Title: Relating Entanglement to Quantum Communication

Date: Dec 12, 2007 04:00 PM

URL: http://pirsa.org/07120015

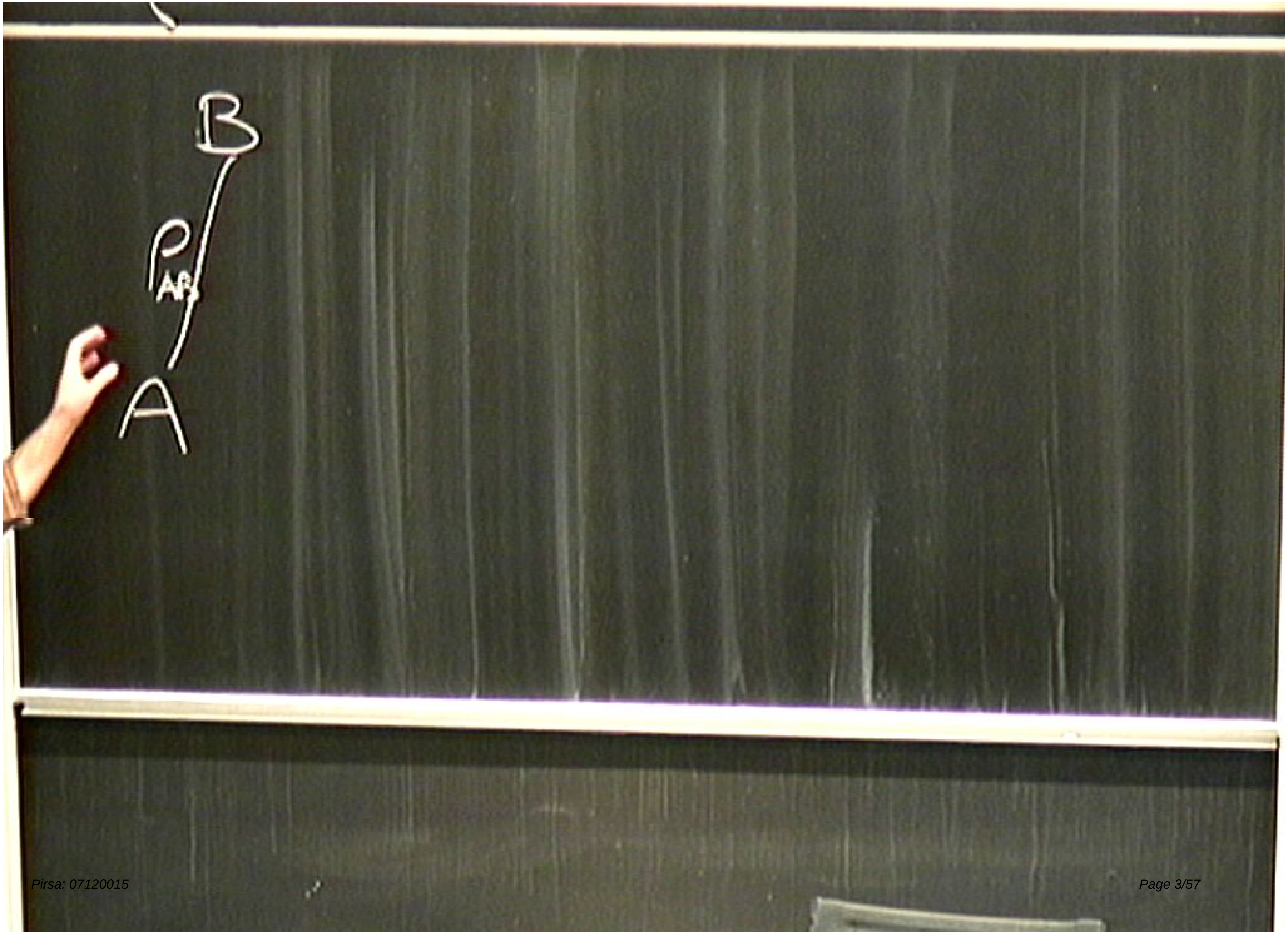
Abstract: Roughly speaking, the more Alice is entangled with Bob, the harder it is for her to send her state to Charlie. In particular, it will be shown that the squashed entanglement, a well known entanglement measure, gives the fastest rate at which a quantum state can be sent between two parties

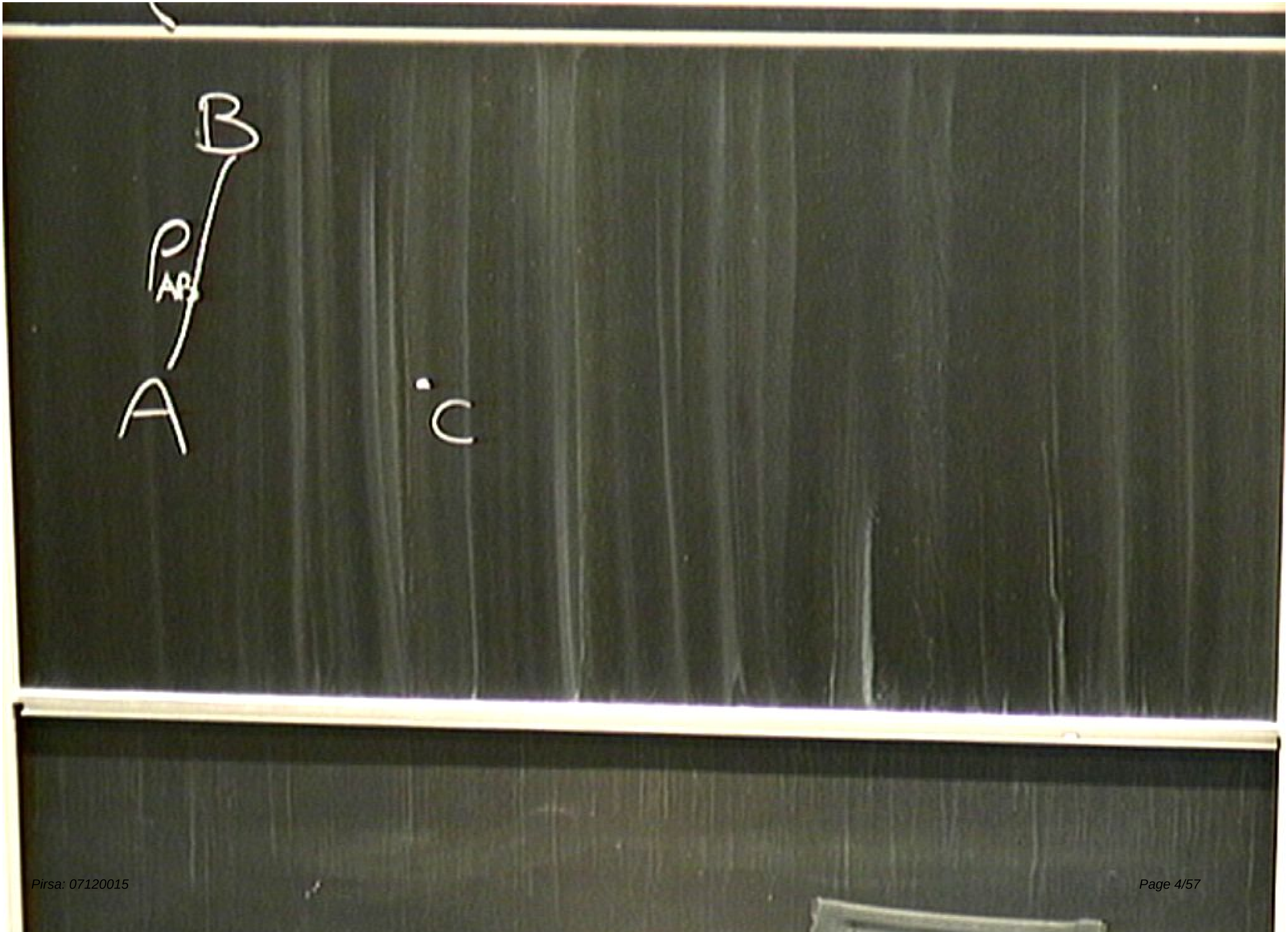
who share arbitrary side information. Likewise, the entanglement of
formation and entanglement cost is shown to be the fastest rate at which a quantum state can be sent when the parties have access to side-information which is maximally correlated. A further restriction on the type of side-information implies that the rate of state transmission is given by
the quantum mutual information. This suggests a new paradigm for understanding entanglement and other correlations in terms of quantum Shannon theroy. Different types of side-information correspond to
different types of correlations with the squashed entanglement and the mutual information being two extremes. Furthermore, there is a dual paradigm: if one distributes the side-information as aliciously as possible so as to make the sending of the state as difficult as possible,
one finds maximum rates which give interpretations to known quantities as well as new ones.

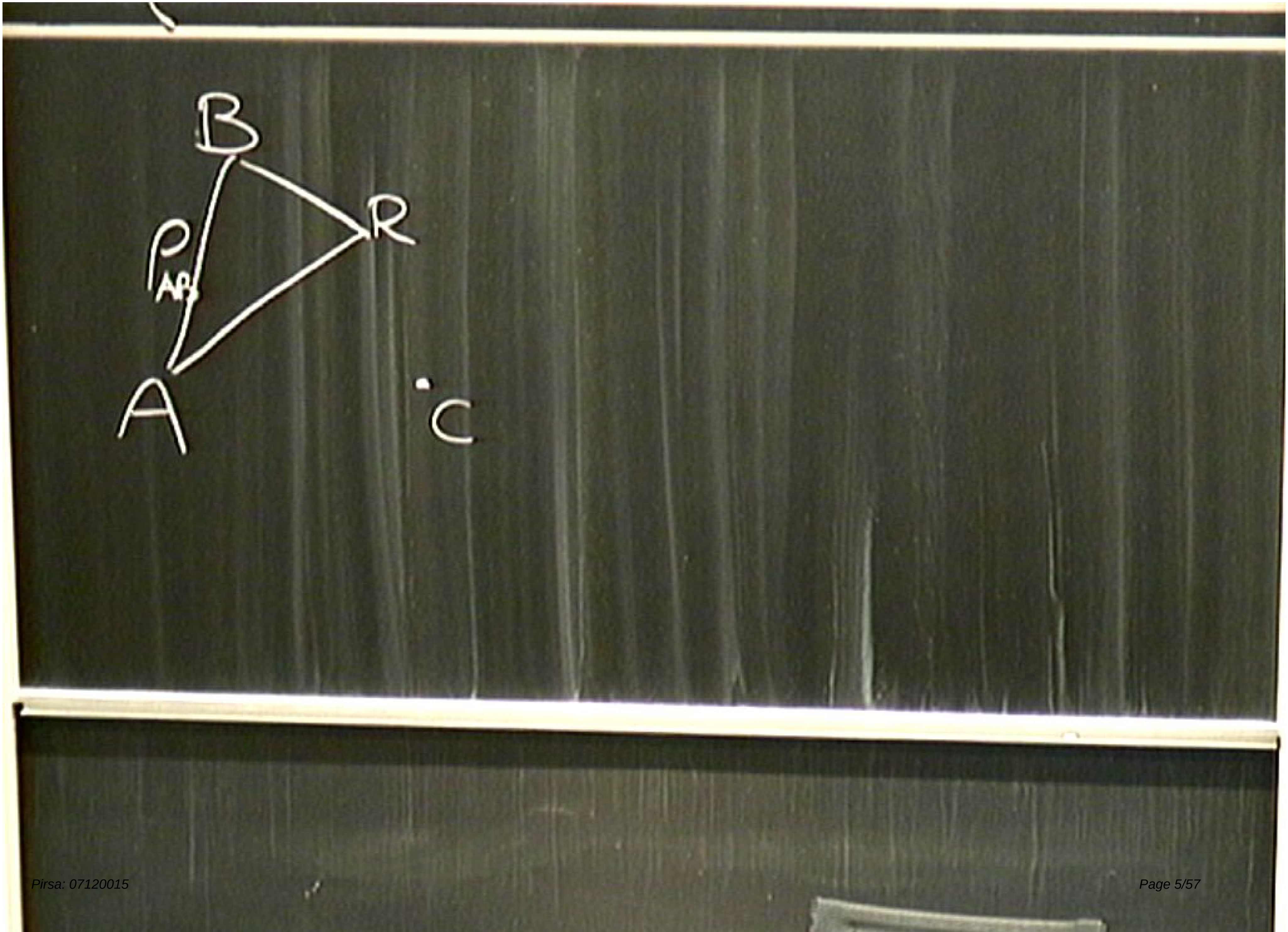
Outline

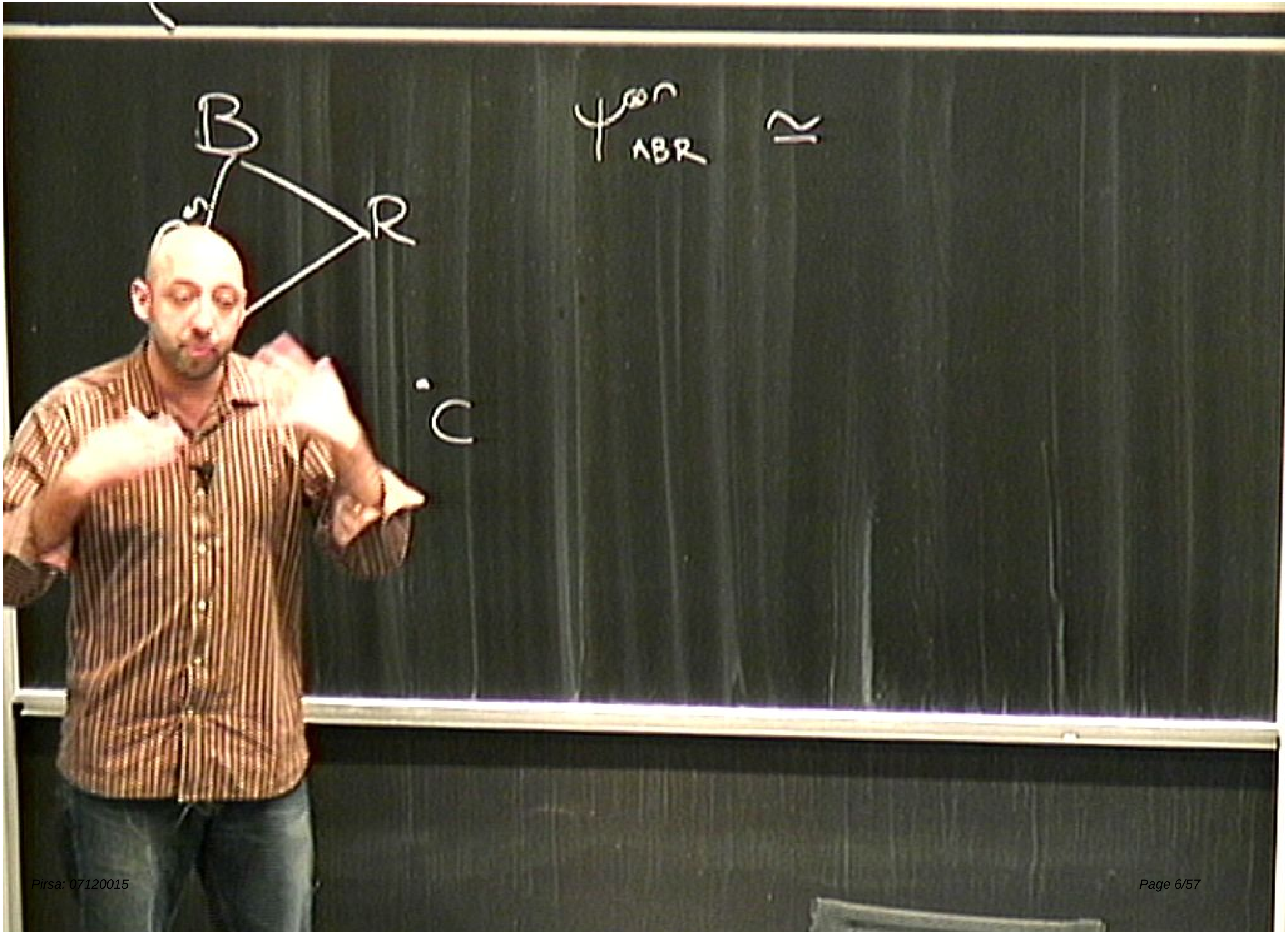
Quantum Shannon Theory vs. Entanglement

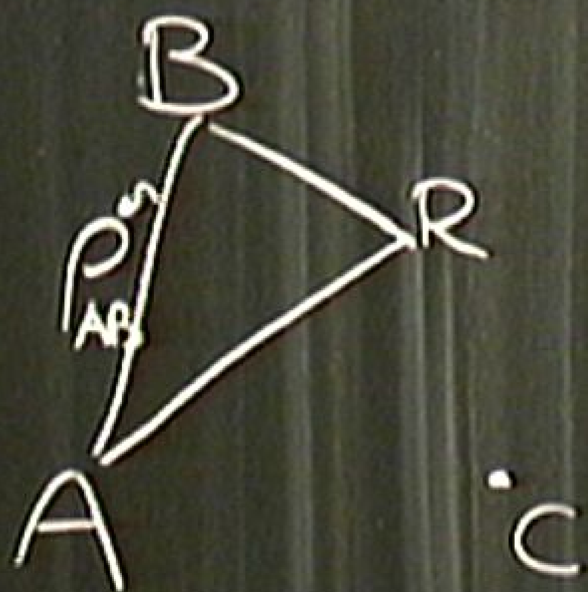
1. Half-hearted attempt to motivate research
2. Vague statements
3. Slightly more precise statements
4. Sloppy proofs
5. Unanswered questions


B
ρ_{AB}
A


B
ρ_{AB}
A
C


B
R
ρ_{AB}
A
C


B
R
C
$\psi^{\otimes n}_{ABR} \cong$



$$\psi^{\otimes n}_{ABR} \simeq \psi^{(n)}_{XR}$$

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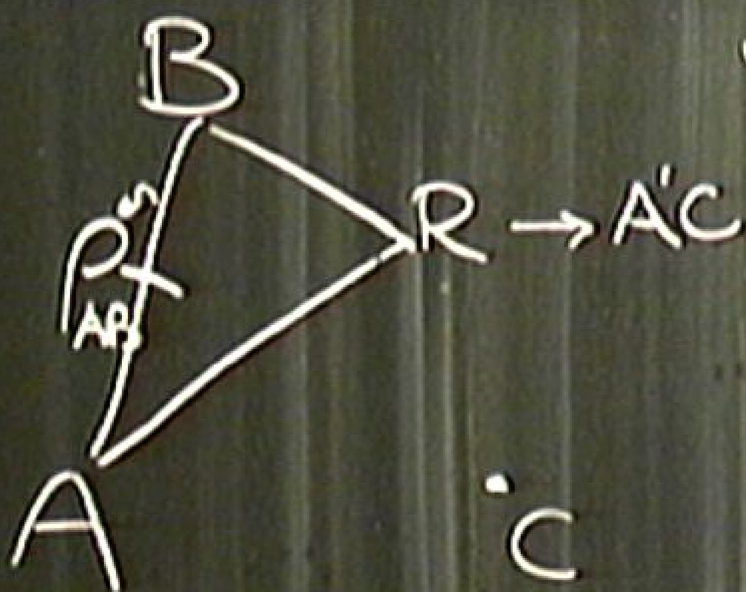
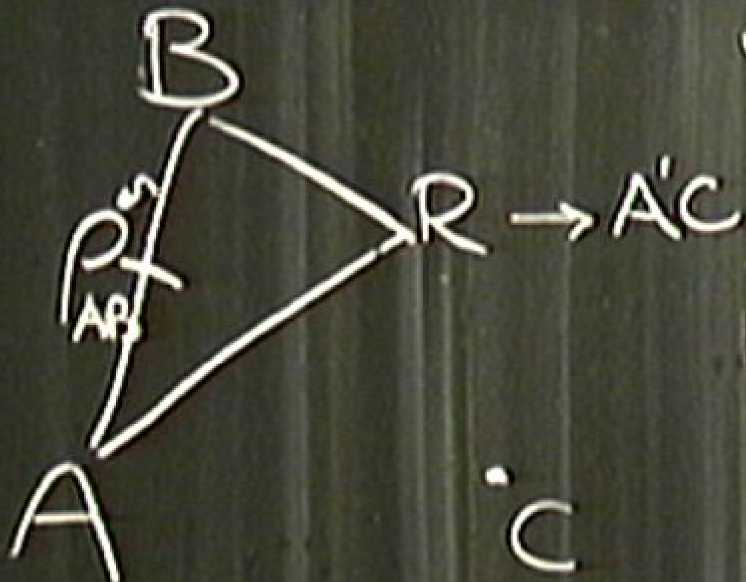
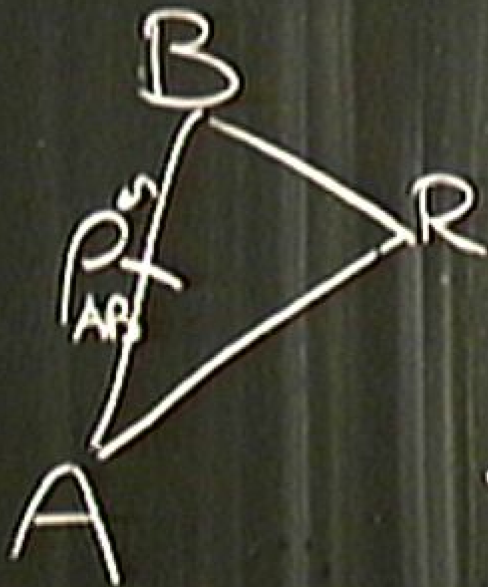




$$\Psi^{\otimes n}_{ABR} \simeq \Psi^{(n)}_{XR}$$

$$E_{sq} = \frac{1}{2} I(A:B|$$

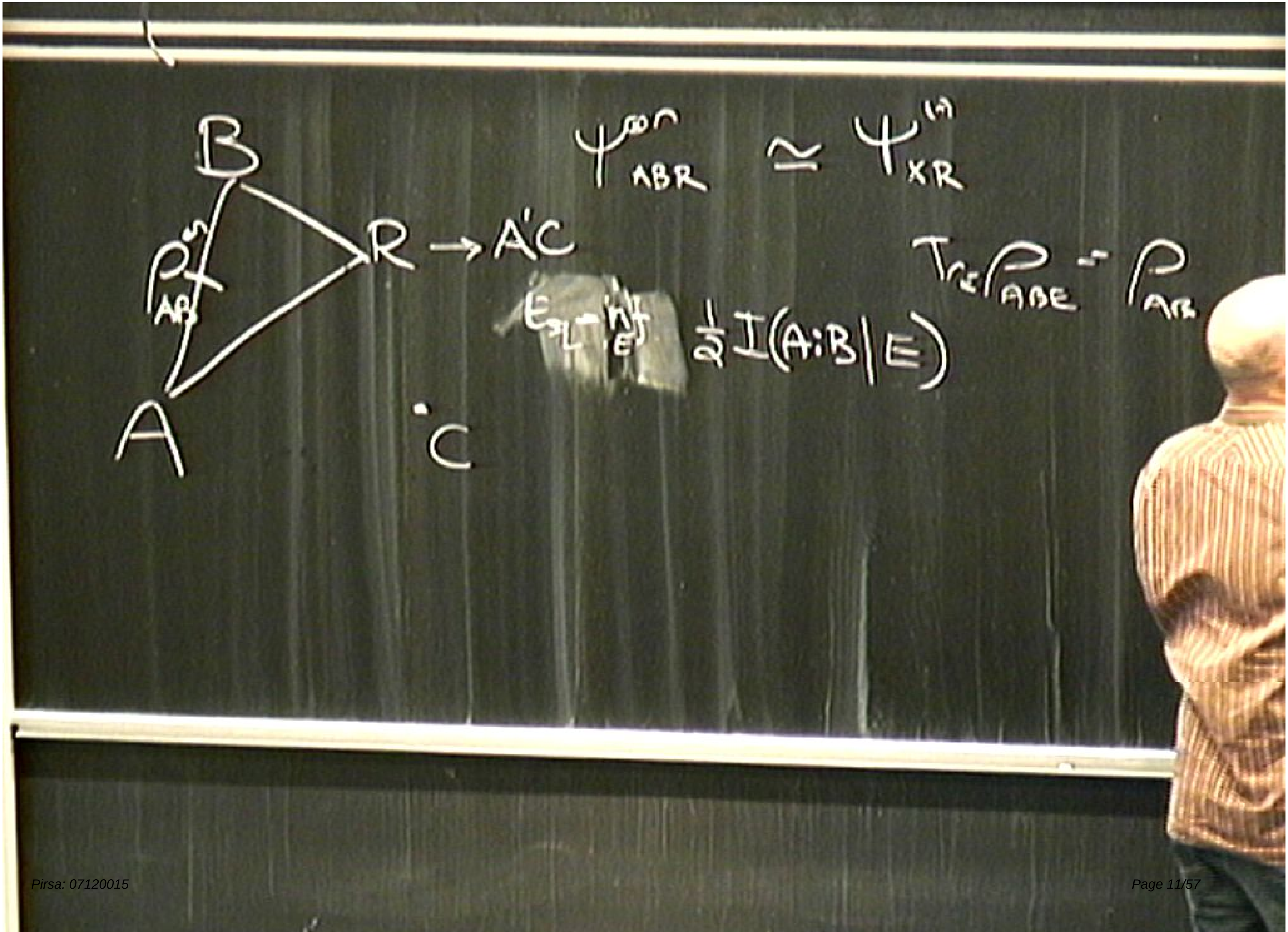


$$\Psi^{\otimes n}_{ABR} \simeq \Psi^{(n)}_{XR}$$

$$R \to A'C$$

$$\mathrm{Tr}_E \, \rho_{ABE} = \rho_{AB}$$

$$E_{sq} = \inf_E \frac{1}{2} I(A:B|E)$$



B
$\Psi^{\otimes n}_{ABR} \simeq \Psi^{(n)}_{XR}$
$\rho^{\otimes n}_{AB}$
$R \rightarrow A'C$
$\mathrm{Tr}_E \rho_{ABE} = \rho_{AB}$
$\frac{1}{2} I(A:B|E)$
A
C

Side Information

$\sigma_{A'C} \in S$

Side Information $\sigma_{A'C} \in S$	Best rate $A \to C$ $\overline{Q}$

Side Information

$\sigma_{A'C} \in \mathcal{S}$

$\mathcal{S}$ = all

Best rate

$A \to C$

$\overline{Q}$

E_{sq}

Side Information

$\sigma_{A'C} \in \mathcal{S}$

Best rate

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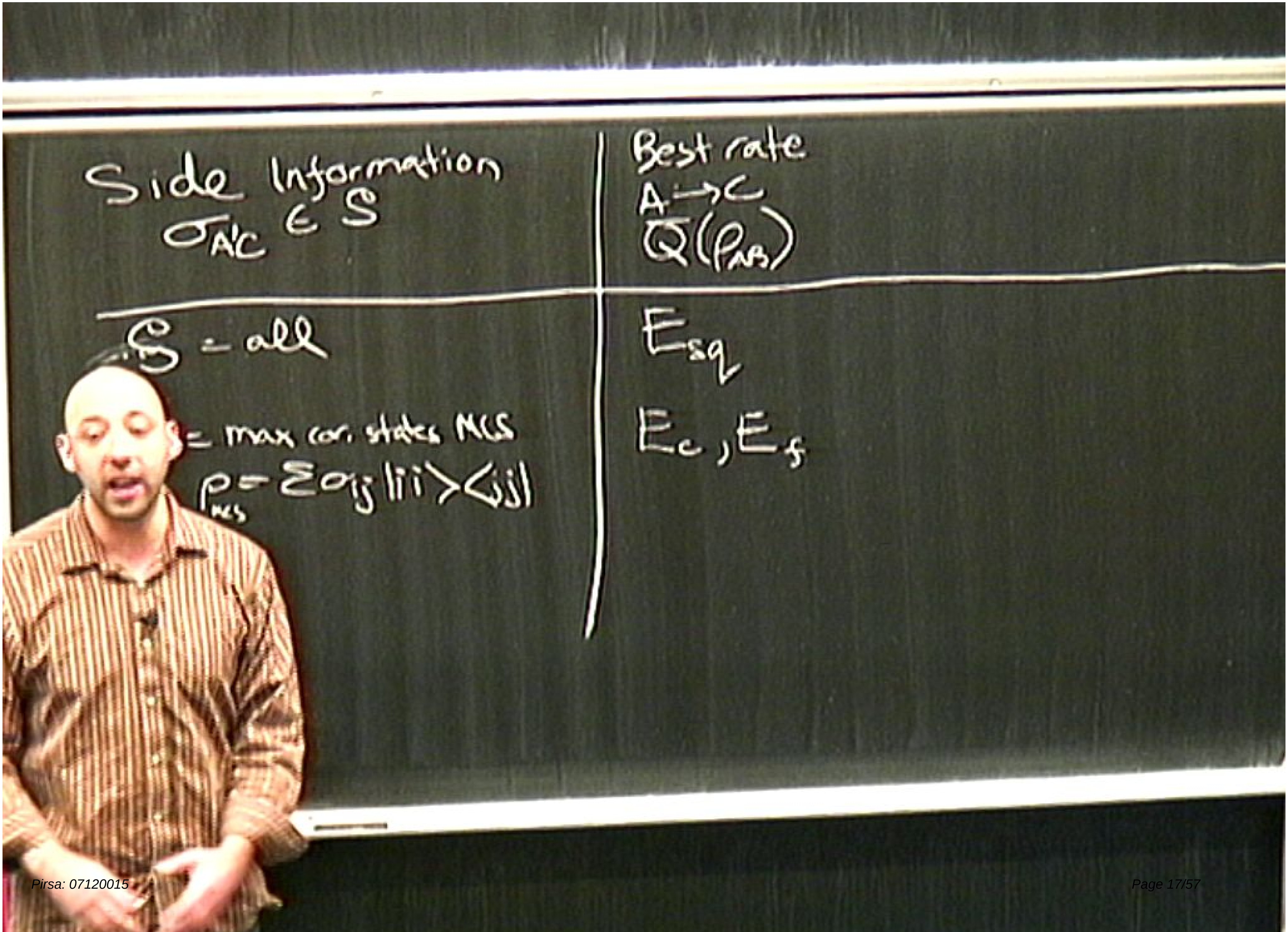
$\overline{Q}$

$\mathcal{S}$ = all

E_{sq}

$\mathcal{S}$ = max cor. states MCS

Side Information $\sigma_{A'C} \in \mathcal{S}$	Best rate $A \to C$ $\overline{Q}$
$\mathcal{S}$ = all	E_{sq}
$\mathcal{S}$ = max cor. states MCS $\rho_{MCS} = \sum a_{ij} \vert ii\rangle\langle jj\vert$	E_c, E_f


Side Information
σA'C ∈ S
Best rate
A → C
Q(ρAB)
S = all
Esq
= max cor. states MCS
ρMCS = Σ αij |ii⟩⟨jj|
Ec , Ef

Side Information $\sigma_{A'C} \in S$	Best rate $A \to C$ $\overline{Q}(\rho_{AB})$
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$\mathcal{S}$ = null on A or null on C	$\frac{1}{2} I(A:B)$

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Best rate $A \to C$

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S = all	E_{sq}
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S = null on A or null on C	$\frac{1}{2} I(A:B)$
S = null	

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$\mathcal{S}$ = null on A or null on C	$\frac{1}{2} I(A:B)$
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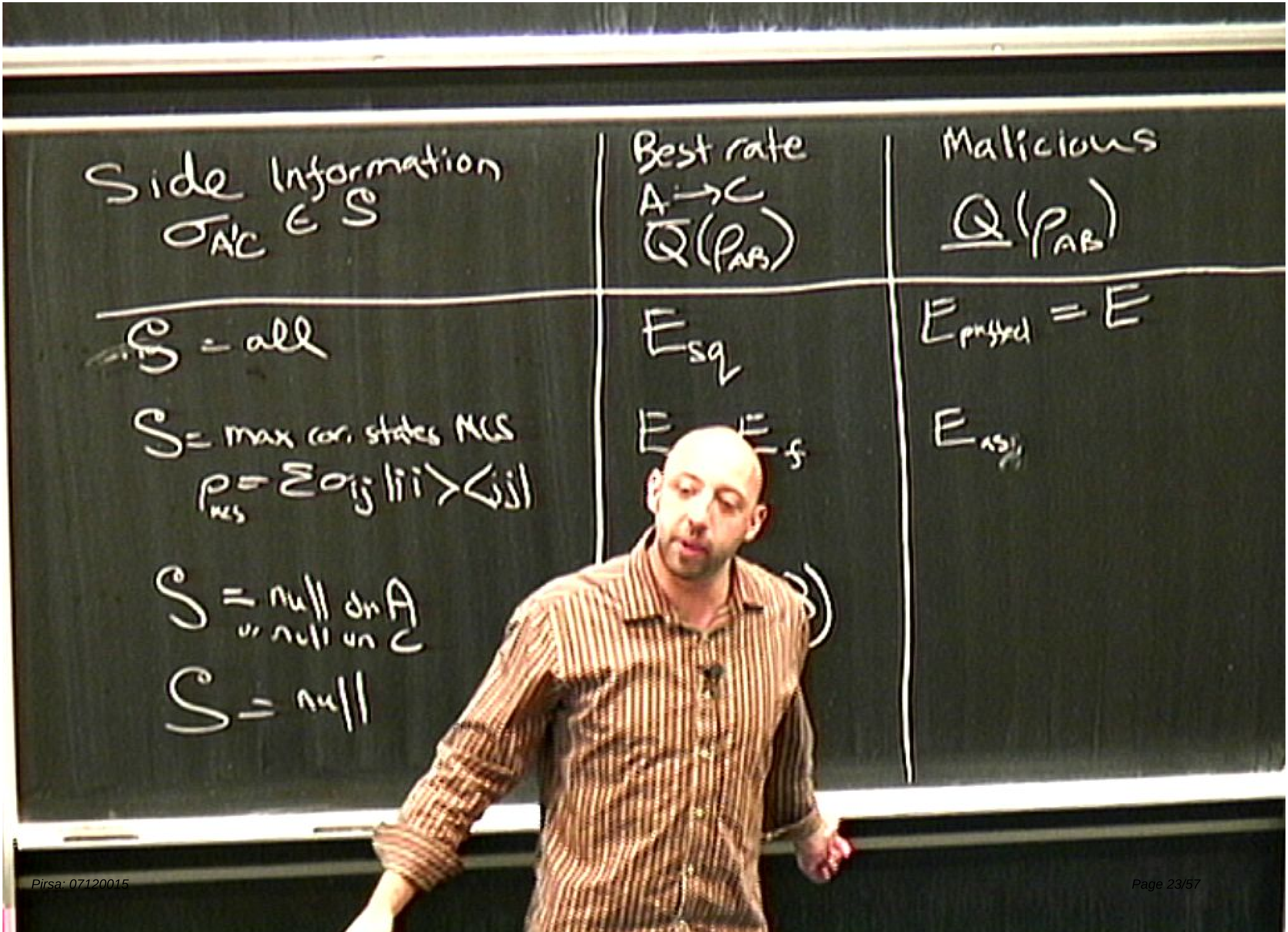
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Ef
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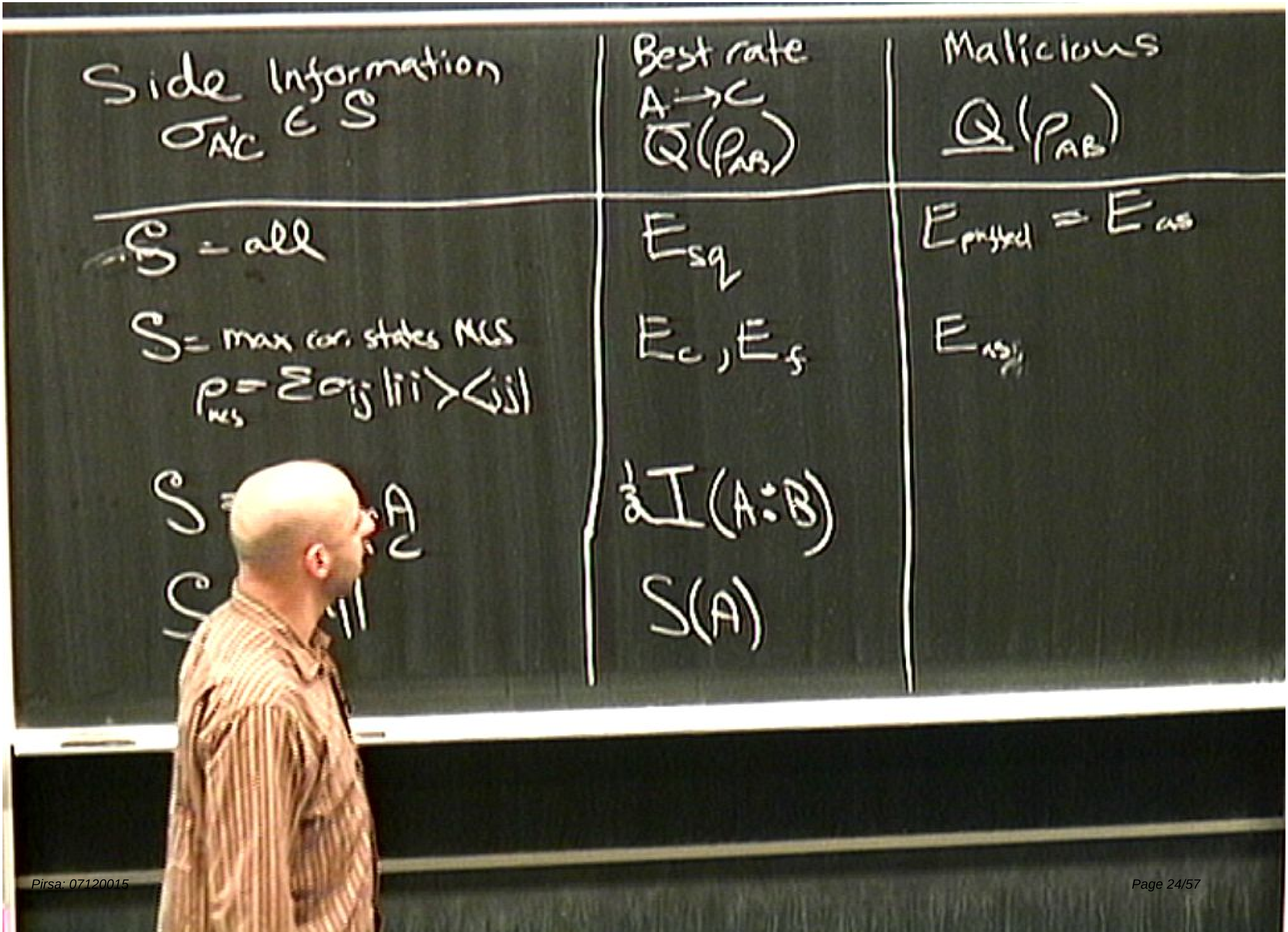
Side Information
σ_{A'C} ∈ S
Best rate
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Q̄(ρ_AB)
Malicious
Q(ρ_AB)
S = all
E_sq
E_pristed = E_as
S = max cor. states MCS
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E_c , E_f
E_asq
½ I(A:B)
S(A)

State ... $\sigma_{A'C} \in S$	$A \to C$ $\overline{Q}(\rho_{AB})$	$\underline{Q}(\rho_{AB})$
S = all	E_{sq}	$E_{prsqd} = E_{as}$
S = max cor. states MCS $\rho_{MCS} = \sum \alpha_{ij} \lvert ii \rangle\langle jj \rvert$	E_c, E_f	E_{as}
S = null on A or null on C	$\frac{1}{2} I(A:B)$	$\frac{1}{2} I(A:B)$
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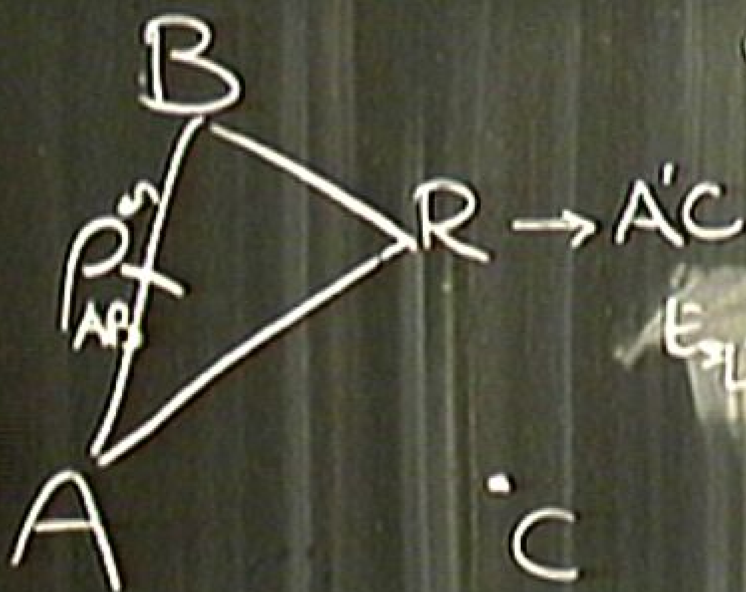
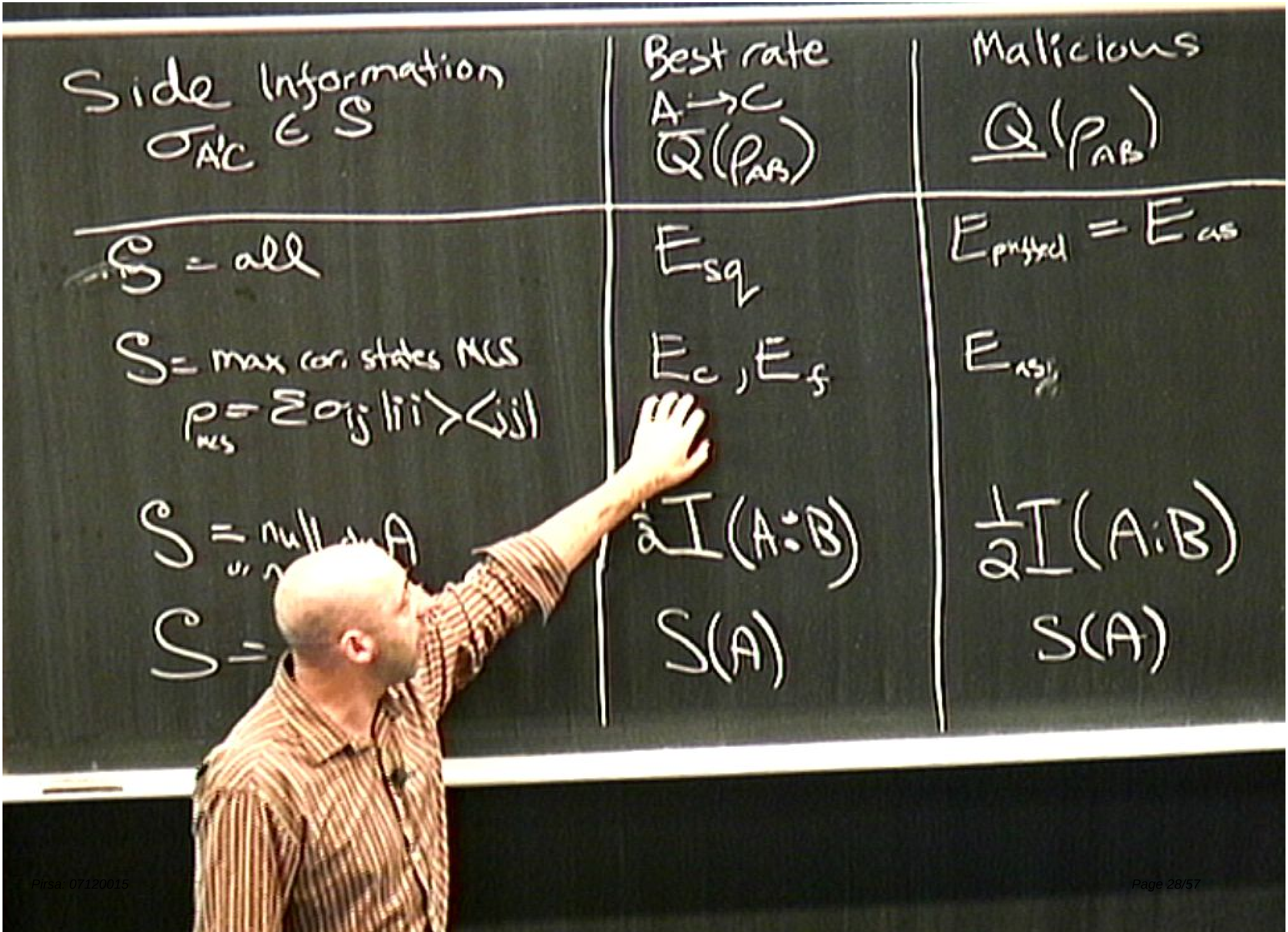


$$\Psi^{\otimes n}_{ABR} \simeq \Psi^{(n)}_{XR}$$

$$\mathrm{Tr}_E \rho_{ABE} = \rho_{AB}$$

$$E_{sq} = \inf_E \frac{1}{2} I(A:B|E)$$

Side Information	Best rate	Malicious
$\sigma_{A'C} \in \mathcal{S}$	$A \to C$ $\overline{Q}(\rho_{AB})$	$\underline{Q}(\rho_{AB})$
$\mathcal{S}$ = all	E_{sq}	$E_{\text{pnfyd}} = E_{cs}$
$\mathcal{S}$ = max cor. states MCS $\rho_{MCS} = \sum \alpha_{ij} \lvert ii\rangle\langle jj\rvert$	E_c, E_f	E_{rsq}
$\mathcal{S}$ = null on A or null on C	$\frac{1}{2} I(A:B)$	$\frac{1}{2} I(A:B)$
$\mathcal{S}$ = null	$S(A)$	$S(A)$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$\mathcal{S}$ = null	$S(A)$	$S(A)$

$$E = S(A|C) - \bar{Q}$$

$$\bar{E} \sup_C S(A|C) - \bar{Q}$$

$\overline{E} \sup_{C} S(A|C) - \overline{Q}$

<u>Thm)</u> If S

$E = S(A|C) - \overline{Q}$

<u>Thm</u>) If $\sigma_0, \sigma_1 \in \mathcal{S}$

$\forall$ purifications $|\psi^0\rangle, |\psi^1\rangle$

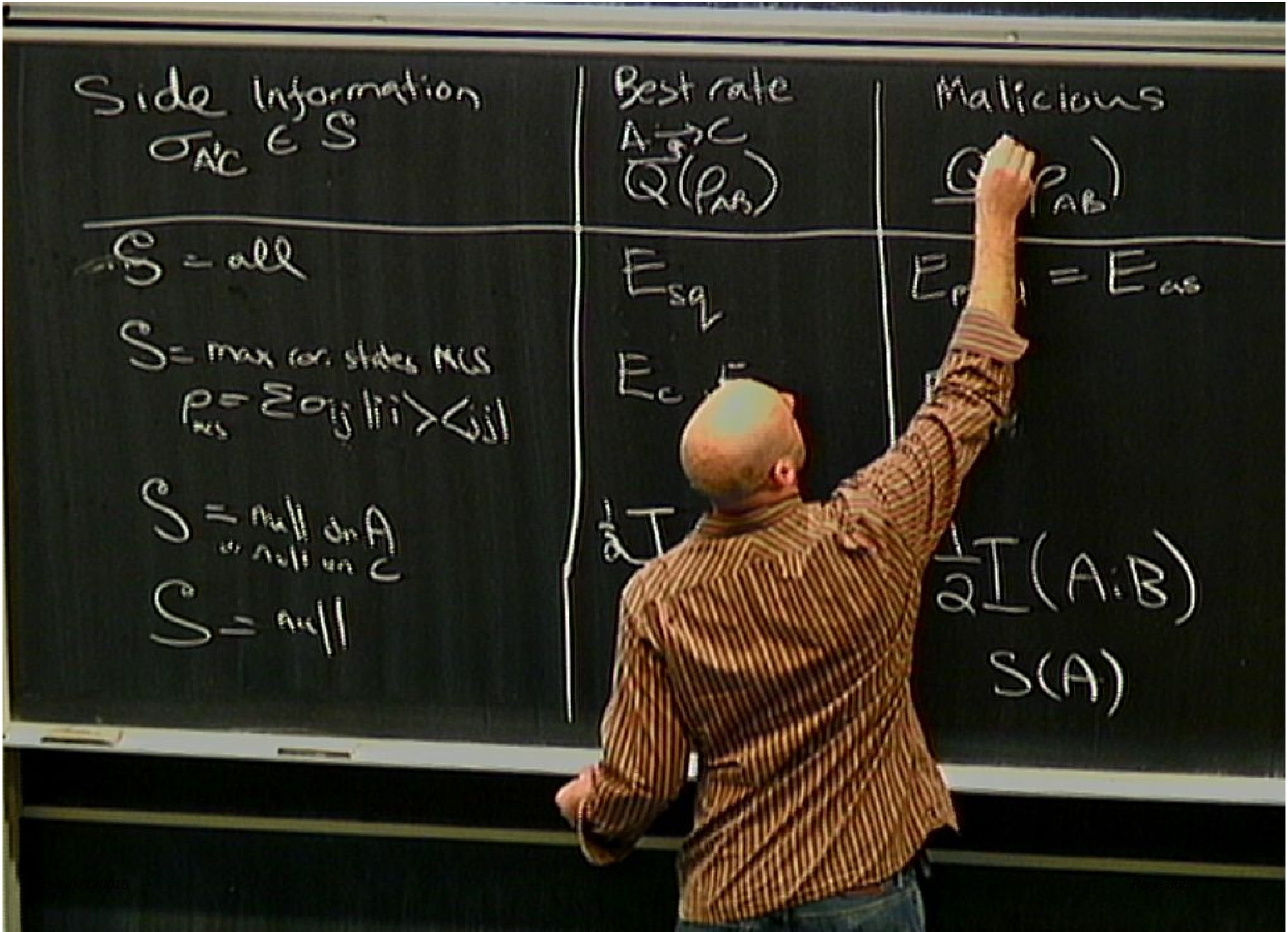
$$E = \sup_C S(A|C) - \overline{Q}$$

Thm) If $\sigma_0, \sigma_1 \in S$

$\forall$ purifications $|\psi^0\rangle, |\psi^1\rangle$

$$\left[|\psi^0\rangle|00\rangle + |\psi^1\rangle|11\rangle\right)' \in S$$

then $\overline{Q}^S$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½I
S = null
½I(A:B)
S(A)

Side Information $\sigma_{A'C} \in \mathcal{S}$	Best rate $A \xrightarrow{\mathcal{S}} C$ $\overline{Q}(\rho_{AB})$	Malicious $\underline{Q}^{\mathcal{S}}(\rho_{AB})$
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$\mathcal{S}$ = null on A or null on C	$\frac{1}{2} I(A:B)$	$\frac{1}{2} I(A:B)$
$\mathcal{S}$ = null	$S(A)$	$S(A)$

$\bar{E} = \sup_{C} S(A|C) - \bar{Q}$

Thm) If $\sigma_0, \sigma_1 \in S$

if $\forall$ purifications $|\psi^0\rangle, |\psi^1\rangle$

$[|\psi^0\rangle|00\rangle + |\psi^1\rangle|11\rangle)' \in S$

then $\bar{Q}^S$ is an entanglement measure

$\overline{E} = \sup_C S(A|C) - \overline{Q}$

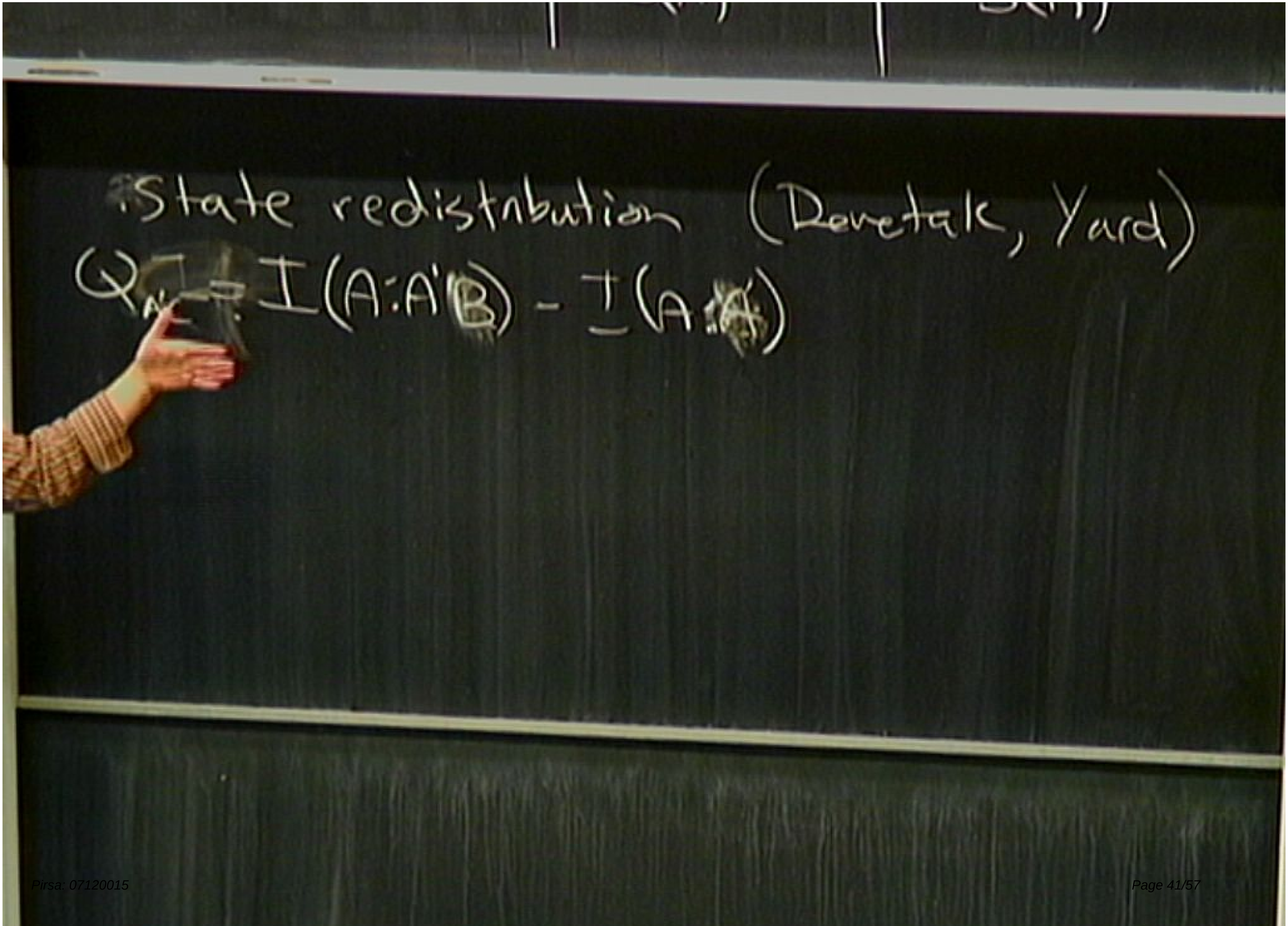
Thm) If $\sigma_0, \sigma_1 \in S$

if $\forall$ purifications $|\psi^0\rangle, |\psi^1\rangle$

$\mathrm{Tr}\left[(|\psi^0\rangle|00\rangle + |\psi^1\rangle|11\rangle)\right] \in S$

then $\overline{Q}^S$ is an entanglement measure

• State redistribution (Devetak, Yard)

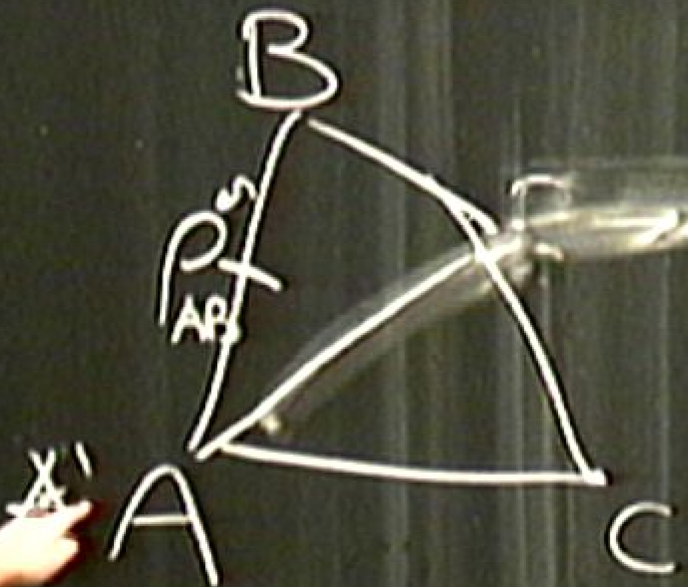

State redistribution (Devetak, Yard)
Q = I(A:A'B) - I(A:A')

State redistribution (Devetak, Yard)

$Q_{NC} = I(A:A'B) - I(A:A')$

State redistribution (Devetak, Yard)

$$Q_{AC} = I(A:A'B) - I(A:A')$$



$$\Psi^{\otimes n}_{ABR} \simeq \Psi^{(n)}_{XR}$$

$$\mathrm{Tr}_E \, \rho_{ABE} = \rho_{AB}$$

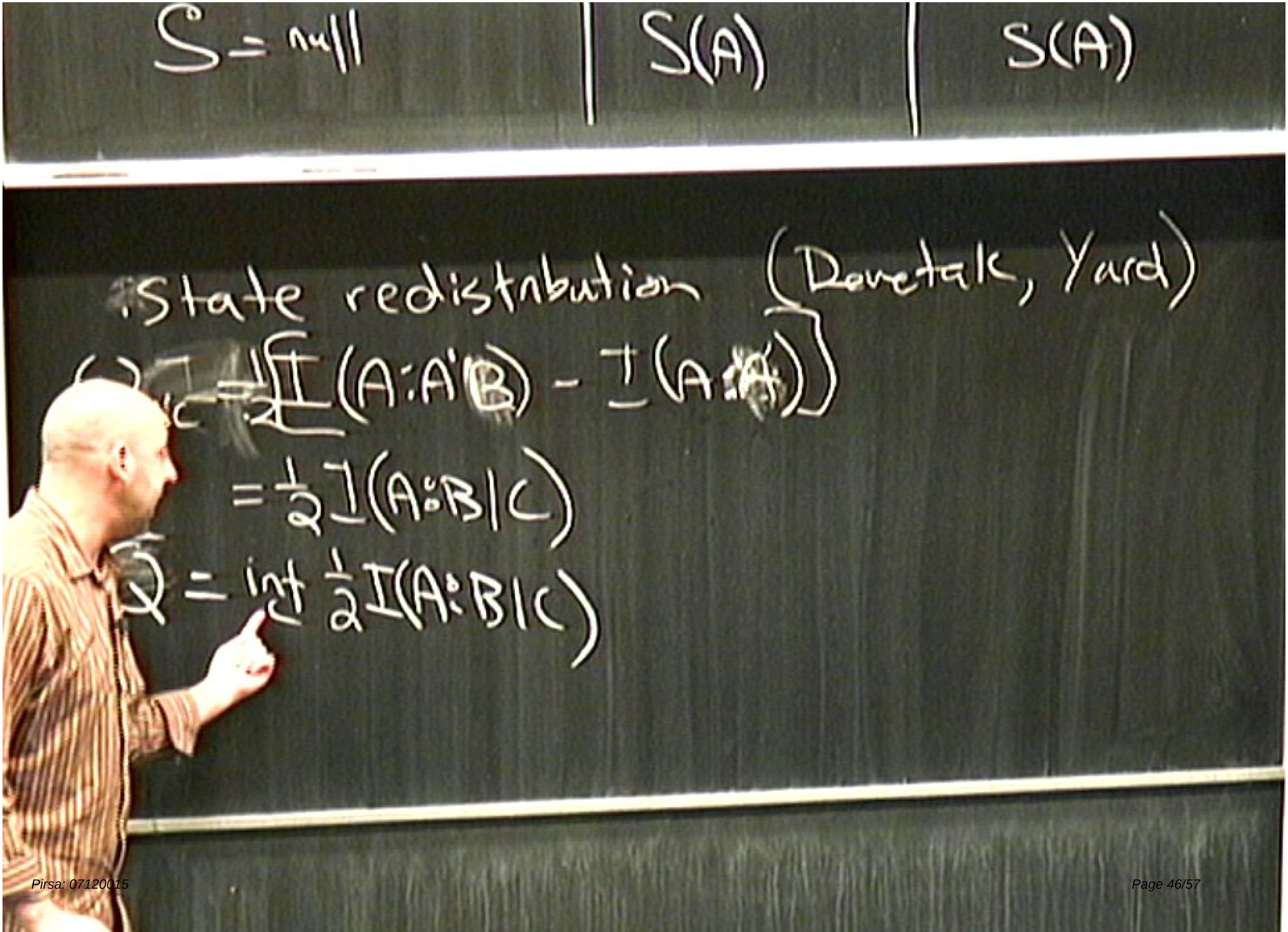
$$E(\rho_{AB}) = \sup_{E} \frac{1}{2} I(A;B|E)$$

$S = \text{null}$ | $S(A)$ | $S(A)$

State redistribution (Devetak, Yard)

$$Q_{AC} = \frac{1}{2}\left\{ I(A:A'B) - I(A:A') \right\}$$

$$= \frac{1}{2} I(A:B|C)$$


S = null
S(A)
S(A)
State redistribution (Devetak, Yard)
$Q_C = \frac{1}{2}\{I(A:A'B) - I(A:A')\}$
$= \frac{1}{2} I(A:B|C)$
$Q = \inf \frac{1}{2} I(A:B|C)$

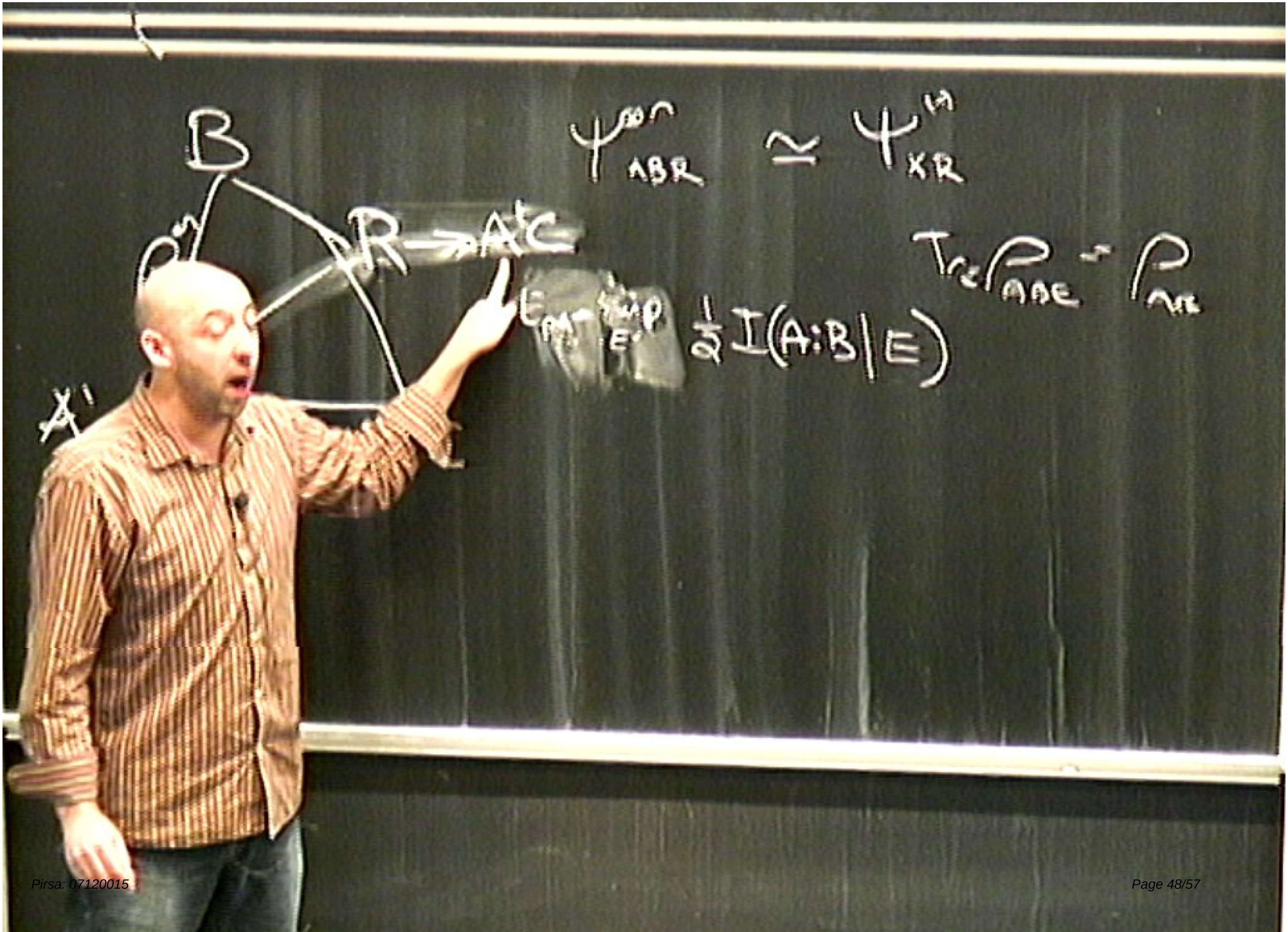
$S =$ null | $S(A)$ | $S(A)$

State redistribution (Devetak, Yard)

$$Q_{AC} = \frac{1}{2}\left[I(A:A'B) - I(A:A') \right]$$

$$= \frac{1}{2} I(A:B|C)$$

$$\overline{Q} = \inf \frac{1}{2} I(A:B|C)$$


B
R → A'C
Tr_E ρ_ABE = ρ_AB
½ I(A:B|E)

$S = \text{null}$ | $S(A)$ | $S(A)$

State redistribution (Devetak, Yard)

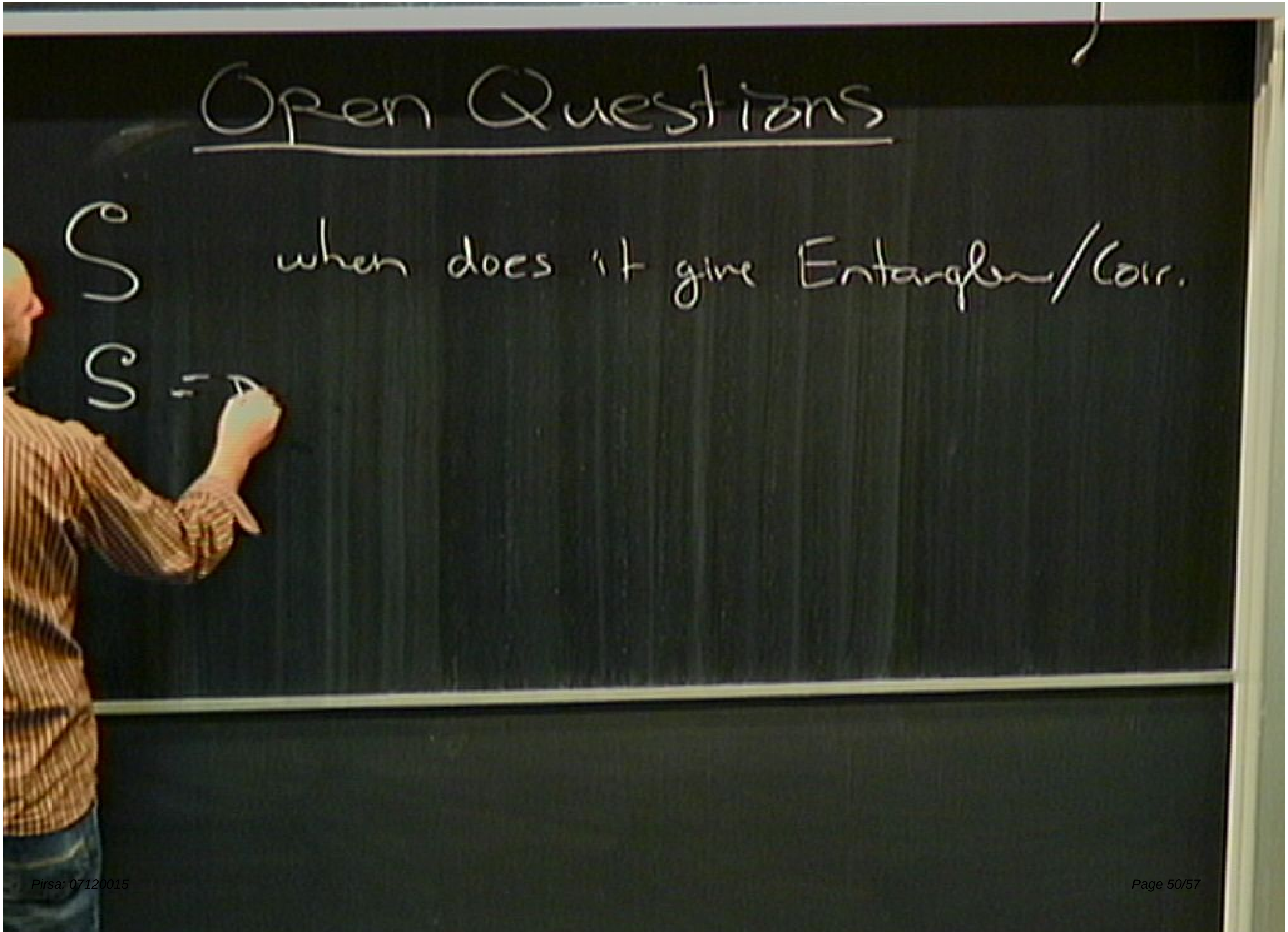
$$Q_{AC} = \frac{1}{2}\left[I(A:A'B) - I(A:A') \right]$$

$$= \frac{1}{2} I(A:B|C)$$

$$\overline{Q} = \inf_{C} \frac{1}{2} I(A:B|C)$$

$$= \inf_{\Lambda} \frac{1}{2} I(A:B|R)$$

$$= E_{sq}$$


Open Questions
S when does it give Entanglem/Corr.
S =

Open Questions

- $\mathcal{S}$ when does it give Entanglm/Corr.
- S = ppt, sep . . .

Open Questions

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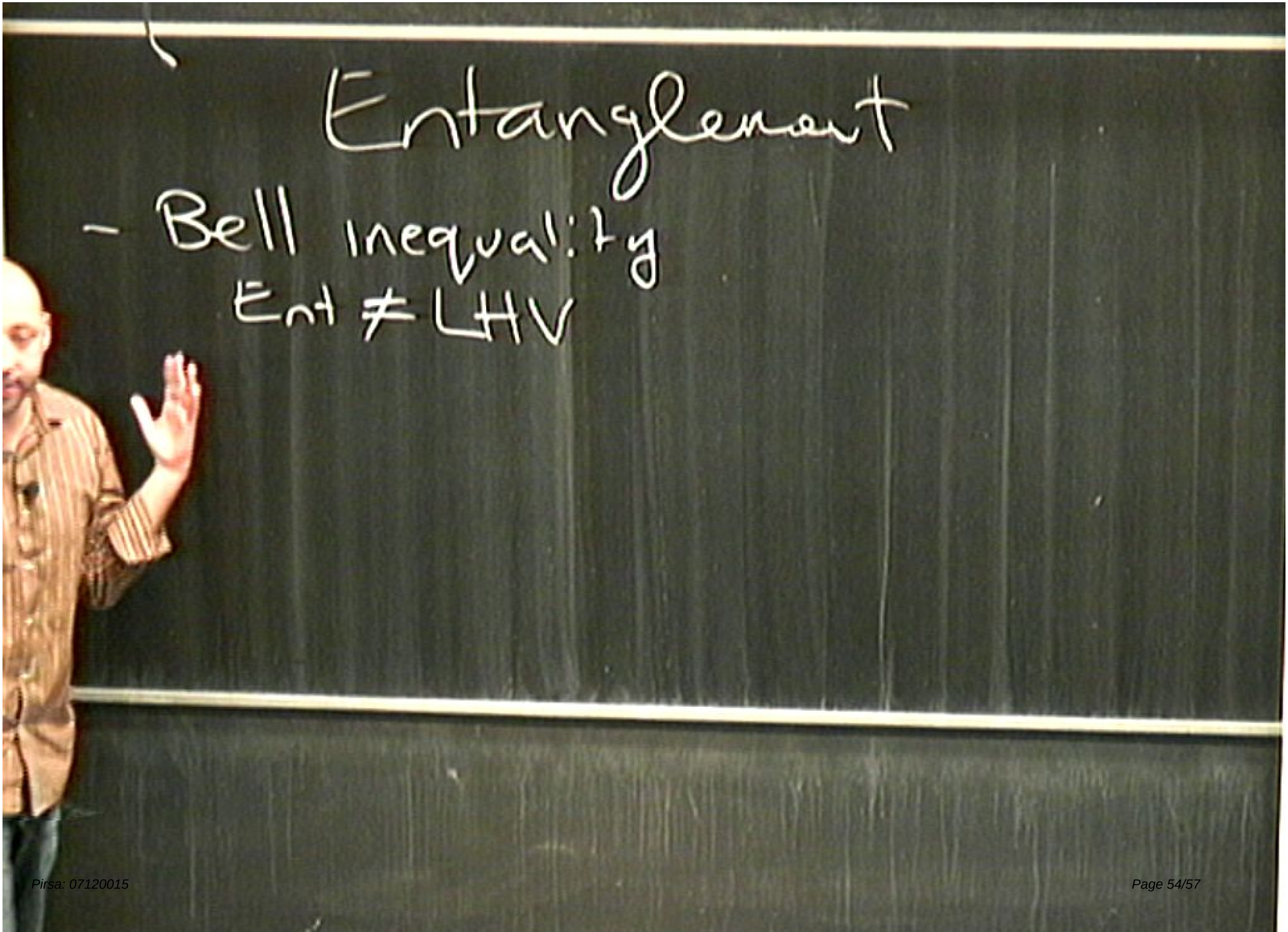
- additivity

Open Questions

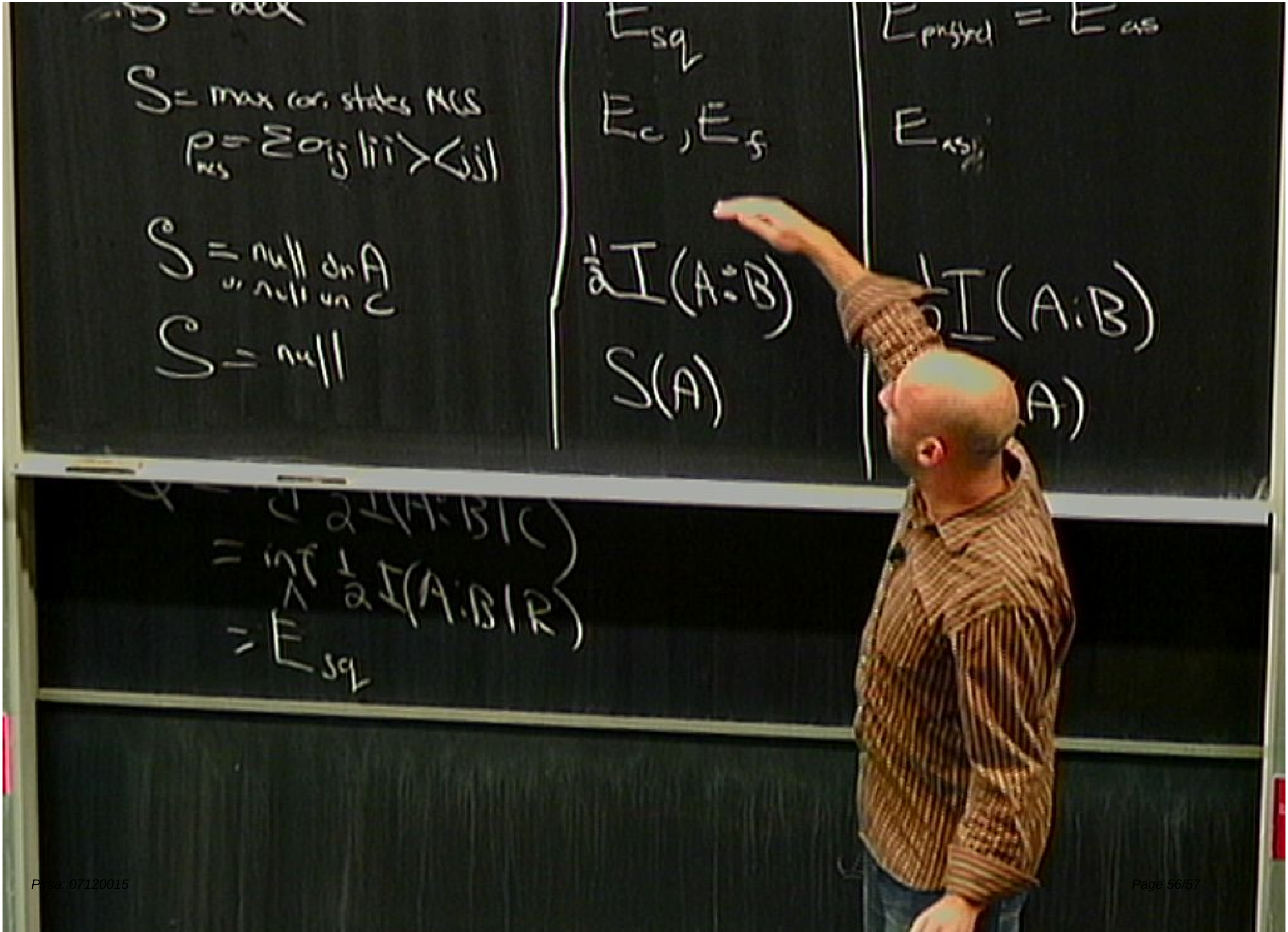
- S when does it give Entanglm/Corr.
- S = ppt, sep . . .
- additivity
- monogamy

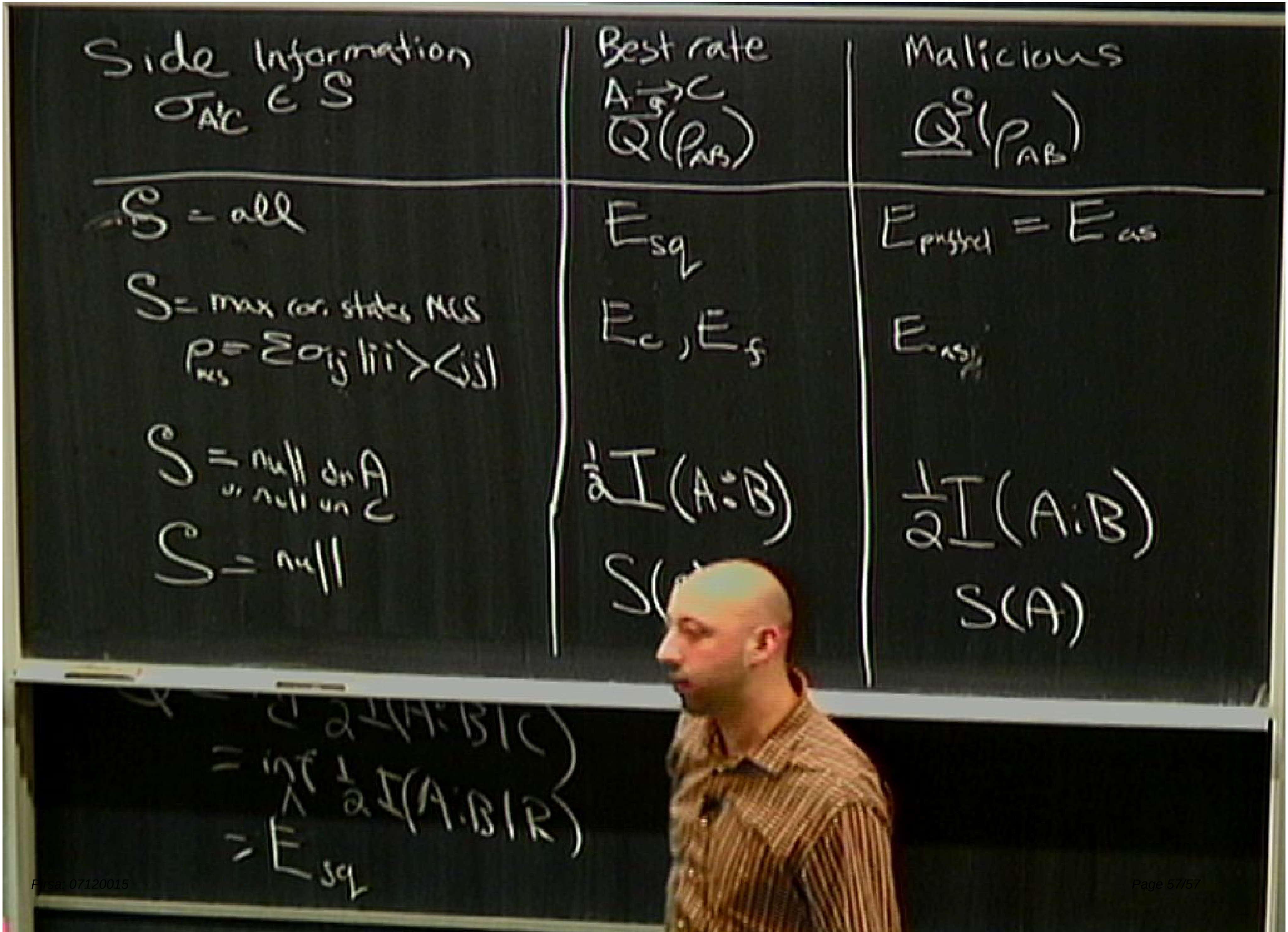

Entanglement
- Bell inequality
Ent ≠ LHV

Entanglement

- Bell inequality
  Ent $\neq$ LHV

- class of ops $\wedge$ monotones
  LOCC


S = max cor. states NCS
ρ_NCS = Σ α_ij |ii⟩⟨jj|
S = null on A or null on C
S = null
E_sq
E_c , E_F
½ I(A:B)
S(A)
½ I(A:B)
½ I(A:B|C)
= inf ½ I(A:B|R)
= E_sq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Q^S(ρ_AB)
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E_pushed = E_cas
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S = null on A
or null on C
½ I(A:B)
½ I(A:B)
S = null
S(A)
= inf ½ I(A:B|R)
= E_sq