Title: A quantum view on locality, realism and information.

Date: Dec 03, 2007 11:30 AM

URL: http://pirsa.org/07120017

Abstract: First, a brief description of Bell\'s theorem will be given.
It states states that there is no classical-like (local realistic) description of all quantum predictions. Next, a plausible class of non-local realistic models will be presented which is incompatible with quantum mechanics, as first shown by Leggett. Experiments confirming the incompatibility will be described. Finally, it will be argued that quantum mechanics can be seen as a theory of systems with limited information resources.

A quantum view on locality, realism and information.

Tomasz Paterek

Institute for Quantum Optics and Quantum Information
Austrian Academy of Sciences

Young Researchers Conference
Perimeter Institute, Waterloo, ON, Canada
3rd December 2007

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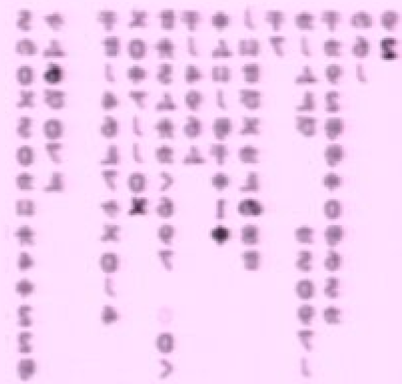
CONTENT

Bell's theorem

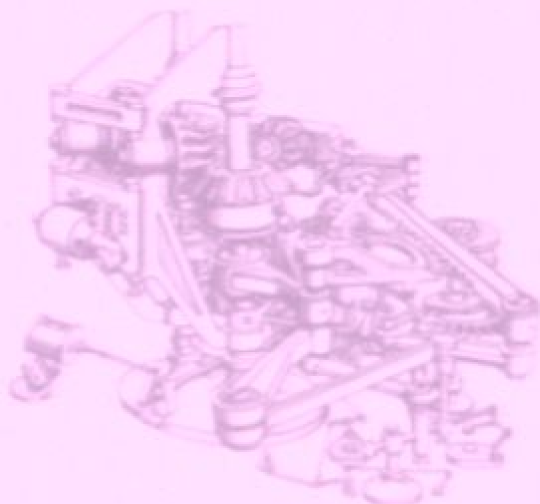
Leggett's theorem



Information approach

Assumptions

- **Realism**: unperformed measurements have well-defined, yet unknown, results
- **Locality**: distant systems are independent



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Thesis

Nature is not like that!

No classical-like description of quantum predictions.

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PROOF

$A(k,x)=\pm 1$ $B(l,x)=\pm 1$

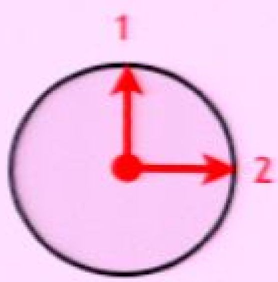
- For a given x (in a single run):

$$A(1,x)[B(1,x) + B(2,x)] + A(2,x)[B(1,x) - B(2,x)] = \pm 2$$

- Averaged over x (in many runs):

$$|E_{11} + E_{12} + E_{21} - E_{22}| \leq 2$$

- Quantum correlations of the Bell singlet state:



$$E_{kl} = -\mathbf{a} \cdot \mathbf{b}$$

Data available to Clauser, Horne, Shimony, and Holt

- C. S. Wu and I. Shaknov,
  The angular correlations of scattered annihilation radiation,
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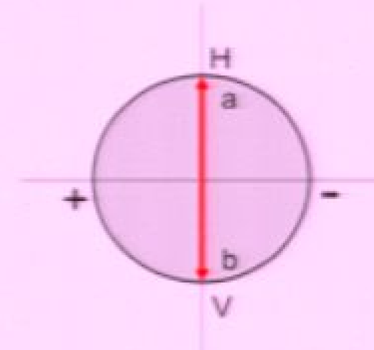

Bell experiments

- S. J. Freedman and J. F. Clauser,
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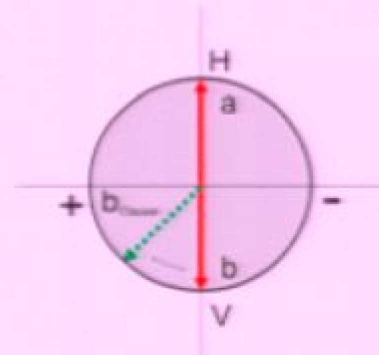

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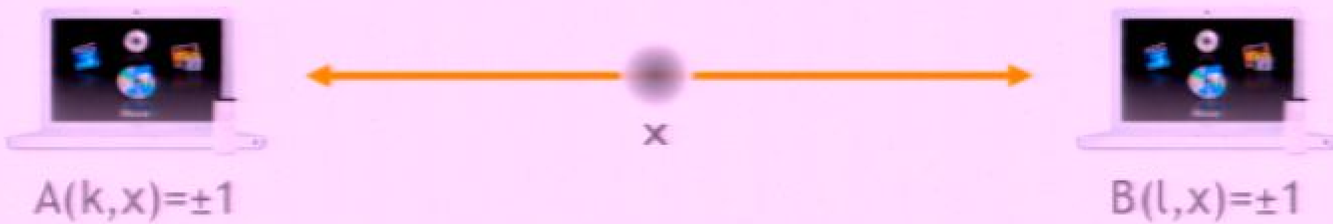

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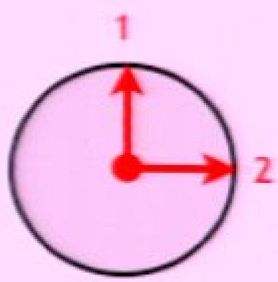

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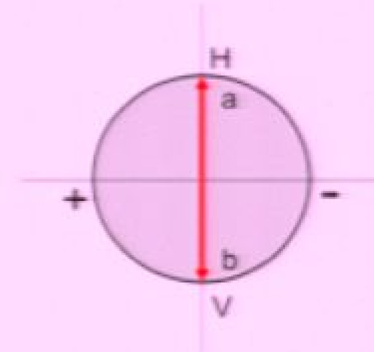

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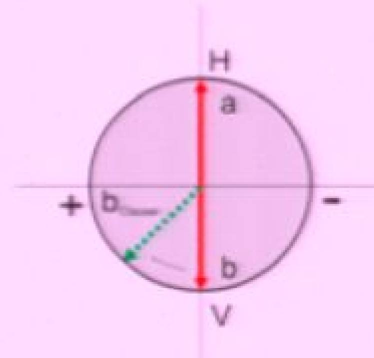
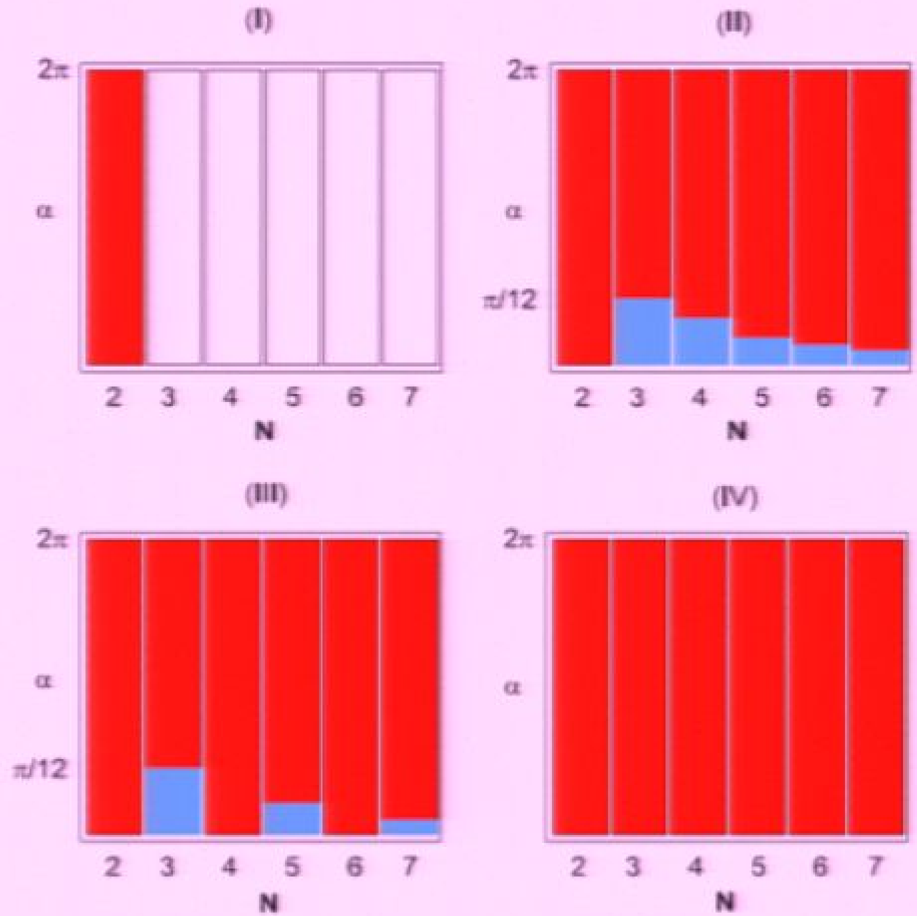


ENTANGLED AND CLASSICAL?

$$|\psi\rangle = \cos\alpha|0...0\rangle + \sin\alpha|1...1\rangle$$



- ✓ K. Nagata, W. Laskowski, and TP, Phys. Rev. A **74**, 62109 (2006)
- ✓ TP, W. Laskowski, and M. Zukowski, Mod. Phys. Lett A **21**, 111 (2006)
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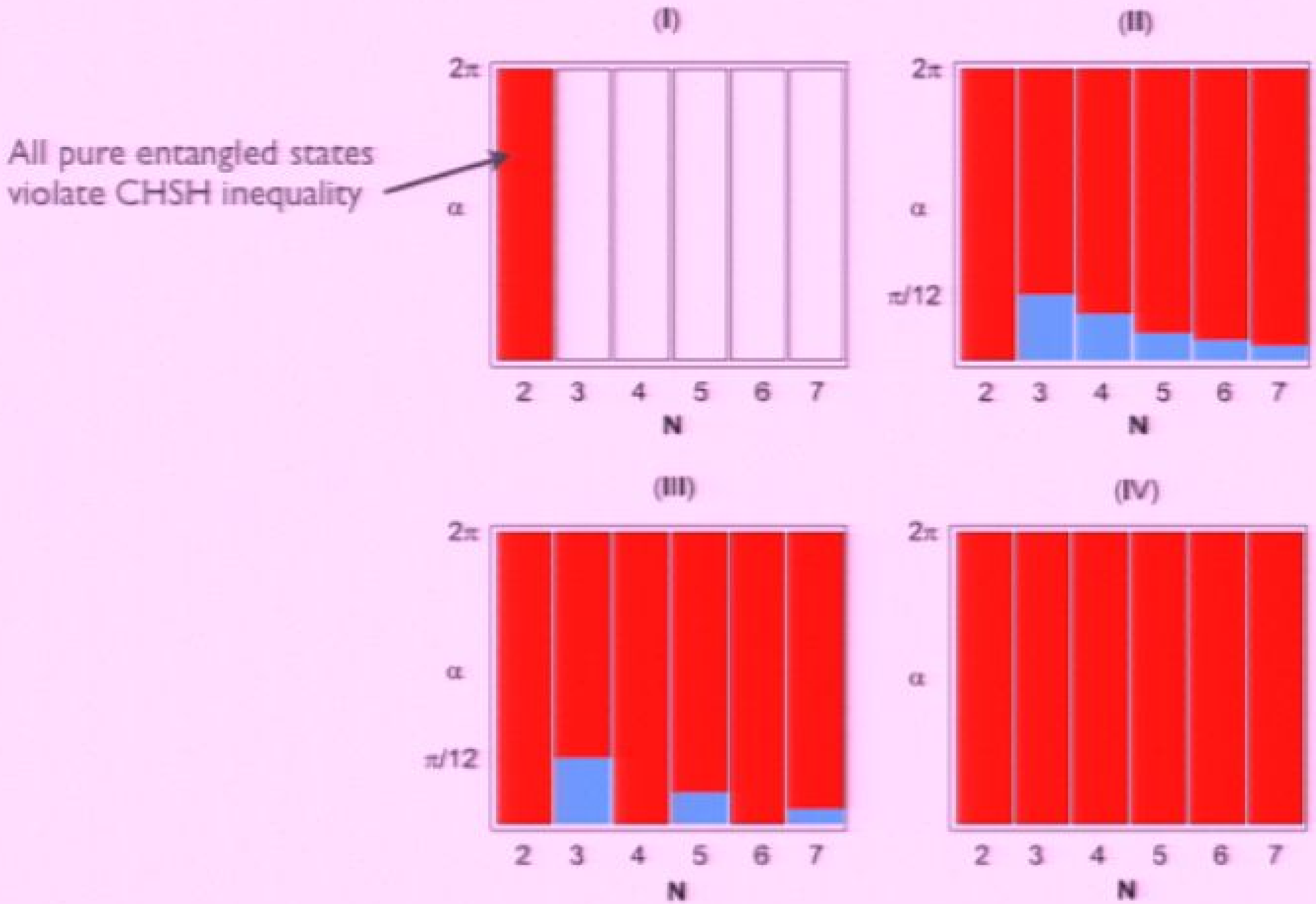

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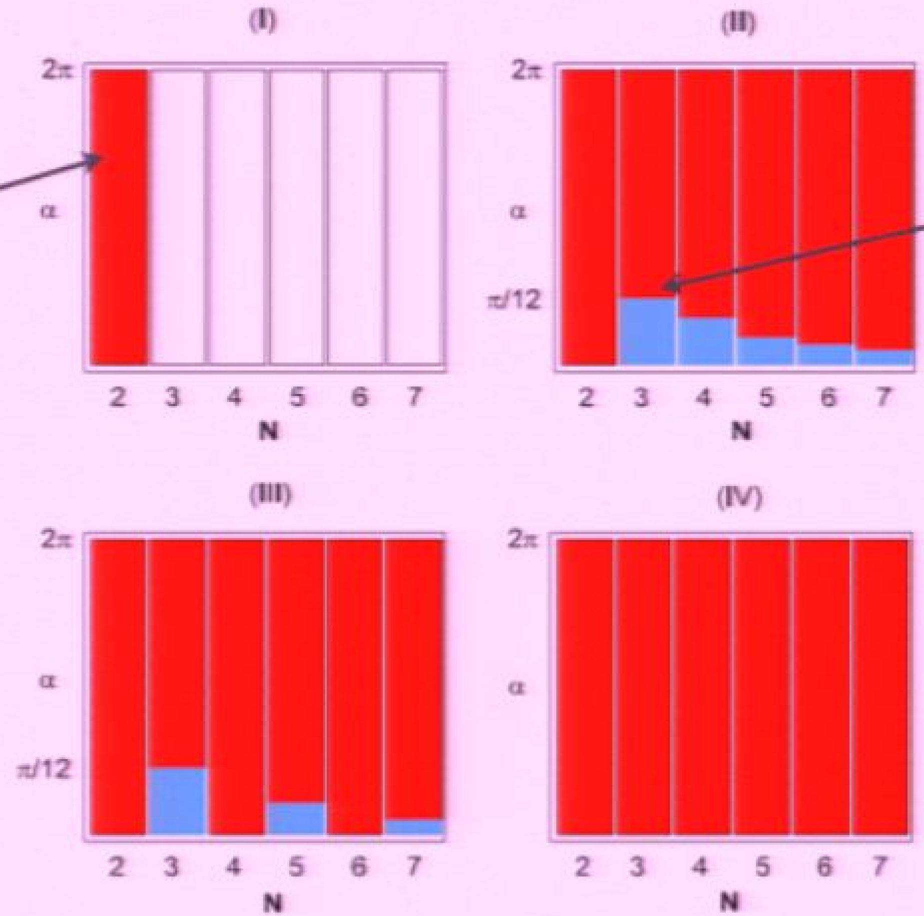
All pure entangled states violate CHSH inequality

There are pure entangled states which *do not* violate the inequalities by Mermin



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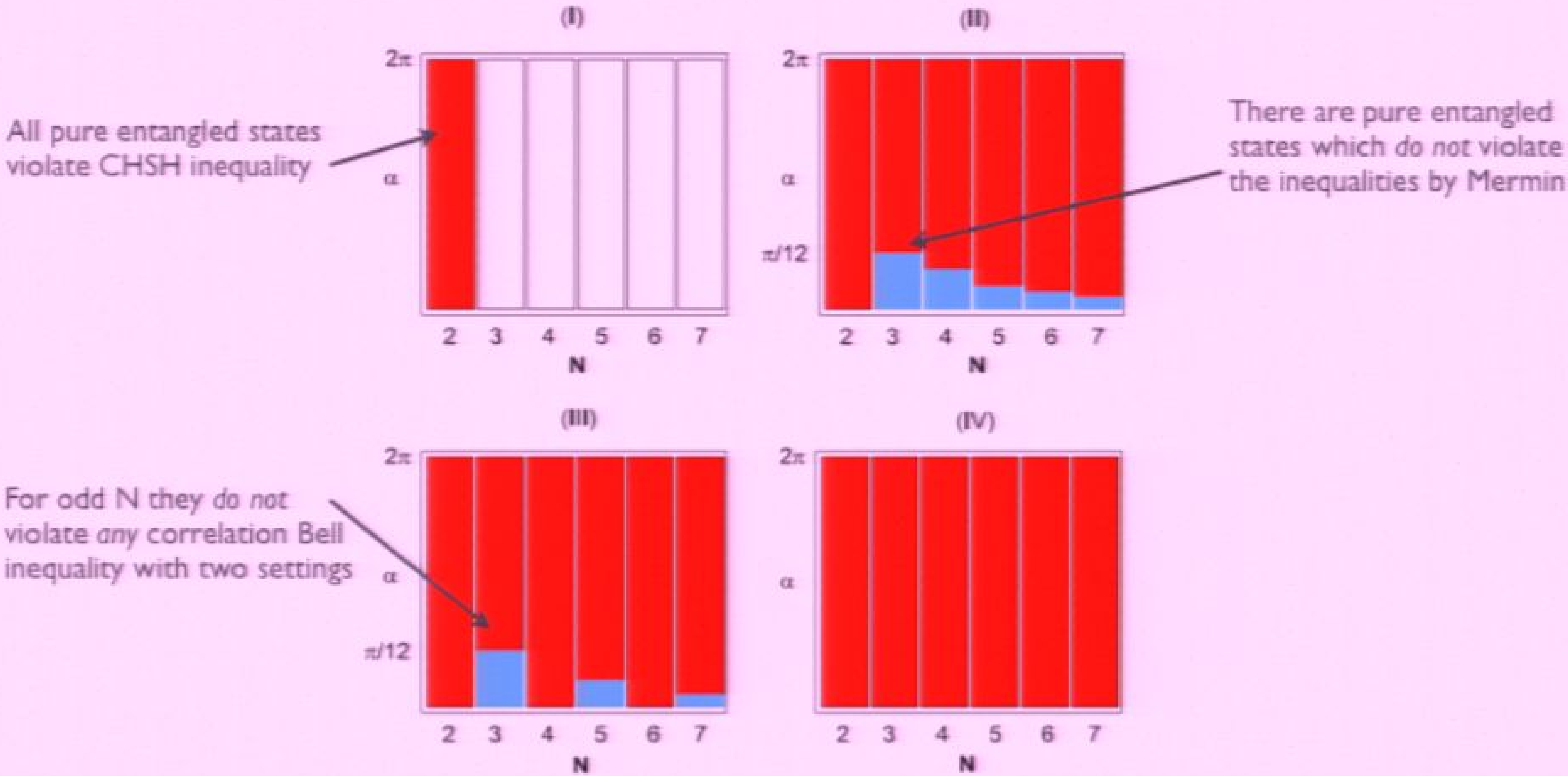

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Bell's theorem

No local realistic explanation of all quantum predictions.

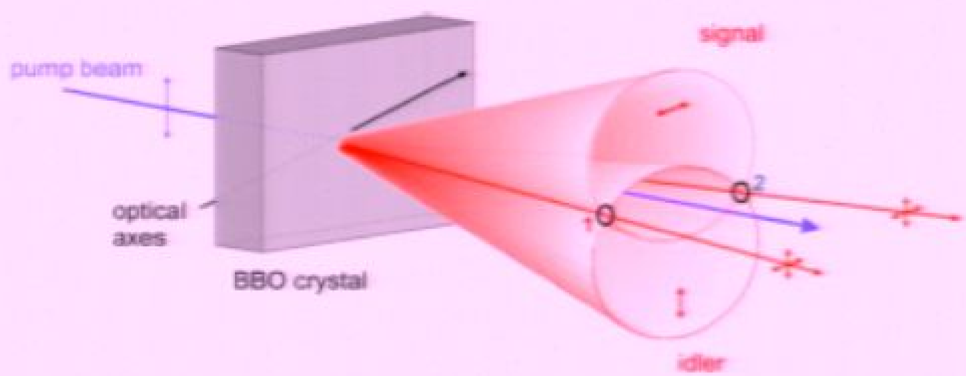
Leggett's theorem

A plausible class of *non-local* realistic models conflicts quantum predictions.

- A. J. Leggett,
  Nonlocal hidden-variable theories and quantum mechanics: An incompatibility theorem,
  Found. Phys. **33**, 1469 (2003).

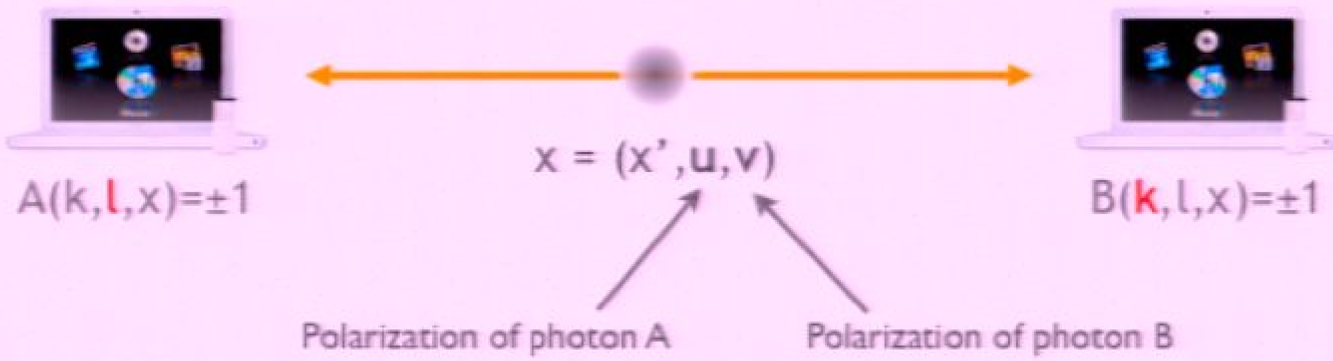
✓ TP, A. Fedrizzi, S. Gröblacher, T. Jennewein, M. Zukowski, M. Aspelmeyer, and A. Zeilinger, Phys. Rev. Lett. **99**, 210406 (2007)
✓ S. Gröblacher, TP, R. Kaltenbaek, C. Brukner, M. Zukowski, M. Aspelmeyer, and A. Zeilinger, Nature **446**, 871 (2007)

Parametric down-conversion



- Photons from the intersections are also well-polarized.
- Measurement outcomes depend on distant parameters.

MATHEMATICAL DESCRIPTION



There are subensembles of definite polarization

$$A(\mathbf{u}) = \mathbf{u} \cdot \mathbf{a}_k \qquad B(\mathbf{v}) = \mathbf{v} \cdot \mathbf{b}_l$$

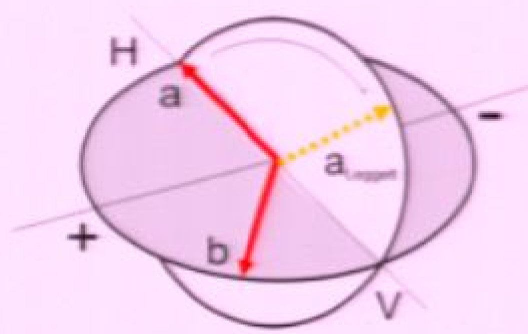
$$AB(\mathbf{u},\mathbf{v}) \neq A(\mathbf{u})\, B(\mathbf{v})$$

Measurable quantities are averaged over the distribution of polarizations

EXPLANATORY POWER

These theories

- explain measurement results obtained with *separable* states
- model all perfect correlations of the Bell singlet state
- rebuild quantum correlations of the singlet for local measurements in one plane
- also model some other correlations of the singlet
- do not allow for faster than light communication



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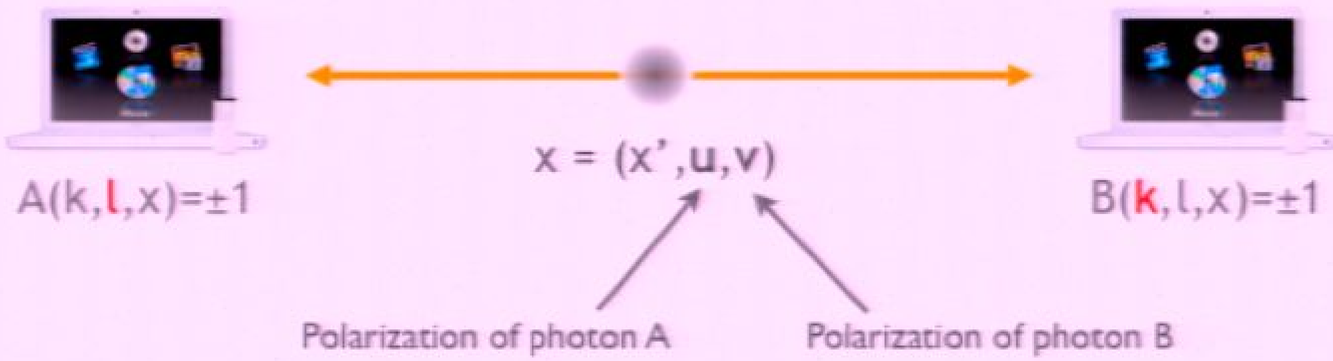

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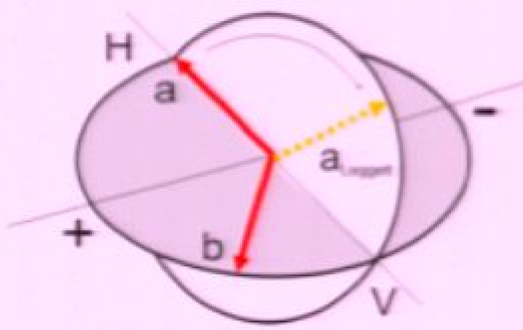

INCOMPATIBILITY

For a given x (in a single run)

$$-1 + | A(k,\mathbf{l},x) + B(\mathbf{k},l,x) | = A(k,\mathbf{l},x)\, B(\mathbf{k},l,x) = 1 - | A(k,\mathbf{l},x) - B(\mathbf{k},l,x) |$$

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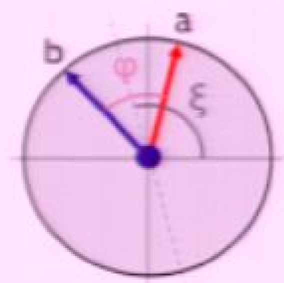
Final inequality

settings from orthogonal planes

$$|\bar{E}_{11}(\varphi) + \bar{E}_{23}(0)| + |\bar{E}^{\perp}_{22}(\varphi) + \bar{E}^{\perp}_{23}(0)| \leq 4 - (4/\pi)\, |\sin(\varphi/2)|$$

valid for the averaged correlations

$$\bar{E}_{kl}(\varphi) = (1/2\pi) \int_0^{2\pi} E_{kl}(\xi,\varphi)\, d\xi$$



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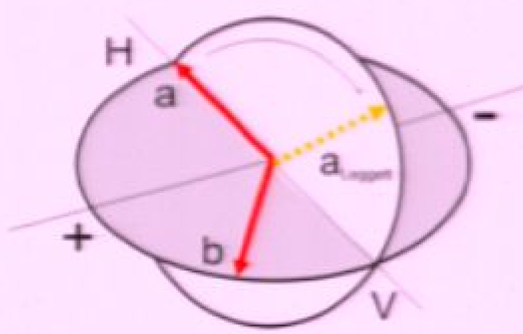

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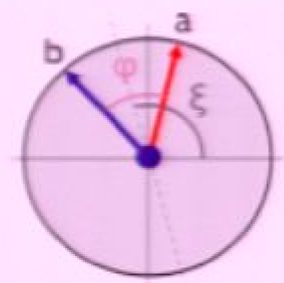
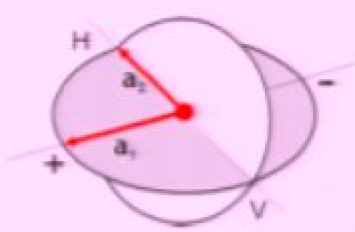
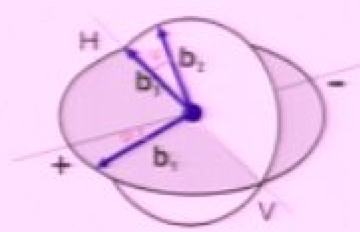


The Bell singlet state is rotationally invariant

$$\overline{E}_{kl}(\varphi) = E_{kl}(\varphi) = -\mathbf{a}_k \cdot \mathbf{b}_l = -\cos(\varphi)$$

Maximal violation





$\varphi_{opt} = 20^\circ$
Bound = **3.792**
Quantum value = **3.893**

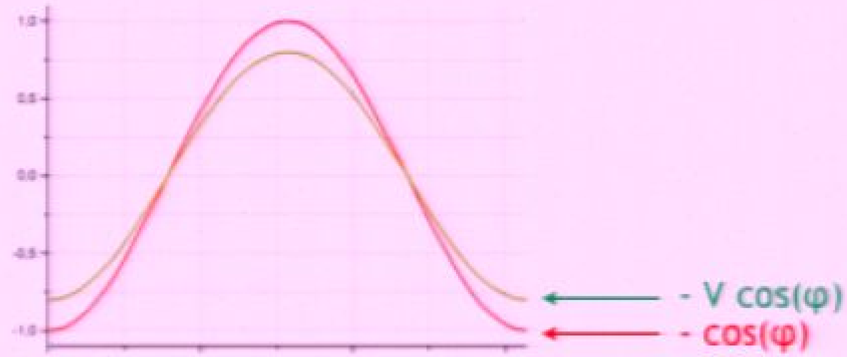
Simultaneously no local realistic model

$$|E_{11}+E_{12}-E_{21}+E_{22}| \leq 2$$

Quantum value = 2.2156

EXPERIMENTAL REQUIREMENTS

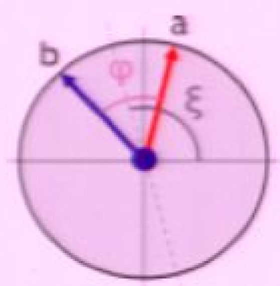
Two-photon interference visibility



$V_{min} \approx 97.4\ \%$

How to measure the averaged correlations?

Additional assumption:
experimentally produced state is rotationally invariant



Is there already data for the "new" settings?

No! Need for a new experiment.

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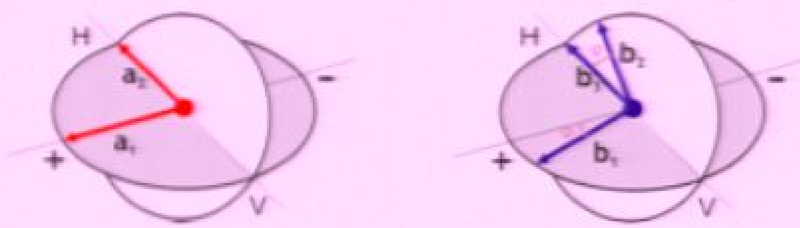

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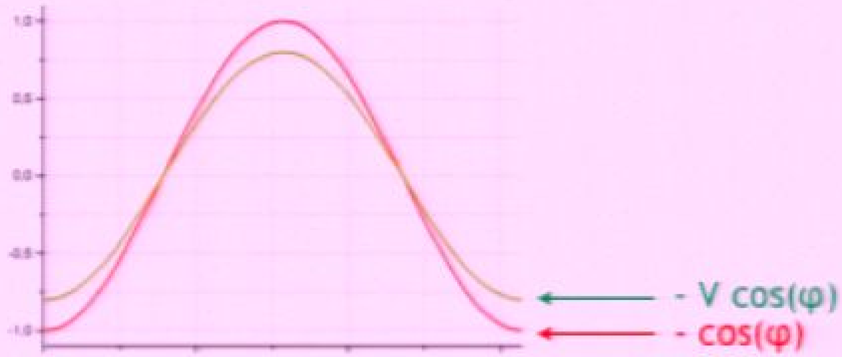

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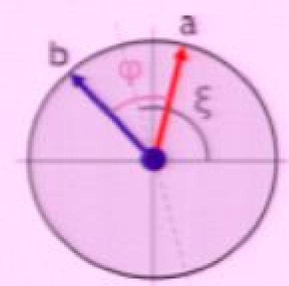

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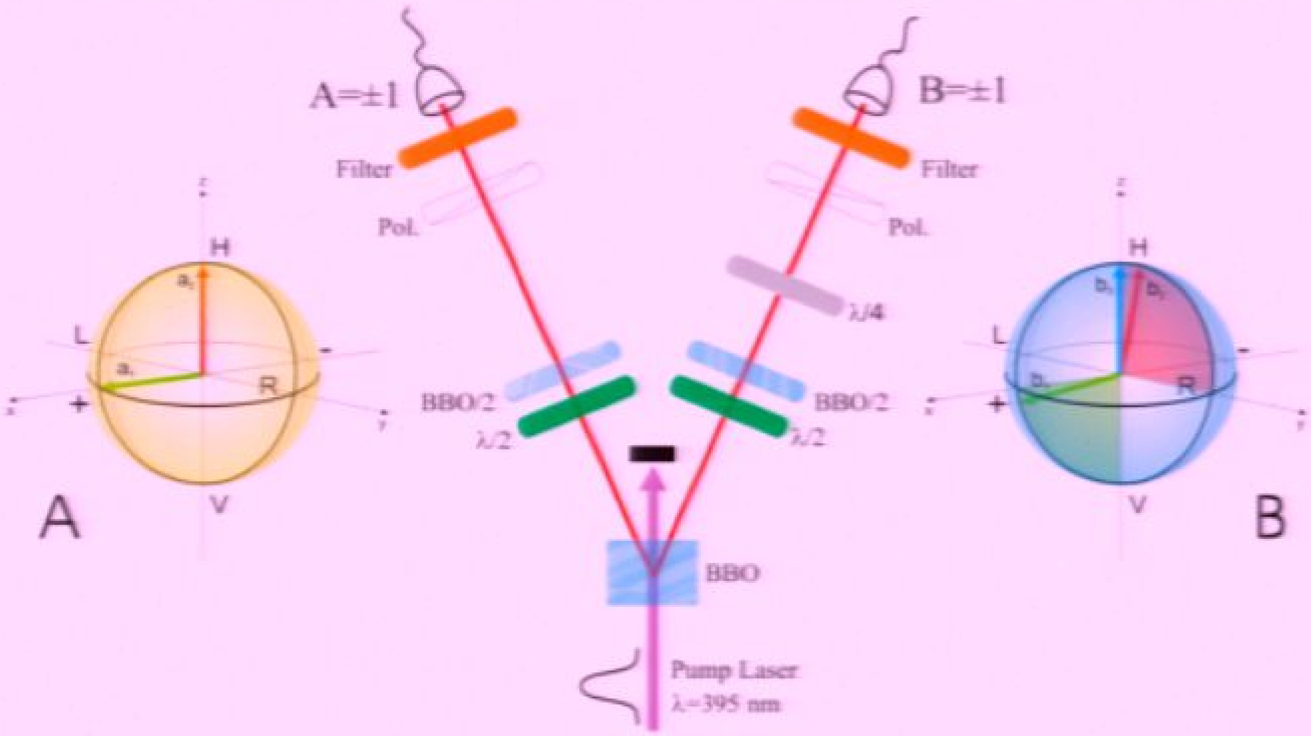


Experimental parameters:

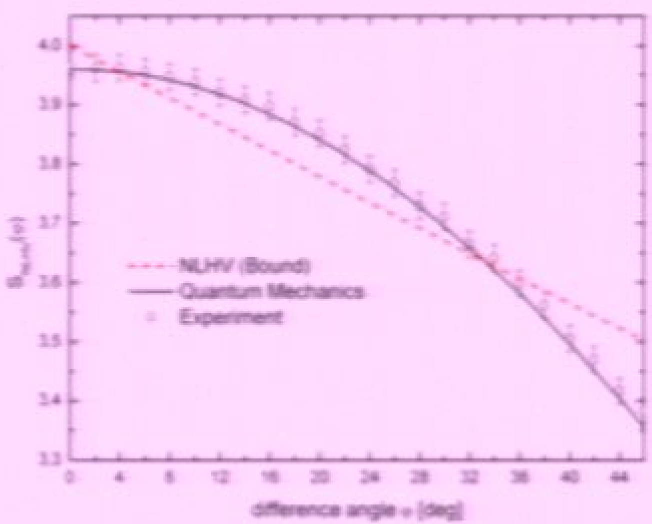
- Thickness of the BBO crystal: **2 mm.**
- Phase matching: **type-II**
- Optical cw-power of the (pulsed) laser: **150 mW**
- Bandwidth of the filters: **1 nm**
- The crystal is aligned to produce "polarization-entangled singlet state"

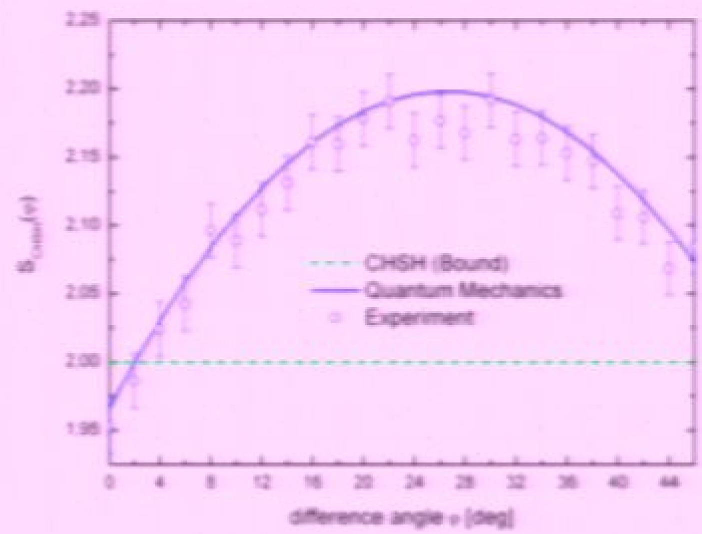
RESULTS

Two-photon interference visibility

$V_{exp} = 99\ \%$ $V_{min} = 97.4\ \%$

Experimental violation





$S_{NLHV} = 3.8521 \pm 0.0227$

Violation by $\approx 3\sigma$

$S_{CHSH} = 2.178 \pm 0.0199$

Violation by $\approx 9\sigma$

Averaged correlations require many measurements

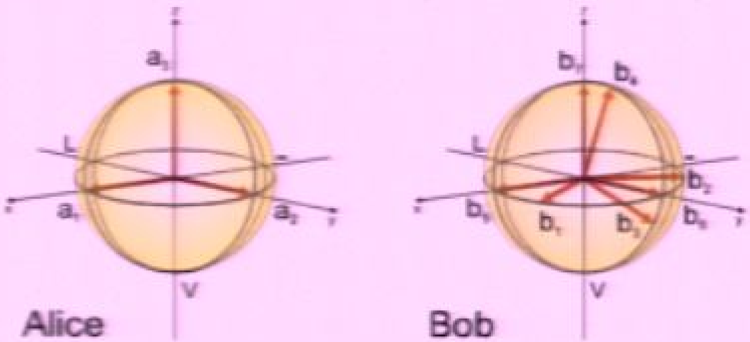
$$\overline{E}_{kl}(\varphi) \longrightarrow \tfrac{1}{2}[E_{kl}(0,\varphi) + E_{kl}(\pi/2,\varphi)]$$

Resulting inequality

$$|\overline{E}_{11}(\varphi) + \overline{E}_{23}(0)| + |\overline{E}^{\perp}_{22}(\varphi) + \overline{E}^{\perp}_{23}(0)| \leq 4 - |\sin(\varphi/2)|$$

No factor $4/\pi$, higher visibility requirement

Experimental requirements



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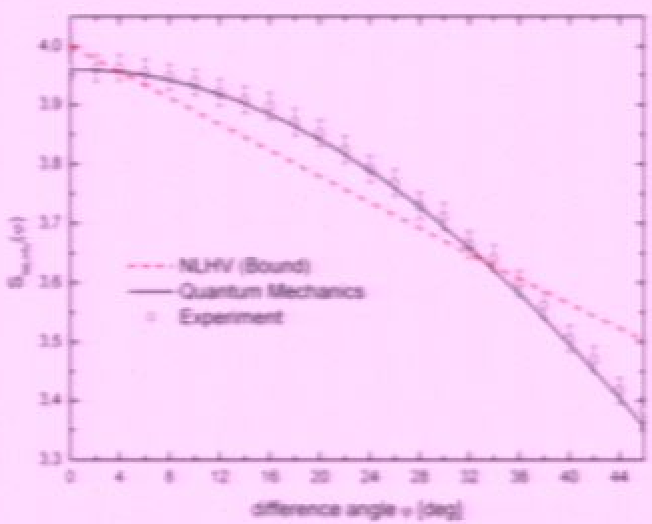

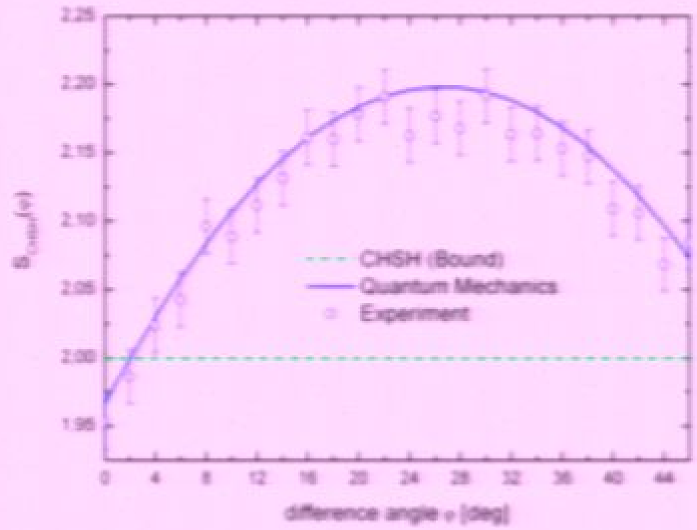

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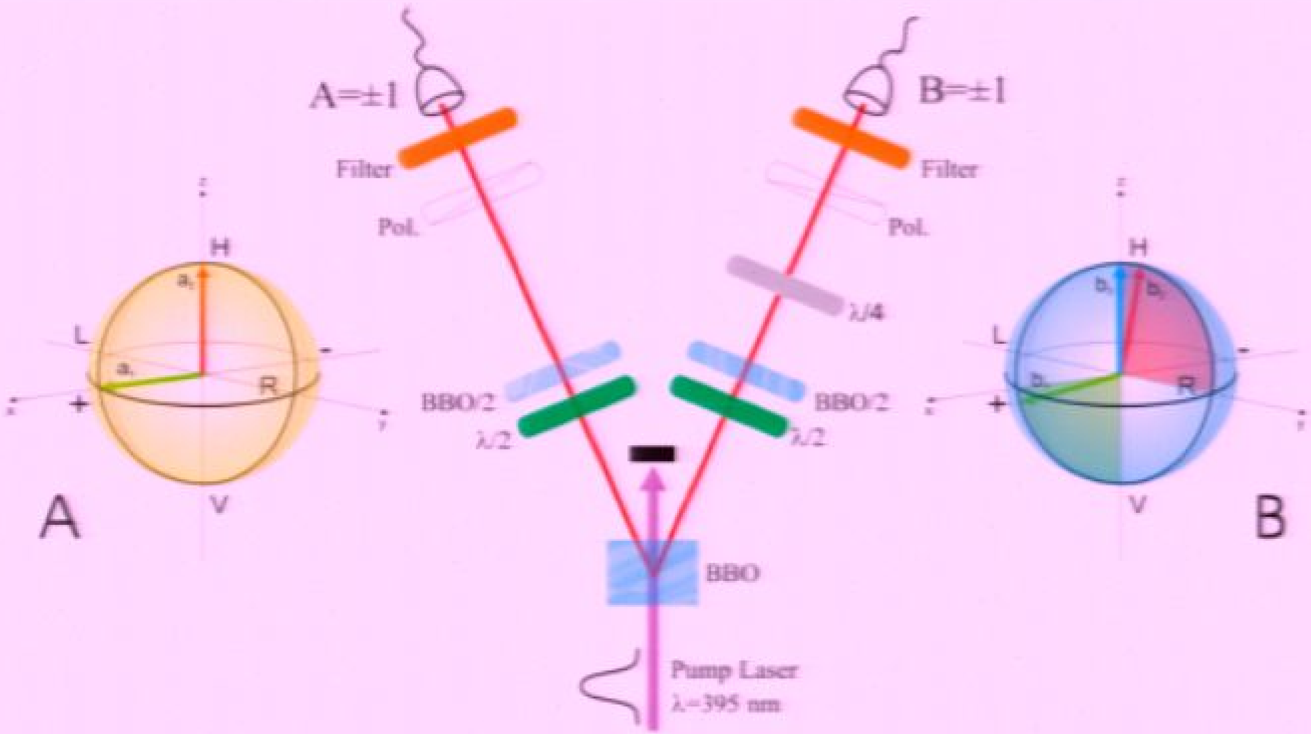

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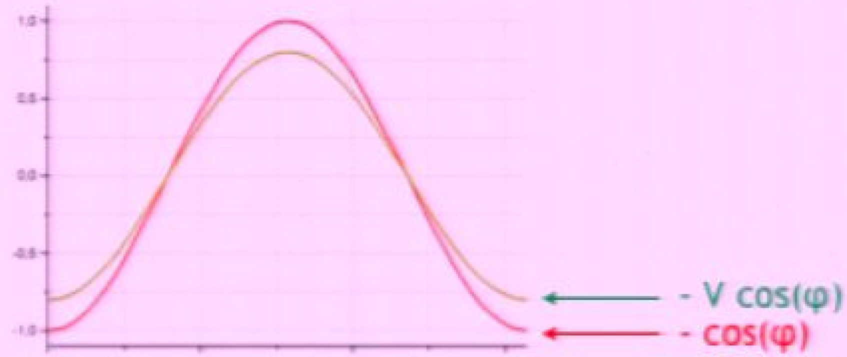

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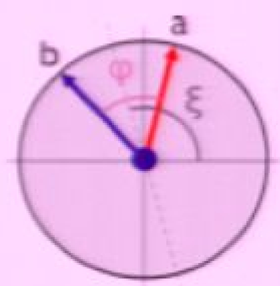
How to measure the averaged correlations?

Additional assumption:
experimentally produced state is rotationally invariant



Is there already data for the "new" settings?

No! Need for a new experiment.

NO ADDITIONAL ASSUMPTION

Averaged correlations require many measurements

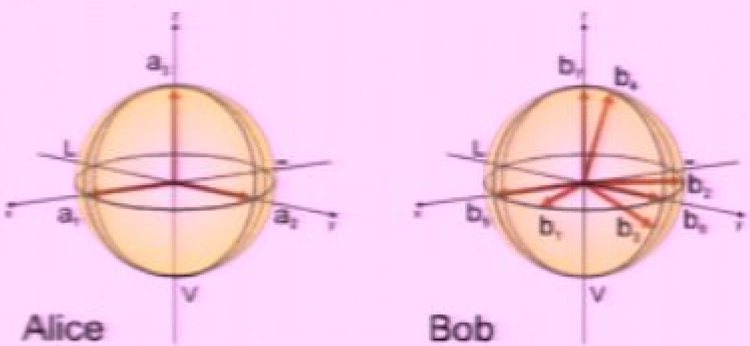
$$\bar{E}_{kl}(\varphi) \longrightarrow \tfrac{1}{2}[E_{kl}(0,\varphi) + E_{kl}(\pi/2,\varphi)]$$

Resulting inequality

$$|\bar{E}_{11}(\varphi) + \bar{E}_{23}(0)| + |\bar{E}^{\perp}_{22}(\varphi) + \bar{E}^{\perp}_{23}(0)| \leq 4 - |\sin(\varphi/2)|$$

No factor $4/\pi$, higher visibility requirement

Experimental requirements

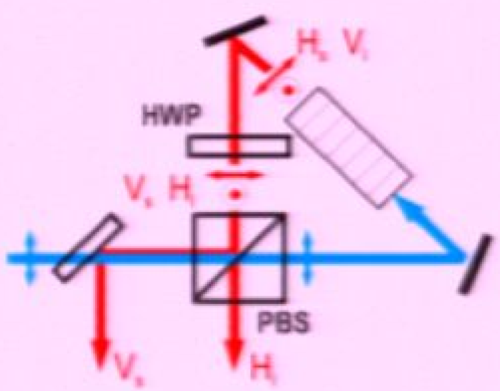


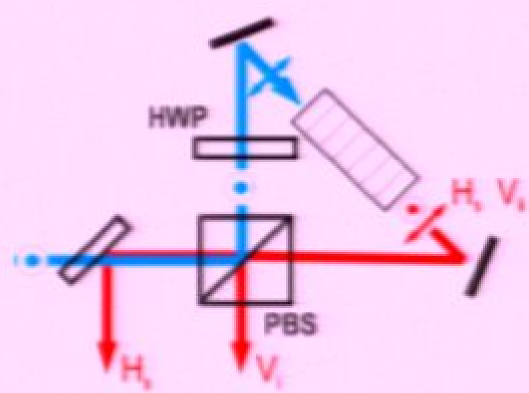
V_{min} = 98.4 %

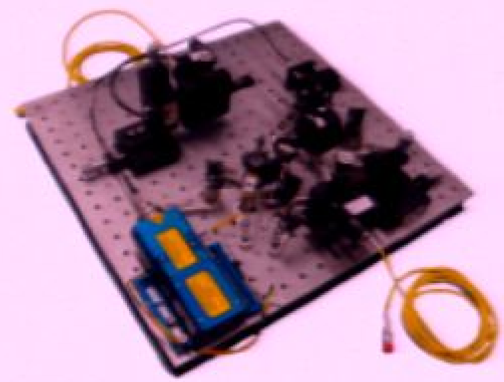
New, extremely efficient source of entanglement

- T. Kim, M. Fiorentino, and F. N. C. Wong
  Phase-stable source of polarization-entangled photons using a polarization Sagnac interferometer,
  Phys. Rev. A **73**, 12316 (2006).

- A. Fedrizzi, T. Herbst, A. Poppe, T. Jennewein, and A. Zeilinger,
  A wavelength-tunable fiber-coupled source of narrowband entangled photons,
  Opt. Express **15**, 15377 (2007).







Experimental violation

$\mathbf{V_{exp} \approx 99.5\,\%}$ $V_{min} \approx 98.4\,\%$ Violation by $\approx 80\sigma$

Bell's theorem

No local realistic explanation of all quantum predictions.

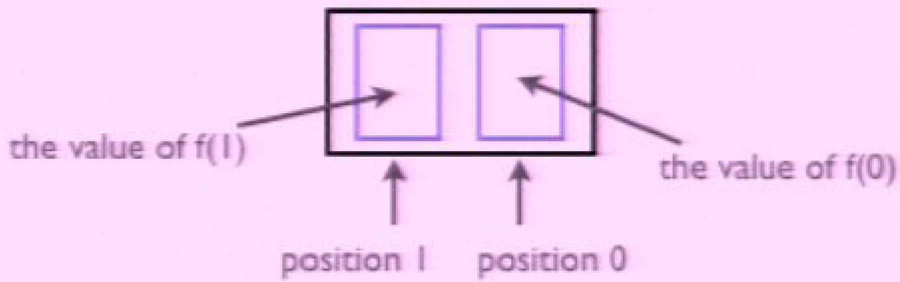
Leggett's theorem

A plausible class of *non-local* realistic models conflicts quantum predictions.

A possible lesson

Give up realism.

How to encode a function in the black box?



x	y_1	y_2	y_3	y_4
0	0	0	1	1
1	0	1	0	1

$y = f(x)$

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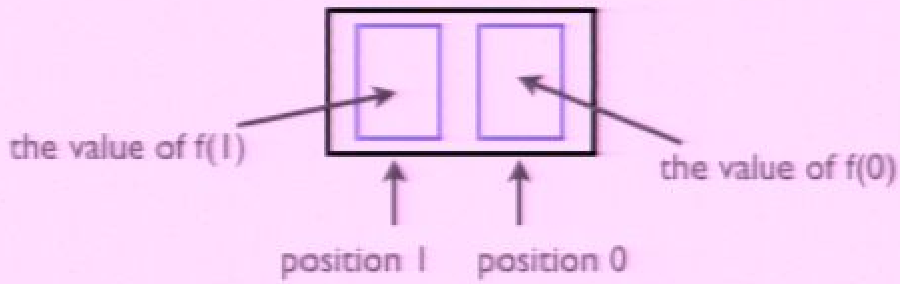

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"Logically complementary" properties of the functions

y_1 y_2	y_3 y_4	f(0)=?
y_1 y_3	y_2 y_4	f(1)=?
y_1 y_4	y_2 y_3	f(0)+f(1)=?

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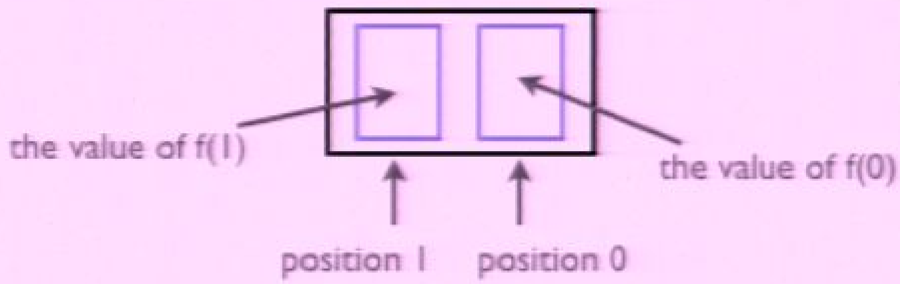

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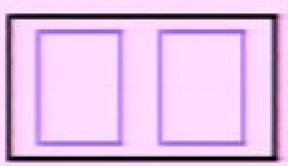
Complementary measurements reveal logically complementary properties



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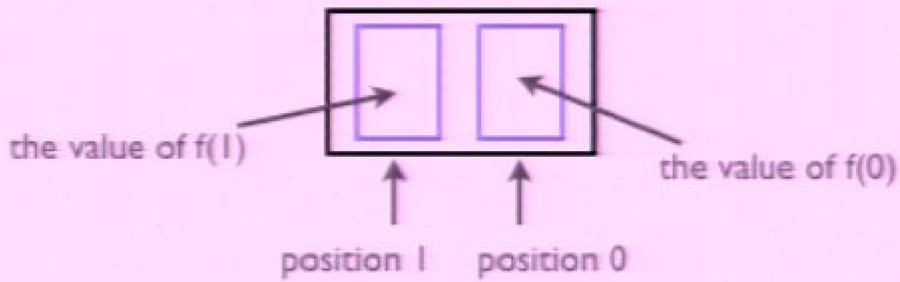

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→ z basis

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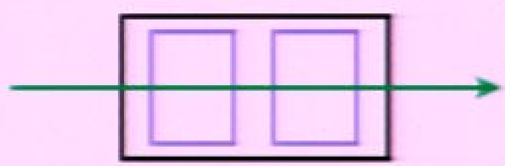

$U = \sigma_x^{f(0)} \sigma_z^{f(1)}$

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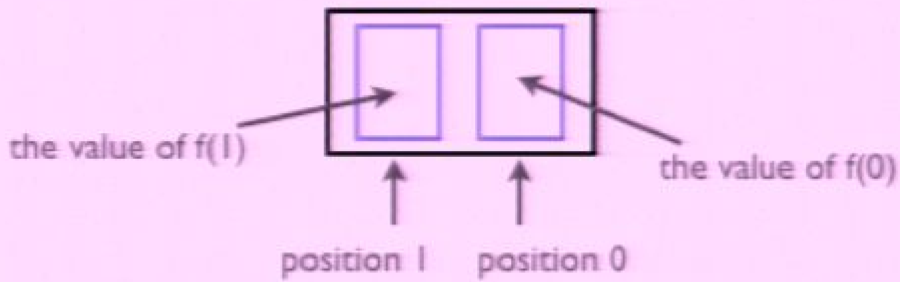

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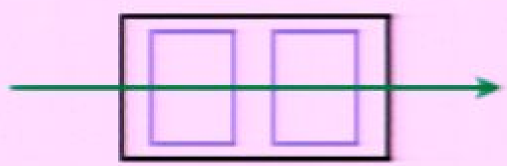

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A QUBIT CARRIES ONE BIT

Quantum states encode properties of functions

$\vert z+\rangle \longrightarrow$ f(0)=0	$\vert x+\rangle \longrightarrow$ f(1)=0	$\vert y+\rangle \longrightarrow$ f(0)+f(1)=0
$\vert z-\rangle \longrightarrow$ f(0)=1	$\vert x-\rangle \longrightarrow$ f(1)=1	$\vert y-\rangle \longrightarrow$ f(0)+f(1)=1

Quantum measurements ask about the properties

z basis $\longrightarrow$ f(0)=? x basis $\longrightarrow$ f(1)=? y basis $\longrightarrow$ f(0)+f(1)=?

Complementarity and randomness

Not enough information to keep deterministic answers to all possible questions

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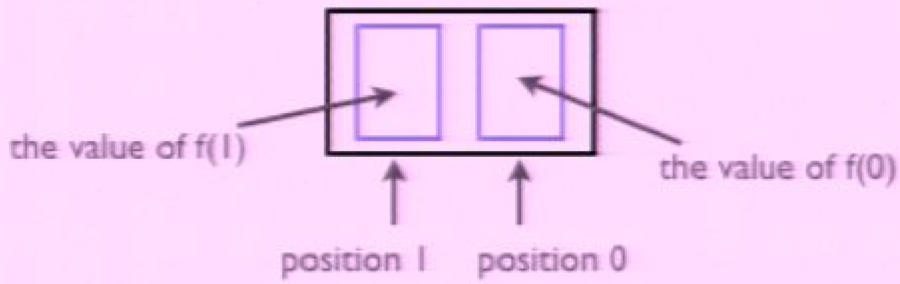

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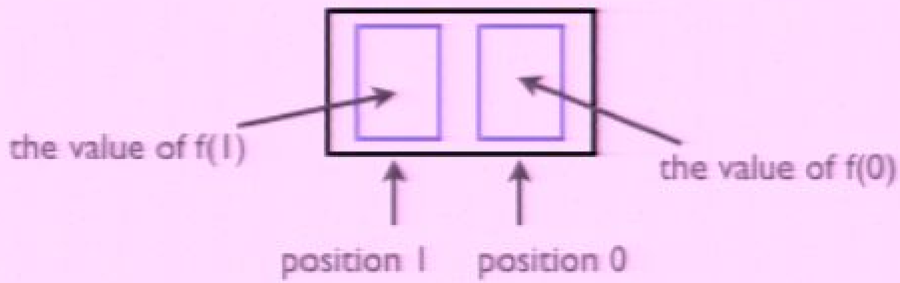

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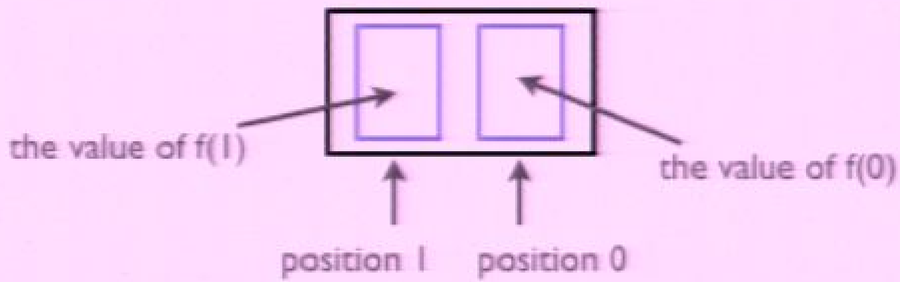

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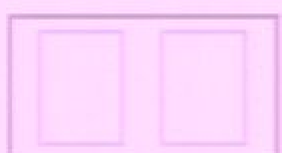

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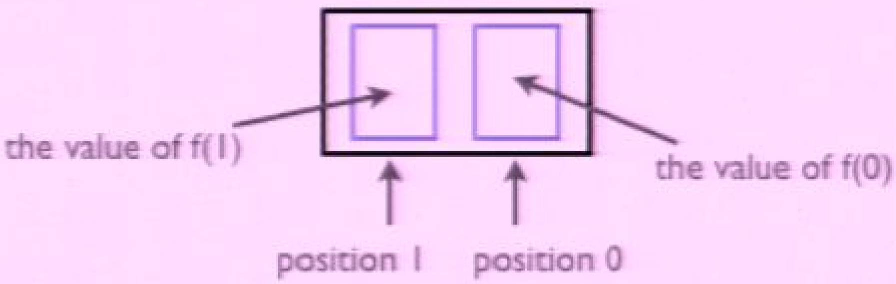

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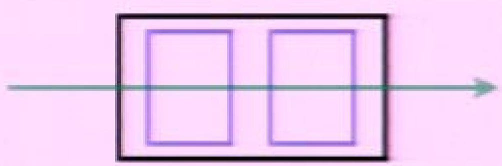

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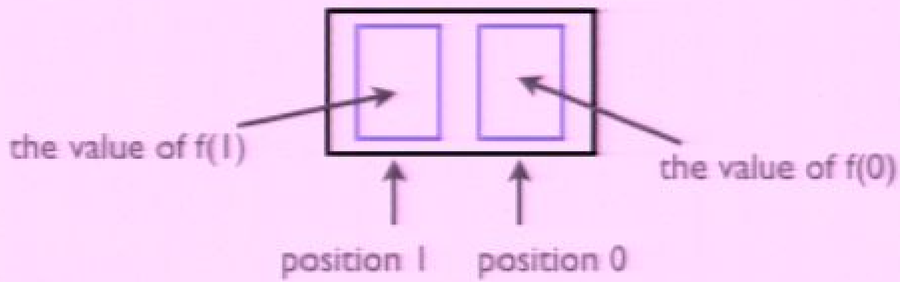

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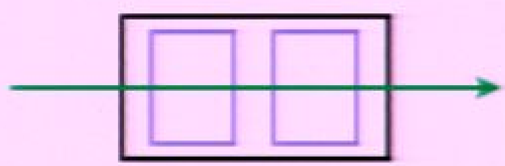

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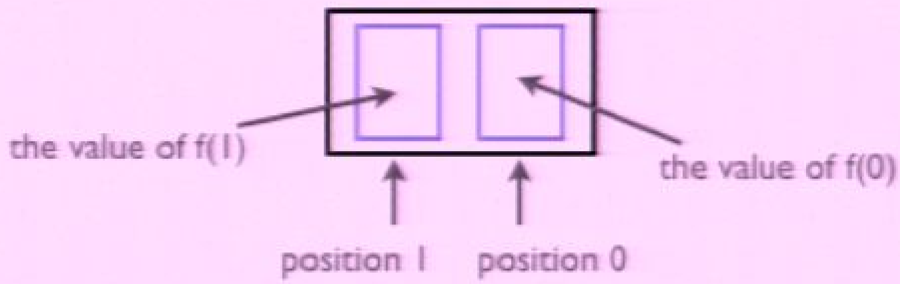

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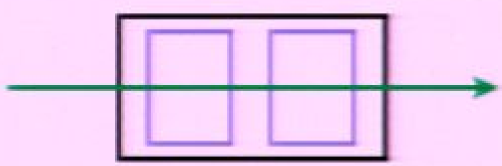

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Mathematical undecidability (Chaitin) and physical randomness

A quantum system encodes axioms.
Quantum measurement checks truth values of certain propositions.
Randomness if the proposition is undecidable in the set of axioms defined in the state.

Loophole-free test of Leggett's models

For many qubits it should be that visibility requirement goes down
and one could perform experimental test with ions.

Bell's theorem

No local realistic explanation of all quantum predictions.

Leggett's theorem

A plausible class of *non-local* realistic models conflicts quantum predictions.

Information approach

A qubit carries one bit of information.

IQI

CONCLUSIONS

Bell's theorem

No local realistic explanation of all quantum predictions.

Leggett's theorem

A plausible class of *non-local* realistic models conflicts quantum predictions.

Information approach

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Thank you!

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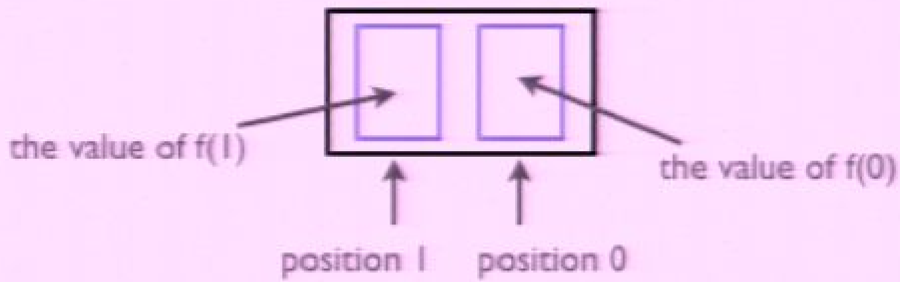

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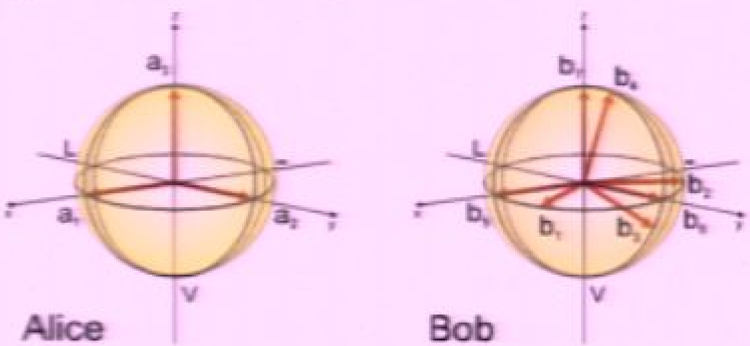
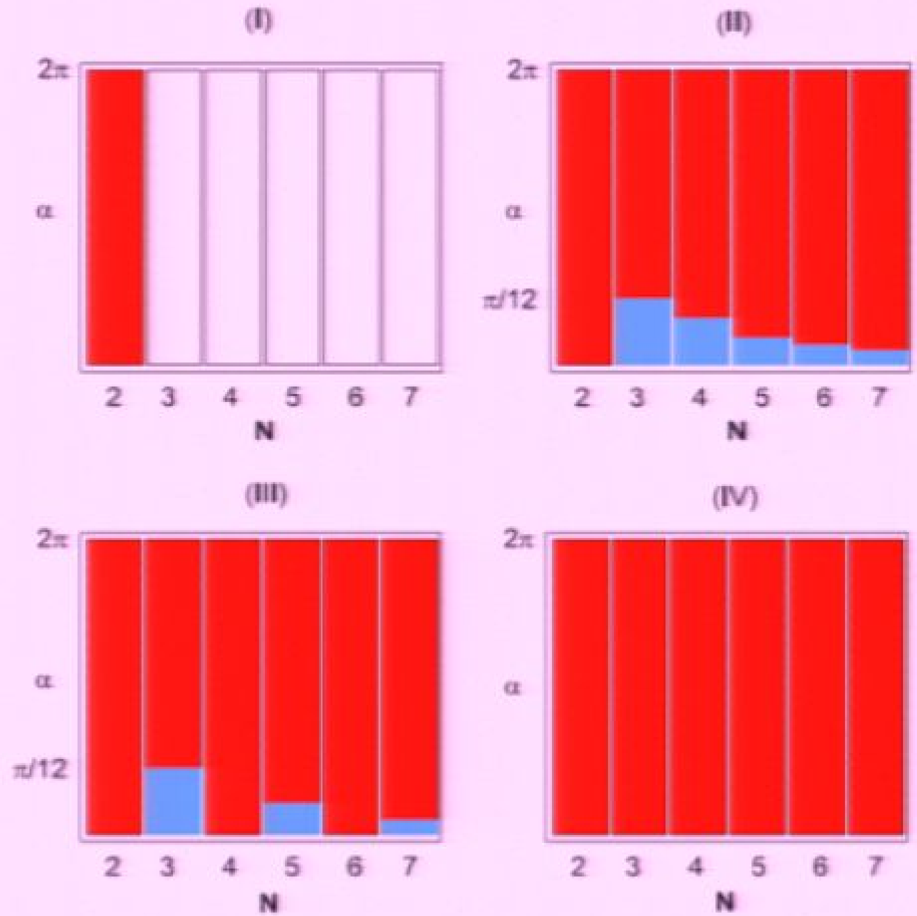
Experimental requirements



$V_{min} = 98.4\ \%$

ENTANGLED AND CLASSICAL?

$$|\psi\rangle = \cos\alpha|0...0\rangle + \sin\alpha|1...1\rangle$$



✓ K. Nagata, W. Laskowski, and TP, Phys. Rev. A **74**, 62109 (2006)
✓ TP, W. Laskowski, and M. Zukowski, Mod. Phys. Lett A **21**, 111 (2006)
✓ W. Laskowski, TP, M. Zukowski, and C. Brukner, Phys. Rev. Lett. **93**, 200401 (2004)

Assumptions

- **Realism**: unperformed measurements have well-defined, yet unknown, results
- **Locality**: distant systems are independent

PROOF

$A(k,x)=\pm 1$ $\qquad$ $B(l,x)=\pm 1$

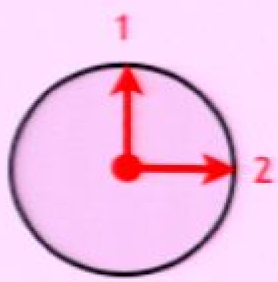
- For a given x (in a single run):

$$A(1,x)[B(1,x) + B(2,x)] + A(2,x)[B(1,x) - B(2,x)] = \pm 2$$

- Averaged over x (in many runs):

$$|E_{11} + E_{12} + E_{21} - E_{22}| \leq 2$$

- Quantum correlations of the Bell singlet state:



$$E_{kl} = -\,\mathbf{a}\cdot\mathbf{b}$$