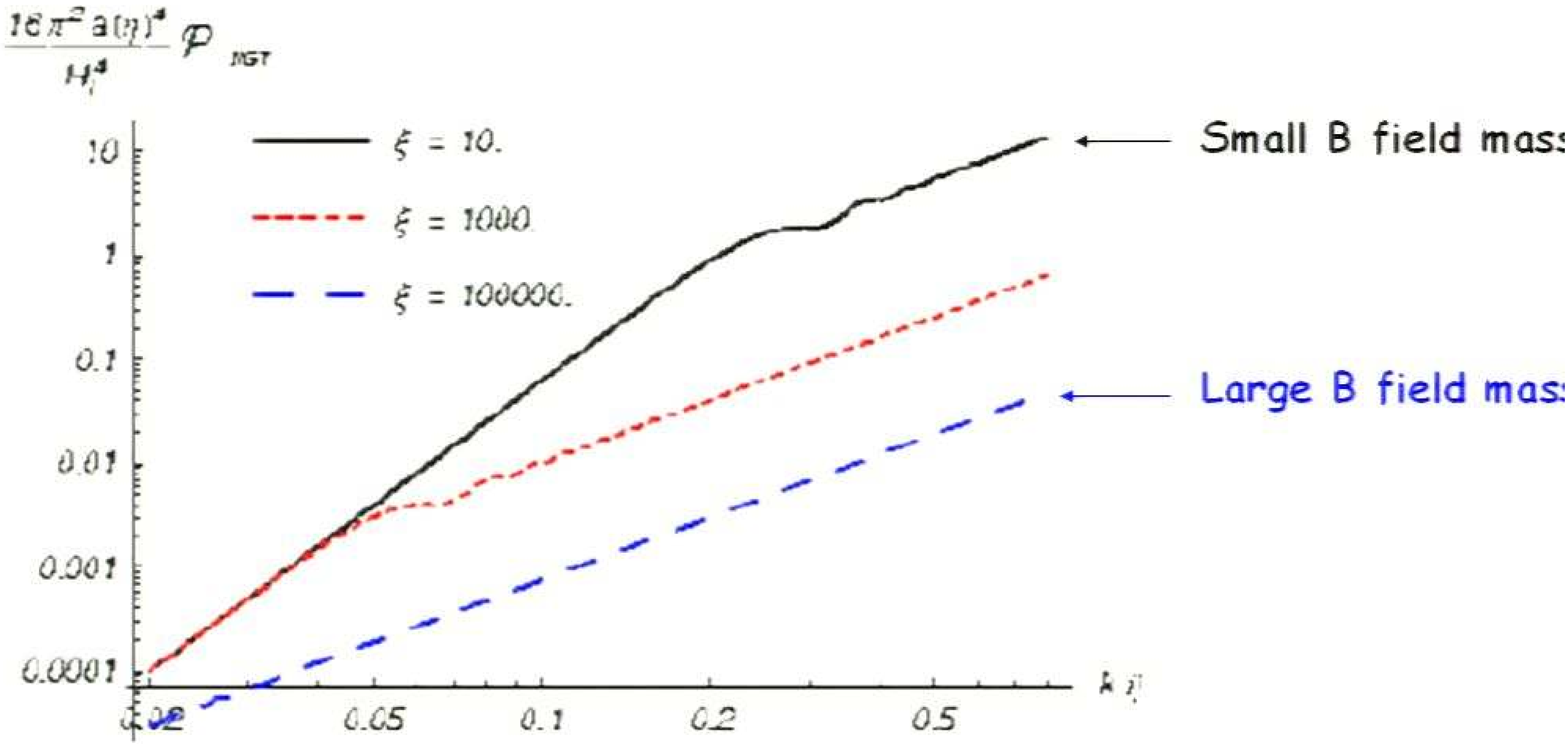
$$P_{NGT} \propto 1/a^3$$

NB2: The log divergent part of the spectrum continues scaling in matter era as, $P_{NGT} \propto \frac{1}{a}$ such that the energy density becomes eventually dominated by the "bump"



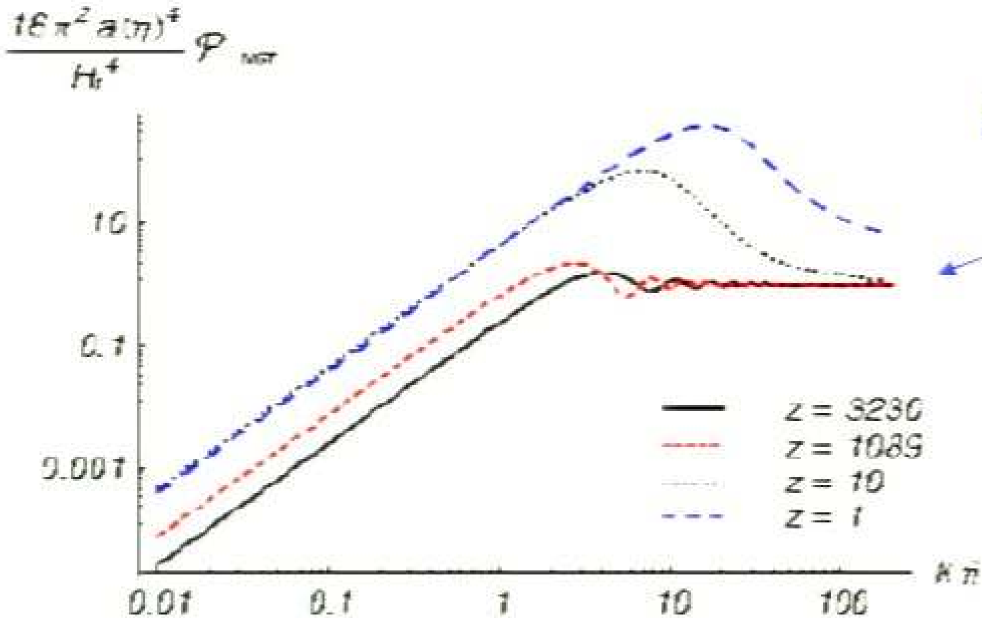

$$\rho_{NGT} \approx \frac{H_I^4}{8\pi^2 a^3 a}$$

# MATTER ERA: MASSIVE B FIELD SPECTRUM

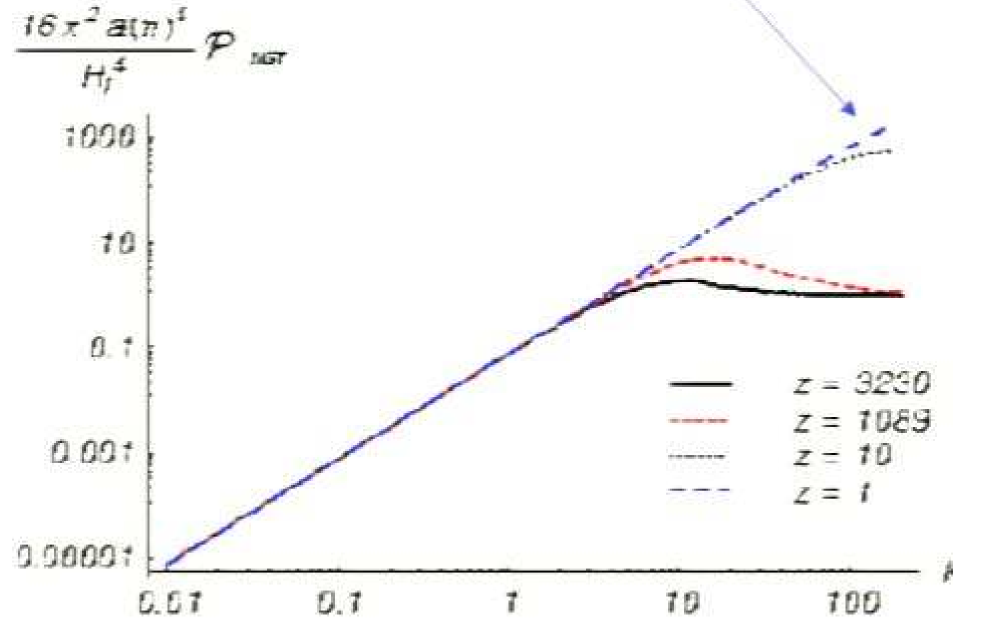


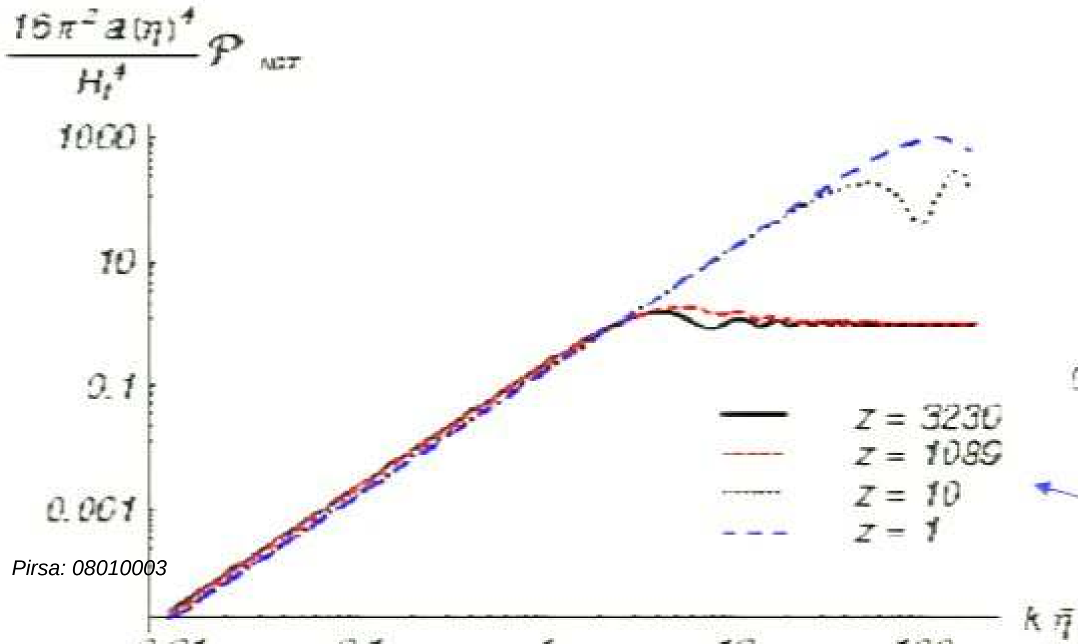
# SNAPSHOTS OF SPECTRA IN MATTER ERA

$m_B H_I \eta_e^2 = 10^{-2} \quad (m_B = 10^{-2} H_e)$




$m_B H_I \eta_e^2 = 10$

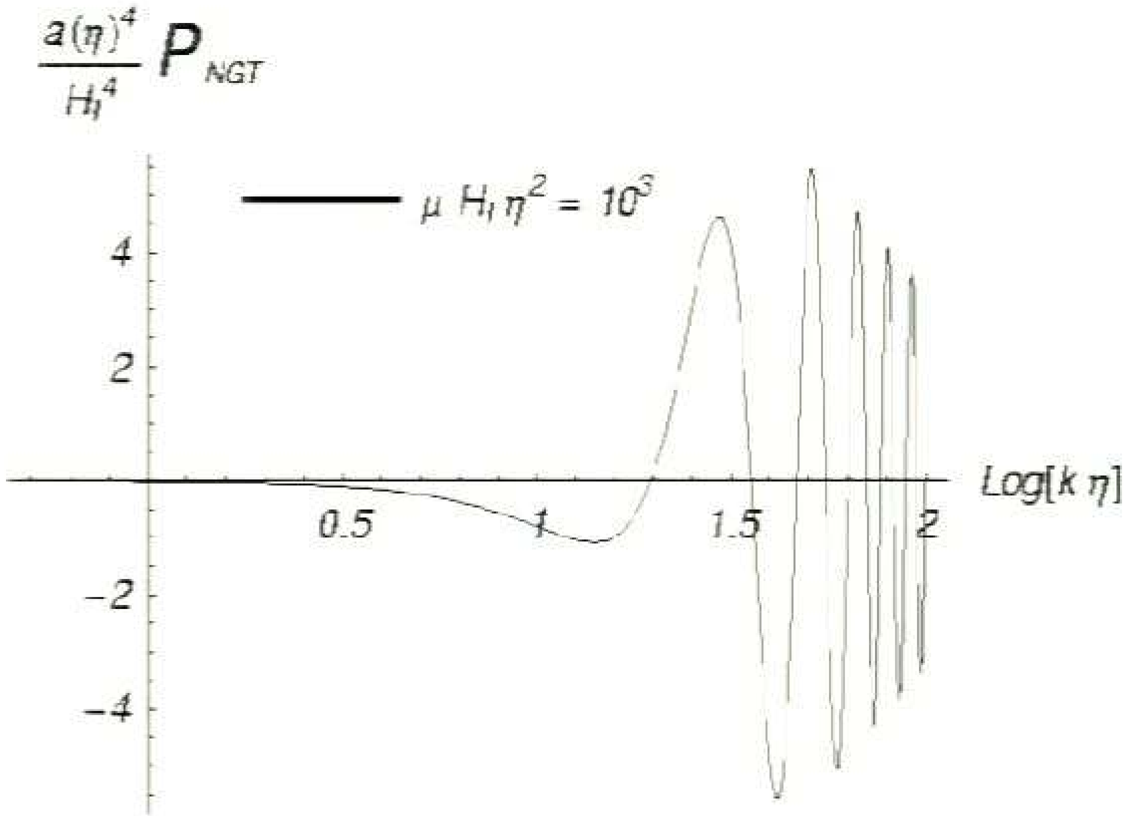



$m_B H_I \eta_e^2 = 1$

# MATTER ERA: PRESSURE OSCILLATIONS






$$\nabla^2\Phi - 3H\Phi' - 3H^2\Phi = 4\pi G a^2 \delta\rho$$

$$\Phi'' + 3H\Phi' + (2H' + H^2)\Phi = 4\pi G a^2 \delta P$$

These momentum space **pressure oscillations** may leave imprint in the relativistic Newton-like gravitational potential $\Phi$, and thus affect CMB & structure formation (but only on very small scales)

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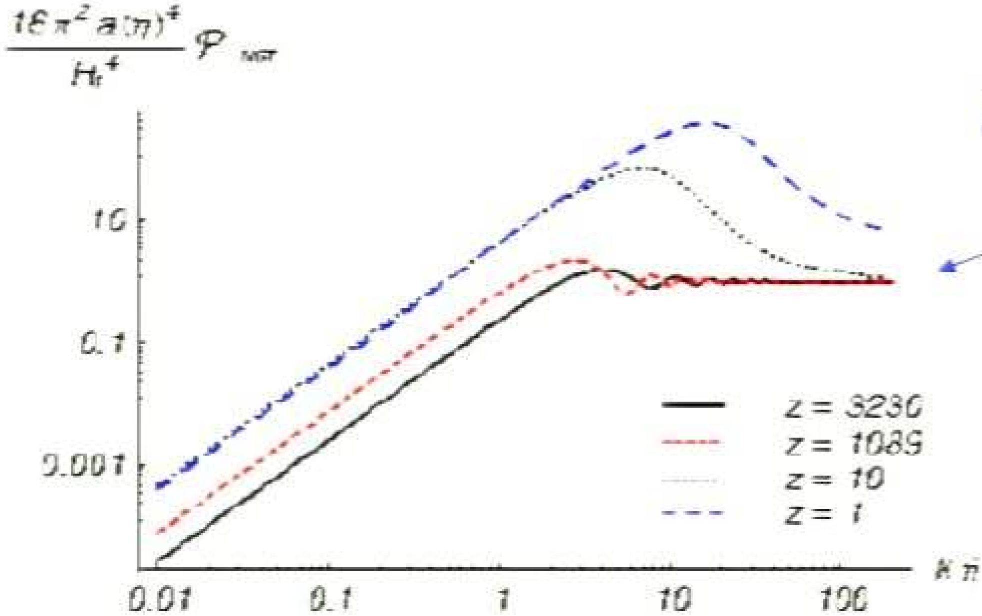


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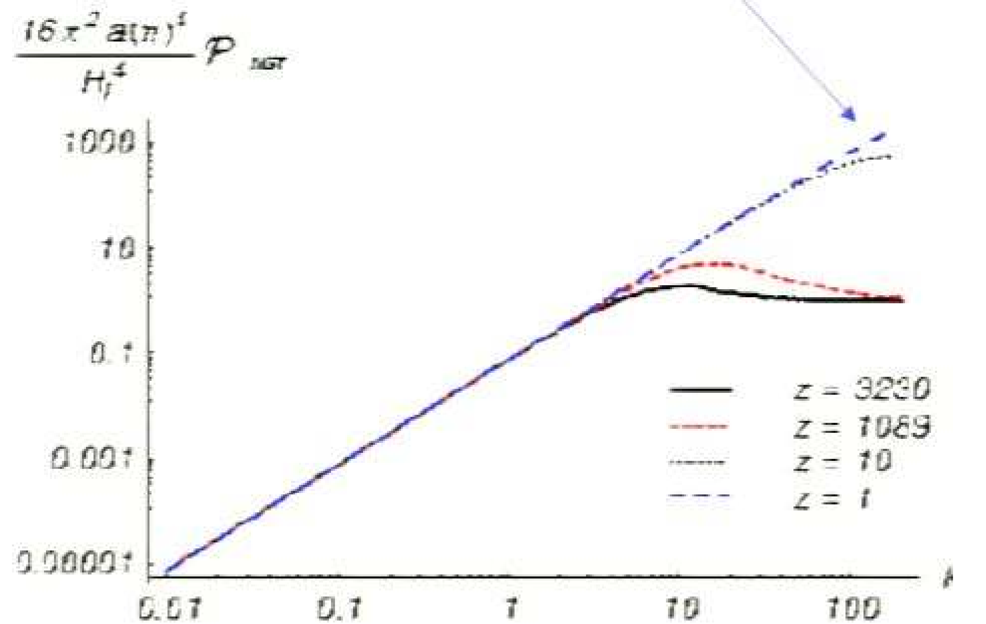


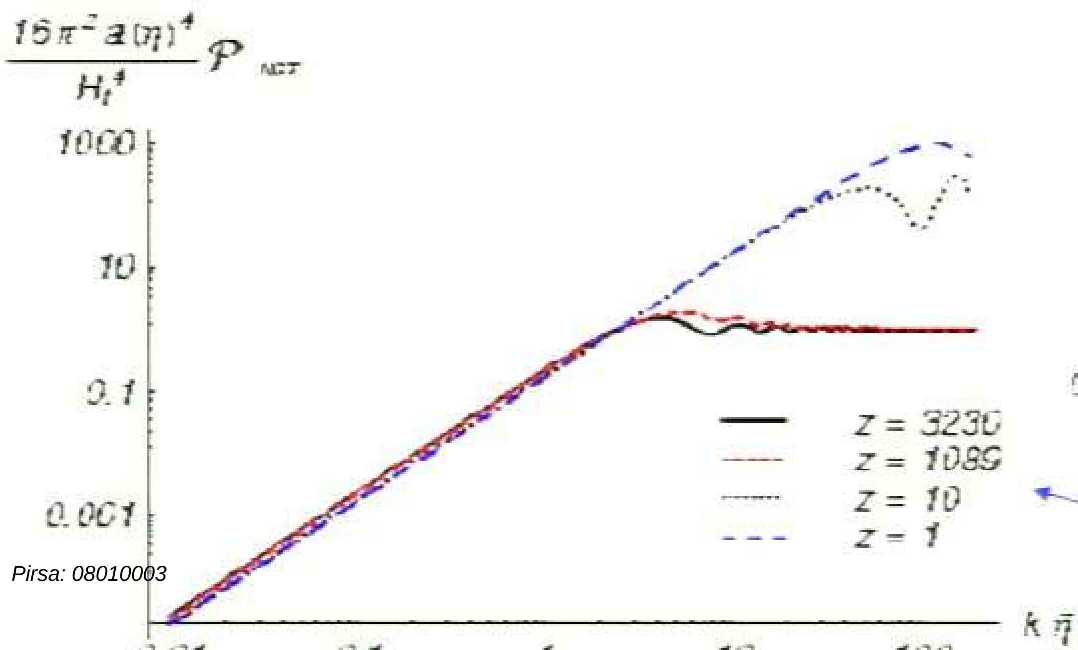


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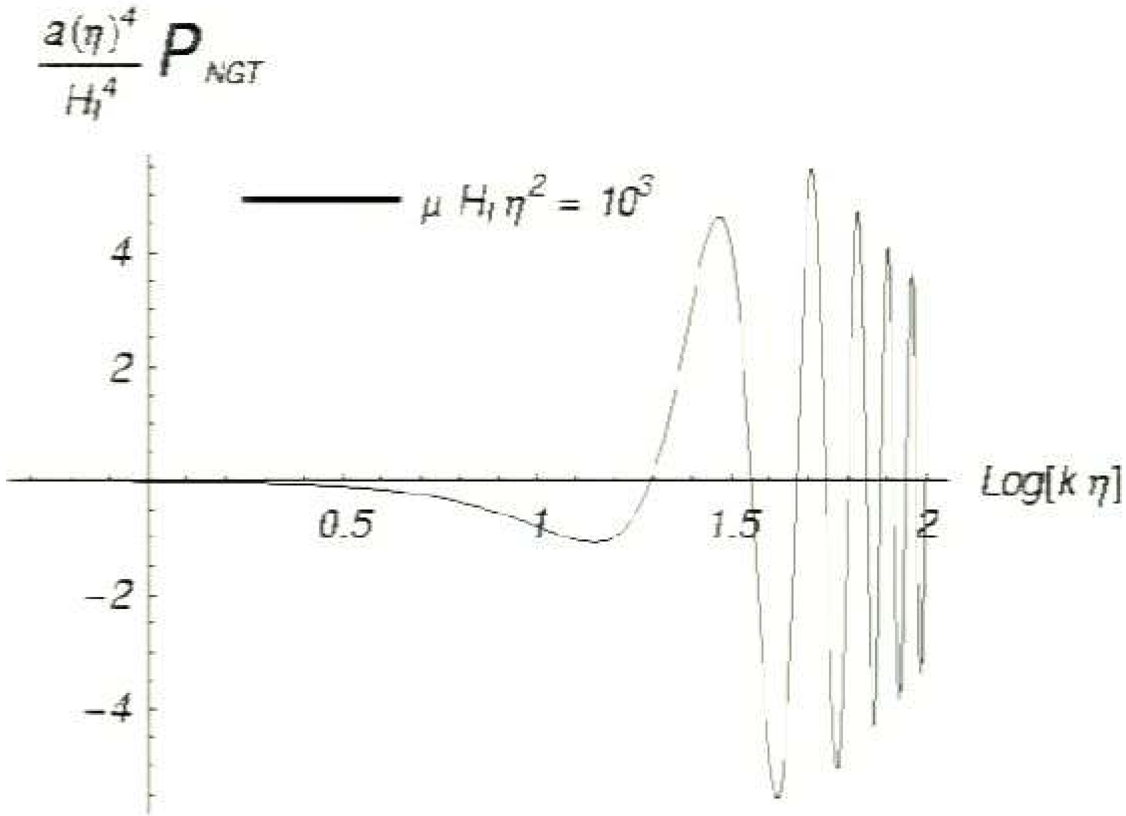


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# SUMMARY & DISCUSSION: DM

- Is the antisymmetric metric field a good DARK MATTER CANDIDATE?
  - → ANSWER: YES, provided it has the right mass,

$$m_B \approx 0.03\left(10^{13}\,GeV / H\right)^4 eV$$

- Massive antisymmetric metric field is NOT equivalent to a massive axion but instead to a massive vector field (only when interactions are excluded)

- Below the mass scale $m_B$, the strength of the gravitational interaction may change $m_B^{-1} \approx 0.1\mu\mathrm{m}\left(H / 10^{13}\,GeV\right)^4$

- **Power is concentrated at a (comoving) scale ~ 1AU**
  (observable consequences for structure formation?)

$$k \approx \sqrt{m_B H_0} / (1+z_{eq})^{1/4}$$

- Goal 1: to construct covariant generalisation of Einstein's theory that includes a dynamical torsion (mediated through antisymmetric tensor field) [work in progress with Christiaan Mantz, master's student]

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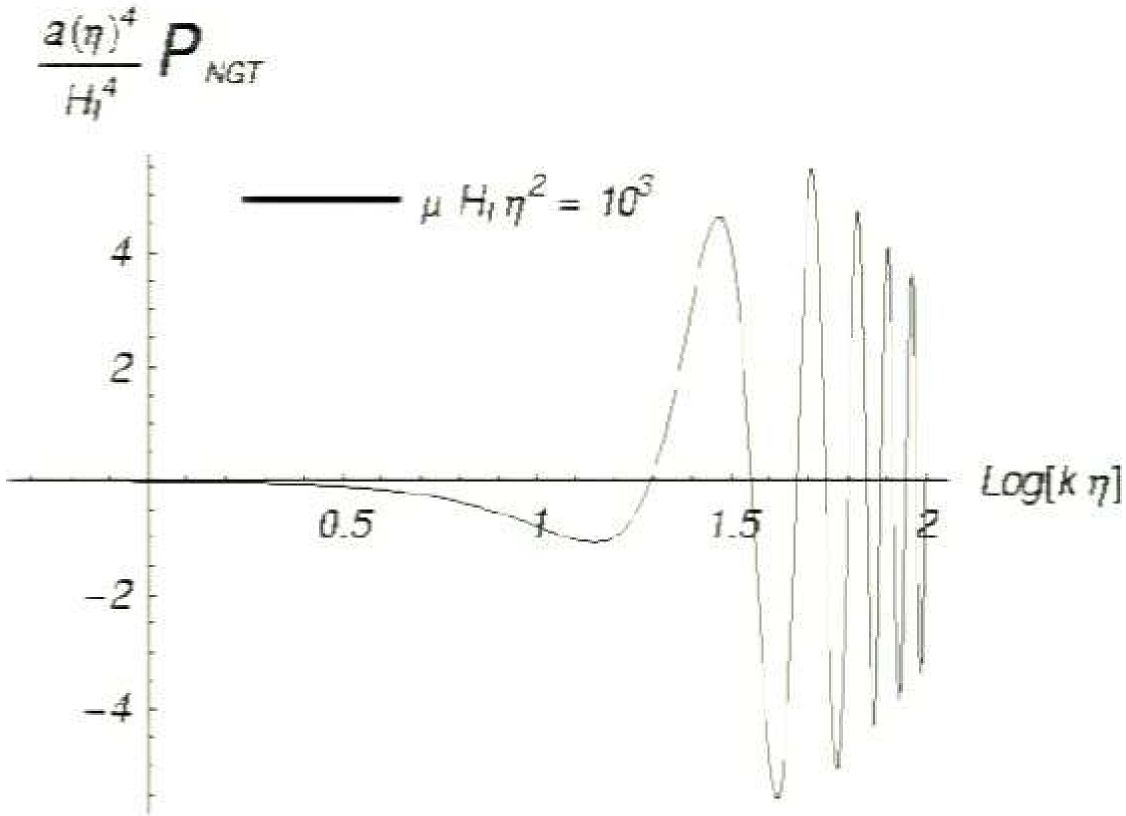


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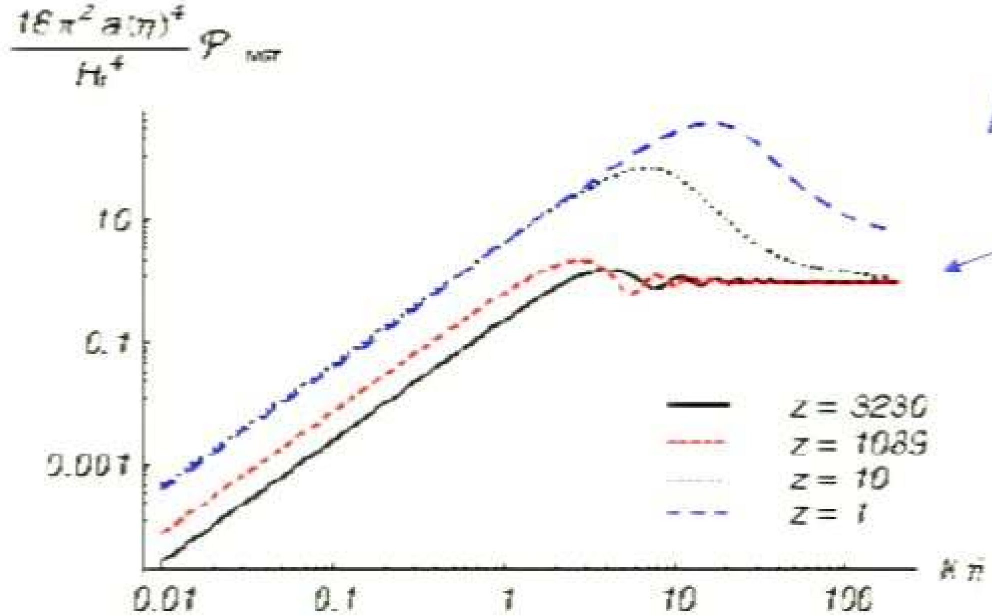


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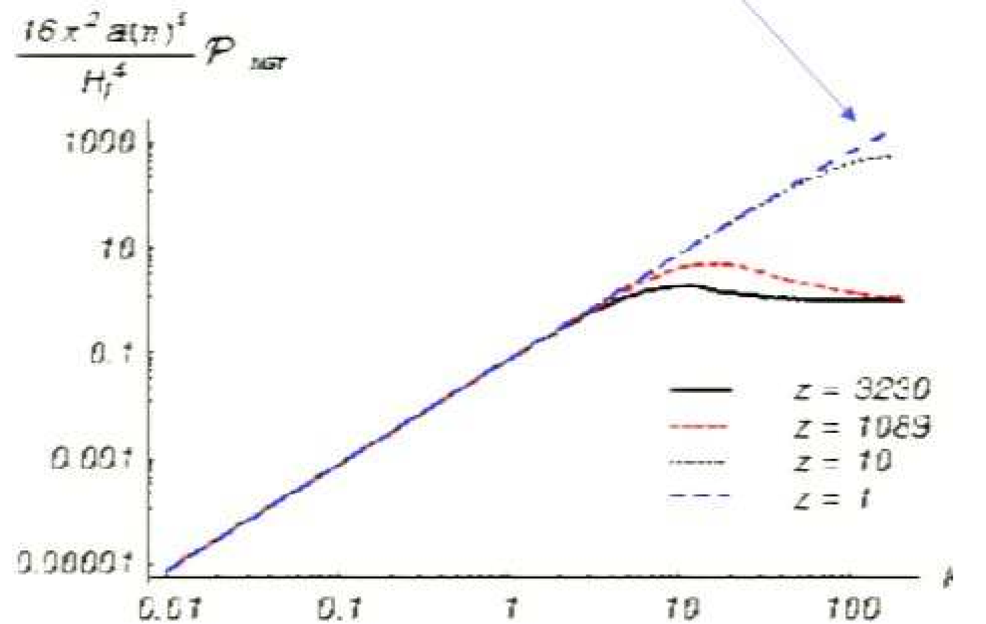


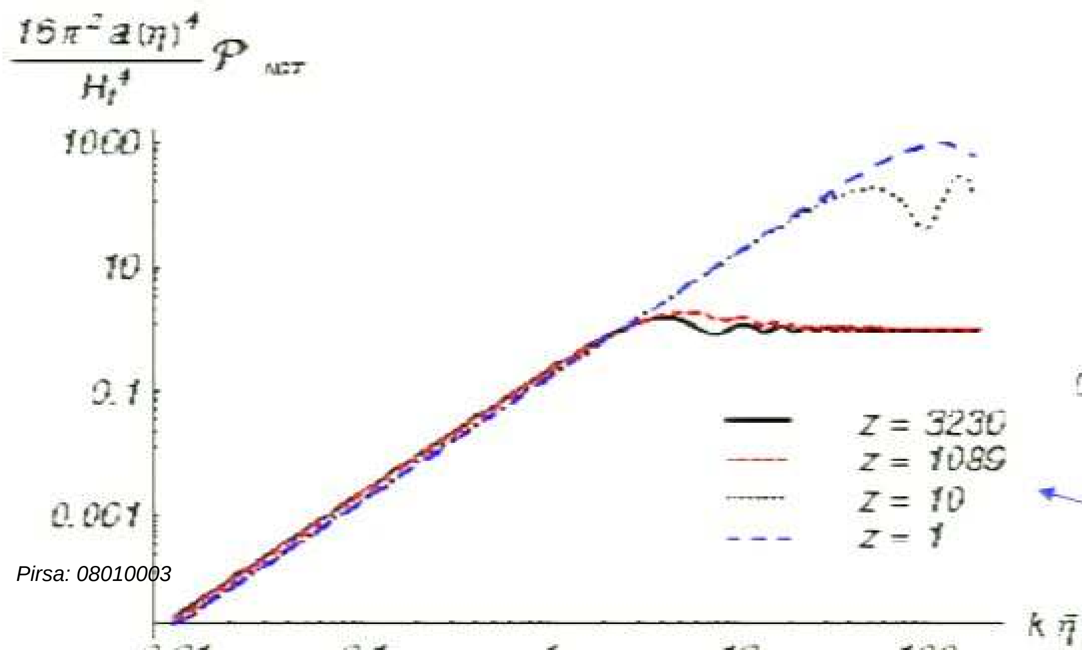


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# GRAVITY ON COMPLEX SPACETIMES

Work with Christiaan Mantz (unpublished

- Fundamental object: holomorphic tetrad

$$e^a_\mu = e^a_\mu(z^\nu), \quad z^\nu = x^\nu + i y^\nu$$

$$y^\nu = \frac{G}{c^3} p^\nu$$

phase space formulation of gravity:

- $x^\nu$: four space-time
- $p^\nu$: four energy-momentum of moving frame

Line element:

$$ds^2 = dz^T \cdot C \cdot dz, \quad C = e^T \cdot e$$

- hermitean case:

$$C_{mn} = \begin{pmatrix} 0 & C_{\mu\bar{\nu}} \\ C_{\bar{\mu}\nu} & 0 \end{pmatrix}$$

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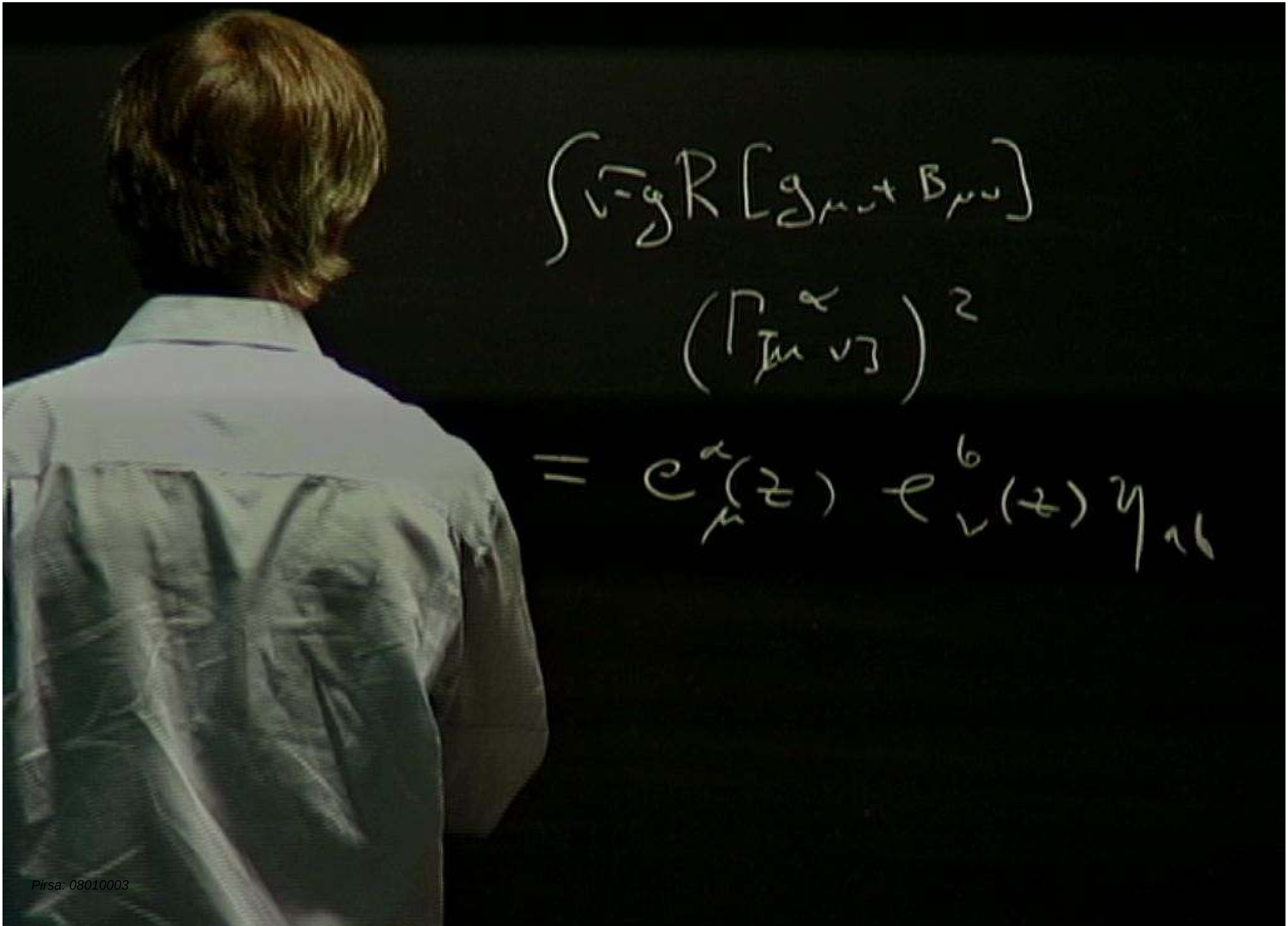
$$C_{mn} = \begin{pmatrix} C_{\mu\nu} & C_{\mu\bar{\nu}} \\ C_{\bar{\mu}\nu} & C_{\bar{\mu}\bar{\nu}} \end{pmatrix}$$

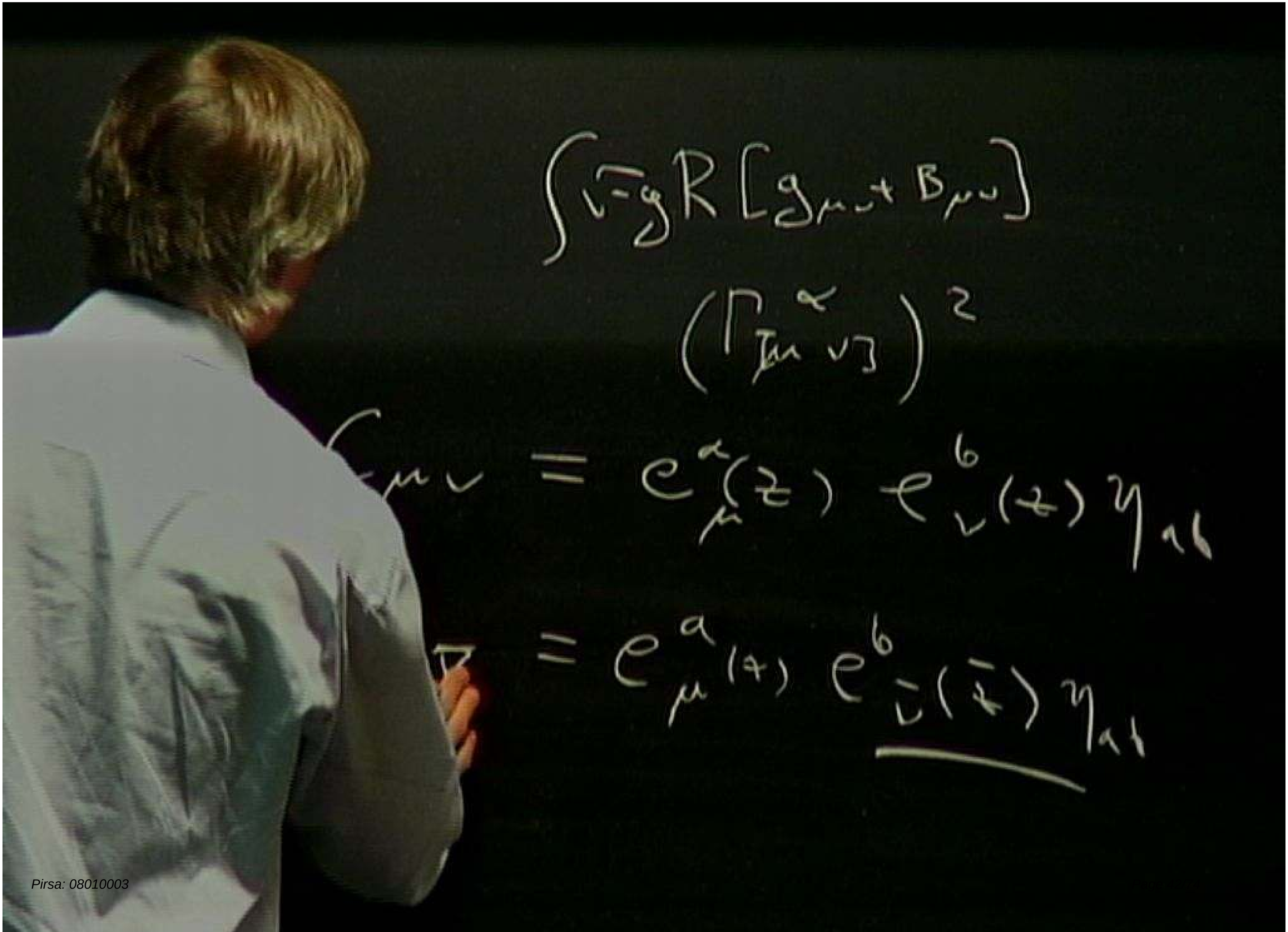
Flat space limit: symmetries

$SU(3,1)$ $\qquad SO(3,1,C)$ $\qquad SO(3,1,C)$



$$ds^2 \to dx^\mu g_{\mu\nu} dx^\nu + dy^\mu g_{\mu\nu} dy^\nu + 2 dx^\mu B_{\mu\nu} dy^\nu$$


$\int \sqrt{-g}\, R[g_{\mu\nu} + B_{\mu\nu}]$
$\left(\Gamma^{\alpha}_{[\mu\nu]}\right)^2$
$= e^{\alpha}_{\mu}(z)\, e^{b}_{\nu}(z)\, \eta_{ab}$

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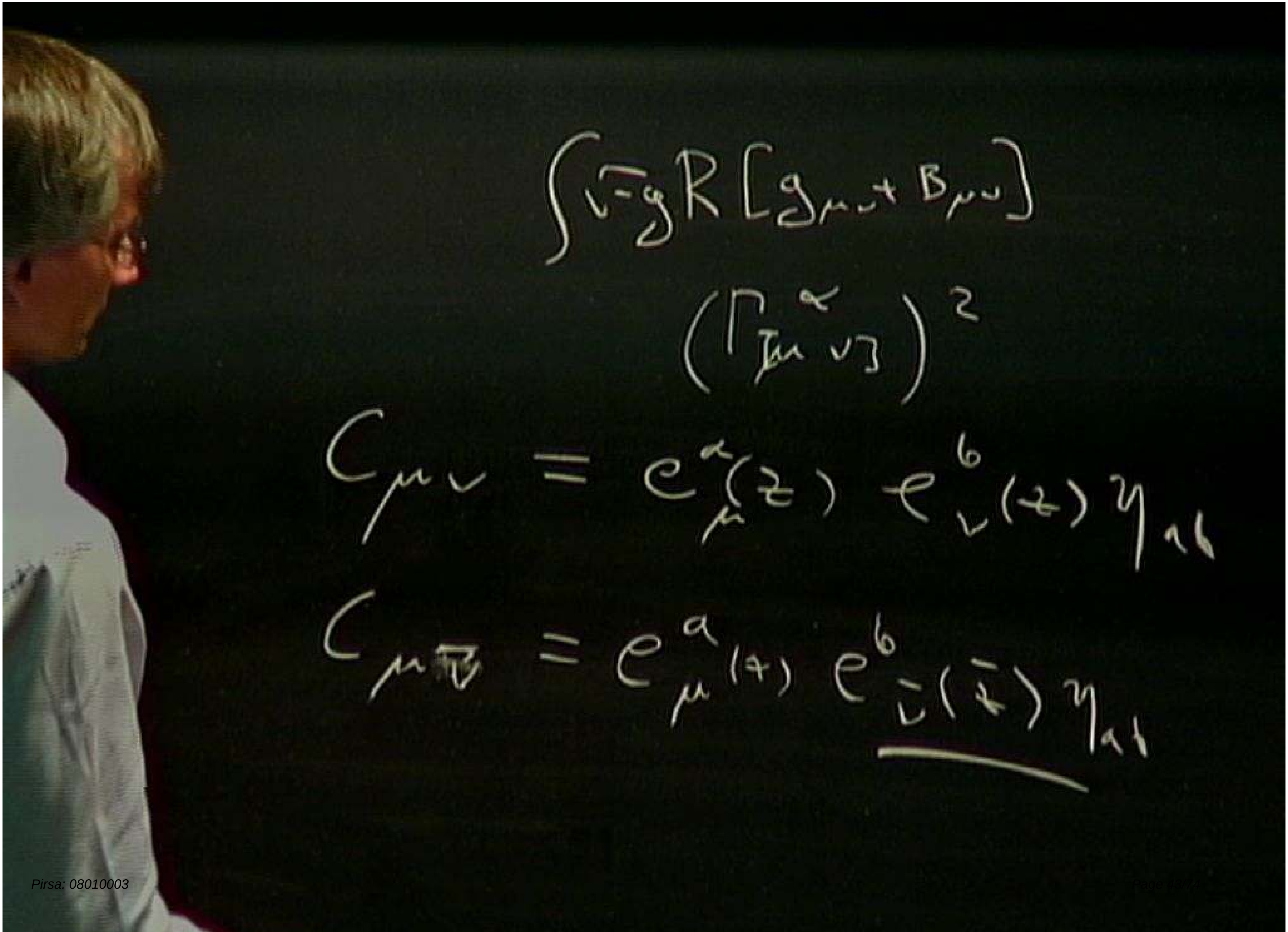
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# HERMITEAN GRAVITY

- Max Born (1938): reciprocity symmetry of physical theories

$$x^\nu \to p^\nu, \quad p^\nu \to -x^\nu$$

- E.g. angular momentum: $M^{\mu\nu} = x^\mu p^\nu - p^\mu x^\nu$
- E.g. commutation relation ($\to$ quantisation possible):

$$\hat{x}^\mu \hat{p}^\nu - \hat{p}^\nu \hat{x}^\mu = i\hbar \eta^{\mu\nu}, \quad \eta^{\mu\nu} = diag[-1,1,1,1]$$

**HERMITEAN SPACE**: Metric tensor **C** invariant under **J**

- An almost complex structure (tensor) J:TpM→TpM

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- Line element:

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$$C_{\bar{\mu}\nu} = e^{a}_{\bar{\mu}}(z^{\bar{\rho}}) \eta_{ab} e^{b}_{\nu}(z^{\rho}), \quad z^{\rho} = x^{\rho} + iy^{\rho}$$

$$ds^2 \rightarrow dx^{\mu} g_{\mu\nu} dx^{\nu} + dy^{\mu} g_{\mu\nu} dy^{\nu} + 2dx^{\mu} B_{\mu\nu} dy^{\nu}$$

B: antisymmetric tensor field

g: Einstein's metric tensor

# FLAT SPACE LIMIT

- Upon rotation:

$$ds^2 = (ds^2)^\dagger = dx^T \cdot g \cdot dx = \begin{pmatrix} dx^\mu & dy^{\bar\nu} \end{pmatrix} \begin{pmatrix} g_{\mu\nu} & g_{\mu\bar\nu} \\ g_{\bar\mu\nu} & g_{\bar\mu\bar\nu} \end{pmatrix} \begin{pmatrix} dx^\mu \\ dy^{\bar\nu} \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} g_{\mu\nu} & B_{\mu\nu} \\ g_{\bar\mu\nu} & g_{\bar\mu\bar\nu} \end{pmatrix} \equiv \begin{pmatrix} g_{\mu\nu} & B_{\mu\nu} \\ -B_{\mu\nu} & g_{\mu\nu} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} C_{\mu\bar\nu} + C_{\bar\mu\nu} & i\left(C_{\bar\mu\nu} - C_{\mu\bar\nu}\right) \\ -i\left(C_{\bar\mu\nu} - C_{\mu\bar\nu}\right) & C_{\bar\mu\nu} + C_{\mu\bar\nu} \end{pmatrix}$$

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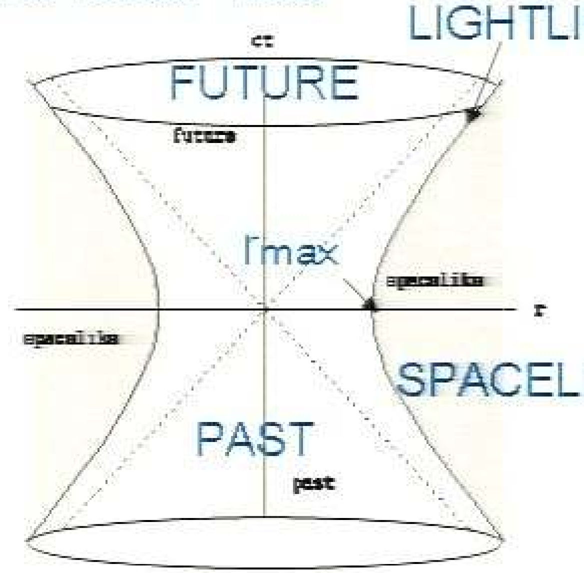



## FLAT SPACE LIMIT: SYMMETRY: SU(3,1)

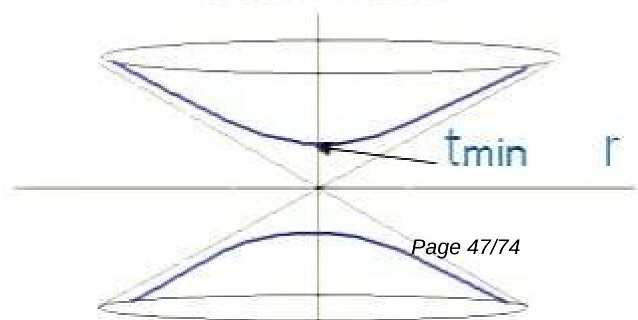
$$ds^2 = -(cdt)^2 + (d\vec{x})^2 + \underbrace{\frac{G_N^2}{c^6}}_{10^{-72} s^2 kg^{-2}\,(tiny)} \left(-(dE/c)^2 + (d\vec{p})^2\right)$$

$$(0) \quad ds^2 = 0 \ \ (\text{LIGHTCONE})\ \&\ -dE = d\vec{p} = 0 \Rightarrow \frac{d\vec{x}}{dt} \le c \quad (GR)$$

## Causality structure changes

$$(1) \quad ds^2 = 0\ \&\ p^2 = p_\mu p^\mu = -\left(\frac{E}{c}\right)^2 + \vec{p}^2 < 0 \Rightarrow r_{\max} = \frac{G_N\sqrt{-p^2}}{c^3}$$

$$(2) \quad ds^2 = 0\ \&\ p^2 > 0 \Rightarrow t_{\min} = \frac{G_N\sqrt{p^2}}{c^4}$$



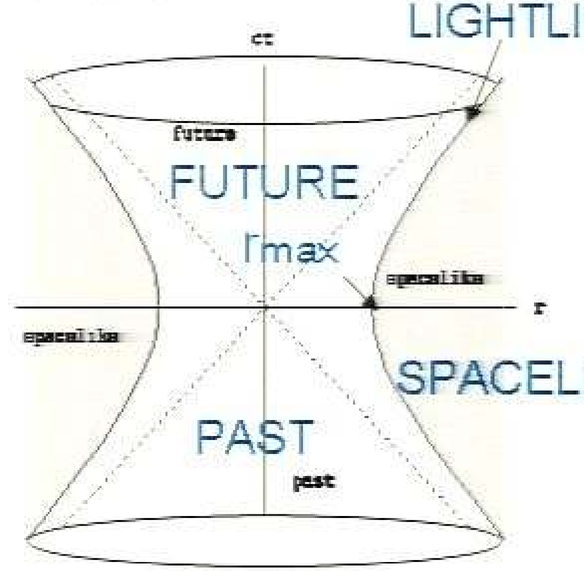
# FLAT SPACE LIMIT: VELOCITIES

- PHASE VELOCITY $v_{\rm ph} = \frac{r}{t} \leq \sqrt{c^2 - \frac{G_N^2 p^2}{c^6 t^2}} \Rightarrow v_{\rm ph} \lessgtr c$

- GROUP VELOCITY $v_g = \frac{dx}{dt} \leq \sqrt{c^2 - \frac{G_N^2 f^2}{c^6}}, \quad f^2 = -\left(\frac{dE}{cdt}\right)^2 + \left(\frac{d\vec{p}}{dt}\right)^2$

- Upper bound on 4-force:

$$f^2 = \frac{c^6}{G_N^2}\left(c^2 - v_g^2 + \underbrace{\left(\frac{ds}{dt}\right)^2}_{\leq 0\,(\text{prop. particles})}\right) \leq \frac{c^6}{G_N^2}\left(c^2 - v_g^2\right) \leq \frac{c^8}{G_N^2} \Rightarrow \exists\,\text{propagation}\,(v_g\ \text{real})$$



# HERMITEAN CONNECTIONS & GEODESIC EQUATIONS

- Nakahara (2003): metric tensor (tetrad) compatible connection

$$\nabla_{\mu} C_{\nu\bar{\sigma}} = 0 = \nabla_{\bar{\mu}} C_{\nu\bar{\sigma}} \quad (\nabla_{\mu} e^{a}_{\nu} = 0 = \nabla_{\mu} e^{a}_{\bar{\nu}} \ \& \ c.c.) \qquad \Rightarrow \Gamma^{\rho}_{\mu\nu} = C^{\rho\bar{\sigma}} \partial_{\mu} C_{\nu\bar{\sigma}} \ \& \ c.c.$$

- Varying the hermitean line element action $S = -m\int ds, \quad ds^2 = dz^T \cdot C \cdot dz, \quad \delta S =$ gives geodesic equations with different connection coefficients:

$$\Gamma^{\rho}_{\mu\nu} = \frac{1}{2} C^{\bar{\lambda}\rho}\left(\partial_{\mu} C_{\nu\bar{\lambda}} + \partial_{\nu} C_{\mu\bar{\lambda}}\right) \qquad \Gamma^{\rho}_{\bar{\mu}\nu} = \frac{1}{2} C^{\bar{\lambda}\rho}\left(\partial_{\bar{\mu}} C_{\nu\bar{\lambda}} - \partial_{\bar{\lambda}} C_{\nu\bar{\mu}}\right)$$

$$\Gamma^{\rho}_{\mu\bar{\nu}} = \frac{1}{2} C^{\bar{\lambda}\rho}\left(\partial_{\bar{\nu}} C_{\mu\bar{\lambda}} - \partial_{\bar{\lambda}} C_{\mu\rho}\right) \qquad \Gamma^{\rho}_{\bar{\mu}\bar{\nu}} = 0 \quad \& \quad c.c.$$

- geodesic equations

$$\ddot{z}^{\rho} + \Gamma^{\rho}_{\mu\nu} \dot{z}^{\mu} \dot{z}^{\nu} + 2\Gamma^{\rho}_{\mu\bar{\nu}} \dot{z}^{\mu} \dot{z}^{\bar{\nu}} = 0 \quad \& \ c.c.$$

# CONCLUSIONS AND DISCUSSION

- We have constructed a theory of gravity on complex manifolds which generalised Einstein's theory

**Hermitean gravity** (which respects the reciprocity symmetry of Born) contains antisymmetric tensor field but it is nondynamical when viewed on complex space

**Holomorphic gravity** (which is antisymmetric with respect to reciprocity symmetry) contains an additional spin 2 metric field tensor

- the theory shows absence of cosmological and black hole singularity for typical observers
- the theory extends (& thus violates) general covariance

Both theories exhibit interesting causal structure:

- In flat space limit: for certain observers small regions of space can appear in instantaneous causal contact, while for some observers there is a minimum time required for establishing a causal contact.



No definite word has been said on viability of either theory.

# HOLOMORPHIC BLACK HOLES

- holomorphic Schwarzschild solution:

$$C_{00} = -\left(1 - \frac{1}{c^2}\frac{2G_N M}{z}\right), \quad C_{rr} = -\left(1 - \frac{1}{c^2}\frac{2G_N M}{z}\right)^{-1}, \quad z \equiv \|\vec{z}\| = r + i\frac{G_N}{c^3}p = \|\vec{x}\| + i\frac{G_N}{c^3}\|\vec{p}\|$$

► For simplicity set dE=0=dp:

$$ds^2 = -\left(1 - \frac{2G_N M}{c^2}\frac{r}{r^2 + \frac{G_N^2}{c^6}p^2}\right)(cdt)^2 - \left(1 - \frac{2G_N M}{c^2}\frac{r^2 + \frac{G_N^2}{c^6}p^2 - \frac{2GM}{c^2}r}{\left(r - \frac{2GM}{c^2}\right)^2 + \frac{G_N^2}{c^6}p^2}\right)^{-1} dr^2 + r^2 d\Omega^2 + O(dp, dE)$$

- Consider a freely falling observed onto a holomorphic black hole: its p will grow very large as it approaches r=0

$$R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} \sim \mathrm{Re}\frac{1}{\left(r + i\frac{G_N}{c^3}p\right)^6} = \frac{r^6 - 15\frac{G_N^2}{c^6}r^4p^2 + 15\frac{G_N^4}{c^{12}}r^2p^4 - \frac{G_N^6}{c^{18}}p^6}{\left(r^2 + \frac{G_N^2}{c^6}p^2\right)^6} \xrightarrow{r\to 0} -\frac{1}{\left(\frac{G_N}{c^3}p\right)^6}$$



**NB**: A freely falling observed does not see a curvature singularity at r=0

# HOLOMORPHIC COSMOLOGY

- holomorphic vielbein $e_\mu^a = a(z^0)\delta_\mu^a$

- HOLOMORPHIC FRIEDMANN EQUATIONS IN FLRW SPACES

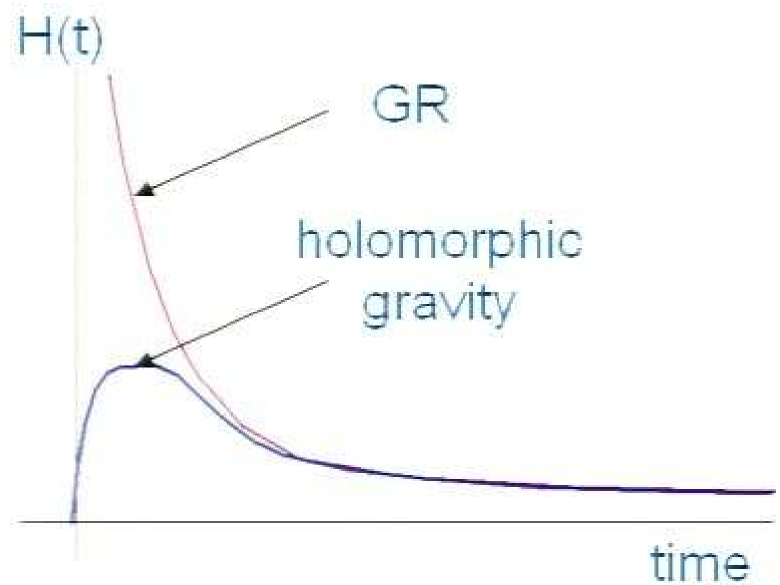
$$H_h^2 = \left(\frac{a'(z^0)}{a^2(z^0)}\right)^2 = \frac{8\pi G_N}{3}\rho, \qquad \frac{\ddot{a}}{a} = -\frac{4\pi G_N}{3}(\rho + 3p)$$

▶ Holomorphic **Hubble parameter**

$$H_h = \frac{2}{3(1+w)\left(t + i(G_N / c^4)E\right)}, \quad w = \frac{p}{\rho}$$

▶ (Real) **Hubble parameter**

$$H = \mathrm{Re}[H_h] = \frac{2t}{3(1+w)\left(t^2 + (G_N^2 / c^8)E^2\right)}$$




**NB**: If E !=0, there is no cosmological singularity for t=0




**NB**: An observer moving backward in time will experience E !=0

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# HOLOMORPHIC DYNAMICS

- HOLOMORPHIC EINSTEIN-HILBERT EQUATIONS

$$S_h[g]=\int d^8 z\sqrt{-C}R,\quad R=C^{\mu\nu}R_{\mu\nu}+C^{\bar{\mu}\bar{\nu}}R_{\bar{\mu}\bar{\nu}},\quad \delta S_h=0\Rightarrow$$

$$G_{\mu\nu}=8\pi G_N T_{\mu\nu},\quad G_{\mu\nu}=R_{\mu\nu}-\frac{1}{2}C_{\mu\nu}R$$

# FLAT SPACE LIMIT: VELOCITIES

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$$ds^2 = -(c\,dt)^2 + (d\vec{x})^2 - \underbrace{\frac{G_N^2}{c^6}}_{10^{-72}\,s^2 kg^{-2})} \left(-(dE/c)^2 + (d\vec{p})^2\right)$$

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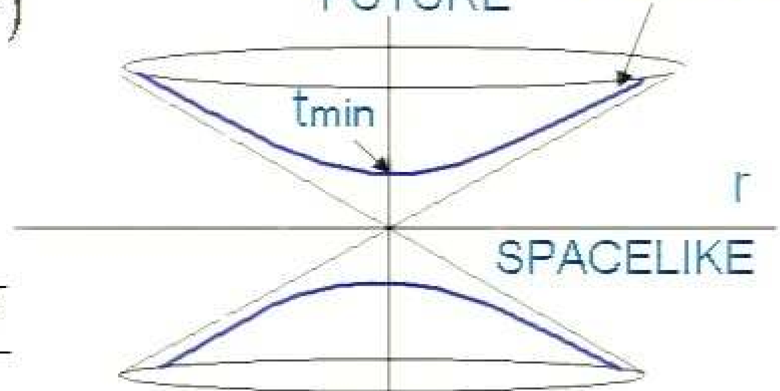
$$(1)\quad ds^2 = 0\ \&\ p^2 < 0 \Rightarrow t_{\min} = \frac{G_N\sqrt{-p^2}}{c^4}$$

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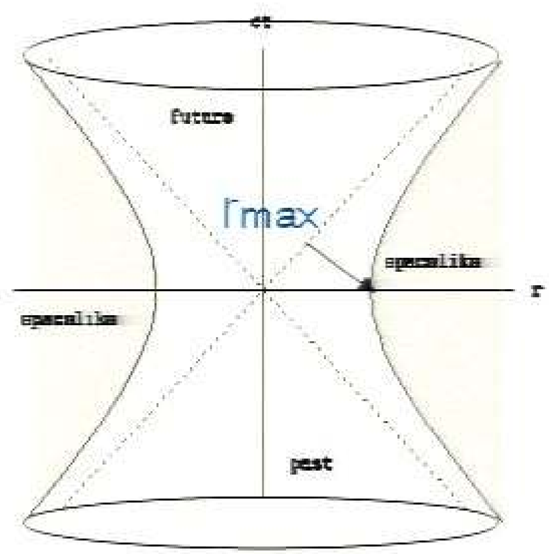

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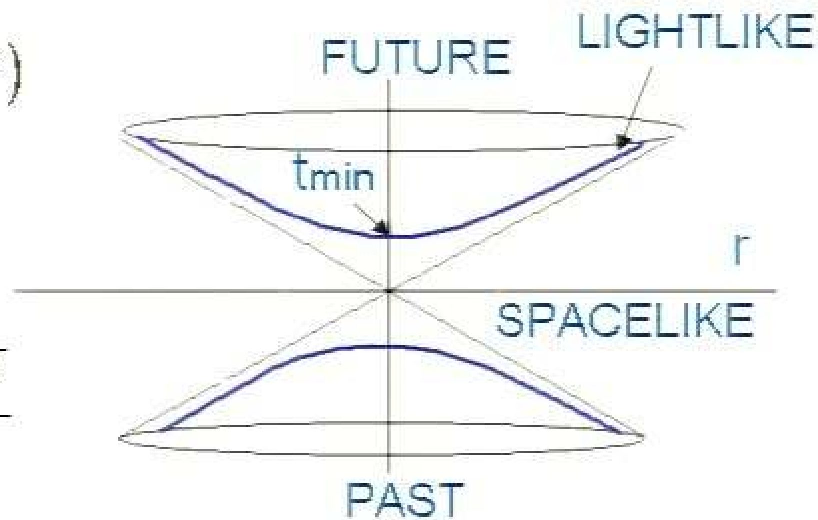


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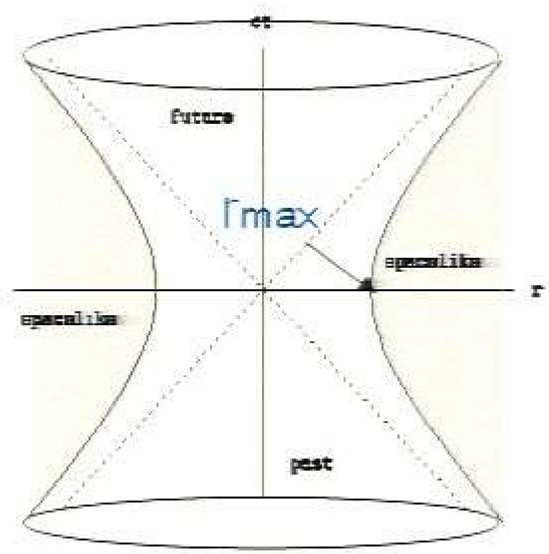

# HOLOMORPHIC GRAVITY

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- holomorphic metric tensor

$$C_{mn} = \begin{pmatrix} C_{\mu\nu} & 0 \\ 0 & C_{\bar{\mu}\bar{\nu}} \end{pmatrix}$$

$$ds^2 = dz^T \cdot C \cdot dz, \quad C = e^T \cdot e$$

$$C_{\mu\nu} = e^a_\mu(z^\sigma)\eta_{ab}e^b_\nu(z^\sigma), \quad z^\sigma = x^\sigma + iy^\sigma, \quad y^\sigma = \frac{G_N}{c^3}p^\sigma$$

• **HOLOMORPHIC SPACE**: Metric tensor **C→-C** under **J**

When rotated: one gets two symmetric tensors (g & K), each 10 dof's

$$g_{mn} \equiv \begin{pmatrix} g_{\mu\nu} & -K_{\mu\nu} \\ -K_{\mu\nu} & -g_{\mu\nu} \end{pmatrix} = \frac{1}{2}\begin{pmatrix} C_{\mu\nu} + C_{\bar{\mu}\bar{\nu}} & i\left(C_{\mu\nu} - C_{\bar{\mu}\bar{\nu}}\right) \\ -i\left(C_{\mu\nu} - C_{\bar{\mu}\bar{\nu}}\right) & -\left(C_{\mu\nu} + C_{\bar{\mu}\bar{\nu}}\right) \end{pmatrix}$$

$$ds^2 = dx^\mu g_{\mu\nu} dx^\nu - \frac{G_N^2}{c^6} dp^\mu g_{\mu\nu} dp^\nu - 2\frac{G_N}{c^3} dx^\mu K_{\mu\nu} dp^\nu$$

# HERMITEAN DYNAMICS

- THE HERMITEAN EINSTEIN-HILBERT EQUATIONS

$$S_H[g] = \int d^8 z \sqrt{-C} R, \quad R = C^{\mu\bar{\nu}} R_{\mu\bar{\nu}}, \quad \delta S_H = 0 \Rightarrow \quad G_{\mu\bar{\nu}} = 8\pi G_N T_{\mu\bar{\nu}}, \quad G_{\mu\bar{\nu}} = R_{\mu\bar{\nu}} - \frac{1}{2} C_{\mu\bar{\nu}} R$$

- PROBLEM with Hermitean Gravity: These equations are nondynamical (do not contain 2nd order time derivatives)

  ▶ More generally: one can show that the Riemann tensor does not contain second order time derivatives.

  ▶ This agrees with the counting of degrees of freedom at second order (in curvature tensors): there is enough freedom in (complex) coordinate transformations to remove the curvature.

$$C^{\mu\bar{\nu}} \to \frac{\partial w^{\mu}(z)}{\partial z^{\rho}} \frac{\partial w^{\bar{\nu}}(\bar{z})}{\partial z^{\bar{\sigma}}} C^{\rho\bar{\sigma}}$$

E.g. cosmology: $\frac{a'(z^0)}{a(z^0)} \frac{\bar{a}'(\bar{z}^0)}{\bar{a}(\bar{z}^0)} \sim \pi G_N(\rho - 3p)$

**Q:** Can one save hermitean gravity?

**A:** Perhaps: projecting onto the $y_\mu=0$ hyperplane may generate dynamics?!?

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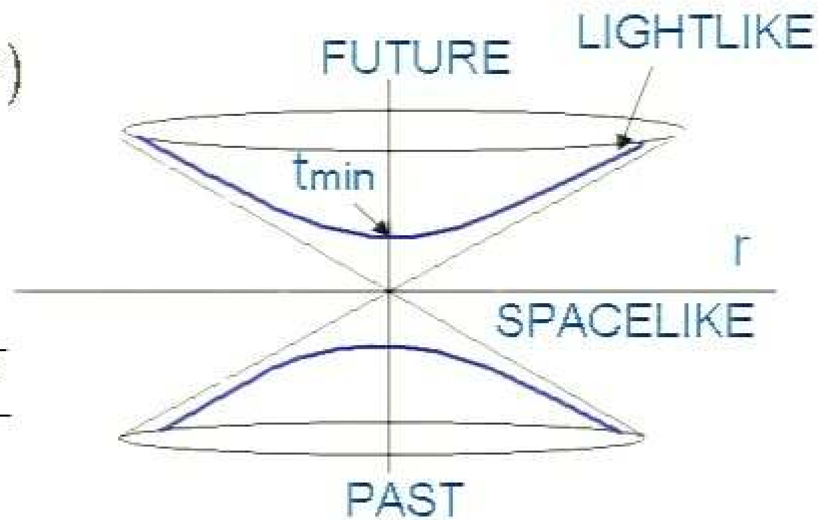


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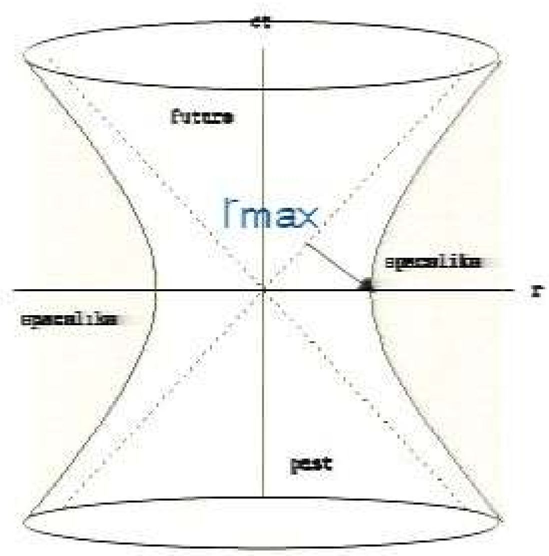

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- holomorphic vielbein $e_\mu^a = a(z^0)\delta_\mu^a$

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$$H_h^2 = \left(\frac{a'(z^0)}{a^2(z^0)}\right)^2 = \frac{8\pi G_N}{3}\rho, \qquad \frac{\ddot{a}}{a} = -\frac{4\pi G_N}{3}(\rho + 3p)$$

► Holomorphic **Hubble parameter**

$$H_h = \frac{2}{3(1+w)\left(t + i(G_N / c^4)E\right)}, \quad w = \frac{p}{\rho}$$

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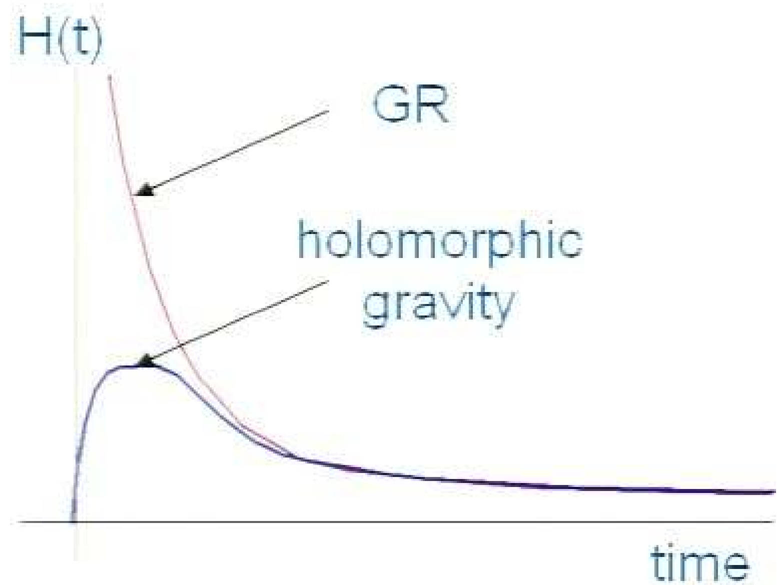


**NB**: If E !=0, there is no cosmological singularity for t=0

**NB**: An observer moving backward in time will experience E !=0

# HOLOMORPHIC BLACK HOLES

- holomorphic Schwarzschild solution:

$$C_{00} = -\left(1 - \frac{1}{c^2}\frac{2G_N M}{z}\right), \quad C_{rr} = -\left(1 - \frac{1}{c^2}\frac{2G_N M}{z}\right)^{-1}, \quad z \equiv \|\vec{z}\| = r + i\frac{G_N}{c^3}p = \|\vec{x}\| + i\frac{G_N}{c^3}\|\vec{p}\|$$

► For simplicity set dE=0=dp:

$$ds^2 = -\left(1 - \frac{2G_N M}{c^2}\frac{r}{r^2 + \frac{G_N^2}{c^6}p^2}\right)(cdt)^2 - \left(1 - \frac{2G_N M}{c^2}\frac{r^2 + \frac{G_N^2}{c^6}p^2 - \frac{2GM}{c^2}r}{\left(r - \frac{2GM}{c^2}\right)^2 + \frac{G_N^2}{c^6}p^2}\right)^{-1} dr^2 + r^2 d\Omega^2 + O(dp, dE)$$

◆Consider a <u>freely falling</u> observed onto a holomorphic black hole: its p will grow very large as it approaches r=0

$$R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} \sim \mathrm{Re}\frac{1}{\left(r + i\frac{G_N}{c^3}p\right)^6} = \frac{r^6 - 15\frac{G_N^2}{c^6}r^4p^2 + 15\frac{G_N^4}{c^{12}}r^2p^4 - \frac{G_N^6}{c^{18}}p^6}{\left(r^2 + \frac{G_N^2}{c^6}p^2\right)^6} \xrightarrow{r\to 0} -\frac{1}{\left(\frac{G_N}{c^3}p\right)^6}$$



**NB**: A freely falling observed does not see a curvature singularity at r=0

# CONCLUSIONS AND DISCUSSION

- **We have constructed a theory of gravity** on complex manifolds **which generalised Einstein's theory**

**Hermitean gravity** (which respects the reciprocity symmetry of Born) contains antisymmetric tensor field but it is nondynamical when viewed on complex space

**Holomorphic gravity** (which is antisymmetric with respect to reciprocity symmetry) contains an additional spin 2 metric field tensor

- the theory shows absence of cosmological and black hole singularity for typical observers
- the theory extends (& thus violates) general covariance

Both theories exhibit interesting causal structure:

- In flat space limit: for certain observers small regions of space can appear in instantaneous causal contact, while for some observers there is a minimum time required for establishing a causal contact.



No definite word has been said on viability of either theory.

$$m_1 \dot{x}_2^\mu$$

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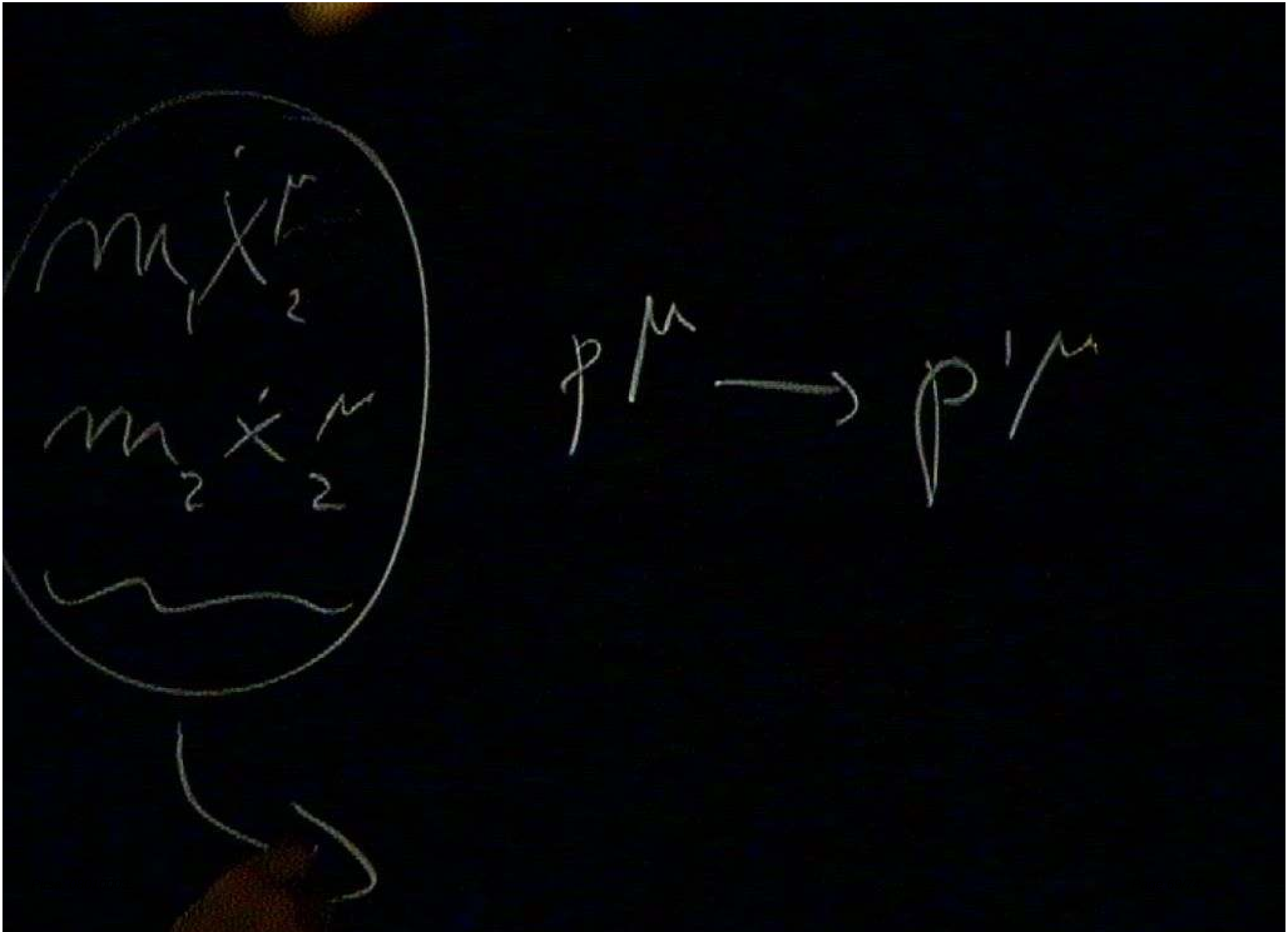

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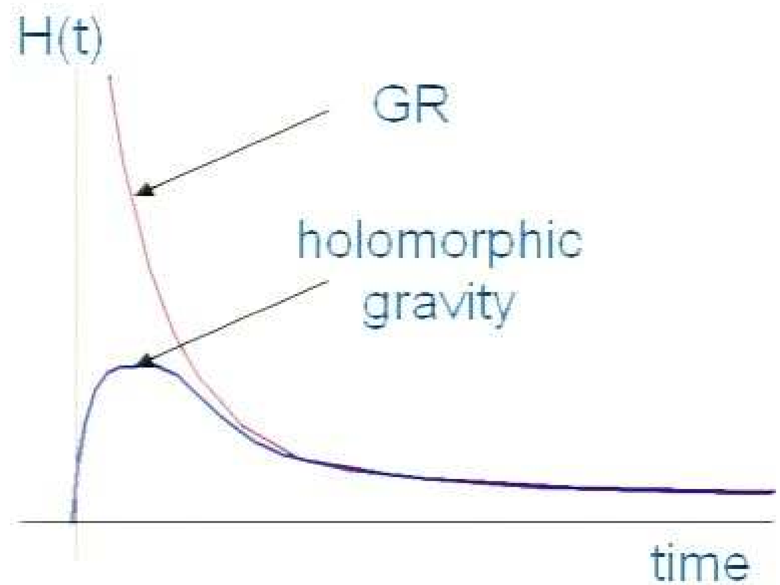


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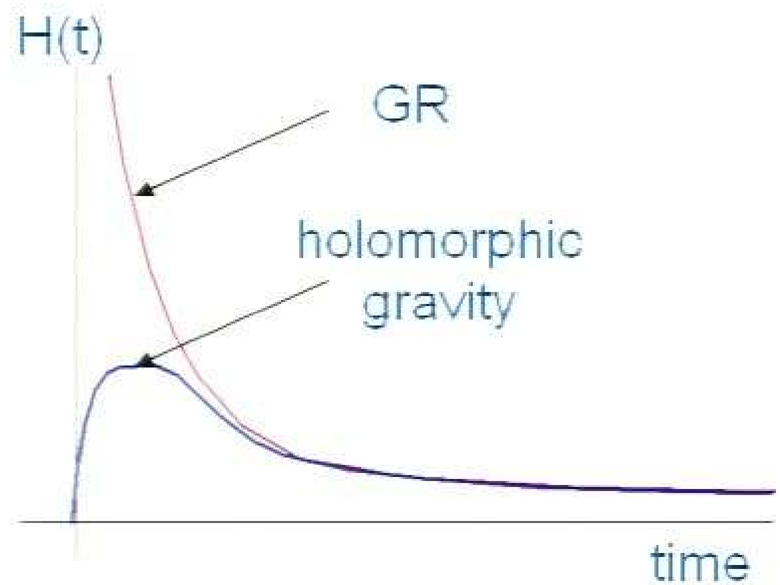


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