Title: Can Classical Description of Physical Reality Be Considered Complete?

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Abstract: A conceptual framework is proposed for understanding the relationship between observables and operators in mechanics. We claim that the transformations generated by the objective properties of a physical system must be strictly interpreted as gauge transformations. It will be shown that this postulate cannot be consistently implemented in the framework of classical mechanics. We argue that the uncertainty principle is a consequence of the mutual intertwining between objective properties and gauge-dependant properties. Hence, in classical mechanics gauge-dependant properties are wrongly considered objective. It follows that the quantum description of objective physical states is not incomplete, but rather that the classical notion is overdetermined.

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- Quantum mechanics provides a complete objective description of physical systems:

$$\psi(q) = n \textit{ objective properties}$$

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- "[...] *objectivity means invariance with respect to the group of automorphisms*."

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- In other words, according to ♣ there should exist a correspondence

$$\textit{Observables} \rightsquigarrow \textit{Operators}$$

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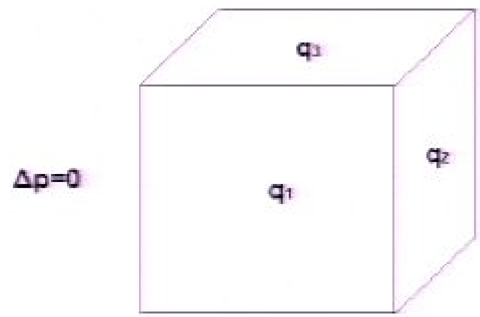

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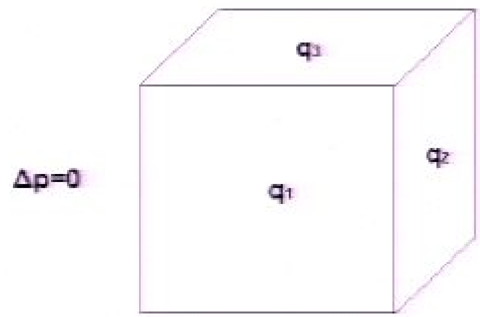

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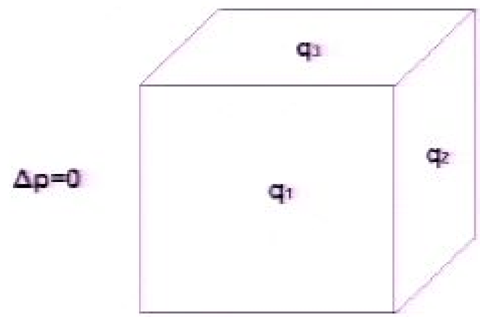

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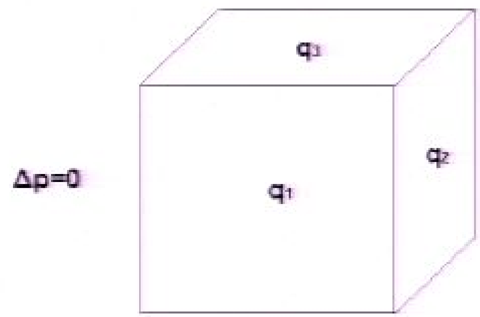

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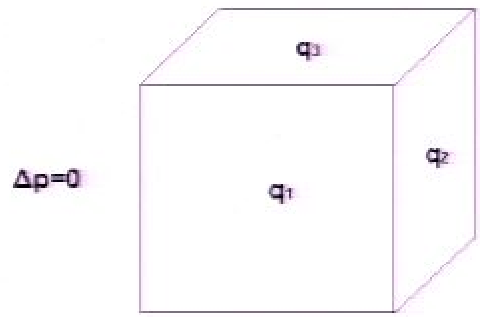

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- Difference between classical and quantum mechanics:

Classical observables $f \in C^\infty(M)$	vs.	Quantum operators $\hat{f}$
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$q \cdot p - p \cdot q = 0$	vs.	$\hat{q}\hat{p} - \hat{p}\hat{q} = i\hbar$

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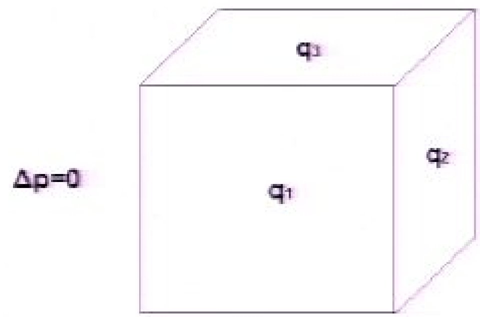

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- Hence, we should compare...

Algebra of classical operators v_f	vs.	Algebra of quantum operators $v_f^\hbar$

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- Classical correspondence between classical observables f and classical operators v_f

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where

$$\omega(v_f, \cdot) = df$$

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- In particular... $q \rightsquigarrow v_q = -\frac{\partial}{\partial p}$ $\qquad p \rightsquigarrow v_p = \frac{\partial}{\partial q}$

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- One-parameter family of symplectic diffeomorphisms:

$$\begin{aligned} \phi_t : M &\rightarrow M \\ (q(0), p(0)) &\mapsto (q(t), p(t)) \end{aligned}$$

Injectivity

- The surjective Lie algebra homomorphism

$$\begin{aligned} \mathcal{C}^\infty(M) &\to \mathcal{H}_M \\ k &\mapsto v_k = \frac{\partial k}{\partial p}\frac{\partial}{\partial q} - \frac{\partial k}{\partial q}\frac{\partial}{\partial p} = 0 \end{aligned}$$

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- $$\mathcal{H}_M \simeq \mathcal{C}^\infty(M)/\mathbb{R}$$

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$$\{q,p\} = 1$$

Dirac Quantization Conditions

- Quantum operators have to represent *faithfully* the Poisson algebra of classical observables:

♠ **Injectivity**: if $f = k \in \mathbb{R}$, then $\hat{f} = kI$, where I is the identity operator.

♠ **Homomorphism of algebras**: if $\{f, g\} = h$, then $\left[\hat{f}, \hat{g}\right] = -i\hbar\hat{h}$.

♠ **Irreducibility**: if $\{f_i\}$ is a complete set of classical observables, then the Hilbert space has to be irreducible under the action of the set $\left\{\hat{f}_i\right\}$.

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- Quantum mechanics can be obtained by forcing this faithfulness.

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  ♠ They can be evaluated on states $f(s) \in \mathbb{R}$ and used for identifying them. Hence, they define properties of the states.

  ♠ They induce classical operators v_f that generate symplectic diffeomorphisms of M.

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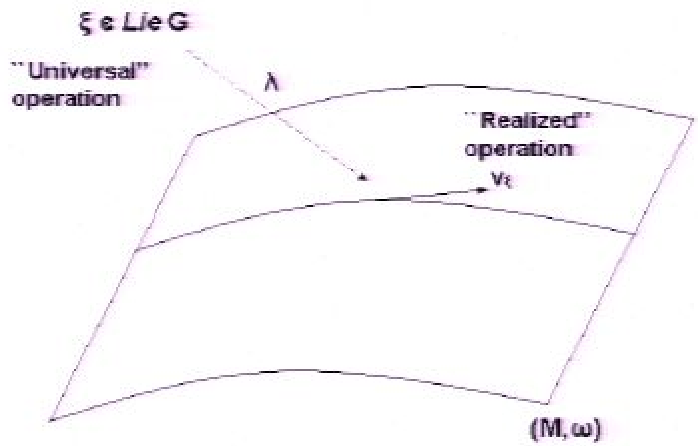
- Why should this correspondence between observables and operators be faithful?

Symplectic Actions



- A symplectic action $\Phi : G \times M \to M$ of a Lie group G on a symplectic manifold (M, ω) (where $\Phi_g^* \omega = \omega$) defines a map

$$\begin{aligned} \lambda : g &\to TM \\ \xi &\mapsto v_\xi = \frac{d}{d\lambda}(exp(-\lambda\xi) \cdot x)|_{\lambda=0} \end{aligned}$$

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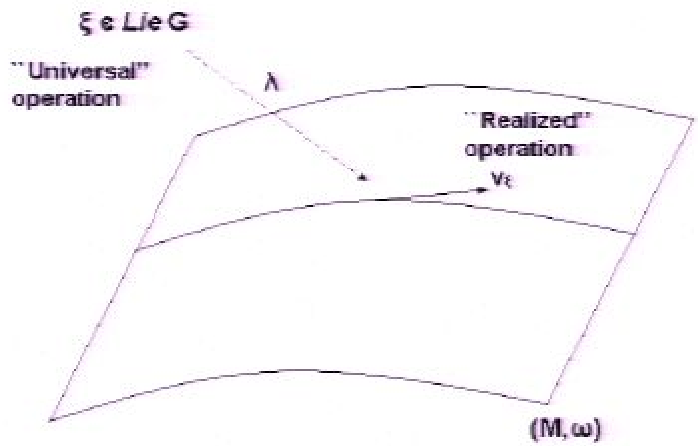

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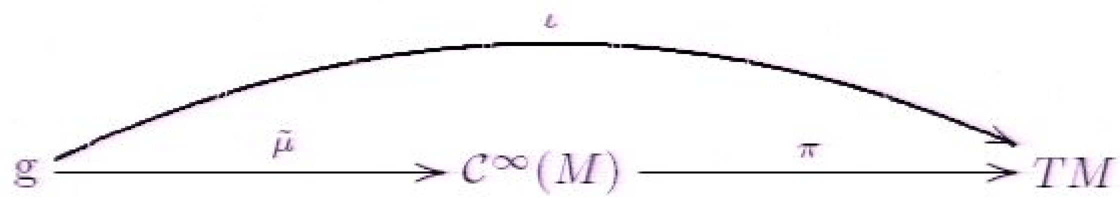
- The *fundamental vector field* v_ξ "*realizes*" on M the *universal operation* $\xi \in \mathfrak{g}$.

Co-Momentum Map

- The symplectic action is *Hamiltonian* if there exists a map

$$\begin{aligned} \tilde{\mu} : \mathrm{g} &\rightarrow \mathcal{C}^{\infty}(M) \\ \xi &\mapsto f_{\xi} \end{aligned}$$

(called *co-momentum map*) such that the following diagram commutes



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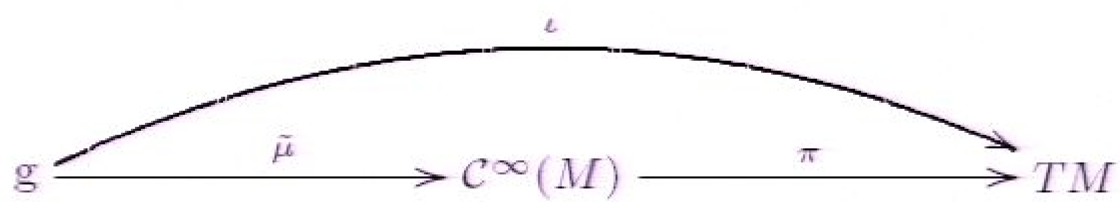


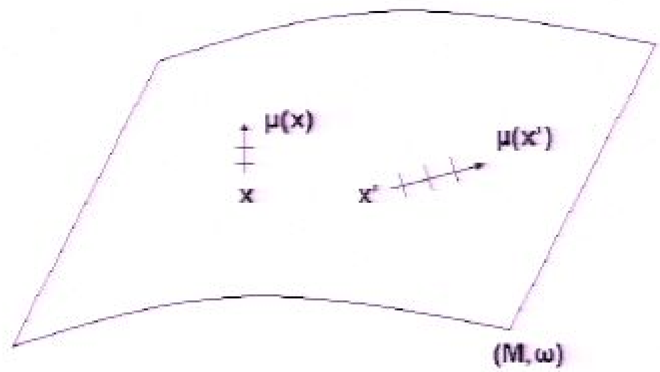
Universal operator ξ ⟶ Observable f_ξ ⟶ Realized operator v_{f_ξ}

- The realization of universal operators is “factorized” through physical observables.

Momentum Map

$$\mu : M \to \mathrm{g}^*$$

$$\langle \mu(x), \xi \rangle = f_\xi(x)$$



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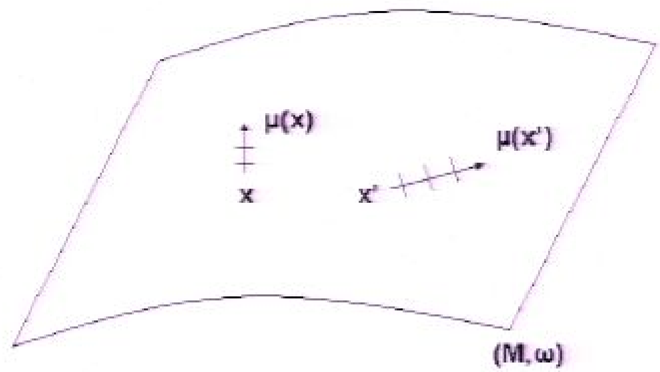

- The *momentum map* μ identifies each state in M with a 1-form $\eta \in g^*$ (or "*universal states*").
- In other words... the contraction of the 1-form $\mu(x) \in g^*$ with a fixed $\xi \in g$ yields a real number *that depends on* $x \in M$. This dependence is given by the observable $f_\xi(x)$.

Equivariance

- If the momentum map $\mu : M \to \mathrm{g}^*$ were equivariant

$$\begin{array}{ccc} M & \xrightarrow{\ \mu\ } & \mathrm{g}^* \\ \Big\downarrow{\scriptstyle \Phi_g} & & \Big\downarrow{\scriptstyle Ad^*_{g^{-1}}} \\ M & \xrightarrow[\ \mu\ ]{} & \mathrm{g}^*. \end{array}$$

(where Ad^* denotes the coadjoint action of G on g^*), then the co-momentum map $\tilde{\mu} : \mathrm{g} \to C^\infty(M)$ would be an homomorphism of Lie algebras:

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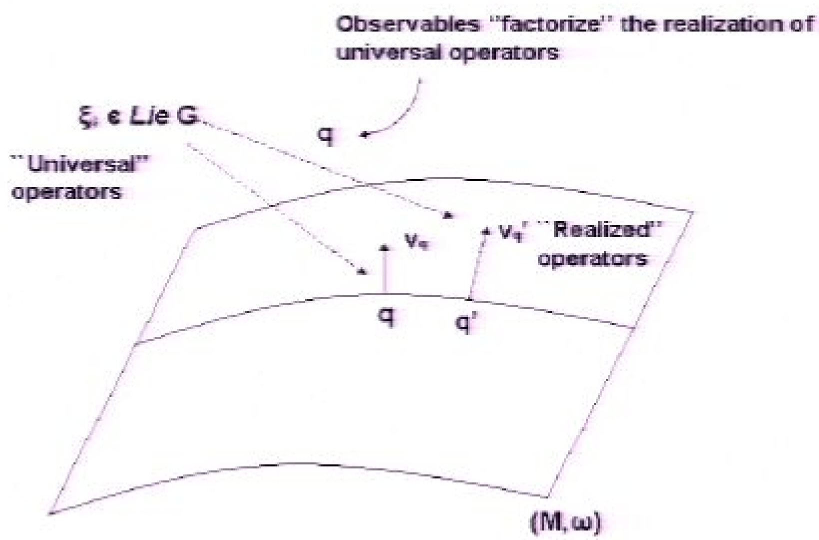
- In this case, g^* is a final object in the category of Poisson G-manifolds (i.e. a "*universal phase space*"), being the momentum map μ the unique morphism from an object in the category (i.e. a "concrete" phase space M) to the universal object g^*.

Realization



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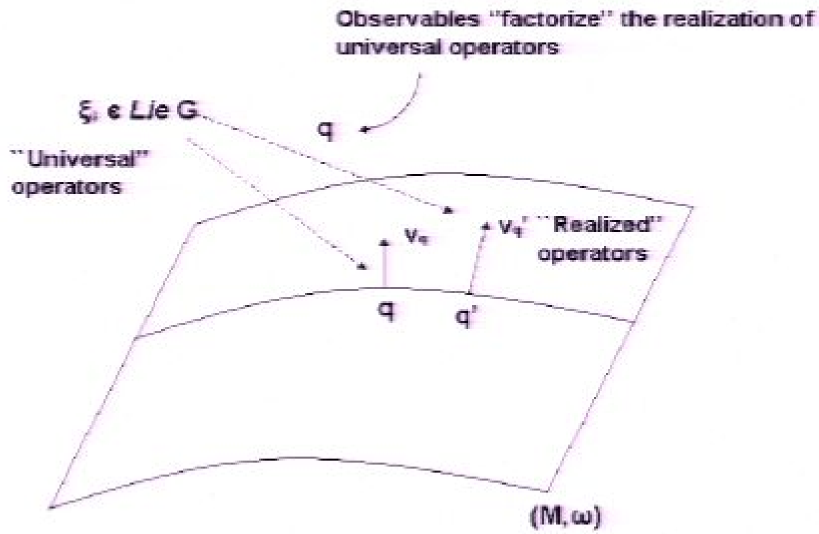

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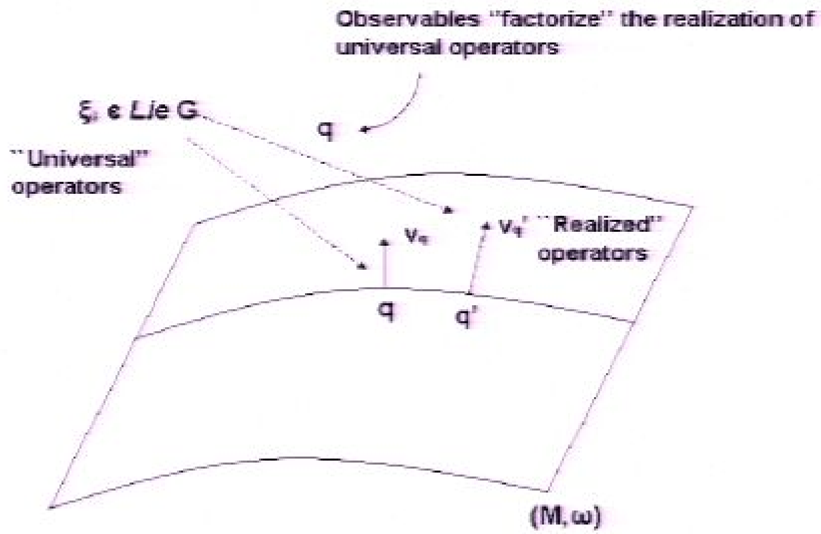

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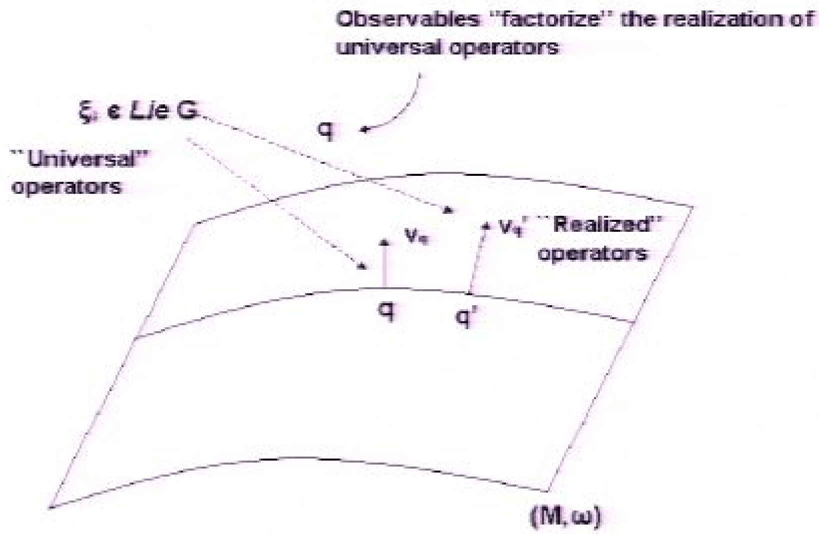

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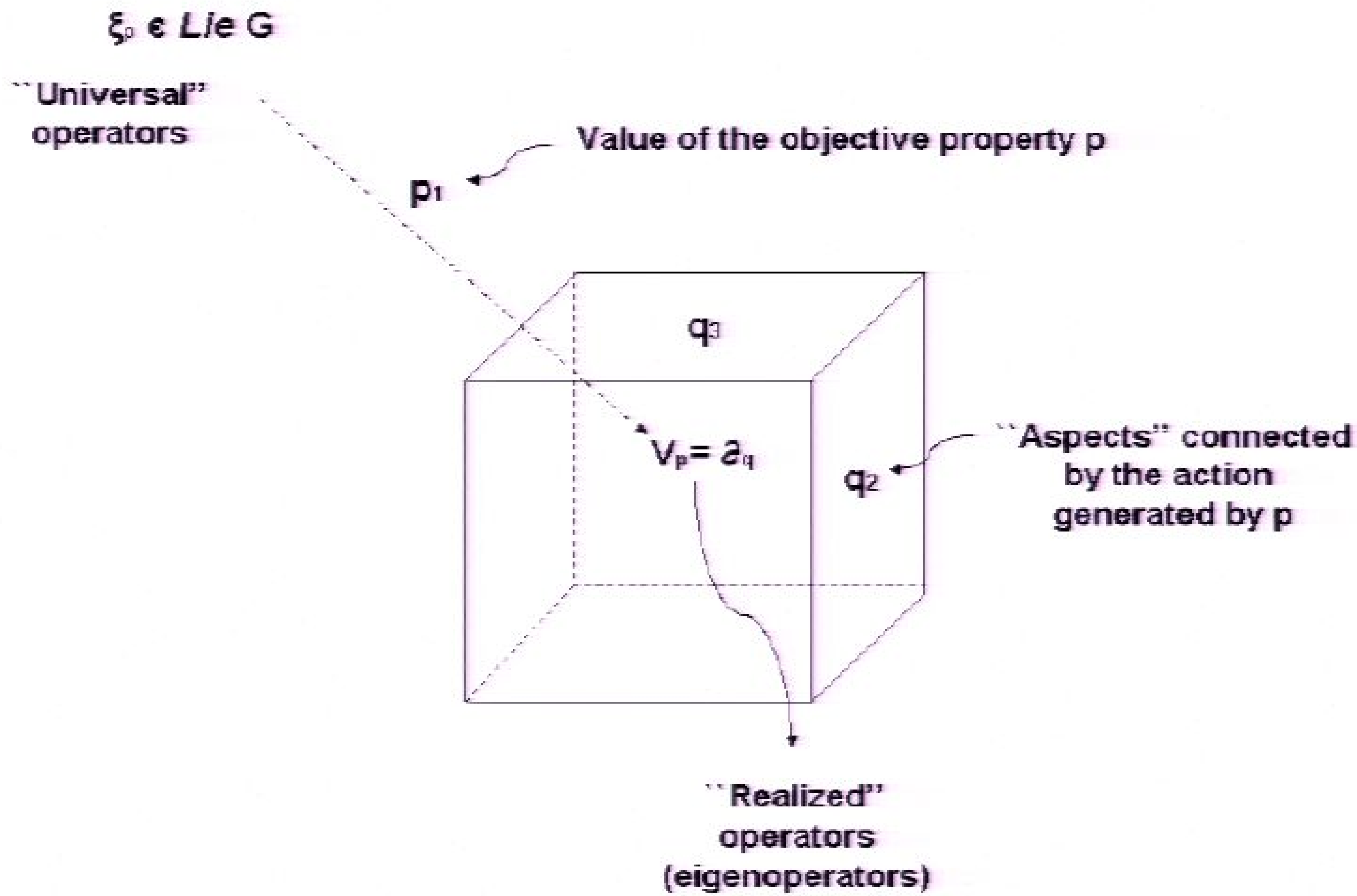
- Observables induce classical operators because observables are nothing but quantities that specify how universal operators are realized differently by different physical systems.

Visualising objects...



Classical Eigenoperators?

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- Classical operators v_f are not "*eigen*-operators":

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  ♠ Moreover, their actions transform classical states in other states that are *physically* different, i.e. they do not generate automorphisms of the states.

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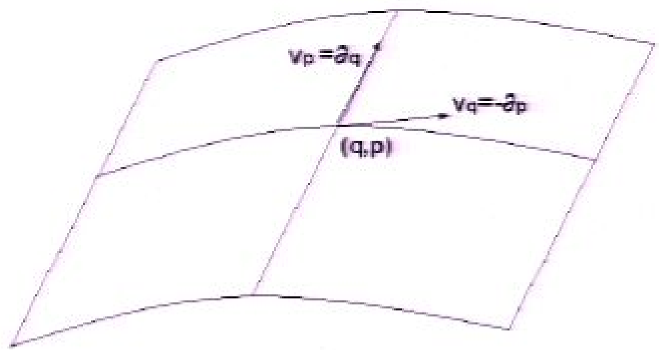
Classical Mechanics



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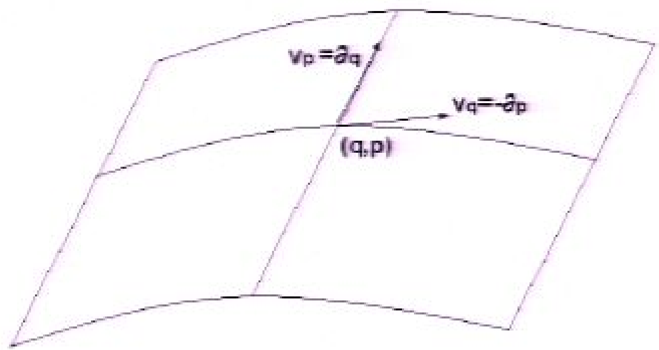

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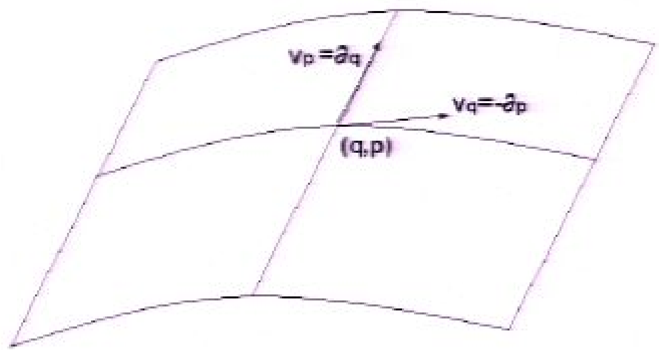

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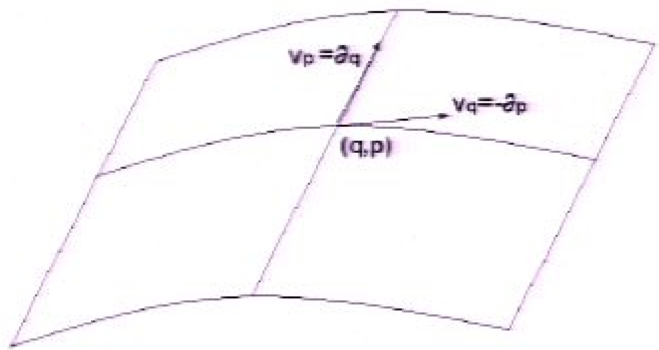

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- Hence, both q and p would be "pure gauge". All states in M would be different representatives of the same physical state.

Canonical Transformations

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- Even if in principle canonical transformations should only be transformations of the variables used for describing a physical state...
- ... in classical mechanics, canonical transformations generated by physical observables have to be interpreted as *active* transformations between physically different states.
- For example, the motion of a mechanical system, even if it is "*the continuous evolution or unfolding of a canonical transformation*" (Goldstein), is interpreted as a transformation between states that are *physically* different.

Conclusions...

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- A physical object is a physical configuration that

  ♠ is defined by a set of objective properties that specify how certain universal operators are realized by the object

  ♠ and such that the realized operators (eigenoperators) generate automorphisms of the object that “permute” its different “aspects” or “profiles” (postulate ♣).

...and more conclusions...

- Classical description of physical systems is not satisfactory since non-objective properties are wrongly considered objective. In other words, classical states (q, p) are *overdetermined*.

- In classical mechanics, the object's automorphisms cannot be generated by the objective properties of the object. Hence, postulate ♣ cannot be consistently implemented in classical mechanics.

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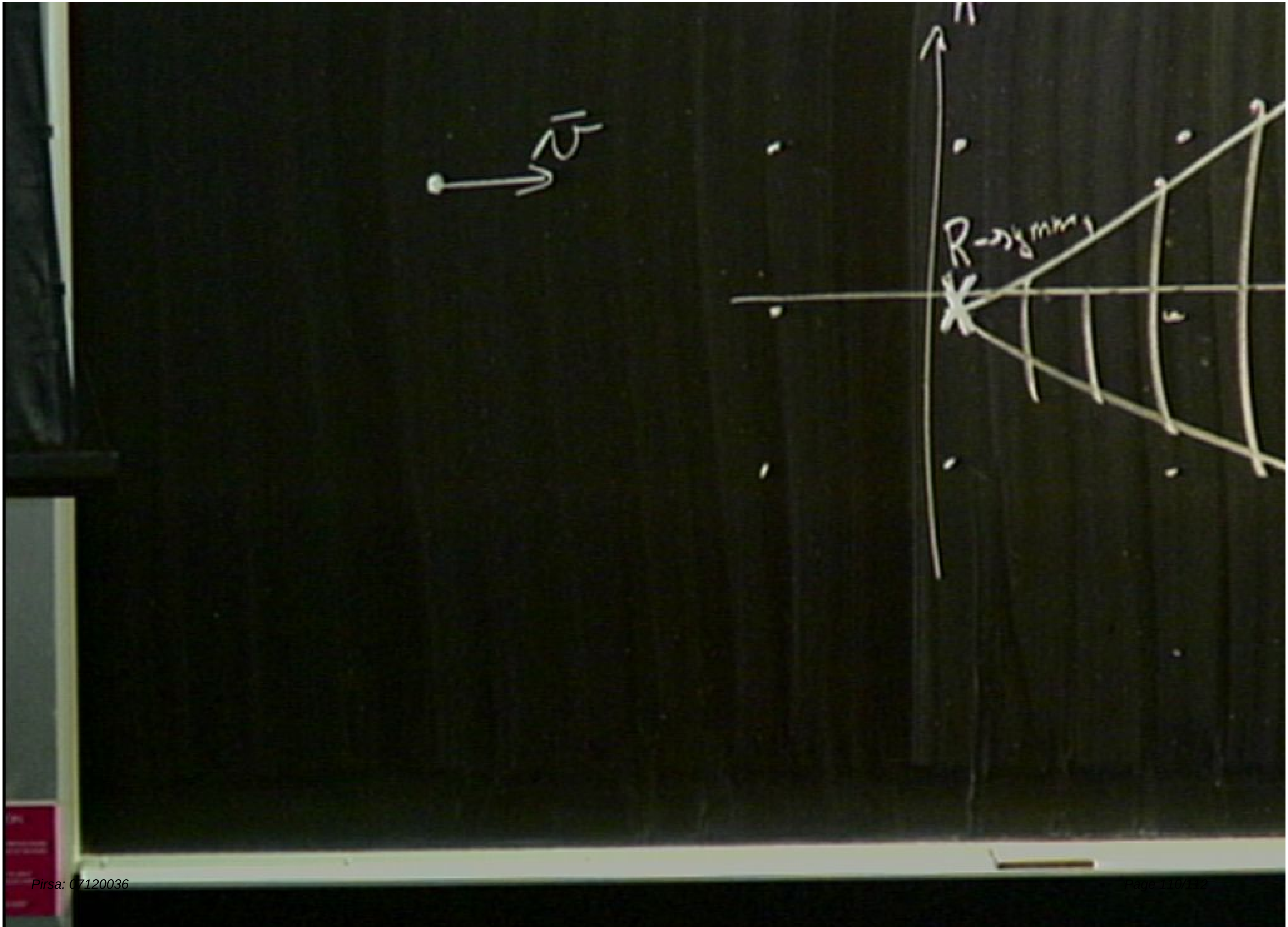

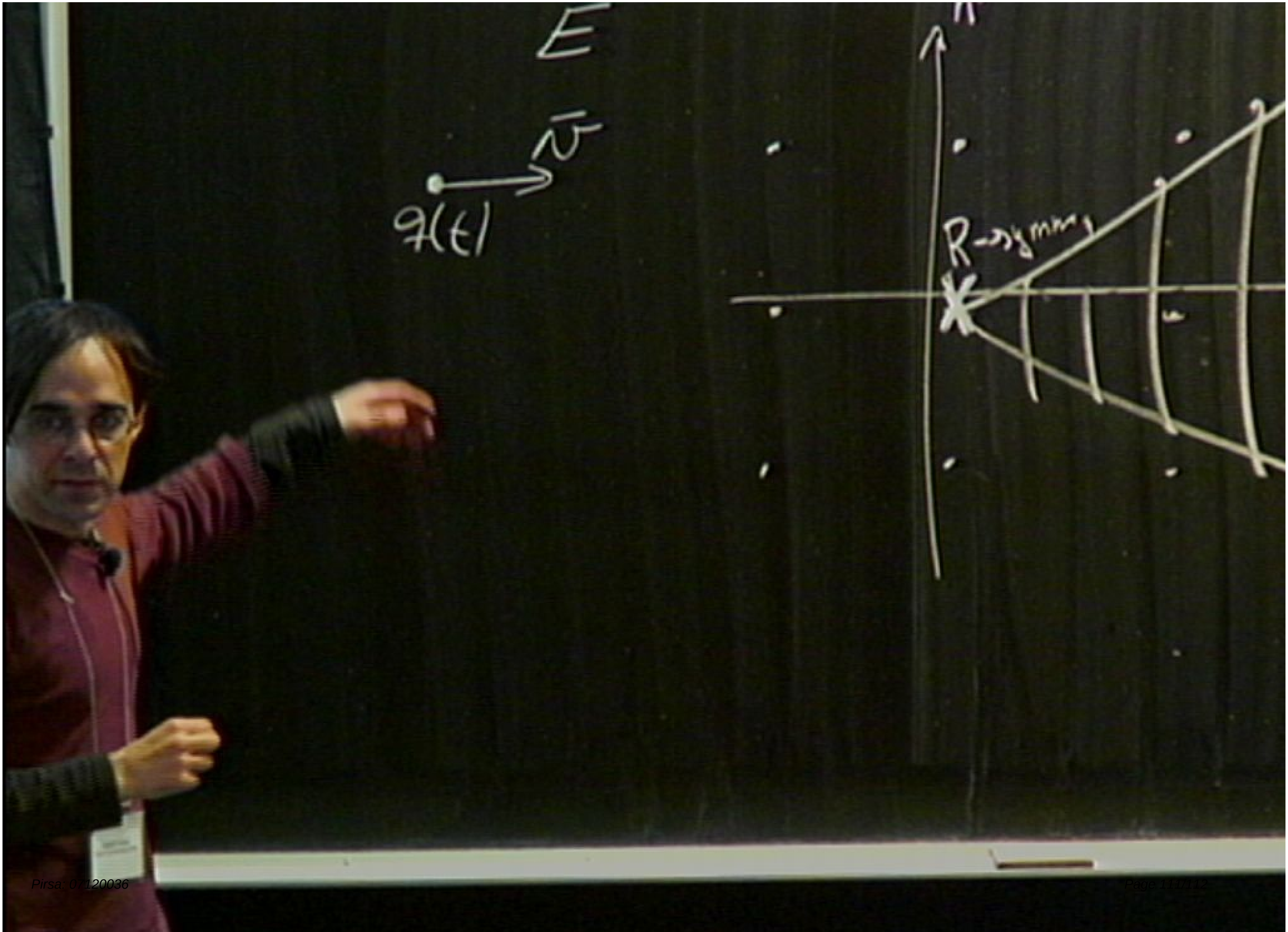
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