Title: Spontaneous Broken Symmetry 6B

Date: Dec 06, 2007 11:00 AM

URL: http://pirsa.org/07120011

Abstract:

$$[H_{red}, \alpha_k] = -E_k \alpha_k$$

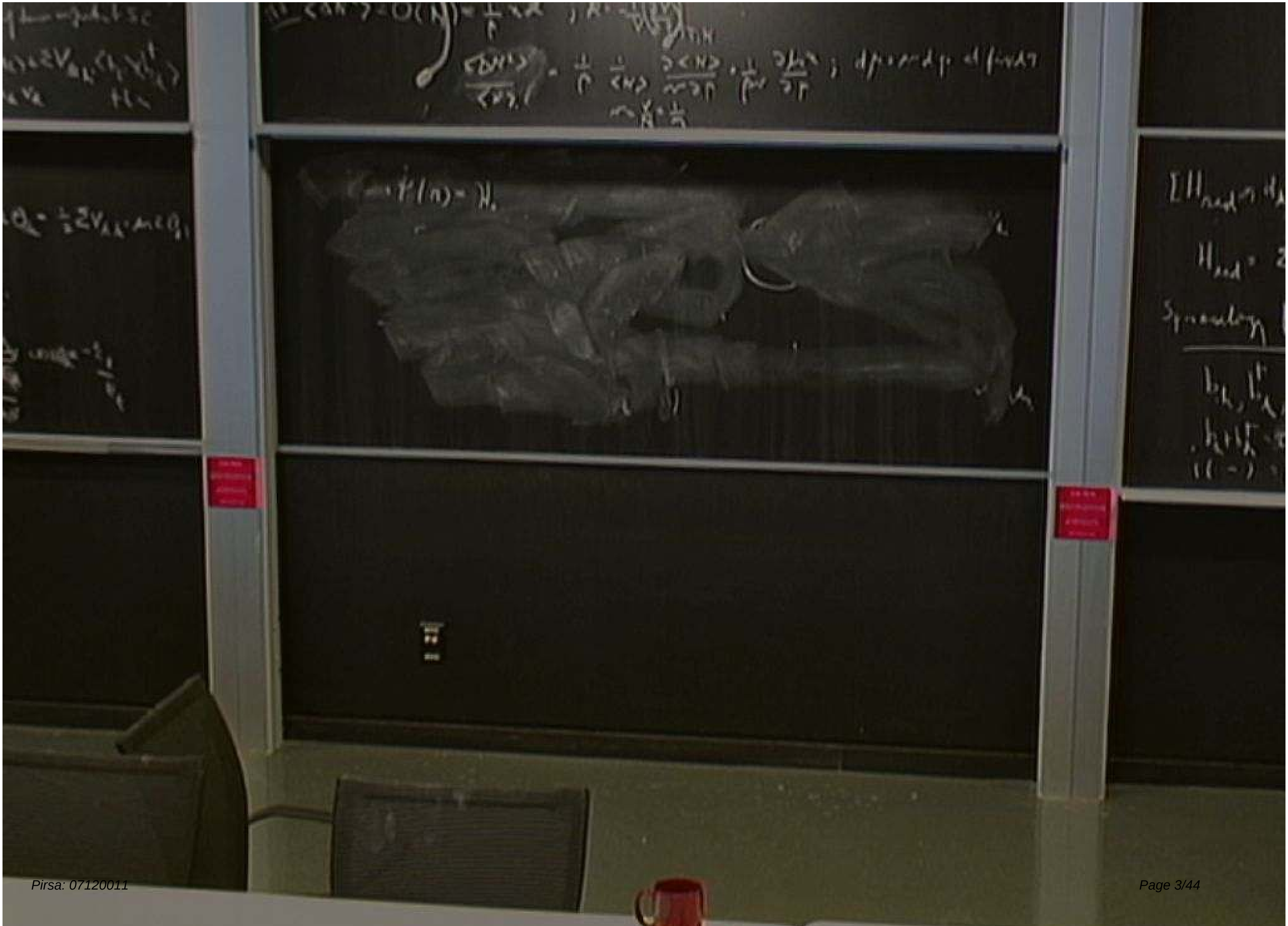
$$H_{red} = \sum \xi_k n_{k\sigma} + \sum_{k,k'} V_{kk'} \left[b_k^\dagger \langle b_{k'} \rangle + h.c. \right]$$

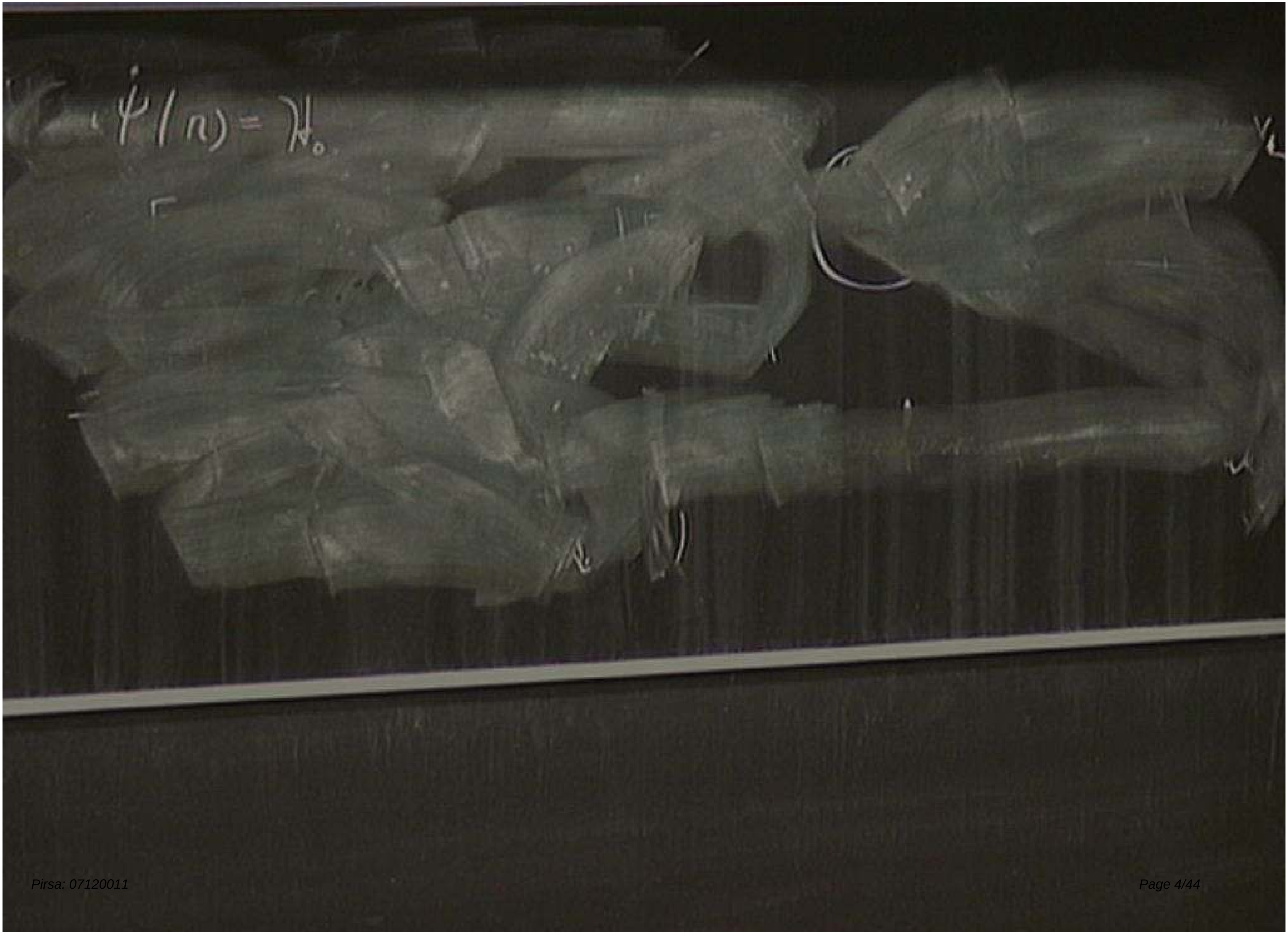
Spin analogy to understand the gap

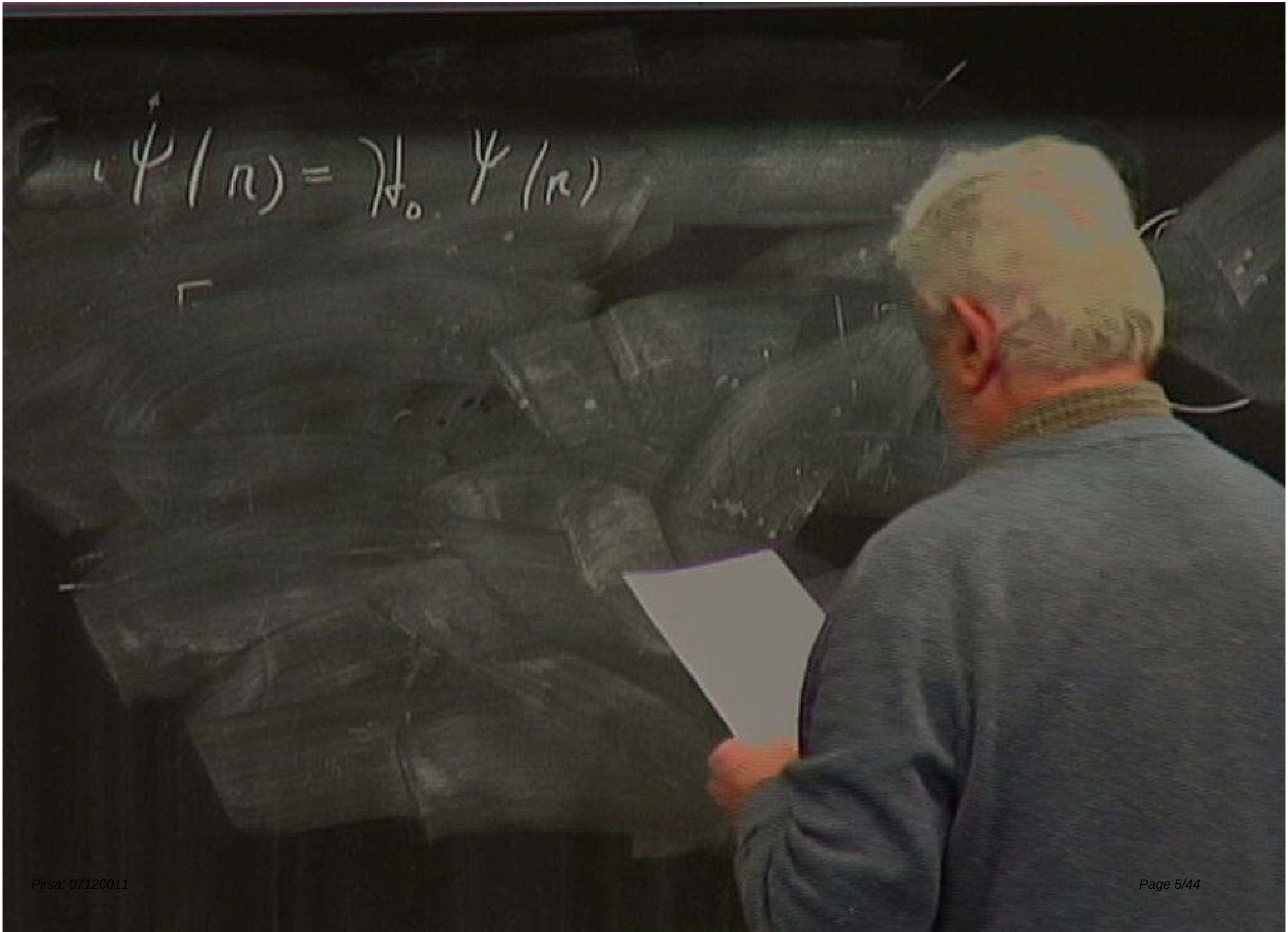
$$b_k, \; b_k^\dagger, \; 1 - n_k - n_{-k}$$

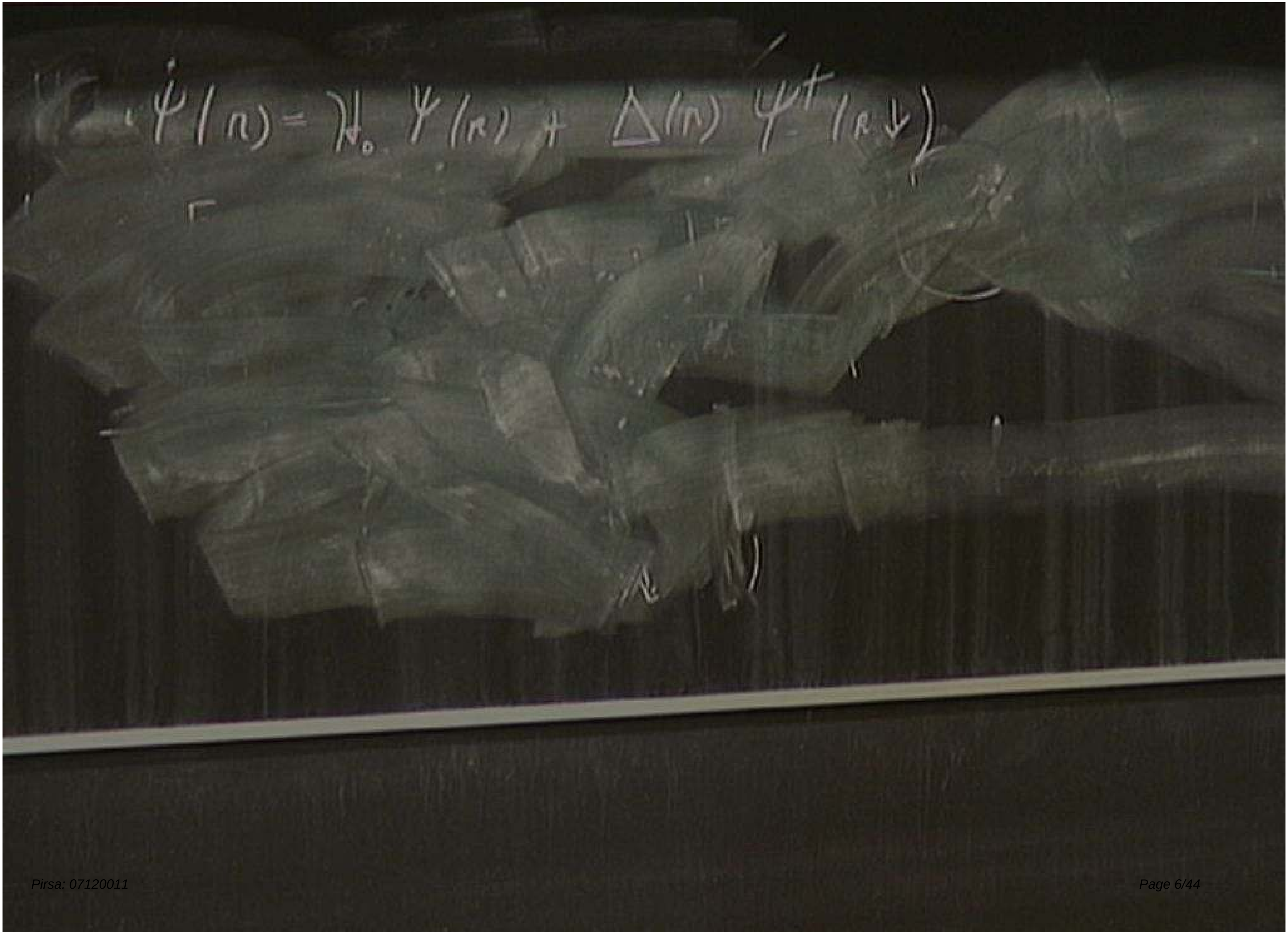
$$b_k + b_k^\dagger = \tau_x \qquad \tau_z$$

$$i(\; - \;) = \tau_y$$






$i\dot{\psi}(r) = \mathcal{H}_0 \psi(r)$


$i\dot{\psi}(r) = \mathcal{H}_0 \psi(r) + \Delta(r) \psi^{\dagger}(r\downarrow)$

Landau Ginzburg Action

(In addition to spin order, where $|\vec{\nabla}\Delta|^2$ comes from)

Gauge Invariance $\vec{\nabla} \to \vec{\nabla} - 2e\vec{A}$

Quantized Flux, Josephson

$$H_{am} = \sum \epsilon_k n_k + \sum V a^\dagger_{k_1} a^\dagger_{k_2} a_{k_3} a_{k_4} \, \delta(k_1+k_2-k_3-k_4)$$

$\langle BCS | H_{am} | BCS \rangle$ = 2 types of terms important S.C.

$$H_{red} = \sum \epsilon_k \langle n_k \rangle + \sum V_{k k'} \langle b_{k'} \rangle \langle b^\dagger_k \rangle + h.c.$$

$n_k = v_k^2$ $\quad \langle b_k \rangle = u_k v_k$

Define $u_k v_k = \sin 2\theta_k$

$n_k = \cos^2 \theta_k$

Soln.

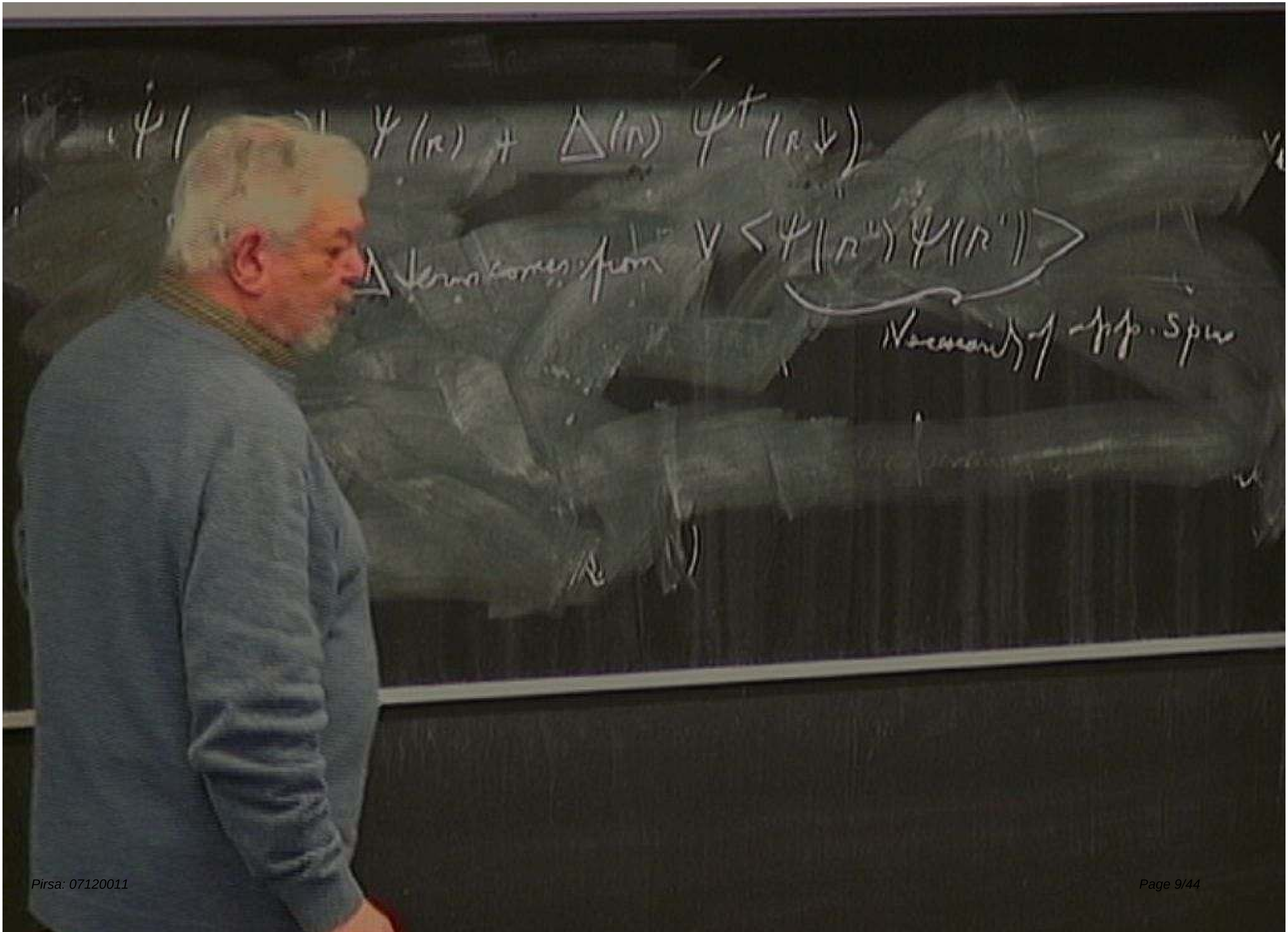
$$\epsilon_k \tan 2\theta_k = \frac{1}{2} \sum V_{k k'} \sin 2\theta_{k'}$$

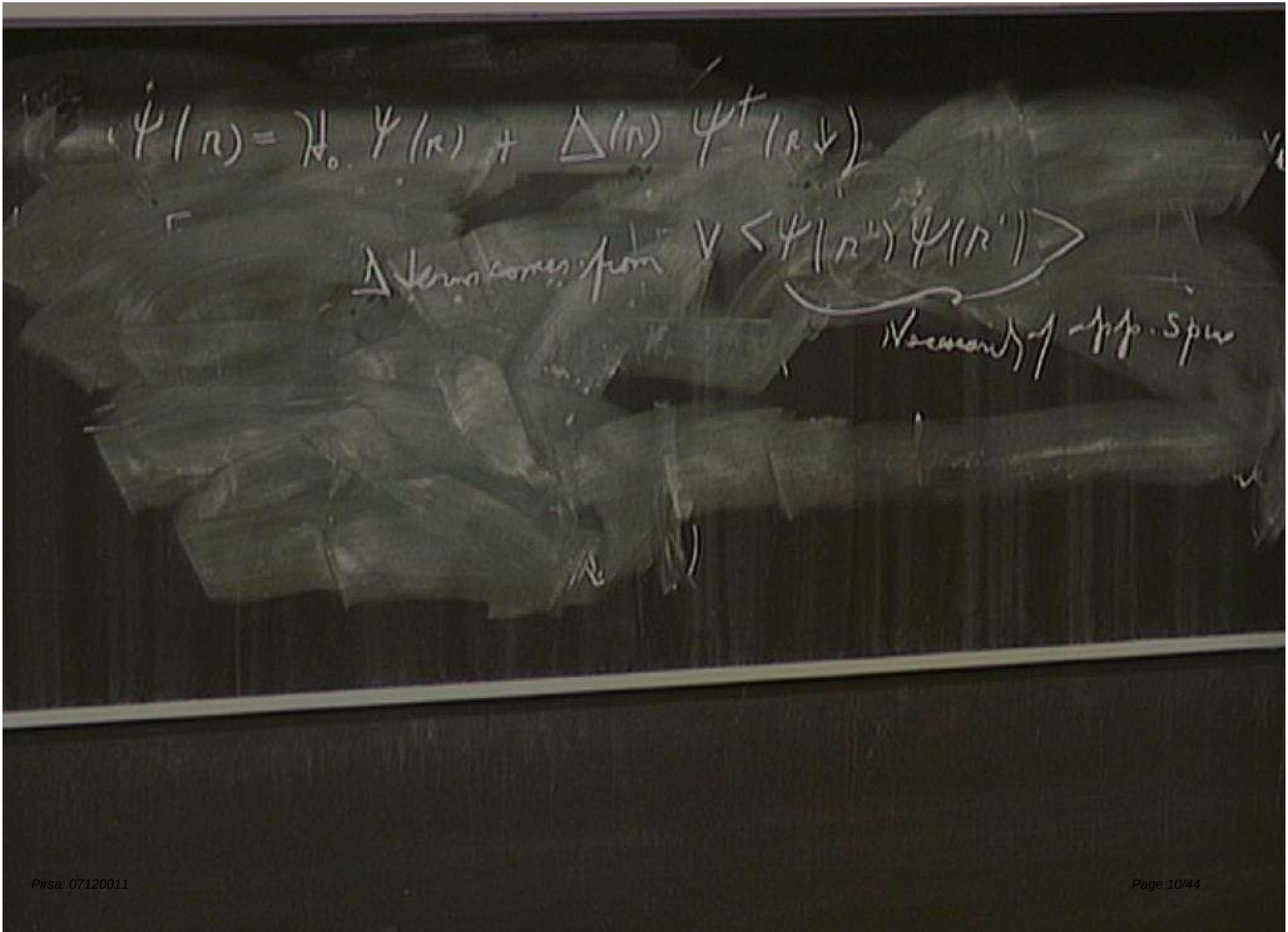
Define $\Delta_k = - \sum_{k'} V(k, k') \dfrac{\sin 2\theta_{k'}}{2}$

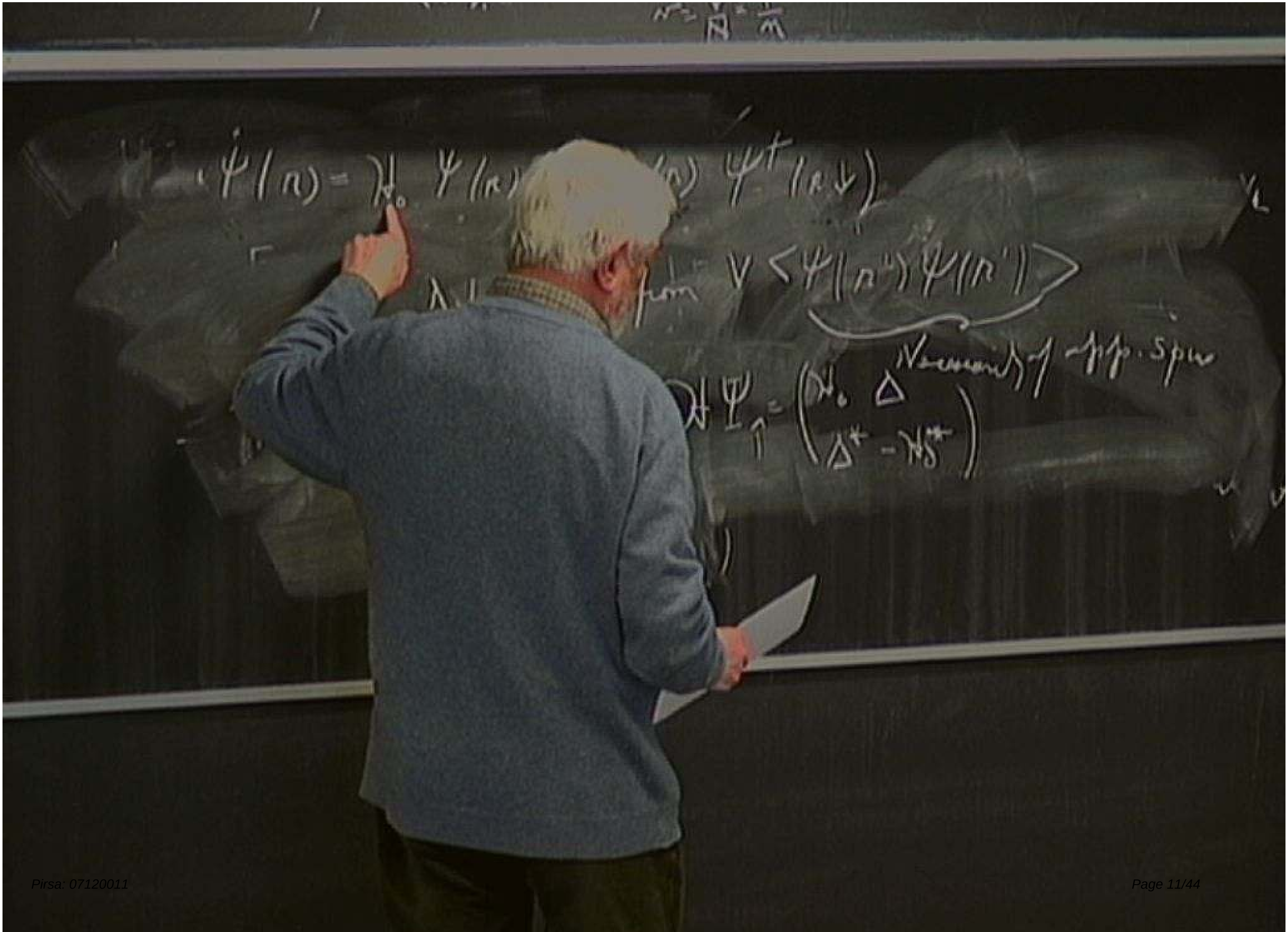
$$\tan 2\theta_k = -\frac{\Delta_k}{\epsilon_k} \qquad \sin 2\theta_k = \frac{\Delta_k}{E_k} \qquad \cos 2\theta_k = -\frac{\epsilon_k}{E_k}$$

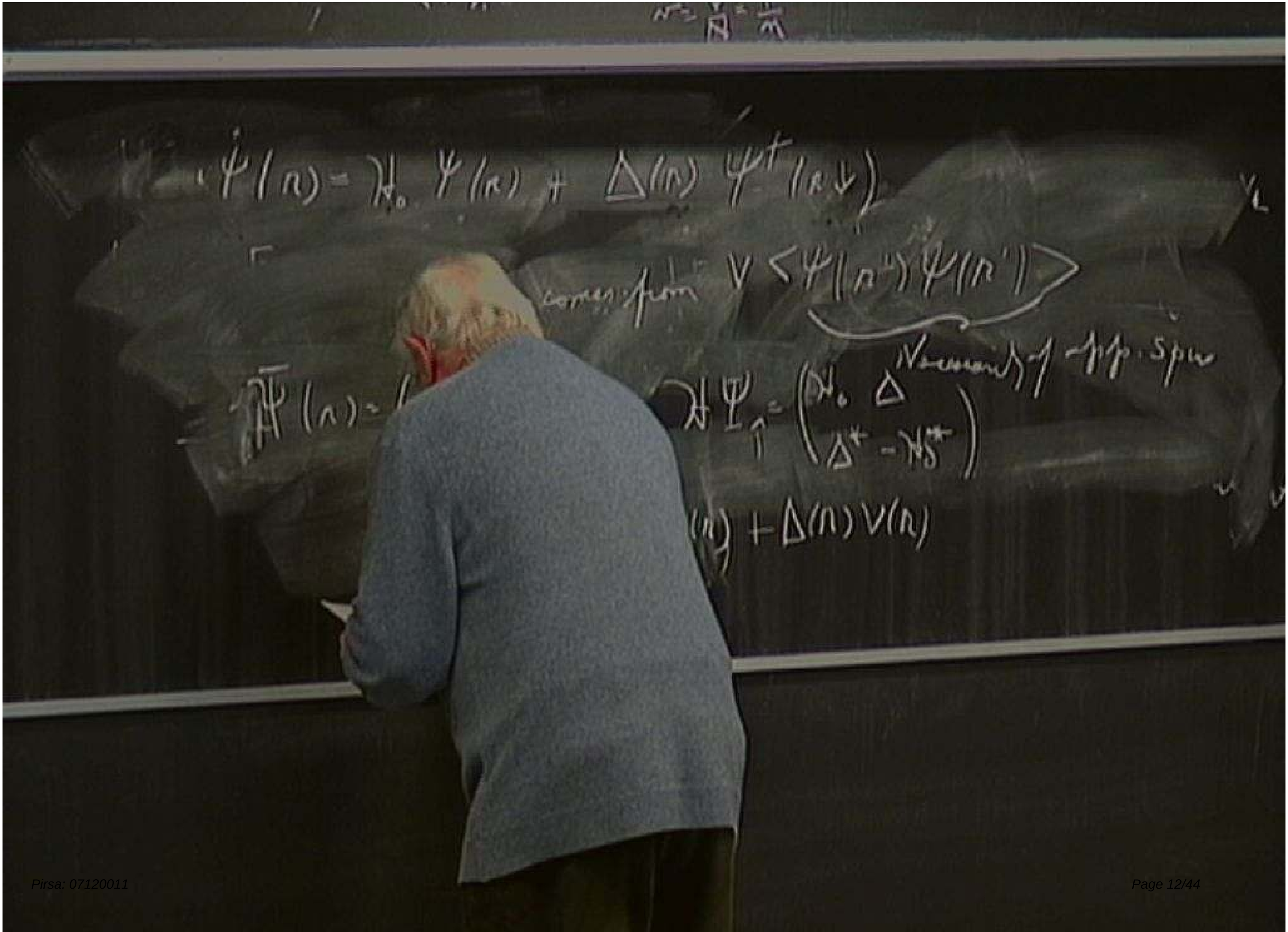
$$E_k = \sqrt{\epsilon_k^2 + \Delta_k^2}$$

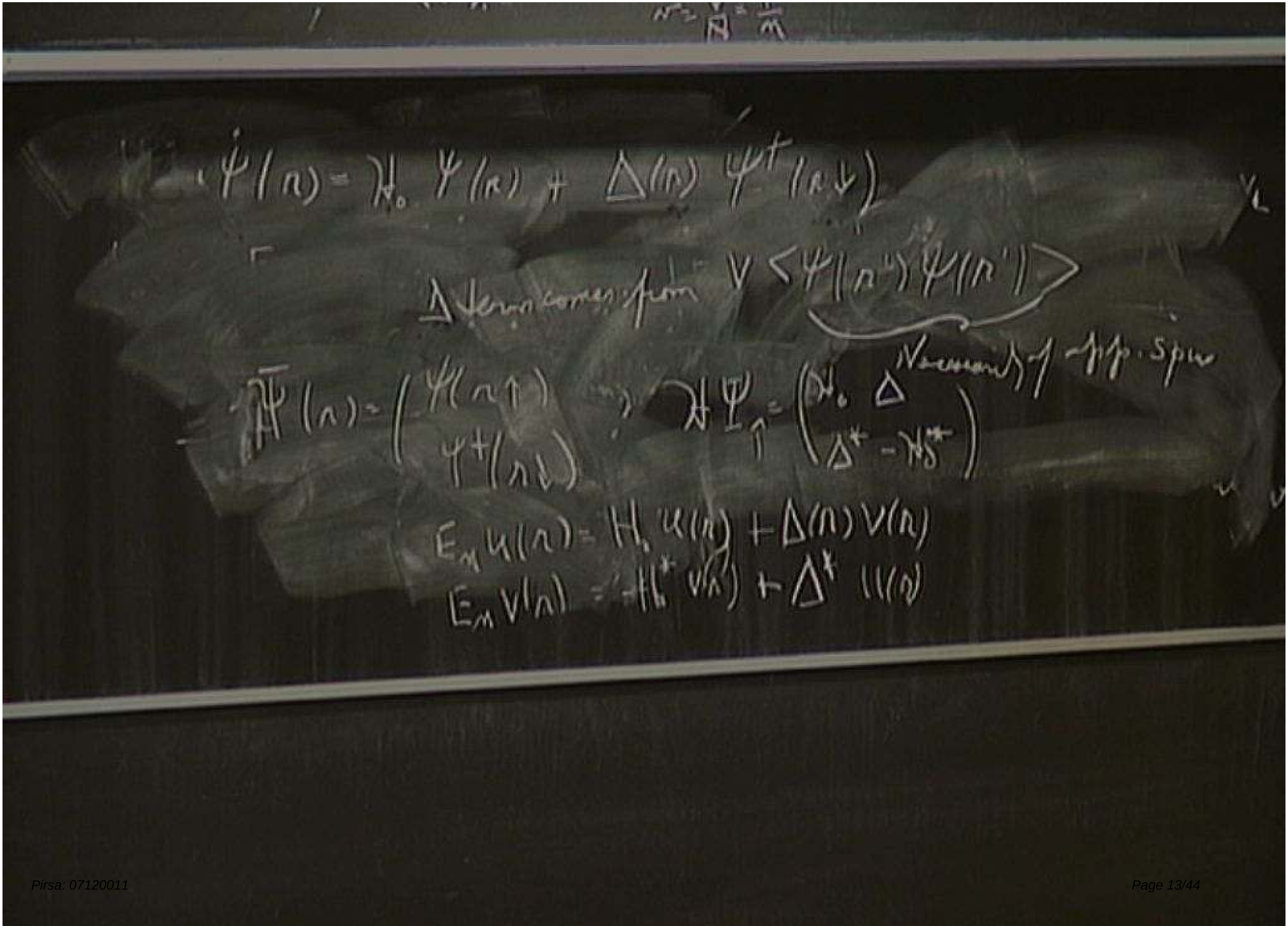
$$i\dot{\psi}(r) = \mathcal{H}_0\, \psi(r) + \Delta(r)\, \psi^{\dagger}(r\downarrow)$$

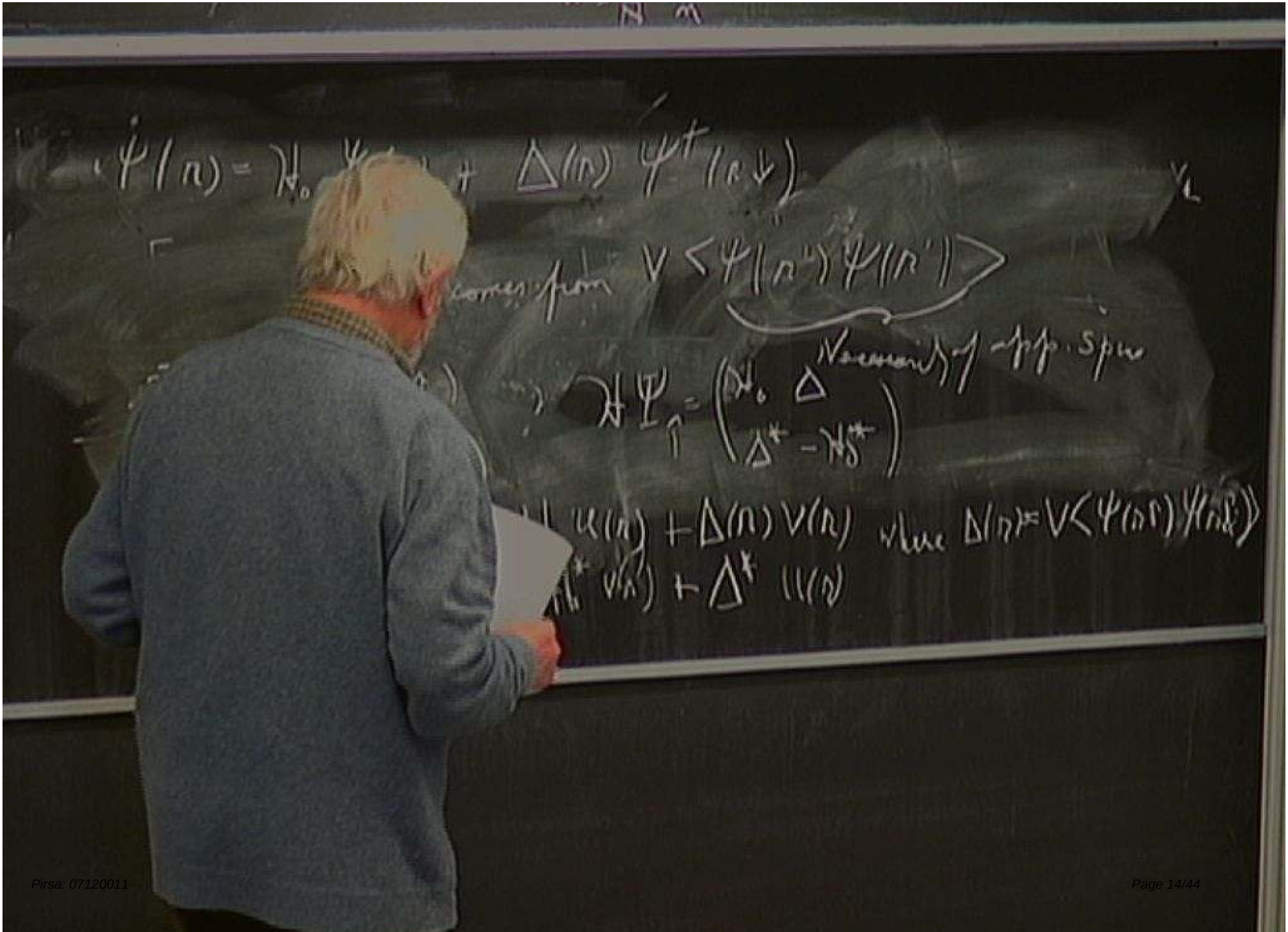


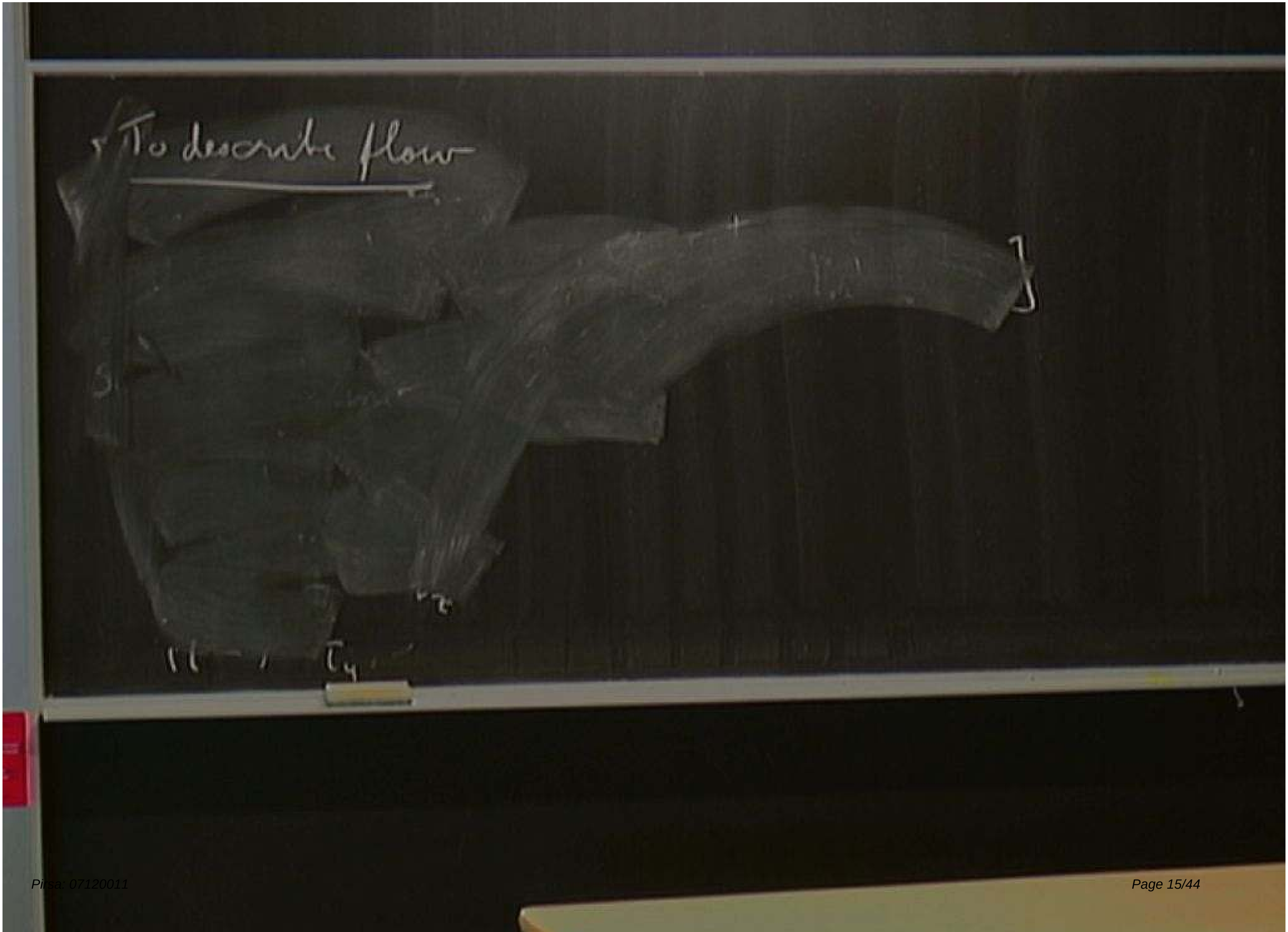

ψ̇(r) = H₀ ψ(r) + Δ(r) ψ†(r↓)
Δ term comes from V ⟨ψ(r↓) ψ(r↑)⟩
Necessarily of opp. spin

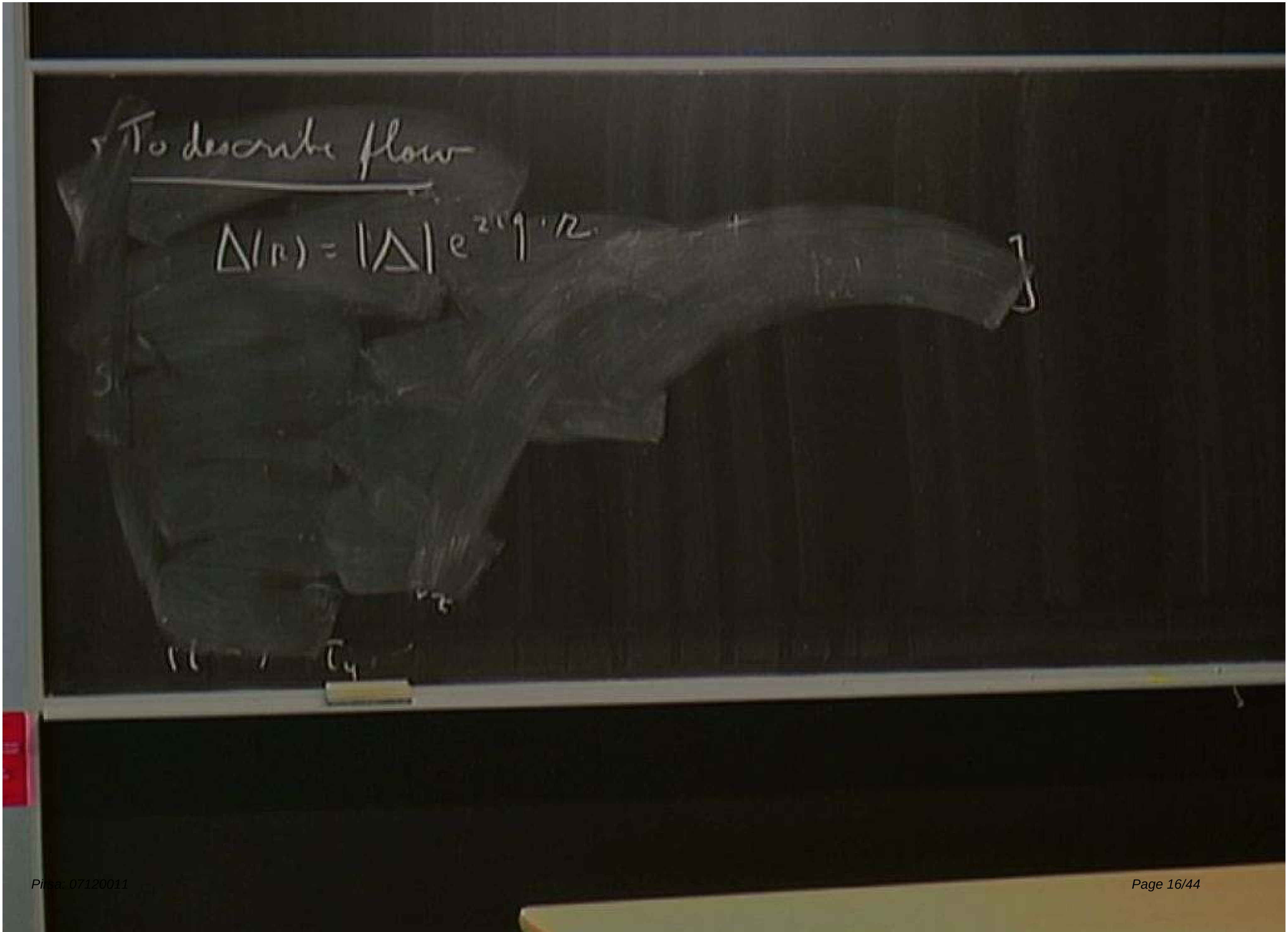


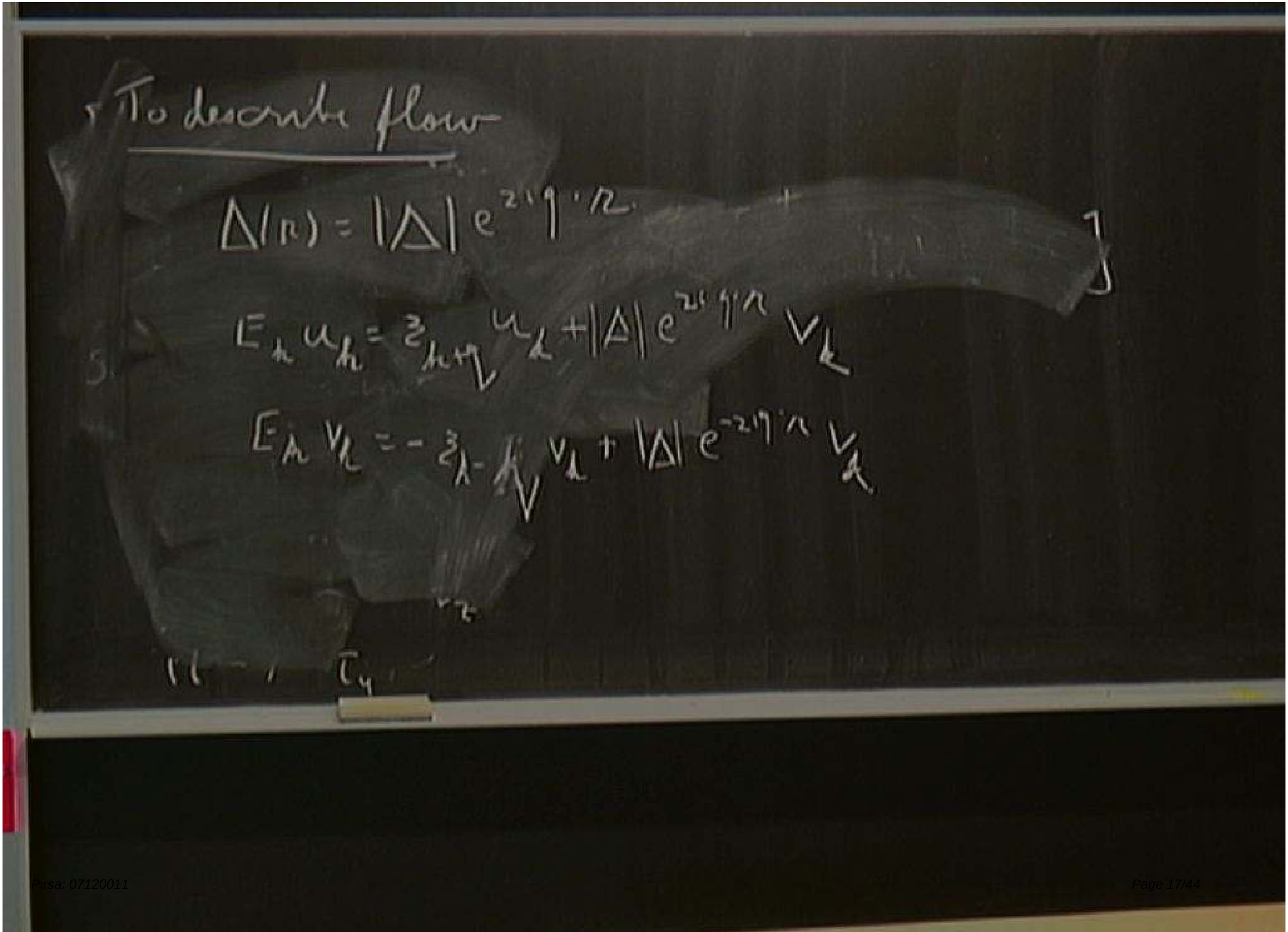


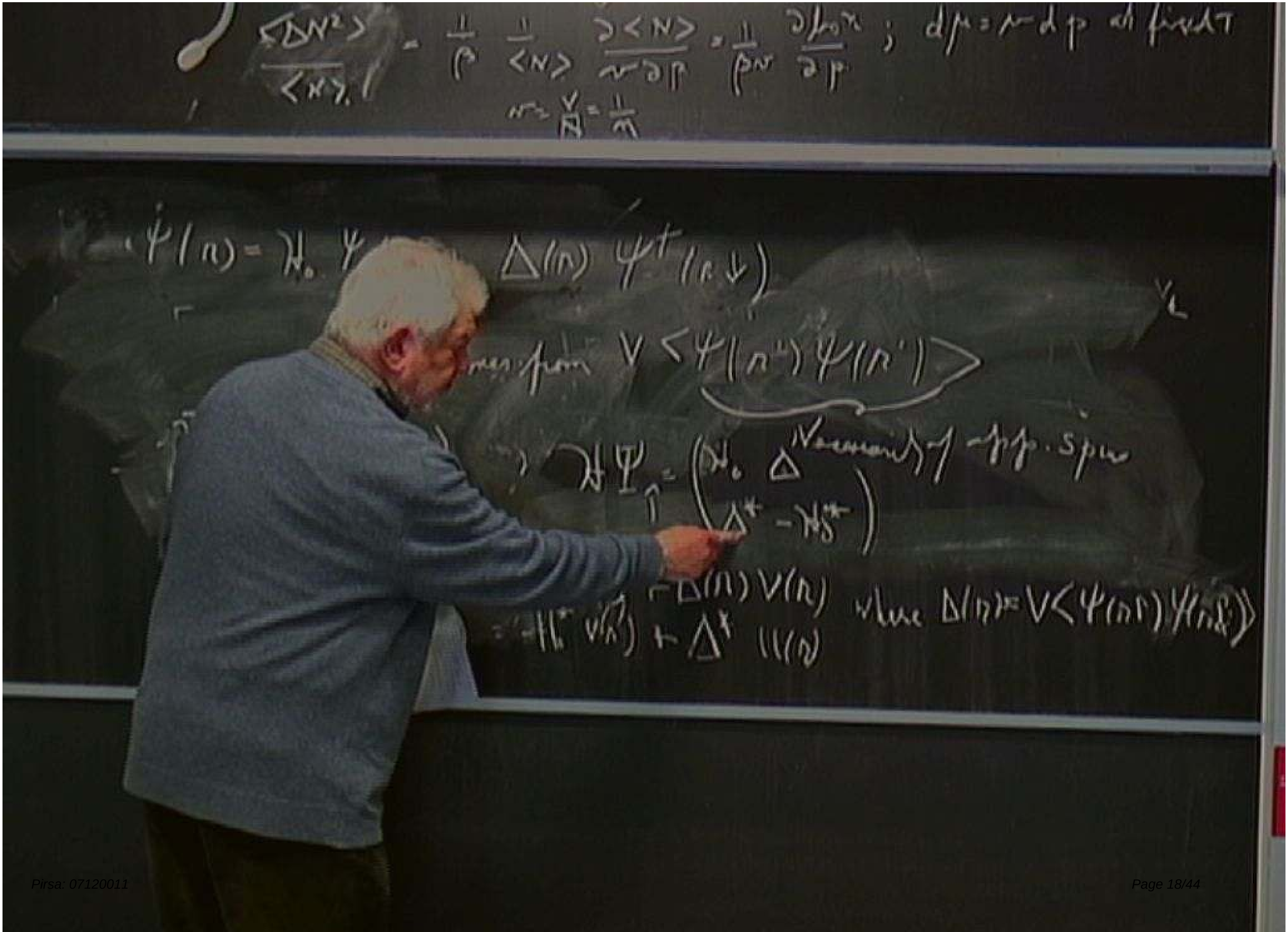

$i\dot{\psi}(r) = H_0 \psi(r) + \Delta(r) \psi^\dagger(r\downarrow)$
Δ term comes from $V \langle \psi(r\uparrow) \psi(r\downarrow) \rangle$
Necessarily of opp. spin
$\Psi(r) = \begin{pmatrix} \psi(r\uparrow) \\ \psi^\dagger(r\downarrow) \end{pmatrix}$; $H\Psi_\uparrow = \begin{pmatrix} H_0 & \Delta \\ \Delta^* & -H_0^* \end{pmatrix}$
$E_n u(r) = H_0 u(r) + \Delta(r) v(r)$
$E_n v(r) = -H_0^* v(r) + \Delta^* u(r)$

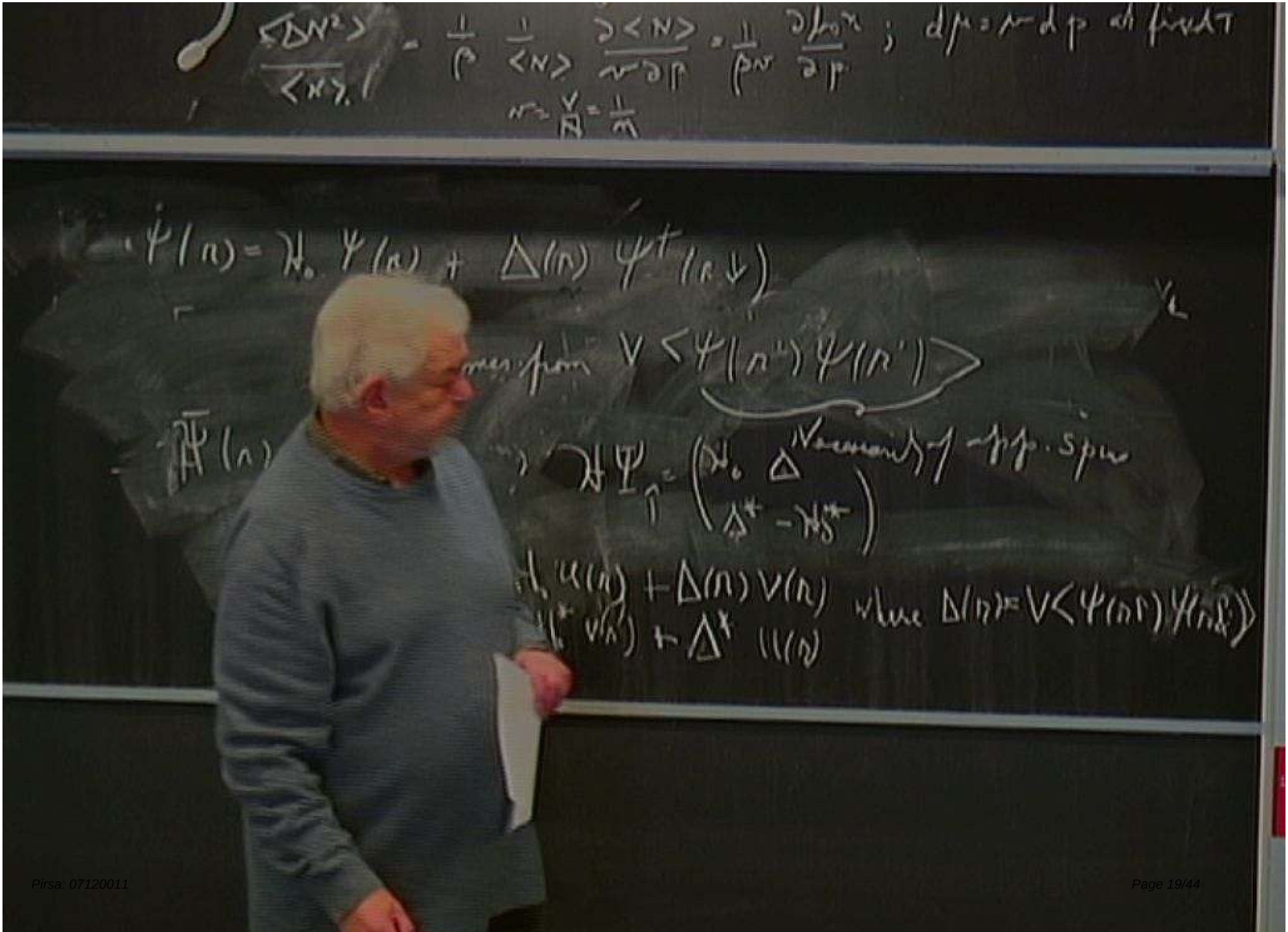

comes from
where


To describe flow


To describe flow
Δ(r) = |Δ| e^{2iq·r}


To describe flow
Δ(r) = |Δ| e^{2iq·r}
E_k u_k = ξ_{k+q} u_k + |Δ| e^{2iq·r} v_k
E_k v_k = − ξ_{k−q} v_k + |Δ| e^{−2iq·r} v_k





$$\frac{\langle \Delta N^2\rangle}{\langle N\rangle} = \frac{1}{\beta}\frac{1}{\langle N\rangle}\frac{\partial\langle N\rangle}{v\,\partial\mu} = \frac{1}{\beta n}\frac{\partial \ln n}{\partial \mu}\ ; \quad d\mu = v\, dp \text{ at fixed } T$$

$$v = \frac{V}{N} = \frac{1}{n}$$

$$i\dot\psi(r) = \mathcal{H}_0\, \psi(r) + \Delta(r)\, \psi^\dagger(r\downarrow)$$

Δ term comes from $V\langle\psi(r\uparrow)\psi(r\downarrow)\rangle$

$$\Psi(r) = \begin{pmatrix}\psi(r\uparrow)\\ \psi^\dagger(r\downarrow)\end{pmatrix}, \quad \mathcal{H}\Psi_r = \begin{pmatrix}\mathcal{H}_0 & \Delta\\ \Delta^* & -\mathcal{H}_0^*\end{pmatrix}$$

Nambu mixing of opp. spin

$$E_n u(r) = H_0\, u(r) + \Delta(r)\, v(r) \quad \text{where } \Delta(r) = V\langle\psi(r\uparrow)\psi(r\downarrow)\rangle$$

$$E_n v(r) = -H_0^*\, v(r) + \Delta^*\, u(r)$$

To describe flow

$$\Delta(r) = |\Delta| e^{2iq\cdot r}$$

$$E_k u_k = \xi_{k+q} u_k + |\Delta| e^{2iq\cdot r} v_k$$

$$E_k v_k = -\xi_{k-q} v_k + |\Delta| e^{-2iq\cdot r} u_k$$

$$u_k = u_k^{(0)} e^{iq\cdot r}$$

$$v_k = v_k^{(0)} e^{-iq\cdot r}$$

To describe flow

$$\Delta(r) = |\Delta| e^{2iq\cdot r}$$

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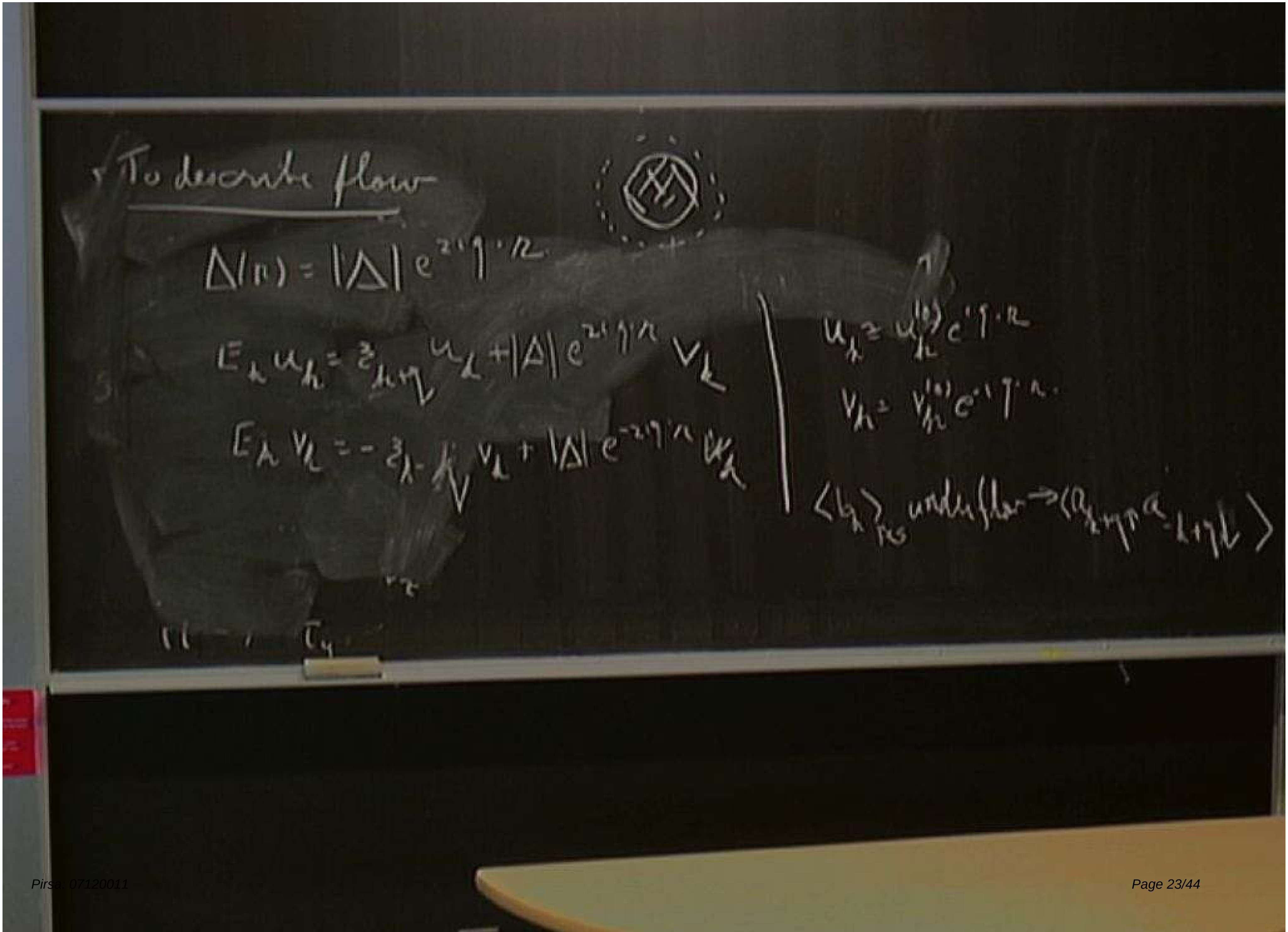
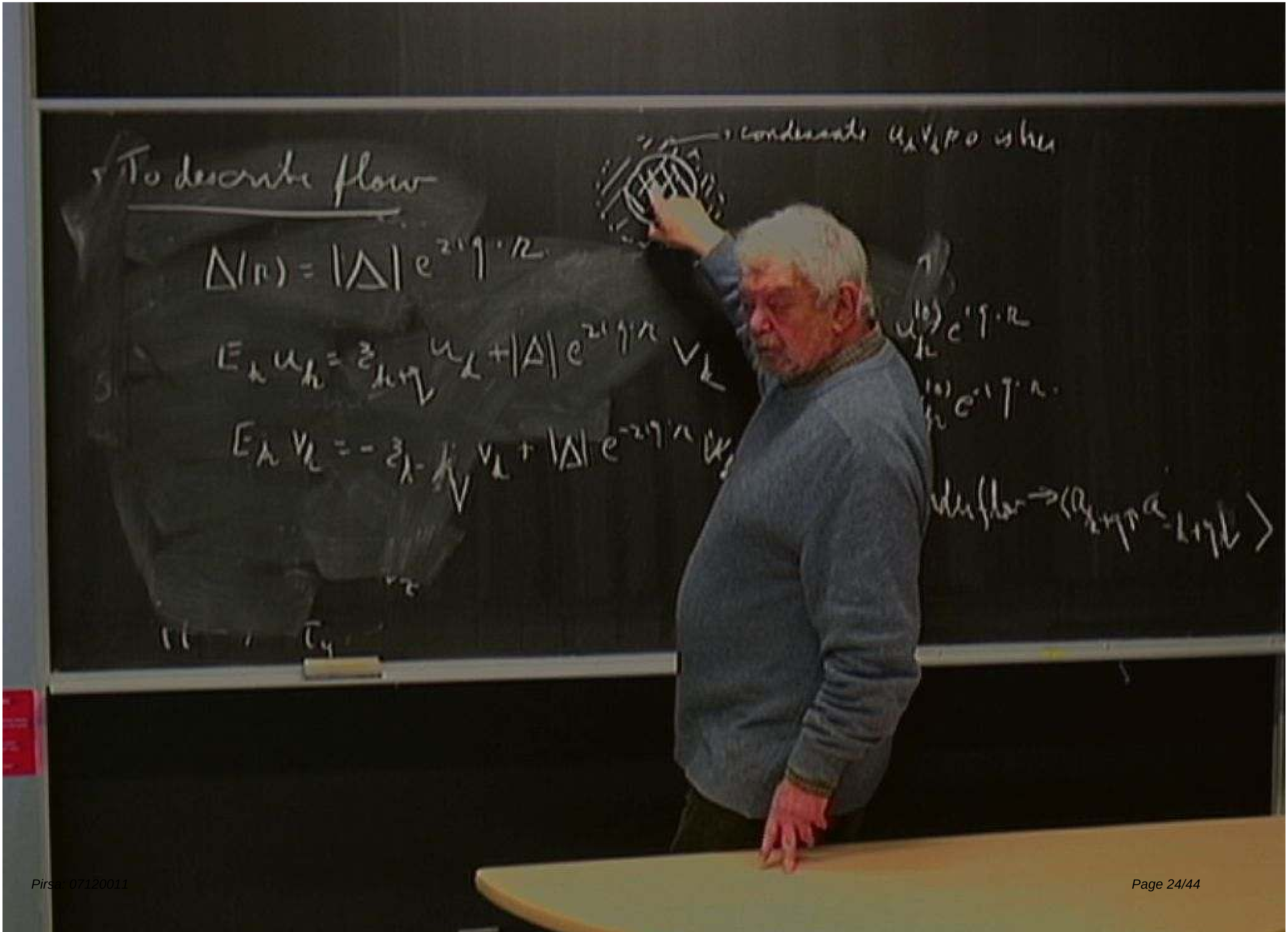
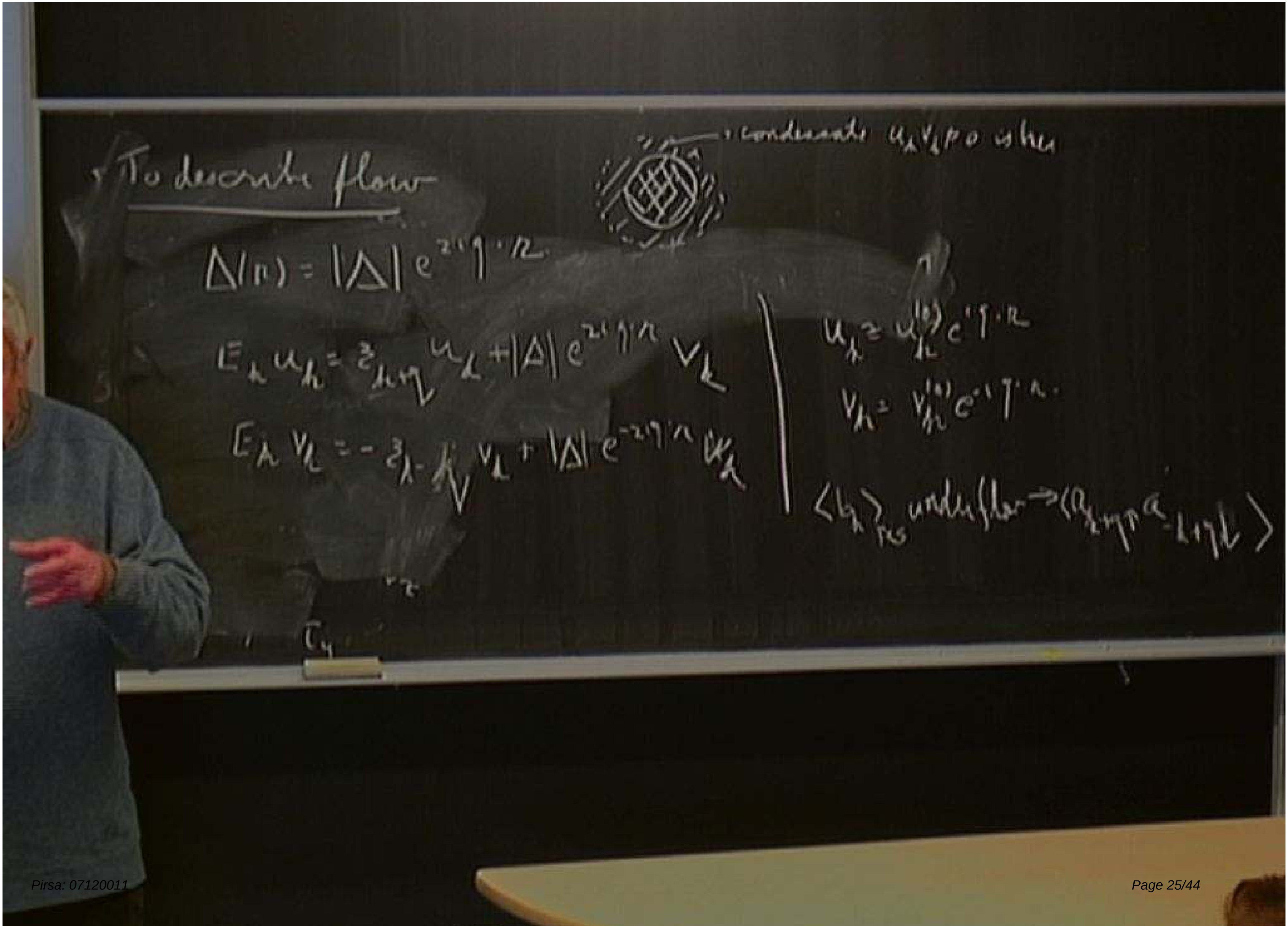
$$u_k = u_k^{(0)} e^{iq\cdot r}$$

$$v_k = v_k^{(0)} e^{-iq\cdot r}$$

$\langle b_k \rangle_{BCS}$ under flow $\rightarrow \langle a_{k+q\uparrow} a_{-k+q\downarrow} \rangle$


To describe flow


To describe flow
condensate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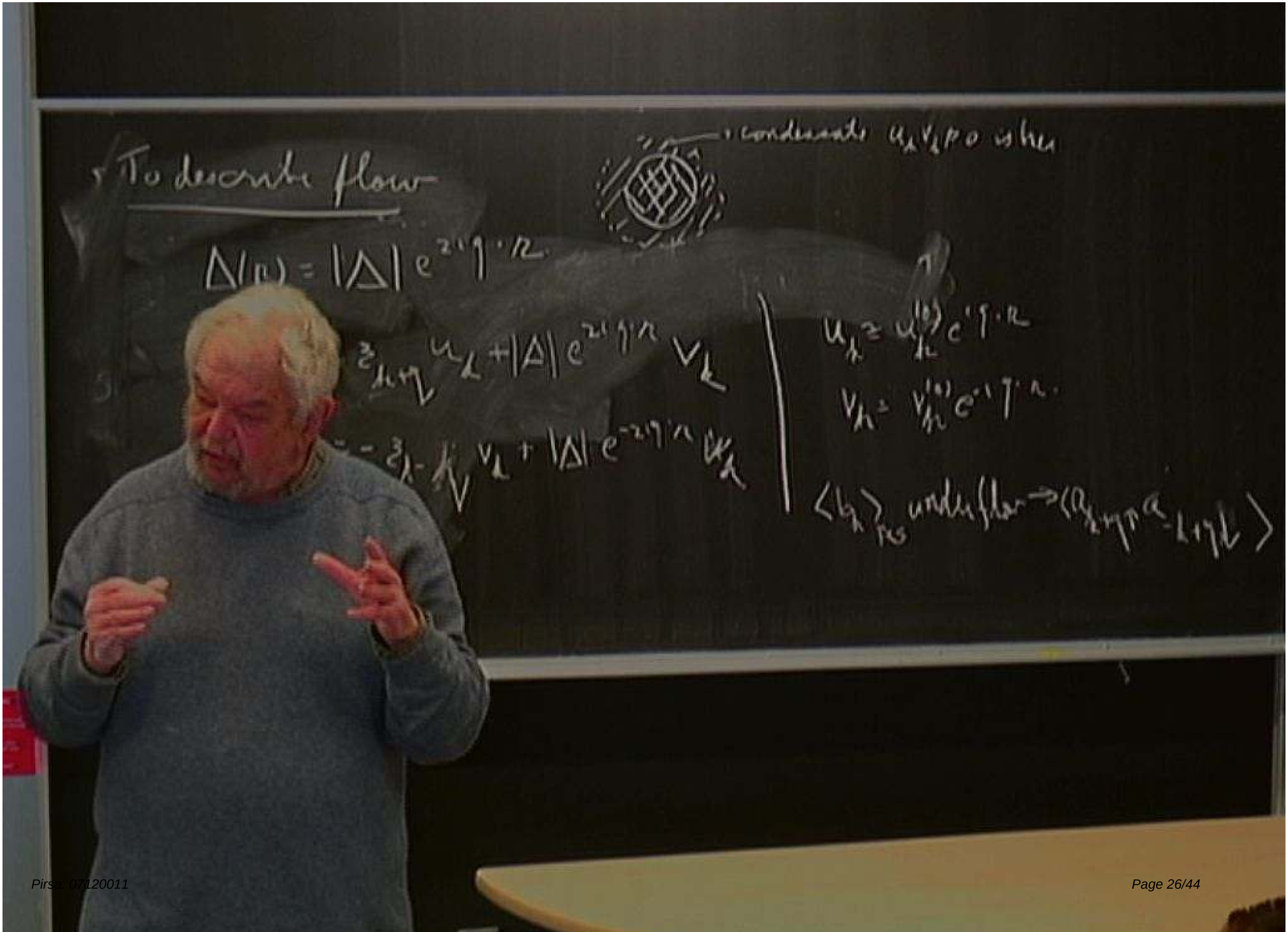
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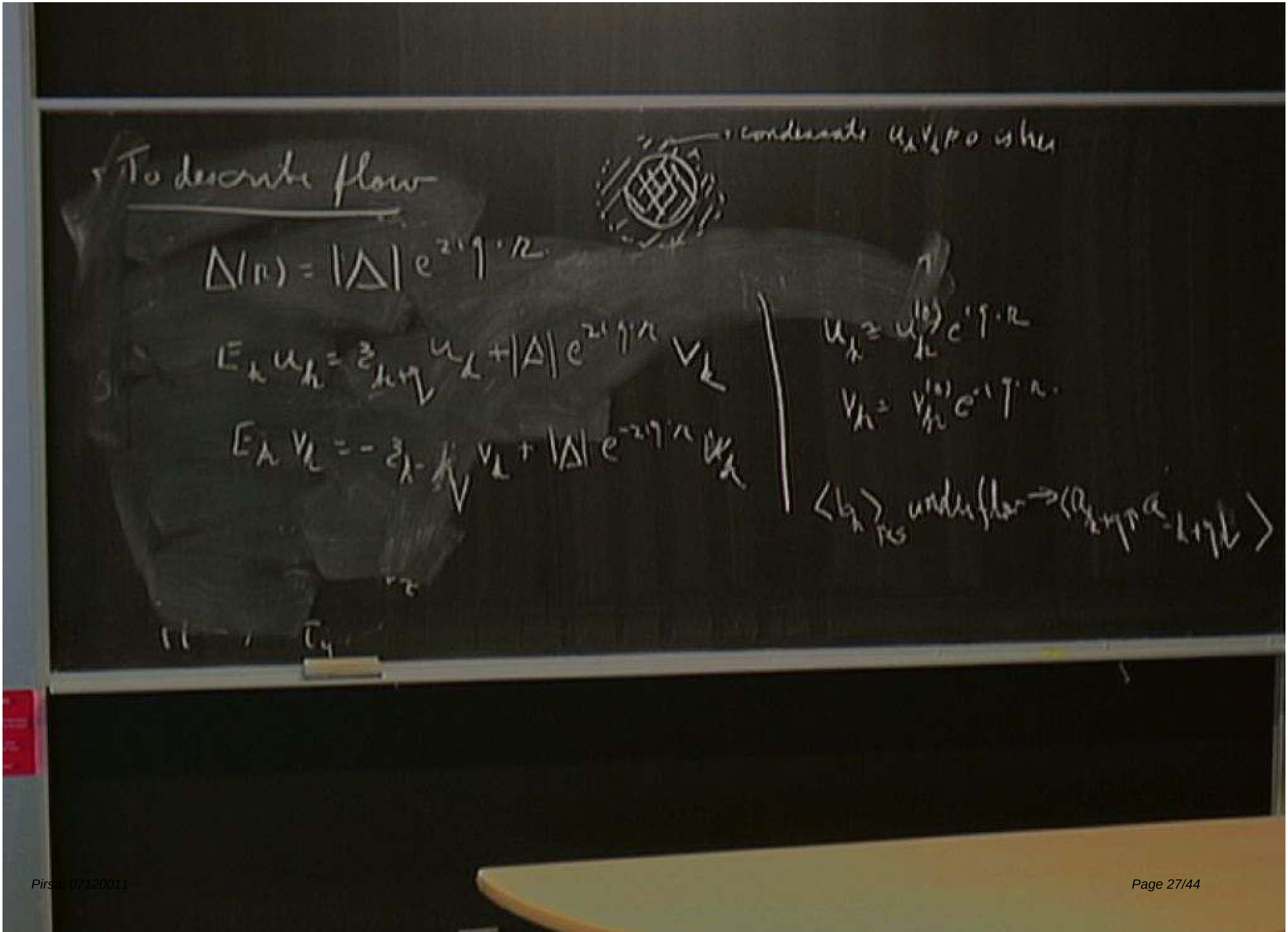
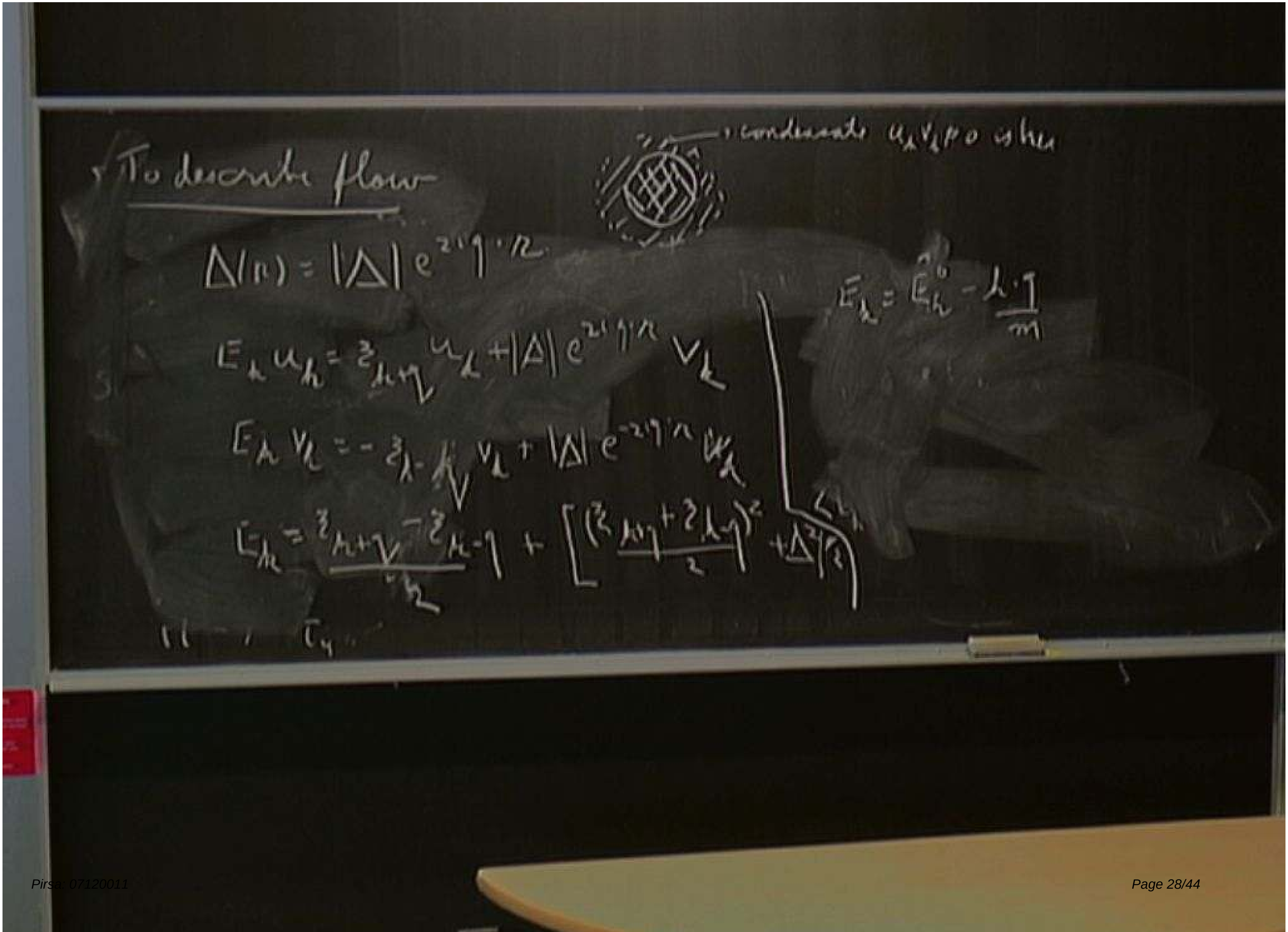
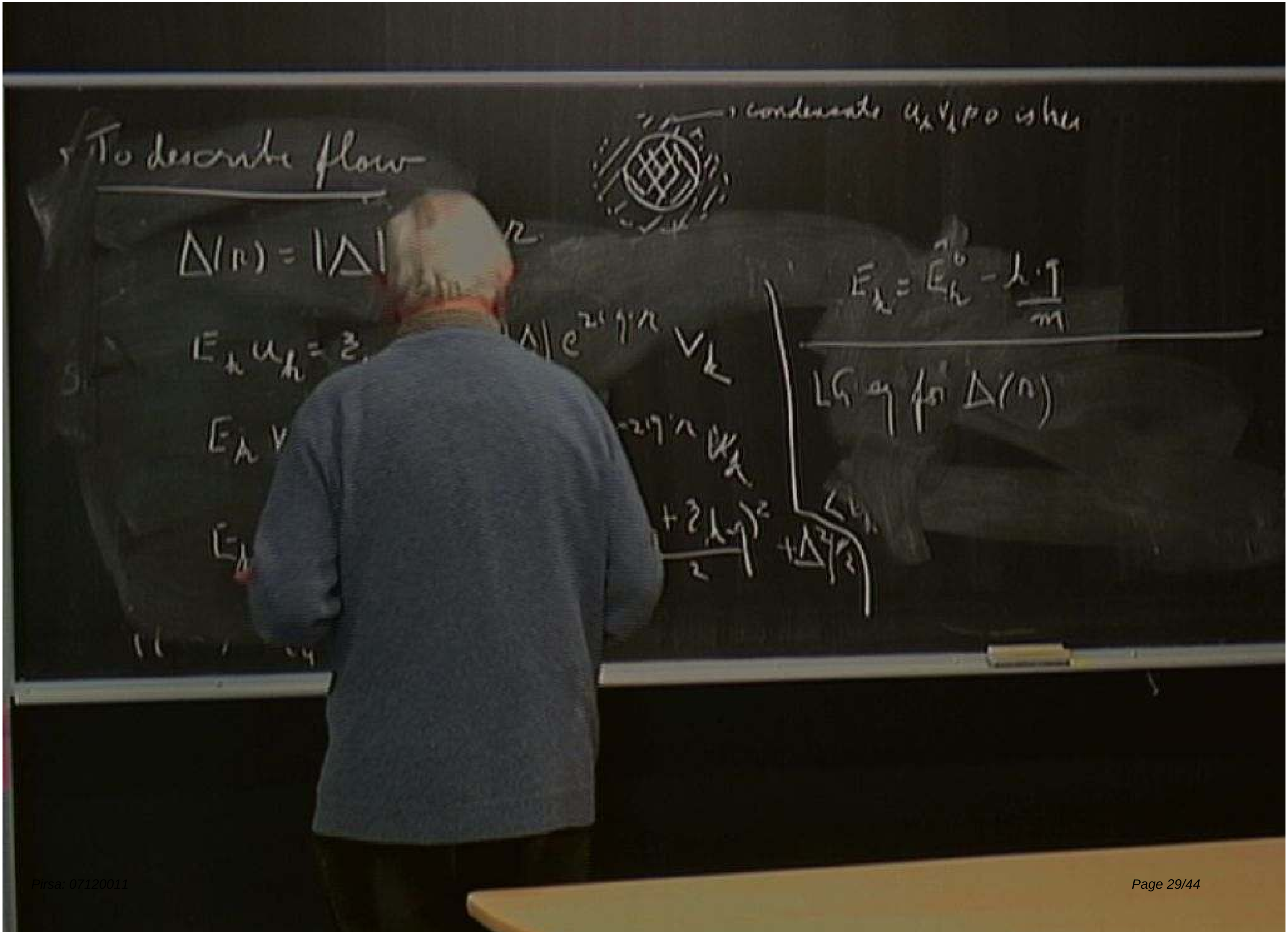
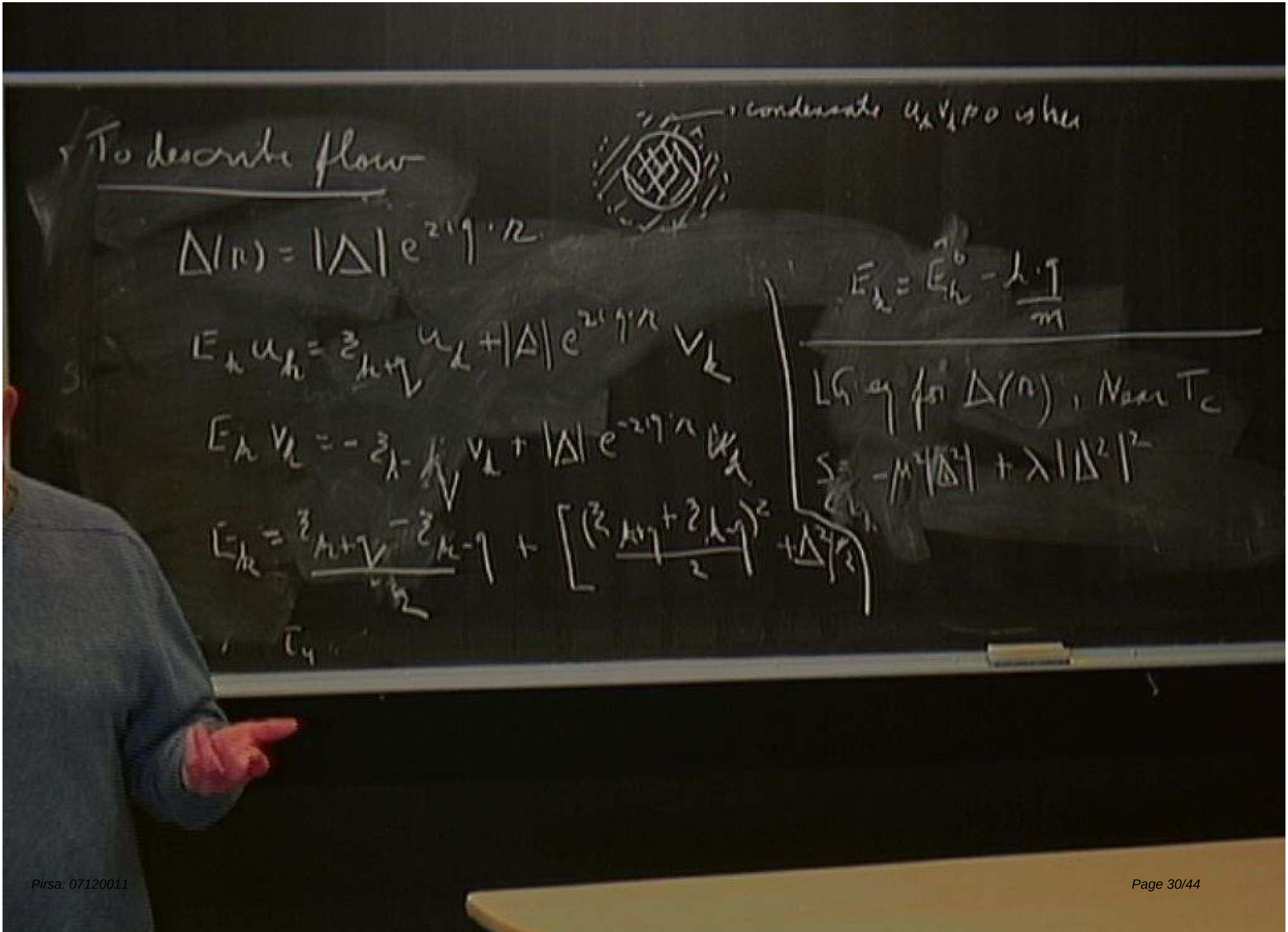
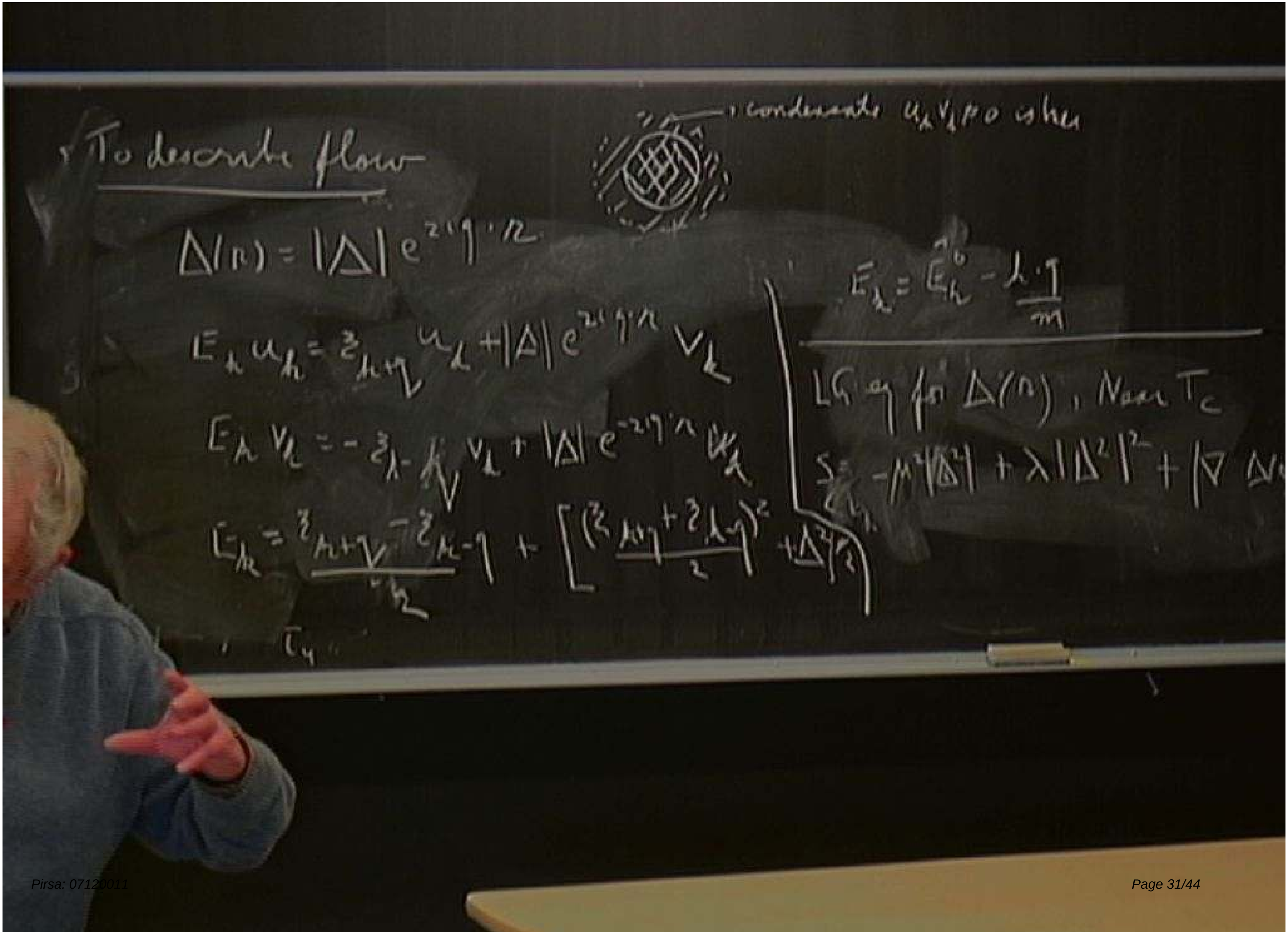
condensate


To describe flow
condensate
Δ(r) = |Δ| e^{2iq·r}
E_k u_k = ξ_{k+q} u_k + |Δ| e^{2iq·r} v_k
E_k v_k = −ξ_{k−q} v_k + |Δ| e^{−2iq·r} u_k
u_k = u_k^{(0)} e^{iq·r}
v_k = v_k^{(0)} e^{−iq·r}
under flow


To describe flow
condensate $u_k v_k \neq 0$ where
$\Delta(r) = |\Delta| e^{2iq\cdot r}$
$E_k u_k = \xi_{k+q} u_k + |\Delta| e^{2iq\cdot r} v_k$
$E_k v_k = -\xi_{k-q} v_k + |\Delta| e^{-2iq\cdot r} u_k$
$E_k = \frac{\xi_{k+q} - \xi_{k-q}}{2} + \left[\left(\frac{\xi_{k+q} + \xi_{k-q}}{2} \right)^2 + \Delta^2 \right]^{1/2}$
$E_k = E_k^0 - k \cdot \frac{J}{m}$


To describe flow
LG eq for Δ(r)


To describe flow
condensate $u_k v_k \neq 0$ is here
$\Delta(r) = |\Delta| e^{2iq\cdot r}$
$E_k = E_k^0 - k \cdot \frac{J}{m}$
LG eq for $\Delta(r)$, Near T_c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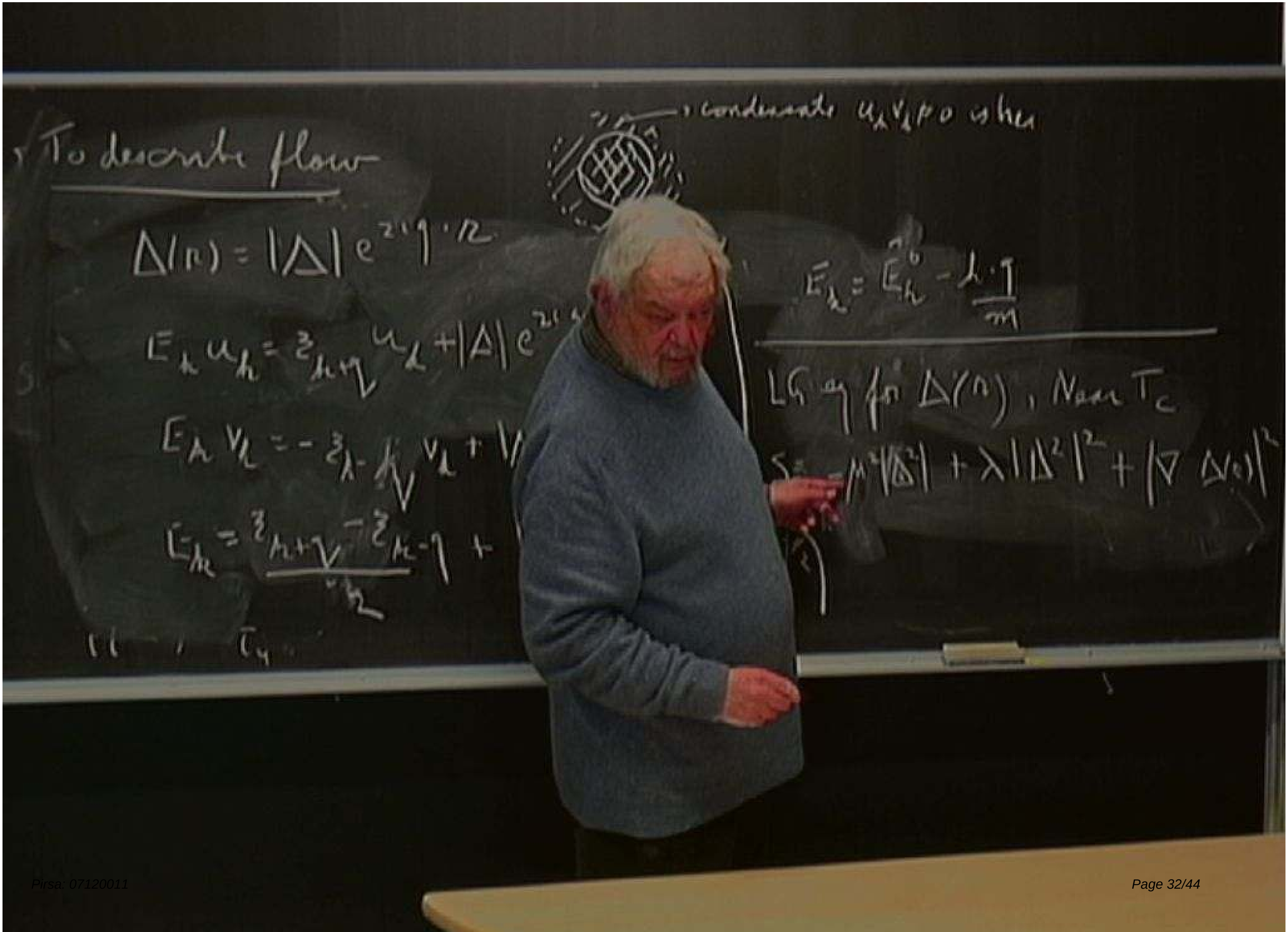
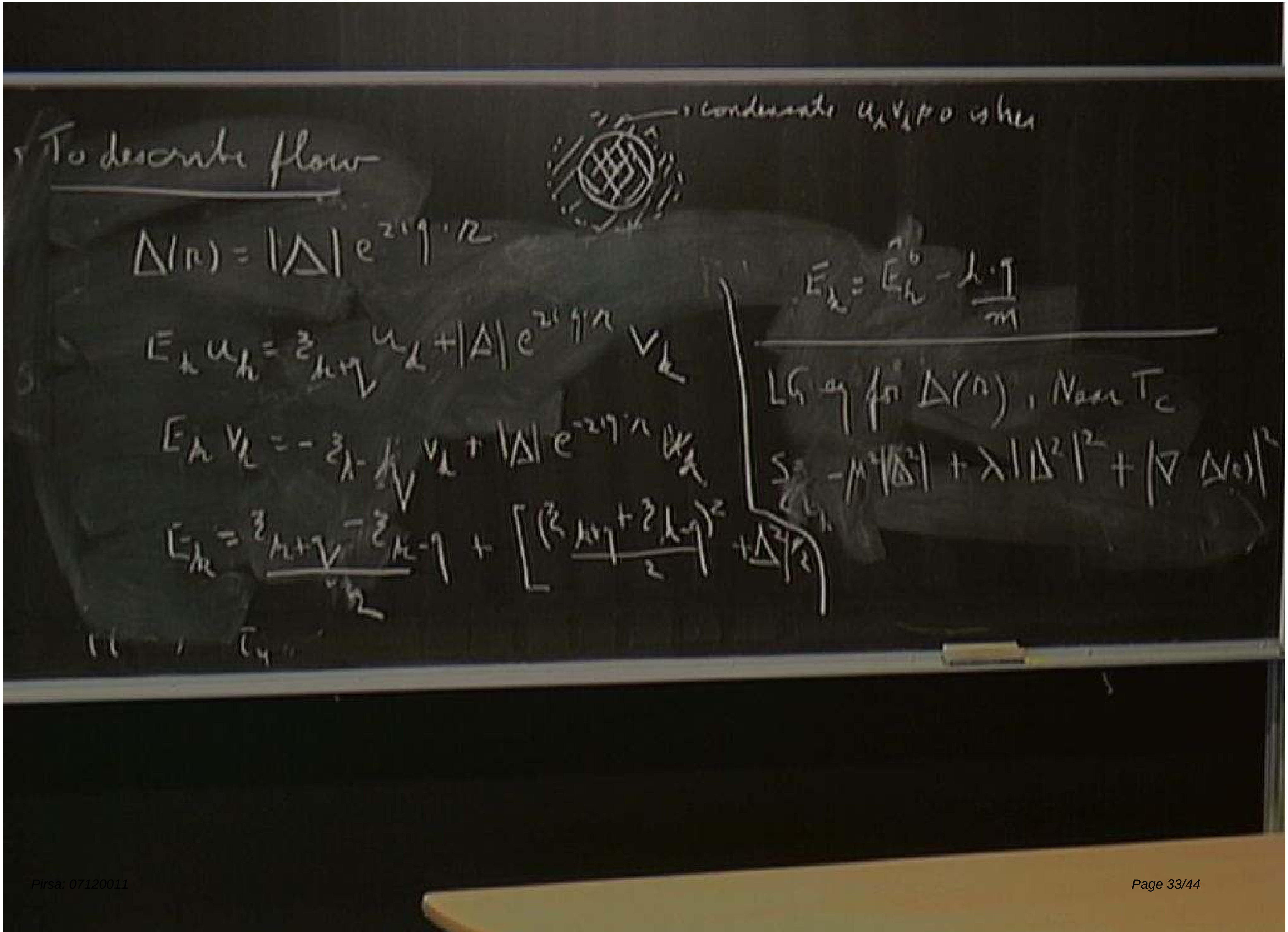
$E_k = E_k^0 - k \cdot \frac{q}{m}$
LG eq for $\Delta(r)$, Near T_c


To describe flow
Δ(r) = |Δ| e^{2iq·r}
LG eq for Δ(r), Near T_c


condensate $u_k v_k \neq 0$ when
To describe flow
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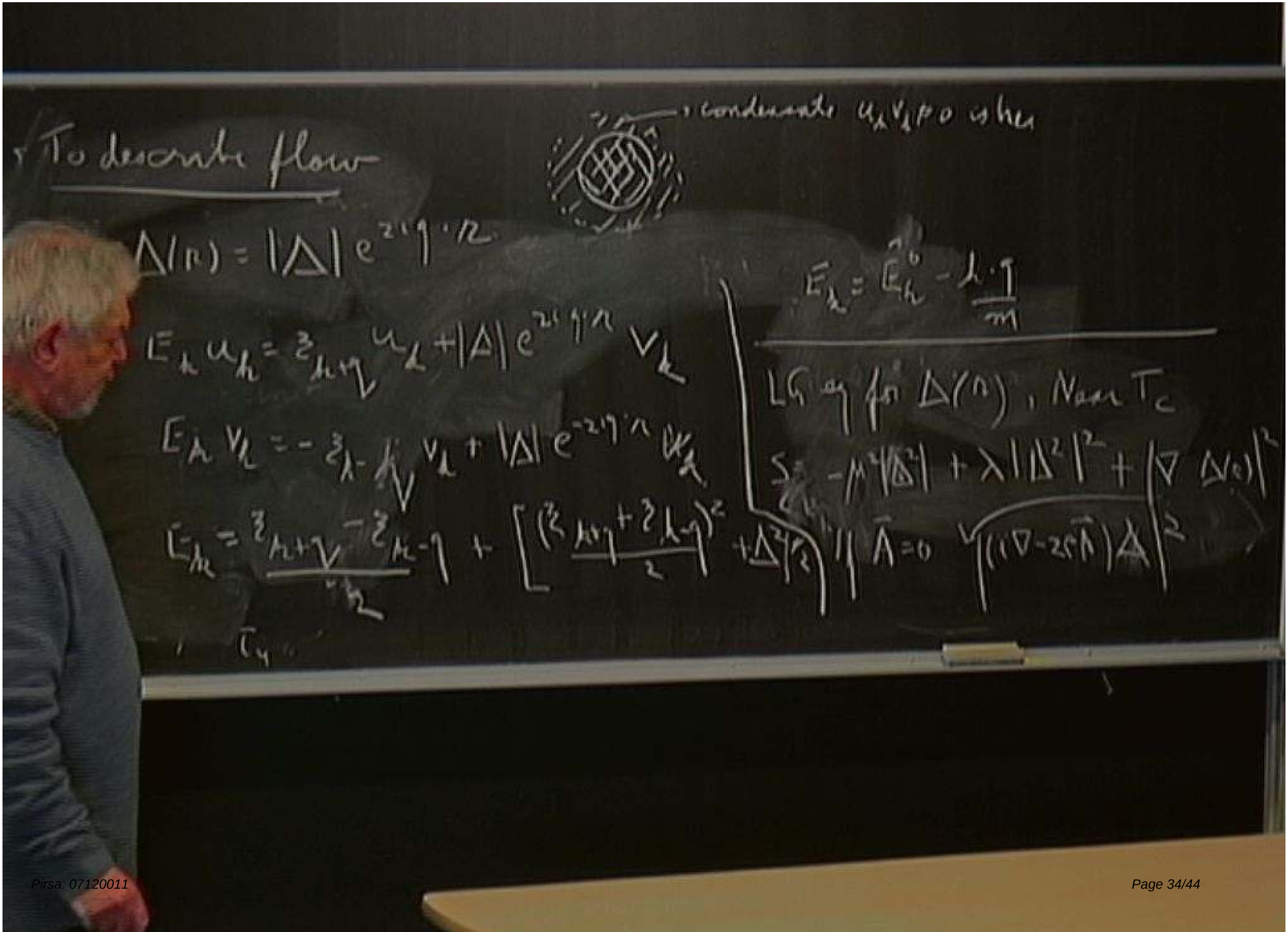
$E_k v_k = -\xi_{k-q} v_k + |\Delta| e^{-2iq\cdot r} u_k$
$E_k = \frac{\xi_{k+q} - \xi_{k-q}}{2} + \left[\left(\frac{\xi_{k+q} + \xi_{k-q}}{2}\right)^2 + \Delta^2\right]^{1/2}$
$E_k = E_k^0 - \frac{k\cdot q}{m}$
LG eq for $\Delta(r)$, Near T_c
$S = -\mu^2|\Delta| + \lambda|\Delta^2|^2 + |\nabla\Delta(r)|^2$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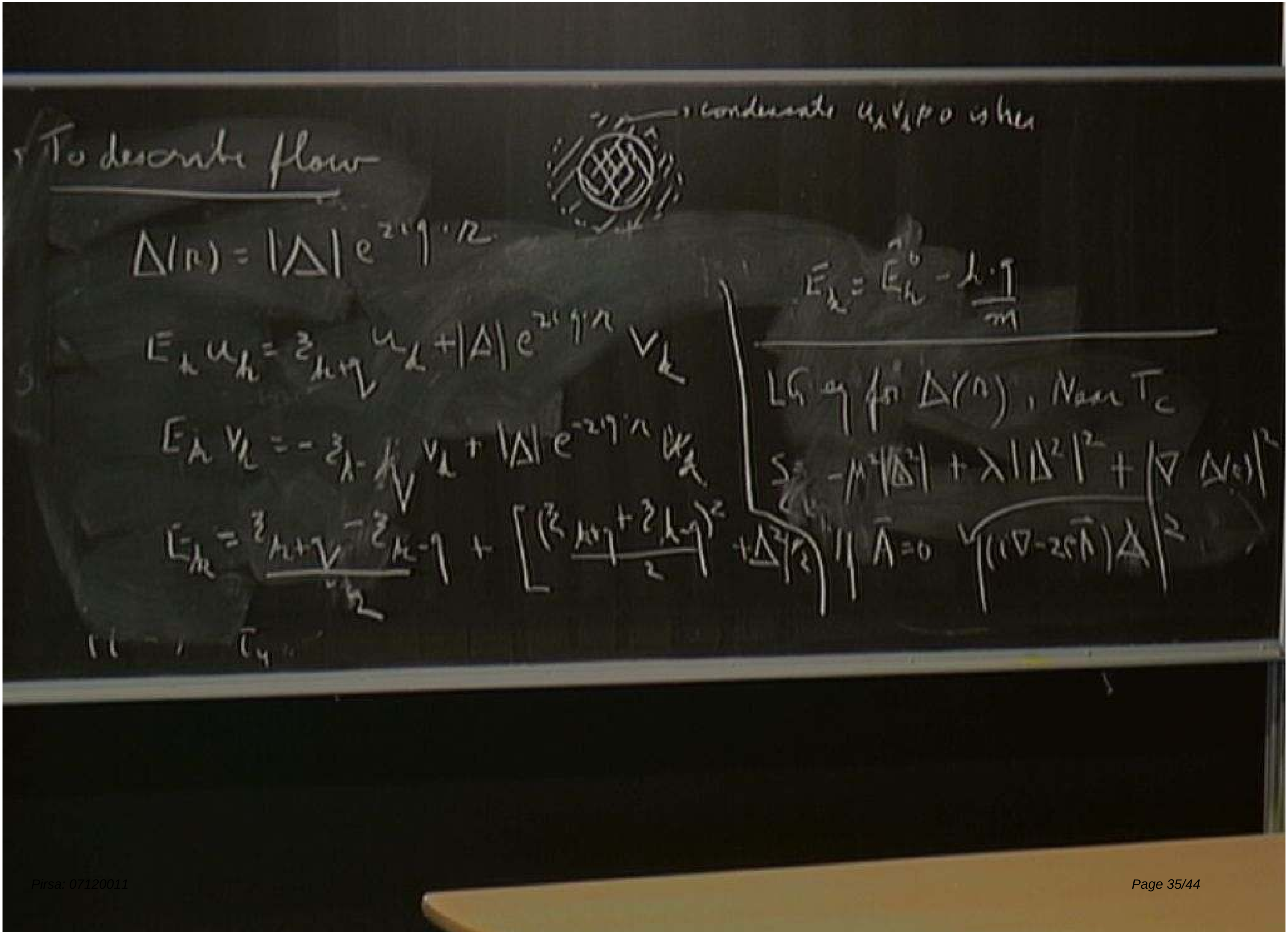
$\Delta(r) = |\Delta| e^{2iq\cdot r}$
$E_k u_k = \xi_{k+q} u_k + |\Delta| e^{2iq\cdot r} v_k$
$E_k v_k = -\xi_{k-q} v_k + |\Delta| e^{-2iq\cdot r} u_k$
$E_k = \frac{\xi_{k+q} - \xi_{k-q}}{2} + \left[\left(\frac{\xi_{k+q}+\xi_{k-q}}{2}\right)^2 + \Delta^2\right]^{1/2}$
$E_k = E_k^0 - \frac{k\cdot J}{m}$
LG eq for $\Delta(r)$, Near T_c
$S = -\mu^2|\Delta|^2 + \lambda|\Delta^2|^2 + |\nabla \Delta(r)|^2$
if $\vec{A} = 0$
$|(i\nabla - 2e\vec{A})\Delta|^2$


To describe flow
condensate
Near

$$\frac{\langle \Delta N^2 \rangle}{\langle N \rangle^2} = \frac{1}{\rho} \frac{1}{\langle N \rangle} \frac{\partial \langle N \rangle}{v \, \partial \mu} = \frac{1}{\rho v} \frac{\partial \ln n}{\partial \mu} \; ; \quad d\mu = v \, dp \text{ at fixed}$$

$$v = \frac{V}{N} = \frac{1}{n}$$

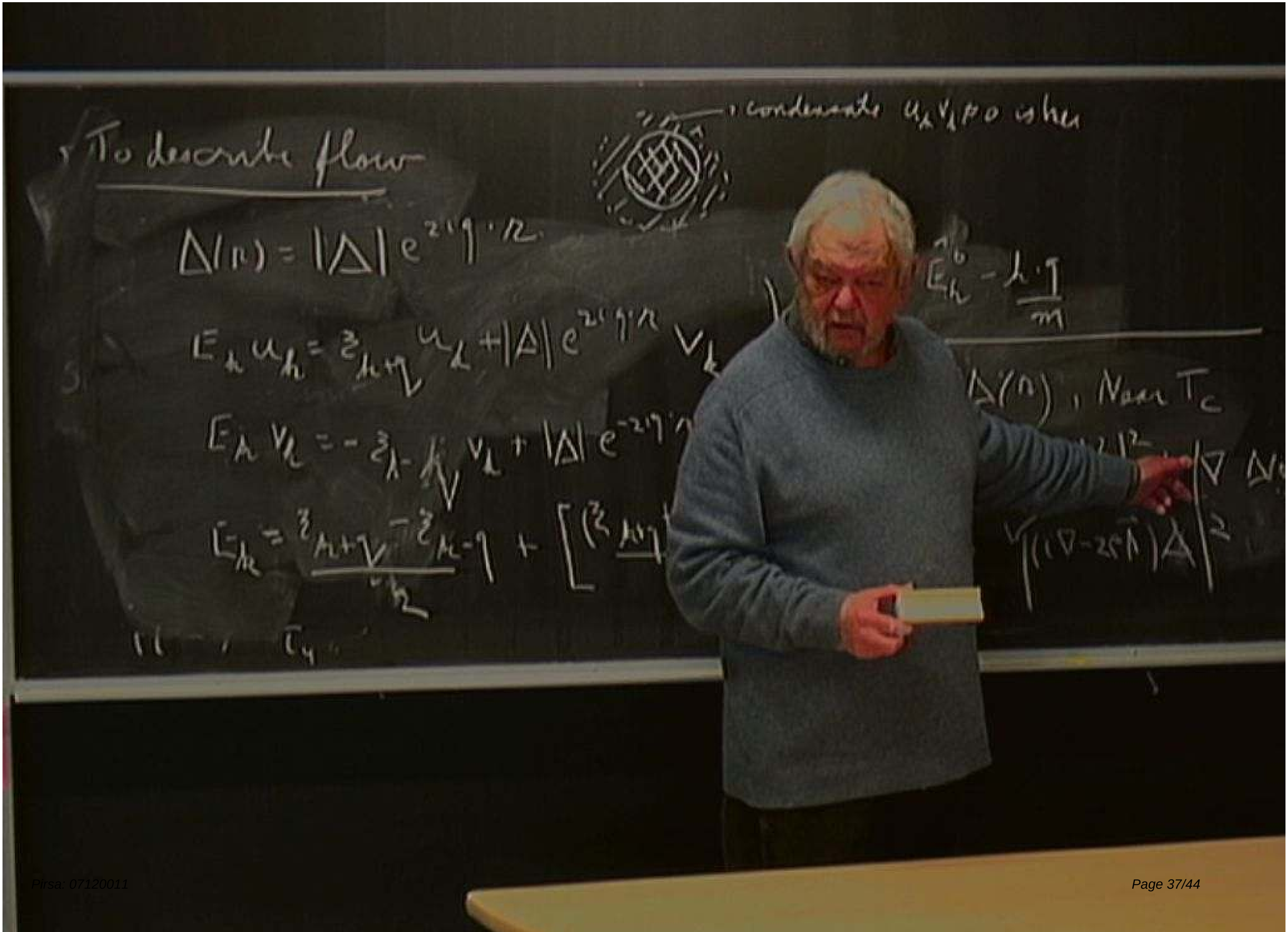
$$\Delta \to \Delta e^{i\chi(r)}$$

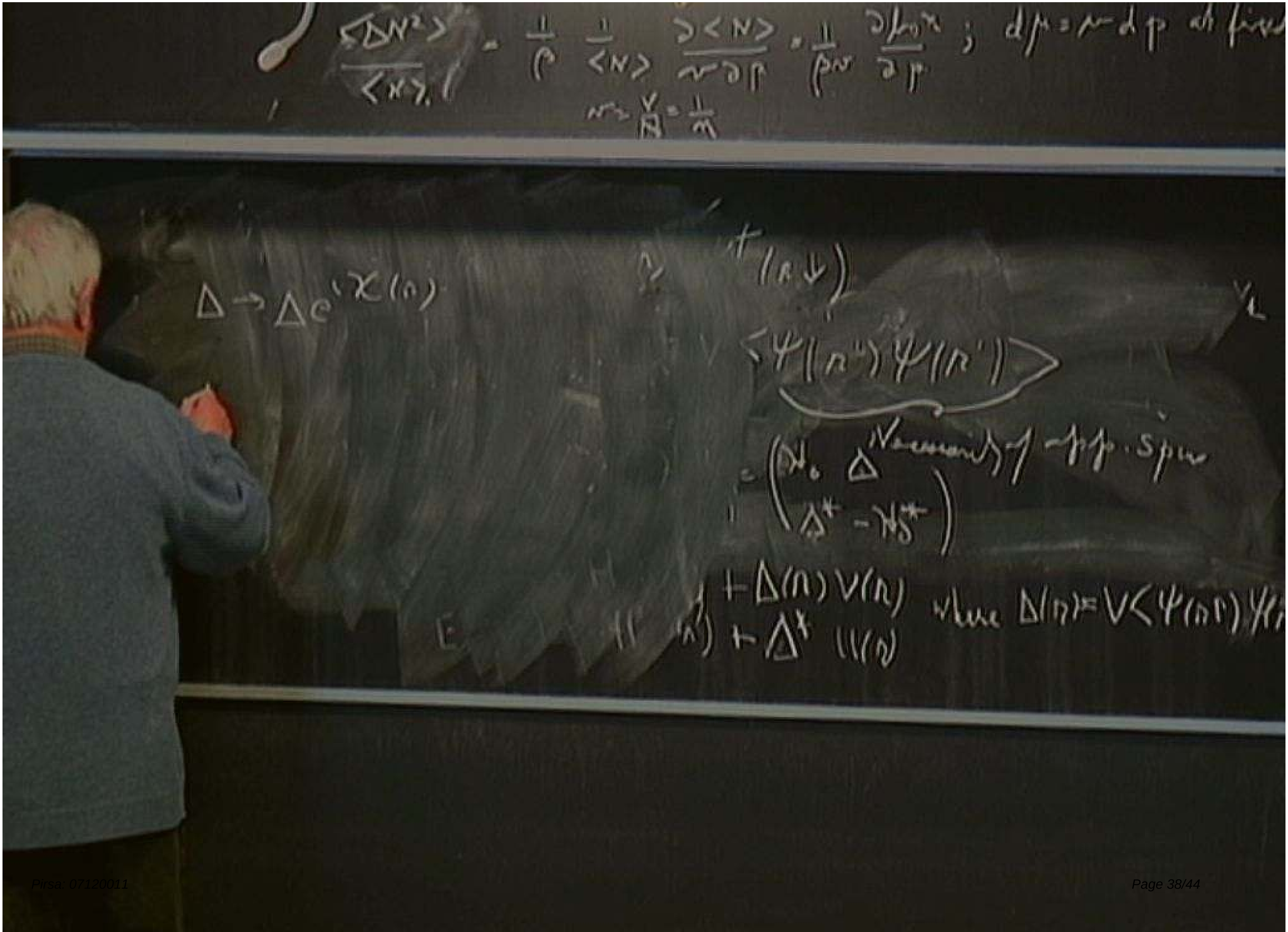
$$\langle \psi(r\uparrow) \psi(r\downarrow) \rangle$$

$$\begin{pmatrix} H_0 & \Delta \\ \Delta^* & -H_0^* \end{pmatrix}$$ Necessarily of spin

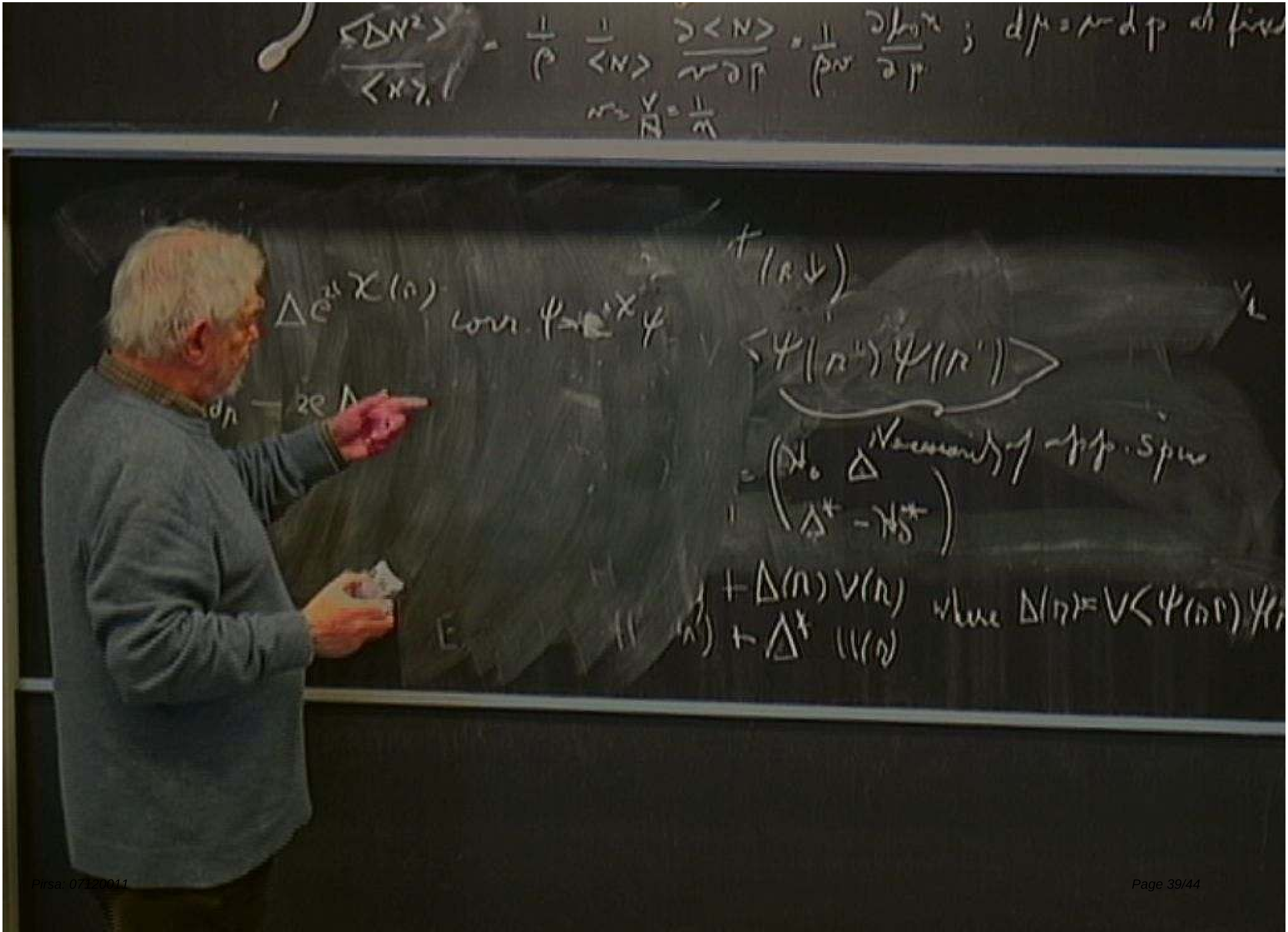
$+ \Delta(r) V(r)$ where $\Delta(r) = V \langle \psi(r\uparrow) \psi(r\downarrow) \rangle$

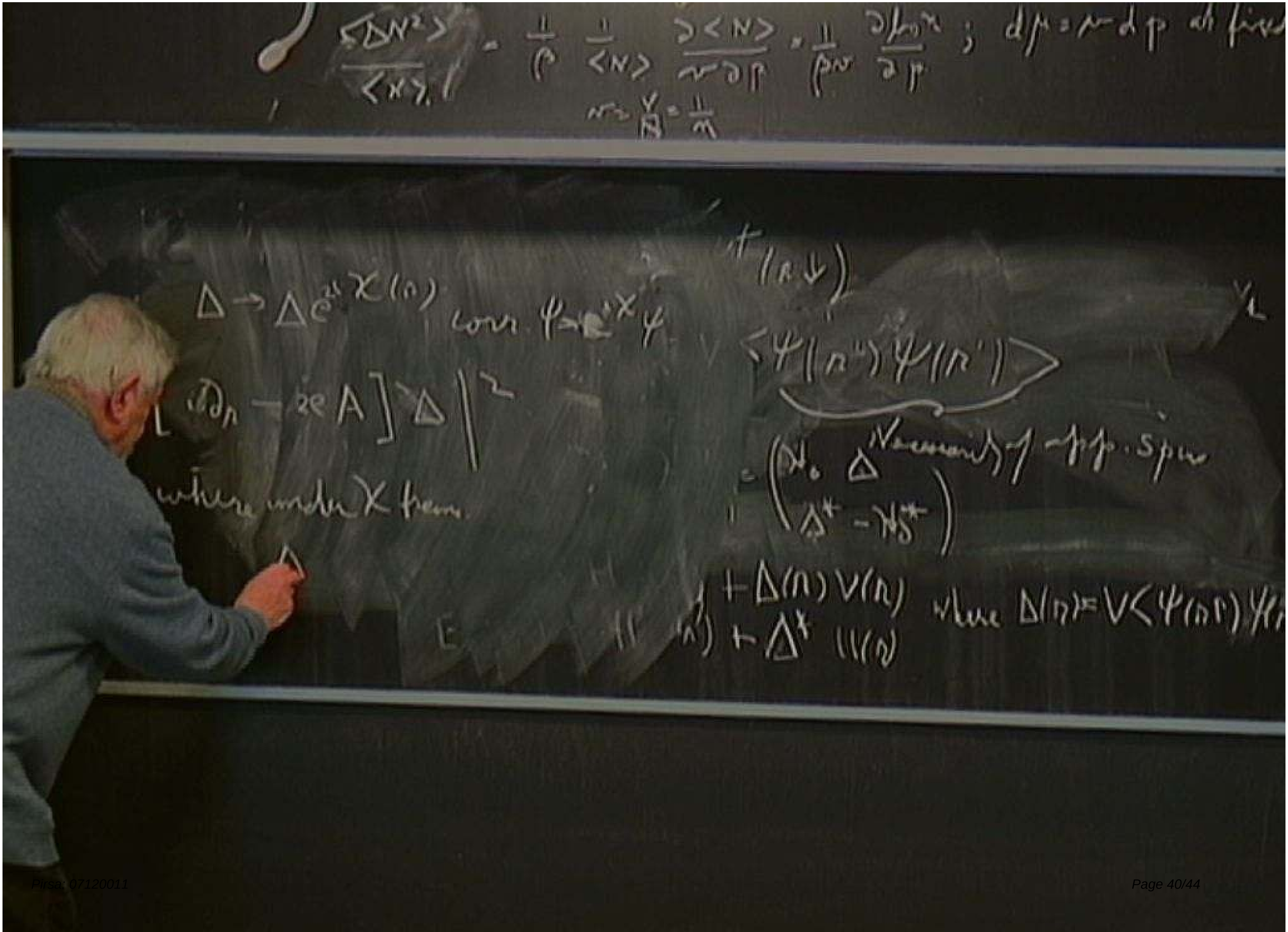
$+ \Delta^* U(r)$

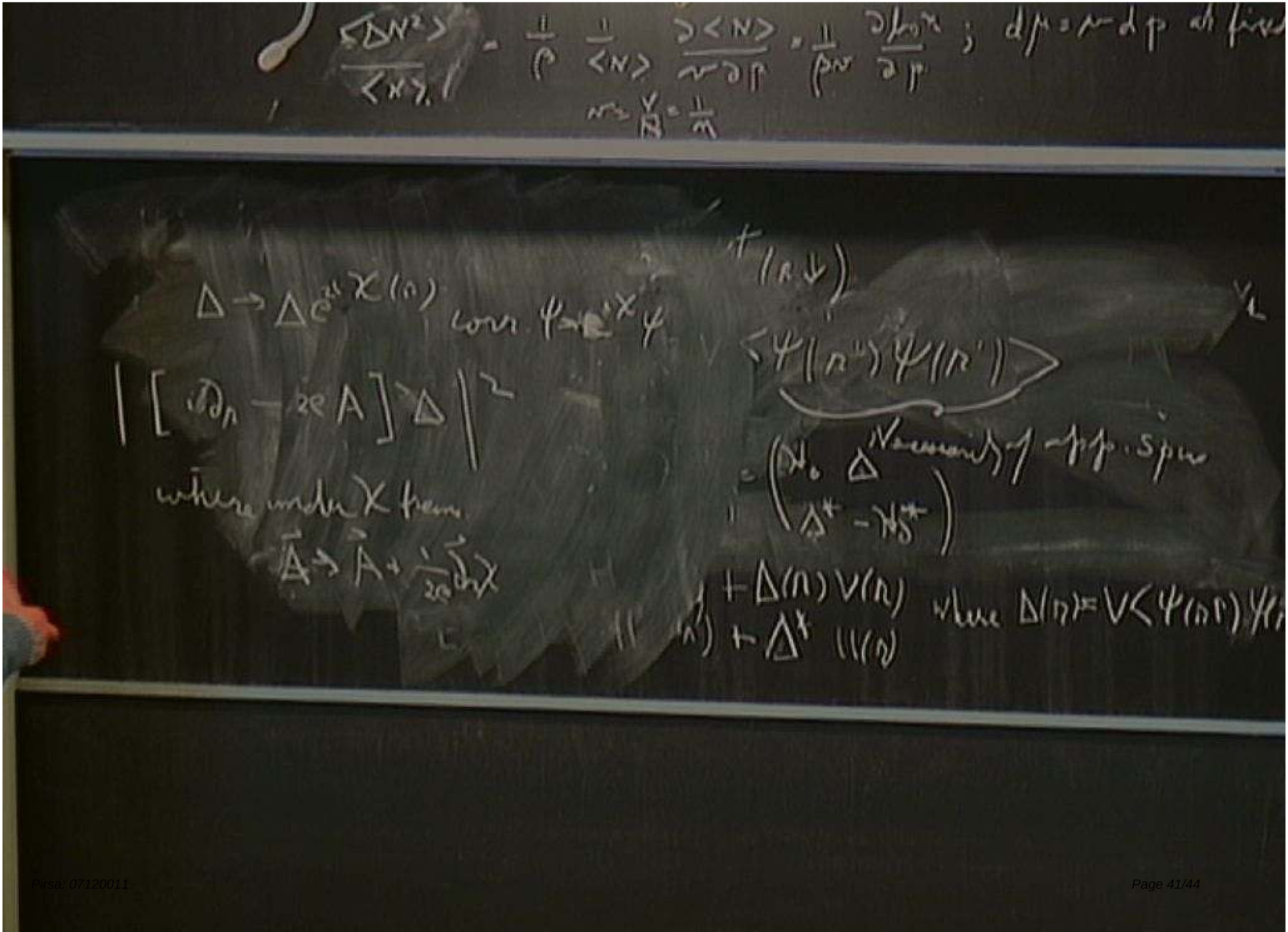

To describe flow
condensate
Near T_c

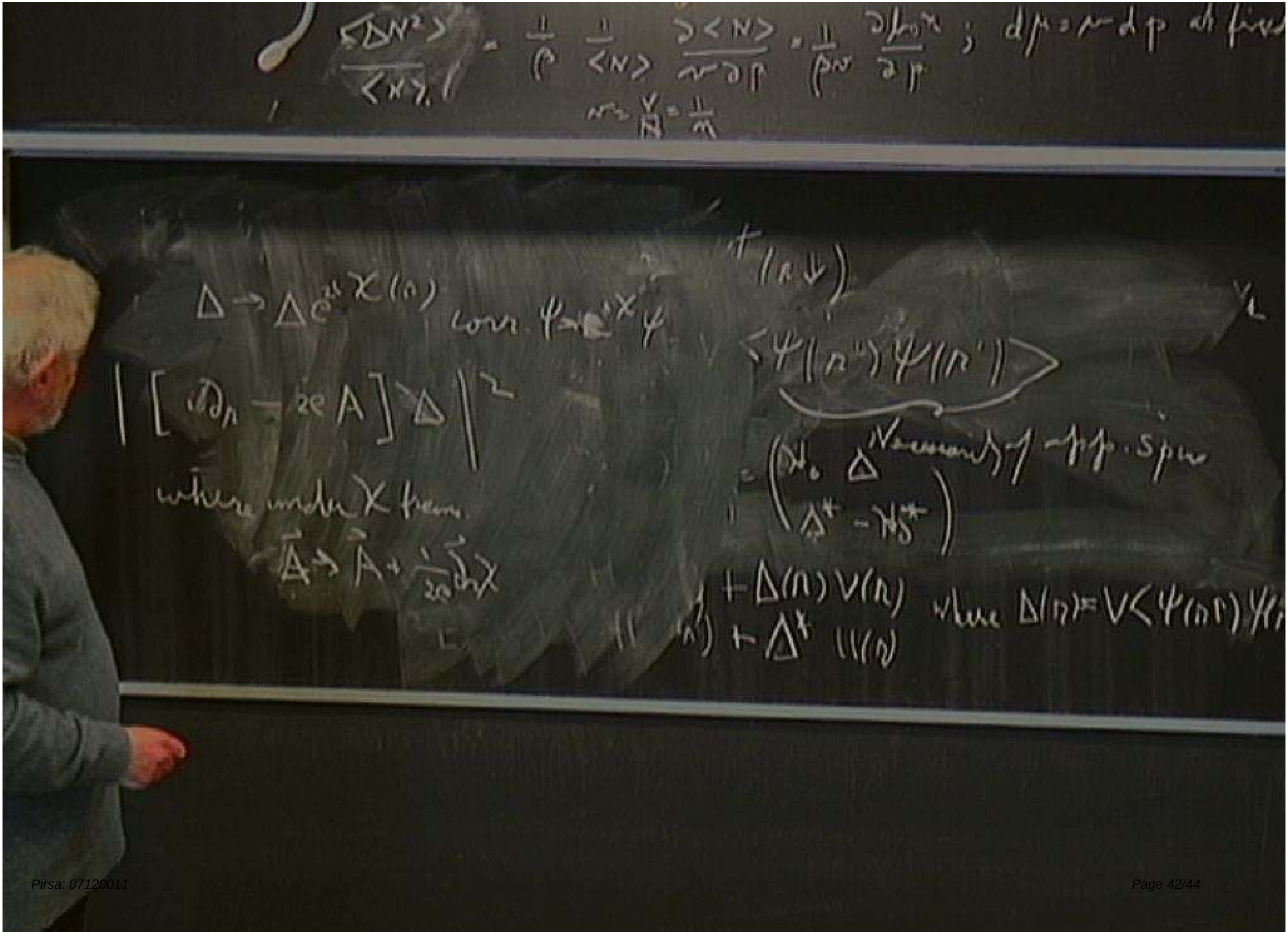


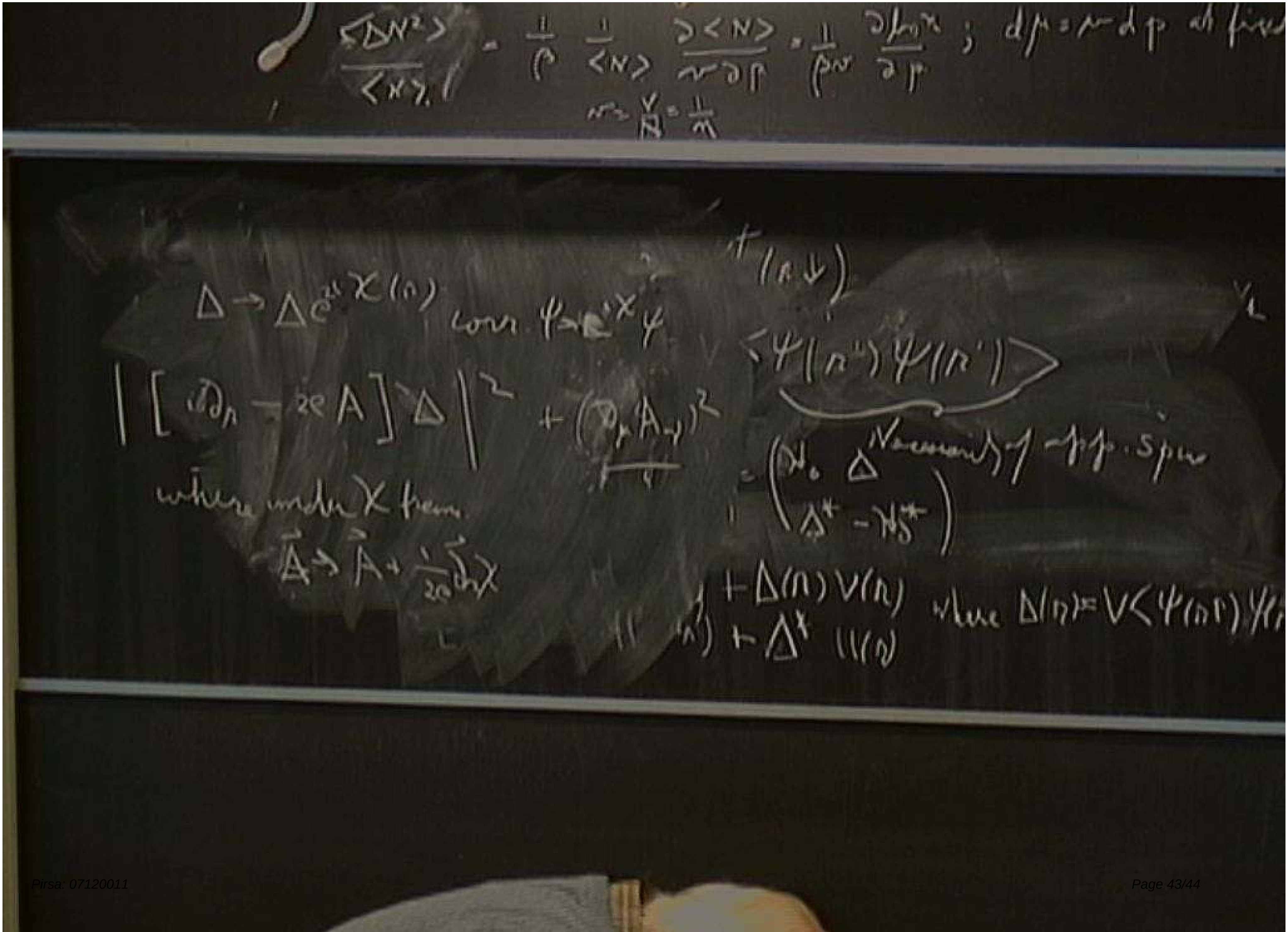
Δ → Δe^{iχ(r)}
⟨ψ(r↑)ψ(r↓)⟩
+ Δ(r) V(r)
where Δ(r) = V⟨ψ(r↑)ψ(r↓)⟩




Δ → Δ e^{2i χ(r)}
where under χ trans.
where Δ(r) = V<ψ(r↑) ψ(r↓)>







where