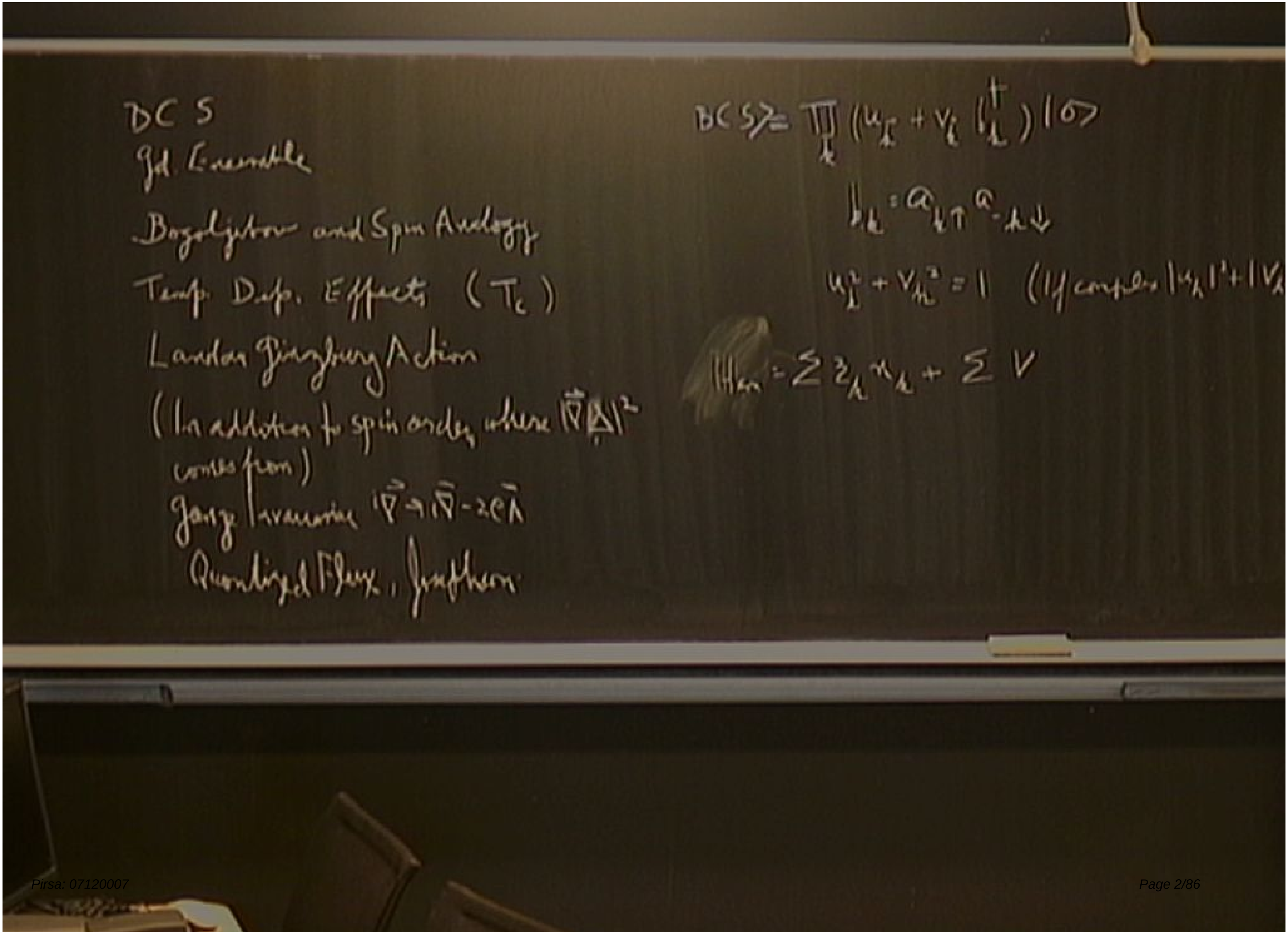
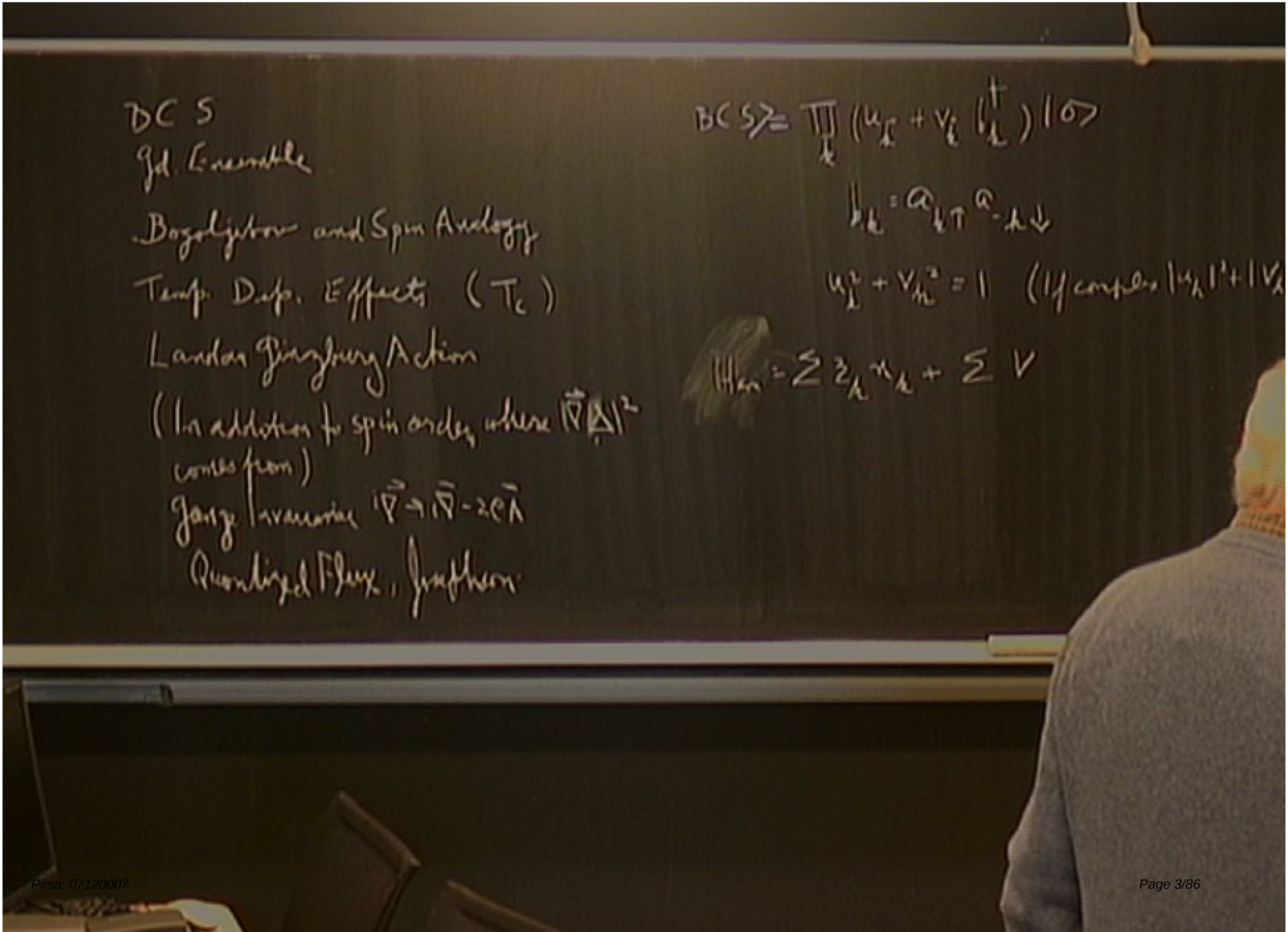
Title: Spontaneous Broken Symmetry 6A

Date: Dec 06, 2007 10:00 AM

URL: http://pirsa.org/07120007

Abstract:


BCS
Gd. Ensemble
Bogoliubov and Spin Analogy
Temp. Dep. Effects (T_c)
Landau Ginzburg Action
((In addition to spin order, where |∇Δ|^2 comes from))
Gauge Invariance ∇ → ∇ - 2eA
Quantized Flux, Josephson
|BCS> = Π_k (u_k + v_k b_k^†) |0>
b_k = a_{k↑} a_{-k↓}
u_k^2 + v_k^2 = 1 ((If complex |u_k|^2 + |v_k
H_m = Σ ε_k n_k + Σ V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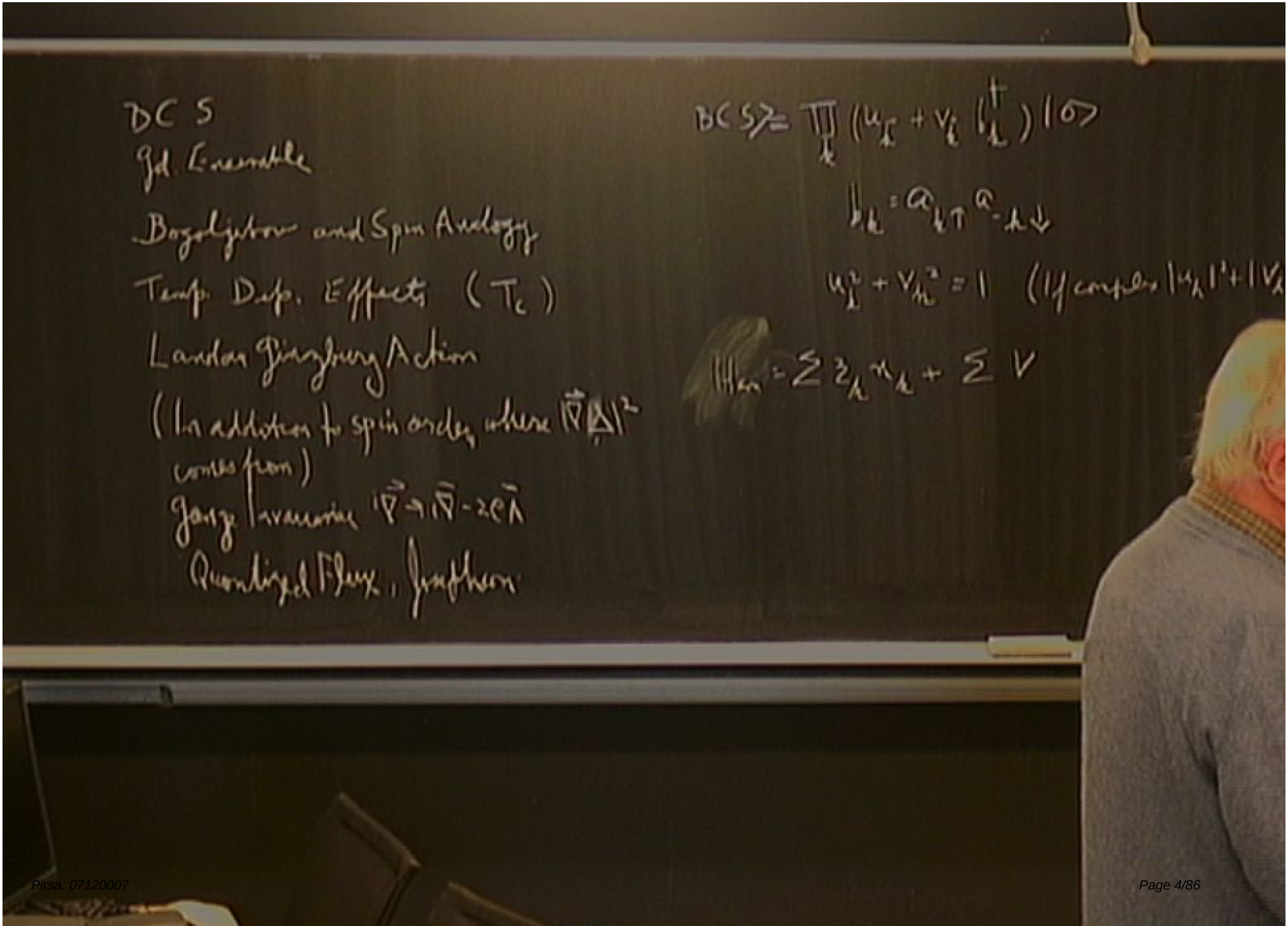
$|BCS\rangle = \prod_k (u_k + v_k b_k^\dagger)|0\rangle$
$u_k^2 + v_k^2 = 1$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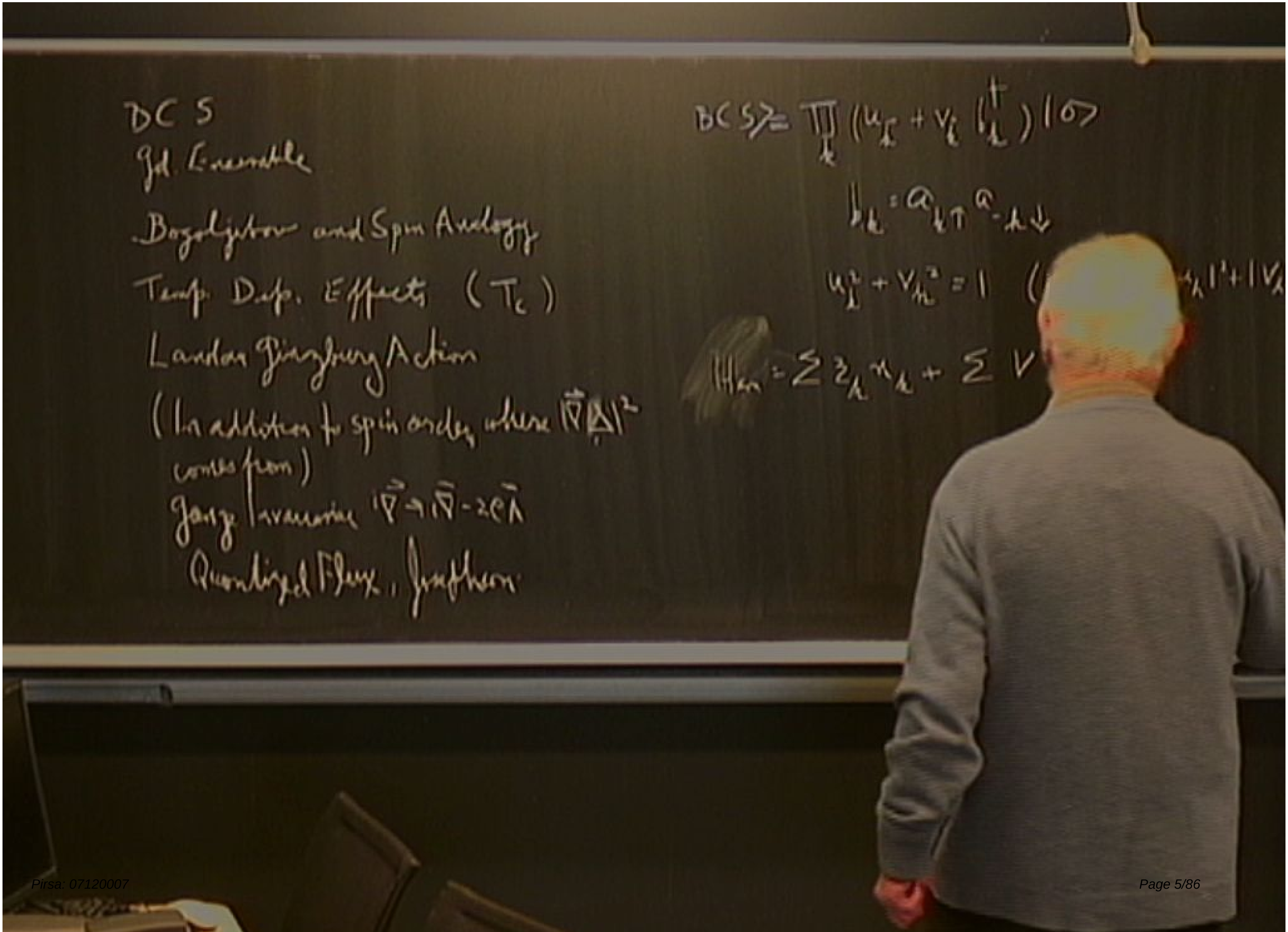
$b_k = a_{k\uparrow} a_{-k\downarrow}$
$u_k^2 + v_k^2 = 1$ ((If complex $|u_k|^2 + |v_k$
$H = \sum \epsilon_k n_k + \sum V$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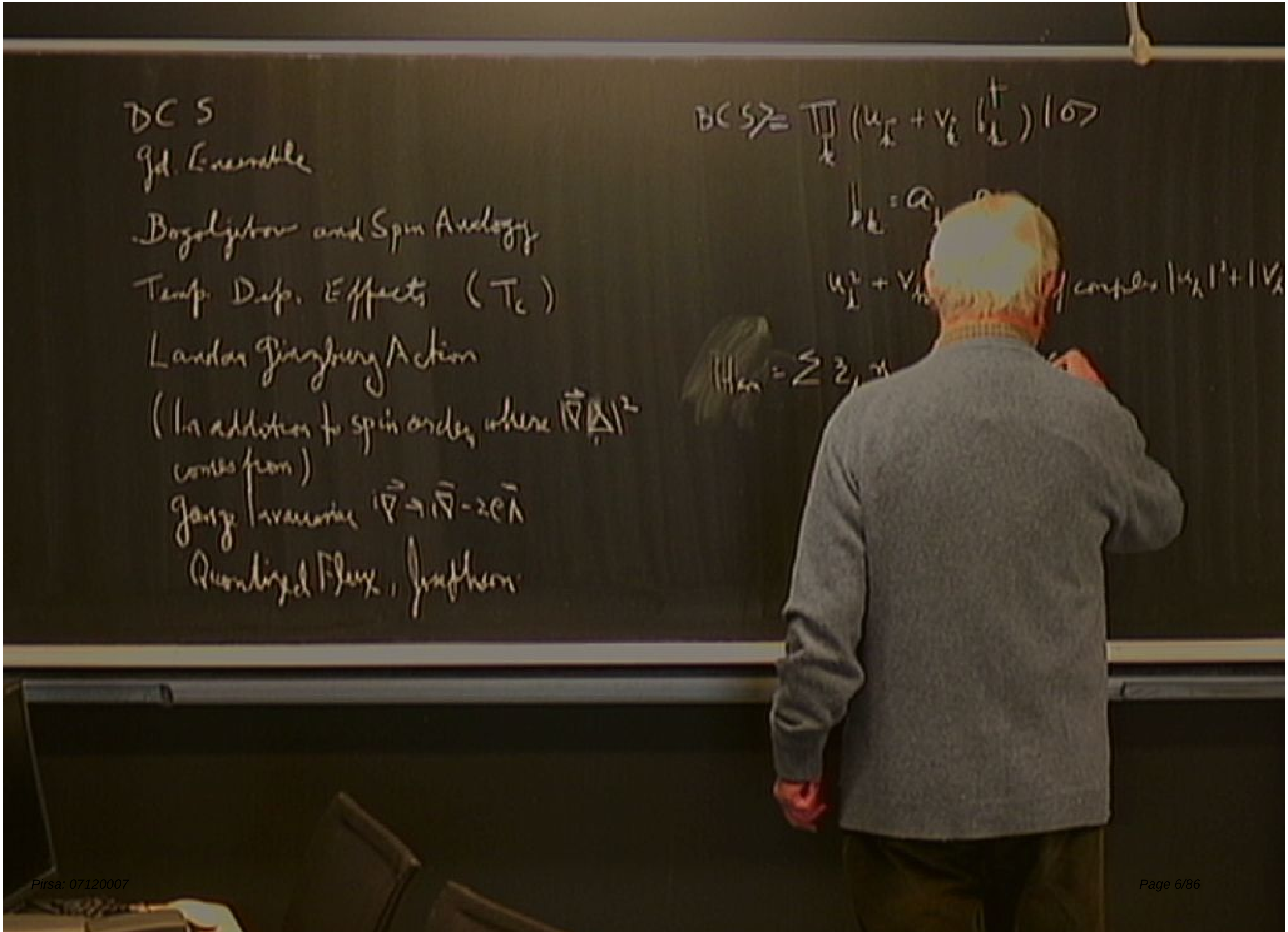
|BCS> = ∏_k (u_k + v_k b_k†) |0>
b_k = a_k↑ a_−k↓
u_k² + v_k² = 1
H = Σ ε_k n_k + Σ V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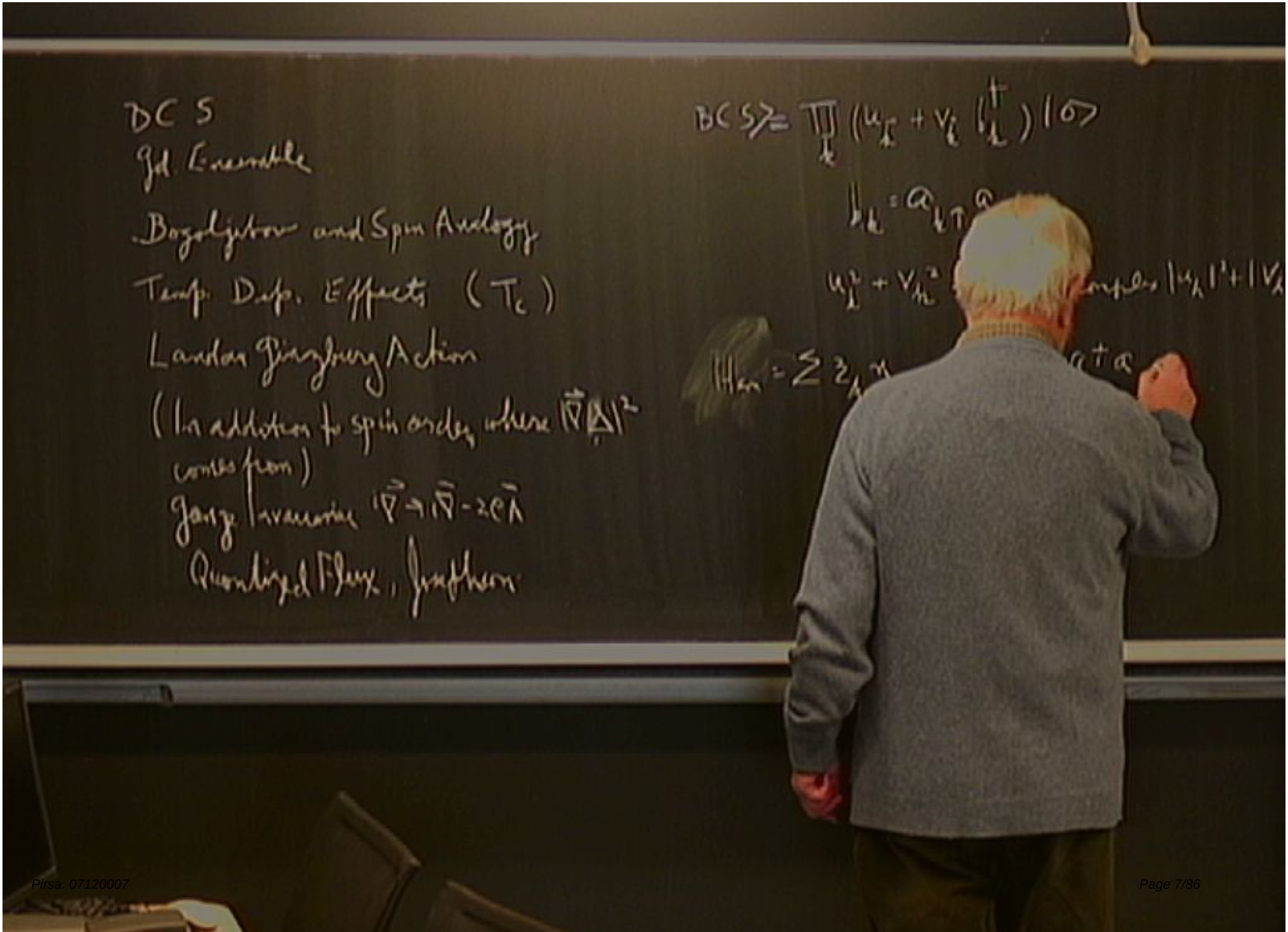
|BCS> = ∏k (uk + vk bk†) |0>
bk = ak
complex |uk|² + |vk|²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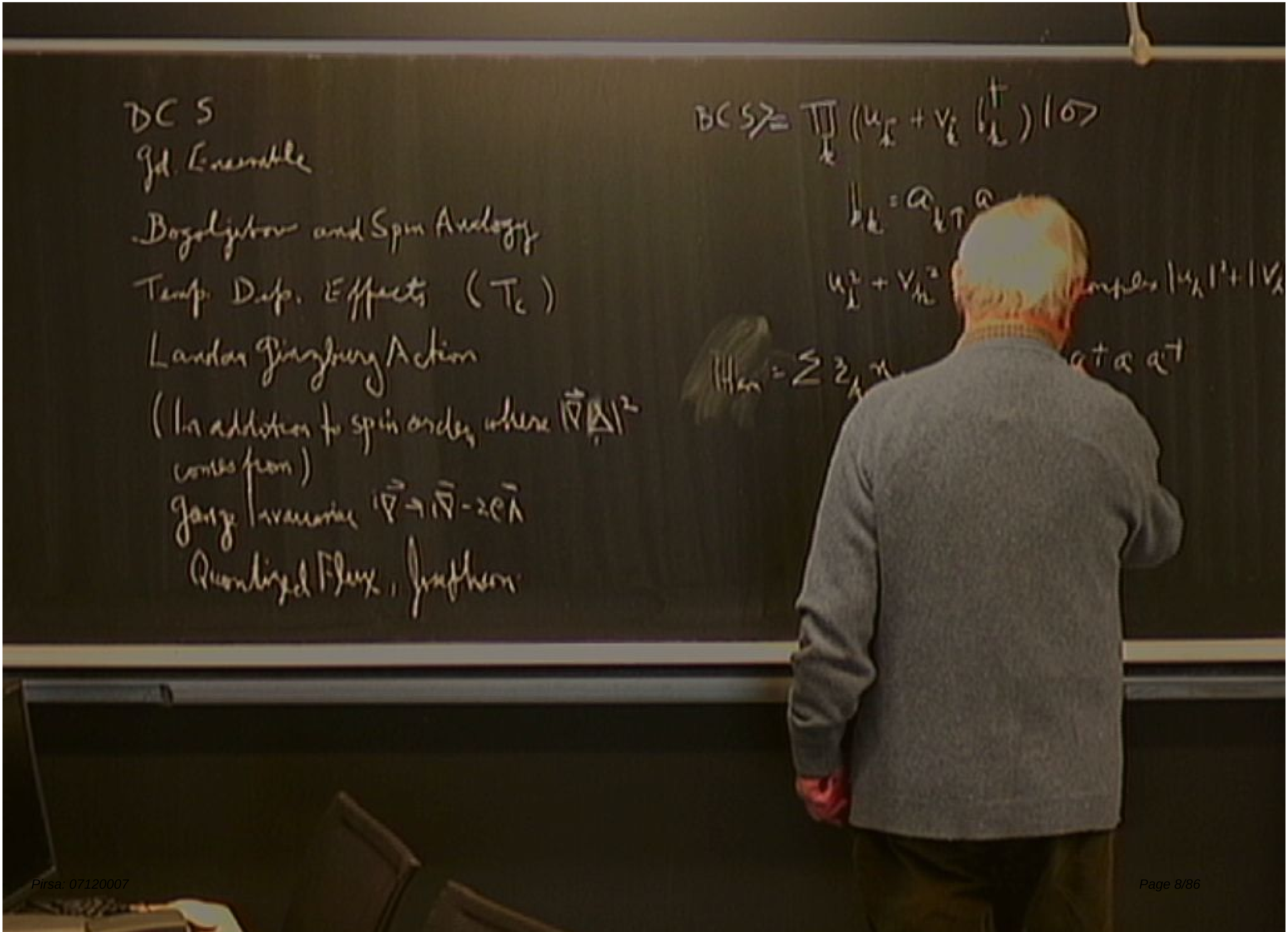
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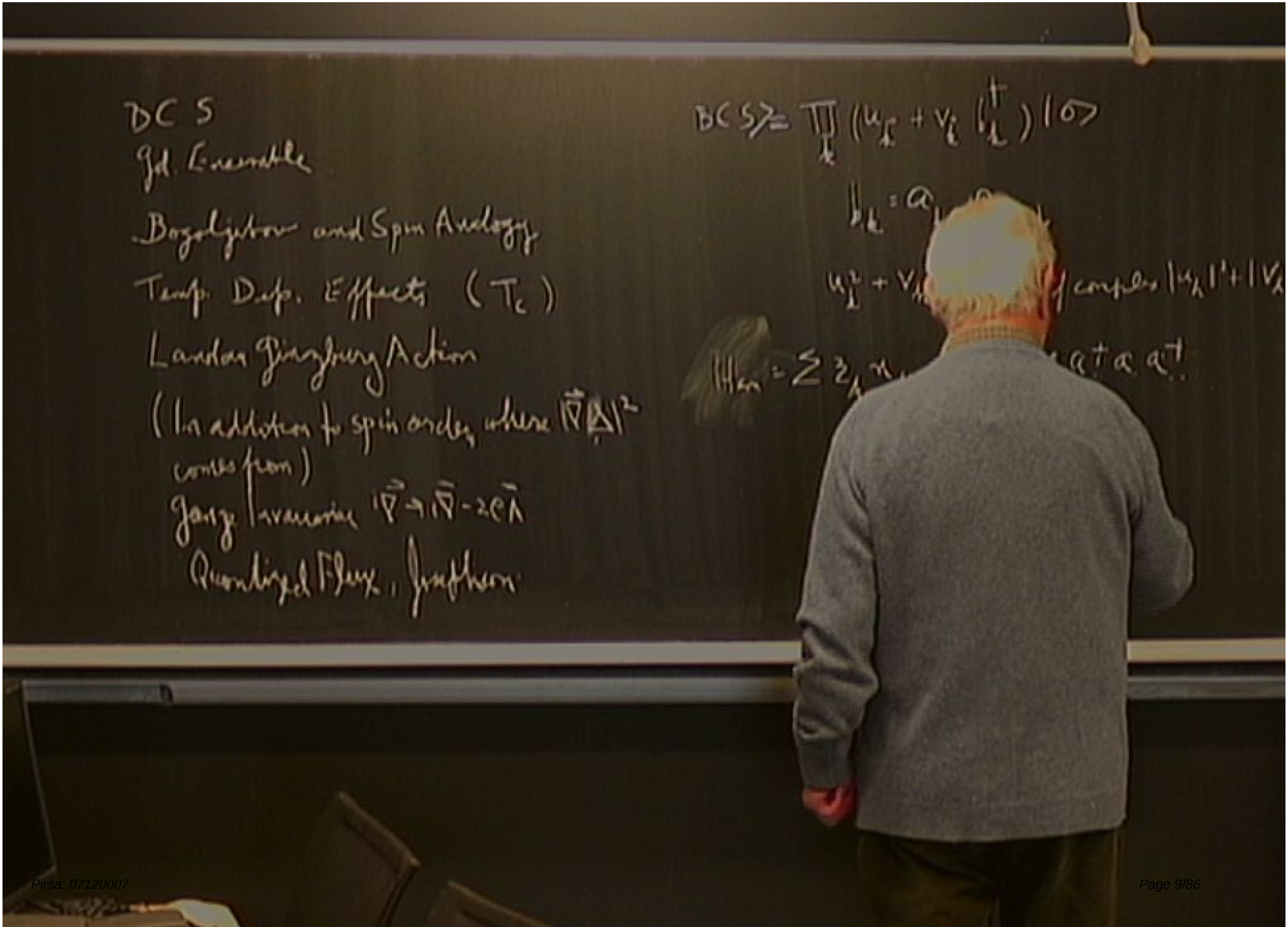
|BCS> = ∏k (uk + vk bk†)|0>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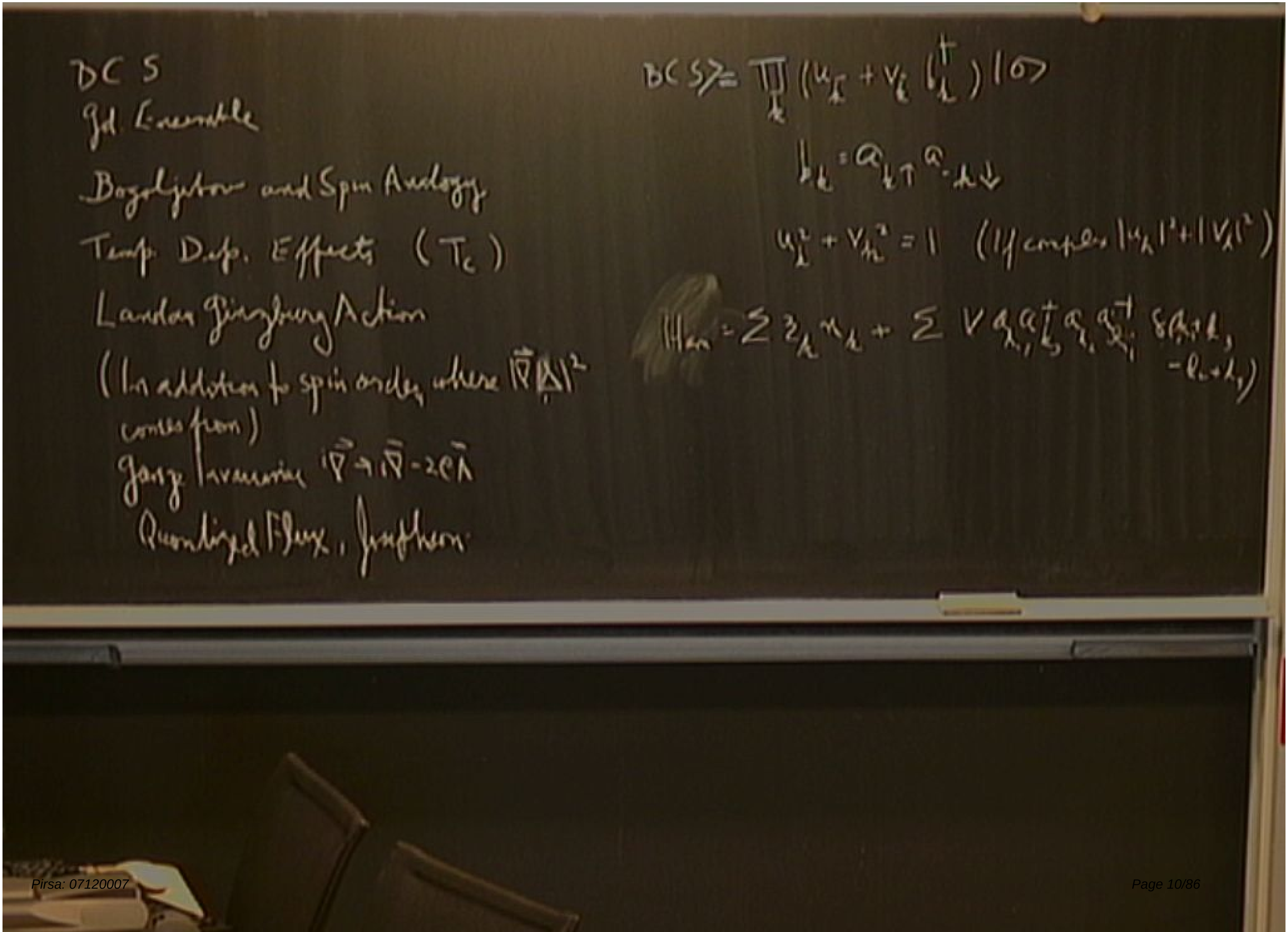
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Gauge Invariance
Quantized Flux, Josephson
|BCS> = ∏ (u + v b†) |0>
u² + v² = 1

BCS

Gd. Ensemble

Bogoliubov and Spin Analogy

Temp. Dep. Effects (T_c)

Landau Ginzburg Action

(In addition to spin order, where $|\vec{\nabla}\Delta|^2$ comes from)

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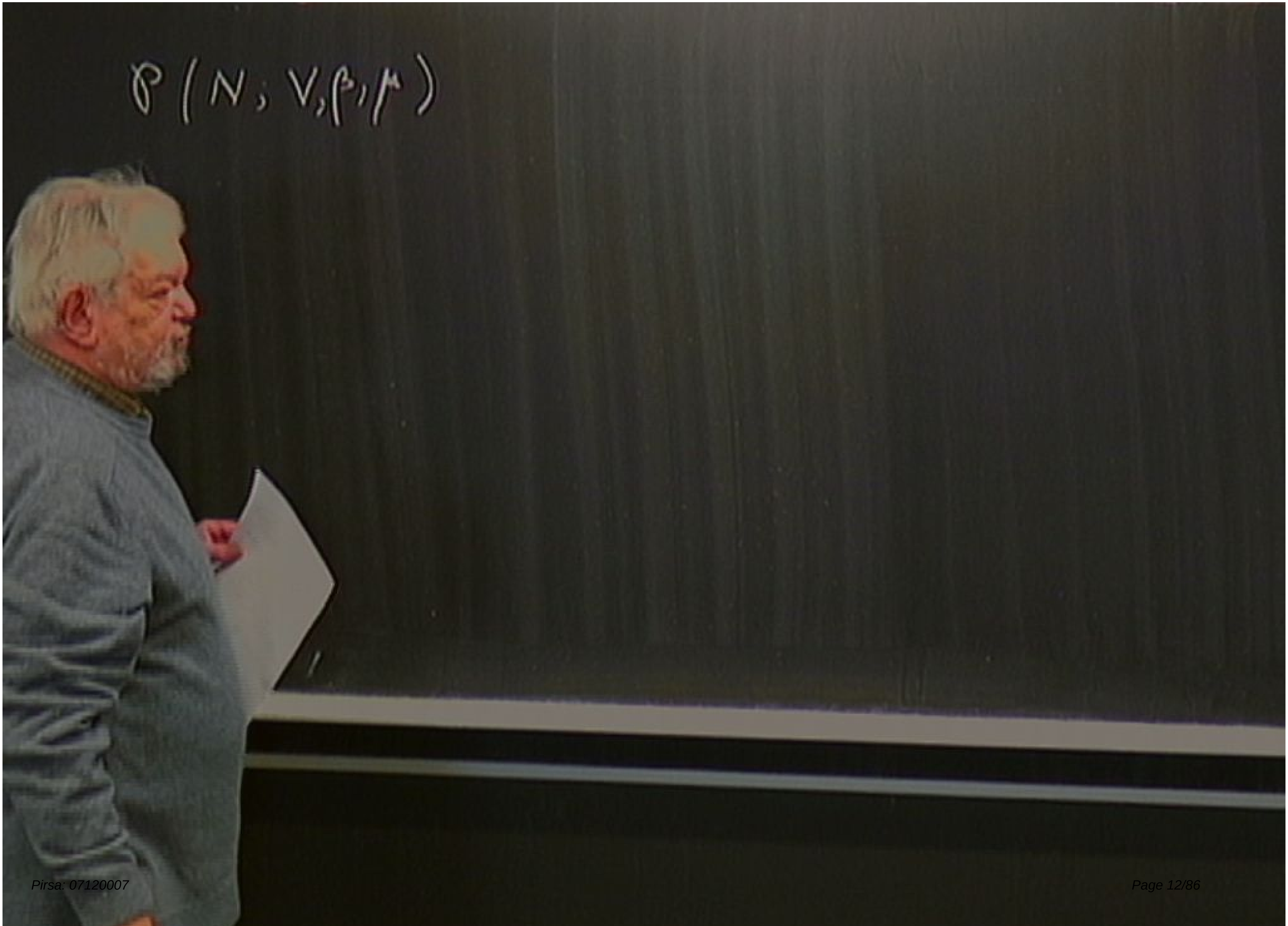
Quantized Flux, Josephson

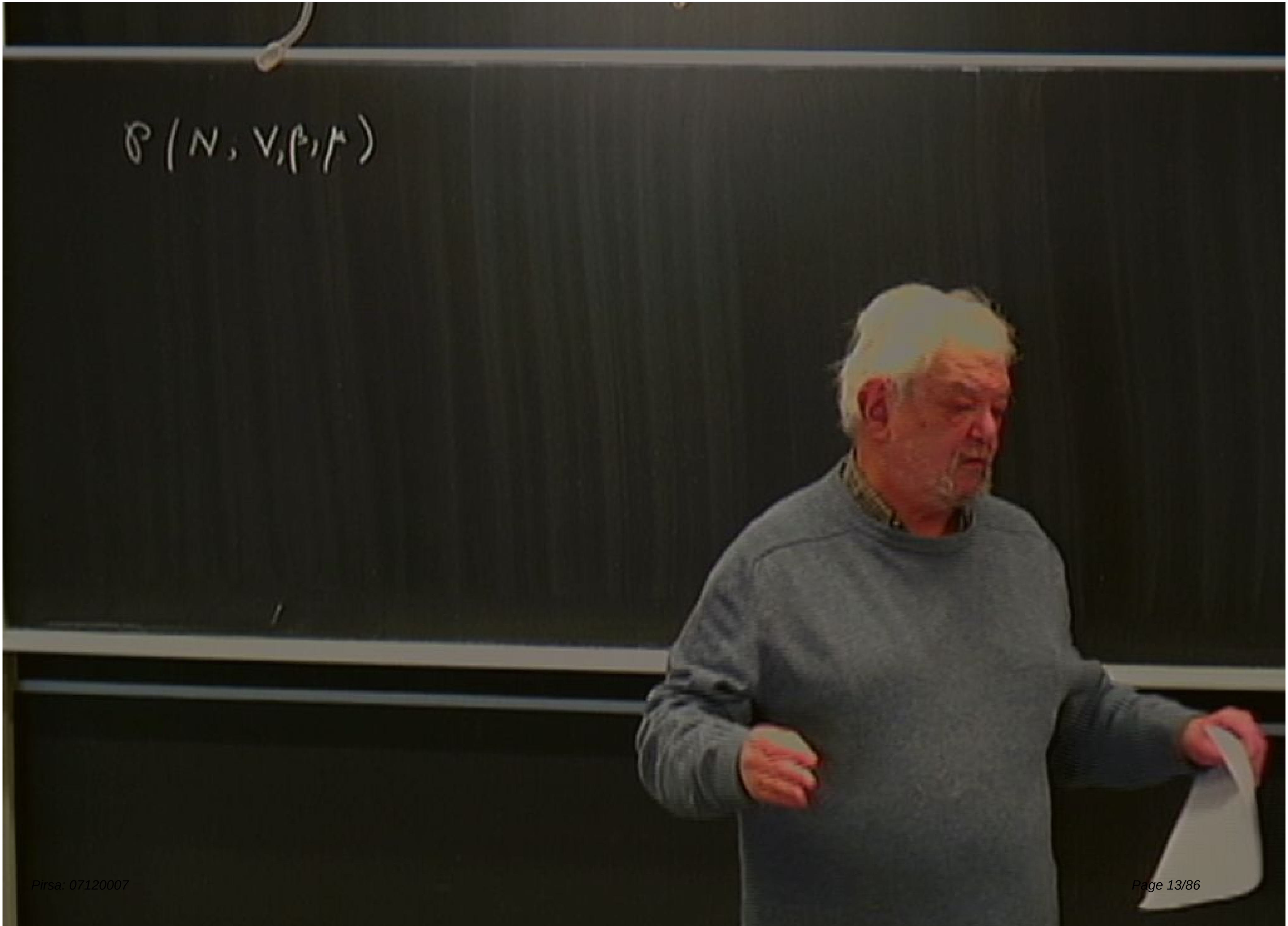
$|BCS\rangle = \prod_k (u_k + v_k b_k^\dagger)|0\rangle$

$b_k = a_{k\uparrow} a_{-k\downarrow}$

$u_k^2 + v_k^2 = 1$ ((If complex $|u_k|^2 + |v_k|^2$))

$H_{eff} = \sum \epsilon_k n_k + \sum V a_{k_1} a^\dagger_{k_2} a_{l_1} a^\dagger_{l_2} \delta(k_1 + k_2 - l_1 + l_2)$


ρ(N; V,β,μ)

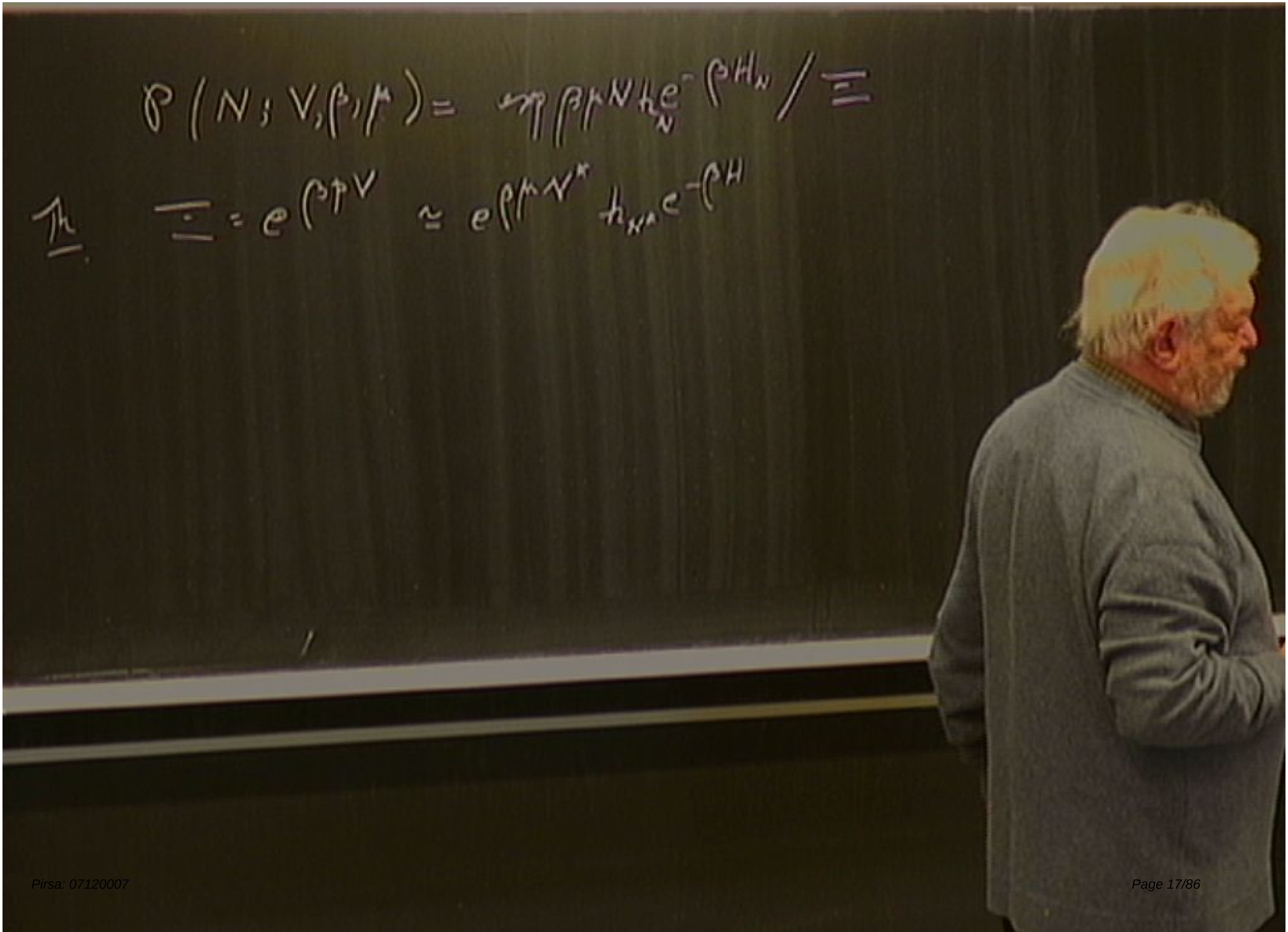

ρ(N, V, β, μ)

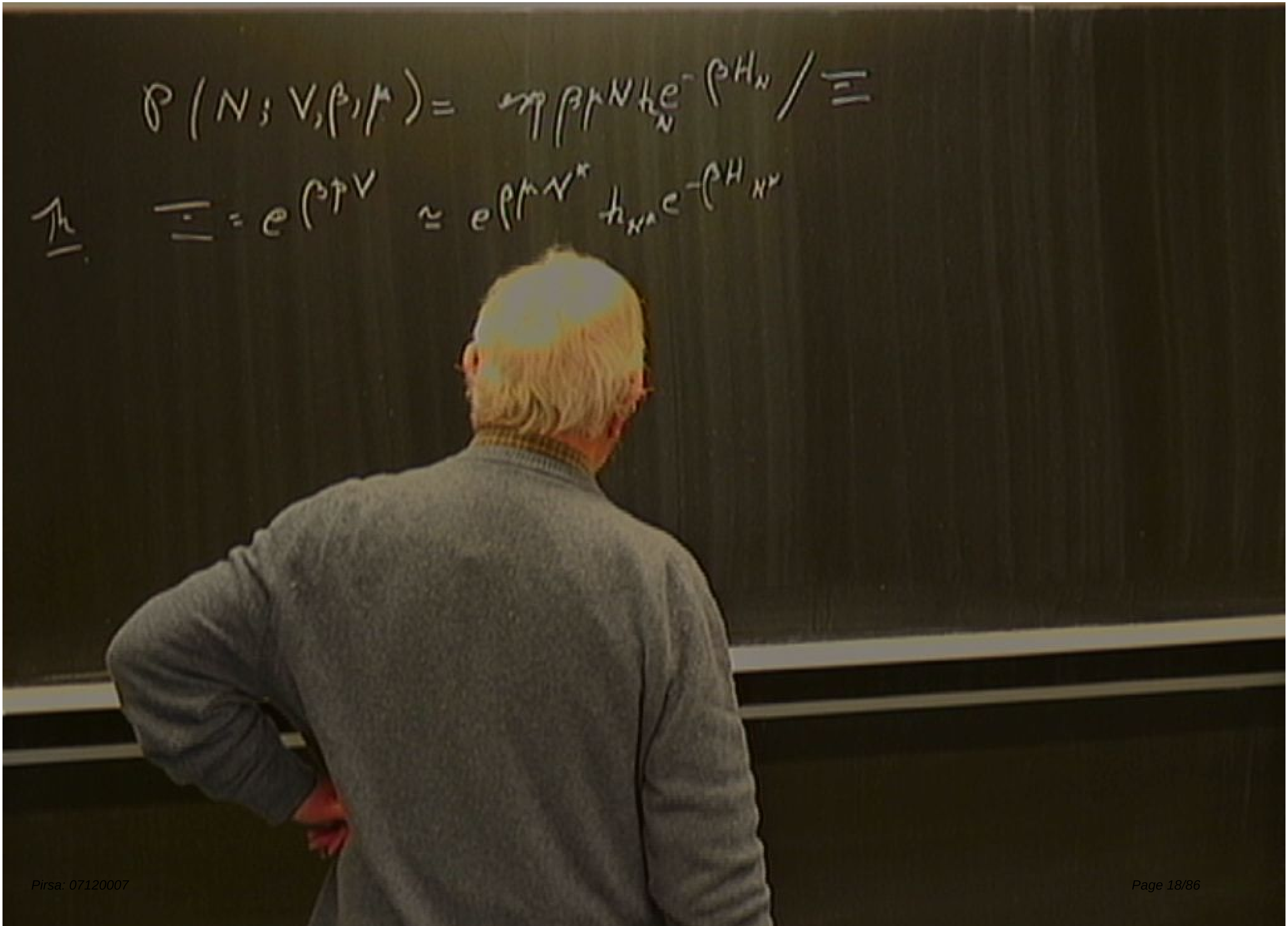
$$\rho(N; V, \beta, \mu) = \beta\mu^{N} e^{-\beta H}$$

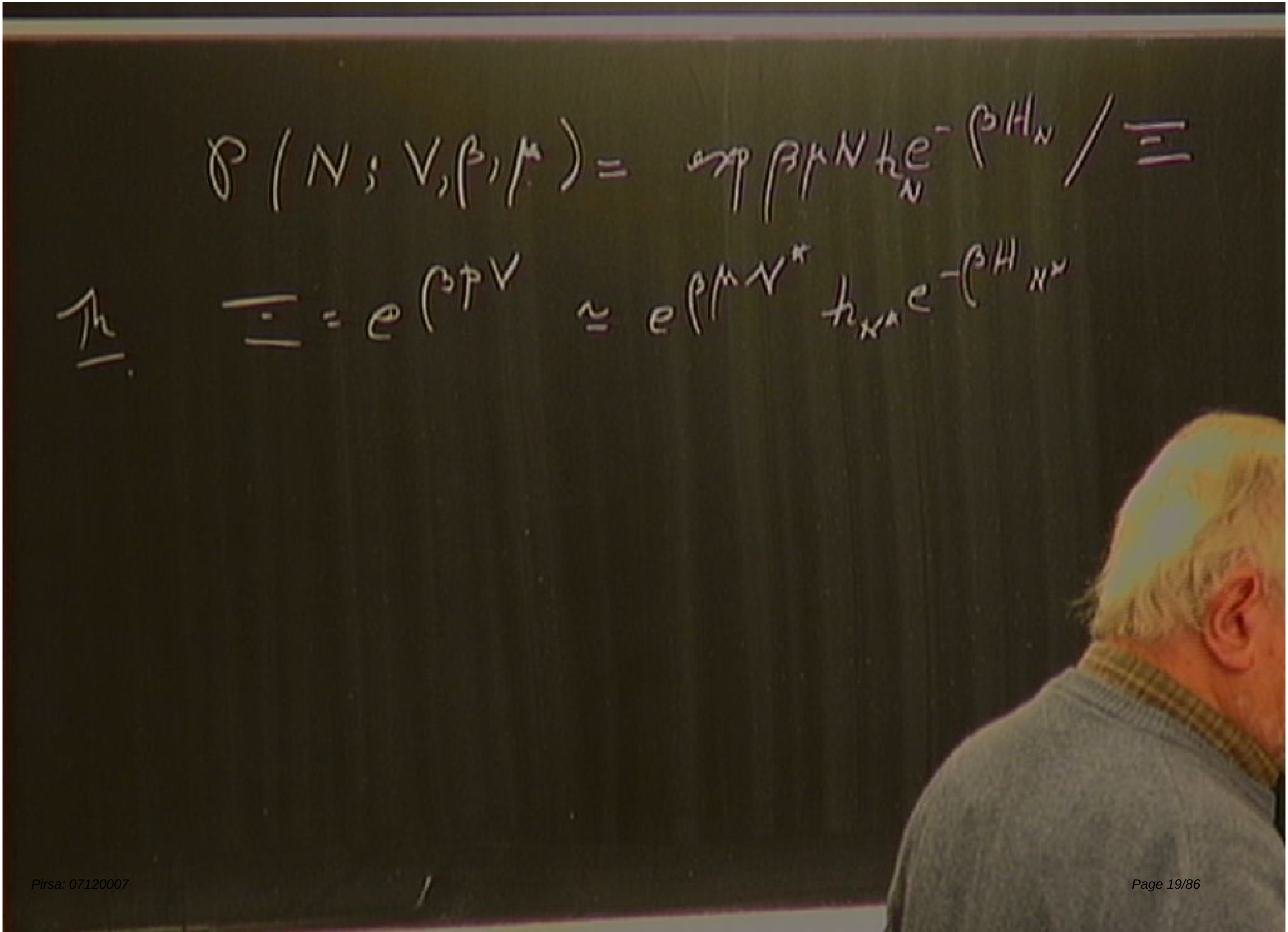
$$\rho(N; V, \beta, \mu) = e^{\beta\mu N}\, \mathrm{tr}_N\, e^{-\beta H} / \Xi$$

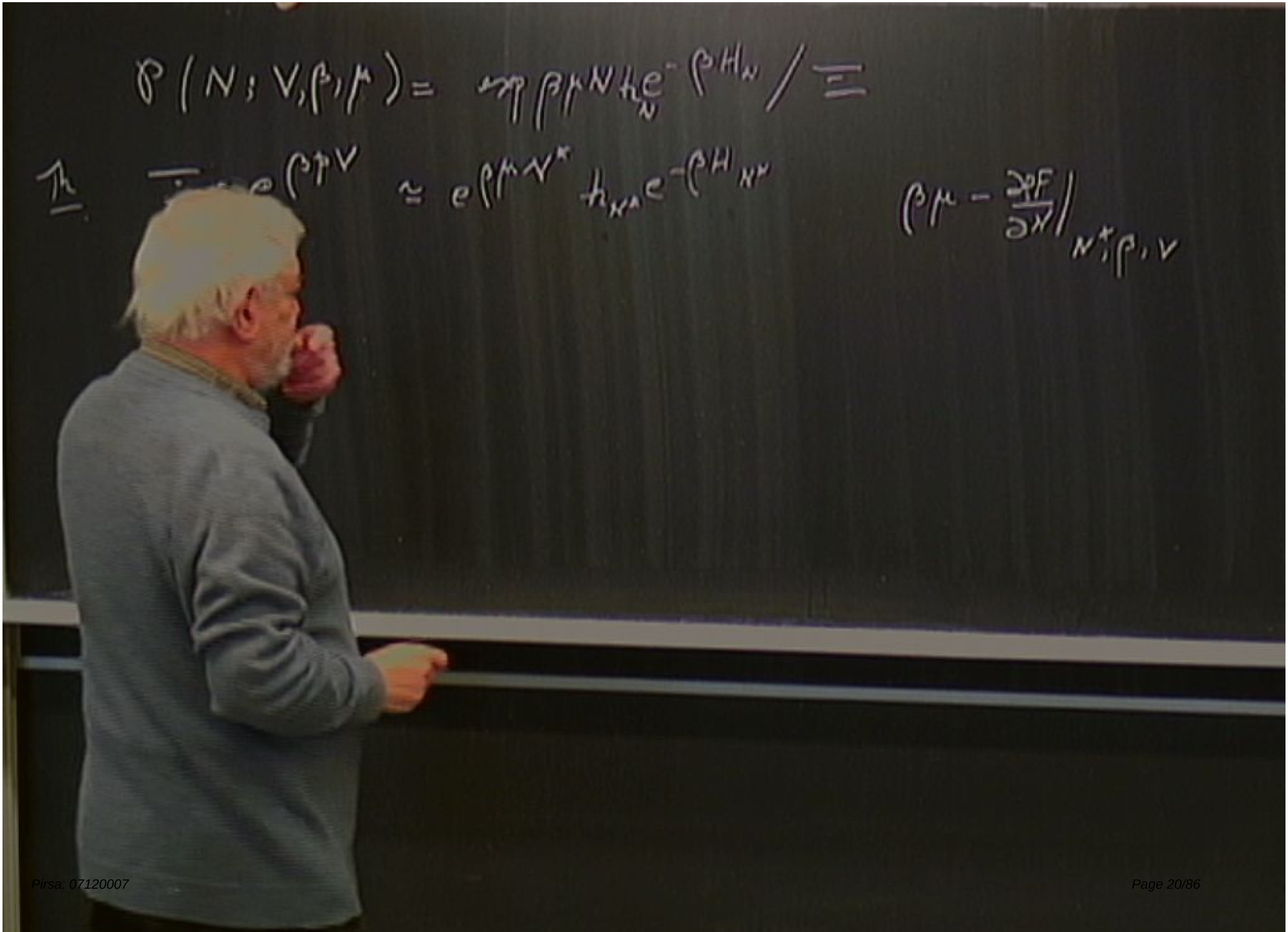
$$\rho(N; V, \beta, \mu) = \exp \beta \mu N \, e^{-\beta H_N} / \Xi$$

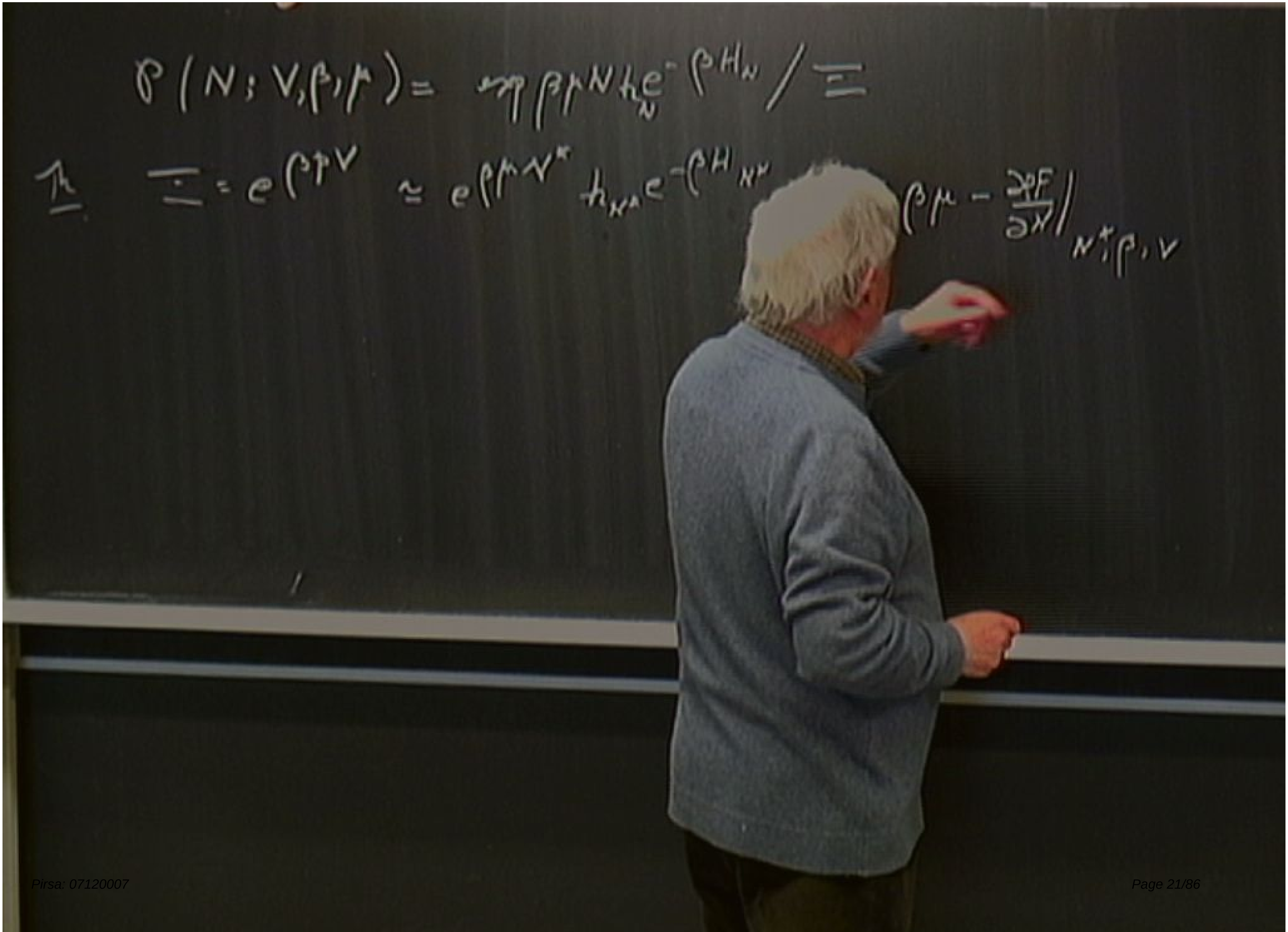
$$\Xi = e^{\beta p V}$$

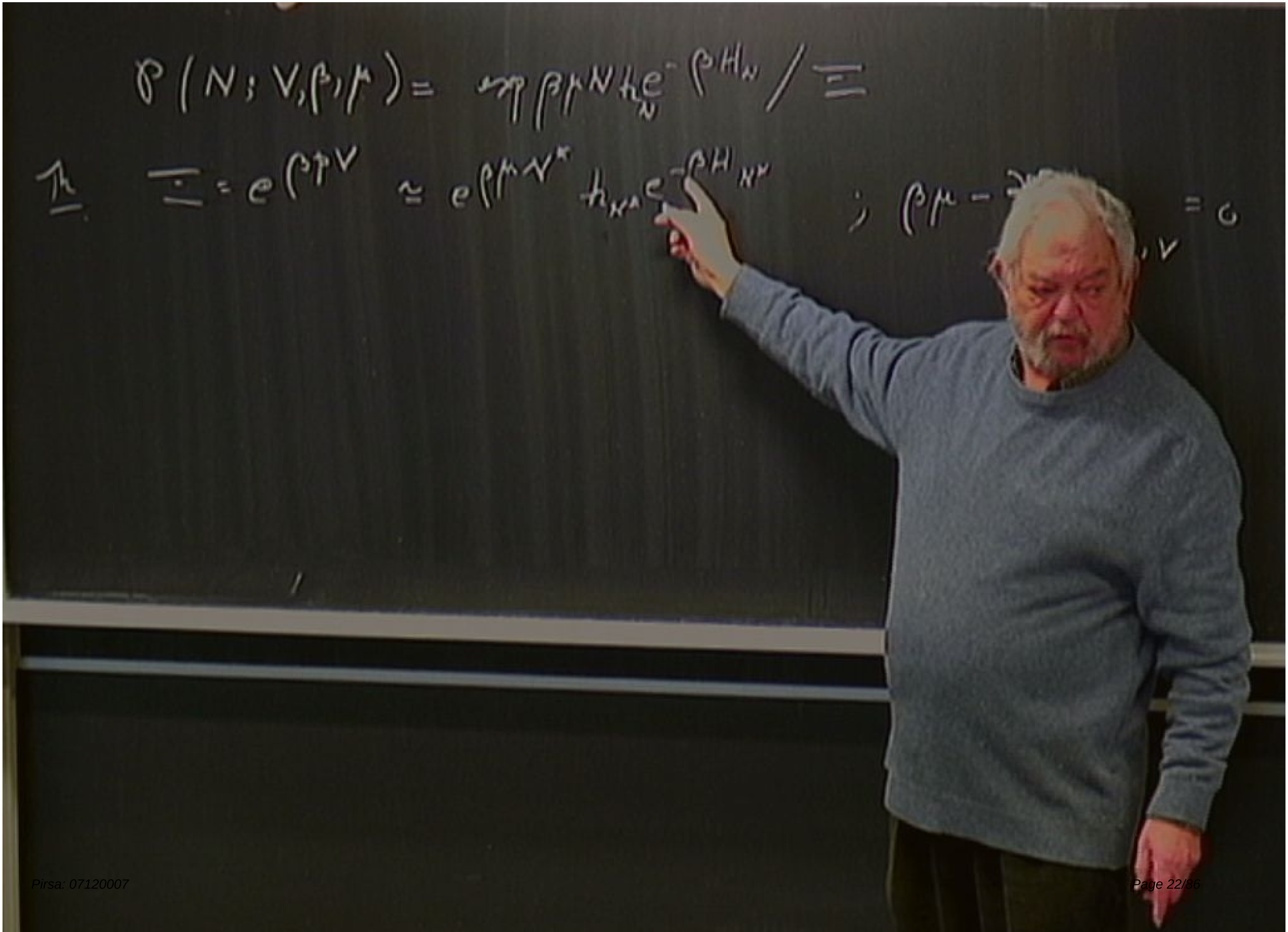

ρ(N; V,β,μ) = βμN h_N e^{-βH_N} / Ξ
Th.
Ξ = e^{βpV} ≈ e^{βμN*} h_{N*} e^{-βH}


ρ(N; V,β,μ) = exp βμN tr_N e^{-βH_N} / Ξ
Ξ = e^{βpV} ≃ e^{βμN*} tr_{N*} e^{-βH_{N*}}


$\rho(N; V, \beta, \mu) = \exp \beta\mu N \, h_N e^{-\beta H_N} / \Xi$
Th.
$\Xi = e^{\beta p V} \simeq e^{\beta\mu N^*} h_{N^*} e^{-\beta H_{N^*}}$


ρ(N; V, β, μ) = exp βμN tr_N e^{-βH_N} / Ξ
Ξ = e^{βpV} ≈ e^{βμN*} tr_{N*} e^{-βH_{N*}}
(βμ − ∂βF/∂N)|_{N*; β, V}

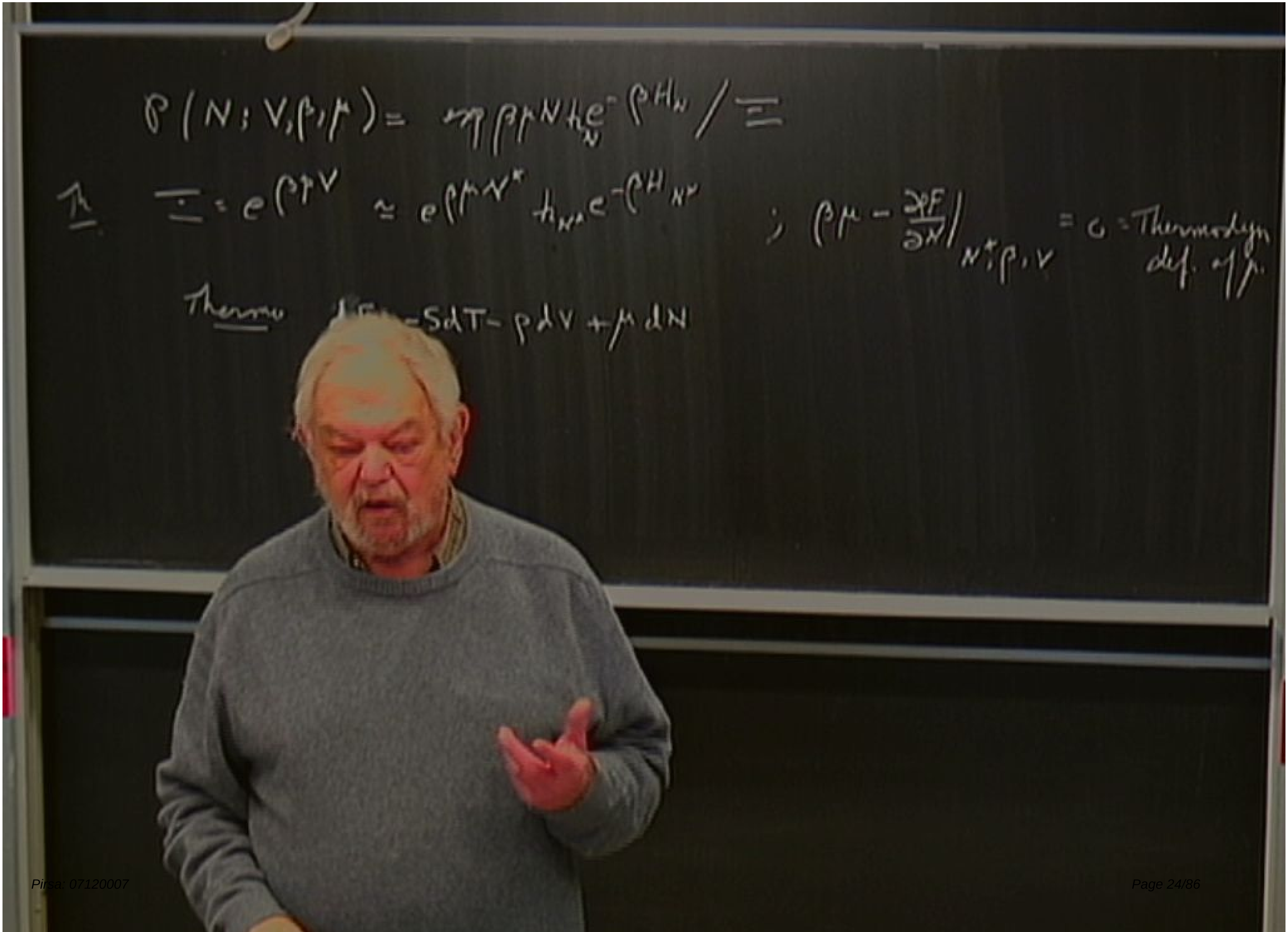


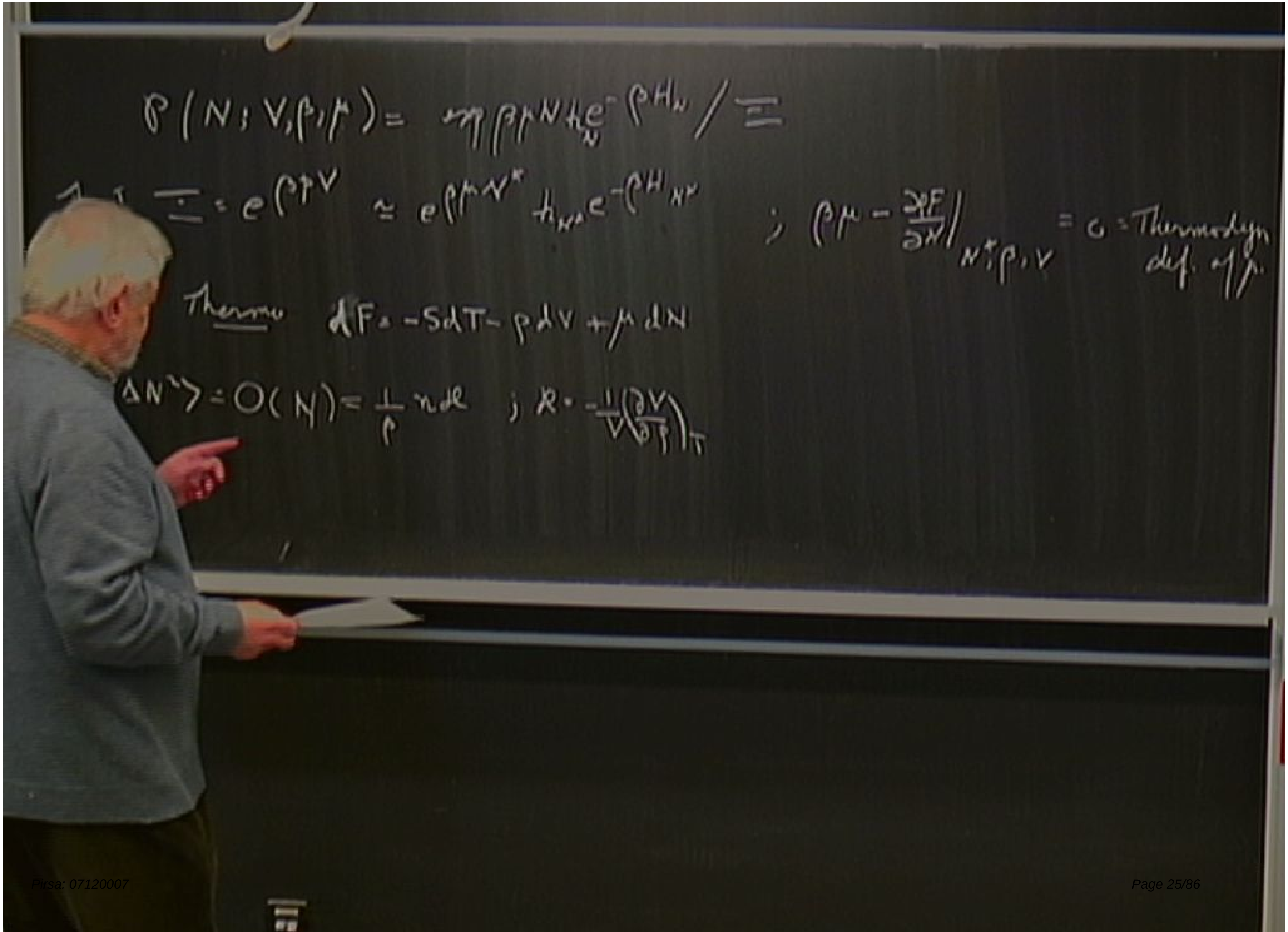


ρ(N; V,β,μ) = exp βμN tr_N e^{-βH_N} / Ξ
Ξ = e^{βpV} ≃ e^{βμN*} tr_{N*} e^{-βH_{N*}}

$$\rho(N; V,\beta,\mu) = \exp \beta\mu N \, \mathrm{tr}_N \, e^{-\beta H_N} / \Xi$$

$$\Xi = e^{\beta p V} \simeq e^{\beta\mu N^*} \mathrm{tr}_{N^*} e^{-\beta H_{N^*}} \quad ; \quad \left(\beta\mu - \frac{\partial \beta F}{\partial N}\right)\Big|_{N^*; \beta, V} = 0$$


ρ(N; V,β,μ) = e^{βμN} tr_N e^{-βH_N} / Ξ
Th. Ξ = e^{βpV} ≃ e^{βμN*} tr_{N*} e^{-βH_{N*}} ; βμ − ∂βF/∂N |_{N*;β,V} = 0 = Thermodyn def. of μ
Thermo dF = −SdT − pdV + μdN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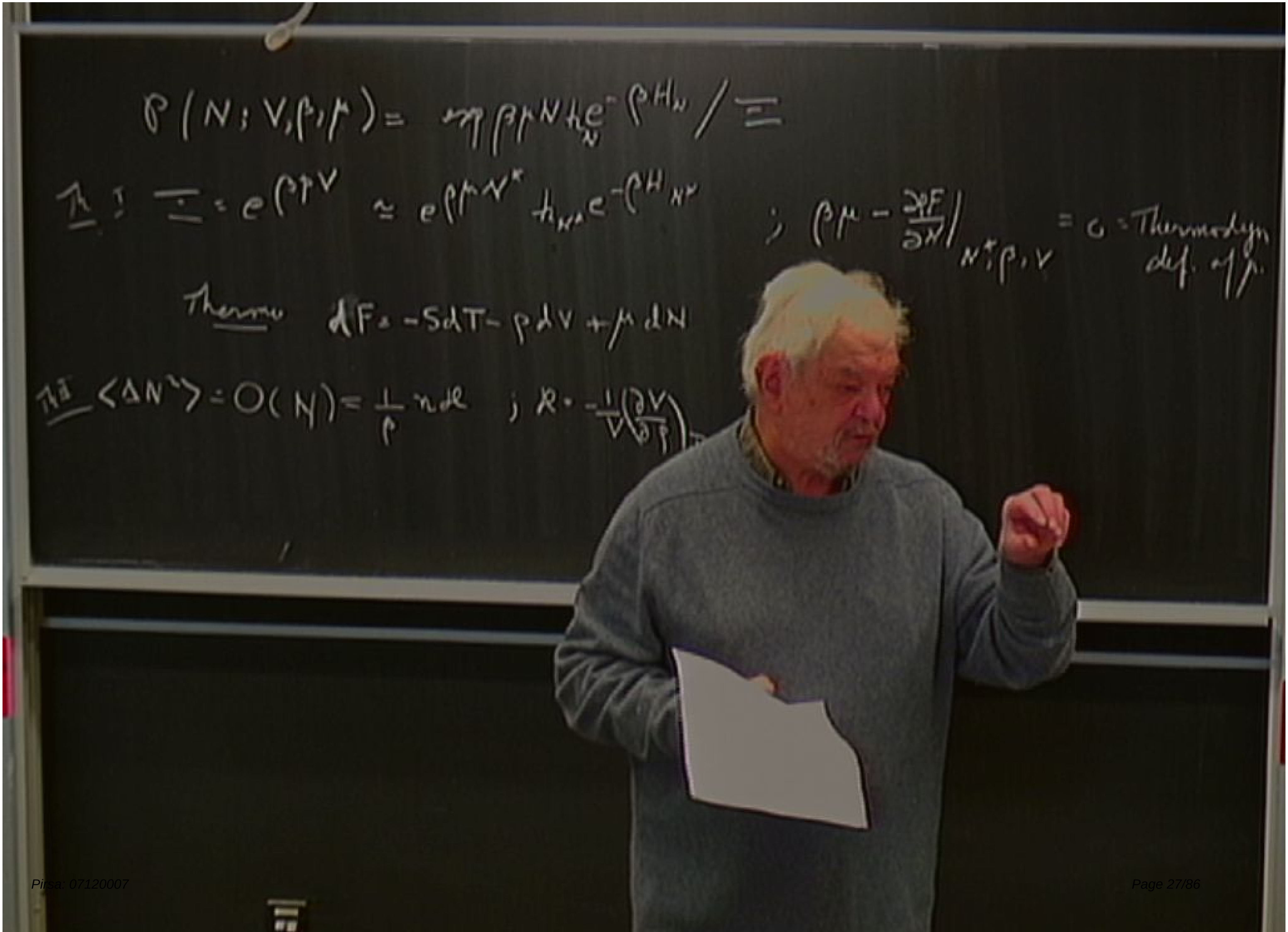
Thermo dF = −SdT − pdV + μdN
⟨ΔN²⟩ = O(N) = (1/ρ) κ ; κ = −(1/V)(∂V/∂p)_T

$$\rho(N; V, \beta, \mu) = e^{\beta\mu N} \mathrm{tr}_N\, e^{-\beta H_N} / \Xi$$

Th: $\Xi = e^{\beta p V} \simeq e^{\beta\mu N^*} \mathrm{tr}_{N^*} e^{-\beta H_{N^*}}$; $\beta\mu - \frac{\partial \beta F}{\partial N}\Big|_{N^*; \beta, V} = 0$: Thermodyn def. of μ.

Thermo $dF = -S\,dT - p\,dV + \mu\,dN$

Th $\langle \Delta N^2 \rangle = O(N) = \frac{1}{\rho} n \kappa$; $\kappa = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_{T,N}$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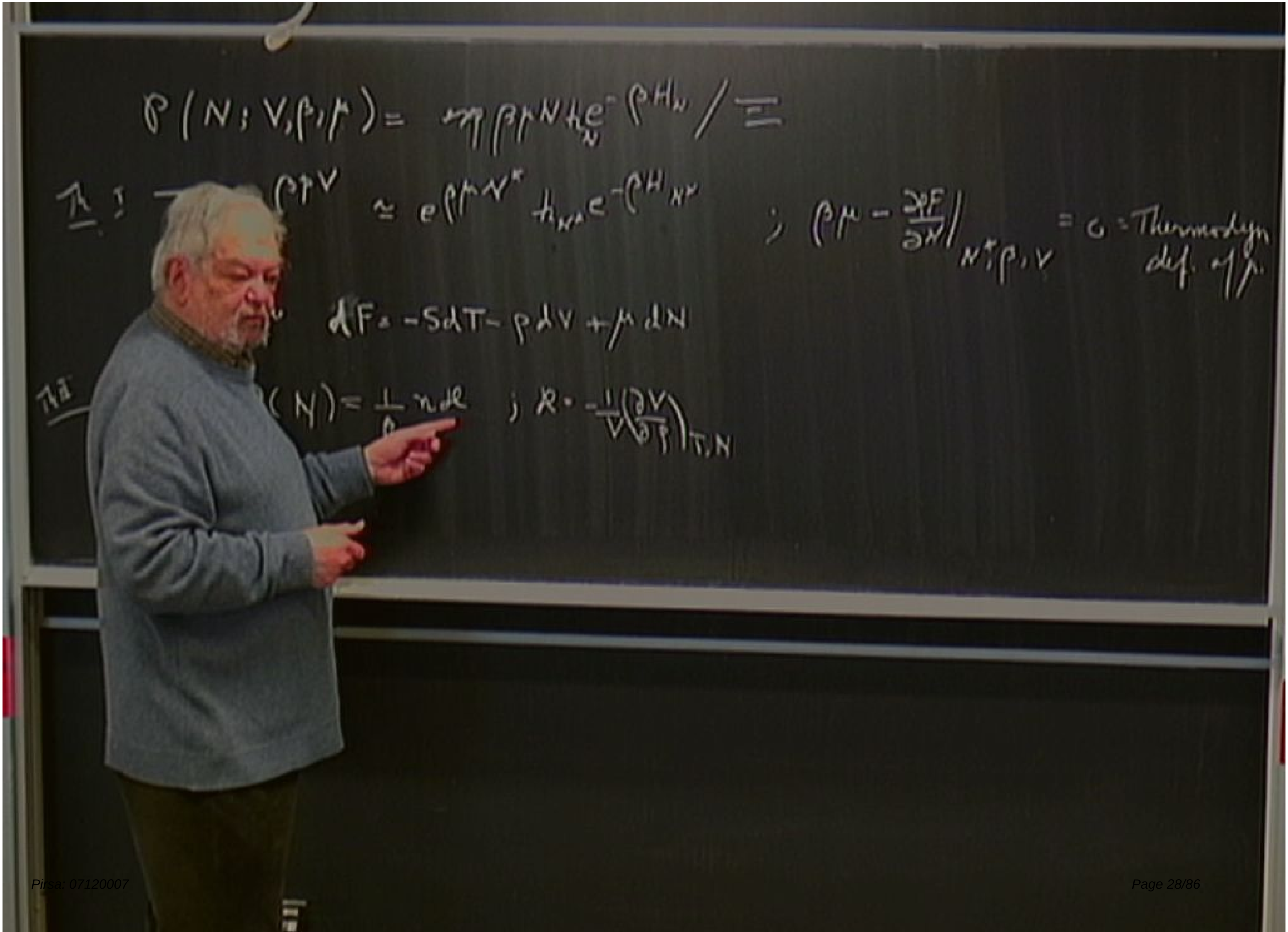
⟨ΔN²⟩ = O(N)
def. of μ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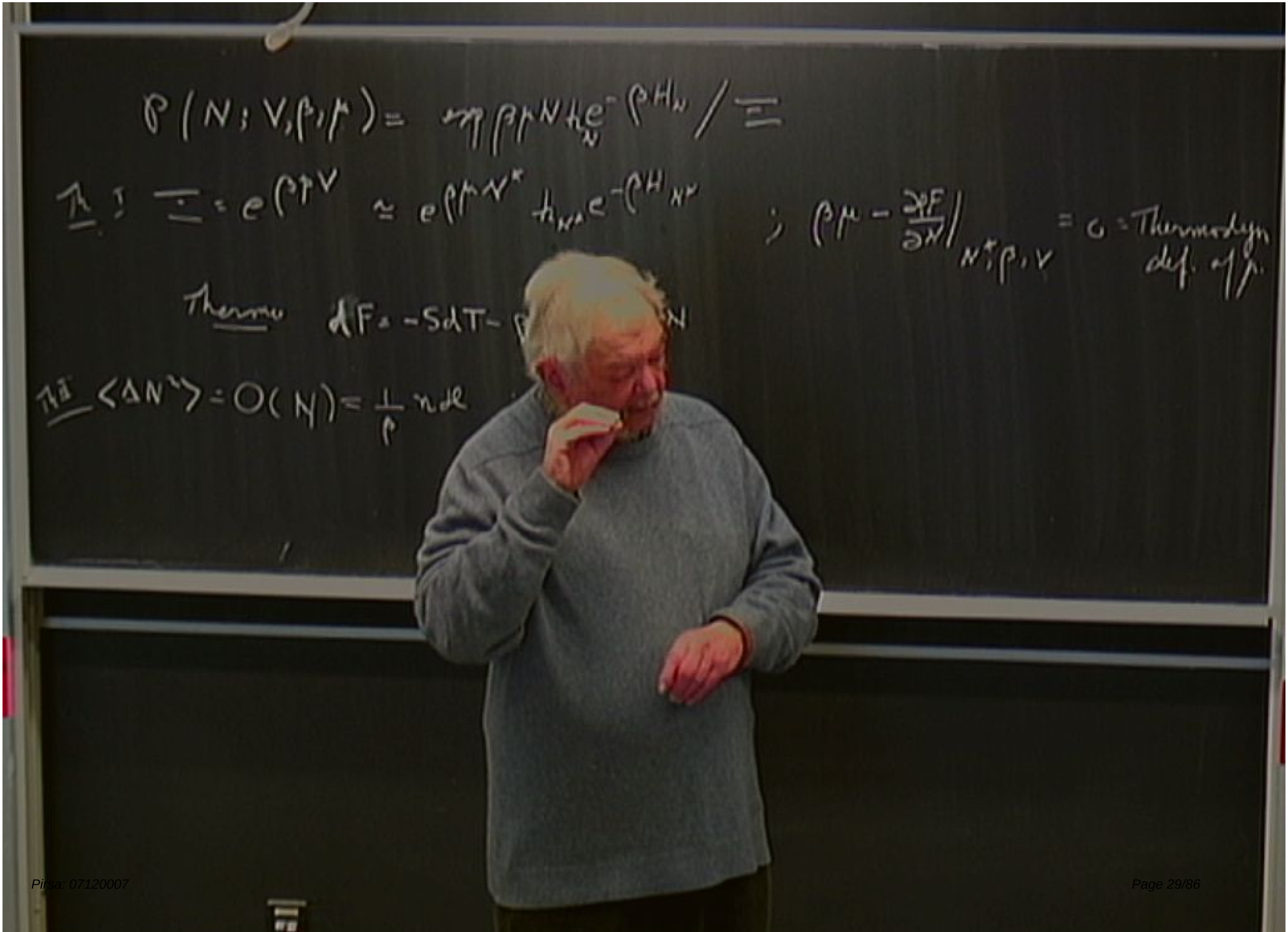
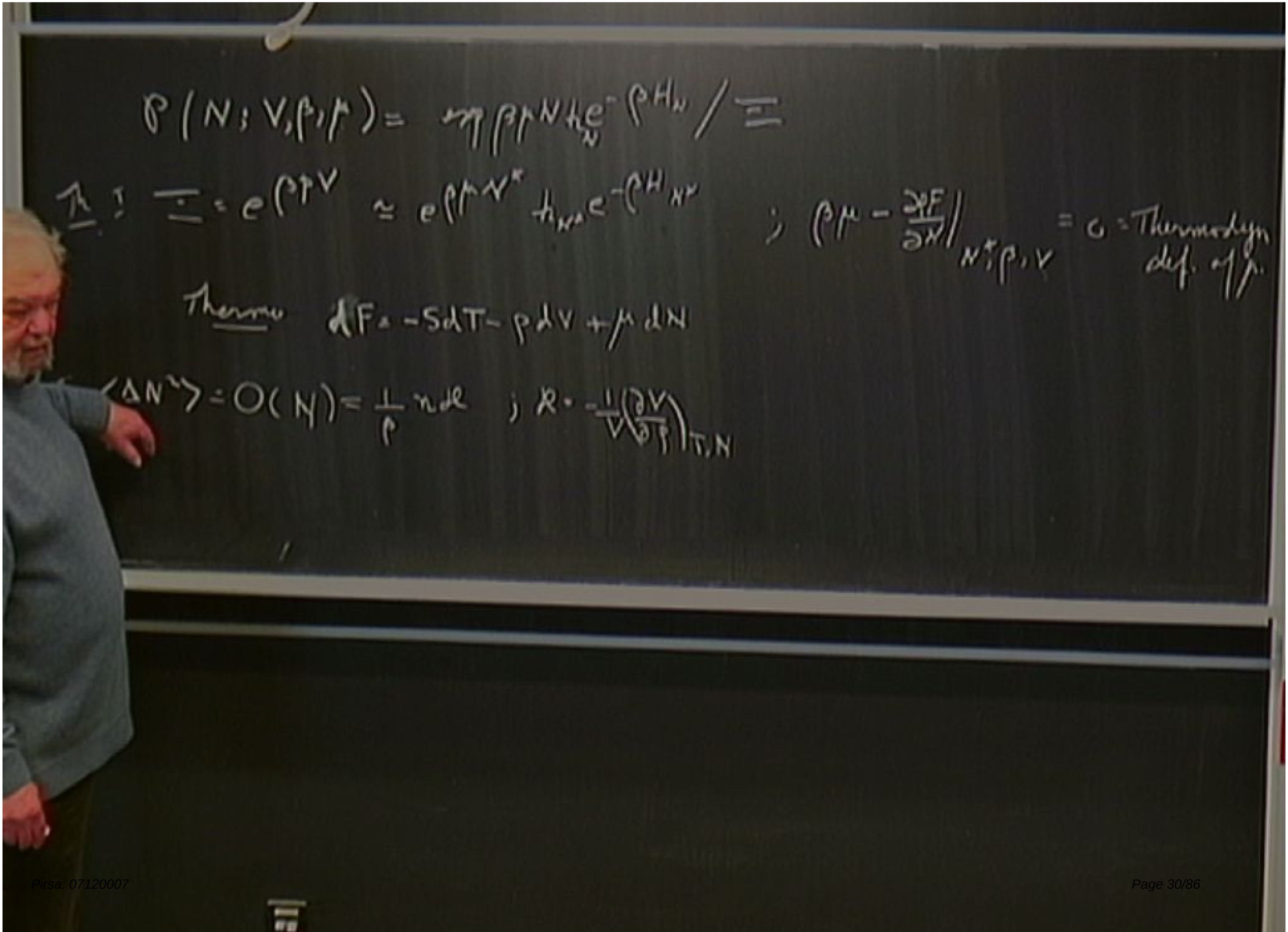

ρ(N; V, β, μ) =
/ Ξ
Th: Ξ = e^βPV ≈ e^βμN*
; βμ − ∂βF/∂N |N*;β,V = 0 = Thermodyn. def. of μ
Thermo dF = −SdT −
Th ⟨ΔN²⟩ = O(N) =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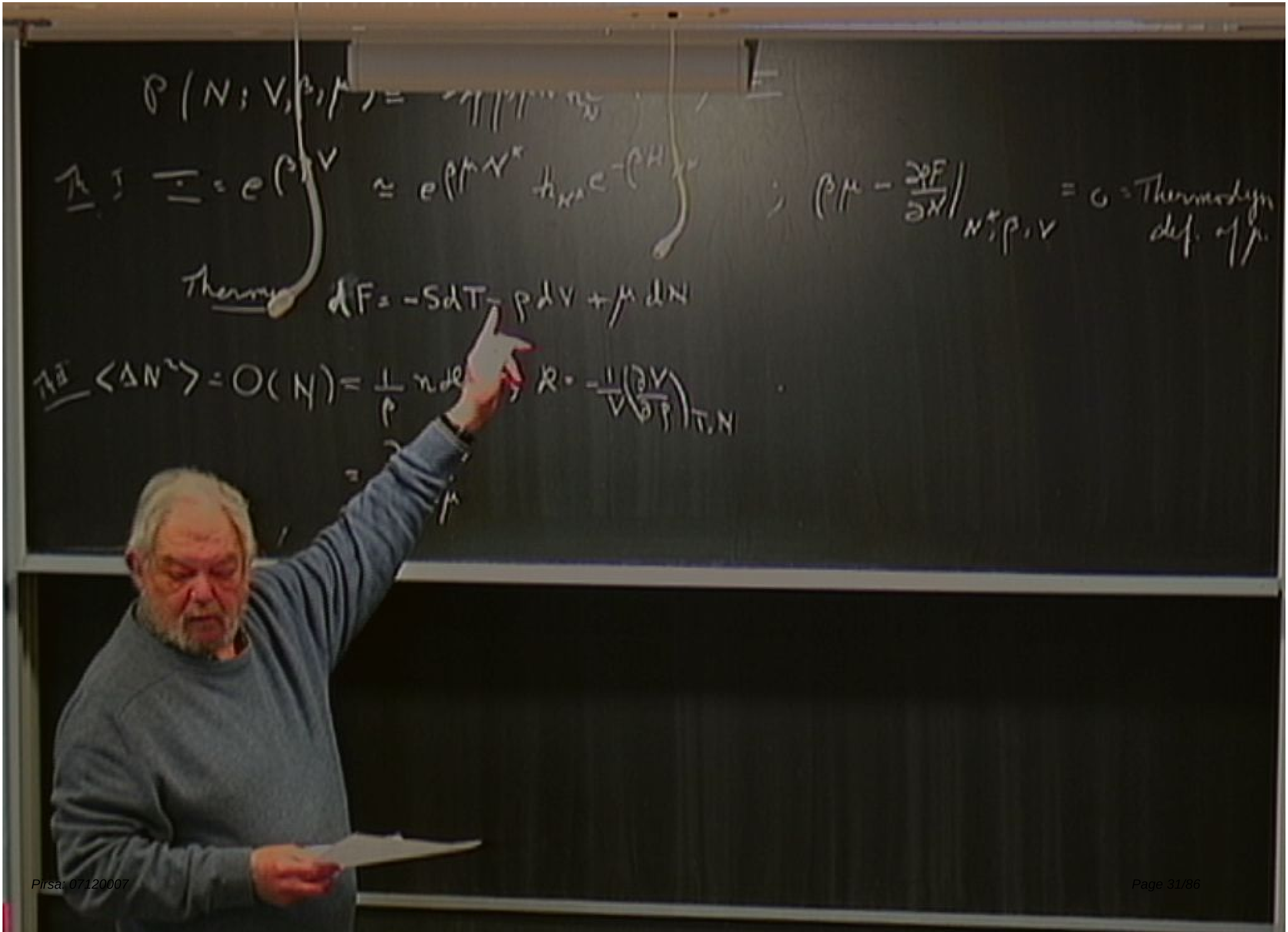
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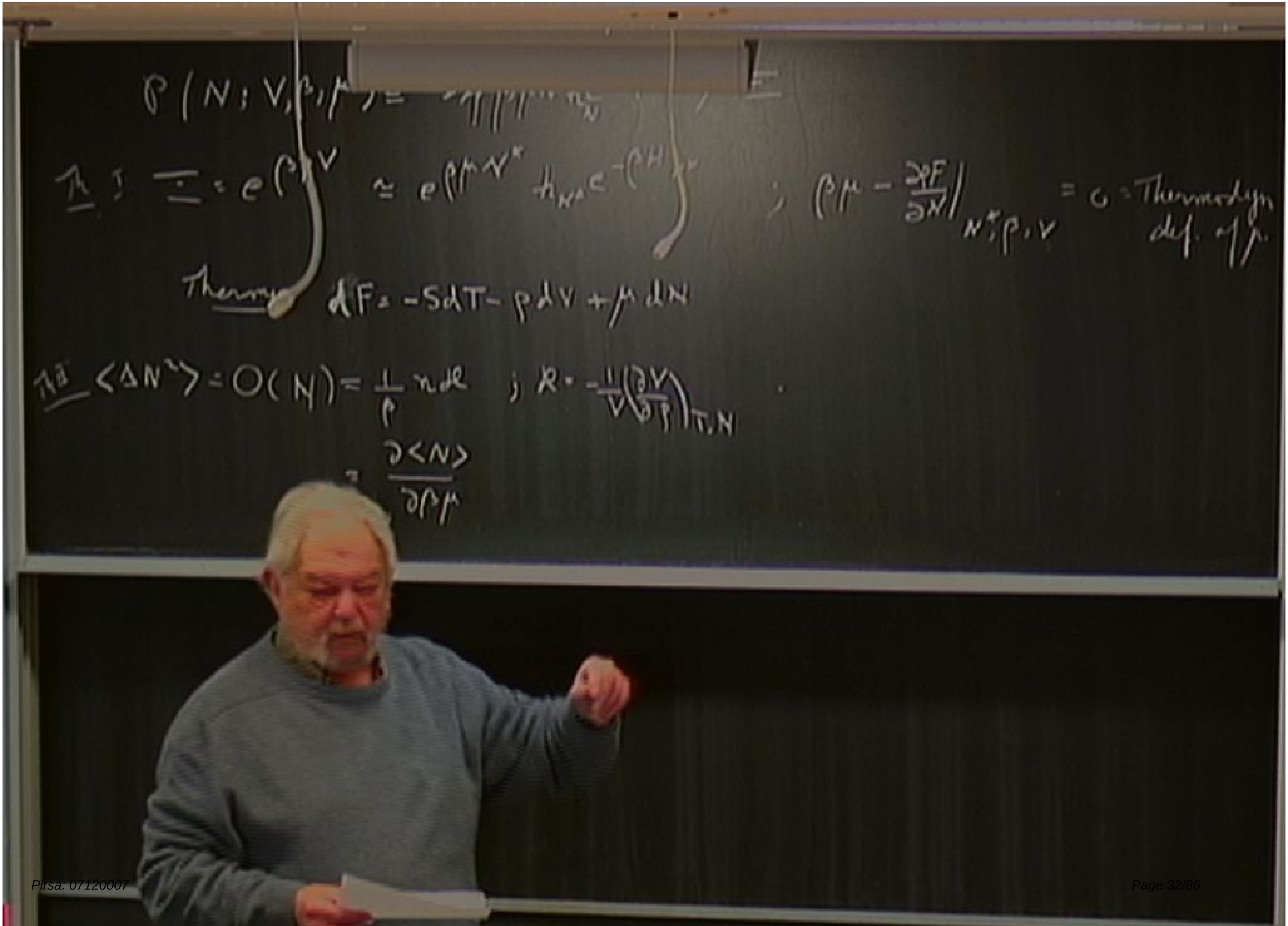
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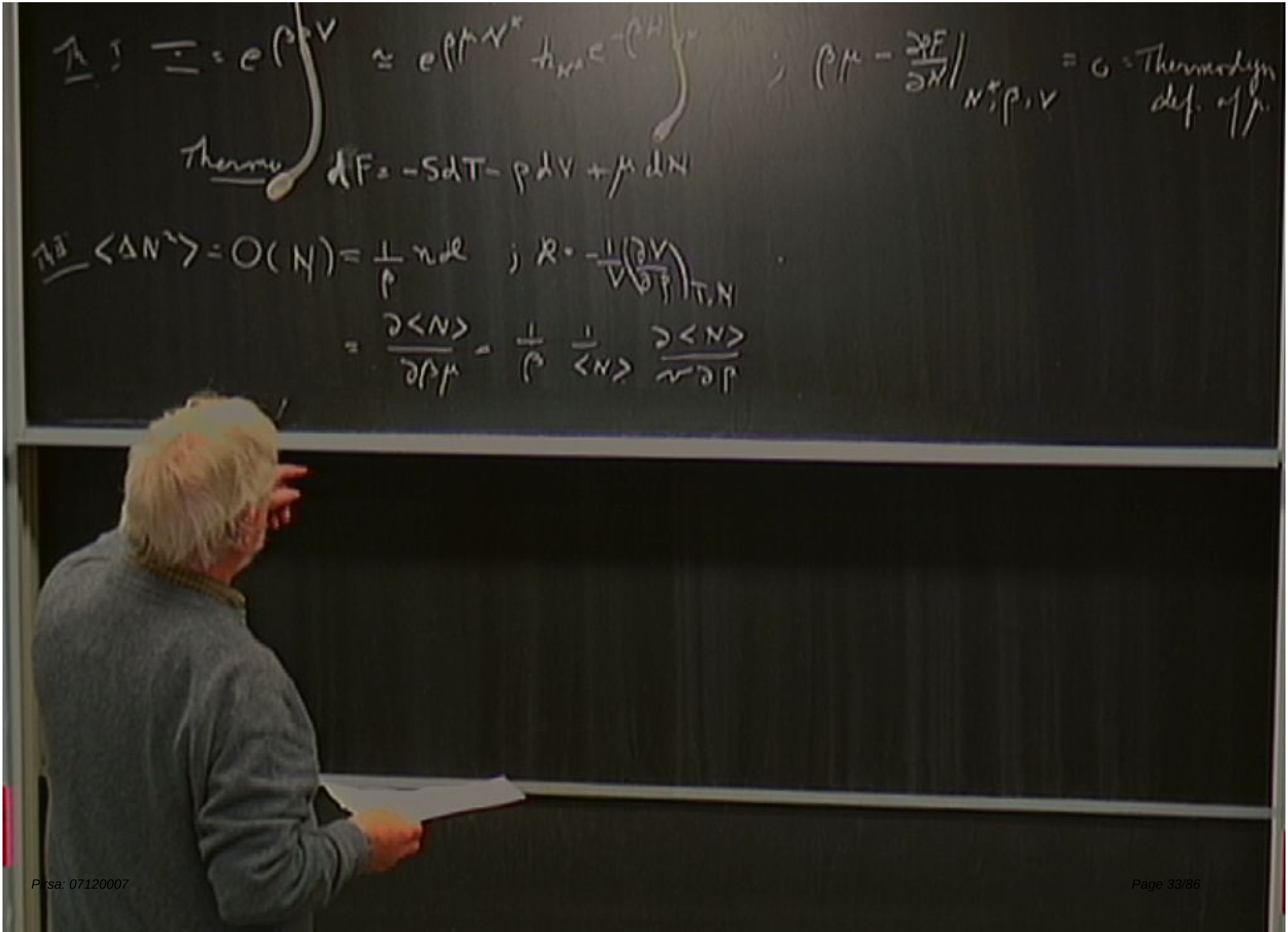
Therm dF = −SdT − pdV + μdN
Th: ⟨ΔN²⟩ = O(N) = (1/ρ) κ ; κ = −(1/V)(∂V/∂p)_{T,N}
= ∂⟨N⟩/∂βμ


Thermo dF = -SdT - pdV + μdN
Thermodyn
def. of μ
⟨ΔN²⟩ = O(N)

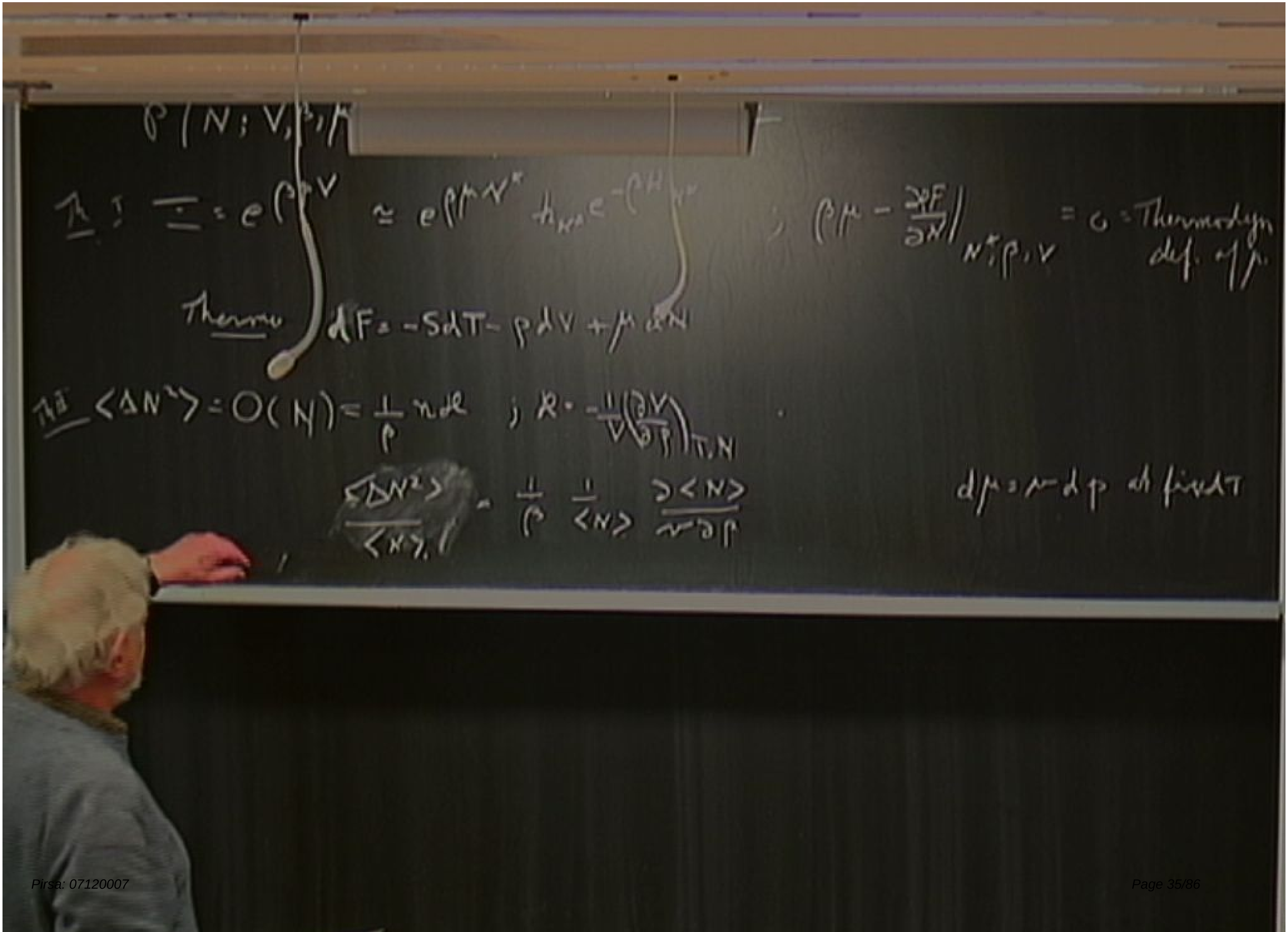
$\underline{\text{TL}}$! $\Xi = e^{(\beta p V} \simeq e^{\beta \mu N} \ldots$; $\left(\beta\mu - \frac{\partial \beta F}{\partial N}\right)\Big|_{N^*; \beta, V} = 0$: Thermodyn. def. of μ

Thermo $dF = -S\,dT - p\,dV + \mu\,dN$

$\underline{\text{TL}}$ $\langle \Delta N^2 \rangle = O(N) = \frac{1}{\beta} \nu \kappa$; $\kappa = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_{T,N}$

$$\frac{\langle \Delta N^2 \rangle}{\langle N \rangle} = \frac{1}{\beta} \frac{1}{\langle N \rangle} \frac{\partial \langle N \rangle}{\nu\, \partial \mu}$$

$d\mu = \nu\, dp$ at fixed T



Thermo dF = −SdT − p dV + μ dN
d μ = v d p at fixed T

$\rho(N; V, \beta, \mu)$

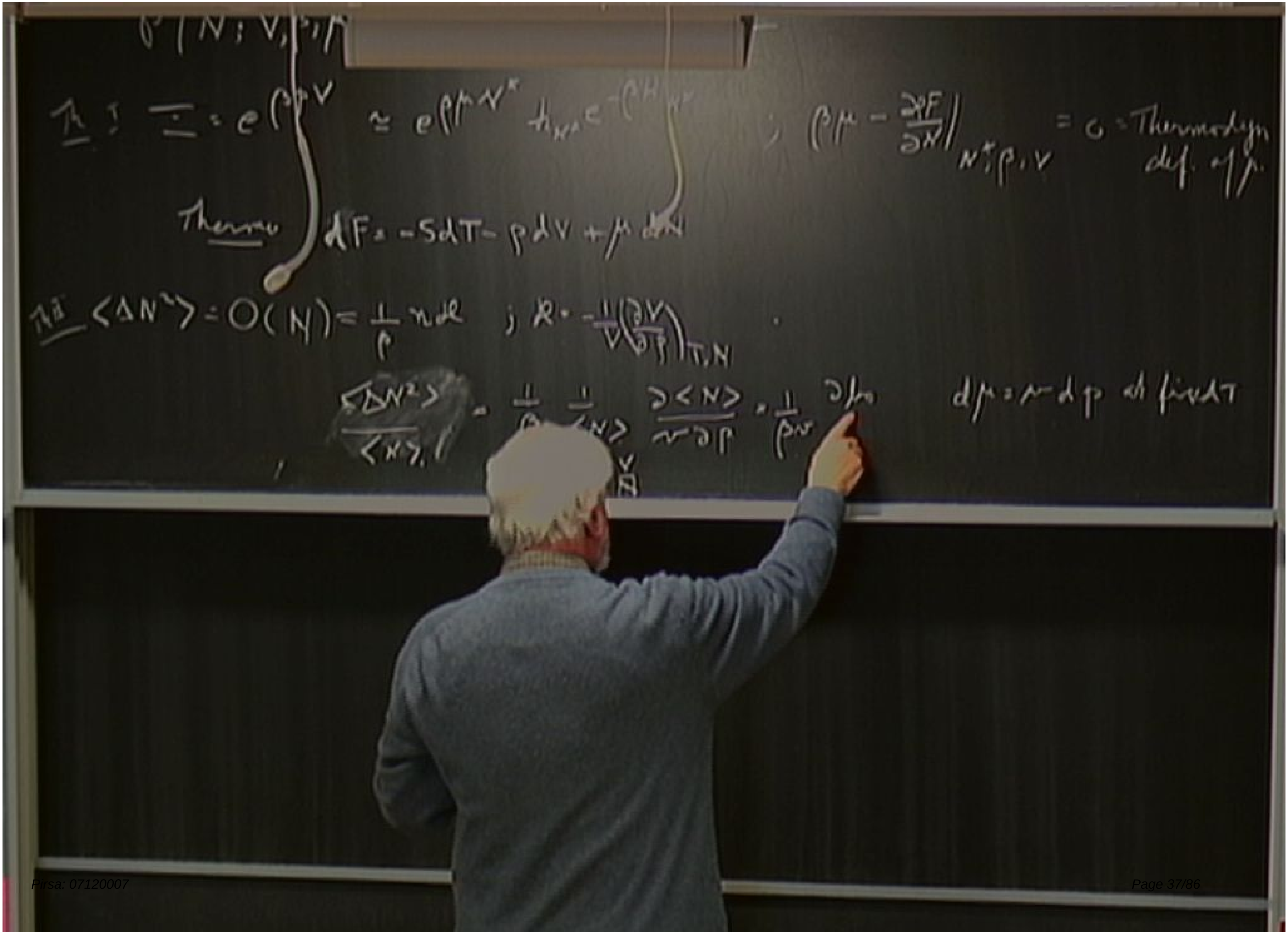
Λ : $\Xi = e^{\beta p V} \simeq e^{\beta \mu N^*} Z_{N^*} e^{-\beta H_{N^*}}$; $\beta\mu = -\frac{\partial F}{\partial N}\Big|_{N^*, \beta, V} = 0$ = Thermodyn. def. of μ.

Thermo: $dF = -S\,dT - p\,dV + \mu\,dN$

$\langle \Delta N^2 \rangle = O(N) = \frac{1}{\beta} n \kappa$; $\kappa = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_{T,N}$

$$\frac{\langle \Delta N^2 \rangle}{\langle N \rangle} = \frac{1}{\beta}\frac{1}{\langle N \rangle}\frac{\partial \langle N \rangle}{v\,\partial p}$$

$d\mu = v\,dp$ at fixed T

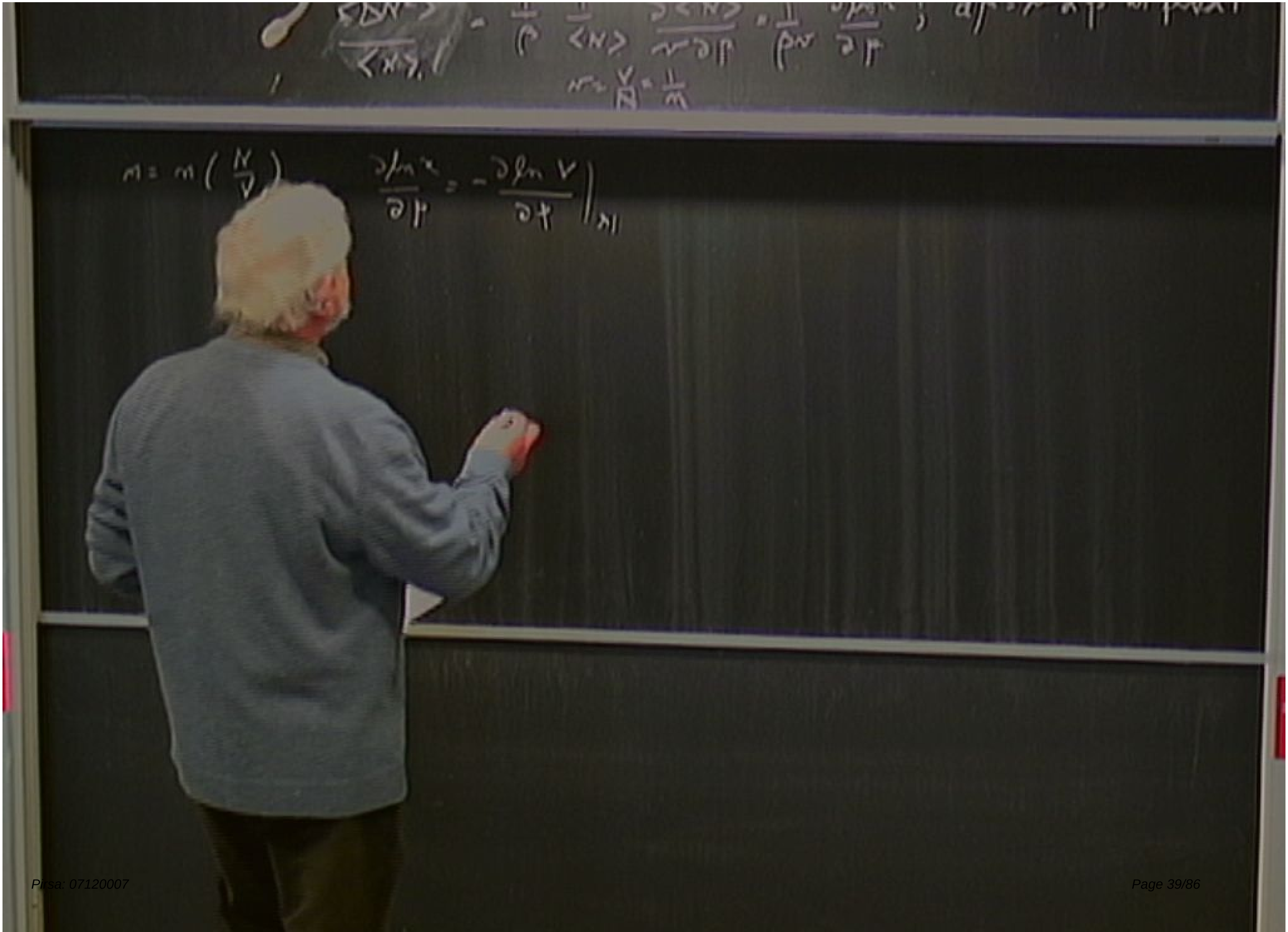

Thermo
dF = -SdT - pdV + μ dN
Thermodyn
def. of μ
dμ = v dp at fixed T

Thermo $dF = -SdT - pdV + \mu dN$

$\langle \Delta N^2 \rangle = O(N) = \frac{1}{\rho}\, n\, \ldots$; $\kappa = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_{T,N}$

$\frac{\langle \Delta N^2 \rangle}{\langle N \rangle} = \frac{1}{\beta} \frac{1}{\langle N \rangle} \frac{\partial \langle N \rangle}{v\, \partial \mu} = \frac{1}{\beta v} \frac{\partial \ln v}{\partial \mu}$; $d\mu = v\, dp$ at fixed T

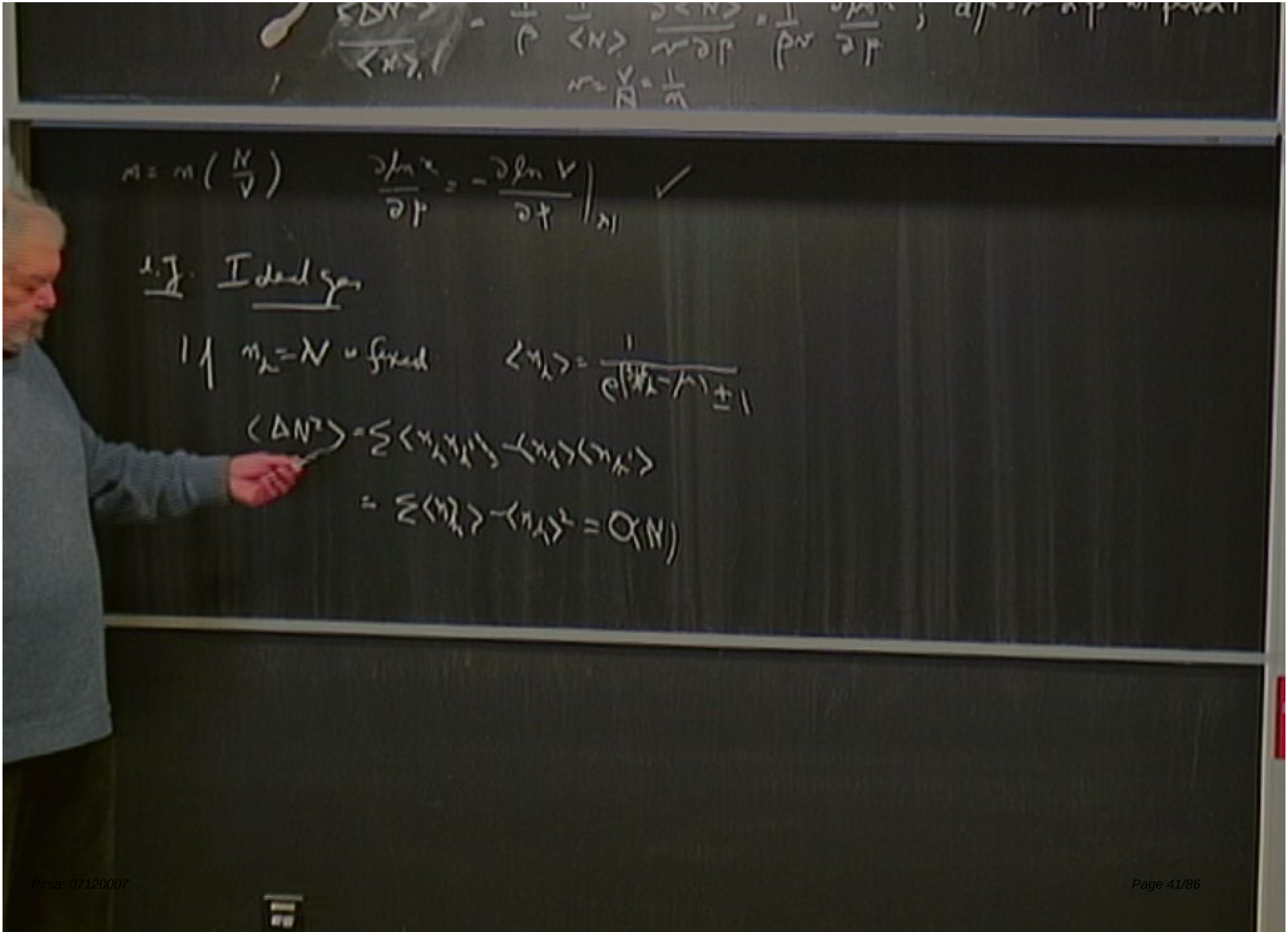
$v = \frac{V}{N} = \frac{1}{n}$

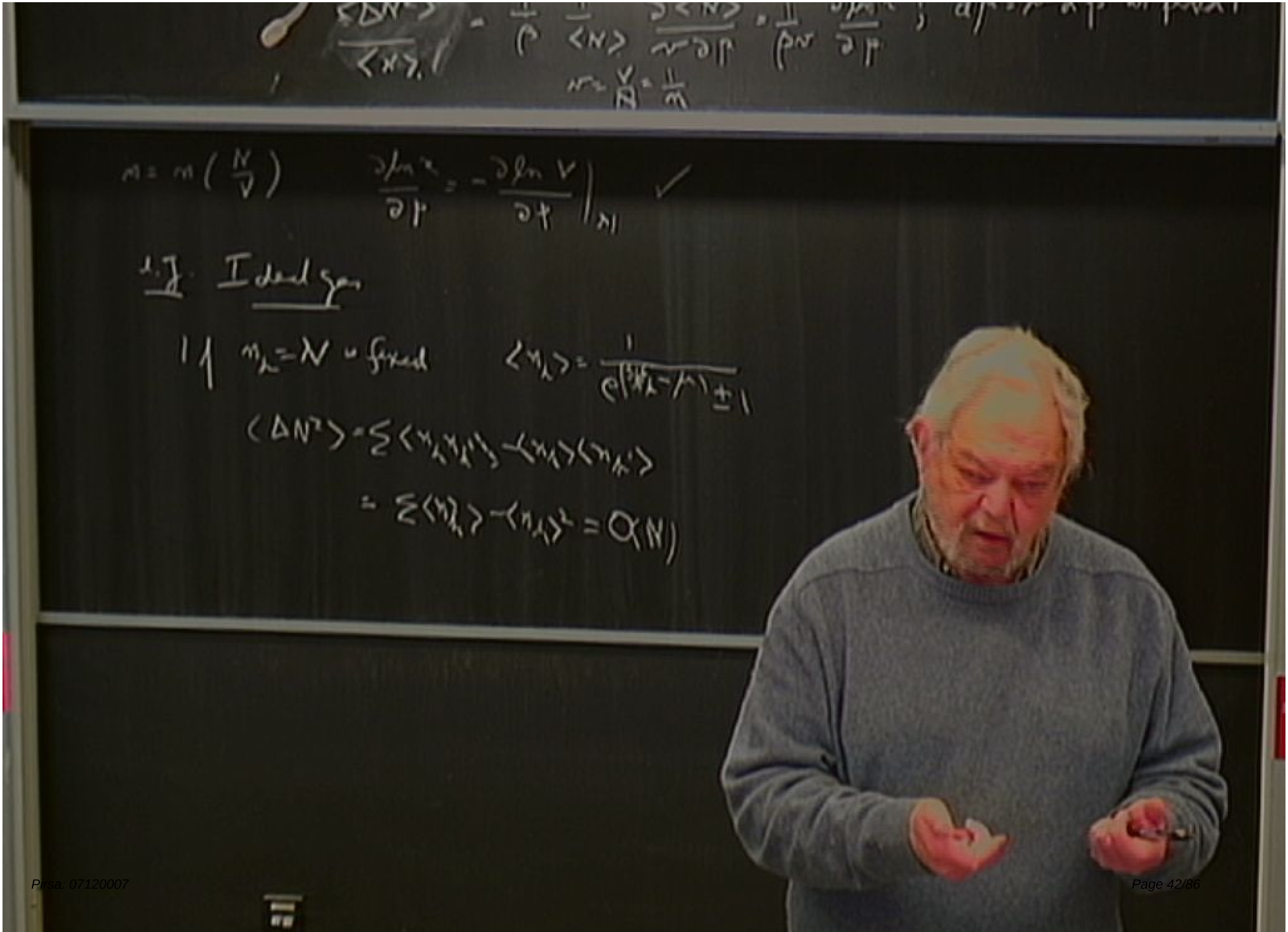


$$\frac{\langle \Delta N^2 \rangle}{\langle N \rangle} = \frac{1}{\beta} \frac{1}{\langle N \rangle} \frac{\partial \langle N \rangle}{\partial \mu} = \frac{1}{\beta n} \frac{\partial \ln n}{\partial \mu} \;;$$

$$v = \frac{V}{N} = \frac{1}{n}$$

$$n = n\left(\frac{N}{V}\right) \qquad \frac{\partial \ln n}{\partial \mu} = - \frac{\partial \ln V}{\partial \mu}\bigg|_N \quad ✓$$


n = n(N/V)
Ideal gas
n_λ = N = fixed
⟨ΔN²⟩ = Σ ⟨n_λ n_λ'⟩ − ⟨n_λ⟩⟨n_λ'⟩
= Σ ⟨n_λ⟩ − ⟨n_λ⟩² = O(N)


Ideal gas

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((In addition to spin order, where $|\vec{\nabla}\Delta|^2$ comes from))

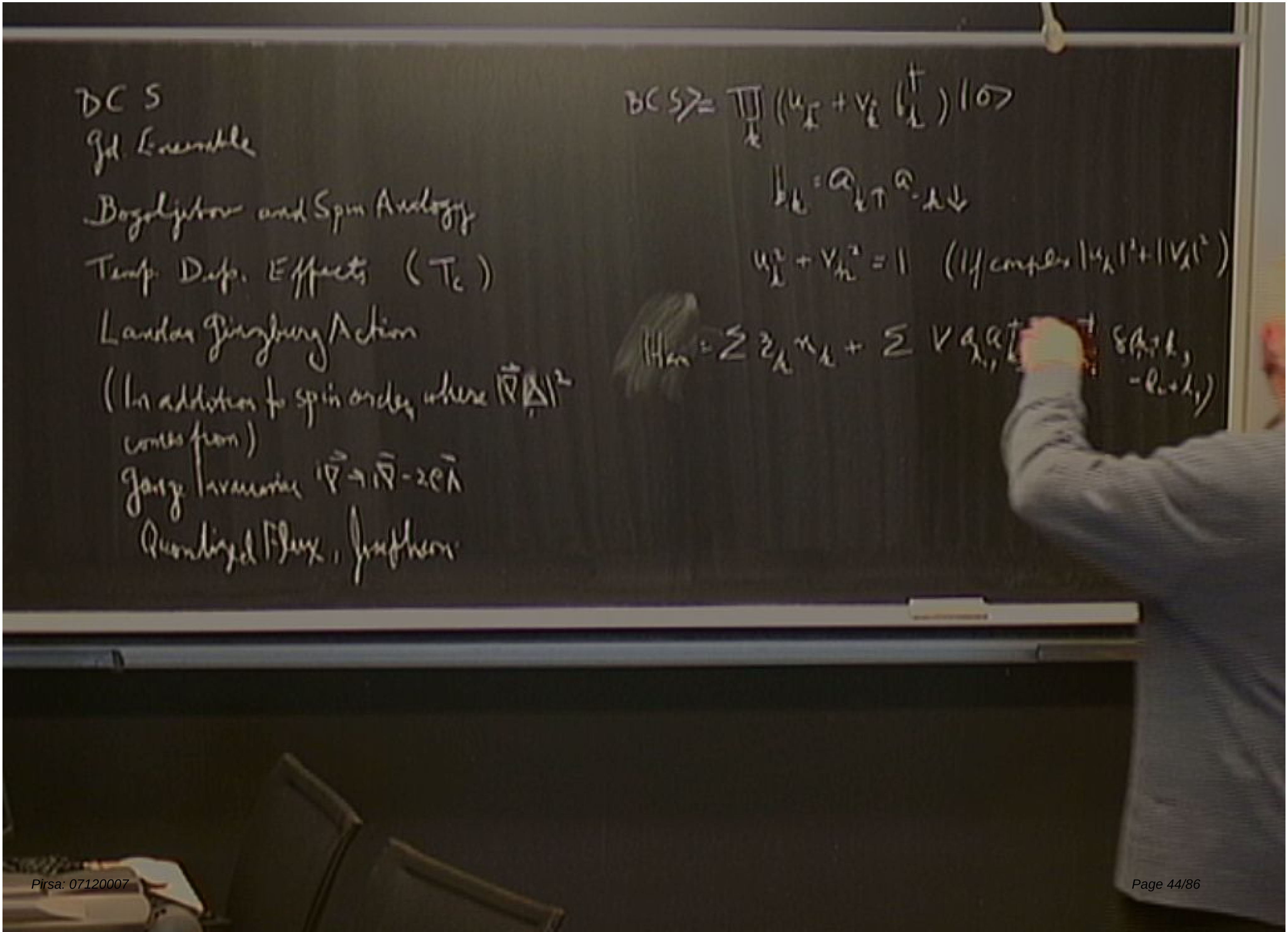
Gauge Invariance $i\vec{\nabla} \to i\vec{\nabla} - 2e\vec{A}$

Quantized Flux, Josephson

$b_k = a_{k\uparrow} a_{-k\downarrow}$

$u_k^2 + v_k^2 = 1$ ((If complex

$H_{eff} = \sum \xi_k n_k + \sum V a_{k_1} a^\dagger_{k_2} a_{k_3} a^\dagger \ldots$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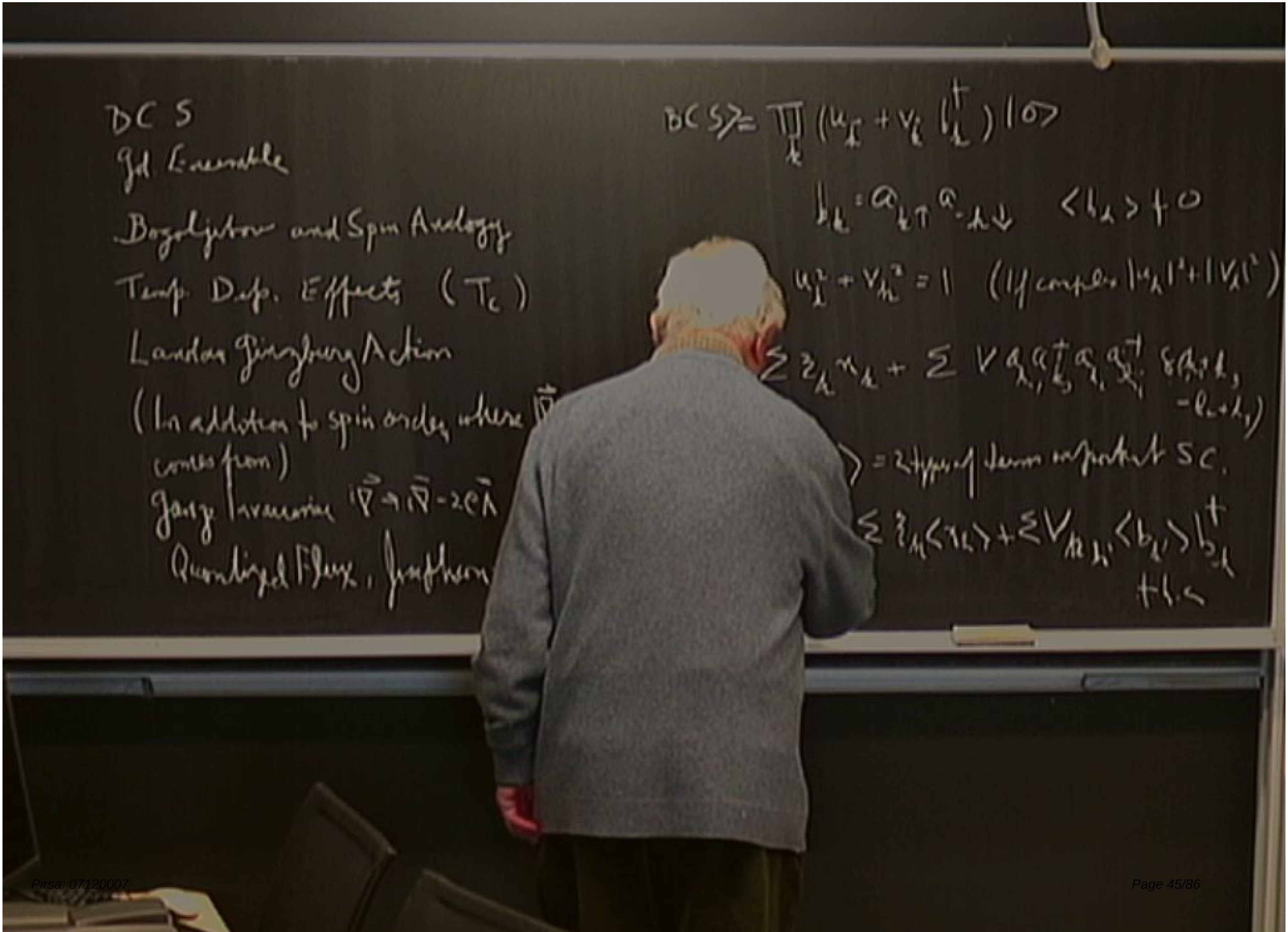

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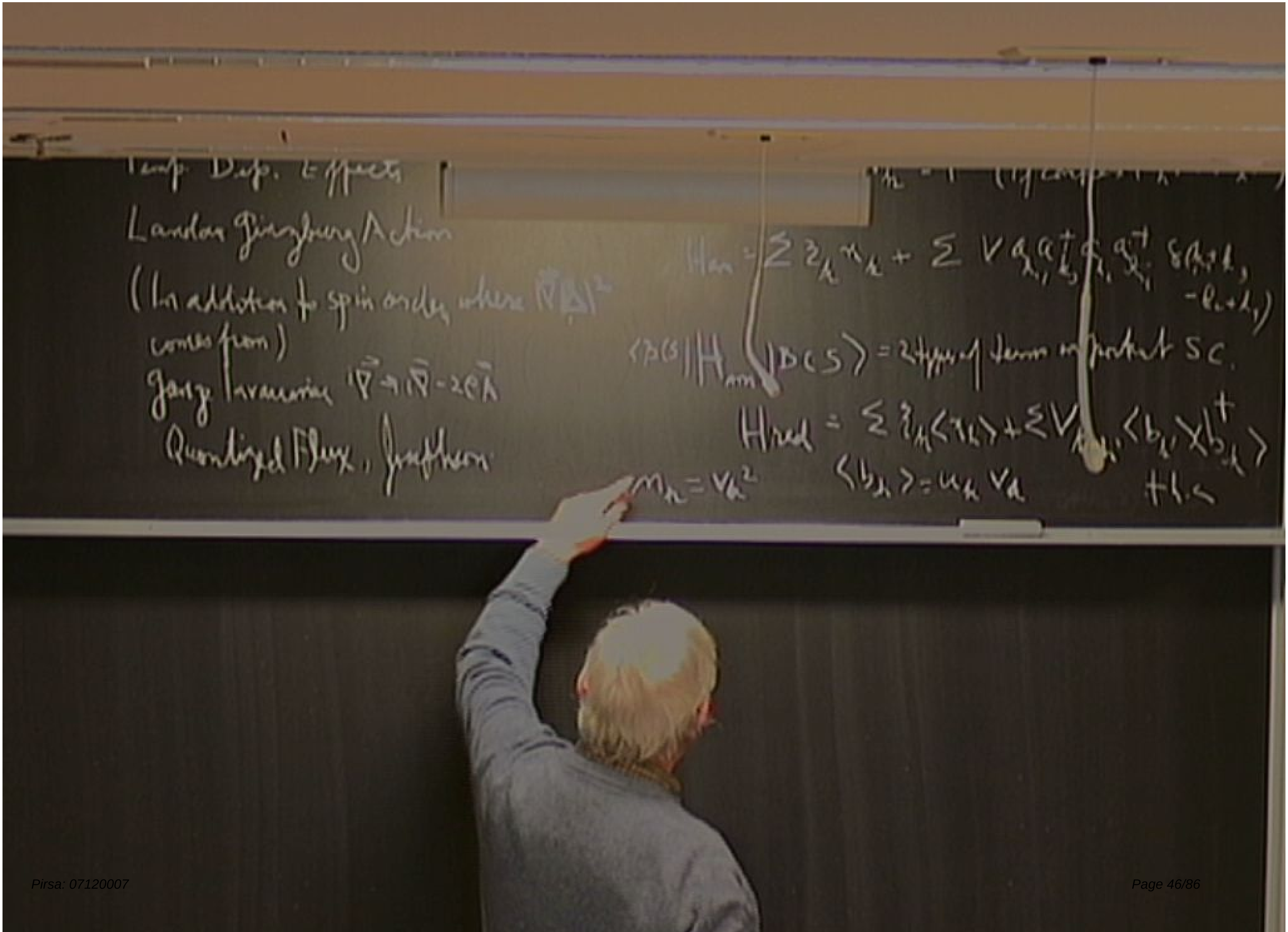
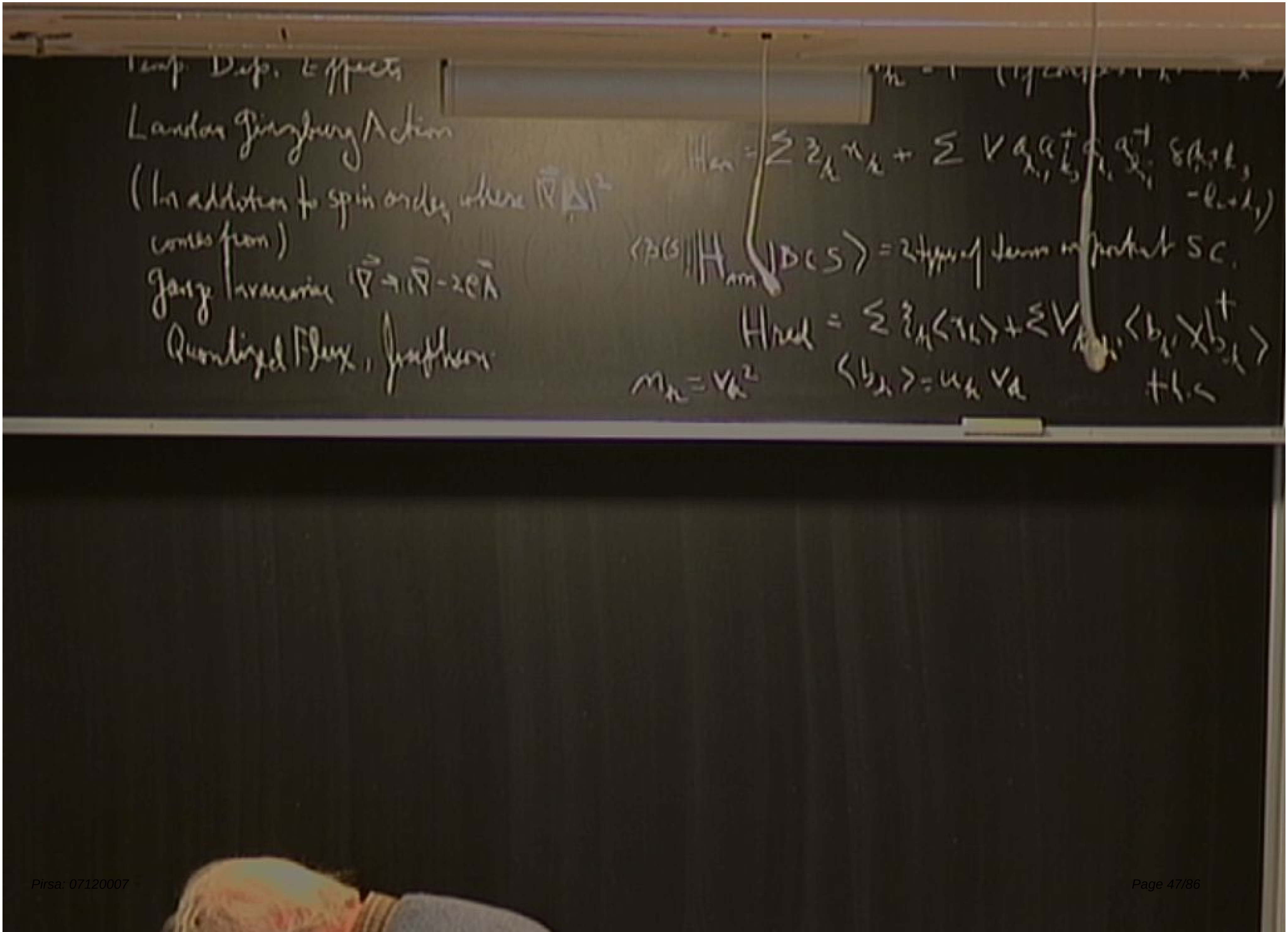
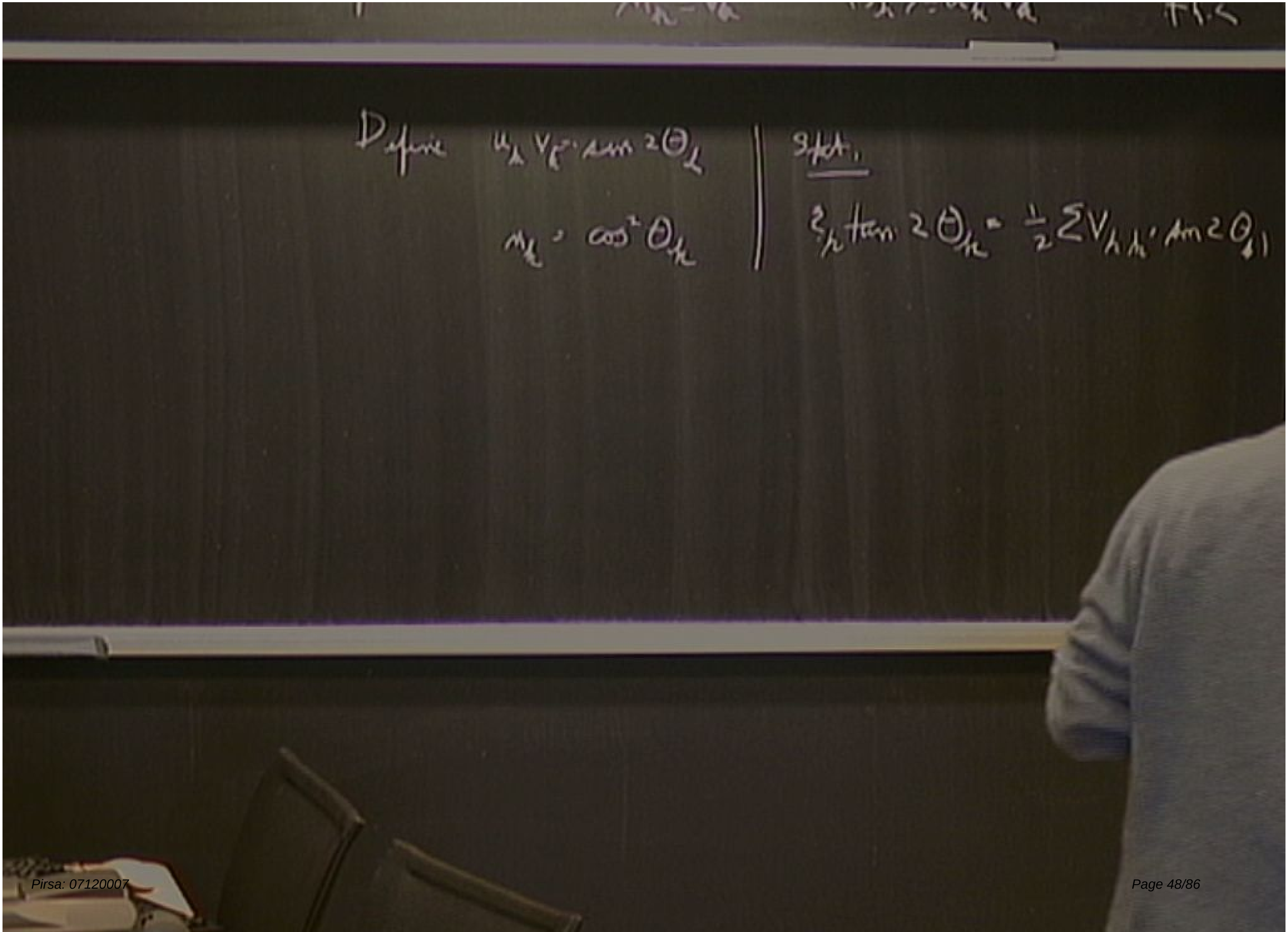
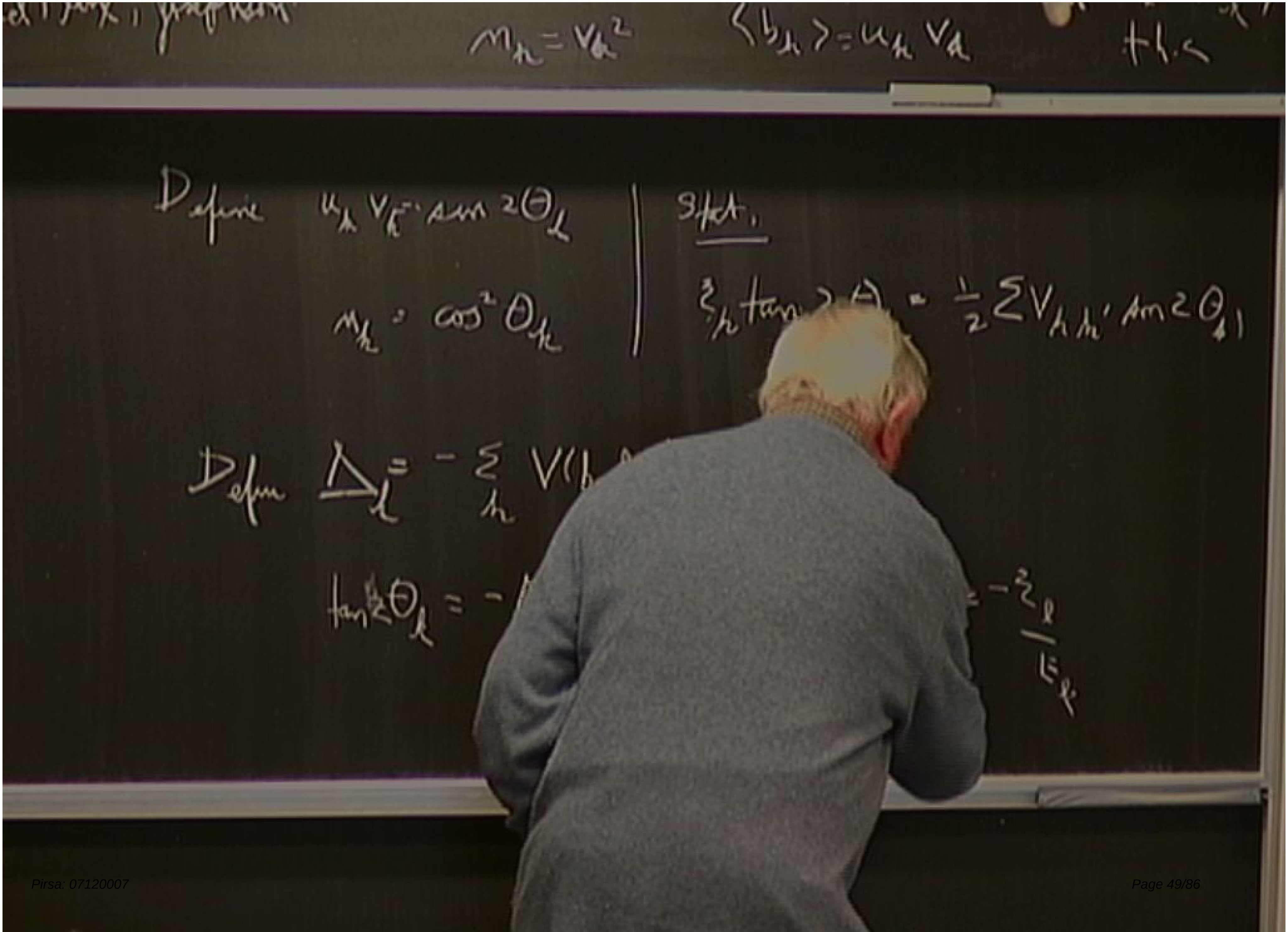
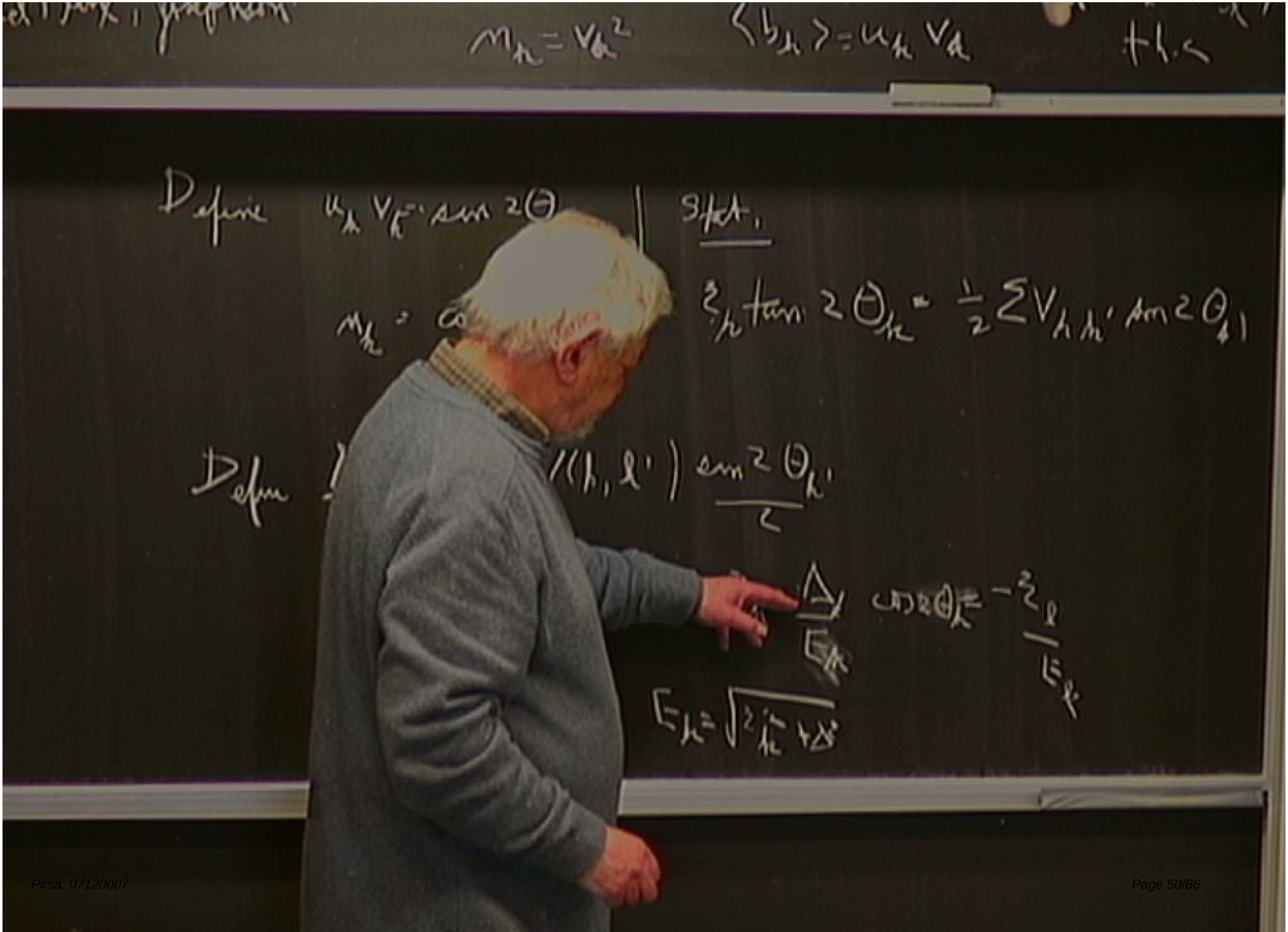
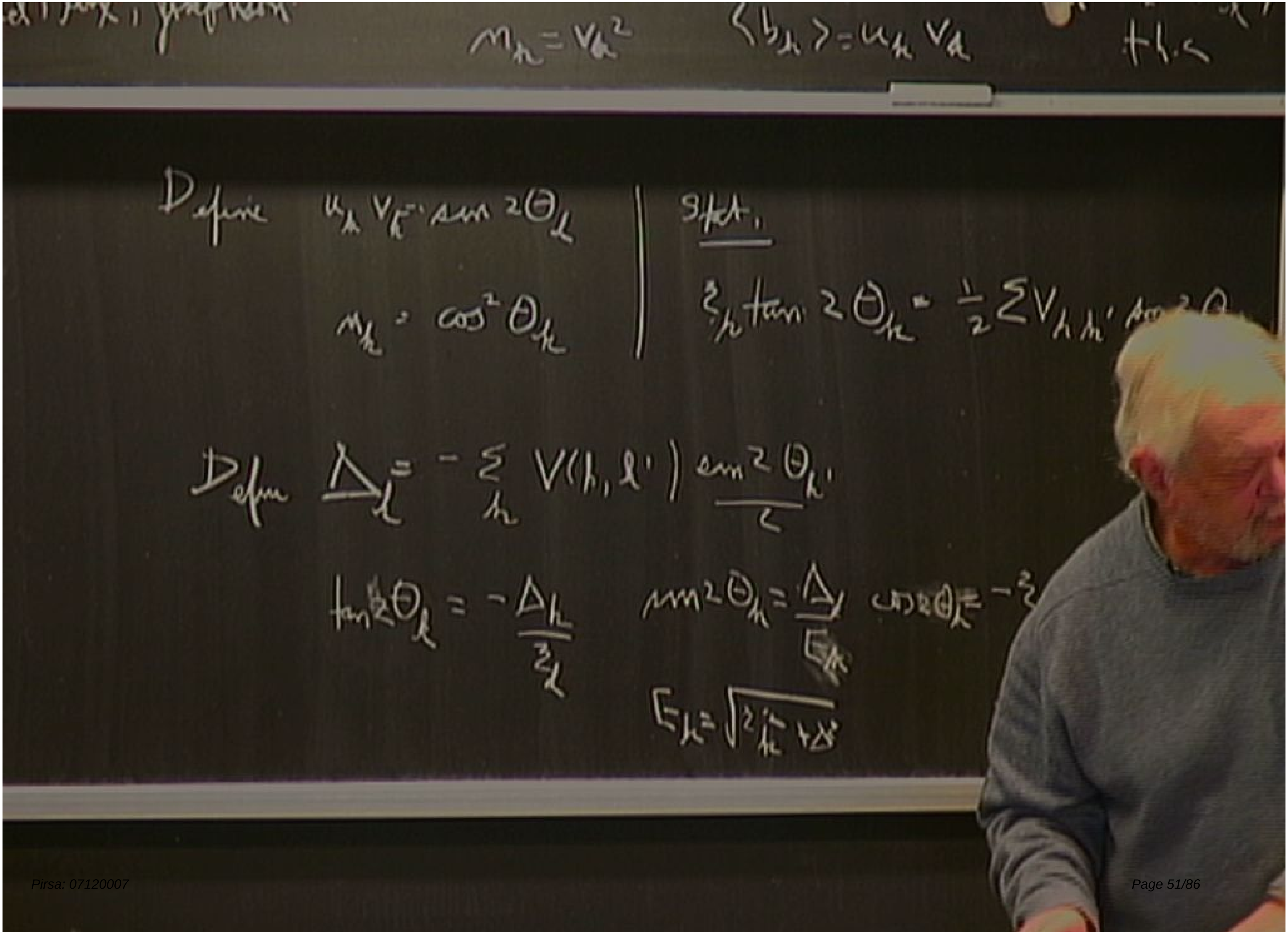

Temp. Dep. Effects
Landau Ginzburg Action
(In addition to spin order where (∇Δ)² comes from)
Gauge Invariance
Quantized Flux, Josephson
2 types of terms important S.C.
Hred
⟨b_k⟩ = u_k v_k
n_k = v_k²
+h.c.


Temp. Dep. Effects
Landau Ginzburg Action
(In addition to spin order, where |∇Δ|² comes from)
Gauge Invariance
Quantized Flux, Josephson
= 2 types of terms in perfect S.C.
Hred
+h.c.


Define $u_k v_k = \sin 2\theta_k$
$n_k = \cos^2\theta_k$
Soln.
$\xi_k \tan 2\theta_k = \frac{1}{2}\sum V_{kk'} \sin 2\theta_{k'}$


Define
Start.
Defin


$m_k = v_k^2$
$\langle b_k \rangle = u_k v_k$
+h.c
Define
Stat.
$\xi_k \tan 2\Theta_k = \frac{1}{2} \sum V_{k k'} \sin 2\Theta_{k'}$
Define
$E_k = \sqrt{\xi_k^2 + \Delta^2}$



$n_k = v_k^2$
$\langle b_k \rangle = u_k v_k$
$+ h.c.$
Define $u_k v_k = \sin 2\Theta_k$
$n_k = \cos^2 \Theta_k$
Soln.
$\xi_k \tan 2\Theta_k = \frac{1}{2} \sum V_{k k'} \sin 2\Theta$
Define $\Delta_k = -\sum_{k'} V(k, k') \frac{\sin 2\Theta_{k'}}{2}$
$\tan 2\Theta_k = -\frac{\Delta_k}{\xi_k}$
$\sin 2\Theta_k = \frac{\Delta_k}{E_k}$
$\cos 2\Theta_k = -\xi$
$E_k = \sqrt{\xi_k^2 + \Delta^2}$

$m_k = v_k^2 \qquad \langle b_k \rangle = u_k v_k \qquad$ this

Define $u_k v_k = \sin 2\Theta_k$

$m_k = \cos^2 \Theta_k$

Subst.

$\xi_k \tan 2\Theta_k = \frac{1}{2} \sum V_{k k'} \sin 2\Theta_{k'}$

Define $\Delta_k = -\sum_{k'} V(k, k') \frac{\sin 2\Theta_{k'}}{2}$

$\tan 2\Theta_k = -\frac{\Delta_k}{\xi_k} \qquad \sin 2\Theta_k = \frac{\Delta_k}{E_k} \qquad \cos 2\Theta_k = -\frac{\xi_k}{E_k}$

$E_k = \sqrt{\xi_k^2 + \Delta^2}$

$n_k = v_k^2 \qquad \langle b_k \rangle = u_k v_k \qquad + h.c.$

Define $u_k v_k = \sin 2\Theta_k$

$n_k = \cos^2 \Theta_k$

Soln.

$\xi_k \tan 2\Theta_k = \frac{1}{2} \sum V_{k k'} \sin 2\Theta_{k'}$

Define $\Delta_k = - \sum_{k'} V(k, k') \frac{\sin 2\Theta_{k'}}{2}$

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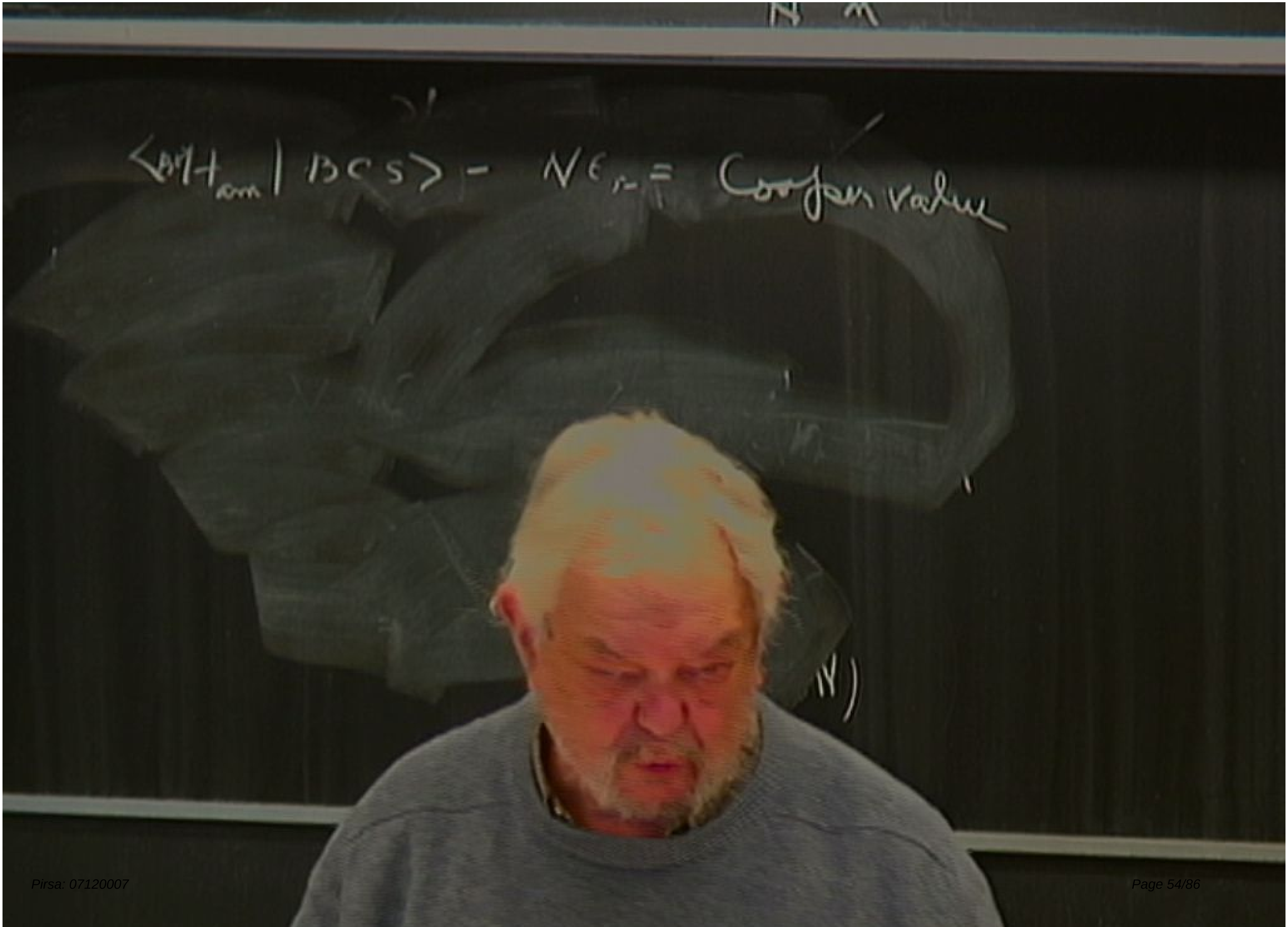
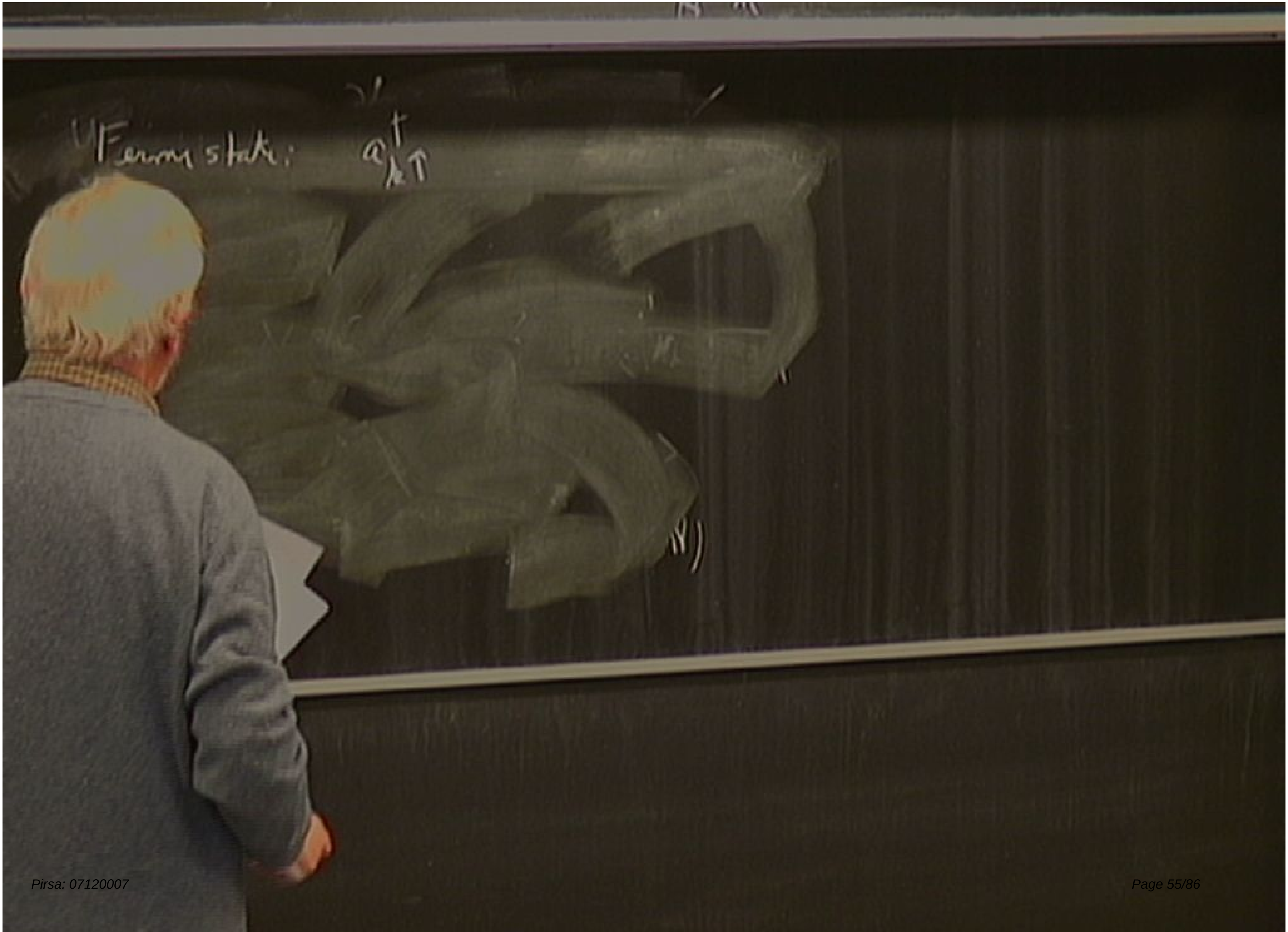
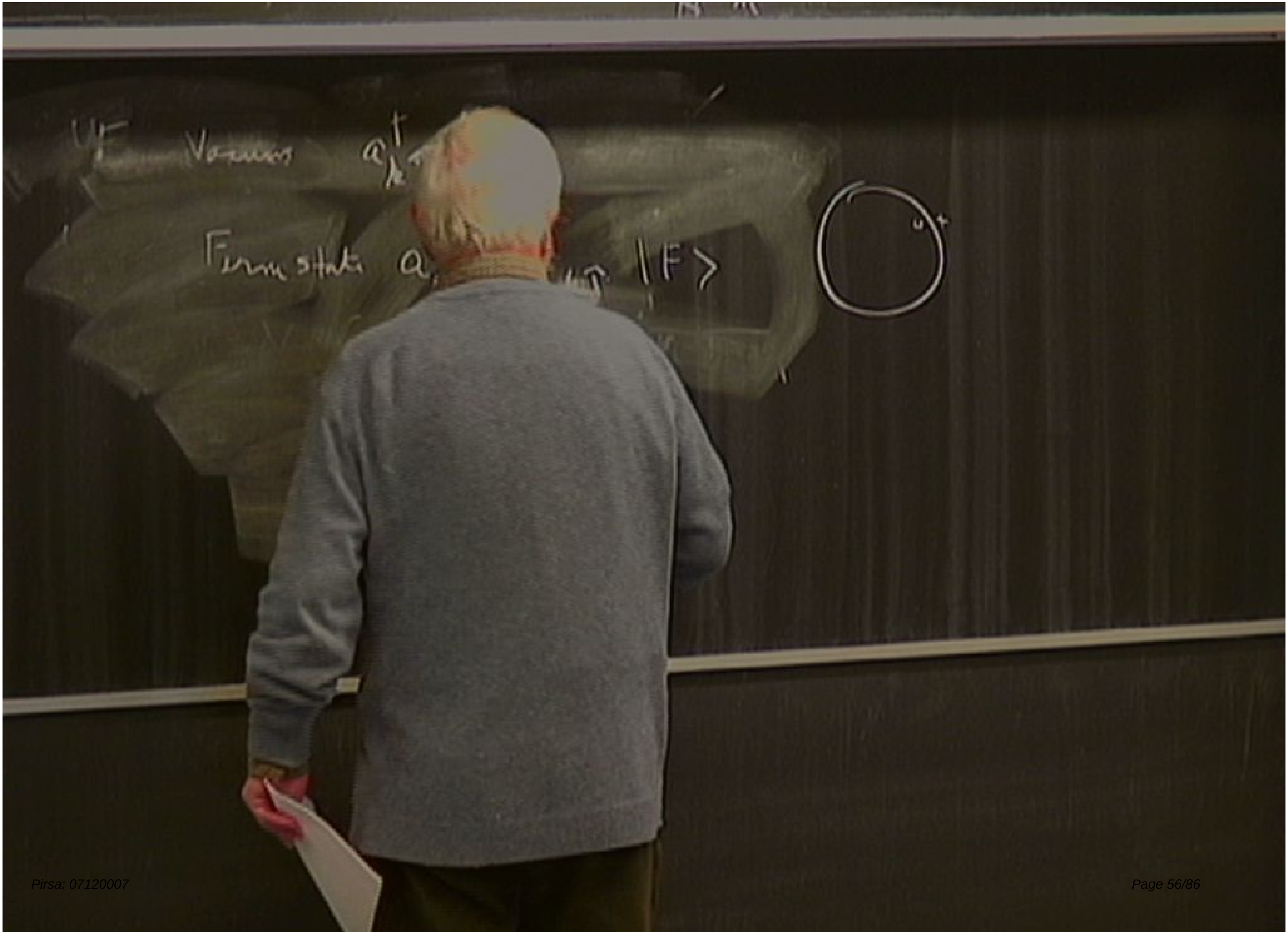
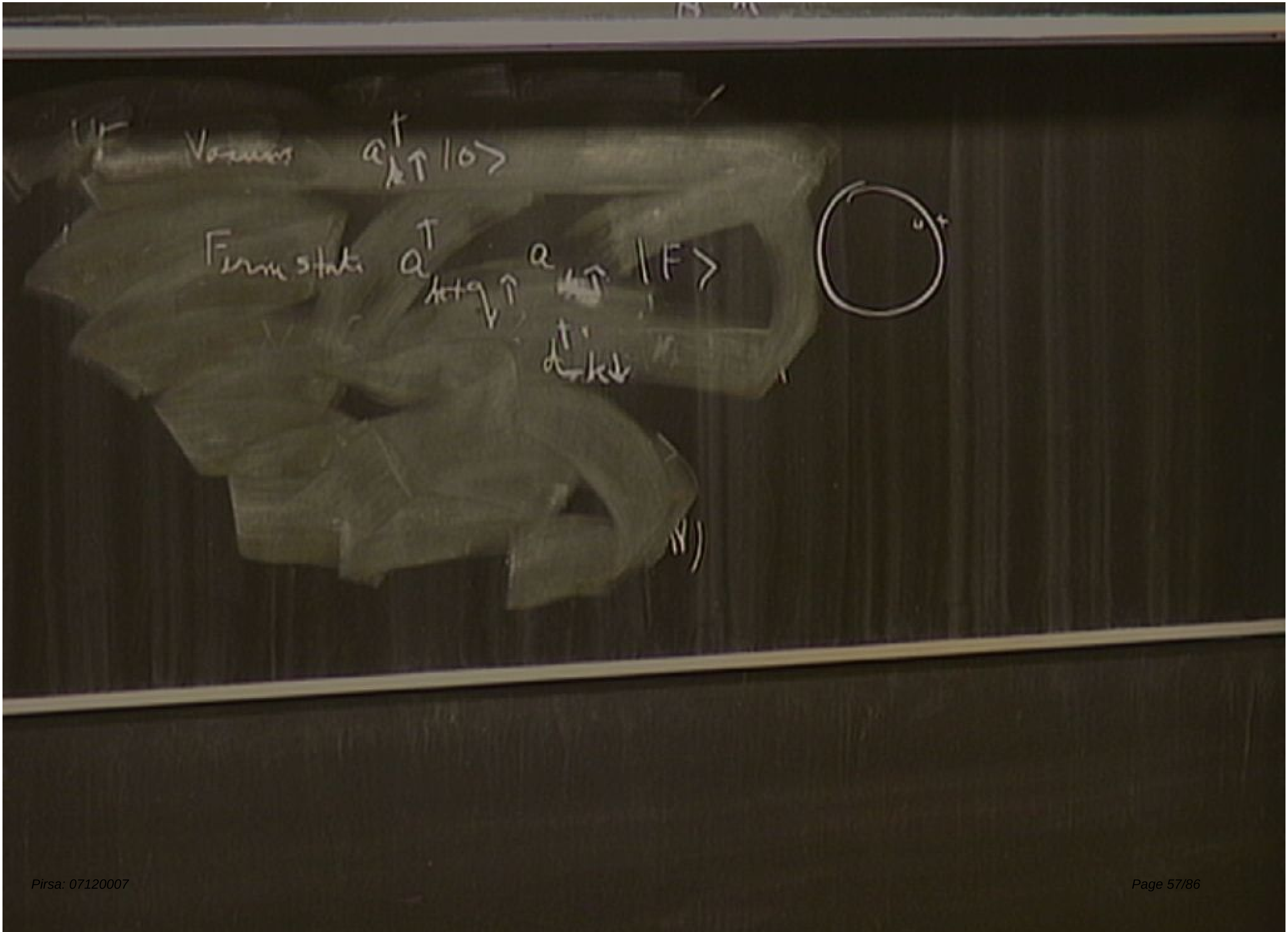
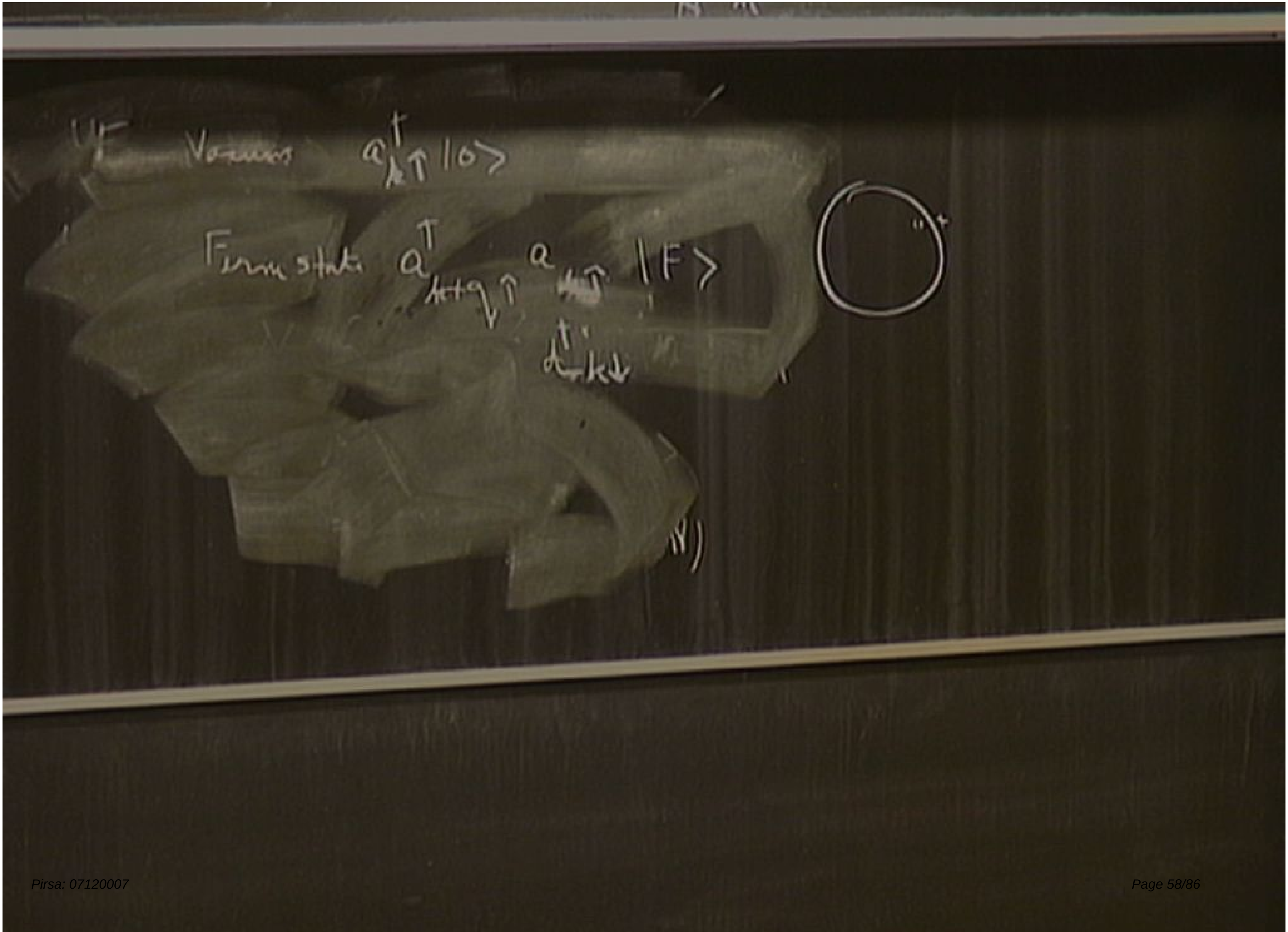
$E_k = \sqrt{\xi_k^2 + \Delta^2}$

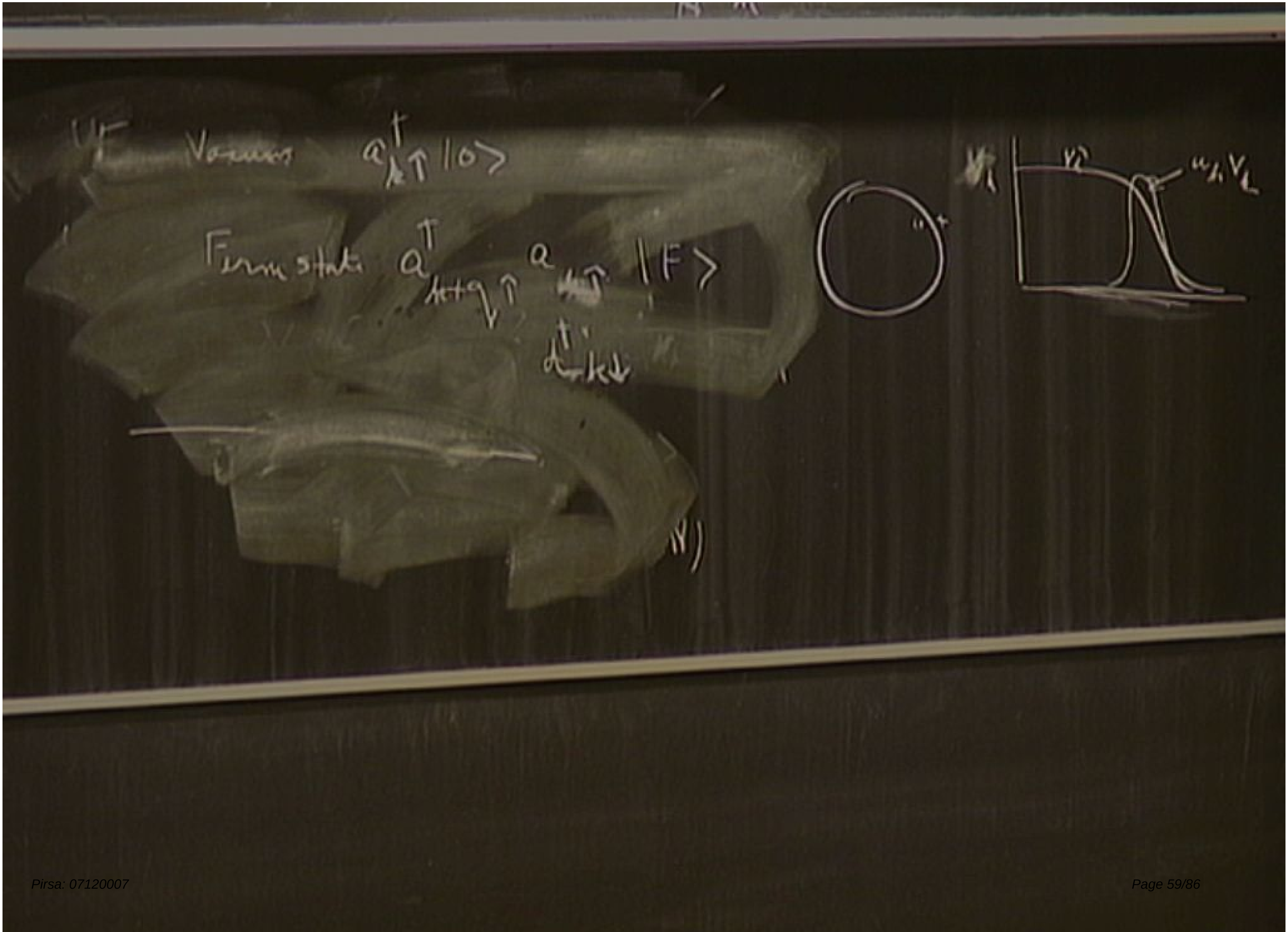


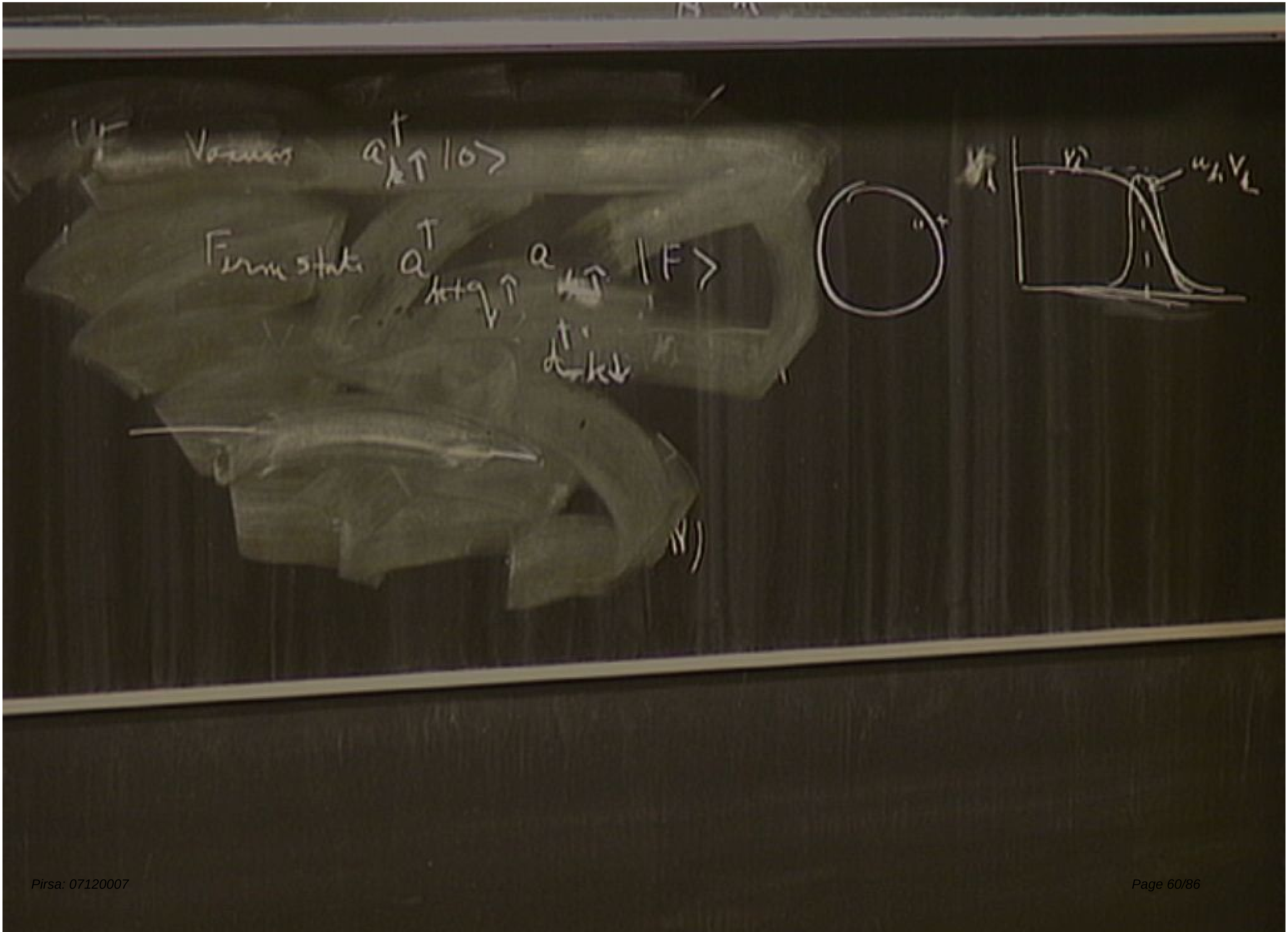

Fermi state: $a^{\dagger}_{k\uparrow}$

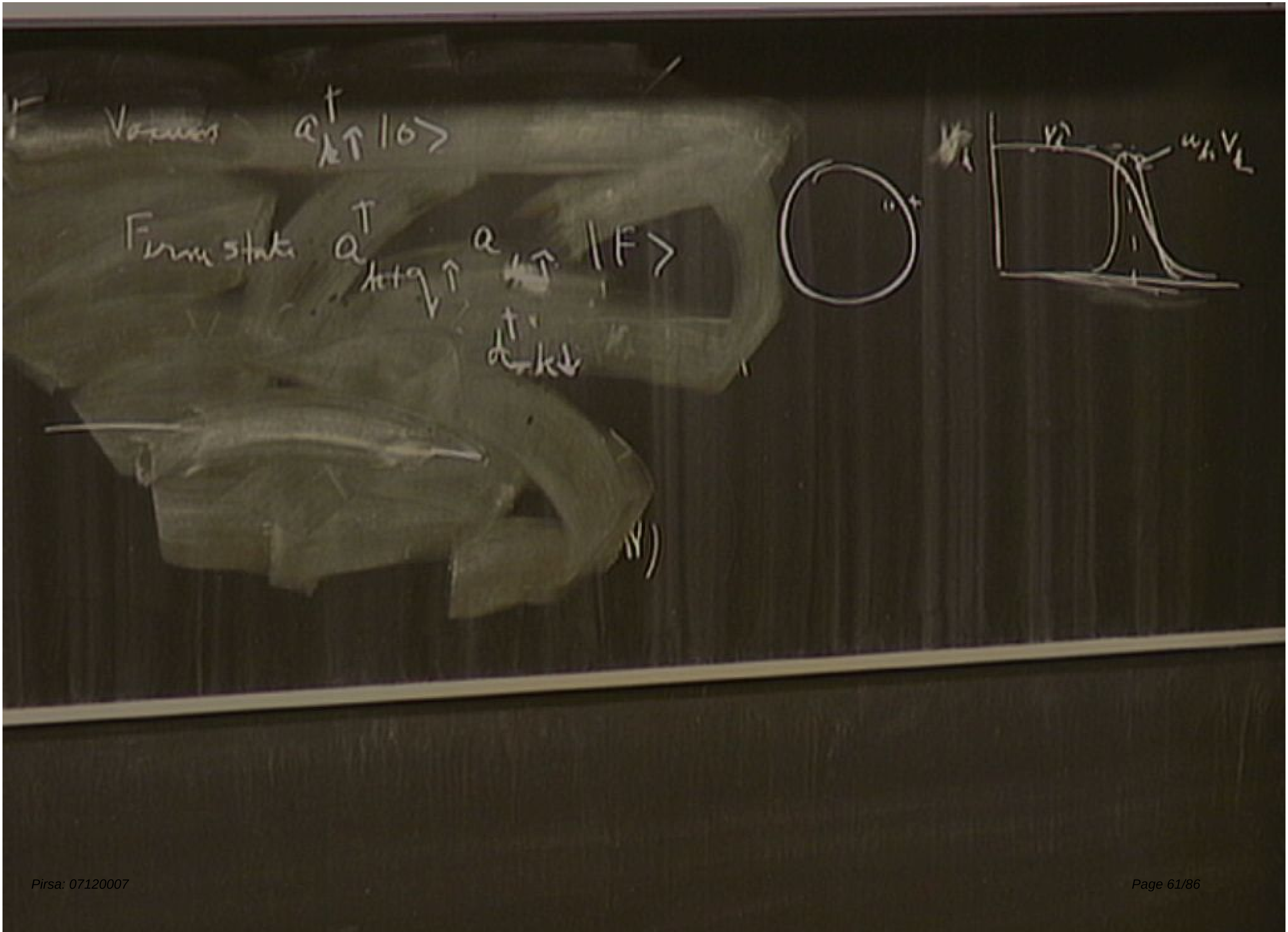

Vacuum
Fermi state
|F>

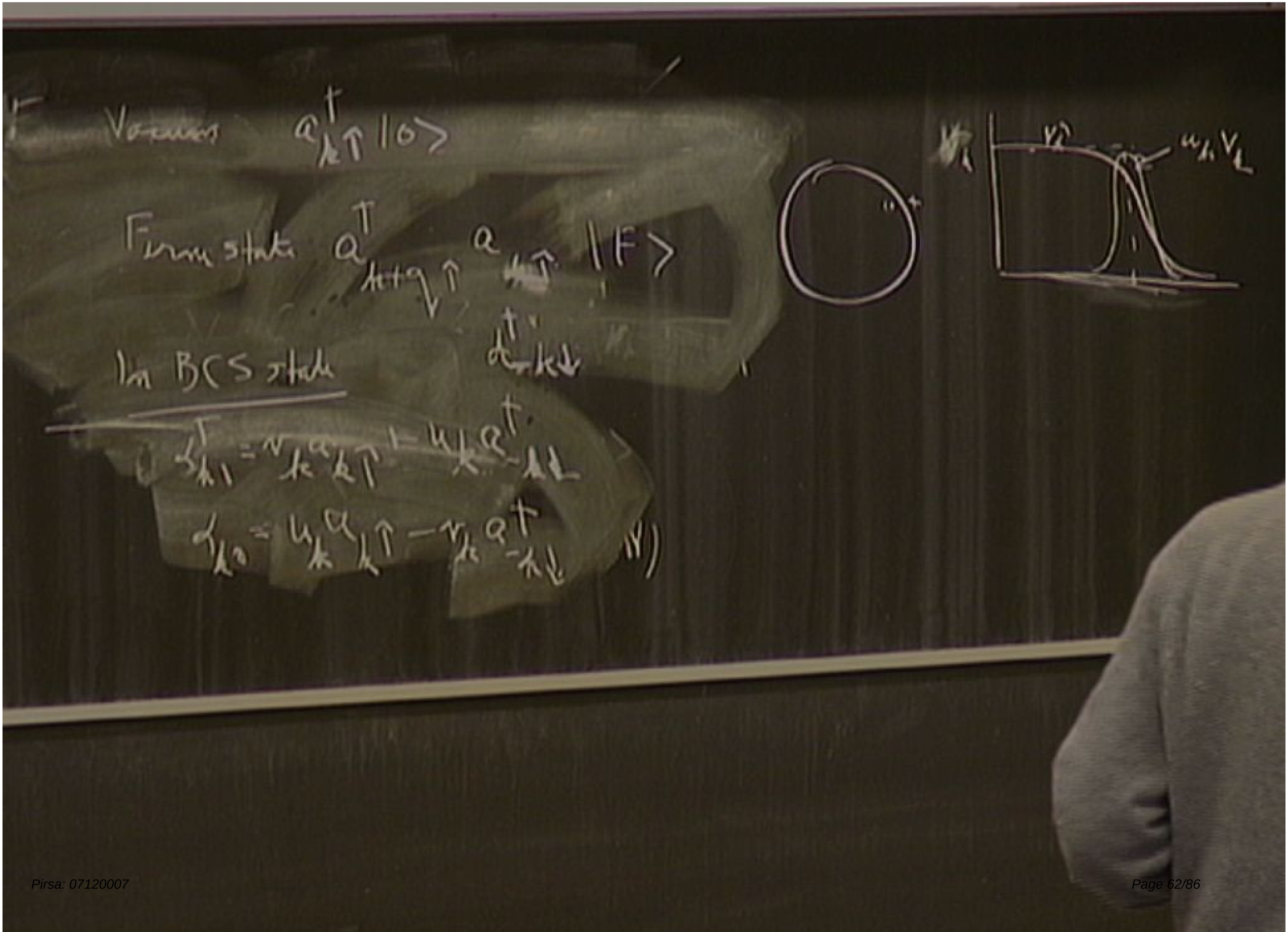

Vacuum $a^\dagger_{k\uparrow}|0\rangle$
Fermi state $a^\dagger_{k+q\uparrow} a_{k\uparrow}|F\rangle$

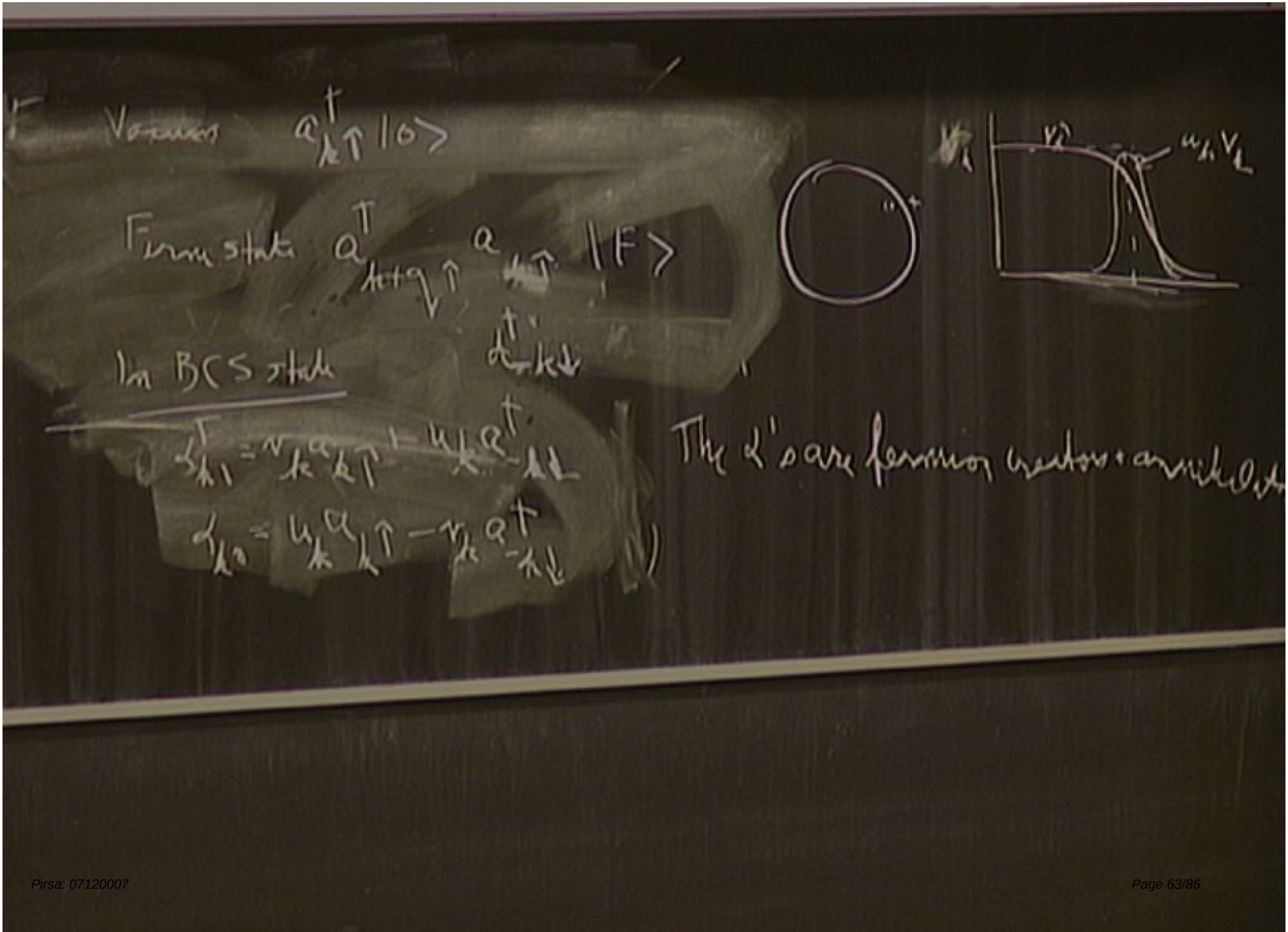

Vacuum
Fermi state
|F>

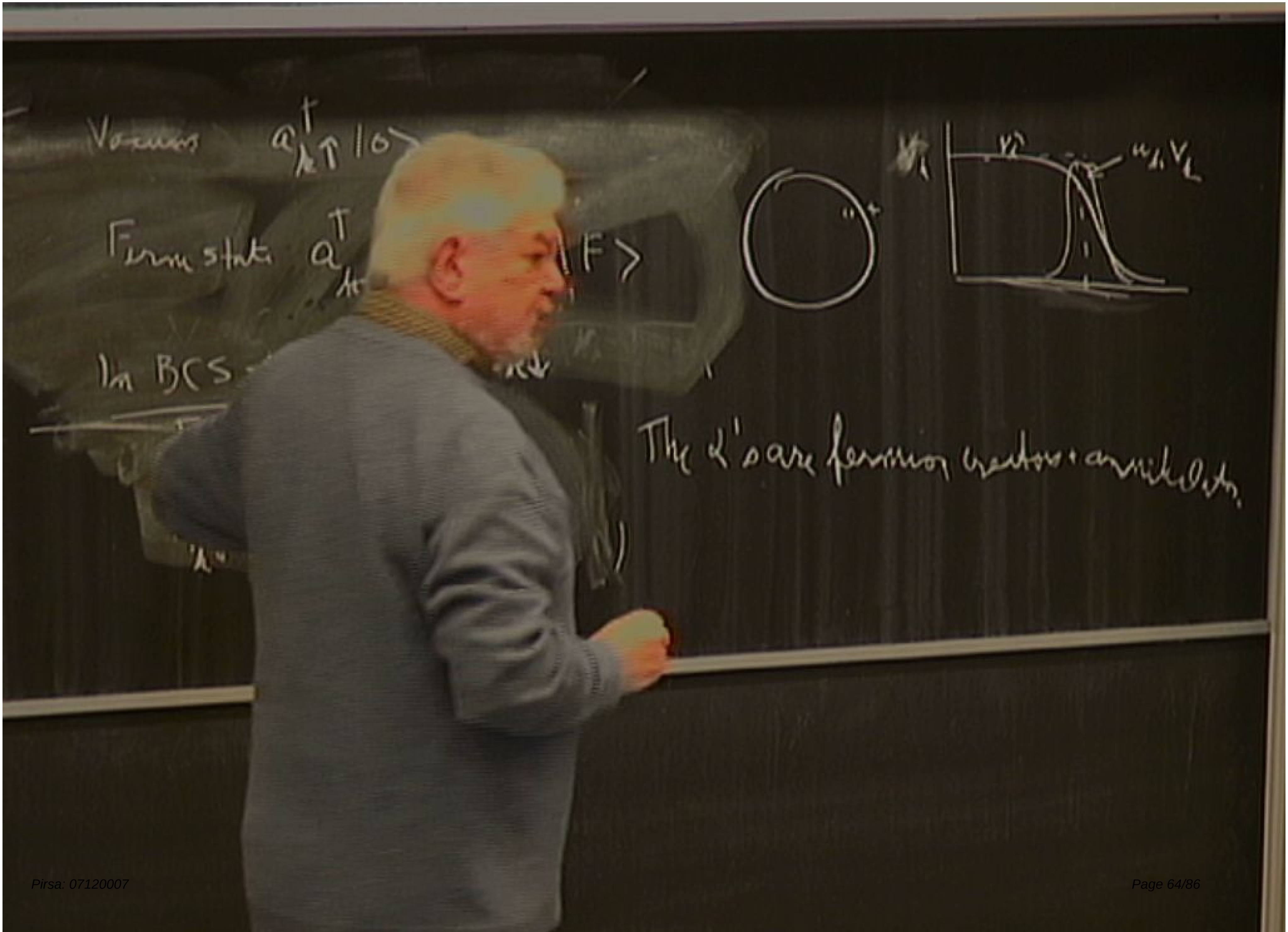

Vacuum
Fermi state


Vacuum
Fermi state


Vacuum
|0>
Fermi state
|F>


Fermi state
In BCS state


Vacuum
Fermi state
In BCS state
The α's are fermion creators + annihilators


Vacuum
Fermi state
In BCS
The α's are fermion creators + annihilators

Define $u_k v_k = \sin 2\theta_k$

$u_k = \cos^2\theta_k$

Stat.

$$\xi_k \tan 2\theta_k = \frac{1}{2}\sum V_{kk'} \sin 2\theta_{k'}$$

Define $\Delta_k = -\sum_{k'} V(k, k') \frac{\sin 2\theta_{k'}}{2}$

$$\tan 2\theta_k = -\frac{\Delta_k}{\xi_k} \qquad \sin 2\theta_k = \frac{\Delta_k}{E_k} \qquad \cos 2\theta_k = -\frac{\xi_k}{E_k}$$

$$E_k = \sqrt{\xi_k^2 + \Delta^2}$$

Temp. Dep. Effects

Landau Ginzburg Action

(In addition to spin order, where $|\vec\nabla\Delta|^2$ comes from)

Gauge Invariance $i\vec\nabla \to i\vec\nabla - 2e\vec A$

Quantized Flux, Josephson

$n_k = 1$ (...)

$$H_{am} = \sum \xi_k n_k + \sum V a_{k_1} a^\dagger_{k_2} a_{k_3} a^\dagger_{k_4} \delta(k_1+k_2-k_3+k_4)$$

$\langle BCS | H_{am} | BCS \rangle$ = 2 types of terms important S.C.

$$H_{red} = \sum \xi_k \langle n_k \rangle + \sum V_{kk'} \langle b_{k'} \rangle \langle b^\dagger_{k} \rangle + h.c.$$

$n_k = v_k^2$ $\quad \langle b_k \rangle = u_k v_k$

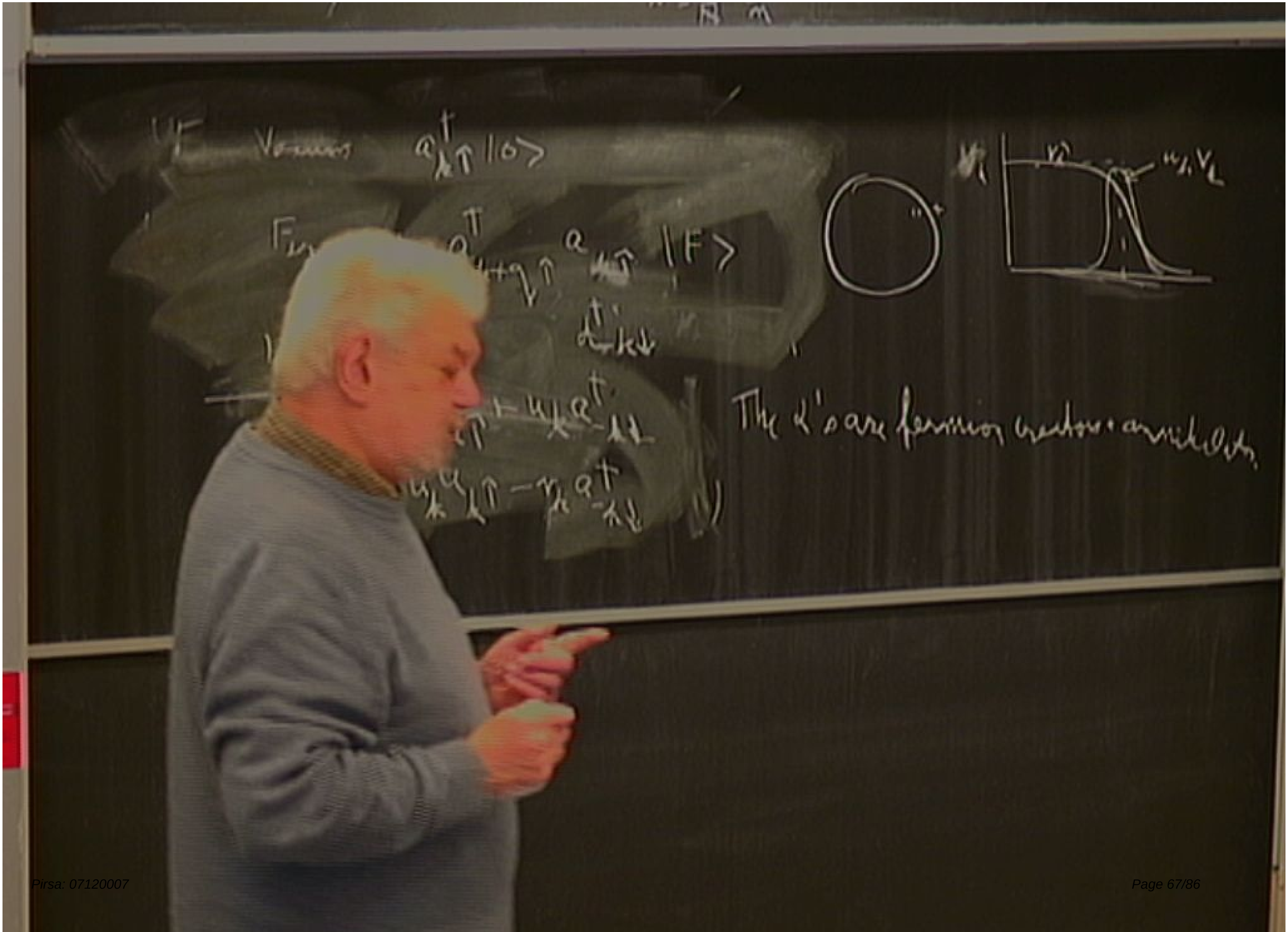
Define $u_k v_k = \sin 2\Theta_k$

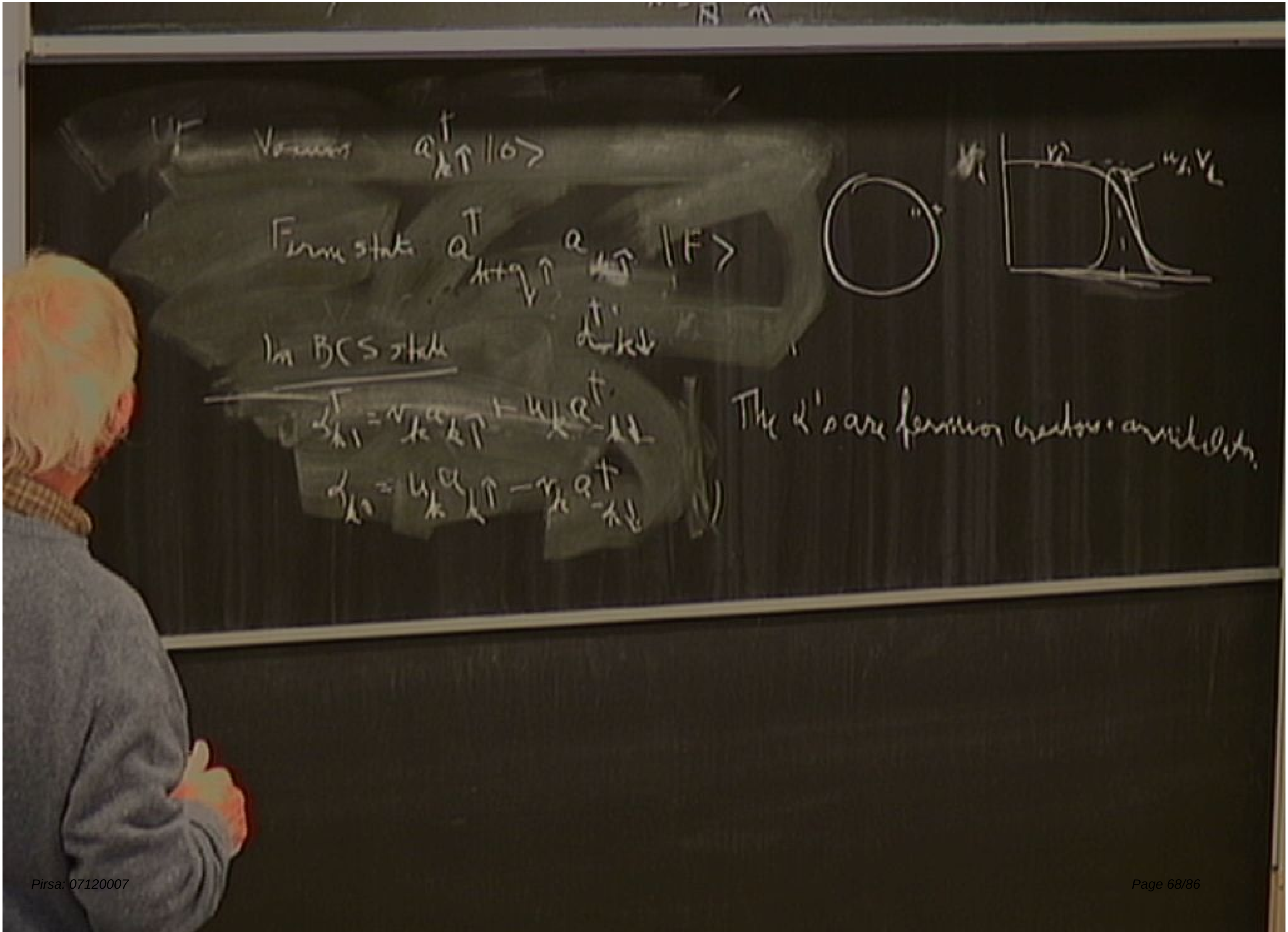
$n_k = \cos^2 \Theta_k$

Stat.

$$\xi_k \tan 2\Theta_k = \frac{1}{2} \sum V_{kk'} \sin 2\Theta_{k'}$$

Define $\Delta_k = - \sum_{k} V(k, k') \frac{\sin 2\Theta_{k'}}{2}$





Vacuum $a^\dagger_{k\uparrow}|0\rangle$
Fermi state
$|F\rangle$
In BCS state
The α's are fermion creators & annihilators

Landau Ginzburg Action

(In addition to spin order where $|\vec{\nabla}\Delta|^2$ comes from)

Gauge Invariance $i\vec{\nabla} \to i\vec{\nabla} - 2e\vec{A}$

Quantized Flux, furthermore

$$H_{am} = \sum \xi_k n_k + \sum V a^{}_{k_1} a^{\dagger}_{k_2} a_{l_1} a^{\dagger}_{l_2} \, \delta(k_1+k_2 - l_1+l_2)$$

$\langle BCS | H_{am} | BCS \rangle$ = 2 types of terms important S.C.

$$H_{red} = \sum \xi_k \langle n_k \rangle + \sum V_{kk'} \langle b_{k'} \rangle \langle b^{\dagger}_{k} \rangle + h.c.$$

$n_k = v_k^2 \qquad \langle b_k \rangle = u_k v_k$

Define $u_k v_k = \sin 2\Theta_k$

$n_k = \cos^2 \Theta_k$

Soln.

$$\xi_k \tan 2\Theta_k = \frac{1}{2} \sum V_{kk'} \sin 2\Theta_{k'}$$

Define $\Delta_k = -\sum_{k} V(k, k') \frac{\sin 2\Theta_{k'}}{2}$

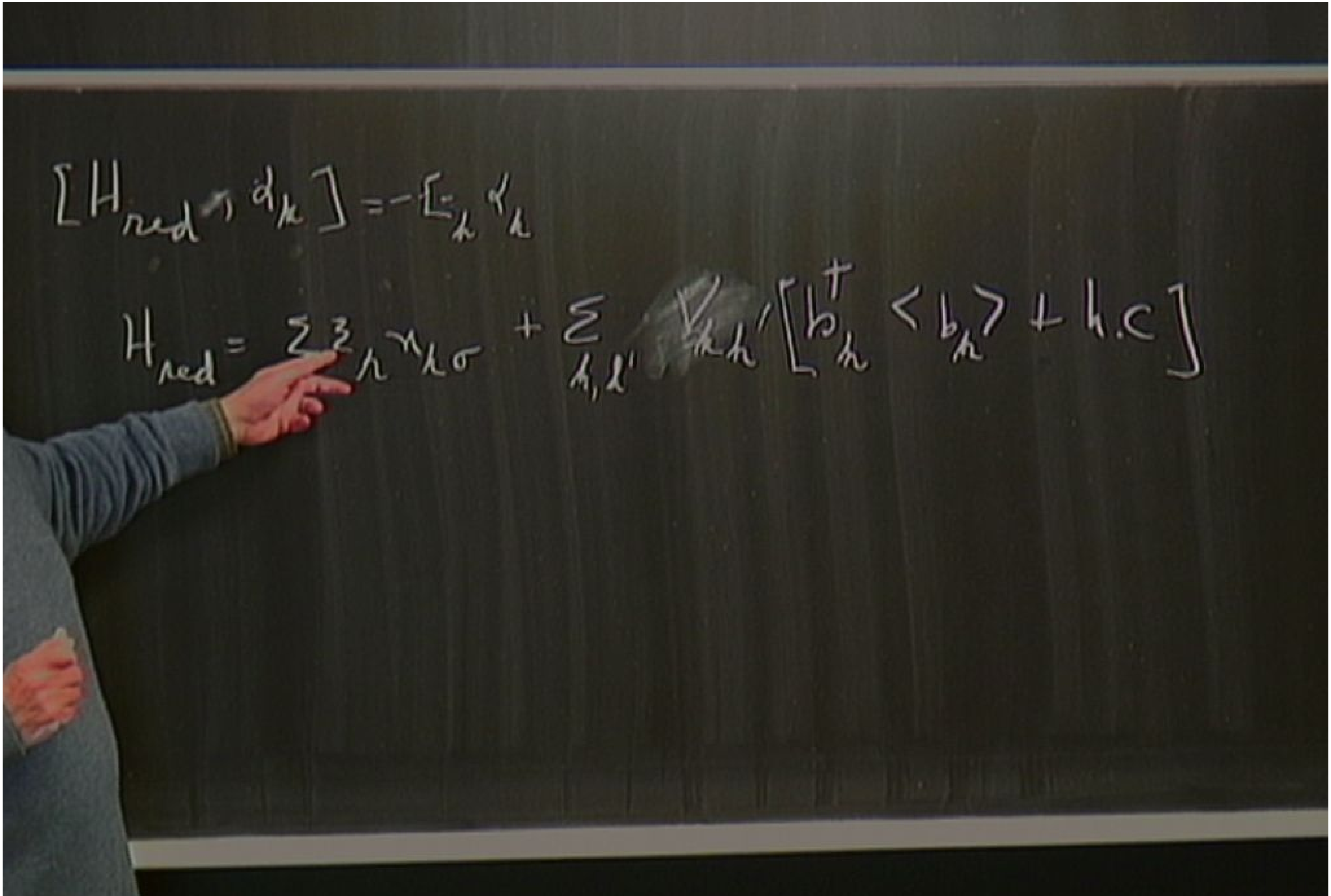
$$\tan 2\Theta_k = -\frac{\Delta_k}{\xi_k} \qquad \sin 2\Theta_k = \frac{\Delta_k}{E_k} \qquad \cos 2\Theta_k = -\frac{\xi_k}{E_k}$$

$E_k = \sqrt{\xi_k^2 + \Delta_k^2}$

$$[H_{red}, \alpha_k] = -\epsilon_k \alpha_k$$

$$[H_{red}, \alpha_k] = -\varepsilon_k \alpha_k$$

$$H_{red} = \sum \varepsilon_k n_{k\sigma} + \sum_{k,k'} V_{kk'} [b_k^\dagger \langle b_k \rangle + h.c]$$


[H_red , α_k] = -ξ_k α_k
H_red = Σ ξ_k n_kσ + Σ_{k,k'} V_kk' [b_k^† <b_k> + h.c]

$$[H_{red}, \alpha_k] = -\xi_k \alpha_k$$

$$H_{red} = \sum \xi_k n_{k\sigma} + \sum_{k,k'} V_{kk'} \left[b_k^\dagger \langle b_k \rangle + h.c \right]$$

$$[H_{red}, \alpha_k] = -\epsilon_k \alpha_k$$

$$H_{red} = \sum \xi_k n_{k\sigma} + \sum_{k,k'} V_{kk'} [b_k^\dagger \langle b_{k'} \rangle + h.c.]$$

Define $u_k v_k = \sin 2\theta_k$

$n_k = \cos^2\theta_k$

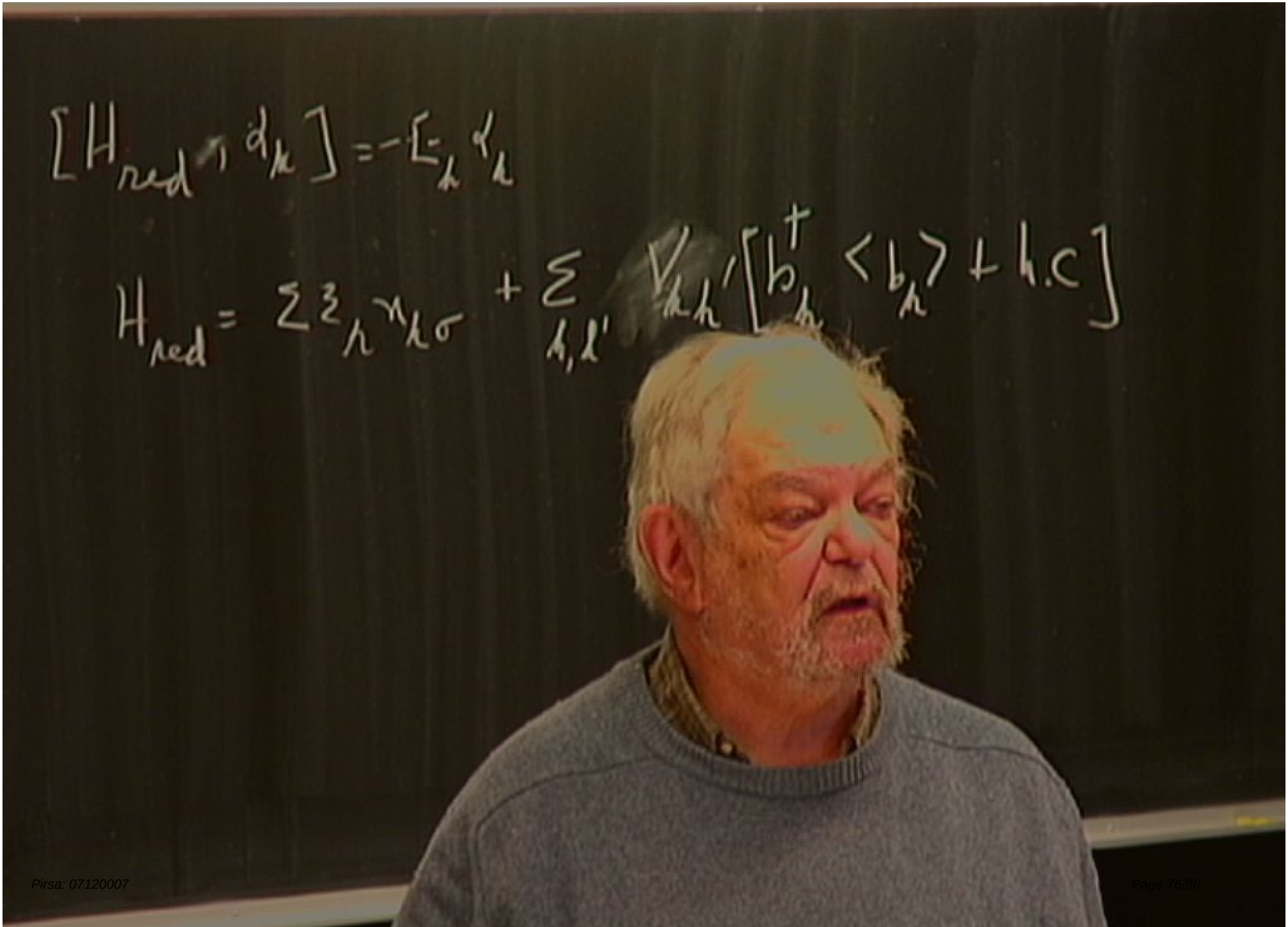
Stat.

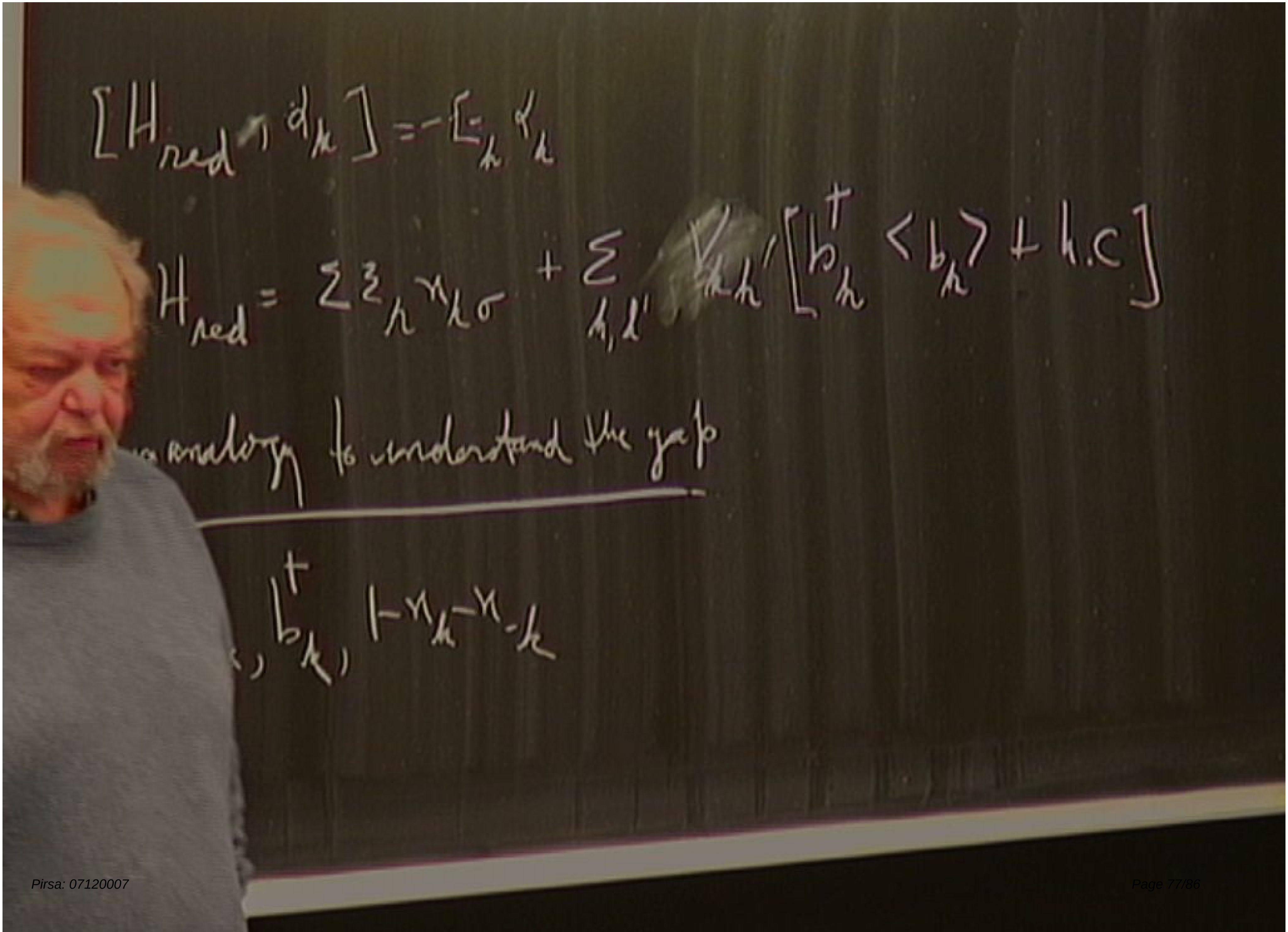
$\xi_k \tan 2\theta_k = \frac{1}{2} \sum V_{k k'} \sin 2\theta_{k'}$

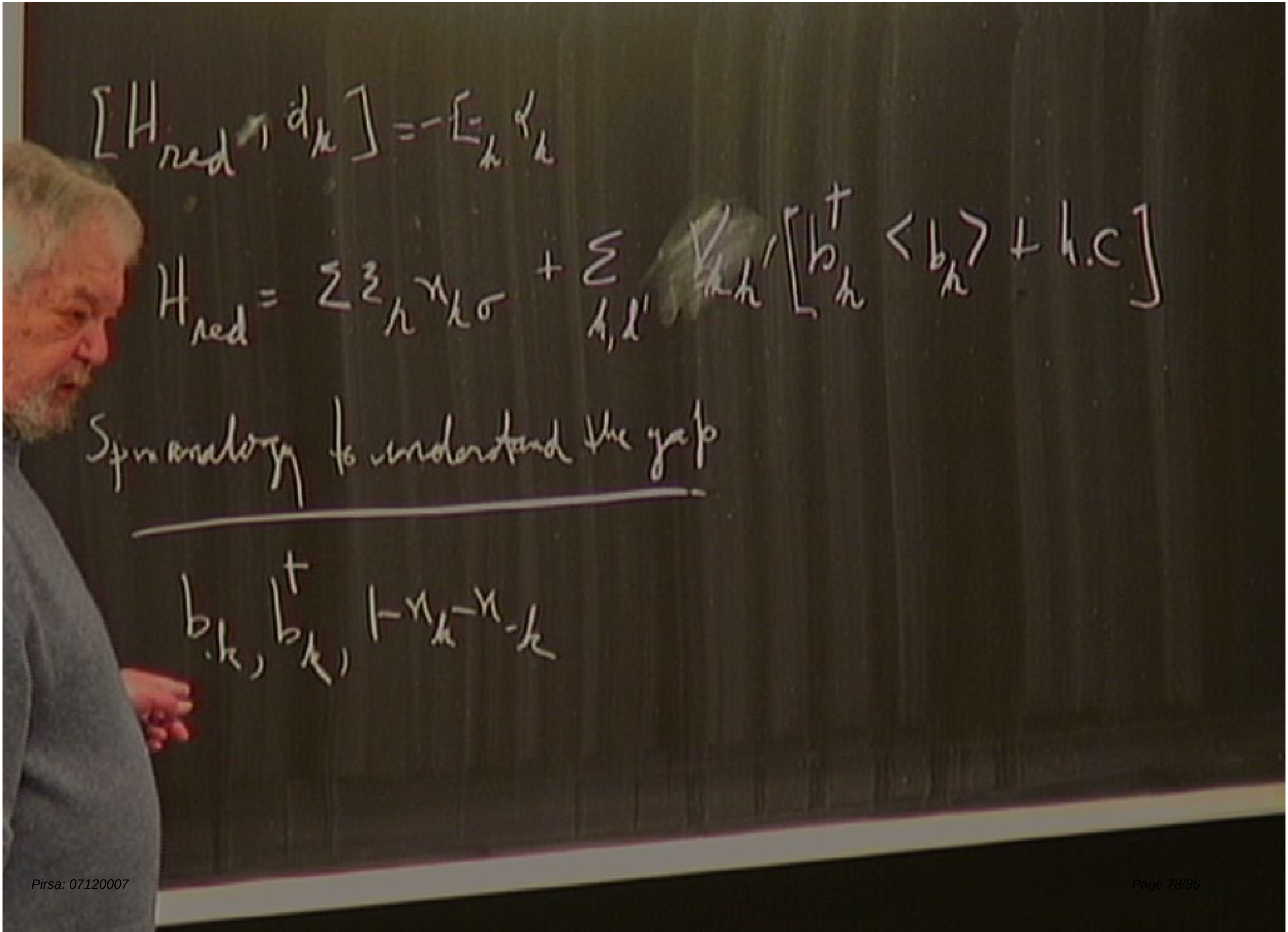
Define $\Delta_k = - \sum_{k} V(k, k') \frac{\sin 2\theta_{k'}}{2}$

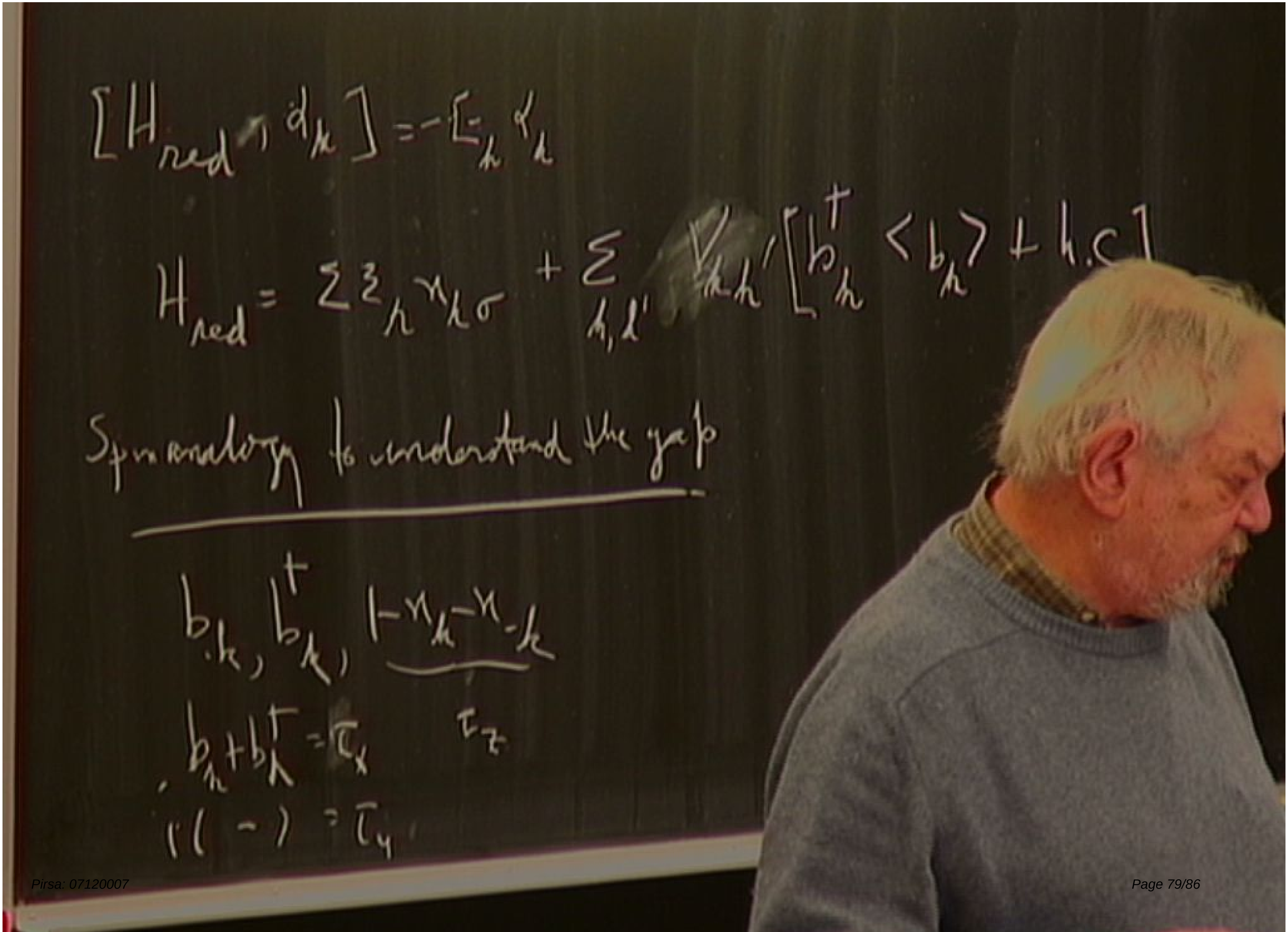
$\tan 2\theta_k = -\frac{\Delta_k}{\xi_k}$ $\quad \sin 2\theta_k = \frac{\Delta_k}{E_k}$ $\quad \cos 2\theta_k = -\frac{\xi_k}{E_k}$

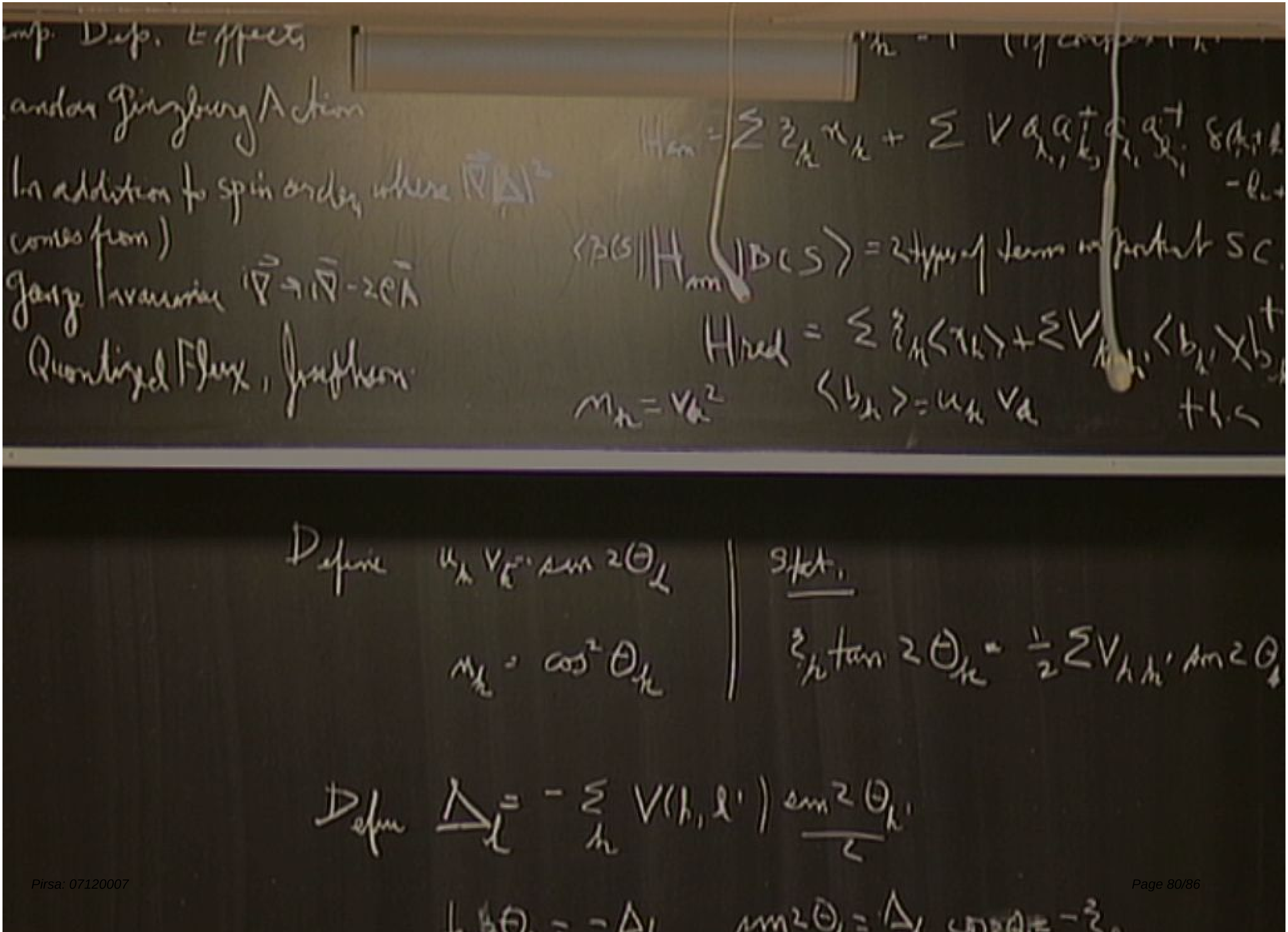
$E_k = \sqrt{\xi_k^2 + \Delta^2}$


$[H_{red}, \alpha_k] = -E_k \alpha_k$
$H_{red} = \sum \xi_k n_{k\sigma} + \sum_{k,k'} V_{kk'}[b_k^{\dagger} \langle b_k \rangle + h.c]$


[H_red , α_k] = −E_k α_k
H_red = Σ ξ_k n_kσ + Σ_{k,k'} V_kk' [b_k^† ⟨b_k⟩ + h.c]
analogy to understand the gap
b_k^† (1 − n_k − n_−k)


[H_red , α_k] = −E_k α_k
H_red = Σ ε_k n_kσ + Σ_{k,k'} V_kk' [b†_k <b_k'> + h.c]
Spin analogy to understand the gap
b_k , b†_k , 1−n_k−n_−k


[H_red , α_k] = −E_k α_k
H_red = Σ ξ_k n_kσ + Σ_{k,k'} V_kk' [b†_k <b_k> + h.c.]
Spin analogy to understand the gap
b_k , b†_k , 1 − n_k − n_−k
b_k + b†_k = τ_x   τ_z
i(−) = τ_y


Landau Ginzburg Action
In addition to spin order, where $|\vec{\nabla}\Delta|^2$
comes from)
Gauge Invariance
Quantized Flux, Josephson
Define
Define

Temp. Dep. Effects

Landau Ginzburg Action

((In addition to spin order, where $|\vec{\nabla}\Delta|^2$ comes from)

Gauge Invariance $i\vec{\nabla} \to i\vec{\nabla} - 2e\vec{A}$

Quantized Flux, Josephson

$$H_{am} = \sum \xi_k n_k + \sum V a_{k_1} a^{\dagger}_{k_2} a_{k_3} a^{\dagger}_{k_4} \delta(k_1+k_2, -k_3+k_4)$$

$\langle BCS | H_{am} | BCS \rangle$ = 2 types of terms important S.C.

$$H_{red} = \sum \xi_k \langle n_k \rangle + \sum V_{kk'} \langle b_{k'} \rangle \langle b^{\dagger}_{-k} \rangle + h.c.$$

$n_k = v_k^2 \qquad \langle b_k \rangle = u_k v_k$

Define $u_k v_k = \sin 2\Theta_k$

$n_k = \cos^2 \Theta_k$

Spect.

$$\xi_k \tan 2\Theta_k = \frac{1}{2} \sum V_{kk'} \sin 2\Theta_{k'}$$

Define $\Delta_k = -\sum_{k'} V(k, k') \frac{\sin 2\Theta_{k'}}{2}$

$$\tan 2\Theta_k = -\frac{\Delta_k}{\xi_k} \qquad \sin 2\Theta_k = \frac{\Delta_k}{E_k} \qquad \cos 2\Theta_k = -\frac{\xi_k}{E_k}$$

$$E_k = \sqrt{\xi_k^2 + \Delta_k^2}$$

$$[H_{red}, \alpha_k] = -E_k \alpha_k$$

$$H_{red} = \sum \xi_k n_{k\sigma} + \sum_{k,k'} V_{kk'} [b_k^\dagger \langle b_k \rangle + h.c.]$$

Spin analogy to understand the gap

$$b_k,\ b_k^\dagger,\ 1 - n_k - n_{-k}$$

$$b_k + b_k^\dagger = \tau_x \qquad \tau_z$$

$$i(\ -\ ) = \tau_y$$

Define $u_k V_k = \sin 2\theta_k$

$n_k = \cos^2\theta_k$

Step 1

$\xi_k \tan 2\theta_k = \frac{1}{2}\sum V_{k k'}$

Define $\Delta_k = -\sum_{k'} V(k, k') \frac{\sin 2\theta_{k'}}{2}$

$\tan 2\theta_k = -\frac{\Delta_k}{\xi_k}$ $\quad \sin 2\theta_k = \frac{\Delta_k}{E_k}$ $\quad \cos 2\theta_k = -\frac{\xi_k}{E_k}$

$E_k = \sqrt{\xi_k^2 + \Delta^2}$

Define $u_k v_k = \sin 2\Theta_k$

$n_k = \cos^2 \Theta_k$

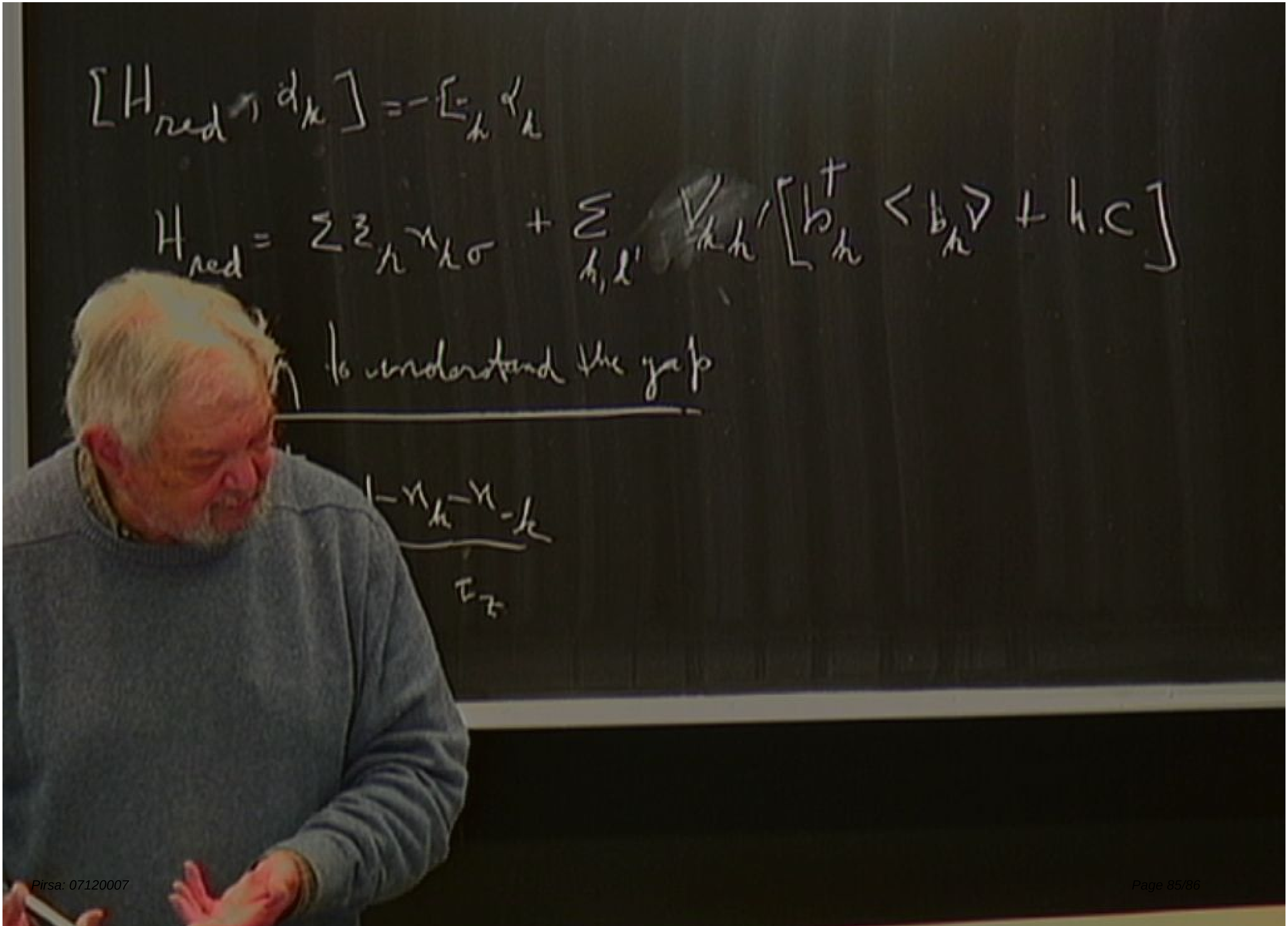
Stat.:

$\xi_k \tan 2\Theta_k = \frac{1}{2} \sum V_{kk'} \sin 2\Theta_{k'}$

Define $\Delta_k = -\sum_{k'} V(k, k') \frac{\sin 2\Theta_{k'}}{2}$

$\tan 2\Theta_k = -\frac{\Delta_k}{\xi_k}$ $\quad \sin 2\Theta_k = \frac{\Delta_k}{E_k}$ $\quad \cos 2\Theta_k = -\frac{\xi_k}{E_k}$

$E_k = \sqrt{\xi_k^2 + \Delta^2}$


to understand the gap

$$[H_{red}, \alpha_k] = -E_k \alpha_k$$

$$H_{red} = \sum \xi_k n_{k\sigma} + \sum_{k,k'} V_{kk'} [b_k^\dagger \langle b_k \rangle + h.c]$$

Spin analogy to understand the gap

$b_k, b_k^\dagger, 1 - n_k - n_{-k}$

$b_k + b_k^\dagger = \tau_x \qquad \tau_z$

$i(-) = \tau_y$