Title: Centrally extended algebras in 3d gravity

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Abstract: After a brief motivation for studying 3d gravity, a review of Strominger argument will be given, showing that 3d quantum gravity with negative cosmological constant is a conformal field theory. Classical phase space tools will be introduced to develop semi-classical analyses of gravity with zero cosmological constant, and, with negative cosmological constant and closed timelike curves.

Centrally extended algebras in $3d$ gravity

Geoffrey Compère
University of California at Santa Barbara

In collaboration with

Glenn Barnich.
U.L.B. & International Solvay Institutes
Brussels, Belgium

Stéphane Detournay.
Universita' degli Studi di Milano
Italy

Outline

1. Motivation
2. Main result
3. Mathematical tools
4. Detailed results
5. Summary

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- Solutions with closed time-like curves (Gödel universe)
- Drawback: Some qualitative differences with $4d$

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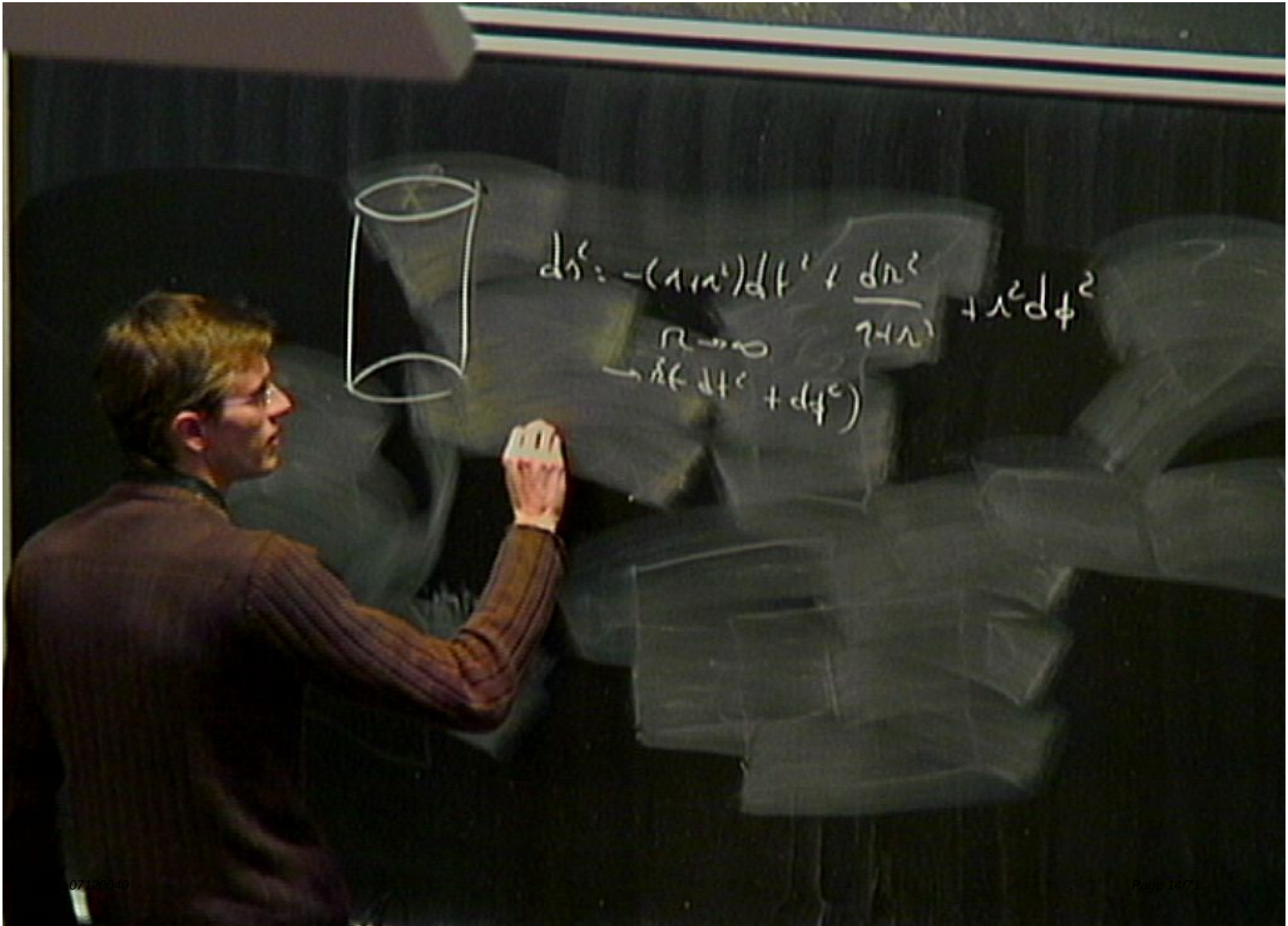
ds² = -(1+r²)dt² + dr²/(1+r²) + r²dφ²
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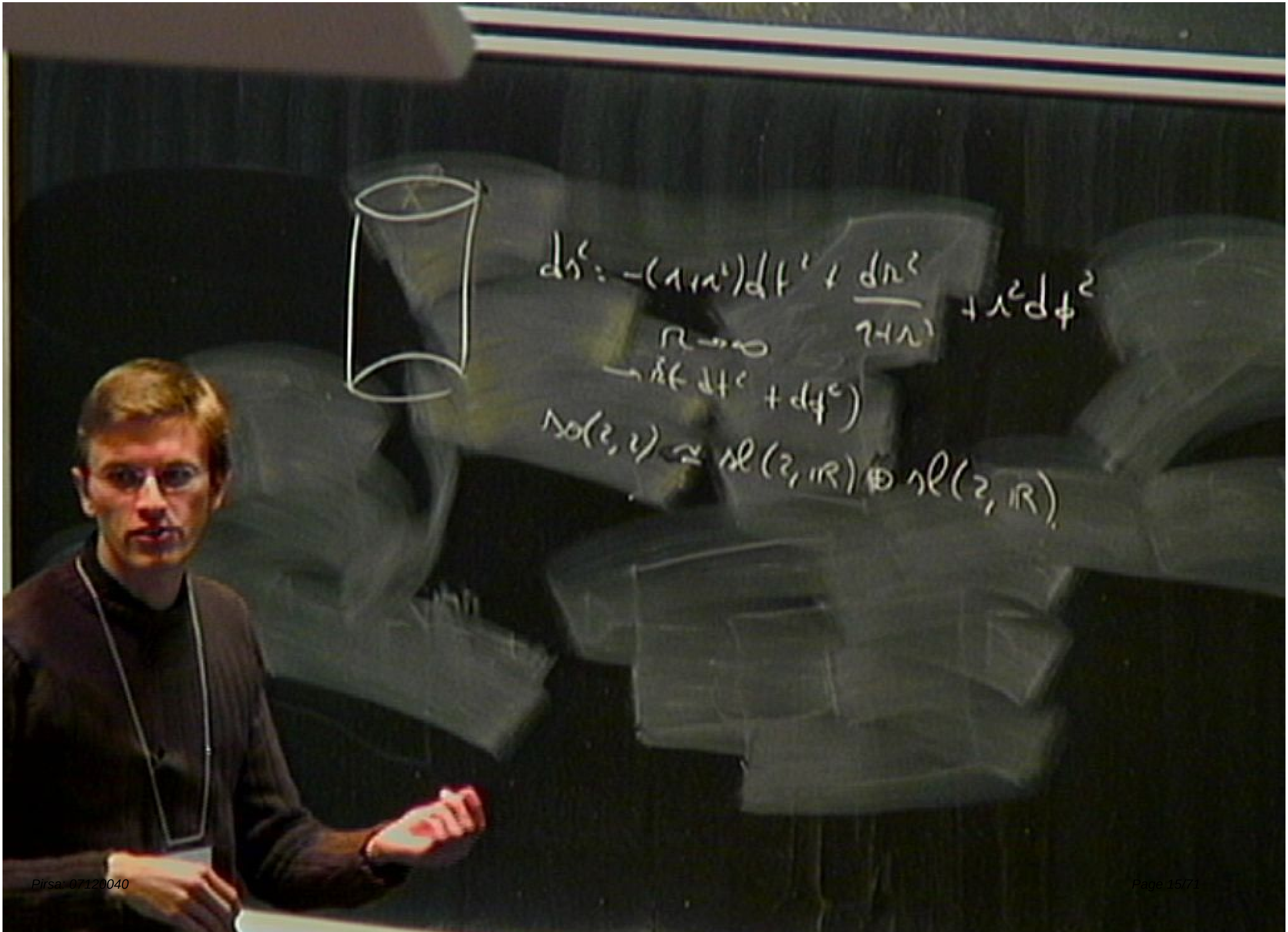
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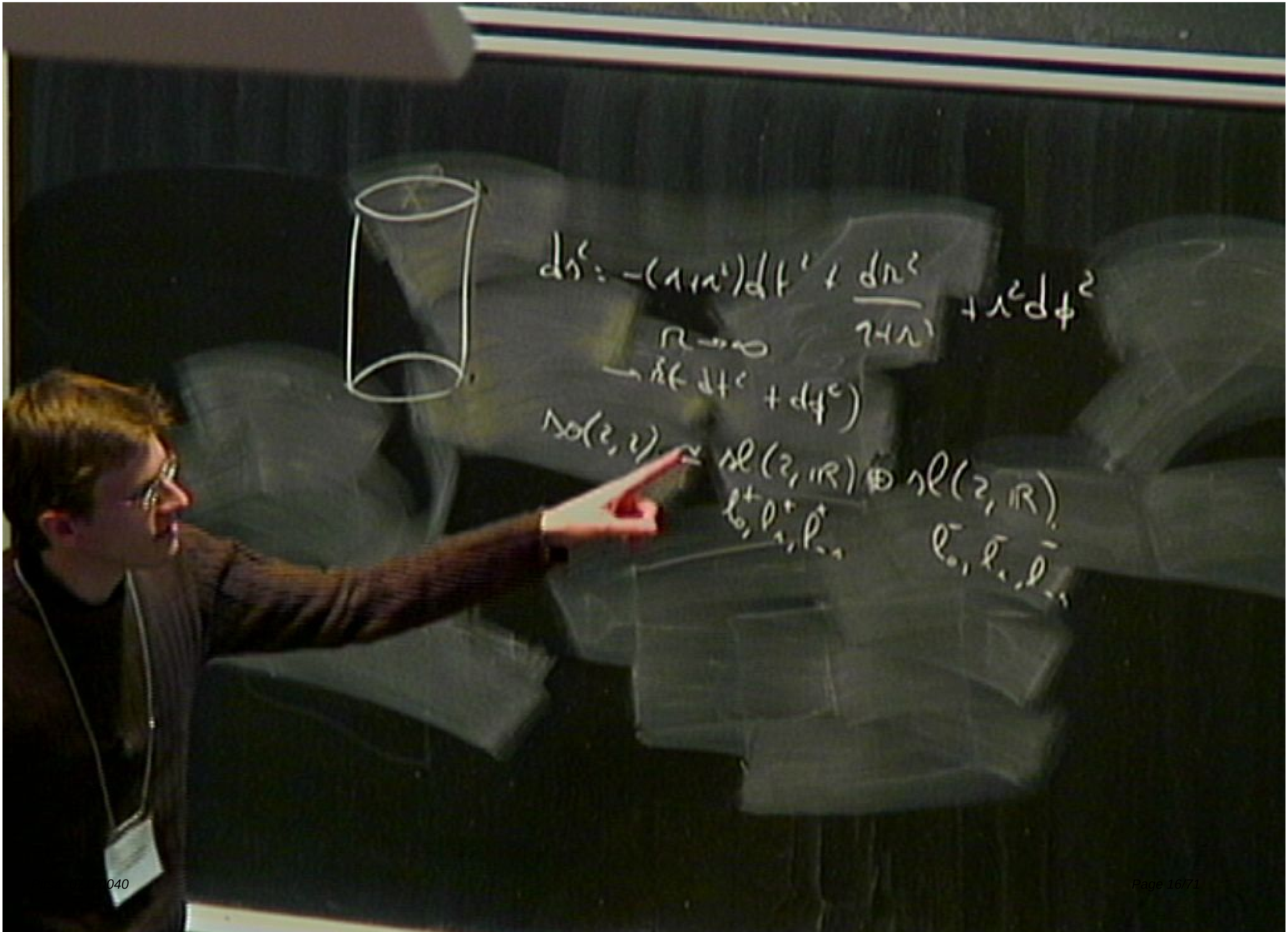
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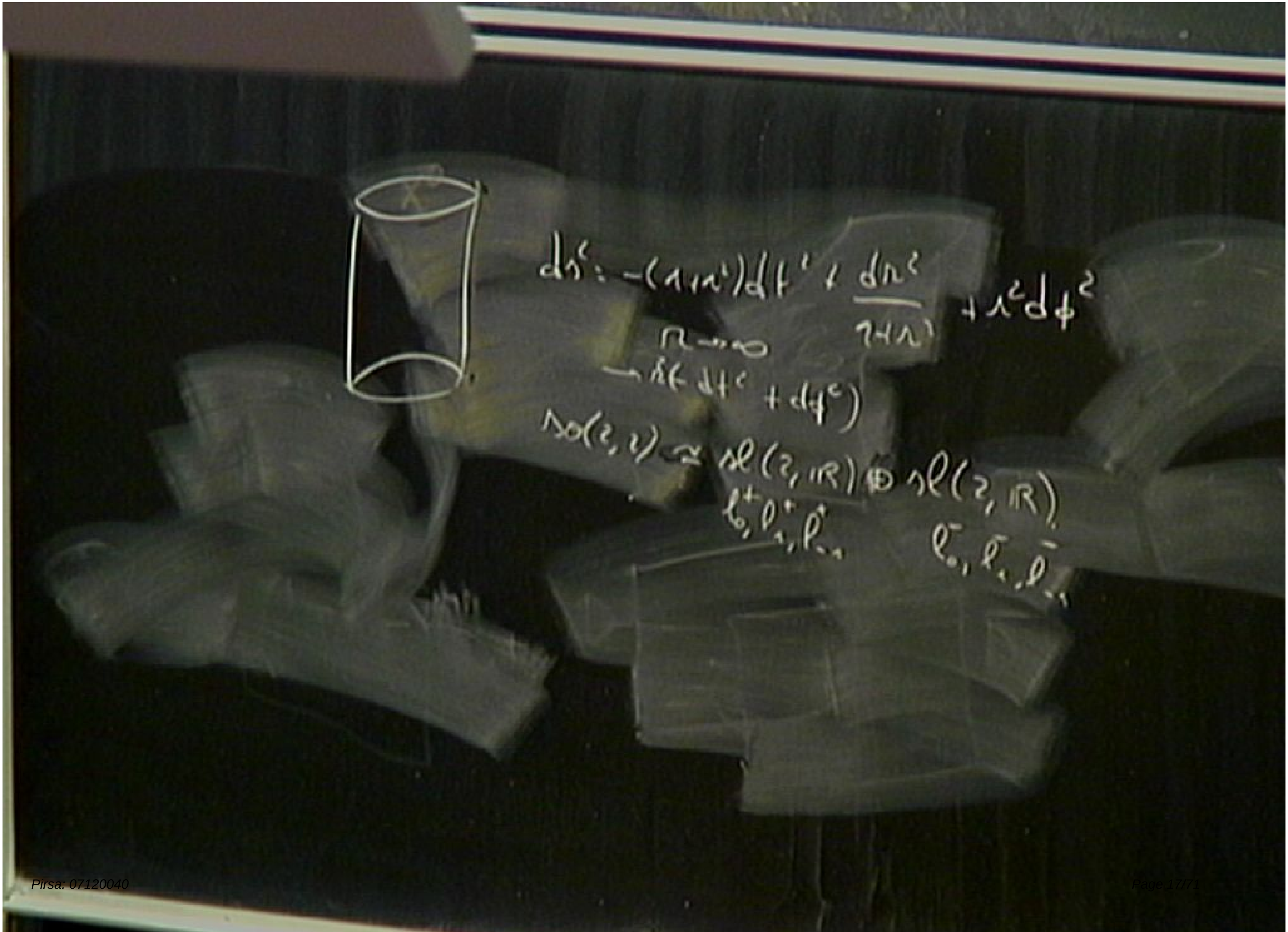
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l_0^+, l_1^+, l_{-1}^+
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Reasoning of *Strominger (1998)* for black holes in $3d$ adS space.

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- The asymptotic symmetry group is isomorphic to the conformal group of the boundary cylinder. It contains two copies of the Witt algebra $i[l_n, l_m] = (n-m)l_{n+m}$.
- The canonical representation of this algebra by asymptotic charges L_n admits a central extension [*Brown, Henneaux*, 86]:

$$i\{\mathcal{L}_n, \mathcal{L}_m\} = (n-m)\mathcal{L}_{n+m} + \frac{c}{12}n(n^2-1), \quad c = \frac{3l}{2G}$$

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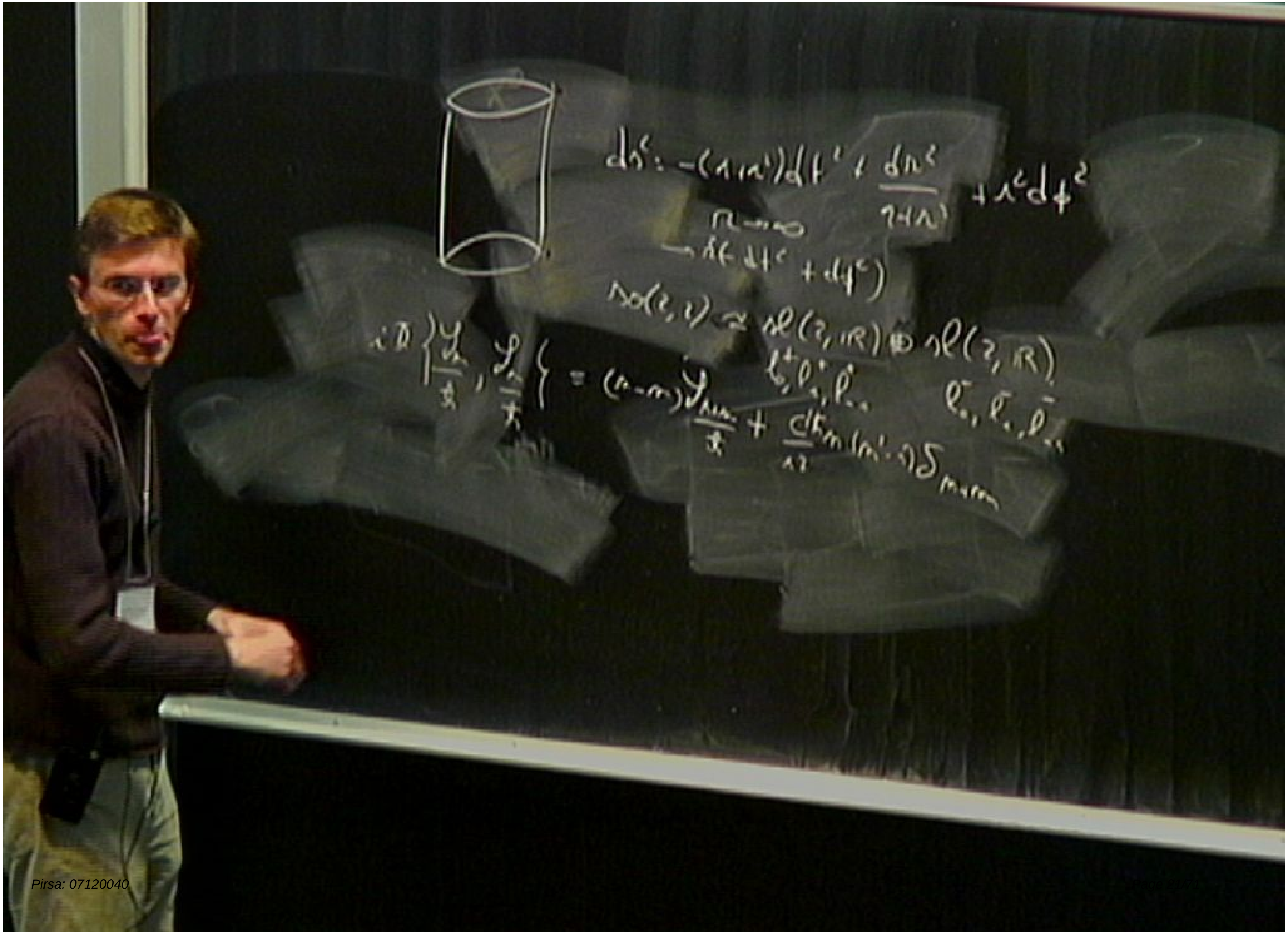
- In semi-classical regime, these charges become operators which are recognized as the Virasoro generators of a conformal field theory.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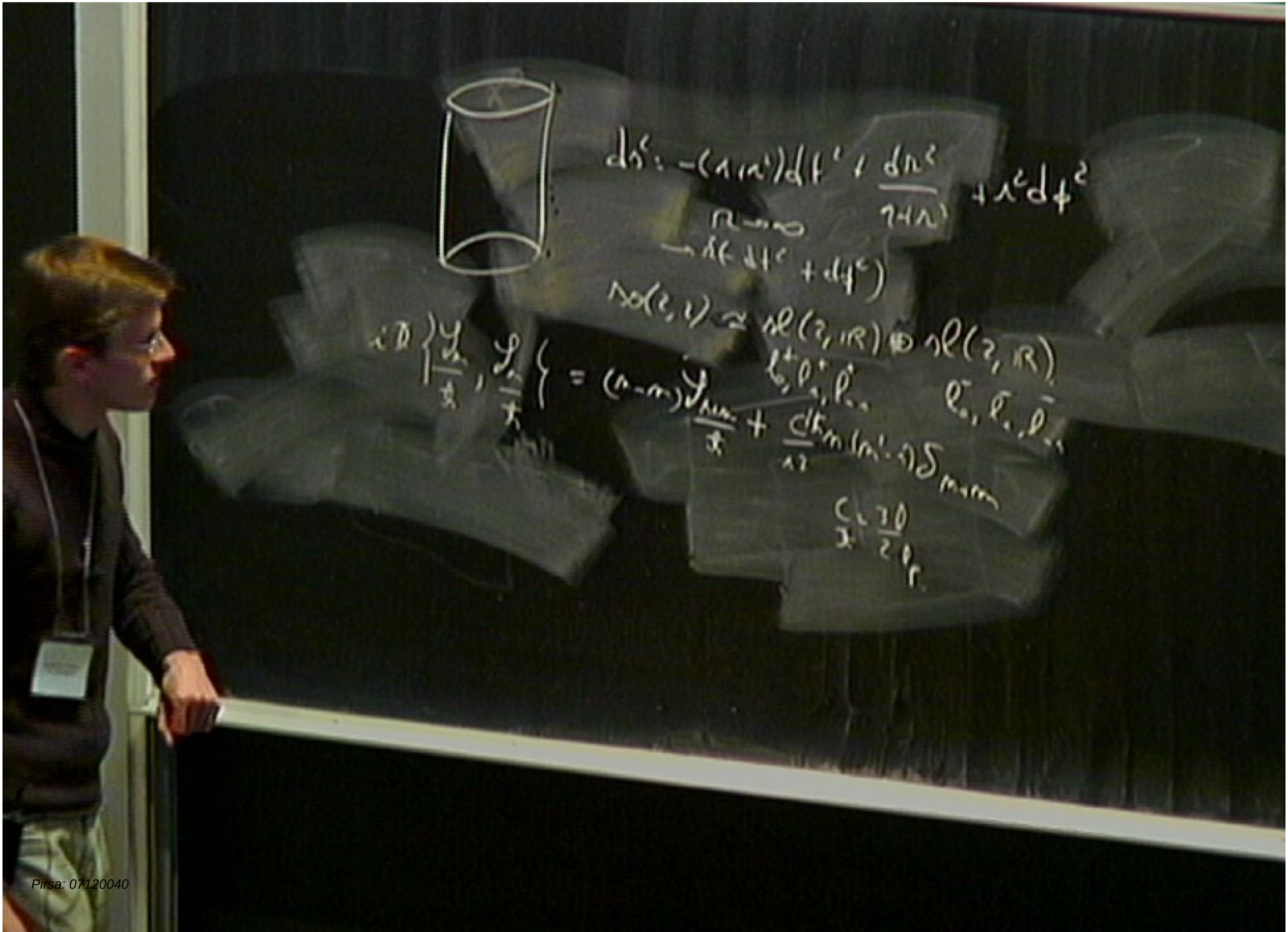

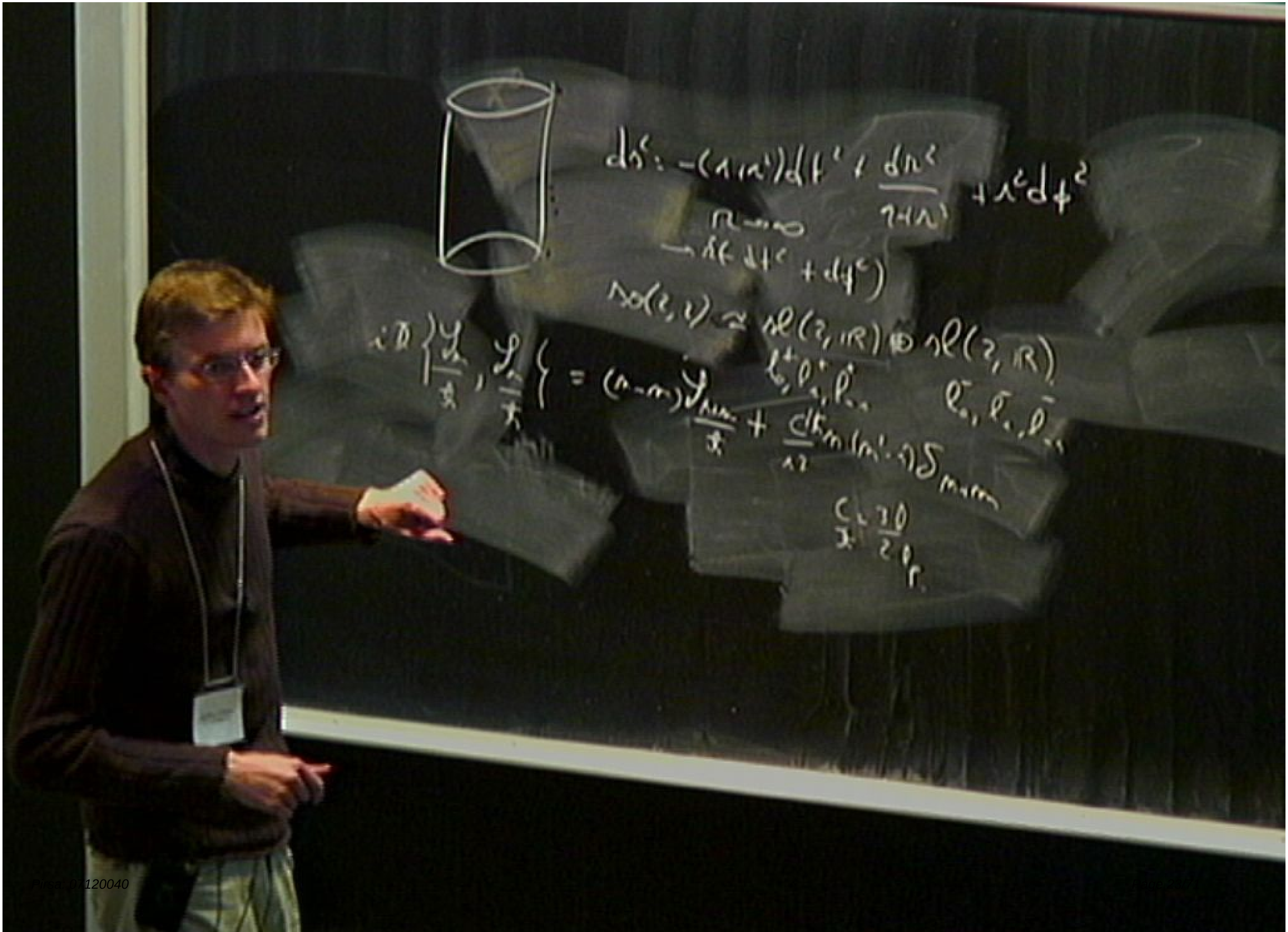

- **Since the states of quantum gravity with** AdS_3 **boundary conditions must form a representation of the conformal algebra, this quantum gravity is a conformal field theory** (AdS_3/CFT_2 correspondence).

- The number of states with given zero modes $\mathcal{L}_0^+ = (M+J)/2$, $\mathcal{L}_0^- = (M-J)/2$, is given by the Cardy formula when $\mathcal{L}_0^+, \mathcal{L}_0^-, c \gg 1$,

$$N = \exp\left(2\pi\sqrt{\frac{c\mathcal{L}_0^+}{6}} + 2\pi\sqrt{\frac{c\mathcal{L}_0^-}{6}}\right) = \exp S_{micro}$$

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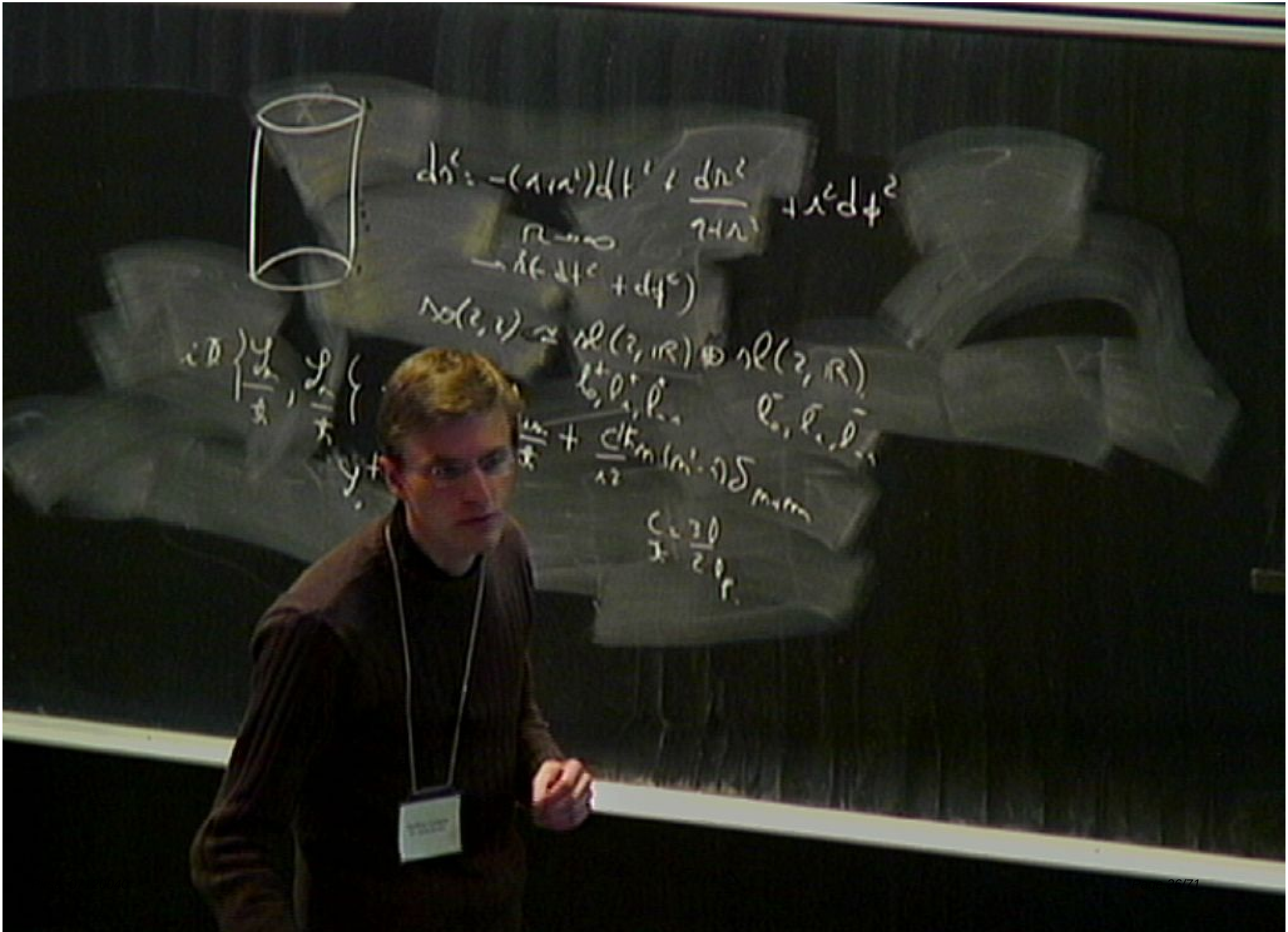

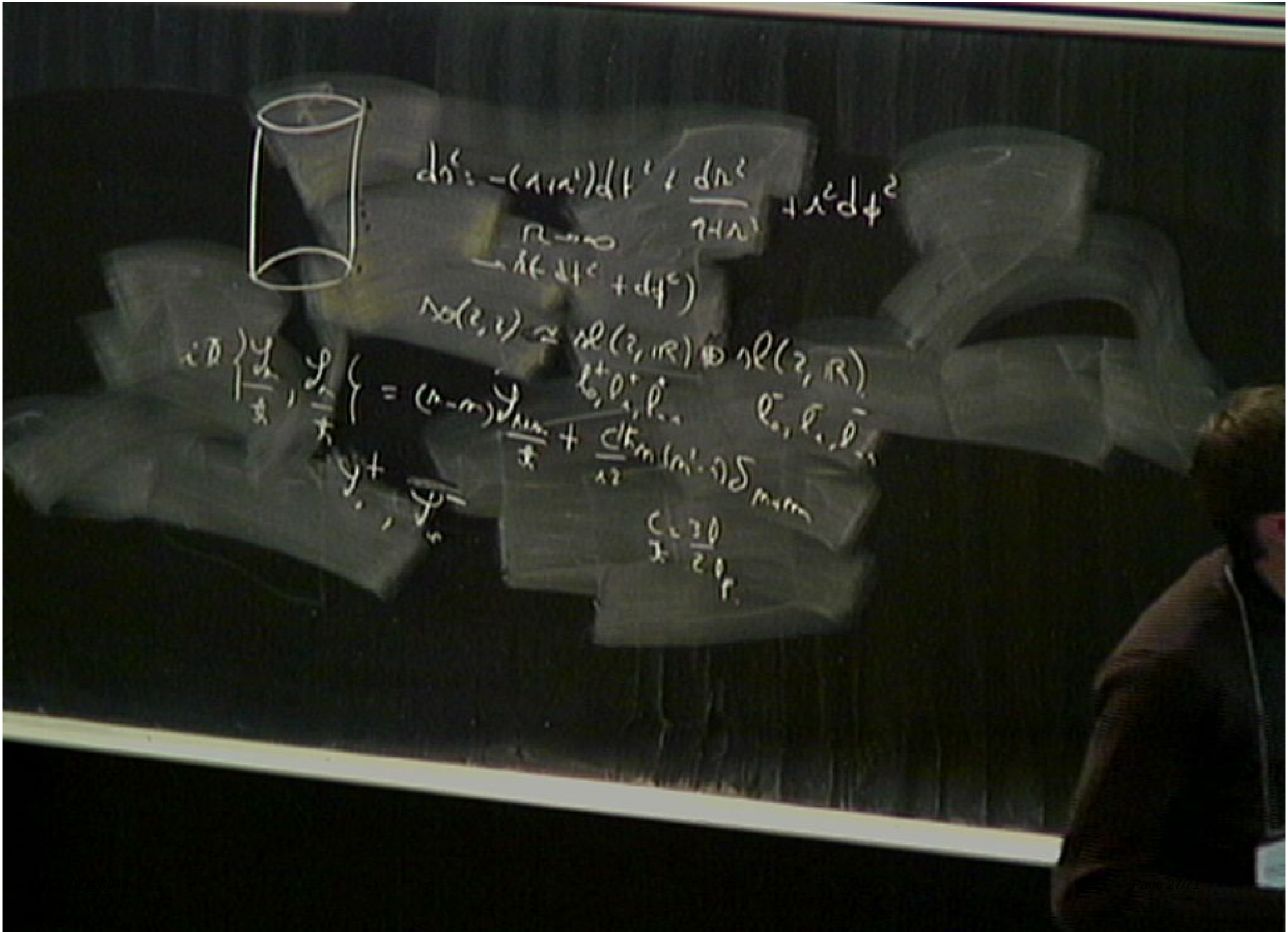
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Two questions addressed here

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- $3d$ Einstein gravity with $\Lambda = 0$?
- $3d$ Einstein gravity with $\Lambda < 0$ with closed-time like curves ?

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- The asymptotic symmetry Poisson algebra is ∞-dimensional and centrally extended. It contains a Virasoro algebra with L_0 bounded from below for black holes, with negative central charge.

Main technical tools

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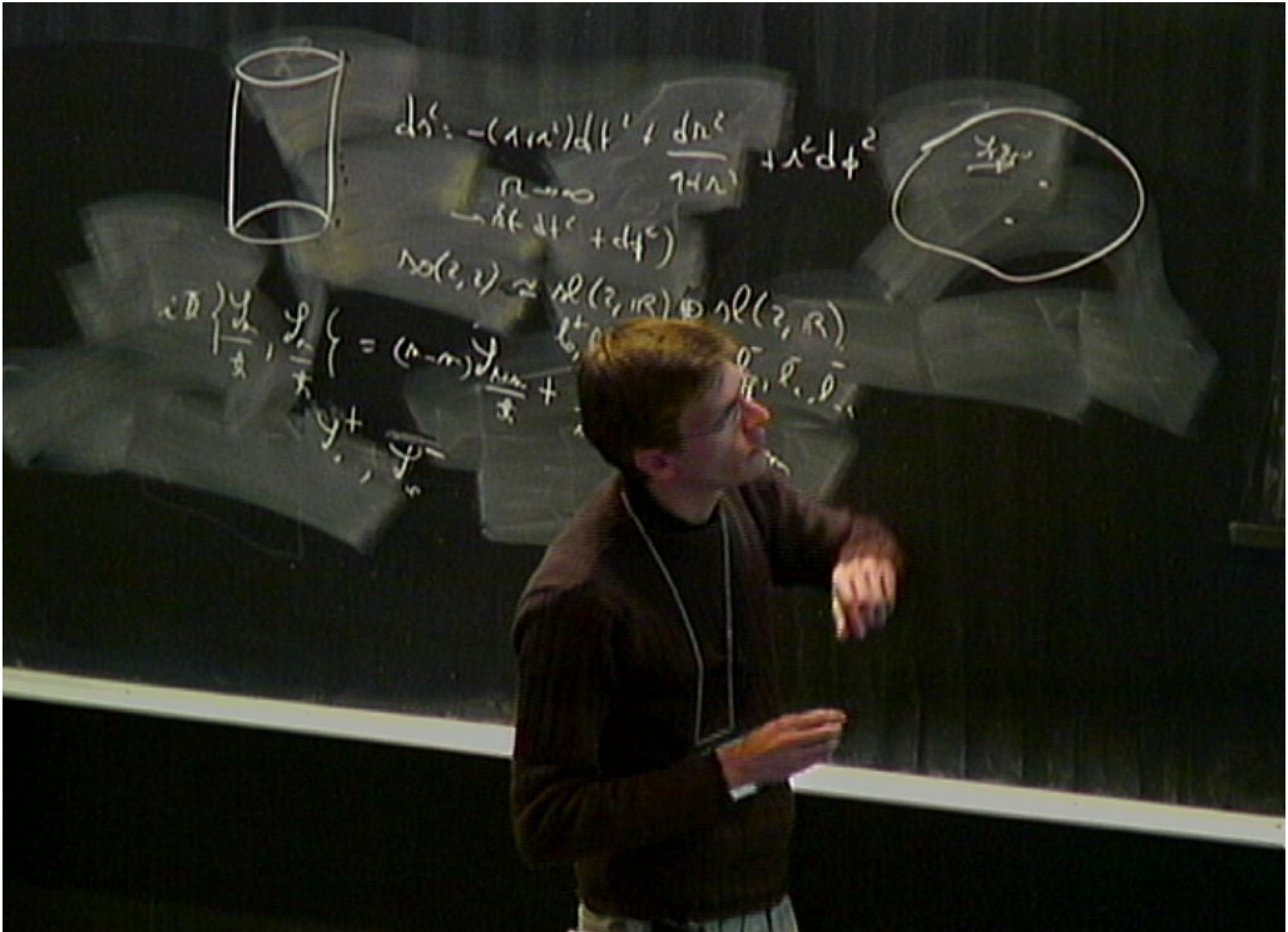
- Algorithm for computing phase space $\{g_{\mu\nu}\}$, allowed diffeomorphisms $\{\xi^\mu\}$.
  - Require $\mathcal{L}_\xi g_{\mu\nu}$ to be tangent to phase space
  - Require $\mathcal{Q}_\xi[g]$ well-defined, finite and conserved
  - Asymptotic symmetries $\equiv$ Diffeomorphisms modulo gauge transformations ($\mathcal{Q}_\xi[g] = 0$)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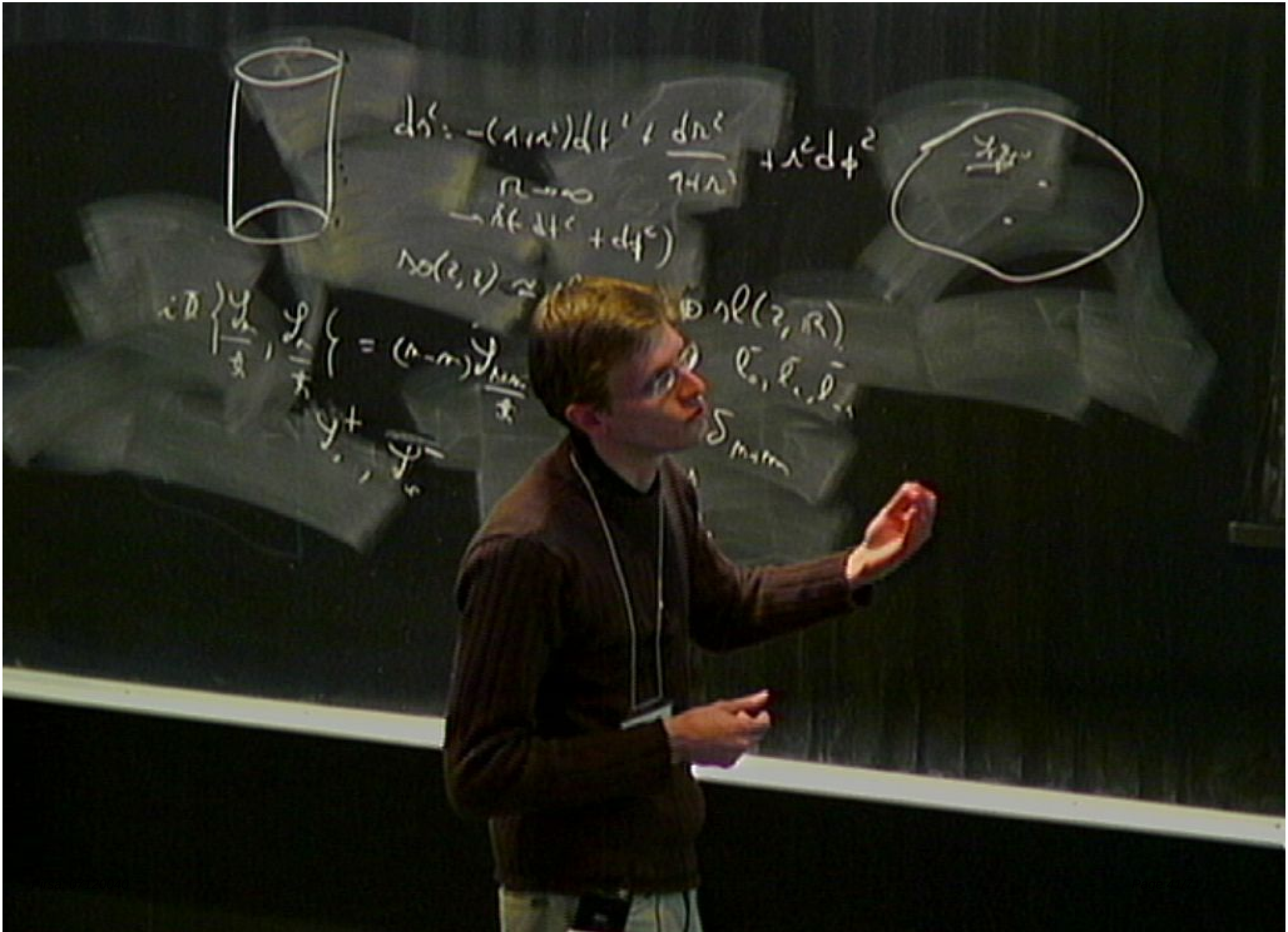

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- Definition of charges (with $\xi[g]$):

$$\mathcal{Q}_\xi[g] = \int_{S^\infty} \int_{\bar{g}}^{g} k_{\xi[g']}[\delta g', g'] + \bar{N}_\xi$$

where k is a $n-2$ form computed by local field cohomology methods from eq. of motion of the Lagrangian.

- Representation theorem :

$$\{Q_{\xi_a}, Q_{\xi_b}\}[g] = Q_{[\xi_a,\xi_b]}[g] + K_{\bar{\xi}_a,\bar{\xi}_b}[\bar{g}] - \bar{N}_{[\bar{\xi}_a,\bar{\xi}_b]}$$

Poisson bracket admits in general a central extension

$$K_{\xi_a,\xi_b} = -K_{\xi_b,\xi_a},$$

$$K_{\xi_a,[\xi_b,\xi_c]} + \text{cyclic} = 0.$$

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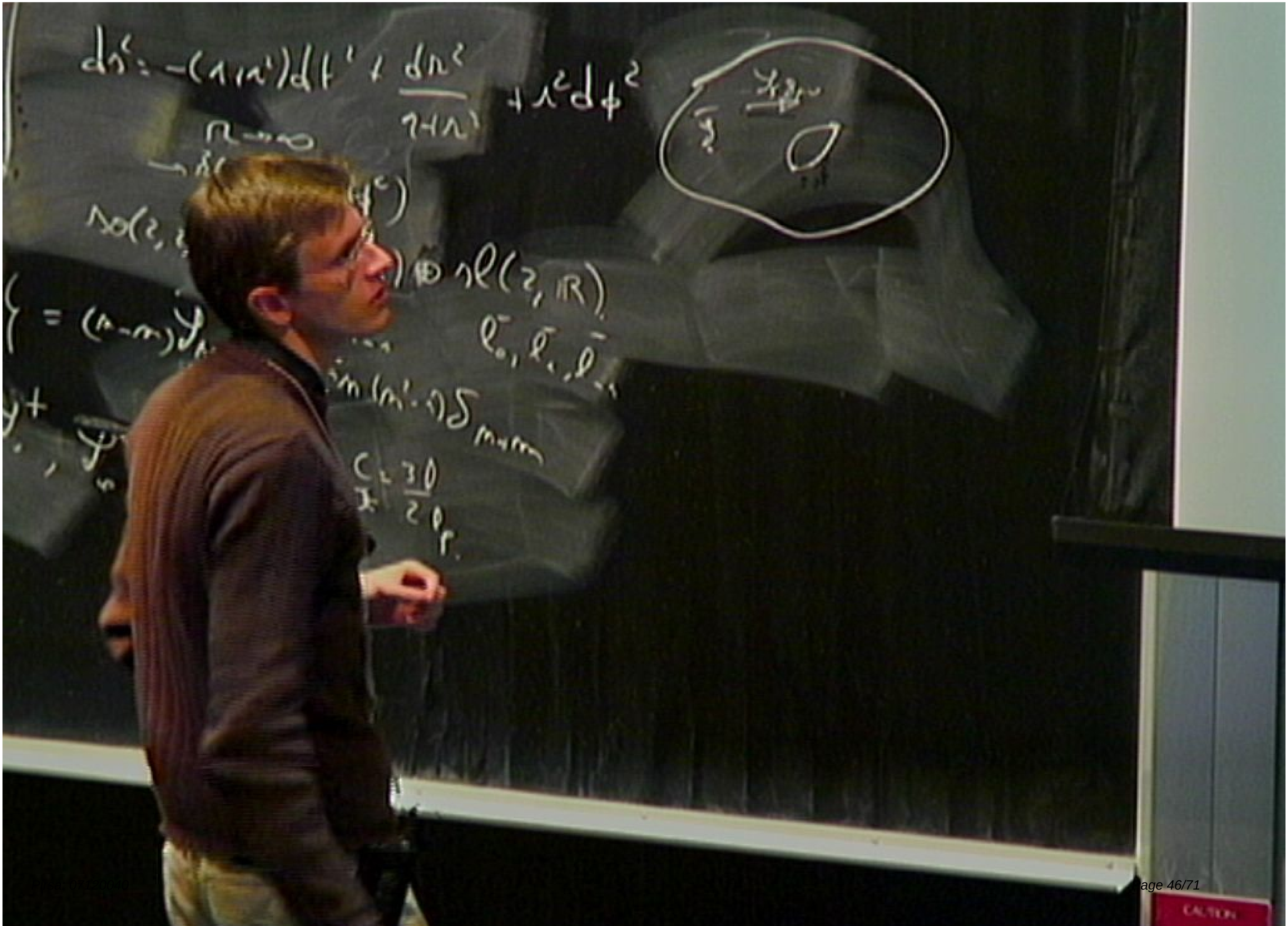
[redacted]



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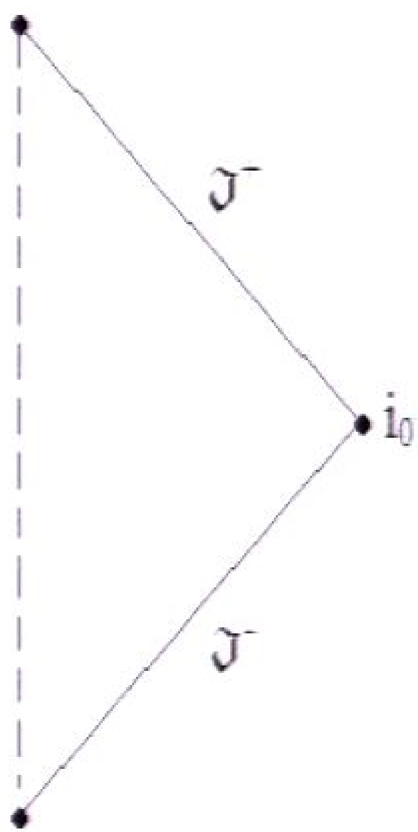
Asymptotic symmetries of $3d$ flat spacetime



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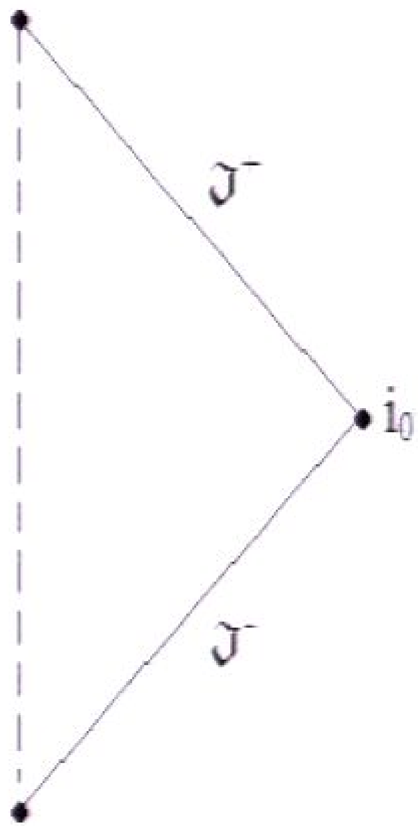

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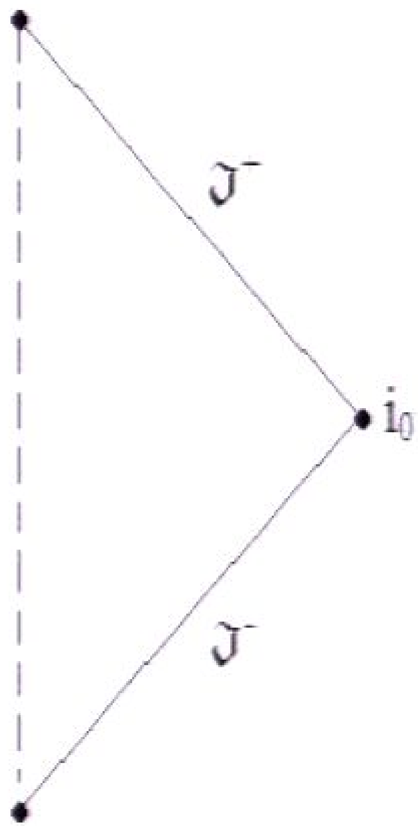

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$\mathfrak{bms}_3$ algebra

$$\begin{aligned} i[J_m, J_n] &= (m-n)J_{m+n}, \qquad i[P_m, P_n] = 0, \\ i[J_m, P_n] &= (m-n)P_{m+n} \end{aligned}$$

Contains $iso(2,1)$ (with $m = -1, 0, 1$)

Poisson bracket representation of $\mathfrak{bms}_3$

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Remark: $[G] = \text{length}$, $[\mathcal{P}_n] = 1/\text{length}$, $[\mathcal{J}_n] = 0$

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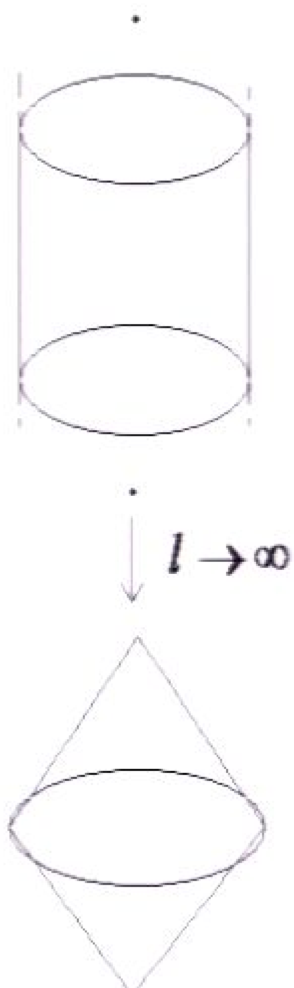
Question: Is there a relation with AdS Virasoro algebras ?

From adS_3 to $Mink_3$

$$i\{\mathcal{L}^{\pm}_m, \mathcal{L}^{\pm}_n\} = (m-n)\mathcal{L}^{\pm}_{m+n} + \frac{c}{12}m(m^2-1), \quad c = \frac{3l}{2G}$$



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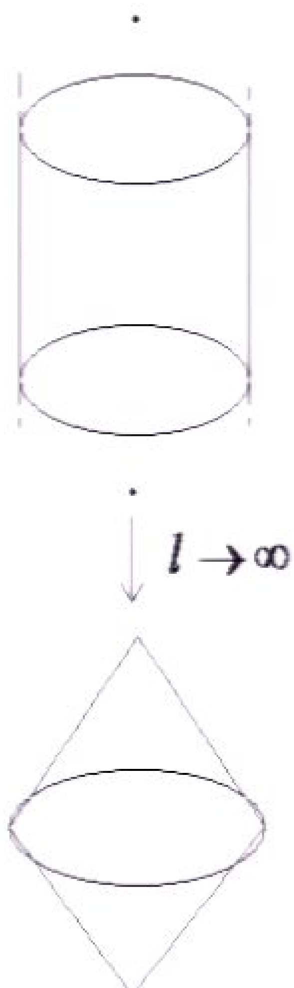

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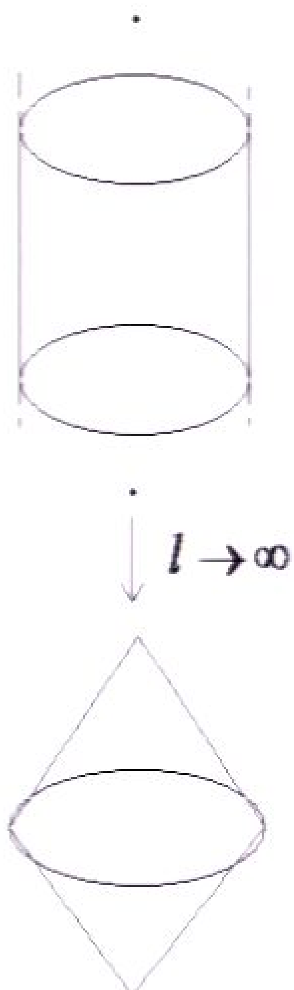

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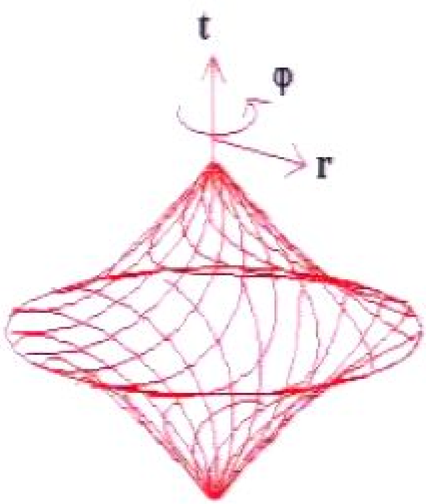
$$\begin{aligned} i\{\mathcal{J}_m, \mathcal{J}_n\} &= (m-n)\mathcal{J}_{m+n}, \\ i\{\mathcal{P}_m, \mathcal{P}_n\} &= \frac{1}{l^2}(m-n)\mathcal{J}_{m+n} \to 0, \\ i\{\mathcal{J}_m, \mathcal{P}_n\} &= (m-n)\mathcal{P}_{m+n} + \frac{1}{4G}m(m^2-1)\delta_{m+n} \end{aligned}$$

$\mathfrak{bms}_3$ obtained after an Inonu-Wigner contraction.

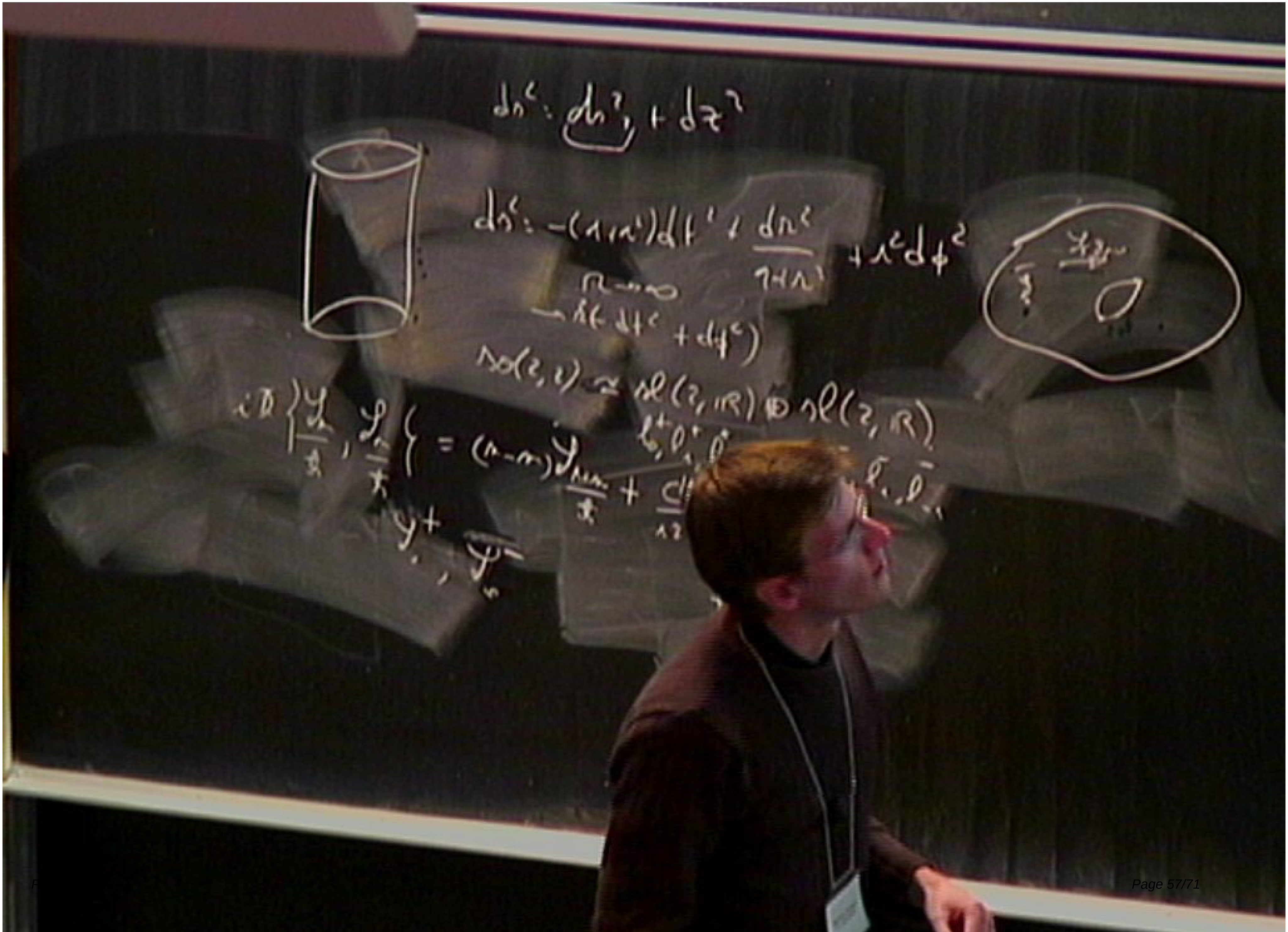
$3d$ Gödel universe

$$ds^2 = -dt^2 - 4\Omega r dt d\varphi + (2r - m^2 r^2) d\varphi^2 + \frac{dr^2}{2r + (4\Omega^2 - m^2) r^2}$$

Closed timelike curves



$g_{\varphi\varphi} < 0$



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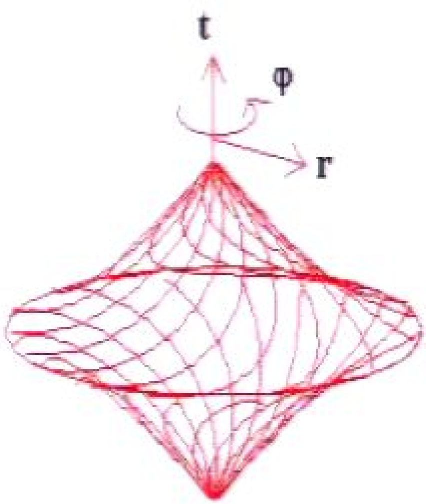

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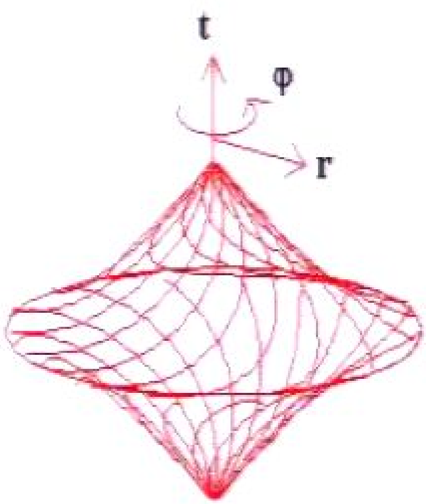

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Solution of Einstein-Maxwell-CS:

$$I = \int d^3x[\sqrt{-g}(R - 2\Lambda - F^2) - \Omega\, A \wedge F]$$

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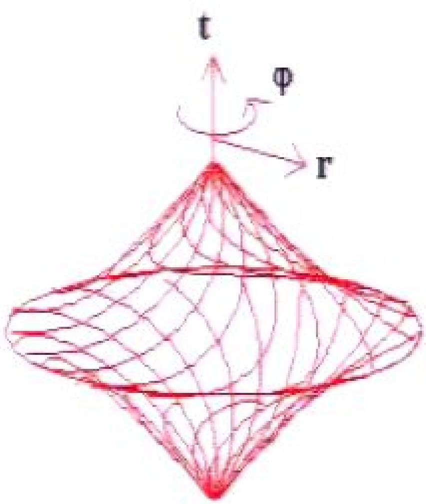

Closed timelike curves

$$g_{\varphi\varphi} < 0$$

Solution of Einstein-Maxwell-CS:

$$I = \int d^3x [\sqrt{-g}(R - 2\Lambda - F^2) - \Omega\, A \wedge F]$$

Exact symmetry algebra

$$u(1) \oplus u(1) \oplus sl(2, \mathbb{R})$$

Asymptotically Gödel spacetimes

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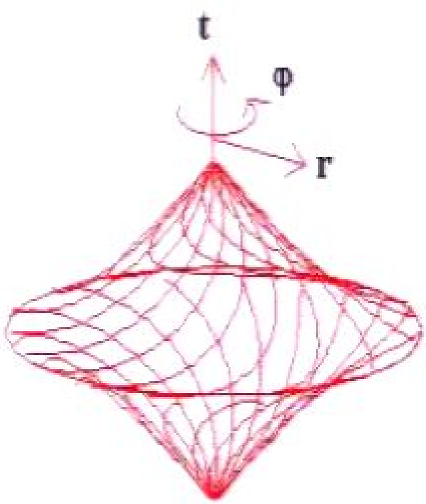

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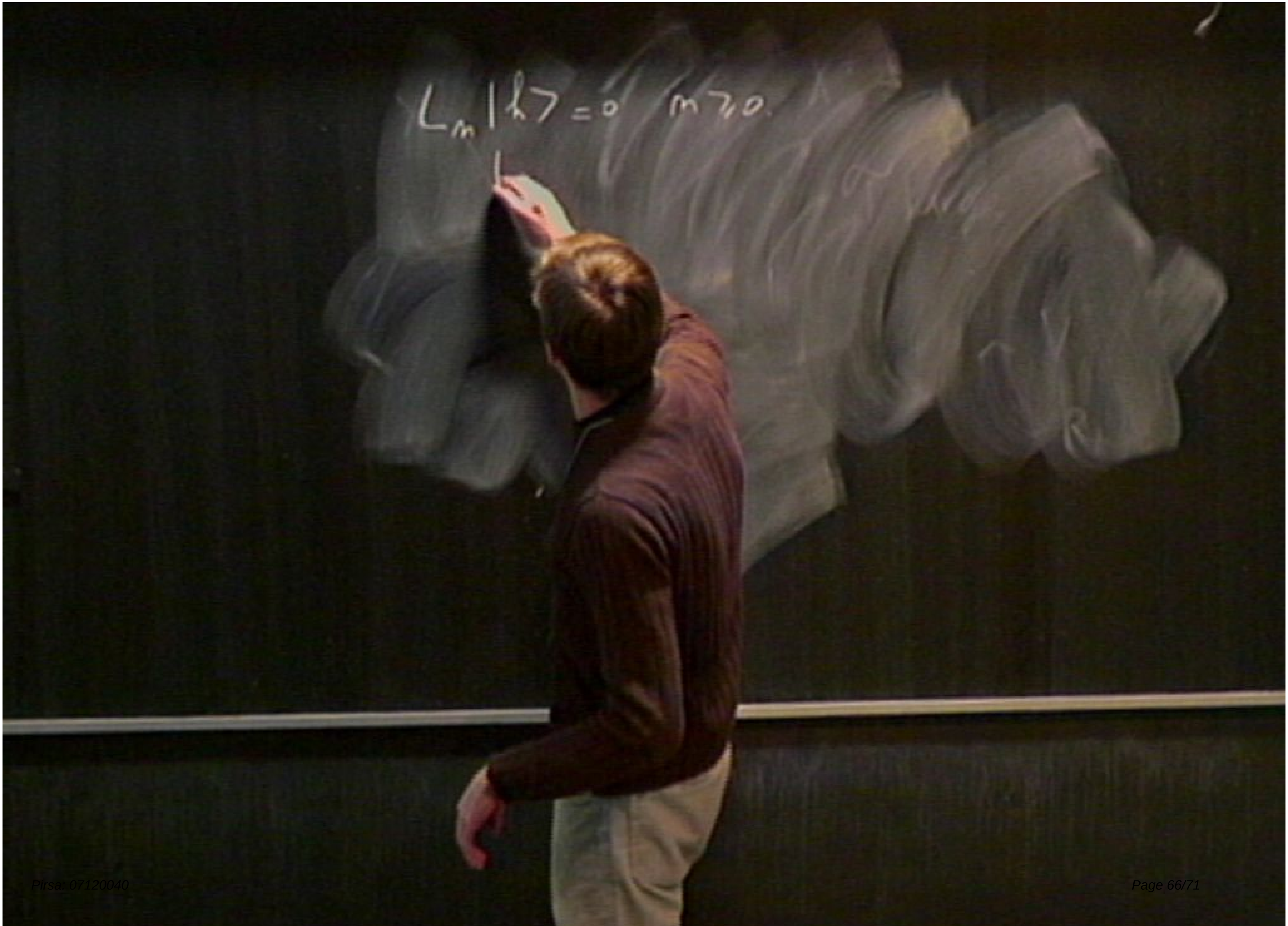
A conformal field theory with $c<0$ is non-unitary.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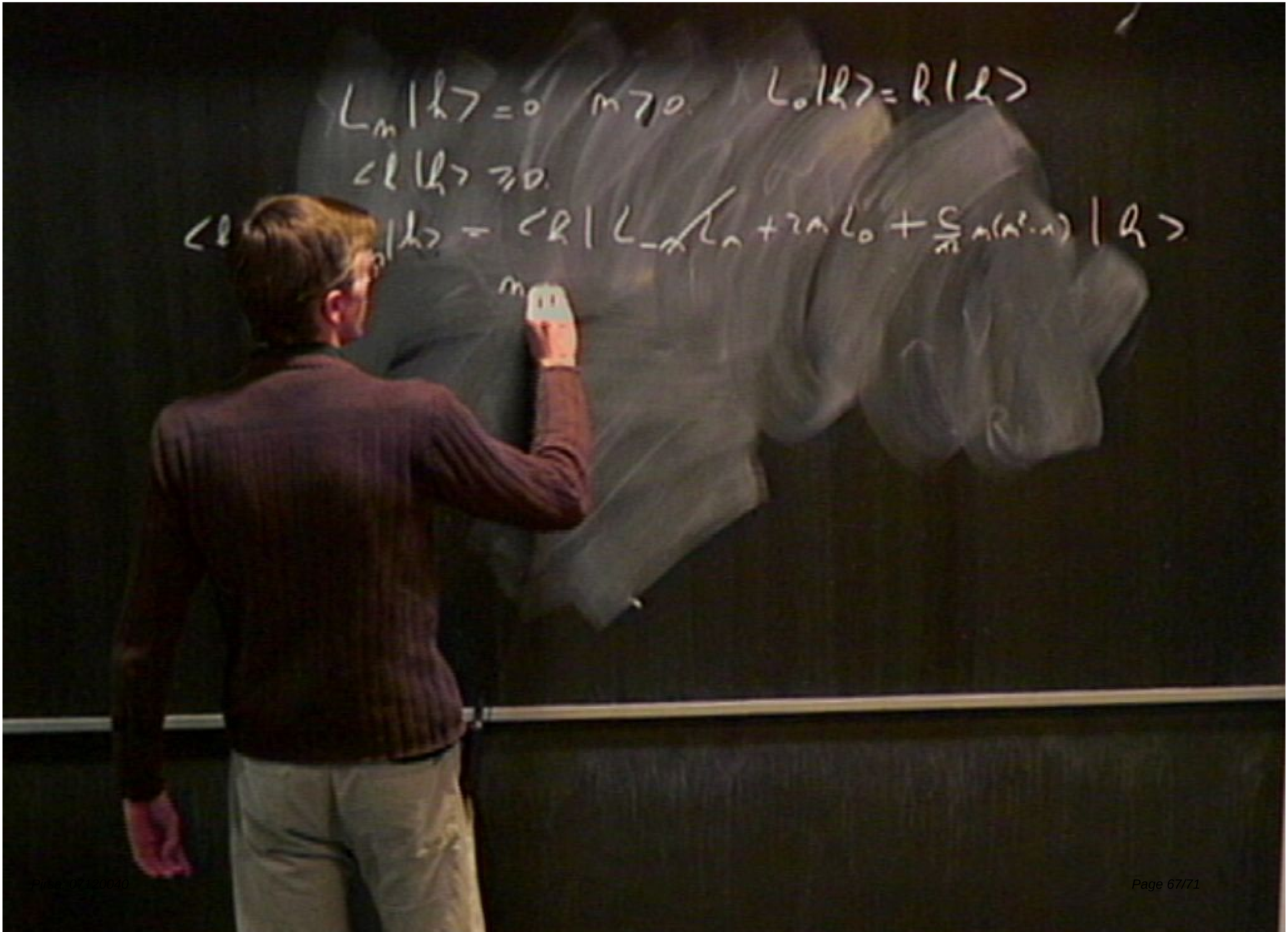
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$L_0 |h\rangle = h |h\rangle$
$\langle h | h \rangle \geq 0$

$$L_n |h\rangle = 0 \quad n > 0. \qquad L_0|h\rangle = h|h\rangle$$

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$$\langle h|L_n L_{-n}|h\rangle = \langle h| L_{-n}L_n + 2n L_0 + \frac{c}{12} n(n^2-1) | h\rangle$$

n large

$$= c\, n^3 \langle h|h\rangle$$

Summary

- Classically, $3d$ gravity has non-trivial asymptotic structures
  - $\Lambda < 0$: Two Virasoro algebras $\supset sl(2,\mathbb{R}) \oplus sl(2,\mathbb{R}) \approx so(2,2)$
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- These structures provide clues to quantum gravity in the semi-classical regime
  - $\Lambda < 0$: Quantum gravity is a conformal field theory
  - $\Lambda = 0$: Quantum gravity linked to the representation of the $\mathfrak{bms}_3$ algebra.
  - $\Lambda < 0$ and closed timelike curves: non-unitarity.