Title: The holographic description of non-relativistic conformal field theories

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Abstract: The AdS/CFT correspondence has recently been extended to field theories satisfying the non-relativistic generalization of conformal symmetry, the Schroedinger symmetry. These holographic descriptions offer the potential to do calculations in the strong coupling regime of experimentally-realized condensed matter systems, such as fermions at unitarity. In this talk, we will outline the holographic formulation of such NRCFTs at zero temperature. We will then discuss the embedding of the appropriate geometry into IIB supergravity, and the finite temperature generalization that results. We will conclude with a brief discussion of the holographic description of non-relativistic conformal hydrodynamics and current research projects.

Holography and non-relativistic conformal field theories

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Outline

- Background and motivation
- The dual geometry
- Non-relativistic conformal hydrodynamics and gravity
- Conclusions

What is holography?

- Class of methods: holography, AdS/CFT, gauge/gravity duality
- 't Hooft, Susskind '93 and '95 - Quantum gravity described by dof living on the boundary of spacetime: "Holography"
- Maldacena '97 - IIB string theory on $AdS_5 \times S^5$ equivalent to $\mathcal{N} = 4$ non-abelian conformal field theory (CFT) living on boundary of AdS: "AdS/CFT"

A tool to calculate in strongly coupled theories

- Last 11 years - many generalizations: "Gauge/gravity duality"
- String calculation is manageable when stringy and quantum effects are small
- On field theory side, this corresponds to large coupling and large N
- Can use classical gravity to calculate strongly-coupled processes in CFT
- Recently applied to condensed matter systems: transport in plasmas, superconductors, superfluids, quantum Hall effect

Non-relativistic Conformal Field Theories (NRCFTs)

- Non-relativistic version of conformal algebra is Schrödinger algebra
- Galilean algebra:
  - $H : t \to t + a \quad P_i : x^i \to x^i + b^i$
  - $M_{ij} : x^i \to O^i{}_j x^j \quad K_i : x^i \to x^i + v^i t$
  - M - central charge, corresponds to conserved mass
- Plus scale and conformal transformations:
  - $D : x^i \to \lambda x^i, \ \ t \to \lambda^2 t$
  - $C : t \to \frac{t}{1+ct}, \ \ x \to \frac{x}{1+ct}$

Example: Fermions at unitarity

- An experimentally-realized system that obeys Schrödinger algebra is fermions at unitarity
- System of fermions interacting through a short-ranged potential that is fine-tuned to have infinite scattering length
- Infinite scattering length and short range implies strong coupling
- Is there a holographic description?

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The Schrödinger metric (Son, Balasubramanian and McGreevy '08)

- Key feature of AdS/CFT: global symmetry group of CFT is isometry group of $AdS_5 \times S^5$
- Need to find metric whose isometry group is the Schrödinger group
- Schrödinger group in d space dimensions is subgroup of relativistic conformal group in $d+2$ spacetime dimensions
- $ds^2 = -\frac{(dx^+)^2}{z^4} + \frac{-2dx^+dx^- + d\vec{x}^2 + dz^2}{z^2}$

Comments on the geometry

- $x^+ = \text{time}, \quad \vec{x} = \text{space}, \quad x^-?$
- $\frac{\partial}{\partial x^-} \to M$ - suggests taking x^- to be compact
- Not solution to pure gravity - need some other fields
- Correlation functions of scalars have correct form:
  $\langle \phi(t,x)\phi(0) \rangle \sim t^{-\Delta_0} \exp\left(\frac{iMx^2}{2t}\right)$

Embedding in string theory

- For $d = 2$, geometry was embedded in string theory by Herzog, Rangamani and Ross - Maldacena, Martelli and Tachikawa - Adams, Balasubramanian and McGreevy in '08
- Solution generating techniques (Null Melvin Twist, TsT transformation) produce Schrödinger geometry from *AdS* geometry

$$AdS_{d+3} \xrightarrow{\text{NMT or TsT}} Sch_d$$

- Non-relativistic theory arises from lightcone reduction of deformed $\mathcal{N} = 4$ theory

NRCFTs at finite temperature

- Gravity dual to finite temperature field theory has black hole and *AdS* asymptotics
- Can apply same solution-generating technique to this geometry to get finite temperature NRCFTs

$$\text{Finite temp } AdS_{d+3} \xrightarrow{\text{NMT or TsT}} \text{Finite temp } Sch_d$$

- $ds^2 = g(\beta, r_+), \quad T = \frac{r_+}{\pi\beta}, \quad \mu = \frac{1}{2\beta^2}$
- $\epsilon = P, \quad \frac{\eta}{s} = \frac{1}{4\pi}, \quad \lim_{T\to 0} E = 0$ at constant density

Fluid-gravity correspondence and NRCFTs (Rangamani, Ross, Son, EGT '08)

- Hydrodynamics is the long-wavelength effective description of field theories
- AdS/CFT suggests that equations of hydro should also appear in gravity
- Bhattacharyya, Hubeny, Minwalla, and Rangamani '07 -

Relativistic conformal hydrodynamics

- Relativistic hydro in $(d+1)+1$: $T, u^\mu, u^\mu u_\mu = -1$

$$\partial_\mu T^{\mu\nu} = 0$$

$$T^{\mu\nu} = (\epsilon_{rel} + P_{rel})\, u^\mu u^\nu + P_{rel}\eta^{\mu\nu} - 2\eta_{rel}\tau^{\mu\nu} \qquad (1)$$

Non-relativistic conformal hydrodynamics

- NR hydro in $d+1$: ρ, T, v^i

$$\partial_t \rho + \partial_i \rho v^i = 0$$
$$\partial_t \rho v^i + \partial_j \Pi^{ij} = 0$$
$$\partial_t \left(\epsilon + \frac{\rho v^2}{2}\right) + \partial_i j^i_\epsilon = 0$$
$$\Pi^{ij} = \rho v^i v^j + \delta^{ij} P - \eta \sigma^{ij}$$
$$j^i_\epsilon = \frac{1}{2}\left(\epsilon + P + \rho v^2\right) v^i + \eta \sigma^{ij} v^j - \kappa \partial_i T$$

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Lightcone reduction of relativistic hydrodynamics

- Assume relativistic dofs do not depend on x^-

$$u^+ = \sqrt{\frac{\rho}{\epsilon_{rel} + P_{rel}}}$$

$$u^i = u^+ \left(v^i + \frac{\eta_{rel}}{\rho} \left(\partial_i u^+ - \frac{u^+}{2\left(\epsilon_{rel} + P_{rel}\right)} \partial_i P_{rel} \right) \right)$$

$$P_{rel} = P, \;\; \eta_{rel} = \frac{\eta}{u^+}, \;\; \epsilon_{rel} = 2\epsilon + P$$

- $\kappa = 2\eta \frac{\epsilon + P}{\rho T}$

Constructing the non-relativistic fluid metric

- Start with finite temp, chemical potential Schrödinger metric
- Change to coordinates regular through event horizon
- Perform Galilean boost
- Promote β, r_+, and v^i to slowly-varying functions of space
- Expand Einstein's equations in boundary derivative expansion

Using the TsT transformation to extract hydrodynamics

- Take *AdS* fluid flow geometry from Bhattacharyya et al
- Demand that r_+, u^μ only depend on $(x^+, \vec{x})$
- Use mapping to get non-relativistic variables
- Perform TsT
- Result reduces to earlier solution when $\beta(x^+, x^i), r_+(x^+, x^i)$ are constant and $v^i(x^+, x^i) = 0$
- Need to interpret these parameters in terms of hydro variables

Fluid flow in AdS_{d+3} $\xrightarrow{\quad TsT \quad}$ Fluid flow in Sch_d

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$$\text{Fluid flow in } AdS_{d+3} \xrightarrow{\quad TsT \quad} \text{Fluid flow in } Sch_d$$

What's been done

- Gravity duals of NRCFTs have been constructed
- These NRCFTs have very particular thermodynamics and hydrodynamics
- First steps towards a gravity dual of fermions at unitarity
- New type of holography has been developed

What hasn't been done

- A gravity dual of fermions at unitarity has not been constructed
  - For string embeddings, $d = 2$ rather than $d = 3$
  - $\lim_{T \to 0} E = 0$ at constant density
  - No superfluidity in our holographic models
  - No parameter N in fermions at unitarity to take large

Things to do

- Bottom-up construction of NRCFT with superfluidity
- Look for other for other geometries that are asymptotically Schrödinger
- Look for AdS_6 solutions to string theory
- Put NRCFTs on a space with a defect

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