Title: Computing the Massless Spectrum of Non-Kahler Compactifications

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Abstract: While Calabi-Yau compactifications of string theory are mathematically elegant, they typically result in many massless scalars in the low-energy, four-dimensional theory. Thus, it is interesting to consider non-Kahler compactifications in the hopes of deriving more phenomenologically interesting models. These models have received little attention in the heterotic theory owing to their mathematical complexity, however in recent work we have found a potential way to derive interesting features of such compactifications using gauged linear sigma models.

COMPUTING THE MASSLESS SPECTRUM OF NON-KÄHLER COMPACTIFICATIONS

Joshua Lapan
Harvard University

based on work with A. Adams and M. Ernebjerg: hep-th/0611084

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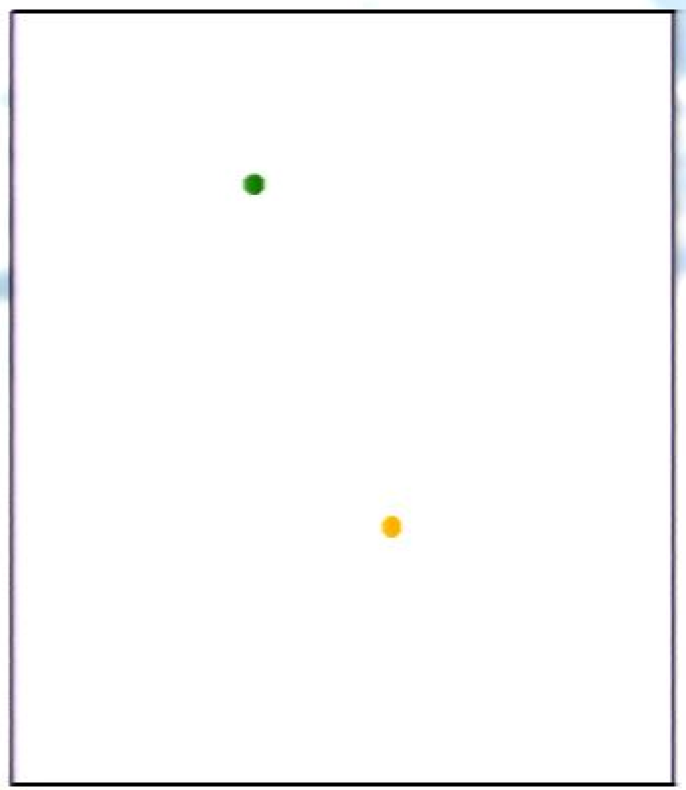
Compactifications:

$$\mathbb{R}^{3,1} \quad \times \quad K$$



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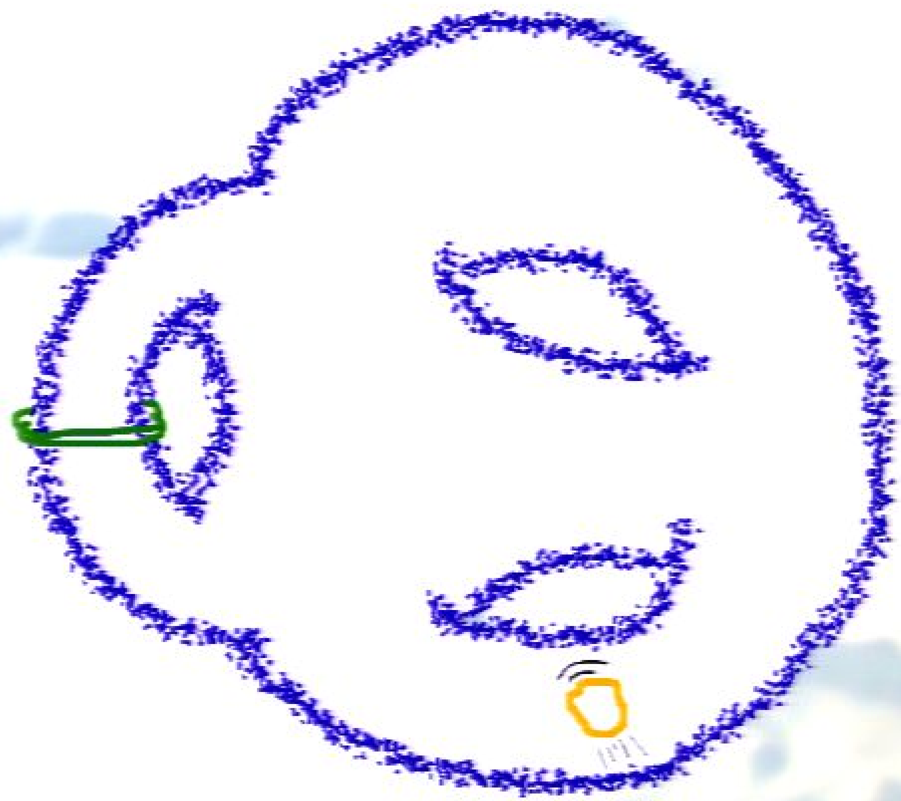

Calabi-Yau

Pros:

- Mathematically "Simple"
- Qualitative Physics $\rightsquigarrow$ Topology

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Cons:

- Many Massless Scalars!
- = Many Moduli = "Continuous" lack of Predictivity

Heterotic String Theory:

- "Natural" Appearance of Chiral, Charged Fermions
- *Exact* Worldsheet CFT (no RR-Fluxes)

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- "Natural" Appearance of Chiral, Charged Fermions
- *Exact* Worldsheet CFT (no RR-Fluxes)

Non-Kähler

- Potentially fewer massless scalars/moduli (*e.g.* Radial mode is fixed)

Worldsheet Sigma Model

$$S \sim \int_\Sigma d^2\sigma \Big\{ \frac{1}{\alpha'} \left(G_{MN} + iB_{MN} \right) \partial X^M \bar{\partial} X^N + \psi^{\hat{M}} \left(\eta_{\hat{M}\hat{N}} \partial + \partial X^P \omega^{(-)}_{P\hat{M}\hat{N}} \right) \psi^{\hat{N}}$$

$$+ \gamma^A \left(\delta^{AB} \bar{\partial} - i \bar{\partial} X^M A^{AB}_M \right) \gamma^B + \frac{1}{2} F^{AB}_{\hat{M}\hat{N}} \psi^{\hat{M}} \psi^{\hat{N}} \gamma^A \gamma^B \Big\}$$

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$V =$ Spacetime Gauge Bundle

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Worldsheet Sigma Model

$$(H_{NS})_{MNP} = 3\partial_{[M}B_{NP]} + \ldots$$

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CALABI-YAU:

- EOMs

$$\Box\Phi = \Box_4\Phi + \Box_{CY}\Phi = 0$$

- Hodge / de Rham / Dolbeault

$$\mathcal{H}^*_{\Box} \cong H^*_d \cong H^*_{\bar{\partial}}$$

- Large radius limit

$$\beta^G_{\mu\nu} \sim R_{\mu\nu} + H_{\mu\rho\sigma}H^{\rho\sigma}{}_{\nu} + \ldots$$

Invariance:

$$G_{\mu\nu} \to \lambda G_{\mu\nu} \quad B_{\mu\nu} \to \lambda B_{\mu\nu}$$

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Non-Kähler:

- EOMs

$$0 \neq \Box\Phi \; ; \qquad \mathcal{H}^*_{\Box}$$

- de Rham / Dolbeault

$$H^*_d \ncong H^*_{\bar\partial}$$

- *No* large radius limit

Bianchi Identity:

$$dH = \alpha'(\mathrm{tr}\mathcal{R}^2 - \mathrm{Tr}F^2)$$

$$\to \lambda dH = \alpha'(\mathrm{tr}\mathcal{R}^2 - \mathrm{Tr}F^2)$$

Standard Calabi-Yau Model

SUSY Variations

$$(1)\, K = complex \text{ and } c_1(TK) = 0 \longrightarrow K = CY_3\,;$$

$$(2)\, i(\bar{\partial} - \partial)J = H_{NS} \longrightarrow H_{NS} = 0$$

$$(3)\, F = HYM$$

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$$\langle A_M \rangle = \left(\begin{array}{c|c} \omega_M(K) & 0 \\ \hline 0 & 0 \end{array}\right) \Rightarrow \mathrm{tr}\mathcal{R}^2 = \mathrm{Tr}F^2$$

NON-Calabi-Yau Model

SUSY Variations

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FU-YAU GEOMETRY

based on hep-th/0604063

Manifold:

$$S^1_{(1)} \times S^1_{(2)} = T^2 \longrightarrow K$$

$$\downarrow \pi$$

$$CY_2 = S$$

Fu-Yau Geometry

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Coordinates: $\vartheta = \vartheta_1 + i\vartheta_2$

Curvature 2-Forms: $\omega_{1,2}$

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$$F^{(2,0)} = F^{(0,2)}$$
$$= F_{MN} J^{MN} = 0$$

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$\int_S$ **Bianchi:** $\int_S \sum_l \omega_l^2 = Ch_2(E) - Ch_2(S)$

Caveats: 1) No explicit metric (even for CY!)
2) Fixed volume $\Rightarrow$ pert. series may break down

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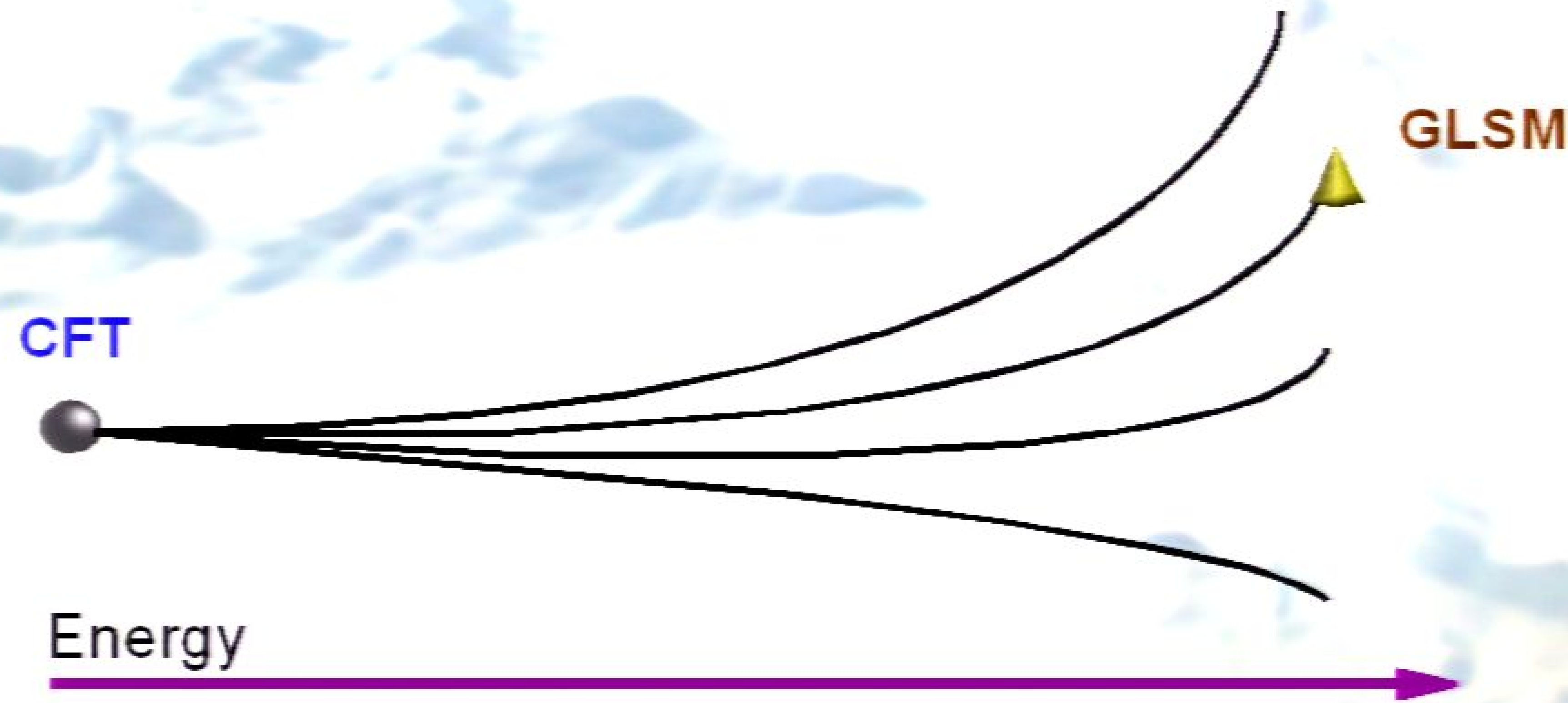
GAUGED LINEAR SIGMA MODEL



$\mathcal{N} = (0,2)$ Fields

$$\overline{D}_+ = -\frac{\partial}{\partial\bar{\theta}^+} + 2i\theta^+\partial_+$$

- **Chiral Superfields:**

$$\overline{D}_+\Phi^i = 0 \quad \Rightarrow \quad \Phi^i = \phi^i + \sqrt{2}\theta^+\psi^i_+ - 2i\theta^+\bar{\theta}^+\partial_+\phi^i$$

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- **Gauge Fields (Real Superfields):**

$$V_+ = 2\theta^+\bar{\theta}^+ v_+ \ , \quad \tfrac{1}{2}V_- = v_- - i\theta^+\bar{\lambda}_- - i\bar{\theta}^+\lambda_- + \theta^+\bar{\theta}^+ D$$

$$\Upsilon = \overline{D}_+(2\partial_- V_+ + iV_-)$$

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- **Gauge Parameter (Chiral):** $\overline{D}_+\Lambda = 0$

$$\Phi^i \to e^{-iQ^i\Lambda}\Phi^i, \quad V_+ \to V_+ + \tfrac{i}{2}(\Lambda - \overline{\Lambda}), \quad \textit{etc.}$$

	$U(1)$
Φ^i	Q^i
P	$-n$
Π	$-d$
Γ^m	q_m

Superpotential

$$\sqrt{2}W = W_S + W_E + W_{FI}$$

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$$W_S = \int d\theta^+ \Pi F(\Phi)$$

$$W_E = \int d\theta^+ P \Gamma^m E_m(\Phi)$$

$$W_{FI} = \frac{it}{4} \int d\theta^+ \Upsilon \,, \qquad \text{where} \quad t = r + i\theta$$

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$$\Longrightarrow \quad \begin{array}{l} F(\Phi) = \text{Homogeneous poly. of degree } d \\ E_m(\Phi) = \text{Homogeneous poly. of degree } n - q_m \end{array}$$

$$\mathcal{L} \sim \mathcal{L}_{kin} + \mathcal{L}_{Yuk} + \underbrace{\mathcal{L}_D + \mathcal{L}_F + \mathcal{L}_E}_{-U} + 2\theta v_{+-}$$

$$\mathcal{L}_D = \frac{1}{2e^2} D^2 + D\left(\sum_i Q^i |\phi^i|^2 - n|p|^2 - r\right)$$

$$\xrightarrow{\int \mathcal{D}D} -\frac{e^2}{2}\left(\sum_i Q^i |\phi^i|^2 - n|p|^2 - r\right)^2$$

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$$\mathcal{L}_F = |\rho|^2 - \rho F(\phi) - \bar{\rho}\overline{F(\phi)}$$

$$\xrightarrow{\int \mathcal{D}\rho} -|F(\phi)|^2$$

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$$\mathcal{L}_E = \sum_m |G_m|^2 - pG^m E_m(\phi) - \bar{p}\overline{G}^m \overline{E_m(\phi)}$$

$$\xrightarrow{\int \mathcal{D}G} -|p|^2 \sum_m |E_m(\phi)|^2$$

$r \gg 1$ Scalar Potential

$$U = \underbrace{\frac{e^2}{2}\left(\sum_i Q^i|\phi^i|^2 - n|p|^2 - r\right)^2}_{\mathcal{L}_D} + \underbrace{|F(\phi)|^2}_{\mathcal{L}_F} + \underbrace{|p|^2\sum_m |E_m(\phi)|^2}_{\mathcal{L}_E}$$

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Manifold:

1. $\{\phi^i\}$, $i = 1, \ldots, N+1$, parameterize $\mathbb{C}^{N+1}$

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Manifold:

1. $\{\phi^i\}$, $i = 1, \ldots, N+1$, parameterize $\mathbb{C}^{N+1}$
2. $\mathcal{L}_D = 0$ and $n, Q^i \geq 0 \quad \Rightarrow \quad \mathbb{C}^{N+1} \rightarrow\sim S^{2N+1}$

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3. $U(1)$ Gauge fixing $\;\Rightarrow\; S^{2N+1} \to \mathbb{WP}^N_{\vec{Q}}$

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3. $U(1)$ Gauge fixing $\quad \Rightarrow \quad S^{2N+1} \to \mathbb{WP}^N_{\vec{Q}}$
4. $\mathcal{L}_F = 0 \quad \Rightarrow \quad \mathbb{WP}^N_{\vec{Q}} \to S = \{F(\phi) = 0 | \vec{\phi} \in \mathbb{WP}^N_{\vec{Q}}\}$

$r \gg 1$ Scalar Potential

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Manifold: (*cont.*)

5. Assume that $F(\phi) = E_1(\phi) = \ldots = 0 \;\Leftrightarrow\; \vec{\phi} = 0$

6. $\mathcal{L}_E = 0$ (given $\mathcal{L}_F = 0$) $\;\Rightarrow\; p = 0$

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$$U = \underbrace{\frac{e^2}{2}\left(\sum_i Q^i|\phi^i|^2 - n|p|^2 - r\right)^2}_{\mathcal{L}_D} + \underbrace{|F(\phi)|^2}_{\mathcal{L}_F} + \underbrace{|p|^2 \sum_m |E_m(\phi)|^2}_{\mathcal{L}_E}$$

Manifold: *(cont.)*

5. Assume that $F(\phi) = E_1(\phi) = \ldots = 0 \;\Leftrightarrow\; \vec{\phi} = 0$

6. $\mathcal{L}_E = 0$ (given $\mathcal{L}_F = 0$) $\;\Rightarrow\; p = 0$

$r \ll -1$ LG Phase

$U = 0$ implies that $p = \sqrt{-\frac{r}{n}}$ and $\vec{\phi} = 0$

$r \gg 1$ Fermions

$\mathcal{L}_{Yuk}$ determines massless fermions

	$U(1)$
Φ^i	Q^i
P	$-n$
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Right-Movers

Cohomology of

$$0 \to \mathcal{O}_S \to \oplus_i \mathcal{O}_S(Q^i) \to \mathcal{O}_S(d) \to 0$$

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Sections of E given by exact sequence

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~~Torsion~~

$$\mathcal{L} \supset 2\theta v_{+-} = \tfrac{1}{2}\theta\epsilon^{\mu\nu}v_{\mu\nu} \sim B_{MN}\epsilon^{\mu\nu}\partial_\mu X^M\partial_\nu X^N$$

$$\Rightarrow B \sim \theta H, \quad \text{where } H \text{ generates } H^2(\mathbb{WP}^N_{\vec{Q}})$$

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$$\Longrightarrow H_{NS} \sim \textstyle\sum_l N_l d\theta^l \wedge H \sim \sum_l d\theta^l \wedge \omega_l$$

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$$\int_S \sum_l \omega_l = Ch_2(E) - Ch_2(S)$$

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(extension of) Silverstein & Witten, hep-th/9403054

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$$\underline{\textbf{Massless Fermions}} \;\equiv\; H_{\overline{Q}}\cap\{|\mathcal{O}\rangle \mid L_0|\mathcal{O}\rangle=0\}$$

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$$\Rightarrow \; p = \sqrt{-r/n}$$

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$\Longrightarrow$ Compute Massless Spectrum in LG Orbifold

RESULTS & OUTLOOK

(1) Still many massless moduli
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(3)# generations =
(4)
(5)

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