Title: Quantum fields and Noether charges for kappa-Minkowski

Date: Dec 07, 2007 11:00 AM

URL: http://pirsa.org/07120047

Abstract: There has been much interest, in the past few years, in the kappa-Poincare\'/kappa-Minkowski framework as a possible scenario for a deformation of Poincare\' symmetries at Planck scale. I will show how it is possible to give a physical characterization of the concept of quantum symmetries described by a nontrivial Hopf algebra. In particular, I will discuss the generalization of the Noether analysis for a scalar field in kappa-Minkowski space-time and derive conserved charges associated with each generator of the kappa-Poincare\' Hopf-algebra. Then I will report on a recent proposal for the quantization of a scalar field enjoying kappa-Poincare\' symmetries, which consists in a construction of the Fock-space of the theory consistent with the structure of deformed symmetries. Finally I will comment on possible applications of deformed symmetries scenarios in cosmology.

Outline

- **Motivations:**
    - quantum κ-Minkowski space-time and modification or breakdown of Poincaré symmetries
    - the fate of Poincaré symmetries at Planck scale and q-dS space-time from a cosmological perspective
- **Translation and Lorentz sector symmetries of κ-Minkowski NCST and Noether charges (classical fields)**
- **Quantum fields in κ-Minkowski**
    - quantization and compatibilty with quantum symmetries
    - Noether charges
- **Outlook**
    - theory
    - phenomenology

Quantum space-time and relativistic symmetries

- Planck length L_p and quantum space-time — S-T symmetries broken or deformed? *(lengths transform non-trivially under boosts in Minkowski spacetime. About L_p?)*
- Various studies support the idea of L_p as a lower bound to any distance measurement. Various pictures of space-time at the Planck-scale:

NON-COMMUTATIVE, “FOAMY”, DISCRETE...

$\Downarrow$

Modification of classical symmetries could appear in the space-time quantization
Deformation of the Poincaré algebra in one parameter proportional to the Planck scale

↓	↓	↓
some arguments within LQG	*N.C. S-T*	*Some strings scenarios*

NOT MERELY ACADEMIC

Low energy limit and first order correction to the symmetries (zero order test).
Phenomenology at the Planck scale (UHECR and GRB)

NCST and algebraic deformation: from Lie to Hopf algebras

An intuitive characterization

Algebra of functions on commutative S-T	NC algebra of functions on NC S-T
$f(x) \cdot g(x) = g(x) \cdot f(x)$	$f(x) \cdot g(x) \neq g(x) \cdot f(x)$
$T[f(x) \cdot g(x)] = T[f(x)] \cdot g(x) + f(x) \cdot T[g(x)]$	$T[f(x) \cdot g(x)] = \sum_i [T_i^{(1)} f(x)] \cdot [T_i^{(2)} g(x)]$

Deformed Lie algebras of S-T coordinates are linked to Hopf algebras of symmetries (deformed Lie algebras).

S-T (deformed Lie algebra) $\longleftrightarrow$ **(deformed) Hopf algebra of symmetries**

- Early 90's (Lukierski et al): use "quantum groups" (non-co-commutative Hopf algebra) to describe "quantization" of standard relativistic symmetries (analogous to Moyal quantization of Poisson manifolds: CM$\rightarrow$QM)
- 1994 (Majid and Ruegg): κ-Poincaré and its relation to κ-Minkowski NCST

κ-Poincaré and κ-Minkowski NCST

NCST: a vast literature using the κ-**Minkowski** /κ-**Poincaré framework**

$$[x_m, t] = \frac{i}{\kappa} x_m \; , \qquad [x_m, x_l] = 0$$

Starting from NC complex exponential $e^{ip\hat{x}}$ reguarded as a Fourier basis

- **Fix** a *normal ordering* for the coordinates (e.g. all time coordinates to the right)

$$: e^{ip\hat{x}} :_R \equiv e^{ip_m \hat{x}_m} e^{-ip_0 \hat{x}_0}$$

- Complex exponentials combine in a non-trivial way: using the BCH formula

$$(: e^{iq_1\hat{x}} :_R)(: e^{iq_2\hat{x}} :_R) =: e^{i(q_1 \dot{+} q_2)\hat{x}} :_R$$

  where $q_1 \dot{+} q_2 = (q_1^0 + q_2^0; \vec{q}_1 + e^{-\frac{q_1^0}{\kappa}} \vec{q}_2)$

- Different orderings: **equivalent descriptions of the same field!**

$$: e^{ik\hat{x}} :_S \equiv e^{-i\frac{k_0}{2}\hat{x}_0} e^{ik_m\hat{x}_m} e^{-i\frac{k_0}{2}\hat{x}_0} \qquad (k^0; \vec{k}) = (p^0; \vec{p} e^{+\frac{p^0}{2\kappa}})$$

- Field on κ-Minkowski

$$: \Phi(\hat{x}) := \int d^4 q \, \tilde{f}(q) : e^{iq\hat{x}} :$$

Ordering prescription is fixed, drop ^ from now on...

κ-Poincaré symmetries

The **action** of Poincaré symmetries on such fields are well defined[1]:

- Translations are **"classical"** on a single plane wave i.e.

$$P_\mu \triangleright : e^{ip\hat{x}} := p_\mu : e^{ip\hat{x}} :$$

Different κ-Poincaré basis acts on the same field in a different way...

$$P^s_\mu \triangleright : e^{ip\hat{x}} : \neq P_\mu \triangleright : e^{ip\hat{x}} :$$

What is going on?

- Boost are **"deformed"** and the same **"ambiguity"** in defining the action of generators appears

- Rotations are **"classical"**

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[1] S. Majid, H. Ruegg, Phys. Lett. B334: 348-354,1994.
A. Agostini, G. Amelino-Camelia, F. D'Andrea Int.J.Mod.Phys. A19: 5187-5220, 2004

Translations and Noether charges for classical fields

Symmetries are characterized in terms of the infinitesimal variations of the fields

- In the κ-deformed case we also **need to specify** dx_μs which must obey

$$[x_j + dx_j, x_0 + dx_0] = \frac{i}{\kappa}(x_j + dx_j)\ , \quad [x_i + dx_i, x_j + dx_j] = 0$$

- Insisting on the **Leibnitz rule**

$$df = i dx_\mu P_\mu f(x)$$

Different P_μ bases leads[2] to the same df!

- κ-Klein-Gordon e.o.m. $C_\kappa(P_\mu)\Phi = 0$

Noether analysis for translations $\Rightarrow$ 4 **independent conserved charges** (i.e. constants)

$$Q^\kappa_\mu = \int d^4p \, \frac{e^{3p_0/\kappa}}{2} p_\mu \tilde{\Phi}(p_0, \vec{p})\tilde{\Phi}(-p_0, -e^{p_0/\kappa}\vec{p})\frac{p_0}{|p_0|}\delta(C_\kappa(p_\mu))$$

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[2] A. Agostini, G. Amelino-Camelia, M. Arzano, A. M., R. A. Tacchi, Mod.Phys.Lett. A22: 1779-1786, 2006.

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Symmetries and symplectic geometry of κ-fields

As an **alternative strategy** [M. Arzano and **A.M.**, Phys. Rev. D 75, 081701 (2007)]

- **Key point**: $\{\phi(x);\pi(x)\} \in \Gamma \longleftrightarrow \Phi \in \mathcal{S}$ **identify the phase space** Γ **with the space of solutions of the equation of motion** $\mathcal{S}$
- On $\mathcal{S}$ is defined a **symplectic 2-form** ω

Our strategy:

(i) **use a map** $\mathfrak{m}$ **between** $\mathcal{S}$ **and** $\mathcal{S}_\kappa$ **to define a symplectic structure on** $\mathcal{S}_\kappa$

(ii) **express the conserved charges associated with** κ**-symmetries through the symplectic structure**

Noether charges associated with translations can be written as

$$Q^\kappa_\mu = \frac{1}{2}(\Phi, P_\mu \triangleright \Phi)$$

One gets the same κ**-Noether charges** Q^κ_μ **!** with a bonus... an **inner product**[3]

$$(\Phi_1, \Phi_2)_\kappa = \int \frac{d^4p}{(2\pi)^3}\, \delta(\mathcal{C}_\kappa(p))\, \mathrm{sign}\left(e^{\frac{p_0}{\kappa}} - 1\right) e^{\frac{3p_0}{\kappa}}\, \tilde{\Phi}^*{}_1(\dot{-}p)\tilde{\Phi}_2(p)\,.$$

[3] Where the antipode $\dot{-}p = (p_0, \vec{p}\, e^{\frac{p_0}{\kappa}})$

Lorentz sector and Noether charges for classical fields

- **Rotation transformations act as classically (trivial co-algebraic structure)**
  - Infinitesimal parameters σ_j commute with x_μ
  - None ambuity in the definition of infinitesimal rotations
  - Noether analysis $\Rightarrow$ 3 **independent conserved charges**[4]

$$Q_j^R = \frac{i}{2}\int d^4k \, \frac{\tilde{k}_0}{|\tilde{k}_0|} \, e^{2\lambda k_0} \, \tilde{\Phi}(-k_0, -e^{\lambda k_0}\vec{k})\epsilon_{jlm}k_m \frac{\partial\tilde{\Phi}(k)}{\partial k_l}\delta(\tilde{k}^2)$$

- **Boosts transformations are highly non-trivial (action and co-algebra "deformed")**
  - No infinitesimal parameters τ_j's acting by associative multiplication on the x_μ's. No pure boost
  - **Consider whole Lorentz sector (τ_j and σ_j noncommutative)**
  - Noether analysis $\Rightarrow$ 3 **independent conserved charges**

$$Q_j^N = \frac{i}{2}\int d^4k \frac{\tilde{k}_0 e^{\lambda k_0}}{|\tilde{k}_0|}\tilde{\Phi}(-k_0, -e^{\lambda k_0}\vec{k})\left\{k_j \frac{\partial\tilde{\Phi}(k)}{\partial k_0} + \frac{\partial}{\partial k_j}\left[\left(\frac{1-e^{-2\lambda k_0}}{2\lambda} - \frac{\lambda}{2}|\vec{k}|^2\right)\tilde{\Phi}(k)\right] - \lambda k_j \tilde{\Phi}(k)\right\}$$

4 G. Amelino-Camelia, G. Gubitosi, A. M., P. Martinetti, F. Mercati 0707.1863 [hep-th]

Intermission

- Most interest in κ-Minkowski/κ-Poincaré scenarios from the "phenomenological" side: **deformations** of energy-momentum **dispersion relation** and deformed Casimir

$$C_\kappa = \left(2\kappa \sinh\left(\frac{P_0}{2\kappa}\right)\right)^2 - \vec{P}^2 e^{\frac{P_0}{\kappa}}$$

  testable predictions (e.g. time-of-flight tests using GRB..)

- HOWEVER ambiguity in defining the action for generators of translations has been cause for **concern** in the recent past...

- First results in defining translations **un-ambigously** and the Noether charges obeying deformed dispersion relation[5]

- At least two different choices: 4D and 5D differential calculus. Discussion on still vivid. Key point: covariance under Hopf-algebras of symmetries: bi-covariance

- All this was done with *classical fields*...need **quantum fields**...

[5] A. Agostini, G. Amelino-Camelia, M. Arzano, A.M., R. A. Tacchi, Mod.Phys.Lett. A22: 1779-1786, 2007.

Quantization: κ-Fock space and κ-Poincaré-symmetries I

Based on [M. Arzano and **AM**, Phys. Rev. D 76, 125005 (2007)]

Assume the physical Hilbert space to be a deformation of the standard bosonic $\mathcal{F}(\mathcal{H})$

"One-particle" κ-Hilbert space of a free quantum (scalar) field theory:

- Complexification of the solutions' space of the κ-deformed EOM $\mathcal{S}_\kappa^{\mathbb{C}}$
- Turn $\mathcal{S}_\kappa^{\mathbb{C}}$ into a Hilbert Space by "κ-deformed" inner product
- For modes on the "positive energy" mass-shell $(\omega^+(\vec{k}))$ with $|\vec{k}| > \kappa$ the inner product is no longer positive definite! Restrict to the subspace $\mathcal{S}^{\kappa+}_{|\vec{k}|\leq\kappa} \subset \mathcal{S}^\kappa$.

Mode components are truncated at the Planck scale κ. Our κ-one particle Hilbert space $\mathcal{H}_\kappa$ will be defined by the completion of $\mathcal{S}^{\kappa+}_{|\vec{k}|\leq\kappa} \subset \mathcal{S}^\kappa$ in the inner product.

- Consider the (normalized) plane wave basis $\{\phi^+_{\vec{p}}\}$ of $\mathcal{S}^{\kappa+}_{|\vec{k}|\leq\kappa} \subset \mathcal{S}^\kappa$.
- Characterize $b^\dagger$ and b relative to this specific basis in terms of their action on the vacuum state they single out in $\mathcal{F}_\kappa(\mathcal{H})$.

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Quantization: κ-Fock space and κ-Poincaré-symmetries II

κ-**bosonic Fock space**: consider first the two-mode tensor product states

Compatibility with symmetries:

$$Y \blacktriangleright (a \circ b) = \circ[\Delta Y \blacktriangleright (a \otimes b)] = \sum (Y_{(1)} \blacktriangleright a) \circ (Y_{(2)} \blacktriangleright b)$$

- $P_\mu \triangleright \phi_{\vec{p}} \otimes \phi_{\vec{q}} = (p \dot{+} q)_\mu \, \phi_{\vec{p}} \otimes \phi_{\vec{q}}$
- Look for momentum labels $\sigma^\kappa_{12}(\vec{p})$ and $\sigma^\kappa_{12}(\vec{q})$ such that

$$P_\mu \triangleright [\phi_{\sigma^\kappa_{12}(\vec{q})} \otimes \phi_{\sigma^\kappa_{12}(\vec{p})}] = (p \dot{+} q)_\mu \, [\phi_{\sigma^\kappa_{12}(\vec{q})} \otimes \phi_{\sigma^\kappa_{12}(\vec{p})}]$$

No standard "flip" of momentum labels

- Verify that is unique!

$$\sigma^\kappa_{12}(\vec{q}) = \tilde{q} = (\tilde{q}_0; \vec{q}\, e^{-p_0/\kappa}) \qquad \sigma^\kappa_{12}(\vec{p}) = \tilde{p} = (\tilde{p}_0; \vec{p}\, e^{\tilde{q}_0/\kappa})$$

with $\tilde{q}_0 = \omega^+(\vec{q}\, e^{-\omega^+(\vec{p})/\kappa})$ and $\tilde{p}_0 = \omega^+(\vec{p}\, e^{\tilde{q}_0/\kappa})$ and $\omega^+(\vec{p}) = -\kappa \log(1 - |\vec{p}|/\kappa)$.

Action of two creation operators on the vacuum: κ-**symmetrized two-particle state**

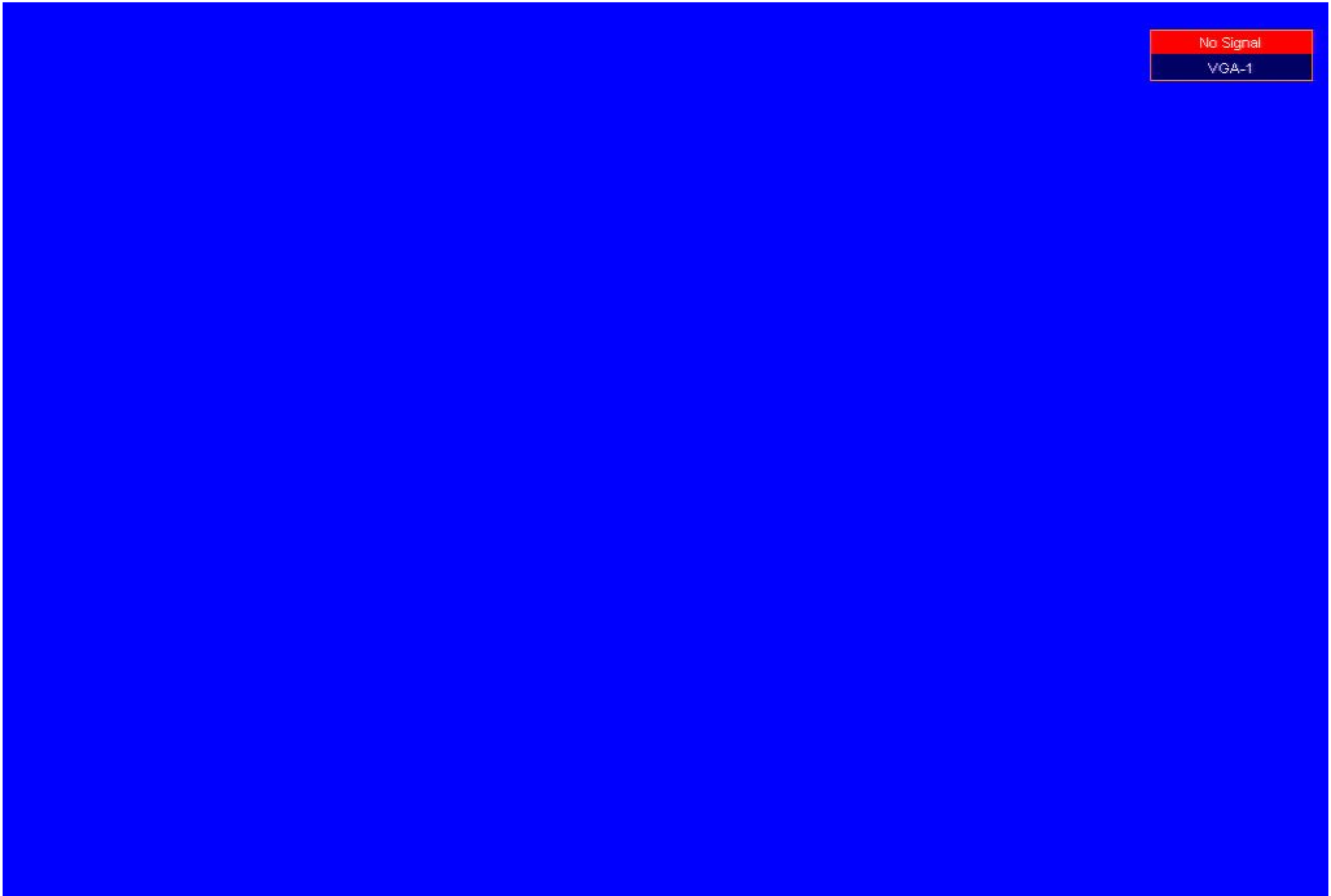
$$b^\dagger_p \star b^\dagger_q |0> = (0, 0, 1/\sqrt{2}(\phi_p \otimes \phi_q + \phi_{\tilde{q}} \otimes \phi_{\tilde{p}}), 0, ...) = b^\dagger_{\tilde{q}} \star b^\dagger_{\tilde{p}} |0>$$

No Signal

VGA-1

No Signal

VGA-1


No Signal
VGA-1

Quantum fields, symplectic-geometry, quantum charges

Deformed field operator acting on the Hilbert space: $\hat{\Phi} = \sum_{\vec{p},\,|\vec{p}|\leq\kappa}\left(\phi_p b_{\vec{p}} + \bar{\phi}_p b^{\dagger}_{\vec{p}}\right)$

- Symplectic-geometry charges as operators on $\mathcal{F}_{\kappa}(\mathcal{H})$

$$\hat{Q}_{\mu} = \frac{1}{2}\sum_{\vec{p},\,|\vec{p}|\leq\kappa}\left[p_{\mu}\, b^{\dagger}_{\vec{p}} \star b_{\vec{p}} - e^{-3\omega^{+}(\vec{p})/\kappa}(\dot{-}p_{\mu}) b_{\vec{p}} \star b^{\dagger}_{\vec{p}}\right]$$

- Mean value over $|\,p>= b^{\dagger}_{\vec{p}}|\,0>=(0,\,\phi_p,\,0...)$

  - Energy-momentum dispersion-relation is modified

$$|\vec{Q}| = \kappa \tanh\left(\frac{Q_0}{\kappa}\right)$$

  - Vacuum energy-momentum turns out to be finite.

$$Q^{Vac}_{\mu} = -\frac{1}{2}\left(\sum_{\vec{k},\,|\vec{k}|\leq\kappa} e^{-3\frac{\omega^{+}(\vec{k})}{\kappa}}(\dot{-}k^{+}_{\mu})\right)$$

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Toward a quantum FRW Universe: q-dS algebra and NCI

Flat noncommutative space-time

⇓

Planck-scale-induced energy dependence of speed of light

$[m^2 = p^\mu \eta_{\mu\nu} p^\nu + F_{flat}(L_p, p^\alpha)]$

⇓

Classical curved space-time + Planck-scale noncommutativity

⇓

no-interplay between Planck scale effects and curvature effects

$[m^2 = p^\mu g_{\mu\nu} p^\nu + F(L_p, p^\alpha)]$

⇓

Noncommutative inflation

⇑

Interplay between curvature and Planck scale [6]

$$[m^2 = p^\mu g_{\mu\nu} p^\nu + F(\Lambda, L_p, p^\alpha)]$$

⇑

Quantization of a curved spacetime **(dS)** ⇒ quantization of symmetries **(q-dS algebra)**

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[6] A. Amelino-Camelia, N.R. Bruno, G. Mandanici and **A.M.**, to appear; **A.M.** Czechoslovak J.P. vol.55,1479 (2005).

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Quantization of a curved spacetime **(dS)** $\Rightarrow$ quantization of symmetries **(q-dS algebra)**

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[6] A. Amelino-Camelia, N.R. Bruno, G. Mandanici and **A.M.**, to appear; **A.M.** Czechoslovak J.P. vol.55,1479 (2005).

Inönü-Wigner contraction of q-dS in 2+1D

1. Assuming the relation between deformation parameter z, Planck-scale L_p and dS constant of curvature H

$$z = (L_p H)^\alpha, \qquad \alpha \in \mathbb{R}$$

$H \to 0$ limit

- $\alpha < 1 \quad \Rightarrow$ singular limit
- $\alpha = 1 \quad \Rightarrow$ in the zeroth order in H one obtains the κ-Poincaré algebra in bicrossproduct basis [7]
- $1 < \alpha < 3 \quad \Rightarrow$ Poincaré algebra in the zeroth order in H
- $\alpha > 3 \quad \Rightarrow$ Classical de Sitter algebra up to second order in H

2. Assuming the relation[8]

$$z_{LQG} = \frac{2\pi}{2 + \frac{1}{L_p H}}$$

the κ-Poincaré algebra is recovered in the limit of "flat space-time" $H \to 0$.

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[7] J.Lukierski, A.Nowicki, H.Ruegg hep-th/9108018; S. Majid, H.Ruegg PLB 334 ,1994

[8] G. Amelino-Camelia L.Smolin A.Starodubtsev Class.Quant.Grav.21 3095-3110, 2004

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Conclusions

- Physical characterization of κ-Poincaré transformations in terms of Noether charges for classical fields.
- Fock space constructed. Next step: interacting theory. (See recent quantum gravity models...)
- Further developments of the quantization scheme...
- Developing applications to cosmology...

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