

Title: Spontaneous Broken Symmetry 6A

Date: Dec 06, 2007 10:00 AM

URL: <http://pirsa.org/07120007>

Abstract:

BCS

gd Ensemble

Bogolyubov and Spin Analogy

Temp. Dep. Effects (T_c)

Landau Ginzburg Action

(In addition to spin order, where $|\vec{\nabla}\Delta|^2$ comes from)

Gauge Invariance $\vec{\nabla} \rightarrow \vec{\nabla} - 2e\vec{A}$

Quantized Flux, Josephson

$$|BCS\rangle = \prod_k (u_k + v_k b_k^\dagger) |0\rangle$$

$$b_k = a_{k\uparrow} a_{-k\downarrow}$$

$$u_k^2 + v_k^2 = 1 \quad (\text{if complex, } |u_k|^2 + |v_k|^2 = 1)$$

$$H_{BCS} = \sum_k \epsilon_k n_k + \sum V$$

BCS

gd. Ensemble

Bogolyubov and Spin Analogy

Temp. Dep. Effects (T_c)

Landau Ginzburg Action

(In addition to spin order, where $|\vec{\nabla} \Delta|^2$ comes from)

Gauge Invariance $\vec{\nabla} \rightarrow \vec{\nabla} - 2e\vec{A}$

Quantized Flux, Josephson

$$|BCS\rangle = \prod_k (u_k + v_k \hat{b}_k^\dagger) |0\rangle$$

$$\hat{b}_k = a_k \uparrow + a_{-k} \downarrow$$

$$u_k^2 + v_k^2 = 1 \quad (\text{if complex, } |u_k|^2 + |v_k|^2 = 1)$$

$$H_{BCS} = \sum_k \epsilon_k n_k + \sum V$$

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$$u_k^2 + v_k^2 = 1 \quad (\text{if complex } |u_k|^2 + |v_k|^2)$$

$$H_{BCS} = \sum_k \epsilon_k n_k + \sum V$$

BCS

Gen. Ensemble

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Temp. Dep. Effects (T_c)

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Quantized Flux, Josephson

$$|BCS\rangle = \prod_k (u_k + v_k b_k^\dagger) |0\rangle$$

$$b_k = a_{k\uparrow} a_{-k\downarrow}$$

$$u_k^2 + v_k^2 = 1 \quad (|u_k|^2 + |v_k|^2 = 1)$$

$$H_{BCS} = \sum_k \epsilon_k n_k + \sum V$$

BCS

gd. Ensemble

• Bogolyubov and Spin Analogy

Temp. Dep. Effects (T_c)

Landau Ginzburg Action

(In addition to spin order, where $|\vec{\nabla}\Delta|^2$ comes from)

Gauge Invariance $\vec{\nabla} \rightarrow i\vec{\nabla} - 2e\vec{A}$

Quantized Flux, Josephson

$$|BCS\rangle = \prod_k (u_k + v_k \hat{b}_k^\dagger) |0\rangle$$

$$\hat{b}_k = a_{k\uparrow} - a_{k\downarrow}$$

$$u_k^2 + v_k^2 = 1 \quad \text{complex } |u_k|^2 + |v_k|^2 = 1$$

$$H_{\text{BCS}} = \sum_k \epsilon_k \hat{b}_k^\dagger \hat{b}_k$$

BCS

Gr. Ensemble

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(In addition to spin order, where $|\vec{\nabla}\Delta|^2$ comes from)

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Quantized Flux, Josephson

$$BCS \hat{=} \prod_k (u_k + v_k \hat{b}_k^\dagger) |0\rangle$$

$$\hat{b}_k = u_k \uparrow + v_k \downarrow$$

$$u_k^2 + v_k^2 = 1$$

$$\text{complex } |u_k|^2 + |v_k|^2$$

$$H_{BCS} = \sum_k \epsilon_k \hat{b}_k^\dagger \hat{b}_k$$

$$a^\dagger a$$

BCS

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(In addition to spin order, where $|\vec{\nabla}\Delta|^2$ comes from)

Gauge Invariance $\vec{\nabla} \rightarrow \vec{\nabla} - 2e\vec{A}$

Quantized Flux, Josephson

$$BCS \approx \prod_k (u_k + v_k \tau) |0\rangle$$

$$b_k = a_k \tau a_k$$

$$u_k^2 + v_k^2$$

$$\text{complex } |u_k|^2 + |v_k|^2$$

$$H_{\text{kin}} = \sum_k \epsilon_k a^\dagger a$$

BCS

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Quantized Flux, Josephson

$$|BCS\rangle = \prod_k (u_k + v_k b_k^\dagger) |0\rangle$$

$$b_k = a_{k\uparrow} - a_{k\downarrow}$$

$$u_k^2 + v_k^2 = 1 \quad \text{complex } |u_k|^2 + |v_k|^2 = 1$$

$$H_{BCS} = \sum_k \epsilon_k a_k^\dagger a_k + \dots$$

BCS

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Quantized Flux, Josephson

$$|BCS\rangle = \prod_k (u_k + v_k b_k^\dagger) |0\rangle$$

$$b_k = a_{k\uparrow} \mp a_{-k\downarrow}$$

$$u_k^2 + v_k^2 = 1 \quad (\text{if complex, } |u_k|^2 + |v_k|^2)$$

$$H_{BCS} = \sum_k \epsilon_k n_k + \sum_{k_1, k_2, k_3, k_4} V_{k_1, k_2, k_3, k_4} a_{k_1}^\dagger a_{k_2}^\dagger a_{k_3} a_{k_4} \delta_{k_1+k_2, -k_3-k_4}$$

BCS

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Quantized Flux, Josephson

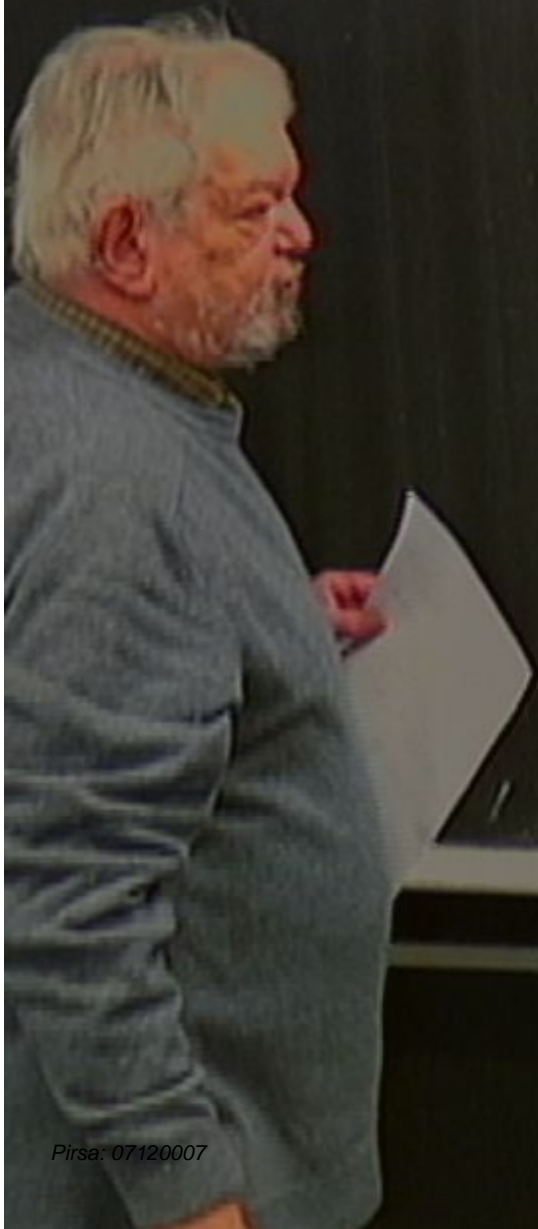
$$BCS \rangle = \prod_k (u_k + v_k b_k^\dagger) |0\rangle$$

$$b_k = a_{k\uparrow} a_{-k\downarrow}$$

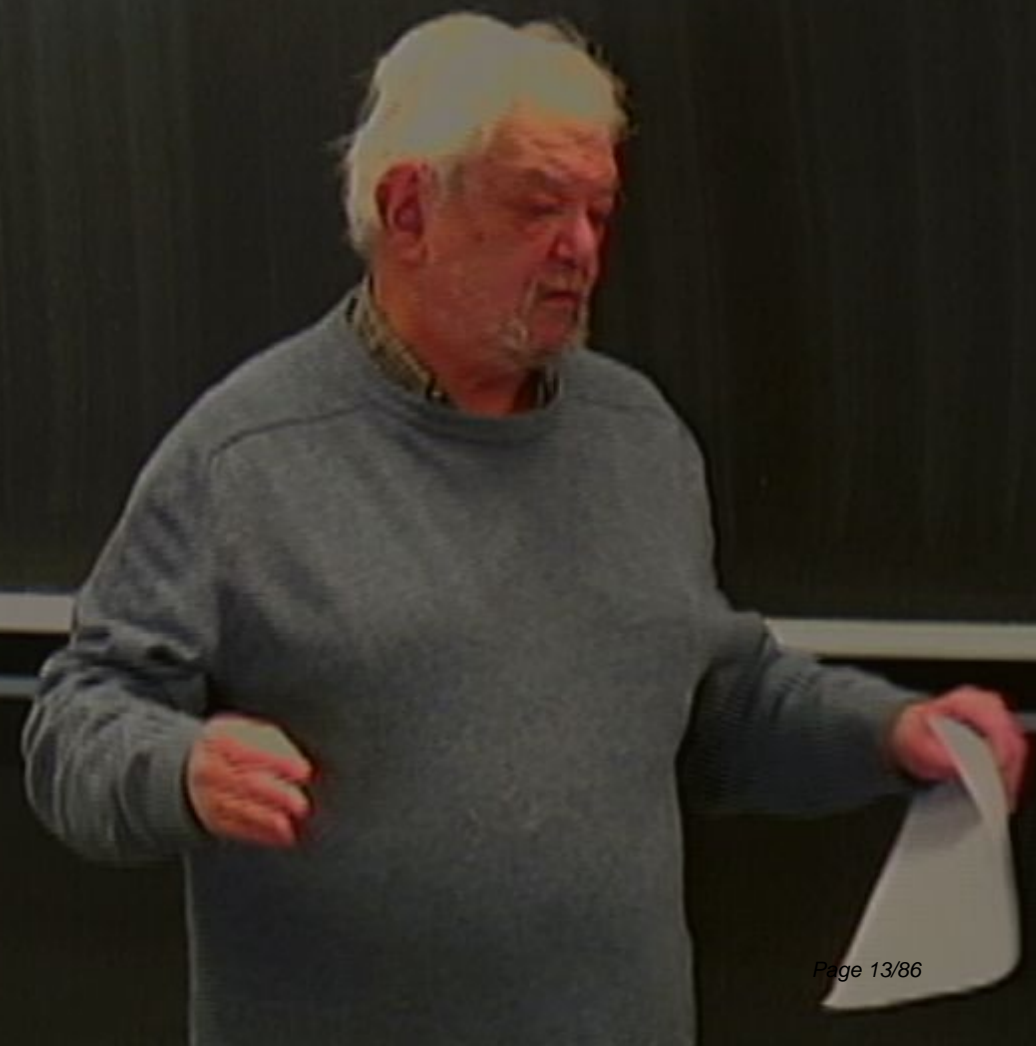
$$u_k^2 + v_k^2 = 1 \quad (\text{if complex } |u_k|^2 + |v_k|^2)$$

$$H_{BCS} = \sum_k \epsilon_k a_{k\uparrow}^\dagger a_{k\uparrow} + \sum_{k_1, k_2, k_3, k_4} V a_{k_1\uparrow}^\dagger a_{k_2\uparrow}^\dagger a_{k_3\downarrow} a_{k_4\downarrow} \delta_{k_1+k_2, -k_3-k_4}$$

$$\mathcal{P}(N, V, \beta, \mu)$$



$$P(N, V, \beta, \mu)$$



$$\mathcal{P}(N; V, \beta, \mu) = \frac{1}{\mathcal{Z}} \beta^N e^{-\beta H}$$

$$P(N; V, \beta, \mu) = \frac{\sum_{\{N_i\}} \prod_k \beta^{\mu N_k} e^{-\beta H}}{\Xi}$$

$$P(N; V, \beta, \mu) = \frac{\sum_{N} \beta \mu^N e^{-\beta H}}{\Xi}$$

$$\Xi = e^{\beta \mu V}$$

$$P(N; V, \beta, \mu) = \frac{\sum \beta^N h_N e^{-\beta H_N}}{\Xi}$$

$$\Xi = e^{\beta P V} \approx e^{\beta \mu N^*} h_{N^*} e^{-\beta H}$$



$$P(N; V, \beta, \mu) = \frac{\sum \beta^N h_N e^{-\beta H_N}}{\Xi}$$

$$\frac{1}{\Xi} = e(\beta P V) \approx e(\beta \mu N^*) h_{N^*} e^{-\beta H_{N^*}}$$

$$P(N; V, \beta, \mu) = \frac{\exp(\beta \mu N) \hbar_N e^{-\beta H_N}}{\Xi}$$

$$\frac{1}{\Xi} = e^{\beta p V} \approx e^{\beta \mu N^*} \hbar_{N^*} e^{-\beta H_{N^*}}$$

$$P(N; V, \beta, \mu) = \frac{1}{Z} \sum_N \beta \mu^N h_N e^{-\beta H_N} / \equiv$$

$$\frac{1}{Z} e^{\beta \mu V} \approx e^{\beta \mu N^*} h_{N^*} e^{-\beta H_{N^*}}$$

$$\left(\beta \mu - \frac{\partial \Phi}{\partial N} \right)_{N^*, \beta, V}$$

$$P(N; V, \beta, \mu) = \frac{1}{\Xi} \sum_N \beta^N h_N e^{-\beta H_N}$$

$$\frac{1}{\Xi} = e^{\beta \mu V} \approx e^{\beta \mu V^*} h_{N^*} e^{-\beta H_{N^*}} \left(\beta \mu - \frac{\partial F}{\partial N} \right)_{N^*, \beta, V}$$

$$\mathcal{P}(N; V, \beta, \mu) = \frac{\exp(\beta \mu N) \int e^{-\beta H_N} \mathcal{D}N}{\Xi}$$

$$\frac{1}{\Xi} = e^{\beta P V} \approx e^{\beta \mu N^*} \int e^{-\beta H_{N^*}} \mathcal{D}N^* ; \quad \beta \mu - \beta P = 0$$

$$P(N; V, \beta, \mu) = \frac{1}{\Xi} \sum_N \beta^N h_N e^{-\beta H_N}$$

$$\frac{1}{\Xi} = e^{\beta P V} \approx e^{\beta \mu N^*} h_{N^*} e^{-\beta H_{N^*}} \quad ; \quad \left(\beta \mu - \frac{\partial \ln \Xi}{\partial N} \right)_{N^*, \beta, V} = 0$$

$$P(N; V, \beta, \mu) = \frac{1}{\Xi} \sum_N \beta^N t_N e^{-\beta H_N} / \Xi$$

$$\frac{1}{\Xi} = e^{\beta P V} \approx e^{\beta P V^*} t_{N^*} e^{-\beta H_{N^*}} ; \quad \beta \mu = \left. \frac{\partial F}{\partial N} \right|_{N^*, \beta, V} = \zeta = \text{Thermodyn def. of } \mu$$

Therm $dF = -SdT - PdV + \mu dN$

$$\mathcal{P}(N; V, \beta, \mu) = \frac{\sum_N \beta^N t_N e^{-\beta H_N}}{\Xi}$$

$$\Xi = e^{\beta PV} \approx e^{\beta PV^*} t_{N^*} e^{-\beta H_{N^*}}$$

$$\beta \mu = \frac{\partial F}{\partial N} \Big|_{N^*, \beta, V} = \zeta = \text{Thermodyn def. of } \mu$$

Thermo $dF = -SdT - PdV + \mu dN$

$$\langle \Delta N^2 \rangle = O(N) = \frac{1}{\beta} \kappa \quad ; \quad \kappa = -\frac{1}{V} \left(\frac{\partial V}{\partial \beta} \right)_T$$

$$P(N; V, \beta, \mu) = \frac{1}{\Xi} \sum_N \beta^N t_N e^{-\beta H_N} / \Xi$$

$$\frac{1}{\Xi} \sum_N \beta^N t_N e^{-\beta H_N} \approx e^{\beta \mu V} t_{N^*} e^{-\beta H_{N^*}} ; \quad \beta \mu = \left. \frac{\partial F}{\partial N} \right|_{N^*, \beta, V} = \mu = \text{Thermodyn def. of } \mu$$

Therm $dF = -SdT - PdV + \mu dN$

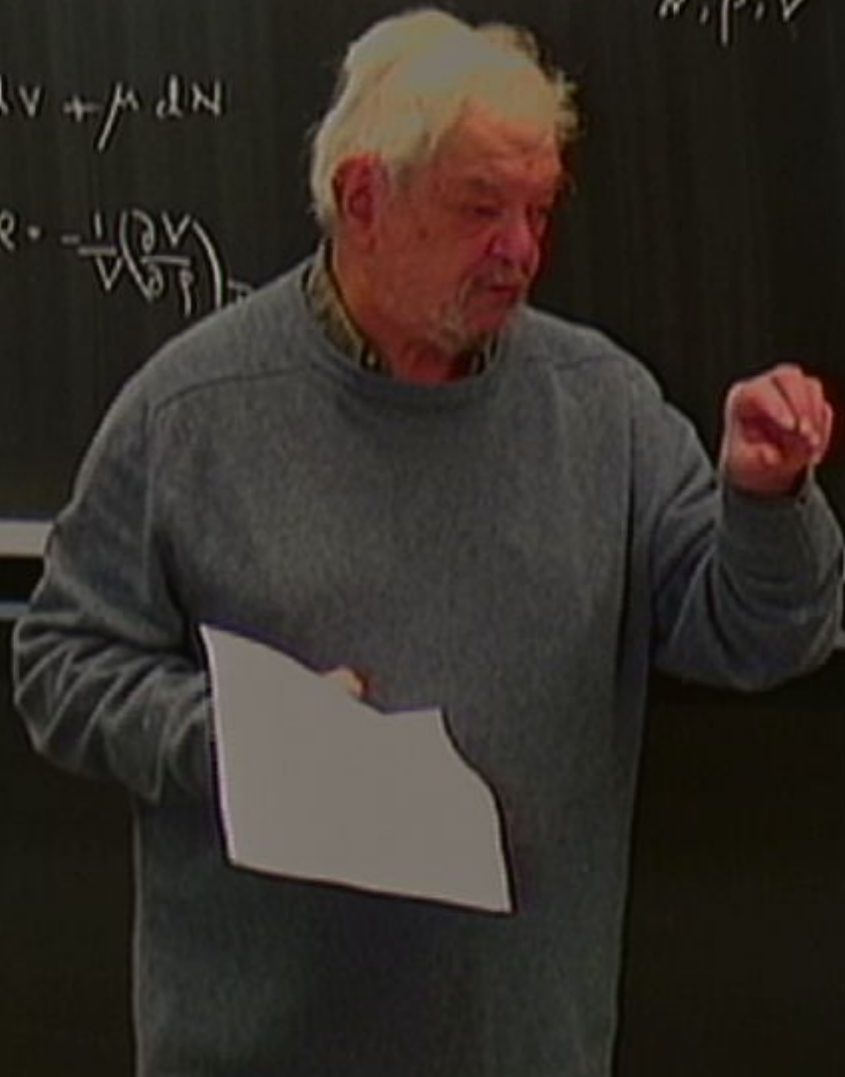
$$\frac{1}{\Xi} \langle \Delta N^2 \rangle = O(N) = \frac{1}{\beta} \kappa = -\frac{1}{V} \left(\frac{\partial V}{\partial \beta} \right)_{T, N}$$

$$P(N; V, \beta, \mu) = \frac{1}{\Xi} \sum_N \beta^N t_N e^{-\beta H_N} / \Xi$$

$$\frac{1}{\Xi} \sum_N \beta^N t_N e^{-\beta H_N} \approx e^{\beta \mu N^*} t_{N^*} e^{-\beta H_{N^*}} ; \left(\beta \mu - \frac{\partial F}{\partial N} \right)_{N^*, \beta, V} = 0 \text{ - Thermodyn def. of } \mu$$

Therm $dF = -SdT - PdV + \mu dN$

$$\frac{1}{\Xi} \langle \Delta N^2 \rangle = O(N) = \frac{1}{\beta} \chi_{NN} ; \chi = -\frac{1}{V} \left(\frac{\partial V}{\partial \beta} \right)$$



$$\rho(N; V, \beta, \mu) = \frac{1}{\Xi} \sum_N \beta^N \frac{1}{N!} e^{-\beta H_N} / \Xi$$

$$\frac{1}{\Xi} \int \rho(N; V, \beta, \mu) \approx e^{\beta \mu} \frac{1}{N!} e^{-\beta H_N} ; \quad \left(\beta \mu - \frac{\partial F}{\partial N} \right)_{N^*, \beta, V} = 0 = \text{Thermodyn def. of } \mu$$

$$dF = -SdT - PdV + \mu dN$$

$$\frac{1}{\rho} \left(\frac{\partial \rho}{\partial N} \right)_{T, V} = \frac{1}{\rho} \frac{\partial \rho}{\partial N} ; \quad \kappa = -\frac{1}{V} \left(\frac{\partial V}{\partial P} \right)_{T, N}$$

$$P(N; V, \beta, \mu) = \frac{z_N \beta^N t_N e^{-\beta H_N}}{\Xi}$$

$$\frac{1}{\Xi} \Xi = e^{\beta P V} \approx e^{\beta P V^*} t_{N^*} e^{-\beta H_{N^*}} ; \quad \beta \mu - \left. \frac{\partial F}{\partial N} \right|_{N^*, \beta, V} = \mu = \text{Thermodyn def. of } \mu$$

Therm $dF = -SdT - PdV + \mu dN$

$$\frac{1}{N} \langle \Delta N^2 \rangle = O(1/N) = \frac{1}{\beta} \kappa_{NN}$$



$$\mathcal{P}(N; V, \beta, \mu) = \frac{z^N \beta^N t_N e^{-\beta H_N}}{\Xi}$$

$$\frac{1}{\Xi} \Xi = e^{\beta P V} \approx e^{\beta P V^*} t_{N^*} e^{-\beta H_{N^*}} ; \quad \left(\beta \mu - \frac{\partial F}{\partial N} \right)_{N^*, \beta, V} = 0 = \text{Thermodyn def. of } \mu$$

Therm $dF = -SdT - PdV + \mu dN$

$$\langle \Delta N^2 \rangle = O(N) = \frac{1}{\beta} \kappa_{NN} ; \quad \kappa = -\frac{1}{V} \left(\frac{\partial V}{\partial \beta} \right)_{T, N}$$

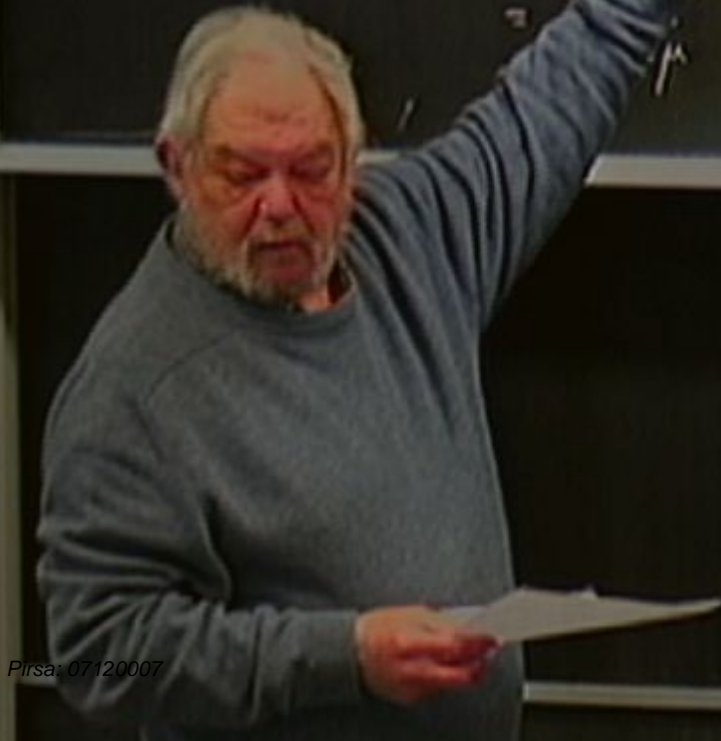
$$P(N; V, \beta, \mu) = \frac{1}{\mathcal{Z}} \sum_{\Omega} e^{-\beta(E - \mu N)}$$

$$\overline{N} = \frac{1}{\mathcal{Z}} \sum_{\Omega} N e^{-\beta(E - \mu N)} \approx e^{\beta \mu} \frac{1}{\mathcal{Z}} \sum_{\Omega} e^{-\beta E}$$

$$\beta \mu = \frac{\partial F}{\partial N} \Big|_{N^*, p, V} = \mu = \text{Thermodyn def. of } \mu$$

Thermodyn $dF = -SdT - PdV + \mu dN$

$$\frac{1}{\mathcal{Z}} \langle \Delta N^2 \rangle = O(N) = \frac{1}{\beta} \frac{\partial^2 \ln \mathcal{Z}}{\partial \mu^2} = -\frac{1}{V} \left(\frac{\partial V}{\partial \mu} \right)_{T, N}$$



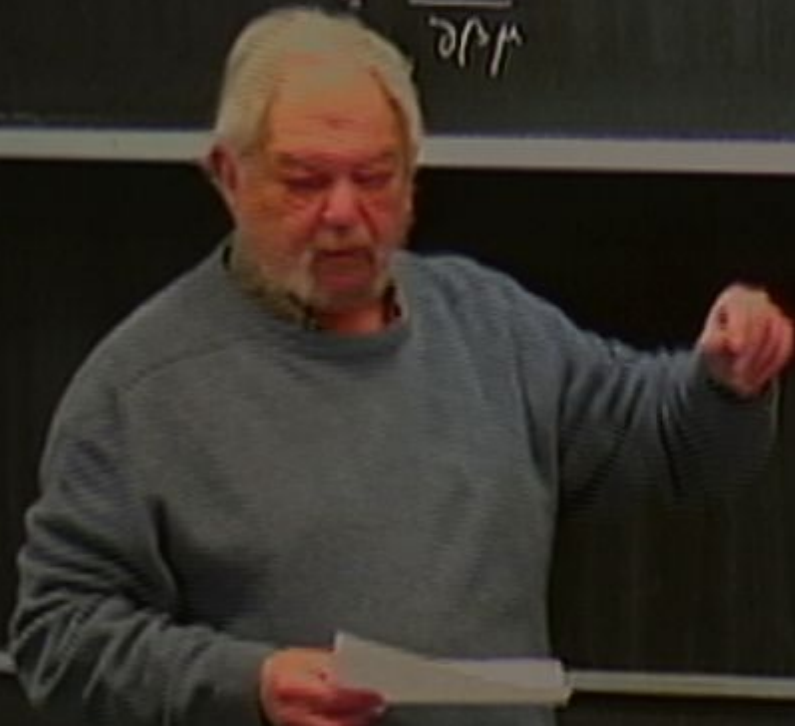
$$P(N; V, \beta, \mu) = \frac{1}{Z} \sum_{N'} \dots$$

$$\bar{N} \approx \frac{1}{Z} e^{\beta \mu N} \approx e^{\beta \mu N^*} \frac{1}{Z} e^{-\beta H(N^*)}$$

$$\beta \mu - \left. \frac{\partial F}{\partial N} \right|_{N^*, \beta, V} = 0 \quad \text{Thermodyn def. of } \mu$$

Thermodyn $dF = -SdT - PdV + \mu dN$

$$\frac{1}{N} \langle \Delta N^2 \rangle = O(N^{-1}) = \frac{1}{N} \frac{\partial \langle N \rangle}{\partial \mu} \quad ; \quad \chi = -\frac{1}{V} \left(\frac{\partial V}{\partial P} \right)_{T, N}$$



$$\underline{\overline{N}} = e^{\beta \mu} \approx e^{\beta \mu} \frac{1}{h_{cl}} e^{-\beta \mu} \quad ; \quad \beta \mu = \frac{\partial F}{\partial N} \Big|_{N^*, \beta, V} = \mu = \text{Thermodyn. def. of } \mu$$

Thermody $dF = -SdT - PdV + \mu dN$

$$\underline{\overline{\langle \Delta N^2 \rangle}} = O(N) = \frac{1}{\beta} \frac{\partial^2 \ln Z}{\partial \mu^2} \quad ; \quad R = -\frac{1}{V} \left(\frac{\partial V}{\partial P} \right)_{T, N}$$

$$= \frac{\partial \langle N \rangle}{\partial \mu} = \frac{1}{\beta} \frac{1}{\langle N \rangle} \frac{\partial \langle N \rangle}{\partial \mu}$$



$$\frac{1}{\Omega} \sum_i = e^{\beta \mu V} \approx e^{\beta \mu V} \frac{1}{N} \sum_i \quad ; \quad \left(\beta \mu - \frac{\partial F}{\partial N} \right)_{N, \beta, V} = 0 \text{ - Thermodyn def. of } \mu$$

Thermody $dF = -SdT - PdV + \mu dN$

$$\frac{1}{\Omega} \langle \Delta N^2 \rangle = O(N) = \frac{1}{\beta} \frac{1}{\Omega} \frac{\partial^2 \ln \Omega}{\partial \mu^2} \quad ; \quad \Omega = \frac{1}{V} \left(\frac{\partial V}{\partial \mu} \right)_{T, N}$$

$$\frac{\langle \Delta N^2 \rangle}{\langle N \rangle} = \frac{1}{\beta} \frac{1}{\langle N \rangle} \frac{\partial \langle N \rangle}{\partial \mu}$$

$$d\mu = \frac{1}{\beta} d\beta \text{ at fixed } T$$

$$\rho(N; V, \beta, \mu)$$

$$\underline{\underline{\mathcal{Z}}} \approx e^{\beta p V} \approx e^{\beta \mu N} h_{N, V} e^{-\beta H_{N, V}} \quad ; \quad \left(\beta \mu - \frac{\partial F}{\partial N} \right)_{N, \beta, V} = 0 = \text{Thermodyn def. of } \mu$$

Therm $dF = -SdT - PdV + \mu dN$

$$\underline{\underline{\mathcal{Z}}} \langle \Delta N^2 \rangle = O(N) = \frac{1}{\beta} \frac{d \ln \mathcal{Z}}{d \mu} \quad ; \quad \kappa = -\frac{1}{V} \left(\frac{\partial V}{\partial P} \right)_{T, N}$$

$$\frac{\langle \Delta N^2 \rangle}{\langle N \rangle} = \frac{1}{\beta} \frac{1}{\langle N \rangle} \frac{d \langle N \rangle}{d \mu}$$

$$d\mu = \mu - d p \text{ at fixed } T$$

$\rho(N; V, \mu)$

$$\underline{\rho} \approx e^{\beta \mu V} \approx e^{\beta \mu V} \frac{1}{N!} e^{-\beta H(N, V)} ; \quad \left(\beta \mu = \frac{\partial F}{\partial N} \right)_{N, \rho, V} = \mu = \text{Thermodyn def. of } \mu$$

Therm $dF = -SdT - PdV + \mu dN$

$$\underline{\rho} \langle \Delta N^2 \rangle = O(N) = \frac{1}{\rho} n de ; \quad R = -\frac{1}{V} \left(\frac{\partial V}{\partial \rho} \right)_{T, N}$$

$$\frac{\langle \Delta N^2 \rangle}{\langle N \rangle} = \frac{1}{\rho} \frac{1}{\langle N \rangle} \frac{\partial \langle N \rangle}{\partial \rho}$$

$$d\mu = \mu - d\rho \text{ at fixed } T$$

$\Omega(N; V, \mu)$

$$\underline{\Omega} = \sum_N e^{\beta \mu N} \approx e^{\beta \mu N^*} \frac{1}{\sqrt{2\pi N^*}} e^{-\frac{1}{2} \beta^2 \mu^2 \chi^2} ; \quad \left(\beta \mu - \frac{\partial F}{\partial N} \right)_{N^*, \beta, V} = 0 \text{ - Thermodyn def. of } \mu$$

Thermody $dF = -SdT - PdV + \mu dN$

$$\underline{\Omega} \langle \Delta N \rangle = O(N) = \frac{1}{\beta} \frac{\partial \ln \Omega}{\partial \mu} ; \quad \chi^2 = -\frac{1}{V} \left(\frac{\partial V}{\partial P} \right)_{T, N}$$

$$\frac{\langle \Delta N^2 \rangle}{\langle N \rangle} = \frac{1}{\beta} \frac{1}{\langle N \rangle} \frac{\partial \langle N \rangle}{\partial \mu} = \frac{1}{\beta \chi^2} \quad \text{w/c} \quad d\mu = \chi^{-1} dP \text{ at fixed } T$$

Therm $dF = -SdT - pdV + \mu dN$

$\frac{1}{T} \langle \Delta N^2 \rangle = O(N) = \frac{1}{\beta} \frac{nd}{\mu} ; \mu = -\frac{1}{V} \left(\frac{\partial V}{\partial \beta} \right)_{T, N}$

$\frac{\langle \Delta N^2 \rangle}{\langle N \rangle} = \frac{1}{\beta} \frac{1}{\langle N \rangle} \frac{\partial \langle N \rangle}{\partial \mu} = \frac{1}{\beta} \frac{\partial \ln Z}{\partial \mu} ; d\mu = \mu - d p \text{ at fixed } T$

$$\frac{\partial \langle N \rangle}{\partial \mu} = \frac{1}{\beta} \frac{\partial \langle N \rangle}{\partial \mu} = \frac{1}{\beta} \frac{\partial \langle N \rangle}{\partial \mu} ; \text{ etc.}$$

$$m = m\left(\frac{N}{V}\right) \quad \left. \frac{\partial \ln Z}{\partial \mu} = -\frac{\partial \ln V}{\partial \mu} \right|_{N, T}$$



$$\frac{d\langle N \rangle}{dt} = \frac{1}{\beta} \left(\frac{\partial \langle N \rangle}{\partial \mu} - \frac{\partial \langle N \rangle}{\partial \beta} \right); \text{ also } \frac{d\langle N \rangle}{dt} = \frac{1}{\beta} \frac{\partial \langle N \rangle}{\partial \mu}$$

$$m = m \left(\frac{N}{V} \right) \quad \left. \frac{d\langle N \rangle}{dt} = - \frac{\partial \langle N \rangle}{\partial \beta} \right|_{N, V} \quad \checkmark$$

$$\frac{\langle \Delta N^2 \rangle}{\langle N \rangle} = \frac{1}{\beta} \frac{\langle N^2 \rangle}{\langle N \rangle} = \frac{1}{\beta} \frac{\partial \langle N \rangle}{\partial \mu} \quad ; \quad \text{the nd. } \frac{\langle N^2 \rangle}{\langle N \rangle} = \frac{1}{\beta} \frac{\partial \langle N \rangle}{\partial \mu}$$

$$m = m\left(\frac{N}{V}\right) \quad \left. \frac{\partial \ln Z}{\partial \mu} = \frac{\partial \ln V}{\partial \mu} \right|_{N, T} \quad \checkmark$$

1.7. Ideal gas

$$| \quad n_k = N = \text{fixed} \quad \langle n_k \rangle = \frac{1}{e^{\beta(\epsilon_k - \mu)} + 1}$$

$$\begin{aligned} \langle \Delta N^2 \rangle &= \sum \langle n_k n_{k'} \rangle - \langle n_k \rangle \langle n_{k'} \rangle \\ &= \sum \langle n_k \rangle - \langle n_k \rangle^2 = O(N) \end{aligned}$$

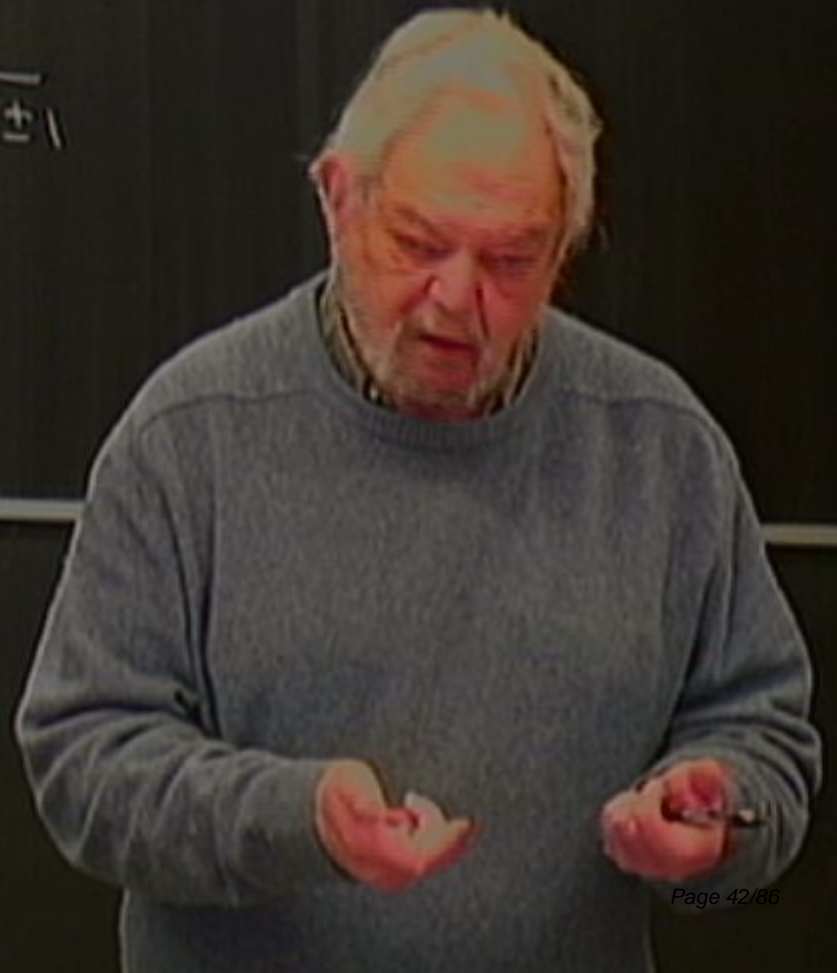
$$\frac{\langle \Delta N^2 \rangle}{\langle N \rangle} = \frac{1}{\beta} \frac{\partial \langle N \rangle}{\partial \mu} = \frac{1}{\beta} \frac{\partial}{\partial \mu} \left(\frac{1}{\beta} \frac{\partial \ln Z}{\partial \mu} \right)$$

$$m = m \left(\frac{N}{V} \right) \quad \left. \frac{\partial m}{\partial \rho} = - \frac{\partial m}{\partial \rho} \right|_{\rho} \quad \checkmark$$

1.7. Ideal gas

$$1 \mid n_k = N = \text{fixed} \quad \langle n_k \rangle = \frac{1}{e^{\beta(\epsilon_k - \mu)} + 1}$$

$$\begin{aligned} \langle \Delta N^2 \rangle &= \sum \langle n_k n_{k'} \rangle - \langle n_k \rangle \langle n_{k'} \rangle \\ &= \sum \langle n_k \rangle - \langle n_k \rangle^2 = O(N) \end{aligned}$$



Gr. Ensemble

Bogolyubov and Spin Analogy

Temp. Dep. Effects (T_c)

Landau Ginzburg Action

(In addition to spin order, where $|\vec{\nabla}\Delta|^2$ comes from)

Gauge Invariance $\vec{\nabla} \rightarrow \vec{\nabla} - 2e\vec{A}$

Quantized Flux, Josephson

$$b_k = a_{k\uparrow} a_{-k\downarrow}$$

$$u_k^2 + v_k^2 = 1 \quad (\text{if complex})$$

$$H_{\text{eff}} = \sum_k \epsilon_k n_k + \sum_{\lambda, \mu} V_{\lambda, \mu} a_{\lambda}^{\dagger} a_{\mu}$$

BCS

Gr. Ensemble

Bogolyubov and Spin Analogy

Temp. Dep. Effects (T_c)

Landau Ginzburg Action

(In addition to spin order, where $|\vec{\nabla}\Delta|^2$ comes from)

Gauge Invariance $\vec{\nabla} \rightarrow i\vec{\nabla} - 2e\vec{A}$

Quantized Flux, Josephson

$$|BCS\rangle = \prod_k (u_k + v_k b_k^\dagger) |0\rangle$$

$$b_k = a_k \uparrow + a_{-k} \downarrow$$

$$u_k^2 + v_k^2 = 1 \quad (\text{if complex, } |u_k|^2 + |v_k|^2)$$

$$H_{BCS} = \sum_k \epsilon_k a_k^\dagger a_k + \sum_{k, l} V_{kl} a_{k, l}^\dagger a_{-k, -l}^\dagger + \text{c.c.}$$

BCS

gd Ensemble

Bogolyubov and Spin Analogy

Temp. Dep. Effects (T_c)

Landau Ginzburg Action

(In addition to spin order, where \vec{D} comes from)

Gauge Invariance $\vec{D} \rightarrow i\vec{D} - 2e\vec{A}$

Quantized Flux, Josephson

$$|BCS\rangle = \prod_k (u_k + v_k b_k^\dagger) |0\rangle$$

$$b_k = a_{k\uparrow} a_{-k\downarrow} \quad \langle b_k \rangle \neq 0$$

$$u_k^2 + v_k^2 = 1 \quad (\text{if complex: } |u_k|^2 + |v_k|^2)$$

$$\sum_k \epsilon_k n_k + \sum_{\lambda, \mu, \nu, \alpha} V_{\lambda, \mu, \nu, \alpha} a_{\lambda}^\dagger a_{\mu}^\dagger a_{\nu} a_{\alpha} \delta_{\lambda+\mu, -\nu-\alpha}$$

$\delta = 2$ type of term important SC.

$$\sum_k \epsilon_k \langle n_k \rangle + \sum_{\lambda, \mu, \nu} V_{\lambda, \mu, \nu} \langle b_{\lambda, \mu} \rangle b_{\nu}^\dagger + h.c.$$

Temp. Dep. Effects

Landau Ginzburg Action

(In addition to spin order, where $|\vec{\Delta}|^2$ comes from)

Gauge Invariance $\vec{\nabla} \rightarrow i\vec{\nabla} - 2e\vec{A}$

Quantized Flux, Josephson

$\mu_n = 1$ (if $\mu_n = 1$)

$$H_{\text{an}} = \sum_k z_k \pi_k + \sum_{\lambda, \lambda'} V_{\lambda, \lambda'} a_{\lambda}^{\dagger} a_{\lambda'}^{\dagger} \delta_{\lambda, \lambda'} - (b_{\lambda} + b_{\lambda'})$$

$$\langle \lambda(S) | H_{\text{an}} | \lambda(S) \rangle = 2 \text{ types of terms in product SC.}$$

$$H_{\text{red}} = \sum_k z_k \langle \pi_k \rangle + \sum_{\lambda, \lambda'} V_{\lambda, \lambda'} \langle b_{\lambda} | X | b_{\lambda'} \rangle + \text{h.c.}$$

$$m_k = v_k^2$$

$$\langle b_{\lambda} \rangle = u_k v_k$$

Temp. Dep. Effects

Landau Ginzburg Action

(In addition to spin order, where $|\vec{\nabla}\Delta|^2$ comes from)

Gauge Invariance $\vec{\nabla} \rightarrow i\vec{\nabla} - 2e\vec{A}$

Quantized Flux, Josephson

$m_{H^*} = 1$ (if $\chi_{H^*} = 1$)

$$H_{em} = \sum_k \tilde{z}_k \pi_k + \sum_{\lambda, \lambda'} V_{\lambda, \lambda'} a_{\lambda}^{\dagger} a_{\lambda'}^{\dagger} \delta_{\lambda, \lambda'} - (l_1 + l_2)$$

$\langle B(S) | H_{em} | D(S) \rangle = 2$ types of terms in product SC.

$$H_{red} = \sum_k \tilde{z}_k \langle \pi_k \rangle + \sum_{\lambda, \lambda'} V_{\lambda, \lambda'} \langle b_{\lambda} | \chi | b_{\lambda'} \rangle + h.c.$$

$$m_{H^*} = v_d^2$$

$$\langle b_{\lambda} \rangle = u_k v_d$$

Define $u_k = v_k \cos 2\theta_k$

$$m_k = \cos^2 \theta_k$$

3/4/11

$$\sum_k \tan^2 \theta_k = \frac{1}{2} \sum_k v_k \cos 2\theta_k$$

at 1/2 π , γ π π

$$m_h = v_h^2$$

$$\langle b_h \rangle = u_h v_h$$

+ h.c.

Define $u_h v_h^{-1} \sin 2\theta_h$

stat.

$$m_h = \cos^2 \theta_h$$

$$\sum_h \tan^2 \theta_h = \frac{1}{2} \sum_h v_h m_h \sin 2\theta_h$$

Define $\Delta_h = - \sum_h v_h$

$$\tan^2 \theta_h = - \frac{\Delta_h}{v_h} = - \frac{\sum_h v_h}{v_h}$$

el 1/2 m v_{rel}² + ...

$$m_k = v_k^2$$

$$\langle b_k \rangle = u_k v_k$$

+ h.c.

Definiere $u_k v_k = \sin 2\theta_k$

Stat.

$$m_k = \dots$$

$$\sum_k \tan 2\theta_k = \frac{1}{2} \sum_k v_k u_k \sin 2\theta_k$$

Definiere !

$$\frac{1}{2} (h, l') \sin 2\theta_k$$

$$\frac{\Delta}{E_k}$$

$$\cos 2\theta_k = -\frac{2l}{E_k}$$

$$E_k = \sqrt{2l^2 + \Delta^2}$$

at 1/2 π , γ π π π

$$m_k = v_k^2$$

$$\langle b_k \rangle = u_k v_k$$

+ h.c.

Define $u_k v_k = \sin 2\theta_k$

$$m_k = \cos^2 \theta_k$$

stat.

$$\sum_k \tan 2\theta_k = \frac{1}{2} \sum_k v_k m_k \sin 2\theta_k$$

$$\text{Define } \Delta_k = - \sum_{l'} V(l, l') \frac{\sin 2\theta_{l'}}{2}$$

$$\tan 2\theta_k = - \frac{\Delta_k}{\xi_k}$$

$$\cos 2\theta_k = \frac{\Delta_k}{E_k} \quad \cos 2\theta_k = - \xi_k$$

$$E_k = \sqrt{\xi_k^2 + \Delta_k^2}$$

$$m_k = v_k^2$$

$$\langle b_k \rangle = u_k v_k$$

+ h.c.

Define $u_k v_k = \sin 2\theta_k$ | g.h.a.

$$m_k = \cos^2 \theta_k \quad \left| \quad \sum_k \tan 2\theta_k = \frac{1}{2} \sum_k v_k u_k \sin 2\theta_k \right.$$

Define $\Delta_k = - \sum_{k'} V(k, k') \frac{\sin 2\theta_{k'}}{2}$

$$\tan 2\theta_k = - \frac{\Delta_k}{\xi_k}$$

$$\sin 2\theta_k = \frac{\Delta_k}{E_k} \quad \cos 2\theta_k = - \frac{\xi_k}{E_k}$$

$$E_k = \sqrt{\xi_k^2 + \Delta_k^2}$$

$$m_k = v_k^2$$

$$\langle b_k \rangle = u_k v_k$$

+ h.c.

$$\text{Define } \begin{array}{l} u_k v_k = \cos 2\theta_k \\ m_k = \cos^2 \theta_k \end{array} \quad \Bigg| \quad \begin{array}{l} \text{Shift,} \\ \sum_k \tan 2\theta_k = \frac{1}{2} \sum_k v_k u_k \tan 2\theta_k \end{array}$$

$$\text{Define } \Delta_k = - \sum_{l'} V(l, l') \frac{\cos 2\theta_{l'}}{2}$$

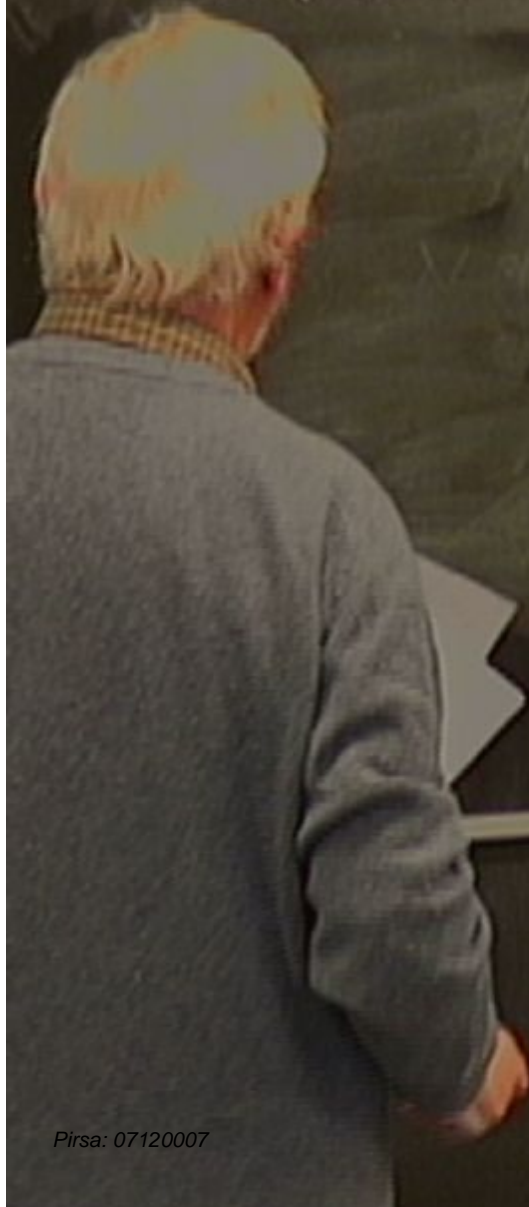
$$\tan 2\theta_k = - \frac{\Delta_k}{\frac{1}{2} \sum_{l'} V(l, l')}$$

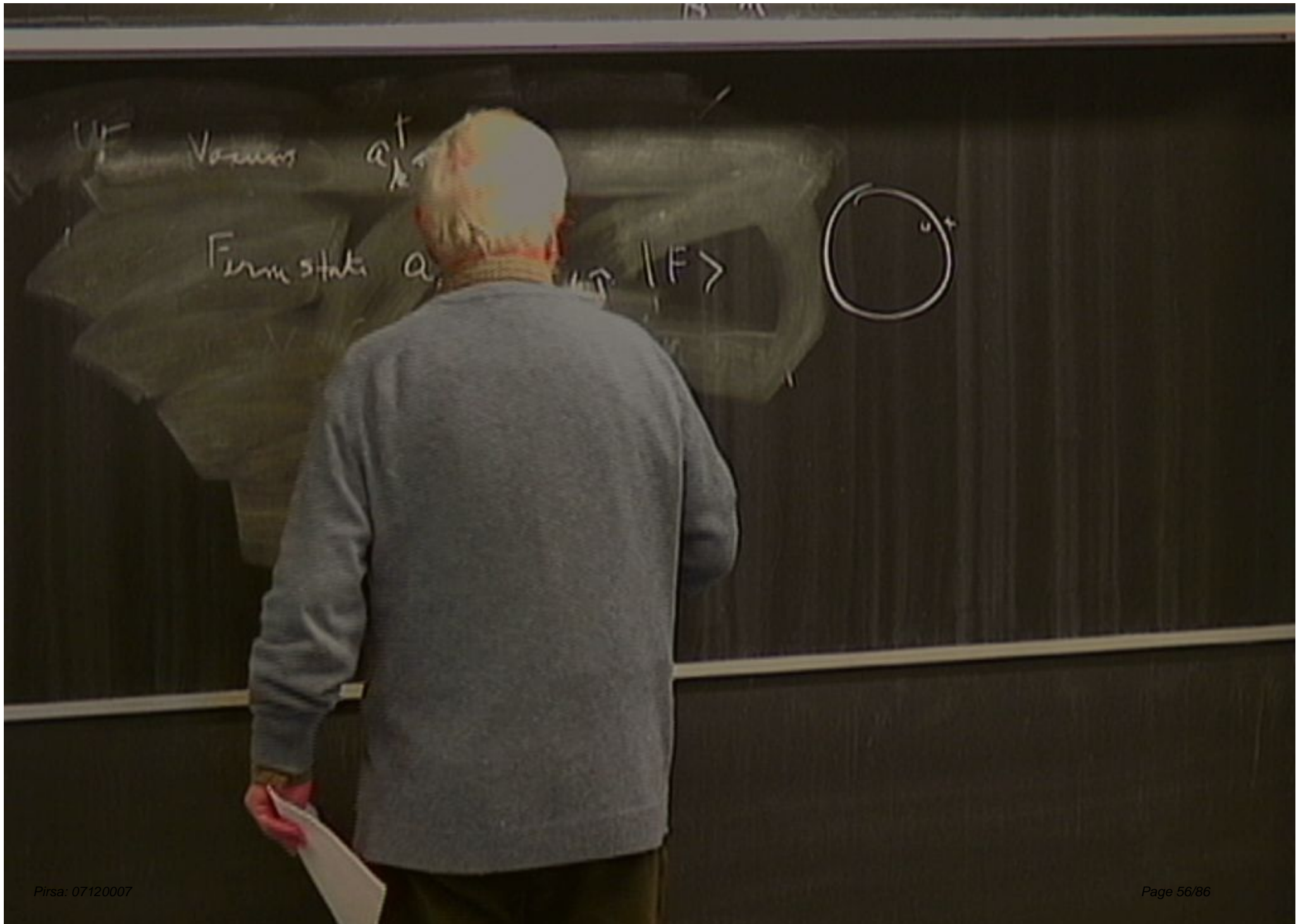
$$\cos 2\theta_k = \frac{\Delta_k}{E_k} \quad \cos \theta_k = - \frac{\sum_{l'} V(l, l')}{E_k}$$

$$E_k = \sqrt{\sum_{l'} V(l, l')^2 + \Delta_k^2}$$

$\langle H_{\text{om}} | BCS \rangle - N\epsilon_n = \text{Cooper value}$

Fermi state: $a_{\mathbf{k}\uparrow}$
 $a_{\mathbf{k}\downarrow}$





Vacuum a^\dagger

Fermi state a $|F\rangle$



Vacuum $a_{k\uparrow}^\dagger |0\rangle$

Fermi state $a_{k+q\uparrow}^\dagger a_{k\uparrow} |F\rangle$
 $a_{k\downarrow}^\dagger$



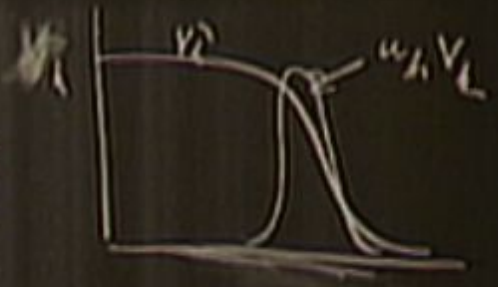
Vacuum $a_{\mathbf{k}\uparrow}^\dagger |0\rangle$

Fermi state $a_{\mathbf{k}\uparrow}^\dagger a_{\mathbf{k}\downarrow}^\dagger |F\rangle$



Vacuum $a_{k\uparrow}^\dagger |0\rangle$

Fermi state $a_{k+q\uparrow}^\dagger a_{k\uparrow} |F\rangle$
 $a_{k\downarrow}^\dagger$



Vacuum $a_{k\uparrow}^\dagger |0\rangle$

Fermi state $a_{k+q\uparrow}^\dagger a_{k\uparrow} |F\rangle$
 $a_{k\downarrow}^\dagger$



Vacuum $a_{\vec{k}}^\dagger |0\rangle$
 Fermi state $a_{\vec{k}+\vec{q}}^\dagger a_{\vec{k}} |F\rangle$
 \downarrow
 $a_{\vec{k}}^\dagger$
 \downarrow
 $a_{\vec{k}}$



Fermion vacuum $a_{k\uparrow}^\dagger |0\rangle$

Fermion state $a_{k\uparrow}^\dagger a_{k\downarrow}^\dagger |F\rangle$

In BCS state

$$d_{k\uparrow} = v_k a_{k\uparrow}^\dagger - u_k a_{k\downarrow}^\dagger$$

$$d_{k\downarrow} = u_k a_{k\uparrow}^\dagger - v_k a_{k\downarrow}^\dagger \quad (1)$$



Fermion vacuum $a_{k\uparrow}^\dagger |0\rangle$

Fermion state $a_{k\uparrow}^\dagger a_{k\downarrow}^\dagger |F\rangle$

In BCS state

$$c_{k\uparrow}^\dagger = v_k a_{k\uparrow}^\dagger + u_k a_{k\downarrow}^\dagger$$

$$c_{k\downarrow}^\dagger = u_k a_{k\uparrow}^\dagger - v_k a_{k\downarrow}^\dagger$$

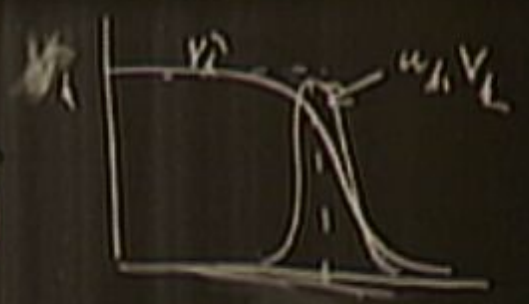


The d 's are fermion creation + annihilation

Vacuum $a_{kT}^\dagger |0\rangle$

Fermi state $a_{kT}^\dagger |F\rangle$

In BCS



The d 's are fermion creation + annihilation.

$$\text{Define } u_k = \frac{1}{\sqrt{2}} \sum_{l'} V_{kl'} \sin 2\theta_{kl'} \quad \left| \quad \text{Stk.} \right.$$

$$m_k = \cos^2 \theta_{kl} \quad \left| \quad \sum_{l'} \tan 2\theta_{kl'} = \frac{1}{2} \sum_{l'} V_{kl} V_{kl'} \sin 2\theta_{kl'}$$

$$\text{Define } \Delta_k = - \sum_{l'} V_{kl} V_{kl'} \frac{\sin 2\theta_{kl'}}{2}$$

$$\tan 2\theta_{kl} = - \frac{\Delta_k}{\sum_{l'} \frac{V_{kl} V_{kl'}}{2}}$$

$$\cos 2\theta_{kl} = \frac{\Delta_k}{E_k} \quad \cos \theta_{kl} = - \frac{\sum_{l'} \frac{V_{kl} V_{kl'}}{2}}{E_k}$$

$$E_k = \sqrt{\sum_{l'} \frac{V_{kl}^2}{2} + \Delta_k^2}$$

Temp. Dep. Effects

Landau Ginzburg Action

(In addition to spin order, where $|\vec{\nabla}\Delta|^2$ comes from)

Gauge Invariance $\vec{\nabla} \rightarrow \vec{\nabla} - 2e\vec{A}$

Quantized Flux, Josephson

$$H_{em} = \sum_k \epsilon_k \pi_k + \sum_{\lambda, l, k} V_{\lambda, l, k} a_{\lambda, l, k}^{\dagger} a_{\lambda, l, k}^{\dagger} \delta_{\lambda, l, k} - l_1 + l_2$$

$\langle BCS || H_{em} || BCS \rangle = 2$ types of terms important SC.

$$H_{red} = \sum_k \epsilon_k \langle \pi_k \rangle + \sum_{\lambda, l, k} V_{\lambda, l, k} \langle b_{\lambda, l, k} | X | b_{\lambda, l, k}^{\dagger} \rangle + h.c.$$

$m_k = v_k^2$ $\langle b_{\lambda, l, k} \rangle = u_k v_k$

Define $u_k v_k = \sin 2\theta_k$

$m_k = \cos^2 \theta_k$

3rd eq.

$$\epsilon_k \tan 2\theta_k = \frac{1}{2} \sum_{\lambda, l, k} V_{\lambda, l, k} \sin 2\theta_{\lambda, l, k}$$

Define $\Delta_l = - \sum_{\lambda, l'} V(\lambda, l') \frac{\sin 2\theta_{\lambda, l'}}{2}$

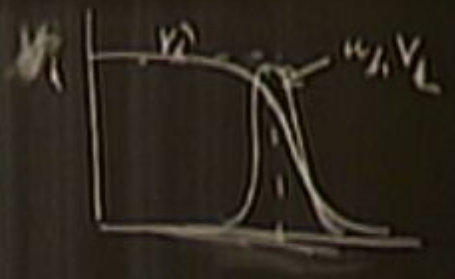
F vacuum $a_{k\uparrow}^\dagger |0\rangle$

$F_{\uparrow} a_{k\uparrow}^\dagger a_{k\downarrow}^\dagger |F\rangle$

$d_{k\downarrow}^\dagger$

$u_k a_{k\uparrow}^\dagger - v_k a_{k\downarrow}^\dagger$

$u_k a_{k\uparrow}^\dagger - v_k a_{k\downarrow}^\dagger$



The d 's are fermion creation + annihilation

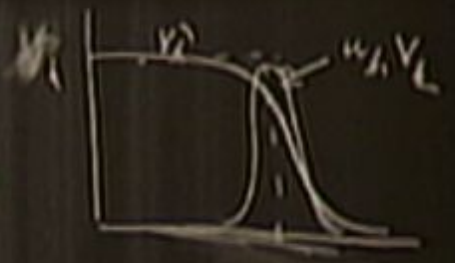
Vacuum $a_{k\uparrow}^\dagger |0\rangle$

Fermi state $a_{k+q\uparrow}^\dagger a_{k\uparrow}^\dagger |F\rangle$

In BCS state

$$d_{k\uparrow}^\dagger = u_k c_{k\uparrow}^\dagger - v_k a_{k\downarrow}^\dagger$$

$$d_{k\downarrow}^\dagger = u_k c_{k\uparrow}^\dagger - v_k a_{k\downarrow}^\dagger$$



The d 's are fermion creation + annihilation

Landau Ginzburg Action

(In addition to spin order, where $|\vec{\Delta}|^2$ comes from)

Gauge Invariance $i\vec{\nabla} \rightarrow i\vec{\nabla} - 2e\vec{A}$

Quantized Flux, Josephson

$$H_{em} = \sum_k z_k n_k + \sum_{\langle l, l' \rangle} V_{ll'} a_{l, \uparrow}^\dagger a_{l', \uparrow}^\dagger a_{l, \downarrow} a_{l', \downarrow} \delta_{\vec{r}_l + \vec{r}_{l'}} - (l \leftrightarrow l')$$

$$\langle BCS || H_{em} || BCS \rangle = 2 \text{ types of terms in product SC.}$$

$$H_{red} = \sum_k z_k \langle n_k \rangle + \sum_{\langle l, l' \rangle} V_{ll'} \langle b_{l, \uparrow}^\dagger b_{l', \uparrow}^\dagger b_{l, \downarrow} b_{l', \downarrow} \rangle$$

$$m_k = v_k^2$$

$$\langle b_{l, \uparrow} \rangle = u_k v_k$$

Define $u_k v_k = \sin 2\theta_k$

$m_k = \cos^2 \theta_k$

3/4/07

$$z_k \tan 2\theta_k = \frac{1}{2} \sum_{l, l'} V_{ll'} \sin 2\theta_{l'}$$

Define $\Delta_l = - \sum_{l'} V(l, l') \frac{\sin 2\theta_{l'}}{2}$

$$\tan 2\theta_k = - \frac{\Delta_k}{z_k}$$

$$\sin 2\theta_k = \frac{\Delta_k}{E_k} \quad \cos 2\theta_k = - \frac{z_k}{E_k}$$

$$E_k^2 = \sqrt{z_k^2 + \Delta_k^2}$$

$$[H_{red} \rightarrow d_k] = -[e_k \leftarrow k]$$

$$[H_{red} \rightarrow d_k] = -[c_k d_k]$$

$$H_{red} = \sum_k \epsilon_k \alpha_k \sigma + \sum_{k, k'} \frac{V_{kk'}}{\hbar k k'} \left[b_k^\dagger \langle b_{k'} \rangle + h.c. \right]$$

$$[H_{red}, d_k] = -\epsilon_k \langle n_k \rangle$$

$$H_{red} = \sum_k \epsilon_k n_{k\sigma} + \sum_{k, k'} V_{kk'} \left[b_k^\dagger \langle b_{k'} \rangle + h.c. \right]$$

$$[H_{red}, d_k] = -\epsilon_k \langle n_k \rangle$$

$$H_{red} = \sum_k \epsilon_k n_{k\sigma} + \sum_{k, k'} V_{kk'} \left[b_k^\dagger \langle b_{k'} \rangle + h.c. \right]$$

$$[H_{red}, d_k] = -\epsilon_k d_k$$

$$H_{red} = \sum_k \epsilon_k n_{k\sigma} + \sum_{k, k'} V_{kk'} \left[b_k^\dagger \langle b_{k'} \rangle + h.c. \right]$$

$$\text{Define } \begin{array}{l} u_h = V_h^{-1} \sin 2\theta_h \\ m_h = \cos^2 \theta_h \end{array} \quad \left| \quad \begin{array}{l} \text{S.H.A.} \\ \sum_h \tan 2\theta_h = \frac{1}{2} \sum V_h m_h^{-1} \sin 2\theta_h \end{array} \right.$$

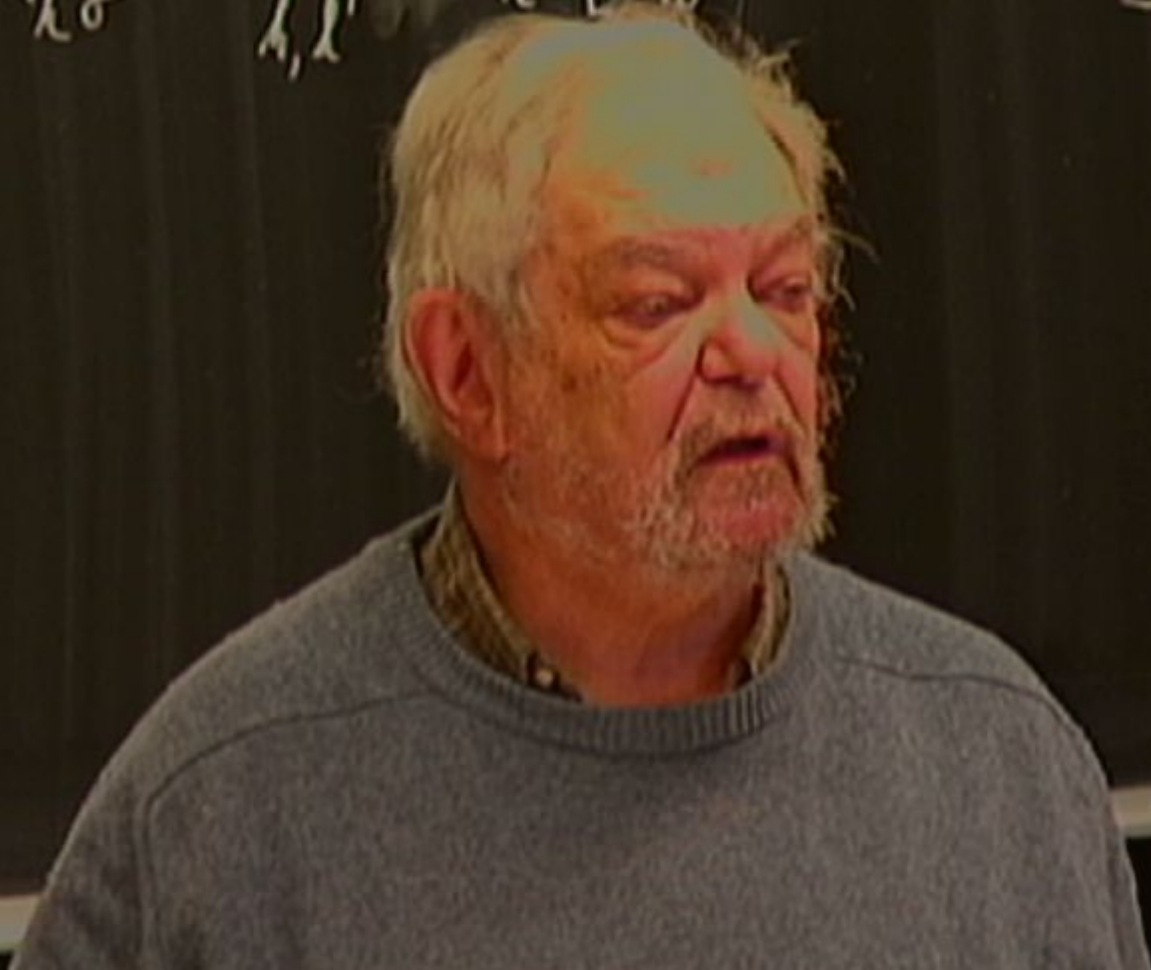
$$\text{Define } \Delta_h = - \sum_{l'} V(h, l') \frac{\sin 2\theta_{l'}}{c}$$

$$\tan 2\theta_h = - \frac{\Delta_h}{\sum_l} \quad \sin 2\theta_h = \frac{\Delta_h}{E_h} \quad \cos 2\theta_h = - \frac{\sum_l}{E_h}$$

$$E_h = \sqrt{\sum_l^2 + \Delta_h^2}$$

$$[H_{red}, d_k] = -\epsilon_k d_k$$

$$H_{red} = \sum_k \epsilon_k n_{k\sigma} + \sum_{k,l} V_{kl} \left[b_k^\dagger b_l + h.c. \right]$$



$$[H_{red}, d_k] = -\epsilon_k d_k$$

$$H_{red} = \sum_k \epsilon_k n_{k\sigma} + \sum_{k, k'} V_{kk'} \left[b_k^\dagger \langle b_{k'} \rangle + h.c. \right]$$

analogy to understand the gap

$$b_k, b_k^\dagger, \epsilon_k, -\epsilon_k$$

$$[H_{red}, d_k] = -\epsilon_k d_k$$

$$H_{red} = \sum_k \epsilon_k n_{k\sigma} + \sum_{k, k'} V_{kk'} \left[b_k^\dagger \langle b_{k'} \rangle + h.c. \right]$$

Symmetry to understand the gap

$$b_{-k}, b_k^\dagger, \epsilon_{-k} = \epsilon_k$$

$$[H_{red}, d_k] = -\epsilon_k d_k$$

$$H_{red} = \sum_k \epsilon_k n_{k\sigma} + \sum_{k,k'} V_{kk'} \left[b_k^\dagger \langle b_{k'} \rangle + h.c. \right]$$

Sp analogy to understand the gap

$$b_k, b_k^\dagger, \underbrace{\epsilon_k - \epsilon_{-k}}_{\epsilon_z}$$

$$b_k + b_k^\dagger = \epsilon_x$$

$$i(-) = \epsilon_y$$

imp. Dip. Effects
and/or Ginzburg Action

In addition to spin order, where $|\vec{\nabla}\Delta|^2$
comes from)

Gauge Invariance $\vec{\nabla} \rightarrow \vec{\nabla} - 2e\vec{A}$

Quantized Flux, Josephson

$$H_{em} = \sum_k \xi_k n_k + \sum_{\lambda, l, k, l'} V_{\lambda, l, k, l'} a_{\lambda, l}^\dagger a_{k, l} a_{k, l'}^\dagger a_{\lambda, l'} - \dots$$

$\langle BCS || H_{em} || BCS \rangle = 2$ types of terms important SC.

$$H_{red} = \sum_k \xi_k \langle n_k \rangle + \sum_{\lambda, l, k, l'} V_{\lambda, l, k, l'} \langle b_{\lambda, l}^\dagger \rangle \langle b_{k, l'} \rangle + h.c.$$

$$m_k = v_k^2$$

Define	$u_k v_k = \sin 2\theta_k$		Stet.
	$m_k = \cos^2 \theta_k$		
			$\xi_k \tan 2\theta_k = \frac{1}{2} \sum_{\lambda, l, l'} V_{\lambda, l, k, l'} \sin 2\theta_{\lambda, l}$

Define $\Delta_l = - \sum_{k, l'} V(l, l') \frac{\sin 2\theta_{k, l'}}{2}$

$\tan 2\theta_l = \frac{\Delta_l}{\xi_l} \Rightarrow \Delta_l = \xi_l \tan 2\theta_l$

Temp. Dep. Effects

Landau Ginzburg Action

(In addition to spin order, where $\vec{\nabla} \Delta^2$ comes from)

Gauge Invariance $\vec{\nabla} \rightarrow i\vec{\nabla} - 2e\vec{A}$

Quantized Flux, Josephson

$$H_{em} = \sum_k z_k \pi_k + \sum_{\lambda, l} V_{\lambda, l} a_{\lambda, l}^\dagger a_{\lambda, l} \delta_{\lambda, l} - (l_1 + l_2)$$

$\langle BCS || H_{em} || BCS \rangle = 2$ types of terms in product SC.

$$H_{red} = \sum_k z_k \langle \pi_k \rangle + \sum_{\lambda, l} V_{\lambda, l} \langle b_{\lambda, l}^\dagger b_{\lambda, l} \rangle$$

$$m_k = v_k^2 \quad \langle b_{\lambda, l} \rangle = u_k v_k \quad + b_{\lambda, l}$$

Define $u_k v_k = \sin 2\theta_k$ | 3/4/11

$$m_k = \cos^2 \theta_k \quad \sum_k \tan 2\theta_k = \frac{1}{2} \sum_{\lambda, l} V_{\lambda, l} \sin 2\theta_{\lambda, l}$$

Define $\Delta_k = - \sum_{l'} V(k, l') \frac{\sin 2\theta_{l'}}{2}$

$$\tan 2\theta_k = - \frac{\Delta_k}{z_k} \quad \sin 2\theta_k = \frac{\Delta_k}{E_k} \quad \cos 2\theta_k = - \frac{z_k}{E_k}$$

$$E_k = \sqrt{z_k^2 + \Delta_k^2}$$

$$[H_{red}, d_k] = -[c_k, d_k]$$

$$H_{red} = \sum_k \epsilon_k n_k \sigma + \sum_{k, k'} \frac{V_{kk'}}{k k'} [b_k^\dagger \langle b_{k'} \rangle + h.c.]$$

Sp analogy to understand the gap

$$b_k, b_k^\dagger, \frac{1 - n_k - n_{-k}}{2}$$

$$b_k + b_k^\dagger = \tau_x \quad \tau_z$$

$$i(-) = \tau_y$$

Define $u_k = V_k \cos 2\theta_k$

$m_k = \cos^2 \theta_k$

split,

$\sum_k \tan 2\theta_k = \frac{1}{2} \sum V_{kl}$

Define $\Delta_k = - \sum_{l'} V_{(k,l')} \frac{\cos 2\theta_{k'}}{c}$

$\tan 2\theta_k = - \frac{\Delta_k}{\sum_l}$

$\cos 2\theta_k = \frac{\Delta_k}{E_k} \quad \cos \theta_k = - \frac{\sum_l}{E_k}$

$E_k = \sqrt{\sum_l^2 + \Delta_k^2}$

$$\text{Define } u_h = V_{h, l} \sin 2\theta_h$$

3/4/11

$$m_h = \cos^2 \theta_h$$

$$\sum_h \tan 2\theta_h = \frac{1}{2} \sum V_{h, l} \sin 2\theta_h$$

$$\text{Define } \Delta_l = - \sum_h V_{(h, l)} \frac{\sin 2\theta_h}{c}$$

$$\tan 2\theta_h = - \frac{\Delta_h}{\sum_l}$$

$$\sin 2\theta_h = \frac{\Delta_h}{E_h} \cos \theta_h = - \frac{\sum_l}{E_h}$$

$$E_h = \sqrt{\sum_l^2 + \Delta_h^2}$$

$$[H_{red}, d_k] = -[d_k, d_k]$$

$$H_{red} = \sum_k \epsilon_k n_k \sigma + \sum_{k, k'} \sqrt{\epsilon_k \epsilon_{k'}} [b_k^\dagger < b_{k'}^\dagger + h.c.]$$

to understand the gap

$$\frac{\epsilon_k - \epsilon_{k'}}{\epsilon_k + \epsilon_{k'}} \approx \frac{\epsilon_k - \epsilon_{k'}}{\epsilon_k + \epsilon_{k'}}$$

$$[H_{red}, d_k] = -[c_k, c_k^\dagger]$$

$$H_{red} = \sum_k \epsilon_k n_{k\sigma} + \sum_{k, k'} V_{kk'} \left[b_k^\dagger c_{k'} + h.c. \right]$$

Spin analogy to understand the gap

$$b_k, b_k^\dagger, \frac{1 - n_{k-} - n_{k+}}{2}$$

$$b_k + b_k^\dagger = \tau_x \quad \tau_z$$

$$i(-) = \tau_y$$