

Title: Gravitons and black holes in loop quantum gravity.

Date: Dec 04, 2007 04:00 PM

URL: <http://pirsa.org/07120034>

Abstract: In the first part of the talk we introduce a technique to compute large scale correlations in LQG and spinfoam models. Using this formalism we calculate some components of the graviton propagator and of the n-points function.

In the second part we apply the ideas suggested by LQG to the black hole interior . The result of the quantum analysis is that the classical singularity disappears in loop quantum gravity. Solving the semiclassical Einstein equation of motion we obtain a metric regular and singularity free in contrast to the classical one. By using the new metric we calculate the Hawking temperature, entropy and we study the mass evaporation process.

## OUTLINE

*Large scale correlations,*

*Non singular black hole,  
Loop quantum black hole*

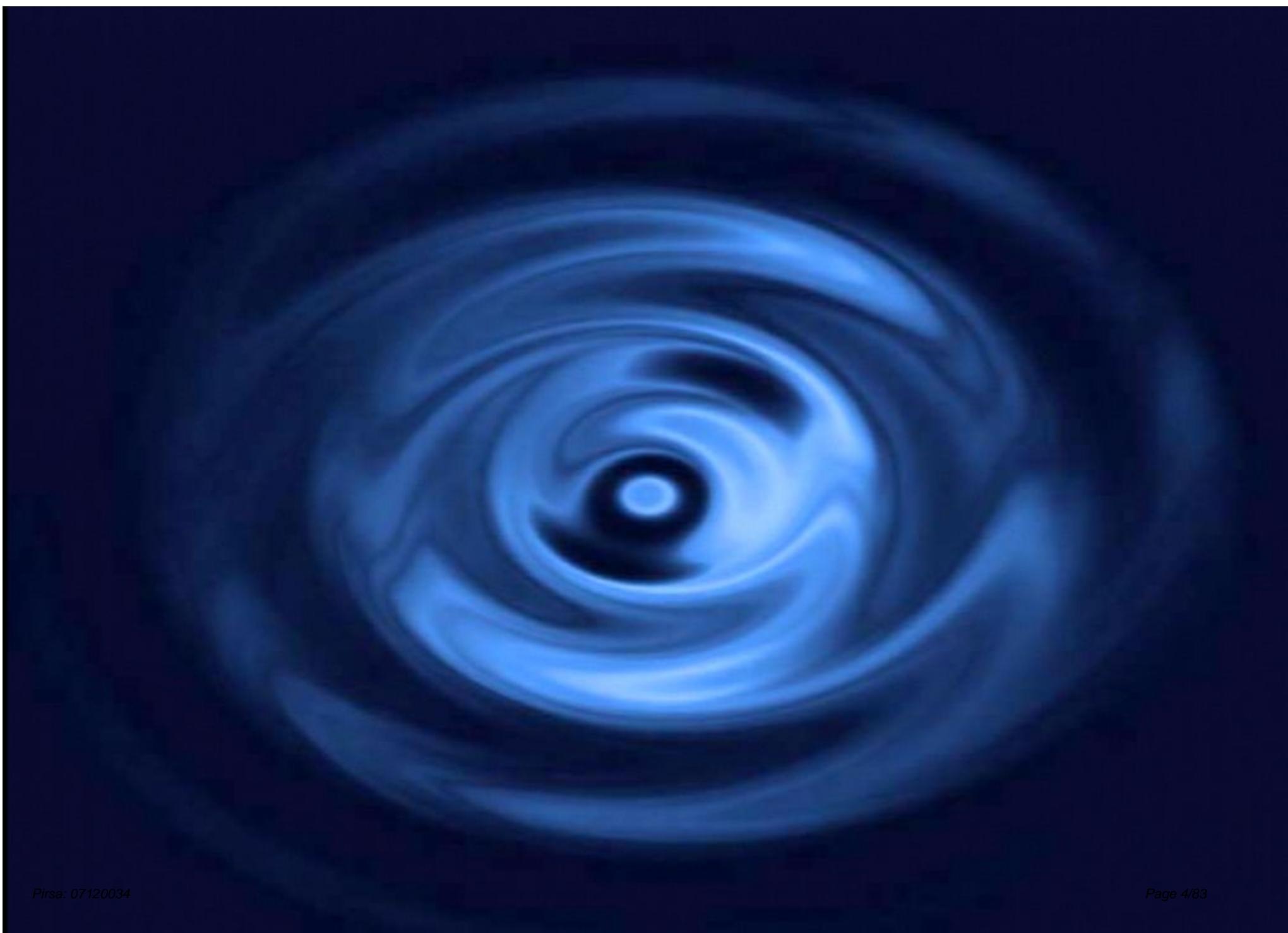
*Loop quantum black hole  
semiclassical analysis.*

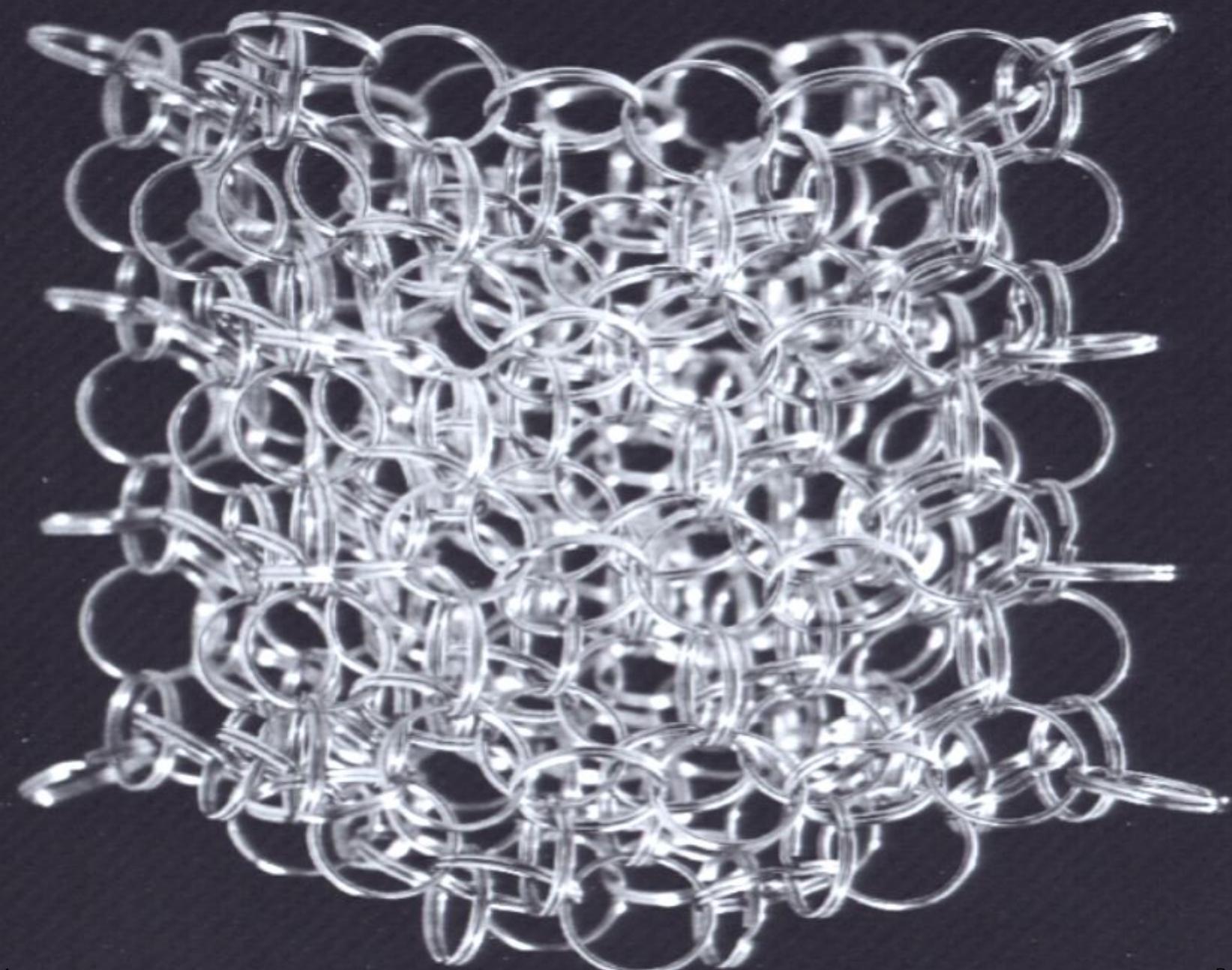
## *OUTLINE*

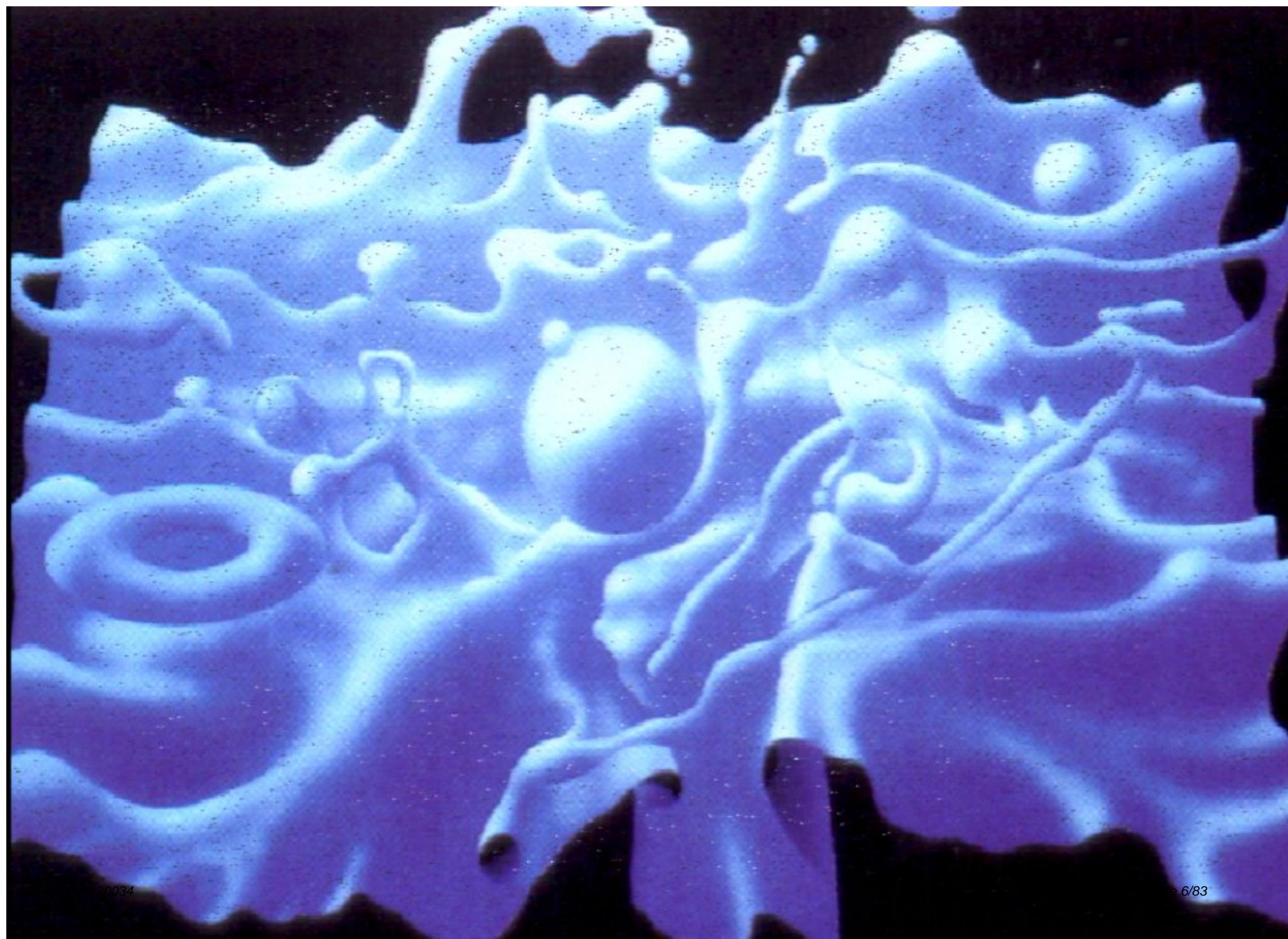
*Large scale correlations.*

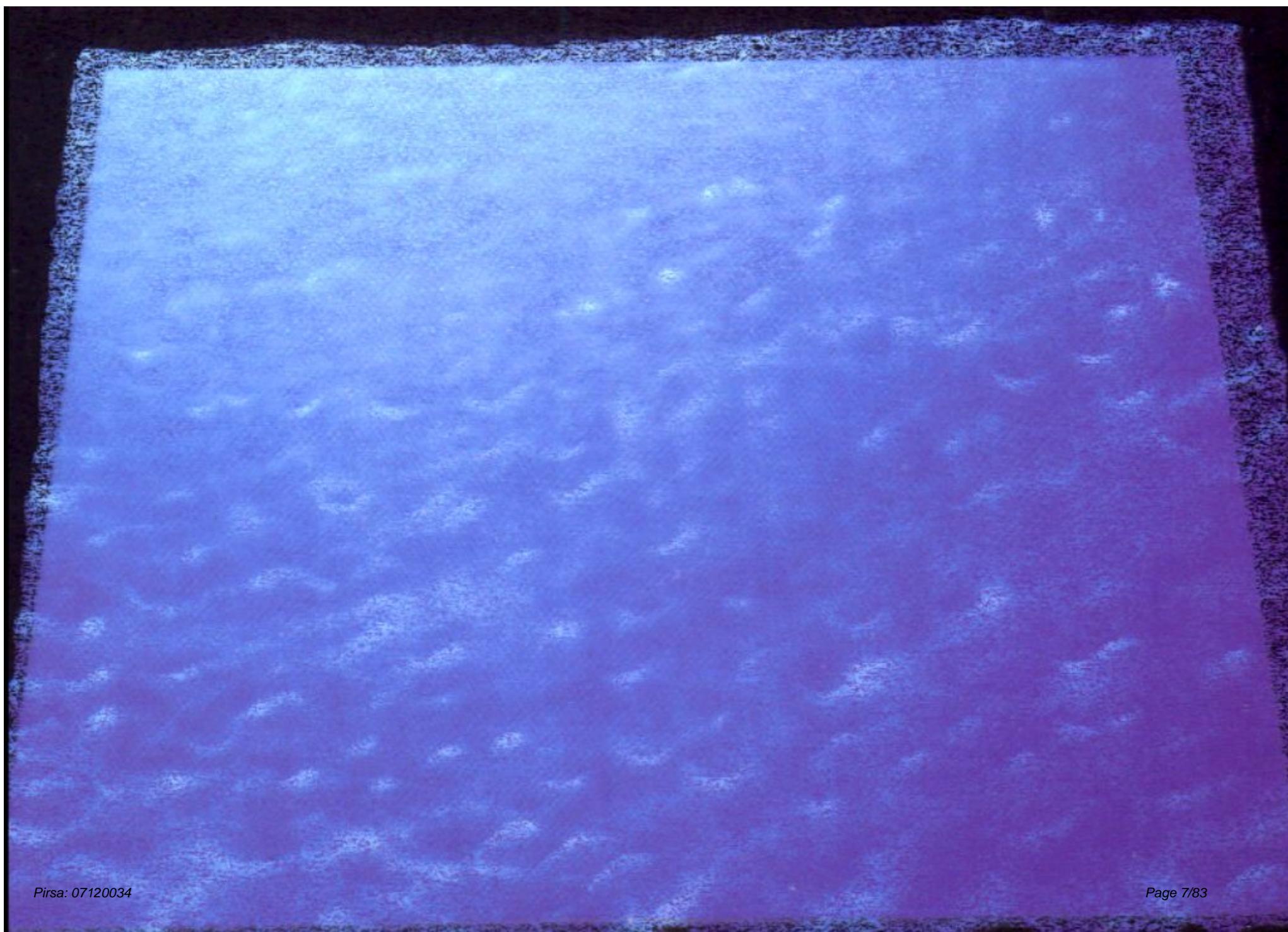
*Non singular black hole,  
Loop quantum black hole.*

*Loop quantum black hole,  
semiclassical analysis.*











## *LOW ENERGY LIMIT*

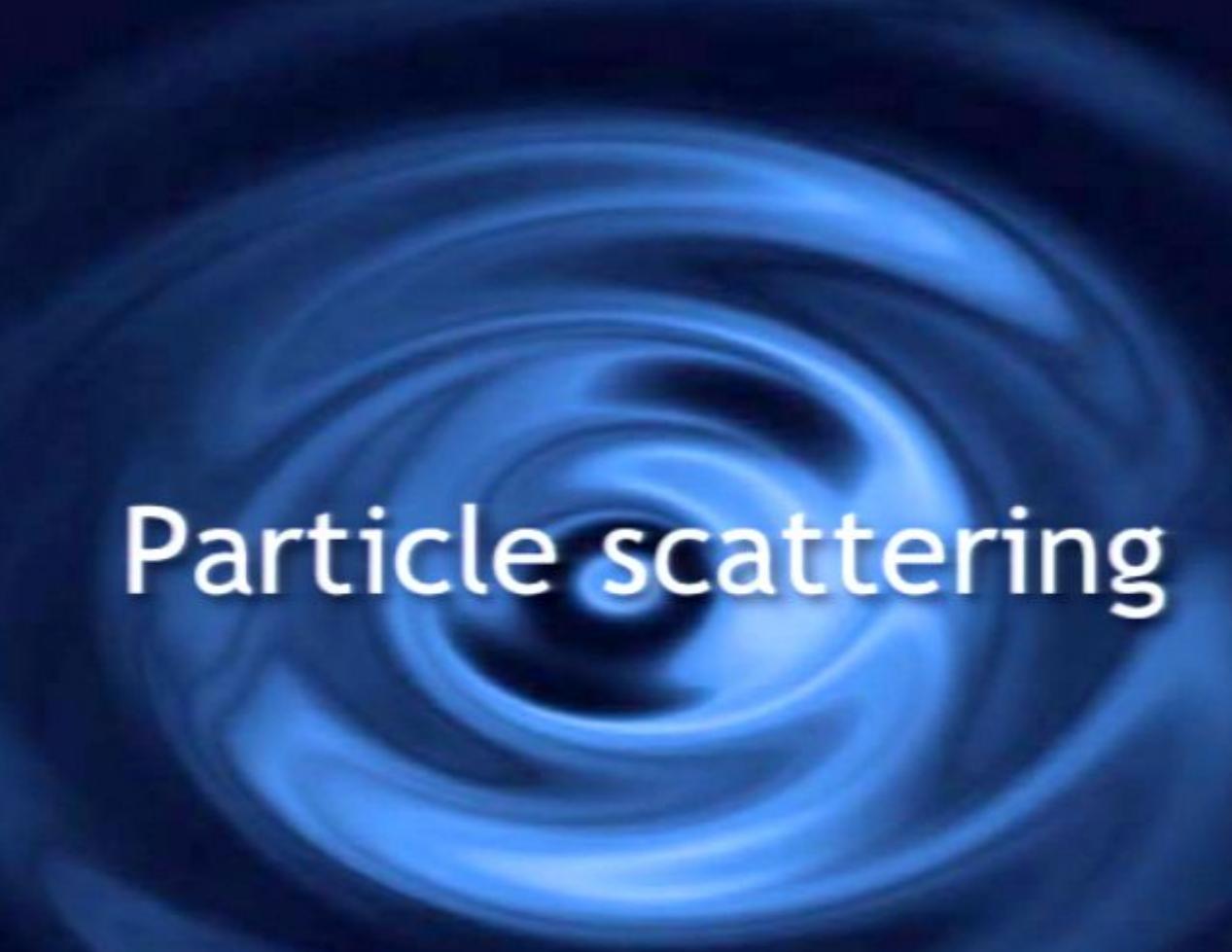
*Emergence of 4-d world from causal quantum gravity,  
J. Ambjorn, R. Loll.*

*Continuum spacetime as a condensed phase of a Group Field Theory,  
Daniele Oriti.*

*Non-commutative quantum field theory from quantum gravity,  
Laurent Freidel, Etera R. Livine.*

*Emergent space-time,  
Fotini Markopoulou, Lee Smolin.*

*Master constraint program, coherent states, algebraic quantum gravity,  
B. Dittrich, K. Giesel, T. Thiemann.*

A large, glowing blue circular pattern resembling a wave or a series of concentric ripples, centered on the slide.

# Particle scattering

## Particle scattering in loop quantum gravity

(Leonardo Modesto, Carlo Rovelli, Phys. Rev. Lett. 95 (2005))

### The $n$ -point function

$$W(x_1, \dots, x_n) = Z^{-1} \int D\phi \phi(x_1) \dots \phi(x_n) e^{-iS[\phi]}$$

In background independent quantum field theory, if we assume to be well-defined with general-covariant measure and action, the  $n$ -point function is constant in spacetime

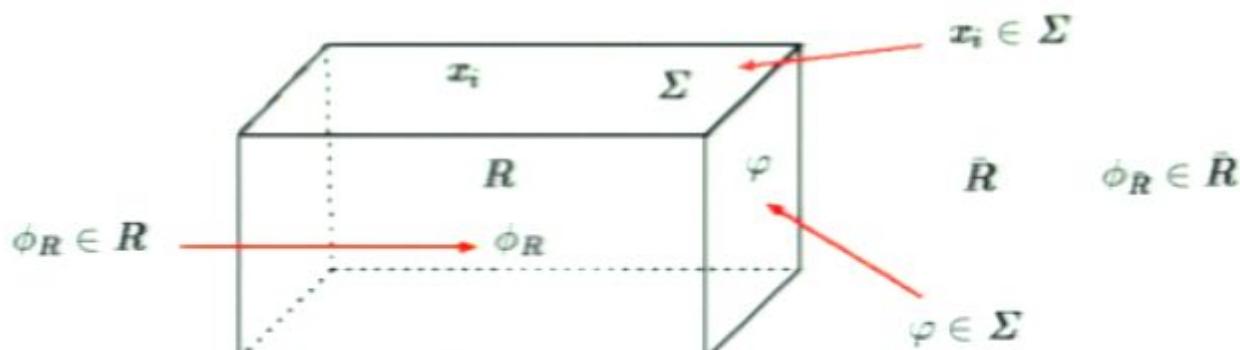
We can write  $W(x_1, \dots, x_n)$  in the form :

$$W(x_1, \dots, x_n) = Z^{-1} \int D\varphi \varphi(x_1) \dots \varphi(x_n) W[\varphi, \bar{\Sigma}] W[\varphi, \Sigma]$$

$$W[\varphi, \Sigma] = \int_{\phi_R|_{\Sigma} = \varphi} D\phi_R e^{-S_R[\phi_R]} \quad W[\varphi, \bar{\Sigma}] = \int_{\phi_R|_{\bar{\Sigma}} = \varphi} D\phi_R e^{-S_R[\phi_R]}$$

We assume that the particle interaction is restricted to  $R$ . Then we approximate  $S_R[\phi]$  with the gaussian theory  $S_R^{(0)}[\phi]$ . Then we can replace  $W[\varphi, \bar{\Sigma}]$  with its free equivalent :

$$W_0[\varphi, \bar{\Sigma}] = \int_{\phi|_{\Sigma} = \varphi} D\phi_R e^{-S_R^{(0)}[\phi]} \equiv \Psi_{\Sigma}[\varphi]$$





### Diffeomorphism invariant theory

- (i) because of diffeomorphism invariance the boundary propagator  $W[\varphi, \Sigma]$  is independent from (local deformations of) the surface  $\Sigma$ .  $W(\varphi, \Sigma)$  reads  $W[\varphi]$ ;
- (ii) the geometry of the boundary surface  $\Sigma$  is not determined by a background geometry (there isn't any), but rather by the boundary gravitational field  $\varphi$  itself.

We may expect an expression of the form :

$$W(x_1, \dots, x_n; q) = Z^{-1} \int D\varphi \varphi(x_1) \dots \varphi(x_n) \Psi_q[\varphi] W[\varphi]$$

$\Psi_q[\varphi]$  is the gaussian boundary state picked around the 3-geometry  $q$  of the boundary surface  $\Sigma$ :

The points  $x_i$  can be defined with respect to the boundary geometry  $q$ .

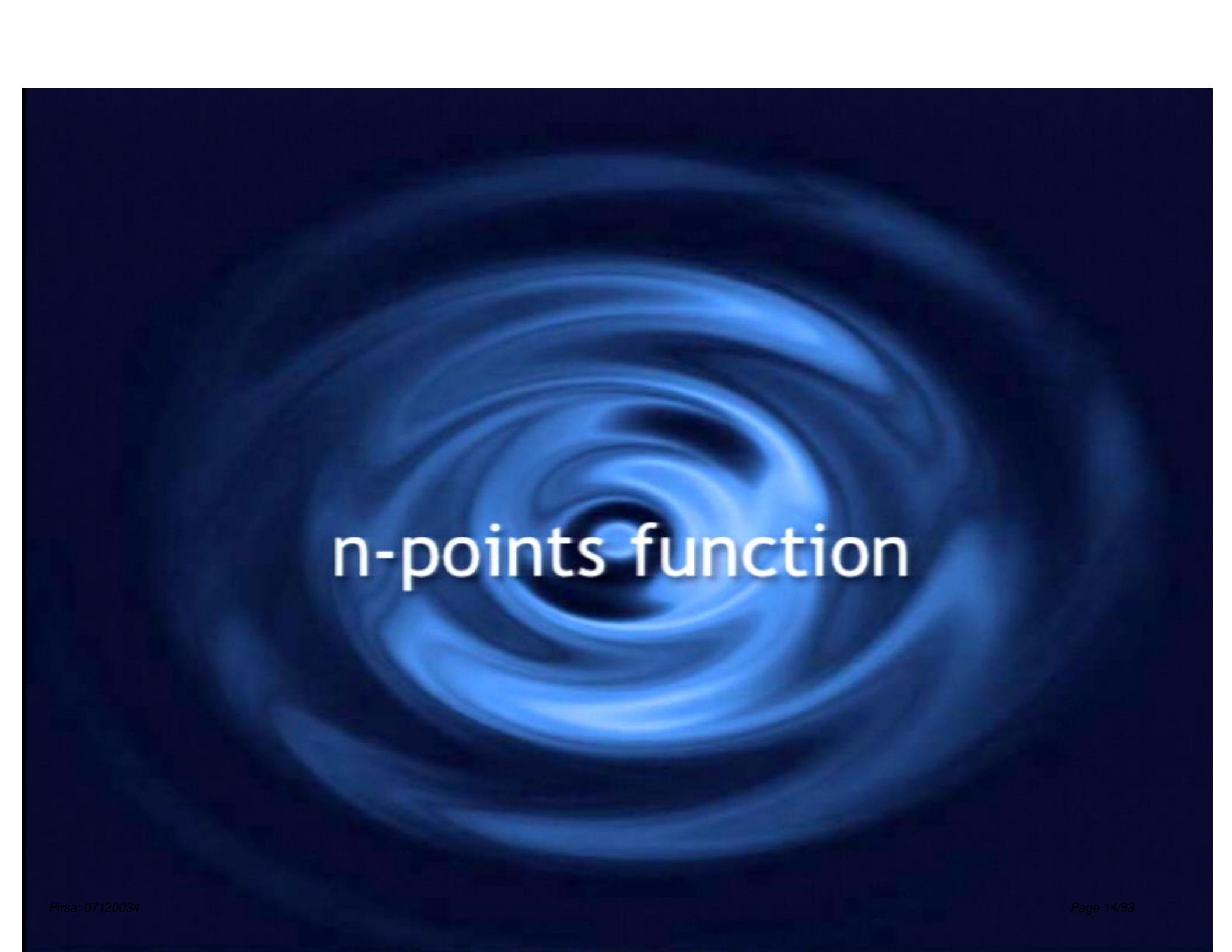
The localization of the arguments of the  $n$ -point function can be defined with respect to the geometry over which the boundary state is picked.

$x_i$  are then metric coordinates: they refer to gravitational field values.

In this manner, we can give meaning to  $n$ -point functions in a background independent context.

### *n*-point functions in loop quantum gravity

$$W(x_1, \dots, x_n; g) = Z^{-1} \sum_s c(s) \varphi_s(x_1) \dots \varphi_s(x_n) \Psi_q[s] W[s]$$



n-points function

## n - points function

$$G_q^{ab\,cd\dots mn}(x_1, x_2, \dots, x_n) = \frac{\sum_s W[s] h_s^{ab}(x_1) h_s^{cd}(x_2) \dots h_s^{mn}(x_n) \Psi_q(s)}{\sum_s W[s] \Psi_q(s)}.$$

*The amplitude from spinfoams :*

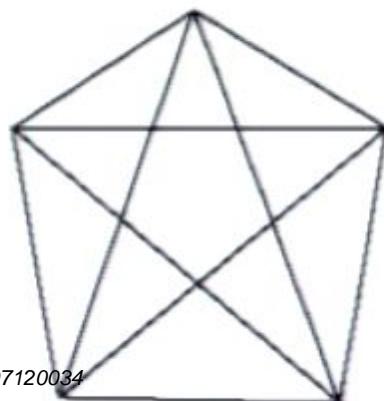
$$W[s] = \sum_{\sigma, \partial\sigma=s} \prod_{f \in \sigma} \dim(j_f) \prod_{v \in \sigma} \lambda B(j_{nm}^v) \left( \prod_{n \in s} \langle i_n | i_{BC} \rangle \right).$$

$\sigma$  : are spinfoams with vertices  $v$  dual to a four-simplex, bounded the spin network  $s$

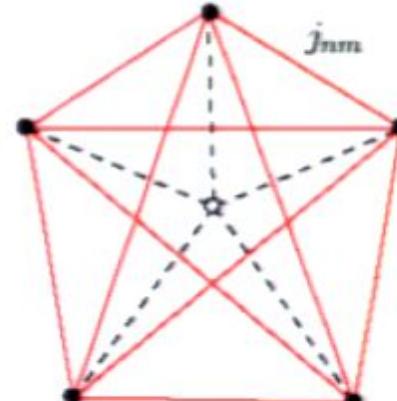
$f$  : are the faces of  $\sigma$

$j_{nm}^v$  : label the representations associated to the ten faces adjacent to the vertex  $v$

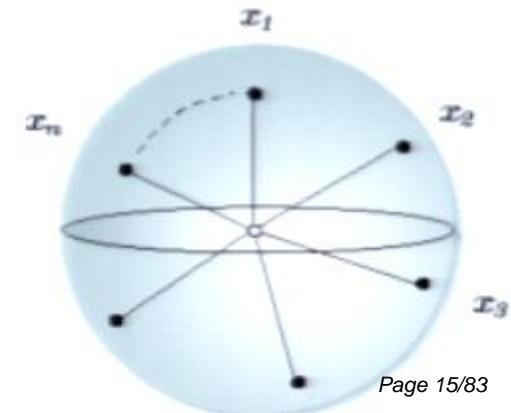
where  $n \neq m = 1, \dots, 5$

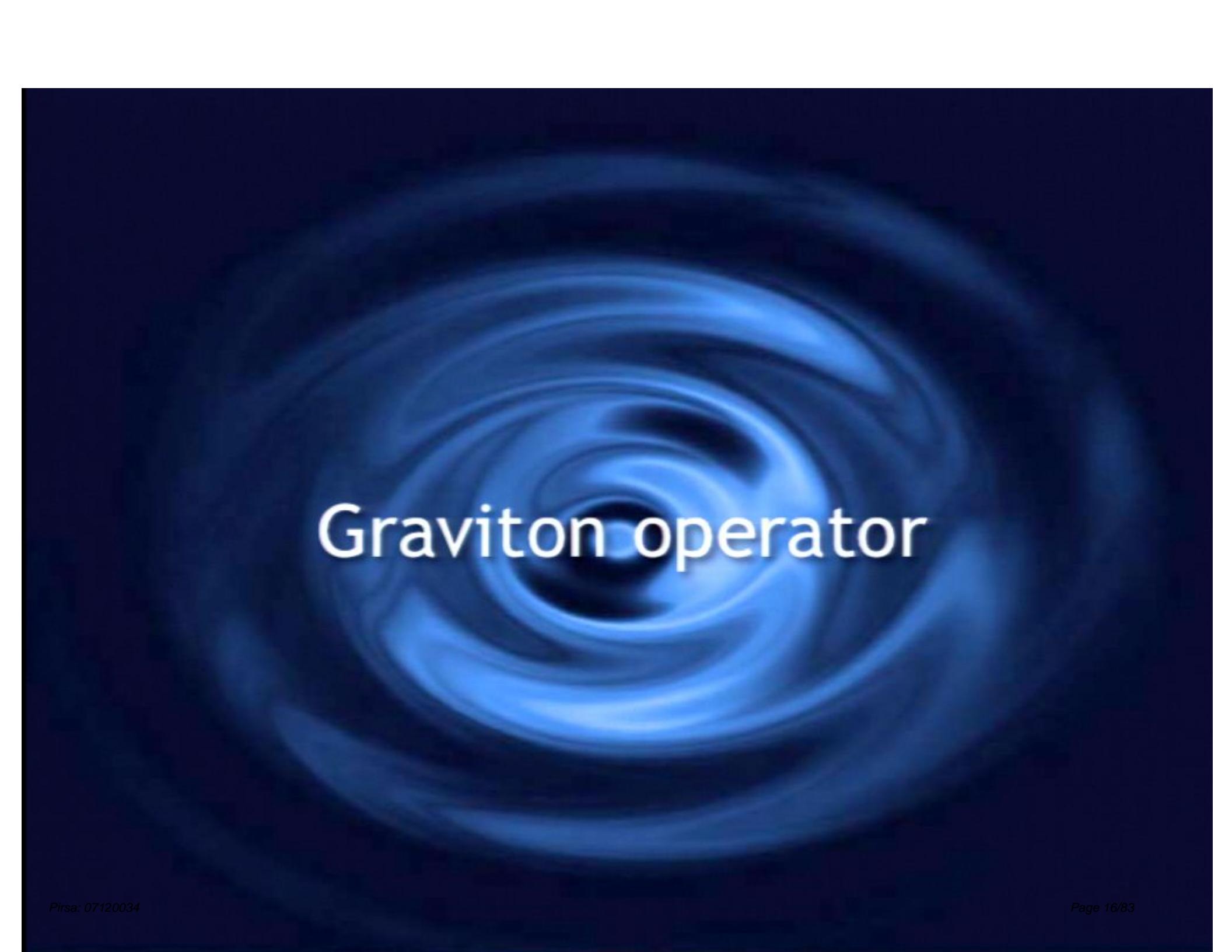


$\xrightarrow{\text{dual spinfoam}}$



boundary spin network





# Graviton operator

## Graviton operator

Fluctuation of the densitized metric operator over the flat metric :

$$\tilde{h}^{ab}(\vec{x}) = (\det g)g^{ab}(\vec{x}) - \delta^{ab} = E^{ai}(\vec{x})E^{bi}(\vec{x}) - \delta^{ab}.$$

The study the action of this operator on a boundary spin network state :  $E^{ai}(\vec{x})E^{bi}(\vec{x})|s\rangle$ .

We identify the point  $\vec{x}$  with one of the nodes  $n$  of the boundary spin network  $s$ .

4 links emerge from this vertex :  $e_I, I = 1, 2, 3, 4$ ; dual to the faces of corresponding tetrahedron.

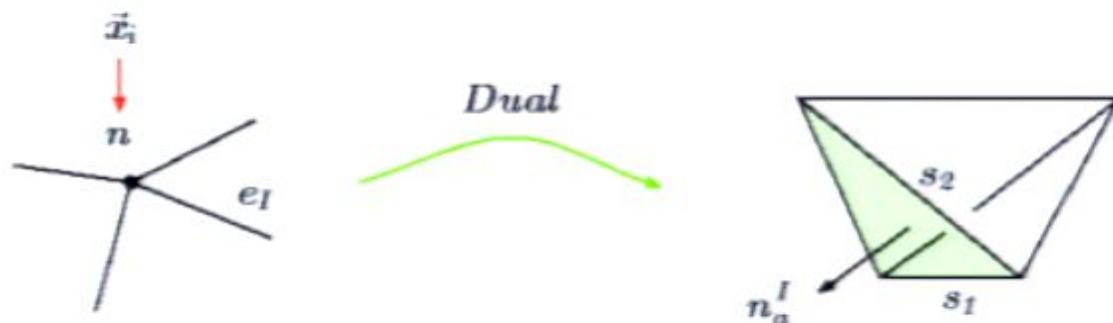
Let  $n_a^I$  be the oriented normal to this face, defined as the vector product of two sides.

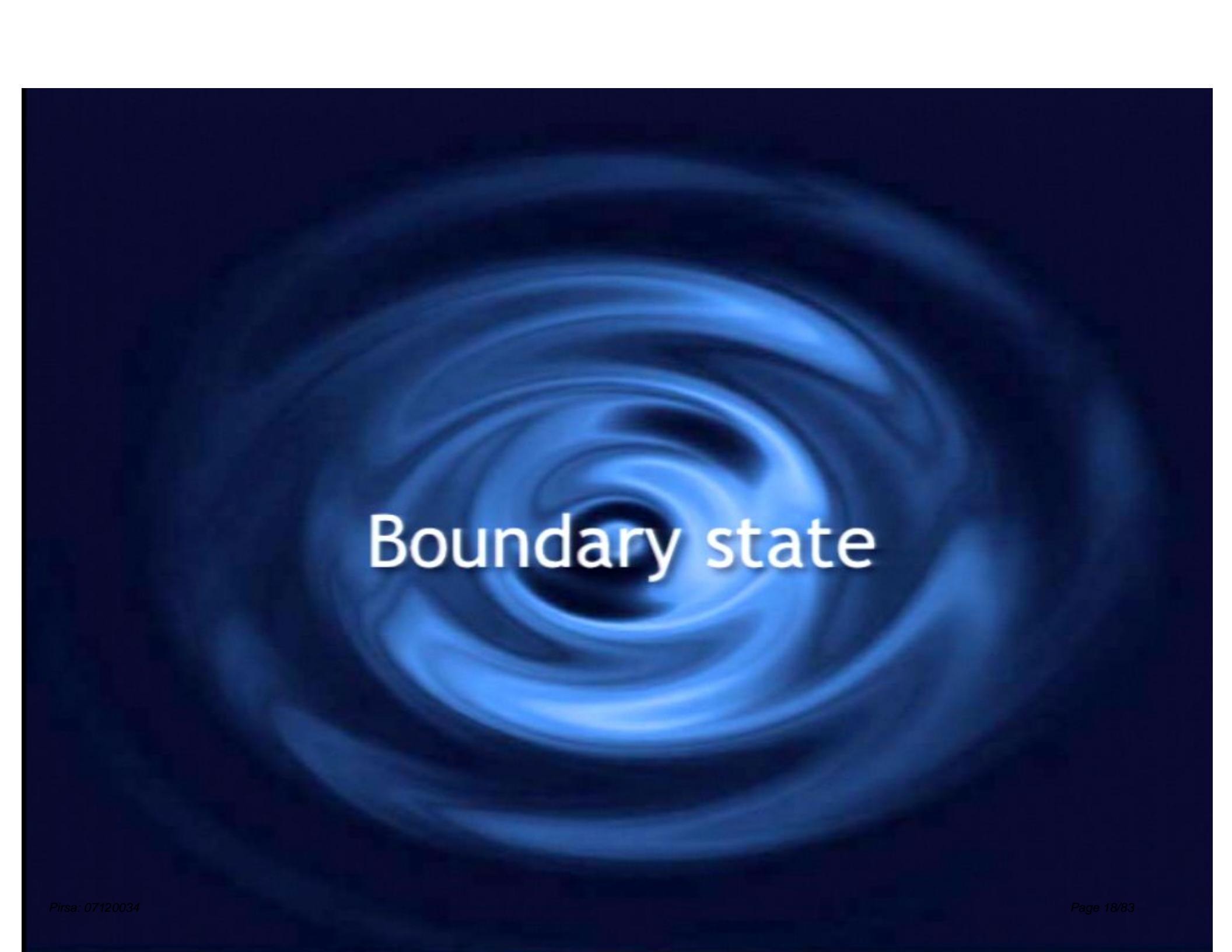
Then  $E(n)^{Ii} = E^{ai}(\vec{x})n_a^I$  can be identified with the action of the  $SU(2)$  generator  $J^i$  on the edge  $e_I$ .

We have then immediately that the diagonal terms define diagonal operators :

$$E^{Ii}(n)E_i^I(n)|s\rangle = (\hbar G)^2 j_I(j_I + 1)|s\rangle$$

where  $j_I$  is the spin of the link in the direction  $I$ .





Boundary state

## The vacuum state $\Psi_q[s]$

$q$  is the metric of a boundary  $\Sigma_q$ .

We assume for simplicity that  $\Psi_q[s]$

depends only on the graph and the spins of  $s$ , and not on the intertwiners.

We consider the state  $\Psi_q[s]$  for the spin networks  $s = (\Gamma, j_{nm}, i_n)$  defined on a graph  $\Gamma$ , dual to 3d triangulations  $\Delta$ .

The areas  $A_{nm}$  of the triangles  $t_{nm}$  of  $\Delta$  determine background values  $j_{nm}^{(\theta)}$  of the spins, via

$$A_{nm}^2 = (\hbar G)^2 j_{nm}^{(\theta)} (j_{nm}^{(\theta)} + 1).$$

We want a state  $\Psi_q[s] = \Psi_q(\Gamma, j_{nm})$  peaked on the background values  $j^{(\theta)}$ .

We choose a Gaussian peaked on these values :

$$\Psi_q[s] = \exp \left\{ -\alpha \sum_{n,m} \frac{(j_{nm} - j_{nm}^{(\theta)})^2}{j_{nm}^{(\theta)}} + i \sum_{n,m} (\Phi_{nm}^{(\theta)} j_{nm}) \right\} \quad \alpha \text{ is a constant}$$

For large  $j$  representations :  $\frac{\Delta A}{A} \sim \frac{\Delta j}{j} \sim \frac{1}{\sqrt{\alpha j^{(\theta)}}} \rightarrow 0 \quad , \quad \frac{\Delta \Phi}{\Phi} \sim \frac{1}{\Phi} \sqrt{\frac{\alpha}{j^{(\theta)}}} \rightarrow 0$

The phase factor determines where the state is peaked on the variables conjugate to the spins  $j_{mn}$ .

We recall the form of the Regge action :  $S = \sum_{n,m} (\Phi_{nm}(j_{mn}) j_{mn}) \quad ,$

$\Phi_{nm}(j_{mn})$  are dihedral angles at the triangles .  $\frac{\partial S_{\text{Regge}}}{\partial j_{nm}} = \Phi_{nm}$

The angles code the extrinsic geometry of the 3-d boundary.

The vacuum state is peaked in the extrinsic curvature of the boundary

in the variables conjugate to the spins



# Graviton propagator

## 2-points function (graviton propagator)

$$G(\mathbf{L})_{(ij)(kl)} = \frac{G_q^{abcd}(x, y) n_a^{(ij)} n_b^{(ij)} n_c^{(kl)} n_d^{(kl)}}{|n|^4}.$$

$$G(\mathbf{L})_{(ij)(kl)} = \frac{1}{8\pi\hbar G j_L^4} \frac{\sum_s \mathbf{W}[s] ((8\pi\hbar G)^2 j_{ij}(j_{ij}+1) - |n|^2) ((8\pi\hbar G)^2 j_{kl}(j_{kl}+1) - |\bar{n}|^2) \Psi_q[s]}{\sum_s \mathbf{W}[s] \Psi_q[s]}.$$

$$\mathbf{W}[s] = \frac{\lambda}{5!} \left( \prod_{n < m} \dim(j_{nm}) \right) B(j_{nm}^u).$$

$$\Psi_q[s] = e^{-\frac{i}{8j_L} \sum_{(nm)(pq)} \alpha_{(nm)(pq)} (j_{nm} - j_L)(j_{pq} - j_L) + i\Phi \sum_{(n,m)} j_{nm}}.$$

For large  $j$  and introducing the variables  $\delta j_{ij} = (j_{ij} - j_L)$ :

$$G(\mathbf{L})_{(ij)(kl)} = \frac{4}{j_L^2} \frac{\sum_{\delta j_{nm}} \delta j_{ij} \delta j_{kl} B(j_L + \delta j_{nm}) e^{-\frac{i}{8j_L} \alpha_{(nm)(pq)} \delta j_{nm} \delta j_{pq} + i\Phi \sum_{nm} j_{nm}}}{\sum_{\delta j_{nm}} B(j_L + \delta j_{nm}) e^{-\frac{i}{8j_L} \alpha_{(nm)(pq)} \delta j_{nm} \delta j_{pq} + i\Phi \sum_{nm} j_{nm}}}.$$

For large representations,  $B(j_{nm}, j_l)$  is :

$$B(j_{nm}) = \sum_{\sigma} P(\sigma) \left[ e^{iS_{\text{Regge}}(\sigma) + k \frac{\pi}{4}} + e^{-iS_{\text{Regge}}(\sigma) + k \frac{\pi}{4}} \right] + D(j_{nm})$$

$D(j_{nm}, j_n)$  does not oscillate (from numerical calculation).

$\sigma$  labels the distinct 4-simplices with triangles having the same area.

$P(\sigma)$  is a slowly varying factor, that grows as  $\lambda^{-9/2}$  when scaling the spins by  $\lambda$ .

Around  $j_{nm} = j_L$  :

$$S_{\text{Regge}}(j_{nm}) = \Phi \sum_{nm} j_{nm} + \frac{1}{2} G_{(mn)(pq)} \delta j_{mn} \delta j_{pq} + \dots \quad G_{(mn)(pq)} = \frac{\partial^2 S_{\text{Regge}}(j_{rs})}{\partial j_{mn} \partial j_{pq}} \Big|_{j_{rs}=j_L}$$

$$G(L)_{(ij)(kl)} = \frac{4}{j_L^2} \frac{\sum_{\delta j_{nm}} \delta j_{ij} \delta j_{kl}}{\langle W | \Psi_q \rangle} \left[ e^{i(\Phi \sum_{nm} j_{nm} + \frac{i}{2} G_{(nm)(pq)} \delta j_{nm} \delta j_{pq})} + h.c. + D(j_{nm}) \right] e^{-\frac{i}{2j_L} \alpha_{(nm)(pq)} \delta j_{nm} \delta j_{pq} + i \Phi \sum_{nm} j_{nm}}$$

rapidly oscillating

$$= \frac{4}{j_L^2} \left[ (j_L^{-1} \alpha + iG)^{-1} \right]_{(ij)(kl)}.$$

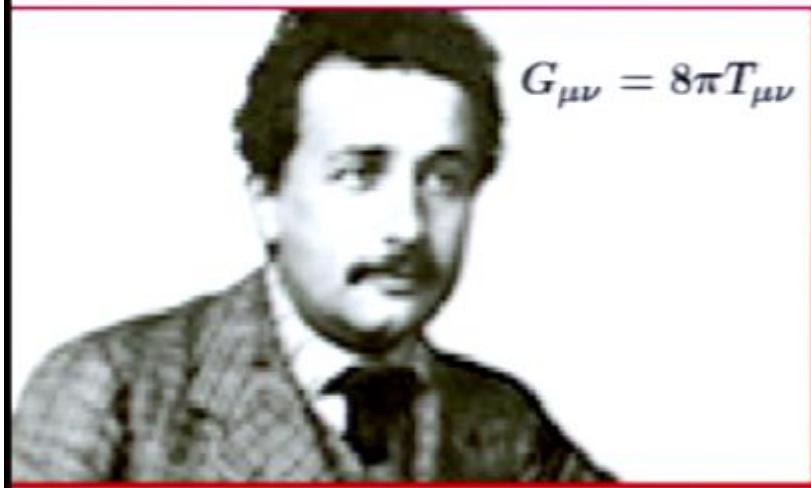
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$$G(L)_{(ij)(kl)} = \frac{32\pi\hbar G}{\sqrt{3}/4 L^2} \left[ \left( \alpha + i\sqrt{3}/4 K \right)^{-1} \right]_{(ij)(kl)}$$

$$G_{(nm)(pq)} = \frac{8\pi\hbar G}{L^2} K_{(nm)(pq)}$$



# Perturbative gravity



$$G_{\mu\nu} = 8\pi T_{\mu\nu}$$



$$S = -\frac{1}{2\kappa} \int R\sqrt{-g} d^4x$$

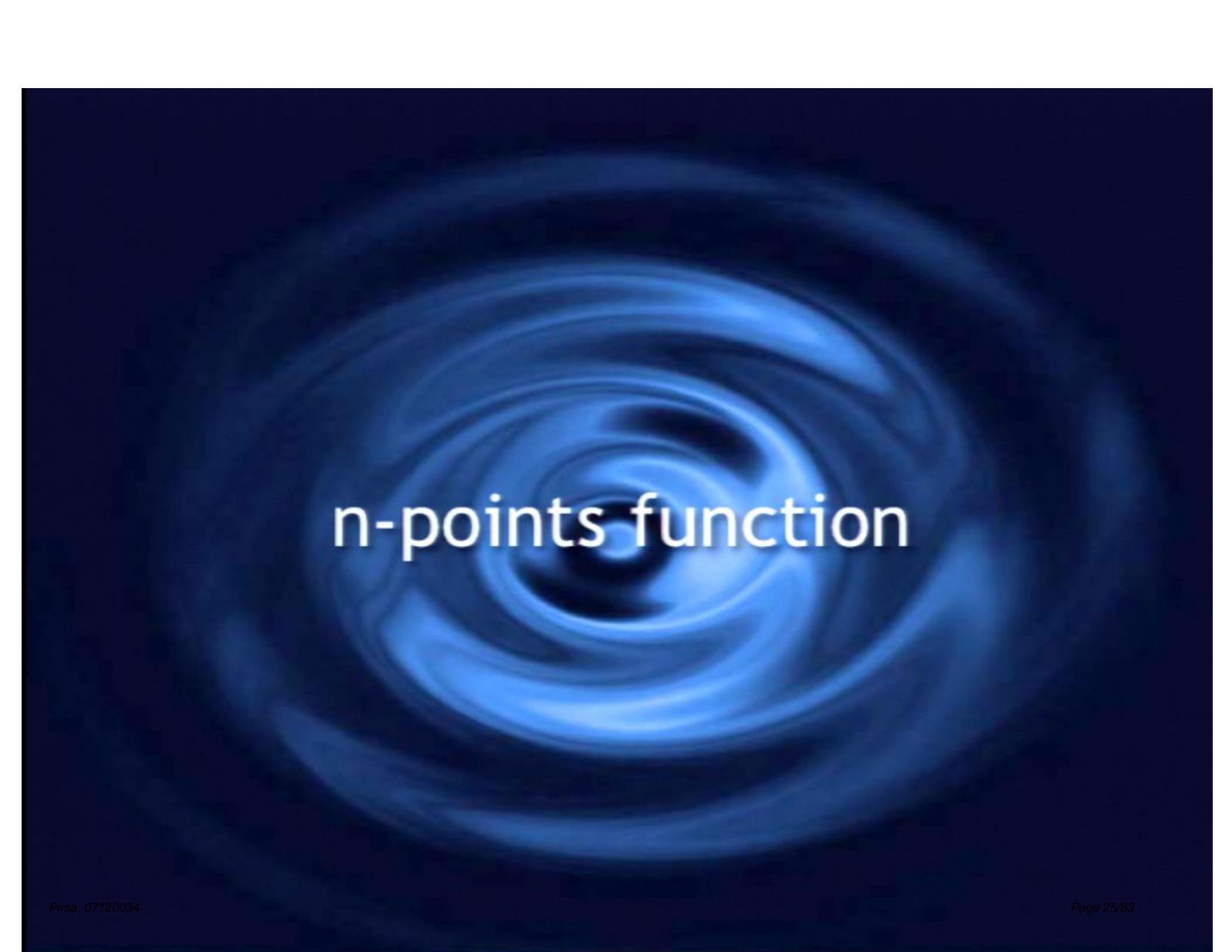
$$g_{\mu\nu} = \delta_{\mu\nu} \times h_{\mu\nu}$$



$$L = \partial h \partial h + G_N^{\frac{l}{2}} \partial h \partial h h + G_N \partial h \partial h h h + G_N^{\frac{3}{2}} \partial h \partial h h h h + \dots$$



$$S = \prod_{n=1}^{\infty} \sum_{m,p,q,r=0}^{\infty} \frac{(-\sqrt{2\kappa})^{p+q+r+m(n-1)} (2\kappa)^{\frac{m}{2}}}{16\pi G_N (2n)^m m!} \int [(h^n)_{\delta}]^m (h^p)^{\mu\nu} (h^q)^{\rho\sigma} (h^r)^{\tau\epsilon} (\Gamma_{\sigma\mu\tau} \Gamma_{\nu\rho} - \Gamma_{\sigma\mu\nu} \Gamma_{\epsilon\rho\tau}).$$

The background of the slide features a dark navy blue gradient. Overlaid on this is a series of concentric, glowing blue circles that radiate from the bottom right corner towards the top left, creating a sense of motion and depth.

# n-points function

### Regge action beyond second order in $\delta j_{mn}$

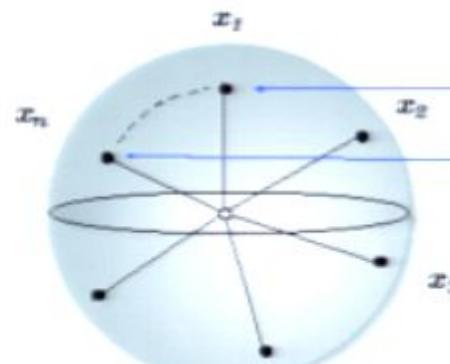
$$S_{\text{Regge}}(j_{nm}) = \Phi \sum_{nm} j_{nm} + \frac{1}{2} G_{(mn)(pq)}^{(2)} \delta j_{mn} \delta j_{pq} + \frac{1}{3!} I_{(mn)(pq)(rs)}^{(3)} \delta j_{mn} \delta j_{pq} \delta j_{rs} + \dots + \frac{1}{n!} I_{(i_1 j_1) \dots (i_n j_n)}^{(n)} \delta j_{i_1 j_1} \dots \delta j_{i_n j_n} \dots$$

### *n*-points function

$$\begin{aligned} G(L)_{(i_1 j_1) \dots (i_n j_n)} &= \frac{(-2)^n i \sum_{\delta \vec{j}} \delta j_{i_1 j_1}^2 \dots \delta j_{i_n j_n}^2 I_{(i_1 j_1) \dots (i_n j_n)}^{(n)} e^{-\frac{i}{2} \delta \vec{j}^T (iG^{(2)} + j_L^{-1} \alpha) \delta \vec{j}}}{n! j_L^n \sum_{\delta \vec{j}} e^{-\frac{i}{2} \delta \vec{j}^T (iG^{(2)} + j_L^{-1} \alpha) \delta \vec{j}} =} \\ &= \frac{(-2)^n i}{n! j_L^n} I_{(i_1 j_1) \dots (j_n j_m)}^{(n)} [(K^{-1})_{(i_1 j_1)(i_1 j_1)} \dots (K^{-1})_{(i_n j_n)(i_n j_m)} + \dots] \sim \boxed{\frac{1}{L^{2n-2}}} \end{aligned}$$

### Perturbative quantum gravity

$$G_{i_1 j_1 \dots i_n j_n}(x_1, \dots, x_n) = \frac{\langle h_{\mu_1 \mu_2}(x_1) \dots h_{\mu_{n-1} \mu_n}(x_n) \rangle_{\text{int}} n_{(i_1 j_1)}^{\mu_1} n_{(i_1 j_1)}^{\mu_2} \dots n_{(i_n j_n)}^{\mu_{n-1}} n_{(i_n j_n)}^{\mu_n}}{|n_{(ij)}|^n} \sim \boxed{\frac{1}{L^{2n-2}} f(\alpha_1, \dots, \alpha_n)}.$$





## Two and three-Area correlations in Spin-foams and perturbative quantum Regge calculus

$$A_{mn} = 8\pi G_N \gamma \sqrt{(j_0 + \delta j_{mn})(j_0 + \delta j_{mn} + 1)} = 8\pi G_N \gamma j_0 \left( 1 + \frac{\delta j_{mn}}{j_0} + \frac{1}{2j_0} + \dots \right)$$

$$\langle A_{m'n'} \dots \rangle_q \approx (8\pi G_N \gamma)^2 j_0^2 \frac{\int \prod d\delta j_{mn} W_v(j_0 + \delta j_{mn}) (j_0 + \delta j_{m'n'}) \dots \Psi_{j_0, \phi_0}(j_0 + \delta j_{mn})}{\int \prod d\delta j_{mn} W_v(j_0 + \delta j_{mn}) \Psi_{j_0, \phi_0}(j_0 + \delta j_{mn})}.$$

*Using the stationary phase approximation for the Barrett-Crane vertex*

$$W_v(j_0 + \delta j_{mn}) \approx N \mu_{j_0}(\delta j_{mn}) e^{iS_{j_0}(\delta j_{mn})} + R(j_0 + \delta j_{mn}),$$

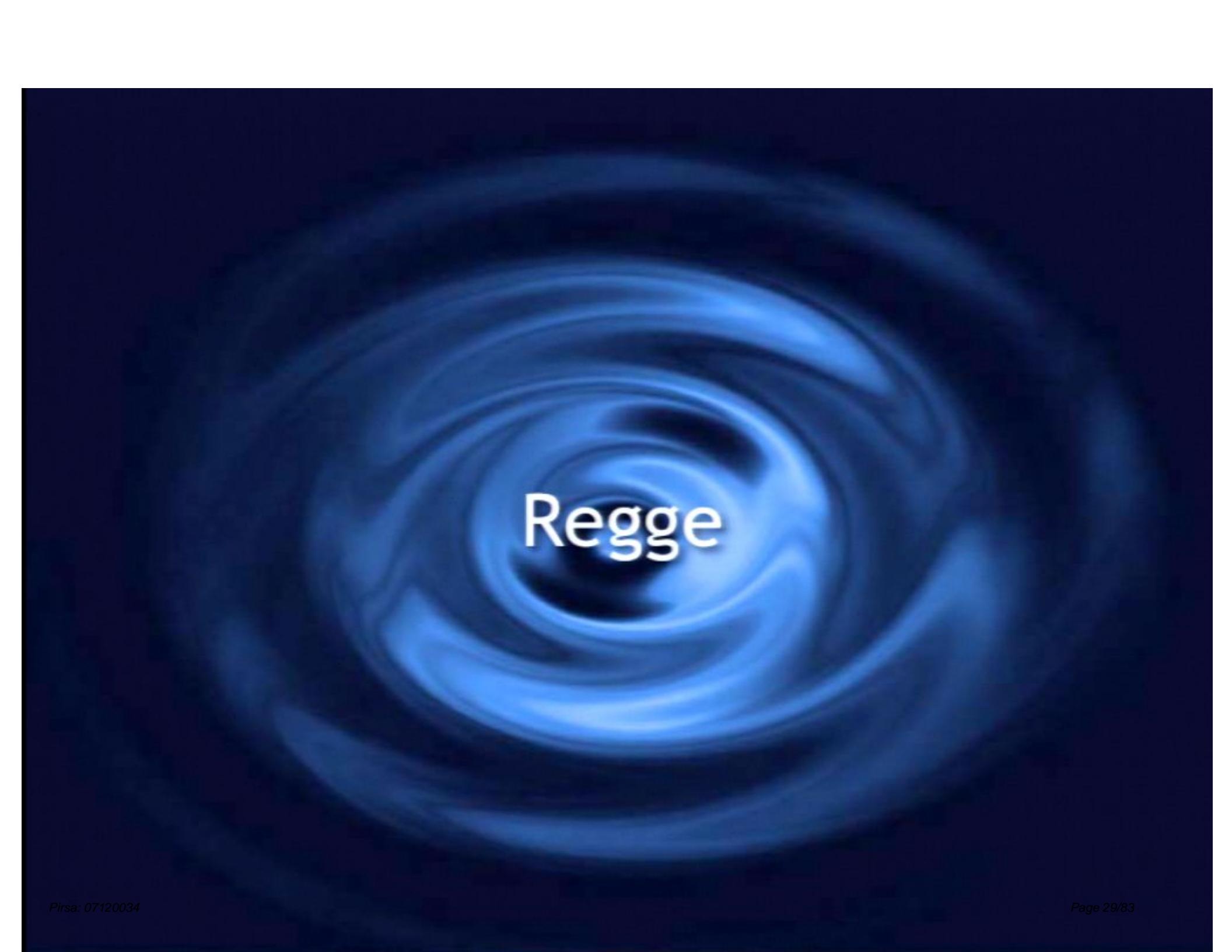
$$S_{j_0}(\delta j_{mn}) = \phi_0 \sum_{m < n} (j_0 + \delta j_{mn}) + \frac{1}{2} \sum_{m < n} \sum_{p < q} \frac{K_{(mn)(pq)}}{j_0} \delta j_{mn} \delta j_{pq} + \frac{1}{3!} \sum_{m < n} \sum_{p < q} \sum_{r < s} \frac{I_{(mn)(pq)(rs)}}{j_0^2} \delta j_{mn} \delta j_{pq} \delta j_{rs} + O(\delta j^4 / j_0^3)$$

$$\langle A_{mn} \rangle_q = 8\pi G_N \gamma j_0 (1 + O(1/j_0))$$

$$\langle A_{mn} A_{pq} \rangle_q - \langle A_{mn} \rangle_q \langle A_{pq} \rangle_q = (8\pi G_N \gamma)^2 j_0^2 \left( \frac{(iK - \alpha)_{(mn)(pq)}^{-1}}{j_0} + O(1/j_0^2) \right)$$

$$\langle A_{mn} A_{pq} A_{rs} \rangle_q - (\langle A_{mn} A_{pq} \rangle_q \langle A_{rs} \rangle_q + \text{perm.}) + 2 \langle A_{mn} \rangle_q \langle A_{pq} \rangle_q \langle A_{rs} \rangle_q =$$

$$= (8\pi G_N \gamma)^3 j_0^3 \left( \frac{1}{j_0^2} \sum_i i I_{(m'n')(p'q')(r's')} (iK - \alpha)_{(m'n')(mn)}^{-1} (iK - \alpha)_{(p'q')(pq)}^{-1} (iK - \alpha)_{(r's')(rs)}^{-1} + O(1/j_0^3) \right)$$



Regge

## Two and three Area correlations in quantum area Regge calculus

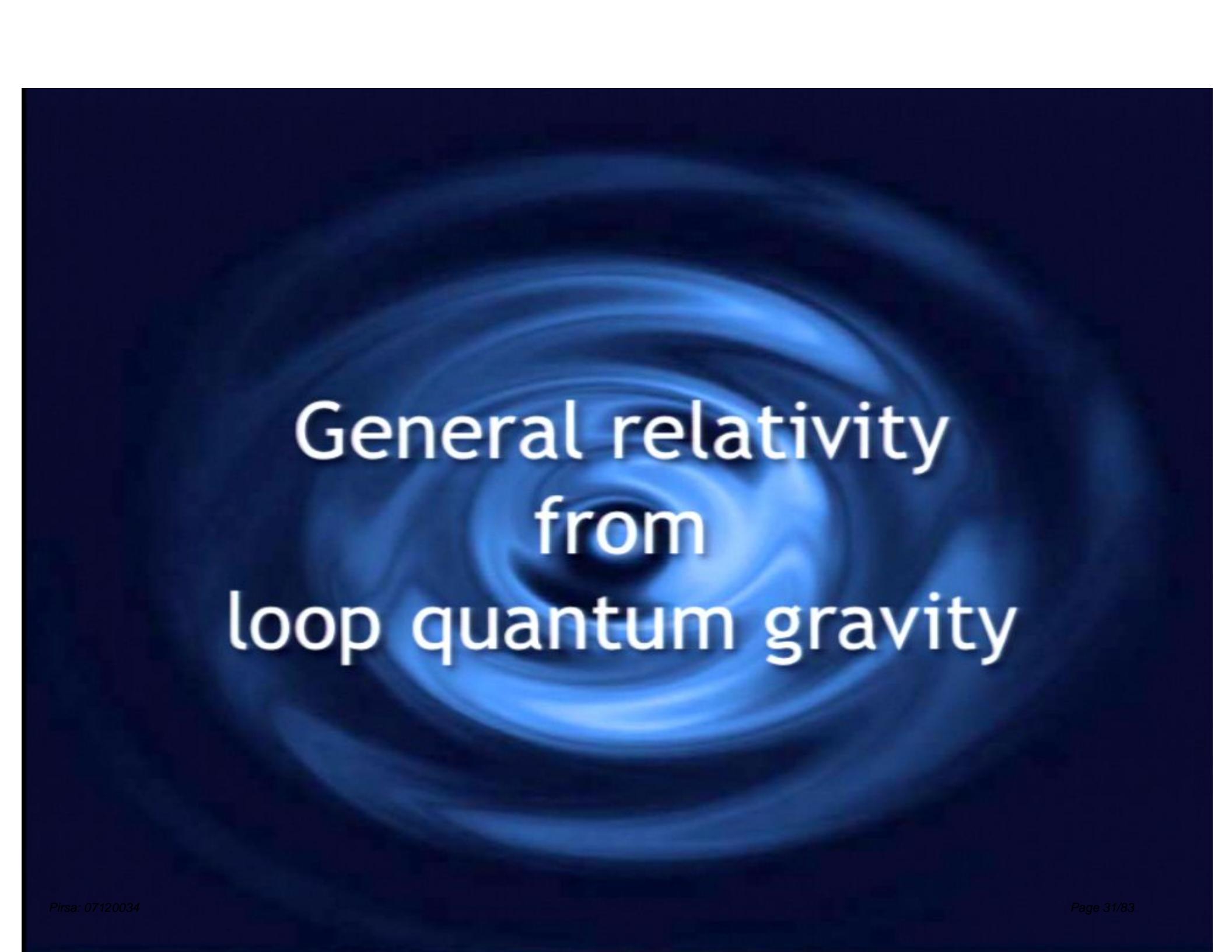
$$\langle \delta \mathbf{A}_{ijk} \dots \rangle = \frac{\int d\delta \mathbf{A} \delta \mathbf{A}_{ijk} \dots e^{iS_{\text{Regge}}(\mathbf{A}_0 + \delta \mathbf{A})} \Psi_0(\delta \mathbf{A})}{e^{iS_{\text{Regge}}(\mathbf{A}_0 + \delta \mathbf{A})} \Psi_0(\delta \mathbf{A})}$$

$$S(\mathbf{A}_0 + \delta \mathbf{A}_{hkl}) = \frac{1}{8\pi G_N} \left( \psi_0 \sum_{h < k < l} (\mathbf{A}_0 + \delta \mathbf{A}_{hkl}) + \frac{1}{2} \sum_{h < k < l} \sum_{r < s < t} \frac{K_{(hkl)(rst)}}{\mathbf{A}_0} \delta \mathbf{A}_{(hkl)} \delta \mathbf{A}_{(rst)} + \right. \\ \left. + \frac{1}{3!} \sum_{h < k < l} \sum_{r < s < t} \sum_{u < v < z} \frac{I_{(hkl)(rst)(rst)}}{\mathbf{A}_0^2} \delta \mathbf{A}_{(hkl)} \delta \mathbf{A}_{(rst)} \delta \mathbf{A}_{(uvz)} + O\left(\frac{\delta \mathbf{A}^4}{\mathbf{A}_0^3}\right) \right)$$

$$K_{(hkl)(rst)} = 8\pi G_N \mathbf{A}_0 \frac{\partial^2 S_{\text{Regge}}}{\partial \mathbf{A}_{hkl} \partial \mathbf{A}_{rst}}(\mathbf{A}_0), \quad I_{(hkl)(rst)(uvz)} = 8\pi G_N \mathbf{A}_0^2 \frac{\partial^3 S_{\text{Regge}}}{\partial \mathbf{A}_{hkl} \partial \mathbf{A}_{rst} \partial \mathbf{A}_{uvz}}(\mathbf{A}_0).$$

$\langle \mathbf{AA} \rangle_{BC} \equiv \langle \mathbf{AA} \rangle_{\text{Regge}}$  ,  $\langle \mathbf{AAA} \rangle_{BC} \equiv \langle \mathbf{AA} \rangle_{\text{Regge}}$  , for  $j_0 \rightarrow +\infty$  and  $\frac{\delta j}{j_0} \ll 1$  ,

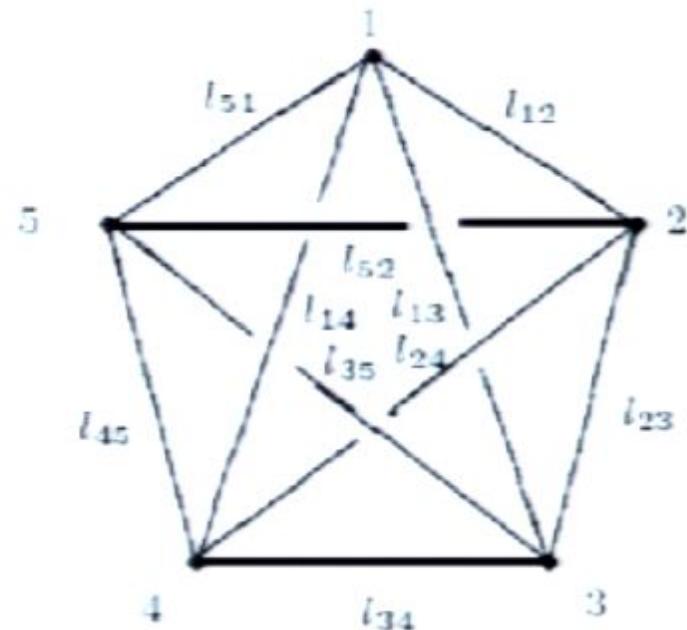
provided that  $j_0 \equiv \frac{\mathbf{A}_0}{8\pi G_N}$  and  $\delta j_{mn} \equiv \frac{\delta \mathbf{A}_{(uvz)}}{8\pi G_N}$



# General relativity from loop quantum gravity

## General relativity action

$$g_{\mu\nu} = \frac{1}{2} (l_{5\mu}^2 + l_{5\nu}^2 - l_{\mu\nu}^2) , \quad \mu, \nu = 1, \dots, 4 \rightarrow$$



**Gravitational fluctuation :**  $\delta g_{\mu\nu} = l_0 (\delta l_{5\nu} + \delta l_{5j} - \delta l_{\mu\nu}) , \delta g_{\mu\nu} \equiv h_{\mu\nu}$ ,

where :  $\delta l_{ij} = \sum_{klm} V_{(ij)(klm)} \delta A_{klm} , \quad i, j = 1, \dots, 5$  and  $V$  is a constant matrix.

$$G_q^{\mu_1 \nu_1 \mu_2 \nu_2 \dots \mu_n \nu_n}(\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n) = \frac{\sum_s \mathbf{W}[s] h_s^{\mu_1 \nu_1}(\mathbf{x}_1) h_s^{\mu_2 \nu_2}(\mathbf{x}_2) \dots h_s^{\mu_n \nu_n}(\mathbf{x}_n) \Psi_q[s]}{\sum_s \mathbf{W}[s] \Psi_q[s]}.$$



# Effective field theory, conclusions



## Conclusions

### Amplitudes

We introduced a technique to compute the  $n$ -points function.

### Graviton propagator

We obtained the graviton propagator (or the Newton law) from a spinfoam model.

### $n$ -points function

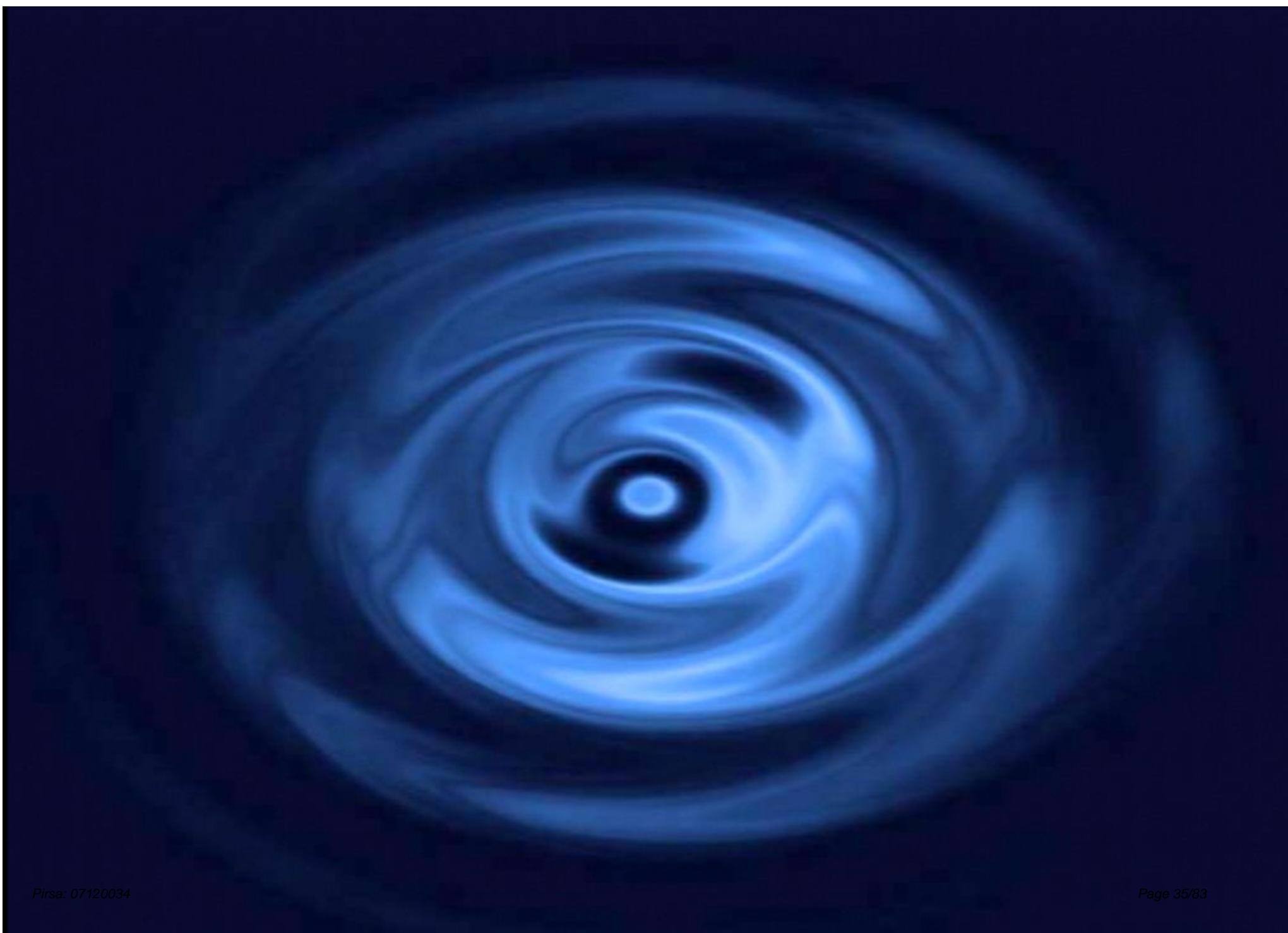
Scaling of the  $n$ -points function.

### Work in progress

We want to compute the full tensorial structure of the propagator and the first corrections to the gravitational potential.

We want to go beyond the first order in the GFT coupling  $\lambda$ .

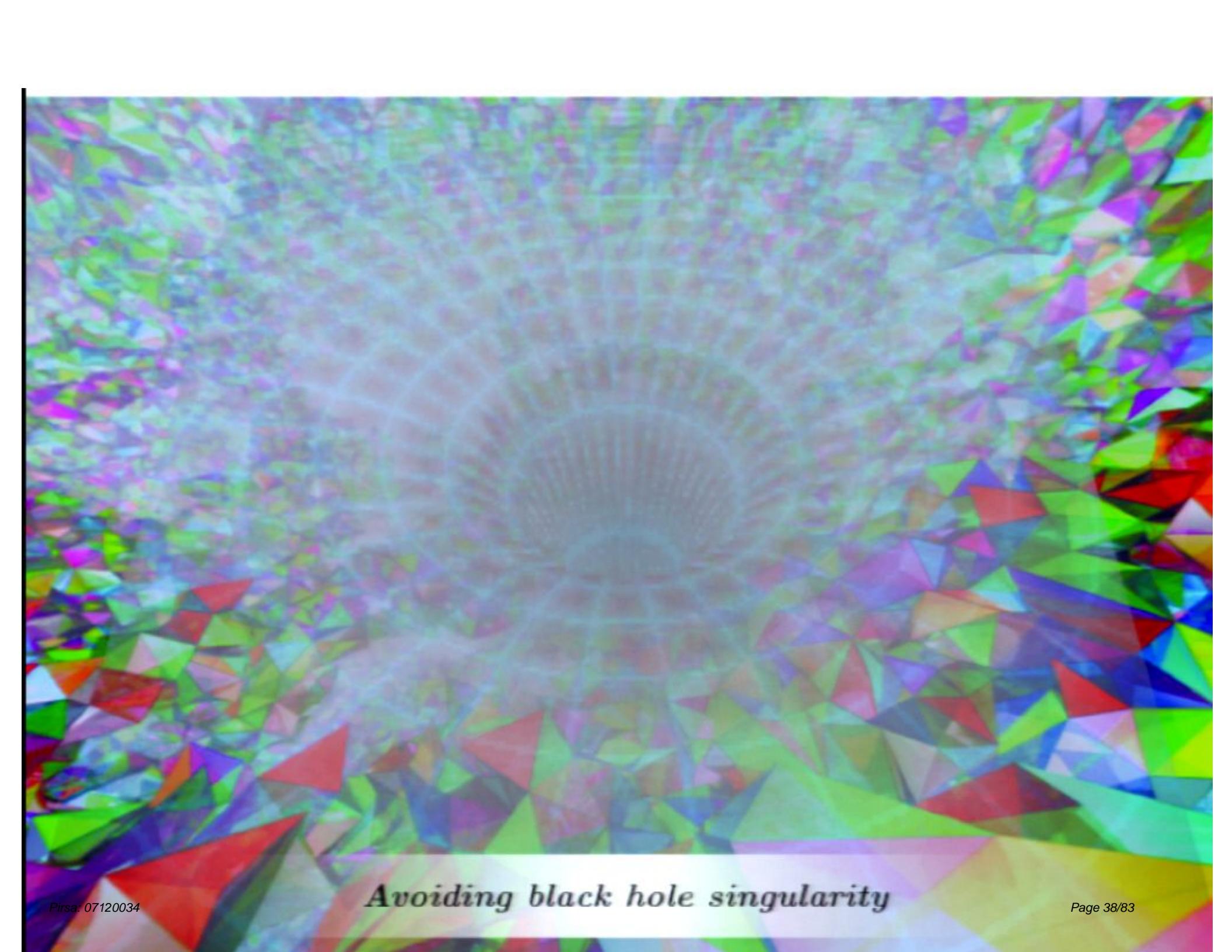
We want to calculate the small scale behavior of the graviton propagator and to deduce the effective dimension.



## *n-points function*

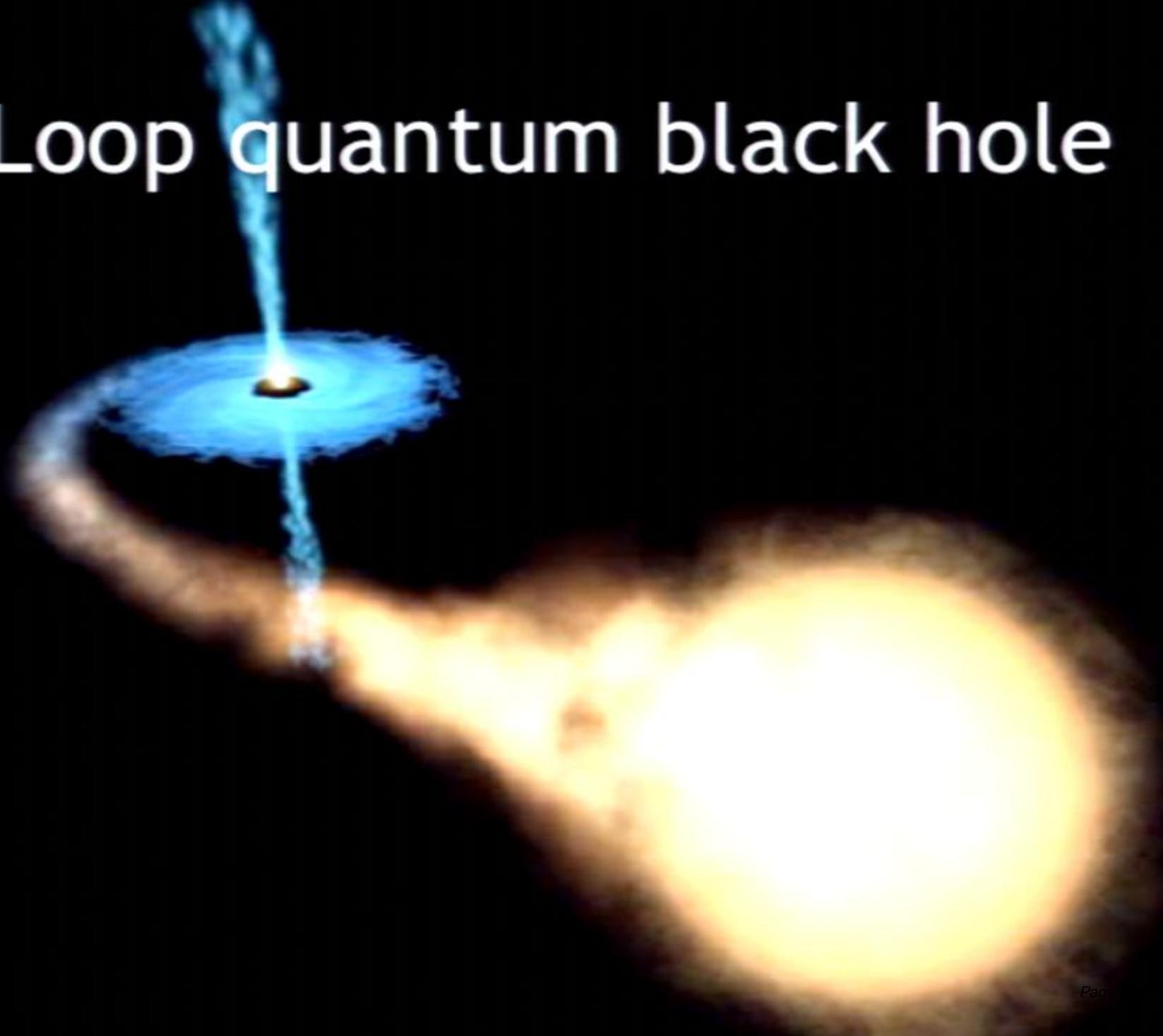
- *F. Conrady, L. Doplicher, R. Oeckl, C. Rovelli, M. Testa,*  
*Minkowski vacuum in background independent quantum gravity,*  
*Phys. Rev. D69, 064019, 2004, gr-qc/0307118;*
- *D. Colosi, L. Doplicher, W. Fairbairn, L. M., K. Noui, C. Rovelli,*  
*Background independence in a nutshell: the dynamics of a tetrahedron.*  
*Class. Quant. Grav. 22 (2005) 2971-2990, gr-qc/0408079;*
- *L. M., C. Rovelli,*  
*Particle scattering in loop quantum gravity.*  
*Phys. Rev. Lett. 95, 191301 (2005), gr-qc/0502036;*
- *C. Rovelli,*  
*Graviton propagator from background-independent quantum gravity.*  
*Phys. Rev. Lett. 97, 151301, 2006, gr-qc/0508124;*
- *E. Bianchi, L. M., C. Rovelli and S. Speziale,*  
*Graviton propagator in loop quantum gravity.*  
*Class. Quant. Grav. 23 (2006) 6989-7028, gr-qc/0604044;*
- *E. Bianchi, L.M.,*  
*Toward perturbative quantum gravity from spinfoams,*  
*in preparation;*
- *S. Speziale,*  
*Towards the graviton from spinfoams: The 3-D toy model,*  
*JHEP 0605, 039, 2006, gr-qc/0512102;*
- *E. Livine, S. Speziale, J. L. Willis,*  
*Towards the graviton from spinfoams: Higher order corrections in the 3-D toy model.*  
*Phys. Rev. D75, 024038, 2007, gr-qc/0605123.*

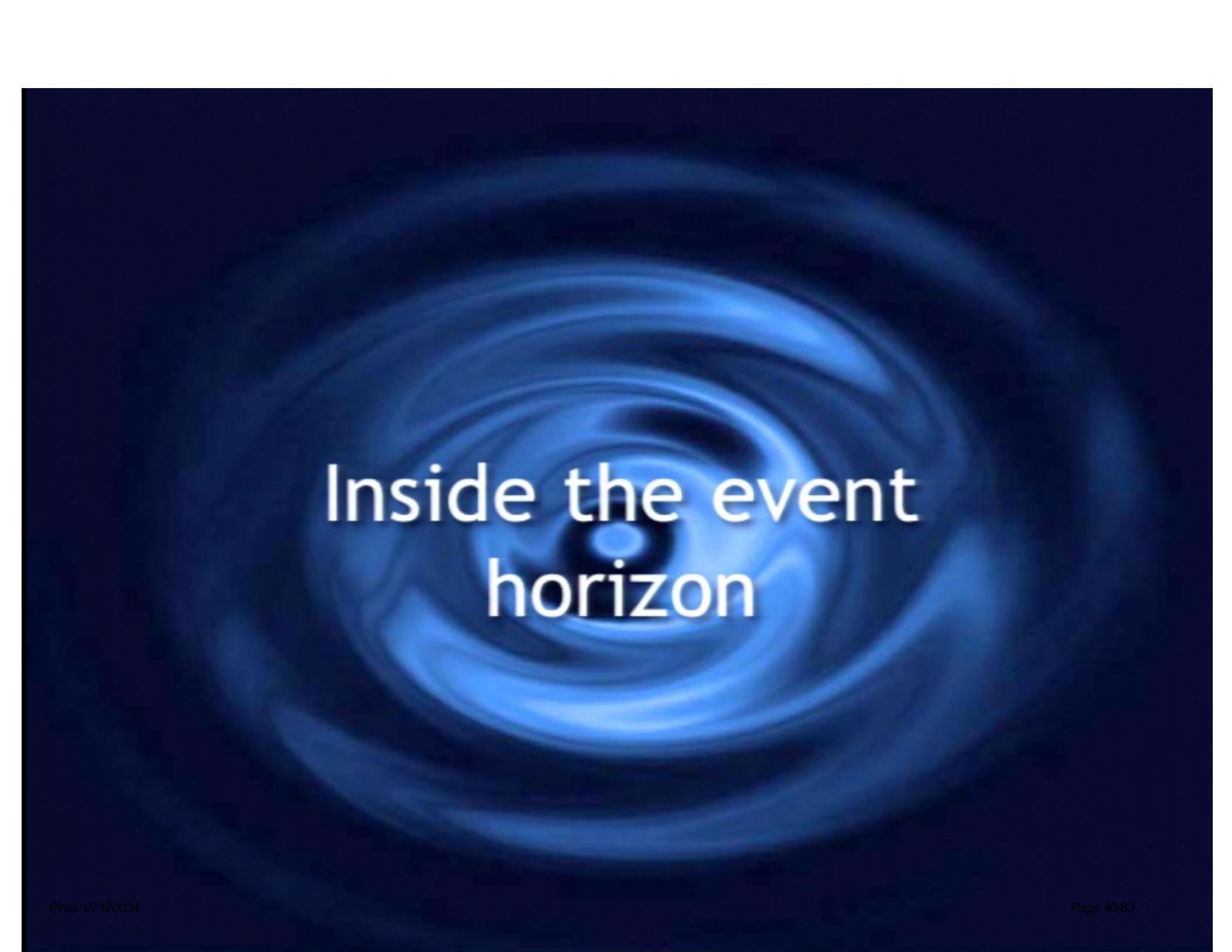
# Avoiding black hole singularity



*Avoiding black hole singularity*

# Loop quantum black hole





Inside the event  
horizon

## *The Schwarzschild solution inside the horizon*

$$ds^2 = -\frac{dT^2}{\left(\frac{2MG_N}{T} - 1\right)} + \left(\frac{2MG_N}{T} - 1\right)dr^2 + T^2(\sin^2 \theta d\phi^2 + d\theta^2)$$

$$T \in ]0, 2MG_N[ \quad , \quad r \in ]-\infty, +\infty[.$$

*The Kantowski-Sachs space-time ( $R \times R \times S^2$ ) :*

$$ds^2 = -N^2(t)dt^2 + a^2(t)dr^2 + b^2(t)(\sin^2 \theta d\phi^2 + d\theta^2)$$

$$R^{\mu\nu\rho\sigma\delta}R_{\mu\nu\rho\sigma\delta} = \frac{48G_N^2 M^2}{b(t)^6}$$



Classical theory

## Classical theory

Invariant 1-form connection  $A_{[1]}$ :

$$A_{[1]} = A_r(t) \tau_3 dr + (A_1(t) \tau_1 + A_2(t) \tau_2) d\theta + (A_1(t) \tau_2 - A_2(t) \tau_1) \sin \theta d\phi + \tau_3 \cos \theta d\phi$$

Invariant densitized triad:

$$E_{[1]} = E^r(t) \tau_3 \sin \theta \frac{\partial}{\partial r} + (E^1(t) \tau_1 + E^2(t) \tau_2) \sin \theta \frac{\partial}{\partial \theta} + (E^1(t) \tau_2 - E^2(t) \tau_1) \frac{\partial}{\partial \phi}$$

Gauss constraint and Hamiltonian constraints:

$$G \sim A_1 E^2 - A_2 E^1$$

$$H_E = \frac{\text{sgn}[\det(E_{[1]})]}{\sqrt{|E^r| |(E^1)^2 + (E^2)^2|}} \left[ 2A_r E^r (A_1 E^1 + A_2 E^2) + ((A_1)^2 + (A_2)^2 - 1) |(E^1)^2 + (E^2)^2| \right]$$

For the Kantowski-Sachs space-time we fix the gauge  $E^2 = E^1$  and so  $A_2 = A_1$

The Hamiltonian constraint becomes:  $H_E = \frac{\text{sgn}(E)}{\sqrt{|E| |E^1|}} \left[ 2AE A_1 E^1 + (2(A_1)^2 - 1) (E^1)^2 \right]$

Volume of the spatial section:  $V = \int dr d\phi d\theta \sqrt{q} = 4\pi \sqrt{2} R \sqrt{|E| |E^1|}$

Background triad and co-triad:  ${}^o e_I^a = \text{diag}(1, 1, \sin^{-1} \theta)$   ${}^o \omega_a^I = \text{diag}(1, 1, \sin \theta)$

$$q_{ab} = \begin{pmatrix} a^2(t) & 0 & 0 \\ 0 & b^2(t) & 0 \\ 0 & 0 & b^2(t) \sin^2 \theta \end{pmatrix} = \begin{pmatrix} \frac{(E^1)^2}{|E|} & 0 & 0 \\ 0 & |E| & 0 \\ 0 & 0 & |E| \sin^2 \theta \end{pmatrix}$$

# Holonomies and Hamiltonian constraint

### Classical phase space

Canonical pairs :  $(A, E)$  and  $(A_I, E^I)$

Symplectic structure :  $\{A, E\} = \frac{\kappa}{l_P}$  ,  $\{A_I, E^I\} = \frac{\kappa}{4l_P}$

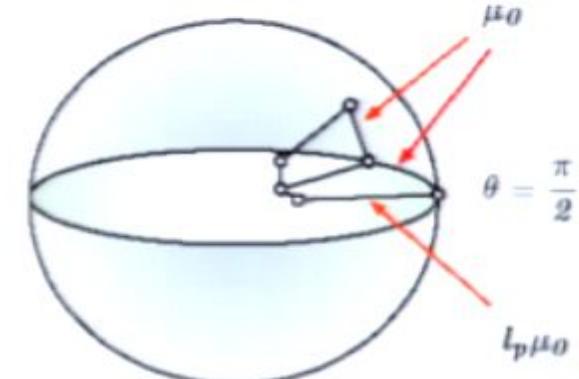
### Holonomies

$$h_I = \exp \int A_I^i \tau_i d\lambda$$

$$h_t = \exp \int A_t^i \tau_i d\lambda = \exp[A \mu_0 l_P \tau_3]$$

$$h_2 = \exp \int A_2^i \tau_i d\lambda = \exp[A_1 \mu_0 (\tau_2 + \tau_I)]$$

$$h_3 = \exp \int A_3^i \tau_i d\lambda = \exp[A_1 \mu_0 (\tau_2 - \tau_I)]$$



Curvature  $F_{ab}$  in terms of holonomies :  $F_{ab}^i \tau_i = {}^o\omega_a^I {}^o\omega_b^J \left[ \frac{h_I h_J h_I^{-1} h_J^{-1} h_{IJ}}{\epsilon(I)\epsilon(J)} - 1 \right]$

### Hamiltonian constraint

$$H^E(N) = \int_{\Sigma} d^3x N \frac{E_i^a E_j^b}{\sqrt{\det(E)}} \epsilon^{ij}{}_k F_{ab}^k, \quad H^E(N) = \int_{\Sigma} dx^3 N \epsilon^{abc} \delta_{ij} F_{ab}^i \{ A_c^j, V \}.$$

$$H_E = -\frac{8\pi}{\mu_0^3} \sum_{IJK} \epsilon^{IJK} \text{Tr} [h_I h_J h_I^{-1} h_J^{-1} h_{IJ} h_K^{-1} \{ h_K, V \}]$$

A dark blue background featuring a bright blue circular pattern resembling a wave or a series of concentric ripples emanating from the center.

# Quantum theory and inverse volume operator

## Quantum theory

*Hilbert space :  $H_E \otimes H_{E^t} \sim L^2(R_{Bohr}^2)$*

*Basis in the Hilbert space :*

$$|\mu_E, \mu_{E^t}\rangle \equiv |\mu_E\rangle \otimes |\mu_{E^t}\rangle \rightarrow \langle A|\mu_E\rangle \otimes \langle A_t|\mu_{E^t}\rangle = e^{\frac{i\mu_E l_P A}{2}} \otimes e^{\frac{i\mu_{E^t} l_P A_t}{2}} \quad \langle \mu_E, \mu_{E^t} | \nu_E, \nu_{E^t} \rangle = \delta_{\mu_E, \nu_E} \delta_{\mu_{E^t}, \nu_{E^t}}$$

*Representation of the momentum operators :*

$$\hat{E} \rightarrow -i l_P \frac{d}{dA} \quad , \quad \hat{E}^t \rightarrow -i \frac{l_P}{4} \frac{d}{dA_t}$$

$$\hat{E}|\mu_E, \mu_{E^t}\rangle = \frac{\mu_E l_P^2}{2} |\mu_E, \mu_{E^t}\rangle \quad , \quad \hat{E}^t|\mu_E, \mu_{E^t}\rangle = \frac{\mu_{E^t} l_P}{4\sqrt{2}} |\mu_E, \mu_{E^t}\rangle$$

## Inverse volume operator

$$\widehat{\frac{sgn(E)}{\sqrt{V}}} = \frac{512 i}{3 l_P^4 \mu_0^3} \epsilon_{ijk} \sum_{IJK} \epsilon^{IJK} \text{Tr} \left[ \tau^i \hat{h}_I^{-1} [\hat{h}_I, \hat{V}^{\frac{i}{2}}] \right] \text{Tr} \left[ \tau^j \hat{h}_J^{-1} [\hat{h}_J, \hat{V}^{\frac{j}{2}}] \right] \text{Tr} \left[ \tau^k \hat{h}_K^{-1} [\hat{h}_K, \hat{V}^{\frac{k}{2}}] \right]$$

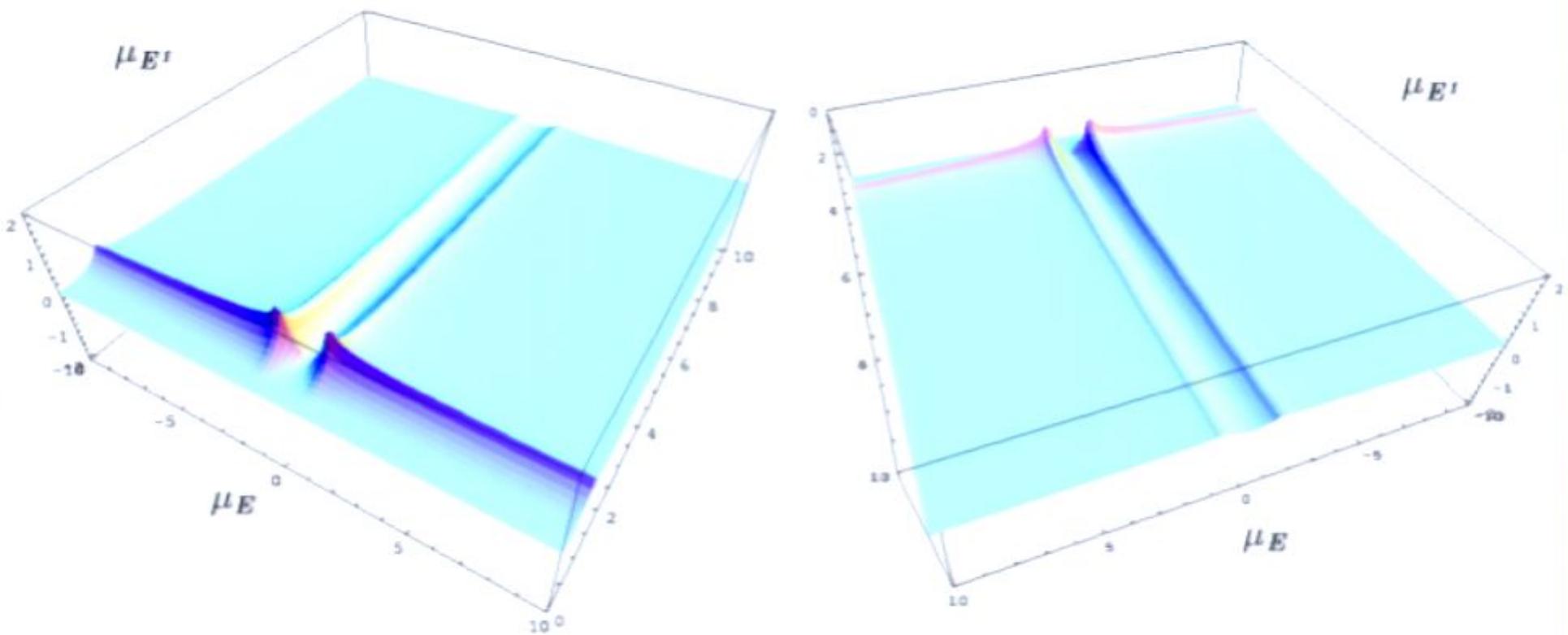
## Spectrum of $\widehat{V}$ and $1/\widehat{V}$

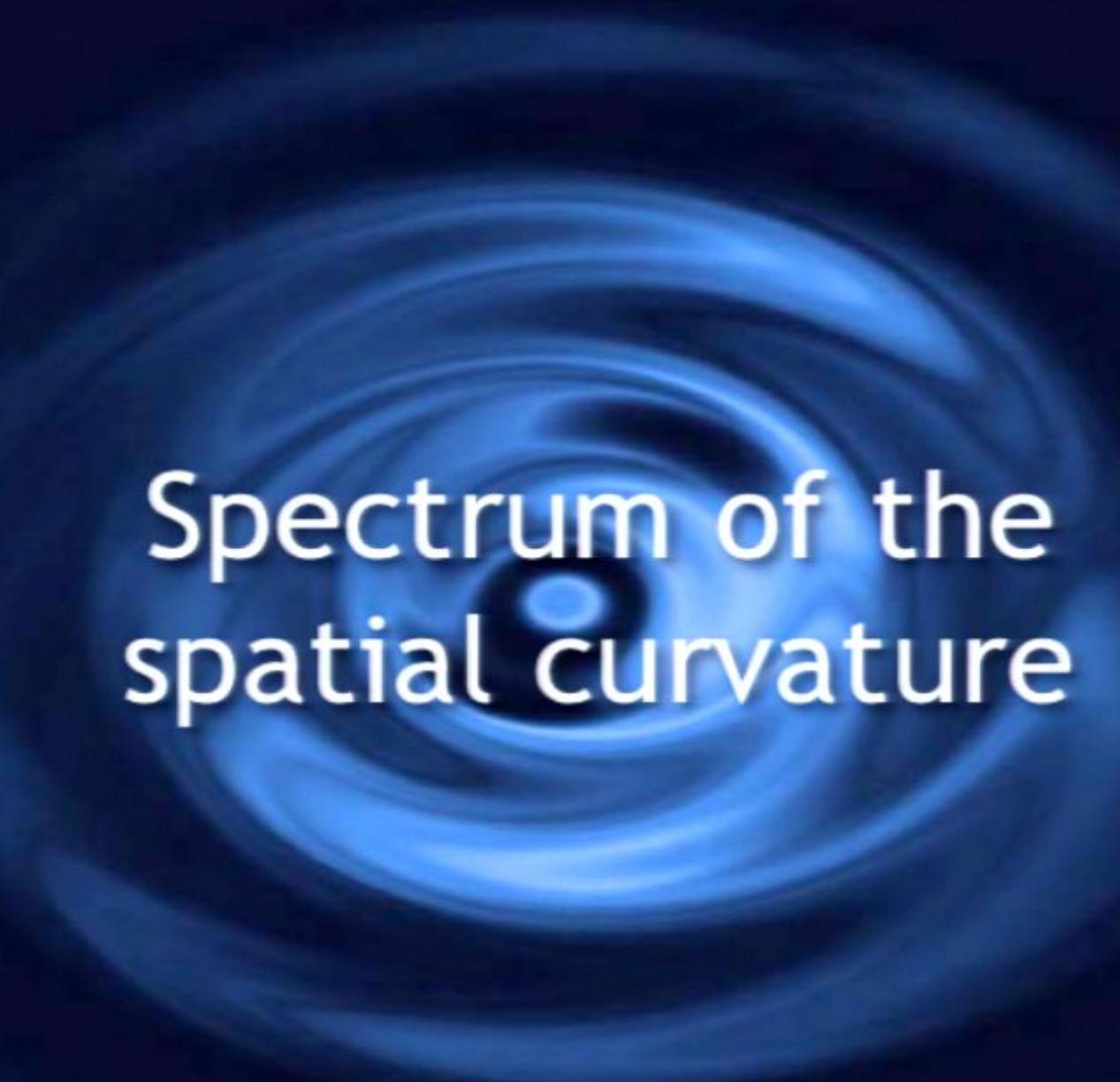
$$\hat{V}|\mu_E, \mu_{E^t}\rangle = \frac{4\pi l_P^3}{\sqrt{2}} \sqrt{|\mu_E|} |\mu_{E^t}| |\mu_E, \mu_{E^t}\rangle$$

$$\widehat{\frac{sgn(E)}{\sqrt{V}}} |\mu_E, \mu_{E^t}\rangle = \frac{8}{\sqrt{2} l_P \mu_0^3} |\mu_E|^{\frac{t}{2}} |\mu_{E^t}|^{\frac{t}{2}} \left( |\mu_E + \mu_0|^{\frac{t}{4}} - |\mu_E - \mu_0|^{\frac{t}{4}} \right) \left( |\mu_{E^t} + \mu_0|^{\frac{t}{2}} - |\mu_{E^t} - \mu_0|^{\frac{t}{2}} \right)^2 |\mu_E, \mu_{E^t}\rangle$$

# Spectrum of the inverse volume operator

*Plot of the inverse volume operator spectrum*



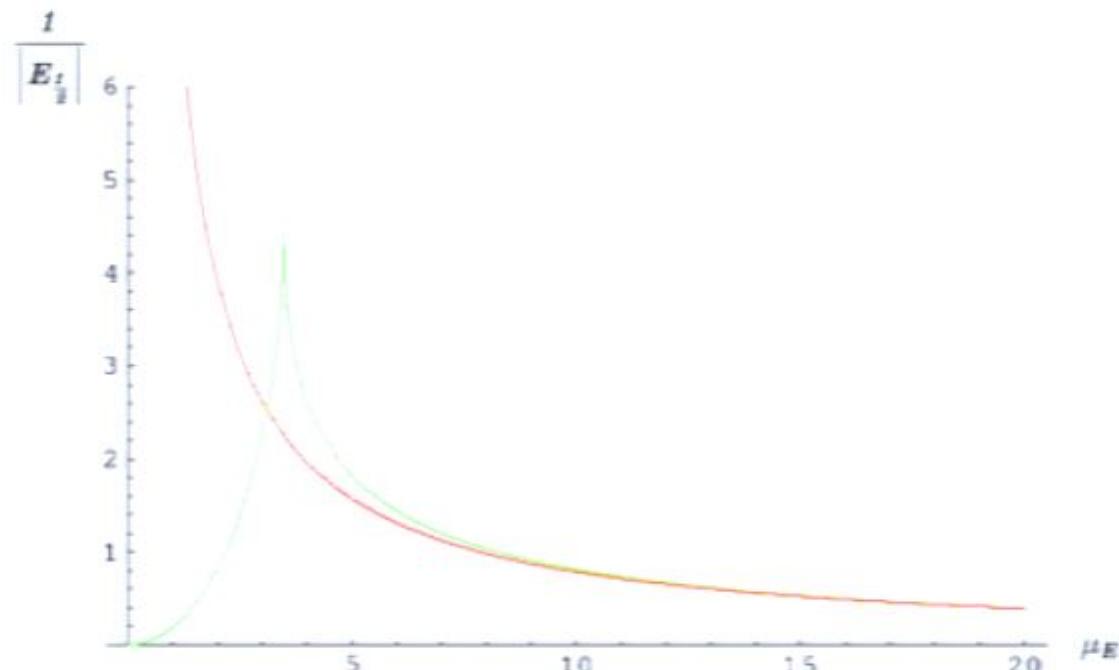
A large, glowing blue circular pattern resembling a black hole's event horizon or a complex wave pattern, centered behind the text.

# Spectrum of the spatial curvature

## *Spatial curvature*

$${}^{(3)}\mathbf{R} = \frac{\mathbf{2}}{|\mathbf{E}|},$$

$$\widehat{\frac{\mathbf{1}}{|\mathbf{E}_j|}} |\mu_E, \mu_{E_t}\rangle = \left( \frac{3}{\gamma^{\frac{j}{2}} \delta \ l_P j(j+1)(2j+1)} \sum_{k=-j}^{k=j} \left[ k \left( \sqrt{|\mu_E|} - \sqrt{|\mu_E - 2k\delta|} \right) \right] \right)^2 |\mu_E, \mu_{E_t}\rangle.$$

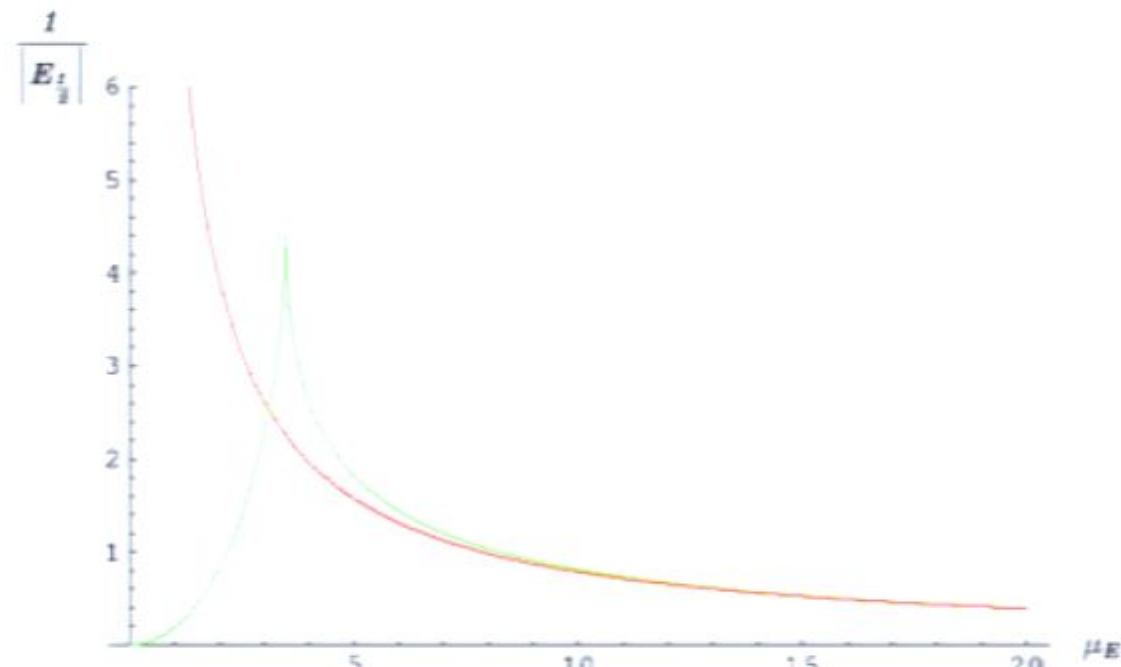


# Hamiltonian constraint and dynamics

## *Spatial curvature*

$${}^{(3)}\mathbf{R} = \frac{\mathbf{2}}{|\mathbf{E}|},$$

$$\widehat{\frac{\mathbf{1}}{|\mathbf{E}_j|}} |\mu_E, \mu_{E_I}\rangle = \left( \frac{3}{\gamma^{\frac{i}{2}} \delta \ l_P j(j+1)(2j+1)} \sum_{k=-j}^{k=j} \left[ k \left( \sqrt{|\mu_E|} - \sqrt{|\mu_E - 2k\delta|} \right) \right] \right)^2 |\mu_E, \mu_{E_I}\rangle.$$



# Hamiltonian constraint and dynamics

### Hamiltonian constraint

The solutions of the Hamiltonian constraint are in  $C^*$  dual of the dense subspace  $C$  of the kinematical space  $H_{\text{kin}}$ .

A generic element of this space is:  $\langle \psi | = \sum_{\mu_E, \mu_{E^\dagger}} \psi(\mu_E, \mu_{E^\dagger}) \langle \mu_E, \mu_{E^\dagger} |$ .

The constraint equation  $\hat{H}_E |\psi\rangle = 0$  gives a relation for the coefficients  $\psi(\mu_E, \mu_{E^\dagger})$ :

$$\begin{aligned} & -\alpha(\mu_E - 2\mu_\theta, \mu_{E^\dagger} - 2\mu_\theta) \psi(\mu_E - 2\mu_\theta, \mu_{E^\dagger} - 2\mu_\theta) + \alpha(\mu_E + 2\mu_\theta, \mu_{E^\dagger} - 2\mu_\theta) \psi(\mu_E + 2\mu_\theta, \mu_{E^\dagger} - 2\mu_\theta) \\ & + \alpha(\mu_E - 2\mu_\theta, \mu_{E^\dagger} + 2\mu_\theta) \psi(\mu_E - 2\mu_\theta, \mu_{E^\dagger} + 2\mu_\theta) - \alpha(\mu_E + 2\mu_\theta, \mu_{E^\dagger} + 2\mu_\theta) \psi(\mu_E + 2\mu_\theta, \mu_{E^\dagger} + 2\mu_\theta) \\ & + \frac{\sin(\mu_\theta^2/2) - \cos(\mu_\theta^2/2)}{2} \left( \beta(\mu_E, \mu_{E^\dagger} - 4\mu_\theta) \psi(\mu_E, \mu_{E^\dagger} - 4\mu_\theta) - \beta(\mu_E, \mu_{E^\dagger}) \psi(\mu_E, \mu_{E^\dagger}) \right. \\ & \quad \left. + \beta(\mu_E, \mu_{E^\dagger} + 4\mu_\theta) \psi(\mu_E, \mu_{E^\dagger} + 4\mu_\theta) \right) \\ & - \sin(\mu_\theta^2/2) \left( \beta(\mu_E, \mu_{E^\dagger} - 2\mu_\theta) \psi(\mu_E, \mu_{E^\dagger} - 2\mu_\theta) + \beta(\mu_E, \mu_{E^\dagger} + 2\mu_\theta) \psi(\mu_E, \mu_{E^\dagger} + 2\mu_\theta) \right) = 0 \end{aligned}$$

$$\alpha(\mu_E, \mu_{E^\dagger}) \equiv |\mu_E|^{\frac{i}{2}} (|\mu_{E^\dagger} + \mu_\theta| - |\mu_{E^\dagger} - \mu_\theta|)$$

$$\beta(\mu_E, \mu_{E^\dagger}) \equiv |\mu_{E^\dagger}| \left( |\mu_E + \mu_\theta|^{\frac{i}{2}} - |\mu_E - \mu_\theta|^{\frac{i}{2}} \right)$$

# Semiclassical analysis

## Semiclassical analysis

$$A = c\tau_3 dx + b\tau_2 d\theta - b\tau_1 \sin \theta d\phi + \tau_3 \cos \theta d\phi,$$

$$E = p_c \tau_3 \sin \theta \frac{\partial}{\partial x} + p_b \tau_2 \sin \theta \frac{\partial}{\partial \theta} - p_b \tau_1 \frac{\partial}{\partial \phi},$$

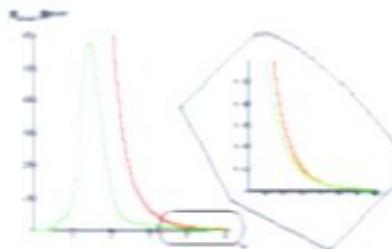
$$h_1 = \cos \frac{\delta c}{2} + 2\tau_3 \sin \frac{\delta c}{2}, \quad h_2 = \cos \frac{\delta b}{2} - 2\tau_1 \sin \frac{\delta b}{2}, \quad h_3 = \cos \frac{\delta b}{2} + 2\tau_2 \sin \frac{\delta b}{2}.$$

$$H^\delta = -\frac{2\hbar N}{\gamma^3 \delta^3 l_p^2} \text{Tr} \left( \sum_{ijk} \epsilon^{ijk} h_i^{(\delta)} h_j^{(\delta)} h_i^{(\delta)-1} h_k^{(\delta)} \left\{ h_k^{(\delta)-1}, V \right\} + 2\gamma^2 \delta^2 \tau_3 h_1^{(\delta)} \left\{ h_1^{(\delta)-1}, V \right\} \right)$$

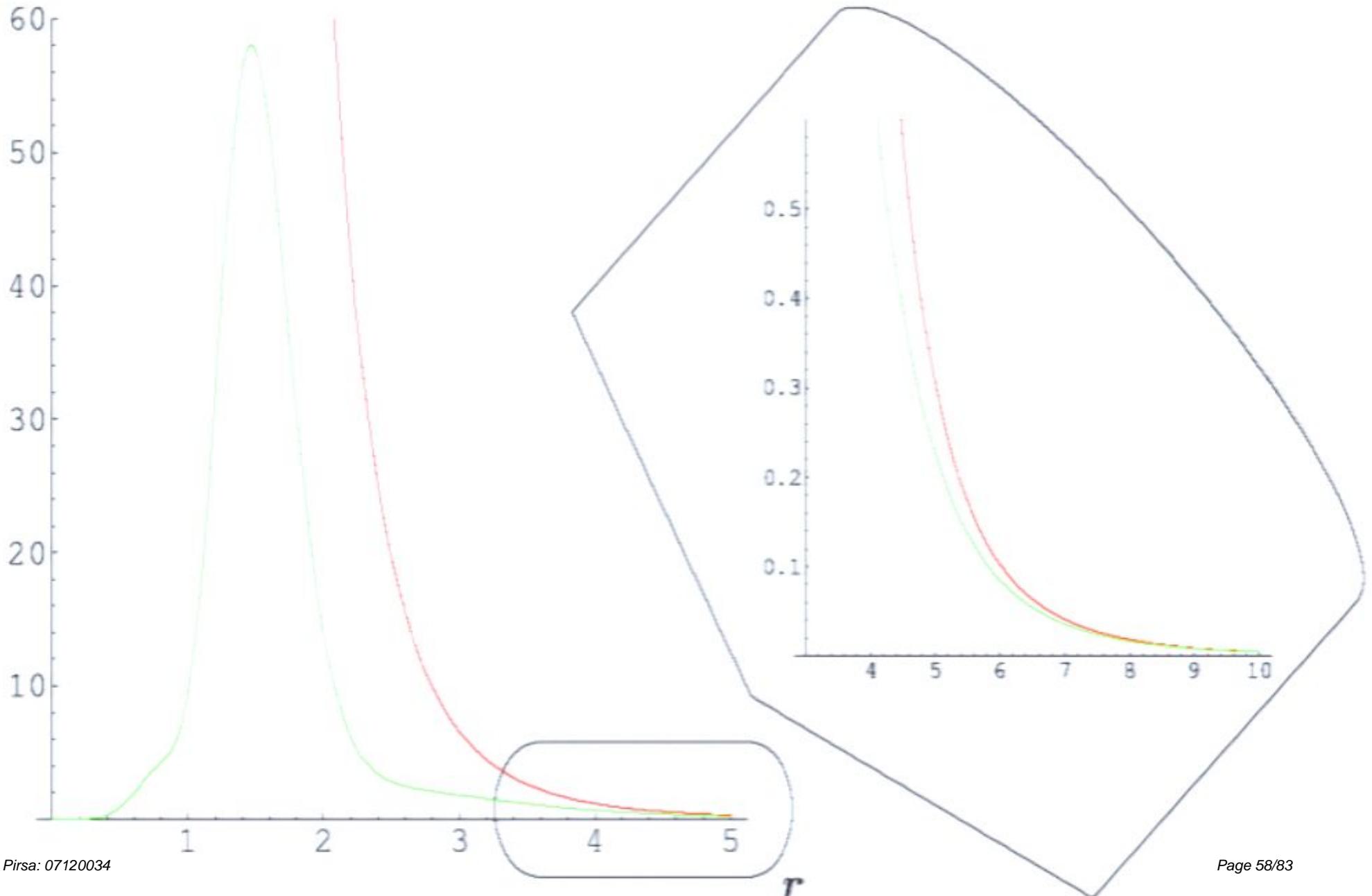
$$H^\delta = -\frac{1}{2\gamma G_N} \left\{ 2 \sin \delta c \ p_c + \left( \sin \delta b + \frac{\gamma^2 \delta^2}{\sin \delta b} \right) p_b \right\}.$$

$$N = \frac{\gamma \sqrt{|p_c|} sgn(p_c) \delta^2}{16\pi G_N \sin \delta b} \quad \& \quad \text{Hamilton e.m.} \rightarrow g_{\mu\nu}.$$

*Regular solution* →



$$R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}$$

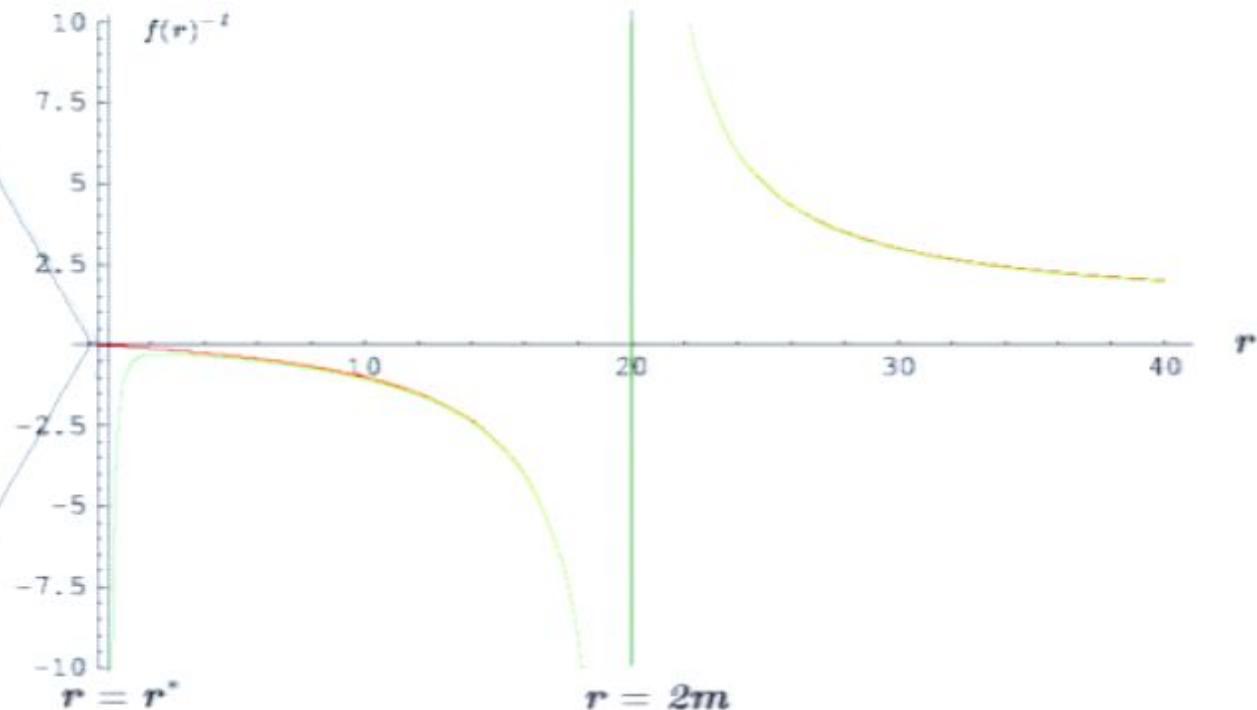
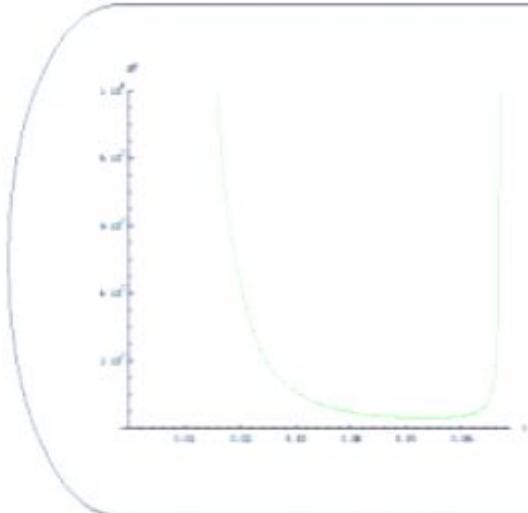




# Semiclassical Schwarzschild Solution

$$f(r)^{-t}$$

$$ds^2 = -f(r)dt^2 + f(r)^{-t}dr^2 + h^2(r)(d\theta^2 + \sin\theta d\phi^2)$$



$$h^2(r)$$



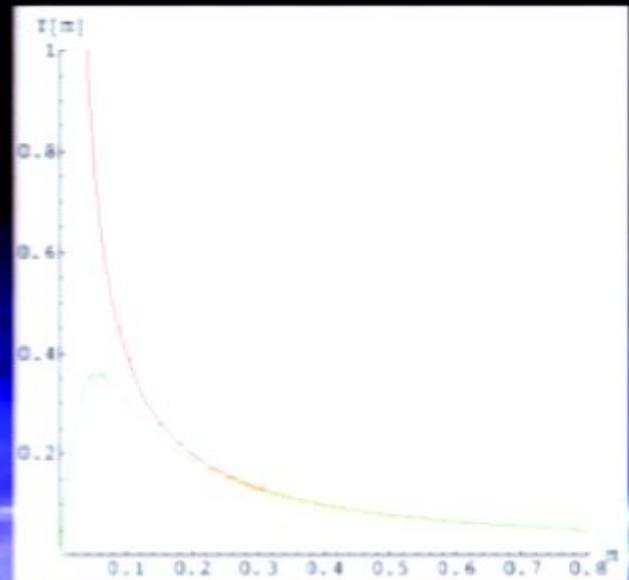


Results

*Temperature :*  $T_{BH} = \frac{8m}{\pi(64m^2 + \gamma^2\delta^2)}.$

*Entropy :*

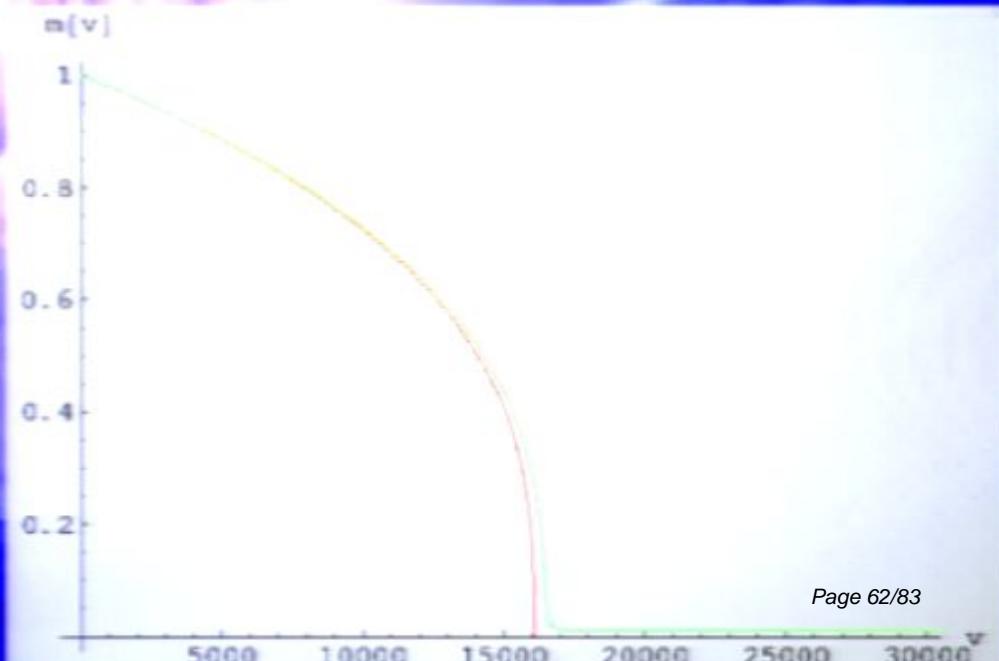
$$S = \frac{A}{4l_P^2} + \frac{\gamma^2}{16} \ln \left( \frac{A}{4l_P^2} \right) + \frac{\gamma^2}{16} \ln \left( 1 - \frac{4\pi\gamma^2 l_P^2}{16A} \right) + \text{const.}$$



*Evaporation process. Luminosity and mass decreasing.*

$$L(m) = \frac{2^{16}m^6 + 2^{10}\gamma^2\delta^2m^4}{60\pi(64m^2 + \gamma^2\delta^2)^4},$$

$$-\frac{dm(v)}{dv} = L[m(v)].$$

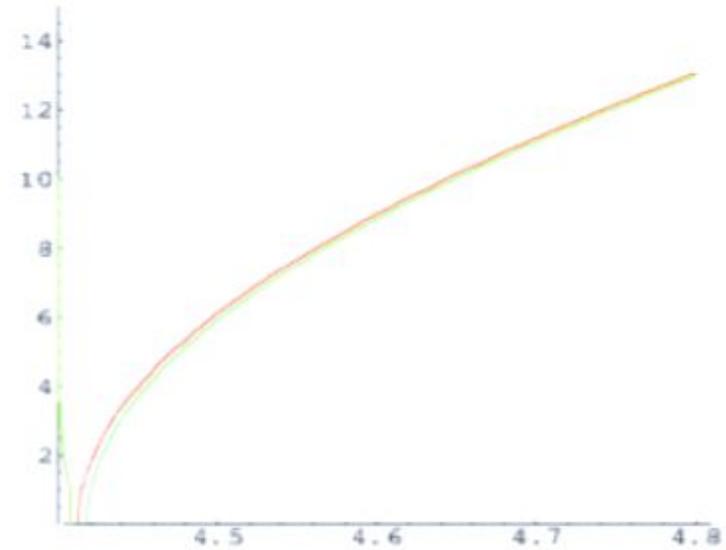
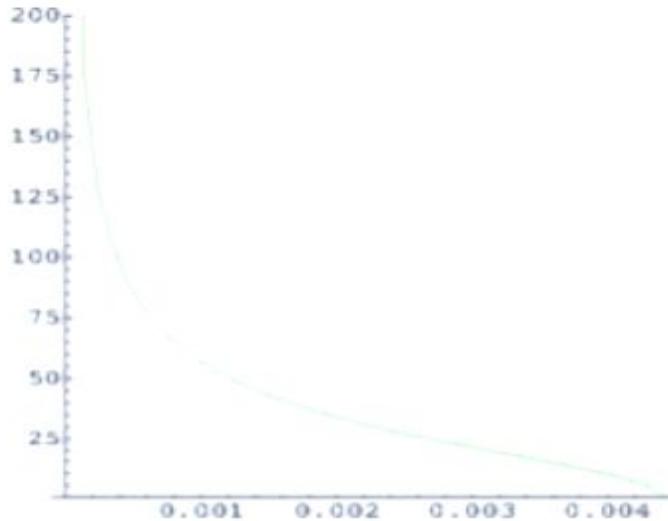


# Quantum Corrections

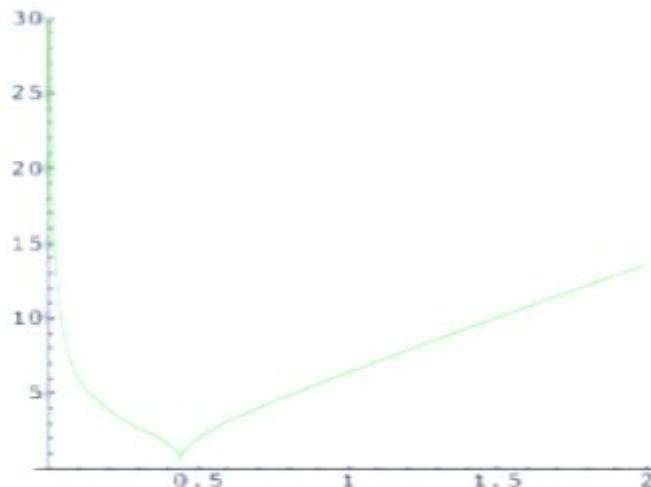
## *Inverse volume operator modifications*

$$\dot{p}_c = -\frac{2\gamma\delta^2 \sqrt{p_c - m^2} (\sqrt{|p_c - \gamma\delta|} - \sqrt{|p_c + \gamma\delta|})}{\sqrt{|p_c - \gamma\delta|} + \sqrt{|p_c + \gamma\delta|}}$$

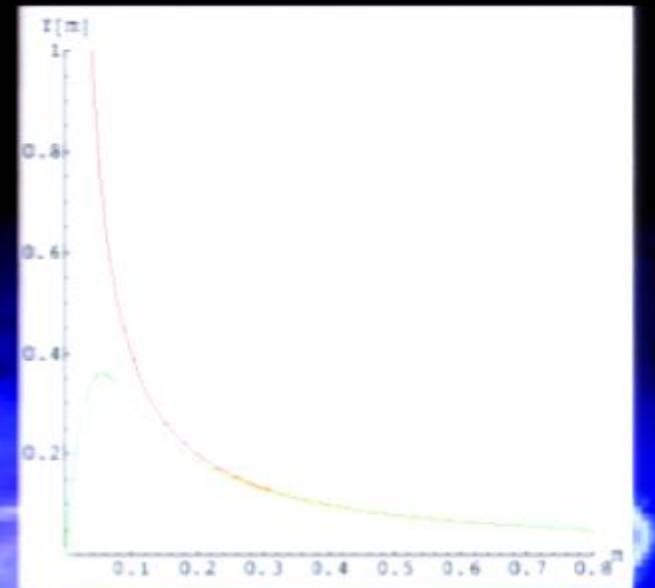
$$m > \frac{1}{\sqrt{2}}$$



$$m \leq \frac{1}{\sqrt{2}}$$



# Quantum Corrections



*Temperature :*  $T_{BH} = \frac{8m}{\pi(64m^2 + \gamma^2\delta^2)}.$

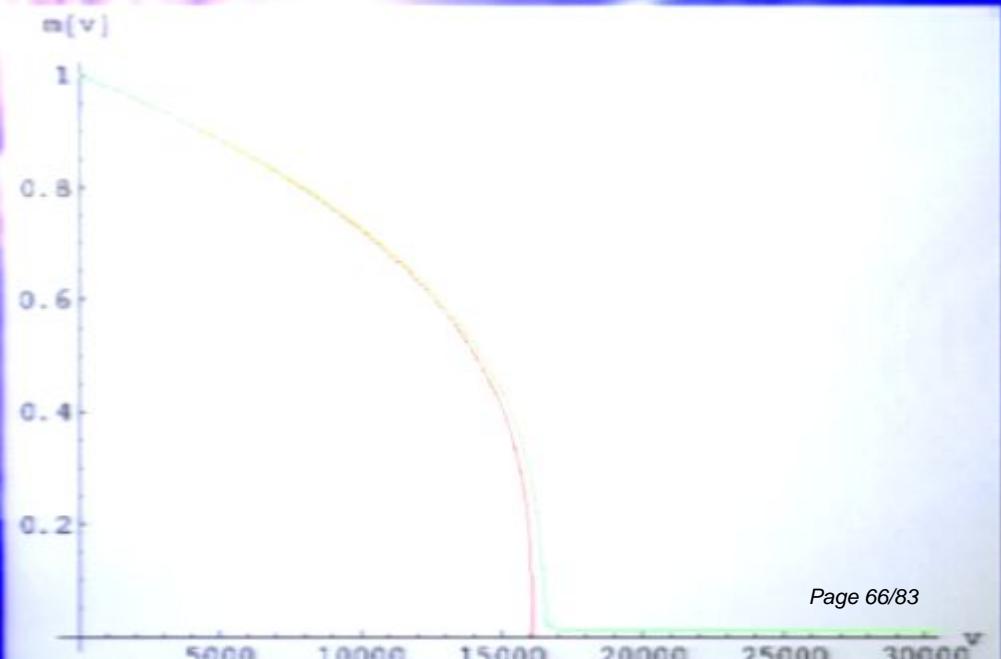
*Entropy :*

$$S = \frac{A}{4l_P^2} + \frac{\gamma^2}{16} \ln \left( \frac{A}{4l_P^2} \right) + \frac{\gamma^2}{16} \ln \left( 1 - \frac{4\pi\gamma^2 l_P^2}{16A} \right) + \text{const.}$$

*Evaporation process. Luminosity and mass decreasing.*

$$L(m) = \frac{2^{16}m^6 + 2^{10}\gamma^2\delta^2m^4}{60\pi(64m^2 + \gamma^2\delta^2)^4},$$

$$-\frac{dm(v)}{dv} = L[m(v)].$$

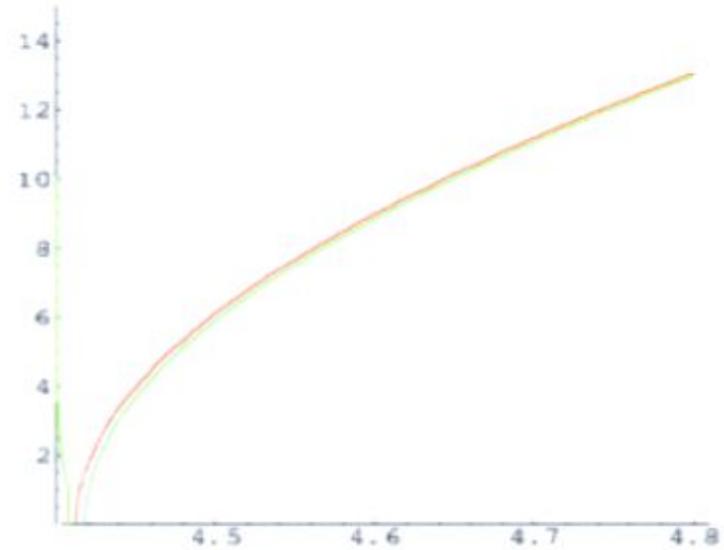
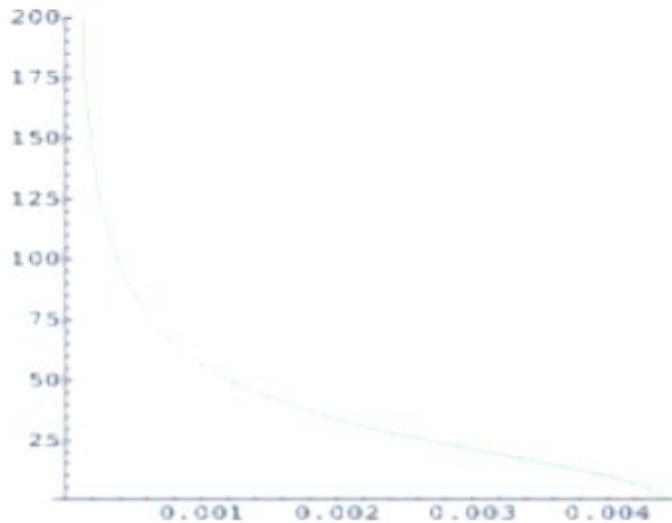


# Quantum Corrections

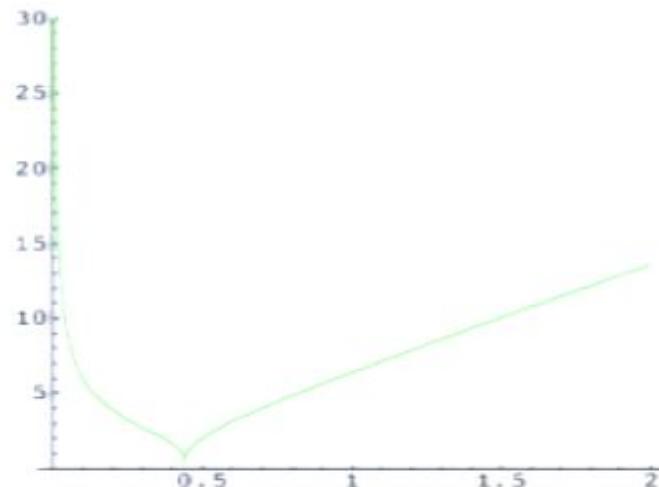
## Inverse volume operator modifications

$$\dot{p}_c = -\frac{2\gamma\delta^2 \sqrt{p_c - m^2} (\sqrt{|p_c - \gamma\delta|} - \sqrt{|p_c + \gamma\delta|})}{\sqrt{|p_c - \gamma\delta|} + \sqrt{|p_c + \gamma\delta|}}$$

$$m > \frac{1}{\sqrt{2}}$$



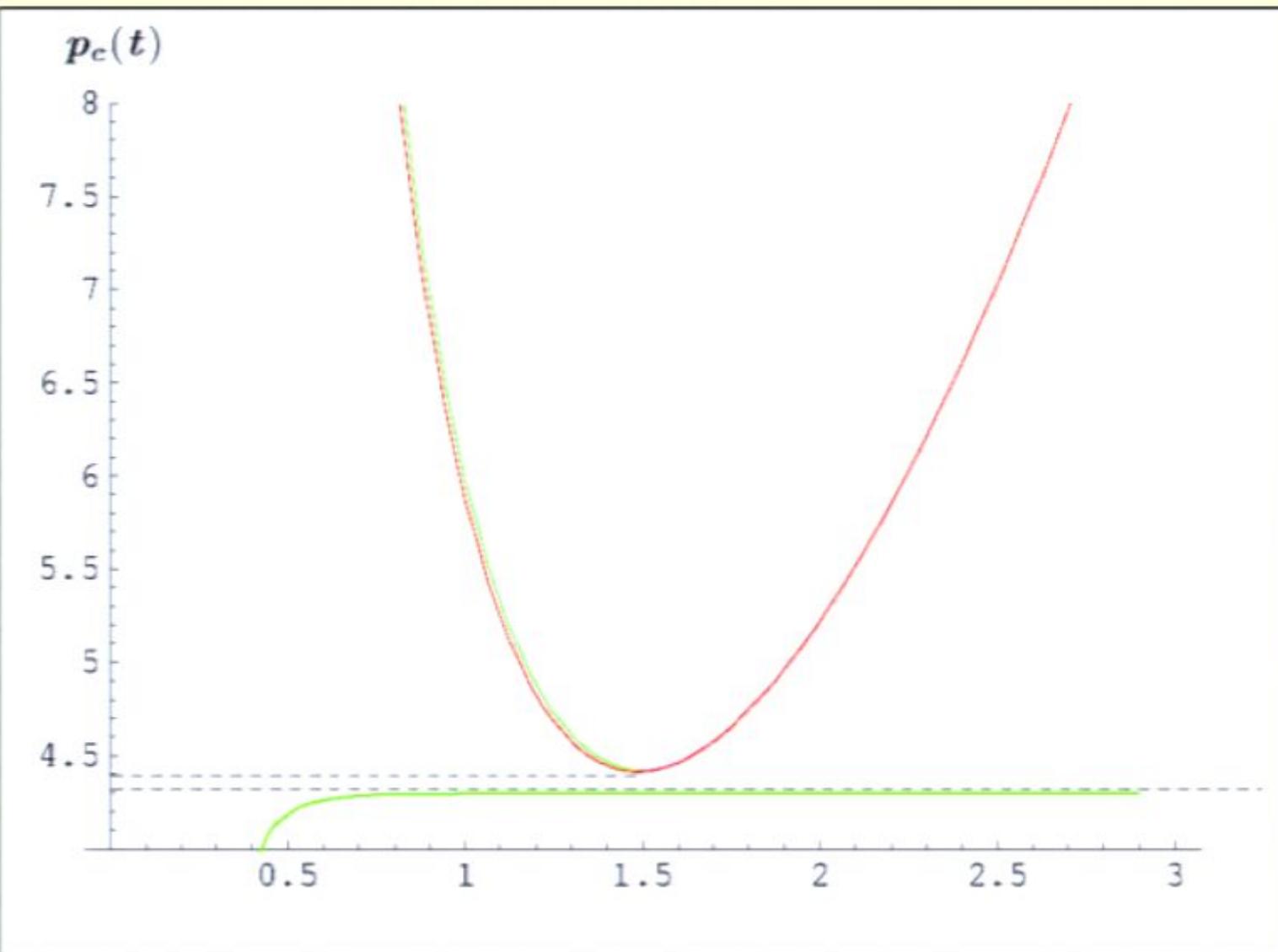
$$m \leq \frac{1}{\sqrt{2}}$$





Large m solution

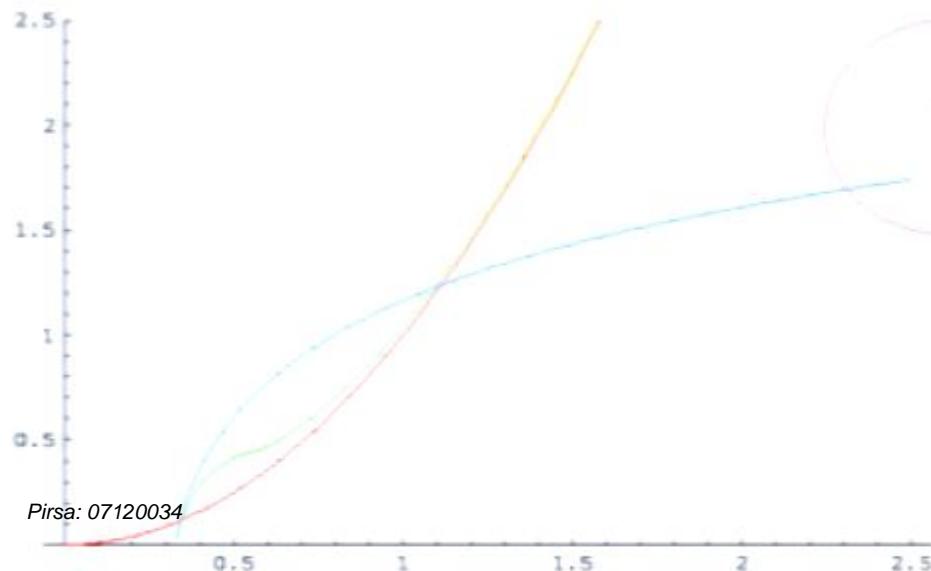
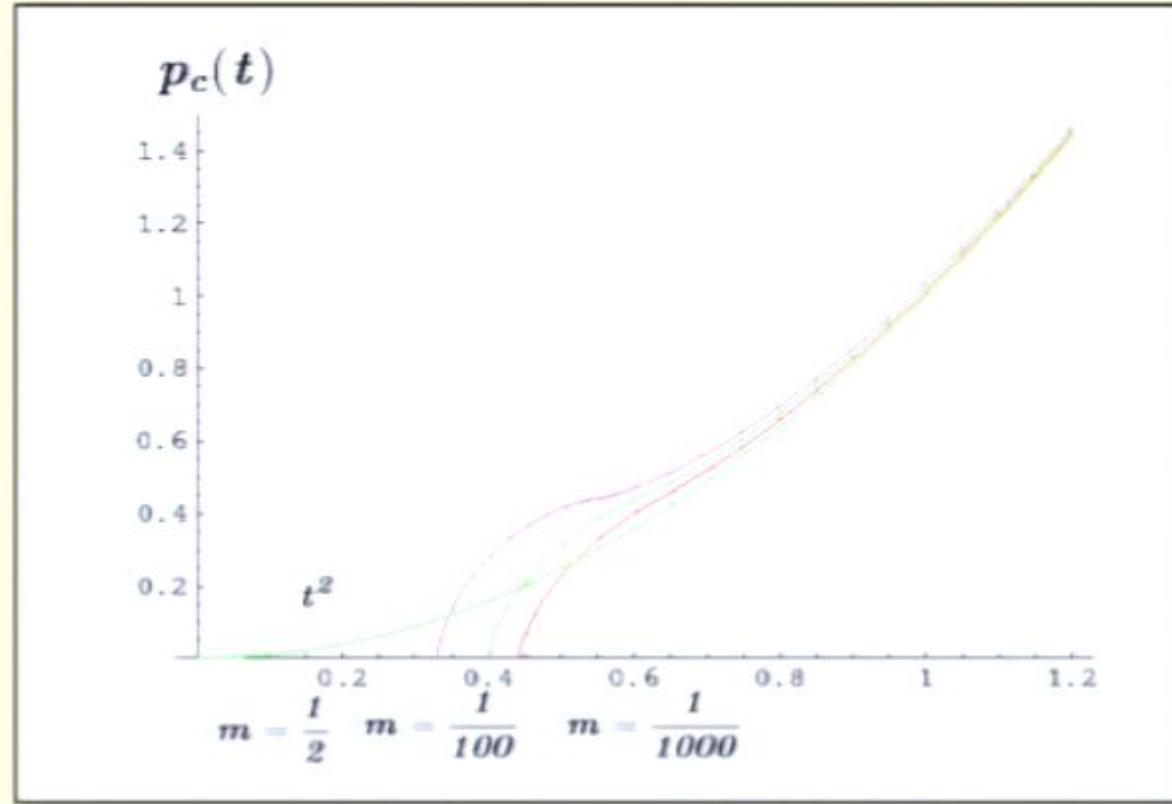
$$m > \frac{1}{\sqrt{2}}$$





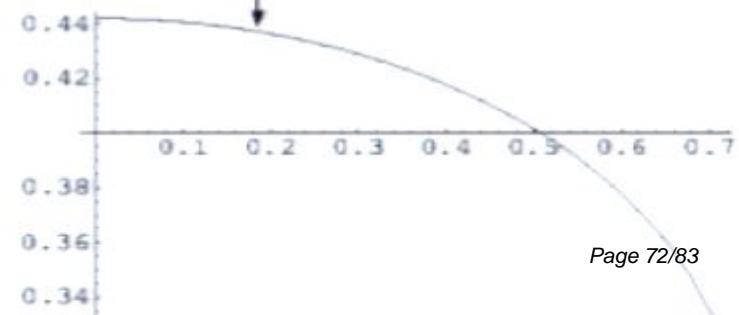
Small m solution

$$m \leq \frac{1}{\sqrt{2}}$$



$$p_c(t) \sim \frac{4}{3}(\gamma\delta)^{\frac{t}{4}} \left( \log \left( \frac{t}{c(m)} \right) \right)^{\frac{2}{3}}$$

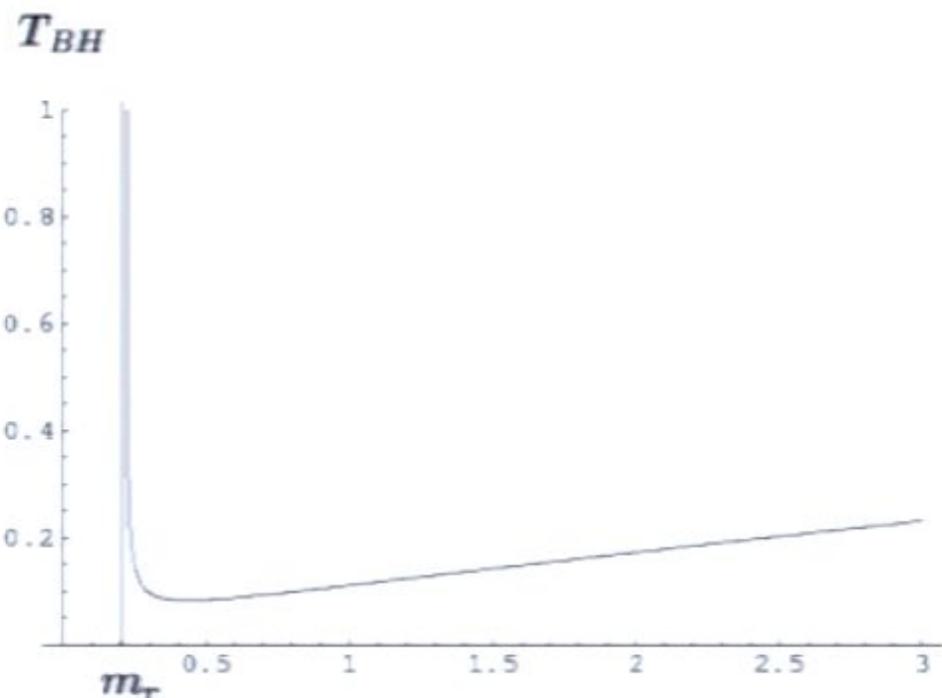
for  $p_c \rightarrow 0$





Temperature for small  $p_c$

## *Temperature for $p_c \rightarrow 0$*



# Conclusions

## CONCLUSIONS

*The classical black hole singularity in  $r = 0$   
disappears from the quantum theory.*

*Classical divergent quantities are bounded in the quantum theory.*

- *Curvature invariant:*  $R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} = \frac{48M^2 G_N^2}{b(t)^6} \rightarrow \widehat{R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}} |\psi\rangle = \frac{48\widehat{M^2} G_N^2}{b^6} |\psi\rangle$   
*is bounded for the Kantowski-Sachs model.*
- *The inverse volume operator  $1/\sqrt{V}$  is bounded.*

*The Hamiltonian constraint gives a difference equation for  
the coefficients of the physical states and we can evolve across the singularity.*

... INSIDE ... ACROSS ... AND BEYOND ...

L. M. , *Disappearance of the black hole singularity in loop quantum gravity*, *Phys. Rev. D70* (2004) 124009,

L. M. , *The Kantowski-Sachs space-time in loop quantum gravity*, *Int. J. of Theor. Phys.* , gr-qc/0411032 ;

L. M. , *Loop quantum black hole*, *Class. Quant. Grav.* 23 (2006) 5587-5601, gr-qc/0509078 ;

A. Ashtekar and M. Bojowald, *Class. Quantum Grav.* 23 (2006) ;

L. M. , *Evaporating loop quantum black hole*, gr-qc/0612084 ;

Pirsa:07120034 L. M. , *Black hole interior from loop quantum gravity*, gr-qc/0611043 ;

L. M. , *Gravitational collapse in loop quantum gravity*, gr-qc/0610074.

## CONCLUSIONS

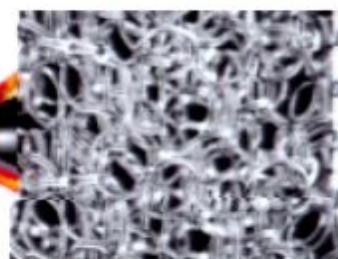
*The classical black hole singularity in  $r = 0$  disappears from the quantum theory.*

*Classical divergent quantities are bounded in the quantum theory.*

- *Curvature invariant:*  $R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} = \frac{48M^2 G_N^2}{b(t)^6} \rightarrow \widehat{R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}} |\psi\rangle = \frac{48\widehat{M^2 G_N^2}}{b^6} |\psi\rangle$  is bounded for the Kantowski-Sachs model.
- *The inverse volume operator  $1/\sqrt{V}$  is bounded.*

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*SS ... AND BEYOND ...*



Quantum gravity, *Phys. Rev. D70* (2004) 124009 ;  
Quantum gravity, *Int. J. of Theor. Phys.* , *gr-qc/0411032* ;  
*gr-qc/0509078* ;  
*gr-qc/0611043* ;  
*gr-qc/0610074* ;

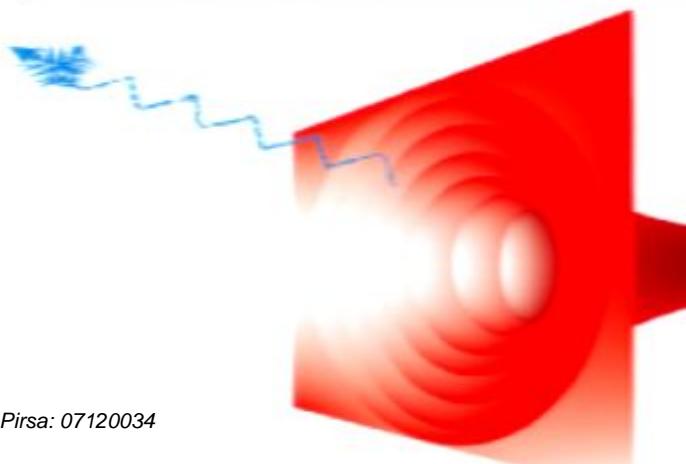
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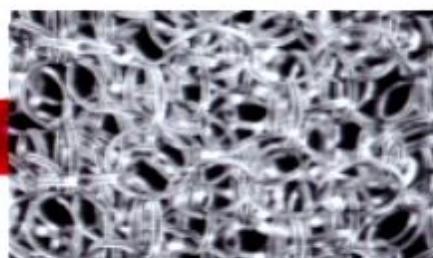
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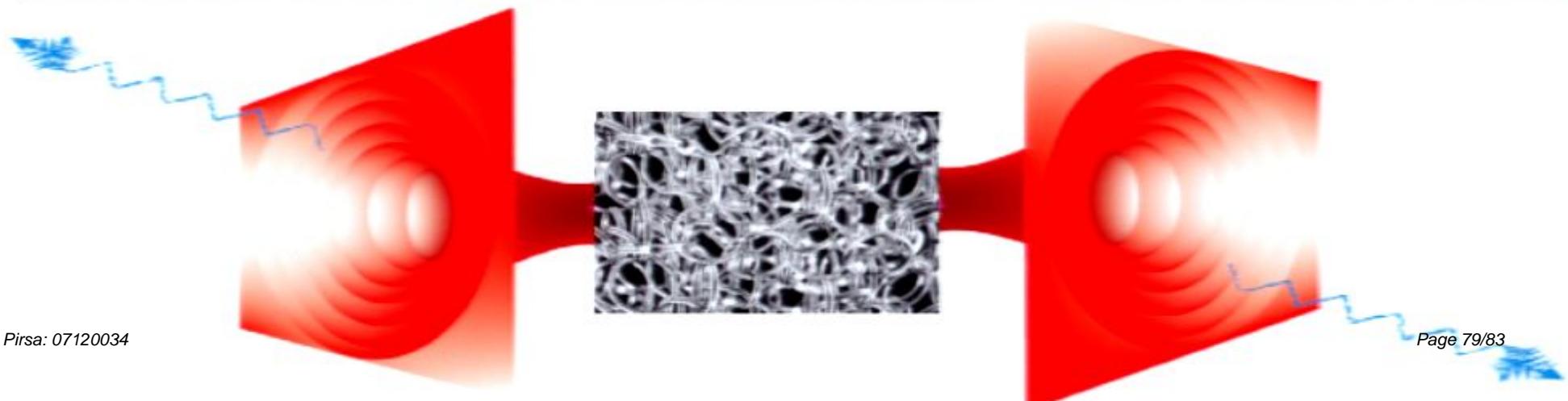
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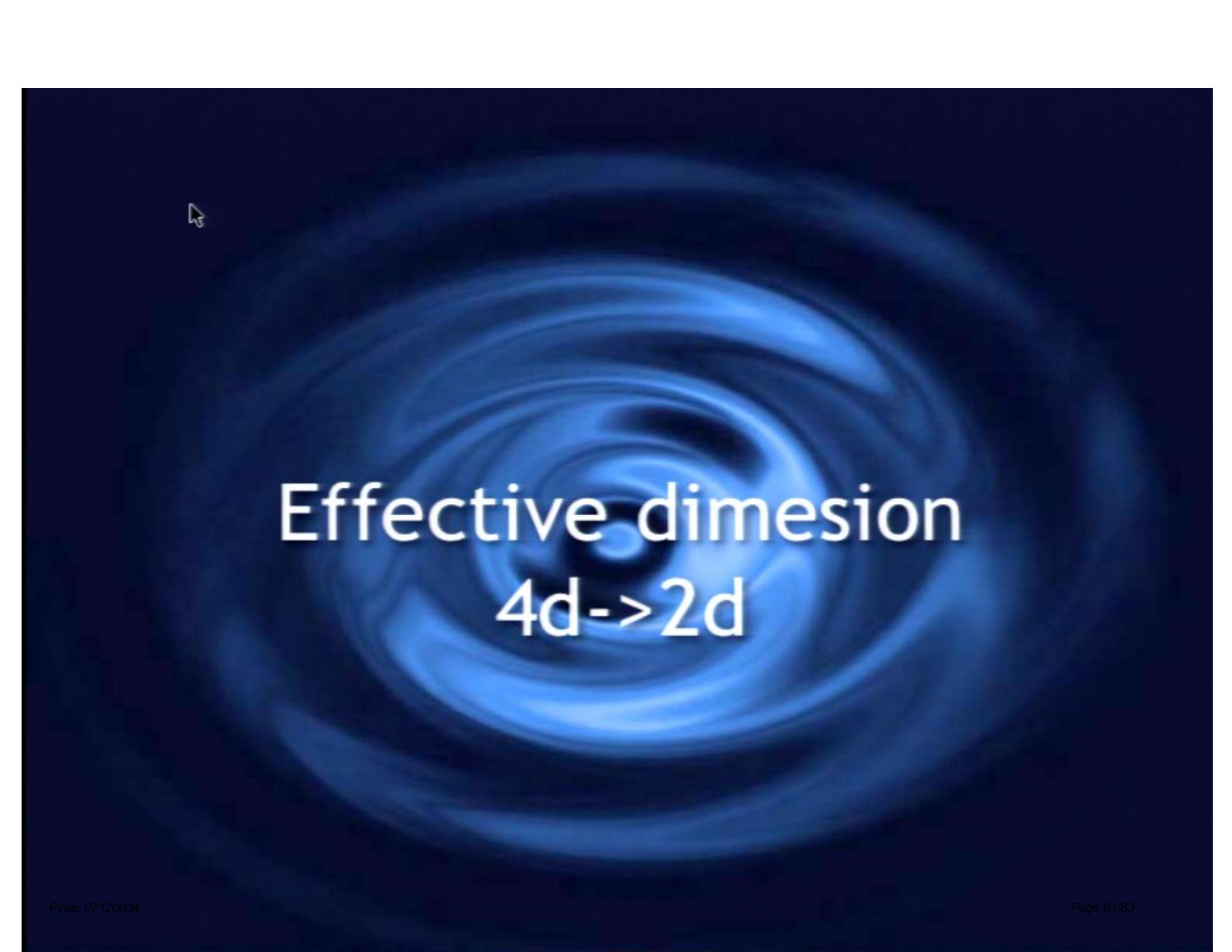
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# Semiclassical analysis conclusions



Effective dimension  
4d->2d

## *Semiclassical analysis and evaporation*

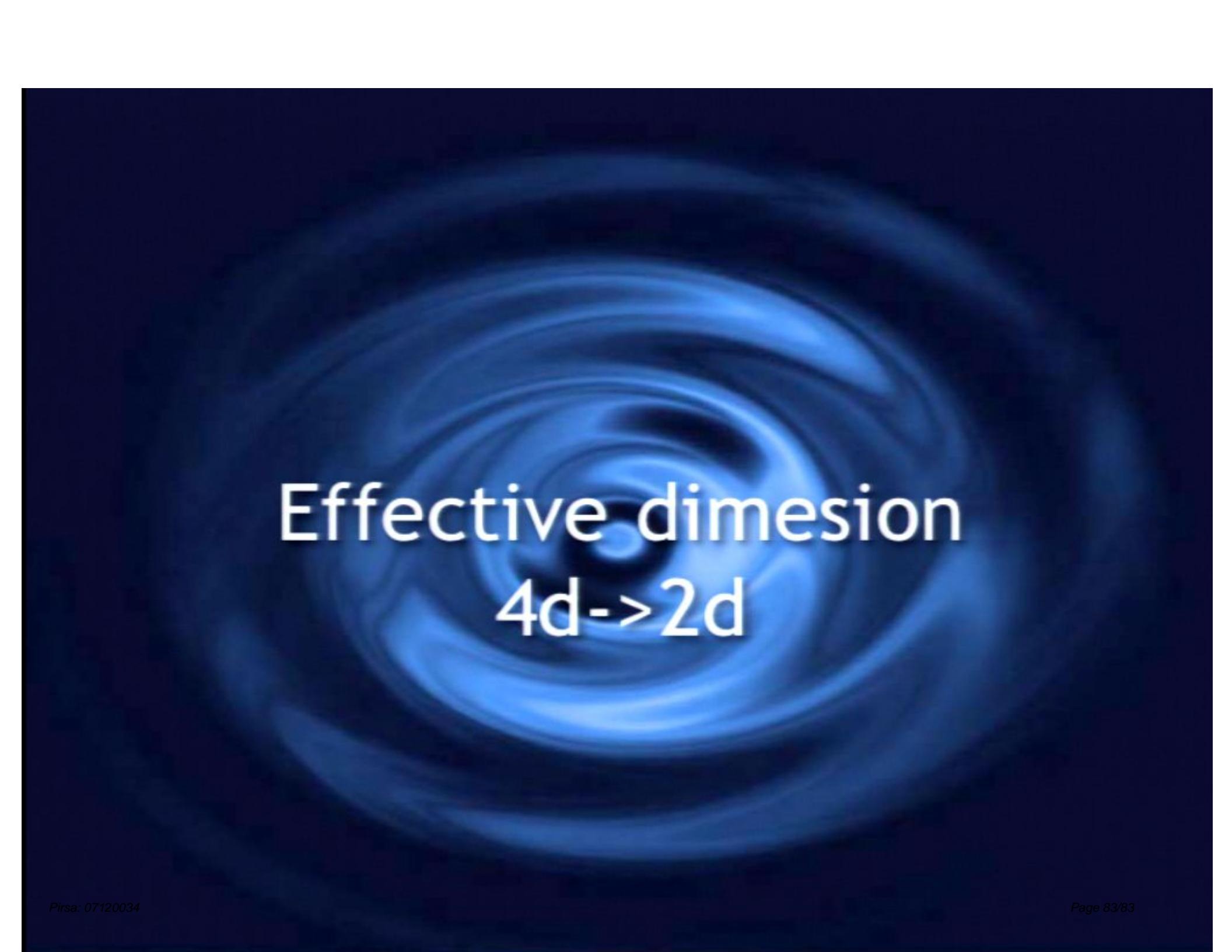
*New regular black hole solution.*

*For*  $m > \frac{l_P}{\sqrt{2}}$

*regular temperature,  
infinite evaporation time.*

*For*  $m \leq \frac{l_P}{\sqrt{2}}$

*Hot remnant.*



Effective dimension  
4d->2d