

Title: Introduction to Quantum Gravity

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Abstract:

Quantum Gravity is not a finished theory yet. There are many approaches to quantum gravity. I can only mention a few (and discuss only one).

The four fundamental forces

?

- quantum electrodynamics
- strong interaction
- weak interaction
- quantum gravity?

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What is so difficult about quantum gravity?

Quantum space-time



- Gravity = property of space-time¹

quantum gravity = quantum theory of space and **time**

Quantum space-time

- Gravity = property of space-time¹

quantum gravity = quantum theory of space and **time**

- Eliminating space and time as fundamental classical attributes and replacing them with something new/quantum.
- Hard part: Reconstruct space-time in low energy limit.
- Different approaches follow this to a different degree.
- Quantum gravity will probably give us new model(s) of time, but may not explain all aspects completely.

Quantum geometry

- gravity encoded in geometry
 - ⇒ measure distances, areas, volumes, ...
 - ⇒ (coordinate dependent) metric

Aim:

- quantum operators/measurements for lengths, areas, volumes
- dynamics for these/ change in time

Background Independence

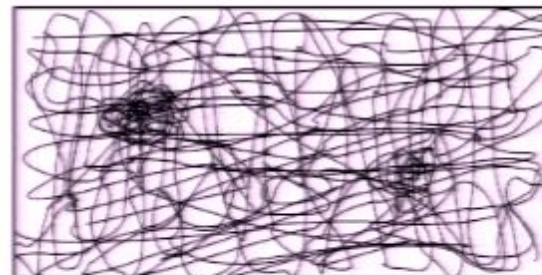
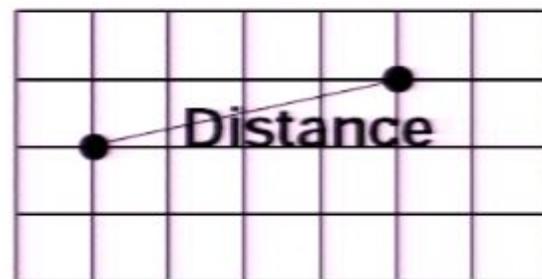


- want to quantize space–time
- treat it as fully dynamical
- (space–time) background has to emerge dynamically in low–energy limit
- post–diction of geometrical features of our universe: dimension, nearly flatness, ...

What is a point?

?

n-point functions: basic tool in standard quantum field theory



What is an observable in (quantum) gravity?²

Quantum Gravity: Early approaches

- perturbative³

- gauge-fixing and Faddeev-Popov ghosts
- background field method, effective action
- constrained systems
- 1986 Goroff and Sagnotti: (non-super) gravity non-renormalizable

³deWitt, Feynman, Veltman, 't Hooft, ...

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⁴Dirac, deWitt, Wheeler, Hartle, Hawking, Kuchař, Isham, ...

⁵Bogga, Hamber, Williams, Ambjørn, Loll

Quantum Gravity: Early approaches

- perturbative³
 - gauge-fixing and Faddeev-Popov ghosts
 - background field method, effective action
 - constrained systems
 - 1986 Goroff and Sagnotti: (non-super) gravity non-renormalizable
- canonical quantization and path integral⁴
 - Dirac quantization, constrained systems
 - problem of time
 - quantum cosmology
 - black hole thermodynamics (saddle point approximation)
 - Regge calculus, dynamical triangulations: effective dimension dynamical (you can get it easily wrong; to get it right implement causality)⁵

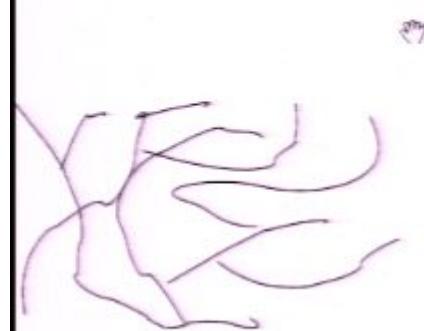
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Effective dimension



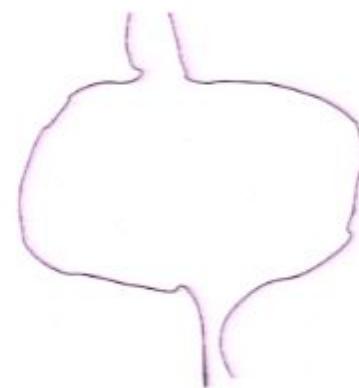
Euclidean dynamical triangulations
1

condensed polymer phase



Euclidean dynamical triangulations
2

$d = \infty$
crumbled phase



Causal dynamical triangulations^a
 $d = 4$

^aAmbjorn, Jurkiewicz, Loll '05, Benedetti, Henson

Canonical Quantization

- Hilbert space representation of metric in a background independent way?
- difficult/impossible⁶ in metric variables

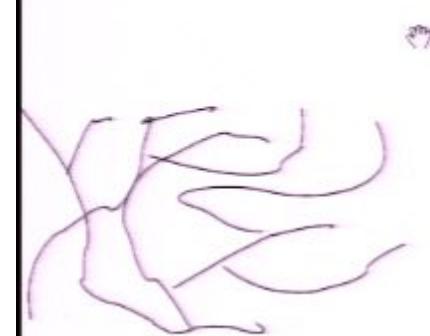
⁶Ashtekar 2007

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Ashtekar 1986, Sen 1982, (complex), Barbero 1995 (real) (canonical)

Samuel, Jacobson, Smolin (1987) (Lagrangian)

Effective dimension



Euclidean dynamical triangulations

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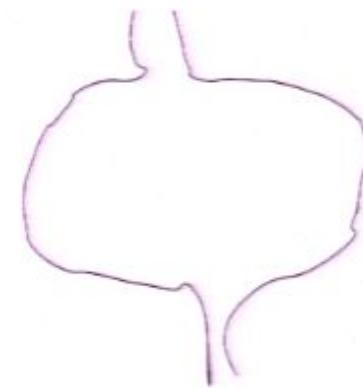


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Use Ashtekar variables instead⁷⁸

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Canonical Quantization

- Hilbert space representation of metric in a background independent way?
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Use Ashtekar variables instead⁷⁸

- allows for gauge theory formulation
 - **connection** A
→ Wilson loops; encodes time derivative of spatial metric
 - **triads (electric field)** E
→ electric flux through surfaces; encodes spatial metric

⁶Ashtekar 2007

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Loop Quantum Gravity

- Hamiltonian hugely simplified (in complex variables)
→ one (formal) solution Kodama state
- background independent Hilbert space formulation⁹
→ (kinematical) spectra of geometrical operators discrete¹⁰
→ black hole entropy¹¹
- (dynamical) lattice gauge theory formulation
→ provides regulator for Hamiltonian¹²
- unique (kinematical) vacuum state/ representation¹³
- motivation for / better connection to path integral approach
→ spin foams

⁹ Ashtekar, Baez, Isham, Jacobson, Lewandowski, Rovelli, Smolin, ...

¹⁰ Rovelli, Smolin, Ashtekar, Lewandowski, ...

¹¹ Ashtekar, Baez, Corichi, Krasnov '97, ...

¹² Thiemann 1996

¹³ Fleischhack, Lewandowski, Okonek, Sahlmann, Thiemann '04/05

The Hamiltonian

$$H = \frac{1}{\sqrt{\det(g)}}(p_{ab}p^{ab} - (p^{ab}g_{ab})^2) - \sqrt{\det(g)}R$$

$$R = g^{ab}(\partial_c\Gamma_{ab}^c - \partial_a\Gamma_{cb}^c + \Gamma_{ab}^d\Gamma_{dc}^c - \Gamma_{bc}^d\Gamma_{ad}^c)$$

$$\Gamma_{ab}^c = \frac{1}{2}g^{cd}(\partial_a g_{bd} + \partial_b g_{ad} - \partial_d g_{ab})$$

- non-polynomial, inverse metric

$$\tilde{H} = F_{ab}^j\epsilon_{jkl}E_k^aE_l^b$$

$$F_{ab}^j = \partial_a A_b^j - \partial_b A_a^j + \epsilon_{jkl}A_a^kA_b^l$$

- polynomial, 4th order
- more complicated in real variables

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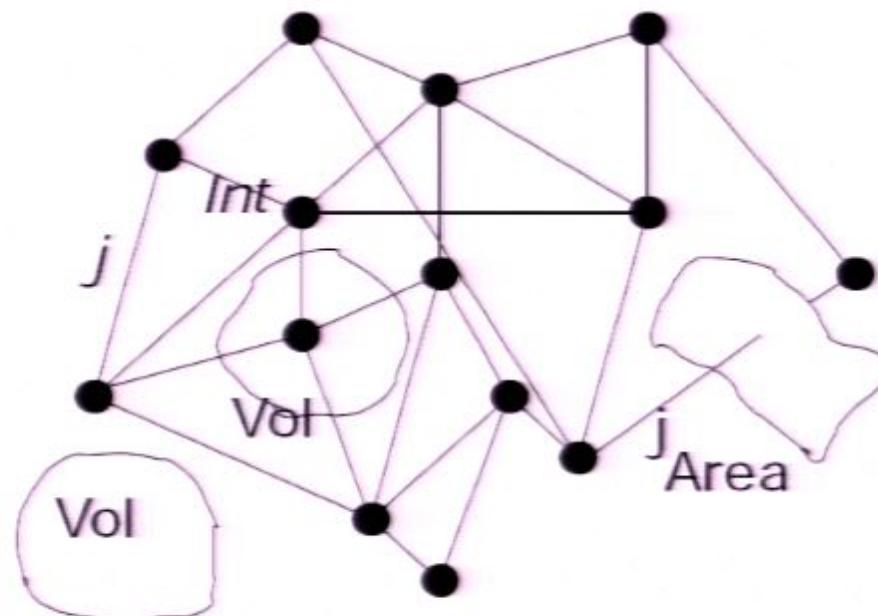
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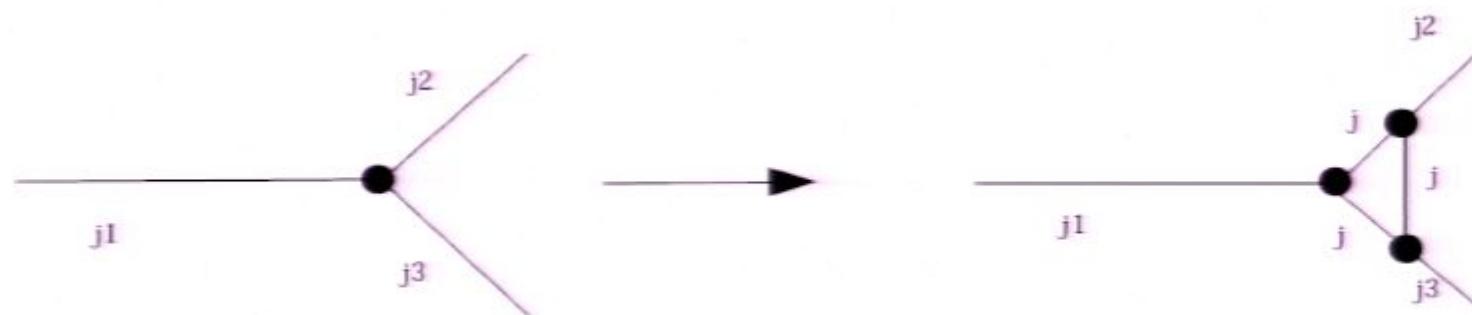
State space of loop quantum gravity: spin networks

- excitations of geometry along lines and at vertices: graphs
- (kinematical) vacuum: nothing is excited
→ no geometry (also no vacuum fluctuations)
- Wilson loop operators: excite/annihilate edges of graph:
dynamical graph/lattice with (spin) labels
- flux operators: discrete spectra for geometric operators



Dynamics

- defined by Hamiltonian
- anomaly-free, uv-finite quantization of Hamiltonian¹⁴
- changes graph and labels → dynamical graph



- ambiguities¹⁵
- action (for this version) might be too local¹⁶
- difficulty: to have a consistent (slicing independent) evolution:
would ensure 4-dim diffeomorphism invariance

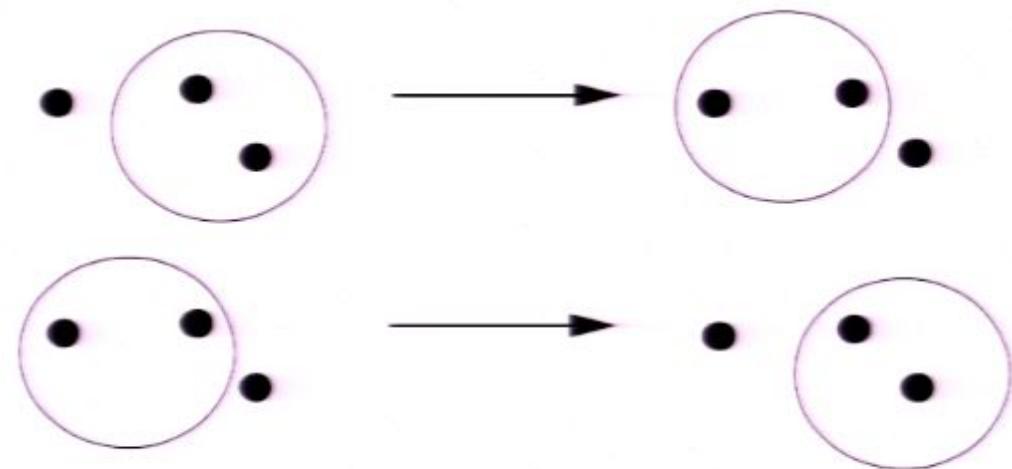
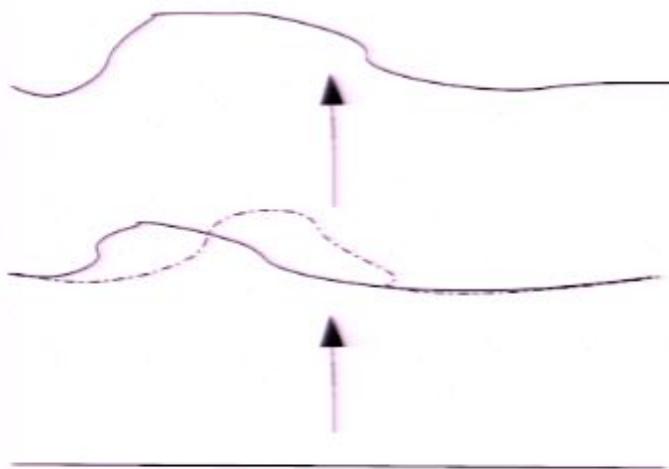
¹⁴ Thiemann '96

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¹⁵ Perez, loop quantum cosmology: Bojowald, ...

¹⁶ Smolin '96, master constraint program; Thiemann '03

Inconsistent evolution

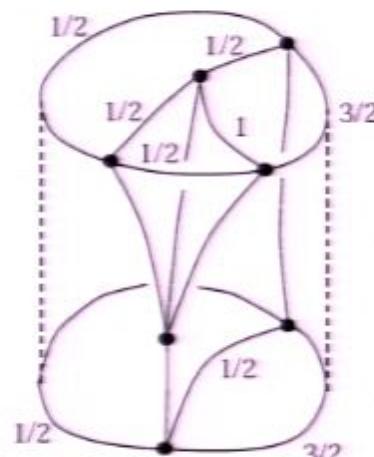


- general covariance: locally independent evolution steps should be possible
- path independence: final state should not depend on the in-between state¹⁷
- this is difficult to achieve even for free scalar field theory in $d > 2$ ¹⁸
- satisfied for topological field theories

n foams

- one motivation: time evolution of spin networks¹⁹
- gravity action → (topological) BF -action plus constraint²⁰ → implement constraint into path integral for BF -action

A spin foam model:



- (dual) triangulations of n-dim manifold with labels
- attach amplitudes to vertices and edges
- sum over labels
- (sum over triangulations)
- Barrett-Crane model '97

Group Field Theory

- spin foam models can be formulated as group field theories²¹
- Feynman diagram expansion for "atoms of space-time": sum over diagrams realizes sum over triangulations
- finiteness result: this expansion can be made finite order by order²²
- general framework for background independent path integrals²³, tool for model building, generalization of matrix models

²¹ Boulatov, Ooguri, DePietri, Freidel, Krasnov, Reisenberger, Rovelli, ..., Ryan

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²² Perez, Rovelli '00

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²³ Oriti

v energy limit

- Hard part: reconstruct semi-classical space-time
- covariant
 - background independent generalization of n -point functions²⁴
 - 2-point function scales correctly with distance for diagonal tensor components²⁵
 - non-diagonal components incorrect for Barret-Crane:
motivated new models²⁶
 - calculations lowest order in group field expansion

²⁴Oeckl, Rovelli, Conrady, Doplicher, Bianchi, Modesto, Speziale

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²⁵Alesci, Rovelli '07

²⁶Engle, Pereira, Rovelli; Freidel, Krasnov; Livine, Speziale 07

v energy limit

- Hard part: reconstruct semi-classical space-time
- canonical
 - weave states²⁷
 - coherent states²⁸
 - can be used to probe semi-classical limit (on fixed graphs)²⁹
 - dynamical semi-classical states?
- loop quantum cosmology: use symmetry to drastically reduce degrees of freedom³⁰
- dynamics solvable, semi-classical limit can be discussed
- singularity resolution

²⁷ Ashtekar, Rovelli, Smolin, Zegwaard,...

²⁸ Thiemann, Ashtekar, Lewandowski, Winkler, Bahr

²⁹ Thiemann, Sahlmann, Giesel

³⁰ Bojowald, Ashtekar, Lewandowski, Singh

v energy limit

- Hard part: reconstruct semi-classical space-time
- this is rather an (extrem) many-body-problem
- we should use statistical physics tools³¹
- space-(time) as emergent/ in a phase transition

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Does a theory of quantum gravity without matter exist?
Or does a quantum theory of gravity have to include matter
couplings?

³²Thiemann '96

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³³Bilson-Thompson, Markopoulou, Smolin '06

³⁴Ijzi 07

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**Does a theory of quantum gravity without matter exist?
Or does a quantum theory of gravity have to include matter
couplings?**

- LQG: add matter as additional fields: Hamiltonians uv–finite³²
- matter degrees of freedom encoded in the topology of the graph:
braiding and knotting³³
- unification of forces based on connection (*BF*–type)
formulation³⁴

³²Thiemann '96

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³³Bilson–Thompson, Markopoulou, Smolin '06

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³⁴Ijzi 07

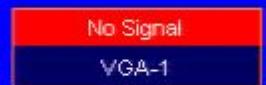
Inclusions

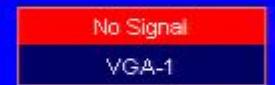
Despite approx. 70 years of research we do not have a quantum theory of gravity.

- but we have learnt a lot
- models for background independent physics
- understand kinematical features very well
- approaching low energy limit
- quantum gravity predictions/phenomenology³⁵
- experimental results might come in pretty soon/ are already there!

No Signal
VGA-1







No Signal
VGA-1

