Title: Emergent gravity from noncommutative gauge theory

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Abstract: We show that the matrix-model for noncommutative U(n) gauge theory actually describes SU(n) gauge theory coupled to gravity. The nonabelian gauge fields as well as additional scalar fields couple to a dynamical metric G_ab, which is given in terms of a Poisson structure. This leads to a gravity theory which is naturally related to noncommutativity, encoding those degrees of freedom which are usually interpreted as U(1) gauge fields. Essential features such as gravitational waves and the Newtonian limit are reproduced correctly.
UV/IR mixing is understood in terms of an induced gravity action. The framework appears suitable for quantizing gravity.

Emergent Gravity from Noncommutative Gauge Theory

Harold Steinacker

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Perimeter Institute, January 2008

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Introduction

- Classical space-time meaningless at Planck scale
  due to gravity $\leftrightarrow$ Quantum Mechanics
  $\Rightarrow$ "quantized" (noncommutative?) spaces:
- Physics on quantized space:
  Noncommutative Quantum Field Theory
  well developed; some problems remain
- What about gravity on/for quantized spaces ??
  should be simple & naturally related to NC

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Main Message:

- $\exists$ simple models for dynamical NC space:

  Matrix Models

  - M. M. known to describe NC gauge theory
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Main result:

The model:

$$S_{YM} = -Tr[X^a, X^b][X^{a'}, X^{b'}]\,\eta_{aa'}\eta_{bb'}$$

where $X^a \in L(\mathcal{H})$ ... matrices (operators), $a = 0, 1, 2, 3$

low-energy effective action:

$$S_{YM} = \int d^4y\, \rho(y)\, tr\left(4\eta(y) - G^{cc'}G^{dd'}F_{cd}F_{c'd'}\right) + 2\int \eta(y)\, trF \wedge F$$

where

$G^{ab}(y) = -\theta^{ac}(y)\theta^{bd}(y)\,\eta_{cd}$ effective dynamical metric

F_{ab} ... field strength

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Outline

- NC gauge theory as Matrix Model
- Effective metric, geometry
- Low-energy effective action and emergent gravity
- Some checks:
  - Gravitational waves, linearized metric
  - Newtonian Limit
- Quantization, UV/IR mixing
- Conclusion

Matrix Models and dynamical space(time)

Consider Matrix Model:

$$S_{YM} = -Tr[X^a, X^b][X^{a'}, X^{b'}]\eta_{aa'}\eta_{bb'}, \qquad a = 0,1,2,3$$

(toy candidate for fundamental theory)

dynamical objects: $X^a \in L(\mathcal{H})$ ... hermitian matrices

equation of motion: $[X^a, [X^{a'}, X^{b'}]]\eta_{aa'} = 0$

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- $[X^a, X^b] = 0$ ...classical objects; ignore here
- $[X^a, X^b] = i\bar{\theta}^{ab}\,\mathbf{1}$, "quantum plane"
  where $\bar{\theta}^{ab}$ ... antisymmetric tensor, nondegenerate
- many more, of type $[X^a, X^b] = i\theta^{ab}(x)$

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fluctuating quantum spaces and gauge fields

consider fluctuations

$$X^a = \overline{Y}^a + A^a \qquad \text{("covariant coordinates")}$$

around solution

$$[\overline{Y}^a, \overline{Y}^b] = i\overline{\theta}^{ab} \qquad \text{"Moyal-Weyl plane"}$$

note

$$[\overline{Y}^a, f(\overline{Y})] \sim i\overline{\theta}^{ab}\partial_b f(\overline{y})$$

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$$\begin{aligned}[X^a, X^b] - i\overline{\theta}^{ab} &= \overline{\theta}^{aa'}\overline{\theta}^{bb'}\left(\partial_{a'}A_{b'} - \partial_{b'}A_{a'} + [A_{a'}, A_{b'}]\right) \\ &= \overline{\theta}^{aa'}\overline{\theta}^{bb'} F_{a'b'}\end{aligned}$$

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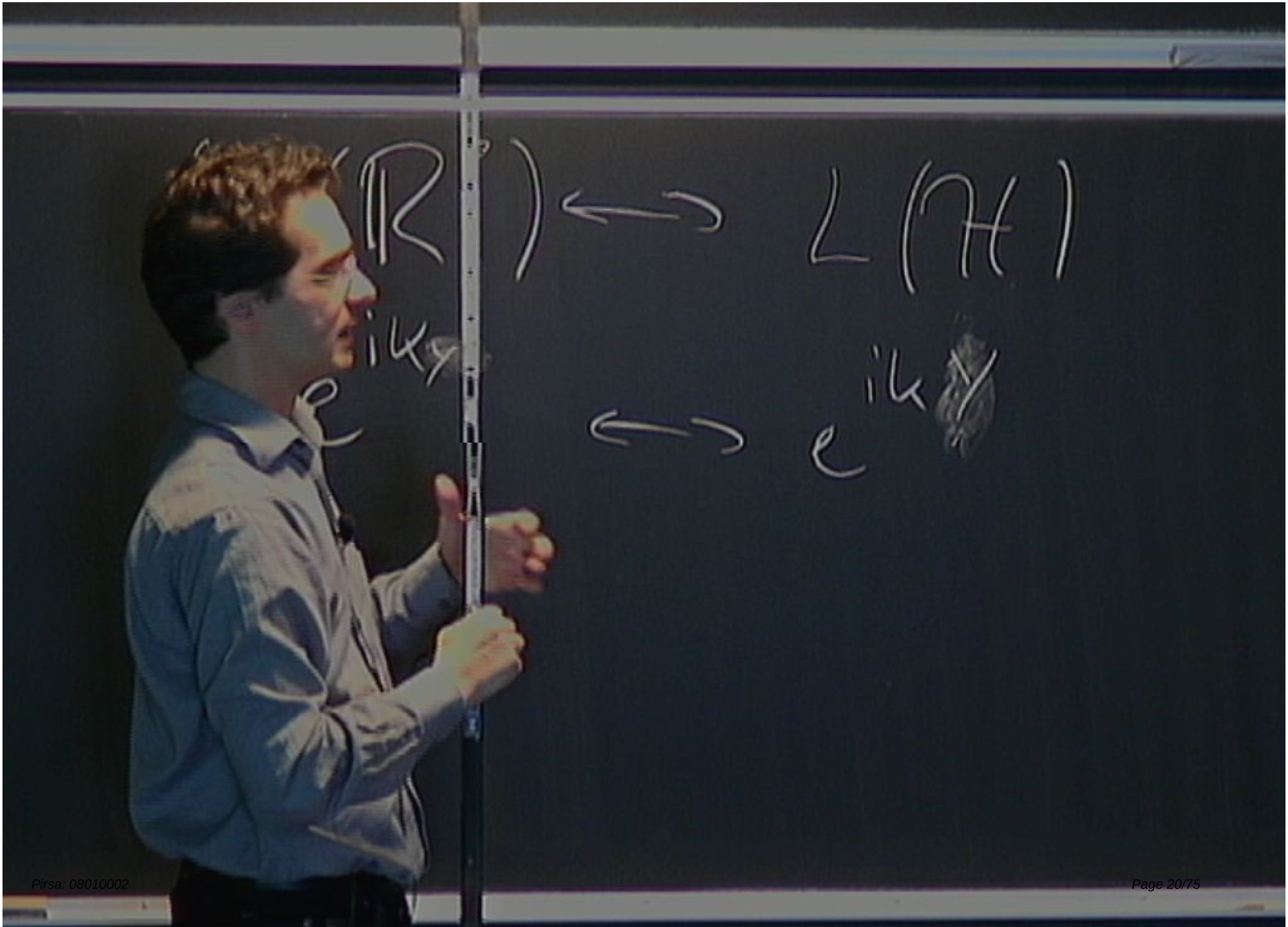
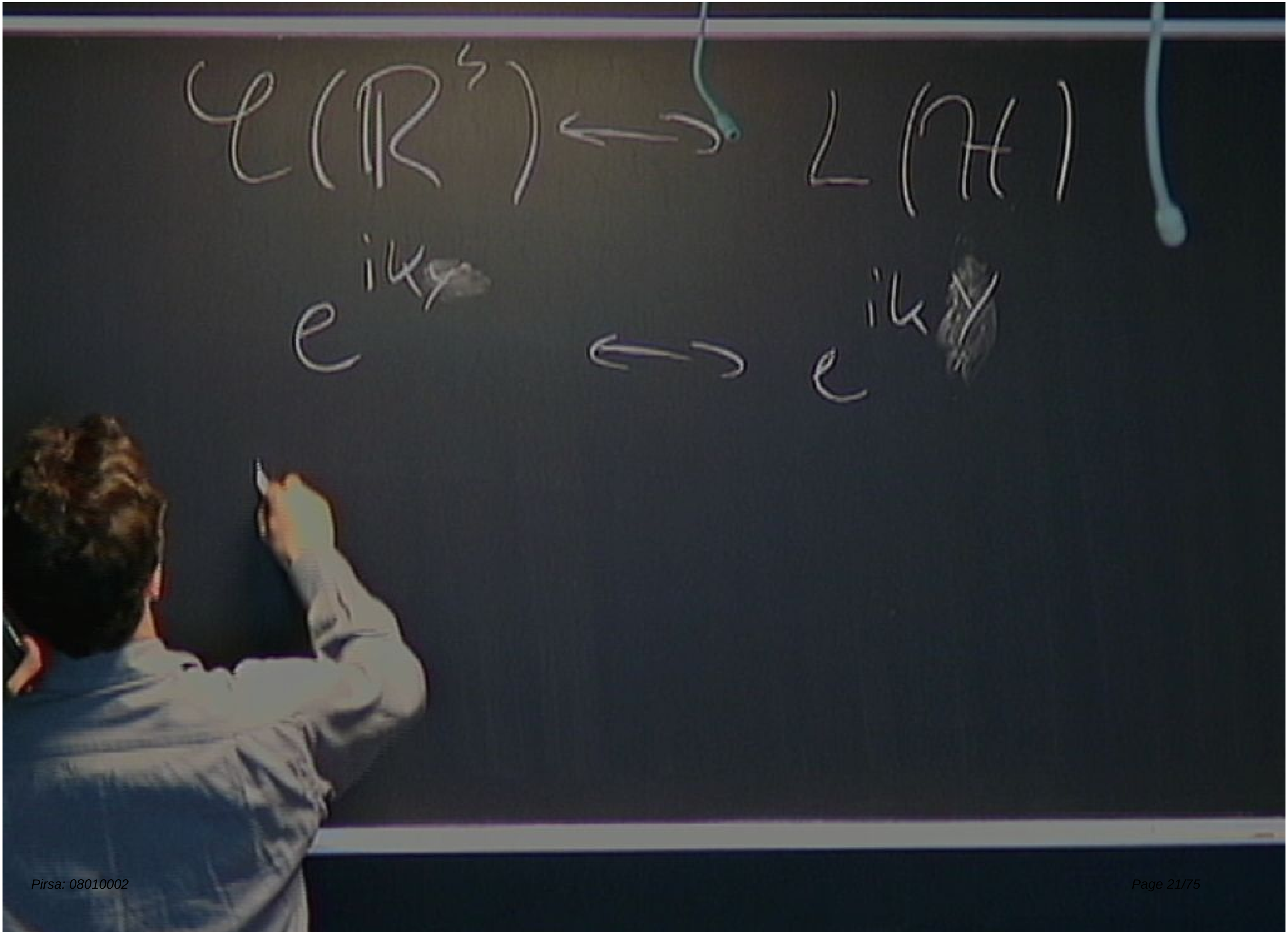
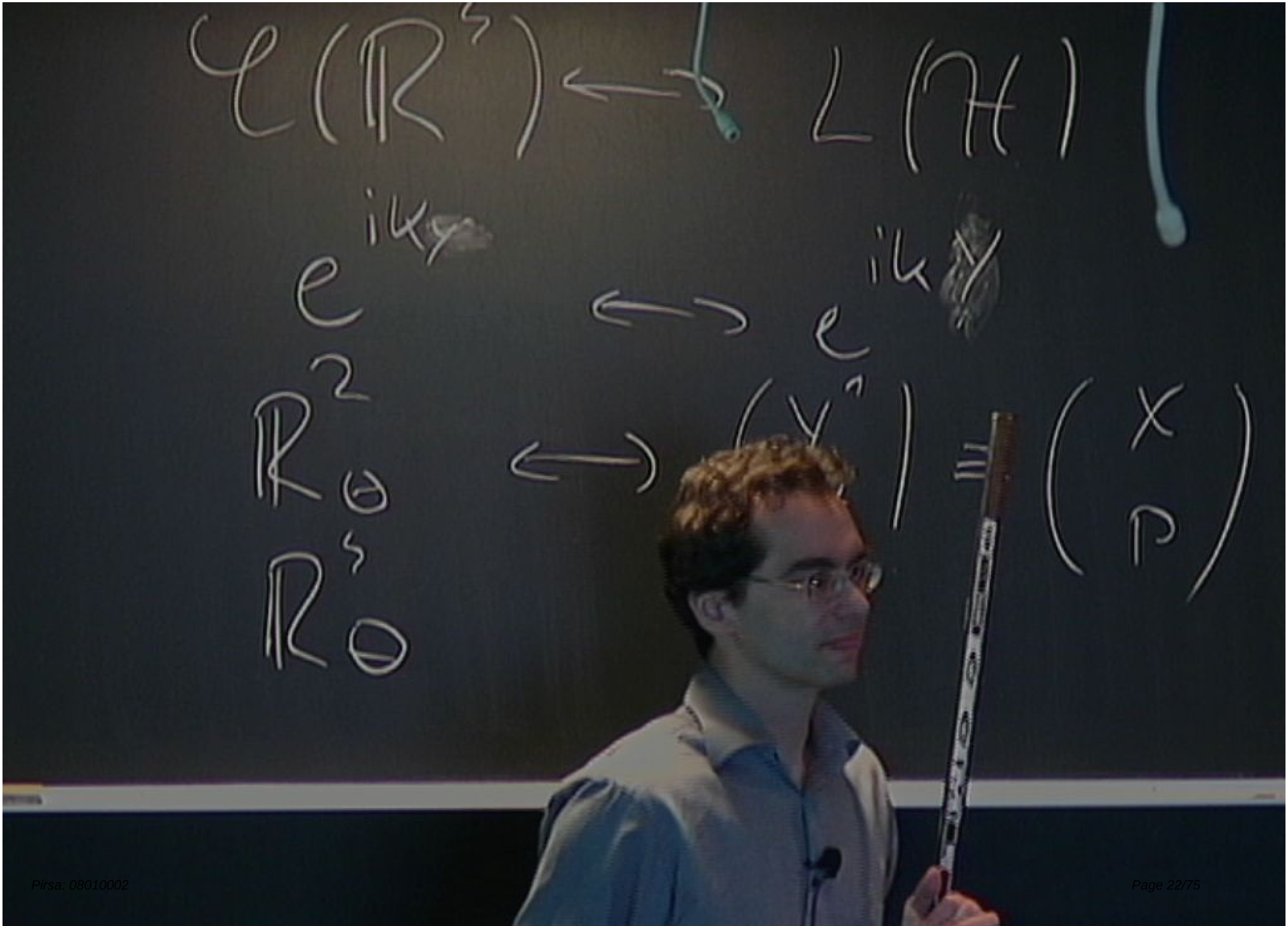
$$\mathcal{C}(\mathbb{R}^4) \hookleftarrow L(\mathcal{H})$$


L(H)
e iky


𝒞(ℝ³) ⟷ L(ℋ)
e^ik ⟷ e^ik


C(R³) ⟷ L(H)
e^iky ⟷ e^iky
R²θ ⟷
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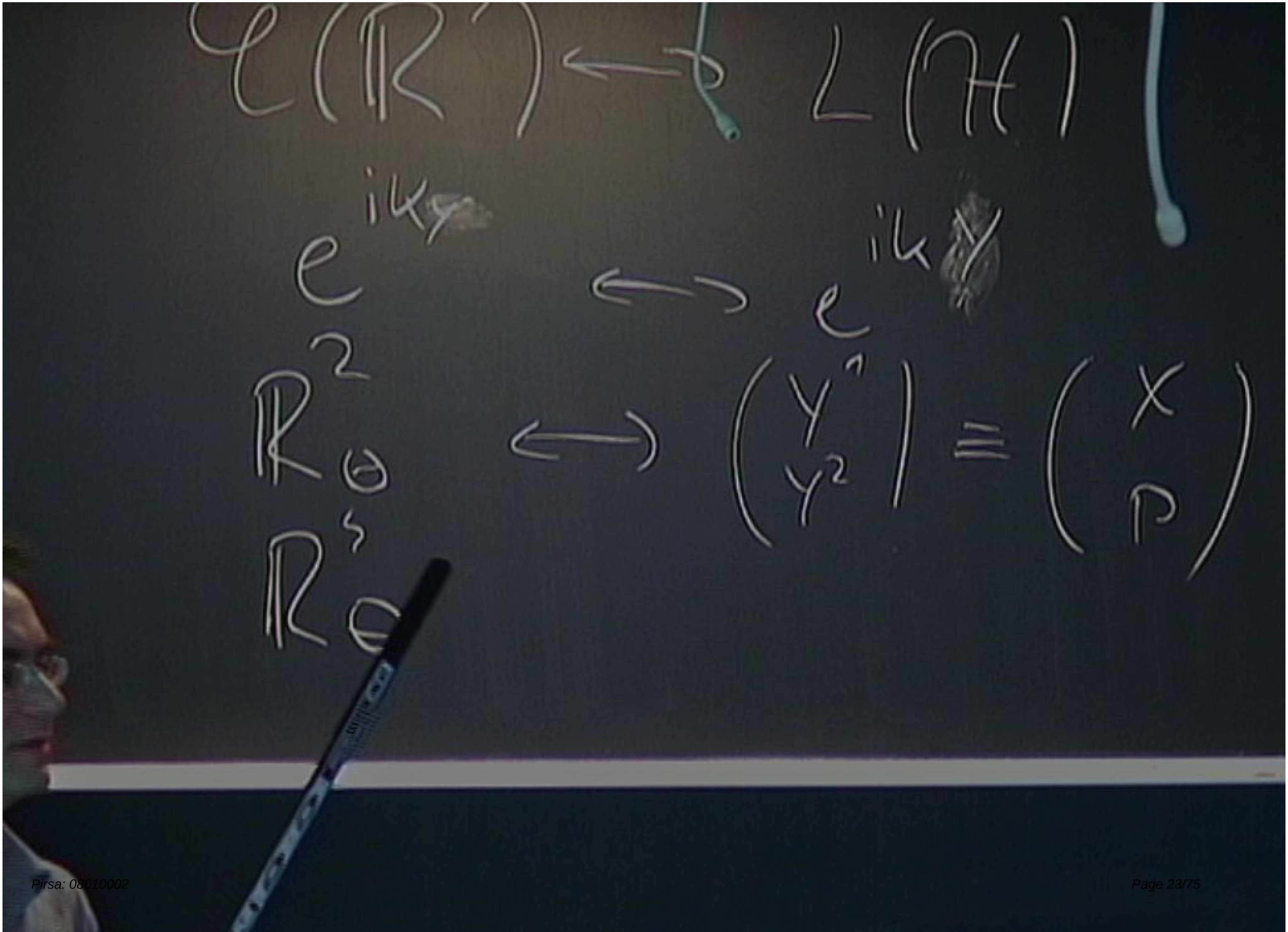

C(R) ← L(H)
e^iky ⟷ e^iky
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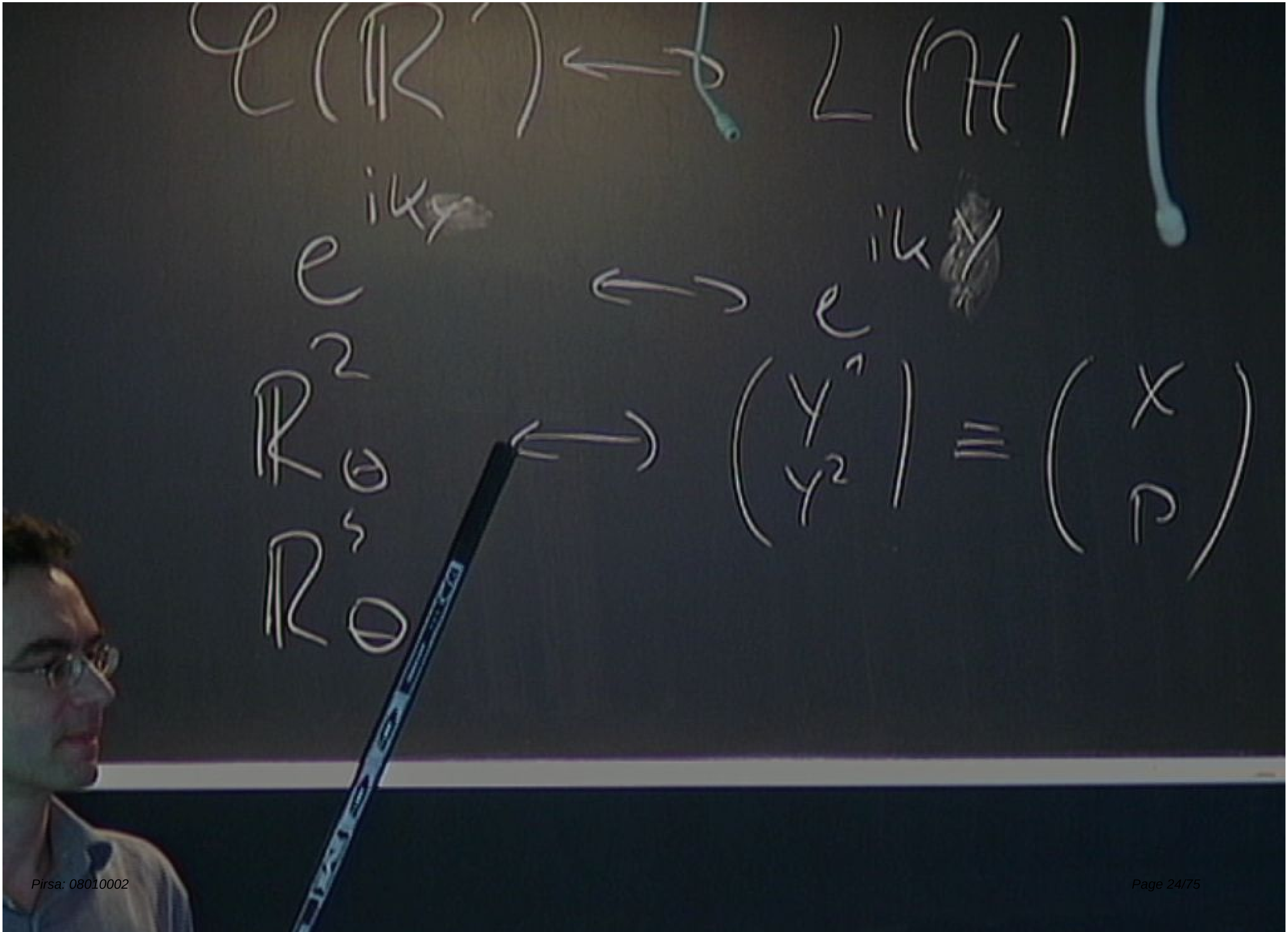
R^3_θ



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nonabelian $U(n)$ case: similar, $Y^a \to \overline{Y}^a \otimes \mathbf{1}_n$

<u>however</u>:

- $U(1)$ sector cannot be disentangled
- space itself obtained as "vacuum", is dynamical;

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consider general quantum space determined by full $\mathfrak{u}(1)$ sector:

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... general quantized Poisson manifold $(\mathcal{M}, \theta^{ab}(x))$

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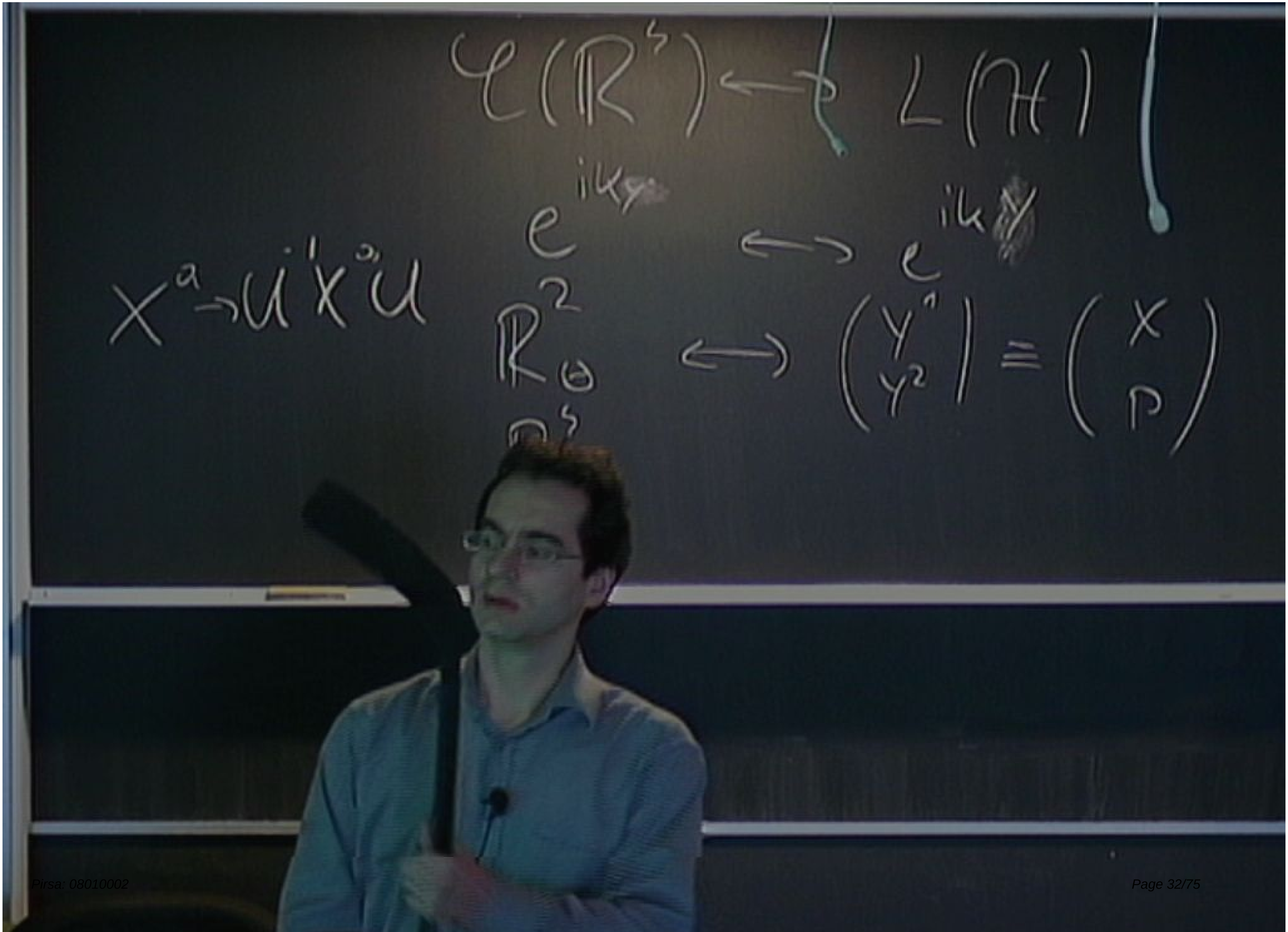
- $\theta^{ac}(x)$ ... vielbein ("gauge-fixed"!)



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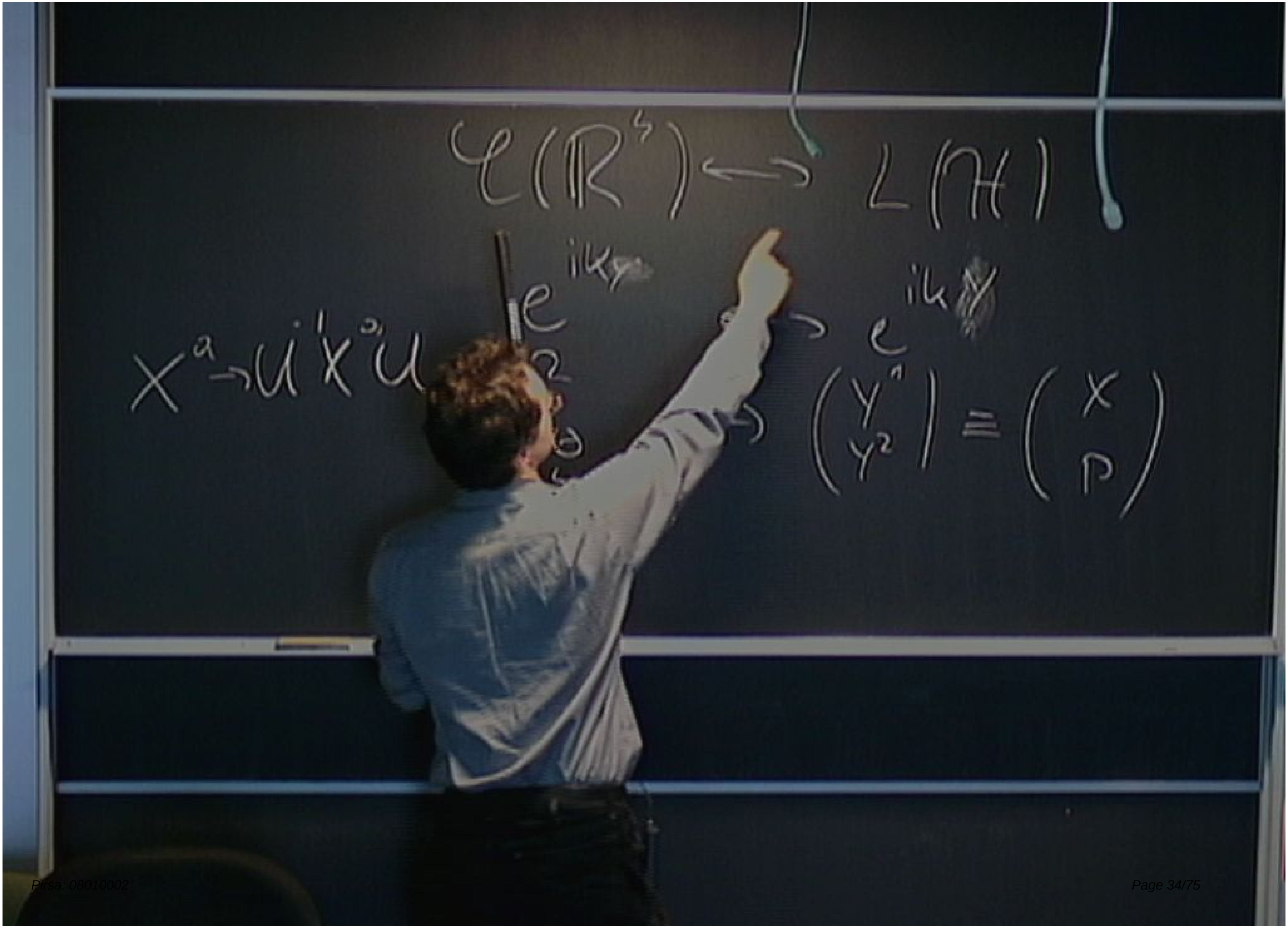

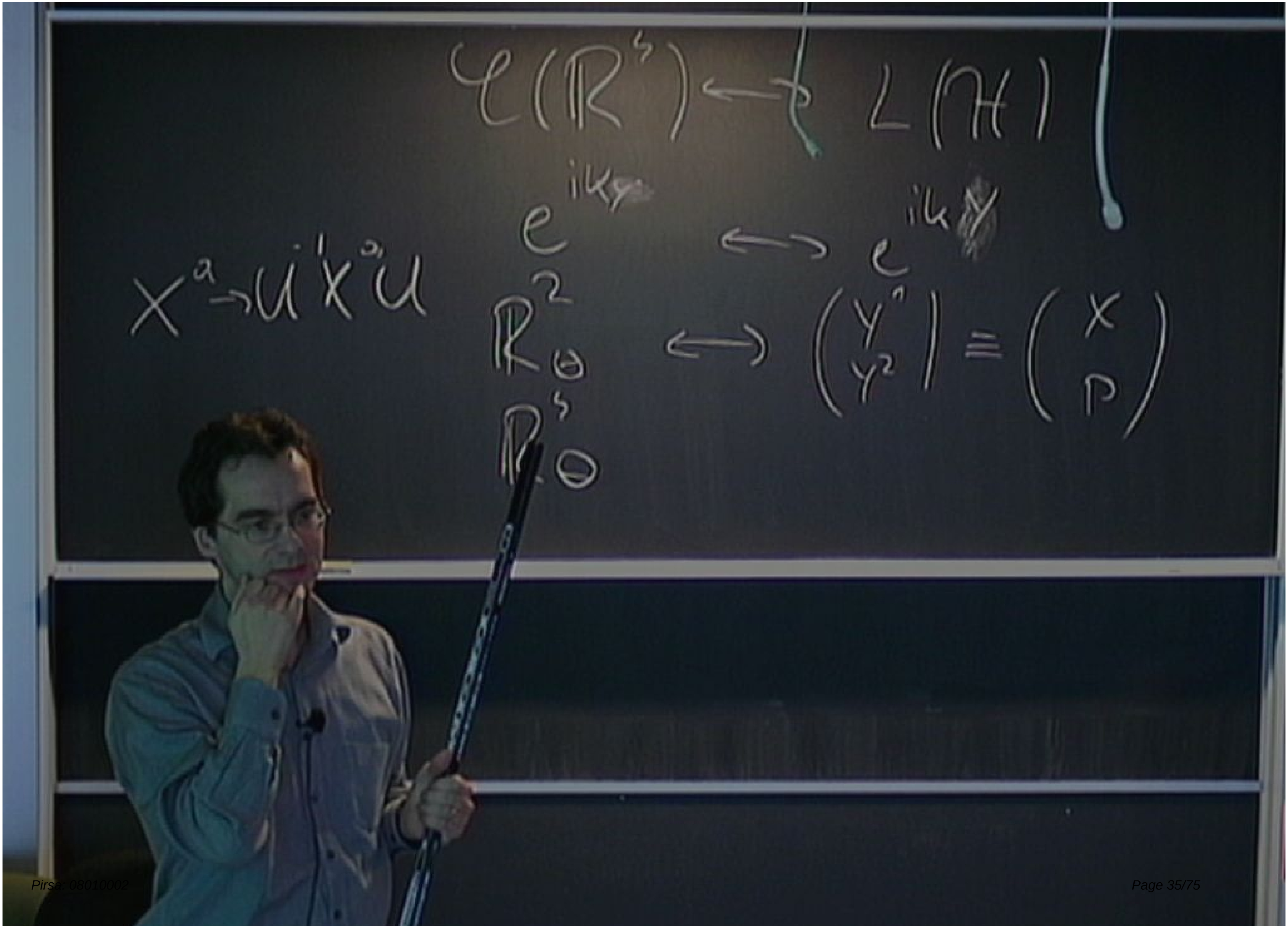
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generalization to $su(n)$ gauge fields

separate $\mathfrak{u}(1)$ and $\mathfrak{su}(n)$ components

$$
\begin{aligned}
X^a &= (\overline{Y}^a + \overline{\theta}^{ab} A_b^0) \otimes \mathbf{1}_n + (\overline{\theta}^{ab} A_b^\alpha \otimes \lambda_\alpha) \\
&=: \quad Y^a \otimes \mathbf{1}_n \quad + \quad \theta^{ab}(y) A_b^\alpha \otimes \lambda_\alpha
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will see:

$\mathfrak{u}(1)$ component Y^a ... dynamical geometry, gravity

$\mathfrak{su}(n)$ components A_a^α ... $\mathfrak{su}(n)$ gauge field coupled to gravity

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Coupling to nonabelian gauge fields (heuristic)

set $X^a = Y^a + \theta^{ab}(y)A_b(y)$ obtain

$$
\begin{aligned}
[X^a, X^b] &= i\theta^{ab}(y) + i\theta^{ac}\theta^{bd}(\partial_c A_d - \partial_d A_c + [A_c, A_d] + O(\theta^{-1}\partial\theta \\
&= i\theta^{ab}(y) + i\theta^{ac}(y)\theta^{bd}(y)F_{cd} + O(\theta^{-1}\partial\theta))
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hence

$$
\begin{aligned}
S_{YM} &= -Tr[X^a, X^b][X^{a'}, X^{b'}]\eta_{aa'}\eta_{bb'} \\
&\approx Tr\left(G^{ab}(y)\eta_{ab} - G^{cc'}(y)\, G^{dd'}(y)\,(F_{cd}\, F_{c'd'} + O(\theta^{-1}\partial\theta))\right)
\end{aligned}
$$

using $Tr(\theta^{ab}(y)F^{ab}) \approx 0$

similar to $\mathfrak{su}(n)$ YM coupled to metric $G^{ab}(y)$

nonabelian gauge fields (correct)

Seiberg-Witten map:

$$X^a = Y^a + \theta^{ab} A_b - \frac{1}{2}(A_c[Y^c, \theta^{ad} A_d] + A_c F^{ca}) \quad + O(\theta^3)$$

- expresses $\mathfrak{su}(n)$ d.o.f. in terms of commutative $\mathfrak{su}(n)$ gauge fields A_a
- relates NC g.t. $i[\Lambda, X^a]$ in terms of standard $\mathfrak{su}(n)$ g.t. of A_a

Volume element:

$$\begin{aligned} (2\pi)^2 Tr\, f(y) &= \int d^4y\, \rho(y)\, f(y), \\ \rho(y) &= \sqrt{\det(\theta^{-1}_{ab})} = (\det(\eta_{ab})\det(G_{ab}))^{1/4} \end{aligned}$$

(cp. Bohr-Sommerfeld quantization)

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flat space: Moyal-Weyl $\quad \bar{\theta}^{ab} = \text{const}$

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small fluctuations: $\quad Y^a = \bar{Y}^a + \bar{\theta}^{ab} A^0_b \quad$ ($\mathfrak{u}(1)$ component)

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implies vacuum equations of motion (linearized)

$$R_{ab} = 0 \; + O(\theta^2)$$

moreover $R_{abcd} = O(\bar{\theta}) \neq 0$ ... nonvanishing curvature

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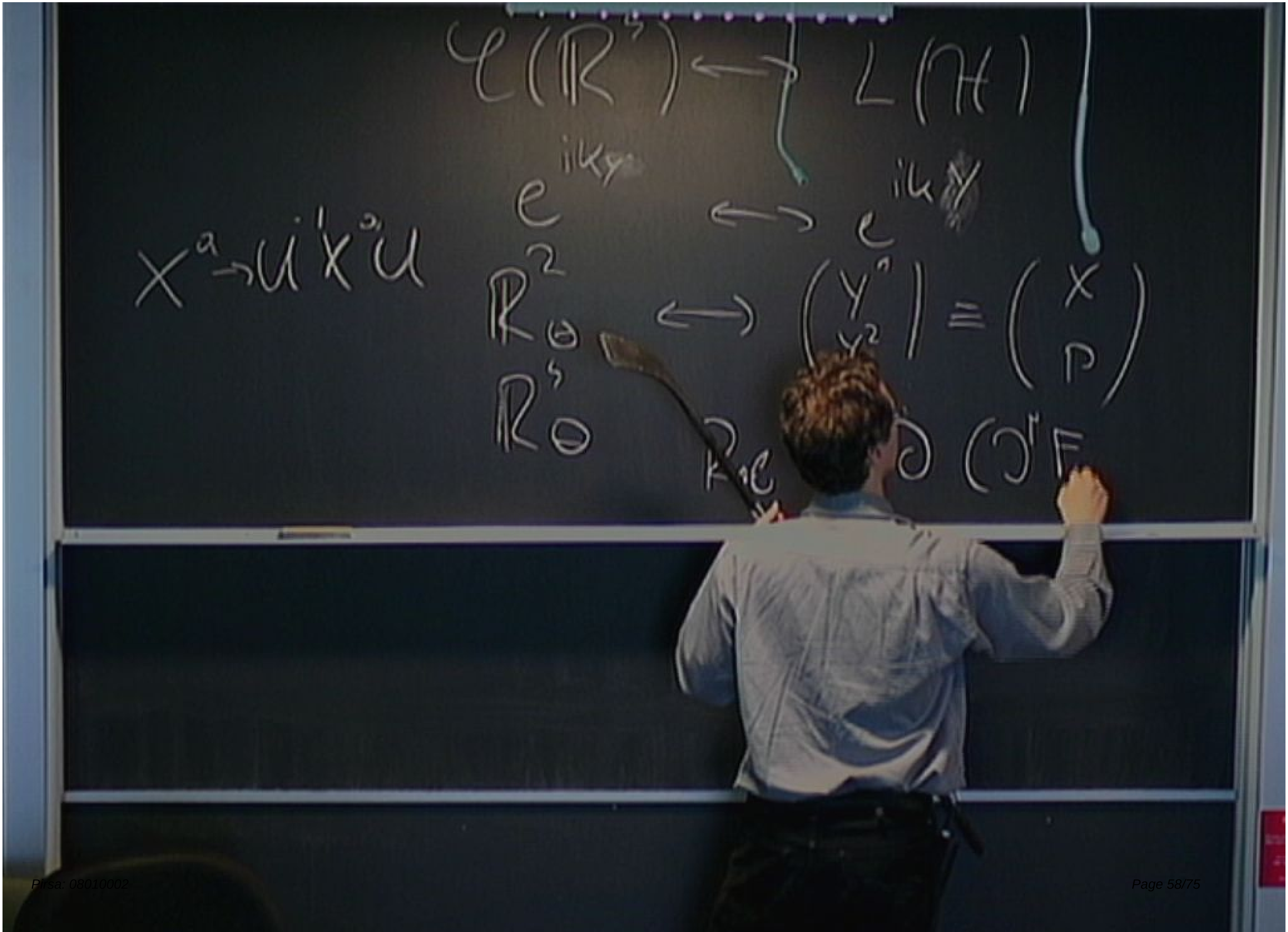

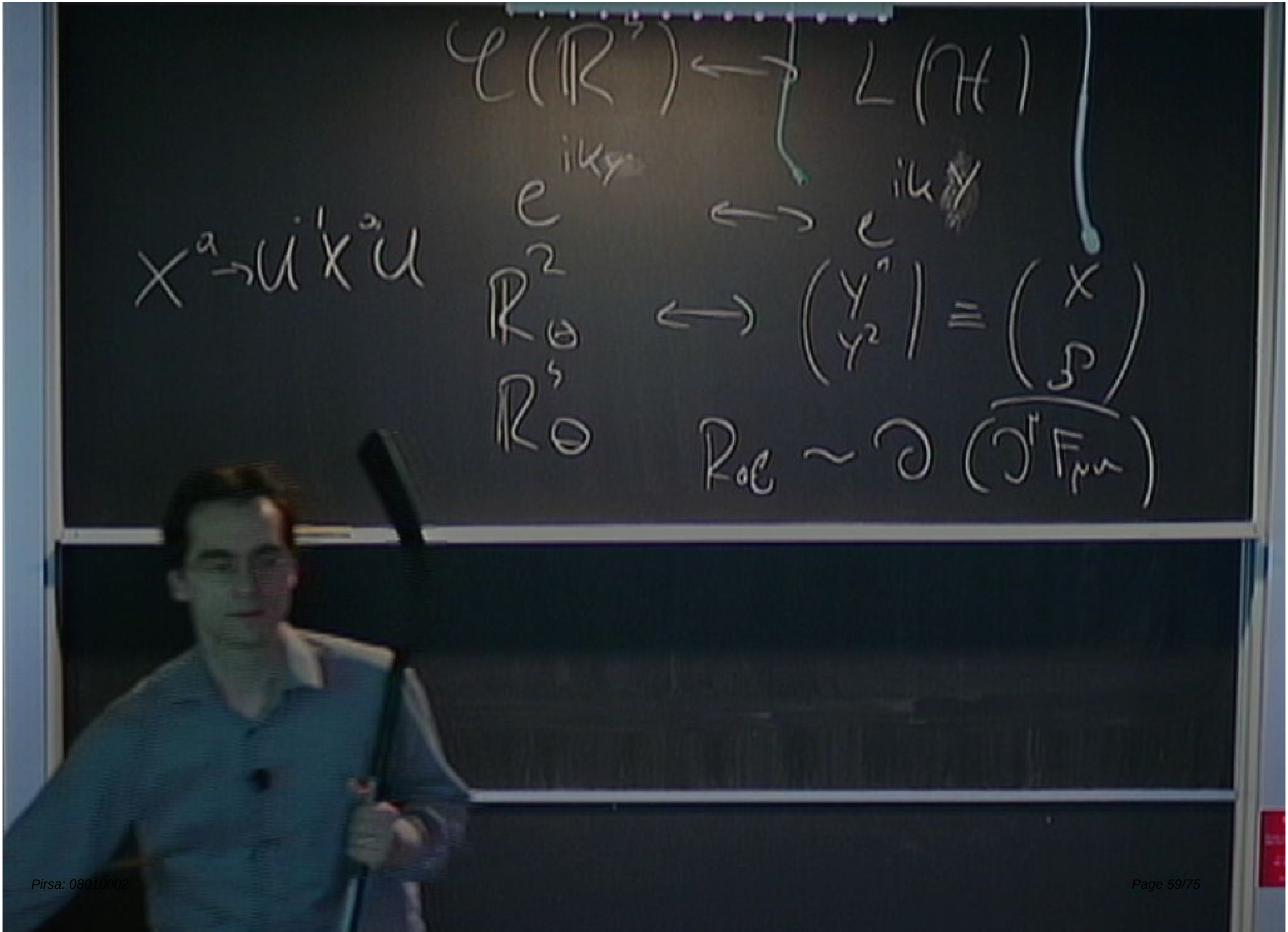

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Newtonian limit

Question: sufficient d. o. f. in G^{ab} for gemetries with matter?

Answer: o.k. at least for Newtonian limit

$$ds^2 = -c^2 dt^2 \Big(1 + \frac{2U(x)}{c^2}\Big) + d\vec{x}^2 \Big(1 + O(\frac{1}{c^2})\Big)$$

where $\Delta_{(3)} U(x) = 4\pi G \rho(x)$ and ρ ...static mass density

can show: $\exists$ sufficient d.o.f. in G^{ab} for arbitrary $\rho(x)$

moreover, vacuum e.o.m. imply

$$ds^2 = -c^2 dt^2 \Big(1 + \frac{2U(x)}{c^2}\Big) + d\vec{x}^2 \Big(1 - \frac{2U(x)}{c^2}\Big)$$

as in G.R.

Question: what about the Einstein-Hilbert action?

Answer:

- tree level: e.o.m. for gravity follow from $\mathfrak{u}(1)$ sector:

$$G^{ac}\partial_c\theta^{-1}_{ab}(y) = 0 \quad \text{implies} \quad \boxed{R_{ab}[\tilde{G}] \sim 0}$$

$$\text{where} \quad \tilde{G}_{ab} = \rho\, G_{ab}, \quad \det(\tilde{G}) = 1$$

  at least for linearized gravity.

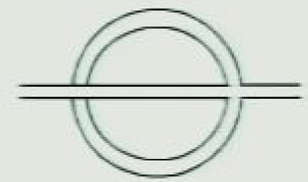
- one-loop: gauge or matter (scalar) fields couple to G_{ab}
  $\Rightarrow$ (Sakharov) induced Einstein-Hilbert action:

$$S_{1-loop} \sim \int d^4y\sqrt{\tilde{G}}\left(c_1\Lambda_{UV}^4 + c_2\Lambda_{UV}^2 R[\tilde{G}] + O(\log(\Lambda_{UV}))\right)$$

Relation with UV/IR mixing

UV/IR mixing of NC gauge theory:

- Quantization of NC field theory $\rightarrow$ new IR - divergences
  nonplanar diagrams: UV-finite, except for $p \rightarrow 0$



$$\Gamma^{NC}[A] \sim g^2 \int d^4p\, (\theta^{ab} F_{ab})^2\, \Lambda^4_{eff}(p) + \ldots$$

$$\Lambda^2_{eff}(p) = \frac{1}{1/\Lambda^2 + \frac{1}{4}p^2/\Lambda^4_{NC}}$$

related to UV divergences; non-renormalizable ?

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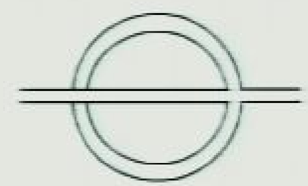

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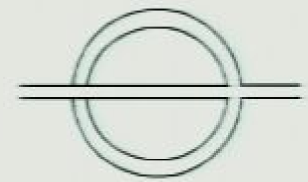

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$\Rightarrow$ expect new divergences in IR due to induced gravity (E-H action) therefore:

explanation for UV/IR mixing in terms of gravitational action

$$\Gamma_{eff}[A] \cong \int d^4y \sqrt{\det \tilde{G}} \left(\Lambda^4 + c\Lambda^2 R[\tilde{G}]\right)$$
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detailed matching UV/IR mixing $\leftrightarrow$ gravity
(H. Grosse, H.S., M. Wohlgenannt, February 2008)

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